

Module

1

Formulas and equations

Lessons

- Lesson 1.1 Substitution into expressions and formulas
- Lesson 1.2 Solving linear equations
- Lesson 1.3 Solving further linear equations
- Lesson 1.4 Variables and formulas
- Lesson 1.5 Travel calculations
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- Using spreadsheets
- Lesson 1.9 Review: Formulas and equations

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Formulas and equations

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Understanding order of operations



Support sheet: Finding powers and square roots



Support sheet: Equivalent fractions



Support sheet: Understanding rates



Support sheet: Expressions and equations



Support sheet: Writing formulas

Syllabus links

- Formulas and equations (MST-11-01)
 - A student selects and applies algebraic techniques to solve problems involving equations and formulas.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 1.1

Substitution into expressions and formulas

Learning intentions

By the end of this lesson, you will be able to ...

→ substitute numbers into expressions, equations and formulas for pronumerals and evaluate the result.

Substitution

- When a **pronumeral** such as x appears in an **expression** like $3x - 5$, we may substitute a number in, for example 2, by replacing every instance of the pronumeral x with a 2. When substituting a number into an expression, we usually place the substituted value within brackets.
 - A multiplication symbol ($\times$) can be included when substituting values to make the implied multiplication clear.
- After substituting in $x = 2$, we can evaluate the above expression as follows:

$$3 \times (2) - 5 = 6 - 5 = 1.$$
- In this example, we say that 2 was substituted in for x , and that the resulting value of the expression is 1.
- A mathematical **formula** may appear in a variety of applications. Usually, a formula has a single **variable** on the left-hand side of an equals sign, followed by the rest of the formula on the right-hand side. The single variable on the left-hand side is called the **subject of the formula** and its value is found by substituting other values into the right-hand side of the formula and evaluating.
 - The words “formulas” and “formulae” both mean more than one formula.

Worked example 1.1A Evaluating expressions by substituting given values for pronumerals

For $a = 5$, $b = -2$ and $c = 3$, find the value of:

a $2a - c$

b $\sqrt{a - 2b}$

c $c - 4b^2$

Part	Think	Write
a	Substitute 5 for a and 3 for c . Evaluate the resulting expression using the order of operations.	$ \begin{aligned} 2a - c &= 2(5) - (3) \\ &= 10 - 3 \\ &= 7 \end{aligned} $
b	Substitute 5 for a and -2 for b . Evaluate under the square root sign first, then find the square root.	$ \begin{aligned} \sqrt{a - 2b} &= \sqrt{(5) - 2(-2)} \\ &= \sqrt{5 + 4} \\ &= \sqrt{9} \\ &= 3 \end{aligned} $
c	Substitute 3 for c and -2 for b . When -2 is squared the answer is $+4$. Evaluate the resulting expression using the order of operations.	$ \begin{aligned} c - 4b^2 &= (3) - 4(-2)^2 \\ &= 3 - 4 \times 4 \\ &= 3 - 16 \\ &= -13 \end{aligned} $



Worked example video: Evaluating expressions by substituting given values for pronumerals

Understanding, fluency and communicating

Answers: Lesson 1.1

Helpful hints

- When asked to evaluate (or find the value of) an expression, substitute all known values into the expression.
- When substituting, put the number in brackets wherever the variable appears in the expression and then use the correct order of operations to determine the value.
- Be careful when using a calculator to ensure the correct order of operations is followed.

1 For $x = 7$ and $y = -3$, complete the following to find the value of:

a $3x + 2y = 3(7) + 2(\quad)$
= $\quad$

b $x - 2y = (\quad) - 2(-3)$
= $\quad$

2 For $a = 2$ and $b = 5$, find the value of:

a $a + b$

b $b - a$

c $2a + 3b$

d $a - 2b$

e b^2

f $2b^2$

g $(2b)^2$

h b^3

i $4a + b$

j $4(a + b)$

k $(a + b)^2$

l $a^2 + b^2$

m $\frac{ab}{20}$

n $5(2a + 3b)$

o $\frac{ab}{4}$

p $\frac{a^2b}{10}$

3 For $p = 2$ and $q = -5$, find the value of:

a $p + q$

b $p - q$

c $2p + 5q$

d $3p - 2q$

e $\frac{1}{3}(p + q)$

f $\frac{p + q}{2}$

g $(p - q)^2$

h $p^2 - q^2$

i $-5(3p - 2q)$

j $4 - (p - q)$

k $8 - (3p - 4q)$

l $(p + q)(p - q)$

4 For $m = 3$ and $n = 7$, find the value of:

a $m + n$

b $m - n$

c $n - m$

d $n + m$

e $m^2 + n^2$

f $n^2 - m^2$

g $3(m + n)$

h $-2(m - n)$

i $\sqrt{6m + n}$

j $\sqrt{10(m + n)}$

k $\sqrt{4(n - m)}$

l $\sqrt{7m + 4n}$

Worked example 1.1B Substituting into a formula to find a value

Given that $v = u + at$, find v when $u = 50$, $a = 10$ and $t = 6$.

Think	Write
Substitute 50 for u , 10 for a and 6 for t in the equation. Evaluate using order of operations or your calculator.	$ \begin{aligned} v &= u + at \\ &= 50 + 10 \times 6 \\ &= 50 + 60 \\ &= 110 \end{aligned} $



Worked example video: Substituting into a formula to find a value

- 5 Complete the following to find t using the formula $t = a + (n - 1)d$ when $a = -3$, $n = 20$ and $d = 2$.

$$t = -3 + (20 - 1) \times (\underline{\quad})$$

$$= \underline{\quad}$$

- 6 For each formula, find the value of the unknown variable.

a Given that $V = IR$, find V when $I = 10$ and $R = 250$.

b Given that $V = IR$, find V when $I = 50$ and $R = 20$.

c Given that $W = Fs$, find W when $F = 100$ and $s = 12$.

d Given that $s = \frac{d}{t}$, find s when $d = 50$ and $t = 10$.

- 7 For each formula, find the value of the unknown variable.

a Given that $v = u + at$, find v when $u = 5$, $a = 10$ and $t = 7$.

b Given that $s = ut + \frac{1}{2}at^2$, find s when $u = -3$, $a = 10$ and $t = 5$.

c Given that $d = 180 - \frac{360}{n}$, find d when $n = 12$.

d Given that $a = \frac{2Rn}{n+1}$, find a when $R = 8$ and $n = 24$.

- 8 The volume, V , of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Find, to one decimal place, the volume of a sphere with radius:

a 6 cm

b 8.5 cm

c 1.3 m

- 9 The amount of money in an investment that pays compound interest is given by $A = P(1 + r)^n$. Find A when:

a $P = 1000$, $r = 0.1$ and $n = 5$

b $P = 600$, $r = 0.05$ and $n = 20$

c $P = 2340$, $r = 0.035$ and $n = 10$

d $P = 625$, $r = 0.112$ and $n = 8$

- 10 Pythagoras' theorem can be written as $c = \sqrt{a^2 + b^2}$. Find c when:

a $a = 3$, $b = 4$

b $a = 5$, $b = 12$

c $a = 15$, $b = 20$

d $a = 16$, $b = 12$

Problem solving and reasoning

- 11 Using $a = 2$ and $b = -5$, and showing all working, check whether these pairs of expressions are the same. For any pairs that are not the same, rewrite the second expression to make it the same as the first.

a $4(a + b)$ and $4a + 4b$

b $4(a + b)$ and $4a + b$

c $2(3a + b)$ and $6a + 3b$

d $3a - 5(a + b)$ and $2a - 5b$

e $3a - 5(a + b)$ and $-2a - b$

f $3a - 5(a + b)$ and $-2a - 5b$

g $(3a - 2b)(4a + 7b)$ and $12a^2 - 14ab$

h $(3a - 2b)(4a + 7b)$ and $12a^2 - 8ab - 14$

i $(a + b)^3$ and $a^3 + b^3$

j $(3a - 2b)(4a + 7b)$ and $12a^2 + 13ab - 14b^2$

- 12 Let $a = 3$ and $b = -4$.

a Evaluate the following by substituting the values of a and b .

i $4(a + b)$

ii $a^2 + b - 9$

iii $4a + 4b$

iv $-10 + 2a$

b What do you notice about the value of all these expressions?

c Does this necessarily mean that all these expressions are always equal?

d Test using other values of a and b to find which, if any, of the expressions are always equal.

- 13** The body mass index, b , is a measure used to determine if a person's mass is within a recommended range. The range is $21 < b < 25$.

The body mass index formula is given by $b = \frac{m}{h^2}$ where m = mass in kilograms and h = height in metres.

- Calculate the value of b for a person of mass 65 kg and
 - height 1.65 m. Is the value within the recommended range?
 - Peter has a mass of 100 kg and is 1.95 m tall. He states that his mass is within the recommended range. By calculating the value of b for Peter, determine if he is correct.
 - Kristy has a mass of 50 kg, is 1.5 m tall and feels that she is below the recommended body mass. Is she correct? Explain using calculations.
- 14** To hire beach chairs a \$50 deposit must be paid as well as \$10 per chair hired.
- Write an algebraic expression for the total hire cost, using n to represent the number of chairs hired.
 - How much would it cost to hire six chairs?
- 15** In basketball, players score points by throwing the ball through the basket, in an act known as “shooting”. Different points are awarded to players based on where they are standing when they shoot.

3 points	Awarded to players successfully shooting from behind the three-point line.
2 points	Awarded to players successfully shooting from anywhere inside the three-point line.
1 point	Awarded to players successfully shooting when playing a foul shot.



- Using appropriate variables, write an algebraic expression to represent the total number of points scored during a basketball game. Remember to carefully define your variables.
 - Calculate the total number of points scored by a team during part of a practice game in which the players successfully scored five goals from behind the three-point line, 12 goals from inside the three-point line and four goals when playing foul shots.
 - Find three different ways of achieving a score of 68 points.
- 16** Emma walks along a beach. She starts 5 kilometres from the lighthouse and travels at 6 kilometres per hour.
- Write an algebraic expression to represent Emma's distance from the lighthouse, using t to represent the number of hours she has been walking.
 - How far must Emma have walked if she has been walking for half an hour?
 - For how long must Emma have walked if she is 14 km from the lighthouse?

Challenge

- 17** Substitute $a = 3$, $b = 4$ and $C = 56$ into the following expression. Write your answer correct to one decimal place.

$$\sqrt{a^2 + b^2 - 2ab \cos(C)}$$

Related resources



1.1 Quick quiz



Worksheet: Substitution



Investigation: Going to the movies

Lesson 1.2

Solving linear equations

Learning intentions

By the end of this lesson, you will be able to ...
→ solve linear equations.

Solving linear equations

- An algebraic **equation** consists of two algebraic expressions that are equal to each other. The aim of solving an equation is to find the value or values of the pronumeral that make the equation a true statement.
- To **solve** a **linear equation**:
 - Keep the equation balanced by performing the same operation to both sides of the equation; that is:
 - add the same number or term to both sides of the equation
 - subtract the same number or term from both sides of the equation
 - multiply both sides of the equation by the same number or term
 - divide both sides of the equation by the same number or term.
 - Continue until you obtain the simplest equivalent equation with the pronumeral on one side of the equation and a number on the other. For example, to solve $x + 4 = 10$, we subtract 4 from both sides of the equation to obtain the **solution** $x = 6$.

$$\begin{array}{l} x + 4 = 10 \quad (-4) \\ x = 6 \end{array}$$

Worked example 1.2A Determining whether a given value is a solution to an equation

Substitute the given value into the equation to see if it is a solution to the equation.

a $3x - 9 = 36$ [$x = 11$]

b $49 - 5x = 29$ [$x = 4$]

Part	Think	Write
a	Substitute $x = 11$ into $3x - 9$ and simplify to get 24. But $24 \neq 36$, so $x = 11$ is not a solution.	$\begin{aligned} 3x - 9 &= (3 \times 11) - 9 \\ &= 33 - 9 \\ &= 24 \\ &\neq 36 \\ x = 11 &\text{ is not a solution.} \end{aligned}$
b	Substitute $x = 4$ into $49 - 5x$ to get 29. This means $x = 4$ is a solution.	$\begin{aligned} 49 - 5x &= 49 - (5 \times 4) \\ &= 49 - 20 \\ &= 29 \\ x = 4 &\text{ is a solution.} \end{aligned}$



Worked example video: Determining whether a given value is a solution to an equation

Understanding, fluency and communicating

Answers: Lesson 1.2

Helpful hints

- To determine whether a given value is a solution to an equation, substitute the value into the expression.
 - If both sides of the equation are the same, then that value is a solution.
 - If the two sides are not the same, then it is not a solution.

1 Complete the following to determine whether:

a $x = 5$ is a solution to $7x - 3 = 28$.

$$\begin{aligned} 7x - 3 &= (7 \times \underline{\quad}) - 3 \\ &= 35 - 3 \\ &= \underline{\quad} \\ x = 5 \text{ is } \underline{\quad}. \end{aligned}$$

b $a = 8$ is a solution to $7 + 3a = 31$.

$$\begin{aligned} 7 + 3a &= 7 + (3 \times \underline{\quad}) \\ &= 7 + \underline{\quad} \\ &= 31 \\ a = 8 \text{ is } \underline{\quad}. \end{aligned}$$

2 Substitute the given value for the pronumeral into the equation to determine if it is a solution to the equation.

a $4x + 8 = 60$ [$x = 13$]

b $5x - 9 = 11$ [$x = 3$]

c $3x + 4 = 13$ [$x = 3$]

d $9 - 2p = -15$ [$p = 12$]

e $10 + 5m = -20$ [$m = -6$]

f $22 + 3c = 46$ [$c = 7$]

g $6d - 5 = 31$ [$d = 9$]

h $11y - 72 = -193$ [$y = -11$]

i $14q + 18 = 116$ [$q = 7$]

Worked example 1.2B Solving two-step linear equations

Solve these equations.

a $5x + 7 = 37$

b $8x - 9 = 79$

Part	Think	Write
a	Subtract 7 from both sides of the equation and simplify the result. Divide both sides of the equation by 5 and simplify the result. Check your answer by substituting $x = 6$ back into the original equation.	$\begin{aligned} 5x + 7 &= 37 && (-7) \\ 5x + 7 - 7 &= 37 - 7 \\ 5x &= 30 && (\div 5) \\ \frac{5x}{5} &= \frac{30}{5} \\ x &= 6 \\ 5(6) + 7 &= 30 + 7 \\ &= 37 \end{aligned}$
b	Add 9 to both sides of the equation and simplify the result. Divide both sides of the equation by 8 and simplify the result. Check your answer by substituting $x = 11$ back into the original equation.	$\begin{aligned} 8x - 9 &= 79 && (+9) \\ 8x - 9 + 9 &= 79 + 9 \\ 8x &= 88 && (\div 8) \\ \frac{8x}{8} &= \frac{88}{8} \\ x &= 11 \\ 8(11) - 9 &= 88 - 9 \\ &= 79 \end{aligned}$



Worked example video: Solving two-step linear equations

Helpful hints

- Perform the same operation on both sides of the equation to keep the equations equivalent from step to step.
- Always substitute the answer back into the equation to check it is correct. Answers are not always whole numbers.

3 Fill in the blanks to solve these equations.

a $7x - 3 = 32$
 $7x - 3 + \underline{\hspace{1cm}} = 32 + \underline{3}$
 $7x = \underline{\hspace{1cm}}$
 $\frac{7x}{7} = \frac{35}{7}$
 $x = \underline{\hspace{1cm}}$

b $4x + 7 = 15$
 $4x + 7 - \underline{7} = 15 - \underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}} = 8$
 $\frac{\underline{\hspace{1cm}}}{4} = \frac{8}{4}$
 $x = 2$

4 Solve the following equations.

a $3x - 9 = 3$

b $4p + 8 = 24$

c $6x - 2 = 82$

d $6x + 7 = 61$

e $4p + 12 = 52$

f $5y - 6 = 44$

g $2x - 32 = 16$

h $11p - 10 = 34$

i $7m + 8 = 50$

j $4q - 21 = 115$

k $8s - 12 = 252$

l $10r - 30 = 100$

m $7t - 8 = 27$

n $11s + 16 = 115$

o $4m + 9 = 73$

Worked example 1.2C Solving linear equations containing a negative pronumeral term

Solve the following linear equations.

a $9 - 7x = 5$

b $17 = 8 - 4x$

Part	Think	Write
a	Subtract 9 from both sides and simplify the result. Divide both sides by -7 and simplify to get $x = \frac{4}{7}$. Check your answer by substituting $x = \frac{4}{7}$ back into the original equation.	$9 - 7x = 5 \quad (-9)$ $9 - 7x - 9 = 5 - 9 \quad (\div 7)$ $-7x = -4$ $\frac{-7x}{-7} = \frac{-4}{-7}$ $x = \frac{4}{7}$ $9 - 7\left(\frac{4}{7}\right) = 9 - 4$ $= 5$
b	Subtract 8 from both sides of the equation and simplify the result. Divide both sides of the equation by -4 and simplify. The answer is $x = -\frac{9}{4}$. The result can also be written as a mixed number as $x = -2\frac{1}{4}$. Check your answer by substituting $x = -\frac{9}{4}$ back into the original equation.	$17 = 8 - 4x \quad (-8)$ $17 - 8 = 8 - 4x - 8 \quad (\div (-4))$ $9 = -4x$ $\frac{9}{-4} = \frac{-4x}{-4}$ $-\frac{9}{4} = x$ $x = -\frac{9}{4}$ $= -2\frac{1}{4}$ $8 - 4\left(-\frac{9}{4}\right) = 8 + 9$ $= 17$



Worked example video: Solving linear equations containing a negative pronumeral term

Helpful hints

- When solving a linear equation, the goal is to have the pronumeral on one side of the equation and the numbers on the other side. Perform the same operation on both sides of the equation to isolate the variable.
- The solving process is the same whether the equation appears as $9 - 7x = 5$ or as $-7x + 9 = 5$: we start by subtracting 9 from each side. Similarly, $17 = 8 - 4x$ can be solved in the same way as $8 - 4x = 17$ or $-4x + 8 = 17$: we subtract 8 from each side.

5 Fill in the blanks to solve these equations.

a $6 - 5x = 14$
 $6 - 5x - 6 = 14 - \underline{\quad}$
 $\frac{-5x}{-5} = \frac{\square}{-5}$
 $\underline{\quad} = \frac{8}{-5}$
 $x = \underline{\quad}$

b $-11 = 7 + 5x$
 $-11 - \underline{\quad} = 7 + 5x - \underline{\quad}$
 $\frac{-18}{\square} = \frac{5x}{\square}$
 $x = \underline{\quad}$

6 Solve the following for x .

a $5 = 3x + 7$

b $6x - 7 = -1$

c $-1 = 2x + 6$

d $6 - x = -5$

e $3 - 4x = 15$

f $3 - 2x = 7$

g $5 - 4x = -7$

h $3 - 7x = -2$

i $17 - 2x = -1$

j $11 = 3 - 2x$

k $15 - 2x = -1$

l $8 = 3 - 2x$

m $6 = -1 - 7x$

n $-15 = 3 - 6x$

o $11 = -4 - 3x$

Worked example 1.2D Solving a linear equation

Solve $\frac{m}{3} - 5 = -2$.

Think	Write
Add 5 to both sides of the equation and simplify the result. Multiply both sides of the equation by 3 and simplify the result to get $m = 9$.	$\frac{m}{3} - 5 = -2 \quad (+5)$ $\frac{m}{3} - 5 + 5 = -2 + 5$ $\frac{m}{3} = 3 \quad (\times 3)$ $\frac{m}{3} \times 3 = 3 \times 3$ $m = 9$
Check your answer by substituting $m = 9$ back into the original equation.	$\frac{9}{3} - 5 = 3 - 5$ $= -2$



Worked example video: Solving a linear equation

Helpful hints

- Isolate the variable on one side of the equation by applying operations to both sides of the equation. Often this means that you can continue to use the same technique of adding or subtracting first before multiplying or dividing.

- 7 Fill in the blanks to solve the equation.

$$\begin{aligned}\frac{x}{5} + 3 &= 7 \\ \frac{x}{5} + 3 - 3 &= 7 - 3 \\ \frac{x}{5} &= 4 \\ \frac{x}{5} \times 5 &= 4 \times 5 \\ x &= 20\end{aligned}$$

- 8 Solve each of these equations for x .

a $\frac{x}{2} + 3 = 8$

b $\frac{x}{2} - 1 = 4$

c $\frac{x}{5} + 2 = -3$

d $\frac{x}{6} + 3 = -4$

e $\frac{x}{7} - 2 = 4$

f $\frac{x}{10} - 6 = -1$

- 9 Check the given solution by substitution and state whether or not it is correct.

a $2x + 8 = 15$ [$x = 7$]

b $7 - 5x = 9$ [$x = -\frac{2}{5}$]

c $-15 = 6 - 7x$ [$x = 3$]

d $\frac{x}{5} - 3 = 6$ [$x = \frac{9}{5}$]

Worked example 1.2E Solving a linear equation resulting from substitution

Given $y = 5x - 3$, find x when $y = -18$.

Think	Write
Substitute the value for y , giving an equation that can be solved.	$y = 5x - 3$
Add 3 to both sides of the equation and simplify the result.	$-18 = 5x - 3$ (+3)
Divide both sides of the equation by 5 and simplify the result to get $x = -3$.	$-18 + 3 = 5x - 3 + 3$ (+3)
	$-15 = 5x$ (+3)
	$\frac{-15}{5} = \frac{5x}{5}$
	$x = -3$
Check your answer by substituting $x = -3$ back into the original equation.	$y = 5(-3) - 3$
	$= -15 - 3$
	$= -18$



Worked example video: Solving a linear equation resulting from substitution

Helpful hints

- Some equations have more than one unknown. Substitute any variables that you know the value of and then solve for the value of the desired variable in the usual way.
- Make sure you substitute a given value for the correct variable!

10 Given $y = 7x + 11$, fill in the blanks in the following to find x when $y = -3$.

$$\begin{aligned}
 y &= 7x + 11 \\
 -3 &= 7x + 11 \\
 -3 - 11 &= 7x + 11 - 11 \\
 -14 &= 7x \\
 -14 \div 7 &= 7x \div 7 \\
 x &= -2
 \end{aligned}$$

11 Substitute the value for y to solve for x .

Given $y = 3x - 5$, find x when $y = -14$.

Given $y = 4x + 2$, find x when $y = 11$.

Given $y = 7 - 5x$, find x when $y = 8$.

Given $y = 4 - 3x$, find x when $y = -3$.

Given $y = 5 - 7x$, find x when $y = -5$.

Given $y = 3x - 5$, find x when $y = 0$.

Worked example 1.2F Developing and solving an equation from a worded description

When five is subtracted from twice a number, the answer is three. Write an equation and solve it to find the original number.

Think	Write
Write an equation based on the description. Let x represent the original number. “Twice a number” means to multiply x by 2 and “five is subtracted” means to subtract 5. “The answer is three” means to write the expression equal to 3.	Let the original number be x . $2x$ $2x - 5$ $2x - 5 = 3$
Solve the equation. Add 5 to both sides of the equation and simplify the result. Divide both sides of the equation by 2 and simplify the result to get $x = 4$.	$2x - 5 = 3$ $(+5)$ $2x - 5 + 5 = 3 + 5$ $2x = 8$ $(\div 2)$ $\frac{2x}{2} = \frac{8}{2}$ $x = 4$
Check your answer by substituting $x = 4$ back into the original equation.	$2(4) - 5 = 8 - 5$ $= 3$



Worked example video: Developing and solving an equation from a worded description

Helpful hints

- Begin by defining the variable so that you can ensure the answer makes sense.
- Form the equation using the information in the question. Pay attention to the order of operations applied to the unknown number.
- After solving, make sure the answer makes sense in the context of the question.

- 12 By first forming an equation, solve each of the following worded questions.
- a When five is subtracted from three times a number, the answer is thirteen. Find the original number.
 - b When seven is subtracted from four times a number, the answer is thirteen. Find the original number.
 - c When nine is added to five times a number, the answer is thirty-four. Find the original number.
 - d When eight is added to seven times a number, the answer is twenty-nine. Find the original number.

Problem solving and reasoning

- 13 The sum of two **consecutive integers** is 275.
- a If the first number is x , write an algebraic expression for the second number.
 - b Write an equation in terms of x to represent this problem.
 - c Solve the equation and find the two numbers.
- 14 For each of these problems, first form an equation then solve.
- a Two consecutive integers add to 127. Find the numbers.
 - b Three consecutive integers have a sum of 27. Find the smallest of the three numbers.
 - c Four consecutive integers have a sum of -6 . Find the largest of them.
 - d Three consecutive even integers have a sum of 132. Find the smallest of the three numbers.
- 15 Tyler is saving to buy a cricket bat that costs \$235. He is able to save \$28 per week and currently has \$67. Follow the steps below to work out when he will be able to buy the bat.
- a Choose a pronumeral to represent the unknown quantity in the problem.
 - b Use the pronumeral to write an equation to represent the problem.
 - c Solve the equation and write your answer.
- 16 For each of these problems, form an equation to solve. Remember to clearly define the pronumeral you use to represent the unknown variable.
- a Rachel bought three equally priced dresses online for a total cost of \$196, which included the delivery charge of \$19. What is the cost of each dress?
 - b The cost of hiring a tennis court is \$9 plus \$14 per hour. For how many hours can you hire the tennis court if you and your friends have a total of \$85?
 - c Andrew and Todd scored a total of 37 goals between them in a basketball match. Todd scored five more goals than Andrew. How many goals did Andrew score?
 - d Tina has a budget of \$1200 for her birthday party. The cost of hiring a local hall is \$325 and catering is \$28 per person. What is the maximum number of people that can attend?
- 17 The length of a rectangular playing field is 14 m longer than its breadth.
- a If b represents the breadth of the field, write an expression for the length of the field in terms of b .
 - b Form an equation for the perimeter of the field in terms of b .
 - c If the perimeter of the field is 156 m, solve an equation to find the dimensions of the playing field.
- 18 In an AFL match, players can score points for their team in two ways. They can kick a goal, which is worth six points, and they can score a behind, which is worth one point.
- a Using appropriate variables, write an algebraic equation to calculate the total number of points scored during an AFL game. Remember to carefully define your variables.
 - b Calculate the number of points scored if a team kicks 12 goals and 3 behinds.



- c** Calculate the number of points scored by both teams in each of the following games and identify the winner. Show all working.
 - i** *Kangaroos*: 6 goals and 12 behinds, *Swans*: 7 goals and 8 behinds
 - ii** *Eagles*: 23 goals and 3 behinds, *Suns*: 17 goals and 25 behinds
- d** The scoring in the following games is incomplete. Calculate the number of behinds scored in each of the games. Show all working.
 - i** *Cats*: total points was 97, number of goals was 15
 - ii** *Power*: total points was 128, number of goals was 19
- e** The scoring for the games below is also incomplete. Calculate the number of goals scored in each of the games. Show all working.
 - i** *Saints*: total points was 87, number of behinds was 9
 - ii** *Hawks*: total points was 131, number of behinds was 17
- f** There has been some discussion about the possibility of changing the scoring system so that players receive three points if they hit the post instead of kicking a goal. How would this change your equation?
- g** If the change described in part **f** was implemented and a team scored 15 goals and 14 behinds, and hit the post a number of times for a total score of 119 points, how many times did the ball hit the goalpost?

Challenge

- 19** Hayley, Davina and Marcel go fishing. Davina catches four more fish than Marcel, and Hayley catches twice as many fish as Marcel.
 - a** Form an equation to find the number of fish each person catches. Explain why this problem cannot be solved.
 - b** Hayley believes they caught a total of 65 fish. Does this seem reasonable? Explain.
- 20** Equations with two unknowns can have an infinite number of solutions. The solutions will be a pair of numbers for x and y . For example,
- 21** consider $x = 3y - 7$.
 - a** Verify that when $x = 5$, $y = 4$.
 - b** Find another pair of integers for x and y which make the equation true.
 - c** Find all the pairs of integers for x and y which make the equation true, where $0 \leq x \leq 20$.

Related resources



1.2 Quick quiz



Worksheet: Solving simple linear equations



Investigation: Household bills

Lesson 1.3

Solving further linear equations

Learning intentions

By the end of this lesson, you will be able to ...
→ solve equations after substituting numbers for pronumerals into formulas.

Solving more complex linear equations

- In the previous lesson, each equation could be solved by performing a simple addition or subtraction, followed by a simple multiplication or division. Solving the equations in this lesson will require different sequences of operations and different types of **terms** will appear.
- Some linear equations have one or more terms that are fractions. Adding or subtracting a number from both sides of an equation follows the same process whether that number is a whole number or a fraction.
- If the pronumeral of interest appears in the numerator of a fraction, multiply both sides of the equation by the fraction's denominator, then expand any brackets and simplify. Once the pronumeral is no longer part of a fraction, solve the resulting equation as normal.
- Sometimes the pronumeral of interest will appear on both sides of an equation. In this situation, the first step is to remove one of the pronumeral terms from both sides of the equation by adding or subtracting. Once the pronumeral only appears on one side of the equation, solve the resulting equation as normal.
- After finding a solution to an equation, the result should always be checked by substituting it back in. If both sides of the evaluated equation are not equal, then the result was incorrect and should be reviewed.

Worked example 1.3A Solving linear equations involving fractions

Solve the following for x .

a $x - \frac{1}{5} = \frac{2}{5}$

b $2x + \frac{2}{3} = \frac{7}{3}$

Part	Think	Write
a	Add $\frac{1}{5}$ to both sides, then simplify.	$\begin{aligned}x - \frac{1}{5} &= \frac{2}{5} \\x - \frac{1}{5} + \frac{1}{5} &= \frac{2}{5} + \frac{1}{5} \\x &= \frac{3}{5}\end{aligned}$
b	Subtract $\frac{2}{3}$ from both sides. Divide both sides by 2, then simplify.	$\begin{aligned}2x + \frac{2}{3} &= \frac{7}{3} \\2x + \frac{2}{3} - \frac{2}{3} &= \frac{7}{3} - \frac{2}{3} \\2x &= \frac{5}{3} \\\frac{2x}{2} &= \frac{5}{3(2)} \\x &= \frac{5}{6}\end{aligned}$



Worked example video: Solving linear equations involving fractions

Understanding, fluency and communicating

Answers: Lesson 1.3

Helpful hints

- A fraction can be added or subtracted from both sides of an equation, just like an integer can.

- 1 Complete the following to solve for x .

$$x - \frac{2}{5} = \frac{7}{5}$$

$$x - \frac{2}{5} + \frac{2}{5} = \frac{7}{5} + \frac{2}{5}$$

$$x = \frac{9}{5}$$

$$\text{or } x = 1\frac{4}{5}$$

- 2 Solve the following linear equations for x .

a $x + \frac{1}{3} = \frac{2}{3}$

d $2x + \frac{1}{5} = \frac{3}{5}$

b $x - \frac{3}{5} = \frac{2}{5}$

e $3x - \frac{1}{4} = \frac{3}{4}$

c $x - \frac{7}{4} = -\frac{5}{4}$

f $5x - \frac{2}{5} = \frac{1}{5}$

Worked example 1.3B Solving more linear equations involving fractions

Solve for x .

a $\frac{x}{3} = \frac{2}{5}$

b $\frac{4x+3}{5} = -2$

Part	Think	Write
a	Multiply both sides by 3, then simplify.	$\frac{x}{3} = \frac{2}{5}$ $\frac{x}{3} \times 3 = \frac{2}{5} \times 3$ $x = \frac{6}{5} \text{ or } x = 1\frac{1}{5}$
b	- Multiply both sides of the equation by the denominator, 5. - Subtract 3 from both sides of the equation, then divide both sides by 4. - Simplify the solution (if required).	$\frac{4x+3}{5} = -2$ $\frac{4x+3}{5} \times 5 = -2 \times 5$ $4x+3 = -10$ $4x+3 - 3 = -10 - 3$ $4x = -13$ $\frac{4x}{4} = \frac{-13}{4}$ $x = -\frac{13}{4} \text{ or } x = -3\frac{1}{4}$



Worked example video: Solving more linear equations involving fractions

Helpful hints

- An expression written as the fraction $\frac{1}{n}$ multiplied by a collection of terms enclosed by brackets can be rewritten so that the terms inside the brackets appear as the numerator of the fraction.
 → For example, $\frac{1}{4}(2x - 7) = \frac{2x - 7}{4}$.

- 3 Complete the following to solve for
- x
- .

$$\frac{x}{5} = \frac{7}{8}$$

$$\frac{x}{5} \times 5 = \frac{7}{8} \times \underline{\hspace{1cm}}$$

$$x = \frac{\square}{\square}$$

$$\text{or } x = \underline{\hspace{1cm}} \frac{3}{8}$$

- 4 Solve the following for
- x
- .

a $\frac{x}{3} = 5$

b $\frac{x}{7} = \frac{1}{3}$

c $\frac{1}{5}x = \frac{1}{2}$

d $\frac{x}{9} = \frac{2}{3}$

e $\frac{1}{7}x = \frac{2}{15}$

f $\frac{x}{5} = \frac{2}{3}$

- 5 Complete the following to solve for
- x
- . Write your answer as a mixed fraction.

$$\frac{1}{4}(3x + 1) = 5$$

$$\frac{1}{4}(3x + 1) \times \underline{\hspace{1cm}} = 5 \times \underline{\hspace{1cm}}$$

$$3x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$3x + 1 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$$

$$3x = \underline{\hspace{1cm}}$$

$$\frac{3x}{\square} = \frac{\square}{\square}$$

$$x = \underline{\hspace{1cm}} \frac{\square}{\square}$$

- 6 Solve the following for
- x
- .

a $\frac{x-2}{4} = -3$

b $\frac{x-1}{2} = 8$

c $\frac{x+5}{3} = -5$

d $\frac{2x+7}{3} = -1$

e $\frac{1}{2}(3x+1) = -3$

f $\frac{1+2x}{7} = 7$

g $\frac{1-2x}{2} = 0$

h $\frac{1}{5}(4-3x) = -2$

i $\frac{1}{4}(5-2x) = -1$

Worked example 1.3C Solving linear equations with a pronumeral term on both sides

Solve these equations.

a $5x + 2 = 3x - 5$

b $15 - 2x = 11 + x$

Part	Think	Write
a	First, subtract $3x$ from both sides of the equation. Then, subtract 2 from both sides. Finally, divide both sides by 2, then simplify.	$5x + 2 = 3x - 5$ $5x + 2 - 3x = 3x - 5 - 3x$ $2x + 2 = -5$ $2x + 2 - 2 = -5 - 2$ $2x = -7$ $\frac{2x}{2} = \frac{-7}{2}$ $x = -\frac{7}{2} \text{ or } x = -3\frac{1}{2}$
b	First, add $2x$ to both sides of the equation. Then, subtract 11 from both sides. Finally, divide both sides by 3, then simplify.	$15 - 2x = 11 + x$ $15 - 2x + 2x = 11 + x + 2x$ $15 = 11 + 3x$ $15 - 11 = 11 + 3x - 11$ $4 = 3x$ $\frac{4}{3} = \frac{3x}{3}$ $x = \frac{4}{3} \text{ or } x = 1\frac{1}{3}$



Worked example video: Solving linear equations with a pronumeral term on both sides

Helpful hints

- When solving equations with pronumerals on both sides, it may be necessary to not only add and subtract numbers from both sides, but also to add and subtract pronumerals from both sides.
- It does not matter which side of the equation you remove a pronumeral term from but sometimes one choice is more convenient than the other. For example, if an equation has $3x$ on the left-hand side and $2x$ on the right-hand side, you may choose to subtract $2x$ from both sides so that the surviving pronumeral term is positive.

7 Complete to solve the following.

$$\begin{aligned}
 4x - 2 &= 4 + 2x \\
 4x - 2 - \underline{\quad} &= 4 + 2x - 2x \\
 2x - 2 &= 4 \\
 2x - 2 + \underline{\quad} &= 4 + \underline{\quad} \\
 2x &= \underline{\quad} \\
 \frac{2x}{\underline{\quad}} &= \frac{\underline{\quad}}{2} \\
 x &= \underline{\quad}
 \end{aligned}$$

8 Solve the following equations with integer solutions.

a $6x + 2 = 2x + 18$

b $3x + 7 = 12 - 2x$

c $5 + x = 9 - 3x$

d $4x - 4 = 5x - 1$

e $3 - x = 2x + 9$

f $9 - 2x = 3 - 5x$

g $3x - 12 = x + 6$

h $5x - 9 = 11 + 7x$

i $3x - 5 = 10 - 2x$

9 Solve these equations, giving your answer as a mixed fraction where appropriate.

a $6x + 7 = 4x - 2$

b $5x + 3 = 2x + 7$

c $5 + 2x = 11 - 3x$

d $2x - 3 = 5x + 7$

e $3 + x = 17 + 5x$

f $15 - 3x = 2 - 5x$

g $2x + 5 = 9 - 3x$

h $4x - 5 = 5x + 9$

i $5 - 7x = 3x + 7$

j $5a + 3 = a - 3$

k $4 - 3s = 2s + 15$

l $9x - 4 = 3 + 5x$

m $11a - 7 = 5a + 15$

n $3y - 5 = -14 - 5y$

o $7p = 15 - 4p$

Worked example 1.3D Checking solutions to linear equations

Substitute the value given in square brackets to check whether it is the solution to each equation.

a $2x - 5 = 10 - 3x$ [$x = 3$]

b $5x + 2 = 2x - 7$ [$x = 2$]

Part	Think	Write
a	Substitute $x = 3$ into the left-hand side of the equation and evaluate, then do the same for the right-hand side. Both sides of the equation are equal to 1, so $x = 3$ is a valid solution.	$ \begin{aligned} 2x - 5 &= 2(3) - 5 \\ &= 6 - 5 \\ &= 1 \\ 10 - 3x &= 10 - 3(3) \\ &= 10 - 9 \\ &= 1 \\ x = 3 &\text{ is a solution.} \end{aligned} $

Part	Think	Write
b	Substitute $x = 2$ into the left-hand side of the equation and evaluate, then do the same for the right-hand side. The left-hand side is 12 and the right-hand side is -3 , so $x = 2$ is not a valid solution.	$5x + 2 = 5(2) + 2$ $= 10 + 2$ $= 12$ $2x - 7 = 2(2) - 7$ $= 4 - 7$ $= -3$ $x = 2 \text{ is not a solution.}$



Worked example video: Checking solutions to linear equations

Helpful hints

- When checking a solution, substitute the value into the equation and check that the two sides of the equation are equal. If they are not equal, the given value is not a valid solution.

10 Complete to show that $x = 3$ is a solution of $7 - 3x = x - 5$.

$$\begin{aligned} \text{LHS: } 7 - 3x &= 7 - 3(\_) \\ &= 7 - \_ \\ &= \_ \\ \text{RHS: } x - 5 &= \_ - 5 \\ &= \_ \end{aligned}$$

Hence, $x = 3$ $\_$ a valid solution.

11 Substitute the value given in square brackets to check whether it is a solution to the equation each time.

a $3x + 9 = 4 - 2x$ [$x = -1$]

b $9a + 2 = 7a - 4$ [$a = -3$]

c $7a - 5 = 3 - a$ [$a = 2$]

d $15 - 2x = 6 + x$ [$x = 3$]

e $2x - 3 = 7 - 4x$ [$x = 1\frac{2}{3}$]

f $5x - 7 = 3 + x$ [$x = 3\frac{1}{2}$]

Worked example 1.3E Solving a problem by first forming an equation to solve

If one more than twice a number is five more than the number, what is the number?

Think	Write
$2x + 1$ is one more than twice the number. $x + 5$ is five more than the number. Form the equation using $2x + 1$ and $x + 5$.	Let the number be x . The equation becomes: $2x + 1 = x + 5$ $2x + 1 - x = x + 5 - x$ $x + 1 = 5$ $x + 1 - 1 = 5 - 1$ $x = 4$ The number is 4.



Worked example video: Solving a problem by first forming an equation to solve

Helpful hints

- When converting a worded problem into an equation, be sure to carefully define any pronumerals used.

12 Form an equation and then solve it to find the number in each of these problems.

- Three more than twice a number, is seven more than the number.
- Eight more than three times a number, is four more than the number.
- Six more than twice a number, is four more than four times the number.
- Nine less than five times a number, is three less than twice the number.

Problem solving and reasoning

- 13 A formula for converting temperature is $C = \frac{5}{9}(F - 32)$ where C represents the temperature in degrees Celsius ($^{\circ}\text{C}$) and F represents the temperature in degrees Fahrenheit ($^{\circ}\text{F}$).
- For $C = 30$, write the formula as a linear equation involving the variable F .
 - Solve your linear equation to find F .
 - What is the equivalent temperature in degrees Fahrenheit for a temperature of 30°C ?
 - Solve a linear equation to find the equivalent temperature in degrees Fahrenheit for a temperature of 18°C .
 - Which temperature is higher: 25°C or 76°F ? Show steps of working to justify your answer.

14 The number of biscuits in a packet is unknown.

However, Lisa can fill the family biscuit barrel with six more biscuits than the contents of two packets, or five less biscuits than the contents of three packets.

- If n represents the number of biscuits in a packet, which equation (A, B, C or D) best matches this situation?

- $2n - 6 = 3n - 5$
- $2n - 6 = 3n + 5$
- $2n + 6 = 3n - 5$
- $2n + 6 = 3n + 5$

- Solve the equation to find the number of biscuits in a packet.

15 Kristen and Daniel have the same amount of money. Kristen buys 5 kg of cherries and has \$2 left over. Daniel buys 2 kg of cherries and has \$20 left over.

- Write an equation to represent this situation.
- Solve the equation to find the cost of 1 kg of cherries.

16 James and Sue have the same amount of money. James buys seven sushi rolls and has \$1.50 left over.

Sue buys four sushi rolls and has \$12 left over.

Form an equation and then solve it to find the cost of a sushi roll.

17 The average of Hayden's four test results for the semester is 78%. He remembers that three of his test results were 91%, 69% and 88%. Form an equation and then solve it to work out the result he received in the fourth test.

18 For three consecutive even integers, the sum of the two smaller numbers is equal to six more than the largest number. Form an equation and then solve it to find the integers.



Challenge

19 Solve the following linear equations for x .

a $\frac{4x+1}{3} = \frac{1}{2}$

b $\frac{3x+1}{5} = \frac{2}{3}$

c $\frac{7x-2}{4} = -\frac{2}{5}$

d $\frac{2x+1}{5} = \frac{3}{4}$

e $\frac{3x+1}{4} = \frac{3}{5}$

f $\frac{3x+2}{5} = x - \frac{1}{3}$

g $\frac{1-x}{5} = x + \frac{1}{3}$

h $\frac{x+1}{2} = \frac{x}{7}$

i $\frac{2x-1}{4} = \frac{3x}{5}$

20 Solve the following problems by first forming an equation.

- a A man is currently three times as old as his son. In 11 years from now, he will be twice as old as his son will be then. How old is his son now?
- b At present Jen is eight years older than Wesley. In three years from now, Jen's age will be double Wesley's age. How old is Wesley?
- c The sum of Kelly and Sam's ages is 45 years. If Kelly's age was doubled, it would be five years more than three times Sam's age. How old is Sam?

21 The formula $A = 2\pi rh + 2\pi r^2$ will be used in a later module. Solve for h when $A = 300$ and $r = 5.5$. Write your answer correct to two decimal places.

Related resources



1.3 Quick quiz



Worksheet: Solving equations with a pronumeral term on both sides



Investigation: Costs at a cinema

Lesson 1.4

Variables and formulas

Learning intentions

By the end of this lesson, you will be able to ...

- change the subject of a formula that is a linear equation.

Finding the value of a variable in a formula

- A formula is an equation that relates two or more variables according to a rule. Each variable can be represented by a pronumeral.
- If the subject of a formula is a pronumeral, its value can be found by substituting numbers in for any remaining pronumerals, as in **Lesson 1.1. To find the value of a pronumeral that is not the subject of the formula**, there are two approaches.
 - Substitute the known values into the formula and solve the resulting equation.
 - Rearrange the formula, so that the subject of the formula is the pronumeral to be found and then substitute any values that are known. Rearranging a formula to change its subject follows the same rules as those used for solving equations.

Worked example 1.4A Substituting known values into a formula and solving the resulting equation

a Find the value of t when $v = 117$, $u = 5$, $a = 8$ and $v = u + at$.

b Find the value of N when $R = 23$, $I = 4$ and $R = \frac{I}{N} + 1$.

Part	Think	Write
a	Substitute $v = 117$, $u = 5$ and $a = 8$ into the formula. Solve the resulting equation by subtracting 5 from both sides then dividing both sides by 8.	$v = u + at$ $117 = 5 + 8t$ $112 = 8t$ $\frac{112}{8} = t$ $t = 14$
b	Substitute $R = 23$ and $I = 4$ into the formula. Solve the resulting equation by subtracting 1 from both sides, multiplying both sides by N and then dividing both sides by 22.	$R = \frac{I}{N} + 1$ $23 = \frac{4}{N} + 1$ $22 = \frac{4}{N}$ $22N = 4$ $N = \frac{4}{22}$ $N = \frac{2}{11}$



Worked example video: Substituting known values into a formula and solving the resulting equation

Understanding, fluency and communicating

Answers: Lesson 1.4

Helpful hints

- Start by substituting all known values into the formula, then solve the resulting equation to make the desired pronumeral the subject.
- If the pronumeral of interest appears in the denominator of a fraction, multiply both sides of the equation by this denominator.

1 If $v = u + at$, find:

a t when $v = 102$, $u = 18$ and $a = 7$

b a when $v = 54$, $u = 12$ and $t = 14$

2 Use the formula $d = \frac{1}{2}ct$ to find the value of:

a c when $d = 100$ and $t = 8$

b t when $d = 320$ and $c = 16$

3 If $M = \frac{5K}{18}$, find K when $M = 15$.

4 If $V^2 = gR$, find R when $V = 12$ and $g = 10$.

5 If $b = \frac{m}{h^2}$, find m when $b = 23$ and $h = 1.63$.

6 Use the formula $I = Prn$ to find the value of:

a P when $I = 19\,500$, $r = 0.03$ and $n = 25$

b n when $I = 2100$, $P = 5000$ and $r = \frac{6}{100}$

7 Given $s = ut + \frac{1}{2}at^2$, find u when $s = 360$, $t = 8$ and $a = 10$.

8 If $S = \frac{n}{2}(a + l)$, find a when $S = 560$, $n = 20$ and $l = 53$.

Worked example 1.4C Changing the subject of a formula (two steps)Make x the subject of each formula below.

a $y = 2x + 3$

b $m = 4 - 3x$

c $y = mx + b$

Part	Think	Write
a	Subtract 3 from both sides of the equation, then divide both sides by 2.	$2x + 3 = y$ $2x + 3 - 3 = y - 3$ $\frac{2x}{2} = \frac{y - 3}{2}$ $x = \frac{y - 3}{2}$
b	Subtract 4 from both sides of the equation, then divide both sides by -3 . The expression $-(m - 4)$ is the same as $-1(m - 4)$ or $-1 \times m - 1 \times -4$ or $-m + 4$.	$4 - 3x = m$ $4 - 3x - 4 = m - 4$ $-3x = m - 4$ $\frac{-3x}{-3} = \frac{m - 4}{-3}$ $x = \frac{-(m - 4)}{3}$ $x = \frac{-m + 4}{3}$
c	Subtract b from both sides of the equation, then divide both sides by m .	$mx + b = y$ $mx + b - b = y - b$ $mx = y - b$ $\frac{mx}{m} = \frac{y - b}{m}$ $x = \frac{y - b}{m}$

**Worked example video:** Changing the subject of a formula (two steps)**Helpful hints**

- Apply the same rules for solving equations to rearrange the formula so that the desired pronumeral is the subject. Any other pronumerals can be treated just like numbers!

16 Make x the subject of each formula below.

a $y = 3x + 2$

b $y = 5x + 1$

c $y = 2x + 5$

d $y = 7x - 4$

e $y = 6x - 5$

f $y = 3x - 5$

g $m = 3 + 2x$

h $m = 5 + 3x$

i $m = 7 + 5x$

j $m = 6 - 5x$

k $m = 4 - 3x$

l $m = -8 - 6x$

m $y = mx + 4$

n $y = mx - 3$

o $y = mx + c$

p $y = mx$

17 Make y the subject of each of these equations.

a $a + by = c$

b $p + qy = r$

c $xy - p = r$

d $a - b = cy$

e $x - ky = t$

f $c = 4 - ry$

g $a + by + r = 0$

h $k = r - py$

i $r + px = qy$

Problem solving and reasoning

18 Given $R = \frac{I}{N} + 1$, find the value of N when:

a $R = 3$ and $I = 7$

b $R = 11$ and $I = 5$

19 Use $t = \sqrt{\frac{l}{g}}$ to find l when $t = 1.3$ and $g = 10$.

20 Use $r = \sqrt[3]{\frac{V}{k}}$ to find V when $r = 3$ and $k = 4.2$.

21 Use the formula $E = \frac{1}{2}mv^2$ to find the value of v when $E = 135$ and $m = 2.8$. Give your answer correct to one decimal place.

Challenge

22 Make r the subject of the following formulas:

a $V = \frac{4}{3}\pi r^3$

b $S = V_0(1 - r)^n$

Related resources



1.4 Quick quiz



Worksheet: Solving more linear equations



Worksheet: Changing the subject of a formula



Investigation: Balanced linear equations

Lesson 1.5

Travel calculations

Learning intentions

By the end of this lesson, you will be able to ...

→ solve problems related to travel, including calculations relating speed, distance and time.

Average speed

- The formula for the **average speed** of an object is given below. This formula can be used to calculate the average speed, distance travelled or time taken.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

→ This is usually written as $S = \frac{D}{T}$, where S is the average speed, D is the distance travelled and T is the time taken.

→ The formula can be rearranged using inverse operations to get

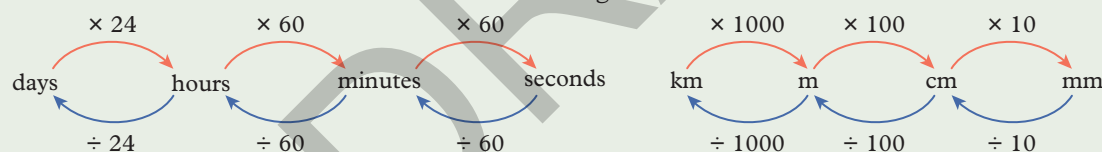
→ $D = S \times T$ and $T = \frac{D}{S}$.

Total stopping distance

- When drivers react to situations in which they need to stop a motor vehicle, the **total stopping distance** of the vehicle depends on the distance travelled, while the driver reacts, and the distance travelled after applying the brakes.
- The **reaction-time distance** is the distance travelled in the time it takes a driver to react to a situation; that is, to realise there is a problem and move their foot to the brake. The usual reaction time for drivers unaffected by alcohol, drugs or fatigue has been found to be about 2.5 seconds.
- The **braking distance** is the distance a car travels after the brakes have been applied. This distance depends on (the square of) the speed of the car.
 - Factors such as the condition of the road (wet or dry) and the condition of the car's brakes and tyres, as well as the state of the driver, all have an effect on the total stopping distance.
 - The braking distance is a function of the square of the speed of the car. For a car with good brakes and tyres, travelling in dry conditions on a good road, the relationship can be approximated by the formula $d = 0.01 v^2$, where d is the braking distance in metres and v is the speed of the car in km/h. For the same car travelling on a slippery road, the formula for braking distance becomes $d = 0.014 v^2$.
 - The distance a car travels in the time it takes to stop is:
Total stopping distance = reaction-time distance + braking distance

Converting units

- Use conversion factors to convert between units of distance and time.
 - Multiply to convert from a larger unit to a smaller unit.
 - Divide to convert from a smaller unit to a larger unit.



Worked example 1.5A Calculating average speed, distance and time

- A car travels 232 km in 4 hours and 17 minutes. Calculate its average speed, correct to the nearest whole number.
- A train averages 83 km/h for 2 hours and 24 minutes. How far does it travel?
- If a motorcyclist can average 52 km/h, how long will it take to travel 34 km, correct to the nearest minute?

Part	Think	Write
a	Convert 17 minutes to hours. Add the result to 4 hours to find the total time in hours. Substitute the values for distance (D) and time (T) into the formula $S = \frac{D}{T}$. Simplify the result and round to the nearest whole number.	$17 \text{ min} = (17 \div 60) \text{ h}$ $= 0.283 \text{ } 33... \text{ h}$ $4 \text{ h } 17 \text{ min} = 4.283 \text{ } 33... \text{ h}$ $S = \frac{D}{T}$ $S = \frac{232 \text{ km}}{4.283 \text{ } 33... \text{ h}}$ $= 54.16... \text{ km/h}$ $\approx 54 \text{ km/h}$
b	Convert 24 minutes to hours. Substitute the values for speed (S) and time (T) into the formula $D = S \times T$. Simplify the result.	$24 \text{ min} = (24 \div 60) \text{ h}$ $= 0.4 \text{ h}$ $D = S \times T$ $D = 83 \text{ km/h} \times 2.4 \text{ h}$ $= 199.2 \text{ km}$
c	Substitute the values for distance (D) and speed (S) into the formula $T = \frac{D}{S}$. Simplify the result. Convert the result from hours to minutes by multiplying by 60. Round to the nearest minute.	$T = \frac{D}{S}$ $T = \frac{34 \text{ km}}{52 \text{ km/h}}$ $= 0.6538... \text{ h}$ $0.6538... \text{ hours} = (0.6538... \times 60) \text{ min}$ $= 39.23... \text{ min}$ $\approx 39 \text{ min (to the nearest minute)}$



Worked example video: Calculating average speed, distance and time

Understanding, fluency and communicating

Answers: Lesson 1.5

Helpful hints

- When using the average speed formula ($S = \frac{D}{T}$), or any rearrangements of it, include the units in your working to help keep track of the units of the result.
- Read questions carefully to make sure you round results when required.

1 Calculate the average speed for these distances and times.

a 185 km is travelled in 4 hours	b 720 km is travelled in 9 hours and 50 min
c 154 km is travelled in 3 hours and 15 min	d 272 km is travelled in 4 hours and 35 min

2 Calculate the distance travelled in:

a 3 hours and 30 min at an average speed of 64 km/h	b 3 hours and 20 min at an average speed of 56 km/h
c 5 hours and 47 min at an average speed of 82 km/h	d 2 hours and 13 min at an average speed of 75 km/h

3 How long will it take to travel:

a 486 km at 60 km/h	b 298 km at 74 km/h
c 365 km at 82 km/h	d 88 km at 95 km/h

Worked example 1.5B Converting units of speed

Convert:

a 65 km/h to m/s (to one decimal place).

b 9.8 m/s to km/h.

Part	Think	Write
a	Write 65 km/h as a fraction. Substitute these conversion factors to convert to the desired units: $1 \text{ km} = 1000 \text{ m}$ $1 \text{ h} = 60 \times 60 \text{ s}$ Simplify the result and write with the units m/s.	$65 \text{ km/h} = \frac{65 \text{ km}}{1 \text{ h}}$ $= \frac{65 \times 1000 \text{ m}}{60 \times 60 \text{ s}}$ $\approx 18.1 \text{ m/s}$
b	Write 9.8 m/s as a fraction. Substitute these conversion factors to convert to the desired units: $\frac{1}{1000} \text{ km} = 1 \text{ m}$ $\frac{1}{60 \times 60} \text{ h} = 1 \text{ s}$ Simplify the result and write with the units km/h.	$9.8 \text{ m/s} = \frac{9.8 \text{ m}}{1 \text{ s}}$ $= \frac{9.8 \times \left(\frac{1}{1000} \text{ km}\right)}{\frac{1}{60 \times 60} \text{ h}}$ $= \left(\frac{9.8}{1000} \text{ km}\right) \div \left(\frac{1}{60 \times 60} \text{ h}\right)$ $= \frac{9.8 \times 60 \times 60 \text{ km}}{1000 \text{ h}}$ $= 35.28 \text{ km/h}$



Worked example video: Converting units of speed

Helpful hints

- When converting units of speed, include the units of distance and time in your calculations. You can then substitute the converted units into the expression and simplify to find the result.

4 Complete the following to convert the units of speed.

a $70 \text{ km/h} = \frac{\square \text{ km}}{\square \text{ h}}$
 $= \frac{\square \times \square \text{ m}}{\square \times \square \text{ s}}$
 $\approx \underline{\hspace{1cm}} \text{ m/s (to one decimal place)}$

b $8 \text{ m/s} = \frac{\square \text{ m}}{\square \text{ s}}$
 $= \frac{\square \times \left(\frac{1}{\square} \text{ km}\right)}{\frac{1}{\square \times \square} \text{ h}}$
 $= \left(\frac{\square}{\square} \text{ km}\right) \div \left(\frac{1}{\square \times \square} \text{ h}\right)$
 $= \frac{\square \times \square \times \square \text{ km}}{\square \text{ h}}$
 $= \underline{\hspace{1cm}} \text{ km/h}$

5 Convert each of the following to m/s (correct to one decimal place).

a 45 km/h

b 76 km/h

c 110 km/h

6 Convert each of the following to km/h.

a 15 m/s

b 12.5 m/s

c 25 m/s

Worked example 1.5C Calculating reaction-time distance

Calculate the reaction-time distance, correct to the nearest whole number, for a car travelling at 60 km/h. Assume a reaction time of 2.5 s.

Think	Write
Convert the speed to m/s. Use the formula $D = S \times T$ where $T = 2.5$ s, where T is the time it takes the driver to react. Multiply the speed in m/s by the reaction time of 2.5 s.	$60 \text{ km/h} = \frac{60 \text{ km}}{1 \text{ h}}$ $= \frac{60 \times 1000 \text{ m}}{60 \times 60 \text{ s}}$ $\text{Reaction-time distance} = \frac{60 \times 1000 \text{ m}}{60 \times 60 \text{ s}} \times 2.5 \text{ s}$ $\approx 42 \text{ m}$



Worked example video: Calculating reaction-time distance

Helpful hints

- Make sure the units of the reaction time and the unit of time in the denominator of the speed are the same. When you multiply the speed by the reaction time, the units of time cancel, and you are left with the unit of distance from the numerator of the speed.

- 7 Calculate the reaction-time distance for a car travelling at 80 km/h. Assume a reaction time of 2.5 s.
- 8 Calculate the reaction-time distance for a car travelling at 100 km/h. Assume a reaction time of 2.5 s.
- 9 Consider again the results of questions 7 and 8.
 - a Use these results to draw a (straight-line) graph of the relationship between reaction-time distance (m) and speed (km/h), given a reaction time of 2.5 seconds.
 - b Use the graph to estimate the reaction-time distances for cars travelling at these speeds.
 - i 120 km/h
 - ii 45 km/h
 - c If the speed of a car increases by 10 km/h, what is the increase in the stopping distance?

Worked example 1.5D Calculating braking distance

Calculate the braking distance for a car travelling in dry conditions at 60 km/h.

Think	Write
Substitute the value of $v = 60$ into the dry conditions formula, $d = 0.01 v^2$.	$\text{Braking distance} = 0.01 v^2$ $= 0.01 \times 60^2$ $= 36 \text{ m}$



Worked example video: Calculating braking distance

Helpful hints

- The wording of the question will tell you which braking distance formula to use.
 - If a question uses words like “dry” or “good” to describe the conditions, use the formula $d = 0.01 v^2$.
 - If a question uses words like “wet”, “poor” or “slippery” to describe the conditions, use the formula $d = 0.014 v^2$.
 - Some questions will give you a new formula to use. Apply the same technique as above with any other braking distance formula.

10 Calculate the braking distance for a car travelling at 80 km/h in good conditions.

11 A car is travelling at 100 km/h.

- Calculate the braking distance if the car is travelling in good conditions.
- What is the braking distance if the car is travelling in wet conditions?
- What is the difference between the braking distances at 100 km/h in good conditions and in wet conditions?

Worked example 1.5E Calculating total stopping distance

Find the total stopping distance, to the nearest metre, for a car travelling at 70 km/h in good conditions, assuming a reaction time of 2.5 s.

Think	Write
Convert 70 km/h to m/s.	$70 \text{ km/h} = \frac{70 \text{ km}}{1 \text{ h}}$ $= \frac{70 \times 1000 \text{ m}}{60 \times 60 \text{ s}}$
Apply the formula $D = S \times T$ to find the reaction-time distance, where S is the speed in m/s and T is the reaction time.	$\text{Reaction-time distance} = \frac{70 \times 1000 \text{ m}}{60 \times 60 \text{ s}} \times 2.5$ $= 48.611 \dots \text{ m}$
Apply the formula $d = 0.01 v^2$ to find the braking distance, where v is the speed in km/h and d is the distance in metres.	$\text{Braking distance} = 0.01 \times 70^2 \text{ m}$ $= 49 \text{ m}$
Add the reaction-time distance to the braking distance to find the stopping distance.	$\text{Total stopping distance} = 48.611 \dots \text{ m} + 49 \text{ m}$ $= 97.611 \dots \text{ m}$ $\approx 98 \text{ m}$



Worked example video: Calculating total stopping distance

Helpful hints

- Pay attention to the units of speed used in each formula. In the example above, the speed is in m/s to find the reaction-time distance but in km/h to find the braking distance.

- 12** Complete the following to find the total stopping distance for a car travelling at 90 km/h in good conditions, assuming a reaction time of 2.5 seconds.

$$90 \text{ km/h} = \frac{\square \text{ km}}{\square \text{ h}}$$

$$= \frac{\square \times \square \text{ m}}{\square \times \square \text{ s}}$$

$$\text{Reaction-time distance} = \frac{\square \times \square \text{ m}}{\square \times \square \text{ s}} \times \text{--- s}$$

$$= \text{--- m}$$

$$\text{Braking distance} = 0.01 \times \square^2 \text{ m}$$

$$= \text{--- m}$$

$$\text{Stopping distance} = \text{--- m} + \text{--- m}$$

$$= \text{--- m}$$

$$\approx \text{--- m (to the nearest metre)}$$

- 13** Find the total stopping distance for a car travelling at 110 km/h in good conditions, assuming a reaction time of 2.5 seconds.

Problem solving and reasoning

- 14** When David is affected by fatigue, he has a reaction time of 3.5 seconds. When not affected by fatigue, his reaction time is only 2.5 seconds. He is driving at 60 km/h.
- What is the reaction-time distance when David is affected by fatigue?
 - What is the difference between the stopping distances of when David is affected by fatigue and when he is not?
- 15** When a given driver is affected by alcohol, they have a reaction time of 4.5 seconds. What difference will this make to the reaction-time distance of a car travelling at 100 km/h compared with the usual reaction time of 2.5 seconds?
- 16** The reaction-time distance, d m, for a car travelling at v km/h, assuming a reaction time of t s, can be approximated using the formula $d = 0.28vt$. Use this formula to check your answers for questions **14** and **15**.
- 17** A car is travelling in good conditions, and the driver has the usual reaction time of 2.5 seconds. If the speed of the car increases from 50 km/h to 60 km/h, what is the increase in total stopping distance?
- 18** Find the stopping distance for a car travelling at 70 km/h in good conditions. Assume a reaction time of 2.5 seconds and use the formula $d = 0.7v + 0.01v^2$, where d is the stopping distance in metres and v is the speed in km/h.
- 19** Another formula for the stopping distance is $d = 0.7v + 0.01v^2$, where d is in metres and v is the speed in km/h.

- a** Use this formula to complete the following table.

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)		$0.7 \times 20 + 0.01 \times 20^2 = 18$				

- Use the information in the completed table from part **a** to graph the relationship between speed and stopping distance. (Join the points with a smooth curve.)
- Using your graph from part **b**, estimate the stopping distance for a car travelling at these speeds.
 - 50 km/h
 - 120 km/h

Challenge

- 20 The formula for the total stopping distance is given by:
- 21 stopping distance = reaction-time distance + breaking distance.
- 22 The average reaction time for a driver is 2 seconds. The breaking distance formula is given by $d = 0.01 v^2$. New South Wales Road and Maritimes Services claim that “If you are driving at 50 km/h, it will take you about 37 metres to stop.” Compare this claim with the result of the stopping distance formula above.

Related resources



1.5 Quick quiz



Investigation: Travelling at constant speed

Lesson 1.6

Blood alcohol content calculations

Learning intentions

By the end of this lesson, you will be able to ...
→ solve problems related to blood alcohol content.

Blood alcohol content

- **Blood alcohol content (BAC)** is a measure of the concentration of alcohol in a person's blood. It is expressed as a percentage mass per unit of volume.
 - A person with a BAC of 0.02 (%) has $\frac{0.02}{100}$ g of alcohol in every millilitre of their blood. This is equivalent to 0.02 g/100 mL or 20 mg/100 mL.
- A BAC measure can be estimated using tables, formulas and on-line calculators; but it is very important to remember that these are only approximations because they are based on average values and do not apply equally to everyone.
- A person's BAC reading is affected by factors such as whether the person is male or female, how much the person drank, the length of time that the person spent drinking and the length of time since his or her last drink, the person's weight, whether the person is physically fit, the state of the person's liver, whether the person is a regular drinker, or the person's mood at the time.
- The only way to measure a person's BAC accurately is with an approved breath analysing unit known as a “breathalyser”.

Standard drinks

- An estimate of a person's BAC can be determined by counting the number of standard drinks consumed. A **standard drink** is the quantity of a beverage that contains 10 g of alcohol. A standard drink always contains the same amount of alcohol irrespective of the container size or the type of drink (beer, wine or spirits).

- The number of standard drinks in a container can be calculated using the formula:

$$N = 0.789 \times V \times A$$

- where:

N = number of standard drinks

V = the capacity of the container in litres

A = percentage of alcohol (% alc/vol) in the drink.

→ The percentage of alcohol in the drink is stated on the container.

→ 0.789 is the specific gravity of ethyl alcohol, the type of alcohol used in beverages.

Estimating blood alcohol content

- An estimate of a person's BAC can be found using these formulas:

$$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

$$\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

- where:

N = number of standard drinks consumed

H = number of hours spent drinking

M = person's mass in kg.

→ One standard drink per hour will raise BAC by between 0.01% and 0.03%.

- A person's BAC will increase at a greater rate if the person:

→ is female

→ is drinking highly carbonated drinks

→ has a low body weight

→ is unfit

→ has not eaten recently

→ has an unhealthy liver.

Reducing blood alcohol content

- After drinking ceases, the only thing that will reduce BAC is the passing of time.
 - Drinking coffee, exercising, taking a cold shower or inducing vomiting will not reduce BAC.
 - Alcohol is eliminated from the body by the liver at a rate between 4 g/h and 12 g/h, or at an average of 7.5 g/h or 0.75 standard drinks per hour (since a standard drink contains 10 g of alcohol), and can vary considerably depending on the person's health. This means that it can take the liver more than an hour to eliminate one standard drink.
- The liver breaks down alcohol at an average rate of 0.75 standard drinks per hour.
- A formula that can be used to calculate the time it takes for BAC to fall to zero is:

$$T = \frac{\text{BAC}}{0.015},$$
 where T is the number of hours the drinker must wait.

Worked example 1.6A Calculating the number of standard drinks

Calculate the number of standard drinks in a 150 mL glass of red wine, correct to one decimal place, given that the alcohol content of the wine is 14.5% alc/vol.

Think	Write
Convert the capacity of the glass to litres using: 1 L = 1000 mL.	$V = 150 \text{ mL}$ $= 150 \times \frac{1}{1000} \text{ L}$ $= 0.15 \text{ L}$
The alcohol content is 14.5% alc/vol, so $A = 14.5$.	
Substitute these values into the formula $N = 0.789 \times V \times A$.	$N = 0.789 \times V \times A$ $= 0.789 \times 0.15 \times 14.5$
Simplify and write the result correct to one decimal place.	≈ 1.7



Worked example video: Calculating the number of standard drinks

Understanding, fluency and communicating

Answers: Lesson 1.6

Helpful hints

- Remember to convert the capacity of the container to litres before substituting it into the formula.

1 Complete the following to calculate the number of standard drinks in these beers, correct to one decimal place.

- a** A 375 mL stubby of full-strength beer with an alcohol content of 4.8% alc/vol

$$\begin{aligned} N &= 0.789 \times V \times A \\ &= 0.789 \times \underline{\quad} \times 4.8 \\ &= \underline{\quad} \end{aligned}$$

- b** A 375 mL stubby of light beer with an alcohol content of 2.7% alc/vol

$$\begin{aligned} N &= 0.789 \times V \times A \\ &= 0.789 \times 0.375 \times \underline{\quad} \\ &= \underline{\quad} \end{aligned}$$

- 2 Calculate, correct to one decimal place, the number of standard drinks in:
- a 120 mL glass of wine with an alcohol content of 12% alc/vol
 - a 90 mL glass of fortified wine with an alcohol content of 16.5% alc/vol
 - a 375 mL can of bourbon and coke with an alcohol content of 6% alc/vol
 - a 750 mL bottle of white wine with an alcohol content of 11.5% alc/vol
 - a 275 mL bottle of vodka and orange with an alcohol content of 5% alc/vol
 - a six-pack (6×330 mL) of full strength beer with an alcohol content of 4.9% alc/vol.

Worked example 1.6B Calculating blood alcohol content (BAC)

Calculate, correct to two decimal places, the BAC of:

- a 78 kg male who has consumed five standard drinks in three hours
- a 46 kg female who has consumed four standard drinks in four hours.

Part	Think	Write
a	Substitute $N = 5$, $H = 3$ and $M = 78$ into the formula $\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ Simplify and write the result, correct to two decimal places.	$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ $= \frac{10 \times 5 - 7.5 \times 3}{6.8 \times 78}$ ≈ 0.05
b	Substitute $N = 4$, $H = 4$ and $M = 46$ into the formula $\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$ Simplify and write the result, correct to two decimal places.	$\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$ $= \frac{10 \times 4 - 7.5 \times 4}{5.5 \times 46}$ ≈ 0.04



Worked example video: Calculating blood alcohol content (BAC)

- 3 Complete the following to calculate the BAC of:
- an 83 kg male who has consumed six standard drinks in three hours

$$\begin{aligned} \text{BAC}_{\text{male}} &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{10 \times \square - 7.5 \times \square}{6.8 \times \square} \\ &= \end{aligned}$$
 - a 58 kg female who has consumed five standard drinks in four hours.

$$\begin{aligned} \text{BAC}_{\text{female}} &= \frac{10N - 7.5H}{5.5M} \\ &= \frac{10 \times \square - 7.5 \times \square}{5.5 \times \square} \\ &= \end{aligned}$$

- 4 Use the formulas for calculating BAC to complete the following table.

	Gender	Mass (kg)	Number of standard drinks consumed	Number of hours spent drinking	BAC (%)
a	Male	70	3	2	
b	Female	50	4	2	
c	Male	95	6	3	
d	Female	57	2	2	

- 5 An 80 kg adult male wants to keep his $BAC \leq 0.05$. Complete the following to find how many drinks he can consume in five hours to achieve this.

Let $BAC = 0.05$.

$$\begin{aligned} 0.05 &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{20N - \square}{\square} \\ \square &= 10N - \square \\ \square &= 10N \\ N &= \square \end{aligned}$$

If he has _____ standard drinks, he will have $BAC = 0.05$.

To have $BAC < 0.05$, he can have up to _____ standard drinks.

- 6 A 56 kg woman wants a BAC that is less than 0.05. How many drinks can she consume in three hours?



- 7 A rule of thumb can be used by the holder of a full drivers' licence to stay under the 0.05 legal limit in NSW:

For males: no more than two standard drinks in the first hour and one standard drink per hour after that.

For females: no more than one standard drink per hour.

Use this rule of thumb to calculate the maximum number of standard drinks that can be consumed by a person who wants to stay under 0.05 BAC if the person is:

- a male and drinking for four hours
 - a male and drinking for six hours
 - a female and drinking for four hours
 - a female and drinking for six hours.
- 8 Use the rule of thumb from question 7 to calculate the answers to questions 5 and 6.

Worked example 1.6C Calculating time needed to eliminate one standard drink

If a person's liver can break down alcohol at the rate of 5 g/h, how long will it take for the person's body to eliminate one standard drink? Write your answer correct to the nearest minute.

Think	Write
Let T be the amount of time required.	$\frac{5 \text{ g}}{1 \text{ h}} = \frac{10 \text{ g}}{T}$
One standard drink contains 10 g of alcohol.	$\frac{10}{5} = 2$
Write the given rate as a fraction.	$\times 2$
Find an equivalent rate with 10 g as the numerator.	$\frac{5 \text{ g}}{1 \text{ h}} = \frac{10 \text{ g}}{2 \text{ h}}$
Divide 10 by 5 to identify the value required to find the equivalent rate. Multiply the numerator and denominator by this value to find the equivalent rate.	$\times 2$
The time taken is the denominator of the equivalent rate.	$T = 2 \text{ h}$



Worked example video: Calculating time needed to eliminate one standard drink

Helpful hints

- If you are unsure where to start with a rate question, write the rate as a fraction and include the units in the numerator and denominator. You can then work with the rate by finding equivalent fractions with the same units.
 - If the rate at which the liver eliminates alcohol is given in g/h, then the time in hours needed to eliminate one standard drink is $\frac{10}{\text{rate of elimination}}$ hours.
 - Check you are using the correct formula to answer the question. If asked to find the amount of time taken for a given BAC to reduce to zero, use $T = \frac{\text{BAC}}{0.015}$.

9 Calculate the time it takes different people's bodies to eliminate one standard drink if their livers break down alcohol at these rates.

- a** 5 g/h **b** 10 g/h **c** 4 g/h **d** 12 g/h **e** 7.5 g/h

10 Complete the following to calculate how long a person must wait for BAC to drop to zero from 0.05%.

$$\begin{aligned}
 T &= \frac{\text{BAC}}{0.015} \\
 &= \frac{\square}{0.015} \\
 &= \square \text{ h} \\
 &= \square \text{ h } \square \text{ min}
 \end{aligned}$$

11 Calculate how long a person must wait for BAC to drop to zero from these levels.

- a** 0.04% **b** 0.02% **c** 0.06% **d** 0.035%

Problem solving and reasoning

- 12** A zero BAC is a requirement of NSW law for all learner and provisional drivers.
- a** Calculate the BAC for an 80 kg male and a 52 kg female, both with provisional licences, who consume four standard drinks in three hours.
 - b** How long would the two people described in part **a** need to wait before they could legally drive a motor vehicle?
- 13** Damien and Nicole go to a party and start drinking at 8 : 00 pm. Damien drinks eight schooners of full-strength beer (12 standard drinks) over the next five hours. Nicole has six mixed drinks (nine standard drinks) in the same time. Damien has a mass of 86.6 kg and Nicole's mass is 56.1 kg.
- a** Calculate the BAC of both Damien and Nicole at 1 : 00 am.
 - b** At what time will they be able to legally drive if they both have provisional licences?
- 14** Ben goes to a party and consumes two stubbies (375 mL) of full-strength beer (4.9% alc/vol) in the first hour and one stubby per hour for the next three hours.
- a** Calculate the number of standard drinks he has consumed.
 - b** Use the formula to calculate his BAC if his mass is 72 kg.
 - c** How long will it be before Ben's BAC drops to zero?

Challenge

- 15** Rebecca goes out with friends for drinks and starts drinking at 8 : 30 pm. Over the next 3 hours, she consumes 4.8 standard drinks. Rebecca's mass is 65 kg.
- a** She stops drinking at 11 : 30 pm. Calculate her BAC at this time correct to two decimal places.
 - b** BAC decreases at a rate of approximately 0.015% per hour. Rebecca decides not to drive home until her BAC should be approximately 0.04. How long should Rebecca wait after she stops drinking?
 - c** How long, correct to the nearest hour, would Rebecca have to wait if she were still on a provisional licence?

Related resources



1.6 Quick quiz



Investigation: Comparing BAC using formulas

Lesson 1.7

Medication calculations

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate medical dosages using Fried's, Young's and Clark's formulas.

Dosage calculations

- This section examines dosages of various medications. Some terms are defined here.
 - The dose is the amount of drug taken at any one time.
 - The dosage regimen is the frequency at which the drug doses are given.
 - The total daily dose is calculated from the dose and the number of times the dose is taken.
 - The dosage form is the physical form of a dose of the drug. Common dosage forms include tablets, capsules, creams, ointments, aerosols and patches.
 - The optimal dosage is the dosage that gives the desired effect with minimal side effects.
- Failing to adhere to the instructions for self-administered medications can have potentially serious consequences. It is important to understand both the maximum frequency and total daily dosage restrictions.
- When administering medication to children, doses must be adjusted appropriately. There are three different formulas that can be used to calculate dosage for children, each depending on the age of the child.
 - Fried's formula for children one to two years old is:

$$\text{Dosage} = \frac{\text{age (in months)} \times \text{adult dosage}}{150} = \frac{mA}{150}$$
 - where:
 - D = infant dosage
 - m = age of infant in months
 - A = adult dosage.
 - Young's formula for children 1 to 12 years old is:

$$\text{Dosage} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12} = \frac{yA}{y + 12}$$
 - where:
 - y = age of child in years
 - A = adult dosage.
 - Clark's formula for children of any age is:

$$\text{Dosage} = \frac{\text{child's weight (in kilograms)} \times \text{adult dosage}}{70} = \frac{kA}{70}$$
 - where:
 - k = mass of child in kilograms
 - A = adult dosage.
- Some medications are administered over a period of time, for example, via intravenous fluid. In these contexts, determining an appropriate dosage regimen requires an understanding of rates and accurate conversion of units.

Worked example 1.7A Understanding maximum dosages

The dosage instructions on a packet of painkillers say that a person 12 years or older can take two tablets every four to six hours, but only up to a maximum of eight tablets in 24 hours.

An adult plans to take two tablets every four hours for 24 hours.

a How many tablets would that person take over 24 hours? Why shouldn't they do this?

b How many doses of two tablets can be taken over 24 hours?

Part	Think	Write
a	Let N be the number of tablets. Write information provided as a rate in fraction form. Find an equivalent rate with 24 hours as the denominator. Divide 24 by 4 to determine there are six 4-hour intervals in a 24-hour period. Multiply the numerator and denominator by 6 to find the equivalent rate. The number of tablets taken over 24 hours is the numerator. Compare the value of N with the maximum daily dose.	$\frac{2 \text{ tablets}}{4 \text{ hours}} = \frac{N}{24 \text{ hours}}$ $\frac{24}{4} = 6$ $\times 6$ $\frac{2 \text{ tablets}}{4 \text{ hours}} = \frac{12 \text{ tablets}}{24 \text{ hours}}$ $\times 6$ $N = 12 \text{ tablets}$ Since the maximum daily dose is 8 tablets, this dosage regimen exceeds the maximum.
b	The maximum daily dose is 8, which can be divided into 4 doses of 2 tablets.	$\text{Doses} = \frac{8}{2}$ $= 4$



Worked example video: Understanding maximum dosages

Understanding, fluency and communicating

Answers: Lesson 1.7

Helpful hints

- Watch out for maximum dosage frequencies that would exceed the maximum daily dosage if followed throughout the day.

1 The dosage for a particular painkiller is as follows.

Age: 7 to 12 years: $\frac{1}{2}$ to 1 tablet every four to six hours (maximum four tablets in 24 hours)

Age: 12 years to adult: one to two tablets every four to six hours (maximum eight tablets in 24 hours)

a How many tablets can an adult take in one dose?

b An adult plans to take two tablets every six hours for 24 hours.

i How many tablets would that person take over the 24 hours?

ii How many doses of one tablet could they take over 24 hours?

c A child takes $\frac{1}{2}$ a tablet every four hours for 24 hours. Has the maximum dosage been exceeded? Explain.

- 2 For a very strong painkiller, dosage for adults and children from 12 years is two caplets, then one to two caplets every four to six hours as necessary. (Maximum six caplets in 24 hours.)
- a** An adult takes two caplets now and then two more after four hours. How many more caplets can that person take in that 24-hour period?
- b** Is two caplets initially, two more after four hours and two more after six hours, then no more, an acceptable dosage? Explain your answer.

Worked example 1.7B Using Fried's formula to calculate medication dosage

The adult dose of a medication is 40 mL. Use Fried's formula for children one to two years old to calculate the dosage for a 20-month-old child.

Think	Write
Use the formula $D = \frac{mA}{150}$ and substitute $m = 20$ months and $A = 40$ mL.	Dose = $\frac{20 \times 40}{150}$ = 5.3 mL



Worked example video: Using Fried's formula to calculate medication dosage

Helpful hints

- Recall that Fried's formula for children one to two years old is:

$$\text{Dosage} = \frac{\text{age (in months)} \times \text{adult dosage}}{150} \text{ or } D = \frac{mA}{150}$$
 where D = infant dosage, m = age of infant in months and A = adult dosage.
- Ensure that the units are correct for the formula; that is, age is in months.

- 3 Use Fried's formula to calculate each child's dosage below.
- a** Adult dose of 50 mL, child's age 15 months **b** Adult dose of 40 mL, child's age 21 months
- c** Adult dose of 30 mL, child's age 18 months **d** Adult dose of 50 mL, child's age 13 months
- e** Adult dose of 100 mL, child's age 17 months **f** Adult dose of 80 mL, child's age 23 months
- 4 Use Fried's formula to calculate the adult dosage in these situations.
- a** A child aged 17 months is given a dosage of 6 mL.
- b** A child aged 11 months is given a dosage of 7 mL.

Worked example 1.7C Using Young's formula to calculate medication dosage

Use Young's formula to calculate the dosage for a $5\frac{1}{2}$ -year-old child if the adult dose is 60 mL.

Think	Write
Use the formula $D = \frac{yA}{y+12}$ where $y = 5.5$ and $A = 60$ mL.	Dose = $\frac{5.5 \times 60}{5.5 + 12}$ ≈ 19 mL



Worked example video: Using Young's formula to calculate medication dosage

Helpful hints

- Recall that Young's formula for children 1 to 12 years old is

$$\text{Dosage} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12} \text{ or } D = \frac{yA}{y + 12}$$
 where y = age of child in years and A = adult dosage.
- Check the units before substituting. Some formulas use age in years, others use age in months.

- 5 Use Young's formula to calculate each child's dosage, to the nearest millilitre.
- Adult dose of 50 mL, child's age 6 years
 - Adult dose of 40 mL, child's age 8 years
 - Adult dose of 80 mL, child's age 4.5 years
 - Adult dose of 20 mL, child's age 7.5 years
 - Adult dose of 100 mL, child's age 6.2 years
 - Adult dose of 75 mL, child's age 8.4 years
- 6 Use Young's formula to calculate the adult dose in these situations.
- The dosage for an 8-year-old child is 10 mL.
 - The dosage for a 6-year-old child is 5 mL.

Worked example 1.7D Using Clark's formula to calculate medication dosage

Use Clark's formula to calculate the dosage for a child weighing 19 kg. The adult dose is 50 mL.

Think	Write
Use the formula $D = \frac{kA}{70}$ where $k = 19$ and $A = 50$.	$\text{Dose} = \frac{19 \times 50}{70}$ $\approx 14 \text{ mL}$



Worked example video: Using Clark's formula to calculate medication dosage

Helpful hints

- Recall that Clark's formula for children of any age is:

$$\text{Dosage} = \frac{\text{child's weight (in kilograms)} \times \text{adult dosage}}{70}$$
 or $D = \frac{kA}{70}$ where k = mass of child in kilograms and A = adult dosage.
- Check units before substituting. The formula uses 70 kg as the average adult weight.

- 7 Use Clark's formula to calculate each child's dosage, to the nearest millilitre.
- a Adult dosage of 18 mL, child's weight 40 kg
 - b Adult dosage of 27 mL, child's weight 60 kg
 - c Adult dosage of 51 mL, child's weight 80 kg
 - d Adult dosage of 39 mL, child's weight 75 kg
 - e Adult dosage of 32 mL, child's weight 100 kg
 - f Adult dosage of 40 mL, child's weight 35 kg
- 8 Use Clark's formula to calculate the adult dose in these situations.
- a A 35 kg child has a dose of 18 mL
 - b A 25 kg child has a dose of 17 mL
- 9 Use Clark's formula to calculate the weight of a child receiving a dose of 40 mL, given that the adult dose is 140 mL.

Problem solving and reasoning

- 10 A sick child requires medication. The adult dose of the medication is 20 mL. The child is 16 months old and has a mass of 7.9 kg.
- a Find the dosage using Fried's rule.
 - b Find the dosage using Young's rule.
 - c Find the dosage using Clark's rule.
 - d How much would you give the child? Explain.
- 11 Repeat question 10 for a 20-month-old child weighing 12 kg, given that the adult dose of the medication is 30 mL.
- 12 A child is 10 years of age and requires a dose of a medicine for which the adult dosage would be 50 mL. The dosage is calculated using Fried's rule, without realising that rule is used for children under two years only. A dose could be an overdose if it is double the correct dose.
- a Calculate the dosage according to Fried's rule.
 - b Calculate the correct dosage using Young's rule.
 - c If the entire amount calculated using Fried's rule is taken at the one time, has the child received an overdose? Explain.

Challenge

- 13 For a certain medication, the adult dosage is 13 mL. Using Young's formula, calculate the age of a child who would receive a dosage of 5 mL.

Related resources



1.7 Quick quiz

Lesson 1.8

Solving simple quadratic equations

Learning intentions

By the end of this lesson, you will be able to ...

→ solve simple quadratic equations and assess whether the solution is reasonable.

Simple quadratic equations

- A **simple quadratic equation** is an equation of the form $y = ax^2 + c$.
- When $y = 0$, a simple quadratic equation can be solved for x using inverse operations.
 - The inverse operation to squaring is taking the square root.
- A quadratic equation can have zero, one or two solutions.
 - The solutions to $0 = ax^2 + c$ are $x = \sqrt{\frac{c}{a}}$ and $x = -\sqrt{\frac{c}{a}}$.
 - If $\frac{c}{a} > 0$, then there are two solutions. These are often written as $x = \pm\sqrt{\frac{c}{a}}$.
 - If $\frac{c}{a} = 0$, then $\sqrt{\frac{c}{a}} = -\sqrt{\frac{c}{a}}$, so the only solution is $x = 0$.
 - If $\frac{c}{a} < 0$, then there are no real solutions.
- Square roots of fractions can be simplified in two ways.
 - The rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where both a and b are positive values, can be used to simplify the square root of a fraction.
 - Square roots of fractions can also be simplified by first writing the fraction in its simplest form before taking the square root.
- In some contexts, a negative solution to an equation may not make sense. For example, measurements like distances, lengths and durations of time are usually positive values.

Worked example 1.8A Solving simple quadratic equations (exact solutions)

Solve the following simple quadratic equations.

a $3x^2 = 27$

b $-4x^2 = -25$

c $\frac{x^2}{2} - 0.08 = 0$

Part	Think	Write
a	Divide both sides of the equation by 3 and simplify the result. Take the square root of both sides of the equation to solve for x . As $9 > 0$, there are two solutions to the equation.	$3x^2 = 27$ $x^2 = \frac{27}{3}$ $x^2 = 9$ $x = \pm\sqrt{9}$ $x = \pm 3$

Part	Think	Write
b	Divide both sides of the equation by -4 and simplify the result. Take the square root of both sides of the equation to solve for x. As $\frac{25}{4} > 0$, there are two solutions to the equation. Use the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ to find the square root of the fraction and simplify the result.	$-4x^2 = -25$ $x^2 = \frac{-25}{-4}$ $x^2 = \frac{25}{4}$ $x = \pm\sqrt{\frac{25}{4}}$ $= \pm\frac{\sqrt{25}}{\sqrt{4}}$ $= \pm\frac{5}{2}$
c	Add 0.08 to both sides of the equation. Multiply both sides of the equation by 2 and simplify the result. Take the square root of both sides of the equation to solve for x. As $0.16 > 0$, there are two solutions to the equation.	$\frac{x^2}{2} - 0.08 = 0$ $\frac{x^2}{2} = 0.08$ $x^2 = 0.16$ $x = \pm\sqrt{0.16}$ $= \pm 0.4$



Worked example video: Solving simple quadratic equations (exact solutions)

Understanding, fluency and communicating

Answers: Lesson 1.8

Helpful hints

- Make sure to only take the square root of a positive number (or zero) when solving simple quadratic equations.

1 Solve the following simple quadratic equations.

a $x^2 = 25$

b $x^2 = 1$

c $2x^2 = 98$

d $8x^2 = 0$

e $-x^2 = -81$

f $-2x^2 = -72$

g $-5x^2 = -125$

h $-10x^2 = -1000$

2 Solve the following simple quadratic equations.

a $4x^2 = 81$

b $9x^2 = 49$

c $-16x^2 = -36$

d $-25x^2 = -121$

e $-100x^2 = -4$

f $-64x^2 = -9$

3 Solve the following simple quadratic equations.

a $x^2 = 0.01$

b $x^2 = 0.25$

c $2x^2 = 0.72$

d $\frac{x^2}{3} = 0.27$

4 Solve the following simple quadratic equations.

a $2x^2 - 1 = 31$

b $-3x^2 + 3 = -9$

c $4x^2 + 9 = 13$

d $5x^2 - 2000 = 0$

e $-6x^2 + 216 = 0$

f $7x^2 - 63 = 0$

g $\frac{x^2}{2} - 3.125 = 0$

h $\frac{x^2}{3} - 0.75 = 0$

i $\frac{x^2}{4} + 1.5 = 2.14$

Worked example 1.8B Solving simple quadratic equations (approximations)

Solve the following simple quadratic equations, correct to two decimal places.

a $2x^2 = 3$

b $0.2x^2 - 1.7 = 0$

Part	Think	Write
a	Divide both sides of the equation by 2. Take the square root of both sides of the equation. As $\frac{3}{2} > 0$, there are two solutions to the equation. Use a calculator to write the solutions, correct to two decimal places.	$2x^2 = 3$ $x^2 = \frac{3}{2}$ $x = \pm\sqrt{\frac{3}{2}}$ $x \approx \pm 1.22$
b	Add 1.7 to both sides of the equation. Divide both sides of the equation by 0.2 and simplify the result. Take the square root of both sides of the equation. As $8.5 > 0$, there are two solutions to the equation. Use a calculator to write the solutions, correct to two decimal places.	$0.2x^2 - 1.7 = 0$ $0.2x^2 = 1.7$ $x^2 = \frac{1.7}{0.2}$ $= 8.5$ $x = \pm\sqrt{8.5}$ $x \approx \pm 2.92$

**Worked example video:** Solving simple quadratic equations (approximations)**Helpful hints**

- Pay attention to the order of operations when typing more complicated expressions into a calculator. Check that the answer you get makes sense.

5 Solve the following simple quadratic equations, correct to two decimal places.

a $3x^2 = 2$

b $5x^2 = 7$

c $4x^2 - 11 = 0$

d $5x^2 - 17 = 0$

e $-10x^2 + 9 = 0$

f $\frac{x^2}{3} + 5 = 10$

6 Solve the following simple quadratic equations.

a $x^2 + 1.5 = 2.9$

b $1.7x^2 - 3.2 = 0$

c $0.3x^2 + 5.1 = 9.2$

d $2.9x^2 - 10 = 0$

7 Solve the following equations.

a $\frac{a^2}{2.7} - 7.2 = -4.5$

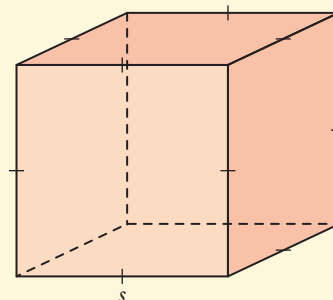
b $\frac{1}{3}(p^2 - 12) = -4$

c $2M^2 + 0.0008 = 0.008$

d $3k^2 - \frac{9}{11} = 0$

Worked example 1.8C Applications of simple quadratic equations

The formula for the surface area of a cube is $SA = 6s^2$, where s is the side length of the cube. Determine the side length of a cube with a surface area of 150 cm^2 .



Think

Substitute $SA = 150 \text{ cm}^2$ into the formula for the surface area of a cube.

Divide both sides of the equation by 6 and simplify the result.

Take the square root of both sides of the equation.

As $25 > 0$, there are two solutions.

The side length of a cube must be a positive value. Take only the positive solution to the equation.

Write

$$\begin{aligned} SA &= 6s^2 \\ 150 &= 6s^2 \\ \frac{150}{6} &= s^2 \\ s^2 &= 25 \\ s &= \pm\sqrt{25} \\ &= \pm 5 \\ s &> 0 \\ s &= 5 \text{ cm} \end{aligned}$$



Worked example video: Applications of simple quadratic equations

Helpful hints

- Make sure the units of measurement in your solutions match what is required in the question.
- Check that your final answer makes sense in the context of the question. For example, a cube cannot have a side length of -5 cm !

- 8 The formula for the surface area of a cube is $SA = 6s^2$, where s is the side length of the cube. Determine the side length of a cube with a surface area of 96 cm^2 .
- 9 The braking distance in metres of a car travelling at $v \text{ km/h}$ in good conditions is $d = 0.01v^2$. What is the speed of a car which has a braking distance of 42.25 m ?
- 10 The braking distance in metres of a car travelling at $v \text{ km/h}$ in poor conditions is $d = 0.014v^2$. What is the speed of a car which has a braking distance of 47.096 m ?
- 11 Two cars are travelling in good conditions at different speeds. One car has a braking distance of 49 m and the other has a braking distance of 56.25 m . What is the difference between the speeds of the two cars in km/h ?

Problem solving and reasoning

- 12 Consider the equation $y = 2x^2 + 3$.
 - a Substitute $y = 2$ into the equation and solve for x .
 - b Substitute $x = 2$ into the equation and solve for y .
 - c How many solutions are there to parts **a** and **b**? Explain why this is the case.
- 13 Consider the equation $y = -3x^2 - 4$.
 - a For which value of y is there only one solution to the equation?
 - b Find a value of y for which there is no solution to the equation.

14 Which of the following equations have no solutions?

a $x^2 + 1 = 0$

b $x^2 - 1 = 0$

c $x^2 = 1$

d $x^2 = 0$

e $2x^2 = 9$

f $2x^2 = -9$

g $-2x^2 = 9$

h $-2x^2 = -9$

i $4x^2 - 1 = 0$

j $4x^2 + 1 = 0$

k $-4x^2 - 1 = 0$

l $-4x^2 + 1 = 0$

15 Write a simple quadratic equation that has no solutions and then show that it has no solutions.

Challenge

16 Solve the following equation after substituting $p = 16$, $n = 5$ and $k = 7$. Write your answer(s) correct to two decimal places.

$$p = n + 12kt^2$$

Related resources



1.8 Quick quiz



Worksheet: Solving simple quadratic equations

DRAFT

Using spreadsheets

Syllabus links

Use a spreadsheet to perform calculations involving formulas (MST-11-01)

Formulas and equations

See Oxford Digital for sample spreadsheets for questions in this lesson.

- 1 Use a spreadsheet program to solve the equation $3x - 7 = 9 - x$, given that the solution is an integer. The spreadsheet needs to have three columns labelled x , $3x - 7$ and $9 - x$.

In cell A1 enter x , in cell B1 enter $3x - 7$, in cell C1 enter $9 - x$, in cell A2 enter 0, in cell B2 enter $= A2 * 3 - 7$ and in cell C2 enter $= 9 - A2$.

Use the **fill down** command to find the values for each side of the equation. The answer is the value of x that gives the same value in columns A and C.

	A	B	C
1	x	$3x - 7$	$9 - x$
2	0	$= A2 * 3 - 7$	$= 9 - A2$
3	1		
4	2		
5	3		
6	4		
7	5		
8	6		

- 2 Change the spreadsheet in question 1 to solve each of these equations with positive integer solutions.

a $5x - 3 = 53 - 2x$

b $3x + 5 = 35 - 2x$

c $19 - 2x = 7x - 44$

d $6x + 11 = 41 - 4x$

e $3x - 17 = 33 - 7x$

f $9x + 15 = 79 - 7x$

- 3 Use a spreadsheet to solve these equations with negative integer solutions.

a $4x - 3 = 13 + 6x$

b $7x - 3 = -25 + 5x$

c $8 - 7x = 2 - 8x$

- 4 Explain how to modify the spreadsheet used in questions 2 and 3 to solve equations that do not have integer solutions.

- 5 Solve these equations using a spreadsheet.

a $3x - 7 = 6 - 2x$

b $5x + 23 = 2x - 8$

c $8 - 7x = 8x + 59$

- 6 Create a spreadsheet to show the blood alcohol content for a 75 kg male and a 55 kg female, where N is the number of standard drinks consumed compared with the number of hours spent drinking. Investigate what the BAC would be for a given number of standard drinks over different periods of time. Remember it is only an approximation!
- 7 Create a spreadsheet to calculate the dosage of a particular medicine for children under 12 years of age. Fried's rule is applicable for children under two years, Young's formula is for children 1 to 12 years and Clark's formula is for children of any age. Set your spreadsheet up so that when you enter the three variables for a child the spreadsheet automatically calculates their dosage.

	A	B	C
1	Calculation of medicine dosage for child 12 years or less		
2	Child's information		
3	Age in months	11	
4	Weight in kilograms	6.2	
5	Adult dosage in millilitres	40	
6	Child dosage in millilitres		
7	Fried's rule	Young's rule	Clark's rule
8	2.9	Not suitable	3.5
9			
10	Enter required information into cells B3, B4 and B5.		

Lesson 1.9

Review: Formulas and equations

Review questions 1.9A Mathematical literacy review

Answers: Lesson 1.9

Key terms

The following key terms are used in this module:

- average speed
- blood alcohol content (BAC)
- braking distance
- consecutive integers
- equation
- expression
- formula
- linear equation
- pronumeral
- reaction-time distance
- simple quadratic equation
- solution
- solve
- standard drink
- subject of the formula
- term (algebra)
- total stopping distance
- variable (algebra)

Check your understanding

- Fill in the blanks using words from the list above.
 - Any letter that appears in a mathematical expression is called a _____.
 - To _____ an equation means to find a value for a _____ that makes the equation true. The value found is a _____ to the equation.
- Write the meaning of the word **formula** in a sentence including the words **equation** and **variable**.
- Give three examples of a **linear equation** and three examples of a **quadratic equation**.
- Label one part of the following equation that is an example of an **expression** and label three different parts that are examples of a **term**.

$$5x - 2y + 1 = 7y^2 + x$$
- Fill in the blanks using words from the list above.
 - Australian adults who consume alcohol responsibly will often take note of how many _____ they consume. This way, they can make a reasonable estimate of their _____ before deciding to engage in risky behaviours, such as driving a car.
 - To evaluate a driver's ability to avoid an accident by coming to a complete stop, we are interested in their _____, which is directly influenced by their _____. The total distance is found by adding together the _____ and _____.

Review questions 1.9B Multiple-choice questions

- Let $a = 7$ and $b = -3$. What is the value of $\frac{a^2 - 5b}{8}$?

A 64
B 3
C $50\frac{7}{8}$
D 8
- What is the value of $s = ut + \frac{1}{2}at^2$ if $u = 0$, $t = 5$ and $a = 10$?

A 125 B 130 C 2500 D 2505
- The volume of a cone with $h = 10$ and $r = 3$ is calculated using $V = \frac{1}{3}\pi r^2 h$. Which value is closest to the volume?

A 31
B 94
C 314
D 30

- 4 What is the solution to $p - 16 = 42$?
- A $p = 26$
 B $p = 36$
 C $p = 48$
 D $p = 58$
- 5 What is the solution to $11x - 10 = 25$?
- A $x = 25$
 B $x = 3\frac{2}{11}$
 C $x = \frac{11}{35}$
 D $x = \frac{15}{11}$
- 6 Which equation has solution $x = -3$?
- A $3x - 7 = 2$
 B $4x - 3 = 15$
 C $x + 5 = 8$
 D $5x + 2 = -13$
- 7 Which equation does *not* have $x = 4$ as a solution?
- A $x - 4 = 0$
 B $\frac{x}{4} = 16$
 C $4x = 16$
 D $x + 5 = 9$
- 8 What is the solution to $\frac{3a + 4}{5} = 2$?
- A $a = 2$
 B $a = 3$
 C $a = 4$
 D $a = 5$
- 9 What is the solution to $7x - 3 = 4x + 9$?
- A $x = 2$
 B $x = 3$
 C $x = 4$
 D $x = 5$
- 10 Using the formula $V = IR - E$, what is the value of I when $V = 13$, $R = 4$ and $E = 7$?
- A $1\frac{1}{2}$
 B $\frac{1}{4}$
 C 5
 D $\frac{2}{3}$
- 11 The formula $V = IR - E$ is rearranged to make R the subject. What is the new formula?
- A $R = \frac{V + E}{I}$
 B $R = \frac{V - I}{E}$
 C $R = \frac{E}{VI}$
 D $R = \frac{E - V}{I}$
- 12 A car travels 280 kilometres in 3 hours and 25 minutes. What is its average speed?
- A 86 km/h
 B 82 km/h
 C 93 km/h
 D 42 km/h
- 13 What is 70 km/h expressed in m/s?
- A 1.2 m/s
 B 0.02 m/s
 C 1167 m/s
 D 19.4 m/s
- 14 What is the distance a car travels in 2.8 seconds at 80 km/h?
- A 6.2 m
 B 7.9 m
 C 62.2 m
 D 373.3 m
- 15 Paul goes to a party and consumes three stubbies (375 mL) of full-strength beer (4.9% alc/vol) in the first hour and one stubby per hour for the next 4 hours. Approximately, how many standard drinks has he consumed?
- A 1.4
 B 4.3
 C 5.8
 D 10.1
- 16 A 75 kg woman consumes seven standard drinks in four hours. What is her BAC?
- A 0.078
 B 0.097
 C 0.010
 D 0.012
- 17 The adult dose of a medication is 50 mL. Using Fried's formula below, what is the dosage for a one-year-old child?
- Child dose = $\frac{\text{age (in months)} \times \text{adult dose}}{150}$
- A 4 mL
 B 0.3 mL
 C 3 mL
 D 0.4 mL
- 18 What are the solutions to the quadratic equation $x^2 - 9 = 7$?
- A $x = 2$
 B $x = \frac{3}{2}$
 C $x = \pm 2$
 D $x = \pm \sqrt{\frac{1}{2}}$

19 How many solutions does the quadratic equation $2x^2 - 3 = -4$ have?

- A** One solution, since $x = \sqrt{\frac{-1}{2}}$
B No solutions, since $-\frac{1}{2} < 0$
C One solution, since $x = -\sqrt{\frac{1}{2}}$
D No solutions, since $-\frac{7}{2} < 0$

Review questions 1.9C Set 1

- If $x = 4$ and $y = 6$, find the value of $3x - 5y$.
- The volume, V , of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Find the volume of a sphere with radius:
 - 7 cm
 - 3.5 cm
 - 0.6 m
- The formula for converting temperature measurements from degrees Celsius, C , to degrees Fahrenheit, F , is $F = \frac{9}{5}C + 32$. Find F when:
 - $C = 200^\circ$
 - $C = 20^\circ$
 - $C = 55^\circ$
- Solve the following equations.
 - $d - 8 = 36$
 - $x + 9 = 20$
 - $\frac{x}{8} = 72$
 - $11x = 55$
 - $x^2 - 3 = 46$
 - $3x - 12 = 60$
 - $10 + 4n^2 = 74$
 - $12 - 6c = 85$
 - $3(2x + 1) = 4$
- Use the formula $E = \frac{1}{2}mv^2$ to find the positive value of v when $m = 50$ and $E = 10\,000$.
- Make x the subject of each formula below.
 - $3x = t$
 - $y = mx - 7$
 - $a = \frac{x+d}{c}$
- For a car travelling on a slippery road, the formula for braking distance becomes $d = 0.015v^2$. What is the braking distance of a car travelling at 100 km/h on this road?
- A train averages 84 km/h for 2 hours and 36 minutes. How far does it travel?
- Calculate how long an average person must wait for BAC to drop to zero from 0.07%.
- Use Fried's formula to calculate the dosage of medicine for a nine-month-old child if the adult dose is 60 mL.
- Use Young's formula to calculate the dosage of medicine for a $6\frac{1}{2}$ -year-old child if the adult dose is 45 mL.

Review questions 1.9D Set 2

- If $x = -2$ and $y = 3$, find the value of $5x - 7y$.
- The surface area, A , of a sphere is found using the formula $A = 4\pi r^2$, where r is the radius. Find the surface area of a sphere with radius:
 - 4 cm
 - 7.5 cm
 - 0.8 m
- The formula for converting temperature measurements from degrees Fahrenheit, F , to degrees Celsius, C , is $C = \frac{5}{9}(F - 32)$. Find C when:
 - $F = 248^\circ$
 - $F = 50^\circ$
 - $F = 32^\circ$
- Solve the following equations.
 - $d - 9 = 23$
 - $x + 4 = 65$
 - $\frac{x}{3} = 5$
 - $x^2 = 121$
 - $3x - 7 = 30$
 - $3x^2 - 12 = 15$
 - $12 - 9c = 43$
 - $7(2x + 5) = 4$
 - $4x - 2 = x + 9$
- Use the area formula $A = \frac{1}{2}bh$ to find the length b , given that $A = 72$ and $h = b$.
- Make x the subject of each formula below.
 - $x + y = m$
 - $n = k - 3x$
 - $m = \frac{x}{k} - 4$

- 7 A car travels 252 km in 4 hours and 19 minutes. Calculate its average speed.
- 8 Calculate the BAC of a 98 kg male who has consumed six standard drinks in 4 hours.
- 9 Use Young's formula to calculate the dosage of medicine for a $7\frac{1}{2}$ -year-old child if the adult dose is 25 mL.
- 10 Use Clark's formula to calculate the dosage of medicine for a child weighing 15 kg. The adult dose is 30 mL.

Review questions 1.9E Set 3

- 1 Given $x = -5$ and $y = 8$, find the value of $4x^2 - 6y$.
- 2 The surface area, A , of a cylinder is found using the formula $A = 2\pi r(r + h)$, where r is the radius and h is the height. Find the surface area of a cylinder with radius 15 cm and height 10 cm.
- 3 The time T seconds for a pendulum of length L m to swing back and forth once is give by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where $g \approx 10 \text{ m/s}^2$. Find how long it takes for a pendulum of length 2.5 m to swing back and forth once.
- 4 Solve the following equations.

a $d - 9 = 11$	b $x + 4 = 14$	c $\frac{x}{3} = 2$
d $11x = 77$	e $3x - 8 = 12$	f $7x - 12 = 19$
g $14 + 3n = 23$	h $23 - 3c^2 = 11$	i $2(4x + 5) = 12$
- 5 Use the formula $T = a + (n - 1)d$ to find n , given that $T = 41$, $a = 3$ and $d = 2$.
- 6 Make x the subject of each formula below.

a $\frac{x}{y} = t$	b $y = k - mx^2$	c $\frac{x-t}{w} = y$
- 7 Find the total stopping distance for a car travelling at 90 km/h in good conditions. Assume a reaction time of 2.5 seconds and a braking distance of $d = 0.01v^2$.
- 8 Calculate the number of standard drinks in a 120 mL glass of red wine, given that the alcohol content of the wine is 14.6% alc/vol.
- 9 Calculate the BAC for:
 - a a 76 kg male who has consumed five standard drinks in three hours
 - b a 52 kg female who has consumed four standard drinks in four hours.
- 10 Calculate how long an average person must wait for BAC to drop to 0 from 0.06%.
- 11 A zero BAC is a requirement of NSW law for all learner and provisional drivers.
 - a Calculate the BAC for a 75 kg male and a 53 kg female, both with provisional licences, who consume four standard drinks in three hours.
 - b How long would the two people have to wait before they could legally drive a motor vehicle?
- 12 Use Clark's formula to calculate the dosage of medicine for a child weighing 38 kg, given the adult dose is 35 mL.



Review questions 1.9F Set 4

- 1 Given $x = 12$ and $y = -4$, find the value of $6x^2 - 11y$.
- 2 The volume, V , of a cylinder is found using the formula $V = \pi r^2 h$, where r is the radius and h is the height. Find the volume of a cylinder with radius 12 cm and height 25 cm.
- 3 A formula for calculating the bend allowance, in millimetres, of sheet metal is $B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$ where B is the bend allowance, T is the thickness in millimetres, A is the number of degrees in the angle of bend, and R is the radius of curvature in millimetres. Find B when $T = 1.5$, $R = 4$ and $A = 113$.

4 Solve the following equations.

a $d - 5 = 11$

b $x + 7 = 14$

c $\frac{x}{6} = 2$

d $7x = 77$

e $3x - 4 = 92$

f $8x^2 - 13 = 11$

g $11 + 7n = 53$

h $27 - 9c = 31$

i $4(3x + 5) = 13$

5 Use the formula $b = \frac{m}{h^2}$ to find positive h if $b = 31.25$ and $m = 80$.

6 Use the formula $a = \frac{2Rn}{n+1}$ to find the value of R when $a = 95$ and $n = 19$.

7 If a cyclist can average 15 km/h, how long will it take them to travel 28 km?

8 Convert the following.

a 55 km/h to m/s

b 21.4 m/s to km/h

9 Calculate the reaction-time distance for a car travelling at 60 km/h. Assume a reaction time of 2.5 seconds.

10 Use the formula $d = 0.01 v^2$ to calculate the braking distance for a car travelling at 80 km/h in good conditions.

11 Find the total stopping distance for a car travelling at 80 km/h in good conditions. Use the formula $d = 0.7v + 0.01 v^2$, where d is the stopping distance in metres and v is the speed in km/h.

12 For a driver under the influence of alcohol and driving in poor road conditions, the formula for the stopping distance of a car becomes $d = 1.1v + 0.018 v^2$.

a Calculate the stopping distance to the nearest metre to complete the table for this driver.

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)						

b Use the table from part **a** to draw a graph of the relationship between speed and stopping distance.

c From the graph drawn for part **b** estimate the stopping distance if the car is travelling:

i 50 km/h

ii 70 km/h

iii 110 km/h

13 Use Young's formula to calculate the dosage of medicine for a 10-year-old child. The adult dose is 20 mL.

Review questions 1.9G Practice examination question

1 Answer the following questions.

a Solve $\frac{x}{3} = -3$. (1 mark)

b Find the value of $\sqrt{10a + 2b}$ when $a = 5$ and $b = 7$. (2 marks)

c The body mass index, b , is used to determine whether a person's mass is within the acceptable range of $21 < b < 25$. To calculate her body mass index, Stella used the formula $b = \frac{m}{h^2}$, where m = mass in kilograms and h = height in metres.

d Here are her calculations.

$$b = \frac{m}{h^2}$$

When $m = 55$ kg and $h = 168$ cm:

$$b = \frac{55}{168^2} \\ = 0.001\ 95$$

i Explain the error in her calculations. (1 mark)

ii Calculate the correct value of b . (1 mark)

iii Is Stella in the healthy mass range? Explain. (1 mark)

e George's solution to the equation $6 - 5x = 8$ is shown below.

$$\begin{aligned} 6 - 5x &= 8 \\ 6 - 5x + 6 &= 8 + 6 \\ -5x &= 14 \\ x &= 8 \end{aligned}$$

He has made some errors. Identify the errors and find the correct solution. (2 marks)

f The total stopping distance, d metres, of a car travelling at v km/h under good conditions is given by the formula $d = 0.7v + 0.01 v^2$.

- i Calculate the stopping distance of a car travelling at 60 km/h. **(1 mark)**
- ii If the speed of the car is increased by 10 km/h, what is the increase in the stopping distance? **(2 marks)**
- g Harry goes to a party and drinks seven standard drinks over four hours.
- i Use the formula $BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ to calculate Harry's blood alcohol content after four hours if his mass is 78 kg. **(2 marks)**
- ii Use the formula $T = \frac{BAC}{0.015}$ to find how long it will take for his BAC to drop to zero. **(1 mark)**
- h Use Young's formula below to calculate the dosage of medicine for a $4\frac{1}{2}$ -year-old child, given the adult dose is 100 mL. **(1 mark)**
- $$\text{Dosage for children 1 to 12 years} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$$

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Formulas and equations

I can do this	I need to review this
☐ substitute numbers into expressions, equations and formulas	Go back to
☐ evaluate the value of the subject of a formula given the value of other pronumerals in the formula	Lesson 1.1 Substitution into expressions and formulas
☐ solve linear equations	Go back to
	Lesson 1.2 Solving linear equations
☐ solve equations by substitution into formulas	Go back to
	Lesson 1.3 Solving further linear equations
☐ change the subject of a formula that is a linear equation	Go back to
	Lesson 1.4 Variables and formulas
☐ solve problems related to speed, distance and time	Go back to
☐ change units when solving problems related to speed, distance and time	Lesson 1.5 Travel calculations
☐ calculate the distance a motor vehicle travels in the time it takes to bring it to a stop	
☐ calculate the number of standard drinks in a serving of alcohol	Go back to
☐ calculate the BAC for males and females	Lesson 1.6 Blood alcohol content calculations
☐ calculate the time it takes for a person's BAC to fall to zero	
☐ identify and explain the limitations of methods used to estimate BAC	
☐ calculate medical dosages using Fried's, Young's and Clark's formulas	Go back to
	Lesson 1.7 Medication calculations
☐ change the subject of a formula that is a simple quadratic equation	Go back to
☐ assess whether a solution is reasonable in the context of the problem	Lesson 1.8 Solving simple quadratic equations
☐ use a spreadsheet to perform calculations involving formulas	Go back to
	Using spreadsheets

Related resources



Module review quiz: Formulas and equations



Worksheet: Challenge 1

Module

2

Linear relationships

Lessons

- Lesson 2.1 Visualising linear models
- Lesson 2.2 Understanding gradient
- Lesson 2.3 The equation $y = mx + c$
- Lesson 2.4 Direct variation
- Lesson 2.5 Analysing linear models
- Using spreadsheets
- Lesson 2.6 Review: Linear relationships

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Linear relationships

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: The Cartesian plane



Support sheet: Plotting linear relationships



Support sheet: Substitution



Support sheet: Writing formulas



Support sheet: Solving linear equations

Syllabus links

- Linear relationships (MST-11-02)
 - A student models and interprets linear relationships to solve problems and make predictions in practical contexts.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 2.1

Visualising linear models

Learning intentions

By the end of this lesson, you will be able to ...

→ sketch straight-line graphs to model real-world linear relationships.

Mathematical models

- A **mathematical model** is a representation of the structure, workings or relationships within a situation. Models can provide insight into the situation and can be used to test or predict outcomes for which there may be no other information.
- Most models are subject to limitations and extending beyond these limitations means any tests or predictions may become less reliable. For example, a model may only be reliable up to a known data value.

Straight-line graphs

- Many situations are modelled using straight-line graphs. These are called **linear models**.
- There are many ways to represent linear models.
 - Points can be plotted on a Cartesian plane with a line drawn through them.
 - A rule or relation describing the relationship between the variables can be given.
 - A list of ordered pairs, which can be represented as a table of values. These can be interpreted as points to plot on a Cartesian plane.
- The graph of a linear relationship between two variables produces a straight line.
- The values of the **dependent variable** depend on the values of the **independent variable**.

Worked example 2.1A Drawing a straight-line graph from a table of values

Bulk washing powder is sold for \$1.40 per kilogram. The table shows weight versus cost for washing powder.

Weight (kg)	1	2	5	10	15
Cost (\$)	1.40	2.80	7.00	14.00	21.00

- Draw the graph of weight versus cost.
- Use the graph to find the cost of 12.5 kg of washing powder.

Part	Think	Write
a	1 The cost depends on the number of kilograms, so it is the dependent variable, and is placed on the vertical axis. The number of kilograms is the independent variable and is placed on the horizontal axis. Choose a suitable scale for each axis by considering the values in each row of the table. 2 Write the values in the table as coordinate points. Plot the points and draw a straight line through them.	Washing powder Cost (\$) Weight (kg) (1, 1.40), (2, 2.80), (5, 7.00), (10, 14.00), (15, 21.00)
b	1 Find the point 12.5 kg on the “Weight” axis. Draw a vertical line to intersect the graph. 2 From this point, draw a horizontal line to the “Cost” axis. Read the value at this point: \$17.50.	The cost of 12.5 kg of washing powder is \$17.50.



Worked example video: Drawing a straight-line graph from a table of values

Understanding, fluency and communicating

Answers: Lesson 2.1

Helpful hints

- Place the dependent variable on the vertical axis and the independent variable on the horizontal axis. This information is useful to remember when looking at a graph of a linear model.
- Remember to label the axes (including units where necessary).
- When choosing a scale for each axis, make sure to consider the values of the points you need to plot and how accurately you need to be able to read information from the graph.

- 1 Oranges are sold for \$3.50 per kilogram. The table shows weight versus cost for the oranges.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	3.50	7.00	10.50	14.00	17.50	21.00



- a Draw the graph of weight versus cost. Which is the dependent variable?
 b Use your graph from part a to find the cost of 2.5 kg of oranges.
- 2 Mobile telephone calls cost 65 cents per minute. The table shows the cost of calls of different duration.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.65	1.30	1.95	2.60	3.25	3.90

- a Draw the graph of time versus cost. Which is the independent variable?
 b Use your graph from part a to find the cost of a call of 2.5 minutes duration.
 c A call cost \$3.00. How long was that call?
- 3 Rema earns \$18 per hour. The table shows her income versus hours worked.

Time (h)	2	4	6	8	10
Income (\$)	36	72	108	144	180

- a Draw the graph of income versus hours worked.
 b Use your graph from part a to find Rema's income for seven hours.
 c How many hours must she work to earn \$90?

Worked example 2.1B Completing a table of values to draw a graph

The cost of travelling in a hire car is \$10 booking fee plus \$3.20 per kilometre.

- a Complete the table of values for the hire car.

Distance (km)	0	10	20	30	40
Cost (\$)					

- b Plot the points and sketch the graph of hire car costs.
 c Find the cost of a journey of 25 km.



d How far can the hire car be driven for \$50?

Part	Think	Write												
a	Calculate the cost by multiplying the cost per kilometre by the number of kilometres and adding this result to the booking fee.	<table><tr><th>Distance (km)</th><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td></tr><tr><th>Cost (\$)</th><td>10</td><td>42</td><td>74</td><td>106</td><td>138</td></tr></table> Even if the hire car is driven 0 km, the cost is still \$10 for the booking fee. For a journey of 10 km: Cost = \$10 + \$3.20 × 10 = \$42 For a journey of 20 km: Cost = \$10 + \$3.20 × 20 = \$74 For a journey of 30 km: Cost = \$10 + \$3.20 × 30 = \$106 For a journey of 40 km: Cost = \$10 + \$3.20 × 40 = \$138	Distance (km)	0	10	20	30	40	Cost (\$)	10	42	74	106	138
Distance (km)	0	10	20	30	40									
Cost (\$)	10	42	74	106	138									
b	Write the values in the table as coordinate points. Put distance on the horizontal axis as it is the independent variable. Put cost on the vertical axis as it is the dependent variable. Choose a suitable scale for each axis. Plot the points and draw a straight line through them.	(0, 10), (10, 42), (20, 74), (30, 106), (40, 138)												
		Hire car charges												
c	Find 25 km on the horizontal axis and move up vertically to find the point on the graph with an x -coordinate of 25. Move horizontally to the vertical axis to find the y -coordinate of that point. This is the cost of the journey.	The cost of a journey of 25 km is \$90.												
d	Find \$50 on the vertical axis and move across horizontally to find the point on the graph with a y -coordinate of 50. Move vertically downwards to the horizontal axis to find the x -coordinate of that point. This is the total distance which can be driven for \$50.	The hire car can be driven 12.5 km for \$50.												



Worked example video: Completing a table of values to draw a graph

Helpful hints

- Recall that the horizontal axis is usually called the x -axis and the vertical axis is usually called the y -axis. In a coordinate point, the first value corresponds to the position along the x -axis and is called the x -coordinate, and the second value corresponds to the position along the y -axis and is called the y -coordinate.

- 4 The cost of hiring a taxi is a \$3.00 flagfall and \$2.40 per kilometre travelled.

- a Copy and complete this table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost (\$)	3	27			

- b Draw the graph showing the cost of hiring the taxi.

- c How much does it cost to travel 35 km?

- d How far could you travel for \$15?

- 5 The cost of hiring a taxi at night is a \$5.00 flagfall and \$3.20 per kilometre travelled.

- a Copy and complete this table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost (\$)					

- b Draw a graph showing the cost of hiring the taxi.

- c How much does it cost to travel 15 km?

- d How far could you travel for \$75?



Problem solving and reasoning

- 6 A person drives non-stop from Taree to Sydney, a distance of 300 km. The rule giving the distance from Sydney is $d = 300 - 100t$ where d is distance in kilometres and t is time in hours.

- a Copy and complete this table of values for the given rule.

Time (h)	0	1	2	3
Distance (km)	300		100	

- b Draw a graph of distance from Sydney versus time.

- c How far is the driver from Sydney after 1.5 h?

- d How far has the driver travelled after 1 h?

- 7 On a particular day, AUD\$100 (Australian dollars) buys USD\$60 (US dollars). Use this and the fact that AUD\$0 = USD\$0 to draw a linear graph relating AUD\$ and USD\$ (this is called a conversion graph). Put AUD\$ on the horizontal axis (values up to \$100) and USD\$ on the vertical axis. How many AUD\$ can be bought for USD\$40?

- 8 Sumeyra is travelling to Europe. At the time Sumeyra is travelling, \$100 (Australian dollars) will buy €70 (euros). Draw a conversion graph with AUD\$ on the horizontal axis (values up to \$100) and euros on the vertical axis. From the graph, how many AUD\$ could be bought for €60?

- 9 Shailesh is filling his car with petrol. He puts in 80 litres of fuel and the cost is \$110.

- a Draw a straight-line conversion graph for this information.

- b How many litres of fuel would \$70 buy?

- c What would be the cost for 55 litres of fuel?

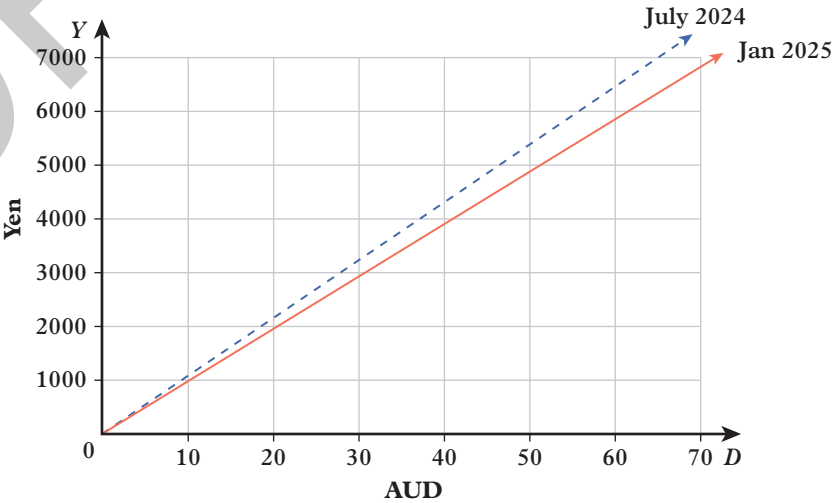
- 10 Karolin is filling her car with petrol. She puts in 50 litres of fuel and the cost is \$65.
- a Draw a straight-line conversion graph for this information.
 - b How many litres of fuel would \$25 buy?
 - c What would be the cost for 35 litres of fuel?
- 11 The table shows the rates for taxi travel in the day and at the special night rate.

Distance (km)	Cost for day rate (\$)	Cost for special night rate (\$)
0	3.60	6.10
5	14.55	19.25
10	25.50	32.40
15	36.45	45.55
20	47.40	58.70
25	58.35	71.85
30	69.30	85.00
35	80.25	98.15
40	91.20	111.30
45	102.15	124.45
50	113.10	137.60

- a Plot the points and draw the graphs for both rates on the same set of axes.
- b Use your graph to determine how much further you can travel for \$75 during the day compared with the special night rate.
- c Use your graph to find the difference in cost between a 42 km journey at the day rate and at the special night rate.

Challenge

- 12 This graph shows the conversion rate between the Australian dollar and Japanese Yen in July 2024 and January 2025.
- 13 Phoebe converted an amount of money in July 2024 from the Australian dollar to the Japanese Yen. In January of 2025, she converted all of this amount back into the Australian dollar. If she got back \$5 more than she converted initially, approximately how much did she originally convert?



Related resources



2.1 Quick quiz



Worksheet: Graphs of linear models



Investigation: Height of an aircraft

Lesson 2.2

Understanding gradient

Learning intentions

By the end of this lesson, you will be able to ...

→ determine the gradient of a straight-line graph.

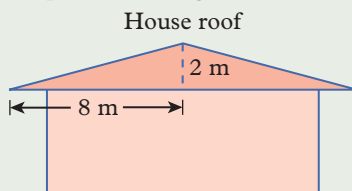
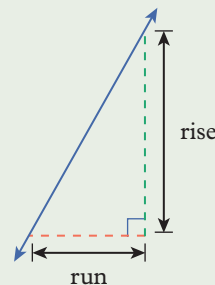
Gradient

- The **gradient (slope)** of a line graph describes how steep the line is. It describes the rate of change in the value of y compared to an increase in the value of x by 1.
 - A positive gradient represents moving upwards from left-to-right. This is like going uphill when you start on the left.
 - A negative gradient represents moving downwards from left-to-right. This is like going downhill when you start on the left.
 - The variable m is usually used to represent the gradient.

Using rise and run to calculate the gradient

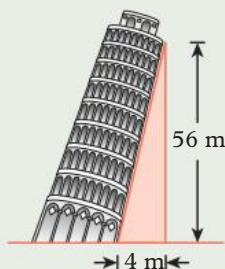
- There are two equivalent formulas which can be used to determine the gradient of a line. Choose which formula to use based on the information provided.
 - The rise between two points is the vertical distance between the two points. The run is the horizontal distance between the two points. The gradient of the line can be found using the formula:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$
 - By convention, moving horizontally to the right is taken as the positive direction, and to the left as the negative direction. Moving vertically, up is the positive direction, and down is the negative direction.
- The following illustrations indicate slopes of varying amounts.



$$\text{slope} = \frac{2}{8} = \frac{1}{4}$$

Leaning Tower of Pisa



$$\text{slope} = \frac{56}{4} = 14$$

- We can determine the gradient of horizontal and vertical lines.
 - For a horizontal line, the rise is 0, therefore the gradient is 0.

$$\text{gradient} = \frac{0}{\text{run}} = 0$$
 - For a vertical line, the horizontal run is 0. Dividing by 0 is undefined, so the gradient of a vertical line is undefined.

Using the two-point formula to calculate the gradient

- When two distinct points (x_1, y_1) and (x_2, y_2) on a line are known, the following formula can be used to calculate the gradient.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
 - Substitute the known values into the formula and simplify to find the gradient. The order of the points (x_1, y_1) and (x_2, y_2) does not affect the result.
 - For a given straight line, both gradient formulas will always give the same value of the gradient. This is because the rise can be seen as the difference between the two y -coordinates, which is the numerator in this formula, and the run can be seen as the difference between the two x -coordinates, which is the denominator.

Worked example 2.2A Finding the gradient of a line segment (positive)

Find the slope of AB using the formula $\text{gradient} = \frac{\text{rise}}{\text{run}}$.



Think

Complete a right-angled triangle using AB as the hypotenuse.

Identify the rise and run and the shorter side lengths of the triangle.

Substitute these values into the gradient formula and simplify the result if possible.

Write



$$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{5}\end{aligned}$$



Worked example video: Finding the gradient of a line segment (positive)

Understanding, fluency and communicating

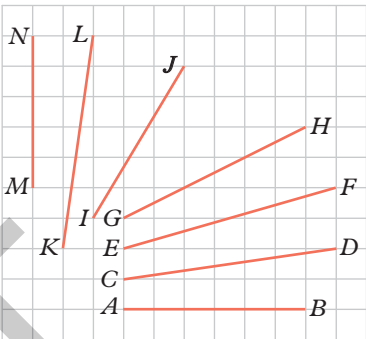
Answers: Lesson 2.2

Helpful hints

- Complete a right-angled triangle using the line segment between two points on the line as the hypotenuse. This triangle can be used to identify the rise and run of the line segment.

1 a Complete the table using the line segments shown.

Line segment	Run	Rise	Gradient
AB			
CD			
EF			
GH			
IJ			
KL			
MN			

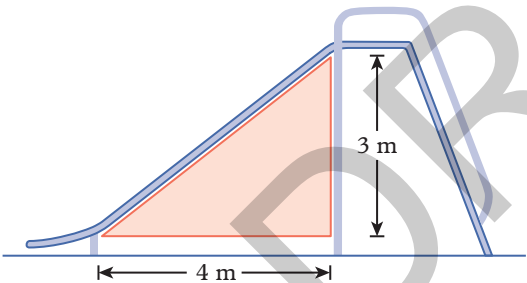


b Complete the following.

c The gradient of a horizontal line is _____. The gradient of a vertical line is _____. As the line segments become steeper, their gradients _____.

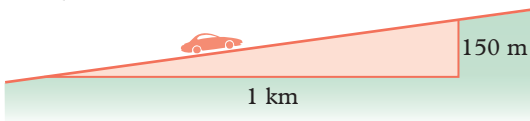
2 Find the slope in each of these situations.

a Playground slide



b

Road up a hill



3 Determine the slope of each of these line segments.

a OA

b OB

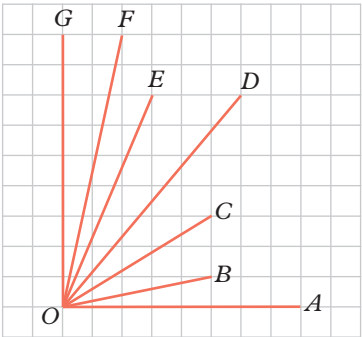
c OC

d OD

e OE

f OF

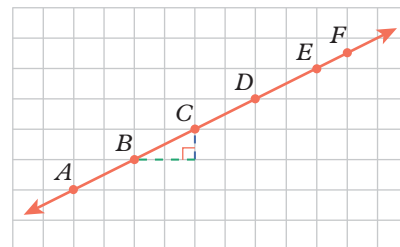
g OG



4 Consider this line, showing points A, B, C, D, E and F .

a Complete the following table.

Line segment	run	rise	$\frac{\text{rise}}{\text{run}}$
BC	2	1	$\frac{1}{2}$
DE			
AC			
BE			
AE			
AF			



b State, in sentence form, any conclusions you can draw from the graph and table.

Worked example 2.2B Finding the gradient of a line (negative)

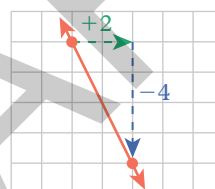


Find the gradient of this line.

Think

Choose two convenient points on the line. To get from the left-hand point to the right-hand point, we move 2 units to the right horizontally and 4 units down vertically.

Write



$$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-4}{+2} \\ &= -2\end{aligned}$$



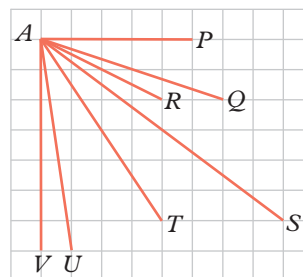
Worked example video: Finding the gradient of a line (negative)

Helpful hints

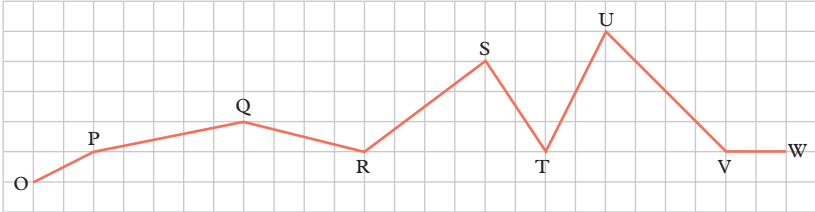
- Moving from left to right, “uphill” lines have positive slopes and “downhill” lines have negative slopes.

5 Determine the gradient of each of these line segments.

- AP
- AQ
- AR
- AS
- AT
- AU
- AV

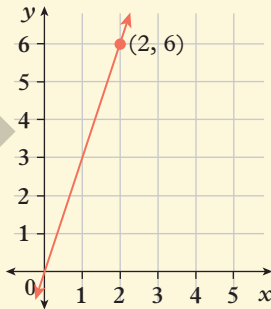


- 6 Imagine you are walking across the countryside from O to W (from left to right). This graph represents the profile of your walk.



- a When are you going uphill?
- b When are you going downhill?
- c Where is the steepest positive slope?
- d Where is the steepest negative slope?
- e Where is the slope zero?
- f Where is the slope the least steep but still positive?

Worked example 2.2C Finding the gradient of a line using the two-point formula (origin)



Find the gradient of this line using the formula $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$.

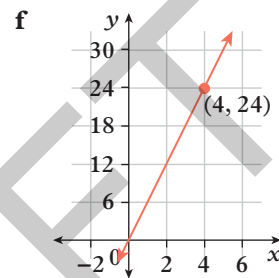
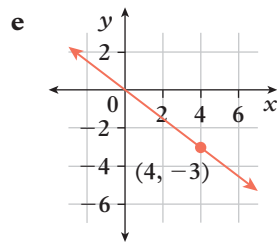
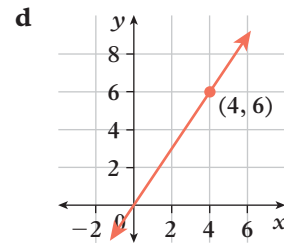
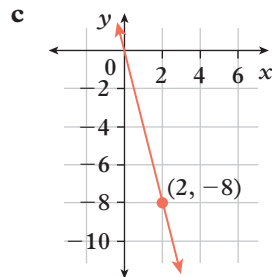
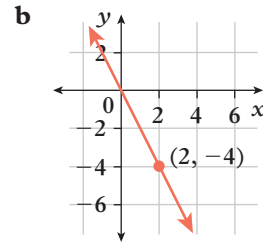
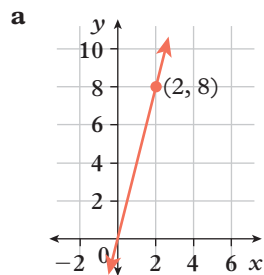
Think	Write
Identify two points on the graph to be (x_1, y_1) and (x_2, y_2) .	$(x_1, y_1) = (0, 0)$ $(x_2, y_2) = (2, 6)$
Substitute the points into the two-point gradient formula and simplify.	$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 0}{2 - 0} \\ &= \frac{6}{2} \\ &= 3\end{aligned}$

 **Worked example video:** Finding the gradient of a line using the two-point formula (origin)

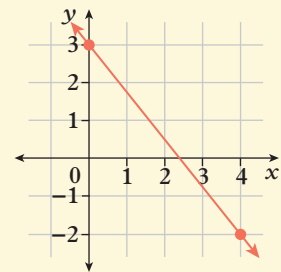
Helpful hints

- When using the two-point formula to find the gradient of a line passing through the origin, set $(x_1, y_1) = (0, 0)$. This makes it easier to simplify the result.

7 Find the gradient of each of these lines using the two-point gradient formula.



Worked example 2.2D Finding the gradient of a line using the two-point formula



Find the gradient of this line using the formula $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$.

Think

Choose any two points on the line to be (x_1, y_1) and (x_2, y_2) . The order of these points doesn't matter.

Substitute the points into the formula and simplify to find the gradient.

Write

$$\begin{aligned}(x_1, y_1) &= (0, 3), \\ (x_2, y_2) &= (4, -2)\end{aligned}$$

$$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 3}{4 - 0} \\ &= -\frac{5}{4}\end{aligned}$$



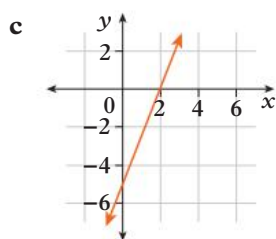
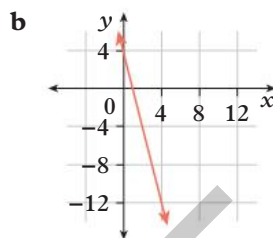
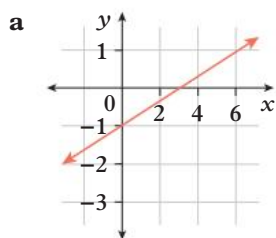
Worked example video: Finding the gradient of a line using the two-point formula

Helpful hints

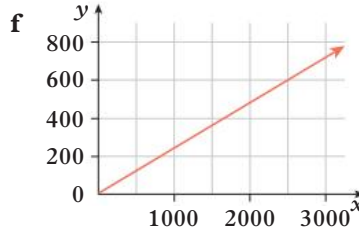
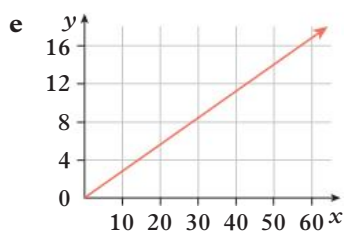
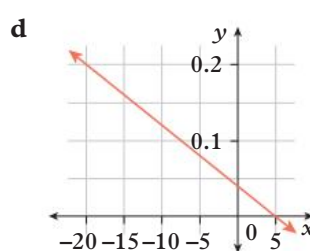
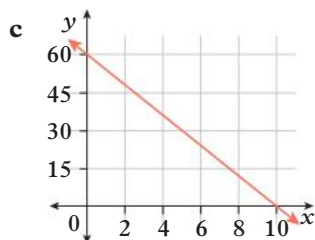
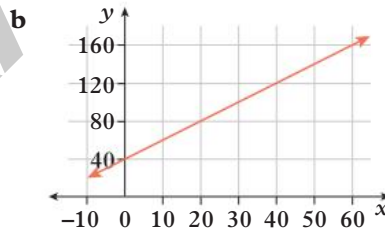
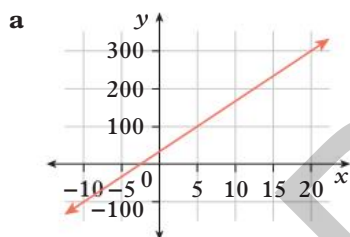
- You can substitute any two points on a line into the two-point gradient formula to find the gradient of the line. The order in which you substitute the points won't affect the result as long as you keep the order in the numerator the same as in the denominator.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

8 Find the gradient of each of these lines.

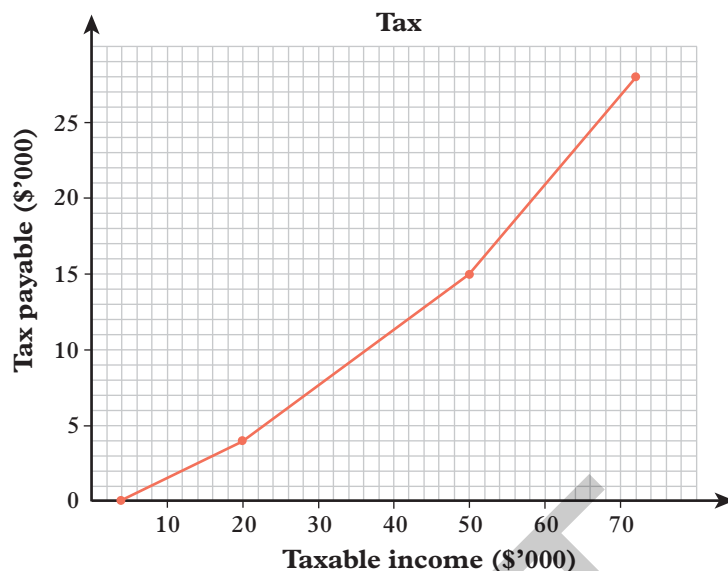


9 Find the gradient of each of these lines. Pay attention to the scale of each axis.



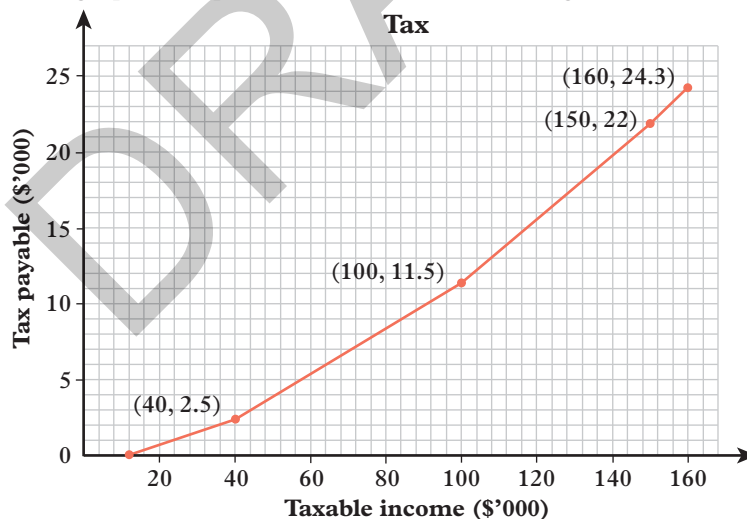
Problem solving and reasoning

10 Consider the tax graph shown. It is a **piecewise graph**. The points where the tax rate changes are marked.



- Find the tax payable on \$40 000.
- Find the income on which tax of \$20 000 is payable.
- Find the gradient of the section between taxable incomes of \$30 000 and \$40 000. What does the gradient represent?
- Find the gradient of the section between taxable incomes of \$60 000 and \$70 000. What does the gradient represent?

11 This is another piecewise graph. The points where the tax rate changes are marked.



- Find the tax payable on \$90 000.
- Find the income on which tax of \$20 000 is payable.
- Find the gradient of the section between taxable incomes of \$60 000 and \$80 000. What does the gradient represent?
- What is the tax rate for taxable incomes between \$100 000 and \$150 000?
- Find the gradient of the section of the graph between taxable incomes of \$20 000 and \$40 000.
- Find the tax rate for incomes over \$150 000.

Challenge

12 A partially completed table of values is given.

x	5	7	10	14
y	97			34

- What is the gradient of the straight-line graph created by plotting the points given in the table and joining them with a line?
- Find the two missing values.

Related resources



2.2 Quick quiz



Worksheet: Finding the gradient of a linear graph

Lesson 2.3

The equation $y = mx + c$

Learning intentions

By the end of this lesson, you will be able to ...

→ translate between the equation of a line and its graph.

The y -intercept

- The **y -intercept** of a graph is the point where the graph crosses the y -axis.
 - The y -intercept can be found by substituting $x = 0$ into the equation of the line.
 - If the line passes through the origin, $(0, 0)$, then the origin is the y -intercept.

Equations in the form $y = mx + c$

- The rules or equations for linear (or straight-line) graphs can all be written in the form $y = mx + c$. In this equation:
 - x is the independent variable, because any value may be used
 - y is the dependent variable, because it depends on the value of x
 - m is the gradient of the line
 - c is the y -intercept. The coordinates of the y -intercept are $(0, c)$.
- This relationship connects every point on a linear graph.
 - To check if a point lies on a linear graph, substitute the x -coordinate into the equation and evaluate the result. If the result is equal to the y -coordinate, then the point lies on the line.

Worked example 2.3A Drawing a linear graph using a table of values

Draw the graph of $y = -3x + 5$ by first completing a table of values.

Think

First set up a table. Choose at least three convenient values of x .
 Substitute the values of x into the equation $y = -3x + 5$.
 Write the points from the table as coordinates and plot them on the Cartesian plane.
 Draw a straight line through the plotted points.

Write

$$x = -2: y = -3 \times -2 + 5 = 11$$

$$x = -1: y = -3 \times -1 + 5 = 8$$

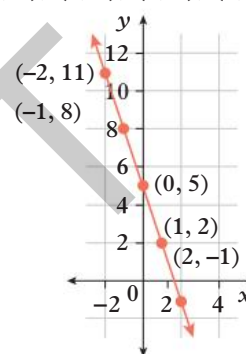
$$x = 0: y = -3 \times 0 + 5 = 5$$

$$x = 1: y = -3 \times 1 + 5 = 2$$

$$x = 2: y = -3 \times 2 + 5 = -1$$

x	-2	-1	0	1	2
y	11	8	5	2	-1

$(-2, 11), (-1, 8), (0, 5), (1, 2), (2, -1)$



Worked example video: Drawing a linear graph using a table of values

Understanding, fluency and communicating

Answers: Lesson 2.3

Helpful hints

- Choose values of x for the table of values which are easy to work with and to plot on the Cartesian plane. For example, if the equation is $y = \frac{x}{2} + 1$, the substitution of even numbers ($\dots, 0, 2, 4, \dots$) will result in needing to plot integer values on the Cartesian plane.

- 1 Draw the graphs of the following linear relationships by first completing tables of values similar to the following.

x	-2	-1	0	1	2
y					

a $y = 3x + 1$

b $y = 2x + 2$

c $y = -2x - 1$

2 Draw the graphs of the linear relationships with the following equations.

a $y = 2x + 7$

b $y = 4x - 6$

c $y = x - 1$

d $y = -x + 3$

e $y = -3x + 2$

f $y = -2x - 2$

g $y = \frac{1}{2}x - 1$

h $y = -\frac{3}{4}x + 1$

3 Identify the gradient and y -intercept of each of the lines in question 2.

Worked example 2.3B Writing the gradient and y -intercept of a line from its equation

Write the gradient and y -intercept of each of the following lines.

a $y = 7x + 8$

b $y = -3x - 1$

c $y = \frac{2x}{3}$

Part	Think	Write
a	The equation is in the form $y = mx + c$. Identify the value of m as the gradient and c as the y -intercept. The y -intercept is located at $(0, c)$.	$y = 7x + 8$ gradient = $m = 7$ $c = 8$ y -intercept: $(0, 8)$
b	The equation is in the form $y = mx + c$. Identify the value of m as the gradient and c as the y -intercept. The y -intercept is located at $(0, c)$.	$y = -3x - 1$ gradient = $m = -3$ $c = -1$ y -intercept: $(0, -1)$
c	The equation is in the form $y = mx + c$. Identify the value of m as the gradient and c as the y -intercept. The y -intercept is located at $(0, c)$.	$y = \frac{2x}{3} = \frac{2}{3}x + 0$ gradient = $m = \frac{2}{3}$ $c = 0$ y -intercept: $(0, 0)$



Worked example video: Writing the gradient and y -intercept of a line from its equation

Helpful hints

- Write the equation of a line in the form $y = mx + c$ and then identify the values of m and c to find the gradient and y -intercept of the line.

4 Write the gradient and y -intercept for each of the following lines.

a $y = 5x - 1$

b $y = -2x + 11$

c $y = \frac{3x}{4}$

d $y = -x - 2$

e $y = \frac{x}{2} + 3$

f $y = x - 15$

g $y = -\frac{7x}{5} - 1$

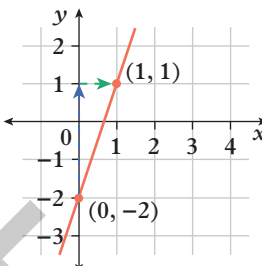
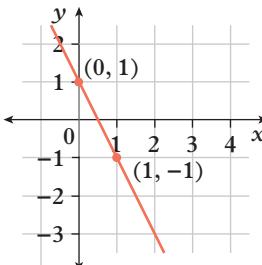
h $y = \frac{x}{6} - \frac{2}{5}$

Worked example 2.3C Sketching a linear graph

Sketch the following straight lines using the method given in brackets.

a $y = 3x - 2$ (Identify rise and run from the gradient)

b $y = -2x + 1$ (Use substitution to identify a second point on the graph)

Part	Think	Write
a	Identify the values of m and c from the equation as the gradient and the y-intercept. Plot the y-intercept on the Cartesian plane. Because the gradient is $3 = \frac{3}{1}$, move up 3 units then right 1 unit. This gives a second point $(1, 1)$ on the line. Rule a line through these two points.	gradient = 3 y-intercept: $(0, -2)$ 
b	Identify the value c from the equation as the y-intercept. Plot the y-intercept on the Cartesian plane. Substitute another value of x into the equation of the line. Choose a value of x that is easy to work with. Plot the resulting point on the Cartesian plane. Rule a line through these two points.	y-intercept: $(0, 1)$ $x = 1$: $y = -2 \times 1 + 1 = -1$ $(1, -1)$ lies on the line. 



Worked example video: Sketching a linear graph

Helpful hints

- When identifying rise and run from the gradient, you can write the gradient as any equivalent fraction. In Worked example 2.3C part **a**, you could write $3 = \frac{6}{2}$ and then move six units up and two units across to find a second point. This will always result in the same line being sketched.
- When using substitution to find a second point, you can substitute any other value of x . Often it is simplest to substitute $x = 1$, but you can substitute any non-zero value to sketch the graph.

5 Sketch each of the lines in question 4 using the following methods.

- Identify rise and run from the gradient.
- Use substitution to identify a second point on the graph.

6 Compare your graphs in each part of question 5. What do you notice?

Problem solving and reasoning

- 7 The KNOT ME Tie Company has fixed costs of \$800 per day and a variable production cost of \$2 per tie. The daily cost can be modelled using the straight-line equation $C = 2n + 800$.
- a Explain each term in the formula $C = 2n + 800$.
 - b Complete this table of values.

n	0	50	100	150	200	250
C						

- c Draw the straight-line graph $C = 2n + 800$.
- d Use your graph from part c to find the number of ties produced when the daily cost is \$940.



- 8 The Chilly Frozen Yoghurt Shop has a monthly fixed expense of \$1500 and the cost of making each serve of yoghurt is \$1.40.
- a Write an equation modelling this information. Use C for monthly cost in dollars and n for the number of serves of yoghurt made.
 - b Complete this table of values.
- | | | | | | | |
|-----|---|-----|------|------|------|------|
| n | 0 | 500 | 1000 | 1500 | 2000 | 2500 |
| C | | | | | | |
- c Draw the straight-line graph.
 - d What is the value of the gradient?
 - e Use the graph to find the number of serves of yoghurt made if the cost is \$3250.
- 9 The Pulp-Free Fruit Juice company has weekly fixed costs of \$750. The cost to produce a cup of fruit juice is \$1.80.
- a Write an equation modelling this information. Use C for the weekly cost in dollars and n for the number of cups of fruit juice.
 - b What value is used for the gradient? Why?

c Where does the graph intersect the vertical axis?

d Complete this table of values.

n	0	200	400	600	800	1000
C						

e Draw the straight-line graph.

f Use the graph to find the number of cups of juice made if the cost is \$1560.

Questions 10 to 12 may be completed manually or using a graphing application.

10 More than one graph can be sketched on the same Cartesian plane.

a Draw graphs of $y = 3x + 1$, $y = 3x - 1$, $y = 3x$ and $y = 3x + 2$ on the same Cartesian plane.

b What do you notice about all four graphs? Explain.

c Without plotting any points, add the graph of $y = 3x + 3$ to the number plane. Explain how you knew what to draw.

11 Consider drawing graphs of lines with the same gradient.

a For each of the following lists of equations, draw the corresponding graphs on the same Cartesian plane.

i $y = 2x + 1$, $y = 2x - 1$, $y = 2x + 3$

ii $y = -3x$, $y = -3x - 1$, $y = -3x + 2$

iii $y = x + 2$, $y = x$, $y = x - 5$

b What do you notice? What conclusion can be drawn about straight lines with the same gradient?

12 Consider drawing graphs of lines with the same constant term.

a Draw the graphs of $y = 2x + 1$, $y = 3x + 1$ and $y = x + 1$ on the same Cartesian plane.

b What do you notice about all three graphs?

c Draw the graphs of $y = 3x - 1$, $y = 2x - 1$ and $y = -4x - 1$ on the same Cartesian plane. What do you notice?

d What conclusion can be drawn about straight lines with the same constant term?

Challenge

13 Consider the equation $y = 5.5x + 7$.

a Show that the point (12, 73) lies on the line above.

b The point also lies on a line with a gradient of $m = -5.5$. What is the y -intercept of this line?

Related resources



2.3 Quick quiz



Worksheet: Linear graphs

Lesson 2.4

Direct variation

Learning intentions

By the end of this lesson, you will be able to ...

→ recognise and describe direct variation relationships.

Direct variation

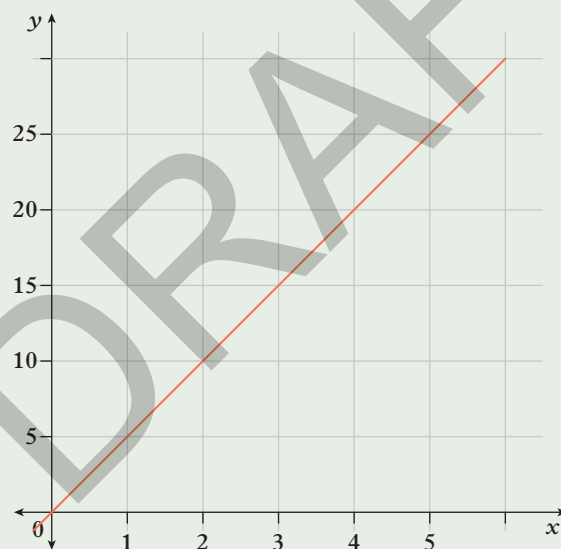
- Consider the two linear relationships shown below.

1 Rule: $y_1 = 5x_1$

Table of values:

x_1	1	2	3	4	5
y_1	5	10	15	20	25

Graph:

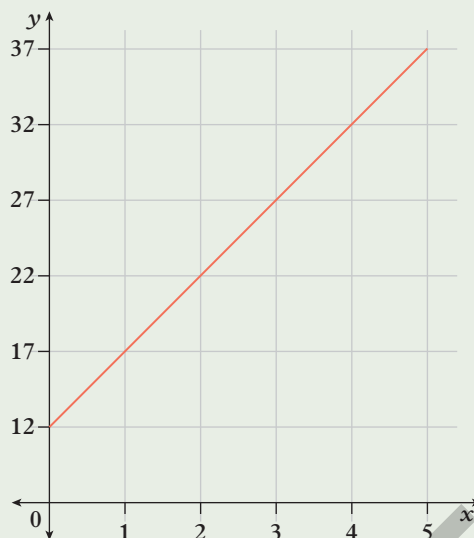


2 Rule: $y_2 = 5x_2 + 12$

Table of values:

x_2	1	2	3	4	5
y_2	17	22	27	32	37

Graph:



- These linear relationships have the same gradient but different y -intercepts.
- The following tables have an extra row added showing $\frac{y}{x}$ for each pair of values.

1

x_1	1	2	3	4	5
y_1	5	10	15	20	25
$\frac{y_1}{x_1}$	5	5	5	5	5

2

x_2	1	2	3	4	5
y_2	17	22	27	32	37
$\frac{y_2}{x_2}$	17	11	9	8	7.4

- These tables show that every pair of x_1 and y_1 are **directly proportional** to each other, while x_2 and y_2 are not.
- Any linear graph of the form $y = mx + c$ with y -intercept $c = 0$ represents **direct variation** between the variables x and y . When a relationship is known to be a direct variation, the rule is often written as $y = kx$, and the gradient k is called the **constant of variation**.
- Direct variation is prevalent in everyday life. You have already seen many examples in this module!
- Examples of direct variation:
 - A person's wage is based on the number of hours they work.
 - The amount spent on petrol is directly proportional to the volume of petrol purchased.
 - The amount of paint required to paint a room depends directly on the surface area of the walls to be painted.
 - When travelling at a constant speed, the distance travelled corresponds directly to the time spent travelling.
 - The total cost of a round of soft drinks is in direct proportion to the number of people in the group who are drinking the soft drinks!
- When a direct variation relationship exists between x and y , we can write $y \propto x$, which is read as either " y is proportional to x " or " y varies directly with x ".

- Direct variation can be recognised in unfamiliar contexts by checking for a constant of variation. The variables x and y are only directly proportional if $\frac{y}{x}$ is the same for every ordered pair (x, y) with $x \neq 0$. You should also check that when $x = 0, y = 0$.
- *Remember:* whenever y and x are known to be directly proportional, they satisfy the equation $y = kx$ with $k = \frac{y}{x}$.

Worked example 2.4A Graphing a direct variation relationship

A train is travelling at a constant speed of 75 km/h.
The distance, d km, the train travels in t hours is shown in the table.

t	1	2	3	4
d	75			

- a Complete the table of values using the information above.
- b Sketch the linear graph that shows how d depends on t . Your axes should make sense in the context of the scenario.
- c Write an equation to describe the relationship between time and the distance travelled by the train.



Part	Think	Write												
a	A speed of 75 km/h means that for each increase in time of 1 h, the distance increases by 75 km. Multiply each value of t by 75 to get the corresponding value of d.	<table><tr><td>t</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>d</td><td>0</td><td>75</td><td>150</td><td>225</td><td>300</td></tr></table>	t	0	1	2	3	4	d	0	75	150	225	300
t	0	1	2	3	4									
d	0	75	150	225	300									
b	To draw a linear graph, we only need to know two points. In a direct variation relationship, we can use the origin (0, 0) as one of these points. Choose (2, 150) as the second point. Draw the line connecting the two points and extend it up and to the right. It does not make sense to extend the line down and to the left, because that would be describing negative distance.	Train travel												

Part	Think	Write
c	The rule describing direct variation between d and t is $d = kt$ where k is the constant of variation. In this case, the constant of variation is the speed of the train. We can also see in the graph that the gradient of the line is 75.	The equation is $d = 75t$.



Worked example video: Graphing a direct variation relationship

Understanding, fluency and communicating

Answers: Lesson 2.4

Helpful hints

- Constant speed s is the gradient of a linear graph relating time t and distance d . Since $d = 0$ when $t = 0$, this graph starts at the origin $(0, 0)$ and hence has equation $d = st$.
- Any straight line of the form $y = mx + c$ with $c = 0$ describes direct variation between the variables x and y .

1 Let N be some number of packets of pencils and let P be the price for a single packet.

a Complete this table, assuming the price per packet of pencils is constant.

Number of packets (N)	1	2	3	4	5
Price (\$ P)	1.50		4.50		7.50

b Sketch the linear graph that shows how P depends on N . Your axes should make sense in the context of the scenario.

c Find the gradient of this line to determine the constant of variation $k = \frac{P}{N}$.

d Write the linear equation for P in terms of N .

e Find the cost of nine packs of pencils.

f If the price for N packs of pencils is \$27, find N .

2 Let C be the circumference of a circle with diameter D .

a Complete this table for the equation $C = \pi D$.

Diameter (D)	1	2	3	4
Circumference (C)				

b Draw a linear graph that represents the information from your completed table.

c Which features of the graph indicate that the circumference of a circle varies directly with the length of its diameter?

d What is the constant of variation in this context?



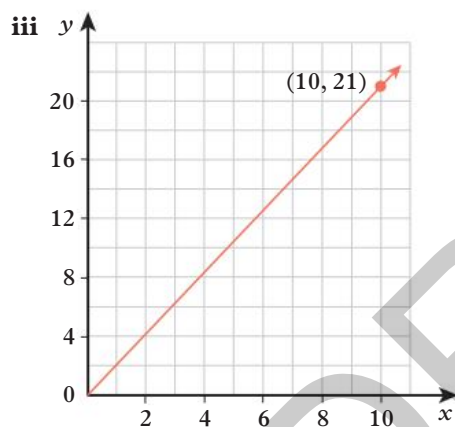
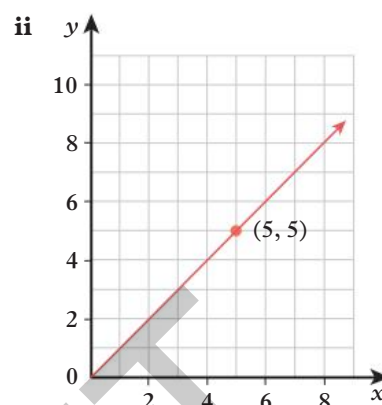
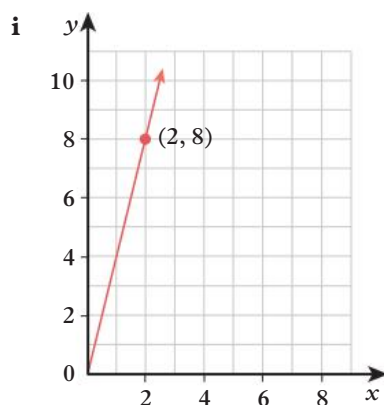
e From the graph, find the approximate value for:

- i** the circumference of a circle with diameter 2.5 cm
- ii** the diameter of a circle with circumference 16 cm.

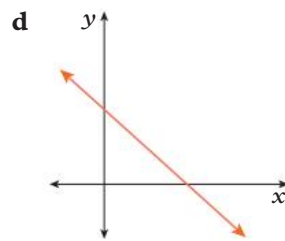
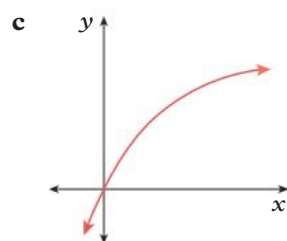
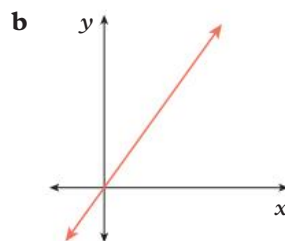
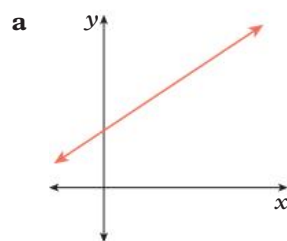
3 Consider the following graphs describing direct variation.

a Draw a right-angled triangle using the given point and the origin as two of the vertices then use the height and base of this triangle to calculate the gradient of the line.

b State the relationship between x and y as a linear equation.



4 Which of the following graphs indicates that y is directly proportional to x ?



Worked example 2.4B Recognising direct variation relationships

By calculating the value of $\frac{y}{x}$ for each (x, y) pair in the following tables, determine if $y \propto x$.

a

x	1	3	5
y	8	24	40

b

x	3	7	9
y	12	21	36

Part	Think	Write												
a	Calculate $\frac{y}{x}$ for each column in the table. Because $\frac{y}{x}$ is the same in every column, y varies directly with x. In this case, $y = 8x$.	<table><tr><td>x</td><td>1</td><td>3</td><td>5</td></tr><tr><td>y</td><td>8</td><td>24</td><td>40</td></tr><tr><td>$\frac{y}{x}$</td><td>8</td><td>8</td><td>8</td></tr></table> $y \propto x$ and the constant of proportionality is 8.	x	1	3	5	y	8	24	40	$\frac{y}{x}$	8	8	8
x	1	3	5											
y	8	24	40											
$\frac{y}{x}$	8	8	8											
b	$\frac{y}{x}$ is not the same number for each column, so y does not vary directly with x.	<table><tr><td>x</td><td>3</td><td>7</td><td>9</td></tr><tr><td>y</td><td>12</td><td>21</td><td>36</td></tr><tr><td>$\frac{y}{x}$</td><td>4</td><td>3</td><td>4</td></tr></table> y does not vary directly with x.	x	3	7	9	y	12	21	36	$\frac{y}{x}$	4	3	4
x	3	7	9											
y	12	21	36											
$\frac{y}{x}$	4	3	4											



Worked example video: Recognising direct variation relationships

Helpful hints

- Direct variation between x and y is only observed if $\frac{y}{x}$ is equal to the same value for all ordered pairs (x, y) with $x \neq 0$.
- A linear graph describing a direct variation relationship must pass through the origin $(0, 0)$.

5 Calculate the values $\frac{y}{x}$ for each column in the following tables then answer parts **i** and **ii** for each table.

a

x	1	2	5
y	4	8	21

c

x	-4	-1	2
y	-8	-2	4

b

x	2	3	6
y	14	21	42

d

x	-7.5	-8.4	11.1
y	2.5	2.8	-3.7

- i** State whether or not direct proportionality exists between x and y .
ii If direct proportionality exists, find the constant of variation and express y in terms of x .

ii

x	4	12	1	b
y	2	6	a	24

iii

x	11.1	-12	34.2	c
y	3.7	a	b	17.1

iv

x	a	5.1	-7.8	c
y	14.4	22.95	b	31.05

- 8** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. Suppose a car uses 6 L of petrol to travel 50 km.

- a** Complete the working below to find an equation relating the amount of petrol used, P , with the distance travelled, d .

Since $P \propto \underline{\hspace{1cm}}$,

$$P = kd$$

$$\underline{\hspace{1cm}} = k \times \underline{\hspace{1cm}}$$

$$k = \underline{\hspace{1cm}}.$$

Hence, $P = \underline{\hspace{1cm}}$.

- b** Use your equation to find the amount of petrol required to travel 180 km at the same speed.

Problem solving and reasoning

- 9** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 6 L of petrol to travel 56 km. How much petrol will be used to travel 140 km at the same speed?
- 10** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 8 L of petrol to travel 75 km. What distance will the car cover if it travels at the same speed and uses 24 L of petrol?
- 11** The amount of paint needed to paint a wall is directly proportional to the area covered. A 4-litre can of paint will cover 14 m^2 . Find the number of litres of paint needed to cover a wall 46 m^2 in area.
- 12** Three packets of chips cost \$6.45.
- a** Find an equation relating the cost, C , with the number of packets, P .
- b** Use the equation from part **a** to find the cost of 17 packets.
- 13** The amount of water dripping into a bucket from a tap is directly proportional to the length of time the tap drips. Water dripping from a tap fills a 5-litre bucket in eight hours. How long will it take to fill a 12-litre bucket?
- 14** The cost of a 2.8 m length of hose is \$4.20.
- a** Find an equation relating the cost, C , to the number of metres of hose, H .
- b** Find the cost of 6.7 m of hose.
- c** How much hose can be purchased for \$30?
- 15** The resistance, R ohms, to the flow of electricity in a wire varies directly with the length, l cm, of the wire and when $l = 20$ cm, $R = 0.08$ ohms.
- a** Find R when $l = 85$ cm.
- b** Find l when $R = 6$ ohms.



- 16 The currency exchange rate on a particular day was $\text{\$AUD1} = \text{\pounds}0.651$ (British pounds).
- Write the rule relating British pounds, P , and Australian dollars, D , on that day.
 - How many Australian dollars could be bought with $\text{\pounds}850$ on that day?
- 17 The current required to operate a car's starter motor varies directly as the voltage. The current is 150 amps when the battery voltage is 12 volts.
- Write the rule relating current, I amps, and voltage, V volts.
 - What current is drawn if the battery voltage drops to 10 volts?
 - A car cannot start if the current falls below 95 amps. What is the lowest battery voltage that will allow the car to start?

Challenge

- 18 For each direct variation scenario, write a sentence to interpret the constant of variation in the context provided.
- A tap is leaking into a bucket. The volume in millilitres, V , of water in the bucket after t minutes is given by $V = 1.2t$.
 - The formula to convert Australian dollars D to British pounds P is $P = 0.51D$.
 - The distance travelled in kilometres d by a train after t hours is given by $d = 72t$.

Related resources



2.4 Quick quiz

Lesson 2.5

Analysing linear models

Learning intentions

By the end of this lesson, you will be able to ...

→ interpret the real-world meaning of a linear model.

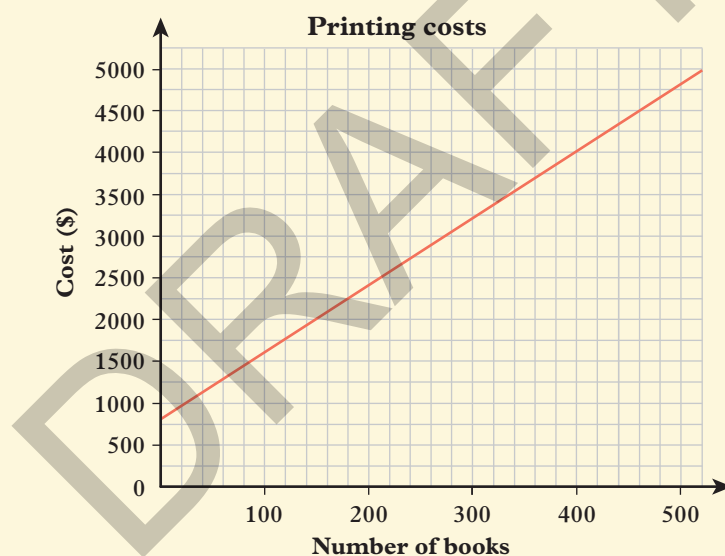
Linear models

- When interpreting a linear model from its graph, the key features to understand are the gradient and vertical intercept.
 - In the context of the scenario being modelled, the gradient describes a rate that is expressed in terms of the vertical units “per” the horizontal units.
 - The vertical intercept represents the value of the dependent variable (on the vertical axis) when the independent variable (on the horizontal axis) is 0.

- To model a relationship between two variables that is known to be linear, only two data points are needed. The graph can be drawn by first plotting the two points, then ruling a straight line that passes through both. In a direct variation relationship, the origin $(0, 0)$ can be used as one of the two data points.
- Connecting two known points by a line allows us to estimate values in between known data points. For example, data may come from a context where measurements can only ever be whole numbers, but a linear model allows you to estimate a value in between two whole numbers. This is known as **interpolation**.
- By extending the straight line in a linear model past the known data points, it is possible to make predictions about values that you would otherwise know nothing about. This is known as **extrapolation**.
- In any model of a real-world scenario, it is important to critique theoretical conclusions in the context they apply to.

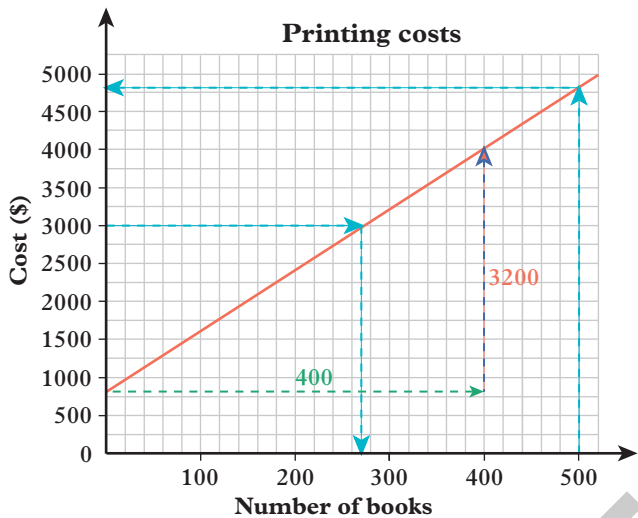
Worked example 2.5A Interpreting the graph of a linear model

The graph shown models printing costs.



- Find the approximate cost of printing 500 books.
- Find the approximate number of books that can be printed for \$3000.
- Find the vertical intercept. What is its meaning in this situation?
- Find the gradient of the line. What is its meaning in this situation?
- Let x represent the number of books and let y represent the cost of printing. Write the equation of the line in the form $y = mx + c$.



Part	Think	Write
a	Find the point on the line whose horizontal coordinate (number of books) is 500. The vertical coordinate (cost) is \$4800. 	\$4800
b	Find the point on the line whose vertical coordinate (cost) is 3000. The horizontal coordinate (number of books) is 270.	270 books
c	The vertical intercept is the cost of printing zero books.	The cost of printing 0 books is \$800. In context, this means that there are \$800 worth of fixed set-up costs.
d	Substitute the points (0, 800) and (400, 4000) into the gradient formula. $\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$ Because the units associated to the vertical axis are \$ and the units for the horizontal axis are books, the gradient encodes the ratio of \$/book.	$\begin{aligned}\text{gradient} &= \frac{4000 - 800}{400 - 0} \\ &= \frac{3200}{400} \\ &= 8\end{aligned}$ This means that, after the initial set-up costs, printing costs a further \$8/book.
e	Substitute the values for the gradient m and the y-intercept c into $y = mx + c$.	The equation is $y = 8x + 800$.



Worked example video: Interpreting the graph of a linear model

Understanding, fluency and communicating

Answers: Lesson 2.5

Helpful hints

- After finding the point on a line with a given coordinate, the second unknown coordinate can be read off the other axis.
- Remember that when the value of one variable depends on the other, it is placed on the vertical axis. Meanwhile, the independent variable is placed on the horizontal axis.

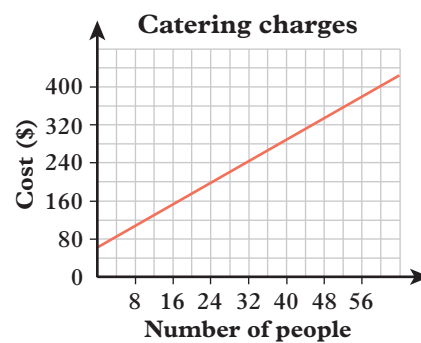
- 1 This graph models the cost of a taxi ride based on a certain company's fees.

- From the graph, find the approximate cost of travelling 22 km by taxi.
- How far can a passenger travel for \$30?
- Find the gradient. What is its meaning in this situation?
- Find the intercept on the vertical axis. What is its meaning in this situation?
- Let x represent the distance in kilometres and let y represent the cost of a taxi ride. Write the equation of the line in the form $y = mx + c$.



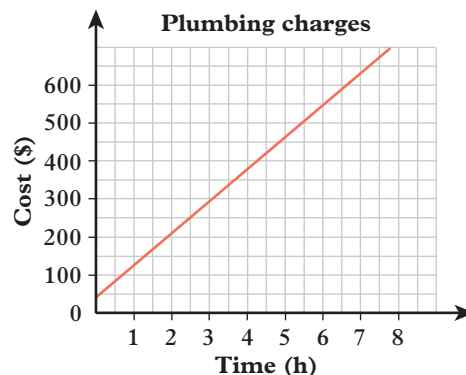
- 2 This graph models catering charges.

- How much would it cost to cater for 24 people?
- How many people could be catered for if \$360 was spent?
- Find the gradient. What is its meaning in this situation?
- Find the intercept on the vertical axis. What is its meaning in this situation?
- Let x represent the number of people and let y represent the cost of catering. Write the equation of the line in the form $y = mx + c$.



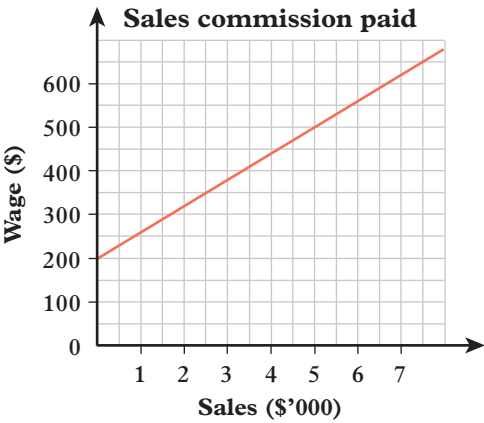
- 3 This graph models plumbing charges.

- Find the cost of the plumber for 4.5 hours.
- Find the number of hours worked if the charge is \$250.
- Find the gradient. What is its meaning in this context?
- Find the intercept on the vertical axis. What is its meaning in this context?
- Let x represent the time in hours and let y represent the amount charged by the plumber. Write the equation of the line in the form $y = mx + c$.



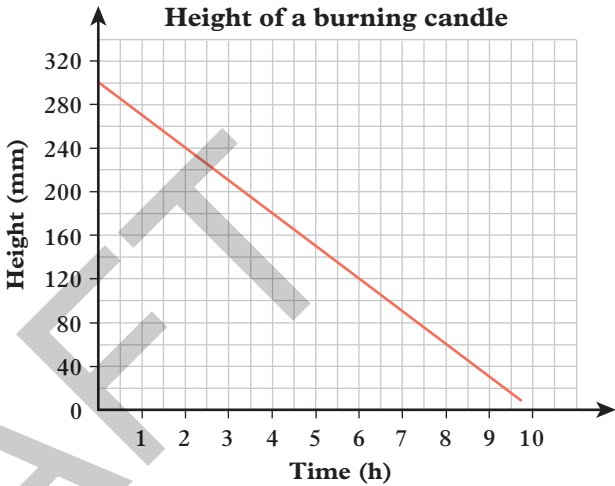
4 This graph models the income of a salesperson.

- a What was the value of sales if the salesperson was paid \$320 for the week?
- b What wage would be paid if the salesperson sold \$3500 worth of goods?
- c Find the gradient. What is its meaning in this context?
- d Find the intercept on the vertical axis. What is its meaning in this context?
- e Let x represent the total value of the salesperson's sales and let y represent their wage. Write the equation of the line in the form $y = mx + c$.



5 This graph models the height of a burning candle.

- a What is the height of the candle:
 - i initially?
 - ii after five hours?
- b Approximately how long does it take for the height to decrease to 210 mm?
- c How long does it take for 60 mm of the candle to be used?
- d Find the gradient. What is its meaning in this context?
- e Find the intercept on the vertical axis. What is its meaning in this context?
- f Let x represent the time the candle has been burning and let y represent the height of the candle. Write the equation of the line in the form $y = mx + c$.



Worked example 2.5B Interpreting linear models in context

The interest, \$ I , earned on a principal of \$ P for 1 year is given by the formula $I = kP$, where k is a constant.

- a If an investment of \$6000 earns interest of \$300, find the value of the constant k .
- b Use the value of k from part a to complete the table.

P	2000	4000	6000	8000	10 000
I			300		

- c Draw a graph of this linear relationship.
- d Calculate the gradient of the linear graph. What is the meaning of the gradient in this context?

Part	Think	Write												
a	Substitute $I = \$300$ when $P = \$6000$ into $I = kP$. Solve the equation by dividing both sides by 6000.	$300 = k \times 6000$ $\frac{300}{6000} = k$ $k = 0.05$												
b	Substitute each value of P into the equation $I = 0.05P$.	<table><tr><td>P</td><td>2000</td><td>4000</td><td>6000</td><td>8000</td><td>10 000</td></tr><tr><td>I</td><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td></tr></table>	P	2000	4000	6000	8000	10 000	I	100	200	300	400	500
P	2000	4000	6000	8000	10 000									
I	100	200	300	400	500									

Part	Think	Write
c	Each column in the previous table gives us a point that can be plotted on a graph. After plotting all points, connect them with a straight line.	
d	Find the gradient of the linear graph using one of the gradient formulas. The gradient can be interpreted as dollars earned per dollar invested; that is, the rate of interest.	$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{200}{4000} = 0.05$ In context, the gradient tells us that the rate of interest being earned on this investment is 5%.



Worked example video: Interpreting linear models in context

Helpful hints

- Recall that the constant of variation in a direct variation relationship between variables in an ordered pair (x, y) is equal to $\frac{y}{x}$. Unknown values can be found by substituting the known coordinate into the equation $y = kx$.
- The meaning of the gradient is determined by the units of each variable on the vertical and horizontal axes.

- 6 The interest, $\$I$, earned on a principal of $\$P$ for one year is given by the formula $I = kP$, where k is a constant.

- a If an investment of $\$8000$ earns interest of $\$240$, find the value of the constant k .
b Use the value of k from part a to complete the table below.

P	2000	4000	6000	8000	10 000
I				240	

- c Draw a graph of this linear relationship.
d What is the meaning of the gradient of this line in this context?
- 7 The distance, d km, travelled by a train in a time of t hours is given by the relationship $d = mt$, where m is a constant.
- a Use the information in the table to find the value of the constant m .
b Use the value of m from part a to complete the table.

t	1	3	5	7	9
d			400		

- c** Draw a graph of this linear relationship.
- d** What is the meaning of the gradient of this line in this context?
- 8** The volume, $V \text{ cm}^3$, of a solid with a fixed cross-section and height $h \text{ cm}$ is given by $V = kh$, where k is a constant.
- a** Use the information in the table to find the value of the constant k .
- b** Use the value of k from part **a** to complete the table.

h	4	8	12	16	20
V		34			

- c** Draw a graph of this linear relationship.
- d** Calculate the gradient of this line. What is its meaning in this context?

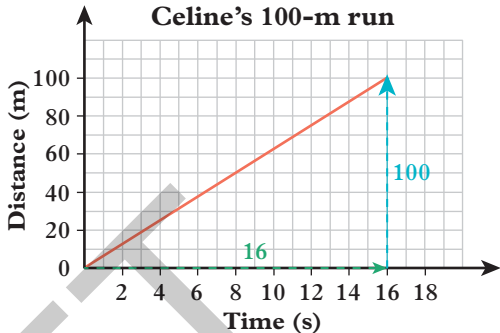
Worked example 2.5C Creating and critiquing linear models

At an athletics carnival, Celine runs 100 m in 16 s.
For the sake of developing a mathematical model, assume that Celine runs at a constant rate.

- a** Draw a straight-line graph to model Celine's run.
- b** Find the gradient of the graph from part **a**. What is the meaning of the gradient in this context?
- c** Write an equation to model the relationship between distance and time for Celine's run.
- d** Extend the model to predict how long it would take Celine to run 200 m. Comment on the reasonableness of this prediction.



Part	Think	Write
a	Put time on the horizontal axis, and distance on the vertical axis. We are told that Celine runs 100 m in 16 s and we can assume that she runs 0 m in 0 s. Hence, we plot the points (0, 0) and (16, 100), and connect them with a straight line.	Celine's 100-m run

Part	Think	Write
b	The vertical difference (rise) between these two points is 100 m and the horizontal difference (run) is 16 s. Hence, the gradient describes Celine's speed in m/s.	$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{100}{16} \\ &= 6.25\end{aligned}$ The gradient tells us that Celine's average speed is 6.25 m/s.
c	Use pronumerals to define the two variables and write an equation in the form $y = mx$.	If d is distance (in m) and t is time (in s) then $d = 6.25t$.
d	Update the graph from part a (or draw a new one) to show the same line extending until it reaches 200 m on the vertical axis. This occurs when $t = 32$.	 The model predicts that Celine could run 200 metres in 32 seconds. In reality, Celine is unlikely to be able to run twice as far while maintaining the same speed, so this model should not be used to predict her performance in races with a longer distance.



Worked example video: Creating and critiquing linear models

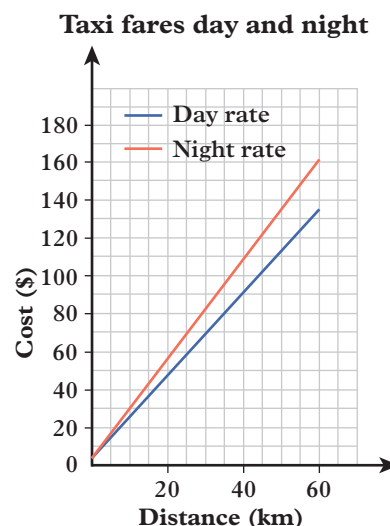
Helpful hints

- When coordinates are only provided for a single point, but the relationship is known to be one of direct variation, the graph can be drawn by connecting the known point to the origin using a straight line.
- The gradient of a line between the point (x, y) and the origin $(0, 0)$ is always $\text{gradient} = \frac{y - 0}{x - 0} = \frac{y}{x}$. The meaning of the gradient is indicated by its units.

- 9 Christopher runs 100 m in 25 s.
- a Draw a straight-line graph to model Christopher's run, assuming he runs at a constant rate.
 - b How long would it take him to run 30 m?
 - c Find the gradient, and hence, Christopher's speed in metres per second.
 - d Write an equation to model the relationship between distance and time.
- 10 Craig drives 200 km in four hours.
- a Draw a straight-line graph to model Craig's drive, assuming he drives at a constant rate.



- b** How far does Craig drive in 1.5 hours?
- c** Find the gradient of the graph you drew in part **a**.
- d** What are the units of the gradient in this case?
- e** Write an equation to model the relationship between distance and time.
- 11** On a particular day, AUD\$100 buys €8 (euros).
- a** Draw a conversion graph with AUD\$ on the x -axis.
- b** How many AUD\$ are needed to purchase €50?
- c** Find the gradient of the graph you drew in part **a**.
- d** What is the meaning of the gradient in this context?
- e** Write an equation to model the relationship between AUD\$ and euros.
- f** Extend your graph to AUD\$500. Is it still accurate as an exchange model?
- 12** Use the information in question **9** and extend the graph to cover a distance of 500 m.
- a** Is the gradient still the same? Why or why not?
- b** The graph is no longer accurate as a model for Christopher's run. Explain, including some reference to the gradient.
- 13** Use the information in question **10** and extend the graph to cover a distance of 1000 km.
- a** Is the gradient still the same? Why or why not?
- b** Is the graph still accurate as a mathematical model for Craig's drive? (*Hint: consider petrol.*)
- 14** The rate for taxi fares varies depending on the time of day. The graph shows the cost in dollars compared to the distance travelled in kilometres for travel during the day and at night.
- a** What is the meaning of the gradient in this context?
- b** Find the gradient of each line. What does the difference in steepness of the two slopes indicate?
- c** For each line, find the intercept on the vertical axis. What is the meaning of the intercept for each situation?
- d** For each line, write an equation which models the linear relationship.



Problem solving and reasoning

- 15** Suppose you use a 4-litre can of paint to paint the first 12 m² section of wall in your new home.
- a** Assume that the number of litres of paint, L , and the area covered, A , are related by direct variation.
- i** Write an equation that models this relationship.
- ii** Use this model to predict how many litres of paint you will need to finish painting the 48 m² area of wall in your home.
- b** Now, consider the possibility that L and A are *not* related by direct variation. It is common practice to paint a “swatch” to check how your chosen paint colour will appear on a particular wall. Assume that a swatch uses 0.4 litres of paint and that this area will still need to be painted over.
- i** Fill in the following table of values relating L and A , using the fact that 0.4 litres of paint were used before beginning your painting “for real”, and the fact that you were able to cover 12 m² by the time you emptied the 4-litre can.

A	0	
L		4

- ii Draw a graph that displays this relationship by plotting the points and connecting them.
 - iii Model this relationship by writing an equation in the form $L = mA + c$.
 - iv Use this new model to answer the same question as in part a ii.
- 16 Choose one of the earlier questions about a direct variation relationship, either from this lesson or the previous one, and challenge any assumptions that were made. Come up with a new and improved model and describe any improvements you make. Be creative!

Challenge

- 17 A salesperson receives a commission of 5% of their total sales, and a retainer of \$200 per week. Write an equation relating their sales s to their income I in a week.

Related resources

 2.5 Quick quiz

 **Worksheet:** Finding the equation of a linear model

Using spreadsheets

Syllabus links

Use a spreadsheet to model a linear relationship in a variety of contexts (MST-11-02)

Linear relationships

See Oxford Digital for sample spreadsheets for questions in this lesson.

- 1 Here is part of a spreadsheet for the data in Lesson 2.1 question 11 showing the formulas for calculating the cost of taxi travel in the day and during the night.

	A	B	C
1	Taxi rates		
2		Day rate	Night rate
3	Rate (per km)	\$ 2.19	\$ 2.63
4	Flagfall (\$)	\$ 3.60	\$ 6.10
5	Distance (km)	Day cost	Night cost
6	0	=B3	=C3
7	5	=A7*\$B\$2+\$B\$3	=A7*\$C\$2+\$C\$3
8	10	=A8*\$B\$2+\$B\$3	=A8*\$C\$2+\$C\$3
9	15	=A9*\$B\$2+\$B\$3	=A9*\$C\$2+\$C\$3

- a Create a spreadsheet or use the one provided on Oxford Digital. Fill down to complete the spreadsheet for the data provided in the table in Lesson 2.1 question 11.
 - b Use the spreadsheet to create a graph showing the corresponding two straight-line graphs on the same Cartesian plane.
 - c Use your spreadsheet to answer Lesson 2.1 question 11.
 - d Edit the kilometre rate and flagfall to see how that affects the graph.
- 2 Use a spreadsheet to create graphs to model the following situations.
- a Picnic Pros is a business that organises picnics for birthday parties. They charge \$50 to set up the picnic, \$50 to pack up and then \$80 per hour for the duration of the party.
 - b The Bean Counter Café delivers cups of coffees to businesses. They charge a delivery fee of \$12 and then \$4.50 per cup of coffee.
 - c Pet Paraders is a dog-walking business. They charge an account fee of \$1 per week and then \$10 per walk. Assume dogs are walked for customers three times per week. Create your model so that you can adjust the number of times dogs are walked per week.
- 3 A 10-litre can of paint costs \$30 and will cover 35 m² of wall. Create a spreadsheet as below to help you answer the following questions.

	A	B	C	D	E
1	Number of cans (C)	1	2	3	4
2	Litres of paint (L)				
3	Area of coverage (A)				
4	Price (P)				

- a Fill in entries as follows:
 - i Row 2: Enter $B2 = 10 \cdot B1$, then drag the formula across the remaining entries.
 - ii Row 3: Assume A varies directly with L and fill in values for Row 3 accordingly, either manually or using an appropriate formula.
 - iii Row 4: Use any method to fill the row so it gives the price of the paint for the corresponding number of cans. Check that each entry makes sense in the context.
- b Write an equation relating each of the following, then enter them as a formula in a new row to check any of the entries that you entered manually in part a.
 - i C and L
 - ii C and A
 - iii C and P
- c Suppose you want to paint some walls with a total area of 175 m².
 - i How many litres of paint will you need?
 - ii How many cans of paint will you need?
- d Now write an equation relating the price $\$P$ of the paint required to cover A m² of wall. Are P and A in direct variation with each other?
- e Suppose you can hire a painter who charges a flat fee of \$100 to paint any room in your house, provided you supply the paint. Update your equation from part d to include this flat fee.
- f Create a graph comparing the linear models described by your equations in parts d and e. Make sure the minimum and maximum values visible on each of the P - and A -axes make sense in context.

Lesson 2.6

Review: Linear relationships

Review questions 2.6A Mathematical literacy review

Answers: Lesson 2.6

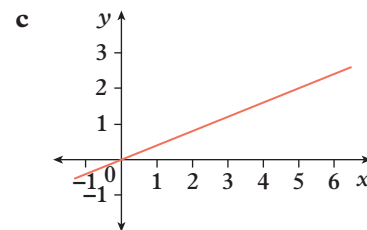
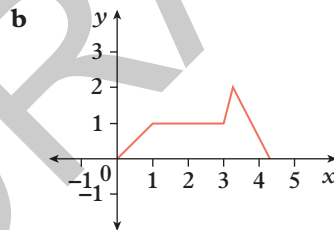
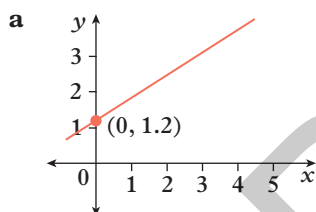
Key terms

The following key terms are used in this module:

- constant of variation
- dependent variable
- direct variation
- directly proportional
- extrapolation
- gradient/slope
- independent variable
- interpolation
- linear model
- mathematical model
- piecewise graph
- y-intercept

Check your understanding

- Fill in the blanks using words from the list above.
 - The _____ describes the rate of change in the dependent variable as the _____ changes.
 - The _____ represents the value of the _____ (y) when the independent variable (x) is 0.
- Explain what a **mathematical model** is by writing a sentence that includes the word **extrapolate**.
- Write the meaning of " $y \propto x$ " in words.
- Define three terms from the list above, using the graphs below as examples. You may choose any three terms, as long as the chosen graph demonstrates the features relevant to your definition.



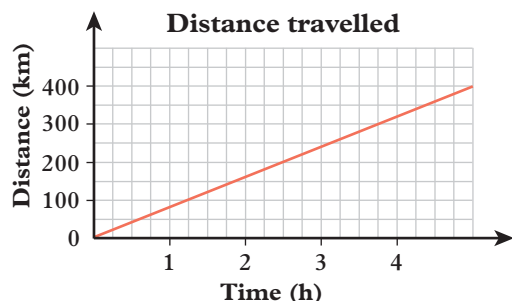
- In the two equations below, add the following labels, making sure to use each term at least once.
constant of variation, y-intercept, dependent variable, independent variable, rise, run

$$y = \frac{5}{8}x + 2$$

$$y = 7x$$

Review questions 2.6B Multiple-choice questions

Use the graph, showing the distance a car travels over a period of time, to answer questions 1 to 3.



- What is the time taken to travel 200 km?
 - 2 h
 - $2\frac{1}{2}$ h
 - 3 h
 - $3\frac{1}{2}$ h

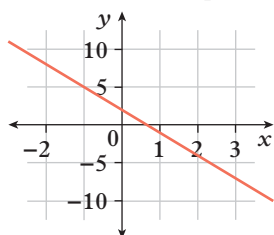
2 What is the distance travelled after $3\frac{3}{4}$ h?

- A 5 km
- B 250 km
- C 300 km
- D 6 km

3 What is the gradient of the graph?

- A $\frac{4}{5}$
- B $\frac{5}{4}$
- C 100
- D 80

Use the graph shown to answer questions 4 and 5.



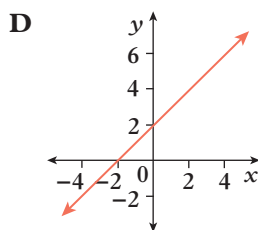
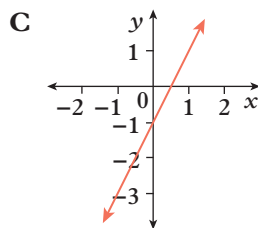
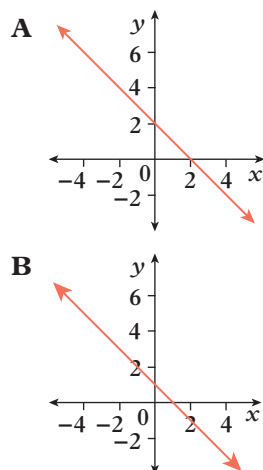
4 What is the gradient of the line?

- A 3
- B $\frac{1}{3}$
- C $-\frac{1}{3}$
- D -3

5 What is the y-intercept?

- A 2
- B 34
- C -3
- D -2

6 Which diagram shows the graph of $y = 2x - 1$?



7 What are the gradient and y-intercept of the line $y = \frac{3x}{2} - 1$?

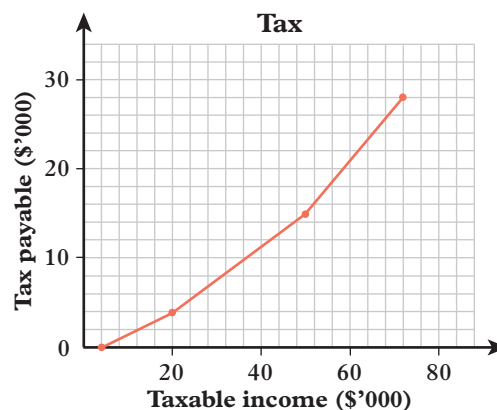
- A 3 and -1
- B $\frac{3}{2}$ and -1
- C $\frac{3}{2}$ and 1
- D $\frac{3}{2}$ and $-\frac{1}{2}$

8 What is the form of the linear equation describing the pattern in this table?

x	1	2	3	4	5
y	3	7	11	15	19

- A $y = 2x + c$
- B $y = 3x + c$
- C $y = 4x + c$
- D $y = 5x + c$

Use the tax graph shown to answer questions 9 and 10.



- 9 The tax payable on \$65 000 is closest to:

A \$20 000
B \$21 000
C \$23 000
D \$25 000

- 10 The income on which tax of \$10 000 is payable is closest to:

A \$2000
B \$36 000
C \$30 000
D \$2500

- 11 The Clear View Water Company purifies water. The weekly fixed costs are \$250 and the cost per glass is \$3. Which of the following equations represents this information?

A $C = 3n + 250$
B $C = 250n + 3$
C $C = 250n - 3$
D $C + 250 = 3n$

- 12 The equation $C = 4n + 520$ models an ice-cream shop's costs. What could the 520 represent?

A number of ice creams sold
B the cost per ice cream
C fixed daily cost
D number of ice creams made

- 13 Which table of values below represents a direct variation relationship?

A

x	1	3	5
y	10	26	42

B

x	1	3	5
y	8	24	40

C

x	1	3	5
y	4	6	8

D

x	1	3	5
y	8	10	12

- 14 The table shows values of x and y which are in direct proportion. What are the values of a and b ?

x	6	12	24	b
y	2	4	a	12

A $a = 8, b = 4$
B $a = 8, b = 36$
C $a = 72, b = 4$
D $a = 72, b = 36$

- 15 The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car used 7 L of petrol to travel 52.5 km. The amount of petrol used to travel 126 km at the same rate is:

A 2.4 L
B 7.5 L
C 16.8 L
D 945 L

- 16 What does the slope of the distance–time graph for question 1 represent?

A speed
B time taken
C distance travelled
D all of these

Review questions 2.6C Set 1

- 1 Dog shampoo is sold in bulk amounts for \$4.50 per litre. This table shows capacity, in litres, versus cost for the liquid.

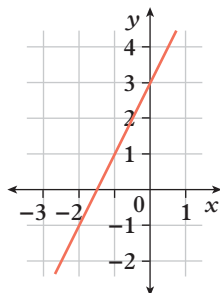
Capacity (L)	1	2	3	4	5	6
Cost (\$)	4.50	9.00	13.50	18.00	22.50	27.00

- a Draw the graph of capacity versus cost. Which is the dependent variable?
b Find the cost of 3.5 L of shampoo.

- 2 A person drives non-stop from Condobolin to Sydney, a distance of 400 km. The linear relationship for finding the distance the driver is from Sydney is $d = 400 - 100t$, where d is distance in kilometres and t is time in hours.
- a Complete this table of values.

Time (h)	0	1	2	3	4
Distance (km)	400		200		

- b Draw the graph of distance to Sydney.
- c How far is it to Sydney after $1\frac{1}{2}$ h of driving?
- d How far has the car travelled after $2\frac{1}{2}$ h?
- 3 Find the gradient and y-intercept of this straight-line graph.



- 4 Sketch the graph of $y = 3x + 5$.
- 5 On a particular day, AUD\$100 buys €68 (euros).
- i Draw a conversion graph with AUD\$ on the horizontal axis.
- ii How many AUD\$ are needed to purchase €50?
- iii Find the gradient.
- iv What is the meaning of the gradient in this context?
- v Write an equation to model the relationship between AUD\$ and euros.
- vi Extend your graph to AUD\$500. Is it still accurate as an exchange model?
- 6 Find a linear equation that describes the pattern shown in this table.

x	0	1	2	3	4
y	-5	-2	1	4	7

- 7 The distance, d kilometres, travelled by a train over time t hours is $d = 70t$.
- a Complete this table of values for t and d .

t	1	2	3	4	5
d					

- b Draw the graph showing the distance travelled by the train versus time.
- c How far away is the train after $3\frac{1}{2}$ hours?
- d When is the train 200 km away?
- e Is this an example of direct variation? Explain.

Review questions 2.6D Set 2

1 Chocolate frogs are sold by weight. This table shows weight, in kilograms, versus cost for the chocolate frogs.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	10	20	30	40	50	60

- a Draw the graph of weight versus cost. Which is the dependent variable?
b Use the graph to find the cost of 4.5 kg of chocolate frogs.
- 2 The distance, d km, travelled by a bus in time t hours is given by $d = 60t$.
- a Complete this table of values.

Time (h)	0	1	2	3	4
Distance (km)	0		120		

- b Draw the graph of distance travelled versus time.
c How far away is the bus after $1\frac{1}{2}$ h?
d When is the bus 100 km away?
- 3 Sketch the straight-line graph of $y = 3 - 2x$.
4 This graph models catering charges.



- a How much would it cost to cater for 35 people?
b For how many people would catering cost \$300?
c Find the gradient. What is its meaning in this context?
d Find the intercept on the vertical axis. What is its meaning in this concept?
- 5 The Hats Are Tops company has fixed costs of \$900 per day and a variable cost of \$5 per hat. The daily cost can be modelled using the straight-line equation $C = 5n + 900$.
- a Explain each term in the formula $C = 5n + 900$.
b Complete the table of values.

n	0	50	150	200	250
C					

- c Draw the straight-line graph $C = 5n + 900$, for $0 \leq n \leq 250$.
d Use the graph to find the number of hats produced when the cost is \$1500.
- 6 By calculating the values for $\frac{y}{x}$ from the table, determine whether $y \propto x$.

x	1	2	5
y	6	14	30

Review questions 2.6E Set 3

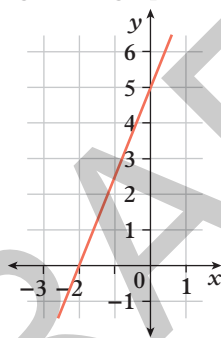
- 1 Cashew nuts are sold by weight. This table shows weight, in kilograms, versus cost for the nuts.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	15	30	45	60	75	90

- Draw the graph of weight versus cost. Which is the dependent variable?
 - Use the graph to find the cost of 3.8 kg of nuts.
 - How many kilograms of nuts can be purchased for \$50?
- 2 The cost of hiring a night-time taxi is \$7.00 flagfall and \$3.80 per kilometre travelled.
- Complete this table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost (\$)					

- Draw the graph showing the cost of hiring the taxi.
 - How much does it cost to travel 15 km?
 - How far can a passenger travel for \$75?
- 3 Find the gradient and y-intercept of this straight-line graph.



- Sketch the straight-line graph of $y = 3x - 4$.
- On a particular day, AUD\$100 buys €87 (euros).
 - Draw a conversion graph with AUD\$ on the horizontal axis.
 - How many AUD\$ are needed to purchase €50?
 - Find the gradient. What is its meaning in this context?
 - Extend your graph to AUD\$500. Is it still accurate as an exchange model?
- Find a linear equation that describes the pattern shown in this table.

x	0	1	2	3	4	5
y	2	-1	-4	-7	-10	-13

- 7 The Socks Are Us company sells socks for \$16 a pair. Sales may be modelled using the straight-line equation $I = 16n$.
- Explain each term in the formula $I = 16n$.
 - Complete the table of values.

n	0	50	150	200	250
I					

- Draw the straight-line graph $I = 16n$.
- Use the graph to find the number of pairs of socks sold when the income is \$1840.
- Is this a direct variation relationship? Explain.

Review questions 2.6F Set 4

- 1 Mobile telephone calls cost 85 cents per minute. This table shows time versus cost for the calls.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.85	1.70	2.55	3.40	4.25	5.10

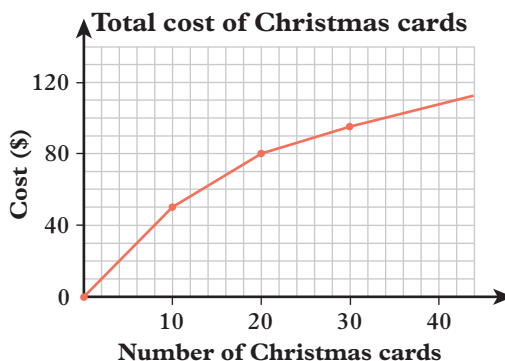
- Draw the graph of time versus cost. Which is the independent variable?
 - Use the graph to find the cost of a 2.5 minute call.
 - What is the maximum length of a call that costs \$3.00?
- 2 The cost of hiring a car is \$10 flagfall and \$3.40/km for each kilometre travelled.
- Complete this table of values for car hire.

Distance (km)	0	10	20	30	40
Cost (\$)	10.00	44.00			

- Draw the graph showing the cost of hiring the car.
 - How much does it cost to travel 35 km?
 - How far can a driver travel for \$15?
 - Is this an example of direct variation? Explain
- 3 Sketch the straight-line graph of $y = -2 - 4x$.
- 4 This graph models taxi charges.



- Find the cost of travelling 65 km.
 - How far can a passenger travel for \$40?
 - Find the gradient. What is its meaning in this context?
 - Find the intercept on the vertical axis. What is its meaning in this context?
- 5 This graph shows the total cost of hand-made Christmas cards.

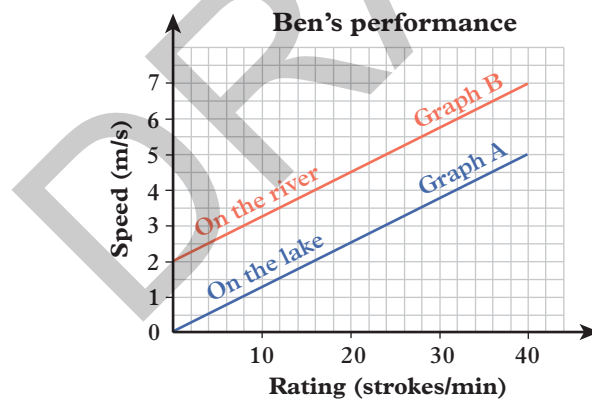


- What is the cost of buying:
 - 6 cards?
 - 18 cards?
 - 30 cards?

- b** How many cards can be bought for:
- i** \$20? **ii** \$65? **iii** \$100?
- c** Consider how the gradients of the line segments in the graph change as you buy more cards.
- i** If you buy 10 cards, what is the cost per card?
- ii** What is the gradient of the line segment for the cost of buying more than 30 cards?
- iii** What does the gradient represent in this situation?
- 6** The Tight Squeeze Belt company has weekly fixed costs of \$400. The cost per belt is \$7.
- a** Write an equation to represent this information. Use A for cost and n for the number of belts.
- b** Complete the table of values.
- | | | | | | | |
|-----|---|-----|-----|-----|-----|------|
| n | 0 | 200 | 400 | 600 | 800 | 1000 |
| A | | | | | | |
- c** Draw a straight-line graph.
- d** Use the graph to find the number of belts made when the cost is \$2000.
- 7** Consider the following table of values.
- | | | | | |
|-----|----|----|-----|-----|
| x | 2 | 3 | 5 | b |
| y | 32 | 48 | a | 8 |
- a** Find the values of a and b in the table, given that $y \propto x$.
- b** From the table, write the relationship between x and y as a linear equation.

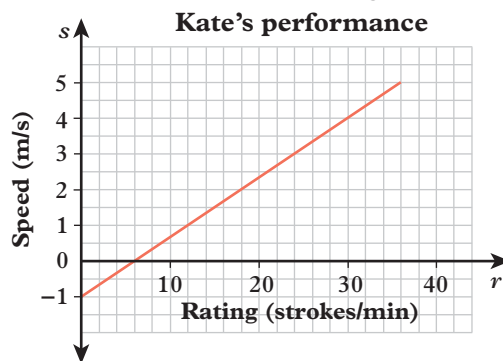
Review questions 2.6G Practice examination question

- 1 Answer the following questions.
- a** The speed, s m/s, at which Ben can row his boat depends on the number of strokes, r , he takes per minute. This is called his rating. Graph A shows his performance when he is training on a lake.



- i Use the graph to find the rating necessary for a speed of 2 m/s. **(1 mark)**
 - ii Find Ben's speed if his stroke rating is 32 strokes/min. **(1 mark)**
 - iii Calculate the time, in minutes and seconds, that it takes to row one kilometre at the speed found in part ii. **(1 mark)**
- b Graph B represents Ben's performance on a river when he rows downstream, with the same rating as his rating on the lake.
 - i Find his speed on the river if his stroke rating is 20 strokes/min. **(1 mark)**
 - ii How fast is the river flowing? **(1 mark)**

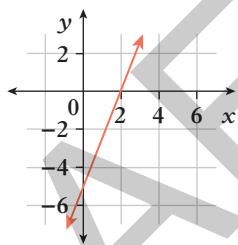
- c Ben's coach records the speed and rating of another rower, Kate. This graph shows Kate's performance when she is rowing on the same river as Ben, but it is flowing at a different speed.



- i Is Kate rowing upstream against the flow or downstream? Give reasons. **(1 mark)**
- ii How fast is this river flowing? **(1 mark)**
- d Find the linear equation that describes the pattern shown in the table. **(2 marks)**

x	0	1	2	3	4	5
y	11	7	3	-1	-5	-9

- e This diagram shows a straight-line graph.



- i Calculate the gradient of the line. **(1 mark)**
- ii What is the y -intercept? **(1 mark)**
- f The cost of pies varies directly with the number purchased. Three pies cost \$13.50.
- i Form an equation linking cost, $\$C$, and the number of pies purchased, n . **(1 mark)**
- ii What is the constant of variation k ? **(1 mark)**
- iii Draw the graph of C versus n for $0 \leq n \leq 10$. **(1 mark)**
- iv How much would 8 pies cost? **(1 mark)**

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Linear relationships

I can do this	I need to review this
☐ construct a straight-line graph on the Cartesian plane using a table of values ☐ complete a table of values using a rule in words ☐ construct a straight-line graph to represent a linear model	Go back to Lesson 2.1 Visualising linear models
☐ determine the gradient of a line using the formula $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ ☐ determine the gradient of a line using the formula $\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$	Go back to Lesson 2.2 Understanding gradient
☐ write the equation of a straight line of the form $y = mx + c$ where m is the gradient and c is the y -intercept ☐ determine the y -intercept and gradient from the equation of a straight line ☐ sketch a linear graph in the form $y = mx + c$ with and without digital tools	Go back to Lesson 2.3 The equation $y = mx + c$
☐ recognise that direct variation relationships in the form $y = kx$ produce a straight-line graph which passes through the origin ☐ explain that k is the constant of variation and represents the gradient ☐ identify and represent situations of direct variation of the form $y = kx$ ☐ determine the value of k and use it to find other pairs of values of x and y ☐ use the formula $y = kx$ to solve problems in a variety of contexts ☐ graph and analyse equations of the form $y = kx$ to solve problems in a variety of contexts	Go back to Lesson 2.4 Direct variation
☐ model a linear relationship and interpret its features ☐ interpret the graph of a linear model ☐ determine the equation of a graph of a linear model ☐ interpolate and extrapolate using a linear model ☐ describe the limitations of a linear model	Go back to Lesson 2.5 Analysing linear models
☐ use a spreadsheet to model linear relationships	Go back to Using spreadsheets

Related resources



Module review quiz: Linear relationships



Worksheet: Challenge 2

Review: Algebra

Review questions

Answers: Review: Algebra

1 Solve each of the following equations.

a $d - 9 = 12$

b $x + 4 = 12$

c $\frac{x}{3} = 6$

d $11x = 55$

e $3x - 8 = 19$

f $7x - 12 = 16$

g $14 + 3n = 29$

h $23 - 4c = 14$

i $\frac{4x+1}{5} = \frac{1}{7}$

2 Substitute the given values into each expression and evaluate the result.

a If $m = 10$, $n = 15$ and $p = -6$, find the value of $\frac{(pm)^2}{n}$.

b If $A = 750$ and $h = 12.6$, find r correct to one decimal place given that $r = \sqrt{\frac{A}{\pi h}}$.

3 Solve each of the following equations.

a $6x = 96$

b $\frac{x}{4} = -2$

c $x - 5 = 8$

d $x + 6 = 9$

e $3x - 5 = 7$

f $4 - 2x = -8$

g $7 - 3x = 4 - 5x$

h $-\frac{4}{x} = 7$

i $\frac{x-5}{2} = 11$

4 Make y the subject of each of these equations.

a $\frac{y}{3} = p$

b $x = 4 - 3y$

c $\frac{y-x}{w} = 2$

5 Find the total stopping distance, to the nearest metre, for a car travelling at 80 km/h in slippery conditions. Assume a reaction time of 2.5 seconds and use the formula $d = 0.014v^2$, where d is the braking distance in metres and v is the speed of the car in km/h. Give your answer correct to the nearest metre.

6 Ada needs to travel 75 km to visit a friend.

a If the journey takes her 45 min in her car, what is her average speed of travel?

b Ada can choose to take a train with an average speed of 120 km per hour. How long does the journey take if Ada chooses this option?

c What is the speed of the train in part **b** in m/s, correct to one decimal place?

7 Lisa goes to a party and drinks four standard drinks over two hours. Use the formulas in Lesson 1.6 to help you answer the following.

a Calculate Lisa's estimated blood alcohol content (BAC) after two hours if her mass is 60 kg.

b Find how long it will be until her BAC drops to zero?

8 The adult dosage for a medication is 50 mg. Use Fried's, Young's and Clark's formulas given in Lesson 1.7 to calculate the dose for children with the following descriptions. Give answers correct to one decimal place where necessary.

a 14 months old

b 8 years old

c Weighs 28 kg

9 Solve the following equations.

a $x^2 = 49$

b $x^2 - 144 = 0$

c $25 - x^2 = 0$

d $2m^2 = 800$

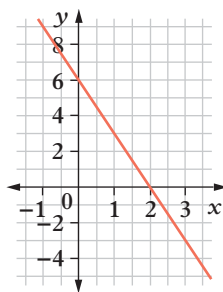
e $4p^2 + 3 = 19$

f $-2r^2 + 3 = -15$

- 10 Mobile telephone calls cost 75 cents per minute. This table shows time versus cost for the calls.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.75	1.50	2.25	3.00	3.75	4.50

- Draw the graph of time versus cost. Which is the dependent variable?
 - Use the graph from part **a** to find the cost of a 2.5 minute call.
 - What is the maximum length of a call that costs \$2.50?
- 11 Find the gradient and y -intercept of the straight-line graph shown.



- 12 Sketch the straight-line graph $y = -3 - 4x$.

- 13 A graph modelling taxi charges is shown.

- Find the cost of travelling 75 km.
- How far can you travel for \$50?
- Find the gradient. What is the meaning of the gradient in this context?
- Find the intercept on the vertical axis. What is its meaning in this context?
- Is this an example of direct variation? Explain.



- 14 Given that $y \propto x$, and $y = 4.5$ when $x = 18$, find:

- the constant of variation
- the equation for y in terms of x
- y , when $x = 48$
- x , when $y = 8$.

- 15 Complete this table of values for $y = 3x - 2$ and draw the straight-line graph on a number plane.

x	-3	-2	-1	0	1	2	3
y							

- 16 For its bow ties, the Polka Dot Tie Company has fixed costs of \$900 per day and a variable cost of \$8 per tie. The daily cost may be modelled using the straight-line equation $C = 8n + 900$.

- Explain each term in the formula.
- Complete this table of values.

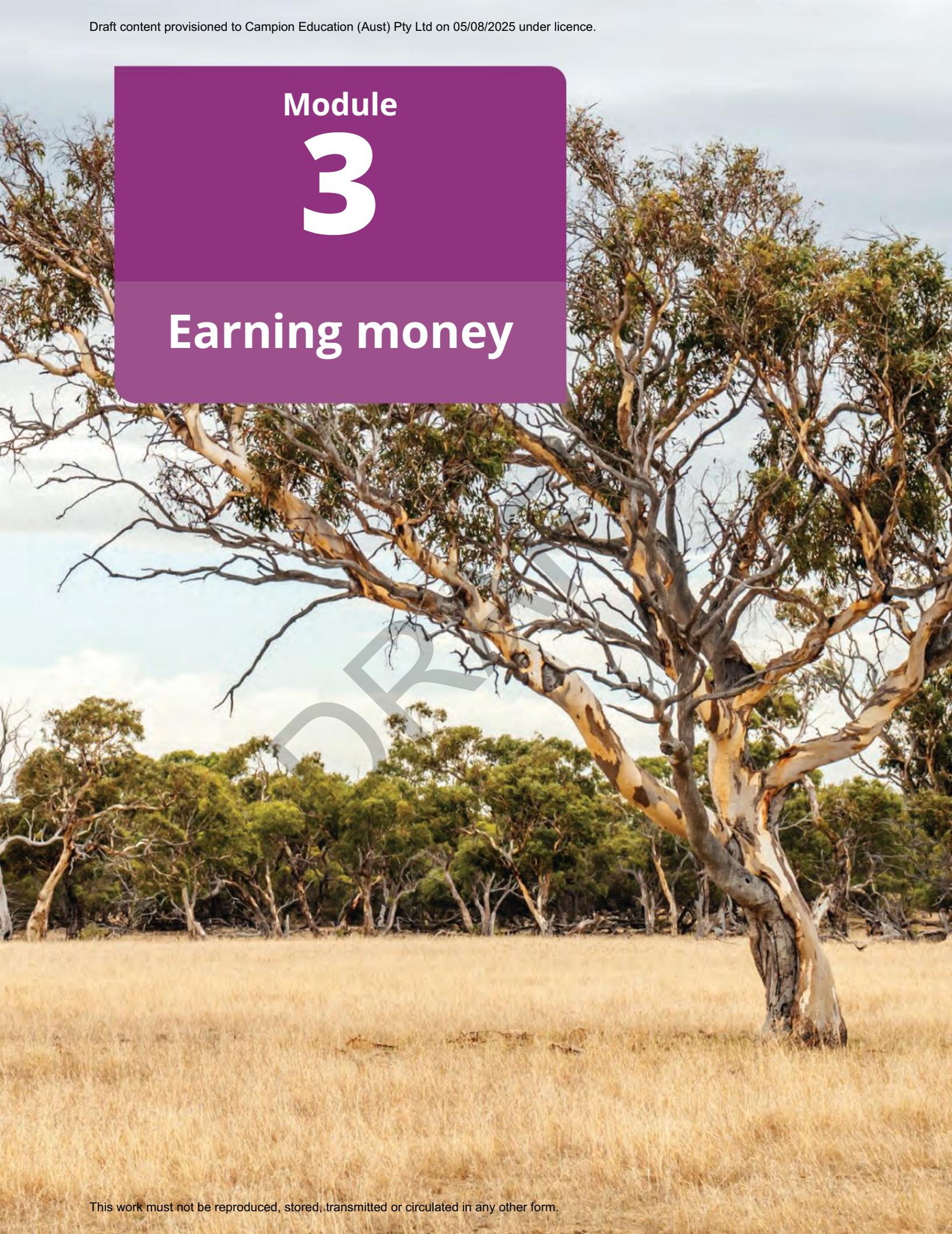
n	0	50	100	150	200	250
c						

- Draw the straight-line graph of $C = 8n + 900$.
- Use your graph from part **c** to find the number of ties produced if the cost is \$1940.

Module

3

Earning money



Lessons

- Lesson 3.1 Earning an income
- Lesson 3.2 Allowances, bonuses and loadings
- Lesson 3.3 Commission, piecework and royalties
- Lesson 3.4 Government allowances and pensions
- Lesson 3.5 Taxable income
- Lesson 3.6 Paying tax
- Lesson 3.7 Tax returns and net earnings
- Using spreadsheets
- Lesson 3.8 Review: Earning money

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Earning money

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Fractions, decimals and percentages



Support sheet: Calculating fractions, decimals and percentages of quantities



Support sheet: Converting units of time



Support sheet: Substitution



Support sheet: Solving equations using inverse operations



Support sheet: Reading and interpreting tables and graphs

Syllabus links

- Earning money (MST-11-03)
 - A student solves financial problems involving earning money and taxation.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 3.1

Earning an income

Learning intentions

By the end of this lesson, you will be able to...

→ solve problems involving salaries, wages and earnings.

Income, wages and salaries

- Income refers to any money that a person receives from any source. Everyone needs some form of income to satisfy their basic needs.
- While some income is passive, such as interest paid on investments, the majority of most people's income is earned through working at a job. Even then, there are many different ways that people receive money for their work.
- Most people work for an employer who pays them either a **wage** or a **salary**. A wage is a form of income that depends on the number of hours worked, whereas a salary is a fixed amount of income earned regardless of any fluctuation in the number of hours worked.
- Salaries are usually specified as an amount **per annum (p.a.)** or per year; however, this amount is divided into smaller payments made at regular intervals throughout the year (for example, weekly, fortnightly or monthly). People who are typically paid salaries include nurses, government employees, teachers and other professionals.
- Wages are set via an hourly rate and are usually paid on a weekly or fortnightly basis, for the hours worked in that period. People who are typically paid wages include factory workers, shop assistants, building workers and mechanics.
 - When converting from yearly salaries to monthly, fortnightly and weekly payment amounts, the following conversions are used:
 $1 \text{ year} = 12 \text{ months} = 26 \text{ fortnights} = 52 \text{ weeks} = 365 \text{ days}$
- Employees may be employed on a permanent/ongoing contract or as a casual staff member. They can work either a full-time or part-time load. Permanent employees receive benefits such as sick leave and holiday pay.

Worked example 3.1A Calculating salary payments

Jenny works as a pastry chef. Her salary is \$72 064 per annum. Determine the amount she would receive in each payment if her salary was paid:

- a** per month
- b** per week
- c** per fortnight.

Part	Think	Write
a	There are 12 months in a year. Divide the annual income by 12. Write the result correct to the nearest cent.	Monthly payment = $\$72\,064 \div 12$ = \$6005.33
b	There are 52 weeks in a year. Divide the annual income by 52. Write the result correct to the nearest cent.	Weekly payment = $\$72\,064 \div 52$ = \$1385.85
c	There are 26 fortnights in a year. Divide the annual income by 26. Write the result correct to the nearest cent.	Fortnightly payment = $\$72\,064 \div 26$ = \$2771.69



Worked example video: Calculating salary payments

Understanding, fluency and communicating

Answers: Lesson 3.1

Helpful hints

- To break an annual salary down into the smaller amounts to be paid at regular intervals throughout the year, divide the annual figure by:
 - 12 intervals if the salary is paid monthly
 - 52 intervals if the salary is paid weekly
 - 26 intervals if the salary is paid fortnightly.
- When working with money, write results correct to the nearest cent.

- Amy earns \$19 884.80 per annum for part-time work. Complete the following to find how much she earns over each interval.
 - Monthly income = $\_\div 12$
= $_\$
 - Weekly income = $\$19\,884.80 \div _\$
= $_\$
 - Fortnightly income = $_\div _\$
= $_\$
- Caterina earns \$86 924 per annum. Calculate her weekly pay.
- Express each of the following annual salaries as an amount:
 - per month
 - per week
 - per fortnight.

a \$38 903	b \$27 106
c \$30 098	d \$109 870
e \$75 076	f \$50 990



Worked example 3.1B Calculating annual and monthly salary payments

Michiko works part-time and earns \$283 per week.

- a** How much does Michiko earn per year?
b How much does Michiko earn per month?

Part	Think	Write
a	There are 52 weeks in a year. Multiply the weekly wage by 52.	Yearly earnings = $\$283 \times 52$ = \$14 716
b	There are 12 months in a year. Divide the annual amount by 12. One month is <i>not</i> 4 weeks, because $12 \times 4 = 48$ weeks, not 52 weeks.	Monthly earnings = $\$14\,716 \div 12$ = \$1226.33



Worked example video: Calculating annual and monthly salary payments

Helpful hints

- To find monthly income based on weekly or fortnightly income, always convert to the annual amount first (by multiplying by 52 for weekly, or 26 for fortnightly) then divide by 12.
- Don't be tempted to use 1 month = 4 weeks or even 1 month = 4.5 weeks. Since 12 months = 52 weeks the correct conversion is:
 $1 \text{ month} = \frac{52}{12} \text{ weeks} = 4\frac{1}{3} \text{ weeks}.$

- 4** Jason earns \$421 per week for weekend work. Complete the following to find:

Jason's annual income = $\$421 \times \underline{\hspace{1cm}}$
 = $\underline{\hspace{1cm}}$

Jason's monthly income = $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}}$
 = $\underline{\hspace{1cm}}$

- 5** Ashley earns \$379 per week for part-time work. How much does Ashley earn:

- a** per year? **b** per month?

- 6** Calculate the annual salary amount for each of these workers.

Name	Weekly wage	Annual salary
C. Cooke	\$1447.94	
D. Moussa	\$1621.06	
B. Boje	\$1230.54	
T. Lambert	\$1170.38	

- 7** Calculate the earnings of the following people.

- a** David works part-time and receives an annual salary of \$41 398.45. How much does he earn each week?
b Debra works part-time and receives a monthly salary of \$2933. How much does she earn each week?
c Ali works part-time and receives an annual salary of \$46 837.25. How much do they earn each fortnight?
d Kristy works full-time and receives an annual salary of \$78 500. How much does she earn each fortnight?
e Iain works part-time and receives a monthly salary of \$3599.60. How much does he earn each fortnight?
f Ngoc works full-time and receives an annual salary of \$74 292. How much does she earn each month?

- 8 Convert each of the following to an annual salary, if necessary, and calculate the difference between the annual salaries.
- Kylie: \$1136.60 a week and Stacey: \$54 642 a year
 - Roland: \$1803 a fortnight and Paul: \$55 271 a year
 - Binh: \$2610 a month and Aaron: \$693.20 a week

Worked example 3.1C Calculating wage payments

Monica is paid \$21.65 per hour for casual work. How much does she earn if she works:

- six hours in one day?
- six hours per day for five days?

Part	Think	Write
a	To find the amount earned, multiply the hourly rate, (\$21.65) by the number of hours (6).	One day's pay = $\$21.65 \times 6$ = \$129.90
b	Multiply the daily rate ($\$21.65 \times 6$) by the number of days (5).	Five days' pay = $\$21.65 \times 6 \times 5$ = \$649.50



Worked example video: Calculating wage payments

Helpful hints

- If an income is stated as an hourly rate, the total income for a given period of time is found by multiplying the hourly rate by the number of hours worked.

- 9 Clare works in a jewellery store earning \$28.45 per hour. Complete the following to find the amount Clare earns:

a in an 8-hour day
Pay = $\_\_ \times 8 = \_\_$

b for a 38-hour week
Pay = $\_\_ \times \_\_ = \_\_$

- 10 Aaron works as an assistant in a clothing store and is paid \$21.75 an hour for a 35-hour week. Calculate his weekly wage.

- 11 Madonna works as a receptionist. If they are paid \$30.90 an hour, how much do they earn in a 38-hour week?

- 12 David works at Pizza Barn delivering pizzas. Calculate his weekly wage if he earns \$27.20 an hour and worked the following number of hours in a given week.

- a 16 h b 24 h c 35 h d 41 h

- 13 Kristie is a junior and earns \$19.43 an hour. Calculate her weekly wage if she works:

- a 18 h b 35 h c 40 h d 48 h.



Worked example 3.1D Calculating annual income

Angus works a 35-hour week and is paid \$32.80 per hour. How much does he earn in a year?

Think	Write
Find the pay per week by multiplying \$32.80 by 35, then multiply by 52 to find the annual income.	$\begin{aligned}\text{Pay per week} &= \$32.80 \times 35 \\ &= \$1148 \\ \text{Annual income} &= \$1148 \times 52 \\ &= \$59\,696 \\ \text{Angus earns } &\$59\,696 \text{ per year.}\end{aligned}$



Worked example video: Calculating annual income

Helpful hints

- Hourly income is the basic building block that can be used to calculate income over any time period using multiplication.

14 Taylor earns \$31.60 per hour for a 38-hour week. Complete to find her weekly pay and her annual income.

$$\begin{aligned}\text{Pay per week} &= \$31.60 \times \_\_ \\ &= \_\_\end{aligned}$$

$$\begin{aligned}\text{Annual income} &= \_\_ \times \_\_ \\ &= \_\_\end{aligned}$$

15 An electrician works a 38-hour week. Find her yearly pay if she earns \$58.40 per hour.

16 Ikbar works a 38-hour week. His hourly rate is \$42.27. Find his pay for one year.

Worked example 3.1E Converting annual salaries to shorter time periods

Convert an annual salary of \$77 424 for a worker who works 35 hours each week to:

a a weekly payment

b an hourly rate.

Part	Think	Write
a	Divide the annual salary of \$77 424 by 52 to convert to a weekly payment.	$\begin{aligned}\text{Weekly payment} &= \$77\,424 \div 52 \\ &= \$1488.92\end{aligned}$
b	Next, divide the weekly payment (\$1488.92) by 35 to find the hourly rate.	$\begin{aligned}\text{Hourly rate} &= \$1488.92 \div 35 \\ &= \$42.54\end{aligned}$



Worked example video: Converting annual salaries to shorter time periods

Helpful hints

- Converting annual salaries to shorter time periods requires division. Divide by 52 for weekly or by 12 for monthly incomes.
- If an answer is in dollars, round to two decimal places (unless otherwise specified).

- 17 Jake works 35 hours per week for an annual income of \$54 527.20. Complete to find his weekly pay and his hourly rate.

a Weekly pay = $\frac{\text{Annual income}}{52}$

= $\frac{\text{Annual income}}{52}$

b Hourly rate = $\frac{\text{Weekly pay}}{35}$

= $\frac{\text{Weekly pay}}{35}$

- 18 Indoo receives an annual salary of \$63 300. Convert this to a weekly salary payment and find Indoo's average hourly rate if she works 52 hours per week.
- 19 Guido works seven hours a day, five days per week. Find his hourly rate if his annual income is \$68 900.

Problem solving and reasoning

- 20 Rodney and Anastasia both work at a shoe factory for an hourly rate of pay of \$36.80, including the time taken for their meal breaks.

- i Calculate the total number of hours worked in one week by each employee.
- ii Calculate each employee's weekly wage.

a Name: Rodney

Day	In	Out	Hours
Monday	7:00 am	4:00 pm	
Tuesday	8:00 am	4:00 pm	
Wednesday	7:00 am	5:00 pm	
Thursday	11:00 am	4:00 pm	
Friday	9:30 am	2:30 pm	
		Total	

b Name: Anastasia

Day	In	Out	Hours
Monday	7:40 am	5:40 pm	
Tuesday	10:20 am	4:20 pm	
Wednesday	9:00 am	3:00 pm	
Thursday	8:30 am	3:30 pm	
Friday	8:35 am	3:35 pm	
		Total	

- 21 Complete the following wage table.

Name	Day					Hours worked	Hourly rate	Weekly rate
	Mon	Tues	Wed	Thurs	Fri			
S. Smith	7	5	–	$4\frac{1}{2}$	9		\$39.40	
D. De Souza	11	4	$3\frac{1}{2}$	$6\frac{1}{2}$	7		\$26.30	
A. Bentley	$6\frac{1}{2}$	–	–	3	$9\frac{1}{2}$			\$801.80
C. Johnson	4	$2\frac{1}{2}$	7	2	$1\frac{1}{2}$		\$30.34	
K. Kernell	3	$7\frac{1}{2}$	5	$4\frac{1}{2}$	–			\$574.80
P. Patrick	$5\frac{1}{2}$	9	$2\frac{1}{2}$	–	$6\frac{1}{2}$		\$29.83	

- 22** Jim is a junior and works 26 days per month. He averages seven hours per day. Find his annual income if his hourly rate is \$18.17.
- 23** Nato is paid \$35.20 per hour. He works four hours per day, seven days a week. Find his pay for one year.
- 24** Isla is paid \$1710 per fortnight.

a Find:

i her weekly pay

ii her monthly pay.

b If Isla works a 6-hour shift five days a week, what is her hourly rate?

- 25** James is a junior and works as a casual employee at the local supermarket. They are paid \$16.05 an hour from Monday to Friday and \$20.35 an hour on weekends. During a week of their school holidays, they worked from 3:00 pm until 6:00 pm Monday to Friday and 8:30 am until 1:30 pm on Saturday and Sunday.

- a** How many weekday hours did they work?
- b** How many weekend hours did they work?
- c** Calculate their income for the week.



- 26** Steve is paid a salary of \$78 904 per annum.

- a** Calculate Steve's weekly pay rate.
- b** What is his hourly rate if he works 45 hours per week?

- 27** Which pay option offers the higher weekly wage? By how much?

- a** Option A: \$903.20 a week or Option B: \$45 103.20 a year
- b** Option A: \$953.29 a fortnight or Option B: \$2405 a month
- c** Option A: \$3276 a month or Option B: \$47 024 a year

- 28** Armin and Richard were comparing their pays. Richard said that he earns more per week than Armin. Richard earns \$43 800 per annum, which he said is \$912.50 per week. Armin earns \$880 per week.

The yearly pay for Richard is correct, but he has made an error in calculating his weekly pay.

- a** Find Armin's annual income.
- b** Find Richard's correct weekly pay.
- c** Who really earns more?
- d** Richard said that he divided the annual income by 12 to find his monthly pay and then by 4 to find his weekly pay. Explain the mistake that Richard made.



Challenge

- 29** Lenora currently earns an hourly rate of \$35 and works on average 37 hours per week. Her employer has offered for her to earn a salary of \$60 000 instead.
- a** If Lenora works 48 weeks of the year, is she financially better off to earn the salary offered?
- b** Are there reasons why she should take the \$60 000 salary even if it is financially worse?

Related resources



3.1 Quick quiz



Worksheet: Salaries

Lesson 3.2

Allowances, bonuses and loadings

Learning intentions

By the end of this lesson, you will be able to...

→ calculate earnings from allowances, bonuses and loadings.

Different rates and extra payments

- Most employees work under an **award** that states the maximum number of normal work hours they should work per day or per week, and also their normal working days. Hours worked in excess of these (including weekend, late night and public holiday work) are considered **overtime** and are paid at a higher hourly rate called a **penalty rate**. The two most common penalty rates are described as **time-and-a-half** and **double-time**.
- Employees who are required to work under difficult, unpleasant or dangerous conditions are often paid an **allowance** that is above the normal rate of pay. Some examples include an allowance for cleaning toilets, working in confined spaces, working in hot or very cold conditions, or dealing with toxic substances.
- Allowances may also be paid for uniforms, dry-cleaning of uniforms, travelling, meals, tools and so on.
- Another kind of extra payment an employee may receive is called a **bonus**. These are sometimes used as an incentive to encourage employees to work harder.
- In Australia, full-time employees receive four weeks of annual leave per year. **Annual leave loading** is an increase in an employee's pay while on holidays. The holiday loading is a fixed percentage of the employee's normal pay. It is usually paid at a rate of $17\frac{1}{2}\%$.

Worked example 3.2A Calculating overtime penalty rates

For an hourly rate of \$26.20, determine an overtime penalty rate of:

a time-and-a-half

b double-time.

Part	Think	Write
a	Multiply the hourly rate of \$26.20 by the overtime loading of 1.5. This is the overtime penalty rate.	Time-and-a-half rate = $\$26.20 \times 1.5$ = \$39.30
b	Multiply the hourly rate of \$26.20 by the overtime loading of 2. This is the overtime penalty rate.	Double-time rate = $\$26.20 \times 2$ = \$52.40



Worked example video: Calculating overtime penalty rates

Understanding, fluency and communicating

Answers: Lesson 3.2

Helpful hints

- To calculate the overtime penalty rate, multiply the normal hourly rate by the overtime factor. For time-and-a-half the factor is 1.5, and for double-time the factor is 2.

1 Jasmine earns \$22.24 per hour. Copy and complete the following to find her overtime penalty rate at:

a time-and-a-half

$$\begin{aligned}\text{Overtime penalty rate} &= \_\_ \times 1.\_\_ \\ &= \_\_\end{aligned}$$

b double-time.

$$\begin{aligned}\text{Overtime penalty rate} &= \_\_ \times \_\_ \\ &= \_\_\end{aligned}$$

2 Convert these normal hourly pay rates to overtime penalty rates at:

a time-and-a-half

b double-time.

i \$16.83

ii \$19.35

iii \$26.59

iv \$28.70

Worked example 3.2B Calculating wages using overtime penalty rates

Janice is a junior and works 10 hours on a Sunday. The Sunday rate of pay is \$16.80 per hour for the first 4 hours, then “time-and-a-half” for additional hours worked as overtime. Calculate Janice’s wage for Sunday.

Think

The first 4 hours are at the normal rate, so \$16.80 is multiplied by 4.
The next 6 hours are at time-and-a-half.
Multiply 6 by 1.5 and \$16.80 to calculate the overtime pay.
Add \$67.20 and \$151.20 to find the total earned.

Write

Number of hours worked is 10.
This is 4 hours at normal rate and 6 hours at time-and-a-half.
Wage at normal rate = $4 \times \$16.80$
 $= \$67.20$
Wage at time-and-a-half = $6 \times 1.5 \times \$16.80$
 $= \$151.20$
Janice’s wage = $\$67.20 + \$151.20$
 $= \$218.40$



Worked example video: Calculating wages using overtime penalty rates

Helpful hints

- The normal hours are multiplied by the hourly rate. The overtime hours are multiplied by the hourly rate as well as by the overtime factor. The total is found by adding the normal and overtime totals together.

- 3 Brian is a junior and is paid \$18.42 per hour. Calculate his weekly wage if he works 38 hours at the normal rate and 6 hours overtime at time-and-a-half.
- 4 Karen earns \$27.70 an hour. Calculate her weekly wage if she worked 33 hours at the normal rate and 6 hours at time-and-a-half.
- 5 Vikash is a junior and is paid \$17.24 per hour for a 35-hour week. He is paid overtime at double-time for the hours he works in excess of 35 hours. Calculate his pay in a week in which he works 44 hours.
- 6 Anna is a junior and earns \$18.35 per hour for the first 30 hours in a week. She receives time-and-a-half for the next 5 hours and double-time after that. Find her pay for a 42-hour week.
- 7 Alexia works 16 hours at \$20.20, 4 hours at time-and-a-half, and 3 hours at double-time. Calculate her weekly wage.



Worked example 3.2C Calculating weekly pay with an allowance

Vicki earns \$29.45 per hour as a cleaner. If she cleans the toilets, she receives an allowance of \$24.00 per day. Find her weekly pay if she works for 4 hours each weekday and cleans the toilets on Wednesday and Friday.

Think	Write
Normal weekly pay is \$29.45 multiplied by 4 hours multiplied by 5 days. She cleans toilets on 2 days. The allowance is then multiplied by 2, giving \$48. Add the normal pay and allowance to get the total pay.	Normal pay = $\$29.45 \times 4 \times 5$ = \$589 Toilet allowance = $\$24 \times 2$ = \$48 Total pay = $\$589 + \$48$ = \$637

 **Worked example video:** Calculating weekly pay with an allowance

Helpful hints

- Allowances are added to normal income. First, calculate the normal pay. Next, calculate the amounts of each allowance. Add the allowance pay to the normal pay to find the total income.

- 8 Jovan earns \$34.20 per hour for an eight-hour day. He is paid a supervisor allowance of \$30 whenever he is the most senior person on the shift. Complete the following to find his pay for a week in which he works five days and is supervisor for three of the days.
- a Normal pay = $\$34.20 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $\hspace{1.5cm} = \underline{\hspace{1cm}}$
- b Supervisor allowance = $\$30 \times \underline{\hspace{1cm}}$
 $\hspace{1.5cm} = \underline{\hspace{1cm}}$
- c Total pay = $\underline{\hspace{1cm}} + \underline{\hspace{1cm}}$
 $\hspace{1.5cm} = \underline{\hspace{1cm}}$
- 9 Rose is paid \$24.70 per hour welding bins. If the temperature is over 30°C, she is paid an extra \$18 per day. Find her pay for a 35-hour week in which there are two days over 30°C.
- 10 Harry works in a timber yard and is paid \$27.40 per hour. He receives a dust allowance of \$12.50 per day. Find his pay for a 5-day, 40-hour week.
- 11 Georgina is a junior who works in a restaurant and receives a uniform allowance of \$11.50 per night. Her hourly rate of pay is \$22.85. Find her pay if she works five nights for six hours per night.

Worked example 3.2D Calculating the value of a bonus

Renee worked extremely hard throughout the year. Her reward was a bonus of 6% of her yearly income. If her normal yearly income was \$68 000, calculate the value of her bonus.

Think	Write
6% is $\frac{6}{100}$. Multiply this by \$68 000 to get \$4080.	Bonus = 6% of \$68 000 $= \frac{6}{100} \times \$68\,000$ $= \$4080$



Worked example video: Calculating the value of a bonus

Helpful hints

- If the bonus is a percentage of annual salary, then find its value by multiplying the salary by the percentage (as a fraction of 100).

- 12 Jeremy receives a bonus of 3% of his annual salary of \$43 600. Complete to find the value of his bonus:

$$\begin{aligned}\text{Bonus} &= \underline{\hspace{1cm}}\% \text{ of } \$43\,600 \\ &= \frac{3}{100} \times \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}}\end{aligned}$$

- 13 Determine the bonus for each of these annual salaries.

- a 5% of \$53 000 b 8% of \$92 500 c 9% of \$123 468
d 10% of \$196 320 e 10% of \$253 452 f 15% of \$271 096

- 14 Zofia earns \$590 per week. What is their total weekly wage in a week that they are awarded the following bonuses?

- a 5% b 12% c 20% d 14%

Worked example 3.2E Calculating holiday pay with a leave loading

Lee is a permanent part-time employee and is entitled to four weeks annual leave with an annual leave loading of $17\frac{1}{2}\%$. If his weekly pay is \$568, find his holiday pay.

Think	Write
Calculate 4 weeks pay by multiplying \$568 by 4. Find $17\frac{1}{2}\%$ of \$2272. Add the 4 weeks pay and the holiday loading to get the holiday pay.	$\begin{aligned} \text{4 weeks pay} &= \$568 \times 4 \\ &= \$2272 \\ \text{Loading} &= 17\frac{1}{2}\% \text{ of } \$2272 \\ &= \left(17\frac{1}{2} \div 100\right) \times \$2272 \\ &= \$397.60 \\ \text{Holiday pay} &= \$2272 + \$397.60 \\ &= \$2669.60 \end{aligned}$



Worked example video: Calculating holiday pay with a leave loading

Helpful hints

- First find four weeks pay; the holiday loading is $17\frac{1}{2}\%$ of that amount.
- Total holiday pay is the normal pay for four weeks plus the holiday loading.
- Total holiday pay can also be calculated by multiplying the normal four weeks pay by $117\frac{1}{2}\%$.

- 15** Caleb earns \$673 per week and is paid a holiday loading of $17\frac{1}{2}\%$ on four weeks pay. Copy and complete the following to find his holiday pay.

$$\begin{aligned} \text{4 weeks pay} &= \$673 \times \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}} \\ \text{Loading} &= \underline{\hspace{1cm}}\% \text{ of } \$2692 \\ &= \frac{17.5}{100} \times \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}} \\ \text{Holiday pay} &= \$2692 + \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}} \end{aligned}$$

- 16** Allie has permanent part-time work as a personal assistant and is paid \$538 per week. If her four weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, find her holiday pay.
- 17** The holiday loading is $17\frac{1}{2}\%$. Find the four weeks holiday pay for employees who earn the following:
- a weekly wage of \$811
 - a fortnightly wage of \$1054
 - an hourly rate of \$28.45 for a 38-hour week.
- 18** If $17\frac{1}{2}\%$ holiday loading is paid on four weeks normal pay, find the holiday pay for:
- John, who earns \$2700 in four weeks
 - Divit, who earns \$1220 in a fortnight
 - Sid, who earns \$430 in a week
 - Riya, who earns \$32.50 an hour for a 30-hour week
 - Angie, who earns \$35.65 an hour for a 35-hour week.

- 19** Ximena was paid 4 weeks holiday pay plus $17\frac{1}{2}\%$ holiday loading. If her normal weekly pay is \$637.52, calculate:
- her normal pay for four weeks
 - her holiday loading for four weeks
 - her total holiday pay.
- 20** Paul earns \$28.60 an hour and works a 38-hour week. Calculate his holiday loading if he is paid $17\frac{1}{2}\%$ of four weeks wages.



Problem solving and reasoning

- 21** Shane is paid \$21.30 per hour as a labourer. His normal working week is 35 hours. He is paid time-and-a-half for the first five hours worked beyond 35 hours and double-time for any hours after that. Calculate Shane's weekly wage if he worked 49 hours.
- 22** Jontey works from 8:00 am to 4:00 pm on Sundays. He is paid time-and-a-half for the morning work and double-time for the afternoon.
- If Jontey's normal hourly rate is \$26.90, calculate his Sunday earnings.
 - Jontey's employer is removing overtime penalty rates and changing the normal rate to \$42 per hour. Is Jontey better off with the change? Explain.
 - What hourly rate would be required for Jontey to receive the same pay for a Sunday?
- 23** This wage table shows the hours worked by the junior employees at Active Sports. Calculate the weekly wage for each employee if:
- the normal hourly rate is \$19.40
 - overtime of time-and-a-half is paid for hours worked beyond four hours from Monday to Friday
 - time-and-a-half is the hourly rate on Saturdays.

	Employee	Day						Normal time (h)	Overtime (h) time-and-a-half	Weekly wage
		Mon	Tue	Wed	Thu	Fri	Sat			
a	M. Goodwyn	8	$8\frac{1}{2}$	8	9	4	6			
b	J. Osbourne	9	9	9	7	8	$3\frac{1}{2}$			
c	G. Dale	10	$9\frac{1}{2}$	9	$9\frac{1}{2}$	10	2			
d	E. Evans	$4\frac{1}{2}$	—	—	10	9	9			
e	M. Harkin	7	6	$4\frac{1}{2}$	5	2	$7\frac{1}{2}$			
f	A. Fenech	10	9	8	9	10	5			

- 24** Raymond earns \$96 325 per year. At the end of the year he is given a bonus of 13% of his yearly wage. Calculate his bonus.
- 25** Louie works as a plumber's apprentice and is paid \$13.80 per hour. He is paid a digging allowance of \$13.50 per day if he has to dig trenches and \$15.80 per day if he works on open sewers. Find his pay in a week in which he works 50 hours, digs on three days and works on an open sewer for two days.
- 26** Benny is a security guard and is paid \$24.25 per hour. If he works on the door he is paid \$25 per night danger pay. Find Benny's wage for a week in which he is on the door for three nights and works 35 hours.

- 27 Nabil works in a factory and is paid \$25.54 per hour. If he works the furnace he is paid a heat allowance of \$5.27 per hour in addition to his normal rate. Find his weekly pay if he works a total of 32 hours of which eight hours are spent working the furnace.
- 28 Jenny works in a bar and is paid \$23.90 per hour. If she makes cocktails she receives an extra \$3.20 per hour. Find her pay for a week in which she works 20 normal hours and a further 10 hours making cocktails.
- 29 Slavo earns \$35 600 per year. At the end of the year his company made a profit of \$129 561. As a reward, Slavo was given a bonus of 1% of the profits.
- What was Slavo's bonus?
 - What was his total income for the year?
- 30 Calculate the normal hourly pay rate for the following two workers.
- Casey works six hours on Saturday at time-and-a-half and is paid \$234.
 - Corey works eight hours on Saturday at time-and-a-half and five hours on Sunday at double-time and is paid \$539.
- 31 Samantha received a $17\frac{1}{2}\%$ holiday loading on four weeks normal wages. She normally works a 35-hour week. Her four weeks holiday pay and loading is \$2533.30.
- Find her normal weekly pay.
 - Find her normal hourly pay rate.

Challenge

- 32 Oscar works at a retail store. He receives the following overtime payments:

- Time-and-a-half for each hour worked over 37 hours in a week
- Double-time for all hours worked on Sunday

He works the following hours in a week:

Day	Start Time	Finish Time
Monday	9 : 00 am	5 : 00 pm
Tuesday	9 : 00 am	5 : 00 pm
Thursday	10 : 00 am	8 : 00 pm
Friday	11 : 00 am	5 : 00 pm
Saturday	9 : 00 am	4 : 00 pm
Sunday	11 : 00 am	4 : 00 pm

If he makes \$1721 this week, what is his normal hourly rate?

Related resources



3.2 Quick quiz



Worksheet: Wages and penalty rates



Worksheet: Leave loading

Lesson 3.3

Commission, piecework and royalties

Learning intentions

By the end of this lesson, you will be able to...

- calculate earnings from commission, piecework and royalties.

Other ways of being paid for work

- The following are some other ways that people might be paid for their work.
 - In industries such as real estate and car sales, employees are often paid by **commission**. This means they are paid a percentage of the total cost of the goods they sell, so the more they sell, the greater their income.
 - A fixed commission means the same commission rate is paid on all sales income, regardless of the total sales, while a sliding commission means different rates are applied, depending on the total sales a person makes.
 - A person who works on commission may also be paid a **retainer**, which is a set amount of money paid similarly to a salary or a wage.
 - It is common for authors and people in similar professions to be paid a **royalty**. This income is determined in the same way as a commission; they receive a percentage of the sale price for each book they have written.
 - People employed to create or produce individual items may be paid a fixed amount for each item produced. This is called **piecework**.
- Not everyone works for a specific employer. Some people run their own small business or are otherwise self-employed. Others may work as a contractor or freelancer, offering their services to different companies at different times. In Australia, an individual running their own business is typically considered a **sole trader**.

Worked example 3.3A Calculating commissions and royalties

Larry sold a new car valued at \$22 800. His commission for selling the car was 6% of the selling price. Calculate Larry's commission.

Think	Write
Larry sold goods worth \$22 800. His commission is 6%, so find 6% of \$22 800.	$\begin{aligned}\text{Commission} &= 6\% \text{ of } \$22\,800 \\ &= \frac{6}{100} \times \$22\,800 \\ &= \$1368 \\ \text{Larry earned } &\$1368 \text{ from his sale.}\end{aligned}$



Worked example video: Calculating commissions and royalties

Understanding, fluency and communicating

Answers: Lesson 3.3

Helpful hints

- Commission is usually a percentage of the value of sales. Calculating the percentage of the amount sold finds the commission.
- Royalties are paid in the same way as a commission. Find the total of a royalty payment by multiplying the royalty percentage by the total value of sales.

- 1 Holly holds a make-up party and sells \$1350 of make-up to her friends. Her commission is 7% of sales. Complete the following to find the amount earned.

$$\begin{aligned}\text{Commission amount} &= \_\% \text{ of } \$1350 \\ &= \_\times \_\ \\ &= \_\end{aligned}$$

- 2 Find the amount of commission earned for each of these sales amounts. Answer to the nearest dollar.

	Commission rate	Sales amount
a	25%	\$42 000
b	18%	\$6782
c	7%	\$840
d	$15\frac{1}{2}\%$	\$132
e	2.7%	\$80 000
f	$17\frac{1}{3}\%$	\$80 000



- 3 Darren sold a house valued at \$650 000. Calculate Darren's commission from the sale if his rate of commission is 4.8%.
- 4 Josephine sold a car valued at \$48 000. Calculate her commission if the rate of commission is 7%.
- 5 An investment advisor receives 6% commission on money invested. How much does he earn for advising on a \$753 000 investment?
- 6 Paula is paid a royalty of 12.3% on all sales of a product she helped to design. Complete the following to find her royalty on sales of \$68 000.

$$\begin{aligned}\text{Royalty} &= \_\% \text{ of } \$68\,000 \\ &= \frac{12.3}{100} \times \_\ \\ &= \_\end{aligned}$$

- 7 Calculate the royalties for these authors.

	Author	Royalty rate	Sales
a	M. Golovchenko	8.7%	\$43 590
b	N. Smythe	7.3%	\$367 098
c	R. Ireland	12.9%	\$23 098
d	P. Goodacre	10.5%	\$358 900

Worked example 3.3B Calculating total earnings including commission

A company pays new employees a retainer of \$800 per week plus 15% commission on sales. After passing probation, they no longer receive a retainer and are instead paid a sliding scale commission according to the table shown.

Total sales	Commission rate
<\$5000	20%
\$5000–\$7500	25%
>\$7500	30%

Find Sampson's income for three different weeks in his few months at the company.

- a** Week 1 (during probation): total sales = \$1200
- b** Week 6 (during probation): total sales = \$6000
- c** Week 12 (passed probation): total sales = \$6300

Part	Think	Write
a	Sampson's total earnings consist of the 15% commission on his \$1200 of sales, plus the \$800 retainer.	$\begin{aligned}\text{Total earnings} &= \text{commission} + \text{retainer} \\ &= \frac{15}{100} \times \$1200 + \$800 \\ &= \$180 + \$800 \\ &= \$980\end{aligned}$
b	We can use the same process as part a , but this time with total sales being \$6000.	$\begin{aligned}\text{Total earnings} &= \text{commission} + \text{retainer} \\ &= \frac{15}{100} \times \$6000 + \$800 \\ &= \$900 + \$800 \\ &= \$1700\end{aligned}$
c	Having passed probation, Sampson is now being paid on a sliding scale commission. This means his earnings all come from commission, and since his total sales were \$6300, the applicable commission rate is 25%.	$\begin{aligned}\text{Total earnings} &= \frac{25}{100} \times \$6300 \\ &= \$1575\end{aligned}$



Worked example video: Calculating total earnings including commission

Helpful hints

- Some salespeople are paid a retainer plus commission. The retainer is an amount paid even if there are no sales. It is independent of any sales. The amount of commission earned from sales is added to the retainer to calculate the total income.
- Some salespeople are paid a sliding commission. The total of a person's sales must be calculated before determining the applicable commission rate and calculating the commission to be paid.

- 12** Emily earns \$350 commission on sales of \$3723. Copy and complete the following to find her percentage rate of commission to one decimal place.

$$\begin{aligned}\text{Rate of commission} &= \frac{\text{commission earned}}{\text{total sale value}} \times 100\% \\ &= \frac{\square}{\$3723} \times 100\% \\ &\approx \underline{\hspace{1cm}}\%\end{aligned}$$

- 13** A car salesperson earned \$7853.20 commission for one week. If his total sales were \$54 160, calculate the rate of commission as a percentage.
- 14** Determine the rate of commission as a percentage if Sue earned \$9348 on sales valued at \$76 000.
- 15** Copy and complete the following table.

	Employee	Rate of commission	Total sales	Commission earned
a	E. Swift	4.4%	\$72 000	
b	M. Keane		\$115 872	\$4634.88
c	S. Wise		\$64 500	\$9481.50
d	G. Walsh	8.2%	\$183 600	
e	G. Potts		\$286 472	\$16 472.14
f	L. Monroe	$12\frac{1}{3}\%$	\$350 700	

- 16** In one week, a used car salesman sold three Ford Focus cars each priced at \$25 360 and earned $4\frac{1}{2}\%$ commission on his total sales. Find his commission for the week.
- 17** A real-estate agent sold a house valued at \$576 000. If she received $2\frac{1}{2}\%$ commission on the first \$150 000 and $\frac{1}{2}\%$ commission on the remainder, find her total commission.
- 18** Rhonda sells paintings. She is paid 3% of the first \$5000 of sales, 2% on the next \$5000 and 1% thereafter. Find her commission on sales of:
- a** \$4500 **b** \$6725 **c** \$9422 **d** \$14 680
- 19** Reginaldo is paid commission of 5% on the first \$1000 of sales, 4% on the next \$3000 and 1% thereafter. Find his commission on sales of:
- a** \$5467 **b** \$4321 **c** \$560 **d** \$9800

Worked example 3.3D Calculating pay for piecework

- a** Enrique paints wooden fire engines. He is paid \$4.38 per fire engine painted. In one week he paints 143 fire engines. How much is he paid?
- b** Antonia makes dolls which she sells at a craft market. It takes Antonia five hours to make a doll and she sells each doll for \$36. What is Antonia's hourly rate?
- c** Ramona produces hankies in a factory and is paid on a piecework basis at the rate of 39 cents per article produced, up to 200 items per day, and 53 cents per article for each piece produced in excess of 200 items per day. Calculate Ramona's income for a day that she produces 224 hankies.

Part	Thinks	Write
a	Multiply \$4.38 by 143.	Pay = $\$4.38 \times 143$ = \$626.34
b	Divide \$36 by 5 to get the hourly rate.	Hourly rate = $\frac{\$36}{5}$ = \$7.20

Part	Thinks	Write
c	Ramona makes more than 200 hankies so we split 224 into $200 + 24$. The first 200 are multiplied by 39 cents and the remainder is multiplied by 53 cents. The total is found by adding these together.	Day's income = $200 \times \\$0.39 + 24 \times \\$0.53$ $= \\$90.72$



Worked example video: Calculating pay for piecework

Helpful hints

- To find total earnings from piecework, multiply the amount per item by the number of items.
- An hourly rate is found by dividing the amount paid per item by the time spent on each item in hours.
- Where more than one rate is paid, find the point where the rate changes. Split the number into the parts required and multiply each part by the correct rate before adding the results together.

- 20 Angelina paints Christmas decorations and is paid \$1.80 per decoration. Complete the following to find her pay for 300 decorations.

$$\text{Pay} = \_ \times 300 = \_$$

- 21 Austin spray paints wooden toys and is paid \$6.90 per toy. Find his pay if he paints 168 toys.

- 22 Shania assembles skateboards. She is paid \$15.00 per assembled board. How much does she earn for a week in which she assembles 43 skateboards?

- 23 Ricky waxes surfboards for the surf hire shop. He is paid \$6.70 per waxed board. Find his pay for a day in which he waxes 15 boards.

- 24 Ellen works in a belt factory. Each belt completed earns her \$4.20. Find her wage for a week in which she makes 157 belts.

- 25 Roberto makes cricket bats and is paid \$130 per bat. In a 35-hour week he makes eight bats. Complete the following to find his weekly pay and hourly rate:

a Weekly pay = $\_ \times 8$
 $= \_$

b Hourly rate = $\frac{\text{weekly pay}}{\text{hours worked}}$
 $= \frac{\square}{35 \text{ h}}$
 $= \$\_/h$

- 26 Ivan is a sole trader who re-felts billiard tables for \$290 per completed table. If he completes eight tables in a 35-hour week, find his:

a weekly pay

b hourly rate.

- 27 Allan strings new tennis racquets and is paid \$7.50 per racquet. In a 40-hour week Allan strings 120 racquets. Find his:

a weekly pay

b hourly rate.



- 28** Stephanie runs a small home business making savouries and charges 80 cents per savoury for the first 50 per day and 95 cents for each savoury in excess of 50. On Monday she makes 45 savouries and on Tuesday she makes 110. Complete the following to find her total takings for the two days.

$$\begin{aligned}\text{Monday} &= \_ \times \$0.80 = \_ \\ \text{Tuesday} &= 50 \times \_ + \_ \times \$0.95 = \_ \\ \text{Total takings} &= \_ + \_ = \_ \end{aligned}$$



- 29** Calculate the weekly wage of the following employees who are paid on a piecework basis at the rate of 57 cents for each piece up to 200 pieces per day and then 65 cents for each piece produced in excess of 200 pieces.

	Name	Monday	Tuesday	Wednesday	Thursday	Friday
a	N. Solo	230	250	186	223	192
b	I. Kuryakin	290	210	175	238	177

Problem solving and reasoning

- 30** Karen and Tony both work for Up Market Realty. Karen sold a home for \$868 000 and Tony sold two units, each valued at \$435 000. The commission rate was 4%.

- Calculate Karen's commission.
- Calculate Tony's commission.
- Who made more from the sales? By how much?

- 31** Bart earned \$446.20 on sales valued at \$4600. Karla earned \$213.18 on sales valued at \$2090.

- Who has the better rate of commission?
- What is their commission rate?



- 32** Madison works in sales. She is given the choice of being paid 8% commission on sales only, or a base salary of \$200 per week plus $2\frac{1}{2}\%$ commission on sales. If Madison sells \$3050 worth of goods in a week, which method of payment would pay her the most? By how much would she be better off?
- 33** Jenny was offered a retainer of \$640 per month, \$140 per month car allowance, plus $1\frac{1}{2}\%$ commission on any houses she sells. If during her first month of work for her new company she sold two units valued at \$349 000 and \$417 000 respectively, what was Jenny's total income for the month?
- 34** A doctor charges each person \$35.80 per consultation. If the doctor sees, on average, four people an hour, calculate her earnings for an eight-hour day.
- 35** Nadia is a sole trader who earns income through her home hairdressing business. She charges \$47 for a haircut, \$92 for a cut and colour, and \$105 for a perm. In one week, she completes seven haircuts, 14 cuts and colours, and two perms.
- Find her income for the week.
 - Calculate her profit for the week, if her expenses were \$687.30.
 - If Nadia worked a total of 32 hours during this week, calculate her hourly rate of pay after expenses.

- 36 A dressmaker charges \$23.00 for major alterations and \$15.20 for minor alterations.
- Calculate her wage if, in one week, she completed 15 major alterations and 23 minor alterations.
 - Determine her hourly rate if she spent a total of 16 hours working on alterations.
- 37 Ayansh sells books and is paid a retainer of \$1275 per month and a percentage commission on total sales made. Calculate the percentage commission rate for a month where he earns a total of \$3562.50 on sales of \$37 500.
- 38 George sells confectionary. He is paid \$2440 per month plus commission on all sales. His commission is calculated using the sliding scale shown in the table.

Sales value	Commission rate
Up to \$15 000	No commission
\$15 001 to \$25 000	4% of sales over \$15 000
\$25 001 to \$40 000	\$600 plus 3% of sales over \$25 000
\$40 001 and over	\$1050 plus 1.5% of sales over \$40 000

Calculate George's earnings for the following months.

- March sales totalled \$24 890
- April sales totalled \$36 002
- May sales totalled \$14 050
- June sales totalled \$53 821

Challenge

- 39 Louisa works as a salesperson and earns a commission based on a sliding scale. Her commission rate is 4.5% for the first \$5000, and $x\%$ for any sales over \$5000. In a week where she makes sales of \$8100, her commission is \$339.70. Find her commission rate $x\%$.

Related resources



3.3 Quick quiz



Worksheet: Commissions and royalties



Worksheet: Piecework



Lesson 3.4

Government allowances and pensions

Learning intentions

By the end of this lesson, you will be able to...

→ calculate income related to government payments.

Payments from the government

- Some people receive a **government payment** in the form of an allowance or a pension.
 - Examples of pensions include the age pension for people over the age of 67 and the disability support pension for those with a long-term condition that prevents them from working.
 - Other government payments that are currently available include Youth Allowance and the JobSeeker payment. Youth Allowance is paid to young people who are studying full-time, undertaking a full-time apprenticeship, training, looking for work or are sick.
- The following tables outline the maximum payments available for people under different circumstances. They are current as of January 2025.

Table 1 Youth Allowance rates

Your situation	Amount your payment reduces by if you earn between \$528 and \$633 per fortnight	Amount your payment reduces by if you earn more than \$633 per fortnight	Maximum income before your payment reduces to \$0
Single, no children, younger than 18, and live at your parent's home	50 cents for each dollar over \$528	\$52.50 plus 60 cents for each dollar over \$633	\$1235.84
Single, no children, 18 or older and live at your parent's home	50 cents for each dollar over \$528	\$52.50 plus 60 cents for each dollar over \$633	\$1340.67
Single or in a couple, no children, and need to live away from your parent's home	50 cents for each dollar over \$528	\$52.50 plus 60 cents for each dollar over \$633	\$1662.67
Member of a couple with children	50 cents for each dollar over \$528	\$52.50 plus 60 cents for each dollar over \$633	\$1755.17
Single with children	50 cents for each dollar over \$528	\$52.50 plus 60 cents for each dollar over \$633	\$1955.17

Table 2 Students and Australian apprentices: Personal income test

Your circumstances	Maximum fortnightly payment
Single, no children	
Under 18, at home	\$410.30
Under 18, away from home	\$663.30
18 and over, at home	\$472.50
18 and over, away from home	\$663.30
Single, with children	\$836.60
Principal carer of a child (under special conditions)	\$1007.50
A couple, no children	\$663.30
A couple, with children	\$718.10

Table 3 Job seekers: Personal income test

For any job seeker, your payment will start to reduce if your income is more than \$150 a fortnight. Your payment will reduce by 50 cents for each dollar of income you have between \$150 and \$250. If your income is over \$250, your payment will reduce by 60 cents for each dollar of income over \$250.

Your situation	Maximum income before payment reduces to \$0
Single, no children	
Under 18, at home	\$857.00
18 and over, at home	\$961.84
Single or a couple, no children, away from home	\$1283.84
A couple, with children	\$1376.34
Single, with children	\$1576.34
Principal carer of a child (under special conditions)	\$1877.17

Worked example 3.4A Calculating Youth Allowance for a student living at home

Jenny is a student living at home. She is 17 and entitled to Youth Allowance.

- What is her Youth Allowance per fortnight if she has no income?
- How much could she earn and still receive the full Youth Allowance payment?
- How much could she earn before she loses her allowance completely?

Part	Think	Write
a	Look in Table 1 “Youth Allowance rates” table for “single, no children, under 18, at home”. The rate is \$410.30.	From the Youth Allowance rates table, Jenny is entitled to \$410.30.
b	Look in Table 2 “Students and Australian apprentices: Personal income test” for “single, under 18, at home”. Consider the meaning of the heading for the second column.	From the personal income test table Jenny could earn up to \$528.
c	Look in the last column of Table 2 “Students and Australian apprentices: Personal income”.	From the personal income test table, Jenny must earn less than \$1235.84.



Worked example video: Calculating Youth Allowance for a student living at home

Understanding, fluency and communicating

Answers: Lesson 3.4

Helpful hints

- Make sure you look in the correct row and column of the correct table!

- Andrea is a student living away from home. She is 18 and entitled to Youth Allowance.
 - What is her Youth Allowance if she has no income?
 - How much could she earn and still keep the full Youth Allowance?
 - How much could she earn before she loses her allowance completely?
- Goran is a student living at home. He is 19 and entitled to Youth Allowance.
 - What is his Youth Allowance if he has no income?
 - How much could he earn and still keep the full Youth Allowance?
 - How much could he earn before he loses his allowance completely?
- Andres is a job seeker and living at home. He is 19 and entitled to Youth Allowance.
 - What is his Youth Allowance if he has no income?
 - How much could he earn and still keep the full Youth Allowance?
 - How much could he earn before he loses his allowance completely?
- Sylvana is a job seeker and living away from home. She is 17 and entitled to Youth Allowance.
 - What is her Youth Allowance if she has no income?
 - How much could she earn and still keep the full Youth Allowance?
 - How much could she earn before she loses her allowance completely?
- Tito is a student living away from home. He is 16 and entitled to Youth Allowance.
 - What is his Youth Allowance if he has no income?
 - If he starts a part-time job earning \$450 per fortnight, would his Youth Allowance be affected? Explain.
 - If he started a part-time job earning \$295 per week, would his Youth Allowance be affected? Explain.
 - How much could he earn before he loses his allowance completely?
- Leia is a job seeker living at home. She is 20 and entitled to Youth Allowance.
 - What is her Youth Allowance if she has no income?
 - If she starts a part-time job earning \$160 per fortnight, would her Youth Allowance be affected? Explain.
 - If she started a part-time job earning \$70 per week, would her Youth Allowance be affected? Explain.
 - How much could she earn before she loses her allowance completely?



Worked example 3.4B Calculating Youth Allowance when earning a small income

Marcella is 19, living at home and a job seeker. She earns \$194 per fortnight and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Marcella will receive.

Think	Write
Full Youth Allowance for Marcella would be \$472.50. She loses some because her income exceeds \$150. Reduction for income between \$150 and \$250 is 50 cents in the dollar.	$\text{Income in excess of } \$150 = \$194 - \$150$ $\text{Reduction} = 44 \times 0.50 = \$22$ $\text{Marcella's allowance} = \$472.50 - \$22 = \$450.50$



Worked example video: Calculating Youth Allowance when earning a small income

Helpful hints

- Find the maximum Youth Allowance rate applicable to the person's situation.
- Compare their income against the reduction rates and calculate any reductions in the form "cents in the dollar" first.
- If applicable, add the fixed reduction to this to find the total reduction. Then subtract the total reduction from the maximum allowance.

7 Amy is 18, living at home and a student entitled to Youth Allowance. She earns \$571 per fortnight. Complete the following to calculate the amount of allowance Amy receives.

a Maximum allowance = __

b Income in excess of \$528 = __ - \$528 = __

c Reduction = __ $\times$ \$0.50 = __

d Amy's allowance = __ - __ = __

8 Ingrid is 17, living at home and a job seeker. She earns \$205 per fortnight and is entitled to Youth Allowance. Calculate the allowance Ingrid will receive each fortnight.

9 George is 19, living away from home and a job seeker. He earns \$211 per fortnight and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that George will receive each fortnight.

10 Theresa is 18, living at home and a student. She earns \$554 per fortnight and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Theresa will receive each fortnight.

11 Christos is 20, living at home and a job seeker. He earns \$85 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Christos will receive each fortnight.

12 Piera is 16, living at home and a student. She earns \$275 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Piera will receive each fortnight.



Worked example 3.4C Calculating Youth Allowance when working part-time

Dukan is 18 and has a partner and one child. He is a full-time student and is entitled to Youth Allowance. Dukan works part-time and earns \$614 per fortnight. Calculate his fortnightly payment.

Think	Write
Look up the full allowance in the “partnered with children” section of Table 1 “Youth Allowance rates”.	Dukan earns more than \$633 but less than \$1755.17 per fortnight, so he is entitled to a part allowance.
Dukan’s income exceeds \$633 and so the fixed reduction applies, along with the 60 cents in the dollar for the amount in excess of \$633.	From the “Youth Allowance rates” table, full allowance in these circumstances is \$718.10.
Subtract \$633 from income of \$723, then multiply the result by 0.60 (being \$0.60 or 60 cents).	Income in excess of \$633 = $\$723 - \$633$ Reduction = $\$90 \times 0.60 = \$90$ = \$54.00
Then add the fixed amount of \$52.50 to that amount. Subtract the total from the maximum allowance of \$718.10.	Total reduction = $\$52.50 + \$54.00$ = \$106.50 Dukan’s allowance = $\$718.10 - \$106.50$ = \$611.60 per fortnight



Worked example video: Calculating Youth Allowance when working part-time

Helpful hints

- Find the maximum Youth Allowance rate applicable to the person’s situation.
- Compare their income against the reduction rates and calculate any reductions in the form “cents in the dollar” first.
- If applicable, add the fixed reduction to this to find the total reduction. Then subtract the total reduction from the maximum allowance.

13 Sami is 19, single and has one child. She is a full-time student entitled to Youth Allowance. Sami’s part-time income is \$664 per fortnight. Complete the following to calculate her Youth Allowance.

- a** Maximum allowance = ____
- b** Income in excess of \$633 = ____ - \$633 = ____
- c** Reduction = ____ $\times$ \$0.60 = ____
- d** Fixed reduction for income over \$633 = ____
- e** Total reduction = ____ + ____ = ____
- f** Sami’s allowance = ____ - ____ = ____



- 14 Venus is 16, living at home and a student. She earns \$680 per fortnight and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Venus will receive.
- 15 Xavier is 17 and lives at home. He is a full-time student and entitled to Youth Allowance. Xavier works part-time and earns \$744 per fortnight. Calculate his Youth Allowance.
- 16 Yoshii is 20, lives away from home and has a partner. He is a full-time student and is entitled to Youth Allowance. He earns \$756.00 per fortnight. Calculate his Youth Allowance.
- 17 Nerita is 20, living at home and a job seeker. She earns \$175 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Nerita will receive each fortnight.

Problem solving and reasoning

The table shows the personal income test for pensioners. Use the table to answer questions 18 to 21.

Table 4 Pensioners: Personal income test

Family situation	Income test for pensions		
	For full pension (per fortnight)*	Reductions	For part pension (per fortnight)
Single	up to \$212	50 cents for each dollar over \$212	less than \$2500.80
Couple (combined)	up to \$372	25 cents for each dollar over \$372	less than \$3822.40
Couple living apart for health reasons	up to \$372	25 cents for each dollar over \$372	less than \$4949.60

- 18 Joan and Don are entitled to the age pension. They are a couple with a combined income of \$553.00 per fortnight. The full pension for each person is \$789.30 per fortnight. Calculate the total pension paid to Joan and Don.
- 19 Maurice is single with an income of \$720 per fortnight. He is entitled to the age pension. If the full pension is \$1047.10 per fortnight, calculate the pension Maurice will receive.
- 20 George and Cheryl are a couple entitled to the age pension but living apart due to health reasons. George earns \$420 per fortnight. The full pension is \$1047.10 each. Calculate the total pension amount they would receive each fortnight.
- 21 Jessica is single and has been receiving the full age pension of \$1047.10 per fortnight. Her financial situation is about to change and she anticipates receiving an income of \$1400 per week. How will this income affect the age pension Jessica has been receiving?
- 22 Consider your own situation and whether you might be eligible to receive Youth Allowance after finishing school.
 - a What would your Youth Allowance be with no extra income?
 - b If you started a part-time job, how much could you earn before your payment would be affected? How many hours of work is this likely to equate to?
 - c How much could you earn before losing your allowance completely? How many hours of work is this likely to equate to?



Challenge

- 23** Youth allowance is only available to Australians aged 24 and under. Austudy is another government allowance that provides financial assistance to people 25 years and over, who are studying or completing an apprenticeship.
- a** By going onto the Services Australia website, find the maximum amount a single person (with no children) can get per fortnight from Austudy.
 - b** If a single person with no children has a part-time job, and earns \$800 per fortnight, how much will they get from Austudy?

Related resources



3.4 Quick quiz

Lesson 3.5

Taxable income

Learning intentions

By the end of this lesson, you will be able to...

- calculate taxable income, the Medicare levy and student loan repayments.

Taxable income

- The **taxable income** is calculated using **total income** and allowable **deductions**.
- Taxable income = total income – allowable deductions
 - Total income includes income from all sources throughout the year. It could include wages and salaries, bonuses, interest earned, commissions and allowances.
 - The total income may be subject to deductions that reduce the amount used to calculate the tax payable. Tax deductions may include the cost of tools used for work, the cost of safety equipment, self-education expenses, union fees, charitable donations over \$2, work-related car travel expenses, the cost of uniforms and their cleaning, and tax agent fees. There are additional allowable deductions for different professions.
- For taxation purposes, income is usually calculated in whole dollars, so always round down to the nearest whole dollar. This is different to the usual rules for rounding.

The Medicare levy

- Medicare is the public hospital medical system available without charge for Australians.
- The **Medicare levy** is an amount you pay in addition to the tax you pay on your taxable income which helps to fund Medicare.
- The Medicare levy is currently calculated at 2% of taxable income.
 - In 2024, the full Medicare levy was payable if a person's taxable income exceeded \$32 500.
 - The Medicare levy on taxable incomes between \$26 000 and \$32 500 is 10 cents for each dollar in excess of \$26 000. These thresholds were higher in some circumstances. For incomes of less than \$26 000, no levy was payable.
 - A Medicare levy surcharge may also apply if a person does not have private hospital health insurance.



Student loans

- For people with loans that assisted them when studying or training, those loans are repayable when taxable income exceeds a threshold. The loan repayment is paid as additional tax. Information about student loans can be found through the Australian Taxation Office and Services Australia.
- The repayment thresholds and rates for the compulsory repayment of debts to the following are reviewed annually:
 - Higher Education Loan Program (HELP)
 - Vet Student Loan (VSL)
 - Student Financial Supplement Scheme (SFSS)
 - Student Start-up Loan (SSL)
 - ABSTUDY Student Start-up Loan (ABSTUDY SSL)
 - Australian Apprenticeship Support Loan (AASL).
- The way student loan repayments are calculated was changed in 2025. The minimum repayment threshold was also increased to \$67 000.
- The Repayment Income (RI) is equal to taxable income plus any total net investment loss (which includes net rental losses), total reportable fringe benefits amounts, reportable superannuation contributions and exempt foreign employment income. For this course, we will assume RI is equal to taxable income.
- The amount to repay is calculated based on RI above \$67 000 at the rates shown in the following table.

Table 1 2025–26 study and training loan repayment income thresholds and rates

Income thresholds	Marginal rate of repayment
0–\$67 000	Nil
\$67 001–\$125 000	15c for each dollar over \$67 000
\$125 001 and over	\$8700 plus 17c for each dollar over \$125 000

Worked example 3.5A Calculating taxable income

Amita uses the following information to calculate her taxable income: salary \$35 980, interest \$569 (joint account), bonus \$200, cost of uniforms \$140, tax agent fee \$80, work-related car costs \$298. Calculate Anita's taxable income.

Think	Write
Interest on a joint account is shared evenly between both account holders (unless told otherwise).	Income = \$35 980 + (\$569 ÷ 2) + \$200
Add all income items and round down to the nearest dollar.	= \$36 464.50
Add all deductions.	= \$36 464 (rounded down)
Subtract the deductions from the income to find the taxable income.	Deductions = \$140 + \$80 + \$298
	= \$518
	Taxable income = Income – Deductions
	= \$36 464 – \$518
	= \$35 946



Worked example video: Calculating taxable income

Understanding, fluency and communicating

Answers: Lesson 3.5

Helpful hints

- Look at the information carefully to work out which items are income, and which are allowable deductions.
- Find the total income and deductions separately and then subtract the deductions from the income to find the taxable income.

- 1 Annie uses the following information to calculate her taxable income: salary \$45 320, interest \$665 (joint account), bonus \$800, cost of uniforms \$340, tax agent fee \$60 and work-related car costs \$45. Complete the following to calculate Annie's taxable income.

$$\begin{aligned} \text{Income} &= \_\_ + (\_\_ \div 2) + \$800 \\ &= \_\_ \\ &= \_\_ \text{ (rounded down to the nearest dollar)} \end{aligned}$$

$$\text{Deductions} = \_\_ + \$60 + \_\_$$

$$\begin{aligned} \text{Taxable income} &= \_\_ - \_\_ \\ &= \_\_ \end{aligned}$$

- 2 Calculate the taxable income for the following people.

Income/deductions	Bryn	Moe	Naomi	Rahul	Rosie
Salary/wages	\$45 996	\$36 899	\$72 665	\$12 990	\$26 773
Allowances	\$590	\$100	–	\$390	\$225
Interest	\$32	\$12	\$366	\$5	\$15
Bonus/tips	\$100	–	\$2000	–	\$25
Uniforms	\$50	–	–	–	\$150
Cleaning	\$35	–	–	–	\$55

Income/deductions	Bryn	Moe	Naomi	Rahul	Rosie
Tax agent fee	\$80	\$60	\$200	\$20	\$45
Union fee	\$300	–	–	\$60	\$190
Donations	–	–	\$100	\$2	\$20
Work-related car costs	\$423	\$19	\$2333	–	–

Worked example 3.5B Calculating the Medicare levy on taxable income

Calculate the Medicare levy payable on a taxable income of \$35 908.

Think	Write
The minimum threshold is \$32 500 so the Medicare levy must be paid on this taxable income. The Medicare levy is 2% of taxable income. Convert 2% to a fraction (or decimal) and multiply by the taxable income. Write the result correct to the nearest cent.	$\\$35\,908 > \\$32\,500$ Medicare levy = 2% of taxable income $= \frac{2}{100} \times \$35\,908$ $= \$718.16$



Worked example video: Calculating the Medicare levy on taxable income

Helpful hints

- Check whether the taxable income is greater than \$32 500 to see if the Medicare levy of 2% must be paid.

- 3 Complete the following to calculate the Medicare levy on a taxable income of \$42 756.

$$\text{Medicare levy} = \frac{\square}{100} \times \text{---}$$

$$= \text{---}$$

- 4 Calculate the Medicare levy on these taxable incomes.

a \$39 021

b \$44 906

c \$43 879

d \$46 110

e \$60 958

f \$10 365

Worked example 3.5C Calculating the Medicare levy on taxable income between \$26 000 and \$32 500

Find the Medicare levy on \$28 536.

Think	Write
Calculate the excess by subtracting \$26 000 from the taxable income. Multiply the excess by 0.10 to find the Medicare Levy owed.	Excess = $\\$28\,536 - \\$26\,000$ $= \$2536$ Medicare levy = $0.10 \times \\$2536$ $= \$253.60$



Worked example video: Calculating the Medicare levy on taxable income between \$26 000 and \$32 500

Helpful hints

- Check whether the taxable income falls between \$26 000 and \$32 500 to see if the reduced Medicare levy of \$0.10 per dollar over \$26 000 is owed.

5 Complete the following to calculate the Medicare levy on a taxable income of \$29 369.

$$\text{Excess} = \underline{\hspace{1cm}} - \$26\,000$$

$$= \underline{\hspace{1cm}}$$

$$\text{Medicare levy} = 0.10 \times \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

6 Calculate the Medicare levy on these taxable incomes.

a \$32 000

b \$26 530

c \$27 450

d \$31 041

Worked example 3.5D Calculating student loan repayments

Christopher has a student loan and a taxable income of \$78 537. Calculate the amount of additional tax he will pay due to the HELP loan.

Think

\$78 537 is greater than \$67 000 so a repayment must be made.

The rate is 15c per dollar over \$67 000.

Find the difference between \$78 537 and \$67 000 and multiply the result by 0.15 to find the amount owed.

Write

$$\$78\,537 > \$67\,000$$

$$\begin{aligned} \text{Repayment} &= (\$78\,537 - \$67\,000) \times 0.15 \\ &= \$11\,537 \times 0.15 \\ &= \$1730.55 \end{aligned}$$



Worked example video: Calculating student loan repayments

Helpful hints

- Determine the repayment income and use the corresponding repayment rate to calculate the amount of extra tax allocated for the repayment.

Exercise questions

7 Use Table 1 to calculate the extra tax allocated to repay the student loan debts of people with taxable incomes of:

a \$75 124

b \$88 223

c \$150 111

d \$111 899

e \$53 000

Problem solving and reasoning

- 8 In the 2024–25 financial year, Gary has an annual salary of \$82 330, earns \$2355 in interest (joint account), spends \$300 on uniforms for work and an extra \$156 cleaning them, hires a tax agent for \$100 and has work-related car costs of \$3220.
- Determine Gary's taxable income.
 - How much does Gary owe for the Medicare levy?
 - Gary has a student loan. What is his compulsory repayment?
- 9 Robert uses the following information to calculate his taxable income: salary \$153 022, interest \$3659, bonus \$1000, tax agent fee \$120, work-related car costs \$1256 and donations to charity \$500.
- Determine Robert's taxable income.
 - Robert has a student loan. What is his compulsory repayment?
 - Determine how much Robert must pay for the Medicare levy.
- 10 In the 2024–25 financial year, Asha uses the following information to calculate her taxable income: salary \$29 555, interest \$243, tax agent fee \$70, work-related car costs \$452 and donations to charity \$120.
- Determine Asha's taxable income.
 - How much does Asha owe for the Medicare levy?
 - Asha has a student loan. Why doesn't she have to make a compulsory repayment?
 - How much does Asha's salary need to increase by for her to be required to make a compulsory repayment to her student loan?

Challenge

- 11 In Australia, people with sufficiently high incomes are encouraged to pay for private hospital insurance to help take some of the burden off Medicare. People in this category who do not take out the required level of hospital cover are required to pay a Medicare levy surcharge (MLS), in addition to the Medicare levy of 2%. The rates for a single person, depending on their income, are shown in the following table.

Threshold	Base tier	Tier 1	Tier 2	Tier 3
Single threshold	\$97 000 or less	\$97 001–\$113 000	\$113 001–\$151 000	\$151 001 or more
MLS	0%	1%	1.25%	1.5%

Robert, from question 9, is single and does not have private hospital insurance.

- How much does Robert owe for the MLS?
- In total, how much is Robert contributing to Medicare through his taxes?
- Research online for possible private hospital insurance options for Robert. Should Robert continue to pay the MLS or should he buy private hospital insurance?

Related resources



3.5 Quick quiz



Investigation: The Medicare Levy Surcharge



Worksheet: Taxable income

Lesson 3.6

Paying tax

Learning intentions

By the end of this lesson, you will be able to...

- determine tax payable using tax tables.

Income tax rates

- In Australia, **tax** is paid on incomes, property value and goods purchased. **Income tax** is a tax paid on an individual’s taxable income.
 - Income tax is a marginal tax, meaning that more tax is paid as the taxable income increases. No tax is paid on the first \$18 200 earned in a financial year. This is called the tax-free threshold.
- Income tax has provided the largest source of Australian Government revenue for over 80 years and helps to fund community services including education.
- Income tax is calculated using a tax table. The following table shows the tax rates for the 2024–25 financial year.

Table 1 Taxation rates for 2024–25 financial year

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	16 cents for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30 cents for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37 cents for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45 cents for each \$1 over \$190 000

- The rates in the table do not include the 2% Medicare levy nor any student loan repayments.
- The tax rates and income thresholds are reviewed each year.

Pay-as-you-go (PAYG)

- Australia uses a “Pay-as-you-go” (**PAYG**) **tax** system where employers withhold a given amount from an employee’s **gross pay** that goes towards the tax owed by the employee that financial year.
 - The employer is responsible for transferring the money to the government after it has been withheld.
- PAYG instalments withheld contribute to the total tax owed by an individual. This includes any income tax, the Medicare levy and any student loan repayments.
- The amount of tax withheld by an employer depends on a number of factors.
 - For the primary income source of an individual, the withholding rate is lower to account for the tax-free threshold of \$18 200. Any other income is subjected to a higher withholding rate.
 - Since employees may be paid in weekly, fortnightly or monthly instalments, the amount of tax withheld depends on this frequency.

- The Australian Tax Office (ATO) publishes annual tables of how much tax employers should withhold based on whether the individual is claiming the tax-free threshold and how frequently they are being paid.
→ Here are some samples from the weekly, fortnightly and monthly tax tables published by the ATO for the 2024–25 financial year. The top row in each of these tables corresponds to an annual income of \$71 812.

Table 2 Weekly, Fortnightly and Monthly PAYG tax tables

Weekly			Fortnightly			Monthly		
	Amount to be withheld			Amount to be withheld			Amount to be withheld	
Weekly earnings	With tax-free threshold	No tax-free threshold	Fortnightly earnings	With tax-free threshold	No tax-free threshold	Monthly earnings	With tax-free threshold	No tax-free threshold
1	2	3	1	2	3	1	2	3
\$	\$	\$	\$	\$	\$	\$	\$	\$
1381.00	266.00	377.00	2762.00	532.00	754.00	5984.33	1153.00	1634.00
1382.00	266.00	377.00	2764.00	532.00	754.00	5988.67	1153.00	1634.00
1383.00	266.00	377.00	2766.00	532.00	754.00	5993.00	1153.00	1634.00
1384.00	267.00	377.00	2768.00	534.00	754.00	5997.33	1157.00	1634.00
1385.00	267.00	378.00	2770.00	534.00	756.00	6001.67	1157.00	1638.00
1386.00	267.00	378.00	2772.00	534.00	756.00	6006.00	1157.00	1638.00
1387.00	268.00	378.00	2774.00	536.00	756.00	6010.33	1161.00	1638.00
1388.00	268.00	379.00	2776.00	536.00	758.00	6014.67	1161.00	1642.00
1389.00	268.00	379.00	2778.00	536.00	758.00	6019.00	1161.00	1642.00
1390.00	269.00	379.00	2780.00	538.00	758.00	6023.33	1166.00	1642.00

Worked example 3.6A Calculating income tax payable using a tax table

Use Table 1 to calculate the income tax payable on these taxable incomes.

a \$53 251**b** \$26 784**c** \$145 631

Part	Think	Write
a	\$53 251 is in the range \$45 001–\$135 000 so the tax owed is \$4288 plus 30 cents for each \$1 over \$45 000. Subtract \$45 000 to find the excess. The tax payable is \$4288 plus 0.3 multiplied by the excess.	Excess = \$53 251 – \$45 000 = \$8251 Tax = \$4288 + 0.3 × \$8251 = \$6763.30
b	\$26 784 is in the range \$18 201–\$45 000 so the tax owed is 16 cents for each \$1 over \$18 200. Subtract \$18 200 to find the excess. The tax payable is 0.16 multiplied by the excess.	Excess = \$26 784 – \$18 200 = \$8584 Tax = 0.16 × \$8584 = \$1373.44
c	\$145 631 is in the range \$135 001–\$190 000 so the tax owed is \$31 288 plus 37 cents for each \$1 over \$135 000. Subtract \$135 000 to find the excess. The tax payable is \$31 288 plus 0.37 multiplied by the excess.	Excess = \$145 631 – \$135 000 = \$10 631 Tax = \$31 288 + 0.37 × \$10 631 = \$35 221.47

**Worked example video:** Calculating income tax payable using a tax table

Understanding, fluency and communicating

Answers: Lesson 3.6

Helpful hints

- Remember that income tax is payable on taxable income. Where required, find the taxable income first and then use Table 1 to calculate the income tax payable.

- 1 Complete to find the income tax payable on a taxable income of \$47 953.
- Excess = $\_\_\_\_\_\_ - \$45\,000$
 = $\_\_\_\_\_\_$
Tax = $\_\_\_\_\_\_ + 0.3 \times \_\_\_\_\_\_$
 = $\_\_\_\_\_\_$
- 2 What is the income tax payable on each of these taxable incomes?
- a \$96 564 b \$48 911 c \$8100 d \$31 000
e \$177 250 f \$211 000 g \$43 802 h \$95 000
- 3 The table below shows the pay rates for state MPs, not including electoral allowances. Calculate the amount of income tax that would be payable on these annual salaries. Round each amount to the nearest dollar.

Office	Pay
Premier	\$416 440
Deputy premier	\$350 329
Senior minister	\$333 072
Parliamentary secretary	\$207 091
Government whip	\$203 640

Worked example 3.6B Calculating tax PAYG deductions from pay

Alan’s annual salary is \$72 072. Use Table 2 to calculate how much Alan’s employer should deduct from his pay if he claims the tax-free threshold and is paid:

- a weekly b fortnightly c monthly.

Part	Think	Write
a	Determine Alan’s weekly pay by dividing his annual salary by 52. Identify the corresponding row in the weekly PAYG tax table and then the corresponding value in the “With tax-free threshold” column.	$\$72\,012 \div 52 = \$1386.00$ Tax withheld = \$267.00
b	Determine Alan’s weekly pay by dividing his annual salary by 26. Identify the corresponding row in the weekly PAYG tax table and then the corresponding value in the “With tax-free threshold” column.	$\$72\,012 \div 26 = \$2772.00$ Tax withheld = \$534
c	Determine Alan’s weekly pay by dividing his annual salary by 12. Identify the corresponding row in the weekly PAYG tax table and then the corresponding value in the “With tax-free threshold” column.	$\$72\,012 \div 12 = \$6006.00$ Tax withheld = \$1157



Worked example video: Calculating tax PAYG deductions from pay

Helpful hints

- Using Table 2, match the pay frequency and then check whether the employee is claiming the tax-free threshold.

- 4 Peishan's annual salary is \$72 228. Use Table 2 to calculate how much Peishan's employer should deduct from her pay if she claims the tax-free threshold and is paid:
- a** weekly **b** fortnightly **c** monthly.
- 5 Peishan decides to no longer claim the tax-free threshold. Repeat question 4 with this new information.
- 6 These tables show some of the 2024 PAYG weekly, fortnightly and monthly tax tables. They do not all correspond to the same annual income.

Weekly			Fortnightly			Monthly		
Amount to be withheld			Amount to be withheld			Amount to be withheld		
Weekly earnings	With tax-free threshold	No tax-free threshold	Fortnightly earnings	With tax-free threshold	No tax-free threshold	Monthly earnings	With tax-free threshold	No tax-free threshold
1	2	3	1	2	3	1	2	3
\$	\$	\$	\$	\$	\$	\$	\$	\$
1121.00	182.00	293.00	4932.00	1226.00	1478.00	5247.67	914.00	1395.00
1122.00	182.00	294.00	4934.00	1226.00	1480.00	5252.00	914.00	1395.00
1123.00	183.00	294.00	4936.00	1226.00	1480.00	5256.33	919.00	1400.00
1124.00	183.00	294.00	4938.00	1228.00	1480.00	5260.67	919.00	1400.00
1125.00	183.00	295.00	4940.00	1228.00	1482.00	5265.00	919.00	1400.00
1126.00	184.00	295.00	4942.00	1228.00	1482.00	5269.33	923.00	1404.00
1127.00	184.00	295.00	4944.00	1230.00	1484.00	5273.67	923.00	1404.00
1128.00	184.00	296.00	4946.00	1230.00	1484.00	5278.00	923.00	1404.00
1129.00	185.00	296.00	4948.00	1230.00	1484.00	5282.33	927.00	1408.00
1130.00	185.00	296.00	4950.00	1232.00	1486.00	5286.67	927.00	1408.00

How much should be deducted for an annual salary of \$58 604 if the employee is paid weekly and:

- a** **i** claims the tax-free threshold?
ii does not claim the tax-free threshold?
- b** How much should be deducted for an annual salary of \$128 648 if the employee is paid fortnightly and:
i claims the tax-free threshold?
ii does not claim the tax-free threshold?
- c** How much should be deducted for an annual salary of \$63 180 if the employee is paid monthly and:
i claims the tax-free threshold?
ii does not claim the tax-free threshold?

Problem solving and reasoning

- 7 Julie is paid an annual salary of \$33 700 and receives an income of \$896 from other sources.
- a** Calculate Julie's total annual income.
- b** Determine the amount of income tax Julie will need to pay on her total income. Assume she has no allowable deductions.

- 8 Joshua's taxable income in the 2024–25 financial year is \$87 500. He also has student loan debt.
- Calculate the amount of income tax he must pay.
 - Calculate the Medicare levy Joshua must pay.
 - How much must Joshua pay towards his student loan debt?
 - What is the total amount of tax Joshua must pay?
- 9 Sam has a few odd jobs. In total, he earns \$22 500 per year. He receives a tax deduction of \$2600. He has no student loans. Calculate:
- his taxable income
 - the income tax payable on his taxable income
 - the Medicare levy payable
 - the total tax payable.
- 10 For this question, the 2024 Weekly, Fortnightly and Monthly tax tables are available on Oxford Digital. Alternatively, you can use the most recent tax tables on the ATO website (Note: This may affect the answers as the tables are reviewed each year).
- 11 This table shows the estimated 2025 starting salaries for some new graduates, their pay frequency and whether they are claiming the tax-free threshold. Determine the amount of tax withheld from each paycheck for each graduate.

Profession	Starting salary	Pay frequency	With or without tax-free threshold?
Dentist	\$125 000	Weekly	With
Doctor	\$75 000	Monthly	With
Optometrist	\$80 000	Fortnightly	With
Engineer	\$82 000	Monthly	Without
Computer scientist	\$59 000	Fortnightly	With
Veterinarian	\$67 000	Weekly	With
Teacher	\$76 000	Monthly	Without
Accountant	\$55 000	Fortnightly	With
Psychologist	\$71 000	Weekly	With



Challenge

- 12 Hamish pays \$20 188 in income tax on his taxable income. This does not include the Medicare levy or any student loan repayments.
- a What is Hamish's taxable income?
 - b What is Hamish's taxable income if the \$20 188 does include the Medicare levy?
- 13 Some people say that people earning over \$190 000 pay 45% of their income in tax. Investigate the truth of this statement. Provide examples supporting your response.

Related resources



3.6 Quick quiz



Worksheet: Calculating income tax



Worksheet: PAYG tax



Additional resource: 2024–25 monthly, fortnightly and weekly tax tables



Weblink: ATO website – Tax tables

Lesson 3.7

Tax returns and net earnings

Learning intentions

By the end of this lesson, you will be able to...

- calculate net earnings and determine whether more tax is payable or if a refund is owed.

Tax returns

- After the end of a financial year, anyone earning an income completes a **tax return**, which is an annual statement of all income, allowable deductions, PAYG tax paid and other personal financial information. The ATO uses this information to determine the amount of income tax owed by the individual for the financial year.
 - Tax returns also include information about whether the person must pay the Medicare levy or is eligible for an exemption or a reduced rate, whether they have private hospital insurance, and whether they have any student loans. Other information about children and dependents is also compiled to ensure other tax benefits or payments are applied.
 - Tax returns can be completed individually online or with the help of a tax agent. The cost of hiring a tax agent is an allowable deduction for the following financial year.
- The ATO compares the tax payable on an individual's income (including the Medicare levy and any compulsory student loan payments), as determined by the information provided in the tax return, and compares this amount with the tax withheld by employers as part of the PAYG system.
 - If the individual has had more tax than they owe withheld, they are refunded the difference by the ATO.
 - If the individual has not had enough tax withheld, they have tax debt and must pay the remaining amount to the ATO.

Net earnings

- Annual **net earnings** can be calculated using the following formula.

Net earnings = taxable income – tax payable – Medicare levy

- Income tax owed can be calculated using **Table 1 Taxation rates** for 2024–25 financial year in Lesson 3.6.
- Tax payable includes any income tax and compulsory student loan payments.
- An exemption from the Medicare levy can be granted to an individual who is not eligible for Medicare. Details of this exemption are provided on the Services Australia website.
- There are other payments that may be collected by the ATO that are not included in this lesson, including the Medicare levy surcharge (MLS). These should all be accounted for when calculating net earnings.

Worked example 3.7A Calculating tax refund or balance payable

Sarah works as a secretary and receives a yearly salary of \$54 479 plus an income of \$1950 per year from babysitting. Her total tax deductions are \$1570. During the year, she had \$8216 withheld by her employer for tax. Sarah has no student loans.

- Find Sarah's taxable income.
- Calculate the Medicare levy she needs to pay.
- Find the income tax payable on her taxable income.
- Find her tax refund or tax balance payable.

Part	Think	Write
a	Add Sarah's salary and babysitting income to find her total income. Subtract her tax deductions from her total income to find her taxable income.	$\begin{aligned}\text{Total income} &= \$54\,479 + \$1950 \\ &= \$56\,429 \\ \text{Taxable income} &= \text{total income} - \text{tax deductions} \\ &= \$56\,429 - \$1570 \\ &= \$54\,859\end{aligned}$
b	The full Medicare levy of 2% applies for taxable incomes in excess of \$32 500.	$\begin{aligned}\text{Medicare levy} &= 2\% \text{ of } \$54\,859 \\ &= \frac{2}{100} \times \$54\,859 \\ &= \$1097.18\end{aligned}$
c	\$54 859 is in the range \$45 001–\$135 000 so the tax owed is \$4288 plus 30 cents for each \$1 over \$45 000. Subtract \$45 000 to find the excess. The tax payable is \$4288 plus 0.3 multiplied by the excess.	$\begin{aligned}\text{Excess} &= \$54\,859 - \$45\,000 \\ &= \$9859 \\ \text{Income tax} &= \$4288 + 0.3 \times \$9859 \\ &= \$7245.70\end{aligned}$
d	Calculate the total tax due. Sarah has already paid \$8216 which is less than the tax owed. Calculate the difference between the tax owed and the amount already paid to find the tax balance payable.	$\begin{aligned}\text{Total tax due} &= \text{Medicare levy} + \text{income tax} \\ &= \$1097.18 + \$7245.70 \\ &= \$8342.88 \\ \text{Tax balance payable} &= \$8342.88 - \$8216 \\ &= \$126.88\end{aligned}$



Worked example video: Calculating tax refund or balance payable

Understanding, fluency and communicating

Answers: Lesson 3.7

Helpful hints

- Keep track of any calculations by labelling results clearly. This will help you put the information together at the end to determine whether a tax refund is owed or if more tax is payable.

- 1 Brittany earns \$89 452 as a company manager and \$5203 as a part-time singer. She has tax deductions of \$2612 and, throughout the year, pays a total of \$20 867.40 in PAYG tax instalments. She has no student loans. Complete the following to find the tax refund she is owed.

$$\text{Total income} = \$89\,452 + \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\text{Taxable income} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\text{Income tax} = \underline{\hspace{1cm}} + 0.3 \times (\underline{\hspace{1cm}} - \$45\,000)$$

$$= \underline{\hspace{1cm}}$$

$$\text{Medicare levy} = 0.02 \times \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\text{Total tax due} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\text{Tax refund} = \$20\,867.40 - \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

- 2 Celine earns \$52 056 per annum as an environmentalist and \$18 159 as a part-time analyst. Celine has student loans. She has tax deductions of \$1903 and has paid \$5123.90 in tax instalments.
- Find her taxable income.
 - Calculate the Medicare levy she needs to pay.
 - Calculate the amount she must pay towards her student loans.
 - Find the income tax payable on her taxable income.
 - Find her tax refund or tax balance payable.

Worked example 3.7B Calculating net earnings

Consider Sarah from Worked example 3.7A. Determine Sarah's net earnings for the year.

Think	Write
Substitute Sarah's total income, tax payable and Medicare levy owed into the Net earnings formula and simplify.	$\begin{aligned} \text{Net earnings} &= \text{taxable income} - \text{tax payable} \\ &\quad - \text{Medicare levy} \\ &= \$54\,859 - \$7245.70 - \$1097.18 \\ &= \$46\,516.12 \end{aligned}$



Worked example video: Calculating net earnings

Helpful hints

- Remember to include any compulsory student loan payments when determining net earnings.

- 3 Consider Brittany from question 1. Determine Brittany's net earnings.
- 4 Consider Celine from question 2. Determine Celine's net earnings. Remember to consider her compulsory student loan payment.

Problem solving and reasoning

- 5 Mr Benton works part time and has a yearly salary of \$39 760. He receives an income of \$2500 from his hobby of wood carving. His tax deductions amount to \$1090 and throughout the year he has paid PAYG instalments of \$3215.90. He has no student loans.
 - a Calculate his taxable income.
 - b Calculate the Medicare levy he needs to pay.
 - c Calculate income tax payable on his taxable income.
 - d Calculate his tax refund or tax balance payable.



- 6 Ricky earns \$84 365 as a cabaret artist and \$4620 from his share portfolio. He has tax deductions of \$13 884 and, throughout the year, pays a total of \$15 988 in tax instalments. Ricky has student loans.
 - a Find his taxable income.
 - b Calculate the Medicare levy Ricky needs to pay.
 - c How much is Ricky's compulsory student loan payment?
 - d Find the income tax payable on his taxable income.
 - e Find the tax refund or tax balance payable.
- 7 Alanna earns \$2776 per fortnight and claims the tax-free threshold. She has student loan debt to repay. She has tax deductions totalling \$1320.
 - a Find Alanna's taxable income.
 - b Calculate the Medicare levy Alanna must pay.
 - c How much is Alanna's compulsory student loan payment?
 - d Use the fortnightly tax table in Lesson 3.6 to determine the total amount of tax withheld by Alanna's employer for the financial year.
 - e Find the tax refund or tax balance payable.

- 8 Brian earns \$2900 each week. Using the information in the weekly tax table, his employer withholds \$773 from each paycheck for tax. Brian claims the tax-free threshold, has student loans totalling \$8200 and must pay the Medicare levy. He has the following deductions: \$2500 self-education, \$250 charitable donation and \$720 in work-related car travel expenses. He earns an additional \$150 per week through private tutoring sessions. He withholds no tax on this extra income but does report an amount earned on his tax return.
- a Determine the amount Brian is either owed as a tax refund or must pay in additional tax. Note that Brian earns enough to full repay his student loan this year.
 - b To the nearest dollar, how much would Brian's employer have needed to withhold each week to ensure Brian is owed a tax refund?
 - c Why did Brian's employer not withhold the amount from part b?
 - d Find information on the ATO website about the amount employers must withhold for employees with student loans. Use this information to determine the total amount Brian's employer should withhold from each weekly paycheck. Remember to only consider the amount paid to Brian by his employer. (Note: The supplied answer uses the ATO Weekly PAYG Study loan withholding rates table 2024–2025, which is available on Oxford Digital.)
 - e Use the new withholding amount from part d to repeat part a.

Challenge

- 9 Eli pays the full Medicare levy of 2% of his taxable income. He has no student loans.
- a If the Medicare levy Eli pays is \$2296, find Eli's taxable income.
 - b Calculate the total tax Eli needs to pay in the year, including the Medicare levy.
 - c Eli has \$1122 withheld from each of his fortnightly pay checks as PAYG tax instalments. Based on this, does he receive a tax refund or owe further tax at the end of the financial year?

Related resources



3.7 Quick quiz



Investigation: The cost of further study



Investigation: Superannuation



Additional resource: ATO Weekly PAYG Study loan withholding rates table 2024–2025

Using spreadsheets

Syllabus links

Use a spreadsheet to model an employee's income including overtime, salary, bonuses, commission, piecework and royalties (MST-11-03)

Use a spreadsheet to model scenarios involving taxation (MST-11-03)

Earning money

See Oxford Digital for sample spreadsheets for questions in this lesson.

- 1 Revisit **Problem solving and reasoning** in Lesson 3.1. Use a spreadsheet, such as the one provided on Oxford Digital, to convert between annual, monthly, fortnightly, weekly and hourly income. Use it to check some of your answers for those questions.

- 2 Revisit **Understanding, fluency and communicating** in Lesson 3.2. A spreadsheet for calculating an employee's wage including overtime at time-and-a-half is provided on Oxford Digital.

The following view shows the formulas used.

	A	B	C	D	E	F
1	Wage calculator					
2	Hours	Mon	Tue	Wed	Thur	Fri
3	Normal	8	8	8	5	8
4	Overtime	2	-	3	-	1
5	Normal hours	=SUM(B3:F3)		Normal pay	=B5*B7	
6	Overtime hours	=SUM(B4:F4)		Overtime pay	=B6*1.5*B7	
7	Hourly rate	\$ 23.60		Total pay	=D5+D6	

- Use this spreadsheet or create your own to check your answers for questions 3 and 4 in Lesson 3.2.
 - How would you change cell D6 in the spreadsheet if the overtime rate was double-time instead of time-and-a-half?
 - Make this change so you can use the spreadsheet to check your answer for question 5.
 - Create a new spreadsheet to check your answers for questions 6 and 7.
- 3 Design a spreadsheet that will allow you to calculate part pensions and part allowances. A prepared spreadsheet is provided on Oxford Digital. Modify this spreadsheet to reflect the latest information from the Australian Government.
- 4 The spreadsheet shown is provided on Oxford Digital. It allows you to calculate the amount of income tax payable on a given taxable income.

	A	B
1	Tax calculator	
2	Taxable income	63995
3	Tax payable	9986.5

The formula typed into cell B3 is:

```
=IF(B2>190000,(B2-190000)*0.45+51638,
  IF(B2>135000,(B2-135000)*0.37+31288,
    IF(B2>45000,(B2-45000)*0.3+4288,
      IF(B2>18200,(B2-18200)*0.16,
        IF(B2>0,"NO TAX",0))))
```

Explain why the formula works. Try using it to calculate tax.

Lesson 3.8

Review: Earning money

Review questions 3.8A Mathematical literacy review

Answers: Lesson 3.8

Key terms

The following key terms are used in this module:

• allowance (employment)	• income tax	• royalty
• annual leave loading	• Medicare levy	• salary
• award	• net earnings	• sole trader
• bonus	• overtime	• tax
• commission	• PAYG tax	• tax return
• deduction	• penalty rate	• taxable income
• double-time	• per annum	• time-and-a-half
• government payments	• piecework	• total income
• gross pay	• retainer	• wage

Check your understanding

- Which key term corresponds to each of the following?
 - a fixed regular payment earned for work or services, usually based on an hourly rate and typically paid daily, weekly or fortnightly
 - base payment of a fixed amount made to employees whose primary payment type is by commission
 - once-off payment paid to an employee for a specific reason, typically determined by the industry award
- Consider annual leave loading.
 - How do you calculate a person's **annual leave loading** for a single week if you know their weekly income is \$1576?
 - What would you be finding if you calculated $1.175 \times \$1576 \times 4$?
- Using a combination of words and symbols, give an example calculation showing a person's income if they are paid:
 - an annual salary in fortnightly instalments
 - a weekly retainer plus a commission of their sales
 - a fixed amount per item they produce in a factory.
- In your own words, explain the meaning of the following key terms and how they relate to each other:
gross pay, income tax, Medicare levy, net earnings, PAYG tax, tax, tax return, taxable income, total income.

Review questions 3.8B Multiple-choice questions

- 1 Katrina is a part-time employee and receives an annual salary of \$19 370. What is her weekly salary?
A \$1616.17 **B** \$403.54 **C** \$372.50 **D** \$745
- 2 Zoltan is paid \$27.90 per hour. What are his earnings for a 26-hour week?
A \$725.40 **B** \$27.90 **C** \$1450.80 **D** \$1088.10
- 3 Andre is an apprentice and works nine hours per day, five days per week. He is paid \$20.86 per hour. What is his annual income?
A \$938.40 **B** \$5423.60 **C** \$45 057.60 **D** \$48 812.40
- 4 Mal receives a salary of \$63 428 per annum. He works six days per week, eight hours per day. What is his hourly rate?
A \$25.41 **B** \$27.52 **C** \$27.53 **D** \$50.82
- 5 Paula is a junior and works during the school holidays. She is paid \$15.70 per hour normal rate. She works 30 normal hours, six hours at time-and-a-half and three hours at double-time. What is her pay?
A \$471 **B** \$612.30 **C** \$642.96 **D** \$706.50
- 6 Zanthia receives a bonus at Christmas of 6.2% of her \$47 695 salary. What is the amount of her bonus?
A \$2957.09 **B** \$17 170.90 **C** \$171.71 **D** \$6.20
- 7 Paulette is a permanent part-time employee and is paid \$456 per week. She receives a holiday loading of $17\frac{1}{2}\%$ on her four weeks holiday pay. What is her total holiday pay?
A \$79.80 **B** \$319.20 **C** \$775.20 **D** \$2143.20
- 8 Marek is paid a weekly retainer of \$400 and 13% commission on sales in excess of \$5000. What is his pay in a week when he has sales of \$8350?
A \$1085.50 **B** \$435.50 **C** \$835.50 **D** \$1685.50
- 9 Alexia is paid \$362 commission on sales of \$4525. What is her percentage rate of commission?
A 0.08% **B** 8% **C** 12.5% **D** 3.62%
- 10 Sue picks mushrooms and is paid \$2.15 per box. What is her pay for picking 53 boxes?
A \$24.65 **B** \$53 **C** \$4.05 **D** \$113.95
- 11 Charles is paid a royalty of 8% on total sales. What is his royalty for sales of \$356 990?
A \$28 559.20 **B** \$3569.90 **C** \$44 623.75 **D** \$35 699
- 12 The fortnightly Youth Allowance payment for a single person under 18 years living at home and studying full time is \$410.30. It reduces by 50 cents for every dollar in excess of \$528 earned per fortnight. Sharon is 17, living at home and earning \$592 per fortnight. What is her allowance?
A \$64 **B** \$378.30 **C** \$346.30 **D** \$474.30
- 13 Ossie works part time for a wage of \$23 879 per annum. He receives a bonus of \$800, pays tax agent fees of \$60, makes donations to charity of \$75 and is entitled to a deduction for work-related car expenses of \$356. What is Ossie's taxable income?
A \$24 415 **B** \$25 397 **C** \$24 188 **D** \$23 961
- 14 The Medicare levy is 2% of taxable income provided the taxable income exceeds \$32 500. What is the Medicare levy on a taxable income of \$34 677?
A \$120.14 **B** \$693.54 **C** \$1201.40 **D** \$43.54
- 15 The student loan repayment on taxable incomes from \$67 001–\$125 000 is 15 cents per dollar above \$67 000. What is the compulsory repayment for an individual with a taxable income of \$97 850?
A \$14 677.50 **B** \$1050 **C** \$4627.50 **D** \$1500

- 16 The income tax on \$100 666 is \$4288 plus 30 cents per dollar over \$45 000. What is the tax payable on \$100 666?

A \$30 199.80 B \$20 987.80 C \$14 765.58 D \$4288

Use the following information to complete questions 17 and 18.

Bernadette earns \$2880 fortnightly. Her employer withholds \$570 from each pay in PAYG tax. Bernadette has allowable deductions totalling \$729. She has no student loans and must pay the Medicare levy.

- 17 How much is Bernadette owed as a tax refund?

A \$14 820 B \$303.68 C \$1483.02 D \$1786.70

- 18 Determine Bernadette's net earnings.

A \$60 363.68 B \$61 846.70 C \$59 634.68 D \$61 117.70

Review questions 3.8C Set 1

- 1 An electrician works a 42-hour week. Find his yearly pay if he earns \$46.50 per hour.
- 2 Indera receives an annual salary of \$67 801. Convert this to a weekly salary and find Indera's average hourly rate if she works 48 hours per week.
- 3 Brian is paid \$27.63 per hour. Calculate his weekly wage if he works 35 hours at normal time and seven hours overtime at time-and-a-half.
- 4 Jack is a security guard and is paid \$29.50 per hour. If he works on the door, he is paid an extra \$36 danger pay. Find Jack's wage for a week in which he is on the door for three nights and works 30 hours.
- 5 Georgina works as a personal assistant and is paid \$1496 per week. If her four weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, find her pay for her holidays.
- 6 Tony is paid 1.2% commission on any sales made. Find his commission on a house that sells for \$876 500.
- 7 Mark waxes surfboards for the surf hire shop. He is paid \$7.40 per waxed board. Find his pay for a day in which he waxes 11 boards.
- 8 Martin is paid a royalty of 10.8% on sales of his book. His sales for a year are \$63 580. Calculate his royalty payment.
- 9 Lorena is a job seeker and living away from home. She is 16 and entitled to Youth Allowance. Use the tables in Lesson 3.4 to answer these questions.
 - a What is her Youth Allowance if she has no income?
 - b How much could she earn before she loses her allowance completely?
- 10 Mary and Ian are entitled to the age pension. They are a couple with a combined income of \$459.00 per fortnight. The full pension for a couple is \$1578.60 per fortnight. Use **Table 4** in Lesson 3.4 to calculate the combined pension paid to them.
- 11 Martha uses the following information to calculate her taxable income: wages \$36 522, interest \$64, tax agent fee \$65, work-related car expenses \$1274 and donations to charity \$225. Calculate Martha's taxable income.
- 12 The Medicare levy is 2% of taxable income over \$32 500. Calculate the Medicare levy payable on a taxable income of \$39 359.



13 Use **Table 1** in Lesson 3.6 to calculate the income tax payable on each of these taxable incomes.

- a** \$57 368 **b** \$31 259 **c** \$6422 **d** \$142 330

14 Gloria earns \$77 366 per annum as a company manager and \$4865 as a part-time tax agent. She has allowable deductions of \$3698 and, throughout the year, pays a total of \$15 600 in PAYG tax instalments. Gloria has student loans. Calculate:

- a** her taxable income
b the Medicare levy she needs to pay
c her compulsory student loan repayment
d the income tax payable on her taxable income
e her tax refund or tax balance payable.

Review questions 3.8D Set 2

- 1** Jan earns \$62 475 per annum. Calculate her weekly pay and her hourly rate if she works 35 hours per week.
- 2** Akbar works a seven-hour day, six days per week. His hourly rate is \$28.56. Find his pay for one year.
- 3** Visnoo is paid \$25.95 per hour for a 30-hour week. He is paid overtime at double-time for all other hours. Calculate his pay for a week in which he works 38 hours.
- 4** Hewey works as an apprentice plumber and is paid \$18.30 per hour. He is paid a digging allowance of \$11.50 per day if he has to dig trenches and \$15.80 per day if he works on open sewers. Find his pay for a week in which he works 42 hours, digs on two days and works on an open sewer for three days.
- 5** Tabitha received a $17\frac{1}{2}\%$ annual leave loading on four weeks of their normal wages. They normally work a 32-hour week. Their four weeks of holiday pay, including annual leave loading, is \$2587.90.
 - a** Find their normal weekly pay.
 - b** What is their normal hourly pay rate?
- 6** Robert sells new cars valued at \$35 675. Find his pay if his commission is 4.5%.
- 7** Mitchell is a student living at home. He is 17 and entitled to Youth Allowance. Use the tables in Lesson 3.4 to calculate the allowance Mitchell receives each fortnight if he earns \$340 per week.
- 8** Giselle uses the following information to calculate her taxable income: wages \$67 220, interest \$985, meal allowance \$612, bonus \$5000, tax agent fee \$180, work-related car expenses \$1255 and donations to charity \$600. Calculate Giselle's taxable income.
- 9** The Medicare levy on taxable incomes between \$26 000 and \$32 500 is 10 cents for each dollar in excess of \$26 000. Calculate the Medicare levy on \$28 317.
- 10** Use **Table 1** in Lesson 3.6 to calculate the income tax payable on each of these taxable incomes.

a \$166 889 **b** \$33 699 **c** \$89 369 **d** \$15 522
- 11** Shania earns \$38 996 per annum as a waitress and \$7560 as a part-time singer. She has tax deductions of \$3561 and, throughout the year, pays a total of \$6963.30 in tax instalments. Shania has no student loans. Find:
 - a** her taxable income
 - b** the Medicare levy she needs to pay
 - c** income tax payable on her taxable income
 - d** her tax refund or tax balance payable.

Review questions 3.8E Set 3

- 1** Erica works part-time and earns \$29 881 per annum. Calculate her weekly pay.
- 2** Ronaldo works five hours a day, five days per week. Find his hourly rate if his annual income is \$36 031.
- 3** Mona earns \$24.88 for the first 35 hours in a week. She receives time-and-a-half for the next 8 hours and double-time after that. Find her pay for a 52-hour week.

- 4 Regina works in a cafe and receives a uniform allowance of \$8.50 per day. Her hourly rate of pay is \$25.23. Find her pay if she works five days for four hours per day.
- 5 Ling works as a personal assistant and is paid \$783 per week. If her four weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, what is her pay for her holidays?
- 6 Robyn is paid a commission of 5.8% of the price of any item of jewellery she sells. If Robyn sells a ring for \$5250, how much is her commission?
- 7 Steven is paid a royalty of 9.8% on sales of his book. If his sales for the year are \$265 800, what is his royalty?
- 8 Fabrice is a job seeker and living at home. He is 17 and entitled to Youth Allowance. Use the tables in Lesson 3.4 to calculate the allowance Fabrice receives each fortnight if he earns \$280 per fortnight.
- 9 Claud is single with an income of \$625 per fortnight. They are entitled to the age pension. If the full pension is \$1047.10 per fortnight, calculate the pension Claud will receive using **Table 4** in Lesson 3.4.
- 10 Vanja uses the following information to calculate his taxable income: wages \$53 005, interest \$134 (joint account), cost of uniforms \$600, cleaning of uniforms \$260, tax agent fee \$100 and work-related car expenses \$2551. Calculate Vanja's taxable income.
- 11 The Medicare levy is 2% of taxable income over \$32 500. Calculate the Medicare levy payable on a taxable income of \$60 080.
- 12 Use **Table 1** in Lesson 3.6 to calculate the income tax payable on each of these taxable incomes.

a \$162 556	b \$108 694	c \$21 211	d \$18 006
- 13 Tania earns \$68 259 per annum as a music teacher and \$9634 in royalties from her recordings. She has tax deductions of \$4980 and throughout the year pays a total of \$8413.44 in tax instalments. Tanya has student loans. Find:
 - a her taxable income
 - b the Medicare levy she needs to pay
 - c her compulsory student loan repayment
 - d income tax payable on her taxable income
 - e her tax refund or tax balance payable.

Review questions 3.8F Set 4

- 1 Pauline earns \$62 408 per annum. Calculate her hourly rate if she works 38 hours per week.
- 2 Fielding is paid \$28.37 per hour. He works five hours per day, six days a week. Find his pay for one year.
- 3 James is an apprentice and is paid \$19.22 per hour. Calculate his weekly wage if he works 35 hours at normal time and five hours overtime at time-and-a-half.
- 4 Ross works in a timber yard and is paid \$25.40 per hour. He receives a dust allowance of \$12.50 per day. Find his pay for a five-day, 38-hour week.
- 5 Endora received a $17\frac{1}{2}\%$ holiday loading on four weeks of her normal wages. She normally works a 30-hour week. Her four weeks holiday pay plus loading is \$3985.70.
 - a Find her normal weekly pay.
 - b Find her normal hourly pay rate.
- 6 A factory worker is paid \$4.35 for each completed garment. Calculate his earnings if he completes 429 garments.
- 7 Willow is paid a royalty of 8.3% on sales of her book. Her sales for a year are \$149 000. Find her royalty.
- 8 Agneska is a student living away from home. She is 19 and entitled to Youth Allowance. Use the tables in Lesson 3.4 to calculate the allowance Agneska receives each fortnight if she earns \$360 per week.
- 9 Carol and John are entitled to the age pension. They are a couple with a combined income of \$428.00 per fortnight. The full pension for each person is \$789.30 per fortnight. Calculate the total pension paid to Carol and John using **Table 4** in Lesson 3.4.

- 10** Ito uses the following information to calculate her taxable income: wages \$28 369, interest \$32 (joint account), bonus \$300, cost of uniforms \$250, tax agent fee \$50 and work-related car expenses \$25. Calculate Ito's taxable income.
- 11** Use **Table 1** in Lesson 3.6 to calculate the income tax payable on each of these taxable incomes.
- a** \$155 233
 - b** \$28 775
 - c** \$12 998
 - d** \$72 225
- 12** Natalie earns \$38 555 per annum as a recording artist and \$23 997 in royalties. She has tax deductions of \$8550 and, throughout the year, pays a total of \$10 044.44 in tax instalments. She has no student loans. Find:
- a** her taxable income
 - b** the Medicare levy she needs to pay
 - c** income tax payable on her taxable income
 - d** her tax refund or tax balance payable.

Review questions 3.8G Practice examination question

- 1** Answer the following questions.
- a** Bimo is paid \$652 for a 38-hour week.
 - i** Convert this to an annual salary. **(1 mark)**
 - ii** What is Bimo's hourly rate? **(1 mark)**
 - iii** Bimo's mother says his monthly pay is \$2608. Is she correct? Explain using calculations. **(2 marks)**
 - b** Leanne received $17\frac{1}{2}\%$ holiday loading for four weeks holiday calculated on her normal weekly wage of \$946. Calculate the holiday pay for her four weeks holiday. **(2 marks)**
 - c** Turay is paid a commission of 8% on all sales and a retainer of \$200 per week. He is offered a new pay deal of 10% on all sales. By calculating Turay's weekly pay based on average weekly sales of \$9000, decide if he should accept the new pay deal. Give reasons for your answer. **(3 marks)**
 - d** Jenny is single and receives the age pension of \$808.60 per fortnight. She takes a part-time job earning \$300 per fortnight. She loses 50 cents of her pension for each dollar she earns in excess of \$164. Find her adjusted pension payment. **(3 marks)**
 - e** Maria is paid \$6058 monthly. Her employer withholds \$1174 from each pay in PAYG tax. Maria has \$2400 in allowable deductions and has no student loans. She must pay the Medicare levy.
 - i** Use **Table 1** in Lesson 3.6 to determine how much income tax Maria owes on her taxable income. **(2 marks)**
 - ii** When Maria files a tax return, how much money will she owe as additional tax or receive as a tax refund? **(3 marks)**

TOTAL: 17 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Earning money

I can do this	I need to review this
☐ solve problems involving salaries ☐ solve problems involving wages, penalty rates for overtime, weekend and public holidays given an hourly rate of pay ☐ calculate weekly, fortnightly, monthly and yearly earnings assuming 1 year = 52 weeks	Go back to Lesson 3.1 Earning an income
☐ calculate leave loading by finding a percentage of eligible normal pay	Go back to Lesson 3.2 Allowances, bonuses and loadings
☐ calculate an individual's earnings from commission, sliding scale commission, piecework, royalties and as a sole trader	Go back to Lesson 3.3 Commission, piecework and royalties
☐ calculate an individual's income from government payments ☐ calculate reductions to government payments depending on employment income	Go back to Lesson 3.4 Government allowances and pensions
☐ determine taxable income by examining allowable deductions ☐ calculate the Medicare levy from taxable income ☐ calculate student loan repayments from taxable income	Go back to Lesson 3.5 Taxable income
☐ determine tax payable by interpreting and applying information in tax tables ☐ calculate the weekly, fortnightly or monthly tax to be deducted from a worker's pay under the PAYG taxation system using tax tables	Go back to Lesson 3.6 Paying tax
☐ calculate net earnings after deductions, taxation and the Medicare levy ☐ determine by calculation if more tax is payable or if a refund is owed after completing a tax return	Go back to Lesson 3.7 Tax returns and net earnings
☐ use a spreadsheet to model an employee's income including overtime, salary, bonuses, commission, piecework and royalties ☐ use a spreadsheet to model scenarios involving taxation	Go back to Using spreadsheets

Related resources



Module review quiz: Earning money



Worksheet: Challenge 3

Module

4

Managing money

Lessons

- Lesson 4.1 Financial calculations
- Lesson 4.2 Payment options
- Lesson 4.3 Purchasing a car
- Lesson 4.4 Ongoing costs of a vehicle
- Using spreadsheets
- Lesson 4.5 Household bills
- Lesson 4.6 Budgeting
- Using spreadsheets
- Lesson 4.7 Review: Managing money

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Managing money

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Percentages



Support sheet: Fractions, decimals and percentages



Support sheet: Writing one amount as a percentage of another



Support sheet: Percentage of a quantity



Support sheet: Interpreting data in graphical form



Support sheet: Working with decimals



Support sheet: Understanding order of operations



Support sheet: Substitution

Syllabus links

- Managing money (MST-11-04)
 - A student solves financial problems involving budgeting and purchasing.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 4.1

Financial calculations

Learning intentions

By the end of this lesson, you will be able to ...

- perform financial calculations involving percentages, profit and loss and GST.

Percentage increase and decrease

- **Percentage increase** is when the amount a quantity is increased by is expressed as a percentage.
 - To increase a quantity by $x\%$, find $(100 + x)\%$ of the quantity.
 - To find the percentage increase between an initial value and a final value of a quantity, use the formula below. Remember to write the result as a percentage.

$$\text{percentage increase} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$
 - The increase in the price of a good or service is called a mark-up. A mark-up is usually given as a percentage increase.
- **Percentage decrease** is when the amount a quantity is decreased by is expressed as a percentage.
 - To decrease a quantity by $x\%$, find $(100 - x)\%$ of the quantity.
 - To find the percentage decrease between an initial value and a final value of a quantity, use the formula below. Remember to write the result as a percentage.

$$\text{percentage decrease} = \frac{\text{initial value} - \text{final value}}{\text{initial value}} \times 100\%$$
 - The decrease in the price of a good or service is called a discount (or sale price). A discount is usually given as a percentage decrease.

Profit and loss

- **Revenue** is the total income generated by a business through the sale of items or services.
 - If the selling price is greater than the **cost**, a **profit** is made.

$$\text{profit} = \text{selling price} - \text{cost}$$
 - The percentage profit, is the profit expressed as a percentage of the cost.

$$\text{percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$
 - If the selling price is less than the cost, a **loss** is made.

$$\text{loss} = \text{cost} - \text{selling price}$$
 - The percentage loss, is the loss expressed as a percentage of the cost.

$$\text{percentage loss} = \frac{\text{loss}}{\text{cost}} \times 100\%$$

GST and VAT

- **GST** stands for goods and services tax, which we have in Australia. Other countries have a tax similar to Australia's GST, called **VAT**, which stands for value-added tax.
- The GST was introduced in 2000 as a 10% tax added to the price of goods and services. The rate of 10% has not been updated.
 - GST is payable on most products (such as all new goods) and services (such as hospitality, electrical work, plumbing, building).
 - GST-free products and services include most basic food and services like water, sewerage and drainage. More details on GST-free products and services can be found on the Australian Taxation Office website.

Worked example 4.1A Increasing or decreasing an amount by a percentage

a Increase \$75 by 20%.

b Decrease \$3 by 12%.

Part	Think	Write
a	To increase \$75 by 20%, find $100\% + 20\% = 120\%$ of \$75. Convert 120% to a fraction and multiply by \$75. Write the result in its simplest form.	$(100\% + 20\%) \times \$75 = 120\% \times \$75$ $= \frac{120}{100} \times \$75$ $= \$90$
b	To decrease \$3 by 12% find $100\% - 12\% = 88\%$ of \$3. Convert 88% to a fraction and multiply by \$3. Write the result in its simplest form.	$(100\% - 12\%) \times \$3 = 88\% \times \$3$ $= \frac{88}{100} \times \$3$ $= \$2.64$



Worked example video: Increasing or decreasing an amount by a percentage

Understanding, fluency and communicating

Answers: Lesson 4.1

Helpful hints

- Worked example 4.1A uses fractions to find the percentage of a quantity. You can also convert the percentage to a decimal and then multiply if you prefer to work with decimals instead of fractions.
- When working with money, round results to the nearest cent where required.

1 Increase the following by the given percentages. Give answers to the nearest cent where required.

a \$450 by 28%

b \$15 by 75%

c \$850 by 8%

d \$160 by 12%

e \$240 by 35%

f \$6 by 200%

g \$2 by 1.5%

h \$40 by $62\frac{1}{2}\%$

i \$300 by $6\frac{2}{3}\%$

2 Decrease the following by the given percentages. Give answers to the nearest cent where required.

a \$115 by 30%

b \$480 by 24%

c \$350 by 40%

d \$4.8 by 19%

e \$86 by 25%

f \$456 by 8.5%

g \$3000 by $2\frac{1}{4}\%$

h \$57 by $33\frac{1}{3}\%$

i \$4.20 by 15.6%

3 Calculate the following mark-ups and discounts.

a \$50 [20% mark-up]

b \$120 [15% discount]

c \$1800 [5% mark-up]

d \$720 [10% discount]

e \$280 [2.5% mark-up]

f \$96 [70% discount]

Worked example 4.1B Calculating percentage increase or decrease

Find the percentage increase or decrease in each of the following situations, correct to two decimal places.

a Jai's pay per hour increased from \$64 to \$68.

b The value of a car decreased from \$18 500 to \$14 900 in one year.

Part	Think	Write
a	Jai's pay per hour has increased so use the percentage increase formula. Substitute the initial value of \$64 and the final value of \$68 into the formula and simplify the result.	percentage increase $= \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$ $= \frac{68 - 64}{64} \times 100\%$ $= \frac{4}{64} \times 100\%$ $= 6.25\%$
b	The value of the car has decreased so use the percentage decrease formula. Substitute the initial value of \$18 500 and the final value of \$14 900 into the formula and simplify the result.	percentage decrease $= \frac{\text{initial value} - \text{final value}}{\text{initial value}} \times 100\%$ $= \frac{18\,500 - 14\,900}{18\,500} \times 100\%$ $= \frac{3600}{18\,500} \times 100\%$ $\approx 19.46\%$



Worked example video: Calculating percentage increase or decrease

Helpful hints

- If the percentage increase formula results in a negative value, then a decrease occurred. The result is the negative of what the result would be using the percentage decrease formula.
- Similarly, if the percentage decrease formula results in a negative value, then an increase occurred. The result is the negative of what the result would be using the percentage increase formula.

4 Find the percentage increase or decrease in each of the following situations, correct to one decimal place.

a An increase in pay per hour from \$52 to \$55.

$$\begin{aligned}\text{percentage increase} &= \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\% \\ &= \frac{\square - \square}{\square} \times 100\% \\ &= \frac{\square}{\square} \times 100\% \\ &\approx \_\_\% \end{aligned}$$

b A decrease in cost from \$186 to \$154.

$$\begin{aligned}\text{percentage decrease} &= \frac{\text{initial value} - \text{final value}}{\text{initial value}} \times 100\% \\ &= \frac{\square - \square}{\square} \times 100\% \\ &= \frac{\square}{\square} \times 100\% \\ &\approx \_\_\% \end{aligned}$$

5 Find the percentage increase (to one decimal place) from:

a \$350 to \$425

b \$7.20 to \$7.80

c \$63 to \$68

d \$80 to \$215

e \$4.20 to \$8.40

f \$480 to \$530

6 Find the percentage decrease (to one decimal place) from:

a \$256 to \$190

b \$55 to \$51

c \$15.80 to \$15.50

d \$430 to \$385

e \$5400 to \$1800

f \$13 to \$12.20

7 Find the percentage mark-up or discount (to one decimal place) on the following price changes.

a \$60 to \$66

b \$72 to \$48

c \$96 to \$112

d \$150 to \$140

e \$320 to \$120

f \$400 to \$480

Worked example 4.1C Calculating profit or loss and percentage profit or loss

For each situation below, calculate the profit or loss and then the percentage profit or loss.

a A signed 2005 West Tigers jersey was purchased for \$800. It was later sold for \$1040.

b A drone was purchased for \$480 and later sold for \$360.

Part	Think	Write
a	To find the profit, subtract the cost price from the selling price. Substitute the profit and cost into the percentage profit formula. Simplify and write the result as a percentage.	$\begin{aligned}\text{profit} &= \text{selling price} - \text{cost price} \\ &= \$1040 - \$800 \\ &= \$240 \\ \text{percentage profit} &= \frac{\text{profit}}{\text{cost}} \times 100\% \\ &= \frac{240}{800} \times 100\% \\ &= 30\%\end{aligned}$
b	To find the loss, subtract the selling price from the cost price. Substitute the loss and cost into the percentage loss formula. Simplify and write the result as a percentage.	$\begin{aligned}\text{loss} &= \text{cost price} - \text{selling price} \\ &= \$480 - \$360 \\ &= \$120 \\ \text{percentage loss} &= \frac{\text{loss}}{\text{cost}} \times 100\% \\ &= \frac{120}{480} \times 100\% \\ &= 25\%\end{aligned}$



Worked example video: Calculating profit or loss and percentage profit or loss

8 Calculate the following for each of the sales listed below:

- i the profit or loss
- ii the percentage profit or loss.
 - a A car purchased for \$22 000 is sold for \$16 500.
 - b A Star Wars collectable purchased for \$100 is sold for \$170.
 - c A piano purchased for \$4200 is sold for \$2520.
 - d A \$50 000 share portfolio is sold for \$57 500.
 - e A surfboard purchased for \$600 is sold for \$280.
 - f A set of golf clubs purchased for \$2100 is sold for \$800.

Worked example 4.1D Calculating multiple percentage increases or decreases

A stamp collection was bought for \$3600. In the first year, its value increased by 5%. In the second year, it increased in value by a further 6%.

- a Calculate the stamp collection's value at the end of the first year.
- b Calculate its value at the end of the second year.
- c What is the overall increase in its value after two years?
- d Find the percentage profit if it is sold for the value at the end of two years.

Part	Think	Write
a	After the first year, the value has increased by 5%. Increase the initial cost by 5% by multiplying by 105%.	$\begin{aligned}\text{value after 1 year} &= 105\% \times \$3600 \\ &= \frac{105}{100} \times \$3600 \\ &= \$3780\end{aligned}$
b	After the second year, the value has increased by another 6%. Increase \$3780 by 6% by multiplying by 106%.	$\begin{aligned}\text{value after 2 years} &= 106\% \times \$3780 \\ &= \frac{106}{100} \times \$3780 \\ &= \$4006.80\end{aligned}$
c	Find the overall increase in value over these 2 years by subtracting the initial cost from \$4006.80.	$\begin{aligned}\text{increase in value} &= \$4006.80 - \$3600 \\ &= \$406.80\end{aligned}$
d	If the collection is sold for \$4006.80, then the profit is equal to the increase in value, which is \$406.80. Substitute the profit and initial cost into the percentage profit formula and simplify the result.	$\begin{aligned}\text{percentage profit} &= \frac{\text{profit}}{\text{cost}} \times 100\% \\ &= \frac{406.8}{3600} \times 100\% \\ &= 11.3\%\end{aligned}$



Worked example video: Calculating multiple percentage increases or decreases

- 9 A painting was bought for \$3600. In the first year, its value increased by 6%. In the second year, it increased by a further 8%. Complete the following.
- a value after first year = $106\% \times \$\_\_\_\_\_\_$
 $= \frac{\square}{100} \times \$\_\_\_\_\_\_$
 $= \$\_\_\_\_\_\_$
- b value after second year = $\_\_\_\_\_\% \times \$\_\_\_\_\_\_$
 $= \frac{\square}{100} \times \$\_\_\_\_\_\_$
 $= \$\_\_\_\_\_\_$
- c overall increase in value = $\_\_\_\_\_\$ - \$3600$
 $= \$\_\_\_\_\_\_$
- d percentage profit after 2 years = $\frac{\square}{\square} \times 100\%$
 $= \_\_\_\_\_\% \text{ (to one decimal place)}$
- 10 A piece of antique jewellery was bought for \$2400. In the first year of ownership, its value increased by 15%. In the second year, it increased in value by a further 8%.
- a Calculate the value of the jewellery at the end of the first year.
- b Calculate its value at the end of the second year.
- c What is the overall increase in the value of the jewellery after two years?
- d Find the percentage profit after two years.
- 11 A \$50 000 share portfolio increased in value by 12% in the first year and decreased by 3% in the second year. Find the percentage profit or loss if it is sold after two years.
- 12 An investor bought \$80 000 worth of gold. In the first year, the gold increased in value by 10%, and then in the second year, it decreased in value by 10%. Find the overall percentage profit or loss of the investment after these two years.

Worked example 4.1E Calculating GST

Calculate the GST payable on:

- a a restaurant meal with a pre-GST cost of \$84
- b a pre-GST bill for \$368 from an electrician.

Part	Think	Write
a	Find 10% of \$84.	$\text{GST} = 10\% \times \$84$ $= \frac{10}{100} \times \$84$ $= \$8.40$
b	Find 10% of \$368.	$\text{GST} = 10\% \times \$368$ $= \frac{10}{100} \times \$368$ $= \$36.80$



Worked example video: Calculating GST

Helpful hints

- GST on an amount is 10% of that amount. You can find the GST payable on an amount by dividing the amount by 10.

- 13** Complete the following to calculate the GST on a plasma television with a pre-GST price of \$750.

$$\begin{aligned}\text{GST} &= 10\% \times \$\_ \\ &= \frac{10}{\square} \times \$\_ \\ &= \$\end{aligned}$$

- 14** Calculate the 10% GST payable on goods with a pre-GST price of:

a	\$46	b	\$150	c	\$8.40	d	\$459
e	\$975.50	f	\$10.99	g	\$1980	h	\$15.42

- 15** Michaela purchased a mobile phone listed at a pre-GST price of \$654.

- Calculate the GST payable on the phone.
- Find the total amount Michaela paid for the phone.

- 16** Aaron had a new bathroom installed in his house. The bill was \$8742.80 and was subject to GST of 10%.

- Calculate the GST payable on the bathroom.
- Find the actual amount Aaron paid for the bathroom.

- 17 Angelica bought a watch listed at a pre-GST price of \$389.

- Calculate the GST payable on the watch.
- Find the total amount Angelica paid for the watch.

Worked example 4.1F Calculating GST when it is included in the total price

Guido buys a sofa for \$720 which includes GST.

- Find the GST paid on the sofa.
- Find the pre-GST price of the sofa.

Part	Think	Write
a	The price with GST added is 110% of the original cost. Since $11 \times 10\% = 110\%$, find the GST by dividing the amount paid by 11.	GST paid = $\$720 \div 11$ = \$65.45
b	Find the pre-GST price by subtracting the GST from the amount paid.	original cost = $\$720 - \$65.45$ = \$654.55



Worked example video: Calculating GST when it is included in the total price

Helpful hints

- To find the GST on a price that already includes GST, divide by 11.
- The unitary method can always be used to find the amount of GST (or VAT). If the GST rate is $r\%$, then divide the total price including GST by $(100 + r)$ to find 1% of the pre-GST price and then multiply by r to find the amount of GST paid.

- 18** Complete the following to find the GST on a laptop computer that is priced at \$1200 including GST.

$$\begin{aligned}\text{GST paid} &= \$\_\_ \div 11 \\ &= \$\end{aligned}$$

19 Find the GST included in the price of goods and services costing a total of:

a \$458	b \$264	c \$34.56	d \$219
e \$19.80	f \$1904	g \$2980	h \$14.90

20 Find the cost of each item in question **19** before the GST was added.

Problem solving and reasoning

21 Keong bought a mobile phone listed at a pre-GST price of \$634. Find the total amount Keong paid for the mobile phone.

22 An antique watch bought for \$15 000 increased in value by 5% in the first year of ownership and increased by 8% in the second year. It is then sold for this value after the second year. Calculate the percentage profit.

23 Shannon is travelling overseas and is being refunded the GST on the purchase of a camera that was \$475 including GST. She calculates the GST refund to be 10% of \$475 which is \$47.50. Is she correct? Explain using calculations.

24 Aimee has two quotes for repairs on her car. The first is a pre-GST price of \$2450. The second is \$2680 including GST. Which quote is cheaper and by how much?

25 The GST in New Zealand is 15%. Jarrod has purchased a kiwi carving for NZ\$95.00 including GST. He is entitled to a refund of the GST when returning to Australia. Calculate Jarrod's GST refund.

26 The VAT rate varies depending on what country you are in. In France, the VAT rate is 20%.

a Find the VAT paid on the following pre-VAT prices in France.

i €50	ii €720	iii €1830

b Find the VAT paid on the following prices which include VAT in France.

i €90	ii €630	iii €3420

c Find the pre-VAT amount for each price in part **b**.

d What number can you divide a VAT-inclusive price by to find the VAT paid when the rate is 20%?

27 In Indonesia, the VAT rate is 12%. The currency in Indonesia is the rupiah (Rp). Give all answers in this question to the nearest rupiah.

a Find the VAT paid on the following pre-VAT prices in Indonesia.

i Rp480	ii Rp12 600	iii Rp30 000

b Find the VAT paid on the following prices which include VAT in Indonesia.

i Rp1008	ii Rp9296	iii Rp70 784

c Find the pre-VAT amount for each price in part **b**.

Challenge

28 A car that cost \$28 900 new, decreased in value by 22% in the first year, by 20% in the second year and by 18% in the third year. What was its value after three years?

29 Alice and Bob work at the same store. Alice reduces the price of an item by 10%. Later, Bob reduces the price he sees by 20%. The price is now \$900. What was the original price of the item?

Related resources



4.1 Quick quiz



Worksheet: Applications of percentages

Lesson 4.2

Payment options

Learning intentions

By the end of this lesson, you will be able to ...

- calculate and compare costs associated with different payment options.

Payment options

- There are several different ways to pay for a good or service.
 - Cash, debit cards and direct debit can be used to directly pay for a good or service.
 - When a credit card is used, the credit card provider pays for the good or service and then the customer repays their debt at a later date. Late fees are payable if the debt payment is overdue.
 - Online payments using a debit or credit card may pass through an online payment platform such as PayPal.
- Different types of payment options may be available to consumers instead of paying the full amount upfront. These include using a **buy now, pay later** service or paying an initial **deposit** and then paying the remaining amount at a later time, either as a lump sum (all at once) or in regular instalments.

Buy now, pay later

- Using a *buy now, pay later* service allows a consumer to buy a product or service and delay the payment. These services are usually interest-free; however, other fees may be charged.
- Payments are usually made in instalments at amounts and intervals agreed upon at the point of purchase. Often there are four payments of 25% of the total price. Payments are usually fortnightly or monthly, but other intervals are possible. The first payment is made at the point of purchase.
- While these services are usually described as interest-free (0% interest), other fees are usually added to the cost. These fees can include late fees, account set-up fees, account-keeping fees (usually paid monthly from the second month onwards until the debt is repaid) and payment processing fees.
- When considering whether to use a *buy now, pay later* service, you should consider whether you will be able to make all repayments (especially if using the service for multiple purchases) and the extra cost of any fees. Using *buy now pay later services* can also impact on loan applications.
- A **layby** agreement is an alternative to using a *buy now, pay later* service. The consumer pays for goods in two or more instalments and does not receive the goods until the full price has been paid. There may be fees associated with using a layby agreement.
 - A consumer can cancel a layby agreement at any time before receiving the product; however, they may have to pay a termination fee.

Buying on terms

- To **buy an item on terms** means to make an initial deposit on the item and then pay the remaining amount in regular instalments. Buying on terms usually has longer repayment periods (months or years) compared to *buy now, pay later* services.
 - The amount paid as the deposit and a payment schedule are agreed upon at the point of purchase. This includes when instalments are due and the payment amounts.
 - Buying an item on terms is usually more expensive than buying an item outright. This is because the sum of the deposit and the payment instalments includes interest paid on the price of the item.
 - Buying on terms is usually used for larger purchases like furniture, household appliances and vehicles.
- Buying an item on terms is different to a layby agreement as the consumer receives the item purchased once the deposit is paid.

Worked example 4.2A Using a *buy now, pay later* service

Linus uses a *buy now, pay later* service to buy a new laptop. The price of the laptop is \$2677. He hasn't used the service before and so must pay an account set-up fee of \$30. He makes four payments of 25% of the total cost and after the first instalment, the payments are fortnightly. There is an \$8 monthly account keeping fee due from the second month onwards. No interest is charged.

- How much does Linus pay for each instalment?
- How much is Linus expecting to pay in fees?
- Linus is late to make the final payment and must pay a late payment fee of \$6. How much does Linus spend in total on this purchase?
- What is the percentage increase on the original cost for using the *buy now, pay later* service?

Part	Think	Write
a	Each instalment is 25% of the total price. Divide the total by 4 and write the result to the nearest cent.	$\$2677 \div 4 = \$669.25$
b	Linus must pay the account-set up fee. Since the payments are fortnightly, he will need the account for at least six weeks. As this is more than one month, he will also need to pay the monthly account keeping fee.	$\begin{aligned} \text{fees} &= \$30 + \$8 \\ &= \$38 \end{aligned}$
c	The total cost includes the price of the laptop, the fees from part b and the late fee.	$\begin{aligned} \text{total} &= \$2677 + \$38 + \$6 \\ &= \$2721 \end{aligned}$
d	Use the percentage increase formula. Substitute the total amount Linus pays as the final value and the laptop price as the initial value. Simplify and write the result correct to one decimal place.	$\begin{aligned} \text{percentage increase} &= \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\% \\ &= \frac{2721 - 2677}{2677} \times 100\% \\ &= \frac{44}{2677} \times 100\% \\ &\approx 1.6\% \end{aligned}$



Worked example video: Using a buy now, pay later service

Understanding, fluency and communicating

Answers: Lesson 4.2

- Sarah uses a *buy now, pay later* service to buy a new TV. The price of the TV is \$1695. Sarah makes four payments of 25% and, after the first payment, the instalments are fortnightly. There is an account set-up fee of \$12 and account keeping fees of \$10 from the second month onwards until the debt is repaid. There is a late fee of \$8 for each late payment. No interest is charged.
 - How much does Sarah pay for each instalment?
 - How much is Sarah expecting to pay in fees?
 - Sarah is late to make two of the instalments and must pay the late fee each time. How much does Sarah spend in total on this purchase?
 - What is the percentage increase on the original cost for using the *buy now, pay later* service?
- Chikako uses a *buy now, pay later* service to pay for \$200 worth of groceries. She makes four payments of 25% with instalments due fortnightly. The service she uses has no set-up or account keeping fees. Late fees are 10% of the price for each instalment, but total late fees are capped at 25% of the price. In addition, a processing fee of 1.5% of the amount being paid is charged on each payment. No interest is charged.
 - If Chikako pays each instalment on time, how much will she pay in total?
 - If Chikako is late for each instalment (except the first payment), how much will she pay in total?
- James uses a *buy now, pay later* payment option to make a purchase of \$600. He makes four equal repayments across six weeks. No interest is charged. James misses his final payment and is charged a late fee of 20% of the instalment amount.
 - Copy the following table and fill in the amount owed for each payment.

Payment schedule	
First payment 1 April 2025	\$__ Paid
Second payment 15 April 2025	\$__ Paid
Third payment 29 April 2025	\$__ Paid
Fourth payment 13 May 2025 – Missed payment	\$__ Owing

- Determine the late fee owed on the fourth payment.
- How much does James pay in total?
- What is the percentage increase on the original cost for using the *buy now, pay later* service?

Worked example 4.2B Buying on terms

Della buys a fridge for a deposit of \$300 and 24 monthly payments of \$250.

- Find the total amount Della pays for the fridge.
- The price of the fridge when paid upfront is \$5999. Find the percentage increase of the amount Della pays compared to the price of the fridge correct to one decimal place.

Part	Think	Write
a	The total amount is equal to the sum of the deposit and the 24 monthly payments.	$\begin{aligned}\text{total} &= \$300 + 24 \times \$250 \\ &= \$300 + \$6000 \\ &= \$6300\end{aligned}$
b	Use the percentage increase formula. Substitute the total amount Della pays as the final value and the price as the initial value. Simplify and write the result correct to one decimal place.	$\begin{aligned}\text{percentage increase} &= \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\% \\ &= \frac{6300 - 5999}{5999} \times 100\% \\ &= \frac{301}{5999} \times 100\% \\ &= 5.0\%\end{aligned}$



Worked example video: Buying on terms

- 4 What is the total payable for each of the following purchasing options?
- a \$75 deposit and 10 payments of \$40 b \$450 deposit and 18 payments of \$199
- c \$2999 deposit and 24 payments of \$899 d \$1200 deposit and 36 payments of \$299
- 5 Brian buys a washing machine for a deposit of \$400 and 12 monthly payments of \$250.
- a Find the total amount Brian pays for the washing machine.
- b The price of the washing machine when paid upfront is \$2995. Find the percentage increase of the amount Brian pays compared to the price of the washing machine correct to one decimal place.
- 6 Arthur buys a robot vacuum for a deposit of \$549 and 24 monthly payments of \$105.
- a Find the total amount Arthur pays for the vacuum.
- b The total amount Arthur pays is a 9.6% increase on the price of the vacuum when paying upfront. Find the price of the vacuum when paying upfront correct to the nearest dollar.

Worked example 4.2C Comparing payment methods

Bryce recently opened a small bakery and wants to purchase a new oven valued at \$24 000. There are three payment options that Bryce is considering.

Option 1: Pay the full amount upfront.

Option 2: Use a *buy now-pay later* service to pay the amount in four equal monthly instalments. There is a \$29.95 account set-up fee, a 10% late fee and a 2.5% processing fee for each transaction. Both late fees and processing fees are calculated as a percentage of the instalment amount (the set-up fee is not included). There are no account-keeping fees, and no interest is charged.

Option 3: Pay a deposit of \$5000 and then pay 36 monthly instalments of \$650.

- a Determine the amount Bryce would pay for each option, assuming she always pays on time.
- b Give one reason why Bryce might choose each of the payment options.
- c Give one reason why Bryce might not choose each of the payment options.
- d Bryce chooses **Option 2** but is late on the third and fourth payments. How much does she pay in total?

Part	Think	Write
a	Option 1: The total cost is just the value of the oven. Option 2: The total cost includes the set-up fee, the four equal payments and the 2.5% processing fee on each payment. Option 3: The total cost includes the deposit of \$5000 plus 36 payments of \$650.	Option 1: \$24 000 Option 2: $\\$24\,000 \div 4 = \\$6000$ $\\$29.95 + 4 \times \\$6000 \times 1.025 = \\$24\,629.95$ Option 3: $\\$5000 + 36 \times \\$650 = \\$28\,400$
b	For each option, consider the total amount Bryce would pay, whether any fees or interest are charged, whether the amount paid in an instalment is manageable and how spread out the payments are.	Option 1: There are no fees or additional interest charged. Option 2: The total payment is spread over four monthly payments. Option 3: The monthly instalment of \$650 is the most easily manageable of the three options.
c	For each option, consider the impact of the payment on the business, any fees or interest charged, how manageable the instalments are and the total amount Bryce would pay.	Option 1: \$24 000 is a lot of money to pay in one instalment, especially for a small business that only recently opened. Option 2: There are additional fees charged, including potential late fees. Option 3: The total amount to pay is the highest.
d	The total amount is equal to the total amount with no late fees plus the late fee of 10% paid on two instalments of \$6000.	$\begin{aligned} \text{total} &= \$29.95 + 4 \times \$6000 \times 1.025 \\ &\quad + 2 \times \$6000 \times 0.01 \\ &= \$24\,629.95 + \$1200 \\ &= \$25\,829.95 \end{aligned}$



Worked example video: Comparing payment methods

- 7 Taylor wants to purchase a new jet ski valued at \$17 000. There are three payment options that Taylor is considering.

Option 1: Pay the full amount upfront.

Option 2: Use a *buy now-pay later* service to pay the amount in four equal monthly instalments. There is a \$10.99 account set-up fee, a 15% late fee and a 1.2% processing fee for each transaction. Both late fees and processing fees are calculated as a percentage of the instalment amount (the set-up fee is not included). There are no account-keeping fees, and no interest is charged.

Option 3: Pay a deposit of \$4500 and then pay 24 monthly instalments of \$575.

- Determine the amount Taylor would pay for each option, assuming he always pays on time.
- Give one reason why Taylor might choose each of the payment options.
- Give one reason why Taylor might not choose each of the payment options.
- Taylor chooses **Option 3**. What is the percentage increase of the amount he pays compared to the upfront price of the jet ski?

Problem solving and reasoning

- 8 Recall Linus from Worked example 4.2A. Linus chooses a different *buy now, pay later* service to buy the laptop where he makes 26 payments in total across a year, with payments made fortnightly. In addition, there is a \$35 set-up fee and a monthly account-keeping fee of \$8 starting from the second month.
 - a Excluding any fees, how much does Linus pay each fortnight, correct to the nearest dollar?
 - b Assuming Linus makes each payment on time, how much does Linus pay in total including fees?
 - c Should Linus have changed service provider? Explain.
- 9 Regina uses a *buy now, pay later* service for online shopping. She uses the service on a purchase of \$350 and two weeks later uses it again on a purchase of \$295. The service requires four equal payments on each purchase, paid fortnightly from the date of purchase. Regina already has an account and has already paid the annual account-keeping fee. There is a 1.2% processing fee on each transaction. The payments on separate purchases are considered separate transactions. Assume Regina makes all payments on time.
 - a Create a payment schedule for the purchases showing how much Regina must pay for each instalment, including the processing fees. Label the day of the first purchase as day 0 and the day of the second purchase as day 14.
 - b Regina budgets \$175 per week to go towards her *buy now, pay later* payments. Is this enough money to cover her payments? Explain.
- 10 George uses a *buy now, pay later* service to pay for some new tools. He uses the service on a purchase of \$588 and two days later, uses it again on a purchase of \$460. He uses the same service as Regina in question 9, but George has to pay an account set-up fee of \$12. Assume George makes all payments on time.
 - a Create a payment schedule showing how much George must pay for each instalment, including the processing fees. Label the day of the first purchase as day 0 and the day of the second purchase as day 2.
 - b George earns \$25 per hour and works 40 hours per week. In a week where he must make two payment instalments, what percentage of George's weekly income goes towards the payments?
 - c George makes the payments across approximately six weeks, starting from the day of the first purchase when the first instalment is paid. What percentage of his income during this period goes towards the payments?
 - d Give George some advice about using *buy now, pay later* services in the future.
- 11 Chrissy is buying a birthday present for her niece. The total price is \$250. Chrissy can't afford the upfront payment, so she chooses a layby service with the store instead. She pays a deposit of 10% of the purchase price and then agrees to make five equal fortnightly payments. There is a \$3 fee for using the layby service.
 - a How much does Chrissy expect to pay for each of the five instalments?
 - b How much does Chrissy expect to pay in total?
 - c Chrissy is struggling financially due to increased costs of living and fails to make the final payment. The store cancels the layby, retains \$11 to cover administrative costs, and refunds Chrissy the rest of the money. How much money is Chrissy refunded?
- 12 Opstrafone is a phone company offering the following payment options for a new phone. The first payment in each option is considered the deposit. The options do not include a phone plan, only the purchase of the phone.

Option 1: 12 monthly payments of \$124.92

Option 2: 24 monthly payments of \$62.46

Option 3: 36 monthly payments of \$41.64

 - a Compare the total cost of the three options.
 - b Give one reason why someone might not choose **Option 3**.

- 13 Find a *buy now, pay later* service online and research their fees and payment structure. How easy is it to find this information?
- 14 A store is offering a gaming computer for a deposit of \$150 and 18 monthly payments of \$120. Buying on terms costs an additional 4.6% compared to paying upfront. What is the price of the computer when paying upfront? Give your answer to the nearest dollar.

Challenge

- 15 A *buy now pay later* service updates the way it structures its payments. Payments are still fortnightly, but instead of each payment being 25%, the first payment is 10% of the total, the second 20%, the third 30% and the last 40%. The cost of late repayments is 1% of the overdue amount per day late.
- Luca buys a new phone for \$1397 using this service. How much does he pay over the first three payments?
 - One day after the final payment was due, Luca still owes the final instalment. How much does he owe, including the late fee?
 - Luca pays the final amount two days after it is due. How much does he pay for the phone altogether?
 - Give one advantage and one disadvantage to the customer of using this new payment structure as opposed to 25% each fortnight.

Related resources



4.2 Quick quiz



Investigation: Payment options

Lesson 4.3

Purchasing a car

Learning intentions

By the end of this lesson, you will be able to ...

- describe and calculate the various costs associated with purchasing a car.

Purchasing a car

- There are a number of costs to consider when purchasing a car.
 - The car can be paid upfront using savings, or money can be borrowed in the form of a loan. Regular loan repayments are required and will include interest charges on the loan.
 - The mandatory additional costs when buying a car are registration, vehicle tax, stamp duty and compulsory third-party insurance.
 - Other costs can include a dealer delivery charge and other insurance costs.
- A car dealer may advertise the “drive-away” price of a vehicle. This price includes all payments that are required before the car can be driven on public roads (including all registration costs, inspection fees, stamp duty, compulsory third-party insurance and dealer delivery fee).

Registration

- All vehicles that are driven or towed on public roads in NSW must be registered. The cost of registration usually includes a registration fee, number plate fee, vehicle tax and an inspection fee.
- The annual cost to register a vehicle in 2025 was \$79. Registrations can also be paid half-yearly (every six months) or quarterly (every three months).
- The fee to transfer the registration of a vehicle from one owner to another was \$39 in 2025 if transferred within 14 days of acquiring the vehicle, and \$182 if any later.
- The fee for a new or replacement standard number plate in 2025 was \$55. There are increased fees for special number plates.
- Vehicle tax is calculated based on the weight of the vehicle without a load (known as the tare weight) and whether the vehicle is being used mostly for private purposes or mostly for business purposes. The tax on vehicles used for business purposes is higher than for vehicles used mostly for private purposes.
→ The table below shows the vehicle tax on some vehicles in 2025.

Table 4.3-1 Vehicle tax in NSW

Size of vehicle	Tare weight (kg)	Private use (\$)	Business use (\$)
Cars, station wagons, trucks	up to 975	266	429
	976 to 1154	308	487
	1155 to 1504	374	588
	1505 to 2504	570	883
Motorcycles		83	83

Source: <https://www.nsw.gov.au/driving-boating-and-transport/vehicle-registration/fees-concessions-and-forms/vehicle-registration-fees>

- An annual safety inspection report, known as a “pink slip”, is required to register most light vehicles more than five years old. In 2025, the inspection fees for light vehicles and motorcycles were \$49 and \$28, respectively.

Stamp duty

- Stamp duty is a state tax that is based on the market value of the vehicle or the price paid for it, whichever is the greater. It is charged at the rate of 3% of the market value up to \$45 000 plus 5% of the value over \$45 000.
→ Stamp duty is always paid when a car is purchased, whether new or used.

Compulsory third-party insurance (CTP)

- Compulsory third-party (CTP) insurance, often referred to as a “green slip”, is necessary to register a vehicle in NSW. It provides compensation to other people injured or killed when the insured driver’s vehicle is involved in an accident.
→ It does not cover damage to other vehicles, property or your vehicle or theft of your vehicle. These events are covered by other types of insurance.
- CTP insurance is paid each time a vehicle is registered and lasts for the duration of the registration.

- The cost of a green slip varies and depends on a number of factors, including the type and age of the vehicle, where it is stored when not in use, and the age and driving record of the drivers.
 - You can find examples of the cost of CTP (premiums) on the www.sira.nsw.gov.au.
 - In July 2024, the average cost of CTP insurance was \$573 for all regions and vehicle types.

Other types of insurance

- There are types of insurance, other than CTP, available for motor vehicles, such as comprehensive insurance, third-party property insurance and roadside assistance. These are not compulsory but are advisable.
 - Comprehensive insurance covers damage to, or theft of, your vehicle as well as damage to other vehicles and property in the case of an accident.
 - Third-party property damage vehicle insurance covers you for damage caused by your vehicle to someone else's vehicle or property. It does not cover damage to, or theft of, your vehicle.
 - Roadside assistance covers help after a mechanical failure whether at home or on the road. This can include jump-starting the engine, changing a flat tyre, battery replacement and vehicle towing.
- Comprehensive insurance is more expensive than third-party property insurance because it covers more types of incidents. Both types of insurance vary in price according to factors such as the driver's age, driving experience and driving record, the type and age of the vehicle and where it is stored when not in use. Many insurance companies will give an online quote for car insurance.

Dealer delivery

- When purchasing a new car, the dealer will often charge a fee for “dealer delivery”. This fee represents the cost to the dealer of preparing the car for delivery to the buyer. The fee varies from dealer to dealer and is sometimes waived or reduced in order to induce people to buy from one dealer rather than another.

Trade-in

- As part of the agreement when you purchase a new car, the dealer may buy your current car, if you have one, at an agreed price. When you sell your current car to the dealer in this way, it is called a trade-in.

Standing costs and loan repayments

- **Standing costs** refer to any costs incurred just by owning the vehicle. These are costs that must be paid to use the vehicle and do not depend on the amount or way it is used.
 - This includes any fees that must be paid to ensure legal use of the vehicle, such as registration and CTP insurance. These are often referred to as on-road costs.
 - Other insurance costs, while not compulsory, must also be paid to ensure coverage in the event of an accident, damage or theft.
 - If a loan is used to purchase the car, then any interest charged on the loan is considered a standing cost of the vehicle.

- Another standing cost is the amount a vehicle decreases in value, since any decrease in the value of the vehicle (e.g. due to age and use) represents money that is impossible to recover by selling the vehicle.
- Any loan payments (excluding interest charged) are not standing costs but must still be paid regularly and on time to continue ownership and use of the car.

Worked example 4.3A Calculating vehicle tax

Use Table 4.3-1 to determine the extra vehicle tax paid on a new Toyota Corolla, which weighs 1250 kg, when used for business purposes compared with private purposes.

Think	Write
In Table 4.3-1, 1250 kg is in the range 1155 kg to 1504 kg. Subtract the amount for private use from the amount for business use.	cost for business use = \$588 cost for private use = \$374 extra tax = \$588 – \$374 = \$214

 **Worked example video:** Calculating vehicle tax

Understanding, fluency and communicating

Answers: Lesson 4.3

- 1 Complete the following to calculate vehicle tax paid, for private use, of a vehicle that weighs 1060 kg.
In Table 4.3-1, 1060 kg is in the range ___ kg to ___ kg.
Cost to register for private use is \$__.
- 2 Use Table 4.3-1 to help you calculate the vehicle tax paid, for private use, of a vehicle that weighs:
a 1820 kg **b** 1150 kg **c** 1450 kg
- 3 How much more expensive is it to register a vehicle for business use than for private use, if the vehicle weighs:
a 2130 kg? **b** 975 kg? **c** 1200 kg?

Worked example 4.3B Calculating the stamp duty on a new car

Calculate the stamp duty to be paid on the purchase of a new car that has a recommended retail price of \$57 000.

Think	Write
The stamp duty payable is 3% of the market value up to \$45 000 plus 5% of the value over \$45 000. Write 3% as 0.03 and 5% as 0.05. Multiply \$45 000 by 0.03. Subtract \$45 000 from \$57 000 and multiply by 0.05.	$0.03 \times \$45\,000 = \$1350$ $0.05 \times (\$57\,000 - \$45\,000) = \$600$ stamp duty = \$1350 + \$600 = \$1950

 **Worked example video:** Calculating the stamp duty on a new car

Helpful hints

- If the price paid for the car is not more than \$45 000, the stamp duty is 3% of its price.
- If the price is more than \$45 000, the stamp duty is 3% of \$45 000 (\$1350) plus 5% of \$(price – 45 000).

4 Complete the following to calculate the stamp duty.

- a** market value = \$17 900
 stamp duty = $3\% \times \$\_\_\_\_\_\_$
 $= 0.03 \times \$\_\_\_\_\_\_$
 $= \$\_\_\_\_\_\_$
- b** market value = \$52 380
 stamp duty = $\_\_\% \times \$45\,000 + \_\_\% \times (\_\_\_\_\_\_ - \$45\,000)$
 $= \_\_\times \$45\,000 + \_\_\times \$\_\_\_\_\_\_$
 $= \$\_\_\_\_\_\_ + \$\_\_\_\_\_\_$
 $= \$\_\_\_\_\_\_$

5 Calculate the stamp duty that would be charged on a vehicle that has a market value of:

- a** \$21 990 **b** \$35 699 **c** \$49 000 **d** \$93 600.

Worked example 4.3C Comparing loan repayments and purchase price

Billie buys a new car for \$45 000. She is unable to pay this amount upfront, so she uses a loan which requires a 20% deposit and monthly payments of \$1075 for three years.

- a** Determine how much money Billie pays for the car in total.
b How much more did Billie pay for the car using a loan than she would have by paying upfront?

Part	Think	Write
a	The amount paid includes the 20% deposit and the 36 monthly payments. Find 20% of the amount paid for the car by multiplying by 0.2 (or dividing by 5). Find the total paid in the monthly payments. Add the deposit and monthly pays total together to find the amount paid towards the loan.	$20\% \times \$45\,000 = 0.2 \times \$45\,000$ $= \$9000$ $3 \times 12 \times \$1075 = \$38\,700$ total = $\$9000 + \$38\,700$ $= \$47\,700$
b	Find the difference between the amount paid towards the loan and the price of the car.	difference = $\$47\,700 - \$45\,000$ $= \$2700$



Worked example video: Comparing loan repayments and purchase price

Helpful hints

- The deposit for a loan is a percentage of the price of the car, not a percentage of the total amount that will be paid towards the loan (the deposit plus payment instalments).

- 6 Determine the total amount paid towards each of the following loans.
- Loan of \$23 000, 20% deposit, monthly payments of \$550 for three years
 - Loan of \$29 000, 10% deposit, monthly payments of \$1200 for two years
 - Loan of \$15 000, 20% deposit, monthly payments of \$700 for 18 months
 - Loan of \$40 000, 15% deposit, monthly payments of \$1020 for three years
 - Loan of \$76 000, 18% deposit, monthly payments of \$1150 for five years
 - Loan of \$49 000, 12% deposit, monthly payments of \$975 for 48 months
- 7 Selma buys a new car for \$52 000. She is unable to pay this amount upfront, so she uses a loan which requires a 20% deposit and monthly payments of \$950 for four years.
- Determine how much money Selma pays for the car in total.
 - How much more did Selma pay for the car using loan than she would have by paying upfront?

Worked example 4.3D Calculating the cost of buying a new car (drive-away price)

Tony buys a new Toyota Camry from a car dealer with a drive-away price of \$39 990. As a special deal, the drive-away price includes one year of roadside assist. Tony uses a loan to pay for the car with a deposit of 15% and 24 monthly payments of \$1550. In addition, Tony pays upfront for comprehensive car insurance for one year for \$1200.

- How much does Tony pay in total for his new car?
- How much would Tony have saved if he had paid for the car upfront instead of using a loan?

Part	Think	Write
a	The total amount Tony pays includes the total he pays towards the loan (which covers the drive-away car price) and the additional insurance. The loan amount is 15% of \$39 990 plus 24 payments of \$1550. Add this value to the cost of comprehensive insurance to find the total amount Tony pays.	$\begin{aligned} \text{loan total} &= 15\% \times \$39\,990 + 24 \times \$1550 \\ &= 0.15 \times \$39\,990 + 24 \times \$1550 \\ &= \$5998.50 + \$37\,200 \\ &= \$43\,198.50 \\ \text{total} &= \$43\,198.50 + \$1200 \\ &= \$44\,398.50 \end{aligned}$
b	Find the difference between the loan total and the price of the car.	$\begin{aligned} \text{savings} &= \$43\,198.50 - \$39\,990 \\ &= \$3208.50 \end{aligned}$



Worked example video: Calculating the cost of buying a new car (drive-away price)

Helpful hints

- The drive-away price usually includes all registration costs, inspection fees, stamp duty, CTP insurance and any dealer delivery fee.

- 8 David buys a new Ford Focus from a car dealer with a drive-away price of \$43 000. David uses a loan to pay for the car with a deposit of 20% and 36 monthly payments of \$1010. In addition, David pays upfront for third-party car insurance for one year for \$420.
- How much does David pay in total for his new car?
 - How much would David have saved if he had paid for the car upfront instead of using a loan?
- 9 Jacqui is planning to buy a new car with a drive-away price of \$36 000. She will buy comprehensive car insurance for \$1056 which she will pay upfront. She needs a loan to pay for the car and has two options to choose from.
- Option 1:** 20% deposit, monthly payments of \$900 for three years
- Option 2:** 15% deposit, monthly payments of \$600 for five years
- Which option should Jacqui choose to save the most money and how much would she save?
- 10 Stuart buys a new car with a drive-away price of \$58 000. He trades in his old car with a trade-in value of \$22 000. He uses a loan to pay for the remainder of the cost with an 18% deposit and monthly payments of \$1350 for two years. Stuart also pays \$850 upfront for one year of comprehensive car insurance. How much does Stuart pay in total for the new car?

Worked example 4.3E Calculating the cost of buying a new car (RRP)

A new car has a recommended retail price (RRP) of \$37 000 and weighs 1704 kg. The car is for private use and the owner decides to take out comprehensive insurance that costs \$960 for the first year.

- Calculate the total cost of purchasing the car, assuming the drive-away price is equal to the purchase price plus the cost of registration (including the registration fee, vehicle tax and new standard number plates), stamp duty, CTP insurance worth \$615 and a dealer delivery fee of \$550.
- If the buyer receives a trade-in value of \$15 000 on her old vehicle, what is the total cost for changing to the new car?

Part	Think	Write
a	The registration fee for all vehicles is \$79. The weight of the car (1704 kg) is in the range 1505 kg to 2504 kg. From Table 4.3-1, the vehicle tax for private use is \$570. The cost for new standard number plates is \$55. As the RRP is less than \$45 000, the stamp duty is 3% of the RRP. The CTP insurance, dealer delivery and comprehensive insurance costs are given. Add all costs to find the total cost of the car.	RRP = \$37 000 registration fee = \$79 vehicle tax = \$570 number plates = \$55 stamp duty = $0.03 \times \\$37\,000 = \\$1110$ CTP insurance = \$615 dealer delivery = \$550 comprehensive insurance = \$960 total = \$40 811
b	The changeover price is the difference between the total and the trade in value.	changeover price = $\\$40\,811 - \\$15\,000$ = \$25 811



Worked example video: Calculating the cost of buying a new car (RRP)

11 Complete the following table to calculate the total cost to purchase each of the following new vehicles.

Costs	Holden	Mazda	Toyota sedan	Toyota SUV	Yamaha motorcycle
RRP (\$)	37 990	31 450	18 000	52 870	14 999
Weight (kg)	1637	1440	1040	1970	N/A
Private (P) or business (B)	B	P	P	B	P
Registration fee (\$)					
Vehicle tax (\$)					
Number plates (\$)	55	55	113	249	53
Stamp duty (\$)					
CTP insurance (\$)	620	487	528	660	280
Dealer delivery (\$)	790	585	499	887	389
Insurance (\$)	1089	790	560	1185	299
Trade-in value (\$)	0	5000	1000	12 000	0
Total					

Worked example 4.3F Calculating the total cost of buying a used car

Find the total cost of purchasing and insuring a seven-year-old Honda hatchback that has an advertised price of \$10 500. The existing registration is transferred to the new owner one week after purchase. The car is still covered by the pink-slip from the previous registration. The purchaser pays for the car upfront and then also pays \$479 for third-party property damage insurance.

Think	Write
The price of the car is \$10 500. As this is paid upfront, there are no loan payments to consider. Transfer of registration fee is \$39 for used vehicles if completed less than two weeks after purchase. The price of the car is less than \$45 000, therefore stamp duty is 3% of its \$10 500 price. The insurance costs \$479. Add all costs to find the total cost of the car.	car price = \$10 500 transfer of registration = \$39 stamp duty = $0.03 \times \\$10\,500$ = \$315 insurance = \$479 total = \$11 333



Worked example video: Calculating the total cost of buying a used car

Helpful hints

- CTP insurance is paid when a car is registered and is transferred with the registration, so it is not due again until the next time registration is due.

- 12** Complete the following to find the total cost of purchasing and insuring a four-year-old car that is advertised for \$43 900. The registration is transferred two days after purchase. Comprehensive insurance is \$1560.

car price = \$__
 registration transfer fee = \$__
 stamp duty = $0.03 \times \$\_ = \$\_$
 insurance = \$__
 total = \$__

- 13** Find the total cost of purchasing and insuring a six-year-old Toyota sedan that has an advertised price of \$23 900. The registration is transferred three weeks after purchase. The previous registration lapsed two months prior to purchase so a new registration is required, including the registration fee, vehicle tax and CTP insurance of \$609. A new pink slip is also required. Third-party property insurance is \$463.
- 14** Find the total cost of purchasing and insuring a two-year-old Kawasaki 1400cc motorcycle that has an advertised price of \$14 890. Comprehensive insurance is \$678.

Problem solving and reasoning

- 15** Stamp duty is calculated as 3% of the price of the vehicle up to \$45 000, plus 5% of the price over \$45 000.

- a** Complete the following table to determine the stamp duty payable for vehicles of different prices.

Price (\$'000)	3% of the price up to \$45 000	5% of the price over \$45 000	Total
10	$0.03 \times \$10\ 000 = \$300$		
20			
30			
40			
45	$0.03 \times \$45\ 000 = \$1350$		
50	$0.03 \times \$45\ 000 = \$1350$	$0.05 \times (\$50\ 000 - \$45\ 000) = \$250$	\$1600
60	$0.03 \times \$45\ 000 = \$1350$	$0.05 \times (\$60\ 000 - \$45\ 000) = \$750$	
70			
80			

- b** Use the information in the table to draw a piecewise graph, with price (the independent variable) on the horizontal axis and stamp duty (the dependent variable) on the vertical axis.
- c** Use the graph to estimate the stamp duty on a vehicle purchased for:
- i** \$35 000 **ii** \$65 000 **iii** \$78 000.

- 16** Here is some information about Richard who is buying a new car and needs to purchase CTP insurance.

Richard is buying a new car. He has an open NSW drivers licence. There is no entitlement to GST input credit. The start date for the insurance will be the 1st of the month/next month/this year. The vehicle is a new Mazda 6 sedan that is normally garaged at Parramatta, postcode 2150 and is expected to travel up to 15 000 km each year. The car will be used two days per week to travel to work. The car will be privately owned and registered for private use. A blue slip (new car safety inspection) is required. It does not have a current CTP insurance policy because it is a new car. It will also be covered by comprehensive insurance with NRMA Insurance. Richard has continuously held comprehensive insurance with NRMA for two years. The policy does not have a no-claim discount. Richard, the sole owner/driver, is 20 years old (Date of birth 01/01/(Current year – 20)). He has not had any accidents in the three years he has had his licence and has not lost any demerit points. Richard is not a member of NRMA roadside service.

- a** Go to the www.greenslips.nsw.gov.au and use Richard's information to get a quote on a green slip.
- b** What is the cheapest quote for Richard's green slip?
- c** What would the cheapest quote be if Richard had had one at-fault accident and lost four demerit points?

17 Here is some information about a person who wants to buy comprehensive car insurance.

Vehicle details: New Toyota Corolla hatchback (2.0 L ascent sport) that is usually garaged at Manly, postcode 2095. Insurance cover required is \$25 990. The car is for private use and the owner has no finance owing on the car. The driver is a 20-year-old female with date of birth 01/01/(Current year – 20)) with two years driving experience and has had no accidents. She expects to drive up to 10 000 km per year. She wants an excess of \$600 on the policy and has no other relevant policies or memberships. As this is her first car there is no previous insurer. She was 17 years old when she got her provisional drivers licence and has never had her licence disqualified, cancelled or suspended. She would not like roadside assist.

- a** Search the Internet for an insurance company (such as NRMA insurance at www.nrma.com.au) and use the information to get a quote for comprehensive insurance for the new car.
 - b** Vary the age of the driver (20, 25, 30, 35 years, etc.) and record and compare costs.
 - c** Vary the gender of the driver for the ages used in part **b** and compare costs.
 - d** Using a map and a list of postcodes, vary the locality where the vehicle is garaged and compare costs.
 - e** Investigate the change in costs for a driver who has had an at-fault accident.
 - f** Vary the type of vehicle and compare costs. For example, compare the costs for small versus large passenger vehicles, 4WDs, people movers and light commercial vehicles.
 - g** Compare the costs from other insurers. (Try www.comparethemarket.com.au/car-insurance/australia/new-south-wales.)
 - h** Is there an age excess to be paid on top of the basic excess?
- 18** Investigate and compare the advantages and disadvantages of comprehensive and third-party property insurance for cars. In what circumstances might one form of insurance be more suitable than the other?
- 19** Investigate and compare the costs of third-party property insurance at www.nrma.com.au/car-insurance. Use the example and investigations suggested in question **17**.
- 20** From the information gathered in the previous questions, make a list of the factors that affect insurance premiums (such as type of vehicle, driver experience).

Challenge

21 The stamp duty of a car that costs less than \$45 000 can be calculated as a formula

$$\text{stamp duty} = 0.03 \times P$$

where P is the price of the car.

- a** Use the above formula to calculate the price of a car where the stamp duty costs \$804.
- b** Write a formula which relates the price of a car over \$45 000 with the stamp duty. Use this formula to find the price of a car where the stamp duty costs \$2095.

Related resources



4.3 Quick quiz

Lesson 4.4

Ongoing costs of a vehicle

Learning intentions

By the end of this lesson, you will be able to ...
→ examine and calculate the ongoing costs of a vehicle.

Operating costs

- Unlike many purchases, the costs of owning a vehicle do not end when you take your purchase home. In addition to annual costs such as registration and insurance, and loan repayments (if relevant), there are a number of other **operating costs** associated with vehicle ownership.
 - Operating costs include things like fuel, tyres, servicing and repairs. These costs will vary depending on the type of vehicle, how often it is used and under what conditions.
 - Before purchasing a vehicle, it is important to understand the type of ongoing costs you can expect and how these might differ between vehicles.



Worked example 4.4A Calculating ongoing costs of a vehicle

Carla makes monthly loan repayments of \$650 for the car she recently purchased. Every year, Carla will need to pay roughly \$2310 to register and insure her car. She works out that her average fuel expenditure is \$45 per week. Between servicing and tyre replacement, Carla spends a further \$580 per year on car maintenance.

How much will Carla spend on her car in the next 12 months?

Think	Write
Carla has four types of payments to make: • loan repayments (monthly) • registration/insurance (annually) • fuel (weekly) • maintenance (annually). Find the yearly amount for each type of payment (call them L , R , F and M), then add these together to find the total.	$\begin{aligned} L &= \$650 \times 12 \\ &= \$7800 \\ R &= \$2310 \\ F &= \$45 \times 52 \\ &= \$2340 \\ M &= \$580 \\ \text{total} &= L + R + F + M \\ &= \$13\,030 \end{aligned}$

 **Worked example video:** Calculating ongoing costs of a vehicle

Understanding, fluency and communicating

Answers: Lesson 4.4

- 1 For each of the following, calculate:
 - a the total amount paid in loan repayments for one year
 - b the total amount paid across the full term of the loan.
 - i A loan with a term of five years, with monthly repayments of \$274
 - ii A loan with a term of three years, with monthly repayments of \$641
 - iii A loan with a term of two years, with monthly repayments of \$533
 - iv A loan with a term of seven years, with monthly repayments of \$897
- 2 For each of the following, calculate:
 - a the total cost of registration and insurance across five years.
 - b the average cost of registration and insurance per month
 - i Registration and insurance costing a total of \$2311 annually
 - ii Registration and insurance costing a total of \$1917 annually
 - iii Registration and insurance costing a total of \$2225 annually
 - iv Registration and insurance costing a total of \$2732 annually
- 3 For each of the following, calculate:
 - a how many times the vehicle is refuelled in one year
 - b the total cost of fuel for the year.
 - i A tank of fuel lasts three weeks and the average cost to refuel is \$72.
 - ii A tank of fuel lasts six weeks and the average cost to refuel is \$65.
 - iii A tank of fuel lasts one week and the average cost to refuel is \$87.
 - iv A tank of fuel lasts 2.5 weeks and the average cost to refuel is \$96.
- 4 For each of the following, calculate:
 - a number of services required over five years if vehicle travels 30 000 km per year
 - b the total cost of servicing across five years.
 - i A vehicle requires servicing every 10 000 km and each service costs \$280, on average.
 - ii A vehicle requires servicing every 15 000 km and each service costs \$175, on average.
 - iii A vehicle requires servicing every 10 000 km and each service costs \$320, on average.
 - iv A vehicle requires servicing every 20 000 km and each service costs \$410, on average.
- 5 Sandy bought a car for \$18 000. She paid an additional \$2800 for 'on-road' costs, including approximately \$2200 for registration and insurance, which she will have to pay annually. Other costs that she expects to pay on an ongoing basis include:
 - petrol: \$114 per month
 - tyre replacements: \$175 per tyre, every two years
 - servicing costs: \$320 per year.
 - a How much did Sandy spend on the purchase, running and maintenance of her car in the first year? Assume the tyres don't need changing in the first year.
 - b How much will Sandy spend on running and maintaining her car in the second year of owning it?

Worked example 4.4B Comparing the ongoing costs of different vehicles

Convert the information in Table 1 about the Suzuki Swift into monthly costs to find the total monthly cost of this vehicle, including loan repayments.

Table 1 Average running costs

	Up-front on-road costs	Five-year loan monthly repayments	Annual registration and insurance	Fuel per week	Annual servicing	Tyres over five years
Suzuki Swift	\$28 930.60	\$572.31	\$2115.72	\$25.35	\$441.00	\$916.20
Mazda 3 Hatchback	\$37 507.98	\$742.00	\$2835.24	\$34.80	\$506.76	\$1020.00
Volkswagen Golf	\$47 796.25	\$945.52	\$4083.36	\$36.76	\$780.24	\$1060.20
Toyota Camry	\$44 170.25	\$873.79	\$3140.64	\$28.52	\$309.96	\$660.00
Nissan X-Trail	\$53 665.60	\$1061.63	\$3535.68	\$38.66	\$756.00	\$1660.20
Ford Everest	\$60 468.20	\$1226.08	\$3474.60	\$40.57	\$600.96	\$1516.20

Think	Write
Loan repayments are already given as a monthly rate.	Monthly loan repayments: $L = \$572.31$
The costs for registration/insurance (R) and servicing (S) are given as annual amounts, so the average monthly rates are found by dividing these figures by 12.	Monthly cost of registration and insurance: $R = \frac{\$2115.72}{12} = \$176.31$ Monthly average of servicing costs: $S = \frac{\$441}{12} = \$36.75$
The cost of fuel (F) per week is \$25.35, so the cost of fuel for one month is found by multiplying \$25.35 by 52 to get the yearly rate, then dividing by 12 to get the monthly rate.	Monthly fuel costs: $F = \frac{\$25.35 \times 52}{12} = \$108.95$
The cost of tyres (T) is given as a total cost over five years. Since there are $5 \times 12 = 60$ months in this period, we divide the total by 60.	Monthly average of tyre costs: $T = \frac{\$916.20}{5 \times 12} = \$15.27$
The total of all ongoing costs per month for the Suzuki Swift is found by adding all of these values together.	Total monthly costs: $L + R + S + F + T$ $= \$572.31 + \$176.31 + \$36.75$ $+ \$108.95 + \$15.27$ $= \$909.59$



Worked example video: Comparing the ongoing costs of different vehicles

Use Table 1 to answer these questions.

- 6 Table 1 shows the average running costs for a selection of vehicles. For each vehicle, find the:
 - a total of all loan repayments in a single year
 - b total cost of fuel for a year
 - c average annual cost of tyres.
- 7 Assuming one year is equal to 365 days, what is the daily cost of registration and insurance (combined) for each vehicle?
- 8 Rank the six vehicles from cheapest to most expensive on each of the following criteria.
 - a Up-front costs
 - b Yearly registration and insurance
 - c Fuel costs
 - d Servicing costs
 - e Cost of tyres
- 9 For each vehicle, calculate:
 - a the total of all loan repayments across five years
 - b the difference between the up-front on-road costs and the total found in part a.



Problem solving and reasoning

- 10 Add another row to Table 1 using the following information.

A Harley-Davidson can be purchased for \$31 394, which includes all on-road costs. Alternatively, the purchase could be financed by a three-year loan with monthly repayments of \$982. Registration and insurance fees are \$2451 per year. On average, this motorcycle will cost \$37 per week in fuel, \$158 per year in servicing, and \$844 across five years for tyres.

Calculate the following:

- a total of all loan repayments in a single year
 - b total cost of fuel for a year
 - c average annual cost of tyres.
- 11 Use the information from Table 1 to answer the following questions.
 - a Calculate the average operating costs per day for the Mazda 3 hatchback, the Toyota Camry, the Ford Everest and the Nissan X-Trail, assuming one year is equal to 365 days.
Hint: you may have already completed some of the necessary calculations in the **Understanding, fluency and communicating** section of this lesson.
 - b Four people, who live in the same general area and work in the same location, drive to work five days per week. Richard drives a Mazda 3 hatchback, Stephanie drives a Toyota Camry, Lilly drives a Ford Everest and Paul drives a Nissan X-Trail.
 - i Over a four-week period, how much does it cost each person to drive to work, assuming the average daily operating cost is completely work related?
 - ii The four drivers decide to start carpooling, so that each person drives everyone to work one week out of every four weeks. How much does each person save every four weeks by carpooling?

12 Use the data in Table 1, which is based on each vehicle travelling 15 000 km per year.

- a** Calculate the average operating costs per kilometre for the Volkswagen Golf, the Suzuki Swift and the Nissan X-Trail.
- b** Three people drive to work over an average distance of 64 km return, from each of their homes. They work a regular five-day week. Helen owns a Volkswagen Golf, Greg owns a Suzuki Swift and Peta owns a Nissan X-Trail.
 - i** Over a three-week period, how much does it cost each person to drive to work?
 - ii** The three drivers start carpooling and take turns driving each other to work on a weekly basis. How much does each person save every three weeks by sharing the driving?



Challenge

13 Research the ongoing costs associated with electric or hybrid vehicles. Summarise any differences between petrol, hybrid and electric vehicles in terms of:

- a** up-front on-road costs
- b** running and maintenance costs.

14 Ewan's current car has a fuel consumption rate of 9.7 L/100 km. He drives an average of 12 000 km each year. The average cost of fuel is 201 c/L.

- a** What is the cost of running his car per year?
- b** Ewan sells his car for \$14 000 and purchases a new car for \$24 990. His new car has a fuel consumption rate of 7.1 L/100 km.

Assuming the price of fuel and his driving distance stays the same, how many years will it take for Ewan to make up the costs of buying the new car?

Related resources



4.4 Quick quiz

Using spreadsheets

Syllabus links

Use a spreadsheet to model the costs of purchasing, running and maintaining a vehicle (MST-11-04)

Using spreadsheets to calculate car costs

See Oxford Digital for sample spreadsheets for questions in this lesson.

- 1** A spreadsheet can be used to calculate the total purchase cost of a vehicle and to compare costs. The spreadsheet below has two vehicles for comparison but using *fill across* will give more information. Enter the RRP, weight and type of use, and then the registration and stamp duty will be calculated automatically. Then enter CTP, dealer delivery and comprehensive insurance. The total cost will sum automatically in row 13.

	A	B	C
1	Comparing the costs of purchasing vehicles		
2	Cost items	Mazda sedan	Holden sedan
3	RRP	\$28,500.00	\$57,888.00
4	Weight (kg)	1440	1720
5	Personal (P) or business (B)	P	B
6	Registration fee	\$79.00	\$79.00
7	Vehicle tax	\$374.00	\$883.00
8	New standard number plates	\$55.00	\$55.00
9	Stamp duty	\$855.00	\$1,994.40
10	CTP insurance		
11	Dealer delivery		
12	Comprehensive insurance		
13	Total	\$29,863.00	\$60,899.40
14			
15	Vehicle tax		
16	Private use	\$374.00	\$570.00
17	Business use	\$588.00	\$883.00

In cell B6, the formula to calculate the registration of the Mazda sedan is:

=IF(B5="P",B16,B17)

In cell B9, the formula to calculate the stamp duty is:

=IF(B3>45000,(B3-45000)*0.05+1350,B3*0.03)

In cell B13, the formula to calculate the total cost is:

=SUM(B6:B12)+B3

In cell B16, the formula to calculate the private use vehicle tax is:

=IF(B4>1505,570,IF(B4>1155,374,IF(B4>976,308,266)))

In cell B17, the formula to calculate the business vehicle tax is:

=IF(B4>1505,883,IF(B4>1155,588,IF(B4>976,487,429)))

Use *fill across* to find the costs for the second vehicle (the Holden sedan).

Enter data for other vehicles.

- a Why is the formula in cell B13 *not* =SUM(B3:B12)?
 - b Look up the vehicle tax costs on vehicles with a tare weight greater than 2504 kg and less than 3564 kg. Edit the formulas in cells B16 and B17 to include this information.
 - c Create a similar spreadsheet to compare the costs of purchasing used cars. In each case, assume that the registration transfer includes current pink and green slips.
- 2 A spreadsheet can be used to compare the costs of different motor vehicles. Extra columns may be added to compare more vehicles. There are tables that calculate fuel cost and registration costs after the variables are entered. This spreadsheet compares the running costs for a Volkswagen Golf with the costs for a large Nissan X-Trail, both used for private purposes.

	A	B	C
1	Comparing the running costs of vehicles driven 15 000 km per year		
2	Average running costs	Volkswagen Golf	Nissan X-Trail (large)
3	Fuel	\$2,042.40	\$3,759.00
4	Tyres	\$184.00	\$455.00
5	Service and repairs	\$670.00	\$1,150.00
6	Green slip (CTP)	\$525.00	\$598.00
7	Registration	\$387.00	\$649.00
8	Comprehensive insurance	\$592.00	\$1,035.00
9	Other		
10			
11	Total	\$4,400.40	\$7,646.00
12	Saving (compared with max. cost)	\$3,245.60	\$0.00
13			
14	Fuel cost		
15	Economy in L/100 km	7.4	14
16	Kilometres travelled	15,000	15,000
17	Fuel cost per litre	\$1.84	\$1.79
18	Total fuel cost	\$2,042.40	\$3,759.00
19			
20	Registration		
21	Weight (kg)	1100	2055
22	Registration fee	\$79.00	\$79.00
23	Vehicle tax	\$308.00	\$570.00

In cell B3, type the formula =B18 to copy the total fuel cost from below.

In cell B7, the formula to calculate the cost of registration is:

=SUM(B22:B23)

In cell B11, the formula to calculate the total cost for the Volkswagen Golf is:

=SUM(B3:B9)

In cell B18, the formula to calculate the fuel cost is:

=B15*B16*B17/100

In cell B23, the formula to calculate vehicle tax is:

=IF(B21>1505,570,IF(B21>1155,374,IF(B21>976,308,266)))

Use *fill across* to find the costs for the second vehicle (the Nissan X-Trail).

Enter the data for other vehicles.

- What would the formula in cell B23 be if the car will be used for business purposes?
- In Excel, the formula =MAX(RANGE) can be used to find the maximum value that appears in a range of cells. The following formula is added to cell B12.
=MAX(B11:C11)-B11

What value appears in cell B12? Explain what this function does.

- A third car is added to the spreadsheet. Edit the formula from part **b** to account for the third car.
- Vary the petrol price in the spreadsheet in row 17. Describe the impact of a price increase of 5 cents per litres versus an increase of 50 cents per litre.

Lesson 4.5

Household bills

Learning intentions

By the end of this lesson, you will be able to ...
→ calculate household costs based on bills for utilities.

Utilities

- Most homes are connected to electricity, water, gas and internet services. Organisations that provide these services are called **utilities**. In this section you will read, interpret and perform calculations relating to various utility bills.

Worked example 4.5A Interpreting and analysing water bills

A small household with two occupants is sent a water bill that says they used 33 kL of water during the 90-day billing period.

- a** If the bill consists of a water usage charge at a rate of \$3.04 per kL and a service charge of \$289.34, calculate the total bill.
- b** The water bill also displays a table of “Targets for water-efficient households”, which suggests that a small household with two occupants should aim to use no more than 356 L/day. Is this household meeting their water-efficiency target?

Part	Think	Write
a	To find the water usage charge multiply the rate (\$3.04 per kL) by the total usage (33 kL). To find the total charge add the water usage charge and the service charge (\$289.34).	$\begin{aligned}\text{water usage charge} &= \$3.04 \times 33 \\ &= \$100.32 \\ \text{total charge} &= \text{water usage charge} \\ &\quad + \text{service charge} \\ &= \$100.32 + \$289.34 \\ &= \$389.66\end{aligned}$
b	To find this household’s average daily water consumption first convert 33 kilolitres into litres by multiplying by 1000. Next, divide the water usage by 90 to get the daily average. Compare this daily rate to the target of 356 L/day.	$\begin{aligned}33 \text{ kL} &= 33\,000 \text{ L} \\ \frac{33\,000 \text{ L}}{90 \text{ days}} &\approx 366.67 \text{ L/day} \\ \text{Since } 366.67 < 356, &\text{ this household is not} \\ &\text{meeting their water efficiency target.}\end{aligned}$



Worked example video: Interpreting and analysing water bills

Understanding, fluency and communicating

Answers: Lesson 4.5

Use the sample water bill to answers questions 1 to 3.

WATER BILL

Mr S Ample
12 Water Street
NORTH WATERVILLE

Customer Number	22988701
Due Date	23 April 2026
Amount	\$543.63

ACCOUNT DETAILS

Account for residential property

Usage Charges (GST does not apply)
For period 01/01/2026 to 31/03/2026 (90 days)

Water Usage	
68 kL @ \$3.12 per kL =	\$212.16
Total Usage Charges	\$212.16

Service Charge Details (GST does not apply)	
Water Service Charge	\$141.46
Wastewater Service Charge	\$190.01
Total Service Charges	\$331.47

Total Current Charges	\$543.63

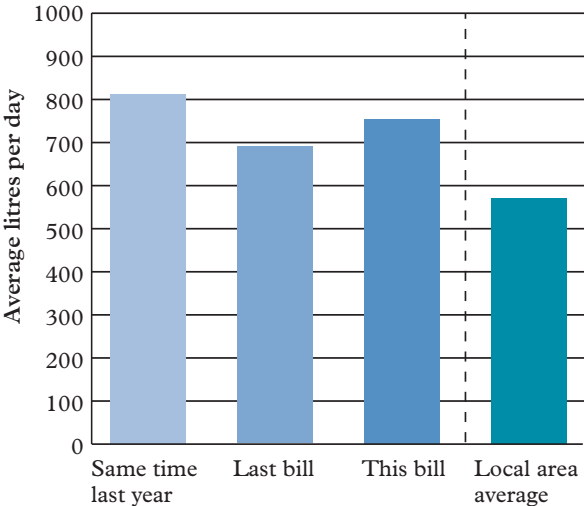
YOUR CHARGES EXPLAINED

Water Usage
Recorded by your water meter, this charge covers the amount of water used at your property. This charge is billed per kilolitre (kL) of water you use.

Service Charges
Your water and wastewater service charges are fixed charges for access to our water supply and wastewater systems. They also help us maintain, renew and expand these systems so we can continue to deliver high quality drinking water and safe wastewater removal now and into the future.

NOTICEBOARD

Your average daily water usage comparison



Targets for water-efficient households

People per household	Property size		
	Small	Medium	Large
1 person	197 L/day	199 L/day	213 L/day
2 people	356 L/day	364 L/day	384 L/day
3 people	456 L/day	470 L/day	490 L/day
4 people	524 L/day	542 L/day	563 L/day
5 people	551 L/day	572 L/day	591 L/day

Note:
A small property has an area less than 500 m².
A medium property has an area between 501 m² and 700 m².
A large property has an area between 701 m² and 900 m².

- 1 Consider the water bill to answer these questions.
 - a Consider the supply period.
 - i What supply period does this bill apply to?
 - ii How many days are there in the supply period?
 - b Fixed charges are those payable for having water and sewerage available to your property.
 - i What is the total of the fixed charges for this account?
 - ii How much water did this household use for the period shown?
 - iii Using the answer for part **a ii**, what is the average daily usage?
 - c The usage charge is what we pay for the amount of water that we use.
 - i What is the cost per kilolitre of water?
 - ii What is the total cost (fixed and usage) for this period?
 - iii Using the answer for part **a ii**, what is the average daily cost?
 - iv Assuming the same fixed and usage charges, was this bill smaller or larger than the last bill?
 - v Was this the bill smaller or larger than for the same period last year?
 - d How did the average daily water consumption for this bill compare with the local area average?
 - e Assuming water usage for the last 90-day billing period averaged 692 L/day, and the fixed and usage charges were the same, calculate the following.
 - i The total water usage for the last bill in kilolitres (kL)
 - ii The water usage charge for the last bill
 - iii The total charges for the last bill
 - f Assume that the water usage for the 90-day billing period at the same time last year averaged 809 L/day. If the water usage charge last year was \$2.67 per kilolitre and the fixed charges were the same, what is the difference between the water usage charge for this bill and that for the bill for the same period last year?
- 2 The table “Targets for water-efficient households” in the sample water bill allows us to compare a household’s water usage with the supplier’s targets for ideal water-efficient households. What is the ideal daily water consumption for the following households?
 - a A large property with four people
 - b A small property with two people
 - c A 600 m² property with three people
 - d A 720 m² property with five people
- 3 Refer to the “Targets for water-efficient households” table on the water bill to answer these questions, assuming this bill is for a medium-sized property with five occupants.
 - a Is this household’s usage smaller or larger than the ideal efficient usage shown in the table?
 - b By how much (to the nearest litre) would this household have to change its daily usage to reach the ideal?
 - c If a bucket can hold 12 L of water, to how many buckets is your answer for part **ii** equivalent? Give your answer to the nearest whole bucket.
- 4 In 2020, Hunter Water charged residential properties \$2.25 per kilolitre for water usage. The water availability charge was \$25.69 per annum. Calculate the total annual cost for a property that used 190 kL of water.
- 5 In 2015, Sandhaven Water charged residential properties \$1.55 per kilolitre for water usage up to 450 kL and \$1.75 per kilolitre for each kilolitre more than 450 kL. The water availability charge was \$92.
 - a What was the annual cost of water for a household that used:
 - i 270 kL?
 - ii 480 kL?

- b** For non-residential properties, the water usage charge was \$1.55 per kilolitre. The water availability charge is based on the size (diameter) of the water meter service connection(s), as shown in the table. If there is more than one meter connection to a property, each is charged separately. What would be the water availability charge for a non-residential property that is supplied by:
- one 32 mm connection?
 - one 25 mm and one 40 mm connection?
 - one 50 mm connection?
 - two 25 mm connections?

Meter connection	Charge (\$)
20 mm	81
25 mm	127
32 mm	207
40 mm	324
50 mm	506
100 mm	2025

- c** A non-residential property has one 20 mm, one 32 mm and one 40 mm connection. If the amount of water used was 560 kL, what was the total annual cost?

Worked example 4.5B Interpreting and analysing electricity bills

Sarsha receives her quarterly electricity bill. It says that she used 399 kWh of electricity during “peak” times and 195 kWh of electricity during “off-peak” times.

- a** The off-peak rate is 38 cents/kWh and the peak rate is 49 cents/kWh. There is an electricity service availability charge of 72 cents/day. What is the total of these charges over the 90-day period?
- b** Sarsha is entitled to a 5% discount on her electricity bill. This discount is applied before GST is calculated. What is her total electricity bill after applying the 5% discount and 10% GST to the answer from part **a**?

Part	Think	Write
a	Find each of the peak and off-peak charges by multiplying the relevant rate by the energy used. Find the service availability charge by multiplying the daily rate by 90 days. To find the total charge add the two previous amounts and the service availability charge.	$\begin{aligned} \text{peak charge} &= \$0.49 \times 399 \\ &= \$195.51 \\ \text{off-peak charge} &= \$0.38 \times 195 \\ &= \$74.10 \\ \text{service charge} &= \$0.72 \times 90 \\ &= \$64.80 \\ \text{total charge} &= \text{usage charges} + \text{service charge} \\ &= \$195.51 + \$74.10 + \$64.80 \\ &= \$334.41 \end{aligned}$
b	To decrease the charges by 5%, find $100\% - 5\% = 95\%$ of \$334.41. To add 10% GST, find $100\% + 10\% = 110\%$ of \$317.69.	$\begin{aligned} (100\% - 5\%) \times \$334.41 &= 95\% \times \$334.41 \\ &= \frac{95}{100} \times \$334.41 \\ &= \$317.69 \\ (100\% + 10\%) \times \$317.69 &= 110\% \times \$317.69 \\ &= \frac{110}{100} \times \$317.69 \\ &= \$349.46 \\ \text{Sarsha's total electricity bill is } &\$349.46. \end{aligned}$



Worked example video: Interpreting and analysing electricity bills

This is Mr Smith's electricity bill.

Energy bill:

Mr P Smith
71 Winter Grove
SUMMERTON

Customer Number 0123456

Due Date 18 March 2026

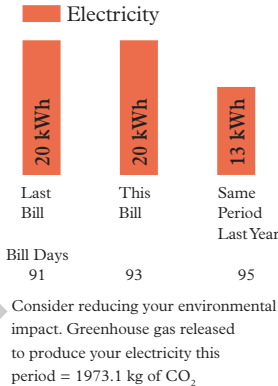
Amount Payable \$819.64

Quarterly Electricity Account

LOCATION: 71 Winter Grove, SUMMERTON

Previous Amount Payable	738.56
Payment Received – Thank you	–738.56 Cr
Electricity (22/11/2025 to 22/02/2026)	784.34
Electricity Discount 5% (22/11/2025 to 22/02/2026)	–39.21 Cr
Subtotal of Charges before GST	745.13
Total GST Payable 10%	74.51
Total Charges including GST	819.64
Total Amount Payable	\$819.64

AVERAGE DAILY USAGE



Energy Used & Costs

METER ID	METER READING	ENERGY USED	RATE	COST
Peak Energy Rate – Contract (22/11/25–22/02/26) EDX009745/001	649.9	649.9 kWh	48.88c/kWh	\$317.67
Shoulder Energy Rate – Contract (22/11/25– 22/11/26) EDX009745/002	940.8	940.8 kWh	34.37c/kWh	\$323.35
Off-Peak (Night) Energy Rate – Contract (22/11/25– 22/11/26) EDX009745/003	270.7	270.7 kWh	24.74c/kWh	\$66.97
Electricity Service Availability Charge			82.1c/day	\$76.35
Total Electricity Before GST		1861.4 kWh		\$784.34

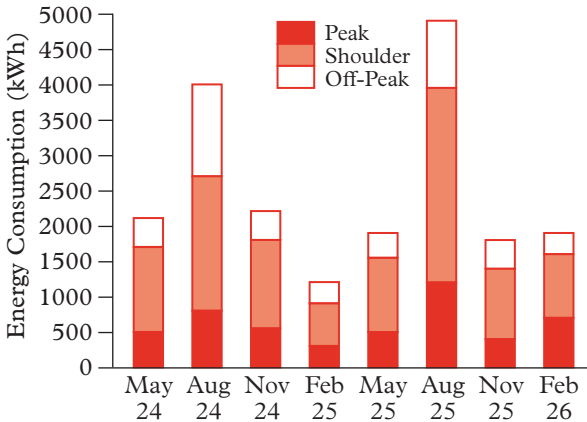
PowerSmart Home Electricity Usage Summary

Supply Period:
22 November 2025 to 22 February 2026 — 93 days

USAGE BREAKDOWN

Peak	650 kWh	34.91%
Shoulder	941 kWh	50.54%
Off-Peak	271 kWh	14.54%
TOTAL ENERGY	1861 kWh	

ELECTRICITY USAGE COMPARISON



- 6 Use Mr Smith's electricity bill to answer the following questions.
- a Identify the following details.
 - i What is the supply period for this bill?
 - ii How many days are in the supply period?
 - b The electricity service availability charge is the charge payable for having electricity available to a property (the cost of supplying and maintaining the poles and wires). Locate this charge on the bill.
 - i What daily rate is used to calculate this charge?
 - ii What is the total electricity service availability charge owed on this account?
 - c The usage charge is a payment for the amount of electricity used.
 - i What was the total amount of electricity used by this household for the period shown?
 - ii What is the average daily usage?
 - d The cost of usage is broken into peak energy rate, shoulder energy charge and off-peak energy rate. What percentage of usage occurred in:
 - i the shoulder period?
 - ii the off-peak period?
 - e What is the peak energy rate per kilowatt-hour?
 - f What is the difference between the peak and off-peak rates per kilowatt-hour?
 - g Determine each of the following.
 - i What is the total energy usage charge for this quarter?
 - ii What is the total cost (availability and usage) for this quarter, after GST?
 - iii What is the average daily cost?
- 7 Use the information provided in Mr Smith's electricity bill to answer the following questions.
- a Consider the bill for the same period last year.
 - i What was the average daily usage?
 - ii Calculate the total amount of electricity used for that period.
 - b Assuming the same fixed and usage charges, was this bill smaller or larger than:
 - i the previous bill?
 - ii the bill for the same period last year?
 - c If the total electricity usage on last year's bill (calculated in part a ii) is broken into the same proportion as the current bill, what would have been the amount of electricity used last year in the:
 - i peak period?
 - ii shoulder period?
 - iii off-peak period?
 - d Using your answers for part c, and assuming the same rates as the current bill, calculate the:
 - i energy usage charge for the same period last year
 - ii service availability charge
 - iii total of the usage and availability charges for this period
 - iv total charges after the discount is applied and the GST is added.
 - e Consider the electricity usage comparison. In which month was the:
 - i most energy consumed?
 - ii least shoulder rate energy used?
 - iii most off-peak rate energy used?

Worked example 4.5C Interpreting and analysing gas bills

Ben's gas bill says that he used 5571 MJ of gas during the 90-day billing period. He is charged at a rate of 2.4 cents/MJ for the first 3500 MJ, then at a rate of 1.8 cents/MJ for anything over this. How much will he be charged for this period?

Think

Start by determining how much of the 5571 is charged at the subsequent rate by subtracting 3500.
Multiply 3500 by the standard rate and 2071 by the subsequent rate, then add these together.

Write

$$5571 - 3500 = 2071$$

$$\text{Total} = (3500 \times \$0.024) + (2071 \times \$0.018)$$

$$= \$121.28$$



Worked example video: Interpreting and analysing gas bills

Consider this sample gas account.

GREEN EDGE ★★

Gas account

Mr A Winters
2 Spring Street
GREENHILL

Your account number 0123456

Due date 27 May 2026

Total amount due \$187.87

Your account summary

Supply period 2 Feb 2026 to 3 May 2026

Previous balance \$183.47
Payment received \$183.47 cr

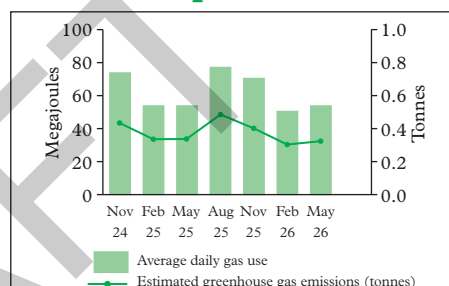
Balance brought forward \$0.00

New charges and credits (see details below)

Usage and supply charges \$170.79
Total GST for new charges \$17.08

Total amount due \$187.87

Your overall picture.



Usage

Average usage per day in this billing period **53.30 MJ**
Same time last year **53.33 MJ**
Average cost per day **\$2.06**

ESTIMATED GREENHOUSE GAS EMISSIONS FOR THIS BILL 0.3 tonnes

Your account in detail

DPI 51408525661

Reading type Actual read on 3 May 2026 for 91 days

Tariff description Residential Standard

Meter number EA186971

Previous reading	Current reading	Units	Multiplier value	Heating factor	Conversion	Usage MJ
7486	7611.01	125.01	1	38.27165	1.013730	4850

Charge
Balance brought forward \$0.00

New charges and credits

Usage and supply charges

Gas Consumption 3740 MJ @ \$0.02967	\$110.97	
Next 1110 MJ @ \$0.01758	\$19.51	
Supply Charge	\$40.31	
Total usage and supply charges		\$170.79
Total GST for new charges		\$17.08
Total amount due		\$187.87

- 8 Use the sample gas account to answer the following questions.
- Locate the “supply period” on the bill.
 - What is the supply period for this account?
 - If the dates are inclusive, how many days is this?
 - Find where the GST is listed.
 - How much GST is included in the total amount due?
 - What percentage is this of the usage and supply charges?
 - What was the total number of megajoules used in this period?
 - What is the cost/MJ for:
 - the first 3740 MJ used?
 - the next 1110 MJ used?
 - What is the supply charge?
 - Assuming that the usage and supply charges were the same for the previous bill, calculate the total amount due if this household used 4510 MJ.
 - For this billing period, what was:
 - the average daily usage?
 - the average cost per day?
 - What were the estimated greenhouse gas emissions for this billing period?
 - Calculate the total usage for the same time last year if the billing period was 90 days.

Problem solving and reasoning

- 9 The table below shows part of the rates and charges notice from a Sydney council.

Details	Rate (\$ p.a.)	Rateable value (\$)	Annual amount (\$)
General rates – Residential	0.000 676 61 per \$ rateable value	1 399 000	946.58
General rates – Residential (Minimum amount payable)	715		715
Stormwater charge	70		70
Domestic waste	471		471
Total of this notice			

- Consider the data in the table.
 - What is the variable residential rate charge, in dollars, for properties in this council area?
 - Multiply the rate from part **ai** by the rateable value of the land. Is the answer the amount in the last column for “General rates – Residential”?
 - Calculate the total charge for domestic waste services.
 - Calculate the total rates payable.
 - Calculate the total rates payable for another residential property in this municipality if this property has a rateable land value of \$1 526 000.
- 10 The average rate of flow of Jenny’s bathroom shower, which has a normal shower hose, is 18 L/min.
- Jenny has two 8-minute showers each day. Calculate the annual cost of her showers if the water usage charge is \$2.45/kL. (Use 1 year = 365 days.)
 - Jenny’s brother Sam has one 20-minute shower each day. What is the annual cost of Sam’s showers?
 - How much could Jenny and Sam save each year by installing a water-efficient shower hose that has a flow rate of 8.5 L/min?

11 This table shows the average amount of water used for flushing a toilet per person per day.

Type of toilet	Litres of water used
Full flush	54 L per person per day
Three-star rated dual-flush	18 L per person per day

- There are four people in the Lee family. Use the table to help you find the number of litres per year they would save by replacing their full-flush toilet with a three-star rated dual-flush toilet.
- What would be the cost savings if the water usage charge is \$2.75/kL?

Challenge

12 A 7.0 kg top-loading washing machine that costs \$556 uses 170 L of water per wash. A 7.0 kg front-loading machine of the same brand costs \$660 and uses 80 L per wash. The Lee family (who use their washing machine four times per week) are considering whether to buy the top-loading or front-loading machine.

- Calculate the annual water usage charges for each machine if water costs \$2.25/kL. (Use 1 year = 52 weeks.)
- What is the annual saving in water costs if they buy the front-loading machine?
- What is the difference in the purchase price of the machines?
- If they purchase the front-loading machine, how long will it take to break even on total costs (purchase price and water usage)?

13 Research the costs associated with home internet connection.

- What types of up-front or set-up costs can you expect to pay when you begin a new internet plan in a new home?
- What types of charges can you expect to see on a typical monthly internet bill?
- Find two internet plans that have different up-front costs and different usage rate charges and compare.

Related resources



4.5 Quick quiz

Lesson 4.6

Budgeting

Learning intentions

By the end of this lesson, you will be able to ...

→ prepare a budget and describe strategies for avoiding financial problems.

Budgets

- A **budget** is a financial plan for the future. It is a means by which we can save for future purchases and avoid overspending. To prepare a budget we need to determine our expected income and estimate our anticipated expenses.

- Expenses include **fixed spending**, such as rent or board, and **discretionary spending**, such as entertainment, clothes and gifts. Our income needs to be greater than our expenses if we are to live within our means.
- To prepare a budget for a given period, such as a week, month or year:
 - 1 Calculate your total income
 - 2 Estimate your total expenses
 - 3 Compute your income minus your expenses
 - 4 Adjust income or expenses if necessary.

Saving money

- When earning enough money to cover fixed expenses and discretionary spending, it is worth setting aside any leftover money as designated savings. Most banks offer at least two types of accounts; “everyday” accounts are typically those connected to a debit card, while “savings” accounts often pay a small amount of interest on the balance of that account.
- Having a separate savings account allows you to keep track of how much money you have allocated to your different expenses. You may even have more than one savings account! For example, if you are saving for a holiday or for a large purchase, you might open a separate savings account dedicated to this goal.

Minimising financial problems

- Understanding how much money you can expect to be earning, spending and saving is an important step when learning to manage money.
- It is easy to lose track of just how much money you are spending. For instance, you may have automated payments set up for subscriptions that you end up forgetting about, or you may not realise how quickly extra transaction surcharges are adding up.
 - It is a good habit to check your bank balance regularly. It is also worth scanning through your transactions to check for anything you may have forgotten about, or to catch any suspicious activity.
- As a person takes on more financial responsibilities, it is advisable to plan ahead and prepare for any unexpected circumstances that impact their financial security.
 - Many people establish a personal “emergency fund”, often as a separate bank account, where they put aside enough savings to cover their expenses in a time of need.
 - If you have a clear idea of your monthly expenses, you can set a target amount for your personal emergency fund. Depending on your situation, a good rule-of-thumb is to save three to six months’ worth of expenses, to allow enough time to regain or replace your sources of income.
 - If you are responsible for something important like a pet or a car, your emergency fund should also account for larger unexpected costs. Many people choose to purchase insurance for these kinds of circumstances, but with insurance comes an extra expense, and it typically will not cover the whole amount if you need to make a claim.
- Some expenses must be paid in one large lump sum, or there may be a significant discount applied if you choose to pay a bill across fewer instalments. Spending throughout the year needs to take these expenses into account so that enough savings are accrued before large bills are due.

Worked example 4.6A Preparing an annual budget

Karen has just started work and lives at home with her parents. Her weekly take-home pay is \$544. Each week she pays her parents \$110 for board and spends approximately \$49 on transport fares and \$60 on lunches. She spends \$120 per week on entertainment, \$95 per fortnight on personal items and \$330 per month on clothes.

- Prepare an annual budget for Karen.
- Karen is saving to go on an overseas holiday in three years. She estimates that she will need to save \$14 500. Determine whether she will be able to afford her holiday.
- If Karen will not have sufficient money to take her holiday, how could she adjust the discretionary expenses in her budget so that she will be able to afford the holiday?

Part	Think	Write
a	To find Karen's annual income, multiply her weekly income (\$544) by 52. To find Karen's annual expenses, multiply each expense by 52 (for weekly expenses), 26 (for fortnightly expenses), or 12 (for monthly expenses). Find the total by adding all her annual expenses together. Calculate Karen's annual savings by subtracting her annual expenses from her annual income.	<i>Income</i> $\$544 \times 52 = \$28\,288$ <i>Expenses</i> $\text{board} = \$110 \times 52 = \$5720$ $\text{fares} = \$49 \times 52 = \$2548$ $\text{lunches} = \$60 \times 52 = \$3120$ $\text{entertainment} = \$120 \times 52 = \$6240$ $\text{personal items} = \$95 \times 26 = \$2470$ $\text{clothes} = \$330 \times 12 = \$3960$ $\text{total} = \$24\,058$ <i>Savings</i> $\text{income} - \text{expenses} = \$4230$
b	Since Karen's net income is greater than her expenses, she can save some money towards her holiday. The amount she can save over the three years is three times the amount she can save in one year. Hence, multiply \$4230 by 3. Compare this amount to the goal amount. To see how far away from her goal she is, we find the difference (using subtraction) between the target amount and her predicted savings.	If Karen saves all her money, in three years she will have $\\$4230 \times 3 = \\$12\,690$. Since $12\,690 < 14\,500$, Karen's projected budget will not allow her to afford the holiday. She is short of her target by $\\$14\,500 - \\$12\,690 = \\$1810$.

Part	Think	Write
c	Karen needs to either increase her income or decrease her expenses. Her total shortfall can be divided into yearly amounts, or even into monthly, fortnightly or weekly amounts, to help decide which expenses can be adjusted to help reach the goal.	Karen must either increase her income or decrease her expenses by at least $\\$1810 \div 3 \approx \\$603$ per year. She could increase her income by finding employment with a higher income or getting a second job. She could decrease her discretionary expenses, for example, by reducing her spending on clothes to \$275 per month. This would save her an extra $\\$55 \times 12 = \\$660$ per year. Or, if she reduced her spending on entertainment to \$105 per week, she would save $\\$15 \times 52 = \\$780$ per year. She would then be able to afford to take the holiday.



Worked example video: Preparing an annual budget

Understanding, fluency and communicating

Answers: Lesson 4.6

Helpful hints

- Find the net income and the total of the expenses for the budget period. Subtract the expenses from the income to determine how much can be saved.

- Alison is in Year 11 and lives at home with her parents. She has a part-time job and earns \$160 per week. She pays \$20 per week for board, spends \$140 per month on clothes, \$45 per fortnight on entertainment, and \$80 per fortnight on driving lessons. She saves the rest.
 - Prepare a budget for Alison.
 - Alison wants to buy a car at the end of the year. How much will she expect to have available to spend?
- Moses is in Year 11 and lives at home with his parents. He has a part-time job and earns \$270 per fortnight. He pays \$25 per week board, spends \$150 per month on personal items, and \$450 per year for his junior golf club membership. He saves the remainder.
 - Prepare a budget for Moses.
 - He wants to buy new golf clubs at the end of the year. The clubs cost \$1200. Will he have enough money saved to buy them?
- Naomi lives at home with her parents. Her weekly take-home pay is \$590. Each week she pays \$100 for board, \$53 for fares and \$42 for lunches. She spends \$150 per week on entertainment, \$84 per fortnight on personal items and \$380 per month on clothes. She saves the rest of her money.
 - Prepare an annual budget for Naomi.
 - Naomi wants to buy a car in three years. Using what she learned in Lesson 4.4, Naomi has determined that her purchase will cost approximately \$16 550 in total. Determine whether Naomi will be able to buy the car.

- 4 Matthew's net earnings are \$940 per week. He shares a house for which he pays \$320 per week rent. Each week he spends \$160 on food, \$145 on entertainment and \$75 on personal items. The loan repayments on his car are \$380 per month. He spends \$95 per week on petrol and the six-monthly car service is \$380. Annual car registration and insurance add to \$2148. His mobile phone costs him \$59 per month.
 - a Prepare an annual budget for Matthew.
 - b How could Matthew adjust his budget so that he can live within his means?
- 5 George is a full-time TAFE student. He receives an allowance of \$663.30 per fortnight from the government and averages net earnings of \$220 per week from his part-time job. His expenses are \$1035 per month for rent, \$190 per week for food, \$120 per quarter for his phone, \$70 per week for entertainment and \$350 per year for books.
 - a Prepare an annual budget for George.
 - b George already has enough savings to pay for the upfront costs of a car. He estimates that the running costs will include approximately \$40 per week for petrol, \$40 per month for maintenance and \$1140 per year for registration and insurance. Does his current budget cover these additional expenses?
- 6 For each of the people in questions 1 to 5, classify their expenses as either fixed or discretionary. Suggest ways that each person could reduce their expenses if need be.

Worked example 4.6B Minimising financial problems

Claudia has a weekly income of \$1153. She prepared a budget for the next financial year and determined that her average weekly expenses should be approximately \$870.

- a What will Claudia's savings be:
 - i on average per week?
 - ii in total for the year?
- b Claudia's expenses include several large bills totalling \$9000 that are all due as lump sums in May and June (at the very end of her budget period).
 - i What would Claudia have calculated as her average weekly expenses if she hadn't included this \$9000?
 - ii How much would Claudia expect to be saving on average each week if she used the weekly expenses from part **bi**?
 - iii What is the difference between how much money Claudia was expecting to have left over on average each week (calculated in part **ai**) and how much she seems to be accumulating each week in the first few months of her budget period (calculated in part **bii**)?
- c After three months, Claudia notices that she has much more money in her bank account than she was expecting. In response to this realisation, Claudia's expenses increase to an average of \$1020 per week for the 25 weeks from October through March.
 - i How much more does Claudia spend in this period than she had budgeted for?
 - ii If she returns to her intended spending habits for the remainder of the budget period, what will Claudia's adjusted savings total be?
 - iii What percentage of her expected yearly savings did Claudia inadvertently spend over these 25 weeks?

Part	Think	Write
a i	Subtract Claudia's average weekly expenses from her average weekly income.	Weekly savings: $\begin{aligned} \text{weekly income} - \text{weekly expenses} \\ &= \$1153 - \$870 \\ &= \$283 \end{aligned}$
a ii	Multiply the previous result by 52 weeks to get the annual amount.	Total savings: $52 \times \$283 = \$14\,716$
b i	First, find Claudia's total annual expenses by multiplying the weekly value by 52 weeks. Subtract the \$9000 from this value, then find the new average weekly expenses by dividing the new total by 52 weeks.	Annual expenses: $\$870 \times 52 = \$45\,240$ Adjusted annual expenses: $\$45\,240 - \$9000 = \$36\,240$ Adjusted weekly expenses: $\frac{\$36\,240}{52} = \$696.92$
b ii	Subtract Claudia's adjusted weekly expenses from her average weekly income.	Adjusted weekly savings: $\$1153 - \$696.92 = \$456.08$
b iii	Subtract the original weekly savings value from the adjusted weekly savings value.	Difference in savings per week: $\$456.08 - \$283 = \$173.08$
c i	Claudia had budgeted for \$870 per week, so the difference per week is $\$1020 - \$870$. The total difference over the 25-week period is found by multiplying the weekly difference by 25.	Extra expenses per week: $\$1020 - \$870 = \$150$ Total extra expenses: $25 \times \$150 = \$3750$
c ii	Claudia was expecting her total savings to be \$14 716, so we subtract \$3750 from this.	Adjusted total savings: $\$14\,716 - \$3750 = \$10\,966$
c iii	To determine the percentage, find $\frac{\text{adjusted total}}{\text{expected total}} \times 100\%$	Percentage difference: $\frac{\text{adjusted total}}{\text{expected total}} = \frac{\$10\,966}{\$14\,716} \approx 0.745$ Claudia inadvertently spent approximately 75% of the money she had expected to save based on her yearly budget.



Worked example video: Minimising financial problems

7 Consider the situation described in Worked example 4.6B.

What could Claudia have done differently to avoid spending 75% of the money she had intended to save, despite having prepared a comprehensive yearly budget?

8 Ramiro is looking ahead at his anticipated income and expenses for the next year. He decides to distinguish between the expenses he will incur on a regular basis and a couple of big bills that he knows will be due in quick succession around the middle of the year.

Fill in the missing values in his budgeting notes:

- My annual salary is \$74 796, so my monthly income is $\frac{\$74\,796}{12} = \$\underline{\hspace{1cm}}$.

- The expenses I can predict regularly per month come to \$2782, so that leaves me with
 $\$ \_\_ - \$2782 = \$ \_\_ \text{ leftover each month.}$
 - I have four months until my mid-year bills are due, and they will be about \$6000 in total.
 - Therefore, I should put an extra $\$ \_\_ \div 4 = \$ \_\_ \text{ aside in my Big Bills account each month.}$
- 9** Suppose you have an average of \$550 in expenses per week. Fill in the blanks below to determine the amount you would need for a three-month emergency fund.
- Yearly expenses = $\$550 \times \_\_$
 $= \_\_$
 Monthly expenses = $\frac{\_\_}{12}$
 $= \_\_$
 3 months of expenses = $3 \times \_\_$
 $= \_\_$
- 10** For each of the following, find how much an emergency fund should contain if it is to cover six months' worth of the person's expenses.
- a** Joel's expenses are approximately \$1803 per week.
 - b** The Marshall family's expenses are approximately \$12 074 per month.
 - c** Henry's expenses are approximately \$2034 per fortnight.
 - d** Pauline's expenses are approximately \$643 per week.
- 11** Find the value of three months' worth of expenses for each person in questions **1 to 5**. In each case, also state whether they should be putting any extra money aside based on their specific circumstances and justify your opinion.
- 12** Shirley's income is \$6204 per month and her expenses on average total \$4580 per month. If Shirley puts all of her unspent income into an emergency fund, how long will it take for her to build up enough money to cover three months' worth of expenses?
- 13** The Jackson family has a combined income of \$7047 per fortnight and an average of \$5118 in expenses per fortnight. If they put aside all of their unspent earnings until they have saved enough money to cover three months' worth of expenses, how many weeks will it take them to achieve their goal?

Problem solving and reasoning

- 14** Julie and Robert are a young couple working their first full-time jobs. They want to prepare a budget for next year and have gathered the following information.
- Income:
 - Julie's take-home pay is \$990 per week.
 - Robert takes home \$880 per week.
 - Interest of \$230 from investments is expected in February and August each year.
 - Expenses:
 - Home loan repayments, \$3230 per month
 - Food, \$275 per week
 - Electricity, \$480 each quarter
 - Phone bills, \$110 per month
 - Council rates, \$491.50 each quarter
 - Water rates, \$286 per quarter
 - Car registration and insurance, \$1180 per year

- Comprehensive car insurance, \$1210 per year
- Car loan repayments, \$180 per week
- Car running expenses, average \$390 per month
- Clothing, average \$350 per month
- Personal items, \$55 per week



- a** Prepare an annual budget for Julie and Robert.
- b** In order to reduce the cost of their loans, Julie and Robert wish to increase their loan repayments. Can they afford to do this? What advice would you give them?
- c** How much would you recommend Julie and Robert put aside in an emergency fund? Justify your answer.
- 15** Tiarne is in Year 11 and lives at home with her parents. She has a part-time job working with a beautician and earns \$180 per week. She assists with the make-up at six weddings per year and is paid \$100 for each. She does not pay board but spends \$200 per month on clothes and entertainment, \$45 per week on make-up and \$30 per fortnight on bus fares. She saves the rest of her money.
- a** Prepare a budget for Tiarne.
- b** If Tiarne completes a specific make-up course she will then be paid \$250 per wedding and her beautician earnings will increase to \$270 per week. The course costs \$1200. Her parents offer to lend her the money so she could do the course at the beginning of the year but Tiarne would have to repay the loan at a rate of \$50 per fortnight.
- i** If Tiarne saved for the whole year, would she be able to pay for herself to do the course next year?
- ii** Modify Tiarne's budget to include her parents' proposed loan repayments and her increased income if she did the course at the start of this year.
- iii** If Tiarne repays \$50 per fortnight for the entire year, how much interest is she paying on the \$1200 loan?
- iv** How much money will Tiarne have at the end of the year if she takes the loan? Is it better to save up and pay for the course herself next year? Give reasons.

Challenge

- 16** Savannah is making a budget for 2027. She is a university student and works much more during university breaks than she does during semester time. She also only needs to pay rent from March to October. Savannah wants to make sure she will have enough money to pay for some of the bigger expenses she needs to cover throughout the year. She makes the following table.

	Jan/Feb (8 weeks)	Mar/Apr (9 weeks)	May (4.5 weeks)	Jun (4.5 weeks)	Jul (4.5 weeks)	Aug/Sep (9 weeks)	Oct (4.5 weeks)	Nov/Dec (8 weeks)
Work (days/ week)	5	1	1	1	5	1	1	5
Income	—	—	—	—	—	—	—	—
Big bills			Car rego \$1180	Car insurance \$923		Car service \$300	QLD trip \$2500	
Rent	No rent	—	—	—	—	—	—	No rent

- a** If Savannah earns an average of \$150 per day of work, enter the total income Savannah is expecting to earn in each column of the row labelled “Income”.
- b** Savannah’s rent is \$350 per week. Enter the total rent she will be paying for each column in the table in the row labelled “Rent”.
- c** Add a row to Savannah’s table that shows her cumulative Income – Expenses by the end of each period (this may be a negative value).
- d** Savannah is worried that she might lose up to one third of her working days before her trip to Queensland.
 - i** How many days of work has Savannah planned for between January and September (inclusive)? Round to the nearest day.
 - ii** If Savannah did lose one third of these workdays, how much less income would she earn?
 - iii** How much money would Savannah need to have set aside to make sure she can afford her trip in October, even if she were to lose one third of her workdays? Hint: Savannah’s Income – Expenses would need to be greater than \$2500 by the start of October.
- e** Will Savannah have enough money to pay rent at the end of October, after her trip? What advice would you give her in this situation?

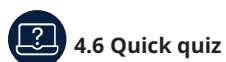
17 Sierra’s expenses are outlined below:

Expense	Amount	Frequency
Rent	\$280	Weekly
Car	\$95	Weekly
Groceries	\$90	Monthly
Entertainment	\$50	Monthly
Clothing	\$100	Monthly

Sierra currently has a part-time job, which earns \$1350 per fortnight.

- a** How much does Sierra save per fortnight?
- b** Sierra plans to quit her current job but wants to accumulate enough savings to last her for six months. She currently has \$2000 in savings. How long will she have to keep working for to save enough money?

Related resources



4.6 Quick quiz

Using spreadsheets

Syllabus links

Use a spreadsheet to model personal and household budgets (MST-11-04)

Using spreadsheets for budgeting and managing money

See Oxford Digital for sample spreadsheets for questions in this lesson.

- 1 Create a functioning spreadsheet that matches the “Account Details” section of Mr Ample’s water bill. Write down and explain the formula you use to generate the amount for “Total Current Charges”.
- 2 Revisit question 10 from Lesson 4.5. Create a spreadsheet that would make it easy for Jenny to answer the same questions if the water usage charge changed or if she or her brother changed their showering routines.

- 3 Enter formulas in the following spreadsheet that will compute the values indicated.

	A	B	C	D	E
1		Per week	Per fortnight	Per month	Per year
2	Income		\$ 9,999.99		=C2*26
3	Expenses	\$ 999.99			
4	Difference	=B2-B3			
5					
6		Emergency	3-month fund	=3*D3	
7			6-month fund		
8					

- 4 The following spreadsheet below shows part of a monthly budget for a year. Set up a similar spreadsheet of your own or download the copy available on Oxford Digital. Enter the formulas as shown and *fill down* and *fill across* to show the entire year. There is a conversion table to convert weekly, fortnightly and monthly payments into annual payments in cells A19 to C23.

	A	B	C	D	E
1	Monthly budget				
2	Item	Annual amount	Jan	Feb	Mar
3	Income				
4			=B4/12	=B4/12	=B4/12
5			=B5/12	=B5/12	=B5/12
6	Total income		=SUM(C4:C5)	=SUM(D4:D5)	=SUM(E4:E5)
7	Expenditure				
8			=B8/12	=B8/12	=B8/12
9			=B9/12	=B9/12	=B9/12
10			=B10/12	=B10/12	=B10/12
11			=B11/12	=B11/12	=B11/12
12			=B12/12	=B12/12	=B12/12
13			=B13/12	=B13/12	=B13/12
14	Total expenditure		=SUM(C8:C13)	=SUM(D8:D13)	=SUM(E8:E13)
15	Monthly difference		=C6-C14	=D6-D14	=E6-E14
16	Cumulative monthly difference		=C15	=C16+D15	=D16+E15
17					
18					
19	Convert to annual amount				
20	Period	Amount	Annual amount		
21	Weekly		=52*B21		
22	Fortnightly		=26*B22		
23	Monthly		=12*B23		

- Revisit the questions in **Problem solving and reasoning** from lesson Lesson 4.6 on budgeting. Use this spreadsheet to prepare each person's budget.
- Enter your own data into the spreadsheet to make an annual budget for yourself.

Lesson 4.7

Review: Managing money

Review questions 4.7A Mathematical literacy review

Answers: Lesson 4.7

Key terms

The following key terms are used in this module:

- budget
- buy an item on terms
- buy now, pay later
- cost
- deposit
- discretionary spending
- fixed spending
- goods and services tax (GST)
- loss
- operating costs
- percentage decrease
- percentage increase
- profit
- revenue
- standing costs
- utilities
- value-added tax (VAT)

Check your understanding

- Is the following formula used to calculate **percentage increase** or **percentage decrease**?

$$\frac{\text{initial value} - \text{final value}}{\text{initial value}} \times 100\%$$
- Write two sentences relating the terms **revenue**, **cost**, **profit** and **loss**.
- Describe what a **deposit** is in your own words.
- Sort the following costs associated with purchasing a vehicle into **standing costs** and **operating costs**.
 - Interest paid on loan
 - Annual registration
 - Comprehensive insurance
 - Servicing
 - Fuel
 - New tyres
 - Compulsory third party insurance
- Fill in the blank: Electricity, gas, water and internet are all examples of _____.
- Which process do the following steps describe?
 - Calculate your total income
 - Estimate your total expenses
 - Compute your income minus your expenses
 - Adjust income or expenses if necessary.
- Sort the following into fixed spending and discretionary spending.
 - Subscription to streaming service
 - Electricity bill
 - Monthly rent
 - Daily take-away coffee
 - Movie tickets
 - Phone bill

Review questions 4.7B Multiple-choice questions

- What is \$60 increased by 5%?
A \$65 **B** \$5 **C** \$63 **D** \$3
- What is the percentage decrease from \$80 to \$74?
A 92.5% **B** 7.5%
C 94% **D** 6%
- Melissa receives a 20% discount on a pair of jeans. She pays \$68 for the jeans. How much does she save because of the discount?
A \$13.60 **B** \$17
C \$85 **D** \$51

- 4 Sharee buys a dining table for \$576 including GST. What is the GST on the table?
- A** \$52.36 **B** \$633.60
C \$523.64 **D** \$57.60
- 5 A laptop is advertised for an upfront cost of \$1899. The laptop can be bought on terms for 20% deposit and monthly payments of \$150 for one year. How much does buying the laptop upfront save?
- A** \$99 **B** \$1420.20
C \$280.80 **D** \$2179.80
- Use the following information for questions 6 and 7.
- Emma uses a buy now, pay later service to pay for a new computer monitor. The price of the monitor is \$799, which is split into four equal payments with instalments due fortnightly across six weeks. Emma must pay a set-up fee of \$15.95 and monthly account keeping fees of \$4. There is a late fee of 2% of the instalment due for any late payments. No interest is charged.
- 6 If Emma pays each instalment on time, how much does she spend in total?
- A** \$199.75 **B** \$799
C \$814.95 **D** \$818.95
- 7 If Emma is late on the last two instalments, how much does she spend in total?
- A** \$826.94 **B** \$835.33
C \$814.98 **D** \$834.93
- 8 What is the vehicle tax paid on a new car that weighs 1504 kg, which is used mostly for business purposes? Use **Table 4.3-1** in Lesson 4.3 to help you.
- A** \$487 **B** \$588
C \$374 **D** \$570
- 9 The stamp duty charged when buying a car is 3% of the market value up to \$45 000, plus 5% of the value over \$45 000. What is the stamp duty to be paid on the purchase of a new car worth \$56 000?
- A** \$2800 **B** \$1900
C \$1680 **D** \$550
- 10 The total annual running costs of a small car were \$8960. The car travelled 14 800 km in the year. What is the average cost/kilometre?
- A** \$1.65/km **B** \$16.50/km
C \$0.61/km **D** \$6.10/km
- 11 It costs an average of \$65 to refuel Andy's car and he fills the tank once a month. How much does he spend on fuel in a year?
- A** \$780 **B** \$715
C \$1690 **D** \$3380
- 12 The ongoing costs for a particular car include \$89 per month on petrol, \$380 per year in servicing/maintenance and \$1908 for annual registration and insurance. How much are the total ongoing costs for this car over a two-year period?
- A** \$2377 **B** \$3356
C \$4754 **D** \$6712
- 13 In Bindora Shire, the water usage charge for residential properties is \$2.06/kL for the first 250 kL and \$2.82/kL for any consumption over 250 kL. If the amount of water consumed is 490 kL, what is the total annual cost?
- A** \$1009.40 **B** \$1381.80
C \$515 **D** \$1191.80
- 14 A front-loading washing machine uses 90 L of water per wash. A family uses the washing machine five times a week. Water costs \$2.26/kL. What is the annual water usage charge for this machine (use 1 year = 52 weeks)?
- A** \$1.02 **B** \$10.58
C \$52.88 **D** \$528.84
- 15 The usage charges set by a gas supplier are:
- \$0.02998/MJ for usage less than or equal to 3560 MJ
 - \$0.01578/MJ for usage greater than 3560 MJ
- What is the cost of using 4236 MJ for the supply period?
- A** \$127 **B** \$117.40
C \$76.44 **D** \$66.84
- 16 If a person's expenses are \$5230 on average per month, and their annual income is \$85 000, what is the most they can afford to put aside as savings in one year?
- A** \$62 760 **B** \$32 700
C \$22 240 **D** \$79 770
- 17 Lena's monthly income is \$6283 and her fixed expenses are \$3952. If she wants to restrict her discretionary spending to 20% of her remaining income, how much will this be each month?
- A** \$1864.80 **B** \$2331
C \$466.20 **D** \$1256.60

- 18** If your average weekly expenses add to \$932, how much would you need in an emergency fund to ensure your expenses could be covered for six months?
- A** \$24 232 **B** \$16 155
C \$12 116 **D** \$5592

Review questions 4.7C Set 1

- 1 Calculate the following percentage increases and decreases.
 - a Increase \$80 by 3%
 - b Decrease \$290 by 15%
 - c Increase \$720 by 18%
 - d Decrease \$1250 by 60%
- 2 A vintage car is bought for \$25 000. Its value increases by 8% in the first year of ownership and then decreases in value by 5% in the second year.
 - a What is the value of the car at the end of the second year?
 - b The car is sold for its value at the end of the second year. Calculate the overall percentage profit made.
- 3 Penny received a discount of \$25.70 on a mobile phone. This was 5% of the original price. Find the original price of the phone.
- 4 Mark has a new kitchen installed in his house. The bill is \$12 546.30. A GST payment of 10% needs to be added.
 - a Calculate the GST payable on the kitchen.
 - b Find the total amount Mark pays for the kitchen.
- 5 Find the GST paid on goods and services costing the following amounts.
 - a \$1235
 - b \$359
 - c \$42.18
 - d \$266
- 6 Antonia buys a new fridge on terms for a 20% deposit and monthly repayments of \$75 for two years. If the original price of the fridge is \$1499, how much does Antonia spend in total?
- 7 Determine the extra vehicle tax paid on a new Toyota sedan, which weighs 1460 kg, when used for business purposes rather than for private purposes.
- 8 Calculate the stamp duty to be paid on the purchase of a new BMW car that has a recommended retail price of \$76 000.
- 9 Dave is buying a new Holden sedan that has a recommended retail price of \$36 000, weighs 1637 kg, with CTP insurance costing \$477 and a dealer delivery charge of \$630. The car is for private use and Dave decides to take out comprehensive insurance that costs \$1150 for the first year.
 - a Calculate the total cost of purchasing and registering the new car. Assume that registration includes the registration fee, vehicle tax and new standard number plates.
 - b Dave receives \$16 500 for the trade-in of his current vehicle, what is the changeover price to purchase the new sedan?
- 10 Find the total cost of purchasing a three-year-old Mazda that has an advertised price of \$11 699, and for which third-party property damage insurance is \$479. Include the cost of registration transfer and stamp duty.
- 11 Jo's monthly repayment on a \$10 000 car loan with a three-year term is \$381.60.
 - a How much did Jo pay in total?
 - b How much of this total was charged as interest?
- 12 The ongoing costs for June's new car include the following:
 - Annual registration/insurance: \$2580
 - Tyre replacements: \$105 per tyre, per year
 - Petrol: \$89 per month
 - Servicing costs: \$287 every 10 000 km
 - a If June travels 10 000 km per year, how much will her ongoing costs be in total for one year?
 - b If June travels 30 000 km in two years, how much will her ongoing costs be for those two years?

- 13** In 2022, Hunter Water charged residential properties \$2.25 per kilolitre for water usage. The water availability charge was \$25.69 per annum. Calculate the total annual cost of water for a property that used 208 kL.
- 14** The usage charges set by a gas supplier are:
- \$0.029 98/MJ for usage less than or equal to 3560 MJ
 - \$0.015 78/MJ for usage greater than 3560 MJ
- What would be the cost of using 3889 MJ in one billing period?
- 15** Herbert receives an income of \$784 a month. On average, his monthly expenses include \$200 on board paid to his parents, \$120 on snacks and meals out, \$85 on entertainment and \$45 on personal items.
- Find Herbert's income minus expenses for an average month.
 - How long would it take Herbert to save up for a new phone, worth \$999?
- 16** For each of the following, find how much an emergency fund should contain if it is to cover six months' worth of the person's expenses.
- Eve's expenses are approximately \$1034 per week.
 - The Cooper family's expenses are approximately \$10 110 per month.
 - Kammy's expenses are approximately \$2487 per fortnight.
 - Sebastian's expenses are approximately \$806 per week.

Review questions 4.7D Set 2

- 1** Kieran is a numismatist and buys a coin collection worth \$8000. In the first year, the collection increases in value by 12% and in the second year by 15%. Calculate the overall percentage increase in the value of the collection.
- 2** Julie's weekly wage of \$850 increased by 3%. Calculate her new weekly wage.
- 3** Harry wants to buy a camera that is listed at the pre-GST price of \$849.
- Calculate the GST payable on the camera.
 - Find the total amount Harry pays for the camera.
- 4** Lucille uses a buy now, pay later service to pay for flights to Jamaica. The price of \$1714 is divided into four equal payments due fortnightly. There is no account set-up fee nor any account-keeping fees. No interest is charged. The late fees are 10% of the instalment due. The late fee for an instalment doubles if the instalment is more than one week late. Lucille pays the first two instalments on time but pays the third instalment two days late and the fourth instalment 10 days late. How much does Lucille pay in total for the flights?
- 5** Calculate the stamp duty that would be charged on a vehicle that has a market value of \$55 600.
- 6** Find the total cost of purchasing a two-year-old Honda 1100cc motorcycle that has an advertised price of \$16 890. Comprehensive insurance is \$685. Include the cost of registration transfer and stamp duty.
- 7** A car has a 55 L petrol tank. Calculate the following:
- The cost of refuelling from empty if the price of petrol is 192c/L.
 - The total amount spent on petrol in one year if the petrol tank is filled up once every month at an average cost of \$72.
- 8** The ongoing costs for Damien's new car include:
- annual registration/insurance: \$2170
 - petrol: \$123 per week
 - tyre replacements: \$445 per year
 - servicing costs: \$350 per year.
- How much does Damien spend on petrol in a year?
 - What are the total ongoing costs associated to Damien's car in one year?

- 9 Refer to the sample water bill in Lesson 4.5.
 - a What is the cost per kilolitre of water?
 - b What are the fixed charges for water availability and wastewater availability?
 - c Calculate the bill for a period in which 72.3 kL of water is consumed.
- 10 Refer to the Sydney council rates and charges notice in question 9 in Lesson 4.5. From that information, calculate the total rates payable for a residential property that has a rateable land value of \$883 000.
- 11 Reggie receives an income of \$2431 a month. On average, his weekly expenses include \$208 in rent, \$130 on food, \$25 on transport and \$100 on discretionary expenses.
 - a Find Reggie's average expenses for a month.
 - b Compare Reggie's monthly income and expenses and comment on whether or not he is living within his means.
 - c Find the value of six months' worth of expenses for Reggie and advise him on a reasonable amount to keep in an emergency fund.

Review questions 4.7E Set 3

- 1 What is the percentage decrease from \$750 to \$735?
- 2 Cedric has a new roof put on his house. The bill is \$22 586 and is subject to GST of 10%.
 - a Calculate the GST payable on the roof.
 - b Find the total amount Cedric pays for the roof.
- 3 Lisa buys a bracelet listed at the pre-GST price of \$425.
 - a Calculate the GST payable on the bracelet.
 - b Find the total amount Lisa pays for the bracelet.
- 4 Find the GST already paid on goods and services costing the following amounts.

a \$7500	b \$1268	c \$25.99	d \$401
- 5 Cyril buys a new dining table on terms for a deposit of 15% and makes monthly payments for 18 months. The original price of the dining table is \$2899.99 and Cyril pays an extra \$200 on top of this for buying on terms. How much does Cyril pay each month correct to the nearest cent?
- 6 Rhea buys a new Ford with a recommended retail price (RRP) of \$54 000 and weight of 1804 kg. The CTP insurance for the car is \$655 and the dealer delivery charge is \$600. The car is for private use and Rhea pays for comprehensive insurance that costs \$1160 for the first year. Rhea also chooses to buy customised number plates for \$242. The drive-away price of the car includes all costs except the comprehensive insurance.
 - a Calculate the total cost of purchasing the new car.
 - b If Rhea receives a trade-in of \$18 000 on her old vehicle, what is the final drive-away price of the car?
 - c Rhea takes out a loan to pay the drive-away price of the car. She pays a 20% deposit and then makes monthly repayments of \$950 for three years. How much more does Rhea pay for the car than if she had paid upfront?
 - d The new Ford can travel 580 km on a full tank of petrol, which costs an average of \$137. Calculate the cost of fuel for:
 - i a 1000 km trip
 - ii a month in which Rhea had to refuel three times
 - iii a whole year, if Rhea buys a full tank of fuel every two weeks, on average.
- 7 Three people, who live in the same general area and work at the same location, drive to work each weekday. Elizabeth owns a Ford Everest, Monique a Suzuki Swift and Tanya a Toyota Camry.
 - a Using the information in **Table 1** from Lesson 4.4, calculate the average operating costs per day for each car (use 1 year = 365 days).

- b** Over a three-week period, how much does it cost each person to drive to work, assuming the average daily operating cost is completely work related?
- c** The three drivers decide to start carpooling, so that each person drives everyone to work one week out of every three weeks. How much does each person save every three weeks by carpooling?
- 8** Refer to the sample electricity bill in Lesson 4.5. Using the “Energy Used & Costs” section, calculate the total electricity cost (before GST) if the energy used for a property was peak 674.6 kW, shoulder 961.3 kW and off-peak 242.8 kW.
- 9** Refer to the Sydney council rates and charges notice in question 9 in Lesson 4.5. Calculate the total rates payable for a residential property which has a rateable land value of \$959 000.
- 10** Allira is in Year 11 and lives at home with her parents.
 She has a part-time job and earns \$170 per week.
 She pays \$30 per week for board and spends \$170 per month on clothes, \$85 per fortnight on entertainment and \$80 per fortnight on driving lessons.
 She saves what she has left.
- a** Prepare a budget for Allira.
- b** Allira wants to buy a car at the end of the year. How much will she expect to have available to spend?



Review questions 4.7F Set 4

- 1** Robert the builder purchased \$2800 worth of materials from the hardware store. He receives a trade discount of 16% and then a further 5% discount if he pays within three days of receiving the account. What is the overall percentage discount if he pays within three days?
- 2** An LED television costs \$578. Paul receives a 15% discount. How much does he pay for the television?
- 3** Lui Min has his house painted. The bill is \$3258.50 and is subject to GST of 10%.
- a** Calculate the GST payable on the cost of the painting.
- b** Find the total amount Lui-Min pays for the painting of his house.
- 4** Gunter wants to buy a freezer that is listed at the pre-GST price of \$520.
- a** Calculate the GST payable on the freezer.
- b** Find the total amount Gunter pays for the freezer.
- 5** Find the GST paid on goods and services costing the following amounts.
- a** \$9953 **b** \$855 **c** \$39.51 **d** \$399
- 6** Frances uses a *buy now, pay later* service to pay for a new cricket bat which costs \$1249. She signs up for a new service and so has to pay the account set-up fee of \$19.95. There are no account keeping fees and no interest charged. She pays four equal instalments, with the first paid at the point of purchase. There is a 2% payment processing fee on each of the four instalments. How much does Frances pay in total for the cricket bat assuming she always pays on time?
- 7** The table below shows the average ongoing costs for the vehicles listed. The calculations are based on the cost of buying the vehicle new and for private use. The full cost of the purchase is assumed to be financed by a five-year loan. It is also assumed that the vehicle travels 15 000 km each year.

Running costs	Kia Cerato	Mazda 6 sedan	Mitsubishi ASX
Monthly loan repayments (\$)	666.73	788.21	660.16
Annual registration and insurances (\$)	2654.16	2512.32	2367.60
Average operating costs:			
Fuel (\$ per week)	43.65	41.29	44.83

Running costs	Kia Cerato	Mazda 6 sedan	Mitsubishi ASX
Tyres (\$ per month)	13.00	13.93	17.67
Service and repairs (\$ per year)	416.16	500.76	441.96
Total cost for 1 year (\$)			
Total cost/week (\$ per week)			
Total cost/kilometre (cents per km)			

- a Convert all costs to a yearly rate and then find the total yearly cost for each vehicle. Enter these values in the relevant cells.
 - b Fill in the last two rows of the table by calculating:
 - i the average cost per week for each vehicle
 - ii the average cost per kilometre for each vehicle (assuming they travel 15 000 km in a year).
- 8 In 2024, Sandhaven Water charged residential properties \$2.15/kL for water usage up to 450 kL and \$2.35/kL for each kilolitre more than 450 kL. The water availability charge was \$81. What is the annual cost of water for a household that uses 510 kL?
- 9 This table shows the amount of water used when flushing the toilet.

Toilet	Litres used
Full-flush	52 L per person per day
Three-star rated dual-flush	18 L per person per day

- a Find the number of litres of water per year that a household of five people would save if they replaced their full-flush toilet with a three-star rated dual-flush toilet.
 - b What would be the cost savings if the water usage charge is \$2.85/kL?
- 10 Xavier's income is \$78 653 per annum. His predictable expenses include \$2342 in monthly mortgage repayments, approximately \$1350 on quarterly utilities bills, \$59 per month for his phone, \$50 a week on public transport and \$100 on discretionary expenses per month.
- a Find the yearly total of the expenses listed.
 - b How much of Xavier's annual income is left over after accounting for these expenses?
 - c What other expenses should Xavier factor into his budget? Consider both fixed expenses and discretionary expenses.
- 11 For each of the following, find how much an emergency fund should contain if it is to cover three months' worth of the person's expenses.
- a Ally's expenses are approximately \$1301 per week.
 - b The Warren family's expenses are approximately \$15 774 per month.
 - c Celeste's expenses are approximately \$897 per fortnight.
 - d Walter's expenses are approximately \$176 per week.

Review questions 4.7G Practice examination question

- 1 Anastasia decides she needs to manage her finances better.
- a First, Anastasia needs to pay her overdue electricity bill.
 - i The fee Anastasia is charged for not paying her bill on time is calculated as a 3% increase on her total electricity costs. If her total electricity costs for the quarter were \$357.23, how much is she charged after applying the late penalty? **(1 mark)**
 - ii Anastasia is eligible for a 5% discount on her electricity bill. Apply her discount to the total found in part i. **(1 mark)**
 - iii The "amount payable" on Anastasia's bill also includes GST. What is Anastasia's total electricity bill, once GST is added to the total in part ii? **(1 mark)**

- b** Looking more closely at her electricity bill, Anastasia realises that she could save money by using less electricity during “peak” periods. Because she works variable hours, Anastasia could easily do more of her chores during “off-peak” periods.

The rates charged by her electricity company are shown in the table below.

	Rate
Peak	45.23 c/kWh
Off-peak	29.87 c/kWh
Service charge	83.2 c/day

- i** Anastasia’s current bill is for a 90-day period in which she used 530.6 kWh during peak periods and 141.8 kWh during off-peak periods. How much of her bill (in \$) was charged for electricity usage during peak periods? **(1 mark)**
- ii** Calculate Anastasia’s total usage costs for a future 90-day period in which she uses 482.3 kWh during peak periods and 190.1 kWh during off-peak periods. Assume the rates stay the same. **(2 marks)**
- iii** How much money would Anastasia save on this future bill, compared to her current one? **(1 mark)**
- c** Anastasia is aiming to buy a new car for private use with an RRP of \$19 990. The drive-away price of the car also includes the following costs. The tare weight of the car is 1255 kg.

New standard number plates	\$55
CTP insurance (green slip)	\$450
Registration fee	\$79
Vehicle tax	\$__
Stamp duty	\$__
Comprehensive insurance	\$1800

- i** Use **Table 4.3-1** in Lesson 4.3 to determine the vehicle tax Anastasia would owe. **(1 mark)**
- ii** Stamp duty is 3% of the purchase price. Calculate the stamp duty payable. **(1 mark)**
- iii** How much would Anastasia pay in total to purchase the new car? **(1 mark)**
- iv** Anastasia would have to use a loan to purchase the car. The loan she is considering requires a 15% deposit and monthly payments of \$350 for five years. How much more would she pay using this loan compared to the amount found in part **iii**? **(1 mark)**
- d** Anastasia’s yearly income is \$35 604 and her current expenses are \$450 for utilities per quarter, \$600 for food each month, \$1473 for rent each month and \$340 a month of discretionary spending.
- i** How much money does she have left over each month? **(1 mark)**
- ii** How much money should Anastasia aim to put into an emergency fund if she would like it to cover three months’ worth of her expenses? **(1 mark)**
- iii** If Anastasia bought a car, she would need to pay for yearly registration and insurance (totalling approximately \$2400) and regular servicing and maintenance (averaging \$400 per year). She would also need to pay the monthly instalment of \$350 towards her loan. She estimates that she would spend at least \$2520 on petrol in a single year. Can she afford to buy a car? Justify your answer. **(2 marks)**

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Managing money

I can do this	I need to review this
☐ apply percentage increase or decrease to determine the cost of goods ☐ calculate profit, loss, percentage profit and percentage loss ☐ calculate GST or VAT on the price of goods and services ☐ solve problems involving GST or VAT	Go back to Lesson 4.1 Financial calculations
☐ examine buy now, pay later services and analyse the costs associated with these schemes for purchasing goods ☐ calculate the cost of buying on terms that involve paying an initial deposit and making regular repayments ☐ compare the total cost of using different methods of purchasing goods and justify which method of purchasing is best suited to a given context	Go back to Lesson 4.2 Payment options
☐ calculate the registration, stamp duty and insurance costs for new and used motor vehicles ☐ compare the purchase price and any loan repayments when buying a car ☐ determine the total up-front costs associated with buying a vehicle	Go back to Lesson 4.3 Purchasing a car
☐ examine and calculate the ongoing costs of a vehicle	Go back to Lesson 4.4 Ongoing costs of a vehicle
☐ use spreadsheets to calculate the costs of purchasing, running and maintaining a vehicle	Go back to Using spreadsheets
☐ calculate household costs including electricity, water and gas usage from household bills	Go back to Lesson 4.5 Household bills
☐ prepare a budget that considers fixed and discretionary spending ☐ describe strategies including budgeting, saving and creating an emergency fund designed to minimise financial problems	Go back to Lesson 4.6 Budgeting
☐ use spreadsheets to model budgets that include a range of personal and household expenses	Go back to Using spreadsheets

Related resources



Module review quiz: Managing money



Worksheet: Challenge 4

Review: Financial mathematics

Review questions

Answers: Review: Financial mathematics

- 1 Calculate each of the following.
 - a Pauline earns \$56 028 per annum. Calculate her weekly pay.
 - b Fielding is a junior and is paid \$24.58 per hour. He works five hours per day, six days a week. Find his pay for one year.
 - c Louise is paid \$34.40 per hour. Calculate her weekly wage if she works 40 hours at normal time and eight hours overtime at time-and-a-half.
 - d Hunter works in the textile industry and is paid \$28.40 per hour. He receives a dust allowance of \$15.17 per week. Find his pay for a 37-hour week.
 - e An art salesperson is paid 6.2% of the selling price of any works he sells. Find his commission on a painting that sells for \$6250.
 - f Endora received a $17\frac{1}{2}\%$ holiday loading on four weeks normal wages. She normally works a 32-hour week. Her four weeks holiday pay with loading is \$4290.92.
 - i Find her normal weekly pay.
 - ii Find her normal hourly pay rate.
 - g A factory worker is paid \$5.35 for each garment completed. Calculate his wage if he completes 324 garments.
 - h Lewis is a full-time student who is over 18 and living at home. He receives Youth Allowance at a maximum rate of \$472.50 per fortnight, but this reduces by 50 cents for every dollar over \$528 he earns. How much Youth Allowance does he receive in a fortnight where he earns \$582?
- 2 Calculate each of the following.
 - a Giselle uses the following information to calculate her taxable income: wages \$67 220, interest \$485, bonus \$4000, tax agent fee \$210, work-related car expenses \$1575 and donations to charity \$600. Calculate Giselle's taxable income.
 - b The Medicare levy is 2% of taxable income for taxable incomes over \$32 500. Calculate the Medicare levy payable on a taxable income of \$61 894.
 - c Use the following table, showing the tax rates for 2024–25, to calculate the tax payable on these incomes.
 - i \$62 889
 - ii \$37 699
 - iii \$139 369
 - iv \$201 522

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	16 cents for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30 cents for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37 cents for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45 cents for each \$1 over \$190 000

- d** Simon earns \$49 996 p.a. as a cocktail attendant and \$17 560 as a part-time singer. He has tax deductions of \$3561 and, throughout the year, pays a total of \$8725 in PAYG tax instalments. He has no student loans. Find:
- his taxable income
 - the Medicare levy he must pay
 - tax payable on his taxable income
 - his tax refund or balance payable.
- 3** Calculate each of the following.
- Increase \$80 by 5%.
 - Find the percentage decrease from \$70 to \$65.80.
 - A Sydney Roosters signed rugby jersey was purchased for \$1200. It was sold for \$425. Calculate the percentage loss.
 - Ollie had a new bedroom built as an extension to his house. The bill for labour and materials was \$18 360 and was subject to GST of 10%.
 - Calculate the GST payable on the bill.
 - Find the amount Ollie paid for the bedroom.
 - Harry wants to buy a camera that is listed at the pre-GST price of \$725.
 - Calculate the GST payable on the camera.
 - Find the total amount Harry will pay for the camera.
 - Find the GST already paid on goods and services costing \$3254.
- 4** Laura uses a *buy now, pay later* service to purchase a new microwave. The price of the microwave is \$649. She makes four instalments of 25%, and after the first payment, the instalments are fortnightly. She already has an account but must pay the account-keeping fee of \$9 for two months. There is a late fee of \$13 for each late payment. There is a transaction processing fee of 1.1% for each instalment.
- How much does Laura pay for each instalment including the transaction processing fee?
 - How much does Laura pay in total, assuming she makes each payment on time? Use your answer from part **a** in this calculation.
 - Laura is late on the second and fourth payments. How much does she pay in total?
 - What is the percentage increase on the price of the microwave for using the *buy now, pay later* service? Give your answer correct to two decimal places.
- 5** The stamp duty on the purchase of a car is 3% of the value up to \$45 000 and 5% for amounts in excess of \$45 000. Calculate the stamp duty on a car valued at:
- \$16 950
 - \$105 000.
- 6** A new car has a recommend retail price (RRP) of \$29 500 and weighs 1150 kg. The CTP insurance for the car is \$612 and the dealer delivery charge is \$575. The car is for private use and the owner decides to purchase third-party property damage insurance for \$400 for the first year.
- Calculate the total cost of purchasing the car, assuming the drive-away price is equal to the purchase price plus the cost of registration (including the registration fee, vehicle tax and new standard number plates), stamp duty, CTP insurance, dealer delivery and the first year of third-party property damage insurance. Refer to **Table 4.3-1** in Lesson 4.3 to determine the vehicle tax owed on this vehicle.
 - If the buyer receives a trade-in value of \$12 500 on their old vehicle, what is the total cost for changing to the new car?

- 7 Consider the following costs associated with owning and driving a car.
- Andi's car requires 14 L of fuel for every 100 km. It also needs to be serviced every 15 000 km and needs new tyres every 45 000 km. If Andi drives 30 000 km a year, calculate:
 - how many litres of fuel she uses in a year
 - how many times her car should be serviced each year
 - how often her tyres need changing.
 - Suppose fuel costs 189 cents per litre on average, and each service costs \$200. How much does Andi spend just on fuel and servicing in one year?
- 8 The table below shows part of the rates and charges notice for a Sydney council.

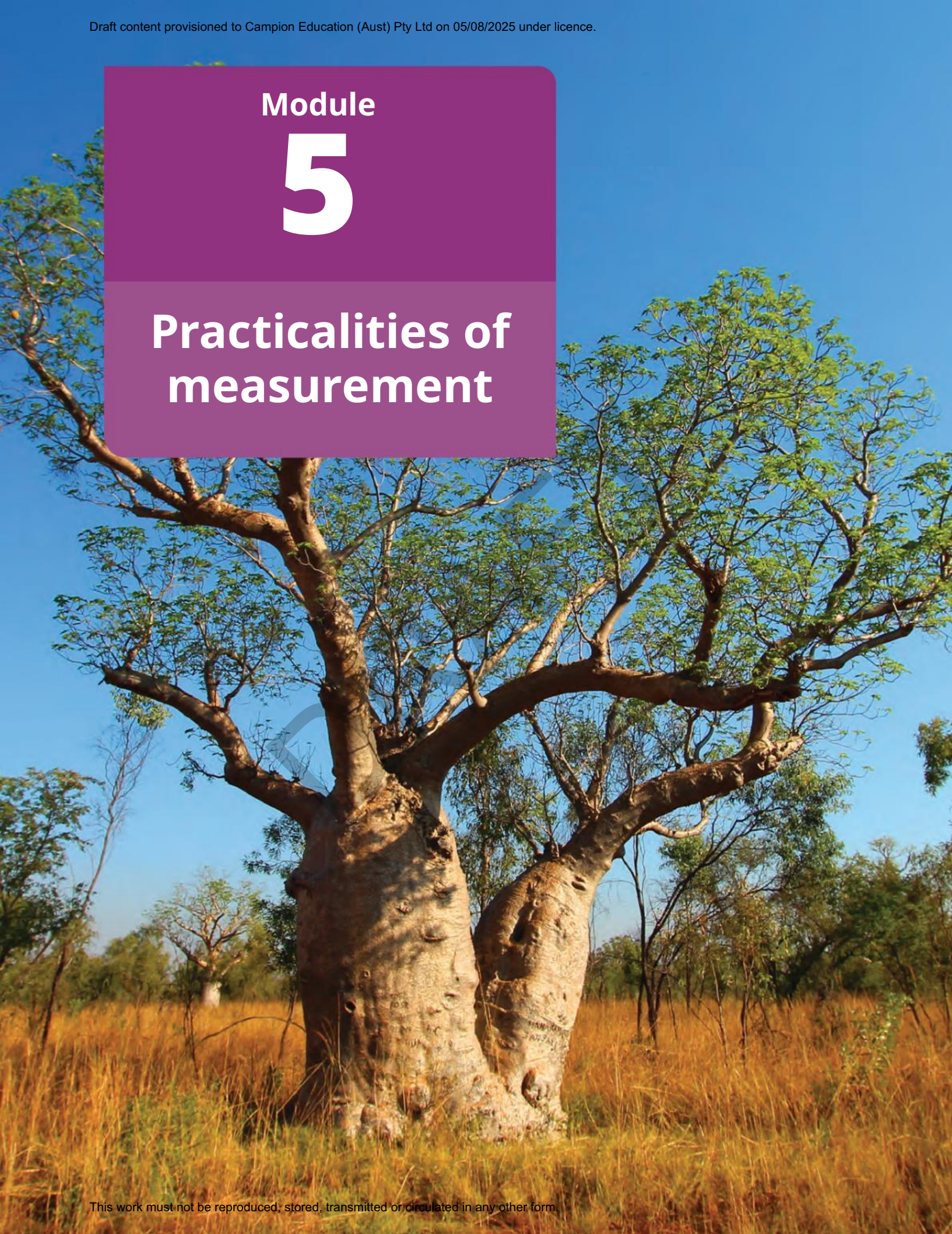
Details	Rate (\$)	Rateable value (\$)	Amount (\$)
General rates – Residential	0.000 974 041 per \$ rateable value	1 480 000	1441.58
General rates – Residential (Minimum amount payable)	617		617
Stormwater charge	25		25
Domestic waste	335		335
Total of this notice			

- What is the rateable value of the land on which this property is situated?
 - Consider the charges for residential land.
 - What is the residential rate, in dollars, for properties in this council area?
 - Multiply the residential rate from part **i** by the rateable value of the land. Is this the amount in the last column for “General rates – Residential”?
 - The minimum general rate payable on a residential property is \$617. Why is this amount excluded when using the above table to calculate the total rates payable on this property?
 - Consider the other charges.
 - What is the stormwater charge?
 - What is the domestic waste charge?
 - Calculate the total charges excluding the general rates.
 - Calculate the total rates payable.
 - Calculate the total rates payable for another residential property in this municipality if that property has a rateable land value of \$1 635 000. Assume the other charges remain the same.
 - Find the maximum rateable land value if the general rates paid are equal to the minimum amount. Give your answer correct to the nearest dollar.
- 9 Fariba is in Year 11 and lives at home with her parents. She has a part-time job and earns \$170 per week. She pays \$40 per week for board, spends \$100 per month on clothes, \$65 per fortnight on entertainment and \$85 per fortnight on driving lessons. She saves whatever is left.
- Prepare a budget for Fariba.
 - Fariba wants to buy a car at the end of the year. How much will she expect to have saved to spend on a car?
 - If Fariba wants to spend more on a car, what aspects of her budget can she modify to help her save more?

Module

5

Practicalities of measurement



Lessons

- Lesson 5.1 Significant figures
- Lesson 5.2 Scientific notation
- Lesson 5.3 Metric units of measurement
- Lesson 5.4 Unit prefixes in the metric system
- Lesson 5.5 Review: Practicalities of measurement

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Practicalities of measurement

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Multiplication and division of decimals by powers of 10



Support sheet: Units of measurement



Support sheet: Rounding decimals

Syllabus links

- Applications of measurement (MST-11-05)
 - A student solves problems involving measurement in practical contexts.
 - Practicalities of measurement
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 5.1

Significant figures

Learning intentions

By the end of this lesson, you will be able to ...

→ round values to a given number of significant figures.

Significant figures

- The **significant figures** in a number are, loosely speaking, the important or meaningful digits. For example, if we say a crowd of 61 348 is approximately 61 000, we are clearly satisfied with an estimate to the nearest thousand, so there are only two significant figures.
- It is impossible to cut a piece of timber precisely to a length of 1.333 333... m. In this case, the digits after the fourth (which represents millimetres) are not important.
- When counting significant figures, work from left-to-right.
- The following rules can be used to determine the significant figures in a number.
 - All non-zero digits are significant.
 - Zeros between non-zero digits are significant.
 - Zeros to the left of the first non-zero digit are not significant.
 - Zeros to the right of the decimal point after the last non-zero digit are significant.
 - Trailing zeros in an integer are not significant.

Worked example 5.1A Rounding to a given number of significant figures

The first significant figure in a number is the first-non-zero digit, reading from left-to-right. Consider the following numbers.

a 293 568

b 0.076 04

Round each number to:

i one significant figure

ii two significant figures

iii three significant figures.

Part	Think	Write
a i	Locate the relevant significant figure and then round appropriately using the next digit. The first non-zero digit is 2. This is the first significant figure. The next digit (9) is greater than 5; so, rounded to one significant figure, $293\,568 \approx 300\,000$. (This is the same as rounding 293 568 to the nearest 100 000 because the first significant figure is in the 100 000s column.)	$293\,568 \approx 300\,000$

Part	Think	Write
a ii	The second significant figure is 9. The next digit (3) is less than 5; so, rounded to two significant figures, $293\,568 \approx 290\,000$. (This is the same as rounding 293 568 to the nearest 10 000 because the second significant figure is in the 10 000s column.)	$293\,568 \approx 290\,000$
a iii	The third significant figure is 3. The next digit is 5, which we round up; so, rounded to three significant figures, $293\,568 \approx 294\,000$. (This is the same as rounding 293 568 to the nearest 1000 because the third significant figure is in the 1000s column.)	$293\,568 \approx 294\,000$
b i	The first non-zero digit is 7. This is the first significant figure. The next digit (6) is greater than 5; so, rounded to one significant figure, $0.076\,04 \approx 0.08$. (This is the same as rounding 0.076 04 to two decimal places because the first significant figure is in the second place after the decimal point; or rounding to the nearest hundredth because the first significant figure is in the hundredths column.)	$0.076\,04 \approx 0.08$
b ii	The second significant figure is 6. The next digit (0) is less than 5; so, rounded to two significant figures, $0.076\,04 \approx 0.076$. (This is the same as rounding 0.076 04 to three decimal places because the second significant figure is in the third place after the decimal point; or rounding to the nearest thousandth because the second significant figure is in the thousandths column.)	$0.076\,04 \approx 0.076$
b iii	The third significant figure is 0. The next digit (4) is less than 5; so, rounded to three significant figures, $0.076\,04 \approx 0.0760$. (This is the same as rounding 0.076 04 to four decimal places because the third significant figure is in the fourth place after the decimal point; or rounding to the nearest ten-thousandth, because the third significant figure is in the ten-thousandths column.)	$0.076\,04 \approx 0.0760$



Worked example video: Rounding to a given number of significant figures

Understanding, fluency and communicating

Answers: Lesson 5.1

1 Fill in the blanks and circle the correct comparison words in order to round:

a 5368 to three significant figures.

The first non-zero digit is _____. This is the first significant figure.

The third significant figure is _____.

The digit after this is *less than* / *equal to* / *greater than* 5, indicating that, when rounded, the number is closer to _____ than to _____.

So, rounded to three significant figures, $5368 \approx$ _____.

- b** 0.062 53 to three significant figures.

The first non-zero digit is __. This is the first significant figure.

The third significant figure is __.

The digit after this is *less than* / *equal to* / *greater than* 5, indicating that, when rounded, the number is closer to __ than to __.

So, rounded to three significant figures, $0.062\ 53 \approx$ __.

- 2** Round each of the following to one significant figure.

a 42 600

b 59

c 4.6

d 108

e 0.6529

f 0.0082

g 0.025

h 990

- 3** Round each of the following to two significant figures.

a 290 365

b 3960

c 24.9

d 2653

e 8.63

f 0.0487

g 0.000 162 8

h 0.003 97

- 4** Round each of the following to three significant figures.

a 3688

b 20 657

c 154 299

d 813.4

e 14.294

f 0.003 508 1

g 0.039 14

h 1.999

- 5** Round each of the following to:

i one significant figure

ii two significant figures

iii three significant figures.

a 17.256

b 0.450 72

c 521 500

d 0.002 095

Problem solving and reasoning

- 6** Express $3\frac{2}{7}$ as a decimal correct to four significant figures.

- 7** Express each of the following numbers as a decimal correct to three significant figures and arrange the numbers in ascending order (from smallest to largest).

$$\sqrt{2} \quad 1.4 \quad 1\frac{2}{5} \quad \frac{10}{7}$$

- 8** A town's average rainfall in summer over six successive years is 246.5 mm, 237.6 mm, 366.9 mm, 287.4 mm, 412.8 mm and 348.2 mm.

Calculate the average rainfall for the town over this six-year period, correct to four significant figures.

Challenge

- 9** A car travels for $3\frac{1}{2}$ hours at 71 km/h and then for $2\frac{1}{4}$ hours at 75 km/h.

a Find the total distance travelled by the car, correct to four significant figures.

b Calculate the average speed for the whole trip, correct to two significant figures.

- 10** A satellite orbits Earth at a height of 32 km above Earth's surface. The diameter of Earth is 12 740 km.

a Find the radius of the satellite's orbit to five significant figures, assuming that the orbit is circular.

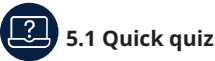
b Calculate the length (l) of the satellite's orbit, using $l = 2\pi r$ where r is the radius. Write your answer to five significant figures.

c If the satellite travels at a speed of 26 000 km/h, calculate how long, in minutes, correct to three significant figures, it will take to complete one orbit.



- 11 A measurement is stated to be 5900 km. Explain why it is possible that this has been rounded to two, three or four significant figures.

Related resources



5.1 Quick quiz

Lesson 5.2

Scientific notation

Learning intentions

By the end of this lesson, you will be able to ...
→ understand and use scientific notation.

Scientific notation

- **Scientific notation** (or standard notation) is a convenient way of writing very large and very small numbers.
- A number written in scientific notation is written in the form $A \times 10^n$ where A is a decimal number between 1 and 10, and n is a positive or negative whole number.
- When rounding a number in scientific notation to a given number of significant figures, all digits in A are significant. For example, in 1.20×10^4 , there are three significant figures.

Worked example 5.2A Identifying correct use of scientific notation

State whether the following expressions satisfy the conditions of scientific notation.

a 5.3×10^7 **b** 78×10^5 **c** $4.9 \times 10\,000$ **d** 3×10^{-4} **e** 294 000

Part	Think	Write
a	The first number in the product (5.3) is between 1 and 10, and the second number (10^7) is a power of 10.	Yes
b	The first number (78) is not between 1 and 10.	No
c	The second number (10 000) is not written as a power of 10.	No
d	The first number in the product (3) is between 1 and 10, and the second number (10^{-4}) is a power of 10.	Yes
e	294 000 is not written as a product of two numbers.	No

 **Worked example video:** Identifying correct use of scientific notation

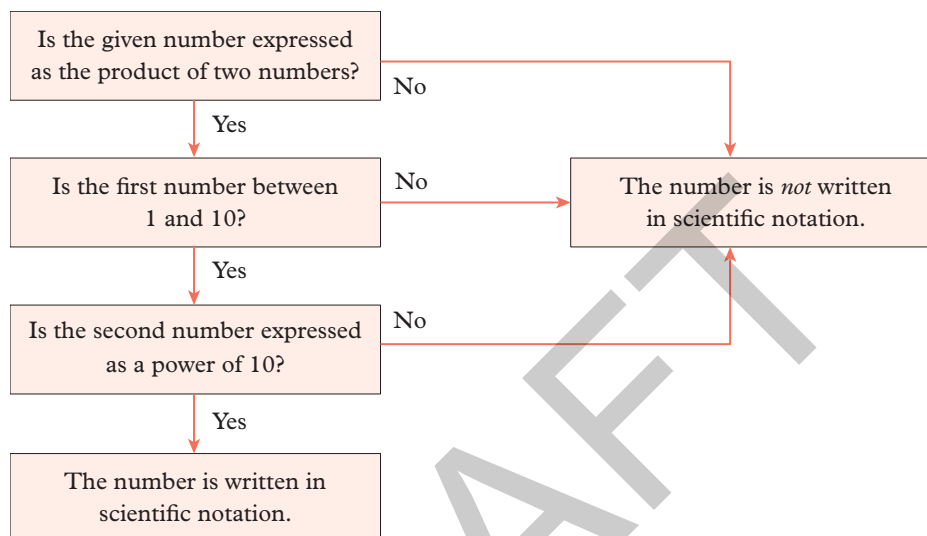
Understanding, fluency and communicating

Answers: Lesson 5.2

Helpful hints

- A number is written in scientific notation if it is written as the product of one number that is between 1 and 10 and another number that is a power of 10.

- 1 Use the flow diagram to determine whether the given numbers are expressed in scientific notation.



a 2.91×10^{-17}

b 53×10^6

c $3.8 \times 100\,000$

d 3^{26}

- 2 State whether the following numbers are written in scientific notation.

a 3.6×10^5

b $5.2 \times 10\,000$

c 21×10^5

d 2.87×10^{-6}

e $6.07 \times \frac{1}{1\,000\,000}$

f 594×10^{-5}

g 70×10^8

h 3.06×10^{-9}

Worked example 5.2B Writing numbers in scientific notation

Write these numbers in scientific notation.

a 138 000

b 0.000 486

Part	Think	Write
a	Move the decimal point so that it is positioned between the 1 and the 3. This produces 1.380 00, which is a number between 1 and 10. Count the number of places back to the original position of the decimal point: 1.38000 Power of 10 = number of places left/right = five to the right = +5	$138\,000 = 1.380\,00 \times 10^5$ $= 1.38 \times 10^5$

Part	Think	Write
b	Move the decimal point so that it is positioned between the 4 and the 8. Count the number of places back to the original position of the decimal point:  00004.86 Power of 10 = number of places left/right = four to the left = -4	$0.000\ 486 = 4.86 \times 10^{-4}$



Worked example video: Writing numbers in scientific notation

Helpful hints

- Move the decimal point so that it is positioned between the two left-most non-zero digits. This always produces a number between 1 and 10.
- Count the number of places back to the original position of the decimal point. This becomes the power of 10.
- When counting back to the original position of the decimal point, counting to the right produces a positive power of 10 and counting to the left produces a negative power of 10.

3 Fill in the blanks and circle the correct direction to write each number in scientific notation.

a 243 000

The number between 1 and 10 found by moving the decimal point to be between the first two non-zero digits is ____.

The number of places back to the original position of the decimal point is ____ to the *left / right*, so the power of 10 is +____.

Hence, in scientific notation, we write

$$243\ 000 = \_\_ \times 10^\square.$$

b 0.000 586

The number between 1 and 10 found by moving the decimal point to be between the first two non-zero digits is ____.

The number of places back to the original position of the decimal point is ____ to the *left / right*, so the power of 10 is -____.

Hence, in scientific notation, we write

$$0.000\ 586 = \_\_ \times 10^\square.$$

4 Write each of the following large numbers in scientific notation.

a 526 000

b 28 000

c 7 000 000

d 49 800

e 28 000 000

f 603 000 000

g 910 000

h 13 200 000 000

5 Write each of the following small numbers in scientific notation.

a 0.000 43

b 0.008 21

c 0.000 007

d 0.000 029

e 0.065

f 0.000 387

g 0.000 008 2

h 0.000 06

Worked example 5.2C Converting from scientific notation to an ordinary number

Write the following as ordinary (or basic) numbers.

a 4.83×10^7

b 9.2×10^{-6}

Part	Think	Write
a	Because the power of 10 is +7, the decimal point is moved 7 places to the right. 	$4.83 \times 10^7 = 48\,300\,000$
b	Because the power of 10 is -6, the decimal point is moved 6 places to the left. 	$9.2 \times 10^{-6} = 0.000\,009\,2$



Worked example video: Converting from scientific notation to an ordinary number

Helpful hints

- The magnitude of the power of 10 tells us how many places to move the decimal point, while the sign of the power tells us which direction to move.
- If the power of 10 is positive, move the decimal point to the right. If the power of 10 is negative, move it to the left.

6 Fill in the blanks and circle the correct direction in the following statements to write each of the numbers as an ordinary number.

a 5.48×10^6

Move the decimal point ___ places to the *left / right*.

$5.48 \times 10^6 = \underline{\hspace{2cm}}$

b 3.09×10^{-5}

Move the decimal point ___ places to the *left / right*.

$3.09 \times 10^{-5} = \underline{\hspace{2cm}}$

7 Express the following as ordinary numbers.

a 3.4×10^6

b 8.3×10^8

c 2.94×10^7

d 2.58×10^5

e 5.26×10^5

f 3.02×10^{12}

g 2.9×10^7

h 8.75×10^8

8 Write the basic number for:

a 5.9×10^{-4}

b 3.2×10^{-6}

c 7.1×10^{-8}

d 2×10^{-3}

e 8×10^{-7}

f 2.64×10^{-5}

g 8.67×10^{-9}

h 2.97×10^{-6}

Problem solving and reasoning

9 Use a calculator to simplify the following. Write your answers in scientific notation, correct to three significant figures.

a $(2.6 \times 10^8) \times (4.1 \times 10^7)$

b $(5.8 \times 10^9) \times (8.2 \times 10^{12})$

c $(9.2 \times 10^{24}) \times (3.5 \times 10^{-8})$

d $(5.8 \times 10^{-6}) \times (2.4 \times 10^{-9})$

e $(8.4 \times 10^{18}) \div (2.5 \times 10^7)$

f $(5.25 \times 10^{12}) \div (4.2 \times 10^{-8})$

g $(1.82 \times 10^{-6}) \times (2.9 \times 10^{-10})$

i $\sqrt{5.76 \times 10^{16}}$

k $\sqrt{(3.1 \times 10^8)^5}$

h $(2.8 \times 10^8)^4 \times (1.6 \times 10^6) \div (2.1 \times 10^{15})$

j $\sqrt{6.8 \times 10^{17}}$

l $(8 \times 10^{-10})^6$

10 Express the following numbers in scientific notation.

- a** The distance of Mars from the Sun is approximately 229 000 000 km.
- b** The diameter of the hydrogen atom is 0.000 000 000 025 4 m.
- c** The Sun produces the same amount of light as 3 000 000 000 000 000 000 000 000 000 candles.
- d** There are approximately 130 000 hairs on a person's head.
- e** There are approximately 10 000 000 000 000 cells in the human body.



11 Change the numbers in the following to ordinary numbers.

- a** There are approximately 3.16×10^7 s in a year.
- b** A molecule's diameter is 8.9×10^{-7} mm.
- c** The number of different possible hands in the card game Poker is approximately 2.6×10^6 .
- d** Swarms of locusts have been known to contain as many as 3×10^{10} locusts.
- e** The size of the influenza virus is approximately 2.6×10^{-4} mm.

12 Light travels at a velocity of 300 000 km/s. Express 300 000 in scientific notation.

- a** How far does light travel in:
 - i** 1 minute?
 - ii** 1 hour?
 - iii** 1 day?
 - iv** 1 year?
- b** If light takes 4.1 min to reach Earth from Mars, what is the distance from Earth to Mars?

13 The radius of Earth is approximately 6400 km.

- a** Calculate the area of Earth's surface, to two significant figures. (Use $A = 4\pi r^2$)
- b** Calculate the volume of Earth, to two significant figures. (Use $V = \frac{4}{3}\pi r^3$)

14 The radius of Earth's orbit around the Sun is approximately 1.49×10^8 km. Assuming that the orbit is circular, calculate to two significant figures the distance travelled by Earth in one orbit. (Use $C = 2\pi r$.)

15 The human brain contains about 10^{10} cells.

- a** Write this value as an ordinary number.
- b** Each human brain cell is about 2.8×10^{-5} m long. If all the brain cells could be placed next to each other, in a straight line, how long would this line be?

16 Measure your pulse to determine the number of times your heart beats in a minute. If you live to be 75 years of age, how many times will your heart have beaten in this time? (Assume your pulse remains constant.)

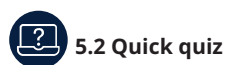
Challenge

17 Light travels at approximately 3×10^5 km/s and sound travels at 330 m/s.

- a** After the starter fires the starting gun, how long does it take:
 - i** the sight of the smoke from the gun to reach a timekeeper standing at the end of a 100 m running track. Give your answer in scientific notation to four significant figures.
 - ii** the sound of the gun to reach the timekeeper, in scientific notation to four significant figures.

- b** What is the difference between the times from your answers to part **a**?
- c** If the timekeeper's stopwatch measures time to the nearest hundredth of a second, does it matter if she uses the sight of the smoke or the sound of the gun to start the stopwatch?
- 18** The number of ways to order a deck of 52 cards is approximately 8.066×10^{67} . The number of atoms on Earth is approximately 1.33×10^{50} . How many Earths would it take so that the number of atoms of all of them would be the same as the number of ways to order a deck of 52 cards? Give your answer in scientific notation to four significant figures.

Related resources



5.2 Quick quiz

Lesson 5.3

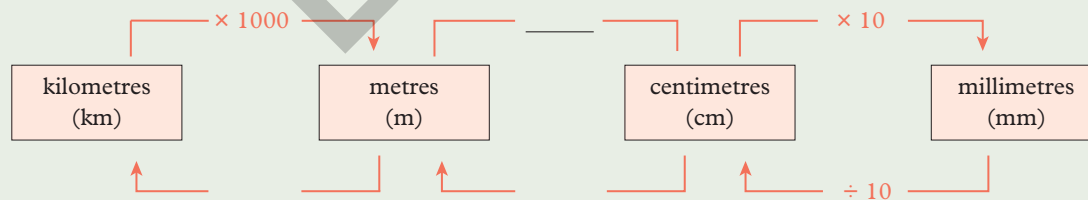
Metric units of measurement

Learning intentions

By the end of this lesson, you will be able to ...
→ convert between commonly used metric units.

Metric units of measurement

- Units in the **metric system** consist of a **base unit**, such as “metre”, and a **unit prefix**, such as “kilo”. This is true for almost all units across all kinds of quantities, including length, mass, capacity, area and volume.
- The conversion factors we use to convert between common metric units of length are summarised in the following conversion diagram.



Worked example 5.3A Converting units of length

Convert these lengths.

a $2.5 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

b $430 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

Part	Think	Write
a	1 km = 1000 m, so the conversion factor is 1000. To convert from km to the smaller unit of m, we multiply by the conversion factor.	$2.5 \text{ km} = (2.5 \times 1000) \text{ m}$ $= 2500 \text{ m}$
b	First convert from mm to cm (conversion factor of 10) and then convert from cm to m (conversion factor of 100). In each case, we divide by the conversion factor because we are converting to a larger unit.	$430 \text{ mm} = (430 \div 10) \text{ cm}$ $= 43 \text{ cm}$ $= (43 \div 100) \text{ m}$ $= 0.43 \text{ m}$



Worked example video: Converting units of length

Understanding, fluency and communicating

Answers: Lesson 5.3

Helpful hints

- To convert to a smaller unit, multiply by the conversion factor.
- To convert to a larger unit, divide by the conversion factor.

1 Convert these lengths.

a $3.6 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

b $8.4 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

c $34.82 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

d $0.56 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

e $2.9 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

f $0.964 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

g $0.658 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

h $45.2 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

i $15.68 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

j $3.69 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

k $16.37 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

l $4.265 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

2 Convert 1 km into the following units, expressing your answer in scientific notation.

a centimetres

b millimetres

3 Convert these lengths.

a $7000 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

b $594 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

c $8930 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

d $6000 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

e $40 \text{ mm} = \underline{\hspace{1cm}} \text{ cm}$

f $85 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

g $800 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

h $328 \text{ mm} = \underline{\hspace{1cm}} \text{ cm}$

i $620 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

j $14\,300 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

k $86 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

l $630 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

m $94 \text{ mm} = \underline{\hspace{1cm}} \text{ cm}$

n $70 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

o $24\,895 \text{ m} = \underline{\hspace{1cm}} \text{ km}$

p $23\,000 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

q $14\,960 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

r $16\,270 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

s $3600 \text{ cm} = \underline{\hspace{1cm}} \text{ m}$

t $72\,945 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

4 What would be a convenient unit (millimetres, centimetres, metres, kilometres) to use for measuring the following?

a breadth of the classroom

b length of a textbook

c height of a student

d length of a baby

e length of your foot

f length of a matchstick

g distance from Sydney to Brisbane

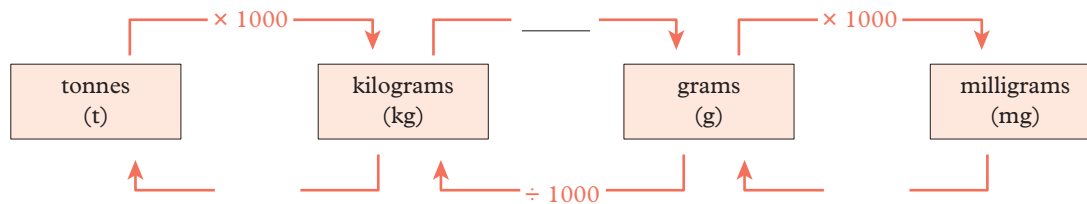
h length of material for a dress

i length of a driveway

j distance between railway stations



5 Complete this conversion diagram for mass.



6 Convert these masses.

a 3.6 t = __ kg	b 7.1 g = __ mg	c 17.84 kg = __ g	d 0.63 t = __ kg
e 4.8 kg = __ g	f 2.465 t = __ kg	g 21.59 t = __ kg	h 0.6 g = __ mg
i 6.06 kg = __ g	j 0.489 kg = __ g	k 1.07 t = __ kg	l 0.03 g = __ mg

7 Convert the following, expressing your answer in scientific notation.

a 1 tonne in grams	b 1 tonne in milligrams	c 1 kilogram in grams

8 Convert the following.

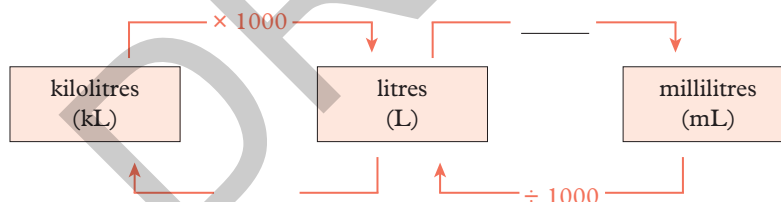
a 8000 kg = __ t	b 4300 g = __ kg	c 2740 mg = __ g	d 690 g = __ kg
e 65 mg = __ g	f 2320 g = __ kg	g 700 kg = __ t	h 460 mg = __ g
i 80 g = __ kg	j 7 mg = __ g	k 9 kg = __ t	l 300 g = __ kg

9 State a convenient unit to use (milligrams, grams, kilograms, tonnes) for measuring the mass of the following.

a an adult	b a jar of honey
c a packet of sugar	d a bag of cement
e a large SUV car	f a rhinoceros
g a headache tablet	h a maths textbook
i a sewing needle	j a tennis ball



10 Complete this conversion diagram for capacity.



11 Convert the following.

a 35 kL = __ L	b 15.9 L = __ mL	c 1.65 L = __ mL	d 0.85 kL = __ L
e 0.06 L = __ mL	f 1.08 kL = __ L	g 0.015 L = __ mL	h 0.005 kL = __ L

12 How many millilitres are there in 1 kL? Express your answer in scientific notation.

13 Convert the following.

a 15 000 mL = __ L	b 8000 L = __ kL
c 7600 mL = __ L	d 800 mL = __ L
e 9280 L = __ kL	f 725 L = __ kL
g 95 mL = __ L	h 40 L = __ kL

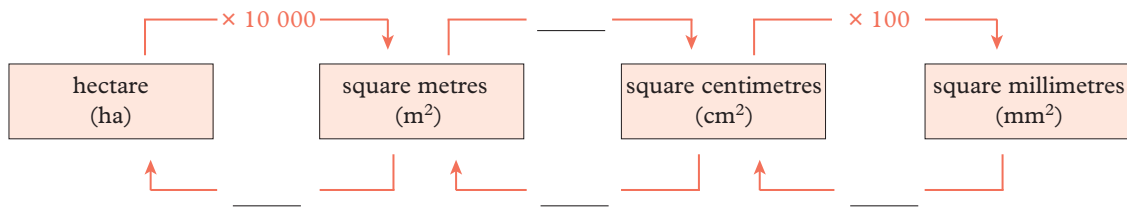
14 State an appropriate unit to use (millimetres, litres, kilolitres) for measuring the capacity of a:



- a** teaspoon
c bucket
e laundry tub
g car's petrol tank
b swimming pool
d fish tank
f farm dam
h kettle.



15 Complete this conversion diagram for area.



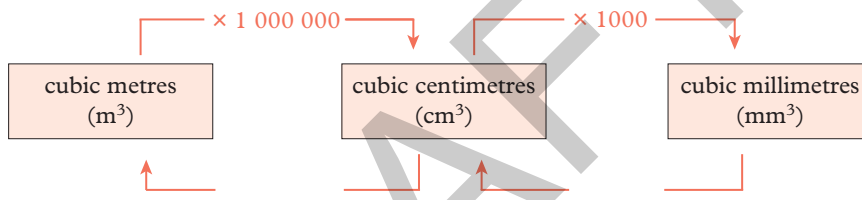
16 Convert these areas.

- a** 2.6 ha = $\square$ m² **b** 4.9 m² = $\square$ cm² **c** 14 cm² = $\square$ mm² **d** 0.752 m² = $\square$ cm²
e 1.65 ha = $\square$ m² **f** 24.8 cm² = $\square$ mm² **g** 8.294 km² = $\square$ m² **h** 5.671 km² = $\square$ cm²

17 Convert the following.

- a** 63 000 m² = $\square$ ha **b** 127 000 cm² = $\square$ m² **c** 810 mm² = $\square$ cm²
d 45 680 cm² = $\square$ m² **e** 298 000 m² = $\square$ km² **f** 2.4 km² = $\square$ ha

18 Complete this conversion diagram for volume.



19 Convert these volumes.

- a** 5 cm³ = $\square$ mm³ **b** 3.9 m³ = $\square$ cm³ **c** 25.6 cm³ = $\square$ mm³
d 0.64 m³ = $\square$ cm³ **e** 0.415 cm³ = $\square$ mm³ **f** 7.39 m³ = $\square$ mm³

20 Convert the following.

- a** 7 400 000 cm³ = $\square$ m³ **b** 56 700 mm³ = $\square$ cm³
c 690 000 cm³ = $\square$ m³ **d** 4258.5 mm³ = $\square$ cm³

Problem solving and reasoning

21 A bottle of wine has a mass of 1140 g.

- a** What would be the mass of a case of one dozen bottles if the cardboard packaging has a mass of 320 g. Write your answer in kilograms.
b If each bottle contains 750 mL of wine, what is the total capacity of one case? Write your answer in litres.



22 The catchment area of a dam is 25.6 km². If 25 mm of rain falls over the catchment area, what will be the increase in capacity of the dam, given that 1 m³ holds 1000 kL of water?

23 The average mass of an adult hippopotamus is 1.5 t. Baby hippopotami, which are born underwater, have an average mass of 37 kg. Express the mass of a baby hippo as a percentage of its mass as an adult.



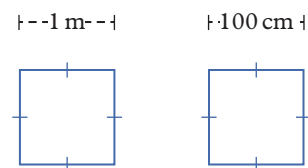
Challenge

24 A house brick has dimensions of $76 \text{ mm} \times 230 \text{ mm} \times 110 \text{ mm}$.

- When travelling to a work site, trucks typically carry six pallets each containing 500 bricks. Calculate the total space in cubic metres occupied by these six pallets.
- A bricklayer wants to know how many bricks they will need for a particular job.
 - Calculate the area of the largest face of a brick.
 - Assume that, when laid, each brick has a 10 mm thickness of mortar along one 230 mm edge and one 110 mm edge. What is the area covered by a brick plus the mortar?
 - How many bricks, surrounded by mortar, are needed to build a wall of area 60 m^2 ?

25 One way to remember the conversion between units of area is by drawing two equivalent pictures and finding the area of each, such as the squares below.

- Draw two squares that could help you remember the conversion between m^2 and km^2 .
- Draw a diagram that could help you remember the conversion between cm^3 and m^3 .



Related resources



5.3 Quick quiz

Lesson 5.4

Unit prefixes in the metric system

Learning intentions

By the end of this lesson, you will be able to ...
→ understand and apply metric prefixes.

Unit prefixes

- The unit prefix attached to a base unit in the metric system always means the same thing. For example, “kilo” always means 1000 copies of the base unit, whether that is “gram”, “metre” or anything else.
- Each unit prefix corresponds to multiplication of the base unit by a different factor of 10, as shown in the table below.

Unit prefix	Multiplying factor
tera (T)	$10^{12} = 1\,000\,000\,000\,000$
giga (G)	$10^9 = 1\,000\,000\,000$
mega (M)	$10^6 = 1\,000\,000$
kilo (k)	$10^3 = 1000$



Unit prefix	Multiplying factor
centi (c)	$10^{-2} = 0.01$
milli (m)	$10^{-3} = 0.001$
micro (μ)	$10^{-6} = 0.000\ 001$
nano (n)	$10^{-9} = 0.000\ 000\ 001$

Worked example 5.4A Converting length measurements

a Convert the following lengths to metres.

i 3.6 Gm

ii 7 μ m

b Convert 5.6 m to:

i kilometres

ii micrometres.

Part	Think	Write
a i	The multiplying factor for the “Giga” prefix is 10^9 , so $1\text{ Gm} = 1 \times 10^9\text{ m}$. To convert <i>to</i> metres, <i>multiply</i> 3.6 by 10^9 , to get the answer in scientific notation, then convert to an ordinary number.	$3.6\text{ Gm} = 3.6 \times 10^9\text{ m}$ $= 3\ 600\ 000\ 000\text{ m}$
ii	The multiplying factor for the “micro” prefix is 10^{-6} , so $1\ \mu\text{m} = 1 \times 10^{-6}\text{ m}$. To convert <i>to</i> metres, <i>multiply</i> 7 by 10^{-6} , to get the answer in scientific notation, then convert to an ordinary number.	$7\ \mu\text{m} = 7 \times 10^{-6}\text{ m}$ $= 0.000\ 007\text{ m}$
b i	The multiplying factor for the “kilo” prefix is 10^3 , so $1\text{ km} = 1 \times 10^3\text{ m}$. To convert <i>from</i> metres, <i>divide</i> 5.6 by 10^3 , which is the same as multiplying by 10^{-3} . Give the answer as an ordinary number.	$5.6\text{ m} = \frac{5.6}{10^3}\text{ km}$ $= 5.6 \times 10^{-3}\text{ km}$ $= 0.0056\text{ km}$
ii	As above, $1\ \mu\text{m} = 1 \times 10^{-6}\text{ m}$. To convert <i>from</i> metres, <i>divide</i> 5.6 by 10^{-6} , which is the same as multiplying by 10^6 . Give the answer as an ordinary number.	$5.6\text{ m} = \frac{5.6}{10^{-6}}\ \mu\text{m}$ $= 5.6 \times 10^6\ \mu\text{m}$ $= 5\ 600\ 000\ \mu\text{m}$



Worked example video: Converting length measurements

Understanding, fluency and communicating

Answers: Lesson 5.4

Helpful hints

- To convert from a very large or very small unit to the base unit, *multiply* by the factor given in the table.
- To convert a measurement in the base unit to one in a larger or smaller unit, *divide* by the factor given in the table.

1 Convert the following:

a lengths to metres

i 5.7 Mm

ii 9 cm

iii 8 nm

b weights to grams

i 8 Gg

ii 4.2 mg

iii 5 μ g

c capacities to litres

i 3 TL

ii 2.8 ML

iii 7 mL

2 Convert

a 4.9 m to:

i kilometres

ii micrometres

iii nanometres.

b 2.4 g to:

i megagrams

ii teragrams

iii micrograms.

c 6.5 L to:

i megalitres

ii millilitres

iii giganlitres.

Worked example 5.4B Converting measurements of mass

Convert the following.

a 3.56 Tg = __ Mg

b 9.4 mg = __ μ g

Part	Think	Write
a	Convert 3.56 Tg into grams using multiplication (by 10^{12}), then convert this value into megagrams using division (by 10^6).	$3.56 \text{ Tg} = \frac{3.56 \times 10^{12}}{10^6} \text{ Mg}$ $= 3.56 \times 10^6 \text{ Mg}$ $= 3\,560\,00 \text{ Mg}$
b	Convert 9.4 mg into grams using multiplication (by 10^{-3}), then convert this value into micrograms using division (by 10^{-6}).	$9.4 \text{ mg} = \frac{9.4 \times 10^{-3}}{10^{-6}} \mu\text{g}$ $= 9.4 \times 10^3 \mu\text{g}$ $= 9400 \mu\text{g}$

**Worked example video:** Converting measurements of mass**Helpful hints**

- First convert to the base unit by multiplying, then convert to the new unit by dividing, each time using the appropriate factor from the table.

3 Convert the following measurements.

a 7.2 Gg = __ Mg

b 2.9 cg = __ ng

c 8 Tg = __ kg

d 4.3 cm = __ μ m

e 8.8 TL = __ ML

f 9 kL = __ mL

g 5.3 ms = __ ns

h 1.2 μ s = __ ns

Problem solving and reasoning

- 4 Light travels approximately 9.46×10^{15} m in a year. Express this in terametres.
- 5 Warragamba dam holds approximately 2 580 000 ML of water at full capacity. How many gigalitres is this?
- 6 The distance from Mars to the Sun is 0.228 Tm. Convert this distance to kilometres.
- 7 The mass of a hydrogen atom is 1.67×10^{-24} g. What is the mass in nanograms of 1 million hydrogen atoms?
- 8 A computer can access its memory in around 5 ns. Convert this to microseconds.

Challenge

- 9 The ocean holds about 1.33 teralitres (TL) of water. Convert this quantity into cubic metres.

Related resources



5.4 Quick quiz

DRAFT

Lesson 5.5

Review: Practicalities of measurement

Review questions 5.5A Mathematical literacy review

Answers: Lesson 5.5

Key terms

The following key terms are used in this module:

- base unit
- metric system
- scientific notation
- significant figures
- unit prefix

Check your understanding

- For each of the following, state the number of significant figures implied by the approximation provided.
 - 76 325 832 is rounded to 76 326 000
 - $8.358\,922 \approx 8.3589$
 - 0.004 637 29 is expressed as 4.6×10^{-3}
- If the following value is expressed in scientific notation, what must be true about the numbers A and n ?
 $A \times 10^n$
- In the United States, they use the imperial system, whereas in Australia, we use the ___ system.
- In each of the following, label the base unit and the unit prefix.

a kilogram	b teralitre	c millisecond	d micrometre
e megametre	f centigram	g nanolitre	h gigasecond

Review questions 5.5B Multiple-choice questions

- When rounded to two significant figures, 3950.628 becomes:

A 3900	B 4000	C 39	D 3950.63
- Which of the following numbers is written in scientific notation?

A $5 \times 10\,000$	B 50 000	C 5×10^4	D 50×10^3
- 7.06×10^{-6} is equivalent to:

A 0.000 007 06	B 0.000 070 6	C 706 000	D 7 060 000
- The result of the calculation $(4 \times 10^5) \div (8 \times 10^{-3})$ is:

A 5×10^8	B 5×10^7	C 5000	D 5×10^2
- Which one of the following is equivalent to 5.06 kg?

A 0.005 06 g	B 5060 g	C 0.0506 g	D 506 g
- Which one of the following is *not* equivalent to 5.3 m?

A 530 cm	B 5300 mm	C 0.0053 km	D 0.053 km
- The capacity of a drinking glass would be closest to:

A 2 mL	B 20 mL	C 200 mL	D 2 L
- 6 ML is equivalent to:

A 60 000 kL	B 6000 kL	C 600 kL	D 60 kL

Review questions 5.5C Set 1

- Round 3659.063 to:
 - the nearest 100
 - the nearest whole number
 - two decimal places
 - three significant figures
 - five significant figures
 - one significant figure.
- State whether each of these numbers is written in scientific notation.
 - 6×1000
 - 15×10^7
 - 2.04×10^{-6}
- Express these numbers in scientific notation.
 - 105 000 000
 - 0.000 062
 - 3179
- Calculate the following, writing the answers in scientific notation.
 - $(4.1 \times 10^8) \times (6 \times 10^5)$
 - $(1.96 \times 10^{-3}) \div (1.4 \times 10^7)$
 - $(8 \times 10^5)^4$
 - $\sqrt{8.41 \times 10^{-12}}$
- Convert the following quantities to the units indicated within parentheses.
 - 5.6 cm^2 [mm²]
 - 43 000 m² [ha]
 - 2.9 m^3 [cm³]
 - 5600 mm³ [cm³]
- Convert the following quantities to the units indicated within parentheses.
 - 2.3 Gm [m]
 - 52 ML [kL]
 - 3 ms [μs]
 - 7.2 Tg [mg]

Review questions 5.5D Set 2

- Round 1472.634 to:
 - the nearest 10
 - two significant figures
 - two decimal places.
- Express each of the following in scientific notation.
 - 749 000
 - 0.000 003
 - 0.0105
- Calculate $(1.4 \times 10^7) \times (4.5 \times 10^8)$, expressing the answer in scientific notation.
- Convert the following quantities to the units indicated within parentheses.
 - 2.1 ha [m³]
 - 780 mm³ [cm³]
 - 9 500 000 cm³ [m³]
 - 72 cm³ [mm³]
- Convert the following quantities to the units indicated within parentheses.
 - 4.3 cm [μm]
 - 2 Tg [Mg]
 - 52 000 kL [ML]
 - 9.1 mm [nm]

Review questions 5.5E Set 3

- Round 0.005 06 to:
 - two significant figures
 - two decimal places
 - one significant figure.
- State whether each of the following numbers is expressed in scientific notation.
 - $4.9 \times 100\ 000$
 - 7.0×10^8
 - 70×10^7
- Convert each of the following numbers to scientific notation.
 - 67 000
 - 0.000 080 9
 - 1230
- Write the basic numeral for:
 - 3.4×10^6
 - 8.7×10^{-5}
 - 2.053×10^4
- Convert the following quantities to the units indicated within parentheses.
 - 11.2 cm^2 [mm²]
 - 129 000 m² [ha]
 - 3.4 m^3 [cm³]
 - 73 000 mm³ [cm³]
- Convert the following quantities to the units indicated within parentheses.
 - 4.5 Mm [m]
 - 2 Mg [t]
 - 7 μs [ns]
 - 3.5 kL [GL]

Review questions 5.5F Set 4

- Round 2.0695 to:
 - one significant figure
 - two significant figures
 - three significant figures
 - four significant figures.
- Convert the following quantities to the units indicated within parentheses.
 - 13.65 m [cm]
 - 3460 kg [t]
 - 276 mL [L]
 - 8.3 m [mm]
- Convert the following quantities to the units indicated within parentheses.
 - 13.65 ha [m²]
 - 1960 mm² [cm²]
 - 3 700 000 cm³ [m³]
 - 6.8 cm³ [mm³]
- Convert the following quantities to the units indicated within parentheses.
 - 6 mL [μL]
 - 4.2 Gg [kg]
 - 8.1 μs [ns]
 - 560 ML [TL]
- Express in scientific notation, correct to two significant figures.
 - 643 700 000
 - 0.000 000 304
- Calculate the following, expressing the answer in scientific notation.
 - $(1.08 \times 10^{-6}) \div (7.2 \times 10^{-5})$
 - $\sqrt{1.96 \times 10^{20}}$

Review questions 5.5G Practice examination question

- Answer the following questions.
 - Write 0.001 306 in scientific notation, rounded to three significant figures. **(2 marks)**
 - How many significant figures are there in the measurement 5.0×10^6 metres? **(1 mark)**
 - Evaluate $\sqrt{6.724 \times 10^{11}}$. Write your answer in scientific notation. **(1 mark)**
 - Convert each measurement:
 - 94 630 mm = __ m
 - 2.5 m³ = __ cm³ **(2 marks)**
 - The Great Barrier Reef spans approximately 344 400 km².
 - Write this area in square metres using scientific notation.
 - How many significant figures does your solution to part i have? **(3 marks)**
 - The “mole” is a unit of measurement most often associated with chemistry.
Given that
1 mol = $6.022\,140\,76 \times 10^{23}$ elementary units,
 - Find the value of 4 moles up to 3 significant figures.
 - Use the standard interpretation of the metric prefix “kilo” to deduce the value of one kilomole:
1 kmol = __ $\times 10^{\square}$ elementary units **(3 marks)**

TOTAL: 12 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.

 **Module checklist:** Practicalities of measurement

I can do this	I need to review this
☐ round numbers to a given number of significant figures	Go back to Lesson 5.1 Significant figures
☐ convert between the basic form and scientific notation for decimal numbers ☐ perform calculations with numbers expressed in scientific notation	Go back to Lesson 5.2 Scientific notation
☐ convert between the commonly used metric units for length, mass, capacity, area and volume	Go back to Lesson 5.3 Metric units of measurement
☐ convert very large and very small quantities to more appropriate metric units ☐ recognise different unit prefixes and how they relate to powers of 10	Go back to Lesson 5.4 Unit prefixes in the metric system

Related resources

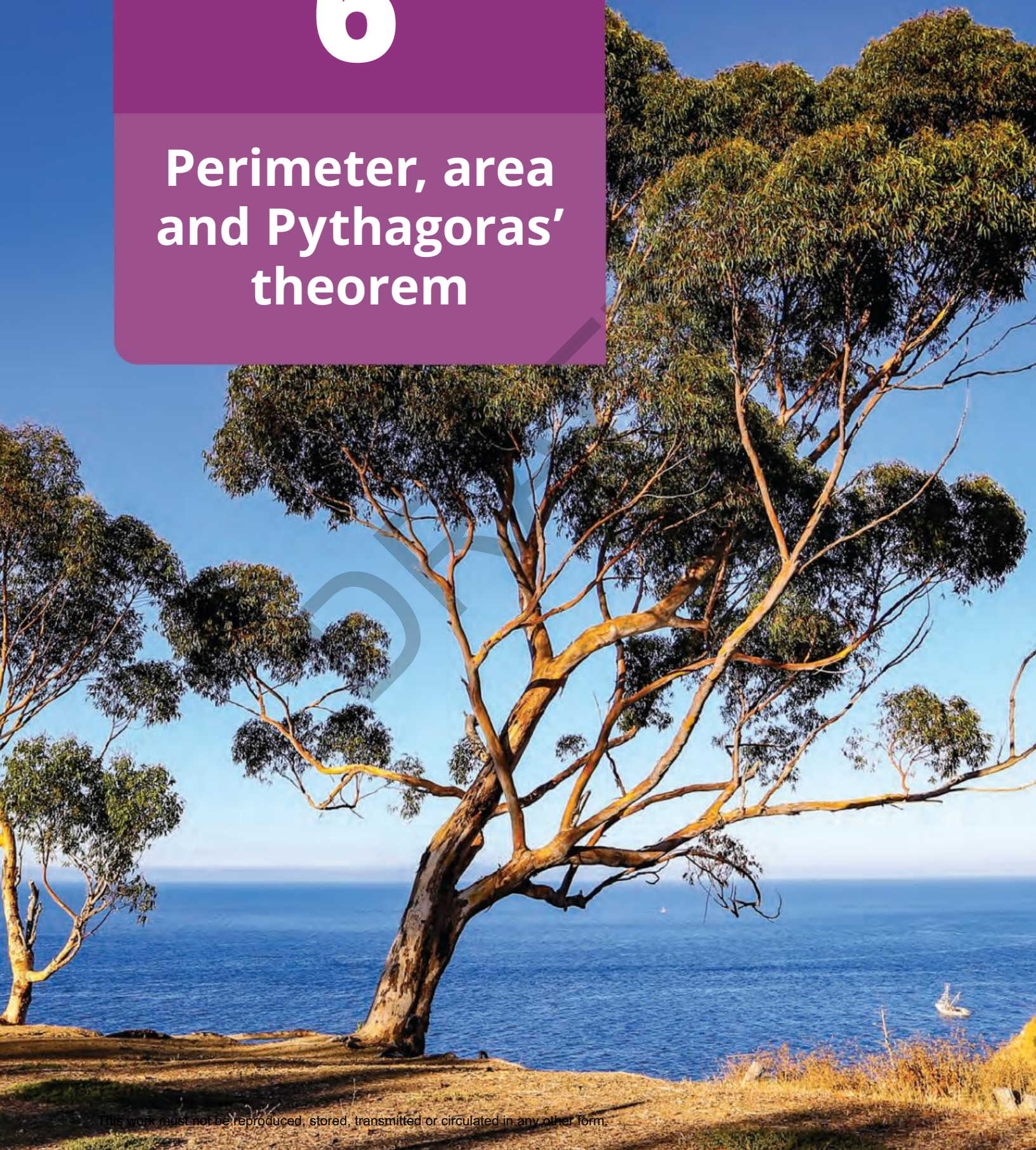
 **Module review quiz:** Practicalities of measurement

 **Worksheet:** Challenge 5 Helpful hints

Module

6

Perimeter, area and Pythagoras' theorem



Lessons

- Lesson 6.1 Perimeter
- Lesson 6.2 Using Pythagoras' theorem
- Lesson 6.3 Area of simple shapes
- Lesson 6.4 Area of composite shapes
- Lesson 6.5 Perimeter and area of irregular shapes
- Lesson 6.6 Review: Perimeter, area and Pythagoras' theorem

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Perimeter, area and Pythagoras' theorem

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Units of measurement



Support sheet: Converting units of length



Support sheet: Identifying simple shapes within composite shapes



Support sheet: Substitution



Support sheet: Rounding decimals



Support sheet: Fraction of a quantity

Syllabus links

- Applications of measurement (MST-11-05)
 - A student solves problems involving measurement in practical contexts.
 - Perimeter, area and volume
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 6.1

Perimeter

Learning intentions

By the end of this lesson, you will be able to ...

→ find the perimeter of simple and composite shapes.

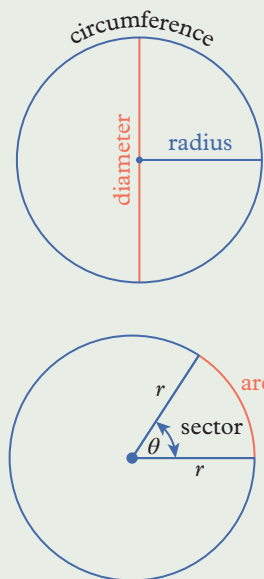
Perimeter

- The **perimeter** of a closed shape is the distance around its boundary. The boundary may consist of straight or curved sides. The perimeter of a shape is found by adding the lengths of all its sides.
 - All measurements must be expressed in the same unit before the side lengths are added.

Circumference of a circle

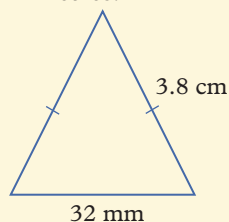
- The perimeter of a circle is called its **circumference**.
 - The distance between the centre of a circle and any point on the circumference is called the **radius**, r , of the circle.
 - The length of a line between two points on the circumference of a circle which passes through the centre of the circle is called the **diameter**, d , of the circle. The diameter is equal to double the radius ($d = 2r$).
- The formula for the circumference, C , of a circle with radius r is $C = 2\pi r$.
 - As $d = 2r$, the circumference can also be found using the formula $C = \pi d$.
- The formulas $C = \pi d$ and $C = 2\pi r$ can be rearranged to make either d or r the subject. This means that you can find the radius and diameter of a circle when given the circumference.
- A **sector** of a circle is the shape enclosed by two radii of a circle and the arc (part of the circumference) between them.
 - A semicircle is a sector which is half of a circle.
 - The arc length of a sector is the length of the curved part of the sector.
 - If the angle between the two radii of a sector is θ , then the perimeter of the sector can be found using the formula

$$P = 2r + \frac{\theta}{360^\circ} \times 2\pi r.$$



Worked example 6.1A Calculating the perimeter after converting units

Calculate the perimeter of this shape, in millimetres.

**Think**

Convert 3.8 cm to mm.
Multiply 38 by 2 because there are two sides marked as equal.

Write

$$\begin{aligned} 1 \text{ cm} &= 10 \text{ mm} \\ 3.8 \text{ cm} &= 3.8 \times 10 \text{ mm} \\ &= 38 \text{ mm} \\ P &= 32 \text{ mm} + 2 \times 38 \text{ mm} \\ &= 108 \text{ mm} \end{aligned}$$



Worked example video: Calculating the perimeter after converting units

Understanding, fluency and communicating

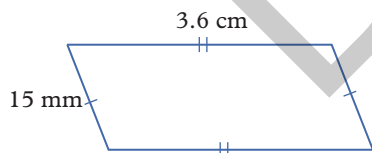
Answers: Lesson 6.1

Helpful hints

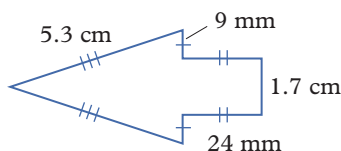
- Convert units so all the sides are expressed in the same unit.
- Sides marked with identical hatch marks are equal in length.

1 Calculate the perimeter of each of these shapes, in millimetres.

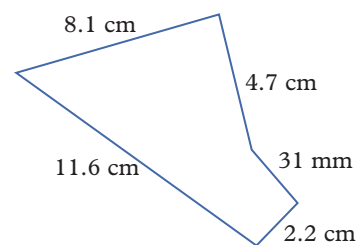
a



b

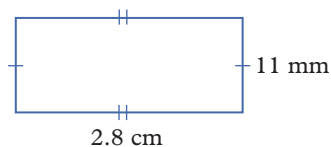


c

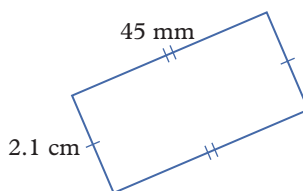


2 Calculate the perimeter of each of these rectangles. Give the answers in centimetres.

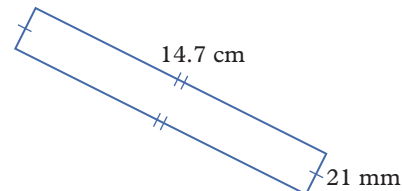
a



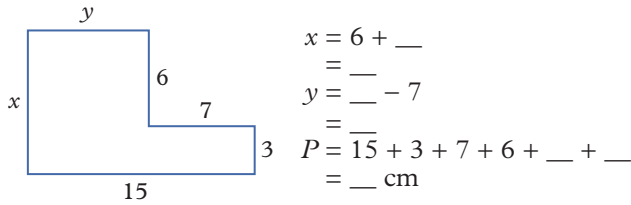
b



c

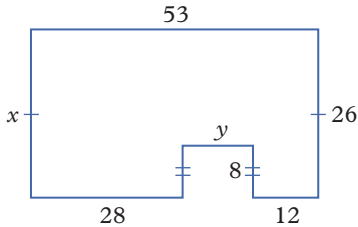


3 Complete the following to find the perimeter of this shape. All measurements are in centimetres.

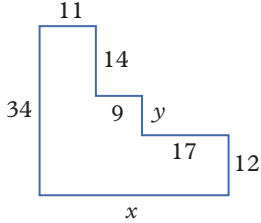


4 Find the unknown side lengths and calculate the perimeter of each of these shapes. All measurements are in centimetres.

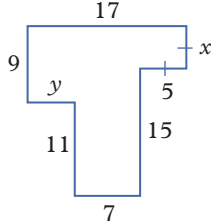
a



b



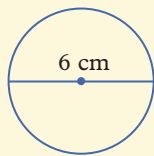
c



Worked example 6.1B Finding the circumference of a circle

Find the circumference of each of these circles, correct to one decimal place.

a



b



Part	Think	Write
a	Use the formula $C = \pi d$ because the diameter is given. Substitute $d = 6 \text{ cm}$ into the formula. Use a calculator to find the circumference, correct to one decimal place.	$C = \pi d$ $= \pi \times 6 \text{ cm}$ $\approx 18.8 \text{ cm}$
b	Use the formula $C = 2\pi r$ because the radius is given. Substitute $r = 4.2 \text{ cm}$ into the formula. Use a calculator to find the circumference, correct to one decimal place.	$C = 2\pi r$ $= 2 \times \pi \times 4.2 \text{ cm}$ $\approx 26.4 \text{ cm}$

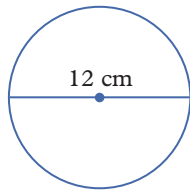


Worked example video: Finding the circumference of a circle

Helpful hints

- The formulas $C = \pi d$ and $C = 2\pi r$ are equivalent because $d = 2r$. You can use either formula by converting between the diameter and radius.

5 Complete the following to find the circumference of this circle in two ways.



$$\begin{aligned} C &= \pi d \\ &= \pi \times \underline{\hspace{1cm}} \text{ cm} \\ &\approx \underline{\hspace{1cm}} \text{ cm} \\ C &= 2\pi r \\ r &= \underline{\hspace{1cm}} \text{ cm} \div 2 \\ &= \underline{\hspace{1cm}} \text{ cm} \\ C &= 2 \times \pi \times \underline{\hspace{1cm}} \text{ cm} \\ &\approx \underline{\hspace{1cm}} \text{ cm} \end{aligned}$$

6 Calculate the circumference of each of these circles, correct to one decimal place.

a

b

c

d

e diameter = 12.6 m

f radius = 7.9 m

g radius = 2.5 m

h diameter = 8.64 m

Worked example 6.1C Finding the perimeter of figures containing semicircles and quarter circles

Find the perimeter of each of these shapes, correct to one decimal place.

a

b

c

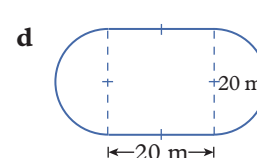
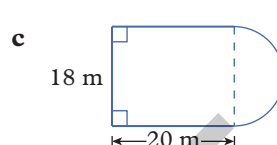
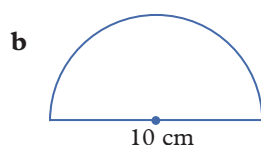
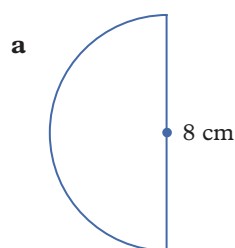
Part	Think	Write
a	A semicircle is half a circle, so divide the circumference of a whole circle by 2 to find its arc length. Add the diameter to this length to find the perimeter.	$\begin{aligned} \text{Curved part} &= \pi d \div 2 \\ &= (\pi \times 6 \text{ cm}) \div 2 \\ &\approx 9.4 \text{ cm} \\ P &\approx 9.4 \text{ cm} + 6 \text{ cm} \\ &= 14.4 \text{ cm} \end{aligned}$
b	The two semicircles make one whole circle. Markings show all sides of the square are equal, so the diameter of the circle is 40 m. Add all lengths to find the perimeter.	$\begin{aligned} \text{Curved part} &= \pi \times 40 \text{ m} \\ &\approx 125.7 \text{ m} \\ P &\approx 125.7 \text{ m} + 2 \times 40 \text{ m} \\ &= 205.7 \text{ m} \end{aligned}$

Part	Think	Write
c	This is a quarter circle, as a right angle is one quarter of 360°. Divide the circumference of a whole circle by 4 to find its arc length. Add the two radii to this length to find the perimeter.	Curved part $= \pi d \div 4$ $= (\pi \times 3 \text{ cm}) \div 4$ $\approx 2.4 \text{ cm}$ $P \approx 2.4 \text{ cm} + 2 \times 3 \text{ cm}$ $= 8.4 \text{ cm}$

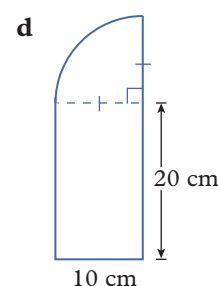
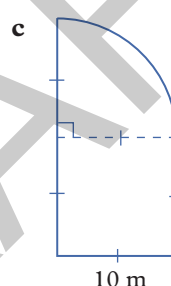
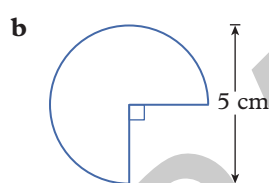
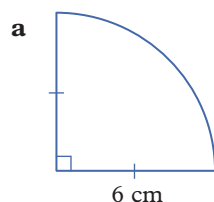


Worked example video: Finding the perimeter of figures containing semicircles and quarter circles

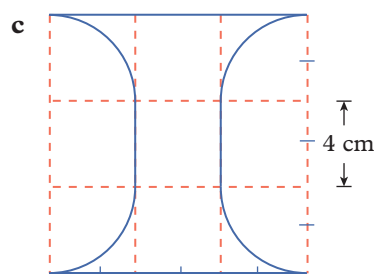
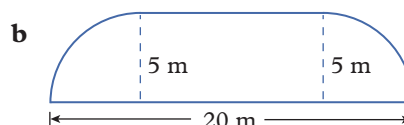
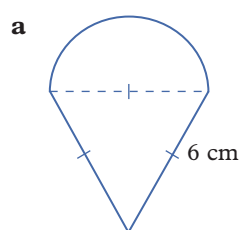
7 Find the perimeter of each of these shapes, correct to one decimal place.



8 Find the perimeter of each of these shapes, correct to one decimal place.

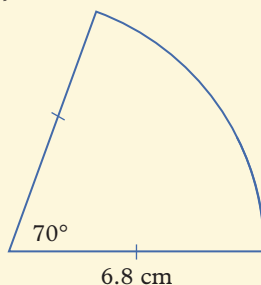


9 Find the perimeter of each of these shapes, correct to four significant figures.



Worked example 6.1D Finding the perimeter of a sector

- a** Determine what fraction of a circle is shown.
b Determine the perimeter of the sector, correct to one decimal place.



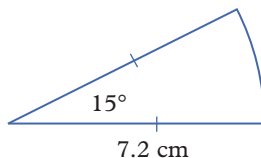
Part	Think	Write
a	There are 360° in a circle. The fraction is the sector angle 70° divided by 360° .	Fraction = $\frac{70^\circ}{360^\circ} = \frac{7}{36}$
b	Multiply the circumference of the circle by the fraction from part a to find the arc length of the sector. Add the two radii to this length to find the perimeter.	Let C be the circumference of the whole circle. Curved part = $\frac{7}{36} \times C$ $= \frac{7}{36} \times 2\pi r$ $= \frac{7}{36} \times 2 \times \pi \times 6.8 \text{ cm}$ $= 8.307 \text{ cm}$ $\approx 8.3 \text{ cm}$ $P \approx 8.3 \text{ cm} + 2 \times 6.8 \text{ cm}$ $= 21.9 \text{ cm}$

**Worked example video:** Finding the perimeter of a sector**Helpful hints**

- The method described in Worked example 6.1D gives the same result as substituting the corresponding values for r and θ into the formula

$$P = 2r + \frac{\theta}{360^\circ} \times 2\pi r.$$

- Use the method you prefer when finding the perimeter of a sector.

10 Consider this sector.

- a** Complete the following to determine what fraction of a circle the sector makes up.

$$\text{Fraction} = \frac{\square^\circ}{360^\circ}$$

$$= \underline{\hspace{1cm}}$$

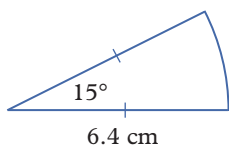
- b** Determine the perimeter of the sector, correct to one decimal place.

$$P = \frac{1}{\square} \times 2 \times \pi \times \underline{\hspace{1cm}} \text{ cm} + 2 \times \underline{\hspace{1cm}} \text{ cm}$$

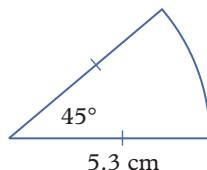
$$\approx \underline{\hspace{1cm}} \text{ cm}$$

- 11** Determine what fraction of a circle is drawn each time, then calculate the perimeter of each sector, correct to one decimal place.

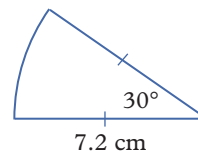
a



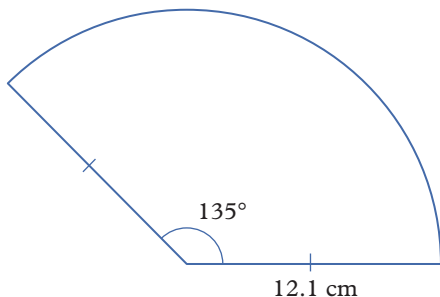
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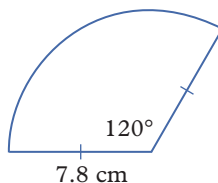
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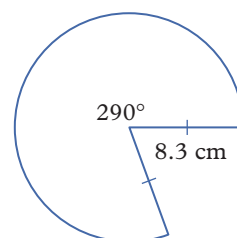
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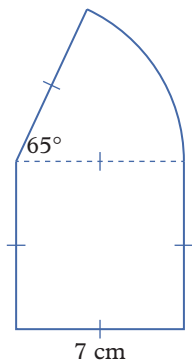


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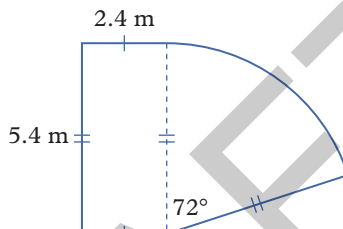


- 12** Find the perimeter of each of these shapes, correct to one decimal place.

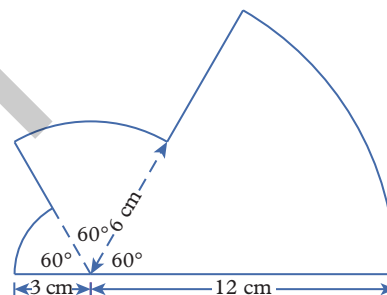
a



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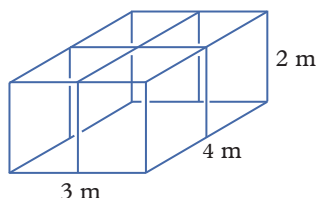


Problem solving and reasoning

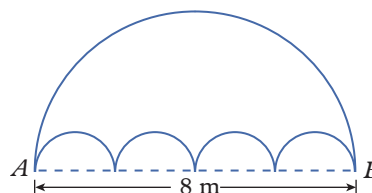
- 13** Find the total length of ribbon used to gift wrap a box as shown. An extra 15 cm is required for the knot and bow.



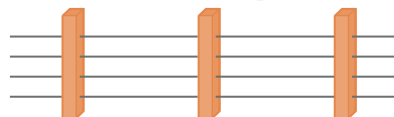
- 14** The frame of a tool shed consists of square galvanised tubing that costs \$4.65 a metre. Find the total cost of the tubing needed to make the frame of this shed.



- 15** Consider the following shape made up of one large semicircle and four small semicircles of equal diameter.



- Find the lengths of the two paths from A to B , correct to two decimal places.
 - Which distance is shorter?
 - What is the difference between the lengths of the two paths?
- 16** A farmer decides to fence a 400 m by 350 m paddock with a four-strand wire fence. Find the total cost of the wire required, given that single strand wire costs 12.4 cents per metre.



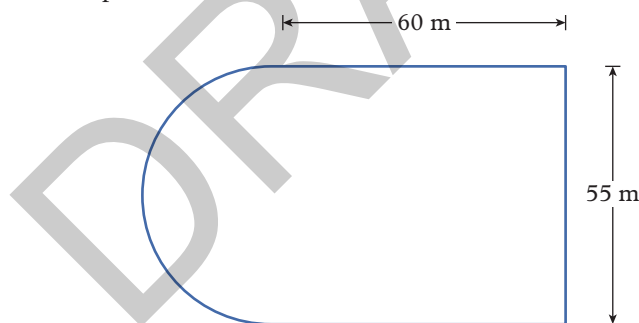
- 17 A circular plate has a circumference of 50 cm. Find its radius, correct to one decimal place.
- 18 Georgette runs around a circular track with a circumference of 200 m. Find the radius of the track, correct to one decimal place.
- 19 A satellite has a circular orbit 800 km above Earth's surface.
- a If the radius of Earth is 6400 km, find the radius of the orbit of the satellite.
 - b Find the circumference of the satellite's orbit, to the nearest kilometre.
 - c If the satellite makes one orbit in a day, find the speed of the satellite.

Challenge

- 20 The production director of a newspaper company decides that a plastic wrapper needs to be placed around its newspapers. Each wrapper is 50% longer than the circumference of the rolled-up paper, and the average diameter of a rolled-up paper is 5 cm. Find the number of kilometres of wrapper required to wrap the 275 000 newspapers printed daily. Give your answer correct to two decimal places.



- 21 Mary regularly jogs around her block, which consists of three straight edges and a semicircle. She starts and finishes her jog at the same point.



- a If she wants to run at least 2 km, what is the minimum number of laps she needs to complete?
- b If her average jogging speed is 7 km/h, find how long it will take her to complete her jog, correct to the nearest minute.

Related resources



6.1 Quick quiz



Worksheet: Perimeter

Lesson 6.2

Using Pythagoras' theorem

Learning intentions

By the end of this lesson, you will be able to ...

→ use Pythagoras' theorem to solve problems involving right-angled triangles.

Pythagoras' theorem

- **Pythagoras' theorem** relates the side lengths of a right-angled triangle.
 - The **hypotenuse** is the longest side of a right-angled triangle and is opposite the right angle. The hypotenuse is usually labelled c .
 - The two shorter sides of a right-angled triangle are usually labelled a and b .
- Pythagoras' theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. This can be written as:

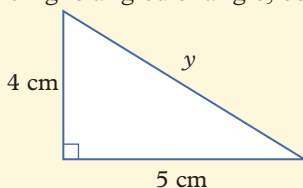
$$c^2 = a^2 + b^2.$$
- Pythagoras' theorem can be used to determine unknown lengths in measurement problems.

Determining whether a triangle is right-angled

- The side lengths of a right-angled triangle always satisfy Pythagoras' theorem.
- Given the side lengths of any triangle, you can check if the triangle is right-angled by substituting the side lengths into Pythagoras' theorem. The longest side length should always be substituted for c .
 - If the side lengths satisfy Pythagoras' theorem, then the triangle is right-angled.
 - If the side lengths do not satisfy Pythagoras' theorem, then the triangle is not right-angled.

Worked example 6.2A Finding the length of the hypotenuse

Find the length of the hypotenuse in this right-angled triangle, correct to one decimal place.



Think

The unknown side, y , is the hypotenuse (as it is opposite the right angle).

Substitute $c = y$, $a = 4$ and $b = 5$ into Pythagoras' theorem then solve for y . Take the positive square root as y represents a length which must be positive.

Use a calculator to write the result correct to one decimal place.

Write

$$\begin{aligned} c^2 &= a^2 + b^2 \\ y^2 &= 4^2 + 5^2 \\ &= 16 + 25 \\ &= 41 \\ y &= \sqrt{41} \\ &\approx 6.4 \end{aligned}$$

The length of the hypotenuse is 6.4 cm.



Worked example video: Finding the length of the hypotenuse

Understanding, fluency and communicating

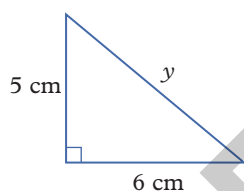
Answers: Lesson 6.2

Helpful hints

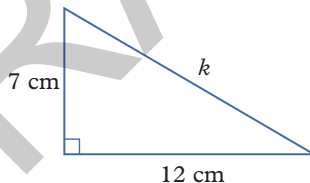
- When solving for the value of c , remember that the length of the hypotenuse must be a positive value.
- Don't forget to include units in your final answer when working with measurements.

1 Find the length of the hypotenuse, correct to one decimal place, in the following right-angled triangles.

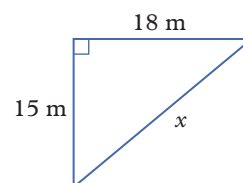
a



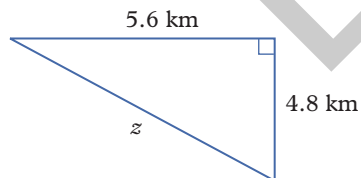
b



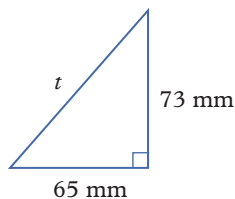
c



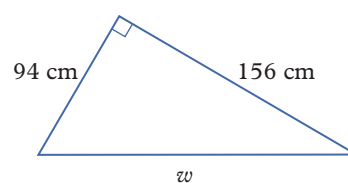
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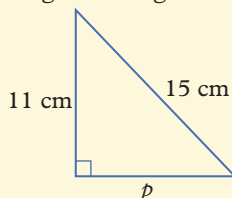


f



Worked example 6.2B Finding the length of a shorter side

Find the unknown side length in this right-angled triangle.



Think

The hypotenuse is 15 cm. The other two sides are p and 11 cm.

Substitute $c = 15$, $a = p$ and $b = 11$ into Pythagoras' theorem.

Solve the resulting equation by subtracting 121 from both sides and then taking the positive square root.

Use a calculator to write the result correct to one decimal place.

Write

$$\begin{aligned} a^2 + b^2 &= c^2 \\ p^2 + 11^2 &= 15^2 \\ p^2 + 121 &= 225 \\ p^2 &= 225 - 121 \\ &= 104 \\ p &= \sqrt{104} \\ &\approx 10.2 \end{aligned}$$

The length of the unknown side is 10.2 cm.

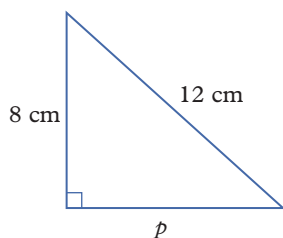
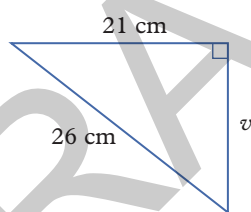
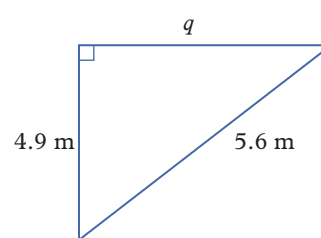
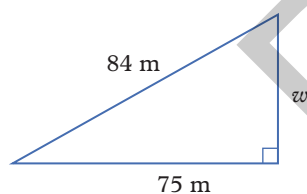
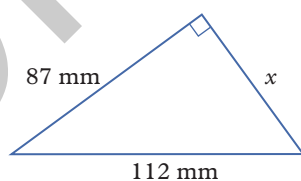
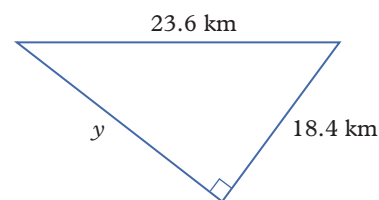


Worked example video: Finding the length of a shorter side

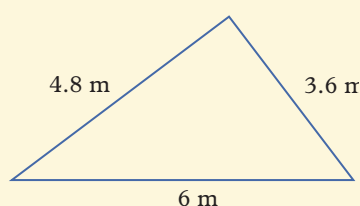
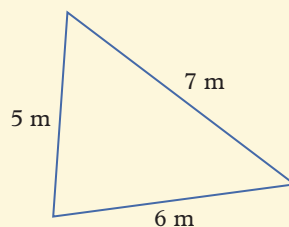
Helpful hints

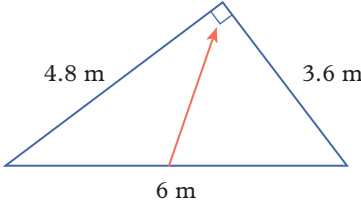
- Make sure to substitute the length of the hypotenuse for c and the known shorter side for either a or b . It may help to label the sides of the triangle with a , b and c before substituting the values into Pythagoras' theorem.

- 2 Find the length of the unknown side in these right-angled triangles. Give the answer correct to one decimal place.

a**b****c****d****e****f****Worked example 6.2C** Determining whether a triangle is right-angled

Determine whether the following triangles are right-angled. If the triangle is right-angled, draw the triangle and mark the right angle on it.

a**b**

Part	Think	Write
a	Calculate the square of the longest side. Calculate the sum of the squares of the other two sides. Compare the results. The results are equal, so the triangle is right-angled. The right angle is opposite the longest side. Draw the triangle and label the right angle.	$c^2 = 6^2 = 36$ $a^2 + b^2 = 4.8^2 + 3.6^2$ $= 23.04 + 12.96$ $= 36$ Since $a^2 + b^2 = c^2$, the triangle is right-angled. 
b	Calculate the square of the longest side. Calculate the sum of the squares of the other two sides. Compare the results. The results are not equal, so the triangle is not right-angled.	$c^2 = 7^2 = 49$ $a^2 + b^2 = 5^2 + 6^2$ $= 25 + 36$ $= 61$ Since $a^2 + b^2 \neq c^2$, the triangle is not right-angled.

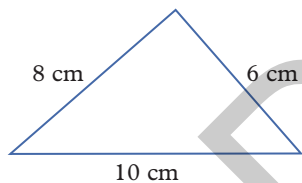


Worked example video: Determining whether a triangle is right-angled

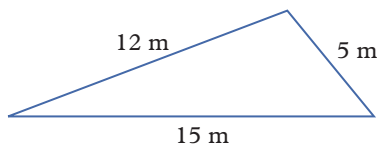
3 Consider the following triangles. For each one:

- state whether the triangle is right-angled
- if the triangle is right-angled, draw the triangle and mark the right angle on it.

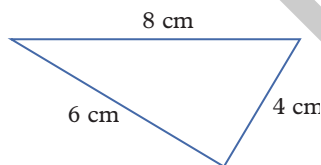
a



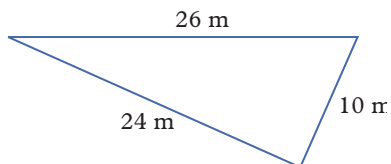
b



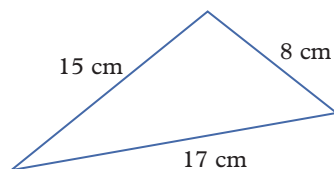
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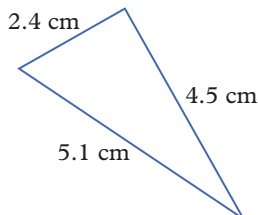
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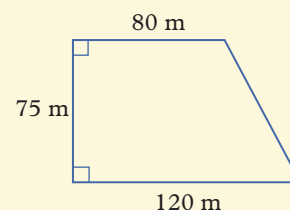
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Worked example 6.2D Using Pythagoras' theorem to calculate the perimeter

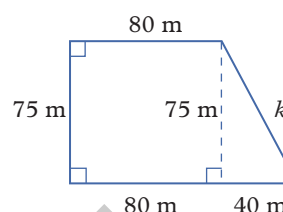


Find the perimeter of the following shape, correct to the nearest metre.

Think

Divide the figure into a rectangle and a right-angled triangle.
Use Pythagoras' theorem to find the unknown side length of the right-angled triangle.
Add the lengths along the boundary of the shape to find the perimeter.

Write



$$\begin{aligned} k^2 &= 40^2 + 75^2 \\ &= 7225 \\ k &= \sqrt{7225} \\ &= 85 \text{ m} \\ \text{Perimeter} &= 120 \text{ m} + 75 \text{ m} + 80 \text{ m} + 85 \text{ m} \\ &= 360 \text{ m} \end{aligned}$$



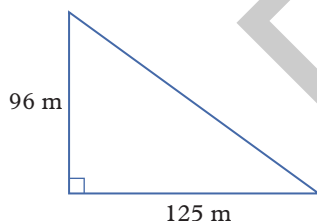
Worked example video: Using Pythagoras' theorem to calculate the perimeter

Helpful hints

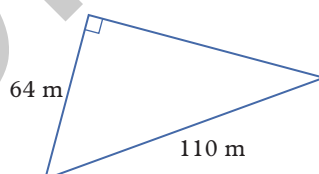
- When dividing a figure into smaller shapes, make sure you have enough information to identify or determine the side lengths of those shapes.

4 Find the perimeter of the following shapes, to the nearest metre.

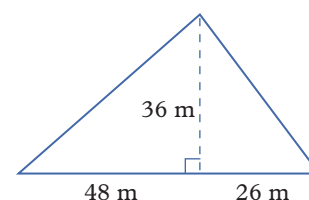
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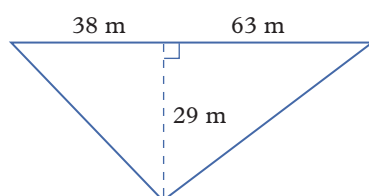
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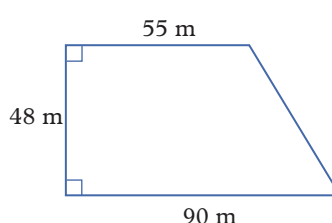
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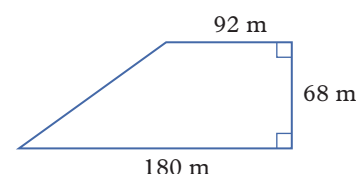
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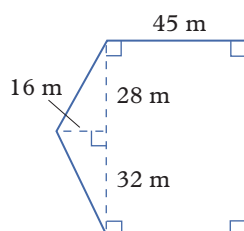
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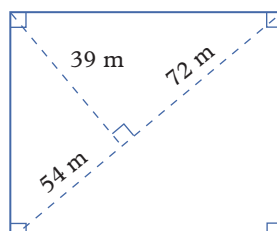
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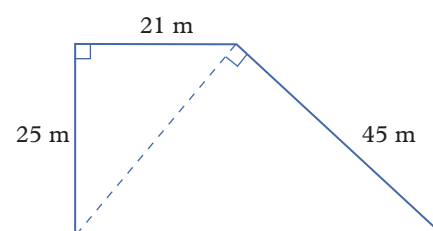
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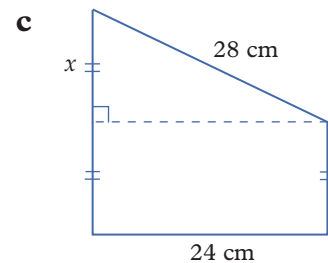
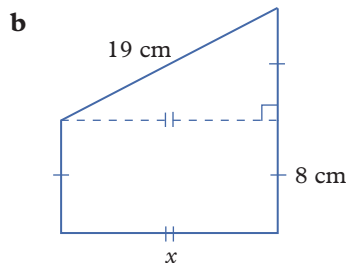
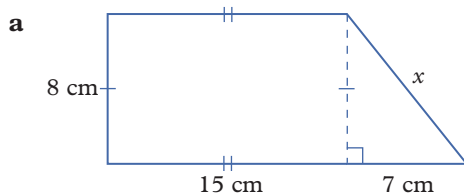
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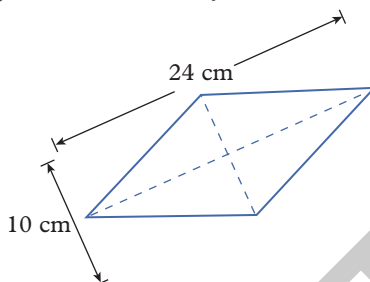
i



- 5 Find x and calculate the perimeter of each of these shapes, correct to one decimal place.



- 6 A rhombus has diagonals measuring 24 cm and 10 cm, as shown.

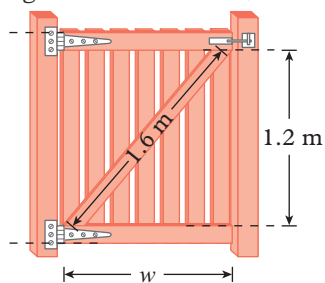


- Calculate the side length of the rhombus.
- Calculate the perimeter of the rhombus.

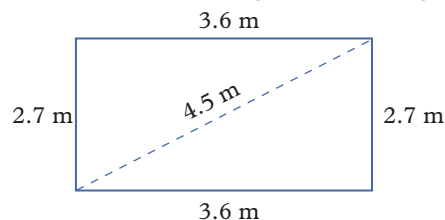
Problem solving and reasoning

- A carpenter builds the frame for the wall of a house. The frame is 2700 mm by 3200 mm and the diagonal measures 4187 mm. Is the frame rectangular?
- A door frame is 2500 mm by 850 mm. A builder measures the diagonal to be 3000 mm. Is the door frame rectangular?
- Pythagoras' theorem can be used to find the length of diagonals in some quadrilaterals.
 - Find the length of the diagonal of a 6 cm by 4 cm rectangle.
 - Find the length of the diagonal of an 8 cm by 8 cm square.

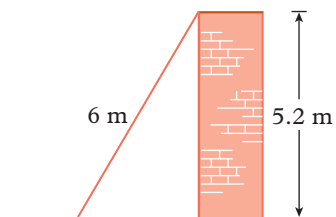
- 10 A rectangular gate of height 1.2 m has a diagonal brace of length 1.6 m. How wide is the gate?



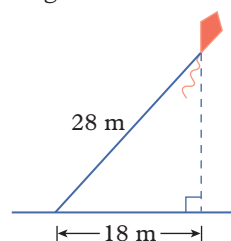
- 11 Kylie measures out an area of ground to be concreted for the floor of a tool shed, as shown in the diagram. To check that the floor will be rectangular, she measures the diagonal and finds it to be 4.5 m. Is the area of ground rectangular?



- 12** A ladder 6 m long reaches 5.2 m up a wall.
How far is the foot of the ladder from the base of the wall?

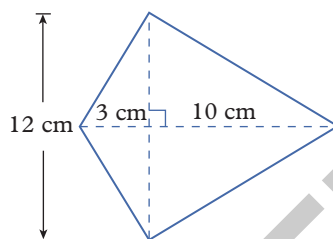


- 13** A kite is flying with 28 m of string let out. At this time, the horizontal distance to the kite is 18 m. Calculate the height of the kite above the ground.



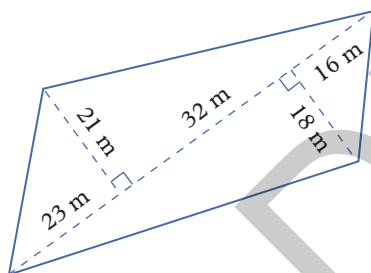
- 14** A ship leaves Sydney Heads and travels 45 km due east. It then turns and travels 86 km due north. How far is the ship from its starting point?

- 15** A kite has diagonals as shown.

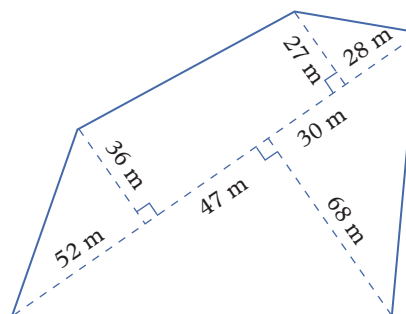


- Calculate the length of one short side of the kite.
 - Calculate the length of one long side of the kite.
 - Calculate the perimeter of the kite.
- 16** A surveyor makes the following measurements of some fields. Calculate the perimeter of each field to the nearest metre.

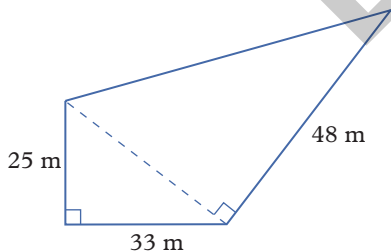
a



b

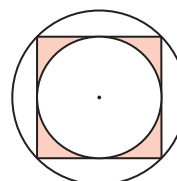


c



Challenge

- 17** The diagram below shows two circles and a square sitting between them. The radius of the smaller circle is 5 cm. Find the circumference of the larger circle, correct to two decimal places.



Related resources

 **6.2 Quick quiz**

 **Investigation:** Pythagoras' theorem with other shapes

 **Worksheet:** Pythagoras' theorem

Lesson 6.3

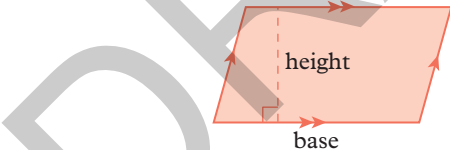
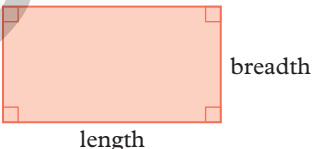
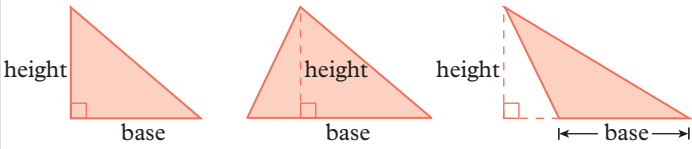
Area of simple shapes

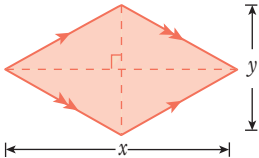
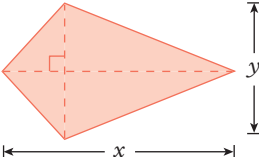
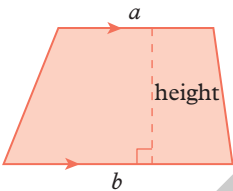
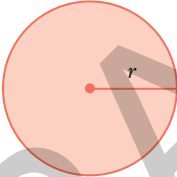
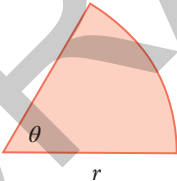
Learning intentions

By the end of this lesson, you will be able to ...
→ find the area of simple and composite shape.

Area formulas

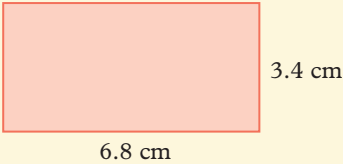
- The areas of the following shapes can be found using the formulas in this table.
 - Lines that are **parallel** are indicated using arrowheads.
 - **Perpendicular** lines meet at a right angle shown using a square instead of an arc for the angle.The height of a shape is perpendicular to its base.

Shape	Example	Area formula(s)
Parallelogram		Area = base × height $A = bh$
Rectangle		Area = length × breadth $A = lb$
Triangle		Area = $\frac{1}{2} \times \text{base} \times \text{height}$ $A = \frac{1}{2}bh$

Shape	Example	Area formula(s)
Rhombus		$A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals. The formula $A = bh$ can also be used if the base and height are known as a rhombus is also a parallelogram.
Kite		$A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals.
Trapezium		$A = \frac{h}{2}(a + b)$, where a and b are the lengths of the parallel sides and h is the height.
Circle		Area = $\pi \times \text{radius}^2$ $A = \pi r^2$
Sector		$A = \frac{\theta}{360^\circ} \times \pi r^2$ This formula finds the fraction of the whole circle made up by the sector and multiplies it by the area of the whole circle.

Worked example 6.3A Finding the area of a rectangle

Find the area of the rectangle shown.



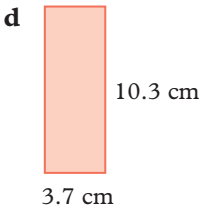
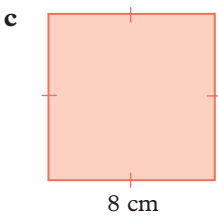
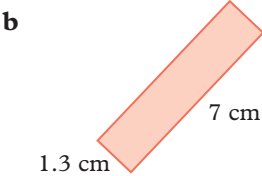
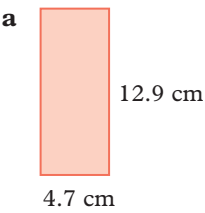
Think	Write
Substitute the given side lengths into the formula $A = lb$. Evaluate and write the result with the correct units.	$A = lb$ $= 6.8 \text{ cm} \times 3.4 \text{ cm}$ $= 23.12 \text{ cm}^2$

 **Worked example video:** Finding the area of a rectangle

Understanding, fluency and communicating

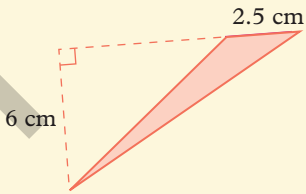
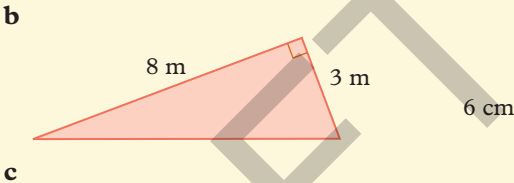
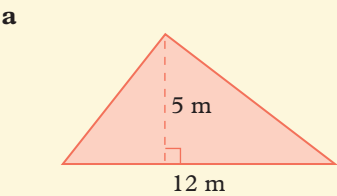
Answer: Lesson 6.3

1 Find the area of each of these rectangles.



Worked example 6.3B Finding the area of a triangle

Find the area of each of these triangles.



Part	Think	Write
a	Identify the base as 12 m and the height as 5 m. Substitute these values into the formula $A = \frac{1}{2}bh$. Evaluate and write the result with the correct units.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 12 \text{ m} \times 5 \text{ m}$ $= 30 \text{ m}^2$
b	Identify the base as 8 m and the height as 3 m. Substitute these values into the formula $A = \frac{1}{2}bh$. Evaluate and write the result with the correct units.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 8 \text{ m} \times 3 \text{ m}$ $= 12 \text{ m}^2$
c	Identify the base as 2.5 cm and the height as 6 cm. Substitute these values into the formula $A = \frac{1}{2}bh$. Evaluate and write the result with the correct units.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 2.5 \text{ cm} \times 6 \text{ cm}$ $= 7.5 \text{ cm}^2$

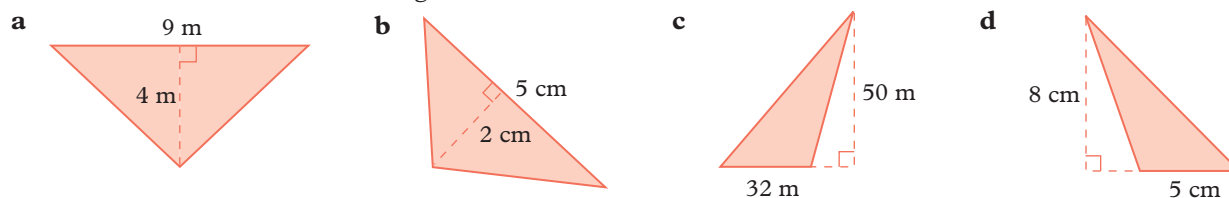


Worked example video: Finding the area of a triangle

Helpful hints

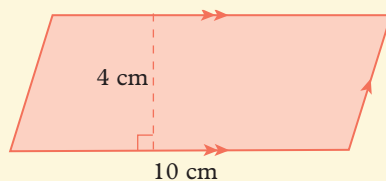
- If the lengths of the two shorter sides of a right-angled triangle are known, the order you substitute the values into the triangle area formula doesn't affect the result.

2 Find the area of each of these triangles.



Worked example 6.3C Finding the area of a parallelogram

Find the area of this parallelogram.



Think

Identify the base and height of the parallelogram and substitute these values into the formula $A = bh$.
Evaluate and write the result with the correct units.

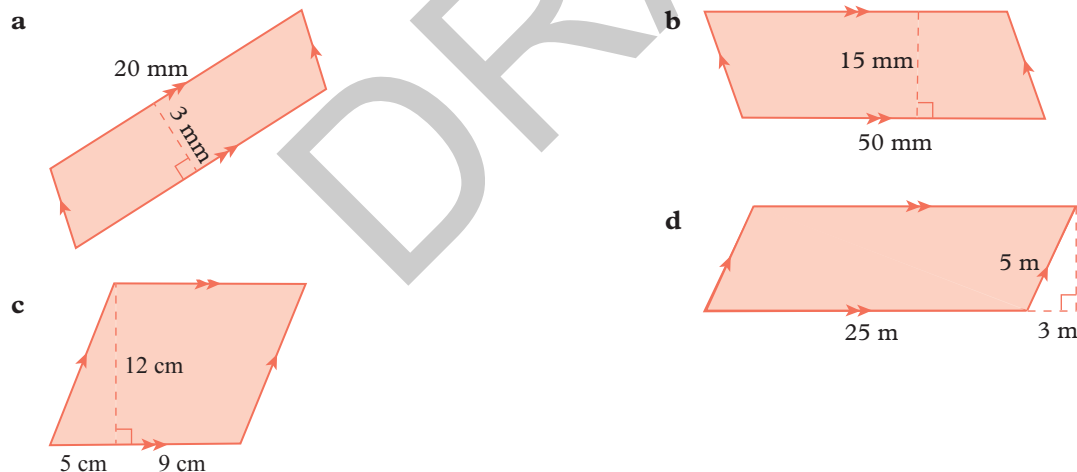
Write

$$\begin{aligned} A &= bh \\ &= 10 \text{ cm} \times 4 \text{ cm} \\ &= 40 \text{ cm}^2 \end{aligned}$$



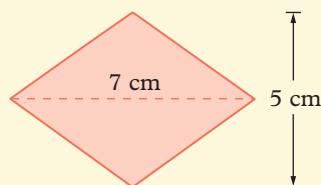
Worked example video: Finding the area of a parallelogram

3 Find the area of each of these parallelograms.



Worked example 6.3D Finding the area of a rhombus

Find the area of a rhombus with diagonals of length 5 cm and 7 cm.

**Think**

Identify the lengths of the two diagonals and substitute these values into the formula $A = \frac{1}{2}xy$.
Evaluate and write the result with the correct units.

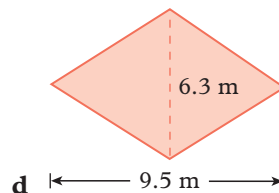
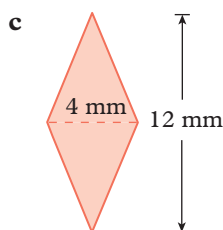
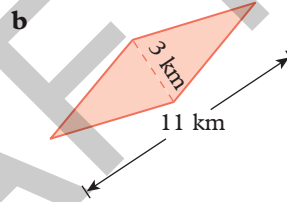
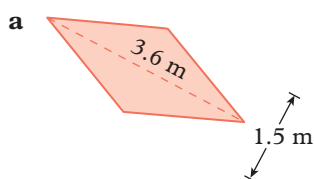
Write

$$\begin{aligned} A &= \frac{1}{2}xy \\ &= \frac{1}{2} \times 5 \text{ cm} \times 7 \text{ cm} \\ &= 17.5 \text{ cm}^2 \end{aligned}$$

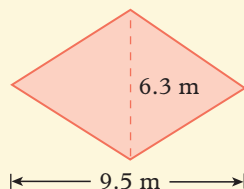


Worked example video: Finding the area of a rhombus

4 Find the area of each rhombus.

**Worked example 6.3E** Finding the area of a kite

Find the area of this kite.

**Think**

Identify the lengths of the two diagonals and substitute these values into the formula $A = \frac{1}{2}xy$.
Evaluate and write the result with the correct units.

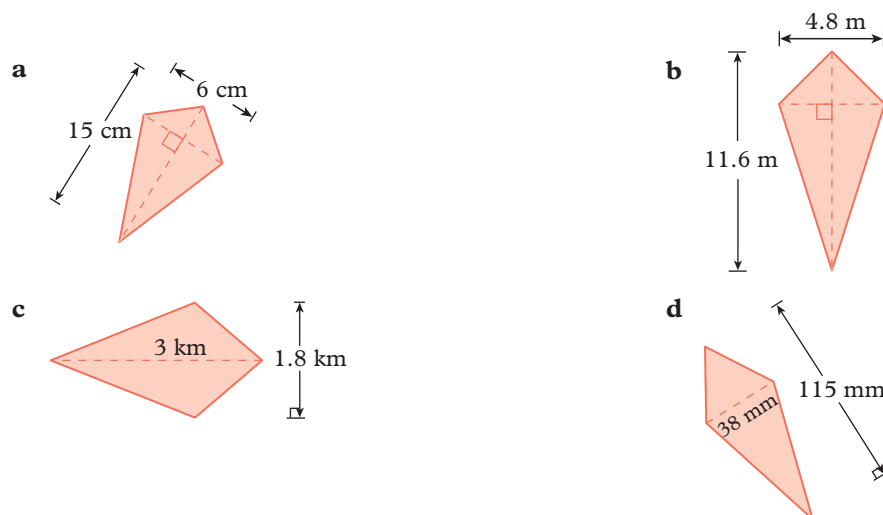
Write

$$\begin{aligned} A &= \frac{1}{2}xy \\ &= \frac{1}{2} \times 5 \text{ cm} \times 8 \text{ cm} \\ &= 20 \text{ cm}^2 \end{aligned}$$



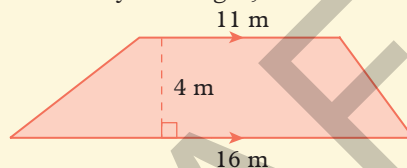
Worked example video: Finding the area of a kite

5 Find the area of each of these kites.



Worked example 6.3F Finding the area of a trapezium

Find the area of this trapezium. First identify the height, then use the formula.



Think

The height of the trapezium is perpendicular distance between the parallel sides, so $h = 4$ m.

The lengths of the parallel sides are $a = 11$ m and $b = 16$ m.

Substitute these values into the formula $A = \frac{h}{2}(a + b)$.

Evaluate and write the result with the correct units.

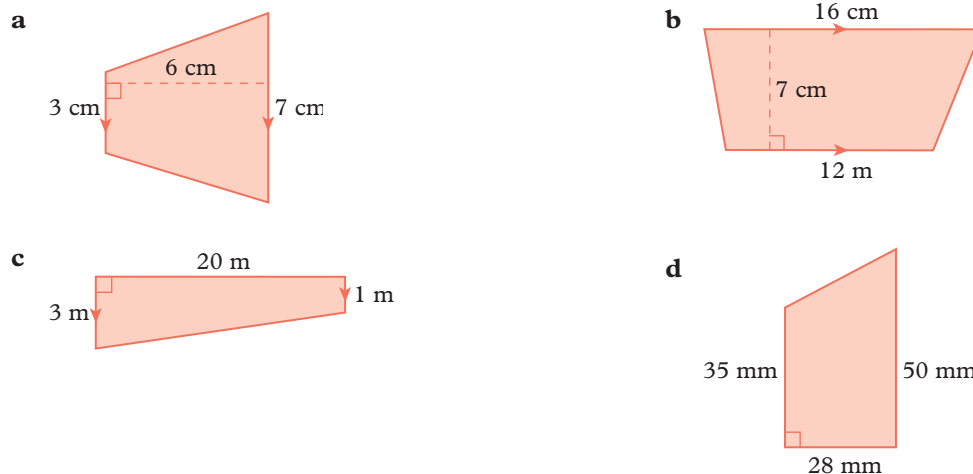
Write

$$\begin{aligned} A &= \frac{h}{2}(a + b) \\ &= \frac{4\text{ m}}{2} \times (11\text{ m} + 16\text{ m}) \\ &= 2\text{ m} \times 27\text{ m} \\ &= 54\text{ m}^2 \end{aligned}$$



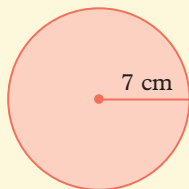
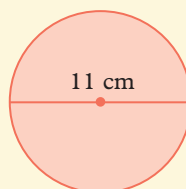
Worked example video: Finding the area of a trapezium

6 Find the area of each of these trapeziums. Identify the height first.



Worked example 6.3G Calculating the area of a circle

Calculate the area of each of these circles, correct to one decimal place.

a**b**

Part	Think	Write
a	The radius of the circle is $r = 7$ cm. Substitute this value into the formula $A = \pi r^2$. Evaluate and write the result with the correct units.	$A = \pi r^2$ $= \pi \times (7 \text{ cm})^2$ $\approx 153.9 \text{ cm}^2$
b	The diameter of the circle is $d = 11$ cm. Substitute this value into the formula $d = 2r$ and solve to find the length of the radius. Substitute the length of the radius into the formula $A = \pi r^2$. Evaluate and write the result with the correct units.	$d = 2r$ $11 \text{ cm} = 2r$ $r = \frac{11 \text{ cm}}{2}$ $= 5.5 \text{ cm}$ $A = \pi r^2$ $= \pi \times (5.5 \text{ cm})^2$ $\approx 95.0 \text{ cm}^2$

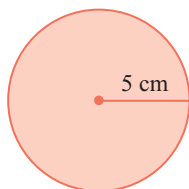
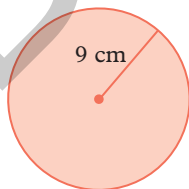
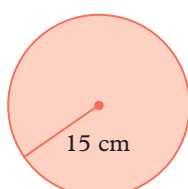
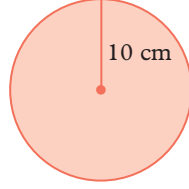
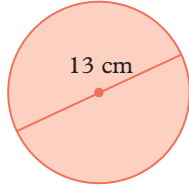
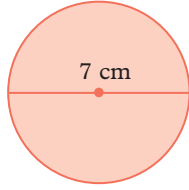
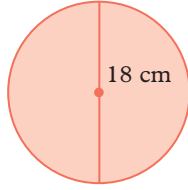
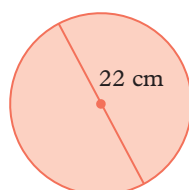


Worked example video: Calculating the area of a circle

Helpful hints

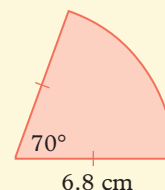
- The formula for the area of a circle requires the radius. If the diameter of the circle is given, remember to use the formula $d = 2r$ to find the radius.

7 Calculate the area of each of these circles, correct to one decimal place.

a**b****c****d****e****f****g****h**

Worked example 6.3H Finding the area of a sector

- a** Determine what fraction of a circle is drawn.
b Determine the area to the nearest square centimetre (cm^2).

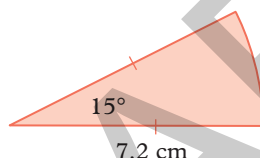


Part	Think	Write
a	There are 360° in a circle. The fraction is the sector angle 70° divided by 360° .	$\text{Fraction} = \frac{70^\circ}{360^\circ}$ $= \frac{7}{36}$
b	Area = fraction of circle $\times$ area of circle	$A = \frac{7}{36} \times \pi r^2$ $= \frac{7}{36} \times \pi \times (6.8 \text{ cm})^2$ $= 28.246... \text{ cm}^2$ $\approx 28 \text{ cm}^2$



Worked example video: Finding the area of a sector

- 8** Consider the following sector.



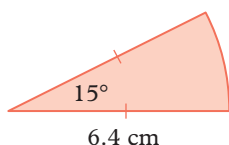
- a** What fraction of the circle is drawn?
 Fraction = $\frac{\square^\circ}{360^\circ}$
 = $\frac{\square}{\square}$
- b** Determine the area of the sector to the nearest square centimetre (cm^2).

$$A = \frac{1}{\square} \times \pi \times (\square \text{ cm})^2$$

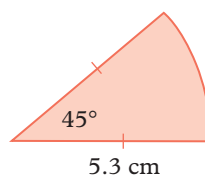
$$\approx \square \text{ cm}^2$$

- 9** Determine the fraction of a circle that is drawn each time, then calculate each area to the nearest cm^2 .

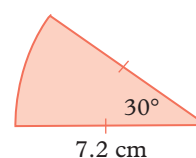
a



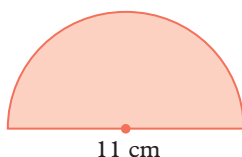
b



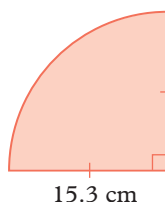
c



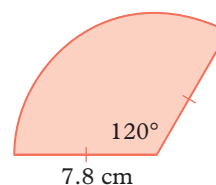
d

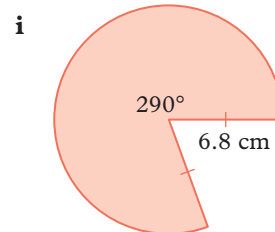
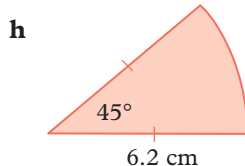
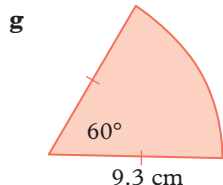


e



f



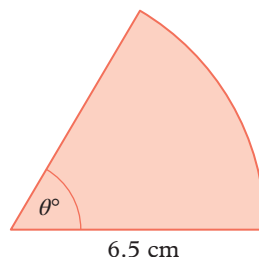


Problem solving and reasoning

- 10** If the values of all variables but one in an area formula are known, you can solve for the unknown value.
- A square has an area of 81 cm^2 . Find its side length.
 - A rectangle has an area of 56 cm^2 . Find the rectangle's length if its breadth is 7 cm .
 - A rhombus has an area of 72 cm^2 . If one diagonal is 18 cm long, how long is the other diagonal?
 - Find the radius of a circle with an area of 100 m^2 .
- 11** A farm covers land in the shape of a trapezium. The length of the northern boundary of the farm is 265 m and the length of the southern boundary is 180 m . The perpendicular distance from the northern end to the southern end is 90 m .
- Find the area of the farm to the nearest square metre.
 - Write your answer for part **a** to the nearest hectare. (*Hint: $1 \text{ ha} = 10\,000 \text{ m}^2$*).
 - The farmer decides to purchase her neighbour's farm, which is in the shape of a rectangle but has the same area as her farm. What are possible dimensions (in metres) of her neighbour's farm?
- 12** A crop duster can carry 480 kg of fertiliser in its tanks. The pilot is required to fertilise a rectangular field of dimensions 2.8 km by 1.4 km . How many flights will be necessary if the fertiliser is to be spread at the rate of 40 kg/ha ?

Challenge

- 13** A circular drink coaster has a circumference of 28 cm . Calculate its area to the nearest square centimetre (cm^2).
- 14** A cafe has square tables of side-length 1.3 m . They are covered by round tablecloths of diameter 1.9 m . What percentage of the tablecloth hangs over the edges of the table? Write your answer correct to one decimal place.
- 15** The sector below has an area of 21 cm^2 . Find the angle of the sector, to the nearest degree.



Related resources



6.3 Quick quiz



Worksheet: Area



Investigation: Maximum area for a vegetable garden

Lesson 6.4

Area of composite shapes

Learning intentions

By the end of this lesson, you will be able to ...

→ solve practical problems involving the area of composite shapes.

Finding the area of composite shapes

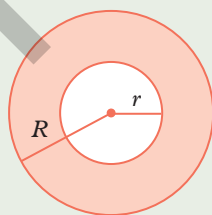
- **Composite shapes** are made up of more than one simple shape. These shapes can be added together to create a figure with more area, or a smaller shape can be removed from a larger shape to create a region with less area.
- To find the area of a composite shape, first divide the figure into its component simple shapes and then add or subtract the areas of the shapes.

Finding the area of an annulus

- The shaded area in this diagram is called an **annulus**. An annulus is the area between two circles with the same centre but different radii. Circles with the same centre are concentric circles. The formula for the area of an annulus is

$$A = \pi R^2 - \pi r^2 \text{ or } A = \pi(R^2 - r^2),$$

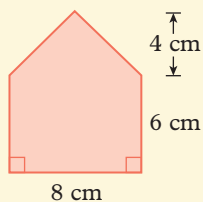
where R is the radius of the large circle, and r is the radius of the small circle.



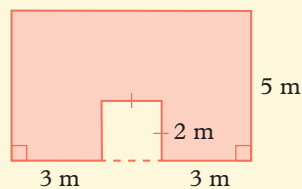
Worked example 6.4A Finding the area of a composite shape

Find the shaded area of each of these shapes.

a



b



	Think	Write
a	Divide the shape into simple shapes that you can find the area of. The shape is made up of a rectangle with length 8 cm and breadth 6 cm and a triangle with base 8 cm and height 4 cm. Substitute these lengths into the sum of the formulas for the area of a rectangle and the area of a triangle. Evaluate and write the result with the correct units.	$A = A_{\text{rectangle}} + A_{\text{triangle}}$ $= 8 \text{ cm} \times 6 \text{ cm} + \frac{1}{2} \times 8 \text{ cm} \times 4 \text{ cm}$ $= 48 \text{ cm}^2 + 16 \text{ cm}^2$ $= 64 \text{ cm}^2$
b	Divide the shape into simple shapes that you can find the area of. The figure is made up of a large rectangle with length 8 m and breadth 5 m and a square with side length 2 m. Substitute these lengths into the difference between the formulas for the area of a rectangle and the area of a square. Evaluate and write the result with the correct units.	$A = A_{\text{large rectangle}} - A_{\text{square}}$ $= 5 \text{ m} \times 8 \text{ m} - 2 \text{ m} \times 2 \text{ m}$ $= 40 \text{ m}^2 - 4 \text{ m}^2$ $= 36 \text{ m}^2$



Worked example video: Finding the area of a composite shape

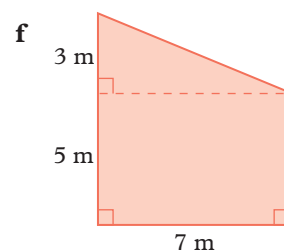
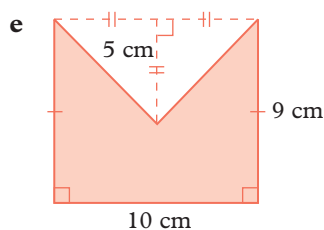
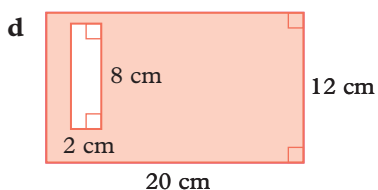
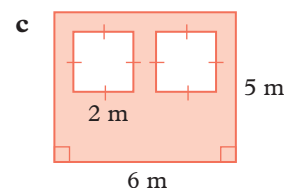
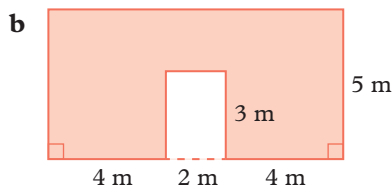
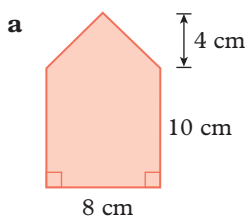
Understanding, fluency and communicating

Answers: Lesson 6.4

Helpful hints

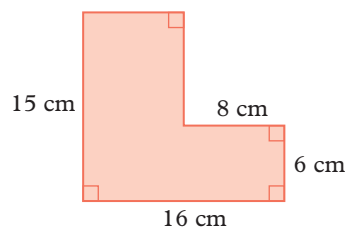
- Add the areas of shapes if they are shaded and included in the region. Subtract the areas of any shapes which are not shaded.

1 Find the shaded area of each of these composite shapes.

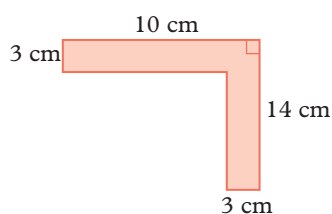


2 Find the area of each of these composite shapes made up of rectangles.

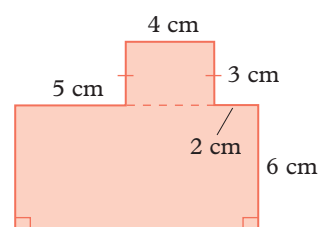
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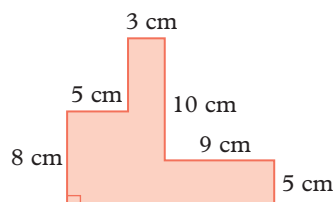
b



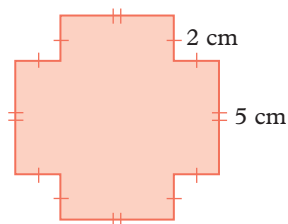
c



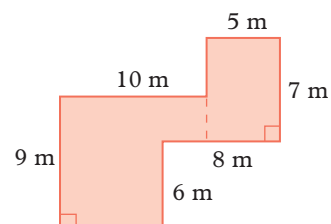
d



e

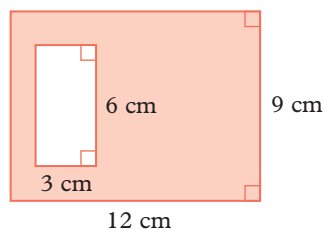


f

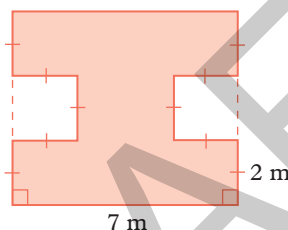


3 Find the shaded area of each of these shapes, correct to two decimal places where necessary.

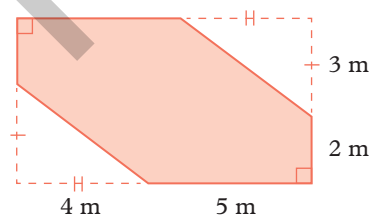
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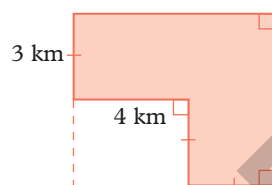
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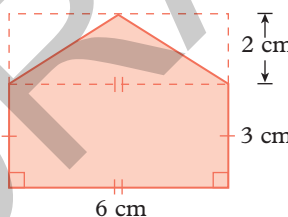
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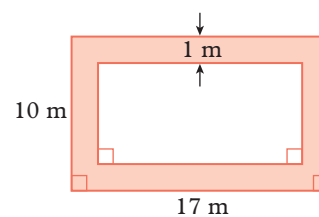
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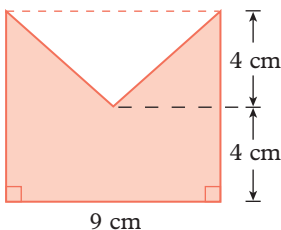
e



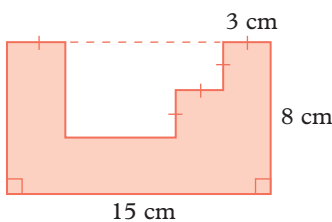
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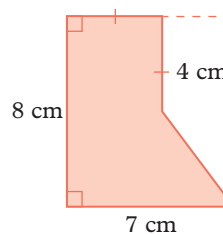
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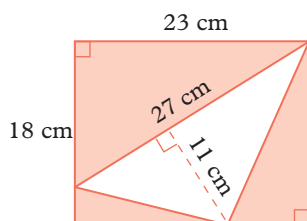
h



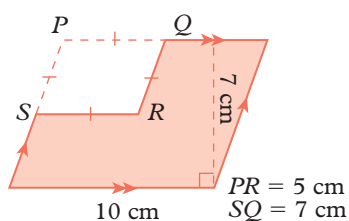
i



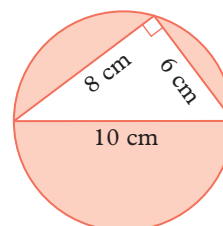
j



k

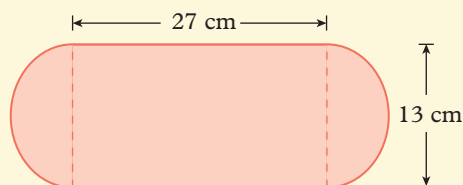


l



Worked example 6.4B Finding the area of a composite shape containing part of a circle

Calculate the area enclosed by this composite shape. Give your answer to the nearest square centimetre (cm^2).

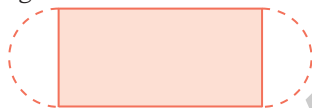


Think

There are two semicircles with diameter 13 cm. The semicircles combine to make a circle with radius 6.5 cm. Substitute the length of the radius into the formula $A = \pi r^2$.



There is one rectangle with length 27 cm and breadth 13 cm. Substitute these lengths into the formula $A = lb$.



Add the areas of the circle (two semicircles) and the rectangle together.

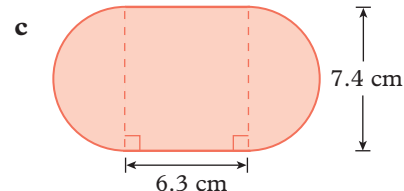
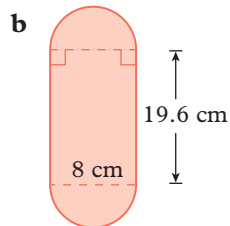
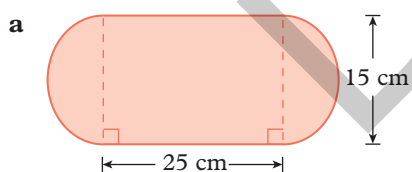
Write

$$\begin{aligned} A_{\text{circle}} &= \pi r^2 \\ &= \pi \times (6.5 \text{ cm})^2 \\ &= 132.732\dots \text{ cm}^2 \\ &\approx 132.7 \text{ cm}^2 \\ A_{\text{rectangle}} &= lb \\ &= 13 \text{ cm} \times 27 \text{ cm} \\ &= 351 \text{ cm}^2 \\ \text{Total area} &= A_{\text{circle}} + A_{\text{rectangle}} \\ &\approx 132.7 \text{ cm}^2 + 351 \text{ cm}^2 \\ &= 483.7 \text{ cm}^2 \\ &\approx 484 \text{ cm}^2 \end{aligned}$$

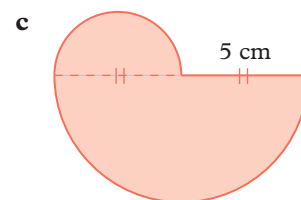
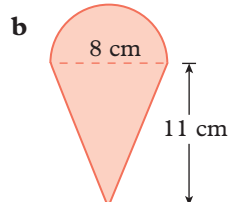
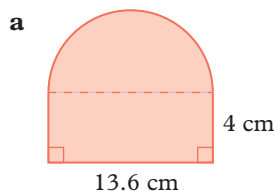


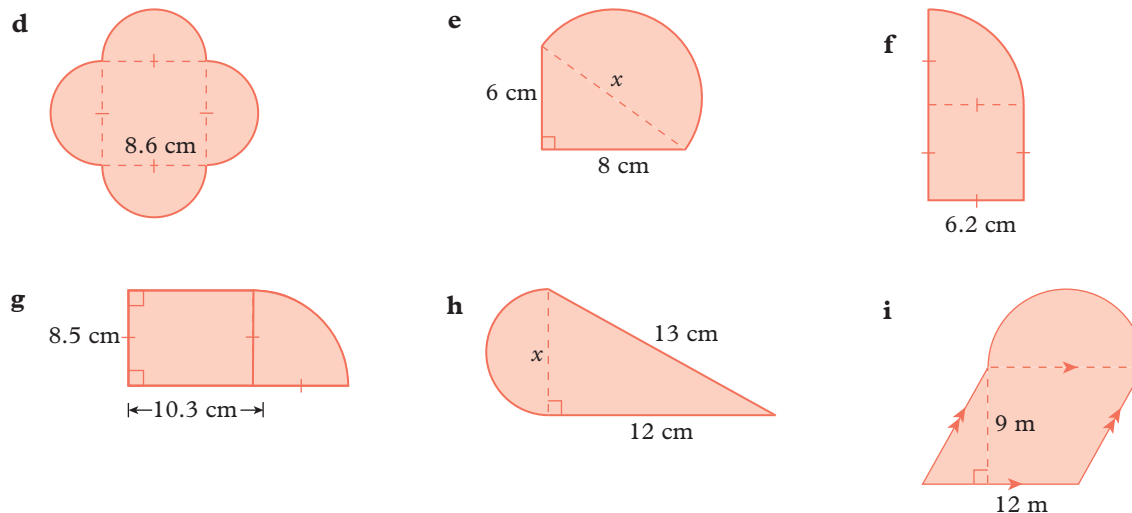
Worked example video: Finding the area of a composite shape containing part of a circle

- 4 Calculate the area enclosed by each of these composite shapes to the nearest square centimetre (cm^2).



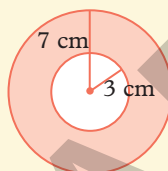
- 5 Find the area of each of these composite shapes, correct to one decimal place if necessary.





Worked example 6.4C Finding the area of an annulus

Calculate the area of this annulus to the nearest square centimetre (cm^2).



Think

The radius of the large circle is $R = 7$ cm.
 The radius of the small circle is $r = 3$ cm.
 Subtract the area of the small circle from the area of the large circle.

Write

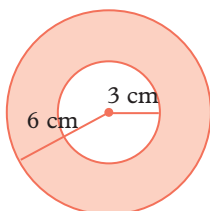
$$\begin{aligned} A &= \pi R^2 - \pi r^2 \\ &= \pi \times 7^2 \text{ cm}^2 - \pi \times 3^2 \text{ cm}^2 \\ &= 125.663... \text{ cm}^2 \\ &\approx 126 \text{ cm}^2 \end{aligned}$$



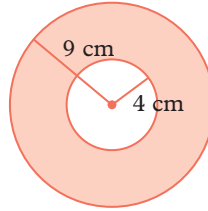
Worked example video: Finding the area of an annulus

6 Calculate the area of each annulus below, to the nearest square centimetre (cm^2).

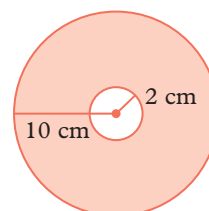
a



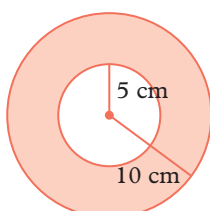
b



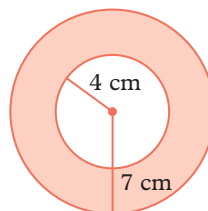
c



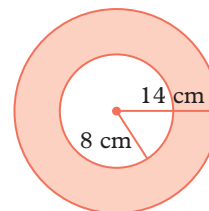
d



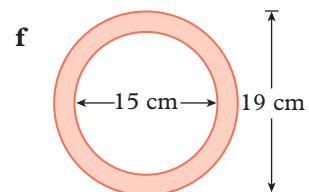
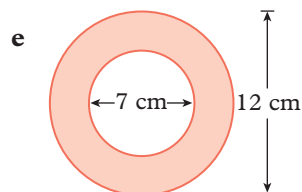
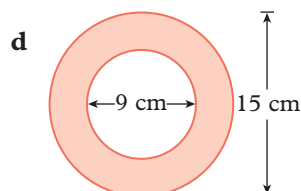
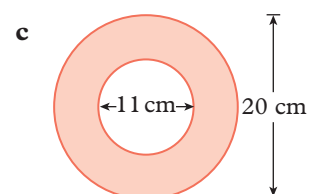
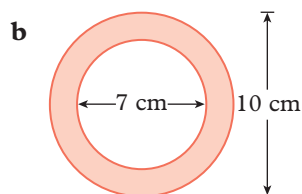
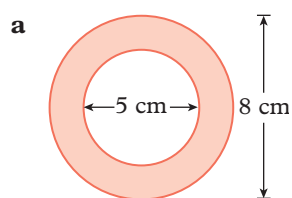
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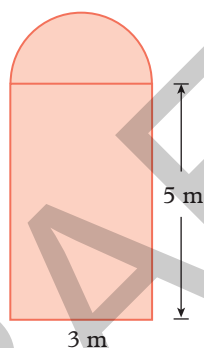


7 Calculate the area of each annulus below, to the nearest square centimetre (cm^2).



Problem solving and reasoning

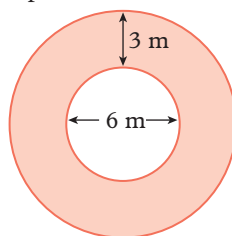
8 The diagram shows a stained-glass window in a church.



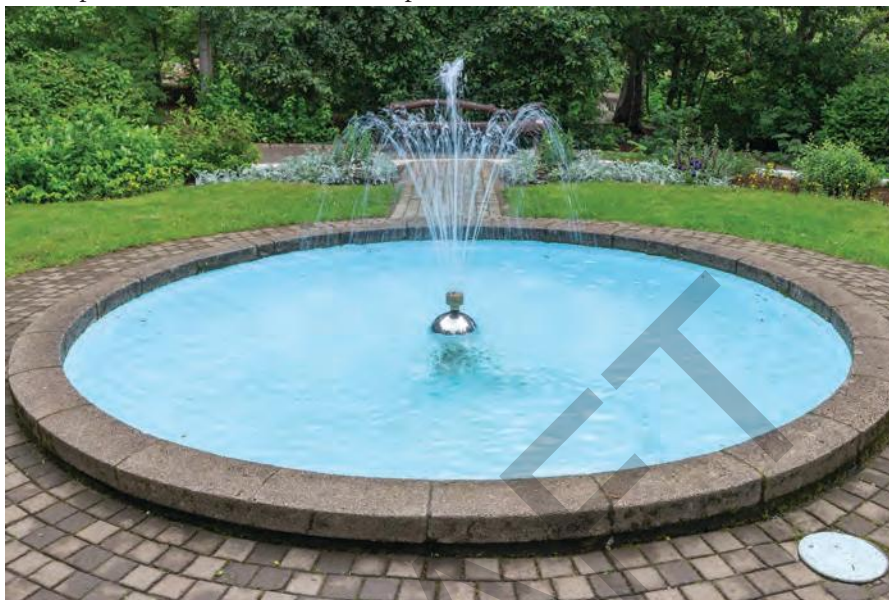
- Calculate the area of the window, correct to two decimal places.
- The cost of replacing a stained-glass window is $\$540/\text{m}^2$. Calculate the cost of replacing this window to the nearest dollar.



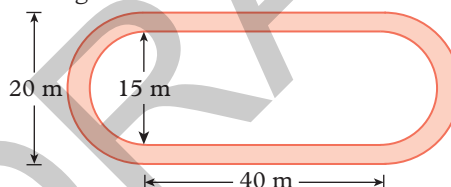
- 9 A 3 m wide path is placed around a circular pond that has a diameter of 6 m.



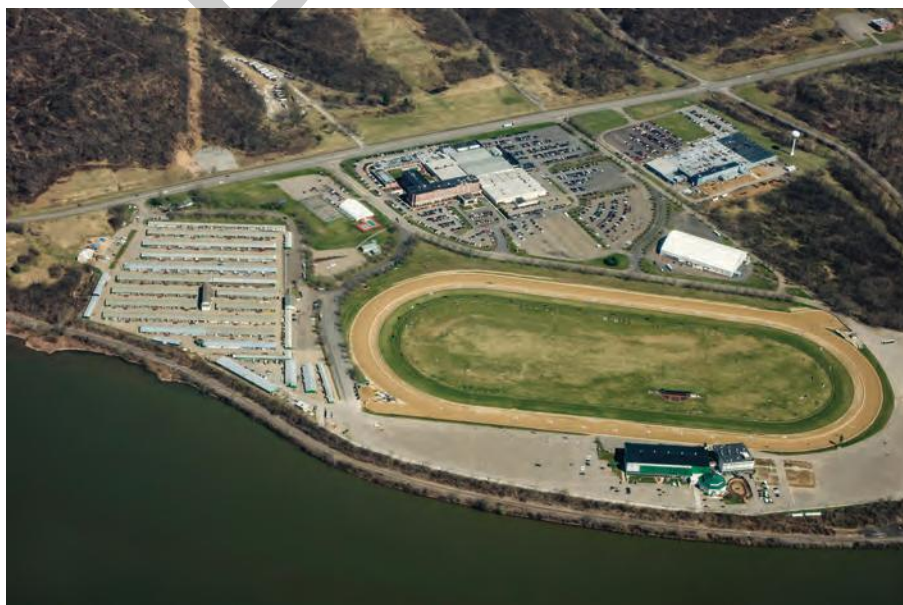
Find the area of the path correct to one decimal place.



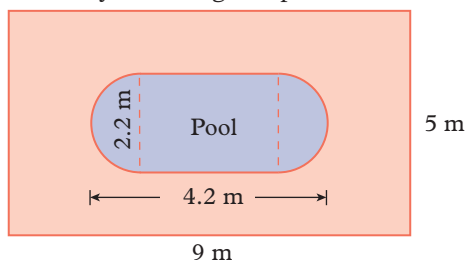
- 10 The racetrack shown shaded in the diagram is to be resurfaced.



- Calculate the area to be resurfaced, to the nearest square metre.
- Calculate the cost of the resurfacing, if the rate is \$9.90/m². Give your answer to the nearest \$10.



11 The diagram shows a pool surrounded by a rectangular paved area.

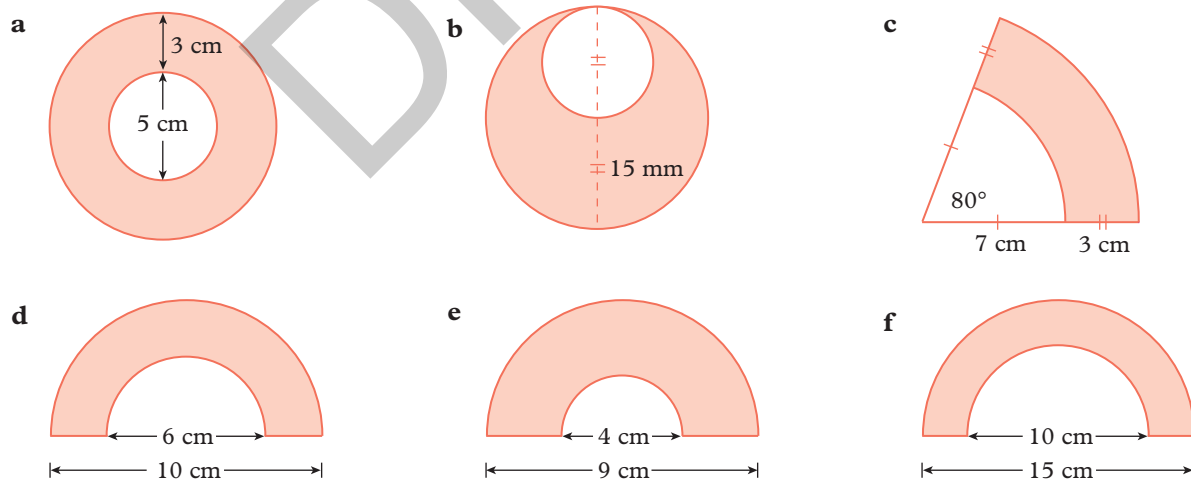


- Calculate the paved area (correct to the nearest square metre).
- If paving costs \$34.90/m², or part thereof, calculate the cost of paving the area surrounding the pool.

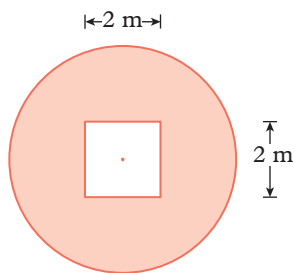


Challenge

12 Determine the area of each shaded region below, correct to one decimal place.



- 13 An arrow shape can be drawn using a rectangle and a triangle. Draw three different arrows that could have an area of 50 cm^2 .
- 14 A couple is designing a courtyard. The design is a circle with a $2 \text{ m} \times 2 \text{ m}$ square missing from the middle of the circle. They have chosen the design so that the circle is paved and is as large as possible.



- 15 Paving costs \$40 per square metre, and the couple will spend exactly \$2000. What is the radius of the circle in metres, correct to one decimal place?

Related resources



6.4 Quick quiz



Investigation: Mouse house



Worksheet: Area of composite figures

Lesson 6.5

Perimeter and area of irregular shapes

Learning intentions

By the end of this lesson, you will be able to ...

→ use the Trapezoidal rule to find approximate areas.

Perimeter and area of irregular shapes

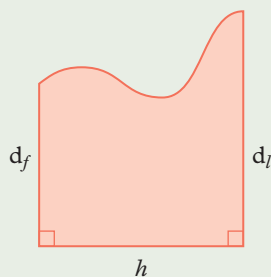
- Add the areas of shapes if they are shaded and included in the region. Subtract the areas of any shapes which are not shaded.
- Although the shapes of paddocks or fields might be plane shapes, they may not be shapes whose perimeter and area can be calculated using simple formulas.
- Irregularly shaped blocks can be dissected into familiar shapes for which formulas can be used to determine the perimeter or area of the block.

The Trapezoidal rule

- If a field has a boundary that is not straight (for example a river), then its area can be approximated by replacing the curved boundary with a straight line which creates a trapezium.
- The area of the trapezium is then an estimate of the area of the field. This is known as the **Trapezoidal rule** and is usually written as:

$$A \approx \frac{h}{2}(d_f + d_l),$$

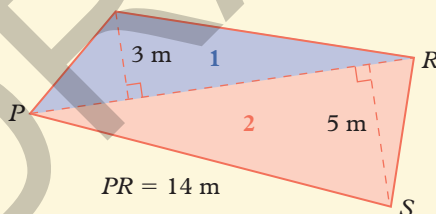
where d_f and d_l are the lengths of the parallel sides of the trapezium and h is the perpendicular distance between them.



- The names d_f and d_l are chosen to represent the “first” length and the “last” length.
- The Trapezoidal rule can be applied multiple times by dividing a field into smaller trapeziums. This results in a more accurate estimate of the area of the field.

Worked example 6.5A Finding the area of a quadrilateral

Find the area of quadrilateral $PQRS$.



Think

The quadrilateral is divided into two triangles. Find the sum of the areas of the two triangles using the diagonal PR as the base of both triangles.

Write the result with the correct units.

Write

$$\begin{aligned} \text{Area} &= A_{\text{triangle 1}} + A_{\text{triangle 2}} \\ &= \frac{1}{2} \times 14 \text{ m} \times 3 \text{ m} + \frac{1}{2} \times 14 \text{ m} \times 5 \text{ m} \\ &= 21 \text{ m}^2 + 35 \text{ m}^2 \\ &= 56 \text{ m}^2 \end{aligned}$$



Worked example video: Finding the area of a quadrilateral

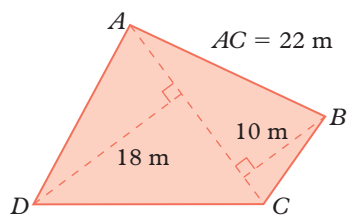
Understanding, fluency and communicating

Answers: Lesson 6.5

Helpful hints

- Using the entire diagonal of the quadrilateral as the base measurement means the areas of only two triangles need to be calculated, instead of four.

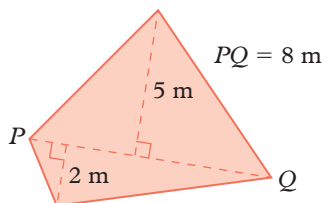
- 1 Fill in the blanks to find the area of quadrilateral $ABCD$.



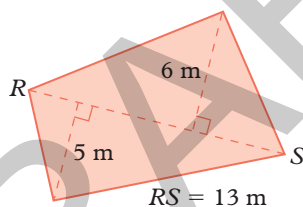
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 22 \text{ m} \times \underline{\hspace{1cm}} \text{ m} + \frac{1}{2} \times \underline{\hspace{1cm}} \text{ m} \times 18 \text{ m} \\ &= \underline{\hspace{1cm}} \text{ m}^2 + \underline{\hspace{1cm}} \text{ m}^2 \\ &= \underline{\hspace{1cm}} \text{ m}^2 \end{aligned}$$

- 2 Find the area of each of these quadrilaterals.

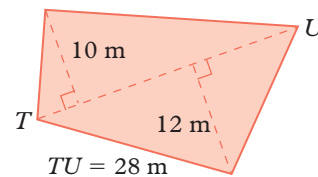
a



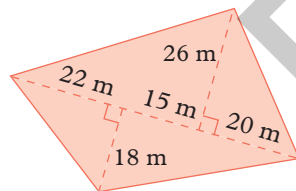
b



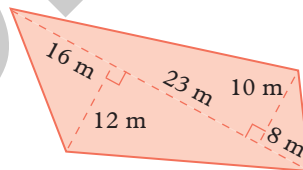
c



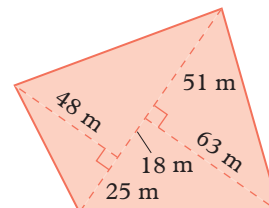
d



e

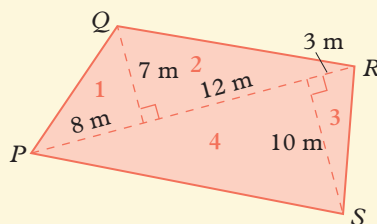


f



Worked example 6.5B Finding the perimeter of a quadrilateral

Use Pythagoras' theorem to help you find the perimeter of quadrilateral $PQRS$, correct to one decimal place.



Think

Divide the quadrilateral into four right-angled triangles. Use Pythagoras' theorem to find the length of the hypotenuse of each triangle.

For triangle 2, first find the length of the third side by adding 12 m and 3 m. Then use Pythagoras' theorem to find OR .

For triangle 4, first find the length of the third side by adding 8 m and 12 m. Then use Pythagoras' theorem to find PS .

Add the lengths of the four hypotenuses together to find the perimeter.

Write

Triangle 1 (hypotenuse PQ):

$$\begin{aligned} 8^2 + 7^2 &= 113 \\ \sqrt{113} &\approx 10.63 \\ PQ &\approx 10.63 \text{ m} \end{aligned}$$

Triangle 2 (hypotenuse QR):

$$7^2 + (12 + 3)^2 = 274$$
$$\sqrt{274} \approx 16.55$$
$$OR \approx 16.55 \text{ m}$$

Triangle 3 (hypotenuse RS):

$$\begin{aligned} 10^2 + 3^2 &= 109 \\ \sqrt{109} &\approx 10.44 \\ RS &\approx 10.44 \text{ m} \end{aligned}$$

Triangle 4 (hypotenuse PS):

$$10^2 + (8 + 12)^2 = 500$$
$$\sqrt{500} \approx 22.36$$
$$PS \approx 22.36 \text{ m}$$

$$\begin{aligned}\text{Perimeter} &\approx 10.63 \text{ m} + 16.55 \text{ m} + 10.44 \text{ m} + 22.36 \text{ m} \\ &= 59.98 \text{ m} \\ &\approx 60.0 \text{ m}\end{aligned}$$

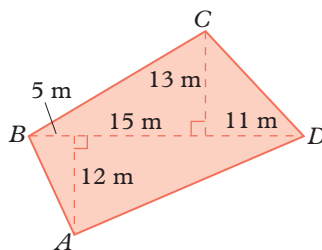


Worked example video: Finding the perimeter of a quadrilateral

Helpful hints

- It is usually best to wait until the final step to round the result. If you need to round results as you work through a question, round to more decimal places than required in the question and then round to the required number of decimal places in the final step.

- 3 Complete the following to find the perimeter of quadrilateral $ABCD$, correct to one decimal place.



Find BC : $13^2 + (5 + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$
 $\sqrt{569} \approx \underline{\hspace{1cm}}$
 $BC \approx \underline{\hspace{1cm}}$ m

Find CD : $3^2 + (\quad)^2 = 290$
 $\sqrt{290} \approx \underline{\hspace{1cm}}$
 $CD \approx \underline{\hspace{1cm}} \text{ m}$

Find AD : $12^2 + (15 + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$
 $\sqrt{\square} \approx \underline{\hspace{1cm}}$
 $AD \approx \underline{\hspace{1cm}}$ m

Find AB : $(\underline{\hspace{1cm}})^2 + (\underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$
 $\sqrt{\square} \approx \underline{\hspace{1cm}}$
 $AB \approx \underline{\hspace{1cm}} \text{ m}$

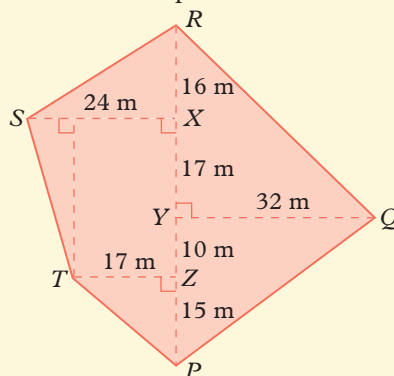
$$\text{Perimeter} \approx 23.85 \text{ m} + \underline{\hspace{1cm}} \text{ m} + \underline{\hspace{1cm}} \text{ m} + \underline{\hspace{1cm}} \text{ m}$$

$$\approx \underline{\hspace{1cm}} \text{ m}$$

- 4** Calculate the perimeter of the quadrilaterals in questions **2d**, **2e** and **2f**.

Worked example 6.5C Finding the area of a field involving triangles and trapeziums

Find the area of the field $PQRST$, to the nearest square metre.



Think	Write
Divide the field into three right-angled triangles, RQP , PTZ and SRX , and a trapezium, $TSXZ$.	$RP = 16\text{ m} + 17\text{ m} + 10\text{ m} + 15\text{ m}$ $= 58\text{ m}$
Use the formula $A = \frac{1}{2}bh$ to find the area of each of the triangles.	$\text{Area}_{RQP} = \frac{1}{2} \times RP \times YQ$ $= \frac{1}{2} \times 58\text{ m} \times 32\text{ m}$ $= 928\text{ m}^2$
Find the length RP by adding the known lengths of its components.	$\text{Area}_{PTZ} = \frac{1}{2} \times PZ \times TZ$ $= \frac{1}{2} \times 15\text{ m} \times 17\text{ m}$ $= 127.5\text{ m}^2$
The area of the trapezium can be found using the formula $A = \frac{h}{2}(a + b)$, where: $h = XY + YZ$ $= 17\text{ m} + 10\text{ m}$ $= 27\text{ m},$ and the parallel sides are 24 m and 17 m .	$\text{Area}_{SRX} = \frac{1}{2} \times RX \times SX$ $= \frac{1}{2} \times 16\text{ m} \times 24\text{ m}$ $= 192\text{ m}^2$
Add all the areas to find the area of the field.	$\text{Area}_{TSXZ} = \frac{27\text{ m}}{2} \times (24\text{ m} + 17\text{ m})$ $= 553.5\text{ m}^2$ $\text{Total area} = 928\text{ m}^2 + 127.5\text{ m}^2 + 192\text{ m}^2 + 553.5\text{ m}^2$ $= 1801\text{ m}^2$

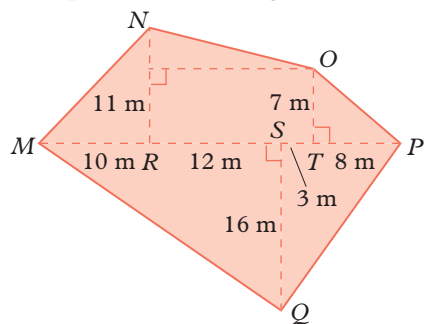


Worked example video: Finding the area of a field involving triangles and trapeziums

Helpful hints

- It is helpful to divide the field into the largest possible simple shapes. For example, in Worked example 6.5C, it is easier to find the area of triangle RQP instead of the areas of the two smaller triangles RQY and PQY . However, the result will be the same regardless of whether you use the largest possible simple shapes or divide them into smaller shapes.

- 5 Complete the following to find the area of the field $MNOPQ$.



$$\text{Area}_{MQP} = \frac{1}{2} \times (10 \text{ m} + 12 \text{ m} + \underline{\hspace{1cm}} \text{ m} + \underline{\hspace{1cm}} \text{ m}) \times 16 \text{ m} = \underline{\hspace{1cm}} \text{ m}^2$$

$$\text{Area}_{MRN} = \frac{1}{2} \times 10 \text{ m} \times \underline{\hspace{1cm}} \text{ m} = \underline{\hspace{1cm}} \text{ m}^2$$

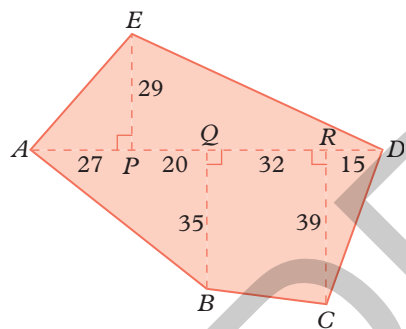
$$\text{Area}_{TOP} = \frac{1}{2} \times \underline{\hspace{1cm}} \text{ m} \times \underline{\hspace{1cm}} \text{ m} = 28 \text{ m}^2$$

$$\text{Area}_{NOTR} = \frac{12 \text{ m} + 3 \text{ m}}{2} \times (\underline{\hspace{1cm}} \text{ m} + \underline{\hspace{1cm}} \text{ m}) = \frac{1}{2} \times \underline{\hspace{1cm}} \text{ m} \times \underline{\hspace{1cm}} \text{ m} = \underline{\hspace{1cm}} \text{ m}^2$$

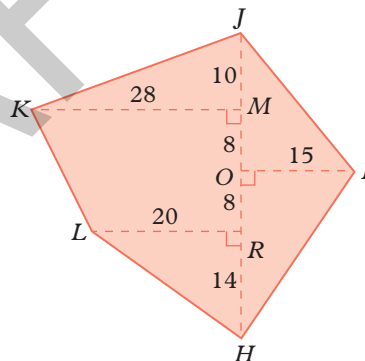
$$\text{Total area} = \underline{\hspace{1cm}} \text{ m}^2 + \underline{\hspace{1cm}} \text{ m}^2 + 28 \text{ m}^2 + \underline{\hspace{1cm}} \text{ m}^2 = \underline{\hspace{1cm}} \text{ m}^2$$

- 6 Find the area of each of the following fields, to the nearest square metre. All measurements are in metres.

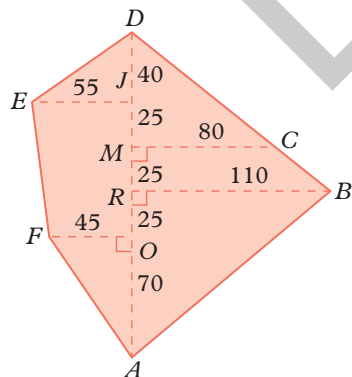
a



b

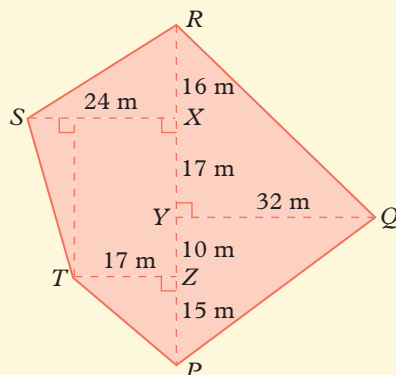


c



Worked example 6.5D Calculating the perimeter of a field involving triangles and trapeziums

Calculate the perimeter of the field $PQRST$, to the nearest metre.



Think	Write
Find the side lengths of each right-angled triangle. Divide the trapezium $TSXZ$ into a rectangle and a right-angled triangle. The length, ZX , of the triangle and the rectangle is $17\text{ m} + 10\text{ m} = 27\text{ m}$. The height of the triangle is $24\text{ m} - 17\text{ m} = 7\text{ m}$. Add all the side lengths to find the perimeter.	Find SR : $24^2 + 16^2 = 832$ $\sqrt{832} \approx 28.84$ $SR \approx 28.84\text{ m}$ Find RQ : $32^2 + (16 + 17)^2 = 2113$ $\sqrt{2113} \approx 45.97$ $RQ \approx 45.97\text{ m}$ Find QP : $32^2 + (10 + 15)^2 = 1649$ $\sqrt{1649} \approx 40.61$ $QP \approx 40.61\text{ m}$ Find PT : $17^2 + 15^2 = 514$ $\sqrt{514} \approx 22.67$ $PT \approx 22.67\text{ m}$ Find ST : $(10 + 17)^2 + (24 - 17)^2 = 778$ $\sqrt{778} \approx 27.89$ $ST \approx 27.89\text{ m}$ Perimeter $\approx 28.84\text{ m} + 45.97\text{ m} + 40.61\text{ m}$ $+ 22.67\text{ m} + 27.89\text{ m}$ $= 165.98$ $\approx 166\text{ m}$



Worked example video: Calculating the perimeter of a field involving triangles and trapeziums

Helpful hints

- Any trapeziums need to be divided into a rectangle and right-angled triangle. Add or subtract measurements to find side lengths.

- 7 Complete the following calculation to find the perimeter of field $MNOPQ$ from question 5.

$$\begin{aligned}\text{Find } MN: 10^2 + (\quad)^2 &= \quad \\ \sqrt{221} &\approx \quad \\ MN &\approx \quad \text{m}\end{aligned}$$

$$\begin{aligned}\text{Find } NO: (11 - \quad)^2 + (12 + \quad)^2 &= \quad \\ \sqrt{\quad} &\approx 15.52 \\ &\approx 15.52 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Find } OP: (\quad)^2 + 8^2 &= \quad \\ \sqrt{\quad} &\approx \quad \\ OP &\approx \quad \text{m}\end{aligned}$$

$$\begin{aligned}\text{Find } PQ: (3 + \quad)^2 + 16^2 &= \quad \\ \sqrt{377} &\approx \quad \\ PQ &\approx \quad \text{m}\end{aligned}$$

$$\begin{aligned}\text{Find } QM: (\quad)^2 + (10 + \quad)^2 &= 740 \\ \sqrt{740} &\approx \quad \\ &\approx \quad \text{m}\end{aligned}$$

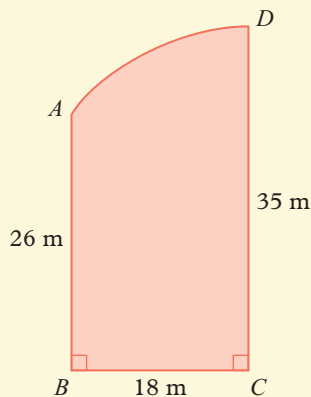
$$\begin{aligned}\text{Perimeter} &= \quad \text{m} + 15.52 \text{ m} + \quad \text{m} \\ &\quad + \quad \text{m} + \quad \text{m} \\ &= \quad \text{m} \\ &\approx \quad \text{m}\end{aligned}$$

- 8 Find the perimeter of the field represented by each field diagram in question 6, to the nearest metre.



Worked example 6.5E Estimating areas using the Trapezoidal rule

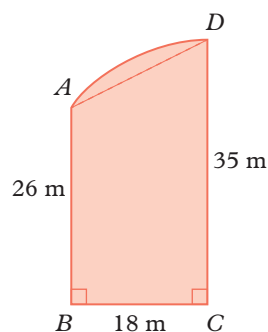
Use the Trapezoidal rule to estimate the area of the field shown.



Think

Draw a straight line from A to D . The required area is approximately equal to the area of trapezium $ABCD$.

The area of the field can be estimated by finding the area of a trapezium. Use the Trapezoidal rule, with $d_f = 26$ m, $d_l = 35$ m and $h = 18$ m.

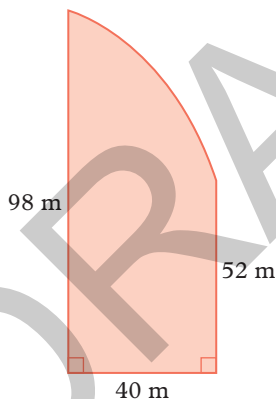
Write

$$\begin{aligned} A &\approx \frac{h}{2}(d_f + d_l) \\ &= \frac{18 \text{ m}}{2}(26 \text{ m} + 35 \text{ m}) \\ &= 9 \text{ m} \times 61 \text{ m} \\ &= 549 \text{ m}^2 \end{aligned}$$



Worked example video: Estimating areas using the Trapezoidal rule

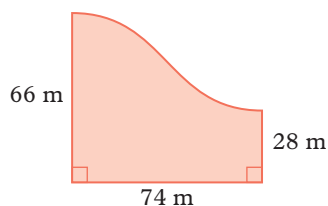
- 9 Complete the following to estimate the area of this field using the Trapezoidal rule.



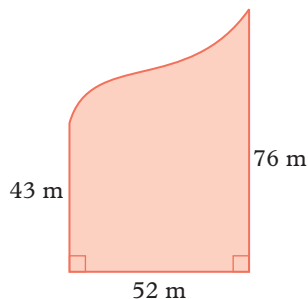
$$\begin{aligned} \text{Area} &\approx \frac{\square \text{ m}}{2}(\square \text{ m} + 52 \text{ m}) \\ &= \square \text{ m} \times \square \text{ m} \\ &= \square \text{ m}^2 \end{aligned}$$

- 10 Use the Trapezoidal rule to estimate the area of each of the following fields.

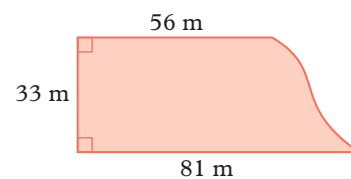
a



b

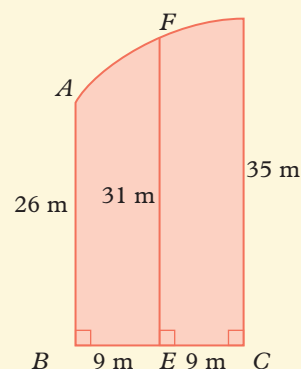


c



Worked example 6.5F Estimating area using two applications of the Trapezoidal rule

The field in Worked example 6.5E has been dissected as shown here. The baseline has been bisected at E and the length of the perpendicular line segment EF is 31 m. Use two applications of the Trapezoidal rule to estimate the area of the field correct to the nearest square metre (m^2).



Think

Divide the total area into two strips and apply the Trapezoidal rule to each strip and add the results.

To divide the area into two or more strips, the baseline BC is divided into two or more equal intervals and the lengths of the perpendicular offsets are measured.

Write

$$\begin{aligned}\text{Area}_{ABCD} &\approx \text{Area}_{ABFE} + \text{Area}_{FECD} \\ &= \frac{9\text{ m}}{2}(26\text{ m} + 31\text{ m}) + \frac{9\text{ m}}{2}(31\text{ m} + 35\text{ m}) \\ &= 4.5\text{ m} \times 57\text{ m} + 4.5\text{ m} \times 66\text{ m} \\ &= 256.5\text{ m}^2 + 297\text{ m}^2 \\ &= 553.5\text{ m}^2 \\ &\approx 534\text{ m}^2\end{aligned}$$



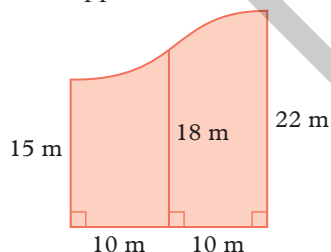
Worked example video: Estimating area using two applications of the Trapezoidal rule

Helpful hints

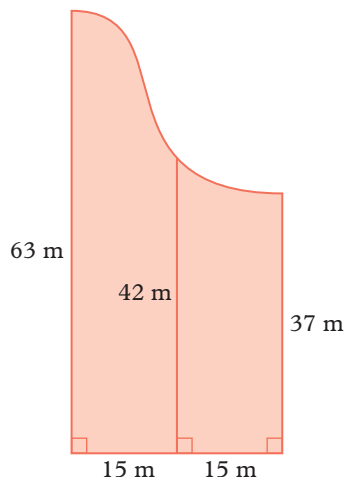
- The more strips the field is divided into, the closer the estimate will be to the actual area.

11 Use two applications of the Trapezoidal rule to estimate the area of each of the following fields.

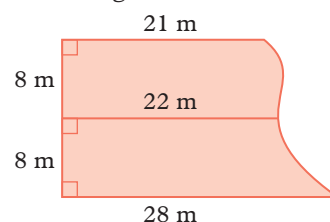
a

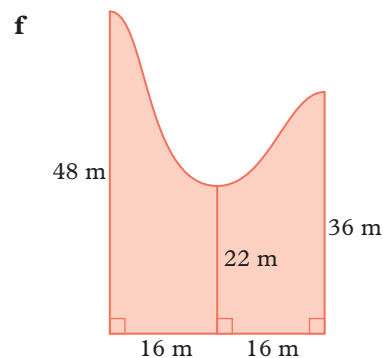
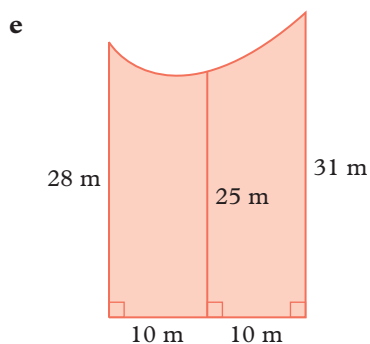
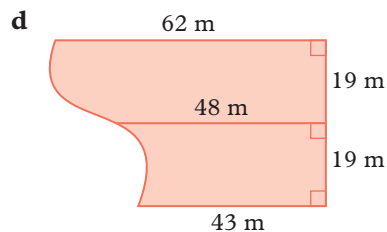


b



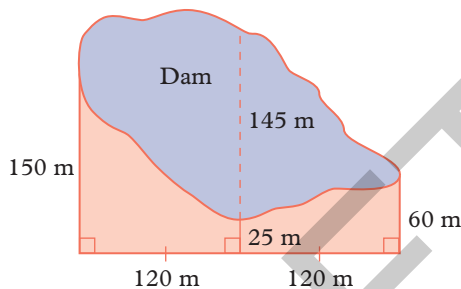
c





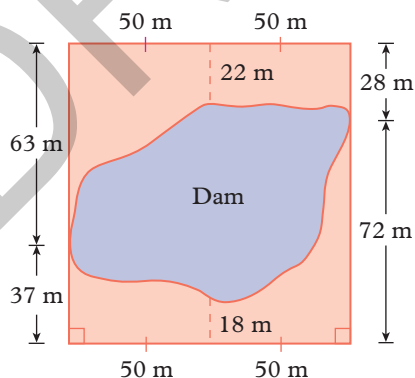
Problem solving and reasoning

12 This diagram shows an aerial view of a dam.



- Use the measurements and two applications of the Trapezoidal rule to find the combined area of the dam and land.
- Use two applications of the Trapezoidal rule to find the land area.
- Find the area of the dam.
- The average depth of water is 10 m. Give an estimate for the volume of water in the dam.

13 This diagram shows an aerial view of a dam.

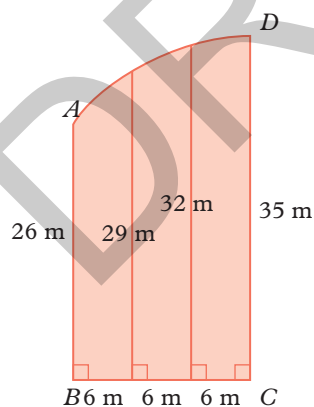


- Use two applications of the Trapezoidal rule to find the area of the dam.
- If the average depth is 18 m, find an estimate for the volume of water in the dam.

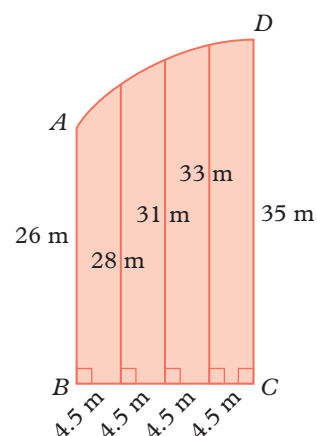
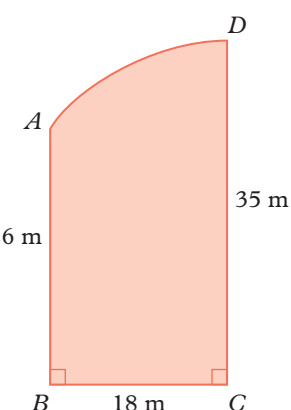


14 We can find a more accurate estimate for the area of the field in Worked example 6.5E and Worked example 6.5F (shown again here) by dividing the field into more strips. To divide the area into two or more strips, the baseline BC is divided into two or more equal intervals and the lengths of the perpendicular offsets are measured.

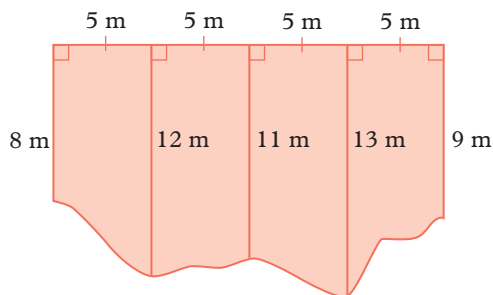
- a** If the field is divided into three strips as shown, use three applications of the Trapezoidal rule to estimate the area of the field.



- b** If the field is divided into four strips as shown, use four applications of the Trapezoidal rule to estimate the area of the field.



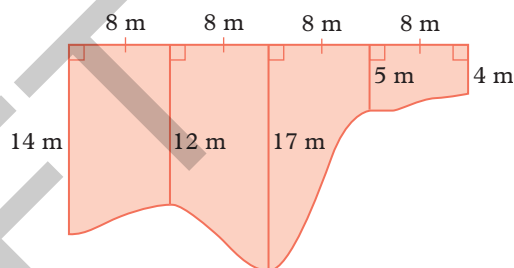
15 The diagram shows the cross-section of a river.



- a Use four applications of the Trapezoidal rule to find an estimate of the area of the cross-section.
- b If the water flows at 10 metres per minute, find the volume of water passing a particular point on the river in one minute.
- c *Hint:* This is the same as finding the volume with a “height” of 10 m.
- d How much water flows past in one hour?

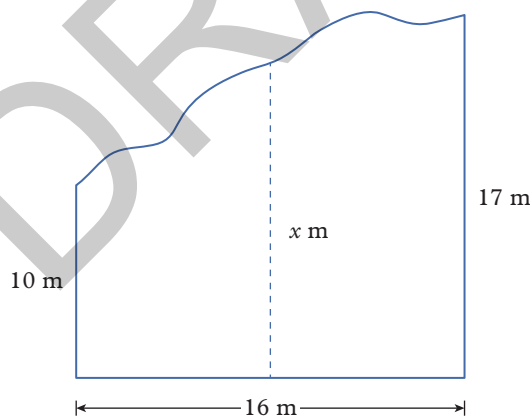
16 The diagram shows the cross-section of a river.

- a Use four applications of the Trapezoidal rule to find an estimate of the area of the cross-section.
- b If water flows past a particular point at 2 metres per minute, find the volume of water passing that point in 10 minutes.
- c How much water flows past in an hour?



Challenge

17 The following figure has an estimated area of 220 m^2 using the Trapezoidal rule. Find the length of the middle parallel.



Related resources



6.5 Quick quiz



Investigation: Area of different parcels of land

Lesson 6.6

Review: Perimeter, area and Pythagoras' theorem

Review questions 6.6A Mathematical literacy review

Answers: Lesson 6.6.

Key terms

The following key terms are used in this module:

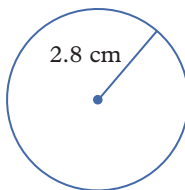
- annulus
- circumference
- composite shape
- diameter
- hypotenuse
- parallel
- perimeter
- perpendicular
- Pythagoras' theorem
- radius
- sector
- trapezium
- Trapezoidal rule

Check your understanding

- 1 Draw a diagram of a circle. Include and label examples of the following key terms: **circumference**, **diameter**, **radius**, **centre**.
- 2 In your own words, explain **Pythagoras' theorem**.
- 3 Give an example of a shape which matches each of the following descriptions. There may be more than one answer.
 - a a quadrilateral with two pairs of parallel lines
 - b a quadrilateral with four sides of equal length
 - c a quadrilateral whose diagonals are perpendicular
- 4 Explain the relationship between the area of a **trapezium** and the **Trapezoidal rule**.

Review questions 6.6B Multiple-choice questions

- 1 What is the circumference of this circle?



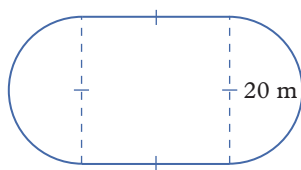
A 5.6 cm

B 8.8 cm

C 14.4 cm

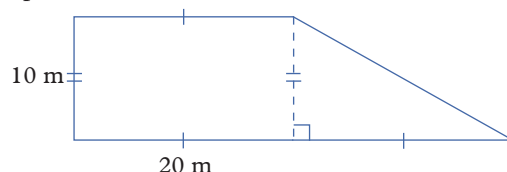
D 17.6 cm

2 What is the perimeter of this shape?



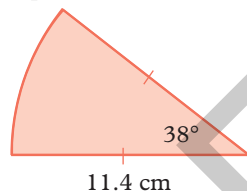
- A** 71 m **B** 80 m
C 103 m **D** 143 m

3 What is the perimeter of this shape?



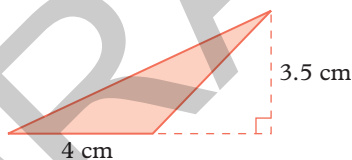
- A** 92 m **B** 70 m
C 50 m **D** 30 m

4 The perimeter of this sector, to the nearest square centimetre is:



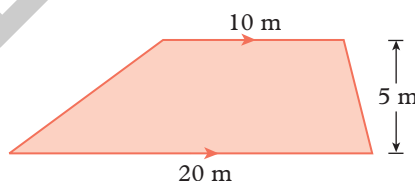
- A** 94 cm **B** 66 cm
C 53 cm **D** 30 cm

5 The area of this triangle is:



- A** 7 cm² **B** 14 cm²
C 1.75 cm² **D** 2 cm²

6 The area of this quadrilateral is:

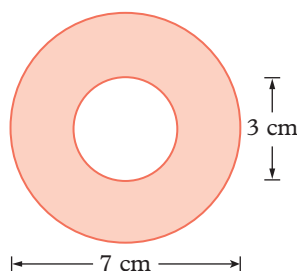


- A** 35 m²
- B** 75 m²
- C** 150 m²
- D** 300 m²

7 The area of the sector shown in question 4 is closest to:

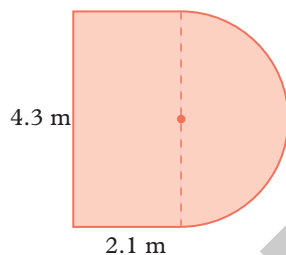
- A** 1.2 cm^2
B 408.3 cm^2
C 43.1 cm^2
D 433.2 cm^2

- 8 The area of the shaded annulus is closest to:



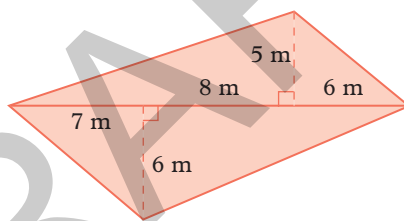
- A** 4 cm²
- B** 31.4 cm²
- C** 125.6 cm²
- D** 125.7 cm²

- 9 The shaded area of this composite shape is closest to:



- A** 16.3 m²
- B** 67.1 m²
- C** 9.03 m²
- D** 11.8 m²

- 10** What is the area of this quadrilateral?



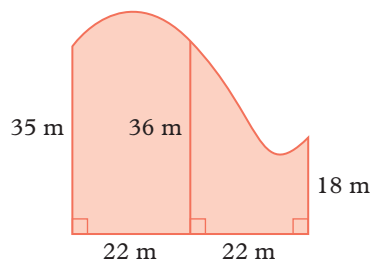
- A** 231 m²
B 168 m²
C 115.5 m²
D 32 m²

- 11** What is the perimeter of the quadrilateral in question 10?

- A** 32 m
B 39 m
C 48 m
D 1521 m

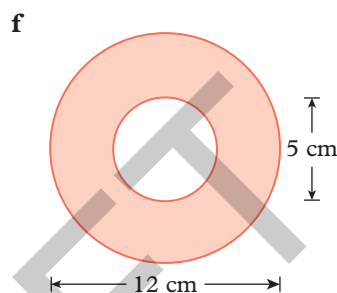
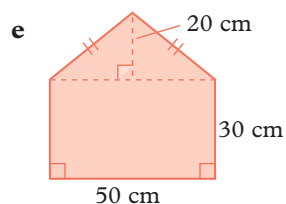
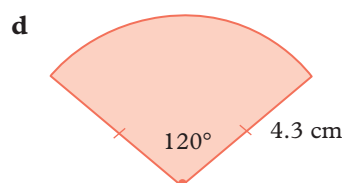
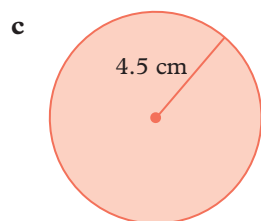
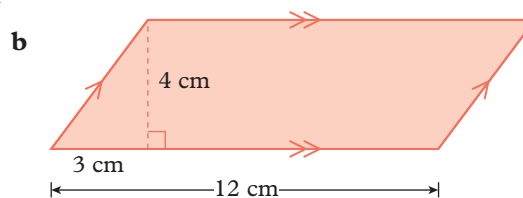
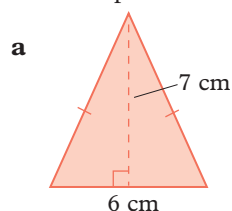
- 12** Using two applications of the Trapezoidal rule, an estimate of the area of the field shown, to the nearest square metre, is:

- A** 688 m²
B 1066 m²
C 1375 m²
D 2750 m²



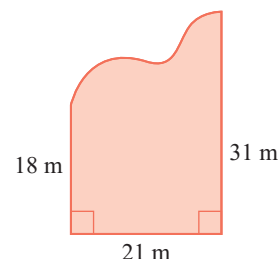
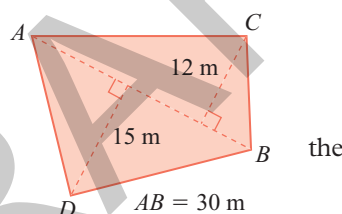
Review questions 6.6C Set 1

1 Find the perimeter and area of each of the following shapes.



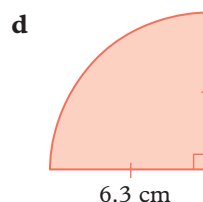
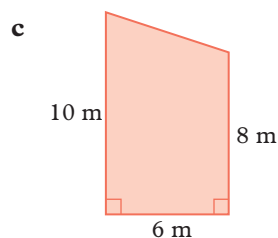
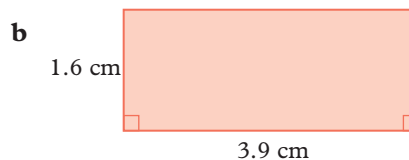
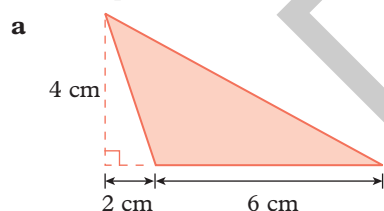
2 Consider the following fields.

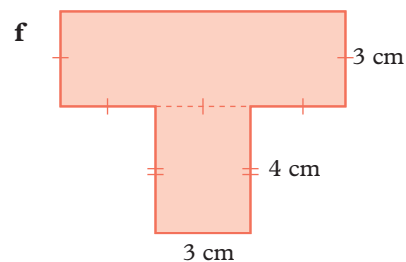
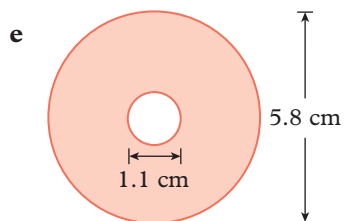
- Calculate the area of the field represented by the diagram below.
- Use the Trapezoidal rule to estimate area of this field.



Review questions 6.6D Set 2

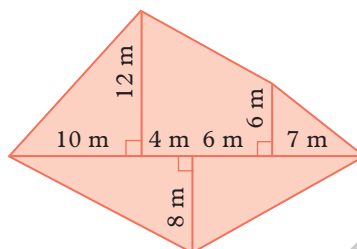
1 Find the perimeter and area of each of these shapes.



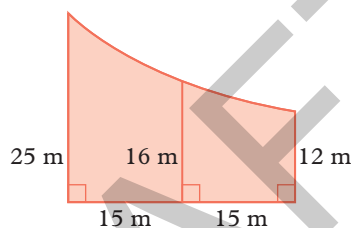


2 Consider the following fields.

a Calculate the perimeter and area of this field.

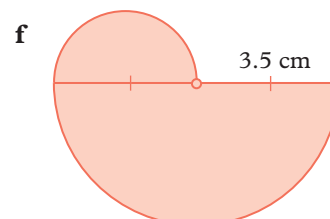
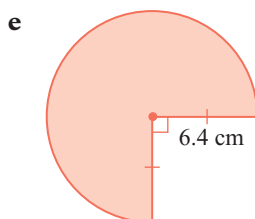
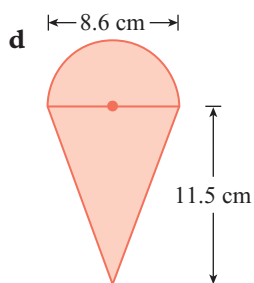
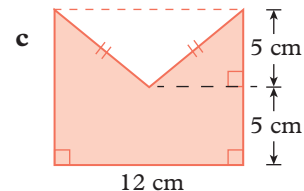
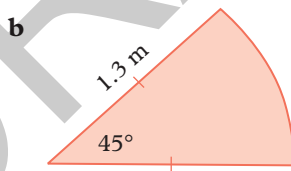
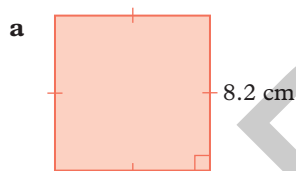


b Use two applications of the Trapezoidal rule to estimate the area of this field.



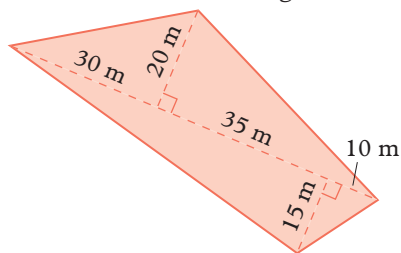
Review questions 6.6E Set 3

1 Find the perimeter and area of each of these shapes.

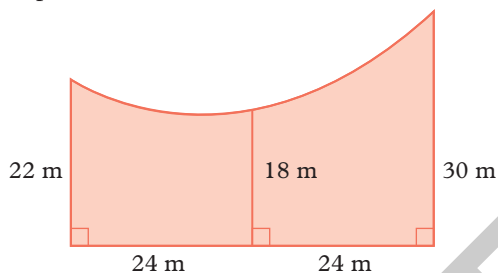


2 Consider the following fields.

a Calculate the perimeter and area of the field from the diagram shown.



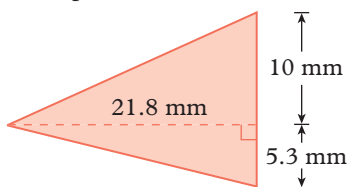
b Use two applications of the Trapezoidal rule to estimate the area of the field shown.



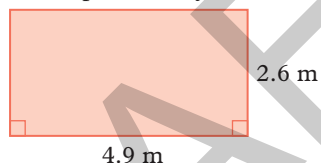
Review questions 6.6F Set 4

1 Find the perimeter and area of each of these shapes. Give your answers correct to one decimal place.

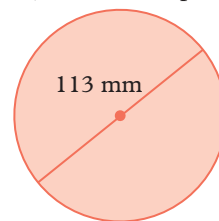
a



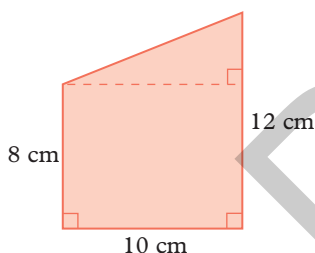
b



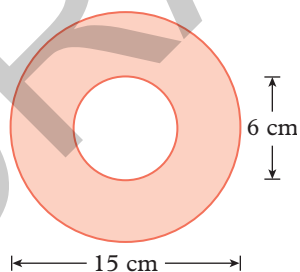
c



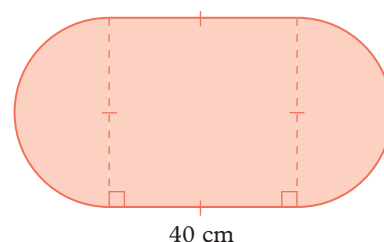
d



e



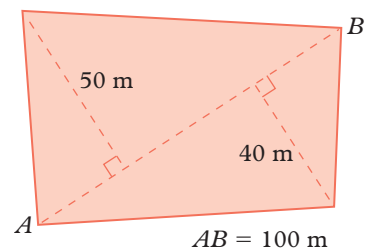
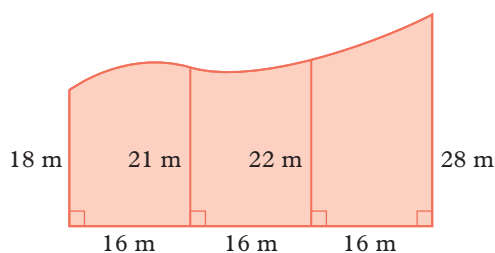
f



2 Consider the following fields.

a Calculate the area of the field from the diagram below.

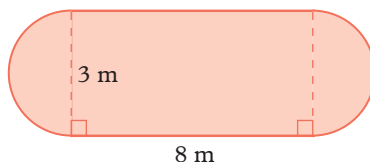
b Use three applications of the Trapezoidal rule to estimate the area of the field below.



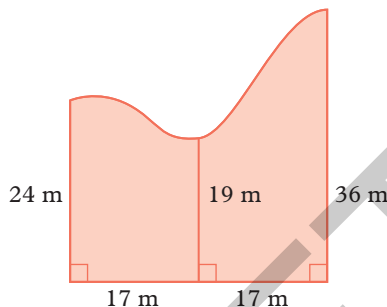
Review questions 6.6G Practice examination question

1 Answer the following questions.

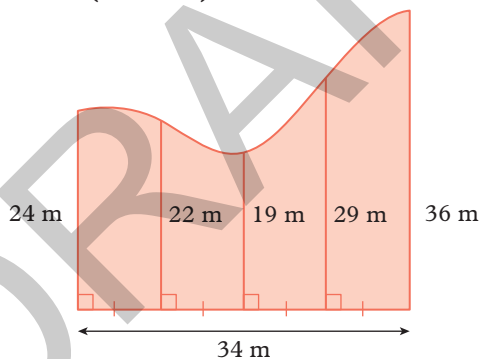
- a** Find the perimeter and area of this shape. Write the area correct to the nearest square metre (m^2).
(4 marks)



- b** Find the perimeter and area of a sector which has radius 8 cm and a sector angle of 66° . (3 marks)
- c** Use two applications of the Trapezoidal rule to estimate the area of the field represented by this diagram.
(2 marks)



- d** The field in part **c** is divided into four sections as shown below. Use four applications of the Trapezoidal rule to estimate the area of the field. (4 marks)



- e** Compare your answers to parts **c** and **d**. Which result is closer to the actual area of the field? Explain your response. (2 marks)

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Perimeter, area and Pythagoras' theorem

I can do this	I need to review this
☐ calculate the perimeter of triangles, quadrilaterals, circles, sectors and composite shapes ☐ solve practical problems involving perimeter	Go back to Lesson 6.1 Perimeter
☐ apply Pythagoras' theorem to solve problems involving right-angled triangles	Go back to Lesson 6.2 Using Pythagoras' theorem
☐ calculate the area of triangles, quadrilaterals, circles and sectors ☐ solve practical problems involving area of simple shapes	Go back to Lesson 6.3 Area of simple shapes
☐ calculate the area of composite shapes ☐ solve practical problems involving the area of composite shapes	Go back to Lesson 6.4 Area of composite shapes
☐ calculate the perimeter and area of irregularly shaped blocks of land ☐ use the Trapezoidal rule to solve practice problems involving area up to four applications	Go back to Lesson 6.5 Perimeter and area of irregular shapes

Related resources



Module review quiz: Perimeter, area and Pythagoras' theorem



Worksheet: Challenge 6 Percentage increase and decrease

Module 7

Surface area, volume and capacity



Lessons

- Lesson 7.1 Surface area of prisms
- Lesson 7.2 Surface area of cylinders and spheres
- Lesson 7.3 Volume of prisms, cylinders and spheres
- Lesson 7.4 Volume of pyramids and cones
- Lesson 7.5 Capacity
- Lesson 7.6 Review: Surface area, volume and capacity

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Surface area, volume and capacity

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Units of measurement



Support sheet: Area of a rectangle



Support sheet: Area of a triangle



Support sheet: Circles and sectors



Support sheet: Naming 3D objects

Syllabus links

- Applications of measurement (MST-11-05)
 - A student solves problems involving measurement in practical contexts.
 - Perimeter, area and volume
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 7.1

Surface area of prisms

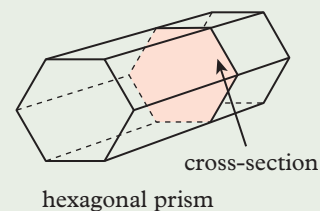
Learning intentions

By the end of this lesson, you will be able to ...

- calculate and solve problems about the surface area of prisms.

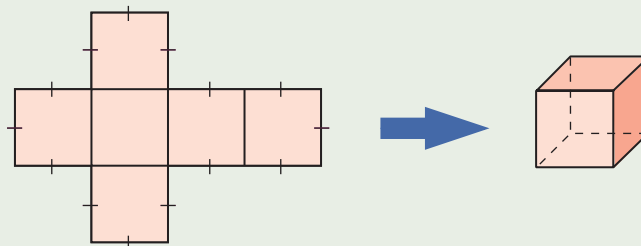
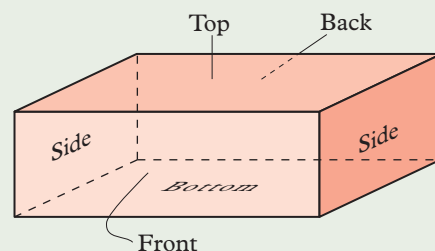
Prisms

- A **prism** is a three-dimensional object with two identical, parallel faces which are **polygons**, and the two faces are connected by straight edges. The shape of these faces gives the prism its name (e.g. triangular prism).
 - All other faces of a prism are rectangles.
 - When a prism is sliced parallel to the two identical faces, the **cross-section** is a shape identical to the two faces. This means that the cross-section of a prism is uniform.



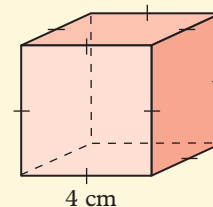
Surface area

- The **surface area** of any shape refers to the sum of the areas of all the faces.
 - For example, this rectangular prism has six faces: two sides, a front and a back, and a top and a bottom.
 - The formulas for simple shapes in Lesson 6.3 can be used to find the area of each face. Some faces may be composite shapes and will need to be divided into simple shapes before the area can be found.
- The **net** of a 3D object is a plane shape that can be folded to form the 3D object. The diagram below shows a net of a cube. The area of a net is equal to the surface area of the corresponding 3D object.



Worked example 7.1A Calculating the surface area of a cube

Calculate the surface area of this cube.



Think

All faces are identical squares. The formula for the area of a square is $A = s^2$.
Multiply the area of one face by 6 to find the surface area of the cube.

Write

$$\begin{aligned}\text{Surface area} &= 6 \times \text{area of square face} \\ &= 6 \times 4^2 \\ &= 96 \text{ cm}^2\end{aligned}$$



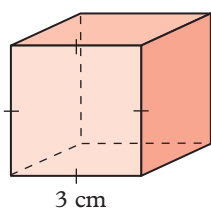
Worked example video: Calculating the surface area of a cube

Understanding, fluency and communicating

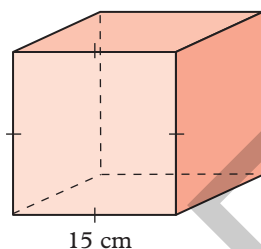
Answers: Lesson 7.1

- 1 Calculate the surface area of each of these cubes.

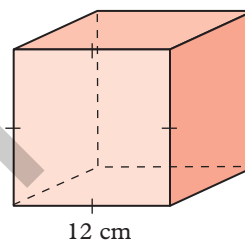
a



b



c



- 2 Calculate the surface area of cubes with sides of the following lengths.

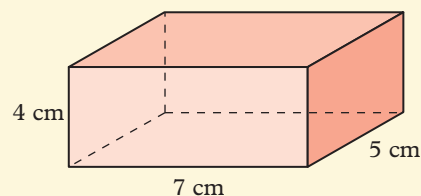
a 4.8 cm

b 17 mm

c 0.31 m

Worked example 7.1B Calculating the surface area of a rectangular prism

Calculate the surface area of this rectangular prism.

**Think**

The prism has six rectangular faces.
The front and back rectangles have length and breadth 7 cm and 4 cm.
The two side rectangles have length and breadth 5 cm and 4 cm.
The top and bottom rectangles have length and breadth 7 cm and 5 cm.
The formula for the area of a rectangle is $A = lb$. Remember that there are two rectangles for each pair of side lengths.
Add the areas of the six rectangles together to find the surface area of the prism.

Write

$$\begin{aligned}\text{Area of front and back} &= 2 \times \text{area of front} \\ &= 2 \times 7 \times 4 \\ &= 56 \text{ cm}^2 \\ \text{Area of sides} &= 2 \times \text{area of one side} \\ &= 2 \times 5 \times 4 \\ &= 40 \text{ cm}^2 \\ \text{Area of top and bottom} &= 2 \times \text{area of top} \\ &= 2 \times 7 \times 5 \\ &= 70 \text{ cm}^2 \\ \text{Surface area} &= 56 + 40 + 70 \\ &= 166 \text{ cm}^2\end{aligned}$$

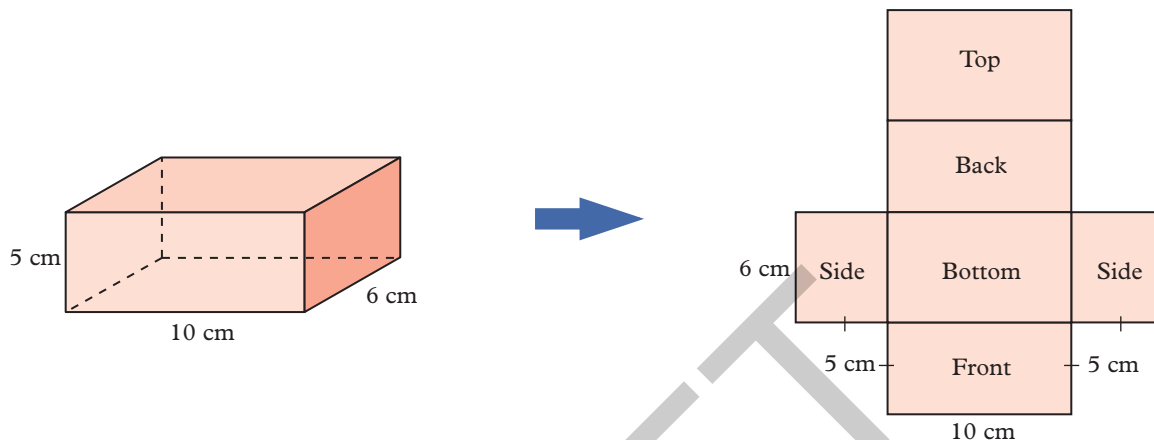


Worked example video: Calculating the surface area of a rectangular prism

Helpful hints

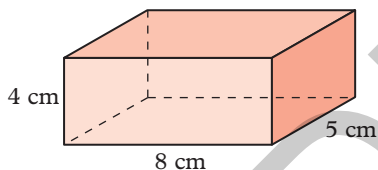
- Count and compare all faces. If some faces have the same dimensions, then the number of calculations is reduced.
- Make sure to account for all the faces when adding together areas to find the surface area.

3 Complete the following to calculate the surface area of this rectangular prism, given its net.



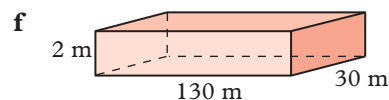
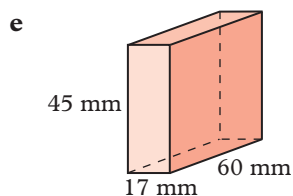
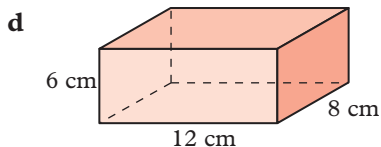
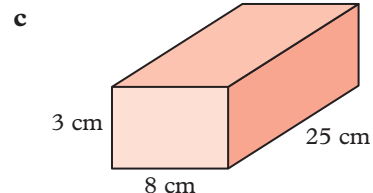
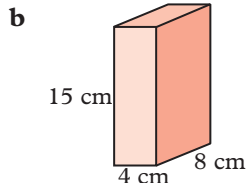
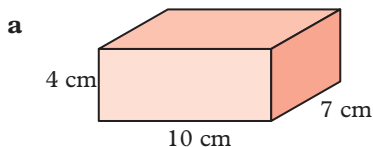
Top and __: $A = 2 \times 10 \text{ cm} \times \text{__ cm} = \text{__ cm}^2$
 __ and back: $A = \text{__} \times 5 \text{ cm} \times \text{__ cm} = \text{__ cm}^2$
 Two sides: $A = 2 \times \text{__ cm} \times \text{__ cm} = \text{__ cm}^2$
 Surface area: $SA = \text{__ cm}^2 + \text{__ cm}^2 + 60 \text{ cm}^2 = \text{__ cm}^2$

4 Complete the following to calculate the surface area of this rectangular prism.



Front and __: $A = 2 \times 8 \text{ cm} \times \text{__ cm} = \text{__ cm}^2$
 Two sides: $A = 2 \times \text{__ cm} \times 5 \text{ cm} = \text{__ cm}^2$
 __ and bottom: $A = 2 \times \text{__ cm} \times 5 \text{ cm} = \text{__ cm}^2$
 Surface area: $SA = 64 \text{ cm}^2 + \text{__ cm}^2 + \text{__ cm}^2 = \text{__ cm}^2$

5 Calculate the surface area of each of these rectangular prisms.



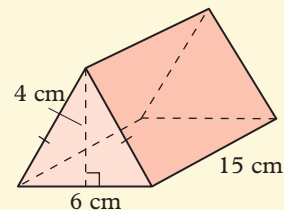
- 6 Calculate the surface area of the rectangular prisms with the dimensions given.

Hint: Draw a diagram of each rectangular prism and then find the surface area.

- a** length 8 m, breadth 9 m and height 4 m
b length 12 mm, breadth 15 mm and height 6 mm
c length 15.3 m, breadth 6.4 m and height 4.3 m

Worked example 7.1C Calculating the surface area of a triangular prism

Calculate the surface area of this triangular prism.



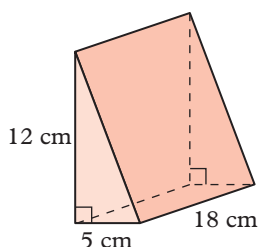
Think	Write
The triangular prism has five faces: a front triangular face (F), a back triangular face (B), a sloping rectangular face on the left (L) and one on the right (R), and a horizontal rectangular face (H) at its base. The two triangular faces F and B have dimensions $b = 6$ cm and $h = 4$ cm, so their area is given by $\frac{1}{2}bh$. The diagram shows that the two sloping faces L and R have length 15 cm, but we need to find their breadth using Pythagoras' theorem. The horizontal face has length 15 cm and breadth 6 cm. Add the areas of the five faces together to find the surface area of the prism.	Front and back faces: $\text{Area}_F = \text{Area}_B = \frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2$ Left and right sloping faces: $\text{breadth} = \sqrt{4^2 + 6^2} \text{ cm} = \sqrt{52} \text{ cm} = 2\sqrt{13} \text{ cm}$ So: $\text{Area}_L = \text{Area}_R = 15 \text{ cm} \times 2\sqrt{13} \text{ cm} = 30\sqrt{13} \text{ cm}^2$ Horizontal face: $\text{Area}_H = 15 \text{ cm} \times 6 \text{ cm} = 90 \text{ cm}^2$ Total surface area: $\begin{aligned} SA &= 2 \times 12 \text{ cm}^2 + 2 \times 30\sqrt{13} \text{ cm}^2 + 90 \text{ cm}^2 \\ &= 24 \text{ cm}^2 + 60\sqrt{13} \text{ cm}^2 + 90 \text{ cm}^2 \\ &= 114 + 60\sqrt{13} \text{ cm}^2 \end{aligned}$



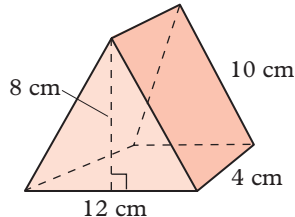
Worked example video: Calculating the surface area of a triangular prism

- 7 Calculate the surface area of each of these triangular prisms.

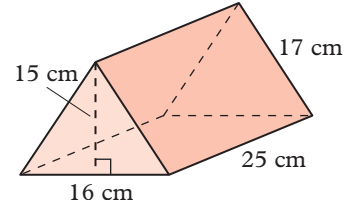
a



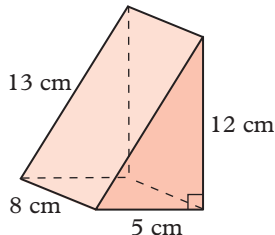
b



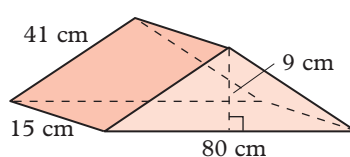
c



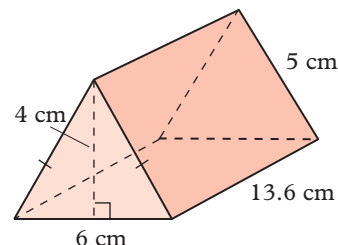
d



e

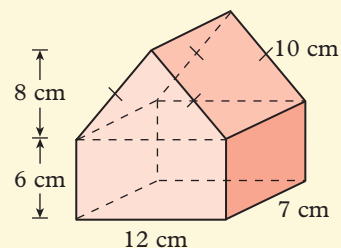


f



Worked example 7.1D Calculating the surface area of a composite prism

Calculate the surface area of this composite prism.

**Think**

This prism has seven faces: the front (F), back (B), horizontal base (H), a vertical face on the left (L), a vertical face on the right (R), a face sloping up (U) and a face sloping down (D).

The faces F and B are the same composite shape made up of a rectangle with length 12 cm and breadth 6 cm and a triangle with base 12 cm and height 8 cm.

Their area is found by adding the areas of the rectangular part (A_1) and the triangular part (A_2).

The rectangular faces L and R both have length 7 cm and breadth 6 cm.

The faces U and D are both rectangular with length 10 cm and breadth 7 cm.

The rectangular base H has length 12 cm and breadth 7 cm.

Add the areas of the seven faces together to find the surface area of the prism.

Write

Front and back faces:

$$A_1 = 12 \text{ cm} \times 6 \text{ cm} = 72 \text{ cm}^2$$

$$A_2 = \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm} = 48 \text{ cm}^2$$

$$\begin{aligned} \text{Area}_F &= \text{Area}_B = A_1 + A_2 \\ &= 72 \text{ cm}^2 + 48 \text{ cm}^2 \\ &= 120 \text{ cm}^2 \end{aligned}$$

Left and right vertical faces:

$$\text{Area}_L = \text{Area}_R = 7 \text{ cm} \times 6 \text{ cm} = 42 \text{ cm}^2$$

Sloping faces:

$$\text{Area}_U = \text{Area}_D = 10 \text{ cm} \times 7 \text{ cm} = 70 \text{ cm}^2$$

Horizontal face:

$$\text{Area}_H = 12 \text{ cm} \times 7 \text{ cm} = 84 \text{ cm}^2$$

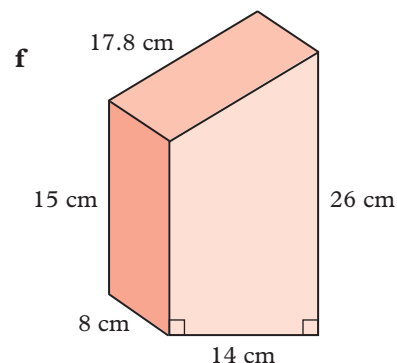
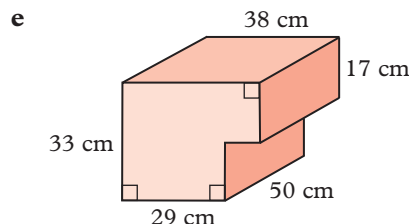
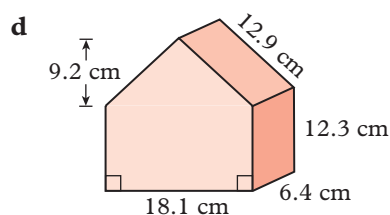
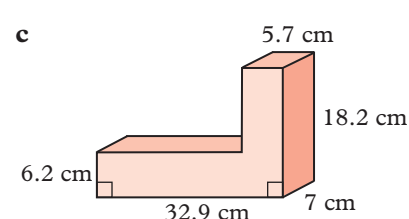
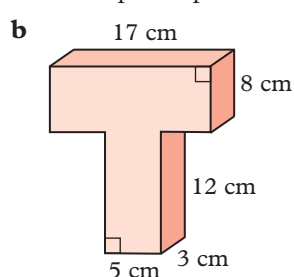
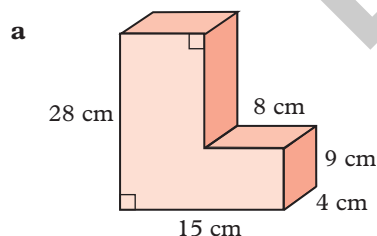
Total surface area:

$$\begin{aligned} SA &= 2 \times \text{Area}_F + 2 \times \text{Area}_L \\ &\quad + 2 \times \text{Area}_U + \text{Area}_H \\ &= 2 \times 120 \text{ cm}^2 + 2 \times 42 \text{ cm}^2 \\ &\quad + 2 \times 70 \text{ cm}^2 + 1 \times 84 \text{ cm}^2 \\ &= 240 \text{ cm}^2 + 84 \text{ cm}^2 + 140 \text{ cm}^2 + 84 \text{ cm}^2 \\ &= 548 \text{ cm}^2 \end{aligned}$$



Worked example video: Calculating the surface area of a composite prism

8 Calculate the surface area of each of these composite prisms.



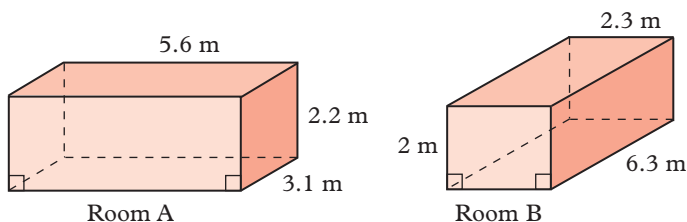
Problem solving and reasoning

9 A gift box measures 10 cm by 12 cm by 20 cm.

- Calculate its total surface area.
- Gift wrapping paper costs \$3.50 a sheet. If each sheet covers 290 cm^2 , calculate the total cost of wrapping the gift box. (*Remember: You must purchase complete sheets.*)

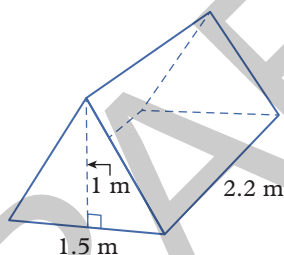


10 Two rooms with the following dimensions are to be painted.



- Which room has the greater wall area?
- A 4 L tin of paint covers approximately 10 square metres and costs \$54.95. Calculate the number of 4 L tins of paint needed to paint room B.
- Calculate the cost of purchasing paint for room B.

11 A floorless tent is to be made with dimensions as shown.



How much material is needed to make the tent? (Ignore overlaps for seams, etc.)

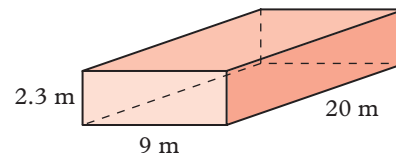


12 Which of these cereal boxes has the greater surface area?



13 The interior walls and floor of an in-ground swimming pool are to be repainted. Its dimensions are shown in the diagram.

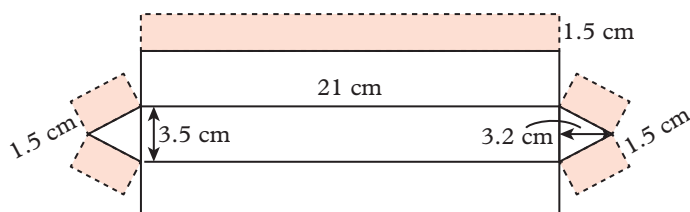
- Calculate the total surface area of the four walls and floor of the pool.
- How many cans of paint are needed if one can covers 70 m^2 ?
- What is the cost of repainting, if each can costs \$82.50?



14 The packaging of a 100 g Toblerone bar has the following dimensions:

base = 3.5 cm, height = 3.2 cm and length = 21.0 cm.

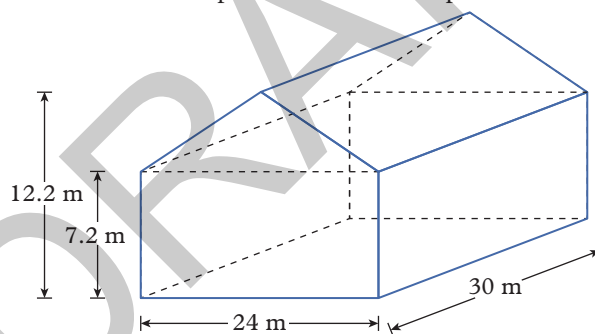
- Calculate the surface area of the packet.
- To make each packet, flaps 1.5 cm wide must be added, as shown below on the net of the solid, so that it can be glued together. How much cardboard is needed to make this packet?



Challenge

15 Find the surface area of the following composite prism.

there are missing dotted lines for the “back” parts of this 3D shape.



Related resources



7.1 Quick quiz



Worksheet: Surface area of prisms

Lesson 7.2

Surface area of cylinders and spheres

Learning intentions

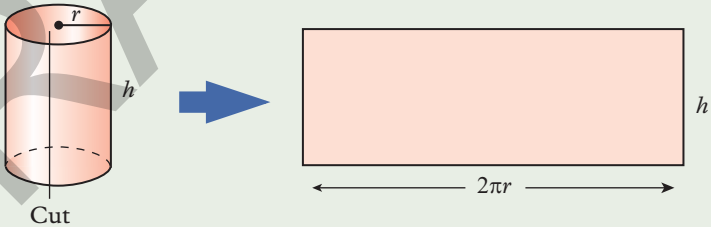
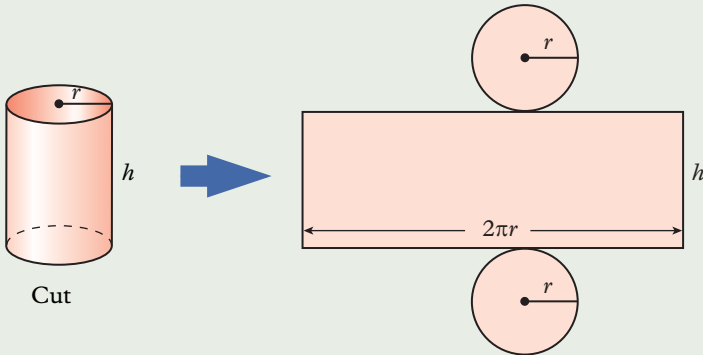
By the end of this lesson, you will be able to ...

→ calculate and solve problems about the surface area of cylinders, spheres and composite solids.

Cylinders

- A **cylinder** is a 3D object which has two identical, parallel circular faces opposite each other which are joined by a curved surface.
 - A closed cylinder includes the two circular faces.
 - An open cylinder does not include the two circular faces.
 - When a cylinder is sliced parallel to the two circular faces, the cross-section is a circle identical to the two faces. This means that the cross-section of a cylinder is uniform (like a prism).

Surface area of a cylinder

- An open cylinder that has been cut perpendicular to the base, and folded out, forms a rectangle. The breadth of the rectangle is equal to the height of the cylinder. The length of the rectangle is equal to the circumference of the circle that formed the top or bottom.
 
- The circumference of a circle is given by $C = 2\pi r$. Therefore, the area of the curved surface of the cylinder is given by $A = 2\pi rh$, which is the area of a rectangle.
 
- A closed cylinder includes the curved surface and the two circular ends. Therefore, the surface area of a closed cylinder is given by:

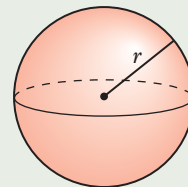
$$SA = 2 \times \text{area of circle} + \text{area of rectangle}$$

$$= 2 \times \pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi rh.$$

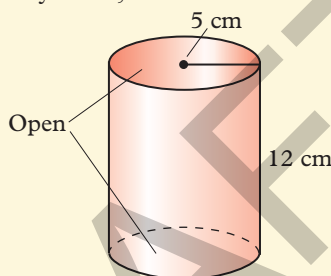
Surface area of a sphere

- A sphere is a 3D object shaped like a ball.
 - The radius of a sphere is the distance between its centre and the surface of the sphere.
 - The diameter of a sphere is the length of a line segment between two points on the sphere which passes through the centre of the sphere. As with circles, the diameter is twice the length of the radius ($d = 2r$).
 - Half of a sphere is called a hemisphere.
- The surface area of a sphere of radius r is given by $SA = 4\pi r^2$.



Worked example 7.2A Finding the surface area of an open cylinder

Find the outer surface area of this open cylinder, correct to one decimal place.



Think

The cylinder is open, so the two circles are not included.
 The radius is $r = 5$ cm and the height is $h = 12$ cm.
 The formula for the area of the curved surface is $SA = 2\pi rh$.
 Substitute the values for the radius and height into the formula. Evaluate and write the result with the correct units.

Write

$$\begin{aligned}
 SA &= 2\pi rh \\
 &= 2 \times \pi \times 5 \text{ cm} \times 12 \text{ cm} \\
 &= 376.991\dots \text{ cm}^2 \\
 &\approx 377.0 \text{ cm}^2
 \end{aligned}$$



Worked example video: Finding the surface area of an open cylinder

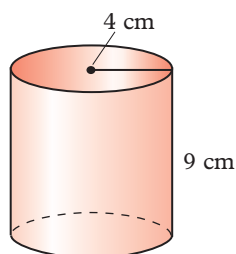
Understanding, fluency and communicating

Answers: Lesson 7.2

Helpful hints

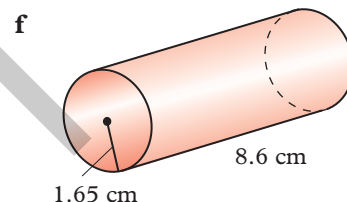
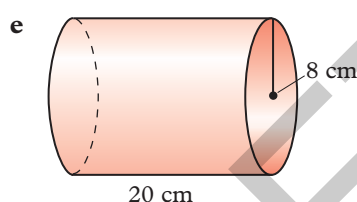
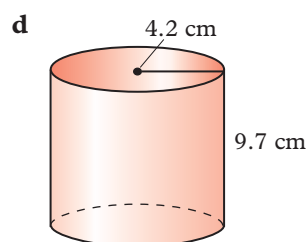
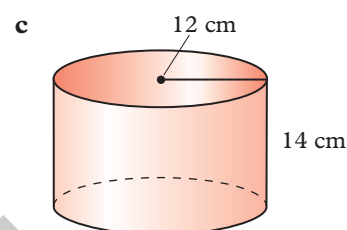
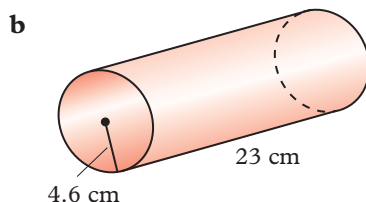
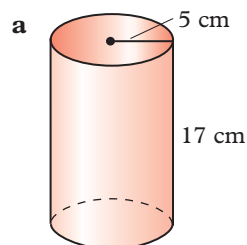
- Remember that an open cylinder does not include the two circular faces.

- 1 Complete the following to find the outer surface area of this open cylinder, correct to one decimal place.

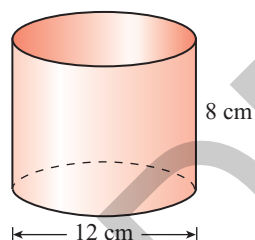


$$\begin{aligned} SA &= 2\pi rh \\ &= 2 \times \pi \times \underline{\hspace{1cm}} \text{ cm} \times \underline{\hspace{1cm}} \text{ cm} \\ &= \underline{\hspace{1cm}} \dots \text{ cm}^2 \\ &\approx \underline{\hspace{1cm}} \text{ cm}^2 \end{aligned}$$

- 2 Find the outer surface area of each of these open cylinders, correct to one decimal place.

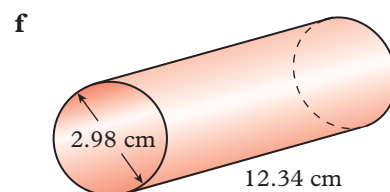
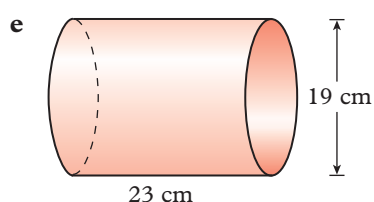
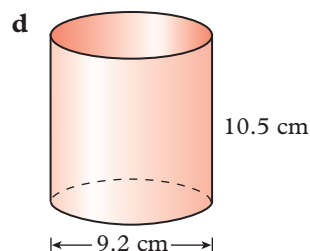
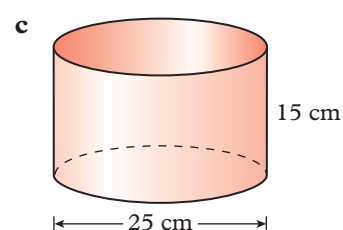
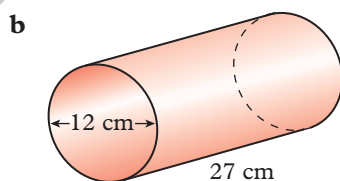
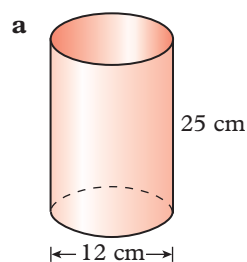


- 3 Complete the following to calculate the outer surface area of this open cylinder, correct to one decimal place.



$$\begin{aligned} \text{diameter} &= \underline{\hspace{1cm}} \text{ cm} \\ \text{radius} &= \underline{\hspace{1cm}} \text{ cm} \\ SA &= 2\pi rh \\ &= 2 \times \pi \times \underline{\hspace{1cm}} \text{ cm} \times \underline{\hspace{1cm}} \text{ cm} \\ &= \underline{\hspace{1cm}} \dots \text{ cm}^2 \\ &\approx \underline{\hspace{1cm}} \text{ cm}^2 \end{aligned}$$

- 4 Find the outer surface area of each of these open cylinders, correct to one decimal place.



5 Find the outer surface area of the open cylinders with these dimensions, correct to one decimal place.

a radius = 4 cm, height = 12 cm

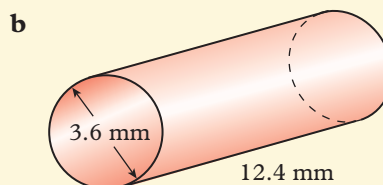
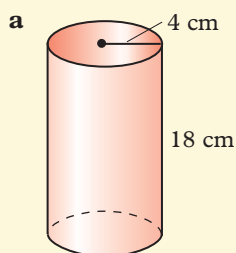
b diameter = 18 cm, height = 13 cm

c radius = 3 cm, height = 5 cm

d diameter = 11.2 cm, height = 23.6 cm

Worked example 7.2B Finding the surface area of a closed cylinder

Calculate the surface areas of these closed cylinders, correct to one decimal place.



Part	Think	Write
a	The cylinder is closed so the two circular faces are included. The radius is $r = 4$ cm and the height is $h = 18$ cm. The formula for the area of the cylinder is $SA = 2\pi r^2 + 2\pi rh$. Substitute the values for the radius and height into the formula. Evaluate and write the result with the correct units.	$SA = 2\pi r^2 + 2\pi rh$ $= 2 \times \pi \times (4\text{ cm})^2$ $+ 2 \times \pi \times 4\text{ cm} \times 18\text{ cm}$ $= 552.920\dots\text{ cm}^2$ $\approx 552.9\text{ cm}^2$
b	The cylinder is closed so the two circular faces are included. Halve the diameter of 3.6 mm to find the radius, $r = 1.8$ mm. The height is $h = 12.4$ mm. The formula for the area of the cylinder is $SA = 2\pi r^2 + 2\pi rh$. Substitute the values for the radius and height into the formula. Evaluate and write the result with the correct units.	Radius: $d = 2r$ $3.6\text{ mm} = 2r$ $r = 1.8\text{ mm}$ Total surface area: $SA = 2\pi r^2 + 2\pi rh$ $= 2 \times \pi \times (1.8\text{ mm})^2$ $+ 2 \times \pi \times 1.8\text{ mm} \times 12.4\text{ mm}$ $= 160.598\dots\text{ mm}^2$ $\approx 160.6\text{ mm}^2$

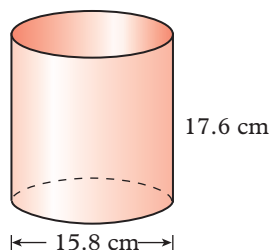


Worked example video: Finding the surface area of a closed cylinder

Helpful hints

- Remember that when a cylinder is closed, the two circular faces are included.
- The formula for the surface area of a closed cylinder can also be written as $SA = 2\pi r(r + h)$.

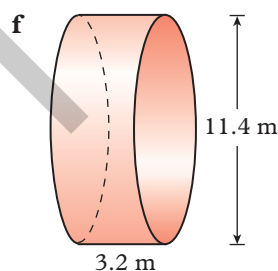
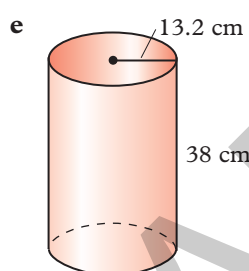
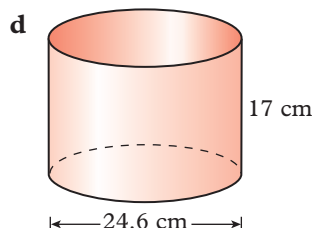
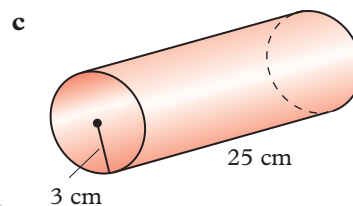
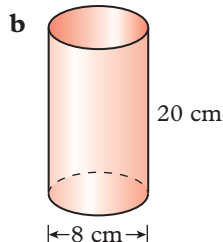
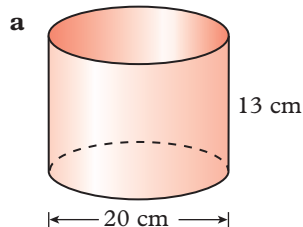
- 6 Complete the following to calculate the surface area of this closed cylinder, correct to one decimal place.



$$\begin{aligned}\text{radius} &= \_\_\text{ cm} \div 2 \\ &= \_\_\text{ cm}\end{aligned}$$

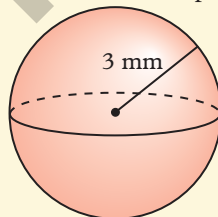
$$\begin{aligned}SA &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times (\_\_\text{ cm})^2 + 2 \times \pi \times \_\_\text{ cm} \times 17.6\text{ cm} \\ &= \_\_\text{ cm}^2 \\ &\approx \_\_\text{ cm}^2\end{aligned}$$

- 7 Find the surface area of each of these closed cylinders, correct to one decimal place.



Worked example 7.2C Finding the surface area of a sphere with given radius

Find the surface area of this sphere, correct to one decimal place.



Think

The radius of the sphere is $r = 3\text{ mm}$.

The formula for the surface area of a sphere is $SA = 4\pi r^2$.

Substitute the radius into the formula. Evaluate and write the result with the correct units.

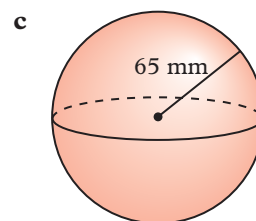
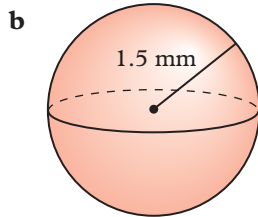
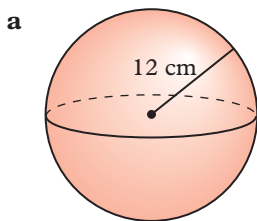
Write

$$\begin{aligned}SA &= 4\pi r^2 \\ &= 4 \times \pi \times (3\text{ mm})^2 \\ &= 113.097\ldots\text{ mm}^2 \\ &\approx 113.1\text{ mm}^2\end{aligned}$$



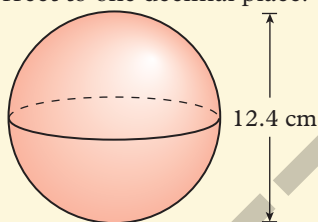
Worked example video: Finding the surface area of a sphere with given radius

8 Find the surface area of each of these spheres, correct to one decimal place.



Worked example 7.2D Finding the surface area of a sphere with given diameter

Find the surface area of this sphere, correct to one decimal place.



Think

The diameter of the sphere is 12.4 cm. Halve the diameter to find the radius.

The formula for the surface area of a sphere is $SA = 4\pi r^2$. Substitute the radius into the formula. Evaluate and write the result with the correct units.

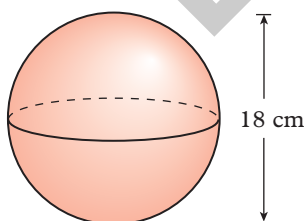
Write

$$\begin{aligned} d &= 2r \\ 12.4 \text{ cm} &= 2r \\ r &= 6.2 \text{ cm} \\ SA &= 4\pi r^2 \\ &= 4 \times \pi \times (6.2 \text{ cm})^2 \\ &= 483.051\dots \text{ cm}^2 \\ &\approx 483.1 \text{ cm}^2 \end{aligned}$$



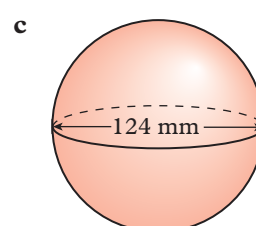
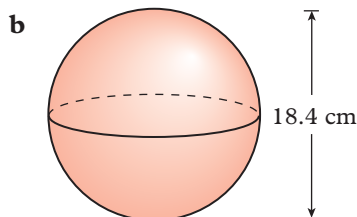
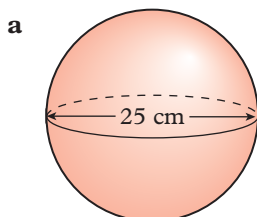
Worked example video: Finding the surface area of a sphere with given diameter

9 Complete the following to find the surface area of this sphere, correct to one decimal place.

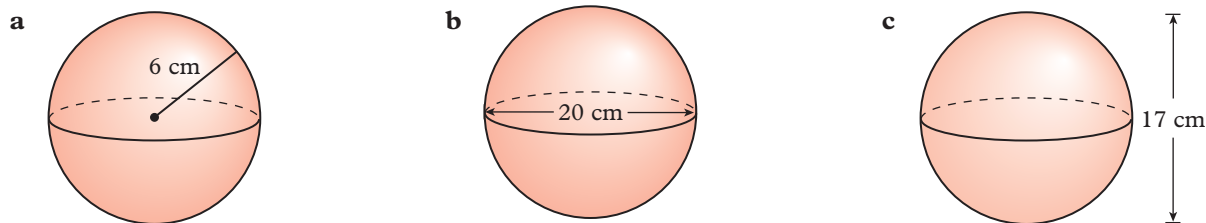


$$\begin{aligned} r &= \text{ } \text{ cm} \div 2 = \text{ } \text{ cm} \\ SA &= 4\pi r^2 \\ &= 4 \times \pi \times (\text{ } \text{ cm})^2 \\ &= \text{ } \text{ cm}^2 \\ &\approx \text{ } \text{ cm}^2 \end{aligned}$$

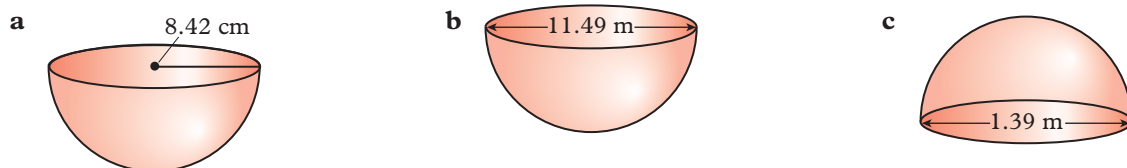
10 Find the surface area of each of these spheres, correct to one decimal place.



11 Find the surface area of each of these spheres, correct to one decimal place.



12 Find the outer surface area of each of these open hemispheres, correct to two decimal places.



Problem solving and reasoning

13 A closed cylindrical tank, 40 m high and 20 m in diameter, sits on a concrete slab.

- Find the outer surface area of the tank.
- If paint covers $8 \text{ m}^2/\text{L}$, find the number of 10 L cans of paint needed to paint the outside of the tank with one coat.
- Paint costs \$115.20 per 10 L can. Find the cost of paint for the tank.

14 How many spheres of 15 cm diameter can be painted with 1 L of glitter paint if the paint covers 10 m^2 ?

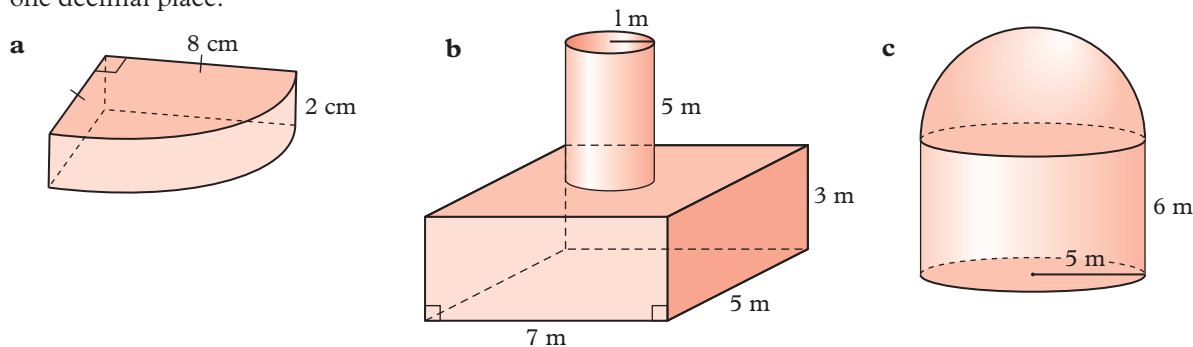
15 Cricket balls can be bought in packs of 12. Determine the total surface area of leather used to cover 20 packets of cricket balls each with diameter 7 cm.

16 Fabian has the choice of two paint rollers. One roller is 25 cm long and has a diameter of 8 cm. The other roller is 30 cm long and has a radius of 3 cm. Assuming the rollers have the same absorbency, which roller would need to be redipped in paint the least often?

17 A can of baked beans has a cylindrical shape with a base diameter of 7 cm and a height of 10 cm. A label is placed around the curved surface of the can, covering it completely. What percentage of the total surface area of the can is covered by the label?

18 If you double the height of a cylinder, do you double its surface area? Explain, using an example.

19 Calculate the total surface area of each of the following closed composite 3D objects to one decimal place.

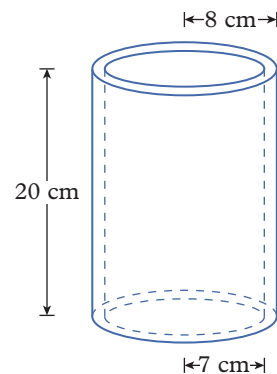


20 Consider the composite 3D object in question **19b**. Explain why you don't need to calculate the area of the circular face at the top of the cylinder when finding the surface area of the 3D object.



Challenge

- 21 A cylinder of 8 cm has a cylinder of 7 cm removed from the middle. Find the surface area of the remaining composite 3D object, correct to one decimal place.



Related resources



7.2 Quick quiz



Worksheet: Surface area of cylinders and spheres

Lesson 7.3

Volume of prisms, cylinders and spheres

Learning intentions

By the end of this lesson, you will be able to ...

- calculate and solve problems about the volume of prisms, cylinders and spheres.

Volume of prisms and cylinders

- The **volume** of a 3D object is the amount of space it occupies.
- A **cross-section** of a 3D object is the shape of a new face formed when the object is sliced by a plane.
- In a prism or a cylinder, any cross-section parallel to the base is exactly the same shape as the base. We call these uniform cross-sections.
- The volume of a prism or cylinder can be determined using:

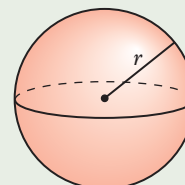
$$\text{Volume} = \text{area of base} \times \text{height}$$
 - The height of a prism or cylinder is measured perpendicular to the base.
 - The volume of a prism of base area A and height h is given by

$$V = Ah.$$
 - The volume of a cylinder of radius r and height h is given by

$$V = \pi r^2 h.$$

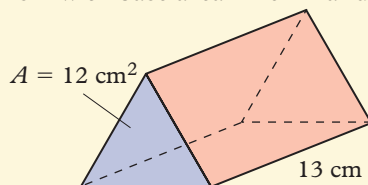
Volume of spheres

- The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.



Worked example 7.3A Finding the volume of a prism using base area and height

Find the volume of this triangular prism with base area 12 cm^2 and height of 13 cm .



Think

The area of the base is 12 cm^2 and the height is 13 cm .
Substitute these values into the formula $V = Ah$. Evaluate and write the result with the correct units.

Write

$$\begin{aligned} V &= Ah \\ V &= 12 \text{ cm}^2 \times 13 \text{ cm} \\ &= 156 \text{ cm}^3 \end{aligned}$$



Worked example video: Finding the volume of a prism using face area and height

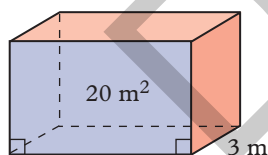
Understanding, fluency and communicating

Answers: Lesson 7.3

Helpful hints

- Remember that the units of volume are cubic like mm^3 , cm^3 or m^3 .
- Convert all lengths to the same unit before substituting them into the volume formula.

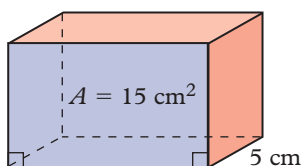
1 Complete the following to find the volume of this prism.



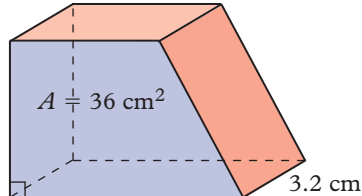
$$\begin{aligned} V &= A \times \underline{\hspace{1cm}} \text{ m} \\ &= \underline{\hspace{1cm}} \text{ m}^2 \times 3 \text{ m} \\ &= \underline{\hspace{1cm}} \text{ m}^3 \end{aligned}$$

2 Calculate the volume of each of the following prisms and cylinders, given the area of the base and the height.

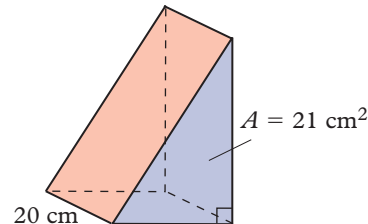
a



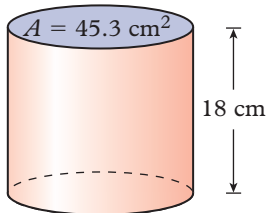
b



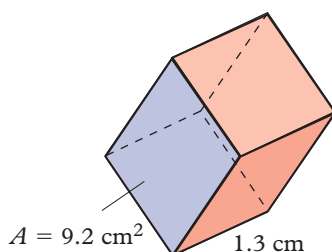
c



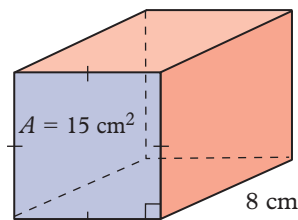
d



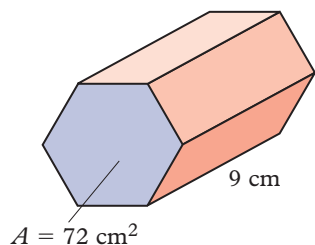
e



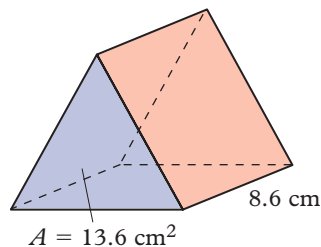
f



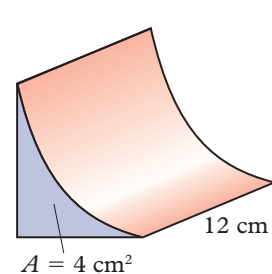
g



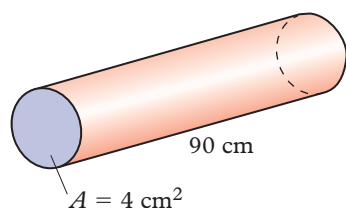
h



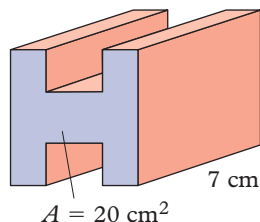
i



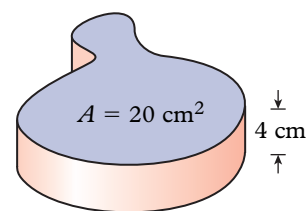
j



k



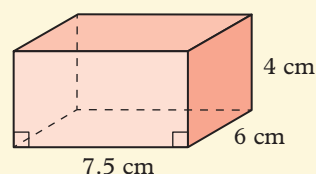
l



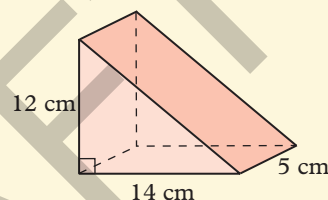
Worked example 7.3B Finding the volume of prisms

Find the volume of each of these prisms.

a



b



Part	Think	Write
a	Any of the faces of a rectangular prism can be chosen as the base. Take the face with length 7.5 cm and breadth 4 cm to be the base. The area of the base can be found using the formula $A = lb$. The height of the prism is 6 cm. The volume can be found using the formula $V = Ah$. Substitute the area of the base and the height into the volume formula. Evaluate and write the result using the correct units.	$A = lb$ $= 7.5 \text{ cm} \times 4 \text{ cm}$ $= 30 \text{ cm}^2$ $V = 30 \text{ cm}^2 \times 6 \text{ cm}$ $= 180 \text{ cm}^3$
b	The base of the triangular prism is a right-angled triangle with base 14 cm and height 12 cm. The area of the base of the prism can be found using the formula $A = \frac{1}{2}bh$. The height of the prism is 5 cm. The volume can be found using the formula $V = Ah$. Substitute the area of the base and the height into the volume formula. Evaluate and write the result using the correct units.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 14 \text{ cm} \times 12 \text{ cm}$ $= 84 \text{ cm}^2$ $V = Ah$ $= 84 \text{ cm}^2 \times 5 \text{ cm}$ $= 420 \text{ cm}^3$

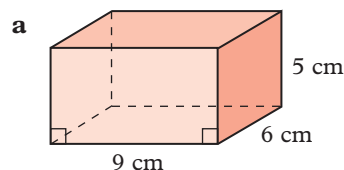


Worked example video: Finding the volume of prisms

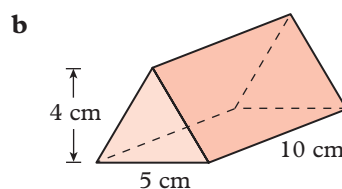
Helpful hints

- The volume of a rectangular prism can be found using the formula
 $V = \text{length} \times \text{breadth} \times \text{height}$.
 → This is equivalent to the formula $V = Ah$.

3 Fill in the blanks to find the volume of each of these prisms.

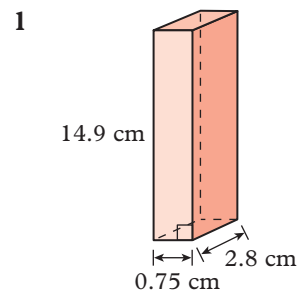
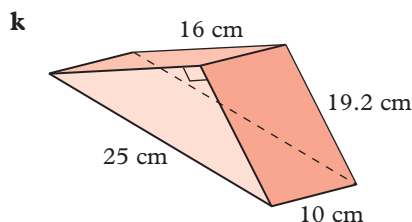
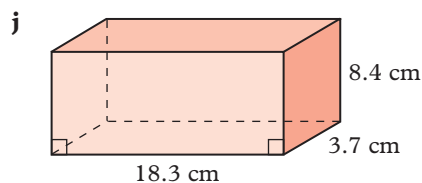
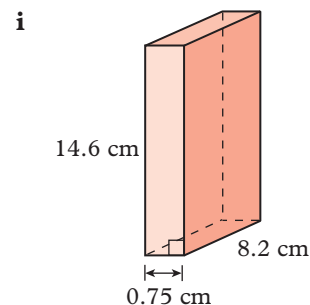
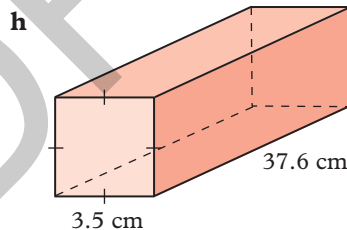
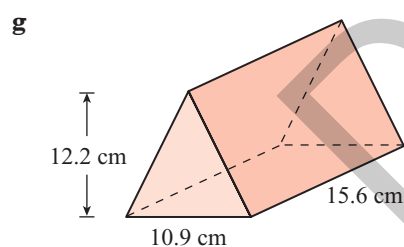
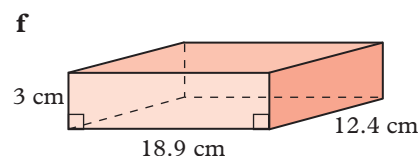
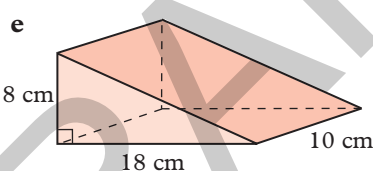
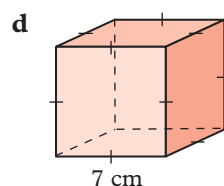
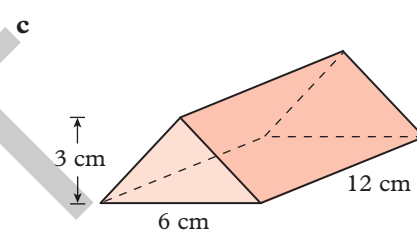
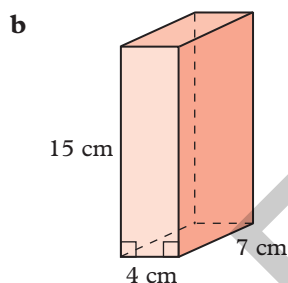
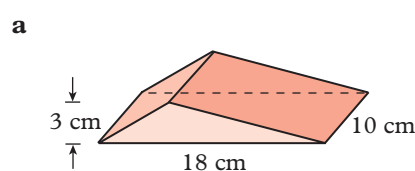


$$\begin{aligned} A &= lb \\ &= 9 \text{ cm} \times 5 \text{ cm} \\ &= \underline{\hspace{1cm}} \text{ cm}^2 \\ V &= Ah \\ &= \underline{\hspace{1cm}} \text{ cm}^2 \times 6 \text{ cm} \\ &= \underline{\hspace{1cm}} \text{ cm}^3 \end{aligned}$$



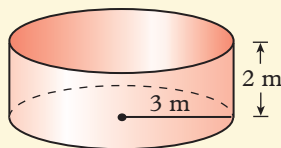
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times \underline{\hspace{1cm}} \text{ cm} \times \underline{\hspace{1cm}} \text{ cm} \\ &= \underline{\hspace{1cm}} \text{ cm}^2 \\ V &= Ah \\ &= \underline{\hspace{1cm}} \text{ cm}^2 \times 10 \text{ cm} \\ &= \underline{\hspace{1cm}} \text{ cm}^3 \end{aligned}$$

4 Calculate the volume of each prism.



Worked example 7.3C Calculating the volume of a cylinder

Calculate the volume of this cylinder, correct to two decimal places.

**Think**

The radius of the cylinder is 3 m and the height is 2 m. The volume of the cylinder can be found using the formula $V = \pi r^2 h$. Substitute the radius and height into the formula. Evaluate, round to two decimal places and write with the correct units.

Write

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (3 \text{ m})^2 \times 2 \text{ m} \\ &\approx 56.55 \text{ m}^3 \end{aligned}$$

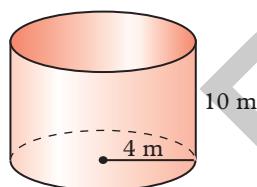


Worked example video: Calculating the volume of a cylinder

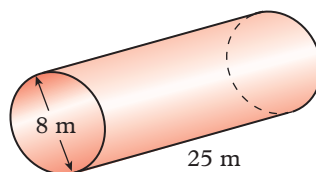
Helpful hints

- The formula for the volume of a cylinder comes from $V = Ah$. The base of a cylinder is a circle and the formula for the area of a circle is $A = \pi r^2$. Substituting this area formula into $V = Ah$ gives the formula $V = \pi r^2 h$.
- If the diameter of the cylinder is given, find the radius and use the volume of the cylinder formula.

5 Complete the following to find the volume of each of these cylinders, correct to two decimal places.

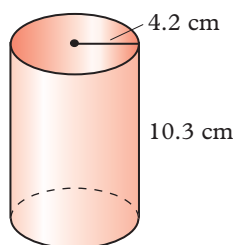
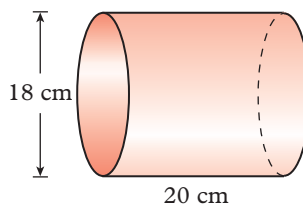
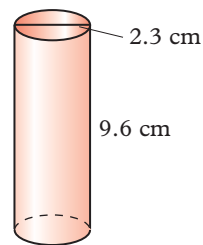
a

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (\text{ } \text{m})^2 \times \text{ } \text{m} \\ &\approx \text{ } \text{m}^3 \end{aligned}$$

b

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (\text{ } \text{m} \div 2)^2 \times \text{ } \text{m} \\ &= \pi \times (\text{ } \text{m})^2 \times \text{ } \text{m} \\ &\approx \text{ } \text{m}^3 \end{aligned}$$

6 Determine the volume of each of these cylinders, correct to the nearest cubic centimetre (cm^3).

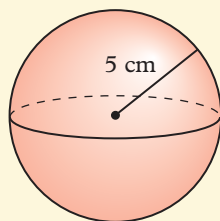
a**b****c**

7 Use the dimensions below to calculate the volume of each cylinder, correct to one decimal place.

a radius = 5 cm, height = 12 cm**b** diameter = 13 cm, height = 25 cm**c** radius = 6.4 cm, height = 16 cm**d** diameter = 9 cm, height = 40 cm**e** diameter = 37.4 cm, height = 100 cm**f** radius = 14.6 cm, height = 70 cm

Worked example 7.3D Finding the volume of a sphere

Find the volume of this sphere, correct to one decimal place.



Think

The radius of the sphere is 5 cm. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Substitute the radius into the volume formula. Evaluate, round to one decimal place and write the correct units.

Write

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times (5 \text{ cm})^3 \\ &= 523.598... \text{ cm}^3 \\ &\approx 523.6 \text{ cm}^3 \end{aligned}$$



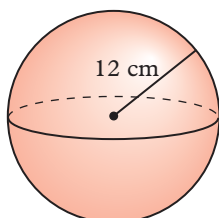
Worked example video: Finding the volume of a sphere

Helpful hints

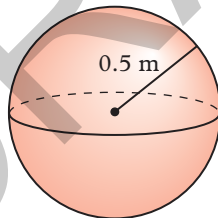
- If the diameter of the sphere is given, find the radius and then use the volume of the sphere formula.

8 Find the volume of each of these spheres, correct to one decimal place.

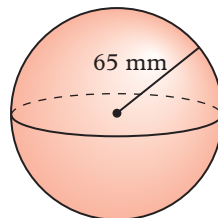
a



b

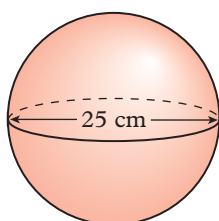


c

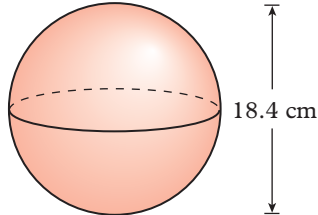


9 Find the volume of each of these spheres, correct to one decimal place.

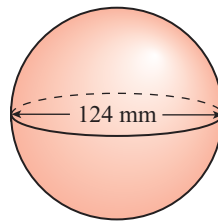
a



b

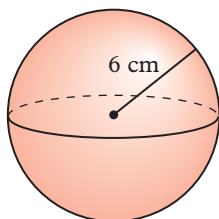


c

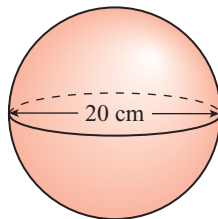


10 Find the volume of each of these spheres, correct to one decimal place.

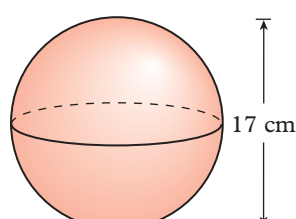
a



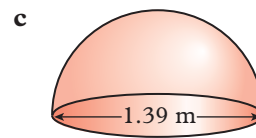
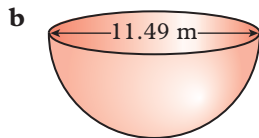
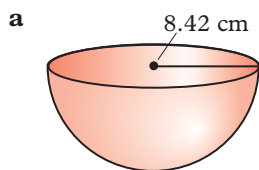
b



c



- 11** Recall that a hemisphere is half a sphere. Find the volume of each of these hemispheres, correct to two decimal places.



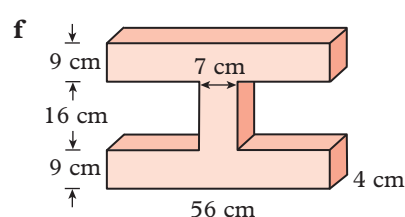
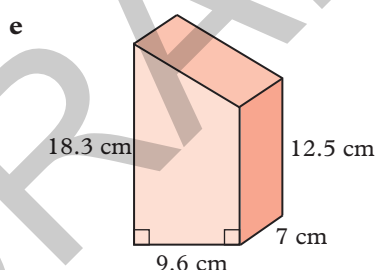
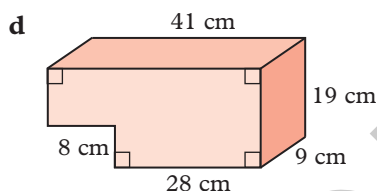
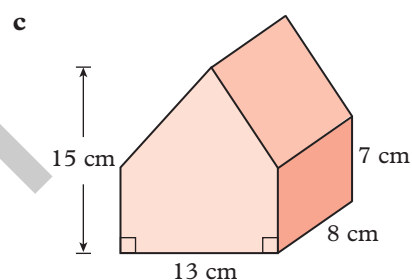
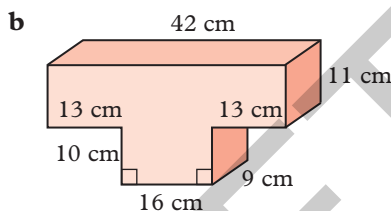
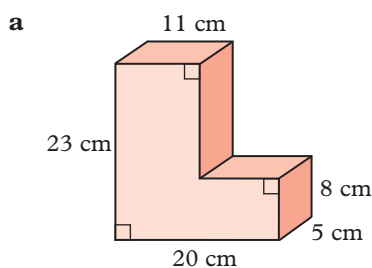
- 12** Use your answers to question 11 to write a formula for the volume of a hemisphere.

Problem solving and reasoning

- 13** Explain why the volume of a rectangular prism can be found using the formula $V = lbh$.

- 14** For each of the composite prisms below, calculate:

- the area of the base
- the volume.



- 15** Which has the greatest volume, a cylinder of radius 3 cm and height 4 cm or a cylinder of radius 4 cm and height 3 cm?

- 16** The formulas for the volumes of 3D objects can be used to solve for an unknown value.

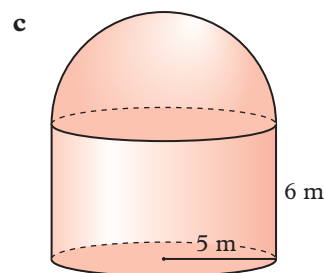
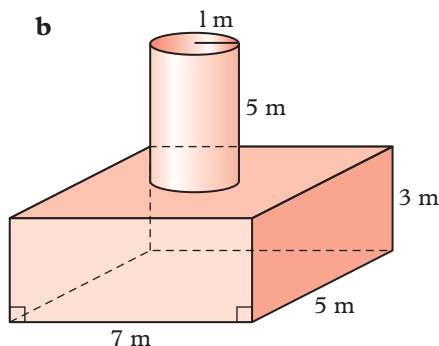
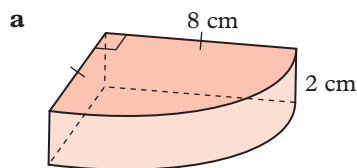
- Find the base area of a prism with height 10 cm and volume 280 cm^3 .
- Find the height of a rectangular prism with length 10 cm and breadth 8 cm if its volume is 560 cm^3 .
- Find the height of a cylinder with radius 8 cm and volume 120 cm^3 .

- 17** A sheet of cardboard 1000 mm by 800 mm has squares of side length 200 mm cut from each corner. The sides are folded up to form an open rectangular box. Calculate the volume of the box.

Hint: Draw a diagram of the cardboard sheet with the corners cut out.



18 Calculate the volume of each of the following composite 3D objects to one decimal place.



Challenge

19 A sphere of radius 1 m fits perfectly inside a cube, touching all faces. What percentage of the cube is filled by the sphere? Write your answer correct to one decimal place.

Related resources



7.3 Quick quiz



Investigation: Rubbish skips



Worksheet: Volume of prisms, cylinders and spheres

Lesson 7.4

Volume of pyramids and cones

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate and solve problems about the volume of pyramids, cones and composite solids.

Volume of pyramids and cones

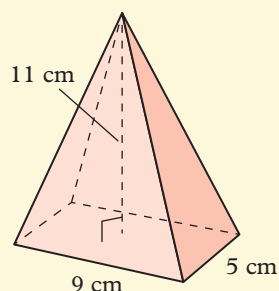
- A **pyramid** is a 3D object with a polygon at its base and all corners of the polygon connected via straight lines to a point called the apex. The shape of the base gives the pyramid its name (e.g. square-based pyramid).
 - All other faces of a pyramid are triangles.
 - Any cross-section parallel to the base is a smaller copy of the base polygon.
- A **cone** is much like a pyramid, except with a circle at its base.
- The volume of a pyramid or a cone is given by

$$\text{Volume} = \frac{1}{3} \times (\text{area of base} \times \text{height}),$$
- which is often written as $V = \frac{1}{3}Ah$.
 - Since the area of a circle is $A = \pi r^2$, the volume of a cone can be written as $V = \frac{1}{3}\pi r^2 h$.

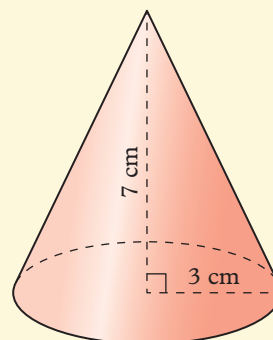
Worked example 7.4A Finding the volume of a pyramid or a cone

Find the volume of each 3D object.

a



b



Part	Think	Write
a	Volume = $\frac{1}{3} \times (\text{area of base} \times \text{height})$ The base is a rectangle, so its area is given by $5 \text{ cm} \times 9 \text{ cm}$. The height is 11 cm. Substitute these values into the formula and simplify.	$A = 5 \text{ cm} \times 9 \text{ cm} = 45 \text{ cm}^2$ $h = 11 \text{ cm}$ $V = \frac{1}{3} \times A \times h$ $= \frac{1}{3} \times 45 \text{ cm}^2 \times 11 \text{ cm}$ $= 165 \text{ cm}^3$
b	Volume = $\frac{1}{3} \times (\text{area of base} \times \text{height})$ Since the base is a circle, area of base = $\pi \times \text{radius}^2$ The radius is 3 cm and the height is 7 cm. Substitute these values into the formula and simplify.	$A = \pi \times (3 \text{ cm})^2 = 9\pi \text{ cm}^2$ $h = 7 \text{ cm}$ $V = \frac{1}{3} \times A \times h$ $= \frac{1}{3} \times 9\pi \text{ cm}^2 \times 7 \text{ cm}$ $= 21\pi \text{ cm}^3$ $\approx 65.97 \text{ cm}^3$



Worked example video: Finding the volume of a pyramid or a cone

Understanding, fluency and communicating

Answers: Lesson 7.4

Helpful hints

- When finding the volume of a pyramid or cone:
 - Find the area of the base first.
 - Don't forget to multiply by $\frac{1}{3}$!

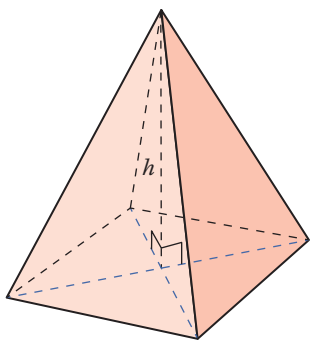
1 Match the following names of objects with the corresponding image below.

a triangular-based pyramid

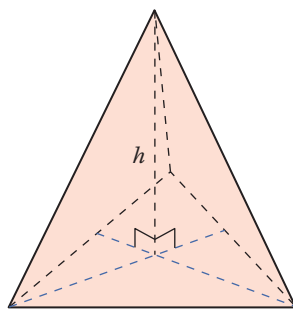
b cone

c square-based pyramid

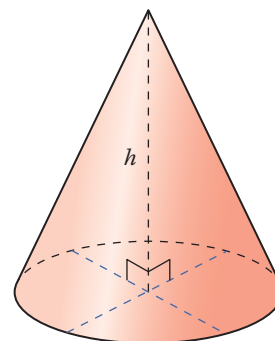
A



B

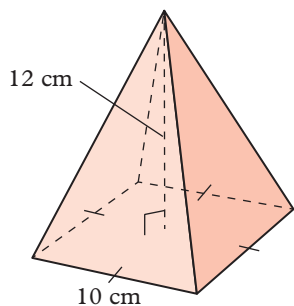


C

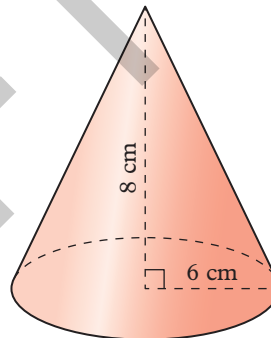


2 Fill in the blanks to determine the volume of the 3D objects shown.

a Volume = $\frac{1}{3} \times \text{area of base} \times \text{height}$
 $= \frac{1}{3} \times 10 \text{ cm} \times \underline{\hspace{1cm}} \text{ cm} \times 12 \text{ cm}$
 $= \underline{\hspace{1cm}} \text{ cm}^3$

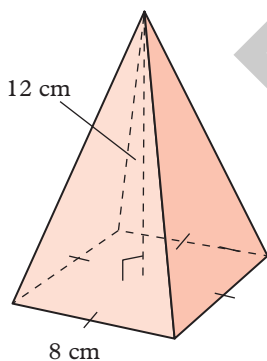


b Volume = $\frac{1}{3} \times \text{area of base} \times \text{height}$
 $= \frac{1}{3} \times \pi \times \underline{\hspace{1cm}} \text{ cm}^2 \times \underline{\hspace{1cm}} \text{ cm}$
 $= \underline{\hspace{1cm}} \pi \text{ cm}^3$
 $\approx \underline{\hspace{1cm}} \text{ cm}^3$

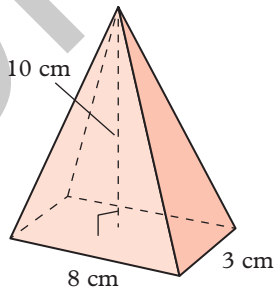


3 Find the volume of each 3D object, correct to one decimal place where necessary.

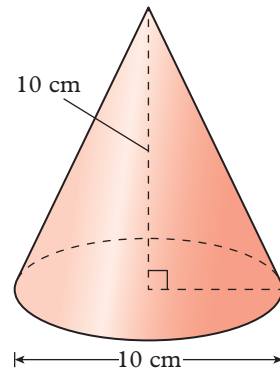
a



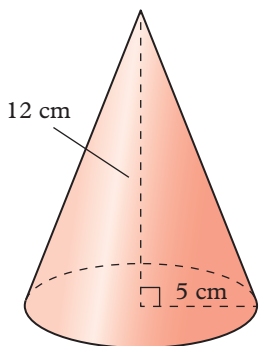
b



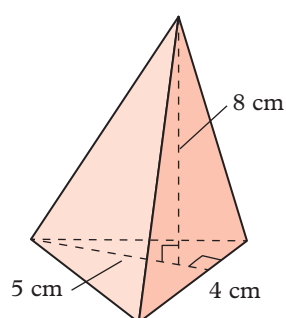
c



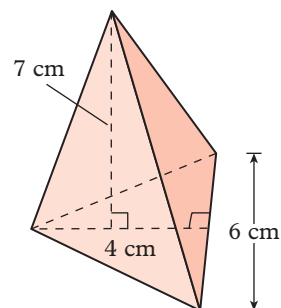
d



e



f

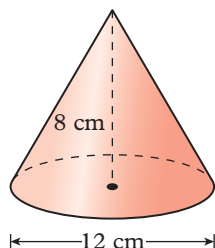


4 Find the volume of the following 3D objects, correct to one decimal place where necessary.

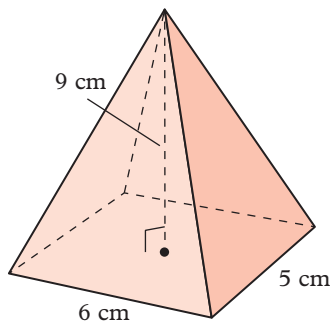
- a a pyramid of rectangular base with base 5 cm wide and 4 cm deep, and height 6 cm
- b a cone with base radius 8 cm and height 12 cm
- c a square-based pyramid of height 12 cm and base edges 6 cm

5 Find the volume of each 3D object to the nearest whole number.

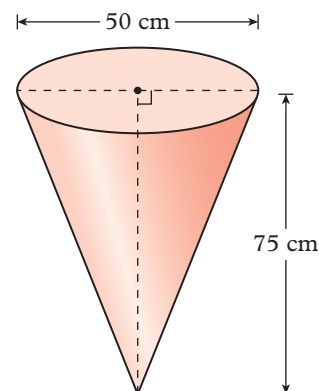
a



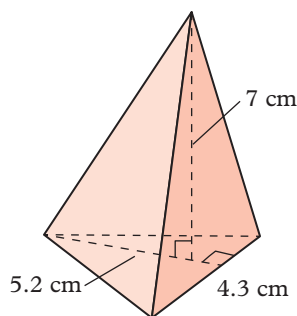
b



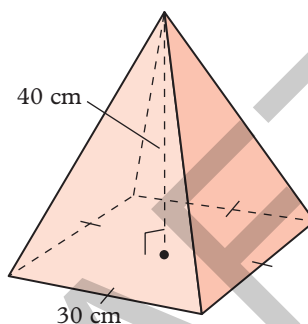
c



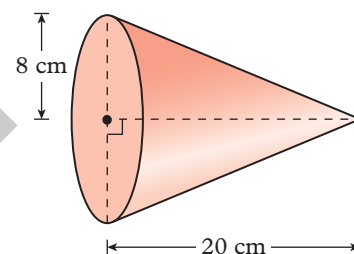
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e



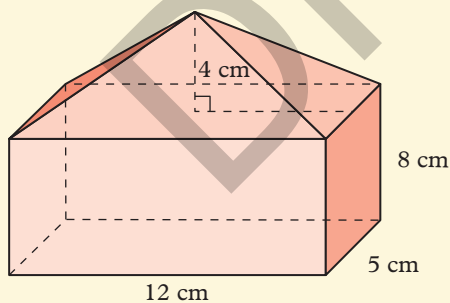
f



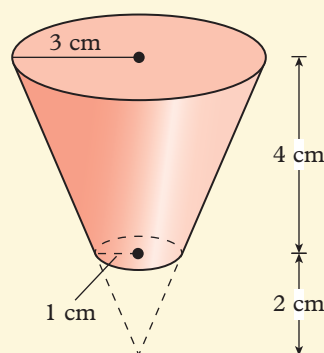
Worked example 7.4B Finding the volume of composite shapes that include pyramids and cones

Find the volume of each 3D object.

a



b



Part	Think	Write
a	Total volume = (volume of rectangular prism) + (volume of pyramid) The rectangular prism has volume $V_{\text{prism}} = \text{base} \times \text{breadth} \times \text{height}$ The pyramid has volume $V_{\text{pyramid}} = \frac{1}{3} \times \text{area of base} \times \text{height}$ Substitute in all known values for each part separately, then add the volumes together.	$V_{\text{prism}} = 12 \text{ cm} \times 5 \text{ cm} \times 8 \text{ cm}$ $= 480 \text{ cm}^3$ $V_{\text{pyramid}} = \frac{1}{3} \times \text{area of base} \times \text{height}$ $= \frac{1}{3} \times (12 \text{ cm} \times 5 \text{ cm}) \times 4 \text{ cm}$ $= 80 \text{ cm}^3$ $V_{\text{total}} = V_{\text{prism}} + V_{\text{pyramid}}$ $= 480 \text{ cm}^3 + 80 \text{ cm}^3$ $= 560 \text{ cm}^3$

Part	Think	Write
b	Total volume = (volume of whole cone) – (volume of removed cone) If the tip had not been removed, the whole cone would have height $4 + 2 = 6$ cm. area of base = $\pi \times \text{radius}^2$ The radius is 3 cm, so we can find the volume of the whole cone V_{whole}. The piece that has been removed is a cone of height 2 cm with a base whose radius is 1 cm. Substitute these values in for V_{removed}. Finally, find $V_{\text{total}} = V_{\text{whole}} - V_{\text{removed}}$ by substituting in both volumes.	$V_{\text{whole}} = \frac{1}{3} \times (\pi \times (3 \text{ cm})^2) \times 6 \text{ cm}$ $= 18\pi \text{ cm}^3$ $V_{\text{removed}} = \frac{1}{3} \times (\pi \times (1 \text{ cm})^2) \times 2 \text{ cm}$ $= \frac{2}{3}\pi \text{ cm}^3$ $V_{\text{total}} = V_{\text{whole}} - V_{\text{removed}}$ $= 18\pi \text{ cm}^3 - \frac{2}{3}\pi \text{ cm}^3$ $\approx 54.45 \text{ cm}^3$

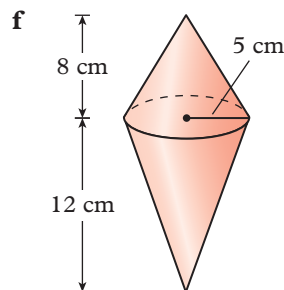
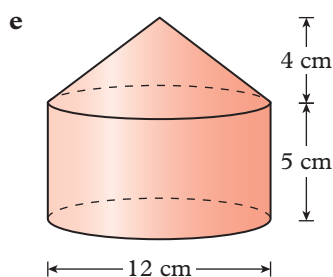
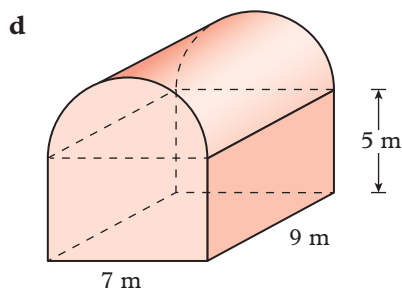
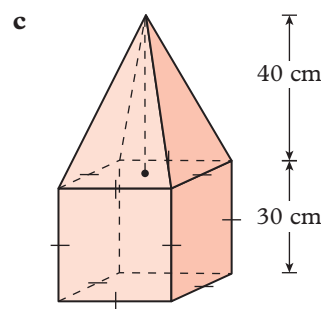
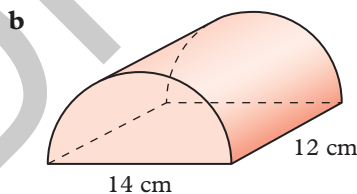
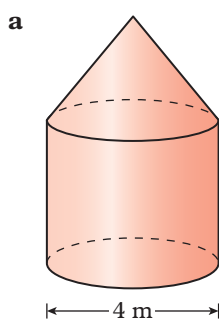


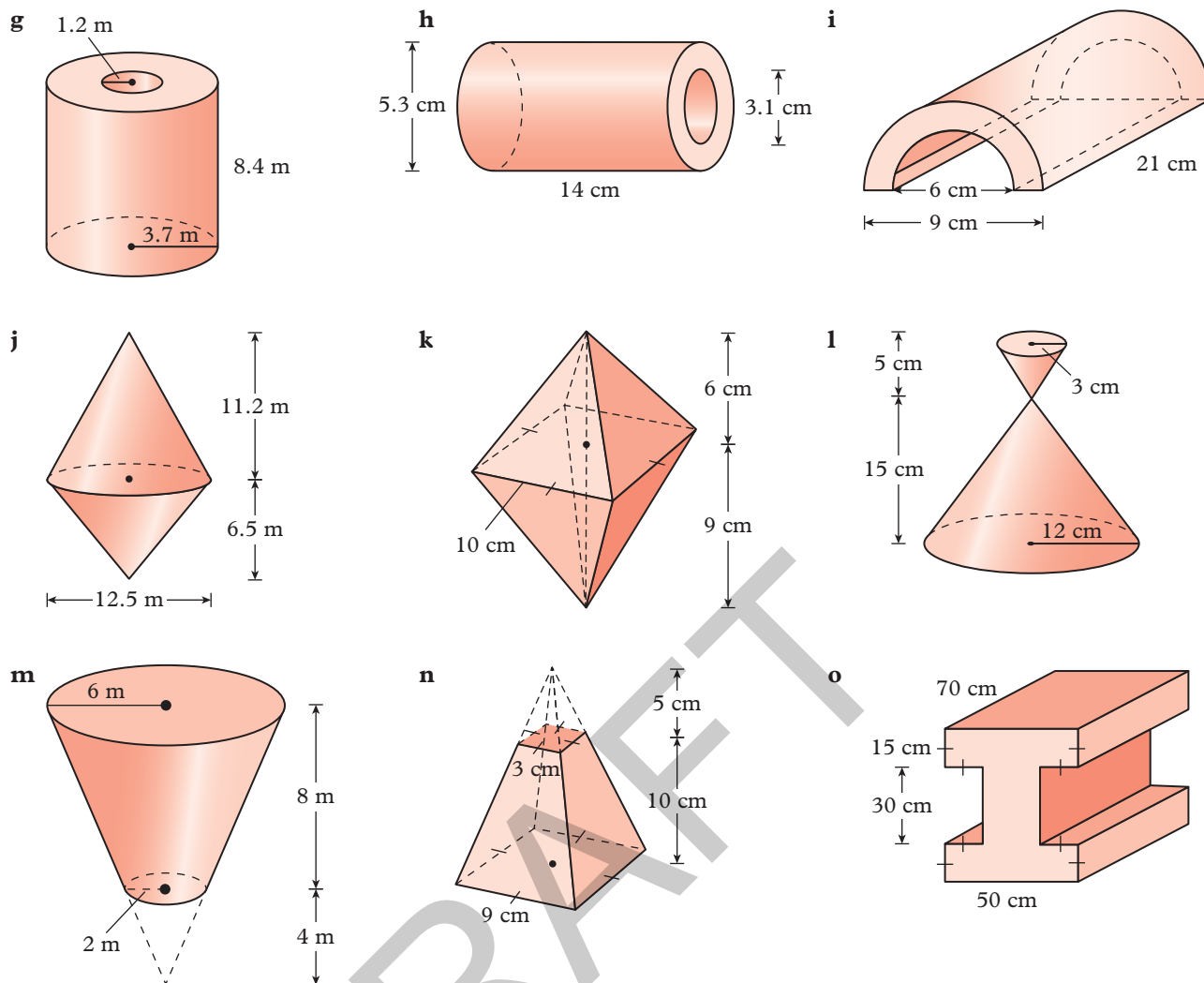
Worked example video: Finding the volume of composite shapes that include pyramids and cones

Helpful hints

- Sometimes it is easier to think of a composite 3D object as multiple separate pieces being glued together, in which case, calculate the volume of each piece and sum them together.
- In other cases, it makes more sense to see a composite 3D object as a whole object that has had a piece removed. In that case, find the volume of the whole piece and the removed piece then subtract the removed volume from the whole volume.

6 Find the volume of each composite 3D object, correct to one decimal place where necessary.





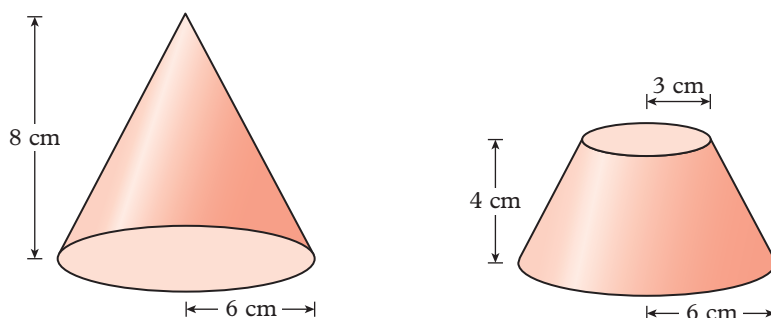
Problem solving and reasoning

- 7 The Great Pyramid of Giza is the largest pyramid in Egypt. It has a square base with side lengths of approximately 230 m, and height approximately equal to 140 m. What is the approximate volume of this pyramid in cubic metres?
- 8 Suppose an ice-cream cone has a height of 10 cm and a radius of 3 cm.
 - a What is the volume of ice cream that would fit inside this ice-cream cone, in cubic centimetres to two decimal places?
 - b If more ice cream can be added on top in the shape of half a sphere, also with radius 3 cm, what is the total volume of ice cream you can have in this cone?

Challenge

- 9 A cone has a radius of 6 cm and a perpendicular height of 8 cm. It is cut halfway along the perpendicular height, and the top half is taken away.

Find the volume of the remaining solid, to four significant figures.



Related resources



7.4 Quick quiz

Lesson 7.5

Capacity

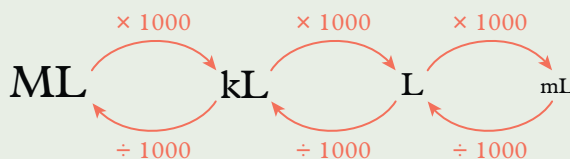
Learning intentions

By the end of this lesson, you will be able to ...

→ solve problems involving capacity.

Capacity

- The **capacity** of a container is the amount of fluid that it can contain. The common units of capacity are millilitre (mL), litre (L), kilolitre (kL) and megalitre (ML).



- The capacity of a container can be found using its volume.
- Capacity and volume are related by the following conversions.
 - $1 \text{ mL} = 1 \text{ cm}^3$
 - $1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$
 - $1 \text{ kL} = 1000 \text{ L} = 1 \text{ m}^3$

Worked example 7.5A Converting between units of volume and capacity

Complete the following conversions between units of volume and capacity.

- a** Calculate the volume of space taken up by 5.2 L of liquid. Answer in cubic centimetres (cm^3).
b Calculate the capacity of a container with volume 1800 cm^3 . Answer in litres (L).

Part	Think	Write
a	The capacity is 5.2 L. To convert to units of volume, use the conversion $1 \text{ L} = 1000 \text{ cm}^3$.	$5.2 \text{ L} = 5.2 \times 1000 \text{ cm}^3$ $= 5200 \text{ cm}^3$ So, the volume of space taken up is 5200 cm^3 .
b	The volume is 1800 cm^3 . To convert to units of capacity, first use the conversion $1 \text{ cm}^3 = 1 \text{ mL}$ to convert to millilitres. Then use $1 \text{ mL} = \frac{1}{1000} \text{ L}$ to convert to litres.	$1800 \text{ cm}^3 = 1800 \times 1 \text{ mL}$ $= 1800 \text{ mL}$ $= 1800 \times \frac{1}{1000} \text{ L}$ $= 1.8 \text{ L}$ So, the capacity of the container is 1.8 L.



Worked example video: Converting between units of volume and capacity

Understanding, fluency and communicating

Answers: Lesson 7.5

Helpful hints

- When converting to a smaller unit of measurement, multiply by the conversion factor. When converting to a larger unit of measurement, divide by the conversion factor.

1 Complete the following to find:

- a** the volume, in cubic centimetres, of 6.5 L of liquid

$$\text{Capacity} = 6.5 \text{ L}$$

$$\begin{aligned} \text{Volume} &= 6.5 \times \underline{\hspace{1cm}} \text{ cm}^3 \\ &= \underline{\hspace{1cm}} \text{ cm}^3 \end{aligned}$$

- b** the capacity, in litres, with volume $25\,600 \text{ cm}^3$

$$\begin{aligned} \text{Volume} &= 25\,600 \text{ cm}^3 \\ \text{Capacity} &= 25\,600 \times \underline{\hspace{1cm}} \text{ mL} \\ &= 25\,600 \text{ mL} \\ &= 25\,600 \times \frac{1}{\square} \text{ L} \\ &= \underline{\hspace{1cm}} \text{ L} \end{aligned}$$

2 Find, in cubic centimetres, the space occupied by the following amounts of liquid.

a 2 L

b 12 L

c 2.6 L

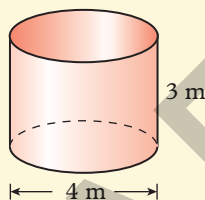
d 450 mL

e 975 mL

- 3 When completely submerged, various solids displaced the following amounts of water. Calculate the volume of each solid in cubic centimetres.
- a 650 mL b 7.5 L c 1.25 L d 3.78 L e 0.845 L
- 4 Calculate the capacity, in millilitres, of containers with the following volumes.
- a 1495 cm³ b 3000 cm³ c 500 cm³ d 24 000 cm³ e 6740 cm³
- 5 Write each of your answers for question 4 in litres.
- 6 Use the conversion 1 m³ = 1000 L to calculate the space occupied by the following amounts of liquid.
- a 2000 L b 5 kL c 6.7 kL d 0.9 kL e 83.200 L
- 7 Convert each of the following to kilolitres, using 1 m³ = 1 kL.
- a 0.02 m³ b 0.7 m³ c 28.4 m³ d 302 m³ e 5660 m³

Worked example 7.5B Finding the volume and capacity of an object

Find the volume in cubic metres and capacity in kilolitres, correct to two decimal places, of a cylindrical rainwater tank with height 3 m and diameter 4 m.



Think

Find the radius of the cylindrical tank using $d = 2r$.
 Find the volume of the cylindrical tank in m³ using $V = \pi r^2 h$.
 Use 1 kL = 1 m³ to find the capacity of the tank.

Write

$$\begin{aligned}
 4 \text{ m} &= 2r \\
 r &= 2 \text{ m} \\
 V &= \pi r^2 h \\
 &= \pi \times (2 \text{ m})^2 \times 3 \text{ m} \\
 &\approx 37.70 \text{ m}^3 \\
 \text{Capacity} &\approx 37.70 \times 1 \text{ kL} \\
 &\approx 37.70 \text{ kL}
 \end{aligned}$$

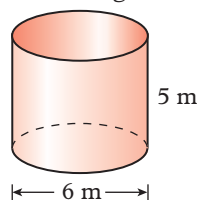


Worked example video: Finding the volume and capacity of an object

Helpful hints

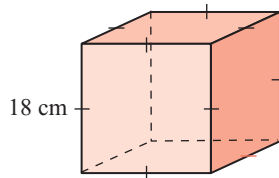
- When finding the capacity of a cylinder or prism, calculate the volume first, using $V = Ah$, then convert to capacity using an appropriate conversion.

- 8 Complete the following to find the volume in cubic metres and capacity in kilolitres, correct to one decimal place, of a cylindrical tank with diameter 6 m and height 5 m.

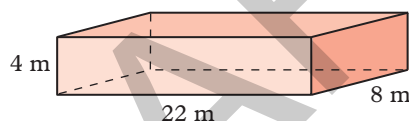


$$\begin{aligned}
 V &= \pi r^2 h \\
 &= \pi \times (\text{ } \text{m})^2 \times \text{ } \text{m} \\
 &\approx \text{ } \text{m}^3 \\
 \text{Capacity} &= \text{ } \text{kL}
 \end{aligned}$$

- 9 For the cube shown, find:



- the volume in cubic centimetres
 - the capacity in millilitres
 - the capacity in litres.
- 10 A rectangular prism measures 22 m by 4 m by 8 m. Find:



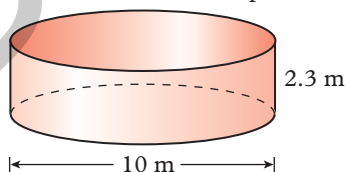
- its volume in cubic centimetres
 - its capacity in millilitres
 - its capacity in litres
 - its capacity in kilolitres.
- Find the capacity of each of the objects in question 6 in Lesson 7.4. Write your answers in litres.
 - How much water can a cylindrical bottle cap hold if it has a diameter of 2.8 cm and a height of 1.1 cm? Write your answer in millilitres correct to two decimal places.
 - A hemispherical bowl has a diameter of 24 cm. What is its capacity in litres correct to two decimal places?
 - Find the capacity, in megalitres, of a dam that has a cross-sectional area of 5000 m² and average depth of 8 m.
 - Aileen has two cylindrical vases. The first vase has a radius of 5 cm and a height of 10 cm. The second vase has a diameter of 6.5 cm and a height of 22 cm. Which vase holds more, and by how much? Write your answers correct to the nearest millilitre.

- 16** A rectangular fish tank is 50 cm long, 20 cm wide and 60 cm tall.
- a** Calculate the volume of the fish tank.
 - b** What is the capacity of the fish tank if it is filled to the brim (the top)? Give your answer in:
 - i** millilitres
 - ii** litres.
 - c** The fish tank can only be filled to a line 6 cm from the top. How many litres of water will be required to fill the tank to this height?



Problem solving and reasoning

- 17** A water trough in the shape of half a cylinder (i.e. the base is half a circle) has radius 40 cm and length 6 m. What is the capacity of the trough, in litres, when it is full?
- 18** The internal dimensions of a refrigerator are height 1.8 m by breadth 84 cm and depth 60 cm. Find:
- a** the internal volume in cubic centimetres (cm^3)
 - b** the capacity in litres.
- 19** A circular swimming pool has a diameter of 10 m and a depth of 2.3 m.



- a** Calculate its volume to the nearest cubic metre (m^3).
 - b** Calculate the number of kilolitres of water required to fill the pool.
- 20** Ten oranges, assumed to be spherical, have a uniform diameter of 6 cm. If 65% of each orange is made up of juice, how much juice could be squeezed from these 10 oranges? Give your answer to the nearest millilitre.
- 21** Suppose a balloon forms a perfect sphere when it is blown up.
- a** What would be the capacity of a balloon of diameter 26 cm? Give your answer in litres.
 - b** By taking a large breath, a teenage boy can expel 1.2 L of air from his lungs. How many of these breaths would be needed to fill the balloon to a diameter of 26 cm?

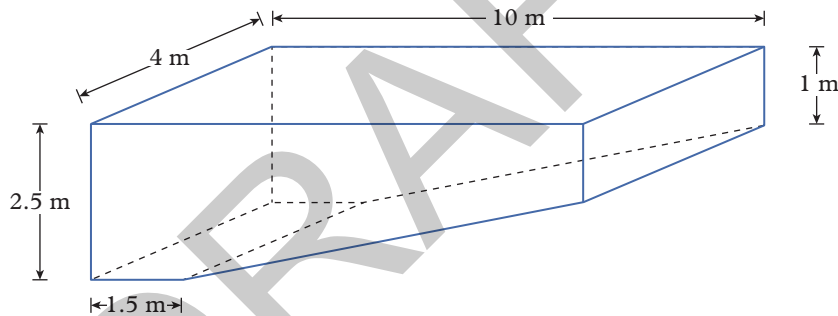
- 22** Suppose the capacity of a cylindrical water tank with radius 10 m is 4000 kL. Calculate the height of the tank to the nearest metre.
- 23** If the fuel tank of a car has a capacity of 60 L, how big is the tank? Suggest some possible dimensions if the tank is in the shape of a rectangular prism.

Challenge

- 24** Suppose the water drops falling from a tap are spheres of diameter 0.4 cm. If 15 drops per minute leak from the tap, how much water is wasted in one day?



- 25** Consider the following design for an in-ground backyard pool.



- a** Find the capacity, in kL, of the pool.
- b** The cost of water is \$2.67 per kilolitre. How much would it cost to fill the pool once? Write your answer to the nearest cent.
- c** Instead of filling the pool to the top, there will be a 15 cm gap between the surface of the water and the top of the pool. How much will the pool owners save when filling the pool compared to filling it to the top?

Related resources



7.5 Quick quiz



Worksheet: Volume and capacity of prisms and cylinders



Investigation: Building a vegetable garden



Investigation: Capacity measurements in cooking and medicine

Lesson 7.6

Review: Surface area, volume and capacity

Review questions 7.6A Mathematical literacy review

Answers: Lesson 7.6

Key terms

The following key terms are used in this module:

- capacity
- cone
- cross-section
- cylinder
- net
- polygon
- prism
- pyramid
- surface area
- volume

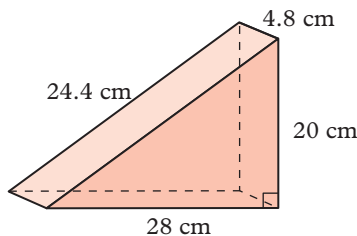
Check your understanding

- Complete the following sentences using words from the key term list.
 - The ____ of an object is how much space it occupies. The ____ of an object is how much fluid it can contain.
 - The ____ of an object is the sum of the areas of all its faces. It can be found by drawing a ____ of the object and labelling all dimensions.
- Why is a **cylinder** not a **prism**?
- Why can the formula $V = Ah$ be used to calculate the volume of a **prism** or **cylinder**, but not a sphere, **pyramid** or **cone**?
- Draw an example of each of the following 3D objects.
 - cone** with base diameter 10 cm and height 12 cm
 - triangular **prism** with height 20 cm and where the triangular face has height 5 cm and base 8 cm
 - square-based **pyramid** where the base side length is 1 m and the apex sits 60 cm above the base

Review questions 7.6B Multiple-choice questions

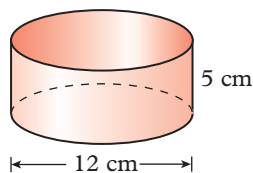
- The surface area of a rectangular prism with length 12 cm, breadth 8 cm and height 5 cm is closest to:

A 480 cm² **B** 392 cm² **C** 296 cm² **D** 196 cm²
- What is the surface area of this triangular prism?

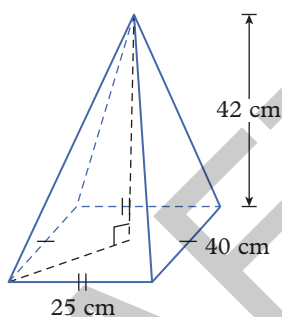


- A** 627 cm² **B** 772 cm² **C** 811 cm² **D** 908 cm²

- 3 The total surface area of a closed cylinder of radius 5 cm and height 12 cm is closest to:
A 377 cm² **B** 456 cm² **C** 534 cm² **D** 603 cm²
- 4 The surface area of a sphere with diameter 17.8 cm, correct to two decimal places, is:
A 995.38 cm² **B** 3981.52 cm² **C** 3981.53 cm² **D** 2952.97 cm²
- 5 The volume of this cylinder is closest to:



- A** 565 cm³ **B** 266 cm³ **C** 188 cm³ **D** 259 cm³
- 6 The volume of a sphere with diameter 8 m is closest to:
A 67.0 m³ **B** 804.2 m³ **C** 268.1 m³ **D** 2144.7 m³
- 7 What is the volume of the following pyramid?



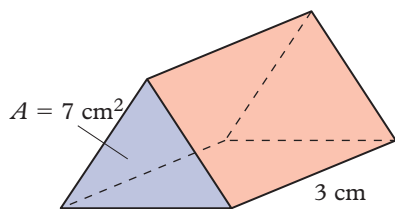
- A** 14 000 cm³ **B** 42 000 cm³ **C** 78 540 cm³ **D** 26 180 cm³
- 8 The volume of a cone with base diameter 15 cm and height 12 cm is closest to:
A 2827 cm³ **B** 2262 cm³ **C** 565 cm³ **D** 707 cm³
- 9 The capacity, in kilolitres, of a cubic tank with side length 8 m is closest to:
A 64 kL **B** 512 kL **C** 640 kL **D** 512 000 kL

Review questions 7.6C Set 1

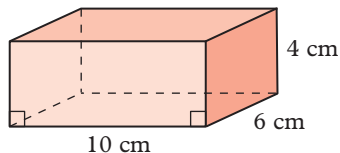
- 1 Calculate the surface area of each of these 3D objects.
- a** a rectangular prism with length 8 m, breadth 4 m and height 3 m
 - b** a closed cylinder with radius 8 cm and height 12 cm
 - c** a sphere of radius 7.5 cm

2 Calculate the volume of each of these 3D objects, correct to one decimal place where necessary.

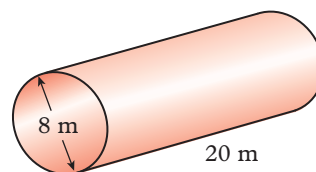
a



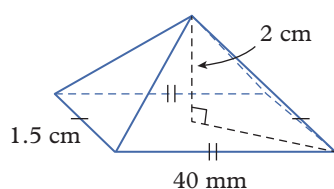
b



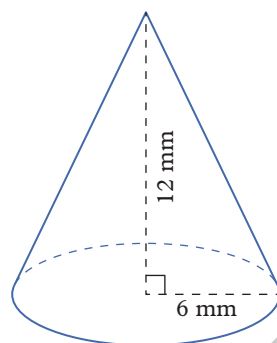
c



d



e



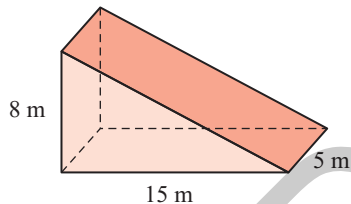
3 Calculate the volume of a sphere with radius 26 mm, to the nearest cubic millimetre (mm^3).

4 Find the capacity in millilitres of a cylinder with radius 7 cm and perpendicular height 14 cm.

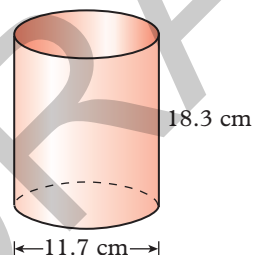
Review questions 7.6D Set 2

1 Find the total surface area of each of these 3D objects, correct to one decimal place where necessary.

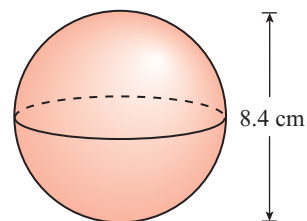
a



b



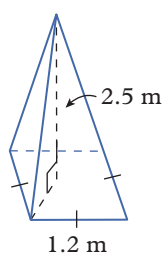
c



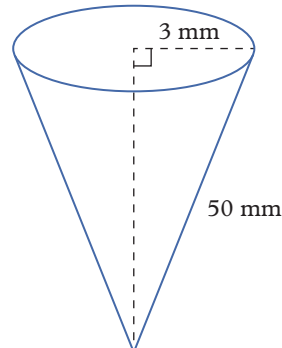
2 Calculate the volume of each 3D object in question 1, correct to one decimal place where necessary.

3 Calculate the volume of the following 3D objects, correct to one decimal place where necessary.

a



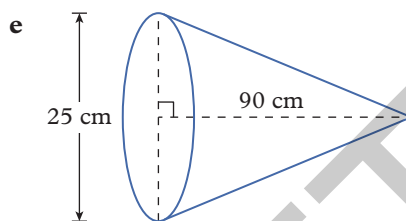
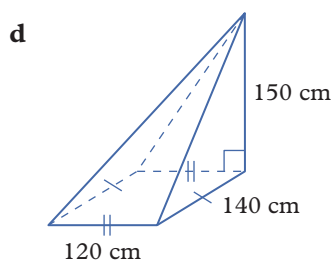
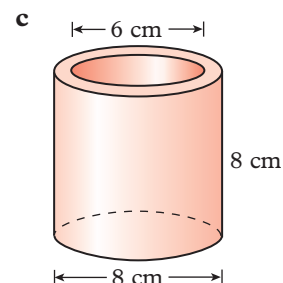
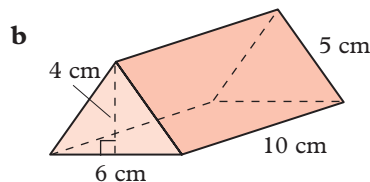
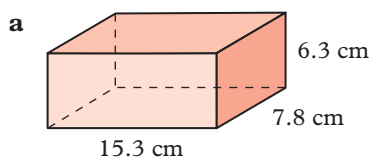
b



4 Find the capacity, in megalitres, of a dam with cross-sectional area of 6000 m^2 and an average depth of 10 m.

Review questions 7.6E Set 3

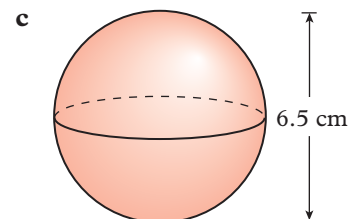
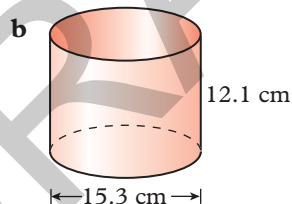
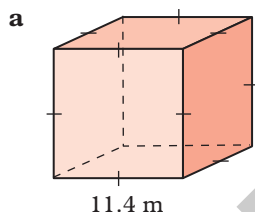
- Calculate the surface area, correct to one decimal place, of:
 - a cube of length 7.2 mm
 - a closed cylinder with diameter 17 cm and height 8 cm
 - a sphere of radius 13 mm.
- Calculate the volume of each of these 3D objects, correct to one decimal place where necessary.



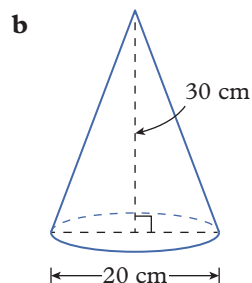
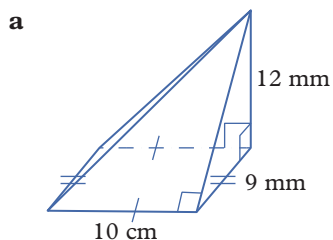
- Find the capacity, in kilolitres, of a spherical tank with diameter 8 m.

Review questions 7.6F Set 4

- Find the total surface area of each of these 3D objects, correct to one decimal place.



- Calculate the volume of each 3D object in question 1, to one decimal place.
- Calculate the capacity of each 3D object in question 1 in litres.
- Calculate the volume of the following 3D objects.

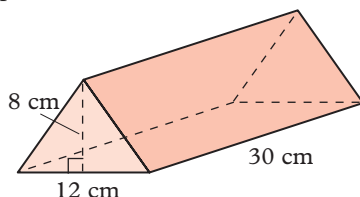


- Find the capacity, in kilolitres, of a rectangular tank with length 8 m, breadth 6 m and height 4 m.

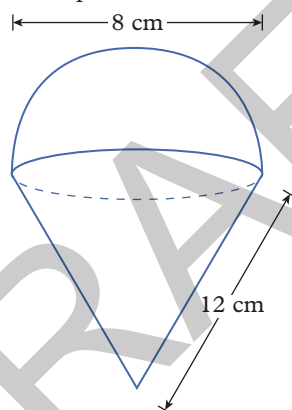
Review questions 7.6G Practice examination question

1 Answer the following questions.

- a Calculate the volume of a cone of diameter 11 cm and height 12 cm, correct to two decimal places. **(2 marks)**
- b Consider a closed cylinder with equal height and radius and a sphere with the same radius.
 - i Show that the cylinder and sphere have the same surface area. **(2 marks)**
 - ii Which object has greater volume? Justify your answer. **(2 marks)**
- c Consider the following triangular prism.



- i Find the volume of the triangular prism. **(2 marks)**
 - ii Find the capacity of the triangular prism in litres. **(1 mark)**
- d An ice-cream cone can be modelled using an inverted cone and a hemisphere as shown in the following diagram. The hemisphere represents a scoop of ice cream sitting on top of the cone.



- i Find the volume of the hemisphere in the model. **(1 mark)**
- ii Use Pythagoras' theorem to find the height of the cone. **(2 marks)**
- iii Find the volume of the cone. **(1 mark)**
- iv Find the capacity of the cone and scoop of ice cream. **(2 marks)**

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Surface area, volume and capacity

I can do this	I need to review this
☐ calculate and solve problems about the surface area of prisms	Go back to Lesson 7.1 Surface area of prisms
☐ calculate and solve problems about the surface area of cylinders and spheres ☐ calculate and solve problems about the surface area of composite solids	Go back to Lesson 7.2 Surface area of cylinders and spheres
☐ calculate and solve problems about the volume of prisms, cylinders, spheres and composite solids	Go back to Lesson 7.3 Volume of prisms, cylinders and spheres
☐ calculate and solve problems about the volume of pyramids and cones ☐ calculate and solve problems about the volume of composite solids	Go back to Lesson 7.4 Volume of pyramids and cones
☐ convert between units of volume and capacity ☐ solve problems involving capacity	Go back to Lesson 7.5 Capacity

Related resources



Module review quiz: Surface area, volume and capacity



Worksheet: Challenge 7

Module

8

**Time and
location**

DRAFT

Lessons

- Lesson 8.1 Latitude and longitude
- Lesson 8.2 Position on Earth's surface
- Lesson 8.3 Time units and intervals
- Lesson 8.4 World time zones
- Lesson 8.5 Australian time zones
- Lesson 8.6 Review: Time and location

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Time and location

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Angles at a point



Support sheet: The Cartesian plane



Support sheet: Understanding a directional compass



Support sheet: Converting units of time



Support sheet: Understanding 24-hour time



Support sheet: Time calculations



Support sheet: Differences in time

Syllabus links

- Time and location (MST-11-06)
 - A student solves problems involving time and location in practical contexts.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 8.1

Latitude and longitude

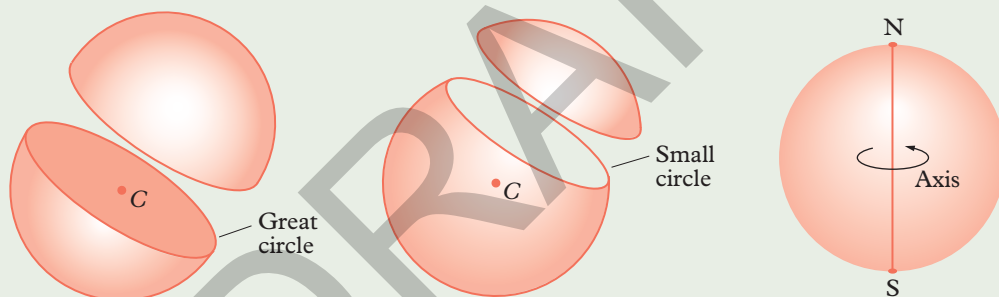
Learning intentions

By the end of this lesson, you will be able to ...

→ identify and describe parallels of latitude and meridians of longitude.

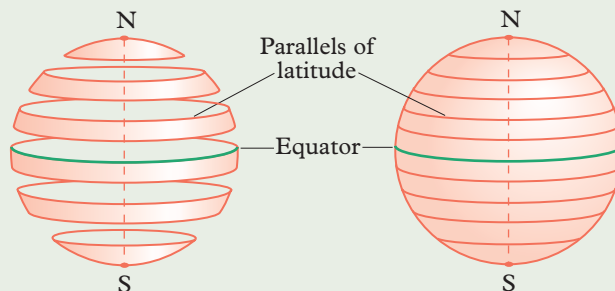
Earth as a sphere

- Earth's surface is essentially a sphere, with radius approximately equal to 6400 kilometres.
- A **great circle** on Earth's surface is any circle formed by slicing the planet in half using a plane that passes through its centre. Great circles appear as the largest possible cross-sections and, as such, have the same radius as the radius of Earth.
- A **small circle** on Earth's surface is any circle formed by slicing through the planet with a plane that does not pass through its centre. The radius of a small circle on Earth's surface is less than the radius of Earth.
- Earth's **axis of rotation** passes through the North and South Poles.



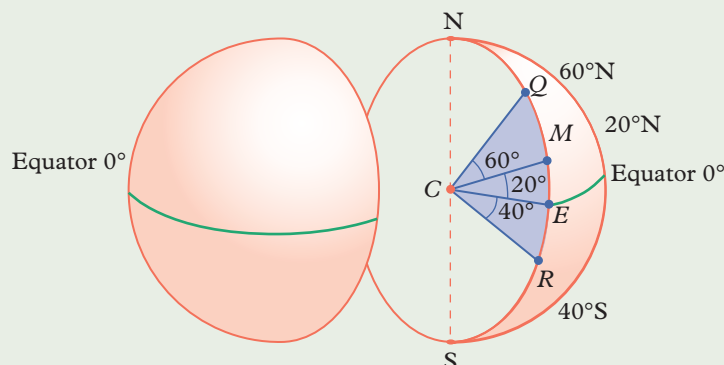
Latitude

- Any plane that slices through Earth, perpendicular to its axis of rotation, forms a **parallel (or circle) of latitude** on Earth's surface.



- The unique parallel of latitude whose centre is the centre of Earth is a great circle called the **equator**.
- All other parallels of latitude are small circles on Earth's surface. They are labelled according to whether they are north or south of the equator and according to the size of an angle made at the centre of Earth.

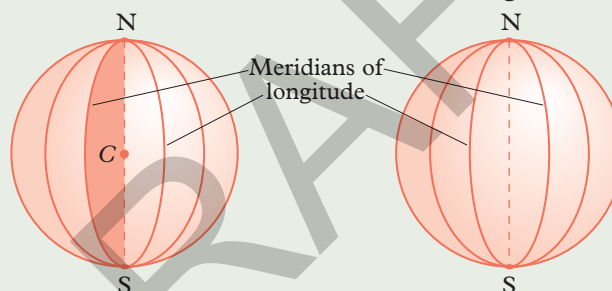
- The diagram below demonstrates how **latitude** is measured. The equator is used as a reference, so the latitude of the point E is defined to be 0° . The latitude of the point M is determined by noticing that M is north of E , then finding the angle ECM , which is 20° . We write the latitude of M as 20°N , which we read as “twenty degrees north”.



- The latitude of the point Q is 60°N , and the latitude of the point R is 40°S (read as “forty degrees south”).

Longitude

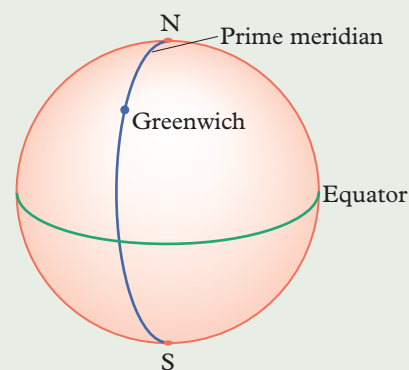
- Earth can also be cut into vertical segments by the **meridians of longitude**. A meridian of longitude is a semicircle from the North Pole to the South Pole that lies on a great circle.



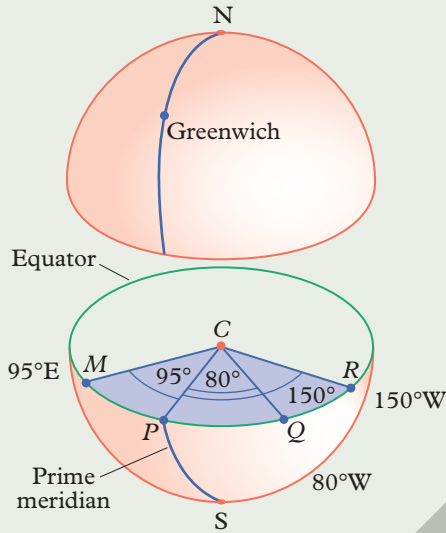
- There is a distinguished meridian of longitude called the **prime meridian**. It was decided in 1884, by international agreement, that the meridian of longitude to be used for this purpose would be the one that passes through Greenwich, England. Hence, it is sometimes referred to as the **Greenwich Meridian**.

- The prime meridian is labelled 0° , and all other meridians of longitude are labelled according to how far to the east or west of the prime meridian they are, as measured by the angle they make at the centre of Earth.

- The diagram below demonstrates how **longitude** is measured. The prime meridian is used as a reference, so the longitude of the point P is defined to be 0° . The longitude of the point Q is determined by first noticing that it lies to the west of P , then finding the angle PCQ , which is 80° . We write the longitude of Q as 80°W , which we read as “eighty degrees west”.



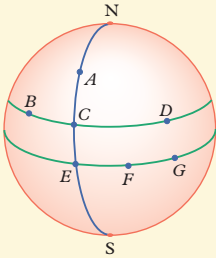
→ The longitude of the point R is 150°W , and the longitude of the point M is 95°E (read as “ninety-five degrees east”).



Worked example 8.1A Identifying features of Earth as a sphere

Consider the diagram of Earth, with the North (N) and South (S) Poles shown. Identify the three points (A – G) that lie on each of the following:

- a the equator
- b a parallel of latitude that is a small circle
- c a meridian of longitude



Part	Think	Write
a	The equator is the great circle that cuts Earth in half horizontally between the North and South Poles. The points that lie on the equator in this diagram are labelled E, F, G .	E, F, G
b	While the equator is a parallel of latitude, it is not a small circle. The other parallel of latitude shown in this diagram passes through the points labelled B, C, D .	B, C, D
c	Meridians of longitude are semicircles that run vertically from the North to South Pole. The only meridian of longitude shown on this diagram, passes through the points labelled A, C, E .	A, C, E .



Worked example video: Identifying features of Earth as a sphere

Understanding, fluency and communicating

Answers: Lesson 8.1

1 Each of the following triples of points corresponds to an arc in the diagram shown. For each one, state whether the arc lies on a parallel of latitude or a meridian of longitude.

- a** *CDI*
- b** *NDS*
- c** *AFG*
- d** *DEF*
- e** *SAB*
- f** *BEH*

2 For the diagram in question 1, state the direction from:

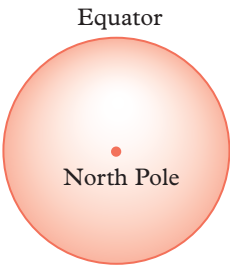
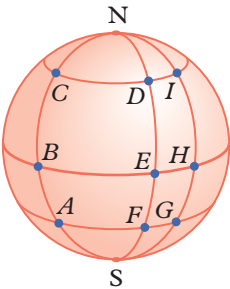
- a** *D to E*
- b** *A to F*
- c** *H to B*
- d** *B to C*

3 Draw two diagrams of the Earth as follows.

- a** On the first, mark the North and South Poles at the top and bottom of a circle, then draw and label four parallels of latitude.
- b** For the second, draw the view looking down on Earth from above the North Pole (copy the diagram shown). Show four parallels of latitude.

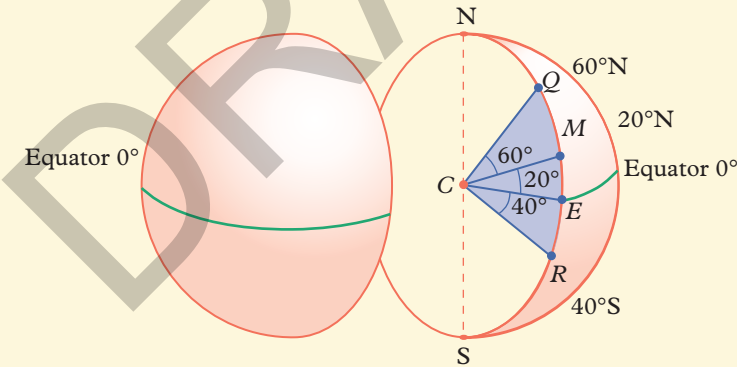
4 Draw another two diagrams of the Earth as follows.

- a** Draw the same diagram as in question 3a, but this time draw and label four meridians of longitude.
- b** Copy the same diagram shown in question 3b, then draw and label four meridians of longitude.



Worked example 8.1B Identifying points on parallels of latitude

This diagram shows parallels of latitude and angles from the equator.



Write the latitude of each of the following points.

- a** *Q*
- b** *R*
- c** *M*

Part	Think	Write
a	Angle $ECQ = 60^\circ$ and Q is north of the equator.	The point Q is on the 60°N parallel of latitude.
b	Angle $ECR = 40^\circ$ and R is south of the equator.	The point R is on the 40°S parallel of latitude.
c	Angle $ECM = 20^\circ$ and M is north of the equator.	The point M is on the 20°N parallel of latitude.

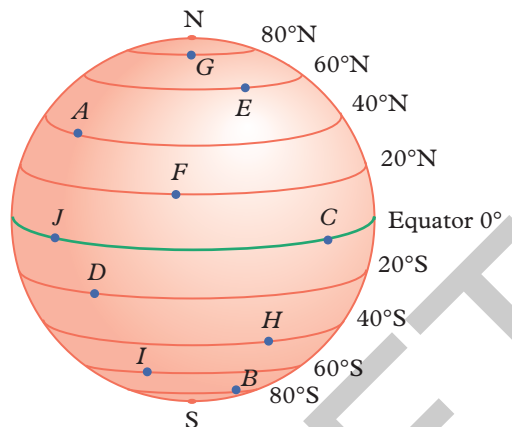


Worked example video: Identifying points on parallels of latitude

Helpful hints

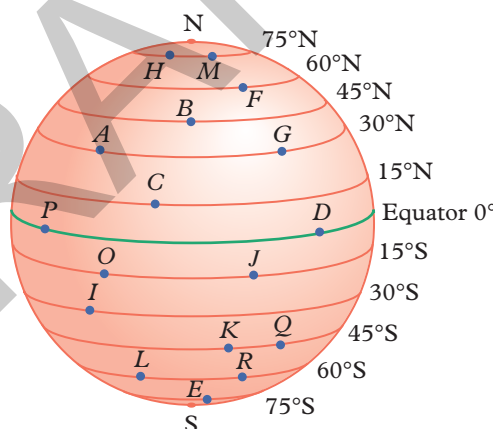
- To label a parallel of latitude, first determine if the parallel of latitude is north or south of the equator. The angle made with the equator combined with “N” (for north) or “S” (for south) names the parallel of latitude.
- To differentiate between latitude and longitude, remember that latitude is “flat”, so parallels of latitude are parallel to the equator.

5 Write the latitude of points *A* to *J*.



6 Use the diagram shown to name the points that lie on the following parallels of latitude.

- 15°N
- 30°S
- 75°N
- 0°
- 45°N
- 60°S
- 30°N
- 15°S
- 45°S
- 60°N
- 75°S



7 What is the latitude of:

- the North Pole?
- the South Pole?

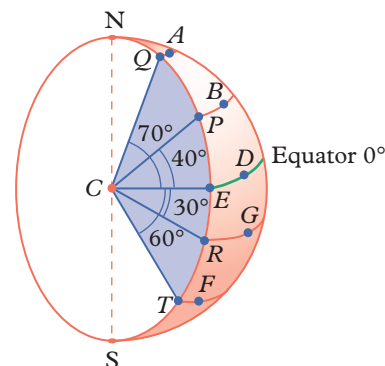
8 Consider this diagram for parts **a** and **b**.

a Write the latitude of the circles through these points from the diagram shown.

i <i>P</i>	ii <i>Q</i>
iii <i>E</i>	iv <i>R</i>
v <i>T</i>	

b Write the latitude of these points from the diagram shown.

i <i>A</i>	ii <i>B</i>
iii <i>D</i>	iv <i>G</i>
v <i>F</i>	



9 Consider this diagram for parts **a** and **b**.

a Write the latitude of these points from the diagram shown.

i H

iii J

v K

vii M

ii I

iv E

vi L

b Write the latitude of these points from the diagram shown.

i P

iii R

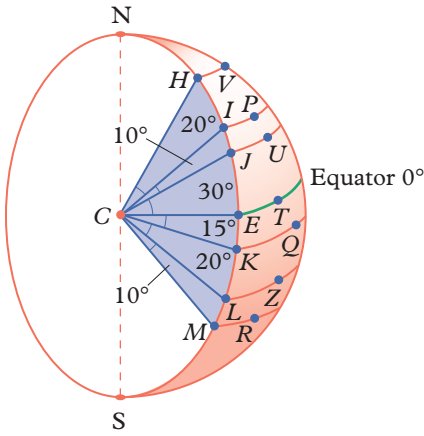
v U

vii Z

ii Q

iv T

vi V



10 Use a globe or map of the world to find three different cities you know that are at approximately the same latitude.

Worked example 8.1C Identifying points on meridians of longitude

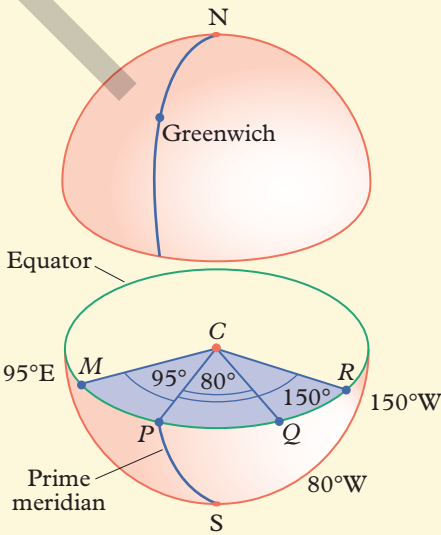
The diagram shows meridians of longitudes and angles from the prime meridian.

a On which meridian of longitude is the point Q ?

b On which meridian of longitude is the point R ?

c Which point lies on the 95°W meridian of longitude?

Part	Think	Write
a	Angle $PCQ = 80^\circ$ and Q is east of the prime meridian.	Q is 80°E .
b	Angle $PCR = 150^\circ$ and R is east of the prime meridian.	R is 150°E .
c	Angle $PCM = 95^\circ$ and M is west of the prime meridian.	The point M .



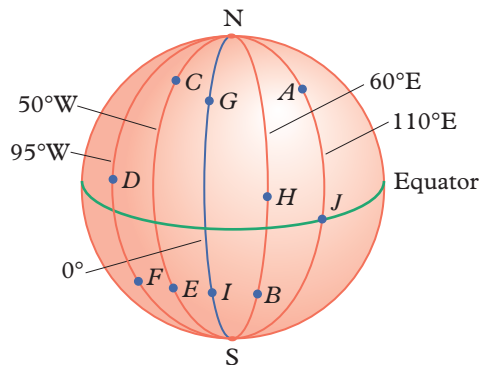
Worked example video: Identifying points on meridians of longitude

Helpful hints

- To label a meridian of longitude, first determine if the meridian of longitude is east or west of the prime meridian. The angle made with the prime meridian combined with “E” (for east) or “W” (for west) names the meridian of longitude.

11 Answer the following questions using the related diagrams.

a Write down the longitude of the points *A* to *J*.



b Name the points that lie on each of the meridians.

i 80°W

ii 70°E

iii 120°E

iv 0°

12 Answer parts **a** and **b** using this diagram.

a Write the longitude of the meridian passing through each of these points from the diagram shown.

i *T*

ii *V*

iii *W*

b Write the longitude of these points from the diagram shown.

i *K*

ii *L*

iii *M*

iv *N*

13 Answer parts **a** and **b** using this diagram.

a Write the longitude of the meridian through these points from the diagram shown.

i *K*

ii *L*

iii *M*

iv *R*

v *Q*

b Write the longitude of these points from the diagram shown.

i *T*

ii *U*

iii *V*

iv *X*

v *Y*

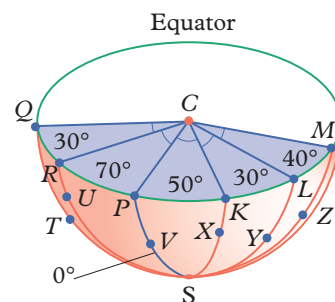
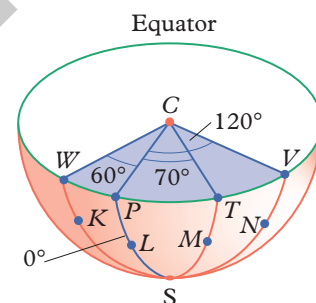
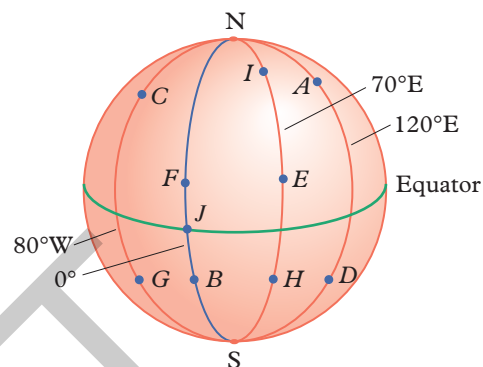
vi *Z*

14 Use a globe or map of the world to find the longitude of:

a Sydney

b London, UK

c New York, US



Challenge

15 Two cities lie on the equator. One is located at 60°W and the other is at 25°E. What is the angle made at the centre of Earth between the two cities?

Related resources



8.1 Quick quiz

Lesson 8.2

Position on Earth's surface

Learning intentions

By the end of this lesson, you will be able to ...
→ use latitude and longitude coordinates to describe specific locations on Earth.

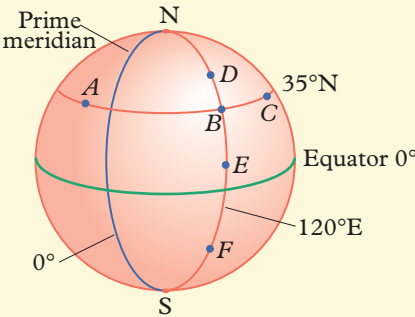
Earth's surface

- Maps of Earth are typically presented as flat projections of its spherical surface, with the North Pole at the top and the South Pole at the bottom. In this format, the parallels of latitude and meridians of longitude form a grid on Earth's surface.
- Lines of latitude run east to west, and meridians of longitude run north to south. The position of a point can be given by the ordered pair of **position coordinates** in the form (latitude, longitude).

Worked example 8.2A Writing position coordinates of a point using latitude and longitude

Use the diagram shown to answer the following.

- a** List all the points that lie on:
- i** the 35°N parallel of latitude
 - ii** the 120°E meridian of longitude
 - iii** both the 35°N parallel of latitude and the 120°E meridian of longitude.
- b** Write the position coordinates of point B .



Part	Think	Write
a i	The 35°N is a horizontal slice.	A, B, C
ii	The 120°E meridian is a vertical slice.	D, B, E, F
iii	This is the point where they intersect.	B
b	Write latitude first, then longitude.	$(35^\circ\text{N}, 120^\circ\text{E})$



Worked example video: Writing position coordinates of a point using latitude and longitude

Understanding, fluency and communicating

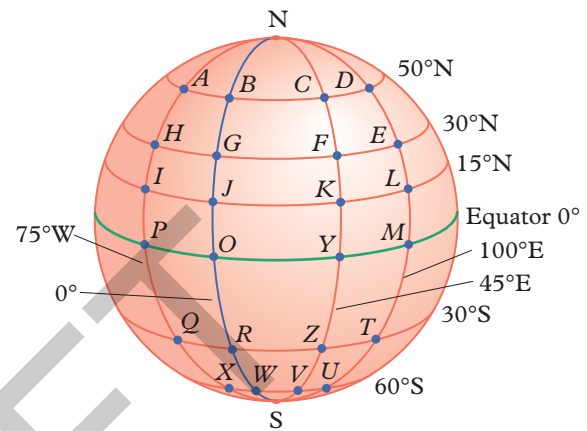
Answers: Lesson 8.2

Helpful hints

- Latitudes correspond to horizontal circles and longitudes correspond to vertical semicircles. The intersection of any parallel of latitude with a meridian of longitude determines the location of a single point.

1 Use **Source 1** to answer the following.

- a** Name the points that are on:
- i** the 50°N parallel of latitude
 - ii** the 45°E meridian of longitude
 - iii** both the 50°N parallel of latitude and the 45°E meridian of longitude.
- b** Name the points that are on:
- i** the 30°S parallel of latitude
 - ii** the 75°W meridian of longitude
 - iii** both the 30°S parallel of latitude and the 75°W meridian of longitude.

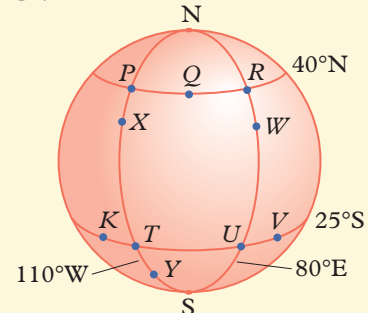


Source 1

Worked example 8.2B Finding and naming points using position coordinates

Answer the following using the diagram shown.

- a** What are the position coordinates of the following points?
- i** P
 - ii** U
- b** Name the points with the following position coordinates.
- i** $(25^{\circ}\text{S}, 110^{\circ}\text{W})$
 - ii** $(40^{\circ}\text{N}, 80^{\circ}\text{E})$



Part	Think	Write
a i	Latitude 40°N , longitude 110°W	$(40^{\circ}\text{N}, 110^{\circ}\text{W})$
ii	Latitude 25°S , longitude 80°E	$(25^{\circ}\text{S}, 80^{\circ}\text{E})$
b i	Intersection of 25°S and 110°W	T
ii	Intersection of 40°N and 80°E	R



Worked example video: Finding and naming points using position coordinates

Helpful hints

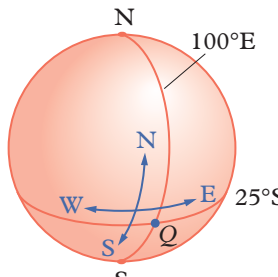
- The position of any point on Earth’s surface is described by an ordered pair with the latitude (how far north or south of the equator) first, then the longitude (how far east or west of the prime meridian) second.
- Directions on the globe are sometimes stated as “due” north/south/east/west. This just means “exactly” in that direction.

- 2 Write the position coordinates of points *A, E, G, I, K, M, O, Z, U, W* as they appear in Source 1.
- 3 Name the points in Source 1 that have the following position coordinates.
- | | | |
|---|---|--|
| a (50°N , 100°E) | b (15°N , 75°W) | c (60°S , 45°E) |
| d (30°S , 0° longitude) | e (15°N , 0° longitude) | f (30°N , 75°W) |
| g (30°S , 100°E) | h (60°S , 100°E) | i (0° latitude, 75°W) |
| j (30°N , 45°E) | k (60°S , 75°W) | l (0° latitude, 0° longitude) |
| m (15°N , 100°E) | n (50°N , 0° longitude) | o (50°N , 45°E) |
- 4 Using a globe or a virtual 3D map of the world, find the country in which these lines of latitude and longitude meet.
- | | | |
|---|---|---|
| a 30°S and 135°E | b 30°N and 105°E | c 75°S and 75°W |
| d 20°N and 100°W | e 0° latitude and 38°E | f 45°N and 0° longitude |
- 5 Suppose a point labelled *A* has position (5°S , 130°W).
- a** State whether the following points are due north or due south of *A*.
- | | | |
|---|---|---|
| i (20°S , 130°W) | ii (0° latitude, 130°W) | iii (15°N , 130°W) |
|---|---|---|
- b** State whether the following points are due east or due west of *A*.
- | | | |
|--|---|---|
| i (5°S , 150°W) | ii (5°S , 120°W) | iii (5°S , 0° longitude) |
|--|---|---|

Worked example 8.2C Determining position coordinates relative to a given point

If *Q* is the point (25°S , 100°E), determine the position coordinates of the point that is:

- a** 20° due north of *Q* **b** 20° due south of *Q* **c** 20° due east of *Q* **d** 20° due west of *Q*

Part	Think	Write
a	Draw a diagram showing the parallel of latitude and meridian of longitude through the point <i>Q</i>. Think of moving north as moving “less south”. So, moving 20° north along the meridian from 25°S takes us to 5°S. The longitude doesn’t change.	 (5°S, 100°E)
b	Move south 20° along the meridian to 45°S . The longitude doesn’t change.	(45°S , 100°E)
c	Move east 20° along the parallel of latitude to 25°S . The latitude doesn’t change.	(25°S , 120°E)
d	Move west 20° along the parallel of latitude to 25°S . The latitude doesn’t change.	(25°S , 80°E)



Worked example video: Determining position coordinates relative to a given point

Helpful hints

- Movements north or south affect the latitude. Movements east or west affect the longitude.

6 For each of the position coordinates **a–d** below, determine the coordinates of the point that is:

i 10° due north of Q

ii 10° due south of Q

iii 10° due east of Q

iv 10° due west of Q .

a $Q = (35^\circ\text{S}, 95^\circ\text{E})$

b $Q = (25^\circ\text{N}, 150^\circ\text{E})$

c $Q = (20^\circ\text{S}, 90^\circ\text{W})$

d $Q = (50^\circ\text{S}, 40^\circ\text{W})$

5 If P is the point $(10^\circ\text{N}, 75^\circ\text{W})$, find the coordinates of the point that is:

a 15° due north of P

b 5° due south of P

c 25° due south of P

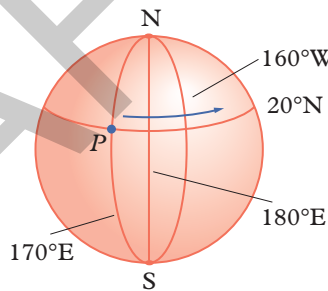
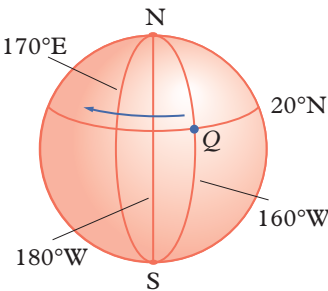
d 60° due south of P .

Worked example 8.2D Determining relative position coordinates close to the 180°E or 180°W meridian

Find the coordinates of the point that is:

a 30° due east of $P(20^\circ\text{N}, 170^\circ\text{E})$

b 30° due west of $Q(20^\circ\text{N}, 160^\circ\text{W})$.

Part	Think	Write
a	Draw a diagram showing the parallel of latitude and meridian of longitude through P. Also label the 180°E meridian, which is the same as the 180°W meridian. First add 10° to move east from 170°E to 180°E, then subtract 20° from 180°W to move east to 160°W. The latitude doesn't change. Hence, the new point has coordinates $(20^\circ\text{N}, 160^\circ\text{W})$.	 $(20^\circ\text{N}, 160^\circ\text{W})$
b	Draw a diagram showing the parallel of latitude and meridian of longitude through Q and the 180°W meridian, remembering this is the same as the 180°E meridian. Break the westward movement into a 20° step towards the 180°W meridian followed by a 10° step away from the 180°E meridian. The latitude doesn't change, so the point that is 30° west of Q is $(20^\circ\text{N}, 170^\circ\text{E})$.	 $(20^\circ\text{N}, 170^\circ\text{E})$



Worked example video: Determining relative position coordinates close to the 180°E or 180°W meridian

Helpful hints

- The 180°E and 180°W meridians are the same. When this meridian is crossed, the longitude counts backwards from 180° . For example, $190^\circ\text{W} = 170^\circ\text{E}$.

8 Find the coordinates of the point that is:

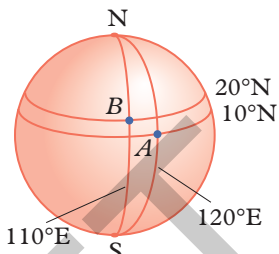
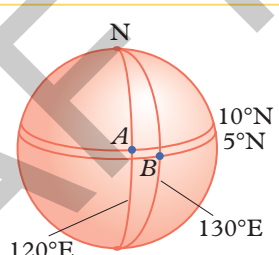
- a** 40° due east of $(15^\circ\text{N}, 155^\circ\text{E})$ **b** 20° due east of $(32^\circ\text{S}, 170^\circ\text{E})$
c 30° due west of $(18^\circ\text{S}, 170^\circ\text{W})$ **d** 50° due west of $(34^\circ\text{N}, 160^\circ\text{W})$

Worked example 8.2E Describing the position of a point relative to another point

Describe the position of point B relative to point A .

a $A(10^\circ\text{N}, 120^\circ\text{E}), B(20^\circ\text{N}, 110^\circ\text{E})$

b $A(10^\circ\text{N}, 120^\circ\text{E}), B(5^\circ\text{N}, 130^\circ\text{E})$

Part	Think		Write
a	Using a sketch and comparing latitudes, B is 10° to the north of A . Comparing longitudes, B is 10° to the west of A .		B is 10° north of A and 10° west of A .
b	Using a sketch and comparing latitudes, B is 5° to the south of A . Comparing longitudes, B is 10° to the east of A .		B is 5° south of A and 10° east of A .



Worked example video: Describing the position of a point relative to another point

Helpful hints

- Drawing a sketch will make finding the solution much easier.
- Take care if the question requires crossing the equator or the 180°E/W meridian.

9 Describe the position of point B relative to point A .

- a** $A(25^\circ\text{N}, 110^\circ\text{E}), B(10^\circ\text{N}, 130^\circ\text{E})$ **b** $A(15^\circ\text{S}, 80^\circ\text{E}), B(20^\circ\text{S}, 70^\circ\text{E})$
c $A(30^\circ\text{S}, 75^\circ\text{W}), B(15^\circ\text{S}, 65^\circ\text{W})$ **d** $A(45^\circ\text{N}, 80^\circ\text{W}), B(30^\circ\text{N}, 120^\circ\text{W})$
e $A(10^\circ\text{S}, 120^\circ\text{E}), B(20^\circ\text{N}, 10^\circ\text{W})$ **f** $A(15^\circ\text{N}, 150^\circ\text{E}), B(25^\circ\text{S}, 150^\circ\text{W})$

Problem solving and reasoning

10 Using a globe or a virtual 3D map of the world, find:

- a** two cities with the same latitude as Paris (France)
b two cities with the same longitude as Berlin (Germany)

c the city closest to:

i (34°S, 151°E)

ii (40°N, 116°E)

iii (42°N, 12°E)

iv (26°S, 28°E)

v (55°N, 37°E)

vi (34°N, 118°W)

d the position coordinates (to the nearest degree) of the following cities.

i Tokyo (Japan)

ii London (UK)

iii New York (US)

iv Tehran (Iran)

v Rio de Janeiro (Brazil)

vi Vancouver (Canada)

11 Use a map of Australia with lines of latitude and longitude marked to answer the following questions.

a Name the city or town that is closest to:

i (35°S, 149°E)

ii (12°S, 131°E)

iii (43°S, 147°E)

iv (17°S, 146°E)

b Which is further south of the equator?

i Sydney or Perth

ii Rockhampton or Alice Springs

c Which is further east of the prime meridian?

i Sydney or Brisbane

ii Cairns or Hobart

12 Angelina is in the desert and heading to Cairo (30°N, 31°E). Her GPS coordinates place her at (28°N, 35°E). What is her current position relative to Cairo?

13 Klaus is sailing to Honolulu (21°N, 58°W). According to GPS navigation, his position is (18°N, 64°W). What is his position relative to Honolulu?

Challenge

14 A ship sails in a straight line from the coordinates (12°N, 26°W) to (4°S, 26°W). Assuming Earth is a sphere with a radius of 6400 km, find how far the ship has sailed.

Related resources



8.2 Quick quiz

Lesson 8.3

Time units and intervals

Learning intentions

By the end of this lesson, you will be able to ...

→ convert between 12- and 24-hour time and solve problems involving elapsed time.

Measuring time

- Measuring time is an important and unavoidable aspect of life.

- The units of time and their conversions are as follows.

→ 60 seconds = 1 minute

→ 24 hours = 1 day

→ 60 minutes = 1 hour

→ 7 days = 1 week

- 2 weeks = 1 fortnight
 - 52 weeks = 1 year (actually 52 weeks and 1 day)
 - 12 months = 1 year
 - 365 days = 1 year (366 for a leap year)
 - 10 years = 1 decade
 - 100 years = 1 century
 - 1000 years = 1 millennium
- To track time within a day we can use either 12-hour time or 24-hour time.
 - In 12-hour time, the day is split into “am” before midday and “pm” after midday. These abbreviations come from the Latin phrases “ante meridiem” and “post meridiem”. In Australia, midday is written as 12:00 pm and midnight is written as 12:00 am, but beware: some countries follow the opposite convention! To avoid ambiguity, midday is sometimes written as 12:00 noon.
 - In 24-hour time, the format used is HHMM. For example, 10:35 am is written as 1035, with no extra symbols or punctuation. The hour is always expressed using two digits, so times before 10 am have a leading 0, such as 8:22 am = 0822. Midnight is written as 0000.

Worked example 8.3A Converting units of time

Convert:

- a** 150 minutes to hours **b** 143 minutes to hours and minutes
c 110 hours to days and hours **d** 3 days 15 hours to hours.

Part	Think	Write
a	There are 60 minutes in an hour, so divide the number of minutes by 60. When expressed in hours, the answer is a decimal. It is preferable to convert to a mixed fraction, since it is more common to say “two and a half hours” than “two point five hours”.	Number of hours = $150 \div 60$ $= 2.5$ Therefore 150 minutes is $2\frac{1}{2}$ hours.
b	Divide the number of minutes by 60 to get a decimal number of hours. There are 2 whole hours in 143 minutes. Subtract 2 whole hours’ worth of minutes from the total number of minutes to determine the number of minutes remaining. On a calculator, you may have a “degrees, minutes, seconds” button that does this automatically.	Number of hours = $143 \div 60$ $= 2.3833\dots$ Additional minutes = $143 - (2 \times 60)$ $= 23$ Therefore, 143 minutes is 2 hours and 23 minutes.
c	There are 24 hours in a day, so divide the number of hours by 24. There are 4 whole days in 110 hours. Subtract 4 whole days’ worth of hours from the total number of hours to determine the number of hours remaining.	Number of days = $110 \div 24$ $= 4.5833\dots$ Additional hours = $110 - (4 \times 24)$ $= 14$ Therefore, 110 hours is 4 days 14 hours.
d	Multiply 3 days by 24 hours to convert to hours and then add the extra 15 hours.	Number of hours = $3 \times 24 + 15$ $= 87$



Worked example video: Converting units of time

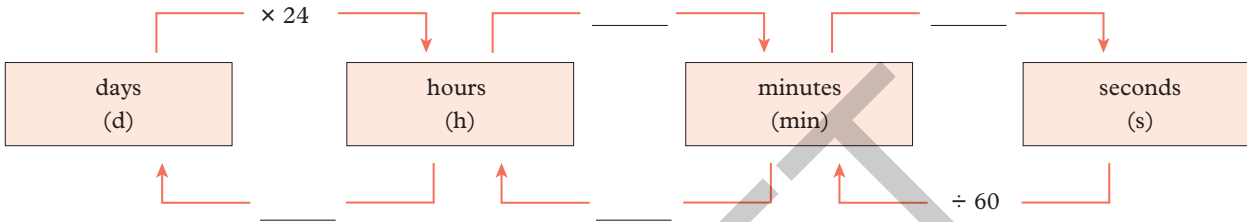
Understanding, fluency and communicating

Answers: Lesson 8.3

Helpful hints

- When converting between units of time:
 - Step 1: Determine the conversion factor.
 - Step 2: Multiply a value expressed in larger units to convert to a value in smaller units, or divide a value expressed in smaller units to convert to a value in larger units.
 - Step 3: Determine any remaining partial units and express them in the appropriate (smaller) units.

1 Complete this time conversion diagram.



2 Convert:

- a 150 minutes to hours
- b 200 minutes to hours
- c 300 seconds to minutes
- d 193 minutes to hours and minutes
- e 485 seconds to minutes
- f 635 minutes to hours and minutes
- g 4 hours 37 minutes to minutes
- h 47 minutes 12 seconds to seconds
- i 5 days 17 hours to hours
- j 100 hours to days and hours
- k 1 week to seconds
- l 10 000 seconds to hours, minutes and seconds

Worked example 8.3B Converting between 12-hour and 24-hour time

- a Write the 24-hour time 1020 in 12-hour time.
- b Write the 24-hour time 1324 in 12-hour time.
- c Convert 8 : 47 am to 24-hour time.
- d Convert 6 : 58 pm to 24-hour time.

Part	Think	Write
a	Because the first two digits are less than 12 this is an “am” time.	1020 is 10 : 20 am.
b	The first two digits are greater than 12 so we subtract 12 hours from the 24-hour time and the result can be written as a “pm” time.	1324 is 1 : 24 pm.
c	Any “am” time has the same digits when expressed in 24-hour time. Since the hour is less than 10, a zero is added at the start.	8 : 47 am is 0847.
d	Any “pm” time has 12 added to the hours when converting to 24-hour time.	6 : 58 pm is 1858.



Worked example video: Converting between 12-hour and 24-hour time

Helpful hints

- 24-hour time always has four digits. If the time is before 10:00 am, a zero is added at the start to satisfy this convention.
- If the 24-hour time is later than 1200, we subtract 12 from the hours to convert to “pm” time.

3 Write these 24-hour times in 12-hour format (including am or pm).

- a** 1406 **b** 0051 **c** 2005 **d** 0122 **e** 0925 **f** 2121
g 1915 **h** 1212 **i** 1254 **j** 1749 **k** 0622 **l** 2254

4 Write these 12-hour times in 24-hour notation.

- a** 7:43 am **b** 5:20 pm **c** 1:35 am **d** 12:43 pm
e 4:20 pm **f** 1:46 am **g** 6:08 am **h** 8:27 pm
i 9:20 am **j** 11:08 pm **k** 12:00 noon **l** 12:00 midnight

Worked example 8.3C Performing calculations involving 24-hour time

- a** Calculate the time 2 hours and 10 minutes after 0920.
b Calculate the time 1 hour and 12 minutes before 1528.
c Calculate the time 5 hours and 26 minutes after 0850.
d Calculate the time 6 hours and 38 minutes before 1524.

Part	Think	Write
a	Add the hours: $9 + 2 = 11$ Add the minutes: $20 + 10 = 30$	2 hours 10 minutes after 0920 is 1130.
b	Subtract the hours: $15 - 1 = 14$ Subtract the minutes: $28 - 12 = 16$	1 hour 12 minutes before 1528 is 1416.
c	Notice that adding 26 minutes to 50 minutes will exceed 60, so perform this addition and convert to hours and minutes first: $50 \text{ minutes} + 26 \text{ minutes} = 76 \text{ minutes}$ $76 \text{ minutes} = 1 \text{ hour and } 16 \text{ minutes}$ This can be added to 0800. Then add the 5 hours.	26 minutes after 0850 is the same as 76 minutes after 0800, which is 0916. 5 hours after 0916 is 1416.
d	We know that 38 minutes before 1524 is sometime in the hour of 1400. Reinterpret 1524 as 60 minutes after 1424. Subtract the 38 minutes from this, leaving us at 22 minutes after 1424, which is 1446. Finally, we find the time 6 hours before this by subtracting 6 from 14.	1524 is 60 minutes after 1424. 38 minutes earlier than this is 22 minutes after 1424, which is 1446. Another 6 hours earlier is 0846.



Worked example video: Performing calculations involving 24-hour time

Helpful hints

- It can be easier to add or subtract minutes before adding or subtracting hours.
 - If by adding minutes you exceed 60, this takes you to the next hour.
 - If by subtracting minutes you end up with a negative value, this takes you to the previous hour.

5 Complete each of the following time calculations, giving the answer in 24-hour time.

a Add 2 hours 30 minutes to the 24-hour time 0600.	f Subtract 3 hours 51 minutes from the 24-hour time 1223.
b Add 1 hour 15 minutes to the 24-hour time 1020.	g Subtract 3 hours 43 minutes from the 24-hour time 1605.
c Subtract 1 hour 40 minutes from the 24-hour time 0916.	h Add 3 hours 46 minutes to the 24-hour time 0854.
d Add 1 hour 48 minutes to the 24-hour time 1422.	i Subtract 4 hours 38 minutes from the 24-hour time 1127.
e Add 2 hours 46 minutes to the 24-hour time 1509.	

Worked example 8.3D Performing calculations involving time intervals

- a** Calculate 2 days 4 hours 55 minutes + 1 day 3 hours 31 minutes.
b Calculate 3 days 7 hours 25 minutes – 1 day 15 hours 12 minutes.

a	If we display the time intervals vertically with matching units aligned, then it is easier to see how they relate to each other. Start by adding the values in each column. Since $31 + 55$ minutes is equal to 86 minutes, we can convert 60 of these minutes to 1 hour and carry this over to the hours column. Simplifying this gives the final answer.	<table> <thead> <tr> <th>day</th><th>hours</th><th>minutes</th></tr> </thead> <tbody> <tr> <td>2</td><td>4</td><td>55</td></tr> <tr> <td>+ 1</td><td>3</td><td>31</td></tr> <tr> <td>= 3</td><td>7⁽⁺¹⁾</td><td>86⁽⁻⁶⁰⁾</td></tr> <tr> <td>= 3</td><td>8</td><td>26</td></tr> </tbody> </table> The answer is 3 days 8 hours 26 minutes.	day	hours	minutes	2	4	55	+ 1	3	31	= 3	7 ⁽⁺¹⁾	86 ⁽⁻⁶⁰⁾	= 3	8	26
day	hours	minutes															
2	4	55															
+ 1	3	31															
= 3	7 ⁽⁺¹⁾	86 ⁽⁻⁶⁰⁾															
= 3	8	26															
b	Since 7 hours is less than 15 hours, we start by borrowing 1 day from the days column (which is equal to 24 hours). We can then simplify and subtract the values in each column to get the final answer.	<table> <thead> <tr> <th>day</th><th>hours</th><th>minutes</th></tr> </thead> <tbody> <tr> <td>3⁽⁻¹⁾</td><td>7⁽⁺²⁴⁾</td><td>25</td></tr> <tr> <td>– 1</td><td>15</td><td>12</td></tr> <tr> <td>= 1</td><td>16</td><td>13</td></tr> </tbody> </table> The answer is 1 day 16 hours 13 minutes.	day	hours	minutes	3 ⁽⁻¹⁾	7 ⁽⁺²⁴⁾	25	– 1	15	12	= 1	16	13			
day	hours	minutes															
3 ⁽⁻¹⁾	7 ⁽⁺²⁴⁾	25															
– 1	15	12															
= 1	16	13															



Worked example video: Performing calculations involving time intervals

Helpful hints

- When adding and subtracting values of time in one unit, it may be necessary to borrow from or carry over to larger units.
- It can be useful to align values expressed in the same units vertically.

6 Perform each of the following calculations.

- a 3 days 16 hours + 4 days 18 hours
- b 2 days 21 hours + 3 days 23 hours
- c 5 days 17 hours – 2 days 10 hours
- d 4 days 2 hours 20 minutes + 1 day 17 hours 11 minutes
- e 6 days 15 hours 38 minutes – 2 days 7 hours 15 minutes
- f 5 days 18 hours 51 minutes + 1 day 13 hours 28 minutes

Problem solving and reasoning

7 Calculate the time (in days/hours/minutes) that elapses between:

- a 4:38 pm Tuesday to 6:30 am on the following Thursday
- b 7:09 am Saturday to 12:16 pm on the following Wednesday
- c 6:52 am Monday to 8:15 pm on the following Friday
- d 1346 on Sunday to 1925 on the following Monday
- e 1047 on Wednesday to 1638 on the following Saturday
- f 0024 Friday to 1915 on the following Wednesday.

8 Pooja leaves for school camp on Friday at 0900. The bus trip takes 3 hours 42 minutes.

- a What time does Pooja arrive at camp?
- b If dinner is at 1800, how long after she arrived at camp does Pooja wait until dinner?
- c Everyone is being woken at 5:45 am to watch the sun rise. Pooja needs 9 hours 25 minutes sleep. She needs 15 minutes to fall asleep. At what time will she need to go to bed?
- d Pooja arrives back at school at 1545 on Monday. How long is it since she left for camp?



Challenge

9 Miko leaves to drive to school at 7:56 am. The trip is 12 km long, and he drives at an average speed of 45 km/h. What time will he arrive at school?

10 George is only allowed to spend 1 hour and 10 minutes watching shows or movies on a weekday and double that time on each day of the weekend. He intends to watch the entire first season of *Small Pop Hypothesis* comprising 35 episodes, each of 24 minutes duration. He starts watching on Monday 7 May at 7:00 pm and continues where he left off at 7:00 pm every subsequent day.

- a At what time and on what day and date does George complete watching Season 1?
- b Due to a poor half-yearly report, George has had his allowed viewing time reduced by 15 minutes per weekday, but thankfully not reduced on weekends. Season 2 of *Small Pop Hypothesis* has 38 episodes of 23 minutes duration and he starts watching it on Wednesday 23 May at 7:00 pm. At what time and on what day and date does he finish watching Season 2?
- c After topping the class in a mathematics assessment, George is told he can watch Season 3 of *Small Pop Hypothesis* over one week, starting on a Friday at 7:00 pm. There are 33 episodes of 24 minutes duration. He is allowed double his weekday viewing time on each day of the weekend. How long must he be allowed to watch on a weekday if he is to finish watching the whole season after exactly one week?



Related resources



8.3 Quick quiz



Investigation: Train timetables

Lesson 8.4

World time zones

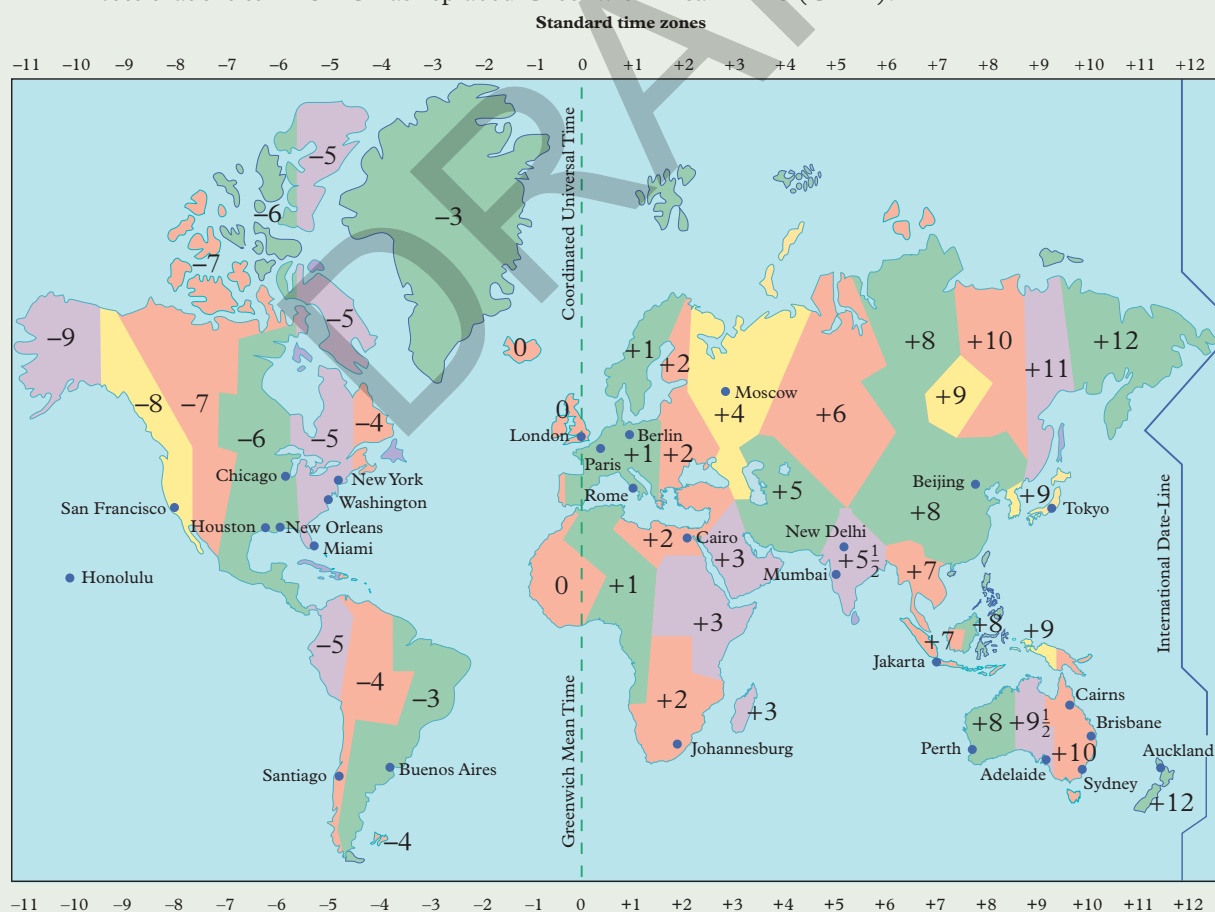
Learning intentions

By the end of this lesson, you will be able to ...

→ solve problems involving different time zones around the world.

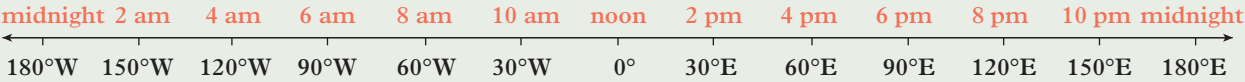
Time zones

- Meridians of longitude are used as the basis of **time zones**. There are 12 time zones to the east of the prime meridian and 12 time zones to its west. Each zone covers about 15° of longitude. Every place within a zone uses the same time. The time within the zone is known as **standard time**.
- The borders of time zones don't follow meridians of longitude too strictly. For convenience, they often follow country or state boundaries. Australia, for example, has three time zones.
- **World Time Zones** are shown on the simplified map below. The numbers in each zone indicate the number of hours to be added to or subtracted from **Coordinated Universal Time (UTC)** to get the standard time for that zone.
→ Note that the term UTC has replaced Greenwich Mean Time (GMT).



Source 1

- This number line shows times and longitudes relative to noon in Greenwich, which is on the prime meridian.



- The meridians of longitude at 180°E and 180°W occur in the same place; however, the standard time to either side (roughly) of this longitude differs by 1 day. The boundary of these time zones is called the **International Date Line**.
- The formula $15^\circ = 1 \text{ hour}$ can be used to calculate the time differences between locations on Earth given the relevant longitudes.

Worked example 8.4A Finding local standard times

Use the world time zone map to find the local standard time in the following cities when UTC is 12:00 noon.

a Sydney

b Houston (US)

Part	Think	Write
a	Sydney is in the time zone marked UTC +10, so local standard time in Sydney is 10 hours ahead of (later than) UTC.	Standard time in Sydney would be 10:00 pm.
b	Houston is in the time zone marked UTC−6, so local standard time in Houston is 6 hours behind (earlier than) UTC.	Standard time in Houston would be 6:00 am.



Worked example video: Finding local standard times

Understanding, fluency and communicating

Answers: Lesson 8.4

Helpful hints

- Find the time zone for the city and add or subtract that number of hours from Coordinated Universal Time (UTC).

- 1** Use the world time zone map to find the local standard time in each of these cities when UTC is 12:00 noon.

- a** Perth
- b** San Francisco
- c** Cairo
- d** Santiago
- e** Johannesburg
- f** Moscow



Worked example 8.4B Comparing standard times involving am and pm

Use the world time zone map to find the local standard time in the following cities when UTC is 10 : 00 am.

- a** Sydney **b** Houston

Part	Think	Write
a	Sydney is in the time zone marked UTC +10, so local standard time in Sydney is 10 hours ahead of (later than) 10 : 00 am.	Standard time in Sydney is 8 : 00 pm.
b	Houston is in the time zone marked UTC –6, so local standard time in Houston is 6 hours behind (earlier than) 10 : 00 am.	Standard time in Houston would be 4 : 00 am.

 **Worked example video:** Comparing standard times involving am and pm

Helpful hints

- Find the time zone for the city and add or subtract that number of hours from the stated UTC time.

- 2 Use the world time zone map to find the local standard time in the following cities when UTC is 2 : 00 pm.
a Tokyo **b** Mumbai **c** New York **d** Houston **e** Rome **f** Adelaide

Worked example 8.4C Finding standard times when days differ

Use the world time zone map to find the local standard time:

- a** in Sydney when UTC is 8 : 00 pm Wednesday **b** in Houston when UTC is 2 : 00 am Monday.

Part	Think	Write
a	Sydney is in the time zone marked UTC +10, so local standard time in Sydney is 10 hours ahead of (later than) 8 : 00 pm Wednesday. This means it is 6 : 00 am Thursday.	Standard time in Sydney is 6 : 00 am Thursday.
b	Houston is in the time zone marked UTC –6, so local standard time in Houston is 6 hours behind (earlier than) 2 : 00 am Monday.	Standard time in Houston would be 8 : 00 pm Sunday.

 **Worked example video:** Finding standard times when days differ

Helpful hints

- Find the time zone for the city and add or subtract that number of hours from the stated UTC time. This may mean that it is the following or the previous day!

- 3 Use the world time zone map to find the local standard time in these cities when UTC is 9 : 00 pm Tuesday.
a Sydney **b** New Delhi **c** Moscow **d** Beijing **e** Brisbane **f** Jakarta

- 4 Use the world time zone map to find the local standard time in these cities when UTC is 4:00 am Monday.
- | | | |
|------------------|-------------------|------------------------|
| a Houston | b Santiago | c San Francisco |
| d Miami | e New York | f Buenos Aires |

Worked example 8.4D Finding the difference in standard time between two cities

What is the difference in standard times between:

- a** New York and San Francisco? **b** New York and Paris?

Part	Think	Write
a	New York is in the time zone marked UTC−5 and San Francisco is in the time zone marked UTC−8. The difference can be calculated as either $-5 - (-8) = 3$, or $-8 - (-5) = -3$	The difference in standard times is 3 hours.
b	New York is in the time zone marked UTC−5 and Paris is in the time zone marked UTC+1. The difference can be calculated as either $1 - (-5) = 6$, or $-5 - 1 = -6$	The difference in standard times is 6 hours.



Worked example video: Finding the difference in standard time between two cities

Helpful hints

- Find the time zone for each city. Subtract the time zones to find the difference.
- Express your answer as a positive value, since it is the magnitude, and not necessarily the direction, of difference that matters.

- 5 What is the difference in standard times between:

a New York and Houston?	b San Francisco and Houston?
c Cairo and Mumbai?	d Beijing and Sydney?
e New York and Moscow?	f Johannesburg and San Francisco?

Worked example 8.4E Finding the standard time in another city

When it is 9:00 am in Cairo, what is the local standard time in:

- a** Mumbai?
b New York?





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Part	Think	Write
a	Sydney standard time is UTC +10. Honolulu standard time is UTC−10. The difference in standard time is 20 hours. Sydney is east of Honolulu. It has already been 11 : 00 am Sydney local time. The time in Honolulu will be 20 hours earlier than 11 : 00 am.	The local standard time in Honolulu is 3 : 00 pm on Thursday. This is due to the International Date Line.
b	Add $11\frac{1}{2}$ hours to 3 : 00 pm on Thursday.	The time is 2 : 30 am Friday.



Worked example video: Finding local standard times involving the International Date Line

Helpful hints

- Find the time zone for each city. Subtract the time zones to find the difference. Add or subtract that number of hours from the stated time.
- Crossing the International Date Line from east to west takes the time back to the previous day.

- 10** Reynold leaves Beijing (31°N , 121°E) at 7 : 00 am Tuesday to fly to San Francisco (38°N , 122°W).
- a What is the local standard time and day in San Francisco when Reynold leaves Beijing?
 - b The flight time is 12 hours. What is the day and time in San Francisco when he arrives?
- 11** Elliot leaves San Francisco (38°N , 122°W) at 1 : 00 pm Sunday to fly to Sydney (34°S , 151°E).
- a What is the local standard time and day in Sydney when Elliot leaves San Francisco?
 - b The flight time is 16 hours. What is the day and time in Sydney when they arrive?
- 12** Kaya leaves Houston (30°N , 95°W) at 6 : 00 pm Saturday to fly to Sydney (34°S , 151°E).
- a What is the local standard time and day in Sydney when Kaya leaves Houston?
 - b The flight time is $15\frac{1}{2}$ hours. What is the day and time in Sydney when she arrives?

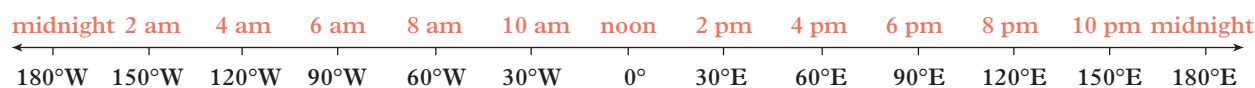
Problem solving and reasoning

- 13** An international company has offices in San Francisco, London and Sydney.
- a Use the world standard time zone map to find the local time in San Francisco and Sydney when it is 12 : 00 noon in London.
 - b Copy and complete the table to compare local standard times in the three cities.

San Francisco	London	Sydney
	12 : 00 noon (Monday)	
	2 : 00 pm	
	4 : 00 pm	
	:	
	12 : 00 noon (Tuesday)	

- c The company directors in the three cities wish to organise a three-way teleconference. What would be a convenient time to hold the conference?

- 14** Using the number line provided, complete the table to show the local time for places east and west of the prime meridian (0°) when it is 12:00 noon Wednesday UTC.



Remember that 180°W and 180°E meridians coincide roughly with the International Date Line, so even though they describe the same location, time differs by 1 day when this line is crossed.

Longitude west	Local time	Longitude east	Local time
0°	12:00 noon Wednesday	0°	12:00 noon Wednesday
45°W		45°E	
90°W		90°E	
135°W		135°E	
180°W		180°E	

- 15** Use your completed table from question **14** to help you answer the following questions.

- On a globe or a virtual 3D map of the world, find the 180°E and 180°W meridians of longitude. What do you notice?
- Compare local times for the 180°E and 180°W meridians from the table.
- Discuss the need for the International Date Line.
- Find the International Date Line on a map. Does it follow the 180° meridian exactly? Discuss why this happens.
- What happens if you cross the International Date Line from:
 - east to west?
 - west to east?
- Compare your completed table with the formula $15^\circ = 1 \text{ hour}$. What do you notice?

- 16** A ship travelling from San Francisco (US) to Brisbane crosses the International Date Line at 8:30 pm on Saturday 25 March. What will be the time and date just after it has crossed?



Challenge

- Joshua is travelling from Brisbane (UTC+10) to New York (UTC-5). The overall trip will take 21 hours and 30 minutes. If he leaves at 8:45 pm on Tuesday, what day and time will he arrive in New York?
- Kelly and Erin travel on a container ship from Perth (32°S , 116°E) to Buenos Aires (34°S , 58°W).
 - They leave Perth at 0700 Monday, 1 December 2025, and travel 12 500 km at an average speed of 20 km/h.
 - What is the local time, day and date in Buenos Aires when they leave Perth?
 - How long is the journey?
 - What is the local time, day and date in Buenos Aires when they arrive?
 - They holiday until the return journey which leaves 25 days and 5 hours after they arrived. The return journey covers 13 000 km at an average speed of 18 km/h.
 - What is the local time, day and date in Perth when they leave Buenos Aires to go back?
 - How long is the journey?
 - What is the local time, day and date in Perth when they arrive?
 - How long was the entire journey?



Related resources



8.4 Quick quiz



Worksheet: Time differences using longitude



Worksheet: Using a time zone map



Investigation: International Date Line

Lesson 8.5

Australian time zones

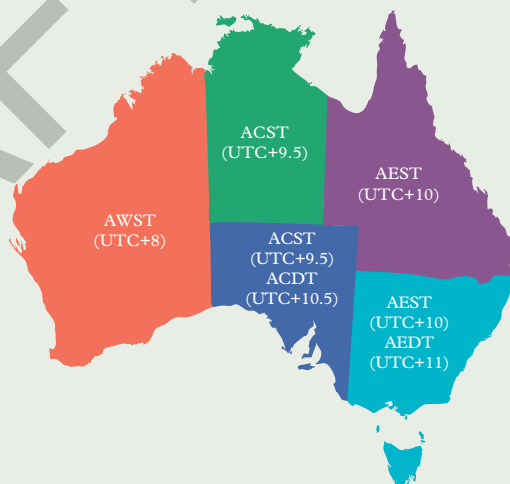
Learning intentions

By the end of this lesson, you will be able to ...

→ solve problems involving both standard and daylight saving time zones in Australia.

Australian time zones

- We observed in the previous section that the borders of time zones do not necessarily follow meridians of longitude. For convenience, they often follow country or state boundaries.
- Australia has three primary time zones, which use:
 - Australian Eastern Standard Time (AEST), equal to UTC +10
 - Australian Central Standard Time (ACST), equal to UTC +9.5
 - Australian Western Standard Time (AWST), equal to UTC +8
- AEST is used by Queensland, Victoria, Tasmania, the ACT and NSW, except for a region around Broken Hill. These states are near the 150° E meridian of longitude.
- ACST, which is only 30 minutes behind AEST, is used by South Australia, the Northern Territory and Broken Hill.
- AWST is 2 hours behind AEST and is used throughout Western Australia, which is roughly centred on the 120° E meridian of longitude.
- Many jurisdictions that lie closer to the North or South Poles follow **daylight saving time (DST)** during summer to maximise daylight hours between favourable times. In Australia, daylight saving time usually starts from the first Sunday in October, when clocks are put forward by one hour, and ends on the first Sunday in April, when clocks are put back by one hour.
 - New South Wales, the Australian Capital Territory, Victoria, Tasmania observe Australian Eastern Daylight Saving Time (AEDT), and South Australia observes Australian Central Daylight Saving Time (ACDT) each year.
 - The diagram below shows the summer times zones in Australia in terms of UTC time.



Source 1

Worked example 8.5A Finding local times in Australia

Find the time in Broome when it is 7 : 30 pm on the Gold Coast.

Think	Write
Broome is in the AWST time zone, which is equal to UTC +8. The Gold Coast is in the AEST time zone, which is equal to UTC +10. Hence, the time difference between these locations is 2 hours. The east coast is ahead of the west coast, so it is earlier in Broome than it is on the Gold Coast.	When it is 7 : 30 pm on the Gold Coast, it is 5 : 30 pm in Broome.



Worked example video: Finding local times in Australia

Understanding, fluency and communicating

Answers: Lesson 8.5

- Use the standard Australian time zones shown in Source 1 to find the time in:
 - Darwin when it is 8 : 30 am in Cairns
 - Hobart when it is 10 : 00 am in Sydney
 - Perth when it is 7 : 00 pm in Brisbane
 - Adelaide when it is 5 : 00 pm in Perth
 - Melbourne when it is 5 : 20 pm in Perth
 - Sydney when it is 2 : 30 pm in Alice Springs.
- Answer the following, assuming Sydney is using AEST.
 - The flight from Sydney to Perth takes 4 hours flying time. If a plane leaves Sydney at 5 : 00 pm, at what time will it arrive in Perth (local time)?
 - A plane leaves Perth at 6 : 00 am for the return trip to Sydney. Due to head winds, the trip takes $4\frac{1}{2}$ hours flying time. At what time will the plane arrive in Sydney (local time)?
- A plane leaves Sydney at 7 : 00 am Saturday, AEST, and flies directly to London. If the trip takes 16 hours, at what time will it be in London when the plane arrives?

Problem solving and reasoning

- Complete the table to compare times in the given cities during the months when daylight saving operates if the time in Sydney is 1 : 00 pm, AEDT.

City	Brisbane	Sydney	Canberra	Melbourne	Hobart	Adelaide	Darwin	Perth
Time		1 : 00 pm						

- Aryun is in Adelaide and rings his mother to say Happy New Year at midnight Adelaide time. If his mother is in Brisbane, at what local time does she receive the call?
- Paula lives in Sydney and rings her friend in Perth during summer when daylight saving time applies. If it is 8 : 00 pm in Sydney, what is the time in Perth?
- The Socceroos are playing in Jakarta in summer and the game is at 8 : 00 pm local time. Jakarta is one hour behind standard Perth time. The match is broadcast live. At what time in Sydney will the game start, using local daylight saving time?



- 8 A summer concert is held in Perth starting at 7 : 00 pm and is live streamed around Australia. Calculate the starting time of the concert as local time in Adelaide, Sydney and Brisbane.
- 9 A plane leaves Adelaide at 8 : 00 am, ACDT, and flies to Hobart. The flight time is $1\frac{1}{2}$ hours. What time will it be in Hobart when the plane arrives?

Challenge

- 10 The train from Perth to Sydney takes 3 days and 2 hours. If the train leaves Perth at 6 pm on Saturday, what day and time does it arrive in Sydney if daylight savings is occurring in Australia?

Related resources



8.5 Quick quiz



Worksheet: Time zones and time difference



Worksheet: Australian time zones and daylight saving



Investigation: Daylight saving time

Lesson 8.6

Review: Time and location

Review questions 8.6A Mathematical literacy review

Answers: Lesson 8.6

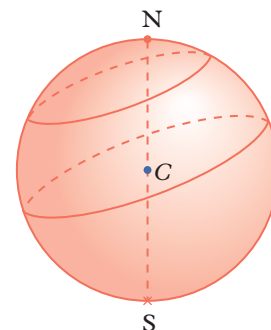
Key terms

The following key terms are used in this module:

- axis of rotation
- Coordinated Universal Time (UTC)
- daylight saving time
- equator
- great circle
- International Date Line
- latitude
- longitude
- meridians of longitude
- parallels of latitude
- prime meridian (Greenwich Meridian)
- small circle
- standard time
- time zone
- World Time Zones

Check your understanding

- 1 In the diagram below, the point N is the North Pole, the point S is the South Pole, and the point C is the centre of Earth. Label the following features:
 - a axis of rotation
 - b great circle
 - c small circle
- 2 There are distinguished lines of **latitude** and **longitude** that are used as references when describing all others.
 - a The **parallel of latitude** used as a reference for all other **latitudes** is called the _____.
 - b The **meridian of longitude** used as a reference for all other **longitudes** is called the _____ or _____.



- 3 Describe the **International Date Line**. Where is it and what happens there?
- 4 Some parts of Australia use **daylight saving time**.
 - a Explain the meaning of daylight saving time.
 - b Which Australian states and territories adopt daylight saving time?
 - c When does daylight saving time start and finish in the places you listed in part b?

Review questions 8.6B Multiple-choice questions

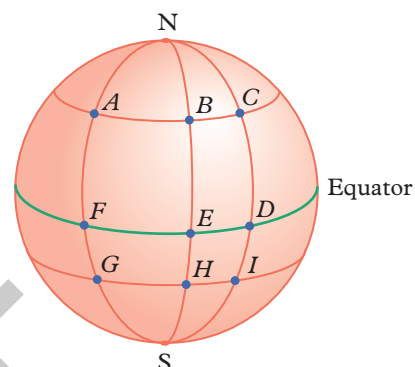
Use the diagram shown for questions 1 and 2.

- 1 Which of the following sets of points lie on the same parallel of latitude?

A A, F, G	B A, E, I
C G, H, I	D C, E, H

- 2 Which of the following sets of points lie on the same meridian of longitude?

A A, F, G	B A, E, I
C G, H, I	D C, E, H



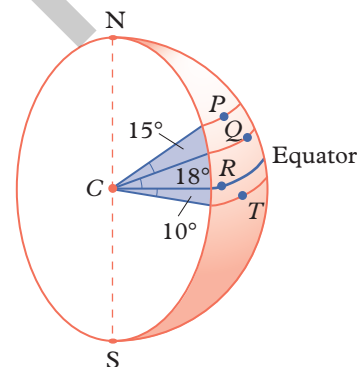
Use the diagram shown for questions 3 and 4.

- 3 What is the latitude of point P?

A 15°N	B 18°N
C 33°N	D 43°N

- 4 What is the difference in latitude between points P and T?

A 5°	B 25°
C 23°	D 43°

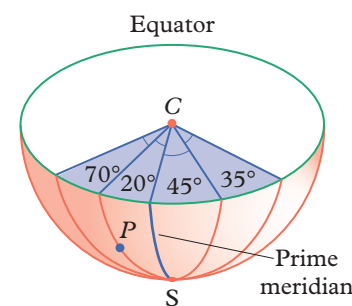


- 5 What is the reference line for longitude on Earth's surface?

- | |
|---|
| A the axis NS |
| B the equator |
| C the prime meridian |
| D the parallel of latitude through Greenwich |

- 6 In the diagram shown, what is the longitude of point P?

A 20°W	B 30°E
C 65°W	D 100°W



- 7 In the diagram shown, what is the point with position coordinates (30°S, 85°E)?

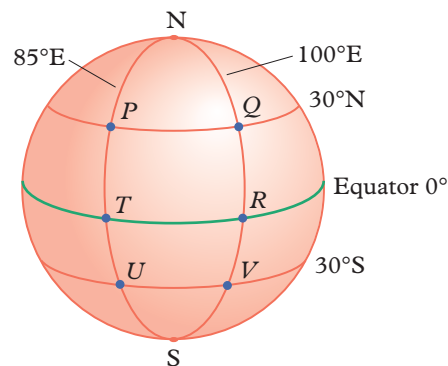
A P	B Q	C U	D V

- 8 What is the point that is 15° north of (10°S, 20°W)?

A (25°N, 20°W)	B (10°S, 35°W)
C (10°S, 5°W)	D (5°N, 20°W)

- 9 If point P has position (12°N, 25°W) and point Q has position (10°S, 30°W), what is the position of Q relative to P?

- | |
|--|
| A Q is 22° south and 5° west of P |
| B Q is 22° south and 5° east of P |
| C Q is 22° north and 5° west of P |
| D Q is 22° north and 5° east of P |



10 What is 1535 expressed in 12-hour time?

A 5:35 pm

B 3:35 pm

C 5:35 am

D 3:35 am

11 What time is it 6 hours 38 minutes before 1625?

A 0947

B 1013

C 2303

D 1047

Use the world standard time zones map (**Source 1**) from Lesson 8.4 to answer questions 12 to 14.

12 What is the time difference between Tokyo and Houston?

A 3:00 hours

B 6:00 hours

C 9:00 hours

D 15:00 hours

13 If it is 2:00 pm in Berlin, what is the standard time in Houston?

A 7:00 am

B 11:00 pm

C 8:00 am

D 3:00 pm

14 When it is 9:00 pm Friday in New York, what is the standard time in Beijing?

A 6:00 pm Friday

B 8:00 am Friday

C midnight Friday

D 10:00 am Saturday

15 Eden leaves Beijing (31°N , 121°E) at 10:00 am Tuesday to fly to Honolulu (21°N , 158°W). The flight takes 13 hours. What day and time is it in Honolulu when Eden arrives?

A Tuesday 11:00 pm

B Wednesday 1:00 am

C Tuesday 6:00 am

D Tuesday 5:00 pm

16 When you cross the International Date Line from east to west, what should you do?

A put the clock forward one day

B put the clock back one day

C put the clock forward 12:00 hours

D put the clock back 12:00 hours

Use the Australian time zones map (**Source 1**) from Lesson 8.5 to answer questions 17 and 18.

17 What is the time in Sydney when it is 3:00 pm in Perth?

A 1:00 pm

B 3:00 pm

C 5:00 pm

D 6:00 pm

18 It is 7:30 am daylight saving time in Adelaide. What is the standard time in Brisbane, which does not have daylight saving time?

A 6:00 am

B 6:30 am

C 7:00 am

D 8:00 am

Review questions 8.6C Set 1

1 Complete the following sentences using either “the same as” or “away from”:

a The centre of a great circle (on Earth’s surface) is _____ the centre of Earth.

b The centre of a small circle (on Earth’s surface) is _____ the centre of Earth.

2 Refer to the diagram shown and write the latitude of the following points.

a J

b K

c L

d M

3 Refer to the diagram shown and write the longitude of the following points.

a D

b E

c F

d G

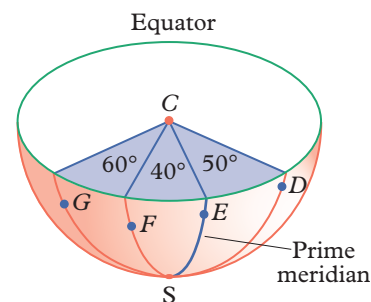
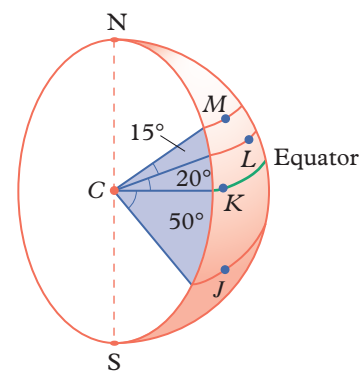
4 Use the diagram shown to name the points with these position coordinates.

a (30°S , 120°W)

b (0° , 80°W)

c (30°S , 80°W)

d (0° , 120°W)



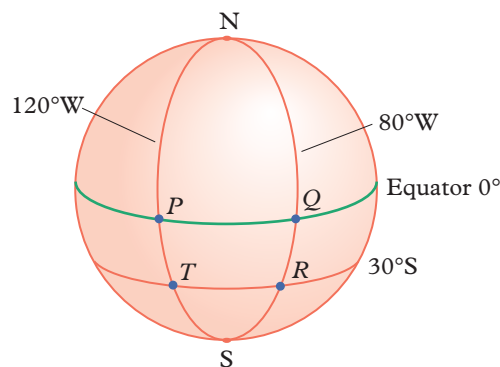
5 Perform each of the following calculations.

- Convert 4 hours 15 minutes to minutes.
- Write the 24-hour time 1046 in 12-hour time.
- Calculate the time 5 hours 49 minutes after 0957.
- Calculate 3 days 9 hours 25 minutes – 1 day 15 hours 37 minutes.
- When it is noon in Greenwich, what is the local time in Canberra?
- If it is 2 : 00 pm in Sydney, what is the standard time in Greenwich?
- When it is 11 : 30 am local time in Darwin, what is the local time in Hobart?

6 Jess leaves Beijing (31°N, 121°E) at 8 : 00 am Wednesday to fly to Honolulu (21°N, 158°W).

- What is the standard time and day in Honolulu when she leaves Beijing?
- The flight takes 11 hours. What is the day and time in Honolulu when she arrives?

7 A plane leaves Perth at 11 : 00 am for Sydney. Due to head winds, the trip takes $4\frac{1}{2}$ hours flying time. At what time will the plane arrive in Sydney (local Sydney time)?

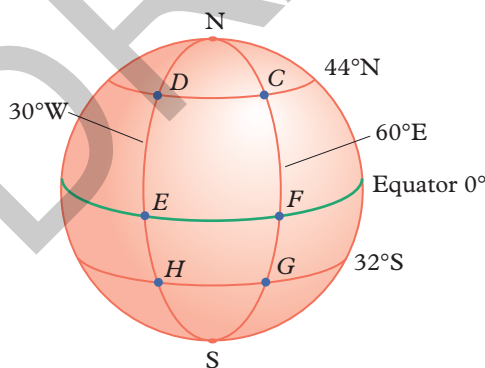


Review questions 8.6D Set 2

1 Explain the meaning of these terms.

- parallel of latitude
- meridian of longitude

2 Use the diagram shown to write the coordinates of these points.



a C	b D	c E
d F	e G	f H

3 Given that point A has position (26°N, 105°E), write the position coordinates of the point that is:

a 15° north of A	b 15° south of A
c 15° east of A	d 15° west of A.

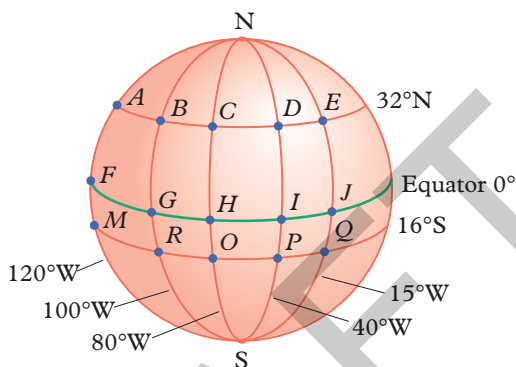
4 Point P has position (16°S, 140°W) and point Q has position (5°N, 120°W). State whether P is:

- to the north or south of Q
- to the east or west of Q.

- 5 Perform each of the following calculations.
 - a Convert 4 days 17 hours to hours.
 - b Convert 9 : 42 pm to 24-hour time.
 - c Calculate 3 days 17 hours 55 minutes + 1 day 14 hours 31 minutes.
- 6 When it is 11 : 00 pm Thursday standard time in Sydney, what is the standard time in Auckland?
- 7 Use the world time zone map (**Source 1**) from Lesson 8.4 to find the standard time in Houston when UTC is 8 : 00 am.
- 8 A plane leaves Sydney at 5 : 00 pm daylight saving time to travel to Perth. The trip takes 3 hours 50 minutes. At what time does the plane arrive in Perth?

Review questions 8.6E Set 3

Use the diagram shown to answer questions **1 to 4**.

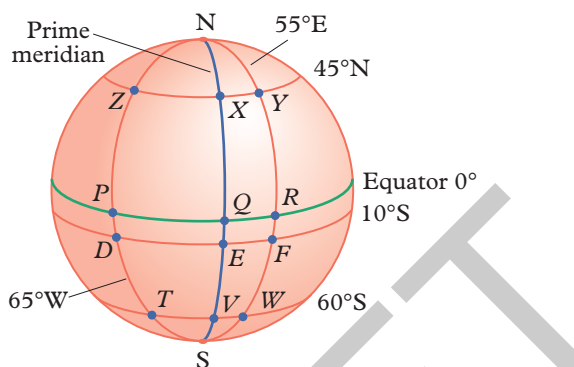


- 1 Write the coordinates of these points.
 - a *E*
 - b *H*
 - c *P*
 - d *F*
- 2 Name a point that is:
 - a due north of *I*
 - b due south of *F*
 - c due east of *C*
 - d due west of *R*.
- 3 State whether:
 - a point *E* is:
 - i north or south of *I*
 - ii east or west of *I*.
 - b point *Q* is:
 - i north or south of *H*
 - ii east or west of *H*.
 - c point *G* is:
 - i north or south of *O*
 - ii east or west of *O*.
 - d point *R* is:
 - i north or south of *C*
 - ii east or west of *C*.
- 4 Write the coordinates of the point that is:
 - a 5° west of *D*
 - b 30° north of *Q*
 - c 12° south of *B*
 - d 15° east of *R*.
- 5 Perform each of the following calculations.
 - a Convert 157 hours to days and hours.
 - b Write the 24-hour time 1534 in 12-hour time.
 - c Convert 3 : 28 pm to 24-hour time.
 - d Calculate the time (in days/hours/minutes) that elapses between 7 : 38 pm Monday and 11 : 30 am Thursday.

- 6 When the time in Chicago is noon Thursday, standard time, what is the standard time in Tokyo?
- 7 A plane leaves Sydney at 11 : 00 am Friday, EST, and flies directly to Paris. If the trip takes 15 hours, at what time does the plane arrive in Paris (Paris standard time)?
- 8 Use the Australian time zones map (**Source 1**) from Lesson 8.5 to find the time in Darwin when it is 11 : 28 am in Cairns.
- 9 A summer concert is held in Perth starting at 6 : 30 pm and it is live streamed around Australia. Calculate the local time in Adelaide, Sydney and Brisbane when the concert starts.

Review questions 8.6F Set 4

Use the diagram shown for questions 1 to 3.



- 1 Write the position coordinates of these points.

a Y	b P	c Q	d D
- 2 Name a point that is:

a due east of T	b due south of E	c due north of P	d due west of F.
- 3 Write the position coordinates of the point that is:

a 20° due north of P	b 30° due west of D	c 15° due south of R	d 50° due east of W.
- 4 Perform each of the following calculations.
 - a Convert 227 minutes to hours and minutes.
 - b Write the 24-hour time 1127 in 12-hour time.
 - c Convert 11 : 23 am to 24-hour time.
 - d Calculate the time 11 hours 33 minutes after 0950.
- 5 When it is noon in Hobart, what is UTC in Greenwich?
- 6 When the time in San Francisco is 5 : 00 pm Monday, standard time, what is the standard time in Moscow?
- 7 A plane leaves Perth at 11 : 00 am and flies to Johannesburg. If the flight takes $12\frac{1}{2}$ hours, at what time will the plane arrive in Johannesburg (Johannesburg standard time)?
- 8 Seo-yeon is in Adelaide and rings her mother to say Happy New Year at midnight Adelaide time. If her mother is in Brisbane, at what local time does she receive the call?

Review questions 8.6G Practice examination question

- 1 Answer the following questions.
 - a What is the reference line for longitude on Earth's surface? (1 mark)
 - b Convert the following times.
 - i Write the 24-hour time 1827 in 12-hour time. (1 mark)
 - ii Write 223 minutes in hours and minutes. (1 mark)
 - c Find the position coordinates of point B, which is 20°N and 15°E of point A(40°N, 45°W). (2 marks)

- d** It is 9:00 am in Adelaide during daylight saving time. What is the time in Brisbane (where there is no daylight saving time)? **(2 marks)**
- e** Perry flew from Rome to Sydney via Singapore where he stopped for 6 hours. He left Rome at 2:40 pm Sunday (Rome time) and arrived at 1:15 am Tuesday (Sydney time).
- i** Use the world times zones map (**Source 1**) in Lesson 8.4 to calculate the time difference between Rome and Sydney. **(1 mark)**
- ii** Calculate the time in Sydney when the plane left Rome. **(1 mark)**
- iii** Calculate the total flight time. **(2 marks)**

TOTAL: 11 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Time and location

I can do this	I need to review this
☐ explain the terms great circle, small circle, axis of rotation, poles, parallels of latitude, meridians of longitude ☐ understand how latitude and longitude are measured	Go back to Lesson 8.1 Latitude and longitude
☐ use the equator and prime meridian as lines of reference for locating position on Earth's surface ☐ describe a location using position coordinates in terms of latitude and longitude	Go back to Lesson 8.2 Position on Earth's surface
☐ convert between seconds, minutes and hours ☐ convert between 12-hour and 24-hour time ☐ calculate elapsed time in practical scenarios	Go back to Lesson 8.3 Time units and intervals
☐ use a world map of time zones to relate time zones to meridians of longitude ☐ determine the local time in different cities around the world ☐ understand the need for the International Date Line and account for date changes in travel questions ☐ Apply $15^\circ = 1\text{ hour}$ time difference to calculate the time differences between locations on Earth given the relevant longitudes	Go back to Lesson 8.4 World time zones
☐ determine the local time in different Australian cities, accounting for daylight saving time ☐ solve practical problems involving Australian time zones	Go back to Lesson 8.5 Australian time zones

Related resources



Module review quiz: Time and location



Worksheet: Challenge 8

Module

9

Networks, paths and trees



Lessons

- Lesson 9.1 Introduction to networks
- Lesson 9.2 Walks, paths and cycles
- Lesson 9.3 Trees
- Lesson 9.4 Minimum spanning trees
- Lesson 9.5 The shortest path
- Lesson 9.6 Review: Networks, paths and trees

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Networks, paths and trees

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Lines and line segments



Support sheet: Interpreting tables

Syllabus links

- Networks, paths and trees (MST-11-07)
 - A student applies network techniques to solve network problems.
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 9.1

Introduction to networks

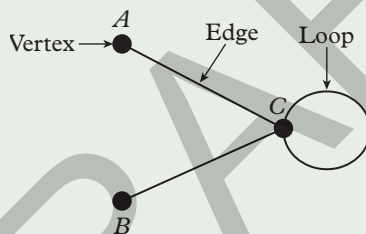
Learning intentions

By the end of this lesson, you will be able to ...

→ identify and construct networks, including directed and weighted networks.

Networks

- This module is about finding the *best* solution to a problem, such as finding the *minimum* spanning tree or the *shortest* path between two places.
- Before we begin solving practical problems, we need to introduce some terminology. A **vertex** is a point, and an **edge** is a line drawn between two vertices. The **degree** of a vertex is the number of edges connected to it.
- An edge drawn from a vertex back to itself is called a **loop**. Since both ends of a loop connect to the vertex, they both count towards the vertex degree.
- Together vertices and edges form a **network**.

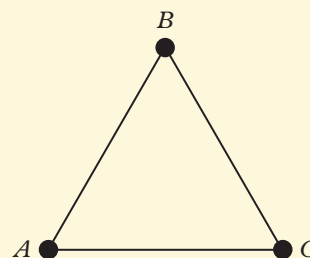


- A **directed network** is a network in which the edges are given a direction indicated by an arrow. A directed edge indicates a *one-sided* relationship between the vertices, such as the direction of a one-way street or the flow of water.
- A **weighted edge** is an edge with a numerical value, such as the length of the street or the rate of flow of water. In practice, the meaning of the edge direction and weight is determined by the context of the network.
- Vertices in a directed network have an indegree and an outdegree, instead of a degree. The **indegree** is the number of edges pointing towards the vertex, and the **outdegree** is the number of edges pointing away from the vertex.

Worked example 9.1A Counting the number of vertices, edges and the degree of each vertex

For the network shown, count:

- the number of vertices
- the number of edges
- the degree of each vertex



Part	Think	Write
a	Vertices are the points in the network. They are denoted by a single letter.	There are three vertices: A , B and C .
b	Edges are the lines joining two vertices together. They are denoted by the letters at either end of the edge, or by their own name.	There are three edges: AB , BC and CA .
c	The degree of a vertex is the number of edges connected to it.	Vertex A is connected to edges AB and CA . It has a degree of 2. Vertex B is connected to edges AB and BC . It has a degree of 2. Vertex C is connected to edges BC and CA . It has a degree of 2.



Worked example video: Counting the number of vertices, edges and the degree of each vertex

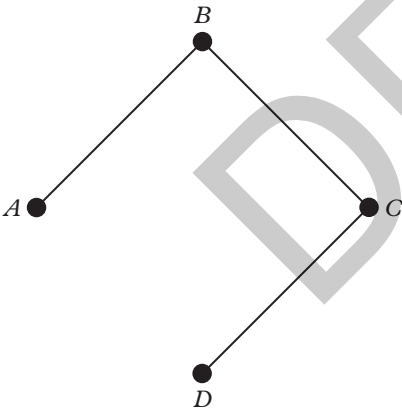
Understanding, fluency and communicating

Answers: Lesson 9.1

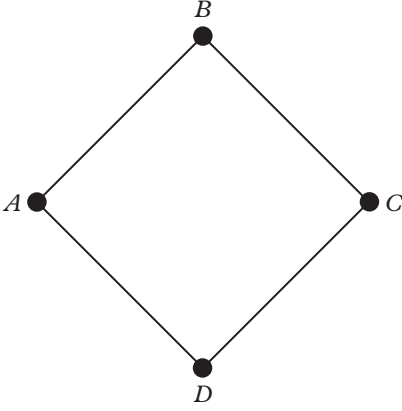
1 For each network:

- i count the number of vertices
- ii count the number of edges
- iii state the degree of each vertex

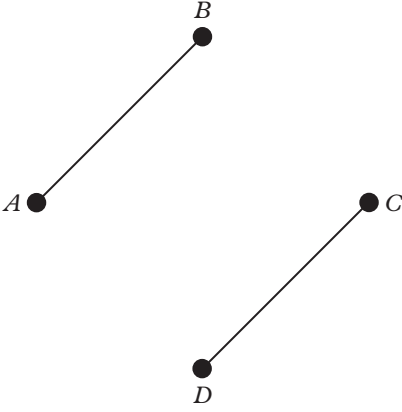
a



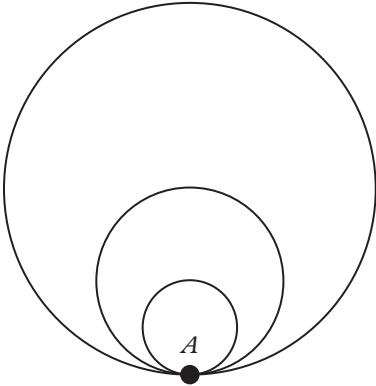
b

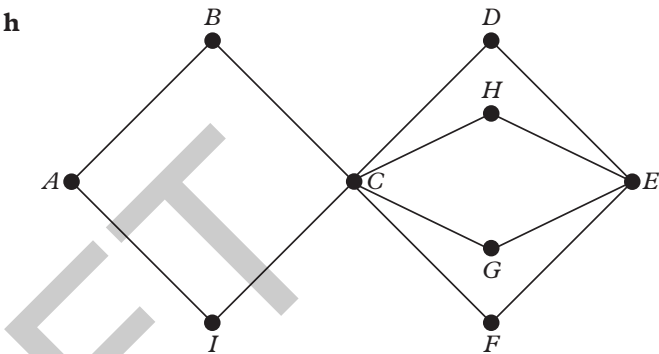
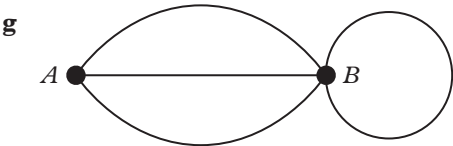
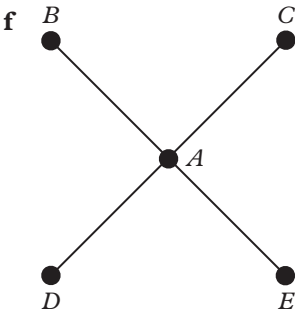
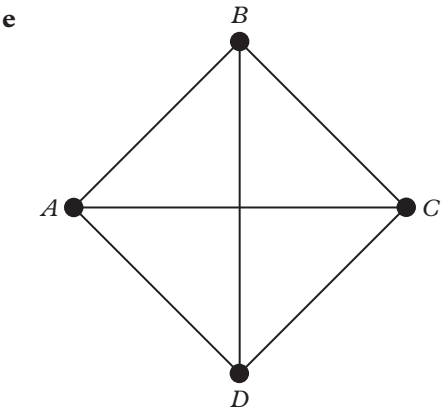


c



d

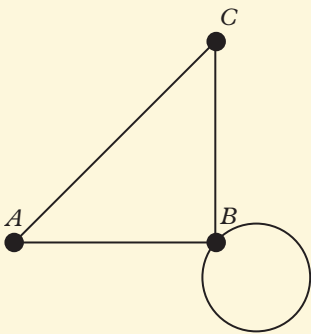




Worked example 9.1B Handshaking lemma

There is a relationship between the number of edges and sum of vertex degrees.

- a For the network shown, make a table of the vertex degrees and the total.
- b How many edges are in this network?
- c Verify that the sum of vertex degrees is twice the number of edges.



Part	Think	Write										
a	Both ends of the loop contribute to the vertex degree of B .	<table><tr><th>Vertex</th><td>A</td><td>B</td><td>C</td><th>Total</th></tr><tr><th>Degree</th><td>2</td><td>4</td><td>2</td><td>8</td></tr></table>	Vertex	A	B	C	Total	Degree	2	4	2	8
Vertex	A	B	C	Total								
Degree	2	4	2	8								
b	Count the number of lines in the network.	The number of edges is 4.										
c	The sum of vertex degrees is always twice the number of edges.	The sum of vertex degrees is $2 + 4 + 2 = 8$ Which is twice the number of edges (4).										

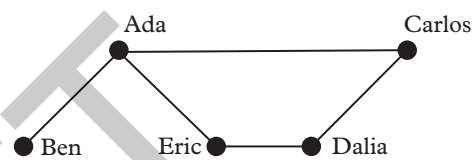
 **Worked example video:** Handshaking lemma

- 2 Consider the networks in question 1 again. For each network:
- i Find the sum of vertex degrees.
 - ii Verify that the sum of the vertex degrees is twice the number of edges.

Worked example 9.1C Drawing a network from a table

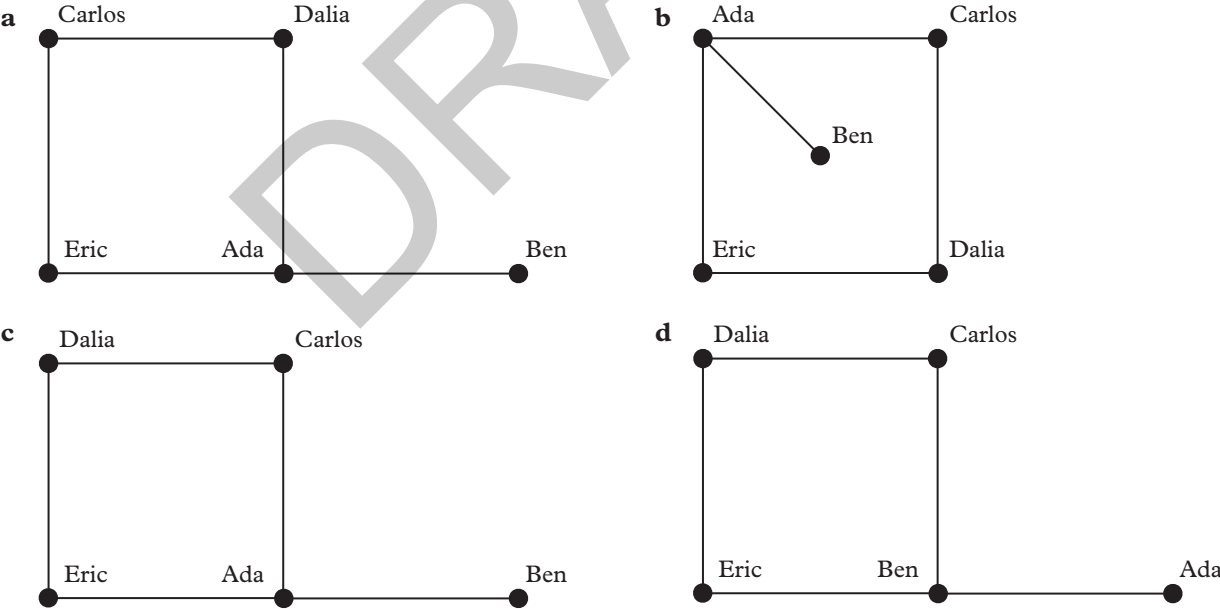
Here is a list of people and their network of friends. Represent this information as a network.

Person	Ada	Ben	Carlos	Dalia	Eric
Friends	Ben, Carlos, Eric	Ada	Ada, Dalia	Carlos, Eric	Ada, Dalia

Think	Write
Use a vertex to represent each person, and use an edge to indicate that they are friends. Note that this is just one of many ways to draw the information in the table.	

 **Worked example video:** Drawing a network from a table

- 3 Determine whether each of the following networks are a drawing of the network from Worked example 9.1C.



- 4 Complete the table of the vertex degrees from Worked example 9.1C.

Vertex	Ada	Ben	Carlos	Dalia	Eric
Degree	3		2		

- 5 The following table represents airports and the destinations serviced by that airport. Draw a network that represents this information.

Airport	Albury (ABX)	Bathurst (BHS)	Coffs Harbour (CFS)	Dubbo City Regional (DBO)	Evans Head (EVH)
Destinations	BHS, DBO	ABX, CFS, DBO	BHS, EVH	ABX, BHS	CFS

- 6 Consider the following table which shows people and their friends.

Person	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Friends	Vanessa, Zhang	Wasim, Uma	Vanessa, Zhang, Yvonne, Xavier	Wasim, Yvonne	Wasim, Xavier	Uma, Wasim

a Draw a network that represents the information in the table.

b Complete the table of vertex degrees.

Vertex	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Degree						

- c Uma is telling Vanessa a story about what happened to a friend of a friend of hers. Use your answer to part a to find out who the story is about.
- d Yvonne is telling Wasim a story about what happened to a friend of a friend of hers. Use your answer to part a to find out who the story is *not* about.

Worked example 9.1D Determining the travel time in a network with weighted edges

The Sydney CBD stations are connected by an underground rail network called the City Circle. Trains start at Central Station and travel in the City Circle clockwise (via Town Hall) or anticlockwise (via Museum). The following table shows the stations in the City Circle (via the Town Hall) line and the time it takes to travel between them.

Station	Central	Town Hall	Wynyard	Circular Quay	St James	Museum
Next stop	Town Hall	Wynyard	Circular Quay	St James	Museum	Central
Time (minutes)	2	2	3	3	2	3

- a Represent the City Circle via the Town Hall line as a directed network with weighted edges.
- b How long does it take to travel from Central to Museum using the City Circle via the Town Hall line?

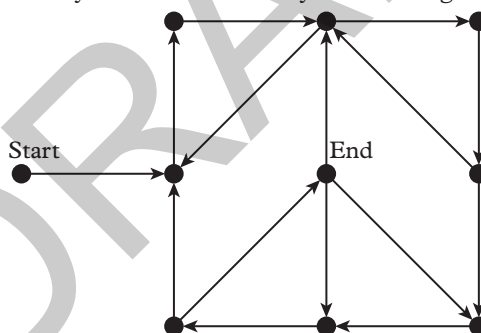


Part	Think	Write
a	Each station is a vertex. The connections between the stations are represented by directed edges and decorated with the time it takes to travel between them.	<pre> graph TD CQ[Circular Quay] -- 3 --> W[Wynyard] W -- 3 --> SJ[St James] SJ -- 2 --> M[Museum] M -- 3 --> C[Central] C -- 2 --> TH[Town Hall] TH -- 2 --> CQ </pre>
b	Travel along this network in the direction of the arrows.	It takes $2 + 2 + 3 + 3 + 2 = 12$ minutes to travel from Central Station to Museum.

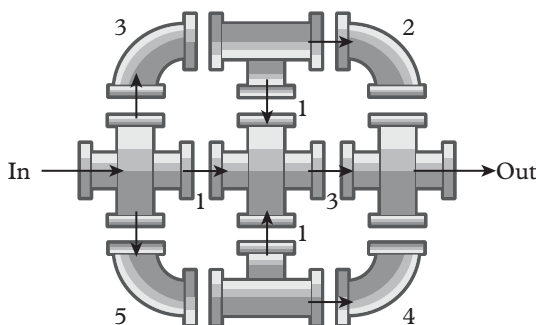


Worked example video: Determining the travel time in a network with weighted edges

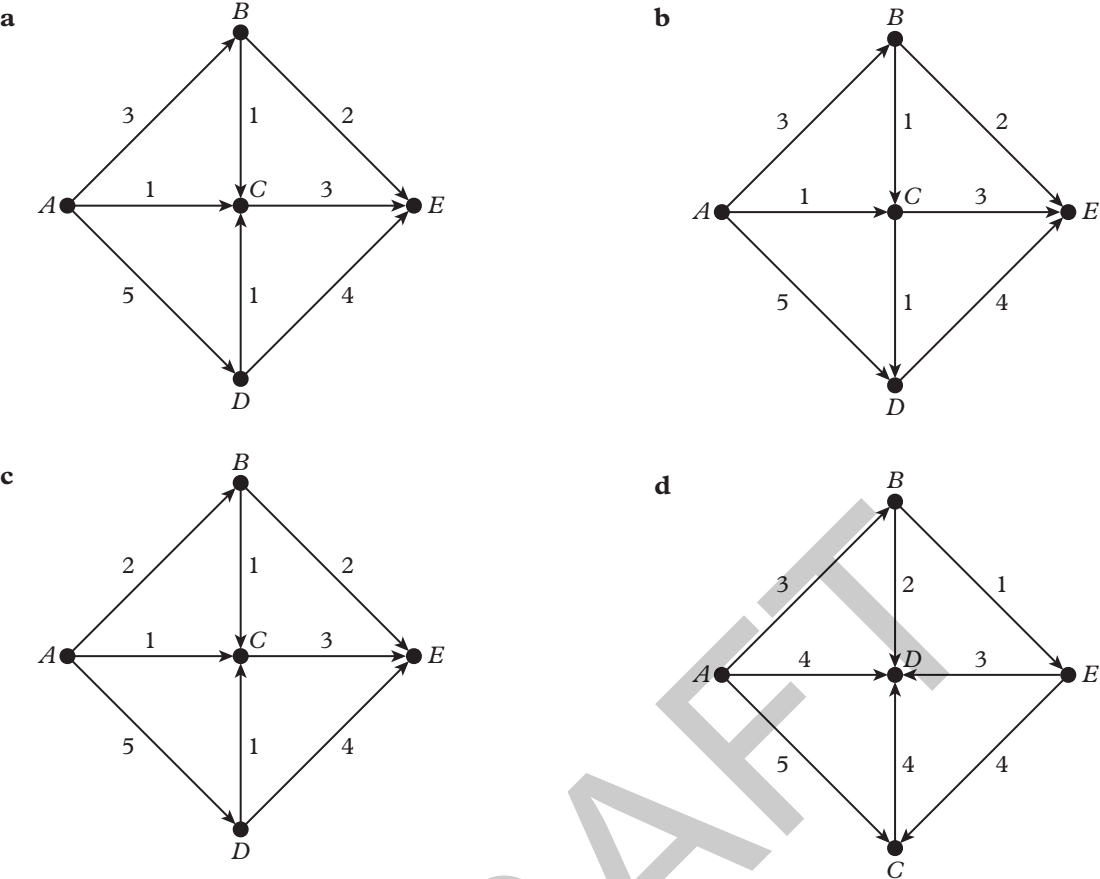
- 7 Using the network from Worked example 9.1D, find the time it takes to travel from Town Hall to Museum using the City Circle (via Town Hall) line.
- 8 Find your way to the centre of the labyrinth. You can only travel along edges in the direction of the arrows.



- a** How many vertices did you touch on your way (including the Start and End)?
 - b** How many edges did you travel along?
 - c** What is the indegree of the End vertex?
 - d** What is the outdegree of the End vertex?
 - e** Is there more than one way from the Start to the End?
- 9** A network of pipes flows from junction to junction as shown. The vertices A , B , C , D and E represent the intersection points of the pipes.



Which of the diagrams below shows the direction of the flow and the amount of the flow, in litres per second, as a directed and weighted network?



10 For the network in question 9, find the total flow, in litres per second:

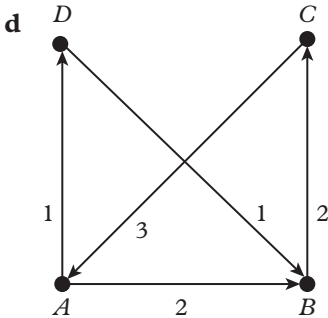
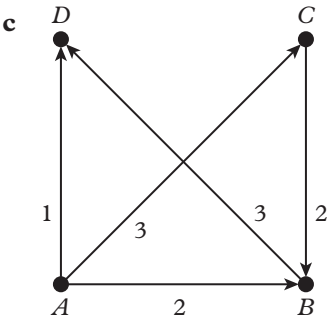
- a** out of vertex *A* **b** into vertex *E* **c** into vertex *B*
d out of vertex *C* **e** out of vertex *D*

11 Here is a table of vertices *A*, *B*, *C* and *D*, and the weight of edges between these vertices.

From vertex	To vertex			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>A</i>	–	2	3	1
<i>B</i>	–	–	–	3
<i>C</i>	–	2	–	–
<i>D</i>	–	–	–	–

Which of the following directed weighted networks represents this information?





12 The table below shows the edge weights from question 9. Fill in the blank edge weights.

	To vertex				
From vertex	A	B	C	D	E
A	–	3			–
B	–	–	1	–	
C	–	–	–	–	3
D	–	–	1	–	
E	–	–	–	–	–

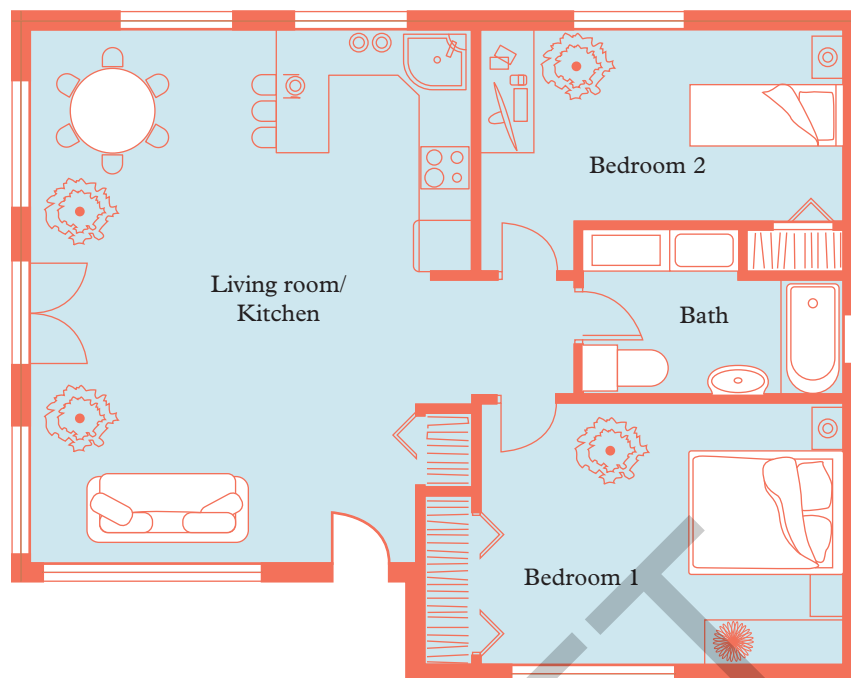
13 Here is a table showing the weights of edges between vertices *A*, *B*, *C* and *D*. Draw a directed weighted network to represent this information.

	To vertex			
From vertex	A	B	C	D
A	–	–	2	5
B	1	–	7	–
C	–	–	–	4
D	–	3	–	–

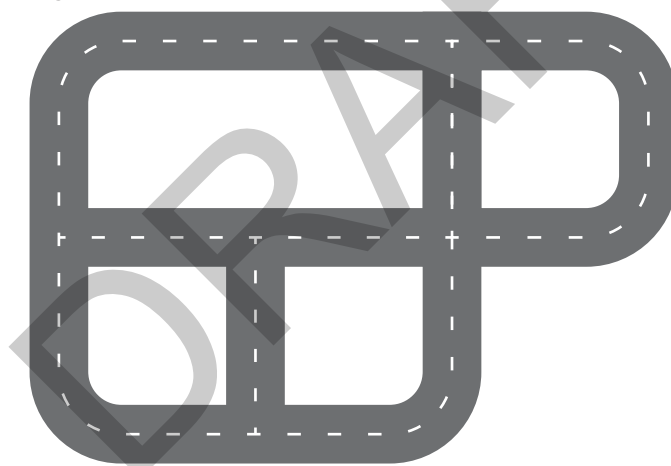
Problem solving and reasoning

- 14 Imagine that the networks in question 1 represent people at a party. The people are vertices, and an edge indicates that the two people have shaken hands (a loop means that the person shakes their other hand, and so counts for two handshakes). How many people shook an odd number of hands in each network?
- 15 A house has a printer, computer, TV and gaming console. All these devices must be connected to a single router via ethernet cables.
- a Draw a network diagram to represent this information. Use vertex names *P*, *C*, *T*, *G* and *R* for the printer, computer, TV, gaming console and router respectively, and use edges to represent an ethernet connection between the devices.
 - b How many ethernet connections are required at *R*?

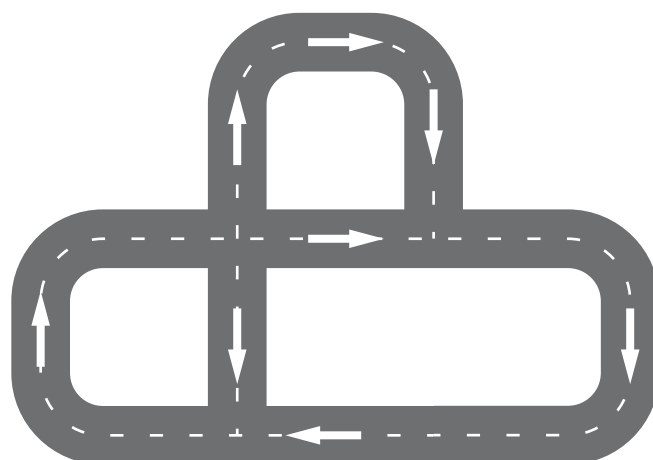
- 16** Draw the following house plan as a network. Use vertices to represent the rooms and edges to represent the doorways that connect the rooms.



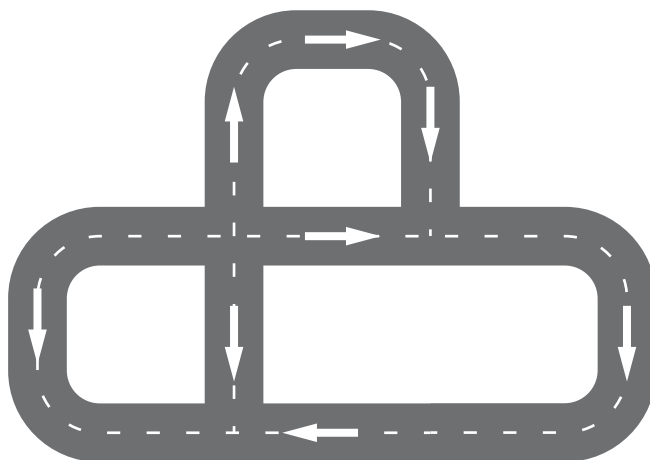
- 17** Redraw the roads below as a network using vertices to represent intersections, and using edges to represent the span of road connecting two intersections.



- 18** Draw a directed network that represents the following street map. Represent the street intersections with vertices, and use directed edges to represent the span of road between intersections.

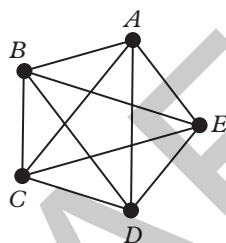


- 19** Here is the same road layout, except that the street orientation has changed. Represent this as a directed network, then use the notions of indegree and outdegree to describe the problem at each intersection.



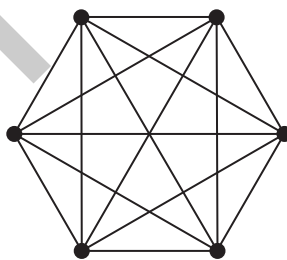
- 20** Colour each edge in the following network either red or blue. Try to do this without drawing a red triangle or a blue triangle.

Hint: first colour as many edges as you can in red without drawing a triangle, then do the same for blue.



Challenge

- 21** Consider the network shown.



- Colour each edge in this network either red or blue.
- There will always be a red triangle or a blue triangle. See if you can find it.
- Suppose that the vertices represent people. The blue edges connect people who have met before, and the red edges connect people who have not met before. If you are hosting a dinner party and want to ensure that three people will all have met before, or three people will have never met before. What is the smallest number of people you should invite?

Hint: compare your answers to questions **20** and **21b**.

Related resources



9.1 Quick quiz



Worksheet: Networks

Lesson 9.2

Walks, paths and cycles

Learning intentions

By the end of this lesson, you will be able to ...

- identify and describe walks, paths and cycles, and distinguish between connected and disconnected networks.

Walks, paths and cycles

- In this section we will explore walks through a network. A **walk** is a journey along the edges of a network from one vertex to another.
- Trace your finger over the network shown from vertex A to vertex D . The vertices and edges that you touched, in the order that you touched them, are:

A, f, B, g, C, h, D .

- This sequence of vertices and edges is a walk. It can be read as follows:

Starting from A , travel along edge f to vertex B , then along edge g to vertex C , then along edge h to vertex D .

- Vertex A is called the “start vertex” and vertex D is called the “end vertex”.

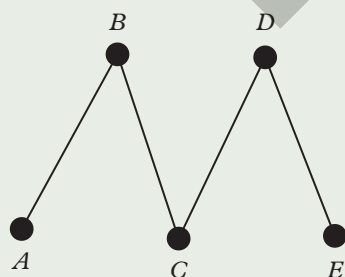
- It is sometimes convenient to abbreviate the walk to just the vertices: $ABCD$.

The edges f, g and h are implicit in the walk that visits vertices A, B, C and D . This is convenient when the edges are un-named, and we will use this style of abbreviation often in this chapter.

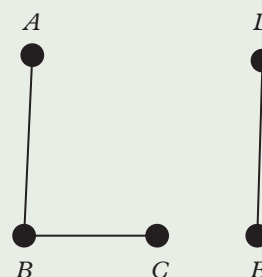
- Another style of abbreviation is to use only the edges: fgh .

The vertices A, B, C and D are implicit in the walk with edges fgh . This is convenient when the edges are named.

- Connected networks** are networks in which there is at least one walk between any two vertices. If there is a pair of vertices with no walk between them, then the network is said to be a **disconnected network**.

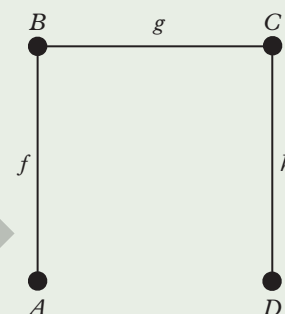


Connected network



Disconnected network

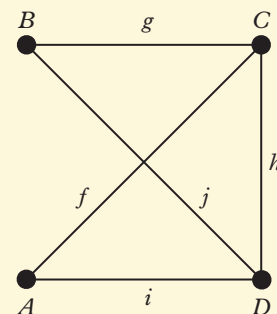
- A **path** is a walk in a network where each vertex is unique. In other words, it does not re-visit any vertices.
- A **cycle** is a walk with the same start and end vertex, and where every other vertex is unique.
 - A cycle is sometimes referred to as a **closed path**. In this case, a path as described above is called an **open path**.



Worked example 9.2A Finding a walk in a network

Which of the following is a walk from A to B ?

- a AB
- b A, f, C, g, B
- c A, f, C, h, D
- d B, j, D, i, A

**Think**

Option A is not a walk since there is no edge between A and B .

Option C ends at D .

Option D is a walk from B to A instead of A to B .

Write

Option B (A, f, C, g, B) is a walk from A to B .

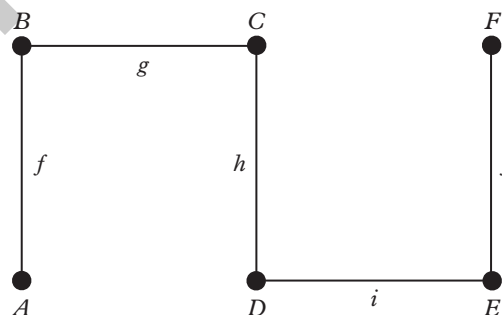


Worked example video: Finding a walk in a network

Understanding, fluency and communicating

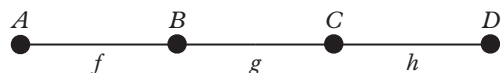
Answers: Lesson 9.2

- Trace out a walk from A to F in the network shown. Write down each vertex and edge that you visit in order.
- In the network from question 1, trace out the walk with edges ihg .
 - Which is the start vertex?
 - Which is the end vertex?
- The streets of Manhattan are divided into East and West by 5th Avenue. The following network is a map of Manhattan between 4th and 6th avenues and 43rd and 42nd streets.

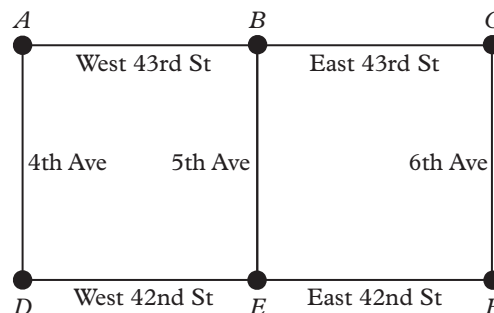


You are given the following directions: Start at corner A . Follow West 43rd St to corner B . Follow 5th Avenue to corner E . Follow East 42nd St to corner F . Then follow 6th Avenue to corner C . Are these valid directions? Where do they lead?

- Consider the network shown.



- Consider the walk fgh . What is the sequence of vertices visited by this walk? What are the start and end vertices?
- Consider the walk $DCBA$. What is the sequence of edges visited by this walk? What are the start and end vertices?
- Consider the walk $ABCD CBA$. What is the sequence of edges visited by this walk? What are the start and end vertices?

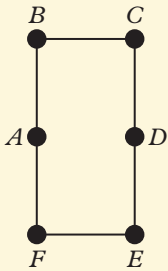


Worked example 9.2B Identifying walks, paths and cycles

Identify the following as either a walk, path, cycle or other, giving reasons for your choice.

- a** $ABCDEDE$
c $ABCDEF A$

- b** $DEFA$
d AD



Part	Think	Write
a	Vertex D appears twice.	This is a walk.
b	This follows the edges of the network and does not visit any vertex more than once.	This is a path.
c	The vertices are all different, except the start and end which are the same.	This is a cycle.
d	There is no edge between A and D .	This is not a valid walk.

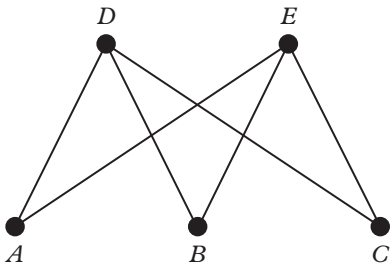


Worked example video: Identifying walks, paths and cycles

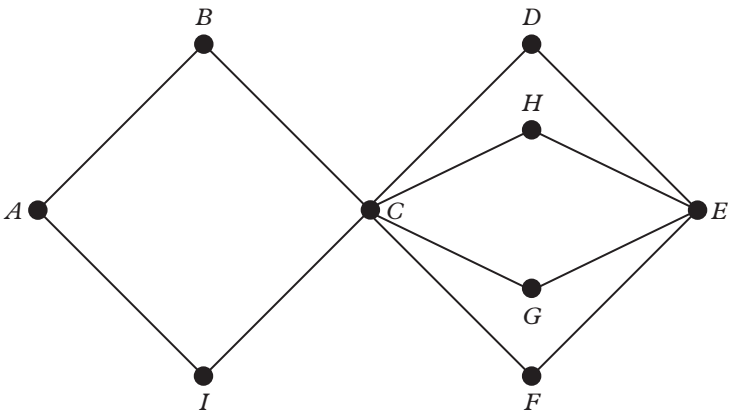
Helpful hints

- In Worked example 9.2B, the walks are referred to as a sequence of vertices. This is common practice when the edges do not have labels.

- 5** Which of the following is not a path from A to C ?
- a** ABC
b ADC
c AEC
d $AEBDC$
- 6** Which of the paths from question **5** visits every vertex?
- 7** Consider the following network.



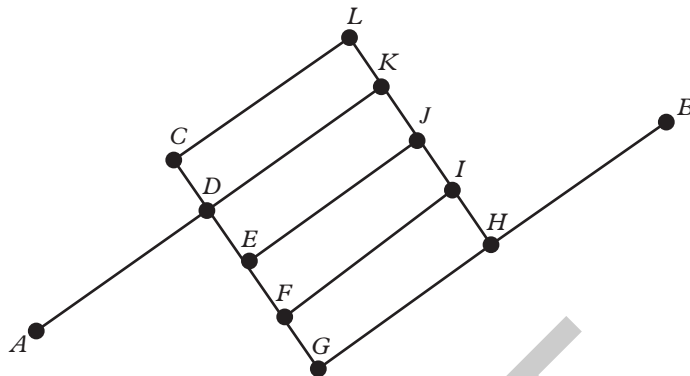
- a** Find a path from A to E .
b Find a cycle that starts from vertex A .
c Find a path from F to B that includes vertex D .
d Find a cycle that starts from vertex H and includes vertex F .



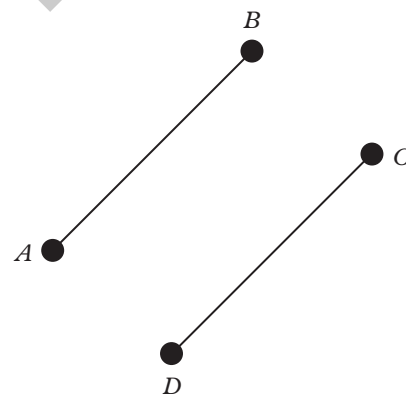
- 8 Consider the network in question 7 again. Find a walk that visits every edge exactly once. Is your walk a cycle? Is your walk a path?

Problem solving and reasoning

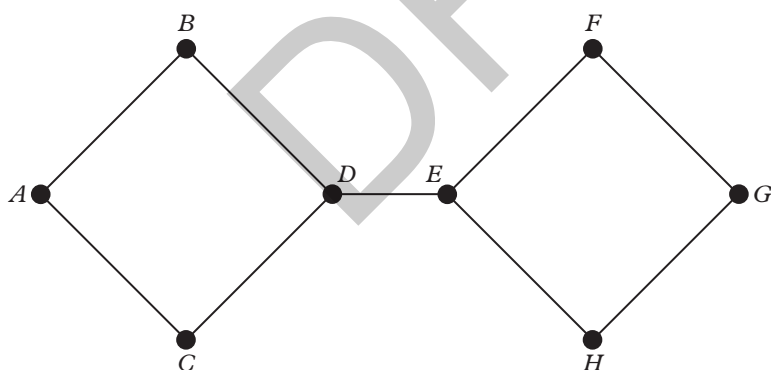
- 9 The following map shows a network of roads between Broken Hill High School (A) and the local park (B).



- a If it takes one minute to walk down the short side of a block, and three minutes to walk down the long side, how long does it take to walk from A to B ?
- b How many different routes from A to B take this amount of time?
- 10 The following network is not connected because there is no path between which two vertices?
- a A and B
- b D and C
- c A and C
- d C and C



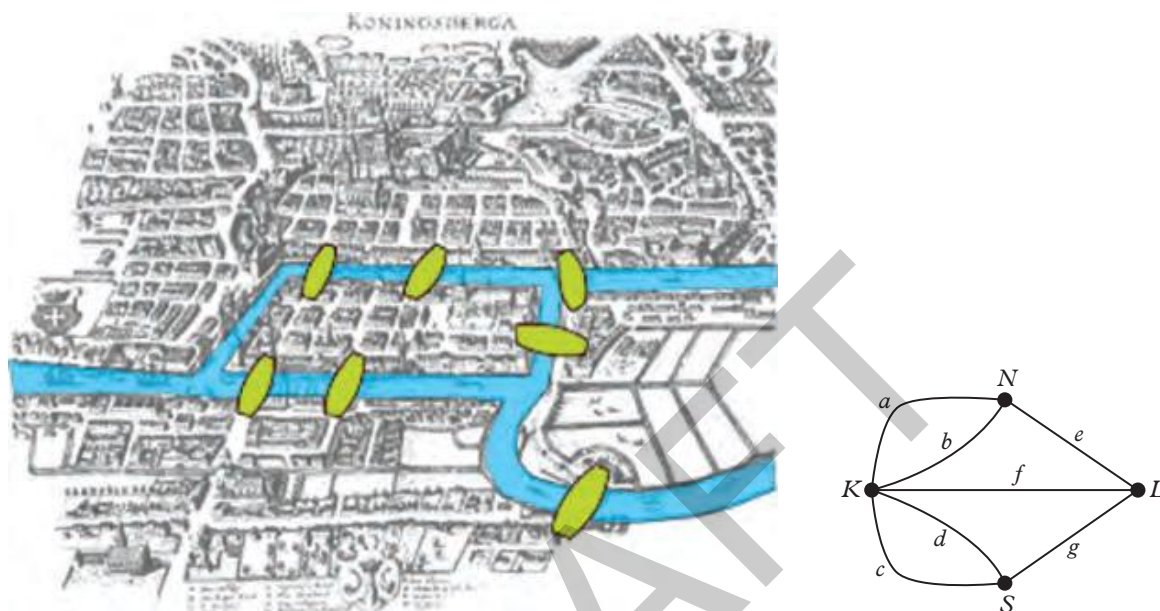
Use this network to answer questions 11 and 12.



- 11 Which single edge should be removed to disconnect the network?
- 12 Find a walk in the network that visits every edge exactly once.

Challenge

- 13** A Hamiltonian path is a path that visits every vertex in a network.
- Find a Hamiltonian path in the network for question 11.
 - Explain why there is no Hamiltonian path in the network from question 7.
- 14** The old city of Königsberg lies on the river Pregel. The city has seven bridges, as shown below. The residents of Königsberg were intrigued by a question: Is it possible to find a walk through the city which crossed each of the seven bridges exactly once?



The mathematician Leonard Euler found the answer to this question, and in doing so he created the branch of mathematics now known as graph theory. Euler realised that the problem was fundamentally about how many bridges were connected to each region of the city.

The city regions are represented by vertices: N for the north side of the river, S for the south side, K for Kneiphof Island and L for Lomse Island. The bridges are represented by edges. The network above shows the city regions as vertices and the bridges between them as edges. Now we need to solve the problem: “Does there exist a walk that visits every edge exactly once?”

Surprisingly, the answer only depends on the number of vertices with odd degree. If there are exactly 2 vertices with odd degree, then there is such a walk. Otherwise, there is no such walk.

- Complete a table of vertex degrees.
- Is there a walk that visits every bridge exactly once?

Related resources



9.2 Quick quiz



Worksheet: Walks, paths and cycles

Lesson 9.3

Trees

Learning intentions

By the end of this lesson, you will be able to ...

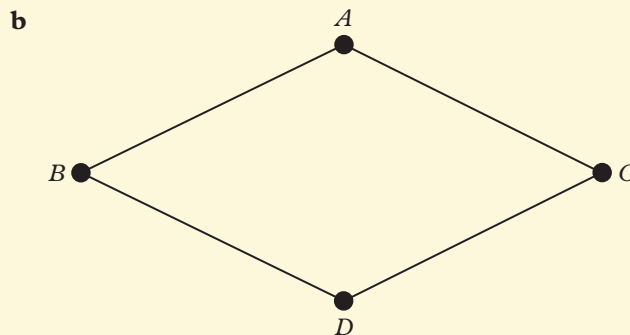
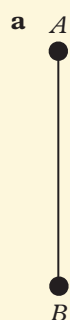
→ identify trees and forests and find spanning trees for a given network.

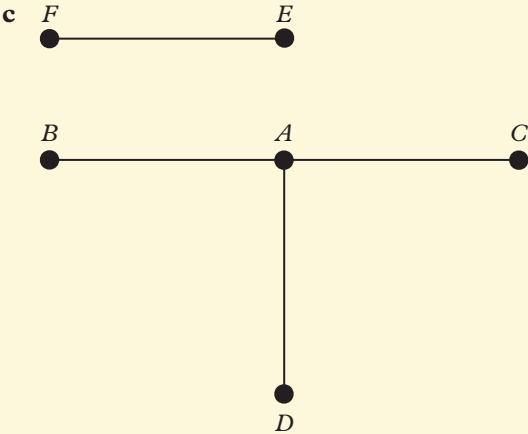
Trees

- A **tree** is a special kind of network with exactly one path between any two vertices. In other words, a tree is a connected network with no cycles.
- Any connected network contains a **spanning tree** consisting of all the vertices from the original network and some of the edges. Every connected network has at least one spanning tree.
 - For a connected network, a spanning tree can be obtained by a “pruning” procedure:
 - Step 1:** Find a cycle.
 - Step 2:** Remove any edge from that cycle.
 - Step 3:** Repeat until there are no more cycles.
 The remaining network will be a spanning tree.
 - Note that a spanning tree always has one less edge than the number of vertices.
- A **forest** is a network with no cycles. In other words, a forest is a network of disconnected trees.

Worked example 9.3A Identifying trees

Identify whether or not the following networks are trees.





Part	Think	Write
a	This network has no cycles and is connected.	This is a tree.
b	This network contains a cycle $ABDCA$.	This is not a tree because it contains a cycle.
c	There is no way to get from A to E .	This is a forest of two trees.

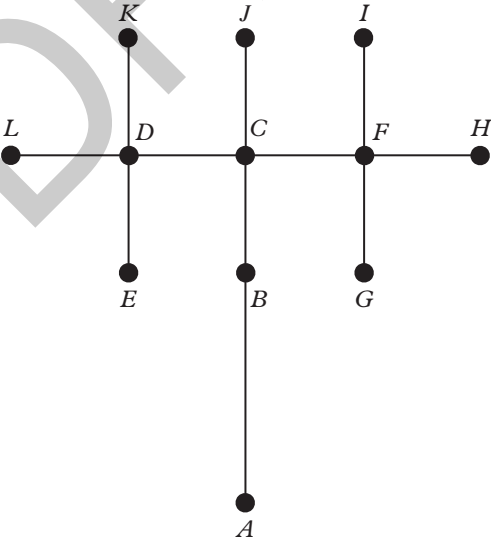


Worked example video: Identifying trees

Understanding, fluency and communicating

Answers: Lesson 9.3

1 The network given here is a tree, because there is exactly one path between any two vertices.

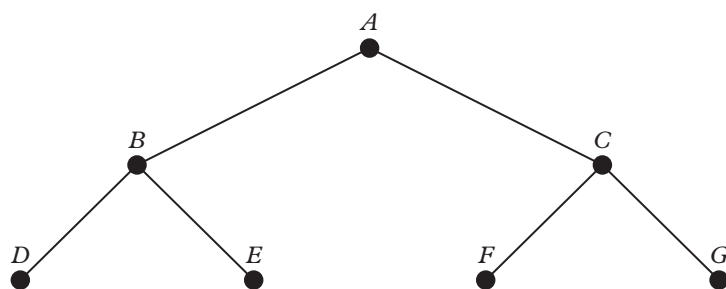


Find the unique path from:

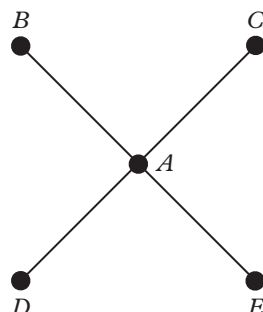
- a A to L
- b L to E
- c B to H
- d G to K .

2 State whether each of the following networks are also trees.

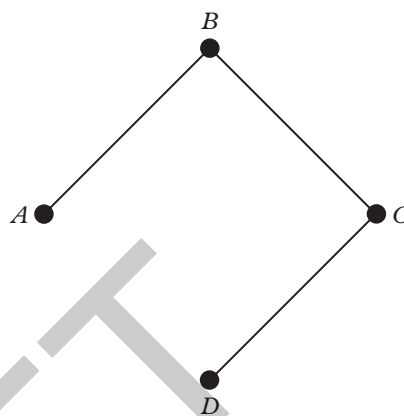
a



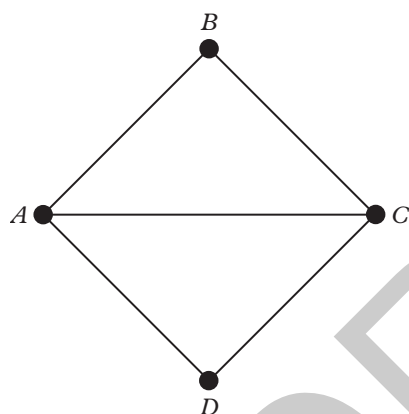
b



c



d



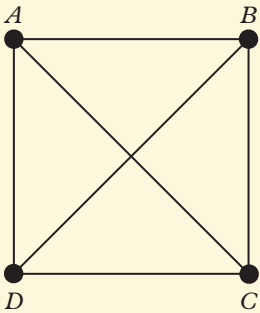
3 Count the number of edges and vertices in each of the trees from question 2.

4 Draw the tree shown in the photo as a network. Use vertices to represent the end of a branch (in network terminology, such a vertex is called a **leaf**), as well as the joins between branches. Draw edges to represent the branches. Is your network a mathematical tree?



Worked example 9.3B Constructing a spanning tree

Construct a spanning tree for the network shown.

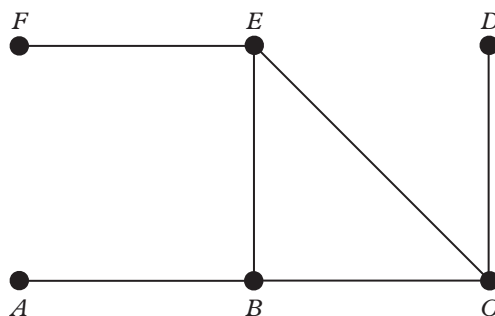


Step	Think	Write
1	Identify that $ABCA$ is a cycle. Remove any edge from the cycle. In this example we have chosen to remove AB .	Remove AB :
2	Identify that $ACDA$ is a cycle. Remove any edge from the cycle. In this example we have chosen to remove AC .	Remove AC :
3	Identify that $BCDB$ is a cycle. Remove any edge from the cycle. In this example we have chosen to remove BC .	Remove BC :
Stop	There are no more cycles.	This is a spanning tree.



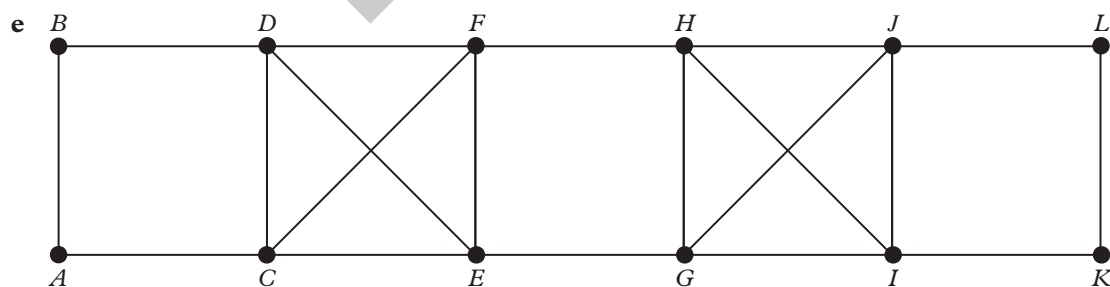
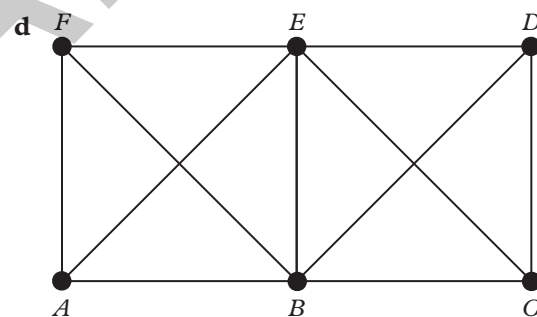
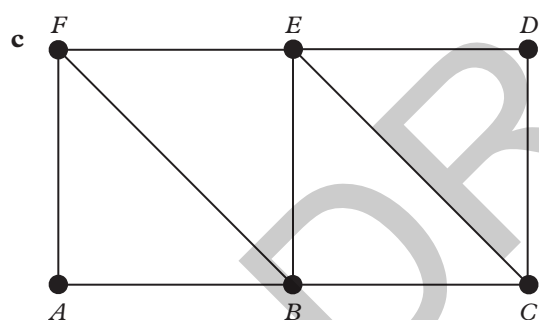
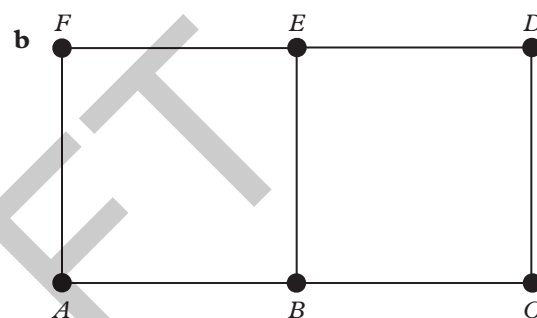
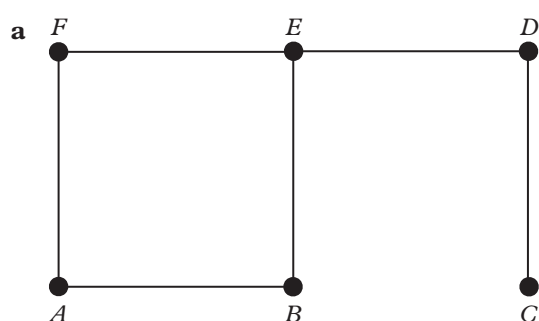
Worked example video: Constructing a spanning tree

5 Which of the following edges should be removed to transform this network into a tree?



- a Edge AB
- b Edge BC
- c Edge CD
- d Edge EF

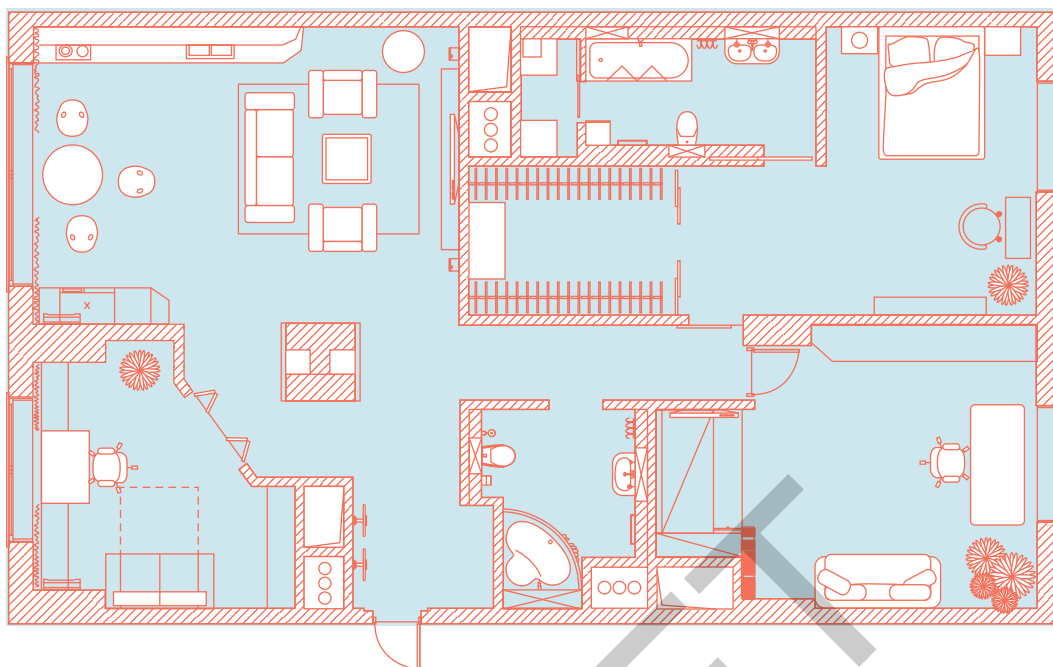
6 Find a spanning tree for the following networks.



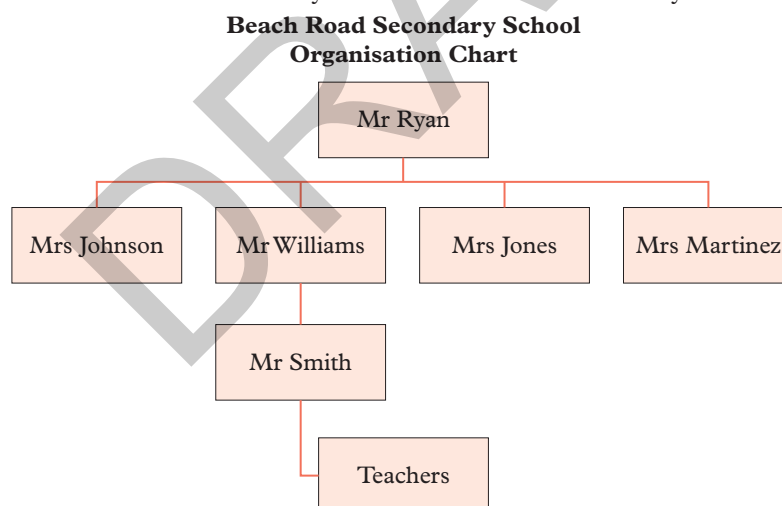
7 Find a different spanning tree to the ones you found for questions 6a and 6e.

Problem solving and reasoning

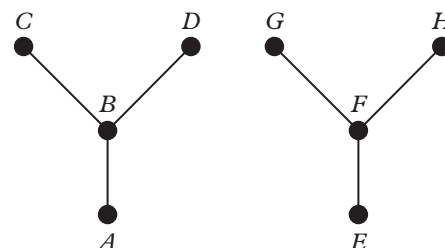
8 Consider the house plan shown.



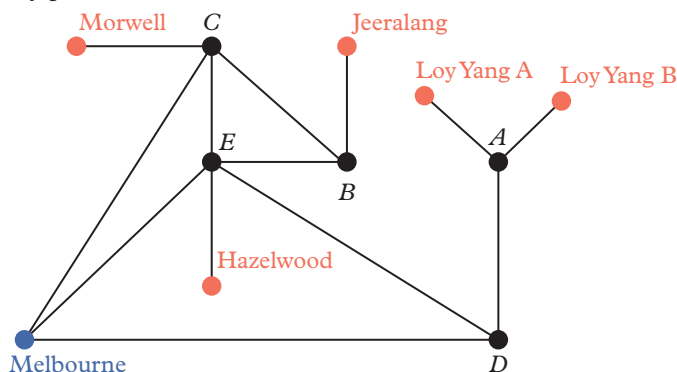
- Draw a network that represents the house plan. Use vertices to represent the rooms and edges to join rooms connected by a door.
 - Is the network a tree? Does that make it a tree house?
- 9 This organisational chart shows the hierarchy within Beach Road Secondary School.



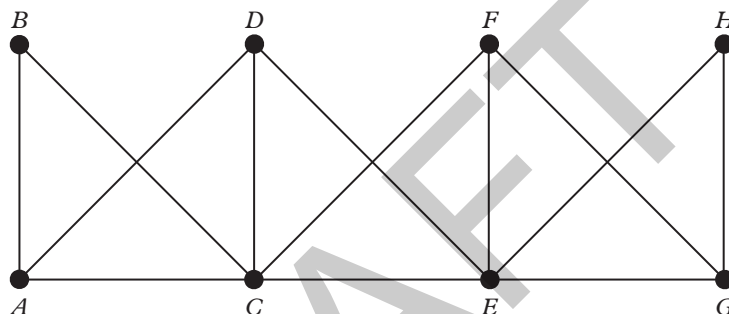
- Draw the organisational chart as a network.
 - Is the network a tree?
 - A teacher reports a serious incident to Mr Smith. How many steps will it take for this report to be handed up to Mr Ryan?
- 10 If a tree is made up of n edges, how many vertices does it have?
- 11 Identify the two trees in the forest shown. Show that the network is not connected by stating two vertices with no path between them, and then turn the forest into a tree by adding an edge between those vertices.



- 12 The network shows several power stations in the Latrobe Valley, Victoria. The power stations, represented by red vertices, are connected to terminals represented by black vertices. Power is transmitted to Melbourne across several transmission lines, represented by edges. Remove as many transmission lines as you can without disconnecting any power station from Melbourne.



- 13 Suppose a spanning tree contains 10 vertices. How many edges are in the spanning tree?
 14 For the network shown, find a spanning tree that is also a path.



- 15 For the network in question 14, find a spanning tree that is *not* a path.
 16 Can a single tree be a forest? Explain your thinking.
 17 Consider a network with five vertices and no edges. Is this a forest?
 18 Consider a network with five vertices and exactly one edge.
 a If the edge is a loop, is this a forest?
 b If the edge is not a loop, is this a forest?

Challenge

- 19 Draw a network that is not a tree but has exactly one spanning tree.

Related resources



9.3 Quick quiz



Worksheet: Trees and forests

Lesson 9.4

Minimum spanning trees

Learning intentions

By the end of this lesson, you will be able to ...

→ describe minimal spanning trees and use algorithms to construct them.

Minimum spanning trees

- Spanning trees in a weighted network inherit the weights of their edges. The spanning tree with the smallest total edge weight is called the **minimum spanning tree**.
- Looking at all possible spanning trees and choosing the one with the lowest total edge weight is impractical for large networks. Fortunately, there are two ways to find a minimum spanning tree that do not require listing every possible tree: Kruskal's algorithm and Prim's algorithm.

Kruskal's algorithm

- Kruskal's algorithm** was developed by Joseph Kruskal in 1956. Given a weighted network, a minimum spanning tree can be constructed as follows:
 - Step 1:** Choose all of the vertices in the network, but none of the edges.
 - Step 2:** List all the edges of the network from smallest weight to largest.
 - Step 3:** Proceed through your list of edges, starting with the edge of smallest weight. Add edges one by one, provided that your selection does not make a cycle.
 - If your original network has n vertices, then you can stop after selecting $n - 1$ edges. The outcome is guaranteed to be a spanning tree of minimum total edge weight.
- Kruskal's algorithm starts with a forest of vertices and adds edges one by one, starting with the edge of smallest weight. At every step, the network is a forest. The final step is when the forest grows into a minimum spanning tree.

Prim's algorithm

- The second procedure for finding a minimum spanning tree is **Prim's algorithm**.
- Starting with an initial seed vertex, Prim's algorithm selects an adjacent edge and adds it to the tree. This produces a minimum spanning tree, which can be constructed as follows:
 - Step 1:** Choose any vertex as the start vertex. This is the "seed" of the tree.
 - Step 2:** Consider the edges adjacent to your tree thus far; that is, the edges with one vertex inside the tree and another outside the tree. Select the edge with *minimum weight*. Grow the tree by extending it along the selected edge (and vertex).

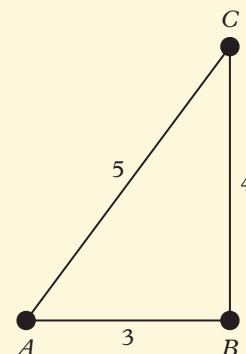
Step 3: Repeat **Step 2** until your tree grows into a spanning tree.

→ In **Step 2**, if there are many edges with the same minimum weight, then any one of them can be chosen.

- Prim's algorithm starts from a single vertex and grows it into a spanning tree.
- In contrast, Kruskal's algorithm includes every vertex from the start and grows the forest into a spanning tree. The two algorithms create minimum spanning trees, but in different ways.

Worked example 9.4A Determining the minimum spanning tree by inspection

The network shows the distances (in kilometres) between three power plants A , B and C . The power plants must be connected by transmission lines. Design the shortest possible transmission network that connects power plants A , B and C .



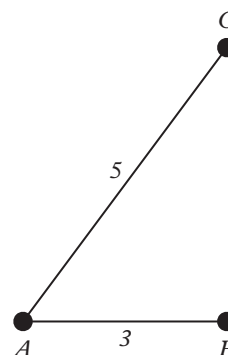
Think

The power plants can be connected by a spanning tree. There are three possible spanning trees.

Write

Option 1

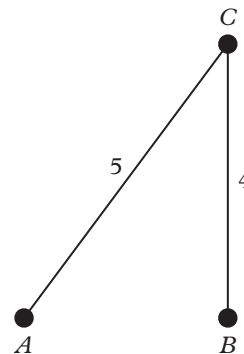
Spanning tree:



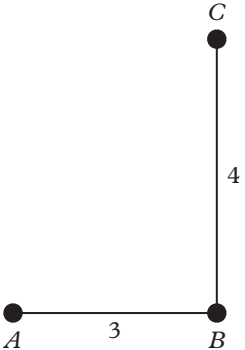
Total distance: $5 + 3 = 8$

Option 2

Spanning tree:



Total distance: $5 + 4 = 9$

Think	Write
	Option 3 Spanning tree:  Total distance: $3 + 4 = 7$
The spanning tree in option 3 covers the least total distance.	Option 3 is the minimum spanning tree.

 **Worked example video:** Determining the minimum spanning tree by inspection

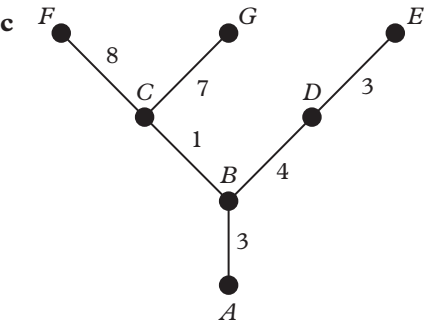
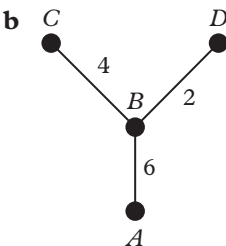
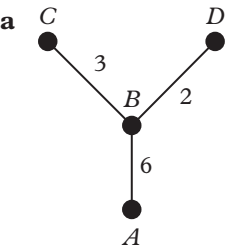
Understanding, fluency and communicating

Answers: Lesson 9.4

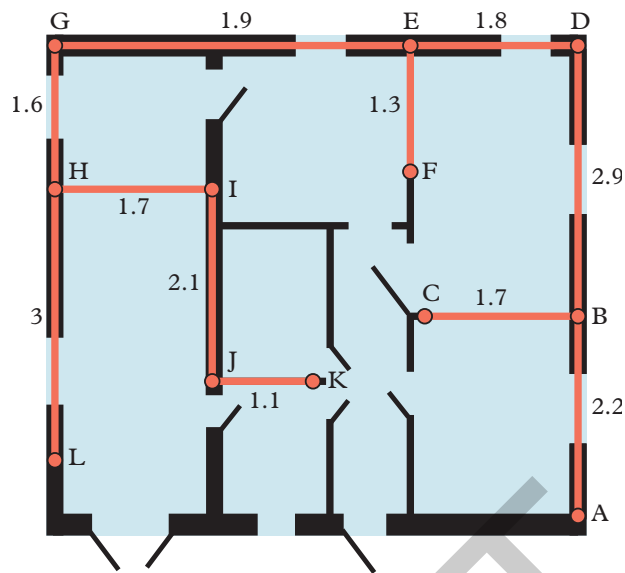
Helpful hints

- For a small network, we can list all possible spanning trees to determine the minimum spanning tree.

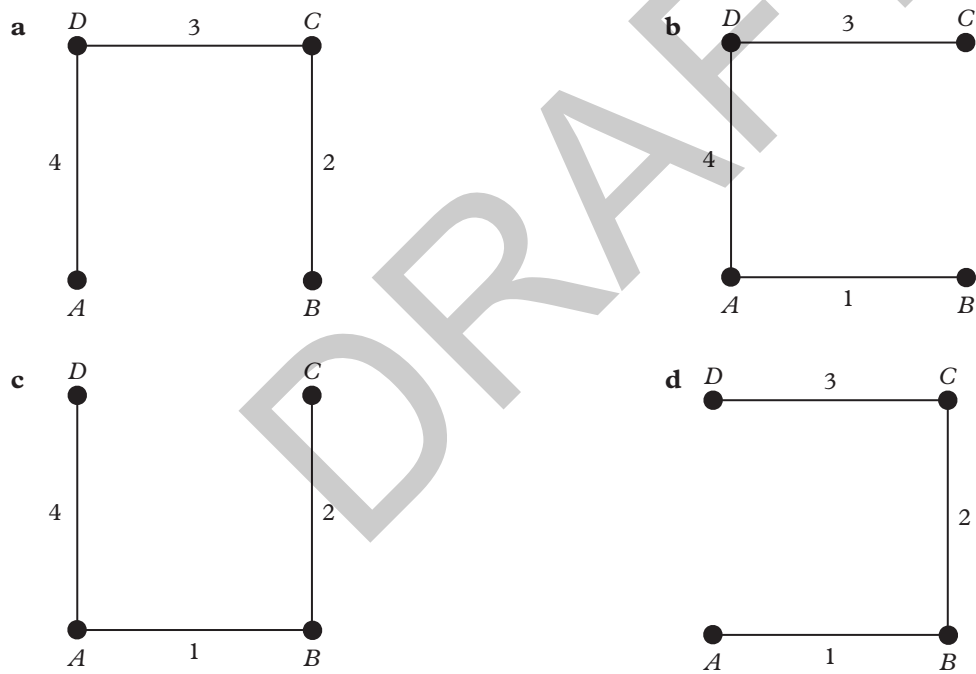
1 What is the total weight of each the following trees?



- 2 The image shows a house plan and the electrical cables that connect the electricity meter at A to the power outlets throughout the house. The length of wire is shown against each edge. What is the total length of wire required?

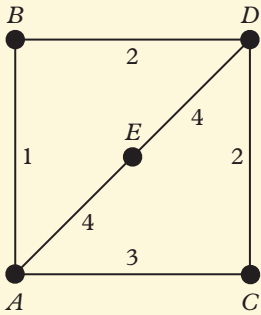


- 3 Four vertices A , B , C and D can be connected in the four configurations shown. What is the total weight of each configuration? Which configuration has the smallest total edge weight?

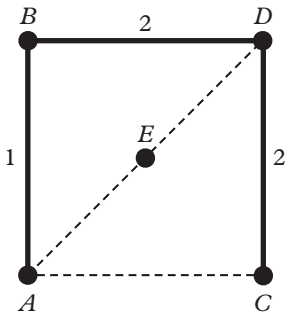
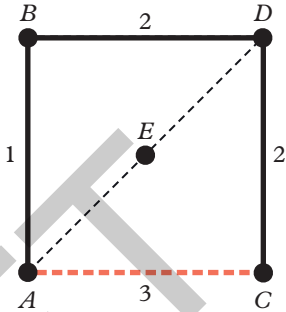
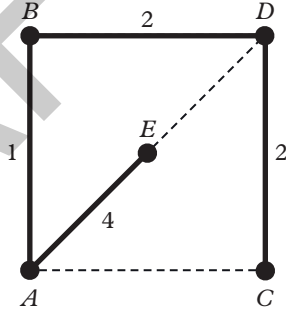


Worked example 9.4B Using Kruskal’s algorithm

Use Kruskal’s algorithm to find the minimum spanning tree for the following weighted network.



Think	Write														
Use the diagram to fill out the table of edge weights. Proceed through this list, adding edges that do not form a cycle. The network has 5 vertices, so we stop once $5 - 1 = 4$ edges have been selected.	The list of edge weights is as follows. <table><tr><th>Edge</th><td>AB</td><td>DC</td><td>BD</td><td>CA</td><td>AE</td><td>ED</td></tr><tr><th>Weight</th><td>1</td><td>2</td><td>2</td><td>3</td><td>4</td><td>4</td></tr></table>	Edge	AB	DC	BD	CA	AE	ED	Weight	1	2	2	3	4	4
Edge	AB	DC	BD	CA	AE	ED									
Weight	1	2	2	3	4	4									
Choose all the vertices.															
Include the first edge AB.															
Include the second edge DC.															

Think	Write
Include the third edge BD .	
Skip the fourth edge CA , since $ABDCA$ would be a cycle.	
Include the fifth edge AE .	
This is a connected network.	The selected vertices and edges form a minimum spanning tree.

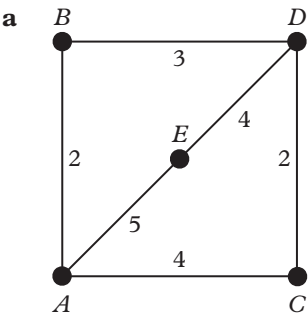


Worked example video: Using Kruskal's algorithm

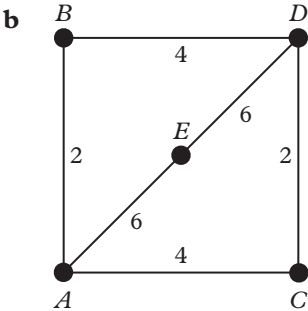
Helpful hints

- The steps in Kruskal's algorithm are:
 - Step 1:** List all the edges of the network from smallest weight to largest weight.
 - Step 2:** Choose all the vertices in the network, but none of the edges.
 - Step 3:** Proceed through your list of edges, starting with the edge of smallest weight. Add edges one by one, provided that your selection does not make a cycle.
- If your network has n vertices, you can stop after adding $n - 1$ edges.

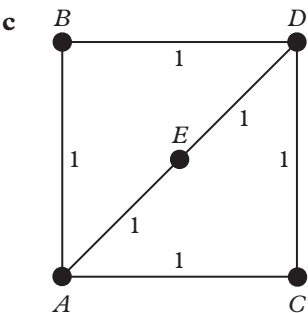
4 Complete the following tables representing the edges and edge weights in each network.



Edge	AB	DC	BD	DE	AC	AE
Weight	2		3			

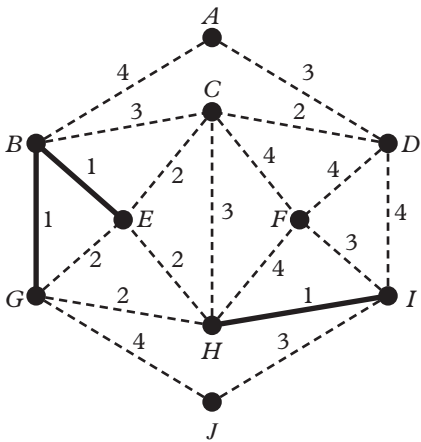


Edge	AB	DC	BD	AC	ED	AE
Weight		2				6

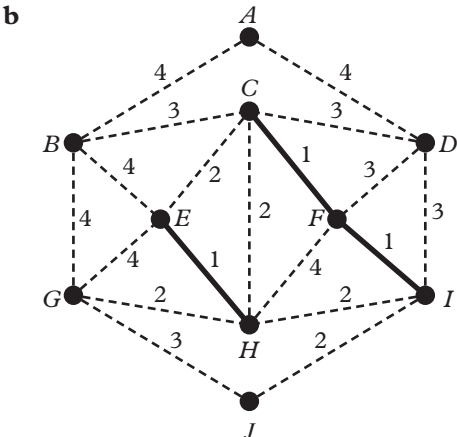
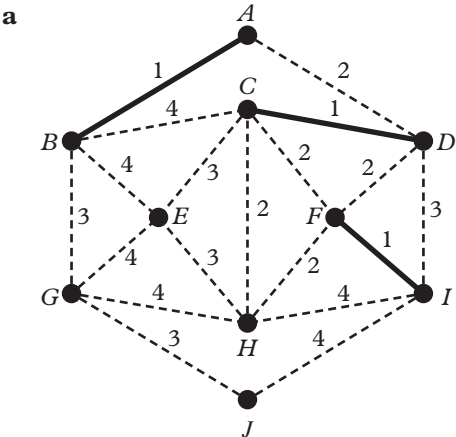


Edge						
Weight						

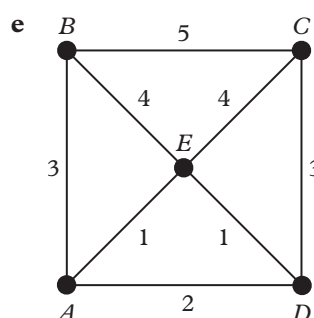
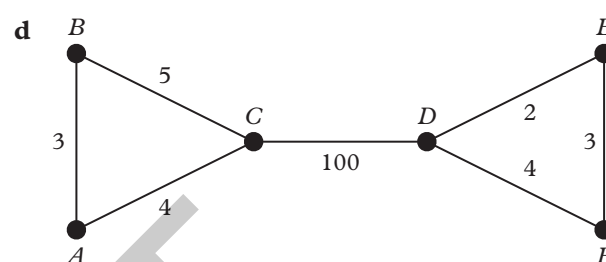
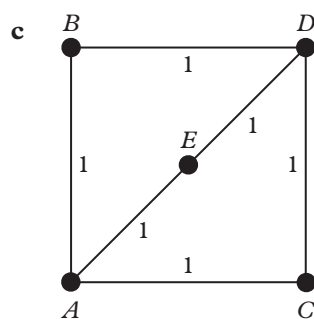
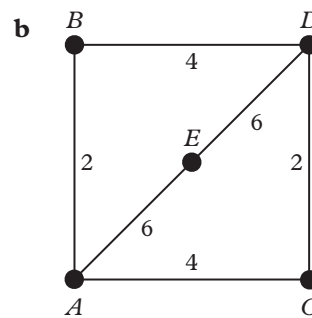
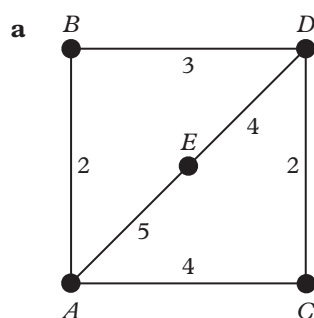
5 At every stage of Kruskal’s algorithm, the selected edges and vertices form a forest of trees. The following network shows the result of the first three edges, all of weight 1, being added to the forest formed during Kruskal’s algorithm. Now proceed through all edges of weight 2, and add as many of them as possible to the forest (i.e. without creating a cycle). Then process all edges of weight 3, and then weight 4, until your forest grows into a single tree. The tree you create will be a minimum spanning tree. Since this network has 10 vertices, your spanning tree should have 9 edges.



6 Repeat question 5 for the following networks.

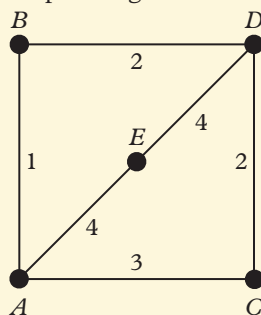


7 Use Kruskal's algorithm to find a minimum spanning tree for the following networks.



Worked example 9.4C Using Prim's algorithm

Use Prim's algorithm to find the minimum spanning tree for the weighted network shown.



Step	Think	Write										
1	Select any vertex as the start vertex. In this example vertex A is chosen.	Let A be the start vertex. 										
2	Edges AB , AC and AE would extend the tree. <table border="1"><thead><tr><th>Edge</th><th>AB</th><th>AC</th><th>AE</th></tr></thead><tbody><tr><th>Weight</th><td>1</td><td>3</td><td>4</td></tr></tbody></table> Choose edge AB because it has the lowest weight. Add vertex B too.	Edge	AB	AC	AE	Weight	1	3	4			
Edge	AB	AC	AE									
Weight	1	3	4									
3.1	Edges BD , AC and AE would extend the tree. <table border="1"><thead><tr><th>Edge</th><th>BD</th><th>AC</th><th>AE</th></tr></thead><tbody><tr><th>Weight</th><td>2</td><td>3</td><td>4</td></tr></tbody></table> Choose edge BD because it has the lowest weight. Add vertex D too.	Edge	BD	AC	AE	Weight	2	3	4			
Edge	BD	AC	AE									
Weight	2	3	4									
3.2	Edges AE , AC , DE and DC would extend the tree. <table border="1"><thead><tr><th>Edge</th><th>AE</th><th>AC</th><th>DE</th><th>DC</th></tr></thead><tbody><tr><th>Weight</th><td>4</td><td>3</td><td>4</td><td>2</td></tr></tbody></table> Connect edge DC because it has the lowest edge weight. Add vertex C too.	Edge	AE	AC	DE	DC	Weight	4	3	4	2	
Edge	AE	AC	DE	DC								
Weight	4	3	4	2								

Step	Think	Write						
3.3	Edges AE and DE (but not AC) would extend the tree. <table><tr><th>Edge</th><td>AE</td><td>DE</td></tr><tr><th>Weight</th><td>4</td><td>4</td></tr></table> As both edges have the minimum weight, we are free to choose either. In this example we chose DE. Add vertex E too.	Edge	AE	DE	Weight	4	4	
Edge	AE	DE						
Weight	4	4						
Stop	The tree now includes every vertex.	The vertices and selected edges form a minimum spanning tree.						



Worked example video: Using Prim's algorithm

Helpful hints

- The steps in Prim's algorithm are:
Step 1: Choose any vertex. This is the initial seed for the tree.
Step 2: Consider the edges adjacent to your tree thus far, i.e. the edges with one vertex inside the tree and another outside the tree. Select the edge with *minimum weight*. Grow the tree by extending it along the selected edge (and vertex).
Step 3: Repeat **Step 2** until the tree grows into a spanning tree.

8 Prim's algorithm is being applied to the following network with vertex E chosen as the initial seed.

a Fill in the table of edges that connect a new vertex to the tree.

Edge	EB	EC	ED	EA
Weight	6			

b Which of these edges has the lowest weight?

9 After a few more steps of Prim's algorithm, the selected tree has grown.

Only vertices B and C remain to be connected.

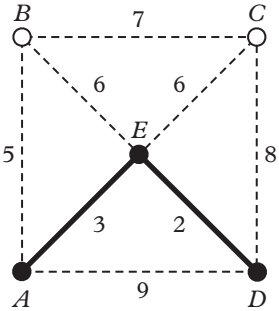
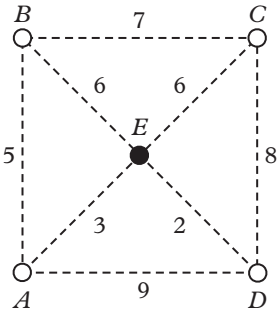
a Complete the table of edges that connect these vertices to the tree.

b Which of these edges has the lowest weight?

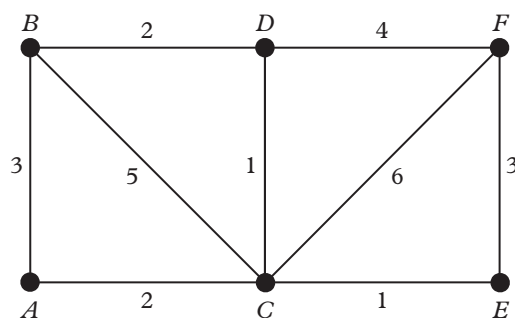
Edge	EB	EC	AB	CD
Weight				8

10 What is the minimum spanning tree for the network in question 8?

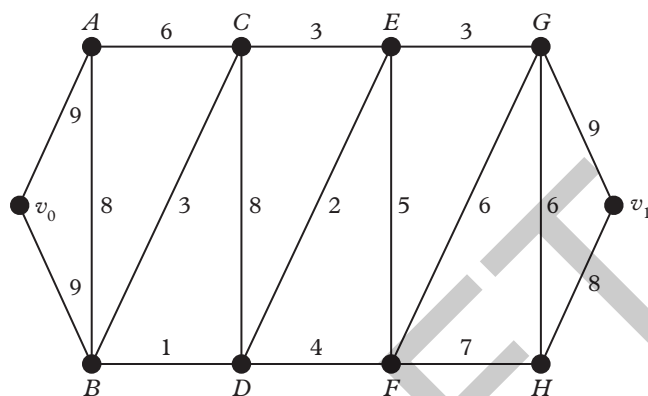
11 Repeat question 7 with Prim's algorithm instead of Kruskal's.



12 Find a minimum spanning tree for this network using any method.



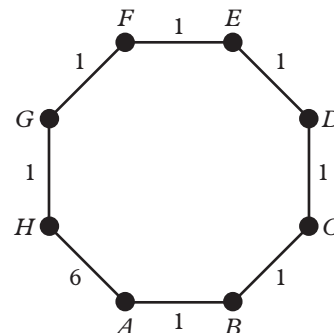
13 Find a minimum spanning tree for this network.



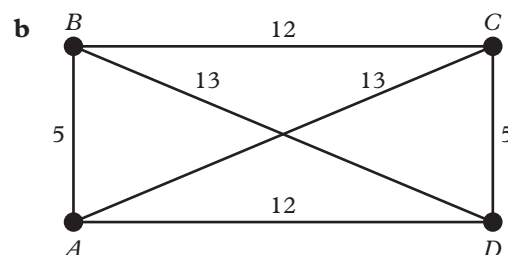
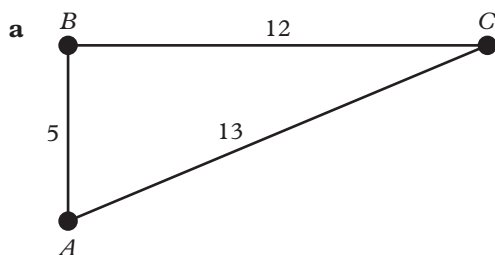
Problem solving and reasoning

14 A network of security cameras must be connected by wires. The costs of each potential connection between pairs of cameras are shown, in dollars, as edge weights in the network.

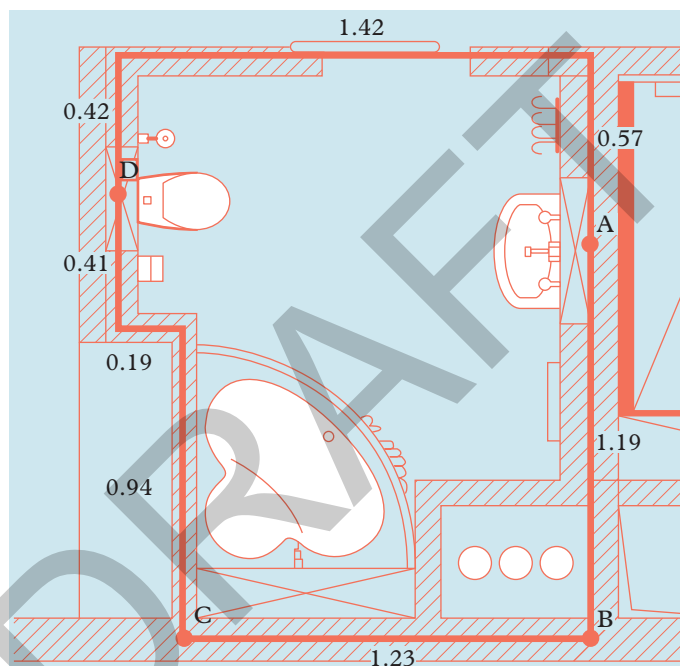
- What is the cheapest way to connect all eight cameras?
- What is the cheapest way to connect only cameras A and H ?
- What is the cheapest way to connect only cameras A and H if the cost of wire between A and H is changed to 8?



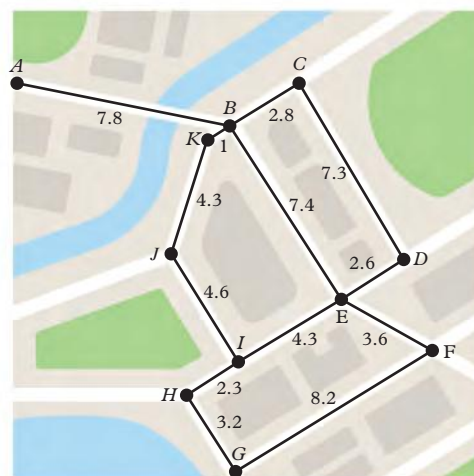
- 15 The following networks represent power plants and the distances between them. The power plants must be connected by transmission lines. Design the shortest possible transmission network in each scenario.



- 16 Consider the following floor plan for a bathroom. Decide where to place the water pipes so that the sink (at vertex A), bath (at vertex C) and toilet (at vertex D) are connected to the water main (at vertex B). The distances along possible pipelines are shown. First represent this information as a network (i.e. add the distances to find the total distances between A and D, then D and C). Then use Kruskal's algorithm to find a spanning tree that minimises the total distance covered by the pipes.



- 17 This network shows the location of fibre optic nodes that form part of a national high-speed internet network. The possible locations of the fibre optic cable segments are given as edges, and the length of each cable segment (in kilometres) is specified as the weight of the corresponding edge. Design the most cost-effective way of connecting each node (i.e. vertex) with fibre optic cable. How many kilometres of cable does your network require?



18 The table below shows the distances between towns A , B , C and D in kilometres.

	A	B	C	D
A	–	3.3	5.8	2.5
B	3.3	–	4.4	1.5
C	5.8	4.4	–	3.6
D	2.5	1.5	3.6	–



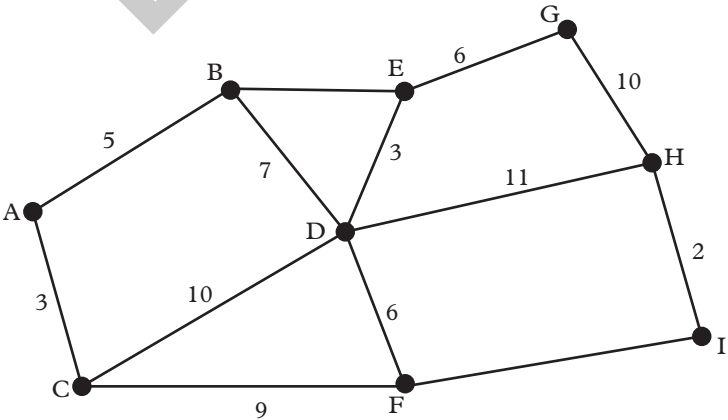
- a Represent the information in the table as a network.
- b What would be the most cost-effective way to connect the towns to each other by rail?
- c If the cost per kilometre of rail is \$5 million, how much would the most cost-effective railway cost?
- d A local politician suggests rail lines along AB , BC and CD . How much would this railway cost?
- e What is the difference in price between the most cost-effective railway and the local politician's suggested railway?

Challenge

19 The following table shows the distances (in kilometres) between mainland capital cities in Australia.

From \ To	Adelaide	Brisbane	Canberra	Darwin	Melbourne	Perth
Brisbane	2075					
Canberra	1209	1267				
Darwin	3041	3435	4034			
Melbourne	732	1813	651	3773		
Perth	2721	4434	3930	4037	3453	
Sydney	1419	978	289	3971	876	3975

- a What is the cheapest way to construct a high-speed rail network that connects the mainland capital cities to each other?
 - b How many kilometres of rail is required?
 - c Given the information in tabular form, would you choose to use Kruskal's or Prim's algorithm?
- 20 For the following network, the weights of the edges BE and FI are missing. If both these sides lie on the only minimum spanning tree, what is the maximum possible weight of the minimum spanning tree?



Related resources

Lesson 9.5

The shortest path

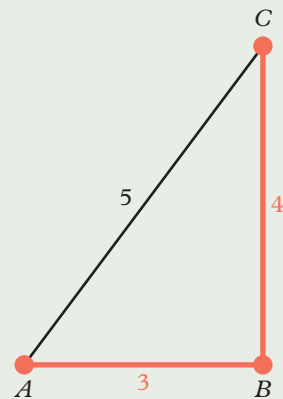
Learning intentions

By the end of this lesson, you will be able to ...

→ calculate the weight of a path and use an algorithm to find the shortest paths.

Shortest path

- In a weighted network, the **shortest path** between two vertices is the path with lowest total edge weight.
- The minimum spanning tree does not necessarily provide the shortest path between two points.
 - For example, in the network shown, the minimum spanning tree contains the path ABC (weight $3 + 4 = 7$), whereas the shortest path from A to C is AC (weight 5).

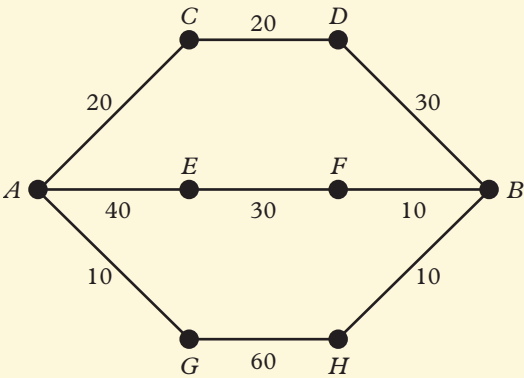


Dijkstra's algorithm

- For larger networks it is difficult to consider all possible paths between two vertices.
- Fortunately, **Dijkstra's algorithm** gives us a method to construct a special spanning tree that contains all the shortest paths emanating from a start vertex.
 - Starting with any vertex, let's call it A , Dijkstra's algorithm grows a tree of "shortest paths from A ". The steps are as follows:
 - Step 1:** Make a tree that only contains the vertex A .
 - Step 2:** Make a list of all paths starting from A that extend the tree by exactly one edge. Choose the path that has the *minimum total weight* and add it to the tree.
 - Step 3:** Repeat **Step 2** until the tree includes all vertices, or until the desired information has been found.
 - This is very similar to Prim's algorithm, except in **Step 2** you select your path based on the total distance from A rather than an individual edge weight.

Worked example 9.5A Finding the shortest path by inspection

The network below shows cities connected by highways. The length of each highway is indicated by an edge weight. What is the shortest path between cities A and B ?



Think	Write
There are three paths from A to B . Find the length of each path and select the path with the shortest length.	Path $ACDB$ has a length of $20 + 20 + 30 = 70$. Path $AEFB$ has a length of $40 + 30 + 10 = 80$. Path $AGHB$ has a length of $10 + 60 + 10 = 80$. The path with the shortest length is $ACDB$.

 **Worked example video:** Finding the shortest path by inspection

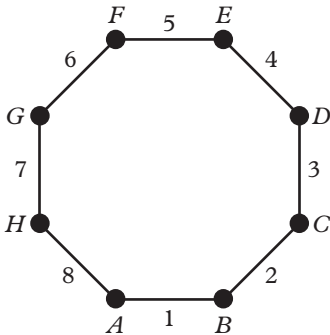
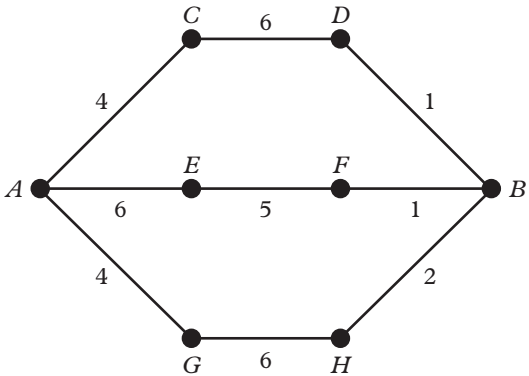
Understanding, fluency and communicating

Answers: Lesson 9.5

Helpful hints

- The steps in Dijkstra’s algorithm are:
Step 1: Make a tree that only contains the initial vertex.
Step 2: Make a list of all paths starting from the initial vertex, that extend the tree by exactly one edge. Choose the path that has the *minimum total weight* and add it to the tree.
Step 3: Repeat **Step 2** until the tree includes all vertices.

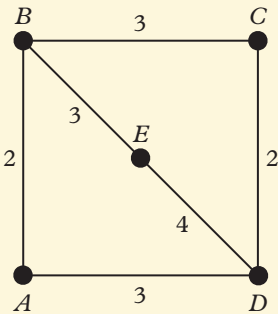
- 1 Consider the network shown.
- a What is the shortest path from A to B ?
- b What is the shortest path from F to G ?
- 2 Consider the network shown.



- a Which path from A to F is shorter: $ABCDEF$ or $AHGF$?
- b Which path from A to G is shorter: $ABCDEFG$ or AHG ?
- c What is the minimum spanning tree for this network?

Worked example 9.5B Using Dijkstra’s algorithm

Use Dijkstra’s algorithm to find the shortest path from A to E in the network shown.



Step	Think	Write						
1	Start with the initial vertex A .							
2	The list of all paths from A that extend the tree by one edge is: <table><tr><th>Path</th><td>AB</td><td>AD</td></tr><tr><th>Total weight</th><td>2</td><td>3</td></tr></table> Path AB has minimum total weight, so add it to the tree.	Path	AB	AD	Total weight	2	3	
Path	AB	AD						
Total weight	2	3						

Step	Think	Write										
3.1	Now the tree contains vertices A and B. The list of all paths starting at A that extend the tree by one edge is: <table><tr><th>Path</th><td>AD</td><td>ABE</td><td>ABC</td></tr><tr><th>Total weight</th><td>3</td><td>5</td><td>5</td></tr></table> Path AD has minimum total weight, so add it to the tree.	Path	AD	ABE	ABC	Total weight	3	5	5			
Path	AD	ABE	ABC									
Total weight	3	5	5									
3.2	The tree contains vertices A, B and D. The list of all paths from A that extend the tree by one edge is: <table><tr><th>Path</th><td>ABE</td><td>ABC</td><td>ADE</td><td>ADC</td></tr><tr><th>Total weight</th><td>5</td><td>5</td><td>7</td><td>5</td></tr></table> Paths ABE, ABC and ADC all have the same minimum total weight, so we are free to choose any one and add it to the tree. In this example we have chosen to add ABE.	Path	ABE	ABC	ADE	ADC	Total weight	5	5	7	5	
Path	ABE	ABC	ADE	ADC								
Total weight	5	5	7	5								
3.3	The tree contains vertices A, B, D and E. Only vertex C remains outside the tree. The paths from A that extend the tree by one edge are: <table><tr><th>Path</th><td>ABC</td><td>ADC</td></tr><tr><th>Total weight</th><td>5</td><td>5</td></tr></table> All paths have the same weight, so we are again free to choose any one and add it to the tree. In this example we have chosen to add ABC.	Path	ABC	ADC	Total weight	5	5					
Path	ABC	ADC										
Total weight	5	5										
Stop	The resulting tree is a spanning tree that contains the shortest path from A to any other vertex.	The shortest path from A to E in this tree is ABE, which has total weight $2 + 3 = 5$.										



Worked example video: Using Dijkstra's algorithm

Helpful hints

- Dijkstra's algorithm creates a spanning tree that contains the shortest paths from the initial vertex.
- The algorithm in Worked example 9.5B could have been stopped as soon as E was added, as the tree already contained the shortest path from A to E .

3 Use Dijkstra’s algorithm to find the shortest path from E to B in the network shown.

- a The first step of Dijkstra’s algorithm is to create a tree with just the start vertex E . Below is a list of paths that extend the tree by one edge. Calculate the total weight of every path that extends this tree by one edge.

Path	EA	EB	EC	ED
Total weight	3			

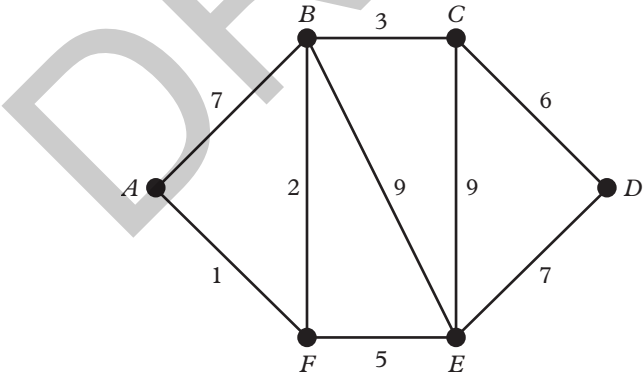
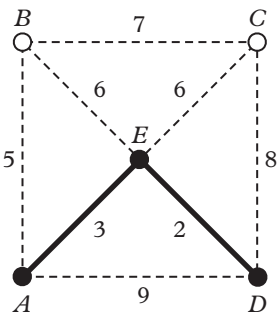
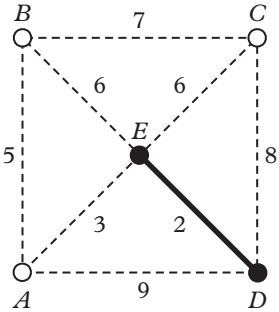
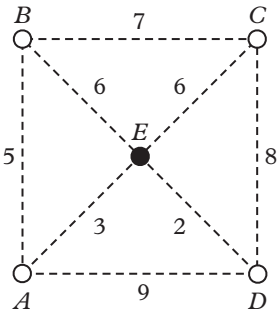
- b In the second step, the tree contains the vertices E and D , and the edge ED . Calculate the total weight of every path that extends this tree by one edge.

Path	EA	EB	EC	EDA	EDC
Total weight				11	

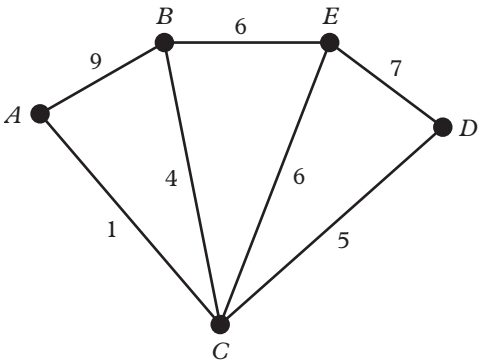
- c In the third step, the tree contains vertices A , E and D , and the edges EA and ED . Calculate the total weight of every path that extends this tree by one edge.

Path	EB	EC	EAB	EDC
Total weight			8	

- i Which of the paths in part c has the smallest total weight?
ii What is the minimum distance from E to B ?
- 4 What is weight of the shortest path from B to E in the networks in question 7 in ?
5 What is weight of the shortest path from A to D in this network?



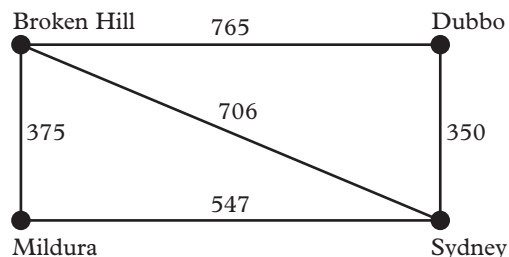
6 Find the shortest distance from A to E in this network.



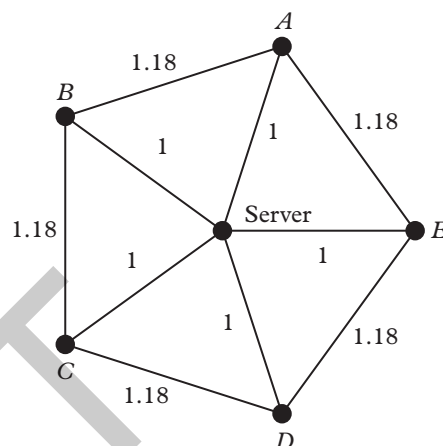
Problem solving and reasoning

- 7 The cost of flying from Sydney to certain regional centres is shown in the network.

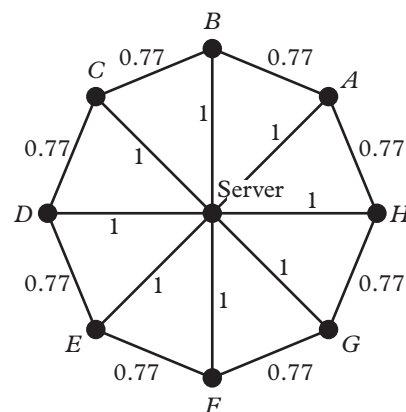
- What is the cheapest route to Broken Hill?
- Assuming the cheapest route is booked out, what is the second cheapest route?



- 8 The network shows the distances between five computers A , B , C , D and E and a central server. Connect each computer to the server, so that the path from the server to the computer is as short as possible. Is your selected network also a minimum spanning tree?

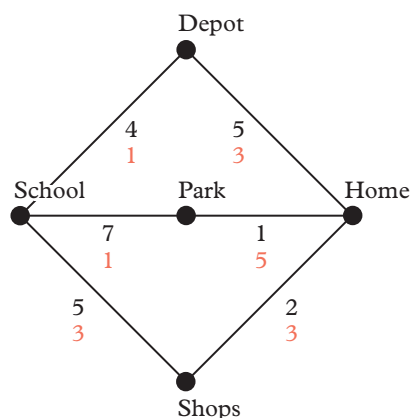


- 9 Repeat question 8 for a network with eight computers. Is your selected network also a minimum spanning tree?



- 10 The map shows the possible routes from school to home. The distance of each route (in kilometres) is given by the black edge weight, and the level of danger is given by the red edge weight.

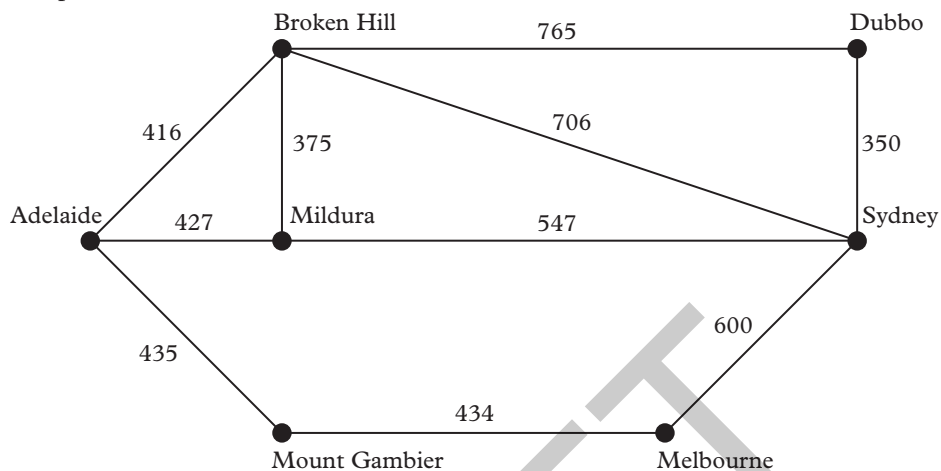
- What is the shortest route from school to home?
- What is the least dangerous route from school to home?



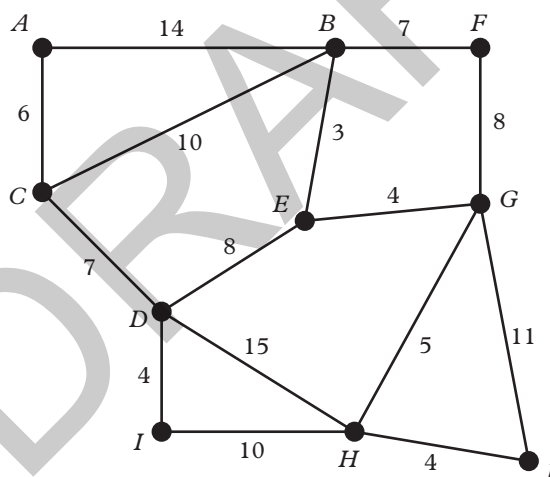
- 11 Apply Prim's algorithm to the network in Worked example 9.5B. Compare the resulting tree with the one in the worked example.

Challenge

- 12 Here is a larger map of regional flights, along with the cost of each route in dollars, given as an edge weight. What is the cheapest route in the network from Dubbo to Mount Gambier?



- 13 Jade is walking through the network shown, where the weights represent the time in minutes it takes to walk between points.



- What is the quickest path from A to J?
- What is the least number of paths that would need to be blocked off for the quickest path from A to J to include the point F?

Related resources



9.5 Quick quiz



Worksheet: The shortest path

Lesson 9.6

Review: Networks, paths and trees

Review questions 9.6A Mathematical literacy review

Answers: Lesson 9.6

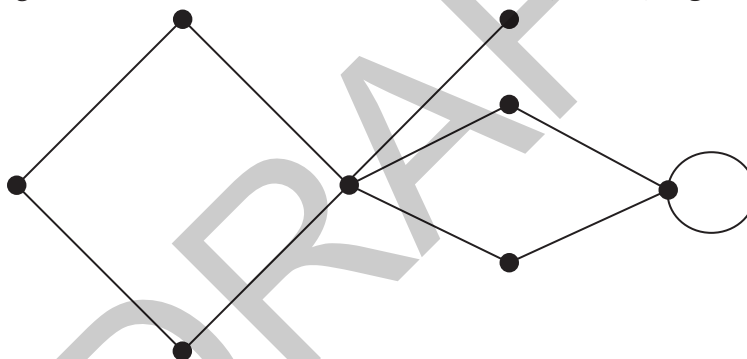
Key terms

The following key terms are used in this module:

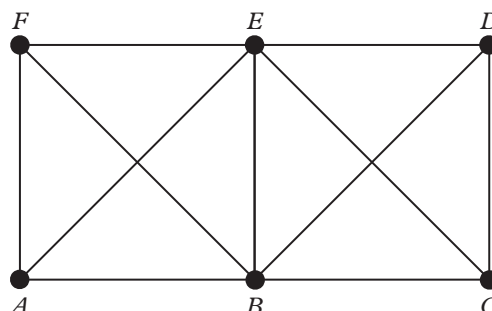
• connected network	• edge	• minimum spanning tree	• spanning tree
• cycle	• forest	• network	• tree
• degree	• indegree	• outdegree	• vertex
• Dijkstra's algorithm	• Kruskal's algorithm	• path	• walk
• directed network	• leaf	• Prim's algorithm	• weighted edge
• disconnected network	• loop	• shortest path	

Check your understanding

- 1 Label parts of the diagram below to demonstrate the terms **network**, **vertex**, **edge** and **loop**.



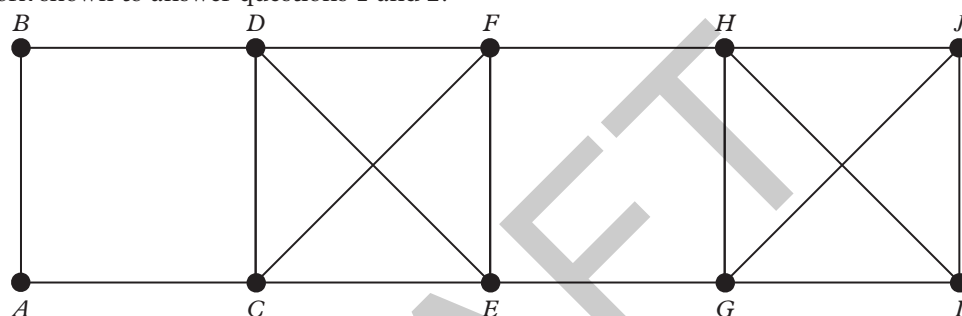
- 2 Draw an example of:
- a **directed network**
 - a **weighted network**
 - a **disconnected network**.
- 3 On the network shown, use different colours to highlight:
- a **path**
 - a **cycle**
 - a **spanning tree**.



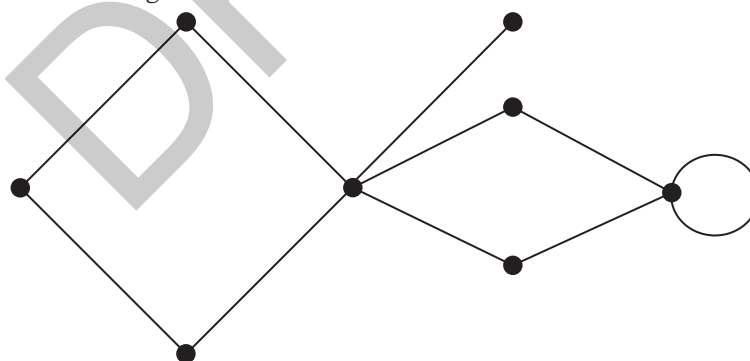
- 4 Write a sentence to explain the difference between a **tree** and a **forest**, using the terms **connected** and **disconnected**.
- 5 Explain the following terms and how they depend on **weighted edges**:
 - a minimum spanning tree
 - b shortest path.
- 6 Match the three algorithms (**Dijkstra's**, **Kruskal's** and **Prim's**) with their descriptions:
 - a A method for finding minimum spanning trees that only adds new edges with the lowest possible weights if they extend a single connected tree.
 - b A method for finding the shortest path in a connected weighted network.
 - c A method for finding the minimum spanning tree of a connected weighted network that adds edges with the lowest possible weights from anywhere in the network.

Review questions 9.6B Multiple-choice questions

Use the network shown to answer questions 1 and 2.

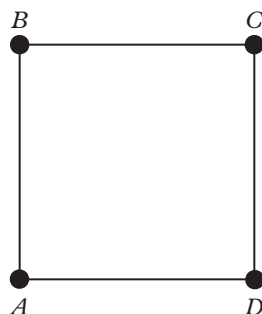


- 1 How many edges does this network have?
 A 18 B 17 C 20 D 21
- 2 Which of the following is *not* the degree of one of the vertices in this network?
 A 2 B 3 C 4 D 5
- 3 What is the sum of the vertex degrees in this network?



- A 8 B 10 C 19 D 20

Use the diagram shown to answer questions 4 to 6.



4 Which of the following walks is a path?

- A $ABCB$
- B ABD
- C DAB
- D $ABCD A$

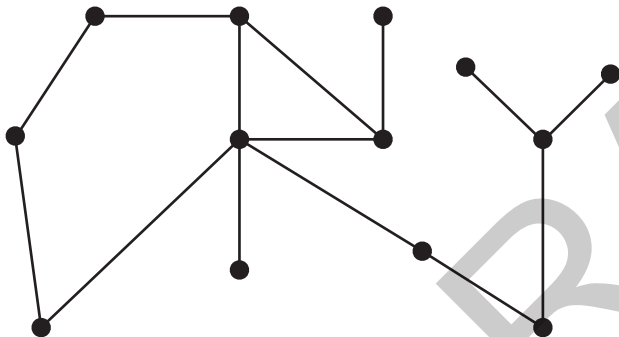
5 Which of the following walks is a cycle?

- A $ABAD$
- B $ABCD$
- C $ABDA$
- D $ABCD A$

6 Which of the following walks is not a path and not a cycle?

- A AB
- B ABA
- C $BCDAB$
- D $ABAD$

7 How many edges need to be removed from the network shown to turn it into a tree?

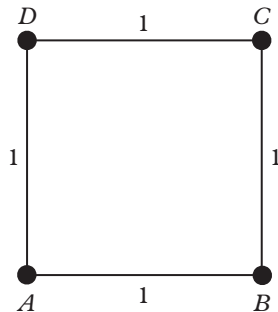


- A 1
- B 2
- C 3
- D 4

8 A spanning tree contains five edges. How many vertices are in the network?

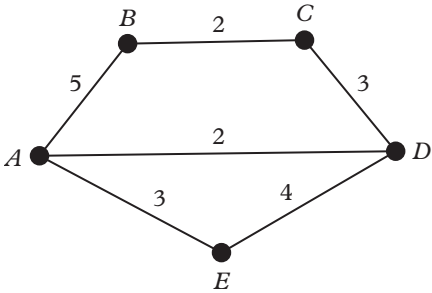
- A 3
- B 4
- C 5
- D 6

9 How many different minimum spanning trees exist in the network shown?



- A one
- B two
- C three
- D four

Use the diagram shown and its description below to answer questions 10 to 13.



The diagram shows the possible paths (in kilometres) for laying gas pipes between various locations. Gas is to be supplied from one location. Any one of the locations can be the source of the supply.

10 What is the minimum total length of the pipes required to provide gas to all locations?

- A 10 km
- B 19 km
- C 12 km
- D 11 km

11 What is the total weight of the cycle $ADCBA$?

- A 7
- B 9
- C 10
- D 12

12 The shortest path from B to E is:

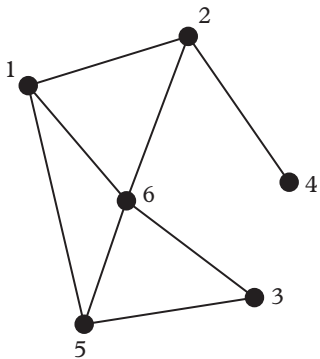
- A BAE
- B BE
- C $BCDE$
- D $BCDAE$

13 The shortest path from A to C is:

- A $AEDC$
- B ABC
- C ADC
- D AC

Review questions 9.6C Set 1

Use the diagram shown to answer questions 1 to 3.



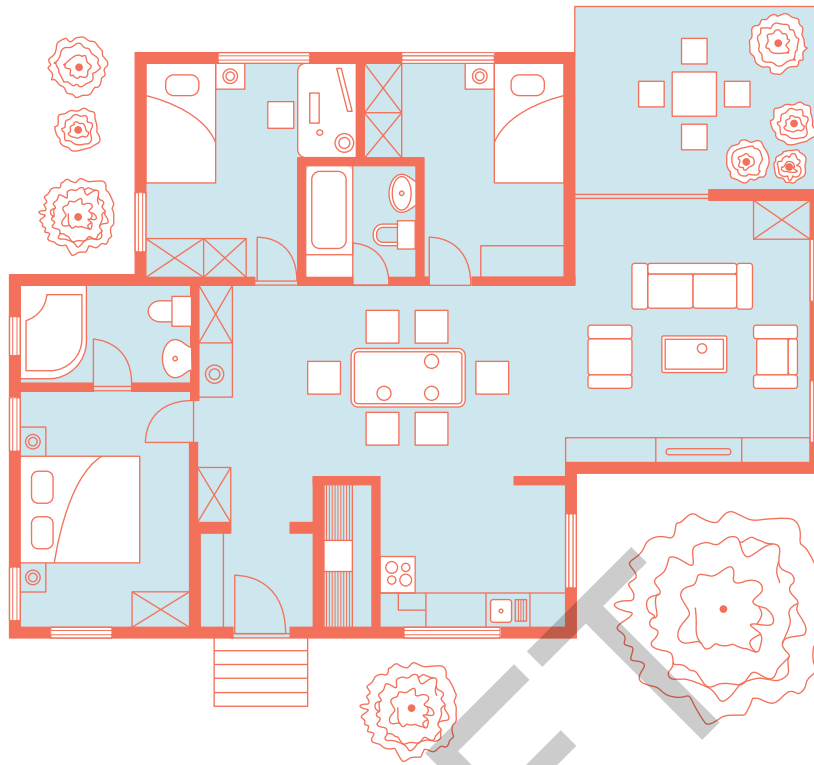
1 Complete the table of vertex degrees for the network.

Vertex	1	2	3	4	5	6
Degree						

2 What is the sum of the vertex degrees in the network?

3 Find a walk in the network that visits every vertex exactly once.

Use the diagram shown to answer questions 4 and 5.



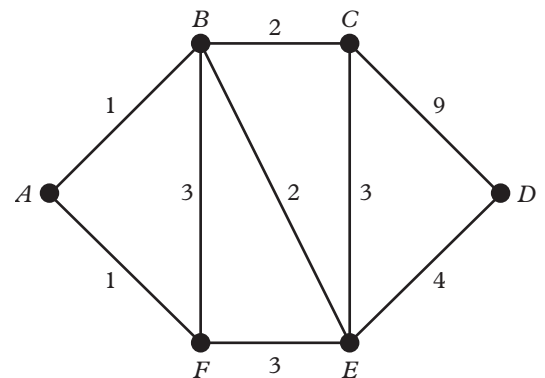
4 Draw a network that represents the floor plan. Use vertices to represent rooms, and represent rooms connected by an internal door with an edge.

5 Is your network in question 4 a tree?

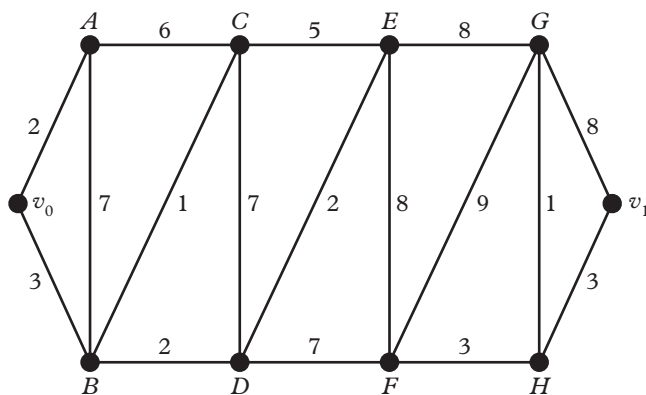
Use the diagram shown to answer questions 6 and 7.

6 Use Prim's algorithm to find the minimum spanning tree for the network.

7 What is the total weight of the path $AFBCD$ for the network?



Use the diagram shown to answer questions 8 and 9.



8 Find the shortest path from v_0 to v_1 .

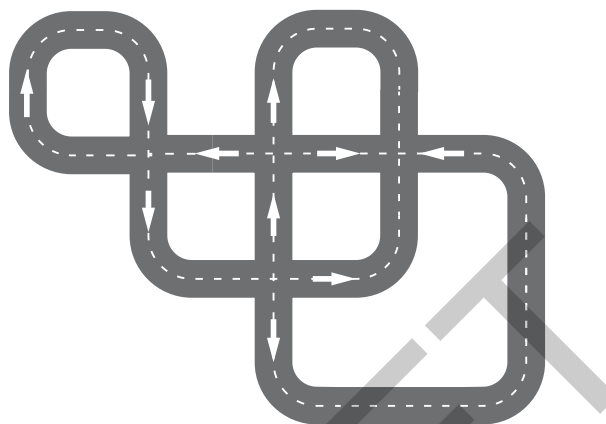
9 Find the shortest path from v_0 to v_1 if the edge FH is deleted.

Review questions 9.6D Set 2

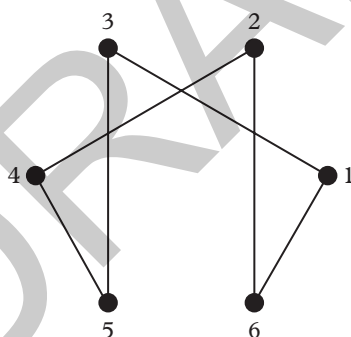
- 1 Represent the following table as a weighted network.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>A</i>	–	2	8	9
<i>B</i>	2	–	7	4
<i>C</i>	8	7	–	1
<i>D</i>	9	4	1	–

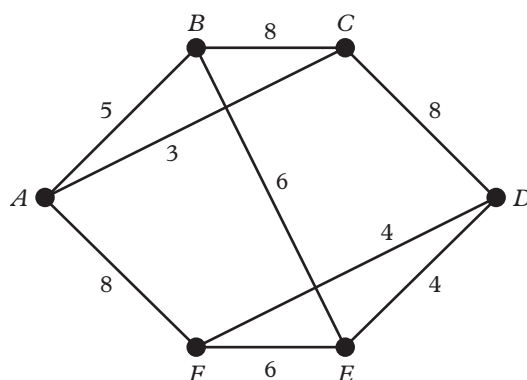
- 2 Draw this system of one-way streets as a directed network.



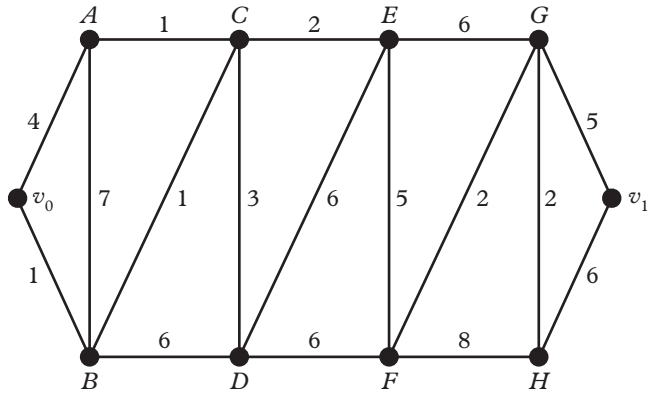
- 3 Using the concepts of indegree and outdegree, show that there is something wrong with the road design in question 2.
- 4 Find a cycle with six edges in the network shown.



- 5 Transform the cycle in question 4 into a spanning tree by removing one or more edges.
- 6 Transform the spanning tree in question 5 into a forest of two trees by removing one or more edges.
- 7 Is there a walk in the network from question 4 that visits every edge exactly once?
- 8 Is there a walk in the network from your answer to question 5 that visits every edge exactly once?
- 9 Find a minimum spanning tree for the network shown.

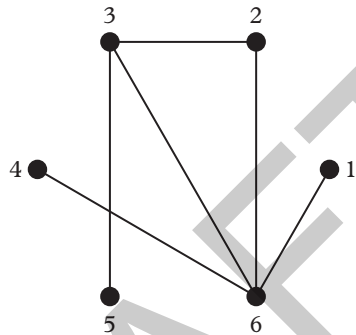


10 Find the shortest path from v_0 to v_1 for the network shown.



Review questions 9.6E Set 3

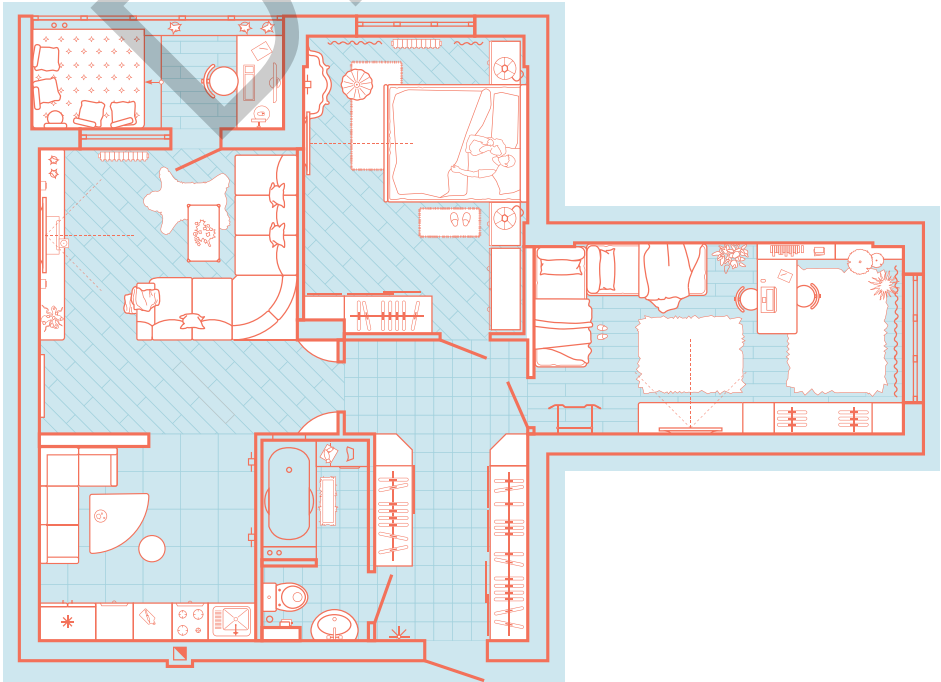
Use the diagram shown to answer questions 1 and 2.



1 Complete the table of vertex degrees for the network.

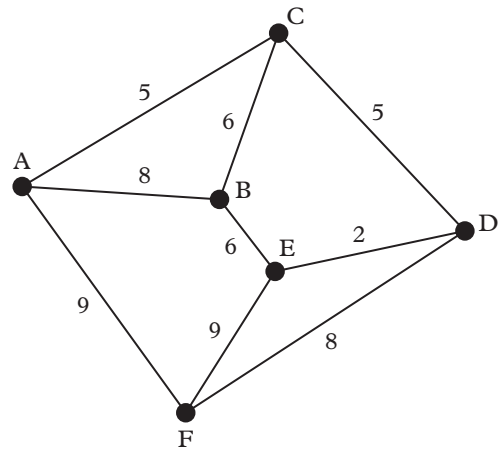
Vertex	1	2	3	4	5	6
Degree						

- 2 Is there a path in the network that visits every edge exactly once?
- 3 Draw a network that represents the floor plan shown. Use vertices to represent rooms, and represent rooms connected by a door with an edge.



4 Is the network from question 3 a tree?

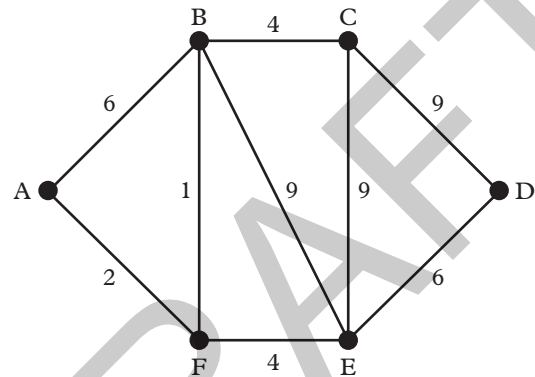
Use the diagram shown to answer questions 5 and 6.



5 Find a minimum spanning tree for the network.

6 What is the weight of the walk $ABEDC$ in the network.

Use the diagram shown to answer questions 7 to 9.



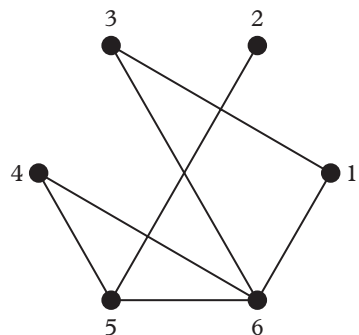
7 Find the shortest path from A to D in the network.

8 Use Prim's algorithm to find the minimum spanning tree for the network.

9 Use Kruskal's algorithm to find the minimum spanning tree for the network.

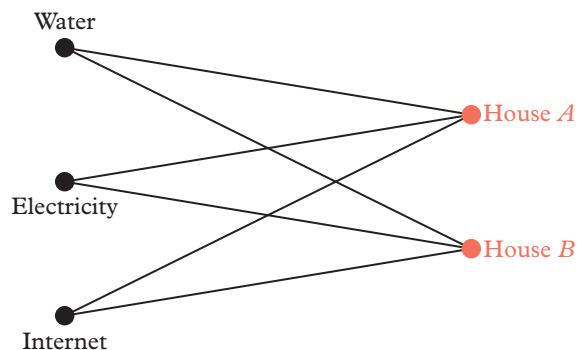
Review questions 9.6F Set 4

1 Complete the table of vertex degrees for the network shown.



Vertex	1	2	3	4	5	6
Degree						

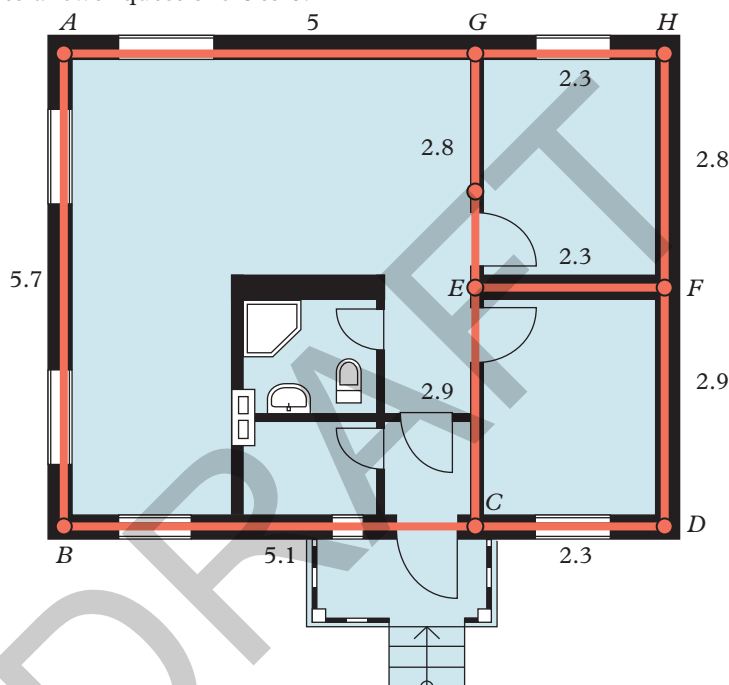
- 2 Three essential services (water, electricity and internet) are to be connected to House *A* and House *B*.



Re-draw this network so that the utility lines do not cross.

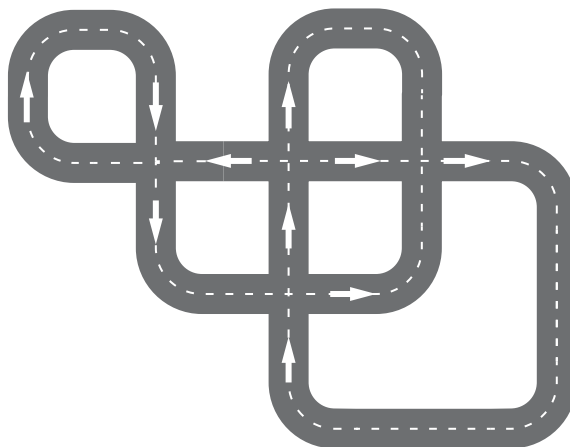
- 3 What is the sum of vertex degrees in the network from question 2?

Use the diagram shown to answer questions 4 to 6.

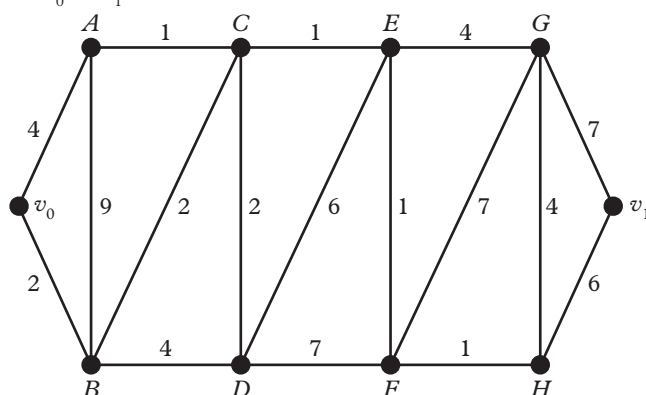


This house plan shows the possible locations of power cables. Assume the edge weights represent distances in metres and that power cabling is sold by the metre.

- 4 What is the most cost-effective way to connect the points *A*, *B*, *C*, *D*, *E* and *F* to the power meter at *G* in the network?
- 5 What is the weight of the path *ABCDFEG* for the network?
- 6 Use Prim's algorithm to find the minimum spanning tree for the network.
- 7 Draw this system of one-way streets as a directed network.



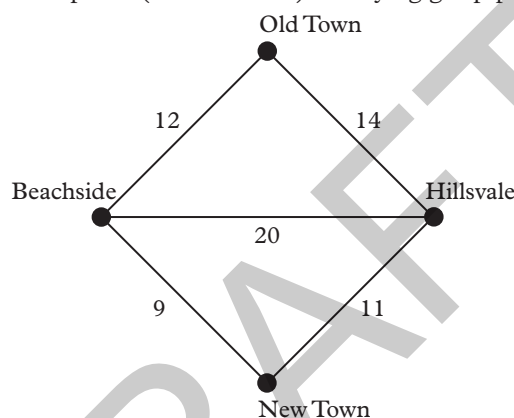
- 8 Find the shortest path from v_0 to v_1 .



Review questions 9.6G Practice examination question

- 1 Answer the following questions.

- a This diagram shows the possible paths (in kilometres) for laying gas pipes between various towns.



Gas is to be supplied from one town. Any one of the towns can be the source of the supply.

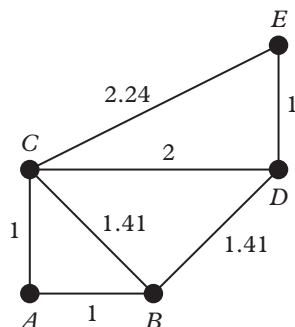
- i What is the minimum total length of the pipes required to provide gas to all the locations? (1 mark)

- a 32 km
- b 33 km
- c 29 km
- d 40 km

- ii How many different paths are there from New Town to Old Town? (1 mark)

- iii What is the shortest path from Beachside to Hillside? (1 mark)

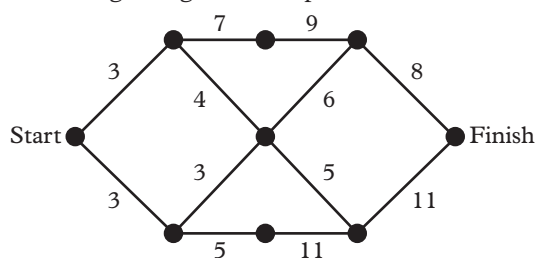
- b This diagram shows the distance by road (in kilometres) between towns as a weighted network.



- i What is the shortest path from town A to town E? (1 mark)

- ii What roads should be resurfaced to minimise the total length of resurfaced road while still connecting each town with newly resurfaced road? (1 mark)

- c This network represents a maze. Edge weights correspond to distances in metres.

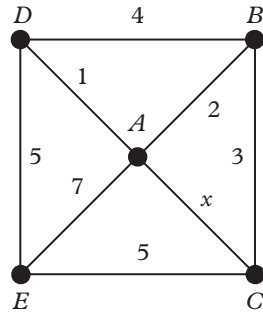


- i What is the shortest path from the start of the maze to the finish? (1 mark)
- a 20 m b 30 m c 40 m d 50 m
- ii Find a minimum spanning tree for this network. (1 mark)
- d Consider the road network shown.



- i Is the graph connected? (1 mark)
- ii Which word describes the road network most accurately and specifically? (1 mark)
- a tree
- b path
- c cycle
- d directed

e Consider the network shown, where the weight of edge AC is x .



- i If x is part of the minimal spanning tree, what is the largest value of x ? (2 marks)
- ii If x is not part of the minimal spanning tree, what is the smallest value of x ? (2 marks)
- iii If the shortest path from D to C has length 6, what is the smallest value of x ? (2 marks)

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Networks, paths and trees

I can do this	I need to review this
☐ explain and use the terms vertex, edge, degree, loop, directed network and weighted edge	Go back to Lesson 9.1 Introduction to networks
☐ construct networks from tables, maps and diagrams	
☐ explain the terms walk, path and cycle, and distinguish between them	Go back to Lesson 9.2 Walks, paths and cycles
☐ identify connected and disconnected networks	
☐ represent a path or cycle on a network or in words	
☐ explain and use the terms tree, forest and spanning tree	Go back to Lesson 9.3 Trees
☐ determine whether a given network is a tree or explain why it is not	
☐ find a spanning tree for a given network	
☐ describe what a minimal spanning tree is	Go back to Lesson 9.4 Minimum spanning trees
☐ find a minimal spanning tree for a given network either by inspection or using Kruskal's or Prim's algorithms	
☐ calculate the weight of a path in a network	Go back to Lesson 9.5 The shortest path
☐ find the shortest path between two vertices in a network either by inspection or using Dijkstra's algorithm	
☐ understand that a minimum spanning tree does not necessarily give the shortest path between two vertices	

Related resources



Module review quiz: Networks, paths and trees



Worksheet: Challenge 9

Review: Measurement and networks

Review questions: Measurement

Answers: Review: Measurement and networks

1 Complete the following questions.

a Round 4263.091 to:

- i the nearest 100
- ii the nearest whole number
- iii two decimal places
- iv three significant figures
- v five significant figures
- vi one significant figure

b Are the following numbers in scientific notation?

- i $6 \times 10\ 000$
- ii 15×10^8
- iii 2.04×10^{-6}

c Express the following in scientific notation.

- i 105 000
- ii 0.000 062 7
- iii 45.6

d Calculate the following, writing the answers in scientific notation.

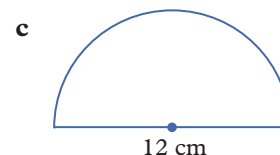
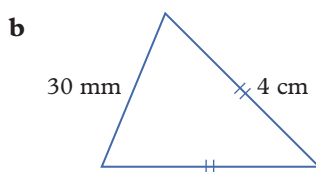
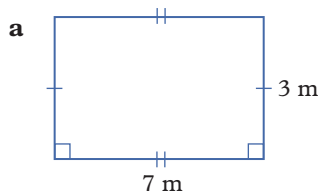
- i $(4.1 \times 10^8) \times (7 \times 10^5)$
- ii $(8 \times 10^5)^3$
- iii $(6.24 \times 10^{-3}) \div (1.43 \times 10^{-7})$

e Convert the following.

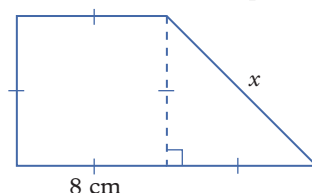
- i 6.73 km = ___ m
- ii 1750 cm = ___ m
- iii 4.3 g = ___ mg
- iv 47 000 kg = ___ t
- v 6.35 L = ___ mL
- vi 90 mL = ___ L
- vii 3.2 Gm = ___ m
- viii 52 ML = ___ kL
- ix $16\ 500\text{ cm}^2 = \text{___ m}^2$

2 The surface area, A , of a cylinder can be found using the formula $A = 2\pi r(r + h)$, where r is the radius and h is the height. Find the surface area of a cylinder with radius 12 cm and height 8 cm.

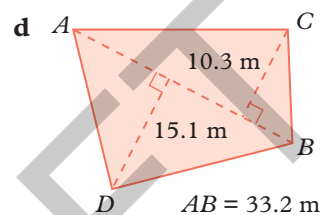
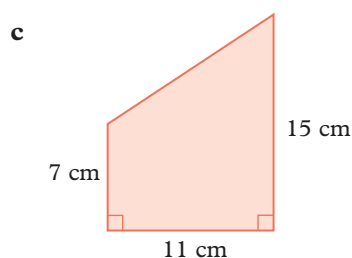
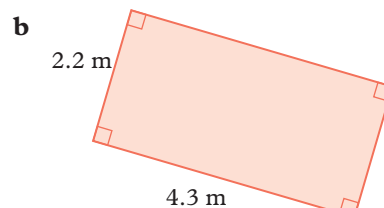
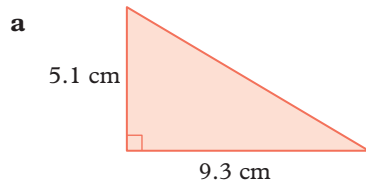
3 Find the perimeter of each of these figures, rounding to one decimal place where necessary.



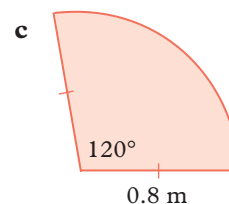
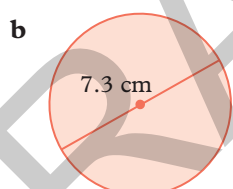
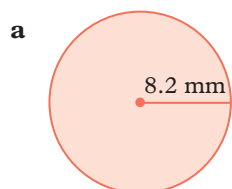
- 4 Consider the figure shown. Calculate, correct to one decimal place:



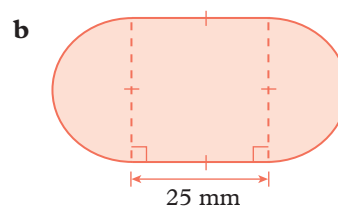
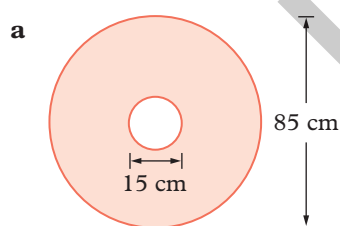
- a the value of x
b the perimeter of the figure.
- 5 Find the area of each of these figures.



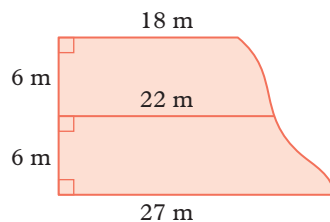
- 6 Find the area of each of these figures, correct to one decimal place.



- 7 Find the shaded area of each of these figures, correct to one decimal place.

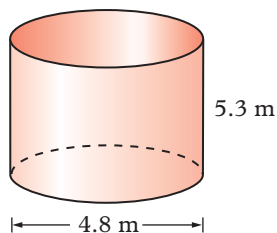


- 8 Consider the diagram of two neighbouring paddocks. Use the Trapezoidal rule to estimate the area of each paddock, then estimate the total area to the nearest square metre.

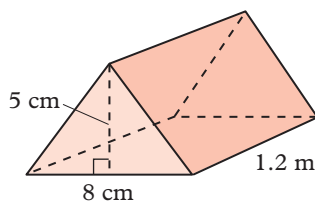


9 Express the following surface areas correct to one decimal place.

a Find the surface area of this cylinder, assuming it is closed at both ends.

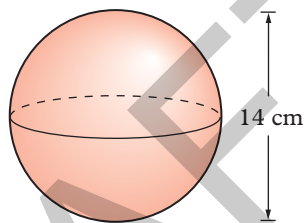


b Find the surface area of this triangular prism.

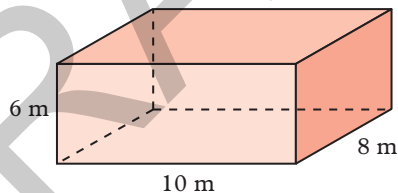


10 Find the following surface areas.

a Find the surface area of this sphere correct to the nearest whole square centimetre.

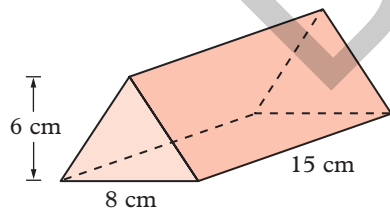


b Find the surface area of this prism.

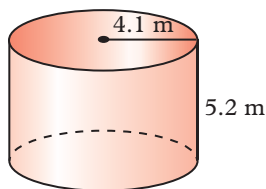


11 Calculate the volume of each of these solids, correct to one decimal place, if necessary.

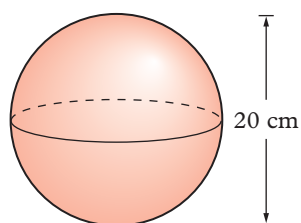
a



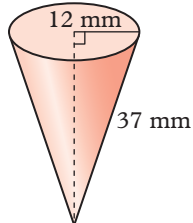
b



c

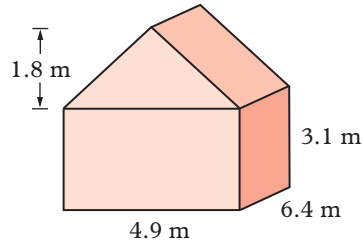


d

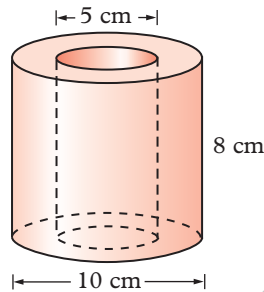


12 Calculate the volumes of the following composite shapes.

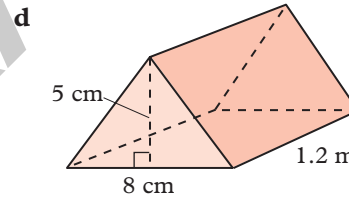
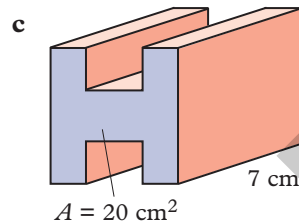
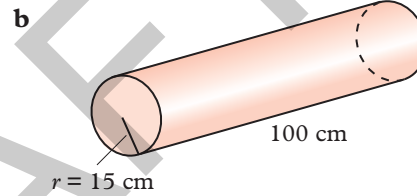
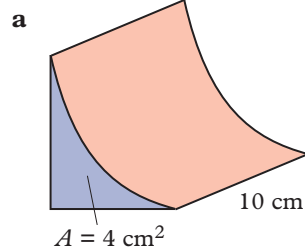
a Find the volume of this barn, which is represented as a prism.



b Find the volume of this hollow cylinder, correct to one decimal place.



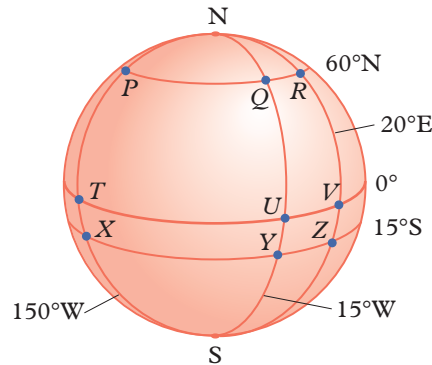
13 Calculate the volume of these solids, rounded to the nearest whole number, if necessary.



14 Find the capacity, in kilolitres, of a rectangular tank with length 10 m, breadth 4 m and height 3 m.

15 Find the capacity, to the nearest millilitre, of a cylindrical drinking glass with diameter 7 cm and height 11 cm.

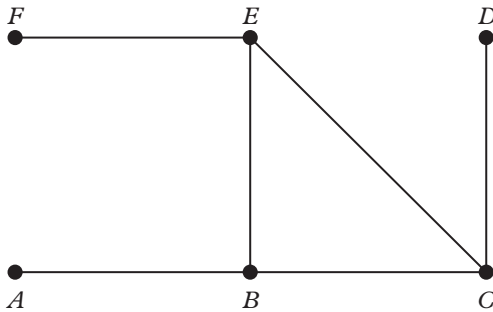
16 Use this diagram of Earth as a sphere to answer the following questions.



- a** List the points on the:
- i** same parallels of latitude
 - ii** equator
 - iii** same meridians of longitude.
- b** Write the latitude and longitude of the points P , U and Y .
- c** Determine whether the following are true or false.
- i** P is west of Q .
 - ii** R is north of Z .
 - iii** X is east of Y .
 - iv** Z is south of X .
- d** Determine the position coordinates of the point that is:
- i** 30° west of Z
 - ii** 8° south of Q .
- 17** Convert and calculate the following times.
- a** Convert 4 days and 19 hours to hours.
 - b** Convert 289 minutes to hours and minutes.
 - c** Calculate the time (in days/hours/minutes) that elapses between 7:38 pm Tuesday and 9:30 am Friday.
- 18** Perform the following time zone calculations.
- a** Moscow is in the UTC +4 time zone and Miami is in the UTC−5 time zone. What time is it in:
 - i** Moscow when it is 4:00 am in Miami?
 - ii** Miami when it is 2:00 pm in Moscow?
 - b** Reynold leaves Beijing (31°N , 121°E) at 11:00 am Friday, flying to San Francisco (38°N , 122°W). Beijing is in the UTC +8 time zone and San Francisco is in the UTC−8 time zone.
 - i** What is the standard time and day in San Francisco when Reynold leaves Beijing?
 - ii** The flight time is $12\frac{1}{2}$ hours. What is the day and time in San Francisco when he arrives?
 - c** Use the Australian summer time zones to find the time in:
 - i** Darwin when it is 8:30 am in Cairns
 - ii** Hobart when it is 10:00 am in Sydney
 - iii** Perth when it is 7:00 pm in Brisbane
 - iv** Adelaide when it is 5:00 pm in Perth.

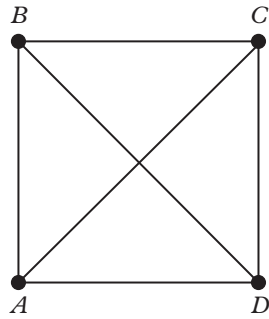
Review questions: Networks

- 1** The network shows vertices connected by edges. Fill out the table, which lists the vertices and the connections between them.



Vertex	A	B	C	D	E	F
Connected to	B	A, E, C				

- 2 Inspector Poirot has four suspects in his criminal investigation: A , B , C and D . Any one suspect may have committed the crime, or the suspects could be working together. Poirot indicates the possibility that two suspects are working together by drawing an edge between the corresponding vertices in the network shown.



New evidence emerges that suspects A and B are not working together, and that suspect D is completely innocent. What does Poirot's network of suspects look like with this new evidence?

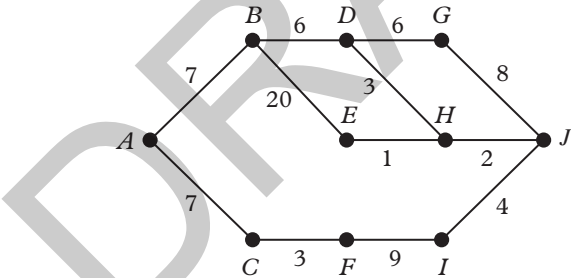
- 3 The five boroughs of New York – Manhattan, the Bronx, Queens, Brooklyn and Staten Island – are connected by many bridges, as listed in the table. Queens and Brooklyn are connected by land, and so for this question are considered the same.

From/To	The Bronx	Queens/Brooklyn	Staten Island
Manhattan	11	8	0
The Bronx		3	0
Queens/Brooklyn			1

Draw a network representing the bridges between these boroughs and highlight a spanning tree.

- 4 Find a spanning tree for the network in your answer to question 3.

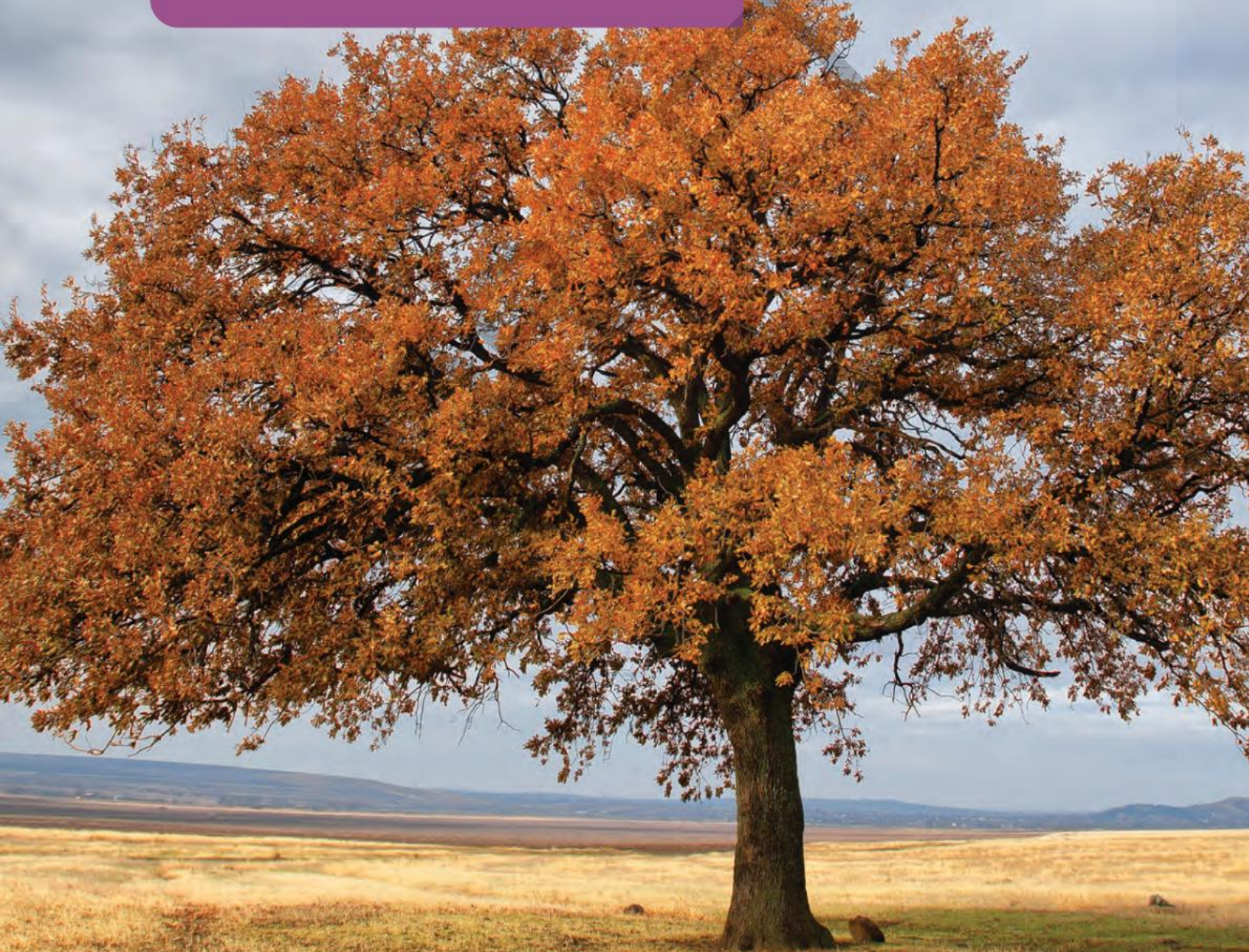
Use the network shown to answer questions 5 to 8.



- 5 Use Prim's algorithm to find a minimum spanning tree for this network.
- 6 Use Kruskal's algorithm to find a minimum spanning tree for this network.
- 7 Use Dijkstra's algorithm to find the shortest path between A and J in this network.
- 8 What is the shortest path between B and E in this network?

Module 10

Collecting and classifying data



Lessons

- Lesson 10.1 Statistical investigations
- Lesson 10.2 Classifying data
- Lesson 10.3 Collecting data
- Lesson 10.4 Random sampling
- Lesson 10.5 Stratified sampling
- Lesson 10.6 Systematic sampling
- Lesson 10.7 Suitability of sample types
- Lesson 10.8 Review: Collecting and classifying data

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Collecting and classifying data

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Writing and simplifying fractions



Support sheet: Multiplying fractions



Support sheet: Calculating percentages



Support sheet: Number patterns

Syllabus links

- Data analysis (MST-11-08)
 - A student displays and analyses datasets using summary statistics and graphical representations.
 - Statistical investigation process
 - Population and sample
 - Data classification
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 10.1

Statistical investigations

Learning intentions

By the end of this lesson, you will be able to ...

→ explain when a statistical investigation is appropriate and what features are important.

Statistical investigations

- **Statistics** is the science of collecting facts and information. Information is organised in a form that can be easily analysed, so that accurate conclusions and predictions can be drawn. Any facts or pieces of information collected are called **data**. Data may be collected by counting, measuring and asking questions.
- The process of statistical inquiry includes the following steps:
 - 1 identifying a problem and posing a statistical question
 - 2 collecting or obtaining data
 - 3 representing and analysing that data
 - 4 communicating and interpreting the findings.
- In this module we will investigate the first two of these steps.

Posing a statistical question

- Statistics can be used to answer a wide range of real-life questions such as:
 - Does Sydney receive more sunshine than Melbourne?
 - Which drug is most effective in the treatment of the common cold?
 - How does time on social media affect concentration in teenagers?
 - Does the location of your school have an impact on your HSC results?
- Posing a question for statistical analysis starts with identifying a broad issue of interest, before distilling the question into one that is concrete and measurable. Part of this process is specifying exactly who or what the question applies to.
- The target **population** is the entire group about whom, or from which, information is sought. For instance, if we were interested in collecting information about public perception of the cleanliness of trains, the target population would be all commuters who use rail transport. If we wanted to determine the most popular colour for new cars in one particular year, the target population would be all new cars sold in that year.
 - The results from a statistical investigation should only ever be used to draw conclusions about the intended population of the study. For example, if you are investigating average tail length in greyhounds, the conclusions you draw will not be applicable to pugs.



Survey design principles

- Surveys are a common way to collect data that can be used to answer a statistical question.
- Surveys can be done by interview, either in person or by phone, or by questionnaire. The usefulness of survey responses will depend on the relevance, clarity and structure of the questions asked.
 - Here are some examples of question types that could be used to start an investigation into students' opinions on their school uniform, with a discussion of some of the advantages and disadvantages of each type of question.

Type of question	Example	Advantage/disadvantage
Free response/ open-ended	What do you like/dislike about the current school uniform?	This question would possibly collect a lot of information, but, because there would be many different responses, it might be difficult to organise and interpret the responses.
Yes or No	Would you like to change the present school uniform?	The responses to this question will indicate whether there is a desire to change the uniform, but will not indicate what change, if any, is wanted. Further questions could be used to determine this.
True or False	"The present school uniform is satisfactory." True or false?	As for the Yes or No question, responses to this question will not reveal the kind of changes wanted.
Tick the box	Which of the following items of school uniform would you like to see changed? ☐ shirt ☐ trousers ☐ shoes ☐ pullover	The responses to this question will give information about which components of the uniform need to be changed, but the overall opinion of the uniform is not captured.
Response scale	Circle the response that most closely aligns with your opinion of the current school uniform. 1 strongly like 2 like 3 no opinion 4 dislike 5 strongly dislike	The responses to this question will give a clearer indication of the depth of feeling concerning a change, but will not indicate what change, if any, is wanted.
Continuum scale	"I like the present school uniform." Place a cross on the line below to indicate your opinion of this statement. strongly agree strongly disagree	Again, the responses will not indicate what changes, if any, are wanted.

- *Note:* when using response scales, some thought should be given to the number of choices. An even number of choices can be used to ensure the response indicates an opinion one way or another, but sometimes it may be desirable to allow a neutral choice by having an odd number of options to choose from.

- When writing questionnaires, the questions should use simple language that any participant can reasonably be expected to understand. Questions should not involve equations or anything needlessly complicated if it is not relevant to the question being investigated. Similarly, there should be no ambiguity about what the question is asking so that there is only one possible interpretation. In general, the simpler the question and the more precise the response required, the more accurate the data will be.
- It is a good idea to trial your questions on a few people to check that the questions are achieving the purpose for which you are designing them.

Observational and experimental studies

- Some statistical questions can be answered by collecting data through observation. This is called an observational study. In this type of study, measurable data is recorded as it already exists in context.
 - For example, the average amount of time that people spend deciding what to buy from a vending machine could be analysed by watching CCTV footage.
- More advanced statistical research is often conducted through an experiment. The key feature of an experiment is that variables of interest are intentionally manipulated, and the effect of this variation is analysed.
 - For example, when testing the effectiveness of a new medication, researchers might give one group of participants the real medication and another group a “placebo”, which looks like the real medication but does not contain the active ingredient. Any differences between participants in the two groups are considered likely to be caused by the medication.

Ethical considerations

- **Ethics** must also be considered when conducting a statistical investigation. Participants should be told ahead of time how their data will be used and shared, and their privacy must always be protected. It is important to be mindful of how your study will be received by people from a wide range of backgrounds.
- **Bias** in statistical investigations can occur when one outcome is favoured above other outcomes. It can be either intentional or unintentional.
 - It is important to avoid any prejudice against a subset of the population you are studying, both for the wellbeing of participants, but also for the integrity of the data you collect.
 - One way bias may occur in the collection of data is if questions are phrased in a way that points the respondent towards a particular response. A survey or interview question that does this is called a leading question.

Worked example 10.1A Posing statistical questions

Which of these questions would be suited to a statistical investigation?

- a** What is the distance between Sydney and Newcastle?
- b** Do students who receive a high mark in Mathematics Standard earn more in their career than students who receive a low mark?
- c** Do teenagers eat more than adults?

- d** What is the average finishing time in the Hobart to Sydney Yacht Race?
- e** Which team scored the least number of tries in the NRL last season?

Part	Think	Write
a	This question has a definitive answer that can be looked up online.	No, statistics are unnecessary to answer this question.
b	This question would be best answered by conducting a statistical investigation.	Yes, there is no way to know the answer to this question without some sort of statistical investigation.
c	This question would be best answered by conducting a statistical investigation.	Yes, a statistical investigation would be the best way to find a reliable answer to this question.
d	The answer to this question would involve calculating the mean or median of recorded finishing times.	Yes, this question can be answered definitively based on a statistical analysis of existing data.
e	The answer to this question can be looked up online.	No, this question has a single correct answer, so statistics are unnecessary here.



Worked example video: Posing statistical questions

Understanding, fluency and communicating

Answers: Lesson 10.1

Helpful hints

- Answering a question using statistics can mean performing statistical calculations on existing data (such as calculating a mean) or collecting new data through a statistical investigation.
- Questions that can be answered without statistical calculations (e.g. by looking up the information) do not need to be answered using statistics.

- 1** For each of the following, decide whether the question is suitable, suitable if restated carefully, or not suitable as the subject of a statistical investigation.
- a** How many more wickets did Shane Warne take in Test cricket than Glenn McGrath?
- b** Are houses more expensive in Sydney or London?
- c** Do tall people earn more than short people?
- d** How hot was the warmest day in Broken Hill last year?
- e** How tall is the tallest building in Sydney?
- f** What is the average height of a skyscraper in Sydney?
- g** How much does it cost to fly from the Central Coast to Perth?
- h** Who is the most famous movie star in the world?

- 2 What would be the target population if we wanted to:
- a collect information about how much pocket money is given to primary school children?
 - b find out about the incomes of homeowners?
 - c investigate the success of a new treatment for the common cold?
 - d determine how common corruption is in professional sports?
 - e investigate the suitability of bus timetables on a certain route?
 - f collect information about the effects of road tax on company profits?
 - g investigate the success of a new treatment for ticks and fleas on dogs?
 - h investigate bank charges?



Worked example 10.1B Designing and developing surveys

Using each of the following question formats, write a survey question that could determine a person’s feelings after participating in a fun activity.

- a free response/open-ended
- b true or false
- c tick the box

Part	Think	Write
a	Ideally this question type should be asked in a way that elicits a multi-word response.	How are you feeling immediately after participating in this activity?
b	This question type requires a statement that the person can either agree or disagree with.	True or False: “I feel elated after participating in this activity.”
c	This question type can ask a similar question to the open-ended one, except we must anticipate the kinds of responses we expect. Sometimes this format allows for an extra option (usually “Other”) where the person can enter their own short response.	How are you feeling immediately after participating in this activity? Tick all that apply. ☐ Elated ☐ Happy ☐ Relaxed ☐ Exhausted ☐ Other (please specify): _____



Worked example video: Designing and developing surveys

Helpful hints

- No matter what question format is being used, survey questions should use simple language and be unambiguous in their meaning.
- When a question offers a list of possible responses, the number of choices provided may influence the type of information you collect (e.g. whether or not participants can choose a “neutral” option in a response-scale style question).

3 List some advantages and disadvantages of the following methods for collecting information from people. You could consider factors such as time, cost, the response rate (the number of people prepared to respond to the questions) and the truthfulness of answers.

- a** face-to-face interviews
- b** phone interviews
- c** online questionnaires

4 Use the prompts below to create a short questionnaire for each of the following statistical questions.

- a** Consider the statistical question, “What changes, if any, would students like to make to your school uniform?”

You may choose to focus on a more specific idea such as how many students want change, whether there are differences in opinion based on gender or year level, or which clothing items are most in need of change.

Write a three-question survey including:

- i** one yes or no question
- ii** one free response/open-ended question
- iii** one tick the box question.

- b** Consider the statistical question, “Which are the most popular audio apps and why?”

You may choose to focus on a more specific idea such as the types of content or range of music genres available, whether there are differences in preferences based on gender or whether the time or frequency of use is relevant.

Write a five-question survey including:

- i** two tick the box questions
- ii** three response scale questions.

- c** Consider the statistical question, “Are students satisfied with the food available at your school canteen?”

You may choose to focus on a more specific idea such as who uses the canteen when and how often, how much they spend or types of food available.

Write a five-question survey including:

- i** one true or false question
- ii** three continuum scale questions
- iii** one free response/open-ended question.



Worked example 10.1C Conducting ethical and unbiased statistical investigations

Explain why the following survey questions may result in a biased statistical investigation:

- a** Do you agree that lowering the company tax rate will be good for society?
- b** Polls show that 90% of customers prefer the new Strawberry Dream frozen yoghurt range. What do you think?

Part	Think	Write
a	It seems like the researcher believes that lowering the company tax rate will be good for society.	By asking “do you agree”, the likelihood that the respondent will agree with the survey question is increased.
b	Many people will not want to express their honest opinion if it goes against the opinion of 90% of others.	By explaining that the majority of people prefer the new frozen yoghurt range, this question is leading people to answer that they also prefer it.



Worked example video: Conducting ethical and unbiased statistical investigations

Helpful hints

- Leading questions lead the respondent to answer in a particular way. Unbiased questions should not lead the respondent in any way.
- Ethical considerations include informing participants of what is involved in the study, ensuring participants’ privacy is protected and taking care to act in a culturally sensitive manner.

- 5** Explain why the following survey questions may result in a biased statistical investigation.
- a** Taking into account the budget deficit, should the government cut military expenditure?
 - b** You don’t support the proposal for the new railway line, do you?
 - c** The Avengers was voted the best film of the 21st century. What’s your favourite film of the 21st century?
 - d** I can’t understand how the new road laws were passed. What is your opinion of them?
 - e** Soccer is the most played sport by Australian teenagers. What’s your favourite sport to play?
- 6** Ryan was researching the health issues that staff at a large company had over the past 10 years. He kept his research in an unlocked spreadsheet, which detailed the names of the staff, as well as their personal information, such as their salary. Explain why there may be a privacy issue with Ryan’s handling of the data.



Problem solving and reasoning

- 7 Return to your questionnaires from question 4.
- a Identify any places where you may have intentionally or unintentionally biased your data. Make any necessary changes to improve them.
 - b Identify any ethical considerations relating to your questionnaires. How could you address these?
 - c Trial your questionnaires from question 4 on a small group and, if necessary, modify the questions.

Challenge

- 8 The Australian Bureau of Statistics (ABS) is introducing some new questions into the 2026 Australian Census.
- a Find some examples of new questions introduced.
 - b Discuss any issues that could arise by including these questions. Has the ABS taken measures to reduce these risks?

Related resources



10.1 Quick quiz

Lesson 10.2

Classifying data

Learning intentions

By the end of this lesson, you will be able to ...

→ distinguish between numerical and categorical data and their subtypes.

Variable

- When collecting data, we are interested in a particular property or characteristic of a group of people or objects. The particular characteristic that we are interested in is called the **variable**.

Categorical data

- A **categorical variable** is one that describes a characteristic that can be divided into categories. For example, colour, quality and gender are categorical variables.
- Categorical variables can be **nominal** or **ordinal**.
 - A **nominal variable** is a qualitative variable that names categories into which the data can be sorted. There is no natural ordering of nominal categories. For example, eye colour could be blue, brown, grey, green or hazel.
 - An **ordinal variable** is a qualitative variable that *does* have an intrinsic order. These variables are often presented as scales or rankings. For example, the quality of work could be poor, satisfactory, good or excellent.

Numerical data

- A **numerical variable** is one that takes a numerical value. Numerical variables can be either **discrete** or **continuous**.
 - A **discrete variable** takes exact numerical values, often obtained as the result of counting. Examples include: the number of children in a family, marks achieved on a test or shoe sizes.
 - A **continuous variable** can take any numerical value up to any number of decimal places. These values are usually the result of measuring something. For example, the weight of an object that is approximately 48 kg could theoretically be found up to the extreme precision of 1000 decimal places, or even more.
- To distinguish between the two types of numerical variables, we can ask whether or not it would be (theoretically) possible to find a new value between any two values of the variable.
 - For example, if we line up a group of people in height order and an extra person joins, we can insert them into the line-up between two people.
 - On the other hand, if we lined the same group of people up based on their shoe sizes, they would form sub-groups, and the additional person would join one of these sub-groups.

Worked example 10.2A Classifying data

Classify the following variables as categorical or numerical, then further classify any categorical variables as nominal or ordinal, and any numerical variables as discrete or continuous.

- a** the number of heads when three coins are tossed
- b** the brands of toothpaste used by the students in a class
- c** the heights of a group of 15-year-old children
- d** the energy star ratings of refrigerators

Part	Think	Write
a	The possible options for this variable are 0, 1, 2 or 3, which are numerical values obtained by counting the number of heads tossed.	numerical, discrete
b	The possible responses a student might provide for the brand of toothpaste they use will not be a numerical value. Brand names form categories for which there is no intrinsic ordering.	categorical, nominal
c	Student heights generate numerical data and are obtained by measuring. These measurements can take any value and are only limited by the degree of accuracy of the measuring device.	numerical, continuous
d	The possible energy ratings are “one star” through to “six stars”. This variable ranks the relative efficiency of refrigerators, which is neither measured nor counted, but there is a clear natural ordering for the options.	categorical, ordinal



Worked example video: Classifying data

Understanding, fluency and communicating

Answers: Lesson 10.2

Helpful hints

- A discrete numerical variable is one that only takes exact or distinct values. They are usually related to counting.
- A continuous numerical variable is one that can take any value in a given range. They are usually associated with measuring.
- A nominal (categorical) variable is one that takes names of categories that cannot be ordered. Toothpaste brands or family pets are two examples.
- An ordinal (categorical) variable is one that takes names of categories that can be ordered. They are usually associated with rankings or things that change over time.

- For each of the following investigations, classify the variable as nominal (categorical), ordinal (categorical), discrete numerical or continuous numerical.
 - the number of goals scored weekly by a netball team
 - the heights of the members of a football team
 - the most popular radio stations
 - the numbers of children in Australian families
 - the number of loaves of bread bought each week by a family
 - the types of pets owned by students in a class
 - the energy ratings of different washing machines
 - the number of leaves on the stems of different plants
 - the amount of sunshine each day
 - the number of people who turn 100 years old each year

- k** students' opinions of their school uniform (like, no opinion, dislike)
- l** the amount of rainfall in each month of the year
- m** the countries of origin of immigrants
- n** the sizes of apples picked (small, medium, large)
- o** the numbers of pets owned by the students in a class
- p** the genders of school principals
- q** the health of patients after being given medication (worse, no different, better)



2 Give three of your own examples of:

- a** nominal (categorical) data
- b** ordinal (categorical) data
- c** discrete numerical data
- d** continuous numerical data.

Note: Be prepared to discuss your examples with the class and justify your answers.

Problem solving and reasoning

- 3 Design a short questionnaire to investigate the use of calculators by students in your class. Include at least one question for collecting information about each of the four types of variables discussed in this lesson.
- 4 Write a short questionnaire to investigate the use of the school canteen by members of your school population and others. Include at least one question for collecting information about each of the four types of variables discussed in this lesson.

Challenge

- 5 Some rating scales, such as rating a movie out of 5 stars, looks like it would be numerical data. However, it is often argued to be categorical, ordinal data. Explain why some people would argue this.
- 6 What statistics does the World Health Organization collect and how are they used? Go to www.who.int to research your answer for this question.
- 7 The United Nations collects statistics. Research what data is collected and how the statistics are used. Go to www.un.org to research your answer for this question.

Related resources



10.2 Quick quiz

Lesson 10.3

Collecting data

Learning intentions

By the end of this lesson, you will be able to ...

→ distinguish between a census and a sample and explain the purpose of sampling.

Collecting data about a population

- There are two overarching ways of collecting information about a population: via a **census** or a **sample**.
 - A census involves collecting data about every individual in the whole population. The individuals may be people or objects. A census is detailed and accurate, but they are expensive, time consuming and often impractical.
 - A sample involves collecting data about a portion of the population. It is cheaper and quicker than a census but is not as detailed or as accurate. Conclusions drawn from samples always involve some degree of error.

Bias in sampling

- Data is more commonly collected from a sample than from a census. The purpose of a sample is to provide an estimate about a particular characteristic of the whole population. Therefore, the challenge in selecting a sample is to make it as free from bias as possible and large enough to be representative of the whole population.
 - People may intentionally bias their data by selecting a sample to support their point of view. For example, if you wanted the local council to upgrade its swimming pools, you might be tempted to survey a sample of people who are already at the swimming pool!
- Statistical investigations should be conducted objectively and honestly. Keep in mind that the data you collect might not always support the outcome that you were expecting to observe.

Self-selected samples

- A **self-selected sample** (or a voluntary response sample) is a sample that all participants have chosen to be a part of. They are not directly approached by the researcher to participate in the survey/research. An opt-in survey shared on social media is an example of a study that relies on a self-selected sample.
 - This type of sampling is useful when we want people (or organisations) to volunteer to take part in research. There may be a wide range of reasons why people do this. They may have strong opinions about the research topic, a specific interest in the research and its findings, or they may simply want to assist a researcher.

- Participants usually select themselves to be part of a study by responding to an advertisement in a newspaper, online or even on a poster in a public place. The volunteers are then checked to make sure that their participation would be relevant to the study. For example, for a study involving 20-to-30-year-old males with Type 1 diabetes, the researcher would first check that the volunteers are indeed between 20 and 30 years old and that they do have a Type 1 diabetes diagnosis.
- The advantages of self-selected samples include:
 - reduced time and expense to find suitable subjects who meet the selection criteria
 - volunteers who are more committed to taking part in the study, which can help ensure attendance
 - volunteers who are more willing to provide insight into the subject of the research by giving full detailed responses to open-ended questions.
- The main disadvantages of self-selected samples come from an increased risk of bias. This type of bias is termed self-selection bias. For example, people volunteering to answer a questionnaire on a proposal to reduce company tax may have extreme political views on the matter and may wish to push their point of view. This can either lead to the sample not being representative of the population being studied or it could exaggerate a particular finding from the study.
- Despite the potential disadvantages of self-selection sampling, it is a popular and effective sampling strategy in experimental research settings.

Worked example 10.3A Deciding whether a census or sample would be used

Would a census or a sample be used to investigate:

- a** the length of time an electric light globe will last?
- b** the causes of car accidents in New South Wales?
- c** the number of people who use Delgate toothpaste?

Part	Think	Write
a	It is obviously impractical to test every light globe produced – there would be none left for sale!	Sample
b	An accurate analysis of all accidents would be required.	Census
c	It would be very time consuming to interview the whole population to find out who uses, or does not use, Delgate toothpaste.	Sample



Worked example video: Deciding whether a census or sample would be used

Understanding, fluency and communicating

Answers: Lesson 10.3

Helpful hints

- A sample collects data about a subset of the population.
- A census collects information about every member of the population.

- 1 For each of the topics listed in question 1 of Lesson 10.2:
 - i identify the target population or write a specific question for a statistical investigation on the topic if it is not already clear
 - ii state whether a census should be conducted or if a sample would be a better choice for the dataset.
- 2 Give three examples of data that would be collected using:
 - a a census
 - b a sample.

Worked example 10.3B Identifying bias in sampling

Identify any possible bias in each of the following surveys.

- a a phone survey during a weekday
- b a survey of people on a train station
- c a survey of a football crowd
- d a survey of 10 people who have been tested with a new drug developed to cure the common cold

Part	Think	Write
a	The sample does not include people who go out to work.	The sample would be biased towards people who are at home during the day.
b	The sample does not include people who use other forms of transport or work at home.	The sample would be biased towards people who catch the train.
c	Most probably more males than females attend football matches.	The sample would be biased towards people who attend football matches.
d	As only 10 people were tested, a valid conclusion cannot be made.	The sample is not large enough.



Worked example video: Identifying bias in sampling

Helpful hints

- The sample is likely to be biased if it is too small or does not represent sufficient diversity within the whole population.

- 3 Identify a potential source of bias in each of the following samples.
 - a a phone survey on a Saturday night
 - b a survey of people at a bus stop
 - c a survey of people at the beach
 - d people selected from the electoral roll
- 4 Explain why each of the following samples are likely to produce biased results.
 - a Year 12 students interviewed about changes to the school uniform
 - b motorists stopped in peak hour interviewed about traffic problems
 - c real-estate agents interviewed about the prices of houses
 - d politicians interviewed about the state of the country's economy

- 5 Explain why we should be sceptical about each of the following statements.
 - a a survey of 20 people indicated that 80% of people watch Channel 9 News
 - b a company marketing team claimed that four out of five dentists recommend its brand of toothbrush
 - c an advertisement claimed that “Dog breeders recommend Buddy dog food”
 - d a survey of hairdressers found in a “Yellow Pages” phone directory revealed that 90% of businesses prefer to make bookings by phone rather than accepting online bookings.
- 6 Identify why the following sampling methods may result in a biased statistical investigation.
 - a Investigating the religious beliefs of people in a town by surveying people at the local shopping mall on Sunday mornings.
 - b Conducting a survey of the views of Australians about the current Prime Minister by interviewing people in a single electorate.
 - c Conducting an investigation on the shopping habits of 17-year-olds by interviewing students from schools in inner Sydney.
 - d Researching favourite types of music by interviewing people outside an Adele concert.
 - e Finding opinions of road users on newly introduced bicycle lanes by interviewing members of the local cycling club.
 - f Understanding the state of the economy by interviewing investment bankers.
 - g Researching the favourite films of teenagers by conducting interviews at Comic-Con.

Worked example 10.3C Self-selected sampling

An ice-cream shop owner wants to trial some new and adventurous flavours by getting volunteers to come in for a taste-testing session. She puts posters up in shop windows all along the street. What type of sample is this?

Think	Write
People will only be part of the sample if they see the poster and contact the ice-cream shop owner themselves.	This is a self-selected sample.



Worked example video: Self-selected sampling

Helpful hints

- A **self-selected sample** is a sample that participants choose to be a part of by responding to a request for volunteers.

- 7 You want to conduct a statistical investigation into the gaming habits of teenagers in your suburb or town.
 - a Describe the process of creating a self-selected sample for this study.
 - b List three advantages of a self-selected sample for this study.
 - c List three disadvantages of a self-selected sample.

- 8 The careers adviser at your school wants to gain an insight into the career goals of the students in their final year.
 - a Write a short notice that the careers adviser could include in the school newsletter to recruit volunteers for this study.
 - b Does this recruitment process introduce any bias into the sample? For example, is one gender more likely to be represented in the study than others? Does this matter? Discuss with your class.
- 9 A poster is advertising a study that is seeking volunteers. It specifies that volunteers should be 15 to 17-year-old girls who use public transport in Sydney. What would happen if a 14-year-old boy applied to participate in the study?

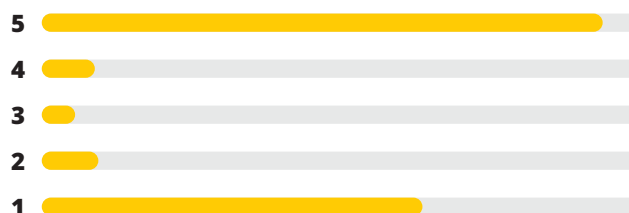
Problem solving and reasoning

- 10 The Australian Bureau of Statistics (ABS) is responsible for administering the Australian census.
 - a How often is the Australian census taken? What information is collected in the census?
 - b What information is openly made available?
 - c How is the information used by governments and businesses?
 - d Why doesn't the ABS just use data from smaller samples to answer the questions they use census data for?
- 11 Melissa wants to investigate the wages of university graduates. If Melissa only interviewed graduates with business degrees, why would her data not be suitable for estimating the wage of a 24-year-old science graduate?
- 12 Discuss and describe a biased sample you could choose if you wanted to gather support to:
 - a stop smoking in public places
 - b improve the local bus service
 - c increase the number of books in the library
 - d improve the school's senior common room.
- 13 Reflect on the following questions about self-selected samples.
 - a Have you ever come across an advertisement asking for volunteers to take part in a research study?
 - b Where was the advertisement you saw, and what was the study for?
 - c Did you take part in the study?
 - d What could be included in an advertisement for a study that would make you more likely to want to participate?

Challenge

- 14 When audiences review media such as movies or music albums, it can be common to see polarising rating summaries such as the one below. Explain why a self-selected sample can lead to ratings that look like these.

Audience rating summary



Related resources



10.3 Quick quiz



Worksheet: Understanding bias



Investigation: Census data

Lesson 10.4

Random sampling

Learning intentions

By the end of this lesson, you will be able to ...

→ generate random numbers and establish random samples of a population.

Random sample

- A **random sample** is a subset of the population in a statistical investigation where each member of the population has an equal chance of being selected.
- There are a number of ways in which a random sample may be taken. For example, suppose we wanted to choose a random sample of 10 students from a class of 30. We could:
 - write the students' names on pieces of paper, place the names in a hat, mix them and draw 10 names
 - write the students' names on cards, shuffle the cards and deal 10
 - assign the numbers 1 to 30 to each member of the class, select 10 random numbers from 1 to 30 using a random number generator, then form a sample from the students whose numbers are selected.



Generating random numbers

- There are many ways to generate a list of random numbers these days. Searching for a “random number generator” on the internet returns hundreds of millions of results. It is also possible to produce random numbers using a scientific calculator or a spreadsheet.
 - Most modern scientific calculators have a random number generator. On Casio calculators this function key is labelled RAN#. In a spreadsheet you can use the RAND function.
 - By default, these functions return random decimal numbers between 0 and 1. Therefore, if you want, for example, a random 3-digit number then you would multiply the randomly generated number by 1000.
 - Alternatively, in a spreadsheet you can use the RANDBETWEEN function to specify what range you want to sample random numbers from.

Worked example 10.4A Using a list of random numbers to select a random sample

A teacher wants to get feedback on an experimental activity they ran with a Year 10 class. Explain how they could choose a sample of 10 students from their class of 25 to obtain some feedback from.

Think	Write
The teacher could use a random number generator to get a list of 10 numbers between 1 and 25. It is important that there are no repeated numbers in the list. They should first decide how to allocate numbers to students. One way to do this would be to number students in the order they appear on the roll or attendance list. The randomly generated list of numbers can be turned into a random list of students using the numbers assigned on the roll.	Step 1: Assign all students a number based on the order they appear on the roll. Step 2: Use a random number generator to produce a list of 10 random numbers between 1 and 25, making sure there are no duplicate numbers. Step 3: Create the sample from students whose numbers appeared in the random list.



Worked example video: Using a list of random numbers to select a random sample

Understanding, fluency and communicating

Answers: Lesson 10.4

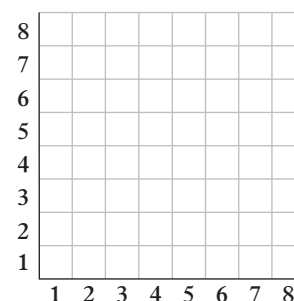
Helpful hints

- When using a random number generator to select a random sample, make sure the range of numbers being selected from matches the population you are sampling from.
- Consider whether or not it makes sense for the same number to be repeated in your sample.
- Decide on the numbering of people/items in your population before generating the list of random numbers.

- 1 Generate a list of 20 random numbers between 0 and 999.
 - a Check the randomness of these numbers by counting the number of odd numbers. Are about half of all the numbers odd numbers?
 - b How many of the numbers are:
 - i less than 500?
 - ii greater than 500?
 - iii equal to 500?
 - c Are there any repeated numbers? Give one example of a statistical investigation where it makes sense to allow the same number to be generated more than once, and one example of a statistical investigation where this would not make sense.
- 2 A factory supervisor wishes to interview a sample of workers about the use of safety equipment. Explain how she could choose a random sample of five from the 25 factory workers.



- 3 A supermarket manager wants a sample of his shop assistants to complete a questionnaire about rosters. Explain how 10 shop assistants could be selected at random from the 70 employed by the supermarket.
- 4 Explain how a random sample of 100 students could be selected from a school of 760 students.
- 5 A medical researcher wants to collect data from a random sample of hospitals in New South Wales that perform heart bypass surgery. Explain how he could select a random sample of six hospitals from the 12 that perform this type of surgery.
- 6 Complete these questions related to two methods of generating a list.
 - a Describe two different methods to generate a list of *pairs* of random numbers.
 - b Using each of the methods described in part **a**, generate two lists of 10 random pairs of numbers between 1 and 8.
 - c Draw two copies of the grid below and shade the squares described by your lists from part **b**.

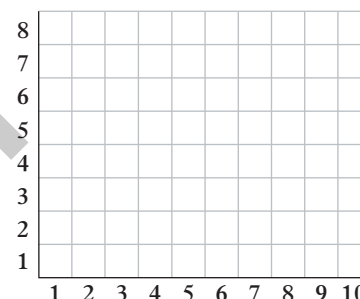


Problem solving and reasoning

- 7 A gardener spreads flower seeds over a large plot of ground. To estimate the number of seeds that germinate, he divides the plot of land into eighty squares, measuring 1 m by 1 m, as shown in this grid.

The gardener selects 10 squares at random and counts the number of seedlings in each square. He uses the numbers he counted to find an average number of seedlings per square, then estimates the total number of seedlings in the garden based on this.

 - a Explain how the gardener could select a random sample of 10 squares.
 - b Use a method of your choice to select 10 squares at random.
 - c If the numbers of seedlings in the squares selected are 21, 17, 18, 13, 27, 9, 22, 10, 18, 25:
 - i calculate the average number of seedlings per square
 - ii estimate the total number of seedlings in the plot.
 - d Now use a random number generator to create a list of ten numbers between 5 and 30 to replace the list in part **c**. Then repeat parts **i** and **ii** using your new list.
- 8 A school principal wishes to select a random sample of teachers to find out the amount of homework that is being set each day for Year 11 students.
 - a Explain two methods that the principal could use to choose a random sample of 10 teachers from the 62 teachers on the staff.
 - b If the number of minutes of homework set by the teachers selected is 20, 45, 30, 35, 40, 20, 25, 15, 50, 25, calculate the average amount of homework set by these teachers.
 - c If each Year 11 student has six teachers, estimate the total amount of homework each student is set each day.
 - d Use a random number generator to create a list of ten numbers between 0 and 60. Round all numbers up to the nearest multiple of 5. Answer part **b** again using this list.
- 9 Complete this class activity.
 - a Assign a number to each student in your class.
 - b Write your age, in years and months on the board, next to the number assigned to you.
 - c Choose a random sample of five students from your class.
 - d Calculate the average age of the five students in your sample.



- e Write your sample average on the board and compare the average of your sample with the averages of the other students' samples.
 - f Calculate the average age of the class using the information on the board.
 - g Compare the sample averages with the actual class average. Discuss and comment on any findings.
 - h Find the average of the sample averages. How does this compare with the actual class average (from part f)? Discuss with the class.
- 10 Find out how geographers use grid sampling to determine percentages of land types.

Challenge

- 11 If 100 students are randomly chosen from a high school to participate in a survey about the school, explain why the sample chosen might still lead to biased results.

Related resources



10.4 Quick quiz



Investigation: What is a good sample size?



Investigation: Types of sampling from a population

Lesson 10.5

Stratified sampling

Learning intentions

By the end of this lesson, you will be able to ...
→ describe and use stratified sampling to collect data.

Stratified random sample

- When we select a random sample of 10 students from a mixed class of 30 students, it is possible to get, for example, 10 boys. Similarly, when we select a random sample of 80 students from a school of 950, it is possible that we could get 80 students from the same year group. If it is important for a particular investigation to have a proportional mix, then simple random sampling may not produce a satisfactory sample. In these cases, we can use a **stratified random sample**.
- For a stratified random sample, the population is divided into subgroups (strata), based on different characteristics such as age, gender or income. The stratified random sample is made up of random samples from each subgroup. The size of each random sample is proportional to the relative size of the subgroup in the population.

Worked example 10.5A Selecting a stratified random sample of school students

A school of 950 students is made up of the following numbers of students at each year level.

Year level	7	8	9	10	11	12
Number of students	130	165	180	190	150	135

- a** If a stratified random sample is to be taken on the basis of these subgroups (school years), how many students should be chosen from each year if we want a stratified random sample of 80 students?
- b** Explain how to obtain this sample.

Part	Think	Write
a	The proportion of students in Year 7 is 130 out of 950 or $\frac{130}{950}$. So, the number of Year 7 students to be selected is $\frac{130}{950} \times 80$. The same calculation can be used for each year group after replacing 130 with the relevant number of students, as shown in the table.	Number of Year 7 students: $\frac{130}{950} \times 80 \approx 11$ Number of Year 8 students: $\frac{165}{950} \times 80 \approx 14$ Number of Year 9 students: $\frac{180}{950} \times 80 \approx 15$ Number of Year 10 students: $\frac{190}{950} \times 80 \approx 16$ Number of Year 11 students: $\frac{150}{950} \times 80 \approx 13$ Number of Year 12 students: $\frac{135}{950} \times 80 \approx 11$
b		We select a random sample of 11 Year 7 students, 14 Year 8 students, 15 Year 9 students, 16 Year 10 students, 13 Year 11 students and 11 Year 12 students.



Worked example video: Selecting a stratified random sample of school students

Understanding, fluency and communicating

Answers: Lesson 10.5

Helpful hints

- Determine the proportion and, therefore, the number of students to be selected from each year category. The stratified sample is then made up of random samples from each of these.

- 1** Complete the following to calculate the proportion of boys and girls that should be chosen in a stratified random sample of students selected on the basis of gender, if there are 75 boys and 65 girls.

Proportion of boys = $\frac{75}{140} = \frac{\square}{28}$ or ____%

Proportion of girls = $\frac{\square}{140} = \frac{\square}{28}$ or ____%

- 2 Calculate the proportion of boys and girls that should be chosen in a stratified random sample of students selected on the basis of gender, if there are 180 students, of which 76 are boys and 104 are girls.
- 3 A school of 859 students is made up of the following numbers of students at each year level.

Year	7	8	9	10	11	12
Number of students	125	142	175	160	134	123

- a If a stratified random sample is to be taken on the basis of these subgroups (school years), what proportion of students should be chosen from each year?
 - b How many students should be chosen from each year if we want a stratified random sample of 60 students?
 - c Explain how to obtain this sample.
- 4 A school of 439 students is made up of the following numbers of students at each year level.

Year	7	8	9	10	11	12
Number of students	96	84	78	65	56	60

- a If a stratified random sample is to be taken on the basis of these subgroups (school years), what proportion of students should be chosen from each year?
- b How many students should be chosen from each year if we want a stratified random sample of 40 students?
- c Explain how to obtain this sample.

Problem solving and reasoning

- 5 In Year 11, at Greengate High School there are 69 girls and 51 boys.
 - a In a stratified random sample that is selected on the basis of gender, what proportion of girls and boys should be in the sample?
 - b How many girls and boys should we select for a stratified random sample of 40 students?
 - c Explain how to obtain this sample.
- 6 The ages of people who live in a small country town are shown in the following table. It is proposed to do a dental check by choosing a stratified random sample of the population on the basis of these age groups.

Age	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70+
Number of people	180	835	260	350	280	250	200	120

- a What proportion of each age group should be included in the sample?
 - b What number of people in each age group should be selected for a sample of 40?
 - c Explain how to obtain this sample.
- 7 Discuss what subgroups could be used in a stratified random sample to investigate:
 - a the number of people with private healthcare cover
 - b the political party that people intend to vote for at the next election.

Challenge

- 8 A researcher wants to find out the average income of adults in a particular suburb. What subgroups would need to be considered in a stratified random sample to avoid skewed results?

9 Of the 150 students in Year 11, 87 travel to school by bus, 23 ride bicycles, 30 travel by private car and 10 walk. In order to find their average time of travel to school, it is decided to take a stratified random sample on the basis of these subgroups (methods of travel).

- a What proportion of each subgroup should be in the sample?
- b How many of each subgroup should be in a sample of 30 students?
- c Explain how to obtain this sample.



Related resources



10.5 Quick quiz

Lesson 10.6

Systematic sampling

Learning intentions

By the end of this lesson, you will be able to ...

→ describe and use systematic sampling to collect data.

Systematic sample

- In systematic sampling, a sample is chosen in a systematic way. To obtain a **systematic sample**, each member of the population is assigned a number, the first number is selected at random, and then we select every n th number; that is, we select numbers at regular intervals. The value of n , the size of the regular interval, depends on the size of the sample wanted and the size of the population.
 - For example, to find the interval size when selecting a systematic sample of 20 items from a population of 240 items, we calculate $240 \div 20 = 12$. That is, we should select every 12th item.

Worked example 10.6A Choosing an interval for systematic sampling

If a population has size 500 and you want to select a sample of size 100 using systematic sampling, what interval should you use?

Think	Write
To find 100 evenly spaced data points in the population of 500 we need to divide 500 by 100.	$500 \div 100 = 5$ So, we should take every 5th data point.



Worked example video: Choosing an interval for systematic sampling

Understanding, fluency and communicating

Answers: Lesson 10.6

- 1 A machine produces 300 items per day. At what interval should we select items to obtain a systematic sample of:
- a 25 items? b 50 items? c 10 items? d 15 items?
- 2 An electoral roll has 1200 names on it. At what interval should we select names to obtain a systematic sample of:
- a 100 names? b 200 names? c 150 names? d 40 names?

Worked example 10.6B Selecting a systematic sample with different starting numbers

A teacher is selecting a systematic sample of 10 students from a year level of 150 students.

- a Explain how the teacher could set up the sample and determine the interval required to select the sample of 10 students.
- b Explain what would happen if the randomly chosen starting point is:
- i 7
- ii 24
- iii 56.

Part	Think	Write
a	Assign the numbers 1 to 150 to all students in the year level. Divide the size of the population by the size of the sample wanted: $150 \div 10 = 15$. Start at any number then select every 15th number.	Assign the numbers 1 to 150 to all students in the year level. $150 \div 10 = 15$, so start at any number then select every 15th number.
b i	Suppose 7 is chosen as the starting number. Starting with 7, choose every 15th student.	If we started at the student who was assigned number 7, then we would select the numbers 7, 22, 37, 52, 67, 82, 97, 112, 127, 142 to form our sample.

Part	Think	Write
b ii	Suppose 24 is chosen as the starting number. Because 159 would have been the next number after 144, but there are only 150 numbers, so we start at the beginning again by subtracting $159 - 150 = 9$.	If we started at the student who was assigned number 24 we would select the numbers 24, 39, 54, 69, 84, 99, 114, 129, 144, 9.
b iii	Suppose 56 is chosen as the starting number. Because 161 would have been the next number after 146, but there are only 150 numbers, we start at the beginning again, $161 - 150 = 11$.	If we started at the student with the number 56, then we would select the numbers 56, 71, 86, 101, 116, 131, 146, 11, 26, 41.



Worked example video: Selecting a systematic sample with different starting numbers

Helpful hints

- To take a systematic sample of a population when the sample size is a divisor of the population size:
 - 1 Assign each member of the population a number.
 - 2 Divide the total size of the population by the desired sample size to find the appropriate interval size.
 - 3 Start at a random number then select participants from the list at regular intervals of the determined size.

- A systematic sample of students from a particular year level is required for a survey.
 - Explain how to obtain a systematic sample of 10 students from a year level of 200 students.
 - Which students would be selected if we started at the student numbered:
 - i 10?
 - ii 15?
 - iii 30?
 - iv 45?
 - v 113?
- A systematic sample of names from a roll are required.
 - Explain how to obtain a systematic sample of 15 names from a roll of 180 names.
 - Which names would be selected if we started at the name numbered:
 - i 5?
 - ii 10?
 - iii 25?
 - iv 50?
 - v 120?

Worked example 10.6C Selecting a systematic sample from different populations

Explain how to select a systematic sample of 20 names from a list of:

- a 146 names
- b 156 names.

Part	Think	Write
a	The population size is 146 and the sample size is 20. Find the appropriate interval size by calculating $146 \div 20 = 7.3$. Round 7.3 to the nearest whole number to get the interval size ($n = 7$). Remember to stop once the required number of participants have been selected.	Assign the numbers 1 to 146 to the names. Randomly select a number to start selecting names from, then select every 7th number until a sample of 20 names is found. For example, starting at 9 would result in a sample of the names in positions 9, 16, 23, 30, 37, 44, 51, 58, 65, 72, 79, 86, 93, 100, 107, 114, 121, 128, 135, 142.
b	The population size is 156 and the sample size is 20. Find the appropriate interval size by calculating $156 \div 20 = 7.8$. Round 7.8 to the nearest whole number to get the interval size ($n = 8$). Remember to stop once the required number of participants have been selected.	Assign the numbers 1 to 156 to the names. Randomly select a number to start selecting names from, then select every 8th number until a sample of 20 names is found. For example, starting at 53 would result in a sample of the names in positions 53, 61, 69, 77, 85, 93, 101, 109, 117, 125, 133, 141, 149, 1, 9, 17, 25, 33, 41, 49.



Worked example video: Selecting a systematic sample from different populations

Helpful hints

- To take a systematic sample of a population when the sample size is *not* a divisor of the population size:
 - 1 Assign each member of the population a number.
 - 2 Divide the total size of the population by the desired sample size and round to the nearest integer to find the appropriate interval size.
 - 3 Start at a random number then select participants from the list at regular intervals of the determined size.
 - 4 Stop when the desired sample size is reached.

- 5 Explain how to obtain a systematic sample of 10 names from a list of:
- a 62 names
 - b 87 names
 - c 124 names
 - d 239 names
 - e 185 names.

- 6** A machine produces 200 numbered engine parts in a day.
- a** At what intervals should a part be selected for a quality check if a systematic sample is required each day of:
 - i** 5 parts?
 - ii** 12 parts?
 - b** Choose a starting number at random and write the numbers of the engine parts you would select for each sample in part **a**.
- 7** A physical education teacher has a class of 30 students. At the start of the day's lesson, he wants to use a systematic sample to select the captains of six teams with equal numbers of students in each team.
- a** Explain how he would do this.
 - b** The teacher tosses two dice and uses the total, 9, as the number for the first captain selected. Which other students will be selected as captains?
 - c** For the second half of the lesson the teacher requires five teams of equal size. If he again starts with student number 9 as the first captain selected, which other students will be selected as captains?

Challenge

- 8** A large factory employs 600 workers.
- a** A systematic sample of 10 employees is to be selected for a health check. Explain how this could be done.
 - b** If the first employee, selected at random, is number 23, list the numbers of the other employees selected.
- 9** Employees at the factory in question **8** are required for a health check.
- a** Which employees should be selected for a health check if a systematic sample of 50 employees is required?
 - b** If the first employee selected is number 17, what will be the number of the last employee selected?
- 10** A researcher wants to measure the average depth of water in a particular spot at the beach. They measure the depth at the spot every 12 hours over the course of a week. Explain why using a systematic approach here may not lead to accurate measurements.

Related resources



10.6 Quick quiz

Lesson 10.7

Suitability of sample types

Learning intentions

By the end of this lesson, you will be able to ...

→ choose appropriate sampling methods and consider whether a sample is representative.

Suitability of sample types

- A good sample is one that has almost exactly the same characteristics as the total population. We say that a good sample is “representative” of the population. In this section, we will decide which sampling methods are most suitable in given situations and investigate the effect the size of a sample has on how well it represents the whole population.

Sampling methods

- Random sampling: each member of the population has an equal chance of being selected in the sample.
- Stratified random sampling: the population is divided into subgroups and then a random sample, proportional to the relative size of each subgroup, is taken from each subgroup.
- Self-selected sampling: the participants choose to be part of the sample by responding to a request for volunteers.
- Systematic sampling: the sample is chosen in a systematic way, with the first number being selected at random from the population and then further numbers being selected at regular intervals from the first number.

Worked example 10.7A Assessing the suitability of different sampling methods

A gardener grows carrots in rows and wants to pull up a small sample to see how well they are growing. Comment on the suitability of each sampling method.

- a** random sampling
- b** stratified random sampling
- c** self-selected sampling
- d** systematic sampling

Part	Think	Write
a	To produce a random sample the gardener might assign each carrot plant a number, then use a random number generator to select which carrots to dig up.	This method would be likely to select a good sample, but there is always a chance that it would only select carrots in the same row or with some other shared characteristic.
b	For this method the gardener would decide what characteristics might be relevant, then apply the random sampling method to each subset that shares the characteristics.	This method allows the gardener to decide what characteristics might be relevant, then ensure that each of these is represented in the sample.
c	The gardener would have to wait a long time for any of the carrots to volunteer to participate.	This method is not suitable, since carrots cannot self-select to participate!
d	In this method, the gardener could start with the first carrot in the first row, then walk along the rows selecting every n th carrot.	This method makes sense given that the carrots are arranged in an orderly fashion. It could be a problem if the interval happens to select only carrots at the same end of each row.



Worked example video: Assessing the suitability of different sampling methods

Understanding, fluency and communicating

Answers: Lesson 10.7

Helpful hints

- Think about what factors are important in the specific context when deciding which sampling method is most suitable.

- Discuss which sampling methods would be suitable for the following situations.
 - A machine produces 5000 bolts in a day. A sample of 50 bolts per day is required for a quality-control check.
 - There are 600 people on a town's electoral roll. A sample of 40 people is required to complete a questionnaire.
 - A sample of people is wanted to investigate average weekly expenditure on food.
 - A sample is wanted of the people attending a football game.
 - The school principal wants a sample of teachers to estimate how much homework is set each night.
 - A shoe company employs five machinists. A weekly quality-control check of their work is required.
 - A medical journal advertises for volunteers from the profession to participate in a research project.
 - A sample of Year 11 students is required to estimate the number of left-handed people in the group.
 - A grid sample is required by a geographer to estimate the proportions of various types of land use (e.g. grazing, crops, forest or buildings).
 - A sample is wanted in order to estimate the average weekly income of senior students.

- 2 The 200 students in Years 11 and 12 of a high school were asked whether or not they had ever used a vape. Using “y” for yes and “n” for no, the replies (as they were received) were as follows.

nnnny nnnyn ynnnn yynyy ynyny ynnyn nyynn yynyn ynnyn
 yynyy nnyyy yyyyy nnnny nnnnn nnyny yynny nynnn ynyyn
 nnyny ynyyy ynnnn yyyyn yynnn nynyn nynnn yynny nyynn
 yynyn ynynn nyyyn yynyy nyyny ynynn nyynn nnnny ynyyn
 yyyny ynnny nnyny ynnnn

- a** Is this a census or a sample?
b Is the data categorical or numerical?
c Calculate the proportion of all students who said they had vaped.
d If each of the following samples were used instead of a census, what would the answer to part **c** have been? Are these representative samples for Years 11 and 12 students at the school?
i the first 5 students
ii the first 10 students
e For each of the following sample sizes, use a table, calculator or computer to generate a set of that many random numbers, then use this to select a random sample of the 200 student responses above. Calculate the proportion of “y” responses in each sample.
i 5
ii 10
iii 20
iv 40
f For each of the following sample sizes, obtain a systematic sample of the 200 student responses above. Calculate the proportion of “y” responses in each sample.
i 5
ii 10
iii 20
iv 40

When separated according to year level, the responses are:

Year 11				Year 12			
nnyny	nnnny	ynyyy	nyynn	nnyyy	yynyy	nnnyy	nynnn
yynyn	ynnyn	yyny	ynyyy	nnyny	yynny	nynnn	ynyyn
yyynn	nnynn	nnnyy	nyynn	nnyny	yynny	nyyny	ynynn
nnyny	nnynn	nyynn	ynyyn	nyynn	nnnyy	ynwyyn	yynny
nnyny	ynnyn	yynyn	ynyyn	nnyny	nn		
nyyy	yyyny	yyn					

- g** For each of the following sample sizes, select a stratified random sample of the 200 student responses, with proportional representation of both Year 11 and Year 12 students. Calculate the proportion of “y” responses in each sample.
i 5
ii 10
iii 20
iv 40
h Comment on whether or not the samples in parts **e**, **f** and **g** are representative of the whole population. What effect does the size of the sample have on estimating the responses of the total population?



Worked example 10.7B Estimating the responses of a population from a representative sample

A sample of 500 people is chosen from a population of 70 000 people. The sample was carefully chosen to be representative of the entire population. The people in the sample were asked the following questions. Their responses are shown in the table.

	“Yes” responses	“No” responses	“Unsure” responses
Should a new cinema be built in the centre of town?	210	80	210
Is the public transport system adequate?	180	270	50

- a Calculate the percentage of the sample who answered “Yes” to a new cinema.
- b Estimate the number of people from the entire population who would want a new cinema.
- c Estimate the number of people from the entire population who would think the public transport system is inadequate.
- d Estimate the number of people from the entire population who would be unsure about whether the public transport system is adequate.

Part	Think	Write
a	210 of the 500 people surveyed said “Yes” to the new cinema.	$\frac{210}{500} \times 100\% = 42\%$
b	The sample is representative of the entire population. Find 42% of the entire population.	$\frac{42}{100} \times 70\,000 = 29\,400$
c	270 people surveyed thought the public transport system was inadequate. Find $\frac{270}{500}$ of the population of 70 000.	$\frac{270}{500} \times 70\,000 = 37\,800$
d	Of the 500 surveyed, 50 people were unsure. Find $\frac{50}{500}$ of the population of 70 000.	$\frac{50}{500} \times 70\,000 = 7000$



Worked example video: Estimating the responses of a population from a representative sample

Helpful hints

- If a sample is chosen so that it is representative of the entire population, then the results of the sample can be used to make predictions or generalisations about population.

- 3 A sample of 20 people is chosen from a population of 4000 people. The sample is believed to be representative of the entire population. The people in the sample were asked the following questions. The results are shown in the table.

	“Yes” responses	“No” responses	“Unsure” responses
Should a new fast-food restaurant be built in the centre of town?	12	6	2
Is the access to childcare adequate?	5	7	8

- a** Calculate the percentage of the sample who said “Yes” to a new fast-food restaurant.

$$\text{Percentage} = \frac{12}{\square} \times 100 = \underline{\hspace{2cm}}$$

- b** Estimate the number of people from the entire population who would want a new fast-food restaurant.

$$\text{Number} = \frac{60}{\square} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- c** Estimate the number of people from the entire population who would think access to childcare is adequate.

$$\text{Adequate childcare} = \frac{5}{\square} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- d** Estimate the number of people from the entire population who would be unsure about the access to childcare.

$$\text{Unsure} = \frac{\square}{20} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- 4** A sample of 400 people is chosen from a population of 50 000 people. The sample is believed to be representative of the entire population. The people in the sample were surveyed on the following questions. The results are shown in the table.

	“Yes” responses	“No” responses	“Unsure” responses
Should a new skate park be built in the town centre?	80	250	70
Are there enough parks?	280	30	90

- a** Calculate the percentage of the sample who said “No” to a new skate park.

- b** Estimate the number of people from the entire population who would not want a new skate park.

- c** Estimate the number of people from the entire population who would think there are enough parks.

- d** Estimate the number of people from the entire population who would answer “No” to the question “Are there enough parks?”

- 5** A sample is selected from a school population of 800. The 40 students chosen are believed to be representative of the entire school population. Of the students surveyed, 28 said they were happy with the school uniform and the rest were not.

- a** Calculate the percentage of the sample who are happy with the school uniform.

- b** How many students from the entire school population would be happy with the school uniform?

- c** How many students from the entire school population would be unhappy with the school uniform?

- 6** A sample is selected from a school population of 600. The 20 students chosen are believed to be representative of the entire school population. Of the students surveyed, 6 said they were happy with the school canteen and the rest were not.

- a** What percentage of the students in the sample are happy with the school canteen?

- b** How many students from the entire school population would be happy with the school canteen?

- c** How many students from the entire school population would be unhappy with the school canteen?

- 7** State whether you think the following samples would be satisfactory in estimating the nature of the whole population.

- a** Using the proportion of boys and girls in your class to estimate the proportion of boys and girls in Year 11 at your school.

- b** Using the proportion of boys and girls in your class to estimate the proportion of boys and girls in Year 11 across the state.

- 8** Write a short analysis of the following statistical statements. Is there anything misleading about them?

- a** Four out of five dentists recommend Delgate toothpaste.

- b** A newspaper reported that 10% of the Citico Company workforce had been dismissed.

- c** Of the two major political parties, one claimed that unemployment was rising and the other claimed that employment was rising. Could both be correct?

Challenge

- 9 Jasmine is collecting data on people's preferred political party in an upcoming local election.
- Which sampling method could she use if she was looking to find true, unbiased results?
 - If she wanted to skew the results towards a particular political party, how could she go about doing that?

Related resources



10.7 Quick quiz



Worksheet: Sample types

Lesson 10.8

Review: Collecting and classifying data

Review questions 10.8A Mathematical literacy review

Answers: Lesson 10.8

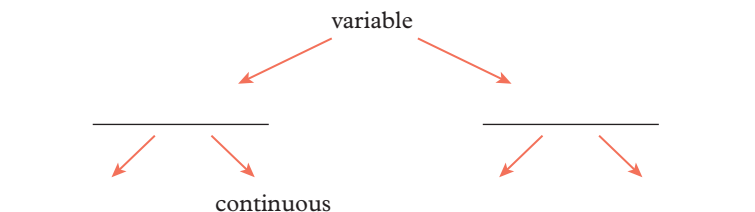
Key terms

The following key terms are used in this module:

- bias
- categorical variable
- census
- continuous variable
- data
- discrete variable
- ethics
- nominal variable
- numerical variable
- ordinal variable
- population
- random sample
- sample
- self-selected sample
- statistics
- stratified random sample
- systematic sample
- variable

Check your understanding

- Fill in the blanks using key terms from the list above.
_____ is the study of collecting, organising, analysing and interpreting ____.
- Draw and annotate a diagram that demonstrates the relationship between the terms **population** and **sample**.
- Fill in the blanks relating the different types of **variables** in the list of key terms:



- Give an example of each type of **variable** that appears in the bottom level of question 3. Try to use examples that you encounter in your own life.
- List three examples of things to consider if you want your statistical investigation to be **ethical**.
- Write a sentence explaining how **bias** might show up when using a **self-selected sample**.

- 7 Describe how each of the following sampling methods are used to select a sample from the population. You may choose to include a diagram for each sampling method.
- a** random sampling
 - b** stratified random sampling
 - c** systematic sampling

Review questions 10.8B Multiple-choice questions

- 1 Which one of the following questions can be answered without a statistical investigation?
 - A** What is the average time it takes Year 12 students to run 100 m?
 - B** Does it rain more in Sydney or Brisbane?
 - C** Which team has won the most NRL premierships?
 - D** Do lawyers earn more than doctors?
- 2 Which one of the following characteristics is *not* indicative of a well-written survey question?
 - A** There is only one way to interpret what the question is asking.
 - B** Simple language is used.
 - C** A neutral option is provided, if relevant.
 - D** The question relates to something that is not relevant to the study.
- 3 Which one of the following is *not* an example of a continuous numerical variable?
 - A** heights of students
 - B** wages of workers
 - C** daily temperatures
 - D** time taken to get to work
- 4 Which one of the following is an example of data that could not be collected by a census?
 - A** the life of a light globe
 - B** the number of vehicle accidents caused by speed
 - C** the number of tries scored by a rugby team
 - D** the age of school teachers in New South Wales
- 5 Which one of the following samples are *not* biased?
 - A** a phone survey on a Friday night
 - B** a survey of the audience at the opera
 - C** 10 students are selected from a class by writing all names on slips of paper, putting them in a hat, shaking them and then selecting 10 slips
 - D** a factory tests a sample of its products every Friday afternoon
- 6 Which one of the following is an example of a self-selected sample?
 - A** A researcher stands at the entrance to a bank and asks the first five people who arrive to participate in her survey.
 - B** From an alphabetical list of names, one is chosen at random and then every tenth name is selected.
 - C** A deck of cards is shuffled and then one card is selected at random.
 - D** Five people respond to a phone-in poll on the radio.
- 7 Which one of the following sampling methods will not result in a random sample?
 - A** Using a random number generator.
 - B** Choosing people who are sitting at the ends of every row.
 - C** Assigning each member of the population to a card in a deck, then shuffling the deck and dealing cards until a sample of the desired size is selected.
 - D** Rolling a 20-sided die.
- 8 Of the 150 students in Year 11, 70 are boys and 80 are girls. In a stratified random sample based on gender, the proportion of boys who should be chosen is:
 - A** $\frac{7}{8}$
 - B** $\frac{7}{15}$
 - C** $\frac{8}{15}$
 - D** $\frac{8}{7}$
- 9 For the information in question 8, the number of girls who should be chosen for a sample of 60 students is:
 - A** 53
 - B** 28
 - C** 32
 - D** 69

- 10** A machine produces 6000 items in a week.
To obtain a systematic sample of 200, we could start with the 50th item and then select the items numbered:
- A** 80, 110, 140, 170, ...
B 200, 400, 600, 800, ...
C 110, 170, 230, 300, ...
D 250, 450, 650, 850, ...
- 11** To investigate the average time it takes for Year 11 students to get to school, the most suitable sample type would be:
- A** a random sample
B a stratified random sample based on mode of travel
C a systematic sample
D a self-selected sample.
- 12** To estimate the number of boys and girls in Year 11 in New South Wales, a suitable sample would be the number of boys and girls in:
- A** your class
B the mathematics classes at your school
C Year 11 at your school
D Year 11 in a selection of schools in different regions.
- 13** A sample is selected from a school population of 900. The 30 students chosen are believed to be representative of the entire school population. Of the students surveyed, 25 said they were happy with the school canteen and the rest were not. How many students from the entire school population do you expect would not be happy with the school canteen?
- A** 5 **B** 25
C 150 **D** 750

Review questions 10.8C Set 1

- 1** Write an example of a leading question that might appear in a survey, then write a similar question that is less likely to bias responses.
- 2** Classify this data as nominal (categorical), ordinal (categorical), discrete numerical or continuous numerical.
- a** number of cloudy days each month
b heights of the girls in a netball team
c dress sizes (small, medium, large, etc.) in a store
d types of fish caught by anglers
- 3** State whether a census or a sample would be used for each of the investigations in question 2.
- 4** Comment on any possible bias in the following situations:
- 1** a phone survey on Saturday night
2 a questionnaire is sent to a sample of people chosen from the electoral roll.
- 5** Explain how a random sample of 12 students could be chosen from a class of 30.
- 6** Use the following list of randomly generated numbers: 5, 3, 1, 4, 5, 5, 3, 1, 1, 2 for these questions.
- A** Make a list of 5 *pairs* of random numbers from 1 to 5. Explain how you created your list.
B Copy the grid shown and shade the squares corresponding to the pairs of numbers in your list from part **a**.

5					
4					
3					
2					
1					
	1	2	3	4	5

- 7** A school population is made up of the following numbers of students at each year level. A stratified random sample on the basis of year groups is to be taken.

Year	7	8	9	10	11	12
Number of students	85	82	89	96	75	73

- a** What proportion of each year should be selected?
b How many from each year should be selected for a sample of 100 students?
- 8** Explain how to obtain a systematic sample of 20 students from a school roll of 300.
- 9** Give two examples of a self-selected sample.



- 10** A sample of 20 students is selected from a school population of 400 students. The sample chosen is believed to be representative of the entire school population. Of the students surveyed, six said they were happy with the school uniform and the rest were not.
- a** What percentage of the students from the sample are happy with the school uniform?
 - b** How many students from the entire school population would be happy with the school uniform?
 - c** How many students from the entire school population would be unhappy with the school uniform?
- 11** The manager of a bus depot wants a sample of drivers to complete a questionnaire about duty rosters. Explain how they could select eight drivers at random from the 74 who work at the depot.

Review questions 10.8D Set 2

- 1 Write two examples of categorical data (one nominal and one ordinal).
- 2 Would a census or a sample be used to investigate:
 - a** the number of people who use Freshspray deodorant?
 - b** the consumption of alcohol by senior students in your school?
- 3 Describe a biased sample you might be tempted to choose if you wanted to demonstrate support for:
 - a** improved netball facilities at the local park
 - b** more lines for serving senior students at the school canteen.
- 4 Describe samples with the opposite bias to those in question 3.
- 5 Suppose your calculator generated the following random numbers between 0 and 1. How would you use this output to create a list of five random three-digit numbers between 100 and 999? Write down your list.
 0.086 95... 0.581 12... 0.960 70...
 0.919 10... 0.130 17...
- 6 This table shows the number of pairs of shoes each of four machinists produces in a week. The factory manager wants to carry out a quality check using a stratified random sample of the shoes produced by each machinist.

Machinist	Vince	Sophie	Talia	Mohammed
Pairs of shoes	56	43	51	50

- a** What proportion of shoes from each machinist should be checked?
 - b** How many shoes produced by each machinist should be selected for a sample of 20 shoes?
- 7 A university administrator places an advertisement on an online notice board asking final year students to volunteer for a series of focus studies on access to tutors. What type of sampling is this?
 - 8 List the steps you would use to produce a list of random one-digit numbers on your calculator.
 - 9 Explain two methods that could be used to choose a random sample of six students from a class of 24.
 - 10 State whether or not you think the size of the following samples would be satisfactory for estimating the characteristics of the whole population.
 - a** using the number of students in your class who approve of the school uniform to estimate approval of the school uniform by all Year 11 students (consider three classes, six classes, etc.)
 - b** using the cars passing the front of the school to estimate the most popular colours of cars

Review questions 10.8E Set 3

- 1 What would be the target population if we wanted to:
 - a** collect information about the income of parents?
 - b** collect information about farm animals?

- 2 Classify the following as nominal (categorical), ordinal (categorical), discrete numerical or continuous numerical.
 - a the eye colours of a group of students
 - b the amount of sunshine each day
 - c the numbers of children in families
 - d the quality of dishwashers (poor, satisfactory, good)
- 3 Give an example of data that could be collected using a:
 - a census
 - b sample.
- 4 Describe a suitable sample to survey if you wanted to collect data about the:
 - a punctuality of the local bus service
 - b cleanliness of the council swimming pool.
- 5 A machine produces 300 numbered engine parts in a day.
 - a At what intervals should a part be selected for a quality check if a systematic sample is required each day of:
 - i 20 parts?
 - ii 12 parts?
 - b Choose a starting number at random and write the numbers of the engine parts you would select for each sample in part a.
- 6 Write two advantages and two disadvantages of a self-selected sample.
- 7 A sample of 15 students is selected from a school population of 300 students. The students chosen are believed to be representative of the entire school population. Of the students surveyed, 12 said they were happy with the school canteen and the rest were not.
 - a What percentage of the students in the sample are happy with the school canteen?
 - b How many students from the entire school population would be happy with the school canteen?
 - c How many students from the entire school population would be unhappy with the school canteen?
- 8 Outline the steps you would use to produce a list of random numbers from 1 to 100 using an Excel spreadsheet.
- 9 Use your calculator to obtain 10 random numbers between, and including, 60 and 80.
- 10 State whether or not you think the size of the following samples would be satisfactory in estimating the characteristic of the whole population.
 - a surveying the people at a bus stop to estimate the suitability of timetables for all users of the bus route (consider the number of bus stops on the route)
 - b surveying the people in a train carriage to estimate the cleanliness of all trains

Review questions 10.8F Set 4

- 1 Give an example of a survey question that could be misinterpreted, then give an alternative version that makes the meaning unambiguous.
- 2 A television station invites viewers to phone in their response to a question. What type of sample is this?
- 3 Write two examples of discrete numerical data.
- 4 State the advantages and disadvantages of using a census to collect information.
- 5 Comment on any possible bias in the following samples.
 - a Car tyre distributors are interviewed about the best-value tyre to buy.
 - b A systematic sample is taken of the people listed on the first five pages of the telephone directory.
- 6 A factory has 120 employees. A sample of 25 employees is to be chosen to complete a questionnaire.
 - a Explain how a random sample using a list of random numbers could be used to select the sample.
 - b Explain how to obtain a random sample without using random numbers.
 - c Explain how to obtain a systematic sample of employees.

- 7 Explain the meaning of the term “bias”.
- 8 What are the characteristics of a good sample and how can their results be used?
- 9 The grid of numbers below was generated randomly. Describe how you might use this to generate a list of five random numbers less than 200 and justify why your method generates a random sample.

08695	58112	96070	91910	18868	52251
99827	32581	90070	98873	89846	50953
92529	68249	54949	83829	10132	27359
13017	41045	13817	65603	87615	55691

Review questions 10.8G Practice examination question

- 1 Answer the following questions.

- a Classify the data obtained from the following question types as nominal or ordinal.

- i A tick the box with the following response options: **(1 mark)**

- ☐ sausages
- ☐ pies
- ☐ pasties
- ☐ veggie patties

- ii A response scale with the following response options: **(1 mark)**

- 1 strongly disagree
- 2 disagree
- 3 neutral
- 4 agree
- 5 strongly agree

- iii A free response box with the prompt: **(1 mark)**

Favourite ice-cream flavour: __

- b How might someone argue that the data collected in part a ii is discrete numerical data? **(1 mark)**

- c State whether a census or a sample would be more appropriate to use to collect the following data.

- i the number of left-handed children in Sydney primary schools **(1 mark)**

- ii the number of motorcyclists killed on NSW roads in a year **(1 mark)**

- d The manager of a bus depot wants a sample of drivers to complete a questionnaire on duty rosters.

- i Explain two methods that could be used to choose six drivers at random from the 48 drivers who work at the depot. **(2 marks)**

- ii Explain the likely bias if a self-selected sample was used to choose the six drivers. **(1 mark)**

- e There are 67 boys and 53 girls in Year 11 at a high school. A stratified random sample is to be selected based on gender.

- i What proportion of boys and girls should be selected? **(1 mark)**

- ii How many boys and how many girls should be selected for a stratified random sample of 30 students? **(2 marks)**

- f Explain how to obtain the sample in part e. **(1 mark)**

- g A machine produces 600 numbered electrical parts each day. A systematic sample of 20 parts each day is required for a quality check.

- i At what intervals should parts be selected? **(1 mark)**

- ii If the first part selected at random is the 37th, what is the number of the last part selected for the sample? **(1 mark)**

- h** A sample is selected from a school population of 700. The 28 students chosen are representative of the entire school population. Of the students surveyed, 11 said they were happy with the school canteen and the rest were not.

How many of the students from the entire school population would be unhappy with the school canteen?
(1 mark)

TOTAL: 16 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Collecting and classifying data

I can do this	I need to review this
☐ determine whether a question is suitable for statistical investigation	Go back to Lesson 10.1 Statistical investigations
☐ identify and produce survey questions that are clear and easy to interpret	
☐ identify ethical considerations in statistical investigations	
☐ classify data as numerical or categorical	Go back to Lesson 10.2 Classifying data
☐ classify numerical data as discrete or continuous	
☐ classify categorical data as nominal or ordinal	
☐ identify whether a census or a sample is appropriate for data collection	Go back to Lesson 10.3 Collecting data
☐ recognise that the purpose of a sample is to estimate a particular characteristic of the whole population	
☐ describe and use self-selected samples to collect data	
☐ generate random numbers using multiple methods and establish random samples	Go back to Lesson 10.4 Random sampling
☐ describe and use random samples to collect data	
☐ describe and use stratified samples to collect data	Go back to Lesson 10.5 Stratified sampling
☐ describe and use systematic samples to collect data	Go back to Lesson 10.6 Systematic sampling
☐ determine the appropriate sample type for a given situation	Go back to Lesson 10.7 Suitability of sample types
☐ justify whether a sample is representative of the population	

Related resources



Module review quiz: Collecting and classifying data



Worksheet: Challenge 10

Module 11

Displaying and interpreting data

DRAFT

Lessons

- Lesson 11.1 Dot plots and stem-and-leaf plots
- Lesson 11.2 Tables and column graphs
- Lesson 11.3 Line graphs
- Lesson 11.4 Histograms and frequency polygons
- Lesson 11.5 Sector graphs and divided bar charts
- Lesson 11.6 Cumulative frequency
- Lesson 11.7 Choosing a data display
- Lesson 11.8 Describing the shape of datasets
- Using spreadsheets
- Lesson 11.9 Review: Displaying and interpreting data

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Displaying and interpreting data

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Presenting data in a frequency table



Support sheet: Calculating percentages



Support sheet: Finding the average of two numbers

Syllabus links

- Data analysis (MST-11-08)
 - A student displays and analyses datasets using summary statistics and graphical representations.
 - Display and interpret grouped and ungrouped data
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 11.1

Dot plots and stem-and-leaf plots

Learning intentions

By the end of this lesson, you will be able to ...

→ construct and interpret dot plots and stem-and-leaf plots for appropriate datasets.

Organising raw data

- In this module we look at recording and displaying data once it has been collected. Raw data collected by observation or experiment is usually in a random order. It is difficult to gain information from the data when it is in this form.
- One aspect of statistics involves rearranging and condensing raw information into a form that is more easily read, so that patterns and characteristics may be identified, conclusions may be drawn, and inferences may be made.

Frequency distribution tables for categorical data

- A **frequency distribution table** displays the frequency, or number of times, that each type of data point occurs in collected data. **Tally marks** are often used to help record the data in the table. Every fifth tally mark is drawn diagonally through the preceding four marks to make bundles of five, which are easy to count.

Dot plots

- A **dot plot** displays data as columns or rows of dots. The number of dots represents the frequency of the variable.
 - A dot plot is a convenient and efficient method for illustrating a small dataset consisting of a small number of categories or possible outcomes. It also has the advantage that it can be used directly for unsorted data.
 - For large datasets, a dot plot is very time-consuming to prepare and, conversely, a completed dot plot is difficult to read. A dot plot can only be used when the outcomes can be counted.
 - Dot plots are useful for drawing attention to clusters, gaps and outliers.

Stem-and-leaf plots

- When raw data consists of a list of numbers, we often want a quick and easy way to get a sense of what the data looks like.
 - If the dataset is small and each data point is a single digit number, with some numbers appearing more than once, then a dot plot is a good option.
 - For datasets consisting of numerical data with more than one digit, another option for visualising the data is to create a **stem-and-leaf plot**.
- Stem-and-leaf plots are used to group and rank data to show the range and distribution.
 - A **leaf** is the final digit of a number, and all other preceding digits form the **stem**.
 - A stem may have any number of digits, but a leaf has exactly one. The stem is written once in the left column and all leaves that share the same stem are listed in numerical order to the right of that stem.
- In a stem-and-leaf plot:
 - all of the data is used and displayed
 - the largest and smallest measurements can be found
 - the clustering of data can be seen more easily
 - the length of the leaf row indicates the number of scores belonging to that stem.
- When we want to compare the stem-and-leaf plots for two datasets, we can use a back-to-back stem-and-leaf plot, which combines two stem-and-leaf plots by putting the second set of leaves to the left of the stem column. The leaves in the left-hand column should be ordered from right to left.
- A key is often provided to indicate the value of data points in a stem-and-leaf plot.
 - For example, the key $1 \mid 3 = 13$ means that a stem of 1 with a leaf of 3 represents the data value 13.

Worked example 11.1A Organising categorical data in a frequency distribution table

The method by which students from a particular class travelled to school on a particular day is recorded below, using the codes walk (W), cycle (C), bus (B), train (T) and motor car (M).

W C B T C B B B W B B B C B T C M C B T M
M T M M M T C C B

Rearrange this information into a frequency distribution table using a tally column.



Think	Write		
Work through the data from left to right using tally marks to record each time a category occurs. Add the tally marks and record the total in the frequency column.	Method of travel	Tally	Frequency
	Walk		2
	Bike		7
	Bus		10
	Car		6
	Train		5
		Total	30



Worked example video: Organising categorical data in a frequency distribution table

Understanding, fluency and communicating

Answers: Lesson 11.1

Helpful hints

- The total of the tally marks for each category gives the frequency for that category.
- Always check that the total of the frequency column is the same as the number of observations recorded.

- 1 The colours of cars that passed the front of a school in a 30-minute period are recorded below using the codes white (W), blue (B), grey (G), red (R) and other (O).

B W R O W B R G W O B G R W W G B R W O G B R W G
B R R G W B R G O W B W G R B W W B R G W B R W B

- Rearrange this information into a frequency distribution table using a tally column.
 - How many cars passed the front of the school in the 30-minute period?
 - What was the most popular colour of car in this survey?
 - Calculate the percentage of cars of each colour.
- 2 The eye colours of a group of students are recorded below using the codes

blue (B), brown (b), green (G) and grey (g).

B b g B b b G b B g b B g b G b b B g B B g b B G
B b g b B

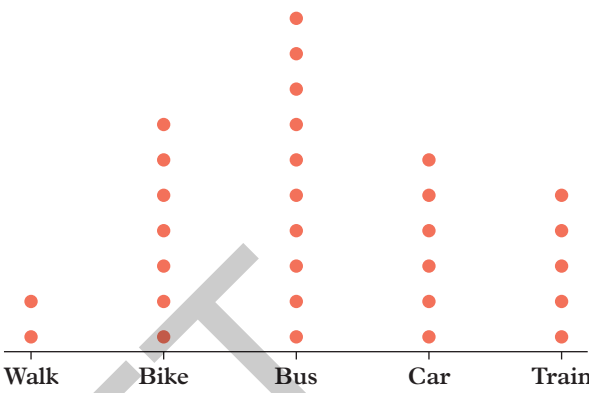
- Organise this data into a frequency distribution table using a tally column.
- How many students were observed?
- What is the most common eye colour in this sample?



Worked example 11.1B Drawing a dot plot

Recall the data from Worked example 11.1A about the transport methods used by students to travel to school.

- a Use the raw data to draw a dot plot.
- b Does the dot plot match the frequency distribution table?

Part	Think	Write
a	Draw a horizontal line and add equally spaced labels for each mode of transport. Work through the list of data from start to finish, placing a dot above each mode of transport as you encounter the corresponding letter in the list.	
b	Count the dots in each column and compare them to the entries in the frequency distribution table.	Yes. The number of dots in each column matches the corresponding entry in the frequency distribution table.



Worked example video: Drawing a dot plot

Helpful hints

- The frequency of each category can be found by counting the dots.

- 3 Draw a dot plot to display the dataset from question 1.
- 4 Draw a dot plot to display the dataset from question 2.
- 5 Data on the number of children in 25 families is recorded below.
2 1 3 5 2 3 1 4 2 2 3 4 1 3 2 5 2 3 2 4 3 2 4 2 3
 - a Draw a dot plot to display this information.
 - b Write the frequency of each number of children.

Worked example 11.1C Drawing a stem-and-leaf plot

The results in a mathematics class test are given below.

43 45 46 22 65 65 23 53 45 26 46 61 51 57 55 55 66 57 42
41 63 70 57 65 48 23 67 62 70 46

- a Draw a stem-and-leaf plot to represent this data.
- b What are the lowest and highest scores?

c How many students scored:

i 46 marks?

ii 50 marks?

iii 70 marks?

iv a mark in the sixties?

Part	Think	Write														
a	In this stem-and-leaf plot, the tens digit forms the stem and the units digit forms the leaf. This means that, for the mark 45, the stem is the 4 and the leaf is the 5.	<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>2</td><td>2 3 6 3</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>3 5 6 5 6 2 1 8 6</td></tr><tr><td>5</td><td>3 1 7 5 5 7 7</td></tr><tr><td>6</td><td>5 5 1 6 3 5 7 2</td></tr><tr><td>7</td><td>0 0</td></tr></table>	Stem	Leaf	2	2 3 6 3	3		4	3 5 6 5 6 2 1 8 6	5	3 1 7 5 5 7 7	6	5 5 1 6 3 5 7 2	7	0 0
	Stem	Leaf														
	2	2 3 6 3														
	3															
4	3 5 6 5 6 2 1 8 6															
5	3 1 7 5 5 7 7															
6	5 5 1 6 3 5 7 2															
7	0 0															
	The first row represents 22, 23, 26 and 23. There were no scores in the thirties.	<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>2</td><td>2 3 3 6</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>1 2 3 5 5 6 6 6 8</td></tr><tr><td>5</td><td>1 3 5 5 7 7 7</td></tr><tr><td>6</td><td>1 2 3 5 5 5 6 7</td></tr><tr><td>7</td><td>0 0</td></tr></table>	Stem	Leaf	2	2 3 3 6	3		4	1 2 3 5 5 6 6 6 8	5	1 3 5 5 7 7 7	6	1 2 3 5 5 5 6 7	7	0 0
Stem	Leaf															
2	2 3 3 6															
3																
4	1 2 3 5 5 6 6 6 8															
5	1 3 5 5 7 7 7															
6	1 2 3 5 5 5 6 7															
7	0 0															
	The leaves can now be put into ascending numerical order.															
	Always remember to include a key to show how leaves should be joined to the stem.	Key: 2 2 = 22														
b	List the first and last entry in the stem-and-leaf plot.	Lowest score = 22 Highest score = 70														
c i	Count how many 6s are in the row with stem 4.	Number of students who scored 46 = 3														
c ii	Count how many 0s are in the row with stem 5.	Number of students who scored 50 = 0														
c iii	Count how many 0s are in the row with stem 7.	Number of students who scored 70 = 2														
c iv	Count how many leaves there are in the row with stem 6.	Number of students who scored a mark in the sixties = 8														



Worked example video: Drawing a stem-and-leaf plot

Helpful hints

- The data is displayed in numerical order using the first digit (or digits) as the stem and the last digit as the leaf.

6 Consider this dataset.

32 45 37 65 48 57 33 42 59 62 47 51 50 47 36 68 60 50 63 47

a Draw a stem-and-leaf plot for the dataset, using the stems 3, 4, 5 and 6.

b What are the lowest and highest data points?

c How many times do the following data points occur?

i 50

ii 40

d Which data point occurs the most often?

e How many data points are in the sixties?

f How many data points are less than 50?

7 Consider this dataset.

132 154 124 156 125 145 130 141 126 151 140 139 128 145 126
146 128 130 140 158 154 142 149 145 150 126 130 148 158 145

a Draw a stem-and-leaf plot for the dataset, using the stems 12, 13, 14 and 15.

b What are the lowest and highest data points?

c How many times do the following data points occur?

i 150

ii 158

iii 120

d Which data point occurs the most often?

e How many data points are:

i less than 130?

ii 150 or more?

8 The results of a class test are shown in this stem-and-leaf plot.

Stem	Leaf
5	0 0 0 1 4 4 4 4 5 6 7 8 8
6	0 0 1 2 3 3 5 7 8 8 9 9
7	4 4 5 7 8 8
8	1 1 5 5 7 7
9	0 0

Key: 1 | 2 = 12

a What are the lowest and highest scores?

b How many times do the following scores occur?

i 50

ii 60

iii 70

iv 80

c Which score occurs the most often?

d How many scores are in the 50s?

e How many scores are:

i less than 60?

ii 80 or more?

9 The stem-and-leaf plot shows the time (in hours) a group of students spent watching television in one particular week.

Stem	Leaf
0	3 5 6 8 9
1	0 2 2 3 5 5 5 9
2	2 4 5 5 5 7 8
3	0 1 1 4 6

Key: 2 | 4 = 24

a How many students were surveyed?

b What was the lowest number and the greatest number of hours spent watching television that week?

c How many students watched fewer than 10 hours of television that week?

d How many students watched more than 30 hours of television that week?

Worked example 11.1D Drawing a stem-and-leaf plot with split stems

The number of people who live in each house in a particular street is recorded below.

4 2 3 4 5 2 4 3 2 5 1 4 5 5 3 7 2 3 4 4

Draw a stem-and-leaf plot for this information.

Think	Write										
<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>0</td><td>1 2 2 2 2 3 3 3 3 4 4 4 4 5 5 5 5 7</td></tr></table> Because all the data points are a less than 10, this plot gives no more information than an ordered list of numbers. This data can be presented in a more meaningful way by splitting the stem into 0 and 0 *, where: The stem 0 contains the leaves 0, 1, 2, 3, 4. The stem 0 * contains the leaves 5, 6, 7, 8, 9.	Stem	Leaf	0	1 2 2 2 2 3 3 3 3 4 4 4 4 5 5 5 5 7	<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>0</td><td>1 2 2 2 2 3 3 3 3 4 4 4 4 4</td></tr><tr><td>0 *</td><td>5 5 5 5 7</td></tr></table> Key : 0 1 = 1 It can be seen from the split stem-and-leaf plot that most households have 4 or fewer people.	Stem	Leaf	0	1 2 2 2 2 3 3 3 3 4 4 4 4 4	0 *	5 5 5 5 7
Stem	Leaf										
0	1 2 2 2 2 3 3 3 3 4 4 4 4 5 5 5 5 7										
Stem	Leaf										
0	1 2 2 2 2 3 3 3 3 4 4 4 4 4										
0 *	5 5 5 5 7										



Worked example video: Drawing a stem-and-leaf plot with split stems

Helpful hints

- If the data has one stem only, it is often useful to split the leaves into two groups, one containing the leaves 0, 1, 2, 3, 4 and one containing the leaves 5, 6, 7, 8, 9.

- 10 The number of hours per week a group of Year 11 students spent doing homework is shown.
- 27 22 38 30 26 34 18 27 32 31 28 33 33 29 24 27 30 31 33 27 28
22 26 32 34 26 26 37 29 28 31 33 23 34 29 26 33 29 32 27
- a How many students were surveyed?
- b Draw a stem-and-leaf plot for this data using stems 1, 2 and 3.
- c Draw another stem-and-leaf plot for the data using stems 1, 1 *, 2, 2 *, 3, 3 *.
- d What information is shown in the second stem-and-leaf plot that is not evident in the first?

Worked example 11.1E Drawing a back-to-back stem-and-leaf plot

- The heights of girls and boys in Year 12 were measured in centimetres. The results are shown in these lists.
- Girls: 152, 162, 167, 170, 180, 193, 174, 157, 163, 172, 169, 174, 167, 160, 170, 171, 167, 174
- Boys: 169, 180, 185, 179, 174, 185, 195, 181, 163, 176, 174, 174, 179, 183, 188, 188, 192, 192
- a Represent this data in a back-to-back stem-and-leaf plot.
- b From the data, what comparisons can be made between the heights of boys and girls?

Part	Think	Write																		
a	Use stems beginning with 15 up to a stem of 19. The values are in order as usual, but for the left-hand side of the plot, the smallest numbers are closest to the stem, which is in the centre.	<table><tr><th>Leaf: Boys</th><th>Stem</th><th>Leaf: Girls</th></tr><tr><td></td><td>15</td><td>2 7</td></tr><tr><td>9 3</td><td>16</td><td>0 2 3 7 7 7 9</td></tr><tr><td>9 9 6 4 4 4</td><td>17</td><td>0 0 1 2 4 4 4</td></tr><tr><td>8 8 5 5 3 1 0</td><td>18</td><td>0</td></tr><tr><td>5 2 2</td><td>19</td><td>3</td></tr></table> Key : 6 15 3 156 cm for Boys 153 cm for Girls	Leaf: Boys	Stem	Leaf: Girls		15	2 7	9 3	16	0 2 3 7 7 7 9	9 9 6 4 4 4	17	0 0 1 2 4 4 4	8 8 5 5 3 1 0	18	0	5 2 2	19	3
Leaf: Boys	Stem	Leaf: Girls																		
	15	2 7																		
9 3	16	0 2 3 7 7 7 9																		
9 9 6 4 4 4	17	0 0 1 2 4 4 4																		
8 8 5 5 3 1 0	18	0																		
5 2 2	19	3																		

Part	Think	Write
b	The largest measurements belong to the boys, while the smallest belong to the girls. The majority of the boys' heights are in the 17 and 18 rows, while the majority of the girls' heights are in the 16 and 17 rows.	The boys are generally taller than the girls, even though some girls are taller than some boys.



Worked example video: Drawing a back-to-back stem-and-leaf plot

11 The average rainfall, to the nearest millimetre, for Adelaide and Melbourne is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adelaide	17	19	22	38	59	52	65	50	43	38	24	23
Melbourne	48	48	52	58	58	50	49	51	59	68	60	59

a Copy and complete this back-to-back stem-and-leaf plot for the rainfall data.

Leaf: Melbourne	Stem	Leaf: Adelaide
	1	
	2	2 3 4
	3	
8 8	4	3
	5	0 2
8 0	6	

Key: 8 | 4 | 3

48 mm for Melbourne
43 mm for Adelaide

b What comparisons can be made between the rainfall of the two cities?

12 The average rainfall, to the nearest millimetre, for Canberra and Hobart is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Canberra	58	57	56	53	49	37	40	47	50	67	63	53
Hobart	48	40	47	52	49	56	54	52	52	64	55	52

a Represent this data in a back-to-back stem-and-leaf plot.

b What comparisons can be made between the rainfall of the two cities?

13 The weights, in kilograms, of 20 five-year-olds in a group are shown.

Girls: 17.6, 18.3, 17.9, 18.5, 17.4, 18.0, 17.7, 18.2, 17.8, 18.1

Boys: 18.2, 19.5, 17.8, 20.1, 18.9, 19.2, 17.5, 18.7, 19.0, 18.4

a Represent the data in a back-to-back stem-and-leaf plot.

b What comparisons can be made about the weights of the girls and boys in this sample?

Problem solving and reasoning

14 Two primary school classes are surveyed and asked to nominate their favourite vegetable from the given list. The results are shown below.

Class 4 Blue	Favourite vegetable	Class 4 Red
.	carrots	
.	peas	
.	potato	
.	broccoli	
.	snow peas	. .
. .	pumpkin	.

- a How many students are there in Class 4 Blue?
- b How many students in Class 4 Red nominated peas as their favourite?
- c Which is the most popular vegetable among students in:
- i Class 4 Blue? ii Class 4 Red?
- d Which is the least popular vegetable among students in:
- i Class 4 Blue? ii Class 4 Red?
- e If the two classes are combined, which vegetable is the:
- i most popular? ii least popular?
- 15 The back-to-back stem-and-leaf plot shows the marks of two Year 11 classes for a mathematics test.

Leaf: Class 11A	Stem	Leaf: Class 11B
	0	2
9 9 9 8 8 5 5 5 4 4 2	1	9
9 9 8 7 6 5 5 5 1 0 0 0 0	2	0 0 2 2 3 3 5 6 6 7 9
9	3	0 0 1 2 2 4 4 4 5 8 8 9

Key: 2 | 1 | 9
12 for Class 11A
19 for Class 11B

- Answer the following questions for:
- i Class 11A ii Class 11B.
- a What was the lowest mark?
- b What was the highest mark?
- c What was the most common mark?
- d In what range are the marks clustered?
- e In this test a mark of 18 or more was considered satisfactory. How many marks were rated unsatisfactory?
- f Based on your analysis of parts i to v, which class performed better on this test?
- 16 The stem-and-leaf plot shows the weights (in kilograms) of all team members in two different school sports teams.

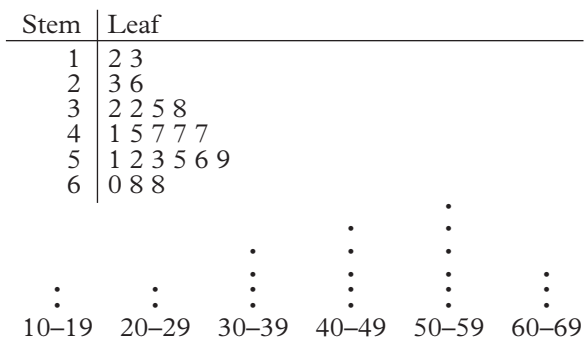
Leaf: Rowers	Stem	Leaf: Tennis players
	4	7 9
7 0	5	3 7 7 7 9
9 9 4 3	6	3 6 7
6 5 3 1 0	7	1 5
9 2 2	8	0

Key: 7 | 5 | 3
57 kg for Rowers
53 kg for Tennis players

- a How many students are in each team?
- b How many rowers weigh less than 60 kg?
- c How many tennis players weigh more than 70 kg?
- d How many students weigh less than 66 kg?

Challenge

17 Explain why the stem-and-leaf plot and dot plot could be considered the same.



Related resources



11.1 Quick quiz



Worksheet: Working with dot plots



Worksheet: Working with stem-and-leaf plots

Lesson 11.2

Tables and column graphs

Learning intentions

By the end of this lesson, you will be able to ...

→ use tables and column graphs to organise and display appropriate datasets.

Grouping numerical data

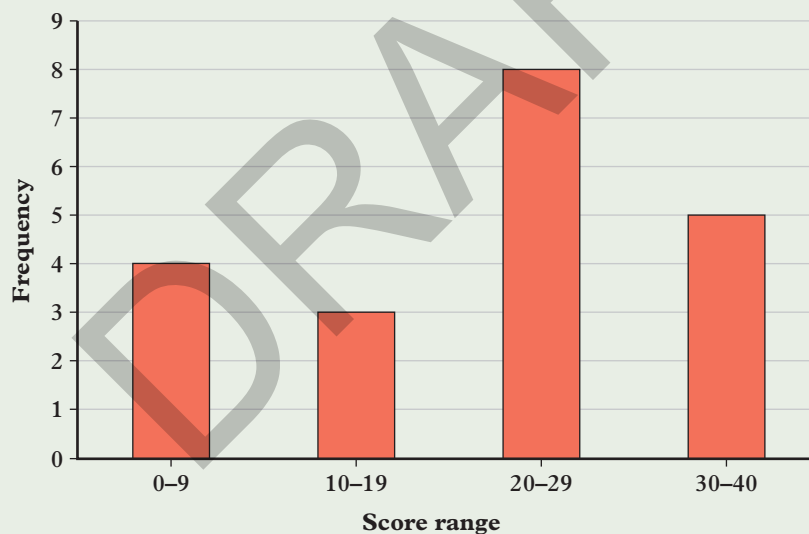
- Large numerical datasets can be difficult to make sense of. When data is numerical, it is often much easier, or even necessary, to group together a range of values. These groups are called **classes**, or class intervals.
- When grouping numerical data, we ensure that:
 - the classes do not overlap. For example, if we formed classes 0–10, 10–20, 20–30, ... then we would not know in which class to put score 10 or score 20.
 - there are no gaps between class intervals. Even if there are no data points observed in a class, the class should still be included.
 - each class interval is the same size. When there are extremely low or high scores it may be convenient to make the bottom and top class intervals open; for example, less than 300, 300–319, 320–329, ..., 390–399, 400 or more. However, this often makes further statistical analysis more difficult.
- Once numerical data has been grouped, we can use all the displays that we would normally use for categorical data.
 - For example, we can create frequency distribution tables just as we did in Lesson 11.1. The frequency of all data points in each class can be tallied in the same row of a frequency distribution table.

Organising and displaying grouped data

- We have already been using tables to summarise the frequency of categorical data. Summary tables can be used to organise any dataset with grouped numerical or categorical data in the first column and any kind of numerical data in the second column.

Correct answers	Frequency
0–9	4
10–19	3
20–29	8
30–40	5

- A frequency distribution table is an example of a summary table.
- In a summary table, the first column (or row, if the table is horizontal) will typically contain the **independent variable**, with the **dependent variable** placed in the second column (or row).
- Data from a summary table can be represented in a **column graph**.
 - In a column graph, the horizontal axis represents the categorical variable (or the numerical variable that has been grouped). The vertical axis represents the numerical variable, which could be a frequency or any other numerical variable, such as a score.
 - The numerical value for each category is represented by a rectangular bar of that height, as measured against the vertical axis. The bars in each column should all be the same width and be evenly spaced, without touching each other.



Worked example 11.2A Grouping numerical data

Sort the following dataset into appropriate classes so that it can be displayed in a meaningful way as a frequency distribution table.

18 29 32 34 23 19 22 12 27 33 42 39 19 37 34 28 18 27

Think

In order to get a sense of this dataset, we can summarise it in a stem-and-leaf plot with the key $1 \mid 2 = 12$:

Stem	Leaf
1	2 8 8 9 9
2	2 3 7 7 8 9
3	2 3 4 4 7 9
4	2

One option would be to group the scores as they are in the stem-and-leaf plot, i.e. in the classes 10–19, 20–29, 30–39, 40–49.

However, in this case, we could also try smaller classes:

Stem	Leaf
1	2
1 *	8 8 9 9
2	2 3
2 *	7 7 8 9
3	2 3 4 4
3 *	7 9
4	2

Write

This dataset can be grouped into the classes 10–14, 15–19, 20–24, 25–29, 30–34, 35–39, 40–44. The frequency distribution table for the dataset with these classes is:

Class	Frequency
10–14	1
15–19	4
20–24	2
25–29	4
30–34	4
35–39	2
40–44	1



Worked example video: Grouping numerical data

Understanding, fluency and communicating

Answers: Lesson 11.2

- 1 The minimum daily temperatures (in °C) recorded for a month were as follows.

22 21 20 22 23 22 21 23 19 21 21 23 21 20 22 21 20 20 22 20 23
19 20 19 20 18 19 17 18 22

- Organise this information into a frequency distribution table using a tally column.
- On how many days was the temperature recorded?
- On how many days did the lowest minimum temperature occur?
- On how many days was the minimum temperature 20°C?
- What month of the year could this be? Discuss.

- 2 Consider the following marks (out of a possible 100) gained by 50 students in a test.

70 31 49 54 55 74 50 34 60 68 68 57 45 56 54 85 44 66 56 60 22
45 53 29 61 20 13 52 41 50 74 51 48 69 75 61 40 41 33 62 65 55
51 42 94 64 38 3 51 79

- a Complete this frequency distribution table, which has been split into three parts, for the data given above.

Score	3	13	20	22	29	31	33	34	38	40	41	42	44
Frequency													
Score	45	48	49	50	51	52	53	54	55	56	57	60	61
Frequency													
Score	62	64	65	66	68	69	70	74	75	79	85	94	
Frequency													

- b How useful is the frequency distribution table you completed? Explain.

- c Consider the following incomplete frequency distribution table in which the data from above is grouped into classes that cover a range of scores.

Class	Tally	Frequency
0–9		1
10–19		1
20–29		3
30–39		
40–49		
50–59		
60–69		
70–79		
80–89		
90–99		
	Total	50

- Explain what a class of 0–9 means.
 - Explain what a class of 60–69 means.
 - Are the classes the same size? What is the size of each class interval?
 - Complete the frequency distribution table for the given data.
 - How many of the test scores lie in the interval 50 to 59?
 - What percentage of students obtained a test score in the range 80 to 89?
- d Consider this incomplete frequency distribution table which groups the data into fewer classes.

Class	Tally	Frequency
0–19		2
20–39		
40–59		
60–79		
80–99		
	Total	50

- What is the size of each class?
- Complete this frequency distribution table for the given data.
- How many of the test scores lie in the interval 20 to 39?
- How many students obtained a test score below 60?

- e List at least one advantage and one disadvantage of grouping data in classes.
- 3 A marriage counsellor asked his clients to keep a record of the number of arguments they had in a one-week period. The results were collected and are shown in this table.

Number of arguments	Frequency
0–2	8
3–5	18
6–8	10
9–11	3
12–14	1

- How many different responses are included in each class interval?
- How many clients responded to the counsellor's request?
- How many clients had fewer than three arguments in the week?
- How many had more than eight arguments?
- Can you determine how many clients had six arguments? Explain.
- Can you determine the lowest number and highest number of arguments? Explain.

- 4 A nursery worker takes a random sample of seedlings and measures their heights to the nearest millimetre. The results are recorded in the frequency distribution table shown.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6

- How many different heights, to the nearest millimetre, are possible in each class interval?
 - How many seedlings are there in this sample?
 - Can you determine how many plants were 350 mm high? Explain.
 - Can you determine the shortest and tallest plants in this sample? Explain.
- 5 Recall the dataset and stem-and-leaf plot from question 9 in Lesson 11.1.
- Draw a grouped frequency distribution table to represent the dataset, using these class intervals:
 - 1–5, 6–10, 11–15, etc.
 - 0–9, 10–19, 20–29, etc.
 - Comment on the advantages and disadvantages of a stem-and-leaf plot compared with a grouped frequency distribution table.



Worked example 11.2B Drawing a column graph from a summary table

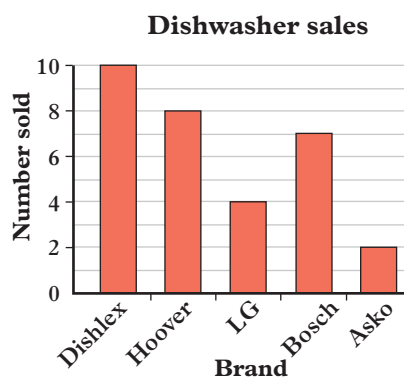
The number of each brand of dishwasher sold by an electrical store in one week is shown in the summary table. Draw a vertical column graph to illustrate this information.

Brand	Dishlex	Hoover	LG	Bosch	Asko
Frequency	10	8	4	7	2

Think

The brands are placed on the horizontal axis and the frequency on the vertical axis. The highest frequency is 10, so a suitable scale is chosen for this axis. A rectangular column 10 units high is drawn to represent Dishlex dishwashers, a column 8 units high is drawn to represent Hoover dishwashers, etc.

Write



Worked example video: Drawing a column graph from a summary table

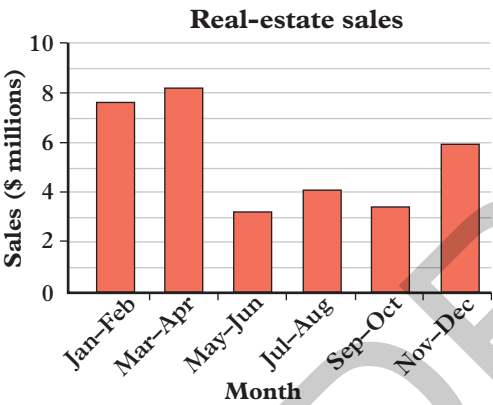
Helpful hints

- To create a column graph, place categories on the horizontal axis and the frequency, with a suitable scale, on the vertical axis. Each bar should be the same width and evenly spaced, with its height representing the frequency of the category.
- Sometimes the axes are swapped, and the bars are drawn horizontally. In this case, the graph is usually called a horizontal bar graph.

6 This table shows the average percentage of protein in certain foods. Draw a column graph to illustrate this information.

Food	Protein (%)
Beef	35
Rice	5
Eggs	13
Fish	30
Milk	3.5

7 This column graph shows the sales for a real-estate company, for the months given. Complete the table below for the data.



Month	Sales (\$ million)
Jan/Feb	
Mar/Apr	
May/Jun	
Jul/Aug	
Sept/Oct	
Nov/Dec	

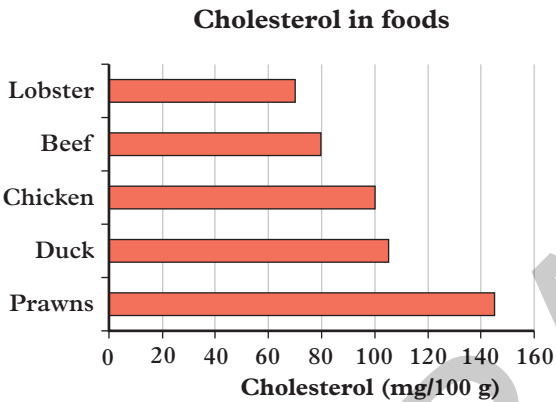
8 The ages of the vehicle occupants killed on NSW roads in one year are shown in the table below. Represent this information using a column graph.

Age group	Number killed
0–9	42
10–19	110
20–29	175
30–39	65
40–49	80
50–59	55
60–69	40
70+	60

9 This table shows the number of injuries in eight of the main sports played in Australia. Illustrate this information using a column graph.

Sport	Injuries (number per year)
AFL	270
Soccer	160
Cricket	99
Rugby League	88
Rugby Union	58
Netball	40
Hockey	32
Indoor Cricket	30
Basketball	23

10 The number of milligrams of cholesterol per 100 g of various foods is shown in the horizontal bar graph below. Complete the given table for the data.

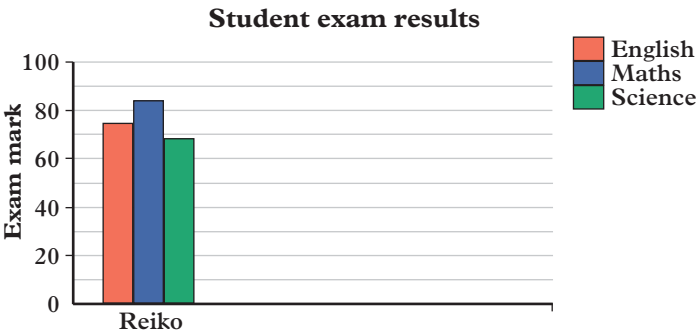


Food	Cholesterol (mg/100 g)
Lobster	
Beef	
Chicken	
Duck	
Prawns	



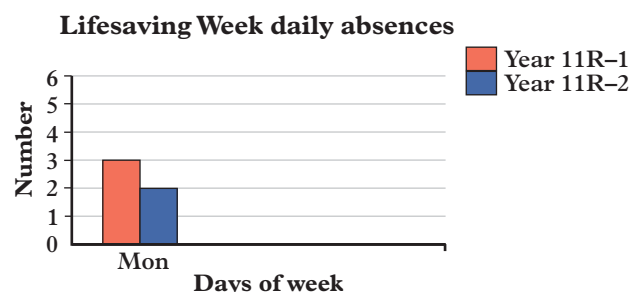
11 The English, mathematics and science exam marks for five students are shown in the table below. Complete the column graph to show these results by grouping the subjects together for each student as has been done for Reiko.

Student	English	Maths	Science
Reiko	75	84	68
James	65	76	84
Wendy	89	65	60
Hassan	54	78	75
Vincent	62	90	88



- 12** The number of absences each day of Lifesaving Week for two roll classes is shown in this table. Complete the column graph, grouping the classes together each day, for comparison purposes.

Day	Number of absences	
	Year 11 R-1	Year 11 R-2
Mon	3	2
Tue	3	3
Wed	5	6
Thu	6	4
Fri	2	3



Problem solving and reasoning

- 13** The marks scored by 30 students in a test are shown below.

28 35 8 61 45 68 47 23 52 44 22 36 20 44 54 51 32 57 24 61 44
34 42 60 39 47 32 76 53 62

- a** Organise this data into frequency distribution tables, using these classes:
- i** 0–4, 5–9, 10–14, 15–19, ...
 - ii** 0–9, 10–19, 20–29, 30–39, ...
 - iii** 0–19, 20–39, 40–59, 60–79
 - iv** 0–49, 50–99
- b** Which of the groupings from part **a** gives the clearest “picture” of the results of the tests? Discuss your choice.
- c** Explain why the use of the following groupings of scores would be unsatisfactory.
- i** 0–5, 5–10, 10–15, ...
 - ii** 0–9, 20–29, 30–39, 40–49, ...
 - iii** 0–4, 5–14, 15–29, 30–34, ...
- d** Create a column graph to display this data, using the groupings 0–19, 20–39, 40–59, 60–79, 80–99.

- 14** A random sample of light globes from a production line was tested and the time ($\times 100$ hours) each globe lasted, to the nearest 100 hours, is recorded in this list.

10 9 13 5 2 11 8 12 4 12 7 13 8 14 10 12 9 7 11 15
10 11 10 4 8 8 11 12 14 13 10 10 9 11 13 12 5 11 8 16

- a** Organise the given data into frequency distribution tables using these class intervals:
- i** 0–4, 5–9, 10–14, 15–19
 - ii** 1–5, 6–10, 11–15, 16–20
- b** Would the following class intervals be suitable to use?
- i** 0–3, 3–6, 6–9, ...
 - ii** 0–3, 4–7, 8–11, ...
 - iii** 0–9, 10–19, 20–29, ...
- c** How many light globes were tested?
- d** Create a column graph to display this data, using the groupings 0–3, 4–7, 8–11, 12–15, 16–19.



15 The top 10 most streamed artists as of 2025 are listed in this table.

Artist	Total Streams
Taylor Swift	96 500 000 000
Drake	82 600 000 000
Bad Bunny	80 400 000 000
The Weeknd	67 400 000 000
Ed Sheeran	51 100 000 000
Ariana Grande	50 600 000 000
Eminem	47 800 000 000
Justin Bieber	46 200 000 000
BTS	45 000 000 000
Billie Eilish	44 500 000 000

- a Add a column to the table and write each number of streams as a number in scientific notation, correct to two significant figures.
 - b Draw a column graph displaying this information. Start at zero on the vertical axis and use a scale of billions of streams.
 - c Redraw the column graph starting at 40 billion sales instead of zero on the vertical axis. How do the two graphs compare?
- 16 The number of fatal crashes in NSW in 2024, grouped by the licence status of the car drivers, is shown in Table 1.
- a Draw a column graph to illustrate this data.
 - b What is the relative frequency of unlicensed drivers involved in fatal accidents?
 - c What percentage of provisional licence holders were involved in fatal accidents?
 - d What percentage of standard licence holders were involved in fatal accidents?
 - e Which licence type do you think should be targeted by safety authorities?
 - f Table 2 shows the frequency of each type of licence held by car drivers in 2024. Use this additional information to answer the following.
 - i How many provisional licence drivers out of every 100 000 were involved in fatal accidents?
 - ii How many standard licence drivers out of every 100 000 were involved in fatal accidents?
 - iii Reassess your answer to part e and change it if necessary.

Table 1 Licence status and fatal crashes

Licence status	Fatal crashes
Learner	30
Provisional	80
Standard	150
Unlicensed	50
Unknown status	56
Total	366

Table 2 Frequency of licence types

Licence type	Frequency
Learner	298 765
Provisional	612 345
Standard	4 523 678

Challenge

17 Investigate the modes of transport the students in your class use to get to school. Present the data in a frequency distribution table and create a column graph to display your findings. Would the results for your class be typical of the whole school?

18 Choose how to summarise and display the following data.

13 32 15 16 27 17 17 28 25 18 19 26 26 26 26 21 27 27 18 28 29
25 30 14 33 20 16 22 23 23 23 24 24

Related resources



11.2 Quick quiz



Investigation: Interpreting data from graphs

Lesson 11.3

Line graphs

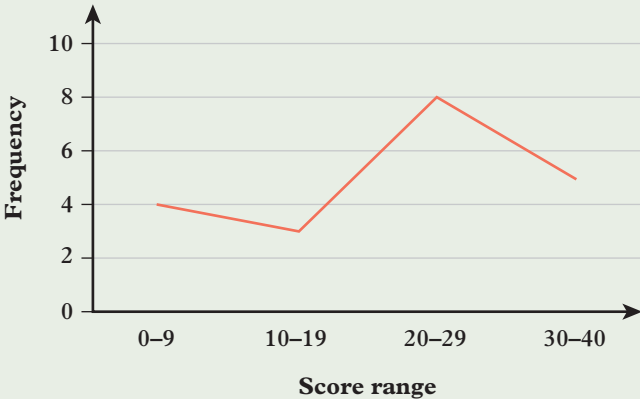
Learning intentions

By the end of this lesson, you will be able to ...

→ use line graphs to display appropriate data and recognise misleading data displays.

Line graphs

- **Line graphs** are useful for showing upward and downward trends in data. They are also useful for finding “in between” values when the numerical variable on the horizontal axis is continuous.
- The data in a summary table can be displayed using a line graph.
 - We assign variables to each axis as follows. The independent variable, which may be grouped or ungrouped, is assigned to the horizontal axis. The dependent variable, which is always an ungrouped numerical variable, is assigned to the vertical axis.
 - For each value or class on the horizontal axis, we place a point at the corresponding height with reference to the vertical axis. These points are then connected to each other by straight lines.
 - For example, the line graph here shows the same data that was presented in a column graph at the beginning of the previous lesson.
 - Line graphs can be used when the independent (horizontal) variable is categorical and ordinal, such as days of the week, but it does not make sense to use a line graph for categorical variables that are nominal.



Misleading data displays

- There are many ways a data display may misrepresent the data it refers to. Misleading displays give a false impression of the data, either by mistake or deliberately.
- Some features to look out for include:
 - the scale on the vertical axis does not start at zero
 - the scale on the vertical or horizontal axis is irregular
 - the scale on the vertical or horizontal axis is missing
 - the use of area or volume creates a false impression.

Worked example 11.3A Drawing a line graph

This table shows the variation in price of a share at “End of Trading” for each day over the course of one week. Draw a line graph to show this information.

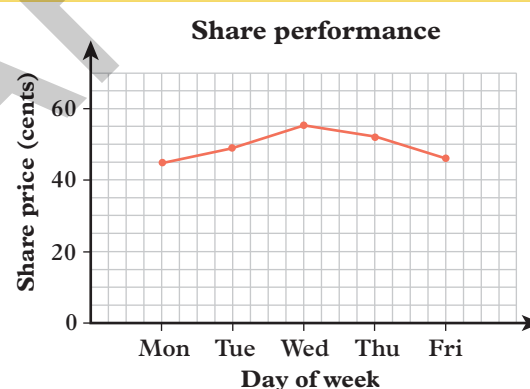
Day	Mon	Tue	Wed	Thu	Fri
Price (cents)	45	48	55	52	46

Think

The days of the week are placed on the horizontal axis and the share prices are placed on the vertical axis.

Each of the five points are plotted then they are joined together by straight lines.

Write



Worked example video: Drawing a line graph

Understanding, fluency and communicating

Answers: Lesson 11.3

Helpful hints

- Plot the points corresponding to the information given in the table. Join the points with straight lines.

- 1 This table shows the temperature variation (°C) on a summer day in Sydney, from 7:00 am to 7:00 pm. Draw a line graph to illustrate this data.

Time of day	Temperature (°C)
7 am	13
8 am	14
9 am	16
10 am	17
11 am	19
12 pm	22
1 pm	25
2 pm	28
3 pm	27
4 pm	27
5 pm	24
6 pm	22
7 pm	18

- 2 The monthly sales figures (in thousands of dollars) for a computer firm are shown in this table. Draw a line graph for this information.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales	54	36	30	30	28	34	25	26	31	38	44	56

- 3 The average yearly health-related expenses for each age group of the Australian population is shown in this table. Illustrate this information using a line graph.

Age group (years)	10–19	20–29	30–39	40–49	50–59	60–69
Health expenses (\$)	1234	1567	2045	2512	3198	4023

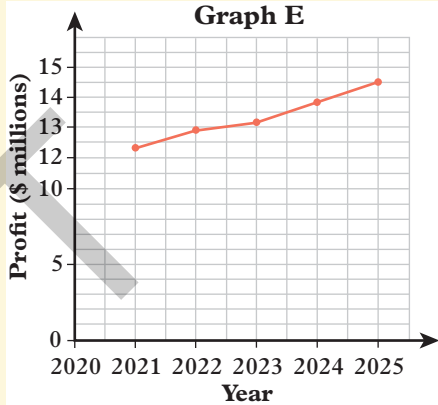
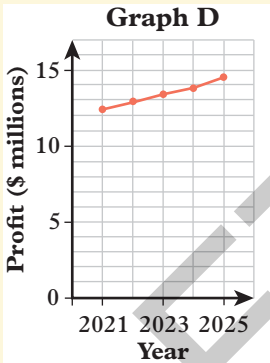
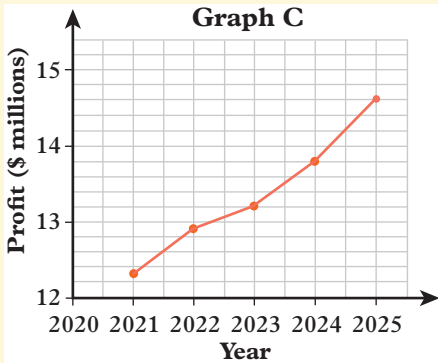
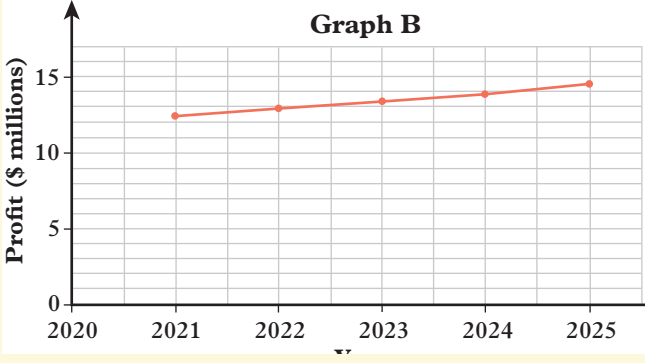
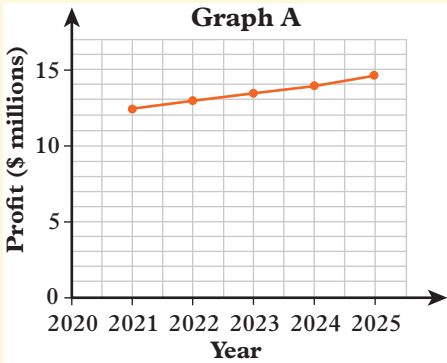
- 4 The weight of a baby at various ages is shown in this table. Illustrate this information on a line graph.

Age (months)	0	3	6	9	12	15	18	21	24
Weight (kg)	3.2	5.1	7.0	8.8	10.0	10.7	11.2	11.8	12.5

Worked example 11.3B Identifying misleading features of line graphs

This table shows the profits of a company over the 5-year period from 2021 to 2025. Graphs A to E show five ways of presenting this information graphically. Which features of these graphs, if any, are misleading?

Year	2021	2022	2023	2024	2025
Profit (\$ millions)	12.3	12.9	13.2	13.8	14.6



Think

Study the scale on the vertical axis. It should start at zero and have regularly spaced intervals with clear labels.

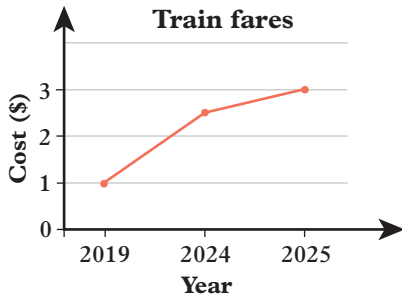
Write

Graphs A, B and D are fair, although each gives a different impression: Graph B makes the increase in profit look smaller because the horizontal scale is enlarged. Graph D makes the increase in profit look larger because the horizontal scale is compressed. Graphs C and E are misleading: Graph C has exaggerated the increase in profit by not starting the vertical scale at 0, thus enlarging this scale. Graph E has an irregular vertical scale, so the graph is incorrect.

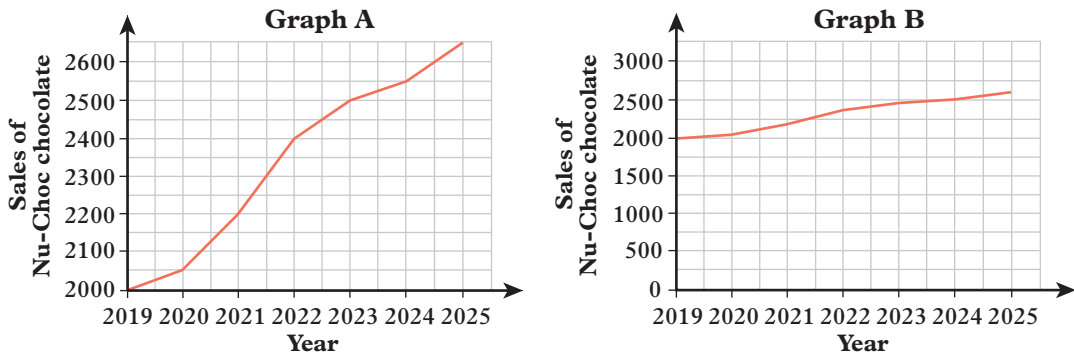


Worked example video: Identifying misleading features of line graphs

5 Describe the misleading or poor features of this graph.



6 Consider Graphs A and B.



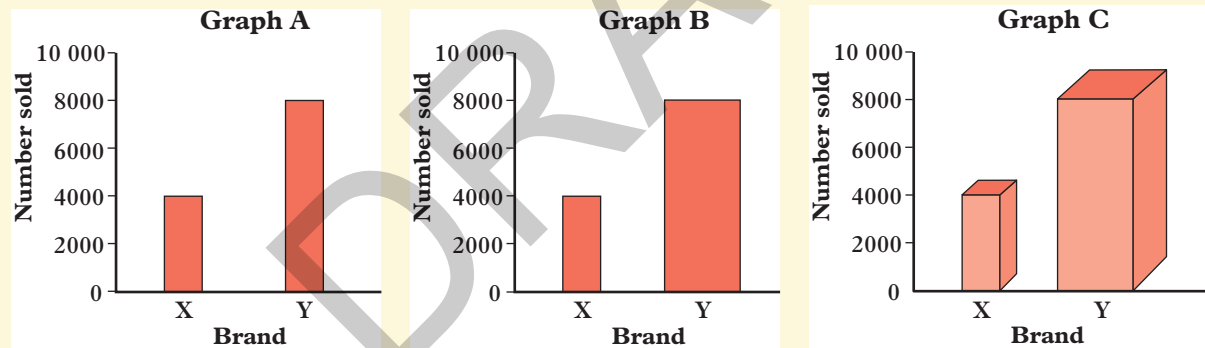
- a Which graph gives the impression of rapidly increasing sales?
- b Have sales in fact increased rapidly over this 7-year period?
- c According to graph A, the sales for 2022 appear to be double those of 2021. Is this true?

Worked example 11.3C Identifying misleading features of column graphs

This table shows the number of televisions of each of two brands sold.

Brand	Number of televisions sold
X	4000
Y	8000

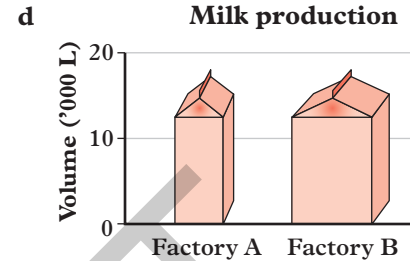
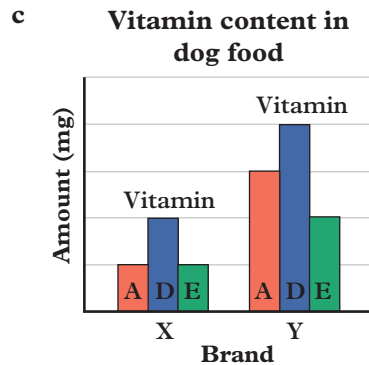
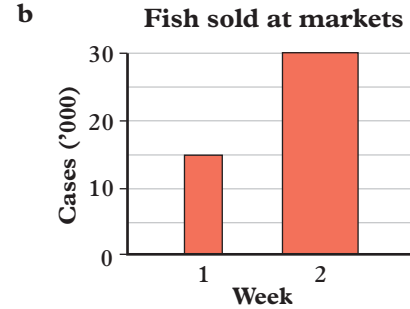
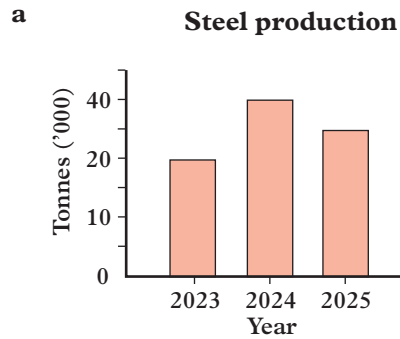
Also shown are three ways of presenting the information in the table graphically. Describe each graph, indicating whether or not it is misleading.



Think	Write
In a column graph, the area of a 2D column or the volume of a 3D column implicitly contribute to the viewer's interpretation of the values. The only difference between the bars in each column should be their height. Check that the area or volume of the bars do not create a false impression of the data.	<i>Graph A:</i> This column graph correctly shows (as the table does) that the sales of brand X are double those of brand Y. The widths of both columns are the same and the height of the second column is twice that of the first. <i>Graph B:</i> In this graph, both the height and the width of the column for brand Y have been doubled, creating the impression that the sales of brand Y are 4 times (2×2 times) those of brand X. <i>Graph C:</i> The three dimensions in this graph have been doubled for brand Y, creating the impression that the sales of brand Y are 8 times ($2 \times 2 \times 2$ times) those of brand X.

Worked example video: Identifying misleading features of column graphs

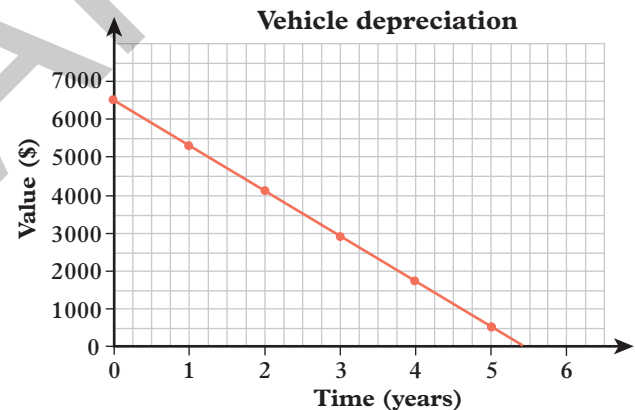
7 Describe the misleading or poor features of these graphs.



Problem solving and reasoning

8 The graph shows the resale value of a motorcycle over time. This is known as the vehicle's "depreciation".

- a** What was the purchase price?
- b** What is the value of the motorcycle after:
- i** 1 year? **ii** $2\frac{1}{2}$ years?
- c** When is the motorcycle worth:
- i** \$4000? **ii** \$1700?
- d** The motorcycle is scrapped after five years. What is its final written-down value after this time?
- e** What is the annual amount of depreciation?



9 Consider the following data about road fatalities in NSW from 1950 to 2020.

Year	1950	1955	1960	1965	1970	1975	1980	1985
Total Road Fatalities	374	556	636	828	1309	1227	1303	1067

Year	1990	1995	2000	2005	2010	2015	2020
Total Road Fatalities	765	581	603	508	405	350	297

- a** Draw a line graph for the data in the table and comment on any trends in these figures.
- b** In which 5-year period did:
- i** the largest increase in fatalities occur? **ii** the largest decrease in fatalities occur?
- c** Discuss why the number of fatalities might have decreased since 1980 even though the number of registered vehicles has increased.

N	1	2	3	4	5	6
BAC						

Lesson 11.4

Histograms and frequency polygons

Learning intentions

By the end of this lesson, you will be able to ...

- construct and interpret frequency histograms and frequency polygons.

Grouping continuous numerical data

- Recall from Lesson 11.2 that, when grouping numerical data, we must ensure that:
 - the classes do not overlap
 - there are no gaps between class intervals
 - each class interval is the same size.
- For *continuous* numerical data, this means we should use half-open intervals. That is, instead of intervals like 0–9 and 10–19, we should use $0-<10$ and $10-<20$.
 - This approach still avoids the confusion of where to put 10 that we would face if we used classes 0–10 and 10–20, since 10 is not included in $0-<10$.
 - We now have somewhere to put the numbers between 9 and 10, which would not be the case if we used 0–9 and 10–19. In particular, we even know where to put a number like 9.999 999 999 999 999, since it is still less than 10.
- For any grouped numerical data, we define the **class centre** as the middle value in the class. Even in a half-open interval, we calculate the class centre by adding the two end values, then dividing by two.
- Continuous numerical data is often collected using rounded values. For example, heights are often rounded to the nearest centimetre. In this case, we know that a data point of 165 cm is actually a value somewhere in the half-open interval $164.5-<165.5$. We can choose to label the horizontal axis of the corresponding histogram either with the values 164.5, 165.5, ... in between the bars, or with the whole number values in the middle of each bar.

Histograms

- A **histogram** looks similar to a column graph, except with no gaps between the columns.
 - The independent (horizontal) variable is always numerical in a histogram, and when the dependent (vertical) variable is frequency, we call the display a frequency histogram.
 - Labels on the horizontal axis may be placed either in the middle of each bar or at the points where two bars meet.

Frequency polygon

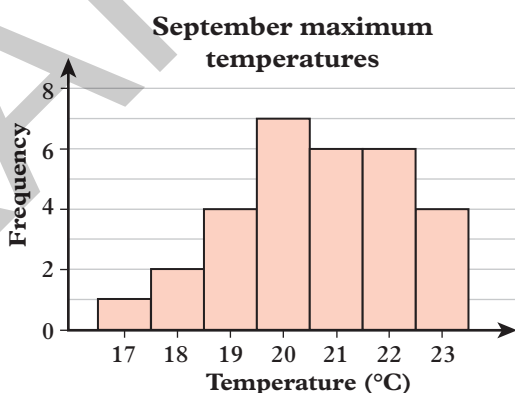
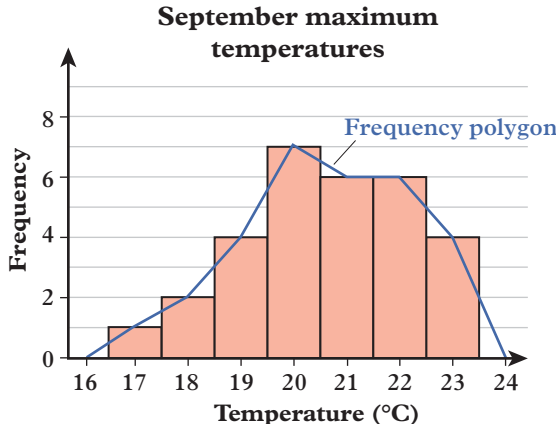
- A **frequency polygon** is a line graph made by joining the midpoints of the tops of the columns of a frequency histogram, with the first and last points joined to the horizontal axis to form a polygon.
→ The first and last point of the polygon are located on the horizontal axis at half the column width before the first column and at half the column width after the last column in the histogram.

Worked example 11.4A Drawing a frequency histogram and frequency polygon

The maximum temperature (in °C) on each day in September was recorded, and the results were summarised in a frequency distribution table, as shown.

Temperature (°C)	17	18	19	20	21	22	23
Frequency	1	2	4	7	6	6	4

- a Draw a frequency histogram of the temperature data.
b Draw a frequency polygon of the distribution.

Part	Think	Write
a	Choose suitable scales. Put temperature on the horizontal axis and frequency on the vertical axis. Draw the columns the same width, joined together, and centred above the temperatures they represent. The width of each column is the distance between successive scores on the horizontal axis. The height of each column is the frequency of each score (temperature).	
b	If the histogram has already been drawn, join the midpoints of the top of each column with straight lines. If the histogram has not been drawn, plot the points and join them. To complete the polygon, the first and last points should be connected to the points (16,0) and (24,0).	



Worked example video: Drawing a frequency histogram and frequency polygon

Understanding, fluency and communicating

Answers: Lesson 11.4

Helpful hints

- A **frequency histogram** is like a column graph with frequency on the vertical axis. The height of each column represents the frequency of data points in each class.
- A **frequency polygon** is a line graph made by joining the midpoints of the tops of the columns of a frequency histogram, with the first and last points joined to the points on the horizontal axis where the scores immediately before and after the given scores would be found.

1 Draw a frequency histogram for each of the following distributions.

a

Number of snacks	0	1	2	3	4	5	6
Frequency	5	7	9	7	6	4	2

b

Score	10	11	12	13	14
Frequency	7	11	17	0	5

2 Draw a frequency polygon for each of the following distributions.

a

Number of goals	0	1	2	3	4
Frequency	8	9	6	3	1

b

Salary (\$)	30 000	40 000	50 000	60 000	70 000	80 000
Frequency	8	6	7	3	0	1

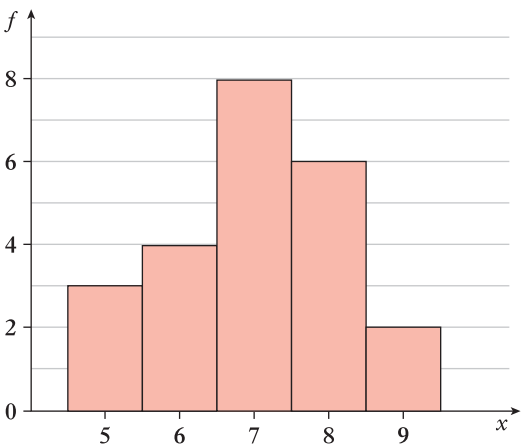
3 On the same diagram, draw a frequency histogram and a frequency polygon for the data given below.

Mark	14	15	16	17	18	19	20
Frequency	4	6	9	0	2	3	1

4 Complete the frequency distribution tables given the following histograms.

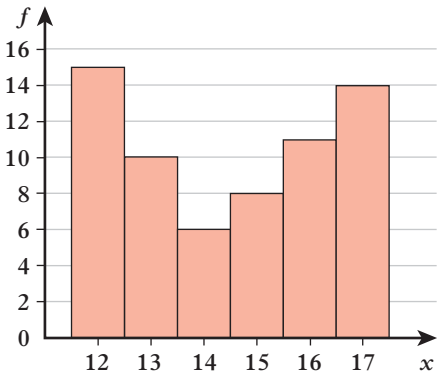
a

Score (x)	Frequency (f)
5	
6	
7	
8	
9	



b

Score (x)	Frequency (f)
12	
13	
14	
15	
16	
17	



Worked example 11.4B Drawing a grouped frequency histogram and frequency polygon

Draw a grouped frequency histogram and polygon for this data about the heights of Year 11 students.

Height (cm)	155–<160	160–<165	165–<170	170–<175	175–<180
Frequency	4	7	10	5	2

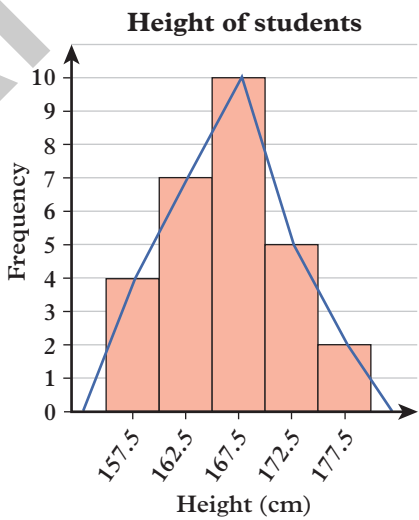
Think

Start by finding the class centre for each class interval.

Height (cm)	Class centre	Frequency
155–<160	$\frac{155 + 160}{2} = 157.5$	4
160–<165	$\frac{160 + 165}{2} = 162.5$	7
165–<170	$\frac{165 + 170}{2} = 167.5$	10
170–<175	$\frac{170 + 175}{2} = 172.5$	5
175–<180	$\frac{175 + 180}{2} = 177.5$	2

The class centres are placed along the horizontal axis and the histogram and polygon completed.

Write



Worked example video: Drawing a grouped frequency histogram and frequency polygon

Helpful hints

- For grouped data, the class centres can be used to label the horizontal axis, then the histogram and polygon are completed as before.

5 For the data in these tables, find the class centre of each class interval. Draw a grouped frequency histogram and polygon, for each.

a

Mass (kg)	Frequency
59.5–<62.5	2
62.5–<65.5	5
65.5–<68.5	7
68.5–<71.5	4
71.5–<74.5	2

b

Score	Frequency
40–<50	3
50–<60	0
60–<70	10
70–<80	8
80–<90	7
90–<100	2

c

Patients per day	Frequency
10–<20	13
20–<30	15
30–<40	16
40–<50	10
50–<60	4
60–<70	2

Problem solving and reasoning

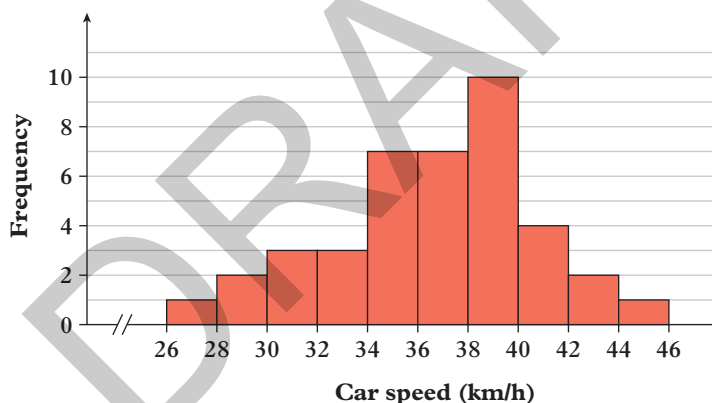
6 The following histograms display data on the number of jet ski accidents involving men and women, broken down by their age and given as a percentage of all jet ski accidents.



- a** According to this data, approximately what percentage of jet ski accidents involve men between the ages of:
 - i** 10 and 20 years old?
 - ii** 60 and 70 years old?
- b** Approximately what percentage of jet ski accidents involve women between the ages of:
 - i** 10 and 20 years old?
 - ii** 60 and 70 years old?
- c** Which age group of male jet ski riders and passengers has the:
 - i** highest number of accidents?
 - ii** lowest number of accidents?
- d** Which age group of female jet ski riders and passengers has the:
 - i** highest number of accidents?
 - ii** lowest number of accidents?
- e** Create a histogram displaying the combined statistics for men and women.
- f** Which age group of all jet ski riders and passengers has the:
 - i** highest number of accidents?
 - ii** lowest number of accidents?

Challenge

- 7 When collecting continuous numerical data, a grouped frequency distribution table should be used.
 - a** Explain why an ungrouped frequency distribution table would generally not be good to collect continuous data.
 - b** Does this mean discrete data should not be collected using groups? Why or why not?
- 8 A speed camera checked the speed of a group of cars going through a school zone over a 5-minute period. The recordings are summarised in this histogram.



- a** How many cars travelled through the school zone in the 5-minute period?
- b** Is it possible to determine the exact speed of any car from the graph? Why or why not?
- c** The speed limit in the school zone is 40 km/h. What percentage of cars were travelling above the speed limit?

Related resources



11.4 Quick quiz



Investigation: Datasets from frequency histograms

Lesson 11.5

Sector graphs and divided bar charts

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate relative frequencies and construct sector graphs and divided bar charts.

Relative frequency

- Sometimes we are more interested in how the frequencies of different variables compare to each other, rather than what their absolute frequencies are. That is, we are interested in their **relative frequency**.
 - The relative frequency of a particular value is its frequency as a *fraction of the total number* of values recorded.
 - By converting this fraction into a percentage, we can also find the **percentage relative frequency** of the value.
- We can determine relative frequencies easily by adding a column to any frequency distribution table.

Sector graphs and divided bar charts

- A **sector graph** (sometimes called a pie chart) is a circular data display that helps to visualise relative frequency.
 - The circle is divided into segments, each proportional in size to the relative frequency of the data it represents. A key shows which variables are represented by each segment.
- A **divided bar chart** is a rectangular data display that represents relative frequencies.
 - The rectangle is divided into pieces that are proportional in length to the relative frequency of the data they represent.
 - The sections are typically ordered from largest to smallest. One exception to this is when the final category includes all “other” options.

Worked example 11.5A Adding relative frequency to a frequency distribution table

- a** Copy this frequency distribution table showing the number of Candy Clicks in a packet and add a relative frequency column.

Number of Candy Clicks	Frequency
36	1
37	5
38	8

Number of Candy Clicks	Frequency
39	13
40	7
41	4
42	2

b What fraction of packets contained:

i 37 Candy Clicks?

ii 40 Candy Clicks?

c What percentage of packets contained:

i 38 Candy Clicks?

ii 39 Candy Clicks?

Part	Think	Write																											
a	We start by adding a row at the bottom of the table and finding the total number of Candy Clicks packets involved in the study by adding all the values in the frequency column. We add a third column for the relative frequency. Each row of the relative frequency column is found by dividing the entry in the frequency column by the total (which is 40 in this case) then multiplying by 100.	<table> <tr> <th>Number of Candy Clicks</th><th>Frequency</th><th>Relative frequency</th></tr> <tr> <td>36</td><td>1</td><td>$\frac{1}{40} = 2.5\%$</td></tr> <tr> <td>37</td><td>5</td><td>$\frac{5}{40} = 12.5\%$</td></tr> <tr> <td>38</td><td>8</td><td>$\frac{8}{40} = 20\%$</td></tr> <tr> <td>39</td><td>13</td><td>$\frac{13}{40} = 32.5\%$</td></tr> <tr> <td>40</td><td>7</td><td>$\frac{7}{40} = 17.5\%$</td></tr> <tr> <td>41</td><td>4</td><td>$\frac{4}{40} = 10\%$</td></tr> <tr> <td>42</td><td>2</td><td>$\frac{2}{40} = 5\%$</td></tr> <tr> <td>Total</td><td>40</td><td>1 = 100%</td></tr> </table>	Number of Candy Clicks	Frequency	Relative frequency	36	1	$\frac{1}{40} = 2.5\%$	37	5	$\frac{5}{40} = 12.5\%$	38	8	$\frac{8}{40} = 20\%$	39	13	$\frac{13}{40} = 32.5\%$	40	7	$\frac{7}{40} = 17.5\%$	41	4	$\frac{4}{40} = 10\%$	42	2	$\frac{2}{40} = 5\%$	Total	40	1 = 100%
Number of Candy Clicks	Frequency	Relative frequency																											
36	1	$\frac{1}{40} = 2.5\%$																											
37	5	$\frac{5}{40} = 12.5\%$																											
38	8	$\frac{8}{40} = 20\%$																											
39	13	$\frac{13}{40} = 32.5\%$																											
40	7	$\frac{7}{40} = 17.5\%$																											
41	4	$\frac{4}{40} = 10\%$																											
42	2	$\frac{2}{40} = 5\%$																											
Total	40	1 = 100%																											
b i	The entry in the relative frequency column in the row for 37 is $\frac{5}{40}$, which can be simplified to $\frac{1}{8}$.	$\frac{1}{8}$ of the packets contained 37 Candy Clicks.																											
b ii	The entry in the relative frequency column in the row for 40 is $\frac{7}{40}$, which cannot be simplified.	$\frac{7}{40}$ of the packets contained 40 Candy Clicks.																											
c i	The entry in the relative frequency column in the row for 38 is equal to 20%.	20% of the packets contained 38 Candy Clicks.																											
c ii	The entry in the relative frequency column in the row for 39 is equal to 32.5%.	32.5% of the packets contained 39 Candy Clicks.																											



Worked example video: Adding relative frequency to a frequency distribution table

Understanding, fluency and communicating

Answers: Lesson 11.5

Helpful hints

- The relative frequency of a value is found by expressing its frequency as a fraction of the total number of values recorded.
- This fraction can be converted to a percentage by evaluating the fraction and multiplying by 100.

- 1 The numbers of children in the families of a class of students are summarised in the given frequency distribution table.

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

- a Copy the frequency distribution table and add a relative frequency column.
- b What fraction of families had:
- i 3 children? ii 4 children?
- c What percentage of families had:
- i 2 children? ii 5 children?
- 2 The number of goals scored in a season by the goal shooter in a netball team is shown in this frequency distribution table.

Goals per match	Frequency
5	4
6	3
7	7
8	5
9	0
10	3
11	2
12	1

- a Copy the frequency distribution table and add a relative frequency column.
- b In what fraction of games did he score:
- i 8 goals? ii 5 goals?
- c In what percentage of games did he score:
- i 6 goals? ii 11 goals?

Worked example 11.5B Displaying relative frequencies in a sector graph

A survey found that 25% of students play basketball, 12.5% play hockey, 50% play soccer and 12.5% play tennis. Represent this data in a sector graph.

Think

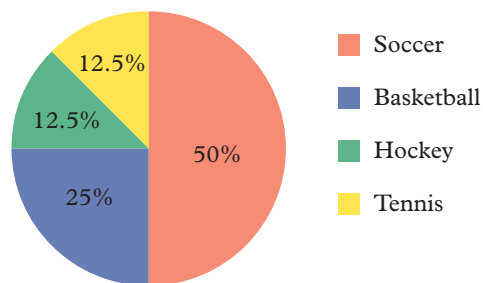
Start with a blank circle.

Determine the smallest and largest segments that are needed. In this case the largest is a 50% segment and smallest are two 12.5% segments.

For the 50% segment, we can divide the circle in half and colour one half one colour to represent the 50% of students who play soccer.

Then we notice that half of the remaining area can be used to represent the 25% of students who play basketball. And finally, the two 12.5% segments are found by splitting the remaining area in half.

Make sure to label each segment with its percentage, and show which colour corresponds to which sport.

Write**Sports played by students**

Worked example video: Displaying relative frequencies in a sector graph

Helpful hints

- Consider all the segments that will be required and start with one that is easier to represent, such as $\frac{1}{2} = 50\%$ or $\frac{1}{4} = 25\%$.
- For proportions that are harder to represent, consider using a spreadsheet to generate a sector graph.

3 Draw a sector graph for each of these datasets.

a

Preferred meal	Relative frequency
Pasta	$\frac{1}{8}$
Pizza	$\frac{5}{8}$
Sushi	$\frac{1}{8}$
Soup	$\frac{1}{8}$

b

Favourite film genre	Relative frequency
Comedy	37.5%
Drama	12.5%
Horror	12.5%
Action	25%
Science fiction	12.5%

c

Favourite instrument	Relative frequency
Drums	$\frac{1}{12}$
Guitar	$\frac{1}{4}$
Piano	$\frac{1}{6}$
Saxophone	$\frac{1}{24}$
Trombone	$\frac{5}{12}$
Violin	$\frac{1}{24}$

4 Draw a sector graph to display the data in each of questions 1 and 2.

Worked example 11.5C Displaying relative frequencies in a divided bar chart

A class of 24 students was asked what they were planning to do on the weekend. The results were that $\frac{1}{3}$ were planning to go to the beach, $\frac{1}{6}$ were going to stay home and play video games, $\frac{1}{4}$ were going to go shopping and $\frac{1}{4}$ were going to spend time at a friend's house. Represent this data in a divided bar chart.

Think

First, sort the fractions into order from largest to smallest. In this case we have $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{6}$.

Use a ruler to draw a rectangle, paying attention to its length. We can choose a convenient length by finding the lowest common multiple of the denominators in our fractions. In this case, if we draw a rectangle that is 12 cm long, then it will be easy to find the proportions for our dataset.

$\frac{1}{3}$ of the 12 cm bar will be 4 cm long, so the first section will go from 0 cm to 4 cm.

$\frac{1}{4}$ of the 12 cm bar will be 3 cm long, so the second section will go from 4 cm to $4 + 3 = 7$ cm.

The next section is also 3 cm, so it will go from 7 cm to $7 + 3 = 10$ cm.

The remaining section is 2 cm long, which is indeed equivalent to $\frac{1}{6}$ of the 12 cm bar.

Make sure to label each segment with its fraction, and show which section corresponds to which weekend plans.

Write**Weekend plans**

Worked example video: Displaying relative frequencies in a divided bar chart

Helpful hints

- For a rectangular bar that will be easier to subdivide, choose a length that is a multiple of all denominators seen in your relative frequencies.
- Order the relative frequencies from largest to smallest before marking in sections within the rectangular bar.

5 Draw a divided bar graph to represent each of the following.

- Glucose is made up of 6 Carbon atoms, 12 Hydrogen atoms and 6 Oxygen atoms.
- A serve of porridge contains approximately 14.4 g of protein, 7 g of fat and 71 g of carbohydrates.
- The average NSW household disposes of different types of waste in the following proportions:

Waste Type	Percentage (%)
Plastics	12%
Paper/Cardboard	27%
Glass	5%
Organics	40%
Metals	6%
Other	10%

6 Draw a divided bar chart to display the data in each of questions 1 and 2.

Problem solving and reasoning

- 7 The numbers of matches in 60 boxes of matches were counted, and the results are shown in this list.

49 51 50 50 48 49 50 49 53 50 48 51 49 50 51 52 50 48 49 51 50
 53 48 49 51 50 52 49 50 52 51 48 50 49 50 51 52 50 49 48 52 50
 51 49 50 50 48 53 52 49 50 51 53 48 50 51 49 50 53 48

- a** Organise this data into a frequency distribution table and add a relative frequency column.

- b** Find the percentage of matchboxes that contained:

i 49 matches

ii 50 matches

iii 51 matches.

- c** Create a divided bar chart to display this data.

- 8 A group of Year 11 golfers had a competition to find out who could hit a golf ball the furthest. Each student's best shot, to the nearest metre, is recorded in this list.

244 220 232 195 242 200 229 205 213 224 196 250 227 186 207 236 205
 212 227 230 209 235 234 251 195 206 221 190 198 210 222 235 250 209
 254 193 203 228 191 242 204 218 212 218 239 223 252 195 220 198
 207 205 226 235 202 196 188 224 255 230

- a** Copy and complete the frequency distribution table for this data.

Distance (m)	Tally	Frequency
180–189		
190–199		
200–209		
210–219		
220–229		
230–239		
240–249		
250–259		
	Total	

- b** How many golfers entered the competition?

- c** Add a relative frequency column to your table from part **a**.

- d** What fraction of golfers had the length of their best shot recorded as:

i 230 m or more?

ii 220 m or more, but less than 230 m?

iii less than 200 m?

- e** Divide the data into the following classes and update their relative frequencies.

Distance (m)	Relative frequency
<200	
200–209	
210–219	
220–229	
230+	

- f** Represent the data from the new table in part **e** using a sector graph.

- 9 This frequency distribution table shows the heights of seedlings, as in question 4 of Lesson 11.2.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6

- Copy the table and add a relative frequency column.
 - What percentage of seedlings were 425 mm or more?
 - Any seedlings that are less than 325 mm tall are discarded. What percentage of seedlings will be discarded?
 - Represent the distribution of seedling heights using your choice of a sector graph or a divided bar chart.
- 10 The maximum daily temperature (in °C) was recorded over the month of April and the results are in this frequency distribution table.

Maximum daily temperature (°C)	Frequency
17	3
18	4
19	8
20	6
21	5
22	4

- Copy the table and add a relative frequency column.
- How many days in April had a maximum daily temperature of 19°C or less?
- What percentage of days in April had a maximum daily temperature of 19°C or less?
- What proportion of days in April had a maximum daily temperature of 21°C or more?
- Represent the distribution of daily maximum temperatures using your choice of a sector graph or a divided bar chart.

Challenge

- 11 Describe how you could draw any sector graph accurately by hand, using only a compass, protractor and ruler.

Related resources



11.5 Quick quiz

Lesson 11.6

Cumulative frequency

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate cumulative frequencies and construct cumulative frequency histograms and polygons.

Cumulative frequency

- In the previous lesson, we added relative frequency columns to our frequency distribution tables. In this lesson, we will add cumulative frequency columns.
 - Cumulative frequency keeps track of the total number of data points as we progress through the rows of a frequency distribution table. The value in the cumulative frequency column for any given row represents the sum of the frequency in that row, plus all of the previously observed frequencies.
 - The final value in a cumulative frequency column will always be the total number of data points in the dataset.
- It makes sense to calculate cumulative frequencies for numerical and ordinal (categorical) data, but not really for nominal (categorical) data, since different choices of order could drastically change the cumulative frequencies observed.

Displaying cumulative frequency

- A **cumulative frequency histogram** is a histogram with cumulative frequency on the vertical axis.
- A **cumulative frequency polygon** is a line graph formed by joining the *upper right-hand corners* of each column of the cumulative frequency histogram. A cumulative frequency polygon is sometimes called an **ogive**.

Worked example 11.6A Adding cumulative frequency to a frequency distribution table

Recall the frequency distribution table showing the number of Candy Clicks in a packet from **Worked Example 11.5A** in Lesson 11.5.

a Copy this frequency distribution table, then add a cumulative frequency column.

Number of Candy Clicks	Frequency
36	1
37	5
38	8
39	13
40	7
41	4
42	2

- b** How many packets contained:
- i** 40 or fewer Candy Clicks? **ii** fewer than 38 Candy Clicks?

Part	Think	Write																											
a	For the cumulative frequency column, we start by copying the frequency in the first row across. For every row after this, we add the number in the cell above to the number in the cell to the right: $\begin{aligned}1 + 5 &= 6 \\6 + 8 &= 14 \\14 + 13 &= 27 \\27 + 7 &= 34 \\34 + 4 &= 38 \\38 + 2 &= 40\end{aligned}$	<table><tr><th>Number of Candy Clicks</th><th>Frequency</th><th>Cumulative frequency</th></tr><tr><td>36</td><td>1</td><td>1</td></tr><tr><td>37</td><td>5</td><td>6</td></tr><tr><td>38</td><td>8</td><td>14</td></tr><tr><td>39</td><td>13</td><td>27</td></tr><tr><td>40</td><td>7</td><td>34</td></tr><tr><td>41</td><td>4</td><td>38</td></tr><tr><td>42</td><td>2</td><td>40</td></tr><tr><td>Total</td><td>40</td><td></td></tr></table>	Number of Candy Clicks	Frequency	Cumulative frequency	36	1	1	37	5	6	38	8	14	39	13	27	40	7	34	41	4	38	42	2	40	Total	40	
Number of Candy Clicks	Frequency	Cumulative frequency																											
36	1	1																											
37	5	6																											
38	8	14																											
39	13	27																											
40	7	34																											
41	4	38																											
42	2	40																											
Total	40																												
b i	The number of packets containing 40 or fewer Candy Clicks can be found by locating 40 in the first column, then reading off the cumulative frequency at that row.	The number of packets containing 40 or fewer Candy Clicks is 34.																											
b ii	The number of packets containing fewer than 38 Candy Clicks is equal to the number of packets containing 36 or 37 Candy Clicks. This value is found in the cumulative frequency column in the row for 37.	The number of packets containing fewer than 38 Candy Clicks is 6.																											



Worked example video: Adding cumulative frequency to a frequency table

Understanding, fluency and communicating

Answers: Lesson 11.6

Helpful hints

- The cumulative frequency of values up to a particular value in a dataset is found by “accumulating” the frequencies of all the values up to and including that value.

1 The numbers of children in the families of a class of students are summarised in the given frequency distribution table.

- a** Copy the frequency distribution table and add a cumulative frequency column.

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

- b** How many families had:

- i** 3 or fewer children?
ii fewer than 5 children?

2 The number of goals scored in a season by the goal shooter in a netball team is shown in this frequency distribution table.

- a** Copy the frequency distribution table and add a cumulative frequency column.

Goals per match	Frequency
5	4
6	3
7	7
8	5
9	0
10	3
11	2
12	1

b In how many games did the goal shooter score:

i 7 or fewer goals?

ii fewer than 10 goals?

Worked example 11.6B Determining frequency from a cumulative frequency table

Given the cumulative frequencies of the data values in this table, determine the frequency of each individual data value.

Data value	Cumulative frequency
7	2
8	5
9	7
10	12
11	16

Think

The cumulative frequency of the first data value is the same as its frequency. So, the frequency of the data value 7 is 2.

The cumulative frequency of the data values up to 8 is 5, which is the sum of the frequencies of 7 and 8. So, the frequency of the data value 8 is $5 - 2 = 3$.

The cumulative frequency of the data values up to 9 is 7, which is the sum of the frequencies of 7, 8 and 9. So, the frequency of 9 is $7 - 5 = 2$.

The cumulative frequency of the data values up to 10 is 12, which is the sum of the frequencies of 7, 8, 9 and 10. So, the frequency of 10 is $12 - 7 = 5$.

The cumulative frequency of the data values up to 11 is 16, which is the sum of the frequencies of 7, 8, 9, 10 and 11. So, the frequency of 11 is $16 - 12 = 4$.

Write

Data value	Cumulative frequency	Frequency
7	2	2
8	5	3
9	7	2
10	12	5
11	16	4



Worked example video: Determining frequency from a cumulative frequency table

Helpful hints

- Use the fact that the cumulative frequency up to a certain value is the sum of the frequencies of all the values up to and including that value.

- 3 Given these cumulative frequency tables for different datasets, determine the frequency of each individual data value.

a

Data value	Cumulative frequency
3	4
4	6
5	10
6	15
7	18

b

Data value	Cumulative frequency
12	5
13	11
14	19
15	29
16	34
17	36

c

Data value	Cumulative frequency
8	11
9	17
10	27
11	27
12	32
13	40

d

Data value	Cumulative frequency
87	23
88	48
89	78
90	106
91	127
92	138

Worked example 11.6C Drawing a cumulative frequency histogram and cumulative frequency polygon

The maximum temperature, in $^{\circ}\text{C}$, on each day in September, (as given in **Worked Example 11.4A** in Lesson 11.4) is shown in this table. Draw a cumulative frequency histogram and a cumulative frequency polygon for this data.

Temperature ($^{\circ}\text{C}$)	17	18	19	20	21	22	23
Frequency	1	2	4	7	6	6	4

Think

Create a table that includes a cumulative frequency column.

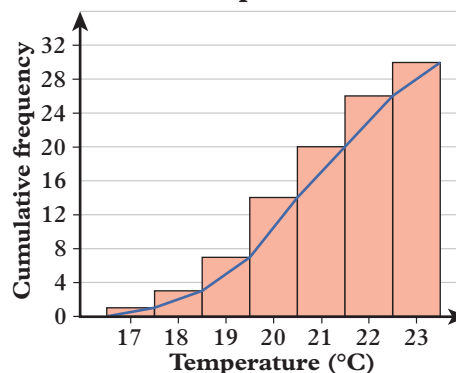
Temperature (°C)	Frequency	Cumulative frequency
17	1	1
18	2	3
19	4	7
20	7	14
21	6	20
22	6	26
23	4	30

Draw a cumulative frequency histogram, using the cumulative frequency of each data value.

Next, use straight lines to join the top right-hand corners of the columns to form the cumulative frequency polygon.

Write

September maximum temperatures



Worked example video: Drawing a cumulative frequency histogram and cumulative frequency polygon

Helpful hints

- When drawing a cumulative frequency polygon, we join the upper-right hand vertices, not the midpoints, of the columns. We do not join the last corner to the horizontal axis (so the line graph is not strictly a polygon).
- To draw a cumulative frequency polygon without first drawing the histogram requires careful consideration of the upper and lower boundaries of each class interval.

4 For each of the frequency distribution tables shown, complete the following.

- Add a cumulative frequency column.
- Draw a cumulative frequency histogram and a cumulative frequency polygon.

a

Number of snacks	Frequency
0	5
1	7
2	9
3	7
4	6
5	4
6	2

b

Score	Frequency
10	7
11	11
12	17
13	0
14	5

c

Number of goals	Frequency
0	8
1	9
2	6
3	3
4	1

d

Salary (\$)	Frequency
30 000	8
40 000	6
50 000	7
60 000	3
70 000	0
80 000	1

Worked example 11.6D Drawing a cumulative frequency histogram and cumulative frequency polygon for grouped data

Recall the frequency distribution table showing the heights of students from **Worked Example 11.4B** in Lesson 11.4, as shown. Add a cumulative frequency column, then draw a cumulative frequency histogram and a cumulative frequency polygon for the data.

Height (cm)	Class centre	Frequency
155–<160	157.5	4
160–<165	162.5	7
165–<170	167.5	10
170–<175	172.5	5
175–<180	177.5	2

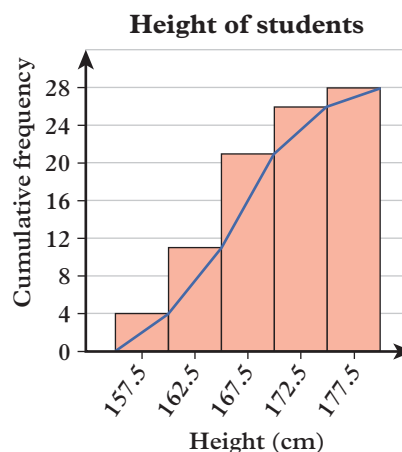
Think

Start by adding and completing the cumulative frequency column.

Height (cm)	Class centre	Frequency	Cumulative frequency
155–<160	157.5	4	4
160–<165	162.5	7	11
165–<170	167.5	10	21
170–<175	172.5	5	26
175–<180	177.5	2	28

Draw the cumulative frequency histogram using the class centres. Join the *top right-hand corners* of the columns of the histogram to draw the cumulative frequency polygon (the ogive).

Write



Worked example video: Drawing a cumulative frequency histogram and cumulative frequency polygon for grouped data

Helpful hints

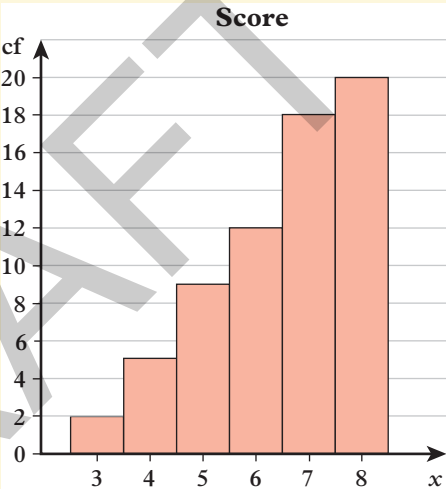
- Use the class centres as the values on the horizontal axis and use the cumulative frequency on the vertical axis. Join the top right-hand corners of the columns of the histogram to form the cumulative frequency polygon.

- 5 For each part of question 5 in Lesson 11.4 :
- i Add a cumulative frequency column to the table.
 - ii Draw a cumulative frequency histogram and cumulative frequency polygon on the same diagram.

Worked example 11.6E Constructing a frequency distribution table from a cumulative frequency histogram

Complete the frequency distribution table given the cumulative frequency histogram.

Score (x)	Cumulative frequency (cf)	Frequency
3		
4		
5		
6		
7		
8		



Think

From the histogram, write the cumulative frequency of each score.

The cumulative frequency up to the first score is the same as its frequency: the frequency of the score 3 is 2.

The cumulative frequency up to the score 4 is 5; this is the sum of the frequencies of the scores 3 and 4.

Hence, the frequency of the score 4 is $(5 - 2)$ or 3.

The cumulative frequency up to the score 5 is 9; this is the sum of the frequencies of 3, 4 and 5. Hence, the frequency of 5 is $(9 - 5)$ or 4, and so on.

Write

Score (x)	Cumulative frequency (cf)	Frequency
3	2	2
4	5	$5 - 2 = 3$
5	9	$9 - 5 = 4$
6	12	$12 - 9 = 3$
7	18	$18 - 12 = 6$
8	20	$20 - 18 = 2$

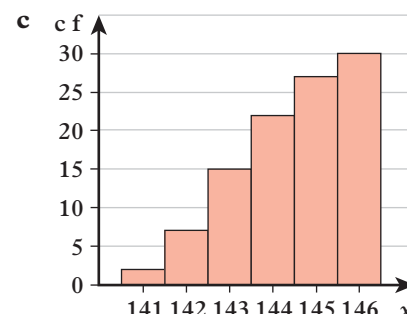
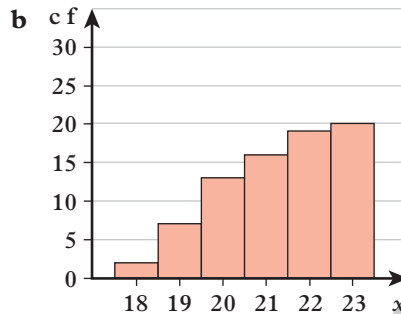
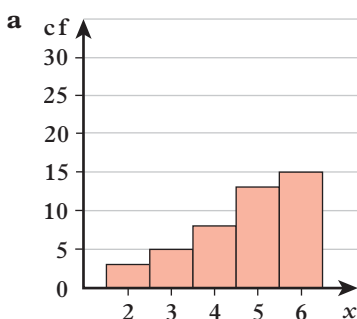


Worked example video: Constructing a frequency table from a cumulative frequency histogram

Helpful hints

- Cumulative frequencies up to a given value can be entered into a table by reading directly from a cumulative frequency histogram. The cumulative frequency column can then be used to find the frequency of each individual value.

6 Complete frequency distribution tables for the data shown in these cumulative frequency histograms.



Problem solving and reasoning

7 Copy your frequency distribution table from question 7 in Lesson 11.5 showing the numbers of matches in 60 boxes of matches that were counted.

- Add a cumulative frequency column to your table.
- How many matchboxes contained:
 - exactly 50 matches?
 - fewer than 50 matches?
 - 50 or fewer matches?
 - more than 50 matches?
- Draw a cumulative frequency histogram and a cumulative frequency polygon on the same diagram to display this data.

8 Copy your frequency distribution table from question 8 in Lesson 11.5 showing the distances that a group of Year 11 golfers could hit a golf ball.

- Add a cumulative frequency column to your table.
- For how many golfers was the length of their best shot recorded as less than:
 - 200 m?
 - 230 m?
- Draw a cumulative frequency histogram and a cumulative frequency polygon on the same diagram to display this data.



Challenge

9 Complete the missing values in the table.

ATAR	Frequency	Cumulative Frequency	Cumulative Percentage
90–<100	3374	3374	33.764%
80–<90	2483		58.611%
70–<80	1712	7569	
60–<70		8684	86.901%
50–<60	675	9359	93.656%
0–<50			100%

Related resources



11.6 Quick quiz

Lesson 11.7

Choosing a data display

Learning intentions

By the end of this lesson, you will be able to ...

→ select and justify appropriate data displays for different datasets.

Data displays

- In this module, we have seen a variety of different ways that data can be displayed. However, not all data displays are suitable for all types of datasets. In this lesson, we will practise choosing the best display for a given dataset.

Types of data

- Recall from Module 10 that data can be classified as numerical or categorical. Numerical data can be either discrete or continuous, and categorical data can be either nominal or ordinal.
 - The type of data we are working with is one of the main considerations when selecting a data display.
 - Frequency is a discrete numerical variable.
 - When a dataset involves an independent and a dependent variable, it is important to know the type of data for each.

- Sometimes the type of data is ambiguous. Discrete numerical data and grouped numerical data can both be interpreted as ordinal categorical data, depending on the circumstances.
 - For example, if the numbers 1, 2, 3, 4 and 5 were used to represent the days on which something occurred, we could also use the name for the day of the week, which would be categorical data.
 - If test scores are sorted into classes <50 , $50-<60$, $60-<70$, $70-<80$ and $80+$, these can be (and often are!) given grade labels, such as Fail, Pass, Credit, Distinction and High Distinction.

Displays with a horizontal and a vertical axis

- Many displays have a horizontal axis representing the independent variable, and a vertical axis representing the dependent variable. Only certain data types make sense on each axis, depending on the type of display.
 - In all the data displays we consider, the vertical axis always represents a numerical variable. Note that we can imagine a vertical axis in a dot plot, which would measure frequency (a numerical variable) by the number of dots in each column.
- Dot plots
 - In a dot plot, each column of dots must have a label, which is a variable on the horizontal axis. This variable can be categorical, numerical or grouped numerical.
 - The main constraint to consider for dot plots is the number of dots that will need to be drawn. Hence, they are best suited to smaller datasets.
- Column graphs
 - Column graphs can be used for all the same datasets as dot plots, but without the restriction on size.
- Line graphs
 - Line graphs are most commonly used with a numerical variable on the horizontal axis. They can be used with grouped numerical or categorical data that is *ordinal*, but *not* with categorical data that is nominal.
- Histograms
 - Histograms are intended for use with a continuous numerical variable on the horizontal axis; however, this axis is sometimes labelled as a discrete numerical variable or even an ordinal (categorical) variable if the class intervals are secretly capturing continuous numerical data.
 - For example, shoe size is a discrete numerical variable that is closely related to the continuous numerical variable of foot length. Similarly, months of the year represent ordinal (categorical) data that is closely related to the continuous numerical variable time.
- Frequency polygons
 - Frequency polygons are used for datasets where a histogram is appropriate.

Other displays

- Stem-and-leaf plots and back-to-back stem-and-leaf plots are only useful in relatively small datasets when we are interested in the frequency of numerical data.
 - For a stem-and-leaf plot to be useful, there should typically be at least three stems, but no more than can reasonably fit on a page.

- The number of leaves in each stem should also be considered, since it is not worth displaying data in a stem-and-leaf plot if every stem has no more than two leaves.
- Sector graphs and divided bar charts are used to display the relative frequencies of a variable, either in the form of a percentage or a fraction.
 - You could expect to see most types of data in these display types, *except* for ungrouped continuous numerical data.
- Finally, cumulative frequency histograms and polygons are exclusively used for datasets where cumulative frequencies have been determined.

Statistical infographics

- **Infographics** are a helpful tool when communicating statistics to a broad audience.
- A typical infographic will include two or more graphs to visualise the relevant data, along with minimal text in eye-catching colours and fonts.
- Because the visual appeal of an infographic is a high priority, data is more likely to be misrepresented, whether intentionally or not.

Worked example 11.7A

For the following dataset:

Grade	Frequency
Fail	2
Pass	37
Credit	68
Distinction	54
High Distinction	23

- a** Classify the variable types.
- b** State whether or not each display type is suitable.
- i** Dot plot **ii** Line graph **iii** Histogram
- c** Add a relative frequency column, then choose and create an appropriate display for the data.

Part	Think	Write
a	Each grade is represented by a word, but there is a logical order to put them in, so this variable is ordinal (categorical). Frequency is always a discrete numerical variable, since the values come from counting data points.	The independent variable, “Grade”, is ordinal (categorical). The dependent variable, “Frequency”, is discrete numerical.

Part	Think	Write																					
b i	The data type is suitable for a dot plot, but because the frequency values are so large, we would not use a dot plot.	A dot plot is not suitable for this dataset.																					
b ii	It is reasonable to put “Grades” on the horizontal axis and “Frequency” on the vertical axis of a line graph. The line between the points for, say, Credit and Distinction will give a sense of how many “high credits” and “low distinctions” there were.	A line graph is suitable for this dataset.																					
b iii	If we assume that “Grades” are determined based on some sort of numerical score, then a histogram is a logical display to use. For example, Fail could represent scores less than 50%, then Pass could represent scores from 50% up to, but not including, 60%, and so on.	A histogram is suitable for this dataset, assuming “Grades” are labels for class intervals of continuous numerical data.																					
c	We find the total of all frequencies by adding them together, which gives 184. The entries in the relative frequency column are found by dividing the frequency in that row by 184 then multiplying by 100. Relative frequency for categorical data is well-suited to being displayed in a sector graph. A spreadsheet can be used to create one.	<table border="1"> <thead> <tr> <th>Grade</th><th>Frequency</th><th>Relative frequency</th></tr> </thead> <tbody> <tr> <td>Fail</td><td>2</td><td>1%</td></tr> <tr> <td>Pass</td><td>37</td><td>20%</td></tr> <tr> <td>Credit</td><td>68</td><td>37%</td></tr> <tr> <td>Distinction</td><td>54</td><td>29%</td></tr> <tr> <td>High Distinction</td><td>23</td><td>13%</td></tr> <tr> <td>Total</td><td>184</td><td>100%</td></tr> </tbody> </table> Grades	Grade	Frequency	Relative frequency	Fail	2	1%	Pass	37	20%	Credit	68	37%	Distinction	54	29%	High Distinction	23	13%	Total	184	100%
Grade	Frequency	Relative frequency																					
Fail	2	1%																					
Pass	37	20%																					
Credit	68	37%																					
Distinction	54	29%																					
High Distinction	23	13%																					
Total	184	100%																					



Worked example video: Choosing a display based on the type of data

Understanding, fluency and communicating

Answers: Lesson 11.7

1 For each of the following tables, classify the variable in the first column as one of:

- nominal (categorical)
- ordinal (categorical)
- discrete numerical
- grouped discrete numerical
- grouped continuous numerical.

a

Shoe sizes	Frequency
2–5	4
6–9	6
10–13	2

b

Day	Frequency
Monday	5
Tuesday	32
Wednesday	45
Thursday	3
Friday	7

c

Score	Frequency
1	6
2	8
3	19
4	23
5	15

d

Suit	Frequency
Clubs	7
Diamonds	9
Hearts	23
Spades	1

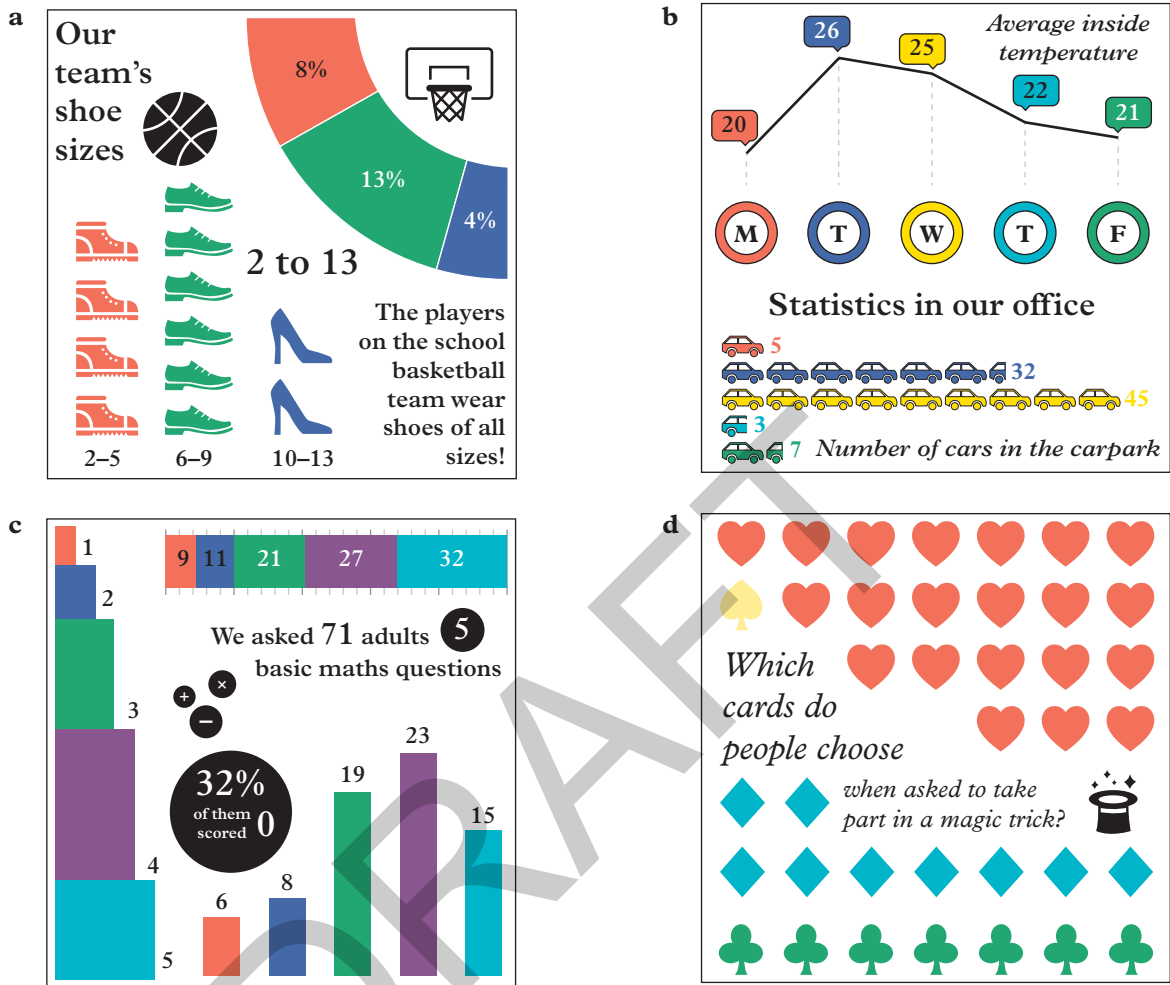
e

Height (cm)	Frequency
125–<140	12
140–<155	32
155–<170	53
170–<185	11
185–<200	5

2 Each of the following refers to the corresponding dataset from question 1.

- Copy the table in question 1a, add a relative frequency column, then choose and create an appropriate display for the data.
- Choose an alternative variable to replace “frequency” in the second column of the table in question 1b and explain a situation that might generate this suggested dataset. Then choose and create an appropriate display for the data.
- Copy the table in question 1c, add a cumulative frequency column, then choose and create an appropriate display for the data.
- Copy the table in question 1d, add a relative frequency column, then choose and create an appropriate display for the data.
- Copy the table in question 1e, add a cumulative frequency column, then choose and create an appropriate display for the data.

- 3 For each of the infographics shown:
- i name and describe any graphs or other data displays used
 - ii list all statistical information that can be interpreted from the graphic
 - iii identify features that are confusing or misleading.



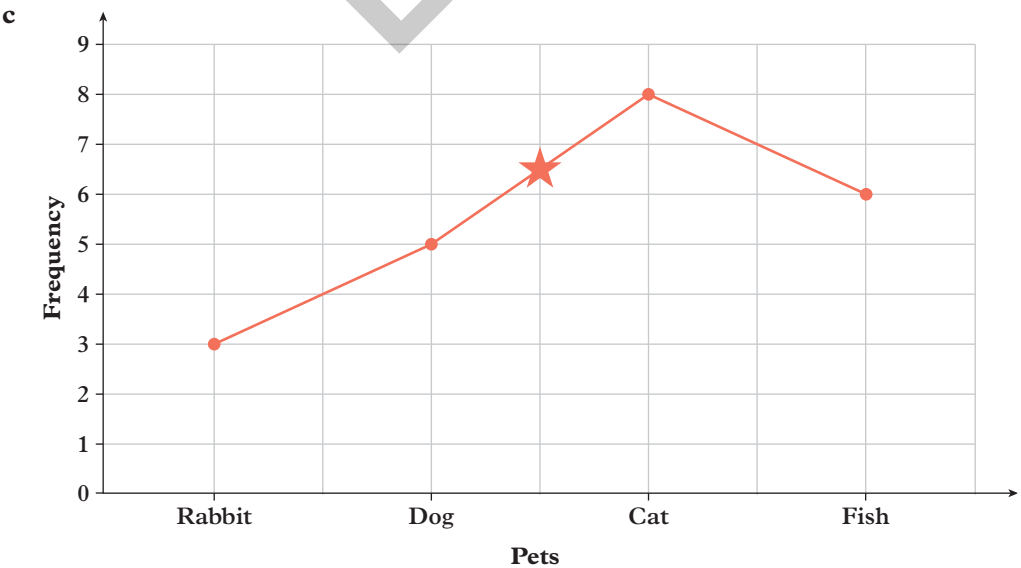
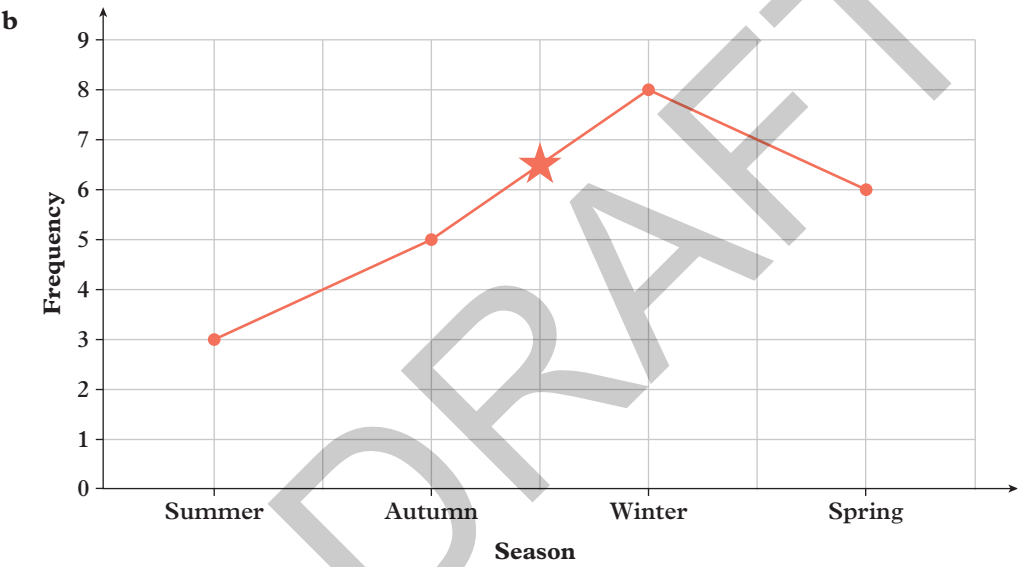
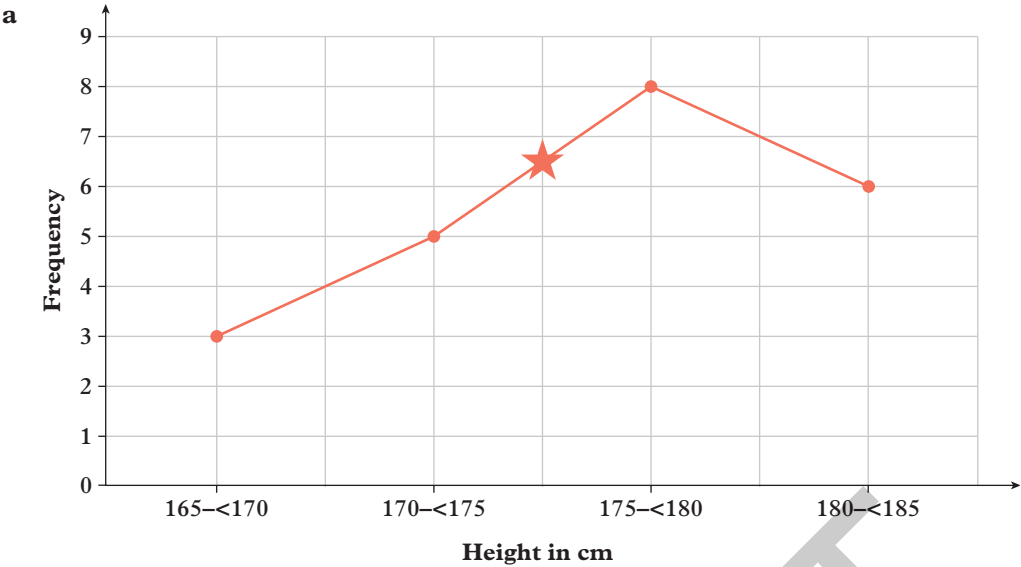
Problem solving and reasoning

- 4 For each of the datasets in question 1:
- i Copy and complete this table by placing a mark in one column for each display type.

	Definitely Suitable	Uncertain	Definitely Unsuitable
Dot plot			
Column graph			
Line graph			
Histogram			
Frequency polygon			

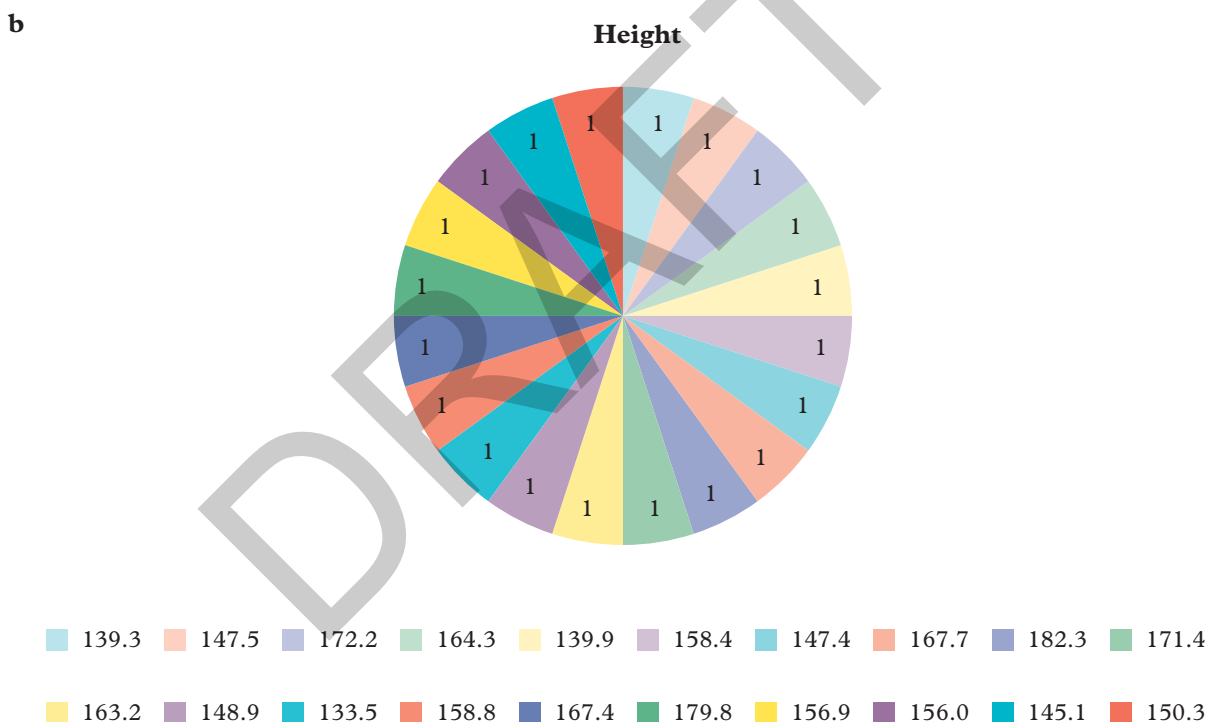
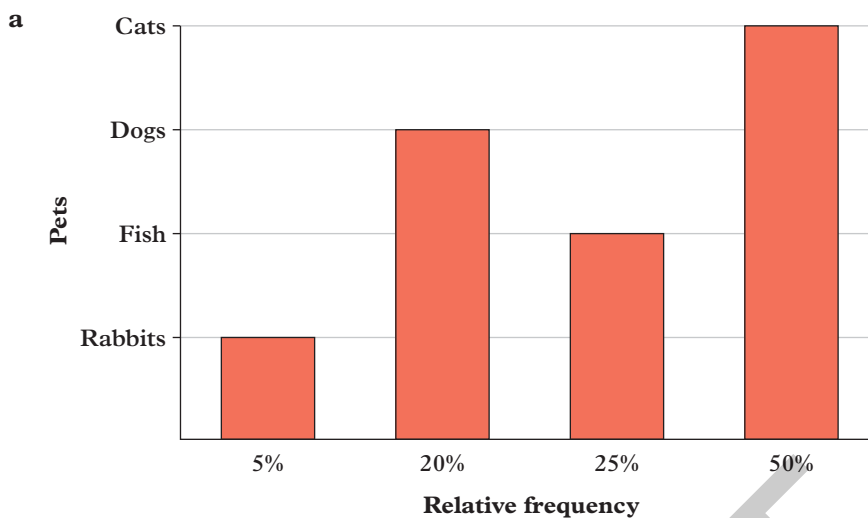
- ii Explain your considerations for any “uncertain” answers.

5 Consider the following line graphs. For each dataset, explain how you would interpret the point on the line indicated by the star.



6 For each of the following data displays

- i give at least one reason why the chosen display is inappropriate for the dataset.
- ii suggest a more appropriate display, giving reasons for your answer.



7 Create an infographic for the dataset in question 1d. You will need to choose a realistic context for the dataset to belong to. You may wish to calculate some summary statistics for the dataset to include in your infographic.

8 Elise is creating an infographic upon doing research into their local suburb.

- a They find the population of the suburb over the past 30 years. Which graph would be suitable to show this information?
- b Elise also finds the breakdown of people in various age groups (e.g. 12–17, 18–24, 25–34). If they wanted to compare the distribution across age groups from 30 years ago versus today, which type of data display could Elise use?
- c They collect data on the house prices, which they believe is a good representation of the suburb. Suggest a graph Elise could use to display this data.

- 9 Find some examples of infographics to share with your class. You may find them online, in print media or even in the resources for some of your other subjects.
- a Discuss what is good about each infographic. For example, does it accurately convey the dataset it represents?
 - b Discuss any issues with each infographic. Is the chosen display inappropriate for the dataset.
 - c Redesign one of the infographics with an inappropriate display. Justify your choices and explain why your version is a better display of the dataset.

Challenge

- 10 Think of some data you could collect from other students in your class or school using a statistical investigation.
- a Design a survey to collect the data. Ensure you follow good survey design principles.
 - b Choose between a census and a sample and justify your choice. If you choose a sample, explain how you will select the sample and ensure the survey results reflect the total population.
 - c Collect the data using your survey.
 - d Represent the collected data using a data display. Justify your choice of display.
 - e Design an infographic to share the results of your investigation with your class. Is your choice of data display the same as in part d?

Related resources



11.7 Quick quiz



Worksheet: Presenting data

Lesson 11.8

Describing the shape of datasets

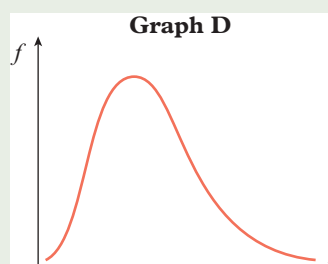
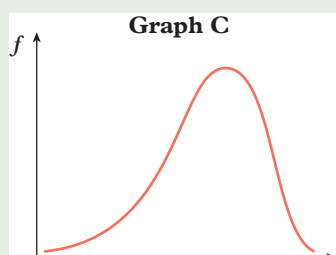
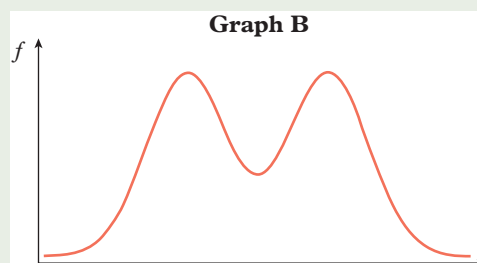
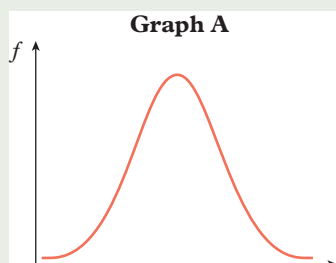
Learning intentions

By the end of this lesson, you will be able to ...

→ describe, compare and interpret different distributions of data.

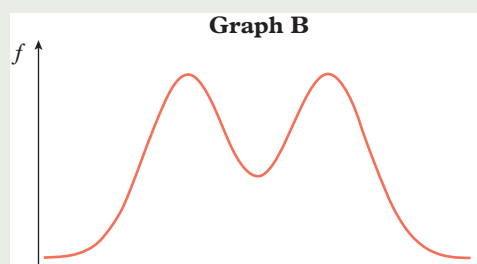
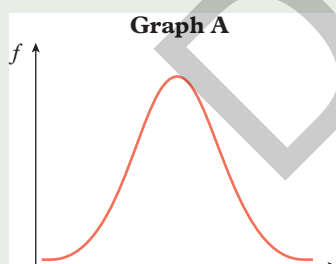
The shape of datasets

- Most of the types of displays discussed in this module allow us to say something about the shape of a dataset. The shape is particularly easy to visualise if we draw a smooth curve that outlines the data.
 - For dot plots and stem-and-leaf plots, we can draw a curve that smoothly connects the dots and leaves at the end of each row or column.
 - For column graphs, we can connect the midpoints on the top of each bar together by a smooth curve, similarly to how a frequency polygon is added to a histogram.
 - If we already have a line graph or a frequency polygon, we can just smooth out the corners to get a smooth curve.



Modality of a dataset

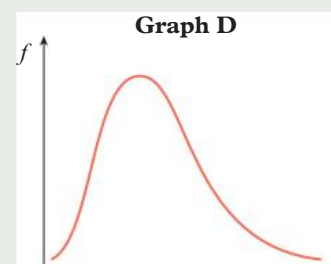
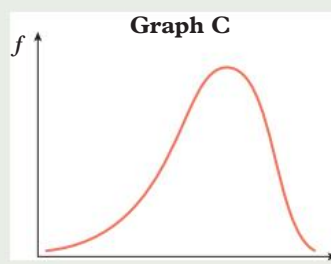
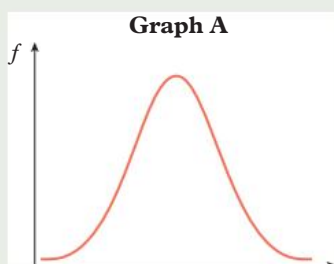
- The shape of a dataset may be described in terms of how many humps and dips it has.
 - The **mode** of a dataset is the value or category recorded with the highest frequency, so it will have the highest peak in a frequency graph.
 - A curve with a clear peak is described as **unimodal**.
 - A curve with two distinct humps of a similar height is called **bimodal**, even if the height of each hump is not exactly the same.
 - If a curve has three or more humps of a similar height it is described as **multimodal**.
 - For example, Graph A is unimodal (has one mode) and Graph B is bimodal (has two modes).



Symmetry and skewness

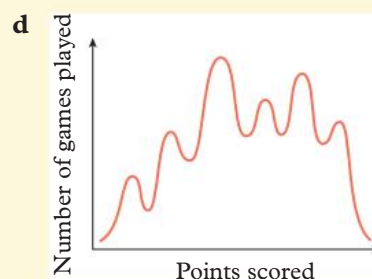
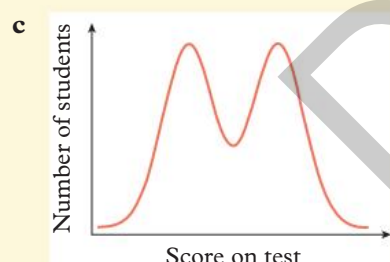
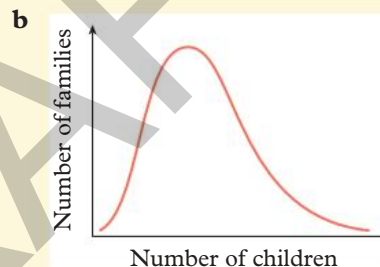
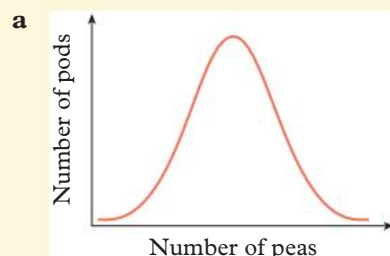
- We also describe the shape of these datasets in terms of their symmetry or “skewness”.
 - For example, Graph A is symmetric, but Graph C is said to be **skewed**.
- Some symmetric distributions can also be described as **bell-shaped**.
 - Bell-shaped curves are often referred to as the **normal distribution**. This is because many naturally occurring characteristics, such as the number of tomatoes on a plant, the number of peas in a pod or the heights of boys in a particular age group, have been found to follow this distribution.

- When a distribution is skewed, it is either negatively skewed, like in Graph C, or positively skewed, like in Graph D.
 - If the longer “tail” of a skewed graph is to the left, then the distribution is **negatively skewed**. This would occur, for example, if we graphed the distribution of the results of a very easy test. Most of the students would score high marks and only a few would score low marks.
 - If the longer tail is to the right, then the distribution is **positively skewed**. This would occur for the distribution of the results of a very hard test.



Worked example 11.8A Describing graphs

Describe the shape of the following curves, including skewness, symmetry and modality.



Think

For each graph, we ask:

- Is the curve symmetric?
 - If yes, is it bell-shaped?
 - If no, is it positively or negatively skewed?
- How many humps are there?

Write

- a** This curve is bell-shaped, symmetric and unimodal.
- b** This curve is positively skewed, so it is not symmetric, and it is unimodal.
- c** This curve is symmetric and bimodal.
- d** This curve is not symmetric and could be considered multimodal.



Worked example video: Describing graphs

Understanding, fluency and communicating

Answers: Lesson 11.8

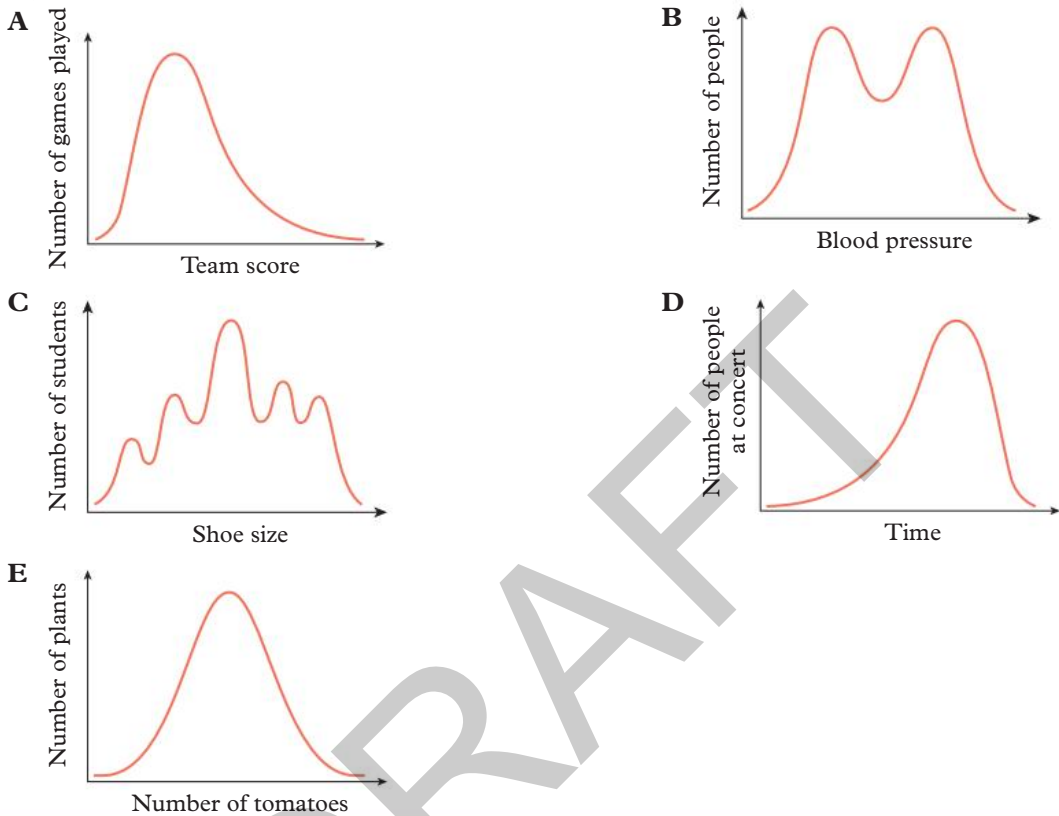
1 Match each description with one of the curves shown.

- a** unimodal, positively skewed

c symmetric, bell-shaped, unimodal

e asymmetric, multimodal
- b** unimodal, negatively skewed

d symmetric, bimodal



Worked example 11.8B Interpreting the shape of a graph in its context

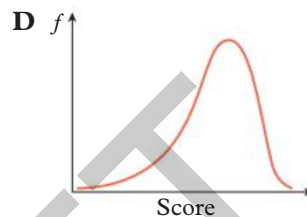
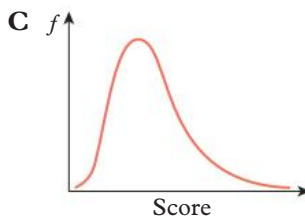
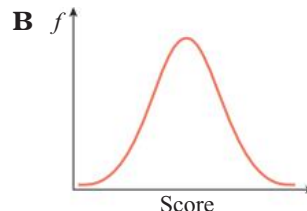
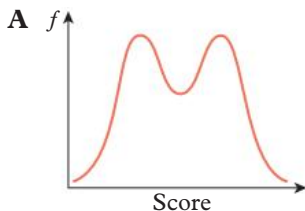
Explain what the shape of each distribution in **Worked example 11.8A** means in the context of the data it represents.

Part	Think	Write
a	This graph shows a symmetric distribution.	The number of peas is evenly spread on both sides of the mode.
b	This graph's positive skew indicates that there are more families with fewer children at the left side of the curve.	More families have small numbers of children and fewer have large families.
c	This graph is symmetric with two modes. The graph could show, for example, that the test was on a topic that students either understood really well or not at all.	A large number of students scored high marks on a test and a large number of students did poorly.
d	No real pattern.	There is no pattern of points scored in the games recorded.



Worked example video: Interpreting the shape of a graph in its context

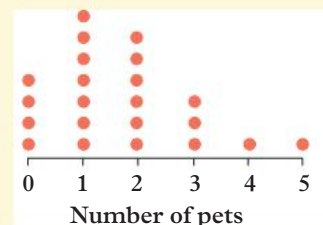
- 2 For each graph in question 1, explain what the shape of each distribution means in the context of the data it represents.
- 3 For graphs **A** and **B** in question 1, place one or more crosses on the horizontal axis to indicate the mode(s).
- 4 Match each of the following datasets to the most likely corresponding curve, **A** to **D**.
- a** the heights of 1000 randomly selected adult women **b** the results in a difficult examination
- c** the weights of a group of men who are all either jockeys or wrestlers **d** the results in an easy examination

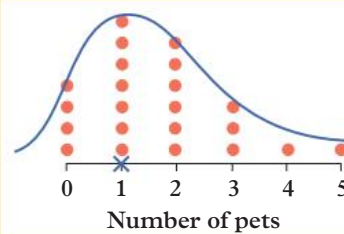


Worked example 11.8C Describing the shape of a distribution from a dot plot

The number of pets that each student in a class owns are displayed in this dot plot.

- a** Draw a curve showing the shape of the distribution.
- b** Describe the shape of the distribution.
- c** Mark the location of the mode on the horizontal axis.



Part	Think	Write
a	Draw a smooth curve that touches or nearly touches the top dot in each column.	
b	The tail points right, so the skew is positive. This means there are more small numbers than large numbers.	Positively skewed distribution. More students have a small number of pets.
c	The highest point on the graph shows the location of the mode.	The mode is marked with a cross on the scale of the graph in part a .



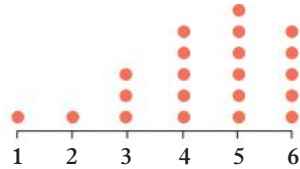
Worked example video: Describing the shape of a distribution from a dot plot

Helpful hints

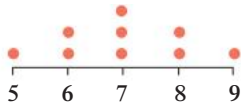
- The smooth curve highlights the shape of the distribution.
- The mode is the score where the graph peaks.

- 5 Complete the following for each dot plot shown.
- i Draw a curve showing the shape of the distribution.
 - ii Plot the position of the mode, if one exists, by placing a cross on the horizontal axis.
 - iii Describe the shape of the distribution.

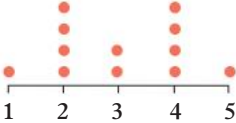
a



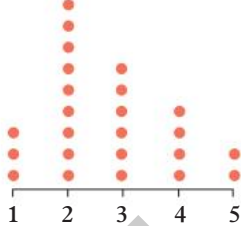
b



c



d



Worked example 11.8D Describing the shape of the distribution from a stem-and-leaf plot

Consider the set of scores given in the stem-and-leaf plot.

Stem	Leaf
0	3
1	
2	8 9
3	1 2 4
4	0 1 1 3 5 5 6 7
5	2 3 3 5 7 8 9
6	4 5

Key: 2 | 8 = 28

- a Determine whether this distribution of scores is symmetric, positively skewed or negatively skewed.
- b Are there any clusters? If so, where are they located?

Part	Think	Write
a	Turn the stem-and-leaf plot on its side and draw a curve to fit the height of each column.	The frequency curve is negatively skewed.
b	The rows with the most numbers can form a cluster of scores.	This set of scores is clustered in the 40s and 50s.



Worked example video: Describing the shape of the distribution from a stem-and-leaf plot

Helpful hints

- Outliers and clusters affect the shape of a distribution.

- 6 Consider the data given in the stem-and-leaf plots below, each with key $2 \mid 8 = 28$. For each one:
- Determine whether the distribution of scores is symmetric, positively skewed or negatively skewed.
 - Identify whether there are any clusters. If yes, where are they located?

a

Stem	Leaf
1	2 3 6 8 8
2	2 5 6 7 7 8 9
3	4 4
4	
5	9

b

Stem	Leaf
0	5
1	3 7
2	2 2 4 6
3	0 1 1 5 8 8 9
4	2 3 3 6
5	1 5
6	4

c

Stem	Leaf
3	9
4	2 5
5	2 2 3
6	0 1 3 4 4 7 8
7	0 0 2 4 5 5 7 9 9
8	1 3

d

Stem	Leaf
8	3
9	0 1 5
10	6 6 7 7 8 9 9
11	0 1 3 5 7 8 9
12	0 0 1 3
13	4

e

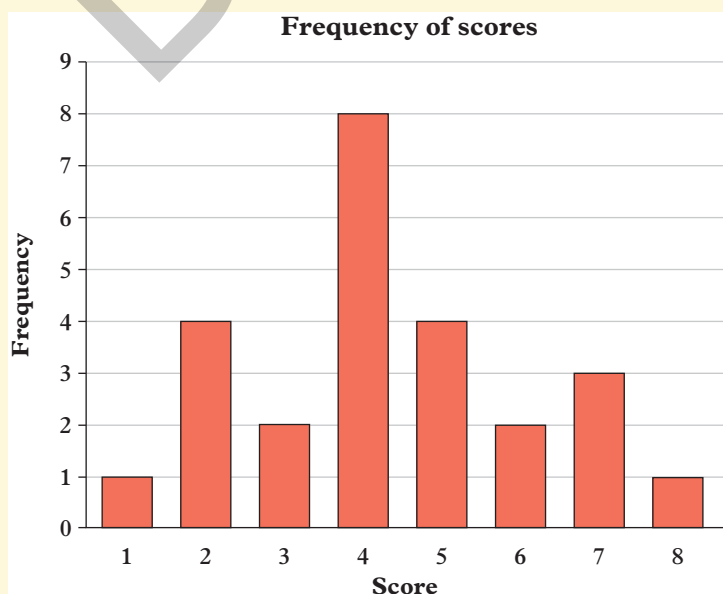
Stem	Leaf
6	5 5 6 7 8 9
7	0 1 3 3 4 6 9 9
8	2 2 5 8
9	1 3
10	0 2
11	5

f

Stem	Leaf
1	9 9
2	0 2 4
3	8 8 8 9
4	0 5 5 9 9
5	2 2 2 6 7 7
6	3 4 5 8 9

Worked example 11.8E Describing the shape of the distribution from a column graph

Consider the set of scores displayed in this column graph.



- a Draw a smooth curve that follows the tops of each column.
- b Describe the shape of the distribution, using the smooth curve.

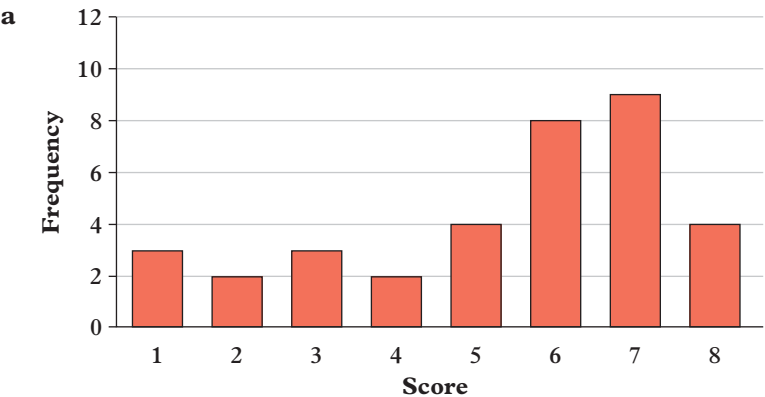
Part	Think	Write																		
a	Make sure the smooth curve touches the top of all the tallest bars.	Update image as follows: Frequency of scores <table><caption>Data for Frequency of scores</caption><tr><th>Score</th><th>Frequency</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td>4</td></tr><tr><td>6</td><td>2</td></tr><tr><td>7</td><td>3</td></tr><tr><td>8</td><td>1</td></tr></table>	Score	Frequency	1	1	2	4	3	2	4	8	5	4	6	2	7	3	8	1
Score	Frequency																			
1	1																			
2	4																			
3	2																			
4	8																			
5	4																			
6	2																			
7	3																			
8	1																			
b	The main hump is to the left of the centre, and there are more lower frequency bars to the right.	This distribution is not symmetric. It is unimodal and somewhat positively skewed.																		

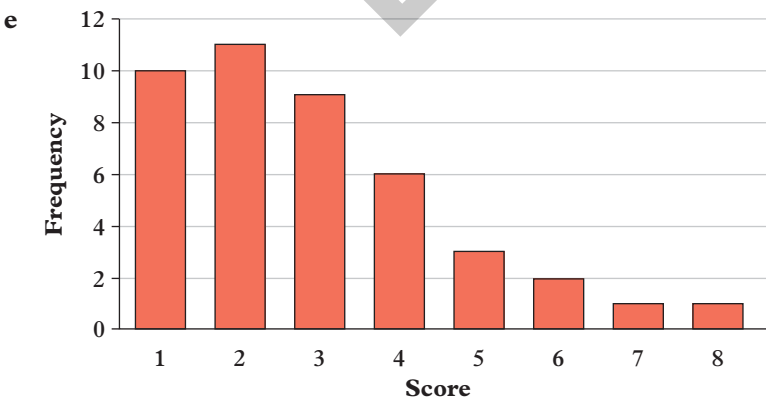
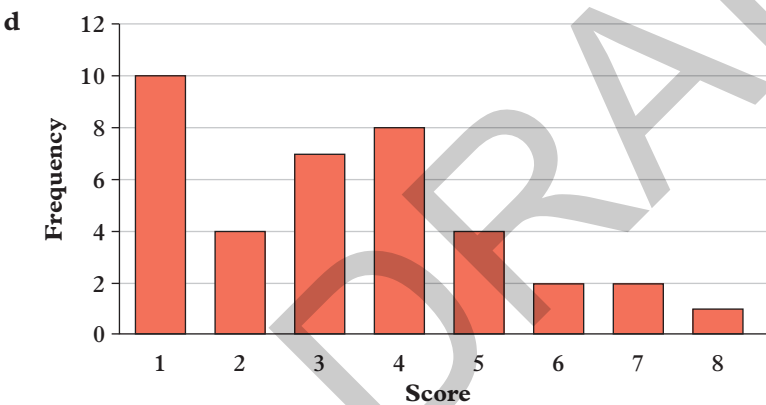
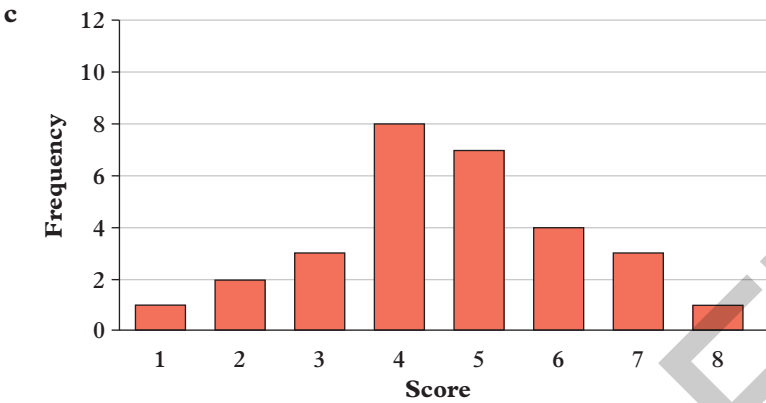
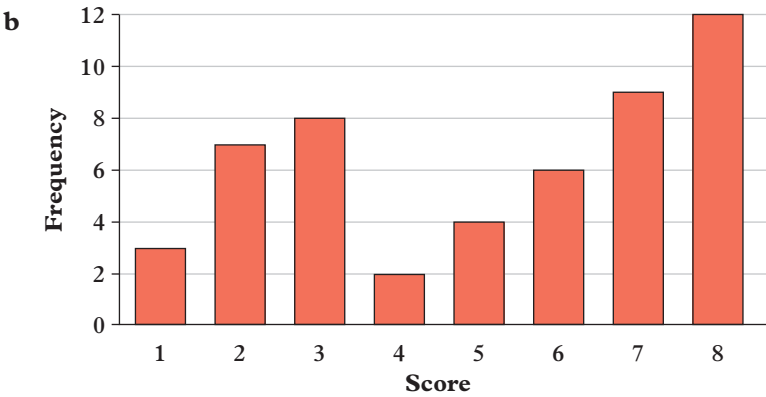
Worked example video: Describing the shape of the distribution from a column graph

Helpful hints

- Your smooth curve should have clear humps for any similarly frequent modal scores, but it does not need to accentuate every small hump.

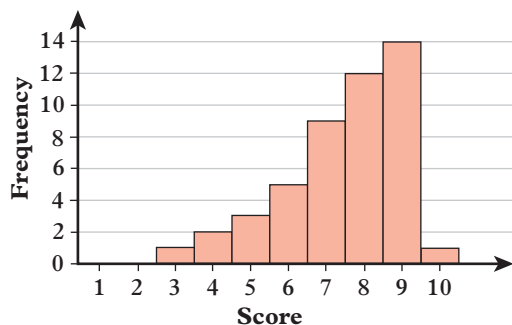
- 7 Consider the sets of scores displayed in the column graphs below. For each one:
- i draw a smooth curve that follows the tops of each column.
 - ii describe the shape of the distribution, using the smooth curve.





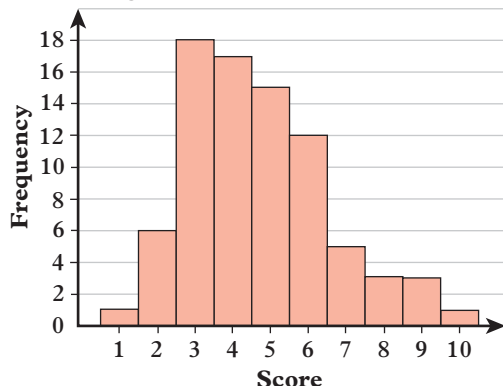
Problem solving and reasoning

8 This histogram shows the results of 47 students for a test marked out of 10.



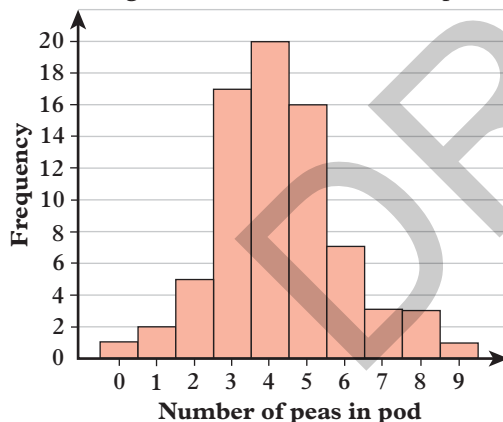
- Draw a smooth curve on the histogram.
- Describe the shape of the curve.
- What comments can be made about the test based on the shape of the curve?

9 This histogram shows the results of 81 students for a test marked out of 10.



- Draw a smooth curve on the histogram.
- Describe the shape of the curve.
- What comments can be made about the test based on the shape of the curve?

10 This histogram shows the number of peas in 75 pods.



- Draw a smooth curve on the histogram.
- Describe the shape of the curve.
- What comments can be made about the number of peas in a pod based on the shape of the curve?

Challenge

11 For each shape of frequency distribution given, sketch what the corresponding *cumulative* frequency polygon would look like.

- Negatively skewed
- Positively skewed
- Bimodal

Related resources



11.8 Quick quiz

Using spreadsheets

Syllabus links

Use a spreadsheet to organise and represent data using appropriate graphs (MST-11-08)

Using spreadsheets

See Oxford Digital for sample spreadsheets for questions in this lesson.

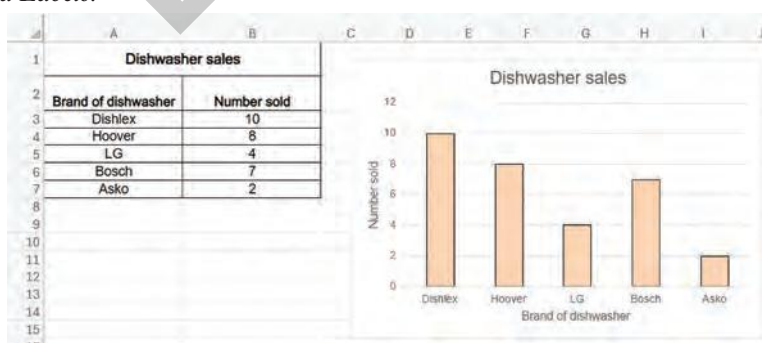
- 1 A spreadsheet can be used to add a cumulative frequency column and a relative frequency column to a frequency distribution table. The following spreadsheet uses the data on the number of Candy Clicks in a packet from **Worked Example 11.5A** in Lesson 11.5 and **Worked example 11.6A** in Lesson 11.6. The relative frequencies are expressed as decimals, set to three decimal places.

	A	B	C	D
1	Frequency distribution table			
2	Score	Frequency	Cumulative frequency	Relative frequency
3	36	1	1	0.025
4	37	5	6	0.125
5	38	8	14	0.200
6	39	13	27	0.325
7	40	7	34	0.175
8	41	4	38	0.100
9	42	2	40	0.050
10	Total:	40		1.000

- a What formulas might have been used to generate the values in columns C and D? See if you can replicate these columns by entering as few numbers manually as possible.
- b Remember you can highlight cells and use *fill down* to complete columns.
- c Return to the exercises in Lesson 11.5 and Lesson 11.6 and use your spreadsheet to repeat some of the questions.

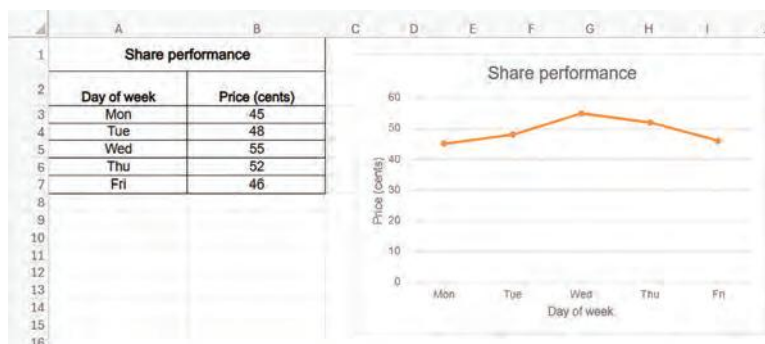
Many different types of graphs can be generated using the *Chart* option (or equivalent) in spreadsheet software.

- 2 The column graph below was generated from the data in **Worked Example 11.2B** of Lesson 11.2. To replicate it, enter the data into two columns of a spreadsheet, highlight all cells (including the column labels), then select *Chart* and choose *Column graph*. Investigate the *Chart Layout* including options such as *Axis Titles* and *Data Labels*.



- a Reproduce this spreadsheet to create your own column graph for the data in **Worked Example 11.2A** of Lesson 11.2.
- b Now use a spreadsheet to draw each of the graphs in questions 1 to 5 of Lesson 11.2.

- 3 The line graph below was generated from the data in **Worked example 11.3A** of Lesson 11.3. To replicate it, enter the data into two columns of a spreadsheet, highlight all cells (including the column labels), then select *Chart* and choose *Line graph*. Investigate the *Chart Layout* including options such as *Axis Titles* and *Data Labels*.



- Reproduce this spreadsheet to create your own line graph for the data in **Worked Example 11.3A** of Lesson 11.3.
 - Now use a spreadsheet to draw each of the graphs in questions 1 to 4 of Lesson 11.3.
- 4 For each of Lesson 11.4 and Lesson 11.5, use a spreadsheet to replicate the worked examples, to practise creating each style of data display.

Lesson 11.9

Review: Displaying and interpreting data

Review questions 11.9A Mathematical literacy review

Answers: Lesson 11.9

Key terms

The following key terms are used in this module:

- bimodal
- bell-shaped
- class centre
- classes (class intervals)
- column graph
- cumulative frequency histogram
- cumulative frequency polygon (ogive)
- divided bar chart
- dot plot
- frequency polygon
- frequency distribution table
- histogram
- leaf
- line graph
- mode
- multimodal
- negatively skewed
- percentage relative frequency
- positively skewed
- relative frequency
- sector graph
- skewed
- stem
- stem-and-leaf plot
- tally marks
- unimodal

Check your understanding

- Draw an example **stem-and-leaf plot** and label the **stem** and **leaf** components.
- Explain the visual similarities and differences between a **column graph** and a **histogram**.
- Choose from the list of key terms to complete the sentence below in as many different ways as possible.
You can use ____ to find the ____.

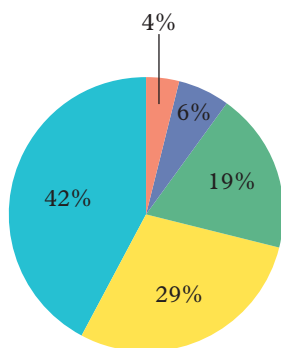
4 Match each of the following types of data displays with one of the key terms in the list.

Stem	Leaf
0	3 6 6 9
1	0 2 9
2	1 5 6 8
3	9

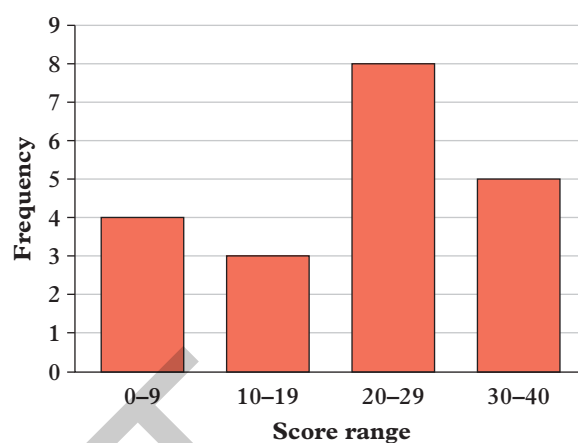
a

b

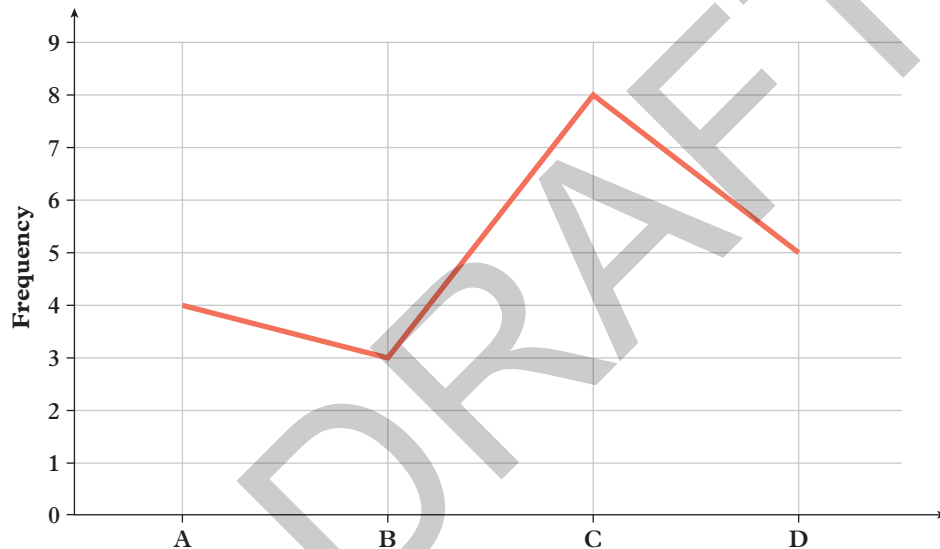
:	:	:	:	:
A	B	C	D	E



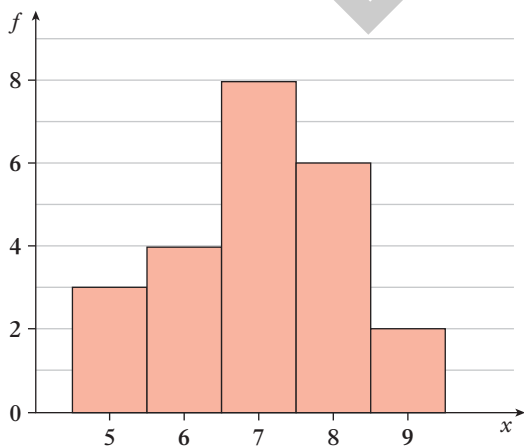
d



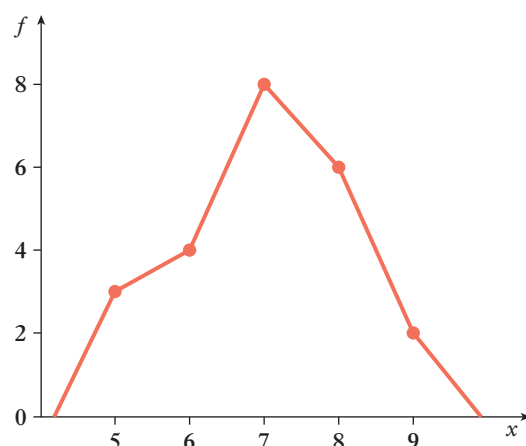
e



f



g



Review questions 11.9B Multiple-choice questions

- 1 The marks out of 10 that 20 students received on a spelling test were:
5 3 9 8 6 4 5 6 6 5 7 8 10 6 7 5 6 8 6 7 Use a tally column to create a frequency distribution table for this dataset. How many students scored less than 6 in the spelling test.
A 6 **B** 5 **C** 4 **D** 3

- 2 This stem-and-leaf plot shows the time, in hours, a group of Year 11 students spent studying during a particular week.

Stem	Leaf
0	2 3 3 6 7 7
1	0 2 2 5 8 8 8 9 9
2	1 1 2 3 4 4 5 6 6 7 8
3	0 2 2 9

Key : 1 | 2 = 12

- The lowest and highest scores are, respectively:
A 7 and 39 **B** 0 and 9 **C** 2 and 9 **D** 2 and 39
- 3 Consider the sales data shown in the table.

Months	Sales (\$ million)
Jan/Feb	3.4
Mar/Apr	3.1
May/Jun	1.8
Jul/Aug	1.4
Sep/Oct	2.8
Nov/Dec	3.6

- If a column graph was used to display the information in this table, with its vertical axis representing millions of dollars, what label on the vertical axis would be in line with the top of the bar representing sales for March/April?
A $\frac{3}{4}$ **B** 3.1 **C** 31 **D** 3.4

- 4 A group of students are asked how far they travel, to the nearest 0.1 km, to get to school each day. Their responses are shown below.
0.6 3.4 0.8 2.8 0.9 1.1 1.9 2.4 3.5 3.6 0.8 1.4 2.7 0.6 1.3 3.9 2.1 2.3
3.0 1.6 1.5 3.2 2.1 1.8 0.7 2.3 3.6 3.5 1.6 1.4 0.4 2.8 3.8 2.9 0.8 1.7
3.7 2.5 1.9 2.0

Organise this data by completing the frequency distribution table.

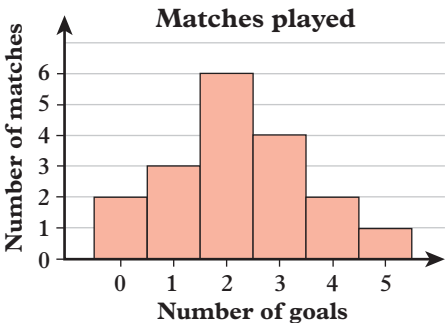
Distance	Frequency
0–0.4	
0.5–0.9	
1.0–1.4	
1.5–1.9	
2.0–2.4	
2.5–2.9	
3.0–3.4	
3.5–3.9	

- The number of students who travel 2.5 km or more is:
A 25 **B** 21 **C** 15 **D** 12
- 5 If the data from question 3 were displayed in a line graph, how many points would be drawn and then connected?
A 5 **B** 8 **C** 6 **D** 12

6 Which of the following is *not* a cause of graphs being misleading?

- A The scale on the vertical axis does not start at zero.
- B The scale on the horizontal axis does not start at zero.
- C The scale on the vertical axis is irregular.
- D The incorrect use of area or volume creates a false impression.

7 Use this frequency histogram to determine the total number of matches played.



- A 17
- B 5
- C 6
- D 18

8 To draw a frequency polygon using the histogram in question 7, the first and last column midpoints should be joined to:

- A any two points on the horizontal axis
- B the two points on the horizontal axis where the previous and the next score would be
- C any two points on the vertical axis
- D the bottom left and right corners of the histogram.

9 Suppose a divided bar chart is used to represent the following dataset. If the total length of the bar chart is 20 cm, how long should the section for category A be?

Category	Relative frequency
A	50%
B	30%
C	20%

- A 50 cm
- B 5 cm
- C 10 cm
- D 20 cm

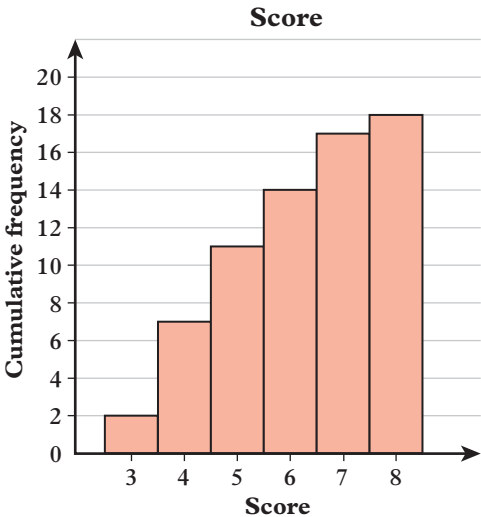
10 For the data in question 1, the relative frequencies are:

- A 5%, 10%, 30%, 60%, 75%, 95%, 100%
- B 1, 1, 4, 6, 3, 3, 1, 1
- C 1, 2, 6, 12, 15, 18, 19, 20
- D 5%, 5%, 20%, 30%, 15%, 15%, 5%, 5%

11 For the data in question 1, the cumulative frequencies are:

- A 5%, 5%, 20%, 30%, 15%, 15%, 5%, 5%
- B 1, 1, 4, 6, 3, 3, 1, 1
- C 1, 2, 6, 12, 15, 18, 19, 20
- D 5%, 10%, 30%, 60%, 75%, 95%, 100%

12 Use the cumulative frequency histogram shown to determine the frequency of the score 5.



- A 3
- B 4
- C 5
- D 11

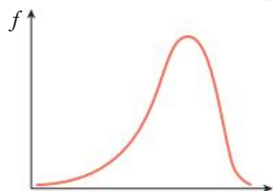
13 Which of the following statements is *not* true?

- A A dot plot is convenient for illustrating small sets of data.
- B A dot plot can be used directly with unsorted data.
- C A dot plot is very time-consuming for large sets of data.
- D A dot plot can be used to show the percentage of each category in the data.

14 Which type of data would *not* make sense on the horizontal axis of a line graph?

- A continuous numerical
- B discrete numerical
- C nominal (categorical)
- D ordinal (categorical)

15 Choose the best description for the graph shown.



- A unimodal and positively skewed
- B unimodal and negatively skewed
- C symmetric and bimodal
- D symmetric and bell-shaped.

16 The graph in question 15 could represent:

- A the class marks for an easy test
- B the class marks for a hard test
- C a uniform distribution of class marks
- D no recognisable pattern of marks.

Review questions 11.9C Set 1

1 The results of a survey of the hair colour of a group of people is shown using the codes black (B), brown (b), blond (w), red (r) and grey (g).

B b b w g b B B r w w b g B w B B b b w w w w b B
g r B b w

- a Organise the information into a dot plot and use it to answer these questions.
 - i How many people were surveyed?
 - ii What was the most common hair colour for this group of people?
- b Create a table with a relative frequency column and use it to create a sector graph displaying the data.



2 This table shows the variation in the price of shares over the course of one week.

Day	Mon	Tue	Wed	Thu	Fri
Price (\$)	2.80	3.50	3.60	2.60	2.55

- a Display this information on a line graph.
 - b Are there any other types of display that would be suitable for this dataset? Why or why not?
- 3 The set of marks out of 10 that a class received for a spelling test were:
- 5 3 9 8 6 4 5 6 6 5 7 8 10 6 7 5 6 8 6 7
- a Draw a column graph to display this data.
 - b Describe the shape of this dataset.
 - c What might the teacher use this graph to argue?

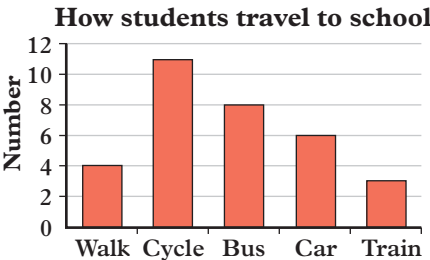
Review questions 11.9D Set 2

Use the following information to answer questions 1 to 7.

The marks scored in a mathematics test by a class are listed below.

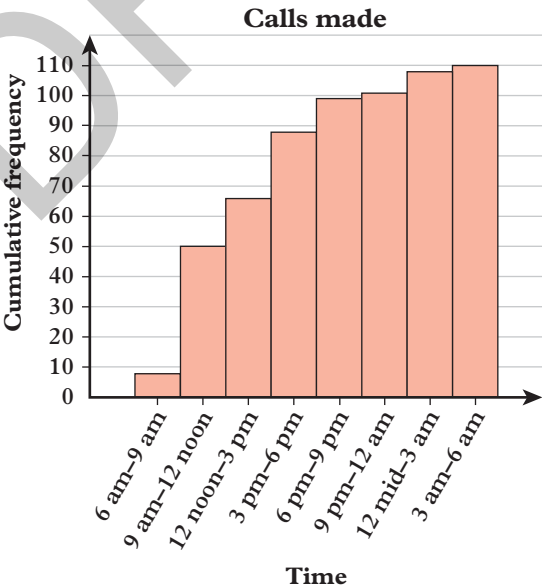
11 12 11 14 18 15 13 14 19 18 17 15 15 12 18 17 13 10 9
14 16 16 13 14 11 14 18 10 14 15

- 1 Organise the data into a frequency distribution table.
- 2 Would a stem-and-leaf plot be suitable for displaying this dataset? Why or why not?
- 3 Add a relative frequency and a cumulative frequency column to your table from question 1.
- 4 How many students scored:
 - a less than 15?
 - b 15 or less?
- 5 What percentage of students scored:
 - a 14?
 - b more than 18?
- 6 Suppose a mark of 12 or lower receives a “Low”, a mark of 16 or greater receives a “High” and any other mark in between receives a “Medium”. Represent the distribution of grades in a divided bar chart.
- 7 Draw the cumulative frequency histogram and polygon for the data.
- 8 From the column graph, how many students:



- a walk to school?
- b don't catch a train?

- 9 What is the shape of the data displayed in question 8? What does the shape of the column graph tell you about this dataset, if anything?
- 10 Using the cumulative frequency graph shown, how many calls were made between 12 :00 noon and 3 :00 pm?



Review questions 11.9E Set 3

- 1 The numbers of each brand of sound system sold by an electrical store in one week are shown in the table.

Brand	Number of sales
Sony	15
National	8
Pioneer	12
Aiwa	10
Phillips	5

- How many sound systems were sold during the week?
- Draw a column graph to display this information.
- Add a relative frequency column, then draw a sector graph to display this data.

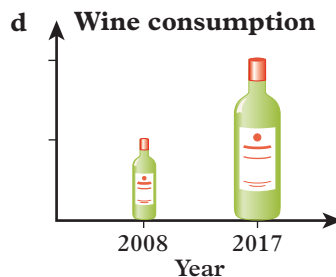
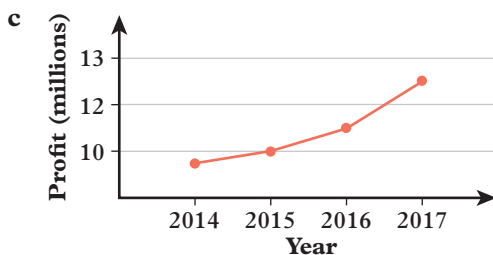
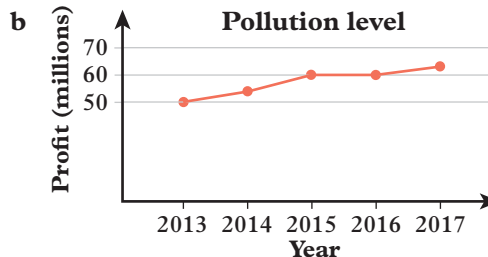
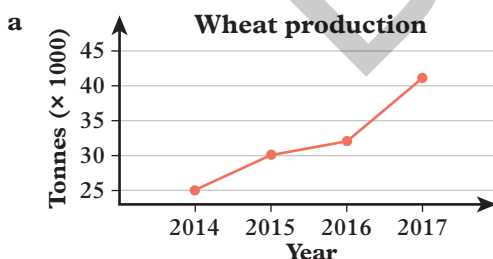
- 2 Consider the following set of scores.

33 46 38 66 49 58 34 43 60 63 48 52 51 48 37 69 61 51 64 48

- Draw a stem-and-leaf plot to display the data, using the stems 3, 4, 5 and 6.
 - What are the lowest and highest scores?
 - How many times do the following scores occur?
 - 51
 - 41
 - Which score occurs the most often?
 - How many scores are in the sixties?
 - How many scores are less than 50?
 - What would the class intervals be if you grouped this data into four classes?
- 3 A bank manager wants to identify the most common reasons for their customers experiencing delays in the processing of credit card applications. A random survey was conducted, and the results are shown in the table. Choose and create an appropriate display for this dataset.

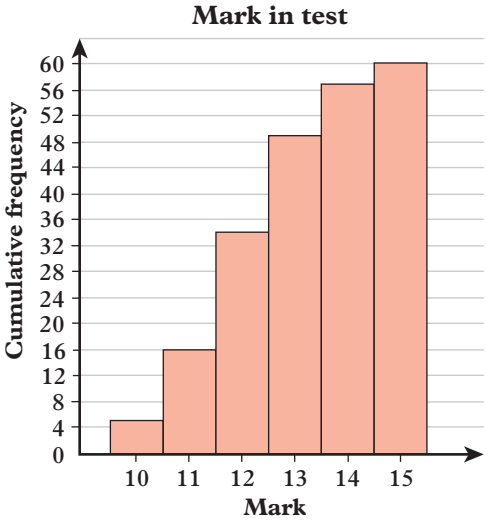
Cause of delay	Frequency
Incorrect address	47
Can't read	22
No signature	92
Wrong form	14
Other	5

- 4 Describe the misleading or poor features of the following graphs.



Review questions 11.9F Set 4

1 Consider the cumulative frequency histogram shown.



- a Copy the histogram and add the cumulative frequency polygon.
- b How many students scored:
 - i less than 12?
 - ii 12 or less?
 - iii more than 12?
- c Describe the shape of the cumulative frequency histogram. What can you say about the shape of cumulative frequency histograms in general?
- d Use the cumulative frequency histogram to recover the frequency distribution table for this dataset.

2 Consider the following dataset.

green blue blue blue red blue green yellow green blue yellow yellow
purple purple green

- a Use a dot plot to organise this data.
- b Describe a reasonable statistical question in response to which this data may have been collected.
- c Use a sector graph to display this dataset, ensuring all relevant features are labelled so that the viewer can understand what is being displayed.
- d Explain why a line graph is or is not suitable for displaying this dataset.

Review questions 11.9G Practice examination question

1 Answer the following questions.

a Consider the dataset given in the following table.

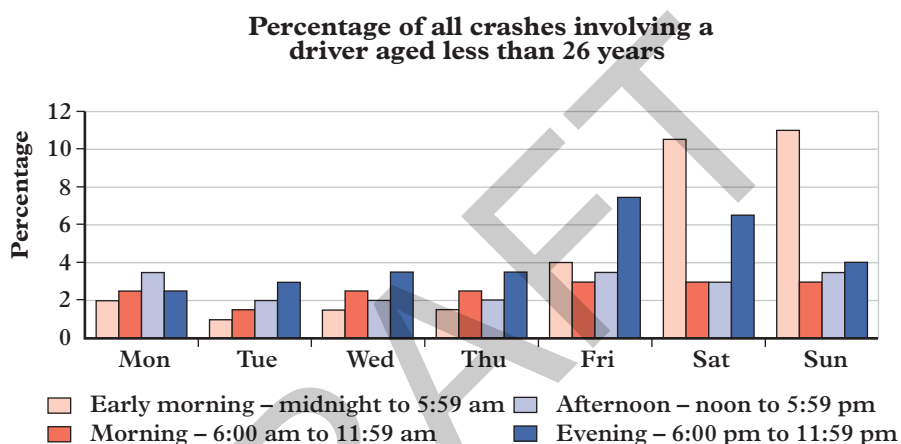
Student score	Class centre	Frequency	Cumulative frequency
0–4		7	
5–9		11	
10–14		15	
15–19		14	
20–24		3	

- i Complete this frequency distribution table. (1 mark)
- ii Describe the shape of this frequency distribution. (1 mark)
- iii What is the relative frequency of students scoring 5, 6, 7, 8 or 9? (1 mark)
- iv What percentage of students scored 15 or more? (1 mark)
- v Draw a cumulative frequency histogram and polygon for this data. (2 marks)

- b** This stem-and-leaf plot shows the time, to the nearest whole hour, the members of a group of university students spent studying per week.

Stem	Leaf
3	1 6
4	0 1 2 2 5 8 9
5	0 1 1 3 4 7 8 9 9
6	2 3 5 8
7	0 1 1
Key 3 1 = 31	

- i** What are the lowest and highest recorded study times? **(1 mark)**
- ii** Would it be more helpful to visualise this dataset after grouping the data into class intervals $24 < 36$, $36 < 48$, $48 < 60$ and $60 < 72$? Why or why not? **(2 marks)**
- iii** Is this numerical data continuous or discrete? Justify your answer. **(2 marks)**
- c** This graph shows the percentage of crashes involving a driver aged less than 26 years, by time of the day and day of the week.



- i** What percentage of crashes involving a driver under 26 occurred between noon and 5 : 59 pm on a Tuesday? **(1 mark)**
- ii** At what time of the day do most crashes occur? On what day of the week do the most crashes occur? **(1 mark)**
- iii** Could this data be displayed in a sector graph? Justify your answer. **(2 marks)**

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Displaying and interpreting data

I can do this	I need to review this
☐ organise categorical data using a frequency distribution table ☐ display categorical data in a dot plot ☐ organise numerical data in a stem-and-leaf plot	Go back to Lesson 11.1 Dot plots and stem-and-leaf plots
☐ group data into classes ☐ organise grouped data in tables ☐ display grouped and categorical data in a column graph	Go back to Lesson 11.2 Tables and column graphs
☐ create and interpret line graphs ☐ identify misleading features of data displays	Go back to Lesson 11.3 Line graphs
☐ group continuous numerical data into classes and find their class centres ☐ construct and interpret frequency histograms and polygons	Go back to Lesson 11.4 Histograms and frequency polygons
☐ add relative frequency to a frequency table ☐ construct and interpret sector graphs and divided bar charts	Go back to Lesson 11.5 Sector graphs and divided bar charts
☐ add cumulative to a frequency table ☐ construct and interpret cumulative frequency histograms and polygons	Go back to Lesson 11.6 Cumulative frequency
☐ select the most appropriate type of graph for a given dataset ☐ justify choices of graphical representations ☐ analyse statistical infographics ☐ interpret and consider the limitations of graphical representations	Go back to Lesson 11.7 Choosing a data display
☐ describe, compare and interpret the distributions of graphical displays ☐ identify clusters in a data display	Go back to Lesson 11.8 Describing the shape of datasets
☐ use a spreadsheet to organise and represent data using appropriate graphs	Go back to Using spreadsheets

Related resources



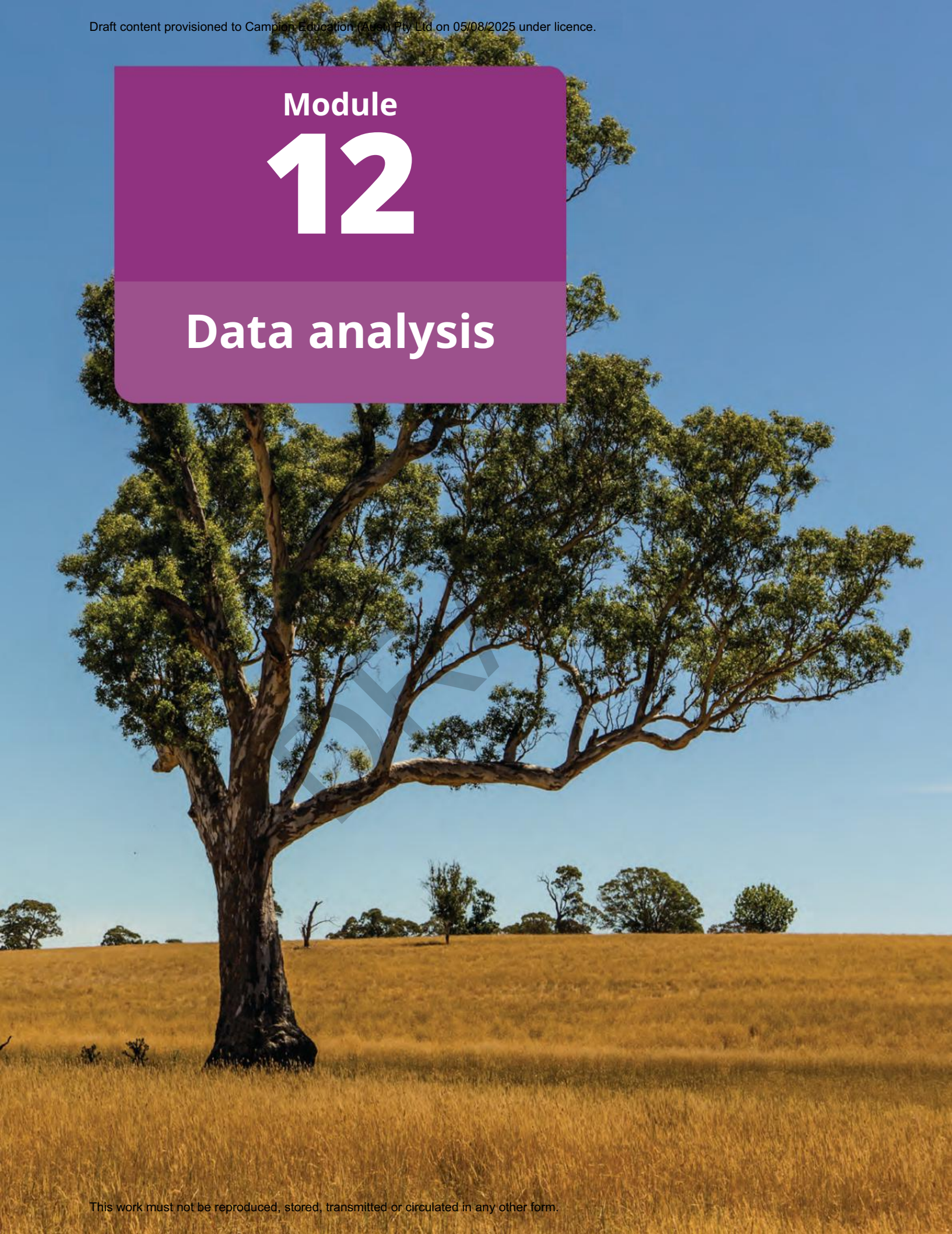
Module review quiz: Displaying and interpreting data



Worksheet: Challenge 11

Module 12

Data analysis



Lessons

- Lesson 12.1 Mean, mode and modality
- Lesson 12.2 Median
- Lesson 12.3 Relative merits of mean, mode and median
- Lesson 12.4 Range and interquartile range
- Lesson 12.5 Box plots
- Lesson 12.6 Interpreting box plots
- Lesson 12.7 Clusters and outliers
- Lesson 12.8 Standard deviation
- Using spreadsheets
- Lesson 12.9 Review: Data analysis

Prerequisite skills

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Diagnostic pre-test: Data analysis

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the downloadable support sheets.



Support sheet: Interpreting tables



Support sheet: Finding the average of two numbers



Support sheet: Mean, median, mode and range

Syllabus links

- Data analysis (MST-11-08)
 - A student displays and analyses datasets using summary statistics and graphical representations.
 - Measures of centre and spread
 - Quartiles and interquartile range
 - Five-number summary and box plots
- Working mathematically (MAO-WM-01)
 - A student develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly.

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Lesson 12.1

Mean, mode and modality

Learning intentions

By the end of this lesson, you will be able to ...
→ calculate the mean and mode of a dataset.

Mean and mode

- In Module 11, we organised raw data into tables and graphs that make the data easy to read, allowing us to identify patterns and characteristics, and draw conclusions about the dataset.
- It is often convenient to find a single number that best represents all the data. This number is sometimes called an “average”, but statisticians prefer to call it a **measure of central tendency** or just a **measure of centre**. The three commonly used measures of central tendency are the **mean**, the mode (which we encountered briefly in Module 11) and the median (which we will study in the following lesson).
- The mean of a dataset is the sum of all the data values divided by the number of data values. The symbol $\bar{x}$ is used to represent the mean.
 - When the data is grouped into classes, we consider all the values to be evenly distributed throughout the class and calculate a class centre as a representative of the class. If a class is made up of values $a - b$ then its centre is $\frac{b-a}{2}$.
 - Then calculate the mean as you would for ungrouped data by using the class centres as the data values. Because we approximated the values for each class, we treat the final answer as an approximation for the real mean.
- The mode is the value that occurs most often; that is, it is the data value with the highest frequency. For grouped data, the class that occurs most often is called the **modal class**.
 - A dataset can have more than one mode.
 - Recall from Module 11 that the shape of a dataset may be described as unimodal, bimodal or multimodal when displayed in an appropriate format. This is called the **modality** of the dataset. We can also use these terms to describe a dataset without relying on the display, based on how many modes we find. For example, 2, 3, 3, 3, 4, 4, 4, 5, 6, 8 is bimodal because it has two modes, namely, 3 and 4. If all data points occur with the same frequency, we say the data is uniform.
- The units for the mean and mode are the same as for the dataset.

Worked example 12.1A Finding the mean from a frequency distribution table

Find the mean of the dataset given in this frequency distribution table.

Value	4	5	6	7	8
Frequency	3	2	4	8	6

Think

Add a row for the subtotals of the 4s, 5s, 6s, 7s and 8s. These values are found by multiplying the data value by its frequency. For example, in the first column we have $4 \times 3 = 12$.

Add an extra column to find the sums all entries in both the Frequency row and the Subtotals row. The sum of the Frequency row is the number of data values and the sum of the Subtotals row is the sum of the data values.

To find the mean we divide the sum of the data values (150) by the number of data values (23).

Write

Value	4	5	6	7	8	Total
Frequency	3	2	4	8	6	23
Subtotals	12	10	24	56	48	150

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of scores}}{\text{number of scores}} \\ &= \frac{150}{23} \\ &= 6.5 \text{ (to one decimal place)}\end{aligned}$$



Worked example video: Finding the mean from a frequency table

Understanding, fluency and communicating

Answers: Lesson 12.1

1 Consider these frequency distribution tables. For each table below:

- copy each table and add a row at the bottom with all subtotals ($f \times x$)
- add a column on the right with the number of data values and the sum of all data values, then use these to find the mean.

a

Value (x)	3	4	5	6	7
Frequency (f)	2	3	6	4	1

b

Value (x)	12	13	14	15	16
Frequency (f)	4	6	5	3	2

c

Value (x)	50	51	52	53	54	55
Frequency (f)	3	5	8	6	2	4

d

Value (x)	8	9	10	11	12
Frequency (f)	12	28	25	26	9

2 Use a spreadsheet or your calculator's statistics function to find the mean of the data given in each part of question 1. *Note:* the function you would use in a spreadsheet is AVERAGE.

Worked example 12.1B Finding the mean from a stem-and-leaf plot

Find the mean of the dataset displayed in this stem-and-leaf plot.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 8
5	1 5 5 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

Key: 1 | 2 = 12

Think

Using a spreadsheet or the statistics function on your calculator, enter the data values 22, 23, 23, 26, 41, etc., then determine their mean using the inbuilt AVERAGE function.

Write

$$\bar{x} = 51.3$$



Worked example video: Finding the mean from a stem-and-leaf plot

Helpful hints

- The mean can be found using the inbuilt function in a spreadsheet or on your scientific calculator.

3 Find the mean for the datasets given in the following stem-and-leaf plots.

a

Stem	Leaf
5	1 2
6	0 2 5
7	1 4 5 5
8	6 7
9	3

Key: 5 | 1 = 51

b

Stem	Leaf
2	4 6 6
3	0 1 1 5
4	5 8 8 9
5	1 6 8
6	7

Key: 2 | 4 = 24

c

Stem	Leaf
10	5 6 8 8 8
11	2 3 3 6
12	0 1

Key: 10 | 5 = 105

d

Stem	Leaf
1	2 3 3 4
1 *	5 6
2	0 1 1 3
2 *	6 6 6 8 9

Key: 1 | 2 = 12

Note: In part **d**, the stems have been split into 1 and 1 *, and 2 and 2 *. The stems of 1 and 2 can have leaves of 0, 1, 2, 3 and 4. The stems of 1 * and 2 * can have leaves of 5, 6, 7, 8 and 9.

Worked example 12.1C Finding the mean for grouped data

This grouped frequency distribution table gives the heights of a sample of plants in a nursery. Find the mean height.

Height (mm)	300–324	325–349	350–374	375–399	400–424	425–449
Frequency	3	18	47	32	14	6

Think

Determine the class centre for each class interval.

Class centre for 300–324 is:

$$\frac{300 + 324}{2} = 312.$$

Class centre for 325–349 is:

$$\frac{325 + 349}{2} = 337, \text{ etc.}$$

Make a new table with the class centres instead of the class intervals.

Add a row and calculate the subtotals $f \times x$ using the class centres for x .

Add a column and calculate the sum of the frequencies and the sum of the subtotals.

Calculate the mean by dividing the sum of the subtotals by the sum of the frequencies.

Write

Class centre	Frequency	Subtotals
312	3	936
337	18	6066
362	47	17 014
387	32	12 384
412	14	5768
437	6	2622
Total	120	44 790

$$\begin{aligned} \text{Mean} &= \frac{\text{sum of subtotals}}{\text{sum of frequencies}} \\ &= \frac{44\,790}{120} \\ &\approx 373.25 \text{ mm} \end{aligned}$$



Worked example video: Finding the mean for grouped data

Helpful hints

- When we use the class centre as a representative for the class, we treat the mean as an approximation for the real mean.
- You can also use a spreadsheet or your calculator to find these means, as for the ungrouped data.

4 Complete the following tables to find an approximation for the mean of the distribution.

a

Mass (kg)	Class centre (x)	Frequency (f)	$f \times x$
60–62	61	2	122
63–65	64	5	
66–68		7	
69–71		4	
72–74		2	
		Total:	Total:

b

Patients per day	Class centre (x)	Frequency (f)	$f \times x$
10–19	14.5	13	188.5
20–29	24.5	15	
30–39		16	
40–49		10	
50–59		4	
60–69		2	
		Total:	Total:

5 Consider these frequency distribution tables. For each table:

- create a copy in your workbook and add a column for the class centres
- use your calculator to find an approximation for the mean.

a

Height (mm)	Frequency (f)
155–159	4
160–164	7
165–169	10
170–174	5
175–179	2

b

Number of arguments	Frequency (f)
0–2	8
3–5	18
6–8	10
9–11	3
12–14	1

c

Score	Frequency (f)
41–50	3
51–60	0
61–70	10
71–80	8
81–90	7
91–100	2

Worked example 12.1D Finding the mode from a frequency distribution table

Find the mode of each dataset shown.

a

Value	Frequency
4	3
5	2
6	4
7	8
8	6

b

Value	Frequency
0–4	1
5–9	3
10–14	6
15–19	4
20–24	2

c

Value	Frequency
15	2
16	6
17	3
18	6
19	6
20	4

Part	Think	Write
a	From the table, the data value with the highest frequency is 7.	Mode = 7
b	For grouped data, the actual data value with the highest frequency cannot be found unless we also have the raw data. For grouped data, we can find the class with the highest frequency. It is called the modal class. From the table, the class with the highest frequency is 10–14.	Modal class = 10 – 14.
c	There are three rows in the table with a frequency of 6, which is the highest frequency.	This dataset is multimodal. Modes = 16, 18, 19



Worked example video: Finding the mode from a frequency table

Helpful hints

- The score that occurs “most often” means the score with the “highest frequency”.
- The modal class is the *class* that occurs most often.
- When more than one value has the same highest frequency, the dataset is multimodal. If there are two modes, we use the more specific description of bimodal. If all values occur equally as often, we say the dataset is uniform.

6 Find the mode, modes or modal class(es), for the following distributions. Describe the dataset as unimodal, bimodal, multimodal or uniform.

a

x	3	4	5	6	7
f	2	3	6	4	1

b

x	3	4	5	6	7
f	4	3	6	4	1

c

x	12	13	14	15	16
f	4	6	5	3	2

d

x	12	13	14	15	16
f	3	6	5	3	3

e

x	50	51	52	53	54	55
f	3	5	8	6	2	4

f

x	50–52	53–55	56–58	59–61	62–64	64–66
f	3	5	8	8	2	4

g

x	8	9	10	11	12
f	12	28	23	28	9

h

x	8	9	10	11	12
f	12	12	12	12	12

i

x	f
24	9
25	11
26	18
27	0
28	12

j

Mass (kg)	Frequency (f)
60–62	2
63–65	5
66–68	7
69–71	4
72–74	2

k	Score (x)	Frequency (f)
	41–50	3
	51–60	0
	61–70	9
	71–80	9
	81–90	7
	91–100	2

l	Patients per day (x)	Frequency (f)
	10–19	13
	20–29	15
	30–39	16
	40–49	10
	50–59	4
	60–69	2

Problem solving and reasoning

7 The marks scored by 30 students in a test are shown.

25 35 8 61 45 70 47 21 52 72
 56 36 20 44 54 51 32 60 16 61
 44 34 42 60 41 47 61 76 53 62

a Calculate the mean of this data.

b Organise the data into frequency distribution tables, using these classes:

i 0–4, 5–9, 10–14, 15–19, ...

ii 0–9, 10–19, 20–29, 30–39, ...

iii 0–19, 20–39, 40–59, ...

iv 0–39, 40–79, ...

c Calculate an approximation for the mean of this data using each of the grouped frequency distribution tables from part **b**.

d Compare the approximations for the mean in part **c** with the actual mean found in part **a**. What is the effect of the size of the class interval on the approximations for the mean?

8 For each of the stem-and-leaf plots in question 3, determine the mode(s) of the dataset.

9 There are two Year 11 maths classes at a school, named 11M1 and 11M2.

a The mean mark in a test for the 24 students in Class 11M1 was 70 out of 100. What was the total of the marks scored by this class?

b The mean mark for the same test done by the 17 students in Class 11M2 was 56. What was the total of the marks scored by Class 11M2?

c What was the combined mean mark of both classes?

10 Consider the frequency of one-digit numbers in a list of random numbers.

a Find the mean of these x -values: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

This is called the “theoretical mean” of the one-digit numbers, because, theoretically, each occurs with the same frequency.

b Use a random number generator or a spreadsheet to make your own set of one-digit values with each of the given numbers of values in the set. For each set of values, find the mean and mode.

i 5

ii 10

iii 20

iv 50

c Compare the means you found in part **b** with the theoretical mean found in part **a**. Is it true that, as the sample gets larger, the sample mean gets closer to the theoretical mean? Discuss this with your class.

d As a class, write the mean you found for the set of 10 random values from part **b ii**.

i Find the mean of all the sample means on the whiteboard.

ii Compare your result from part **i** with the theoretical mean from part **a**. Is it true that the mean of a large number of sample means approaches the value of the theoretical mean? Discuss this with your class.

e Comment on the modes for each of your sets of values from part **b**.

Challenge

- 11 A rugby league team scores the following number of points in each of its first 5 games: 18, 14, 26, 24, 18.

After 3 more games, the mean has increased by 2, and the mode has increased by 8.

What were the 3 scores the team recorded in those games?

- 12 The distance from Broken Hill to Tibooburra is 360 km. A motorist travels from Broken Hill to Tibooburra at an average (mean) speed of 90 km/h and returns at an average speed of 60 km/h. What is the average speed of the motorist for the whole journey?



Related resources



12.1 Quick quiz

Lesson 12.2

Median

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate the median of a dataset.

Median

- The **median** is a value that divides a dataset into two parts, with equal numbers of values below it and above it.
 - It is the middle data value after the dataset has been arranged in ascending order; that is, from the smallest to the largest. For an odd number of data values, there is one middle value. For an even number of data values, the median is the number midway between the two middle data values.
- In grouped data, the class containing the median is sometimes called the median class.
- To find the location of the median in a set of n data values arranged in numerical or ascending order:
 - 1 cross off equal numbers of values from each end until there is one or two middle values, or
 - 2 calculate the value of $\frac{n+1}{2}$ to determine where the middle value is placed within the set.

Worked example 12.2A Finding the median of an odd number of data values

Find the median of the following dataset.

7 9 3 9 8

Think	Write
First arrange the five values in ascending order. For an odd number of data values, there is one middle value. By observation, the middle value is 8 because there are two data values below it and two scores above it. We also could have found the position of the median by calculating $\frac{5+1}{2} = 3$. The number in the third position is indeed 8. Alternatively, we can repeatedly cross off one value from each end of the ordered list until we reach the middle: ~~3~~ 7 8 9 ~~9~~ ~~3~~ 7 8 ~~9~~ ~~9~~	3, 7, 8, 9, 9 Median = 8



Worked example video: Finding the median of an odd number of data values

Helpful hints

- Sort the dataset into ascending order and locate the middle data value either by crossing off values from both ends or by finding which position is the middle position.
- When there are an odd number of data values, say n , the middle position is found by calculating $\frac{n+1}{2}$.

Understanding, fluency and communicating

Answers: Lesson 12.2

1 Find the median of the following sets of data by:

- rearranging the datasets into numerical order, where necessary
- crossing off equal numbers of data values from each end.

a 2, 4, 5, 6, 9, 9, 10

b 2, 3, 3, 4, 5, 6, 7, 8, 9

c 11, 13, 13, 16, 17

d 27, 28, 29, 27, 30, 31, 27, 31, 30

e 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1

2 For the following sets of data:

- rearrange the datasets, into ascending order, where necessary.
- calculate the value of $\frac{n+1}{2}$, where n is the number of data values.
- find the median.

a 6, 6, 7, 7, 8, 9, 10, 10, 11

b 20, 21, 21, 22, 22, 23, 23

c 48, 49, 50, 51, 53, 49, 50, 50, 51, 55, 53

Worked example 12.2B Finding the median of an even number of data values

Find the median of the dataset: 3, 4, 8, 9, 8, 9, 3, 5.

Think	Write
Arrange the dataset in ascending order:	3 3 4 5 8 8 9 9
Cross off equal numbers of data values from each end:	~~3~~ ~~3~~ ~~4~~ 5 8 ~~8~~ ~~9~~ ~~9~~
In this case there are two middle data values, 5 and 8. The median is the number midway between 5 and 8, so it is the mean of 5 and 8.	Median = $\frac{5+8}{2}$ = 6.5



Worked example video: Finding the median of an even number of data values

Helpful hints

- Sort the dataset into ascending order and, because there is an even number of data values, locate the *two* middle data values. The median is the number midway between the two middle scores, which is the mean of these values.
- The median is not always one of the data values.

3 Find the median of the following sets of data by:

- rearranging the datasets into numerical order, where necessary
- crossing off equal numbers of data values from each end.

a 20, 20, 20, 23, 25, 27

b 51, 52, 54, 55, 57, 57, 58, 59

c 1, 1, 2, 4, 4, 4, 4, 7, 7, 8, 9, 10

d 240, 243, 245, 246, 244, 243

e 104, 101, 104, 102, 104, 105, 106, 101

4 For the following sets of data:

- rearrange the datasets, into ascending order, where necessary.
- calculate the value of $\frac{n+1}{2}$, where n is the number of data values.
- find the median.

a 15, 15, 16, 16, 17, 17, 18, 19

b 48, 49, 49, 49, 50, 50, 50, 51, 52, 52

c 2, 1, 0, 3, 4, 5, 4, 4, 3, 1, 0, 2

d 34, 35, 38, 40, 37, 36, 33, 38, 34, 39, 36, 37, 38, 37

Worked example 12.2C Finding the median from a frequency distribution table

Find the median of the dataset given below.

x	20	21	22	23	24
f	5	7	11	9	4

Think

Add a cumulative frequency column to the table.

x	f	Cumulative frequency	Positions of ordered values
20	5	5	1st, 2nd, 3rd, 4th, 5th
21	7	12	6th, 7th, 8th, ..., 11th, 12th
22	11	23	13th, 14th, 15th, ..., 22nd, 23rd
23	9	32	24th, 25th, 26th, ..., 31st, 32nd
24	4	36	33rd, 34th, 35th, 36th

From the table, the sum of all frequencies is $n = 36$, so, $\frac{n+1}{2} = 18.5$.
This indicates that the median is the number midway between the 18th and 19th values.

From the cumulative frequency column, we can see that the 18th and 19th values are both 22. Calculating their mean gives the median.

Write

x	f	Cumulative frequency
20	5	5
21	7	12
22	11	23
23	9	32
24	4	36

$$\text{Median} = \frac{22 + 22}{2} = 22$$



Worked example video: Finding the median from a frequency distribution table

Helpful hints

- Add a cumulative frequency column to the table and use it to find the middle data value(s).

5 Complete the following tables to find the median of each dataset.

a

x	f	Cumulative frequency	Positions of ordered values
3	2	2	1st, 2nd
4	3	5	3rd, 4th, 5th
5	7	12	6th, 7th, ..., 12th
6	4	—	13th, —, —, —
7	1	—	—

b

x	f	Cumulative frequency	Positions of ordered values
12	4	4	1st, 2nd, 3rd, 4th
13	7	11	5th, 6th, ..., 11th
14	5	16	12th, ..., —
15	3	—	—
16	2	—	—

6 Find the median of each dataset.

a

x	50	51	52	53	54	55
f	3	5	8	6	2	4

b

x	33	34	35	36	37
f	2	4	9	10	5

c

x	8	9	10	11	12
f	12	28	25	26	9

d

x	24	25	26	27	28
f	9	11	18	0	12

Worked example 12.2D Finding the median for grouped data

Find the median of the data given in this frequency distribution table.

Height (mm)	300–<325	325–<350	350–<375	375–<400	400–<425	425–<450
Frequency	3	18	47	32	14	6

Think

Add a cumulative frequency column to the table and use the data to draw a cumulative frequency polygon.

So the median is the mean of the 60th and 61st data values. The 60th and 61st scores are in the 350–<375 class so an exact value for the median cannot be found.

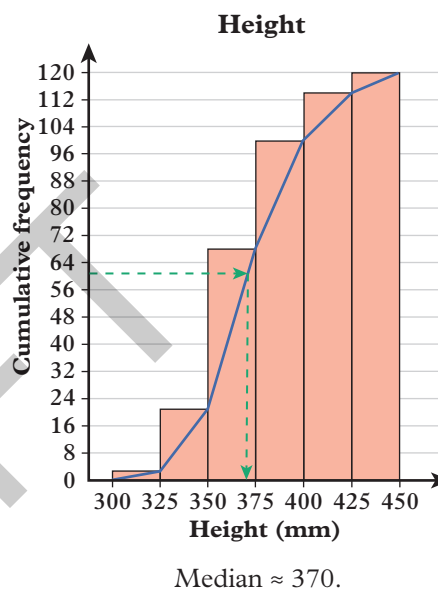
The class containing the median is sometimes called the median class. We could estimate the median to be at the class centre (362.5), but a more accurate estimate can be found by using the class centres and cumulative frequency to help you draw the cumulative frequency histogram and polygon.

To find the median, locate the point halfway up the cumulative frequency axis, $\frac{1}{2} \times 120 = 60$, and draw a horizontal line across to the polygon. Draw a vertical line to the data value axis (Height) and use it to read the median, on the horizontal scale.

Write

Height (mm)	Frequency	Cumulative frequency
300–<325	3	3
325–<350	18	21
350–<375	47	68
375–<400	32	100
400–<425	14	114
425–<450	6	120

$$\frac{n+1}{2} = \frac{120+1}{2} = 60.5$$



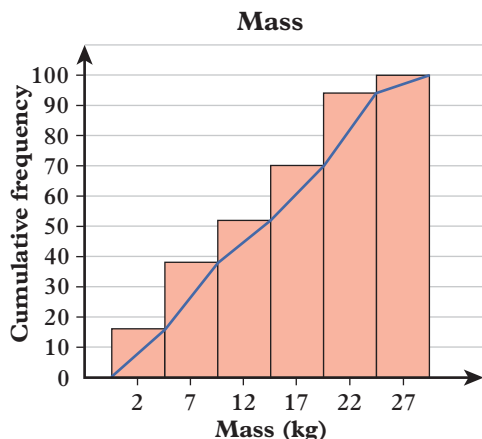
Worked example video: Finding the median for grouped data

Helpful hints

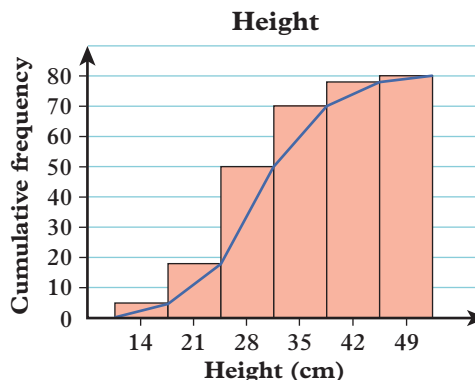
- Draw a cumulative frequency polygon. Estimate the median by locating the point halfway (vertically) up the polygon.

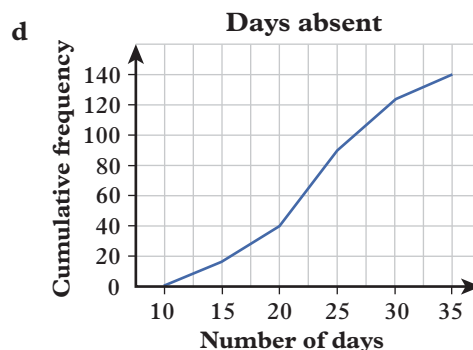
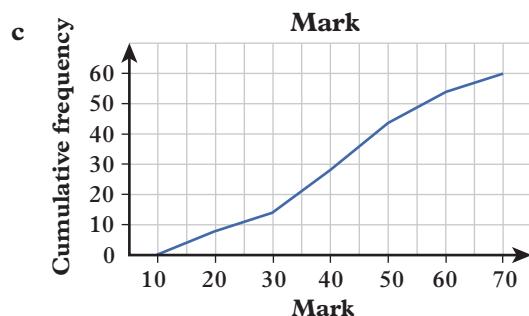
7 Estimate the median from the cumulative frequency polygons drawn on each graph.

a



b





8 Consider each of the grouped frequency distributions shown.

- i Copy each table and add columns for class centre and cumulative frequency.
- ii Draw the cumulative frequency polygon for each distribution.
- iii Use your cumulative frequency polygons from part **b** to help you estimate the median data value for each set of data.

a

Score	Frequency
41–50	5
51–60	0
61–70	12
71–80	10
81–90	9
91–100	4

b

Mass (kg)	Frequency
59.5–<62.5	4
62.5–<65.5	7
65.5–<68.5	9
68.5–<71.5	6
71.5–<74.5	4

c

Height (cm)	Frequency
154.5–<159.5	19
159.5–<164.5	21
164.5–<169.5	24
169.5–<174.5	19
174.5–<179.5	17

d

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

Worked example 12.2E Finding the median from a stem-and-leaf plot

Find the median for the data in this stem-and-leaf plot.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

Key: 2 | 3 = 23

Think

Cross off numbers in pairs, starting with the lowest and highest numbers, 22 and 70. The two middle data values remaining are 55 and 55.

The median is the mean of the two middle values.

Or, because there are 30 scores, the median is the average of the 15th and 16th data values. The values are already in order, and the 15th and 16th scores are both 55.

Write

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 3 3 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 3 3 3 6 7
7	0 0
Median = $\frac{55 + 55}{2}$	
= 55	



Worked example video: Finding the median from a stem-and-leaf plot

Helpful hints

- The median can be found by crossing off pairs of numbers from the top and bottom of the stem-and-leaf plot or by identifying the middle data value(s).

9 Find the median of the data in each of these stem-and-leaf plots.

a

Stem	Leaf
5	1 2
6	0 2 5
7	1 4 5 5
8	6 7
9	3

Key: 5 | 1 = 51

b

Stem	Leaf
2	4 6 6
3	0 1 1 5
4	5 8 8 9
5	1 6 8
6	7

Key: 2 | 4 = 24

c

Stem	Leaf
10	5 6 8 8 8
11	2 3 3 6
12	0 1
13	8
14	3 5 5 6
15	1 1 3 7

Key: 10 | 5 = 105

d

Stem	Leaf
1	2 3 3 4
1 *	5 6
2	0 1 1 3
2 *	6 6 6 8 9

Key: 1 | 2 = 12

Problem solving and reasoning

10 Use the data from the stem-and-leaf plot to answer the following questions.

Stem	Leaf
3	0 2 2 3 8
4	1 3 6
5	2 4 4 4 8 8 9
6	1 5 5 6 7

Key: 3 | 2 = 32

- Find the mean, mode and median of the data.
- Complete a frequency distribution table using the classes 30–39, 40–49, etc.

- c** Calculate the mean from the grouped data. Compare this with the mean found in part **a**.
 - d** Add a cumulative frequency column to your table from part **b**.
 - e** Draw the cumulative frequency polygon and estimate the median. Compare this with the exact median value found in part **a**.
 - f** Why are some later answers different from those in part **a**?
- 11** Repeat question **10** using classes of 30–34, 35–39, etc. Explain any differences from the question **10** answers in your results this time.
- 12** The values of the mean, median and mode(s) of a dataset can be compared.
- a** Show that the median of the data values 2, 5, 5, 5, 8, 11, 14, 17 and 44 lies between the mode and the mean.
 - b** Show that the mean of the data values 1, 3, 3, 3, 5, 5, 6, 6, 7 and 7 lies between the mode and the median.
 - c** Find a dataset for which the mode lies between the median and the mean.
 - d** Find a dataset for which the mean lies between the median and the mode.
 - e** Find a dataset that has the same mean, mode and median.

Challenge

- 13** Using the following cumulative frequency table, find the smallest and largest values that can go in the last box so that the median is 18.

x	Cumulative frequency
15	7
16	9
17	15
18	19
19	

Related resources



12.2 Quick quiz

Lesson 12.3

Relative merits of mean, mode and median

Learning intentions

By the end of this lesson, you will be able to ...

→ choose the most appropriate measure of centre for a dataset.

Mean, mode and median

- The mean, the mode and the median are all measures of central tendency; that is, single numbers that are central to or typical of all the numbers under consideration. But which of these measures is the most appropriate in a given situation? There is no general rule; rather it depends on the nature of the data and the relative merits of the measures. In this lesson, we will investigate some of the properties of the mean, mode and median.

Properties of the mean

- The mean is the most commonly used, is easy to understand and is easy to calculate.
- Its value depends on every point in the dataset.
- Its main disadvantage is that its value is easily distorted by very large or very small, or otherwise atypical scores.
- It has the best sampling stability: if we choose random samples of the same size from a large population, the mean is the measure that varies the least from sample to sample.
- It is not suitable for categorical data.

Properties of the mode

- The mode is the most frequent value in a dataset.
- Its value is easy to determine, and it is not affected by outliers.
- There may be no mode, or more than one mode, in a dataset.
- It may not be a central value.
- When comparing random samples of the same size taken from a large population, the mode is the measure that varies the most.
- It is the only measure suitable for categorical data.

Properties of the median

- The median is simple to describe: there are equal numbers of points above and below it.
- It is not affected by outliers, since its value depends on the number of scores above and below it, not on the values of these points.
- For some unusual patterns of data points, it may not be a central value; e.g. 0, 1, 1, 5, 5, 5, 5
- When comparing samples, it varies more than the mean but much less than the mode.
- It is not suitable for categorical data.

Worked example 12.3A Finding the mode and comparing it to the mean and median

a Find the mode of each of these datasets.

i 14, 15, 15, 16, 16, 17, 18

ii 14, 15, 16, 17, 19, 21

iii 14, 15, 16, 17, 26, 26, 26

b Consider the relative merits of the mode, the mean and the median for each of the cases above.

Part	Think	Write
a i	The mode is the number that occurs with the highest frequency.	Mode = 16, because the number 16 occurs the most times.
a ii	The mode is the number that occurs with the highest frequency.	There is no mode because all the numbers have the same frequency.
a iii	The mode is the number that occurs with the highest frequency.	Mode = 26, because the number 26 occurs the most times.
b i	Calculate the mean and median of the dataset and compare these values to the mode. Determine which values are good measures of central tendency.	The mode (16) is central and typical of the data values. The mean (15.9) and the median (16) are also good measures of central tendency.
b ii	Calculate the mean and median of the dataset and compare these values to the mode. Determine which values are good measures of central tendency.	There is no mode, so the mode is not a satisfactory measure of central tendency. The mean (17) and the median (16.5) are both good measures of central tendency.
b iii	Calculate the mean and median of the dataset and compare these values to the mode. Determine which values are good measures of central tendency.	The mode (26) occurs at one end of the dataset; it is not a central value. The mean (20) and the median (17) are both good measures of central tendency.



Worked example video: Finding the mode and comparing it to the mean and median

Understanding, fluency and communicating

Answers: Lesson 12.3

Helpful hints

- There are cases in which the mode is important even if it is not a central value. For example, if the numbers are the sizes of hats sold by a shop, then the size that is sold most often would be important to the shop owner and hat manufacturer.

1 For each of the following datasets:

i find the mode

ii determine whether the mode is a satisfactory measure of central tendency

iii find the mean and median and determine their relative merits as measures of central tendency.

a 5, 6, 6, 7, 7, 7, 8, 9

b 2, 3, 4, 5, 7, 9

c 23, 24, 25, 26, 35, 35, 35

Worked example 12.3B Finding the mean and comparing it to the mode and median

a Find the mean of each of these datasets.

i 5, 5, 6, 7, 8

ii 5, 5, 6, 7, 80



Helpful hints

- 2 For each of the following datasets:

- ### Worked example 12.3C Finding the median and comparing it to the mean and mode

- a** Find the median of this dataset about the number of goals scored by a footballer over 10 games:

1 2 2 3 6 6 6 6 6 6

- b** Consider the relative merits of the median, mode and mean for this dataset.

Part	Think	Write
a	The median is the middle score, after the scores have been arranged in ascending order.	Median = average of 5th and 6th scores $= \frac{6 + 6}{2}$ $= 6$

Part	Think	Write
b	Calculate the mean and median for the dataset. The most appropriate measure is the one that is a central value.	For this pattern of scores the median happens to be the end score (which is also the mode). In this case, the mean (4.4) would be a better measure of central tendency.



Worked example video: Finding the median and comparing it to the mean and mode

- 3 Consider the following dataset: 2, 3, 3, 3, 5, 5, 5, 5, 5.
 - a Find the median of the dataset.
 - b Is the median a satisfactory measure of central tendency for this dataset?
 - c Consider the relative merits of the mode and the mean for this dataset.
- 4 Which measure of central tendency depends on every value in a dataset?
- 5 Which is the only measure suitable for categorical data?
- 6 A shoe store had a discount sale of women's running shoes and sold shoes of the following sizes in one day.
6 10 4 7 8 7 6 5 7 8 7 5 6 4 3 7
 - a For the sizes of shoes sold, find the:
 - i mean
 - ii mode
 - iii median.
 - b Which of these measures would be of most use to the shop owner?

Problem solving and reasoning

- 7 The Beacon Lamp Company has a total of 30 employees whose annual salaries are listed.

1 general manager	\$260 000
1 marketing manager	\$100 000
1 accountant	\$120 000
1 engineer	\$100 000
1 warehouse manager	\$100 000
15 production workers	\$50 000 each
10 tradespeople	\$60 000 each

- a What is the total annual wage bill for this company?
- b Calculate the mean wage for the employees.
- c How many employees earn:
 - i less than the mean wage from part **b**?
 - ii more than the mean wage from part **b**?
- d What is the median wage?
- e What is the modal wage?
- f In a wage determination case for the employees of this company, which measure of central tendency would you use to support your argument if you were the representative for:
 - i the general manager?
 - ii the production workers?
- g Which measure of central tendency is the most appropriate to represent the wages of the employees of this company? Give reasons for your answer.

- 8 The times to the nearest minute that 10 patients, chosen at random, spent in the waiting room of a hospital were:

18 19 19 20 20 27 28 29 29 29

- a** Find the mean, mode and median of these waiting times.
b Consider the relative merits of these three measures for this dataset.

- 9 Geoff scored these marks in a series of topic tests:

6 3 6 3 6 2 6 1

- a** Find the mean, mode and median for Geoff's marks.
b Which of these “averages” would Geoff prefer to tell his parents?

- 10 The prices of land sold by a real-estate agent in a particular suburb in one month were:

\$510 000 \$630 000 \$960 000 \$540 000 \$560 000

- a** The real-estate agent claimed that, for this month, the “average” price of houses in the suburb was \$640 000. To which “average” is the estate agent referring?
b Is the “average” from part **a** an appropriate “average” to use? Give reasons for your answer.

- 11 The eye colours of a group of 10 students are recorded below.

brown blue brown green blue
 grey brown brown blue green

Which, if any, of the mean, mode and median can be used with this categorical (qualitative) data?

- 12 The number of mistakes made by 100 students in a spelling test is shown (no-one made more than 9 mistakes).

4 8 0 4 7 4 5 3 8 1 3 3 2 3 2 3 5 1 7 8
 4 6 9 7 1 8 5 8 7 9 3 1 4 5 8 2 2 0 1 6
 0 8 6 9 4 5 8 1 1 2 9 6 0 7 0 9 1 9 1 0
 1 8 8 6 8 5 2 2 5 1 9 9 8 2 7 3 2 6 8 1
 9 0 0 7 0 9 8 8 7 3 8 9 8 4 6 5 0 9 5 3

- a** Suppose each row represents a different English class.
i Complete the frequency distribution table for the 20 students in the first English class.

Number of mistakes	Tally	Frequency
0		
1		
2		
:		
9		

- ii** Use your calculator to find the mean, mode and median of the number of mistakes made by the 20 students in the first class.

- b** Repeat part **a** for the remaining four English classes.

- c** Complete a table similar to the one shown to summarise the results for each English class.

	Class A	Class B	Class C	Class D	Class E
Mean					
Mode					
Median					

- d** Which of these measures (mean, mode or median) varies the most between samples?
e Which of the measures varies the least?

- 13 The weights, in kilograms, of the players in a women's hockey team are:

55, 56, 63, 48, 58, 56, 48, 54, 55, 62, 64, 157

Barbie	Hidden Figures
8.5	9.5
7.8	8.8
9.0	9.2
6.5	8.0
8.2	9.0
7.0	8.5
8.8	9.3
7.5	8.7
9.2	9.1
6.8	8.3
8.0	9.4
7.3	8.6
8.7	9.0
7.9	8.9
9.1	9.2
6.9	8.4
8.4	9.3
7.6	8.7
8.9	9.1
7.2	8.5

- a** Create a back-to-back stem-and-leaf plot to organise this data.
- b** Use your calculator to find the mean, mode and median for each dataset.
- c** For each measure of centre calculated in part **b**, explain one reason why it is or isn't a useful measure in this context. If possible, give reasons for both arguments.

16 An outlier is a data value that is significantly different from the rest of the dataset.

- a** Create a dataset with an outlier by taking a dataset from an earlier question and adding a data value that is much lower than the rest.
- b** Calculate the mean of the dataset before and after adding the outlier.
- c** Explain why the median would be the best choice as a measure of centre.

12.3 Quick quiz

Lesson 12.4

Range and interquartile range

Learning intentions

By the end of this lesson, you will be able to ...

→ find the five-number summary for a dataset.

Range and interquartile range

- Consider these two sets of data: 3, 4, 5, 6, 7 and 0, 1, 5, 9, 10.
 - For both sets, the mean is 5. However, it is obvious that these datasets are quite different in nature. The first dataset is closely grouped around the mean, but the second set is quite scattered about the mean. So, as well as having a central value for a dataset, it is useful to have some measure of how spread out the data values are. Such measures are called **measures of spread**.
- The first two measures of spread we will investigate are the **range** and the **interquartile range**.
- The range is the difference between the highest data value and the lowest data value in a dataset.

$$\text{Range} = \text{highest data value} - \text{lowest data value}$$
- A major disadvantage in using the range as a measure of spread is that it depends only on the extreme values of the data. These are the data values that are most likely to be atypical, and hence outliers.
 - In practical contexts, such as measuring temperature, height, length or blood pressure, an outlier is often caused by an error of measurement and, in these cases, the range is not a good measure to rely on.
- To avoid this problem, we can find the difference between two numbers that are not at the extremes of the data. The numbers we will use are called **quartiles**. In the same way that the median divides a dataset in half, the quartiles divide a dataset into four equal sized parts. Each set of data has three quartiles: the lower quartile, the middle quartile (this is equal to the median) and the upper quartile, usually denoted by Q_1 , Q_2 and Q_3 , respectively.
- Using these values, we can define a new measure of spread called the **interquartile range**. The interquartile range (IQR) is the difference between the upper and lower quartiles:

$$\text{IQR} = Q_3 - Q_1$$
- The interquartile range gives an indication of the spread of the data values from the median. The greater its value, the greater the spread. A smaller value indicates that the data values are more closely clustered around the median.
- Because of the way quartiles are calculated, we can say that:
 - 25% of the data values are less than Q_1 and 75% are greater than Q_1
 - 50% of the data values are less than Q_2 and 50% are greater than Q_2
 - 75% of the data values are less than Q_3 and 25% are greater than Q_3
 - the interquartile range contains the middle 50% of the data values.
- The advantage of using the interquartile range instead of the range is that it is not affected by outliers.

Worked example 12.4A Finding the range of a dataset

Find the range of each of these datasets.

a 3, 4, 4, 5, 5, 5, 7

b 1, 9, 0, 5, 10

Part	Think	Write
a	By inspection, the highest data value is 7 and the lowest data value is 3. The range is the difference between these values.	$\text{Range} = 7 - 3$ $= 4$
b	By inspection, the highest data value is 10 and the lowest data is 0. The range is the difference between these values.	$\text{Range} = 10 - 0$ $= 10$



Worked example video: Finding the range of a dataset

Understanding, fluency and communicating

Answers: Lesson 12.4

Helpful hints

- Range = highest data value – lowest data value

1 Find the range of each of these datasets.

a 2, 4, 5, 6, 9, 9, 10

b 2, 3, 3, 4, 5, 6, 7, 8, 9

c 11, 13, 13, 16, 170

d 27, 28, 29, 27, 30, 31, 27, 31, 30

e 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1

f 20, 20, 20, 23, 25, 27

g 51, 52, 54, 55, 57, 57, 58, 59

h 104, 101, 104, 102, 104, 105, 106, 101

2 Find the range of the each of these datasets.

a

x	f
3	2
4	3
5	6
6	4
7	1

b

x	f
50	3
51	5
52	8
53	6
54	2
55	4

c

x	f
12	4
13	3
14	5
15	6
16	2

Worked example 12.4B Finding the interquartile range when quartiles are in the set

Find the lower, middle and upper quartiles and the interquartile range for this dataset:

2 2 3 5 6 6 7

Think

Write the dataset in ascending order, if necessary.

Find the median (2nd quartile): $\frac{n+1}{2}$

The median is the 4th data value: $Q_2 = 5$

Note: the median divides the data into two parts with three data values in each.

The lower part contains the data values 2, 2, 3. The lower quartile is the data value that divides this set into two equal parts; it is the middle data value of this set. So, $Q_1 = 2$.

The upper part contains the data values 6, 6, 7. The upper quartile is the data value that divides this set into two equal parts; it is the middle data value of this set. So, $Q_3 = 6$.

We can see that the quartiles have divided the data into four parts with one data value in each.

Use the values of Q_3 and Q_1 to calculate the interquartile range.

Write

$$\begin{array}{ccccccc} 2 & 2 & 3 & 5 & 6 & 6 & 7 \\ & \uparrow & & \uparrow & & \uparrow & \\ & Q_1 & & Q_2 & & Q_3 & \end{array}$$

$$\begin{aligned}\text{Interquartile range} &= Q_3 - Q_1 \\ &= 6 - 2 \\ &= 4\end{aligned}$$



Worked example video: Finding the interquartile range when quartiles are in the set.

Helpful hints

- Find the median (Q_2).
- This divides the data into two groups with equal numbers in each.
- The middle data value of the lower group is the lower quartile (Q_1). The middle data value of the upper group is the upper quartile (Q_3).
- The interquartile range is $IQR = Q_3 - Q_1$.

3 Identify Q_1 in the following, then find the interquartile range of this dataset.

7 8 10 12 13 13 14

Q_2 Q_3

$$\begin{aligned} Q_1 &= \_\_\_\_\_\_ \\ \text{IQR} &= \_\_\_\_\_\_ - \_\_\_\_\_\_ \\ &= \_\_\_\_\_\_ \end{aligned}$$

4 Find the interquartile range for each of these datasets.

- a** 15, 16, 17, 20, 22, 23, 25

- b** 10, 11, 11, 13, 13, 14, 14, 15, 18, 20, 20

Worked example 12.4C Finding the interquartile range when quartiles are not in the set

Find the lower, middle and upper quartiles and the interquartile range for these datasets:

- a** 1 2 2 3 4 5 5 6 6 **b** 23 27 25 23 24 25 **c** 1 2 2 3 4 5 6 6

Part	Think	Write
a	The data values are already in ascending order. In this case $n = 9$, so $\frac{n+1}{2} = 5$. So, the median is the 5th data value, which is 4. Q_1 is the middle data value of the set 1, 2, 2, 3. Q_3 is the middle data value of the set 5, 5, 6, 6. Use the values of Q_3 and Q_1 to calculate the interquartile range.	$ \begin{array}{ccccccccc} 1 & 2 & 2 & 3 & 4 & 5 & 5 & 6 & 6 \\ & & \uparrow & & \uparrow & & \uparrow & & \\ & & Q_1 & & Q_2 & & Q_3 & & \end{array} $ $Q_2 = 4$ $Q_1 = \frac{2+2}{2} = 2, Q_3 = \frac{5+6}{2} = 5.5$ $ \begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 5.5 - 2 \\ &= 3.5 \end{aligned} $
b	Arrange the data values in ascending order. In this case $n = 6$, so $\frac{n+1}{2} = 3.5$. The median is the number midway between the 3rd and 4th data values. Q_1 is the middle data value of the set 23, 23, 24. Q_3 is the middle data value of the set 25, 25, 27. Use the values of Q_3 and Q_1 to calculate the interquartile range.	$ \begin{array}{ccccccc} 23 & 23 & 24 & 25 & 25 & 27 \\ & \uparrow & & \uparrow & \uparrow & \\ & Q_1 & & Q_2 & Q_3 & \end{array} $ $Q_2 = \frac{24+25}{2} = 24.5$ $Q_1 = 23, Q_3 = 25$ $\text{IQR} = 25 - 23 = 2$
c	The data values are already in ascending order. In this case $n = 8$, so $\frac{n+1}{2} = 4.5$. The median is the mean of the 4th and 5th data values. Q_1 is the middle data value of the set 1, 2, 2, 3. Q_3 is the middle data value of the set 4, 5, 6, 6. Use the values of Q_3 and Q_1 to calculate the interquartile range.	$ \begin{array}{ccccccccc} 1 & 2 & 2 & 3 & 4 & 5 & 6 & 6 \\ & & \uparrow & & \uparrow & & \uparrow & \\ & & Q_1 & & Q_2 & & Q_3 & \end{array} $ $Q_2 = \frac{3+4}{2} = 3.5$ $Q_1 = \frac{2+2}{2} = 2$ $Q_3 = \frac{5+6}{2} = 5.5$ $\text{IQR} = 5.5 - 2 = 3.5$



Worked example video: Finding the interquartile range when quartiles are not in the set

Helpful hints

- First find the median (Q_2). This divides the data into two groups with equal numbers in each. The middle data value of the lower group is the lower quartile (Q_1).
- The middle data value of the upper group is the upper quartile (Q_3).
- $\text{IQR} = Q_3 - Q_1$

5 Complete the following to find the interquartile range of the following dataset.

42 44 45 48 53 61 64 68 71

$\uparrow$ $\uparrow$
 Q_1 Q_2

$Q_1 = \underline{\hspace{2cm}}$
 $Q_3 = \underline{\hspace{2cm}}$
 $\text{IQR} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$
 $\quad = \underline{\hspace{2cm}}$

6 Find the interquartile range of each of these datasets.

a 170, 170, 170, 185, 188, 189, 194, 196, 203

b 2, 3, 5, 8, 8, 8, 9, 9, 9, 11, 15, 17, 18

- 7 Complete the following to find the interquartile range of this dataset.

17 19 20 22 24 26
 $\uparrow$
 Q_2

$$Q_2 = \underline{\hspace{1cm}}$$

$$Q_1 = \underline{\hspace{1cm}}$$

$$Q_3 = \underline{\hspace{1cm}}$$

$$\text{IQR} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} \\ = \underline{\hspace{1cm}}$$

- 8 Find the interquartile range for each of the following datasets.

a 50, 50, 52, 55, 55, 57, 57, 58, 60, 60

b 33, 35, 38, 42, 43, 44, 52, 53, 55, 58, 61, 64, 66, 68

- 9 Complete the following to find the interquartile range of these datasets.

4 4 5 6 7 8 10 12
 $\uparrow$ $\uparrow$
 Q_1 Q_2

$$Q_1 = \underline{\hspace{1cm}}$$

$$Q_3 = \underline{\hspace{1cm}}$$

$$\text{IQR} = \underline{\hspace{1cm}}$$

- 10 Find the interquartile range for each of the following datasets.

a 17, 20, 20, 21, 24, 26, 27, 28

b 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 4, 5

- 11 Find the interquartile range for each of the following datasets.

a 34, 31, 33, 31, 32, 35, 30

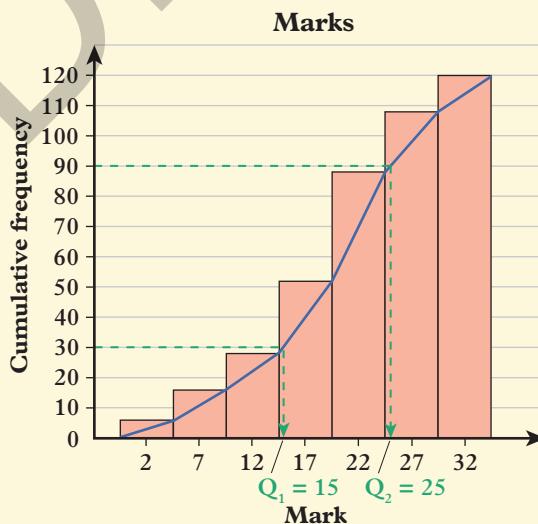
b 16, 16, 14, 15, 14, 13, 16, 14, 10

c 5, 6, 6, 3, 7, 4, 5, 6, 7, 4

d 80, 82, 81, 81, 80, 83, 82, 83

Worked example 12.4D Finding the interquartile range from a cumulative frequency polygon

Estimate the lower quartile and upper quartile from the data graphed on the cumulative frequency polygon shown. Find the interquartile range of the scores.



Think

For large numbers of data values, the lower and upper quartiles can be estimated from the cumulative frequency polygon.

The lower quartile can be estimated by locating the point one-quarter of the way up the cumulative frequency axis and drawing a horizontal line across to the polygon. Next draw a vertical line from this point to the horizontal axis and read the value of the lower quartile.

The upper quartile can be estimated by locating the point three-quarters of the way up the cumulative frequency axis and drawing a horizontal line across to the polygon. Next draw a vertical line from this point to the horizontal axis and read the value of upper quartile.

Use the values of Q_3 and Q_1 to calculate the interquartile range.

Write

From the cumulative frequency polygon:

$$Q_1 \approx 15$$

$$Q_3 \approx 25$$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &\approx 25 - 15 \\ &= 10 \end{aligned}$$

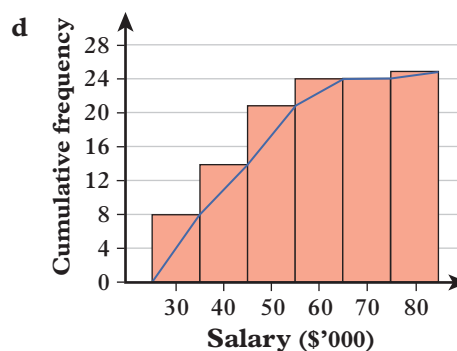
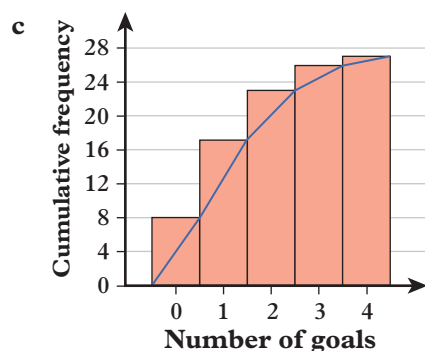
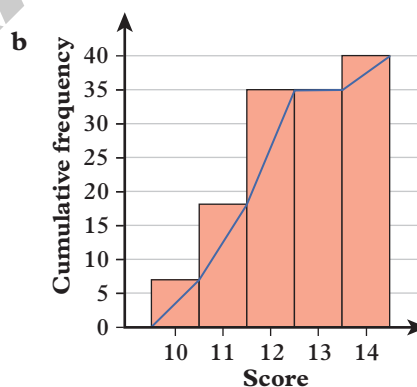
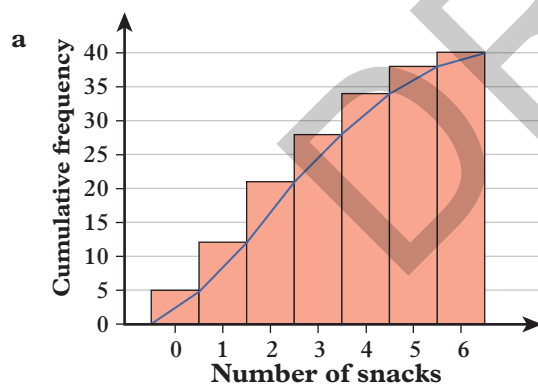


Worked example video: Finding the interquartile range from a cumulative frequency polygon

Helpful hints

- Estimate the lower quartile by finding the point on the polygon that is one-quarter of the way up the vertical scale. Estimate the upper quartile by locating the point on the polygon that is three-quarters of the way up the vertical scale.
- Then $\text{IQR} = Q_3 - Q_1$.

12 By first estimating the values of the lower quartile and upper quartile, find the interquartile range from each of the cumulative frequency polygons shown.



13 Estimate the interquartile range for each of the following datasets, to the nearest whole number.

a

x	f
33	2
34	4
35	9
36	10
37	5

b

x	f
8	12
9	28
10	25
11	26
12	9

c

x	f
24	9
25	11
26	18
27	0
28	12

14 Estimate the interquartile range (to the nearest whole number) for each dataset in question 8 of Lesson 12.2.

Hint: use the cumulative frequency polygons.

Worked example 12.4E Interpreting the meaning of an interquartile range

In the term test, the median result for Kelly's class was 65 and the interquartile range was 20. For Sian's class, the median was 65 and the interquartile range was 12. What conclusions could be made about these results?

Think

The median mark for both classes is 65: half of the students scored less than 65 and half scored more than 65. However, the interquartile range for Sian's class is smaller, indicating that the marks for Sian's class are more closely clustered around the median than the marks for Kelly's class (or the marks for Kelly's class are more spread out from the median).

Write

The marks for Sian's class are more closely clustered about the median than those for Kelly's class.



Worked example video: Interpreting the meaning of an interquartile range

Helpful hints

- The smaller the value of the interquartile range, the closer the scores are clustered about the median. The greater its value, the greater the spread.
- The interquartile range contains the middle 50% of the scores.
- The interquartile range is not affected by outliers.

15 In a test, the median result for Lydia's class was 58 and the interquartile range was 21. In Lara's class, the median was also 58 but the interquartile range was 10. From this information, what conclusions could be made about the results?

16 At the end of the cricket season, Tadashi's median batting score was 31 with an interquartile range of 8. Matthew's median score was 31 and the interquartile range was 24. On the basis of the information given, answer the following questions.

- Which batsman was more consistent?
- Who is more likely to make a high score?
- Who is more likely to make a low score?



Problem solving and reasoning

17 Can you find the range for the dataset described in this table? Explain your answer.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6

18 The summary statistics of two sets of data are given.

	A	B
Mean	20.6	21.9
Median	22	18
Range	26	50
IQR	7	9
Upper quartile	24	25
Lower quartile	17	16
Lowest score	4	15
Highest score	30	65

- a By comparing the range and interquartile range, determine which set of data, **A** or **B**, is more likely to contain an atypical score, possibly due to a measurement error.
- b Which measures are most affected by an atypical or erroneous score?

Challenge

19 Write down a dataset of 10 values with the following summary statistics:

Mean = 9, Median = 9, Mode = 12, Range = 11, Interquartile range = 6

Related resources



12.4 Quick quiz



Worksheet: Summary statistics



Investigation: Effects of varying data on the interquartile range

Lesson 12.5

Box plots

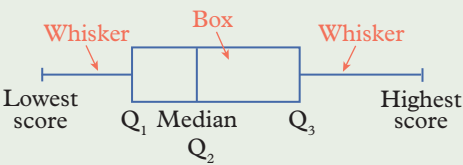
Learning intentions

By the end of this lesson, you will be able to ...

→ draw box plots and identify the corresponding five-number summary.

Box plots

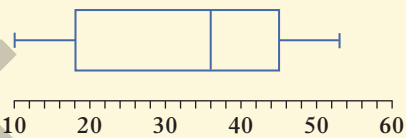
- A useful way of displaying and comparing datasets is by means of a **box plot** (or a **box-and-whisker plot**). A box plot is a diagram that displays the highest and lowest data values, the median, and the upper and lower quartiles of a dataset. These five values are also known as the **five-number summary**.



Worked example 12.5A Finding the range, median and interquartile range from a box plot

For the box plot shown, find the:

- a highest and lowest data values, then use these to find the range of the dataset
- b median, upper quartile and lower quartile, then use these to find the interquartile range.



Part	Think	Write
a	The highest data value is at the right-hand end of the right-hand whisker. Using the scale, this is 53. The lowest data value is the first point on the left-hand whisker. Using the scale, this is 10. The range is the highest data value minus the lowest data value.	Highest score = 53 Lowest score = 10 Range = $53 - 10$ = 43
b	The median is the data value indicated by the vertical line drawn inside the box. This is 36. The upper quartile is the data value indicated by the right-hand end of the box. This is 45. The lower quartile is the data value indicated by the left-hand end of the box. This is 18. The interquartile range is the upper quartile minus the lower quartile.	Median = 36 $Q_3 = 45$ $Q_1 = 18$ IQR = $45 - 18$ = 27



Worked example video: Finding the range, median and interquartile range from a box plot

Understanding, fluency and communicating

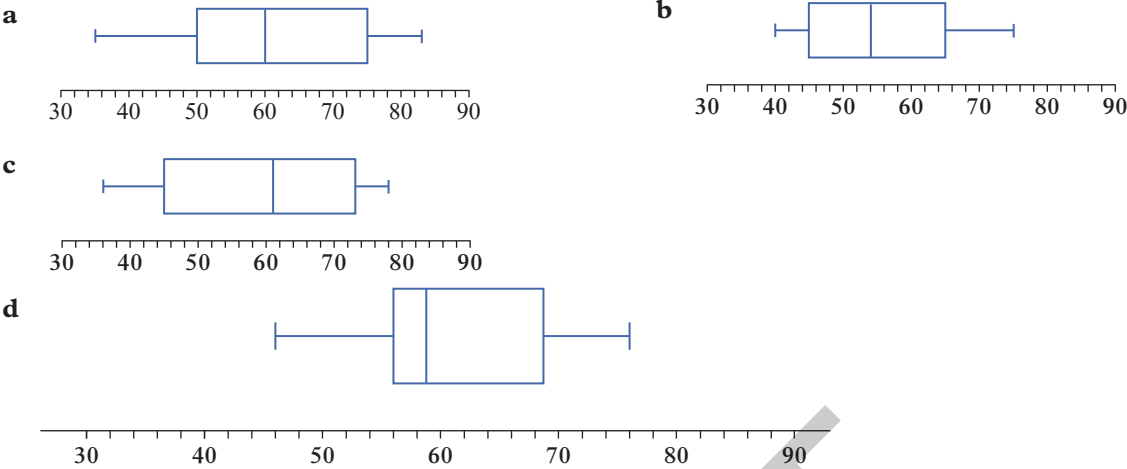
Answers: Lesson 12.5

Helpful hints

- The ends of the “whiskers” indicate the lowest and highest data values. The ends of the “box” indicate the lower quartile and upper quartile. The vertical line drawn inside the box indicates the median.
- The five numbers used (lowest, Q_1 , Q_2 , Q_3 and highest) are referred to as a five-number summary of the dataset.

1 For each of the box plots shown, find these values.

- i highest data value ii lowest data value iii range of the dataset
iv median v upper quartile vi lower quartile
vii interquartile range.



Worked example 12.5B Five-number summaries from different data displays

Find the five-number summary for the following set describing people's ages in a particular group.

Stem	Leaf
0	8 9
1	0 1 2 5 7 9
2	2 4 7 9
3	1 1

Key: 0 | 8 = 8

Think

We can read the highest and lowest data values off directly.
For the quartiles, we need to know how many people are in the group.
There are 14 people.
Hence, the median is the average of the 7th and 8th people's ages. The 7th age in the ordered list is 17 and the following age is 19. The average of these numbers is the number halfway between, which is 18.
The median divides the set into two halves with 7 ages in each half.
Therefore, Q_1 is the middle age in the lower half:
8, 9, 10, 11, 12, 15, 17
and Q_3 is the middle age in the upper half:
19, 22, 24, 27, 29, 31, 31.

Write

Highest age = 31
Lowest age = 8
 $Q_2 = 18$
 $Q_1 = 11$
 $Q_3 = 27$
The five-number summary for this dataset is:
Min = 8
 $Q_1 = 11$
 $Q_2 = 18$
 $Q_3 = 27$
Max = 31

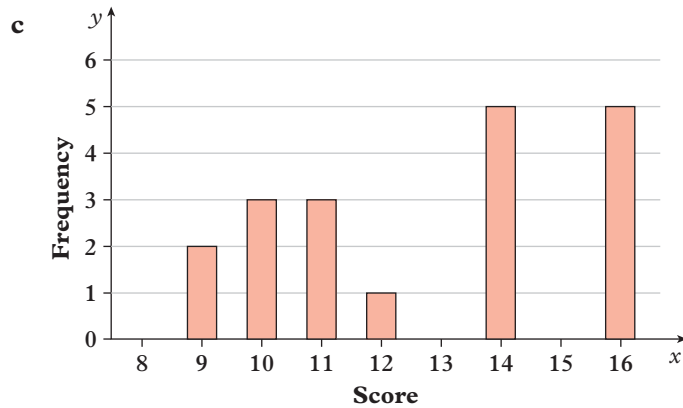
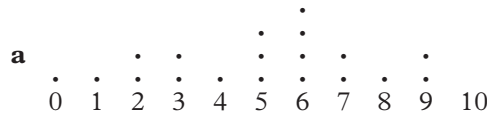


Worked example video: Five-number summaries from different data displays

Helpful hints

- If you can rewrite the data from a data display in an ordered list of numbers, then you can find the five-number summary from the ordered list.

2 Find the five-number summary for each of the following datasets.



b

Stem	Leaf
16	7 9
17	3 7 7 7 9
18	3 6 7
19	1 2 3 5
20	0 1

Key: 16 | 7 = 167

Worked example 12.5C Drawing a box plot from a five-number summary

Draw a box plot using the following five-number summary of a dataset.

Highest data value = 82 Lowest data value = 25 Median = 64 Lower quartile = 49 Upper quartile = 71

Think	Write
Draw a number line to include values from 25 to 82. On the number line, mark the position of each of the five values given. Draw a box using 49 and 71 as its endpoints and draw a vertical line at 64. Draw horizontal lines connecting 25 to 49 and 71 to 82 for the whiskers.	

Worked example video: Drawing a box plot from a five-number summary

Helpful hints

- Mark the positions of the five values on a number line. Draw the box using the lower quartile and upper quartile as endpoints and draw the median line inside the box. Draw the whiskers by joining the lowest data value and highest data value to the box using straight lines.

3 Draw a box plot using the five-number summary given for each dataset.

	Lowest score	Highest score	Lower quartile	Median	Upper quartile
a	15	40	23	28	32
b	130	153	139	141	148
c	6	28	7	10	18
d	71	83	73	78	80
e	1	9	3	5	7

4 Draw a box plot for each of the datasets in question 2.

Problem solving and reasoning

5 Draw a box plot for each of the following datasets.

a 4, 5, 8, 8, 10, 12, 12, 14, 15, 19

b 21, 21, 23, 24, 24, 24, 26, 28, 30

c 89, 90, 92, 95, 95, 98, 102, 103

d 18, 20, 22, 23, 25, 29, 30, 30, 30, 31

6 Draw a box plot for each of the following datasets. Each dataset has the same key:

Key: 5 | 3 = 53.

a	Stem	Leaf
	5	3
	6	1 2 5 8 9
	7	0 2 3 6 7 9
	8	4 4 6

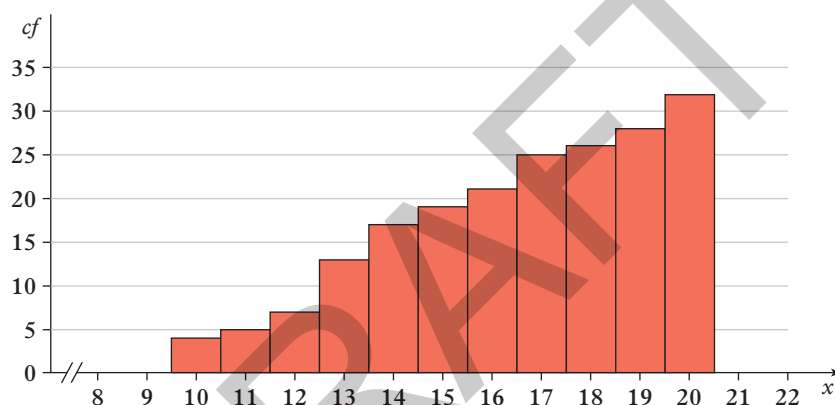
b	Stem	Leaf
	1	7 7 9
	2	0 1 1 4
	3	2 5 6 6 7
	4	1 3 3 8
	5	4 5

c	Stem	Leaf
	4	2
	5	4 6
	6	3 3 7
	7	1 8 8 9
	8	5 6

d	Stem	Leaf
	2	1 4
	3	0 2 2 5 8
	4	1 3 4 6 9
	5	4 7 8

Challenge

7 Draw a box plot based on the cumulative frequency graph shown.



Related resources



12.5 Quick quiz



Worksheet: Box plots



Investigation: The Olympic Games men's 100 m sprint

Lesson 12.6

Interpreting box plots

Learning intentions

By the end of this lesson, you will be able to ...

→ interpret box plots and use parallel box plots to compare datasets.

Describing box plots

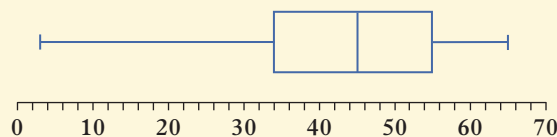
- In Module 11, we described the shapes of datasets using a variety of displays. Now that we understand box plots, we can do the same thing.
 - A distribution is said to be bell-shaped if most of the data is clustered in the middle, while the rest of the data tapers off symmetrically in both directions.
 - If there is a cluster of data to one side of the distribution and a longer tail of data tapering off in only one direction, we say the distribution is skewed. If the longer tail goes to the left, the data is negatively skewed, and if the longer tail goes to the right, the data is positively skewed.
- In a box plot, the shape of the distribution is determined by the width of each section; that is, the left and right whiskers, and the two halves of the box.
 - Because quartiles divide a dataset into 4 equally sized subsets, we can say that:
 - 25% of the data lies between the lowest value and the first quartile
 - 25% of the data lies between the first quartile and the median
 - 25% of the data lies between the median and the third quartile
 - 25% of the data lies between the third quartile and the highest value.
 - When one of the four segments of the box plot is very wide, this means that 25% of the data is spread out over a wider range.
 - On the other hand, when a segment of the box plot is very narrow, this indicates that 25% of the data is densely clustered within a smaller range.

Comparing box plots

- We can compare two datasets using their box plots by putting them on a shared axis. We call these parallel box plots.
 - minimums, maximums and medians
 - range and interquartile range
 - the shape or skew of each dataset.
- Parallel box plots can be used to compare:

Worked example 12.6A Describing the shape of a distribution from a box plot

The data from the stem-and-leaf plot in Worked example 11.8D has been drawn as a box plot. Describe the shape of the distribution.



Think

Each of the four parts of the box plot contain 25% of the data values.
 The first 25% of values are spread from 3 to 34. The first 50% are spread from 3 to 45.
 The last 25% of values are spread from 55 to 65. The second 50% are spread from 45 to 65.
 Therefore, the first 25% and 50% of values are spread out much further than the last 25% and 50% of values. This gives a tail pointing left; that is, it is negatively skewed.

Write

The distribution is negatively skewed.



Worked example video: Describing the shape of a distribution from a box plot

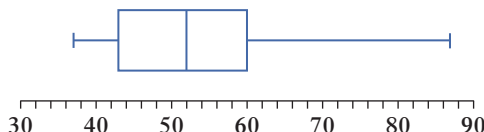
Understanding, fluency and communicating

Answers: Lesson 12.6

Helpful hints

- Check the first and last 25% and 50% of values to see how spread out they are. This will indicate skewness. If all four parts of the box plot are evenly spread, then the distribution is symmetrical. If not, then it is skewed.

- 1 The box plot is drawn from the data in the stem-and-leaf plot shown. Explain why the distribution is positively skewed.

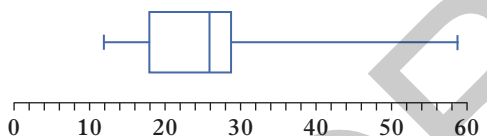


Stem	Leaf
3	7 8 9
4	0 3 5 7 8
5	2 2 2 6 9 9
6	0 1 8
7	4
8	7

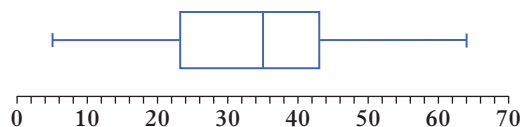
Key: 3 | 7 = 37

- 2 These box plots are drawn from the stem-and-leaf plots in Lesson 11.8 question 6. What is the shape of each distribution? What aspect of the box plot shows this?

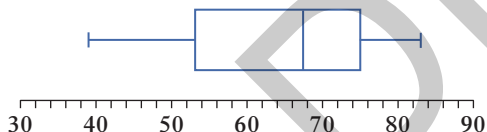
a



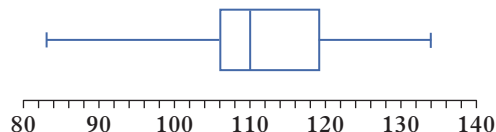
b



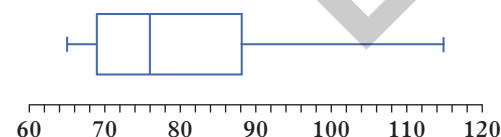
c



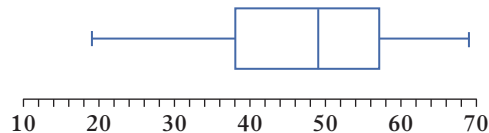
d



e



f

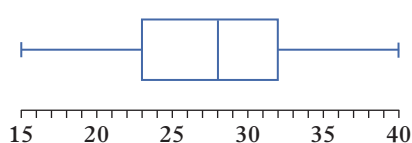


- 3 Answer the following questions for each of the box plots shown.

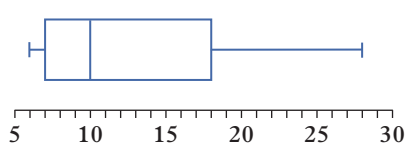
i Describe the shape of the data distribution in each box plot.

ii Determine Q_1 , Q_2 , Q_3 and IQR.

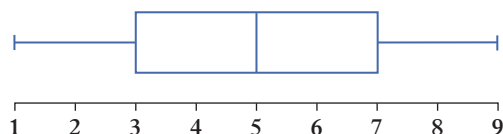
a



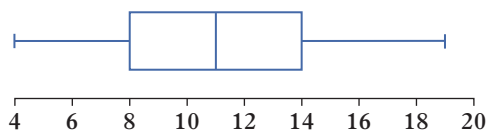
b

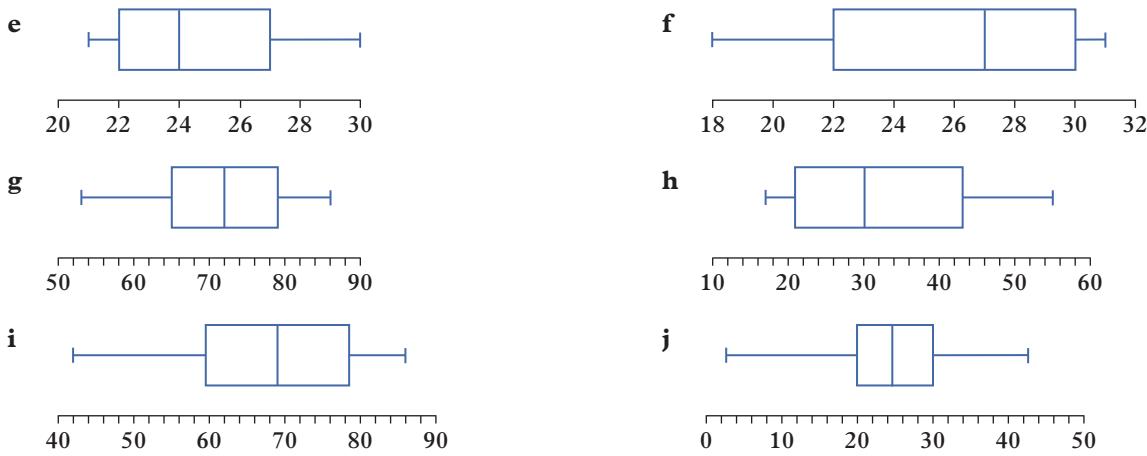


c



d

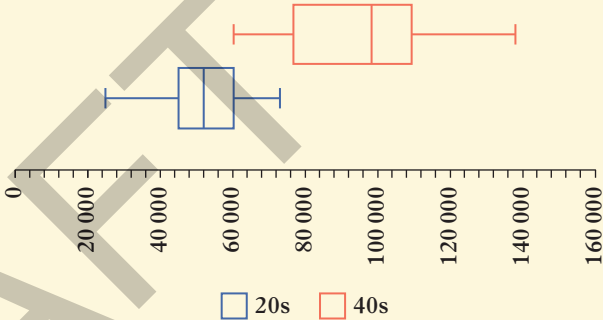




Worked example 12.6B Comparing datasets using parallel box plots

These parallel box plots show the annual income (in AUD) for 50 Australians in their 20s and 50 Australians in their 40s. Comment on how these datasets compare in terms of their:

- a centre
- b spread
- c shape.



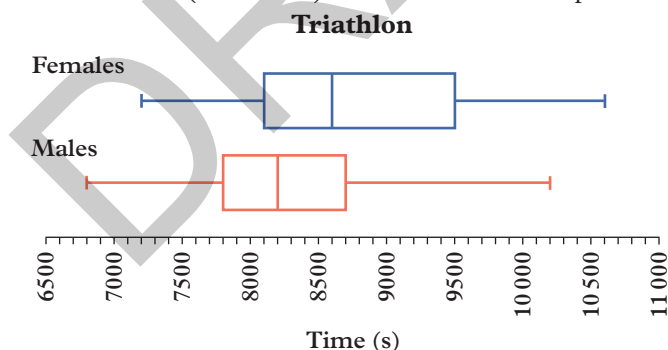
Part	Think	Write
a	Box plots tell us the median of a dataset, so we use the median as our measure of centre. The median annual income for 20-somethings is approximately \$52 000. The median annual income for 40-somethings is approximately \$98 000. The difference is $\$98\,000 - \$52\,000 = \$46\,000$.	The median annual income for the Australians in their 40s was \$98 000, which is approximately \$46 000 higher than the median annual income for the Australians in their 20s (at \$52 000).
b	We find the five-number summary for both datasets. For 20-somethings, we have: Min = \$24 000 Q_1 = \$44 000 Median = \$52 000 Q_3 = \$60 000 Max = \$72 000 For 40-somethings, we have: Min = \$60 000 Q_1 = \$76 000 Median = \$98 000 Q_3 = \$108 000 Max = \$138 000	The spread of annual incomes was greater for the Australians in their 40s than it was for the Australians in their 20s. The highest and lowest annual incomes of the Australians in their 20s had a difference of \$48 000, while this difference was \$78 000 for the Australians in their 40s. The interquartile range for the Australians in their 40s was twice the size of the interquartile range for the Australians in their 20s.

Part	Think	Write
	We can use these values to find the range and interquartile range for each dataset: 20s: Range = $\\$72\ 000 - \\$24\ 000 = \\$48\ 000$ IQR = $\\$60\ 000 - \\$44\ 000 = \\$16\ 000$ 40s: Range = $\\$138\ 000 - \\$60\ 000 = \\$78\ 000$ IQR = $\\$108\ 000 - \\$76\ 000 = \\$32\ 000$	
c	We find the range of each 25% for both datasets. For 20-somethings, we have: First 25%: $\\$44\ 000 - \\$24\ 000 = \\$20\ 000$ Second 25%: $\\$52\ 000 - \\$44\ 000 = \\$8\ 000$ Third 25%: $\\$60\ 000 - \\$52\ 000 = \\$8\ 000$ Fourth 25%: $\\$72\ 000 - \\$60\ 000 = \\$12\ 000$ For 40-somethings, we have: First 25%: $\\$76\ 000 - \\$60\ 000 = \\$16\ 000$ Second 25%: $\\$98\ 000 - \\$76\ 000 = \\$22\ 000$ Third 25%: $\\$108\ 000 - \\$98\ 000 = \\$10\ 000$ Fourth 25%: $\\$138\ 000 - \\$108\ 000 = \\$30\ 000$	The distribution of annual incomes for Australians in their 20s was slightly negatively skewed, but the middle 50% was symmetric about the median. Meanwhile, the annual incomes for the Australians in their 40s saw a cluster between $\\$98\ 000$ and $\\$108\ 000$, but more of an even spread across the bottom 50%.



Worked example video: Comparing datasets using parallel box plots

- 4 This parallel box plot shows the times (in seconds) male and female competitors take to complete a triathlon.

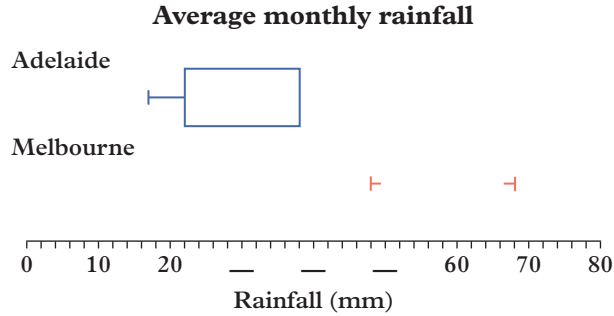


- What was the fastest time for the event?
- What was the difference in time between the slowest male and slowest female competitor?
- How long did it take the first half of the male competitors to finish?
- How long did it take the top 75% of the males to finish?
- How long did it take the first 25% of the females to finish?
- Which group had the greatest range of times?
- Which group had the smallest interquartile range of times?
- Which group had the fastest overall times?

5 The average monthly rainfall, in millimetres, for Adelaide and Melbourne is shown in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adelaide	17	19	22	38	59	52	65	50	43	38	24	23
Melbourne	48	48	52	58	58	50	49	51	59	68	60	59

Copy and complete the parallel box plots for this data, as started below.



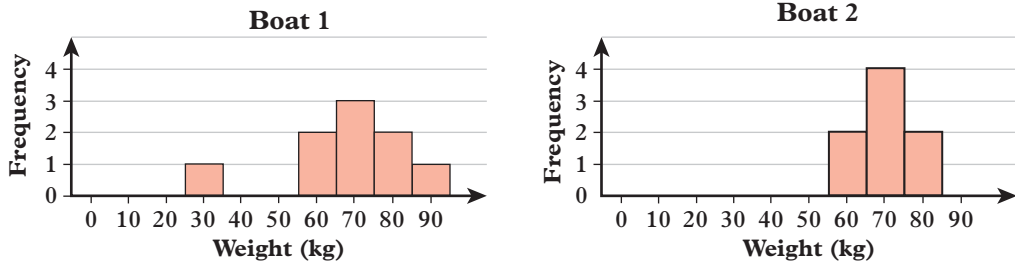
6 The average monthly rainfall, to the nearest millimetre, for Canberra and Hobart is shown in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Canberra	58	57	56	53	49	37	40	47	50	67	63	53
Hobart	48	40	47	52	49	56	54	52	52	64	55	52

- a Use the given rainfall data to draw two box plots on the same scale.
- b What comparisons can be made using the box plots?
- 7 The average number of minutes spent moving each day for two different groups of students are recorded below. The first group represents students who live within walking distance from their school, while the students in the second group are those who must rely on alternative methods of transport.
- Within walking distance: 68, 75, 84, 77, 86, 70, 55, 68, 67, 69
- Not within walking distance: 62, 45, 53, 58, 50, 48, 56, 65, 78, 54
- a Display the data for minutes of movement using two box plots on the same scale.
- b Make comparisons between the minutes of movement between the two groups, using the fact that box plots divide the data into four groups of 25%.

Problem solving and reasoning

8 These histograms show the weights, to the nearest 10 kg, of the people in two row boats. Row boats can also carry a non-rower called a cox. Generally, the cox will be as light as possible.



- a Connect the top of each bar in the histogram using a smooth curve.
- b Describe the shape of the curve on each histogram.
- c Which boat has a cox? Explain in terms of the shape of the distribution and any atypical data points.



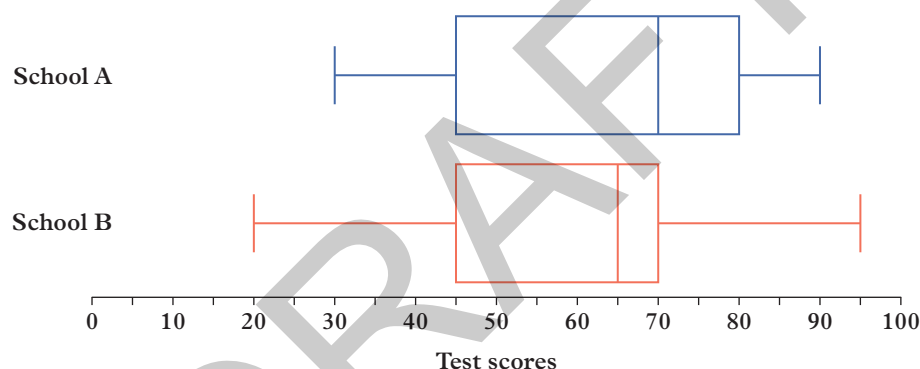
- 9 The towns of Karuah and Buladelah both have a speed limit of 60 km/h. The speeds of the first 100 cars travelling through these towns, from 9:00 am to 10:00 am on Saturday, were measured and the results are shown in the table.

	Karuah	Buladelah
Mean	59	60
Median	58	58
Lower quartile	52	50
Interquartile range	18	9
Highest speed	85	105
Lowest speed	35	40

- a Draw box plots to display this data.
- b Use this data to give a reason why the police might target Karuah for speeding. Also give a reason why police might target Buladelah.

Challenge

- 10 The Year 11 students from two schools sat the same test, and their test results are shown in the parallel box plots.



If 14 students from School B got a score below 45, and School B had 6 more students get a score between 45 and 70 than School A did, find the percentage, correct to one decimal place, of students across both schools that achieved a score of 70 or more.

Related resources



12.6 Quick quiz



Investigation: Distributions and cumulative frequency

Lesson 12.7

Clusters and outliers

Learning intentions

By the end of this lesson, you will be able to ...
→ identify clusters and gaps in datasets and describe the effects of outliers.

Clusters and outliers

- **Outliers** are data values which appear to be abnormally higher or lower than the rest of the dataset. It is up to statisticians to determine what is considered abnormal.
- The five-number summary for a dataset can be used to formally define what constitutes an outlier.
 - Outliers are data values which lie outside the range:
 - $Q_1 - 1.5 \times \text{IQR}$ to $Q_3 + 1.5 \times \text{IQR}$.
 - Notice that this calculation only relies on the first and third quartiles, which means it is typically uninfluenced by the presence of outliers. Hence, this method can be used even when the data is skewed.
- Sometimes data appears in **clusters** around certain values. These clusters might be more or less noticeable depending on the way the data is being displayed.

Worked example 12.7A Formally identifying outliers from a dataset

Consider the dataset 30, 50, 50, 52, 55, 55, 57, 57, 58, 60, 60.
Use the five-number summary to identify any outliers in the dataset.

Think	Write
30 appears to be much lower than the other data values which are clustered from 50 to 60. For this method, outliers are data values which lie outside the range $Q_1 - 1.5 \times \text{IQR}$ to $Q_3 + 1.5 \times \text{IQR}$	$\begin{array}{cccccccccccc} 30 & 50 & 50 & 52 & 55 & 55 & 57 & 57 & 58 & 60 & 60 \\ & & \uparrow & & & \uparrow & & & \uparrow & & \\ & & Q_1 & & & Q_2 & & & Q_3 & & \end{array}$ $\begin{aligned} \text{Median} &= Q_2 = 55 \\ Q_1 &= 50 \\ Q_3 &= 58 \\ \text{IQR} &= Q_3 - Q_1 = 58 - 50 = 8 \\ \text{Lower limit} &= Q_1 - 1.5 \times \text{IQR} \\ &= 50 - 1.5 \times 8 \\ &= 38 \end{aligned}$ Since $30 < 38$ that means 30 is an outlier.



Worked example video: Formally identifying outliers from a dataset

Understanding, fluency and communicating

Answers: Lesson 12.7

Helpful hints

- Once you have the five-number summary for a dataset, you can calculate the values $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$, which represent the lowest and highest values a data value can be before it is considered an outlier.

1 Use the five-number summary to identify any outliers in the following datasets.

a 3, 24, 25, 26, 26, 26, 28, 29, 30, 32

b 5, 5, 6, 7, 7, 7, 8, 10, 12, 13, 21

c 17, 31, 32, 32, 34, 35, 35, 36, 38, 41, 44, 45

d 49, 50, 51, 51, 51, 53, 54, 56, 60, 63, 64, 75

2 Use the five-number summary to identify any outliers in the following datasets.

a

x	f
3	1
6	1
7	4
8	8
9	5
10	3

b

x	f
12	1
13	4
14	12
15	5
18	1

3 Consider the data in the stem-and-leaf plots shown. All the stem-and-leaf plots share the same key:

Key: 1 | 2 = 12.

i Calculate the median, Q_1 , Q_3 and IQR.

ii Calculate the limits for outliers in this data.

iii Which, if any, data values are outliers?

a

Stem	Leaf
1	2 3 6 8 8
2	2 5 6 7 7 8 9
3	4 4
4	
5	9

b

Stem	Leaf
0	5
1	3 7
2	2 2 4 6
3	0 1 1 5 8 8 9
4	2 3 3 6
5	1 5
6	4

c

Stem	Leaf
3	9
4	2 5
5	2 2 3
6	0 1 3 4 4 7 8
7	0 0 2 4 5 5 7 9 9
8	1 3

d

Stem	Leaf
8	3
9	0 1 5
10	6 6 7 7 8 9 9
11	0 1 3 5 7 8 9
12	0 0 1 3
13	4

e

Stem	Leaf
6	5 5 6 7 8 9
7	0 1 3 3 4 6 9 9
8	2 2 5 8
9	1 3
10	0 2
11	5

f

Stem	Leaf
1	5 8 9 9 9
2	0 2 2 2 4 8 9
3	8
4	0 5
5	2 6 7 7
6	3 4 5 8 9 9

4 For each stem-and-leaf plot in question 3, comment on any clustering in the dataset.

5 For each box plot in question 3 of Lesson 12.6, determine whether or not the dataset contains outliers. Formally justify your answer.

Problem solving and reasoning

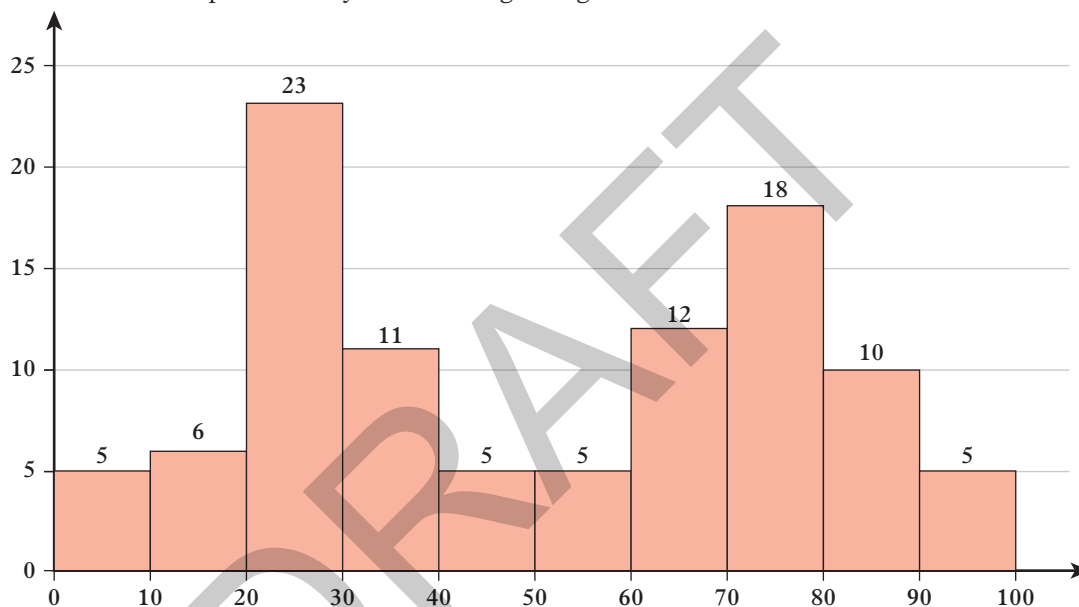
6 For each box plot in question 2 of Lesson 12.6, draw a copy of the box plot, then draw copies of the box section over the top of the whiskers to make a visual argument as to whether the dataset contains outliers or not.

- 7 The heights, to the nearest centimetre, of the players in a women's hockey team are recorded as:
65, 170, 168, 172, 160, 175, 167, 169, 173, 166, 174, 171, 162, 176, 164, 177
- a** Show formally that 65 is an outlier. **b** Discuss possible reasons for the score of 65.
- 8 For parts **a-d** below, consider each of these versions of the dataset in question 7 and then answer part **e**.
- i** exactly as given **ii** with the outlier removed
- iii** with 65 replaced by 165

- a** Calculate the mean.
b Calculate the median.
c Calculate the range.
d Calculate the interquartile range.
e Describe the heights of the team members using the most appropriate measures of centre and spread.



- 9 Consider the dataset represented by the following histogram.



- a** Describe the shape of this distribution.
b Determine Q_1 , Q_2 and Q_3 for this dataset, using the class centres as approximate values.
c Draw a box plot for this dataset, assuming the minimum value is 0 and the maximum value is 100.
d Describe the shape of the distribution, based on the box plot.
e Comment on any clusters in this dataset and how they appear in each display.

Challenge

- 10 The interquartile range of a set of data is 10. If there are no outliers in this dataset, what is the largest possible range?

Related resources



12.7 Quick quiz



Investigation: Outliers in box plots

Lesson 12.8

Standard deviation

Learning intentions

By the end of this lesson, you will be able to ...

→ calculate the standard deviation of a dataset and compare measures of spread.

Standard deviation

- The interquartile range is a better measure of the spread of a dataset than the range, but it still only depends on two values, the upper quartile and lower quartile.
- A more commonly used measure, whose value depends on all the data values, is the **standard deviation**.
→ Standard deviation, denoted by σ , is determined by finding the distances of all the data values from the mean and then calculating an “average” of these distances.
- In the same way as the mean, mode and median are “typical” values of the dataset, the standard deviation is a “typical” distance of the values from the mean. The smaller the standard deviation, the smaller the “typical” distance of the values from the mean, and hence, the smaller the spread of the dataset.
- Note that the interquartile range is a measure of the spread about the median, whereas the standard deviation is a measure of the spread about the mean. If the median gives a better measure of centre than the mean for a particular dataset, then the interquartile range will typically give a better measure of spread than the standard deviation, and vice versa.
- Scientific calculators have built-in functions to calculate standard deviation, as do spreadsheets. The formula they are using “behind the scenes” is
$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}},$$
- where $\bar{x}$ is the mean of the dataset and n is the number of data values in the dataset. The symbol $\sum$ and the term x_i mean that this calculation is performed for every data value x in the dataset, then the answers are added together.
- The units for standard deviation are the same as the units for the dataset.

Worked example 12.8A Finding the standard deviation for a dataset

Find the standard deviation of the following dataset. 2 3 5 1 4 6 5 2 1 4

Think	Write
Use your calculator’s in-built “standard deviation” function. You will need to enter the dataset and then follow your calculator’s instructions to calculate the standard deviation. Note: Make sure to read the value for the population standard deviation. This is usually represented by σx on a calculator.	$\sigma = 1.7$ (to one decimal place)



Worked example video: Finding the standard deviation for a dataset

Understanding, fluency and communicating

Answers: Lesson 12.8

Helpful hints

- You may need to put your calculator into “statistics mode” in order to enter the data.
- Most calculators will have two options for the standard deviation of a dataset. In this course, you should always choose the population standard deviation, σ .

1 Use your calculator to find the mean and standard deviation of each of the following datasets.

- a 4, 5, 6, 7, 8

c 0, 3, 6, 9, 12

e 88, 89, 89, 91, 91, 92

g 7, 5, 9, 9, 3, 3
- b 2, 4, 6, 8, 10

d 25, 26, 27, 28, 29, 33

f 239, 245, 248, 250, 253, 253, 254, 258

h 5, 5, 5, 5, 5, 5

2 Determine the mean and standard deviation of each of the following datasets.

a	Score	2	3	4	5	6
	Frequency	5	7	4	8	1

b	Score	15	16	17	18	19
	Frequency	3	8	11	5	3

c	Score	0	1	2	3	4
	Frequency	12	15	24	14	5

d	Score	121	122	123	124	125
	Frequency	4	6	6	3	5

Worked example 12.8B Finding the mean and standard deviation for grouped data

Find the mean and standard deviation for the following grouped data.

Height (mm)	300–324	325–349	350–374	375–399	400–424	425–449
Frequency	3	18	47	32	14	6

Think	Write																					
For grouped data, we again assume that the data values are evenly distributed throughout each class, and we use the class centres. Apply the appropriate steps to enter the data into your calculator and determine the mean and standard deviation.	<table><tr><th>Height (mm)</th><th>Class centre (x)</th><th>Frequency (f)</th></tr><tr><td>300–324</td><td>312</td><td>3</td></tr><tr><td>325–349</td><td>337</td><td>18</td></tr><tr><td>350–374</td><td>362</td><td>47</td></tr><tr><td>375–399</td><td>387</td><td>32</td></tr><tr><td>400–424</td><td>412</td><td>14</td></tr><tr><td>425–449</td><td>437</td><td>6</td></tr></table> $\bar{x} = 373.3$ mm (to one decimal place) $\sigma = 27.7$ mm (to one decimal place)	Height (mm)	Class centre (x)	Frequency (f)	300–324	312	3	325–349	337	18	350–374	362	47	375–399	387	32	400–424	412	14	425–449	437	6
Height (mm)	Class centre (x)	Frequency (f)																				
300–324	312	3																				
325–349	337	18																				
350–374	362	47																				
375–399	387	32																				
400–424	412	14																				
425–449	437	6																				



Worked example video: Finding the mean and standard deviation for grouped data

Helpful hints

- Enter the class centre as the value to represent each class.

3 Find the mean and standard deviation for each of these datasets.

a

Mass (kg)	Frequency
60–62	2
63–65	5
66–68	7
69–71	4
72–74	2

b

Score	Frequency
41–50	3
51–60	0
61–70	10
71–80	8
81–90	7
91–100	2

c

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

d

Height	Frequency
155–159	4
160–164	7
165–169	10
170–174	5
175–179	2

4 Consider the data values 3, 3, 4, 5, 6, 6.

a Find the mean and standard deviation.

b Add 5 to each of the data values above.

i Recalculate the mean and standard deviation.

ii What do you think would happen to the mean and standard deviation if we added 10 to each data value? Check by calculation.

c Multiply each of the data values above by 5.

i Recalculate the mean and standard deviation.

ii What do you think would happen to the mean and standard deviation if we multiplied each data values by 10? Check by calculation.

5 For a dataset, the mean is 17 and the standard deviation is 4. Determine the new mean and new standard deviation if:

a 3 is added to each value in the dataset

b each data value is multiplied by 5

c 5 is subtracted from each data value.

6 If the standard deviation of a dataset is zero, describe the data.

Worked example 12.8C Comparing datasets using mean and standard deviation

The length of 16 teenagers' feet (in millimetres) was measured. The measurements are in the following table. Calculate the mean and standard deviation for the boys and girls separately, then compare the data.

Boys	255	260	265	270	268	272	263	275	267	271	269	273	264	270	266	274
Girls	225	230	235	228	232	229	234	227	231	226	233	229	230	235	228	232

Think	Write
Use a calculator or a spreadsheet. In a spreadsheet, use the function STDEV.P and not STDEV.S.	Boys: Mean = 267.8 mm Standard deviation = 5.1 mm Girls: Mean = 230.05 mm Standard deviation = 3.0 mm
Compare the values of the mean and standard deviation for each dataset. Interpret what these values mean for the datasets.	The mean for the boys, 267.8 mm, is greater than the mean for the girls, 230.05 mm. The standard deviation for the boys, 5.1 mm, is greater than the standard deviation for the girls, 3.0 mm. This means that on average the boys' feet are longer than the girls' feet and that the lengths of the boys' feet are more spread out compared to the girls' feet.



Worked example video: Comparing datasets using standard deviation

- 7 The shoe sizes of students in a Year 11 class are recorded below. Find the standard deviation in shoe sizes for the boys and girls separately, then compare the two datasets.
Girls: 5, 4, 7, 5, 6, 7, 4, 8, 5, 6, 4, 6, 7, 5, 9
Boys: 9, 5, 8, 8, 9, 9, 7, 11, 8, 9, 5, 10, 10, 6, 10
- 8 Two different sets of 20 families were chosen at random from two different suburbs in Sydney. The numbers of children in each family are recorded in this table. Calculate the mean and standard deviation for each suburb separately, then compare the results.

Number of children	Suburb A (frequency)	Suburb B (frequency)
0	2	12
1	3	5
2	10	2
3	4	1
4	0	0
5	1	0

- 9 Two coworkers measure the time it takes them to get to work each day. Their times, to the nearest minute, for a two-week period are summarised in this table. Determine the mean and standard deviation of each of their travel times, then describe any differences.

Time	Coworker 1 (frequency)	Coworker 2 (frequency)
24–26	2	2
27–29	3	2
30–32	3	2
33–35	1	2
36–38	1	2

Problem solving and reasoning

- 10** The weights (in kilograms) of a group of 17-year-old soccer players were:

69, 64, 58, 77, 85, 90, 61, 73, 59.

- a** For this dataset, find the following measures of spread:

i range

ii interquartile range

iii standard deviation.

- b** Replace the weight of 90 kg with a weight of 110 kg, then recalculate each of the measures of spread from part **a**.

- c** Based on your answers to parts **a** and **b**, comment on the strengths and weaknesses of each measure of spread.

- 11** Two machines produce chocolates with an advertised weight of 60 g. Some chocolates outputted by these machines were weighed. The weights of the chosen chocolates, in grams, are listed.

Machine 1: 50, 60, 60, 60, 60, 60, 60

Machine 2: 50, 50, 55, 55, 60, 60, 60

- a** For *Machine 1* and *Machine 2*, find the:

i range

ii interquartile range

iii (population) standard deviation.

- b** Where are the scores clustered for *Machine 1*?

- c** Which machine produces chocolates of a more consistent weight? Explain your answer.

- 12** In a game of golf, the player with the lowest score wins the hole. Usually the player with the lowest total score wins the match. Carrie and Laura play nine holes of golf. Their scores are listed in the table below.

Hole	Carrie	Laura
1	5	4
2	8	6
3	6	5
4	3	3
5	5	4
6	8	8
7	4	4
8	7	6
9	8	15
Total	54	55

- a** For each player, find the:

i mean score

ii median score.

- b** How many holes did each player win?

- c** For each set of scores, compare the:

i range

ii interquartile range

iii standard deviation.

- d** Use the data to present an argument that Laura is the better player.

- e** Use the data to present an argument that Carrie is the better player.



- 13** The marks scored by Jimena and Alison in the eight topic tests they did in mathematics over the year are shown.

Test	1	2	3	4	5	6	7	8
Jimena	72	71	81	74	72	65	78	44
Alison	71	70	76	73	78	62	76	69

- a** Find the total marks scored by each student.
 - b** For the marks scored by each student, find the:
 - i** mean mark
 - ii** median mark.
 - c** In how many tests did Jimena score higher marks than Alison?
 - d** For each student's marks, compare and comment on the:
 - i** range
 - ii** interquartile range.
 - e** Calculate the standard deviation of each student's marks.
 - f** Which student's performances were more consistent?
 - g** The Mathematics Prize is to be given to the better of these two students. Use the data to present a case for each student to receive the award.
- 14** Consider the following datasets.
- i** 5, 6, 6, 7, 8, 9
 - ii** 5, 6, 6, 7, 8, 90
 - a** Find the standard deviation for each of the datasets.
 - b** Discuss the effect of outliers on the standard deviation.
 - c** Find the range and interquartile range for each dataset.
 - d** Does the outlier affect the range or the interquartile range?
- 15** Find examples in the media to illustrate appropriate and inappropriate use or misuse of each of the following.
- a** The mean as a measure of centre.
 - b** The range as a measure of spread.

Challenge

- 16** Consider the dataset: 12, 14, 15, 16, 18.
- a** Show that the mean of this dataset is 15 and the standard deviation is 2.
 - b** Explain what will happen to the standard deviation when an extra data value of 15 is added to the dataset.
 - c** Explain what will happen to the standard deviation when an extra data value of 17 is added to the dataset.
 - d** Compare the outcomes in part **b** and **c**. Explain any differences.

Related resources



12.8 Quick quiz



Worksheet: Measures of centre and spread



Investigation: Analysing team data for the Opals

Using spreadsheets

Syllabus links

Use a spreadsheet to analyse data including calculating measures of centre and spread (MST-11-08)

Using spreadsheets

Spreadsheets have in-built functions to find various statistics for a set of data. The functions given here are used in Microsoft Excel. Similar functions can be used in other spreadsheet software.

- Measures of centre
 - To find the mean of data in the cells A1 to A5, use =AVERAGE(A1:A5).
 - To find the median of data in the cells A1 to A5, use =MEDIAN(A1:A5).
 - To find the mode of data in the cells A1 to A5, use =MODE.MULT(A1:A5).
 - Five-number summary
 - To find the lowest value in the cells A1 to A5, use =MIN(A1:A5) or =QUARTILE.INC(A1:A5, 0).
 - To find the first quartile for the data in cells A1 to A5, use =QUARTILE.INC(A1:A5, 1).
 - To find the second quartile (i.e. the median) for the data in cells A1 to A5, use =MEDIAN(A1:A5) or =QUARTILE.INC(A1:A5, 2).
 - To find the third quartile for the data in cells A1 to A5, use =QUARTILE.INC(A1:A5, 3).
 - To find the highest value in the cells A1 to A5, use =MAX(A1:A5) or =QUARTILE.INC(A1:A5, 4).
 - Measures of spread
 - To find the range of data in the cells A1 to A5, use =MAX(A1:A5)-MIN(A1:A5) or =QUARTILE.INC(A1:A5, 4)-QUARTILE.INC(A1:A5, 0).
 - To find the interquartile range of data in the cells A1 to A5, use =QUARTILE.INC(A1:A5, 3)-QUARTILE.INC(A1:A5, 1).
 - To find the standard deviation for the data in cells A1 to A5, use =STDEV.P(A1:A5).
 - Some other useful functions include those listed below.
 - To count how many cells in the cells A1 to A5 contain numerical values, use =COUNT(A1:A5).
 - To determine the sum of the values in the cells A1 to A5, use =SUM(A1:A5).
- 1 Cells B2 down to B6 in the spreadsheet below left contains a student's marks (out of 50) on a series of tests. The total of their marks is entered in cell B7 using the formula =SUM(B2:B6). Their mean mark is entered in cell B8 using the formula =AVERAGE(B2:B6). The standard deviation of their marks is entered in cell B9 using the formula =STDEV.P(B2:B6). The spreadsheet below right contains the results after evaluating these formulas.

	A	B	C	D	E
1	Test	Mark		Test	Mark
2	1	33		1	33
3	=A2+1	23		2	23
4	=A3+1	45		3	45
5	=A4+1	18		4	18
6	=A5+1	44		5	44
7	Total	=SUM(B2:B6)		Total	163
8	Mean	=AVERAGE(B2:B6)		Mean	32.6
9	Standard deviation	=STDEV.P(B2:B6)		Standard deviation	10.86

- a Enter some marks of your own and find the total, mean and standard deviation.
 - b Try the formula =COUNT(B2:B6) and explain what it does.
 - c Use the functions SUM and COUNT to write your own formula that calculates the mean of a set of data.
- 2 Create your own dataset by entering =RANDARRAY(20,1,20,30,TRUE) into cell A1. Select the cells A1 to A20. Copy then Paste “Values” only in column B. The entries in column A can then be deleted.
- a Set up the following summary statistics next to your dataset. Enter the formulas at the start of this section into the shaded cells to find each statistic for your dataset.

	A	B	C	D
1				Formulas
2			Measures of centre	
3			Mean	
4			Median	
5			Mode	
6				
7			Measures of spread	
8			Range	
9			Interquartile range	
10			Standard deviation	

- b Enter the value of the first entry in your dataset into cell E1, then copy the evaluated summary statistics from column D, and paste “Values” only in column E.
 - c Change the value of the first entry in your dataset to a number less than 10 and repeat part **b** in column F.
 - d Change the value of the first entry in your dataset to a number greater than 40 and repeat part **b** in column G.
 - e Which of the statistics change, and by how much?
 - f Repeat this experiment using a dataset of 100 randomly generated values.
 - g Repeat the experiment using 20 randomly generated values between 100 and 120, changing the first value to a number less than 10, then changing it to a number greater than 200.
- 3 Repeat question 5 from Lesson 12.5 using a spreadsheet.
- a After entering and selecting each set of data, use the Insert Chart option to create a “Box & Whisker” chart.
 - b Randomly generate your own dataset and create a box plot to display it. Describe the shape and spread of your dataset using the box plot.
 - c *For fun:* If your spreadsheet software only produces box plots that are oriented vertically, find out how to construct a horizontal box plot using different types of charts.

Lesson 12.9

Review: Data analysis

Review questions 12.9A Mathematical literacy review

Answers: Lesson 12.9

Key terms

The following key terms are used in this module:

- box plot (box-and-whisker plot)
- cluster
- five-number summary
- interquartile range
- mean
- measure of central tendency (measure of centre)
- measure of spread
- median
- modal class
- modality (of a dataset)
- mode
- outlier
- quartile
- range
- standard deviation

Check your understanding

- Match the following descriptions to the terms **mean**, **median** and **mode**.
 - The middle value in an ordered list of numbers
 - The value that appears most frequently in a dataset
 - The value found by adding all numbers in a dataset, then dividing by the size of the dataset
- Classify the following terms as either a **measure of centre** or a **measure of spread**.

a mode	b range
c standard deviation	d modal class
e mean	f modality of a dataset
g interquartile range	h median
- Draw a **box plot** and label the following features:

a box	b whiskers
c minimum	d maximum
e first quartile	f second quartile
g third quartile	h median
i interquartile range	j range.
- Explain what a **parallel box plot** is and why you might use one.

Review questions 12.9B Multiple-choice questions

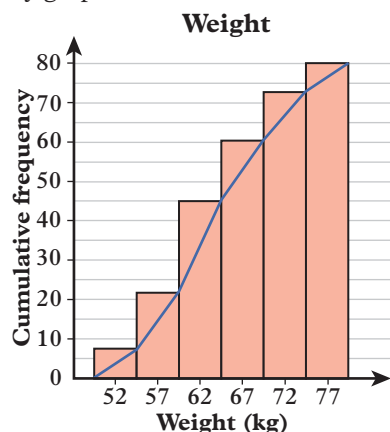
- 1 Which are the correct values for the dataset in the frequency distribution table shown?

Value	Frequency
23	4
24	11
25	9
26	6

- A Mean = 36.9, Mode = 11
 B Mean = 36.9, Mode = 24
 C Mean = 24.6, Mode = 11
 D Mean = 24.6, Mode = 24
- 2 The mean of the data in the table is:
- | Class | Frequency |
|-------|-----------|
| 0–4 | 3 |
| 5–9 | 4 |
| 10–14 | 6 |
| 15–19 | 2 |
- A 2.5 B 9.3 C 9.5 D 35
- 3 The modal class of the data in the table in question 2 is:
- A 0–4 B 5–9
 C 10–14 D 15–19
- 4 The median of the data in the table from question 1 is:
- A 24 B 25 C 24.5 D 10
- 5 For the dataset 7, 8, 8, 8, 8, 9, 10, 11, 12 and 12, which of the following lists the correct values?

- A Mean = 9.3, Mode = 8, Median = 8.5
 B Mean = 9, Mode = 8.5, Median = 8
 C Mean = 9.3, Mode = 8, Median = 9
 D Mean = 9, Mode = 9, Median = 8.5

- 6 The median of the data shown in the cumulative frequency graph is:



- A 64.5 B 62
 C 67 D 63.5

- 7 The data in this stem-and-leaf plot has:

Stem	Leaf
3	0 2 4
4	1 3 6 6 6
5	0 4 8 8

Key: 3 | 2 = 32

- a Mode = 6, Median = 6
 b Mode = 6, Median = 4
 c Mode = 46, Median = 46
 d Mode = 46, Median = 4
- 8 The most appropriate measure of central tendency for the data values 5, 5, 5, 9, 10 and 44 is the:
- A mode B mean
 C median D range.
- 9 If we choose random samples from a large population, which of the following is the measure that varies the most?
- A mean
 B mode
 C median
 D all vary the same amount
- 10 For the cumulative frequency graph shown in question 6, the interquartile range is approximately:
- A 15 B 40 C 69 D 10
- 11 Which of the following statements is *not* true for the data in this back-to-back stem-and-leaf plot?

Leaf: Class A	Stem	Leaf: Class B
8 5 2 2 0	4	2 5
9 7 6 3 1	5	3 3 6 9 9
8 7 4 3 2	6	0 1 4 8 8 8
9 8 6 5	7	1 1 3 7
	8	0 1 1

Key: 1 | 5 | 3

51 for Class A
 53 for Class B

- A The median for Class B is greater than the median for Class A.
 B The mean for Class B is greater than the mean for Class A.
 C The interquartile range for Class B is greater than the interquartile range for Class A.
 D The range for Class B is greater than the range for Class A.

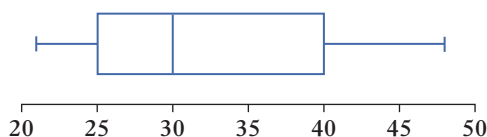
12 For the box plot shown:

A Range = 20, Interquartile range = 23

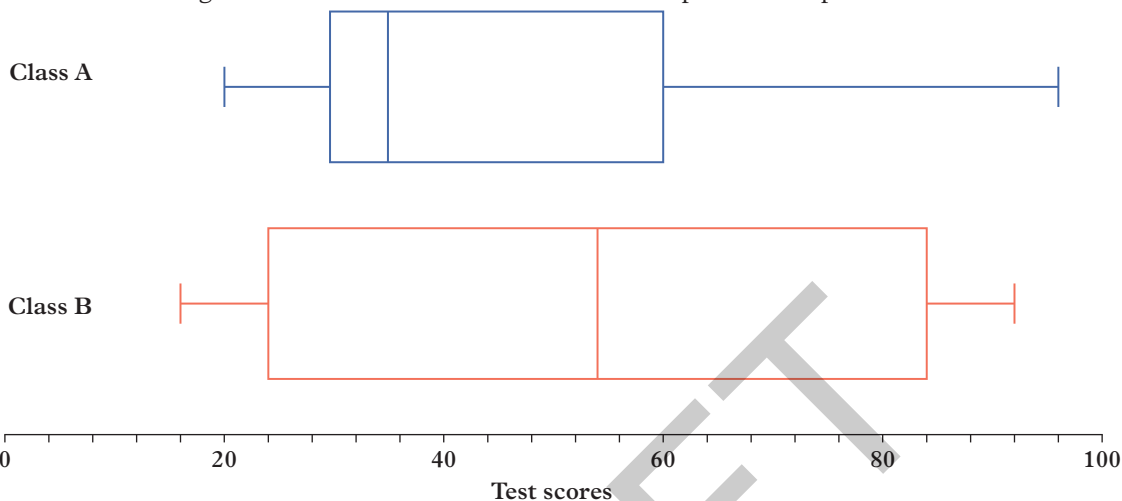
B Range = 27, Interquartile range = 15

C Range = 10, Interquartile range = 5

D Range = 15, Interquartile range = 27



13 Which of the following statements is *not* true for the data in this parallel box plot?



A The scores for Class B are symmetrically distributed, while the scores for Class A are positively skewed.

B The range of scores is equal for both Class A and Class B.

C There is a cluster of scores around 32 for Class A and a cluster around 54 for Class B.

D The interquartile range for Class B is twice the size of the interquartile range for Class A.

14 Consider the box plot for the Class A test scores in question 13. Which of the following is an accurate statement about the distribution of the data?

A 25% of the scores were between 30 and 35.

B 50% of the scores were between 35 and 60.

C 25% of the scores were above 50.

D 50% of the scores were above 60.

15 Which of the following can be used to formally identify outliers in a dataset?

A $\text{mean} - 1.5 \times \text{IQR}$

B $Q_3 + 1.5 \times \text{IQR}$

C $\text{median} + 1.5 \times \text{IQR}$

D $Q_1 + 1.5 \times \sigma$

16 Which of the following measures is *least* affected by the presence of outliers?

A range

B standard deviation

C mean

D median

17 What is the standard deviation for the dataset in the following table?

Mark	5	6	7	8	9
Number of students	5	5	8	7	2

A 1.21

B 1.41

C 2.28

D 2.55

18 The standard deviation of the data values 23, 23, 24, 25, 25, 25 and 26 is:

A 1.13

B 1.05

C 171

D 7

Review questions 12.9C Set 1

- Find the mean, mode, median and range of this dataset: 6, 6, 6, 7, 8, 8, 9, 9, 10.
- Consider the grouped data in the table.

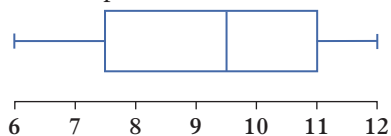
Class	Frequency
1–5	12
6–10	8
11–15	18
16–20	31
21–25	11

- Copy the table and add a cumulative frequency column.
 - Draw a cumulative frequency histogram and cumulative frequency polygon.
 - From the cumulative frequency polygon you drew in part **b**, determine the:
 - median
 - first quartile
 - interquartile range.
- For each of the following datasets, find the:
 - 1st, 2nd and 3rd quartiles
 - interquartile range.
 - 9, 9, 10, 11, 12, 12, 13
 - 8, 9, 9, 10, 11, 12, 12, 13, 14
 - 8, 9, 9, 10, 11, 12, 12, 12, 14, 14
 - 8, 8, 9, 10, 10, 11, 12, 12, 13, 14, 14, 15
 - Draw a box plot for this five-number summary:
Highest score = 42, Lowest score = 25, Upper quartile = 36, Lower quartile = 30, Median = 32.
 - Find the mean and standard deviation of this dataset: 17, 25, 19, 20, 23, 19, 21.
 - For the data values 5, 5, 6, 7, 7, 8, 8, 8 and 10, find the:
 - mean
 - median
 - standard deviation.
 - The prices of apartments sold in one month by a real-estate agent are listed below.
\$250 000 \$280 000 \$240 000 \$300 000 \$290 000 \$550 000
 - Calculate the:
 - mean price
 - median price.
 - Identify any outliers.
 - Which of these measures (mean or median) is the more appropriate average to use for these prices? Give reasons for your answer.
 - The maximum temperature (to the nearest °C) recorded each month for two towns is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Town A	31	30	28	26	23	18	17	19	24	27	28	30
Town B	36	34	30	26	22	17	13	20	25	27	32	35

- For each town, find the:
 - mean temperature
 - median temperature.
- For each town's temperatures, find the:
 - range
 - interquartile range
 - standard deviation.
- Write a short description comparing the temperatures for the two towns. In your description include the shape of each distribution and any clustering or outliers.

- 9 Consider the dataset represented by this box plot.



- Describe the shape of the distribution.
- State whether or not there are any outliers and justify your answer.

Review questions 12.9D Set 2

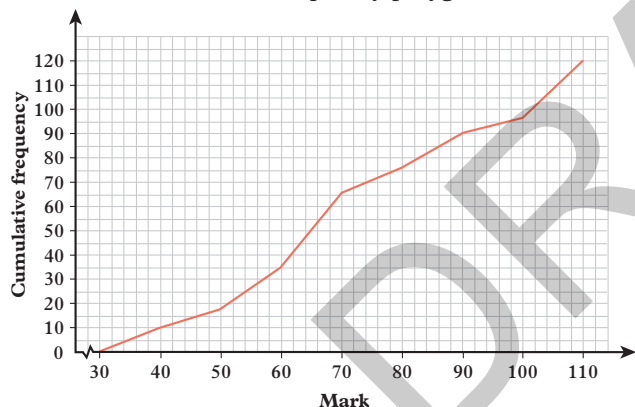
- 1 Find the mean, mode, median and range for the data in this table.

Score	85	86	87	88
Frequency	9	15	24	12

- 2 Consider each of the following datasets.

- Find the mean, mode and median.
- Discuss the relative merits of each measure of central tendency for representing the data.
 - 4, 5, 5, 6, 100
 - 4, 4, 4, 5, 6, 6, 7
 - 1, 1, 1, 1, 5, 6

- 3 Use the cumulative frequency polygon to find the:



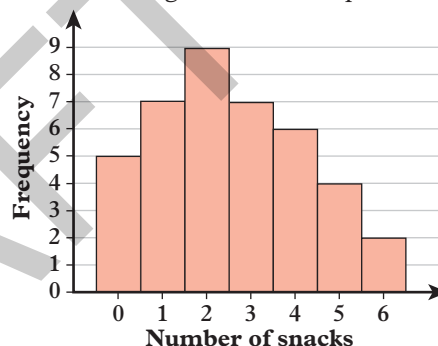
- median
 - upper quartile
 - lower quartile
 - interquartile range.
- 4 Which summary statistic is suitable for analysing categorical data?
- 5 Calculate the mean and standard deviation of the data given in this table.

Weight	Frequency
4.3	6
4.4	9
4.5	4
4.6	5
4.7	6

- 6 For the data values 22, 23, 23, 25, 26, 26, 27, 29, 31 and 31, find the:

- mean
- median
- range
- interquartile range
- standard deviation.

- 7 Consider this histogram for these questions.



- Draw a smooth curve representing the data in the histogram.
 - Describe the curve in terms of skewness.
- 8 Two groups of students are given pairs of shoes to wear to school. The students in group X are given rubber-soled shoes and those in group Y are given shoes with soles made of a new synthetic material. The thickness (in millimetres) of the soles of the shoes after 6 months is shown below.

Group X: 5, 3, 4, 5, 4, 3, 5, 6, 2, 6, 4, 4, 5, 3, 5, 6, 7, 6, 6, 3, 4, 5, 4, 5

Group Y: 3, 4, 5, 6, 5, 6, 4, 8, 5, 3, 4, 7, 6, 5, 2, 8, 5, 6, 5, 6, 8, 7, 3, 4

- For each group of results, find the:
 - mean
 - median.
- For each group of results, determine the:
 - range
 - standard deviation.
- For the given data, draw parallel box plots on the same scale.
- Is the new synthetic sole an improvement on the rubber soles? Justify your answer.

Review questions 12.9E Set 3

- 1 Consider the data in the table.

Class	Frequency
30–39	5
40–49	10
50–59	14
60–69	13
70–79	0
80–89	8

- Determine the modal class.
- Complete the table by adding a column for the class centres.
- Find the mean.

- 2 Draw a box plot for this dataset:

6 6 7 8 8 9 10 10 11 11 11 12

- 3 Use the data given in the table in question 1.

- Draw the cumulative frequency polygon.
- Using your cumulative frequency polygon from part **a**, estimate the:
 - lower quartile
 - upper quartile
 - interquartile range
 - median.

- 4 The number of goals scored by two footballers in 20 games are shown below:

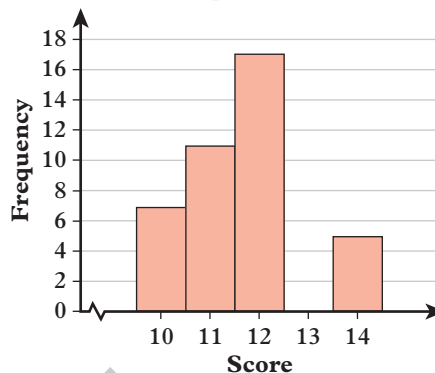
Adrian: 8, 9, 8, 6, 7, 7, 9, 4, 8, 8, 6, 7, 5, 7, 8, 8, 8, 9, 9, 10

Zoltan: 8, 7, 5, 7, 8, 9, 9, 9, 8, 5, 7, 10, 6, 6, 9, 5, 7, 8, 8, 10

- For each player, calculate the:
 - mean
 - median
 - range
 - interquartile range
 - standard deviation.
- Draw two box plots of the basketballers data on the same scale.

- Compare the box plots.
- Who is the better player, Adrian or Zoltan? Given reasons for your answer.

- 5 Consider the dataset represented in this histogram.



- Draw a smooth curve representing the shape of this data.
 - Describe the curve in terms of skewness.
- 6 The following are the scores in a language test:
- 60 61 59 50 65 77 63 58 36 88
66 74 61 43 58
- Draw a stem-and-leaf plot for this data.
 - Describe the shape of the distribution.
 - Use your stem-and-leaf plot to determine:
 - the range
 - the mode
 - the median.

- 7 Consider the following frequency distribution table.

x	f
3	1
4	1
5	1
6	3
7	5
8	1

- Find the five-number summary for this dataset.
- Formally identify any outliers.

Review questions 12.9F Set 4

- 1 Find the mean, mode, median, range and standard deviation of this dataset:

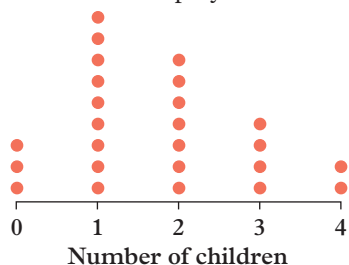
2 2 3 3 4 5 5 5 6

- 2 These are the results in a test given to two classes.

Class A: 24, 34, 16, 18, 30, 26, 36, 40, 22, 26, 32, 20, 8, 38, 14, 28, 32, 30, 24

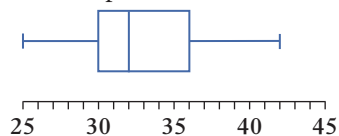
Class B: 6, 16, 28, 24, 24, 34, 38, 26, 22, 20, 28, 18, 24, 12, 36, 36, 22, 10, 2

- Construct back-to-back stem-and-leaf plots for this data.
 - For each class find the:
 - mean
 - median
 - range
 - standard deviation.
 - Describe the shape of the distribution for each class.
 - Use the median and interquartile range to formally identify any outliers.
 - Which class performed better? Explain.
- 3 Find the range and interquartile range of this dataset: 5, 5, 6, 7, 7, 8, 8, 8, 10.
- 4 Consider the dataset displayed in this dot plot.



- Draw a smooth curve representing the shape of this data.
- Describe the curve in terms of skewness.
- What does the shape of the curve tell you about the data?

- 5 Consider this box plot.



- From the box plot, determine the value of the:
 - lowest data value
 - lower quartile
 - median
 - upper quartile
 - highest data value.
 - Describe the shape of the distribution.
 - Comment on any clustering or outliers.
- 6 Consider the following stem-and-leaf plot.

Stem	Leaf
5	0
6	
7	9 9
8	1 2 8
9	1 1 4 6 6 6 7 8
10	0 0 1 3 4 5 8
11	3 6

- Calculate the median, Q_1 , Q_3 and IQR for the data shown.
- Calculate the limits for outliers in this data.
- Is 50 an outlier? Explain your answer.

Review questions 12.9G Practice examination question

- 1 Answer the following questions.

- a The following information is given about a set of 12 data value:

Lowest score = 3, Highest score = 17, Mean = 9.5, Median = 10, Mode = 11.

The highest score (17) is replaced by 71. For the new dataset, find the:

- median
 - mode
 - mean. (3 marks)
- b Calculate the standard deviation for the dataset given in this stem-and-leaf plot. (1 mark)

Stem	Leaf
1	0 6
2	1 1 2 5 8 8
3	3 5 6
4	1 1

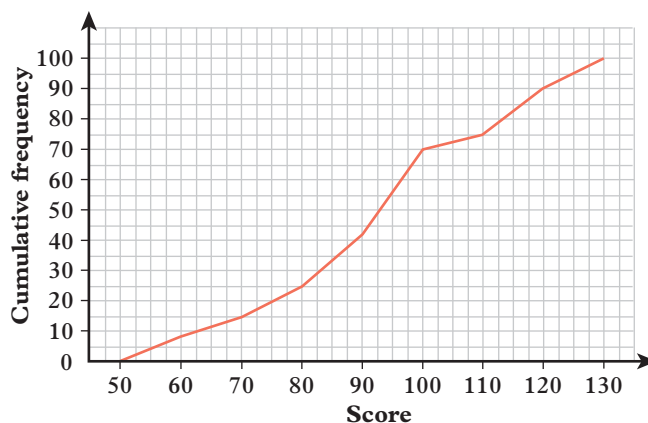
Key: 1 | 6 = 16

- c The annual incomes (\$) for a group of 12 people are given below.

40 000 43 000 32 000 29 000 37 000 75 000 60 000 51 000 52 000 45 000
38 000 25 000

- Determine the lower quartile, middle quartile and upper quartile for this data. (3 marks)
- Construct a box plot for the data. (2 marks)

- iii Use the median and interquartile range to formally identify any outliers. (3 marks)
- iv Calculate the standard deviation. (1 mark)
- d The cumulative frequency polygon for a set of data is shown below.
From the graph determine the:



- i first quartile (1 mark)
- ii interquartile range. (1 mark)

TOTAL: 15 marks

Checklist

Now that you have completed this module, reflect on your ability to do the following.



Module checklist: Data analysis

I can do this	I need to review this
☐ calculate mean and mode	Go back to Lesson 12.1 Mean, mode and modality
☐ calculate median	Go back to Lesson 12.2 Median
☐ compare datasets using measures of centre ☐ select the appropriate measure of centre to use for different datasets ☐ explain why a measure of centre is inappropriate for a given dataset	Go back to Lesson 12.3 Relative merits of mean, mode and median
☐ determine the five-number summary for a dataset ☐ calculate the range and interquartile range for a set of data	Go back to Lesson 12.4 Range and interquartile range
☐ recognise the five-number summary for data displayed in a box plot ☐ draw a box plot from a raw dataset ☐ draw a box plot from a displayed dataset	Go back to Lesson 12.5 Box plots
☐ describe the shape and spread of data displayed in a box plot ☐ draw parallel box plots to compare two datasets on the same axis ☐ compare measures of centre, spread and shape using parallel box plots	Go back to Lesson 12.6 Interpreting box plots

I can do this	I need to review this
☐ formally identify outliers using quartiles and the interquartile range	Go back to Lesson 12.7 Clusters and outliers
☐ describe the effect of outliers on different summary statistics	
☐ recognise clusters and gaps in data as they appear in different types of displays	
☐ calculate the standard deviation of a dataset	Go back to Lesson 12.8 Standard deviation
☐ compare the spread of different datasets using standard deviation, range and interquartile range	
☐ use a spreadsheet to calculate measures of centre	Go back to Using spreadsheets
☐ use a spreadsheet to calculate measures of spread	

Related resources



Module review quiz: Data analysis



Worksheet: Challenge

DRAFT

Review: Statistics

Review questions

Answers: Review: Statistics

1 Answer the following questions.

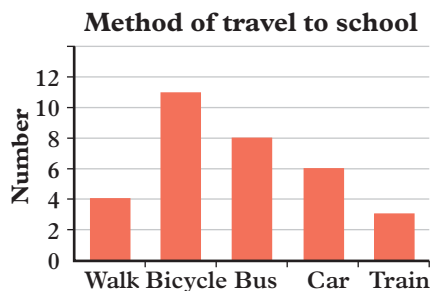
- a Classify the following data as nominal, ordinal, discrete or continuous.
 - i number of rainy days each month
 - ii the hair colours of students in your class
 - iii the lengths of fish in a pond
- b Explain how a random sample of 10 students may be chosen from a class of 120 students.
- c Explain what is meant by a “biased sample” in a statistical investigation.
- d An office supervisor wants to interview a sample of workers about a new computer system.
 - i Explain how four workers could be selected from the 27 people who work in the office.
 - ii Name three characteristics that might be relevant if the supervisor wants the sample to be representative of all 27 workers.
- e The numbers of students in each year at a school are shown in this table. In a stratified random sample selected on the basis of year groups, what proportion of students should be chosen from each year level?

Year	7	8	9	10	11	12	Total
Number of students	135	144	170	120	94	68	731

- f A sample is selected from a school population of 700. The 35 students chosen are believed to be representative of the entire school population. Of the students surveyed, 16 said they were happy with the school uniform and the rest were not.
 - i What percentage of students in the sample said they were happy with the uniform?
 - ii How many students from the entire school population should we expect to be happy with the uniform?
 - iii Suggest a potential flaw in the way the survey was written that could have led to results that do not reflect the true opinions of the whole school population.
 - g An electoral roll has 1800 names on it. At what intervals should we select names to obtain a systematic sample of:
 - i 100 names?
 - ii 200 names?
 - iii 150 names?
 - iv 40 names?
- 2 The results of a survey of the hair colours of a group of people are shown below using the codes black (B), brown (b), blonde (w), red (r) and grey (g).
- B B w w b b B B r w w b g B w B B b b w w w w b B
g g g B B
- a Organise the information into a frequency distribution table.
 - b How many people were surveyed?
 - c What was the most common hair colour in this group of people?

- d** Add a relative frequency column to your table from part **a** and find the relative frequency of people with:
- i** black hair
 - ii** blonde hair.
- e** Display the data above using:
- i** a dot plot
 - ii** a column graph.

- 3** Use this column graph to answer the following questions.



- a** How many students catch a bus to school?
 - b** How many students don't travel by car?
- 4** The results in a test given to two classes are shown below.
- Class A: 29, 39, 21, 23, 35, 31, 41, 45, 27, 31, 37, 25, 13, 43, 19, 33, 37, 35, 29
- Class B: 11, 21, 33, 29, 29, 39, 43, 31, 27, 25, 33, 23, 29, 17, 41, 41, 27, 15, 7
- a** Construct a back-to-back stem-and-leaf plot for this data.
 - b** Which class performed better? Explain your answer.
- 5** Consider the following frequency distribution table.

Score	Class centre	Frequency	Cumulative frequency
1–4	2.5	1	
5–8		4	
9–12		7	
13–16		12	
17–20		9	
21–24		8	
25–28		4	
29–32		5	

- a** Enter the missing values in the “Class centre” and “Cumulative frequency” columns.
 - b** Draw a cumulative frequency histogram and polygon to display the data.
- 6** A coffee tin was tossed 100 times and the way it landed was recorded as either “upright” or “sideways”. The total frequency of each is recorded in this frequency distribution table.

Landing position	Frequency
Upright	23
Sideways	77

- a** What was the percentage relative frequency of the tin landing upright?
- b** Construct a divided bar chart to display the data from this investigation.

- c Classify each variable as nominal, ordinal, discrete or continuous.
 - i landing position
 - ii frequency
- d Name two other data displays that would be suitable for this dataset.
- e Identify a type of data display that would *not* be suitable for this dataset and explain why.

7 Consider the following datasets.

- a Find the mean, mode, median and range of these scores:

6 6 6 7 8 8 8 8 10

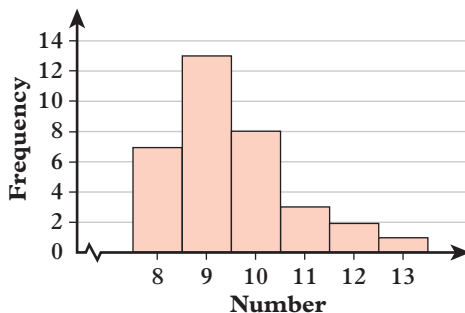
- b Consider the grouped data shown in the table.

Class	Frequency
0–<5	16
5–<10	8
10–<15	22
15–<20	30
20–<25	8

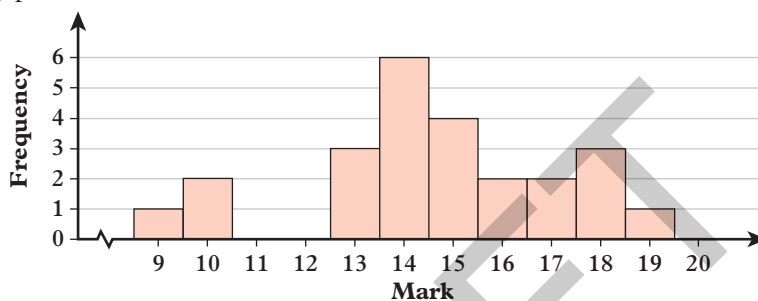
- i Copy the table and add a cumulative frequency column.
 - ii Draw a cumulative frequency histogram and polygon.
 - iii The cumulative frequency polygon can be used to estimate values for the median and quartiles of this dataset. Which three values on the (vertical) “Cumulative frequency” axis would you use in this process?
 - iv Use the cumulative frequency polygon to estimate the interquartile range.
- c For the values 8, 9, 10, 10, 10, 11, 11, 12, 12, 12, 12 and 13, find the:
 - i five-number summary
 - ii interquartile range.
 - d Organise the following dataset into a stem-and-leaf plot:
56, 58, 74, 62, 54, 50, 51, 73, 60, 71, 59, 65, 49, 78, 65, 56, 48, 48, 64, 57
 - e Use your stem-and-leaf plot from part d to determine the:
 - i range
 - ii mode
 - iii median
 - iv interquartile range.
 - f Draw a box plot for this set of scores: 6, 6, 7, 8, 8, 9, 10, 10, 10, 10, 11, 12.
 - g If the mean of a set of data is 11.3 and the range is 2.9, what is the new mean and range when:
 - i 5 is added to each score?
 - ii each score is multiplied by 5?
 - h Find the mean, mode, median, range and standard deviation (σ) of this set of scores: 2, 2, 3, 3, 3, 5, 5, 5, 6. Round your answer to three decimal places where necessary.

8 Consider the following histograms.

- a** Sketch a smooth curve on top of this histogram, then describe the shape of the distribution in terms of symmetry and skewness.



- b** Describe the shape of this dataset represented in this histogram by commenting on the range, as well as any clusters or gaps.



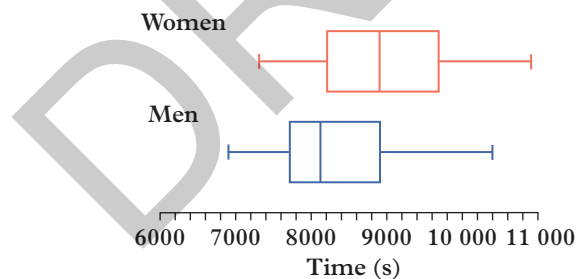
- c** Classify each of the distributions in parts **a** and **b** as unimodal, bimodal, multimodal or uniform.

9 The following scores are from a mathematics test: 40, 41, 39, 30, 45, 57, 43, 38, 16, 68, 36, 54, 41, 23, 38.

- a** Draw a stem-and-leaf plot of this data.

- b** Determine whether the distribution of scores is symmetric, positively skewed or negatively skewed.

10 These parallel box plots show the times (in seconds) for men and women competitors to complete a triathlon.



- a** What was the fastest time for this event, to the nearest 100 seconds?
- b** How long did it take the first half of the men to finish?
- c** How long did it take the first quarter of the women to finish?
- d** Which group had the greatest range of times?
- e** Which group had the smallest interquartile range of times?

Exam revision

Revision questions

Answers: Exam revision

- 1 The surface area of Venus is approximately $4.602 \times 10^8 \text{ km}^2$. Assuming Venus is a perfect sphere, find its volume, correct to 4 significant figures.

- 2 The “lower fence” L for a score to be considered an outlier is given by the formula:

$$L = Q_1 - 1.5 \times \text{IQR}.$$

- a Rewrite this formula using the variables L , Q_3 and Q_1 .

- b Solve the equation for Q_3 when $L = 16$ and $Q_1 = 25$.

- 3 Make b the subject of the formula $A = \frac{h}{2}(a + b)$.

- 4 The following figure is a pentagonal prism, where the pentagonal face is regular (all side lengths are the same).

- a The area of a regular pentagon can be approximated by the formula $A = 1.72s^2$, where s is the side length of the pentagon.

Solve this quadratic equation using $A = 10$ to find the side length s to two decimal places for the regular pentagon face in the given prism. Explain why there is a unique solution.

- b Using your answer to part a, find the surface area of this pentagonal prism.

- 5 This sector graph represents Lucas' expenses over the last fortnight. For this question, you do *not* need to consider any income tax.

- a The angle at the centre of the “utilities” sector is what percentage of 360° ?

- b Given that Lucas spends \$160 on groceries each fortnight, what are his total fortnightly expenses?

- c Lucas works a casual job while he is at university. He works 18 hours per week, at a rate of \$25 per hour. Based on your answer to part b, does Lucas earn more or less than his expenses, and by how much?

- d Lucas wants his rent to be no more than 30% of his income.

- i Using your answer to part b, and the information in the sector graph, determine the fortnightly cost of Lucas' rent.

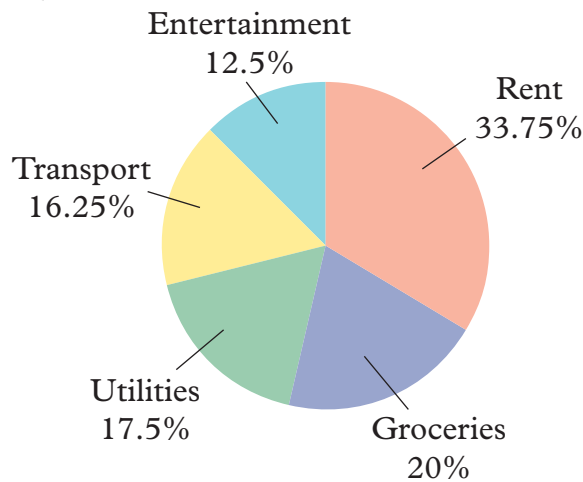
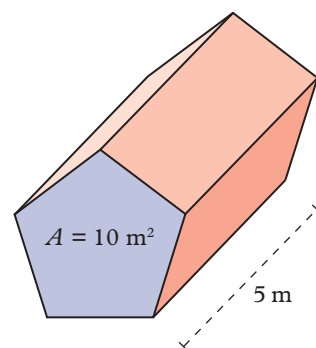
- ii Assuming his hourly rate from part c, how many (whole) hours per week would Lucas need to work for his rent to be no more than 30% of his income?

- 6 The GST of an item is 10% of the pre-GST price.

- a If an item costs \$275, including the GST, how much is the GST charge?

- b Let G represent the GST charged on an item and let C represent the total cost of the item, including GST. Explain how we know that G and C are in direct variation with each other.

- c Sketch the graph relating G and C . What is the gradient of this line?



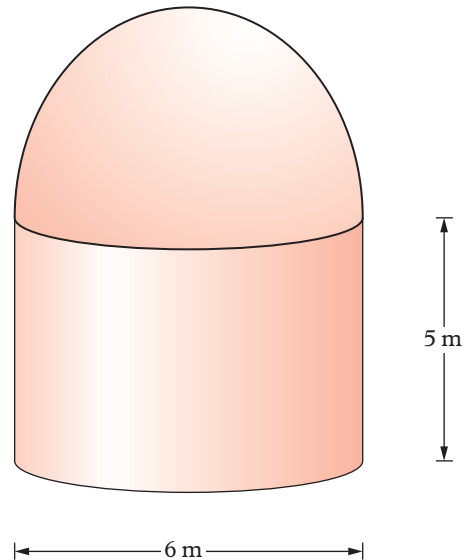
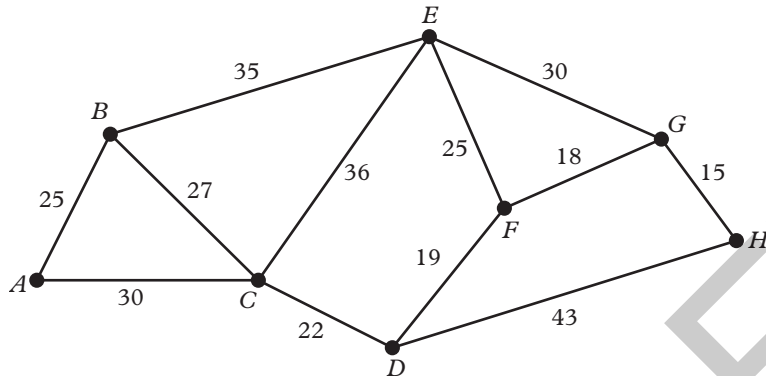
- 7 A water storage tank is designed to consist of a cylinder with a circular base and a hemisphere on top.

- a** The metal sheeting used to construct the tank costs \$50 per square metre. To account for cutting and wastage, it is recommended that the total area of metal sheeting purchased should be 10% more than the surface area of the tank.

How much will these materials cost for the water storage tank with dimensions shown in the diagram?

- b** If the tank contains 60 kL of water, what percentage of the tank is full? Round your answer to the nearest whole percent.

- 8 This network shows the distances (in metres) between eight buildings in a school.



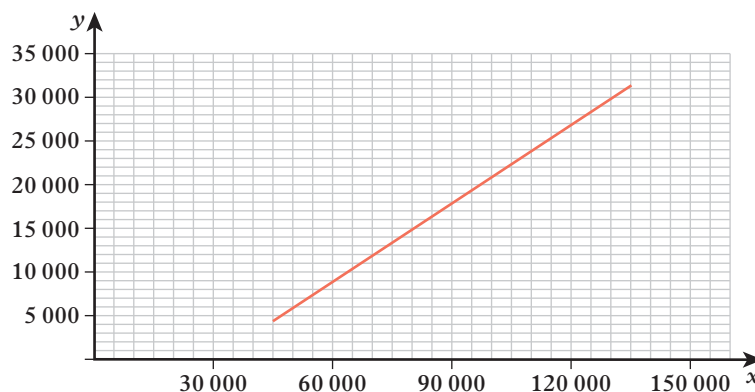
- a** The school must install wiring that will connect to all buildings. If the wiring costs \$7 per metre, and the minimal length of wiring is used, how much will it cost the school to connect all buildings with this wiring?
- b** Maylene is walking from A to H at an average speed of 1.6 m/s. Will she reach building H in less than 1 minute?
- 9 A plane flies from Sydney (UTC +10) to Singapore (UTC +8) nonstop. The flight leaves at 11 : 00 am Sydney time and arrives in Singapore at 5 : 10 pm Singapore time. The distance that the plane travels is approximately 6300 km.

What is the average speed of the plane?

- 10 Here is part of the Australian Tax Rates table for the 2024–25 financial year.

Taxable income	Tax on this income
\$45 001–\$135 000	\$4288 plus 30 cents for each \$1 over \$45 000

The graph below represents the linear relationship between taxable income in the given range (x) and the tax paid on this income (y).



- a** What is the gradient of this line, and what does it represent?
- b** Find the equation of the line in the form $y = mx + c$.
- c** Using your equation, find the taxable income someone earns when they pay \$9538 of income tax.

Answers

Module 1 Formulas and equations

Lesson 1.1 Substitution into expressions and formulas

- 1 a $-3, 15$ b $7, 13$
 2 a 7 b 3 c 19 d -8
 e 25 f 50 g 100 h 125
 i 13 j 28 k 49 l 29
 m $\frac{1}{2}$ n 95 o $2\frac{1}{2}$ p 2
 3 a -3 b 7 c -21 d 16
 e -1 f $-\frac{3}{2}$ g 49 h -21
 i -80 j -3 k -18 l -21
 4 a 10 b -4 c 4 d 10
 e 58 f 40 g 30 h 8
 i 5 j 10 k 4 l 7
 5 2, 35
 6 a 2500 b 1000 c 1200 d 5
 7 a 75 b 110 c 150 d 15.36
 8 a 904.8 cm^3 b 2572.4 cm^3 c 9.2 m^3
 9 a \$1610.51 c \$3300.80
 b \$1591.98 d \$1461.23
 10 a 5 b 13 c 25 d 20
 11 a Yes
 b No, $4a + 4b$
 c No, $6a + 2b$
 d No, $-2a - 5b$
 e No, $-2a - 5b$
 f Yes
 g No, $12a^2 + 13ab - 14b^2$
 h No, $12a^2 + 13ab - 14b^2$
 i No, $a^3 + 3a^2b + 3ab^2 + b^3$
 j Yes
 12 a i -4 ii -4 iii -4 iv -4
 b All of the expressions equal -4 .
 c No
 d Only expressions a i and iii are always equal.
 13 a 23.9, yes b 26.3, no c 22.2, yes
 14 a $10n + 50$
 b \$110
 15 a $3t + 2f + p$ where t represents the number of goals scored from behind the three-point line, f represents the number of goals scored inside the three-point line and p represents the number of goals scored on a foul shot
 b 43
 c One possible answer is $t = 10, f = 17$ and $p = 4$.

16 a $6t + 5$ where t represents time in hours

b 8 km c 1.5 h

17 2.1

Lesson 1.2 Solving linear equations

- 1 a 5, 32, not a solution b 8, 24, a solution
 2 a Yes b No c Yes
 d Yes e Yes f No
 g No h Yes i Yes
 3 a 3, 35, 7, 5 b $7, 4x, 4x, 4$
 4 a $x = 4$ b $p = 4$ c $x = 14$
 d $x = 9$ e $p = 10$ f $y = 10$
 g $x = 24$ h $p = 4$ i $m = 6$
 j $q = 34$ k $s = 33$ l $r = 13$
 m $t = 5$ n $s = 9$ o $m = 16$
 5 a 6, 8, $x, -1\frac{3}{5}$ b $7, 7, 5x, 5, 5, -3\frac{3}{5}$
 6 a $x = -\frac{2}{3}$ b $x = 1$ c $x = -3\frac{1}{2}$
 d $x = 11$ e $x = -3$ f $x = -2$
 g $x = 3$ h $x = \frac{5}{7}$ i $x = 9$
 j $x = -4$ k $x = 8$ l $x = -2\frac{1}{2}$
 m $x = -1$ n $x = 3$ o $x = -5$
 7 3, 3, 4, 5, 20
 8 a $x = 10$ b $x = 10$ c $x = -25$
 d $x = -42$ e $x = 42$ f $x = 50$
 9 a No b Yes c Yes d No
 10 $-3, 11, -14, 7, -2$
 11 a $x = -3$ b $x = \frac{9}{4} = 2\frac{1}{4}$
 c $x = -\frac{1}{5}$ d $x = \frac{7}{3} = 2\frac{1}{3}$
 e $x = \frac{10}{7} = 1\frac{3}{7}$ f $x = \frac{5}{3} = 1\frac{2}{3}$
 12 a $3x - 5 = 13, x = 6$ c $9 + 5x = 34, x = 5$
 b $4x - 7 = 13, x = 5$ d $7x + 8 = 29, x = 3$
 13 a $x + 1$
 b $x + x + 1 = 275$ or $2x + 1 = 275$
 c $x = 137$
 The two consecutive numbers are 137 and 138.
 14 a Let x be the first integer, hence the second integer is $x + 1$.
 $x + x + 1 = 127$ or $2x + 1 = 127, x = 63$.
 The two consecutive integers are 63 and 64.
 b Let x be the first integer, hence the second integer is $x + 1$ and the third integer is $x + 2$.
 $x + x + 1 + x + 2 = 27$ or $3x + 3 = 27, x = 8$
 The smallest number is 8.
 c Let x be the first integer, hence the second integer is $x + 1$, the third integer is $x + 2$ and the fourth integer is $x + 3$.
 $x + x + 1 + x + 2 + x + 3 = -6$ or $4x + 6 = -6$,
 $x = -3$
 The largest number is 0.

- d** Let x be the first even integer, hence the second even integer is $x + 2$ and the third even integer is $x + 4$.
 $x + x + 2 + x + 4 = 132$ or $3x + 6 = 132$, $x = 42$
 The smallest number is 42.
- 15 a** Let x be the number of weeks.
b $67 + 28x = 235$
c $x = 6$
 Tyler can buy the bat in 6 weeks.
- 16 a** Let d be the cost of one dress in dollars.
 $3d + 19 = 196$
 $d = 59$
 The cost of each dress is \$59.
b Let t be the time in hours.
 $14t + 9 = 85$
 $t = 5.4$
 The tennis court can be hired for 5 hours (for a total cost of \$79).
c Let n be the number of goals scored by Andrew.
 $n + n + 5 = 37$
 $2n + 5 = 37$
 $n = 16$
 Andrew scored 16 goals.
d Let n be the number of people.
 $28n + 325 = 1200$
 $n = 31.25$
 31 people can attend the party.
- 17 a** $b + 14$ **b** $P = 4b + 28$ **c** $b = 32$
 The length is 46 m and the breadth is 32 m.
- 18 a** Let g be the number of goals scored, b the number of behinds scored and n be the total number of points scored.
 $n = 6g + b$
b 75 points
c i Kangaroos: 48 points, Swans: 50 points
ii The winner is the Swans.
iii Eagles: 141 points, Suns: 127 points
iv The winner is the Eagles.
d i 7 **ii** 14
e i 13 **ii** 19
f $n = 6g + b + 3p$ where p is the number of times a player hits the goal post instead of kicking a goal.
g 5
- 19** Let n be the number of fish caught by Marcel.
 Davina catches $(n + 4)$ fish, and Hayley catches $2n$ fish.
a The total number of fish caught is unknown so we cannot form an equation to solve.
b $n + 4 + 2n + n = 65$ or $4n + 4 = 65$
 $n = 15.25$
- 20 a** For $x = 5$, $y = 4$. **b** For $x = 2$, $y = 3$.
c The pairs of integers are: (2, 3), (5, 4), (8, 5), (11, 6), (14, 7), (17, 8), (20, 9).

Lesson 1.3 Solving further linear equations

- 1** 2, 5, 9, 5, 4
2 a $x = \frac{1}{3}$ **b** $x = 1$ **c** $x = \frac{1}{2}$
d $x = \frac{1}{5}$ **e** $x = \frac{1}{3}$ **f** $x = \frac{3}{25}$
- 3** 5, 35, 8, 4
4 a $x = 15$ **b** $x = \frac{7}{3}$ or $2\frac{1}{3}$
c $x = \frac{5}{2}$ or $2\frac{1}{2}$ **d** $x = 6$
e $x = \frac{14}{15}$ **f** $x = \frac{10}{3}$ or $3\frac{1}{3}$
- 5** 4, 4, 1, 20, 1, 1, 19, 3, 19, 3, $6\frac{1}{3}$
6 a $x = -10$ **b** $x = 17$ **c** $x = -20$
d $x = -5$ **e** $x = -2\frac{1}{3}$ **f** $x = 24$
g $x = \frac{1}{2}$ **h** $x = 4\frac{2}{3}$ **i** $x = 4\frac{1}{2}$
- 7** $2x$, 2, 2, 6, 2, 6, 3
8 a $x = 4$ **b** $x = 1$ **c** $x = 1$
d $x = -3$ **e** $x = -2$ **f** $x = -2$
g $x = 9$ **h** $x = -10$ **i** $x = 3$
- 9 a** $x = -4\frac{1}{2}$ **b** $x = 1\frac{1}{3}$ **c** $x = 1\frac{1}{5}$
d $x = -3\frac{1}{3}$ **e** $x = -3\frac{1}{2}$ **f** $x = -6\frac{1}{2}$
g $x = \frac{4}{5}$ **h** $x = -14$ **i** $x = -\frac{1}{5}$
j $a = -1\frac{1}{2}$ **k** $s = -2\frac{1}{5}$ **l** $x = 1\frac{3}{4}$
m $a = 3\frac{2}{3}$ **n** $y = -1\frac{1}{8}$ **o** $p = 1\frac{4}{11}$
- 10** 3, 9, -2, 3, -2, is
11 a Yes **b** Yes **c** No
d Yes **e** Yes **f** No
- 12** Let x be the unknown number in each case.
a $2x + 3 = x + 7$ **b** $3x + 8 = x + 4$
 $x = 4$ $x = -2$
c $2x + 6 = 4x + 4$ **d** $5x - 9 = 2x - 3$
 $x = 1$ $x = 2$
- 13 a** $30 = \frac{5}{9}(F - 32)$ **b** $F = 86$
c 86°F **d** 64.4°F
e 25°C because $25^\circ\text{C} = 77^\circ\text{F}$
- 14 a** C
b $n = 11$, so there are 11 biscuits in a box.
- 15 a** Let x be the cost (\$) for 1 kg of cherries.
 $5x + 2 = 2x + 20$
b $x = 6$
 1 kg of cherries costs \$6.
- 16** Let x be the cost (\$) for a sushi roll.
 $7x + 1.5 = 4x + 12$
 $x = 3.5$
 A sushi roll costs \$3.50.
- 17** Let x be the result (%) Hayden received in the fourth test.
 $\frac{91 + 69 + 88 + x}{4} = 78$
 $x = 64$
 The fourth test result is 64%.

18 Let x be the first even integer.

$$x + x + 2 = x + 4 + 6 \text{ or } 2x + 2 = x + 10$$

$$x = 8$$

The integers are 8, 10 and 12.

19 a $x = \frac{1}{8}$ b $x = \frac{7}{9}$ c $x = \frac{2}{35}$
 d $x = 1\frac{3}{8}$ e $x = \frac{7}{15}$ f $x = 1\frac{5}{6}$
 g $x = -\frac{1}{9}$ h $x = -1\frac{2}{5}$ i $x = -2\frac{1}{2}$

20 a Let x be the current age (in years) of the son.

$$3x + 11 = 2(x + 11)$$

$$x = 11$$

The son is currently 11 years old.

b Let x be the current age (in years) of Wesley.

$$x + 8 + 3 = 2(x + 3)$$

$$x = 5$$

Wesley is currently 5 years old.

c Let x be the current age (in years) of Sam.

$$2(45 - x) = 3x + 5$$

$$x = 17$$

Sam is currently 17 years old.

21 $h = 8.50$

Lesson 1.4 Variables and formulas

- 1 a $t = 12$ b $a = 3$
 2 a $c = 25$ b $t = 40$
 3 $K = 54$
 4 $R = 14.4$
 5 $m = 61.1$ (to one decimal place)
 6 a $P = 26\,000$ b $n = 7$
 7 $u = 5$
 8 $a = 3$
 9 $a = 2$
 10 $n = 6$
 11 a $R = 6\frac{1}{4}$ b $R = 10$
 12 a $E = 144$ b $R = 9$
 13 a $x = y - 2$ b $x = y - 5$ c $x = y - 8$
 d $x = \frac{p}{5}$ e $x = \frac{q}{7}$ f $x = \frac{r}{2}$
 g $x = 3t$ h $x = 5r$ i $x = 8m$
 j $x = p + 3$ k $x = m + 7$ l $x = z + 1$
 14 a $x = z - y$ b $x = m - n$ c $x = p + q$
 d $x = n + k$ e $x = \frac{c}{a}$ f $x = \frac{e}{d}$
 g $x = vw$ h $x = kt$ i $x = p - q$
 j $x = k + m$ k $x = \frac{a}{b}$ l $x = bt$
 15 a $x = \frac{mp}{q}$ b $x = \frac{kt}{m}$ c $x = \frac{ay}{b}$
 d $x = \frac{k}{at}$ e $x = \frac{d}{ez}$ f $x = \frac{m}{kn}$
 16 a $x = \frac{y-2}{3}$ b $x = \frac{y-1}{5}$ c $x = \frac{y-5}{2}$
 d $x = \frac{y+4}{7}$ e $x = \frac{y+5}{6}$ f $x = \frac{y+5}{3}$
 g $x = \frac{m-3}{2}$ h $x = \frac{m-5}{3}$ i $x = \frac{m-7}{5}$

j $x = \frac{6-m}{5}$ k $x = \frac{4-m}{3}$ l $x = \frac{m+8}{6}$
 m $x = \frac{y-4}{m}$ n $x = \frac{y+3}{m}$ o $x = \frac{y-c}{m}$
 p $x = \frac{y}{mz}$
 17 a $y = \frac{c-a}{b}$ b $y = \frac{r-p}{q}$ c $y = \frac{r+p}{x}$
 d $y = \frac{a-b}{c}$ e $y = \frac{x-t}{k}$ f $y = \frac{4-c}{r}$
 g $y = \frac{r+a}{b}$ h $y = \frac{r-k}{p}$ i $y = \frac{r+px}{q}$
 18 a $N = 3.5$ b $N = 0.5$

19 $l = 16.9$

20 $V = 113.4$

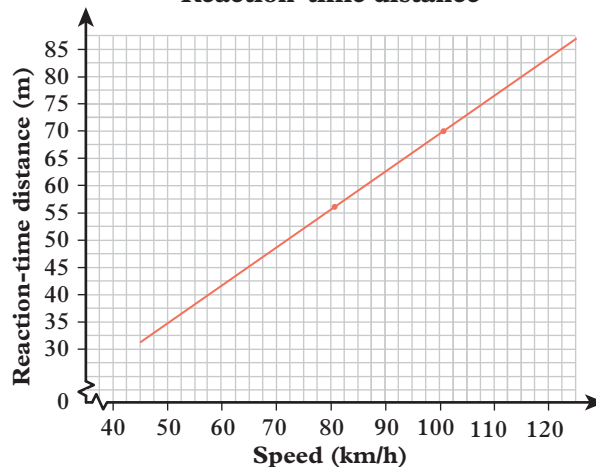
21 $v = \pm 9.8$

22 a $r = \sqrt[3]{\frac{3V}{4\pi}}$ b $r = 1 - \sqrt[n]{\frac{S}{V_0}}$

Lesson 1.5 Travel calculations

- 1 a 46 km/h b 73 km/h
 c 47 km/h d 59 km/h
 2 a 224 km b 187 km
 c 474 km d 166 km
 3 a 8 h 6 min b 4 h 2 min
 c 4 h 27 min d 56 min
 4 a $70 \text{ km/h} = \frac{70 \times 1000}{60 \times 60} \text{ m/s} = 19.4 \text{ m/s}$ (to one decimal place)
 b $8 \text{ m/s} = \frac{8 \times 60 \times 60}{1000} \text{ km/h} = 28.8 \text{ km/h}$
 5 a 12.5 m/s b 21.1 m/s c 30.6 m/s
 6 a 54 km/h b 45 km/h c 90 km/h
 7 55.6 m (to one decimal place)
 8 69.4 m (to one decimal place)
 9 a

Reaction-time distance



- b i 83 m ii 31 m
 c About 7 m
 10 64 m
 11 a 100 m b 140 m c 40 m
 12 90, 1, 90, 1000, 60, 60, 90, 1000, 60, 60, 2.5, 62.5, 90, 81, 62.5, 81, 143.5, 144
 13 197.4 m
 14 a 58.3 m b 16.7 m
 15 55.6 m

16 58.8 m, 16.8 m, 56 m

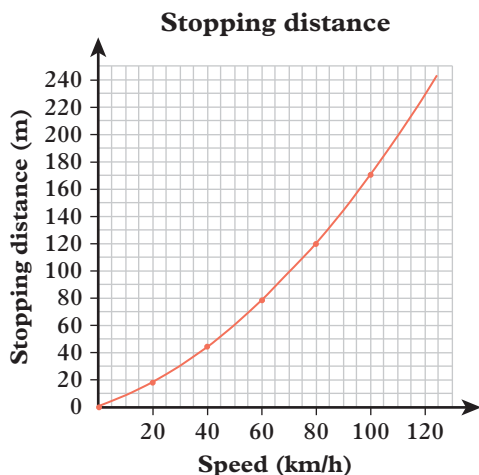
17 18 m

18 98 m

19 a

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)	0	18	44	78	120	170

b

c i ≈ 60 mii ≈ 230 m

20 d = 60 m

The stopping distance is 60 m instead of 37 m.

Lesson 1.6 Blood alcohol content calculations

1 a 0.375, 1.4

b 2.7, 0.8

2 a 1.1

b 1.2

c 1.8

d 6.8

e 1.1

f 7.7

3 a $BAC_{\text{male}} = \frac{10 \times 6 - 7.5 \times 3}{6.8 \times 83} \approx 0.07$ b $BAC_{\text{female}} = \frac{10 \times 5 - 7.5 \times 4}{5.5 \times 58} \approx 0.06$ 4 a $BAC \approx 0.03$ b $BAC \approx 0.09$ c $BAC \approx 0.06$ d $BAC \approx 0.02$ 5 $0.05 = \frac{10N - 7.5 \times 5}{6.8 \times 80} = \frac{10N - 37.5}{544}$ $27.2 = 10N - 37.5$ $64.7 = 10N$ $N = 6.47$

If he has 6.47 standard drinks, his BAC = 0.05.

For his $BAC < 0.05$ he can have up to 6 standard drinks.

6 Up to 3 standard drinks

7 a 5

b 7

c 4

d 6

8 6 standard drinks, 3 standard drinks

9 a 2 h

b 1 h

c 2 h 30 min

d 50 min

e 1 h 20 min

10 $T = \frac{0.05}{0.015} = 3.33... \text{ h} = 3 \text{ h } 20 \text{ min}$

11 a 2 h 40 min

b 1 h 20 min

c 4 h

d 2 h 20 min

12 a Male: 0.0321... ≈ 0.03 Female: 0.0611... ≈ 0.06

b Male: 2 h 9 min

Female: 4 h 5 min

13 a Damien: 0.14

b Damien: 10:20 am

Nicole: 0.17

Nicole: 12:20 pm

14 a 7.2

b 0.0857... ≈ 0.09

c 5 h 43 min

15 a 0.07

b 2 hours

c 5 hours

Lesson 1.7 Medication calculations

1 a 2

b i 8

ii 6

c No, as the child took three tablets in 24 hours, the maximum allowed of four tablets has not been exceeded.

2 a 2

b Yes, the maximum is 6.

3 a 5 mL

b 5.6 mL

c 3.6 mL

d 4.3 mL

e 11.3 mL

f 12.3 mL

4 a 53 mL

b 95 mL

5 a 17 mL

b 16 mL

c 22 mL

d 8 mL

e 34 mL

f 31 mL

6 a 25 mL

b 15 mL

7 a 10 mL

b 23 mL

c 58 mL

d 42 mL

e 46 mL

f 20 mL

8 a 36 mL

b 48 mL

9 20 kg

10 a 2.1 mL

b 2 mL

c 2.3 mL

d 2 mL

all about 2 mL

11 a 4 mL

b 3.7 mL

c 5.1 mL

d 4 mL

closest to the mean of all three

12 a 40 mL

b 23 mL

c No, but close to overdose

13 $y = 7.5$ years old

Lesson 1.8 Solving simple quadratic equations

1 a $x = \pm 5$ b $x = \pm 1$ c $x = \pm 7$ d $x = 0$ e $x = \pm 9$ f $x = \pm 6$ g $x = \pm 5$ h $x = \pm 10$ 2 a $x = \pm \frac{9}{2}$ b $x = \pm \frac{7}{3}$ c $x = \pm \frac{3}{2}$ d $x = \pm \frac{11}{5}$ e $x = \pm \frac{1}{5}$ f $x = \pm \frac{3}{8}$ 3 a $x = \pm 0.1$ b $x = \pm 0.5$ c $x = \pm 0.6$ d $x = \pm 0.9$

- 4 **a** $x = \pm 4$ **b** $x = \pm 2$ **c** $x = \pm 1$
d $x = \pm 20$ **e** $x = \pm 6$ **f** $x = \pm 3$
g $x = \pm 2.5$ **h** $x = \pm 1.5$ **i** $x = \pm 1.6$
- 5 **a** $x \approx \pm 0.82$ **b** $x \approx \pm 1.18$ **c** $x \approx \pm 1.66$
d $x \approx \pm 1.84$ **e** $x \approx \pm 0.95$ **f** $x \approx \pm 3.87$
- 6 **a** $x \approx \pm 1.18$ **b** $x \approx \pm 1.37$
c $x \approx \pm 3.70$ **d** $x \approx \pm 1.86$
- 7 **a** $a \approx \pm 2.7$ **b** $p = 0$
c $M = \pm 0.06$ **d** $k = \pm 0.52$

- 8 $s = 4$ cm
 9 $v = 65$ km/h
 10 $v = 58$ km/h
 11 $v_1 - v_2 = 5$ km/h

12 **a** No solutions

b $y = 11$

c There are zero solutions to part **a**, since $-\frac{1}{2} < 0$.
 There is one solution to part **b**, which makes sense because y is the subject of the equation and can therefore be evaluated directly.

13 **a** $y = -4$ **b** Any value $y > -4$

14 **a, f, g, j, k**

15 Any equation of the form $y = ax^2 + c$, where $-\frac{c}{a} < 0$ has no real solutions, since we cannot take the square root of a negative number.

16 $t \approx 0.36$

Using spreadsheets

- 1 $x = 4$
- 2 **a** $x = 8$ **b** $x = 6$ **c** $x = 7$
d $x = 3$ **e** $x = 5$ **f** $x = 4$
- 3 **a** $x = -8$ **b** $x = -11$ **c** $x = -6$
- 4 Find the integers between which the answer lies. Then use smaller increments between these values. Otherwise guess, check and refine.
- 5 **a** $x = 2.6$ **b** $x = -7.75$ **c** $x = -3.4$

Lesson 1.9 Review: Formulas and equations

Review questions 1.9A

- 1 **a** pronumeral
b solve, pronumeral/variable, solution
- 2 A formula is an equation which includes two or more variables.
- 3 Answers will vary.
 Linear equations: $x = 2$, $3x + 4 = 7$, $7x - 2 = 9x + 3$
 Quadratic equations: $x^2 = 4$, $3 - x^2 = 0$,
 $2x^2 - 1 = 3$

4
$$\underbrace{\frac{\text{term}}{5x} - \frac{\text{term}}{2y} + 1}_{\text{expression}} = \underbrace{\frac{\text{term}}{7y^2} + \frac{\text{term}}{x}}_{\text{expression}}$$

- 5 **a** standard drinks, blood alcohol content
b total stopping distance, average speed, reaction-time distance, braking distance

Review questions 1.9B

- 1 **D** 2 **A** 3 **B** 4 **D** 5 **B**
 6 **D** 7 **B** 8 **A** 9 **C** 10 **C**
 11 **A** 12 **B** 13 **D** 14 **C** 15 **D**
 16 **B** 17 **A** 18 **C** 19 **B**

Review questions 1.9C

- 1 -18
- 2 **a** 1437 cm³ **b** 180 cm³ **c** 0.9 m³
- 3 **a** $F = 392^\circ$ **b** $F = 68^\circ$ **c** $F = 131^\circ$
- 4 **a** $d = 44$ **b** $x = 11$ **c** $x = 576$
d $x = 5$ **e** $x = \pm 7$ **f** $x = 24$
g $n = \pm 4$ **h** $c = -12\frac{1}{6}$ **i** $x = \frac{1}{6}$
- 5 $v = 20$
- 6 **a** $x = \frac{t}{3}$ **b** $x = \frac{y+7}{m}$ **c** $x = ac - d$
- 7 150 m
- 8 218.4 km
- 9 4 h 40 min
- 10 3.6 mL
- 11 15.8 mL

Review questions 1.9D

- 1 -31
- 2 **a** 201 cm² **b** 707 cm² **c** 8 m²
- 3 **a** $C = 120^\circ$ **b** $C = 10^\circ$ **c** $C = 0^\circ$
- 4 **a** $d = 32$ **b** $x = 61$ **c** $x = 15$
d $x = \pm 11$ **e** $x = 12\frac{1}{3}$ **f** $x = \pm 3$
g $c = -3\frac{4}{9}$ **h** $x = -2\frac{13}{14}$ **i** $x = 3\frac{2}{3}$
- 5 $b = 12$
- 6 **a** $x = m - y$ **b** $x = \frac{k-n}{3}$
c $x = k(m+4)$
- 7 58.4 km/h
- 8 0.045
- 9 15.8 mL
- 10 6.4 mL

Review questions 1.9E

- 1 52
- 2 2356 cm²
- 3 3.1 s
- 4 **a** $d = 20$ **b** $x = 10$ **c** $x = 6$
d $x = 7$ **e** $x = 6\frac{2}{3}$ **f** $x = 4\frac{3}{7}$
g $n = 3$ **h** $c = \pm 2$ **i** $x = \frac{1}{4}$
- 5 $n = 20$
- 6 **a** $x = ty$ **b** $x = \pm \sqrt{\frac{k-y}{m}}$ **c** $x = wy + t$
- 7 143.5 m
- 8 1.4
- 9 **a** 0.053 **b** 0.035

10 4 h

11 a Male: 0.0343, female: 0.06

b Male: 2 h 17 min, female: 4 h

12 19 mL

Review questions 1.9F

1 908

2 11 310 cm³3 $B = 9.4$ mm4 a $d = 16$ b $x = 7$ c $x = 12$ d $x = 11$ e $x = 32$ f $x = \pm\sqrt{3}$ g $n = 6$ h $c = -\frac{4}{9}$ i $x = -\frac{7}{12}$ 5 $h = 1.6$ 6 $R = 50$

7 1 h 52 min

8 a 15.3 m/s

b 77 km/h

9 42 m

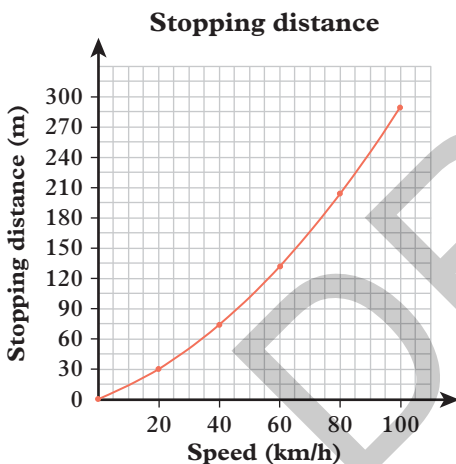
10 64 m

11 120 m

12 a

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)	0	29	73	131	203	290

b



c i 100 m

ii 165 m

iii 340 m

13 9 mL

Review questions 1.9G1 a $x = -9$

b 8

c i The height must be in metres; that is, 1.68 m not 168 cm.

ii 19.5

iii No, her BMI is too low.

d He added 6 instead of subtracting 6, then divided by 14 not -5 . Solution is $x = -\frac{2}{5}$.

e i 78 m

ii 20 m

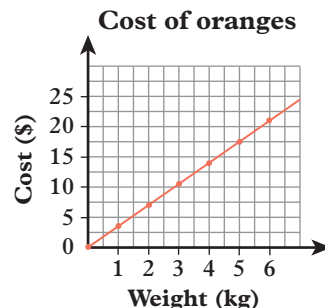
f i 0.075... $\approx$ 0.08

ii 5 h

g 27 mL

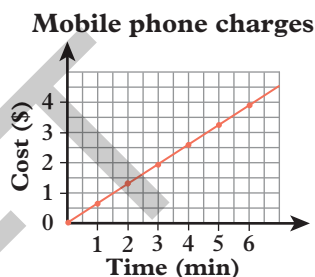
Module 2 Linear relationships**Lesson 2.1** Visualising linear models

1 a Dependent variable is cost.



b About \$9.00

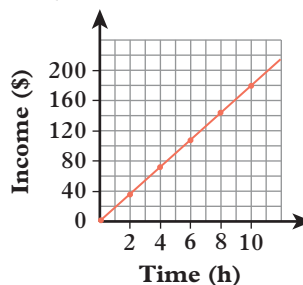
2 a Independent variable is time.



b \$1.60

c 4.7 min

3 a

Rema's income

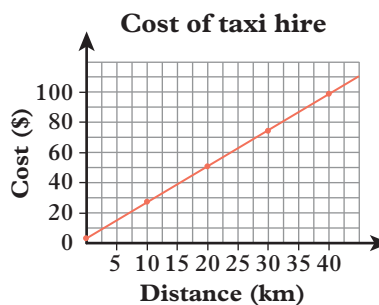
b \$130

c 5 h

4 a

Distance (km)	0	10	20	30	40
Cost (\$)	\$3.00	\$27.00	\$51.00	\$75.00	\$99.00

b



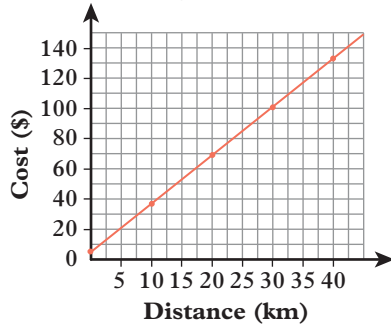
c \$87.00

d 5 km

5 a

Distance (km)	0	10	20	30	40
Cost (\$)	\$5.00	\$37.00	\$69.00	\$101.00	\$133.00

b Cost of night-time taxi hire



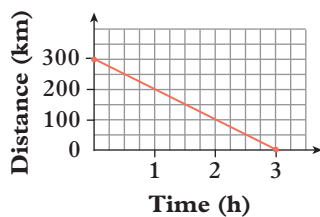
c \$53

d 22 km

6 a

Time (h)	0	1	2	3
Distance (km)	300	200	100	0

b Distance to Sydney

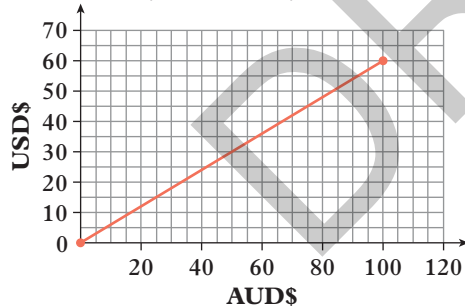


c 150 km

d 200 km

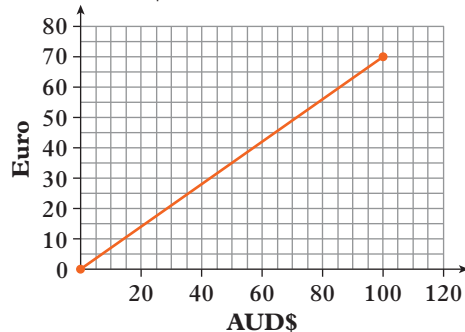
7 USD\$40 is about AUD\$67.

AUD\$ and USD\$ conversion



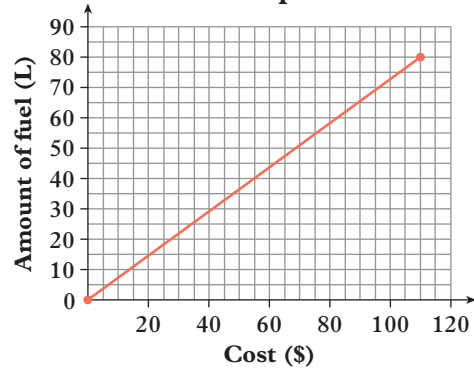
8 €60 (Euro) is about AUD \$85.

AUD\$ and Euro conversion



9 a

Cost of petrol

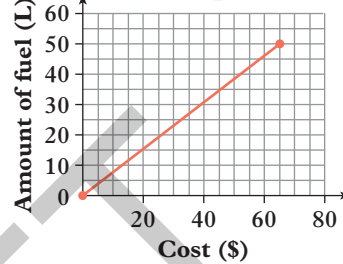


b About 50 L

c About \$75

10 a

Cost of petrol

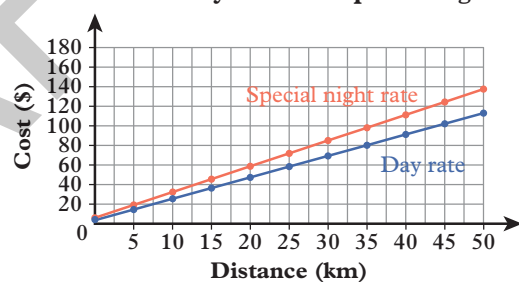


b About 20 L

c About \$45

11 a

Taxi fares at day rate and special night rate



b About 8 km

c About \$20

12 About \$46

Lesson 2.2 Understanding gradient

1 a

Line segment	Horizontal run	Vertical rise	Slope
AB	6	0	0
CD	7	1	$\frac{1}{7}$
EF	7	2	$\frac{2}{7}$
GH	6	3	$\frac{1}{2}$
IJ	3	5	$\frac{5}{3}$
KL	1	7	7
MN	0	5	undefined

b 0; undefined; increase

2 a $\frac{3}{4}$

b $\frac{150}{1000} = \frac{3}{20}$

- 3 a 0 b $\frac{1}{5}$ c $\frac{3}{5}$
 d $\frac{7}{6}$ e $\frac{7}{3}$ f $\frac{9}{2}$
 g Undefined

4 a

Line segment	Run	Rise	$\frac{\text{rise}}{\text{run}}$
BC	2	1	$\frac{1}{2}$
DE	2	1	$\frac{1}{2}$
AC	4	2	$\frac{1}{2}$
BE	6	3	$\frac{1}{2}$
AE	8	4	$\frac{1}{2}$
AF	9	$4\frac{1}{2}$	$\frac{1}{2}$

b No matter which pair of points on a straight line are used, the gradient is the same.

- 5 a 0 b $-\frac{1}{3}$ c $-\frac{1}{2}$
 d $-\frac{3}{4}$ e $-\frac{3}{2}$ f -7
 g Undefined

- 6 a OP, PQ, RS, TU b QR, ST, UV
 c TU d ST
 e VW f PQ

- 7 a 4 b -2 c -4
 d $1\frac{1}{2}$ e $-\frac{3}{4}$ f 6

- 8 a Gradient = $\frac{1}{3}$
 b Gradient = -4
 c Gradient = $2\frac{1}{2}$

- 9 a $13\frac{1}{3}$ b 2 c -6
 d -0.008 e $\frac{7}{25}$ f $\frac{6}{25}$

- 10 a About \$11 500
 b Just over \$58 000
 c Gradient = $\frac{4}{10} = 0.4$; that is, 40 cents per dollar in this tax bracket
 d Gradient = $\frac{6}{10} = 0.6$; that is, 60 cents per dollar in this tax bracket

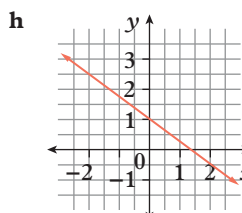
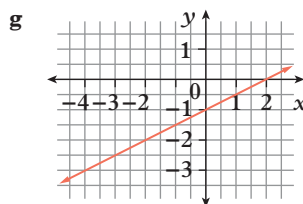
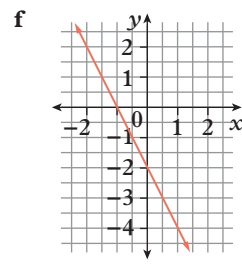
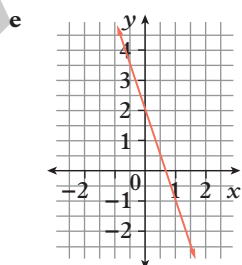
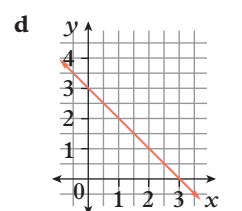
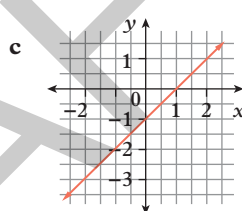
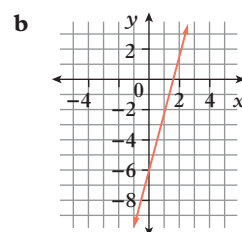
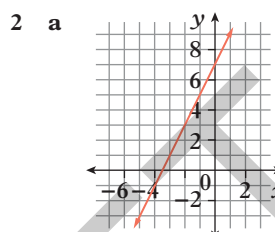
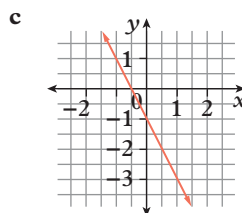
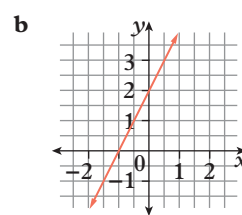
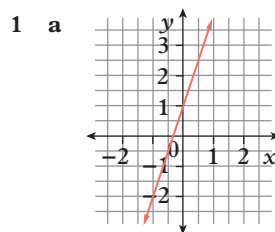
- 11 a \$10 000
 b Just over \$140 000
 c Gradient = 0.15; that is, 15 cents per dollar in this tax bracket
 d Gradient = 0.21; that is, 21 cents per dollar in this tax bracket
 e Gradient = 0.09; that is, 9 cents per dollar in this tax bracket
 f Approx. 23 cents in the dollar

- 12 a Gradient = -7

b

x	5	7	10	14
y	97	83	62	34

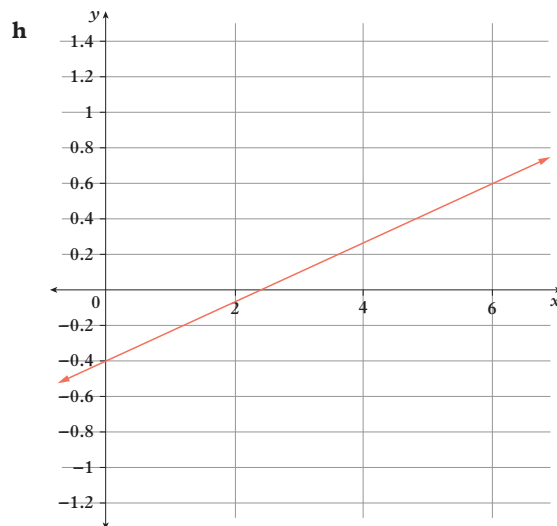
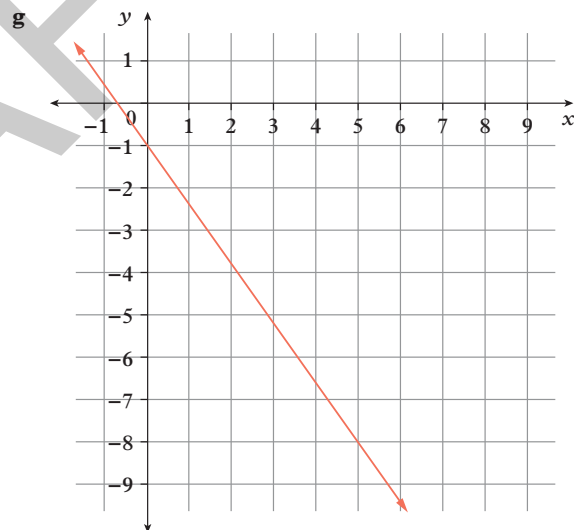
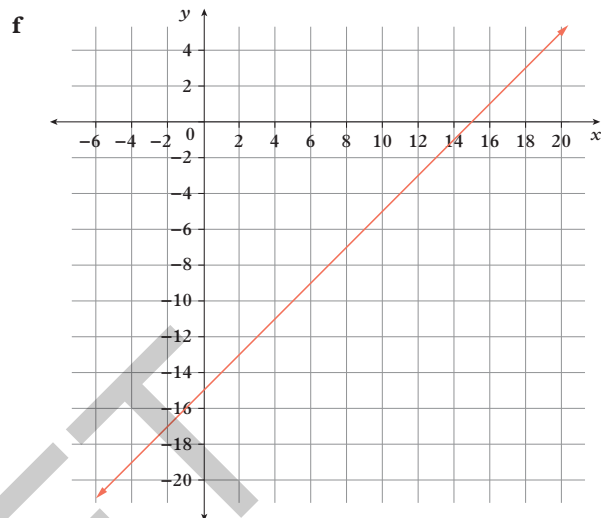
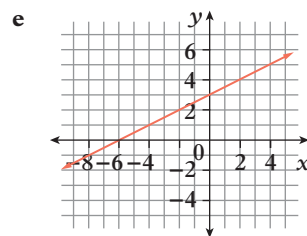
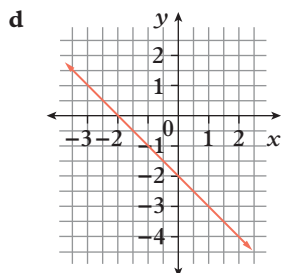
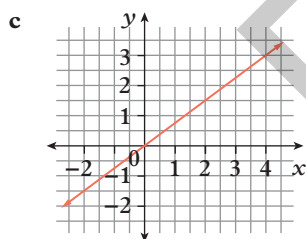
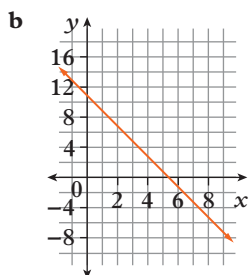
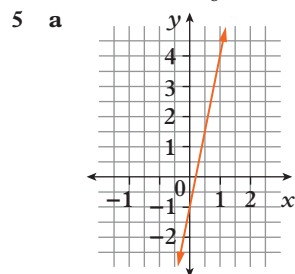
Lesson 2.3 The equation $y = mx + c$



- 3 a Gradient = 2, y-intercept = (0, 7)
 b Gradient = 4, y-intercept = (0, -6)

- c Gradient = 1, y -intercept = $(0, -1)$
- d Gradient = -1 , y -intercept = $(0, 3)$
- e Gradient = -3 , y -intercept = $(0, 2)$
- f Gradient = -2 , y -intercept = $(0, -2)$
- g Gradient = $\frac{1}{2}$, y -intercept = $(0, -1)$
- h Gradient = $-\frac{3}{4}$, y -intercept = $(0, 1)$

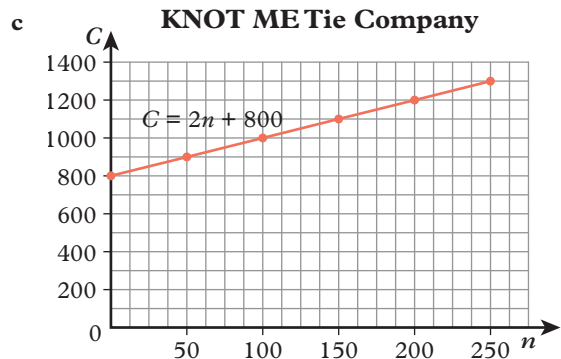
- 4 a Gradient = 5, y -intercept = $(0, -1)$
- b Gradient = -2 , y -intercept = $(0, 11)$
- c Gradient = $\frac{3}{4}$, y -intercept = $(0, 0)$
- d Gradient = -1 , y -intercept = $(0, -2)$
- e Gradient = $\frac{1}{2}$, y -intercept = $(0, 3)$
- f Gradient = 1, y -intercept = $(0, -15)$
- g Gradient = $-\frac{7}{5}$, y -intercept = $(0, -1)$
- h Gradient = $\frac{1}{6}$, y -intercept = $(0, -\frac{2}{5})$



- 6 The two methods result in the same graph.
- 7 a C is the daily cost in dollars, n is the number of ties sold each day and 800 is the fixed cost per day in dollars.

b

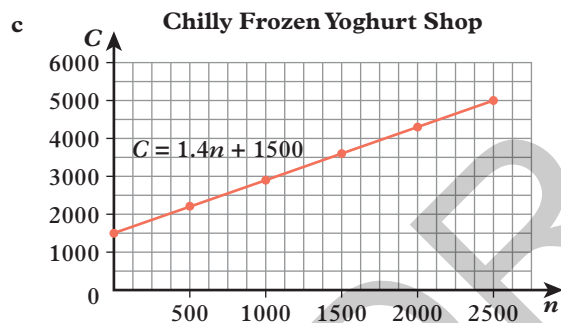
n	0	50	100	150	200	250
C	800	900	1000	1100	1200	1300



- d 70 ties
- 8 a $C = 1.4n + 1500$

b

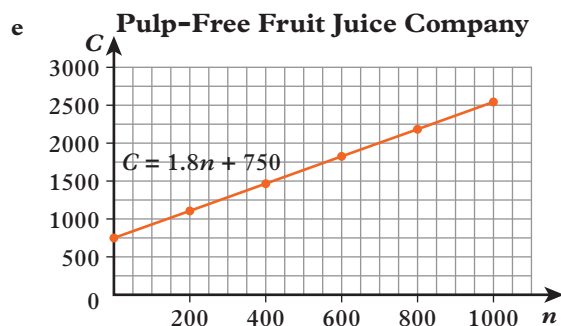
n	0	500	1000	1500	2000	2500
C	1500	2200	2900	3600	4300	5000



- d 1.4
- e 1250 serves of yoghurt
- 9 a $C = 1.8n + 750$
- b 1.8 is cost per cup
- c 750

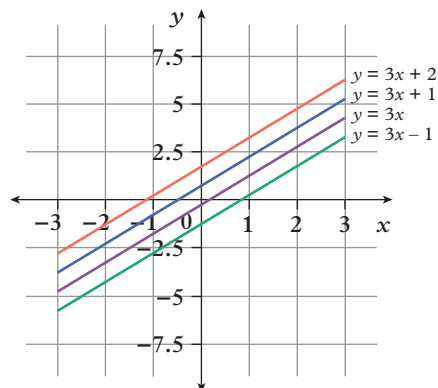
d

n	0	200	400	600	800	1000
C	750	1110	1470	1830	2190	2550



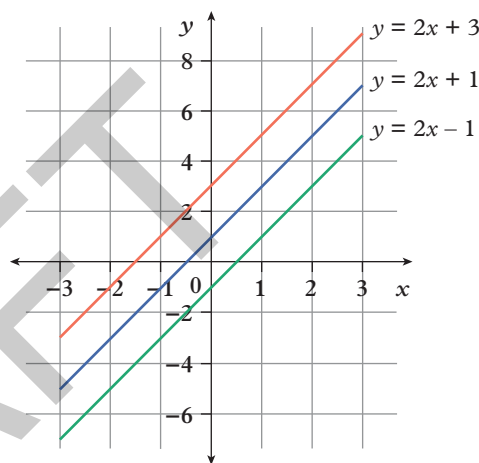
- f 450 cups of juice

10 a

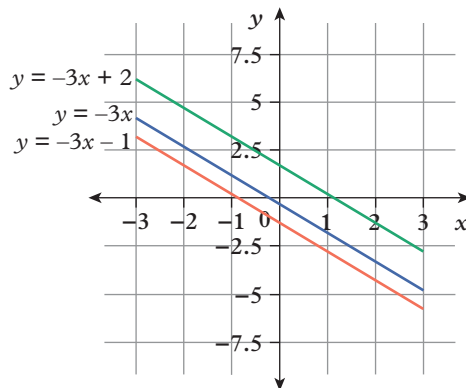


- b All parallel
- c Parallel to $y = 3x + 2$ and one unit further up, intersecting the y -axis at $(0, 3)$

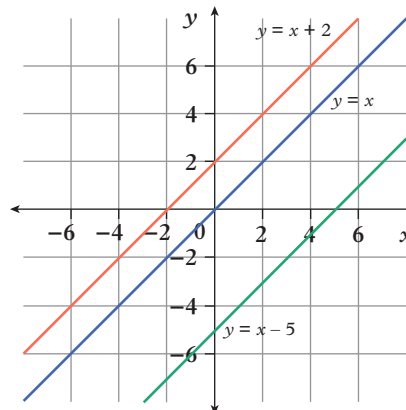
11 a i



ii

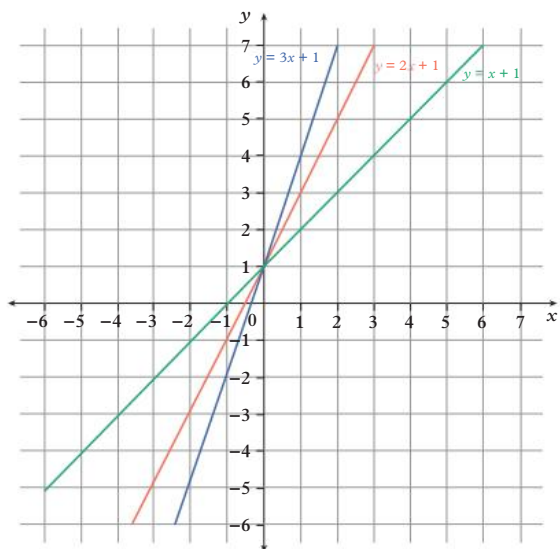


iii



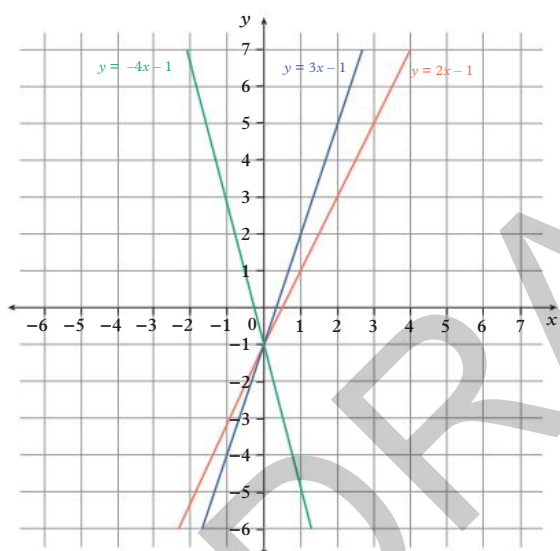
- b Lines with the same gradient are parallel.

12 a



b Not parallel, but all lines have the same y-intercept of (0, 1).

c



Not parallel, but all lines pass through (0, -1); that is, all lines have a y-intercept of (0, -1).

d All lines pass through the y-axis at the same point.

13 a $5.5 \times 12 + 7 = 66 + 7 = 73$

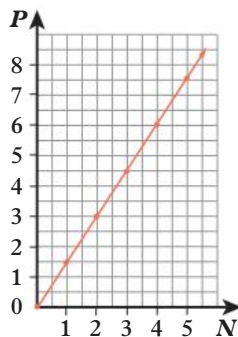
b (0, 139)

Lesson 2.4 Direct variation

1 a

Number of packets (N)	1	2	3	4	5
Price (\$P)	1.50	3.00	4.50	6.00	7.50

b



The model should not include negative values of N because it does not make sense to have a negative number of packets of pencils.

c 1.5

d $P = 1.5N$

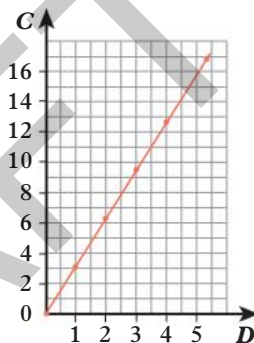
e \$13.50

f 18

2 a

D	1	2	3	4
C	3.1	6.3	9.4	12.6

b



c Yes, it is a straight line through the origin.

d $\pi \approx 3.1$

e i 7.9 cm

ii 5 cm

3

a i 4

ii 1

iii 2.1

b i $y = 4x$

ii $y = x$

iii $y = 2.1x$

4

Graph B

5

a i Not proportional

b i $\frac{y}{x} = 7$, proportional

ii $y = 7x$

c i $\frac{y}{x} = 2$, proportional

ii $y = 2x$

d i $\frac{y}{x} = -\frac{1}{3}$, proportional

ii $y = -\frac{x}{3}$

6

a i 8.4

ii $y = 8.4x$

iii 58.8

iv $\frac{51}{8.4} \approx 6.07$

b i 6

ii $a = 6b$

iii 126

iv $1\frac{1}{6} \approx 1.83$

7

a i $a = 90, b = 1\frac{1}{9}$

ii $a = \frac{1}{2}, b = 48$

iii $a = -4, b = 11.4, c = 51.3$

iv $a = 3.2, b = -35.1, c = 6.9$

b i $y = 18x$

ii $y = \frac{x}{2}$

iii $y = \frac{x}{3}$

iv $y = 4.5x$

- 8 a $P \propto d$, $P = kd$, $6 = k \times 50$, $k = \frac{3}{25}$, $P = \frac{3d}{25}$
 b 21.6 L
- 9 15 L
- 10 225 km
- 11 13.1 L
- 12 a $C = 2.15P$ b \$36.55
- 13 19.2 hours
- 14 a $C = 1.5H$ b \$10.05 c 20 m
- 15 a 0.34 ohms b 1500 cm
- 16 a $P = 0.651D$ b \$1305.68
- 17 a $I = 12.5V$ b 125 amps c 7.6 volts
- 18 a The tap is leaking at a constant rate of 1.2 mL per minute.
 b Each British Pound is worth 0.51 Australian Dollars.
 c The train travels at 72 km per hour.

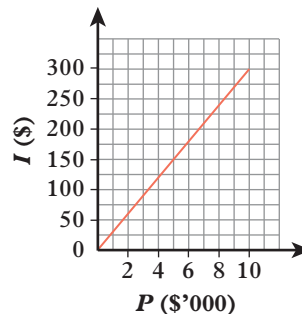
Lesson 2.5 Analysing linear models

- 1 a \$60
 b 10 km
 c Gradient = 2.5; cost per km after the flagfall is \$2.50
 d Vertical intercept = 5; flagfall = \$5
 e $y = 2.5x + 5$
- 2 a \$200
 b 52 people
 c Gradient $\approx \frac{160}{28} = \frac{40}{7} \approx \$5.71$ (using the points in parts a and b); \$5.71 per person
 d Vertical intercept = 60; set-up cost = \$60
 e $y = 5.71x + 60$
- 3 a \$420
 b $2\frac{1}{2}$ hours
 c Gradient ≈ 85 ; cost/hour $\approx \$85$
 d Vertical intercept ≈ 40 ; call-out charge = \$40
 e $y = 85x + 40$
- 4 a \$2000
 b \$410
 c The gradient is 0.06, as it is the commission as a percentage (6%).
 d The vertical intercept is 200, as it is a retainer of \$200.
 e $y = 0.06x + 200$
- 5 a i 300 mm ii 150 mm
 b 3 h
 c 2 h
 d The gradient is -30 , as the height decreases at 30 mm/h.
 e The vertical intercept is 300, as it is the initial height of the candle.
 f $y = -30x + 300$
- 6 a $k = 0.03$

b

P	2000	4000	6000	8000	10 000
I	60	120	180	240	300

c Interest earned



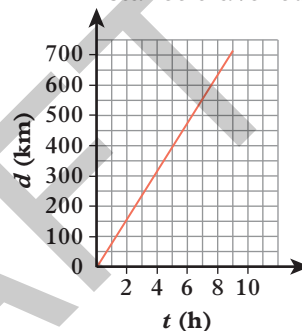
- d The gradient is 0.03, as it is the rate of interest as a decimal.

7 a 80

b

t	1	3	5	7	9
d	80	240	400	560	720

c Distance travelled



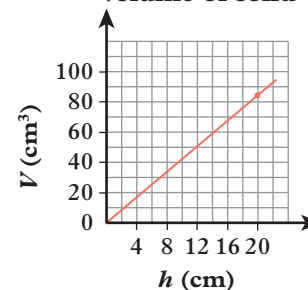
- d The gradient is 80, as the speed of the train is 80 km/h.

8 a 4.25

b

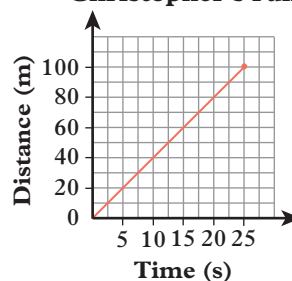
h	4	8	12	16	20
V	17	34	51	68	85

c Volume of solid



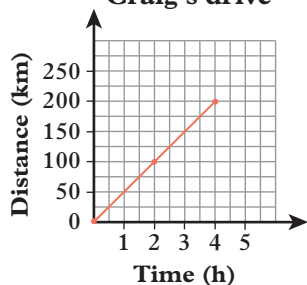
- d The gradient is 4.25, as it is the cross-sectional area of the solid.

9 a Christopher's run



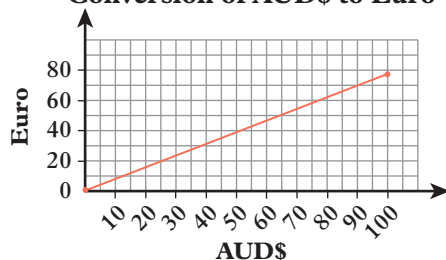
- b 7.5 s
 c Gradient = 4, speed = 4 m/s
 d If d is distance (in m) and t is time (in s), then $d = 4t$.

10 a **Craig's drive**



- b 75 km
 c Gradient = 50
 d km/h
 e If d is distance (in km) and t is time (in h), then $d = 50t$.

11 a **Conversion of AUD\$ to Euro**

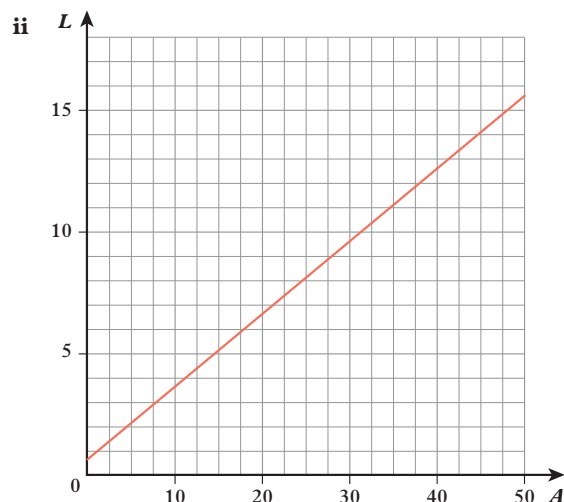


- b \$65 AUD
 c Gradient ≈ 0.77
 d It is the conversion rate.
 e If e is the amount of euros and d is the amount of Australian dollars then $e = 0.78d$.
 f Yes
- 12 a Yes, the line has the same gradient as it is straight.
 b The runner would get tired and so their speed would decrease. A longer race would have to be run at a slower speed to cover the distance.
- 13 a Yes, the line is straight, so the speed has not changed.
 b Not necessarily, as Craig may need petrol stops, there may be traffic, he should take a rest, etc.
- 14 a Cost (in dollars) per kilometre
 b Day rate: gradient = 2.2, Night rate: gradient ≈ 2.6 ; The cost of a taxi trip at night is higher than during the day.
 c 5; fixed cost of \$5 for each trip
 d If C is the total cost in dollars to travel d km, then:
 e Day rate: $C = 2.2d + 5$; Night rate: $C = 2.6d + 5$

- 15 a i $L = \frac{A}{3}$
 ii $L = 16$ litres

b i

A	0	12
L	0.4	4



iii $L = 0.3A + 0.4$

iv $L = 14.8$ litres

- 16 Consider adding a constant term or even using a piecewise linear graph with multiple segments of different gradients.

17 $I = 200 + 0.05s$

Using spreadsheets

- 1 Answers will vary.
 2 Answers will vary.
 3 a Completed table:

	A	B	C	D	E
1	Number of cans (C)	1	2	3	4
2	Litres of paint (L)	10	20	30	40
3	Area of coverage (A)	35	70	105	140
4	Price (P)	30	60	90	120

i $L = 10C$, use formula B2=10*B1

ii $A = 35C$, use formula B3=35*B1

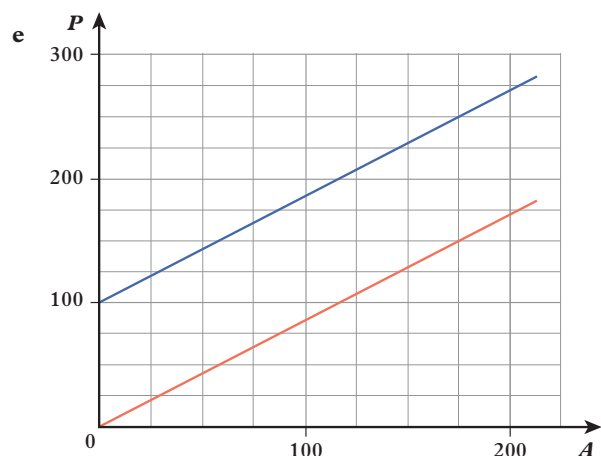
iii $P = 30C$, use formula B4=30*B1

b i 50L

ii 5 cans

c $P = \frac{6}{7}A$, yes this is a direct variation relationship

d $P = \frac{6}{7}A + 100$



Lesson 2.6 Review: Linear relationships**Review questions 2.6A**

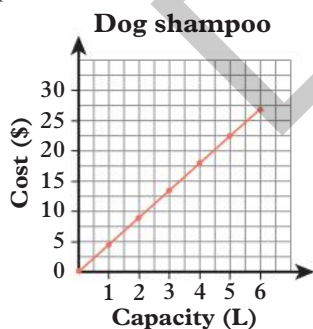
- gradient, independent variable
 - y -intercept, dependent variable
- Mathematical models are representations of situations that are often used to extrapolate and make predictions about unknown or future information.
- " y is proportional to x " or " y varies directly with x "
- Answers will vary.
 - The **y -intercept** is the point on a graph where the line crosses the y -axis, for example, the coordinates of the y -intercept in this graph are $(0, 1.2)$.
 - This is an example of a **piecewise graph**, which consists of two or more connected line segments with different gradients.
 - Any linear graph that passes through the origin, as shown in this graph, describes a relationship of **direct variation** between the dependent and independent variables.
- constant of variation label " 7 "
 y -intercept label " 2 "
 dependent variable label " y "
 independent variable label " x "
 rise, run label " 5 ", " 8 "

Review questions 2.6B

1 B	2 C	3 D	4 D
5 A	6 C	7 B	8 C
9 C	10 B	11 A	12 C
13 B	14 B	15 C	16 A

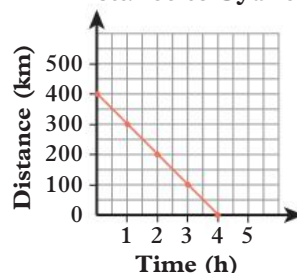
Review questions 2.6C

- The dependent variable is cost.



- About \$16

Time (h)	0	1	2	3	4
Distance (km)	400	300	200	100	0

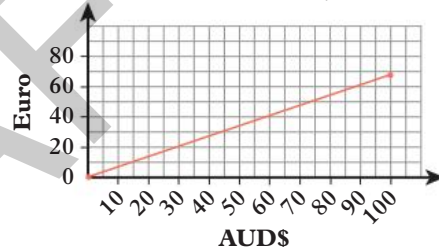
b Distance to Sydney

- 250 km

- 150 km

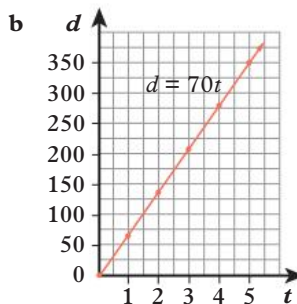
- Gradient = 2, y -intercept = 3

-

5 a Conversion of AUD\$ to Euro

- About \$73 AUD
 - Gradient = 0.68
 - Conversion rate of \$1 AUD to Euro
 - If e is the amount of euros and d is the amount of Australian dollars then $e = 0.68d$.
 - Yes
- $y = 3x - 5$

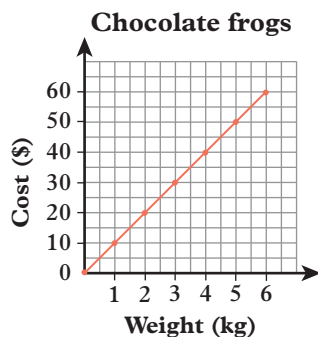
t	1	2	3	4	5
d	70	140	210	280	350



- 245 km
 - 2 h 51 min
- Yes, as it is a straight-line graph passing through the origin.

Review questions 2.6D

- 1 a The dependent variable is cost.

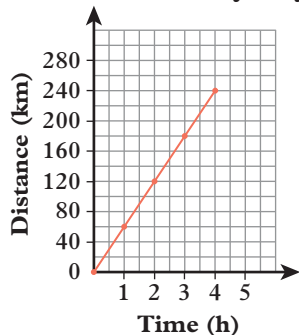


- b \$45

- 2 a

Time (h)	0	1	2	3	4
Distance (km)	0	60	120	180	240

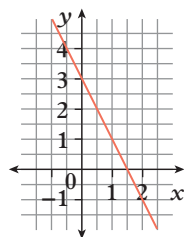
- b
- Distance to Sydney**



- c 90 km

- d 1 h 40 min

- 3



- 4 a Just under \$260

- b 42 people

- c Gradient
- ≈ 5.7
- ; this is the cost per person

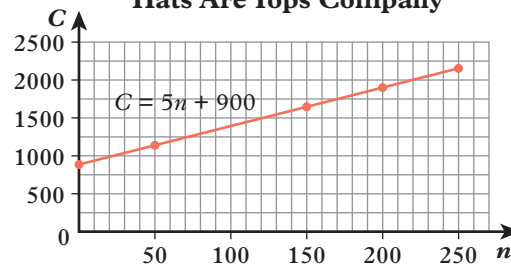
- d 50; this is the set up cost of \$50

- 5 a
- C
- is the daily cost in dollars,
- n
- is the number of hats produced each day, 5 is the cost per item in dollars and 900 is the fixed cost in dollars.

- b

n	0	50	150	200	250
C	900	1150	1650	1900	2150

- c

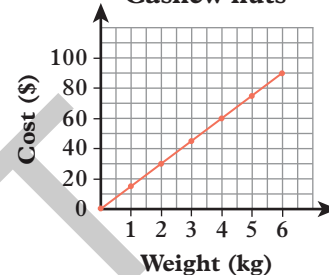
Hats Are Tops Company

- d 120 hats

- 6 No, as
- $\frac{y}{x}$
- does not produce a constant value.

Review questions 2.6E

- 1 a The dependent variable is cost.

Cashew nuts

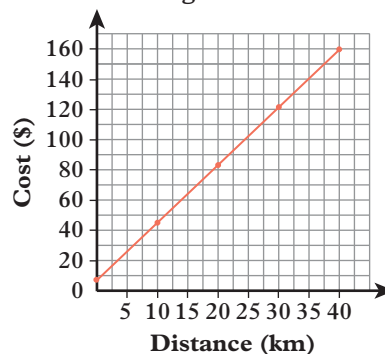
- b About \$57

- c About 3.3 kg

- 2 a

Distance (km)	0	10	20	30	40
Cost (\$)	7.00	45.00	83.00	121.00	159.00

- b
- Cost of night-time taxi hire**

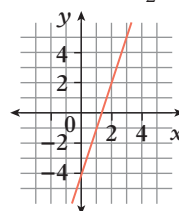


- c \$64

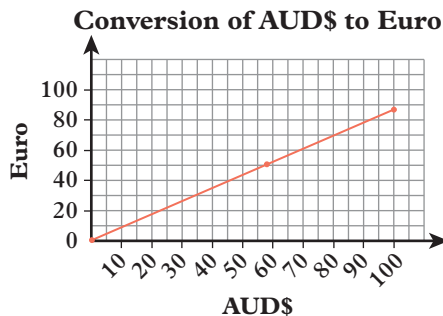
- d 17.89 km

- 3 Gradient =
- $2\frac{1}{2}$
- ,
- y
- intercept = 5

- 4



5 a



b About AUD \$58

c Gradient is 0.87; the conversion rate of AUD \$1 to Euro.

d Yes

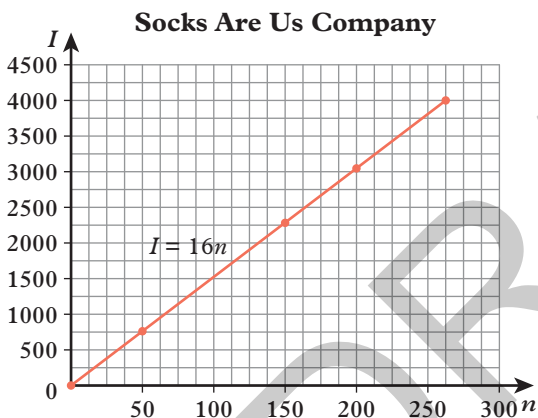
6 $y = -3x + 2$

a I is the income in dollars, n is the number of pairs of socks produced and 16 is the cost (in dollars) per pair of socks.

b

n	0	50	150	200	250
I	0	800	2400	3200	4000

c



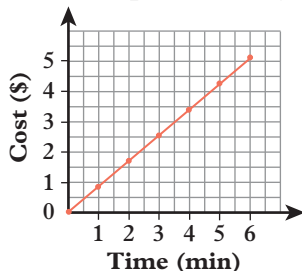
d 115 pairs of socks

e Yes, as it is a straight line passing through the origin.

Review questions 2.6F

1 a The independent variable is time.

Mobile phone charges



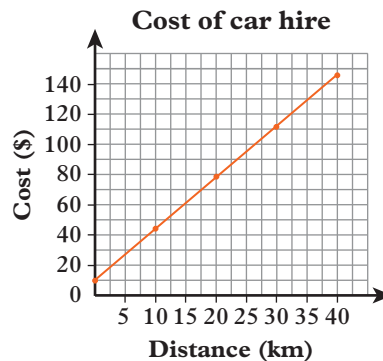
b About \$2.10

c About 3.5 min

2 a

Distance (km)	0	10	20	30	40
Cost (\$)	10.00	44.00	78.00	112.00	146.00

b

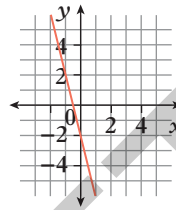


c \$129

d 1.47 km

e No, as the straight line does not pass through the origin.

3



4

a About \$32

b About 85 km

c About $\frac{2}{5}$; the charge in \$ per km

d \$8; the flagfall is \$8

5

a i \$30

ii \$75

iii \$95

b i 4

ii 15

iii 34

c i \$5

ii 1.25

iii The cost for each extra card, having already bought 30 cards

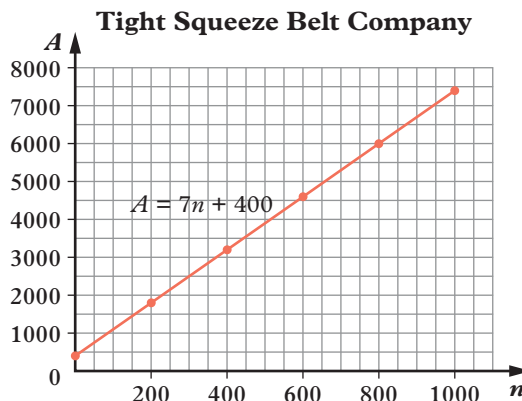
6

a $A = 7n + 400$

b

n	0	200	400	600	800	1000
A	400	1800	3200	4600	6000	7400

c



d About 230 belts

7

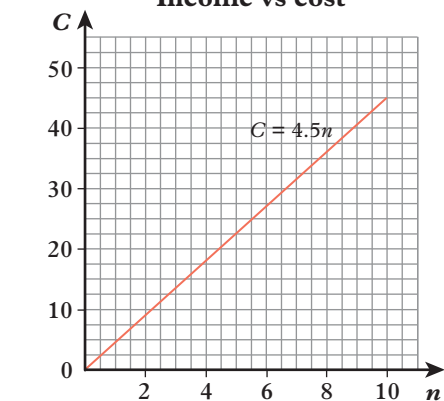
a $a = 80$, $b = 0.5$

b $y = 16x$

Review questions 2.6G

- 1 a i 16 strokes/min ii 4 m/s
 iii 4 min 10s
 b i 4.5 m/s ii 2 m/s
 c i Upstream. When $r = 0$, s is negative, she is going backwards.
 ii 1 m/s
 d $y = -4x + 11$
 e i $\frac{5}{2}$ or $2\frac{1}{2}$ ii -5
 f i $C = 4.5n$ ii 4.5

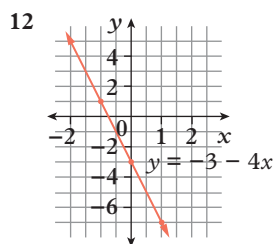
iii Income vs cost



iv \$36

- b About \$1.90 c About $3\frac{1}{3}$ min

- 11 Gradient = -3, y -intercept = 6

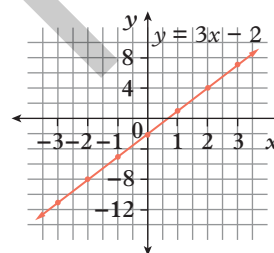


- 13 a \$40
 b 100 km
 c Gradient is 0.4; cost per km
 d Intercept is \$10; flag fall
 e No, as the y -intercept is not at the origin.

- 14 a $\frac{1}{4}$ b $y = \frac{x}{4}$ c 12 d 32

15

x	-3	-2	-1	0	1	2	3
y	-11	-8	-5	-2	1	4	7

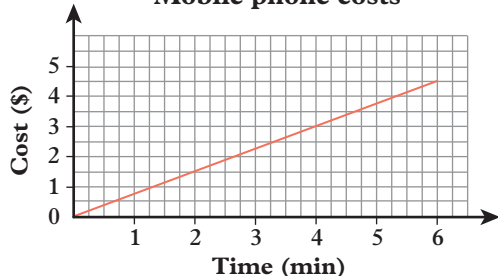


Review: Algebra

Review questions

- 1 a $d = 21$ b $x = 8$ c $x = 18$
 d $x = 5$ e $x = 9$ f $x = 4$
 g $n = 5$ h $c = \frac{9}{4} = 2\frac{1}{4}$ i $x = -\frac{1}{14}$
 2 a 240 b 4.4
 3 a $x = 16$ b $x = -8$ c $x = 13$
 d $x = 3$ e $x = 4$ f $x = 6$
 g $x = -1\frac{1}{2}$ h $x = -\frac{4}{7}$ i $x = 27$
 4 a $y = 3p$ b $y = \frac{(4-x)}{3}$ c $y = 2w + x$
 5 145 m
 6 a 100 km/h b 37.5 min c 33.3 m/s
 7 a 0.08 b 5 h 20 min
 8 a 4.7 mg b 20 mg c 20 mg
 9 a $x = \pm 7$ b $x = \pm 12$ c $x = \pm 5$
 d $m = \pm 20$ e $p = \pm 2$ f $r = \pm 3$
 10 a The dependent variable is cost.

Mobile phone costs

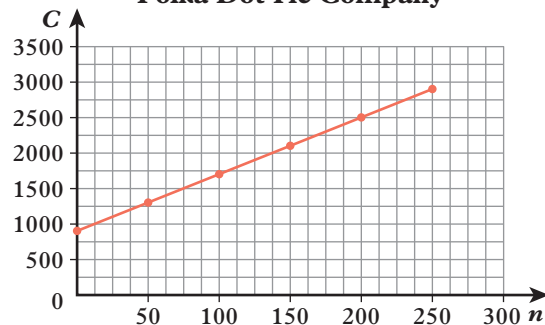


- 16 a C is the total cost, \$8 is the cost per tie, n is the number of ties and \$900 is the fixed cost.

b

n	0	50	100	150	200	250
C	900	1300	1700	2100	2500	2900

c Polka Dot Tie Company



- d About 130 ties

Module 3 Earning money

Lesson 3.1 Earning an income

- 1 a \$19 884.80, \$1657.07
 b 52, \$382.40
 c \$19 884.80, 26, \$764.80
 2 \$1671.62

- 3 **a** i \$3241.92 **ii** \$748.13 22 \$39 683.28
 iii \$1496.27 23 \$51 251.20
b i \$2258.83 **ii** \$521.27 24 **a** i \$855 **ii** \$3705
 iii \$1042.54 **b** \$28.50
c i \$2508.17 **ii** \$578.81 25 **a** 15 **b** 10 **c** \$444.25
 iii \$1157.62 26 **a** \$1517.38 per week
d i \$9155.83 **ii** \$2112.88 **b** \$33.72 per hour
 iii \$4225.77 27 **a** Option A by \$35.83
e i \$6256.33 **ii** \$1443.77 **b** Option B by \$78.36
 iii \$2887.54 **c** Option B by \$148.31
f i \$4249.17 **ii** \$980.58 28 **a** \$45 760
 iii \$1961.15 **b** \$842.31
 4 **a** 52, \$21 892.00 **b** \$21 892, 12, \$1824.33 **c** Armin
 5 **a** \$19 708 **b** \$1642.33 **d** Richard assumed that 1 month = 4 weeks, but this
 would mean there are 48 weeks in a year and not 52!
 6 Cooke: \$75 292.88, Moussa: \$84 295.12, Boje:
 \$63 988.08, Lambert: \$60 859.76 29 **a** Lenora currently earns approximately \$2160 more
 than the proposed salary.
 7 **a** \$796.12 **b** \$676.85 **c** \$1801.43 **b** On a salary she is guaranteed the full amount,
 whereas on an hourly wage her yearly income relies
 on enough work being available throughout the year.
d \$3019.23 **e** \$1661.35 **f** \$6191
 8 **a** Kylie earns \$4461.20 more than Stacey.
 b Paul earns \$8393 more than Roland.
 c Aaron earns \$4726.40 more than Joshua.
 9 **a** \$28.45, \$227.60
 b \$28.45, 38, \$1081.10
 10 \$761.25
 11 \$1174.20
 12 **a** \$435.20 **b** \$652.80
 c \$952.00 **d** \$1115.20
 13 **a** \$349.74 **b** \$680.05 **ii** \$29.03
 c \$777.20 **d** \$932.64 **iv** \$43.05
 ii \$38.70
 iv \$57.40
 14 **a** 38, \$1200.80
 b \$1200.80, 52, \$62 441.60
 15 \$115 398.40
 16 \$83 525.52
 17 **a** \$54 527.20, \$1048.60 **b** \$1048.60, 35, \$29.96
 18 \$23.41
 19 \$37.86
 20 **a** Rodney:
 i 37 **ii** \$1361.60
 b Anastasia:
 i 36 **ii** \$1324.80

21

Name	Hours worked	Hourly rate	Weekly wage
S. Smith	25 $\frac{1}{2}$	\$39.40	\$1004.70
D. De Souza	32	\$26.30	\$841.60
A. Bentley	19	\$42.20	\$801.80
C. Johnson	17	\$30.34	\$515.78
K. Kernell	20	\$28.74	\$574.80
P. Patrick	23.5	\$29.83	\$701.01

Lesson 3.2 Allowances, bonuses and loadings

- 1 **a** \$22.24, 5, \$33.36
 b \$22.24, 2, \$44.48
 2 **a** i \$25.25 **ii** \$29.03
 iii \$39.89 **iv** \$43.05
 b i \$33.66 **ii** \$38.70
 iii \$53.18 **iv** \$57.40
 3 \$865.74
 4 \$1163.40
 5 \$913.72
 6 \$945.03
 7 \$565.60
 8 **a** 8, 5, \$1368
 b 3, \$90
 c \$1368, \$90, \$1458
 9 \$900.50
 10 \$1158.50
 11 \$743
 12 3, \$43 600, \$1308
 13 **a** \$2650 **b** \$7400
 c \$11 112.12 **d** \$19 632
 e \$25 345.20 **f** \$40 664.40
 14 **a** \$619.50 **b** \$660.80
 c \$708 **d** \$672.60
 15 4, \$2692; 17 $\frac{1}{2}$, \$2692, \$471.10; \$471.10, \$3163.10
 16 \$2528.60
 17 **a** \$3811.70 **b** \$2476.90 **c** \$5081.17
 18 **a** \$3172.50 **b** \$2867 **c** \$2021
 d \$4582.50 **e** \$5864.43

- 19 a \$2550.08 b \$446.26 c \$2996.34
 20 \$760.76
 21 \$1288.65
 22 a \$376.60
 b No, \$336
 c \$47.08 per hour

23	Normal hours	Overtime hours	Weekly wage
a	20	$23\frac{1}{2}$	\$1071.85
b	20	$25\frac{1}{2}$	\$1130.05
c	20	30	\$1261.00
d	12	$20\frac{1}{2}$	\$829.35
e	18	14	\$756.60
f	20	31	\$1290.10

- 24 \$12 522.25
 25 \$762.10
 26 \$923.75
 27 \$859.44
 28 \$749
 29 a \$1295.61 b \$36 895.61
 30 a \$26 b \$24.50
 31 a \$539 b \$15.40
 32 \$34.42 per hour

Lesson 3.3 Commission, piecework and royalties

- 1 $7, \frac{7}{100}, \$1350, \$94.50$
 2 a \$10 500 b \$1221 c \$59
 d \$20 e \$2160 f \$13 867
 3 \$31 200
 4 \$3360
 5 \$45 180
 6 12.3, \$68 000, \$8364
 7 a \$3792.33 b \$26 798.15
 c \$2979.64 d \$37 684.50
 8 a 9, \$2800, \$2800, \$252
 b \$252, \$532
 9 a \$278.02 b \$357.51 c \$555.12
 d \$685.84 e \$701.08 f \$906.40
 10 a i \$917.50 ii \$902.50
 b i \$1010 ii \$987.50
 c Samir earned a combined total of \$1820, which is more than Travis's total of \$1997.50.
 11 \$19 736
 12 \$350, 9.4
 13 14.5%
 14 12.3%

- 15 a \$3168 b 4% c 14.7%
 d \$15 055.20 e 5.75% f \$43 253
 16 \$3423.60
 17 \$5880
 18 a \$135 b \$184.50
 c \$238.44 d \$296.80
 19 a \$184.67 b \$173.21
 c \$28 d \$228
 20 \$1.80, \$540
 21 \$1159.20
 22 \$645
 23 \$100.50
 24 \$659.40
 25 a \$130, \$1040 b \$1040, \$29.71
 26 a \$2320 b \$66.29
 27 a \$900 b \$22.50
 28 45, \$36; \$0.80, 60, \$97; \$36, \$97, \$133
 29 a \$624.41 b \$632.34
 30 a \$34 720 b \$34 800
 c Tony by \$80
 31 a Karla b 10.2%
 32 The second method is better by \$32.25.
 33 \$12 270
 34 \$1145.60
 35 a \$1827 b \$1139.70
 c \$35.62 per hour
 36 a \$694.60 b \$43.41
 37 6.1%
 38 a \$2835.60 b \$3370.06
 c \$2440 d \$3697.32
 39 Commission rate = 3.7%

Lesson 3.4 Government allowances and pensions

- 1 a \$663.30 per fortnight
 b \$528 per fortnight
 c \$1662.67 per fortnight
 2 a \$472.50 per fortnight
 b \$528 per fortnight
 c \$1340.67 per fortnight
 3 a \$472.50 per fortnight
 b \$150 per fortnight
 c \$961.84 per fortnight
 4 a \$663.30 per fortnight
 b \$150 per fortnight
 c \$1283.84 per fortnight
 5 a \$663.30 per fortnight
 b No
 c Yes, fortnightly payment reduces by \$31.
 d \$1662.67 per fortnight

- 6 a \$472.50 per fortnight
b Yes, fortnightly payment reduces by \$5.00.
c No
d \$961.84 per fortnight
- 7 a \$472.50
b \$571, \$43
c \$43, \$21.50
d \$472.50, \$21.50, \$451
- 8 \$382.80
- 9 \$632.80
- 10 \$459.50
- 11 \$462.50 per fortnight
- 12 \$399.30 per fortnight
- 13 a \$836.60
b \$664, \$31
c \$31, \$18.60
d \$52.50
e \$18.60, \$52.50, \$71.10
f \$836.60, \$71.10, \$765.50
- 14 \$329.60 per fortnight
- 15 \$291.20 per fortnight
- 16 \$537 per fortnight
- 17 \$412.50 per fortnight
- 18 \$1533.35 per fortnight
- 19 \$793.10 per fortnight
- 20 \$2082.20
- 21 Jessica will no longer receive an age pension.
- 22 Answers will vary.
- 23 a \$663.30 basic rates, \$783.30 other rates
b \$510.60

Lesson 3.5 Taxable income

- 1 \$45 320, \$665, \$46 452.50, \$46 452; \$340, \$45, \$445;
\$46 452, \$445, \$46 007
- 2 Bryn: \$45 830, Moe: \$36 932, Naomi: \$72 398, Rahul:
\$13 303, Rosie: \$26 578
- 3 2, \$42 756, \$855.12
- 4 a \$780.42 b \$898.12 c \$877.58
d \$922.20 e \$1219.16 f \$0
- 5 \$29 369, \$3369, \$3369, \$336.9
- 6 a \$600 b \$53
c \$145 d \$504.10
- 7 a \$1218.60 b \$3183.45
c \$12 968.87 d \$6734.85
e \$0
- 8 a \$79 731.50 b \$1594.63
c \$1909.73
- 9 a \$155 805 b \$13 936.85
c \$3116.10

- 10 a \$29 156
b \$315.60
c Her RI is lower than \$67 000.
d \$37 844
- 11 a \$2337.08
b \$5453.18
c Example: HBF gold hospital cover: \$3021.71/year,
AHM advanced gold hospital cover: \$3287.65/year,
Bupa gold hospital: \$3669.84/year. Prices for private
hospital insurance cost less than the Medicare levy
and the MLS Robert is paying.

Lesson 3.6 Paying tax

- 1 \$47 953, \$2953, \$4288, \$2953, \$5173.90
- 2 a \$19 757.20 b \$5461.30
c \$0 d \$2048
e \$46 920.50 f \$61 088
g \$4096.32 h \$19 288
- 3 Premier: \$153 536, Deputy: \$123 786, Minister:
\$116 020, Secretary: \$59 329, Whip: \$57 776
- 4 a \$268 b \$536 c \$1161
5 a \$379 b \$758 c \$1642
- 6 a i \$184 ii \$295
b i \$1230 ii \$1484
c i \$919 ii \$1400
- 7 a \$34 596 b \$2623.36
- 8 a \$17 038 b \$1750
c \$3075 d \$21 863
- 9 a \$19 900 b \$272
c \$0 d \$272
- 10 Dentist: \$593, Doctor: \$1235, Optometrist: \$632,
Engineer: \$1902, Computer scientist: \$372,
Veterinarian: \$236, Teacher: \$1742, Accountant: \$322,
Psychologist: \$261
- 11 a \$98 000
b \$91 875
- 12 Example: For a pay of \$200 000, 45% of
their income is $0.45 \times \$200\,000 = \$90\,000$
. Using the 2024–25 tax table, the tax to pay is
 $\$51\,638 + 0.45(\$200\,000 - \$190\,000) = \$56\,138$. As
\$56 138 is less than \$90 000, the statement is not true.

Lesson 3.7 Tax returns and net earnings

- 1 \$5203, \$94 655, \$94 655, \$2612, \$92 043, \$4288,
\$92 043, \$18 400.90, \$92 043, \$1840.86, \$18 400.90,
\$1840.86, \$20 241.76, \$20 241.76, \$625.64
- 2 a \$68 312
b \$1366.24
c \$196.80
d \$11 281.60
e \$7720.74 payable
- 3 \$71 801.24

- 4 \$55 467.36
- 5 a \$41 170 b \$823.40
c \$3675.20 d \$1282.70 payable
- 6 a \$75 101 b \$1502.02
c \$1215.15 d \$13 318.30
e \$47.47 payable
- 7 a \$72 176 b \$1443.52
c \$776.40 d \$13 936
e \$724.72 payable
- 8 a \$9842.70 payable b At least \$963
c The amount the employer withholds does not cover the study loan. Also, Brian's private tutoring was part of his taxable income that his employer is not responsible for withholding tax for.
d \$1049 e \$4509.30 refund
- 9 a \$114 800 b \$27 524
c \$1648 refund

Using spreadsheets

- 1 See example spreadsheet on Oxford Digital and answers to questions 20–28 in Lesson 3.1.
- 2 a See example spreadsheet on Oxford Digital and answers to questions 3–7 in Lesson 3.2
b $=B6*2*B7$
c See answers to questions 3–7 in Lesson 3.2
d See answers to questions 3–7 in Lesson 3.2
- 3 See example spreadsheet on Oxford Digital
- 4 The formula in cell B3 uses a nested IF statement, beginning with the last row of the tax table.
If the value is greater than \$190 000, then the first statement is **true** and the formula will calculate the tax payable on that income, using a formula matching the last row of the tax table, $(B2-190\,000)*0.45+51\,638$. If the statement is **false** (i.e. the taxable income is less than \$190 000), then the formula tests for the next IF statement.
If the taxable income is greater than \$135 000, then the second statement is **true** and the formula will calculate the tax payable on that income, using a formula matching the second last row of the tax table, $(B2-135\,000)*0.37+31\,288$.
The formula in cell B3 continues upwards through the tax table, displaying “NO TAX” in cell B3 instead of evaluating a formula to match the first row of the tax table. This formula returns the correct output as any taxable income can be entered into cell B2 and the formula in cell B3 will work out which row of the tax table it corresponds to and deliver the tax payable as an output in cell B3.

For example, an income of 47 000 entered into cell B2 is not greater than 190 000, is not greater than 135 000, but is greater than 45 000. Thus, $(47\,000 - 45\,000) \times 0.3 + 4288 = 4888$.

Lesson 3.8 Review: Earning money

Review questions 3.8A

- 1 a wage b retainer c allowance
- 2 a 1 week of annual leave loading = $0.175 \times \$1576$
b This gives the person's total pay for 4 weeks of annual leave.
- 3 a fortnightly pay = annual salary $\div 26$
b weekly income = retainer + $\frac{\text{commission rate}}{100} \times$ total sales
c income = amount per item $\times$ number of items
- 4 Answers will vary

Review questions 3.8B

- 1 C 2 A 3 D 4 A 5 D 6 A
7 D 8 C 9 B 10 D 11 A 12 B
13 C 14 B 15 C 16 B 17 B 18 C

Review questions 3.8C

- 1 \$101 556
2 \$1303.87, \$27.16
3 \$1257.17
4 \$993
5 \$7031.20
6 \$10 518
7 \$81.40
8 \$6866.64
9 a \$663.30 per fortnight
b \$1662.67 per fortnight
10 \$1556.85 per fortnight
11 \$35 022
12 \$787.18
13 a \$7998.40 b \$2089.44
c \$0 d \$34 000.10
14 a \$78 533 b \$1570.66
c \$1729.95 d \$14 347.90
e \$2048.51 payable

Review questions 3.8D

- 1 \$1201.44, \$34.33
2 \$62 375.04
3 \$1193.70
4 \$839
5 a \$550.62 b \$17.21
6 \$1605.38
7 \$329.60 per fortnight
8 \$71 782
9 \$231.70

- 10 **a** \$43 086.93 **b** \$2479.84
 c \$17 598.70 **d** \$0
 11 **a** \$42 995 **b** \$859.90
 c \$3967.20 **d** \$2136.20 refund

Review questions 3.8E

- 1 \$574.63
 2 \$27.72
 3 \$1617.20
 4 \$547.10
 5 \$3680.10
 6 \$304.50
 7 \$26 048.40
 8 \$392.30 per fortnight
 9 \$840.60
 10 \$49 561
 11 \$1201.60
 12 **a** \$41 483.72 **b** \$23 396.20
 c \$481.76 **d** \$0
 13 **a** \$72 913 **b** \$1458.26
 c \$886.95 **d** \$12 661.90
 e \$6593.67 payable

Review questions 3.8F

- 1 \$31.58
 2 \$44 257.20
 3 \$816.85
 4 \$1027.70
 5 **a** \$848.02 **b** \$28.27
 6 \$1866.15
 7 \$12 367
 8 \$558.60 per fortnight
 9 \$1564.60 per fortnight
 10 \$28 360
 11 **a** \$38 774.21 **b** \$1692
 c \$0 **d** \$12 455.50
 12 **a** \$54 002 **b** \$1080.04
 c \$6988.60 **d** \$1975.80 refund

Review questions 3.8G

- 1 **a i** \$33 904 **ii** \$17.16
 iii No. $\$33\,904 \div 12 = \$2825.33$
b \$4446.20
c No; he would receive \$20 less per week.
d \$740.60 per fortnight
e i \$11 876.80 **ii** \$805.28 refund

Module 4 Managing money**Lesson 4.1** Financial calculations

- 1 **a** \$576 **b** \$26.25 **c** \$918
 d \$179.20 **e** \$324 **f** \$18
 g \$2.03 **h** \$65 **i** \$320
 2 **a** \$80.50 **b** \$364.80 **c** \$210
 d \$3.89 **e** \$64.50 **f** \$417.24
 g \$2932.50 **h** \$38 **i** \$3.54
 3 **a** \$60 **b** \$102 **c** \$1890
 d \$648 **e** \$287 **f** \$28.80
 4 **a** Percentage increase = $\frac{55 - 52}{52} = \frac{3}{52} \times 100\% \approx 5.8\%$
 b Percentage decrease = $\frac{186 - 154}{186} = \frac{32}{186} \times 100\% \approx 17.2\%$
 5 **a** 21.4% **b** 8.3% **c** 7.9%
 d 168.8% **e** 100% **f** 10.4%
 6 **a** 25.8% **b** 7.3% **c** 1.9%
 d 10.5% **e** 66.7% **f** 6.2%
 7 **a** 10.0% mark-up **b** 33.3% discount
 c 16.7% mark-up **d** 6.7% discount
 e 62.5% discount **f** 20% mark-up
 8 **a i** Loss \$5500 **ii** 25%
 b i Profit \$70 **ii** 70%
 c i Loss \$1680 **ii** 40%
 d i Profit \$7500 **ii** 15%
 e i Loss \$320 **ii** $53\frac{1}{3}\%$
 f i Loss \$1300 **ii** 61.9%

Value after first year = $106\% \times \$3600$

$$9 \text{ a } = \frac{106}{100} \times \$3600 \\ = \$3816$$

Value after second year = $108\% \times \$3816$

$$\text{b } = \frac{108}{100} \times \$3816 \\ = \$4121.28$$

c Overall increase in value = $\$4121.28 - \$3600 = \$521.28$ **d** Percentage profit = $\frac{521.28}{3600} \times 100\% \approx 14.5\%$

- 10 **a** \$2760 **b** \$2980.80
 c \$580.80 **d** 24.2%

11 8.64% profit

12 1% loss

$$13 \text{ GST} = 10\% \times \$750 = \frac{10}{100} \times \$750 = \$75$$

- 14 **a** \$4.60 **b** \$15
 c \$0.84 **d** \$45.90
 e \$97.55 **f** \$1.10
 g \$198 **h** \$1.54

15 **a** \$65.40 **b** \$719.4016 **a** \$874.28 **b** \$9617.0817 **a** \$38.90 **b** \$427.90

18 GST paid = \$109.09

- 19 a \$41.64
c \$3.14
e \$1.80
g \$270.91
- 20 a \$416.36
c \$31.42
e \$18
g \$2709.09
- 21 \$697.40
- 22 13.4%
- 23 No, GST = $\$475 \div 11 = \$43.18$
- 24 The second quote (\$2680 including GST) is cheaper by \$15.
- 25 \$12.39
- 26 a i €10 ii €144 iii €366
b i €15 ii €105 iii €570
c i €75 ii €525 iii €2850
d 6
- 27 a i Rp 58 ii Rp 1512
iii Rp 3600
b i Rp 108 ii Rp 996
iii Rp 7584
c i Rp 900 ii Rp 8300
iii Rp 63 200
- 28 \$14 787.55
- 29 \$1250

Lesson 4.2 Payment options

- 1 a \$423.75 b \$22
c \$1733 d 2.2%
- 2 a \$203 b \$218
- 3 a \$150, \$150, \$150, \$180
b \$30
c \$630
d 5.0%
- 4 a \$475 b \$4032
c \$24 575 d \$11 964
- 5 a \$3400 b 13.5%
- 6 a \$3069 b \$2800
- 7 a Option 1: \$17 000, Option 2: \$17 214.99, Option 3: \$18 300
b Option 1: There are no fees or additional interest charged. Option 2: The total payment is spread over four monthly payments. Option 3: The monthly instalment of \$575 is the most easily manageable of the three options.
c Option 1: \$17 000 is a lot of money to pay in one instalment. Option 2: There are additional fees charged, including potential late fees. Option 3: The total amount to pay is the highest.
d 7.6%

- 8 a \$103
b \$2800
c If Linus changed providers he could have a longer term to pay off the laptop; however, it would cost more to do so, by $\$2800 - \$2721 = \$79$.

9 a

Payment schedule	
First payment	
Day 0	\$88.55
Second payment	
Day 14	$\$88.55 + \$74.64$
Third payment	
Day 28	$\$88.55 + \$74.64$
Fourth payment	
Day 42	$\$88.55 + \$74.64$
Fifth payment	
Day 56	\$74.64

- b Yes. The total amount Regina needs to pay is \$652.76 over 8 weeks. As $\$175 \times 8 = \$1400$, she will have enough to cover her payments.

10 a

Payment schedule	
First payment	
Day 0	$\$148.76 + \$12$
Second payment	
Day 2	\$116.38
Third payment	
Day 14	\$148.76
Fourth payment	
Day 16	\$116.38
Fifth payment	
Day 28	\$148.76
Sixth payment	
Day 30	\$116.38
Seventh payment	
Day 42	\$148.76
Eighth payment	
Day 44	\$116.38

- b 26.5%
- c 17.9%
- d Spread out the purchases with at least a week in between to help with weekly budgeting.
- 11 a \$45 b \$253 c \$194
- 12 a Option 1: \$1499.04, Option 2: \$1499.04, Option 3: \$1499.04
b If you think you may want a new phone before 3 years is up, option 3 may not be a good choice.

13 Answers will vary

14 \$2208

15 a \$838.2

b \$564.39

c \$1408.18

d Example: Advantage: If you miss the first payment, the late fee is less. Disadvantage: If you miss a later payment, the fee is higher.

4 a \$17 900, \$17 900, \$537

b 3%, 5%, \$52 380, 0.03, 0.05, \$7380, \$1350, \$369, \$1719

5 a \$659.70

b \$1070.97

c \$1550

d \$3780

6 a \$24 400

b \$31 700

c \$15 600

d \$42 720

e \$82 680

f \$52 680

7 a \$56 000

8 a \$45 380

b \$4000

b \$2380

9 Option 2; she will save \$1800.

10 \$39 730

Lesson 4.3 Purchasing a car

1 976 to 1154, \$308

2 a \$570

b \$308

c \$374

3 a \$313

b \$163

c \$214

Costs	Holden	Mazda	Toyota sedan	Toyota SUV	Yamaha motorcycle
RRP (\$)	37 990	31 450	18 000	52 870	14 999
Weight (kg)	1637	1440	1040	1970	
Private (P) or business (B)	B	P	P	B	P
Registration fee (\$)	79	79	79	79	79
Vehicle tax (\$)	883	374	308	883	83
Number plates (\$)	55	55	113	249	53
Stamp duty (\$)	1139.70	943.50	540	1586.10	449.97
CTP insurance (\$)	620	487	528	660	280
Dealer delivery (\$)	790	585	499	887	389
Insurance (\$)	1089	790	560	1185	299
Trade-in value (\$)	0	5000	1000	12000	0
Total (\$)	42 645.70	34 763.50	20 627.00	58 399.10	16 631.97

12 \$43 900, \$39, \$43 900, \$1317, \$1560, \$46 816

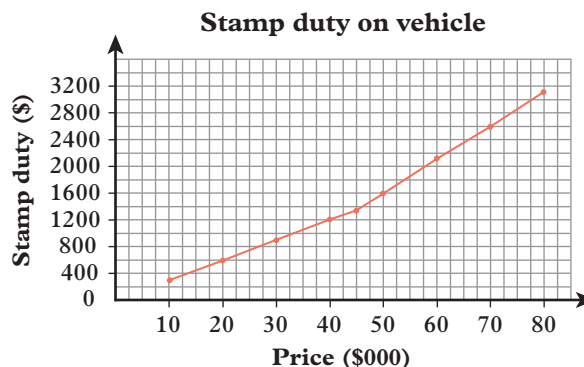
13 \$25 920

14 \$16 053.70

15 a

Price (\$000)	3% of the price up to \$45 000	5% of the price over \$45 000	Total
10	\$300	–	\$300
20	\$600	–	\$600
30	\$900	–	\$900
40	\$1200	–	\$1200
45	\$1350	–	\$1350
50	\$1350	\$250	\$1600
60	\$1350	\$750	\$2100
70	\$1350	\$1250	\$2600
80	\$1350	\$1750	\$3100

b



c i \$1050

ii \$2350

iii \$3000

16 Answers will vary.

17 Answers will vary.

18 Answers will vary.

19 Answers will vary.

20 Answers will vary.

21 a \$26 800

$$b \quad P = \frac{\text{Stamp Duty} - 1350}{0.05} + 45\,000, \$59\,900$$

Lesson 4.4 Ongoing costs of a vehicle

- 1 **a i** \$3288 **ii** \$7692
iii \$6396 **iv** \$10 764
b i \$16 440 **ii** \$23 076
iii \$12 792 **iv** \$75 348
- 2 **a i** \$11 555 **ii** \$9585
iii \$11 125 **iv** \$13 660
b i \$192.58 **ii** \$159.75
iii \$185.42 **iv** \$227.67
- 3 **a i** 18 **ii** 9 **iii** 52 **iv** 21
b i \$1296 **ii** \$585
iii \$4524 **iv** \$2016
- 4 **a i** 15 **ii** 10 **iii** 15 **iv** 8
b i \$4200 **ii** \$1750
iii \$4800 **iv** \$3280
- 5 **a** \$22 488
b \$4588
- 6 **a** Suzuki Swift: \$6867.72, Mazda 3 Hatchback: \$8904, Volkswagen Golf: \$11 346.24, Toyota Camry: \$10 485.48, Nissan X-Trail: \$12 739.56, Ford Everest: \$14 712.96
b Suzuki Swift: \$1318.20, Mazda 3 Hatchback: \$1809.60, Volkswagen Golf: \$1911.52, Toyota Camry: \$1483.04, Nissan X-Trail: \$2010.32, Ford Everest: \$2109.64
- 7 Suzuki Swift: \$5.80, Mazda 3 Hatchback: \$7.77, Volkswagen Golf: \$11.19, Toyota Camry: \$8.60, Nissan X-Trail: \$9.69, Ford Everest: \$9.52
- 8 **a** Suzuki Swift, Mazda 3 Hatchback, Toyota Camry, Volkswagen Golf, Nissan X-Trail, Ford Everest
b Suzuki Swift, Mazda 3 Hatchback, Toyota Camry, Ford Everest, Nissan X-Trail, Volkswagen Golf
c Suzuki Swift, Toyota Camry, Mazda 3 Hatchback, Volkswagen Golf, Nissan X-Trail, Ford Everest
d Toyota Camry, Suzuki Swift, Mazda 3 Hatchback, Ford Everest, Nissan X-Trail, Volkswagen Golf
e Toyota Camry, Suzuki Swift, Mazda 3 Hatchback, Volkswagen Golf, Ford Everest, Nissan X-Trail
- 9 **a** Suzuki Swift: \$34 338.60, Mazda 3 Hatchback: \$44 520, Volkswagen Golf: \$56 731.20, Toyota Camry: \$52 427.40, Nissan X-Trail: \$63 697.80, Ford Everest: \$73 564.80
b Suzuki Swift: \$5408, Mazda 3 Hatchback: \$7012.02, Volkswagen Golf: \$8934.95, Toyota Camry: \$8257.15, Nissan X-Trail: \$10 032.20, Ford Everest: \$13 096.60

	Up-front on-road costs	3-year loan monthly repayments	Annual registration and insurance	Fuel per week	Annual servicing	Tyres over 5 years
Harley-Davidson	\$31394	\$982	\$2451	\$37	\$158	\$844

a \$11 784**b** \$1924**c** \$171

11

	Richard	Stephanie	Lilly	Paul
a	\$6.91	\$5.27	\$8.26	\$8.49
b i	\$138.20	\$105.40	\$165.20	\$169.80
b ii	\$103.65	\$79.05	\$123.90	\$127.35

12

	Helen	Greg	Peta
a	19.4c	12.9c	20.7c
b i	\$186.24	\$123.84	\$198.72
b ii	\$124.16	\$82.56	\$132.48

13 Answers will vary.

14 **a** \$2339.64**b** 6.4 years**Using spreadsheets**

- 1 **a** The formula =SUM(B3:B12) would include the cells B4 and B5, which should not be included in the sum of all costs.
b Answers will vary.
c Answers will vary.
- 2 **a** \$487
b \$3301.10, finds the amount saved by the Volkswagen compared to the Nissan.
c =MAX(B11:D11)-B11
d \$2042.40, \$2541.90

Lesson 4.5 Household bills

- 1 **a i** 1/01/26 to 31/03/26 **ii** 90 days
b i \$331.47 **ii** 68 kL **iii** 756 L
c i \$3.12 **ii** \$543.63
iii \$6.04 **iv** Larger
v Smaller

- d** This bill was 185 L greater than the local average.
- e i** 62.28 kL **ii** \$194.31 **iii** \$525.78
- f** \$17.76
- 2 a** 563 L/day **b** 356 L/day
- c** 470 L/day **d** 551 L/day
- 3 a** Larger **b** 184 L **c** 15 buckets
- 4** \$453.19
- 5 a i** \$510.50 **ii** \$842
- b i** \$207 **ii** \$451
- iii** \$506 **iv** \$254
- c** \$1480
- 6 a i** 22/11/25 to 22/02/26 (inclusive)
- ii** 93 days
- b i** 82.1 c/day **ii** \$76.35
- c i** 1861.4 kWh **ii** 20 kWh
- d i** 50.54% **ii** 14.54%
- e** 48.88 c/kWh
- f** 24.14 c/kWh
- g i** \$784.34 **ii** \$819.64
- iii** \$8.81/day
- 7 a i** 13 kWh **ii** 1235 kWh
- b i** Larger **ii** Larger
- c i** 431.2 kWh **ii** 624.2 kWh
- iii** 179.6 kWh
- d i** \$469.74 **ii** \$78.00
- iii** \$547.74 **iv** \$572.39
- e i** August 2025 **ii** February 2025
- iii** August 2024
- 8 a i** 2/02/26 to 3/05/26 **ii** 91
- b i** \$17.08 **ii** 10%
- c** 4850 MJ
- d i** \$0.029 67 **ii** \$0.017 58
- e** \$40.31
- f** \$181.29
- g i** 53.30 MJ **ii** \$2.06
- h** 0.3 t
- i** 4799.7 MJ
- 9 a i** \$0.000 676 61 **ii** \$946.58, yes
- b** \$541
- c** \$2202.58
- d** \$2288.51
- 10 a** \$257.54 **b** \$321.93 **c** \$305.83
- 11 a** 52 560 L **b** \$144.54
- 12 a** Top-loader \$79.56, front-loader \$37.44
- b** \$42.12
- c** \$104
- d** About $2\frac{1}{2}$ years
- 13** Answers will vary.

Lesson 4.6 Budgeting

- 1 a** *Income:* $\$160 \times 52 = \$8320$
- Expenses:*
- Board = $\$20 \times 52 = \$1040$
- Lessons = $\$80 \times 26 = \$2080$
- Entertainment = $\$45 \times 26 = \$1170$
- Clothes = $\$140 \times 12 = \$1680$
- Total = \$5970
- Income – expenses = \$2350
- b** \$2350
- 2 a** *Income:* $\$270 \times 26 = \$7020$
- Expenses:*
- Board = $\$25 \times 52 = \$1300$
- Golf = \$450
- Personal items = $\$150 \times 12 = \$1800$
- Total = \$3550
- Income – expenses = \$3470
- b** Yes
- 3 a** *Income:* $\$590 \times 52 = \$30\,680$
- Expenses:*
- Board = $\$100 \times 52 = \$5200$
- Fares = $\$53 \times 52 = \$2756$
- Lunches = $\$42 \times 52 = \$2184$
- Entertainment = $\$150 \times 52 = \$7800$
- Personal items = $\$84 \times 26 = \$2184$
- Clothes = $\$380 \times 12 = \$4560$
- Total = \$24 684
- Income – expenses = \$5996
- b** Yes, as she will have \$17 988.
- 4 a** *Income:* $\$940 \times 52 = \$48\,880$
- Expenses:*
- Rent = $\$320 \times 52 = \$16\,640$
- Food = $\$160 \times 52 = \$8320$
- Entertainment = $\$145 \times 52 = \$7540$
- Personal items = $\$75 \times 52 = \$3900$
- Car loan repayments = $\$380 \times 12 = \$4560$
- Petrol = $\$95 \times 52 = \$4940$
- Car servicing = $\$380 \times 2 = \$760$
- Car registration and insurance = \$2148
- Mobile phone = $\$59 \times 12 = \$708$
- Total = \$49 516
- Income – expenses = –\$636

- b** He needs to reduce his expenses by at least \$12.23 per week; for example, cut entertainment to \$130 per week.

5 a *Income:*

$$\text{Allowance} = \$663.30 \times 26 = \$17\,245.80$$

$$\text{Job} = \$220 \times 52 = \$11\,440$$

$$\text{Total} = \$28\,685.80$$

Expenses:

$$\text{Rent} = \$1035 \times 12 = \$12\,420$$

$$\text{Food} = \$190 \times 52 = \$9880$$

$$\text{Phone} = \$120 \times 4 = \$480$$

$$\text{Entertainment} = \$70 \times 52 = \$3640$$

$$\text{Books} = \$350$$

$$\text{Total} = \$26\,770$$

$$\text{Income} - \text{expenses} = \$1915.80$$

- b** Car costs = \$3700, no

6 Alison

Fixed expenses: board, driving lessons

Discretionary expenses: clothes, entertainment

Spend less on clothes and entertainment; lower the number of paid driving lessons

Moses

Fixed expenses: board, golf

Discretionary expenses: personal items

Spend less on personal items

Naomi

Fixed expenses: board, fares, lunches

Discretionary expenses: entertainment, personal items, clothes

Spend less on entertainment, personal items, clothes or lunch

Matthew

Fixed expenses: rent, food, loans repayments, petrol, services, registration and insurance, mobile phone

Discretionary expenses: entertainment, personal items

Spend less on entertainment, personal items

George

Fixed expenses: rent, food, phone

Discretionary expenses: entertainment, books

Spend less on entertainment or books (or find cheaper versions of the books, e.g. second-hand, eBook)

- 7** Claudia could have planned to put aside savings for the end-of-budget bills each time she got paid, ideally using a separate savings account.

8 \$6233, \$6233, \$3451, \$6000, \$1500

9 52, \$28 600, \$28 600, \$2383.33, \$2383.33, \$7150

10 a \$46 878

b \$72 444

c \$26 442

d \$16 718

11

Alison	\$1492.50
Moses	\$887.50
Naomi	\$6171
Matthew	\$12379
George	\$6692.50

12 Approximately 8 and a half months

13 Approximately 34 and a half weeks

14 a *Income:*

$$\text{Julie} = \$990 \times 52 = \$51\,480$$

$$\text{Robert} = \$880 \times 52 = \$45\,760$$

$$\text{Interest} = \$230 \times 2 = \$460$$

$$\text{Total} = \$97\,700$$

Expenses:

$$\text{Home loan repayments} = \$3230 \times 12 = \$38\,760$$

$$\text{Food} = \$275 \times 52 = \$14\,300$$

$$\text{Electricity} = \$480 \times 4 = \$1920$$

$$\text{Phone} = \$110 \times 12 = \$1320$$

$$\text{Council rates} = \$491.50 \times 4 = \$1966$$

$$\text{Water rates} = \$286 \times 4 = \$1144$$

$$\text{Car registration and insurance} = \$1180 \text{ per year}$$

$$\text{Comprehensive car insurance} = \$1210 \text{ per year}$$

$$\text{Car loan repayments} = \$180 \times 52 = \$9360$$

$$\text{Car running expenses} = \$390 \times 12 = \$4680$$

$$\text{Clothes} = \$350 \times 12 = \$4200$$

$$\text{Personal items} = \$55 \times 52 = \$2860$$

$$\text{Total} = \$82\,900$$

$$\text{Income} - \text{expenses} = \$14\,800$$

- b** Yes, they could increase them by at most \$1233 per year. They should leave some savings for emergencies, and look at reducing expenses.
- c** As they are responsible for a car, a larger emergency fund is a good idea to account for larger unexpected costs, for example, 6 months (\$41 550) may be good.
- 15 a** *Income:*

$$\text{Beautician} = \$180 \times 52 = \$9360$$

$$\text{Wedding} = \$100 \times 6 = \$600$$

$$\text{Total} = \$9960$$

Expenses:

$$\text{Bus fares} = \$30 \times 26 = \$780$$

$$\text{Make up} = \$45 \times 52 = \$2340$$

$$\text{Clothes and entertainment} = \$200 \times 12 = \$2400$$

$$\text{Total} = \$5520$$

$$\text{Income} - \text{expenses} = \$4440$$

b i Yes**ii** *Income:*

$$\text{Beautician} = \$270 \times 52 = \$14\,040$$

$$\text{Wedding} = \$250 \times 6 = \$1500$$

$$\text{Total} = \$15\,540$$

Expenses:

$$\text{Bus fares} = \$30 \times 26 = \$780$$

$$\text{Make up} = \$45 \times 52 = \$2340$$

$$\text{Clothes and entertainment} = \$200 \times 12 = \$2400$$

$$\text{Loan repayment} = \$50 \times 26 = \$1300$$

$$\text{Total} = \$6820$$

$$\text{Income} - \text{expenses} = \$8720$$

iii \$100**iv** \$8720; not based on earnings/savings. She will have \$4280 more at the end of the year by taking the loan.**16 a**

	Jan/Feb (8 weeks)	Mar/Apr (9 weeks)	May (4.5 weeks)	Jun (4.5 weeks)	Jul (4.5 weeks)	Aug/Sep (9 weeks)	Oct (4.5 weeks)	Nov/Dec (8 weeks)
Work (days/ week)	5	1	1	1	5	1	1	5
Income	\$6000	\$1350	\$675	\$675	\$3375	\$1350	\$675	\$6000

b

	Jan/Feb (8 weeks)	Mar/Apr (9 weeks)	May (4.5 weeks)	Jun (4.5 weeks)	Jul (4.5 weeks)	Aug/Sep (9 weeks)	Oct (4.5 weeks)	Nov/Dec (8 weeks)
Rent	No rent	\$3150	\$1575	\$1575	\$1575	\$3150	\$1575	No rent

c

	Jan/Feb (8 weeks)	Mar/Apr (9 weeks)	May (4.5 weeks)	Jun (4.5 weeks)	Jul (4.5 weeks)	Aug/Sep (9 weeks)	Oct (4.5 weeks)	Nov/Dec (8 weeks)
Income- expenses	\$6000	\$4200	\$2120	\$297	\$2097	-\$3	-\$3403	\$2597

d i 90**iii** \$6978**ii** \$4475**e** It depends on her savings going into 2027. Good advice would be to prioritise her rent and look for ways to save money on the trip. She could try to pick up more hours of work in the lead up to the trip.**17 a** \$489.23**b** About 38 weeks or nearly 9 months

Using spreadsheets

- See example spreadsheet on Oxford Digital and the water bill from Lesson 4.5
- See example spreadsheet on Oxford Digital

3

	A	B	C	D	E
1		per week	per fortnight	per month	per year
2	Income	=C2/2	\$ 9,999.99	=(C2*26)/12	=C2*26
3	Expenses	\$ 999.99	=B3*2	=(C2*52)/12	=B2*52
4	Difference	=B2-B3	=C2-C3	=D2-D3	=E2-E3
5					
6		Emergency	3-month fund	=3*D3	
7			6-month fund	=6*D3	
8					

4 See example spreadsheet on Oxford Digital and answers to Lesson 4.6 **Problem solving and reasoning**

Lesson 4.7 Review: Managing money

Review questions 4.7A

1 percentage decrease

- 2 Revenue is the total income generated by a business through the same of items or services. A profit is made if the items or services are sold for more than the cost to the business, while a loss is made if they are sold for less than the cost to the business.
- 3 A deposit is an initial payment made when purchasing something, with the remaining amount to be paid later.
- 4 Standing costs: interest paid on loan, annual registration, comprehensive insurance, compulsory third party insurance
Operating costs: servicing, fuel, new tyres
- 5 utilities
- 6 budgeting
- 7 Fixed spending: electricity bill, monthly rent, phone bill
Discretionary spending: subscription to streaming service, daily take-away coffee, movie tickets
- 5 \$1880
- 6 \$18 120.70
- 7 **a** \$105.60 **b** \$864
- 8 **a** \$6396 **b** \$9361
- 9 **a** \$3.12 **b** \$331.47
c \$557.05
- 10 \$1853.45
- 11 **a** \$2006.33
b \$424.67, yes
c \$12 037.98

Review questions 4.7E

Review questions 4.7B

- 1 **C** 4 **A** 7 **A** 10 **C** 13 **D** 16 **C**
2 **B** 5 **C** 8 **B** 11 **A** 14 **C** 17 **C**
3 **B** 6 **D** 9 **B** 12 **D** 15 **B** 18 **A**

Review questions 4.7C

- 1 **a** \$82.40 **b** \$246.50
c \$849.60 **d** \$500
- 2 **a** \$25650 **b** 2.6%
- 3 \$514
- 4 **a** \$1254.63 **b** \$13 800.93
- 5 **a** \$112.27 **b** \$32.64
c \$3.83 **d** \$24.18
- 6 \$2099.80
- 7 \$214
- 8 \$2900
- 9 **a** \$40 041 **b** \$23 541
- 10 \$12 567.97
- 11 **a** \$13 737.60 **b** \$3737.60
- 12 **a** \$4355 **b** \$8997
- 13 \$493.69
- 14 \$111.92
- 15 **a** \$334 **b** 3 months
- 16 **a** \$26 884 **b** \$60 660
c \$32 331 **d** \$20 956

Review questions 4.7D

- 1 \$10 304
- 2 \$875.50
- 3 **a** \$84.90 **b** \$933.90
- 4 \$1842.55

- 1 2%
- 2 **a** \$2258.60 **b** \$24 844.60
- 3 **a** \$42.50 **b** \$467.50
- 4 **a** \$681.82 **b** \$115.27
c \$2.36 **d** \$36.45
- 5 \$148.06
- 6 **a** \$59 106
b \$41 106
c 2243.20
d **i** 236.21 **ii** \$411 **iii** \$3562
- 7 **a** Elizabeth \$8.26, Monique \$5.32, Tanya \$5.27
b Elizabeth \$123.90, Monique \$79.80, Tanya \$79.05
c Elizabeth \$82.60, Monique \$53.20, Tanya \$52.70
- 8 \$720.21
- 9 \$1904.87
- 10 **a** *Income:* $\$170 \times 52 = \$8840$
Expenses:
Board = $\$30 \times 52 = \$1560$
Lessons = $\$80 \times 26 = \$2080$
Entertainment = $\$85 \times 26 = \$2210$
Clothes = $\$170 \times 12 = \$2040$
Total = \$7890
Income – expenses = \$950
b \$950

Review questions 4.7F

- 1 20.2%
- 2 \$491.30
- 3 **a** \$325.85 **b** \$3584.35
- 4 **a** \$52 **b** \$572
- 5 **a** \$904.82 **b** \$77.73
c \$3.59 **d** \$36.27
- 6 \$1293.93

7 a

Running costs	Kia Cerato	Mazda 6 sedan	Mitsubishi ASX
Yearly loan repayments (\$)	8000.76	9458.52	7921.92
Annual registration and insurances (\$)	2654.16	2512.32	2367.60
<i>Average operating costs:</i>			
Fuel (\$ per year)	2269.80	2147.08	2331.16
Tyres (\$ per year)	156	167.16	212.04
Service and repairs (\$ per year)	416.16	500.76	441.96
Total cost for 1 year (\$)	13 496.88	14 785.84	13 274.68

b i

	Kia Cerato	Mazda 6 sedan	Mitsubishi ASX
Total cost/week (\$ per week)	259.56	284.34	255.28

ii

	Kia Cerato	Mazda 6 sedan	Mitsubishi ASX
Total cost/kilometre (cents per km)	0.90	0.99	0.88

8 \$1189.50

9 a 62 050 L

b \$176.84

10 a \$38 012

b \$40 641

c Example: Any car payments or loans, food costs, entertainment, personal items, clothing

11 a \$16 913

b \$47 322

c \$5382

d \$2288

Review questions 4.7G

1 a i \$367.95 ii \$349.55 iii \$384.51

b i \$239.99 ii \$274.93 iii \$7.42

c i \$374 ii \$599.70

iii \$23 347.70 iv \$1154.46

d i \$404

ii \$7689

iii No, she would need an extra \$9520 – \$4848 = \$4672 in savings a year.

2 a \$69 320

b \$1237.88

c i \$9654.70

ii \$3119.84

iii \$32 904.53

iv \$56 822.90

d i \$63 995

ii \$1279.90

iii \$9986.50

iv \$2541.40 payable

3 a \$84

b 6%

c 64.6%

d i \$1836

e i \$72.50

ii \$20 196

ii \$797.50

f \$295.82

4 a \$164.03

b \$674.14

c \$700.14

d 7.88%

5 a \$508.50

b \$4350

6 a \$32 414

b \$19 914

7 a i 4200 L

ii Twice

iii Every 18 months

b \$8338

8 a \$1 480 000

b i \$0.000 974 041

ii Yes

c The general rate payable in the row above is greater than the minimum amount of \$617.

d i \$25

ii \$335

iii \$360

e \$1801.58

f \$1952.56

g \$633 444

9 a Income: $170 \times 52 = \$8840$

Expenses:

Board: $40 \times 52 = \$2080$ Clothes: $100 \times 12 = \$1200$ Entertainment: $65 \times 26 = \$1690$ Lessons: $85 \times 26 = \$2210$

Total expenses: \$7180

Excess: Income – Expenses = \$1660

b \$1660

c Reduce expenses or increase income

Review: Financial mathematics

Review questions

1 a \$1077.46

b \$38 344.80

c \$1788.80

d \$1065.97

e \$387.50

f i \$912.96

ii \$28.53

g \$1733.40

h \$445.50

Module 5 Practicalities of measurement

Lesson 5.1 Significant figures

- a** 5; 6; bigger than, 5370, 5360; 5370

b 6; 5; smaller than, 0.0625, 0.0626; 0.0625
- a** 40 000 **b** 60

c 5 **d** 100

e 0.7 **f** 0.008

g 0.03 **h** 1000
- a** 290 000 **b** 4000

c 25 **d** 2700

e 8.6 **f** 0.049

g 0.000 16 **h** 0.0040
- a** 3690 **b** 20 700

c 154 000 **d** 813

e 14.3 **f** 0.003 51

g 0.0391 **h** 2.00
- a** **i** 20 **ii** 17 **iii** 17.3

b **i** 0.5 **ii** 0.45 **iii** 0.451

c **i** 500 000 **ii** 520 000 **iii** 522 000

d **i** 0.002 **ii** 0.0021

iii 0.002 10
- 3.286
- $1\frac{2}{5}(\approx 1.40)$, $\sqrt{2}(\approx 1.41)$, $\frac{10}{7}(\approx 1.42)$, $1.4(\approx 1.44)$
- 316.6 mm
- a** 417.3 km **b** 73 km/h
- a** 6402.0 km **b** 40 225 km **c** 92.8 min
- The original number could have been, for example, 5890 or 5900.3.

Lesson 5.2 Scientific notation

- a** Yes **b** No **c** No **d** No
- a** Yes **b** No **c** No **d** Yes

e No **f** No **g** No **h** Yes
- a** 2.430 00; 5, right, 5; 2.43×10^5

b 5.86; 4, left, 4; 5.86×10^{-4}
- a** 5.26×10^5 **b** 2.8×10^4

c 7×10^6 **d** 4.98×10^4

e 2.8×10^7 **f** 6.03×10^8

g 9.1×10^5 **h** 1.32×10^{10}
- a** 4.3×10^{-4} **b** 8.21×10^{-3}

c 7×10^{-6} **d** 2.9×10^{-5}

e 6.5×10^{-2} **f** 3.87×10^{-4}

g 8.2×10^{-6} **h** 6×10^{-5}
- a** 6, right, 5 480 000 **b** 5, left, 0.000 030 9
- a** 3 400 000 **b** 830 000 000

c 29 400 000 **d** 258 000

e 526 000 **f** 3 020 000 000 000

g 29 000 000 **h** 875 000 000

- a** 0.000 59 **b** 0.000 003 2

c 0.000 000 071 **d** 0.002

e 0.000 000 8 **f** 0.000 026 4

g 0.000 000 008 67 **h** 0.000 002 97
- a** 1.07×10^{16} **b** 4.76×10^{22}

c 3.22×10^{17} **d** 1.39×10^{-14}

e 3.36×10^{11} **f** 1.25×10^{20}

g 5.28×10^{-16} **h** 4.68×10^{24}

i 2.40×10^8 **j** 8.25×10^8

k 1.69×10^{21} **l** 2.62×10^{-55}
- a** 2.29×10^8 **b** 2.54×10^{-11}

c 3×10^{27} **d** 1.3×10^5

e 1×10^{13}
- a** 31 600 000 **b** 0.000 000 89

c 2 600 000 **d** 30 000 000 000

e 0.000 26
- a** **i** 1.8×10^7 km **ii** 1.08×10^9 km

iii 2.592×10^{10} km **iv** 9.4608×10^{12} km

b 7.38×10^7 km
- a** 5.1×10^8 km² **b** 1.1×10^{12} km³
- 9.4 $\times 10^8$ km
- a** 10 000 000 000 **b** 280 km
- Answers vary. For a 65 year old with a pulse of 60 bpm we have that:
beats $\approx 3.15 \times 10^8$
- a** **i** 3.333×10^{-7} s **ii** 3.030×10^{-1} s

b 3.030×10^{-1}

c If the timekeeper starts the stopwatch when they see the smoke, they will actually start the stopwatch 0.000 000 333 3 seconds late (because this is the time it takes for the light to travel this distance). Since the stopwatch only measures to the nearest 0.01 of a second, this will make no difference to the timing of the race. If the timekeeper starts the stopwatch when they hear the gun, they will actually start the stopwatch 0.0303 seconds late (because this is the time it takes for the sound to travel this distance). This will make the timing of the race 0.03 seconds less than the real time.

18 6.065×10^{17}

Lesson 5.3 Metric units of measurement

- a** 3600 m **b** 8400 mm **c** 3482 cm

d 560 m **e** 290 cm **f** 964 mm

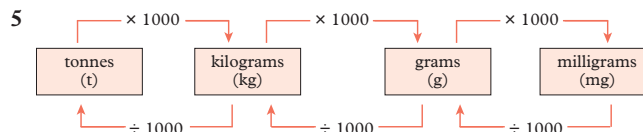
g 65.8 cm **h** 452 mm **i** 15 680 m

j 36.9 mm **k** 16 370 mm **l** 4265 m
- a** 1×10^5 cm

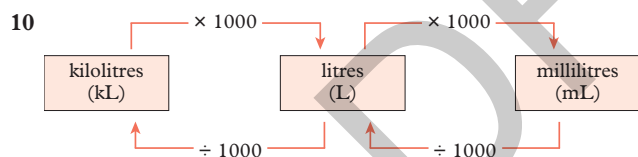
b 1×10^6 mm

- 3 a 7 km b 5.94 m c 8.93 km
 d 6 m e 4 cm f 0.085 km
 g 8 m h 32.8 cm i 0.62 m
 j 14.3 km k 0.86 m l 0.63 km
 m 9.4 cm n 0.07 m o 24.895 km
 p 23 m q 14.96 m r 162.7 m
 s 36 m t 72.945 m

- 4 a m b cm or mm
 c cm d cm
 e cm or mm f mm
 g km h cm
 i m j km



- 6 a 3600 kg b 7100 mg c 17 840 g
 d 630 kg e 4800 g f 2465 kg
 g 21 590 kg h 600 mg i 6060 g
 j 489 g k 1070 kg l 30 mg
- 7 a 1×10^6 g b 1×10^9 mg c 1×10^3 g
- 8 a 8 t b 4.3 kg c 2.74 g
 d 0.69 kg e 0.065 g f 2.32 kg
 g 0.7 t h 0.46 g i 0.08 kg
 j 0.007 g k 0.009 t l 0.3 kg
- 9 a kg b g c g
 d kg e t or kg f t or kg
 g mg h g i mg
 j g

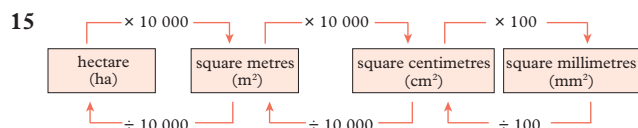


- 11 a 35 000 L b 15 900 mL
 c 1650 mL d 850 L
 e 60 mL g 15 mL
 f 1080 L h 5 L

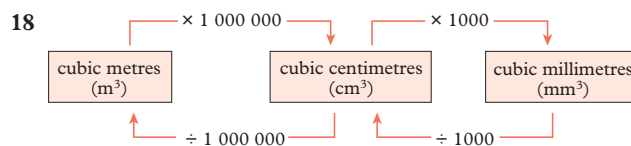
12 $1\,000\,000 = 1 \times 10^6$ mL

- 13 a 15 L b 8 kL
 c 7.6 L d 0.8 L
 e 9.28 kL f 0.725 kL
 g 0.095 L h 0.04 kL

- 14 a mL b kL c L d L
 e L f kL g L h L

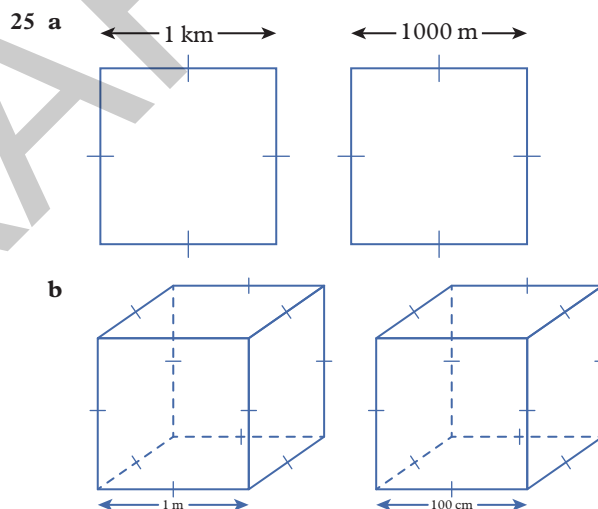


- 16 a 26 000 m² b 49 000 cm²
 c 1400 mm² d 7520 cm²
 e 16 500 m² f 2480 mm²
 g 8 294 000 m² h 56 710 000 000
 17 a 6.3 ha b 12.7 m²
 c 8.1 cm² d 4.568 m²
 e 0.298 km² f 240 ha



- 19 a 5000 mm³ b 3 900 000 cm³
 c 25 600 mm³ d 640 000 cm³
 e 415 mm³ f 7 390 000 000 mm³
 20 a 7.4 m³ b 56.7 cm³
 c 0.69 m³ d 4.2585 cm³
 21 a 14 kg b 9 L

- 22 6.4×10^8 kL
 23 2.5%
 24 a 5.8 m³ (to one decimal place)
 b i 0.0253 m² ii 0.0288 m²
 iii 2084



Lesson 5.4 Unit prefixes in the metric system

- 1 a i 5.7×10^6 (5 700 000) m
 ii 9×10^{-2} (0.09) m
 iii 8×10^{-9} (0.000 000 008) m
 b i 8×10^9 (8 000 000 000) g
 ii 4.2×10^{-3} (0.0042) g
 iii 5×10^{-6} (0.000 005) g
 c i 3×10^{12} (3 000 000 000 000) L
 ii 2.8×10^6 (2 800 000) L
 iii 7×10^{-3} (0.007) L
- 2 a i 4.9×10^{-3} (0.0049) km
 ii 4.9×10^6 (4 900 000) μ m
 iii 4.9×10^9 (4 900 000 000) nm

- b i** 2.4×10^{-6} (0.000 002 4) Mg
ii 2.4×10^{-12} (0.000 000 000 002 4) Tg
iii 2.4×10^6 (2 400 000) μg
- c i** 6.5×10^{-6} (0.000 006 5) ML
ii 6.5×10^3 (6500) mL
iii 6.5×10^{-9} (0.000 000 006 5) GL
- 3 a** 7.2×10^3 (7200) Mg
b 2.9×10^7 (29 000 000) ng
c 8×10^9 (8 000 000 000) kg
d 4.3×10^4 (43 000) μm
e 8.8×10^6 (8 800 000) ML
f 9×10^6 (9 000 000) mL
g 5.3×10^6 (5 300 000) ns
h 1.2×10^3 (1200) ns
- 4** 9.46×10^3 (9460) Tm
5 2580 GL
6 228 000 000 km
7 1.67×10^{-9} ng
8 5.00×10^{-3} μs
9 1 330 000 000 m^3

Lesson 5.5 Review: Practicalities of measurement

Review questions 5.5A

- 1 a** 5 **b** 5 **c** 2
- 2** A is a decimal number between 1 and 10. n is a positive or negative whole number.
- 3** Metric
- 4 a** Gram, k **b** Litre, T
c Second, m **d** Metre, μ
e Metre, M **f** Gram, c
g Litre, n **h** Second, G

Review questions 5.5B

- 1 B** **2 C** **3 A** **4 B**
5 B **6 D** **7 C** **8 B**

Review questions 5.5C

- 1 a** 3700 **b** 3659
c 3659.06 **d** 3660
e 3659.1 **f** 4000
- 2 a** No **b** No **c** Yes
- 3 a** 1.05×10^8 **b** 6.2×10^{-5} **c** 3.179×10^3
- 4 a** 2.46×10^{14} **b** 1.4×10^{-10}
c 4.096×10^{23} **d** 2.9×10^{-6}
- 5 a** 560 mm^2 **b** 4.3 ha
c 2 900 000 cm^3 **d** 5.6 cm^3
- 6 a** 2 300 000 000 m
b 52 000 kL
c 3000 μs
d 7 200 000 000 000 000 mg

Review questions 5.5D

- 1 a** 1470 **b** 1500 **c** 1472.63
2 a 7.49×10^5 **b** 3×10^{-6} **c** 1.05×10^{-2}
3 6.3×10^{15}
4 a 21 000 m^2 **b** 7.8 cm^2
c 9.5 m^3 **d** 72 000 mm^3
5 a 43 000 μm **b** 2 000 000 Mg
c 52 ML **d** 9 100 000 nm

Review questions 5.5E

- 1 a** 0.0051 **b** 0.01 **c** 0.005
2 a No **b** Yes **c** No
3 a 6.7×10^4 **b** 8.09×10^{-5} **c** 1.23×10^3
4 a 3 400 000 **b** 0.000 087 **c** 20 530
5 a 1120 mm^2 **b** 12.9 ha
c 3 400 000 cm^3 **d** 73 cm^3
6 a 4 500 000 m **b** 2 t
c 7000 ns **d** 0.000 003 5 GL

Review questions 5.5F

- 1 a** 2 **b** 2.1
c 2.07 **d** 2.070
2 a 1365 cm **b** 3.46 t
c 0.276 L **d** 8300 mm
3 a 136 500 m^2 **b** 19.6 cm^2
c 3.7 m^3 **d** 6800 m^3
4 a 6000 μL **b** 4 200 000 kg
c 8100 ns **d** 0.000 56 TL
5 a 6.4×10^8 **b** 3.0×10^{-7}
6 a 1.5×10^{-2} **b** 1.4×10^{10}

Review questions 5.5G

- 1 a** 1.31×10^{-3} **b** Two **c** 8.2×10^5
d i 94.63 m **e i** $3.444 \times 10^{11} \text{ m}^2$
ii 2 500 000 cm^3 **ii** Four
f i 2.41×10^{24} elementary units
ii $6.022\,140\,76 \times 10^{26}$

Module 6 Perimeter, area and Pythagoras' theorem

Lesson 6.1 Perimeter

- 1 a** 102 mm **b** 189 mm **c** 297 mm
2 a 7.8 cm **b** 13.2 cm **c** 33.6 cm
3 $x = 6 + 3 = 9$
 $y = 15 - 7 = 8$
 $P = 15 + 3 + 7 + 6 + 8 + 9 = 48 \text{ cm}$
4 a $x = 26 \text{ cm}, y = 13 \text{ cm}, P = 174 \text{ cm}$
b $x = 37 \text{ cm}, y = 8 \text{ cm}, P = 142 \text{ cm}$
c $x = 5 \text{ cm}, y = 5 \text{ cm}, P = 74 \text{ cm}$

5 a $C = \pi d = \pi \times 12 \text{ cm} \approx 37.7 \text{ cm}$

$C = 2\pi r$

b $r = (12 \text{ cm} \div 2) = 6 \text{ cm}$

$C = 2 \times \pi \times 6 \text{ cm} \approx 37.7 \text{ cm}$

6 a 28.3 m b 20.1 m

c 56.9 m d 42.7 m

e 39.6 m f 49.6 m

g 15.7 m h 27.1 m

7 a 20.6 cm b 25.7 cm

c 86.3 m d 102.8 m

8 a 21.4 cm b 16.8 cm

c 55.7 m d 75.7 m

9 a 21.42 cm b 45.71 m c 57.13 cm

10 a Fraction $= \frac{15^\circ}{360^\circ} = \frac{1}{24}$

b $P = \frac{1}{24} \times 2 \times \pi \times 7.2 \text{ cm} + 2 \times 7.2 \text{ cm}$
 $\approx 16.3 \text{ cm}$

11 a $\frac{1}{24}$, $P \approx 14.5 \text{ cm}$ b $\frac{1}{8}$, $P \approx 14.8 \text{ cm}$

c $\frac{1}{12}$, $P \approx 18.2 \text{ cm}$ d $\frac{3}{8}$, $P \approx 52.7 \text{ cm}$

e $\frac{1}{3}$, $P \approx 31.9 \text{ cm}$ f $\frac{29}{36}$, $P \approx 58.6 \text{ cm}$

12 a 35.9 cm b 22.4 cm c 46.0 cm

13 125 cm

14 \$237.15

15 a 12.57 m, 12.57 m b Same distance

c No difference

16 \$744

17 8.0 cm

18 31.8 m

19 a 7200 km b 45 239 km c 1885 km/h

20 64.80 km

21 a 8 laps b 18 min

Lesson 6.2 Using Pythagoras' theorem

1 a 7.8 cm b 13.9 cm

c 23.4 m d 7.4 km

e 97.7 mm f 182.1 cm

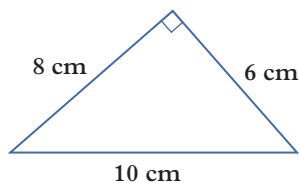
2 a 8.9 cm d 37.8 m

b 15.3 cm e 70.5 mm

c 2.7 m f 14.8 km

3 a i Yes

ii

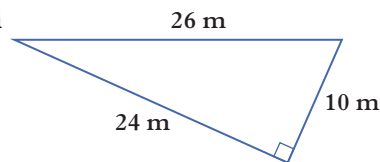


b i No

c i No

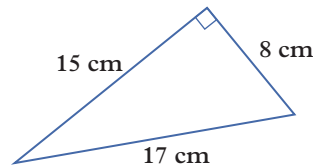
d i Yes

ii



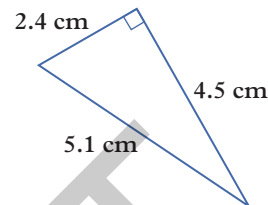
e i Yes

ii



f i Yes

ii



4 a 379 m b 263 m c 178 m

d 218 m e 252 m f 451 m

g 218 m h 297 m i 147 m

5 a $x \approx 10.6 \text{ cm}$, $P \approx 55.6 \text{ cm}$

b $x \approx 17.2 \text{ cm}$, $P \approx 60.2 \text{ cm}$

c $x \approx 14.4 \text{ cm}$, $P \approx 95.3 \text{ cm}$

6 a 13 cm b 52 cm

7 Yes ($4187^2 = 2700^2 + 3200^2$)

8 No ($3000^2 \neq 2500^2 + 850^2$)

9 a 7.2 cm b 11.3 m

10 1.1 m

11 Yes

12 3.0 m

13 21.4 m

14 97.1 km

15 a 6.7 cm

b 11.7 cm

c $P \approx 36.7 \text{ cm}$

16 a 136 m b 256 m c 169 m

17 44.43 cm

Lesson 6.3 Area of simple shapes

1 a 60.63 cm^2

b 9.1 cm^2

c 64 cm^2

d 38.11 cm^2

2 a 18 m^2

b 5 cm^2

c 800 m^2

d 20 cm^2

3 a 60 mm^2

b 750 mm^2

c 168 cm^2

d 100 m^2

4 a 30 cm^2

b 98 m^2

c 40 m^2

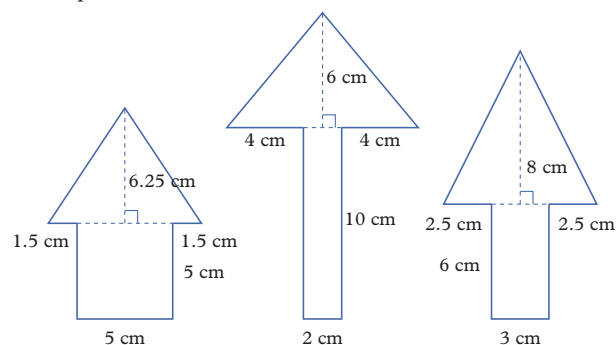
d 1190 mm^2

- 5 a 78.5 cm^2 b 254.5 cm^2
 c 706.9 cm^2 d 314.2 cm^2
 e 132.7 cm^2 f 38.5 cm^2
 g 254.5 cm^2 h 380.1 cm^2
- 6 a Fraction $= \frac{15^\circ}{360^\circ} = \frac{1}{24}$
 b $A = \frac{1}{24} \times \pi \times (7.2 \text{ cm})^2 \approx 7 \text{ cm}^2$
- 7 a 5 cm^2 b 11 cm^2 c 14 cm^2
 d 48 cm^2 e 184 cm^2 f 64 cm^2
 g 45 cm^2 h 15 cm^2 i 117 cm^2
- 8 a 9 cm b 8 cm
 c 4 cm d 5.6 m
- 9 a $20\,025 \text{ m}^2$ b 2 ha
 c One possible set of dimensions is 225 m by 89 m .
- 10 33
 11 62 cm^2
 12 40.4%
 13 57°

Lesson 6.4 Area of composite shapes

- 1 a 96 cm^2 b 44 m^2 c 22 m^2
 d 224 cm^2 e 65 cm^2 f 45.5 m^2
- 2 a 168 cm^2 b 63 cm^2 c 78 cm^2
 d 130 cm^2 e 65 cm^2 f 107 m^2
- 3 a 90 cm^2 b 34 m^2 c 33 m^2
 d 30 km^2 e 24 cm^2 f 50 m^2
 g 54 cm^2 h 57 cm^2 i 38 cm^2
 j 265.50 cm^2 k 52.50 cm^2 l 54.54 cm^2
- 4 a 552 cm^2 b 207 cm^2 c 90 cm^2
 5 a 127.0 cm^2 b 69.1 cm^2 c 49.1 cm^2
 d 190.1 cm^2 e 63.3 cm^2 f 68.6 cm^2
 g 144.3 cm^2 h 39.8 cm^2 i 164.5 m^2
- 6 a 85 cm^2 b 204 cm^2 c 302 cm^2
 d 236 cm^2 e 104 cm^2 f 415 cm^2
- 7 a 31 cm^2 b 40 cm^2 c 219 cm^2
 d 113 cm^2 e 75 cm^2 f 107 cm^2
- 8 a 18.53 m^2 b $\$10\,006$
- 9 84.8 m^2
- 10 a 337 m^2 b $\$3340$
 11 a 37 m^2 b $\$1291.30$
 12 a 75.4 cm^2 b 530.1 mm^2
 c 35.6 cm^2 d 25.1 cm^2
 e 25.5 cm^2 f 49.1 cm^2

13 Some possible answers are:



14 4.1 m

Lesson 6.5 Perimeter and area of irregular shapes

- 1 $A = \frac{1}{2} \times 22 \text{ m} \times 10 \text{ m} + \frac{1}{2} \times 22 \text{ m} \times 18 \text{ m}$
 $= 110 \text{ m}^2 + 198 \text{ m}^2$
 $= 308 \text{ m}^2$
- 2 a 28 m^2 b 71.5 m^2 c 308 m^2
 d 1254 m^2 e 517 m^2 f 5217 m^2
- 3 $13^2 + (5 + 15)^2 = 569$
 $\sqrt{569} \approx 23.9$
 $BC \approx 23.9 \text{ m}$
 $13^2 + 11^2 = 290$
 $\sqrt{290} \approx 17.0$
 $CD \approx 17.0 \text{ m}$
 $12^2 + (15 + 11)^2 = 820$
 $\sqrt{820} \approx 28.6$
 $AD \approx 28.6 \text{ m}$
 $12^2 + 5^2 = 169$
 $\sqrt{169} = 13$
 $AB = 13 \text{ m}$
 Perimeter $\approx 23.85 + 17.03 + 28.64 + 13$
 $\approx 82.5 \text{ m}$
- 4 a 145.8 m b 106.3 m
 c 295.5 m
- 5 Area $\triangle MQP = \frac{1}{2} \times (10 \text{ m} + 12 \text{ m} + 3 \text{ m} + 8 \text{ m}) \times 16 \text{ m}$
 $= 264 \text{ m}^2$
 Area $\triangle MRN = \frac{1}{2} \times 10 \text{ m} \times 11 \text{ m} = 55 \text{ m}^2$
 Area $\triangle TOP = \frac{1}{2} \times 7 \text{ m} \times 8 \text{ m} = 28 \text{ m}^2$
 Area $\triangle NOTR = \frac{1}{2} \times (12 \text{ m} + 3 \text{ m}) \times (7 \text{ m} + 11 \text{ m})$
 $= \frac{1}{2} \times 15 \text{ m} \times 18 \text{ m}$
 $= 135 \text{ m}^2$
 Area $MNOPQ = 264 \text{ m}^2 + 55 \text{ m}^2 + 28 \text{ m}^2 + 135 \text{ m}^2$
 $= 482 \text{ m}^2$
- 6 a 3662 m^2 b 964 m^2 c $16\,625 \text{ m}^2$

7 $(MN)^2 = 10^2 + 11^2 = 221$

$MN = \sqrt{221} \approx 14.87 \text{ m}$

$(NO)^2 = (11 - 7)^2 + (12 + 3)^2 = 241$

$NO = \sqrt{241} \approx 15.52 \text{ m}$

$(OP)^2 = 7^2 + 8^2 = 113$

$OP = \sqrt{113} \approx 10.63 \text{ m}$

$(PQ)^2 = (3 + 8)^2 + 16^2 = 377$

$PQ = \sqrt{377} \approx 19.42 \text{ m}$

$(QM)^2 = 16^2 + (10 + 12)^2 = 740$

$QM = \sqrt{740} \approx 27.20 \text{ m}$

Perimeter $\approx 14.87 \text{ m} + 15.52 \text{ m} + 10.63 \text{ m} +$

$19.42 \text{ m} + 27.20 \text{ m}$

$= 87.64 \text{ m}$

$\approx 88 \text{ m}$

8 a $245.2 \text{ m} \approx 245 \text{ m}$

b 122 m

c $514.4 \text{ m} \approx 514 \text{ m}$

9 Area $\approx \frac{40 \text{ m}}{2}(98 \text{ m} + 52 \text{ m}) = 20 \text{ m} \times 150 \text{ m} = 3000 \text{ m}^2$

10 a 3478 m^2 b 3094 m^2 c 2260.5 m^2

11 a 365 m^2 b 1380 m^2 c 372 m^2

d 1909.5 m^2 e 545 m^2 f 1024 m^2

12 a $33\,000 \text{ m}^2$ b $15\,600 \text{ m}^2$

c $17\,400 \text{ m}^2$ d $174\,000 \text{ m}^3$

13 a 3000 m^2 b $54\,000 \text{ m}^3$

14 a 549 m^2 b 551.25 m^2

15 a 222.5 m^2 b 2225 m^3 c $133\,500 \text{ m}^3$

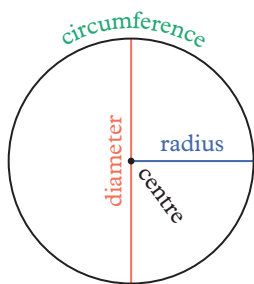
16 a 344 m^2 b 6880 m^3 c $41\,280 \text{ m}^3$

17 14 m

Lesson 6.6. Review: Perimeter, area and Pythagoras' theorem

Review questions 6.6A

1



2 Pythagoras' theorem states, that in a right-angled triangle, the square of the hypotenuse (c) is equal to the sum of the squares of the other two sides (a and b).

3 a parallelogram (or rectangle or square)

b square (or rhombus)

c kite (or square or rhombus)

4 The Trapezoidal rule uses two opposite sides of the shape which are parallel (like in a trapezium) and then approximates the other two sides using straight lines. This approximates the shape as a trapezium and so its area formula can be used.

Review questions 6.6B

1 D

2 C

3 A

4 D

5 A

6 B

7 C

8 B

9 A

10 C

11 C

12 C

Review questions 6.6C

1 a $P = 21.2 \text{ cm}, A = 21 \text{ cm}^2$

b $P = 34 \text{ cm}, A = 48 \text{ cm}^2$

c $P = 28.3 \text{ cm}, A = 63.6 \text{ cm}^2$

d $P = 17.6 \text{ cm}, A = 19.4 \text{ cm}^2$

e $P = 174.0 \text{ cm}, A = 2000 \text{ cm}^2$

f $P = 53.4 \text{ cm}, A = 93.5 \text{ cm}^2$

2 a 405 m^2

b 514.5 m^2

Review questions 6.6D

1 a $P = 19.4 \text{ cm}, A = 12 \text{ cm}^2$

b $P = 11 \text{ cm}, A = 6.24 \text{ cm}^2$

c $P = 30.3 \text{ m}, A = 54 \text{ m}^2$

d $P = 22.5 \text{ cm}, A = 31.2 \text{ cm}^2$

e $P = 21.7 \text{ cm}, A = 25.5 \text{ cm}^2$

f $P = 32 \text{ cm}, A = 39 \text{ cm}^2$

2 a $P = 67.9 \text{ m}, A = 279 \text{ m}^2$

b 517.5 m^2

Review questions 6.6E

1 a $P = 32.8 \text{ cm}, A = 67.24 \text{ cm}^2$

b $P = 3.6 \text{ m}, A = 0.7 \text{ m}^2$

c $P = 47.6 \text{ cm}, A = 90 \text{ cm}^2$

d $P = 38.1 \text{ cm}, A = 78.5 \text{ cm}^2$

e $P = 43.0 \text{ cm}, A = 96.5 \text{ cm}^2$

f $P = 20.0 \text{ cm}, A = 24.1 \text{ cm}^2$

2 a $P = 170.0 \text{ m}, A = 1312.5 \text{ m}^2$

b 1056 m^2

Review questions 6.6F

1 a $P = 61.7 \text{ mm}, A = 166.8 \text{ mm}^2$

b $P = 15 \text{ m}, A = 12.7 \text{ m}^2$

c $P = 355.0 \text{ mm}, A = 10028.7 \text{ mm}^2$

d $P = 40.8 \text{ cm}, A = 100 \text{ cm}^2$

e $P = 66.0 \text{ cm}, A = 148.4 \text{ cm}^2$

f $P = 205.7 \text{ cm}, A = 2856.6 \text{ cm}^2$

2 a 4500 m^2

b 1056 m^2

Review questions 6.6G

1 a $P = 25.4 \text{ m}, A = 31 \text{ m}^2$

b $P = 25.2 \text{ cm}, A = 36.9 \text{ cm}^2$

c 833 m^2

d $A = 850 \text{ m}^2$

e The area in part d is greater than the area in part c and is closer to the actual area of the field. This is because more applications of the Trapezoidal rule will result in a better estimate of the area of the field. The estimated area increased in part d because more of the area of the field is included in the estimate.

Module 7 Surface area, volume and capacity

Lesson 7.1 Surface area of prisms

- a** 54 cm^2 **b** 1350 cm^2 **c** 864 cm^2

a 138.24 cm^2 **b** 1734 mm^2 **c** 0.5766 m^2
- Top and bottom: $A = 2 \times 10 \times 6 = 120 \text{ cm}^2$

Front and back: $A = 2 \times 5 \times 10 = 100 \text{ cm}^2$

Two sides: $A = 2 \times 5 \times 6 = 60 \text{ cm}^2$

Surface area: $SA = 120 + 100 + 60 = 280 \text{ cm}^2$
- Front and back: $A = 2 \times 8 \times 4 = 64 \text{ cm}^2$

Two sides: $A = 2 \times 4 \times 5 = 40 \text{ cm}^2$

Top and bottom: $A = 2 \times 8 \times 5 = 80 \text{ cm}^2$

Surface area: $SA = 64 + 80 + 40 = 184 \text{ cm}^2$
- a** 276 cm^2 **b** 424 cm^2 **c** 598 cm^2

d 432 cm^2 **e** 8970 mm^2 **f** 8440 m^2
- a** 280 m^2 **b** 684 mm^2 **c** 382.46 m^2
- a** 600 cm^2 **b** 224 cm^2 **c** 1490 cm^2

d 300 cm^2 **e** 3150 cm^2 **f** 241.6 cm^2
- a** 880 cm^2 **b** 614 cm^2

c 1260.16 cm^2 **d** 1050.18 cm^2

e 9320 cm^2 **f** 1156.40 cm^2
- a** 1120 cm^2 **b** $\$14$
- a** Room A = 38.28 m^2 , Room B = 34.4 m^2 .
Room A has the greater area.

b 4 tins

c $\$219.80$
- 7 m^2
- Weet-Bix: 1161 cm^2 , Sultana Bran: 1262.5 cm^2 .
The Sultana Bran box has the greater surface area.
- a** 313.4 m^2 **b** 5 cans **c** $\$412.50$
- a** 231.7 cm^2 **b** 285.1 cm^2
- 2397.6 m^2

Lesson 7.2 Surface area of cylinders and spheres

- $SA = 2\pi rh$
 $= 2 \times \pi \times 4 \text{ cm} \times 9 \text{ cm}$
 $= 226.19... \text{ cm}^2$
 $\approx 226.2 \text{ cm}^2$
- a** 534.1 cm^2 **b** 664.8 cm^2 **c** 1055.6 cm^2

d 256.0 cm^2 **e** 1005.3 cm^2 **f** 89.2 cm^2
- Diameter = 12 cm

Radius = 6 cm

$SA = 2\pi rh$
 $= 2 \times \pi \times 6 \text{ cm} \times 8 \text{ cm}$
 $= 301.592... \text{ cm}^2$
 $\approx 301.6 \text{ cm}^2$
- a** 942.5 cm^2 **b** 1017.9 cm^2 **c** 1178.1 cm^2

d 303.5 cm^2 **e** 1372.9 cm^2 **f** 115.5 cm^2

- a** 301.6 cm^2 **b** 735.1 cm^2

c 94.2 cm^2 **d** 830.4 cm^2
- Radius = $15.8 \text{ cm} \div 2 = 7.9 \text{ cm}$

$SA = 2\pi r^2 + 2\pi rh$
 $= 2 \times \pi \times (7.9 \text{ cm})^2 + 2 \times \pi \times 7.9 \text{ cm} \times 17.6 \text{ cm}$
 $= 1265.747... \text{ cm}^2$
 $\approx 1265.7 \text{ cm}^2$
- a** 1445.1 cm^2 **b** 603.2 cm^2

c 527.8 cm^2 **d** 2264.4 cm^2

e 4246.4 cm^2 **f** 318.7 m^2
- a** 1809.6 cm^2 **b** 3.1 m^2

c $53\,092.9 \text{ mm}^2$
- $r = 18 \text{ cm} \div 2 = 9 \text{ cm}$

$SA = 4\pi r^2$
 $= 4 \times \pi \times (9 \text{ cm})^2$
 $= 1017.876... \text{ cm}^2$
 $\approx 1017.9 \text{ cm}^2$
- a** 1963.5 cm^2 **b** 1063.6 cm^2

c $48\,305.1 \text{ mm}^2$
- a** 452.4 cm^2 **b** 1256.6 cm^2

c 907.9 cm^2
- a** 445.46 cm^2 **b** 207.38 m^2

c 3.03 m^2
- a** 2827.4 m^2 **b** 36 cans

c $\$4147.20$
- 141 spheres
- $36\,945.13 \text{ cm}^2$
- The roller with a diameter of 8 cm and length 25 cm has a larger surface area and so would not need to be re-dipped as often.
- 74%
- No
- a** 157.7 cm^2 **b** 173.4 m^2

c 424.1 m^2
- The area of the top surface of the cylinder is the same as the area not counted when calculating the surface area of the top of the rectangular prism.
- 1979.2 cm^2

Lesson 7.3 Volume of prisms, cylinders and spheres

- $V = A \times h$ $\text{m} = 20 \text{ m}^2 \times 3 \text{ m} = 60 \text{ m}^3$
- a** 75 cm^3 **b** 115.2 cm^3 **c** 420 cm^3

d 815.4 cm^3 **e** 11.96 cm^3 **f** 120 cm^3

g 648 cm^3 **h** 116.96 cm^3 **i** 48 cm^3

j 360 cm^3 **k** 140 cm^3 **l** 80 cm^3
- a** $A = lb$
 $= 9 \text{ cm} \times 5 \text{ cm}$
 $= 45 \text{ cm}^2$
 $V = Ah$
 $= 45 \text{ cm}^2 \times 6 \text{ cm}$
 $= 270 \text{ cm}^3$

$$\begin{aligned} \text{b } A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 5 \text{ cm} \times 4 \text{ cm} \\ &= 10 \text{ cm}^2 \\ V &= Ah \\ &= 10 \text{ cm}^2 \times 10 \text{ cm} \\ &= 100 \text{ cm}^3 \end{aligned}$$

- 4 a 270 cm³ b 420 cm³ c 108 cm³
 d 343 cm³ e 720 cm³ f 703.08 cm³
 g 1037.244 cm³ h 460.6 cm³ i 89.79 cm³
 j 568.764 cm³ k 1536 cm³ l 31.29 cm³

$$\begin{aligned} 5 \text{ a } V &= \pi r^2 h \\ &= \pi \times (4 \text{ m})^2 \times 10 \text{ m} \\ &\approx 502.65 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{b } V &= \pi r^2 h \\ &= \pi \times (8 \text{ m} \div 2)^2 \times 25 \text{ m} \\ &= \pi \times (4 \text{ m})^2 \times 25 \\ &\approx 1256.64 \text{ m}^3 \end{aligned}$$

- 6 a 571 cm³ b 5089 cm³ c 40 cm³

- 7 a 942.5 cm³ b 3318.3 cm³
 c 2058.9 cm³ d 2544.7 cm³
 e 109 858.4 cm³ f 46 876.3 cm³

- 8 a 7238.2 cm³ b 0.5 m³
 c 1 150 346.5 mm³

- 9 a 8181.2 cm³ b 3261.8 cm³
 c 998 306.0 mm³

- 10 a 904.8 cm³ b 4188.8 cm³
 c 2572.4 cm³

- 11 a 1250.24 cm³ b 397.13 m³
 c 0.70 m³

$$12 V = \frac{2}{3}\pi r^3$$

- 13 The base of a rectangular prism is given by $A = lb$.
 Substituting this into the formula $V = Ah$ gives $V = lbh$.

- 14 a i 325 cm² ii 1625 cm³
 b i 622 cm² ii 5598 cm³
 c i 143 cm² ii 1144 cm³
 d i 675 cm² ii 6075 cm³
 e i 147.84 cm² ii 1034.88 cm³
 f i 1120 cm² ii 4480 cm³

- 15 If $r = 3 \text{ cm}$, $h = 4 \text{ cm}$, then $V = 113.1 \text{ cm}^3$.

If $r = 4 \text{ cm}$, $h = 3 \text{ cm}$, then $V = 150.8 \text{ cm}^3$.

The cylinder of radius 4 cm and height 3 cm has the greater volume.

- 16 a 28 cm² b 7 cm c 0.6 cm
 17 48 000 000 mm³ or 48 000 cm³
 18 a 100.5 cm³ b 120.7 m³ c 733.0 m³
 19 52.4%

Lesson 7.4 Volume of pyramids and cones

- 1 a B b C c A

$$\begin{aligned} 2 \text{ a } \text{Volume} &= \frac{1}{3} \times \text{area of base} \times \text{height} \\ &= \frac{1}{3} \times 10 \text{ cm} \times 10 \text{ cm} \times 12 \text{ cm} \\ &= 400 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Volume} &= \frac{1}{3} \times \text{area of base} \times \text{height} \\ &= \frac{1}{3} \times (\pi \times 6^2 \text{ cm}^2) \times 12 \text{ cm} \\ &= 96\pi \text{ cm}^3 \\ &\approx 301.6 \text{ cm}^3 \end{aligned}$$

- 3 a 256 cm³ b 80 cm³ c 261.8 cm³
 d 314.2 cm³ e 26.7 cm³ f 28 cm³
 4 a 40 cm³ b 804.2 cm³ c 144 cm³
 5 a 302 cm³ b 90 cm³ c 49 087 cm³
 d 26 cm³ e 12 000 cm³ f 1340 cm³
 6 a 62.8 m³ b 923.6 cm³ c 39 000 cm³
 d 488.2 m³ e 716.3 cm³ f 523.6 cm³
 g 323.3 m³ h 203.2 cm³ i 371.1 cm³
 j 724.0 m³ k 500 cm³ l 2309.1 cm³
 m 435.6 m³ n 390 cm³ o 147 000 cm³
 7 2 468 666.67 m³
 8 a 94.25 cm³ b 150.80 cm³
 9 263.9 cm³

Lesson 7.5 Capacity

$$\begin{aligned} 1 \text{ a } \text{Volume} &= 6.5 \times 1000 \text{ cm}^3 \\ &= 6500 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Capacity} &= 25\,600 \times 1 \text{ mL} \\ &= 25\,600 \text{ mL} \\ &= 25\,600 \times \frac{1}{1000} \text{ L} \\ &= 25.6 \text{ L} \end{aligned}$$

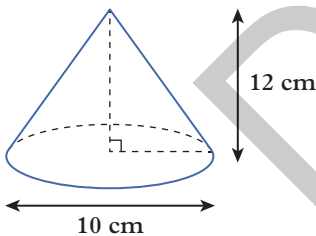
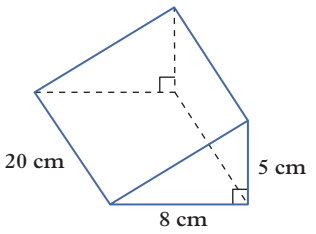
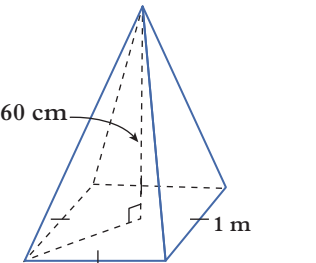
- 2 a 2000 cm³ b 12 000 cm³ c 2600 cm³
 d 450 cm³ e 975 cm³
 3 a 650 cm³ b 7500 cm³ c 1250 cm³
 d 3780 cm³ e 845 cm³
 4 a 1495 mL b 3000 mL c 500 mL
 d 24 000 mL e 6740 mL
 5 a 1.495 L b 3 L c 0.5 L
 d 24 L e 6.74 L
 6 a 2 m³ b 5 m³ c 6.7 m³
 d 0.9 m³ e 83.2 m³
 7 a 0.02 kL b 0.7 kL c 28.4 kL
 d 302 kL e 5660 kL
 8 $V = \pi r^2 h = \pi \times (3 \text{ m})^2 \times 5 \text{ m} \approx 141.4 \text{ m}^3$
 Capacity = 141.4 kL
 9 a 5832 cm³ b 5832 mL c 5.832 L
 10 a 704 000 000 cm³ b 704 000 000 mL
 c 704 000 L d 704 kL
 11 a 62 800 L b 0.9236 L c 39 L
 d 488 200 L e 0.7163 L f 0.5236 L
 g 323 300 L h 0.2032 L i 0.3711 L
 j 724 000 L k 0.5 L l 2.3091 L
 m 435 600 L n 0.390 L o 147 L
 12 6.77 mL

- 13 3.62 L
 14 40 ML
 15 Vase 1: 785 mL; Vase 2: 730 mL; Vase 1 holds 55 mL more.
 16 a 60 000 cm³
 b i 60 000 mL ii 60 L
 c 54 L
 17 1508 L
 18 a 907 200 cm³ b 907.2 L
 19 a 181 m³ b 181 kL
 20 735 mL
 21 a 9.2 L b 8 breaths
 22 13 m
 23 Inside volume of tank is 60 000 cm³.
 Possible dimensions are 100 cm by 60 cm by 10 cm.
 24 724 mL
 25 a 74.5 kL b \$198.92 c \$16.02

Lesson 7.6 Review: Surface area, volume and capacity

Review questions 7.6A

- 1 a volume, capacity b surface area, net
 2 The two identical, parallel faces of a prism are polygons, which are 2D shapes with straight edges. The two identical, parallel faces of a cylinder are circles which are not polygons.
 3 The formula $V = Ah$ can be used whenever the object has a uniform cross-section. The cross-sections of spheres, pyramids and cones are not uniform.

- 4 a 
 b 
 c 

Review questions 7.6B

- 1 B 2 D 3 C
 4 A 5 A 6 C
 7 A 8 D 9 B

Review questions 7.6C

- 1 a 136 m² b 1005.3 cm² c 706.9 cm²
 2 a 21 cm³ b 240 cm³ c 1005.3 m³
 d 4 cm³ e 452.4 mm³
 3 73 622 mm³
 4 2155 mL

Review questions 7.6D

- 1 a 320 m² b 887.7 cm² c 221.7 cm²
 2 a 300 m³ b 1967.5 cm³ c 310.3 cm³
 3 a 1.2 m³ b 471.2 mm³
 4 60 ML

Review questions 7.6E

- 1 a 311.0 mm² b 881.2 cm² c 2123.7 mm²
 2 a 751.8 cm³ b 120 cm³ c 175.9 cm³
 d 840 000 cm³ e 14 726.2 cm³
 3 268.1 kL

Review questions 7.6F

- 1 a 779.8 m² b 949.3 cm² c 132.7 cm²
 2 a 1481.5 m³ b 2224.6 cm³ c 143.8 cm³
 3 a 1 481 500 L b 2.2246 L
 c 0.1438 L
 4 a 3.6 cm³ b 3141.6 cm³
 5 192 kL

Review questions 7.6G

- 1 a 380.13 cm³
 b i $SA_{\text{sphere}} = 4\pi r^2$
 $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$
 $= 2\pi r^2 + 2\pi r^2$
 $= 4\pi r^2$
 ii $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
 $V_{\text{cylinder}} = \pi r^2 h = \pi r^3$
 $\frac{4}{3}\pi r^3 > \pi r^3$ as $\pi r^3 > 0$ and $\frac{4}{3} > 1$, so $V_{\text{sphere}} > V_{\text{cylinder}}$
 c i 1440 cm³ ii 1.44 L
 d i 134.04 cm³ ii 11.31 cm
 iii 189.50 cm³ iv 324 mL

Module 8 Time and location

Lesson 8.1 Latitude and longitude

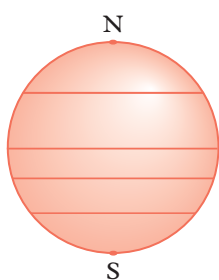
- 1 a Parallel of latitude b Meridian of longitude
 c Parallel of latitude d Meridian of longitude
 e Meridian of longitude f Parallel of latitude

2 a S b E

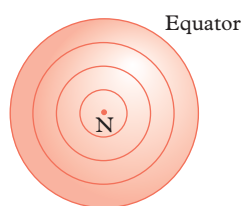
c W

d N

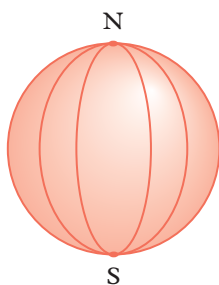
3 a



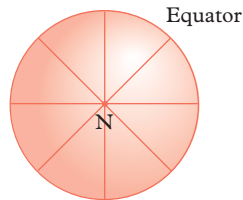
b



4 a



b



5 A 40°N, B 80°S, C 0°, D 20°S, E 60°N, F 20°N, G 80°N, H 40°S, I 60°S, J 0°

6 a C b I c H, M d D, P
e B f L, R g A, G h J, O
i K, Q j F k E

7 a 90°N b 90°S

8 a i 40°N ii 70°N iii 0°
iv 30°S v 60°S

b i 70°N ii 40°N iii 0°
iv 30°S v 60°S

9 a i 60°N ii 40°N iii 30°N
iv 0° v 15°S vi 35°S
vii 45°S

b i 40°N ii 15°S iii 45°S
iv 0° v 30°N vi 60°N
vii 35°S

10 Answers will vary.

11 a A 110°E, B 60°E, C 50°W, D 95°W, E 50°W, F 95°W, G 0°, H 60°E, I 0°, J 110°E

b i C, G iii A, D
ii E, H, I iv B, F, J

12 a i 70°E ii 120°E iii 60°W

b i 60°W ii 0°
iii 70°E iv 120°E

13 a i 50°E ii 80°E iii 120°E

iv 70°W v 100°W

b i 100°W ii 70°W iii 0°
iv 50°E v 80°E
vi 120°E

14 a 151°E b 0° c 74°W

15 85°

Lesson 8.2 Position on Earth's surface

1 a i A, B, C, D

ii C, F, K, Y, Z, V

iii C

b i Q, R, Z, T

ii A, H, I, P, Q, X

iii Q

2 A(50°N, 75°W), E(30°N, 100°E), G(30°N, 0°),
I(15°N, 75°W), K(15°N, 45°E), M(0°, 100°E), O(0°, 0°),
Z(30°S, 45°E), U(60°S, 100°E), W(60°S, 0°)

3 a D

b I

c V

d R

e J

f H

g T

h U

i P

j F

k X

l O

m L

n B

o C

4 a Australia

b China

c Antarctica

d Mexico

e Kenya

f France

5 a i S

ii N

iii N

b i W

ii E

iii E

6 a i (25°S, 95°E)

ii (45°S, 95°E)

iii (35°S, 105°E)

iv (35°S, 85°E)

b i (35°N, 150°E)

ii (15°N, 150°E)

iii (25°N, 160°E)

iv (25°N, 140°E)

c i (10°S, 90°W)

ii (30°S, 90°W)

iii (20°S, 80°W)

iv (20°S, 100°W)

d i (40°S, 40°W)

ii (60°S, 40°W)

iii (50°S, 30°W)

iv (50°S, 50°W)

7 a (25°N, 75°W)

b (5°N, 75°W)

c (15°S, 75°W)

d (50°S, 75°W)

8 a (15°N, 165°W)

b (32°S, 170°W)

c (18°S, 160°E)

d (34°N, 150°E)

9 a B is 15°S and 20°E of A.

b B is 5°S and 10°W of A.

c B is 15°N and 10°E of A.

d B is 15°S and 40°W of A.

e B is 30°N and 130°W of A.

f B is 40°N and 60°E of A.

10 a Lancaster (United States), Florence (Italy)

b Budapest (Hungary), Diepholz (Germany)

c i Sydney

ii Beijing

iii Rome

iv Johannesburg

v Moscow

vi Los Angeles

d i (36°N, 140°E)

ii (52°N, 0°)

iii (41°N, 74°W)

iv (36°N, 51°E)

v (23°S, 43°W)

vi (49°N, 123°W)

11 a i Canberra

ii Darwin

iii Hobart

iv Cairns

b i Sydney

ii Alice Springs

c i Brisbane

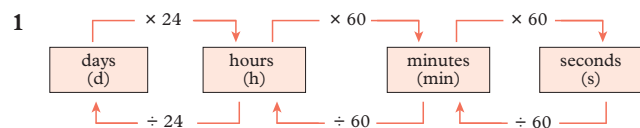
ii Hobart

12 (2°S, 4°E)

13 (3°S, 6°W)

14 1787.2 km

Lesson 8.3 Time units and intervals



- 2
- a 2.5 hours b $3\frac{1}{3}$ hours
- c 5 minutes d 3 hours 13 minutes
- e 8 minutes 5 seconds f 10 hours 35 minutes
- g 277 minutes h 2832 seconds
- i 137 hours j 4 days 4 hours
- k 604 800 seconds l 2 hours 46 minutes 40 seconds

- 3
- a 2:06 pm b 12:51 am c 8:05 pm
- d 1:22 am e 9:25 am f 9:21 pm
- g 7:15 pm h 12:12 pm i 12:54 pm
- j 5:49 pm k 6:22 am l 10:54 pm

- 4
- a 0743 b 1720 c 0135
- d 1243 e 1620 f 0146
- g 0608 h 2027 i 0920
- j 2308 k 1200 l 0000

- 5
- a 0830 b 1135 c 0736
- d 1610 e 1755 f 0832
- g 1222 h 1240 i 0649

- 6
- a 8 days 10 hours
- b 6 days 20 hours
- c 3 days 7 hours
- d 5 days 19 hours 31 minutes
- e 4 days 8 hours 23 minutes
- f 7 days 8 hours 19 minutes

- 7
- a 1 day 13 hours 52 minutes
- b 4 days 5 hours 7 minutes
- c 4 days 13 hours 23 minutes
- d 1 day 5 hours 39 minutes
- e 3 days 5 hours 51 minutes
- f 5 days 18 hours 51 minutes

- 8
- a 1242
- b 5 hours 18 minutes
- c 2005
- d 3 days 6 hours 45 minutes

- 9 8:12 am

- 10
- a 8:10 pm Wednesday 16 May
- b 7:14 pm Sunday 3 June
- c 1 hour 27 minutes

Lesson 8.4 World time zones

- 1
- a 8:00 pm b 4:00 am c 3:00 pm
- d 8:00 am e 2:00 pm f 4:00 pm
- 2
- a 11:00 pm b 7:30 pm c 9:00 am
- d 8:00 am e 3:00 pm f 11:30 pm

- 3
- a 7:00 am Wednesday b 2:30 am Wednesday
- c 1:00 am Wednesday d 5:00 am Wednesday
- e 7:00 am Wednesday f 4:00 am Wednesday

- 4
- a 10:00 pm Sunday
- b 12:00 midnight end of Sunday
- c 8:00 pm Sunday
- d 11:00 pm Sunday
- e 11:00 pm Sunday
- f 1:00 am Monday

- 5
- a 1 hour b 2 hours c $3\frac{1}{2}$ hours
- d 2 hours e 9 hours f 10 hours

- 6
- a 5:00 pm b 7:00 pm c 11:00 am
- d 4:00 am e 1:00 am f 6:00 am

- 7
- a 6:00 pm b 11:00 pm c 8:00 am
- d 7:30 pm e 10:00 am f 9:00 pm

- 8
- a 2:00 am Sunday
- b 3:00 am Sunday
- c 9:00 am Sunday

- 9
- a 9:00 pm Monday
- b 7:00 pm Monday
- c 2:00 pm Monday

- 10
- a 3:00 pm Monday b 3:00 am Tuesday

- 11
- a 7:00 am Monday b 11:00 pm Monday

- 12
- a 10:00 am Sunday b 1:30 am Monday

- 13
- a San Francisco 4:00 am, Sydney 10:00 pm

b

San Francisco	London	Sydney
4:00 am	12:00 noon (Monday)	10:00 pm
6:00 am	2:00 pm	12:00 am
8:00 am	4:00 pm	2:00 am
10:00 am	6:00 pm	4:00 am
12:00 pm	8:00 pm	6:00 am
2:00 pm	10:00 pm	8:00 am
4:00 pm	12:00 midnight	10:00 am
6:00 pm	2:00 am	12:00 pm
8:00 pm	4:00 am	2:00 pm
10:00 pm	6:00 am	4:00 pm
12:00 am	8:00 am	6:00 pm
2:00 am	10:00 am	8:00 pm
4:00 am	12:00 noon (Tuesday)	10:00 pm

- c 10:00 pm London time

14	Longitude west	Local time	Longitude east	Local time
	0°	12:00 noon Wed	0°	12:00 noon Wed
	45°W	9:00 am Wed	45°E	3:00 pm Wed
	90°W	6:00 am Wed	90°E	6:00 pm Wed
	135°W	3:00 am Wed	135°E	9:00 pm Wed
	180°W	12:00 midnight Tue	180°E	12:00 midnight Wed

- 15 **a** Same meridian
b Differ by one day
c Change of one day
d No, due to countries needing to be on the same day.
e **i** Add one day
ii Subtract one day
f The table aligns with the formula. For example, from 0° west to 45°W, is a difference of 45°, which is $3 \times 15^\circ$. Applying the formula tells us that this should be a time difference of 3 hours, which is what we see in the table.
- 16 Sunday 26 March 8:30 pm
- 17 3:15 am Wednesday
- 18 **a** **i** 8:00 pm Sunday 30 November 2025
ii 26 days 1 hour
iii 9:00 pm Friday 26 December 2025
b **i** 1:00 pm Wednesday 21 January 2026
ii 30 days 2 hours 13 minutes
iii 3:13 pm Friday 20 February 2026
c 81 days 8 hours 13 minutes

Lesson 8.5 Australian time zones

- 1 **a** 8:00 am **b** 10:00 am
c 5:00 pm **d** 6:30 pm
e 7:20 pm **f** 3:00 pm
- 2 **a** 7:00 pm **b** 12:30 pm
- 3 2:00 pm Saturday

Exercise 8.5B Problem solving and reasoning

4	City	Time
	Brisbane	12:00 pm
	Sydney	1:00 pm
	Canberra	1:00 pm
	Melbourne	1:00 pm
	Hobart	1:00 pm
	Adelaide	12:30 pm
	Darwin	11:30 am
	Perth	10:00 am

- 5 11:30 pm
6 5:00 pm

- 7 12:00 midnight
8 Adelaide 9:30 pm, Sydney 10:00 pm, Brisbane 9:00 pm
9 10:00 am
10 11:00 pm Tuesday

Lesson 8.6 Review: Time and location

Review questions 8.6A

- 1 **a** line between *N* and *S*
b the bigger circle
c the smaller circle
- 2 **a** equator
b prime meridian
- 3 The International Date Line is located roughly along the meridian of longitude at 180°E, which is the same as the meridian of longitude at 180°W. When crossing the International Date Line, not only does the time change by one hour, but so too does the date.
- 4 **a** Daylight saving time is a 1-hour adjustment to a standard time zone that is used to maximise daylight during favourable hours of the day.
b Australian Capital Territory, New South Wales, South Australia, Tasmania, Victoria
c Starts: first Sunday in October; Ends: first Sunday in April

Review questions 8.6B

- 1 **C** 2 **A** 3 **C** 4 **D** 5 **C** 6 **A**
7 **C** 8 **D** 9 **A** 10 **B** 11 **A** 12 **D**
13 **A** 14 **D** 15 **C** 16 **A** 17 **C** 18 **C**

Review questions 8.6C

- 1 **a** the same as **b** away from
2 **a** 50°S **b** 0° **c** 20°N **d** 35°N
3 **a** 50°E **b** 0°
c 40°W **d** 100°W
4 **a** *T* **b** *Q* **c** *R* **d** *P*
5 **a** 255 minutes
b 10:46 am
c 1546
d 1 day 17 hours 48 minutes
e 10:00 pm
f 4:00 am

g 12:00 pm

6 a 3:00 pm Tuesday

b 2:00 am Wednesday

7 5:30 pm

Review questions 8.6D

1 a Circle parallel to the equator

b Circle passing through both poles

2 a (44°N , 60°E)b (44°N , 30°W)c (0° , 30°W)d (0° , 60°E)e (32°S , 60°E)f (32°S , 30°W)3 a (41°N , 105°E)b (11°N , 105°E)c (26°N , 120°E)d (26°N , 90°E)

4 a South

b West

5 a 113 hours

b 2142

c 5 days 8 hours 26 minutes

6 1:00 am Friday

7 2:00 am

8 5:50 pm

Review questions 8.6E1 a (32°N , 15°W)b (0° , 80°W)c (16°S , 40°W)d (0° , 120°W)

2 a D

b M

c D or E

d M

3 a i North

ii East

b i South

ii East

c i North

ii West

d i South

ii West

4 a (32°N , 45°W)b (14°N , 15°W)c (20°N , 100°W)d (16°S , 85°W)

5 a 6 days 13 hours

b 3:34 pm

c 1528

d 2 days 15 hours 52 minutes

6 3:00 am Friday

7 5:00 pm Friday

8 11:58 am

9 Adelaide 9:00 pm, Sydney 9:30 pm, Brisbane 8:30 pm

Review questions 8.6F1 a (45°N , 55°E)b (0° , 65°W)c (0° , 0°)d (10°S , 65°W)

2 a V, W

b V

c Z

d E, D

3 a (20°N , 65°W)b (10°S , 95°W)c (15°S , 55°E)d (60°S , 105°E)

4 a 3 hours 47 minutes

b 11:27 am

c 1123

d 2123

5 2:00 am

6 5:00 am Tuesday

7 5:30 pm

8 11:30 pm

Review questions 8.6G

1 a prime meridian (or Greenwich meridian)

b i 6:27 pm

ii 3 hours 43 minutes

c (60°N , 30°W)

d 8:30 am

e i 9 hours

ii 11:40 pm Sunday

iii 19 hours 35 minutes

Module 9 Networks, paths and trees**Lesson 9.1 Introduction to networks**

1 a i 4

ii 3

iii

Vertex	A	B	C	D
Degree	1	2	2	1

b i 4

ii 4

iii

Vertex	A	B	C	D
Degree	2	2	2	2

c i 4

ii 2

iii

Vertex	A	B	C	D
Degree	1	1	1	1

d i 1

ii 3

iii

Vertex	A
Degree	6

e i 4

ii 6

iii

Vertex	A	B	C	D
Degree	3	3	3	3

f i 5

ii 4

iii

Vertex	A	B	C	D	E
Degree	4	1	1	1	1

g i 2

ii 4

iii

Vertex	A	B
Degree	3	5

h i 9

ii 12

iii

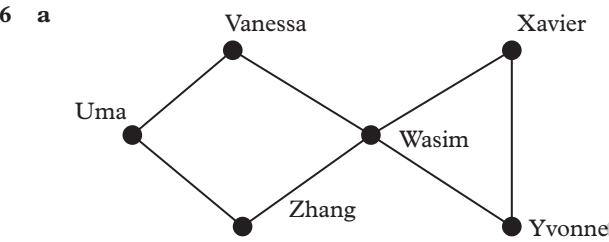
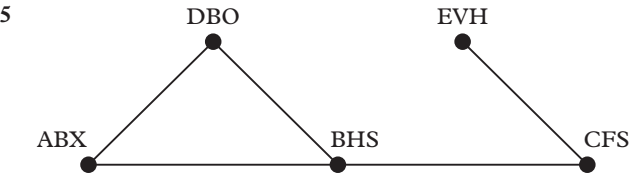
Vertex	A	B	C	D	E	F	G	H	I
Degree	2	2	6	2	4	2	2	2	2

- 2 a i 6 ii Yes
b i 8 ii Yes
c i 4 ii Yes
d i 6 ii Yes
e i 12 ii Yes
f i 8 ii Yes
g i 8 ii Yes
h i 24 ii Yes

- 3 a No b Yes c Yes d No

4

Vertex	Ada	Ben	Carlos	Dalia	Eric
Degree	3	1	2	2	2



b

Vertex	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Degree	2	2	4	2	2	2

- c Wasim or Uma
7 10 minutes
8 a 10 (including the start and end)
b 9
c 1
d 3
e Yes (look for a cycle)

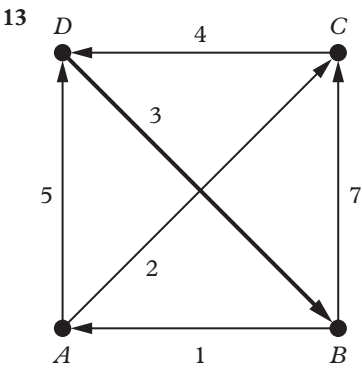
9 A

- 10 a 9 litres per second b 9 litres per second
c 3 litres per second d 3 litres per second
e 5 litres per second

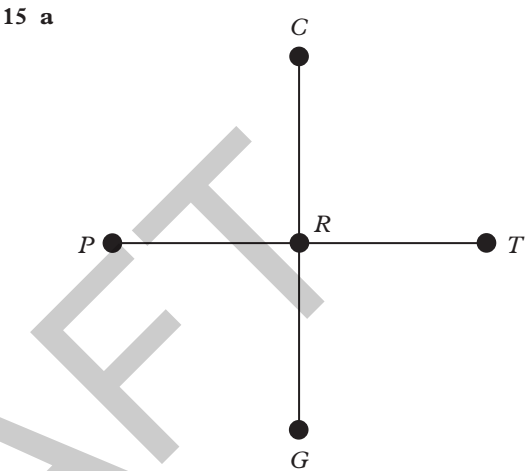
11 C

12

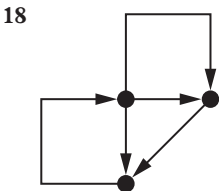
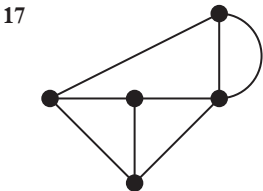
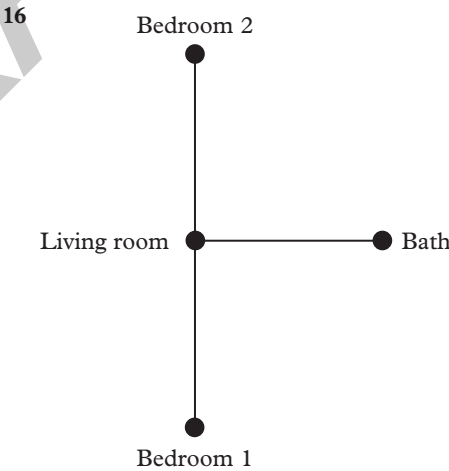
To vertex		A	B	C	D	E
From vertex		A	B	C	D	E
A		–	3	1	5	–
B		–	–	1	–	2
C		–	–	–	–	3
D		–	–	1	–	4
E		–	–	–	–	–



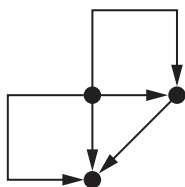
- 14 a 2 b 0 c 4 d 0
e 4 f 4 g 2 h 0



b 4

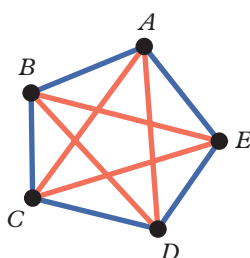


19



One vertex has zero indegree (cars cannot enter the intersection) and another has zero outdegree (cars cannot exit the intersection).

20



- 21 **a** Answers will vary. **b** Answers will vary.
c 6

Lesson 9.2 Walks, paths and cycles

- $A, f, B, g, C, h, D, i, E, j, F$
- a** E **b** B
- Corner C
- a** A, B, C, D ; The start vertex is A and the end vertex is D .
b h, g, f ; The start vertex is D and the end vertex is A .
c f, g, h, h, g, f ; A is both the start and end vertex.
- A
- D
- a** $ABCDE$ **b** $ABCI A$
c $FEDCB$ **d** $HCFEH$
- $ABCDEFCHGECI A$. This is neither a cycle nor a path.
- a** 12 minutes
b There are four paths that each take 12 minutes.
- C
- DE
- $DBACDEFGH$
- a** $BACDEFGH$
b Consider how the path would include vertices D, H, G and F . These four vertices can only be accessed via vertices C and E . In a Hamiltonian cycle, you would have part of the path looking like $XCYEZ$ (perhaps in reverse), where X, Y and Z are other vertices in the path (or C or E are the start/end of the path). X, Y and Z represent the only possible positions for D, H, G and F . This means that there are three positions for four vertices, which is impossible.

14

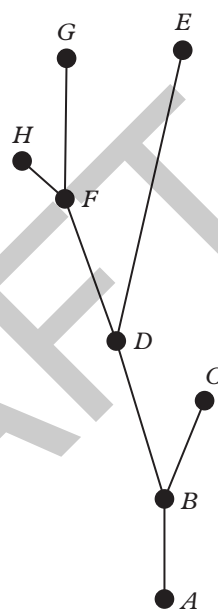
Vertex	K	L	N	S
Degree	5	3	3	3

- b** No. (There are more than two vertices with odd degree.)

Lesson 9.3 Trees

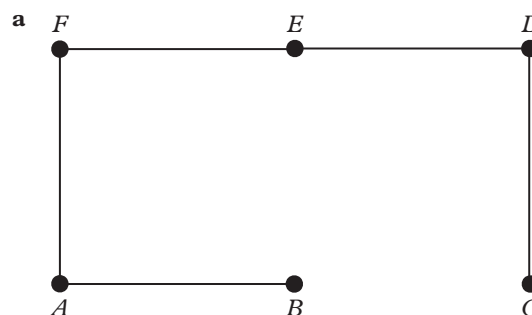
- a** $ABCDL$ **b** LDE
c $BCFH$ **d** $GFCDK$
- a, b** and **c** are trees
- A**: 7 vertices and 6 edges
B: 5 vertices and 4 edges
C: 4 vertices and 3 edges

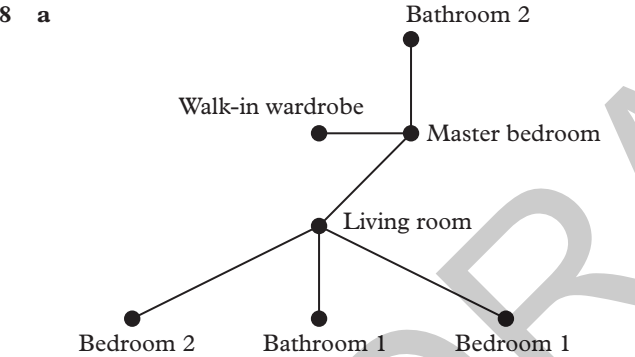
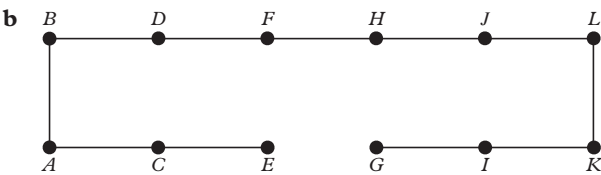
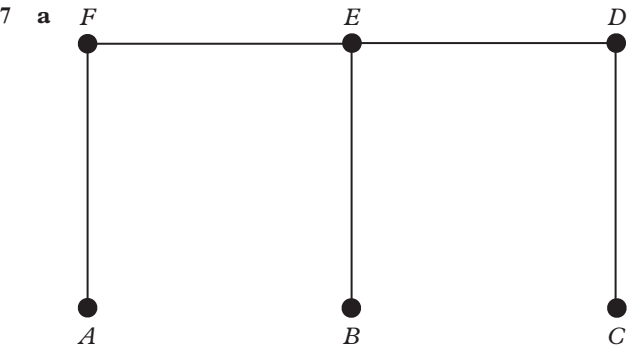
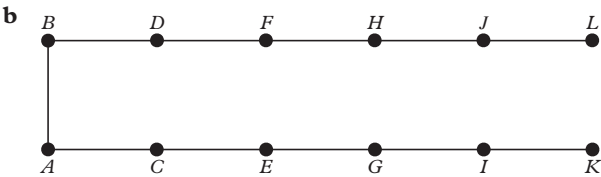
4



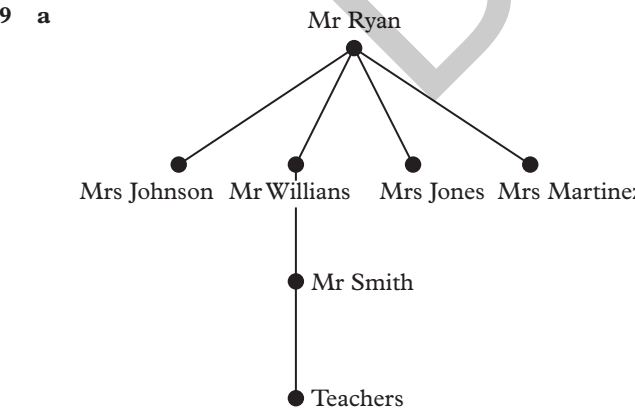
It is a tree.

- B
- There are many correct answers. For parts **a, b, c** and **d** the same spanning tree can be chosen.





b Yes



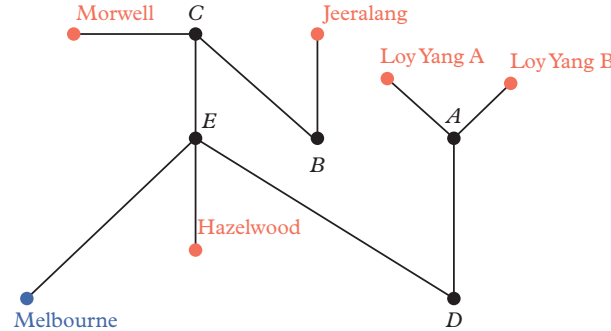
b Yes

c 2

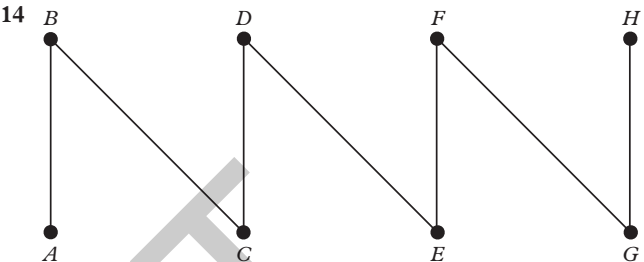
10 $n + 1$

11 There is a left tree (with vertices A, B, C and D) and a right tree (with vertices E, F, G, H).
Connecting D and G with an edge will turn the forest into a tree (for example).

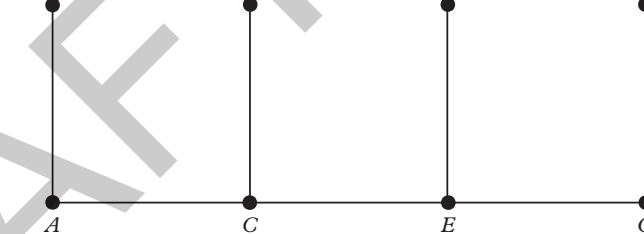
12 At most 3 edges can be removed.



13 9



15



16 Yes. Every tree is a forest, but not every forest is a tree.

17 Yes

18 a No

b Yes



Lesson 9.4 Minimum spanning trees

1 a 11 **b** 12 **c** 26

2 21.3 m

3 a 9 **b** 8 **c** 7

d 6 (smallest total edge weight)

4 a

Edge	AB	DC	BD	DE	AC	AE
Weight	2	2	3	4	4	5

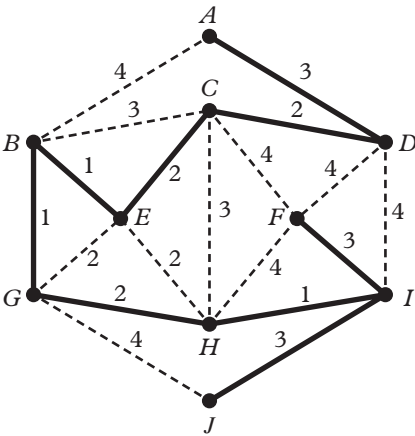
b

Edge	AB	DC	BD	AC	ED	AE
Weight	2	2	4	4	6	6

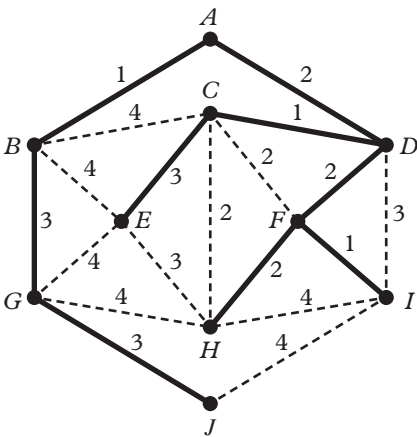
c

Edge	AB	AC	BD	CD	AE	ED
Weight	1	1	1	1	1	1

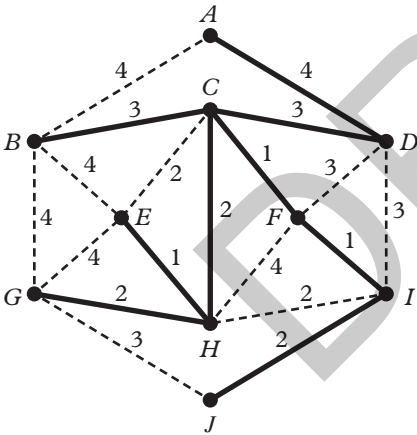
5



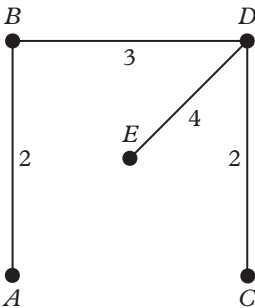
6 a



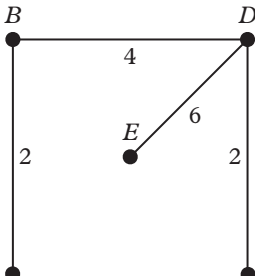
b



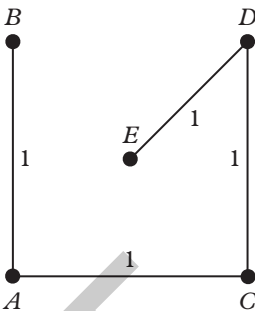
7 a



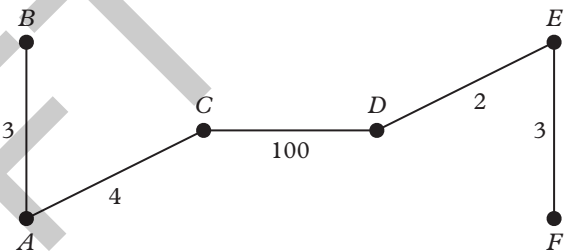
b



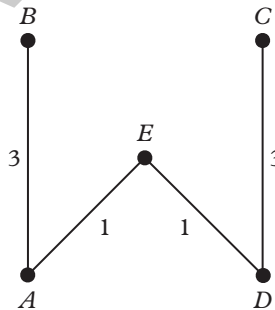
c



d



e



8 a

Edge	EB	EC	ED	EA
Weight	6	6	2	3

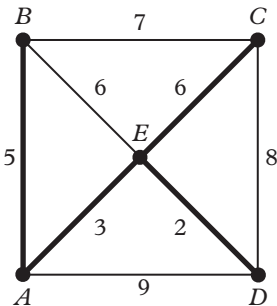
b Edge ED has the smallest weight.

9 a

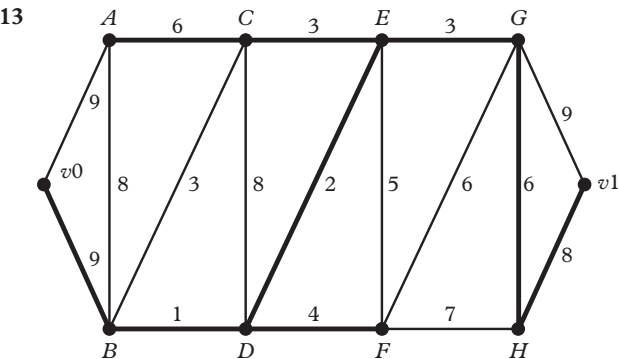
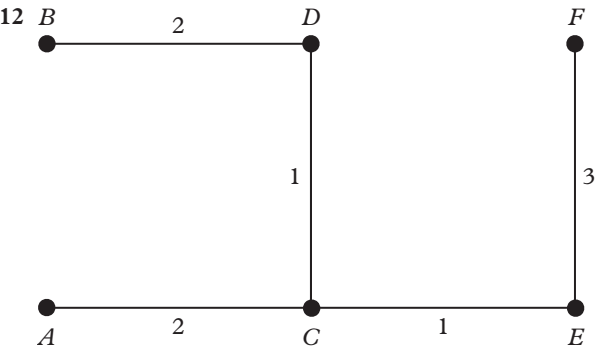
Edge	EB	EC	AB	CD
Weight	6	6	5	8

b Edge AB has the smallest weight.

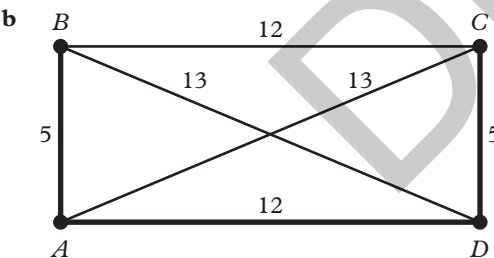
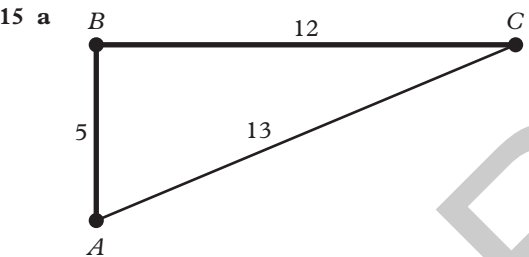
10



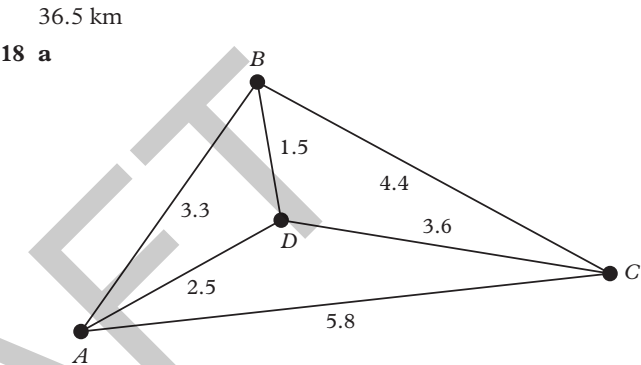
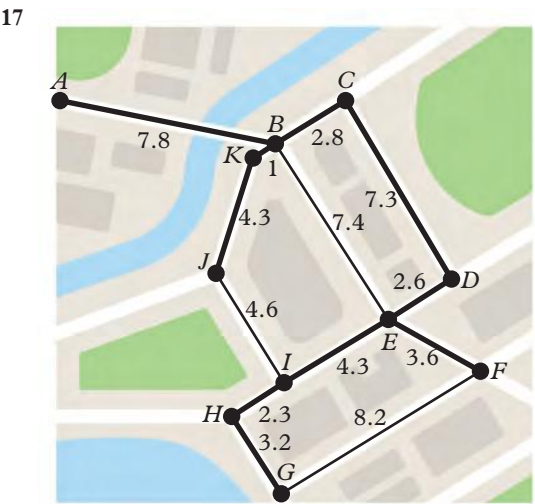
11 See answer to question 7.



14 a \$7 b \$6 c \$7



16 Connect AB , BC and CD .



b Connect rail lines along AD , BD and DC .
c \$38 million d \$56.5 million
e \$18.5 million

19 a Perth to Adelaide, Adelaide to Melbourne, Melbourne to Canberra, Canberra to Sydney, Sydney to Brisbane, Adelaide to Darwin

b 8412 km
c Use Kruskal's algorithm as you can work through the distances from shortest to longest instead of only considering edges adjacent to the tree at each step.

20 41

Lesson 9.5 The shortest path

1 a $ACDB$ b $FBHG$
2 a $ABCDEF$ b AHG
c $ABCDEF GH$

3 a

Path	EA	EB	EC	ED
Total weight	3	6	6	2

b

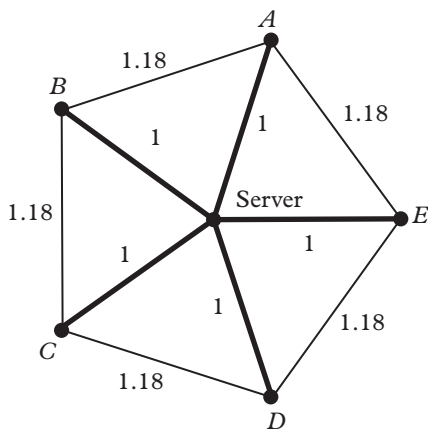
Path	EA	EB	EC	EDA	EDC
Total weight	3	6	6	11	10

c

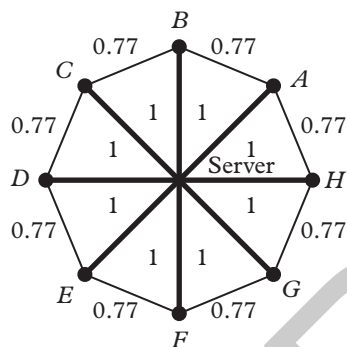
Path	EB	EC	EAB	EDC
Total weight	6	6	8	10

i EB and EC both have the smallest weight.
ii 6

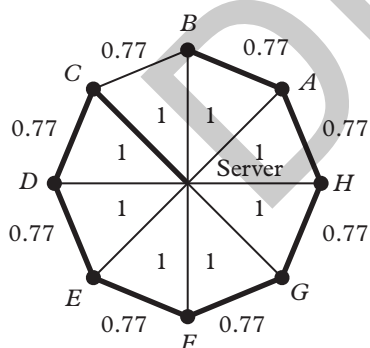
- 4 **a** 7 **b** 8 **c** 2
d 107 **e** 4
 5 12
 6 7
 7 **a** Direct flight (\$706) **b** Via Mildura (\$922)
 8 Yes



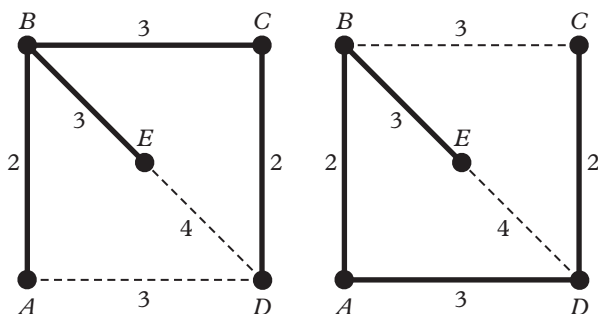
9 No



A minimum spanning tree is given below.



- 10 **a** Via the shops **b** Via the depot
 11 There are two possible minimum spanning trees:



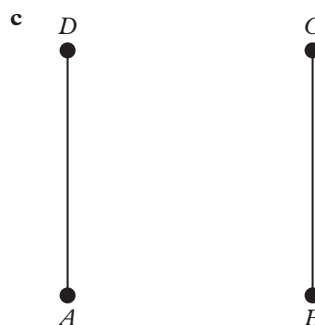
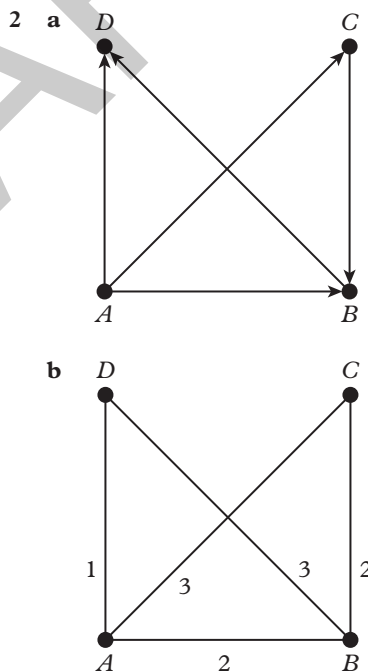
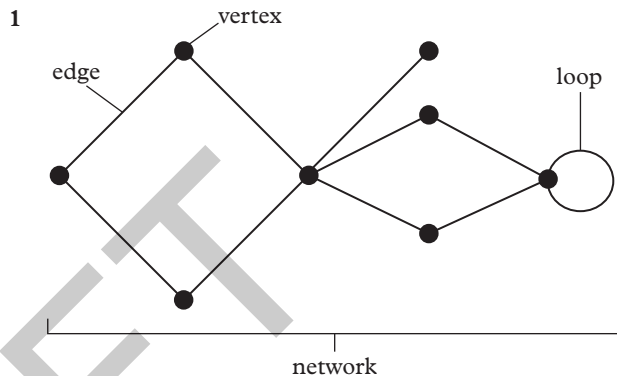
Both are different to the tree found using Dijkstra's algorithm in the worked example. However, if path *ADC* is added in **Step 3.3** instead of path *ABC* in the worked example, the resulting tree is the same as the second minimum spanning tree above.

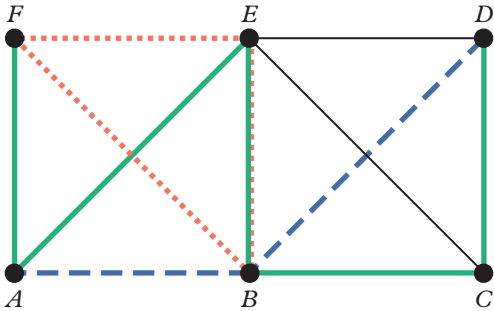
- 12 Dubbo, Sydney, Melbourne, Mount Gambier and it costs \$1384.

- 13 **a** *ABEGHJ* **b** 2

Lesson 9.6 Review: Networks, paths and trees

Review questions 9.6A



- 3 a The dashed blue line is a path (or an open path).
b The dotted red line is a cycle (or a closed path).
c The solid green line is a spanning tree.
- 
- 4 A tree is a connected network, whereas a forest is a disconnected network consisting of more than one tree.
- 5 a A minimum spanning tree is defined as the spanning tree with smallest total edge weight, so the choice of spanning tree depends on the sums of edge weights.
b The shortest path between two vertices in a weighted network is the path for which the sum of edge weights is as small as possible.
- 6 a Prim's algorithm b Dijkstra's algorithm
c Kruskal's algorithm

Review questions 9.6B

- 1 B 2 D 3 D 4 C 5 D
6 D 7 B 8 D 9 D 10 A
11 D 12 A 13 C

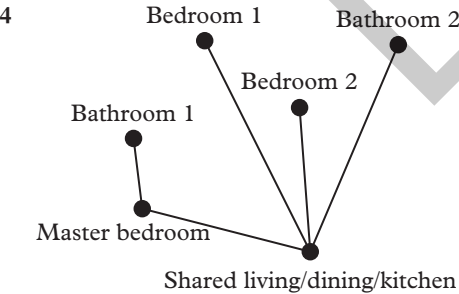
Review questions 9.6C

1

Vertex	1	2	3	4	5	6
Degree	3	3	2	1	3	4

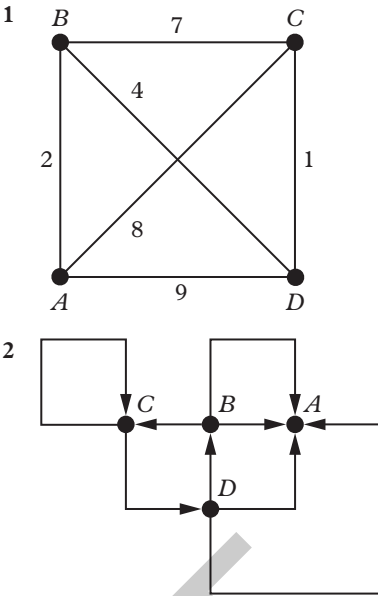
2 16

3 4, 2, 1, 6, 5, 3

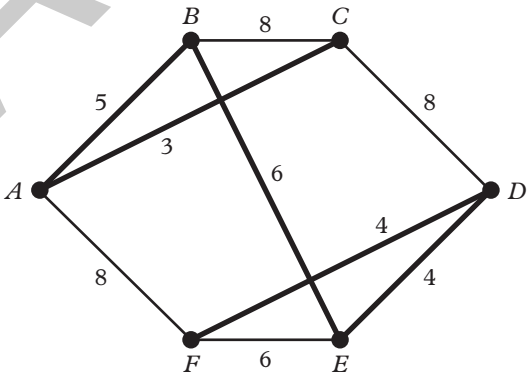


- 5 Yes
6 If you start with vertex A , then Prim's algorithm chooses edges: AF , AB , BC , BE , ED .
7 15
8 The shortest path is v_0, B, D, F, H, v_1 .
9 The shortest path is v_0, B, D, E, G, H, v_1 .

Review questions 9.6D



- 3 Vertex A has indegree 4 but outdegree 0, so cars cannot exit this intersection.
4 1, 3, 5, 4, 2, 6, 1
5 Remove any single edge.
6 Remove another edge (any one will do).
7 Yes
8 Yes
9



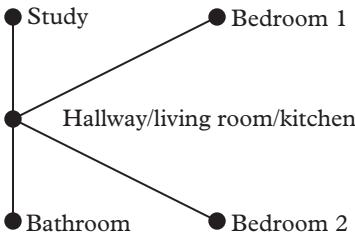
- 10 The shortest path is v_0, B, C, E, G, v_1 .

Review questions 9.6E

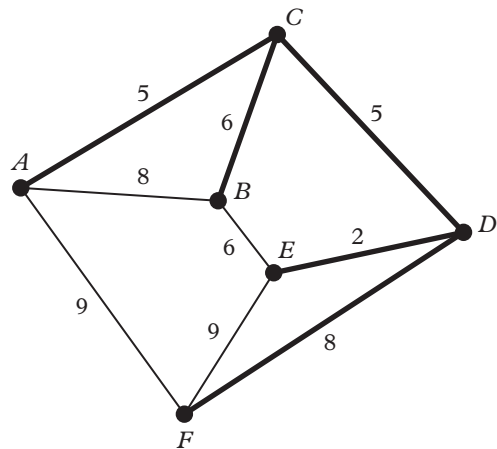
1

Vertex	1	2	3	4	5	6
Degree	1	2	3	1	1	4

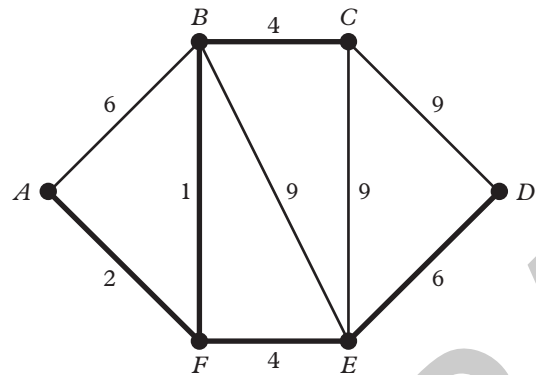
- 2 No, there are more than two vertices with odd degree.
3



4 Yes
5



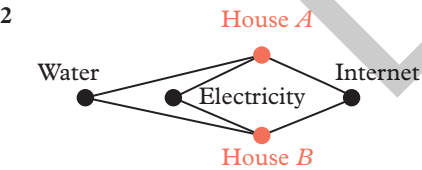
6 21
7 AFED
8 Both algorithms will result in the following:



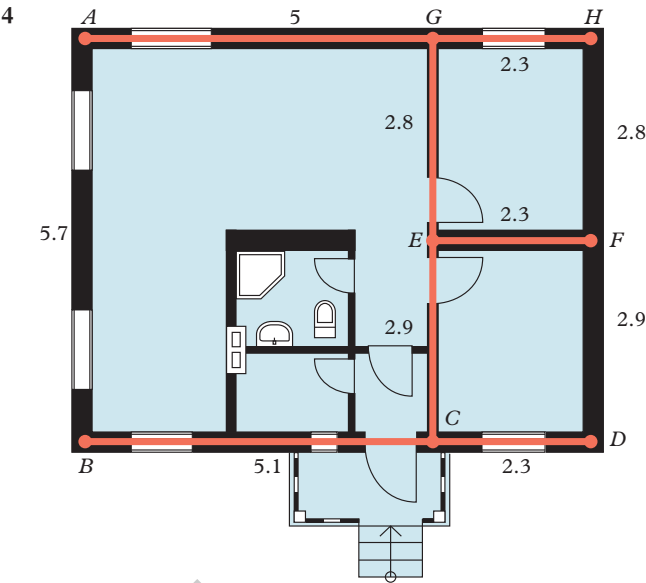
9 See answer for question 8.
Review questions 9.6F

1

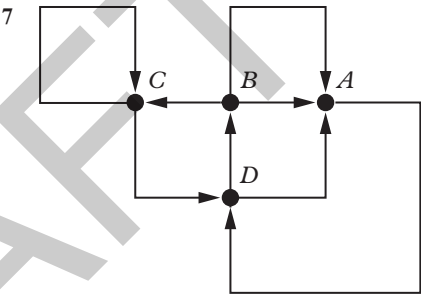
Vertex	1	2	3	4	5	6
Degree	2	1	2	2	3	4



3 12

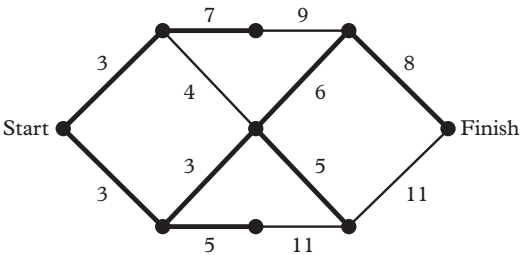


5 21.1
6 See answer to question 4.



8 v_0, B, C, E, F, H, v_1
Review questions 9.6G

1 a i A ii 4
iii Via the edge between Beachside and Hillsvale
b i ACE
ii AC, AB, BD and DE
c i A
ii



d i Yes ii A
e i 2 iii 5
ii 4

Review: Measurement and networks

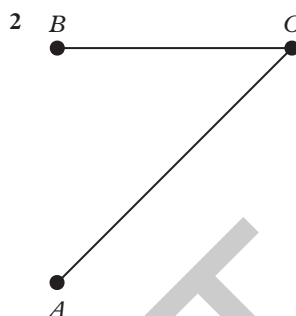
Review questions: Measurement

- 1 **a** **i** 4300 **ii** 4263 **iii** 4263.09
iv 4260 **v** 4263.1 **vi** 4000
b **i** No **ii** No **iii** Yes
c **i** 1.05×10^5
ii 6.27×10^{-5}
iii 4.56×10^1
d **i** 2.87×10^{14}
ii 5.12×10^{17}
iii 4.36×10^4
e **i** 6730 m **ii** 17.5 m
iii 4300 mg **iv** 47 t
v 6350 mL **vi** 0.09 L
vii 3.2×10^9 m **viii** 52 000 kL
ix 1.65 m^2
- 2 1508 cm²
- 3 **a** 20 m **b** 11 cm or 110 mm
c 30.8 mm
- 4 **a** $x = 11.3$ **b** 43.3 cm
- 5 **a** 23.715 cm² **b** 9.46 m²
c 121 cm² **d** 421.64 m²
- 6 **a** 211.2 mm² **b** 41.9 cm²
c 0.7 m²
- 7 **a** 5497.8 cm² **b** 1115.9 mm²
- 8 267 m²
- 9 **a** 116.1 m² **b** 2536.7 cm²
- 10 **a** 616 cm² **b** 376 m²
- 11 **a** 360 cm³ **b** 274.6 m³
c 4188.8 cm³ **d** 5277.9 mm³
- 12 **a** 125.44 m³ **b** 471.2 cm³
- 13 **a** 40 cm³ **b** 70686 cm³
c 140 cm³ **d** 2400 cm³
- 14 120 kL
- 15 423 mL
- 16 **a** **i** PQR, TUV, XYZ
ii TUV
iii PTX, QUY, RVZ
b P(60° N, 150° W), U(0°, 15° W), Y(15° S, 15° W)
c **i** True **ii** True
iii False **iv** False
d **i** (15° S, 10° W) **ii** (52° N, 15° W)
- 17 **a** 115 hours
b 4 hours 49 minutes
c 2 days 13 hours 52 minutes

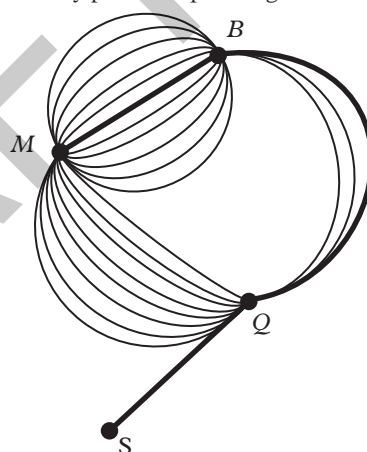
- 18 **a** **i** 1:00 pm **ii** 5:00 am
b **i** 7:00 pm Thursday **ii** 7:30 am Friday
c **i** 8:00 am **ii** 10:00 am
iii 5:00 pm **iv** 7:30 pm

Review questions: Networks

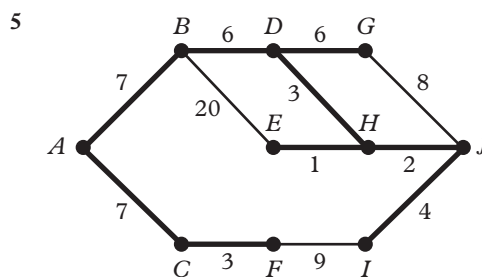
1	Vertex	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
	Connected to	<i>B</i>	<i>A, E, C</i>	<i>D, E, B</i>	<i>C</i>	<i>F, B, C</i>	<i>E</i>



- 3 There are many possible spanning trees.



- 4 There are many possible answers, for example, edges AB, BC and CD.



- 6 See answer to question 5.
 7 ABDHJ
 8 BDHE

Module 10 Collecting and classifying data

Lesson 10.1 Statistical investigations

- 1
 - a not suitable
 - b suitable
 - c suitable
 - d not suitable
 - e not suitable
 - f suitable
 - g suitable if restated carefully
 - h suitable if restated carefully
- 2
 - a Primary school children
 - b Homeowners
 - c People with a cold who use the new treatment
 - d Staff and athletes of professional sports
 - e Commuters who use buses on the bus route
 - f Companies that pay road tax
 - g Dog owners who have used the new treatment
 - h Bank customers
- 3 Example responses provided. Many responses are valid.
 - a Advantages: The interviewer can collect more detailed responses and clarify any questions that the participant might not understand.
Disadvantages: Conducting face-to-face interviews can be expensive when factoring in the interviewer's time and any required venue use.
 - b Advantages: Phone interviews have similar advantages to face-to-face interviews, without the costs associated with using a physical venue.
Disadvantages: Requires participants to answer their phone, which is not guaranteed!
 - c Advantages: Easy and relatively cheap to implement.
Disadvantages: It is difficult to identify and correct participants' misunderstanding or misinterpretation of questions.
- 4 Example responses provided. Many responses are valid.
 - a
 - i Do you like our school uniform as it is?
 - ii If you could change one thing about our school uniform, what would it be?
 - iii Which uniform items do you wear? Tick all that apply.

☐ Shirt and tie	☐ Rugby shirt
☐ Long pants	☐ Shorts
☐ Dress	
 - b
 - i Which audio apps do you use on your phone? Tick all that apply.

☐ Spotify
☐ Apple Music
☐ YouTube Music

- ☐ Bandcamp
- ☐ Other (please specify): _____

ii How often do you use audio apps to listen to podcasts?

- ☐ never ☐ rarely
- ☐ sometimes ☐ often
- ☐ regularly

c i True or False: The food at our school canteen is satisfactory.

ii Place a cross on the continuum to indicate how strongly you agree with the statement, "There are enough nutritious options available at our school canteen".

|-----|

strongly agreestrongly disagree

iii What lunch options would you like to see added to our school canteen menu?

- 5
 - a By mentioning the budget deficit, this question is leading people to answer that they should cut military expenditure.
 - b By using "don't ... do you", the likelihood that the respondent will not support the railway line is increased.
 - c By explaining that Avengers was voted best film, this question is leading people to answer that it is their favourite film.
 - d By giving your opinion first, the likelihood that the respondent will echo your opinion is increased.
 - e By explaining that soccer is the most played sport, the likelihood that the respondent's favourite sport is soccer is increased.
- 6 By leaving the file unlocked, anyone in the company could see the information. As the information contained names and personal information (i.e. Not deidentified), whoever sees it could use it against others in the company.
- 7 Answers will vary.
 - a Watch out for leading questions.
 - b Consider, for example, whether your questions collect personal or identifying information about the respondent.
 - c Teacher to review.
- 8 Answers will vary.

Lesson 10.2 Classifying data

1 a Discrete numerical	b Continuous numerical
c Nominal	d Discrete numerical
e Discrete numerical	f Nominal
g Ordinal	h Discrete numerical
i Continuous numerical	j Discrete numerical
k Ordinal	l Continuous numerical

- m** Nominal
o Discrete numerical
q Ordinal

- 2 Answers will vary. Compare responses with answers to question 1.
 3 Answers will vary. An example question for each type of variable type is provided.

Discrete: How many different calculators do you own?

Continuous: How much time per day do you spend using a calculator?

Ordinal: How often do you use a calculator during math lessons?

- ☐ Never ☐ Rarely
☐ Sometimes ☐ Often
☐ Always

Nominal: What type of calculator do you use in class?

- ☐ Scientific ☐ Basic
☐ Graphing ☐ None

- 4 Answers will vary. An example question for each type of variable type is provided.

Discrete: How many times did you use the school canteen last week?

Continuous: On average, how much money do you spend at the canteen per visit?

Ordinal: How would you rate the food at the school canteen?

- ☐ Very poor ☐ Poor
☐ Fair ☐ Good
☐ Excellent

Nominal: What is your role in the school?

- ☐ Student ☐ Teacher
☐ Visitor ☐ Staff

- 5 Stars can represent a scale from poor (1 star) to excellent (5 stars).
 6 Answers will vary.
 7 Answers will vary.

Lesson 10.3 Collecting data

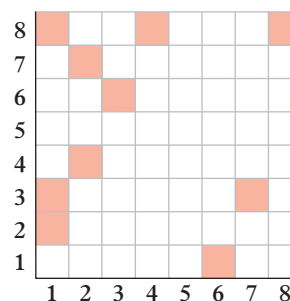
- 1 **a i** Netball team members
ii Census
b i Football team members
ii Census
c i Example: "What is the most popular radio station of the teachers at this school?"
ii Sample
d i Children in Australian families
ii Sample
e i A family **ii** Sample
f i Students of a class **ii** Census

- g i** Example: "What is the average energy rating of the washing machines owned by the people on my street?"
ii Census or sample
h i Example: "What is the average number of leaves on the saplings at the front of the school?"
ii Sample
i i Example: "How many hours of sunshine are there in January compared to July?"
ii Census
j i People turning 100 **ii** Census
k i Students **ii** Census or sample
l i Monthly rainfall measurements
ii Census
m i Immigrants **ii** Sample
n i Apples **ii** Census or sample
o i Students of the class
ii Census
p i School principals **ii** Sample
q i Patients who've been given medicine
ii Sample
 2 Answers will vary.
 3 **a** The sample would be biased towards people who are at home on a Saturday night.
b The sample would be biased towards people who catch a bus.
c The sample would be biased towards people who go to the beach.
d The sample would be biased towards people on the electoral roll (Australian citizens 18 years old and over).
 4 **a** Year 12 students are less likely to be invested in the outcome of the study, given that they will be leaving the school very soon.
b Motorists in peak hour traffic are likely to be frustrated about their present traffic problems and might describe the problems as worse than they would at other times.
c Real-estate agents often get paid a commission for selling a house, so they may have personal reasons for either over- or under-stating house prices.
d Politicians rely on data about their country's economy for many reasons, including in relation to being elected, so they may have personal reasons for describing the economy in a certain positive or negative way.
 5 **a** This sample is not large enough.
b The sample is likely biased towards the company's product (they may have only asked for recommendations from dentists they partner with).
c The sample is likely biased as there is no mention of how many dog breeders recommend the dog food, nor what other dog food dog breeders may recommend.

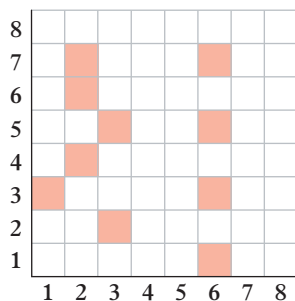
- d** Hairdressers who prefer online bookings are probably less likely to list their business in the Yellow pages phone directory.
- 6 a** Christians may attend a church on Sunday morning, so the survey is likely to miss a large segment of the Christian religion.
- b** As each electorate has different representatives, views may be biased towards the view of that representative.
- c** Shopping habits are likely to be different between students in inner-Sydney and other regions, especially when looking at rural areas with less access to the same shopping opportunities.
- d** People just outside the concert are likely to have attended the concert and have similar views.
- e** Opinions would be biased towards those using the bicycle lane.
- f** As investment bankers generally work for companies and governments, they may be biased towards those they work for.
- g** Opinions would be biased on those who attend Comic-Con.
- 7 a** Answers will vary. For example, you could create an opt-in survey to be shared on social media.
- b** They can reduce the time and expense of finding suitable subjects that meet the selection criteria. Volunteers are likely to be committed to take part in the study. They may also show a greater willingness to provide more insight into the subject of the research by giving full detailed responses to open-ended questions.
- c** Because the people chosen are volunteers, there is likely to be a degree of self-selection bias. This can either lead to the sample not being representative of the population being studied or exaggerating some particular finding from the study.
- 8 a** Answers will vary. The notice should specify who is eligible to participate (students in their final year of school) and should also outline what participants will be required to do (for example, answer several short questions about their career plans and aspirations).
- b** Answers will vary.
- 9** Volunteers are to be checked to make sure that they satisfy the criteria of the study. As the boy does not fit the selection criteria, his response will not be used in the survey findings.
- 10 a** Every 5 years
- b** Snapshot of Australia's key demographic, social and economic information (age, country of birth, religion, ancestry, language used at home, work and education)
- c** Used in planning for schools, health care, transport and infrastructure
- d** To limit bias; more detailed and accurate
- 11** Graduates with different degrees often work in different areas related to their skills and education.
- 12 a** Example: Mothers of young children
- b** Example: Commuters who use the bus service
- c** Example: People who have borrowed books from the library
- d** Example: Seniors who use the common room
- 13** Answers will vary.
- 14** Volunteers are likely to have strong opinions about the research topic, some with strong positive opinions and some with strong negative opinions.

Lesson 10.4 Random sampling

- 1** Answers will vary.
- 2** Write the workers' names on pieces of paper, place the names in a hat, mix and draw 5 names. Write the workers' names on cards, shuffle the cards and deal 5.
- 3** Assign the numbers 1 to 70 to each shop assistant. Select 10 random numbers from 1 to 70 using a table of random numbers. The shop assistants whose numbers are selected form the sample.
- 4** Assign the numbers 1 to 760 to each student. Select 100 random numbers from 1 to 760 using a table of random numbers. The students whose numbers are selected form the sample.
- 5** Number the hospitals 1 to 12. Use a table of random numbers to select 6 random numbers less than or equal to 12. The hospitals whose numbers are selected form the sample.
- 6 a** Answers will vary.
- Method 1:* Roll two standard dice for each pair, if dice are different colours pick one colour to be the first number in each pair.
- Method 2:* Use the random number function on your calculator, listing the first two numbers between 1 to 8 in each decimal as a pair.
- b** *Method 1:* (8, 8), (3, 6), (1, 3), (2, 7), (7, 3), (1, 8), (6, 1), (4, 8), (2, 4), (1, 2)
- Method 2:* (2, 4), (6, 3), (6, 7), (1, 3), (2, 6), (6, 1), (2, 7), (6, 5), (3, 5), (3, 2)
- c** *Method 1:*



Method 2:



- 7 a Example: He could roll a D10 and a D8 in pairs 10 times, to find the position of 10 squares on the grid.
 b Methods will vary.
 c i 18 seedlings ii 1440 seedlings
 d Answers will vary.
- 8 a *Method 1:* Put all the teachers' names in a hat, mix and select 10 names.
Method 2: Assign the numbers 1 to 62 to the teachers. Use a table of random numbers to select 10 random numbers less than or equal to 62. The teachers whose numbers are selected form the sample.
 b 30.5 minutes
 c 3 hours and 3 minutes
 d Answers will vary.
- 9 Answers will vary.
- 10 Answers will vary.
- 11 Example: Any sample has a risk of bias; there may be significant differences in each year grouping.

Lesson 10.5 Stratified sampling

- 1 15, 53.6%, 65, 13, 46.4%
- 2 Boys: $\frac{19}{45}$ or 42.2%, girls: $\frac{26}{45}$ or 57.8%
- 3 a Year 7: $\frac{125}{859}$, Year 8: $\frac{142}{859}$, Year 9: $\frac{175}{859}$, Year 10: $\frac{160}{859}$, Year 11: $\frac{134}{859}$, Year 12: $\frac{123}{859}$
 b 9, 10, 12, 11, 9, 9
 c Choose at random 9 students from Year 7, 10 from Year 8, 12 from Year 9, 11 from Year 10, 9 from Year 11, 9 from Year 12.
- 4 a $\frac{96}{439}$, $\frac{84}{439}$, $\frac{78}{439}$, $\frac{65}{439}$, $\frac{56}{439}$, $\frac{60}{439}$
 b 9, 8, 7, 6, 5, 5
 c Choose at random the required number of students from each year.
- 5 a Girls: $\frac{69}{120}$, boys: $\frac{51}{120}$
 b 23 girls, 17 boys
 c Choose at random 23 girls and 17 boys.
- 6 a 0–9: $\frac{180}{2475} = \frac{4}{55}$, 10–19: $\frac{835}{2475} = \frac{167}{495}$, 20–29: $\frac{260}{2475} = \frac{52}{495}$, 30–39: $\frac{350}{2475} = \frac{14}{99}$, 40–49: $\frac{280}{2475} = \frac{56}{495}$, 50–59: $\frac{350}{2475} = \frac{10}{99}$, 60–69: $\frac{200}{2475} = \frac{8}{99}$, 70+: $\frac{120}{2475} = \frac{8}{165}$

- c 0–9: 3, 10–19: 13, 20–29: 4, 30–39: 6, 40–49: 5, 50–59: 4, 60–69: 3, 70+: 2
- d Choose at random the required number of people from each age group.
- 7 a Examples: Age group, Income level, Employment status, Geographic location
 b Examples: Age group, Education level, Geographic location, Ethnic or cultural background
- 8 Examples: Age group, Employment status, Education level, Occupation
- 9 a Bus: $\frac{87}{150} = \frac{29}{50}$, ride: $\frac{23}{150}$, car: $\frac{30}{150} = \frac{1}{5}$, walk: $\frac{10}{150} = \frac{1}{15}$
 b Bus: 17, ride: 5, car: 6, walk: 2
 c Choose at random 17 students who travel by bus, 5 students who ride bicycles, 6 students who travel by car and 2 students who walk.

Lesson 10.6 Systematic sampling

- 1 a Every 12th item b Every 6th item
 c Every 30th item d Every 20th item
- 2 a Every 12th name b Every 6th name
 c Every 8th name d Every 30th name
- 3 a Start at any random number and select every 20th student.
 b i 10, 30, 50, 70, 90, 110, 130, 150, 170, 190
 ii 15, 35, 55, 75, 95, 115, 135, 155, 175, 195
 iii 30, 50, 70, 90, 110, 130, 150, 170, 190, 10
 iv 45, 65, 85, 105, 125, 145, 165, 185, 5, 25
 v 113, 133, 153, 173, 193, 13, 33, 53, 73, 93
- 4 a Start at any random number and select every 12th name.
 b i 5, 17, 29, 41, 53, 65, 77, 89, 101, 113, 125, 137, 149, 161, 173
 ii 10, 22, 34, 46, 58, 70, 82, 94, 106, 118, 130, 142, 154, 166, 178
 iii 25, 37, 49, 61, 73, 85, 97, 109, 121, 133, 145, 157, 169, 1, 13
 iv 50, 62, 74, 86, 98, 110, 122, 134, 146, 158, 170, 2, 14, 26, 38
 v 120, 132, 144, 156, 168, 180, 12, 24, 36, 48, 60, 72, 84, 96, 108
- 5 Start at any number and select every:
 a 6th name b 9th name
 c 12th name d 24th name
 e 19th name
- 6 a i Select every 40th part.
 ii Select every 17th part.
 b Answers will vary.
- 7 a Assign the numbers 1 to 30 to the students, choose the first number at random and then select every 5th number until a sample of 6 students has been chosen.
 b 9, 14, 19, 24, 29, 4 c 9, 15, 21, 27, 3

- 8 a Assign the numbers 1 to 600 to the employees, choose the 1st number at random and then select every 60th number until a sample of 10 has been chosen.
b 23, 83, 143, 203, 263, 323, 383, 443, 503, 563
- 9 a Every 12th employee b 5
- 10 Example: High and low tides affect the water depth during the day and night. Storms can also change the water depth. Systematic sampling does not take either into account.

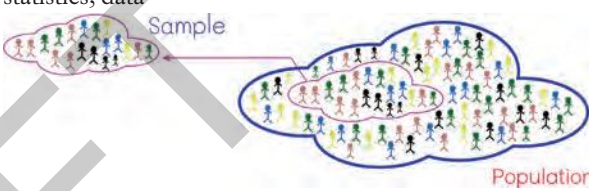
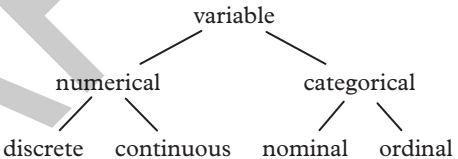
Lesson 10.7 Suitability of sample types

- 1 Possible answers are:
a Systematic sampling
b Random sampling
c Stratified random sampling
d Random sampling
e Stratified random sampling
f Systematic sampling
g Self-selected sampling
h Random sampling
i Random sampling
j Stratified random sampling
- 2 a Census b Categorical
c $\frac{96}{200} = \frac{12}{25}$
d i $\frac{1}{5}$, no ii $\frac{1}{5}$, no
e Answers will vary.
f i Every 40th student, answers will vary
ii Every 20th student, answers will vary
iii Every 10th student, answers will vary
iv Every 5th student, answers will vary
g i ≈ 3 Year 11; ≈ 2 Year 12, answers will vary
ii ≈ 6 Year 11; ≈ 4 Year 12, answers will vary
iii ≈ 11 Year 11; ≈ 9 Year 12, answers will vary
iv ≈ 23 Year 11; ≈ 17 Year 12, answers will vary
h The larger the sample, the closer it approximates to the population response.
- 3 a Percentage = $\frac{12}{20} \times 100 = 60\%$
b Number = $\frac{60}{100} \times 4000 = 2400$
c Adequate childcare = $\frac{5}{20} \times 4000 = 1000$
d Unsure = $\frac{8}{20} \times 4000 = 1600$
- 4 a 62.5% b 31 250
c 35 000 d 3750
- 5 a 70% b 560 c 240
- 6 a 30% b 180 c 420
- 7 a It may or may not be representative, as it depends on how students are allocated to classes in your school.
b The sample is too small to be representative of the whole population of Year 11 students in your state. It may also be biased.

- 8 a Dentists may be paid by Delgate. The sample is not representative of the whole population.
b Is the 10% across all positions from senior management down or is it 10% of the lowest paid workers? This could be misleading as, for example, if it was 10% of the 10 managers, then it would be 1 person. If it was 10% of the 500 factory workers, then it would be 50 people.
c Yes, depending on how the sample was grouped.
- 9 a Example: She could use systematic sampling.
b Example: She could use a self-selected sampling by advertising her survey near an office of that political party.

Lesson 10.8 Review: Collecting and classifying data

Review questions 10.8A

- 1 statistics, data
- 2 
- 3 
- 4 Examples are provided.
Discrete: Number of siblings
Continuous: Height in cm
Nominal: Hair colour
Ordinal: Survey responses (Strongly disagree, Disagree, Neutral, Agree, Strongly agree)
- 5 How data will be used and shared, the privacy of participants, perception of study from a wide range of backgrounds and perspectives.
- 6 The people who choose to participate in the study may not be representative of the population. For example, if you want to know whether a community facility should be upgraded through a survey, the only respondents may be those who would use the facility. This may not represent the views of the entire community.
- 7 a A random sample is a subset of the population in a statistical investigation where each member of the population has an equal chance of being selected. The sample is chosen randomly (e.g. using a random number generator).
b A stratified random sample is a random sample where the population is divided into subgroups (strata), based on different characteristics (e.g. age, gender, income). The proportion of each subgroup in the sample is the same as in the population.

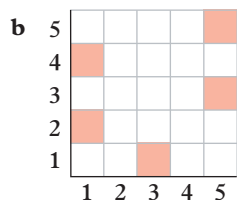
- c A systematic random sample is a random sample chosen in a systematic way. Each member of the population is assigned a number, then the first number is selected at random. Every n th person after the first is selected. The value of n depends on the desired size of the sample and the size of the population.

Review questions 10.8B

- 1 C 2 D 3 B 4 A
5 C 6 D 7 B 8 B
9 C 10 A 11 B 12 D
13 C

Review questions 10.8C

- 1 Example: The previous survey found that 85% of people chose chocolate as the best ice-cream flavour. What do you think? What is your favourite ice-cream flavour?
- 2 a Discrete numerical
b Continuous numerical
c Ordinal (categorical)
d Nominal (categorical)
- 3 a Census b Census
c Census d Sample
- 4 a Only people at home on Saturday night are surveyed.
b Only people on the electoral roll are surveyed.
- 5 Write the names of the students on cards, shuffle the cards and select 12.
- 6 a Moving left across the list gives (5, 3), (1, 4), (5, 5), (3, 1), (1, 2).



- 7 a Year 7: $\frac{85}{500} = \frac{17}{100}$
Year 8: $\frac{82}{500} = \frac{41}{250}$
Year 9: $\frac{89}{500}$
Year 10: $\frac{96}{500} = \frac{24}{125}$
Year 11: $\frac{75}{500} = \frac{3}{20}$
Year 12: $\frac{73}{500}$
b 17, 16, 18, 19, 15, 15
- 8 Assign the numbers 1 to 300 to the students, choose the first number at random and then select every 15th number.
- 9 Answers will vary.
- 10 a 30% b 120 c 280
- 11 Assign the numbers 1 to 74 to the drivers. Use a table or calculator to select 8 random numbers less than or equal to 74. The drivers whose numbers are selected form the sample.

Review questions 10.8D

- 1 Answers will vary.
- 2 a Sample b Census
- 3 a Members of the local netball club
b Senior students
- 4 a Members of another sporting club who may want their own facilities improved or non-sporting people.
b Junior students
- 5 Example: Start at the second decimal place of each number, take that value and the next two values to make each three-digit number. This gives: 869, 811, 607, 191, 301
- 6 a Vince: $\frac{56}{200} = \frac{7}{25}$, Sophie: $\frac{43}{200}$, Talia: $\frac{51}{200}$,
Mohammed: $\frac{50}{200} = \frac{1}{4}$
b Vince: 6, Sophie: 4, Talia: 5, Mohammed: 5
- 7 Self-selected sampling
- 8 Use the random number generator function: RAN#. Multiply the randomly generated number by 10.
- 9 *Method 1:* Write names of students on pieces of paper and put in a box. Select 6 pieces of paper at random from the box.
Method 2: Assign the numbers 1 to 24 to the students. Use a table of random numbers to select 6 numbers which are less than or equal to 24.
- 10 a Yes, if the number of classes is small (2 or 3).
b No

Review questions 10.8E

- 1 a Parents b Farm animals
- 2 a Nominal (categorical) b Continuous numerical
c Discrete numerical d Ordinal (categorical)
- 3 a Example: heights of students in a class
b Example: money spent on petrol
- 4 a People who use the bus service
b People who use the pool
- 5 a i Every 15th part ii Every 25th part
b Answers will vary.
- 6 Advantages: It can reduce the time and expense of finding suitable subjects that meet the selection criteria. Volunteers are likely to be committed to take part in the study. They may also show a greater willingness to provide more insight into the subject of the research by giving full detailed responses to open-ended questions. Disadvantages: Because the people chosen are volunteers, there is likely to be a degree of self-selection bias. This can either lead to the sample not being representative of the population being studied or exaggerating some particular finding from the study.
- 7 a 80% b 240 c 60
- 8 Use the RANDBETWEEN function and specify numbers between 1 and 100.

- 9 Answers will vary.
- 10 a If there are only a few bus stops on the route, then the number of people at one bus stop would be suitable. If there are many bus stops, then one bus stop is not satisfactory.
- b No. One carriage on one train out of many trains in service is not suitable.

Review questions 10.8F

- 1 Answers will vary.
- 2 Self-selected sample
- 3 Answers will vary.
- 4 Advantage: It collects data about every member of the population so is detailed and accurate.
Disadvantage: It is often expensive, time-consuming and impractical.
- 5 a The tyre distributor would only recommend tyres that they sell.
- b The sample is biased towards people whose names start with one (or two) letters of the alphabet.
- 6 a Number every employee from 1 to 120. Use a table of random numbers to choose 25 different numbers from 1 to 120.
- b Put names in a hat and draw names at random. Other answers are possible.
- c Start at any number and select every 5th employee.
- 7 “Biased” means “not free from prejudice”.
- 8 A good sample is one whose characteristics are representative of the whole population.
- 9 Answers will vary. Choose the first two or three digits from five numbers.

Review questions 10.8G

- 1 a i Nominal ii Ordinal iii Nominal
- b Because the response options have been explicitly assigned a numerical value, the values 1 to 5 could be interpreted as agreement ‘scores’. In this case, it would be reasonable to add together or find the average of the scores across a range of questions with the same response options.
- c i Sample ii Census
- d i *Method 1:* Write names of drivers on pieces of paper and put them in a hat. Shake and select 6 names.
Method 2: Assign the numbers 1 to 48 to the drivers and select 6 numbers less than or equal to 48 from a table of random numbers (or use a calculator to generate the random numbers).
- ii Drivers who self-select to complete the questionnaire are likely to be those who have strong opinions about the duty roster. The responses may be skewed towards very strong negative opinions, for example.

e i Boys: $\frac{67}{120}$, girls: $\frac{53}{120}$

ii Boys: 17, girls: 13

f Take a random sample of 17 boys and a random sample of 13 girls.

g i Intervals of 30 ii 7

h 425

Module 11 Displaying and interpreting data

Lesson 11.1 Dot plots and stem-and-leaf plots

1 a

Colour	Frequency
White	14
Blue	12
Grey	9
Red	11
Other	4

b 50

c White

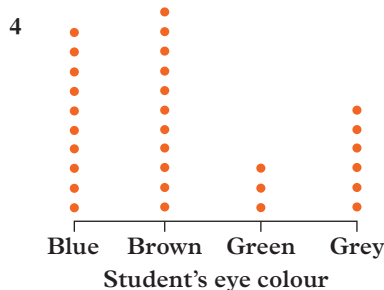
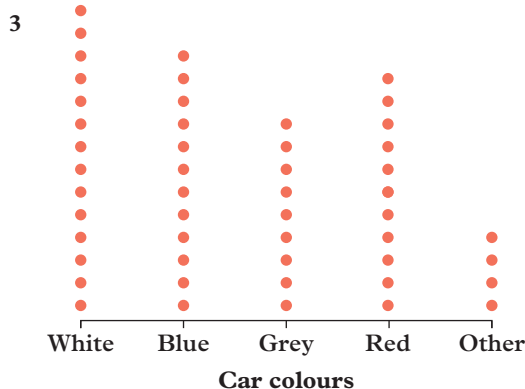
d W 28%, B 24%, G 18%, R 22%, O 8%

2 a

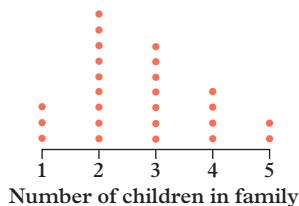
Colour	Frequency
Blue	10
Brown	11
Green	3
Grey	6

b 30

c Brown



5 a



b

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

6 a

Stem	Leaf
3	2 3 6 7
4	2 5 7 7 7 8
5	0 0 1 7 9
6	0 2 3 5 8

Key: 3 | 2 = 32

b Lowest = 32, highest = 68

c i 2

ii 0

d 47

e 5

f 10

7 a

Stem	Leaf
12	4 5 6 6 6 8 8
13	0 0 0 2 9
14	0 0 1 2 5 5 5 5 6 8 9
15	0 1 4 4 6 8 8

Key: 12 | 4 = 124

b Lowest = 124, highest = 158

c i 1

ii 2

iii 0

d 145

e i 7

ii 7

8 a Lowest = 50, highest = 90

b i 3

ii 2

iii 0

iv 0

c 54

d 14

e i 14

ii 8

9 a 25

b Lowest = 3, greatest = 36

c 5

d 4

10 a 40

b

Stem	Leaf
1	8
2	2 2 3 4 6 6 6 6 6 6 7 7 7 7 7 8 8 8 9 9 9 9
3	0 0 1 1 1 1 2 2 2 3 3 3 3 3 4 4 4 7 8

Key: 1 | 8 = 18

c

Stem	Leaf
1	
1 *	8
2	2 2 3 4
2 *	6 6 6 6 6 7 7 7 7 7 8 8 8 9 9 9 9
3	0 0 1 1 1 2 2 2 2 3 3 3 3 4 4 4
3 *	7 8

Key: 1 | 8 = 18

d Most students spend between 25 and 35 hours per week doing homework.

11 a

Leaf: Melbourne	Stem	Leaf: Adelaide
	1	7 9
	2	2 3 4
	3	8 8
9 8 8	4	3
9 9 8 8 2 1 0	5	0 2 9
8 0	6	5

Key: 8 | 4 | 3

48 mm for Melbourne

43 mm for Adelaide

b Melbourne has more rain than Adelaide. Melbourne's rainfall is clustered more than Adelaide's.

12 a

Leaf: Hobart	Stem	Leaf: Canberra
	3	7
9 8 7 0	4	0 7 9
6 5 4 2 2 2 2	5	0 3 3 6 7 8
4	6	3 7

Key: 2 | 5 | 0

52 mm for Hobart

50 mm for Canberra

b Hobart has a more constant rainfall than Canberra. Canberra has a greater range.

13 a

Leaf: Boys	Stem	Leaf: Girls
8 5	17	4 6 7 8 9
9 7 4 2	18	0 1 2 3 5
5 2 0	19	
1	20	

Key: 5 | 17 | 4

17.5 kg for Boys

17.4 kg for Girls

b The boys in this sample are heavier than the girls.

14 a 30

b 6

c i Carrots

d i Peas

ii Potato

ii Pumpkin

e i Potato

ii Pumpkin

15 a i 12

b i 39

ii 2

ii 39

c i 20

d i 10s and 20s

ii 34

ii 20s and 30s

e i 6

f Class 11B

ii 1

16 a Rowers = 14, Tennis players = 13

b 2

c 3

d 12

17 The frequency of each group in the dot plot equates to the number of leaves in the corresponding stems.

Lesson 11.2 Tables and column graphs

1 a

Temperature	Number of days
17	1
18	2
19	4
20	7
21	6
22	6
23	4

2 a

Score	3	13	20	22	29	31	33	34	38	40	41	42	44
Frequency	1	1	1	1	1	1	1	1	1	1	2	1	1

Score	45	48	49	50	51	52	53	54	55	56	57	60	61
Frequency	2	1	1	2	3	1	1	2	2	2	1	2	2

Score	62	64	65	66	68	69	70	74	75	79	85	94
Frequency	1	1	1	1	2	1	1	2	1	1	1	1

- b We are unable to obtain much information from this table apart from getting a rank order of the scores. It is also very time consuming to prepare.

- c i The scores from 0 to 9
 ii The scores from 60 to 69
 iii Yes, ten possible scores

iv

Class	Tally	Frequency
0–9		1
10–19		1
20–29		3
30–39		4
40–49		9
50–59		14
60–69		11
70–79		5
80–89		1
90–99		1
	Total	50

v 14

vi 2%

- d i Twenty possible scores

ii

Class	Tally	Frequency
0–19		2
20–39		7
40–59		23
60–79		16
80–99		2
	Total	50

iii 7

iv 32

b 30

c 1

d 7

- e There are 30 temperatures recorded, so the month must have 30 days. The most realistic month for these temperatures to have been recorded would depend on the location.

- e Advantage: Grouping data into classes makes it easier to organise and summarise.

Disadvantage: Grouping can result in loss of accuracy because individual data values within each class are not retained.

3 a 3

b 40

c 8

d 4

- e No, the grouped data only shows how many clients had 6, 7 or 8 arguments.

- f No, the grouped data shows three responses in each class interval, so the lowest could be 0, 1 or 2 and highest could be 12, 13 or 14.

4 a 25

b 120

- c No, the grouped data only shows how many plants were 350–374 mm high.

- d No, the grouped data only shows that the shortest plant is in the class interval 300–324 and the tallest plant is in the class interval 425–449.

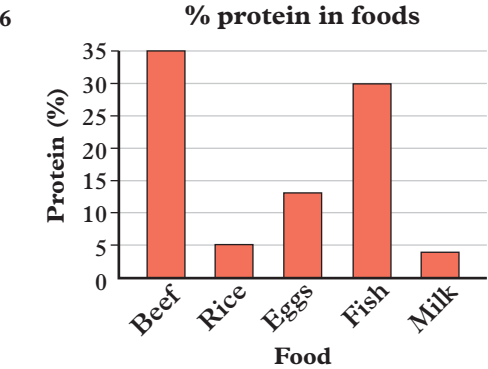
5 a i

Time	Frequency
1–5	2
6–10	4
11–15	6
16–20	1
21–25	5
26–30	3
31–35	3
36–40	1

ii

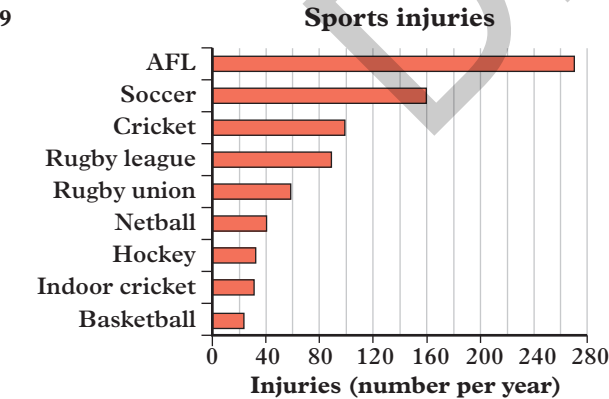
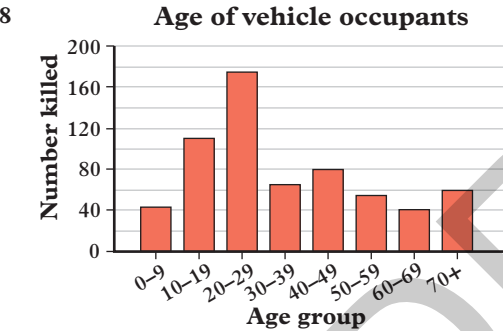
Time	Frequency
0–9	5
10–19	8
20–29	7
30–39	5

- b Advantages: Largest and smallest measurements can be found.
Disadvantages: Not suitable for larger amounts of data.



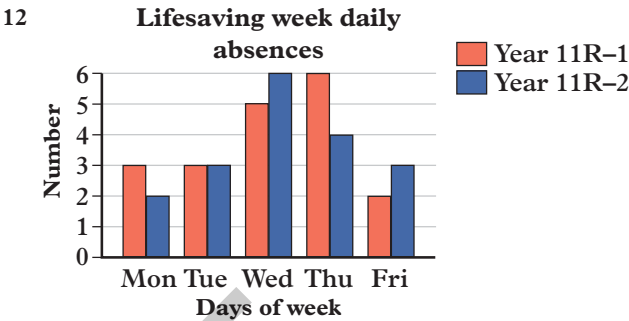
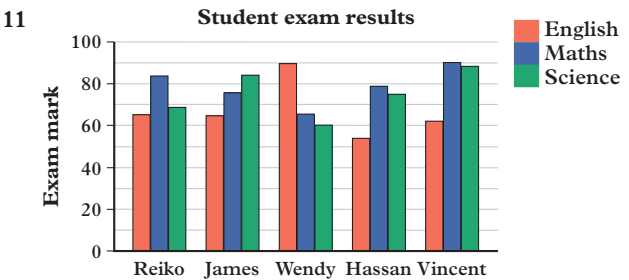
7

Month	Sales (\$ million)
Jan/Feb	7.7
Mar/Apr	8.2
May/Jun	3.2
Jul/Aug	4.1
Sep/Oct	3.4
Nov/Dec	6.0



10

Food	Cholesterol (mg/100 g)
Lobster	70
Beef	80
Chicken	100
Duck	105
Prawns	145



13 a i

Mark	Frequency
0-4	0
5-9	1
10-14	0
15-19	0
20-24	4
25-29	1
30-34	3
35-39	3
40-44	4
45-49	3
50-54	4
55-59	1
60-64	4
65-69	1
70-74	0
75-79	1

ii

Mark	Frequency
0-9	1
10-19	0
20-29	5
30-39	6
40-49	7
50-59	5
60-69	5
70-79	1

iii

Mark	Frequency
0–19	1
20–39	11
40–59	12
60–79	6

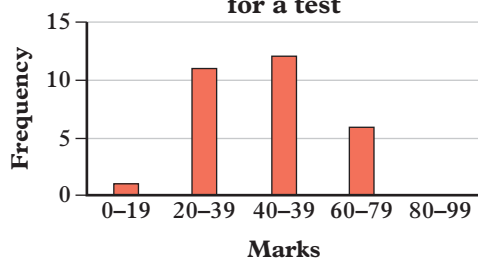
iv

Mark	Frequency
0–49	19
50–99	11

- b Answers will vary. Example: The classes in part ii give a good picture of the test results because there are enough classes to see how spread out the test results are, without there being too many classes to deal with, as in part i.

- c i Classes overlap.
 ii Class 10–19 is missing.
 iii Classes are different sizes.

d **Frequency of students marks for a test**



14 a i

Time ($\times 100$ h)	Number of globes
0–4	3
5–9	12
10–14	23
15–19	2

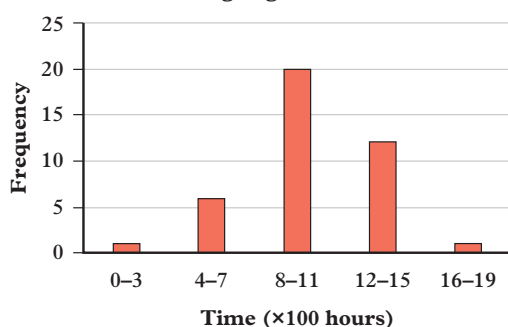
ii

Time ($\times 100$ h)	Number of globes
1–5	5
6–10	16
11–15	18
16–20	1

- b i No, classes overlap. ii Yes
 iii Yes, but class size is too big.

c 40

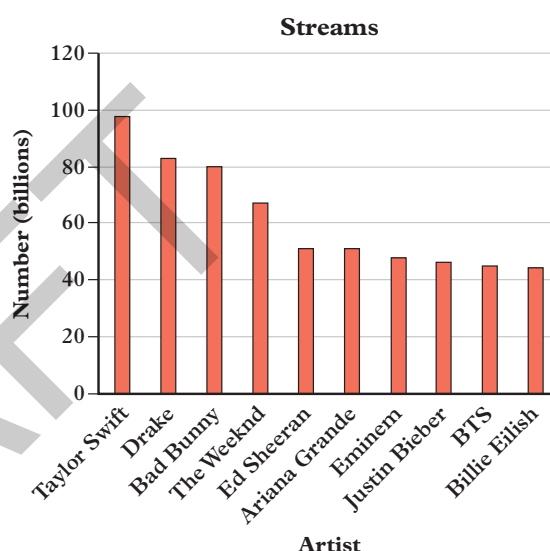
d **Light globe times**



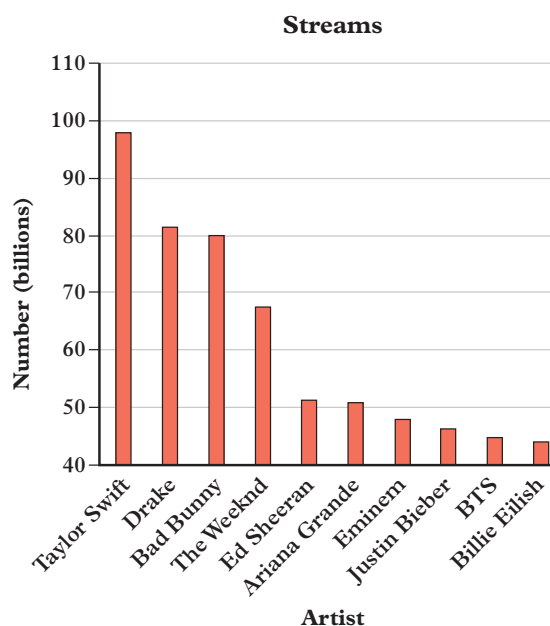
15 a

Artist	Total Streams	Scientific notation
Taylor Swift	96 500 000 000	9.7×10^{10}
Drake	82 600 000 000	8.3×10^{10}
Bad Bunny	80 400 000 000	8.0×10^{10}
The Weeknd	67 400 000 000	6.7×10^{10}
Ed Sheeran	51 100 000 000	5.1×10^{10}
Ariana Grande	50 600 000 000	5.1×10^{10}
Eminem	47 800 000 000	4.8×10^{10}
Justin Bieber	46 200 000 000	4.6×10^{10}
BTS	45 000 000 000	4.5×10^{10}
Billie Eilish	44 500 000 000	4.5×10^{10}

b

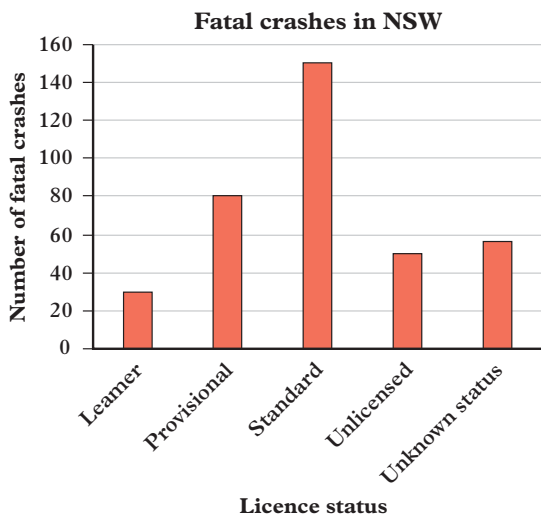


c



The differences are exaggerated in the second graph.

16 a



- b $\frac{25}{183} \approx 0.137$ c 21.9% d 41.0%
- e This data could be used to argue that standard licence holders should be targeted by safety authorities.
- f i 13.1 ii 3.3
- iii The additional context suggests that provisional licence holders should be targeted by safety authorities.

17 Answers will vary.

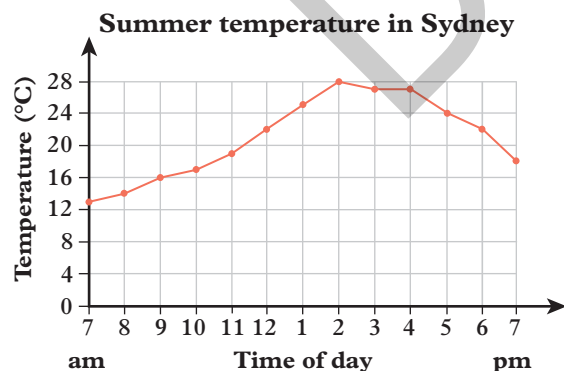
18 One option is to use a split-stem stem-and-leaf plot. This has the advantage of simultaneously summarising and displaying the data.

Stem	Leaf
1	3 4
1 *	5 6 6 7 7 8 8 9
2	0 1 2 3 3 3 4 4
2 *	5 5 6 6 6 6 7 7 7 8 8 9
3	0 2 3

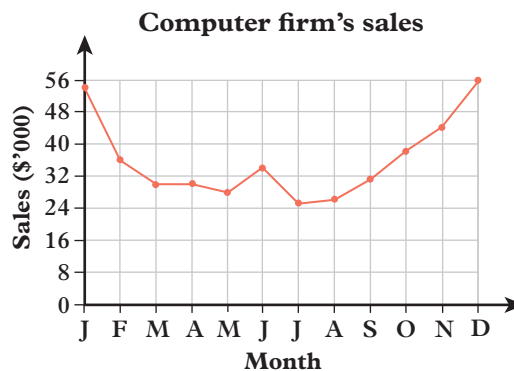
Key: 1 | 3 = 13

Lesson 11.3 Line graphs

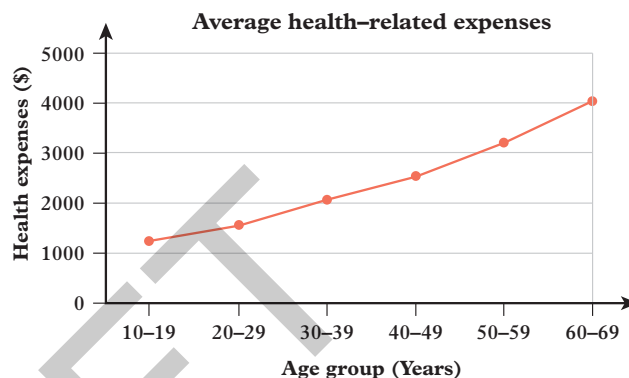
1



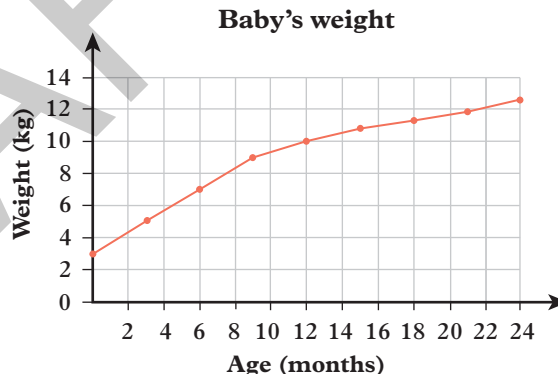
2



3



4



5 The scale on the horizontal axis is irregular.

6 a Graph A b No c No

7 a The scale on the vertical axis is irregular.

b Use of area

c No scale on vertical axis

d Use of volume

8 a \$6500

b i \$5300

ii \$3500

c i After 2 years

ii After 4 years

d \$500

e \$1200

9 a



The number of fatalities rose sharply from about 400 in 1950 to 1300 in 1970, where it remained steady for the next decade. It then decreased from 1980 to 2020, quickly at first and then at a relatively slower rate.

b i 1965–70

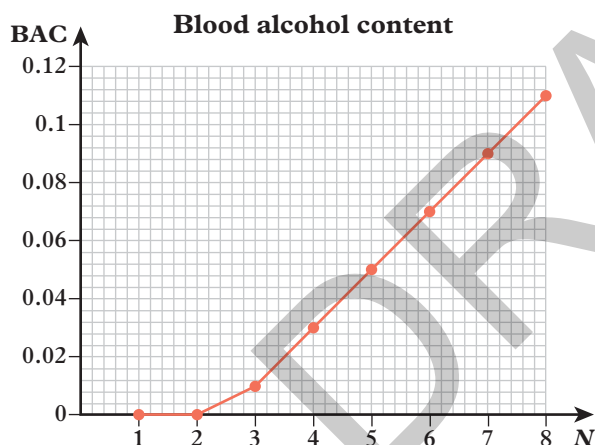
ii 1985–90

c Answers will vary. For example, consider changes in vehicle safety features.

10 a

<i>N</i>	1	2	3	4	5	6
<i>BAC</i>	0	0	0.01	0.03	0.05	0.07

b



c i 0.09

ii 0.11

d 4

e 2

11 Answers will vary.

12 Answers will vary.

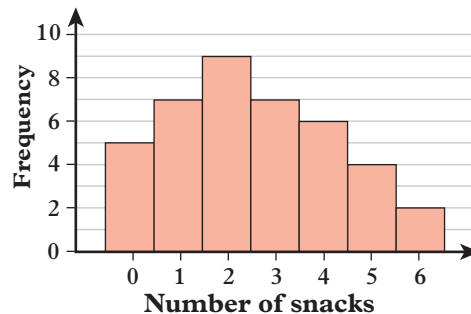
13 a 2020 and 2021. Covid-19 pandemic lockdowns and social distancing resulted in decreased numbers of people gathering in public places.

b The NRL crowds remained consistent from 2015 to 2019. They then decreased in 2020 and increased from 2021 to 2023, where they remained relatively consistent.

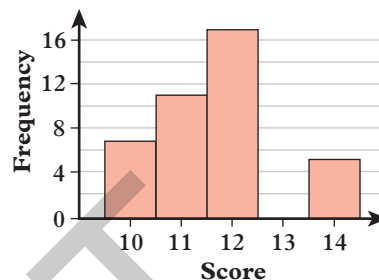
c 20%

Lesson 11.4 Histograms and frequency polygons

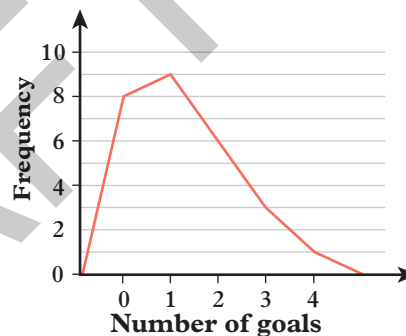
1 a



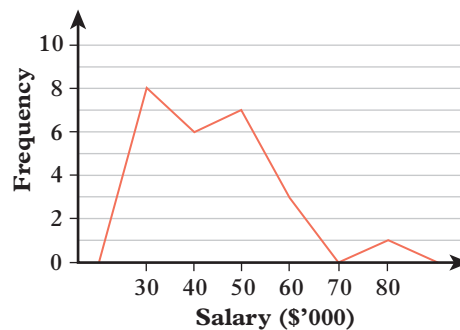
b



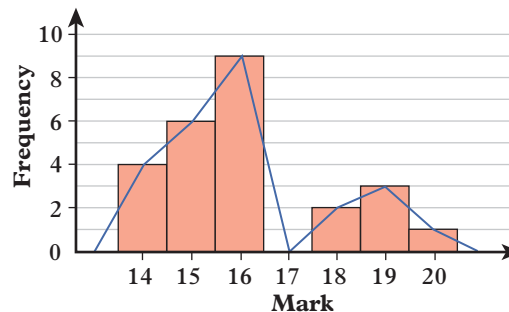
2 a



b



3

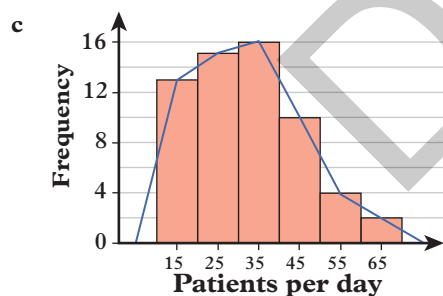
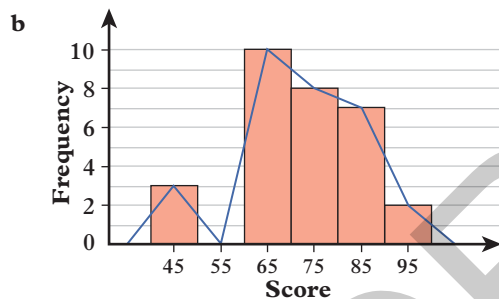
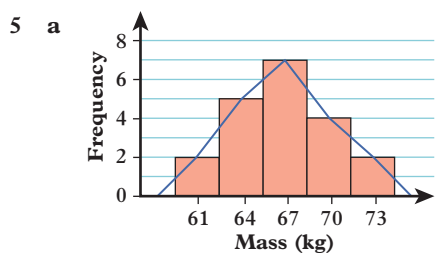


4 a

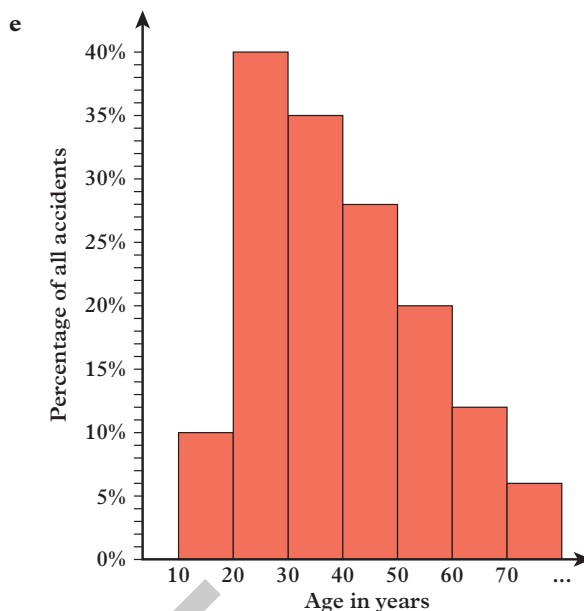
Score (x)	Frequency (f)
5	3
6	4
7	8
8	6
9	2

b

Score (x)	Frequency (f)
12	15
13	10
14	6
15	8
16	11
17	14



- 6 a i 8%
 ii 10%
 b i 3%
 ii 2%
 c i 20–30
 ii 70+
 d i 20–30
 ii 70+



- f i 20–30 ii 70+

- 7 a There would end up being too many rows and columns in the table, most with frequency equal to 1.
 b No, for large ranges of data, using groups makes sense.
 8 a 40
 b No, as the data is grouped.
 c 17.5%

Lesson 11.5 Sector graphs and divided bar charts

1 a

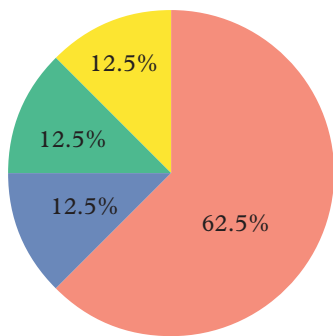
Number of children	Frequency	Relative frequency
1	3	12%
2	9	36%
3	7	28%
4	4	16%
5	2	8%

- b i $\frac{7}{25}$ c i 36%
 ii $\frac{4}{25}$ ii 8%

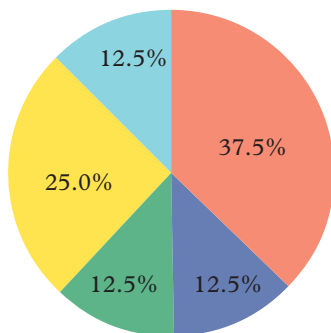
2 a

Goals/match	Frequency	Relative frequency
5	4	16%
6	3	12%
7	7	28%
8	5	20%
9	0	0%
10	3	12%
11	2	8%
12	1	4%

4 Number of children

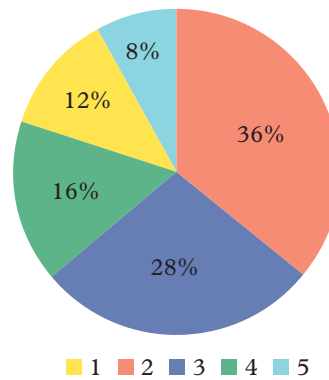
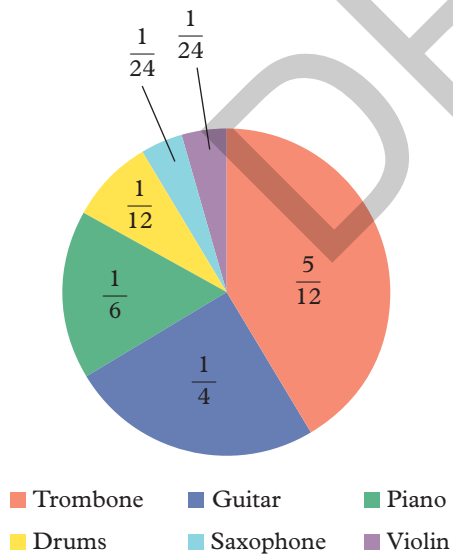
Preferred meal

■ Pizza ■ Sushi ■ Soup ■ Pasta

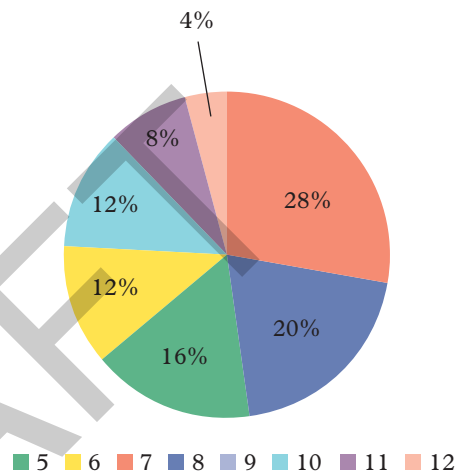
b Favourite film genre

■ Comedy
 ■ Drama
 ■ Horror
■ Action
 ■ Science fiction

c Favourite instrument

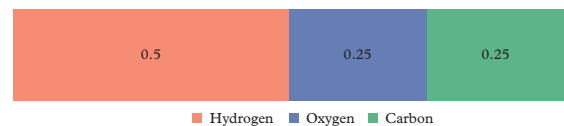


Goals per match



5 a

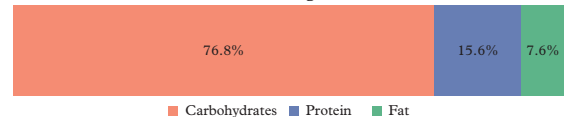
Glucose



■ Hydrogen
 ■ Oxygen
 ■ Carbon

b

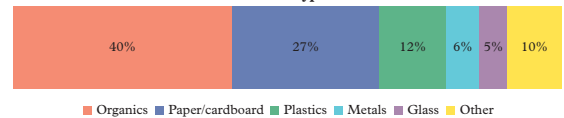
Porridge



■ Carbohydrates ■ Protein ■ Fat

c

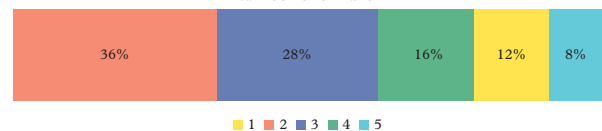
Waste type



Organics Paper/cardboard Plastics Metals Glass Other

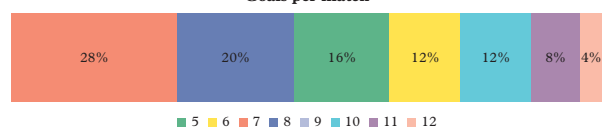
6

Number of children



1 2 3 4 5

Goals per match



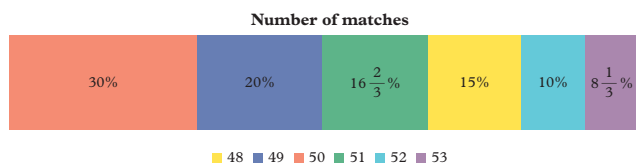
5 6 7 8 9 10 11 12

7 a

Number of matches	Frequency	Relative frequency
48	9	15%
49	12	20%
50	18	30%
51	10	$16\frac{2}{3}\%$
52	6	10%
53	5	$8\frac{1}{3}\%$

b i 20% ii 30% iii $16\frac{2}{3}\%$

c



8 a

Distance	Tally	Frequency	Relative frequency
180–189		2	$\frac{1}{30}$
190–199		10	$\frac{1}{6}$
200–209	 	12	$\frac{1}{5}$
210–219		6	$\frac{1}{10}$
220–229	 	12	$\frac{1}{5}$
230–239		9	$\frac{3}{20}$
240–249		3	$\frac{1}{20}$
250–259		6	$\frac{1}{10}$

b 60

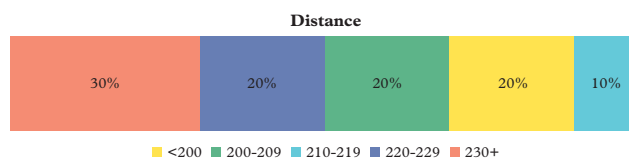
c See answer for part a.

d i $\frac{3}{10}$ ii $\frac{1}{5}$ iii $\frac{1}{5}$

e

Distance (m)	Relative frequency
<200	$\frac{1}{5}$
200–209	$\frac{1}{5}$
210–219	$\frac{1}{10}$
220–229	$\frac{1}{5}$
230+	$\frac{3}{10}$

f



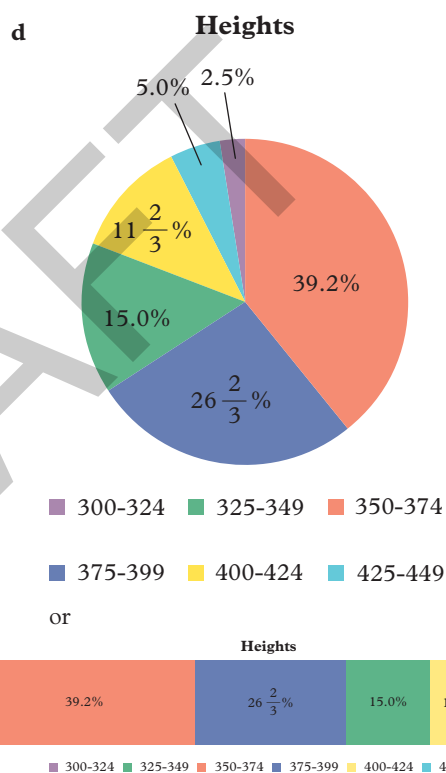
9 a

Height	Frequency	Relative frequency
300–324	3	2.5%
325–349	18	15%
350–374	47	39.2%
375–399	32	$26\frac{2}{3}\%$
400–424	14	$11\frac{2}{3}\%$
425–449	6	5%

b 5%

c 2.5%

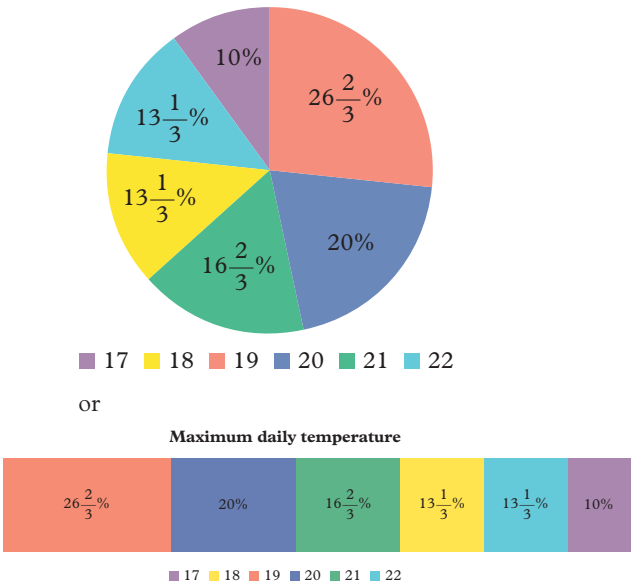
d



10 a

Maximum daily temperature (°C)	Frequency	Relative frequency
17	3	10%
18	4	13.3%
19	8	26.7%
20	6	20%
21	5	16.7%
22	4	13.3%

- b 15 days
- c 50%
- d 9 days
- e **Maximum daily temperature**



11 Draw a circle using a compass and mark the centre of the circle. Multiply each relative frequency by 360 to find the size of the angle at the centre of the circle of each segment. Use a protractor and ruler to divide the circle into the segments required.

Lesson 11.6 Cumulative frequency

1 a

Number of children	Frequency	Cumulative frequency
1	3	3
2	9	12
3	7	19
4	4	23
5	2	25

- b i 19
- ii 23

2 a

Goals/match	Frequency	Cumulative frequency
5	4	4
6	3	7
7	7	14
8	5	19
9	0	19
10	3	22
11	2	24
12	1	25

- b i 14
- ii 19

3 a

Score	Frequency
3	4
4	2
5	4
6	5
7	3

b

Score	Frequency
12	5
13	6
14	8
15	10
16	5
17	2

c

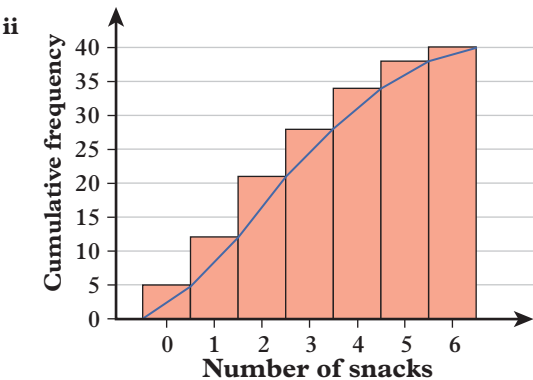
Score	Frequency
8	11
9	6
10	10
11	0
12	5
13	8

d

Score	Frequency
87	23
88	25
89	30
90	28
91	21
92	11

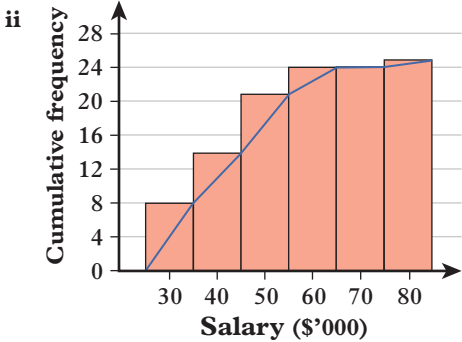
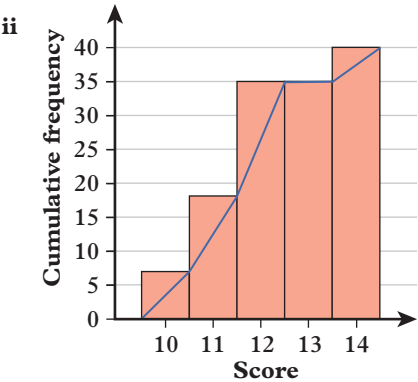
4 a i

Number of snacks	Frequency	Cumulative frequency
0	5	5
1	7	12
2	9	21
3	7	28
4	6	34
5	4	38
6	2	40



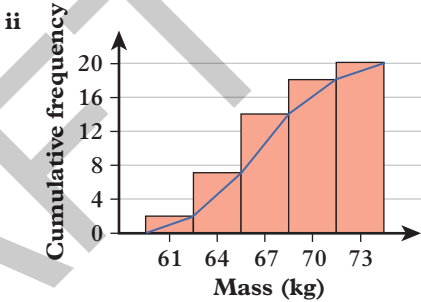
b i

Score	Frequency	Cumulative frequency
10	7	7
11	11	18
12	17	35
13	0	35
14	5	40



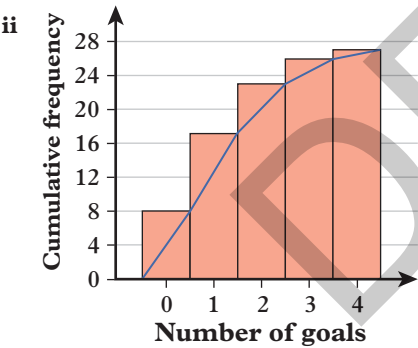
5 a i

Mass (kg)	Frequency	Cumulative frequency
59.5–<62.5	2	2
62.5–<65.5	5	7
65.5–<68.5	7	14
68.5–<71.5	4	18
71.5–<74.5	2	20



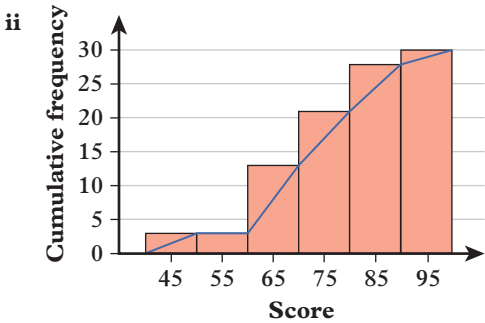
c i

Number of goals	Frequency	Cumulative frequency
0	8	8
1	9	17
2	6	23
3	3	26
4	1	27



b i

Score	Frequency	Cumulative frequency
40–<50	3	3
50–<60	0	3
60–<70	10	13
70–<80	8	21
80–<90	7	28
90–<100	2	30



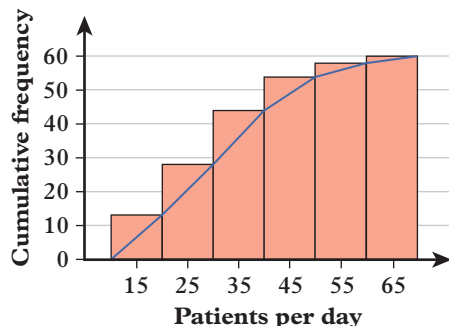
d i

Salary (\$)	Frequency	Cumulative frequency
30 000	8	8
40 000	6	14
50 000	7	21
60 000	3	24
70 000	0	24
80 000	1	25

c i

Patients/ day	Frequency	Cumulative frequency
10–<20	13	13
20–<30	15	28
30–<40	16	44
40–<50	10	54
50–<60	4	58
60–<70	2	60

ii



6 a

Score	Cumulative frequency	Frequency
2	3	3
3	5	2
4	8	3
5	13	5
6	15	2

b

Score	Cumulative frequency	Frequency
18	2	2
19	7	5
20	13	6
21	16	3
22	19	3
23	20	1

c

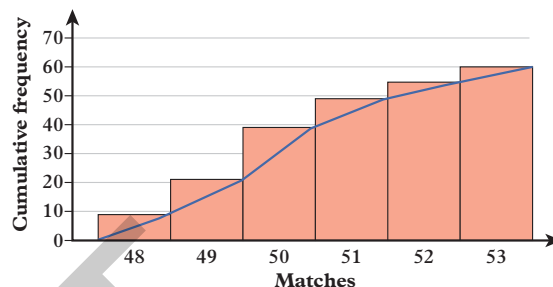
Score	Cumulative frequency	Frequency
141	2	2
142	7	5
143	15	8
144	22	7
145	27	5
146	30	3

7 a

Number of matches	Frequency	Cumulative frequency
48	9	9
49	12	21
50	18	39
51	10	49
52	6	55
53	5	60

b i 18 ii 21 iii 39 iv 21

c

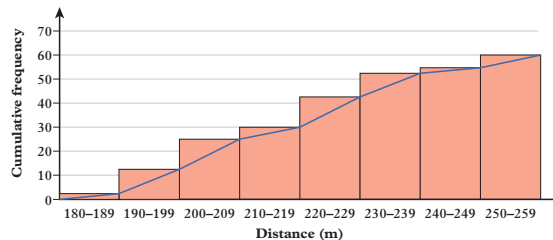


8 a

Distance (m)	Frequency	Cumulative frequency
180–189	2	2
190–199	10	12
200–209	12	24
210–219	6	30
220–229	12	42
230–239	9	51
240–249	3	54
250–259	6	60

b i 12 ii 42

c



9

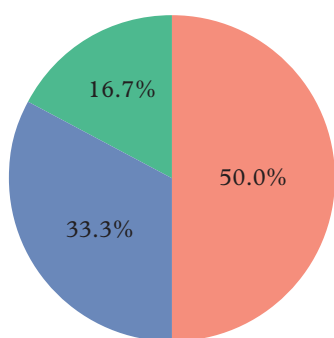
ATAR	Frequency	Cumulative Frequency	Cumulative Percentage
90–<100	3374	3374	33.764%
80–<90	2483	5857	58.611%
70–<80	1712	7569	75.743%
60–<70	1115	8684	86.901%
50–<60	675	9359	93.656%
0–<50	634	9993	100%

Lesson 11.7 Choosing a data display

- 1 a Grouped discrete numerical
 b Ordinal categorical
 c Discrete numerical
 d Nominal categorical
 e Grouped continuous numerical

2 a

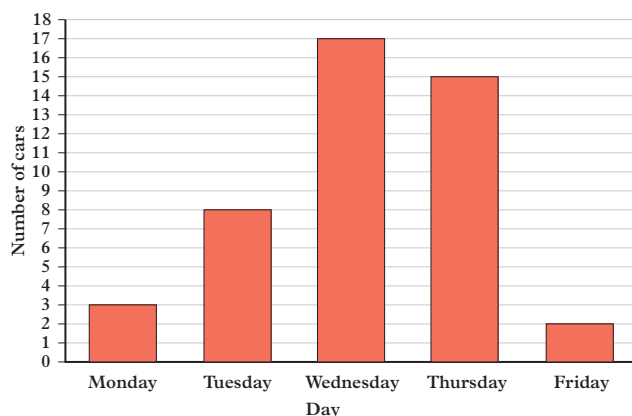
Shoe sizes	Frequency	Relative frequency
2–5	4	$\frac{1}{3} \approx 33.3\%$
6–9	6	$\frac{1}{2} = 50\%$
10–13	2	$\frac{1}{6} \approx 16.7\%$

Shoe sizes

■ 2–5 ■ 6–9 ■ 10–13

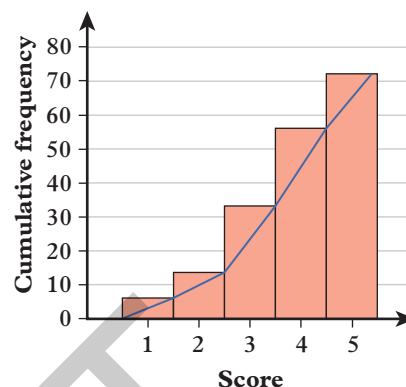
- b For example, suppose someone counts the number of cars parked in their office carpark each day for a week and records these numbers in the table as shown.

Day	Number of cars
Monday	3
Tuesday	8
Wednesday	17
Thursday	15
Friday	2



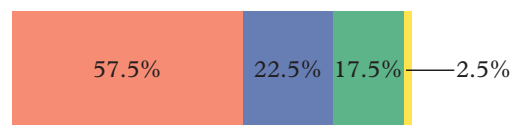
c

Score	Frequency	Cumulative frequency
1	6	6
2	8	14
3	19	33
4	23	56
5	15	71



d

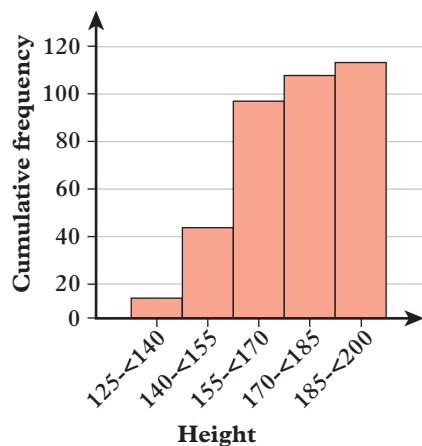
Suit	Frequency	Relative frequency
Clubs	7	$\frac{7}{40} = 17.5\%$
Diamonds	9	$\frac{9}{40} = 22.5\%$
Hearts	23	$\frac{23}{40} = 57.5\%$
Spades	1	$\frac{1}{40} = 2.5\%$

Suits

■ Hearts ■ Diamonds ■ Clubs ■ Spades

e

Height (cm)	Frequency	Cumulative frequency
125–<140	12	12
140–<155	32	44
155–<170	53	97
170–<185	11	108
185–<200	5	113



- 3 a i** Modified dot plot (shoe icons are used in place of dots), modified sector graph/divided bar chart
- ii** Assuming one shoe icon corresponds to one team member, this infographic appears to represent the data from questions **1a** and **2a**.
- iii** The different shoe icons have different heights, so the total heights of each column misrepresent the frequency of each size bracket. The percentages only add up to 25%.
- b i** Line graph and modified dot plot/column graph
- ii** The line graph appears to show the average inside temperatures in an office on each day of the work week. The car icons seem to represent the number of cars in the office carpark each day, with each car icon representing 5 cars.
- iii** The units of measurement for the temperature are not specified. It is also unclear whether the data is from a single week or averaged across many weeks.
- c i** Divided bar chart, column graph, and a third visual that is unclear
- ii** There are five categories that appear to be represented by the same colour in the three different displays. Using all available information, we can deduce that the five categories probably represent the number of basic maths question the 71 adults got incorrect.
- iii** These charts and graphs are almost meaningless because they are not labelled clearly enough.
- d i** This data is not displayed in a standard graph or chart.
- ii** There appears to be one icon for each time the different suits are selected by magic show participants.
- iii** This data is not summarised at all, and it is not clear exactly what each icon represents.

4 a i

	Definitely suitable	Uncertain	Definitely unsuitable
Dot plot	X		
Column graph	X		
Line graph	X		
Histogram		X	
Frequency polygon		X	

- ii** Shoe size is a discrete numerical variable, but because it depends on the continuous variable of foot length, any displays used for continuous variables can also be interpreted meaningfully in this context.

b i

	Definitely suitable	Uncertain	Definitely unsuitable
Dot plot			X
Column graph	X		
Line graph		X	
Histogram		X	
Frequency polygon		X	

- ii** Even though the day of the week is a categorical variable, it is related to the continuous variable of time, so any displays used for continuous variables can also be interpreted meaningfully in this context.

c i

	Definitely suitable	Uncertain	Definitely unsuitable
Dot plot		X	
Column graph	X		
Line graph	X		
Histogram		X	
Frequency polygon		X	

- ii** This is probably too many dots to put in a dot plot, but not completely unreasonable. Even though only whole number scores are recorded, it would be reasonable to treat scores as a continuous numerical variable, so a histogram or frequency polygon could make sense in this context.

d i

	Definitely suitable	Uncertain	Definitely unsuitable
Dot plot		X	
Column graph	X		
Line graph			X
Histogram			X
Frequency polygon			X

- ii The column for Hearts requires 23 dots, which makes for quite a tall dot plot.

e i

	Definitely suitable	Uncertain	Definitely unsuitable
Dot plot			X
Column graph			X
Line graph	X		
Histogram	X		
Frequency polygon	X		

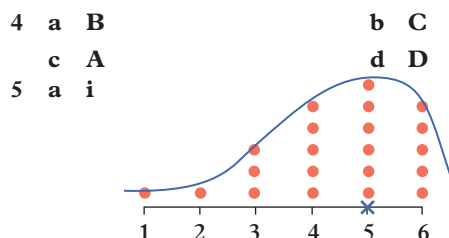
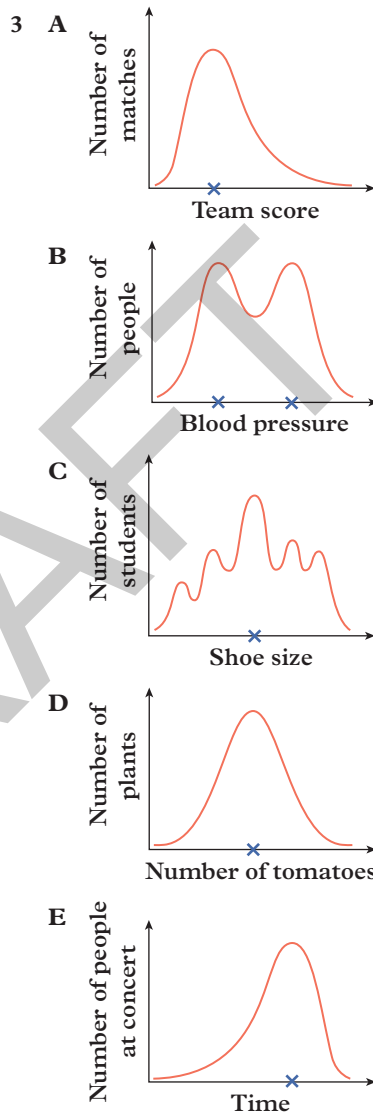
ii N/A

- 5 a The frequency of people who are approximately 175 cm in height
b The frequency of what is being measured around the time autumn turns to winter
c There is no way to interpret this data point as the data on the horizontal axis is categorical. There is no way to interpret this data point as being halfway between a dog and a cat.
- 6 a i The vertical axis should be numerical and the horizontal categorical.
ii Choose a sector graph, as relative frequencies are given and there is one categorical and one numerical data.
b i As there is one of each height, the sector graph doesn't simplify the visual at all.
ii A histogram with heights grouped would give a better understanding of the dataset.
- 7 Answers will vary. Example: Improve the infographic and elaborate on the context used in question 3d.
- 8 a Line graph b Column graph
c Histogram
- 9 Answers will vary. 10 Answers will vary.

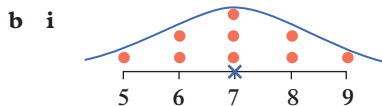
Lesson 11.8 Describing the shape of datasets

- 1 a A b D c E
d B e C

- 2 A More matches will result in lower scores than higher scores.
B The two humps might represent the clustering of blood pressures for males and females.
C There is no pattern of shoe sizes except that there are fewer pairs of small and large sizes.
D More people arrive closer to the concert starting time than earlier than the starting time.
E The number of tomatoes/plant is evenly distributed about the mode.

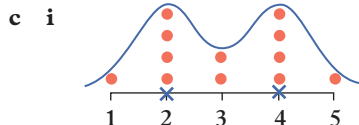


- ii See answer for part i.
iii Negatively skewed



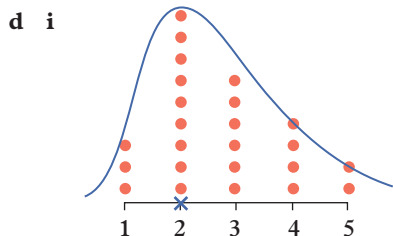
ii See answer for part i.

iii Bell-shaped or symmetric



ii See answer for part i.

iii Bimodal



ii See answer for part i.

iii Positively skewed

6 a i Positively skewed

ii 10s and 20s

b i Symmetric about mode

ii 30s

c i Negatively skewed

ii 60s and 70s

d i Symmetric

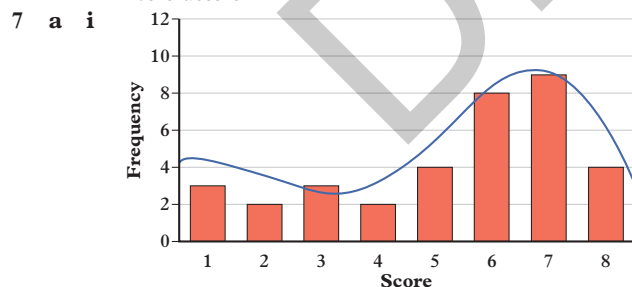
ii 100s and 110s

e i Positively skewed

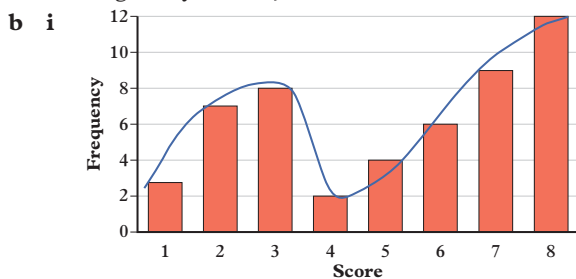
ii 60s and 70s

f i Negatively skewed

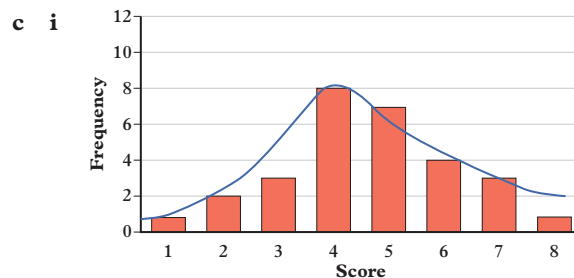
ii No clusters



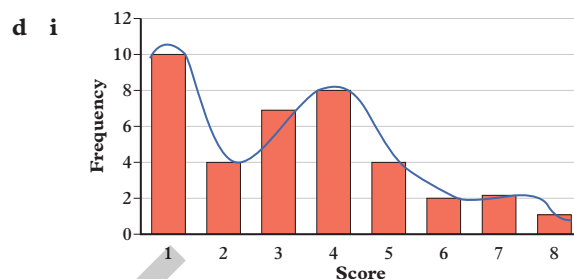
ii Negatively skewed, unimodal



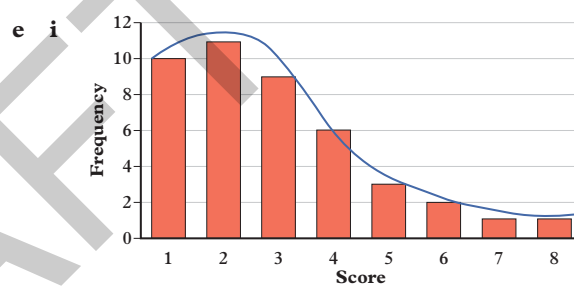
ii Negatively skewed, bimodal



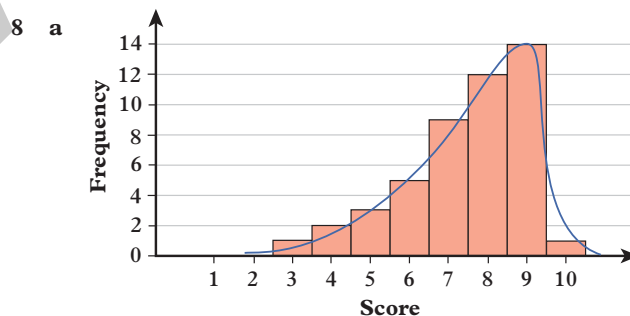
ii Symmetric, bell curved



ii Positively skewed, bimodal

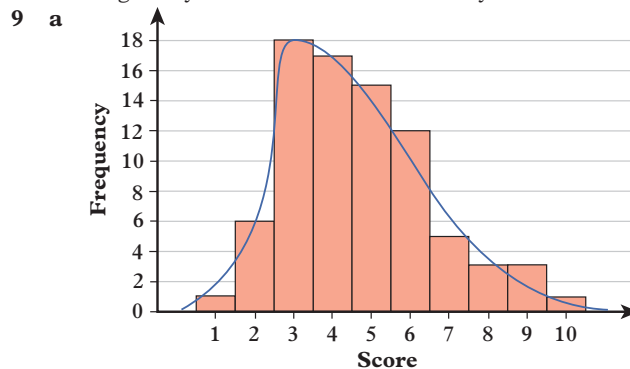


ii Positively skewed, unimodal



b Negatively skewed

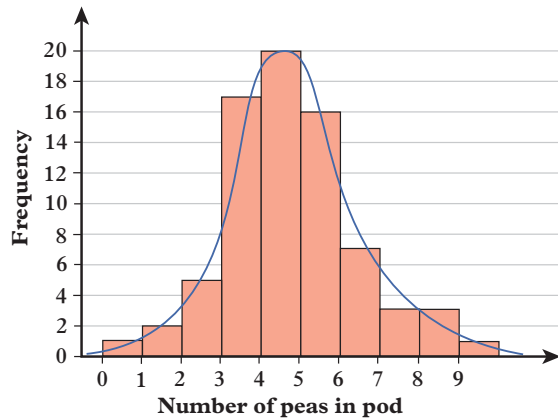
c An easy test



b Positively skewed

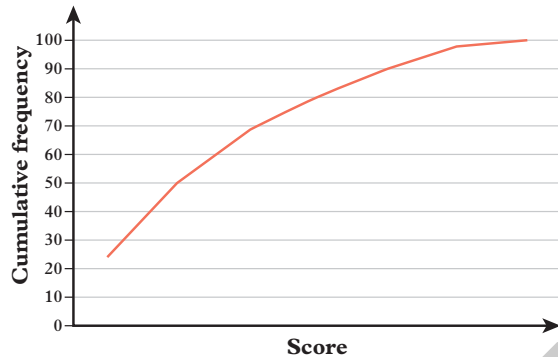
c A hard test

10 a

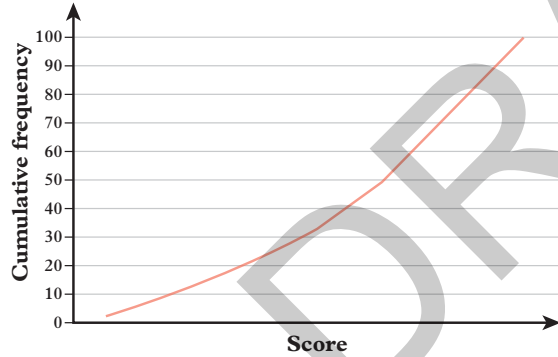


- b Symmetric
c The number is centred around 4 peas in a pod.

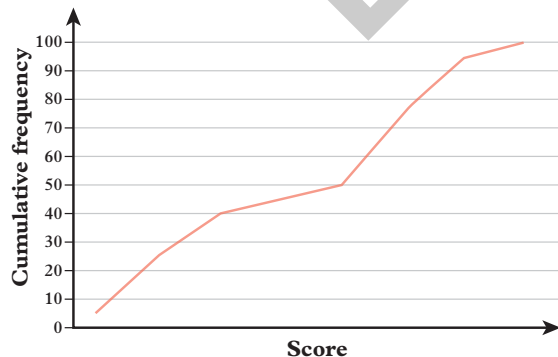
11 a



b



c



Using spreadsheets

- 1 a Column C: enter $=\text{SUM}(\$B\$3:B3)$ in C3, then **fill down** to C9.
Column D: enter $=B3/\$B\$10$ in D3, then **fill down** to D9
- b Answers will vary.
- 2 Answers will vary.
- 3 Answers will vary.
- 4 Answers will vary.

Lesson 11.9 Review: Displaying and interpreting data

Review questions 11.9A

1	Stem	Leaf
	0	7
	1	1 5 6
Stems →	2	0 2 9 9 ← Leaves
	3	
	4	3 7

- 2 Both column graphs and histograms have a horizontal and a vertical axis. They both have vertical bars; however, the bars in a column graph are never touching, whereas the bars in a histogram always touch.
- 3 For example:

First blank	Second blank
class intervals	class centre
frequency	relative frequency
frequency	cumulative frequency
tally marks	frequency
cumulative frequency	cumulative frequency
histogram	polygon

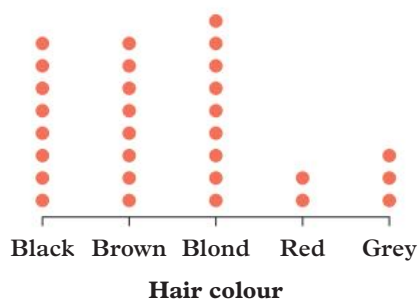
- 4 a Stem-and-leaf plot
b Dot plot
c Sector graph
d Column graph
e Line graph
f Histogram
g Frequency polygon

Review questions 11.9B

1 A	2 D	3 B	4 C
5 C	6 B	7 D	8 B
9 C	10 D	11 C	12 B
13 D	14 C	15 B	16 A

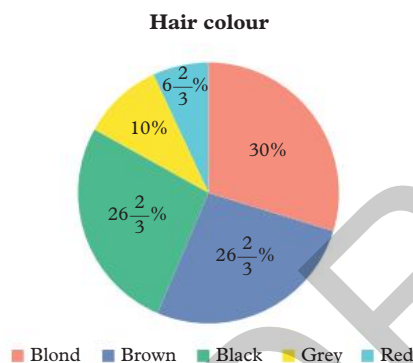
Review questions 11.9C

1 a Survey of hair colour

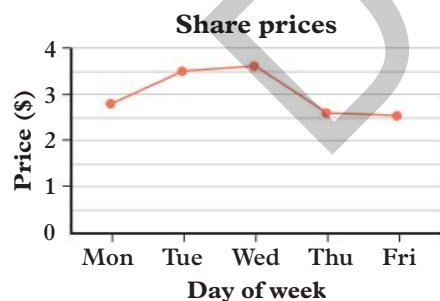


i 30 ii Blond

Hair colour	Frequency	Relative frequency
Black	8	$26\frac{2}{3}\%$
Brown	8	$26\frac{2}{3}\%$
Blond	9	30%
Red	2	$6\frac{2}{3}\%$
Grey	3	10%

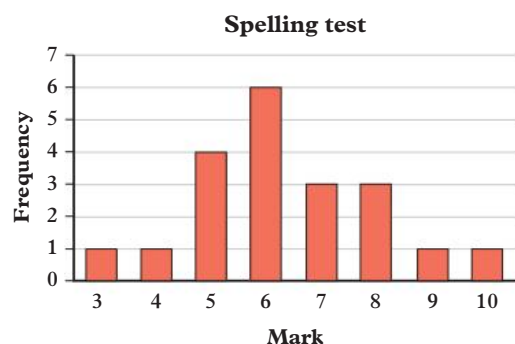


2 a



b Bar chart

3 a



b Positively skewed, unimodal

c The students need to review their spelling.

Review questions 11.9D

Mark	Frequency	Cumulative frequency	Relative frequency
9	1	1	$3\frac{1}{3}\%$
10	2	3	$6\frac{2}{3}\%$
11	3	6	10%
12	2	8	$6\frac{2}{3}\%$
13	3	11	10%
14	6	17	20%
15	4	21	$13\frac{1}{3}\%$
16	2	23	$6\frac{2}{3}\%$
17	2	25	$6\frac{2}{3}\%$
18	4	29	$13\frac{1}{3}\%$
19	1	30	$3\frac{1}{3}\%$

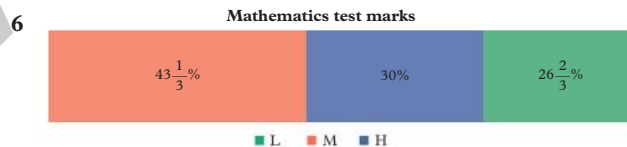
2 No, as almost all data would be connected to the one stem.

3 See answer for question 1.

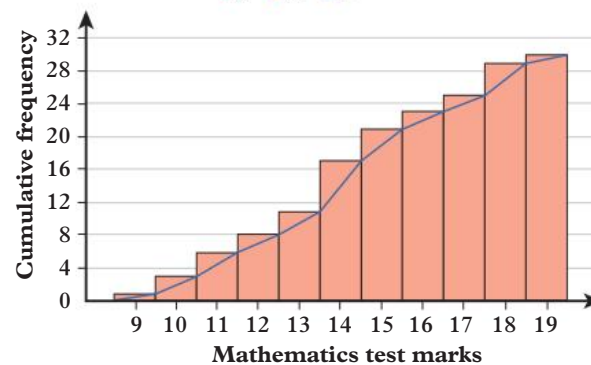
4 a 17

b 21

5 a 20%

b $3\frac{1}{3}\%$ 

7



8 a 4

b 29

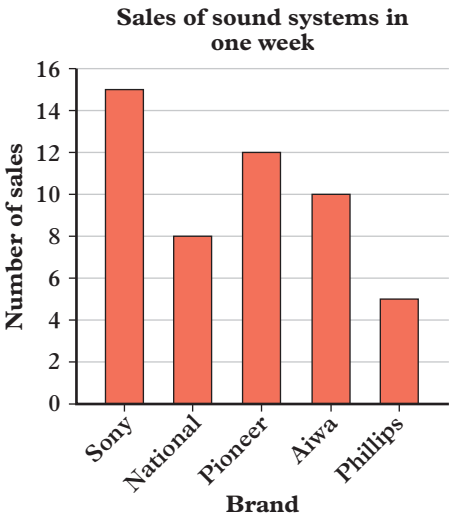
9 It is positively skewed, as the horizontal axis shows nominal categorical data, the shape does not tell us anything.

10 15

Review questions 11.9E

1 a 50

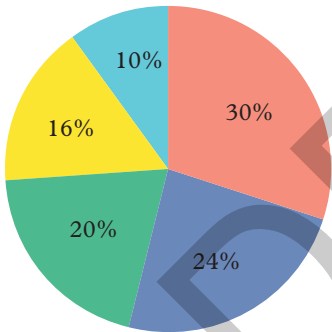
b



c

Brand	Number of sales	Relative frequency
Sony	15	30%
National	8	16%
Pioneer	12	24%
Aiwa	10	20%
Phillips	5	10%

Number of sales



■ Sony ■ Pioneer ■ Aiwa ■ National ■ Phillips

2

a

Stem	Leaf
3	3 4 7 8
4	3 6 8 8 8 9
5	1 1 2 8
6	0 1 3 4 6 9

Key: 3 | 3 = 33

b 33, 69

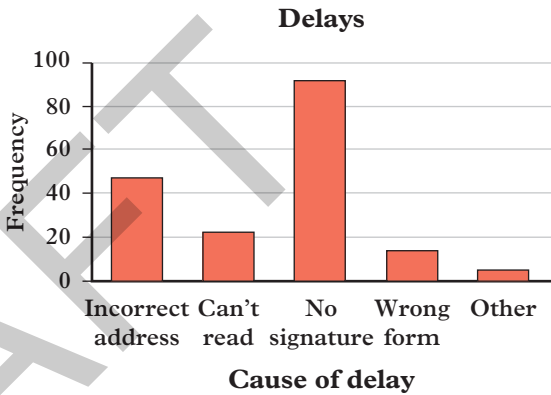
c i 2 times ii 0 times

d 48 e 6 f 10

g Multiple answers are valid. For example: 31–40, 41–50, 51–60, 61–70

3

Cause of delay	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
No signature	92	51	51
Incorrect address	47	26	77
Can't read	22	12	89
Wrong form	14	8	97
Other	5	3	100
Total	180		

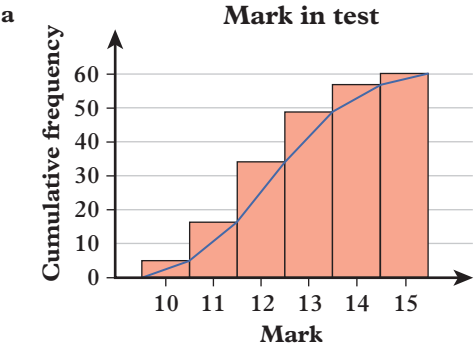


4

- a The vertical axis does not start at zero.
b The vertical axis does not start at zero.
c Irregular scale on vertical axis
d Use of area

Review questions 11.9F

1

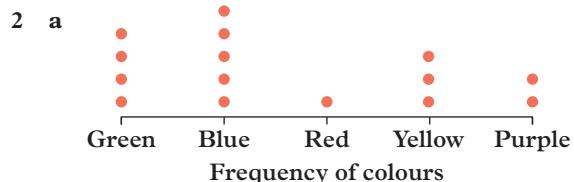


b i 16 ii 34 iii 26

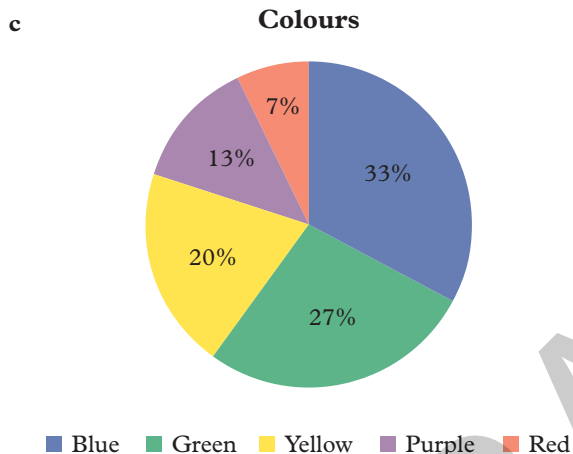
- c It is a slight S curve that increases from 0 to the total score. Cumulative frequency histograms always increase, though the shape of their curves tell you its shape/distribution; for example, a slight S curve (such as this question) indicates a symmetric distribution.

d

Mark	Frequency
10	5
11	11
12	18
13	15
14	8
15	3



b Answers will vary.



d It is not suitable as this dataset is nominal categorical.

Review questions 11.9G

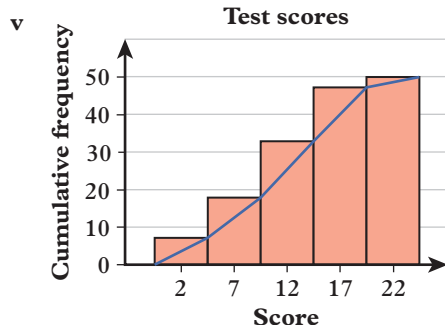
1 a i

Class centre	Cumulative frequency
2	7
7	18
12	33
17	47
22	50

ii Unimodal, negatively skewed

iii $\frac{11}{50}$ or 22%

iv 34%



b i Lowest = 31 hours, highest = 71 hours

ii By starting the class grouping at 24, the dataset will end up looking negatively skewed as opposed to symmetrical. This wouldn't be a good representation of the dataset.

iii Discrete numerical. While time in general is a continuous variable, this survey asked for time spent studying to the nearest whole hour, so it is discrete.

c i $\approx 2\%$

ii Midnight to 5:59 am, Saturday

iii There are too many variables to reasonably display in a single sector graph. A sector graph could display the percentage of crashes throughout the day showing (midnight to 11:59 pm) each day.

Module 12 Data analysis

Lesson 12.1 Mean, mode and modality

1 a i

Value (x)	3	4	5	6	7	Total
Frequency (f)	2	3	6	4	1	16
$f \times x$	6	12	30	24	7	79

ii 4.9

b i

Value (x)	12	13	14	15	16	Total
Frequency (f)	4	6	5	3	2	20
$f \times x$	48	78	70	45	32	273

ii 13.65

c i

Value (x)	50	51	52	53	54	55	Total
Frequency (f)	3	5	8	6	2	4	28
$f \times x$	150	255	416	318	108	220	1467

ii 52.4

d i

Value (x)	8	9	10	11	12	Total
Frequency (f)	12	28	25	26	9	100
$f \times x$	96	252	250	286	108	992

ii 9.92

2 See answers to question 1

3 a 70.9

b 41.7

c 111.8

d 20.2

4 a

Mass (kg)	Class centre (x)	Frequency (f)	$f \times x$
60–62	61	2	122
63–65	64	5	320
66–68	67	7	469
69–71	70	4	280
72–74	73	2	146
	Total	20	1337

$$\text{Mean} = \frac{1337}{20} = 66.85$$

b

Patients per day	Class centre (x)	Frequency (f)	$f \times x$
10–19	14.5	13	188.5
20–29	24.5	15	367.5
30–39	34.5	16	552.0
40–49	44.5	10	445.0
50–59	54.5	4	218.0
60–69	64.5	2	129.0
	Total	60	1900

$$\text{Mean} = \frac{1900}{60} \approx 31.7$$

5 a i

Height (mm)	Class centre (x)	Frequency (f)
155–159	157	4
160–164	162	7
165–169	167	10
170–174	172	5
175–179	177	2

ii 165.9 mm

b i

Number of arguments	Class centre (x)	Frequency (f)
0–2	1	8
3–5	4	18
6–8	7	10
9–11	10	3
12–14	13	1

ii 4.8

c i

Score	Class centre (x)	Frequency (f)
41–50	45.5	3
51–60	55.5	0
61–70	65.5	10
71–80	75.5	8
81–90	85.5	7
91–100	95.5	2
		Total: 30

ii 72.83

- 6 a 5, unimodal
 c 13, unimodal
 e 52, unimodal
 f 56–58, 59–61, bimodal
 g 9, 11, bimodal
 h 8, 9, 10, 11, 12, uniform
 i 26, unimodal
 j 66–68, unimodal
 k 61–70, 71–80, bimodal
 l 30–39, unimodal
- 7 a 46.2

b i

Mark	Frequency
0–4	0
5–9	1
10–14	0
15–19	1
20–24	2
25–29	1
30–34	2
35–39	2
40–44	4
45–49	3
50–54	4
55–59	1
60–64	6
65–69	0
70–74	2
75–79	1

ii

Mark	Frequency
0–9	1
10–19	1
20–29	3
30–39	4
40–49	7
50–59	5
60–69	6
70–79	3

iii

Mark	Frequency
0–19	2
20–39	7
40–59	12
60–79	9

iv

Mark	Frequency
0–39	9
40–79	21

v 46.7

vi 47.5

vii 48.2

viii 47.5

- c The smaller the class interval, the closer the approximation to the real mean.

8 a 75

b 26, 31 and 48

c 108

d 26

9 a 1680

b 952

c ~ 64.2

10 a 4.5

b Answers will vary.

c Answers will vary.

d Answers will vary.

e Answers will vary.

11 24, 26, 26

12 72 km/h

Lesson 12.2 Median

1 a i already ordered

ii 6

b i already ordered

ii 5

c i already ordered

ii 13

d i 27, 27, 27, 28, 29, 30, 30, 31, 31

ii 29

e i 0, 0, 1, 2, 4, 4, 4, 5, 5, 6, 7

ii 4

2 a i already ordered

b i already ordered

ii 5

ii 4

iii 8

iii 22

c i 48, 49, 49, 50, 50, 50, 51, 51, 53, 53, 55

ii 6

iii 50

3 a i already ordered

ii 21.5

b i already ordered

ii 56

c i already ordered

ii 4

d i 240, 243, 243, 244, 245, 246

ii 243.5

e i 101, 101, 102, 104, 104, 104, 105, 106

ii 104

4 a i already ordered

b i already ordered

ii 4.5

ii 5.5

iii 16.5

iii 50

c i 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 4, 5

ii 6.5

iii 2.5

d i 33, 34, 34, 35, 36, 36, 37, 37, 37, 38, 38, 38, 39, 40

ii 7.5

iii 37

5 a

x	f	Cumulative frequency	Positions of ordered values
3	2	2	1st, 2nd
4	3	5	3rd, 4th, 5th
5	7	12	6th, 7th, ..., 12th
6	4	16	13th, 14th, 15th, 16th
7	1	17	17th

Median = 5 (9th score)

b

x	f	Cumulative frequency	Positions of ordered values
12	4	4	1st, 2nd, 3rd, 4th
13	7	11	5th, 6th, ..., 11th
14	5	16	12th, ..., 16th
15	3	19	17th, 18th, 19th
16	2	21	20th, 21st

Median = 13 (11th score)

6 a 52

b 35.5

c 10

d 26

7 a 14

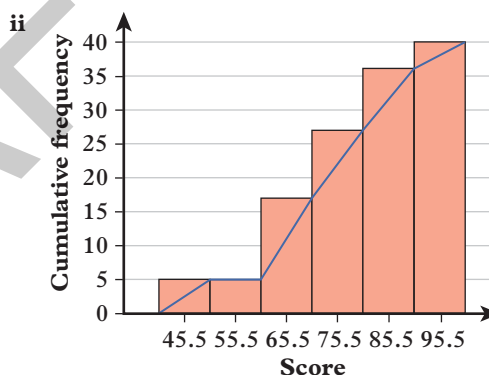
b 29

c 41

d 23

8 a i

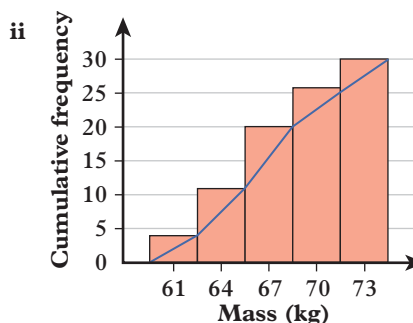
Score	Frequency	Class centre	Cumulative frequency
41–50	5	45.5	5
51–60	0	55.5	5
61–70	12	65.5	17
71–80	10	75.5	27
81–90	9	85.5	36
91–100	4	95.5	40



iii 74

b i

Mass (kg)	Frequency	Class centre	Cumulative frequency
59.5–<62.5	4	61	4
62.5–<65.5	7	64	11
65.5–<68.5	9	67	20
68.5–<71.5	6	70	26
71.5–<74.5	4	73	30

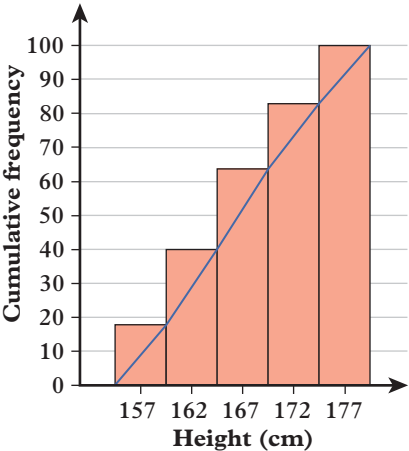


iii 67

c i

Height (cm)	Frequency	Class centre	Cumulative frequency
154.5–<159.5	19	157	19
159.5–<164.5	21	162	40
164.5–<169.5	24	167	64
169.5–<174.5	19	172	83
174.5–<179.5	17	177	100

ii

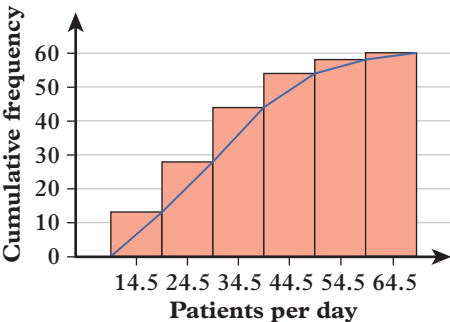


iii 167

d i

Patients per day	Frequency	Class centre	Cumulative frequency
10–19	13	14.5	13
20–29	15	24.5	28
30–39	16	34.5	44
40–49	10	44.5	54
50–59	4	54.5	58
60–69	2	64.5	60

ii



iii 32

9 a 72.5 b 45 c 120.5 d 21

10 a Mean = 50.4, mode = 54, median = 54

b

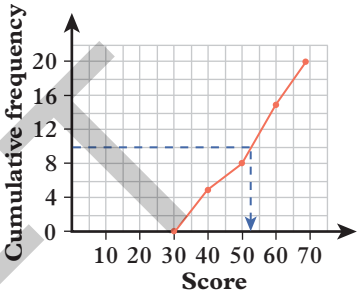
Scores	Frequency
30–39	5
40–49	3
50–59	7
60–69	5

c 50.5; reasonably close

d

Scores	Frequency	Cumulative frequency
30–39	5	5
40–49	3	8
50–59	7	15
60–69	5	20

e



The median is approximately 52.

This is smaller than the exact median of 54.

f The median found from a stem-and-leaf plot is exact, but the median found from grouped data is an estimate.

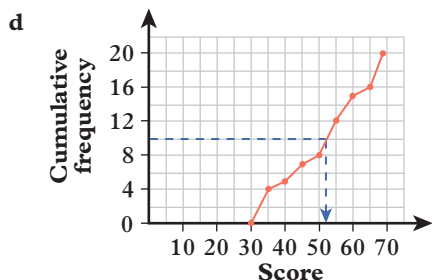
11 a

Scores	Frequency
30–34	4
35–39	1
40–44	2
45–49	1
50–54	4
55–59	3
60–64	1
65–69	4

b The mean is 50.25, which is close to 50.4.

c

Scores	Frequency	Cumulative frequency
30–34	4	4
35–39	1	5
40–44	2	7
45–49	1	8
50–54	4	12
55–59	3	15
60–64	1	16
65–69	4	20



Median = 52

- e** Smaller than the exact median
- f** The median of a stem-and-leaf plot is exact, but the median of grouped data is an estimate.
- 12 a** For this dataset, Mean = $12\frac{1}{3}$, Median = 8 and Mode = 5. Hence, the median lies between the mode and the mean.
- b** For this dataset, Mean = 4.6, Median = 5 and Mode = 3. Hence, the mean lies between the mode and the median.
- c** Example: 1, 2, 2, 3, 3, 4, 4, 4, 20
- d** Example: 1, 2, 2, 3, 3, 4, 4, 4, 10
- e** Example: 1, 3, 3, 3, 5
- 13** Smallest value = 31, largest value = 37

Lesson 12.3 Relative merits of mean, mode and median

- 1 a i** 7 **ii** Yes
- iii** Mean = 6.9 and median = 7 are both good measures.
- b i** No mode **ii** No
- iii** Mean = 5 and median = 4.5 are both good measures.
- c i** 35
- ii** No, not a central value
- iii** Mean = 29 and median = 26 are both good measures.
- 2 a i** 9.2
- ii** 29
- b i** Yes
- ii** No, it is affected by outlier 110.
- c i** The median = 9 is a good measure. The mode = 8 is not a central value.
- ii** The median = 9 is satisfactory. The mode = 8 is not a central value.
- 3 a** 5
- b** No, it is not a central value.
- c** The mode = 5 is not central. The mean = 4 is satisfactory.
- 4** Mean
- 5** Mode
- 6 a i** 6.25 **ii** 7 **iii** 6.5
- b** Mode, as it is the shoe size with highest sales.

- 7 a** \$2 030 000
- b** \$67 667
- c i** 25 **ii** 5
- d** \$55 000
- e** \$50 000
- f i** Mean **ii** Mode
- g** Median: half the employees earn more and half earn less
- 8 a** Mean = 23.8, mode = 29, median = 23.5
- b** Mean and median are central and typical. Mode is not central but is important because more people had to wait this length of time than any other.
- 9 a** Mean = 4.1, mode = 6, median = 4.5
- b** Mode
- 10 a** Mean = \$640 000
- b** The mean has been affected by the high price of \$960 000. No mode. The median = \$560 000, so it is the most appropriate.
- 11** Mode
- 12 a** Results are summarised in part **c**.
- b** Results are summarised in part **c**.
- c**

	Class A	Class B	Class C	Class D	Class E
Mean	4.05	4.8	4.3	5.05	5.4
Mode	3	1, 8	0, 1, 9	8	0, 8, 9
Median	3.5	5	4.5	5.5	6.5

- d** Mode **e** Mean
- 13 a** 157 is an outlier possibly caused by an error in measurement or recording of measurement.
- b** 56.3 kg (without), 64.7 kg (with)
- c** Median = 56 kg, mode: 48, 55, 56 kg
- d** Median or mean without outlier
- 14 a i** 1.3 **ii** 0 **iii** 1
- b** Median or mean; number of injuries of main concern and need to be minimised
- 15 a**
- | Leaf: Barbie | Stem | Leaf: Hidden Figures |
|---------------|------|----------------------|
| 9 8 5 | 6 | |
| 9 8 6 5 3 2 0 | 7 | |
| 9 8 7 5 4 2 0 | 8 | 0 3 4 5 5 6 7 7 8 9 |
| 2 1 0 | 9 | 0 0 1 1 2 2 3 3 4 5 |
- Key: 8 | 0 = 8.0
- b** Barbie: mean = 7.965, median = 7.95, no mode
Hidden Figures: mean = 8.875, median = 8.95, mode = 8.5, 8.7, 9, 9.1, 9.2, 9.3
- c** Barbie: Both the mean and the median are appropriate as they are central.
Hidden Figures: Both the mean and the median are appropriate as they are central. We have multiple modes so they are not appropriate.

16 a Example: from question 2 i: 1, 8, 8, 9, 10, 11

b Example: mean: 9.2 (without), 7.83 (with)

c Not affected by the outlier

Lesson 12.4 Range and interquartile range

1 a 8 b 7 c 159 d 4

e 7 f 7 g 8 h 5

2 a 4 b 5 c 4

3 $Q_1 = 8$, $IQR = 13 - 8 = 5$

4 a 7 b 7

5 $Q_1 = 44.5$, $Q_3 = 66$, $IQR = 66 - 44.5 = 21.5$

6 a 25 b 6.5

7 $Q_2 = 21$, $Q_1 = 19$, $Q_3 = 24$, $IQR = 5$

8 a 6 b 19

9 $Q_1 = 4.5$, $Q_3 = 9$, $IQR = 4.5$

10 a 6.5 b 3

11 a 3 b 2.5 c 2 d 2

12 a $IQR \approx 3$ b $IQR \approx 1$

c $IQR \approx 2$ d $IQR \approx 20$

13 a $IQR \approx 1$ b $IQR \approx 2$ c $IQR \approx 1$

14 See answers to question 8 in Lesson 12.2 for the corresponding cumulative frequency polygons.

a $IQR \approx 20$ b $IQR \approx 6$

c $IQR \approx 10$ d $IQR \approx 20$

15 The IQR is smaller for the results for Lara's class, so there is less spread than for Linda's class.

16 a Tom b Matthew c Matthew

17 No, as the individual heights are not given.

18 a Set B b Range, mean

19 Example: 3, 5, 6, 8, 9, 9, 12, 12, 12, 14

Lesson 12.5 Box plots

1 a i 83 ii 35 iii 48 iv 60

v 75 vi 50 vii 25

b i 75 ii 40 iii 35 iv 54

v 65 vi 45 vii 20

c i 78 ii 36 iii 42 iv 61

v 73 vi 45 vii 28

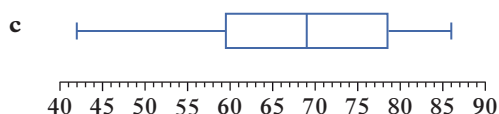
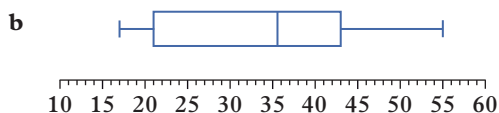
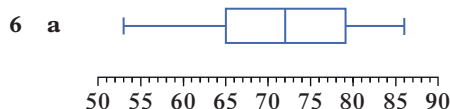
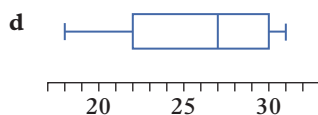
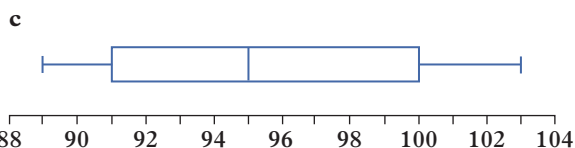
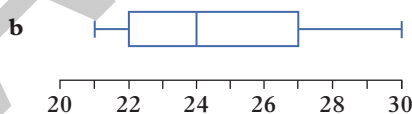
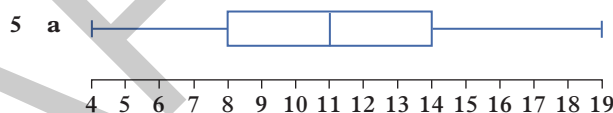
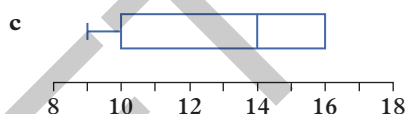
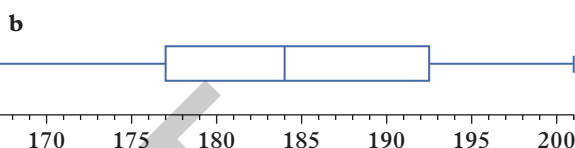
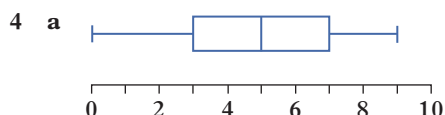
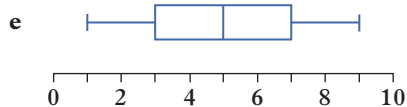
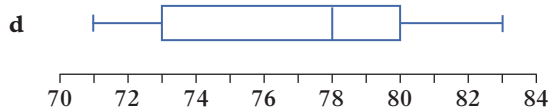
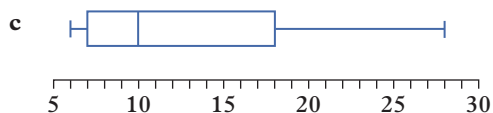
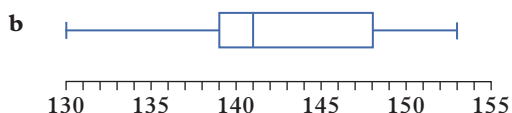
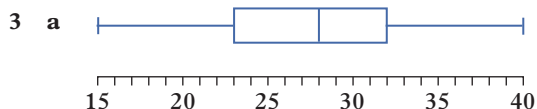
d i 76 ii 46 iii 30 iv 59

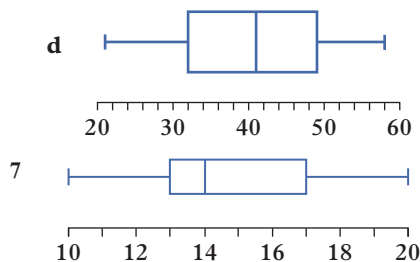
v 69 vi 56 vii 13

2 a $\text{Min} = 0$, $Q_1 = 3$, $Q_2 = 5$, $Q_3 = 7$, $\text{Max} = 9$

b $\text{Min} = 167$, $Q_1 = 177$, $Q_2 = 184.5$, $Q_3 = 192.5$, $\text{Max} = 201$

c $\text{Min} = 9$, $Q_1 = 10$, $Q_2 = 14$, $Q_3 = 16$, $\text{Max} = 16$





Lesson 12.6 Interpreting box plots

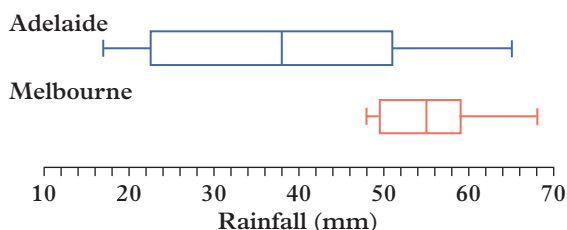
- The last 25% of scores are more spread out than the first 25% of scores.
- Positively skewed. The last 25% of scores are more spread out than first 25% of scores.
 - Symmetric. There is an even spread of scores.
 - Negatively skewed. The first 25% of scores are more spread out than last 25% of scores.
 - Symmetric. There is an even spread of scores.
 - Positively skewed. The last 25% of scores are more spread out than first 25% of scores.
 - Negatively skewed. The first 25% of scores are more spread out than last 25% of scores.

3

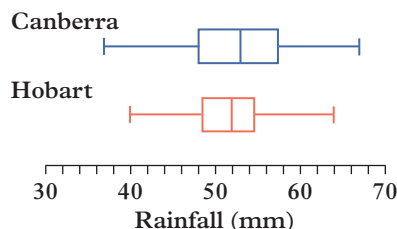
	Shape	Q_1	Q_2	Q_3	IQR
a	Symmetric	23	28	32	9
b	Positively skewed	7	10	18	11
c	Symmetric	3	5	7	4
d	Symmetric	8	11	14	6
e	Positively skewed	22	24	27	5
f	Negatively skewed	22	27	30	8
g	Negatively skewed	65	72	79	14
h	Positively skewed	21	30	43	22
i	Negatively skewed	60	69	79	19
j	Symmetric	20	25	30	10

- 6800 s
 - 400 s
 - 8200 s
 - 9500 s
 - 8100 s
 - The ranges are the same.
 - Males
 - Males
- Adelaide: Median = 38, $Q_1 = 22.5$, $Q_3 = 51$;
Melbourne: Median = 55, $Q_1 = 49.5$, $Q_3 = 59$

Average monthly rainfall

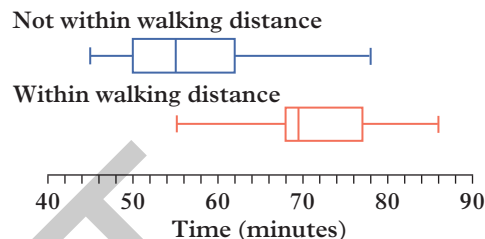


6 a Average monthly rainfall



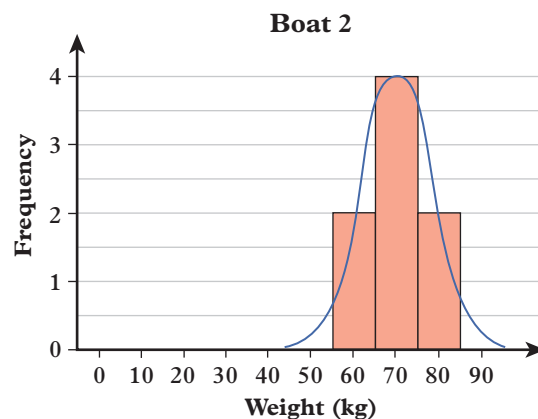
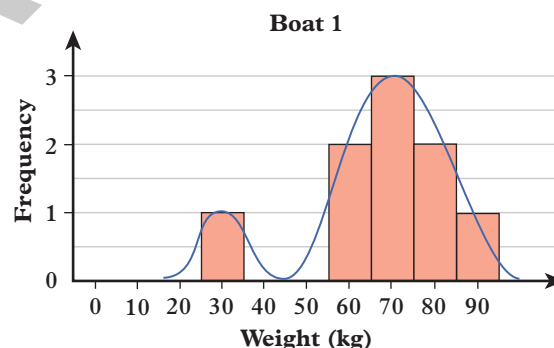
- Canberra has a greater spread of rainfall than Hobart. The two cities have similar rainfall in the middle 50% of values.

7 a Minutes of movement



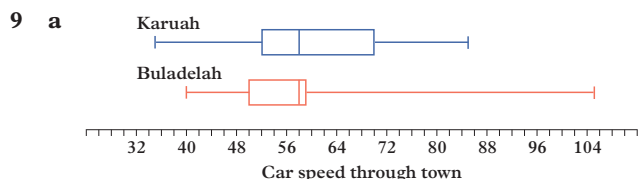
- 50% of students who do not live within walking distance of their school spend less time moving each day than any of the students who live within walking distance of their school. About 25% of students who live within walking distance of their school spend more time moving each day than any of the students who do not live within walking distance.

8 a



- Weights for boat 1 show a bimodal distribution and weights for boat 2 are symmetrically distributed around 70 kg.

- c Boat 1, as there is an outlier of 30 kg, which represents the weight of the cox.



- b Karuah has more drivers exceeding the speed limit and so more drivers would be fined for speeding. Buladelah has fewer speeding drivers but the highest speed is very dangerous, so it would be targeted for safety.

10 40.3%

Lesson 12.7 Clusters and outliers

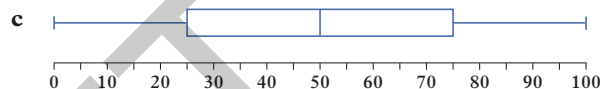
- 1 a 3 b 21
c 17 d No outlier
- 2 a 3
b 12, 12, 13, 15, 15, 15, 18 are all outliers.
- 3 a i $Q_1 = 18$, $Q_2 = 26$, $Q_3 = 29$, IQR = 11
ii 1.5, 45.5
iii 59
b i $Q_1 = 23$, $Q_2 = 35$, $Q_3 = 43$, IQR = 20
ii -7, 73
iii No outlier
c i $Q_1 = 56.5$, $Q_2 = 67.5$, $Q_3 = 75$, IQR = 18.5
ii 28.75, 102.75
iii No outlier
d i $Q_1 = 106$, $Q_2 = 110$, $Q_3 = 119$, IQR = 13
ii 86.5, 138.5
iii 83
e i $Q_1 = 69$, $Q_2 = 76$, $Q_3 = 88$, IQR = 19
ii 40.5, 116.5
iii No outlier
f i $Q_1 = 21$, $Q_2 = 38$, $Q_3 = 60$, IQR = 39
ii -37.5, 118.5 iii No outlier
- 4 a Clustered around 27
b Clustered around 38
c Clustered around 75
d Clustered around 107
e Clustered around 68
f Clustered around 20
- 5 a No, limits for outliers: 9.5, 45.5
b No, limits for outliers: -9.5, 34.5
c No, limits for outliers: -3, 13
d No, limits for outliers: -1, 23
e No, limits for outliers: 14.5, 34.5
f No, limits for outliers: 10, 42
g No, limits for outliers: 44, 100
h No, limits for outliers: -12, 76

- i No, limits for outliers: 31.5, 107.5

- j Yes, limits for outliers: 5, 45

- 6 a Yes b No c No
d Yes e Yes f No
- 7 a Lower limit for outlier = $165 - 8.5 \times 1.5 = 152.25$
b There is probably an error in measuring or recording the correct height.
- 8 a i 163.1 cm b i 169.5 cm
ii 169.6 cm ii 170 cm
iii 169.3 cm iii 169.5 cm
c i 112 d i 8.5
ii 17 ii 8
iii 17 iii 8
e Ignore the outlier because it is an obvious error. Mean = 169.6, median = 170 and IQR = 8

- 9 a Bimodal
b $Q_1 = 27$, $Q_2 = 50$, $Q_3 = 74$



- d Symmetric
e Clusters around 20–<30 and 70–<80 are shown in the histogram, but are not apparent in the box plot.

10 40

Lesson 12.8 Standard deviation

- 1 a 6, 1.4 b 6, 2.8
c 6, 4.2 d 28, 2.6
e 90, 1.4 f 250, 5.6
g 6, 2.5 h 5, 0
- 2 a 3.72, 1.22 b 16.9, 1.11
c 1.79, 1.16 d 122.96, 1.37
- 3 a 66.85, 3.35 b 72.83, 12.89
c 31.67, 13.30 d 165.93, 5.57
- 4 a 4.5, 1.26
b i 9.5, 1.26
ii $(4.5 + 10)$, 1.26
c i 22.5, 6.29
ii (4.5×10) , (1.26×10)
- 5 a 20, 4 b 85, 20 c 12, 4
- 6 The scores are all the same.
- 7 Girls: 1.45, boys: 1.77. Girls have a smaller spread of data values.
- 8 Suburb A: mean = 2, s.d. = 1.1
Suburb B: mean = 0.6, s.d. = 0.9
Suburb B has a smaller mean and smaller spread of data.
- 9 Coworker 1: mean = 29.8, s.d. = 3.6
Coworker 2: mean = 31, s.d. = 4.24
Coworker 2 has a slightly bigger mean and spread of data.

- 10 a** i 32
ii 21
iii 10.9
- b** i 52
ii 21
iii 15.6
- c** The range is most affected by the change from 90 kg to 110 kg, which is an outlier.
The IQR is unchanged by the addition of an outlier as it only considers the middle 50% of a dataset.
The standard deviation increased due to the data being more spread out from the mean.
- 11 a** i Range: machine 1 is 10, machine 2 is 10.
ii Interquartile range: machine 1 is 0, machine 2 is 10.
iii Standard deviation: machine 1 is 3.5, machine 2 is 4.2.
- b** Around 60
- c** Machine 1 as it has a lower standard deviation, with data values clustered around 60.
- 12 a** i Carrie: 6; Laura: $6\frac{1}{9}$
ii Carrie: 6; Laura: 5
- b** Carrie 1; Laura 5
- c** i Carrie: 5; Laura: 12
ii Carrie: 3.5; Laura: 3
iii Carrie: 1.76; Laura: 3.45
- d** Laura won more holes and had a lower median.
- e** Carrie played fewer strokes and her mean was lower.
- 13 a** Jimena: 557; Alison: 575
- b** i Jimena: 69.6; Alison: 71.9
ii Jimena: 72; Alison: 72
- c** 6
- d** i Jimena: 37; Alison: 16
ii Jimena: 8; Alison: 6.5
- e** Jimena: 10.7; Alison: 4.8
- f** Alison
- g** Jimena had higher marks than Alison in 6 of the 8 tests. Alison had a higher mean and lower standard deviation.
- 14 a** i 1.3
ii 31.2
- b** Outliers have a large effect on the standard deviation.
- c** i Range = 4, IQR = 2
ii Range = 85, IQR = 2
- d** Range not IQR
- 15** Answers will vary.
- 16 a** Use a calculator or spreadsheet to find the mean, 15, and the standard deviation, 2.
- b** Lowers, 1.83
- c** Lowers, 1.97
- d** The closer to the median new data values are added, the smaller the standard deviation becomes.

Using spreadsheets

- 1 a** Answers will vary.

- b** The COUNT function counts how many of the highlighted cells contain data.
- c** To find the mean of data contained in cells A1 to A9, for example, use the formula
=SUM(A1:A9)/COUNT(A1:A9)
- 2** Answers will vary.
- 3** Answers will vary.

Lesson 12.9 Review: Data analysis

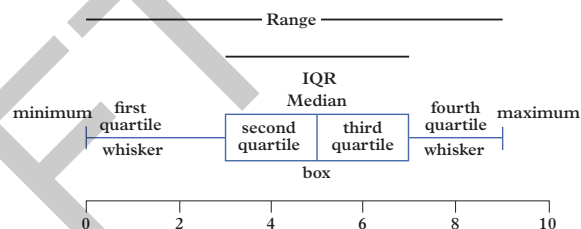
Review questions 12.9A

- 1 a** Median **b** Mode **c** Mean

2

Measure of centre	Measure of spread
mode	range
modal class	standard deviation
mean	interquartile range
modality of a dataset	
median	

3



4

- Two box plots on the same axes; to compare datasets.

Review questions 12.9B

- 1 D** **2 B** **3 C** **4 C** **5 A** **6 D**
7 C **8 C** **9 B** **10 D** **11 D** **12 B**
13 C **14 A** **15 B** **16 D** **17 A** **18 B**

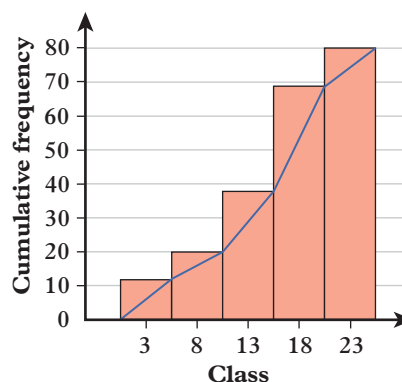
Review questions 12.9C

- 1** Mean = 7.7, mode = 6, median = 8, range = 4

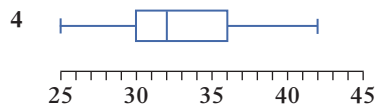
2 a

Class centre	Cumulative frequency
3	12
8	20
13	38
18	69
23	80

b



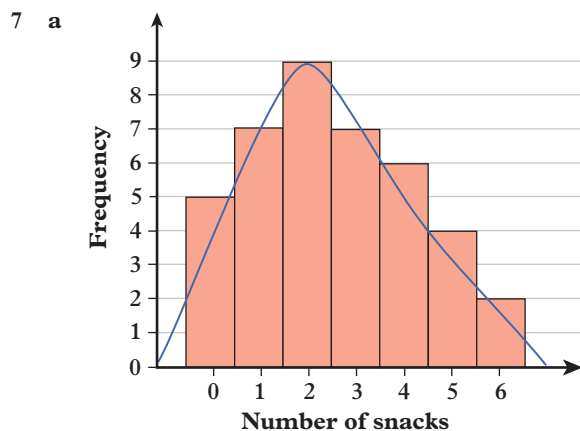
- c i 16 ii 10.5 iii 9.5
- 3 a i 9, 11, 12 b i 9, 11, 12.5
ii 3 ii 3.5
- c i 9, 11.5, 12 d i 9.5, 11.5, 13.5
ii 3 ii 4



- 5 20.6, 2.5
- 6 a 7.1 b 7 c 1.5
- 7 a i \$318 333 ii \$285 000
b \$550 000
c The median is better, as it is not affected by the outlier of \$550 000.
- 8 a i Town A: 25; Town B: 26
ii Town A: 26.5; Town B: 26.5
b i Town A: 14; Town B: 23
ii Town A: 8; Town B: 12
iii Town A: 4.7; Town B: 7.1
c Town A: negatively skewed, cluster above median
Town B: symmetric, no clustering
- 9 a Negatively skewed b No outliers

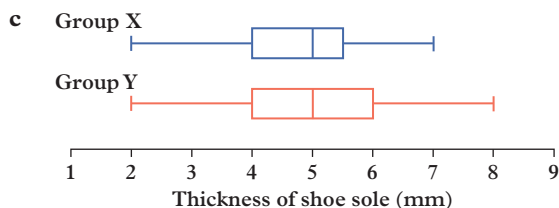
Review questions 12.9D

- 1 Mean = 86.65, mode = 87, median = 87, range = 3
- 2 a i Mean = 24, mode = 5, median = 5
ii The mean has been affected by the outlier 100. The mode and median are appropriate measures.
b i Mean = 5.1, mode = 4, median = 5
ii The mode is not a central score. The mean and median are appropriate measures.
c i Mean = 2.5, mode = 1, median = 1
ii The mode and median are not central scores. The mean is an appropriate measure.
- 3 a 68 b 90 c 57 d 33
- 4 Mode 5 4.49, 0.14
- 6 a 26.3 b 26 c 9
d 6 e 3.07



- b Slightly positively skewed with mode of 2 snacks

- 8 a i Group X: 4.6, Group Y: 5.2
ii Group X: 5, Group Y: 5
b i Group X: 5, Group Y: 6
ii Group X: 1.2, Group Y: 1.6

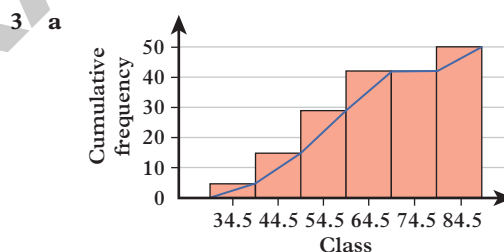
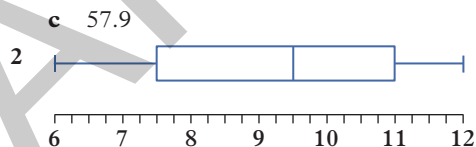


- d This is generally the same for the lower 50%, but group Y students have more shoes that last longer.

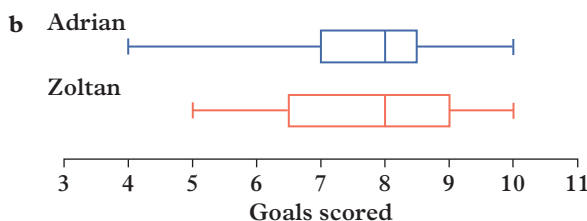
Review questions 12.9E

- 1 a 50–59
b

Class	Class centre	Frequency
30–39	34.5	5
40–49	44.5	10
50–59	54.5	14
60–69	64.5	13
70–79	74.5	0
80–89	84.5	8



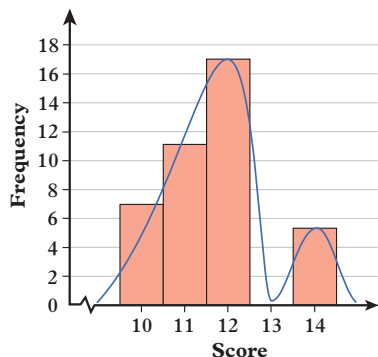
- b i 47 ii 66 iii 19 iv 57
- 4 a i Adrian: 7.6; Zoltan: 7.6
ii Adrian: 8; Zoltan: 8
iii Adrian: 6; Zoltan: 5
iv Adrian: 1.5; Zoltan: 2.5
v Adrian: 1.4; Zoltan: 1.5



- c The scores for Adrian are negatively skewed and clustered between 7 and 8.5. The scores for Zoltan are almost symmetric.

- d Zoltan has more games with a higher number of goals than Adrian. Zoltan's lowest number of goals per game was 5, whereas Adrian's was 4.

5 a



- b Distribution is bimodal.

6 a

Stem	Class A
3	6
4	3
5	0 8 8 9
6	0 1 1 3 5 6
7	4 7
8	8

Key: 3 | 6 = 36

- b Distribution is symmetrical.

c i 52

ii 58 and 61

iii 61

- 7 a Min = 3, $Q_1 = 5.5$, Median = $Q_2 = 6.5$, $Q_3 = 7$, Max = 8

- b Lower limit = $Q_1 - 1.5 \times \text{IQR} = 5.5 - 1.5 \times 1.5 = 3.25$. Any score below 3.25 is an outlier, so 3 is an outlier. Upper limit = $Q_3 + 1.5 \times \text{IQR} = 7 + 1.5 \times 1.5 = 9.25$. So, 8 is not an outlier.

Review questions 12.9F

- 1 Mean = 3.9, Mode = 5, Median = 4, Range = 4, Standard deviation = 1.37

2 a

Class B	Stem	Class A
6 2	0	8
8 6 2 0	1	4 6 8
8 8 6 4 4 4 2 2 0	2	0 2 4 4 6 6 8
8 6 6 4	3	0 0 2 2 4 6 8
	4	0

Key: 1 | 4 = 14

- b i Class A: 26.2, Class B: 22.4

ii Class A: 26, Class B: 24

iii Class A: 32, Class B: 36

iv Class A: 8.3, Class B: 9.9

- c Class A: slightly negatively skewed

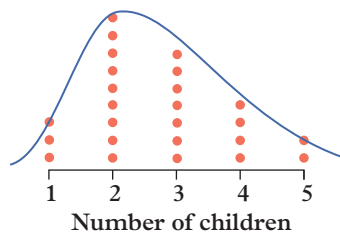
Class B: almost symmetric, slight negative skew

- d No outliers

- e Class A had more high scores than class B, and Class A had the highest score.

- 3 Range = 5, IQR = 2.5

4 a



- b Positively skewed

- c There are more families with a smaller number of children.

- 5 a i 25 ii 30 iii 32
iv 36 v 42

- b Positively skewed

- c No outliers, as the data is clustered between 30 and 32.

- 6 a Median = 96, $Q_1 = 88$, $Q_3 = 103$, IQR = 15

- b Lower limit = 65.5, Upper limit = 125.5

- c Since $50 < 65.5$, it is an outlier.

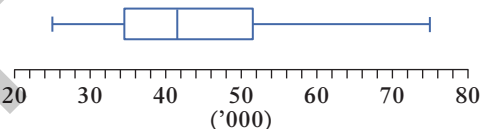
Review questions 12.9G

- 1 a i 10 ii 11 iii 14

- b 9.1

- c i \$34 500, \$41 500, \$51 500

ii



- iii No outliers, as all incomes sit within the interval \$9000 and \$77 000.

- iv Approximately \$13 425

- d i 80 ii 30

Review: Statistics

Review questions

- 1 a i Discrete

- ii Nominal

- iii Continuous

- b Select 10 numbers randomly from 120.

- c A sample that is biased is one in which data has been unfairly influenced by the collection process and is not truly representative of the whole population.

- d i Assign numbers 1 to 27 to the workers. Use a calculator, spreadsheet or a random number generator to choose 4 random numbers from 1 to 27.

- ii For example: seniority, time spent working at the company, age

- e The proportions are provided as percentages, correct to one decimal place.

Year	7	8	9	10	11	12
Number of students	18.5%	19.7%	23.3%	16.4%	12.9%	9.3%

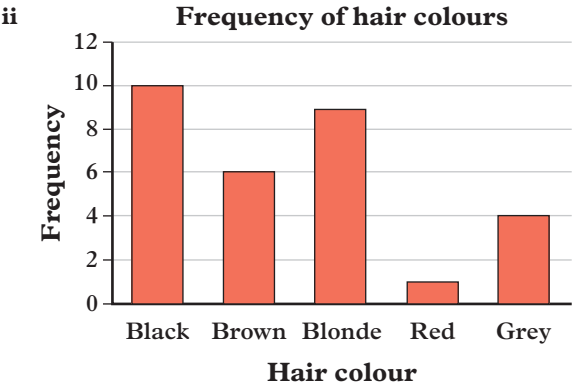
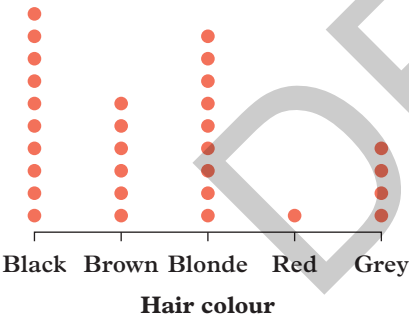
- f i 46% ii 320
- iii The survey could have included leading questions that biased the participants in one direction or the other.
- The questions could have been written in complex or confusing language resulting in participants misunderstanding their meaning.

- g i 18 ii 9
- iii 12 iv 45

2 a

Hair colour	Number	Relative frequency
Black	10	$\frac{10}{30} = \frac{1}{3}$
Brown	6	$\frac{6}{30} = \frac{1}{5}$
Blonde	9	$\frac{9}{30} = \frac{3}{10}$
Red	1	$\frac{1}{30}$
Grey	4	$\frac{4}{30} = \frac{2}{15}$

- b 30
- d i $\frac{1}{3}$
- e i
- c Black ii $\frac{3}{10}$



3 a 8

b 26

4 a

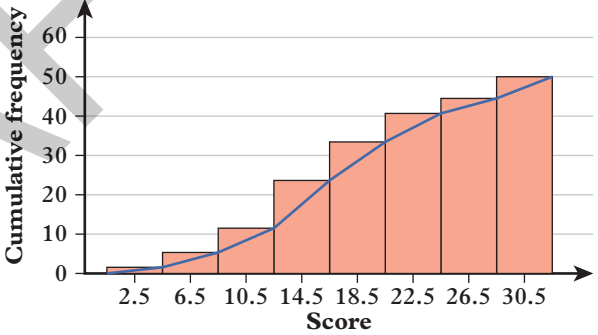
Class B	Stem	Class A
7	0	
7 5 1	1	3 9
9 9 9 7 7 5 3 1	2	1 3 5 7 9 9
9 3 3 1	3	1 1 3 5 5 7 7 9
3 1 1	4	1 3 5

- b Class A is better because it had more high scores than Class B.

5 a

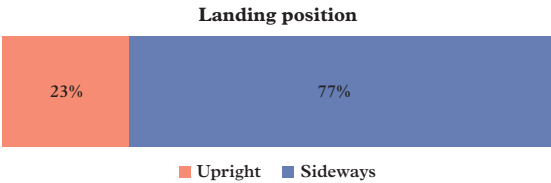
Score	Class centre	Frequency	Cumulative frequency
1–4	2.5	1	1
5–8	6.5	4	5
9–12	10.5	7	12
13–16	14.5	12	24
17–20	18.5	9	33
21–24	22.5	8	41
25–28	26.5	4	45
29–32	30.5	5	50

b



6 a 23%

b



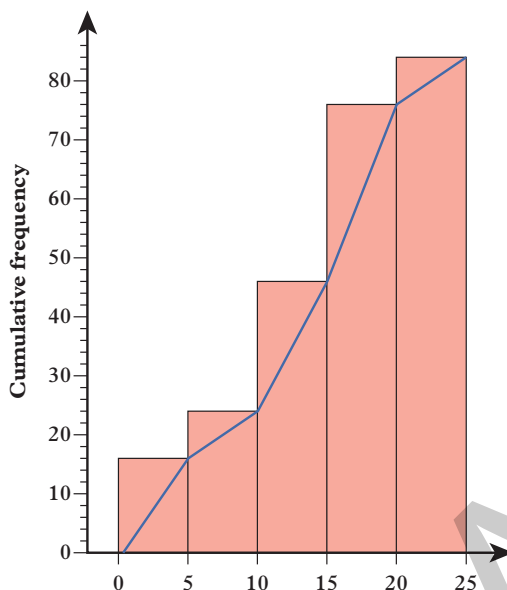
- c i Nominal ii Discrete
- d Sector graph, column graph
- e A line graph would not be suitable for this dataset. The labels on the horizontal axis would be “Upright” and “Sideways”, which are nominal variables. The line connecting the two points would have no meaning, as there is no reasonable interpretation of the tin landing somewhere between “Upright” and “Sideways”.

- 7 a Mean = 7.4, Mode = 8, Median = 8

b i

Class	Frequency	Cumulative frequency
$0 \leq 5$	16	16
$5 \leq 10$	8	24
$10 \leq 15$	22	46
$15 \leq 20$	30	76
$20 \leq 25$	8	84

ii



iii 21, 42, 63

iv IQR ≈ 10

c i 8, 10, 11, 12, 13

ii 2

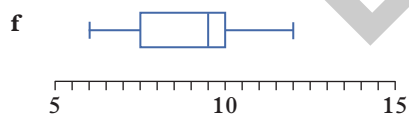
Stem	Leaf
4	8 8 9
5	0 1 4 6 6 7 8 9
6	0 2 4 5 5
7	1 3 4 8

e i Range = 30

ii 48, 56, 65

iii 58.5

iv 12.5

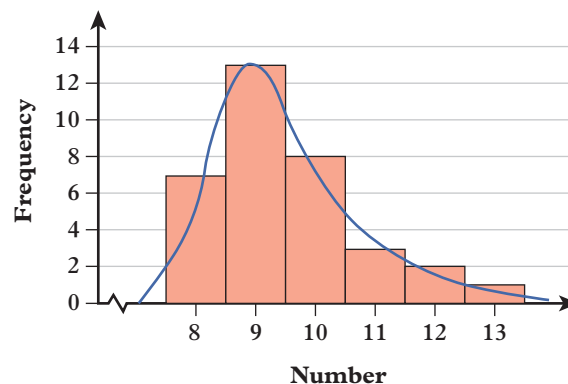


g i Mean = 16.3, Range = 2.9

ii Mean = 56.5, Range = 14.5

h Mean = 3.778, Mode = 3 and 5, Median = 3, Range = 4, $\sigma \approx 1.397$

8 a



The distribution is positively skewed, and hence not symmetric.

b The range of this dataset is 11; however, there is a gap in the data with a small cluster of marks between 9 and 11, which are separated from the main cluster of marks that lie between 13 and 20.

c Both distributions are unimodal. The distribution in part b might have been considered multimodal if the modal class was not so clearly more frequent than the other classes.

9 a

Stem	Leaf
1	6
2	3
3	0 6 8 8 9
4	0 1 1 3 5
5	4 7
6	8

Stem	Leaf
1	6
2	3
3	0 6 8 8 9
4	0 1 1 3 5
5	4 7
6	8

b Symmetric

10 a 6900 s

b 8100 s

c 8200 s

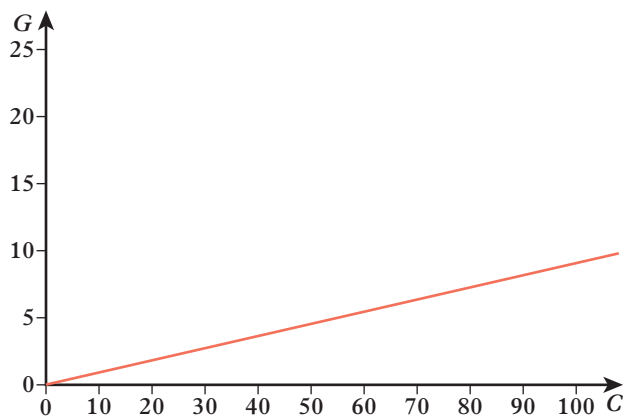
d Women

e Men

Exam revision

Revision questions

- 1 9.283×10^{11}
- 2 **a** $L = Q_1 - 1.5(Q_3 - Q_1) = 2.5Q_1 - 1.5Q_3$
b $Q_3 = 31$
- 3 $b = \frac{2A}{h} - a$
- 4 **a** $s = 2.41$ m
 Even though the quadratic equation has two solutions, only the positive one makes sense in the context of side lengths of a pentagon.
b $SA = 80.25 \text{ m}^2$
- 5 **a** 17.5%
b \$800
c Lucas earns \$100 more than his expenses per fortnight.
d **i** \$270 **ii** 6 h/week
- 6 **a** \$25
b $G = \frac{1}{11}C$, which is a direct variation relationship with a constant of proportionality of $\frac{1}{11}$.
c The gradient is $\frac{1}{11}$.



- 7 **a** \$9848.89 **b** 30%
- 8 **a** \$1057 **b** Yes
- 9 771.43 km/h
- 10 **a** The gradient is $m = 0.3$, which represents the increase of 30 cents in the amount of tax payable for every increase of \$1 in taxable income.
b $y = 0.3x - 9212$
c \$62 500

Glossary

pay-as-you-go (PAYG) tax

system for making tax instalments which are removed from gross pay towards the expected income tax liability for that financial year

allowance (employment)

once-off payment paid to an employee for a specific reason, typically determined by the industry award

annual leave loading

extra payment added to the holiday pay of employees taking their annual holidays

annulus

the area between two circles with the same centre but different radii

average speed

calculated by dividing the distance travelled during a time interval by the length of the time interval; e.g. average speed of a car that travels 120 km in 2 hours is 60 km/h

award

legal document that specifies the conditions under which employees in different industries can be expected to work, including minimum pay rates

axis of rotation

the straight line through an object in three dimensions, around which the object rotates; e.g. Earth rotates around the straight line with endpoints at the North and South Poles

base unit

fundamental quantity used to describe all other quantities of the same type

bell-shaped

description of a distribution graph for a dataset that is normally distributed; also see *normal distribution*

bias

systematic prejudice that means certain outcomes or groups are unfairly favoured (whether knowingly or otherwise)

bimodal

description of a distribution graph for a dataset with two modes

blood alcohol content (BAC)

percentage concentration of alcohol in a person's blood; e.g. a BAC of 0.05 (%) is 0.05 g of alcohol in every 100 mL of blood

bonus

extra payment paid as an incentive to employees who work hard

box plot (box-and-whisker plot)

data display consisting of a box and two "whiskers" that show the five-number summary for a dataset; also see *five-number summary*

braking distance

distance a motor vehicle travels between the time the brakes are first applied and when the vehicle comes to a complete stop

budget

financial plan listing all expected income and expenses over a certain time period; aims to ensure expenses remain less than or equal to income

buy on terms

to make an initial deposit on an item, and then pay the remaining amount in regular instalments

buy now, pay later

service that allows a consumer to purchase a product and pay the cost in instalments; interest is not charged but there are usually additional costs such as account set-up fees and late payment fees

capacity

the amount that a container can hold

categorical variable

variable that can take values from exactly one of a number of categories; usually describes a quality or a characteristic

census

collection of data from every member or element in a population

circumference

perimeter of a circle

class centre

average value of the upper and lower bounds for a class interval

classes (class intervals)

equal-sized ranges of numerical data with no overlap that, together, cover all possible values

cycle

walk in a network that starts and ends at the same vertex, and where every other vertex is unique; sometimes called a *closed path*

cluster

group of data points concentrated around a particular value

column graph

data display consisting of vertical bars for each category, value or class interval, with gaps between each bar; can also be displayed as a horizontal bar graph

commission

payment for services determined as a percentage of the associated sale price

composite shape

shape made up of two or more simple shapes

cone

3D object with a circular base and a curved surface connecting the base to a single point (the apex)

connected network

network in which there is a walk between every pair of vertices

consecutive integers

whole numbers that follow one another in order, with a difference of 1 between them; e.g. 7 and 8, or -4 , -3 and -2

constant of variation (or proportionality)

value $k = \frac{y}{x}$ (for $x \neq 0$) that is constant when a direct variation relationship exists between y and x ; also the gradient of the graph $y = kx$ relating y and x

continuous variable

type of numerical variable that can take any value within an interval, usually associated with measurement; e.g. heights of students

coordinated universal time (UTC)

reference time used for standardising clocks and time zones, with 1 hour difference for every 15° longitude east or west of the Prime Meridian; previously Greenwich Mean Time (GMT)

cost

price paid to acquire, produce, accomplish or maintain anything

cross-section

the shape of the face formed when a 3D object is sliced by a plane

cumulative frequency histogram

histogram displaying the cumulative frequency of grouped data using bars the width of each class interval

cumulative frequency polygon (ogive)

type of line graph formed by connecting the upper right-hand corners of each bar in a cumulative frequency histogram

cycle

walk in a network that starts and ends at the same vertex, and where every other vertex is unique; sometimes called a *closed path*

cylinder

3D object with two circular faces of equal radius which are connected by a curved surface

data

pieces of collected information that can be interpreted and analysed

daylight saving time (DST)

the practice of setting clocks forward one hour from standard time during the summer months (from October to April in Australia)

deduction

expense incurred by a worker in relation to their job or profession which can be subtracted from total income to obtain the taxable income

degree

(of a vertex) the number of edge ends connected to a vertex

dependent variable

variable whose value depends upon that of another; usually represented on the vertical axis of a graph; e.g. for the relationship $y = 2x + 1$, the dependent variable is y (as it depends on the value of x)

deposit

an initial payment made when purchasing something, with the remaining amount to be paid later

diameter

the length of a line segment between two points on the circumference of a circle which passes through the centre and is equal to double the radius

dijkstra's algorithm

a method for finding the shortest path in a connected weighted network

direct variation relationship

relationship between two quantities or variables in which one of the quantities can be expressed as a constant multiplied by the other quantity

directed network

a network in which the edges have a direction; the orientation of the edge is represented by an arrow

directly proportional

two quantities or variables are directly proportional when an increase (or a decrease) in one causes a proportional increase (or decrease) in the other; e.g. y is directly proportional to x if $\frac{y}{x}$ is a constant value

disconnected network

a network that is not connected

discrete variable

type of numerical variable that takes exact values, usually resulting from counting; e.g. number of pets in a family

discretionary spending

inessential or non-mandatory expenses that can be adjusted; e.g. entertainment, clothes and gifts

divided bar chart

data display representing the relative frequency of different sets or categories in proportionally sized subsets of a horizontal bar

dot plot

display showing frequency of data using one dot per data point, arranged in columns above the observed category labels or values

double-time

penalty rate where employees are paid two times their normal hourly rate

edge

line between two vertices in a network, often indicating a relationship; sometimes called an arc

equation

two algebraic expressions related to each other using an equals sign; e.g. $2x + 1 = 7$ or $3x^2 = 5x - 2$

equator

great circle that is a parallel of latitude and whose centre is the centre of Earth

ethics

moral principles that guide the way that one should act in a given situation

expression

numbers and pronumerals connected using operations such as addition, subtraction, multiplication and division but not including an equals sign

extrapolation

calculation based on known information and additional assumptions used to make predictions or estimate unknown information outside the range of known values

five-number summary

five key values for a dataset; lowest score, highest score, median, lower quartile and upper quartile

fixed spending

essential or mandatory expenses that cannot be varied; e.g. rent or board

forest

a network with no cycles

formula

relationship or rule between two or more variables that contains an equals sign; e.g. $v = u + at$

frequency distribution table

table that displays the number of times each type of data point occurs

frequency polygon

type of line graph made by connecting the midpoints of the tops of each bar in a frequency histogram, with the first and last points joined to the horizontal axis

government payment

form of income paid by the government to an individual who satisfies certain eligibility criteria

gradient (slope)

steepness of a line (defined as m) that can be measured using any two points

on the line or interval; $m = \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

where (x_1, y_1) and (x_2, y_2) are points on the line

great circle

largest possible circle on the surface of a sphere; for example, Earth's equator

prime meridian

(Greenwich Meridian)

reference line for longitude (0°) that passes through Greenwich in London, England

gross pay

total amount of money earned by an employee before any deductions including PAYG tax are taken out

GST (goods and services tax)

indirect tax levied at a flat percentage (rate) of 10% on most goods and services in Australia

histogram

data display that represents the distribution of numerical data across multiple class intervals, with each class interval represented by a vertical bar of varying heights; similar to a column graph but without gaps between the columns

hypotenuse

longest side in a right-angled triangle; always opposite the right angle

income tax

tax paid on taxable income; amount of tax paid increases as the taxable income increases

indegree

number of edges pointing towards a vertex in a directed network

independent variable

variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship $y = 2x + 1$, the independent variable is x

infographic

chart, diagram or illustration used to present statistical information in a visually striking way, often seen in the media and advertising

International Date Line

line on Earth's surface from the North Pole to the South Pole where the date changes; located approximately along 180°E meridian of longitude

interpolation

calculation based on known information and additional assumptions used to make predictions or estimate unknown information within the range of known values

interquartile range

measure of spread found by calculating the difference between the upper quartile and lower quartile; that is, $IQR = Q_3 - Q_1$

kruskal's algorithm

a method for finding the minimum spanning tree of a connected, weighted network by growing a forest into a minimal spanning tree

latitude

angular distance of a position on Earth's surface north or south of the equator

layby

payment option where the consumer pays for goods in two or more instalments but does not receive the goods until payment is complete

leaf

last digit in a piece of numerical data, as displayed in a stem-and-leaf plot; e.g. for 256, the leaf would be 6

line graph

data display consisting of a point for each category, value or class interval, and line segments connecting adjacent points

linear equation

equation in which the highest positive integer power of the pronumeral is 1; e.g. $2x + 1 = 7$ is a linear equation but $3x^2 = 5x - 2$ is not because its highest power of x is 2

linear model

a mathematical model whose graph is a straight line

longitude

angular distance of a position on Earth's surface to the east or west of the prime meridian

loop

edge in a network that starts and ends at the same vertex

loss

difference between the total cost and the total revenue of producing and selling goods where the cost is greater than the revenue

mathematical model

a mathematical representation of the structure, workings or relationships within a situation

mean

the sum of all the data values divided by the number of data values, also known as the average

measure of centre

a single number that indicates the location of the centre or most typical data value in a dataset; e.g. the mean, the mode and the median

measure of spread

a single number that indicates the dispersion of data values, also known as a measure of dispersion, e.g. the range, the interquartile range and the standard deviation

median

the middle data value in a dataset that has been arranged in ascending order

medicare levy

extra payment to fund the health care system, calculated as a percentage of taxable income

meridians of longitude

semicircles on Earth's surface that are part of great circles with endpoints at the North Pole and South Pole

metric system

measuring system, based on decimal numbers, that uses units such as metres, litres and grams

minimum spanning tree

spanning tree for a network that has the smallest total edge weight

modal class

class in a grouped dataset that occurs the most often

modality (of a dataset)

description of a dataset referring to the number of different data points that occur equally most frequently; a dataset may be unimodal (one mode), bimodal (two modes), multimodal (many modes), or uniform if all data points occur with the same frequency

mode

data value that occurs the most often in a dataset; there may be more than one

multimodal

description of a distribution graph for a data set with many modes

negatively skewed

description of a skewed dataset whose graph has a longer tail to the left

net

2D plan that can be folded to form a 3D object

net earnings

amount of gross pay remaining after tax and other deductions have been made

network

collection of vertices and edges

nominal variable

type of categorical variable for which the categories are named but not ordered; e.g. hair colour

normal distribution

type of data distribution commonly observed amongst naturally occurring characteristics; the mean, median and mode are all equal and the distribution graph is bell-shaped

numerical variable

quantitative characteristic of data that can be counted or measured; it makes sense to find the average of data points for a numerical variable

operating costs

(for a vehicle) variable running costs that depend on how a vehicle is driven, e.g. cost of fuel, tyres, servicing and repairs

ordinal variable

type of categorical variable for which the categories have a natural order, such as responses collected on a scale or as a ranking; e.g. rating system from 1–5

outdegree

number of edges pointing away from a vertex in a directed network

outlier

an extreme piece of data that is much higher or much lower than the rest of the data in the dataset

overtime

extra time worked by employees in excess of their normal working hours

parallel

two lines, rays or line segments in the same plane that have no points of intersection

parallels of latitude (circles of latitude)

circles on Earth's surface formed by slicing Earth along any plane perpendicular to its axis of rotation

path

walk in a network for which each vertex is unique (sometimes called an *open path*); also see *cycle* (or *closed path*)

penalty rate

higher pay rate that applies to hours worked on certain days or times (such as on the weekend)

per annum (p.a.)

for each year (used in financial contexts)

percentage decrease

reduction in a value compared to its initial value, expressed as a percentage

percentage increase

growth in a value compared to its initial value, expressed as a percentage

percentage relative frequency

relative frequency expressed as a percentage

perimeter

the distance around the boundary of a closed shape

perpendicular

two lines, rays or line segments that intersect at a 90° angle

piecewise graph

graph made up of two or more joined straight lines with different gradients

piecework

type of employment paid at a fixed rate for each item produced, regardless of the time taken

polygon

closed 2D shape with straight edges

population

entire dataset relevant to a statistical investigation

position coordinates

pair of coordinates expressed in degrees of longitude and latitude that represent a precise location on Earth's surface; e.g. the point (118°N, 12°W) has latitude 118°N and longitude 12°W

positively skewed

description of a skewed dataset whose graph has a longer tail to the right

prime meridian

(Greenwich Meridian)

reference line for longitude (0°) that passes through Greenwich in London, England

prim's algorithm

method for finding the minimum spanning tree of a connected, weighted network by growing a single tree into a spanning tree

prism

3D object with two parallel faces that are identical polygons joined by straight edges of equal length

profit

excess of total revenue above the total cost of selling goods or services

pronumeral

letter or symbol that takes the place of a number

pyramid

3D object whose base is a polygon, with all other faces being triangles that meet at a single point (the apex)

Pythagoras' theorem

square of the length of the hypotenuse, c , of a right-angled triangle equals the sum of the squares of the lengths of the other two sides, a and b ; that is, $a^2 + b^2 = c^2$

quartiles

three values that divide a numerical dataset (in ascending order) into four equally sized parts; includes the lower quartile (Q_1), middle quartile or median (Q_2) and upper quartile (Q_3)

radius

distance between the centre and the boundary of a circle

random sample

sample in which each member or element of the population has an equal chance of being selected

range

difference between the highest data value and the lowest data value in a dataset

reaction-time distance

distance travelled in the time it takes a driver to react and move their foot to apply the brakes

relative frequency

number of times a data value occurs as a fraction of the total number of data values

retainer

base payment of a fixed amount made to employees whose primary payment type is by commission

revenue

total income generated from normal business operations, such as from the sale of items produced

royalty

payment based on a percentage of the continuing sales of a product that someone has made or created

salary

fixed regular payment made by an employer to an employee, often expressed as an annual sum that assumes a certain average of hours are worked per week; payment is typically made weekly, fortnightly or monthly

sample

subset of a population used to make estimates about the entire population

scientific notation

representation of a very large or very small value as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a (positive or negative) power of 10

sector

shape enclosed by two radii of a circle and the arc (part of the circumference) between them

sector graph

data display representing the relative frequency of different sets or categories in proportionally sized sectors of a circle; sometimes called a pie chart

self-selected sample

sample in which participants choose to take part by responding to a request for volunteers

shortest path

(in a network) path between two vertices where the sum of the weights of the edges is minimised

significant figures

digits in a number that are necessary to express it to the desired level of accuracy

simple quadratic equation

equation of the form $y = ax^2 + c$

skewed

description of a dataset whose distribution graph is not symmetrical

small circle

circle on the surface of a sphere found by intersecting with a plane that does not pass through the centre of the sphere; for example, parallels of latitude for Earth

sole trader

individual who is solely and personally responsible for all aspects of running a business, including financial liability

solution

value(s) of the pronumeral(s) that makes an equation a true statement; e.g. the solution of $2x + 1 = 7$ is $x = 3$

solve

to find the solution(s) of an equation involving one or more pronumerals

spanning tree

(of a network) tree consisting of all vertices and some edges of the original network; must be connected

standard deviation

measure of spread whose value depends on all the data values; describes the "average" distance of each data point from the mean

standard drink

quantity of a beverage which contains 10 g of alcohol

standard time

adopted for a particular region within a time zone

standing costs

(for a vehicle) fixed costs that do not depend on use; e.g. loan interest, on-road costs such as registration and insurance

statistics

area of study dedicated to collecting, organising, analysing and interpreting data

stem

digit or digits preceding the last digit in a piece of numerical data, as displayed in a stem-and-leaf plot; e.g. for 256, the stem would be 25

stem-and-leaf plot

data display where each piece of numerical data is split into a stem and a leaf; leaves with the same stem are listed in a single row

stratified random sample

made up of random samples taken from subgroups (strata) of the population, proportional to the relative size of each subgroup

subject (of a formula)

single variable that appears only on one side of a formula (usually on the left-hand side); e.g. v is the subject of the formula $v = u + at$

surface area

sum of the areas of all faces of a 3D object

systematic sample

sample chosen methodically by assigning numbers to all members of the population, choosing one number at random, then selecting every n th number until the desired sample size is reached

tally marks

lines or marks that help record data in a table; every fifth tally mark is drawn across the preceding four marks

tax

compulsory monetary contribution to support government; levied on incomes, property value and goods purchased

tax return

annual statement of all income, allowable deductions, PAYG tax paid and other personal financial information; used by the Australian Taxation Office to calculate the amount of income tax owed by an individual for the financial year

taxable income

yearly income that is used to determine amount of tax owed; calculated by subtracting allowable deductions from total income

term

(algebra) either a single number or pronumeral, or a product of numbers and pronumerals; e.g. the expression

$3x + a - \frac{1}{2}$ has three terms, which are $3x$, a and $-\frac{1}{2}$

time zone

region of the world that observes the same time throughout; roughly correspond to 24 equal wedges of the globe, where each wedge spans 15° of longitude and neighbouring wedges differ by one hour

time-and-a-half

penalty rate where employees are paid $1\frac{1}{2}$ times their normal hourly rate

total income

all income received by a person during one financial year

total stopping distance

total distance travelled in the time it takes for a driver to react, apply the brakes and for the car to come to a stop

trapezium

quadrilateral with at least one pair of parallel sides

Trapezoidal rule

used to estimate the area of an irregular shape that can be approximated by one or more trapeziums (trapezia); the total area is approximately equal to the sum of the areas of each trapezium

tree

(networks) connected network with no cycles; any two vertices are connected by exactly one path

unimodal

description of a distribution graph for a data set with one mode

unit prefix

prefix added to a base unit that indicates multiples or fractions of the base unit

utilities

organisations that provide services such as electricity, gas, water and internet

variable

(algebra) concepts or quantities that can take on different values, often represented by pronumerals; e.g. time or t (statistics) measurable or observable characteristic that is expected to differ when collecting data

VAT (value-added tax)

sales tax added to the purchase price of an item, usually as a percentage of the purchase price

vertex

(node) point in a network, typically met by one or more edge; often used to represent an object, location or concept in a network diagram

volume

amount of space a 3D object occupies

wage

payment earned for work or services, usually based on an hourly rate and typically paid daily, weekly or fortnightly

walk

(in a network) sequence of vertices and the edges between them

weighted edge

edges with a weight that represents a numerical quantity associated with the relationship between the vertices

world time zones

time zones around the world that are expressed as positive or negative offsets from UTC

y-intercept

(of a graph) point where a line crosses the y -axis of the Cartesian plane