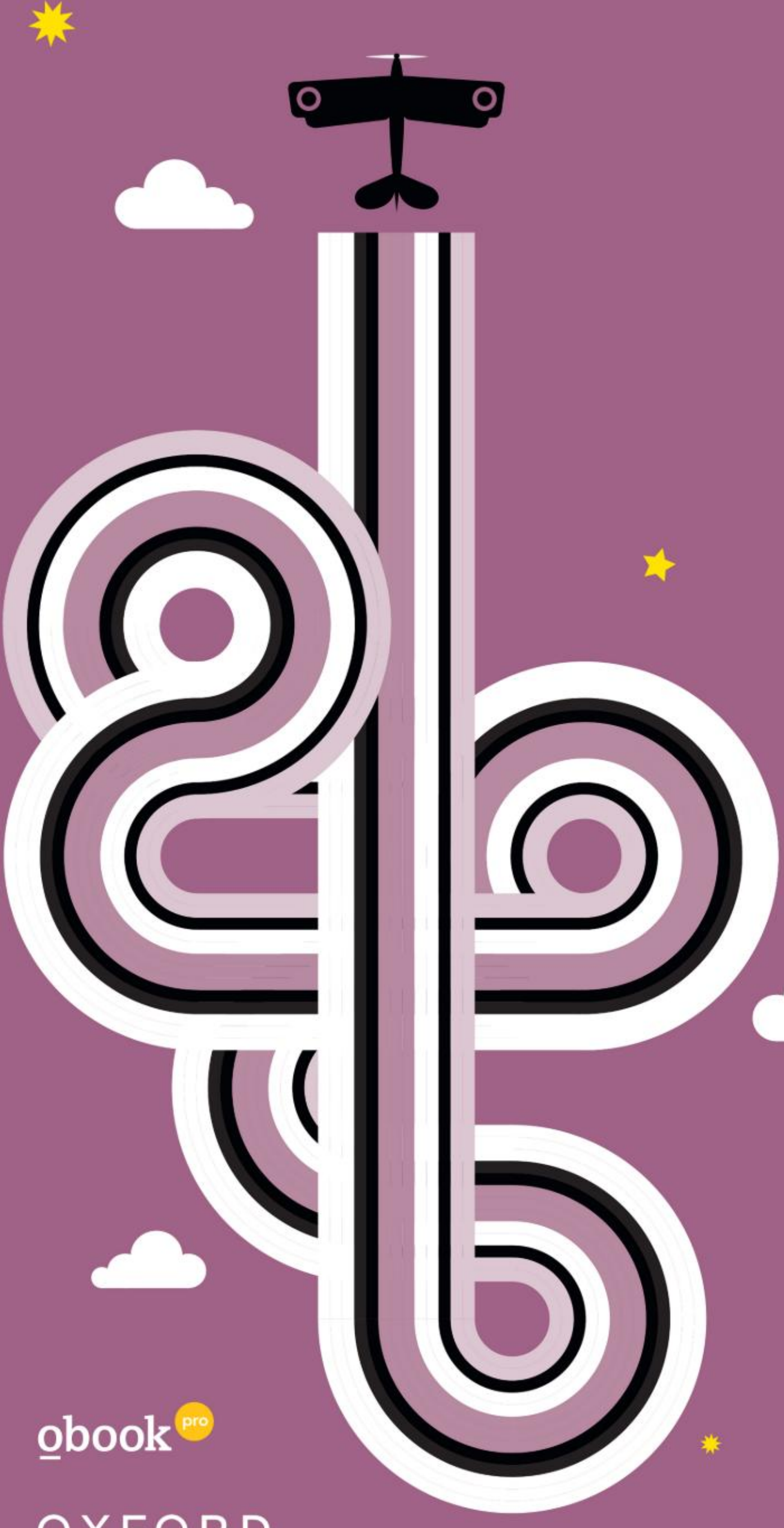
MATHEMATICS

OXFORD MATHS 9



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AUSTRALIAN CURRICULUM

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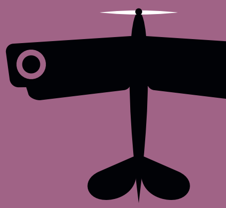
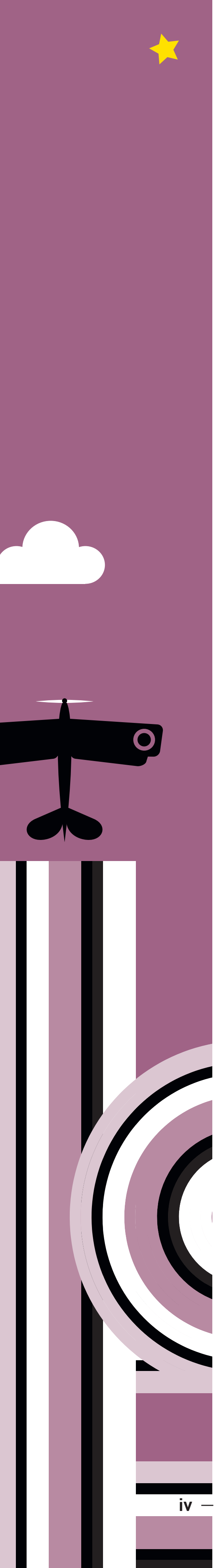
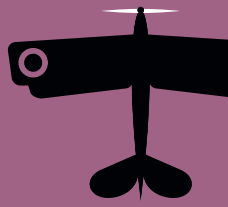
Warning to First Nations Australians
Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

CONTENTS





CONTENTS

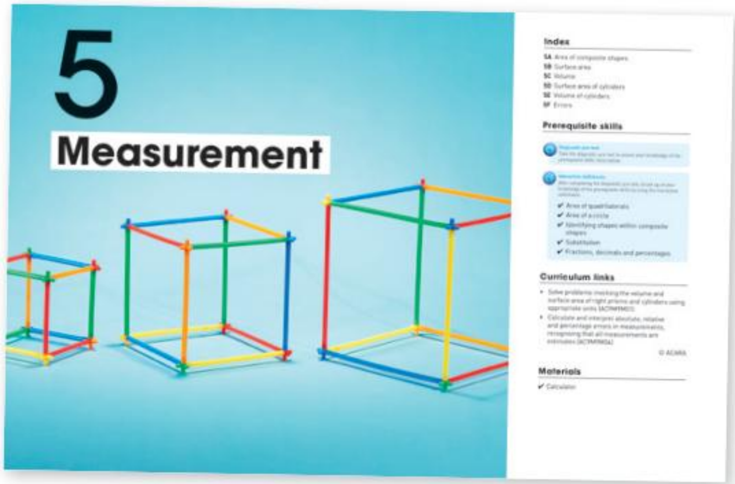


Introducing Oxford Maths 7–10

Oxford Maths 7–10 Australian Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

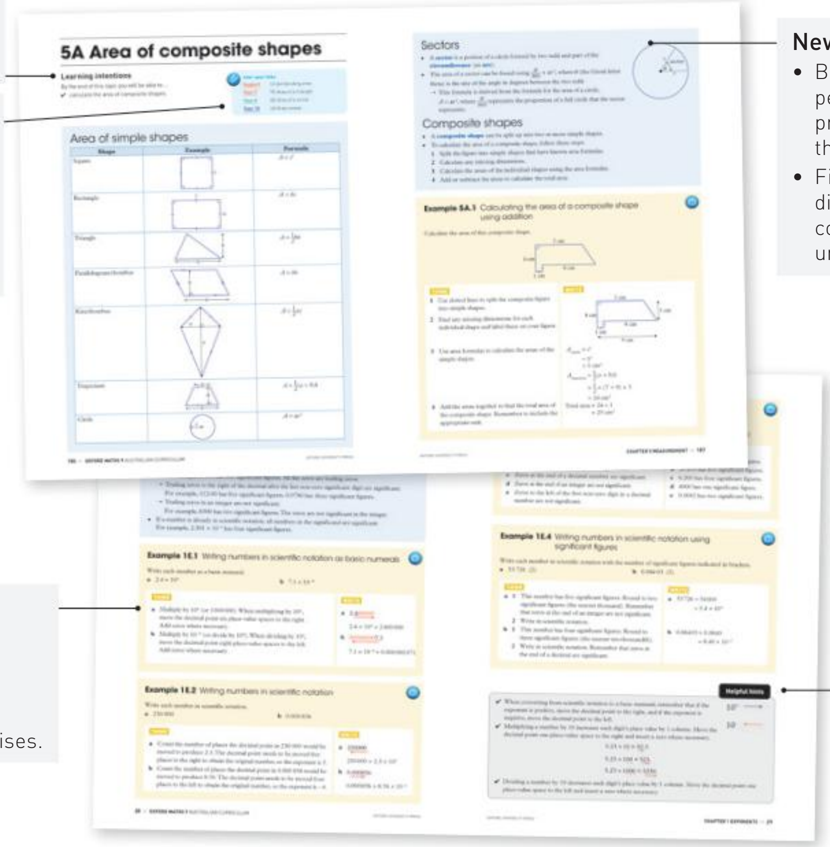
Key features of Student Books

- Complete access to all digital resources available on Student obook pro.
- Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- STEAM projects encourage inter-disciplinary thinking.
- Semester reviews provide an opportunity to revise key concepts from each semester.
- NAPLAN practice allows students to revise numeracy skills for the National Assessment Program.



Each chapter opens with:

- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the Australian Curriculum.
- **Materials** used to complete the exercises.



Learning intentions

- Signpost the foundational skills being developed in each section.

Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.

New theory

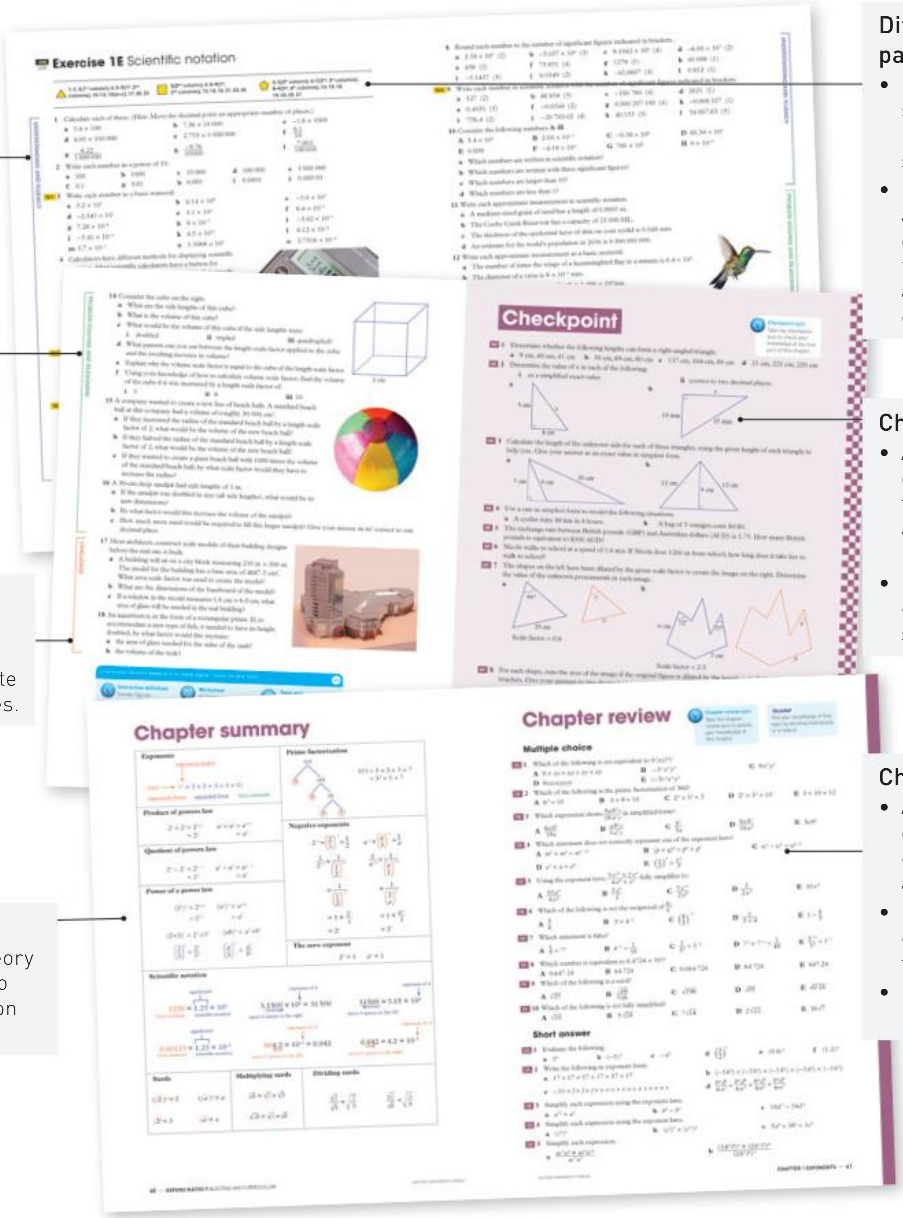
- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life, and aid understanding.

Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

Helpful hints

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.



Exercise 1E Scientific notation

Checkpoint

Chapter summary

Chapter review

Multiple choice

Short answer

Understanding and fluency

- Basic exercises dedicated to practising key concepts.

Differentiated learning pathways

- Each exercise is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Checkpoint

- A section in the middle of each chapter dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.

Chapter review

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Chapter summary

- Condenses all the theory from each section into one accessible revision page.

[STEAM project 1]

How can we use sustainable farming practices so that no one goes hungry in the future?

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.

Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

Key features of Student obook pro

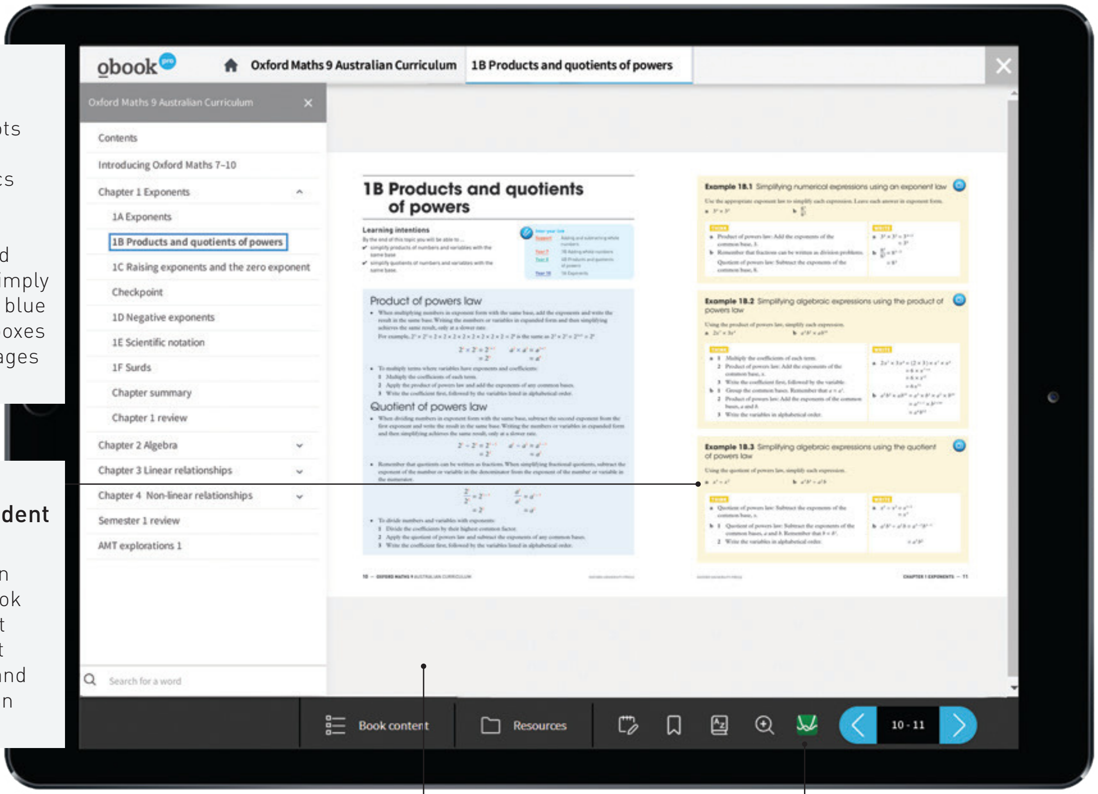
- Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.

Integrated digital resources

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.

Complete digital version of the Student Book

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.



Toolbar features

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- *Australian Concise Oxford Dictionary* can provide immediate definitions to any word within the text.

Desmos integration

- Our partnership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

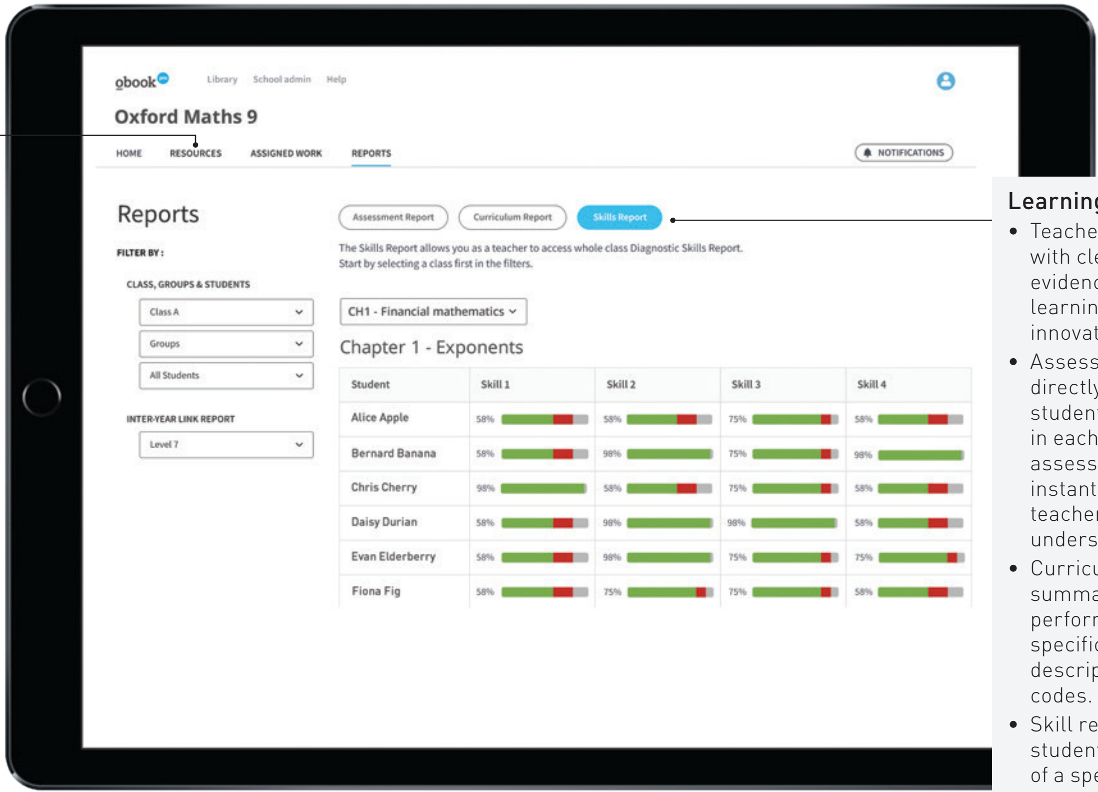
Benefits for students

- Integrated *Australian Concise Oxford Dictionary* look-up feature
- Targeted instructional videos for every worked example question
- Groundwork resources to support assumed knowledge
- Interactive assessments to consolidate understanding
- Auto-marked practice exam question sets
- Integrated Quizlet sets, including real-time online quizzes with live leaderboards
- Access to online assessment results to track progress

obook pro

Key features of Teacher obook pro

- Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.



Oxford Maths 9

Reports

The Skills Report allows you as a teacher to access whole class Diagnostic Skills Report. Start by selecting a class first in the filters.

Chapter 1 - Exponents

Student	Skill 1	Skill 2	Skill 3	Skill 4
Alice Apple	58%	58%	75%	58%
Bernard Banana	58%	98%	75%	98%
Chris Cherry	98%	58%	75%	58%
Daisy Durian	58%	98%	98%	58%
Evan Elderberry	58%	98%	75%	75%
Fiona Fig	58%	75%	75%	58%

Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students' understanding of a specific skill in mathematics.

Additional resources

- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

Benefits for teachers

- Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill
- Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- Ability to set-up classes, monitor student progress and graph results
- Worked solutions for every Student Book question
- Detailed teacher notes, teaching programs and lesson plans

1

Exponents

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Prime factorisation
- ✔ Multiplying and dividing negative numbers
- ✔ Equivalent fractions

Curriculum links

- Recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers using digital tools (AC9M9N01)
- Apply the exponent laws to numerical expressions with integer exponents and extend to variables (AC9M9A01)
- Solve problems involving very small and very large measurements, time scales and intervals expressed in scientific notation (AC9M9M02)

Materials

- ✔ Calculator

1A Exponents

Learning intentions

By the end of this topic you will be able to …

✔ convert integers and terms between exponent and expanded form

✔ calculate the value of numbers in exponent form

✔ express integers as a product of prime factors.

Inter-year links

Year 7	1G Exponents and square roots
Year 8	4A Exponents
Year 10	1A Exponents

Exponent notation

- **Exponent notation** or **exponent form** is used to represent repeated multiplication.
  - → 3^4 is read as '3 to the power of 4'.
  - → a^3 is read as 'a to the power of 3'.
- The **base** is the number or variable that is multiplied repeatedly.
- The **exponent** or **index** indicates the number of times the base is multiplied.
- If no exponent is shown, then the base has an exponent of 1.

$$2 = 2^1$$
$$x = x^1$$

exponent/index

$$\text{base} \longrightarrow 3^4 = 3 \times 3 \times 3 \times 3 = 81$$

exponent form   expanded form   basic numeral

exponent/index

$$\text{base} \longrightarrow a^3 = a \times a \times a$$

exponent form   expanded form

- Exponent notation is also used to represent powers of negative numbers.

exponent/index

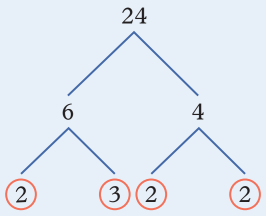
$$\text{base} \longrightarrow (-2)^3 = (-2) \times (-2) \times (-2) = -8$$

exponent form   expanded form   basic numeral

  - → If the base is negative and the exponent is an even number, then the basic numeral is positive.
  - → If the base is negative and the exponent is an odd number, then the basic numeral is negative.

Prime factorisation

- The **prime factorisation** of a positive integer is the product of all prime factors of that integer.
  - → Prime factorisation is often expressed in exponent form with the bases listed in ascending order. For example, the prime factorisation of 24 is:
$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned}$$
- Prime factorisation can be performed using **factor trees**. In factor trees, composite numbers are broken down into pairs of factors until all factors are prime numbers.



Example 1A.1 Calculating the value of a number in exponent form

Write the following in expanded form and then evaluate.

a 2^5 **b** $(-4)^3$ **c** $\left(\frac{2}{5}\right)^4$

THINK	**WRITE**
a 1 Identify the base and the exponent. The base is 2 and the exponent is 5, so 2 is multiplied by itself 5 times.	**a** $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
2 Perform the multiplication.	$= 4 \times 2 \times 2 \times 2$ $= 8 \times 2 \times 2$ $= 16 \times 2$ $= 32$
b 1 Identify the base and the exponent. The base is −4 and the exponent is 3, so −4 is multiplied by itself 3 times.	**b** $(-4)^3 = (-4) \times (-4) \times (-4)$
2 Perform the multiplication. Recall that if the base is negative and the exponent is an odd number, then the basic numeral is negative.	$= 16 \times (-4)$ $= -64$
c 1 Identify the base and the exponent. The base is $\frac{2}{5}$ and the exponent is 4, so $\frac{2}{5}$ is multiplied by itself 4 times.	**c** $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
2 Perform the multiplication. Recall that to multiply fractions, you multiply the numerators together and the denominators together.	$= \frac{2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5}$ $= \frac{16}{625}$

Example 1A.2 Writing variables in expanded form

Write the following in expanded form.

a x^4 **b** $(-ab)^3$ **c** $2xy^2z$

THINK	**WRITE**
a Identify the base and the exponent. The base is *x* and the exponent is 4, so *x* is multiplied by itself 4 times.	**a** $x^4 = x \times x \times x \times x$
b Identify the base and the exponent. The base is −*ab* and the exponent is 3, so −*ab* is multiplied by itself 3 times.	**b** $(-ab)^3 = (-ab) \times (-ab) \times (-ab)$
c There are four bases in this term. Identify the bases and the matching exponent. Recall that if a base doesn't appear to have an exponent, then it has an exponent of 1. Therefore, 2, *x* and *z* each have an exponent of 1, and *y* has an exponent of 2.	**c** $2xy^2z = 2 \times x \times y \times y \times z$

Example 1A.3 Prime factorisation using factor trees

Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 20 **b** 315

THINK

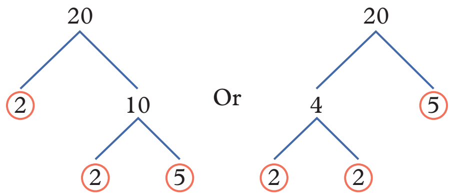
1 Identify a factor pair by dividing the composite number by its lowest prime factor. The lowest prime factor of an even number is always 2. Remember that if the sum of all the digits in a number is divisible by 3, then that number is also divisible by 3, and any number ending in 0 or 5 is divisible by 5.

2 Continue to split factors into further factor pairs until all factors are prime.

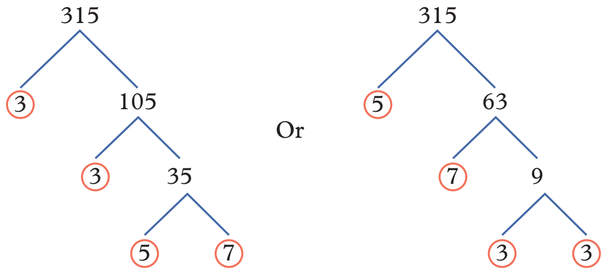
3 Write the composite number as a product of its prime factors. Write the answer in exponent form and list the bases in ascending order.

WRITE

a $20 = 2 \times 2 \times 5$
$= 2^2 \times 5$



b $315 = 3 \times 3 \times 5 \times 7$
$= 3^2 \times 5 \times 7$



Helpful hints

- ✔ Remember that raising a number to an exponent and multiplying are two different operations. For example: $2^4 \neq 2 \times 4$, $2^4 = 2 \times 2 \times 2 \times 2$
- ✔ Take care when writing exponents – they should be a smaller size than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ When creating factor trees, remember that a branch always ends on a prime number. If the number is composite, keep dividing!
- ✔ Recall that the first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

ANS p404

Exercise 1A Exponents

△ 1–8, 9(a, c, e, g), 10, 11, 13, 14, 17(a, b)

■ 1–3(b, d, f, h), 4, 6–8, 9(b, d, f, h), 12, 13, 15, 17, 19

⬟ 1–3(g, h), 4, 6(e–h), 8(e, f), 12–14, 16, 17(c, d), 18–20

UNDERSTANDING AND FLUENCY

1A.1 **1** Write the following in expanded form and then evaluate.

a 6^4 **b** 8^3 **c** $(-2)^5$ **d** $(-3)^6$

e $\left(\frac{5}{4}\right)^3$ **f** $\left(\frac{1}{2}\right)^7$ **g** $\left(-\frac{2}{3}\right)^4$ **h** $\left(-\frac{3}{5}\right)^5$

1A.2 **2** Write the following in expanded form.

a b^6 **b** $(-n)^5$ **c** $(-cd)^2$ **d** $(2pq)^4$

e $2pq^4$ **f** $-4a^2b^3c$ **g** $(3m^2)^5$ **h** $3(m^2)^5$

3 Write the following in exponent form.

a $5 \times 5 \times 5 \times 5$

b $a \times a \times a \times a$

c $v \times k \times k \times v \times k \times v \times v \times 7$

d $qu \times qu \times qu \times qu \times qu$

e $(-h) \times (-h) \times (-h)$

f $-(h \times h \times h)$

g $n^3 \times n^3 \times n^3 \times n^3 \times n^3 \times n^3$

h $5b^3d^4 \times 5b^3d^4$

1A.3 **4** Express each number as the product of its prime factors. Write your answers in exponent form.

a 50 **b** 72 **c** 135 **d** 378

e 152 **f** 812 **g** 550 **h** 1665

5 Evaluate the following.

a $(0.2)^2$ **b** $(-0.2)^2$ **c** $(0.02)^2$

d $(0.2)^3$ **e** $(-0.2)^3$ **f** $(0.02)^3$

g $(0.2)^4$ **h** $(-0.2)^4$ **i** $(0.02)^4$

6 Write the following in exponent form without brackets.

a $(-5)^4$ **b** $(-5)^3$ **c** $(ab)^4$ **d** $(5xy)^8$

e $5(xy)^8$ **f** $\left(\frac{11}{2}\right)^6$ **g** $(-3abc)^5$ **h** $(-3abc)^8$

7 Substitute the given values and evaluate the expression.

a x^3, where $x = 7$

b $6a^4b^2$, where $a = -2$ and $b = \frac{1}{4}$

c $\frac{p^4}{qr^3}$, where $p = 3$, $q = 5$ and $r = -4$

d $2x^3 + 8x^2 + x + 7$, where $x = 10$

8 Write the following in exponent form.

a $2 \times 2 \times 2 \times 3 \times 3$

b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 6$

c $13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17 \times 17 \times 17$

d $101 \times 101 \times 103 \times 103 \times 103 \times 103 \times 103$

e $4 \times 4 \times 4 \times x \times x \times x \times x$

f $7 \times 7 \times xy \times xy \times xy \times xy \times xy \times xy \times xy$

9 Use the fact that $300 = 2^2 \times 3 \times 5^2$ to help find the prime factors of each of the following numbers and then write in exponent form.

a 600
b 150
c 900
d 1500
e 3000
f 1800
g 2100
h 2400

10 Express each of the following in exponent form.

a $xxxxxxx$
b $aaabb$
c $3rssttt$
d $4eeeeeeeff$

PROBLEM SOLVING AND REASONING

11 Explain the mistake in each of the following. Change the right-hand side so that the equation is correct.

a $tk^5 = t \times k \times t \times k \times t \times k \times t \times k \times t \times k$

b $(2rw)^4 = 2 \times r \times w \times w \times w \times w$

c $-3 \times (-2)^4 = 6^4$

12 Substitute the given values and evaluate each expression.

a $(2x + 3)^8$, where $x = -2$

b $\left(\frac{y}{3}\right)^3 + 4\sqrt{y}$, where $y = 9$

c $ab^3 - ba^2$, where $a = 5$ and $b = -3$

d $2r^3 + 8r^2 - 3r$, where $r = -\frac{3}{2}$

13 a Evaluate each of the following.

i $(-1)^2$
ii $(-1)^3$
iii $(-1)^4$
iv $(-1)^5$
v $(-1)^6$
vi $(-1)^7$
vii $(-1)^8$
viii $(-1)^9$
ix $(-1)^{10}$
x $(-1)^{11}$

b Copy and complete the following sentences.

i When the exponent n is odd, the basic numeral of $(-1)^n$ is __________.

ii When the exponent n is even, the basic numeral of $(-1)^n$ is __________.

c Decide for each of the following whether the basic numeral is positive or negative. Do not evaluate.

i $(-2)^{15}$
ii $(-4)^{27}$
iii $(-24)^{30}$
iv $(-17)^{198}$
v $(-16)^7 \times (-34)^{11}$
vi $(-8)^{14} \times (-5)^{27}$
vii $(-78)^{99} \times (-81)^{45} \times (-21)^{68}$
viii $\left(-\frac{77}{101}\right)^{108} \times \left(-\frac{301}{22}\right)^{404}$

14 Consider each pair of numbers in exponent form.

i Using a calculator, evaluate each pair.

ii Describe how the two numbers are similar and how they are different in their exponent form and as a basic numeral.

a $(0.7)^3$ and $(0.07)^3$

b $(-0.4)^3$ and $-(0.4)^3$

c $(-1.2)^3$ and $(-1.2)^4$

d $(2.1)^3$ and $(2.01)^3$

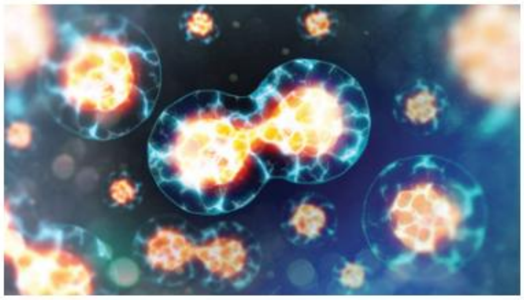
15 A farmer's herd of cattle grows by approximately 20% each year. In 2020, the farmer had 20 cows.

a By what number can the number of cows be multiplied by to increase it by 20%?

b Predict the size of the farmer's herd in 2021, 2022 and 2025. Round to the nearest whole number.

PROBLEM SOLVING AND REASONING

16 Three different groups of bacteria, Bacteria A, Bacteria B and Bacteria C, reproduce at different rates.



a Bacteria A splits into two bacteria every day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

b Bacteria B splits into two bacteria twice each day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

c Bacteria C splits into two bacteria once every two days. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in exponent form.

d Populations of Bacteria A, B, and C each have 3 bacteria initially. Determine the size of each bacteria population after 3 days.

17 The lowest common multiple is the product of the largest exponent of each prime factor or pronumeral in each term. The highest common factor is the product of the smallest exponent of each prime factor or pronumeral in each term. Find the lowest common multiple and highest common factor of each pair of terms. Write your answers in exponent form.

a $2^8 \times 3^5 \times 5^2 \times 7$ and $2^4 \times 3^{15} \times 5^2 \times 7^4$

b $a^8 b^5 c^2 d$ and $a^4 b^{15} c^2 d^4$

c $pq^5 r^7 s^2$ and $pq^3 r^{10} s^4$

d $8x^3 y^9 z^4$ and $12xy^3 z^4$

CHALLENGE

18 For each of the following, state how many different sequences of answers there are. Write your answers in exponent notation.

a A quiz that has 10 true or false questions.

b A quiz that has 10 multiple choice questions each with options A, B, C, D, E.

c A quiz that has 12 true or false questions and 8 multiple choice questions with options A, B, C, D, E.

19 Using positive and negative whole numbers (integers), see how many different exponent expressions that equal 64 you can find.

$$\square^{\square} = 64$$

20 Evaluate each of the following.

a $\frac{ab^2}{c^3}$ where $a = 6$, $b = \frac{1}{3}$, and $c = -2$

b $\frac{p^3}{q^2}$ where $p = \frac{3}{2}$ and $q = \frac{2}{3}$

c $\frac{r^4}{(mn)^3}$ where $m = -0.5$, $n = 0.2$, and $r = -0.7$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Exponents

Investigation
Using exponents to determine how card tricks work

Topic quiz
1A

1B Products and quotients of powers

Learning intentions

By the end of this topic you will be able to …

✔ simplify products of numbers and variables with the same base

✔ simplify quotients of numbers and variables with the same base.

Inter-year link

Support	Adding and subtracting whole numbers
Year 7	1B Adding whole numbers
Year 8	4B Products and quotients of powers
Year 10	1A Exponents

Product of powers law

- When multiplying numbers in exponent form with the same base, add the exponents and write the result in the same base. Writing the numbers or variables in expanded form and then simplifying achieves the same result, only at a slower rate.

  For example, $2^3 \times 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$ is the same as $2^3 \times 2^5 = 2^{3+5} = 2^8$

$$\begin{aligned} 2^3 \times 2^5 &= 2^{3+5} \\ &= 2^8 \end{aligned} \qquad \begin{aligned} a^3 \times a^5 &= a^{3+5} \\ &= a^8 \end{aligned}$$

- To multiply terms where variables have exponents and coefficients:
  1. Multiply the coefficients of each term.
  2. Apply the product of powers law and add the exponents of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

Quotient of powers law

- When dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent and write the result in the same base. Writing the numbers or variables in expanded form and then simplifying achieves the same result, only at a slower rate.

$$\begin{aligned} 2^5 \div 2^3 &= 2^{5-3} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} a^5 \div a^3 &= a^{5-3} \\ &= a^2 \end{aligned}$$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the exponent of the number or variable in the denominator from the exponent of the number or variable in the numerator.

$$\begin{aligned} \frac{2^5}{2^3} &= 2^{5-3} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} \frac{a^5}{a^3} &= a^{5-3} \\ &= a^2 \end{aligned}$$

- To divide numbers and variables with exponents:
  1. Divide the coefficients by their highest common factor.
  2. Apply the quotient of powers law and subtract the exponents of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

Example 1B.1 Simplifying numerical expressions using an exponent law

Use the appropriate exponent law to simplify each expression. Leave each answer in exponent form.

a $3^4 \times 3^2$ **b** $\dfrac{8^5}{8^2}$

THINK	WRITE
a Product of powers law: Add the exponents of the common base, 3.	**a** $3^4 \times 3^2 = 3^{4+2}$ $= 3^6$
b Remember that fractions can be written as division problems. Quotient of powers law: Subtract the exponents of the common base, 8.	**b** $\dfrac{8^5}{8^2} = 8^{5-2}$ $= 8^3$

Example 1B.2 Simplifying algebraic expressions using the product of powers law

Using the product of powers law, simplify each expression.

a $2x^7 \times 3x^4$ **b** $a^3b^2 \times ab^{10}$

THINK	WRITE
a 1 Multiply the coefficients of each term. 2 Product of powers law: Add the exponents of the common base, x. 3 Write the coefficient first, followed by the variable.	**a** $2x^7 \times 3x^4 = (2 \times 3) \times x^7 \times x^4$ $= 6 \times x^{7+4}$ $= 6 \times x^{11}$ $= 6x^{11}$
b 1 Group the common bases. Remember that $a = a^1$. 2 Product of powers law: Add the exponents of the common bases, a and b. 3 Write the variables in alphabetical order.	**b** $a^3b^2 \times ab^{10} = a^3 \times b^2 \times a^1 \times b^{10}$ $= a^{3+1} \times b^{2+10}$ $= a^4b^{12}$

Example 1B.3 Simplifying algebraic expressions using the quotient of powers law

Using the quotient of powers law, simplify each expression.

a $x^5 \div x^2$ **b** $a^5b^3 \div a^2b$

THINK	WRITE
a Quotient of powers law: Subtract the exponents of the common base, x.	**a** $x^5 \div x^2 = x^{5-2}$ $= x^3$
b 1 Quotient of powers law: Subtract the exponents of the common bases, a and b. Remember that $b = b^1$. 2 Write the variables in alphabetical order.	**b** $a^5b^3 \div a^2b = a^{5-2}b^{3-1}$ $= a^3b^2$

Helpful hints

✔ ‘Simplify’ and ‘evaluate’ are two different command terms.
  → To simplify in this chapter, use exponent laws to combine the terms and hence reduce the complexity of the calculation or numerical expression.
  → To evaluate or ‘find the value’ of a calculation or numerical expression, convert the expression from exponent form into a basic numeral.

✔ Exponents only apply to the number or pronumeral immediately to the left of the exponent. For example, in the term $4gh^3$, the exponent of 3 only applies to the variable h, so $4gh^3 = 4 \times g \times h \times h \times h$.

✔ When an algebraic term immediately follows a division symbol, the entire term is the divisor in the division. For example, $x^2 \div 2x = \frac{x^2}{2x} = \frac{x}{2}$ and $x^2 \div 2x \neq \frac{x^2}{2} \times x = \frac{x^3}{2}$.

ANS p405

Exercise 1B Products and quotients of powers

▲ 1–5, 6–8(1st, 3rd columns), 9, 11(a, b), 14

■ 1(f, h, j, l), 3–9(2nd, 4th columns), 10, 11(b, c), 12(a, b), 14, 15(a–d)

⬟ 5–9(2nd, 4th columns), 10, 11(c, d), 12(b, c), 13, 15, 16

UNDERSTANDING AND FLUENCY

1B.1 **1** Use the appropriate exponent law to simplify each expression. Leave each answer in exponent form.

a $3^5 \times 3^4$ **b** $7^8 \div 7^2$ **c** $(-2)^7 \times (-2)^5$ **d** 6×6^2

e $(-8)^{13} \div (-8)^6$ **f** $10^2 \times 10^9$ **g** $3^6 \div 3^5$ **h** $5^3 \div 5$

i $\frac{4^7}{4^5}$ **j** $\frac{(-9)^{18}}{(-9)^9}$ **k** $\frac{13^9}{13^6}$ **l** $2^5 \times 2^2 \times 2^3$

2 Using a calculator, calculate the basic numeral for question **1** parts **a** to **d**.

1B.2 **3** Using the product of powers law, simplify each expression.

a $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$ **d** $-6k^5 \times 2k^8$

e $-2b^8 \times (-3b^3)$ **f** $-5g^5 \times (-2g) \times (-8g^5)$ **g** $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times (-3p^2) \times (-5p^2)$

1B.3 **4** Using the quotient of powers law, simplify each expression.

a $a^6 \div a^4$ **b** $d^7 \div d^6$ **c** $g^{11} \div g$ **d** $p^{10} \div p^7$

e $a^8 \div a^3$ **f** $n^{14} \div n^{11}$ **g** $r^9 \div r$ **h** $8x^{17} \div x^6$

5 Use the exponent laws to simplify each expression.

a $3x^5 \times 4x^6$ **b** $5x^4 \times 2x^3$ **c** $-8x^2 \times 3x^7$ **d** $-6x^{10} \times (-9x)$

e $6x^7 \div 2x^3$ **f** $-20x^6 \div (-5x^2)$ **g** $4x^8 \div 10x^7$ **h** $15x^{12} \div 9x^4$

i $\frac{-24t^{18}}{3t^6}$ **j** $\frac{-20r^8}{-32r^2}$ **k** $\frac{10c^7}{2c^3}$ **l** $\frac{15y^{12}}{6y^5}$

6 Use the exponent laws to simplify:

a $\frac{ax^{13}}{ax^4}$ **b** $\frac{a^2b}{a^3}$ **c** $\frac{m^5n}{m^5}$ **d** $\frac{b^{20}d}{b^{12}d}$

e $\frac{x^4 \times x^3}{x^2}$ **f** $\frac{m^7 \times m^6}{m^9}$ **g** $\frac{6a^2 \times a^8}{a^4}$ **h** $\frac{n^5 \times n^7}{n^3 \times n^4}$

i $\frac{5d^6 \times d^3}{d^9}$ **j** $\frac{8t^2 \times t^3}{-2t^3}$ **k** $\frac{-4k \times 3k^9}{-6k^{10}}$ **l** $\frac{15e^{13}}{3e^8 \times 5e^5}$

7 Simplify each expression.

a $a^3b^4 \times a^6b^2$ **b** $6m^5n^2 \times (-3m^6n)$ **c** $\frac{c^2d^9}{d^7}$ **d** $\frac{k^3m^8}{km^5}$

e $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$ **g** $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$

i $9w^4x^8 \times 6x^5y^4$ **j** $\frac{1}{tu^3} \times \frac{tu^7}{5}$ **k** $\frac{-6e^5f^{11}}{8e^4f}$ **l** $-4v^9 \times (-9y^3) \times (-3v^8y^7)$

8 Use the exponent laws to simplify each expression.

a $\dfrac{x^7 \times x^3}{x^4}$ **b** $\dfrac{2k^4 \times k^5}{k^6}$ **c** $\dfrac{4a^2 \times 3a^6}{2a^7}$ **d** $\dfrac{5m^2 \times 2x^4}{10x^6}$

e $\dfrac{a^5b^7 \times a^3b^6}{a^8b^{10}}$ **f** $\dfrac{n^{17}p^{13}}{n^3p^2 \times np^8}$ **g** $\dfrac{-6jq^5 \times 5j^7q^2}{15j^3q}$ **h** $\dfrac{6w^9x^6 \times 3w^4x^5}{9w^5x^4 \times w^6x^3}$

PROBLEM SOLVING AND REASONING

9 Use the exponent laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $x^3 \times x^4 = x^{12}$ **b** $k^3 + k^3 = k^6$ **c** $y^7 \div y = y^6$ **d** $a^5 \times a \times a^5 = a^{10}$

e $m^3n^5 \times m^2n^4 = m^{14}n^{14}$ **f** $100^8 \div 100^2 = 100^4$ **g** $\dfrac{m^3 \times m^7}{m^{11}} = \dfrac{1}{m}$ **h** $\dfrac{a^5b^6}{a^2b^4} \times \dfrac{a^3b^5}{a^4b} = a^2$

10 If the exponent of the denominator is greater than the exponent of the numerator, we can instead subtract the numerator's exponent from the denominator's exponent, leaving the base on the denominator. For example:
$\dfrac{2^3}{2^5} = \dfrac{1}{2^{5-3}} = \dfrac{1}{2^2}$

a Simplify the following. Write your answers in exponent form.

i $\dfrac{3^4}{3^{10}}$ **ii** $\dfrac{5^2}{5^8}$ **iii** $\dfrac{2^5 \times 3^2}{2^9 \times 3^7}$ **iv** $\dfrac{2^5 \times 3^7}{2^9 \times 3^2}$ **v** $\dfrac{2^9 \times 3^2}{2^5 \times 3^7}$

b Copy and complete the following.

i $\dfrac{2^3}{2^5} = \dfrac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \dfrac{1}{\square \times \square} = \dfrac{1}{2^{\square}} = \dfrac{1}{2^{\square - \square}}$

ii $\dfrac{2^4}{2^8} = \dfrac{\square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square \times \square \times \square \times \square} = \dfrac{\square}{2^{\square}} = \dfrac{\square}{2^{\square - \square}}$

iii $\dfrac{5^6}{5^7} = \dfrac{\square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square} = \dfrac{\square}{\square^{\square}} = \dfrac{\square}{\square^{\square - \square}}$

c Explain why subtracting the exponents on the numerator or denominator when the base is the same gives the exponent of the number on the numerator or denominator when simplified.

11 Write the following products in exponent form with prime number bases.

a $2 \times 4 \times 8 \times 16 \times 32$ **b** $3 \times 9 \times 27 \times 81 \times 243$ **c** $6 \times 36 \times 216$ **d** $4 \times 16 \times 64 \times 256 \times 1024$

12 Determine the values of the unknowns.

a $2^5 \times 3^{4x} \times 5^{12} \times 7^{z+3} = 2^w \times 3^{12} \times 5^{6y} \times 7^{11}$

b $(5^x \times 7^{4y} \times 11^z) \times (3^9 \times 5^6 \times 11) = 3^9 \times 5^{15} \times 7^{24} \times 11^5$

c $\dfrac{11^a \times 13^b \times 17^{2c} \times 19^8}{11^5 \times 13^6 \times 17^3 \times 19^d} = 11^{11} \times 13^3 \times 17^5 \times 19^2$

13 Do the exponent laws for multiplying and dividing terms in exponent form work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

CHALLENGE

14 Fill in the box to make the statement true.

a $2^{\square} = 8$ **b** $3^{\square} = 27$ **c** $5^{\square} = 25$ **d** $10^{\square} = 10\,000$

15 Fill in the box to make the statement true. Start by writing the base on the right as a power of the base on the left. For example: $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{12}$

a $2^{\square} = 8^4$ **b** $3^{\square} = 27^5$ **c** $5^{\square} = 25^9$ **d** $10^{\square} = 10\,000^3$

e $4^{\square} = 16^7$ **f** $2^{\square} = 32^6$ **g** $6^{\square} = 216^2$ **h** $3^{\square} = 243^6$

16 Simplify each of the following expressions.

a $a^mb^x \times a^nb^y$ **b** $a^mb^x \div (a^nb^y)$

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Interactive skillsheet
Products and quotients of powers

Topic quiz
1B

1C Raising exponents and the zero exponent

Learning intentions

By the end of this topic you will be able to …

✔ raise a number or a variable to two exponents using an exponent law

✔ evaluate calculations involving the zero exponent.

Inter-year links

Support	Multiplying and dividing whole numbers
Year 7	1D Multiplying whole numbers
Year 8	4C Raising exponents and the zero exponent
Year 10	1A Exponents

Power of a power law

- When raising a number or a variable to two exponents, multiply the exponents. Writing the numbers or variables in expanded form and applying the product of powers law achieves the same result, only at a slower rate. For example, $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{15}$ is the same as $(2^3)^5 = 2^{3\times5} = 2^{15}$

$$(2^3)^5 = 2^{3\times5} = 2^{15} \qquad (a^2)^3 = a^{2\times3} = a^6$$

- To raise an exponent by another exponent:
  1. Multiply the exponent of every base inside the brackets by the exponent outside the brackets. If there is no exponent applied to a number or a variable, the exponent is 1 and still must be multiplied.
  2. Write the coefficient first, followed by the variables listed in alphabetical order.
- Every base number or variable inside brackets should have its exponent multiplied by the exponent outside the brackets.

$$(2 \times 3)^5 = 2^5 \times 3^5 \qquad (ab)^3 = a^3 \times b^3$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \qquad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

The zero exponent

- Excluding 0, any number or variable with an exponent of 0 is equal to 1. This is because for any non-zero base: an exponent indicates the number of times we multiply 1 by the base. If we multiply 1 by the base zero times, we haven't multiplied so we are left with 1.

  For example, $1 = \frac{a^m}{a^m} = a^{(m-m)} = a^0$

  Therefore, $a^0 = 1$

$$2^0 = 1 \qquad (-k)^0 = 1$$

- The order of operations also applies to simplification. Calculations in grouping symbols should be simplified first. Remember **BIDMAS**: Brackets, Indices (exponents), Division and Multiplication, Addition and Subtraction.

Example 1C.1 Simplifying numerical expressions using the power of a power law

Use the power of a power to simplify the following products. Give your answer in exponent form.

a $(3^4)^5$ **b** $(4^3 \times 7)^2$ **c** $\left(\frac{5}{2}\right)^4$ **d** $3(2^2)^4$

THINK	**WRITE**
a Multiply the exponent of 3 by the exponent outside the brackets.	**a** $(3^4)^5 = 3^{4\times5}$ $= 3^{20}$
b Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that base numbers that do not appear to have an exponent have an exponent of 1, so $7 = 7^1$.	**b** $(4^3 \times 7)^2 = 4^{3\times2} \times 7^{1\times2}$ $= 4^6 \times 7^2$
c Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that $5 = 5^1$.	**c** $\left(\frac{5}{2}\right)^4 = \frac{5^{1\times4}}{2^{1\times4}}$ $= \frac{5^4}{2^4}$
d Multiply the exponent of 2 by the exponent outside the brackets. The exponent outside the brackets only applies to the term inside the brackets.	**d** $3(2^2)^4 = 3 \times 2^{2\times4}$ $= 3 \times 2^8$

Example 1C.2 Simplifying algebraic expressions using the power of a power law

Using the power of a power law, simplify each expression.

a $(2x^4)^3$ **b** $(3a^2b^3)^2$ **c** $\left(\frac{-x}{y^2}\right)^3$ **d** $4(a^2b)^4$

THINK	**WRITE**
a **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that $2 = 2^1$.	**a** $(2x^4)^3 = 2^{1\times3}x^{4\times3}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 2^3x^{12}$ or $8x^{12}$
b **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets.	**b** $(3a^2b^3)^2 = 3^{1\times2}a^{2\times2}b^{3\times2}$
2 Write the coefficient first, followed by the variables listed in alphabetical order. Simplify where possible.	$= 3^2a^4b^6$ $= 9a^4b^6$
c Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Recall that if the base is negative and the exponent is an odd number, then the basic numeral is negative.	**c** $\left(\frac{-x}{y^2}\right)^3 = \frac{(-x)^{1\times3}}{y^{2\times3}}$ $= \frac{-x^3}{y^6}$ $= -\frac{x^3}{y^6}$
d **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets. The exponent only applies to the terms inside the brackets.	**d** $4(a^2b)^4 = 4 \times a^{2\times4}b^{1\times4}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 4a^8b^4$

Example 1C.3 Simplifying expressions using the zero exponent

Use the property $a^0 = 1$ to simplify each expression.

a 23^0 **b** $3x^0$ **c** $(x^2)^0$

THINK	WRITE
a Any number, excluding 0, to the power of 0 is equal to 1.	**a** $23^0 = 1$
b Any variable to the power of 0 is equal to 1. Recall that an exponent only applies to the number or pronumeral immediately to its left.	**b** $3x^0 = 3 \times x^0$ $= 3 \times 1$ $= 3$
c **1** Multiply the exponent of every base inside the brackets by the exponent outside the brackets. **2** Any variable to the power of 0 is equal to 1.	**c** $(x^2)^0 = x^{2\times 0}$ $= x^0$ $= 1$

Example 1C.4 Simplifying expressions using multiple exponent laws

Use the appropriate exponent law to simplify each expression.

a $(x^3)^5 \times x^2$ **b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$

THINK	WRITE
a **1** Use the power of a power law to simplify the first term. Multiply the exponent of every base inside the brackets by the exponent outside the brackets.	**a** $(x^3)^5 \times x^2 = x^{3\times 5} \times x^2$ $= x^{15} \times x^2$
2 Apply the product of powers law and add the exponents of the common base, *x*.	$= x^{15+2}$ $= x^{17}$
b **1** Simplify the brackets using the power of a power law. Remember BIDMAS.	**b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \dfrac{4x^8 \times 3x^5}{2x^4 \times x^9}$
2 Simplify the numerator and simplify the denominator.	$= \dfrac{12x^{13}}{2x^4 \times x^9}$
3 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract the exponents.	$= \dfrac{12x^{13}}{2x^{13}}$ $= 6x^0$
4 Use the property $a^0 = 1$ to simplify further.	$= 6 \times 1$ $= 6$

Helpful hints

✔ Take care not to mix up the exponent laws.
  → Across a multiplication sign, add exponents.
  → Across a division sign, subtract exponents.
  → Across brackets, multiply exponents.

✔ Remember that $2^0 = 1$, not 0.

Exponent law	Example
Product of powers law	$a^5 \times a^3 = a^{5+3}$
Quotient of powers law	$a^5 \div a^3 = a^{5-3}$
Power of a power law	$(a^5)^3 = a^{5\times 3}$ $(ab)^3 = a^3b^3$ $\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$
The zero exponent	$a^0 = 1$

ANS p405

Exercise 1C Raising exponents and the zero exponent

▲ 1–10(1st, 3rd columns), 11, 12(a–e), 14(a, b)

■ 1–8(2nd, 4th columns), 9, 10(c, h, j, l), 12–14, 15(b), 16

⬟ 2–4(e, f) 6–8(2nd, 4th columns), 9, 10, 12, 14, 15(c, d), 17–19

1C.1 1 Use the power of a power law to simplify the following. Give your answer in exponent form.

a $(6^4)^3$ **b** $(3^2)^2$ **c** $(3^3 \times 4)^2$ **d** $(2^6)^4$

e $(5 \times 2^7)^4$ **f** $\left(\frac{3}{4}\right)^2$ **g** $\left(\frac{5^3}{2^2}\right)^4$ **h** $\left(\frac{1}{8^5}\right)^5$

i $(-3^4)^7$ **j** $(-3^5)^4$ **k** $(-7^4 \times -11^3)^7$ **l** $\left(\frac{13^8}{-17^4}\right)^6$

1C.2 2 Using the power of a power law, simplify each expression.

a $(b^5)^2$ **b** $(m^4)^2$ **c** $(j^5)^2$ **d** $(j^2)^5$ **e** $(n^{10})^8$ **f** $(p^{11})^9$

3 Using the power of a power law, simplify each expression. Give your answer in exponent form.

a $(xy)^6$ **b** $(2d)^3$ **c** $(-5k)^7$ **d** $(9p)^{10}$

e $(-3m)^4$ **f** $\left(\frac{8}{p}\right)^2$ **g** $\left(\frac{x}{y}\right)^6$ **h** $(gh)^2$

i $(ab)^5$ **j** $\left(\frac{k}{m}\right)^3$ **k** $(-2x)^8$ **l** $\left(-\frac{d}{3}\right)^5$

4 Using the power of a power law, simplify each expression. Give your answer in exponent form.

a $(3x^6)^4$ **b** $5(a^4b)^7$ **c** $\left(\frac{2m}{n}\right)^3$ **d** $\left(\frac{a^2}{b^5}\right)^4$

e $-2(u^3)^4$ **f** $\frac{4}{3}(v^7w^3)^{10}$ **g** $\frac{7}{9}\left(\frac{p}{q^6}\right)^7$ **h** $\frac{1}{2^3}(3^2r^9)^5$

i $8\left(\frac{1}{5^6t^{11}}\right)^4$ **j** $7(3i^{17})^2$ **k** $2^3(2^4c^5)^8$ **l** $\frac{5}{7}\left(\frac{5x^{25}}{7y^{30}}\right)^3$

1C.3 5 Use the property $a^0 = 1$ to simplify each expression.

a 34^0 **b** $(18)^0$ **c** y^0 **d** $(7a)^0$

6 Use the property $a^0 = 1$ to simplify each expression.

a $2x^0$ **b** $(2x)^0$ **c** $-7y^0$ **d** $(-7y)^0$

e $-(-3c)^0$ **f** $8^0 + 4^0$ **g** $2 \times 5^0 - 3^0$ **h** $m^0 + m^0$

i $n^0 + p^0$ **j** $a^0 + b^0 + c^0$ **k** $(x + y)^0$ **l** $(-a^0)^4$

m $(5^3)^0$ **n** $(-8)^0$ **o** -8^0 **p** $-(-3)^0$

1C.4 7 Use the exponent laws to simplify each expression.

a $(x^2)^4 \times x^5$ **b** $(x^5)^3 \times x^7$ **c** $x^3 \times (x^4)^6$ **d** $(x^3)^2 \times (x^7)^3$

e $\frac{x^4 \times (x^3)^5}{x^9}$ **f** $\frac{(w^2)^4 \times (w^5)^2}{(w^4)^3}$ **g** $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$ **h** $\frac{e^5 \times e^8}{e^3 \times e^4}$

i $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$ **j** $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$ **k** $\frac{t^8}{(t^2)^5} \times \frac{(t^6)^7}{t^{15}}$ **l** $(f^6)^9 \times \left(\frac{f^7}{f^2}\right)^{11}$

8 Use the exponent laws to simplify each expression.

a $a^3 \div a^3$ **b** $-7x^9 \div x^9$ **c** $(m^2)^3 \div m^6$ **d** $-18(b^4)^5 \div [-6(b^5)^4]$

e $y^7 \times y \div y^8$ **f** $(k^6)^0 \times k^2$ **g** $5g^4 \times 2(-g^7)^0$ **h** $3(w^5)^2 \div (w^2)^5$

i $x^8 \times (x^2)^5 \div x^3$ **j** $4p^7 \times 3p^2 \div (6p^9)$ **k** $16(b^3)^3 \div [-2(b^2)^4]$ **l** $4m^5 \times m \div [10(m^3)^2]$

9 Use the exponent laws to simplify each expression.

a $\frac{5(n^7)^2 \times [-6(n^2)^3]}{15n^2 \times (n^3)^6}$ **b** $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$ **c** $(x^4)^2y^7 \times x^3y^2$

d $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$ **e** $\frac{-3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times [-6h(k^4)^2]}$

PROBLEM SOLVING AND REASONING

10 Use the exponent laws to simplify each expression.

a $(xy)^3 \times x^6y^4$ **b** $(2k)^5 \times (7k)^2$ **c** $(-3x^6)^4$ **d** $-5(a^4b)^7$

e $\frac{x^4}{y^5} \times \left(\frac{x}{y}\right)^6$ **f** $\left(\frac{2m}{n}\right)^3$ **g** $\left(\frac{a^2}{b^5}\right)^4$ **h** $\left(\frac{w^5x^3}{y^4}\right)^2$

i $\left(\frac{k^3m}{n^2}\right)^5 \times \left(\frac{n^3}{k^2m}\right)^4$ **j** $\left(\frac{t^4}{r^2p^3}\right)^5 \times \frac{(4r^5)^2}{p^6t^7}$ **k** $\frac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$ **l** $\frac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

11 Use the exponent laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $(3g)^4 = 3^4 \times g^4$ **b** $-8^0 = -1$ **c** $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y}$ **d** $\frac{(k^3)^2 \times k^4}{k^2} = k^5$

e $6 + k^0 = 7$ **f** $100^9 \div 100^9 = 0$ **g** $\frac{m^3 \times m^8}{m^{11}} = 1$

12 Find the value of x that makes each statement true.

a $2^x = 2^7$ **b** $5^x \times 5^2 = 5^6$ **c** $4^x = 1$ **d** $7^x \div 7^3 = 7^5$

e $(9^x)^2 = 9^6$ **f** $\left(\frac{2}{3}\right)^x = \frac{32}{243}$ **g** $\frac{6^x \times 6^3}{6^5} = 6^5$ **h** $(3a^x)^4 = 81a^{20}$

13 Eden simplified $3^4 \times (3^5)^3$ as $(3^9)^3 = 3^{27}$. Explain and correct her mistake.

14 The power of a power law can be explained in terms of the product of powers law. Complete the following.

a $(2^3)^5 = (2^3) \times (2^3) \times$ ____ $\times$ ____ $\times$ ____
$= 2^{3+3+\square+\square+\square}$
$= 2^{3\times\square}$

b $(x^7)^4 = (x^7) \times$ ____ $\times$ ____ $\times$ ____
$= x^{7+\square+\square+\square}$
$= x^{7\times\square}$

c $(2 \times 3)^4 = (2 \times 3) \times$ ____ $\times$ ____ $\times$ ____
$= 2 \times$ __ $\times$ __ $\times$ __ $\times\ 3 \times$ __ $\times$ __ $\times$ __
$= 2^{\square} \times 3^{\square}$

d $\left(\frac{2}{3}\right)^6 = \frac{2}{3} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square}$
$= \frac{2^{\square}}{3^{\square}}$

15 We can describe multiplication in terms of repeated addition, $2 \times 3 = 2 + 2 + 2$, and raising to the power of an exponent in terms of repeated multiplication, $2^3 = 2 \times 2 \times 2$. However, repeatedly raising a number to the same exponent does not require a new operation as it can be simplified to exponent form.

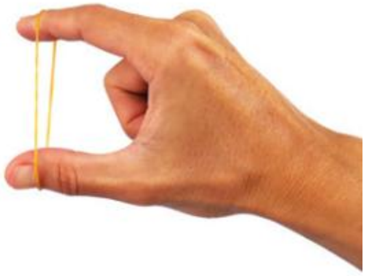
For example: $(((2^5)^5)^5) = 2^{5\times5\times5} = 2^{5^3} = 2^{125}$

Write the following in exponent form with a single exponent.

a $(((((3^2)^2)^2)^2)^2)^2$ **b** $(((5^3)^3)^3)^3$ **c** $((((7^4)^4)^4)^4)^4$ **d** $((((10^5)^5)^5)^5)^5$

16 A cube has side lengths of 8^5 cm. What is the volume of the cube in cm^3? Write your answer in exponent form.

17 A rubber band is stretched to $\frac{4}{3}$ of its current length, and this is repeated another four times until it snaps. How many times longer was the rubber band when it snapped than it was originally?



CHALLENGE

18 Use the power of a power to show that $(a^m)^n = (a^n)^m$.

19 Solve the following equation for x.

$$\frac{12x^6y^7}{7x^2y^{10}z^4} \times \frac{35y^3z^4}{3x^3} = 3$$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Raising exponents and the zero exponent

Topic quiz
1C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

1A **1** Write the following in expanded form then evaluate.

a 2^6 **b** $(-3)^4$ **c** $(-4)^3$ **d** $\left(\frac{5}{6}\right)^3$

1A **2** Write the following in expanded form.

a a^6 **b** $(-b)^4$ **c** $(3y)^5$ **d** $3(xy)^5$

1A **3** Write the following in exponent form.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $u \times u \times u \times u$

c $4b \times 4b \times 4b \times 4b \times 4b$ **d** $-7 \times k \times k \times k \times h \times h \times h \times h \times h$

1A **4** Write the following numbers as a product of their prime factors. Express your answer in exponent form.

a 28 **b** 72 **c** 484 **d** 270

1B **5** Use the exponent laws to simplify each expression. Express your answers in exponent form.

a $8^5 \times 8^6$

b $5^7 \times 7^4 \times 5^3 \times 7^8$

c $\dfrac{6^8}{6^3}$

d $\dfrac{3^{14} \times 10^{12}}{3^6 \times 10^5}$

1B **6** Use the exponent laws to simplify each expression.

a $a^3 \times a^9$

b $4b^{11}c^8 \times (-3b^7c^{13})$

c $\dfrac{u^{14}}{u^9}$

d $\dfrac{-15p^{17}q^{21}}{-21p^3q^4}$

1B **7** Use the exponent laws to simplify each expression. Write your answer in exponent form.

a $\dfrac{3^7 \times 3^{12}}{3^9}$

b $\dfrac{k^{23}}{k^7 \times k^8}$

c $\dfrac{c^3t^8}{c^{14}t^7}$

d $\dfrac{9d^7w^4}{10d^{12}w^7} \times \left(-\dfrac{25d^{17}w^{12}}{6dw^2}\right)$

1C **8** Use the exponent laws to simplify each expression.

a 87^0 **b** $t^5 \div t^5$ **c** $-(4g)^0$ **d** $7a^0 + (8b)^0$

1C **9** Use the exponent laws to simplify each expression. Write your answers in exponent form.

a $(3^4)^6$ **b** $(j^5)^9$ **c** $(-5a^3b^7)^6$ **d** $-\left(\dfrac{3p^5}{2q^7}\right)^8$

1C **10** Use the exponent laws to simplify each expression. Write your answers in exponent form.

a $\dfrac{t^4 \times (t^2)^3}{t^{10}}$

b $\dfrac{3(g^2)^8 \times (3g^5)^3}{(3g)^{11}}$

c $\dfrac{(5m^{11}n^{10})^8 \times (5mn^6)^6}{(5m^9n^7)^2 \times (5m^2n^3)^5}$

d $\dfrac{(8j^5p)^0 \times 6(j^0p^4)^3}{(j^7p^2)^6}$

1D Negative exponents

Learning intentions

By the end of this topic you will be able to ...

- ✔ write a term with a negative exponent as a term with a positive exponent
- ✔ write a term with a positive exponent as a term with a negative exponent
- ✔ apply exponent laws to simplify and evaluate numerical and algebraic expressions with negative exponents.

Inter-year links

Year 7	3F Multiplying fractions
Year 8	2C Multiplying and dividing fractions
Year 10	1B Negative exponents

Negative exponents

$$2^{-1} = \left(\frac{2}{1}\right)^{-1} = \frac{1}{2}$$

$$a^{-1} = \left(\frac{a}{1}\right)^{-1} = \frac{1}{a}$$

- A **negative exponent** is the reciprocal of the base with the positive exponent.
- The reciprocal of a number or a variable n is $\frac{1}{n}$ so the reciprocal of $\frac{m}{n}$ is $\frac{n}{m}$.
  The reciprocal of a fraction is found by swapping the numerator and denominator.
- A negative exponent can be used to write a fraction in exponent form.
- For numbers and variables with a negative exponent, any non-zero number a, and positive integer m:

$$2^{-3} = \left(\frac{2}{1}\right)^{-3} = \frac{1}{2^3} \qquad a^{-m} = \left(\frac{a}{1}\right)^{-m} = \frac{1}{a^m}$$

- For numbers and variables in the denominator of a fraction with a negative exponent, any non-zero number a, and positive integer m:

$$\frac{1}{2^{-3}} = \left(\frac{1}{2}\right)^{-3} = 2^3 \qquad \frac{1}{a^{-m}} = \left(\frac{1}{a}\right)^{-m} = a^m$$

- The exponent laws apply to expressions containing terms with negative exponents.
- The above rules can be simplified to the following.

  For any non-zero a, and positive integer m: $a^{-1} = \frac{1}{a}$, $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$

Example 1D.1 Determining the reciprocal

Determine the reciprocal of each of the following.

a $\frac{3}{2}$ **b** $\frac{1}{4}$ **c** 3 **d** $2x$ **e** $\frac{1}{2x}$

THINK

1. Write the base with an exponent of −1 as a fraction if it is not already.
2. Find the reciprocal of the fraction. Swap the numerator and denominator.
3. Simplify the result.

WRITE

a $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$

b $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1}$
$= 4$

c $3^{-1} = \left(\frac{3}{1}\right)^{-1}$
$= \frac{1}{3}$

d $(2x)^{-1} = \left(\frac{2x}{1}\right)^{-1}$
$= \frac{1}{2x}$

e $\left(\frac{1}{2x}\right)^{-1} = \frac{2x}{1}$
$= 2x$

Example 1D.2 Writing a term with a positive exponent

Write each term with a positive exponent.

a 3^{-3} **b** 7^{-4} **c** x^{-3} **d** $(2x)^{-2}$

THINK

1. Write the base with a negative exponent as a fraction if it is not already.
2. Find the reciprocal of the fraction and write the exponent as a positive number.
3. Use the power of a power law to remove the brackets.
4. Simplify the result.

WRITE

a
$$\begin{aligned}3^{-3} &= \left(\frac{3}{1}\right)^{-3}\\ &= \left(\frac{1}{3}\right)^{3}\\ &= \frac{1}{3^3}\end{aligned}$$

b
$$\begin{aligned}7^{-4} &= \left(\frac{7}{1}\right)^{-4}\\ &= \left(\frac{1}{7}\right)^{4}\\ &= \frac{1}{7^4}\end{aligned}$$

c
$$\begin{aligned}x^{-3} &= \left(\frac{x}{1}\right)^{-3}\\ &= \left(\frac{1}{x}\right)^{3}\\ &= \frac{1}{x^3}\end{aligned}$$

d
$$\begin{aligned}(2x)^{-2} &= \left(\frac{2x}{1}\right)^{-2}\\ &= \left(\frac{1}{2x}\right)^{2}\\ &= \frac{1}{(2x)^2}\\ &= \frac{1}{4x^2}\end{aligned}$$

Example 1D.3 Writing fractions with positive exponents

Write each fraction in exponent form with a positive exponent.

a $\frac{1}{3^{-2}}$ **b** $\frac{1}{(2x)^{-1}}$ **c** $\left(\frac{2}{5}\right)^{-3}$ **d** $\left(\frac{3x}{y}\right)^{-2}$

THINK

1. Write the base with a negative exponent as a fraction if it is not already.
2. Find the reciprocal of the fraction and write the exponent as a positive number.
3. Use the power of a power to remove the brackets.
4. Simplify the calculation. Dividing by a fraction is the same as multiplying by the reciprocal.
5. Simplify the result.

WRITE

a
$$\begin{aligned}\frac{1}{3^{-2}} &= \frac{1}{\left(\frac{3}{1}\right)^{-2}}\\ &= \frac{1}{\left(\frac{1}{3}\right)^{2}}\\ &= 1 \times \frac{3^2}{1}\\ &= 3^2\end{aligned}$$

b
$$\begin{aligned}\frac{1}{(2x)^{-1}} &= \frac{1}{\left(\frac{2x}{1}\right)^{-1}}\\ &= \frac{1}{\left(\frac{1}{2x}\right)^{1}}\\ &= 1 \times \frac{2x}{1}\\ &= 2x\end{aligned}$$

c
$$\begin{aligned}\left(\frac{2}{5}\right)^{-3} &= \left(\frac{5}{2}\right)^{3}\\ &= \frac{5^3}{2^3}\end{aligned}$$

d
$$\begin{aligned}\left(\frac{3x}{y}\right)^{-2} &= \left(\frac{y}{3x}\right)^{2}\\ &= \frac{y^2}{3^2x^2}\end{aligned}$$

Example 1D.4 Writing terms with positive exponents

Write each term with positive exponents.

a $x^{-5}y^3$ **b** $\frac{5x^{-4}}{y^{-7}}$ **c** $\frac{6a^7b^{-2}}{3c^{-4}}$

THINK	WRITE
a 1 Write the term as a product of two factors and write the base with a negative exponent as a fraction.	**a** $x^{-5}y^3 = \left(\frac{x}{1}\right)^{-5} \times y^3$
2 Find the reciprocal of the fraction and write the exponent as a positive number.	$= \frac{1}{x^5} \times y^3$
3 Simplify the result.	$= \frac{y^3}{x^5}$
b 1 Write the term as a product of three factors and write the base with a negative exponent as a fraction.	**b** $\frac{5x^{-4}}{y^{-7}} = 5 \times \left(\frac{x}{1}\right)^{-4} \times \frac{1}{\left(\frac{y}{1}\right)^{-7}}$
2 Find the reciprocal of the fraction and write the exponent as a positive number.	$= 5 \times \frac{1}{x^4} \times 1 \times \frac{y^7}{1}$
3 Simplify the result.	$= \frac{5y^7}{x^4}$
c 1 Cancel 6 and 3 by a common factor of 3.	**c** $\frac{6a^7b^{-2}}{3c^{-4}} = \frac{\cancel{6}^{2}a^7b^{-2}}{\cancel{3}^{1}c^{-4}}$ $= \frac{2a^7b^{-2}}{c^{-4}}$
2 Write the term as a product of four factors and write the base with a negative exponent as a fraction.	$= 2 \times a^7 \times \left(\frac{b}{1}\right)^{-2} \times \frac{1}{\left(\frac{c}{1}\right)^{-4}}$
3 Find the reciprocal of the fraction and write the exponent as a positive number.	$= 2 \times a^7 \times \frac{1}{b^2} \times c^4$
4 Simplify the result.	$= \frac{2a^7c^4}{b^2}$

Example 1D.5 Simplifying expressions with negative exponents using exponents laws

Use an appropriate exponent law to simplify each expression. Write your answers using positive exponents.

a $3^5 \times 3^{-7}$ **b** $2^4 \div 2^{-3}$ **c** $(5^{-6})^2 \times 5^3$

THINK	WRITE
a 1 Apply the product of powers law to multiply the terms. Write the base and add the exponents.	**a** $3^5 \times 3^{-7} = 3^{5+(-7)}$ $= 3^{-2}$
2 Find the reciprocal of the fraction and write the exponent as a positive number.	$= \frac{1}{3^2}$
b Apply the quotient of powers law to divide the terms. Write the base and subtract the exponents.	**b** $2^4 \div 2^{-3} = 2^{4-(-3)}$ $= 2^7$
c 1 Apply the power of a power to simplify the first term. Multiply the exponent of every base inside the brackets by the exponent outside the brackets.	**c** $(5^{-6})^2 \times 5^3 = 5^{-6\times2} \times 5^3$ $= 5^{-12} \times 5^3$
2 Apply the product of powers law to multiply the terms. Write the base and add the exponents.	$= 5^{-12+3}$ $= 5^{-9}$
3 Find the reciprocal of the fraction and write the exponent as a positive number.	$= \frac{1}{5^9}$

Helpful hints

✔ Keep the exponent laws and the rules for negative exponents close by until they become second nature.

$$a^{-1} = \frac{1}{a} \qquad a^{-m} = \frac{1}{a^m} \qquad \frac{1}{a^{-m}} = a^m$$

✔ If you want to move a number or a variable from the numerator to the denominator, remember that a 1 is left in its place, not zero.

For example: $\frac{a}{b} = \frac{a}{1} \times \frac{1}{b}$

$$= \frac{1}{a^{-1}} \times \frac{1}{b}$$

$$= \frac{1}{a^{-1}b}$$

✔ Don't confuse negative exponents with negative numbers. For example: $2^{-3} = \frac{1}{2^3}$ and $2^{-3} \neq -(2^3)$

ANS p406

Exercise 1D Negative exponents

△ 1–12(1st, 2nd columns), 14, 15(a–d), 16, 19, 21, 22

■ 1–12(2nd, 4th columns), 14, 15(e–h), 17, 20–23, 25

⬠ 1(c, f, h), 6–12(4th column), 13, 14, 15(e–h), 18, 22, 24–28

UNDERSTANDING AND FLUENCY

1D.1 **1** Determine the reciprocal of each of the following.

a $\frac{8}{7}$
b $\frac{1}{2}$
c -3
d $3w$
e $\frac{1}{d}$
f $\frac{r}{m}$
g $\frac{1}{5y}$
h u^2

1D.2 **2** Write each term with a positive exponent.

a 5^{-1}
b 8^{-1}
c $(-2)^{-1}$
d x^{-1}
e p^{-1}
f $(3w)^{-1}$

3 Write each of the following in exponent form with a negative exponent.

a $\frac{1}{5}$
b $-\frac{1}{13}$
c 5
d -8
e $\frac{1}{m}$
f x
g $\frac{1}{5y}$
h $3w$
i $\frac{4}{3}$
j $\frac{5}{6}$
k $\frac{w}{a}$
l v^3

4 Write each term with a positive exponent.

a 4^{-2}
b 2^{-6}
c $(-9)^{-3}$
d $(-5)^{-4}$
e -7^{-8}
f 10^{-5}
g a^{-4}
h x^{-7}
i k^{-10}
j m^{-2}
k u^{-9}
l g^{-11}

5 Write each fraction in exponent form with a negative exponent.

a $\frac{1}{3^4}$
b $\frac{1}{4^7}$
c $\frac{1}{6^5}$
d $\frac{1}{(-5)^3}$
e $\frac{1}{(-9)^2}$
f $-\frac{1}{11^6}$
g $\frac{1}{n^2}$
h $\frac{1}{g^{11}}$
i $\frac{1}{x^8}$
j $\frac{1}{a^9}$
k $\frac{1}{p^4}$
l $\frac{1}{w^7}$

UNDERSTANDING AND FLUENCY

1D.3 **6** Write each fraction in exponent form with positive exponents.

a $\frac{1}{2^{-3}}$ **b** $\frac{1}{5^{-6}}$ **c** $\frac{1}{(-8)^{-4}}$ **d** $\frac{1}{3^{-9}}$

e $\frac{1}{(-7)^{-5}}$ **f** $-\frac{1}{4^{-2}}$ **g** $\frac{1}{x^{-7}}$ **h** $\frac{1}{y^{-3}}$

i $\frac{1}{c^{-4}}$ **j** $\frac{1}{z^{-35}}$ **k** $\frac{1}{(5t)^{-4}}$ **l** $\frac{1}{(uv)^{-15}}$

7 Write each fraction in exponent form with positive exponents.

a $\left(\frac{4}{5}\right)^{-2}$ **b** $\left(\frac{7}{3}\right)^{-1}$ **c** $\left(-\frac{3}{4}\right)^{-3}$ **d** $\left(-\frac{a}{2}\right)^{-4}$

e $\left(\frac{6}{m}\right)^{-1}$ **f** $\left(\frac{144}{u}\right)^{-1}$ **g** $\left(\frac{x}{y}\right)^{-6}$ **h** $\left(-\frac{9}{7}\right)^{-11}$

i $\left(-\frac{13}{17}\right)^{-14}$ **j** $\left(\frac{500}{43}\right)^{-11}$ **k** $\left(\frac{u}{97}\right)^{-198}$ **l** $\left(\frac{123}{g}\right)^{-654}$

1D.4 **8** Write each term with positive exponents.

a $x^{-2}y^{3}$ **b** $m^{6}n^{-4}$ **c** $a^{-1}c^{7}$ **d** $2k^{5}p^{-3}$

e $5a^{-8}b^{2}$ **f** $-4x^{-6}w^{-2}$ **g** $a^{4}b^{-5}c^{7}$ **h** $k^{-3}m^{5}n^{-8}$

i $7b^{9}c^{-6}d$ **j** $3x^{-2}y^{-7}z^{-4}$ **k** $-12p^{17}q^{-18}u^{-21}$ **l** $-34j^{-65}b^{-78}$

9 Write each term with positive exponents.

a $\frac{a^{-5}}{b^{-3}}$ **b** $\frac{k^{-6}}{p^{-2}}$ **c** $\frac{h^{-4}}{m^{-7}}$ **d** $\frac{e^{8}}{d^{-5}}$

e $\frac{u^{3}}{w^{-8}}$ **f** $\frac{3x^{2}}{y^{-4}}$ **g** $\frac{8h^{5}}{g^{-6}}$ **h** $\frac{-6d^{-5}}{3c^{-9}}$

i $\frac{-4m^{-7}}{-10n^{-3}}$ **j** $\frac{y^{-8}}{6e^{5}}$ **k** $\frac{6g^{-99}}{-12h^{11}}$ **l** $\frac{14r^{-72}}{35c^{-101}}$

10 Write each term with positive exponents.

a $\frac{m^{-3}n^{4}}{p^{-6}}$ **b** $\frac{8c^{2}d^{-5}}{2e^{-6}}$ **c** $\frac{4k^{-1}n^{-3}}{6p^{-4}}$ **d** $\frac{3^{-1}a^{2}c^{-5}}{b^{-3}d^{4}}$

e $\frac{m^{-6}n}{7^{-1}k^{-2}}$ **f** $\frac{5^{-1}k^{-2}x^{3}}{2u^{-5}w^{8}}$ **g** $\frac{-5q^{4}r^{-4}}{p^{-1}s^{3}}$ **h** $\frac{7b^{14}u^{10}}{3^{-2}g^{-15}k^{-17}}$

i $\frac{t^{-4}u^{-3}}{5v^{7}w^{12}}$ **j** $\frac{-4^{-3}l^{-88}n^{41}}{7^{-2}m^{-59}v}$ **k** $\frac{a^{97}b^{-105}c^{153}}{d^{-167}e^{111}f^{-89}}$ **l** $\frac{k^{402}l^{812}}{m^{-999}n^{-571}}$

1D.5 **11** Use an appropriate exponent law to simplify each expression. Write your answers in exponent form with positive exponents.

a $4^{-5} \times 4^{2}$ **b** $7^{3} \times 7^{-4}$ **c** $2^{-6} \times 2^{8}$ **d** $(-3)^{-1} \times (-3)^{-5}$

e $5^{7} \times 5^{-3}$ **f** $(-2)^{-4} \div (-2)^{3}$ **g** $9^{5} \div 9^{7}$ **h** $3^{6} \div 3^{-2}$

i $4^{-1} \div 4^{8}$ **j** $10^{-7} \div 10^{-4}$ **k** $211^{-9} \times 211^{-5}$ **l** $13^{-87} \div 13^{13}$

12 Use an appropriate exponent law to simplify each expression. Write your answers in exponent form with positive exponents.

a $(5^{-3})^{2}$ **b** $(3^{-2})^{4}$ **c** $(-2^{-4})^{-1}$ **d** $(3^{-1})^{4} \times 3^{2}$

e $(6^{-5})^{3} \times 6^{11}$ **f** $(4^{-2})^{3} \times (4^{-5})^{-1}$ **g** $9^{3} \times 9^{-6} \times 9^{2}$ **h** $\frac{5^{4} \times 5^{-2}}{5^{-6}}$

i $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$ **j** $\frac{2^{8} \times (2^{-2})^{3}}{2^{5}}$ **k** $\frac{(99^{-12})^{-6} \times 99^{15}}{(99^{8})^{-5}}$ **l** $\frac{(15^{-9})^{8} \times (15^{7})^{6}}{(15^{11})^{12}}$

13 Using a calculator, calculate the basic numeral for parts **a** to **f** in question **11**. Write your answer as a whole number or fraction.

14 Use an appropriate exponent law to write each expression without brackets using positive exponents only.

a $(a \times b)^{-7}$ **b** $(3 \times x)^{-5}$ **c** $(4 \times y)^{-1}$

d $(7k)^{-2}$ **e** $(2p)^{-3}$ **f** $(mk)^{-11}$

15 Find the value of x that makes each statement true.

a $2^{x} = \frac{1}{2^{3}}$ **b** $5^{x} = \frac{1}{5^{7}}$ **c** $3^{x} = \frac{1}{3}$ **d** $6^{x} = \frac{1}{6^{-2}}$

e $4^{x} = \frac{1}{16}$ **f** $3^{x} = \frac{1}{27}$ **g** $5^{x} = \frac{1}{25}$ **h** $10^{x} = \frac{1}{10\,000}$

16 A microscopic worm is 4^{-3} mm in length. Using a calculator, write this length in millimetres:

a as a fraction

b as a decimal.

17 The time for light to travel 3 m is about 10^{-8} s. Using a calculator, write this time in seconds:

a as a fraction

b as a decimal.

18 The diameter of a strand of human hair is about 5^{-6} m. Using a calculator, write this measurement in metres:

a as a fraction

b as a decimal.

19 a Complete this table.

Exponent form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8			1		$\frac{1}{4}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 2^{-6} as a fraction.

d Given 2^{10} is 1024, write the value of 2^{-10} as a fraction.

e Given 2^{-7} is $\frac{1}{128}$, write the value of 2^7.

20 a Complete this table.

Exponent form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral		81	27			1		$\frac{1}{9}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 3^{-6} as a fraction.

d Given that 3^8 is 6561, write the value of 3^{-8} as a fraction.

e Given that 3^{-7} is $\frac{1}{2187}$, write the value of 3^7.

21 a Complete this table.

Exponent form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral			100			$\frac{1}{10}$	$\frac{1}{100}$		

b Describe the pattern you can see in the table.

c Write the value of each term as a whole number.

i 10^5 **ii** 10^6 **iii** 10^7 **iv** 10^8 **v** 10^9

d Write the value of each term as a fraction.

i 10^{-5} **ii** 10^{-6} **iii** 10^{-7} **iv** 10^{-8} **v** 10^{-9}

e Write 10^{-1} as:

i a fraction **ii** a decimal.

f Write each term as a decimal. Hint: Use the matching fractions from your table.

i 10^{-2} **ii** 10^{-3} **iii** 10^{-4}

g Write each fraction in part **d** as a decimal.

h Explain any shortcuts you have used to obtain your answers to parts **c** to **g**.

PROBLEM SOLVING AND REASONING

22 a Without using a calculator, find the whole number value of each of the following. Hint: What shortcut can you use when multiplying by a positive power of 10?

i 2×10^4 **ii** 7×10^3 **iii** 3×10^5

iv 4×10^{11} **v** 9×10^7

b Write each expression as a fraction involving positive exponents.

i 5×10^{-2} **ii** 8×10^{-5} **iii** 2×10^{-3}

iv 7×10^{-4} **v** 6×10^{-9}

c Without using a calculator, find the decimal value of each result in part **b**. Hint: What shortcut can you use when dividing by a positive power of 10?

d Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of 10.

23 a Complete the following by writing the missing numerals and operation.

i $2 \times 3^{-1} = 2 \times \frac{\square}{\square} = \frac{2}{\square} = 2 \square 3$

ii $2 \div 3^{-1} = \frac{2}{\square} = 2 \times \frac{\square}{\square} = 2 \square 3$

b Explain the connection between multiplication, division, and reciprocals.

24 a Evaluate the following.

i $(3^{-1})^{-1}$ **ii** $(5^{-1})^{-1}$ **iii** $\left(\left(\frac{5}{4}\right)^{-1}\right)^{-1}$

b Use exponent laws to explain why $(a^{-1})^{-1} = a$.

c Explain what $(a^{-1})^{-1} = a$ means in terms of reciprocals.

25 Write the following as products without fractions by using negative exponents.

For example: $\frac{9x^3}{y^2} = 9x^3y^{-2}$

a $\frac{2}{a^2b}$ **b** $-\frac{3t^2}{v^3}$ **c** $\frac{x^4}{5y^4}$ **d** $\frac{1}{pq^5r^{-2}}$

CHALLENGE

26 Use an appropriate exponent law to simplify each expression. Write your answer using positive exponents only.

a $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$

e $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$

i $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$

m $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$

27 Write all answers from question **26** that have a positive exponent using a negative exponent.

28 Write $a^m \div a^n$ in each of the following different forms.

a $a^{\square} \times a^{\square} = a^{\square + \square}$

b $\frac{1}{a^{\square} \times a^{\square}} = \frac{1}{a^{\square + \square}}$

c Two variations of $a^{\square} \div a^{\square} = a^{\square - \square}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Negative exponents

Topic quiz
1D

1E Scientific notation

Learning intentions

By the end of this topic you will be able to ...

✔ convert between scientific notation and basic numerals

✔ identify significant figures in a number and express it in scientific notation.

Inter-year links

Support	Place value
Year 7	1A Place value
Year 8	1A Rounding and estimating
Year 10	6F Logarithmic scales

Scientific notation

- **Scientific notation** (or **standard form**) is a simple way of writing very large and very small numbers.
- A number is written in scientific notation if it is the product of a number, the significand, a, and a power of 10, written in the form $a \times 10^m$ where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer.
  - → If m is a positive integer, the number is greater than or equal to 10.
  - → If m is a negative integer, the number is between 0 and 1.
  - → If m is zero, the number is either between 1 and 10 or −1 and −10.

$$\underset{\text{basic numeral}}{1230} = \underset{\text{scientific notation}}{\overbrace{1.23}^{\text{significand}} \times 10^3}$$

$$\underset{\text{basic numeral}}{0.00\,123} = \underset{\text{scientific notation}}{\overbrace{1.23}^{\text{significand}} \times 10^{-3}}$$

- ‘Approximate value’ is a term referring to a value obtained by a calculation that uses rounded values. Numbers in scientific notation are usually approximate values.
- The exponent laws can be used to perform operations on numbers in scientific notation.
- To convert a number in scientific notation to a basic numeral, the exponent indicates the number of places the decimal point is moved.
  - → If the exponent is positive, move the decimal point to the right.
  - → If the exponent is negative, move the decimal point to the left.

$$10^{+} \longrightarrow$$

$$10^{-} \longleftarrow$$

- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10.

$$\underbrace{31500}_{\text{move 4 spaces to the left}} = 3.15 \times 10^4 \longleftarrow \text{exponent of 4}$$

$$\underbrace{0.042}_{\text{move 2 spaces to the right}} = 4.2 \times 10^{-2} \longleftarrow \text{exponent of } -2$$

- Scientific notation takes advantage of our base 10 number system, as each place value represents a power of 10. Where we can write the expanded form of a numeral using powers of 10, scientific notation uses only the highest power of 10 from the expansion.

Place value	**Ten thousands**	**Thousands**	**Hundreds**	**Tens**	**Ones**	**.**	**Tenths**	**Hundredths**	**Thousandths**
Exponent form	10^4	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}
Basic numeral	10 000	1000	100	10	1	.	0.1	0.01	0.001

Significant figures

- **Significant figures** are the number of digits in a number that contribute to the level of **accuracy**.
- When counting significant figures, count the first non-zero digit from left to right.
  - → All non-zero digits are significant.
    For example, 7.789 has four significant figures as all are non-zero.
  - → Zeros between two non-zero digits are significant.
    For example, 4056 has four significant figures including the zero between 4 and 5.
  - → Leading zeros are not significant.
    For example, 0.051 has two significant figures. All the zeros are leading zeros.
  - → Trailing zeros to the right of the decimal after the last non-zero significant digit are significant.
    For example, 112.00 has five significant figures. 0.0780 has three significant figures.
  - → Trailing zeros in an integer are not significant.
    For example, 8300 has two significant figures. The zeros are not significant in the integer.
- If a number is already in scientific notation, all numbers in the significand are significant.
  For example, 2.301×10^{-2} has four significant figures.

Example 1E.1 Writing numbers in scientific notation as basic numerals

Write each number as a basic numeral.

a 2.4×10^6 **b** 7.1×10^{-8}

THINK	WRITE
a Multiply by 10^6 (or 1 000 000). When multiplying by 10^6, move the decimal point six place-value spaces to the right. Add zeros where necessary.	**a** 2.400000 $2.4 \times 10^6 = 2\,400\,000$
b Multiply by 10^{-8} (or divide by 10^8). When dividing by 10^8, move the decimal point eight place-value spaces to the left. Add zeros where necessary.	**b** 000000007.1 $7.1 \times 10^{-8} = 0.000\,000\,071$

Example 1E.2 Writing numbers in scientific notation

Write each number in scientific notation.

a 230 000 **b** 0.000 856

THINK	WRITE
a Count the number of places the decimal point in 230 000 would be moved to produce 2.3. The decimal point needs to be moved five places to the right to obtain the original number, so the exponent is 5.	**a** 230000 $230\,000 = 2.3 \times 10^5$
b Count the number of places the decimal point in 0.000 856 would be moved to produce 8.56. The decimal point needs to be moved four places to the left to obtain the original number, so the exponent is −4.	**b** 0.000856 $0.000\,856 = 8.56 \times 10^{-4}$

Example 1E.3 Identifying significant figures

How many significant figures are shown in each number?

a 5.42 **b** 20 803 **c** 6.200 **d** 4000 **e** 0.0082

THINK	WRITE
a All non-zero digits are significant.	**a** 5.42 has three significant figures.
b Zeros between non-zero digits are significant.	**b** 20 803 has five significant figures.
c Zeros at the end of a decimal number are significant.	**c** 6.200 has four significant figures.
d Zeros at the end of an integer are not significant.	**d** 4000 has one significant figure.
e Zeros to the left of the first non-zero digit in a decimal number are not significant.	**e** 0.0082 has two significant figures.

Example 1E.4 Writing numbers in scientific notation using significant figures

Write each number in scientific notation with the number of significant figures indicated in brackets.

a 53 726 (2) **b** 0.084 03 (3)

THINK	WRITE
a **1** This number has five significant figures. Round to two significant figures (the nearest thousand). Remember that zeros at the end of an integer are not significant.	**a** $53\,726 \approx 54\,000$
2 Write in scientific notation.	$= 5.4 \times 10^{4}$
b **1** This number has four significant figures. Round to three significant figures (the nearest ten-thousandth).	**b** $0.084\,03 \approx 0.0840$
2 Write in scientific notation. Remember that zeros at the end of a decimal are significant.	$= 8.40 \times 10^{-2}$

Helpful hints

✔ When converting from scientific notation to a basic numeral, remember that if the exponent is positive, move the decimal point to the right, and if the exponent is negative, move the decimal point to the left.

10^{+} →

10^{-} ←

✔ Multiplying a number by 10 increases each digit's place value by 1 column. Move the decimal point one place-value space to the right and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$

$$5.23 \times 100 = 523.$$

$$5.23 \times 1000 = 5230.$$

✔ Dividing a number by 10 decreases each digit's place value by 1 column. Move the decimal point one place-value space to the left and insert a zero where necessary.

ANS
p408

Exercise 1E Scientific notation

△ 1, 2, 3(1st column), 4, 5–9(1st, 2nd columns), 10–12, 14(a–c), 17, 20, 22

■ 3(2nd column), 4, 5–9(1st, 2nd columns), 13, 14, 16, 21, 23, 26

⬠ 4, 5(4th column), 6–7(2nd, 3rd columns), 8–9(3rd, 4th columns), 14, 15, 18, 19, 24, 25, 27

UNDERSTANDING AND FLUENCY

1 Calculate each of these. Hint: Move the decimal point an appropriate number of places.

a 5.4×100 **b** $7.36 \times 10\ 000$ **c** -1.8×1000

d $4.05 \times 100\ 000$ **e** $2.753 \times 1\ 000\ 000$ **f** $\frac{6.1}{10}$

g $\frac{8.22}{1\,000\,000}$ **h** $\frac{-9.76}{10\,000}$ **i** $\frac{7.003}{100\,000}$

2 Write each number as a power of 10.

a 100 **b** 1000 **c** 10 000 **d** 100 000 **e** 1 000 000

f 0.1 **g** 0.01 **h** 0.001 **i** 0.0001 **j** 0.000 01

1E.1 **3** Write each number as a basic numeral.

a 3.2×10^5 **b** 8.14×10^9 **c** -5.0×10^2

d -2.345×10^7 **e** 1.1×10^4 **f** 6.4×10^{-3}

g 7.28×10^{-6} **h** 9×10^{-7} **i** -3.02×10^{-5}

j -5.41×10^{-2} **k** 4.5×10^{11} **l** 6.12×10^{-9}

m 5.7×10^{-1} **n** 1.3068×10^3 **o** 2.7316×10^{-4}

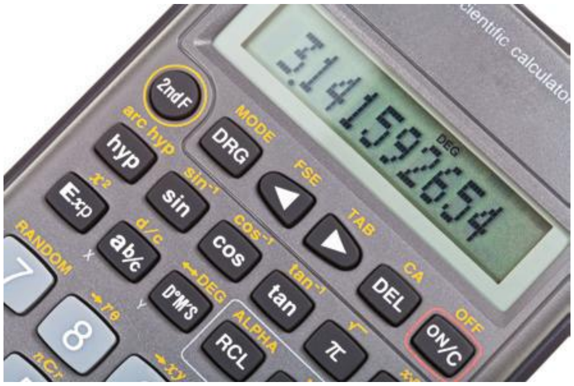
4 Calculators have different methods for displaying scientific notation. Most scientific calculators have a button for entering numbers in scientific notation quickly. It is usually labelled with a bold **E**, Exp, $\times 10^x$ or $\times 10^n$.

Check with your teacher if you cannot find this button.

To use the button, type the significand, press the scientific notation button, and then type the exponent of 10.

a Use a calculator to verify each number in question **3**.

b Were there any numbers that you could not easily obtain on your calculator? Explain.



1E.2 **5** Write each number in scientific notation.

a 4500 **b** 7 320 000 **c** 200 000 **d** −190

e 3216 **f** 0.0063 **g** 0.000 000 18 **h** 0.05

i −0.000 0702 **j** 0.427 **k** 11 220 **l** 0.000 004

m −568.2 **n** 0.000 249 **o** 679 300 **p** −0.0102

1E.3 **6** How many significant figures are shown in each number?

a 345 **b** 25 000 **c** 5072

d 400 **e** −809 **f** 0.59

g −0.003 **h** 1.472 **i** 48.062

j −7.300 **k** 36 020 **l** 0.009 04

7 How many significant figures are shown in each number?

a 2.4×10^3 **b** 5.06×10^{-4} **c** 1.900×10^7

d 8.0×10^5 **e** -3.206×10^{-9} **f** 7.00×10^5

8 Round each number to the number of significant figures indicated in brackets.

- **a** 2.58×10^5 (2)
- **b** -5.037×10^4 (3)
- **c** 9.1042×10^6 (4)
- **d** -6.00×10^3 (2)
- **e** 458 (2)
- **f** 73 051 (4)
- **g** 1279 (1)
- **h** 40 008 (1)
- **i** −5.1437 (3)
- **j** 0.0349 (2)
- **k** −42.0607 (4)
- **l** 0.852 (1)

1E.4 **9** Write each number in scientific notation with the number of significant figures indicated in brackets.

- **a** 327 (2)
- **b** 48 654 (3)
- **c** −190 760 (4)
- **d** 2621 (1)
- **e** 0.4031 (3)
- **f** −0.0544 (2)
- **g** 0.000 207 193 (4)
- **h** −0.008 327 (1)
- **i** 758.4 (2)
- **j** −20 703.02 (4)
- **k** 40.155 (3)
- **l** 54 007.63 (5)

10 Consider the following numbers **A–H**.

- **A** 3.4×10^4
- **B** 2.03×10^{-3}
- **C** -0.58×10^6
- **D** 60.34×10^2
- **E** 0.009
- **F** -4.19×10^3
- **G** 700×10^5
- **H** 9×10^{-4}

- **a** Which numbers are written in scientific notation?
- **b** Which numbers are written with three significant figures?
- **c** Which numbers are larger than 10?
- **d** Which numbers are less than 1?

11 Write each approximate measurement in scientific notation.

- **a** A medium-sized grain of sand has a length of 0.0005 m.
- **b** The Cooby Creek Reservoir has a capacity of 23 000 ML.
- **c** The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.
- **d** An estimate for the world's population in 2050 is 9 300 000 000.

12 Write each approximate measurement as a basic numeral.

- **a** The number of times the wings of a hummingbird flap in a minute is 6.4×10^3.
- **b** The diameter of a virus is 8×10^{-5} mm.
- **c** The distance from the Sun to Earth is 1.496×10^8 km.
- **d** The radius of an electron is 2.8×10^{-13} cm.

13 Complete the table below by writing the numbers as a product with each of the powers of 10. Underline the answers that are in scientific notation. The first row has been completed for you.

	1234.56	**4.0191**	**0.0492**	**0.007 40**
$\times 10^3$	$\underline{1.234\ 56 \times 10^3}$	$0.004\ 0191 \times 10^3$	$0.000\ 0492 \times 10^3$	$0.000\ 007\ 40 \times 10^3$
$\times 10^2$				
$\times 10^1$				
$\times 10^0$				
$\times 10^{-1}$				
$\times 10^{-2}$				
$\times 10^{-3}$				

14 We can perform arithmetic operations in scientific notation. Multiplication and division can be performed by multiplying or dividing the significands and then using the product and quotient of powers laws to multiply or divide the powers of 10. For example:

$$\begin{aligned} &(2.1 \times 10^7) \times (8.4 \times 10^3) \\ &= (2.1 \times 8.4) \times (10^7 \times 10^3) \\ &= 17.64 \times 10^{10} \\ &= (17.64 \div 10) \times 10^{10+1} \\ &= 1.764 \times 10^{11} \end{aligned}$$

$$\begin{aligned} &(2.1 \times 10^7) \div (8.4 \times 10^3) \\ &= (2.1 \div 8.4) \times (10^7 \div 10^3) \\ &= 0.25 \times 10^4 \\ &= (0.25 \times 10) \times 10^{4-1} \\ &= 2.5 \times 10^3 \end{aligned}$$

Evaluate the following products and quotients. Write your answer in scientific notation.

- **a** $(1.7 \times 10^5) \times (4 \times 10^2)$
- **b** $(8 \times 10^7) \div (4 \times 10^5)$
- **c** $(-5 \times 10^{-5}) \times (-9 \times 10^8)$
- **d** $(-6 \times 10^9) \div (1.5 \times 10^5)$
- **e** $(4.1 \times 10^{-6}) \times (-3 \times 10^4)$
- **f** $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

PROBLEM SOLVING AND REASONING

15 Addition and subtraction require digits with the same place value to be added together. Therefore, in scientific notation, both numbers must be written using the same power of 10 so that the digits in the significands have the same place value relative to the decimal point. For example:

$$\begin{aligned}&(2.1 \times 10^5) + (8.4 \times 10^3)\\ &= (2.1 \times 10^5) + (0.084 \times 10^5)\\ &= (2.1 + 0.084) \times 10^5\\ &= 2.184 \times 10^5\end{aligned} \qquad \begin{aligned}&(2.1 \times 10^{-2}) - (8.4 \times 10^{-3})\\ &= (2.1 \times 10^{-2}) - (0.84 \times 10^{-2})\\ &= (2.1 - 0.84) \times 10^{-2}\\ &= 1.26 \times 10^{-2}\end{aligned}$$

Evaluate the following sums and differences. Write your answer in scientific notation.

a $(3.4 \times 10^2) + (7.3 \times 10^5)$ **b** $(8.52 \times 10^4) - (1.6 \times 10^3)$ **c** $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$

d $(8.2 \times 10^{-3}) - (3.5 \times 10^{-2})$ **e** $(-9.8 \times 10^3) + (-7.7 \times 10^2)$ **f** $(1.01 \times 10^5) - (7.5 \times 10^3)$

16 Light travels at a speed of approximately 3.00×10^{10} cm/s. How many kilometres does it travel in 1 hour? Give your answer in scientific notation.

17 Earth revolves around the Sun at an average speed of 10^5 km/h.

a What distance does Earth travel in 1 day?

b How many days would it take Earth to travel 9.6×10^8 km?

18 The Australian \$1 coin has a mass of 9 g and a thickness of 3×10^{-1} cm.

a Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.

i How many coins are in the stack?

ii What would be the mass of these coins?

b Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a \$1 coin?

19 The Sun is 1.52×10^8 km from Earth. Light from the Sun travels towards Earth at a speed of 3×10^8 m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.

20 a Round each of the following to one, two and three significant figures.

i 1.901 **ii** 1.994 **iii** 1.997

iv 2.003 **v** 2.006 **vi** 2.098

b Explain how the significant trailing zeros are important in determining to how many significant figures a number is rounded.

21 Consider 0.41, 0.000 000 000 0012 and −0.000 034.

a Round each number to one significant figure.

Assume, only for this question, that we included leading zeros as significant.

b Round each number to one significant figure.

c Explain why not including leading zeros as significant is more useful than including them. Consider how including them is similar to rounding to a place value or number of decimal places.

22 Explain the mistake each student made.

a Jane rounded 4.1025 to three significant figures as 4.103.

b Kaleb rounded 0.0432 to three significant figures as 0.04.

c Lisa rounded 102 948.3618 to three significant figures as 102 900.

d Marius rounded 102 948.3618 to three significant figures as 103.

23 a Write the following in seconds in scientific notation.

i 47 minutes **ii** 14 days

iii 40 weeks **iv** 1 year (not a leap year)

b Write the following in the unit in brackets correct to three significant figures.

i 10 000 seconds (hours) **ii** 1 000 000 seconds (days)

iii 1 000 000 000 seconds (years) **iv** 1 000 000 000 000 seconds (millennia)

24 We use metric prefixes for very large and very small measurements. These are closely related to engineering notation or engineering form, which is similar to scientific notation.

A number written in engineering notation is the product of a number, the significand, a, between positive or negative 1 (inclusive) and positive or negative 1000 (exclusive) and a power of 1000 written as a power of 10 in exponent form. That is, $a \times 10^{3m}$ where $1 \le a < 1000$ or $-1000 < a \le -1$ and m is an integer.

For example, 3.456×10^9, 34.56×10^6, 345.6×10^{-6} are in engineering form but 0.3456×10^9, 3456×10^6, 345.6×10^{-5} are not in engineering form.

Prefix	Abbreviation	Meaning	Power of 10
tera	T	one trillion of the unit	10^{12}
giga	G	one billion of the unit	10^9
mega	M	one million of the unit	10^6
kilo	k	one thousand of the unit	10^3
milli	m	one-thousandth of the unit	10^{-3}
micro	µ	one-millionth of the unit	10^{-6}
nano	n	one-billionth of the unit	10^{-9}
pico	p	one-trillionth of the unit	10^{-12}

a Write the following in seconds in engineering form.

i 7.3 kiloseconds (7.3 ks)

ii 9.1 microseconds (9.1 µs)

iii 54 nanoseconds (54 ns)

iv 82 teraseconds (82 Ts)

v 129 megaseconds (129 Ms)

vi 974 picoseconds (974 ps)

b Write the following times in engineering form using the appropriate prefix.

i 5.601×10^7 seconds

ii 9.2×10^5 seconds

iii 4.31×10^{-5} seconds

iv 7.88×10^{-7} seconds

v 8×10^{-2} seconds

vi 1.0356×10^{14} seconds

25 Sound travels at 330 m/s, whereas light travels at 3×10^5 km/s.

a Compare the speed of light and the speed of sound.

A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.

b How long does it take:

i for the sight of the smoke to reach the timekeeper?

ii for the sound of the gun to reach the timekeeper?

c What advice should you give the timekeeper in order to have an accurate recording of the time of the race?

26 The circumference of a hydrogen atom is 7.98×10^{-9} cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?

27 Consider the multiplication problem $2^{350} \times 3^2 \times 4^3 \times 5^{355}$. Write the exact answer in scientific notation.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Scientific notation

Investigation
Measuring large units in our solar system

Investigation
Accuracy in science experiments

CAS instructions
Scientific notation

Topic quiz
1E

1F Surds

Learning intentions

By the end of this topic you will be able to ...

- ✔ simplify roots to surds
- ✔ multiply surds
- ✔ divide surds.

Inter-year links

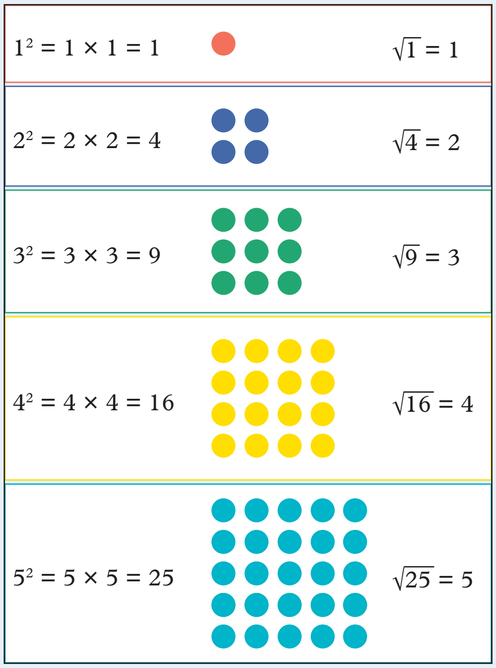
Year 7	1G Exponents and square roots
Year 8	4E Roots
Year 10	2A Rational and irrational numbers

Surds

- A **perfect square** is the square of an integer.
  For example, 1, 4, 9, 16, 25… are perfect squares.
- A **surd** is an irrational number that cannot be simplified.
  - → $\sqrt{2}$ cannot be written as a fraction, integer or recurring or terminating decimal, so it is a surd.
  - → $\sqrt{4}$ can be simplified to 2, so it is not a surd.

$$\begin{aligned}\sqrt{1} &= 1\\ \sqrt{2} &\approx 1.414\,2135\ldots\\ \sqrt{3} &\approx 1.732\,0508\ldots\\ \sqrt{4} &= 2\end{aligned}$$

- Surds can be other types of roots such as the square root, $\sqrt{2}$, the cube root, $\sqrt[3]{2}$, or fourth root, $\sqrt[4]{2}$.
- Exact value refers to a value given precisely as measured or recorded, not rounded off. Surds are a way to present numbers as exact values that have not been rounded.



Basic rules for square roots

- The inverse operation of taking a square root is an exponent of 2.
- The inverse operation of an exponent of 2 is taking a square root.
- A square root can be written as the product of the square roots of its positive factors.
- A square root divided by a square root can be written as the square root of the quotient.

$$(\sqrt{2})^2 = 2 \qquad (\sqrt{a})^2 = a$$

$$\sqrt{2^2} = 2 \qquad \sqrt{a^2} = a$$

$$\sqrt{6} = \sqrt{2} \times \sqrt{3} \qquad \sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Simplifying square roots to surds

- To simplify a square root with a perfect square factor:
  1. Rewrite the number as a product of its factors, including the square number.
  2. Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.
  3. Simplify the perfect square roots.
  4. Multiply the remaining square roots and simplify the expression.

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3}\\ &= \sqrt{4} \times \sqrt{3}\\ &= 2 \times \sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

Multiplying and dividing surds

- To multiply square roots, use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ in reverse, then simplify.

  For example, $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8}$
  $= \sqrt{16}$
  $= 4$
- To divide square roots, use the rule $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ to first simplify the fraction under the root, and then simplify the square root.

  For example, $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}}$
  $= \sqrt{4}$
  $= 2$

Example 1F.1 Simplifying surds

Simplify the following surds.

a $\sqrt{8}$ **b** $\sqrt{22}$ **c** $\sqrt{108}$

THINK	WRITE
a 1 Rewrite the number as a product of its factors, including the square number 4.	**a** $\sqrt{8} = \sqrt{4 \times 2}$
2 Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.	$= \sqrt{4} \times \sqrt{2}$
3 Simplify the perfect square roots.	$= 2 \times \sqrt{2}$
4 Multiply the remaining square roots and simplify the expression.	$= 2\sqrt{2}$
b Rewrite the number as a product of its factors. There are no square number factors so $\sqrt{22}$ is already in its simplest form.	**b** $\sqrt{22} = \sqrt{2 \times 11}$ $= \sqrt{2} \times \sqrt{11}$ Therefore, $\sqrt{22}$ is already in its simplest form.
c 1 Rewrite the number as a product of its factors, including the square numbers 4 and 9.	**c** $\sqrt{108} = \sqrt{4 \times 9 \times 3}$
2 Write the square root as a product of the square root of the factors using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.	$= \sqrt{4} \times \sqrt{9} \times \sqrt{3}$
3 Simplify the perfect square roots and multiply together.	$= 2 \times 3 \times \sqrt{3}$ $= 6 \times \sqrt{3}$
4 Multiply the remaining square roots and simplify the expression.	$= 6\sqrt{3}$

Example 1F.2 Writing simplified surds as square roots

Write the simplified surd, $4\sqrt{3}$, as the square root of a single number.

THINK	WRITE
1 Write the simplified surd as a product of the integer and square root.	$4\sqrt{3} = 4 \times \sqrt{3}$
2 Rewrite the integer as a square root using $a = \sqrt{a^2}$.	$= \sqrt{16} \times \sqrt{3}$
3 Multiply the square roots together using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.	$= \sqrt{16 \times 3}$ $= \sqrt{48}$

Example 1F.3 Multiplying surds

Simplify the following expressions containing surds.

a $\sqrt{15}\times\sqrt{6}$ **b** $\sqrt{8}\times\sqrt{75}$

THINK	WRITE
a 1 Multiply the square roots together using $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$.	**a** $\sqrt{15}\times\sqrt{6}=\sqrt{15\times 6}$ $=\sqrt{90}$
2 Identify a perfect square factor and write the number as a product of the factors.	$=\sqrt{9\times 10}$ $=\sqrt{9}\times\sqrt{10}$
3 Simplify the square root.	$=3\times\sqrt{10}$ $=3\sqrt{10}$
b 1 Multiply the square roots together using $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$.	**b** $\sqrt{8}\times\sqrt{75}=\sqrt{8\times 75}$ $=\sqrt{600}$
2 Identify a perfect square factor and write the number as a product of the factors.	$=\sqrt{100\times 6}$ $=\sqrt{100}\times\sqrt{6}$
3 Simplify the square root.	$=10\times\sqrt{6}$ $=10\sqrt{6}$

Example 1F.4 Dividing surds

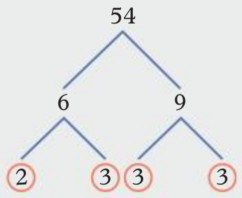
Simplify the following expressions containing square roots.

a $\frac{\sqrt{27}}{\sqrt{3}}$ **b** $\frac{\sqrt{35}}{\sqrt{15}}$

THINK	WRITE
a 1 Divide the square roots using $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$.	**a** $\frac{\sqrt{27}}{\sqrt{3}}=\sqrt{\frac{27}{3}}$
2 Simplify the fraction.	$=\sqrt{9}$
3 Simplify the square root.	$=3$
b 1 Divide the square roots using $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$.	**b** $\frac{\sqrt{35}}{\sqrt{15}}=\sqrt{\frac{35}{15}}$
2 Simplify the fraction by cancelling a common factor of 5.	$=\sqrt{\frac{7}{3}}$
3 Simplify the square roots. $\sqrt{7}$ and $\sqrt{3}$ are surds. Write in the form $\frac{\sqrt{a}}{\sqrt{b}}$.	$=\frac{\sqrt{7}}{\sqrt{3}}$

Helpful hints

- ✔ Remember that factor trees can help to determine square number factors as a number that is multiplied by itself to give a square number. For example, $3\times 3=3^2=9$ is a perfect square factor of 54.
- ✔ If there are no repeated prime factors in a surd, then the surd cannot be simplified.
- ✔ Simplifying surds is an important skill for Year 11 and 12 Maths.



ANS p409

Exercise 1F Surds

△ 1(a, c, e), 2–7(1st, 3rd columns), 8–11, 14–16

□ 1(b, d, f), 2–7(2nd, 4th columns), 8–13, 15–18

⬠ 3(e–h), 5–7(f–h), 8–11, 14–21

UNDERSTANDING AND FLUENCY

1 Evaluate the following products.

a **i** 4×2 **ii** 4×3 **iii** 4×5 **iv** 4×6 **v** 4×7

b **i** 9×2 **ii** 9×3 **iii** 9×5 **iv** 9×6 **v** 9×7

c **i** 25×2 **ii** 25×3 **iii** 25×5 **iv** 25×6 **v** 25×7

d **i** $4 \times 4 \times 2$ **ii** $4 \times 4 \times 3$ **iii** $4 \times 4 \times 5$ **iv** $4 \times 4 \times 6$ **v** $4 \times 4 \times 7$

e **i** $4 \times 9 \times 2$ **ii** $4 \times 9 \times 3$ **iii** $4 \times 9 \times 5$ **iv** $4 \times 9 \times 6$ **v** $4 \times 9 \times 7$

f **i** $4 \times 4 \times 4 \times 2$ **ii** $4 \times 4 \times 4 \times 3$ **iii** $4 \times 4 \times 4 \times 5$ **iv** $4 \times 4 \times 4 \times 6$ **v** $4 \times 4 \times 4 \times 7$

1F.1 **2** Simplify the following surds.

a $\sqrt{20}$ **b** $\sqrt{27}$ **c** $\sqrt{28}$ **d** $\sqrt{50}$

e $\sqrt{72}$ **f** $\sqrt{800}$ **g** $\sqrt{147}$ **h** $\sqrt{384}$

i $\sqrt{432}$ **j** $\sqrt{847}$ **k** $\sqrt{486}$ **l** $\sqrt{891}$

1F.2 **3** Write each of the following simplified surds as the square root of a single number.

a $3\sqrt{5}$ **b** $4\sqrt{6}$ **c** $2\sqrt{3}$ **d** $2\sqrt{11}$

e $5\sqrt{10}$ **f** $8\sqrt{7}$ **g** $13\sqrt{3}$ **h** $12\sqrt{5}$

1F.3 **4** Write each of the following products of square roots as the square root of a single number.

a $\sqrt{5} \times \sqrt{6}$ **b** $\sqrt{8} \times \sqrt{12}$ **c** $\sqrt{9} \times \sqrt{15}$ **d** $\sqrt{24} \times \sqrt{14}$

e $\sqrt{13} \times \sqrt{27}$ **f** $\sqrt{50} \times \sqrt{12}$ **g** $\sqrt{101} \times \sqrt{7}$ **h** $\sqrt{123} \times \sqrt{45}$

1F.4 **5** Simplify each of the following quotients of square roots.

a $\frac{\sqrt{24}}{\sqrt{8}}$ **b** $\frac{\sqrt{65}}{\sqrt{5}}$ **c** $\frac{\sqrt{126}}{\sqrt{6}}$ **d** $\frac{\sqrt{144}}{\sqrt{16}}$

e $\frac{\sqrt{132}}{\sqrt{12}}$ **f** $\frac{\sqrt{136}}{\sqrt{8}}$ **g** $\frac{\sqrt{360}}{\sqrt{15}}$ **h** $\frac{\sqrt{420}}{\sqrt{14}}$

6 Simplify the following products of square roots.

a $\sqrt{6} \times \sqrt{8}$ **b** $\sqrt{20} \times \sqrt{30}$ **c** $\sqrt{18} \times \sqrt{6}$ **d** $\sqrt{35} \times \sqrt{15}$

e $\sqrt{32} \times \sqrt{20}$ **f** $\sqrt{63} \times \sqrt{54}$ **g** $\sqrt{60} \times \sqrt{96}$ **h** $\sqrt{75} \times \sqrt{105}$

7 Simplify the following quotients of square roots.

a $\frac{\sqrt{960}}{\sqrt{3}}$ **b** $\frac{\sqrt{160}}{\sqrt{20}}$ **c** $\frac{\sqrt{882}}{\sqrt{7}}$ **d** $\frac{\sqrt{4752}}{\sqrt{12}}$

e $\frac{\sqrt{3456}}{\sqrt{8}}$ **f** $\frac{\sqrt{2520}}{\sqrt{10}}$ **g** $\frac{\sqrt{2016}}{\sqrt{24}}$ **h** $\frac{\sqrt{3600}}{\sqrt{75}}$

8 Consider the following numbers.

$\sqrt{9}, \sqrt{6}, 6^2, \sqrt[3]{6}, \sqrt{\frac{4}{9}}, \sqrt{8}, \sqrt{27}, 4^3, \sqrt{0.16}, \sqrt[4]{16}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$

a List the roots.

b Of the square roots, list the surds.

9 Evaluate the following.

a $(\sqrt{3})^2$ **b** $\sqrt{5^2}$ **c** $\sqrt{8 \times 8}$ **d** $\sqrt{7} \times \sqrt{7}$

e $\frac{\sqrt{2}}{\sqrt{2}}$ **f** $5\sqrt{2} \times \sqrt{50}$ **g** $\frac{3\sqrt{6}}{\sqrt{54}}$ **h** $\frac{\sqrt{24}}{2\sqrt{6}}$

10 The coefficient of a surd represents the number of surds that have been added together. This is similar to the coefficient of a variable. For example:

$\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$

a Write each of the following sums in the form $a\sqrt{b}$.

i $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$ **ii** $\sqrt{5} + \sqrt{5} + \sqrt{5}$ **iii** $\sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6}$ **iv** $\sqrt{10} + \sqrt{10}$

b Write the following as repeated sums.

i $5\sqrt{3}$ **ii** $2\sqrt{6}$ **iii** $8\sqrt{2}$ **iv** $3\sqrt{11}$

PROBLEM SOLVING AND REASONING

11 When simplifying a surd, write the number as a product of its prime factors to help you find the square factors. You can do this by pairing repeated prime factors. For example:

$\sqrt{3 \times 7 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11}$
$= \sqrt{3 \times (7 \times 7) \times (7 \times 7) \times (11 \times 11) \times 11}$
$= \sqrt{3} \times \sqrt{7 \times 7} \times \sqrt{7 \times 7} \times \sqrt{11 \times 11} \times \sqrt{11}$
$= 7 \times 7 \times 11 \times \sqrt{3 \times 11}$
$= 539\sqrt{33}$

Simplify the following by pairing repeated prime factors.

a $\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$ **b** $\sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 11 \times 11}$

c $\sqrt{2^4 \times 3^2 \times 3 \times 5 \times 7}$ **d** $\sqrt{2^3 \times 3 \times 5^5}$

12 a Factorise the following numbers by writing them as a product of their prime factors.

i 288 **ii** 270 **iii** 980 **iv** 1125

b Hence, simplify the following surds.

i $\sqrt{288}$ **ii** $\sqrt{270}$ **iii** $\sqrt{980}$ **iv** $\sqrt{1125}$

13 When simplifying products of surds, writing the numbers as products of their prime factors before multiplying can help you to find the square factors. Pair repeated factors from both sets of products. For example:

$\sqrt{15} \times \sqrt{6} = \sqrt{3 \times 5} \times \sqrt{2 \times 3}$
$= \sqrt{2 \times 3 \times 3 \times 5}$
$= \sqrt{9 \times 2 \times 5}$
$= \sqrt{9} \times \sqrt{10}$
$= 3\sqrt{10}$

Simplify the following by first writing the numbers as a product of their prime factors.

a $\sqrt{21} \times \sqrt{14}$ **b** $\sqrt{27} \times \sqrt{33}$ **c** $\sqrt{30} \times \sqrt{18}$ **d** $\sqrt{42} \times \sqrt{231}$

14 To multiply or divide surds with coefficients, multiply or divide the coefficients and then multiply or divide the numbers in the surd.

For example: $2\sqrt{3} \times 4\sqrt{5} = 2 \times 4 \times \sqrt{3 \times 5} = 8\sqrt{15}$ and $\frac{8\sqrt{6}}{2\sqrt{3}} = \frac{8}{2}\sqrt{\frac{6}{3}} = 4\sqrt{2}$

Simplify the following products and quotients.

a $4\sqrt{3} \times 7\sqrt{2}$ **b** $5\sqrt{6} \times 8\sqrt{15}$ **c** $\frac{12\sqrt{56}}{3\sqrt{8}}$ **d** $\frac{7\sqrt{72}}{14\sqrt{12}}$

e $12\sqrt{70} \times 7\sqrt{50}$ **f** $\frac{24\sqrt{120}}{88\sqrt{24}}$ **g** $9\sqrt{91} \times 16\sqrt{78}$ **h** $\frac{196\sqrt{252}}{16\sqrt{42}}$

15 Consider the product $\sqrt{8} \times \sqrt{11}$. All decimals in this question should be rounded to two decimal places.

a Write the product as the square root of a single number.

b Use a calculator to find a decimal approximation of your answer to part **a**.

c Find decimal approximations of $\sqrt{8}$ and $\sqrt{11}$.

d Multiply your answers to part **c** together. Compare the result with your answer to parts **a** and **b**.

e Repeat parts **a** to **d** with the following products:

i $\sqrt{20} \times \sqrt{5}$

ii $\sqrt{30} \times \sqrt{12}$

f Explain the impact of using approximations in a calculation compared to using exact values.

PROBLEM SOLVING AND REASONING

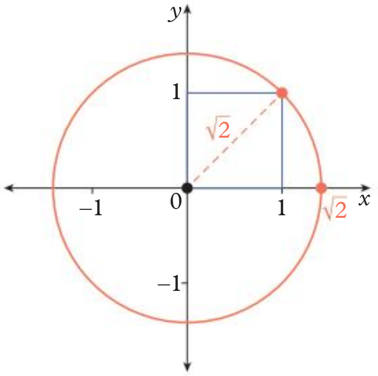
16 Evaluate the following products and quotients.

a $\sqrt{120} \times \sqrt{270}$ **b** $\frac{\sqrt{96}}{\sqrt{24}}$ **c** $\sqrt{35} \times \sqrt{28} \times \sqrt{45}$ **d** $\frac{\sqrt{2700}}{\sqrt{12}}$

17 Geometric constructions can be used to locate surds on the number line. For example, draw a square on the Cartesian plane with side length equal to 1 and the bottom left corner at the origin as shown. Pythagoras' theorem tells us that the length of the diagonal is equal to:

$\sqrt{(1^2 + 1^2)} = \sqrt{(1 + 1)} = \sqrt{2}$.

Sketch the circle with radius equal to $\sqrt{2}$ centred at the origin. The point of intersection between the circle and the positive half of the x-axis shows the location of $\sqrt{2}$ on the number line.



a Copy the given construction to locate $\sqrt{2}$ on the number line.

b Use a square with side length equal to 2 to locate $\sqrt{8}$ on the same number line.

c Use a square with side length equal to 3 to locate $\sqrt{18}$ on the same number line.

d Simplify $\sqrt{8}$ and $\sqrt{18}$.

e How does your answer to part **d** relate to the locations of $\sqrt{2}$, $\sqrt{8}$ and $\sqrt{18}$ on the number line?

CHALLENGE

18 Surds can be added or subtracted in the same way we add like terms. You can add or subtract the coefficients of surds that have the same surd factor after simplifying.

For example: $7\sqrt{2} + 3\sqrt{2} = 10\sqrt{2}$ and $7\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$

Simplify the following sums and differences.

a $5\sqrt{3} + 2\sqrt{3}$ **b** $5\sqrt{3} - 2\sqrt{3}$

c $12\sqrt{5} + \sqrt{5}$ **d** $9\sqrt{11} - 8\sqrt{11}$

e $4\sqrt{7} + 2\sqrt{7} - 6\sqrt{3} + 2\sqrt{7} - 5\sqrt{3}$ **f** $12\sqrt{3} - 4\sqrt{5} - 7\sqrt{3} + 9\sqrt{2} + 8\sqrt{2} + 7\sqrt{5} - 4\sqrt{2} + 2\sqrt{3}$

g $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32}$ **h** $\sqrt{12} + \sqrt{20} + \sqrt{45} + \sqrt{48}$

19 Evaluate the following.

a $\frac{\sqrt{2} \times \sqrt{10}}{\sqrt{5}}$ **b** $\frac{\sqrt{24}}{\sqrt{5}} \times \frac{\sqrt{15}}{\sqrt{6}}$ **c** $\frac{\sqrt{8 + 13}}{\sqrt{4 + 3}}$

20 Evaluate the following.

a $(5\sqrt{3})^2$ **b** $(3\sqrt{5})^2$ **c** $(4\sqrt{7})^2$ **d** $(8\sqrt{6})^2$

21 Determine the smallest surd you would need to multiply (or divide) these surds by so that the product (or quotient) is not a surd.

a $\sqrt{8}$ **b** $\sqrt{45}$ **c** $\sqrt{12}$ **d** $\sqrt{50}$ **e** $\sqrt{75}$ **f** $\sqrt{54}$

Check your Student obook pro for these digital resources and more:

pro

Interactive skillsheet
Simplifying surds

Interactive skillsheet
Multiplying and dividing surds

CAS instructions
Simplifying surds

Topic quiz
1F

Chapter summary

Exponents

exponent/index

$$\text{base} \longrightarrow 3^4 = 3 \times 3 \times 3 \times 3 = 81$$

exponent form expanded form basic numeral

Product of powers law

$$2^3 \times 2^5 = 2^{3+5} = 2^8 \qquad a^3 \times a^5 = a^{3+5} = a^8$$

Quotient of powers law

$$2^5 \div 2^3 = 2^{5-3} = 2^2 \qquad a^5 \div a^3 = a^{5-3} = a^2$$

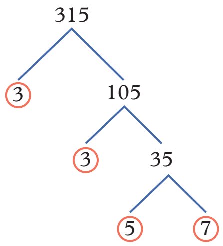
Power of a power law

$$(2^3)^5 = 2^{3\times5} = 2^{15} \qquad (a^2)^3 = a^{2\times3} = a^6$$

$$(2\times3)^5 = 2^5 \times 3^5 \qquad (ab)^3 = a^3 \times b^3$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \qquad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Prime factorisation



$$315 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7$$

Negative exponents

$$2^{-1} = \left(\frac{2}{1}\right)^{-1} = \frac{1}{2} \qquad a^{-1} = \left(\frac{a}{1}\right)^{-1} = \frac{1}{a}$$

$$\frac{1}{2^{-3}} = \frac{1}{\left(\frac{1}{2}\right)^{-3}} = \frac{1}{\left(\frac{1}{2}\right)^{-3}} = 1 \times \frac{2^3}{1} = 2^3$$

$$\frac{1}{a^{-m}} = \frac{1}{\left(\frac{a}{1}\right)^{-m}} = \frac{1}{\left(\frac{1}{a}\right)^{m}} = 1 \times \frac{a^m}{1} = 2^3$$

The zero exponent

$$2^0 = 1 \qquad a^0 = 1$$

Scientific notation

significand

$$1230 = 1.23 \times 10^3$$

basic numeral scientific notation

exponent of 4

$$3.1500 \times 10^4 = 31\,500$$

move 4 spaces to the right

exponent of 4

$$31500 = 3.15 \times 10^4$$

move 4 spaces to the left

significand

$$0.00123 = 1.23 \times 10^{-3}$$

basic numeral scientific notation

exponent of -2

$$004.2 \times 10^{-2} = 0.042$$

move 2 spaces to the left

exponent of -2

$$0.042 = 4.2 \times 10^{-2}$$

move 2 spaces to the right

Surds

$$(\sqrt{2})^2 = 2 \qquad (\sqrt{a})^2 = a$$

$$\sqrt{2^2} = 2 \qquad \sqrt{a^2} = a$$

Multiplying surds

$$\sqrt{6} = \sqrt{2} \times \sqrt{3}$$

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

Dividing surds

$$\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

1A **1** Which of the following is *not* equivalent to $9(xy)^4$?

A $9 \times xy \times xy \times xy \times xy$ **B** $-3^2x^4y^4$ **C** $9x^4y^4$
D $9xxxxyyyy$ **E** $(-3)^2x^4y^4$

1A **2** Which of the following is the prime factorisation of 360?

A $6^2 \times 10$ **B** $4 \times 9 \times 10$ **C** $2^3 \times 3^2 \times 5$ **D** $2^2 \times 3^2 \times 10$ **E** $3 \times 10 \times 12$

1B **3** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{3a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $3ab^2$

1B **4** Which statement does *not* correctly represent one of the exponent laws?

A $m^5 \times m^2 = m^{5+2}$ **B** $(p \times q)^8 = p^8 \times q^8$ **C** $w^7 \div w^5 = w^{7-5}$
D $a^5 \times a = a^6$ **E** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$

1C **5** Using the exponent laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ fully simplifies to:

A $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$ **C** $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$ **E** $10x^9$

1D **6** Which of the following is *not* the reciprocal of $\frac{4}{3}$?

A $\frac{3}{4}$ **B** 3×4^{-1} **C** $\left(\frac{4}{3}\right)^{-1}$ **D** $\frac{1}{3 \times 4}$ **E** $1 \div \frac{4}{3}$

1D **7** Which statement is false?

A $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$ **C** $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$ **E** $\frac{5^{-3}}{5^4} = 5^{-7}$

1E **8** Which number is equivalent to 6.4724×10^2?

A 0.647 24 **B** 64.724 **C** 0.064 724 **D** 64 724 **E** 647.24

1F **9** Which of the following is a surd?

A $\sqrt{25}$ **B** $\sqrt{\frac{49}{36}}$ **C** $\sqrt{196}$ **D** $\sqrt{91}$ **E** $\sqrt{0.16}$

1F **10** Which of the following is *not* fully simplified?

A $\sqrt{10}$ **B** $5\sqrt{24}$ **C** $7\sqrt{14}$ **D** $2\sqrt{22}$ **E** $16\sqrt{7}$

Short answer

1A **1** Evaluate the following.

a 3^4 **b** $(-5)^3$ **c** -4^3 **d** $\left(\frac{3}{2}\right)^5$ **e** $(0.6)^3$ **f** $(1.2)^4$

1A **2** Write the following in exponent form.

a $17 \times 17 \times 17 \times 17 \times 17 \times 17$
b $(-5b^2) \times (-5b^2) \times (-5b^2) \times (-5b^2) \times (-5b^2)$
c $-10 \times f \times f \times f \times v \times v \times v \times v \times v \times v \times v$
d $\frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3}$

1B **3** Simplify each expression using the exponent laws.

a $a^{11} \times a^5$ **b** $b^9 \div b^8$ **c** $18d^7 \div 54d^4$

1C **4** Simplify each expression using the exponent laws.

a $(c^8)^2$ **b** $(e^5)^5 \times (e^{11})^2$ **c** $5a^0 + 3b^0 + 1c^0$

1C **5** Simplify each expression.

a $\frac{m^3n^4 \times m^9n^{11}}{m^7n^7}$ **b** $\frac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$

1D **6** Write each term with positive exponents.

a 4^{-1}
b b^{-1}
c m^{-7}
d $\frac{6x^3}{2y^{-3}}$
e $\frac{f^{-4}}{g^{-5}}$
f $\frac{a^{-3}b}{5^{-2}c^{-4}}$
g $(p^{-2})^5 \times p^{-5}$
h $\left(\frac{k}{l}\right)^{-7}$

1D **7** **a** If $4^8 = 65\ 536$, write the value of 4^{-8} as a fraction.

b If $7^{-3} = \frac{1}{343}$, write the value of 7^3.

1E **8** Write each number as a basic numeral.

a 5.876×10^4
b 9.02×10^{-6}

1E **9** State the number of significant figures in each part of question **8**.

1E **10** Write each number in scientific notation.

a 540 000
b 0.000 76

1E **11** Round each of the following to the number of significant figures indicated.

a 879 (2)
b 2.58×10^5 (1)

1E **12** A scientist estimates that there are 3.40×10^4 bacteria in one sample and 4.6×10^3 in the second. Write the total number of bacteria:

a as a basic numeral
b rounded to two significant figures
c in scientific notation.

1F **13** Simplify the following square roots.

a $\sqrt{72}$
b $\sqrt{49}$
c $\sqrt{243}$
d $\sqrt{256}$
e $\sqrt{1000}$
f $\sqrt{350}$

1F **14** Simplify the following products and quotients.

a $\sqrt{5} \times \sqrt{7}$
b $\sqrt{60} \times \sqrt{70}$
c $\frac{\sqrt{180}}{\sqrt{36}}$
d $\frac{\sqrt{1625}}{\sqrt{13}}$

1F **15** Determine a simplified surd expressing the side length of a square with an area of:

a 30 m^2
b 71 m^2
c 44 m^2
d 189 m^2

1F **16** Simplify the following surds.

a $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$
b $\sqrt{2^4 \times 2 \times 3^6 \times 3 \times 5}$
c $\sqrt{2^3 \times 5^4 \times 7}$
d $\sqrt{2^2 \times 3^2 \times 5^3 \times 7}$

Analysis

1 In September 2020, the population of each Australian state was recorded. The figure for each state is shown in the table.

State	Population at 30 June 2020 ('000)
NSW	8166.4
Vic	6680.6
Qld	5184.8
SA	1770.6
WA	2667.1
Tas	541.4
NT	246.5
ACT	431.2

Source: http://www.abs.gov.au/ausstats/abs@.nsf/mf/3101.0/

a Which states and territories have a population listed to:

- **i** four significant figures?
- **ii** five significant figures?

b Copy the table and add three additional columns.

c In the first new column, write the population of each state and territory in full.

d In the second new column, round each population to its leading digit.

e In the third new column, write each population in scientific notation to one significant figure.

f Use your answers from part **d** to determine the following. Write the values in scientific notation.

- **i** Which state or territory has the highest population?
- **ii** Which state or territory has the lowest population?
- **iii** Calculate the difference between the highest and the lowest population.
- **iv** Calculate the total population of SA, Tas, ACT and NT.
- **v** Calculate the total population of Australia.

g The actual total population value recorded at the end of September 2020 was 25 693 100. Calculate the difference between your answer to **f** part **v** and the actual value. Why is there a difference?

2 Thy and Asha are playing a game. They are using die rolls and a coin flip to generate the product of three numbers in exponent form per round. They each roll the die to determine the value of the bases and exponents and flip the coin to determine if each exponent is positive or negative.

The products generated after each round are multiplied together with the goal to end up with the lowest number of remaining factors after three rounds.

The table below shows the numbers Thy and Asha got in their three rounds.

	Thy	Asha
Round 1	$2^4 \times 4^3 \times 5^{-3}$	$1^3 \times 3^6 \times 5^{-5}$
Round 2	$3^2 \times 5^6 \times 6^{-3}$	$2^{-3} \times 3^1 \times 6^4$
Round 3	$2^{-6} \times 2^3 \times 3^4$	$3^{-5} \times 4^2 \times 5^3$

a Use the facts that $4 = 2^2$ and $6 = 2 \times 3$ to write Thy and Asha's round 1, 2 and 3 numbers in exponent form with positive exponents using only the bases 2, 3 and 5.

b Determine Thy and Asha's final number for their game by multiplying their round 1, 2 and 3 numbers together and simplifying the products in exponent form with positive exponents.

c Who won the game with the lowest number of factors (find the sum of the positive exponents)?

d Did the winner have the smaller value? Explain.

Thy and Asha decide to play one more round of the game.

e What products do Thy and Asha need to generate to end up with a total product of 1?

Thy and Asha decide to change the rules so that they can choose which base gets which exponent. Their products from the first two rounds are given in the table below.

	Thy	Asha
Round 1	$4^{-3} \times 5^4 \times 6^5$	$2^{-3} \times 5^{-2} \times 6^4$
Round 2	$1^5 \times 2^3 \times 3^{-4}$	$3^{-3} \times 4^4 \times 5^2$

f Determine the product of rounds 1 and 2 for Thy and Asha. Write the products in exponent form using only the bases 2, 3 and 5.

For round 3:

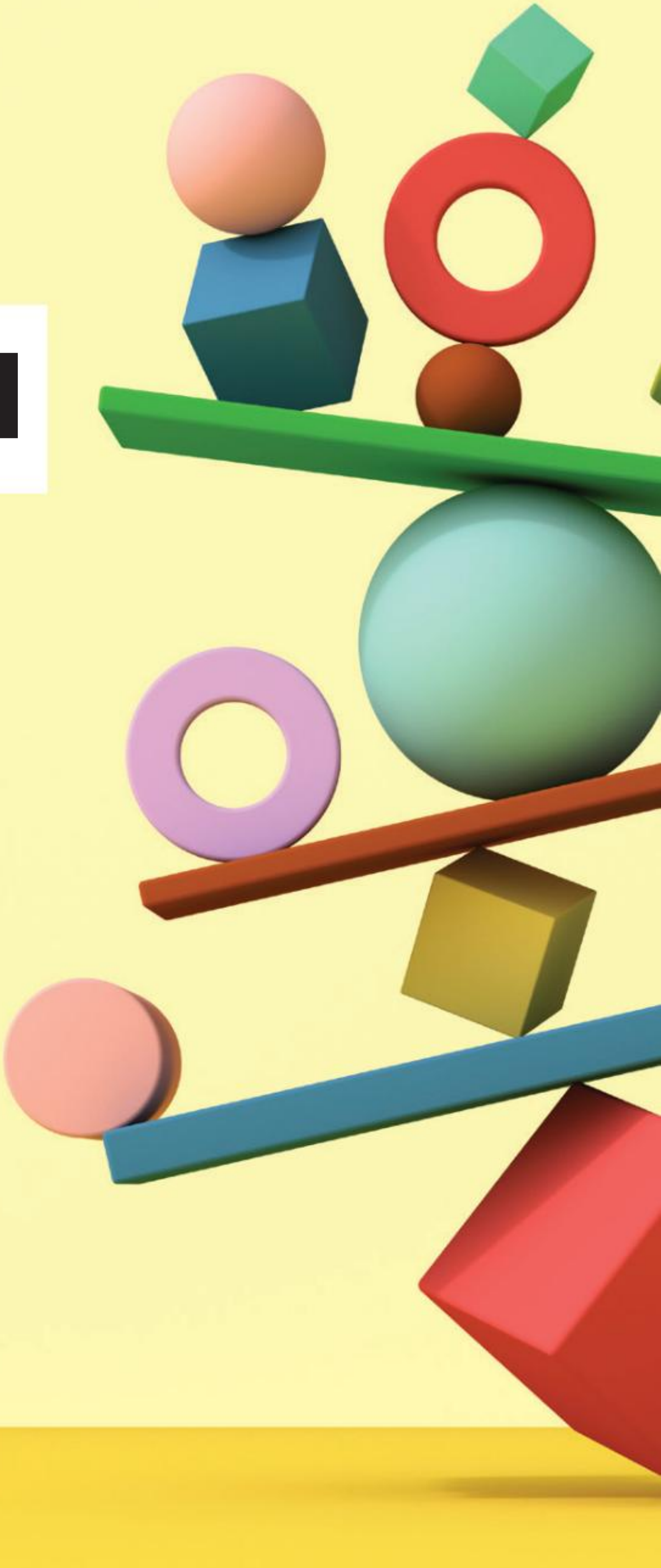
- Thy gets the bases 2, 4 and 6 and the exponents −6, −4 and 1.
- Asha gets the bases 1, 5 and 5 and the exponents −4, 3 and 6.

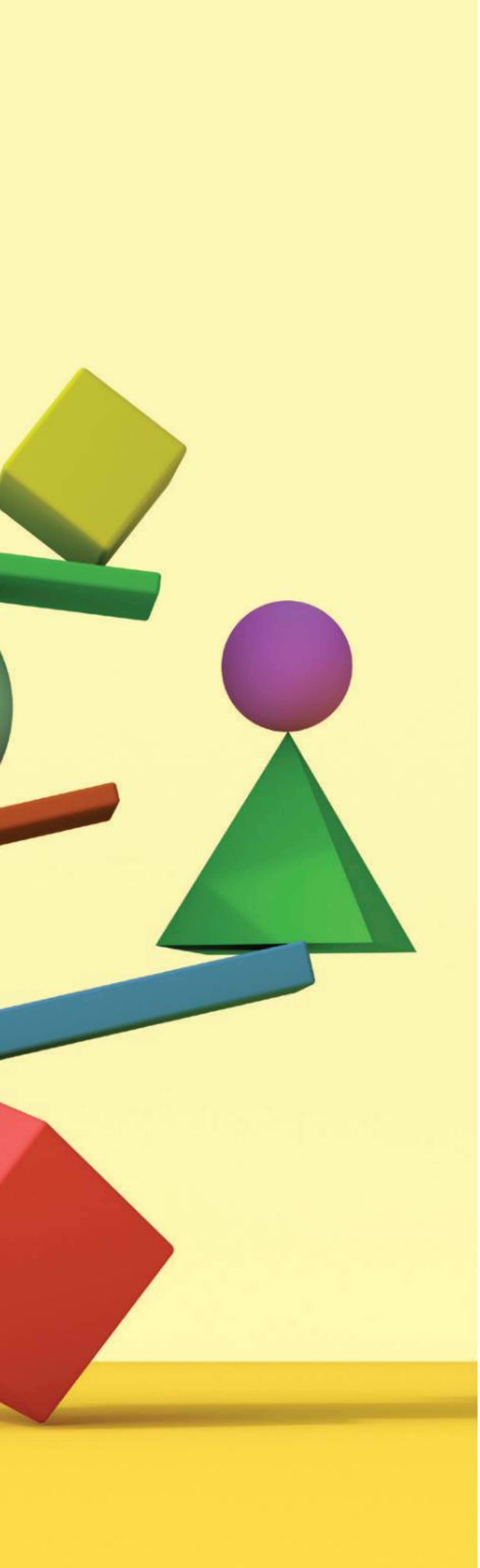
g Determine which exponent should go with which base so that Thy and Asha get the minimum number of factors in their final products.

h Who wins this game and by how many factors?

2

Algebra





Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Multiplying and dividing with negative numbers
- ✓ Grouping symbols
- ✓ Exponent laws
- ✓ Highest common factor
- ✓ Expressions and equations

Curriculum links

- Simplify algebraic expressions, expand binomial products and factorise monic quadratic expressions [AC9M9A02]

2A Simplifying

Learning intentions

By the end of this topic you will be able to ...

✔ simplify algebraic terms involving addition and subtraction

✔ simplify algebraic terms involving multiplication and division.

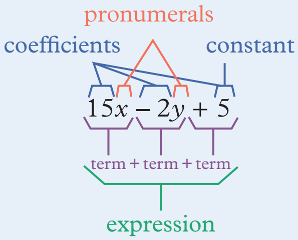
Inter-year links

Year 8	5C Adding and subtracting algebraic terms
Year 10	1C Simplifying

Terms and expressions

- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an **unknown** or a **variable**.
- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is part of an expression that is separated from the other parts by a plus or minus sign (as long as the plus or minus sign is not inside any brackets).

  Note: If the term is separated on the left by a minus sign, then the term is negative.
- A **coefficient** is the number acting as a multiplier in an algebraic term, usually written before the pronumeral. A pronumeral without a number preceding it has a coefficient of 1.
- A **constant** is a term without any pronumerals. It also counts as a coefficient.



Adding and subtracting algebraic terms

- **Like terms** contain the same pronumerals with the same exponents.
  - → The order of pronumerals can be different in two like terms. For example, xyz, $4yxz$, and $7zxy$ are all like terms.
  - → You can write terms containing exponents in **expanded form** to determine whether they are like terms. For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$, so a^2b and ab^2 are not like terms.

$$\underbrace{3a^2b}_{\text{exponent form}} = \underbrace{3 \times a \times a \times b}_{\text{expanded form}}$$

- Like terms can be added or subtracted by adding or subtracting the coefficients of the terms. For example, $a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$

Multiplying algebraic terms

- To multiply algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.
  2. Multiply the coefficients together.
  3. Apply the product of powers law to multiply the numbers in exponent form. Keep the base and add the exponents.
  4. Simplify by leaving out the multiplication signs. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times (-2) \times a^{3+2} \times b^{1+2} \\ &= -6a^5b^3 \end{aligned}$$

Dividing algebraic terms

- Remember that quotients can be expressed as fractions.
  For example, $x \div 7 = \frac{x}{7}$
- To divide algebraic terms:
  1. Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.
  2. Divide the coefficients by the HCF.
  3. Apply the quotient of powers law for the division of numbers in exponent form. Keep the base and subtract the exponents.
  4. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\begin{aligned}\frac{6a^5b^3}{3a^3b} &= \frac{\cancel{6}^2 \times a^5 \times b^3}{\cancel{3}^1 \times a^3 \times b}\\ &= 2 \times a^{5-3} \times b^{3-1}\\ &= 2a^2b^2\end{aligned}$$

Example 2A.1 Adding and subtracting algebraic terms

Simplify each expression where possible.

a $10a - 6a + a$

b $4xy + 2x - 5xy$

c $-5bc + 2b - 3cb - \frac{b}{2}$

d $6x^2y - 3y^3 - 2y^2x + y^3$

THINK	WRITE
a Identify like terms and simplify by adding and subtracting the coefficients.	**a** $10a - 6a + a = (10 - 6 + 1)a$ $= 5a$
b 1 Rearrange the expression so that like terms are grouped together. 2 Simplify by adding and subtracting the coefficients.	**b** $4xy + 2x - 5xy = 2x + 4xy - 5xy$ $= 2x + (4 - 5)xy$ $= 2x - xy$
c 1 Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term. 2 Simplify by adding and subtracting the coefficients. Remember that fractional coefficients can be written in two ways, so $\frac{b}{2} = \frac{1}{2}b$.	**c** $-5bc + 2b - 3cb - \frac{b}{2} = -5bc - 3cb + 2b - \frac{b}{2}$ $= (-5 - 3)bc + \left(2 - \frac{1}{2}\right)b$ $= -8bc + \frac{3}{2}b$
d 1 Rearrange the expression so that like terms are grouped together. Check that the + or − sign in front of each term has moved with that term. Note that xy^2 and x^2y are not like terms. 2 Simplify by adding and subtracting the coefficients. Rearrange pronumerals in alphabetical order.	**d** $6x^2y - 3y^3 - 2y^2x + y^3 = 6x^2y - 3y^3 + y^3 - 2y^2x$ $= 6x^2y + (-3 + 1)y^3 - 2xy^2$ $= 6x^2y - 2y^3 - 2xy^2$

Example 2A.2 Multiplying algebraic terms

Simplify the following products.

a $4de \times 7ab$　　**b** $5x^2y^5 \times (-2kwxy^3)$

THINK	**WRITE**
a **1** Write in expanded form.	**a** $4de \times 7ab = 4 \times d \times e \times 7 \times a \times b$
2 Multiply the coefficients together.	$= 28 \times d \times e \times a \times b$
3 Simplify by leaving out the multiplication signs. Write the pronumerals in alphabetical order.	$= 28abde$
b **1** Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.	**b** $5x^2y^5 \times (-2kwxy^3)$ $= 5 \times x^2 \times y^5 \times (-2) \times k \times w \times x \times y^3$
2 Multiply the coefficients together.	$= -10 \times x^{2+1} \times y^{5+3} \times k \times w$
3 Apply the product of powers law for the multiplication of exponents, so $x^2 \times x = x^{(2+1)}$ and $y^5 \times y^3 = y^{(5+3)}$.	$= -10 \times x^3 \times y^8 \times k \times w$
4 Simplify by leaving out the multiplication signs.	$= -10kwx^3y^8$

Example 2A.3 Dividing algebraic terms

Simplify the following quotients.

a $\frac{-15xy}{10x}$　　**b** $\frac{8a^3b}{2a^2b^2}$

THINK	**WRITE**
a **1** Write in expanded form.	**a** $\frac{-15xy}{10x} = \frac{-15 \times x \times y}{10 \times x}$
2 Cancel the coefficients, and divide −15 and 10 by the HCF of 5. Cancel any common pronumerals from the numerator and denominator.	$= \frac{-\cancel{15}^{3} \times \cancel{x}^{1} \times y}{\cancel{10}^{2} \times \cancel{x}^{1}}$
3 Simplify the numerator and the denominator.	$= -\frac{3y}{2}$
b **1** Write the coefficients and pronumerals for each term in expanded form, without expanding exponent form.	**b** $\frac{8a^3b}{2a^2b^2} = \frac{8 \times a^3 \times b}{2 \times a^2 \times b^2}$
2 Divide each coefficient by the HCF.	$= \frac{\cancel{8}^{4}}{\cancel{2}^{1}} \times \frac{a^3}{a^2} \times \frac{b}{b^2}$
3 Apply the quotient of powers law to cancel out repeated pronumerals that appear in both the numerator and denominator. Keep the base and subtract the exponents.	$= 4 \times a^{3-2} \times b^{1-2}$
4 Simplify the numerator and the denominator. Use $a^{-m} = \frac{1}{a^m}$ to write the pronumeral as a fraction with a positive exponent.	$= 4ab^{-1}$ $= \frac{4a}{b}$

Helpful hints

✔ Recall the rules for writing algebraic notation.
- → Products are simplified by leaving out the multiplication sign and placing the coefficient first.
  For example: $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$
- → When a pronumeral is multiplied by 1, the 1 is not shown.
  For example: $1 \times x = x$
- → Quotients are represented by fractions.
  For example: $x \div 7 = \frac{x}{7}$ and $(x + 2) \div 7 = \frac{x + 2}{7}$
- → Terms with fractional coefficients can be written in two ways.
  For example: $\frac{x}{7} = \frac{1}{7}x$
- → Write pronumerals in a term in alphabetical order
  For example: $ba^2c = a^2bc$

✔ Recall the rules for multiplying and dividing positive and negative numbers.

$$+ \times - = - \qquad + \times + = +$$
$$- \times + = - \qquad - \times - = +$$

✔ Take care not to mix up the exponent laws and definitions.
- → Across a multiplication sign, add exponents.
- → Across a division sign, subtract exponents.
- → Across brackets, multiply exponents.
- → Zero exponent: $a^0 = 1$
- → Negative exponent: $a^{-1} = \frac{1}{a}$

Exponent law	Example
Product of powers law	$a^5 \times a^3 = a^{5+3}$
Quotient of powers law	$a^5 \div a^3 = a^{5-3}$
Power of a power law	$(a^5)^3 = a^{5 \times 3}$

ANS
p411

Exercise 2A Simplifying

△ 1, 2, 3–4(1st column), 5, 6–8(1st column), 9–12, 13(a, d, e, f), 14(a, c, f), 15, 16, 21, 23(a, b)

■ 4, 5, 6–8(2nd column), 9, 10, 14(a, c, f), 17–20, 22, 23

⬟ 5, 7, 8(2nd column), 9, 10, 14(a, c, f), 17–20, 22–24

UNDERSTANDING AND FLUENCY

1 Consider these terms: $3x$, $7xy$, $-x$, $2x^2$, xw, $20x$
- **a** Which are like terms?
- **b** Explain how you can tell.
- **c** What is the coefficient of each term?

2 List each group of like terms from these terms: $2ba^2$, $3a$, $2b^2a$, $6a^3$, aaa, $6a$, $3aab$, $6ab^2$, $6a^2a$, $6a^2b$, $3abb$

2A.1 3 Simplify each expression where possible.
- **a** $6a - 4a + 8a$
- **b** $4k - 5k - 7k$
- **c** $x^2 + 3x^2 + 2x^2$
- **d** $3cd + cd - 9cde$
- **e** $3x + 4y + 9x + 2y$
- **f** $7a + 5b - 3a + b$
- **g** $m - 2p + 4p + 8m$
- **h** $3 + 5k - 2 - 6k$
- **i** $4xy + 3x^2 - xy + 2x^2$
- **j** $d + de^2 + d - 5de^2$
- **k** $5m^3 + 7 - m^3 - 5$
- **l** $abc + ab + ac + 3ab$

4 Simplify each expression.
- **a** $6x + 3y - x + 2y + 5x - 4y$
- **b** $8ab - 4b - b + b^2 + a - 3ab$
- **c** $2k + 3km - 6k + 4 + 4k - km$
- **d** $4x^2 - 7x^2 - 3x + 5 + 6x - 9$
- **e** $9a - 4a^2 + a^3 + 5a^2 - 3 - 7a$
- **f** $m^2n + 3m^2 + 5nm^2 - 2n^2 + 4mn^2 - 3m^2$

UNDERSTANDING AND FLUENCY

5 Simplify each expression by first using exponent laws.

a $7a^2 + 5(3a)^2 + (-5a)^2 - 9aa$

b $-5bbb + 40b^2b - (2b)^3 + 2(-3b)^3$

c $9c^2d^2 - (6cd)^2 + 15(cd)^2 + 9d^2c^2$

d $r(pq)^2 + \dfrac{q^2}{(pr)^{-1}} + ppqqr + p^2q^2r + pq^2r$

e $\dfrac{2}{f^{-1}} + \dfrac{3}{e^{-1}} + \dfrac{4}{(ef)^0} + \dfrac{5}{e^{-1}} + \dfrac{6}{f^{-1}}$

f $\dfrac{4y}{x^{-2}} + \dfrac{1}{(3xy)^{-1}} - \dfrac{2x}{y^{-1}} + \dfrac{1}{2x^{-1}y^{-1}} + \dfrac{3x^2}{y^{-1}}$

2A.2 **6** Simplify each expression.

a $2ab \times 3cd$

b $-5xy \times 4mp$

c $9gh \times g$

d $4km \times (-6kn)$

e $7jp \times 8bpt$

f $-x^2y \times (-ay)$

g $6a^2b \times 3acd$

h $-10hk \times 2hkp$

i $3b \times (-2b) \times b$

j $m^2n \times 4n \times kn$

k $-5xy \times x^2 \times (-3xy)$

l $8abc \times 7a^3c \times b^2$

7 Simplify each expression by first using exponent laws.

a $(5a^3z^8)^2 \times 3(a^3z^5)^4$

b $(-3by^4)^3 \times (-2yb^7)^4$

c $5(cmx)^5 \times c(-2mx)^3 \times mx(-2c)^2$

d $4(d^3n^{-2}w)^5 \times 11(d^{-2}n^5w^{-3})^4$

e $(5e^3p^{12}t^5)^{-1} \times (e^{10}p^4t^2)^0 \times (-3e^4p^6t^7)^{-2}$

f $-6(g^{-2}q^5u^3)^5 \times 4(-g^4q^{-4}u^5)^{-3} \times -7(g^{-2}qu^7)^{-4}$

2A.3 **8** Simplify each expression.

a $\dfrac{abc}{ac}$

b $\dfrac{kmn}{kp}$

c $\dfrac{12cd}{3d}$

d $\dfrac{-16w}{8xw}$

e $\dfrac{7ef}{-14ef}$

f $\dfrac{-5xy}{-20x}$

g $\dfrac{18ab}{15ac}$

h $\dfrac{4mn}{22mn}$

i $\dfrac{18a^2}{3a}$

j $\dfrac{6x^2y}{-2x}$

k $\dfrac{15mn^2}{9m}$

l $\dfrac{-3a^2bc}{12ab}$

9 Simplify each expression using exponent laws.

a $\left(\dfrac{12d^8}{48d^3}\right)^{-1}$

b $\left(\dfrac{c^3d^{-4}}{c^{-5}d^{-9}}\right)^{-3}$

c $\dfrac{(-7u^6r^{-5})^{-2}}{(-5u^5r^{-2})^{-3}}$

d $\dfrac{(2k^{-3}p^4)^5}{18(k^{-3}p^6)^{-3}}$

e $\dfrac{1.6xy^{-4}z^0}{0.8(x^2z^3)^{-4}(y^{-2}z^3)^5}$

f $\dfrac{4.4f^{-8}}{1.1(f^5)^2} \times \dfrac{0.35(f^{-4})^{-3}}{0.07f^{-7}}$

10 Simplify each expression. Hint: First write each as an algebraic fraction.

a $m^4p^3q \div m^4$

b $-2a^8de^7 \div (a^8e^7)$

c $6ax \div (-2ac)$

d $-5km \div (-10mp)$

e $7a^2bc \div (abd)^2$

f $3mn^2w^2 \div (9n^3w)^2$

g $-12x^2y \div [8(xyz)^3]$

h $-ab^3cd \div (-2abc^2)^{-4}$

i $8km^2n \div [-12(k^2mn)^{-2}]$

11 Answer true or false to each statement.

a Two like terms can be added to form one new term.

b Any term can be subtracted from another term to form one new term.

c Two terms can be multiplied to form one new term only if they are like terms.

d Any term can be divided by another term to form one new term.

12 Students in a class were asked to simplify three algebraic expressions. Three sets of working for each expression are shown below. One set is correct and the other two sets contain errors.

For each expression, choose the correct set and then identify the errors in the other two sets of working.

a **Expression 1:** $4a - 3b + 2 + 2a + 8b - 7$

Set A	**Set B**	**Set C**
$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$
$= 4a + 2a + 2 + 7 + 3b + 8b$	$= 4a + 2a + 2 - 7 - 3b + 8b$	$= 4a - 2a + 2 - 7 - 3b + 8b$
$= 6a + 9 + 11b$	$= 6a - 5 + 5b$	$= 2a + 9 + 5b$

b **Expression 2:** $-3ab \times 4bc$

Set A	**Set B**	**Set C**
$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= -12ab^2c$	$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= 12 \times a \times b^2 \times c$ $= 12abc$	$-3ab \times 4bc$ $= -3 \times 4 \times a \times b^{1+1} \times c$ $= -12 \times a \times b \times 2 \times c$ $= -24abc$

c **Expression 3:** $4a^2bc \div (8abd)$

Set A	**Set B**	**Set C**
$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^1}{8^2} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times a^{2-1} \times 1 \times c \times \frac{1}{d}$ $= 2acd$	$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^1}{8^2} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times 1 \times 1 \times c \times \frac{1}{d}$ $= \frac{c}{2d}$	$4a^2bc \div (8abd)$ $= \frac{4a^2bc}{8abd}$ $= \frac{4 \times a^2 \times b \times c}{8 \times a \times b \times d}$ $= \frac{4^1}{8^2} \times \frac{a^2}{a} \times \frac{b}{b} \times c \times \frac{1}{d}$ $= \frac{1}{2} \times a^{2-1} \times 1 \times c \times \frac{1}{d}$ $= \frac{ac}{2d}$

13 If $x = 3$ and $y = -2$, evaluate each expression. To make it easier, simplify each expression first.

a $5x - 6y + 7y + 3x$

b $3xy - 8xy - xy$

c $5x \times 3y$

d $\frac{10x}{xy}$

e $xy \times xy^2$

f $6x^2y \div (2xy)$

g $x + y - 2xy + 5y - x$

h $x \times 3x - 2x^2 + 4x - y$

i $xy^{12} \div (x^4y^8)$

14 Evaluate each expression if $a = 2$, $b = -1$ and $c = 5$. Remember to simplify each expression first.

a $3a + 2b + 7c - a - 5c + b$

b $7ab + 4a - 5a + ab$

c $a^2b + ab^2 + ac - 3a^2b + 2ac$

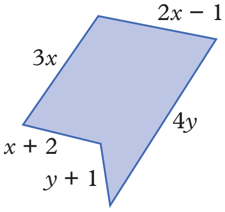
d $2abc \times bc \times 5a$

e $18ab^2c \div (6bc)$

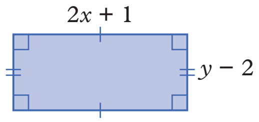
f $3ac^2 \times 4ab \div (9abc)$

15 Write the perimeter of each shape as an algebraic expression in simplest form.

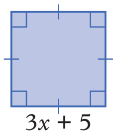
a



b



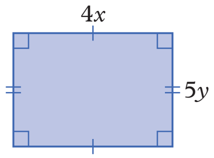
c



16 Calculate the perimeter of each shape in question **15** for $x = 4$ cm and $y = 5$ cm.

17 Write the shaded area in each shape as an algebraic expression in simplest form.

a



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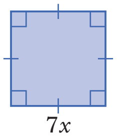

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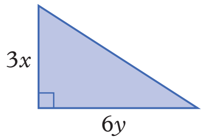

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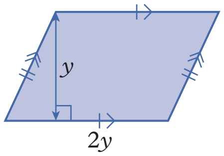


18 Calculate the area shaded in each shape in question **17** for $x = 3$ m and $y = 2$ m.

19 The area of a rectangle is $16xy$.

a If the length is $8x$, write an expression for the width of the rectangle.

b If the length of the rectangle is 16 m and y is 5 m, calculate the width and the area of the rectangle using the result from part **a**.

20 The area of a right-angled triangle is $6x^2$.

a If the base length is $4x$, write an expression for the height of the triangle.

b If the height of the triangle is 12 cm, calculate the area and the base length of the triangle using the result from part **a**.

21 A rectangle has a width of k.

a If the length of the rectangle is twice the width, write an expression for:

i the perimeter of the rectangle

ii the area of the rectangle.

b Calculate the perimeter and the area of the rectangle when $k = 5$ cm.

22 Lana plants a 1-metre-wide flowerbed around a square section of lawn.

a If the lawn has a length of x metres, write an expression for:

i the perimeter of the lawn

ii the area of the lawn

iii the perimeter around the outer edge of the flowerbed

iv the area of the flowerbed, given that the total area of the lawn and flowerbed is $(x^2 + 4x + 4)$ m^2.

b When $x = 8$, calculate:

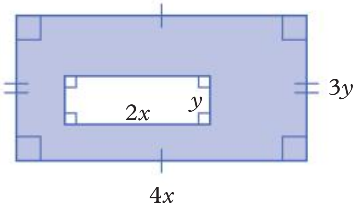
i the area of the flowerbed

ii the length of edging needed around the inner edge of the flowerbed

iii the length of edging needed around the outer edge of the flowerbed

iv the area to be mown.

23 Consider the shaded region of this shape.



a Write an algebraic expression for the area of the shaded region.

b If $x = 4$ cm and $y = 5$ cm, calculate the area of the shaded region.

c Write an expression for the total length of the outer and inner edges of the shape.

d Use your sets of values from part **b** to calculate the total length of the outer and inner edges of the shape.

e If $x = 12$ cm and the area of the shaded region is 210 cm^2, determine the total length of the outer and inner edges of the shape.

24 Write $((2x)^7 \times (4x)^{-3} \times (8x)^3 \div (16x)^4)^8$ in the form $2^{\square}x^{\square}$.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Like terms

Interactive skillsheet
Multiplying terms

Interactive skillsheet
Dividing terms

Topic quiz
2A

2B Expanding

Learning intentions

By the end of this topic you will be able to ...

✔ expand algebraic expressions of the form $a(b + c)$

✔ expand algebraic expressions of the form $(a + b)(c + d)$.

Inter-year links

Year 8	5F Expanding
Year 10	1D Expanding

The distributive law

- A **binomial** is an expression containing two terms that are either added or subtracted. For example, $x - 7$, $2x + y$, and $x^2 + y^2$ are all binomials.
- The **distributive law** is used to distribute or **expand** products over addition and subtraction.

$$a(b + c) = ab + ac$$

Note: The distributive law only applies if the expression inside the brackets is not raised to the power of any exponent (other than 1).

- To expand algebraic expressions with one pair of brackets:
  1. Multiply each term inside the brackets by the term outside the brackets.
  2. Simplify the results by performing any multiplication, addition and subtraction.
- The distributive law for expanding over a **binomial product** is: $(a + b)(c + d) = ac + ad + bc + bd$

$$(a + b)(c + d) = ac + ad + bc + bd$$

- To expand binomial products:
  1. Multiply each term inside the second pair of brackets, c and d, by each term in the first pair of brackets, a and b.
  2. Simplify the results by performing any multiplication, addition and subtraction.
- Where possible, expanded expressions should be simplified.
- The distributive law for expanding a binomial product can be derived by treating the first binomial as a single term and then applying the distributive law for expanding over one pair of brackets.

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= c(a + b) + d(a + b) \\ &= ca + cb + da + db \\ &= ac + ad + bc + bd\end{aligned}$$

- The **difference of two squares** is a specific form of expansion with a simplified rule:

$$(a + b)(a - b) = a^2 - b^2$$

- The expansion of a perfect square is a specific form of expansion with a simplified rule:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Example 2B.1 Expanding over binomial expressions

Expand and simplify each expression using the distributive law.

a $3(k + 2)$ **b** $-5(3a + 8)$ **c** $-3a(5a - 4b)$

THINK	**WRITE**
a **1** Multiply each term inside the brackets by the term outside the brackets.	**a** $3(k + 2) = 3 \times k + 3 \times 2$
2 Simplify the results by performing the multiplications.	$= 3k + 6$
b **1** Multiply each term inside the brackets by the term outside the brackets.	**b** $-5(3a + 8) = -5 \times 3a + (-5) \times 8$
2 Simplify the results by performing the multiplications. Take care with + and – signs when simplifying.	$= -15a - 40$
c **1** Multiply each term inside the brackets by the term outside the brackets. Enclose the second term in brackets to separate the plus sign and the negative coefficient of the *b* term.	**c** $-3a(5a - 4b) = -3a \times 5a + (-3a \times -4b)$
2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.	$= -15a^2 + 12ab$

Example 2B.2 Expanding binomial products

Expand each algebraic expression to remove the brackets.

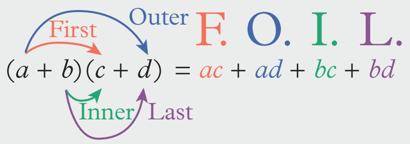
a $(a + 9)(b + 2)$ **b** $(x + 4)(x - 6)$ **c** $(3 - a)(t + 6)$

THINK	**WRITE**
a **1** Multiply each term inside the second pair of brackets, *b* and 2, by the first term in the first pair of brackets, *a*, and then the second term in the first pair of brackets, 9.	**a** $(a + 9)(b + 2) = a \times b + a \times 2 + 9 \times b + 9 \times 2$
2 Simplify each term.	$= ab + 2a + 9b + 18$

b **1** Multiply each term inside the second pair of brackets, x and -6, by the first term in the first pair of brackets, x, and then the second term in the first pair of brackets, 4.	**b** $(x + 4)(x - 6) = x \times x + x \times (-6) + 4 \times x + 4 \times (-6)$
2 Simplify each term.	$= x^2 - 6x + 4x - 24$
3 Simplify any like terms.	$= x^2 - 2x - 24$
c **1** Multiply each term inside the second pair of brackets, t and 6, by the first term in the first pair of brackets, 3, and then the second term in the first pair of brackets, $-a$.	**c** $(3 - a)(t + 6) = 3 \times t + 3 \times 6 + (-a) \times t + (-a) \times 6$
2 Simplify each term.	$= 3t + 18 - at - 6a$

Helpful hints

- ✔ When multiplying by a negative term, use brackets for clarity.
  For example: $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
- ✔ Show all your working to avoid arithmetical errors and to ensure all signs are correct.
- ✔ After expanding expressions containing brackets, always simplify your results by looking for like terms. Simplify, simplify, simplify!
- ✔ A good way of remembering which terms to multiply when expanding a binomial product is to memorise 'FOIL' (First terms, Outer terms, Inner terms, Last terms).



Note: This rule only works for expressions of the form $(a + b)(c + d)$.

ANS p412

Exercise 2B Expanding

△ 1(1st column), 2, 3–4(1st, 2nd column), 5–8(1st, 2nd columns), 9–13, 21, 22
□ 2–8(2nd column), 11–17, 19(a, c, d), 20, 24
⬠ 4(b, d, f), 5–8(2nd column), 11–18, 23, 24(d, f), 25–27

2B.1 **1** Expand each algebraic expression to remove the brackets.

a $4(a + 3)$	**b** $7(b + 5)$	**c** $3(c - 2)$
d $5(d - 1)$	**e** $6(4 + e)$	**f** $-2(f + 8)$
g $-3(g + 4)$	**h** $-8(h - 5)$	**i** $-4(x - 9)$
j $-5(2 - j)$	**k** $k(p + 6)$	**l** $a(b - 4)$
m $6(3m + k)$	**n** $n(2p + 4q)$	**o** $x(x - 7y)$

UNDERSTANDING AND FLUENCY

2 Expand and simplify each algebraic expression. Remember, $\frac{x}{4} = \frac{1}{4}x$ and $\frac{x+3}{4} = \frac{1}{4}(x+3)$.

a $\frac{1}{2}(8x + 12)$
b $\frac{8x + 12}{4}$
c $\frac{18 - 6n}{6}$
d $\frac{7xy + 5xz}{x}$
e $\frac{35ab - 50ac}{-5a}$
f $\frac{-21t^2 - 49t}{-7t}$

3 Expand and simplify each algebraic expression.

a $3(x + 2) + 8x$
b $11 + 5(p - 1)$
c $a(b + 4) - 2a$
d $-7(1 - y) + 4y + 3$
e $5k + 2 + 4(h - k)$
f $m(m - 6) - m^2$

4 Expand and simplify each algebraic expression.

a $2(x + 5) + 3(x - 6)$
b $8(k - 3) + 5(k + 4)$
c $3(p + 7) - 4(5 - p)$
d $x(x + 1) + 3(x + 4)$
e $m(m + 2) + 3(m + 2)$
f $y(y - 5) - 2(y - 5)$

5 Expand and simplify each expression.

a $3a(4z + 5)$
b $8b(7 + 5y)$
c $2c(7 - 3x)$
d $-5d(4w + 5)$
e $-6e(8v - 9)$
f $-10e(4t + 3u)$
g $4g(2g - 7t)$
h $-6h(5r - 7h)$
i $3ij(4ik + 5jk)$
j $3m^3(4m^2 + 5m^5)$
k $\frac{20n^6 - 12n^5}{4n^2}$
l $\frac{p^5(pq^4 - p^4q^8)}{q^4}$

2B.2 **6** Expand each algebraic expression to remove the brackets.

a $(a + 3)(b + 4)$
b $(c + 2)(d + 7)$
c $(m + 5)(n + 1)$
d $(x + 9)(y + 3)$
e $(k + 6)(p - 2)$
f $(f + 4)(g - 1)$
g $(a - 5)(c + 3)$
h $(w - 7)(f + 2)$
i $(x - 4)(y - 8)$
j $(j - 9)(k - 5)$
k $(2a + 7)(b + 3)$
l $(5c + 2)(3d - 4)$

7 Expand each algebraic expression to remove the brackets.

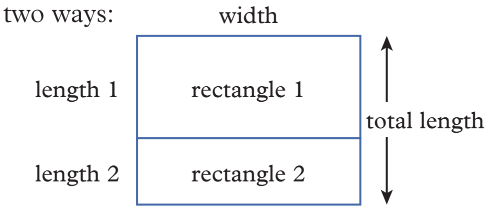
a $(a + 2)(a + 3)$
b $(x + 5)(x + 10)$
c $(d + 4)(d - 6)$
d $(y + 3)(y - 8)$
e $(k - 7)(k + 9)$
f $(m - 6)(m + 3)$
g $(5e - 2)(e - 4)$
h $(7a - 8)(3a - 1)$
i $(3y - 5)(2y - 1)$

8 Expand each algebraic expression to remove the brackets.

a $(5 - a)(b + 4)$
b $(x + 6)(3 - x)$
c $(2c + 3)(5 - 4c)$
d $(11d - 9e)(4d - 6e)$
e $(6p + 5q)(8q - 7p)$
f $(x + 2)(x - 3) + 3x^2 - 8x + 5$
g $(x^3 + x^2)(x^5 + x)$
h $(4x^2 + 7)(12 - 5x^2)$
i $(x + 2)(x + 3) + (x + 1)(x + 5)$

PROBLEM SOLVING AND REASONING

9 The area of the large rectangle in this diagram can be determined in two ways:



Area of large rectangle = width × total length

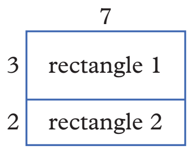
Area of large rectangle = area of rectangle 1 + area of rectangle 2
= width × length 1 + width × length 2

For each diagram below:

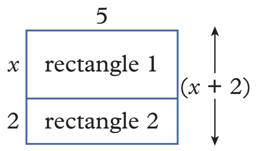
i write an expression for the area of the large rectangle by multiplying the total length by the width

ii write an expression for the area of the large rectangle by adding the area of rectangle 1 and rectangle 2.

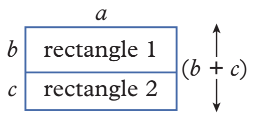
a



b



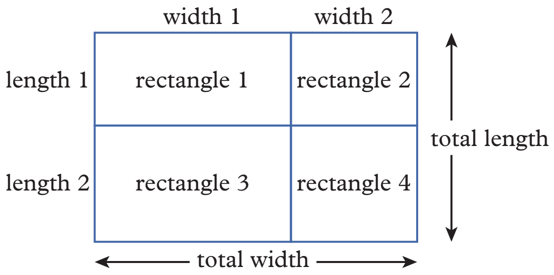
c Use the rectangle below to explain how $a(b + c)$ can be expanded to obtain $ab + ac$.



10 The area of the largest rectangle in this diagram can be determined in two ways:

Area of largest rectangle = total width × total length

Area of largest rectangle = area of rectangle 1 + area of rectangle 2 + area of rectangle 3 + area of rectangle 4
= width 1 × length 1 + width 2 × length 1 + width 1 × length 2 + width 2 × length 2

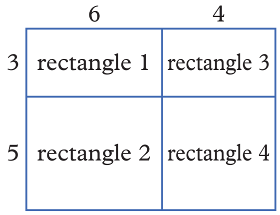


For each diagram below:

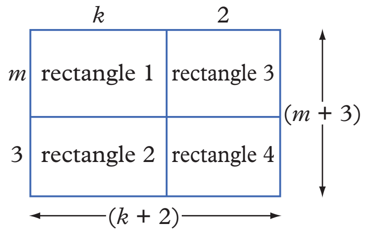
i write an expression for the area of the large rectangle by multiplying the total length by the total width

ii write an expression for the area of the large rectangle by adding the area of rectangle 1, rectangle 2, rectangle 3 and rectangle 4.

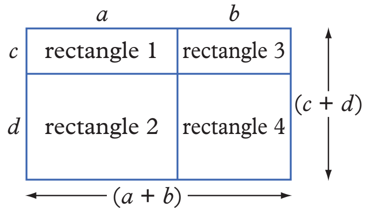
a



b



c Use this rectangle to explain how $(a + b)(c + d)$ can be expanded to obtain $ac + ad + bc + bd$.



11 Expand each algebraic expression to remove the brackets.

a $(a + 3)(a - 3)$ **b** $(b + 2)(b - 2)$ **c** $(c + 5)(c - 5)$

d $(3p - 8)(3p + 8)$ **e** $(h - 1)(h + 1)$ **f** $(k + m)(k - m)$

12 Describe the pattern or shortcut you can see in question **11**. What is special about the two binomial factors that are multiplied together?

13 The pattern you observed in question **11** is known as the difference of two squares. This rule can be written as: $(a + b)(a - b) = a^2 - b^2$

a Why do you think the rule is called the difference of two squares?

b Does it matter whether the product is $(a + b)(a - b)$ or $(a - b)(a + b)$? Explain.

c Use the rule (or shortcut) to expand each algebraic expression.

i $(x - 2)(x + 2)$ **ii** $(y + 9)(y - 9)$

iii $(m + 6)(m - 6)$ **iv** $(3d - 10)(3d + 10)$

v $(m + n)(m - n)$ **vi** $(3 + x)(3 - x)$

PROBLEM SOLVING AND REASONING

14 a Use the difference of two squares expansion rule to expand $(100 + 3)(100 - 3)$.

b Use your answer to work out the result for 103×97.

c Work out the result for each product without using a calculator.

i 102×98 **ii** 95×105

iii 1001×999 **iv** 994×1006

15 Expand each algebraic expression to remove the brackets and simplify.

a $(a + 2)(a + 2)$ **b** $(b + 7)(b + 7)$

c $(c + 4)(c + 4)$ d $(3p - 5)(3p - 5)$

e $(h - 1)(h - 1)$ **f** $(m + n)(m + n)$

16 Describe the pattern or shortcut you can see in question **15**. What is special about the two binomial factors that are multiplied together?

17 The pattern you observed in question **15** is known as the expansion of a perfect square. This rule can be written as: $(a + b)^2 = a^2 + 2ab + b^2$.

a Why do you think the rule is called the expansion of a perfect square?

b Use the rule (or shortcut) to expand each algebraic expression.

i $(x + 3)^2$ **ii** $(y + 6)^2$

iii $(m + 2)^2$ **iv** $(4b + 11)^2$

v $(m + n)^2$ **vi** $(5 + x)^2$

c Is $(a - b)^2 = a^2 - 2ab + b^2$ also an expansion of a perfect square? Explain.

d Expand each algebraic expression.

i $(a - 2)^2$ **ii** $(b - 4)^2$

iii $(c - 7)^2$ **iv** $(5w - 6)^2$

v $(k - p)^2$ **vi** $(3 - x)^2$

18 A binomial is an algebraic expression with two terms.

a Write three examples of a binomial.

b $(x + 1)(x - 2)$ is an example of a binomial product. Explain what the term 'binomial product' means.

c How many terms do you think a trinomial has?

d Write three examples of a trinomial.

e Find out what a quadratic trinomial is and provide three examples.

19 We can also expand products of sums and differences with more than two terms; it just involves adding more products together. For example:

$$\begin{aligned} 2(3x + 5y - 7) &= 2 \times (3x) + 2 \times (5y) + 2 \times (-7) \\ &= 6x + 10y - 14 \end{aligned}$$

and

$$\begin{aligned} (a + b + c)(x + y) &= x(a + b + c) + y(a + b + c) \\ &= ax + bx + cx + ay + by + cy \end{aligned}$$

It is recommended to expand in two stages as shown above to ensure that no products are missed.
Expand each algebraic expression.

a $4(3a - 5b + 10)$ **b** $2x(3w - 5x + 7y - 4z)$

c $5[(x + 3)(x + 4)]$ **d** $(3a + 5b - 6)(2x + 3y)$

e $(x + 3y + 6)(2x + y + 3)$ **f** $(x + 2)(x^2 + 6x + 7)$

20 Which of the following is not equivalent to $3(x + 2)(x + 8)$?

A $(3x + 6)(x + 8)$ **B** $(x + 2)(3x + 24)$ **C** $3(x^2 + 10x + 16)$

D $3x^2 + 30x + 48$ **E** $(3x + 6)(3x + 24)$

PROBLEM SOLVING AND REASONING

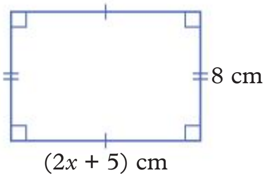
21 **a** For the rectangle on the right, write the area as a product of two factors.

b Expand the expression to remove brackets.

c When $x = 3$, calculate the area using your answer to:

i part **a** **ii** part **b**.

d Compare your answers in part **c**. How does this show that you have expanded the area expression correctly?



22 For each of the following rectangles:

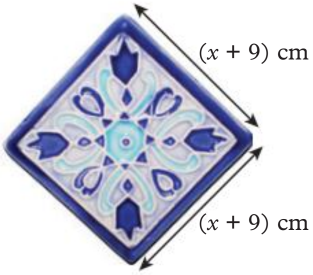
i write the area as a binomial product

ii expand the binomial product to remove brackets

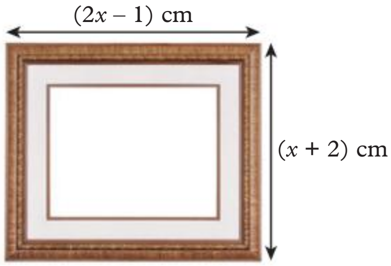
iii calculate the area when $x = 5$.

a

b



c



23 A rectangular trampoline is 6 m long and 4 m wide. Safety matting that is p metres wide is to be placed around the perimeter of the trampoline.

a Draw a labelled diagram of the top of the trampoline and the matting around it.

b Write an expression for:

i the total length of the trampoline and matting

ii the total width of the trampoline and matting.

c Write an expression for the total area taken up by the trampoline and matting. Simplify the expression by expanding to remove any brackets.

d Write an expression for the area of the matting only.

e If $p = 2$, calculate the area taken up by:

i the trampoline **ii** the trampoline and matting **iii** the matting.

24 Expand and simplify each expression.

a $(10y - 7)^2$ **b** $x^4(x^3 - x^2)$ **c** $(x^2 - 5)(x^2 + 5)$

d $(y^6 + 3y)^2$ **e** $a^3(a^2 + 4) - a^2(a^3 + 9a)$ **f** $x^5y^2(xy^4 + 2w)$

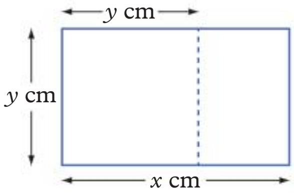
CHALLENGE

25 Show that the expression $(a - b)^2 + (c - d)^2$ is equivalent to $(b - a)^2 + (d - c)^2$.

26 A rectangular piece of paper is x cm long and y cm wide. The paper is torn along a line parallel to its width, forming a square of side length y cm, and another rectangle.

a Write the dimensions of the newly formed square and rectangle.

b Prove that the area of the original rectangle is the same as the total area of the two new shapes.



27 Paul has invented a new form of expansion as: $a \dashv b = b - a + ab$.

If, using this expansion, $3 \dashv 8$ has the same value as $6 \dashv x$, what is the value of x?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Expanding over one pair of brackets

Interactive skillsheet
Expanding binomial products

Investigation
It's a mystery

CAS instructions
Expanding

Topic quiz
2B

2C Factorising using the HCF

Learning intentions

By the end of this topic you will be able to ...

✔ find the HCF of two or more algebraic terms
✔ factorise algebraic expressions by taking out the HCF
✔ factorise algebraic expressions by grouping terms.

Inter-year links

Support	Multiples and factors
Year 7	2D Factors and the highest common factor
Year 8	5G Factorising
Year 10	1F Factorising

Factorising terms

- **Factorising** an algebraic expression is the opposite to expanding an algebraic expression.
- An expression is in simplest factorised form if all the terms inside the brackets have no common factors.
- An expression is in expanded form if it has no brackets and is simplified.

expanding →

$$7(a+2) = 7a + 14$$

← factorising

$$\underbrace{2x^2 - 6x}_{\text{expanded form}} = \underbrace{2x(x-3)}_{\text{factorised form}}$$

The highest common factor (HCF) of an expression

- The **highest common factor (HCF)** of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients, pronumerals and algebraic expressions that are factors of every term.
- The HCF of an expression can have numerical and pronumeral terms.
- The HCF can be a binomial factor.
- To identify the HCF of an expression:
  1 Find the HCF of the coefficients.
  2 Find the HCF of the pronumerals.
  3 Multiply the factors from steps 1 and 2, and then simplify.

Algebraic expressions	HCF
$x + 7x$	x
$6m + 12mn + 8mp$	$2m$
$15a^3bc + 9a^2c$	$3a^2c$
$x(x-1) + 2(x-1)$	$(x-1)$

Factorising algebraic expressions

- To factorise an expression:
  1 Identify the HCF of the expression.
  2 Write the HCF in front of the brackets. Inside the brackets, divide each term in the original expression by the HCF.
  3 Simplify the bracketed expression.

$$\begin{aligned} 7a + 14 &= 7\left(\frac{\cancel{7}^{1}a}{\cancel{7}_{1}} + \frac{\cancel{14}^{2}}{\cancel{7}_{1}}\right) \\ &= 7(a+2) \end{aligned}$$

Factorising by grouping terms

- To factorise an expression with four terms by grouping:
  1 Identify pairs of terms with common factors. Group the pairs of terms, remembering to move the positive or negative sign with the term.
  2 Factorise each pair of terms by dividing out the HCF.
  3 Factorise using the binomial factor and simplify the expression.

$$\begin{aligned} x^2 + 2x - 3x - 6 &= x(x+2) - 3(x+2) \\ &= (x+2)(x-3) \end{aligned}$$

Example 2C.1 Identifying the HCF

Find the highest common factor (HCF) of each pair of terms.

a 24 and 84　　**b** a^2b and b^2ac　　**c** $12x^2y^3$ and $-18x^3y$

THINK	**WRITE**
a 1 Write the prime factorisation for each number.	**a** $24 = 2 \times 2 \times 2 \times 3$ $84 = 2 \times 2 \times 3 \times 7$
2 Multiply the common factors to find the HCF.	$\text{HCF} = 2 \times 2 \times 3$ $= 12$
b 1 Write each term in expanded form.	**b** $a^2b = a \times a \times b$ $b^2ac = b \times b \times a \times c$
2 Multiply the common factors to find the HCF.	$\text{HCF} = a \times b$ $= ab$
c 1 Find the HCF of the coefficients. The negative sign can be represented using the factor '−1'.	**c** The HCF of 12 and −18 is 6.
2 Find the HCF of the pronumerals.	The HCF of x^2y^3 and x^3y is x^2y.
3 Multiply the common factors to find the HCF. Simplify the product.	The HCF of $12x^2y^3$ and $-18x^3y$ is $6x^2y$.

Example 2C.2 Factorising algebraic expressions

Factorise each expression.

a $6a + 18$　　**b** $x^2 - 7x$　　**c** $10k^3m^2 + 15k^2m$

THINK	**WRITE**
a 1 Identify the HCF.	**a** HCF = 6
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$6a + 18 = 6\left(\frac{\cancel{6}^1 a}{\cancel{6}^1} + \frac{\cancel{18}^3}{\cancel{6}^1}\right)$
3 Simplify the bracketed expression.	$= 6(a + 3)$
b 1 Identify the HCF.	**b** HCF = x
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$x^2 - 7x = x\left(\frac{x^2}{x} - \frac{7x}{x}\right)$
3 Simplify the bracketed expression. Use the exponent laws to help you.	$= x(x^{2-1} - 7x^{1-1})$ $= x(x - 7)$
c 1 Identify the HCF.	**c** HCF = $5k^2m$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$10k^3m^2 + 15k^2m$ $= 5k^2m\left(\frac{\cancel{10}^2k^3m^2}{\cancel{5}^1k^2m} + \frac{\cancel{15}^3k^2m}{\cancel{5}^1k^2m}\right)$
3 Simplify the bracketed expression. Use the exponent laws to help you.	$= 5k^2m(2k^{3-2}m^{2-1} + 3k^{2-2}m^{1-1})$ $= 5k^2m(2km + 3)$

Example 2C.3 Factorising using a HCF that is a binomial factor

Factorise each expression.

a $y(x+3)+7(x+3)$ **b** $4k(2-m)-5(2-m)$

THINK	WRITE
a **1** Identify the HCF. It is the binomial factor of $(x+3)$.	**a** HCF $= (x+3)$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$y(x+3)+7(x+3) = (x+3)\left(\frac{y\cancel{(x+3)}^1}{\cancel{(x+3)}^1} + \frac{7\cancel{(x+3)}^1}{\cancel{(x+3)}^1}\right)$
3 Simplify the bracketed expression.	$= (x+3)(y+7)$
b **1** Identify the HCF. It is the binomial factor of $(2-m)$.	**b** HCF $= (2-m)$
2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.	$4k(2-m)-5(2-m) = (2-m)\left(\frac{4k\cancel{(2-m)}^1}{\cancel{(2-m)}^1} - \frac{5\cancel{(2-m)}^1}{\cancel{(2-m)}^1}\right)$
3 Simplify the bracketed expression.	$= (2-m)(4k-5)$

Example 2C.4 Factorising by grouping terms

Factorise $xy + 2x + 3y + 6$ by grouping terms.

THINK	WRITE
1 Check for a HCF of all four terms. Group the terms in pairs and identify the HCF for each pair.	$xy+2x+3y+6 = (xy+2x)+(3y+6)$ HCF of xy and $2x$ is x. HCF of $3y$ and 6 is 3.
2 Factorise each pair of terms by dividing out the HCF.	$xy+2x+3y+6 = x(y+2)+3(y+2)$
3 Factorise using the binomial factor of $(y+2)$. Simplify the expression.	$= (y+2)(x+3)$

Helpful hints

- ✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2, if the number is even
  - → divisible by 3, if the number's digits add to a multiple of 3
  - → divisible by 5, if the number ends in 0 or 5.
- ✔ The great thing about factorising is that you can always check your answer by expanding the brackets!

$$2x-16 = 2(x-8) = 2x-16$$

ANS p413

Exercise 2C Factorising using the HCF

△ 1–4(1st, 2nd columns), 5, 6, 7–10(1st, 2nd columns), 11, 12, 15, 16(a, b), 17

□ 2–4(3rd column), 5–6(e–h), 7–10(2nd, 3rd columns), 13–16, 18, 20, 24

⬠ 4(g–i), 7–10(2nd, 3rd columns), 15, 16, 19–25

UNDERSTANDING AND FLUENCY

2C.1 **1** Find the highest common factor (HCF) of each pair of terms.

a 4 and 28 **b** 6 and 10 **c** 15 and 35

d d^2 and $3d$ **e** $2e$ and $2k$ **f** 3 and -6

g 12 and -8 **h** $9h$ and $-15h^2$ **i** $24x^2$ and $36x$

2 Find the HCF of each pair of terms.

a bc and cd **b** $2xy$ and $2y$ **c** $18mn$ and $-9m$

d $abcd$ and bdf **e** $8xy$ and $28y$ **f** $6k^2$ and $-10k$

g p and $11p^2$ **h** $45ab$ and $-40cd$ **i** $3pq$ and $6p$

3 Factorise each expression using the HCFs from question **2**.

a $bc + cd$ **b** $2xy + 2y$ **c** $18mn - 9m$

d $abcd + bdf$ **e** $8xy + 28y$ **f** $6k^2 - 10k$

g $p + 11p^2$ **h** $45ab - 40cd$ **i** $3pq + 6p$

4 Find the HCF of each pair of terms.

a a^4 and a^7 **b** $5b^6$ and $3b^2$ **c** $16c^5$ and $-2c^3$

d $6d^4z^7$ and $6d^3z^5$ **e** $7e^6y^5$ and $14e^7y^8$ **f** $-20fx^{12}$ and $6f^3x^9$

g $-2g^2$ and $-2gh^5$ **h** $12i^6k^5$ and $-18j^3k^8$ **i** $x^9y^{11}z^{13}$ and $-x^{11}y^7z^{16}$

2C.2 **5** Factorise each expression.

a $10x + 5$ **b** $3y - 21$ **c** $8k + 12$ **d** $15 - 6d$

e $28x + 21y$ **f** $20n - 50$ **g** $x^2y + 3xy^4$ **h** $n^3m^2 - 9n^2m$

6 Factorise each expression.

a $20ab - 5b$ **b** $8d + 8cde$ **c** $15x^2 + 10x$ **d** $4k^2 - 22k$

e $30n - 18n^2$ **f** $16a^2 + a$ **g** $2h^2 - 14h$ **h** $6p + 6p^2$

7 Factorise each expression. Remember to use the exponent laws to help you simplify.

a $24m^6 + 16m^4$ **b** $21q^5r^2 + 35q^3r^6$ **c** $15t^4u^2 - 5t^8u^7$

d $12c^9d^5 + 6c^4d^4$ **e** $7e^{11}f^3 - 7e^5f^3$ **f** $3i^5j^2k^6 + 27ij^7k$

g $a^7b^{12}c^7 + b^3c^9\mathrm{d}^4$ **h** $5mpq - 3np^{10}q^5$ **i** $u^4w^5y^2z^3 + v^4w^5x^2z^3$

8 Factorise each expression by factoring out a negative HCF. Remember that $-5mn = -1 \times 5 \times m \times n$.

a $-5mn - 10n$ **b** $-14xy - 7x$ **c** $-6c + 6cd$

d $-a^2 - 3a$ **e** $-4k^2 - 2k$ **f** $-8x^2 + 8x$

g $-12 - 3xy$ **h** $-16m - 10m^2$ **i** $-9x^2y + 18xy$

2C.3 **9** Factorise each expression.

a $x(w + 4) + 2(w + 4)$ **b** $y(x - 1) + 7(x - 1)$ **c** $a(a + 6) - 3(a + 6)$

d $p(5 - n) + 8(5 - n)$ **e** $3k(4 - k) - 5(4 - k)$ **f** $2x(3x - 4) + 9(3x - 4)$

g $4g(2g + 1) + (2g + 1)$ **h** $2h(8n - d) - (8n - d)$ **i** $3a(7x + 6y) + 2b(7x + 6y)$

2C.4 **10** Factorise each expression by grouping terms.

a $ab + 5b + 4a + 20$ **b** $xy - 6x + 7y - 42$ **c** $mn + 4m - 2n - 8$

d $y^2 + 3y + 5y + 15$ **e** $k^2 - 7k + 2k - 14$ **f** $6x + 18 + x^2 + 3x$

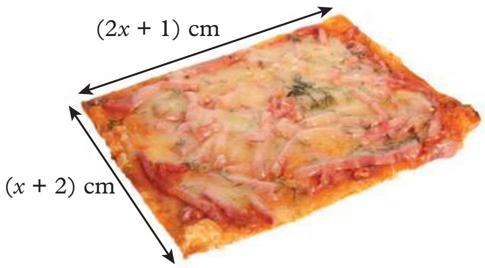
g $a^2 - 7a - 2a + 14$ **h** $p^2 + 5p - 2p - 10$ **i** $6c^2 - 2c + 9c - 3$

PROBLEM SOLVING AND REASONING

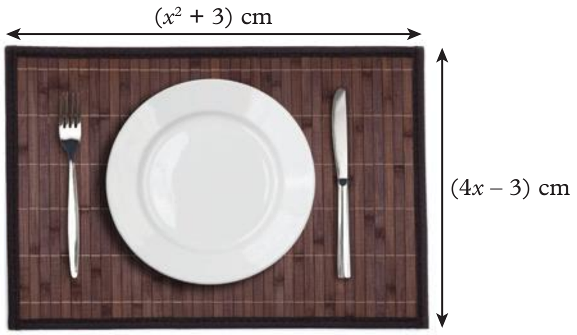
11 Explain the difference between expanding an expression and factorising an expression.

12 **a** Write an expression in factorised form for the perimeter of each rectangular shape.

i



ii



b Calculate the perimeter of each rectangle when $x = 5$.

13 Write an expression for the missing side length of each rectangular object, given the area shown.

a Area of rug is $(8x + 20)$ m^2.

b Area of stained glass panel is $(m^2 + 15m)$ cm^2.

14 Write an expression for the perimeter of each item in question **13** in factorised form.

15 Expressions with more than two terms can be factorised in the same way as those with two terms. For example, to factorise $8a + 12b - 6c$ write the HCF, 2, in front of the brackets and divide each term inside the brackets by the HCF. So, $2\left(\frac{8a}{2} + \frac{12b}{2} - \frac{6c}{2}\right) = 2(4a + 6b - 3c)$. Factorise each expression.

a $27x - 9y + 15z$

b $45p - 50q - 5$

c $4 - 20i + 40j - 60k$

d $a^5 + a^3 + a^2$

e $18b^3c^5 - 36b^4c^4 + 24b^8c^5$

f $84r^{12}t^8 + 7r^5t^7 + 49r^8t^{14} + 14r^9t^8$

16 We can take fractions out as a factor so that all terms in the brackets no longer involve fractions. For example:

$$\begin{aligned}\frac{5}{3}a + \frac{5}{6}b &= \frac{10}{6}a + \frac{5}{6}b\\ &= \frac{1}{6}\left(\frac{10}{6}a \times 6 + \frac{5}{6}b \times 6\right)\\ &= \frac{1}{6}(10a + 5b)\\ &= \frac{5}{6}\left(\frac{\cancel{10}^{2}a}{\cancel{5}} + \frac{\cancel{5}^{1}b}{\cancel{5}}\right)\\ &= \frac{5}{6}(2a + b)\end{aligned}$$

Factorise each expression.

a $\frac{5}{2}x + \frac{3}{2}y$

b $\frac{9}{5}g + \frac{3}{5}h$

c $\frac{5}{2}p + \frac{15}{4}q$

d $\frac{7}{3}m - \frac{5}{4}n$

PROBLEM SOLVING AND REASONING

17 A rectangle has an area of $(10x - x^2)$ mm^2.

a Write the area in factorised form.

b List possible expressions for the length and width of the rectangle.

c Suggest values for the length and width of three different rectangles that have the given area.

18 A triangle has an area of $(21x - 3x^2)$ cm^2.

a List possible expressions for the height and base length of the triangle.

b Suggest numerical values for the height and base length of three different triangles that have the given area.

19 A square-based prism has a volume of $(9x^3 + 45x^2)$ cm^3 and a height of $(x + 5)$ cm. Determine the length of the square base of the prism in terms of x.

20 Consider the expression $x^2 + 3x - 4x - 12$.

a Factorise the expression by grouping the first two terms together, and the last two terms together.

b Perform the factorisation again by grouping the first and third terms together, and the second and fourth terms together.

c Compare your answers to parts **a** and **b**.

21 A number is represented by the pronumeral n.

a Write down the next two consecutive numbers in terms of n.

b Write the sum of these three consecutive numbers.

c Factorise your expression from part **b**. Explain the answer.

d Investigate to see whether the same outcome results from the sum of three consecutive even numbers.

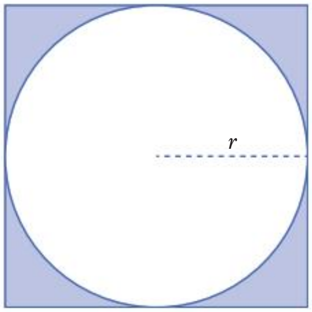
CHALLENGE

22 In question **21**, an odd number (three) of consecutive numbers were added together. Investigate to see the outcome when an even number of consecutive numbers are added together. Write a factorised expression for the sum of 10 consecutive numbers, where the first number is n.

23 Completely factorise each expression.

a $x(y + 5) - (3y + 15)$ **b** $p(2q - 3) - 2q + 3$ **c** $2(a^2 - 3a) + (9 - 3a)$

24 A circle of radius r cm fits within a square as shown in the diagram. Write a factorised expression for the area of the square not covered by the circle. Leave your answer in terms of r and π.



25 It is possible to take any term or expression as a factor of another expression by dividing it out. For example, to take 7 out as a factor of $3x + 5$, write the 7 in front of the brackets; inside the brackets, divide each term by 7.

So, $3x + 5 = 7\left(\frac{3x}{7} + \frac{5}{7}\right)$.

a Take 5 out as a factor of $9x + 17$.

b Take 12 out as a factor of $12x^2 + 3x + 6$.

c Take x out as a factor of $3x^2 + 5xy + y^2$.

d Take $x + 2$ out as a factor of $7(x + 2)^2 + 5(x + 2) + 9$.

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Factorising using the HCF

Interactive skillsheet
Factorising by grouping terms

Investigation
Surface area of a soft drink can

Topic quiz
2C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

2A **1** Simplify the expressions.

a $-3a + 6b - 5a - 8b + 12a - 5$

b $9t - t^2 + 4t - 6 + 5t^2 + 2 - t$

c $7x^3 - 4x^2 - 3x + 2 + 8x - 3x^2 - 5x^3$

d $7cd^2 - 7d^2 + 2c - 8d^2 - 3d^2c + 5dd$

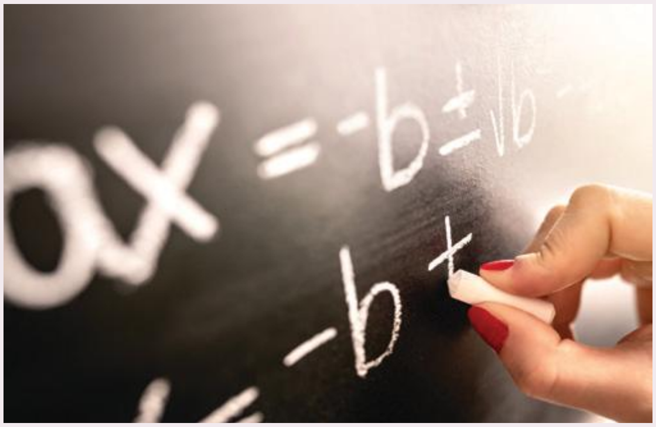
2A **2** Simplify the expressions.

a $5a \times (-7b)$

b $-8ac \times 3a \times (-2bc)$

c $\dfrac{45gh}{20g}$

d $\dfrac{18t^2u}{54tu^2}$



2A **3** Simplify the expressions.

a $(-5a^3b^6)^2 \times (-2a^2b^4)^5$

b $\left(-\dfrac{72k^4p^3}{16k^5p}\right)^3$

c $(8r^{-3}t^4)^{-1} \times \left(\dfrac{1}{3}r^5t^{-4}\right)^{-2}$

d $-42(x^3y^{-5}z^4)^{-4} \div [-63(x^{-5}y^3z^{-2})^8]$

2A **4** Expand the following.

a $8(a + 3)$

b $-4(2b - 5)$

c $3c(7 - y)$

d $9d(2d - 1)$

2B **5** Expand and simplify the following.

a $5(w + 7) - 2w + 6$

b $8 - 3(x - 9) + 2x$

c $10y - (12 - 5y) + 3$

d $z(z + 2) + 3(z - 5)$

2B **6** Expand and simplify the following.

a $(p + 2)(v + 5)$

b $(4 - q)(u + 3)$

c $(r + 6)(r - 2)$

d $(2t - 5)(3t - 7)$

2B **7** Expand and simplify the following.

a $(a^4 + 2a^2)(a^3 + 5)$

b $(b^2 - 3b)(b + 8) + 2b^2 - 4b + 1$

c $(c + 2)(c - 9) - 4(c - 2)$

d $(5 - d)(4 - d) + (2d + 1)(3d - 1)$

2C **8** Factorise the following.

a $6a + 9$

b $12b - 36$

c $-28c - 7$

d $8d^2 + 36d$

2C **9** Factorise the following.

a $2m(3g + 2) + 5(3g + 2)$

b $8(h - 3) - n(h - 3)$

c $p(q + 7r) - (q + 7r)$

d $2x(x + 3) - 5(x + 3)$

2C **10** Factorise the following by grouping.

a $ab + 5a + 4b + 20$

b $12cd - 8c - 21d + 14$

c $15ef - 5e + 3f - 1$

d $x^2 + 6x + 5x + 30$

2D Factorising the difference of two squares

Learning intentions

By the end of this topic you will be able to …

✔ factorise expressions in the form $a^2 - b^2$.

Inter-year links

Year 7	1G Exponents and square roots
Year 8	5G Factorising
Year 10	1F Factorising

The difference of two squares

- Binomials that have the same terms but one different sign, such as $a + b$ and $a - b$, are **conjugates** of each other.
- If a pair of conjugates are multiplied together, $(a + b)(a - b)$, then the middle terms add to zero and the solution is the difference of two squares:

$$\begin{aligned}(a + b)(a - b) &= a^2 + ab - ab - b^2\\ &= a^2 - b^2\end{aligned}$$

The difference of two squares rule can be used to factorise expressions of the form $a^2 - b^2$:

$$a^2 - b^2 = (a - b)(a + b)$$

Factorising the difference of two squares

1 Check for common factors. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor.

2 Write the expressions in the brackets as the difference of two squares.

3 Factorise using the difference of two squares rule.

$$\begin{aligned}3x^2 - 12 &= 3(x^2 - 4)\\ &= 3(x^2 - 2^2)\\ &= 3(x - 2)(x + 2)\end{aligned}$$

Remember the first 10 square numbers so that they can be easily identified.

	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
Perfect square	1	4	9	16	25	36	49	64	81	100

Example 2D.1 Factorising simple expressions using the difference of two squares rule

Factorise $x^2 - 25$.

THINK	WRITE
1 There are no common factors. Write the expressions as a difference of two squares.	$x^2 - 25 = x^2 - 5^2$
2 Factorise using the difference of two squares rule with $a = x$ and $b = 5$.	$= (x - 5)(x + 5)$

Example 2D.2 Factorising expressions using the difference of two squares rule

Factorise each quadratic expression.

a $9y^2 - 16$ **b** $4k^2 - 9m^2$

THINK	WRITE
a 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.	**a** $9y^2 - 16 = (3y)^2 - 4^2$
2 Factorise using the difference of two squares rule with $a = 3y$ and $b = 4$.	$= (3y - 4)(3y + 4)$
b 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the squares of the terms with a coefficient and a pronumeral.	**b** $4k^2 - 9m^2 = (2k)^2 - (3m)^2$
2 Factorise using the difference of two squares rule with $a = 2k$ and $b = 3m$.	$= (2k - 3m)(2k + 3m)$

Example 2D.3 Factorising complex expressions using the difference of two squares rule

Factorise each quadratic expression.

a $2x^2 - 8y^2$ **b** $(m + 4)^2 - 1$

THINK	WRITE
a 1 Check for common factors: 2. Write the factor in front of the brackets. Inside the brackets, divide each term by the factor.	**a** $2x^2 - 8y^2 = 2(x^2 - 4y^2)$
2 Write the expressions as a difference of two squares.	$= 2(x^2 - (2y)^2)$
3 Factorise using the difference of two squares rule with $a = x$ and $b = 2y$. Write the factor of 2 before the brackets.	$= 2(x - 2y)(x + 2y)$
b 1 There are no common factors. Write the expressions as a difference of two squares. Use brackets to show the square of the term with a coefficient and a pronumeral.	**b** $(m + 4)^2 - 1 = (m + 4)^2 - 1^2$
2 Factorise using the difference of two squares rule with $a = m + 4$ and $b = 1$.	$= (m + 4 - 1)(m + 4 + 1)$
3 Simplify the expression in each pair of brackets.	$= (m + 3)(m + 5)$

Helpful hints

- ✔ Memorise the first 15 square numbers to help you with factorising and mental arithmetic.
- ✔ You can check your factorisation by expanding the brackets!
- ✔ Simplify the expression inside the brackets before you give your final answer.
- ✔ The difference of two squares rule only applies to two square terms that are subtracted, not added together. For example, $x^2 - 4$ can be factorised using the rule but $x^2 + 4$ cannot.

$$\begin{aligned} a^2 - 9^2 &= (a-3)(a+3) \\ &= a^2 - 3a + 3a - 9 \\ &= a^2 - 9 \end{aligned}$$

ANS p414

Exercise 2D Factorising the difference of two squares

△ 1, 2, 3(1st, 2nd columns), 4–5(1st column), 6(1st, 2nd columns), 7–9(1st column), 10, 13–15, 20(a, b)

■ 3(2nd, 3rd columns), 4–5(2nd column), 6(2nd, 3rd columns), 7–9(2nd column), 12, 15, 16, 18, 19(a, b, e), 20(a, c), 21

⬠ 4–5(2nd column), 6(3rd column), 7–9(2nd column), 11, 15–17, 18, 19(b, d, f, h), 20(c, d), 21–23

1 **a** Expand $(3x + 7)(3x - 7)$.

b Expand $(3x + 7)(3x - 7)$ using the difference of two squares rule.

c Explain how you factorise $9x^2 - 49$.

2D.1 **2** Factorise each expression.

a $x^2 - 36$ **b** $a^2 - 100$ **c** $9 - y^2$

2D.2 **3** Factorise each expression using the difference of two squares rule.

a $25m^2 - 4$ **b** $49 - 64c^2$ **c** $(ab)^2 - 9$

d $x^2y^2 - w^2$ **e** $p^2 - q^2r^2$ **f** $b^2n^2 - d^2m^2$

g $(5jk)^2 - (6mn)^2$ **h** $81a^2b^2 - 49c^2d^2$ **i** $144p^2q^2 - 121r^2t^2$

2D.3 **4** Factorise each expression.

a $3x^2 - 12$ **b** $18p^2 - 50$

c $500 - 245g^2$ **d** $7 - 28u^2$

e $36r^2 - 100$ **f** $6400 - 12100z^2$

g $6x^2 - 6y^2$ **h** $pq^2 - pr^2$

i $u^2t^2 - d^2u^2$ **j** $36dk^2 - 16dj^2$

k $162a^2b^2 - 242a^2d^2$ **l** $9x - 81x^3$

5 Factorise each expression.

a $(k - 3)^2 - 16$ **b** $(7 - m)^2 - 1$

c $9 - (y + 2)^2$ **d** $4 - (a + 2)^2$

e $(y + 5)^2 - x^2$ **f** $u^2 - (t - 9)^2$

g $(2p + 4)^2 - p^2$ **h** $(9 - 4m)^2 - m^2$

i $(x + 3)^2 - x^2$ **j** $(q + 6)^2 - (r - 5)^2$

k $(7x - 4)^2 - (3x - 5)^2$ **l** $(z + 1)^2 - (z - 1)^2$

6 Factorise each expression.

a $-x^2 + 36$ **b** $-16 + b^2$ **c** $-25 + 9h^2$

d $-49d^2 + 64n^2$ **e** $-(rp)^2 + 1$ **f** $-t^2u^2 + w^2x^2$

UNDERSTANDING AND FLUENCY

7 Factorise each expression.

a $t^2 - \frac{4}{9}$ **b** $r^2 - \frac{25}{49}$

c $\frac{81}{16} - u^2$ **d** $\frac{9}{100}d^2 - \frac{121}{144}$

e $\frac{1}{64}k^2 - \frac{1}{25}$ **f** $\frac{w^2}{169} - \frac{x^2}{196}$

8 Factorise each expression. Hint: It might help to convert each decimal to a fraction first.

a $z^2 - 0.16$ **b** $y^2 - 1.21$

c $0.04 - x^2$ **d** $c^2 - 0.0025$

e $0.49g^2 - 0.01h^2$ **f** $0.0144 - 0.0121k^2$

9 Factorise each expression as the difference of two squares. Hint: Use the exponent laws.

a $x^6 - 9$ **b** $36 - y^{10}$

c $z^{22} - 49$ **d** $4p^{14} - 25$

e $\frac{q^{18}}{16} - \frac{81}{121}$ **f** $-r^{30}t^{50} + 1$

PROBLEM SOLVING AND REASONING

10 a Write the following differences as a product.

i $14^2 - 4^2$ **ii** $10^2 - 15^2$

iii $99^2 - 98^2$ **iv** $16 - 64$

v $169 - 9$ **vi** $225 - 196$

b Evaluate each difference in part **a** using the product.

c Write the following products as a difference of two square numbers.

i $(8 + 5)(8 - 5)$ **ii** $(12 + 18)(12 - 18)$

iii $(30 + 1)(30 - 1)$ **iv** 12×6

v 40×-20 **vi** 13×15

d Evaluate each product in part **c** using the difference.

11 Without using a calculator, work out the value of each problem.

a $63^2 - 37^2$

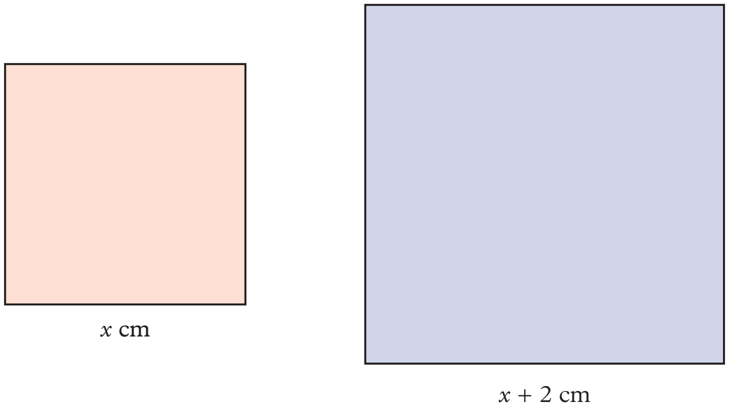
b $15.19^2 - 14.81^2$

c $\left(\frac{13}{25}\right)^2 - \left(\frac{12}{25}\right)^2$

12 a Write a simplified expression to represent the difference between the squares of two consecutive numbers n and $(n + 1)$.

b If n is always an integer, what type of number is the difference of the squares of two consecutive numbers?

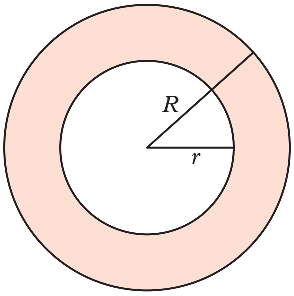
13 A square has side lengths of x cm. A second square has side lengths 2 cm longer than the first square. Write a factorised expression to represent the difference between the areas of the two perfect squares.



14 Use the difference of two squares rule to fully factorise each expression.

a $x^4 - 16$ **b** $x^8 - y^8$

15 An annulus (plural annuli) is a 2D shape that looks like a donut, with a circle cut out of the centre of another circle. The radius of the larger circle is labelled as R and the radius of the smaller circle is labelled as r.

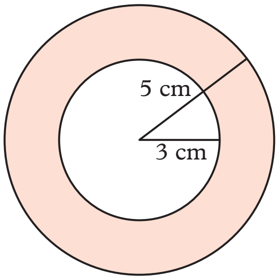


a Write an expression, in terms of r and R, for the area of the annulus. Remember, the area of a circle with radius of r units is πr^2.

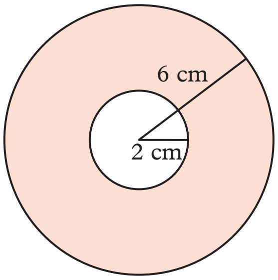
b Factorise the expression from part **a**.

c Calculate the area of the following annuli. Write your answers correct to two decimal places.

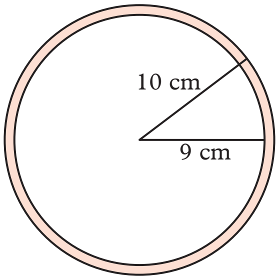
i



ii

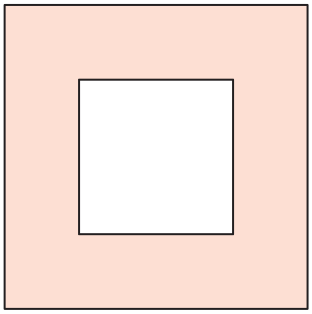


iii



16 A square annulus is similar to an annulus but uses squares instead of circles.

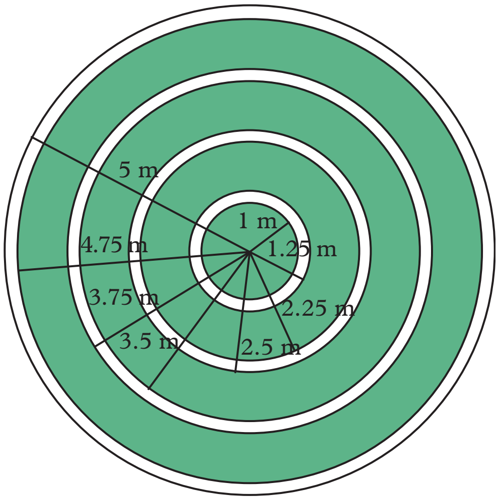
The square annulus shown has integer side lengths where the larger square has a side length 2 cm greater than the smaller square. If the shaded area of the square annulus is 24 cm², determine the side lengths of the two squares.



17 A landscaper has designed a concentric circle pattern for the lawn at a university. The innermost circle of lawn has a diameter of 1 m and is surrounded by a concrete ring that is 0.25 m wide. The concrete ring is surrounded by a 1 m-wide ring of lawn. Three more rings of concrete and two more rings of lawn of the same width surround the original circle.

a Write a calculation using the radii lengths for the area of concrete needed.

b By factorising, show that the area of concrete needed is given by $0.25\pi(5 + 4.75 + 3.75 + 3.5 + 2.5 + 2.25 + 1.25 + 1)$ m²



18 Show how $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$ can be written in its simplest factorised form of $4xy(x + 1)$.

19 We can factorise the difference of any two terms by using square roots. For example, $x - y$ can be factorised to $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$. Factorise the following to the product of conjugates. Remember to evaluate square roots and simplify surds where possible.

a $a - 5$

b $b^2 - 7$

c $c - 8$

d $3d - 16$

e $4e - 45$

f $2f^2 - 75$

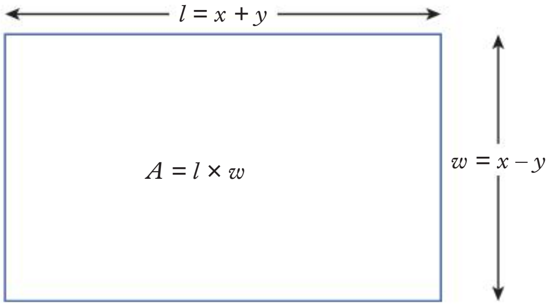
g $x^3 - y^5$

h $8x^2 - 12y^2$

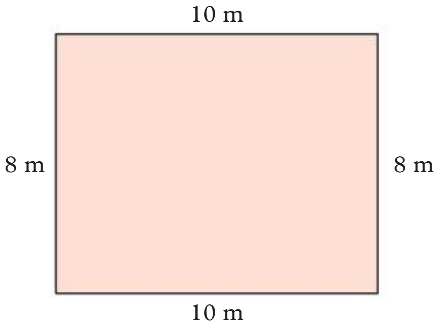
CHALLENGE

20 If the length of a rectangle is given by $l = x + y$ and the width is given by $w = x - y$, then the area of any rectangle, $A = l \times w$, can be expressed as the difference of two squares $A = x^2 - y^2$.

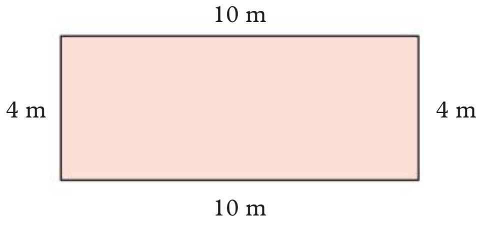
For each of the following rectangles, state the side lengths (x and y) of two squares, where the difference in the areas of the squares is the same as the area of the rectangle.



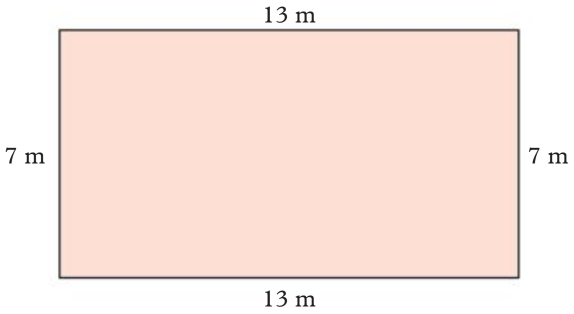
a



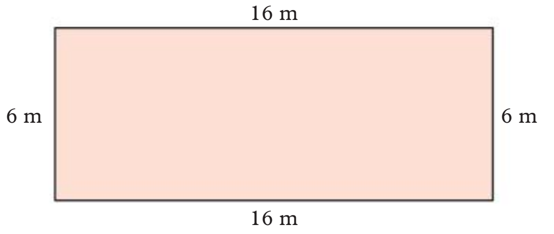
b



c



d



21 A rectangle has a length 6 cm longer than its width. The rectangle's area is $y^2 - x^2$. What is the value of x^2 in square centimetres?

22 a Expand $(x + y)(x^2 - xy + y^2)$.

b Use your result in part **a** to factorise $x^3 + 8$.

c Predict the factorised form of $x^3 - 8$.

23 These are the known facts about two numbers: The difference between two numbers is 5 and the difference between the squares of the two numbers is 155. What is the sum of the two numbers?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Factorising the difference of two squares

CAS instructions
Factorising

Topic quiz
2D

2E Factorising quadratic expressions

Learning intentions

By the end of this topic you will be able to ...

✔ factorise monic quadratic trinomials

✔ factorise quadratic trinomials by first taking out a common factor.

Inter-year links

Support	Multiples and factors
Year 7	2D Factors and the highest common factor
Year 8	5G Factorising
Year 10	1G Factorising quadratic expressions

Quadratic trinomials

- A **quadratic expression** is an algebraic expression that contains a squared pronumeral, with no exponents greater than 2 in the expression.
  For example, $6t^2$, $x^2 + 5$, $k^2 + 14k + 30$, and $4b^2 - a^2$ are all quadratic expressions.
- An expression with three terms is called a **trinomial**.
- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where a, b and c are constants.

$$ax^2 + bx + c$$

  → a is the **leading coefficient**
  → b is the coefficient of the **linear term**
  → c is the **constant term**
- Many quadratic trinomials can be factorised to produce a binomial product.

expanding

$$\xleftarrow{\qquad\qquad}$$

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

quadratic trinomial
expanded form

binomial product
factorised form

factorising

$$\xrightarrow{\qquad\qquad}$$

Factorising monic quadratics of the form $x^2 + bx + c$

- A **monic quadratic trinomial** is a quadratic trinomial in which the leading coefficient is equal to 1.

$$x^2 + bx + c$$

- Expanding a binomial product of the form $(x + m)(x + n)$, where m and n are constants, gives a quadratic trinomial of the form $x^2 + bx + c$.

$$\begin{aligned}(x + m)(x + n) &= x(x + m) + n(x + m)\\ &= x^2 + mx + nx + mn\\ &= x^2 + (m + n)x + mn\\ &= x^2 + bx + c\end{aligned}$$

where $b = m + n$ and $c = m \times n$

- The process can be reversed to factorise quadratic trinomials of the form $x^2 + bx + c$, by finding two numbers (m and n) that add to give b and multiply to give c.

  For example, $\begin{aligned}x^2 + 4x + 3 &= x^2 + (3 + 1)x + (3 \times 1)\\ &= (x + 3)(x + 1)\end{aligned}$

Example 2E.1 Finding the numbers for a given product and sum

Identify which two numbers add to give the first number and multiply to give the second number.

a sum: 5, product: 6 **b** sum: −2, product: −8 **c** sum: 1, product: −6

THINK	WRITE
a 1 List the factor pairs of 6. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.	**a** $1 \times 6 = 6 \rightarrow 1 + 6 = 7$ $-1 \times (-6) = 6 \rightarrow -1 + (-6) = -7$ $2 \times 3 = 6 \rightarrow 2 + 3 = 5$ $-2 \times (-3) = 6 \rightarrow -2 + (-3) = -5$
2 Add the factor pairs together and determine which pair adds to 5.	The numbers are 2 and 3.
b 1 List the factor pairs of −8. Remember that a negative multiplied by a positive is a negative, so there are two combinations of factor pairs with the same numerals.	**b** $-1 \times 8 = -8 \rightarrow -1 + 8 = 7$ $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$ $-2 \times 4 = -8 \rightarrow -2 + 4 = 2$ $2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$
2 Add the factor pairs together and determine which pair adds to −2.	The numbers are 2 and −4.
c 1 List the factor pairs of −6. Remember that a negative multiplied by a positive is a negative, so there are two combinations of factor pairs with the same numerals.	**c** $-1 \times 6 = -6 \rightarrow -1 + 6 = 5$ $1 \times (-6) = -8 \rightarrow 1 + (-6) = -5$ $-3 \times 2 = -8 \rightarrow -3 + 2 = -1$ $3 \times (-2) = -8 \rightarrow 3 + (-2) = 1$
2 Add the factor pairs together and determine which pair adds to 1.	The numbers are 3 and −2.

Example 2E.2 Factorising simple quadratic trinomials

Factorise the quadratic trinomial $x^2 + 7x + 10$.

THINK	WRITE
1 List the factor pairs of the constant term, 10. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.	$1 \times 10 = 10 \rightarrow 1 + 10 = 11$ $-1 \times (-10) = 10 \rightarrow -1 + (-10) = -11$ $2 \times 5 = 10 \rightarrow 2 + 5 = 7$ $-2 \times (-5) = 10 \rightarrow -2 + (-5) = -7$
2 Add the factor pairs together and identify which pair adds to the linear coefficient, 7.	The numbers are 2 and 5.
3 Write the expression in factorised form.	$x^2 + 10x + 7 = (x + 2)(x + 5)$
4 Check your result by expanding.	Check: $(x + 2)(x + 5) = x^2 + 2x + 5x + 10$ $= x^2 + 7x + 10$

Example 2E.3 Factorising more complex quadratic trinomials (+ and –)

Factorise each quadratic trinomial.

a $m^2 + 2m - 3$

b $x^2 - 7x - 8$

THINK

1 List the factor pairs of the constant term. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.

2 Add the factor pairs together and identify which pair adds to the linear term.

3 Write the expression in factorised form.

4 Check your result by expanding.

WRITE

a $1 \times (-3) = -3 \rightarrow 1 + (-3) = -2$

$-1 \times 3 = -3 \rightarrow -1 + 3 = 2$

$m^2 + 2m - 3 = (m + 3)(m - 1)$

Check: $(m + 3)(m - 1) = m^2 - m + 3m - 3$

$= m^2 + 2m - 3$

b $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$

$-1 \times 8 = -8 \rightarrow -1 + 8 = 7$

$2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$

$-2 \times 4 = -8 \rightarrow -2 + (4) = 2$

$x^2 - 7x - 8 = (x - 8)(x + 1)$

Check: $(x - 8)(x + 1) = x^2 + x - 8x - 8$

$= x^2 - 7x - 8$

Example 2E.4 Factorising quadratic trinomials by first taking out a common factor

Factorise by first taking out a common factor.

a $2x^2 - 14x + 12$

b $-x^2 + 4x - 3$

THINK

1 If the coefficient of x^2 is not 1, check for a common factor of the three terms. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF. A common factor can be –1.

2 Factorise the expression inside the brackets. List the factor pairs of the constant term.

3 Add the factor pairs together and identify which pair adds to the linear term.

4 Write the expression in factorised form.

5 Check your result by expanding.

WRITE

a $2x^2 - 14x + 12 = 2(x^2 - 7x + 6)$

$1 \times 6 = 6 \rightarrow 1 + 6 = 7$

$-1 \times -6 = 6 \rightarrow -1 + (-6) = -7$

$2 \times 3 = 6 \rightarrow 2 + 3 = 5$

$-2 \times -3 = 6 \rightarrow -2 + (-3) = -5$

$2(x^2 - 7x + 6) = 2(x - 6)(x - 1)$

Check: $2(x - 6)(x - 1) = 2(x^2 - x - 6x + 6)$

$= 2(x^2 - 7x + 6)$

$= 2x^2 - 14x + 12$

b $-x^2 + 4x - 3 = -(x^2 - 4x^2 + 3)$

$1 \times 3 = 3 \rightarrow 1 + 3 = 4$

$-1 \times -3 = 3 \rightarrow -1 + (-3) = -4$

$-(x^2 - 4x + 3) = -(x - 3)(x - 1)$

Check: $-(x - 3)(x - 1) = -(x^2 - x - 3x + 3)$

$= -(x^2 - 4x + 3)$

$= -x^2 + 4x - 3$

Helpful hints

- ✔ The order of the binomial products doesn't matter. This is because of the commutative law for multiplication. For example: $2 \times 3 = 3 \times 2$ and $(x - 2)(x + 3) = (x + 3)(x - 2)$
- ✔ The order of the terms in a quadratic trinomial does not change the value of the expression. This is because of the commutative law for addition. For example: $x^2 + 6x + 8 = 6x + 8 + x^2 = 8 + x^2 + 6x$
- ✔ Always look for the HCF of the three terms before factorising. You might get lucky and find that it makes factorising easier.
- ✔ You can check your factorisation of the binomial products by expanding the brackets!

$$
\begin{aligned}
x^2 + 4x + 3 &= (x + 1)(x + 3) \\
&= x^2 + 3x + x + 3 \\
&= x^2 + 4x + 3
\end{aligned}
$$

- ✔ You don't always need to consider all the factor pairs of the constant term, c, to factorise a quadratic of the form $x^2 + bx + c$.

  If the constant term, c, is positive, then both factors will have the same sign. So:
  - → if b is positive, then both factors will be positive
  - → if b is negative, then both factors will be negative.

ANS p415

Exercise 2E Factorising quadratic expressions

△ 1–3(1st, 2nd columns), 4, 5–6(1st, 2nd columns), 7–13, 16(a–d), 18

□ 4, 6–11, 13, 14, 16(b, e, g, h, i), 18, 19, 21(a, c, d, f)

⬠ 4(c, f, i, l), 6(c, f, i, l), 7–11, 14, 15, 16(d, g, h, i), 17, 20, 21(b, c, d, f, i), 22, 23

UNDERSTANDING AND FLUENCY

2E.1 **1** Identify which two numbers add to give the first number and multiply to give the second number.

a Sum: 5, product: 4 **b** Sum: 6, product: 8 **c** Sum: 13, product: 22

d Sum: 9, product: 20 **e** Sum: 10, product: 24 **f** Sum: 7, product: 12

g Sum: 13, product: 42 **h** Sum: 12, product: 35 **i** Sum: 8, product: 16

2 Identify which two numbers add to give the first number and multiply to give the second number.

a Sum: 2, product: −8 **b** Sum: −1, product: −6 **c** Sum: −8, product: 12

d Sum: 3, product: −10 **e** Sum: −8, product: −9 **f** Sum: −5, product: 6

g Sum: 5, product: −6 **h** Sum: −6, product: −27 **i** Sum: −12, product: 11

3 Use your results from question **1** to factorise each of these quadratic trinomials.

a $x^2 + 5x + 4$ **b** $x^2 + 6x + 8$ **c** $x^2 + 13x + 22$

d $x^2 + 9x + 20$ **e** $x^2 + 10x + 24$ **f** $x^2 + 7x + 12$

g $x^2 + 13x + 42$ **h** $x^2 + 12x + 35$ **i** $x^2 + 8x + 16$

2E.2 **4** Factorise each quadratic trinomial.

a $a^2 + 4a + 3$ **b** $b^2 + 9b + 14$

c $c^2 + 7c + 6$ **d** $d^2 + 10d + 21$

e $e^2 + 8e + 7$ **f** $f^2 + 8f + 15$

g $g^2 + 11g + 28$ **h** $h^2 + 13h + 36$

i $x^2 + 9x + 18$ **j** $j^2 + 14j + 45$

k $k^2 + 11k + 30$ **l** $y^2 + 13y + 40$

5 Use your results from question **2** to factorise each of these quadratic trinomials.

a $x^2 + 2x - 8$
b $x^2 - x - 6$
c $x^2 - 8x + 12$
d $x^2 + 3x - 10$
e $x^2 - 8x - 9$
f $x^2 - 5x + 6$
g $x^2 + 5x - 6$
h $x^2 - 6x - 27$
i $x^2 - 12x + 11$

2E.3 6 Factorise each quadratic trinomial.

a $a^2 + 2a - 3$
b $b^2 - 2b - 15$
c $c^2 - 5c + 4$
d $d^2 + 5d - 14$
e $e^2 - 10e + 24$
f $f^2 - 3f - 10$
g $g^2 + g - 12$
h $h^2 - 8h + 15$
i $x^2 - 5x - 24$
j $j^2 - 10j + 16$
k $k^2 + 3k - 18$
l $y^2 - y - 2$

2E.4 7 Factorise each quadratic trinomial by first taking out a common factor.

a $3x^2 + 9x + 6$
b $2x^2 + 16x + 30$
c $5x^2 + 15x - 20$
d $-4x^2 - 20x - 24$
e $-6x^2 + 36x - 30$
f $-x^2 - 2x + 35$

8 Factorise each quadratic trinomial by first taking out a common factor.

a $ax^2 + 8ax + 12a$
b $bx^2 - 11bx + 28b$
c $10cx^2 - 10cx - 300c$
d $-2dx^2 + 34dx - 120d$
e $p^2x^2 + 13p^2x + 30p^2$
f $q^2x^2 + 15qx^2 + 44x^2$

9 Factorise each quadratic.

a $x^2 - 6x + 9$
b $x^2 + 0x - 9$
c $x^2 - 6x + 0$

10 Factorise each quadratic.

a $x^2 - 25$
b $x^2 - 10x + 25$
c $x^2 - 25x$
d $x^2 + 14x + 49$
e $x^2 - 49x$
f $x^2 - 49$

11 Factorise each quadratic by first reordering and simplifying the terms of each expression.

a $21 + 10a + a^2$
b $2b - 35 + b^2$
c $3c + c^2 - 18$
d $48 + 2d - d^2$
e $-24 - 10e - e^2$
f $10f - f^2 - 16$
g $10g + 70 + g^2 + 7g + 2$
h $-2h + 5 + h^2 - 19 - 3h$
i $50 - 20x - x^2 + 7x - 86$

12 **a** Determine all the non-negative integers (including zero) that can be substituted into the constant term of $x^2 + 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the positive integers up to 40 that can be substituted into the constant term of $x^2 + 6x - \square$ so that it can be factorised. Write both the expanded and factorised form for each possible value.

c When the coefficient of x is positive, how can you tell if the factors are both positive or if one is positive and one negative?

13 **a** Determine all the positive integers that can be substituted into the constant term of $x^2 - 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the non-negative (including zero) integers up to 40 that can be substituted into the constant term of $x^2 - 6x - \square$ so that it can be factorised. Write both the expanded and factorised form for each possible value.

c When the coefficient of x is negative, how can you tell if the factors are both negative or if one is positive and one negative?

14 **a** Determine all the positive integers that can be substituted into the x term of $x^2 + \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the positive integers that can be substituted into the x term of $x^2 - \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

c When the constant term is positive:

i how can you tell if the factors need to be positive or negative?

ii is the coefficient of x the sum or difference of the positive values of the factors?

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

15 a Determine all the non-negative integers (including zero) that can be substituted into the x term of $x^2 - \square x - 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the non-negative integers (including zero) that can be substituted into the x term of $x^2 + \square x - 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

c When the constant term is negative:

- **i** how can you tell which factor must be positive and which must be negative?
- **ii** is the coefficient of x the sum or difference of the positive values of the factors?

16 Use the expansion of a perfect square rule, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$, to completely factorise each quadratic trinomial as a perfect square.

a $x^2 + 8x + 16$
b $y^2 - 10y + 25$
c $v^2 + 2v + 1$
d $(2z)^2 - 12z + 9$
e $9w^2 + 42w + 49$
f $3h^2 - 12h + 12$
g $-k^2 - 18k - 81$
h $f^2 + 2fg + g^2$
i $(5p)^2 - 40pq + (4q)^2$

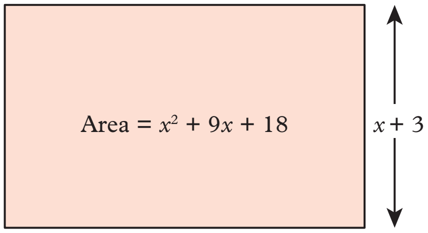
17 The following quadratic trinomials cannot be factorised as a perfect square.

i $x^2 + 6x - 9$
ii $-x^2 + 6x + 9$
iii $x^2 + 3x + 9$
iv $x^2 + 6x + 3$

a Describe how you can check if a quadratic trinomial is a perfect square.

b State why each of the above quadratic trinomials are not perfect squares.

18 A rectangle has an area of $x^2 + 9x + 18$ and a width of $x + 3$.



a Determine the expression of the length of the rectangle.

b Write the area of the rectangle as a product of its length and width.

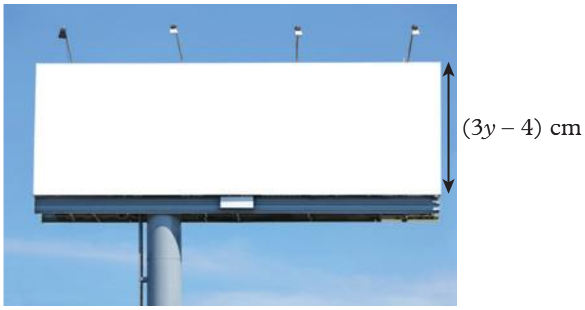
c If $x = 2$ m, calculate the area using:

- **i** the product of the length and width
- **ii** the expression $x^2 + 9x + 18$.

19 a Write an expression for the missing side length for each rectangular object.

i area within frame is $(x^2 + 7x - 18)$ cm^2

ii area of billboard is $(9y^2 - 16)$ m^2



b Find the value of x that gives an area of 152 cm^2 for part **i**.

c Find the value of y that gives an area of 425 m^2 for part **ii**.

d Determine the positive value of x and y such that the area of the rectangular objects is zero.

e Substitute $x = 1$ and $y = 1$ into the length, width, and area expression and state why they are not appropriate for the items.

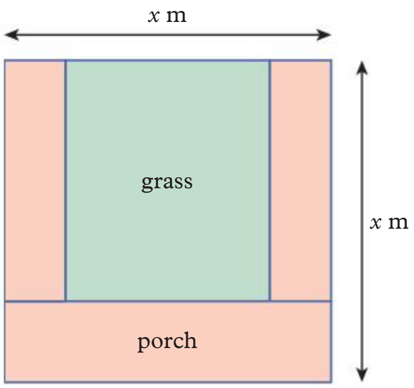
20 Two friends, Melissa and Lena, want to renovate their square-shaped backyards by adding a full-length rectangular porch and planting a rectangular section of grass with garden beds on either side. Both friends want to have the same width for their porches and their garden beds but have different-sized backyards, so they decide to use algebra to determine the area of grass that they need for both. They've already decided on the width of their porches, and have narrowed down the area of grass to four options:

i $(x^2 - 5x + 6)$ m^2

ii $(x^2 - 7x + 10)$ m^2

iii $(x^2 - 4x + 4)$ m^2

iv $(x^2 - 6x + 8)$ m^2



a Factorise each quadratic expression.

b Consider the factors of each equation and relate them to the possible side lengths of the grass. Judging from Melissa and Lena's four options, what is the width of Melissa and Lena's porches?

c Lena also suggested the grass areas $(x^2 - x - 6)$ m^2 and $(x^2 + x - 6)$ m^2. Explain why Melissa said these areas wouldn't be possible.

d Melissa and Lena decide to have 2 m of space either side of the lawn for their garden beds. Which quadratic expression did they decide on?

e If Melissa's backyard is 6 m by 6 m, determine what area of grass she will plant.

f If Lena's backyard is 7.5 m by 7.5 m, determine what area of grass she will plant.

g Determine how much area Melissa and Lena will each have for planting flowers.

21 Factorise each quadratic expression.

a $(3x)^2 + 5(3x) - 14$
b $(11x)^2 + 2(11x) - 80$
c $(5x)^2 - 12(5x) + 32$
d $(x + 2)^2 + 3(x + 2) - 4$
e $(x - 4)^2 - 11(x - 4) + 30$
f $(x - 9)^2 + 14(x - 9) + 45$
g $(3x)^2 - (3x) - 42$
h $(2x)^2 + 7(2x) - 30$
i $(8x)^2 + 32(2x) - 48$

22 Fully factorise each expression.

a $(x^2)^2 - 13x^2 + 36$
b $(x^2)^2 - 12x^2 - 64$
c $(x^2)^2 + 24x^2 - 25$

23 We can factorise quadratic trinomials by splitting the linear term once we have determined the two factors and then using grouping. For example, two numbers that add to 7 and multiply to 12 are 3 and 4 so:

$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 3)(x + 4) \end{aligned}$$

Factorise each of the following by splitting the linear term and using grouping.

a $x^2 + 5x + 4$
b $x^2 + 6x + 8$
c $x^2 + 13x + 22$
d $x^2 + 3x - 10$
e $x^2 - 8x - 9$
f $x^2 - 5x + 6$

Check your Student obook pro for these digital resources and more:

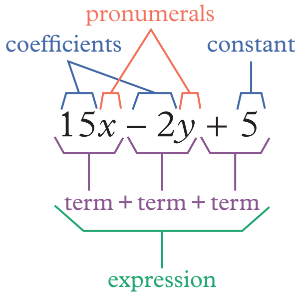
Interactive skillsheet Factorising quadratic expressions

Worksheet Factorising quadratic expressions

Topic quiz 2E

Chapter summary

Terms and expressions



Multiplying algebraic terms

$$3a^3b \times (-2a^2b^2) = 3 \times (-2) \times a^{3+2} \times b^{1+2}$$
$$= -6a^5b^3$$

Dividing algebraic terms

$$\frac{6a^5b^3}{3a^3b} = \frac{\overset{2}{\cancel{6}} \times a^5 \times b^3}{\cancel{3}^{1} \times a^3 \times b}$$
$$= 2 \times a^{5-3} \times b^{3-1}$$
$$= 2a^2b^2$$

Expanding

- Use the distributive law to expand brackets

$$a(b + c) = ab + ac$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

Difference of two squares

$$\xrightarrow{\text{expanding}}$$
$$(a + b)(a - b) = a^2 - b^2$$

binomial product factorised form

difference of two squares expanded form

$$\xleftarrow[\text{factorising}]{}$$

Expansion of a perfect square

$$\xrightarrow{\text{expanding}}$$
$$(a + b)^2 = a^2 + 2ab + b^2$$

quadratic trinomial expanded form

perfect square factorised form

$$\xleftarrow[\text{factorising}]{}$$

Adding and subtracting like terms

- Like terms contain the exact same pronumerals with the same exponents.

Like terms: xyz, $4yzx$, $7zxy$
Not like terms: ab^2, a^2b

- Like terms can be added and subtracted by adding and subtracting their coefficients.

$$a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$$

Factorising using the HCF

$$\xrightarrow{\text{factorising}}$$
$$2x^2 - 6x = 2x(x - 3)$$

expanded form factorised form

Factorising by grouping terms

$$x^2 + 2x - 3x - 6 = x(x + 2) - 3(x + 2)$$
$$= (x + 2)(x - 3)$$

Factorising quadratic trinomials

To factorise a quadratic trinomial of the form $x^2 + bx + c$:

- find two numbers that add to give b and multiply to give c

$$\begin{aligned}(x + m)(x + n) &= x(x + m) + n(x + m)\\ &= x^2 + mx + nx + mn\\ &= x^2 + (m + n)x + mn\\ &= x^2 + bx + c\end{aligned}$$

where $b = m + n$ and $c = m \times n$

- substitute those two numbers into the binomial product factorised form.

$$\xrightarrow{\text{factorising}}$$
$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

quadratic trinomial expanded form

binomial product factorised form

$$\xleftarrow[\text{expanding}]{}$$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

2A **1** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{12a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $\frac{b^2}{3a}$

2A **2** Which is not a like term to $4ab^3$?

A $3abbb$ **B** $4ba^3$ **C** $-4b^3a$ **D** $2bab^2$ **E** $\frac{ab^3}{7}$

2A **3** If $x = -3$ and $y = 2$ then $-5y^3x^2$ is equal to:

A -360 **B** 360 **C** -540 **D** 540 **E** -9000

2B **4** Which expression is equivalent to $8 - 2(3 - 5g)$?

A $18 - 30g$ **B** $10g + 2$ **C** $10g + 10$ **D** $2 - 10g$ **E** $10 - 10g$

2B **5** Which expression is **not** equivalent to $(3x + 4)(2y - 5)$?

A $2y(3x + 4) - 5(3x + 4)$ **B** $6xy - 20$

C $6xy - 15x + 8y - 20$ **D** $3x \times 2y + 3x \times (-5) + 4 \times 2y + 4 \times (-5)$

E $3x(2y - 5) + 4(2y - 5)$

2B **6** Which expression cannot be expanded using the difference of two squares rule?

A $(x + 6)(x - 6)$ **B** $(7 - p)(7 + p)$ **C** $(2x - 7)(2x + 7)$

D $(q + 9)(-q + 9)$ **E** $(d + 5)(d - 2)$

2B **7** Which statement is **incorrect**?

A $(d + 3)(d - 7) = d^2 + 4d - 21$ **B** $-3(b + 5) = -3b - 15$

C $(m - 4)(m + 4) = m^2 - 16$ **D** $(2b + 5)(3b - 2) = 6b^2 + 11b - 10$

E $(e - 6)^2 = e^2 - 12e + 36$

2C **8** When fully factorised, the expression $9x^2 - 3x^3$ factorises to:

A $3(3x^2 - x^3)$ **B** $3x^2(3 - x)$ **C** $x^2(9 - 3x)$

D $3x(3x - x^2)$ **E** $-3x(x^2 - 3x)$

2C **9** Which of the following is **not** a factor of $12x^2y - 24xy^2$?

A x^2 **B** $12xy$ **C** 12 **D** $6y$ **E** $3x$

2C **10** The expression $3x - 2$ is **not** a factor of which expression?

A $24xy + 15x - 18y - 10$ **B** $6(3x - 2) - y(3x - 2)$ **C** $3x(5y + 9) - 2(5y + 9)$

D $21x - 14$ **E** $3x - 2$

2D **11** The expression $w^2 - 49$ factorises to:

A $(w - 7)^2$ **B** $(w + 7)^2$ **C** $(w + 7)(w - 7)$

D $(7 + w)(7 - w)$ **E** $(w - 7)(7 - w)$

2D **12** When fully factorised, the expression $36x^2z - 100y^2z$ factorises to:

A $4z(5y + 3x)(5y - 3x)$ **B** $z(6x - 10y)(6x - 10y)$ **C** $4z(6x - 10y)(6x - 10y)$

D $4z(3x + 5y)(3x - 5y)$ **E** $z(6x + 10y)(6x - 10y)$

2D **13** The expression $(a - 3)^2 - 9$ factorises to:

A $(a + 6)(a - 12)$ **B** $(a - 12)(a + 6)$ **C** $a^2 - 6a$

D $a(a + 6)$ **E** $a(a - 6)$

2E **14** The expression $t^2 + 3t - 18$ factorises to:

A $(t + 9)(t - 2)$ **B** $(t - 6)(t + 3)$ **C** $(t + 6)(t - 3)$

D $(t - 9)(t + 2)$ **E** $(t + 5)(t - 2)$

2E **15** When fully factorised, the expression $3y^2 - 33y + 84$ factorises to:

A $3(y - 7)(y - 4)$ **B** $(3y - 7)(3y - 4)$ **C** $(3y - 21)(y - 4)$

D $3(y + 7)(y + 4)$ **E** $(y - 7)(3y - 12)$

2E **16** Which of the following is not equivalent to $b^2 - 8b + 16$?

A $(b - 4)^2$ **B** $(4 - b)^2$ **C** $(b - 4)(4 - b)$

D $(4 - b)(4 - b)$ **E** $(b - 4)(b - 4)$

Short answer

2A **1** Simplify each expression.

a $15t - 7t + 8t$ **b** $a - 7p - 11p + 12a$

c $3k + 5km - 7k - 15 + 2k - 4km$ **d** $6m^2n - 2m^2 + 7nm^2 + 11n^2 - 4mn^2 - 3m^2$

2A **2** Simplify each expression.

a $4xy \times 11xyz$ **b** $9mnp \times 2m^3p \times 4n^2$

c $15de \div (18df)$ **d** $11klmn \div (-22klm^2)$

2B **3** Expand each product.

a $4(z - 7)$ **b** $-8(5 - 3y)$ **c** $5x(7 - 6w)$

d $(u + 3)(t - 4)$ **e** $(5r + 6)(8 - 3v)$ **f** $(9p + 11q)(7m - 3n)$

2B **4** Expand and simplify each expression to remove the brackets.

a $5(a + 2) - 3(7 - a)$ **b** $(b - 11)(b + 2)$ **c** $(3c - 2)(4c - 5)$

d $(d + w)(d - w)$ **e** $(6 + e)^2$ **f** $(f - 9)^2$

2B **5** Expand and simplify each product.

a $x^4(x^3 - x^2)$ **b** $y^3z^4(y^9 - z^2)$ **c** $(x^5 - y^3)(x^6 + y^4)$

2C **6** Factorise each expression.

a $4a - 24$ **b** $36pq^2 + 144pq$

c $7d(8 - d) - 4(8 - d)$ **d** $5e + 15ef + 2 + 6f$

2C **7** Factorise each expression.

a $6z^5 - 5z^4$ **b** $9d^2y + 15dx^2$

c $-36r^{10}t^8u^6 - 96r^7t^8u^3$ **d** $a^5b^2c^7d^8 + a^6b^5c^9d^5 + a^4b^2c^8d^7$

2C **8** Factorise each expression.

a $24x^2(x^2 + 2) - 8x(x^2 + 2)$ **b** $(r + q)(t + 5) + (r + q)(p - 3)$ **c** $4x^2 + 16x - 6x - 24$

2D **9** Factorise each expression using the difference of two squares rule.

a $a^2 - 64$ **b** $121 - b^2$ **c** $36m^2 - 49n^2$

d $(p + 1)^2 - 4$ **e** $2e^2 - 32$ **f** $(f + 4)^2 - (f - 5)^2$

2D **10** Factorise each expression.

a $y^{10} - z^{10}$ **b** $q^2 - (q + 4)^2$ **c** $-r^2 + 100$ **d** $-64 + 81j^2$

2D **11** Factorise each expression.

a $\frac{196}{25}x^2 - \frac{169}{4}y^2$ **b** $\frac{144}{c^2} - \frac{225}{d^2}$

c $-\frac{u^2}{16} + \frac{36}{49}$ **d** $0.04v^2 - 1.21$

2E **12** Factorise each quadratic trinomial.

a $a^2 + 6a + 5$ **b** $b^2 - 7b + 12$ **c** $c^2 + 4c - 21$ **d** $d^2 - 16d - 36$

2E **13** Factorise each quadratic trinomial.

a $15 + 8x + x^2$ **b** $60 + 20x - 5x^2$

c $32x - 2x^2 - 120$ **d** $240 - 6x - 3x^2$

2E **14** Factorise each quadratic.

a $98 - 2x^2$ **b** $16y^2 + 64x^2$ **c** $x^2 - 10x + 25$ **d** $9x^2 - 36x$

Analysis

1 Expressions in the form $(x + a)^2 - b^2$ can be factorised using the difference of two squares rule.

a Factorise each of the following using the difference of two squares rule.

i $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

b Describe the connection between the numbers in the brackets in factor form and the values of a and b in $(x + a)^2 - b^2$ form.

c Expand each of the following.

i $(x + 8)^2 - 9$ **ii** $(x - 10)^2 - 16$ **iii** $(x + 11)^2 - 25$

d Describe the connection between the coefficient of x in the expanded form and the value of a in $(x + a)^2 - b^2$ form.

e Describe the connection between the constant in the expanded form and the values of a and b in $(x + a)^2 - b^2$ form.

f Fill in the spaces so that the expressions are equivalent.

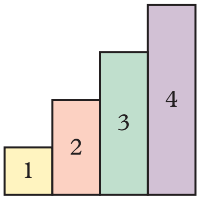
i $(x + \boxed{})^2 - 4 = x^2 + 6x + 5 = (x + \boxed{})(x + \boxed{})$

ii $(x - 5)^2 - \boxed{} = x^2 - \boxed{}x + \boxed{} = (x - 9)(x - 1)$

iii $(x + \boxed{})^2 - \boxed{} = x^2 + 14x - 51 = (x - \boxed{})(x + \boxed{})$

2 Malak and Samara are investigating how to quickly add the positive integers from 1 to n:
1, 2, 3, 4, 5, ..., n

Malak finds the expression $\frac{1}{2}n(n + 1)$ and Samara finds the expression $\frac{(n + 1)^2 - (n + 1)}{2}$, which will give the sum of the integers from 1 to 4, if $n = 4$. Visually, the first four positive integers can be represented using bars.



a Fully expand both of Malak's and Samara's expressions to show that they are equivalent.

b Calculate the sum of the first five positive integers:

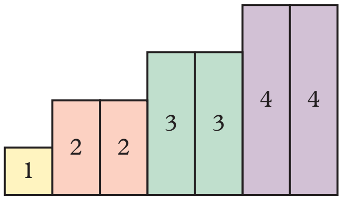
i by manually computing the sum 1 + 2 + 3 + 4 + 5

ii by using one of the expressions.

c Calculate the sum of the first 100 positive integers.

Malak decides to investigate the sum of the first n positive even numbers:
2, 4, 6, 8, 10,..., n

Malak finds the expression $n^2 + n$, which will give the sum of the positive even numbers from 2 to 8, if $n = 4$. Visually, the first four even numbers can be represented using double bars.



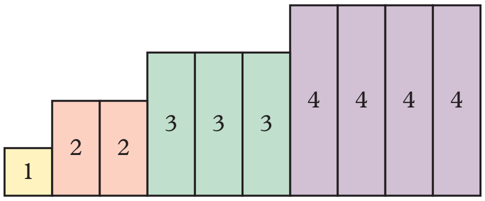
d Factorise $n^2 + n$.

e Calculate the sum of the first 100 positive even numbers.

f Describe the connection between the sum of the first n positive even numbers and the sum of the first n positive integers.

Samara decides to investigate the sum of the first n square numbers: 1, 4, 9, 16, 25,..., n

Samara finds the expression $\frac{1}{6}(n^2 + n)(2n + 1)$, which will give the sum of the square numbers from 1 to 16, if $n = 4$. Visually, the first four square numbers can be represented using squares of bars.



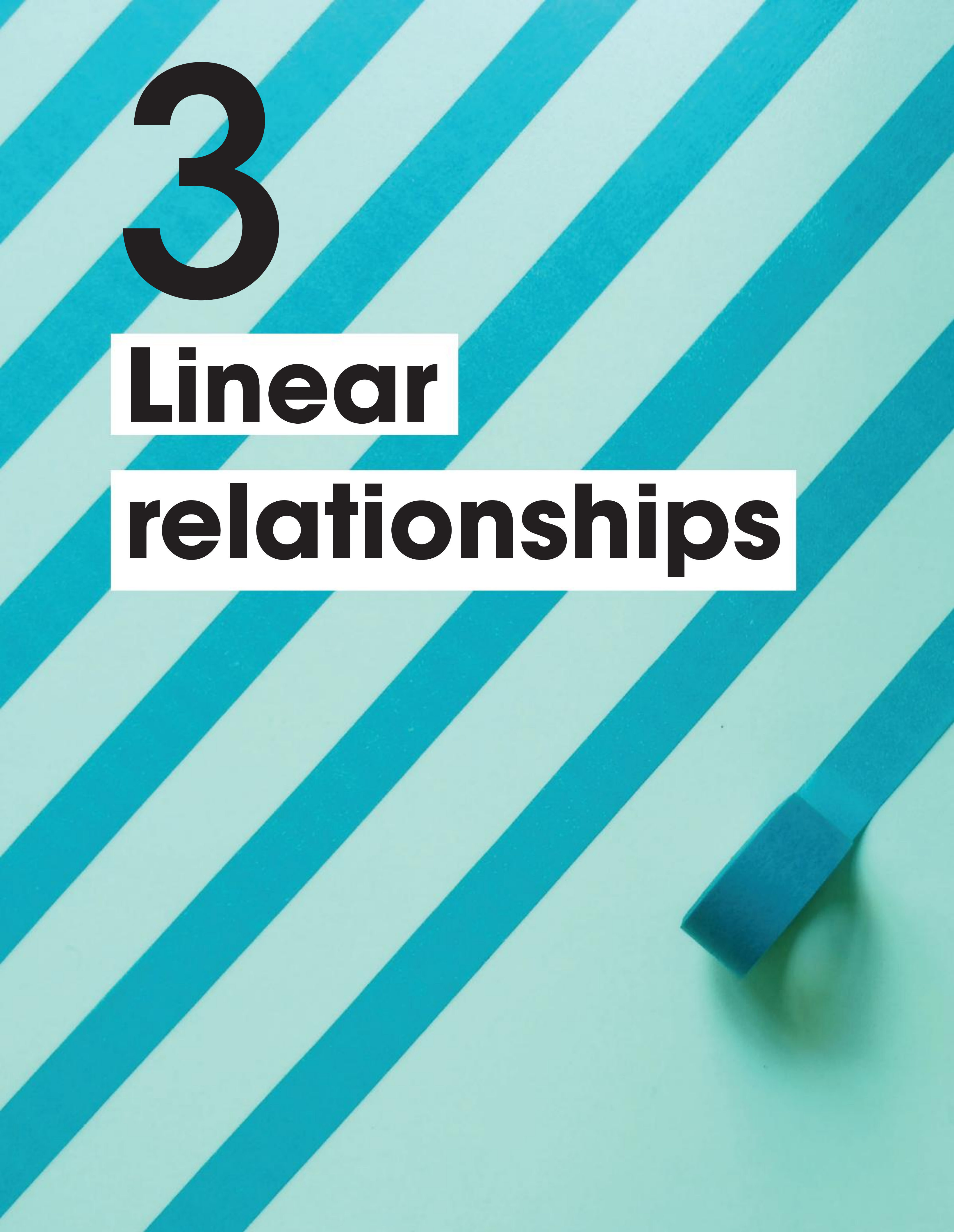
g Fully expand $\frac{1}{6}(n^2 + n)(2n + 1)$.

h Calculate the sum of the first five square numbers:

i by manually computing the sum 1 + 4 + 9 + 16 + 25

ii by using one of the expressions.

i Calculate the sum of the first 100 square numbers.



3

Linear relationships

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Number lines
- ✔ Order of operations
- ✔ Expanding over one pair of brackets
- ✔ The Cartesian plane
- ✔ Plotting graphs

Curriculum links

- Recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers using digital tools (AC9M9N01)
- Find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane (AC9M9A03)
- Use mathematical modelling to solve applied problems involving change including financial contexts; formulate problems, choosing to use either linear or quadratic functions; interpret solutions in terms of the situation; evaluate the model and report methods and findings (AC9M9A05)
- Experiment with the effects of the variation of parameters on graphs of related functions, using digital tools, making connections between graphical and algebraic representations, and generalising emerging patterns (AC9M9A06)
- Solve spatial problems, applying angle properties, scale, similarity, Pythagoras' theorem and trigonometry in right-angled triangles (AC9M9M03)

Materials

✔ Calculator ✔ Graph paper ✔ Ruler

3A Solving linear equations

Learning intentions

By the end of this topic you will be able to …

✔ solve linear equations using inverse operations

✔ solve linear equations with the unknown on both sides using inverse operations.

Inter-year links

Year 7	6G Solving equations using inverse operations
Year 8	6C Solving equations with the unknown on both sides
Year 10	3A Solving linear equations

Linear equations

- An **equation** is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) and the right-hand side (RHS) of the equation.
- An equation may contain one or more pronumerals (such as x, y, a or b) that represent values, sometimes referred to as unknowns. If the pronumeral represents an unknown that can have more than one value, then it is called a variable.

$$\underbrace{3n + 10}_{\text{left-hand side of equation (LHS)}} \overset{\substack{\text{equals symbol}\\ \downarrow}}{=} \underbrace{40}_{\text{right-hand side of equation (RHS)}}$$

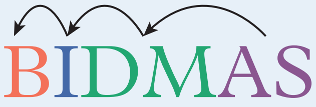
- A **linear equation** is an equation containing only pronumerals that are raised to a power of 1 and no two such pronumerals are multiplied together.

Linear equations	Non-linear equations
$y = x$	$y = x^2$
$\frac{n}{4} = 2$	$\frac{mn}{4} = 2$
$2a - \frac{3}{5}b = 10$	$a^3 + 6b = 0$

Solving linear equations using inverse operations

- A **solution** is a value for an unknown that makes the equation a true statement.
- To check whether a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.
- To solve equations using **inverse operations**, identify and apply the inverse operation(s) required to reverse the operation(s) and isolate the unknown on the LHS of the equation ($x = \ldots$).
  - → For equations involving more than one operation, inverse operations must be performed in the reverse order to BIDMAS.
  - → A useful shorthand is to put the inverse operation in brackets to the right of the equation for each line of working out.
  - → To solve an equation in which the unknown appears on both sides of the equation, use inverse operations to eliminate the pronumeral term from one side of the equation, then solve the equation using inverse operations.

Operation	Inverse operation
+3	−3
−3	+3
×3	÷3
÷3	×3



For example,
$$\begin{aligned} 4x - 2 &= 3x + 1 \quad && (-3x) \\ x - 2 &= 1 && (+2) \\ x &= 3 && \end{aligned}$$

Example 3A.1 Solving two-step equations using inverse operations

Solve the following equations using inverse operations.

a $\frac{x}{3} - 5 = 7$ **b** $10 = -2(x + 6)$

THINK	WRITE
a Identify and apply the inverse operations to both sides of the equation in the reverse order of BIDMAS, and then write the solution to the equation.	**a** $\frac{x}{3} - 5 = 7 \quad (+5)$ $\frac{x}{3} = 12 \quad (\times 3)$ $x = 36$
b Divide both sides by −2, remembering that the sign changes when you multiply or divide by a negative number. Subtract 6 from both sides. Note that $-11 = x$ is the same as $x = -11$.	**b** $10 = -2(x + 6) \quad (\div (-2))$ $-5 = x + 6 \quad (-6)$ $x = -11$

Example 3A.2 Solving three-step equations using inverse operations

Solve:

a $12 = \frac{7x}{2} - 9$ **b** $\frac{-x - 4}{3} = 1$

THINK	WRITE
a Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, and then write the solution to the equation.	**a** $12 = \frac{7x}{2} - 9 \quad (+9)$ $21 = \frac{7x}{2} \quad (\times 2)$ $42 = 7x \quad (\div 7)$ $x = 6$
b Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, then write the solution to the equation. Note that $-x = 7$ is not the solution, as the $-x$ has a coefficient of −1.	**b** $\frac{-x - 4}{3} = 1 \quad (\times 3)$ $-x - 4 = 3 \quad (+4)$ $-x = 7 \quad (\div (-1))$ $x = -7$

Example 3A.3 Solving equations with the unknown on both sides

Solve each equation for x.

a $4x + 7 = 2x - 3$ **b** $3(2x + 1) = -17 - 4x$

THINK	WRITE
a 1 Eliminate the pronumeral term from one side of the equation by subtracting the pronumeral with the smaller coefficient, $2x$, from both sides of the equation.	**a** $4x + 7 = 2x - 3 \quad (-2x)$
2 Solve the equation using inverse operations.	$2x + 7 = -3 \quad (-7)$ $2x = -10 \quad (\div 2)$ $x = -5$

b **1** Remove the brackets by expanding the expression on the left-hand side of the equation.

2 Eliminate the negative pronumeral term by adding $4x$ to both sides of the equation.

3 Solve the equation using inverse operations.

b $3(2x + 1) = -17 - 4x$

$$\begin{aligned} 6x + 3 &= -17 - 4x && (+4x) \\ 10x + 3 &= -17 && (-3) \\ 10x &= -20 && (\div 10) \\ x &= -2 \end{aligned}$$

Helpful hints

- ✔ Writing the inverse operation beside the appropriate line of working out is a great way to keep track of your calculations! $\frac{x}{3} - 5 = 7 \quad (+5)$
- ✔ Remember that you have to apply the inverse operation to both sides of the equation.
- ✔ Remember to define your pronumerals before writing equations to represent the variables in worded problems. For example, let n = number of eggs in a carton or let w = weight of eggs (grams).

  Note: 'n = eggs in a carton' would not be correct, as a pronumeral must always represent a quantity. Similarly, 'w = weight of eggs' would not be correct because it doesn't specify a unit of measurement.

ANS p418

Exercise 3A Solving linear equations

▲ 1–6, 7(a, c, f, g), 8–14, 16(a, b, c), 17, 21

■ 2–13, 15, 16, 18, 19, 23(a, b)

⬟ 2–5, 7(e–h), 9, 11–13, 16(c, d), 20–23

UNDERSTANDING AND FLUENCY

3A.1 **1** Solve the following equations using inverse operations.

a $4x + 5 = 29$ **b** $\frac{x+3}{2} = 4$ **c** $\frac{x}{4} - 2 = 7$ **d** $\frac{2x}{5} = 4$

e $-2(x + 6) = 28$ **f** $-17 = -3x - 5$ **g** $9 = \frac{x}{5} + 6$ **h** $\frac{x-4}{5} = -1$

i $-20 = 4(x - 2)$ **j** $-5x + 1 = 16$ **k** $\frac{-3x}{2} = -18$ **l** $\frac{x}{2} + 7 = 4$

3A.2 **2** Solve the following equations using inverse operations.

a $\frac{3x+4}{5} = 2$ **b** $\frac{-3x}{4} + 1 = 7$ **c** $\frac{5(x-1)}{2} = 20$ **d** $4(x + 3) - 2 = 30$

e $\frac{x-2}{6} - 3 = 0$ **f** $1 = \frac{11x}{4} + 23$ **g** $3 = \frac{-7x+6}{2}$ **h** $-5(x + 2) - 7 = 3$

i $2 = \frac{2(x+8)}{3}$ **j** $-16 = 4(5 - x) + 4$ **k** $6 = \frac{-x}{2} - 4$ **l** $\frac{2-3x}{5} = -2$

3 Solve:

a $\frac{x-2.9}{4} = 1$ **b** $\frac{x-4}{3} + 11.2 = 9$ **c** $\frac{3x}{4} - 1.9 = 6.2$ **d** $\frac{5x+2}{3} = -4.6$

4 **a** Solve $5(x - 2) = 20$ by first dividing both sides by 5.

b Another way to solve this equation is first to expand the expression on the left-hand side. Try this method. Do you obtain the same solution?

c Solve $5(x - 2) = 18$ by first dividing both sides by 5.

d Solve $5(x - 2) = 18$ by first expanding the expression on the left side.

e Which method did you find easier to use when solving $5(x - 2) = 18$? Explain.

UNDERSTANDING AND FLUENCY

5 Solve the following equations using the appropriate method from question **4**. Where relevant, write the solution as an improper fraction.

a $4(x - 1) = 8$ **b** $3(x + 7) = -6$ **c** $2(x - 3) = 5$

d $5(x + 4) = 8$ **e** $-4(x + 2) = -24$ **f** $-6(x - 2) = 1$

6 **a** What value of x would make $\frac{12}{x} = 3$ a true statement?

b What is the first step to solving this equation using inverse operations?

c Solve $\frac{12}{x} = 3$ using inverse operations. Use substitution to check that your solution is correct.

7 Solve the following equations using inverse operations. Use substitution to check that your solution is correct.

a $\frac{10}{x} = 2$ **b** $\frac{21}{x} = -7$ **c** $\frac{-18}{x} = 3$ **d** $\frac{-24}{x} = -6$

e $\frac{11}{x} = 2$ **f** $\frac{8}{x} = -5$ **g** $\frac{9.6}{x} = 2$ **h** $-\frac{1}{x} = 3$

3A.3 **8** Solve each equation for x.

a $6x + 5 = 4x + 9$ **b** $3x - 11 = x + 3$ **c** $3x - 8 = 6x - 5$ **d** $-15 - 2x = 4x + 3$

e $3x + 7 = -3 - 2x$ **f** $9x - 4 = 10x - 11$ **g** $10 - x = 7x - 22$ **h** $-5x - 5 = 11 - x$

9 Solve each equation for x. Use substitution to check that your solution is correct.

a $3(x - 2) = 8x - 1$ **b** $2x - 1 = 5(x - 2)$ **c** $2(3x - 4) = 5x - 1$ **d** $-3(-2x - 1) = -18 - x$

e $4(x + 3) = 5(x + 1)$ **f** $5(x + 9) = -3(x - 7)$ **g** $-6(1 - x) = 3(x - 8)$ **h** $-6(x + 1) = -10(x - 3)$

10 Consider the equation $\frac{2x + 5}{3} = \frac{x - 4}{3}$.

a The first step to solving this equation using inverse operations is to multiply both sides by 3. Multiply both sides of the equation by 3 to obtain an equivalent equation.

b Solve the equation obtained in part **a**. Use substitution to check that your solution is correct.

11 Solve:

a $\frac{3x - 4}{7} = \frac{x + 6}{7}$ **b** $\frac{5x + 2}{4} = \frac{2x - 7}{4}$ **c** $\frac{2x + 3}{11} = \frac{15 - 4x}{11}$ **d** $\frac{4(2x + 1)}{9} = \frac{x - 17}{9}$

12 Consider the equation $\frac{5x - 1}{2} = \frac{x + 5}{3}$.

a The first step to solving this equation using inverse operations is to find a common denominator. Write an equivalent equation where the fractions have a common denominator.

$$\frac{\_\_(5x - 1)}{\_\_} = \frac{\_\_(x + 5)}{\_\_}$$

b Solve the equation obtained in part **a**, using the method from question **10**. Use substitution to check that your solution is correct.

13 Use the method in question **12** to solve the following equations. Use substitution to check that your solution is correct.

a $\frac{4x + 1}{5} = \frac{3x + 2}{4}$ **b** $\frac{13x - 8}{10} = \frac{4x - 2}{3}$ **c** $\frac{2x - 1}{3} = \frac{4x + 11}{9}$ **d** $\frac{x + 15}{4} = \frac{x + 9}{6}$

PROBLEM SOLVING AND REASONING

14 Trent is sharing a bag of jellybeans equally with three of his friends and finds that there are two left over. Consider the number of jellybeans that each person receives, including Trent, if there were 34 jellybeans in the bag.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d How many jellybeans did each person receive?

15 Lily is saving to buy a pair of sneakers that cost \$395. She is able to save \$70 per month. If she currently has \$115, consider the number of months it will take for Lily to buy the shoes.

a Define a pronumeral to represent the unknown quantity in the problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d In how many months can Lily buy the shoes?

PROBLEM SOLVING AND REASONING

16 For each problem, set up an equation and solve it using inverse operations.

a Darnell buys three model planes online for a total cost of \$590, which includes the delivery charge of \$35. What is the cost of each model plane?

b Emma and Maggie score a total of 35 goals in a basketball match. Maggie scores seven more goals than Emma. How many goals did Emma score?

c The perimeter of a rectangular playing field is 100 m. If the length is 12 m longer than the width, what are the dimensions of the playing field?

d The cost of hiring a party venue is \$500. There is a \$26 per person charge for food. If Kasey has a budget of \$2700 for the party, what is the maximum number of people that can attend?

17 Nylah and Josh have the same amount of money. Nylah buys seven sushi rolls and has \$1.50 left over. Josh buys four sushi rolls and has \$12 left over.

a If x represents the cost of one sushi roll (\$), which equation fits this situation?

A $7x + 150 = 4x + 12$ **B** $7x + 1.5 = 4x + 12$ **C** $7x - 1.5 = 4x - 12$

D $7x - 150 = 4x - 12$ **E** $7x + 12 = 4x + 1.5$

b Solve the equation to find the cost of one sushi roll.

18 The length, l, of a rectangle is twice its width, w.

a Write an expression for the perimeter, P, of the rectangle in terms of l.

b If the perimeter of the rectangle is 60 m, find the width of the rectangle.

19 Violetta cooked sausages for the school sausage sizzle. Each sausage was placed in bread with tomato sauce. Twenty of these were sold with mustard. Half of those left were sold with fried onions. If there were 18 sausages sold with fried onions, how many sausages did Violetta cook?

20 One angle in a triangle is 30°. The second angle is twice the size of the third angle in the triangle. Find the size of the largest angle.

21 The sum of three consecutive integers is 13 more than the smallest of the three numbers. Identify the three numbers using algebra.

CHALLENGE

22 The linear equations you have dealt with have sometimes been equal. That is, for a particular value of the pronumeral, the left- and right-hand sides are equal. The linear equations $2x + 5 = 2x + 5$, $3x + 5 = 3x + 5$ and $3x + 6 = 3x + 6$ are equal for all values of x. That is, regardless of the value of x, the equations are always equal. The linear equations $2x + 5 = 2x + 6$, $3x + 5 = 3x + 6$ and $3x + 4 = 3x + 6$ are equal for no value of x. That is, regardless of the value of x, the equations are never equal.

a For each of the following, determine whether the equation is always, sometimes or never equal.

i $2x + 5 + 3x = 1 + 5x + 4$ **ii** $7x - 2 + 4 = 3x + 4x - 6$ **iii** $2x + 3x - 5 = 6x - 9$

iv $15 - 12x = -3(4x + 5)$ **v** $4(6x + 10) = 8(3x + 5)$ **vi** $4(3 - 4x) + x = 4 - 3(5x - 3)$

b When are the two sides of a linear equation with one pronumeral equal?

c In which cases are the two sides of a linear equation with one pronumeral never equal?

23 Solve the following equations for x. Use substitution to check your solution.

a $\frac{3x + 2}{2} + \frac{5 - x}{3} = \frac{5x + 1}{4} + \frac{7x - 2}{6}$ **b** $\frac{5x - 1}{2} = \frac{x + 5}{3} + 1$ **c** $\frac{5x - 1}{2} = \frac{x + 5}{3} + x$

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Interactive skillsheet
Solving equations using inverse operations

Interactive skillsheet
Solving equations with the unknown on both sides

Worksheet
Solving linear equations

CAS instructions
Solving equations

Topic quiz
3A

3B Solving linear inequalities

Learning intentions

By the end of this topic you will be able to ...

✔ represent linear inequalities using a number line

✔ solve linear inequalities using inverse operations.

Inter-year links

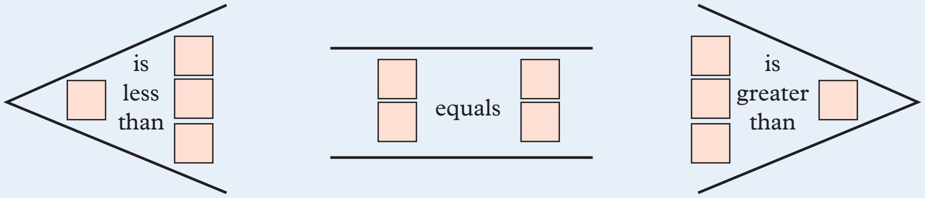
Year 7	6G Solving equations using inverse operations
Year 8	6G Solving linear inequalities
Year 10	3D Solving linear inequalities

Linear inequalities

- An **inequality** is a mathematical statement that compares the values of two unequal expressions. Inequalities are written using inequality signs, like the ones shown in the table below.

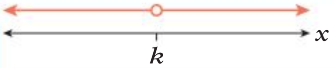
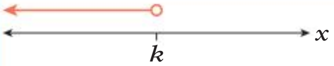
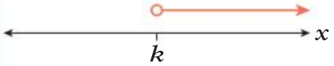
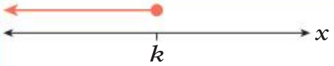
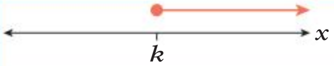
Inequality sign	Meaning	Example	Meaning of example
$\neq$	'is **not** equal to'	$x \neq 2$	x is **not** equal to 2
$<$	'is less than'	$x < 2$	x is less than 2
$>$	'is greater than'	$x > 2$	x is greater than 2
$\leq$	'is less than or equal to'	$x \leq 2$	x is less than or equal to 2
$\geq$	'is greater than or equal to'	$x \geq 2$	x is greater than or equal to 2

- A **linear inequality** is an inequality containing pronumerals only raised to the power of 1 and no two pronumerals are multiplied together.



Representing linear inequalities on a number line

- Inequalities involving a single variable can be represented on a number line, using rays and line segments to represent all of the values that satisfy the inequality (see the table on the next page). On a number line:
  - → a line or line segment indicates that all the values the line passes through are included in the possible range of values for the variable
  - → a ray (arrowed line) indicates that the possible values for the variable have no limit and continue to positive or negative infinity in the direction indicated
  - → a closed circle (solid dot ●) indicates that the endpoint value is included, meaning the value is either 'less than or equal to' (≤) or 'greater than or equal to' (≥) the value represented by that point on the number line
  - → an open circle (hollow dot ○) indicates that the value is not included, meaning the value is either 'less than' (<) or 'greater than' (>) the value represented by that point on the number line.

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k

Solving linear inequalities

- In most cases, if the same operation is performed on both sides of an inequality sign, the result is an equivalent linear inequality of a different form and the possible values of the unknown remain the same.
- Inverse operations can be used to solve linear inequalities. However, the inequality sign is reversed when both sides of the inequality are multiplied or divided by a negative number.

  For example, $5 - 2x \leq 1$ $\quad (-5)$

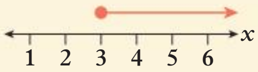
  $-2x \leq -4$ $\quad (\div (-2))$

  $x \geq 2$

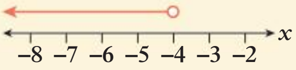
Example 3B.1 Representing linear inequalities on a number line

Write the inequality represented by each of these number lines.

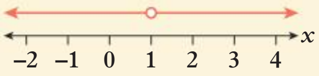
a



b



c



THINK	WRITE
a The line has a closed circle at 3, indicating that x can be equal to 3. The ray points to the right, indicating that x can be any value greater than 3. So x is greater than or equal to 3.	**a** $x \geq 3$
b The line has an open circle at –4, indicating that x cannot be equal to –4. The ray points to the left, indicating that x can be any value less than –4. So x is less than –4.	**b** $x < -4$
c The line has an open circle at 1, indicating that x cannot be equal to 1. A ray continues on in both directions, indicating that x can be any other value. So x is not equal to 1.	**c** $x \neq 1$

Example 3B.2 Solving linear inequalities

Solve each of these inequalities.

a $\frac{x}{4} - 8 \le 0$ **b** $-2 > 10x + 3$ **c** $3 > -1 - \frac{x}{7}$

THINK	**WRITE**
a 1 Apply the inverse operations to both sides in the correct order.	**a** $\frac{x}{4} - 8 \le 0$ (+ 8) $\frac{x}{4} \le 8$ (× 4) $x \le 32$
2 Check your solution. Substitute $x = 32$ into the LHS of the inequality. The result should equal the right-hand side (RHS). Then substitute any value for x that is less than 32 into the original inequality to check the inequality sign. The result should be less than or equal to the RHS.	Check by substituting $x = 32$ into $\frac{x}{4} - 8 \le 0$. $\text{LHS} = \frac{32}{4} - 8$ $= 0$ $= \text{RHS}$ (For $x = 32$, LHS = RHS as required.) Check by substituting a value less than 32 (for example, $x = 8$). $\text{LHS} = \frac{8}{4} - 8$ $= -7$ $-7 < 0$ (For $x < 32$, LHS < RHS as required.)
b 1 Apply the inverse operations to both sides in the correct order.	**b** $-2 > 10x + 3$ (− 3) $-5 > 10x$ (÷ 10)
2 Write the solution with x on the LHS, reverse the inequality sign if needed.	$-\frac{1}{2} > x$ $x < -\frac{1}{2}$
3 Check your solution. Substitute $x = -\frac{1}{2}$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is less than $-\frac{1}{2}$ into the original inequality to check the inequality sign. The result should be less than the LHS.	Check by substituting $x = -\frac{1}{2}$ into $-2 > 10x + 3$. $\text{RHS} = 10(-\frac{1}{2}) + 3$ $= -2$ $= \text{LHS}$ (For $x = -\frac{1}{2}$, LHS = RHS as required.) Check by substituting a value less than $-\frac{1}{2}$ (for example, $x = -2$). $\text{RHS} = 10(-2) + 3$ $= -17$ $-2 > -17$ (For $x < -\frac{1}{2}$, LHS > RHS as required.)
c 1 Apply the inverse operations to both sides in the correct order.	**c** $3 > -1 - \frac{x}{7}$ (+ 1) $4 > -\frac{x}{7}$ (× (−7))
2 Write the solution with x on the LHS, reverse the inequality sign if needed.	$-28 < x$ $x > -28$
3 Check your solution. Substitute $x = -28$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is greater than −28 into the original inequality to check the inequality sign. The result should be less than the LHS.	Check by substituting $x = -28$ into $3 > -1 - \frac{x}{7}$. $\text{RHS} = -1 - \frac{-28}{7}$ $= -1 + 4$ $= 3$ $= \text{LHS}$ (For $x = -28$, LHS = RHS as required.) Check by substituting a value greater than −28 (for example, $x = 0$). $\text{RHS} = -1 - \frac{0}{7}$ $= -1$ $3 > -1$ (For $x > -28$, LHS > RHS as required.)

Example 3B.3 Solving inequalities with the unknown on both sides

Solve each of these inequalities.

a $7x - 2 \geq 3x - 10$ **b** $4 - 5x < 3 - 8x$

THINK

1 Use inverse operations to remove the pronumeral on the side where the coefficient is a number less than the coefficient on the other side.

2 Identify and apply the correct inverse operations, in order, to both sides of the inequality.

3 Check your solution by substituting appropriate values for x into the original inequality to see if true inequalities result.

WRITE

a $7x - 2 \geq 3x - 10$ $(-\ 3x)$

$4x - 2 \geq -10$ $(+\ 2)$

$4x \geq -8$ $(\div\ 4)$

$x \geq -2$

Check by substituting $x = -2$ into $7x - 2 \geq 3x - 10$.

$\text{LHS} = 7(-2) - 2$

$= -16$

$\text{RHS} = 3(-2) - 10$

$= -16$

$-2 > -10$ (For $x > -2$, LHS = RHS as required.)

Check by substituting a value greater than -2 (for example, $x = 0$).

$\text{LHS} = 7(0) - 2$

$= -2$

$\text{RHS} = 3(0) - 10$

$= -10$ (For $x > -2$, LHS > RHS as required.)

b $4 - 5x < 3 - 8x$ $(+\ 8x)$

$4 + 3x < 3$ $(-\ 4)$

$3x < -1$ $(\div\ 3)$

$x < -\frac{1}{3}$

Check by substituting $x = -\frac{1}{3}$ into $4 - 5x < 3 - 8x$.

$\text{LHS} = 4 - 5\left(-\frac{1}{3}\right)$

$= \frac{17}{3}$

$\text{RHS} = 3 - 8\left(-\frac{1}{3}\right)$

$= \frac{17}{3}$ (For $x = -\frac{1}{3}$, LHS = RHS as required.)

Check by substituting a value less than $-\frac{1}{3}$ (for example, $x = -1$).

$\text{LHS} = 4 - 5(-1)$

$= 9$

$\text{RHS} = 3 - 8(-1)$

$= 11$

$9 < 11$ (For $x < -\frac{1}{3}$, LHS < RHS as required.)

Helpful hints

- ✔ Be careful when multiplying and dividing inequalities by negative numbers.
- ✔ If you need to swap the LHS and RHS of an inequality, remember to reverse the inequality sign so that it is still pointing towards the lesser expression. For example, if $5 \leq x$, then $x \geq 5$.
- ✔ When solving linear inequalities with the unknown on both sides, always remove the pronumeral on the side where the coefficient is less. That way you will always have a positive value for the remaining coefficient. Remember, $4 > 2$ but $-4 < -2$.

ANS
p418

Exercise 3B Solving linear inequalities

△ 1-4, 5(1st, 2nd columns), 6, 7, 8 (a-e), 9, 11, 13

■ 2-3(2nd column), 5-7(3rd, 4th columns), 8(e-i), 9, 10, 12, 14

⬠ 2-3(2nd column), 6-8(e-h), 9, 12, 14-17

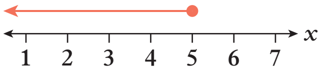
UNDERSTANDING AND FLUENCY

1 Write each of the following statements as an inequality.

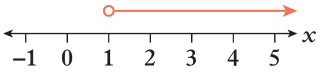
- **a** Two is less than three.
- **b** Three is greater than two.
- **c** Three is greater than or equal to two.
- **d** Negative three is less than or equal to negative two.
- **e** Three is not equal to negative three.
- **f** Negative three is not equal to three.

3B.1 **2** Write the inequality represented on each of these number lines.

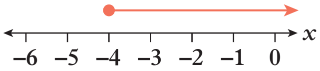
a



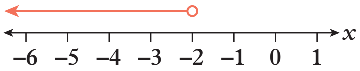
b



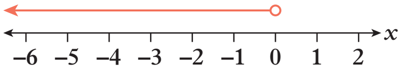
c



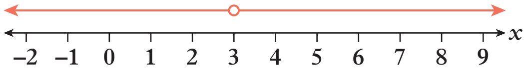
d



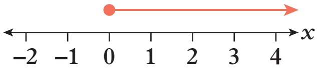
e



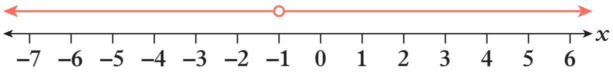
f



g



h



3 For each of these inequalities, show the possible values of x on a number line.

- **a** $x > 4$
- **b** $x \leq 3$
- **c** $x \geq -2$
- **d** $x < 0$
- **e** $x \neq 2$
- **f** $x \neq -2$

4 Apply the following operations to each side of the following inequalities. Ensure the correct inequality symbol is written between the two numbers.

- **a** $-4 < 9$
- **b** $x \geq 6$
  - **i** Add five
  - **ii** Subtract nine
  - **iii** Multiply by three
  - **iv** Multiply by one-half
  - **v** Multiply by negative three
  - **vi** Multiply by negative one-half
  - **vii** Divide by three
  - **viii** Divide by negative three

3B.2 **5** Solve each of these inequalities.

- **a** $x + 5 > 19$
- **b** $\frac{x}{10} \leq 30$
- **c** $-20 > 4x$
- **d** $-3x \geq -15$
- **e** $7 < -\frac{x}{10}$
- **f** $\frac{x}{5} + 10 \leq 11$
- **g** $2x - 9 \geq -11$
- **h** $26 > 14 - 4x$
- **i** $-x + 3 \geq 12$
- **j** $5(x + 3) \leq 50$
- **k** $\frac{x - 8}{15} > -3$
- **l** $1 < 10x + 14$
- **m** $12 - 14x > 14$
- **n** $-17 \leq -6x + 12$
- **o** $-6(x - 4) < 13$
- **p** $-11(3x + 5) \geq 40$

6 Solve each of these inequalities and show the solution on a number line each time.

- **a** $3x + 2 \geq -13$
- **b** $1 - 7x < -6$
- **c** $5 - \frac{x}{6} \leq 2$
- **d** $\frac{x - 4}{2} < 5$
- **e** $\frac{7 - x}{3} \geq -1$
- **f** $\frac{2 + x}{5} > 3$
- **g** $\frac{3x}{2} - 5 \leq 4$
- **h** $\frac{8 - x}{7} - 3 < -2$

UNDERSTANDING AND FLUENCY

3B.3 **7** Solve each of these inequalities. Use substitution to check your solutions.

a $4x - 3 > 2x + 5$ **b** $7x + 1 \leq 3x - 7$ **c** $5x - 9 \geq 9 - x$ **d** $2x + 4 < 5x - 8$

e $3 - x > 4x - 2$ **f** $x + 11 \geq 7 - 3x$ **g** $8 - 3x \leq 18 - 5x$ **h** $1 - 6x > -3x - 5$

8 Solve each of these inequalities by first expanding to remove brackets.

a $2(x - 3) > x + 5$ **b** $4x - 7 \leq 3(x + 2)$ **c** $5(x + 1) \geq -3x - 11$

d $3(2x - 5) < 7x + 4$ **e** $9(x + 5) < 10(x + 6)$ **f** $3(3x - 1) \leq -2(x - 4)$

g $4(x + 3) \geq 2(5x - 3)$ **h** $2(2 - x) > 3(1 - 2x)$ **i** $5(3 - 2x) \leq -4(2x - 7)$

9 Solve each of these inequalities. Use substitution to check your solutions.

a $\frac{3x + 2}{7} \leq \frac{2x - 5}{7}$ **b** $\frac{2 - x}{3} < \frac{6x - 5}{3}$ **c** $\frac{1 - 4x}{2} \geq \frac{11 - 5x}{4}$

d $\frac{x + 4}{5} > \frac{x + 2}{3}$ **e** $\frac{x - 4}{7} \geq \frac{4 - x}{2}$ **f** $\frac{4x - 1}{6} < \frac{3x + 1}{4}$

PROBLEM SOLVING AND REASONING

10 Write an inequality statement to represent each of the following situations. Use a pronumeral for the unknown quantity each time.

a A boat is sold for at least $750 000. What could the boat have sold for?

b To fit in an economy class seat on an aeroplane, a person must be less than 200 cm tall. What could be the height of a person in such a seat?

c A Vespa cannot travel faster than 55 km/h. At what speeds could a Vespa travel?

d To fit in a new sleeping bag, a person must be less than 196 cm tall. How tall could a person be to fit in the sleeping bag?

11 Emily and Klaus are selling watermelons at a market. They start with 20 melons and agree to share any that are left at the end of the day.

a Write an expression for the number of melons they each will take home if they sell x watermelons.

b Emily and Klaus aim to take home no more than three watermelons each. Use your expression from part **a** to write an inequality for this situation and then solve it.

c How many watermelons can they sell to meet their goal?

12 Ella runs a business selling candles where customers can pay for delivery or pick them up from her store. She sells each candle for $2.50 and can mail a package containing one candle for $3, two candles for $5.50 and four candles for $10.

A customer has $26 to spend on candles. They live too far away to visit the store, so they must pay the delivery fee.

a How many candles could they buy if each candle is packaged individually?

b Ella decides to pack the candles in packages of two where possible and adjust the delivery fee.

  i Can the customer afford an extra candle in comparison to part **a** (excluding any delivery fees)?

  ii Can the customer afford an extra candle and the delivery fee if they still use packages of two where possible?

The customer decides to prioritise buying packs of four candles instead. If they have enough money left over, they will buy a pack of two candles or a single candle.

c How many candles can the customer afford now? Make sure to include the delivery fee in your calculations.

PROBLEM SOLVING AND REASONING

13 Hassan is deciding how many packs of playing cards he should buy. Each pack costs \$3 and he has \$25 in his wallet.

a Write an inequality to represent this situation and then solve it.

b List the number of packs of playing cards Hassan could possibly buy.

14 **a** Square all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $2 < 3$ **ii** $-2 > -3$ **iii** $2 > -3$ **iv** $-2 < 3$

v $\frac{1}{2} > \frac{1}{3}$ **vi** $-\frac{1}{2} < \frac{1}{3}$ **vii** $-2 \neq 2$

b Explain why we need to be careful when squaring both sides of an inequality.

c Apply the reciprocal to all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $3 > \frac{1}{2}$ **ii** $-3 < \frac{1}{2}$

iii $-3 < -\frac{1}{2}$ **iv** $3 > -\frac{1}{2}$

v $\frac{1}{3} < \frac{1}{2}$ **vi** $-\frac{1}{3} < \frac{1}{2}$

d Explain why we need to be careful when applying the reciprocal to both sides of an inequality.

CHALLENGE

15 A company makes and sells two products: X and Y. It costs the company \$4 for each of product X to be made and \$5 for each of product Y. The company needs the cost of producing the two products to be no more than \$100 per day. Let x be the number of product X the company makes and let y be the number of product Y the company makes.

a Write an inequality describing the cost of producing products X and Y for one day.

b State two other inequalities that must be true given that x and y represent the number of each product.

c Determine the greatest number of product X that could be produced in one day.

d Determine the greatest number of product Y that could be produced in one day.

e Determine the greatest number of product X that could be produced in one day if 8 of product Y are produced.

16 Solve each of these inequalities.

a $\frac{2(4-x)}{5} - 3 \le 3$ **b** $\frac{3(7x-4)}{2} - 8 < 6x + 4$

c $\frac{5x-1}{3} + \frac{x-2}{4} \ge 3$ **d** $\frac{x-4}{8} - \frac{x+3}{2} > 1$

e $\frac{4(2x+1)}{5} + \frac{7-2x}{3} \ge 2x + 1$ **f** $\frac{3x+4}{2} - \frac{x+4}{6} \le \frac{x-5}{3}$

17 Solve each of these inequalities.

a $-3 \le 5 - 2x \le 7$ **b** $15 < \frac{5(1-2x)}{2} < 25$

c $-2 < 4 - \frac{x}{2} < 8$ **d** $1 \le \frac{5}{2} - \frac{5x}{3} < 10$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Representing linear inequalities on a number line

Interactive skillsheet
Solving linear inequalties

Worksheet
Solving linear equations and inequalities

Topic quiz
3B

3C Plotting linear relationships

Learning intentions

By the end of this topic you will be able to …

✔ plot linear relationships from tables of values and equations

✔ identify independent and dependent variables.

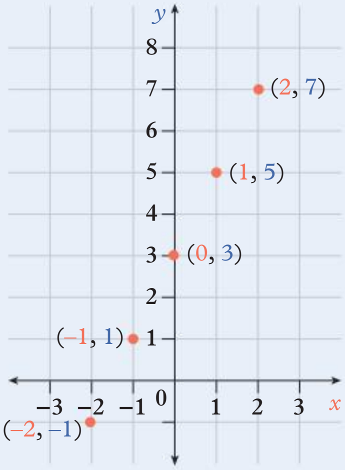
Inter-year links

Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 8	6D Plotting linear relationships
Year 10	3B Sketching linear graphs

Linear relationships

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of **coordinate points** or a graph.
- A **linear relationship** is a relationship between two variables, the independent variable and the dependent variable, which produces a linear graph.
- A **linear graph** is a straight line on a **Cartesian plane**.
- The **independent variable** is the quantity that does not depend on the other variable.
- The value of the **dependent variable** depends on the value of the independent variable.

  For example, if a car is moving at a constant speed, then *time* is an independent variable, the *distance travelled by the car* is a dependent variable, and the relationship between them is linear.



- It is conventional for the independent variable to be:
  - → listed in the top row of a table of values

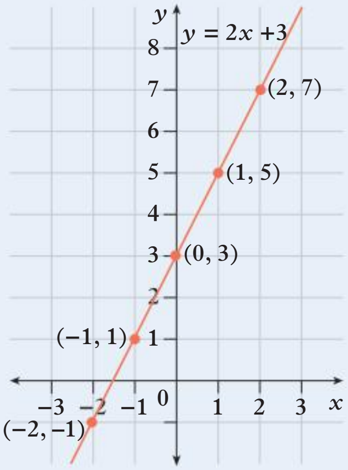
x	−2	−1	0	1	2
y	−1	1	3	5	7

  - → listed first in a pair of coordinates

    (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7)
  - → shown on the horizontal axis.

    Independent variable: x

    Dependent variable: y



Plotting linear relationships

- A **plot** is composed of individual coordinate points.

  A linear graph is a continuous line made up of an infinite number of coordinate points.
- To sketch a graph from an equation follow these steps:
  1. Construct a table of values by selecting values for x, then substituting each value of x into the equation to find the corresponding value of y.
  2. Write out the coordinate points listed in the table.
  3. Plot the coordinate points on the Cartesian plane.
  4. Join the points using a straight line.

Example 3C.1 Plotting linear relationships from a table of values

Use the table of values to construct a plot of the relationship between x and y. Is the relationship linear or non-linear?

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

THINK

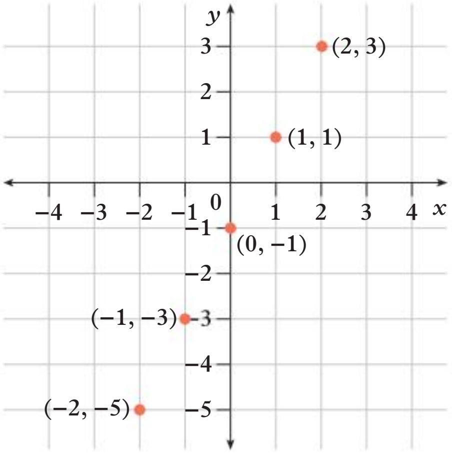
1 Write out the coordinate points listed in the table.

2 Plot the points on the Cartesian plane.

3 Consider whether the points form a straight line. As the points form a straight line, the relationship is linear.

WRITE

(−2, −5), (−1, −3), (0, −1), (1, 1), (2, 3)



The relationship is linear.

Example 3C.2 Graphing linear relationships from an equation

Plot a graph of $y = -x - 3$ by first completing a table of values for x from −3 to 2.

THINK

1 Construct a table of values for x from −3 to 2. Substitute each value of x into the equation to find the corresponding value of y.

2 Write out the coordinate points listed in the table.

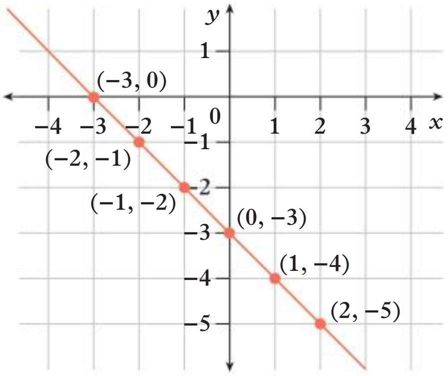
3 Plot the points on the Cartesian plane.

4 Join the points with a straight line.

WRITE

x	−3	−2	−1	0	1	2
y	0	−1	−2	−3	−4	−5

(−3, 0), (−2, −1), (−1, −2), (0, −3), (1, −4), (2, −5)



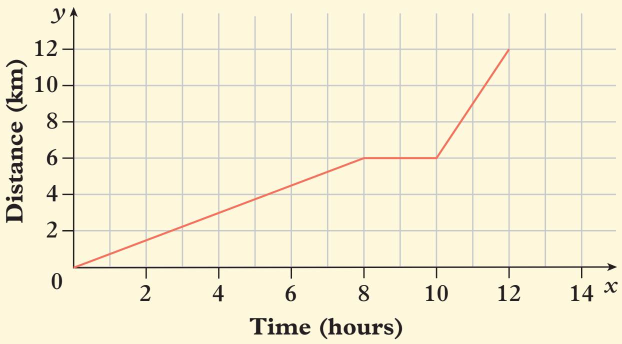
Example 3C.3 Identifying independent and dependent variables

Identify the independent and dependent variables in the following relationships. Explain why for each case.

a

Age (years)	10	11	12	13
Height (cm)	140	148	154	160

b

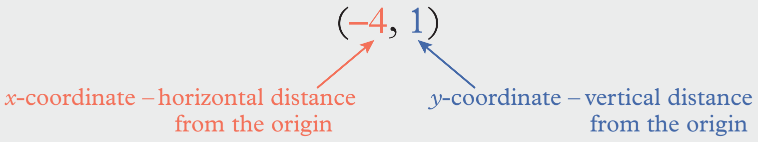


c The number of oranges you put in a juicer and the volume of orange juice produced by the juicer

THINK	WRITE
a In a table of values, the values of the independent variable are listed in the first row and the values of the dependent variable are listed in the second row. To explain why height depends on age, think about which variable cannot change without the other.	**a** Independent variable: **Age** because a person ages constantly, but their height might not change. Dependent variable: **Height** because as a person ages their height changes.
b In a graph, the independent variable is shown on the horizontal axis and the dependent variable is shown on the vertical axis. To explain why distance depends on time, think about which variable cannot change without the other.	**b** Independent variable: **Time** because time passes regardless of an object's movement. Dependent variable: **Distance** because something cannot move without time passing.
c The volume of orange juice *depends* on the number of oranges you put in a juicer. To explain why, think about which one is required to make the other.	**c** Independent variable: **Number of oranges** because oranges cannot be made from orange juice. Dependent variable: **Volume of orange juice** because the amount of orange juice depends on the number of oranges put in the juicer.

Helpful hints

- ✔ Remember that in Cartesian coordinates, the x-coordinate is always listed first, followed by the y-coordinate.
- ✔ When constructing plots and sketches, always label your x- and y-axes and label your graph with the equation of the graph.



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Exercise 3C Plotting linear relationships

△ 1, 2(a–e), 3–8, 9, 12 □ 2(f, g, h, i), 3–10, 12–14 ⬠ 2(h, i), 3, 5–9, 10, 11, 13–15

3C.1 **1** **a** Use the following tables of values to construct a plot of each relationship between x and y.

i

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

ii

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11

iii

x	−3	−2	−1	0	1	2	3
y	10	9	8	7	6	5	4

iv

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

b Classify each relationship as linear or non-linear.

3C.2 **2** For each of the following linear relationships, construct a table of values for x from −3 to 3 and then write out the coordinate points listed in the table.

a $y = x + 2$
b $y = x - 4$
c $y = 3 - x$
d $y = 2 - x$
e $y = -x - 3$
f $y = 4x$
g $y = 2x + 1$
h $y = 3x - 2$
i $y = 4 - 2x$

3C.3 **3** Identify the independent and dependent variables in the following relationships. Explain why for each case.

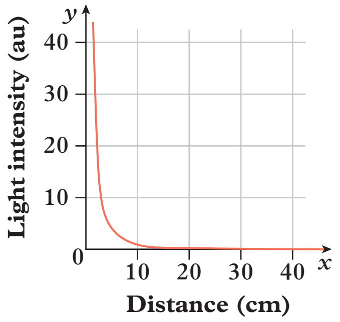
a

Radius (cm)	1	2	3	4
Area (cm^2)	3.14	12.57	28.27	50.27

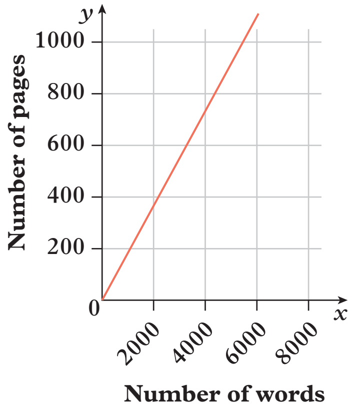
b

Number sold	20	40	60	80
Revenue ($)	100	200	300	400

c



d



e The *amount of breath* used to play a note on a flute and the *volume* of the note produced

f The *chance* of drawing the ace of spades from a standard deck after removing cards that are not the ace of spades and the *number of cards* remaining in the deck

4 **a** Complete the following tables of values using the equations provided.

i $x + y = 24$

x	0	3	6	9	12	15	18
y							

ii $xy = 24$

x	−12	−6	−2	2	6	12
y						

iii $y = x(x + 2)$

x	−3	−2	−1	0	1	2	3
y							

iv $x = 4y - 2$

x							
y	−3	−2	−1	0	1	2	3

b Plot the coordinates from the tables of values and classify each relationship as linear or non-linear. Do not attempt to join the points with curves or lines.

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

5 **a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = 2x$ **iii** $y = 3x$ **iv** $y = 4x$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the coefficient of x and the steepness of the corresponding graph.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = 5x$ **ii** $y = 3.5x$

e Use your answers to **b** and **c** to describe the graph of $y = 0.5x$.

6 **a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = -x$ **ii** $y = -2x$ **iii** $y = -3x$ **iv** $y = -4x$

b How do these graphs differ from those sketched in question **5**?

c Describe the relationship between the coefficient of x and the feature of the graph identified in part **b**.

d Without using a table of values, sketch the graphs of the following relationships on the same set of axes.

i $y = -5x$ **ii** $y = -1.5x$

7 **a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = x + 1$ **iii** $y = x + 2$ **iv** $y = x + 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x + 4$ **ii** $y = x + 1.5$

8 **a** Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$ **ii** $y = x - 1$ **iii** $y = x - 2$ **iv** $y = x - 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

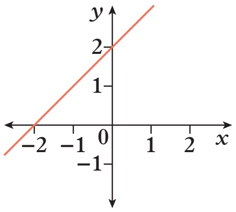
d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x - 4$ **ii** $y = x - 5$

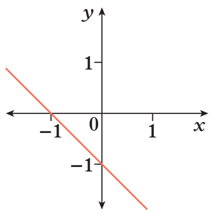
9 Use your answers to questions **5** to **8** to match each equation with its corresponding graph.

a $y = x + 2$ **b** $y = 2x$ **c** $y = -x - 1$ **d** $y = -0.5x$

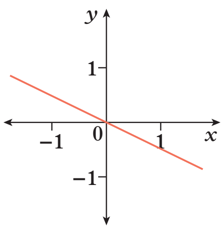
A



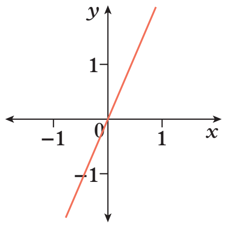
B



C



D



10 Use your answers to questions **5** to **8** to sketch the graphs of the following linear relationships. In each part, sketch the two graphs on the same set of axes.

a **i** $y = 2x$ **ii** $y = 2x + 1$

b **i** $y = -x$ **ii** $y = -x - 1$

c Describe the relationship between the first and second graph in parts **a** and **b**.

11 Javier is training for the cross-country race. He runs laps of the oval every other day and records the number of laps he runs.

Day	1	3	5	7	9	11	13
Number of laps	3	3.5	4	4.5	5	5.5	6

a Identify the independent variable and dependent variable.

b **i** What feature(s) of the table of values indicates that the graph is likely to be linear?

ii Hence, what is the minimum number of points you need to plot to sketch the line?

c Sketch the relationship on a Cartesian plane and extend the line to day 25.

d Use the graph to determine:

i the number of laps Javier should run on day 17 **ii** the day Javier should run 8 laps.

12 A bus is hired for a school trip to the snow. The school pays \$250 towards the bus hire and each student is charged \$40.

a If m is the total amount of money collected for bus hire and n students go on the trip, write an equation for the relationship between m and n.

b Identify the independent and dependent variables in the relationship.

c Calculate how much money would be collected for bus hire if:

i 0 students go on the trip

ii 20 students go on the trip.

d Use your answer to part **c** to sketch a graph of this relationship using a scale from 0 to 30 along the horizontal axis.

e Is the relationship linear? Explain.

f Use the graph to find the total amount of money collected if 30 students go on the trip.

g Use the graph to determine how many students need to go on the trip to collect a total of \$850 for bus hire.

h The hire cost of the bus is \$1300. Use the graph to determine the minimum number of students who need to go on the trip to cover the hire cost.

13 The formula $F = 1.8C + 32$ describes the relationship between temperatures in degrees Celsius, C, and temperatures in degrees Fahrenheit, F.

a Plot the graph of this relationship. Show a scale from −50 to 50 along the horizontal axis.

b Use the graph to find the temperature in °F for 30°C.

c Use the graph to find the temperature in °C for −22°F.

14 Use the graph in question **13** to find where the temperature in °C has the same numerical value as the matching temperature in °F.

15 For each of the following pairs of linear relationships, plot two graphs on the same Cartesian plane to determine the coordinates where the two lines intersect.

a $y = x + 1$ and $y = 2x$ **b** $y = 5x$ and $y = \frac{1}{3}x$

c $y = \frac{1}{2}x - 1$ and $y = -x + 2$ **d** $y + 2x = 12$ and $3y + x = 16$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Plotting linear relationships

Interactive skillsheet
Classifying variables

Investigation
How accurate is a 'rule of thumb' temperature formula?

CAS instructions
Graphing functions

Topic quiz
3C

3D Gradient and intercepts

Learning intentions

By the end of this topic you will be able to ...

✔ identify the x- and y-intercepts of a linear graph

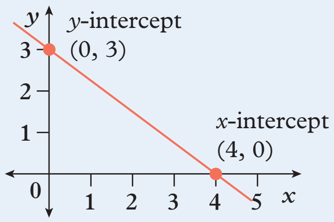
✔ determine the gradient of a line segment and a graph.

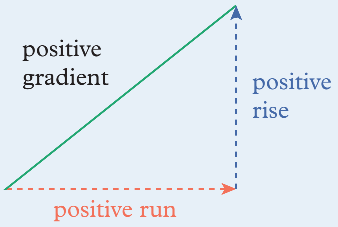
Inter-year links

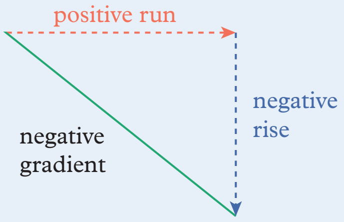
Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 8	6F Finding linear equations
Year 10	3B Sketching linear graphs

Features of linear graphs

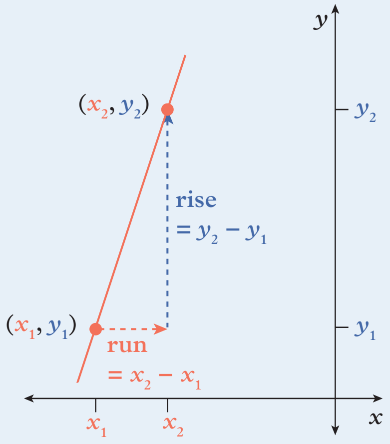
- The features of a linear graph include its x-intercept, y-intercept and gradient.
- The **x-intercept** is the point where the graph crosses the x-axis.
- The **y-intercept** is the point where the graph crosses the y-axis.
- The **gradient** is a numerical measure of the slope of the graph. It is also referred to as the rate of change between the two variables.



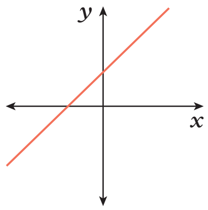
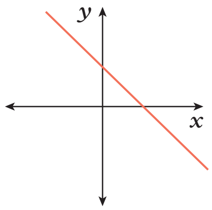
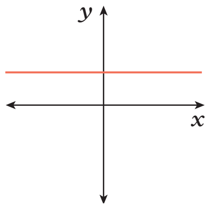
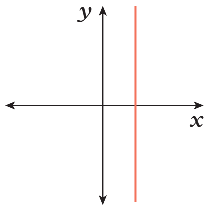




- The gradient of a linear graph is a constant because the gradient between any two points on the line is the same. This means that the rate of change between two variables in a linear relationship does not change.
- The value of the gradient is the number of units that the graph increases in the vertical direction for every 1 unit that it increases in the horizontal direction. So, if the gradient is 3, then the linear graph increases 3 units up in the vertical direction for every 1 unit that it increases to the right in the horizontal direction.



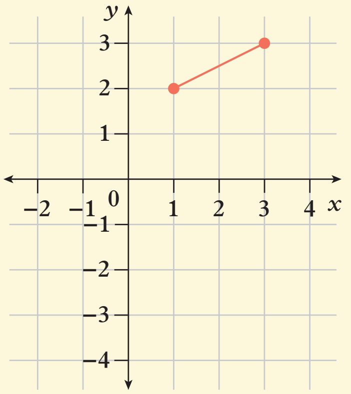
- The formula for the gradient, m, between any two points, (x_1, y_1) and (x_2, y_2), is: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Gradients can be positive, negative, zero or undefined.

Positive	Negative	Zero	Undefined
			

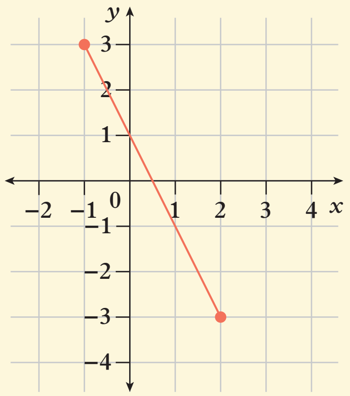
Example 3D.1 Determining the gradient of a line segment by identifying rise and run

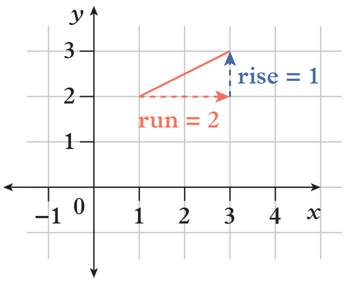
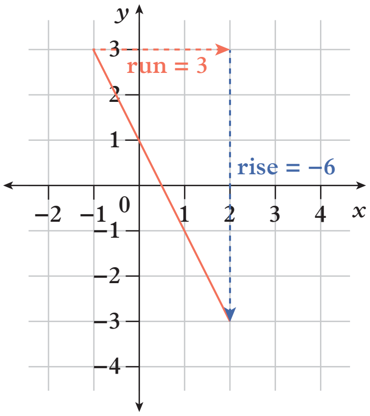
Find the gradient of each line segment.

a



b



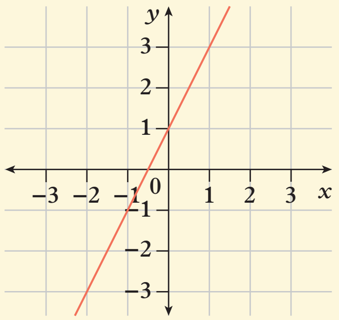
THINK	WRITE
a **1** Determine the run, the horizontal distance between the endpoints of the line segment. The run is 2 units.	**a** 
2 Determine the rise, the vertical distance between the endpoints of the line segment. The rise is 1 unit.	
3 Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{1}{2}$
b **1** Determine the run, the horizontal distance between the endpoints of the line segment. The run is 3 units.	**b** 
2 Determine the rise, the vertical distance between the endpoints of the line segment. The rise is –6 units.	
3 Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{-6}{3}$ $= -2$

Example 3D.2 Determining the gradient, *x*-intercept and *y*-intercept

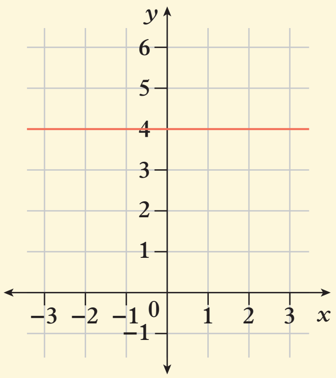
For each of the linear graphs shown, determine the:

i gradient **ii** *x*-intercept **iii** *y*-intercept.

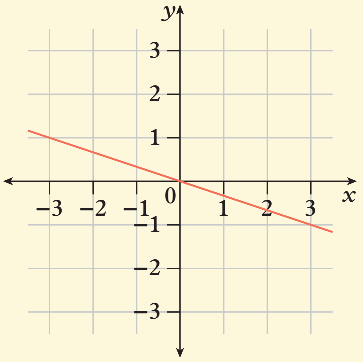
a

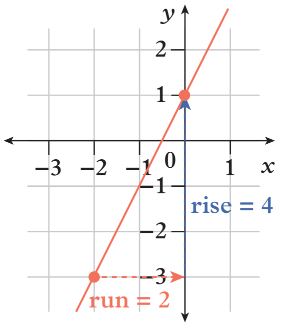
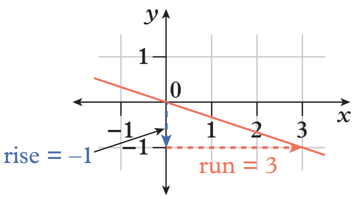


b



c



THINK	**WRITE**
a i Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient.	**a i**  $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{4}{2}$ $= 2$
ii State the coordinates of the point at which the graph crosses the *x*-axis.	**ii** *x*-intercept: $\left(-\frac{1}{2}, 0\right)$
iii State the coordinates of the point at which the graph crosses the *y*-axis.	**iii** *y*-intercept: (0, 1)
b i The graph is a horizontal line, so it has a zero gradient.	**b i** Gradient = 0
ii The graph does not cross the *x*-axis, so it does not have an *x*-intercept.	**ii** no *x*-intercept
iii State the coordinates of the point at which the graph crosses the *y*-axis.	**iii** *y*-intercept: (0, 4)
c i Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient. Remember that if the graph is sloping down to the right then the gradient is negative.	**c i**  $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{-1}{3}$ $= -\frac{1}{3}$
ii State the coordinates of the point at which the graph crosses the *x*-axis.	**ii** *x*-intercept: (0, 0)
iii State the coordinates of the point at which the graph crosses the *y*-axis.	**iii** *y*-intercept: (0, 0)

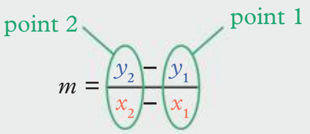
Example 3D.3 Determining gradient using two coordinate points

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining the points (3, 2) and (9, 5).

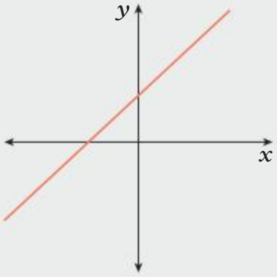
THINK	WRITE
1 Define the points (x_1, y_1) and (x_2, y_2). The order of the coordinates does not affect the value of the gradient.	Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (9, 5)$
2 Substitute the x- and y-coordinates into the gradient formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - 2}{9 - 3}$
3 Calculate and simplify the gradient.	$= \frac{3}{6}$ $= \frac{1}{2}$

Helpful hints

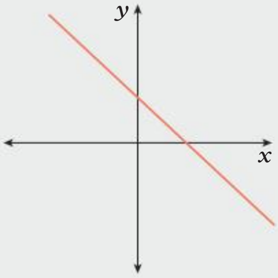
- ✔ The order in which you substitute points into the formula for the gradient of a line won't affect your final value – you just need to make sure the x- and y-coordinates of a given point match up vertically!
- ✔ Graphs with positive gradients are described as increasing as the value of y increases from left to right.
- ✔ Graphs with negative gradients are described as decreasing as the value of y decreases from left to right.



Positive gradient: Increasing



Negative gradient: Decreasing



ANS
p423

Exercise 3D Gradient and intercepts

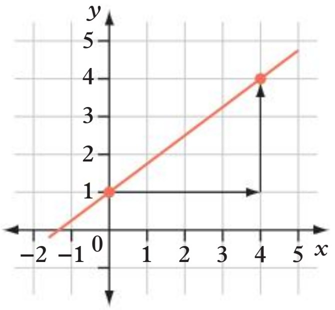
▲ 1–5, 6(1st column), 7–10, 12, 16

■ 3–5, 6(2nd column), 8(b, c, f), 9–11, 13, 14, 15(a–c)

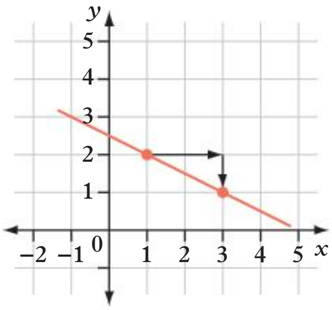
⬟ 3, 4(c, e, h), 6(2nd column), 7, 9, 13–16

1 Use the rise and run marked on the graphs below to determine the gradient of each line segment.

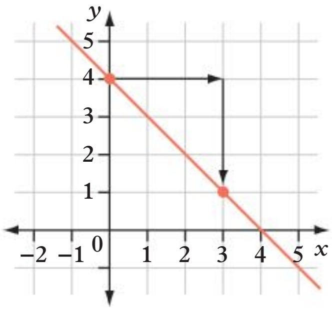
a



b

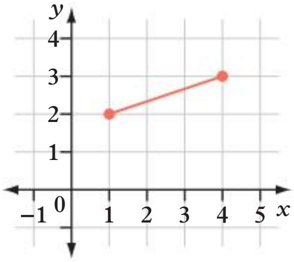


c

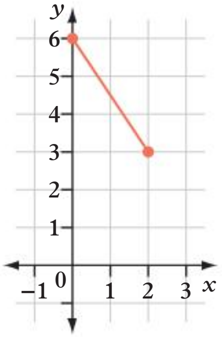


3D.1 **2** Find the gradient of each line segment.

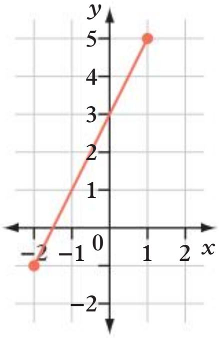
a



b

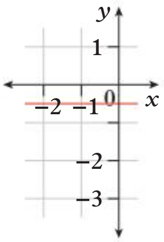


c

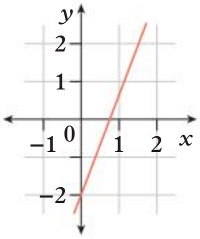


3 Classify the gradients of the following lines as positive, negative, zero or undefined.

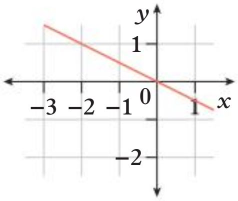
a



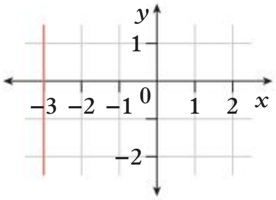
b



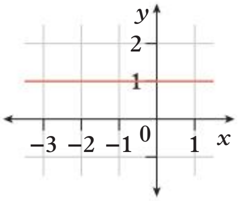
c



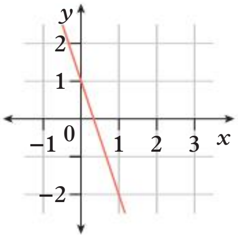
d



e



f



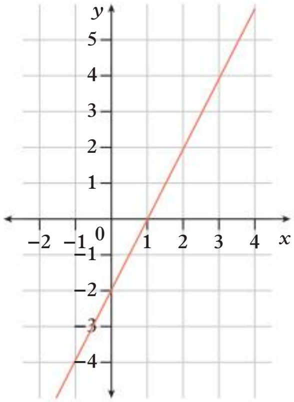
3D.2 **4** For the linear graph shown, determine the:

i gradient

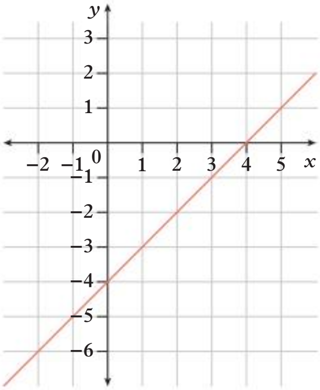
ii x-intercept

iii y-intercept

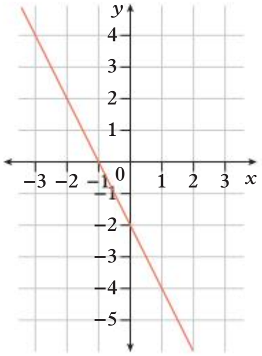
a



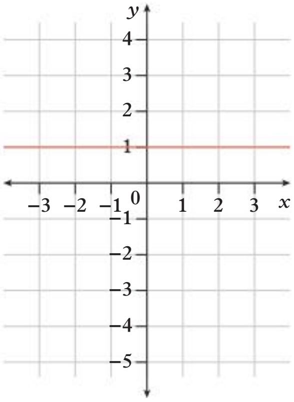
b



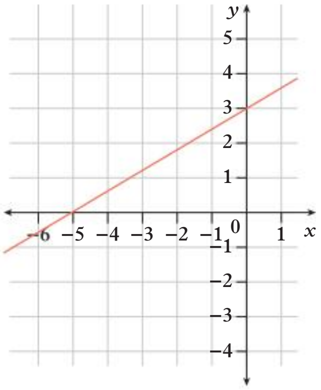
c



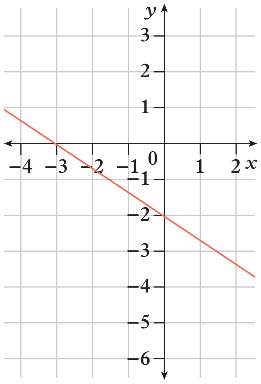
d



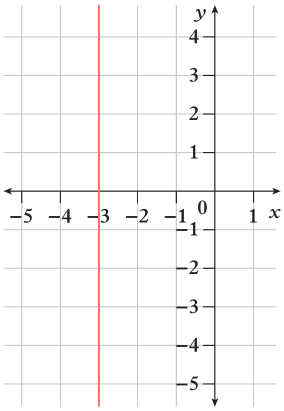
e



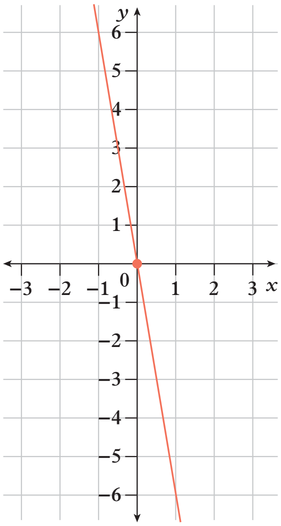
f



g



h



5 **i** Plot each pair of points on the Cartesian plane and join them with a straight line to form a line segment.

ii Determine the gradient of the line segment.

a (2, 3) and (6, 8)

b (1, 2) and (3, 6)

c (3, 7) and (4, 4)

d (−4, 5) and (2, −3)

3D.3 **6** Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining each pair of points.

a (2, 4) and (5, 6)

b (3, 1) and (7, 4)

c (1, 2) and (4, 8)

d (4, 6) and (7, 5)

e (−2, 5) and (3, 7)

f (0, 8) and (1, 5)

g (−1, −6) and (−1, −1)

h (0, 5) and (1, 0)

i (−4, −3) and (−1, 4)

j (−3, 3) and (−3, −5)

k (−5, −6) and (−4, −8)

l (−9, 3) and (−6, 3)

7 Consider the following pairs of points.

i Plot each pair on the Cartesian plane and then join them with a straight line. Ensure your lines cross both axes.

ii Determine the *x*- and *y*-intercept of the line, if they exist. Write the intercepts as coordinates.

a (2, 2) and (5, 8)

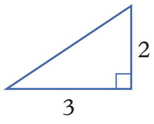
b (1, 2) and (3, 6)

c (−6, 6) and (−4, 3)

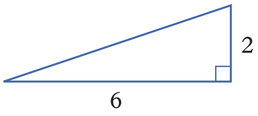
d (−2, 1) and (−2, −3)

8 Calculate the gradient of the hypotenuse of these right-angled triangles.

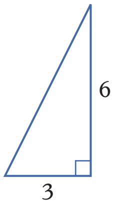
a



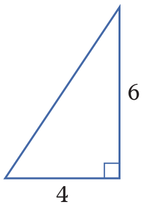
b



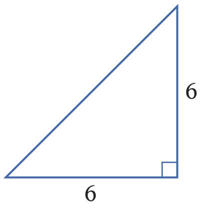
c



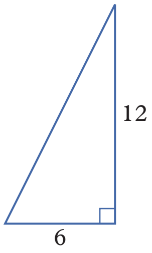
d



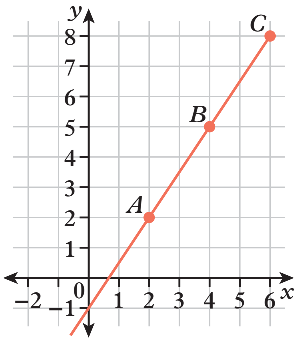
e



f



9 Consider the graph below.

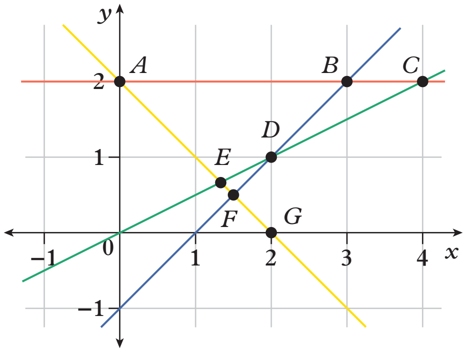


a Determine the gradient of the following line segments:

i *AB* **ii** *AC*

b Compare your answers to part **a** and describe the relationship between the gradient of a line and the gradient of line segments on that line.

10 Consider the four linear graphs on the Cartesian plane below.



a Calculate the gradient for the following line segments:

i *AB* **ii** *BD* **iii** *CD* **iv** *AE*

b Use your answers from part **a** and your observations in question **9** to determine the gradient for the following line segments.

i *DE* **ii** *BC* **iii** *DF* **iv** *FG*

11 Kane calculates the gradient of the line segment joining (2, 8) and (6, 5), as shown below. Identify Kane's error and calculate the correct value for the gradient of the line segment.

Kane

$$m = \frac{8-5}{6-2}$$
$$= \frac{3}{4}$$

12 Todd and Bridget calculated the gradient of the line segment joining (−1, 9) and (3, −3), as shown below. Explain why both students have produced correct calculations.

Todd

$$m = \frac{-3-9}{3-(-1)}$$
$$= \frac{-12}{4}$$
$$= -3$$

Bridget

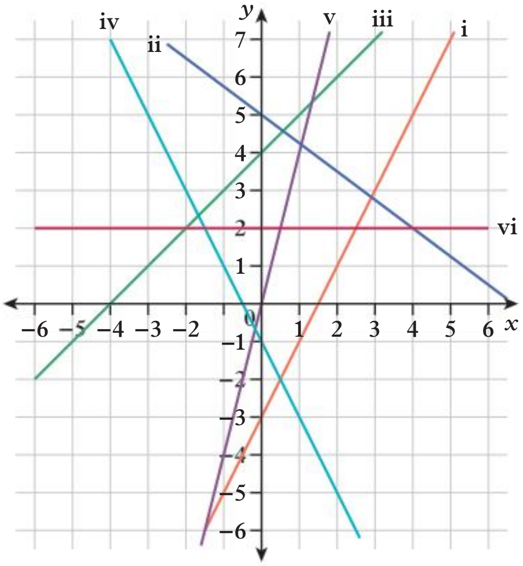
$$m = \frac{9-(-3)}{-1-3}$$
$$= \frac{12}{-4}$$
$$= -3$$

13 Complete the table below.

Run	8	−20			4	
Rise			−4	−16		30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	−5

14 Consider these linear graphs.

a Complete the following table by calculating the gradient and identifying the coordinates of the *y*-intercept of each graph.



	Equation	Gradient	*y*-intercept
i	$y = 2x - 3$		
ii	$y = -\frac{3}{4}x + 5$		
iii	$y = x + 4$		
iv	$y = -2x - 1$		
v	$y = 4x$		
vi	$y = 2$		

b What pattern can you see in the table that allows you to identify the gradient and the *y*-intercept from the equation for the graph?

c Predict the gradient and *y*-intercept of the linear graphs with the following rules:

i $y = 6x + 4$

ii $y = x - 5$

iii $y = -3x$

15 Determine the value of the unknowns in each of the following.

a A line with a gradient of 3 passes through the points (1, 4) and (x, 10).

b A line with a gradient of –2 passes through the points (1, 4) and (3, y).

c A line with a gradient of 2 passes through the points (1, 4) and (3, y).

d A line with a gradient of 2 passes through the points (0, 4) and (3, y).

e A line with a gradient of $-\frac{1}{2}$ passes through the points (1, 4) and (3, y).

f A line with a gradient of $\frac{3}{2}$ passes through the points (−1, −4) and (x, 8).

16 The gradient of a line is 2 and it passes through the points (0, 4) and (x, y).

a Write an equation for the gradient of this line.

b Rearrange the equation in part **a**, to solve for y in terms of x.

The gradient of another line is 2 and it passes through the points (1, 4) and (x, y).

c Write an equation for the gradient of this line.

d Rearrange the equation in part **c**, to solve for y in terms of x.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

pro

Interactive skillsheet
Gradients

Interactive skillsheet
Intercepts

Worksheet
Identifying features of a linear graph

Topic quiz
3D

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

3A **1** Solve the following equations for x.

a $\frac{x}{3} + 2 = -7$ **b** $4(x + 2) = 28$

c $\frac{4 - x}{9} = -2$ **d** $5(2x - 3) - 8x = -1$

3A **2** Solve the following equations for x.

a $\frac{24}{x} = 4$ **b** $\frac{3}{x} = 18$ **c** $\frac{5}{x} = 0.8$ **d** $\frac{10}{x} = 0.25$

3A **3** Solve the following equations for x (variables on both sides).

a $5x + 7 = 2x - 5$ **b** $7 - 4x = 6x - 13$ **c** $7(x - 3) = -56 - x$ **d** $8(2x + 7) = 4(3x + 11)$

3A **4** Jamie orders five equally priced video games online for a total cost of \$409.70, which includes the delivery charge of \$9.95.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d What is the cost of each game?

3B **5** Represent the values of x for each inequality on a number line.

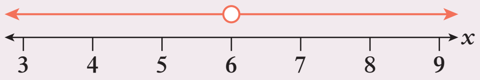
a $x \leq 5$ **b** $x > -3$

c $x \neq 2$ **d** $x \geq 4$

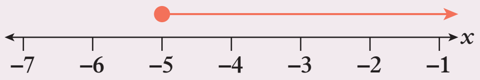
3B **6** Write the inequality that is represented on each of these number lines.

a

b



c



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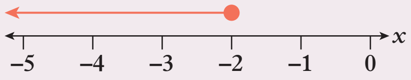


3B **7** Solve the following inequalities.

a $6x > 18$ **b** $-4x \geq 20$

c $-13 < \frac{x}{5} - 8$ **d** $7 - 3x < 4$

e $4x + 7 \neq 6x + 13$ **f** $4(3x - 4) \leq 3(3x + 2)$

3C **8** Consider the following tables of coordinate points.

i Plot the points from the following tables.

ii State whether the graph is linear or non-linear.

a

x	−3	−2	−1	0	1	2	3
y	−10	−8	−6	−4	−2	0	2

b

x	−3	−2	−1	0	1	2	3
y	4.5	4	3.5	3	2.5	2	1.5

c

x	−3	−2	−1	0	1	2	3
y	5	2.25	0	−1.75	−3	−3.75	−4

d

x	−3	−2	−1	0	1	2	3
y	−2	0	2	4	6	8	12

3C **9** Sketch a graph of each of the following linear relationships by first completing a table of values for x from −3 to 3.

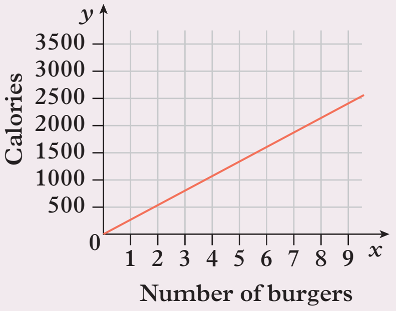
a $y = 2x - 5$ **b** $y = 5 - x$ **c** $y = -3x + 1$ **d** $y = -2x - 3$

3C **10** Identify the independent and dependent variables in the following relationships.

a

Length (cm)	5	10	15	20
Volume (cm^3)	125	1000	3375	8000

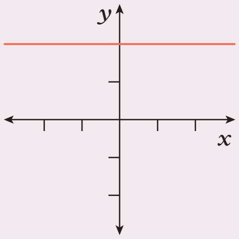
b



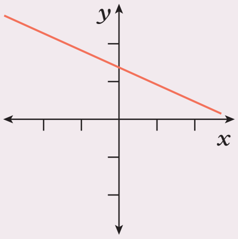
c Time spent playing a video game and the number of achievements unlocked

3D **11** State whether the following lines have a gradient that is positive, negative, zero or undefined.

a



b



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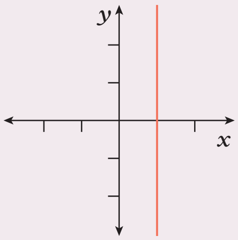
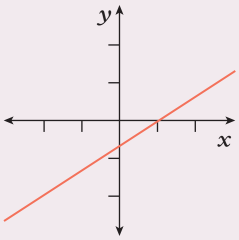


d



3D **12** Determine the gradients of the lines that pass through the following pairs of points.

a $(1, 5)$ and $(3, 11)$

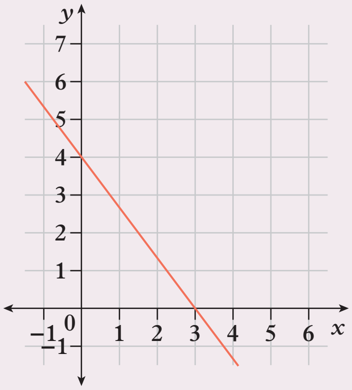
b $(-1, 3)$ and $(2, -6)$

c $(0, 5)$ and $(3, 0)$

d $(-6, -2)$ and $(-1, -8)$

3D **13** State the x- and y-intercepts of the following lines as coordinates.

a



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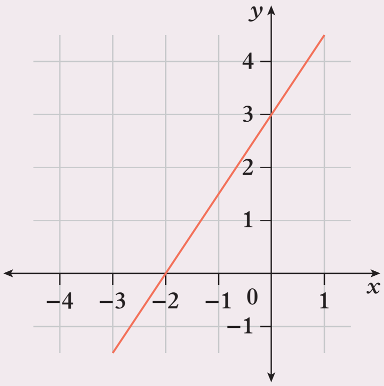

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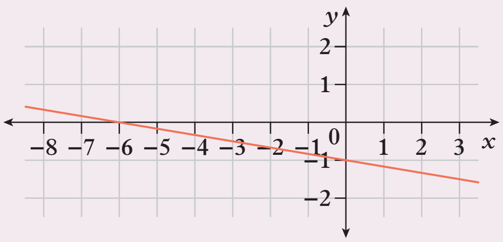


3E Sketching linear graphs using intercepts

Learning intentions

By the end of this topic you will be able to ...

✔ determine the *x*- and *y*-intercepts of a linear graph from its equation

✔ sketch linear graphs with two intercepts using the *x*- and *y*-intercepts

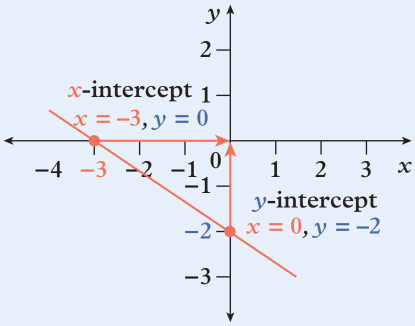
✔ sketch linear graphs with one intercept.

Inter-year links

Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 8	6D Plotting linear relationships
Year 10	3B Sketching linear graphs

Sketching linear graphs with two intercepts

- A minimum of two coordinate points are required to sketch a linear graph.
- The *x*-intercept is the point where the linear graph crosses the *x*-axis and $y = 0$.
- The *y*-intercept is the point where the linear graph crosses the *y*-axis and $x = 0$.
- Sketch a linear graph with two intercepts by first finding the *x*- and *y*-intercepts:
  1. Determine the *x*-intercept by substituting $y = 0$ into the equation of the line and solving for *x*.
  2. Determine the *y*-intercept by substituting $x = 0$ into the equation of the line and solving for *y*.
  3. Plot and label the *x*- and *y*-intercepts on the Cartesian plane.
  4. Draw a straight line through the two points.



For example, to sketch the graph of $y = 2x + 3$:

For *x*-intercept, let $y = 0$:

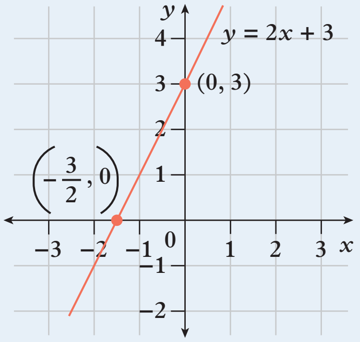
$$0 = 2x + 3$$
$$-3 = 2x$$
$$x = -\frac{3}{2}$$

x-intercept: $\left(-\frac{3}{2}, 0\right)$

For *y*-intercept, let $x = 0$:

$$y = 2 \times 0 + 3$$
$$y = 0 + 3$$
$$y = 3$$

y-intercept: (0, 3)



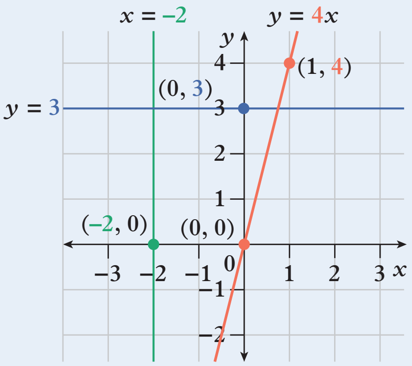
Sketching linear graphs with one intercept

- There are three cases in which a linear graph has only one intercept:

Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	*x*-intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	*y*-intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: (0, 0)	m

where a, b and m are constants.

For example, the graphs of $x = -2$, $y = 3$ and $y = 4x$ are shown on the Cartesian plane below.



- To sketch a linear graph that passes through the origin $(0, 0)$, determine a second point on the graph by substituting any value of x into the equation and then solve for y.
- For example, in the relationship $y = 4x$, $y = 4 \times 1 = 4$ for $x = 1$. Therefore $(1, 4)$ is another point on $y = 4x$.

Example 3E.1 Calculating x- and y-intercepts

Determine the coordinates of the x- and y-intercepts of the graphs of the linear relationships:

a $x + 5y = 10$

b $y = 3x - 4$

THINK	**WRITE**
a **1** To determine the x-coordinate of the x-intercept, substitute $y = 0$ into the equation and solve for x. The coordinates of the x-intercept have the form $(x, 0)$.	**a** For x-intercept, let $y = 0$: $x + 5(0) = 10$ $x + 0 = 10$ $x = 10$ x-intercept: $(10, 0)$
2 To determine the y-coordinate of the y-intercept, substitute $x = 0$ into the equation and solve for y. The coordinates of the y-intercept have the form $(0, y)$.	For y-intercept, let $x = 0$: $0 + 5y = 10$ $5y = 10 \quad (\div 5)$ $y = 2$ y-intercept: $(0, 2)$
b **1** To determine the x-coordinate of the x-intercept, substitute $y = 0$ into the equation and solve for x. The coordinates of the x-intercept have the form $(x, 0)$.	**b** For x-intercept, let $y = 0$: $0 = 3x - 4 \quad (+4)$ $4 = 3x \quad (\div 3)$ $x = \frac{4}{3}$ x-intercept: $\left(\frac{4}{3}, 0\right)$
2 To determine the y-coordinate of the y-intercept, substitute $x = 0$ into the equation and solve for y. The coordinates of the y-intercept have the form $(0, y)$.	For y-intercept, let $x = 0$: $y = 3(0) - 4$ $= 0 - 4$ $= -4$ y-intercept: $(0, -4)$

Example 3E.2 Sketching linear graphs with two intercepts

Sketch a graph of $4x - y = 8$ by first finding the x- and y-intercepts.

THINK

1 Determine the x-intercept by substituting $y = 0$ into the equation and solving for x. Determine the y-intercept by substituting $x = 0$ into the equation and solving for y.

2 Plot and label the x- and y-intercepts on the Cartesian plane.

3 Rule a straight line through the points. Label the graph with its equation.

WRITE

For x-intercept, let $y = 0$:

$4x - 0 = 8$

$4x = 8 \quad (\div 4)$

$x = 2$

x-intercept: $(2, 0)$

For y-intercept, let $x = 0$:

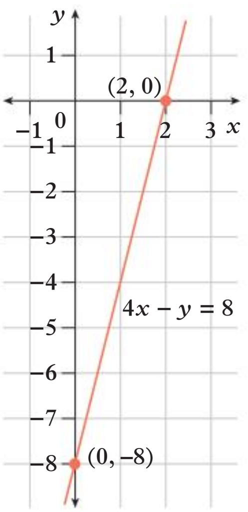
$4(0) - y = 8$

$0 - y = 8$

$-y = 8 \quad (\div -1)$

$y = -8$

y-intercept: $(0, -8)$



Example 3E.3 Sketching vertical and horizontal lines

Sketch a graph of the following linear relationships.

a $x = 4$

b $y = -2$

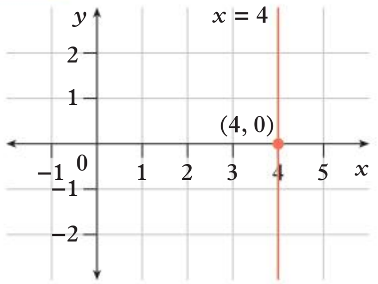
THINK

a The graph of $x = 4$ is a vertical line that passes through the point $(4, 0)$. Rule a vertical line through $(4, 0)$ and label the x-intercept. Label the graph with its equation.

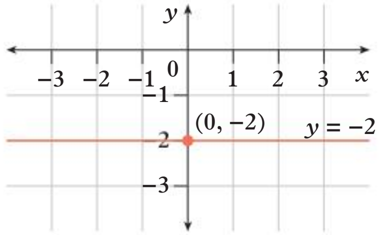
b The graph of $y = -2$ is a horizontal line that passes through the point $(0, -2)$. Rule a horizontal line through $(0, -2)$ and label the y-intercept. Label the graph with its equation.

WRITE

a

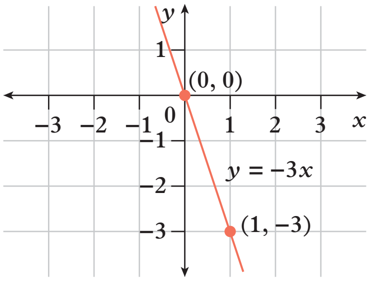


b



Example 3E.4 Sketching linear graphs that pass through the origin

Sketch the graph of $y = -3x$.

THINK	**WRITE**
1 The x- and y-intercepts are both (0, 0) so the graph passes through the origin (0,0). Determine a second point on the graph by substituting any value of x into the equation and solving for y.	For x-intercept, let $y = 0$: $0 = -3x \ (\div -3)$ $x = 0$ x- and y-intercept: (0, 0) For a second point, let $x = 1$: $y = -3(1)$ $= -3$ Second point: (1, −3)
2 Plot and label the two points on the Cartesian plane. 3 Rule a straight line through the points and label the graph with its equation.	

Helpful hints

- ✔ Find the x- and y-intercepts before you start sketching so you can plan the scale on the axes of your Cartesian plane.
- ✔ Always label your graph with the following information:
  - → the equation of the graph
  - → any x- and y-intercepts.

ANS p426

Exercise 3E Sketching linear graphs using intercepts

△ 1–7, 9, 11, 13, 14 □ 3–8, 10, 12–14, 16(a, b) ⬠ 3, 6, 7, 12–17

3E.1 **1** Determine the coordinates of the x- and y-intercepts of the graphs of the following linear relationships.

a $x + 4y = 12$
b $y = x + 4$
c $2x + y = 6$
d $y = x - 5$
e $y = -2x + 8$
f $y = 3x - 6$
g $5x + y = -10$
h $y = -x + 7$
i $y = 4 - x$

2 Sketch the linear graphs that have the following x- and y-intercepts.

a x-intercept: $(2,0)$, y-intercept: $(0,1)$
b x-intercept: $(-3,0)$, y-intercept: $(0,5)$
c x-intercept: $(-1,0)$, y-intercept: $(0,-1)$
d x-intercept: $\left(\frac{1}{2},0\right)$, y-intercept: $\left(0,-\frac{5}{2}\right)$

UNDERSTANDING AND FLUENCY

3E.2 **3** Sketch a graph of each of the following linear relationships by first finding the x- and y-intercepts.

a $4x + y = 4$ **b** $y = x - 2$ **c** $-2x + 3y = 6$

d $y = -3x + 3$ **e** $5x + y = -5$ **f** $y = 2 - x$

g $2y = 2 - 4x$ **h** $3y - 9x + 12 = 0$ **i** $4x - 3y - 8 = 0$

3E.3 **4** Sketch a graph of the following linear relationships.

a $x = 1$ **b** $y = 4$ **c** $x = -3$ **d** $y = 0$

3E.4 **5** Sketch a graph of the following linear relationships.

a $y = 3x$ **b** $y = -2x$ **c** $y = 6x$ **d** $y = -x$

6 Use the most appropriate method to sketch the graph of each linear relationship.

a $2x - 5y = 10$ **b** $y = 4x + 2$ **c** $x + y = 6$

d $y = -3x$ **e** $y = 7$ **f** $y = 6 - 3x$

g $x = 1$ **h** $y = 4(x - 1)$ **i** $x + 3y - 9 = 0$

j $y = x$ **k** $y = -3x + 5$ **l** $y = -7$

PROBLEM SOLVING AND REASONING

7 Without sketching, determine how many axis intercepts the graph of each relationship has.

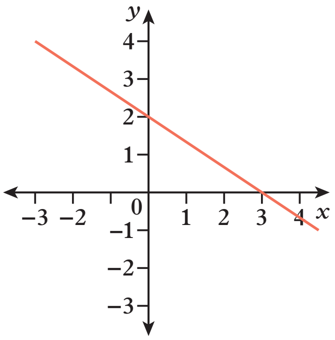
a $y = 10$ **b** $y = 2x + 1$ **c** $x = -12$

d $y = 15x$ **e** $5x - y = 25$ **f** $x + y = 1$

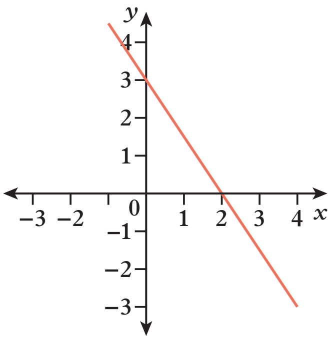
g $y = x$ **h** $x = 1$ **i** $-2y = -7x$

8 Which of these three graphs is a correct sketch of $3x + 2y = 6$? Identify the errors in the other two options.

A



B



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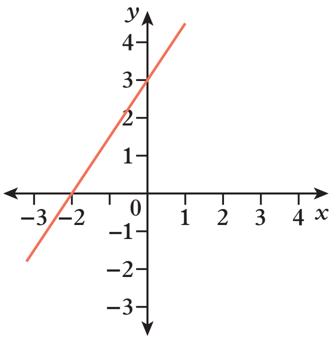


9 Decide whether each statement about the graph of $y = 3x + 6$ is true or false. Correct each false statement.

a The y-intercept is 6. **b** The x-intercept is 2.

c The relationship is linear. **d** The point (1, 9) lies on the line.

e The line passes through the origin. **f** The gradient of the line is 3.

10 Decide whether each statement about the graph of $y = -4x$ is true or false. Correct each false statement.

a The x-intercept is −4. **b** The y-intercept is 0.

c The line passes through the origin. **d** The point (1, 4) lies on the line.

e The gradient is negative. **f** The gradient of the line is $-\frac{1}{4}$.

11 **a** Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x + 3y = 18$, $2x + 3y = 0$ and $2x + 3y = -12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

12 **a** Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x - 3y = 12$, $-2x + 3y = 12$ and $-2x - 3y = 12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

13 Determine the gradient of the following lines by finding and using the x- and y-intercepts.

a $3x - 5y = 30$ **b** $7x + 6y + 9 = 0$ **c** $10y = 3 - 5x$ **d** $8x = 4y - 3$

PROBLEM SOLVING AND REASONING

14 Tony is buying a skateboard on a purchase plan where he makes a regular payment each week. He creates the formula $y = 300 - 25x$ to describe the relationship between the amount still owed in dollars (y) after a number of weeks (x).

- **a** Explain why Tony has chosen to solve for y (amount still owed in dollars) in terms of x (number of weeks), rather than the other way around. Hint: Think about the relationship between the variables.
- **b** Sketch the graph of this relationship by first determining the coordinates of the x- and y-intercepts.
- **c** What does the y-intercept represent on this graph?
- **d** What does the x-intercept represent on this graph?
- **e** Describe the purchase plan Tony is using. When will he be able to bring his skateboard home?

15 A rainwater tank has a capacity of 1500 L and feeds a drip system to water the garden. At the beginning of April, the tank is full, but it is empty at the end of the last day of the month. Let x represent the number of days from the start of April and y represent the number of litres of water in the tank. Assume a constant rate of water use and no further rain during April.

- **a** Identify the independent variable and dependent variable in this situation. Explain your reasoning.
- **b** Explain why this relationship can be represented by a linear graph.
- **c** Determine the coordinates of the x-intercept for this relationship.
- **d** Determine the coordinates of the y-intercept for this relationship.
- **e** Use these intercepts to sketch a graph of the relationship.
- **f** Use the graph to estimate the number of litres of water in the tank at the end of the day on 10 April.
- **g** Use the graph to estimate when there is 600 L of water left in the tank.

CHALLENGE

16
- **a** Explain why the reciprocals of a and b are respectively the x- and y-intercepts of $ax + by = 1$.
- **b** Explain how you could find the x- and y-intercepts of linear equations in the form $ax + by = d$.
- **c** For the equation $ax + by = d$, write an expression for d in terms of a and b such that the x-intercept is equal to b and the y-intercept is equal to a.

17 Determine the axis intercepts of the following lines.

- **a** $3x - 5y = -7$
- **b** $\frac{2}{3}x + \frac{5}{2}y = \frac{4}{5}$
- **c** $\sqrt{2}x - \sqrt{3}y = \sqrt{6}$
- **d** $\sqrt{8}x + \sqrt{18}y = \sqrt{2}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Sketching linear relationships using intercepts

Investigation
Making a stained glass window

Topic quiz
3E

3F Determining linear equations

Learning intentions

By the end of this topic you will be able to ...

✔ determine the equation of a linear graph.

Inter-year links

Year 8	6F Finding linear equations
Year 10	3C Determining linear equations

Gradient-intercept form

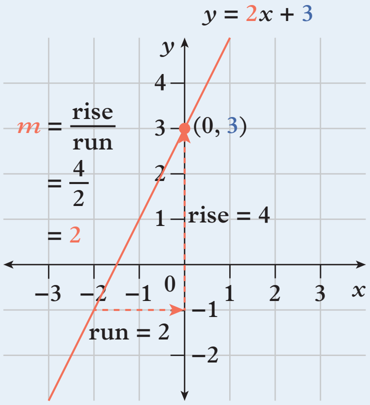
- The equation of a linear graph can be expressed in **gradient-intercept form**:

$$y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{y\text{-coordinate of }y\text{-intercept}}$$

→ m is the gradient of the line

→ c is the y-coordinate of the y-intercept

For example, the equation of the linear graph below is $y = 2x + 3$. It has a gradient of 2 and y-intercept at (0, 3) so $m = 2$ and $c = 3$.



Determining the equation of a linear graph

- To find the equation of a linear graph in the form of $y = mx + c$, determine the value of the:

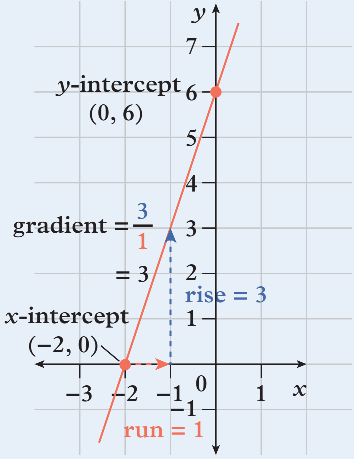
→ gradient, for any two points (x_1, y_1) and (x_2, y_2): $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

→ y-coordinate of the y-intercept: $(0, c)$

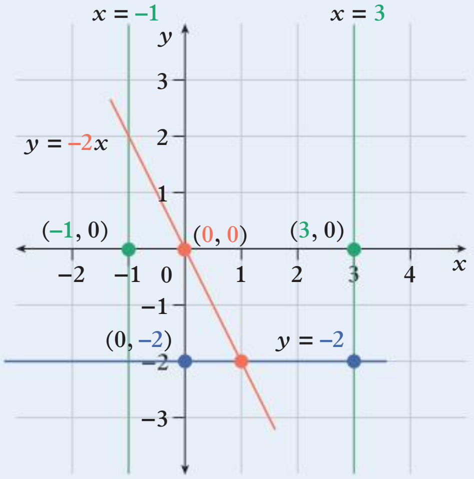
For example, this linear graph has a gradient of 3 and a y-intercept at (0, 6).

$m = 3$ and $c = 6$ so the equation of the graph is $y = 3x + 6$.

Note: The x-intercept at (–2, 0) is not used in this calculation.



- For lines with only one intercept, the equations can be determined using the following general equations where a, b and m are constants.
  - → A vertical line with an x-intercept at $(a, 0)$ is given by $x = a$.
    For example, the graph of the equation $x = -1$ is a vertical line with an x-intercept at $(-1, 0)$.
  - → A horizontal line with an y-intercept at $(0, b)$ is given by $y = b$.
    For example, the graph of the equation $y = -2$ is a horizontal line with a y-intercept at $(0, -2)$.
  - → A line that passes through the origin $(0, 0)$ with gradient m is given by $y = mx$.
    For example, the graph of the equation $y = -2x$ passes through the origin and has a gradient of -2.



Example 3F.1 Determining the gradient and y-intercept of a linear equation

Identify the gradient, m, and the coordinates of the y-intercept of the linear graphs with the following equations.

a $y = 3x - 8$

b $y = 4 - \frac{3x}{5}$

THINK	WRITE
a Compare the equation to the gradient-intercept form $y = mx + c$ to identify the values of m and c. The y-intercept is $(0, c)$.	**a** $y = 3x - 8$ $m = 3, c = -8$ y-intercept: $(0, -8)$
b Rearrange the equation so that it is in gradient-intercept form $y = mx + c$. Note that the coefficient of x is the gradient and the constant term is the y-coordinate of the y-intercept.	**b** $y = 4 - \frac{3x}{5}$ $y = -\frac{3}{5}x + 4$ $m = -\frac{3}{5}, c = 4$ y-intercept: $(0, 4)$

Example 3F.2 Determining the equation of a linear graph given the gradient and the y-intercept

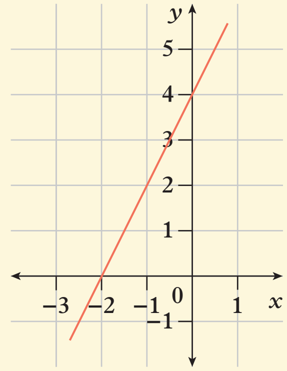
Determine the equation of a linear graph with a gradient of -3 and a y-intercept of $(0, 7)$.

THINK	WRITE
Identify the values of gradient m and the y-coordinate of the y-intercept c, and then substitute into the gradient-intercept form, $y = mx + c$.	$m = -3$ $c = 7$ $y = -3x + 7$ The y-intercept is $(0, c)$.

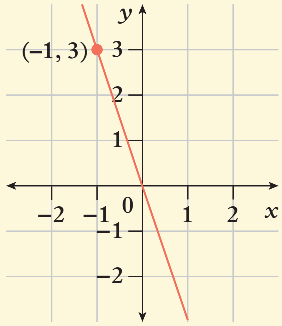
Example 3F.3 Determining the equation of a linear graph given the y-intercept and a second point

Determine the equation of each of the following linear graphs.

a



b



THINK

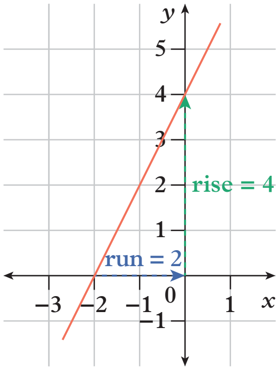
a **1** Determine the value of the gradient, m, by identifying the rise and run between any two integer coordinates on the graph.

WRITE

a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{4}{2}$

$= 2$



2 Determine the value of the constant, c, by identifying the y-coordinate of the y-intercept.

y-intercept: $(0, 4)$

3 Substitute the values of m and c into the general equation for a straight line.

$c = 4$

$y = mx + c$

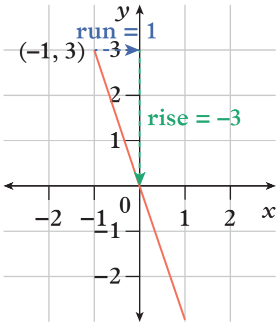
$y = 2x + 4$

b **1** Determine the value of the gradient, m, by identifying the rise and run between any two integer coordinates on the graph.

b $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{1}$

$= -3$



2 Determine the value of the constant c by identifying the y-coordinate of the y-intercept.

y-intercept: $(0, 0)$

3 Substitute the values of m and c into the general equation for a straight line.

$c = 0$

$y = mx + c$

$y = -3x + 0$

$y = -3x$

Example 3F.4 Determining the equation of a linear graph given two points

Find the equation of the linear graph that passes through the points (6, 5) and (2, −3).

THINK	WRITE
1 Calculate the value of the gradient, m.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - (-3)}{6 - 2}$ $= \frac{8}{4}$ $= 2$
2 Substitute the value of the gradient, m, into the general equation for a straight line, $y = mx + c$.	$y = 2x + c$
3 Select one of the two coordinates, substitute the values of x and y into the partially completed equation, and solve for c.	From the point (6, 5): Let $x = 6$ and $y = 5$: $5 = 2(6) + c$ $5 = 12 + c \quad (-12)$ $c = -7$
4 State the equation of the line.	$y = 2x - 7$

Helpful hints

✔ The different methods for finding the equation of a line can be overwhelming, so remember that you just need to find the gradient m and the y-intercept $(0, c)$, which you can then substitute into the formula for a linear graph, $y = mx + c$.

ANS p429

Exercise 3F Determining linear equations

△ 1(1st, 2nd columns), 2, 3, 4(a, c, e, g), 5, 6(a–e), 7(a, c, e), 8, 11(a, b), 13

■ 1(2nd, 3rd columns), 3(b, e, g, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 9, 10, 11(a, b, d), 13, 14(a, b)

⬠ 1(3rd, 4th columns), 3(f, h), 4(b, d, f, h), 5, 6(e–h), 7(b, d, f), 10, 11(c, d), 12–15

3F.1 **1** Identify the gradient, m, and the coordinates of the y-intercept of the linear graphs with the following equations.

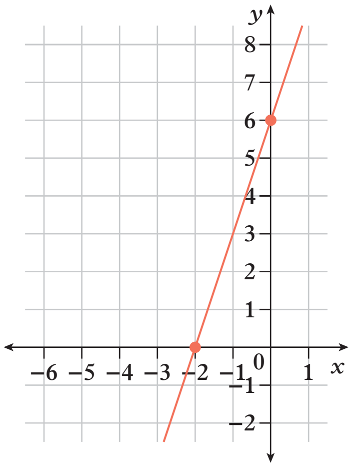
a $y = 2x + 5$
b $y = 4x + 1$
c $y = -3x + 7$
d $y = -5x - 3$
e $y = x - 6$
f $y = 1 - x$
g $y = \frac{4}{3}x + 2$
h $y = \frac{x}{2} - 8$
i $y = -\frac{4x}{3} + \frac{1}{4}$
j $y = 9$
k $y = \ -7x$
l $y = 5 - \frac{2}{5}x$

3F.2 **2** Determine the equations of the lines in the following graphs in the form $y = mx + c$.

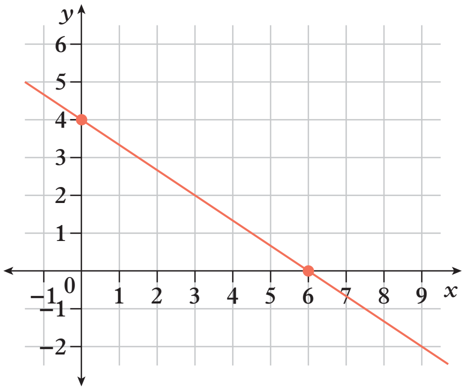
a gradient: 3, y-intercept: (0,4)
b gradient: −2, y-intercept: (0,10)
c gradient: 1, y-intercept: (0,−7)
d gradient: −12, y-intercept: (0, −1)
e gradient: −1, y-intercept: (0,20)
f gradient: 5, y-intercept: (0,0)
g gradient: $\frac{1}{3}$, y-intercept: $\left(0, \frac{4}{5}\right)$
h gradient: $-\frac{3}{4}$, y-intercept: (0,0)

3F.3 **3** Determine the equation of each of the following linear graphs.

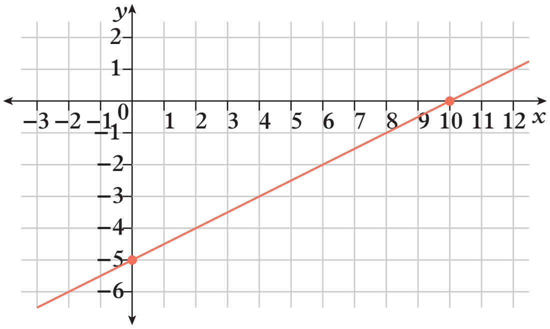
a



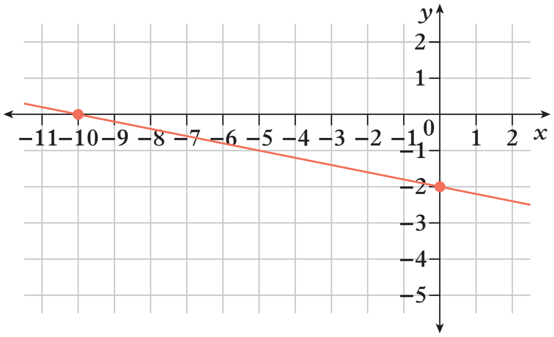
b



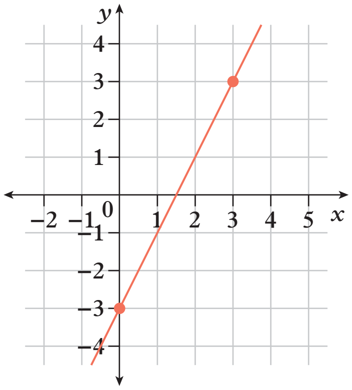
c



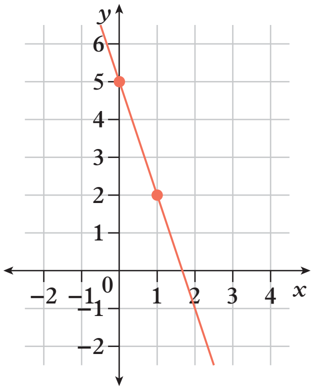
d



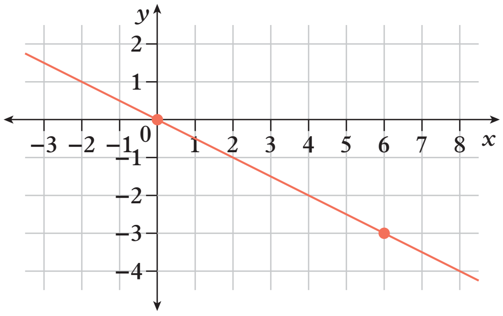
e



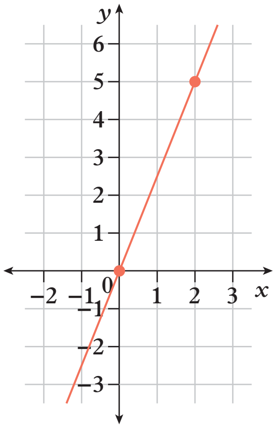
f



g



h

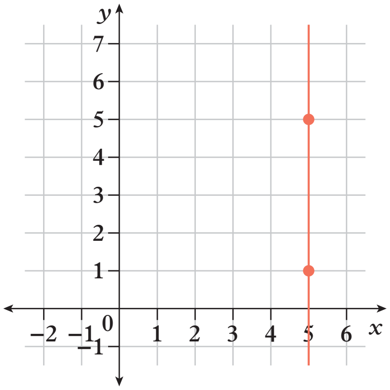


3F.4 **4** Find the equations of the linear graphs that pass through each pair of points.

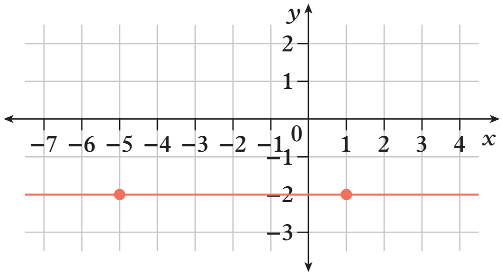
a	$(1,-20)$ and $(11,0)$	**b**	$(2,4)$ and $(3,2)$
c	$(-1,6)$ and $(3,2)$	**d**	$(-4,-12)$ and $(-2,-6)$
e	$(2,5)$ and $(1,3)$	**f**	$(9,-4)$ and $(11,6)$
g	$(-1,-2)$ and $(5,-5)$	**h**	$(6,-3)$ and $(9,1)$

5 Find the equation of the linear graphs below. Recall that the general equation for a vertical line is $x = a$ and the general equation for a horizontal line is $y = b$, where a and b are constants.

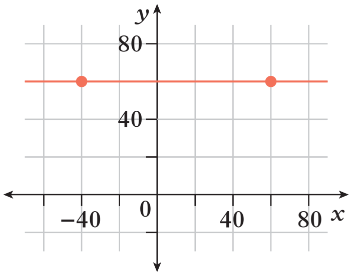
a



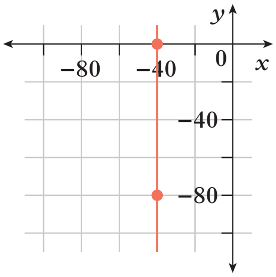
b



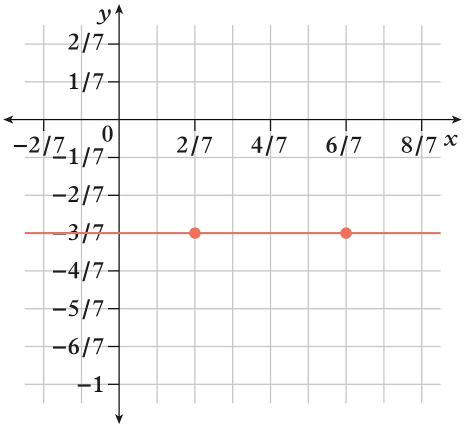
c



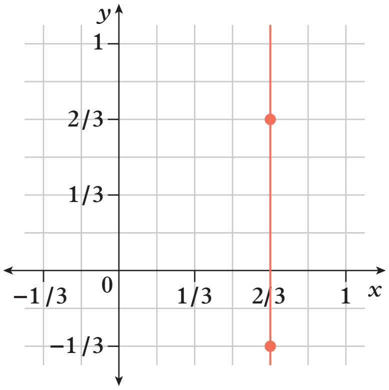
d



e



f



6 Determine the equations of the linear graphs that have the following intercepts.

a x-intercept: $(4,0)$, y-intercept: $(0,12)$

b x-intercept: $(-5,0)$, y-intercept: $(0,10)$

c x-intercept: $(3,0)$, y-intercept: $(0,-21)$

d x-intercept: $(-6,0)$, y-intercept: $(0,-24)$

e x-intercept: $(-1,0)$, y-intercept: $(0,10)$

f x-intercept: $(18,0)$, y-intercept: $(0,6)$

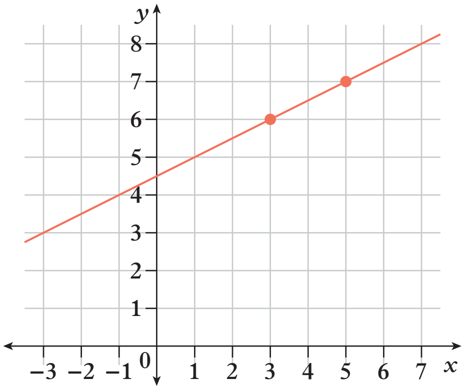
g x-intercept: $(5,0)$, y-intercept: $(0,-2)$

h x-intercept: $(-6,0)$, y-intercept: $(0,9)$

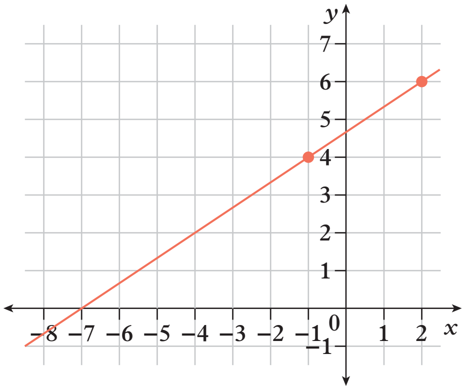
UNDERSTANDING AND FLUENCY

7 Identify two integer coordinates on each linear graph and use these to determine their equations.

a



b



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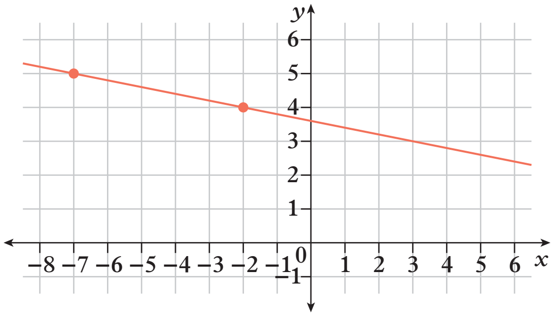

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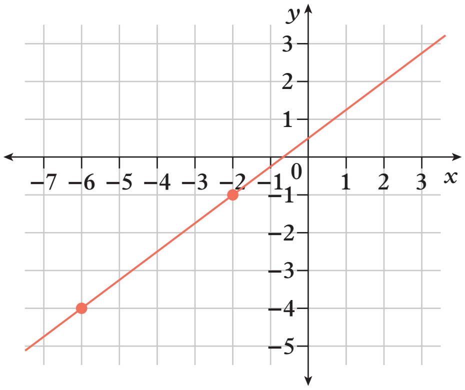
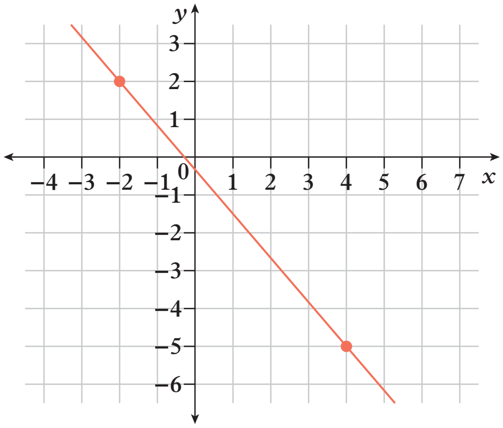


e



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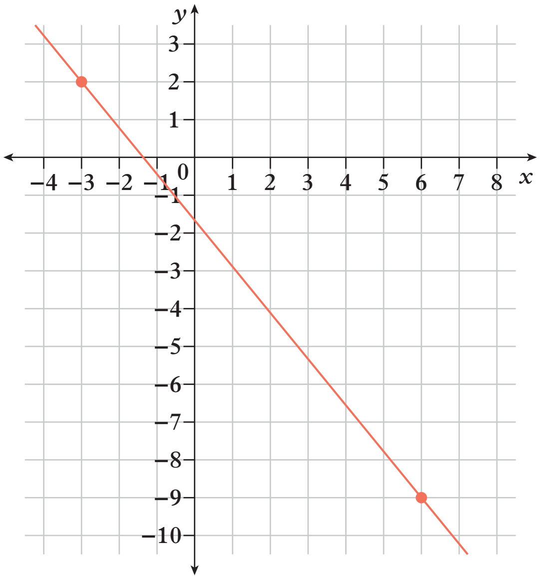


8 Azami invests some money into a simple interest account. After 2 years the account has \$2500 and after 5 years the account has \$3400.

a Identify the independent variable and the dependent variable.

b Determine the equation that gives the amount in Azami's account, \$$A$, after n years.

9 Roland recently started a business and realised his profts were increasing at a constant rate. After 7 days of being open, he had made a profit of \$450, and after 14 days, he had made \$870 in profit.

a Write a linear equation that gives the amount of profit, \$$P$, that Roland earns after n days.

Being overly optimistic about his launch, Roland uses his model to predict the profit he will earn 1 year (365 days) after opening.

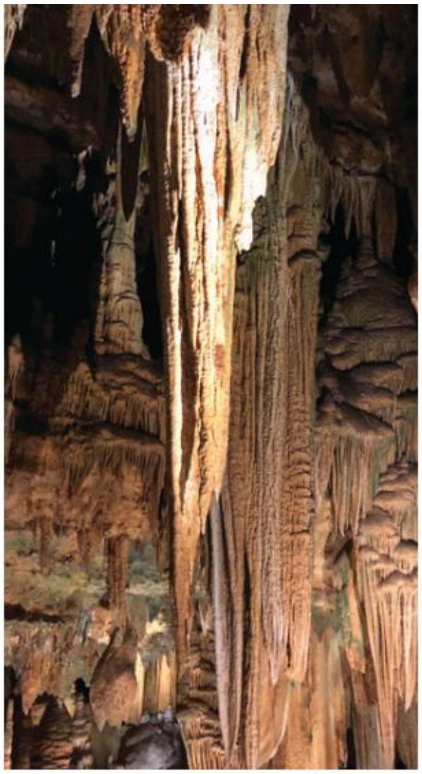
b What is the predicted profit Roland will have earned exactly one year (365 days) after opening?

10 As water drips from the ceiling of a cave it creates a long crystalline formation that hangs from the ceiling called a stalactite. A specific stalactite is measured to be 85 centimetres long in 2010 and then 10 years later it is measured to be 87 centimetres long.

a Identify the independent variable and dependent variable.

b Determine a linear equation for the growth of the stalactite, g, in centimeters n years after 2010.

c Use your equation to estimate the age of the stalactite in 2010. What year did it begin to form?



11 Determine the equation of the lines with the following tables of values.

a

x	−3	−2	−1	0	1	2	3
y	−32	−23	−14	−5	4	13	22

b

x	−3	−1	1	3	5	7	9
y	−2	1	4	7	10	13	16

c

x	10	20	30	40	50	60	70
y	48	51	54	57	60	63	66

d

x	1	4	9	16	25	36	49
y	28	22	12	−2	−20	−42	−68

12 A carpenter sells his wares based on the cost of the materials and the time spent crafting. A particular collection all cost the same in materials but take different amounts of time to craft. A piece that takes 16 hours to craft costs \$845 and a piece that takes 28 hours to craft costs \$1385. The carpenter then marks up the total cost by 120% to sell.

a Write the equation that gives the total cost, \$$C$, for a piece in this collection that takes n hours to craft.

b Write the equation that gives the revenue, \$$R$, for a piece in this collection that takes n hours to craft.

c Write the equation that gives the profit, \$$P$, for a piece in this collection that takes n hours to craft.

13 **a** Factorise the right-hand side of each of the following equations.

b Find the x-intercept of each of the following equations.

c Describe the connection between the factor form in part **a** and the x-intercept in part **b**.

i $y = 3x + 12$ **ii** $y = -5x - 15$

iii $y = 2x - 10$ **iv** $y = 6x - 8$

v $y = 21 - 7x$ **vi** $y = 16x - 40$

14 Find the equations of the linear graphs that pass through each pair of points.

a $\left(\frac{2}{3}, 0\right)$ and $\left(0, \frac{7}{5}\right)$

b $\left(-\frac{2}{5}, \frac{3}{4}\right)$ and $\left(\frac{1}{5}, -\frac{1}{4}\right)$

c $\left(-\frac{9}{2}, -\frac{7}{4}\right)$ and $\left(-\frac{9}{8}, -\frac{5}{6}\right)$

15 The product of the gradient, m, and y-intercept, c, for a particular line is 99. The sum of the gradient and y-intercept is 20.

a Write the two possible equations for the line.

b Find the coordinates of the point where the graphs of the two equations intersect.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Determining linear equations

Investigation
The human body

Investigation
Parallel and perpendicular lines

Topic quiz
3F

3G Midpoint and length of a line segment

Learning intentions

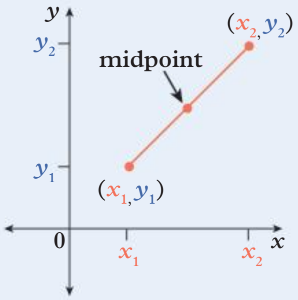
By the end of this topic you will be able to ...

- ✔ determine the midpoint of a line segment on the Cartesian plane
- ✔ calculate the length of a line segment on the Cartesian plane.

Inter-year links

Year 7	1G Exponents and square roots
Year 8	7C Pythagoras' theorem

Midpoint of a line segment



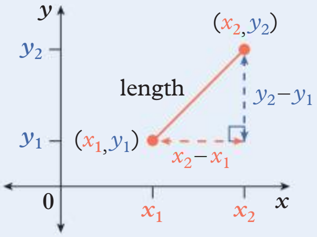
- The **midpoint** of a line segment is the point located exactly halfway between the endpoints of the line segment. If the coordinates of two endpoints of a line segment are (x_1, y_1) and (x_2, y_2), then:

  $$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

  → The *x*-coordinate of the midpoint is the average of the *x*-coordinates of the endpoints.

  → The *y*-coordinate of the midpoint is the average of the *y*-coordinates of the endpoints.

Length of a line segment



- The formula for the length of a line segment can be determined using Pythagoras' theorem, which is covered in Chapter 6. If the coordinates of two endpoints are (x_1, y_1) and (x_2, y_2), then the length of the line segment is:

  $$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 3G.1 Determining the midpoint of a line segment

Find the coordinates of the midpoint of the line segment joining (−1, 4) and (7, 9).

THINK	WRITE
1 Calculate the *x*-coordinate of the midpoint by averaging the *x*-coordinates of the endpoints.	*x*-coordinate of the midpoint $= \frac{-1 + 7}{2} = 3$
2 Calculate the *y*-coordinate of the midpoint by averaging the *y*-coordinates of the endpoints.	*y*-coordinate of the midpoint $= \frac{4 + 9}{2} = \frac{13}{2}$
3 Write the coordinates of the midpoint.	Coordinates of the midpoint: $\left(3, \frac{13}{2}\right)$

Example 3G.2 Calculating the length of a line segment

Calculate the length of the line segment joining (−1, 4) and (7, 9), correct to one decimal place.

THINK	**WRITE**
1 Define the points (x_1, y_1) and (x_2, y_2). The order of the coordinates does not affect the result.	Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (7, 9)$ length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2 Substitute the x- and y-coordinates into the formula for the length of a line segment.	$= \sqrt{(7 - (-1))^2 + (9 - 4)^2}$ $= \sqrt{8^2 + 5^2}$
3 Use a calculator to evaluate the root and round the answer to one decimal place as specified by the question.	$= \sqrt{64 + 25}$ $= \sqrt{89}$ ≈ 9.4 units

Helpful hints

- ✔ Take care when substituting negative values into the formula for the length of a line segment – watch carefully for any changes in sign!
- ✔ If you are already familiar with Pythagoras' theorem, it may help to draw the line segment as the longest side of a right-angled triangle and identify the lengths of the two shorter sides.

ANS p430

Exercise 3G Midpoint and length of a line segment

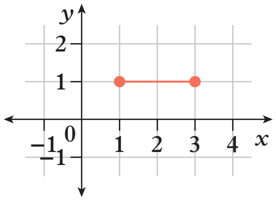
△ 1, 2–3(1st, 2nd columns), 4–7, 9, 11

■ 2–3(2nd, 3rd columns), 4, 8–10, 12, 13(a), 15

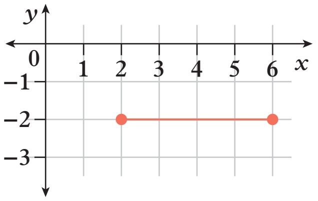
⬟ 2–3(3rd column), 4, 8, 10, 12, 13(b), 14, 16–18

1 State the coordinates of the midpoint of the following line segments.

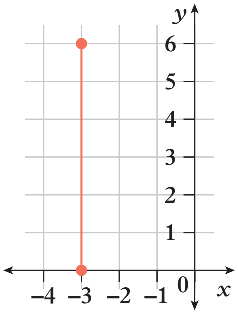
a



b



c



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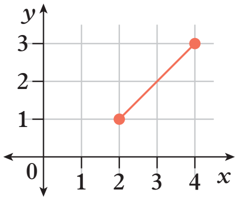


3G.1 **2** Find the coordinates of the midpoint of the line segment joining each pair of points.

a (1, 4) and (3, 10)
b (2, 5) and (8, 3)
c (1, 0) and (5, 2)
d (2 , 6) and (2 , 10)
e (0, 5) and (8, 9)
f (3, −4) and (7, 6)
g (2, −1) and (6, 7)
h (−3, −4) and (5, −4)
i (−4, −2) and (−2, 2)
j (3, 9) and (4, 8)
k (5, 0) and (8, 11)
l (−5, 7) and (5, −7)

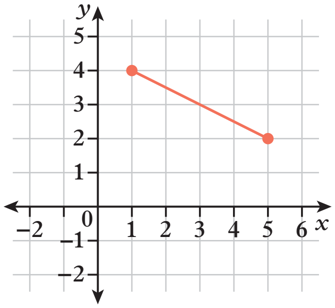
3G.2 **3** Calculate the length of the line segment joining each pair of points correct to one decimal place.

a (2, 5) and (3, 7)
b (3, 4) and (5, 8)
c (6, 2) and (9, 3)
d (−4, 5) and (−4, 9)
e (2, −4) and (4, 2)
f (5, 0) and (8, −4)
g (0, −1) and (1, −2)
h (7 , 8) and (−7 , 8)
i (4, −3) and (6, 0)
j (−3, 6) and (−2, 2)
k (−5, −4) and (−1, −2)
l (6, −5) and (−6 , 5)

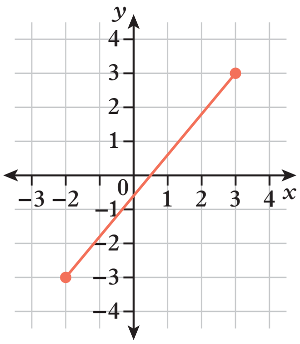
4 For each line segment, find:

i the midpoint
ii the length (to one decimal place)
iii the gradient.

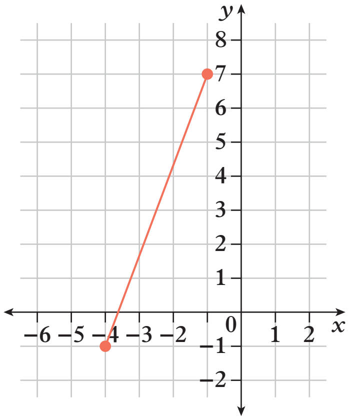
a



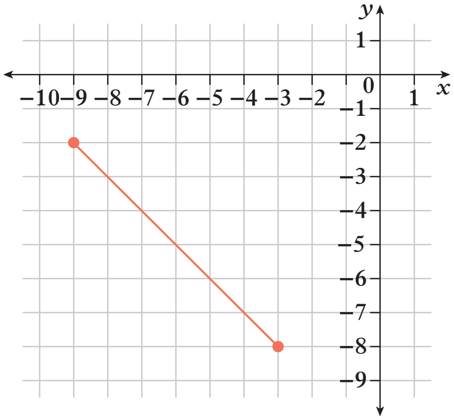
b



c



d



5 Calculate the distance between the midpoint and one of the endpoints of each of the line segments in question **3**.

6 The midpoint of a line segment *AB* has the coordinates (6, 4). If point *A* has the coordinates (2, 3), find the coordinates of point *B*.

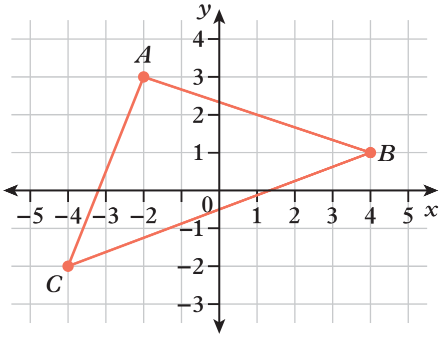
7 The midpoint of a line segment *CD* has the coordinates (−5, 1). If point *D* has the coordinates (4, −7), find the coordinates of point *C*.

8 A point, A, on a circle has the coordinates $(-1, -1)$.

a If the centre of the circle is at $(2, 3)$, calculate the radius of the circle.

b Identify the coordinates of another point on the circle that forms a diameter with point A.

9 Calculate the perimeter of ΔABC correct to one decimal place.



10 Calculate the perimeter of each shape correct to one decimal place.

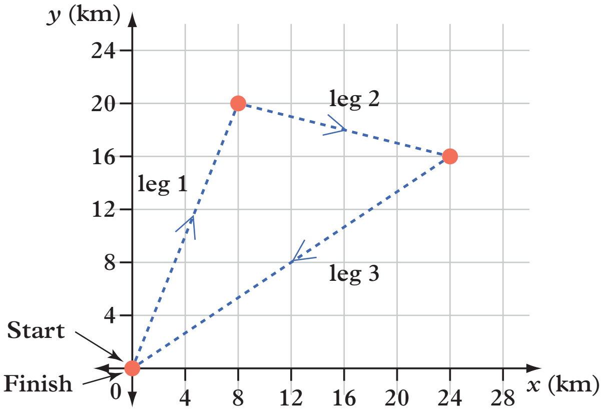
a Triangle with vertices at $(-3, -2)$, $(-2, 4)$ and $(4, 2)$

b Square with vertices at $(-1, 2)$, $(2, 5)$, $(5, 2)$ and $(2, -1)$

c Rectangle with vertices at $(-4, -2)$, $(2, 4)$, $(4, 2)$ and $(-2, -4)$

d Trapezium with vertices at $(-3, 3)$, $(1, 5)$, $(3, 3)$ and $(2, -2)$

11 A yacht race follows a triangular course that has been mapped onto the Cartesian plane. The scales on the axes represent distances in kilometres. The race begins and ends at the origin.



a Calculate the length of each leg of the race correct to one decimal place.

b Calculate the total distance covered during the race correct to one decimal place.

c An observer's boat is located close to the midpoint of the second leg of the race. Determine the distance between the observer's boat and the finishing point, correct to one decimal place.

12 Consider this parallelogram drawn on the Cartesian plane.

a List the coordinates of the vertices of the parallelogram.

b Find the coordinates of the midpoint of:

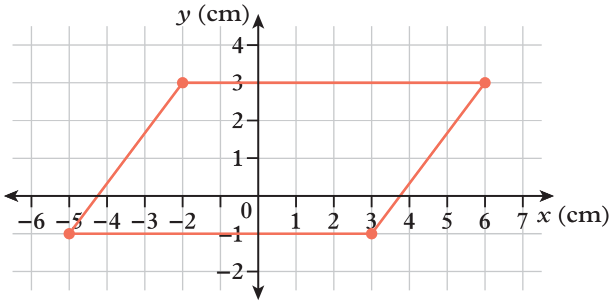
  i the longer diagonal

  ii the shorter diagonal.

c What do you notice about your answers to part **b**?

d Calculate the perimeter of the parallelogram.

e Find the difference in length of the two diagonals correct to one decimal place.



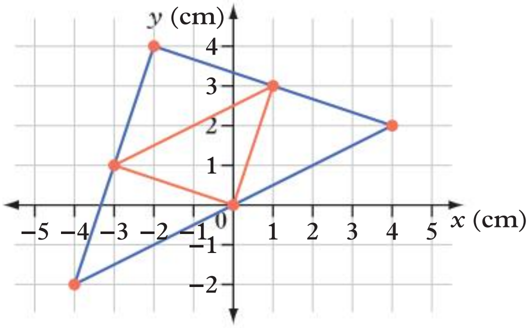
PROBLEM SOLVING AND REASONING

13 A shape (in blue) is drawn on the Cartesian plane. A smaller shape (in orange) is then formed by joining the midpoints of the vertices of the original shape.

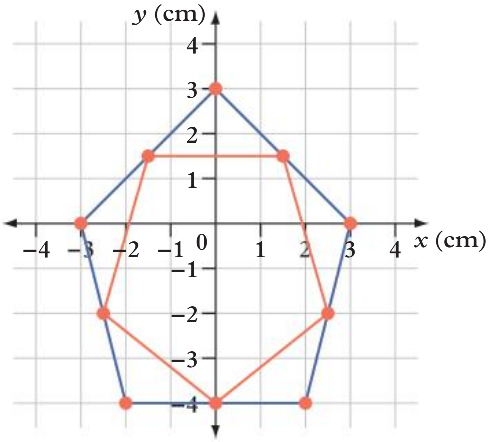
In each case, find the perimeter (correct to one decimal place) of:

- **i** the blue shape
- **ii** the orange shape.
- **iii** Compare the perimeter of the orange shape to the perimeter of the blue shape.

a



b



14 A quadrilateral $ABCD$ has vertices at $A(-4, 2)$, $B(-1, 4)$, $C(3, -2)$ and $D(0, -4)$. The scales on the axes represent distances in metres.

- **a** Determine the length of each side of the quadrilateral correct to one decimal place.
- **b** Determine the coordinates of the midpoint of each diagonal.
- **c** Determine the distance between each vertex and the midpoint correct to one decimal place.
- **d** Use your answers for parts **a–c** to identify the shape of the quadrilateral $ABCD$. Explain your reasoning.
- **e** Hence use the correct formula to determine the area of the quadrilateral, correct to one decimal place.

CHALLENGE

15 a Prove that triangle ABC with vertices at $A(3, 6)$, $B(-1, -2)$ and $C(-5, 2)$ is an isosceles triangle. State any dimensions in simplified surd form.

b Calculate the area of the triangle using your knowledge of the midpoint and length of a line segment.

16 Prove that quadrilateral $ABCD$ with vertices at $A(-5, 3)$, $B(-1, 5)$, $C(3, -2)$ and $D(-1, -4)$ is a parallelogram. State any dimensions in simplified surd form.

17 A line segment has endpoints at $A(-1, -1)$ and $B(2, 5)$. If point C lies between A and B such that AC is $\frac{1}{3}$ the length of AB, determine the exact coordinates of C.

18 A line segment has a midpoint at the origin, a length of 10 units and a gradient of 2. Determine the coordinates of the two endpoints. Write your answer in simplified surd form. Hint: Sketch a diagram and then consider only one half of the line segment.

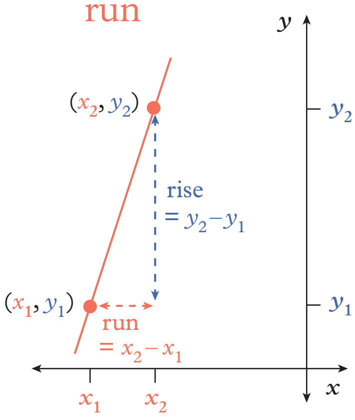
Check your Student obook pro for these digital resources and more:

Interactive skillsheet Midpoint of a line segment	Interactive skillsheet Length of a line segment	Investigation From square to triangle	Topic quiz 3G

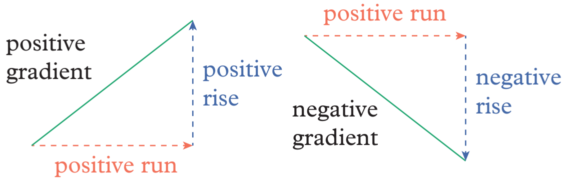
Chapter summary

Gradient

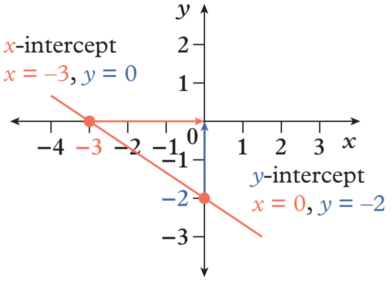
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$



$$m = \frac{\overset{\text{point 2}}{y_2} - \overset{\text{point 1}}{y_1}}{x_2 - x_1}$$



Intercepts

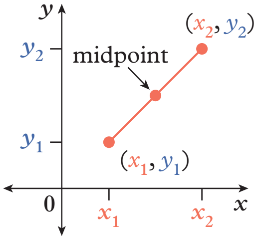


Gradient-intercept form

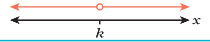
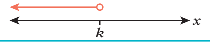
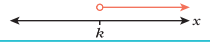
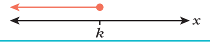
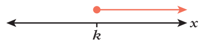
$$y = \underset{\text{gradient}}{m}x + \underset{\text{y-coordinate of y-intercept}}{c}$$

Midpoint

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

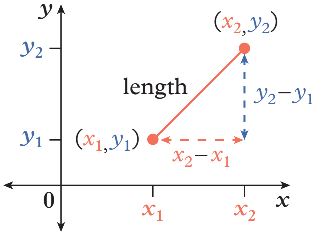


Linear inequalities

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k

Length

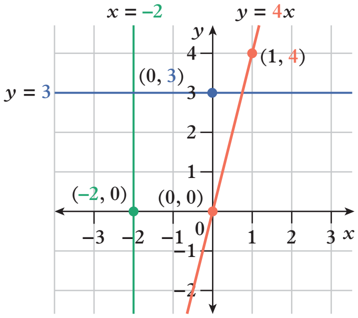
$$\text{Length} - \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$



Linear graphs with one intercept

Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	x-intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	x-intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: $(0, 0)$	m

where a, b and m are constants.



Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

3A **1** The solution to the equation $3x - 9 = 12$ is:

A $x = 1$ **B** $x = 7$ **C** $x = 12$ **D** $x = 13$ **E** $x = 63$

3B **2** Which of these statements is incorrect?

A $-7 < -4$ **B** $\frac{2}{3} < \frac{3}{4}$ **C** $-3.5 < -4.7$ **D** $\frac{8}{9} < \frac{9}{8}$ **E** $\frac{3}{4} \neq \frac{8}{16}$

3B **3** If $x \geq -6.2$, which of these could *not* be a value for x?

A 0 **B** -6.2 **C** 6.2 **D** -6.3 **E** -4.9

3B **4** The value of x in the equation $5x - 2 = 3x + 7$ is:

A $\frac{5}{8}$ **B** $1\frac{1}{8}$ **C** $2\frac{1}{2}$ **D** $4\frac{1}{2}$ **E** $5\frac{1}{2}$

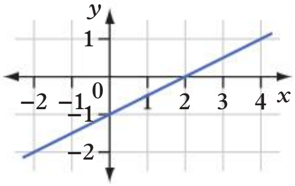
3C **5** Which point does *not* lie on the graph of $y = 2x - 4$?

A $(0, -4)$ **B** $(1, -2)$ **C** $(4, 4)$ **D** $(-1, -2)$ **E** $(-2, -8)$

3D **6** The gradient of the linear graph shown to the right is:

A -2 **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$

D 1 **E** 2



3D **7** The gradient of the line segment joining the points $(-2, 4)$ and $(7, -4)$ is:

A $\frac{8}{9}$ **B** $-\frac{8}{9}$ **C** $\frac{9}{8}$ **D** $-\frac{9}{8}$ **E** 0

3D **8** A line with a gradient of 0:

A is vertical **B** is horizontal **C** is undefined

D increases from left to right **E** decreases from left to right.

3E **9** At the point where a line crosses the x-axis:

A $x = y$ **B** $y = -x$ **C** $y = 1$ **D** $x = 0$ **E** $y = 0$

3E **10** The y-intercept for the graph of $2y = 3x - 4$ is:

A $(0, -4)$ **B** $(0, -2)$ **C** $(0, 0)$ **D** $(0, 2)$ **E** $(0, 4)$

3F **11** In the general equation of a linear graph, $y = mx + c$, the pronumeral c represents the:

A y-intercept **B** x-intercept **C** rise **D** run **E** gradient

3F **12** A linear graph has a gradient of 3 and a y-intercept of $(0, -2)$. The equation of the graph is:

A $y = -2x + 3$ **B** $y = \frac{1}{3}x - 2$ **C** $y = 3x - 2$ **D** $y = -3x + 2$ **E** $3x - 2y = 1$

3G **13** The coordinates of the midpoint of the line segment joining $(3, 5)$ and $(7, 9)$ are:

A $(5, 7)$ **B** $(7, 5)$ **C** $(2, 2)$ **D** $(4, 8)$ **E** $(1, 2)$

3G **14** The length of the line segment joining $(3, 5)$ and $(7, 9)$ is closest to:

A 4 **B** 5 **C** 6 **D** 7 **E** 8

Short answer

3A **1** Solve each equation using inverse operations.

a $3x - 4 = 2$ **b** $2 = 4 - 3x$ **c** $\frac{x + 2}{3} = 4$ **d** $\frac{x}{3} - 2 = -1$

e $\frac{x - 2}{3} + 5 = 4$ **f** $\frac{4}{x} + 3 = -2$ **g** $4(x - 3) = 3$ **h** $2(3 - x) - 7 = -2$

3A **2** Solve each equation for x.

a $7x - 3 = 4x + 9$ **b** $x + 8 = 1 - 6x$

3A **3** One more than three-fifths of the class is 16 people.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation.

d How many people are in the class?

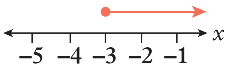
3B **4** For each inequality below, choose the x values from this list that make it a true statement:

-3.9 $\quad 1\frac{5}{8}$ $\quad 2.6$ $\quad 8.5$ $\quad -8.5$ $\quad \frac{3}{4}$

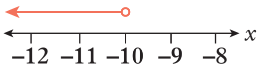
a $x > -1$ **b** $x \le 0.75$ **c** $x \ge 2.6$

3B **5** Write the inequality that is represented on each of these number lines.

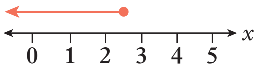
a



b



c



3B **6** Solve each of these inequalities.

a $5 - 2x < 7$ **b** $\frac{2 - 3x}{5} > 2$

3C **7** Complete a table of values from $x = -2$ to $x = 2$ for each linear relationship, and then plot the relationship between x and y.

a $y = x + 5$ **b** $y = x - 5$ **c** $y = -x + 5$ **d** $y = -x - 5$

3C **8** Identify the independent and dependent variables in the following relationships.

a

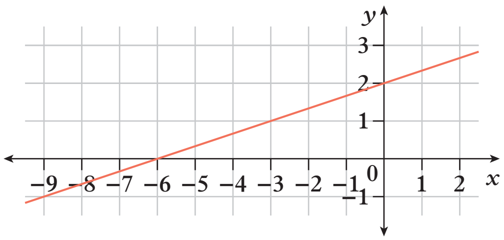
Accuracy (%)	75	80	90	98
Number of attempts	3	8	12	15

b The number of victories and the number of chess matches played

3D **9** For each linear graph shown, determine the:

i gradient **ii** x-intercept **iii** y-intercept.

a



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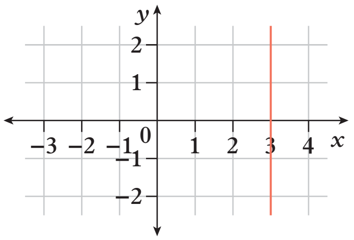

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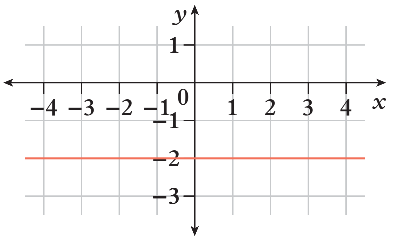

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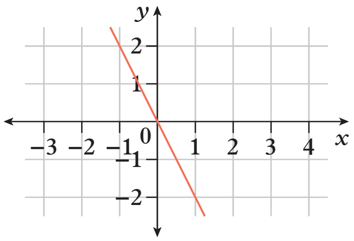


3D **10** Find the gradient of the line segment joining each pair of points.

a $(2, 3)$ and $(-2, -3)$ **b** $(-3, -2)$ and $(-2, -3)$

3E **11** For the graph of each linear relationship below, determine the coordinates of the:

i x-intercept **ii** y-intercept.

a $2x + 3y = 18$ **b** $3x - y = 6$ **c** $y = 4x - 2$

d $2y = 5x - 3$ **e** $x - 2y = 4$ **f** $-y = 4 - x$

3E **12** Use your answers from question **11** to sketch each linear graph.

3E **13** Use the most appropriate method to sketch the graph of each linear relationship.

a $3x + 2y = 4$ **b** $4 - 2x = 3y$ **c** $y = \frac{1}{3}x$ **d** $y = 5 - \frac{2}{3}x$

e $y = 5$ **f** $2x + 3y + 6 = 0$ **g** $y = \frac{3}{4}x - 2$ **h** $x = -7$

3F **14** Write the equations of the linear graphs with the following properties.

a gradient: 4, y-intercept: $(0, -2)$

b gradient: $\frac{1}{4}$, y-intercept: $(0, 0)$

c gradient: 0, y-intercept: $\left(0, -\frac{1}{2}\right)$

3F **15** Water flows into an empty tank at a constant rate. After 4 hours, the tank contains 2000 litres of water.

a **i** Sketch a graph of the volume of water in the tank, V, against time, t from $t = 0$ to 4 hours.

ii Determine the gradient of the graph.

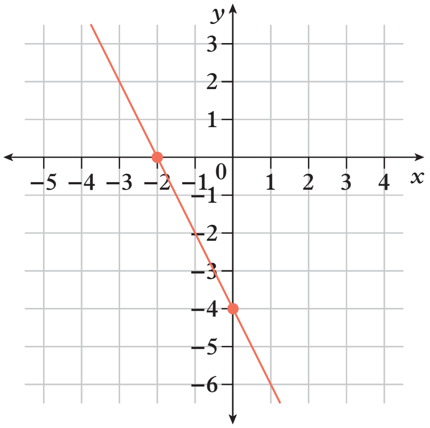
b Determine the flow rate of the water.

c Determine the equation for the relationship between V and t.

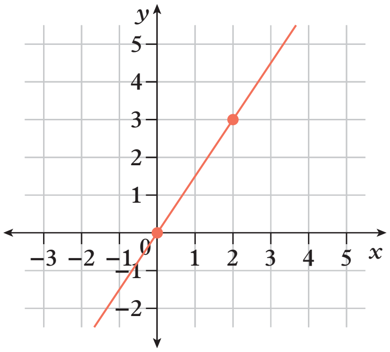
d If the tank has a total capacity of 10 000 litres, how long will it take to fill the tank to full capacity?

3F **16** Find the equation of each of the following linear graphs.

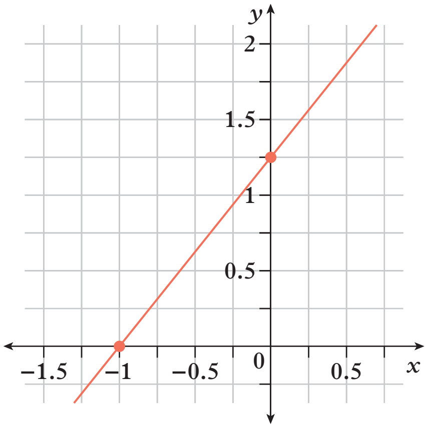
a



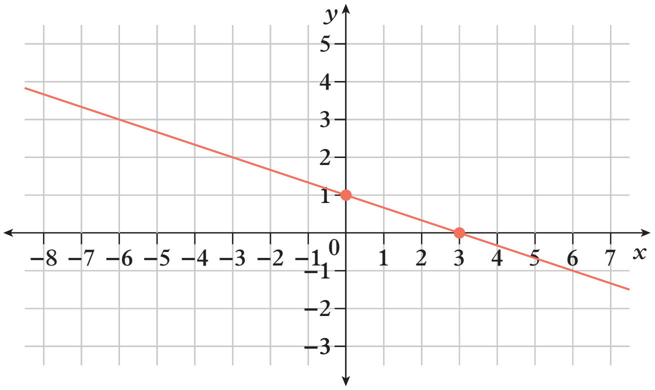
b



c



d



3F **17** Determine the equations of the linear graphs that pass through each pair of points.

a $(-5,2)$ and $(1,-4)$

b $(100,39)$ and $(-47,39)$

c $(-3,4)$ and $(6,-8)$

d $(7,12)$ and $(-3,4)$

3G **18** For the line segment that joins each of the following pairs of points, find the:

i midpoint

ii length (correct to one decimal place).

a $(2, 3)$ and $(8, 7)$

b $(-4, 6)$ and $(-3, 5)$

c $(2, -4)$ and $(-2, 4)$

d $(-3, -1)$ and $(2, -9)$

Analysis

1 The cross-section of a building is drawn on a Cartesian plane with the scale on the axes showing length in metres. The x-axis represents ground level.

a On the same Cartesian plane, sketch the graph of:

i $4y - x = 12$ **ii** $y = 5 - \frac{1}{4}x$

b To represent the cross-section of the building, shade the area between the graph of $4y - x = 12$ and the x-axis from $x = 0$ to $x = 4$, as well as the area between the graph of $y = 5 - \frac{1}{4}x$ and the x-axis between $x = 4$ and $x = 8$.

c How tall is the building at its highest point?

d What is the distance from the top of the roof to the lower edge of the roof, correct to one decimal place?

e What is the positive gradient of the roof?

f If a chimney is to be placed halfway along the slope of the roof on the side with the positive gradient, describe its position on the Cartesian plane.

2 A rectangle $DEFG$ has vertices at $D(-2, -1)$, $E(0, 1)$, $F(3, -2)$ and $G(1, -4)$.

a Draw the rectangle on the Cartesian plane.

b Calculate the lengths of all the sides of the rectangle, correct to one decimal place.

c Using your sketch, identify the coordinates of the y-intercepts of the line segments:

i $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$

d Find the gradients of the lines through:

i $\overline{DE}$ **ii** $\overline{EF}$ **iii** $\overline{DG}$ **iv** $\overline{FG}$

e If point P is the midpoint of $\overline{DE}$, point Q is the midpoint of $\overline{EF}$, point R is the midpoint of $\overline{FG}$ and point S is the midpoint of $\overline{DG}$. Find the coordinates of:

i P **ii** Q **iii** R **iv** S

f Describe the shape of the figure $PQRS$. Justify the statements you make.

g If the original figure $DEFG$ had been a square instead of a rectangle, explain how this would affect the shape of $PQRS$. Support your answer with mathematical evidence.

3 For a hot food stall, Sophie buys and then sells sausage rolls and party pies. The sausage rolls cost 71 cents each and the party pies cost 26 cents each. Sophie sells the sausage rolls for \$1.20 each and the party pies for 50 cents each. Sophie has a budget of \$60 per day. Let s be the number of sausage rolls Sophie buys and p be the number of party pies Sophie buys.

a Write an inequality that describes how Sophie can spend her budget each day.

b If Sophie buys 50 sausage rolls, write an inequality that describes the number of party pies she can buy.

c If Sophie buys 150 party pies, write an inequality that describes the number of sausage rolls she can buy.

d Assume Sophie is able to sell all the sausage rolls and party pies she buys on a particular day. Let P be the profit Sophie will make that day.

Write an equation that describes the profit Sophie will make.

e On another day, Sophie made \$50 profit.

Sketch the graph of the number of sausage rolls against the number of party pies Sophie could have bought and sold. Write the coordinates of the intercepts, correct to two decimal places. Hint: Put sausage rolls on the vertical axis.

f State the maximum number of party pies Sophie could have bought and sold on the day she made \$50 profit.

g On the day Sophie made \$50 profit, the price she paid for the sausage rolls and party pies was \$59.90. Determine the number of sausage rolls and party pies Sophie bought and sold that day.

4

Non-linear relationships

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Solving equations using inverse operations
- ✔ Factorising quadratic expressions
- ✔ Factorising the difference of two squares
- ✔ The Cartesian plane
- ✔ Plotting graphs

Curriculum links

- Identify and graph quadratic functions, solve quadratic equations graphically and numerically, and solve monic quadratic equations with integer roots algebraically, using graphing software and digital tools as appropriate (AC9M9A04)
- Use mathematical modelling to solve applied problems involving change including financial contexts; formulate problems, choosing to use either linear or quadratic functions; interpret solutions in terms of the situation; evaluate the model and report methods and findings (AC9M9A05)
- Experiment with the effects of the variation of parameters on graphs of related functions, using digital tools, making connections between graphical and algebraic representations, and generalising emerging patterns (AC9M9A06)

Materials

- ✔ Calculator
- ✔ Graph paper

4A Solving quadratic equations

Learning intentions

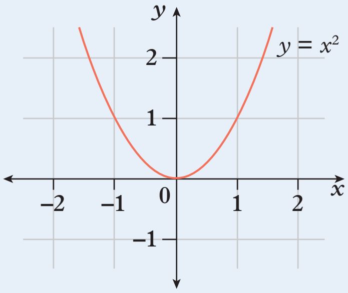
By the end of this topic you will be able to ...

✔ solve simple quadratic equations.

Inter-year links

Year 7	6C Terms, expressions and equations
Year 8	6A Equations
Year 10	4A Solving quadratic equations

Quadratic equations



- A **quadratic equation** is a single-variable equation in which the highest exponent is 2.
- The general form of a quadratic equation is: $ax^2 + bx + c = 0$ where a, b and c are constants.
- Similarly, the general form of a quadratic relationship is: $y = ax^2 + bx + c$
- A quadratic relationship produces a parabola-shaped graph, as shown on the right.

The Null Factor Law

- The **Null Factor Law** states that if the product of two factors is 0, then one or both of the factors must be 0.

  Given the factors a and b, if $a \times b = 0$, then $a = 0$ or $b = 0$.
- Applying the Null Factor Law to quadratic equations in **factor form**, $(x - p)(x - q) = 0$:

  If $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$.

Solving quadratic equations

- To solve a quadratic equation in general form:
  1. Factorise the quadratic expression.
  2. Apply the Null Factor Law by equating each factor to 0.
  3. Solve each linear equation.

  For example, $x^2 + 2x - 3 = 0$

  $$(x + 3)(x - 1) = 0$$
  $$x + 3 = 0 \text{ or } x - 1 = 0$$
  $$x = -3 \text{ or } x = 1$$
- Quadratic equations can have 0, 1 or 2 solutions.

Type of quadratic equation	Example	Number of solutions
Factorised form is $(x + a)(x + b) = 0$, where $a \neq b$	$x^2 + 3x + 2 = 0$ $(x + 1)(x + 2) = 0$ $x + 1 = 0$ or $x + 2 = 0$ $x = -1$ or $x = -2$	2
Factorised form is a perfect square: $(x + a)^2 = 0$	$x^2 + 2x + 1 = 0$ $(x + 1)^2 = 0$ $x + 1 = 0$ $x = -1$	1
$ax^2 + bx + c = 0$ cannot be factorised for some values of a, b and c	$x^2 + x + 1 = 0$	0

Example 4A.1 Solving factorised quadratic equations

Solve each quadratic equation.

a $(x-6)(x+2)=0$ **b** $x(x-4)=0$ **c** $(2x+1)(4x-3)=0$

THINK	WRITE
1 Check that the quadratic equation is in factor form and is equal to 0.	**a** $(x-6)(x+2)=0$ $x-6=0$ or $x+2=0$ $x=6$ or $x=-2$
2 Apply the Null Factor Law by equating each factor to 0.	**b** $x(x-4)=0$ $x=0$ or $x-4=0$ $x=0$ or $x=4$
3 Solve each linear equation.	**c** $(2x+1)(4x-3)=0$ $2x+1=0$ or $4x-3=0$ $2x=-1$ or $4x=3$ $x=-\frac{1}{2}$ or $x=\frac{3}{4}$

Example 4A.2 Factorising and solving quadratic equations

Solve each quadratic equation.

a $x^2-3x-10=0$ **b** $x^2-9=0$

THINK	WRITE
1 Factorise the quadratic equation. 2 Apply the Null Factor Law by equating each factor to 0. 3 Solve each linear equation.	**a** $x^2-3x-10=0$ $(x+2)(x-5)=0$ $x+2=0$ or $x-5=0$ $x=-2$ or $x=5$
	b Using the difference of two squares rule: $x^2-9=0$ $(x+3)(x-3)=0$ $x+3=0$ or $x-3=0$ $x=-3$ or $x=3$

Example 4A.3 Rearranging quadratic equations before solving

Solve $x^2+12=8x$

THINK	WRITE
1 Rearrange the equation, so that the right-hand side of the equation is equal to 0.	$x^2+12=8x$ $x^2-8x+12=0$ $(-\,8x)$
2 Factorise the quadratic equation.	$(x-2)(x-6)=0$
3 Apply the Null Factor Law by equating each factor to 0.	$x-2=0$ or $x-6=0$
4 Solve each linear equation.	$x=2$ or $x=6$

Example 4A.4 Solving quadratic equations with common factors

Solve each quadratic equation.

a $2x^2 + 22x + 36 = 0$ **b** $-x^2 + 10x = 25$

THINK	WRITE
a **1** Take the highest common factor (HCF) out of the quadratic expression.	**a** $2x^2 + 22x + 36 = 0$ $2(x^2 + 11x + 18) = 0$
2 Divide both sides by the HCF. Note that you can never divide both sides by the variable, x, in case $x = 0$.	$\frac{\cancel{2}(x^2 + 11x + 18)}{\cancel{2}} = \frac{0}{2}$
3 Factorise the quadratic equation.	$x^2 + 11x + 18 = 0$ $(x + 2)(x + 9) = 0$
4 Apply the Null Factor Law by equating the factors to 0.	$x + 2 = 0$ or $x + 9 = 0$
5 Solve each linear equation.	$x = -2$ or $x = -9$
b **1** Rearrange so that the right-hand side is 0.	**b** $-x^2 + 10x = 25$ $-x^2 + 10x - 25 = 0$
2 Take out the HCF. Remember, the HCF can be −1.	$-1(x^2 - 10x + 25) = 0$
3 Divide both sides by the HCF.	$\frac{\cancel{-1}(x^2 - 10x + 25)}{\cancel{-1}} = \frac{0}{-1}$
4 Factorise and apply the Null Factor Law. Remember, there is only one solution when the factorised form is a perfect square.	$x^2 - 10x + 25 = 0$ $(x - 5)^2 = 0$ $x - 5 = 0$
5 Solve the linear equation.	$x = 5$

Helpful hints

- ✔ Always check the coefficients for a highest common factor (HCF). You can factor out the HCF from any quadratic equation, $ax^2 + bx + c = 0$, by dividing on both sides. This is because $\frac{0}{\text{HCF}} = 0$ for any HCF.
- ✔ Watch for changes in sign when solving quadratic equations for x.

$$\begin{aligned}(x + 6)(x - 2) &= 0\\ x + 6 = 0 \text{ or } x - 2 &= 0\\ x = -6 \text{ or } x &= +2\end{aligned}$$

- ✔ Although it is possible to add and subtract multiples of x, you **must not** divide both sides of an equation by x in case $x = 0$, since dividing anything by 0 is **undefined**.

For example,

$x^2 = 5x$ $(-5x)$	$x^2 = 5x$ $(\div x)$
$x^2 - 5x = 0$	$x = 5$
$x(x - 5) = 0$	Solution $x = 0$ is missing
$x = 0$ or $x = 5$	

ANS
p433

Exercise 4A Solving quadratic equations

△ 1, 2(a, c, e, g), 3–11, 13, 16 ■ 1, 2(f, g, h), 4–10, 12, 13, 15, 18 ⬠ 1, 5–7, 9, 10, 12, 15–18

1 Which of the following are quadratic equations?
a $x^2 - 2 = 0$ **b** $3(x + 1) = 0$ **c** $x^2 + x = 5$ **d** $x^2 + 5x + 6$
e $2x^2 + x - 4 = 0$ **f** $x^3 + 8 = 0$ **g** $6x + 1 = 2x - 5$ **h** $x^2 + 7x = x - 3$

2 Use substitution to check whether the value in brackets is a solution to the given quadratic equation.
a $(x - 4)(x - 5) = 0$ $(x = 5)$ **b** $(x + 2)(x - 8) = 0$ $(x = 2)$
c $x(x - 6) = 0$ $(x = 3)$ **d** $x^2 + 8x + 7 = 0$ $(x = -1)$
e $x^2 - 4x + 4 = 0$ $(x = -2)$ **f** $x^2 - 49 = 0$ $(x = 7)$
g $x^2 - 2x - 15 = 0$ $(x = -3)$ **h** $x^2 - 8x + 12 = 0$ $(x = -4)$

3 Solve each of the following linear equations.
a $x + 3 = 0$ **b** $x - 6 = 0$ **c** $4x - 8 = 0$ **d** $3x + 1 = 0$

4A.1 **4** Solve each of the following quadratic equations.
a $(x + 2)(x - 3) = 0$ **b** $(x - 7)(x - 1) = 0$ **c** $(x + 4)(x - 4) = 0$
d $x(x - 6) = 0$ **e** $(x + 5)(x + 1) = 0$ **f** $x(x + 2) = 0$
g $(x - 8)(x + 8) = 0$ **h** $(2x + 2)(x - 7) = 0$ **i** $x(x - 11) = 0$
j $(2x + 3)(x - 5) = 0$ **k** $(4x - 2)(4x - 2) = 0$ **l** $(3x + 5)(2x + 3) = 0$

4A.2 **5** Factorise and then solve each quadratic equation.
a $x^2 + 7x + 10 = 0$ **b** $x^2 - 3x + 2 = 0$ **c** $x^2 + 5x = 0$
d $x^2 - 3x = 0$ **e** $x^2 - 36 = 0$ **f** $x^2 + 10x + 21 = 0$

4A.3 **6** Rearrange, factorise and then solve each quadratic equation.
a $x^2 - 2x = 8$ **b** $x^2 = 1$ **c** $x^2 = -8x$
d $x^2 + 3 = 4x$ **e** $6x = -x^2 -9$ **f** $1 = 2x - x^2$

7 Use substitution to check that your solutions for questions **5** and **6** are correct.

8 Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).
a $2(x + 8)(x - 2) = 0$ **b** $-3(x - 1)(x - 4) = 0$
c $-7(x + 6)(x - 6) = 0$ **d** $-5x(x + 9) = 0$

4A.4 **9** Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).
a $3x^2 + 9x + 6 = 0$ **b** $2x^2 + 8x = 0$ **c** $2x^2 - 4x - 16 = 0$
d $3x^2 - 12x + 12 = 0$ **e** $-5x^2 - 5x + 10 = 0$ **f** $-4x^2 + 8x = 0$
g $-x^2 - 10x - 21 = 0$ **h** $-3x^2 - 24x - 48 = 0$ **i** $-2x^2+32 = 0$

10 Solve each equation.
a $x^2 + 14x + 48 = 0$ **b** $3x^2 - 12x = 0$ **c** $x - x^2 = 0$
d $x^2 - 64 = 0$ **e** $-x^2 - 2x + 3 = 0$ **f** $-x^2 + 8x - 16 = 0$
g $-x^2 + 5x = 6$ **h** $2x^2 + 50 = -20x$ **i** $20 = 5x^2$

11 a Determine how many solutions each of the following quadratic equations has.
i $(x - 4)(x - 7) = 0$ **ii** $(x - 4)(x - 4) = 0$ **iii** $x^2 + 4 = 0$

b Identify and describe the feature(s) of each equation in part **a** that result in the number of solutions that the equation has.

PROBLEM SOLVING AND REASONING

12 State how many solutions each equation has.

a $x^2 + 3x - 10 = 0$ **b** $x^2 - 4 = 0$ **c** $x^2 - 6x + 9 = 0$ **d** $x^2 + 1 = 0$

e $x^2 + 7x = 0$ **f** $x^2 + 12x + 32 = 0$ **g** $x^2 - x - 72 = 0$ **h** $x^2 + 2x + 5 = 0$

13 The length of a rectangular mouse pad is 8 cm longer than its width (w).

a Write an expression for the area of the mouse pad in terms of w.

b Expand the expression.

c The area of the mouse pad is estimated to be 560 cm^2. Write an equation for the area of the mouse pad.

d Factorise and solve the equation. Which value of w is a feasible solution in this scenario? Explain.

e State the dimensions of the mouse pad.

14 The area of a rectangular sand pit is 35 m^2. The length is 2 m longer than the width (w).

a Write a quadratic equation to represent this scenario.

b Solve the quadratic equation.

c State the dimensions of the sand pit.

15 The width of a laptop screen is 12 cm less than its length. If the area of the screen is 640 cm^2, use algebra to determine the dimensions of the screen.

16 Alec throws a tennis ball back on to the court from the spectator stand. The height of the ball above the surface of the tennis court can be represented by the quadratic relationship $h = -4(t + 1)(t - 2)$, where h is the height in metres after t seconds in the air.

a What is the height of the ball after:

i 1 second? **ii** 2 seconds?

b What is the height of the ball when Alec releases it from his hand?

c How long does it take for the ball to hit the tennis court after it is thrown?

d Explain why there is only one time value for your answer to part **c** even though you have solved a quadratic equation that has two solutions.

CHALLENGE

17 The height above the ground, h, of a firework rocket x seconds after it is launched is given by the equation $h = -5x^2 + 40x$.

a When is the rocket at a height of 60 m above the ground?

b Why are there two solutions? Explain the significance of each solution.

18 An equation containing a pronumeral that is taken to the power of three (and no higher) is called a cubic equation. Use the Null Factor Law to solve the following cubic equations.

a $(x - 1)(x - 3)(x - 5) = 0$

b $x(x + 6)(x - 6) = 0$

c $(x + 11)(x - 4)^2 = 0$

d $(2x - 4)(3x + 1)(4x - 6) = 0$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving quadratic equations

CAS instructions
Solving equations

Topic quiz
4A

4B Plotting quadratic relationships

Learning intentions

By the end of this topic you will be able to ...

✔ identify the key features of a quadratic graph

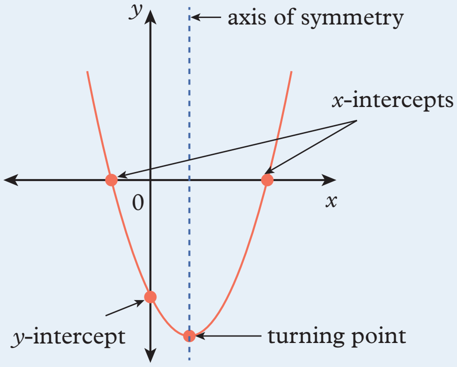
✔ plot quadratic relationships from tables of values.

Inter-year links

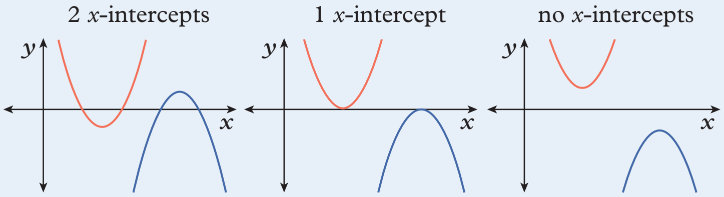
Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 8	6D Plotting linear relationships

Key features of quadratic graphs

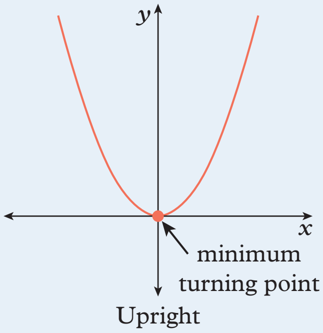
- The graph of a quadratic relationship is called a **parabola**.

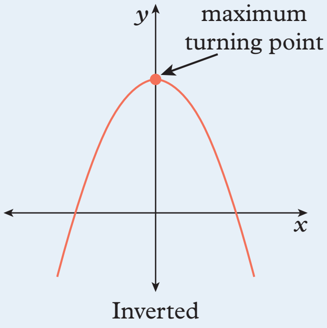


- The key features of a parabola include:
  → x- and y-intercepts
    – all parabolas have one y-intercept
    – parabolas can have no, one or two x-intercept(s)



  → a **turning point** or vertex; the turning point can be a **minimum turning point** or **maximum turning point**



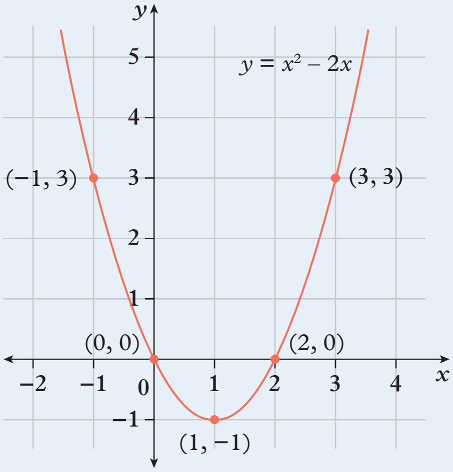


  → an **axis of symmetry**, $x = a$, where a is the x-coordinate of the turning point.
- If there are two x-intercepts, then the axis of symmetry will be at the midpoint between them.

Plotting quadratic graphs

- To plot the graph of a quadratic relationship, create a table of coordinate points and join the points on a Cartesian plane with a smooth curve. In section 4C you will **sketch** parabolas using the key features of the graphs.
  1. Construct a table of x- and y-values by selecting values for x, then substituting each value of x into the relationship to find the corresponding value of y.
  2. Write the coordinate points in the table.
  3. Plot the coordinate points on a Cartesian plane.
  4. Join the points using a smooth curve to form a parabola and label the graph with its equation.

x	−1	0	1	2	3
$y = x^2 - 2x$	3	0	−1	0	3



Example 4B.1 Identifying the key features of a quadratic graph

For the quadratic graph shown, identify:

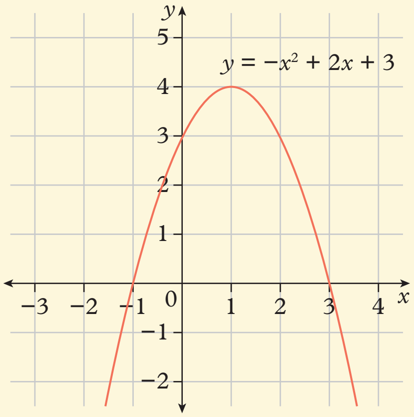
a the coordinates of the x-intercepts

b the coordinates of the y-intercept

c the coordinates of the turning point

d whether the turning point is a maximum or minimum

e the equation of the axis of symmetry.



THINK	WRITE
a State the coordinates of the points where the graph crosses the x-axis.	**a** x-intercepts: $(-1, 0)$ and $(3, 0)$
b State the coordinates of the point where the graph crosses the y-axis.	**b** y-intercept: $(0, 3)$
c State the coordinates of the point where the graph changes direction.	**c** turning point: $(1, 4)$
d The turning point is the highest point on the parabola, so it is a maximum turning point.	**d** maximum turning point
e State the equation of the axis of symmetry.	**e** axis of symmetry: $x = 1$

Example 4B.2 Plotting quadratic relationships

Sketch a graph of $y = x^2 - 2x - 1$ by first completing a table of values for x from -1 to 3.

THINK

1 Construct a table of values for x from -1 to 3. Substitute each value of x into the equation to find the corresponding value of y.

2 List the coordinate points.

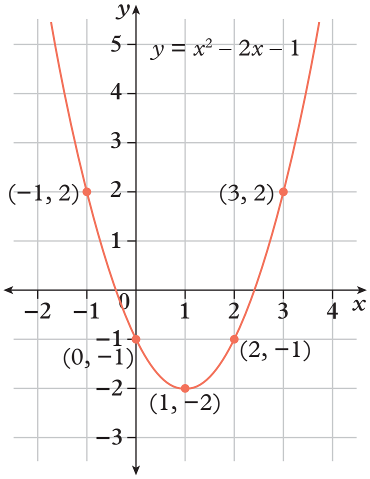
3 Plot the points on a Cartesian plane.

4 Join the points with a smooth curve. Label the graph with its equation.

WRITE

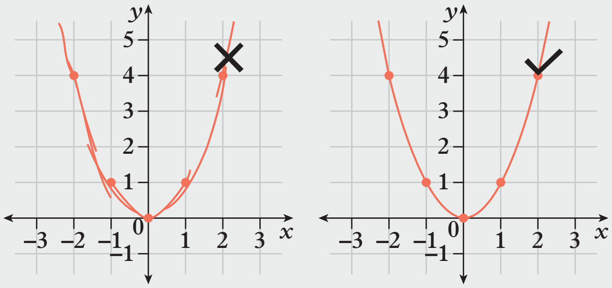
x	-1	0	1	2	3
y	2	-1	-2	-1	2

$(-1, 2), (0, -1), (1, -2), (2, -1), (3, 2)$

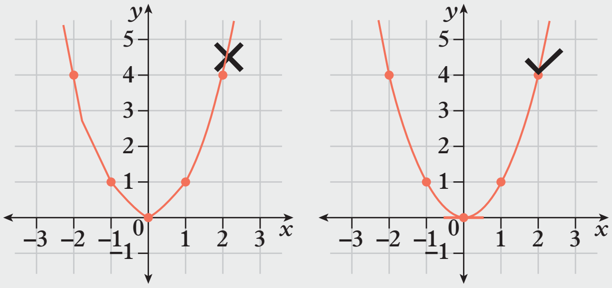


Helpful hints

✔ Rather than drawing ‘hairy’ parabolas, try starting at the turning point and then drawing two smooth, continuous and symmetrical curves through the other plotted points.



✔ Draw a short horizontal dash where the turning point is to ensure the parabola does not end up with a pointy turning point.



✔ Take care when reading a question. If you are asked to find the x- and y-intercepts, then you must give your answer as coordinates. So if the coordinates of the intercepts are $(5, 0)$ and $(0, -1)$, **do not** write $x = 5$ and $y = -1$.

ANS p433

Exercise 4B Plotting quadratic relationships

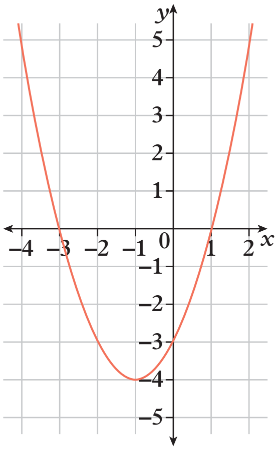
△ 1–4, 7, 9, 10, 12　　□ 1(b, d, f, h), 2, 3, 5, 6, 8, 9, 11, 13　　⬠ 1(d, f, h), 2, 3, 6, 8, 9, 11, 13, 14

UNDERSTANDING AND FLUENCY

4B.1 **1** For each quadratic graph, identify:

- **i** the coordinates of the x-intercepts, if any
- **ii** the coordinates of the y-intercept
- **iii** the coordinates of the turning point
- **iv** whether the turning point is a maximum or minimum
- **v** the equation of the axis of symmetry.

a



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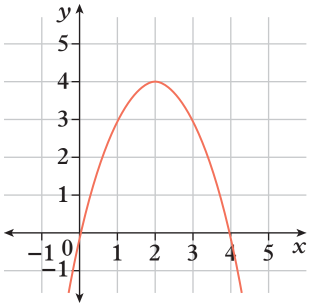

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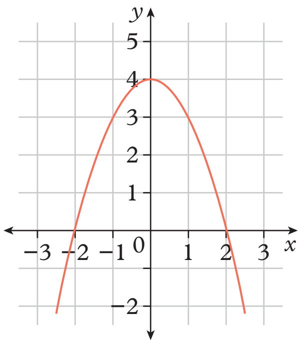

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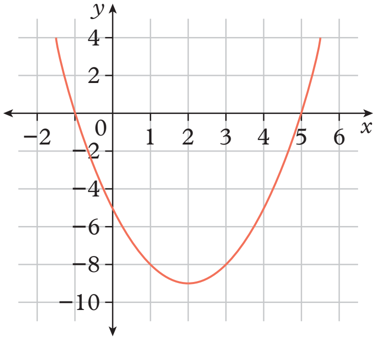

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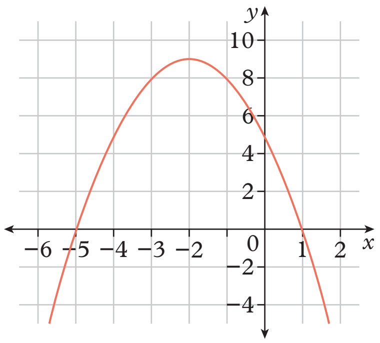

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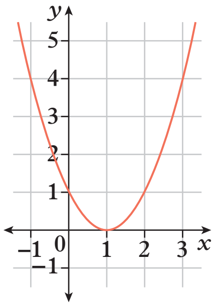

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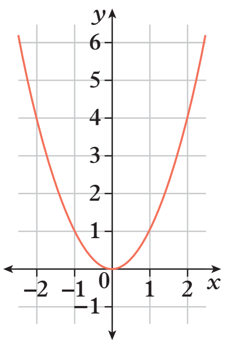

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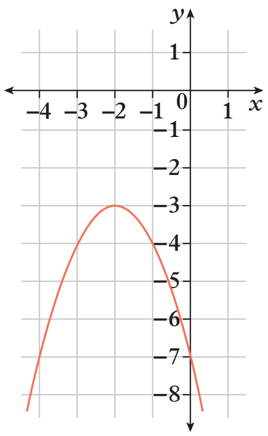
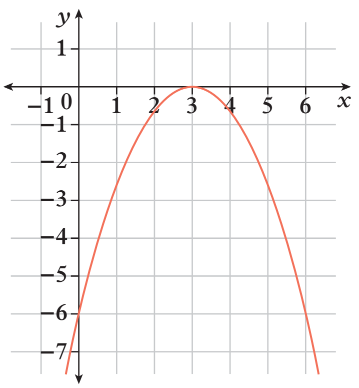


i



UNDERSTANDING AND FLUENCY

2 For each quadratic relationship:

- **i** complete the table of values
- **ii** plot the graph
- **iii** identify the axis intercepts and the turning point, and then state the equation of the axis of symmetry
- **iv** state whether the turning point is a maximum or minimum.

a $y = x^2 + 2x - 8$

x	-5	-4	-3	-2	-1	0	1	2	3
y									

b $y = 9 - x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

4B.2 **3** For each quadratic relationship:

- **i** plot its graph by first completing a table of coordinate points, using the x-values given in brackets
- **ii** identify the coordinates of the x- and y-intercepts, and the turning point, and state the equation of the axis of symmetry.

a $y = -x^2 - 6x - 5$ (−6 to 0)

b $y = x^2 - 4$ (−3 to 3)

c $y = x^2 - 2x - 15$ (−4 to 6)

d $y = x^2 + 4x$ (−5 to 1)

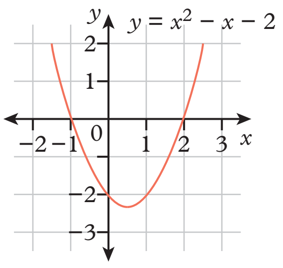
e $y = -x^2 + 2x$ (−1 to 3)

f $y = x^2 - 6x + 9$ (0 to 6)

PROBLEM SOLVING AND REASONING

4 **a** How many x-intercepts do each of the following parabolas have?

i



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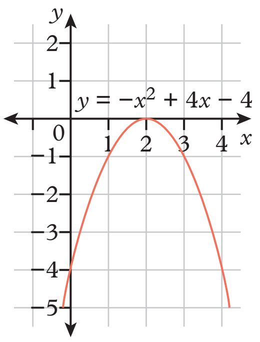
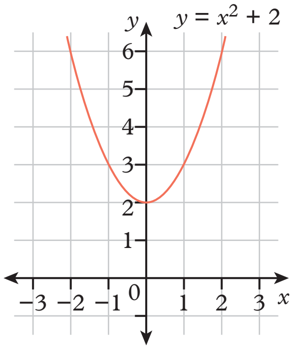


iii



b Can a parabola have more than two x-intercepts? Explain.

c How many y-intercepts does each parabola in part **a** have?

d Describe the link between the y-intercept of a parabola and the general equation, $y = ax^2 + bx + c$.

5 For each of the following parabolas state the number of:

i x-intercepts **ii** y-intercepts.

a Parabola with a minimum turning point at (−3, 1)

b Parabola with a maximum turning point at (10, 0)

6 Find the equation of the axis of symmetry if the x-intercepts of a quadratic graph are:

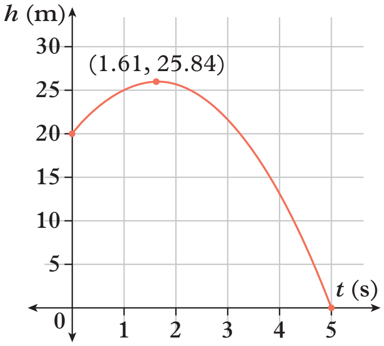
a (−3, 0) and (5, 0)

b (5, 0) and (6, 0)

c (−2, 0) and (1, 0)

d (−2, 0) and (−1, 0).

7 The graph below shows the height, h (in metres), of a firework rocket t seconds after it is launched.



a From what height is the rocket launched?

b What is the maximum height of the rocket to the nearest metre?

c For how long is the rocket in the air?

8 Alani's hair clip falls to the ground while she is on a roller coaster. The position of the clip as it falls can be described by the relationship $h = 100 - 4t^2$, where h is the height of the clip above the ground in metres after t seconds.

a Plot the graph of the relationship for t-values from 0 to 5.

b Why didn't you draw the parabola for values less than 0 or greater than 5?

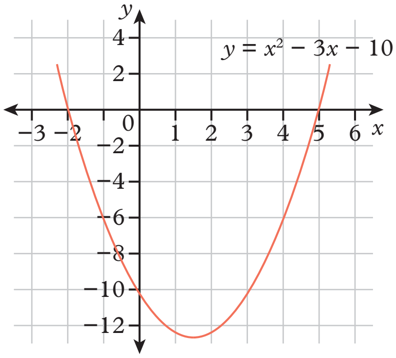
c What is the height of the clip above the ground after:

i 2 s? **ii** 3 s?

d From what height above the ground did the hair clip start to fall?

e How long did it take for the hair clip to hit the ground?

9 Consider the graph of $y = x^2 - 3x - 10$ shown below.



a Identify the x-coordinates of the x-intercepts from the graph.

b Solve the quadratic equation $x^2 - 3x - 10 = 0$ by first factorising the quadratic expression, and then applying the Null Factor Law.

c Compare your answers for parts **a** and **b**. What do you notice?

d Explain how you can use the graph of $y = x^2 - 3x - 10$ to solve $x^2 - 3x - 10 = 0$.

10 Determine the equation for the axis of symmetry of a parabola that has:

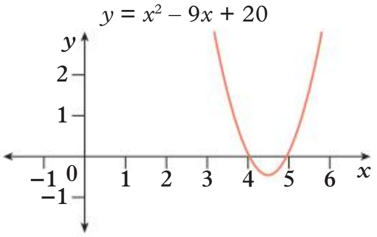
a x-intercepts at (2, 0) and (8, 0)

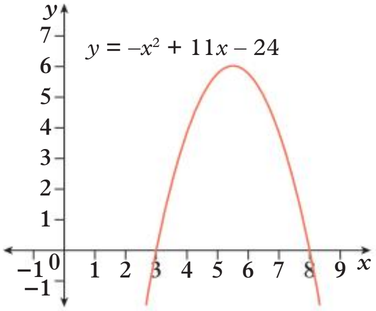
b x-intercepts at (−3, 0) and (3, 0)

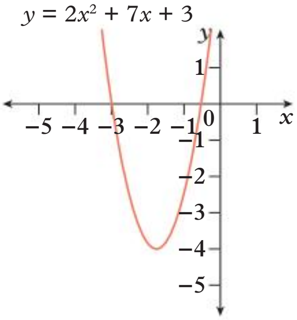
c one x-intercept at (−4, 0). Hint: Draw a rough sketch of the parabola and consider the midpoint.

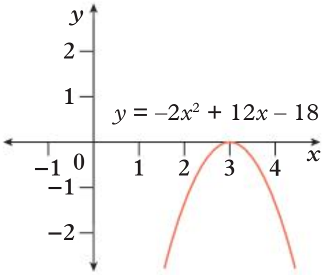
PROBLEM SOLVING AND REASONING

11 Use your observations from question **9** to solve each quadratic equation using the following graphs.









a $x^2 - 9x + 20 = 0$

b $-x^2 + 11x - 24 = 0$

c $2x^2 + 7x + 3 = 0$

d $-2x^2 + 12x - 18 = 0$

12 The axis of symmetry of a quadratic graph is $x = -4$. If the graph passes through the x-axis at $(-8, 0)$, determine the coordinates of the second x-intercept.

CHALLENGE

13 The equation for a graph with x-intercepts at $(-7, 0)$ and $(5, 0)$ is written as $y = (x - p)(x - q)$, where p and q are constants.

a Determine the x-intercepts of $y = (x - p)(x - q)$ in terms of p and q.

b Hence identify appropriate values for p and q and write the equation for the graph.

c Determine the minimum value of y.

d Determine the equation for a graph with x-intercepts at $(-7, 0)$ and $(5, 0)$ and a maximum value of 36.

14 Kim throws a javelin. The position of the tip of the javelin can be represented by the quadratic relationship $y = -0.05x^2 + 1.5x + 1.55$, where y is the height above the ground and x is the horizontal distance from where the javelin was thrown. Both x and y are in metres.

a Plot the graph of this relationship. Use 0, 5, 10, 15, …, 30 as the x-values in the table.

b At what height off the ground was the javelin thrown?

c Let $y = 0$ and solve the quadratic equation for x. Hint: Start by multiplying both sides of the equation by −20.

d What horizontal distance did the javelin travel before hitting the ground?

e What was the maximum height reached by the javelin?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Plotting quadratic relationships

Investigation
Difference between consecutive values

CAS instructions
Graphing functions

Topic quiz
4B

4C Sketching parabolas using intercepts

Learning intentions

By the end of this topic you will be able to ...

✔ find the x- and y-intercepts of a quadratic graph from its equation

✔ find the coordinates of the turning point of a quadratic graph given the two x-intercepts of a quadratic relationship

✔ sketch quadratic graphs using the x- and y-intercepts and the turning point.

Inter-year links

Support	The Cartesian plane
Year 7	6E Substitution
Year 8	5B Substitution
Year 10	4C Graphing parabolas using intercepts

Sketching quadratic graphs

- Before sketching quadratic relationships of the form $y = ax^2 + bx + c$, determine the coordinates of any x-intercepts, the y-intercept and the turning point.
- To find the x-intercepts of $y = ax^2 + bx + c$:
  1. Substitute $y = 0$ into the quadratic relationship.
  2. Factorise the quadratic equation.
  3. Solve the equation by applying the Null Factor Law. (Note: A parabola can have 0, 1 or 2 x-intercepts)

$y = x^2 + 2x - 3$

Let $y = 0$:

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \text{ or } x - 1 = 0$$

$$x = -3 \text{ or } x = 1$$

x-intercepts: $(-3, 0)$ and $(1, 0)$

- To find the y-intercept, substitute $x = 0$ into the quadratic relationship.

$y = x^2 + 2x - 3$

Let $x = 0$:

$y = (0)^2 + 2(0) - 3$

$y = -3$

y-intercept: $(0, -3)$

- The turning point is located on the axis of symmetry, which is halfway between the x-intercepts. So to find the coordinates of the turning point for a parabola with two x-intercepts:
  1. Calculate the x-coordinate of the turning point by finding the midpoint of the x-intercepts.

$$\begin{aligned} x &= \frac{-3 + 1}{2} \\ &= \frac{-2}{2} \\ &= -1 \end{aligned}$$

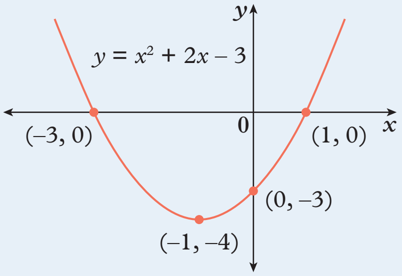
  2. Calculate the y-coordinate of the turning point by substituting the x-coordinate into the quadratic relationship.

$y = x^2 + 2x - 3$

Let $x = -1$:

$$\begin{aligned} y &= (-1)^2 + 2(-1) - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

Turning point: $(-1, -4)$



Example 4C.1 Finding the coordinates of the *x*- and *y*-intercepts

For each quadratic relationship, find the coordinates of the:

i *x*-intercepts **ii** *y*-intercept.

a $y = x^2 - 6x$ **b** $y = x^2 + 7x + 10$

THINK	WRITE	
i **1** To find the *x*-coordinate of the *x*-intercept, substitute $y = 0$ into the quadratic relationship.	**a** **i** $y = x^2 - 6x$ For *x*-intercepts, let $y = 0$:	**b** **i** $y = x^2 + 7x + 10$ For *x*-intercepts, let $y = 0$:
2 Factorise the quadratic equation.	$x^2 - 6x = 0$ $x(x - 6) = 0$	$x^2 + 7x + 10 = 0$ $(x + 2)(x + 5) = 0$
3 Apply the Null Factor Law to solve the equation.	$x = 0$ or $x - 6 = 0$ $x = 0$ or $x = 6$ *x*-intercepts: (0, 0) and (6, 0)	$x + 2 = 0$ or $x + 5 = 0$ $x = -2$ or $x = -5$ *x*-intercepts: (−2, 0) and (−5, 0)
ii To find the *y*-coordinate of the *y*-intercept, substitute $x = 0$ into the quadratic relationship and solve for *y*.	**ii** $y = x^2 - 6x$ For *y*-intercept, let $x = 0$: $y = (0)^2 - 6(0)$ $= 0$ *y*-intercept: (0, 0)	**ii** $y = x^2 + 7x + 10$ For *y*-intercept, let $x = 0$: $y = (0)^2 + 7(0) + 10$ $= 10$ *y*-intercept: (0, 10)

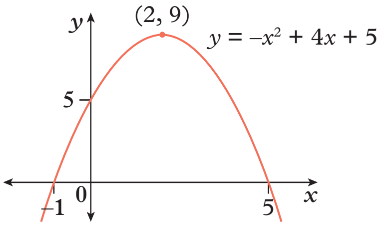
Example 4C.2 Finding the coordinates of the turning point using the *x*-intercepts

Find the coordinates of the turning point of $y = (x + 3)(x - 9)$.

THINK	WRITE
1 Identify the coordinates for the *x*-intercepts.	*x*-intercepts: (−3, 0) and (9, 0)
2 Calculate the *x*-coordinate of the turning point by finding the midpoint of the *x*-intercepts.	$x = \frac{-3 + 9}{2}$ $= \frac{6}{2}$ $= 3$
3 Calculate the *y*-coordinate of the turning point by substituting the *x*-coordinate into the quadratic relationship.	Let $x = 3$: $y = (x + 3)(x - 9)$ $= (3 + 3)(3 - 9)$ $= 6 \times (-6)$ $= -36$ Turning point: (3, −36)

Example 4C.3 Sketching parabolas using the intercepts

Sketch the graph of $y = -x^2 + 4x + 5$ by first finding the x- and y-intercepts. Label the turning point with its coordinates.

THINK	**WRITE**
1 Find the x-intercepts by substituting $y = 0$ into the relationship.	$y = -x^2 + 4x + 5$ For the x-intercepts, let $y = 0$:
2 Divide both sides of the equation by the coefficient of x^2, so that the equation is of the form $x^2 + bx + c = 0$.	$-x^2 + 4x + 5 = 0$ $-(x^2 - 4x - 5) = 0$ $x^2 - 4x - 5 = 0$
3 Factorise the quadratic equation.	$(x + 1)(x - 5) = 0$
4 Apply the Null Factor Law to solve each linear equation for x.	$x + 1 = 0$ or $x - 5 = 0$ $x = -1$ or $x = 5$ x-intercepts: $(-1, 0)$ and $(5, 0)$
5 Find the y-intercept by substituting $x = 0$ into the relationship and solving for y.	For the y-intercept, let $x = 0$: $y = -(0)^2 + 4(0) + 5$ $= 5$ y-intercept: $(0, 5)$
6 Find the coordinates of the turning point.	For the turning point: $x = \frac{-1 + 5}{2}$ $= \frac{4}{2}$ $= 2$ Let $x = 2$: $y = -(2)^2 + 4(2) + 5$ $= -4 + 8 + 5$ $= 9$ Turning point: $(2, 9)$
7 Mark the x- and y-intercepts on the axes, plot the turning point, then draw the parabola. Label the equation of the graph and the coordinates of the turning point.	

Helpful hints

- ✔ Label your working out so that it is clear that you are calculating the x-intercepts, the y-intercept and the turning point. This can make it easier to check your calculations on a test.
- ✔ Always label the key features of a graph such as the x-intercepts, the y-intercept and the turning point when sketching parabolas.
- ✔ For any given quadratic relationship, $y = ax^2 + bx + c$:
  - if $a > 0$, then the parabola has a minimum turning point
  - if $a < 0$, then the parabola has a maximum turning point.

ANS
p436

Exercise 4C Sketching parabolas using intercepts

△ 1, 2, 3(a, b, d, i), 4–8, 12, 14

□ 2–6, 8–11, 13, 16(a)

⬠ 4–6, 8, 10, 11, 13, 15, 16, 17

UNDERSTANDING AND FLUENCY

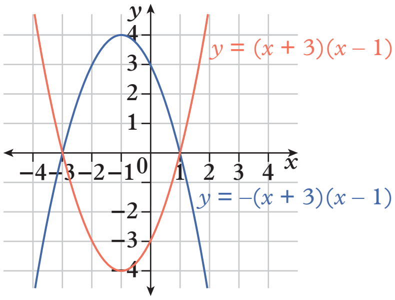
1 For each quadratic relationship, find the coordinates of the:
i x-intercepts **ii** y-intercept.
a $y = (x + 7)(x - 5)$ **b** $y = 3(x - 2)(x - 6)$ **c** $y = (3x + 12)(4x + 8)$

4C.1 **2** For each quadratic relationship, find the coordinates of the:
i x-intercepts **ii** y-intercept.
a $y = x^2 - 2x$ **b** $y = x^2 + 8x$ **c** $y = x^2 + 6x + 8$
d $y = x^2 - 8x + 12$ **e** $y = x^2 - 4x - 5$ **f** $y = x^2 - 9$

4C.2 **3** Find the coordinates of the turning point for each quadratic relationship.
a $y = x(x - 4)$ **b** $y = (x - 3)(x - 5)$ **c** $y = (x + 4)(x - 6)$
d $y = -(x + 1)(x + 3)$ **e** $y = (x - 2)(x + 2)$ **f** $y = -(x + 2)(x - 4)$
g $y = x(x + 3)$ **h** $y = -x(x + 7)$ **i** $y = 2(x - 2)(x - 8)$

4 Sketch the graph of each quadratic relationship by first finding the x- and y-intercepts. Label the turning point with its coordinates.
a $y = (x + 2)(x + 4)$ **b** $y = (x + 3)(x - 3)$ **c** $y = x(x - 2)$
d $y = -(x - 2)(x - 6)$ **e** $y = (x - 4)^2$ **f** $y = (2x - 4)(x - 6)$

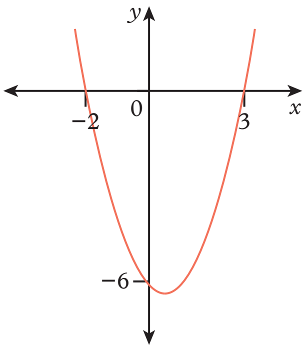
5 Consider the graphs of $y = (x + 3)(x - 1)$ and $y = -(x + 3)(x - 1)$.
a Identify the x-intercepts for each graph.
b Find the turning point of each graph.
c Describe the similarities and differences between the two graphs.



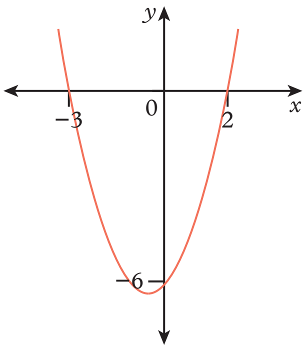
4C.3 **6** Sketch the graph of each quadratic relationship by first finding the x- and y-intercepts. Label the turning point with its coordinates.
a $y = x^2 - 6x + 5$ **b** $y = x^2 + 4x - 12$ **c** $y = -x^2 + 6x + 7$
d $y = x^2 + 4x$ **e** $y = -x^2 - 8x - 12$ **f** $y = x^2 - 4$
g $y = x^2 + 2x - 15$ **h** $y = 10x - x^2$ **i** $y = x^2 - 6x - 7$
j $y = x^2 - 5x$ **k** $y = 16 - x^2$ **l** $y = -x^2 + x$

7 Match each graph with its rule from the list below.

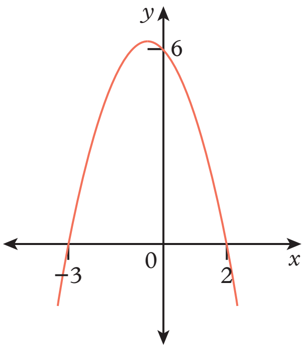
a



b



c



A $y = x^2 + x - 6$ **B** $y = -x^2 - x + 6$ **C** $y = x^2 - x - 6$

PROBLEM SOLVING AND REASONING

8 A parabola with a maximum turning point is called an inverted parabola. Explain how you can tell whether a parabola will be upright or inverted from its equation. Refer to your answers to question **6** as examples in your explanation.

9 Without sketching the graph, decide whether these equations will produce upright or inverted parabolas.

a $y = x^2 - 2x - 3$ **b** $y = x^2 - 100$ **c** $y = -x^2 - 8x + 15$

d $y = -x^2 + 6x$ **e** $y = 9x^2 - 10x$ **f** $y = 81 - 4x^2$

10 Consider the graph shown to the right.

a Is the parabola upright or inverted?

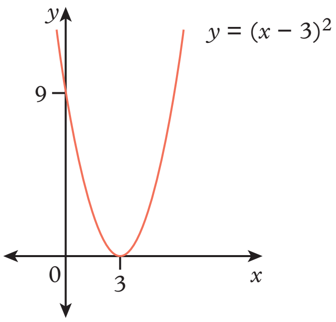
b How many y-intercepts does the parabola have? List the coordinates of the y-intercept(s).

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept(s).

d What are the coordinates of the turning point?

e Describe the relationship between the x-intercept(s) and the turning point.

f Describe the common features between quadratic relationships of the form $y = a(x - p)^2$ (where a and p are constants), and the coordinates of the turning point.



11 Sketch the graph of each quadratic relationship. Label the turning point with its coordinates.

a $y = (x - 1)^2$

b $y = -(x + 2)^2$

c $y = x^2 + 8x + 16$

d $y = x^2 - 4x + 4$

e $y = -x^2 - 6x - 9$

12 Consider the graph shown to the right.

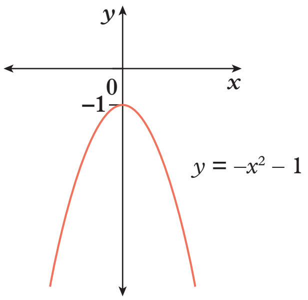
a Is the parabola upright or inverted?

b How many y-intercepts does the parabola have? List the coordinates of the y-intercept(s).

c How many x-intercepts does the parabola have? List the coordinates of the x-intercept(s).

d What are the coordinates of the turning point?

e Describe the relationship between the equations of the form $y = ax^2 + c$ (where a and c are constants), and the coordinates of the turning point.



13 Rhys fires an arrow from a bow. The path of the arrow can be represented by the quadratic relationship $h = -0.1(d + 1)(d - 15)$ where h is the height above the ground and d is the horizontal distance from where the arrow was fired. Both h and d are in metres.

a Sketch the graph of this relationship by first finding the intercepts and turning point.

b Between what two values of d does the graph represent the path of the arrow?

c How high does the arrow reach?

d At what height above the ground was the arrow fired?

e What horizontal distance did the arrow fly before hitting the ground?

14 A soccer ball is kicked off the ground. The path of the ball can be represented by the quadratic relationship $y = -0.2x^2 + 2.4x$, where x is the horizontal and y is the vertical distance. Both x and y are in metres.

a Sketch the graph of this relationship by first finding the intercepts and turning point.

b What was the maximum height of the soccer ball?

c What horizontal distance had the soccer ball travelled when it was at its maximum height?

d What horizontal distance did the soccer ball travel before hitting the ground?

15 During a game of netball, Ayca throws the ball to her teammate Siegrid. The ball's position is described by the equation $h = -4t^2 + 2t + 2$, where h is the height of the ball above the ground in metres and t is the time the ball spends in the air in seconds.

a If Siegrid catches the ball 0.5 seconds after Ayca throws it to her, from what height does Siegrid catch the ball?

b List the coordinates that represent the point at which the ball is thrown and the point at which it is caught. What do you notice about the height of the ball in each case?

c Use your observations from part **b** and the symmetry of the parabola to determine the coordinates for the turning point.

d Hence sketch the graph from $t = 0$ seconds to $t = 0.5$ seconds.

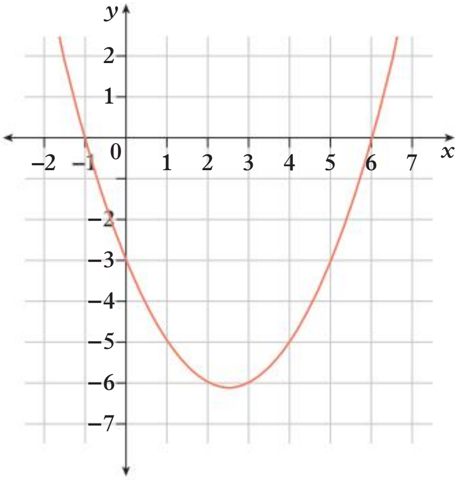
e What is the maximum height of the netball?

16 The equation for a quadratic graph with at least one x-intercept can be expressed in factor form:

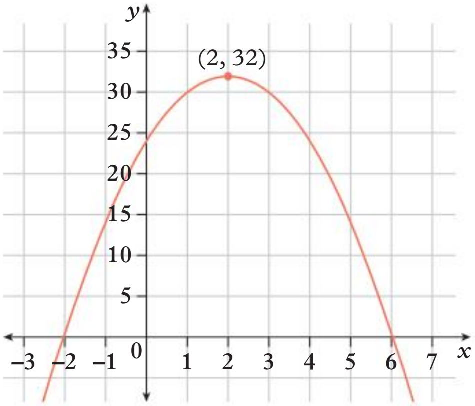
$$y = a(x - p)(x - q)$$

The values of p and q are equal to the x-coordinates of the x-intercepts. The value of a can be determined by substituting the coordinates of any other points on the graph into the values for x and y. Use the factor form to determine the equation of each of the following quadratic graphs.

a



b



17 Find the equation of the parabola that has only one x-intercept at (3, 0) and passes through the point (−1, 6).

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Sketching parabolas using intercepts

Investigation
Fencing the chicken run

Topic quiz
4C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

4A **1** Solve each quadratic equation using the Null Factor Law.

a $(x-4)(x-3)=0$ **b** $(x-2)(x+2)=0$

c $(x+9)(x+9)=0$ **d** $x(x+4)=0$

4A **2** Solve:

a $x^2 - 12x + 35 = 0$ **b** $x^2 + 3x - 28 = 0$

c $x^2 - 12x + 36 = 0$ **d** $x^2 - 81 = 0$

4A **3** Solve the following quadratic equations by first identifying a HCF.

a $2x^2 - 32 = 0$ **b** $8x^2 - 16x - 64 = 0$

c $-2x^2 - 12x - 10 = 0$ **d** $45x - 3x^2 = 0$

4B **4** For the quadratic graph shown identify:

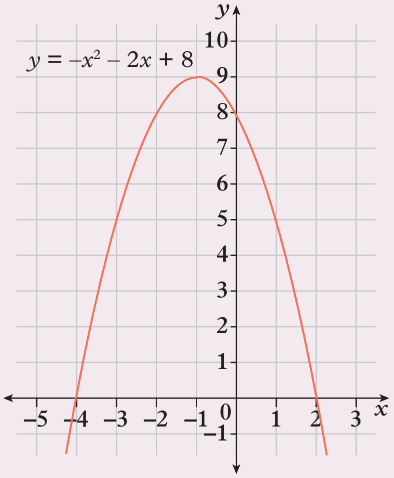
a the coordinates of the x-intercepts

b the coordinates of the y-intercept

c the coordinates of the turning point

d whether it is a maximum or minimum turning point

e the equation of the axis of symmetry.



4B **5** Sketch a graph of $y = 2x^2 - 8$ by first completing a table of values for x from −3 to 3.

4B **6** A quadratic graph has a minimum turning point at (5, 0).

a What is the equation of the axis of symmetry?

b What are the coordinates of the x-intercept(s)?

4C **7** For each quadratic relationship, find the coordinates of the:

i x-intercept(s)

ii y-intercept.

a $y = x^2 + 3x$

b $y = 2x^2 - 50$

c $y = x^2 + 2x - 8$

d $y = x^2 - 16x + 64$

4C **8** Find the coordinates of the turning point for each quadratic relationship.

a $y = (x-2)(x+4)$ **b** $y = x(x-4)$

c $y = (x-6)(x+6)$ **d** $y = -(x-8)(x-12)$

4C **9** Sketch the graph of $y = x^2 - 4x + 3$ by first finding the x- and y-intercepts. Label the turning point with its coordinates.

4C **10** A cannon is fired up into the air as an experiment. The height, h, in metres of the cannonball above the ground is given by $h = 15d - d^2$, where d is the horizontal distance in metres from where the cannonball is fired.

a Sketch the graph of this relationship by first finding the intercepts. Label the turning point with its coordinates.

b How far from the cannon does the cannonball land?

c What is the maximum height reached by the cannonball?

4D Sketching parabolas using transformations

Learning intentions

By the end of this topic you will be able to …

✔ sketch quadratic graphs of the form $y = a(x - h)^2 + k$ using reflections, dilations, vertical translations and horizontal translations.

Inter-year links

Support	The Cartesian plane
Year 7	8E Rotations and reflections
Year 8	7E Transformations
Year 10	4D Graphing parabolas using transformations

Turning point form

- The graph of $y = x^2$ is an upright parabola with a minimum turning point at the origin, (0, 0).
- Quadratic relationships can be written in **turning point form**:

$$y = a(x - h)^2 + k$$

vertical translation of k units

$k > 0$ up

$k < 0$ down

dilation

$a > 0$ upright

$a < 0$ inverted

horizontal translation of h units

$h > 0$ right

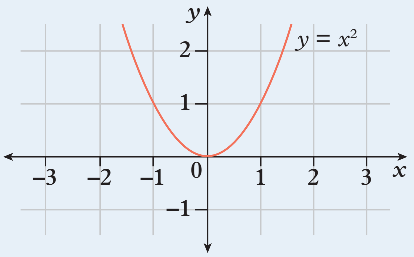
$h < 0$ left

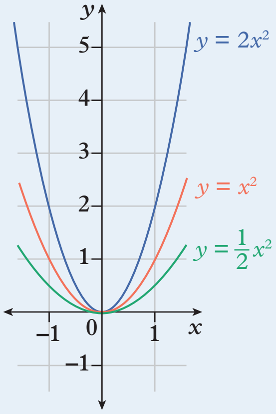
where:

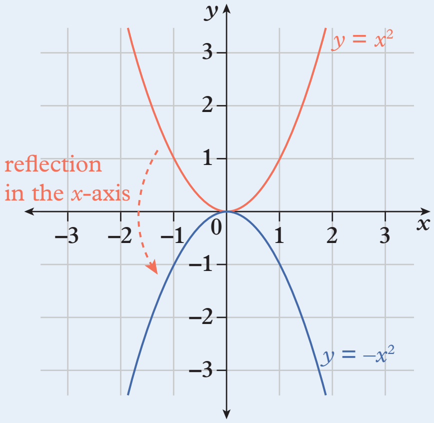
→ the dilation in the y-direction is by a factor of a

→ the coordinates of the turning point are (h, k).

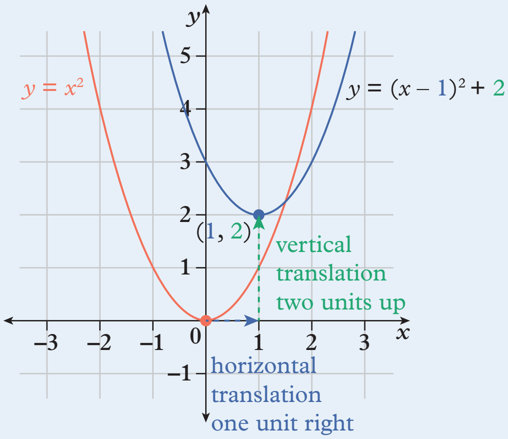
- **Transformations** can be performed on the graph of $y = x^2$ to sketch a graph of $y = a(x - h)^2 + k$.
- A **dilation** by a factor of a in the y-direction will:
  - → *enlarge* the parabola in the y-direction if $a > 1$
  - → *compress* the parabola in the y-direction if $0 < a < 1$.
- A **reflection** in the x-axis will take place if $a = -1$.
- A reflection and a dilation will take place if $a < 0$ and $a \neq -1$.
- Note: Reflections and dilations must be applied before translations.







- A **horizontal translation** will move the parabola h units:
  - → *right* if $h > 0$
  - → *left* if $h < 0$.
- A **vertical translation** will move the parabola k units:
  - → *up* if $k > 0$
  - → *down* if $k < 0$.



Example 4D.1 Sketching a parabola using translations

Sketch the graph of $y = x^2$ on a Cartesian plane, and then perform a translation to sketch the graph of each quadratic relationship. Label the y-axis at the intercept and label the turning point with its coordinates.

a $y = x^2 - 3$

b $y = (x - 4)^2$

THINK

a **1** Identify the transformation. The relationship is in the form $x^2 + k$ where $k = -3$, so the graph of $y = x^2$ undergoes a vertical translation of 3 units down. So the y-intercept is at the turning point.

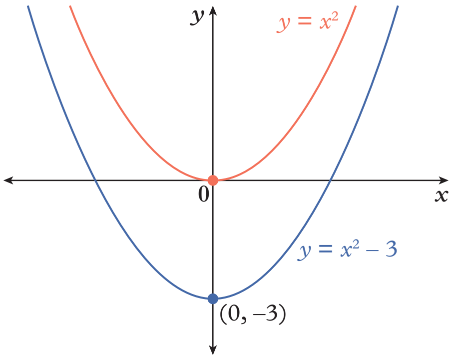
$$y = x^2 \boxed{-\,3}$$

vertical translation of 3 units down

2 Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Translate the turning point 3 units down to find the turning point of $y = x^2 - 3$.

WRITE

a



THINK

b **1** Find the y-coordinate of the y-intercept and mark it on the y-axis.

2 Identify the transformation. The relationship is in the form $(x - h)^2$ where $h = 4$, so the graph of $y = x^2$ undergoes a horizontal translation of 4 units to the right.

$$y = (x \boxed{-\,4})^2$$

horizontal translation of 4 units to the right

3 Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Translate the turning point 4 units to the right to find the turning point of $y = (x - 4)^2$.

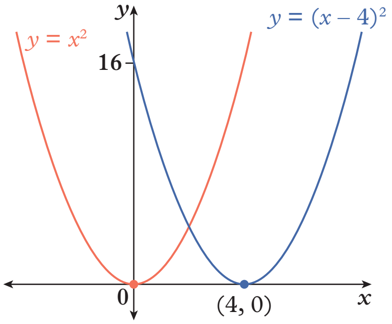
WRITE

b $y = (x - 4)^2$

Let $x = 0$:

$y = (0 - 4)^2$

$= 16$



Example 4D.2 Sketching a parabola using multiple transformations

Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y-axis at the intercept and label the turning point with its coordinates.

a $y = -(x + 3)^2$

b $y = (x - 2)^2 - 1$

THINK

a **1** Find the y-coordinate of the y-intercept and mark it on the y-axis.

2 Identify the transformations. The relationship is in the form $a(x - h)^2$ so the graph of $y = x^2$:

- is inverted as $a = -1$
- is translated 3 units to the left as $h = -3$.

The turning point (h, k) is $(-3, 0)$.

$$y = \underbrace{-}_{\text{inverted}}(x \underbrace{+\ 3}_{\text{horizontal translation of 3 units to the left}})^2$$

3 Sketch the graph.

b **1** Find the y-coordinate of the y-intercept and mark it on the y-axis.

2 Identify the transformations. The relationship is in the form $(x - h)^2 + k$ so the graph of $y = x^2$:

- is translated 2 units to the right as $h = 2$
- is translated 1 unit down as $k = -1$.

The turning point (h, k) is $(2, -1)$.

$$y = (x \underbrace{-\ 2}_{\text{horizontal translation of 2 units to the right}})^2 \underbrace{-\ 1}_{\text{vertical translation of 1 unit down}}$$

3 Sketch the graph.

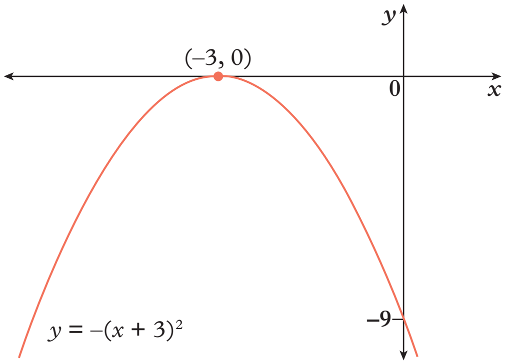
WRITE

a $y = -(x + 3)^2$

Let $x = 0$:

$y = -(0 + 3)^2$

$= -9$



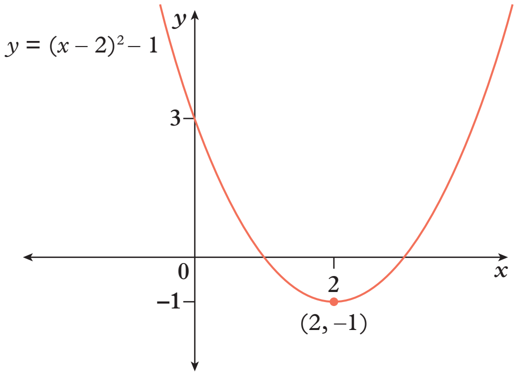
b $y = (x - 2)^2 - 1$

Let $x = 0$:

$y = (0 - 2)^2 - 1$

$= 4 - 1$

$= 3$



Helpful hints

- ✔ In turning point form:
  - → vertical translations are determined by the constant *outside* the brackets
  - → horizontal translations are determined by the constant *inside* the brackets with x.

$$y = a(x - \underbrace{h}_{\text{horizontal translation of } h \text{ units}})^2 + \underbrace{k}_{\text{vertical translation of } k \text{ units}}$$

- ✔ It is always helpful to label the key features of your graph before sketching. The turning point, y-intercept, and a dashed line for the axis of symmetry will help you sketch an accurate parabola.
- ✔ Remember that if a is negative, you must reflect the graph in the x-axis before translating.

ANS p439

Exercise 4D Sketching parabolas using transformations

▲ 1–7, 8(a–d), 9(a, c, e, f), 10, 14, 17(a) ■ 2–6, 8, 9(b, d, f, h), 10, 13–15, 16(a, e), 17(a–c) ⬟ 5, 6, 8(f–i), 9(b, d, f, h), 11, 13, 14, 16(b, d, f, h, j), 17, 18

UNDERSTANDING AND FLUENCY

1 Complete the descriptions of transformations below.

a To sketch the graph of $y = x^2 - 10$, the graph of $y = x^2$ is translated __________ units __________.

b To sketch the graph of $y = (x - 5)^2$, the graph of $y = x^2$ is translated __________ units __________.

c To sketch the graph of $y = (x + 12)^2$, the graph of $y = x^2$ is translated __________ units __________.

d To sketch the graph of $y = x^2 + 8$, the graph of $y = x^2$ is translated __________ units __________.

4D.1 2 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of each quadratic relationship on the same set of axes. Label the y-axis at the intercept and label the turning point on each parabola with its coordinates.

a $y = x^2 + 3$ **b** $y = x^2 + 1$ **c** $y = x^2 - 2$

d $y = x^2 + 6$ **e** $y = x^2 - 4$ **f** $y = x^2 + 9$

3 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a horizontal translation to sketch the graph of each quadratic relationship on the same set of axes. Label the y-axis at the intercept and label the turning point on each parabola with its coordinates.

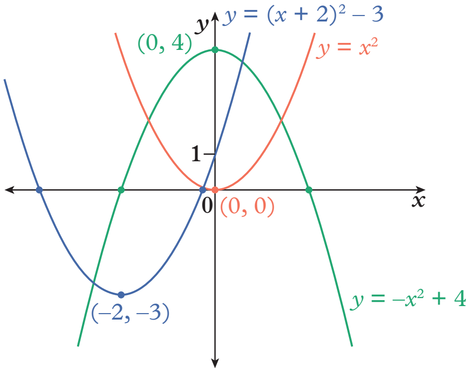
a $y = (x - 3)^2$ **b** $y = (x - 1)^2$ **c** $y = (x + 2)^2$

d $y = (x + 4)^2$ **e** $y = (x - 5)^2$ **f** $y = (x + 7)^2$

4 Complete the following descriptions of transformations.

a To sketch the graph of $y = (x + 2)^2 - 3$, the graph of $y = x^2$ is translated __________ units __________ and __________ units __________.

b To sketch the graph of $y = -x^2 + 4$, the graph of $y = x^2$ is __________ across the x-axis and translated __________ units __________.



4D.2 5 Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y-axis at the intercept and label the turning point on each parabola with its coordinates.

a $y = -(x - 2)^2$ **b** $y = (x - 2)^2 + 3$

c $y = -(x + 6)^2$ **d** $y = -x^2 + 4$

e $y = (x - 1)^2 - 2$ **f** $y = -x^2 - 3$

g $y = -(x + 4)^2 + 6$ **h** $y = -(x - 7)^2 - 5$

6 For each quadratic relationship:

i identify whether its graph will be an upright or inverted parabola

ii find the coordinates of the x- and y-intercepts

iii write the coordinates of the turning point

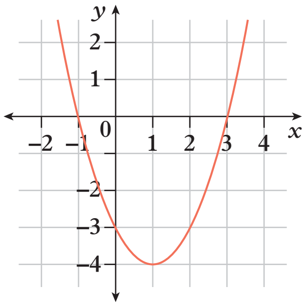
iv sketch the graph of the relationship.

a $y = (x - 2)^2 - 1$ **b** $y = -(x + 1)^2 + 9$

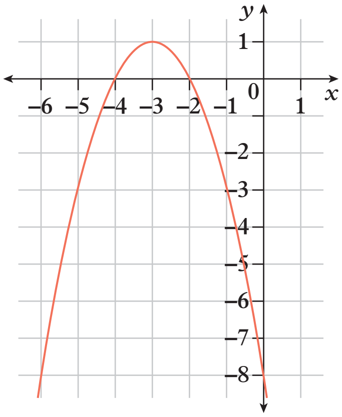
c $y = (x - 3)^2 - 4$ **d** $y = -(x + 4)^2 + 1$

7 Consider the quadratic graphs below.

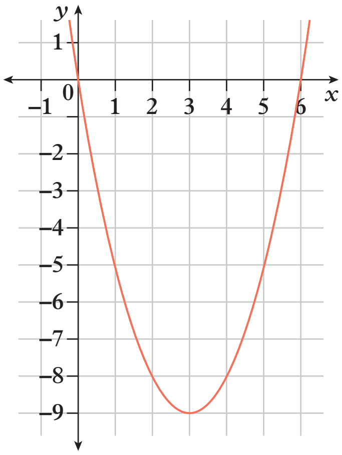
a



b



c



i State whether the parabola is upright or inverted.

ii Identify the coordinates of the turning point.

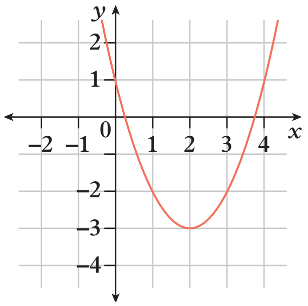
If each graph has an equation of the form $y = a(x - h)^2 + k$:

iii decide whether a is positive or negative

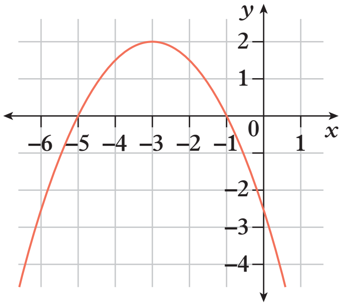
iv determine the values of h and k.

8 Match each graph with its equation from the list provided below.

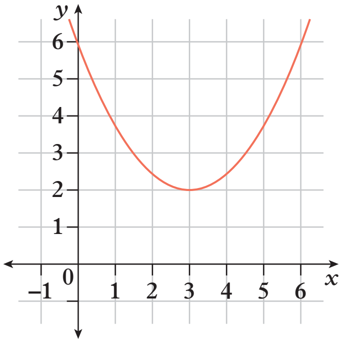
a



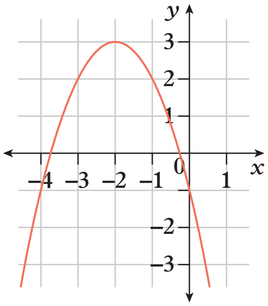
b



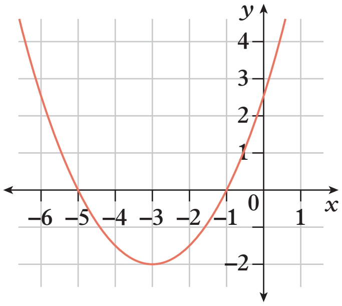
c



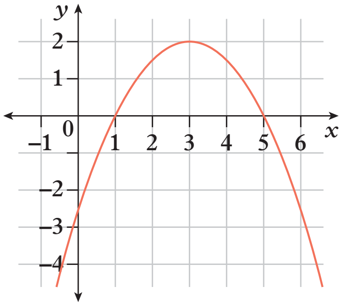
d



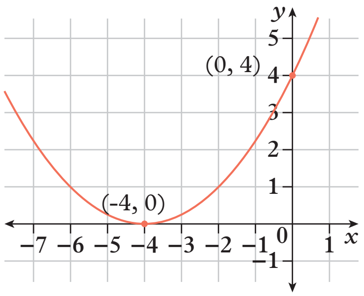
e



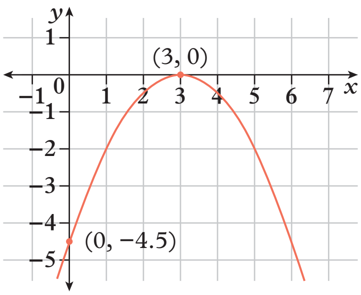
f



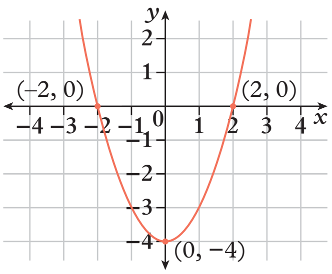
g



h



i



A $y = (x - 3)^2 + 2$

B $y = -(x - 3)^2 + 2$

C $y = (x + 3)^2 - 2$

D $y = -(x + 3)^2 + 2$

E $y = (x - 2)^2 - 3$

F $y = -(x + 2)^2 + 3$

G $y = x^2 - 4$

H $y = \frac{1}{4}(x + 4)^2$

I $y = -\frac{1}{2}(x - 3)^2$

9 Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is +1 or −1.

a Upright, turning point at (3, 7)

b Upright, turning point at (−2, 5)

c Inverted, turning point at (2, 4)

d Inverted, turning point at (6, −1)

e Upright, turning point at (9, 0)

f Inverted, turning point at (0, 4)

g Inverted, turning point at (−1, −2)

h Upright, turning point at (0, −5)

10 Consider the quadratic relationship $y = (x - 4)^2 + 5$.

a What are the coordinates of the turning point?

b What is the smallest y-value that this relationship can have?

c Is there a maximum value for y? Explain your answer.

11 Consider the quadratic relationship $y = -(x + 1)^2 + 2$. What is the maximum y-value in this relationship? Explain.

12 Jenna throws a netball to a teammate. The height of the ball can be represented by the relationship $h = -(d - 2)^2 + 6$, where h is the height in metres and d is the horizontal distance from where Jenna throws the ball.

a What are the coordinates of the turning point of this relationship?

b What is the value of h when:

  i $d = 0$?

  ii $d = 4$?

c Sketch the graph of this relationship from $d = 0$ to $d = 4$.

d Use the graph to find:

  i the height at which the ball left Jenna's hands

  ii the maximum height of the ball during the pass to her teammate.

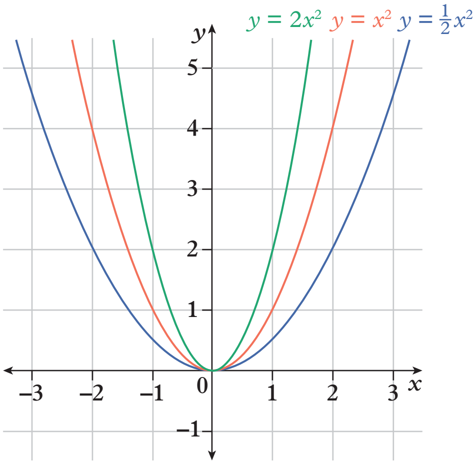
13 The path of a firework follows the relationship $y = -(x - 10)^2 + 100$, where y is the height of the firework for a horizontal distance x from where it was launched. Both x and y are in metres.

a Sketch the graph of this relationship from when the firework was launched to when it landed on the ground after exploding. Hint: Use the fact that a parabola is symmetrical.

b What was the maximum height of the firework?

c How far from the launch pad did the firework land?

14 Consider the graphs of the form $y = ax^2$.



a What is the value of the dilation factor, a, for $y = x^2$?

b Which graph is narrower than the graph of $y = x^2$? State the value of the graph's dilation factor.

c Which graph is wider than the graph of $y = x^2$?

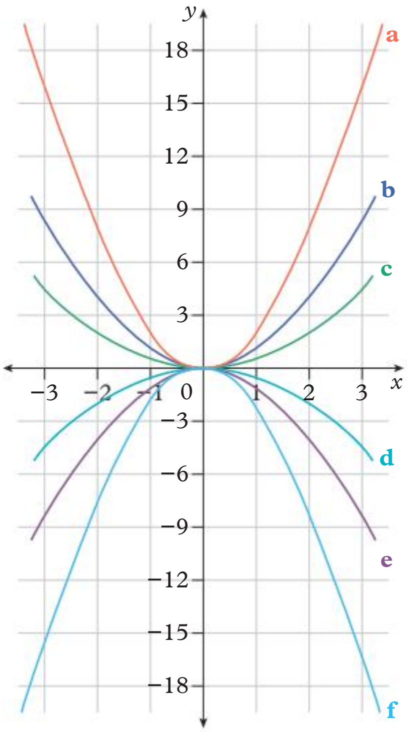
d Describe how the value of the dilation factor affects the width of the corresponding parabola compared to the parabola $y = x^2$.

PROBLEM SOLVING AND REASONING

15 Match each graph on the Cartesian plane to the right with its equation from the list provided below.

A $y = x^2$ **B** $y = -2x^2$ **C** $y = -\frac{1}{2}x^2$

D $y = \frac{1}{2}x^2$ **E** $y = 2x^2$ **F** $y = -x^2$



16 For each quadratic relationship, identify:

- **i** whether the graph will have wider, narrower or equal width to the graph of $y = x^2$
- **ii** whether the parabola will be upright or inverted
- **iii** the coordinates of the turning point.

a $y = 2(x - 4)^2 - 3$

b $y = \frac{1}{2}(x + 2)^2 + 6$

c $y = 4(x + 2)^2$

d $y = -5x^2 - 4$

e $y = -\frac{1}{2}(x - 7)^2$

f $y = -3(x + 1)^2 + 5$

g $y = -\frac{1}{3}(x - 3)^2 - 4$

h $y = \frac{1}{4}x^2 + 3$

i $y = -(x - 5)^2 + 4$

j $y = 4(x + 3) - 7$

17 Write an equation for the parabola produced after performing each set of transformations on the graph of $y = x^2$.

a Dilation by a factor of 3, then a translation of 2 units right

b Reflection in the x-axis, then a translation of 4 units down and 5 units left

c Dilation by a factor of $\frac{1}{2}$, then a reflection in the x-axis

d Dilation by a factor of 4, reflection in the x-axis, then a translation of 2 units left

CHALLENGE

18 A ball is kicked during a competition to see who can kick it the highest. The height of the ball, h, in metres over a horizontal distance, d, in metres is given by $h = -(d - 3)^2 + 9$.

a Sketch a graph showing the path of the ball. Label the turning point with its coordinates.

b What is the maximum height reached by the ball?

c When the ball lands, how far away is it from where it was kicked?

A second ball is kicked from the same point. It travels 4 metres further than the first ball.

d Assuming the second ball also follows a parabolic pathway with a dilation factor of 1, determine the equation for the path of the second basketball.

e On the same set of axes used in part **a**, sketch the path of the second ball. Label the turning point with its coordinates.

f How much higher does the second ball reach than the first ball?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Sketching parabolas using transformations

Worksheet Quadratic relationships

Topic quiz 4D

4E Other non-linear relationships

Learning intentions

By the end of this topic you will be able to ...

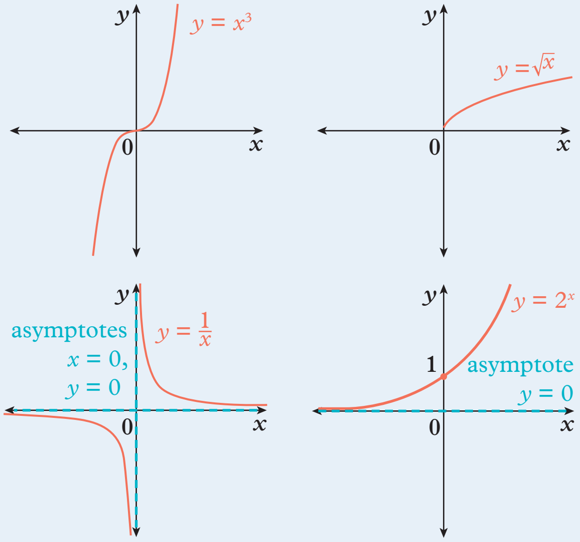
✔ sketch non-linear relationships using transformations.

Inter-year links

Support	The Cartesian plane
Year 7	8D Translations
Year 8	7E Transformations
Year 10	4E Graphing circles

Non-linear relationships

- Linear relationships produce straight-line graphs. **Non-linear relationships** produce curved-line graphs.



- For all linear and non-linear relationships:
  - → The x-intercept(s) are the point(s) where the graph crosses the x-axis and $y = 0$.
  - → The y-intercept(s) are the point(s) where the graph crosses the y-axis and $x = 0$.

Square root of a number

- Many **non-linear equations** are solved by taking the square root of a number. In these cases, it is important to understand that there are two possible solutions for the square root of a positive number.
- The square root of a number has a positive and negative solution.

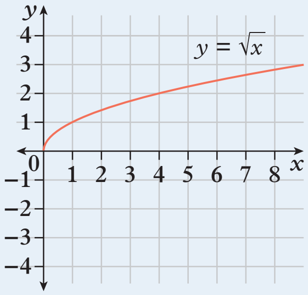
  For example, $2 \times 2 = 4$ and $-2 \times (-2) = 4$, so the equation $x^2 = 4$ has two solutions: $x = 2$ and $x = -2$
- The **plus-minus symbol**, ±, is used as shorthand to write both results in an efficient way.

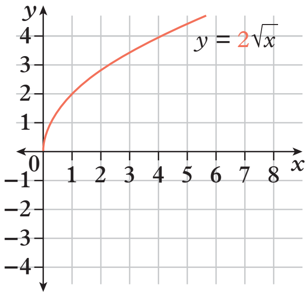
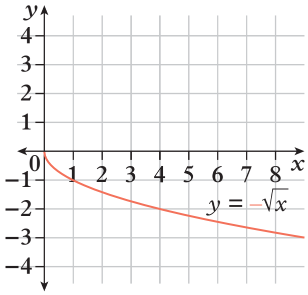
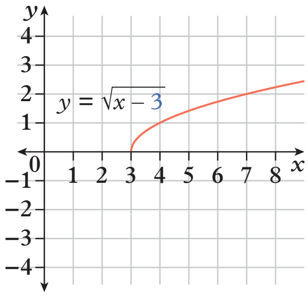
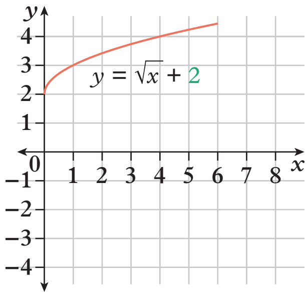
  For example, $x^2 = 4$

  $$x = \pm\sqrt{4}$$
  $$= \pm 2$$
- The graph of $y^2 = x$ is a sideways parabola, where $y = \sqrt{x}$ represents its upper half with positive y values and $y = -\sqrt{x}$ represents the lower half with negative y values.
- The graph of $y = \sqrt{x}$ does not have a plus-minus symbol and so it implicitly takes the positive root of x.

General transformations

- Transformations can be performed on any non-linear relationship of the form $y = \ldots$, much like the transformations applied to the turning point form of a quadratic graph.



Transformation on equation	Transformation on graph	Example
Multiply the RHS by a constant, a.	Dilate by a factor of a in the y-direction.	
Multiply the RHS by -1.	Reflect in the x-axis.	
Substitute $x - h$ for x.	Translate h units in the x-direction.	
Add k to the RHS.	Translate k units in the y-direction.	

Example 4E.1 Finding the coordinates of the x- and y-intercepts

For each non-linear relationship, find the coordinates of the:

i x-intercept **ii** y-intercept.

a $y = 2x^3$ **b** $y = 2 + \sqrt{x + 3}$

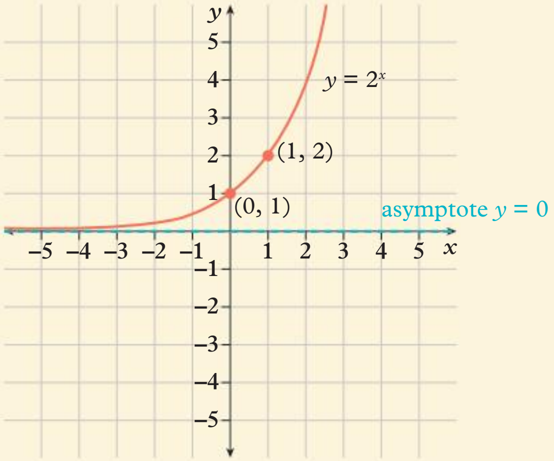
THINK	**WRITE**
a i To find the x-coordinate of the x-intercept, substitute $y = 0$ into the given equation and solve for x.	**a i** For x-intercept, let $y = 0$: $2x^3 = 0$ $x^3 = 0$ $x = 0$ x-intercept: $(0, 0)$
ii To find the y-coordinate of the y-intercept, substitute $x = 0$ into the given equation and solve for y.	**ii** For y-intercept, let $x = 0$: $y = 2(0)^3$ $y = 0$ y-intercept: $(0, 0)$
b i To find the x-coordinate of the x-intercept, substitute $y = 0$ into the given equation and solve for x. The LHS of the equation is a non-negative number, which means the equation has no solution.	**b i** For x-intercept, let $y = 0$: $2 + \sqrt{x + 3} = 0$ $\sqrt{x + 3} = -2$ There's no real solution for x. $y = \sqrt{x + 3}$ does not have any x-intercept.
ii To find the y-coordinate of the y-intercept, substitute $x = 0$ into the given equation and solve for y.	**ii** For y-intercept, let $x = 0$: $y = 2 + \sqrt{(0 + 3)}$ $y = 2 + \sqrt{3}$ y-intercept: $(0, 2 + \sqrt{3})$

Example 4E.2 Sketching graphs using transformations

Sketch the following non-linear relationships by transforming the graph of $y = 2^x$.

a $y = 1 + 2^x$

b $y = -2^x$



THINK

a **1** Identify the transformations. 1 is added to the right-hand side of the equation, so the graph undergoes a vertical translation of 1 unit up.

2 Identify the corresponding coordinates of the key points. The whole graph has been moved up by 1 unit, so for the same x-coordinate, add 1 unit to the y-coordinate. The corresponding coordinates for (0, 1) and (1, 2) on the graph of $y = 1 + 2^x$ are (0, 2) and (1, 3).

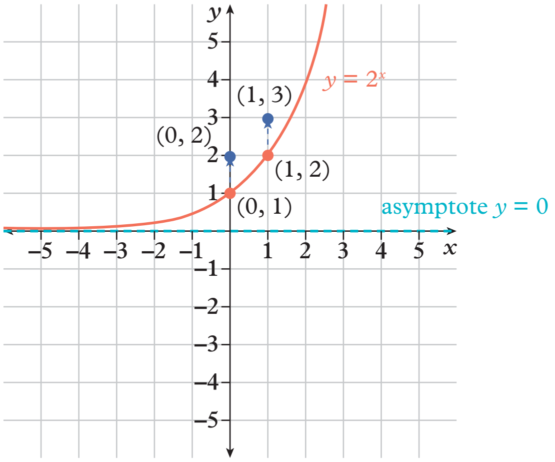
3 Sketch the new graph following the shape of $y = 2^x$, ensure the graph passes through the points (0, 2) and (1, 3) and that the asymptote is also move up one unit.

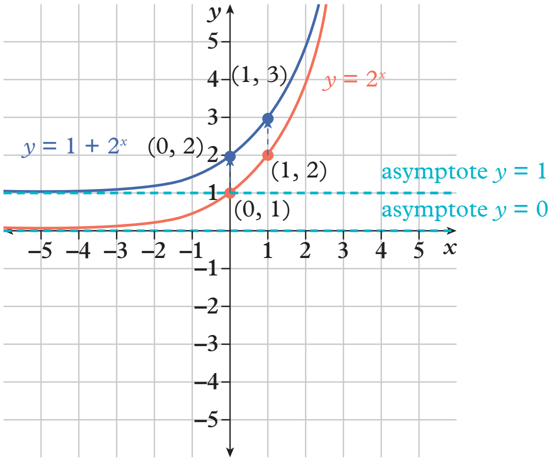
b **1** Identify the transformations. The right-hand side of the equation is multiplied by −1, so the graph is reflected in the x-axis.

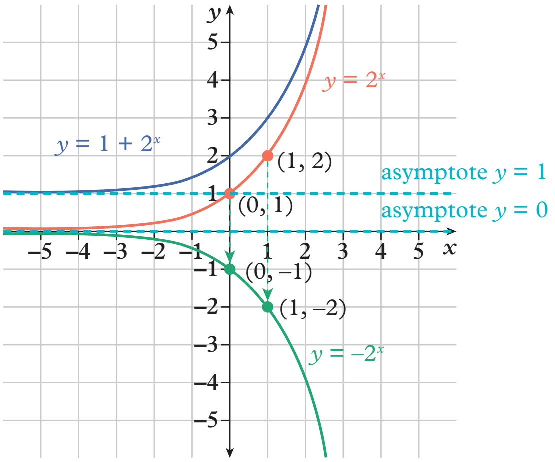
2 Identify the corresponding coordinates of the key points. The whole graph has been reflected in the x-axis, so for the same x-coordinate, the y-coordinate is multiplied by −1. The corresponding coordinates for (0, 1) and (1, 2) on the graph of $y = -2^x$ are (0, −2) and (1, −3)

3 Reflect the graph of $y = 2^x$ in the x-axis, ensuring the graph passes through the points (0, −2) and (1, −3).

WRITE







Helpful hints

✔ Remember that the values of h and k are the opposite sign to how they appear in the equation. For example, in the equation $y = \sqrt{x + 2} + 4$, $h = -2$ and $k = +4$.

✔ Non-linear relationships may have 0, 1 or multiple x-intercepts and y-intercepts.

ANS p441

Exercise 4E Other non-linear relationships

△ 1–6, 8, 9 □ 2–5, 8–10 ⬠ 3–5, 7–9, 11–13

UNDERSTANDING AND FLUENCY

1 Match the table of values with the correct graph.

a

x	-2	-1	0	1	2
y	undefined	undefined	0	1	$\sqrt{2}$

b

x	-2	-1	0	1	2
y	-8	-1	0	1	8

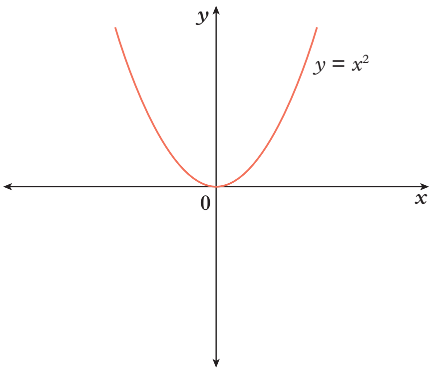
c

x	-2	-1	0	1	2
y	4	1	0	1	4

d

x	-2	-1	0	1	2
y	-0.5	-1	undefined	1	0.5

A



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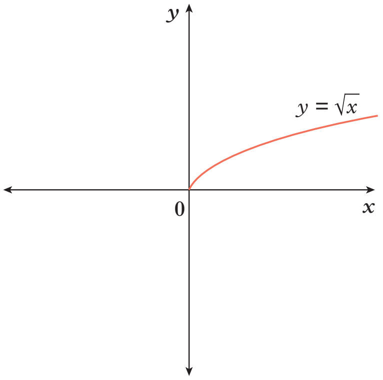
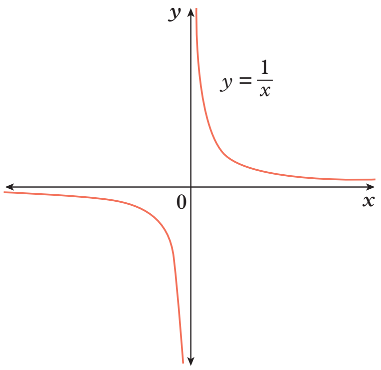
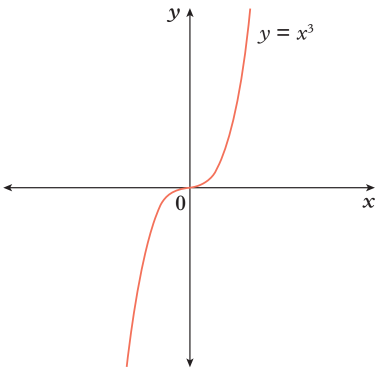


C



D



2 Use the graph and table of values in question **1** to write the coordinates of the x- and y-intercept(s) of the following non-linear relationships (if there are any).

a $y = x^2$ **b** $y = \sqrt{x}$ **c** $y = \frac{1}{x}$ **d** x^3

4E.1 **3** For each non-linear relationship, find the coordinates of the:

i x-intercept **ii** y-intercept.

a $y = 3x^4$ **b** $y = \sqrt{x} + 5$ **c** $y = -2x^3$ **d** $y = \sqrt{2x}$

4 Write the coordinates of the corresponding points you found in question **3** if the following operations are performed on the graphs.

i Reflected in the x-axis

ii Translated 2 units in the x-direction

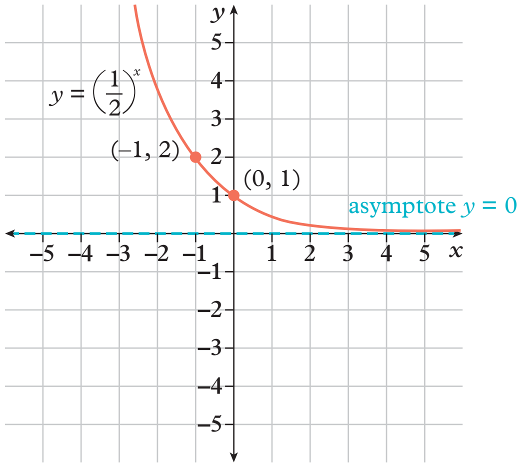
iii Translated -5 units in the y-direction

iv Translated 3 units in the y-direction first, then 3 units in the x-direction

UNDERSTANDING AND FLUENCY

4E.2 **5** Sketch the following non-linear relationships by transforming the graph of $y = \left(\frac{1}{2}\right)^x$.

a $y = -\left(\frac{1}{2}\right)^x$ **b** $y = \left(\frac{1}{2}\right)^x + 1$ **c** $y = \left(\frac{1}{2}\right)^{x+1}$



PROBLEM SOLVING AND REASONING

6 Complete the descriptions of transformations below.

a To sketch the graph of $y = \frac{1}{x} + 20$, the graph of $y = \frac{1}{x}$ is translated __________ units __________.

b To sketch the graph of $y = \frac{1}{x - 15}$, the graph of $y = \frac{1}{x}$ is translated __________ units __________.

c To sketch the graph of $y = -1 + \sqrt[3]{x}$, the graph of $y = \sqrt[3]{x}$ is translated __________ units __________.

d To sketch the graph of $y = \sqrt[3]{2 + x}$, the graph of $y = \sqrt[3]{x}$ is translated __________ units __________.

7 Write the equation of the new non-linear relationship if the following transformations are performed on $y = 5^x$.

a The graph is stretched by a factor of 3 in the y-direction.

b The graph is reflected in the x-axis.

c The graph is translated 3 units in the x-direction.

d The graph is translated −2 units in the y-direction.

8 Consider the non-linear relationship $y = \sqrt{x}$, where only the positive roots of x are taken.

x	0	1	4	9	16	25
y	0					

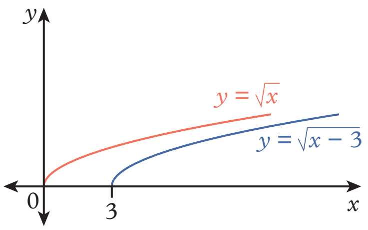
a Complete the table of values for the relationship $y = \sqrt{x}$.

b Use the table of values to construct a plot of $y = \sqrt{x}$. Join the plotted points with a smooth curve.

c Describe the shape of the graph.

d An important feature of the graph of $y = \sqrt{x}$ is the point on the graph where y is a minimum. Label this point with its coordinates on your graph.

e The graphs of $y = \sqrt{x}$ and $y = \sqrt{x - 3}$ are shown to the right.



i Identify the coordinates of the point on the graph of $y = \sqrt{x - 3}$ where y is a minimum.

ii Describe the translation required to produce the graph of $y = \sqrt{x - 3}$ from the graph of $y = \sqrt{x}$.

f Use your understanding of translations to describe how the graphs of these relationships can be produced from the graph of $y = \sqrt{x}$.

i $y = \sqrt{x} + 2$ **ii** $y = \sqrt{x} - 1$ **iii** $y = \sqrt{x - 1}$ **iv** $y = \sqrt{x + 4}$ **v** $y = \sqrt{x - 2} + 3$

g Use your answers to part **f** to sketch each relationship. Label the point on each graph where y is a minimum with its coordinates on each graph.

PROBLEM SOLVING AND REASONING

9 Consider the basic cubic relationship, $y = x^3$.

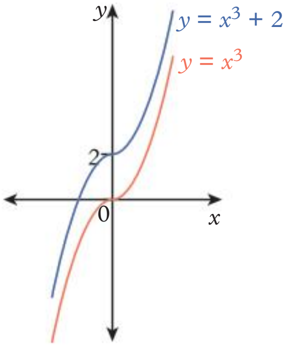
x	-3	-2	-1	0	1	2	3
y	-27						

a Complete the table of values for the relationship $y = x^3$.

b An important feature of the graph of $y = x^3$ is the stationary point at $(0, 0)$, where the slope of the graph is flat (gradient = 0) at a single point. Use the table of values to construct a plot of $y = x^3$. Label the point of inflection with its coordinates on your graph and join the plotted points with a smooth curve.

c Describe the shape of the graphs.

d The graphs of $y = x^3$ and $y = x^3 + 2$ are shown below.



i Identify the coordinates of the point of inflection for $y = x^3 + 2$.

ii Describe the translation required to produce the graph of $y = x^3 + 2$ from the graph of $y = x^3$.

e Use your understanding of translations to describe how the graphs of these cubic relationships can be produced from the graph of $y = x^3$.

i $y = x^3 + 1$ **ii** $y = x^3 - 3$ **iii** $y = (x - 2)^3$ **iv** $y = (x + 4)^3$ **v** $y = (x - 1)^3 + 2$

f Use your answers to part **e** to sketch each relationship. Label the point of inflection with its coordinates on each graph.

10 Consider the relationships in questions **8–9**.

a Sketch the graphs of $y = -x^3$, $y = -x^3 + 4$ and $y = -(x - 3)^3$ on the same Cartesian plane.

b Sketch the graphs of $y = -\sqrt{x}$, $y = -\sqrt{x} + 3$ and $y = -\sqrt{x - 4}$ on the same Cartesian plane.

11 Use your knowledge of transformations to sketch the graph of each of the following relationships by first finding any x- and y-intercepts. Label all intercepts with their coordinates.

a $y = 3\sqrt{x + 1} - 6$ **b** $y = (x + 2)^3 - 1$

CHALLENGE

12 a Sketch a graph of $y = \sqrt{x}$ and $y = -\sqrt{x}$ on the same Cartesian plane.

b Write one equation to describe all the points on both graphs. What is the shape of the graph for this equation?

c Sketch a graph of $y = x^2$ on the same Cartesian plane and label the coordinates where the two graphs intersect each other.

d Draw a dotted line that passes through the points where the two graphs intersect each other. Write an equation for this line.

e Describe the transformation required to turn the graph of $y = x^2$ to the graph in part **b**.

13 Use your observations from question **12** to sketch the graph for $x = y^3$.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Non-linear relationships

Worksheet
Non-linear relationships

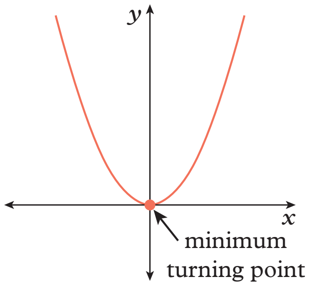
Topic quiz
4E

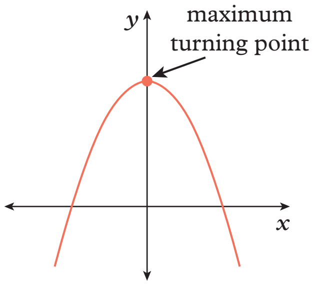
Chapter summary

The Null Factor Law

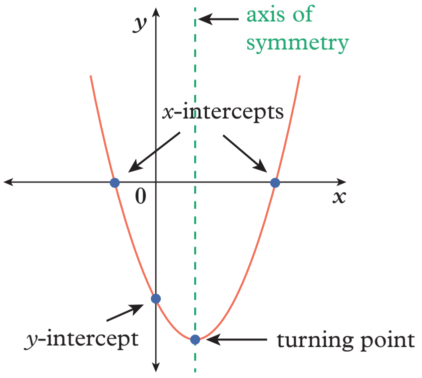
- If $a \times b = 0$, then $a = 0$ or $b = 0$.
- If $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$.

Turning points





Parabolas



Turning point form

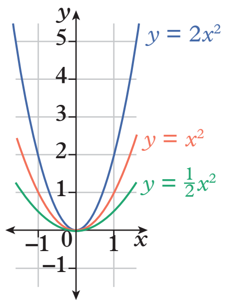
$$y = a(x - h)^2 + k$$

vertical translation of k units
If $k > 0$, up
If $k < 0$, down

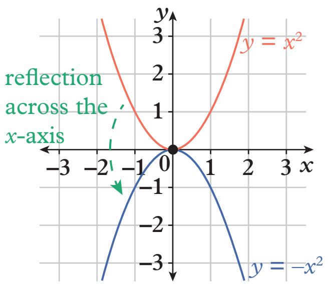
$a > 0$ upright
$a < 0$ inverted

horizontal translation of h units
If $h > 0$, right
If $h < 0$, left

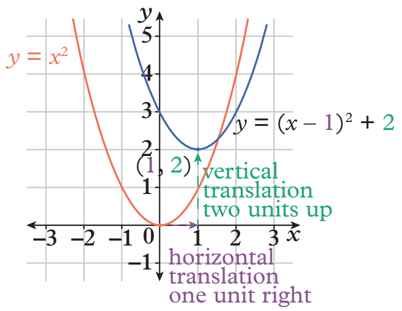
Dilations in the y-direction



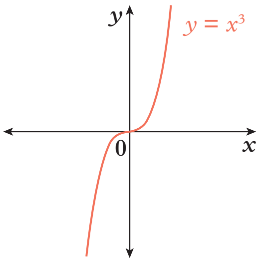
Reflection

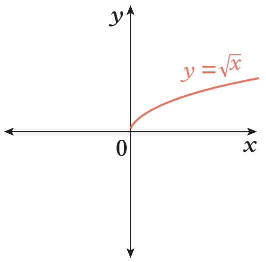


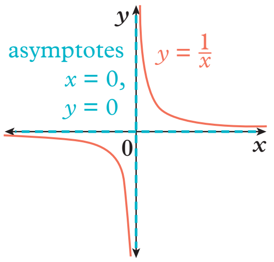
Translation

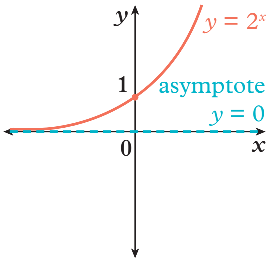


Non-linear relationships









Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

4A **1** The solution(s) to the quadratic equation $x(x + 1) = 0$ are:

A $x = -1$ **B** $x = 1$ **C** $x = -1$ or $x = 1$ **D** $x = 0$ or $x = -1$ **E** $x = 0$ or $x = 1$

4A **2** In the quadratic equation $x^2 - 13x - 48 = 0$, $x =$

A -3 or 16 **B** -16 or 3 **C** -12 or 4 **D** -6 or 8 **E** -8 or 6

4B **3** If a parabola intersects the x-axis when $x = -1$ and -5, the turning point could be:

A $(-3, 7)$ **B** $(-2, 2)$ **C** $(0, -3)$ **D** $(2, -6)$ **E** $(3, 4)$

4C **4** The coordinates of the x-intercepts of the graph of $y = x^2 - 4x - 12$ are:

A $(0, -6), (0, 2)$ **B** $(0, 6), (0, -2)$ **C** $(-6, 0), (2, 0)$ **D** $(6, 0), (-2, 0)$ **E** $(6, 0), (2, 0)$

4C **5** The y-coordinate of the y-intercept of the graph of $y = (x - 5)(x + 2)$ is:

A 0 **B** -2 **C** -10 **D** 5 **E** 10

4D **6** The graph of $y = x^2$ is translated 5 units down. The equation of the translated graph is:

A $y = x^2 + 5$ **B** $y = 5x^2$ **C** $y = x^2 - 5$ **D** $y = (x + 5)^2$ **E** $y = \frac{1}{5}x^2$

4D **7** Which of the following transformations is **not** required to sketch the graph of $y = -4(x + 2)^2 + 1$?

A $y = x^2$ must be dilated.
B $y = x^2$ must be reflected in the x-axis.
C $y = x^2$ must be translated up.
D $y = x^2$ must be translated to the left.
E $y = x^2$ must be rotated 90° in the clockwise direction.

4D **8** The coordinates of the turning point of the graph of $y = -(x - 4)^2$ are:

A $(0, 0)$ **B** $(4, 0)$ **C** $(-4, 0)$ **D** $(0, 4)$ **E** $(0, -4)$

4E **9** The y-intercept of the relationship $y = 4^x$ is $(0, 1)$. What is the y-intercept of $y = 2 + 4^x$?

A $(0, 1)$ **B** $(0, 2)$ **C** $(0, 3)$ **D** $(0, -1)$ **E** $(2, 1)$

4E **10** What is the y-intercept of $y = \frac{1}{(x - 3)}$?

A $(0, 3)$ **B** $(3, 0)$ **C** $(0, \frac{1}{3})$ **D** $(0, -\frac{1}{3})$ **E** $(\frac{1}{3}, 0)$

Short answer

4A **1** Solve:

a $x^2 - 5x + 6 = 0$ **b** $x^2 + x - 30 = 0$ **c** $x^2 + 9 = 0$ **d** $3x^2 - 36x = 0$

4B **2** Sketch a graph of the following relationships by first completing a table of values for x from -3 to 3. Use your graph to identify:

i the coordinates of the x- and y-intercepts
ii the coordinates of the turning point
iii whether there is a maximum or a minimum turning point
iv the equation of the axis of symmetry.

a $y = 4x^2 - 4$ **b** $y = -4x^2 - 4x$ **c** $y = x^2 - 4x + 4$

4C **3** Complete the following for each quadratic relationship below.

i Find the coordinates of the x- and y-intercepts.
ii Find the coordinates of the turning point.
iii Sketch the graph.

a $y = -x^2 - 4x$ **b** $y = x^2 - x - 12$
c $y = (x + 5)(x - 4)$ **d** $y = -(x + 2)(x + 1)$

4D **4** Sketch a graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y-axis at the intercept and label the turning point with its coordinates.

a $y = x^2 + 3$ **b** $y = (x - 1)^2$ **c** $y = x^2 - 5$ **d** $y = (x + 4)^2$

4D **5** Sketch the graph by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.

a $y = (x + 1)^2 - 1$ **b** $y = -(x + 3)^2 - 3$ **c** $y = (x - 4)^2 + 4$ **d** $y = -(x - 2)^2 - 2$

4E **6** For each of the relationships below, describe the translation performed on $y = 3^x$.

a $y = 3 + 3^x$ **b** $y = 3^{x-2}$

4E **7** For each of the relationships below, describe the translation performed on $y = \frac{2}{x}$.

a $y = -\frac{2}{x}$ **b** $y = 2 + \frac{2}{x}$

Analysis

A parabolic through-arch bridge has an upper arch and a lower arch that are closely modelled by the following quadratic relationships:

$$\text{Lower arch: } h = -\frac{1}{120}d^2 + d$$

$$\text{Upper arch: } h = -\frac{1}{180}(d - 60)^2 + 40$$

where h is the height of the arch at a horizontal distance of d from the left side of the bridge. Both h and d are in metres.

a Sketch the two arches on the same Cartesian plane from $d = 0$ m to $d = 120$ m.

b For each parabola, determine:

  i the coordinates of the y-intercepts

  ii the coordinates of the turning points

  iii the equation of the axis of symmetry.

c What is the height of the upper arch above the lower arch at each end of the bridge?

d What is the span of the bridge? Show this:

  i graphically, by referring to your graphs in part **a**

  ii algebraically, by solving a quadratic equation.

The distance between the arches does not remain constant over the span of the bridge.

e Find the height of the highest point of the:

  i lower arch **ii** upper arch.

f How far apart are the two arches at their highest point?

g How far apart are the two arches at a horizontal distance of 30 m from the left side of the bridge?

h Describe the distance between the two arches over the span of the bridge.

Semester 1 review

Short answer

1 Round the following as specified.

a 20.040 194 (3 decimal places)

b 20.040 194 (3 significant figures)

c 0.000 782 718 (4 decimal places)

d 0.000 782 718 (4 significant figures)

2 Simplify the following.

a $\sqrt{32}$

b $\sqrt{180}$

c $\sqrt{84}$

d $\sqrt{1008}$

3 Simplify each of the following. (Write as an expression with a single surd.)

a $\sqrt{3} \times \sqrt{11}$

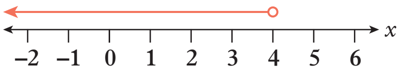
b $\sqrt{45} \div \sqrt{15}$

c $7\sqrt{2} \times 4\sqrt{6}$

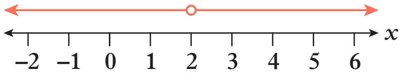
d $\dfrac{24\sqrt{200}}{8\sqrt{20}}$

4 State the inequality represented on each of the following number lines.

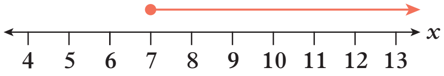
a



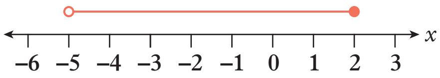
b



c



d



5 Solve each of the following inequalities. Show the solution on a number line.

a $4x + 5 \neq 17$

b $-6x + 4 > 28$

c $8 - 4x \leq 5$

d $5(2x - 3) \leq 4x$

6 Write the following numbers in exponent form (as the product of powers of prime factors).

a 81

b 500

c 216

d 2700

7 Write the following as a basic numeral.

a 7^3

b 3×5^3

c 8.213×10^4

d 1.0530×10^{-2}

e $2^{-3} \times 3^3$

f $\left(\dfrac{5}{7}\right)^{-2}$

8 Write the following in exponent form with positive exponents.

a $(2^3 \times 7^2)^4 \times (2^9 \times 7^5)^2$

b $\dfrac{11^{12} \times 13^9 \times 17^6}{11^4 \times 13^9 \times 17^{15}}$

c $101^{-5} \times 97^5 \times 101^{-7} \times 97^{-9}$

d $\left(\dfrac{41^{-3}}{57^5}\right)^{-8} \times \left(\dfrac{57^{-4}}{41^{-7}}\right)^6$

9 Simplify the following. Write your answer using positive exponents.

a $(ab^3)^4 \times (ba^6)^2$

b $\dfrac{c^2de^8}{c^{12}de^7}$

c $f^{-4}g^6 \times f^3g^{-2}$

d $\dfrac{(p^8q^{-4})^3 \times (p^{-5}q^3)^8}{(p^{-3}q^{-6})^2 \times (p^{-2}q^{-9})^3}$

10 State the number of significant figures.

a 946 025

b 120 000 000

c 0.002 520 20

d 630.00

11 Write the following numbers in scientific notation.

a 751 425 963, correct to four significant figures

b 25.360 141, correct to five significant figures

c 0.000 085 945 6, correct to two significant figures

d 75 000, correct to three significant figures

12 Expand the following products.

a $8(7 - 2x)$

b $-2x^3(3x^2 - 8)$

c $(x + 2)(x - 7)$

d $(5x - 2)(2y + 3)$

e $5 - 4(3x + 7)$

f $2x(x + 9) + 3(x^2 - 4x + 2)$

13 Factorise the following expressions.

a $12a + 18b^2 - 6cd$

b $21g^2 - 7g$

c $a^2 - 25$

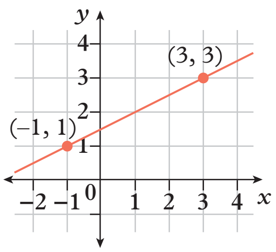
d $8m^2 - 18n^2$

e $(t + 2)^2 - 49$

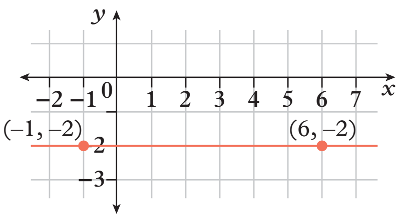
f $r^2 + 5r - 36$

14 Calculate the gradient of the following.

a



b



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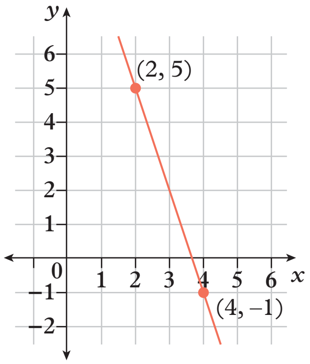

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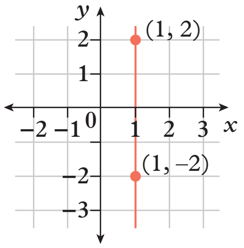

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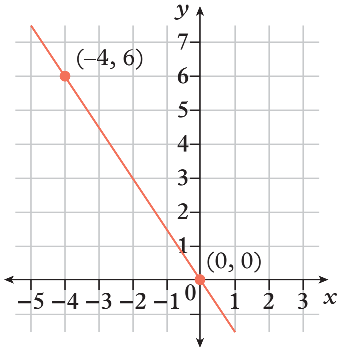


15 Solve the following equations for x.

a $12x - 7 = 4$

b $9 - 2x = 15$

c $4x + 9 = 6x - 7$

d $\frac{21}{x} = 3$

e $\frac{4}{x} = 5$

f $(x + 3)(x - 7) = 0$

g $(9x - 1)(5x + 3) = 0$

h $x^2 + 12x + 35 = 0$

16 Plot the graphs of the following equations by completing each table of values and plotting the coordinates.

a $y = 3x - 4$

x	0	1	2	3	4
y					

b $y = -\frac{5}{2}x$

x	−4	−2	0	2	4
y					

c $y = (x + 1)^2 - 5$

x	−4	−3	−2	−1	0	1	2
y							

d $y = -x^2 + 6x - 2$

x	0	1	2	3	4	5	6
y							

17 Determine the x- and y-intercepts for the graphs of each of the following equations.

a $y = 3x - 6$

b $5x + 8y = -60$

c $y = (x - 5)(x + 9)$

d $y = x^2 - 6x - 40$

18 Sketch the graphs of the following equations.

a $2x + 3y = 12$

b $y = 2$

c $y = 2x$

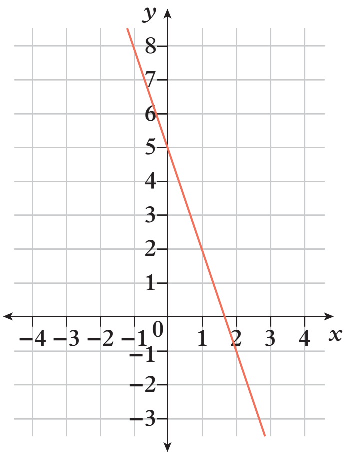
d $y = x^2$

e $y = (x - 2)(x + 3)$

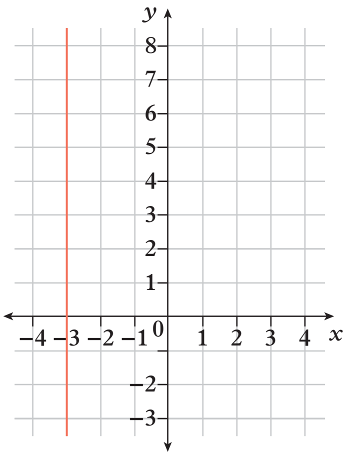
f $y = x^2 - 5x - 6$

19 Determine the equations of the following graphs.

a

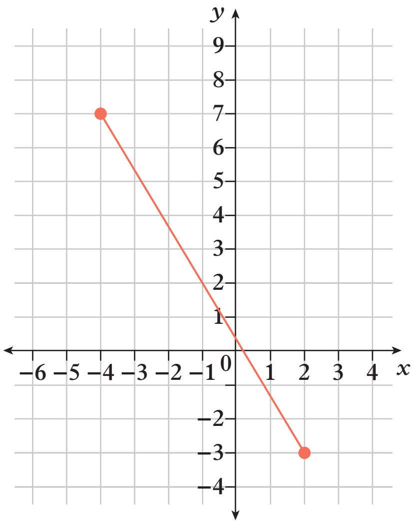


b

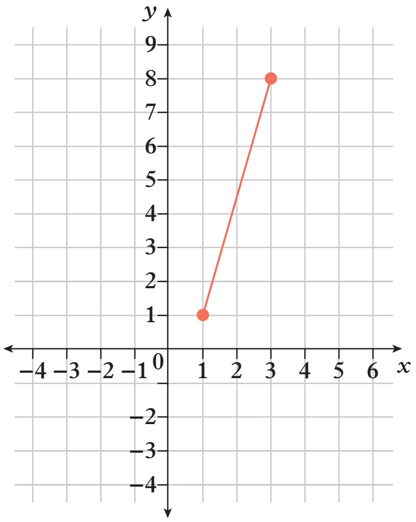


20 Find the midpoint of the following straight-line segments.

a

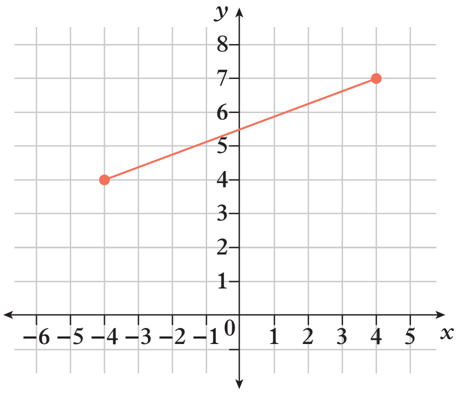


b

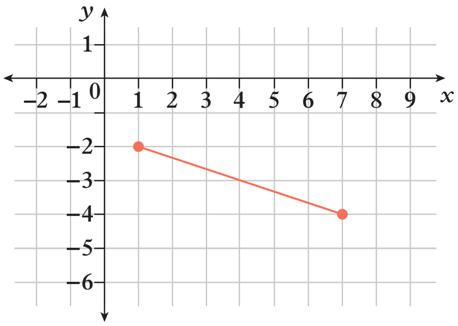


21 Determine the length of the following line segments, correct to two decimal places.

a



b



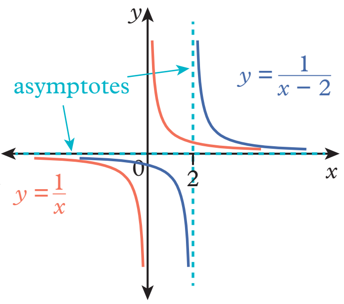
22 Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is +1 or −1.

a Upright, turning point at (7, 3)

b Upright, turning point at (−12, 5)

c Inverted, turning point at (4, 3)

d Inverted, turning point at (3, −2)



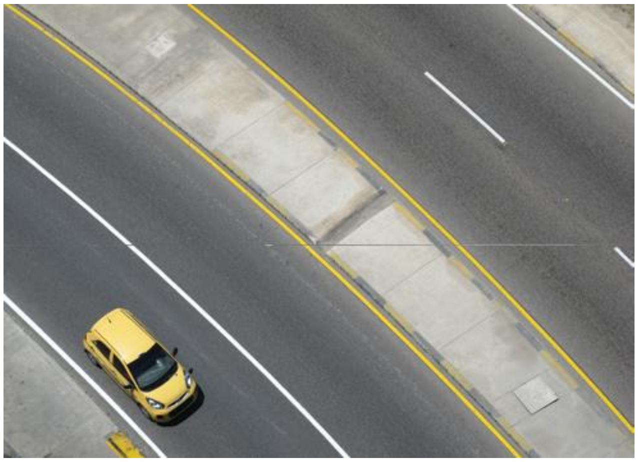
23 The graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$ are shown on the right. Describe the translation required to produce the following graphs from the graph of $y = \frac{1}{x}$.

a $y = \frac{1}{x-2}$

b $y = \frac{1}{x+5}$

c $y = \frac{1}{x+3} + 1$

Analysis



1 Jane is driving home from work, which is 30 km from where she lives. It takes her 36 minutes to drive home. Assume Jane drives at a constant speed throughout her journey. Jane's distance from home, d kilometres, at time t minutes after leaving work can be described using a linear equation.

- **a** Which two features of the graph of d are given in the information above?
- **b** Determine Jane's speed throughout her journey in kilometres per minute.
- **c** How does the gradient of d relate to your answer to part **b**?
- **d** Write the equation of d in gradient–intercept form.
- **e** Sketch the graph of d and label the axis intercepts.

Jane's daughter Vanessa attends a school located at the exact midpoint of Jane's journey home. Occasionally, Jane picks up Vanessa on her way home. It takes three minutes for Vanessa to get into the car and for Jane to be on her way again at the same constant speed as before.

- **f** Find the coordinates of the point on your graph that represents Jane stopping to pick up Vanessa.
- **g** How should you represent the time spent picking up Vanessa from school on your graph? How does this affect the key features of the graph of d?
- **h** Sketch the graph representing Jane's journey home on a day when she picks up Vanessa. Label any key coordinates.
- **i** Your sketch from part **h** can be broken into three linear graph segments. Find the equation of each segment in terms of d and t.

2 To analyse the path of the ball during a soccer match, Lisa records some short pieces of video so measurements can be taken. The measurements (in metres) indicate the height of the ball (h) for a given horizontal distance (d) that the ball travels.

She chooses two plays where the ball follows a parabolic path after contact with a player. One is when the ball was kicked by David and the other when Nick hit the ball with his head. Each path can be modelled by a quadratic relationship involving d and h.

David: $h = -\frac{1}{100}d(d - 44)$

Nick: $h = -\frac{1}{20}(d - 8)^2 + 5$

Answer the below questions for each player.

i David **ii** Nick

- **a** What is the height of the ball when the player made contact with it?
- **b** What is the maximum height of the ball during the play?
- **c** What's the horizontal distance from where the player made contact to where the ball hits the ground?
- **d** Show the trajectory of the ball graphically.

During training, each player dribbles the ball along the ground for the same distance from a marked point to the goal line. Lisa records measurements for the average speed of the ball in metres per second (s) and the time in seconds (t) for the ball to travel this distance (see table below).

Time (t)	2	3	4	6	8
Speed (s)	24	16	12	8	6

- **e** Use the table of values to plot the relationship between t and s with t on the horizontal axis and s on the vertical axis. Is the relationship linear or non-linear?

EXPLORATIONS 1

1 Overtake

A car, a van, a truck and a bicycle are all travelling in the same direction on the same road, each at its own constant speed. At 10 am, the car overtakes the van; at noon, it overtakes the truck; at 2 pm it overtakes the bicycle. At 4 pm, the truck overtakes the bicycle. At 6 pm, the van overtakes the truck.

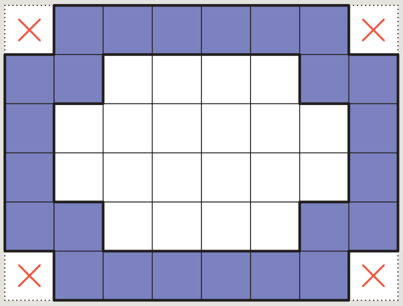
a Suppose the speed of the car is 120 km/h and the speed of the truck is 80 km/h.

- **i** Find the speeds of the van and the bicycle. Then find the time at which the van overtakes the bicycle.
- **ii** A motorbike overtakes the van at 10 am and the bicycle at 3 pm. What is its speed?
- **iii** A semi-trailer travelling at 80 km/h in the opposite direction passes the van at 6 pm. At what time does it pass the bicycle?

b Let c and t represent the speeds, in km/h, of the car and truck, respectively. Find the speeds of the van and the bicycle in terms of c and t. Then show that the time at which the van overtakes the bicycle is always the same, regardless of the speeds of the car and the truck.

2 Borders

The design shown is formed by removing one square from each corner of a rectangular grid and shading a border of thickness 1 unit. In this 6 × 8 example, there are 24 shaded squares in the border and 20 unshaded squares in the interior.



a Ignoring rotations, we want to find the number of such designs that have the same numbers of border and interior squares. Follow these steps:

- **i** Let the dimensions of the original rectangular grid be $x \times y$. Find expressions, in terms of x and y, for the number of border squares and the number of interior squares.
- **ii** Setting the expressions equal, rearrange the equation to get 0 on the right-hand side.
- **iii** The left-hand side can almost be fully factorised as a binomial product of the form $(x - \_\_\_)(y - \_\_\_)$. What number do you need to add to both sides of the equation to make this work?
- **iv** Remember that x and y are positive integers, so the factorised expression on the left-hand side of your equation should match up with a factorisation of the number on the right-hand side. Deduce that there are six possible pairs of positive integer solutions for x and y.
- **v** Why does this mean that there are only three possible designs?

b Adapt the above methods to find, ignoring rotations, the number of such designs in which there are:

- **i** twice as many interior squares as border squares
- **ii** twice as many border squares as interior squares.

3 Absurd surds

What does the infinite surd expression $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ equal? Does it even equal anything? Maybe as you add more and more sixes, the result gets larger and larger without ever settling down.

a One way to get an idea of what's going on is to use the 'answer' button on your scientific calculator, as follows. The instructions for your model might be slightly different.

- ▶ Enter √ 6 = – this calculates the first approximation of $\sqrt{6}$ and stores it as the most recent answer.
- ▶ Enter √ 6 + Ans = – this calculates the next approximation $\sqrt{6 + \text{Ans}}$ based on the previous answer.
- ▶ Now repeatedly enter = – this repeats the previous calculation as often as you like.
- ▶ What number do the approximations appear to get closer to?

b Now let's prove that your observation in part **a** is correct. Let x be the final answer, then notice that $x = \sqrt{6 + x}$. Square both sides and solve the quadratic equation. Which solution can you ignore and why?

c **i** Adapt the method in part **b** to calculate the exact values of $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$ and $\sqrt{2-\sqrt{2-\sqrt{2-\ldots}}}$

- **ii** Find two different infinite surd expressions that are equal to 5.

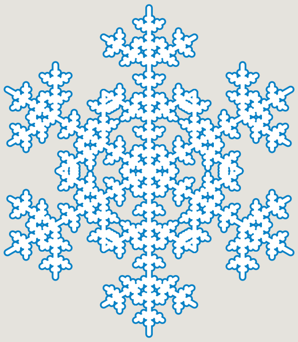
4 Snowflakes

There is a famous conjecture in mathematics about so-called *hailstone* numbers. Starting with any positive integer, we halve the number if it is even, otherwise we triple it and add 1. Now repeat this process to form a sequence of numbers called hailstones, because of the way they rise and fall like real hailstones forming in a cloud. The conjecture states that every number will eventually ‘fall to the ground’ by reaching the number 1. Even though this idea dates back to 1937, when it was first introduced by Lothar Collatz, mathematicians still do not know whether this will always happen. You might like to check what happens when you start with 27; it takes a surprisingly long time for the hailstones to fall!

Let’s look at a gentler version of this problem: *snowflake* numbers. Again, if a number is even we halve it, otherwise we triple it and *subtract* 1. For example, the snowflakes of 11 are 32, 16, 8, 4, 2, 1, 2, 1 . . . and so on. The sequence starting with 11 has a cycle of length 2 that repeats indefinitely, which we can represent as follows:

$$\begin{tikzcd} 11 \arrow[r] & 32 \arrow[r] & 16 \arrow[r] & 8 \arrow[r] & 4 \arrow[r] & 2 \arrow[r] & 1 \arrow[l, bend right] \end{tikzcd}$$

We call them snowflakes because, being lighter than hailstones, they might never fall to the ground!



a Show that the snowflakes of 12 also result in a cycle of length 2.

b Show that the snowflakes of 13 result in a cycle of length 5.

c Find a cycle of snowflakes with length greater than 5.

d Explain why no cycle of snowflakes can contain a multiple of 3.

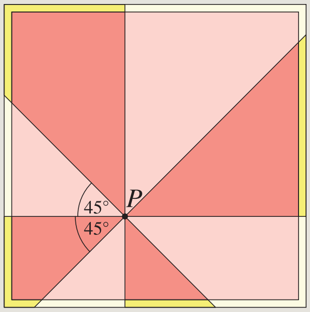
e Find all cycles of length 5.

f Find an odd number whose snowflakes are alternately even and odd, which do not form a cycle for at least 1000 terms.

5 Square pizzas

Angelo likes square pizzas because they fit inside square boxes more efficiently than round pizzas do. However, not having as many symmetries, he worries that it might not be possible to share square pizzas as fairly as round ones. By ‘sharing fairly’, Angelo means that everyone gets the same amount of topping (total area of their slices) and the same amount of crust (total length of boundary shared with the original square).

Angelo makes a cut parallel to one pair of sides, and then makes a second cut parallel to the other pair of sides. Through the point P where these two cuts meet, he makes two more cuts at 45° to the others, as shown.



a A pizza has side length 20 cm and the point P is 8 cm from the left edge and 6 cm from the bottom edge. Find the length of each piece of crust (ignoring its thickness) and the area of each slice (including the crust).

b Verify that the pizza in part **a** will be shared fairly among two people if they take alternate slices, shaded lighter and darker in the diagram.

c Show that, regardless of the size of the pizza and the location of the point P, the method above will always share a pizza fairly among two people. Hint: Use coordinates $(0, 0)$, $(c, 0)$ and (a, b) for the bottom-left corner, bottom-right corner and intersection point P, respectively. What if P is on a diagonal? What if it isn’t?

d Next, Angelo is interested in sharing a square pizza fairly among three people. He wants to do this by making three straight cuts from a common point somewhere inside the square. Is this possible?

e For what values of n is it possible to share a square pizza fairly among n people by making n straight cuts from a common point?

Explorations inspired by the Australian Maths Trust’s competitions and programs: www.amt.edu.au

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Algorithmic Thinking

Algorithmic thinking: Number and algebra

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Once the actions in an algorithm have been identified, it may be possible to encode the algorithm in a coding language so that it can be automated by a computer or machine.

The term iteration refers to repetitive actions in algorithms as shown in Examples 1 and 2, which use the "For each" directive.

Some algorithms contain actions that are only done if certain conditions are true; these are called conditional actions. Conditions and iteration can be combined to create more intelligent algorithms as shown in Example 1.

Snap! is a free block-based coding language and is used in Examples 1 and 2.

Example 1

The following algorithm combines iterative and conditional actions and is defined in pseudocode and coded in Snap!

```
For each integer in the set {1, 2, 3, 4, 5}:
    If the integer is even then:
        print the integer
```

```
for each (integer) in list 1 2 3 4 5
    if (integer mod 2) = 0
        say integer for 1 secs
```

Notice that indenting is used in Example 1 to group the actions that belong to the iterations and the conditions. These correspond to the blocks in Snap! The results of algorithmic actions can be stored in structures called variables, which contain values that can change according to a rule.

Example 2

The following algorithm contains variables and is defined in pseudocode and coded in Snap!

```
Create a table variable to store a list of $(x, y)$ values
For each integer $x$ in the set $\{-3, -2, -1, 0, 1, 2, 3\}$:
    set the variable $y = x^2$
    print $(x, y)$
    add $(x, y)$ to the table
```

```
when clicked
set table to list
for each (x) in list -3 -2 -1 0 1 2 3
    set y to x ^ 2
    say join for·x= x then·y= y for 2 secs
    add list x y to table
```

Task 1

1 What are the algorithms in Examples 1 and 2 printing out?

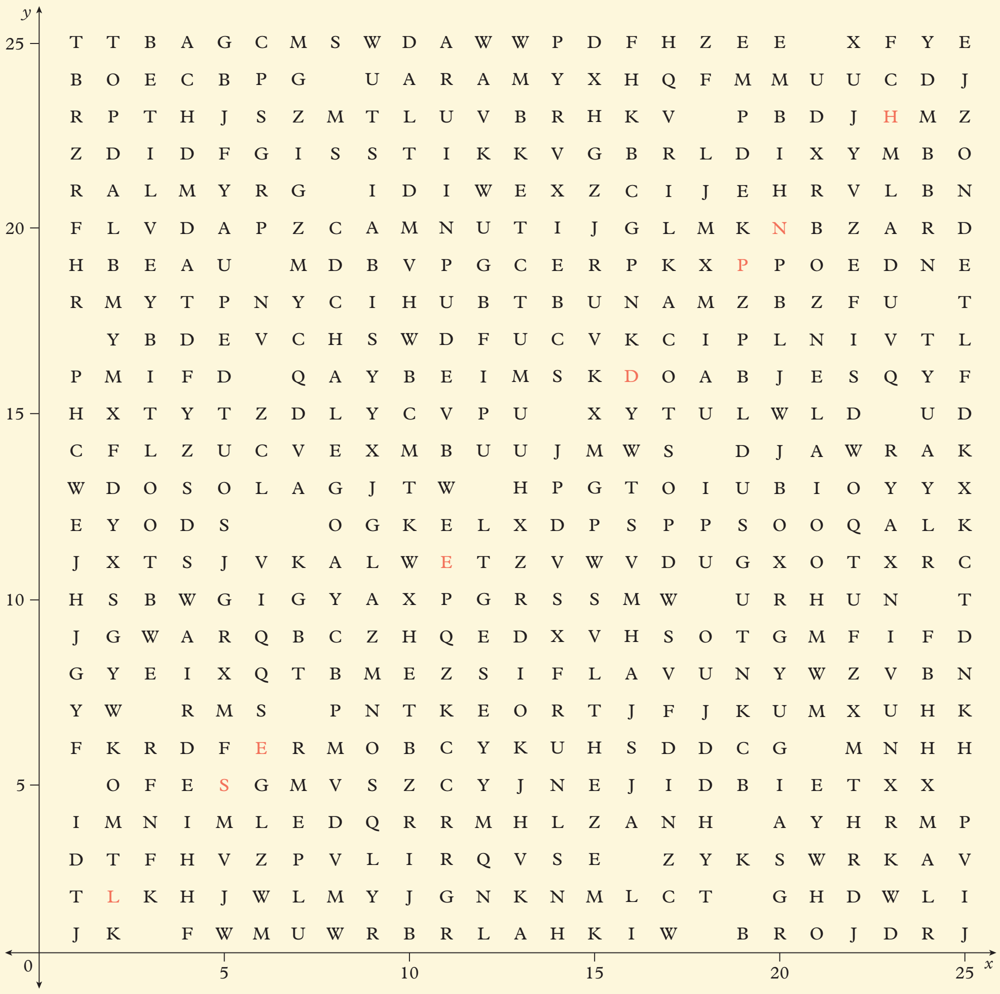
Cryptography is used to send and receive secret encrypted messages. Ciphers are special algorithms that are applied to encrypt messages. The goal of cryptography is to make the deciphering of an encrypted message extremely difficult and to keep messages secret.

Example 3

There is a secret message hidden on the following Cartesian plane. The linear equation to use to decipher the message is $y = x$, with the x-values:

```
for each (secretposition) in list 5 11 20 16 7 23 6 2 19
```

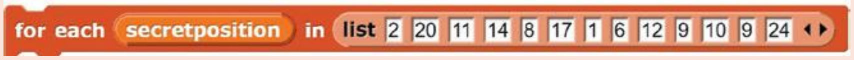
This works out to be the coordinates (5, 5), (11, 11), (20, 20), (16, 16), (7, 7), (23, 23), (6, 6), (2, 2) and (19, 19), which can be looked up on the Cartesian plane below. The secret message is 'SEND HELP'; the letters are shown in orange.



25	T	T	B	A	G	C	M	S	W	D	A	W	W	P	D	F	H	Z	E	E		X	F	Y	E
	B	O	E	C	B	P	G		U	A	R	A	M	Y	X	H	Q	F	M	M	U	U	C	D	J
	R	P	T	H	J	S	Z	M	T	L	U	V	B	R	H	K	V		P	B	D	J	H	M	Z
	Z	D	I	D	F	G	I	S	S	T	I	K	K	V	G	B	R	L	D	I	X	Y	M	B	O
	R	A	L	M	Y	R	G		I	D	I	W	E	X	Z	C	I	J	E	H	R	V	L	B	N
20	F	L	V	D	A	P	Z	C	A	M	N	U	T	I	J	G	L	M	K	N	B	Z	A	R	D
	H	B	E	A	U		M	D	B	V	P	G	C	E	R	P	K	X	P	P	O	E	D	N	E
	R	M	Y	T	P	N	Y	C	I	H	U	B	T	B	U	N	A	M	Z	B	Z	F	U		T
		Y	B	D	E	V	C	H	S	W	D	F	U	C	V	K	C	I	P	L	N	I	V	T	L
	P	M	I	F	D		Q	A	Y	B	E	I	M	S	K	D	O	A	B	J	E	S	Q	Y	F
15	H	X	T	Y	T	Z	D	L	Y	C	V	P	U		X	Y	T	U	L	W	L	D		U	D
	C	F	L	Z	U	C	V	E	X	M	B	U	U	J	M	W	S		D	J	A	W	R	A	K
	W	D	O	S	O	L	A	G	J	T	W		H	P	G	T	O	I	U	B	I	O	Y	Y	X
	E	Y	O	D	S			O	G	K	E	L	X	D	P	S	P	P	S	O	O	Q	A	L	K
	J	X	T	S	J	V	K	A	L	W	E	T	Z	V	W	V	D	U	G	X	O	T	X	R	C
10	H	S	B	W	G	I	G	Y	A	X	P	G	R	S	S	M	W		U	R	H	U	N		T
	J	G	W	A	R	Q	B	C	Z	H	Q	E	D	X	V	H	S	O	T	G	M	F	I	F	D
	G	Y	E	I	X	Q	T	B	M	E	Z	S	I	F	L	A	V	U	N	Y	W	Z	V	B	N
	Y	W		R	M	S		P	N	T	K	E	O	R	T	J	F	J	K	U	M	X	U	H	K
	F	K	R	D	F	E	R	M	O	B	C	Y	K	U	H	S	D	D	C	G		M	N	H	H
5		O	F	E	S	G	M	V	S	Z	C	Y	J	N	E	J	I	D	B	I	E	T	X	X	
	I	M	N	I	M	L	E	D	Q	R	R	M	H	L	Z	A	N	H		A	Y	H	R	M	P
	D	T	F	H	V	Z	P	V	L	I	R	Q	V	S	E		Z	Y	K	S	W	R	K	A	V
	T	L	K	H	J	W	L	M	Y	J	G	N	K	N	M	L	C	T		G	H	D	W	L	I
	J	K		F	W	M	U	W	R	B	R	L	A	H	K	I	W		B	R	O	J	D	R	J
0					5					10					15					20					25

Task 2

1 There is another secret message hidden on this Cartesian plane. The linear equation to use to decipher the message is $y = x + 1$ with the following x-values.



```
for each (secretposition) in list 2 20 11 14 8 17 1 6 12 9 10 9 24
```

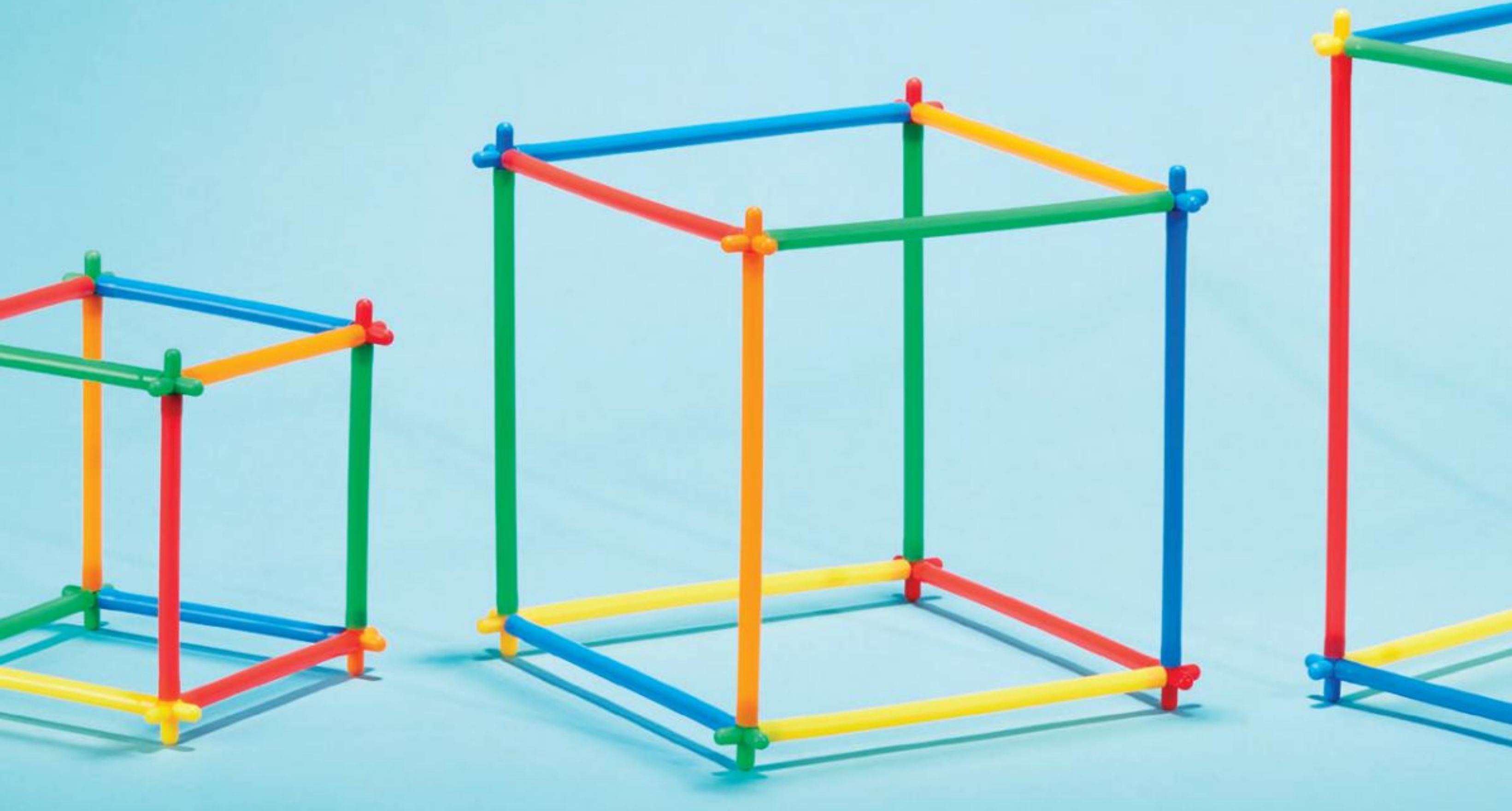
Decipher this hidden message.

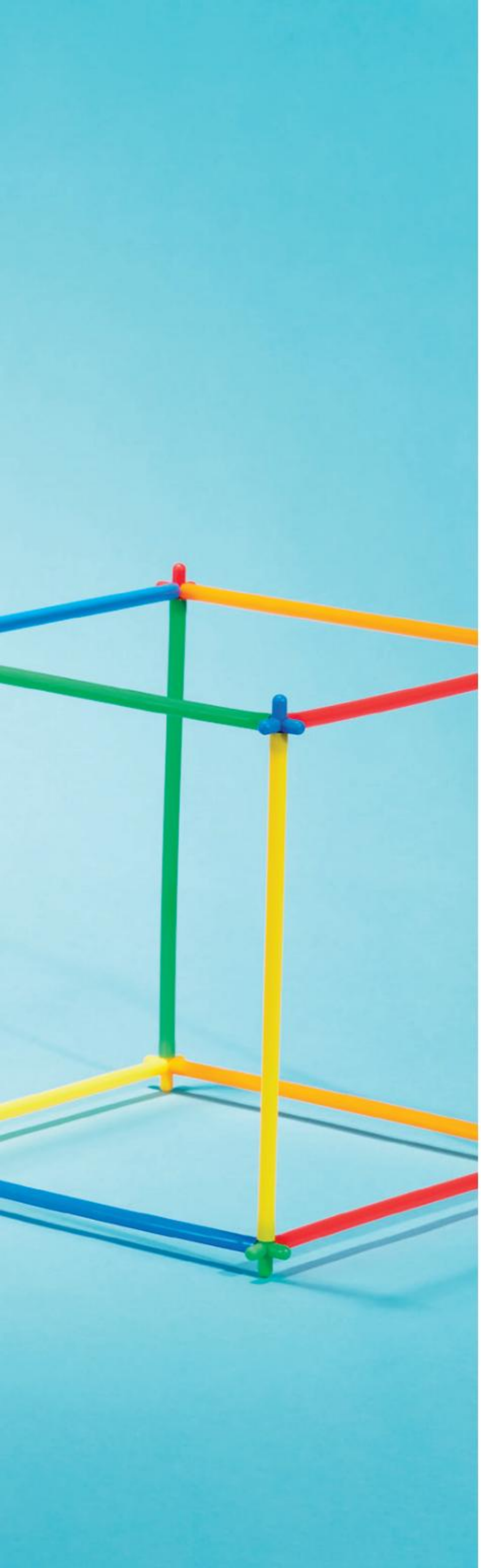
2 Modify the algorithm in Example 2 to use the linear function $y = x + 1$ to print the y-values into a table with the given x-values.

3 **CHALLENGE:** Code your modified algorithm from Example 2 into a coding language such as Snap!

5

Measurement





Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Area of quadrilaterals
- ✔ Area of a circle
- ✔ Identifying shapes within composite shapes
- ✔ Substitution
- ✔ Fractions, decimals and percentages

Curriculum links

- Solve problems involving the volume and surface area of right prisms and cylinders using appropriate units (AC9M9M01)
- Calculate and interpret absolute, relative and percentage errors in measurements, recognising that all measurements are estimates (AC9M9M04)

Materials

✔ Calculator

5A Area of composite shapes

Learning intentions

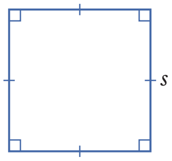
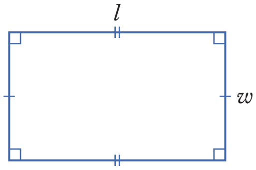
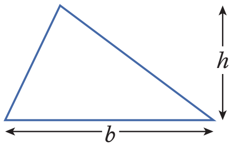
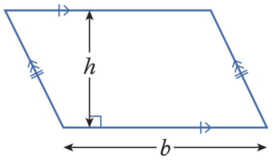
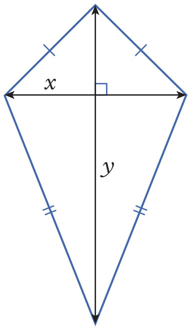
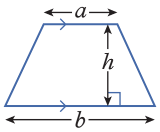
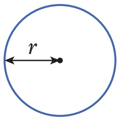
By the end of this topic you will be able to ...

✔ calculate the area of composite shapes.

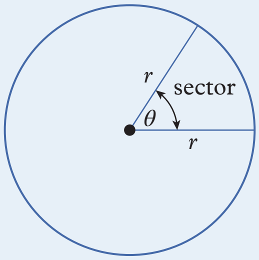
Inter-year links

Support	Understanding area
Year 7	9E Area of a triangle
Year 8	8D Area of a circle
Year 10	6A Area review

Area of simple shapes

Shape	Example	Formula
Square		$A = s^2$
Rectangle		$A = lw$
Triangle		$A = \frac{1}{2}bh$
Parallelogram/rhombus		$A = bh$
Kite/rhombus		$A = \frac{1}{2}xy$
Trapezium		$A = \frac{1}{2}(a + b)h$
Circle		$A = \pi r^2$

Sectors



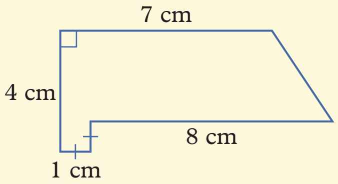
- A **sector** is a portion of a circle formed by two radii and part of the **circumference** (an **arc**).
- The area of a sector can be found using $\frac{\theta}{360} \times \pi r^2$, where θ (the Greek letter theta) is the size of the angle in degrees between the two radii.
  - → This formula is derived from the formula for the area of a circle, $A = \pi r^2$, where $\frac{\theta}{360}$ represents the proportion of a full circle that the sector represents.

Composite shapes

- A **composite shape** can be split up into two or more simple shapes.
- To calculate the area of a composite shape, follow these steps:
  1. Split the figure into simple shapes that have known area formulas.
  2. Calculate any missing dimensions.
  3. Calculate the areas of the individual shapes using the area formulas.
  4. Add or subtract the areas to calculate the total area.

Example 5A.1 Calculating the area of a composite shape using addition

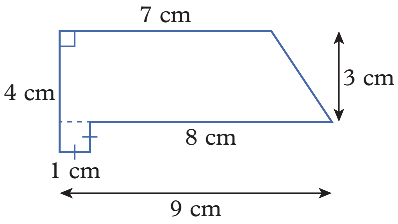
Calculate the area of this composite shape.



THINK

1. Use dotted lines to split the composite figure into simple shapes.
2. Find any missing dimensions for each individual shape and label these on your figure.
3. Use area formulas to calculate the areas of the simple shapes.
4. Add the areas together to find the total area of the composite shape. Remember to include the appropriate unit.

WRITE

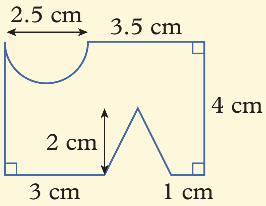


$$A_{\text{square}} = s^2$$
$$= 1^2$$
$$= 1 \text{ cm}^2$$
$$A_{\text{trapezium}} = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2} \times (7 + 9) \times 3$$
$$= 24 \text{ cm}^2$$

Total area = 24 + 1
= 25 cm^2

Example 5A.2 Calculating the area of a composite shape using subtraction

Calculate the area of this composite shape, correct to one decimal place.



THINK

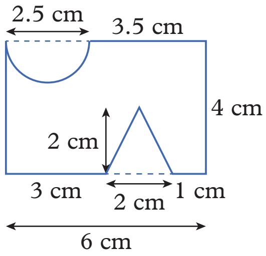
1 Use dotted lines to split the composite figure into simple shapes.

2 Find any missing dimensions for each individual shape and label these on your figure.

3 Use area formulas to calculate the areas of the simple shapes.

4 Subtract the smaller areas from the main area to find the total area of the composite shape. Remember to round your answer and include the appropriate unit.

WRITE



$$A_{\text{rectangle}} = lw$$
$$= 6 \times 4$$
$$= 24 \text{ cm}^2$$

$$A_{\text{triangle}} = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 2 \times 2$$
$$= 2 \text{ cm}^2$$

$$A_{\text{semicircle}} = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2} \times \pi \times 1.25^2$$
$$= 2.45\ldots \text{ cm}^2$$

$$\text{Total area} = 24 - 2 - 2.45\ldots$$
$$= 19.54\ldots$$
$$\approx 19.5 \text{ cm}^2$$

Helpful hints

- ✔ When calculating the areas of composite shapes, a useful technique can be to number the different parts, e.g. A_1 = Area of shape 1. This is useful when there is more than one of a particular type of simple shape in the composite figure.
- ✔ Many composite shapes can be split up in more than one way. There is not a correct way to split up a composite shape, but you should look for the easiest method. However the shape is split up, the total area should be the same!
- ✔ Remember that area is a measure of two-dimensional space so the units of measurement must be squared: mm^2, cm^2, m^2, km^2

ANS p449

Exercise 5A Area of composite shapes

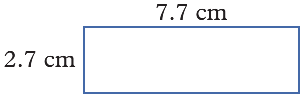
▲ 1–3, 4(b, c, e), 5–7, 8(c, d, f), 9, 12, 13(a, b)

■ 1, 2, 4, 5, 7, 8(b, d, e, f), 10, 12, 13(c, d, f), 15, 18(a, b)

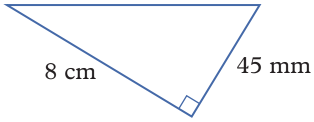
⬟ 2, 4, 5, 8(b, e, f), 10, 11, 13(d, e, f), 14, 16, 17, 18(b, c)

1 Calculate the area of these simple shapes. Give your answers to two decimal places where necessary.

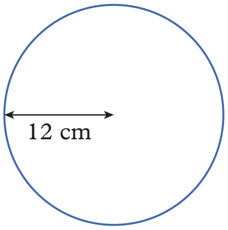
a



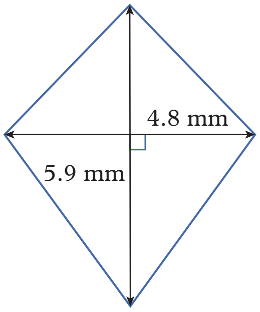
b



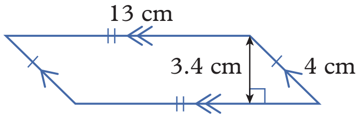
c



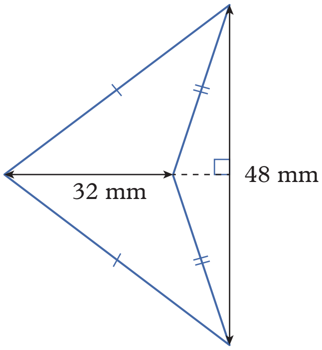
d



e

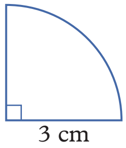


f

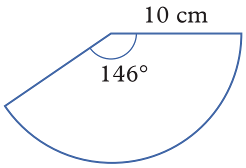


2 Calculate the area of each sector correct to two decimal places.

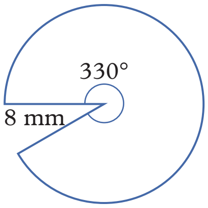
a



b

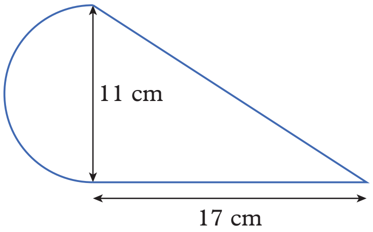


c

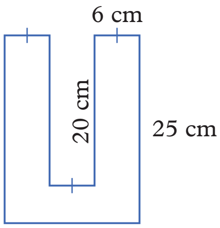


3 Identify the basic shapes within each composite shape.

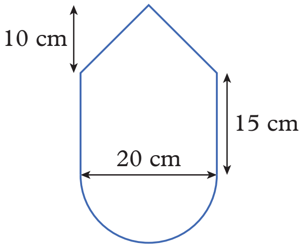
a



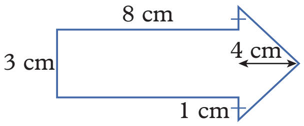
b



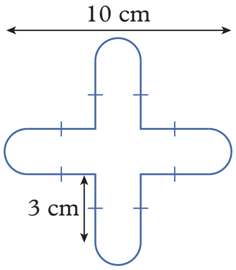
c



d



e



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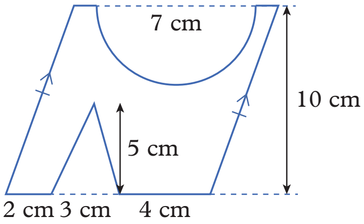
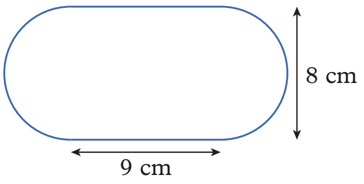
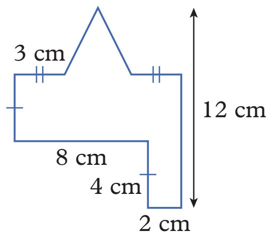


5A.1 **4** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

a



b



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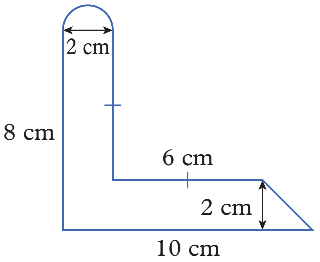

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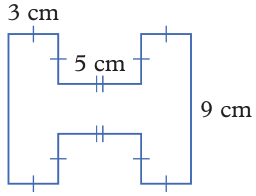

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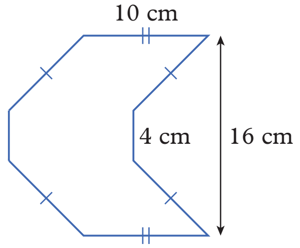

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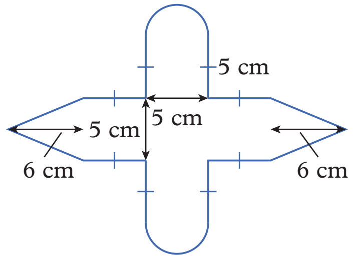
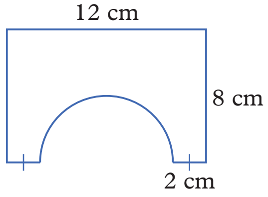
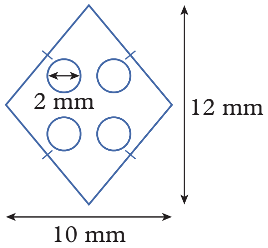


5A.2 **5** Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

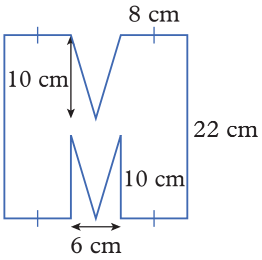
a



b



c



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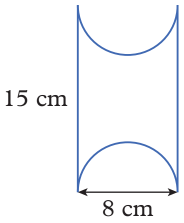

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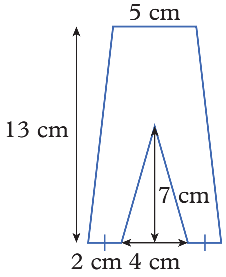

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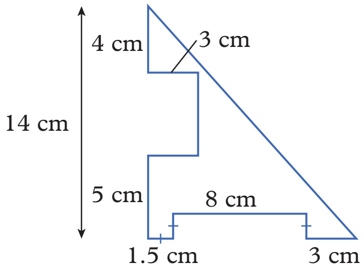
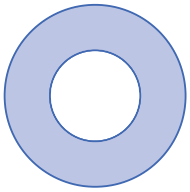


6 Calculate the area of each composite shape in question **3**. Give your answers to two decimal places where necessary.

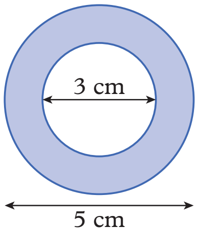
7 This figure is known as an annulus (plural annuli).



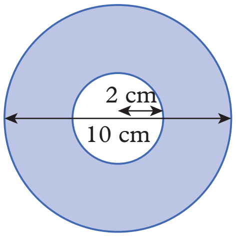
- **a** What is an annulus?
- **b** Explain how you can find the area of this annulus using a formula.
- **c** Find the area of this annulus if it has an outer diameter of 16 cm and an inner diameter of 9 cm. Give your answer to two decimal places.

8 Calculate the area of each annulus correct to two decimal places.

a



b



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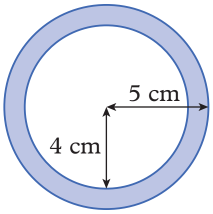

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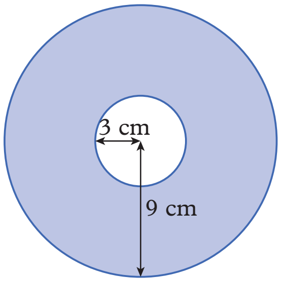

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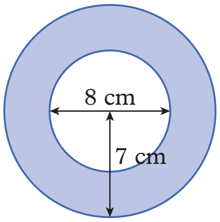

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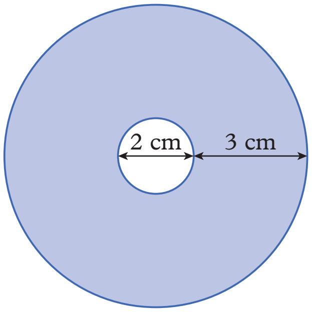
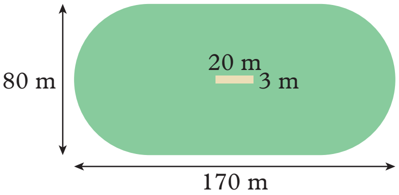
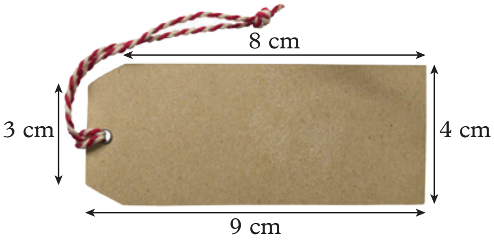


9 What area of grass needs to be mowed on this sports oval, assuming that the cricket pitch in the middle is artificial and does not require mowing? Give your answer to two decimal places.



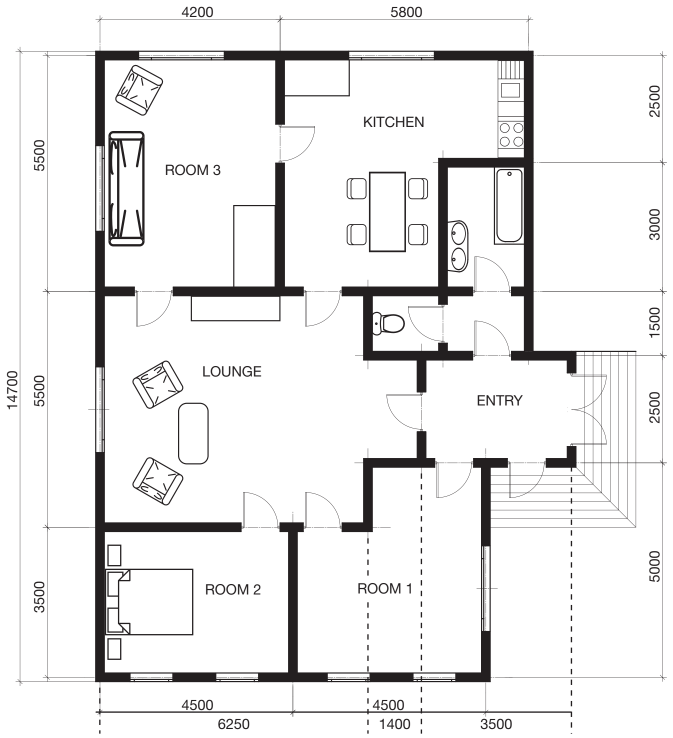
10 Emily makes and sells custom tags through the website *Etsy*.



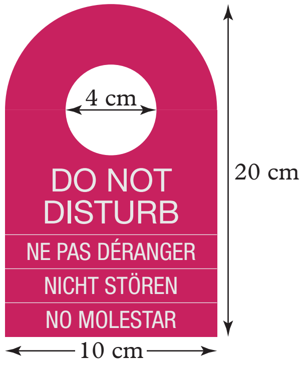
- **a** What is the area of cardboard in a tag with the dimensions shown?
- **b** What is the total area of cardboard required to make 550 tags with the dimensions shown? Explain why Emily might need to buy more than this amount of cardboard in order to make the tags.

11 Shannon wants to lay new carpet in his house. Consider the floor plan with measurements in millimetres.

- **a** What area of the house would he need to cover if everything except the bathroom, kitchen and toilet was to be carpeted? Write this in square metres. Hint: You may find it easier to convert the measurements to metres.
- **b** How much would it cost if the carpet is priced at $45/m²?
- **c** How much money would he save if he chose a cheaper carpet at $30/m²?

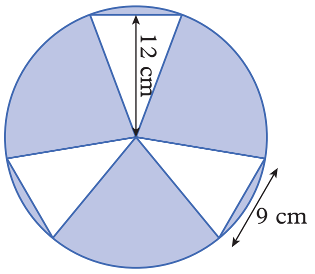


12 Find the amount of cardboard in this DO NOT DISTURB sign correct to two decimal places.

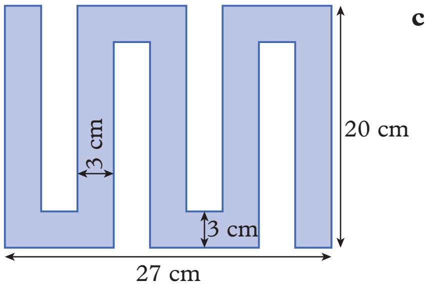


13 Calculate the shaded area of each shape. Give your answers to two decimal places where necessary.

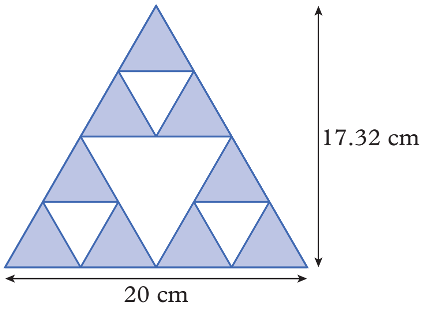
a



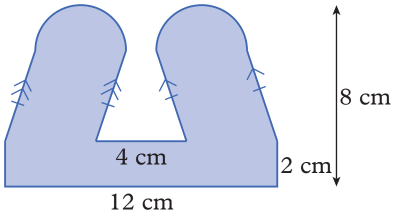
b



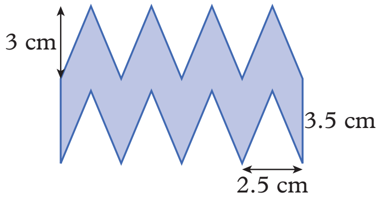
c



d



e



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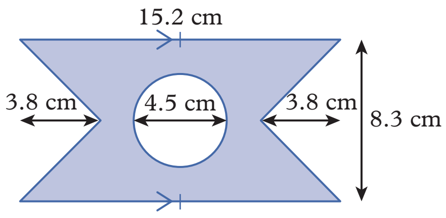
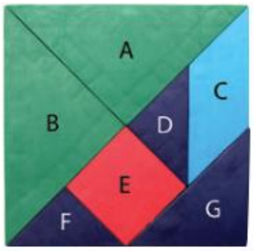


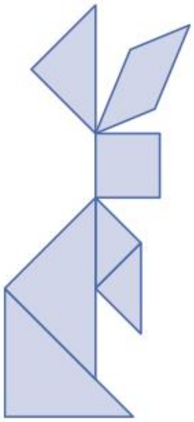
14 The tangram is an ancient Chinese puzzle that consists of seven pieces (usually placed in a square as shown) that can be rearranged to form a variety of shapes.



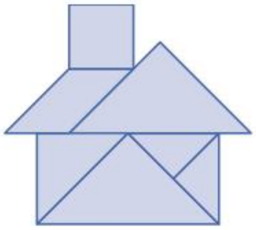
a What is the area of each individual piece in this tangram square if the entire square has side lengths of 10 cm?

b What is the total area of each of these shapes that can be made from the seven pieces of the tangram?

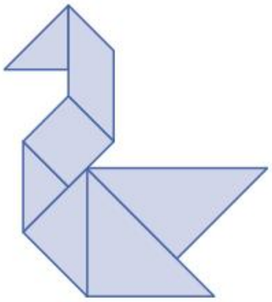
i



ii



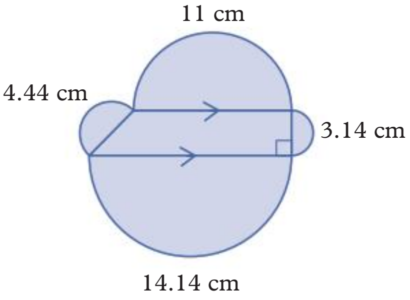
iii



15 Find the area of an annulus with an outer circumference of 35 cm and an inner circumference of 25 cm. Give your answer to two decimal places.

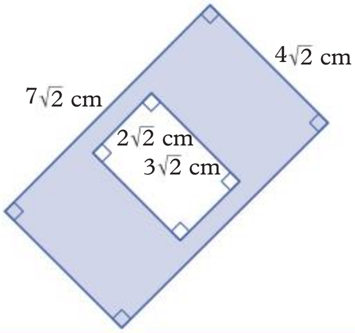
16 A circular skirt, without hems, is cut from a circle of material with a hole for the waist. If a particular circular skirt had an inner circumference of 70 cm and was 60 cm in length when worn, what was the area of material used to create the skirt? Give your answer to two decimal places.

17 Calculate the shaded area correct to two decimal places, given the number shown on each arc is the length of the corresponding semicircle.

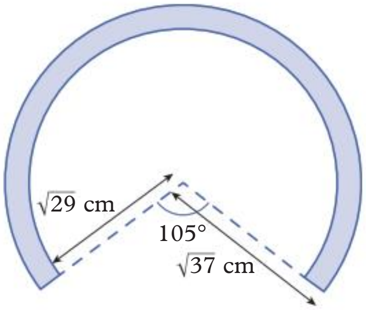


18 Calculate the shaded area of each shape. Give your answers correct to two decimal places where necessary.

a



b



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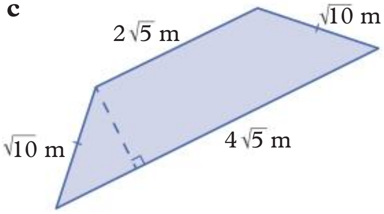


Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Area of composite shapes

Worksheet
Identifying simple shapes within composite shapes

Topic quiz
5A

5B Surface area

Learning intentions

By the end of this topic you will be able to ...

✔ draw the net for right prisms

✔ calculate the surface area of right prisms.

Inter-year links

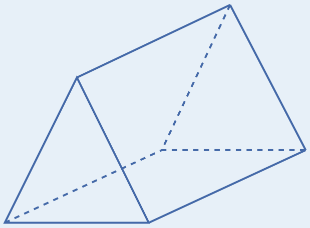
Year 7	9F Surface area
Year 8	8C Area of quadrilaterals
Year 10	6B Surface area

Surface area

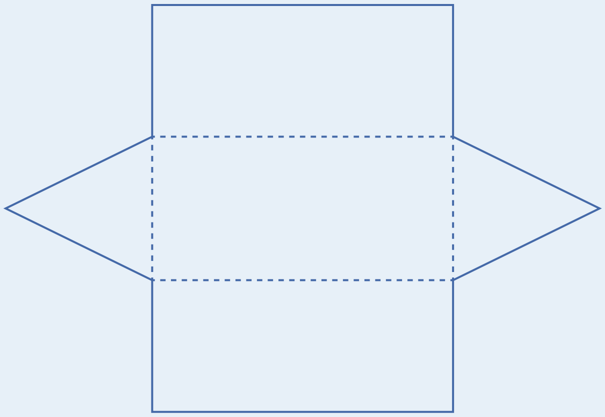
- The **total surface area** (TSA) of a three-dimensional (3D) object is the total area of the outer surface of that object.
- To calculate the total surface area of a 3D object:
  1. Determine the number of **faces**.
  2. Calculate the area of each face.
  3. Add the areas of the faces together.

Nets

- A **net** is a 2D plan that can be folded to form a 3D object. For example, the net shown is the 2D plan of the triangular prism.
- Nets can be used to show all the faces of 3D objects and hence to help calculate the surface areas of those objects.



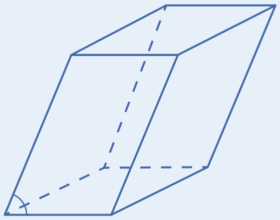
Triangular prism



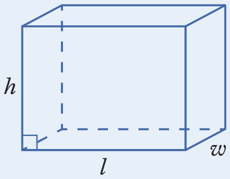
Net

Right prisms

- A **prism** is a 3D object with straight sides, two identical bases and a constant cross-section.
- A **right prism** has right angles between the base and the sides.
  → All the sides (non-bases) of a right prism are rectangles.

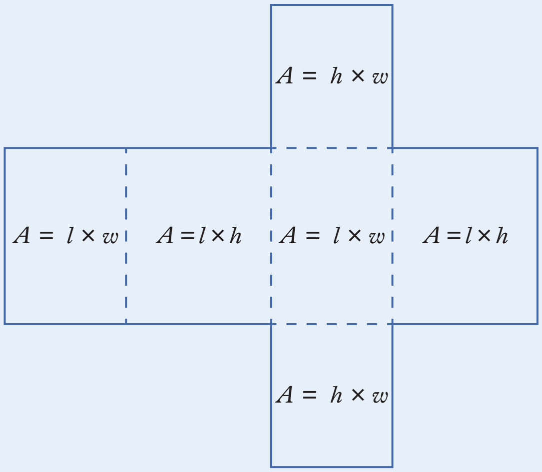


Non-right prism



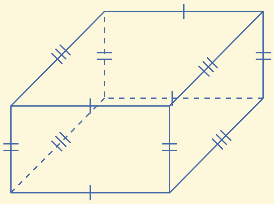
Right prism

- → The surface area of a rectangular prism can be calculated using the formula: TSA = $2lw + 2lh + 2hw$, where: l is the length of the base rectangle, w is the width of the base rectangle and h is the height of the prism.



Example 5B.1 Drawing the net of a right prism

Draw a net for this 3D object.



THINK

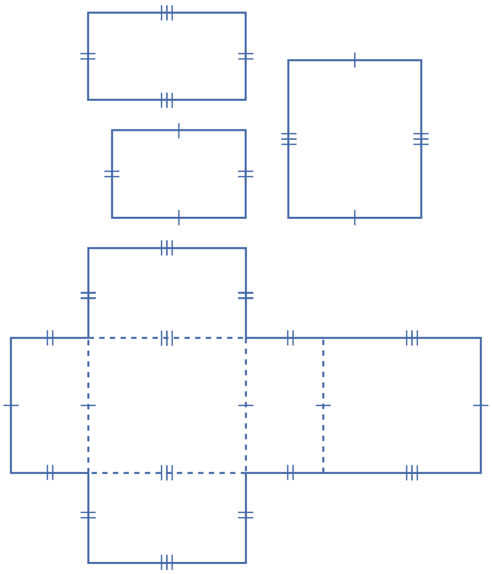
1 A rectangular prism has six faces, made up of three different kinds. Draw each different one.

2 Try to visualise the rectangular prism unfolding – which face is on the bottom? This will form the centre of the net.

3 Arrange the remaining faces in a hopscotch style net. Look carefully at which faces are touching the centre face.

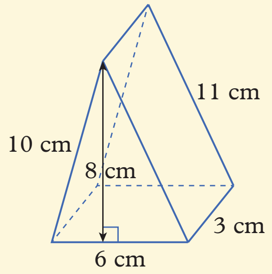
4 Add tabs if you wish to construct the object.

WRITE



Example 5B.2 Calculating the surface area of a triangular prism

Calculate the surface area of this right triangular prism.



THINK	**WRITE**
1 Determine the number of faces. A right triangular prism has two identical triangular faces and three rectangular faces.	
2 Calculate the area of each face.	Area of triangular face $= \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times 6 \times 8$ $= 24\text{ cm}^2$ Area of bottom rectangular face $= l \times w$ $= 6 \times 3$ $= 18\text{ cm}^2$ Area of left-hand rectangular face $= 10 \times 3$ $= 30\text{ cm}^2$ Area of right-hand rectangular face $= 11 \times 3$ $= 33\text{ cm}^2$
3 Add the areas of the faces together. Include the appropriate unit.	TSA $= 2 \times 24 + 18 + 30 + 33$ $= 129\text{ cm}^2$

Helpful hints

- ✔ The number of faces in a prism is equal to the number of sides in the matching ends plus two.
- ✔ All of the non-matching end faces in a right prism are rectangles.
- ✔ A prism can be orientated in any way. Identify the matching ends to determine the shape of the base of a prism.

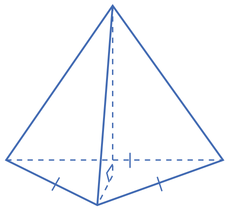
ANS p449

Exercise 5B Surface area

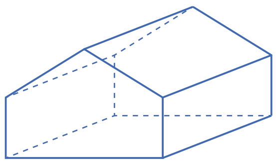
△ 1, 2, 3(a, b), 4(a, b), 6, 7, 9, 11 ■ 3(c, d), 4 (c, d), 5, 6, 8, 11 ⬠ 6, 9, 10,12, 13

1 Which of the following objects are right prisms?

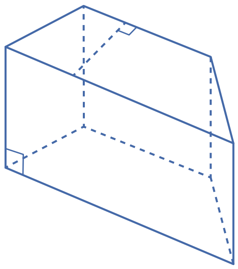
a



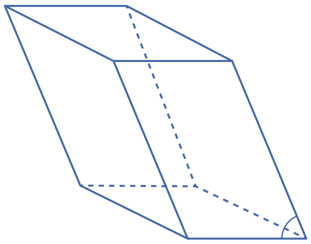
b



c

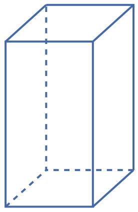


d



5B.1 **2** Draw a net for each 3D object.

a



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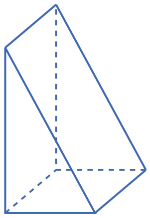

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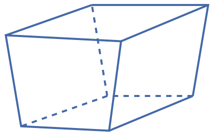

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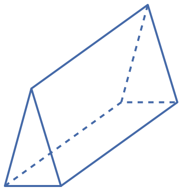
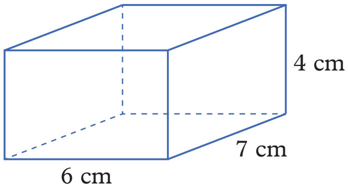


3 Calculate the surface area of each right prism.

a



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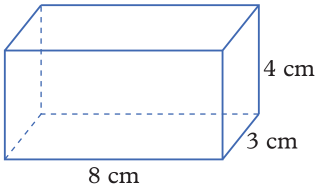

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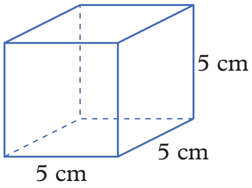

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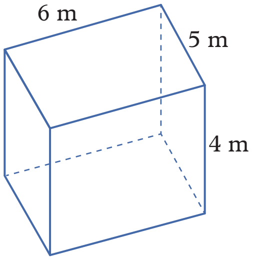
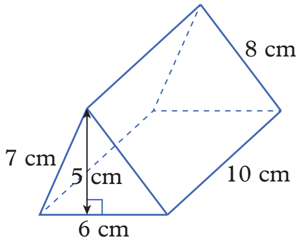


5B.2 **4** Calculate the surface are of each triangular prism.

a



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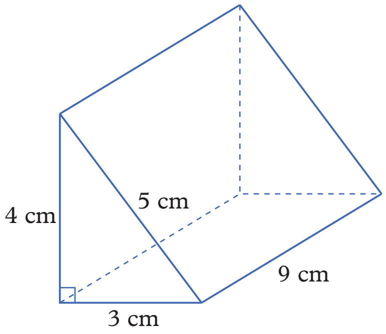

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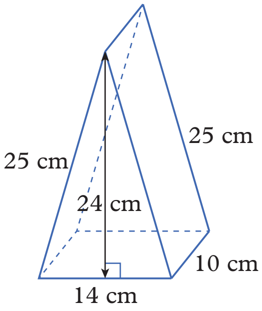
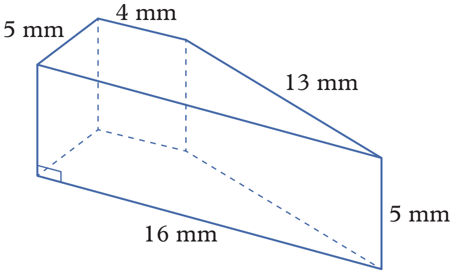


d



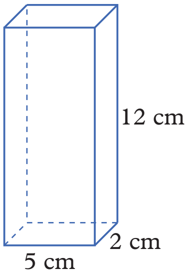
UNDERSTANDING AND FLUENCY

5 What's the surface area of each of the following objects?

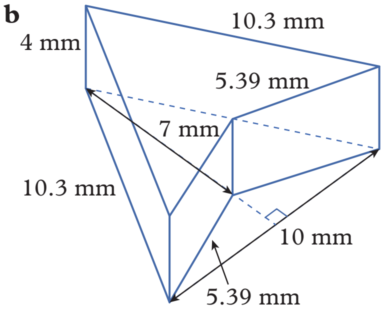
- **a** A cube with side length 3 cm
- **b** A rectangular prism with length 4 cm, width 2 cm and height 3 cm
- **c** A cube with side length $\frac{5}{6}$ cm
- **d** A rectangular prism with length $\frac{2}{3}$ cm, width $\frac{1}{4}$ cm and height $\frac{1}{3}$ cm

6 Calculate the surface area of each right prism.

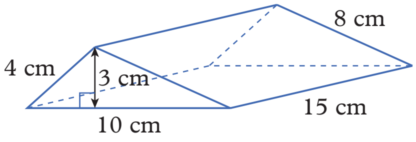
a



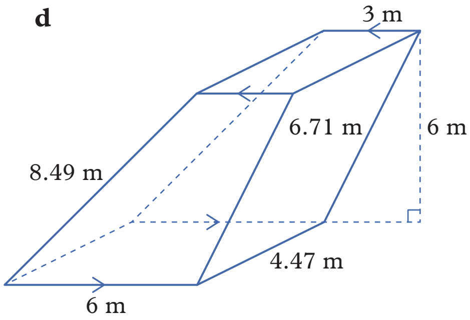
b



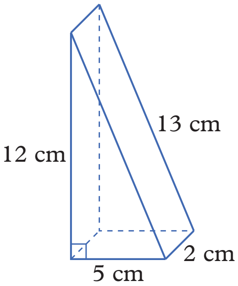
c



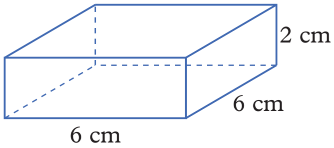
d



e



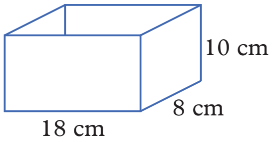
f



PROBLEM SOLVING AND REASONING

7 Find the surface area of these boxes if you were to consider:

a



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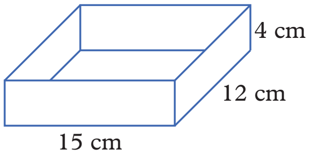
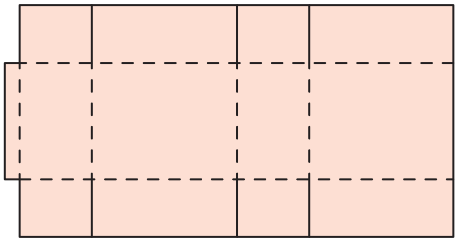
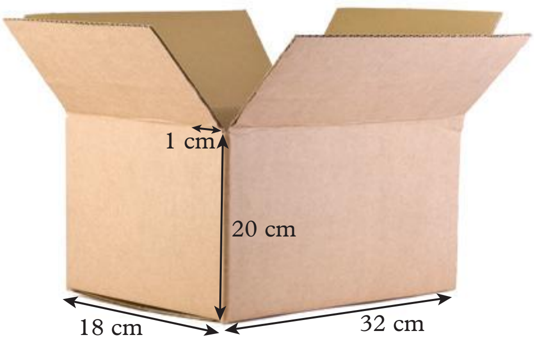


- **i** the amount of material required to build the boxes
- **ii** the amount of paint (in cm^2) required to paint all surfaces of the boxes (inside and outside).

8 The net of a basic cardboard box is shown. The dashed lines represent folds, and the solid lines represent cuts. The tab on the left is used to join the central section of the box, and the flaps at the top and bottom are all half of the width of the box. Calculate the amount of cardboard required to make these boxes.



a



b

9 A block of butter is in the shape of a rectangular prism.

- **a** If the block was 7 cm wide, 16 cm long and 6 cm high, calculate its total surface area.
- **b** If the block was cut halfway along the length (that is, to give two pieces 8 cm long), what is the surface area:
  - **i** of each piece?
  - **ii** in total?
- **c** How is your answer to part **b ii** different from your answer to part **a**?
- **d** Imagine that you cut the block of butter into 1-cm cubes. What would be the total surface area of the butter now?
- **e** If somebody wanted to melt butter quickly, what would you recommend to them? Why?

10 Adnan is having his room painted. His room is 6 m long, 5 m wide and 2.5 m high and has a large window on one wall that measures 150 cm × 95 cm. The walls are to be painted blue and the ceiling is to be painted cream.

- **a** What area is to be painted blue and what area is to be painted cream?
- **b** How many litres of each colour paint is needed if 1 L of paint covers about 15 m^2 and the room will take two coats?

11 A cube has a surface area of 150 cm^2.

- **a** What is the surface area of each face?
- **b** What is the side length of the cube?
- **c** Write a formula that will help you determine the side length of any cube if you know its surface area.

12 Explain why knowing the surface area of a cube is enough to determine its side lengths but knowing the surface area of a rectangular prism is not.

13 Consider a cube with side length 5 cm.

- **a** What is its surface area?
- **b** What is the surface area if two of these cubes are stacked, one on top of the other?
- **c** What is the surface area if there is a stack of:
  - **i** three cubes?
  - **ii** four cubes?
  - **iii** five cubes?
- **d** Is there a pattern? Describe a shortcut to calculate the surface area of a stack of 17 cubes.
- **e** If the cube is cut halfway both vertically and horizontally to form 8 small cubes of the same size, what is the surface area of each small cube?
- **f** If the 8 small cubes from part **e** are stacked on top of each other, what is the total surface area?
- **g** Some small cubes from part **e** have been stacked on top of each other to create a tower with a total surface area of 87.5 cm^2. How many small cubes have been stacked?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Surface area of prisms

Investigation
Dissecting cubes

Topic quiz
5B

5C Volume

Learning intentions

By the end of this topic you will be able to ...

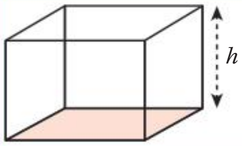
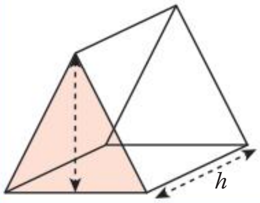
✔ calculate the volume of right prisms.

Inter-year links

Support	Volume and capacity
Year 7	9G Volume of prisms
Year 8	8E Volume and capacity
Year 10	6C Volume

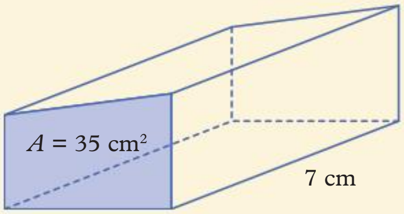
Volume

- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The volume of a prism can be determined by multiplying the base area by the height of the prism. $V = Ah$, where A is the area of the base and h is the height of the prism.
- Remember that volume is a measure of three-dimensional space so the units of linear measures must be cubed: mm^3, cm^3, m^3.

Shape	Diagram	Formula
Rectangular prism		$V = (\text{area of rectangle}) \times h$
Triangular prism		$V = (\text{area of triangle}) \times h$

Example 5C.1 Calculating the volume of a right prism

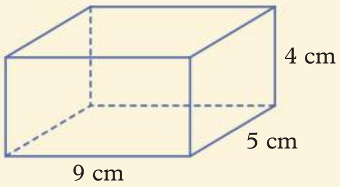
Calculate the volume of this prism.



THINK	WRITE
1 Identify the values for A and h.	$A = 35$ cm^2, $h = 7$ cm
2 Substitute the values for A and h into the formula for the area of the prism.	$V = Ah$ $= 35 \times 7$
3 Calculate the volume of the prism. Remember to include the appropriate unit in the answer.	$= 245$ cm^3

Example 5C.2 Calculating the volume of a rectangular prism

Calculate the volume of this rectangular prism using the formula $V = l \times w \times h$.



THINK

1 Identify the measurements for l, w and h. Check that each measurement is in the same unit.

2 Substitute the values for l, w and h into the formula.

3 Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.

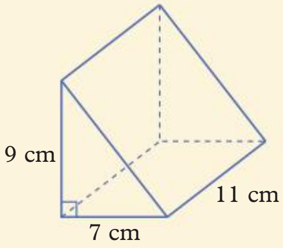
WRITE

$l = 9$ cm
$w = 5$ cm
$h = 4$ cm

$$\begin{aligned} V &= l \times w \times h \\ &= 9 \times 5 \times 4 \\ &= 180 \text{ cm}^3 \end{aligned}$$

Example 5C.3 Calculating the volume of a triangular prism

Calculate the volume of this triangular prism.



THINK

1 Identify the shape of the base of the right prism (triangle with base length b and height h_1) and write the appropriate formula for the area: $A = \frac{1}{2}bh_1$.

2 Substitute the values for the base area (A) and the height of the prism (h_2) into the formula for the volume of the prism: $V = Ah_2$.

3 Calculate the result. Remember to include the appropriate unit.

WRITE

$$\begin{aligned} A &= \frac{1}{2}bh_1 \\ &= \frac{1}{2} \times 7 \times 9 \\ &= 31.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V &= Ah_2 \\ &= 31.5 \times 11 \\ &= 346.5 \text{ cm}^3 \end{aligned}$$

Helpful hints

✔ For rectangular prisms, any of the sides can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.

ANS p450

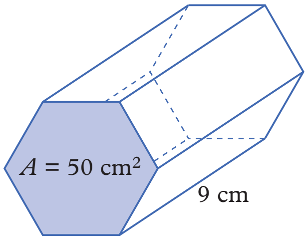
Exercise 5C Volume

△ 1–3, 4(a), 5, 6, 11 ■ 2, 3, 4(b), 7, 9, 11, 13 ⬟ 2–3(1st column), 4, 8, 10, 12,13

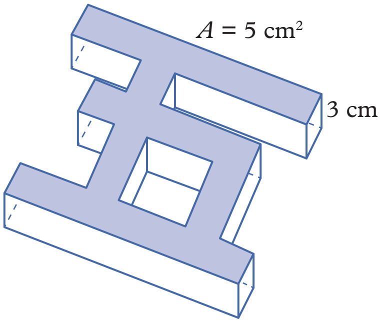
UNDERSTANDING AND FLUENCY

5C.1 **1** Calculate the volume of each prism by using the formula $V = Ah$.

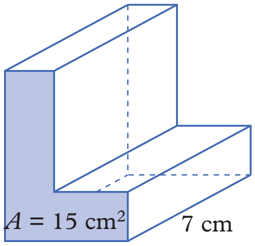
a



b

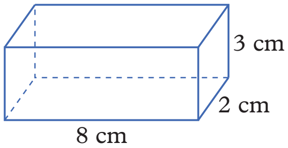


c

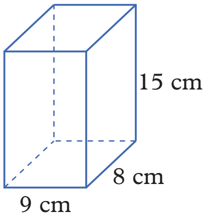


5C.2 **2** The volume of a rectangular prism can be calculated using the formula $V = lwh$. Calculate the volume of each rectangular prism.

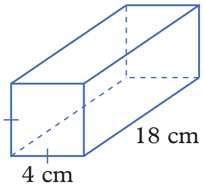
a



b

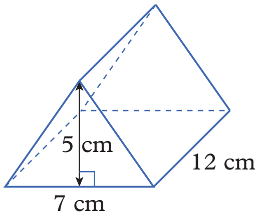


c



5C.3 **3** Calculate the volume of each triangular prism.

a



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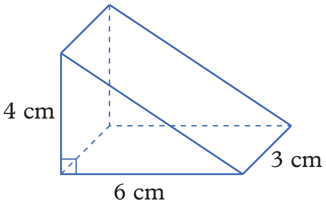
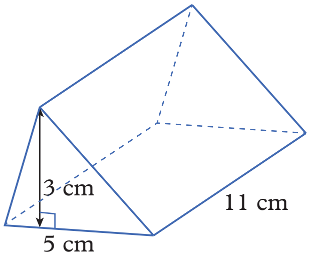
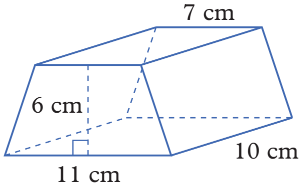


c

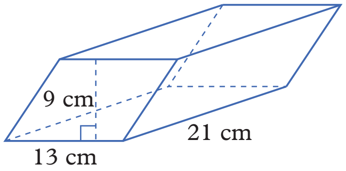


4 Calculate the volume of each prism.

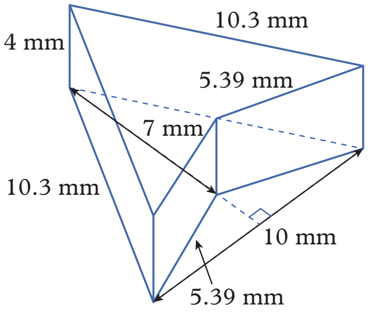
a



b



c



5 A prism has a volume of 240 cm^3.

a If the prism has a height of 8 cm, what is the area of its base?

b If the prism has a base area of 60 cm^2, find its height.

6 A prism has a volume of 360 cm^3.

a If the prism has a height of 60 mm, what is the area of its base in cm^2?

b If the prism has a base area of 90 cm^2, find its height in mm.

7 The height of a cube of side length x is increased 4 cm. The surface area of the new rectangular prism formed is 80 cm^2 more than the surface area of the cube.

- **a** Write an expression to represent the surface area of the cube.
- **b** Write an expression to represent the volume of the cube.
- **c** What are the dimensions of the rectangular prism?
- **d** What is the surface area of the rectangular prism?
- **e** What is the value of x?
- **f** What is the volume of the rectangular prism?

8 The height of a cube is increased 6 cm and the surface area of the new rectangular prism formed is 96 cm^2 more than the surface area of the cube. What is the volume of the rectangular prism?

9 The height of a rectangular prism with square base is increased 2 cm to form a cube of side length x cm. The surface area of the new cube formed is 48 cm^2 more than the surface area of the rectangular prism.

- **a** Write an expression to represent the surface area of the cube.
- **b** Write an expression to represent the volume of the cube.
- **c** What are the dimensions of the rectangular prism?
- **d** What is the surface area of the rectangular prism?
- **e** What is the value of x?
- **f** What is the volume of the rectangular prism?

10 The height of a rectangular prism with square base is increased 1 cm to form a cube and the surface area of the new cube formed is 12 cm^2 more than the surface area of the rectangular prism. What is the volume of the rectangular prism?

11 The length and width of a rectangular prism are the same, and the height of the prism is double the length. The sum of the length, the width and height is 12 cm. What is the volume of the prism?

12 The dimensions of a small container are 5.9 m × 2.35 m × 2.39 m. A large container has the same width and height as the small container, but the length is doubled.

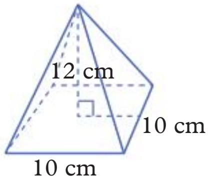
- **a** What is the volume of the small container?
- **b** What is the difference between the volume of the small container and the larger container?

Cubic boxes of side length 1 m are used to move furniture.

- **c** How many boxes can fit into the small container?
- **d** How many more boxes can fit into the large container?
- **e** Explain why you cannot double the number of boxes that would fit into a small container to calculate the total boxes that would fit into the large container.

13 The volume of any pyramid is equal to $\frac{1}{3}$ the volume of the corresponding prism with the same base and height, $V = \frac{1}{3}Ah$. Calculate the volume for each pyramid shown below.

a



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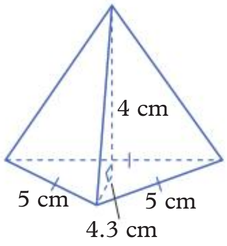
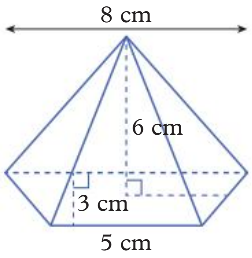


c



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Volume of prisms

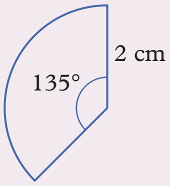
Topic quiz
5C

Checkpoint

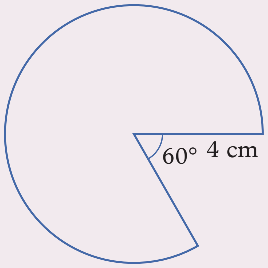
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

5A **1** Calculate the area of these sectors correct to two decimal places.

a

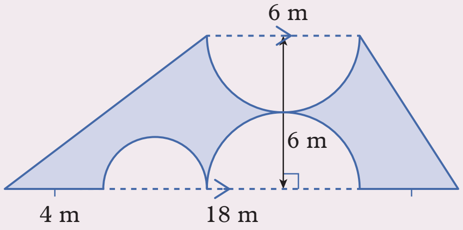


b

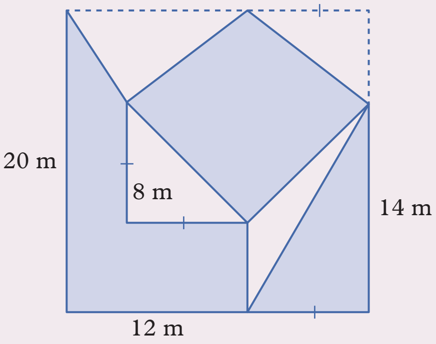


5A **2** Calculate the shaded area of these composite shapes correct to two decimal places.

a

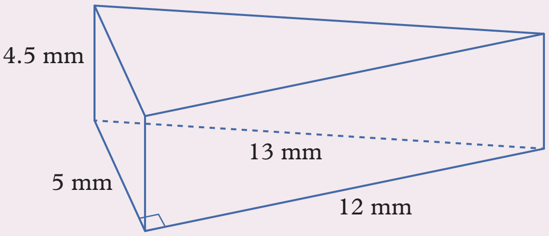


b

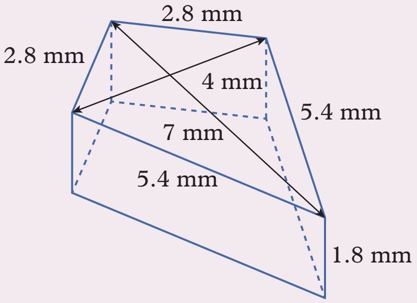


5B **3** Calculate the surface area of these prisms.

a

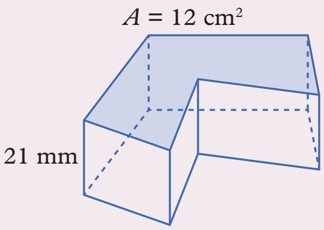


b

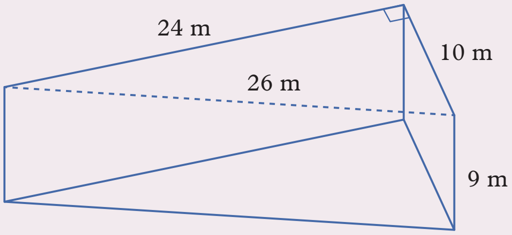


5C **4** Calculate the volume of these prisms.

a



b



c A cube with side lengths 5 cm

d A rectangular prism with width 3 mm, length 4 mm and height 5 mm

5D Surface area of cylinders

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the surface area of cylinders

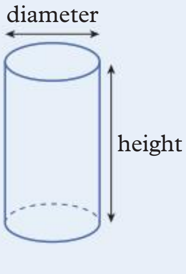
✔ draw the net of cylinders.

Inter-year links

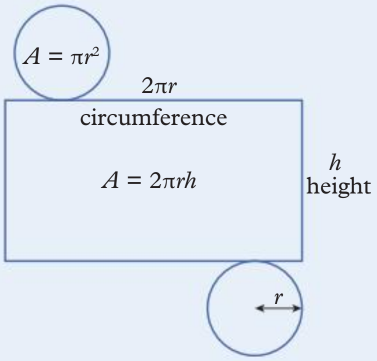
Year 7	9F Surface area
Year 8	8D Area of a circle
Year 10	6B Surface area

Cylinders

- A **cylinder** is a 3D object with a circular base and a constant circular cross-section.
  - → The net of a cylinder is formed by a rectangle and two identical circular bases.
  - → The surface area of a cylinder can be calculated using the formula TSA $= 2\pi rh + 2\pi r^2$, where $2\pi rh$ is the area of the rectangle, $2\pi r^2$ is the area of the two base circles.

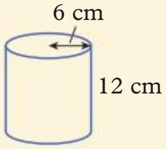


Cylinder



Example 5D.1 Drawing the net of a cylinder

Draw the net of this cylinder, labelling dimensions accurately.



THINK

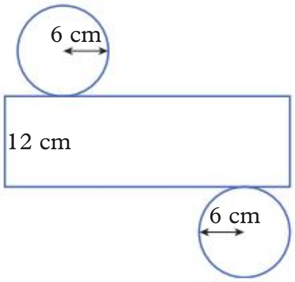
1 Draw a net of a cylinder, remembering that it consists of a rectangle and two circles.

2 Label the radius of the circles and the height of the rectangle. Include appropriate units.

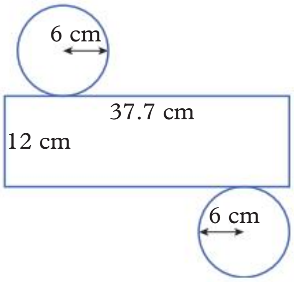
3 The length of the rectangle is the circumference of the circular ends. Use the formula $C = 2\pi r$ to calculate this length, correct to one decimal place.

4 Label the length of the rectangle to complete your net. Include appropriate units.

WRITE

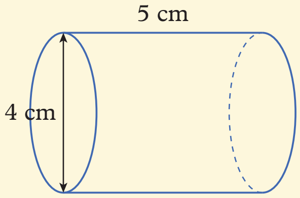


$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 6 \\ &\approx 37.7 \text{ cm} \end{aligned}$$



Example 5D.2 Calculating the surface area of a cylinder

Calculate the surface area of this cylinder correct to two decimal places.



THINK

1 Identify the measurements for the radius (r) and the height (h) and substitute these into the formula.

2 Write the formula for surface area of a cylinder.

3 Substitute the values for r and h into the formula and calculate the result correct to two decimal places. Include the appropriate unit.

WRITE

$$\begin{aligned} r &= d \div 2 \\ &= 4 \div 2 \\ &= 2 \text{ cm} \\ h &= 5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{TSA} &= 2\pi rh + 2\pi r^2 \\ &= 2 \times \pi \times 2 \times 5 + 2 \times \pi \times 2^2 \\ &= 20\pi + 8\pi \\ &= 28\pi \\ &= 87.964\ldots \\ &\approx 87.96 \text{ cm}^2 \end{aligned}$$

Helpful hints

✔ If you can't remember the formula for the surface area of a cylinder, remember that a cylinder consists of two circles (the ends) and a rectangle (the curved surface). Calculate the area of each individual face and add the areas together.

ANS p450

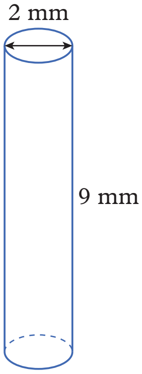
Exercise 5D Surface area of cylinders

△ 1–5(a, e, f), 7, 9 ▢ 5, 6, 8, 11, 14 ⬠ 5, 8, 10, 12–16

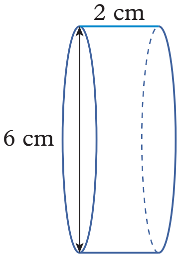
UNDERSTANDING AND FLUENCY

5D.1 **1** Draw a net for each cylinder, labelling the dimensions correct to two decimal places where appropriate.

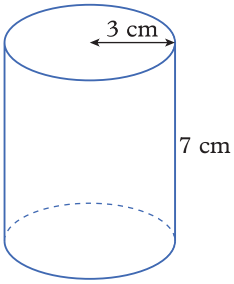
a



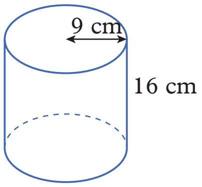
b



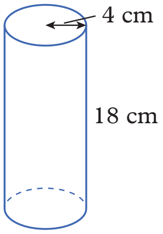
c



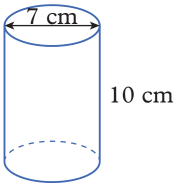
d



e



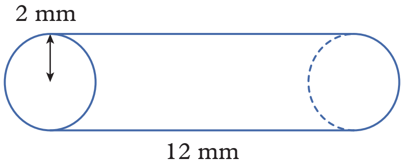
f



5D.2 **2** Calculate the surface area of each cylinder in question **1**. Correct your answers to two decimal places.

3 Draw a net for each cylinder, labelling the dimensions correct to two decimal places where appropriate.

a



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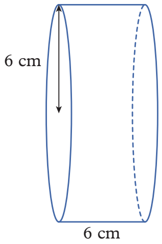

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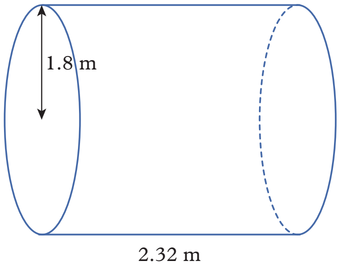
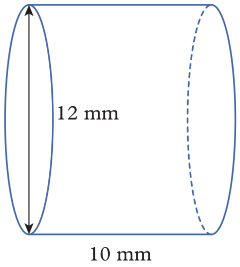
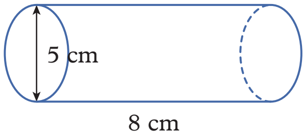


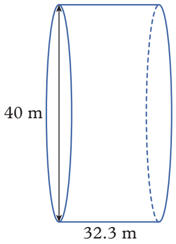
d



e



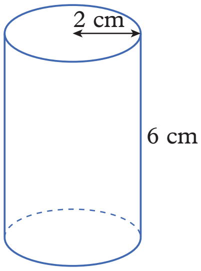
f



4 Calculate the surface area of each cylinder in question **3**. Give your answers to two decimal places.

5 Calculate the total surface area for each cylinder correct to two decimal places.

a



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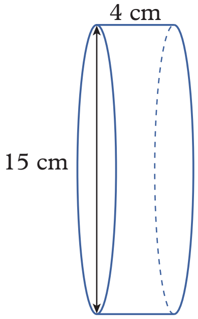
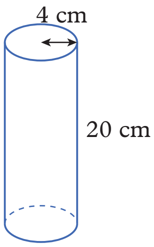
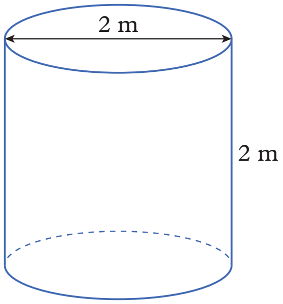


c



d



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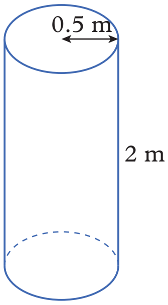
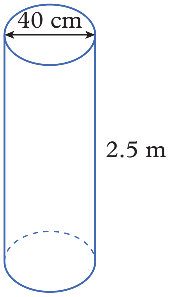


f



6 Calculate the total surface area for each cylinder correct to two decimal places.

a $h = 8$ cm, $r = 10$ cm
b $h = 2$ cm, $d = 3.1$ cm
c $h = 2.1$ m, $r = 3$ m
d $h = 3.2$ m, $d = 7.3$ m
e $h = \frac{1}{2}$ mm, $r = 1\frac{1}{3}$ mm
f $h = \frac{3}{2}$ mm, $d = \frac{4}{3}$ mm

PROBLEM SOLVING AND REASONING

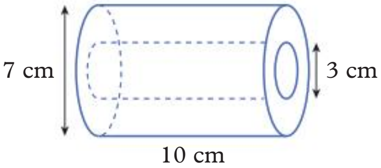
7 Calculate the outer surface area of this box and its lid, if the box is 22 cm tall and has a radius of 15 cm. The lid is 4 cm tall and has a diameter of 31 cm. Give your answer to two decimal places.

8 A cylindrical pool is 2 m deep and has a radius of 6.5 m. How much would it cost, to the nearest dollar, to paint its interior if it needs two coats of paint that costs $50 per litre? Assume that 1 L covers 15 m^2.

9 Maria has the choice of two paint rollers. One roller is 25 cm long and has a radius of 4 cm. The other roller is 30 cm long and has a diameter of 6 cm. Assuming they have the same absorbency, which roller would need to be re-dipped in paint the least often?

10 a What is the surface area of the tube below, including the internal exposed surface? Give your answer correct to two decimal places.

b Lucian answered part **a** as 296.88 cm^2 and Curtis said it was 282.74 cm^2. Explain where they went wrong.



11 A cylinder has a radius of 4 cm and a surface area of 300 cm^2. What is its height to the nearest centimetre?

12 The surface area for any prism can be calculated using the formula:

SA = 2 × area of base + perimeter of base × height

Explain why this formula works.

CHALLENGE

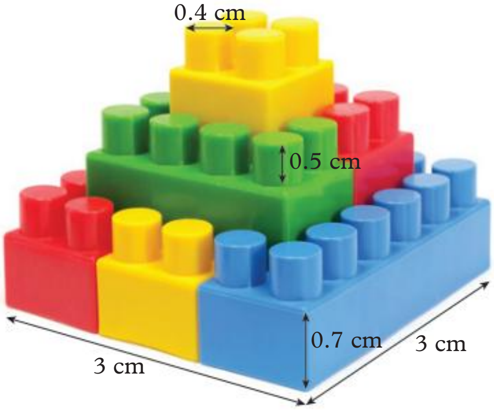
13 If you double the height of a cylinder, does its surface area also double? Explain, using an example.

14 The surface area of a sphere can be calculated using the formula TSA = $4\pi r^2$. Calculate the surface area of a sphere, correct to two decimal places, that has:

a radius 4 cm
b radius 7 mm
c diameter 10 cm
d circumference 20 cm.

15 Write a formula that will calculate the height of a cylinder, given its surface area and radius.

16 Calculate the exterior surface area, correct to two decimal places, of the structure shown in the image on the right (do not include the base).



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Interactive skillsheet
Surface area of cylinders

Investigation
Making can-coolers

Topic quiz
5D

5E Volume of cylinders

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the volume of cylinders

✔ calculate the capacity of cylindrical objects.

Inter-year links

Support	Volume and capacity
Year 7	9G Volume of prisms
Year 8	8E Volume and capacity
Year 10	6C Volume

Volume of cylinders

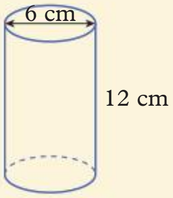
- The volume of a 3D object is the amount of three-dimensional space that it occupies.
- The volume of a cylinder can be found by multiplying the base area by the height. As the base area is $A = \pi r^2$, this gives the following formula:
  $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

Capacity

- The **capacity** of a 3D object is a measure of how much the object can hold.
  A container with an inside volume of:
  → 1 cm^3 holds 1 mL of liquid
  → 1000 cm^3 holds 1 L of liquid
  → 1 m^3 holds 1 kL of liquid.

Example 5E.1 Calculating the volume of a cylinder

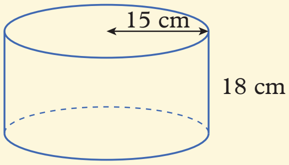
Calculate the volume of this cylinder correct to two decimal places.



THINK	**WRITE**
1 Identify the values for the radius of the base (r) and the height of the cylinder (h).	$r = d \div 2$ $= 6 \div 2$ $= 3$ cm $h = 12$ cm
2 Substitute the values for r and h into the formula for the volume of the cylinder.	$V = \pi r^2 h$ $= \pi \times 3^2 \times 12$ $= 108\pi$
3 Calculate the result using the π button on your calculator and round your answer to two decimal places. Remember to include the appropriate unit.	$= 333.292...$ $\approx 333.29 \text{ cm}^3$

Example 5E.2 Calculating the capacity of a cylinder

Calculate the capacity of this cylinder correct to the nearest litre.



THINK	**WRITE**
1 Identify the values for the radius of the base (r) and the height of the cylinder (h).	$r = 15$ cm $h = 18$ cm
2 Calculate the volume of the cylinder, using the π button on your calculator.	$V = \pi r^2 h$ $= \pi \times 15^2 \times 18$ $= 4050\pi$
3 Convert the volume into capacity by using an appropriate conversion factor.	$= 12\,723.45\ldots$ cm^3 $= (12\,723.45\ldots \div 1000)$ L ≈ 13 L

Helpful hints

✔ Remember that volume and capacity are not the same thing! Capacity is a measure of how much an object can hold (mL, L, kL), whereas volume is a measure of how much space an object occupies (mm^3, cm^3, m^3).

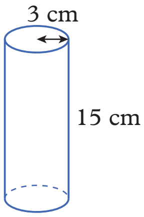
ANS p452

Exercise 5E Volume of cylinders

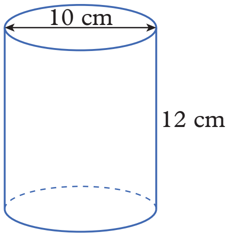
△ 1–3(a, e, f), 4, 5, 7, 10, 13(a, b) ■ 1–3(b, c, d), 4, 5, 8, 10, 13, 15 ⬟ 1–3(c, f), 4, 6, 7, 9, 11–15

5E.1 **1** Calculate the volume of each cylinder. Give your answers to two decimal places.

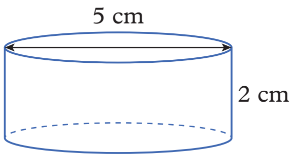
a



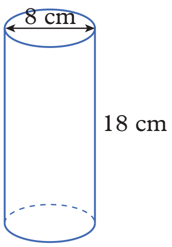
b



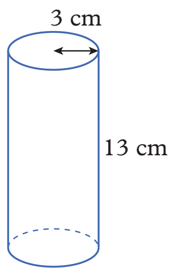
c



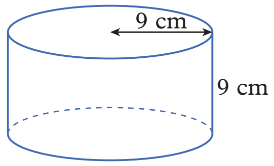
d



e

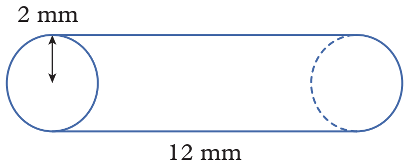


f

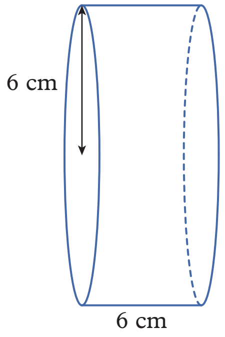


2 Calculate the volume of each cylinder. Give your answers to two decimal places.

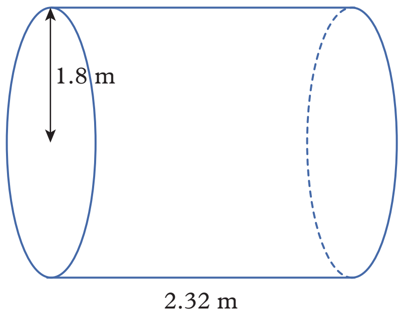
a



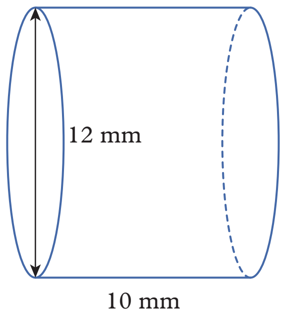
b



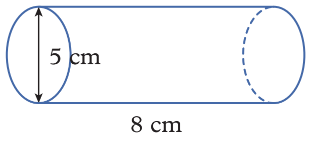
c



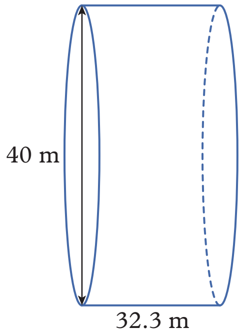
d



e



f



3 Calculate the volume of each cylinder. Give your answers to two decimal places.

a $h = 8$ cm, $r = 10$ cm

b $h = 2$ cm, $d = 3.1$ cm

c $h = 2.1$ m, $r = 3$ m

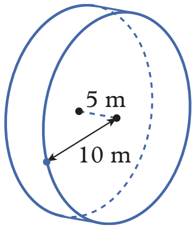
d $h = 3.2$ m, $d = 7.3$ m

e $h = 120$ mm, $r = 13$ mm

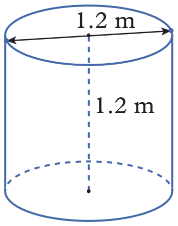
f $h = 23$ cm, $d = 31$ cm

5E.2 **4** Calculate the capacity of each cylinder, correct to the nearest litre.

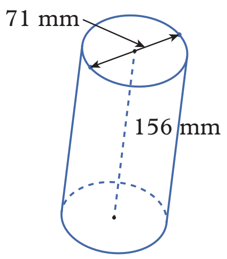
a



b



c



5 Calculate the capacity of each cylinder in question **3**, correct to the nearest litre.

PROBLEM SOLVING AND REASONING

6 A cylinder has a volume of 150 cm^3.

a If it has a radius of 6 cm, find its height.

b If it has a height of 6 cm, find its radius.

7 A can of condensed milk has an internal diameter of 7 cm and a height of 12 cm.

a What is the volume of condensed milk in the can? Give your answer to the nearest cm^3.

b If condensed milk has a density of 1.3 g/cm^3, what is the weight of the milk if the can is filled? Give your answer to the nearest gram. (Weight = volume × density)

8 A $1 coin has a diameter of 2.5 cm and a thickness of 3 mm.

a How much metal is in a stack of 30 $1 coins? Write your answer in both cubic millimetres and cubic centimetres, correct to two decimal places.

b How many $1 coins could be made from 1000 cm^3 of metal?

9 The capacity of a cylinder with diameter 7 mm and height 15 mm correct to the nearest litre is 0 L. Explain why rounding to this degree of accuracy does not make sense.

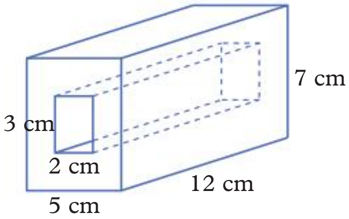
10 Oliver has two cylindrical glasses. The first glass has a diameter of 6.5 cm and a height of 22 cm. The second glass has a radius of 5 cm and a height of 10 cm. Which glass has the greatest capacity and by how much? Give your answer in millilitres correct to two decimal places.

11 What is the capacity of a cylindrical bottle cap if it has an internal diameter of 2.8 cm and a height of 1.1 cm? Give your answer in mL correct to two decimal places.

12 Explain why if the radius of a cylinder is doubled it will have twice the volume of the same cylinder with its height doubled.

13 Find the volume of these hollow objects. Where necessary, give your answers to two decimal places.

a



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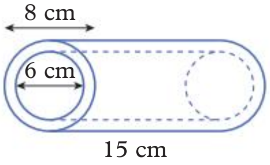
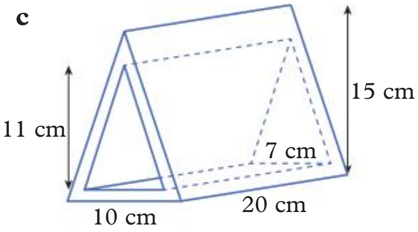


c

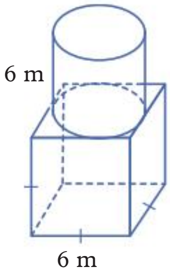


14 A candle has a diameter of 6 cm and a height of 16 cm. If the wick can be thought of as a cylinder with a diameter of 3 mm, what is the amount of wax contained within a single candle? Give your answer in both cubic centimetres and in millilitres correct to two decimal places.

CHALLENGE

15 Find the volume of the following composite solids.

a



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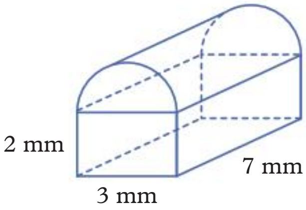

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Interactive skillsheet
Volume of cylinders

Interactive skillsheet
Capacity

Topic quiz
5E

5F Errors

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate absolute, relative and percentage errors
- ✔ calculate the upper and lower bounds for measurement values
- ✔ interpret the meaning of errors.

Inter-year links

Year 7	4G Fractions, decimals and percentages
Year 8	1A Rounding and estimating
Year 10	6D Measurement errors and accuracy

Errors

- An **error** is the difference between the exact value and the estimated value of a quantity.

  $$\text{error} = \text{exact value} - \text{estimated value}$$

  → When the estimated value is smaller than the exact value, the error is positive.

  → When the estimated value is larger than the exact value, the error is negative.

- The **absolute value** of a number is the distance of the number from zero on the number line. It measures the magnitude of the number, which is never negative. The absolute value is indicated using two straight brackets around the number.

  → The absolute value of a non-negative number is itself. For example, $|5| = 5$, $|0| = 0$.

  → The absolute value of a negative number is equal to the number multiplied by -1. For example, $|-5| = 5$.

- An **absolute error** is the absolute value of an error.

  $$\text{absolute error} = |\text{exact value} - \text{estimated value}|$$

- A **relative error** is the ratio between the absolute error and the exact value of a quantity.

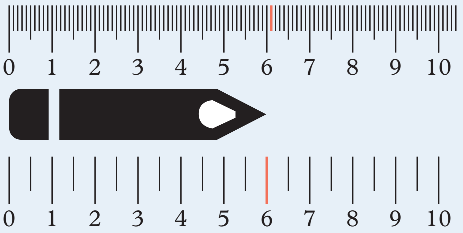
  $$\text{relative error} = \frac{\text{absolute error}}{\text{exact value}}$$

- A **percentage error** is the relative error expressed as a percentage.

  $$\text{percentage error} = \frac{\text{absolute error}}{\text{exact value}} \times 100\%$$

Upper and lower bounds

- All measurements are estimates of exact values. The accuracy of a measurement depends on the smallest measuring unit of the instrument being used.
- For example, when measuring to the nearest millimetre, the length of the pencil on the right is 6.1 cm, and when measuring to the nearest half centimetre, the length of the pencil is 6.0 cm.



- For a given level of accuracy:

  → the **lower bound** is the boundary of values that can round up to the given measurement

  → the **upper bound** is the boundary of values that can round down to the given measurement

  → usually,

  $$\text{lower bound} = \text{measurement} - 0.5 \times \text{smallest unit of measurement}$$

  $$\text{upper bound} = \text{measurement} + 0.5 \times \text{smallest unit of measurement.}$$

Example 5F.1 Calculating errors in financial problems

The budget for a company's media department is \$520 per month. In June, the media department spent \$500 in total. Between the amount spent in June and the budget, find:

a the absolute error

b the relative error

c the percentage error.

THINK	WRITE
a Identify the estimated value as the budget \$520, and the exact value as the amount spent \$500. The absolute error is the absolute value of the difference between \$520 and \$500.	$\text{exact value} = \$500$ $\text{estimated value} = \$520$ **a** $\text{absolute error} = \lvert \text{exact value} - \text{estimated value} \rvert$ $= \lvert 500 - 520 \rvert$ $= \lvert -20 \rvert$ $= 20$
b Divide the absolute error found in part **a** by the exact value.	**b** $\text{relative error} = \frac{20}{500}$ $= 0.04$
c Convert the relative error in part **b** to a percentage.	**c** $\text{percentage error} = \frac{20}{500} \times 100\%$ $= 4\%$

Example 5F.2 Calculating errors in approximation

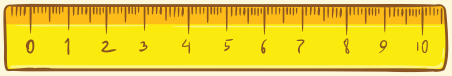
When calculating the circumference of a circle with a 5 cm radius, 3.14 is used as an approximate value of π. Find the percentage error in this calculation, round your answer to one significant figure.

THINK	WRITE
1 Calculate the circumference of the circle using both π and the approximate value 3.14.	$\text{exact circumference} = 2\pi \times 5$ $= 10\pi \text{ cm}$ $\text{approximated circumference} = 2 \times 3.14 \times 5$ $= 31.4 \text{ cm}$
2 Calculate the absolute error in the approximation by finding the absolute value of the difference between the exact circumference and approximated circumference.	$\text{absolute error} = \lvert \text{exact value} - \text{estimated value} \rvert$ $= \lvert 10\pi - 31.4 \rvert$
3 Calculate the percentage error by representing the absolute error as a percentage of the exact value.	$\text{percentage error} = \frac{\lvert 10\pi - 31.4 \rvert}{10\pi} \times 100\%$ $\approx 0.05\%$

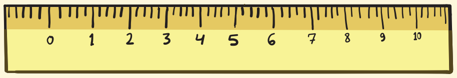
Example 5F.3 Finding the upper and lower bounds

Mitch measures the length of a pen as 9 cm. Find the lower and upper bounds for this measurement value if the ruler has:

a 1 mm as the smallest unit of measurement



b 2 mm as the smallest unit of measurement.



THINK	**WRITE**
a 1 The measurement 9 cm is an estimation of the exact length of the pen. The error of this measurement depends on the accuracy of the ruler being used. Identify the smallest unit of measurement of this ruler as 1 mm. Convert 1 mm to 0.1 cm.	**a** 1 mm = 0.1 cm measurement value = 9 cm
2 Calculate the lower and upper bounds by substituting the measurement value and the smallest unit of measurement into the lower and upper bound formulas.	lower bound = measurement – 0.5 × smallest unit = 9 – 0.5 × 0.1 = 8.95 cm upper bound = measurement + 0.5 × smallest unit = 9 + 0.5 × 0.1 = 9.05 cm
b 1 Identify the smallest unit of measurement as 2 mm. Convert 2 mm to 0.2 cm.	**b** 2 mm = 0.2 cm measurement value = 9 cm
2 Calculate the lower and upper bounds by substituting the measurement value and the smallest unit of measurement into the lower and upper bound formulas.	lower bound = measurement – 0.5 × smallest unit = 9 – 0.5 × 0.2 = 8.9 cm upper bound = measurement + 0.5 × smallest unit = 9 + 0.5 × 0.2 = 9.1 cm

Helpful hints

✔ Interpret the scenario to identify the exact value and estimated value to calculate errors.
✔ To calculate percentage errors, remember to multiply relative errors by 100% rather than just 100.
✔ Convert values to the same unit of measurement before substituting them into the formulas.

ANS
p452

Exercise 5F Errors

△ 1–5, 8, 9, 12, 14, 16 ▢ 2, 3, 5, 6, 9–10, 13, 16–17 ⬠ 2, 3, 5, 7, 11, 13, 15, 17

UNDERSTANDING AND FLUENCY

1 State the absolute value of the following numbers.

a 3 **b** $\frac{2}{5}$ **c** −8 **d** −4.2 **e** 0 **f** $-\sqrt{2}$

2 Find the absolute value of the following:

a $|5 - 3|$ **b** $|3 - 5|$ **c** $|2.4 - 1|$

d $|-1 - 2.4|$ **e** $|0.24 - 0.172|$ **f** $|0.172 - 0.24|$

3 a What is the smallest decimal that could result in an answer of 1.7 when rounded to one decimal place?

b What is the smallest decimal that could result in an answer of 2.33 when rounded to two decimal places?

4 A cafe estimates 80 croissants will be sold in a day. Find the absolute error if the actual number of croissants sold is:

a 92 **b** 72.

5F.1 **5** A worker expects to receive a salary of $5400 per month according to their contract. As a reward of his great performance last month, the company gave him $1000 bonus. Determine:

a the absolute error

b the relative error

c the percentage error.

6 A theatre anticipates 180 audience members to come to a show. In the end, there were only 160 audience members in attendance. For the size of the actual audience and the anticipated audience, determine:

a the absolute error

b the relative error

c the percentage error.

7 Adam measured his height as 188 cm. Later he realised that he did not take his shoes off during the measurement. Adam measured his height again with bare feet and found that he is 185 cm tall. Determine:

a the relative error

b the percentage error.

5F.2 **8** When 3.14 is used as an approximate value of π, determine the percentage error in calculating the circumference and the area of a circle with radius:

a 2 cm **b** 10 cm.

5F.3 **9** A scale shows that a block of cheese weighs 430 g. Calculate the lower and upper bounds for this measurement value if the scale has:

a 1 g as the smallest unit of measurement **b** 5 g as the smallest unit of measurement.

10 An electronic scale needs to tare before each use, that is, to reset the scale to zero. A chef uses a scale to measure the weight of an apple. The chef didn't notice that the weight shown initially on the scale was 8 g and used it without pressing the tare key first. The scale showed the apple weighs 92 g.

a State the absolute error of this measurement.

b What is the actual weight of the apple?

c Calculate the percentage error.

11 A student mistakenly copied the question 35 × 128 as 35 × 182 in an exam and did the remainder of the calculation correctly. Find the percentage error of this calculation.

PROBLEM SOLVING AND REASONING

12 $\frac{22}{7}$ is used as an approximate value of π.

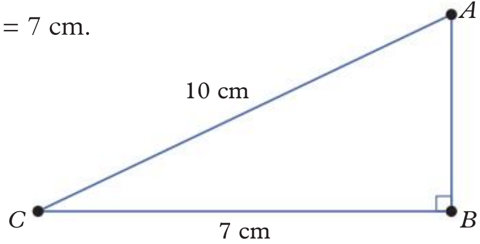
a Find the percentage error in calculating the circumference of a circle with radius:

i 2 cm **ii** 10 cm **iii** r cm

b Comment on your findings from the results above. Explain if this approximation is an overestimate or underestimate.

c Compare your results above with answers from question **8**. Which approximation of π is more accurate?

13 In the right-angled triangle ABC, $\angle ABC = 90°$, $AC = 10$ cm and $BC = 7$ cm.



a Find the exact length of AB using Pythagoras' theorem.

b Find the exact area of $\triangle ABC$.

c Find the approximate length of AB, correct to one decimal place.

d Use your answer from part **c** to estimate the area of $\triangle ABC$

e Calculate the relative error of the estimation in part **d**.

14 A piece of string is 14.2 cm in length. A recorder measured the length of the string as 14 cm using a ruler that can only read 1 cm as the smallest unit. Find:

a the absolute error in this measurement

b the lower and upper bounds for the measurement value

c the percentage error corresponding to the lower and upper bounds respectively.

15 Swetha buys contact lenses to correct her eyesight. The contact lenses purchased have strengths of −2.5 and −2.4 for the left and right eye respectively and work well for her.

a Contact lenses are available with strength increments of 0.1. Find the lower and upper bounds for her left and right eye strengths.

b The optometrist measured her eyesight to be of strength −2.53 and −2.37 for the left and right eye. Calculate the relative error for both eyes' strength when wearing the contact lenses.

16 Deb used a ruler to measure the length of a rope. She wrote down the measurement as 24 cm. Paul suggests that it's better to record it as 24.0 cm. Is Paul's suggestion always valid? Explain.

CHALLENGE

17 Five customers, Andy, Bill, Carol, Daniel and Emily, made purchases at a supermarket and paid cash. The amount of money they spent is recorded in the table below.

	Andy	Bill	Carol	Daniel	Emily
Exact total ($)	25.65	99.79	64.77	44.94	12.27
Cash payment ($)					

Because cash payments must be rounded to the nearest 5 cents, the amount of money they paid differs from the table above.

a Find the exact amount of cash each customer paid, fill in the table above.

b For each customer, the error is the difference between the exact total and the amount they paid in cash.

i Calculate the error for each customer, and then find the average of the errors.

ii Calculate the absolute error for each customer, and then find the average of the absolute errors.

c Compare the two averages you find in part **b**. Which average is more suitable to measure the errors in cash payments? Explain your reasoning.

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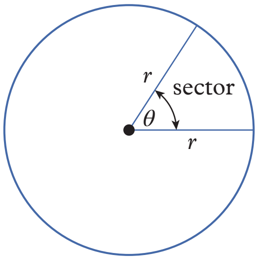
Interactive skillsheet
Errors

Interactive skillsheet
Upper and lower bounds

Topic quiz
5F

Chapter summary

Area of a sector

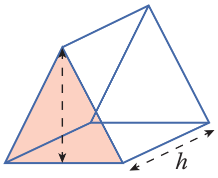


$$A = \frac{\theta}{360} \times \pi r^2$$

Area of composite shapes

1. Split the figure into simple shapes that have known area formulas.
2. Calculate any missing dimensions.
3. Calculate the areas of the individual shapes using the area formulas.
4. Add or subtract the areas to calculate the total area.

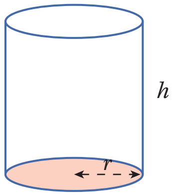
Volume of prisms



$V = Ah$

Where A is the area of the base and h is the height

Volume of cylinder

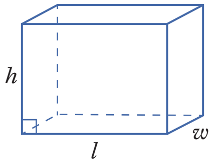


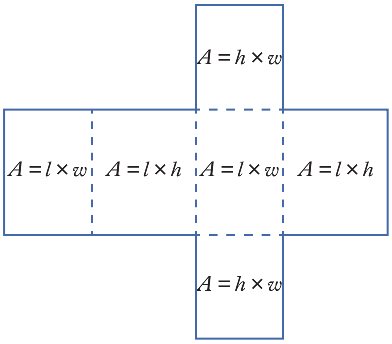
$V = \pi r^2 h$

Surface area

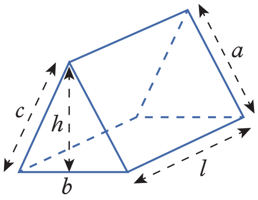
1. Determine the number of faces.
2. Calculate the area of each face.
3. Add the areas of the faces together.

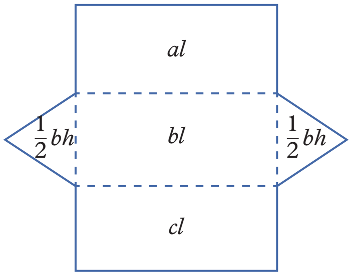
- Rectangular prism TSA = $2lw + 2lh + 2hw$



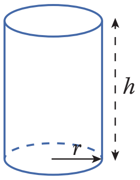


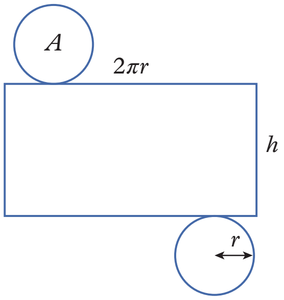
- Triangular prism TSA = $bh + (a + b + c)l$





- Cylinder TSA = $2\pi rh + 2r^2$





Capacity

The capacity of a 3D object is a measure of how much the object can hold. A container with an inside volume of:

- 1 cm³ holds 1 mL of liquid
- 1000 cm³ holds 1 L of liquid
- 1 m³ holds 1 kL of liquid.

Errors

error = exact value − estimated value

absolute error = |exact value − estimated value|

lower bound = measurement − 0.5 × smallest unit of measurement

upper bound = measurement + 0.5 × smallest unit of measurement

$$\text{relative error} = \frac{\text{absolute error}}{\text{exact value}}$$

$$\text{percentage error} = \frac{\text{absolute error}}{\text{exact value}} \times 100\%$$

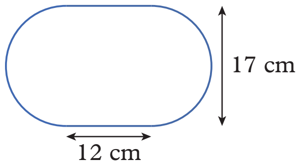
Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

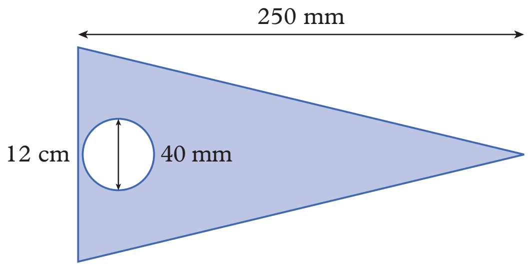
Multiple choice

5A **1** The area of this shape is closest to:



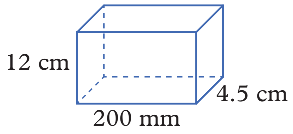
A 77 cm^2 **B** 204 cm^2 **C** 431 cm^2 **D** 1112 cm^2 **E** 641 cm^2

5A **2** The shaded area is closest to:



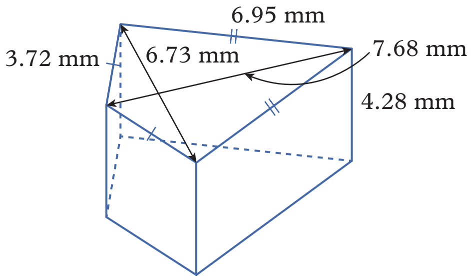
A 99.7 cm^2 **B** 137.43 cm^2 **C** 143.72 cm^2 **D** 150 cm^2 **E** 162.57 cm^2

5B **3** The surface area of this rectangular prism is:



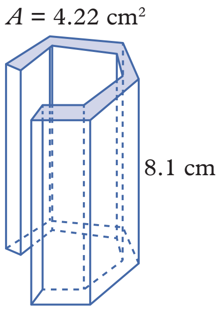
A 6708 cm^2 **B** 1080 cm^2 **C** 768 cm^2 **D** 678 cm^2 **E** 384 cm^2

5B **4** The surface area of this prism is:



A 150.7444 mm^2 **B** 143.0216 mm^2 **C** 110.655 12 mm^2 **D** 221.217 792 mm^2 **E** 194.708 mm^2

5C **5** The volume of this prism is:

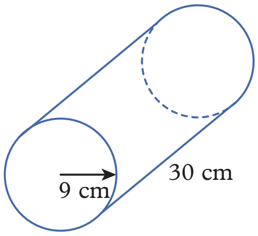


A 154.062 cm^3 **B** 80.2644 cm^3 **C** 114.4464 cm^3 **D** 650.141 64 cm^3 **E** 34.182 cm^3

5C **6** The volume of the prism in question **4** is:

A 150.7444 cm^3 **B** 143.0216 cm^3 **C** 110.608 896 cm^3 **D** 221.217 792 cm^3 **E** 194.708 cm^3

5D **7** The surface area of the following cylinder is:



A 270 cm^2 **B** 7634.1 cm^2 **C** 2205.4 cm^2 **D** 7351.3 cm^2 **E** 678.6 cm^2

5D **8** The surface area of a cylinder increased 12π cm^2 after the height of the cylinder is increased 3 cm^2. What is the radius of the base?

A 4π cm **B** 2π cm **C** 4 cm **D** 2 cm **E** 1 cm

5E **9** Which of the following options is closest to the volume of the cylinder in question **7**?

A 7700 cm^3 **B** 7600 cm^3 **C** 700 cm^3 **D** 800 cm^3 **E** 2000 cm^3

5E **10** What is the capacity of a cylinder with height 25 cm and radius 35 cm?

A 9.6 L **B** 6.7 L **C** 1.7 L **D** 38 L **E** 2.4 L

5F **11** The lower and upper bounds of a measurement are 14.95 km and 15.05 km. What is the actual measurement?

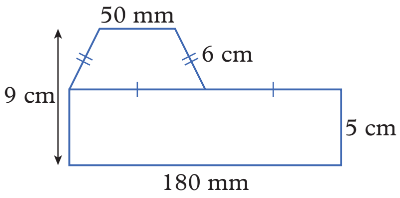
A 14.9 **B** 14.90 **C** 15.0 **D** 15 **E** 15.00

5F **12** Find the percentage error in calculating the area of a circle with a radius of 10 cm, using 3.14 as an approximate value of π.

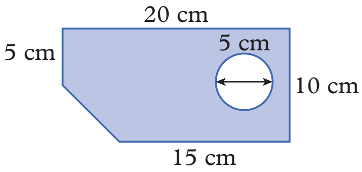
A 0.0005 **B** 0.05 **C** 5% **D** 0.05% **E** 0.5%

Short answer

5A **1** Calculate the area of this composite shape in square centimetres.

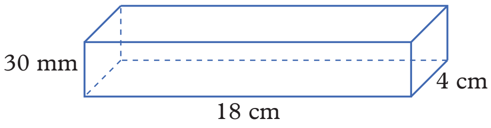


5A **2** Calculate the shaded area correct to two decimal places.

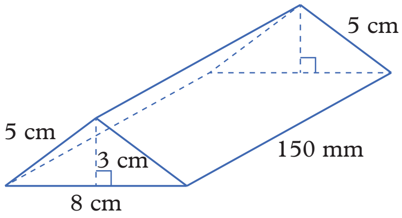


5B **3** Calculate the surface area of each prism.

a



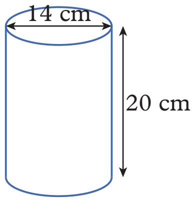
b



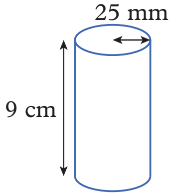
5C **4** Calculate the volume of each prism in question **3**.

5D **5** Calculate the surface area of each cylinder correct to two decimal places.

a



b



5E **6** Calculate the volume of each cylinder in question **5**. Correct your answers to two decimal places.

5E **7** The volume of a cylindrical water tank with radius 10 m is 4000 m^3.

a What is the capacity of the water tank?

b Calculate the height of the tank to the nearest metre.

5F **8** Calculate the upper and lower bounds for the following measurements.

a 55.0 cm using a rule with 1 mm as the smallest unit of measurement

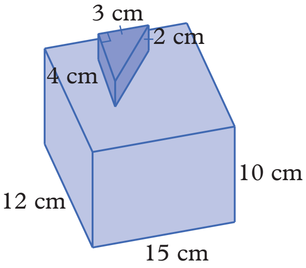
b 32.5 cm using a tape measure with 5 mm as the smallest unit of measurement

c 30 g using a scale with 1 g as the smallest unit of measurement

d 108 mL using a measuring cylinder with 1 mL as the smallest unit of measurement

Analysis

1 Angelique is making a sculpture. The rectangular prism will be painted purple and the triangular prism will be painted pink. The triangular prism has a height of 2 cm and the cross-section is a right-angled triangle with a base length of 3 cm and a perpendicular height of 4 cm.



a What is the length of the longest side of the triangle?

b Calculate the area to be painted in each colour (cm^2), assuming all sides, including the bottom, will be painted before assembly.

c Angelique has a tin of purple paint that will cover 1 m^2. Does she have enough purple paint for two coats?

d After painting two purple coats, what area would the remaining purple paint cover?

e Pink paint comes in smaller tins that contain enough paint to cover 90 cm^2. What is the maximum number of coats possible from one tin?

2 George is interested in finding out the thickness of a piece of A4 paper. As one sheet of paper is too thin to be measured alone, he decides to measure a stack of papers using a ruler and calculate the thickness for each piece from that. George records his measurement in a table below.

	Stack A	**Stack B**	**Stack C**	**Stack D**
Number of sheets	20	50	200	500
Thickness (mm)	2	4	17	42

a Stack C has 10 times as many papers as stack A, and stack D has 10 times as many papers as Stack B. Explain why the thicknesses recorded for stack C and D are not exactly 10 times the thicknesses of stack A and B.

b Which measurement is likely to give the most accurate data for calculating the thickness of a piece of paper? Calculate the thickness of a piece of paper from this measurement.

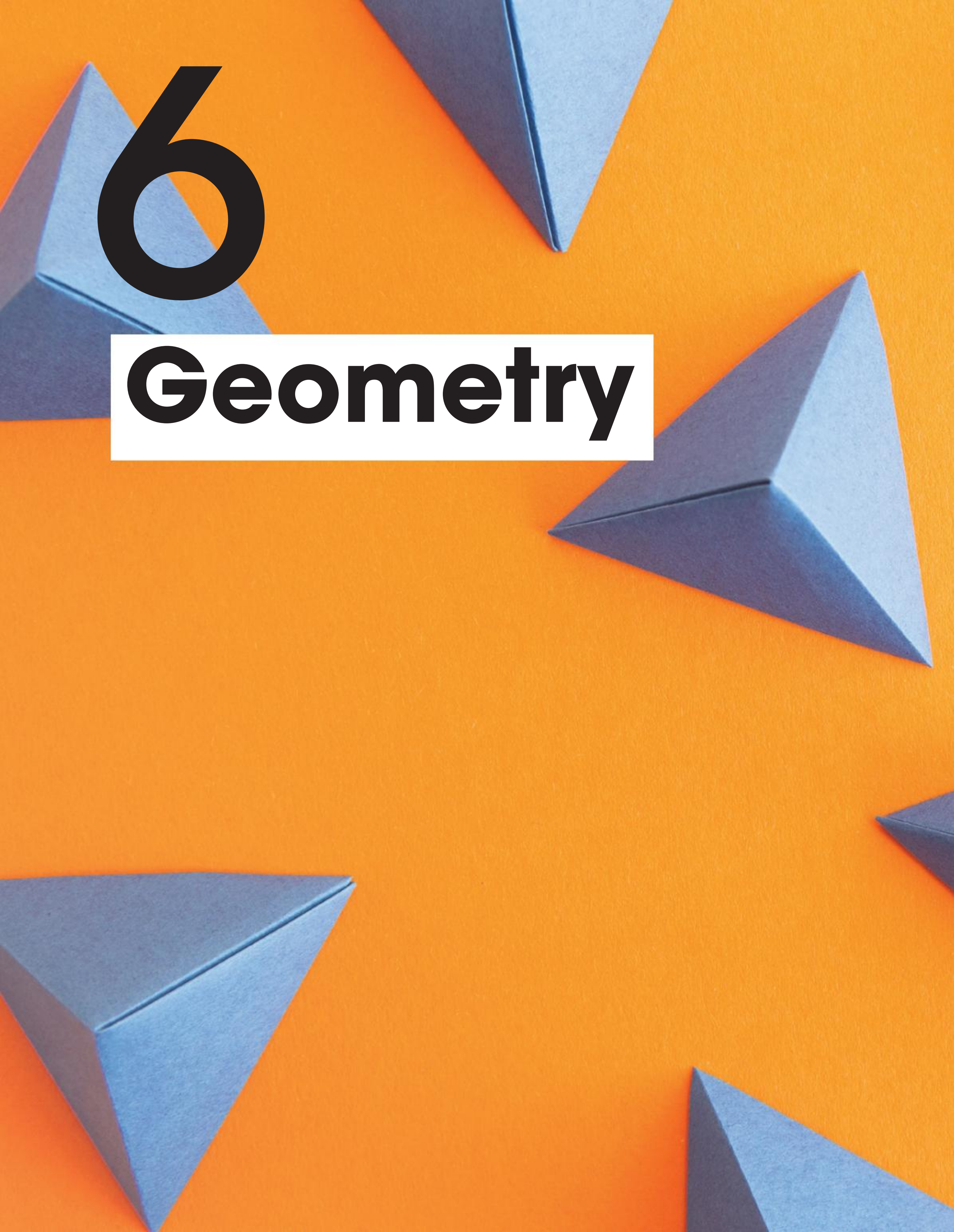
The actual thickness of this printing paper is 0.085 mm per sheet.

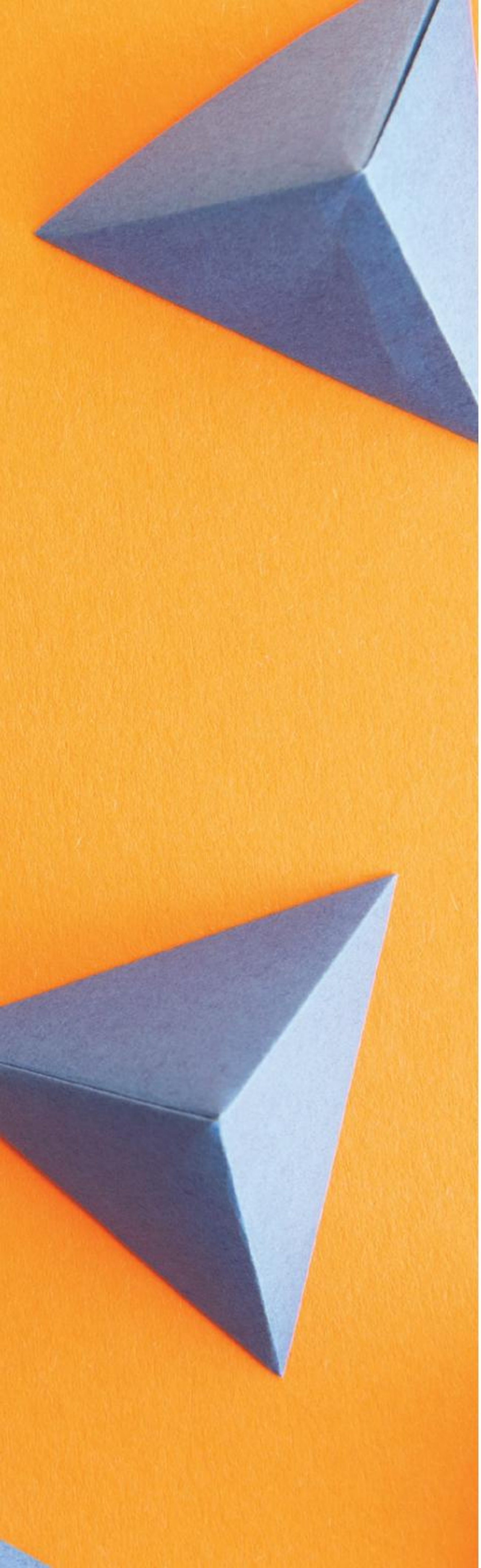
c Find the absolute error for each of the four measurements. Which measurement is most accurate?

d Find the percentage error for each of the four measurements. Does smaller absolute error lead to smaller percentage error? Explain your reasoning in this context.

6

Geometry





Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Triangle properties
✔ Simplifying surds
✔ Multiplying and dividing surds
✔ Substitution
✔ Solving equations using inverse operations

Curriculum links

- Solve spatial problems, applying angle properties, scale, similarity, Pythagoras' theorem and trigonometry in right-angled triangles (AC9M9M03)
- Use mathematical modelling to solve practical problems involving direct proportion, rates, ratio and scale, including financial contexts; formulate the problems and interpret solutions in terms of the situation; evaluate the model and report methods and findings (AC9M9M05)
- Recognise the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles using properties of similarity (AC9M9SP01)
- Apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain aspects that remain the same and those that change (AC9M9SP02)

Materials

✔ Calculator

6A Pythagoras’ theorem

Learning intentions

By the end of this topic you will be able to …

- ✔ use Pythagoras’ theorem to determine whether a triangle is right-angled
- ✔ use Pythagoras’ theorem to determine unknown side lengths.

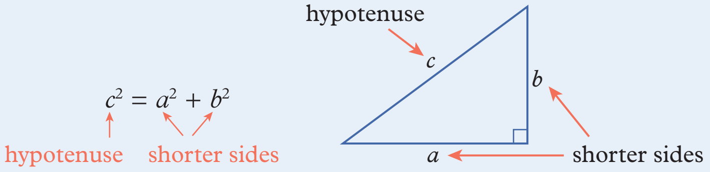
Inter-year links

Year 8 7C Pythagoras’ theorem

Year 10 7A Pythagoras’ theorem

Pythagoras’ theorem

- The longest side of a right-angled triangle is called the hypotenuse. It is always opposite the right angle.
- **Pythagoras’ theorem** states the relationship between the three side lengths of a right-angled triangle: the sum of the squares of the shorter sides that meet at the right angle is equal to the square of the hypotenuse. The relationship for the triangle shown below is:

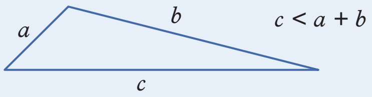


- Any set of three whole numbers that satisfies Pythagoras’ theorem is called a **Pythagorean triple** (or Pythagorean triad).

  For example, 3, 4, 5 is a Pythagorean triple because $5^2 = 3^2 + 4^2$.
- If the length of the unknown side of a right-angled triangle is an irrational number, the solution can be written as either an approximate decimal value or an exact value in simplified surd form.

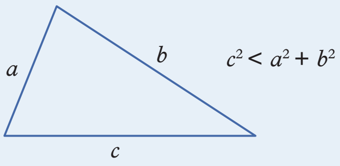
The triangle inequality

- In every triangle the length of any one side must be less than the sum of the lengths of the other two sides.

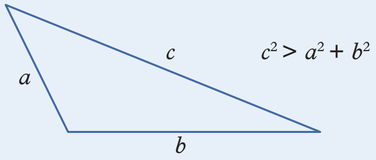


Inequalities for acute and obtuse triangles

- In an acute triangle, the square of the longest side will be less than the sum of the squares of the other two sides.



- In an obtuse triangle, the square of the longest side will be greater than the sum of the squares of the other two sides.



→ The longest side in an obtuse triangle will be the side opposite the obtuse angle.

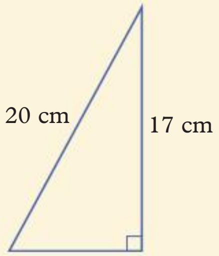
Example 6A.1 Using Pythagoras’ theorem to determine whether a triangle is right-angled

Determine whether a triangle with side lengths of 10 cm, 15 cm, and 12 cm is right-angled.

THINK	**WRITE**
1 Identify the length of the longest side and call it c.	If $c = 15$ cm:
2 Calculate the value of c^2.	$c^2 = (15)^2$ $= 225$
3 Identify the lengths of the two shorter sides and call them a and b.	$a = 10$ cm and $b = 12$ cm
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = (10)^2 + (12)^2$ $= 100 + 144$ $= 244$
5 Compare c^2 to $a^2 + b^2$. If these two values are equal, the triangle is right-angled. If the two values are not equal, the triangle is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

Example 6A.2 Using Pythagoras’ theorem to calculate an unknown side (decimal value)

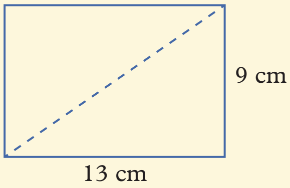
Calculate the length of the unknown side of this triangle, correct to one decimal place.



THINK	**WRITE**
1 Since the triangle is right-angled, state Pythagoras’ theorem and define a, b and c. Remember that c is the hypotenuse and use b for the unknown side.	$c^2 = a^2 + b^2$, where $c = 20$ cm and $a = 17$ cm
2 Substitute the values for a and c into the equation for Pythagoras’ theorem and simplify the equation.	$(20)^2 = (17)^2 + b^2$ $400 = 289 + b^2$
3 Solve the equation for b using inverse operations.	$b^2 = 400 - 289$ $= 111$ $b = \sqrt{111}$
4 Use a calculator to find the approximate length of the unknown side. Round to one decimal place and include the units.	$b \approx 10.5$ cm

Example 6A.3 Using Pythagoras' theorem to calculate an unknown length (exact value)

Calculate the length of the diagonal of this rectangle. Give your answer as an exact value in simplest form.



THINK	WRITE
1 Because the shape is a rectangle, the corners are right angles. State Pythagoras' theorem and define *a*, *b* and *c*. Remember that the diagonal of a rectangle will be the hypotenuse of the triangle including the two side lengths.	$c^2 = a^2 + b^2$, where $a = 13$ cm and $b = 9$ cm
2 Substitute the values for *a* and *b* into the equation for Pythagoras' theorem. Let *c* be the unknown side and solve the equation for *c*.	$c^2 = (13)^2 + (9)^2$ $= 169 + 81$ $= 250$ $c = \sqrt{250}$
3 Simplify the surd by identifying any factors of 250 that are perfect squares. Include the units.	$= \sqrt{25 \times 10}$ $= \sqrt{25} \times \sqrt{10}$ $= 5\sqrt{10}$ cm

Helpful hints

- ✔ Make sure you correctly identify the hypotenuse. It is always the longest side, and is opposite the right angle on the triangle. It may not be obvious, so always look at the numbers carefully.
- ✔ It doesn't matter which of the shorter sides you assign to be *a* or *b*, but the hypotenuse must always be *c*.
- ✔ Pythagoras' theorem is an excellent formula to memorise. You will use it many times during your mathematics studies.
- ✔ Remember to always give your answer with the correct units.

ANS p453

Exercise 6A Pythagoras' theorem

△ 1, 3–7, 9(a, c, f), 10, 12, 13

■ 1, 2, 3(b, d, f, h), 4(d–f), 5, 8, 9, 11, 15, 16, 18

⬟ 1, 2, 3(e, g, i), 6(b, d), 9(c, d, f), 11, 14, 16, 17, 19

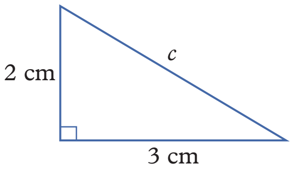
6A.1 **1** Determine whether each set of side lengths form a right-angled triangle.

a 26 cm, 24 cm, 10 cm
b 100 cm, 96 cm, 28 cm
c 3.5 cm, 8.5 cm, 9.3 cm
d 1 cm, 2.4 cm, 2.6 cm
e 7.5 cm, 4.2 cm, 8.5 cm
f 5 cm, 7 cm, 6 cm

2 For the triangles in question **1** which do not form a right-angled triangle, determine if they form an acute or an obtuse triangle.

6A.2 **3** Calculate the length of the unknown side in each of these triangles, correct to one decimal place.

a



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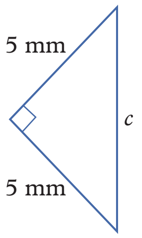
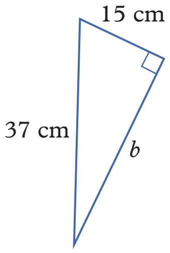
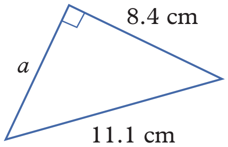


c



d



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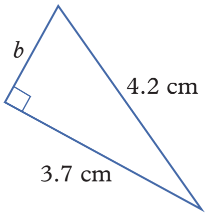

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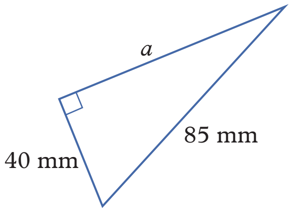

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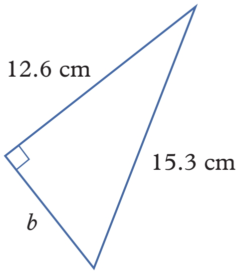

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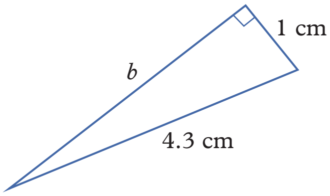

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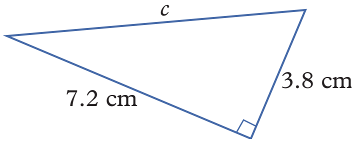
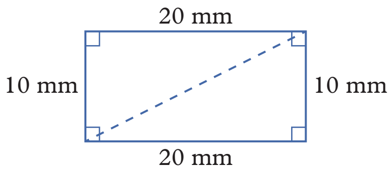
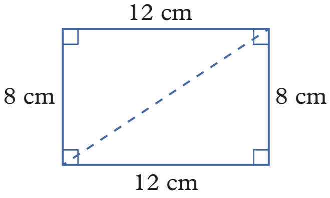


6A.3 **4** Calculate the length of the diagonal in each of these rectangles. Give your answer as an exact value in simplest form.

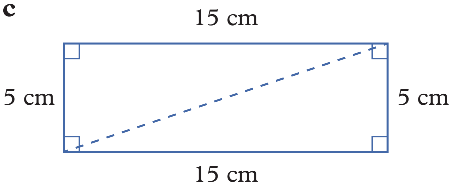
a



b



c



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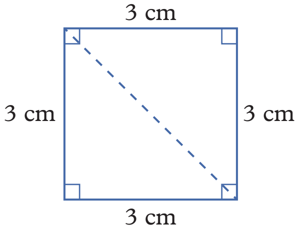

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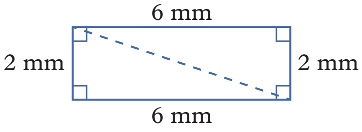
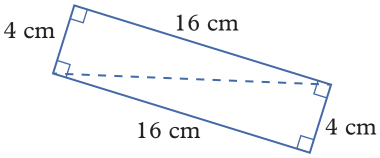


f



5 Rewrite the lengths of the diagonals of the rectangles in question **4**, correct to one decimal place.

6 Calculate the length of the diagonal of a square with each of these side lengths. Give your answer as an exact value in simplest form.

a 5 cm **b** 20 mm **c** 8 cm **d** 25 cm

7 Rewrite the lengths of the diagonals of the squares in question **6**, correct to one decimal place.

8 An equilateral triangle has side lengths of 8 cm.

a Draw a sketch of the triangle with two vertices forming the base of the triangle. From the top vertex, draw a line that is perpendicular (at right angles) to the base. Label the lengths of all sides of the figure.

b Use Pythagoras' theorem to calculate the height of the triangle. Give your answer as:

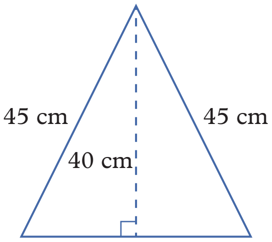
i an exact value **ii** an approximate value, correct to one decimal place.

UNDERSTANDING AND FLUENCY

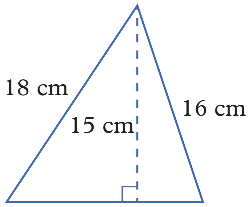
9 Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answers as:

i exact values **ii** approximate values correct to one decimal place.

a



b



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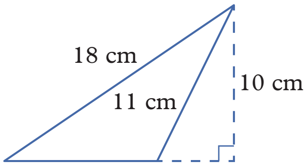

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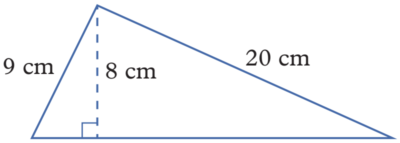

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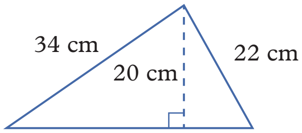

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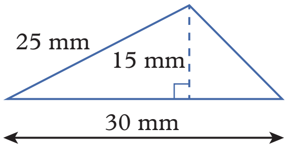


PROBLEM SOLVING AND REASONING

10 A squaring tool is used when making picture frames, to ensure that the corners of the frames are true right angles.

Imagine you are constructing a square picture frame with side lengths of 54 cm.

a One of the diagonals of the frame measures 78 cm. Explain how you can be sure that the frame is not truly square.

b The other diagonal measures 76 cm. Draw a labelled sketch of the frame, showing its true shape.

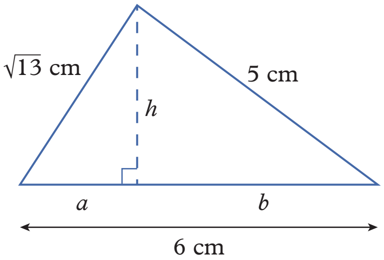
11 A sheet of writing paper is 21 cm wide and 30 cm long. The envelope used for this writing paper has a length of 22 cm and a diagonal of 24.6 cm.

a What is the width of the envelope?

b The writing paper is folded into equal-sized strips along its longer side. What is the minimum number of folds necessary to ensure the paper will fit in the envelope?

c If the paper was folded along its shorter side as well as its longer side, what would be the minimum number of folds needed for it to fit the envelope?

12 Afra wanted to determine the perpendicular height to the 6 cm edge of the triangle on the right but does not know exactly where along the 6 cm edge it intersects. She was able to write the following three equations based on the information in the triangle.



$a^2 + h^2 = 13$ (1)

$b^2 + h^2 = 25$ (2)

$a + b = 6$ (3)

a Explain how Afra was able to write the three equations.

b **i** Rearrange equation (1) to make h^2 the subject.

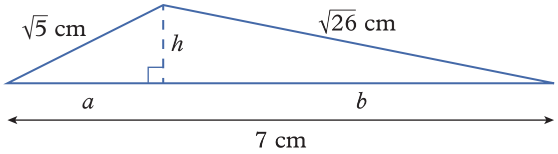
ii Rearrange equation (3) to make b the subject.

c Use substitution of the rearranged equations in part **b** to show that equation (2) can be written as $49 - 12a = 25$.

d Solve the equation in part **c**.

e Hence, determine the values of b and h.

f Repeat this process to find the exact perpendicular height to the 7 cm edge of the triangle shown on the right.



PROBLEM SOLVING AND REASONING

13 We know that a triangle with side lengths 3 cm, 4 cm and 5 cm is right-angled. The numbers 3, 4, 5 are one example of a Pythagorean triple. There are many more.

a To investigate other triples, we can start by finding out if multiples of the numbers 3 : 4 : 5 also satisfy Pythagoras' theorem. Do each of the following sets of numbers satisfy Pythagoras' theorem?

i 30 : 40 : 50 **ii** 6 : 8 : 10 **iii** 12 : 16 : 20

b We can use the Pythagorean triple 3, 4, 5 to check whether sets of side lengths that are not whole numbers are also Pythagorean triples. Use the 3, 4, 5 triple to show that triangles with the following side lengths are indeed right-angled triangles.

i 1.5 cm, 2 cm, 2.5 cm **ii** 0.6 cm, 0.8 cm, 1 cm **iii** 1.2 cm, 1.6 cm, 2 cm

c What do you conclude from your investigations in parts **a** and **b**?

d All the triangles in parts **a** and **b** are similar triangles. Explain why this is so.

14 There are different formulas that can be used to generate sets of Pythagorean triples. For example, three numbers in a Pythagorean triple can be written as: $x^2 - y^2$, $2xy$ and $x^2 + y^2$, where integer x is greater than integer y.

a Which of the given expressions will represent the hypotenuse? Will this always be the case?

b Choose values for x and y. Note the restriction that has been placed on these values.

c Use the values you chose in part **b** to calculate values for:

i $x^2 - y^2$ **ii** $2xy$ **iii** $x^2 + y^2$

d Verify that the set of numbers you generated in part **c** does, in fact, satisfy Pythagoras' theorem.

e Explain the restriction $x > y$.

f Repeat the procedure from parts **b–d** to generate another Pythagorean triple.

g What values of x and y will generate the triple 3, 4, 5 from the expressions given in part **c**?

15 The two sides of a right-angled triangle, other than the hypotenuse, are such that one of those sides is twice as long as the other. The hypotenuse is 25 cm long. What are the lengths of the two shorter sides? Write your answer:

a as a simplified exact value **b** correct to one decimal place.

16 Explain why it is impossible to draw a triangle where the length of one side is equal to the sum of the lengths of the other two sides.

17 Romesh has an extension ladder that is 9.8 m long. He needs to clean the windows on the outside of a building. The lower part of the windows are 9.8 m from the base of the building, and the windows are 1.2 m tall. Unfortunately, bushes prevent Romesh from placing his ladder any closer to the base of the building than 3 m. Romesh is 1.7 m tall.

a Draw a diagram to show the situation.

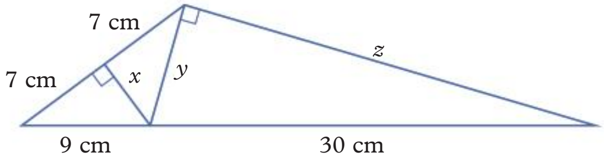
b Calculate the height the ladder reaches up the building correct to one decimal place.

c Explain whether Romesh will be able to clean the windows.

18 A cubby house is built on two levels in a tree. The lower level is 10 m above the ground, and the upper level is a further 5 m higher vertically. The ladder from the ground to the lower level is 12 m long. Sam wants to construct another ladder to go from the ground, 1 m further along the ground from the base of the tree than the first ladder, to the top level. How long should he make this ladder?

CHALLENGE

19 Determine the exact simplified values of x, y, and z.



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Interactive skillsheet
Pythagoras' theorem

Interactive skillsheet
Finding the length of the hypotenuse

Interactive skillsheet
Finding the length of a shorter side

Topic quiz
6A

6B Ratios, rates, direct proportion and scale

Learning intentions

By the end of this topic you will be able to ...

✔ use ratios to model situations

✔ solve problems involving direct proportion

✔ interpret scale drawings.

Inter-year links

Year 7	3H Ratios
Year 8	3D Ratios

Ratios

- A **ratio** expresses a relationship between two or more different quantities. Ratios are written with a colon (:) between the quantities. For example, if there are 3 red balls and 2 pink balls in a bag, this could be expressed using the ratio 3 : 2.
- Ratios can be simplified by dividing each part of a ratio by a common factor. A ratio is in simplest form when the only common factor between the parts is 1.

Rates

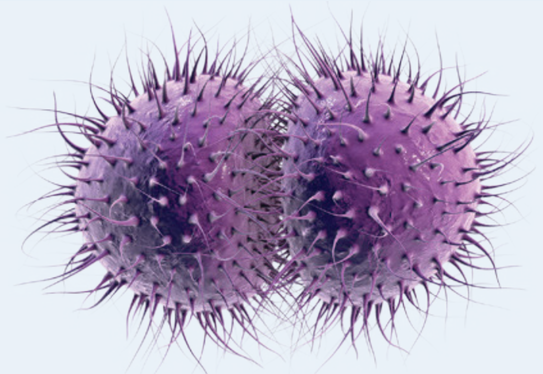
- A **rate** is a measure of how much one quantity changes as each unit of another quantity changes. Rates are written using the symbol / (meaning 'per' or 'for each'). For example, 70 km/h is a speed of 70 kilometres per hour.
- A rate is written in simplest form when the second of the two quantities has a value of 1.

Direct proportion

- Two quantities are in **direct proportion** if, as they increase or decrease, the ratio between the two quantities remains the same.
  - → An hourly pay rate is an example of direct proportion. As the number of hours worked increases, the overall pay will increase in direct proportion.
- If two quantities are in direct proportion, when the value of one quantity is 0, the value of the other quantity must also be 0.

Scale drawings

- A **scale drawing** is a drawing that represents a real-life object. Scale drawings can be used to accurately represent objects that are difficult to draw at their actual size; for example, house plans.
- In a scale drawing, the **scale** represents the ratio 'drawing length : actual length'. For example, if the scale was 1 : 100 each cm in the drawing would represent 100 cm in the real-life object.
- Scale drawings can also be used to represent objects that are too small to see without using a microscope, for example, bacteria and viruses.
  - → If the scale drawing was 10 000 : 1, each cm in the drawing would represent 0.0001 cm in real life.



Scale 10 000 : 1

Example 6B.1 Using rates to model situations

Harry can wash 24 dishes in 4 minutes. Use a rate in its simplest form to model this situation.

THINK	**WRITE**
Divide the number of dishes Harry can wash in 4 minutes by the number of minutes (4) to determine the number of dishes Harry can wash each minute.	$\frac{24}{4}$ = 6 dishes/minute

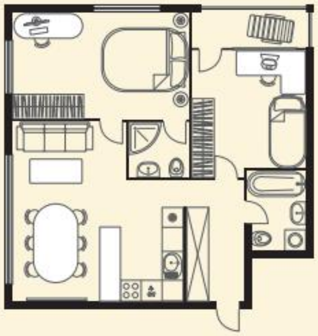
Example 6B.2 Solving problems using direct proportion

As a barista, Kayla is paid an hourly rate of $31.50. If Kayla earns $756 in a week, how many hours did she work?

THINK	**WRITE**
1 The amount Kayla is paid increases in direct proportion with the number of hours Kayla works, so if we divide the total amount earned by the hourly rate, we can determine the number of hours Kayla worked.	$\frac{756}{31.5} = 24$
2 Write the answer.	Kayla worked for 24 hours.

Example 6B.3 Interpreting scale drawings

The plan for the two-bedroom apartment at right is drawn to a scale of 1 : 60. If, in the scale drawing, one of the bedrooms measures 8 cm by 5 cm, what is the size of the bedroom in real life? Give your answer in metres.



THINK	**WRITE**
1 The scale is 1 : 60, so each cm in the scale drawing represents 60 cm in real life.	
2 Multiply the scale drawing dimensions by 60 to determine the real-life dimensions.	$8 \times 60 = 480$ cm $5 \times 60 = 300$ cm
3 Convert the units to metres by dividing by 100.	$\frac{480}{100} = 4.8$ m $\frac{300}{100} = 3$ m
4 Write the answer.	The real-life size of the bedroom is 4.8 m by 3 m.

Helpful hints

✔ Remember that for two quantities to be in direct proportion, if one quantity is 0, then the other quantity must be 0.

ANS p454

Exercise 6B Ratios, rates, direct proportion and scale

△ 1(a–c), 2, 3, 5, 8, 11, 14, 15 ☐ 1(b, d, f), 2, 4, 6, 9–11, 13, 16, 17 ⬠ 2, 6, 7, 9–12, 16–19

UNDERSTANDING AND FLUENCY

6B.1 1 Use a rate in simplest form to model the following situations.

- **a** Mark drives 225 km in 3 hours.
- **b** Steph can run 10 km in 50 minutes.
- **c** Pablo can type 660 words in 12 minutes.
- **d** A box of 6 donuts costs \$15.
- **e** It costs \$78.12 to fill a 42-litre petrol engine.
- **f** The energy bill came to \$160.50 for 30 days.

6B.2 2 Michaela charges \$120 for a one-hour personal training session. If Michaela earned \$3120 in a week, how many personal training sessions did she give?

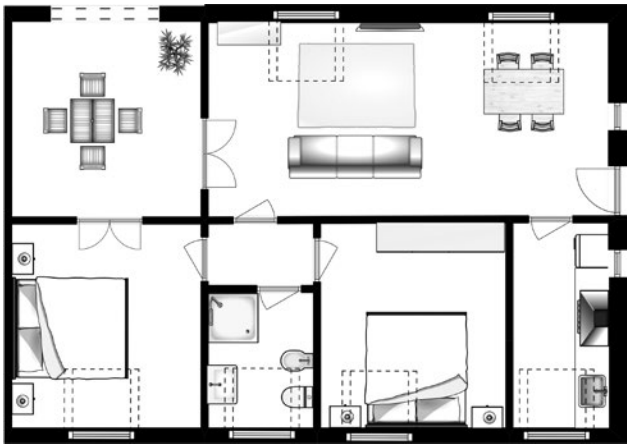
3 The exchange rate between Australian dollars (AUD) and United States dollars (USD) is 0.68. This means that each Australian dollar is worth 0.68 US dollars. How many US dollars is equivalent to \$3500 AUD?

4 Ariel is an office manager and needs to order some more stationary. If it costs \$1.69 for each pen, how much would it cost to order 80 pens?

6B.3 5 The plan for the two-bedroom apartment to the right is drawn to a scale of 1 : 80. If, in the scale drawing, the living room measures 9.8 cm by 4.5 cm, what is the size of the living room in real life? Give your answer in metres.

6 Ayesha creates a scale drawing of a house with a scale of 1 : 120. How big are each of the following rooms given their sizes in the scale drawing?

- **a** Bathroom (3 cm by 2.3 cm)
- **b** Bedroom (3.5 cm by 2.8 cm)
- **c** Kitchen (3.8 cm by 2.2 cm)
- **d** Living room (4.2 cm by 4.05 cm)



7 A photo of a red blood cell is printed at a scale of 5000 : 1. In the photo, the diameter of the red blood cell is 3.5 cm. What is the diameter of the red blood cell in micrometres (μm)?
Note: 1 cm = 10 000 μm

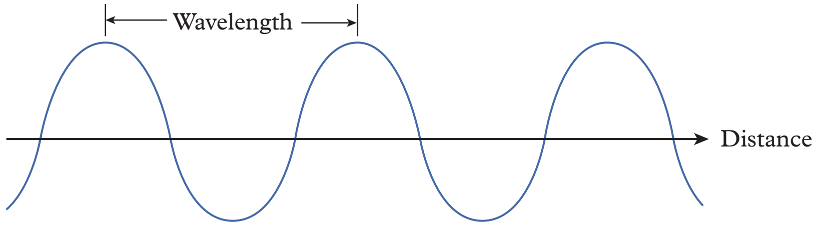
8 A physiotherapist sees 54 patients every week.

- **a** If the physiotherapist works 36 hours each week, on average, how many patients does the physiotherapist see per hour?
- **b** On average, how many minutes does the physiotherapist spend with each patient?

9 Jasmine lives 1200 m from school. She walks to school each day at a speed of 1 m/s. Rajesh lives 4.5 km from school. He rides his bike to school each day at a speed of 15 m/s.

- **a** If both Jasmine and Rajesh arrive at school at 8.30 am, what time did each student leave home?
- **b** Jasmine and Rajesh live 4.8 km apart. They plan to meet each other between their homes one morning and both leave their homes at 8 am. At what time will they meet?

10 Sound waves carry noise to our ears. The wavelength of a sound is the distance between identical parts of the wave, as shown in the diagram below.

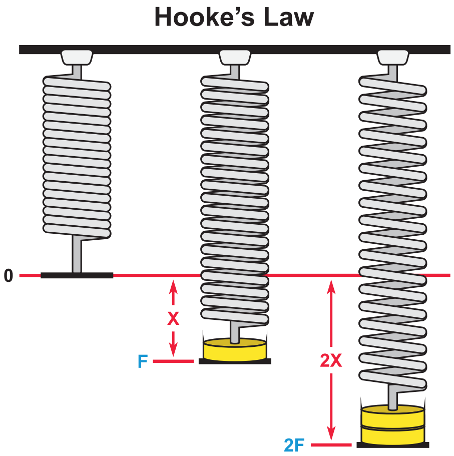


The frequency of a sound wave is measured in Hertz (Hz), which is the number of wavelengths per second. Humans can hear frequencies between 20 Hz and 20 000 Hz.

- **a** If a sound has a frequency of 500 Hz, what is the duration of each wavelength?
- **b** How many wavelengths does a sound with a frequency of 2000 Hz go through in 4 seconds?

11 Hooke's law states that, up to a certain distance, the distance an elastic material extends is directly proportional to the force applied. It requires a force of 2 Newtons to extend a spring by 8 cm.

- **a** How much force is required to extend the spring by 20 cm?
- **b** How much force is required to extend the spring by 15 cm?
- **c** How much would the spring extend by if 6.2 Newtons of force was applied?
- **d** Give a reason why Hooke's law only applies up to a certain distance.



12 In Australia, buildings must be made to specific standards. Staircases are made up of risers and treads, with the risers being the vertical sections and the treads being the steps. The risers and treads must be uniform. Riser heights must be between 150 mm and 180 mm, and treads must be at least 240 mm in depth.

- **a** If the height of a staircase is 3.06 m and there are 18 risers, what is the height of each riser?
- **b** If there are 18 risers, how many steps are needed?
- **c** If the tread depth is 240 mm, what is the overall horizontal distance covered by the staircase? Give your answer in centimetres.

13 Martin is looking to book an end-of-lease clean for his rental property. He sources three quotes for the job.

- Quote 1: $120 per hour, estimate 3.5 hours
- Quote 2: $130 per hour, estimate 3 hours
- Quote 3: $120 per hour + $80 call-out fee, estimate 2.5 hours

- **a** Calculate the total amount for each quote.
- **b** Explain why quotes 1 & 2 are both examples of direct proportion and quote 3 is not.

14 The measurement of 1 inch is equal to 2.54 cm, and there are 12 inches in a foot.

- **a** Explain why the relationship between inches and centimetres is an example of direct proportion.
- **b** Sara is 5 foot and 7 inches tall. Express Sara's height in inches.
- **c** Convert Sara's height into centimetres.

PROBLEM SOLVING AND REASONING

15 Karl takes a photo on his phone with a resolution of 1200 × 675 pixels. He uploads the photo to a social media platform, which converts the image to a size of 800 × 475 pixels.

a Determine the aspect ratio (height : width) of both the original image and the resized image.

b Was the ratio between the side lengths maintained in the image conversion?

16 Direct proportion between two variables can be identified using a table of values. To determine whether two variables (x and y) are directly proportional:

- check that when $x = 0, y = 0$
- check that the rate of change is constant.

The rate of change can be determined by dividing y by x.

Determine whether y is proportional to x for each of the following tables of values.

a

x	0	1	2	3	4
y	0	4	8	12	16

b

x	0	1	2	3	4
y	0	1	8	27	64

c

x	0	1	2	3	4
y	0	−9	−18	−27	−36

d

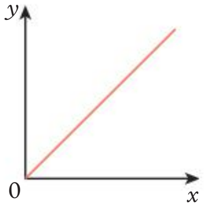
x	0	1	2	3	4
y	1	2	4	6	8

17 Direct proportion can also be identified by graphing the relationship between two objects. If two objects are directly proportional:

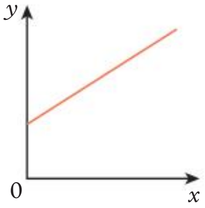
- the graph will be linear (a straight line)
- the graph will pass through the origin.

Determine whether y is directly proportional to x for each of the following graphs.

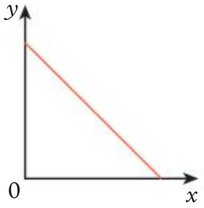
a



b



c



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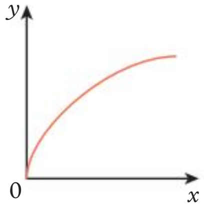

CHALLENGE

18 A direct proportion relationship between two quantities can be written as an equation. For example, the relationship between centimetres and inches can be written as $c = 2.54i$, where c is the number of centimetres and i is the number of inches.

The constant value in this equation (2.54) is known as the constant of proportionality. Identify and state the constant of proportionality for each of these direct proportion relationships.

a $y = 4x$ **b** $m = -10n$ **c** $h = 3.5t$ **d** $y = \frac{x}{3}$

19 The cost, C dollars, of building a house is directly proportional to the area, A, of the floor space in square metres. It costs $90 000 to build a house with a floor space of 150 m^2.

a Determine the constant of proportionality.

b Write an equation for the relationship between C and A.

c Use your answer from part **b** to calculate the floor space of a house that you could build with a budget of $126 000.

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pro

Interactive skillsheet Simplifying ratios

Interactive skillsheet Rates

Interactive skillsheet Scale drawings

Investigation Birth rates

Topic quiz 6B

6C Dilations and similar figures

Learning intentions

By the end of this topic you will be able to ...

✔ solve problems involving dilations
✔ perform dilations
✔ determine scale factors and area scale factors.

Inter-year links

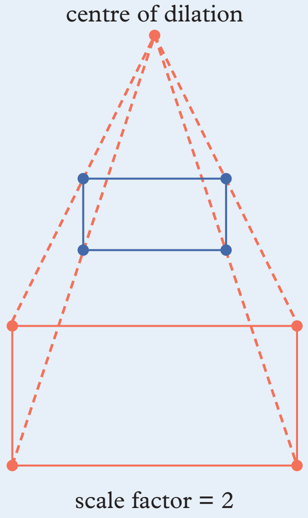
Year 8	7H Dilations and similar figures
Year 10	5C Congruence and similarity

Dilations

- A **dilation** is a transformation that changes the size of a shape. After a dilation, the shape and orientation of the image is the same as the original shape.
  - → The internal angles of a dilated image will be the same as in the original shape.
- To perform a dilation, you need a **centre of dilation** and a scale factor.
- A dilation can be either an **enlargement** or a **reduction**.
  - → If the scale factor is between 0 and 1, the image will be a reduction (smaller than the original shape).
  - → If the scale factor is greater than 1, the image will be an enlargement (greater than the original shape)
- The scale factor of a dilation can be found using the formula:

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

- The side length of a dilated image can be found by multiplying the original side length by the scale factor.



Similar figures

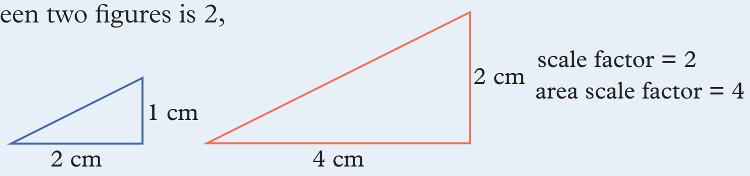
- Two figures are **similar** if they have the same shape. Similar figures can be of a different size.
  - → If two figures are similar, corresponding angles will be equal in size.
- After a dilation, the image will be a similar figure with the original shape.
- If two figures are known to be similar, an unknown side length can be found by multiplying (or dividing) by the scale factor between the shape and image.

Area scale factor

- The **area scale factor** can be found by using the formula:

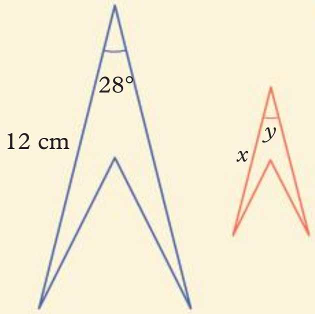
$$\text{Area scale factor} = \frac{\text{image area}}{\text{original area}}$$

- The area scale factor between two similar figures is equal to the square of the scale factor between the figures.
  - → For example, if the scale factor between two figures is 2, the area scale factor will be $2^2 = 4$.



Example 6C.1 Solving problems involving dilations

The shape on the left has been dilated by a scale factor of $\frac{1}{2}$ to create the image on the right. Find the values of x and y.



THINK

1 As the image has been created by a dilation these are similar figures.

2 12 cm and x are corresponding sides, so the value of x can be found by multiplying 12 cm by the scale factor.

3 28° and y are corresponding angles. Corresponding angles in similar shapes are equal.

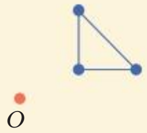
WRITE

$$x = 12 \text{ cm} \times \frac{1}{2}$$
$$= 6 \text{ cm}$$

$$y = 28°$$

Example 6C.2 Performing a dilation

Dilate this figure by a scale factor of 3 using the centre of dilation O.



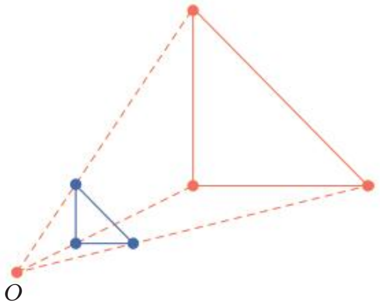
THINK

1 Use dashed lines to connect the centre of dilation with each of the vertices on the original shape.

2 The scale factor is 3, so extend the dashed lines by a factor of 3 and mark points at the end of these lines. These are the corresponding vertices of the image.

3 Label the vertices of the image and join them together to complete the dilation.

WRITE



Example 6C.3 Finding the area scale factor

A figure is dilated by a scale factor of 4. What is its area scale factor?

THINK

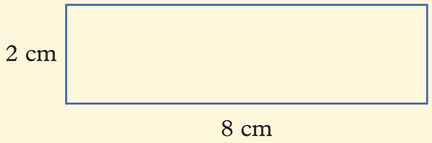
Calculate the square of the scale factor to determine the area scale factor.

WRITE

$$\text{Area scale factor} = 4^2$$
$$= 16$$

Example 6C.4 Finding area using the area scale factor

If this rectangle is dilated by a scale factor of 2, what will be area of the image?



THINK	**WRITE**
1 Find the area of the original rectangle.	$A = lw$ $= 2 \times 8$ $= 16\text{ cm}^2$
2 Find the area scale factor (the square of the length scale factor).	Area scale factor $= 2^2$ $= 4$
3 Multiply the original area by the area scale factor to find the area of the image.	$16 \times 4 = 64\text{ cm}^2$
4 Write your answer.	The area of the image will be 64 cm^2.

Helpful hints

✔ Dilations will create similar figures in the same orientation; however, two figures do not need to be in the same orientation to be similar.

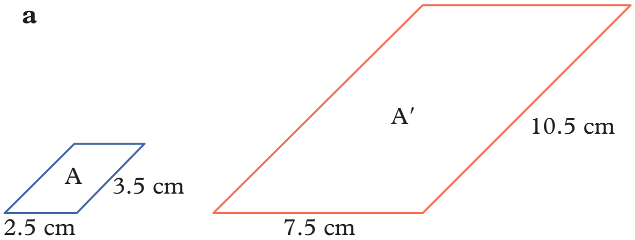
ANS p454

Exercise 6C Dilations and similar figures

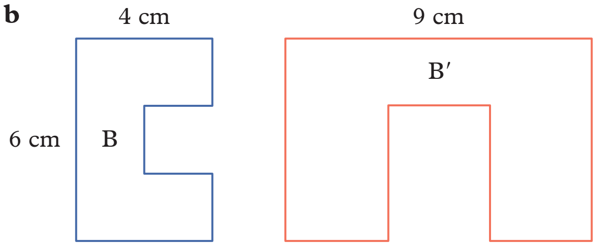
△ 1–2(a, b), 3(a), 4(a–d), 6(a, b), 7, 10, 14 ■ 1–2(b, c), 3(b), 5, 6(b, c), 8, 9, 11, 14, 15 ⬟ 2(c, d), 3(c), 6, 8–10, 12–18

1 Find the scale factor for each pair of similar figures.

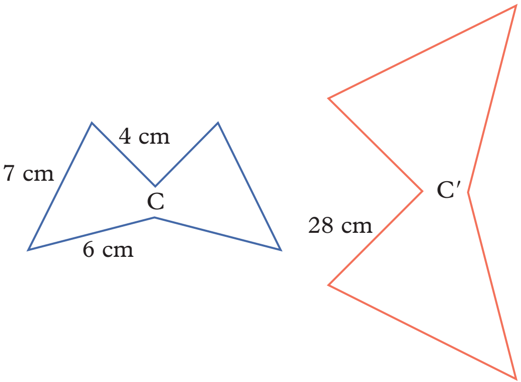
a



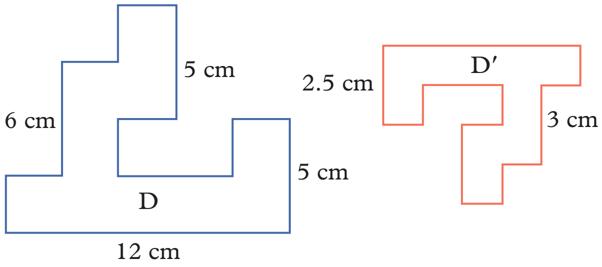
b



c

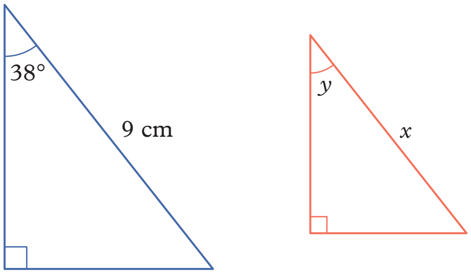


d



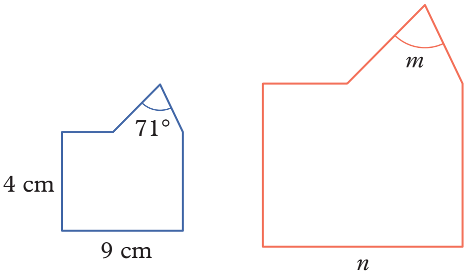
6C.1 **2** In each pair of shapes, the shape on the left has been dilated by the given scale factor to create the image on the right. Determine the values of the unknown pronumerals in each image.

a



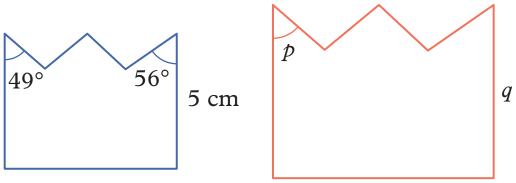
Scale factor = $\frac{2}{3}$

b



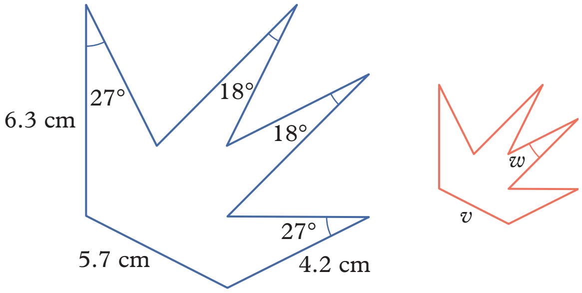
Scale factor = 3

c



Scale factor = 1.4

d

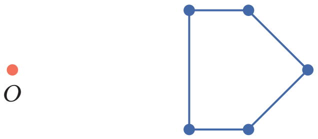


Scale factor = $\frac{1}{3}$

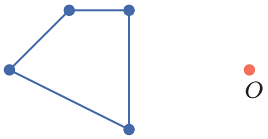
6C.2 **3** Use the worksheet available on your obook pro to dilate each figure using the centre of dilation O and a scale factor of:

i 2 **ii** $\frac{1}{2}$

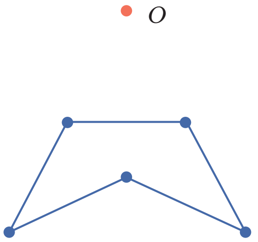
a



b



c



6C.3 **4** For each dilation scale factor, what is the area scale factor?

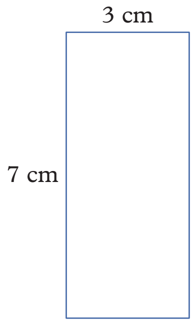
a 2 **b** 3 **c** 4 **d** $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $\frac{1}{4}$

5 Copy and complete this table.

	Original area	Dilation scale factor	New area
a	12 cm^2	2	
b	5 cm^2	3	
c	10 cm^2	5	
d	40 cm^2	$\frac{1}{2}$	
e	27 cm^2	$\frac{1}{3}$	
f	100 cm^2	$\frac{1}{5}$	

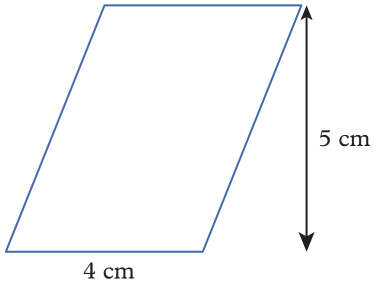
6C.4 **6** For each shape, state the area of the image if the original figure is dilated by the given scale factor. Give your answers to two decimal places where necessary.

a



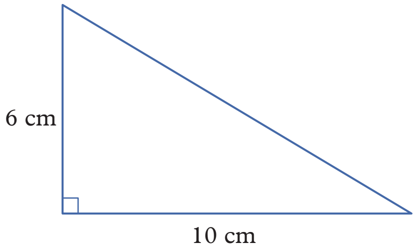
Scale factor = 4

b



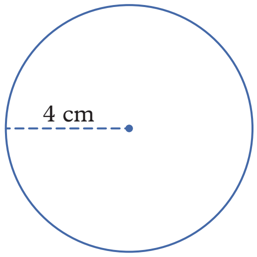
Scale factor = 3

c



Scale factor = 5

d



Scale factor = $\frac{1}{2}$

UNDERSTANDING AND FLUENCY

7 Ethan needed to increase the size of the area of his working space to nine times its previous size in order to accommodate a new project. Explain why he does not need to increase the length measurements of his working space by nine.

8 Explain how, if you know the area scale factor, you can find the length scale factor.

9 Calculate the length scale factor for these area scale factors.

a 49 **b** 25 **c** 100 **d** 64 **e** 4 **f** 400

10 Which of the following dimensions of rectangle is similar to a rectangle with side lengths of 11.7 cm and 9 cm?

A 4 cm × 5.5 cm
B 7 cm × 9.1 cm
C 12 cm × 15.5 cm
D 8.2 cm × 6 cm
E 10 cm × 7.5 cm

11 In low light, the pupils of our eyes dilate to let in more light. In normal light, Chantelle's pupils had a radius of 1.2 mm. When she walked into a dark room, they dilated to have a radius of 3.6 mm.

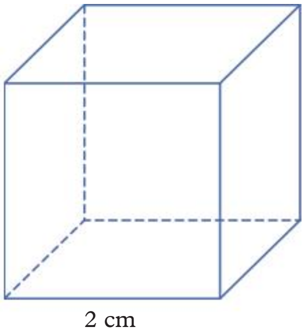
a What is the length scale factor?
b What is the area scale factor?
c Using the formula $A = \pi r^2$ and your answer to part **b**, find these areas correct to two decimal places.
  i The original pupil area
  ii The dilated pupil area
  iii The difference in these two areas

12 A farmer bought new land that was next to a paddock he already owned. If the area that he owned increased by a factor of 4, by what scale factor did the length of the land increase by?

13 A radar has a scanning radius of roughly 5 km. If the owner wanted to cover an area of 100π km^2, by how much would the radius of the radar need to increase?

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING



14 Consider the cube on the right.

a What are the side lengths of this cube?

b What is the volume of this cube?

c What would be the volume of this cube if the side lengths were:

i doubled **ii** tripled **iii** quadrupled?

d What pattern can you see between the length scale factor applied to the cube and the resulting increase in volume?

e Explain why the volume scale factor is equal to the cube of the length scale factor.

f Using your knowledge of how to calculate volume scale factor, find the volume of the cube if it was increased by a length scale factor of:

i 5 **ii** 6 **iii** 10

15 A company wanted to create a new line of beach balls. A standard beach ball at this company had a volume of roughly 30 000 cm^3.

a If they increased the radius of the standard beach ball by a length scale factor of 2, what would be the volume of the new beach ball?

b If they halved the radius of the standard beach ball by a length scale factor of 2, what would be the volume of the new beach ball?

c If they wanted to create a giant beach ball with 1000 times the volume of the standard beach ball, by what scale factor would they have to increase the radius?

16 A 30-cm deep sandpit had side lengths of 1 m.

a If the sandpit was doubled in size (all side lengths), what would be its new dimensions?

b By what factor would this increase the volume of the sandpit?

c How much more sand would be required to fill this larger sandpit? Give your answer in m^3 correct to one decimal place.

CHALLENGE

17 Most architects construct scale models of their building designs before the real one is built.

a A building will sit on a city block measuring 250 m × 300 m. The model for the building has a base area of 4687.5 cm^2. What area scale factor was used to create the model?

b What are the dimensions of the baseboard of the model?

c If a window in the model measures 1.8 cm × 6.0 cm, what area of glass will be needed in the real building?

18 An aquarium is in the form of a rectangular prism. If, to accommodate a new type of fish, it needed to have its height doubled, by what factor would this increase:

a the area of glass needed for the sides of the tank?

b the volume of the tank?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Similar figures

Worksheet Dilations

Topic quiz 6C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

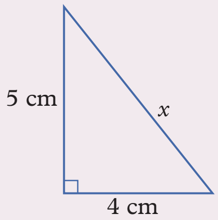
6A **1** Determine whether the following lengths can form a right-angled triangle.

a 9 cm, 40 cm, 41 cm **b** 39 cm, 89 cm, 80 cm **c** 137 cm, 104 cm, 89 cm **d** 21 cm, 221 cm, 220 cm

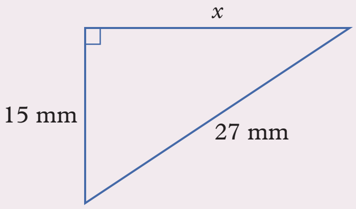
6A **2** Determine the value of x in each of the following:

i as a simplified exact value **ii** correct to two decimal places.

a

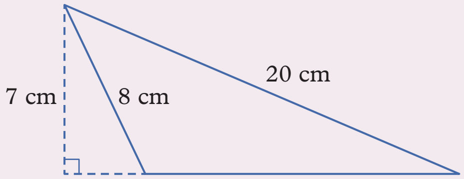


b

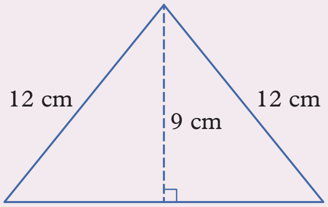


6A **3** Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answer as an exact value in simplest form.

a



b



6B **4** Use a rate in simplest form to model the following situations.

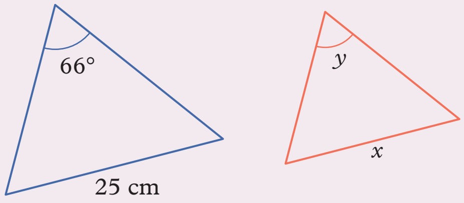
a A cyclist rides 86 km in 4 hours. **b** A bag of 5 oranges costs $4.80.

6B **5** The exchange rate between British pounds (GBP) and Australian dollars (AUD) is 1.75. How many British pounds is equivalent to $500 AUD?

6B **6** Nicole walks to school at a speed of 1.6 m/s. If Nicole lives 1200 m from school, how long does it take her to walk to school?

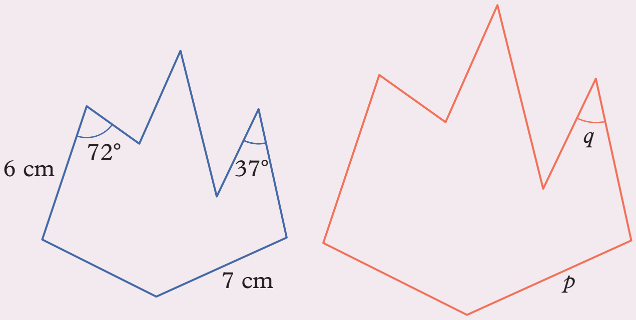
6C **7** The shapes on the left have been dilated by the given scale factor to create the image on the right. Determine the value of the unknown pronumerals in each image.

a



Scale factor = 0.8

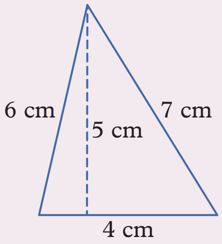
b



Scale factor = 2.5

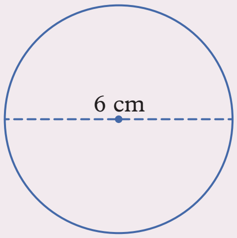
6C **8** For each shape, state the area of the image if the original figure is dilated by the length scale factor shown in brackets. Give your answers to two decimal places where necessary.

a



Scale factor = $\frac{1}{3}$

b



Scale factor = 2

6C **9** A triangular flag needs its dimensions increased by a factor of 5. If the flag has a 30 cm base and a 45 cm height, find the difference in the area of material needed to make the two flags.

6D Trigonometric ratios

Learning intentions

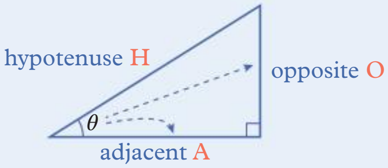
By the end of this topic you will be able to ...

- ✔ identify the hypotenuse, opposite and adjacent sides of a right-angled triangle containing a reference angle
- ✔ calculate sine, cosine and tangent ratios of an angle using side length relationships.

Inter-year links

Year 7	3H Ratios
Year 8	3D Ratios
Year 10	7B Trigonometry

Trigonometry



- **Trigonometry** is the study of relationships between angles and side lengths in triangles.
- In a right-angled triangle with a reference angle:
  - → the **opposite side** is opposite the reference angle
  - → the **adjacent side** is next to or adjacent the reference angle that is not the hypotenuse
  - → the hypotenuse is always the longest side and is opposite the right angle.
- The symbol θ (*theta*, a Greek letter) is often used to represent the reference angle.

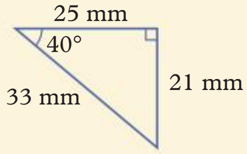
Trigonometric ratios

- For any right-angled triangle with a reference angle of θ:
  - → the **sine** of angle θ is the ratio of the lengths of the opposite side to the hypotenuse
  - → the **cosine** of angle θ is the ratio of the lengths of the adjacent side to the hypotenuse
  - → the **tangent** of angle θ is the ratio of the lengths of the opposite side to the adjacent side.
- The ratios can be written as fractions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Example 6D.1 Labelling the sides of a right-angled triangle with respect to a reference angle

Label the sides of this triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the angle of 40°.

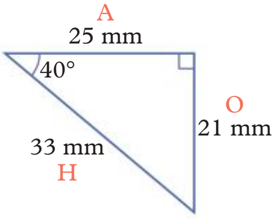


THINK

1 Label the hypotenuse with H. It is the longest side and is opposite the right angle.

2 Identify the opposite side and the adjacent side with respect to the reference angle of 40°. Label the side opposite 40° as O and the side next to 40° (adjacent) as A.

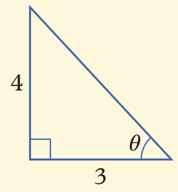
WRITE



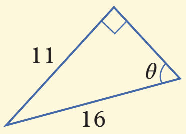
Example 6D.2 Determining trigonometric ratios

For each triangle, identify which trigonometric ratio can be used and then determine its value, based on the side lengths provided.

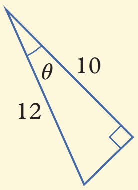
a

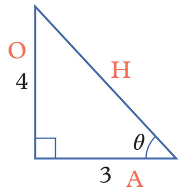
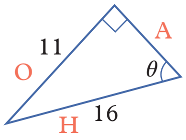
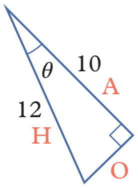


b



c



THINK	**WRITE**
a 1 Identify the given sides with respect to the reference angle. The side length opposite the angle is 4. The side length adjacent to the angle is 3. Label the sides accordingly.	**a**  O = 4, A = 3
2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.	$\tan(\theta) = \frac{O}{A}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{4}{3}$
b 1 Identify the given sides with respect to the angle. The side length opposite the angle is 11. The hypotenuse is 16. Label the sides accordingly.	**b**  O = 11, H = 16
2 Decide which trigonometric ratio to use. As O and H are involved, use sine.	$\sin(\theta) = \frac{O}{H}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{11}{16}$
c 1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 10. The hypotenuse is 12. Label the sides accordingly.	**c**  A = 10, H = 12
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.	$\cos(\theta) = \frac{A}{H}$
3 Substitute the values into the equation to determine the ratio.	$= \frac{10}{12}$
4 Simplify the fraction.	$= \frac{5}{6}$

Helpful hints

- ✔ Trigonometry can involve a lot of calculator work. You will need to know how to enter the trigonometric function in your scientific calculator.
- ✔ To remember the ratio of the two different sides according to the reference angle, you can use SOH CAH TOA. Like with Pythagoras' theorem, writing the equation from memory before substituting the values in will help you to memorise it quicker.

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A} \longrightarrow \text{SOH CAH TOA}$$

- ✔ The sine, cosine or tangent values do not have units. The units cancel out in the ratio.
- ✔ Do not write sin =, cos = or tan = without including the angle. Sine, cosine and tangent do not have a value without an angle.
- ✔ The tangent of the angle is the gradient of the hypotenuse of a triangle in the first quadrant with the reference angle placed at the origin.
- ✔ The prefix 'co-' in cosine is short for complementary, as cosine is the sine of the complementary angle, the other non-right-angle in a right-angle triangle, which is opposite the side adjacent to the reference angle.

ANS p456

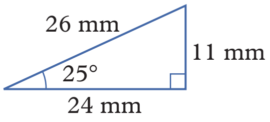
Exercise 6D Trigonometric ratios

▲ 1–5, 6(a, c, e), 7(b, d, f), 8–11, 14, 15, 19 ■ 2–5, 7–10, 12, 16, 17, 19, 20 ⬟ 3–5, 8–10, 12, 13, 16, 18, 20, 21

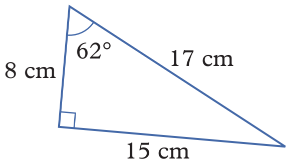
UNDERSTANDING AND FLUENCY

6D.1 **1** Label the sides of each triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the given reference angle.

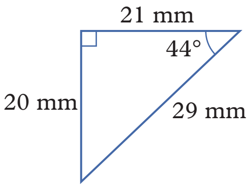
a



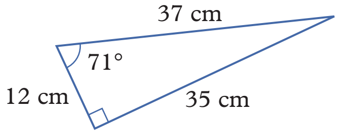
b



c

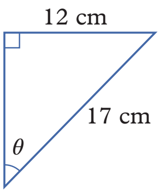


d

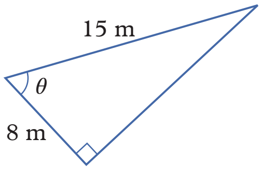


6D.2 **2** For each triangle, determine which trigonometric ratio can be used based on the side lengths provided.

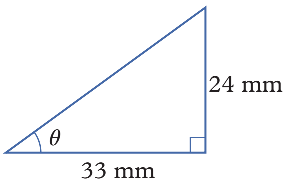
a



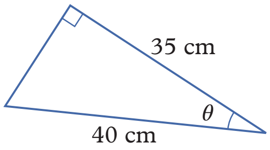
b



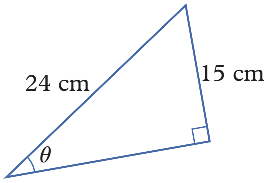
c



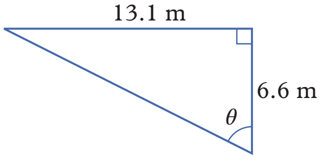
d



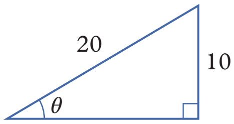
e

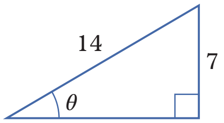


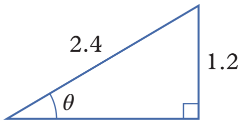
f



3 For each of the triangles, determine the ratio equal to the sine of angle θ. Simplify your fractions where possible.

a


b


c


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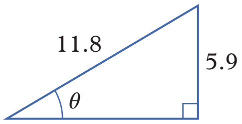
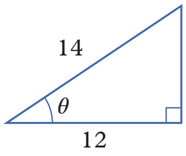


4 For each of the triangles, determine the ratio equal to the cosine of angle θ. Simplify your fractions where possible.

a


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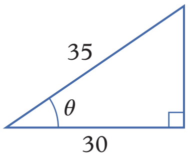

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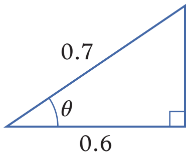

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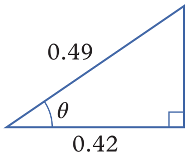
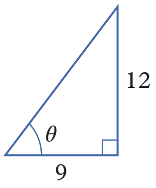


5 For each of the triangles, determine the ratio equal to the tangent of angle θ. Simplify your fractions where possible.

a


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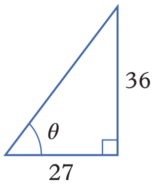

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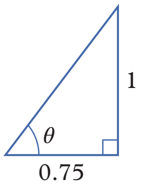

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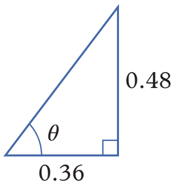


6 Evaluate the following using a calculator correct to four decimal places. Ensure your calculator is in degree mode.

a $\sin(20^\circ)$ **b** $\sin(36^\circ)$ **c** $\cos(36^\circ)$

d $\tan(36^\circ)$ **e** $\tan(63^\circ)$ **f** $\cos(63^\circ)$

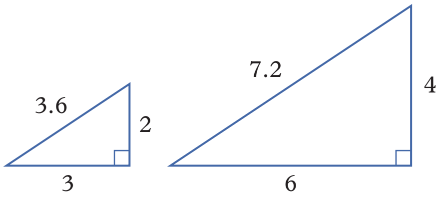
7 Evaluate the following using a calculator correct to four significant figures. Ensure your calculator is in degree mode.

a $6\sin(20^\circ)$ **b** $\dfrac{6}{\sin(20^\circ)}$ **c** $\dfrac{\sin(20^\circ)}{6}$

d $(\cos(20^\circ))^2 + (\sin(20^\circ))^2$ **e** $(\cos(20^\circ))^2 - (\sin(20^\circ))^2$ **f** $\dfrac{\sin(20^\circ)}{\cos(20^\circ)}$

8 **a** Describe the relationship between the triangles in questions **3**, **4** and **5**.

b Determine which pairs of triangles will have the same corresponding sine, cosine and tangent values for their non-right angles.

i


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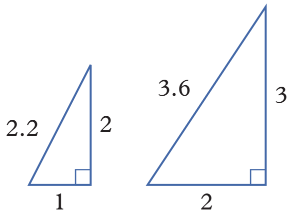
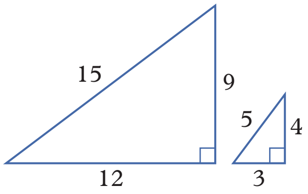


iii


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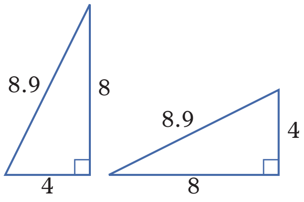
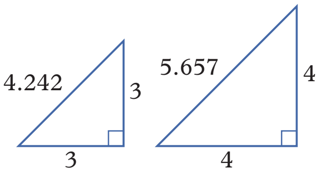
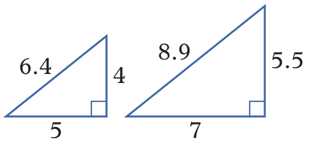


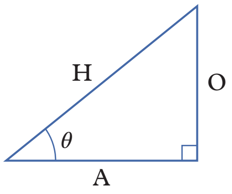
v


vi


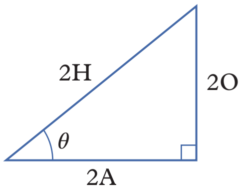
UNDERSTANDING AND FLUENCY

9 **a** For each of these similar triangles, determine the ratio equal to the sine, cosine and tangent of angle θ. Simplify your fractions where possible.

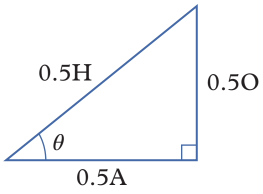
i



ii



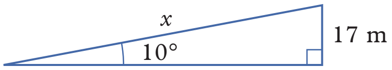
iii



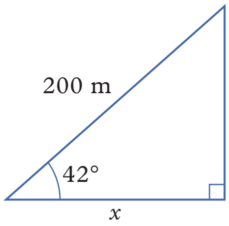
b Write a sentence about the values of the sine, cosine and tangent ratios of angles in similar triangles.

10 For each of the following, use the given approximate trigonometric ratio to determine the value of x.

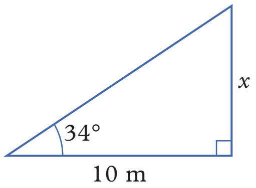
a $\sin(10°) = 0.17$



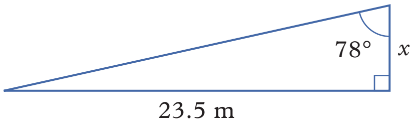
b $\cos(42°) = 0.74$



c $\tan(34°) = 0.67$



d $\tan(78°) = 4.7$



PROBLEM SOLVING AND REASONING

11 **a** Using your calculator, complete the table below. Write your answers correct to four decimal places.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\frac{1}{\tan(\theta)}$
1°				
5°				
15°				
30°				
45°				
60°				
75°				
85°				
89°				

b Which column(s) increase as the angle, θ, increases? Which column(s) decrease as the angle, θ, increases?

c Fill in the blanks of the following equations.

i $\sin(27°) = \cos(\underline{\quad}°)$

ii $\sin(\underline{\quad}°) = \cos(56°)$

iii $\tan(12°) = \frac{1}{\tan(\underline{\quad}°)}$

iv $\tan(\underline{\quad}°) = \frac{1}{\tan(47°)}$

d Describe the relationship between sine and cosine. Use the word *complementary* in your description.

e Describe the relationship between tangent and the reciprocal of tangent. Use the word *complementary* in your description.

12 Two special triangles can help us determine exact values of trigonometric ratios for common angles.

Diagram 1 shows a square with side lengths of 1 unit. If we cut the square in half along a diagonal, we obtain a right-angled isosceles triangle, as shown in Diagram 2. This is the first special triangle.

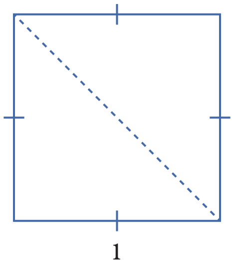


Diagram 1

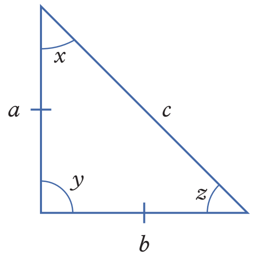


Diagram 2

a Determine the value of all side lengths and angles in Diagram 2 using geometry and Pythagoras' theorem. Write answers exactly, as a square root, where required.

b Using the values determined in part **a**, write down the exact values of the sine, cosine and tangent ratios of 45°.

The second special triangle is formed by cutting an equilateral triangle of side length 2 units in half down the middle, as shown in Diagrams 3 and 4.

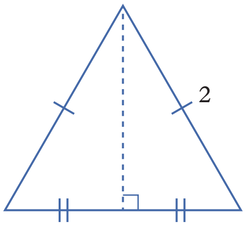


Diagram 3

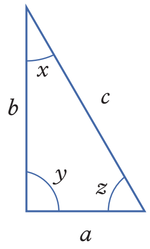


Diagram 4

c Determine the value of all side lengths and angles in Diagram 4 using geometry and Pythagoras' theorem. Write answers exactly, as a fraction or a square root, where required.

d Using the values determined in part **c**, write down the exact values of the sine, cosine and tangent ratios of 30° and 60°.

e Evaluate the following on a calculator.

i $(\sin(45°))^2$ **ii** $(\cos(30°))^2$ **iii** $(\tan(30°))^2$ **iv** $(\sin(60°))^2$

v $(\sin(30°))^2$ **vi** $(\cos(60°))^2$ **vii** $(\tan(45°))^2$ **viii** $(\cos(45°))^2$

f Describe how you could use the calculations in part **e** to obtain the exact values of the sine, cosine and tangent ratios of 30°, 45° and 60° without the special triangles.

13 a Explain, by referring to the hypotenuse as the longest side of a right-angled triangle, why the values of sine and cosine of θ are never greater than 1 for $0° < \theta < 90°$.

b Explain why the values of tangent, unlike sine and cosine, can be greater than 1 for $0° < \theta < 90°$

c Under what condition(s) is the value of tangent greater than 1?

14 a Add another column to the table in question **11** with the heading $\frac{\sin(\theta)}{\cos(\theta)}$. Complete the column by performing the division $\sin(\theta) \div \cos(\theta)$ for each angle.

b Fill in the blanks of the following equations.

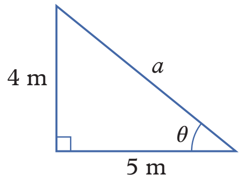
i $\frac{\sin(27°)}{\cos(27°)} = \tan(\_\_\_°)$ **ii** $\frac{\sin(\_\_\_°)}{\cos(56°)} = \tan(56°)$

iii $\frac{\sin(12°)}{\cos(\_\_\_°)} = \tan(12°)$ **iv** $\frac{\sin(\_\_\_°)}{\cos(\_\_\_°)} = \tan(47°)$

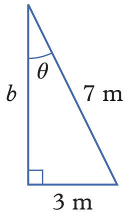
c Describe the relationship between $\frac{\sin(\theta)}{\cos(\theta)}$ and $\tan(\theta)$.

15 a For each of the triangles below, use Pythagoras' theorem to determine the missing length correct to two decimal places.

i



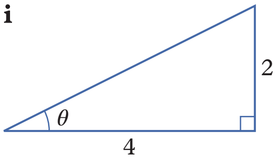
ii



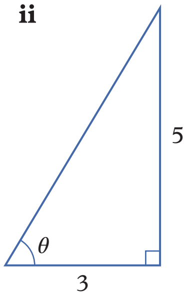
b Write the sine, cosine and tangent ratios of θ as a decimal correct to four decimal places where required.

16 a Calculate the gradient of the hypotenuse in each right-angled triangle.

i



ii



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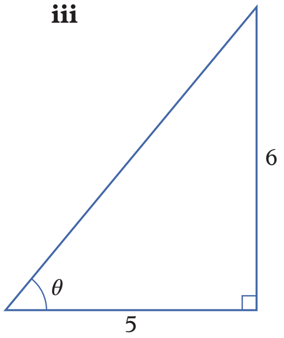
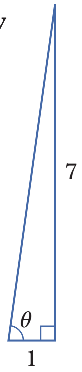


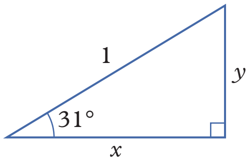
iv



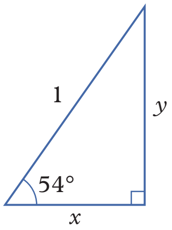
b Write the value of the tangent of θ for each right-angled triangle in part **a**.

c Describe the relationship between the gradient of a line segment and the tangent of the angle from the horizontal.

17 a Consider the following right-angled triangle, with a base of x and an altitude (height) of y. Write two equations using the sine ratio and cosine ratio for this triangle.



b Write two equations using the sine ratio and cosine ratio for this triangle.



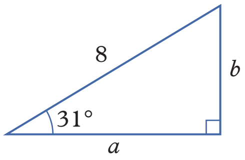
c Complete the following sentences by using the words 'base' and 'altitude'.

i The sine of an angle is the length of the ________ of a right-angled triangle with a hypotenuse of length 1.

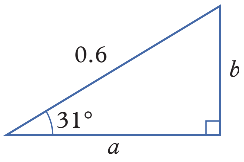
ii The cosine of an angle is the length of the ________ of a right-angled triangle with a hypotenuse of length 1.

d Use your answer to part **c** and the fact that trigonometric ratios are equal in similar triangles to determine the lengths of a and b in these similar triangles correct to four significant figures.

i

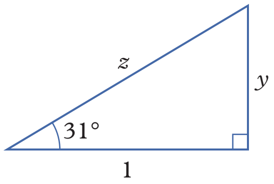


ii



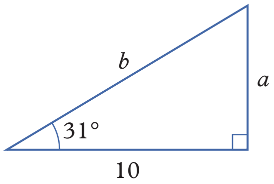
PROBLEM SOLVING AND REASONING

18 a Write two equations using the cosine ratio and tangent ratio for this triangle.

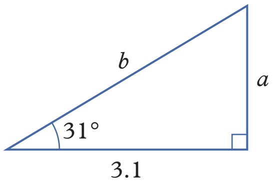


b Determine the lengths of a and b in these similar triangles correct to four significant figures.

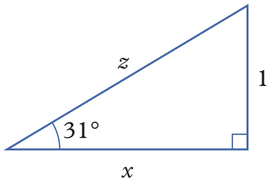
i



ii

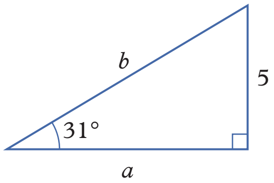


c Write two equations using the sine ratio and tangent ratio for this triangle.

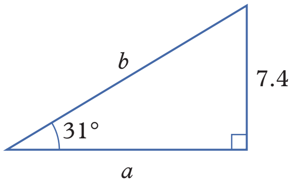


d Determine the lengths of a and b in these similar triangles correct to four significant figures.

i

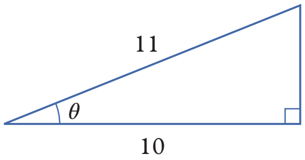


ii



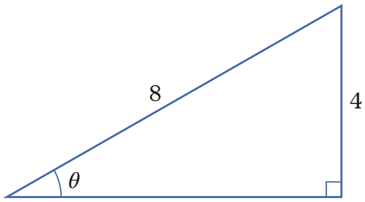
19 Explain and correct the error each person has made when writing the trigonometric ratio.

a



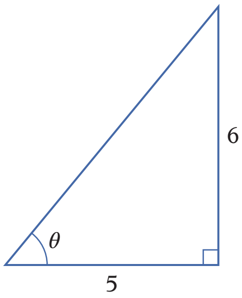
$\cos = \frac{10}{11}$

b



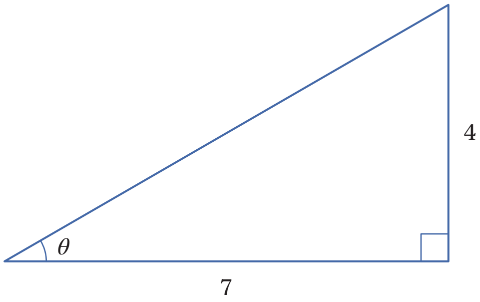
$\sin(\theta) = \frac{8}{4}$

c



$\cos(\theta) = \frac{6}{5}$

d



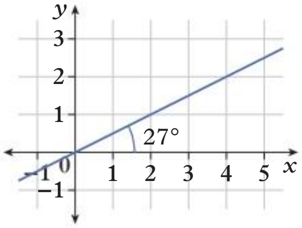
$\tan(\theta) = \frac{7}{4}$

CHALLENGE

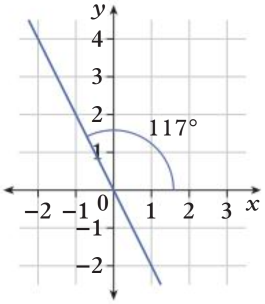
20 In question **16**, we saw that the tangent of the angle from the horizontal is equal to the gradient of the hypotenuse. This is also true for lines inclined at any angle, not just $0° < \theta < 90°$.

a State the tangent of the angle for each by determining the gradient of the line.

i



ii



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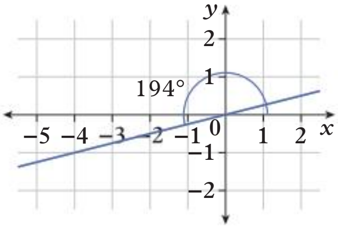

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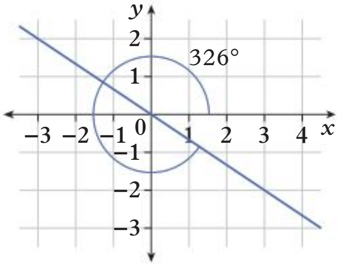


b Evaluate the following with a calculator.

i $\tan(45°)$ **ii** $\tan(135°)$ **iii** $\tan(225°)$ **iv** $\tan(315°)$

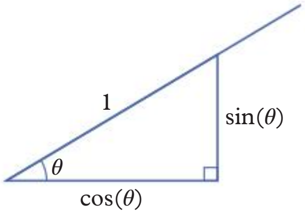
c Explain the similarity of the values in part **b**. Drawing a to-scale diagram may help.

d Complete the following sentences:

i The gradient of a horizontal line is ____________. Therefore, the tangent of 0° is ____________.

ii The gradient of a vertical line is ____________. Therefore, the tangent of 90° is ____________.

21 We saw in question **14** that $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$. The tangent of the angle from the horizontal is equal to the gradient, so $\dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{\text{rise}}{\text{run}}$. Considering the ray:



- the sine of the angle is the amount of rise when the hypotenuse is 1
- the cosine of the angle is the amount of run when the hypotenuse is 1.

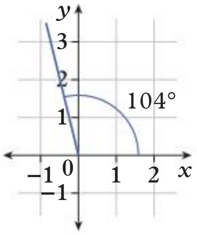
a Evaluate the following with a calculator.

i $\sin(30°)$ **ii** $\sin(150°)$ **iii** $\sin(210°)$ **iv** $\sin(330°)$

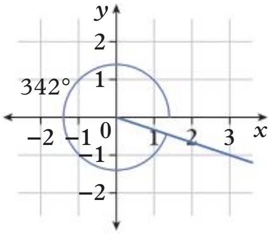
v $\cos(60°)$ **vi** $\cos(120°)$ **vii** $\cos(240°)$ **viii** $\cos(300°)$

b State whether the sine and cosine values will be positive or negative for the angles shown.

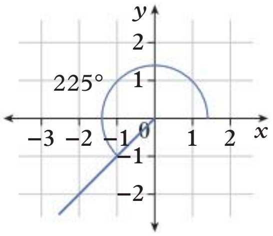
i



ii



iii



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Trigonometry

Investigation Nested similar triangles

CAS instructions Trigonometry

Topic quiz 6D

6E Using trigonometry to find lengths

Learning intentions

By the end of this topic you will be able to ...

✔ find the unknown lengths in a right-angled triangle.

Inter-year links

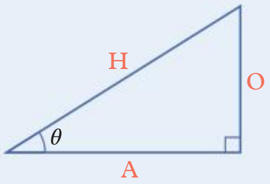
Year 7	6G Solving equations using inverse operations
Year 8	6B Solving linear equations
Year 10	7C Applications of trigonometry

Using trigonometry to find a side length

- Trigonometric ratios can be used to find an unknown side length in a right-angled triangle if an angle (other than the right angle) and one side length are known.
- To find a side length:
  1 Identify the given sides with respect to the angle θ. Label the sides accordingly.
  2 Decide which trigonometric ratio to use.
  3 Substitute in the known side length and angle.
  4 Solve the equation for the unknown.
- Use a calculator to perform the final calculation and, where appropriate, round your answer.

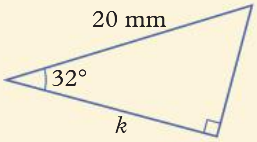
SOH CAH TOA

$\sin(\theta) = \frac{O}{H}$ $\cos(\theta) = \frac{A}{H}$ $\tan(\theta) = \frac{O}{A}$



Example 6E.1 Identifying which trigonometric ratio to use to write an equation

Determine which trigonometric ratio should be used, based on the side lengths provided, and hence write an equation that involves k.



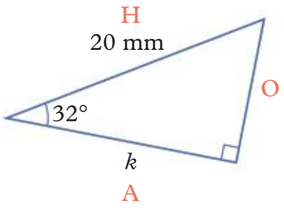
THINK

1 Identify the given sides with respect to the angle 32°. The side length adjacent to the angle is k. The hypotenuse is 20 mm. Label the sides accordingly.

2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.

3 Substitute for θ, A and H to write an equation involving k.

WRITE



A = k, H = 20 mm

$$\cos(\theta) = \frac{A}{H}$$

$$\cos(32°) = \frac{k}{20}$$

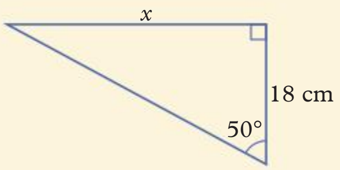
Example 6E.2 Solving equations involving trigonometric ratios

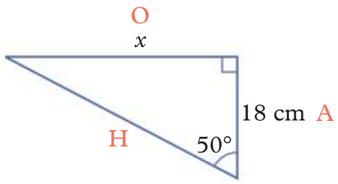
Solve the equation $\sin(23°) = \frac{m}{4}$. Round your answer to two decimal places.

THINK	**WRITE**
1 Rearrange the equation to make m the subject of the formula.	$m = 4 \times \sin(23°)$
2 Use a calculator to evaluate the trigonometric ratio $\sin(23°)$ and perform the multiplication operation $(\times 4)$.	$= 1.5629...$
3 Round the answer to two decimal places.	≈ 1.56

Example 6E.3 Using trigonometry to find an unknown side length in the numerator

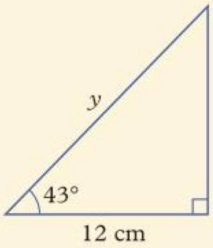
Use trigonometry to find the side length x in this triangle, correct to two decimal places.

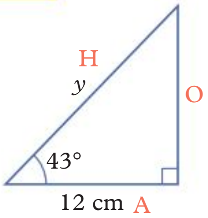


THINK	**WRITE**
1 Identify the given sides with respect to the angle 50°. The side length adjacent to the angle is 18 cm. The side length opposite the angle is x. Label the sides accordingly.	 $O = x, A = 18$ cm
2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.	$\tan(\theta) = \frac{O}{A}$
3 Substitute for θ, O and A.	$\tan(50°) = \frac{x}{18}$ $18 \times \tan(50°) = x$
4 Use a calculator to solve the equation for x.	$21.4515... = x$
5 Round the value of x to two decimal places.	$x \approx 21.45$ cm

Example 6E.4 Using trigonometry to find an unknown side length in the denominator

Use trigonometry to find the side length y in this triangle, correct to two decimal places.



THINK	WRITE
1 Identify the given sides with respect to the angle 43°. The side length adjacent to the angle is 12 cm. The hypotenuse is y. Label the sides accordingly.	 A = 12 cm, H = y
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.	$\cos(\theta) = \frac{A}{H}$
3 Substitute for θ, A and H.	$\cos(43°) = \frac{12}{y}$
4 Solve the equation for y. Multiply both sides of the equation by y, and then divide both sides of the equation by cos(43°).	$y \times \cos(43°) = 12$ $y = \frac{12}{\cos(43°)}$
5 Use a calculator to divide 12 by cos(43°).	$\approx 16.4079...$
6 Round the value of y to two decimal places.	≈ 16.41 cm

Helpful hints

- ✔ Make sure your calculator is in degree mode, not radian mode. Otherwise, all your answers will be wrong! Check the DEG or D symbol is on screen, not RAD or R or GRAD or G.
- ✔ Remember when solving an equation for an unknown, you can check your answer by substituting it back into the original equation.
- ✔ Drawing your diagram close to scale (making all angles roughly the correct size) will let you check that your answer is reasonable by referring to the relative length of the other sides. Always ensure that at the end of the question the hypotenuse is still the longest side of the triangle.
- ✔ SOH CAH TOA is still a useful mnemonic device for this section. You may find it helpful to write it at the top of your page before starting the exercise.

$$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$$

ANS p457

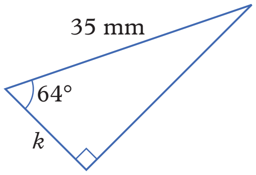
Exercise 6E Using trigonometry to find lengths

△ 1–6, 8, 9, 13, 14(a–d)   □ 2, 3, 5–7, 10, 11, 13, 14, 18, 19   ⬠ 3, 5–7, 12, 13, 15–18, 20

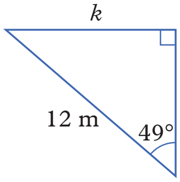
UNDERSTANDING AND FLUENCY

6E.1 **1** Determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves k.

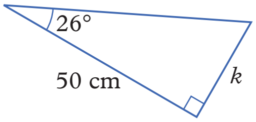
a



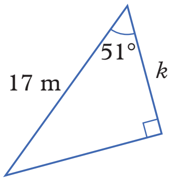
b



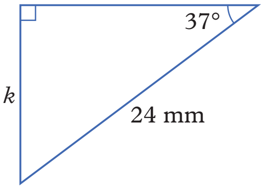
c



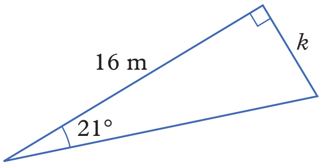
d



e



f



6E.2 **2** Solve the following equations for the unknown, correct to two decimal places.

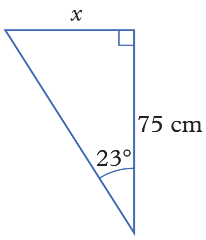
a $0.4226 = \frac{x}{5}$ **b** $0.4226 = \frac{5}{x}$ **c** $\sin(25°) = \frac{x}{5}$ **d** $\sin(25°) = \frac{5}{x}$

e $\cos(53°) = \frac{x}{11}$ **f** $\cos(53°) = \frac{11}{x}$ **g** $\tan(11°) = \frac{x}{20}$ **h** $\tan(79°) = \frac{20}{x}$

3 Solve each equation formed in question **1** to calculate the value of k, correct to two decimal places.

6E.3 **4** Use trigonometry to find the side length x in each triangle, correct to two decimal places.

a



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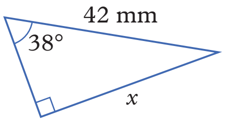

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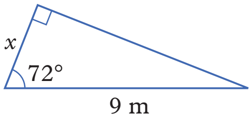

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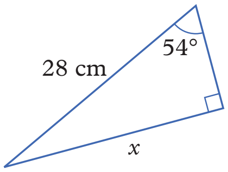

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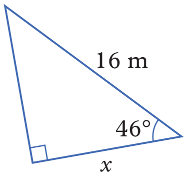

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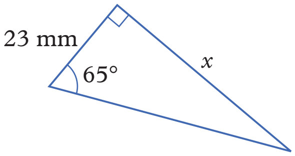

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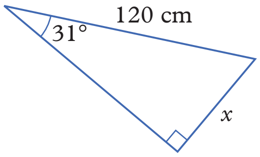

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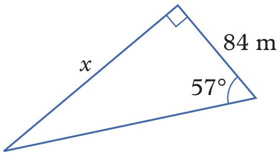
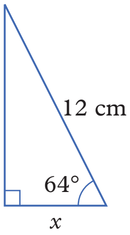


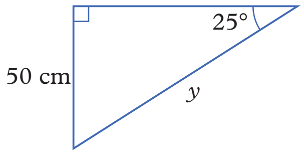
i



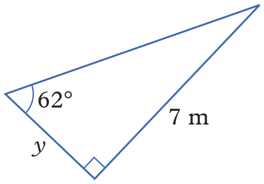
UNDERSTANDING AND FLUENCY

6E.4 **5** Find the side length y in each triangle, correct to two decimal places.

a



b



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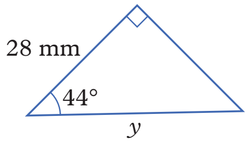

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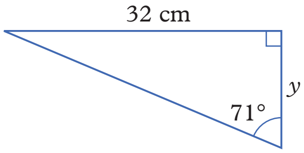

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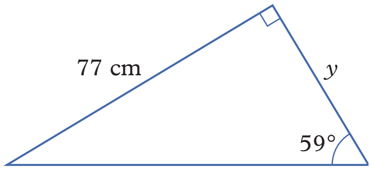

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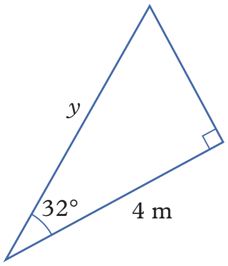
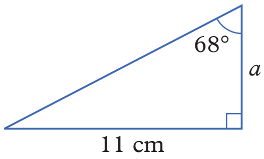


6 Find the value of the pronumeral in each triangle, correct to two decimal places.

a



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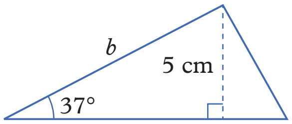

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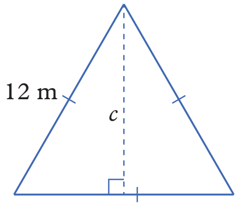

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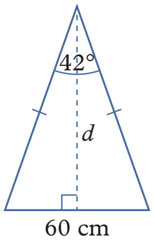
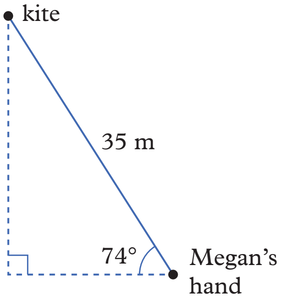


PROBLEM SOLVING AND REASONING

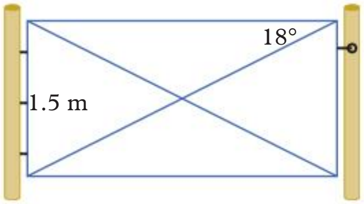
7 Megan holds the string attached to her kite at a height of 1 m above the ground. The 35 m long string makes an angle of 74° with the horizontal.



a Use trigonometry to calculate, correct to two decimal places, the vertical distance from one end of the string to the other.

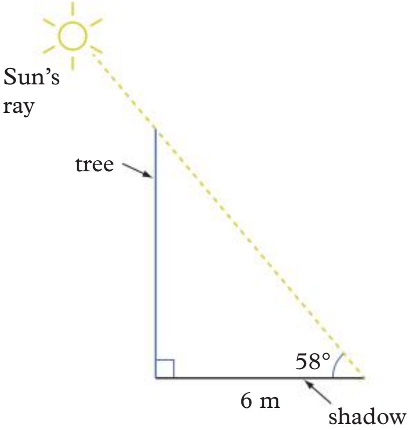
b What is the height of the kite above the ground, correct to two decimal places?

8 A farm gate, 1.5 m high, has diagonal supporting braces that make an angle of 18° with the horizontal. How wide is the gate, correct to two decimal places?



9 A shadow is formed when the Sun's rays are blocked by an object. The angle of the Sun's rays determine how long a shadow will be.

At a certain time of day, the Sun's rays make an angle of 58° with the ground and a tree forms a shadow that is 6 m long. Find the following correct to two decimal places.



a Use the diagram to calculate the height of the tree.

b At the same time, another tree forms a shadow that is 10 m long. How high is this tree?

10 The two identical sides of this ladder meet at an angle of 50° and are 1.8 m apart where they touch the ground. Find the following correct to two decimal places.

a How high is the top of the ladder above the ground?

b How long is each side of the ladder?

11 The Great Hall of Parliament House in Canberra displays a tapestry based on a painting by the Australian artist Arthur Boyd. The tapestry measures 20 m wide and is said to be one of the largest tapestries in the world. The tapestry hangs from the ceiling, and reaches all the way to the floor, with the angle between the diagonal and the floor being 24.2°.

a Draw a diagram to display this information.

b How high is the ceiling in the Great Hall, correct to the nearest metre?

c What is the perimeter of the tapestry, correct to the nearest metre?

12 A ramp for wheelchair access to a building is to be built at an angle of 3° to the horizontal. The front door of the building is 45 cm above ground level. How long should the ramp be? Give your answer in metres, correct to two decimal places.

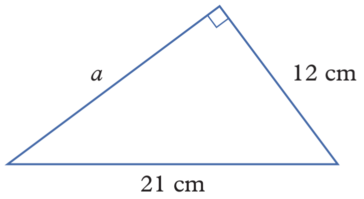
13 **a** What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using trigonometry?

b What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using Pythagoras' theorem?

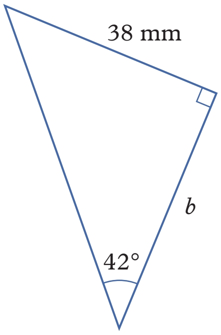
PROBLEM SOLVING AND REASONING

14 Find the length of the unknown side correct to one decimal place.

a



b



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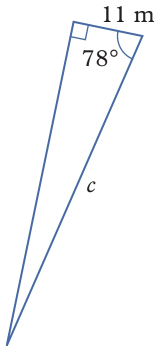

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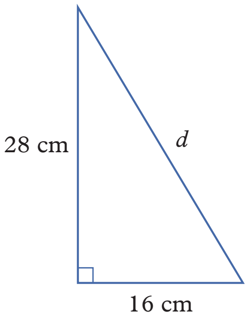

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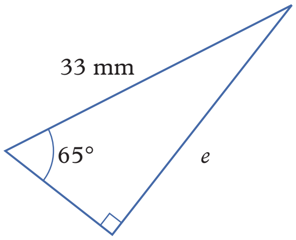

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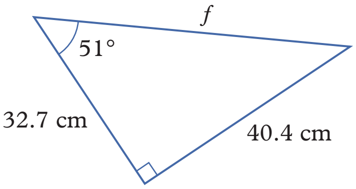
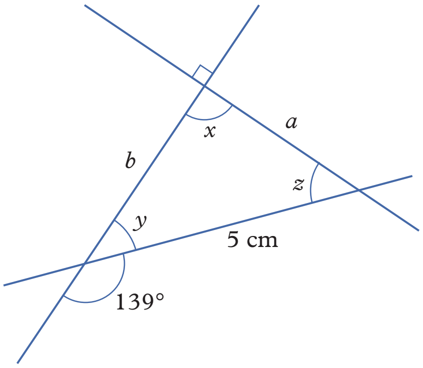
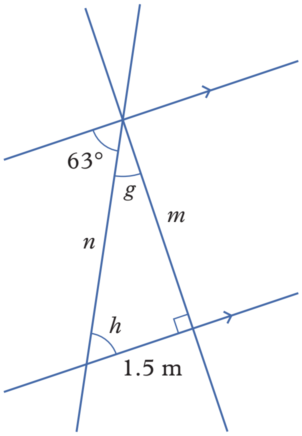


15 Determine the values of the pronumerals correct to two decimal places.

a

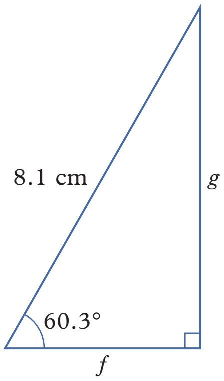


b

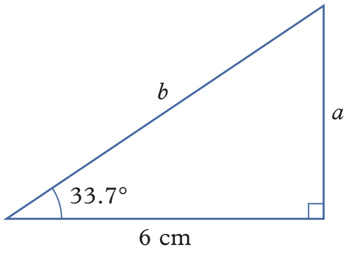


16 Use the specified ratio, geometry and Pythagoras' theorem to determine the value of the pronumerals correct to two decimal places.

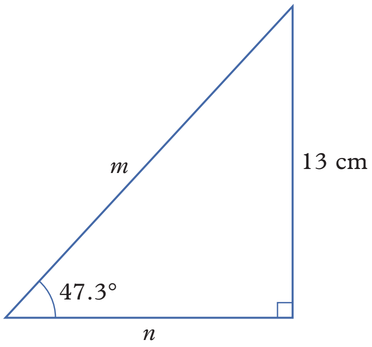
a sine



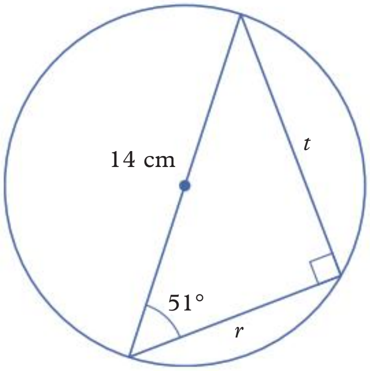
b tangent



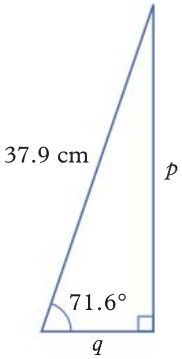
c sine



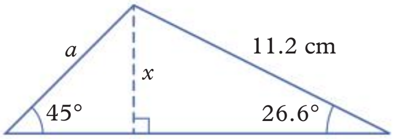
17 Determine the lengths *r* and *t* correct to two decimal places.



18 Use only the tangent ratio and Pythagoras' theorem to determine the value of the pronumerals correct to two decimal places. Hint: Start by writing *p* in terms of *q*.



19 We can find lengths in non-right-angled triangles by constructing right-angled triangles made up of known lengths. Consider the non-right-angled triangle shown with a perpendicular height added as a dotted line.



a Calculate the length of *x* correct to two decimal places.

b Using part **a**, calculate the length of *a* correct to two decimal places.

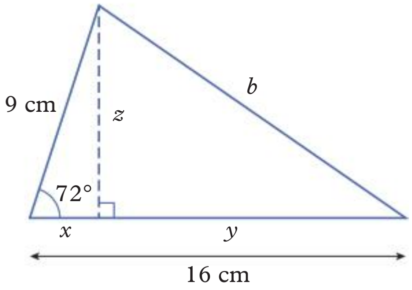
20 Consider the non-right-angled triangle shown on the right.

a Calculate the length of *x* correct to two decimal places.

b Calculate the length of *y* correct to two decimal places.

c Calculate the length of *z* correct to two decimal places.

d Hence, calculate the length of *b*.



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Finding lengths by trigonometry

Investigation
Using trigonometry to find an approximate value for π

Topic quiz
6E

6F Using trigonometry to find angles

Learning intentions

By the end of this topic you will be able to ...

✔ find an unknown angle in a right-angled triangle.

Inter-year links

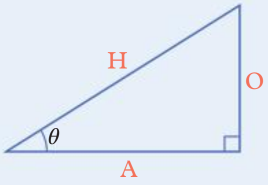
Year 7	6G Solving equations using inverse operations
Year 8	6B Solving linear equations
Year 10	7C Applications of trigonometry

Using trigonometry to find an angle

- Trigonometric ratios can be used to find an unknown angle in a right-angled triangle if two side lengths are known.
- A calculator can be used to obtain the angle from its sine, cosine or tangent value. This is known as finding the **inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) or **inverse tangent** ($\tan^{-1}$) of a value.
- To find an angle:
  1. Identify the given sides with respect to the angle. Label the sides accordingly.
  2. Decide which trigonometric ratio to use.
  3. Substitute in the known side lengths.
  4. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
  5. Use the calculator to find the result.

SOH CAH TOA

$\sin(\theta) = \frac{O}{H}$ $\quad\cos(\theta) = \frac{A}{H}$ $\quad\tan(\theta) = \frac{O}{A}$



Example 6F.1 Solving trigonometric equations to find the value of an angle

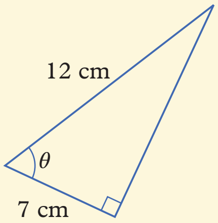
Solve each equation to find the value of θ, correct to the nearest degree.

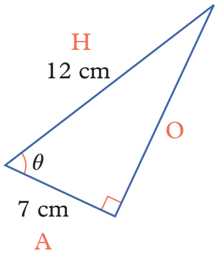
a $\tan(\theta) = 2.4$ **b** $\sin(\theta) = \frac{22}{35}$

THINK	WRITE
a 1 Rearrange to make θ the subject of the equation using the inverse tangent.	$\tan(\theta) = 2.4$ $\theta = \tan^{-1}(2.4)$
2 Use the $\tan^{-1}$ key on the calculator to find the inverse tangent of 2.4. Round the value of θ to the nearest degree.	$\approx 67.3801...$ $\approx 67°$
b 1 Rearrange to make θ the subject of the equation using the inverse sine.	$\sin(\theta) = \frac{22}{35}$ $\theta = \sin^{-1}\left(\frac{22}{35}\right)$
2 Use the $\sin^{-1}$ key on the calculator to find the inverse sine of $\frac{22}{35}$. Round the value of θ to the nearest degree.	$= 38.9448...$ $\approx 39°$

Example 6F.2 Using trigonometry to find an unknown angle

Use trigonometry to find angle θ in this triangle, correct to the nearest degree.



THINK	**WRITE**
1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 7 cm. The hypotenuse is 12 cm. Label the sides accordingly.	 A = 7 cm, H = 12 cm
2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.	$\cos(\theta) = \frac{A}{H}$
3 Substitute for A and H.	$\cos(\theta) = \frac{7}{12}$
4 Rearrange to make θ the subject of the equation using the inverse cosine.	$\theta = \cos^{-1}\left(\frac{7}{12}\right)$
5 Use the $\cos^{-1}$ key on the calculator to find the inverse cosine of $\frac{7}{12}$. Round the value of θ to the nearest degree.	$= 54.3146...$ $\approx 54°$

Helpful hints

- ✔ While a superscript of −1, like 2^{-1}, usually indicates 2 to the power of −1 (the reciprocal), $\sin^{-1}$ does *not* mean sine to the power of −1 (the reciprocal of the value of sine); instead it is the inverse of sine.
- ✔ It can be easy to choose the wrong trigonometric function when determining the angle. Make sure you take your time and label the sides of the triangles. This can help to minimise errors.
- ✔ If you get the ratio upside-down for sine and cosine when determining the angle, your calculator will display an error as no real angle has a sine or cosine ratio greater than 1. However, if you get the ratio upside-down for tangent, your calculator will not show an error.

ANS
p458

Exercise 6F Using trigonometry to find angles

△ 1–3(a, c, e, f), 4–6, 8, 10, 12 □ 2–3(b, d, f, h), 4–7, 11–13, 17 ⬟ 2–3(c, e, f), 4(b, d, f), 5, 7, 9, 13–17

UNDERSTANDING AND FLUENCY

6F.1 **1** Calculate each of these, correct to the nearest degree.

a $\sin^{-1}(0.23)$ **b** $\cos^{-1}(0.72)$ **c** $\tan^{-1}(1.46)$ **d** $\sin^{-1}(0.95)$

e $\sin^{-1}\left(\frac{2}{3}\right)$ **f** $\cos^{-1}\left(\frac{24}{25}\right)$ **g** $\tan^{-1}\left(\frac{39}{11}\right)$ **h** $\cos^{-1}\left(\frac{11}{17}\right)$

2 Find the unknown angle θ, correct to the nearest degree.

a $\sin(\theta) = 0.34$ **b** $\cos(\theta) = 0.81$ **c** $\tan(\theta) = 0.65$ **d** $\tan(\theta)= 0.47$

e $\cos(\theta) = 0.99$ **f** $\sin(\theta) = 0.01$ **g** $\tan(\theta) = 3.14$ **h** $\sin(\theta) = 0.71$

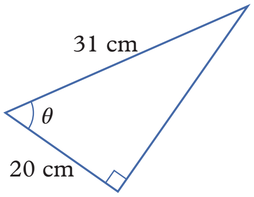
3 Solve each equation to find the value of θ, correct to the nearest degree.

a $\sin(\theta) = \frac{4}{5}$ **b** $\cos(\theta) = \frac{15}{19}$ **c** $\tan(\theta) = \frac{8}{21}$ **d** $\sin(\theta) = \frac{23}{26}$

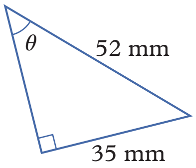
e $\cos(\theta) = \frac{11}{31}$ **f** $\sin(\theta) = \frac{9}{23}$ **g** $\cos(\theta) = \frac{17}{31}$ **h** $\tan(\theta) = \frac{26}{3}$

6F.2 **4** Use trigonometry to find angle θ in each triangle, correct to the nearest degree.

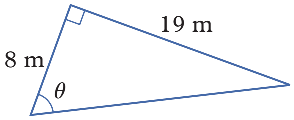
a



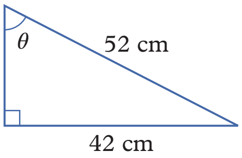
b



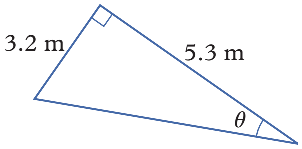
c



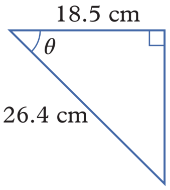
d



e

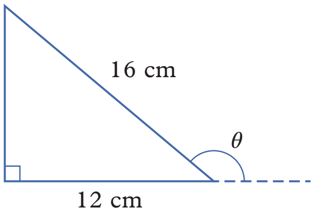


f

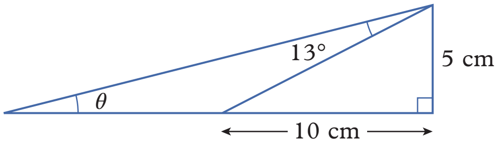


5 Determine the value of θ in each of the following, correct to one decimal place.

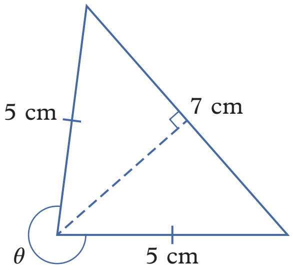
a



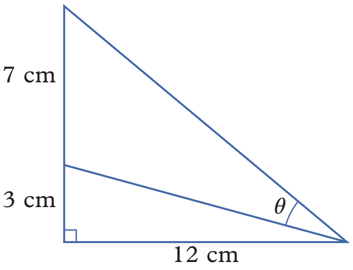
b



c



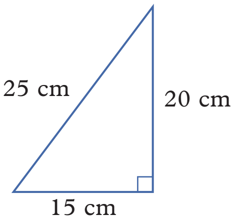
d



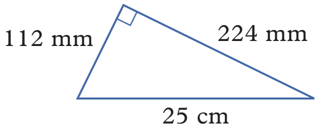
UNDERSTANDING AND FLUENCY

6 Calculate the size of the non-right angles in the following right-angled triangles. Give your answers in degrees correct to one decimal place.

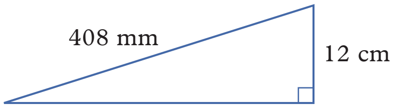
a



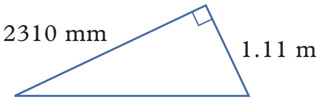
b



c

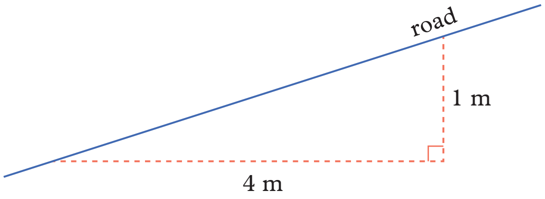


d



PROBLEM SOLVING AND REASONING

7 A road has a gradient of 1 in 4; that is, it rises 1 m vertically for every 4 m horizontally.



a What angle does the road surface make with the horizontal? Give your answer correct to the nearest degree.

b How far have you travelled along the road if you are now 1.7 m higher than when you started? Give your answer correct to two decimal places.

8 The roof of a holiday house has the dimensions shown.

Calculate, correct to the nearest degree:

a the angle the roof makes with the horizontal

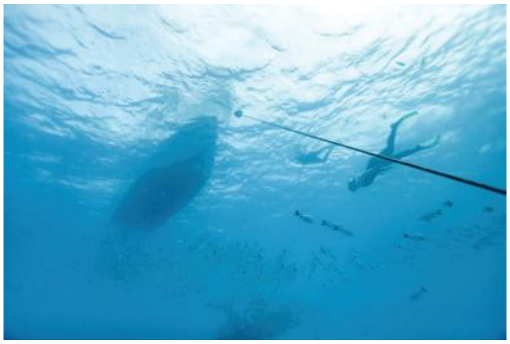
b the angle formed where the two sections of roof meet.

9 An anchor holding a boat in position lies on the seabed at a depth of 7.5 m. It is attached to the boat by a chain that is 8.4 m long.

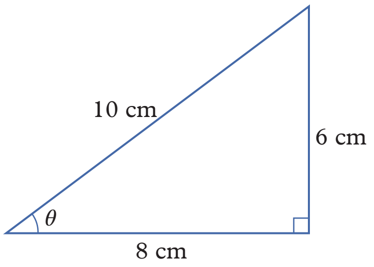
a Draw a diagram of this scenario.

b What angle does the chain make with the vertical? Give your answer correct to the nearest degree.

c If the chain was longer, would this angle be larger or smaller? Explain.



10 Consider the right-angled triangle shown.



a Calculate the value of θ to the nearest degree using a calculator and:

  i the sine ratio

  ii the cosine ratio

  iii the tangent ratio.

b Draw your own scale diagram of the triangle and measure angle θ with a protractor.

c Compare your answers to parts **a** and **b**. Comment on the advantages and disadvantages of using:

  i trigonometry

  ii measurements from a scale diagram.

11 Nadia and Alex are set a problem-solving task by their teacher. Their challenge is to work out the height of a tree in their school yard without climbing it or using a ladder. The only allowable equipment is a 1 m ruler, a tape measure and a calculator.

Nadia and Alex discuss some possible ideas, take some measurements simultaneously and draw these diagrams.

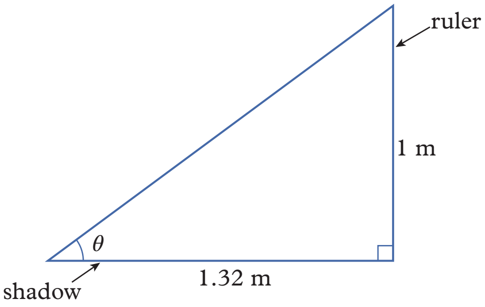


Diagram 1

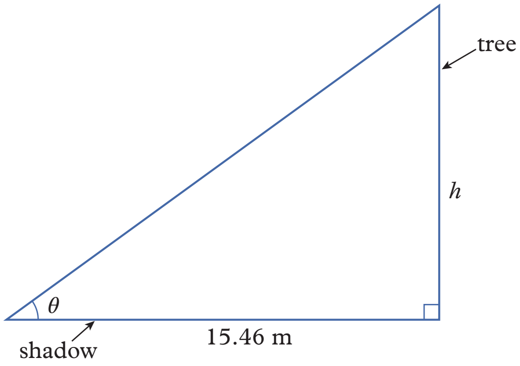


Diagram 2

a What you think their strategy is?

b Use trigonometry to calculate the value of θ to the nearest degree in Diagram 1.

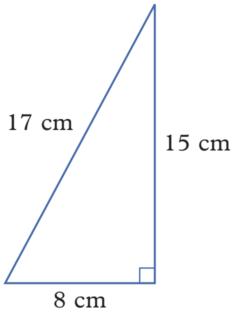
c Explain why this value of θ can be used in Diagram 2.

d Use trigonometry to calculate the height of the tree correct to two decimal places.

e Instead of using trigonometry, Nadia and Alex could have used their knowledge of similar triangles. Explain how this strategy could be used to calculate the height of the tree.

f Can you think of any other strategies that could have been used? Try them.

12 Consider the right-angled triangle shown.



a Correct to the nearest degree, calculate the size of the non-right angles using a calculator and:

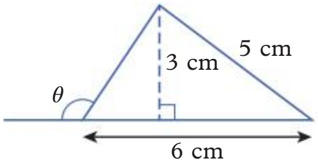
  i the sine ratio **ii** the cosine ratio **iii** the tangent ratio.

b After the size of one non-right angle has been determined, how else could the size of the other non-right angle be determined?

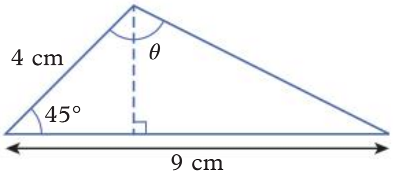
PROBLEM SOLVING AND REASONING

13 Determine the value of θ in each of the following correct to one decimal place.

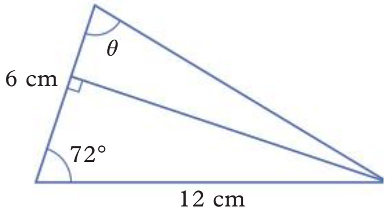
a



b

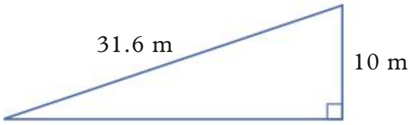


c



14 Calculate the size of the non-right angles and the missing side length in the following right-angled triangles using the sine ratio and Pythagoras' theorem correct to one decimal place.

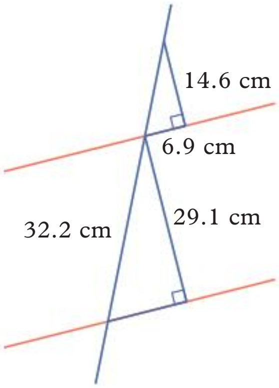
a



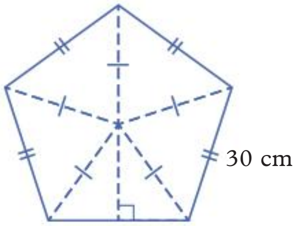
b

CHALLENGE

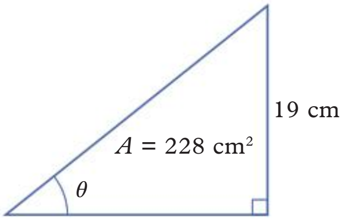
15 Determine if the two red lines are parallel. Explain why or why not.



16 Calculate the area of this regular pentagon correct to one decimal place.



17 The area of the triangle shown is 228 cm^2. Determine the size of the angle θ correct to one decimal place.



Check your Student obook pro for these digital resources and more:

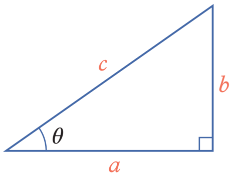
Interactive skillsheet
Finding angles by trigonometry

Topic quiz
6F

Chapter summary

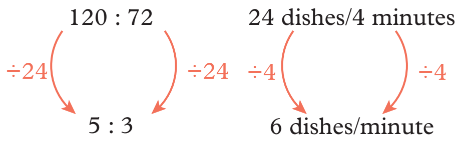
Pythagoras' theorem

$$\underset{\text{hypotenuse}}{c^2} = \underbrace{a^2 + b^2}_{\text{shorter sides}}$$



- Any set of three whole numbers that satisfy Pythagoras' theorem is called a Pythagorean triple.
  For example, 3, 4, 5 is a Pythagorean triple as $3^2 + 4^2 = 5^2$.

Simplifying ratios and rates



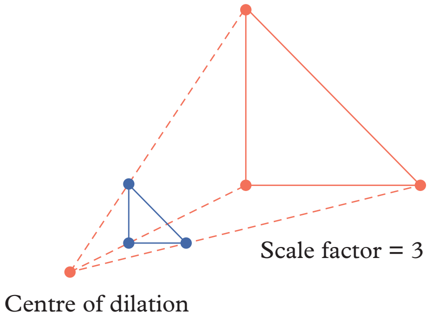
Direct proportion and scale

- Ratio between quantities remains the same
- When one quantity = 0, other quantity = 0

Scale drawings

- Scale is 'drawing length : actual length'
- 1 : 100 means 1 cm represents 100 cm

Dilations



- Creates similar figures
- Scale factor between 0 and 1: reduction
- Scale factor greater than 1: enlargement

Scale factor

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}} \qquad \text{Area scale factor} = \frac{\text{image area}}{\text{original area}}$$

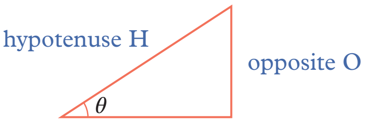
Similar figures

- Side lengths are in proportion
- Corresponding angles are equal

Trigonometry

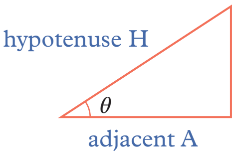
SOH

$$\sin(\theta) = \frac{O}{H}$$



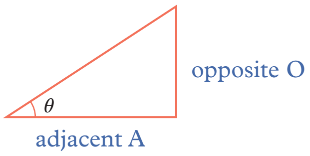
CAH

$$\cos(\theta) = \frac{A}{H}$$



TOA

$$\tan(\theta) = \frac{O}{A}$$



To find a side length:

1. Identify given sides with respect to the angle θ.
2. Decide which trigonometric ratio to use.
3. Substitute in known side length and angle.
4. Solve for the unknown.

To find an angle:

1. Identify given sides with respect to the angle.
2. Decide which trigonometric ratio to use.
3. Substitute in known side lengths.
4. Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
5. Solve for the unknown.

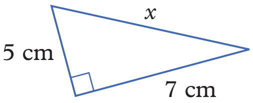
Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

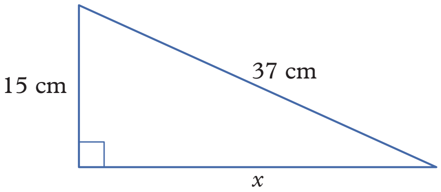
Multiple choice

6A **1** Which equation can be used to find the length of the hypotenuse in this triangle?



A $x = 7 + 5$ **B** $7 = x + 5$ **C** $7^2 = x^2 + 5^2$ **D** $5^2 = x^2 + 7^2$ **E** $x^2 = 5^2 + 7^2$

6A **2** The unknown side length x in this triangle, correct to the nearest cm, is:



A 22 cm **B** 33 cm **C** 34 cm **D** 40 cm **E** 52 cm

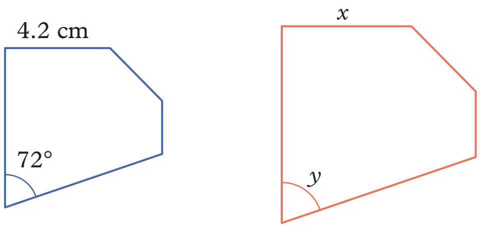
6B **3** The exchange rate is 1 AUD = 0.66 EUR. How much is \$800 AUD worth?

A €528 **B** €800 **C** €1212 **D** €2424 **E** €5280

6B **4** If a scale drawing is made with a scale of 1 : 500, how much does 1.5 cm in the drawing represent in real life in metres?

A 5 m **B** 7.5 m **C** 75 m **D** 500 m **E** 750 m

6C **5** The shape on the left has been dilated by a scale factor of 2 to create the image on the right. What are the values of x and y?



A $x = 2.1$ cm, $y = 72°$ **B** $x = 2.1$ cm, $y = 144°$ **C** $x = 4.2$ cm, $y = 144°$
D $x = 8.4$ cm, $y = 72°$ **E** $x = 8.4$ cm, $y = 144°$

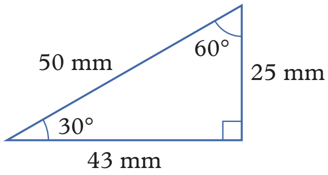
6C **6** A figure is dilated by a scale factor of $\frac{1}{2}$. What is its area scale factor?

A $\frac{1}{8}$ **B** $\frac{1}{4}$ **C** $\frac{1}{2}$ **D** 2 **E** 4

6D **7** Sine is the trigonometric ratio of which side to which side of a right-angled triangle?

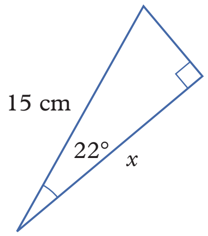
A Hypotenuse to opposite side **B** Opposite side to adjacent side
C Adjacent side to hypotenuse **D** Opposite side to hypotenuse
E Hypotenuse to adjacent side

6D **8** Using the lengths on the triangle shown, the cosine of 30° is:



A $\frac{43}{50}$ **B** $\frac{25}{50}$ **C** $\frac{25}{43}$ **D** $\frac{50}{43}$ **E** $\frac{50}{25}$

6E **9** Which equation can be used to find x in this triangle?

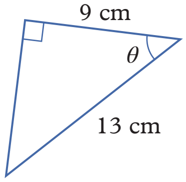


A $\cos(22°)=\frac{x}{15}$
B $\tan(22°)=\frac{x}{15}$
C $\sin(22°)=\frac{x}{15}$
D $\cos(22°)=\frac{15}{x}$
E $\sin(22°)=\frac{15}{x}$

6F **10** If $\sin(\theta)=\frac{12}{15}$, then θ is equal to:

A $\sin\left(\frac{15}{12}\right)$
B $\sin^{-1}\left(\frac{12}{15}\right)$
C $\sin^{-1}\left(\frac{15}{12}\right)$
D $\sin\left(\frac{12}{15}\right)^{-1}$
E $\frac{\sin^{-1}(12)}{15}$

6F **11** The angle θ in this triangle is closest to:



A 35°
B 44°
C 46°
D 55°
E 0.81°

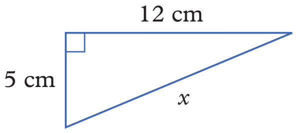
Short answer

6A **1** A triangle has side lengths 30 mm, 40 mm and 50 mm.

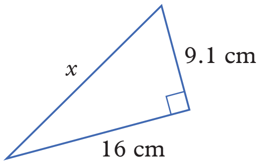
- **a** Show that the triangle is right-angled.
- **b** Are the side lengths of the triangle an example of a Pythagorean triad? Briefly explain why or why not.

6A **2** Calculate the length of the hypotenuse in each triangle, correct to two decimal places.

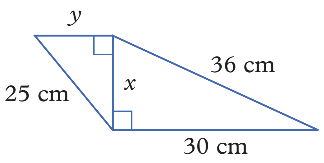
a



b



6A **3** Find the length of the sides represented by x and y in this diagram, correct to two decimal places.



6B **4** Use a rate in simplest form to model the following situations.

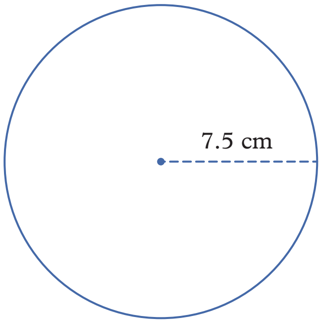
- **a** It costs $7.20 to buy 4 avocados.
- **b** A heart beats 510 times in 6 minutes.
- **c** A train travels 546 km in 6.5 hours.

6B **5** Up to certain point, the distance an elastic material extends is directly proportional to the force applied. It requires 5 Newtons of force to extend a spring by 18 cm.

- **a** How much force is required to extend the spring by 7.92 cm?
- **b** How much would the spring extend by if 3.1 Newtons of force was applied?

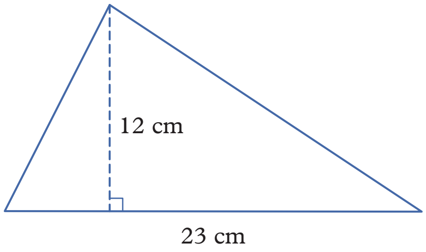
6C **6** For each shape, state the area of the image if the original figure is dilated by the given scale factor. If necessary, give your answers to two decimals.

a



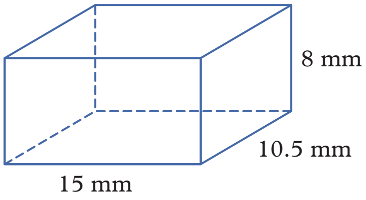
Scale factor = 3

b



Scale factor = $\frac{1}{3}$

6C **7** Consider this foam block.

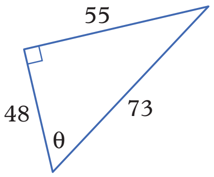


a What is the volume of the foam block?

b If the foam block was dilated by a scale factor of 3, what are the dimensions of the dilated foam block?

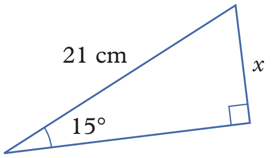
c What is the volume of the dilated foam block?

6D **8** Write equations for the sine, cosine and tangent of θ for this triangle.

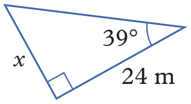


6E **9** Find the side length x in each triangle, correct to two decimal places.

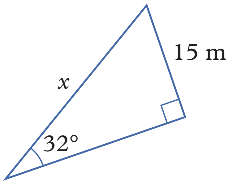
a



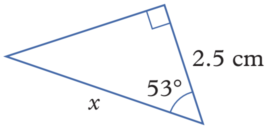
b



c

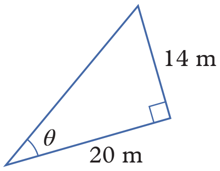


d

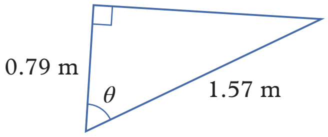


6F **10** Find the angle θ in each triangle, correct to the nearest degree.

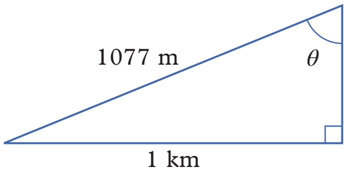
a



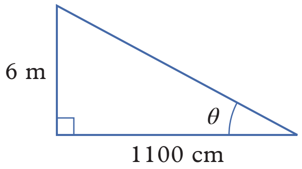
b



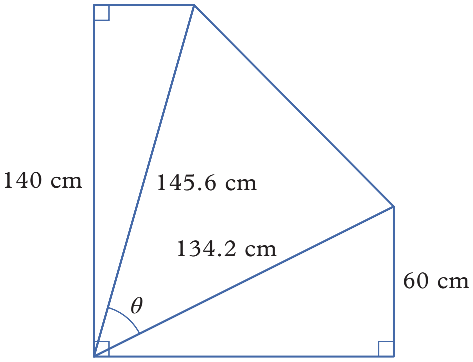
c



d

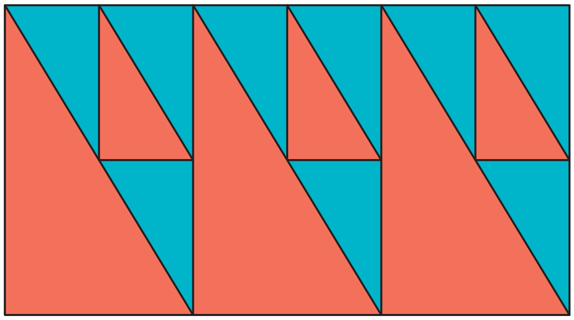


6F **11** Calculate the size of the angle θ, correct to one decimal place.



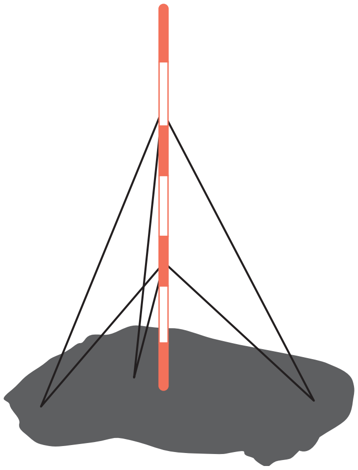
Analysis

1 Julie is making a design for a blanket using right-angled triangles. The blanket design consists of small and large triangles in a repeating pattern, as shown on the right.



- **a** Each large triangle is 8 cm high and 4.8 cm wide. Calculate the length of the hypotenuse of a large triangle, correct to one decimal place.
- **b** Each small triangle is 4 cm high and 2.4 cm wide. Calculate the length of the hypotenuse of a small triangle, correct to one decimal place.
- **c** Explain why the large and small triangles are similar triangles.
- **d** If the large triangle was dilated to create the small triangle, what scale factor was used?
- **e** What is the area scale factor between the large and small triangles?
- **f** What is the ratio of red to blue material used in each blanket section?
- **g** The blanket will be 168 cm long and 144 cm wide. How many large red triangles will be in the blanket?
- **h** Calculate the area of **i** red material and **ii** blue material in the blanket.
- **i** The red material costs $12 per square metre and the blue material costs $10 per square metre. Calculate the total cost of material used in the blanket.

2 A 150 m tall AM radio tower is supported by six guy wires equally spaced around the tower connecting to the ground at three points. Three of the wires are connected one-third of the way up the tower and the other three are connected one-third of the way down from the top of the tower.



- **a** If the shorter wires are each 90 m long, how far from the base of the tower are they connected, correct to one decimal place?
- **b** Determine the total length of wire being used to support the tower, correct to the nearest metre.
- **c** Determine the straight-line distance between two of the connecting points on the ground, correct to one decimal place.
- **d** Determine the angles the wires make with the ground, correct to two decimal places.
- **e** Determine the angle between the two wires where they meet at the ground, correct to two decimal places.

Algorithmic Thinking

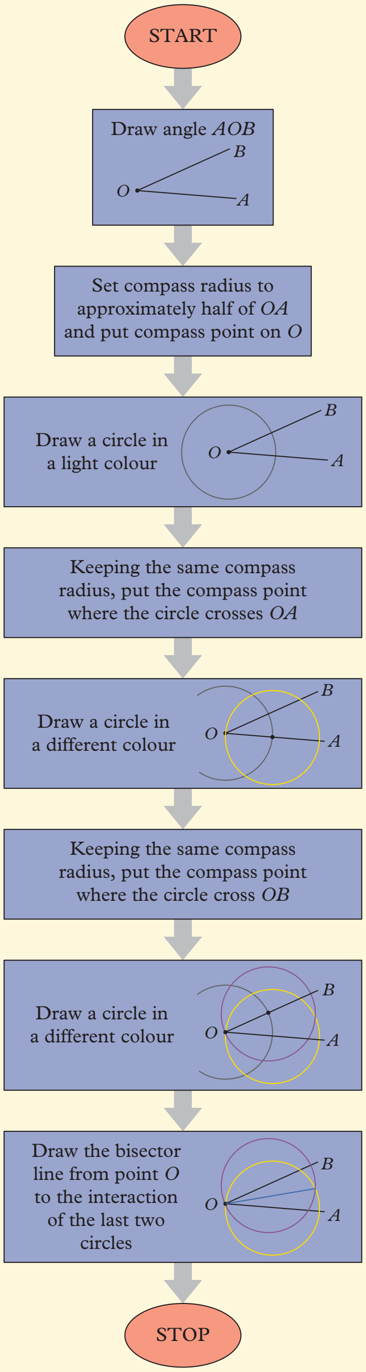
Algorithmic thinking: Measurement and space

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Algorithms can be described using diagrams called flowcharts, which have actions written inside shapes such as boxes and diamonds, and arrows to indicate which actions need to be completed next. The simplest flowcharts use sequential steps.

A bisector is a straight line that divides a line segment or angle into two equal parts. A perpendicular bisector forms a right angle with the line segment it bisects.

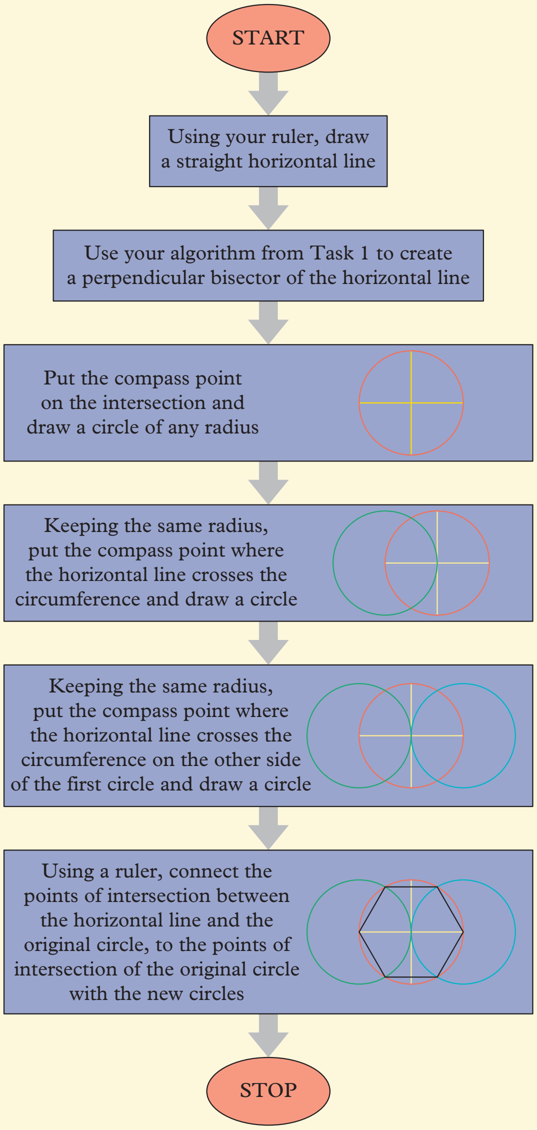
Example 1

This flowchart shows how to bisect an angle using a compass and ruler. Diagrams have been included to help follow the flowchart.



Example 2

This flowchart shows how to inscribe a hexagon within a circle using a compass and ruler. To avoid repetition, it refers to the flowchart from Task 1 to create a perpendicular bisector.



Example 3

This is an algorithm to create a mystery geometry construction using a compass, ruler, blue pencil, red pencil and black pencil.

1 Label a point *O*.

2 Using a compass, draw a circle centred on point *O* with a blue pencil.

3 Label a point *A* anywhere on the circumference of the circle.

4 Place the compass point on *A*.

5 Keeping the same radius, draw a circle with a red pencil.

6 Label the points where the blue and red circles intersect *B* and *C*.

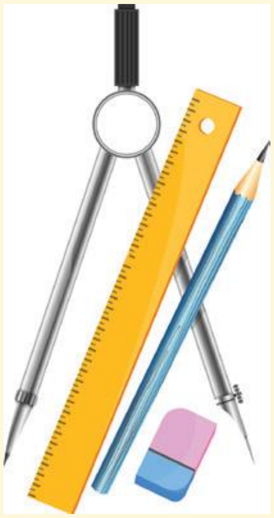
7 Using a ruler, connect *B* and *C* with a straight line in black pencil.

8 Adjust the compass radius to the length of *BC*.

9 Place the compass point on point *B* and draw a circle with a red pencil.

10 Label the point of intersection with the first circle as *D*.

11 Using a ruler, draw the straight lines *BD* and *CD* in black pencil.



Task 1

1 Follow the flowchart in Example 1 using a compass, a ruler and an acute angle.

2 Using ideas from Example 1, create your own flowchart with instructions and diagrams for using a compass and ruler to create a perpendicular bisector of a straight line from point *A* to point *B*.

Task 2

1 Follow the algorithm in Example 2 using a compass and a ruler.

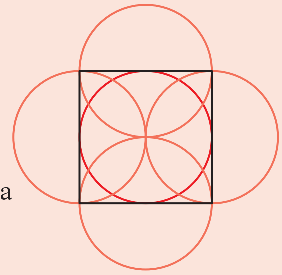
2 Identify the sequential actions and turn the flowchart in Example 2 into a numbered step-by-step algorithm.

3 Follow the mystery algorithm in Example 3 using a compass, ruler and coloured pencils. What geometric shapes were created?

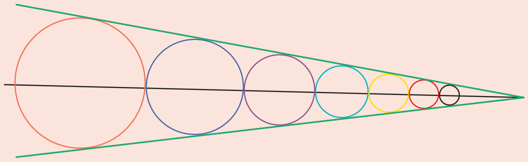
4 Using the ideas and the algorithms from Task 1 and Examples 1–3, create a numbered step-by-step algorithm using a compass and ruler to inscribe a square in a circle. Check your algorithm with someone else.

Task 3

1 Using ideas in the algorithms from previous tasks and examples, create a numbered step-by-step algorithm using a compass and ruler to create the geometric construction on the right.



2 Using ideas in the algorithms from previous tasks and examples, create a numbered step-by-step algorithm using a compass and ruler to create the following geometric construction.



7

Statistics

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Interpreting graphs
✔ Dot plots
✔ Stem-and-leaf plots

Curriculum links

- Analyse reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians (AC9M9ST01)
- Analyse how different sampling methods can affect the results of surveys and how choice of representation can be used to support a particular point of view (AC9M9ST02)
- Represent the distribution of multiple data sets for numerical variables using comparative representations; compare data distributions with consideration of centre, spread and shape, and the effect of outliers on these measures (AC9M9ST03)
- Choose appropriate forms of display or visualisation for a given type of data; justify selections and interpret displays for a given context (AC9M9ST04)
- Plan and conduct statistical investigations involving the collection and analysis of different kinds of data; report findings and discuss the strength of evidence to support any conclusions (AC9M9ST05)

7A Surveys and sampling methods

Learning intentions

By the end of this topic you will be able to ...

✔ evaluate the appropriateness of sampling methods

✔ evaluate statistical reports.

Inter-year links

Year 8	9A Collecting data and sampling methods

Surveys

- In statistics, a **population** is every potential piece of data under consideration.
- A **survey** is a list of questions designed to extract information about a population.
- Surveys usually take a **sample** of a population, and use the sample data to make **inferences** (conclusions drawn from evidence and reasoning) about the population.
  - → A **census** is a survey of an entire population and gives perfectly accurate data. However, taking a census is often impractical, expensive and time-consuming.
- Population means and medians can be estimated by using the information obtained in surveys.

Sampling methods

- The goal of selecting a sample is to get an accurate representation of the population, which means it is important to select the sample randomly to avoid possible **bias**.
  - → Bias is an inclination towards one person, group of people, thing, or idea, over another.
- When collecting a random sample from a population, natural variation means the sample statistics are unlikely to exactly match the population parameters.
  - → Large samples are more accurate and should better reflect the population parameters than small samples (assuming they are random).
- **Stratified sampling** divides the population into small groups known as strata (for example, based on age) and takes a random sample from each stratum. The size of each sample is proportional to how large the category is.
- **Systematic sampling** selects data at fixed intervals (for example, every fifth person). The starting point should be random.
- Non-random sampling methods include **convenience sampling** and **quota sampling**.
  - → Convenience sampling is when a sample that is easily accessible is selected.
  - → Quota sampling is similar to stratified sampling, but with the researcher selecting the size of each group.

Evaluating statistical reports

- When evaluating a statistical report or claim, you need to consider how the information was collected and how the information has been interpreted.
- Data should be fairly represented, so as not to skew the outcome. Presenting only some of the information, or selecting inaccurate statistics to describe the data, can be misleading.

Example 7A.1 Identifying the sampling method

Jordan is conducting a survey to find out people's favourite foods. She asks people on the main street to complete her survey and finishes when she has 30 respondents. Determine the sampling method Jordan used.

THINK	**WRITE**
This sample was taken from people who were easily accessible and is not randomly selected, so it is an example of convenience sampling.	This is a convenience sample.

Example 7A.2 Evaluating surveys

Decide whether each of these surveys will provide fair or biased results, giving a reason for your answer.

a Asking residents of every 20th household from a local community register if they enjoy running, to determine how many people in the local community exercise.

b Surveying 1000 random households in Sydney in order to find the average number of people living in residential homes in NSW.

THINK	**WRITE**
a **1** Consider the surveying technique. Is the sample large enough and gathered in an unbiased way so that it theoretically represents the population (the local community)?	**a** The sample is randomly selected from the community and is of sufficient size. However, this survey is likely to provide biased results, because the question does not ask if people exercise, just if they enjoy running.
2 Consider the question asked. Does it answer what the questioner is trying to discover (how many people exercise)?	
b Consider the surveying technique. Is the sample large enough and gathered in an unbiased way so that it theoretically represents the population (people in NSW)?	**b** This survey is likely to provide biased results because, although the sample is of a reasonable size, it only samples from one city in NSW and is therefore unlikely to be representative of the whole population.

Example 7A.3 Identifying how a statement might misrepresent data

Explain how the statement 'Cricket is the world's most popular sport' might be a misrepresentation of the data that 67% of people surveyed at cricket matches in 15 countries answered that cricket was their favourite sport.

THINK	**WRITE**
1 Look at the sample chosen. Does it reflect the population (the world)?	This statement misrepresents the data by using a biased sample (people at cricket matches), who are much more likely to answer that cricket is their favourite sport than people not attending a cricket match. The question was also only asked in 15 countries, not every country in the world.
2 Look at the question or result given. Is the conclusion fair?	

Example 7A.4 Analysing surveys and results

Caleb wanted to find information about the age of people at a skate park. He surveyed every 10th person he saw there over one week, surveying a total of 20 people. Summary statistics gave a mode of 17, a median of 18, a mean of 22 and a range of 70. Caleb concluded that the mean age of people at the skate park of 22 was the most appropriate summary statistic to represent the data.

a Decide whether Caleb's data collection method is fair.
b Decide whether the interpretation of the data is fair.
c If appropriate, provide a suggestion to improve the survey.

THINK	WRITE
a Is the sampling method random, with an equal chance of selecting every person in the population?	**a** The method appears fair, although it could be biased depending on the times that Caleb visited the skate park. The sample could be larger.
b Does the conclusion fairly interpret the statistics?	**b** The interpretation could be improved. The sample size is small and the mean is distinctly larger than the median, which, along with a large range, implies the presence of an outlier. The median would be a better statistic to use.
c Are there any ways to improve this survey?	**c** Caleb could take a larger sample and ensure that he visits the skate park at different times of the day. His interpretation could also be improved by using the median as the measure of centre.

Helpful hints

✔ Make sure samples are taken from the whole population, not from a subset of the population. A sample will only be random if it is randomly selected from the whole population.

ANS p459

Exercise 7A Surveys and sampling methods

△ 1, 2–4(a, b), 6–8, 12 ■ 1, 2–4(c, d), 5, 8–10, 13, 14 ⬠ 1, 2, 4(c, d), 5, 9–12, 14–16

UNDERSTANDING AND FLUENCY

7A.1 **1** Determine the sampling method used in the following situations.

a Xavier calls every 100th phone number from a register to complete a survey.
b Mia surveys 20 students from public and private schools in proportions that reflect the town's population.
c Ali surveys the first 50 people who will answer his questionnaire about school dinners.
d Tina surveys 10 adults from each age group (20–29, 30–39, etc.) about their voting intentions.

7A.2 **2** Decide whether each survey below will provide fair or biased results, giving reasons for your answers.

- **a** Asking 500 random people at an AFL match their favourite sport, in order to determine the country’s favourite sport
- **b** Asking everybody at your school their opinion on a school issue
- **c** Asking every 10th person on the electoral roll if they like dogs, to determine the most popular pet
- **d** Asking a random sample of 100 people (stratified by age) from around the country who they think should be prime minister at the next election

7A.3 **3** Explain how each of the following headlines might be misrepresenting the data described in brackets.

- **a** Hip-hop is the most popular music genre.’ (70% of people surveyed said they enjoy listening to hip-hop.)
- **b** ‘New Apple tablet tops popularity stakes.’ (48 people out of 50 surveyed in an Apple store said the Apple tablet was their favourite.)
- **c** ‘Women now earn more than men.’ (10 people were randomly surveyed on a street.)
- **d** ‘New anti-ageing cream is unbeatable.’ (100% of the anti-ageing cream company’s shareholders were surveyed and agreed.)

7A.4 **4** For each scenario below:

- **i** decide whether the data collection method is fair
- **ii** decide whether the interpretation of the data collected is fair
- **iii** if appropriate, provide a suggestion to improve the survey.

- **a** Tess wanted to know which is the most popular beach in NSW. She asked every 10th person on the electoral roll in her town, ‘What is your favourite beach in NSW?’ Seventy-five per cent of 200 people said that Bondi was their favourite beach, so Tess concluded that Bondi was the most popular beach in NSW.
- **b** Carlos wanted to find information about the average height of basketball players in his town. He surveyed 10 players at random. Summary statistics gave a mode of 1.82 m, a median of 1.80 m, a mean of 1.79 m and a range of 25 cm. Carlos concluded the median height of basketball players in his town of 1.80 m was the most appropriate summary statistics to represent the data.
- **c** Isaac wanted to know the most popular computer brands. He asked a random sample of 1000 people, ‘Do you like Macs?’ He found that 60% of people said yes. Isaac concluded that Mac computers were the most popular brand of computer.
- **d** Asha wanted to know the most popular brand of soft drink in town. She asked all the local supermarkets which brand they sold the most of, and found that Coke made up 37% of the sales, Pepsi 29%, Schweppes 17% and all other brands made up the remainder. She concluded that no single brand was the most popular.

UNDERSTANDING AND FLUENCY

5 The marketing manager of a drug company claims that the company’s new pain relief drug blocks pain for up to 18 hours. This is based on a sample of 15 people recording the number of hours they were pain-free. Here is that data:

2	18	3	2	2
1	2.5	2	1.5	2.5
2	1.5	2	1.5	1.5

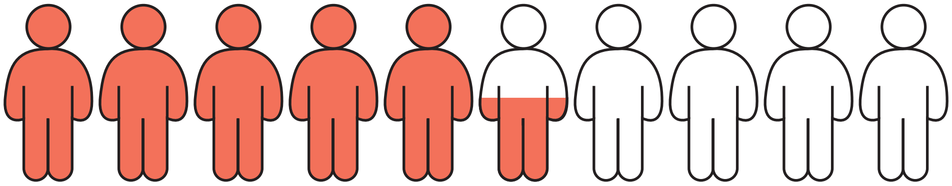
- **a** Why is the company’s marketing manager making this claim? Explain why it is misleading.
- **b** Calculate the three measures of centre (mean, median and mode) for the given data.
- **c** The marketing manager changes tactics and decides to claim that the pain relief drug blocks pain for an average of 3 hours. Comment on this claim. Is the claim now fair? Explain.

6 What questions should you ask yourself when you are presented with a claim that uses statistics?

7 Each of these media statements is followed by a 'fine print' statement in brackets qualifying the original claim. Explain why the claims are not so 'amazing' when you read the fine print.

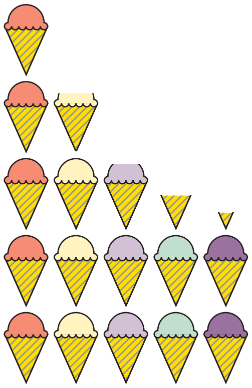
- **a** An advertisement for a phone's battery life claims it 'lasts up to 7 days' (based on standby power consumption).
- **b** An advertisement for toilet paper claims it is 'Voted Australia's favourite!' (a total of 42 people were surveyed).
- **c** An advertisement for a moisturising lotion states '67% of women notice a difference after just one week!' (compared to the use of regular soap).

8 The following pictogram was created for an infographic about the popularity of the current government.



- **a** Using the pictogram, what percentage of people do you think are satisfied with the current government?
- **b** Discuss the strengths and weaknesses of using this pictogram to display the data.

9 A gelateria commissioned the following pictogram as part of their internal research into their most popular flavours.



- **a** What is the purpose of the pictogram?
- **b** What information does the pictogram provide?
- **c** What information is missing in the pictogram?
- **d** How well do you think the pictogram communicates the data?

10 Over the past 20 years, there has been a steady decrease in the use of landline phones, and an increase in the screening of phone calls from unknown numbers. Discuss the impact that both of these factors may have had on collecting data from surveys.

11 Online surveys are very common. However, for an online survey to have any credibility, many issues need to be considered. For example:

- How can we ensure that a random sample has been surveyed that is representative of the population?
- How can we ensure that people completing the survey are who they claim to be?
- How do we deal with the non-response rate?

Write some guidelines you feel would be important to follow when choosing a sample for an online survey.

12 Use the statistics shown in the following table to comment on how the operating profit before tax of different Australian industries varied during the 2010s.

	Industry				
Year	**Mining**	**Manufacturing**	**Construction**	**Retail**	**Media**
2010–11	$85 156 m	$27 986 m	$30 282 m	$19 860 m	$10 430 m
2012–13	$54 374 m	$16 353 m	$26 340 m	$19 417 m	$8116 m
2014–15	$16 295 m	$20 652 m	$36 027 m	$18 724 m	$9400 m
2016–17	$33 258 m	$24 639 m	$36 043 m	$18 847 m	$7054 m
2018–19	$98 551 m	$33 822 m	$43 485 m	$17 582 m	$2935 m

(Source: ABS)

Note: figures provided for every other year of the decade.

13 Consider the table and text below, which were published online.

> 'Australia has been ranked among the top 10 countries for its handling of the coronavirus pandemic, with New Zealand taking out the top spot in a poll conducted by a prominent Australian think tank.
>
> The Lowy Institute assessed the response of 98 countries in how they managed the pandemic in the 36 weeks following their hundredth confirmed case of the virus.
>
> It judged countries that had fewer reported cases and deaths (both in aggregate and per capita basis), as well as nations where testing rates were high.
>
> China was excluded from the ranking because of a lack of publicly available data on testing.
>
> The United States, Brazil, Iran and Mexico were ranked the worst handlers of the pandemic.
>
> Countries that proved more successful in containing the virus were largely in the Asia-Pacific region.'

Rank	Country
1	New Zealand
2	Vietnam
3	Taiwan
4	Thailand
5	Cyprus
6	Rwanda
7	Iceland
8	Australia
9	Latvia
10	Sri Lanka

(Source: SBS news)

a From the table, where is Australia ranked in terms of their response to handling the coronavirus pandemic?

b Why would it be misleading to say that Australia ranked 8th out of 10 countries?

c Was the information collected a sample or a census? Justify your answer.

d Does this data support the claim that 'Australia has been ranked among the top 10 countries for its handling of the coronavirus pandemic'?

e The article does not directly provide the data to support its claims. What could you do to research this?

14 Find a media article that uses statistics to support its claims (including at least one visual display). Write a paragraph analysing these claims and commenting on how trustworthy you think the article is.

15 Write an informative guide to teach somebody how and what to look for when reading statistics and the claims that they are supposedly supporting.

16 Conduct your own statistical investigation using data taken from the ABS (Australian Bureau of Statistics). Ask some questions in your investigation, and provide answers after you have analysed the data. Provide a conclusion at the end of your report.

PROBLEM SOLVING AND REASONING

Check your Student obook pro for these digital resources and more:

Interactive skillsheet	Worksheet	Investigation	Topic quiz
Sampling methods	Analysing statistics	Comparing annual rainfall	7A

7B Classifying and displaying data

Learning intentions

By the end of this topic you will be able to …

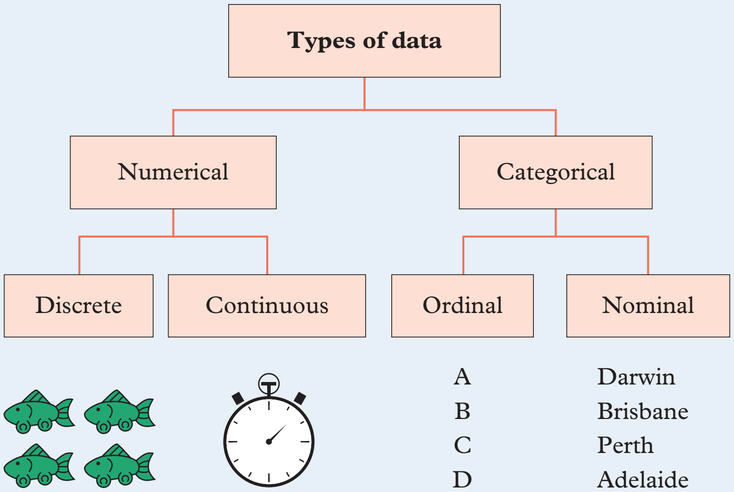
- ✔ classify data involving more than one variable
- ✔ display data involving more than one variable
- ✔ interpret data displayed in charts and graphs.

Inter-year links

Support	Interpreting graphs
Year 7	5E Interpreting graphs
Year 10	9E Scatterplots and bivariate data

Classifying data

- **Numerical data** can be classified as either **discrete** (countable) or **continuous** (measured).
- **Categorical data** can be classified as either **ordinal** (can be ordered) or **nominal** (unordered categories).

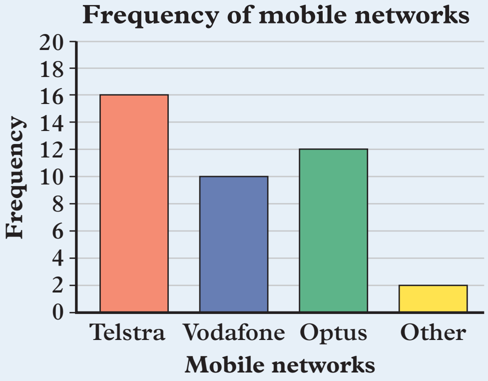


- **Univariate data** is data that only concerns a single variable.
- **Bivariate data** is data that shows the relationship between two variables. Bivariate data often falls within more than one classification. For example, comparing the amount of rainfall in different cities could involve a numerical variable, the amount of rainfall, and a categorical variable, the city.
- **Multivariate data** is data that shows the relationship between three or more variables.

Bar charts

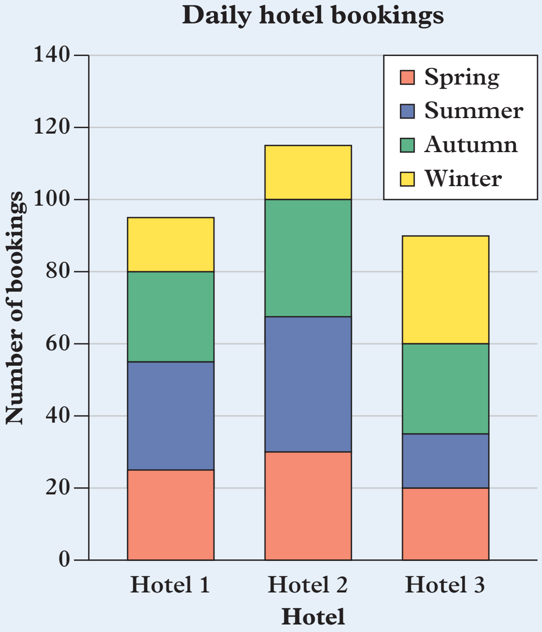
- **Bar charts** (or column graphs) can be used to display either univariate or bivariate data.
  - → The horizontal axis will display either a discrete numerical or categorical variable.
  - → The vertical axis will display either the frequency (for univariate data) or a numerical variable (for bivariate data).

For example, the bar chart on the right is displaying one categorical variable, with the frequency detailed in the vertical axis.

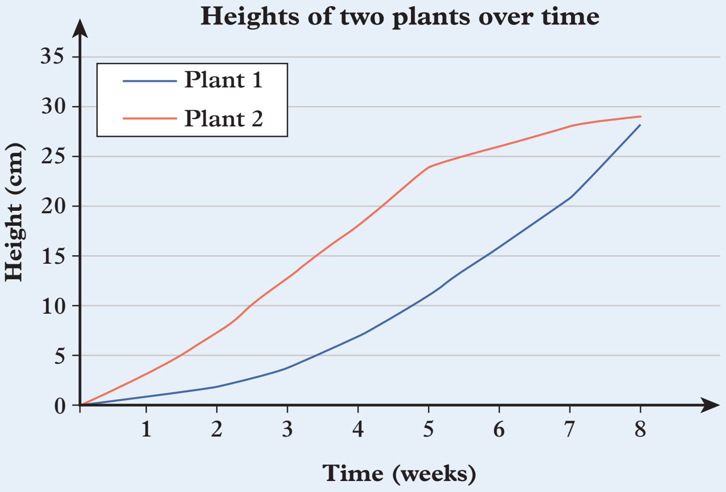


- A **stacked bar chart** displays an additional categorical variable within each bar.
  - → Each bar is divided into sections based on the additional categorical variable, which is detailed in a legend.

  For example, the stacked bar chart on the right is displaying two categorical variables (one on the horizontal axis and one in the legend), with a numerical variable on the vertical axis.

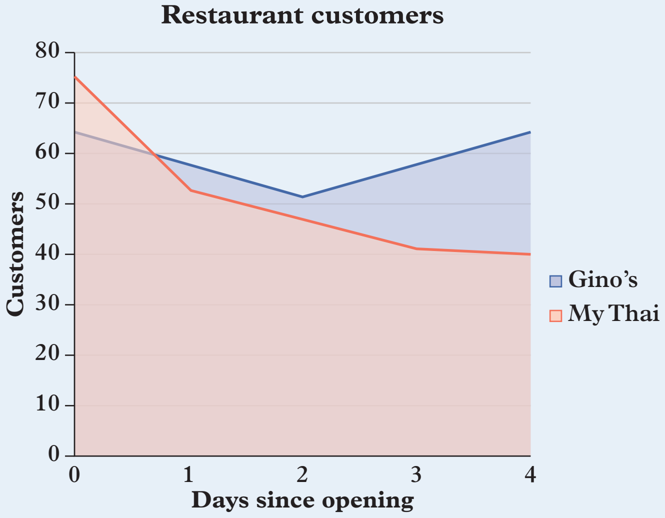


- **Line graphs** can be used to display data involving two numerical variables.
  - → The horizontal axis must be a continuous variable and usually displays the independent variable. The vertical axis usually displays the dependent variable. For example, the height of a plant depends on the length of time it has been growing.
  - → Travel graphs are an example of line graphs.
  - → If necessary, a break in the scale on the vertical axis, indicated by a zigzag, can be used to show the focus of the graph is a distance away from the origin.
- **Multiple line graphs** can be used to display data involving two numerical variables and one categorical variable. The categorical variable is detailed in the legend.
- All graphs should include clearly labelled axes with evenly spaced scales, and a title and legend where necessary.



Area charts

- **Area charts** shade the area between the lines and axes of a multiple line graph to create a sense of quantity over time.
- Area charts can be used to display data with two numerical variables (on the axes) and one categorical variable (in the legend).
- Area charts should only be used when it makes sense for the numerical variable in the vertical axis to accumulate over time. For example, in the multiple line graph above you cannot determine the total height of the plants by summing the values at each week, so an area chart should not be used for this data.



Example 7B.1 Classifying bivariate data

A journalist is researching the time spent commuting to work from different suburbs. Identify each variable in this situation as either numerical or categorical.

THINK	**WRITE**
1 Identify the two variables: time duration and name of suburb. **2** Classify each of these variables from the previous step. Time is a numerical variable and the suburb is a categorical variable.	Time: numerical Suburb: categorical

Example 7B.2 Interpreting line graphs

Consider the line graph on the right.

a Classify the variables in the graph.

b What time period does it cover?

c Between which weeks does the number of customers increase the fastest?

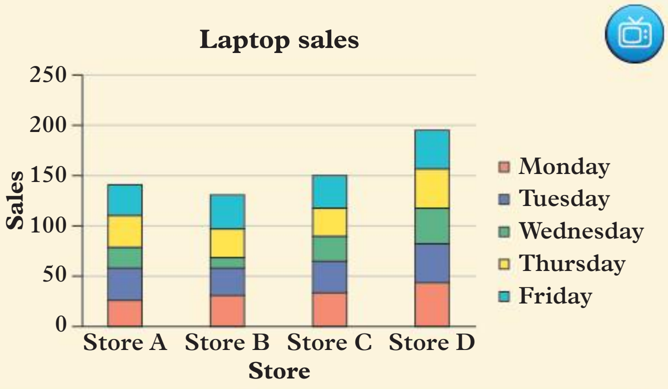
THINK	**WRITE**
a There are two variables in a line graph: a numerical variable on the vertical axis, and a different numerical variable on the horizontal axis.	**a** Time (weeks): continuous Number of customers: discrete
b Look at the range on the horizontal axis.	**b** 10 weeks
c Identify between which two points was the greatest increase on the vertical axis.	**c** Weeks 2 and 3

Example 7B.3 Interpreting stacked bar charts

Consider the stacked bar chart on the right.

a Which variables are displayed in the graph?

b Which store sold the most laptops on the Monday?



THINK	**WRITE**
a Look at the title of the axes and the legend.	**a** laptop sales, store and day
b Identify the tallest section representing Monday, which is the bottom part of each bar. Use the horizontal axis to interpret which store this represents.	**b** Store D

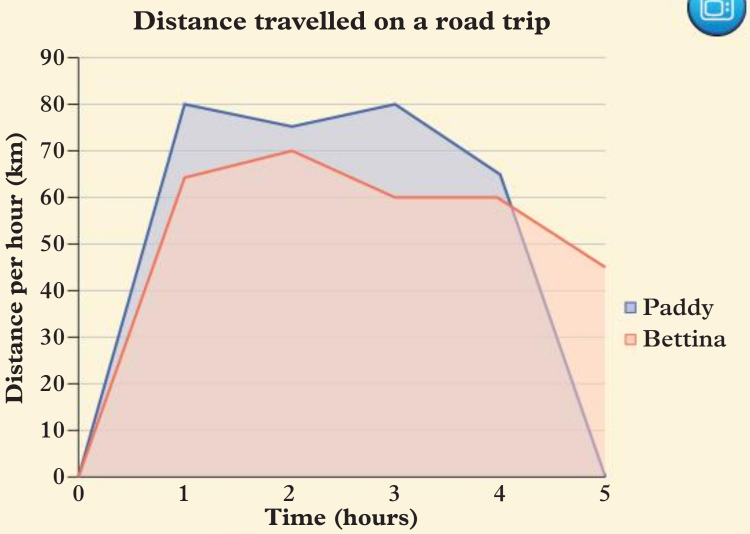
Example 7B.4 Interpreting area charts

Consider this area chart.

a Classify the variables in this chart.

b Who covered the most distance in an hour?

c Who completed their road trip first?



THINK

a There are three variables in an area chart: two numerical variables on the axes and a categorical variable in the legend.

b Identify the line with the highest point on the chart. Use the legend to identify who this represents.

c Identify which line reached 0 km first. Use the legend to identify who this represents.

WRITE

a Distance: continuous
Time: continuous
Person: categorical

b Paddy covered the most distance in an hour.

c Paddy completed his road trip first.

Example 7B.5 Drawing a line graph

The data in the following table was recorded over 10 days.

Day	0	1	2	3	4	5	6	7	8	9
Weight of puppy (g)	92	100	110	125	123	131	142	150	156	164

Draw a line graph to represent this data.

THINK

1 The independent variable (days) should be placed on the horizontal axis and the dependent variable (weight) should be placed on the vertical axis.

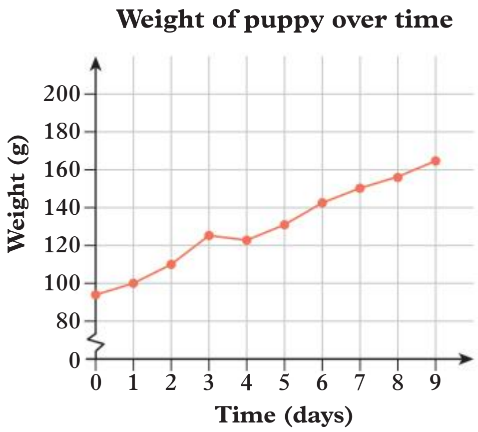
2 Draw the vertical axis. As the values start above 80, use a break in the axis to focus the graph above here.

3 Draw the horizontal scale, providing enough space to fit in all the independent values.

4 Plot the points from the table and join subsequent points with straight lines.

5 Include a title and axis labels.

WRITE



Helpful hints

✔ Univariate (single variable) data is usually displayed in a column graph with the frequency of the categories on the vertical axis. Bivariate data consists of two distinct variables.

ANS p460

Exercise 7B Classifying and displaying data

△ **1–5, 7, 8, 12, 14, 16** ■ **2–4, 6, 7, 9–11, 15, 17, 18** ⬟ **3, 4, 6, 8, 11, 13, 15, 17–19**

UNDERSTANDING AND FLUENCY

1 Match each of the following variables with the data classification.

a Eye colour of students	**A** Numerical – discrete
b Number of students in each class	**B** Numerical – continuous
c Time (in minutes) spent travelling to school	**C** Categorical – ordinal
d Final grade on an end-of-year exam (A+, A, B+, B, etc.)	**D** Categorical – nominal

2 Consider the following four variables.

1 Height (in cm) of AFL players

2 Method of transport for commuting to school (car, bus, train, etc.)

3 Mark (out of 40) on a Mathematics test

4 Belt level in karate (white, orange, blue, etc.)

a Classify each variable as numerical or categorical.

b Classify each numerical variable as either discrete or continuous, or each categorical variable as either ordinal or nominal.

7B.1 **3** In each of the following situations, identify the two variables and classify each as either numerical or categorical.

a A car company hires an independent researcher to find the fuel consumption (kilometres per litre of petrol) for each of their car models.

b A business owner investigates the relationship between the number of ice cream sales per day in her shop and the maximum daily temperature (in °C).

c On each student's report, a school lists the student's exam score, as a percentage, for each subject.

7B.2 **4** Consider the line graph on the right.

a What does the graph show?

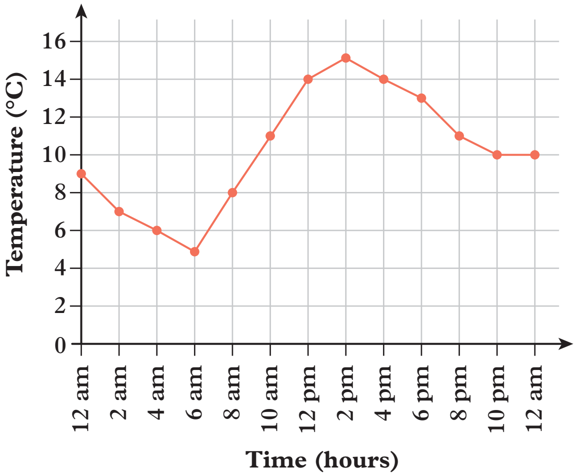
b Classify the variables in the graph.

c What is the temperature at 10 am?

d When is the temperature 15°C?

e What is the coolest temperature recorded? At what time does it occur?

Temperature over 24-hour period

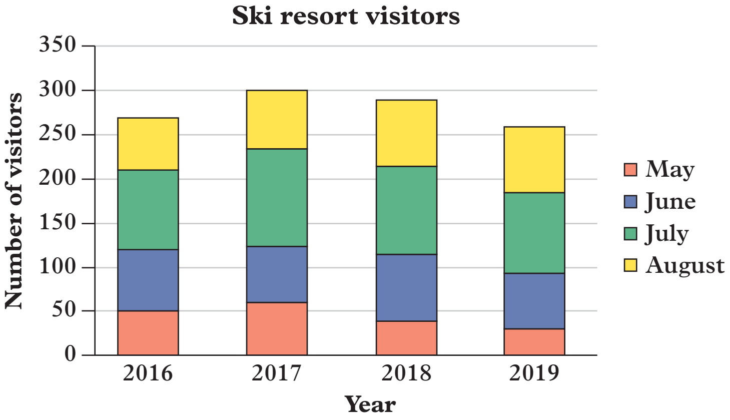


7B.3 5 Consider the stacked bar chart on the right, which shows the number of people who visited a ski resort during the months it was open from 2016 to 2019.

a What are the different variables in the graph?

b In which month of which year did the resort have the most visitors?

c In which month of which year did the resort have the least visitors?



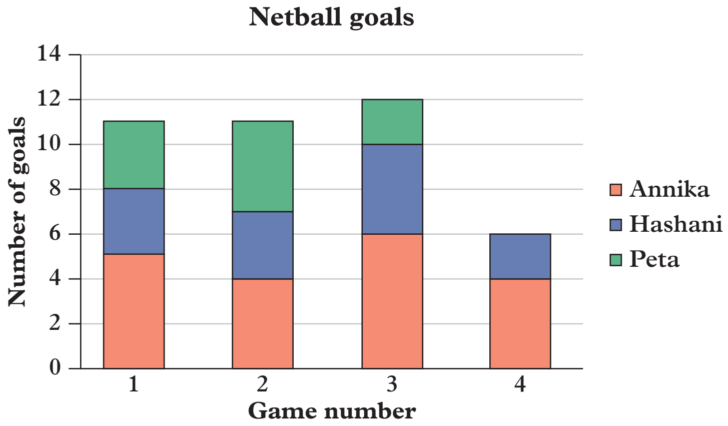
6 Consider the stacked bar chart on the right.

a What are the different variables in the graph?

b In which game did one of the three players not score a goal?

c In which game did the three players score the most goals combined?

d Which of the three players scored the least number of goals across the four games?



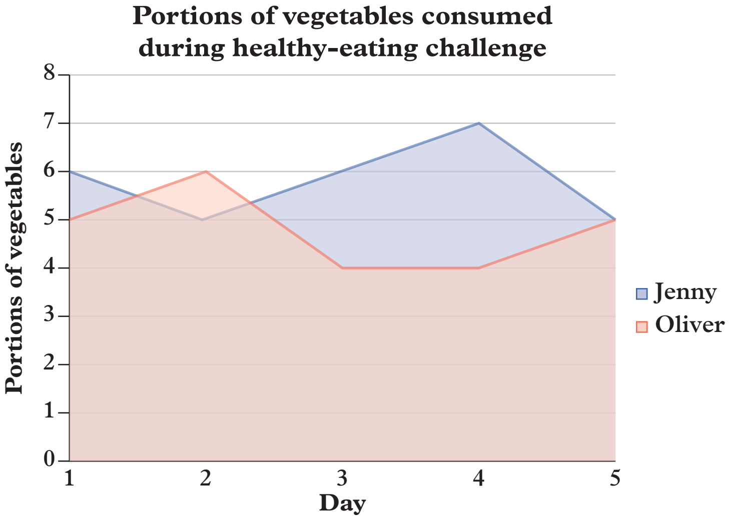
7B.4 7 Consider the area chart on the right.

a Classify the variables in this chart.

b On which day did Oliver consume more portions of vegetables than Jenny?

c How many more portions of vegetables did Jenny consume than Oliver on day 4 of the challenge?

d How many portions of vegetables did Jenny and Oliver consume during the challenge?



7B.5 8 The data in the following table shows the estimated market value of a gaming console for 10 months after its purchase. Draw a line graph to represent this data.

Time since purchase (months)	0	1	2	3	4	5	6	7	8	9	10
Market value ($)	600	400	350	320	300	270	250	250	220	220	220

9 The following data shows the number of kilometres driven by Antoine over a period of 9 days.

Day	1	2	3	4	5	6	7	8	9
Number of km driven	18	11	0	18	15	8	10	6	13

a Draw a line graph to represent this data.

b On which day(s) did Antoine drive the most kilometres?

c On how many days did Antoine drive more than 10 km?

10 The following data is the monthly profit (rounded to the nearest $100) of a small business over the course of a year.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	4000	5300	3800	4100	4300	4000	3800	3400	5100	5400	4900	4600

a Draw a line graph to represent this data, using an appropriate scale.

b In which month was the greatest profit?

c In which month was the least profit?

d In which month was the greatest increase in profit over the previous month? How much was this increase?

e In which month was the greatest decrease in profit over the previous month? How much was this decrease?

11 Students in years 8, 9 and 10 were asked to choose their preferred ice-cream flavour from three choices given. The data is shown below.

	Year 8	**Year 9**	**Year 10**
Chocolate	140	135	130
Strawberry	100	90	70
Vanilla	60	75	60

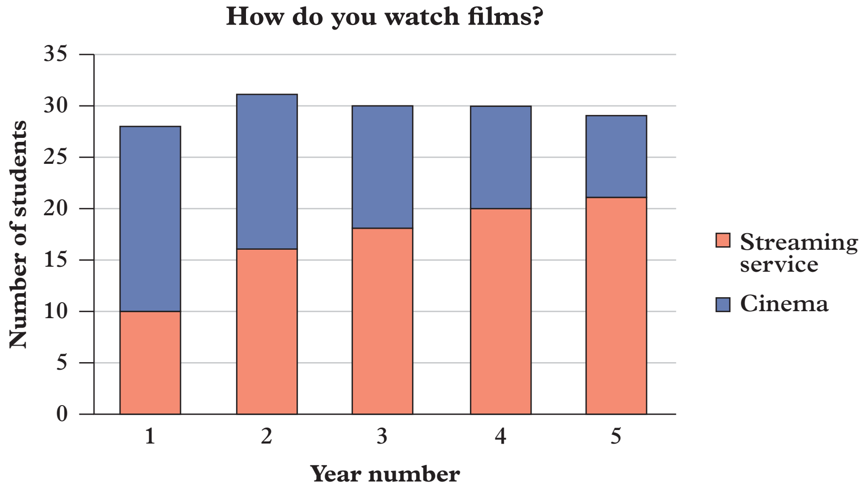
a Create a stacked bar chart for the set of data if the category on the horizontal axis is year level.

b Create a stacked bar chart for the set of data if the category on the horizontal axis is ice-cream flavour.

c Which of the two graphs is better for showing which ice-cream flavour is the most popular with Year 9 students?

d Is it accurate to say that, in general, Year 9 students prefer chocolate ice-cream more than Year 10 students? Why or why not?

12 Each year a Mathematics teacher has a Year 9 class with exactly 25 students. He asks each student 'Do you regularly watch films on an online streaming service (like Netflix)?' and 'Do you go to the cinema regularly (once or more a month)?' The following stacked bar chart shows the data collected over a 5-year period.



a How many students regularly watched films on streaming service in year 4?

b In which year was the number of students in each category closest to being equal?

c Explain why the values for each year can add to more than the number of students in the class.

d What would you expect the two columns to look like in 10 years, assuming the teacher continues collecting data? Explain your answer.

13 A specialty store sells Christmas items all year round. The table below shows the average number of customers per day, rounded to the nearest 10 customers, in each month over the course of 12 months.

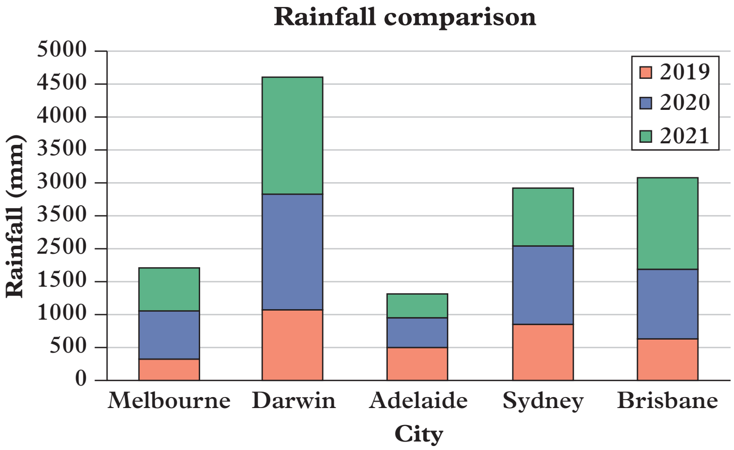
Month	1	2	3	4	5	6	7	8	9	10	11	12
Average daily customers	10	20	20	30	60	170	300	40	20	10	0	10

a Draw a line graph to represent this data.

b It is safe to assume that, in this data set, month 12 does not represent December? Which month number do you think represents December? Explain your answer.

c Is it accurate to say that the store had no customers at all in month 11? Explain your answer.

14 The following stacked bar chart shows the total rainfall (in mm) in different cities in Australia over the years 2019, 2020 and 2021.



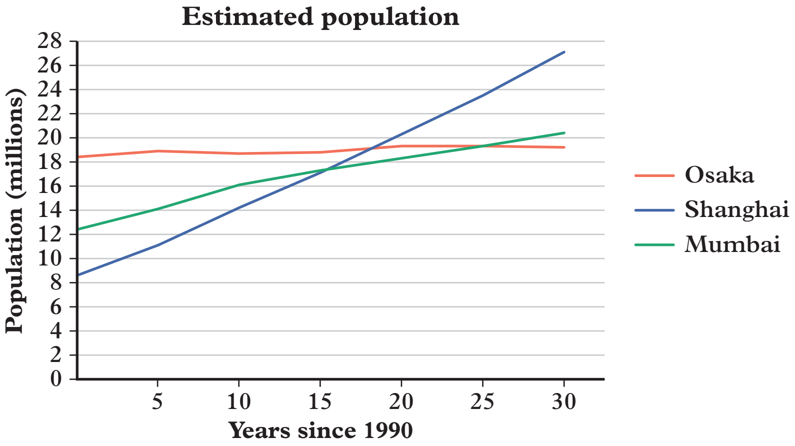
a Based on this data, which of the five cities is the wettest?

b Based on this data, which of the five cities is the driest?

c In which city did the rainfall decrease across the three years?

d Which city had the least rainfall in 2019?

15 The following multiple line graph shows the estimated metro population of three Asian cities since 1990.



a Which city had the highest population in 1990?

b Which city had the highest population in 2010?

c Approximately how many people were living in Mumbai in 2000?

d Which city had the highest population in 2020?

e In which year, approximately, did the population of Shanghai surpass the population of Osaka?

f Describe the change in each city's population between 1990 and 2020.

PROBLEM SOLVING AND REASONING

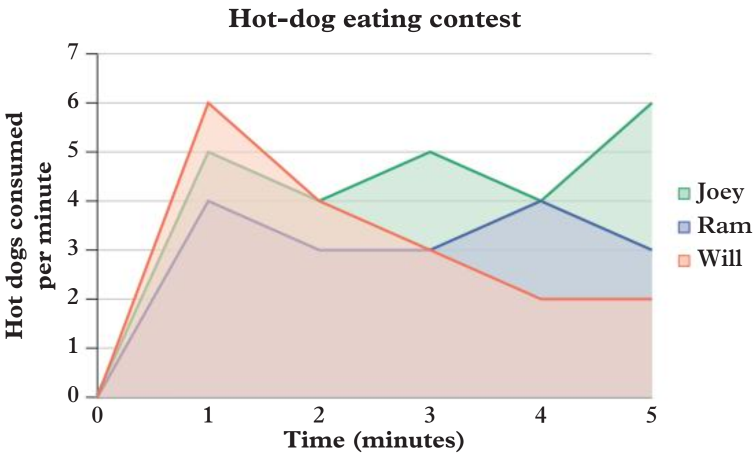
16 Consider the area chart on the right.

a Who won the hot-dog eating contest?

b Who came in second place?

c How many hot dogs were consumed by the three contestants?

d Who was most consistent over the 5 minutes? Justify your answer.



17 Otto is trying to decide which graph to use for several different research subjects. Decide which graph type (stacked bar chart, multiple line graph, area chart) Otto should choose in each instance, justifying your decisions.

a Number of steps taken by colleagues each day in a work stepathon challenge

b Favourite pizza toppings of children and adults

c Height of three children measured each year

CHALLENGE

18 The table below shows the monthly profit, rounded to the nearest $1000, of a small ice-cream store in Geelong. In this case, January is represented by month 1.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	9000	8000	6000	3000	1000	−1000	−2000	0	3000	4000	7000	9000

a Draw a line graph to represent this data. Remember to make space below the horizontal axis for the negative values.

b Explain what the negative values in month 6 and month 7 mean.

c Explain what the 0 in month 8 means.

d What was the total profit of the ice-cream store over the course of the year?

e From month 8 to month 12 the profit is increasing each month. Do you expect this trend to continue forever? Explain your answer.

f How would you expect the monthly profit of an ice-cream store to be different for a similarly sized store in the northern hemisphere (for example, in a small town in Spain)? Explain your answer.

19 The following table shows the height (in cm) of a person over the first 15 years of their life.

Age (years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Height (cm)	52	75	85	92	100	108	112	120	128	135	140	147	154	160	165	172

a Draw a line graph to represent this data.

b The person had a height of 52 cm when they were born and 172 cm 15 years later. Using these two points, find the average height they gained per year.

c Assuming the person continues growing at this average rate per year, how tall would they be at 25 years old?

d Instead, suggest a reasonable height that the person might be at 25 years old.

e Explain what you would expect the rest of the graph to look like if it continued to when the person was 25 years old.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Classifying data

Interactive skillsheet
Stacked bar charts

CAS instructions
Line graphs

Topic quiz
7B

7C Grouped data and histograms

Learning intentions

By the end of this topic you will be able to ...

✔ group numerical data into class intervals

✔ represent grouped numerical data in a histogram.

Inter-year links

Support	Data displays
Year 7	10C Stem-and-leaf plots
Year 10	9C Distributions of data

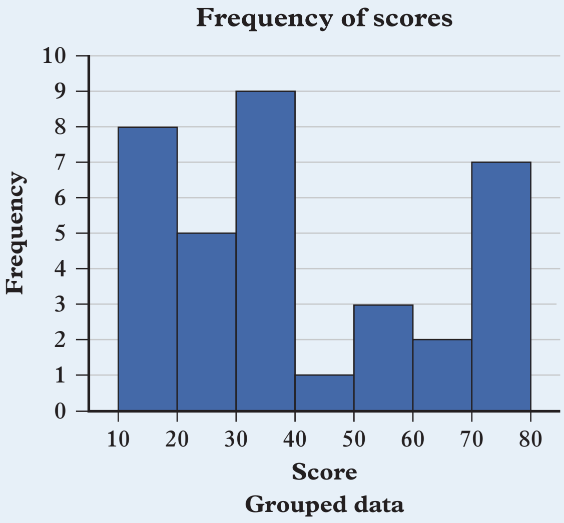
Grouping data into class intervals

- **Grouped data** is numerical data that has been sorted into groups or **class intervals**. **Ungrouped data** is raw data that has not been placed into class intervals.
- When grouping data, class intervals should be chosen so that a frequency table contains 5 to 10 classes.
  - → Class intervals should start at a round number, for example 0 or 10.
- Before constructing a frequency table with class intervals, identify whether the numerical data is discrete or continuous.
  - → Discrete data uses class intervals such as 0–9, which means any value from 0 to 9, including both 0 and 9.
  - → Continuous data uses class intervals such as 0–<10, which means any value from 0 to 10, including 0 but not including 10.

Discrete interval	Continuous interval
0–9	0–<10
10–19	10–<20

Histograms

- A **histogram** is a special kind of bar chart that can be used to represent grouped continuous or discrete numerical data.
  - → Ungrouped discrete numerical data is usually represented by using a standard bar chart.
- Histograms have no gap between columns, but there is a small gap between the vertical axis and the first column when the intervals do not start at 0.
- When displaying grouped data in a histogram, the **lower bound** of the class interval is shown on the left of each column and the **upper bound** of the class interval is shown on the right of each column.
  - → For discrete data, where the upper and lower bounds of subsequent classes are not equal, the lower bound of the higher class interval should be shown. For example, for the discrete intervals 0–9, 10–19, 20–29, the intervals on the horizontal axis are labelled 0, 10, 20.



Example 7C.1 Understanding histograms

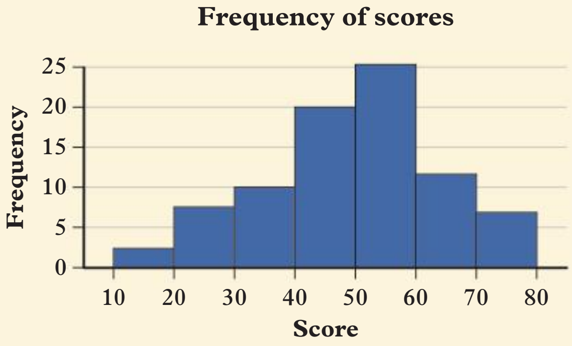
Consider this histogram representing continuous data.

a What is the width of each class interval?

b What is the frequency of the class interval 30–<40?

c What is the most common class interval? What is its frequency?

d Which class has a frequency of 20?



THINK

a Look at the horizontal axis. The width of the class intervals is the width of each column.

b Look at the column covering the marks 30 to 40 and read off the frequency from the vertical axis.

c Identify the tallest column. Use the horizontal axis to identify the class interval and read off the frequency from the vertical axis.

d Identify the column with a frequency of 20 on the vertical axis, and then use the horizontal axis to identify the class interval.

WRITE

a The width of each class interval is 10.

b The class interval 30–<40 has a frequency of 10.

c The most common class interval is 50–<60 and it has a frequency of 25.

d The class interval with a frequency of 20 is 40–<50.

Example 7C.2 Drawing a frequency table to represent grouped data

Use the frequency table below with class intervals of width 10 to represent this continuous data.

4.5, 11.6, 67.3, 33.7, 28.1, 36.4, 22.6, 54.8, 1.4, 66.8, 36.4, 29.3, 37.8, 42.3, 52.1, 38.3

Class							
Frequency							

THINK

1 Identify the minimum and maximum scores in the data set. The minimum score is 1.4 and the maximum score is 67.3.

2 The data is continuous so the class intervals must be in the form of 0–<10, etc.

3 Draw the frequency table and group the raw data into each class interval.

WRITE

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	2	1	3	5	1	2	2

Example 7C.3 Drawing a histogram

Draw a histogram to represent the data in this frequency table.

Class	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	7	5	3	4	3	2

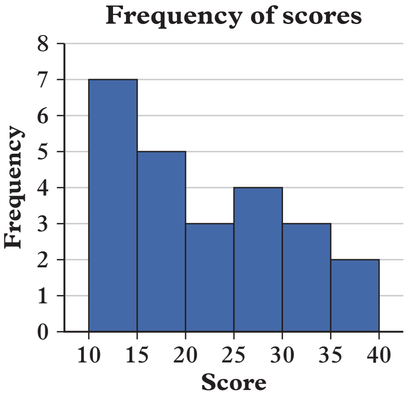
THINK

1 Draw the axes with an even scale that allows the minimum and maximum values to be shown. Ensure that there is a half space between the vertical axis and first class interval.

2 Use the frequency table to draw the columns of the histogram.

3 Label both axes and provide a title.

WRITE



Helpful hints

- ✔ The class interval 0–9 contains 10 values. If this doesn't make sense to you, try counting the values using your hands: 0, 1, 2 … 8, 9.
- ✔ When grouping raw data, a tally column can help to ensure you don't miss any scores.
- ✔ You don't need to order the values in a data set before grouping data. Avoiding this unnecessary step can save you time when answering questions.

ANS p462

Exercise 7C Grouped data and histograms

△ 1–3, 4(a, c), 5(a, b), 6(a, d), 8, 10, 13, 16

□ 2, 4(b, c), 5(a, c), 6(d, e), 7, 11, 14, 16, 19

⬠ 4(d), 5(a, c), 6(e, f), 9, 12, 15, 17–20

7C.1 **1** Consider this histogram representing continuous data.

a What is the width of each class interval?

b What is the frequency of class interval 20–<25?

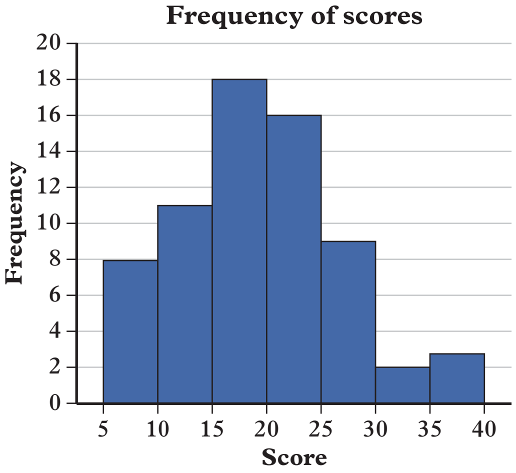
c What is the most common class interval? What frequency does it have?

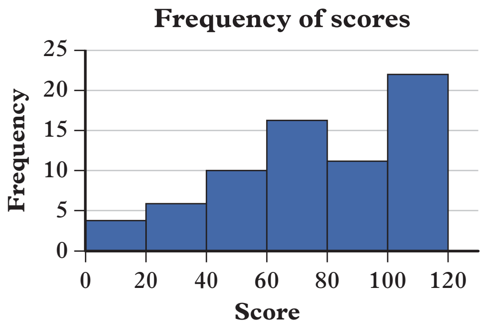
d Which class interval has a frequency of 9?

e What is the total frequency of scores that are larger than 25?

f What is the total frequency of scores that are less than or equal to 15?

g What percentage of the scores are more than 35?





2 Consider the histogram on the right representing continuous data.

a How many class intervals are there?

b What is the least common class interval? What frequency does it have?

c How many people were surveyed?

3 Joseph collected the following data on the heights of the people in his class.

145, 183, 167, 172, 161, 158, 153, 168, 165, 174, 157, 152, 173, 166, 158, 159, 160, 171, 171, 161, 165, 172, 165, 158, 154

a Identify the minimum value in the data set.

b Identify the maximum value in the data set.

c Calculate the range of the data.

d Group the data into class intervals of 5 (e.g. 150–<155) in a frequency table.

7C.2 **4** Draw a frequency table with the given class interval widths to represent each of these continuous data sets.

a 5, 16, 28, 24, 31, 39, 3, 18, 13, 11, 25, 33, 8, 12, 19, 21, 31, 28 [class interval width = 5]

b 14.5, 73.2, 22.1, 43.9, 42.0, 58.4, 19.8, 37.6, 62.1, 29.4, 34.5, 72.1, 59.1, 52.3, 63.1, 26.3, 34.0, 41.9, 48.5, 16.4, 31.2, 52.9 [class interval width = 10]

c 1.2, 5.4, 9.3, 11.4, 3.3, 4.7, 3.3, 3.9, 4.8, 6.6, 2.9, 1.9, 10.6, 9.7, 10.8, 3.6, 4.8, 2.7, 2.1, 1.7, 1.9, 11.9, 6.7, 5.4, 5.1, 1.6, 1.8 [class interval width = 2]

d 42, 79, 56, 49, 77, 50, 51, 46, 48, 72, 61, 78, 63, 45, 58, 53, 73, 58, 49, 61, 68, 67, 43, 49, 75, 77, 58, 54, 67, 72, 51, 56, 53, 48, 76, 78, 72, 42, 48, 53 [class interval width = 5]

7C.3 **5** Use each frequency table to draw a histogram.

a

Class	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40	40–<45	45–<50
Frequency	8	6	7	2	3	1	5	6	9

b

Class	0–<20	20–<40	40–<60	60–<80	80–<100	100–<120
Frequency	14	21	18	13	8	2

c

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	5	8	12	3	11	9	6

6 Use each data set to draw an appropriate histogram with the given class interval widths.

a 13, 46, 13, 17, 35, 9, 22, 15, 8, 2, 35, 42, 42, 17, 16, 22, 29, 31, 47, 29, 13, 20, 36, 47, 28, 23, 30, 38 [class interval width = 10]

b 18.1, 24.5, 32.1, 15.6, 22.5, 29.1, 34.6, 16.7, 19.4, 17.5, 21.8, 27.5, 29.2, 30.1, 20.0, 33.1, 32.8, 31.9, 33.8, 14.3 [class interval width = 5]

c 64, 18, 120, 7, 29, 40, 145, 38, 72, 38, 18, 29, 2, 56, 49, 87, 99, 104, 59, 5, 29, 112, 118, 34, 59, 29, 19, 13 [class interval width = 20]

d 125, 726, 632, 465, 428, 257, 283, 399, 619, 402, 132, 196, 183, 743, 120, 703, 336, 652, 349, 402, 560, 144, 759, 717, 588, 185, 464, 685, 268, 352, 310, 408, 114, 782, 189 [class interval width = 100]

e 1.25, 1.89, 1.09, 1.76, 1.15, 1.36, 1.55, 1.67, 1.99, 1.32, 1.08, 1.14, 1.17, 1.62, 1.88, 4.9, 1.68, 1.49, 1.08, 1.16, 1.24, 1.19, 1.26, 1.83, 1.52, 1.18, 1.07, 1.42, 1.01, 1.19 [class interval width = 0.2]

f 25, 58, 48, 33, 26, 53, 42, 49, 58, 53, 46, 24, 58, 53, 46, 41, 38, 47, 44, 58, 53, 57, 39, 21, 48, 46, 42, 58, 52, 43, 42, 37, 36, 27, 46, 42, 49, 53, 57, 59 [class interval width = 5]

7 Data was collected on the time spent listening to music in hours per week, as shown below.

7	10	2	4	24	3	7	9	5	15
17	16	19	20	5	3.5	5	10	14	7
7	5	9	10	17	7	4	10	11	16
4	5.5	12	14	6	7	12	14	16	3

a Create a frequency table with class intervals of width 5 to collate the data.

b Draw a histogram to represent this data.

8 Data was collected on the weights (in kg) of various dogs, as shown below.

11.2	14.8	6.4	19.7	22.1	4.8	5.4	12.6	18.2	9.2
13.1	20.1	8.6	4.9	8.4	17	6.2	14.3	26.3	16.6
18.6	19.2	14.5	28.3	16.2	9	11.4	14.9	11.3	13.2

a Fill in the following frequency table.

Class	Frequency
0–<5	
5–<10	
10–<15	
15–<20	
20–<25	
25–<30	

b Based on your frequency table, draw a histogram to represent this data.

9 Data was collected on the ages of people in a train carriage one day.

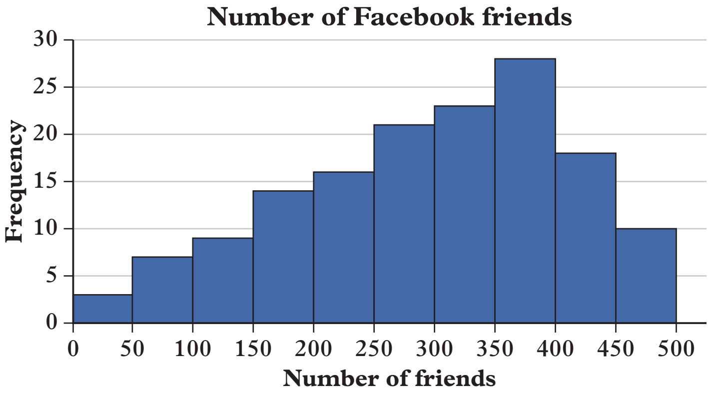
16	22	19	28	25	14	18	19	21
32	27	22	14	15	15	23	22	17
14	18	26	24	39	31	20	15	13
16	16	24	29	30	19	20	21	21

a Arrange this data in a frequency table with class intervals of width 5.

Note: consider age as a continuous variable.

b Draw a histogram to represent this data.

10 Consider the following histogram.



a What does it show?

b What is the size of its class intervals?

c State the most common class interval and its frequency.

d If you were to add a piece of data with a value of 140 to the histogram, to which class interval would you add it?

UNDERSTANDING AND FLUENCY

11 Data collected on the ages of customers in a clothes store in a day is shown in the table below.

a Why is this table difficult to read?

b Redraw the table with class intervals of width 10 so that it is easier to read.

Class	Frequency
<10	2
10–<12	6
12–<14	12
14–<16	16
16–<18	14
18–<20	13
20–<22	11
22–<24	9
24–<26	10
26–<28	8
28–<30	6

Class	Frequency
30–<32	4
32–<34	6
34–<36	5
36–<38	3
38–<40	2
40–<42	3
42–<44	1
44–<46	1
46–<48	0
48–<50	1
≥50	6

PROBLEM SOLVING AND REASONING

12 Explain why you can't accurately decrease the size of the class intervals in this table.

Class	Frequency
0–<20	16
20–<40	48
40–<60	42

13 The following ordered data shows the number of kilometres travelled by different people to attend a conference.

4, 4, 11, 12, 12, 13, 19, 19, 20, 21, 21, 25, 28, 31, 33, 34, 42, 44, 52, 55, 57, 58

a Create a histogram with class intervals of width 20 to represent this data.

b Create a histogram with class intervals of width 10 to represent this data.

c Create a histogram with class intervals of width 5 to represent this data.

d Which of the three histograms do you think displays the data most appropriately? Explain your answer.

14 This data was collected on how many thousands of people were present at AFL matches at Marvel Stadium, Melbourne.

10	51	35	22	25	46	34	21	8	28	11	19
22	31	38	42	22	22	48	30	25	26	15	21
18	18	41	39	33	33	38	25	28	10	17	13

a Create a histogram with class intervals of width 5 to represent this data.

b Create a histogram with class intervals of width 10 to represent this data.

c What is similar and what is different about your graphs for parts **a** and **b**?

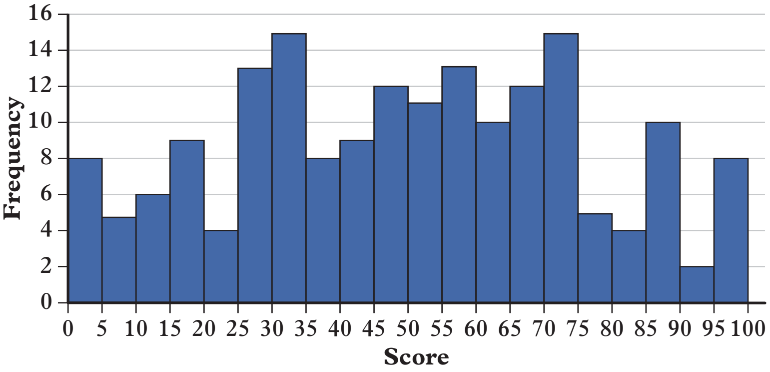
15 Consider the histogram on the right.

a What is the width of the class intervals?

b What can you say about the number of classes? Is it easy to read? Can you see a pattern?

c Redraw the histogram with class intervals of width 10.

d What patterns can you see now?



16 Stem-and-leaf plots can be used to display discrete data, where each piece of data is split into two parts (a stem and a leaf). The key indicates the value of the data. Consider this stem-and-leaf plot.

Stem	Leaf
1	2 6 7 8 8 9
2	0 1 5 6 7
3	3 6 8
4	0 2
5	1

Key: 1 | 2 = 12

- **a** Which part of the plot represents the class intervals?
- **b** What is the size of the class intervals?
- **c** How many people were surveyed?
- **d** What advantage does a stem-and-leaf plot have over a histogram?

17 Use the stem-and-leaf plot shown to draw a histogram.

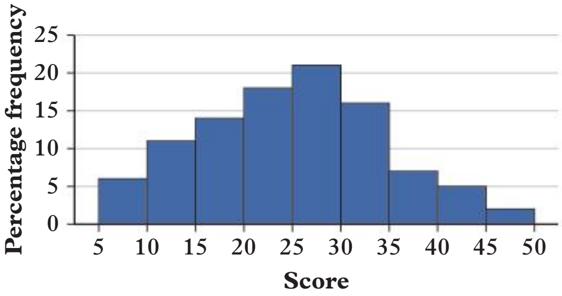
Stem	Leaf
1	0 0 1 1 3 4
1*	5 5 5 6 6 7 9
2	0 1 2 2 3 3 4 4 6
2*	5 6 6 7 8 8 9
3	0 1 1 2 2 3 4
3*	5 6 7 8

Key: 1 | 3 = 13

18 Explain why you can't accurately construct a stem-and-leaf plot from a histogram.

19 Consider this percentage frequency histogram.

- **a** How is it different from a normal histogram?
- **b** What percentage of scores are between 30 and 35?
- **c** What percentage of scores are greater than 35?
- **d** Without performing a calculation, state the sum of the percentage frequency columns. Explain how you know the answer.
- **e** If there were 400 scores in total, calculate:
  - **i** the number of scores between 15 and 20
  - **ii** the number of scores less than 25
  - **iii** the number of scores between 20 and 40.



20 Create a percentage frequency histogram with class intervals of width 0.2 to represent this data set.

Weights of newborn babies at a particular hospital in one week (in kg):

3.25, 4.15, 2.75, 3.60, 3.95, 3.05, 2.85, 4.20, 1.95, 3.50, 3.65, 3.15, 3.70, 3.95, 4.10, 4.85, 2.90, 3.10, 3.30, 3.25, 3.50, 4.05, 3.45, 3.85, 3.75, 3.15, 3.45, 3.20, 3.25, 4.25, 2.55, 2.95, 3.40, 3.85, 3.80, 3.55, 3.20, 3.00, 3.20, 3.75, 4.00, 4.15, 3.80, 3.75, 3.40, 3.25, 3.15, 3.05, 3.85, 2.95

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Histograms	Investigation Pocket money	CAS instructions Histograms	Topic quiz 7C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

7A **1** Determine the sampling method used in the following situations.

- **a** Ms Walker surveys 10 students from each class about their opinions on school assembly.
- **b** April surveys 20 people in her class about their opinions on the new school sports centre.
- **c** Julio surveys 100 people from suburbs in his local government area in proportions that reflect their populations.
- **d** Trent selects every 10th person from a list to complete a survey.

7A **2** For each scenario below:

- **i** decide whether the data collection method is fair
- **ii** decide whether the interpretation of the data collected is fair
- **iii** if appropriate, provide a suggestion to improve the survey.

- **a** Sienna wanted to know the average number of hours people in her town spent sitting down every day. She asked more than 100 people, chosen at random from people she saw walking through a mall, how many hours they sat down per day, and came up with an average of 8.5 hours.
- **b** Kane wanted to know the most popular party foods to serve at an upcoming school dance. He surveyed every second person in his neighbourhood about their favourite party foods, and decided to serve party pies, chocolates and sandwiches.

7A **3** Decide whether each of these surveys will provide fair or biased results, giving a reason for your answer.

- **a** Asking 10 000 people randomly selected around the country their opinion on an issue in your community
- **b** Asking everybody you see at a local shopping mall their opinion on a global issue

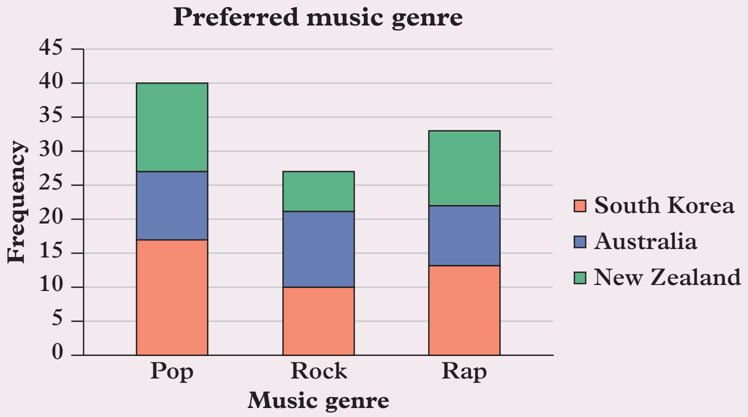
7B **4** Classify the following variables as numerical or categorical.

- **a** Time spent watching television
- **b** Most used smartphone app
- **c** Programming skill level (Beginner, Intermediate, Advanced)
- **d** Number of members in a chess club

7B **5** Consider the following table of data. Draw a line graph to represent this data.

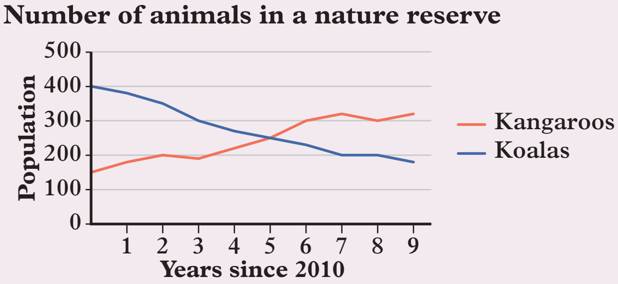
Hours since midnight	0	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°C)	8	9	9	11	10	12	13	15	17	18	17	20	22

7B **6** Thirty teenagers from three different countries were asked to choose their preferred music genre. The results are shown in the following stacked bar chart graph.



- **a** Which music genre was most preferred by South Koreans?
- **b** The teenagers of which country preferred rock to rap?
- **c** What was the most popular music genre, overall?

7B **7** Consider the following multiple line graph, which shows the number of two different animals in a nature reserve since 2010.

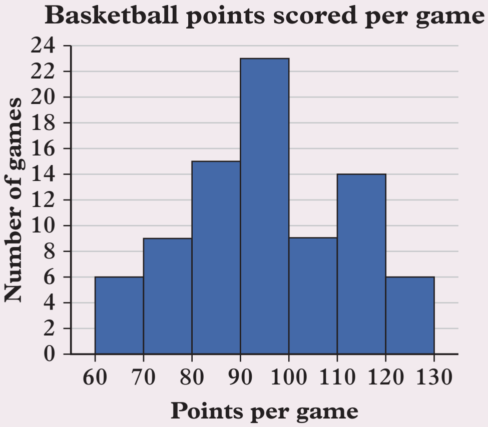


a The population of which animal in the reserve is increasing?

b During which year was the population of each animal the same?

c Between which two years did the population of koalas remain steady?

7C **8** Teams in the NBA play 82 games of basketball in a regular season. The number of points scored per game for a certain team is displayed in the following histogram.



a Which interval contains the most values?

b In how many games did the team score at least 120 points?

c In how many games did the team score less than 80 points?

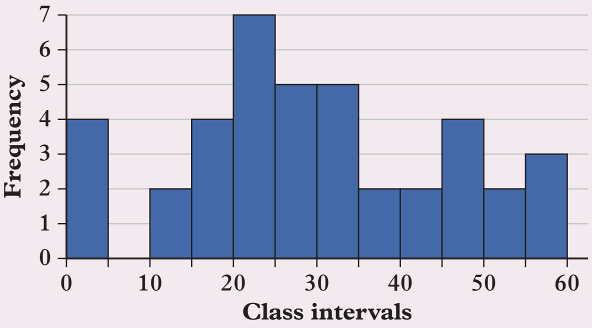
7C **9** A researcher asked 24 people how many minutes they spend in the shower on average. The results are shown below.

7	12	14	8	6	4	4	9	8	7	6	11
16	18	21	3	4	9	8	10	12	13	14	20

a Draw a frequency table with class intervals of 5 to represent this data.

b Use the frequency table to draw a histogram for this data.

7C **10** Redraw the following histogram with class intervals of 10, starting at 0.



7D Summary statistics from tables and displays

Learning intentions

By the end of this topic you will be able to ...

✔ calculate summary statistics from frequency tables

✔ calculate summary statistics from displays.

Inter-year links

Support	Understanding data
Year 7	10A Summary statistics
Year 8	9B Summary statistics
Year 10	9A Five-number summary and interquartile range

Measures of centre and spread

- The **mean**, **median** and **mode** are **measures of centre**, as they each represent a central point in a data set.

$$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

Median = middle value when data is listed in order (divides a data set in half)

Mode = most common value (there can be more than one mode)

- The **range** is a **measure of spread**, as it gives an indication of the spread of the data.

Range = maximum value – minimum value

For example: The numbers of humpback whales passing the Portland coast each week during a period of the migration season were counted. The results are shown below.

2, 13, 16, 28, 35, 35, 46, 29, 16, 24

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all data points}}{\text{total number of data points}}\\ &= \frac{2 + 13 + 16 + 28 + 35 + 35 + 46 + 29 + 16 + 24}{10}\\ &= \frac{244}{10}\\ &= 24.4\end{aligned}$$

List data in order: 2, 13, 16, 16, 24, 28, 29, 35, 35, 46

The number of data values is even, so the median is the mean of the two middle values.

$$\begin{aligned}\text{Median} &= \frac{24 + 28}{2}\\ &= 26\end{aligned}$$

The highest frequency is 2 for both values 16 and 35.

Modes = 16 and 35

$$\begin{aligned}\text{Range} &= 46 - 2\\ &= 44\end{aligned}$$

Calculating summary statistics from data displays

- To calculate the mean from a frequency table, insert a column multiplying each individual score by its frequency, and divide the total of this column by the total of the frequencies.
- To calculate the median from a frequency table, insert a column for the **cumulative frequency** (a running total of the frequencies) to help identify where the middle value lies.
- To calculate summary statistics from a display that details individual scores (e.g. stem-and-leaf plots, dot plots or bar charts) the data can be treated as either a raw list or as a frequency table.

Example 7D.1 Summary statistics from a frequency table

For the data shown in the following frequency table, find the:

a mean **b** median **c** mode **d** range.

Score (x)	**Frequency (f)**
1	4
2	7
3	8
4	8
5	2
6	1

THINK

a **1** Insert a 'Score × frequency' column and a 'Total' row and then complete the table. The total of the 'Frequency' column is the number of data values in the table.

2 To find the mean, divide the 'Score × frequency' total by the 'Frequency' total.

b **1** Insert a 'Cumulative frequency' column in the table. Check that the uppermost number matches the number of data values.

2 Divide the number of data values by 2 to find out how many values lie below the median. Then identify the row that contains the median score by using the cumulative frequency column.

c Identify the score with the highest frequency.

d Identify the minimum and maximum values in the score column of the data set. Subtract the minimum value from the maximum value.

WRITE

Score (x)	**Frequency (f)**	**Score × frequency ($x \times f$)**	**Cumulative frequency**
1	4	$1 \times 4 = 4$	4
2	7	$2 \times 7 = 14$	$4 + 7 = 11$
3	8	$3 \times 8 = 24$	$11 + 8 = 19$
4	8	$4 \times 8 = 32$	$19 + 8 = 27$
5	2	$5 \times 2 = 10$	$27 + 2 = 29$
6	1	$6 \times 1 = 6$	$29 + 1 = 30$
Total	**30**	**90**	

a Mean $= \frac{90}{30}$

$= 3$

b There are 30 values in the data set. $\frac{30}{2} = 15$ so 15 data values lie below the median.

The number of data values is even so the median is between the 15th and 16th value.

The 12th to 19th scores are 3, so both the 15th and 16th score are 3.

Median = 3

c The highest frequency is 8 for both scores 3 and 4.

Modes = 3, 4

d Range $= 6 - 1$

$= 5$

Example 7D.2 Finding summary statistics from a dot plot

For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.

THINK

a **1** Create a frequency table by counting the number of dots in each column.

2 Insert a 'Total' row, a 'Score × frequency' column and a 'Cumulative frequency' column in the table.

3 Find the mean by dividing the 'Score × frequency' total by the 'Frequency' total.

b Divide the number of values by 2 to find out how many values lie below the median.

c Find the mode by identifying the score with the highest frequency.

d Find the range by subtracting the minimum score from the maximum score.

WRITE

Score (x)	**Frequency (f)**	**Score × frequency ($x \times f$)**	**Cumulative frequency**
7	6	42	6
8	9	72	15
9	13	117	28
10	14	140	42
11	8	88	50
Total	**50**	**459**	

a $\text{Mean} = \frac{459}{50}$

$= 9.18$

b $\frac{50}{2} = 25$, so 25 data values lie below the mean.

The number of data values is even so the median is between the 25th and 26th score.

The 16th to 28th scores are 9, so both the 25th and 26th score are 9.

Median = 9

c The highest frequency is 14 for the score 10.

Mode = 10

d Range = 11 − 7

= 4

Example 7D.3 Finding summary statistics from a stem-and-leaf plot

For the data shown in the following stem-and-leaf plot, find the:

a mean

b median

c mode

d range.

Stem	**Leaf**
1	5 9
2	3 7 8 8 9
3	0 1 2 2 6 9 9
4	0 2 3 3 6 6 6 7 8
5	1 5

Key 2|3 = 23

THINK

1 Find the mean by dividing the sum of scores by the number of scores.

2 Divide the number of values by 2 to find out how many values lie below the median.

3 Find the mode by identifying the most common scores.

4 Find the range by calculating the difference between the minimum score and the maximum score.

WRITE

a $\text{Mean} = \dfrac{915}{25}$

$= 36.6$

b $\dfrac{25}{2} = 12.5$, so 12 data values lie below the mean.

The number of data values is odd so the median is the 13th score, which is 39.

Median = 39

c The highest frequency is 3 for the score 46.

Mode = 46

d Range = 55 − 15

= 40

Helpful hints

- ✔ When calculating summary statistics from a stem-and-leaf plot make sure that you use the full data values by combining the stem and leaf and using the key.
- ✔ Remember that if the median falls between two different values, the median will be the average (mean) of these values.
- ✔ A data set can have more than one mode.

ANS p466

Exercise 7D Summary statistics from tables and displays

△ 1(a, c, e), 2, 4, 5(a, b), 6(a, b), 9, 11(a, c–e), 13(a–c)

□ 1(b, d, f), 3, 5, 6(b, c), 7, 10, 13–15

⬠ 1(d, f), 3, 5–6(b, c), 10, 12–14, 16

UNDERSTANDING AND FLUENCY

7D.1 **1** For the data shown in each frequency table, find the **i** mean, **ii** median, **iii** mode and **iv** range. Give your answers to two decimal places where necessary.

a

Score (x)	Frequency (f)
1	6
2	7
3	5
4	3
5	1

b

Score (x)	Frequency (f)
10	8
20	6
30	8
40	2

c

Score (x)	Frequency (f)
13	3
14	4
15	8
16	11
17	12
18	4

d

Score (x)	Frequency (f)
0	11
1	13
2	6
3	3
4	1

e

Score (x)	Frequency (f)
15	29
20	41
25	58
30	72

f

Score (x)	Frequency (f)
1	6
2	11
3	9
4	4
5	3
19	1

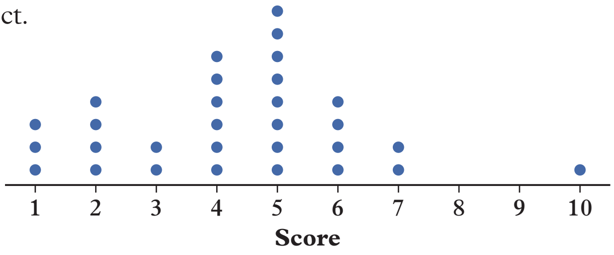
2 This dot plot shows the time in hours to complete a project.

a How many people were surveyed?

b State the mode and range.

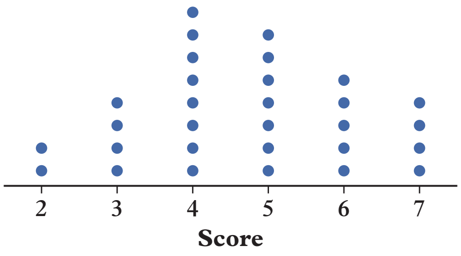
c Create a frequency table.

d Use the frequency table to calculate the median and mean of project completion time.

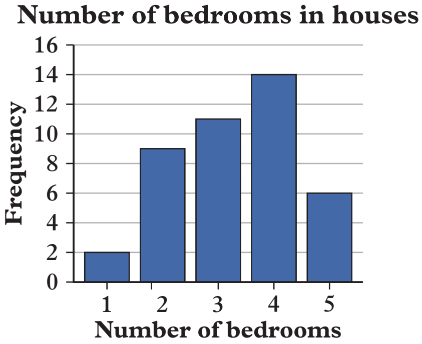


7D.2 **3** For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.



4 This column graph shows the number of bedrooms in a survey of houses.



a What is the most common number of bedrooms in the houses surveyed?

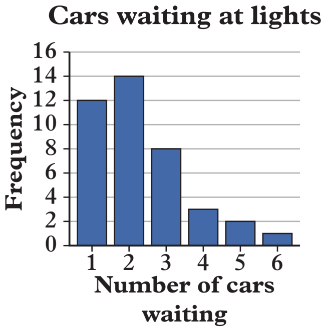
b What is the range of the number of bedrooms?

c Create a frequency table to represent the data shown in the column graph.

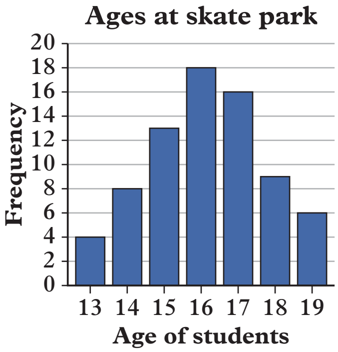
d Use the frequency table to calculate the median and the mean number of bedrooms. Give your answers to two decimal places where necessary.

5 Find the mean, median, mode and range for the data shown in each column graph. Give your answers to two decimal places where necessary.

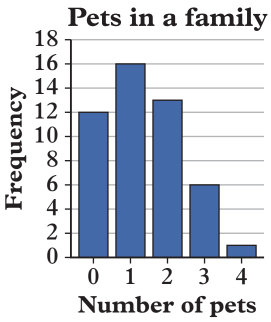
a



b



c



7D.3 6 Find the mean, median, mode and range for the data displayed in each stem-and-leaf plot. Give your answers to two decimal places where necessary.

a

Stem	Leaf
3	2 4 7
4	2 2 2 6 9
5	3 6 8 9 9
6	1 1 4 6 7 7 8 8
7	0 4

Key 3 | 2 = 32

b

Stem	Leaf
1	2 3 8 8 8 9
2	0 0 1 2 3 6 7 8
3	2 3 7 9
4	4 6
5	2

Key 1 | 3 = 1.3

c

Stem	Leaf
1	0 1 2 4 5 6 7 7 9
2	2 3 4 4 6 8 8
3	0 1 1 1 3
4	3 7 9
5	1
6	
7	7

Key 1 | 2 = 120

7 A number of people were surveyed on how many pairs of shoes they bought in a year. Use the results shown in the table to find the mean, median, mode and range. Give your answers to two decimal places where necessary.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	9	13	21	18	13	11	7	4	1	3

8 Explain why the only summary statistic that you can find for categorical data is the mode.

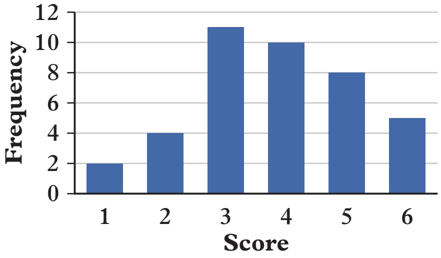
9 Match these summary statistics with these graphs.

a mean = 3.3, median = 3, mode = 3, range = 5

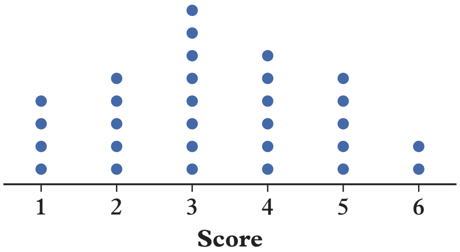
b mean = 3.825, median = 4, mode = 3, range = 5

c mean = 40.74, median = 36.5, mode = 74, range = 97

A



B



C

Stem	Leaf
0	1 2 6
1	2 6 7 7 8 9
2	0 1 1 2 4 6 7 7 8 9
3	0 3 4 4 5 6 7 9
4	0 1 2 2 5 9
5	0 1 2 3 5
6	0 1 4 8 9
7	4 4 4 4
8	1 9
9	8

Key 2 | 1 = 21

10 Draw a dot plot on a number line as shown below using a different data set consisting of exactly 9 numbers for each of the following scenarios. In each case, write a sentence explaining your reasoning for placing the dots in the positions you chose.

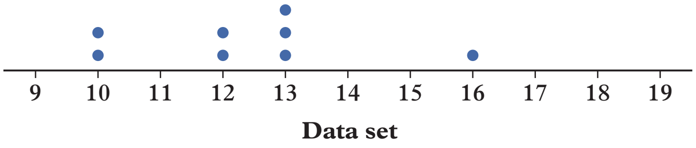
a The minimum value is 2 and the range is 7.

b The median and the maximum value are both 6.

c The mode includes four different numbers.

11 The dot plot on the right is missing one dot. Find a possible value of the missing dot for each of the following scenarios.

In each case, state whether your answer is unique or if it is one answer out of several that are possible.



a The maximum value is 17.

b The mean is 12.

c The median is 13.

d The modes are 10 and 13.

e The range is 7.

f The mean is a whole number.

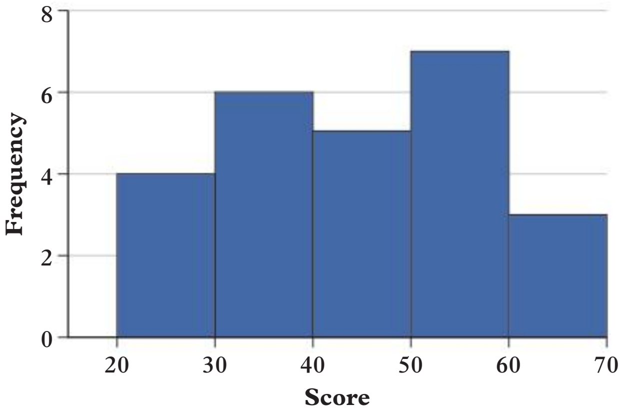
12 A large data set contains 457 data points that are listed in a spreadsheet. The numbers are ordered from smallest to largest and are listed in rows of 30. This means the number in column 1 of row 1 is the smallest number in the set, the number in column 1 of row 2 is the 31st smallest number in the set, and so on.

a Find the row number and column number of the greatest number in the data set.

b Find the row number and column number of the median of the data set.

PROBLEM SOLVING AND REASONING

13 A researcher has collected discrete data and arranged it into the following histogram. The data is discrete, so the first interval is 20–29, the second interval is 30–39, etc. The raw data (i.e. the list of numbers that make up the data set) is not available.



a What is the maximum possible value of the range? Explain your reasoning.

b What is the minimum possible value of the range? Explain your reasoning.

c Explain why it is not possible to find the median.

d In which interval is the median contained?

e Explain why it is not possible to calculate the mean or even to solve for the interval in which it is contained.

14 For the histogram given in question **13**, it is not possible to calculate the exact value of the mean. However, it is possible to find the minimum and maximum possible values of the mean.

a Find the minimum possible value of the mean by assuming every data point occurs at the lowest possible value of its interval.

b Find the maximum possible value of the mean by assuming every data point occurs at the highest possible value of its interval.

c Find the average value of the minimum possible mean and the maximum possible mean. Is this a good way to estimate the mean for data presented in a histogram? Explain your answer.

CHALLENGE

15 Elena has completed four Maths tests this year with an average score of 78%.

a What is the maximum value her average could increase to after completing the next test?

b What is the minimum value her average could drop to after completing the next test?

c What percentage score must she achieve on her next test to increase her average to 80%?

16 Another measure of spread is the standard deviation. It measures the average spread of each data value from the mean. A small standard deviation means that most values are close to the mean and a large standard deviation means that the values are spread far from the mean. Standard deviation is best found using a calculator with the appropriate function. It can also be found (for a sample) using the formula:

$$s = \sqrt{\frac{\Sigma(\overline{x} - x)^2}{n - 1}}$$

where s represents standard deviation, Σ means ‘the sum of’, $\overline{x}$ represents the mean, x represents an individual score and n represents the number of scores.

a Use your calculator or the standard deviation formula to find the standard deviation for each data set, correct to two decimal places.

i 4, 8, 2, 6, 4, 9, 7, 7, 4, 1, 2, 4, 6, 9, 7, 4, 6, 7, 8, 1, 2, 3, 1, 6, 2, 6

ii 22, 27, 35, 64, 12, 74, 37, 93, 27, 33, 11, 71, 64, 42, 81, 37, 13, 19, 88, 50

iii 103, 118, 109, 111, 117, 116, 117, 117, 105, 107, 116, 113, 108, 109, 112, 113

b Use your results to state whether each data set from part **a** has a large or small spread from the mean.

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Interactive skillsheet
Summary statistics from displays

Investigation
Estimating population means and medians

CAS Instructions
Measures of centre

Topic quiz
7D

7E Describing data

Learning intentions

By the end of this topic you will be able to ...

✔ identify whether a data set is symmetric, skewed or bi-modal

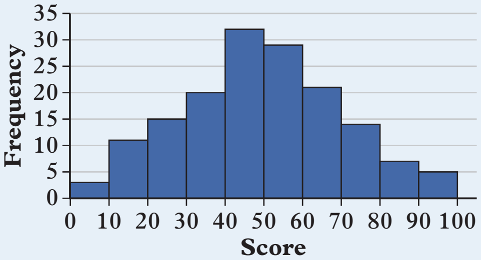
✔ determine which measure of centre best describes a data set.

Inter-year links

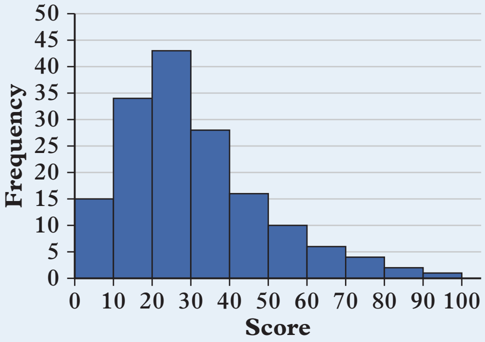
Year 7	10B Describing data with dot plots
Year 8	9C Analysing data
Year 10	9C Distributions of data

Describing data

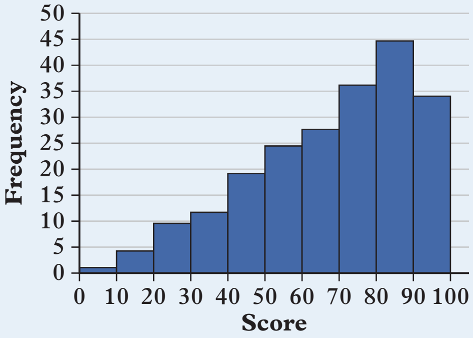
- By displaying numerical data, you can identify the shape of the distribution of the data and use this to describe data sets. **Symmetric** distributions have a peak in the middle and are roughly evenly spread on either side.



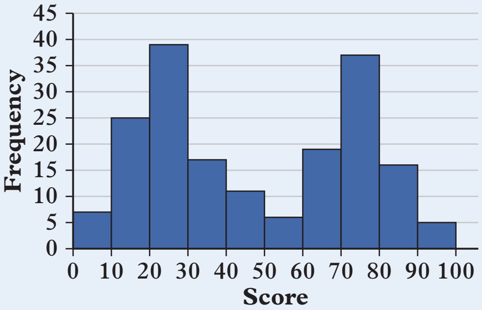
- A distribution is **skewed** when the data points are clustered on one side of the distribution. There is a tail on the other side of the distribution.
  - → **Positively skewed** distributions are skewed towards the vertical axis with a tail on the right of the distribution.



  - → **Negatively skewed** distributions are skewed away from the vertical axis with a tail on the left of the distribution.



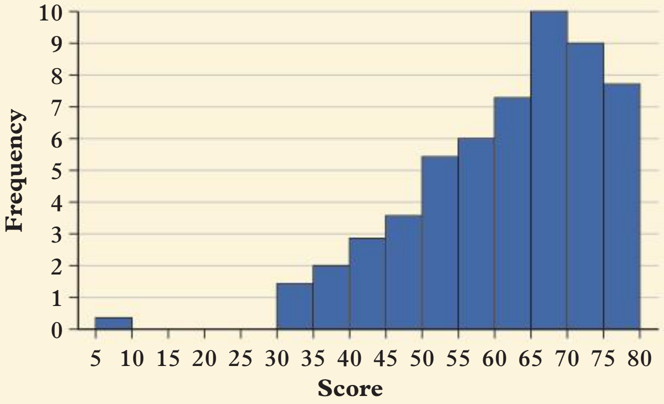
- A distribution is **bi-modal** if there are two clear peaks in the data.



- The mean, median and mode can all be used to represent a central point of a data set, but the shape of the distribution should be considered before deciding which measure of centre to use for this purpose.
- When describing symmetric distributions, either the mean or the median are appropriate representations of the centre. In perfect symmetric distributions, the mean, mode and median are equal.
- When describing skewed distributions, the median is a better representation of the centre than the mean, as the mean is skewed by the values in the tail.
- Bi-modal distributions are best described using the mode of each peak.
- An **outlier** is a data point that is significantly higher or lower than the other values in a data set.
  - → The presence of outliers can have a significant impact on the mean, so distributions with outliers are best described by using the median.
  - → Outliers can be determined using several different methods; in Year 10 you will be given a formula for determining whether a data point is an outlier. At this stage, only clear outliers will be considered.

Example 7E.1 Describing distributions

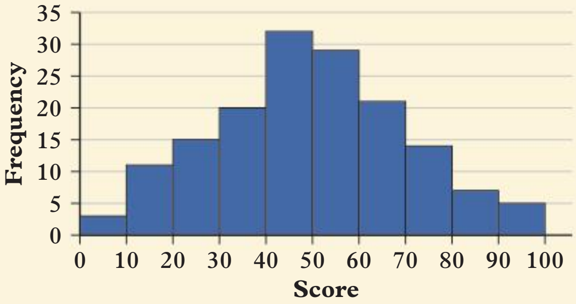
Describe the distribution of this histogram.



THINK	WRITE
1 Identify whether the distribution is symmetric, skewed or bi-modal. **2** Identify any outliers. **3** Write your answer.	The distribution is negatively skewed with an outlier between 5 and 10.

Example 7E.2 Deciding which measure of centre best describes a distribution

Decide which measure(s) of centre would best describe this distribution.



THINK	WRITE
1 Identify whether the distribution is symmetric, skewed or bi-modal. **2** Identify how the shape of the distribution determines which measure of centre should be used to describe the distribution. **3** Write your answer.	The distribution is symmetric so the centre of the distribution could be described by the mean or the median.

Example 7E.3 Describing a histogram

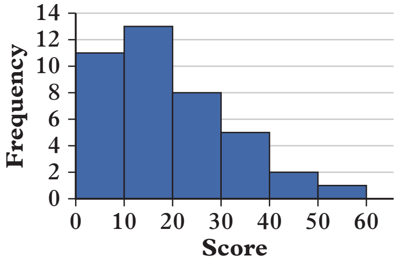
Use the data below to:

a draw a histogram with class intervals of width 10

b describe its distribution

c state the best measure of centre to describe the data, providing a reason.

49, 3, 16, 12, 20, 49, 22, 37, 32, 18, 34, 13, 4, 7, 17, 9, 13, 59, 25, 1, 15, 30, 23, 27, 4, 17, 26, 3, 10, 5, 8, 2, 31, 11, 8, 20, 27, 13, 16, 11

THINK	WRITE
a Use a frequency table to draw a histogram with an even scale and labels on both axes.	**a** 
b Decide whether the data is skewed.	**b** The distribution is positively skewed.
c Identify the best measure of centre to describe the data. The data is positively skewed, not symmetric or bi-modal, so the median should be used.	**c** The median should be used to describe the centre, because the distribution is skewed, which affects the value of the mean.

Helpful hints

✔ The peaks in a bi-modal distribution do not need to be of equal height.

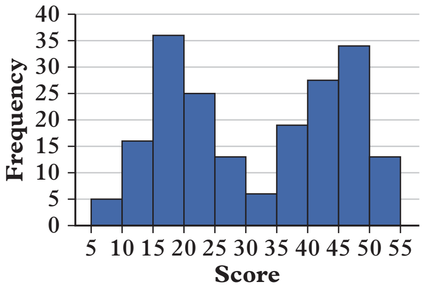
ANS p467

Exercise 7E Describing data

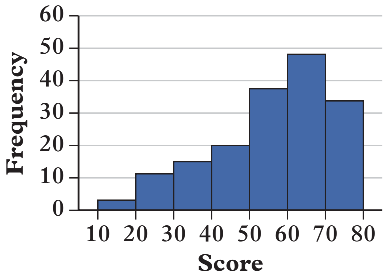
△ 1–3, 5(a, b), 6, 8, 9 □ 1, 2, 4, 5(b, c), 6, 9–11, 12(a, b) ⬠ 2, 3, 5(b, c), 7, 9, 11–13

7E.1 **1** Describe the distribution of each histogram.

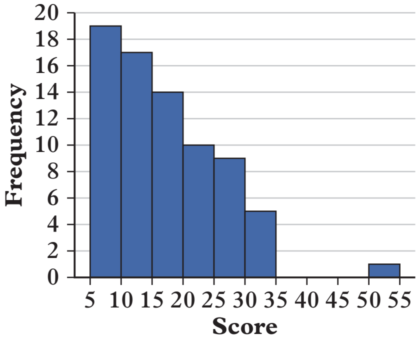
a



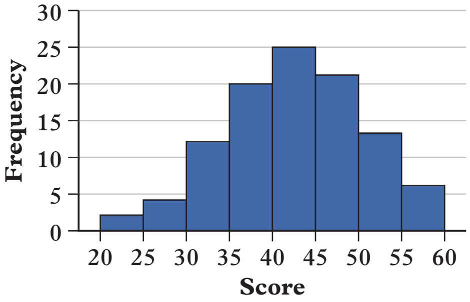
b



c



d



2 Describe the distribution of each stem-and-leaf plot.

a

Stem	Leaf
1	0 2
2	1 1 4
3	0 4 5 6
4	4 5
5	3

Key 1 | 2 = 12

b

Stem	Leaf
0	1 4 6 8
1	3 4 5 5 6
2	1 2 4 4
3	4 8 9
4	3
5	1

Key 1 | 3 = 13

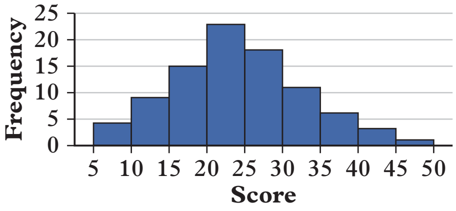
c

Stem	Leaf
0	9
1	
2	
3	1 9
4	4 5 6
5	4 5 5 6
6	0 1 1 2 4

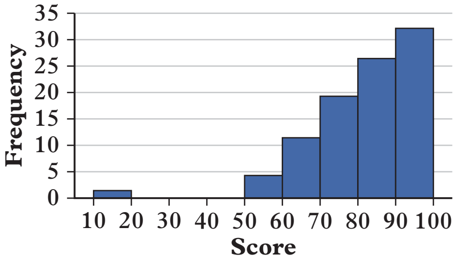
Key 3 | 9 = 39

7E.2 **3** Decide which measure(s) of centre would best describe each distribution.

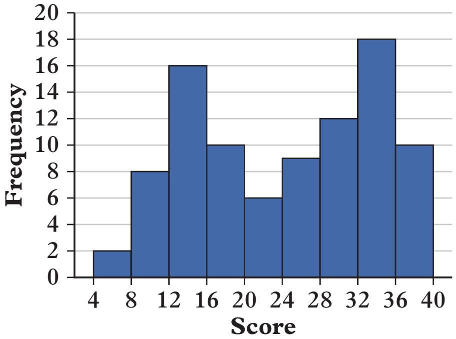
a



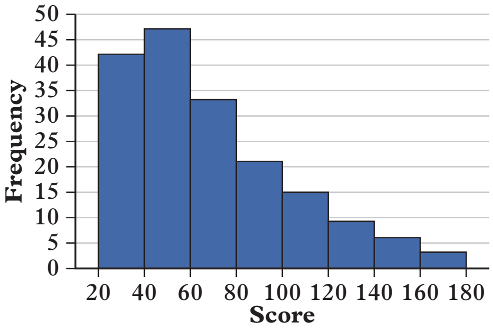
b



c



d



4 Decide which measure(s) of centre would best describe each distribution in question **1**.

7E.3 **5** For each data set:

- **i** draw a histogram with the given class interval width
- **ii** describe its distribution
- **iii** state the best measure of centre to describe the data, providing a reason.

a 35, 33, 42, 99, 54, 68, 4, 91, 97, 55, 99, 86, 40, 58, 41, 95, 38, 62, 35, 88, 82, 98, 77, 69, 78, 78, 82, 81, 98, 57, 88, 41, 60, 85, 82, 85, 91, 90, 80, 49, 58, 66, 97, 95, 82, 84, 78, 91, 62, 42 [width = 10]

b 9, 2, 3, 9, 43, 8, 2, 15, 12, 17, 10, 10, 14, 9, 34, 7, 12, 18, 18, 47, 2, 12, 24, 34, 19, 1, 12, 18, 35, 47, 6, 14, 8, 35, 7, 9, 4, 17, 2, 20, 8, 12, 21, 24, 48, 6, 7, 8, 17, 41 [width = 5]

c 49, 23, 45, 23, 31, 77, 62, 21, 52, 51, 60, 54, 46, 69, 27, 60, 142, 41, 32, 80, 52, 21, 80, 65, 37, 33, 74, 45, 48, 78, 70, 21, 55, 64, 33, 42, 59, 67, 32, 79, 30 [width = 10]

6 This stem-and-leaf plot does not seem to have a noticeable pattern. However, it is possible to redraw this stem-and-leaf plot with more stems, just like histograms can be redrawn with different class sizes. For example, the number 1 can be the stem for two leaves, with the numbers 10–<15 listed in the first stem labelled 1 and the numbers 15–<20 listed in the second stem labelled 1*.

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 2 3 3 5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3 9

Key 1 | 3 = 13

a Redraw it as a split stem-and-leaf plot with:

i four stems

ii ten stems.

b Describe the distributions that you see from the split stem-and-leaf plots.

7 The following tables show the data that Ava collected on the weight of dogs (in kilograms) in her community.

Class	0–<2	2–<4	4–<6	6–<8	8–<10	10–<12	12–<14	14–<16	16–<18	18–<20
Frequency	1	6	3	5	6	5	8	6	7	3

Class	20–<22	22–<24	24–<26	26–<28	28–<30	30–<32	32–<34	34–<36	36–<38	38–<40
Frequency	7	2	5	3	4	5	4	2	1	4

a Draw a histogram to represent the data.

b What pattern can you see?

c Reorder the data into class intervals of width 4 and redraw your histogram.

d What pattern can you now see?

e Write a sentence describing the weight distribution of dogs as shown in your second histogram.

8 The histogram on the right shows the number of people at a skate park recorded at various times.

a Describe the shape of the distribution.

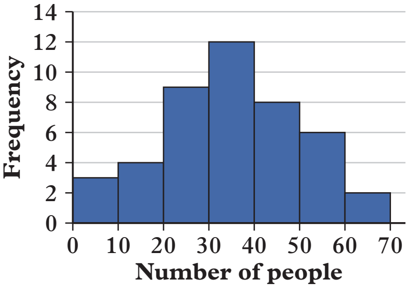
b What measure(s) of centre would be the most appropriate to use?

c Create a frequency table that represents the histogram.

d Use the frequency table to determine in which class interval the median lies.

e Write a sentence that describes the distribution in terms of the number of people at the skate park.

Number of people at a skate park



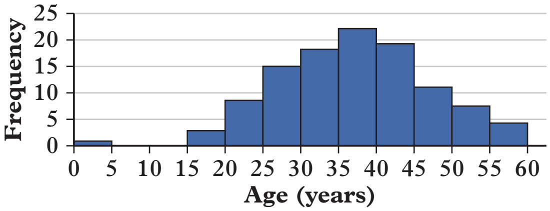
9 For each data distribution:

i describe its shape

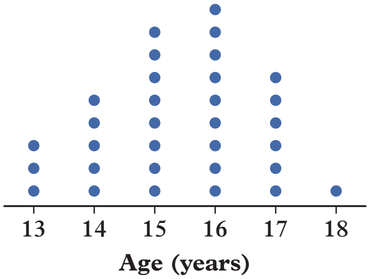
ii calculate the appropriate measure(s) of centre

iii write a sentence that summarises the graph in its context.

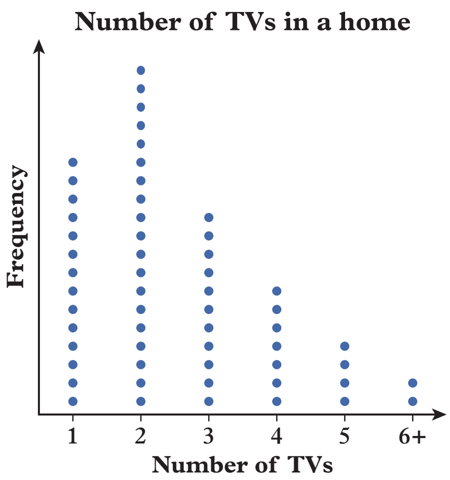
a **Ages of people at a hairdresser in a week**



b **Ages of students at a pool**



c

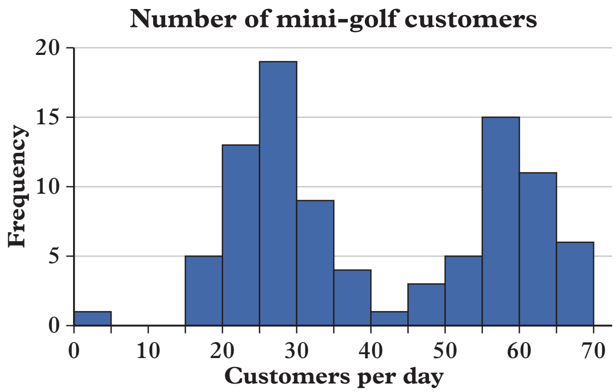


d **Length of hair in Year 9 students (cm)**

Stem	Leaf
0	1 2 2 2 3 3 3 4 4 6 6 6 6 7 8 8 8 9
1	0 2 4 6 8
2	0 1 5 8 8 9 9
3	0 1 2 2 2 2 2 4 5 5 5 5 6 7 8 9 9
4	1 2 2 4 4 5 6 6 8 9
5	0 4 5 6 8 8 8
6	2 9

Key 1 | 2 = 12

10 A mini-golf course is open every day of the year except Christmas day. The number of customers who visited the course each day in the months of October, November and December has been placed in the frequency table on the right. The histogram for this data is also shown.



Class	Frequency
0–<5	1
5–<10	0
10–<15	0
15–<20	5
20–<25	13
25–<30	19
30–<35	9
35–<40	4
40–<45	1
45–<50	3
50–<55	5
55–<60	15
60–<65	11
65–<70	6

a Suggest a reason why the class interval 0–<5 has a frequency of 1.

b Suggest a reason why the number of daily customers at the mini-golf course has a bi-modal distribution.

c Give examples of other businesses that will most likely have a bi-modal distribution for the number of daily customers.

11 A boat tour company offers ocean cruises where customers have a chance to see different wildlife, including orcas (killer whales). The following data set is the number of orcas that were spotted during tours for each day of February.

0	0	0	0	10	0	0	8	0	0	0	26	0	0
0	13	0	41	0	0	7	7	0	0	0	0	0	0

The whale watching company decides to run an advertisement during March on social media with the claim "Last month we saw an average of 4 killer whales per tour!"

a Explain why this claim, while technically true, is misleading.

b Why would the company not want to advertise the median number of whales they saw per tour?

c Create a histogram with class intervals of size 10 to display the data.

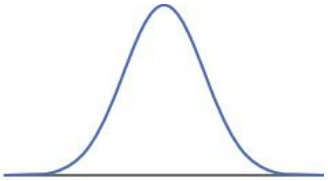
d Explain why the class interval 0–9 is misleading in the histogram.

e The company decides to change the advertisement to be more honest. The new advertisement will instead highlight the percentage of tours on which killer whales were spotted. Complete the sentence below to help the company write their new advertisement.

"Last month we saw killer whales on ___% of tours!"

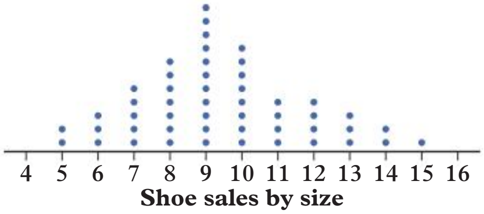
CHALLENGE

12 A common distribution that arises from many sets of real-life data is called the normal distribution or bell curve. A perfect normal distribution has many nice features, including that it is symmetric and that its mean, median and mode are all equal. When a real-life data set follows this pattern closely, the data is described as ‘approximately normally distributed’.



An example of this phenomenon is the number of sales of shoes by size, with the most common sales for adult males being of size 9 and the number of sales decreasing for shoe sizes on either side of this centre.

The dot plot below shows one day’s worth of sales by size in a shoe store that sells adult male shoes.



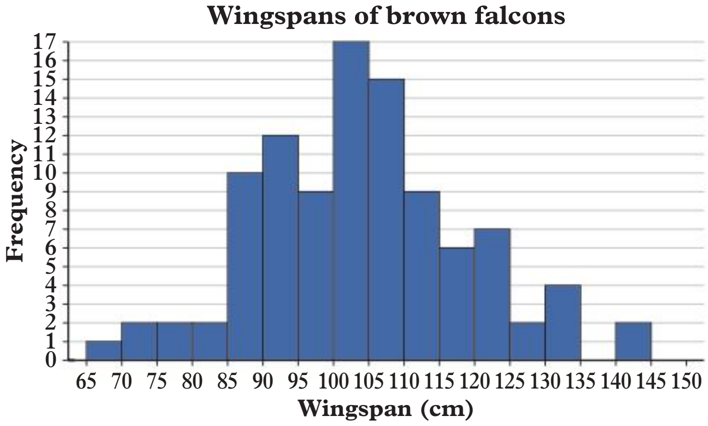
a Find the mean, median and mode of this data set. Round the value of the mean to one decimal place.

b Explain why, of the three values you found in part **a**, the mode is the most relevant statistic for the owner of the shoe store.

c What percentage of shoe sales were of sizes between 7 and 11, including 7 and 11?

d What percentage of shoe sales were of sizes between 5 and 13, including 5 and 13?

13 The bell-curve pattern can also be seen in histograms of continuous data. Approximately normally distributed data appears often in nature, from the heights and weights of most species of animals to the maximum daily temperatures in a city.



The histogram above was created from data collected on the wingspans of 100 adult brown falcons and placed in class intervals of size 5. The brown falcon is a bird species native to Australia and New Guinea, which is known to have a mean wingspan of approximately 105 cm.

a Redraw this histogram with class intervals of size 10, starting at 60.

b Redraw this histogram with class intervals of size 15, starting at 60.

c Which class interval size produces the histogram that most clearly shows the bell shape? Explain your answer.

d What percentage of the studied brown falcons had a wingspan between 90 cm and 120 cm?

e What percentage of the studied brown falcons had a wingspan between 75 cm and 135 cm?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Describing data

Topic quiz
7E

7F Comparing data

Learning intentions

By the end of this topic you will be able to ...

- ✔ compare data sets in different graphical displays
- ✔ compare data sets using summary statistics
- ✔ construct back-to-back stem-and-leaf plots.

Inter-year links

Year 7	10C Stem-and-leaf plots
Year 8	9C Analysing data

Comparing data sets

- Data sets are easier to compare when displayed in the same format; for example, when two or more data sets are displayed in histograms.
- By placing the data in graphical displays, the range and shape of the distributions can be compared.
- To make a more thorough comparison of two or more data sets, summary statistics should be calculated so the centre and spread of each data set can be compared.

Back-to-back stem-and-leaf plots

- **Back-to-back stem-and-leaf plots** share the same stems with one data set on the right of the stems and the other data set on the left. For example, these two ordered stem-and-leaf plots show the ages of people swimming in two different pools.

Stem	Leaf Pool A
0	2 7 9
1	1 4 5 6 7 9
2	0 1 1 5 6 8 8
3	1 2 2 4 9 9 9
4	2 4 5 7
5	2 3
6	
7	9

Key: 1 | 4 = 14

Stem	Leaf Pool B
0	
1	7 7 7 7 8 9
2	2 3 4 4 6 7 8 9
3	2 2 4 5 5 7 9
4	2 2 6 7
5	1 9
6	2
7	

Key: 1 | 4 = 14

- The two stem and leaf plots can be combined into this back-to-back stem-and-leaf plot.

Leaf Pool B	Stem	Leaf Pool A
	0	2 7 9
9 8 7 7 7 7	1	1 4 5 6 7 9
9 8 7 6 4 4 3 2	2	0 1 1 5 6 8 8
9 7 5 5 4 2 2	3	1 2 2 4 9 9 9
7 6 2 2	4	2 4 5 7
9 1	5	2 3
2	6	
	7	9

Key: 1 | 4 = 14

Example 7F.1 Drawing a back-to-back stem-and-leaf plot to compare data sets

Draw an ordered back-to-back stem-and-leaf plot and make a brief comparison of the two data sets detailing the ages of people at a pool.

Winter: 45, 23, 15, 36, 57, 31, 9, 38, 44, 56, 52, 13, 36, 27, 48, 44, 48, 14, 27, 45

Summer: 31, 16, 14, 15, 23, 56, 24, 18, 17, 8, 11, 13, 16, 21, 17, 36, 20, 17, 14, 15

THINK

1 Identify the minimum and maximum numbers for both data sets.

2 The youngest is 8 and the oldest is 57, so use six stems: 0, 1, 2, 3, 4, 5

3 Draw the back-to-back stem-and-leaf plot with these stems, and place the winter leaves on the left and the summer leaves on the right. Write the leaves in order and include a key.

4 Look at the centre and spread of the two data sets. Where does the centre appear to be for each set? What does this indicate about the ages at different times of year?

WRITE

Leaf Winter	Stem	Leaf Summer
9	0	8
5 4 3	1	1 3 4 4 5 5 6 6 7 7 7 8
7 7 3	2	0 1 3 4
8 6 6 1	3	1 6
8 8 5 5 4 4	4	
7 6 2	5	6

Key: 1|4 = 14

The two data sets have a similar range of age brackets, but they are skewed in different directions and have different measures of centre. The data suggests that the pool attracts younger people in summer and older people in winter.

Example 7F.2 Comparing data sets using summary statistics

Rachel investigated the average age of customers in two different cafes in her local area. Use her results to compare the ages of the customers in cafes.

Cafe	Mean	Median	Range
Gumtree	21	22	17
Jinx	27	21	45

THINK

1 Compare the given measure of spread (range).

2 Compare the given measures of centre (mean and median). The medians are similar, but Jinx has a higher mean than Gumtree. This difference combined with Jinx's large range suggests that Jinx's distribution may be skewed or affected by an outlier.

3 Summarise your findings.

WRITE

Jinx (45) covers a much wider range of ages than Gumtree (17).

The average ages are similar but Jinx (27) has a slightly higher mean than Gumtree (21). Jinx may have a positively skewed distribution or a data value for an older customer that is an outlier.

The mean customer age at the cafes is similar but Jinx has a larger range of ages.

Helpful hints

- ✔ When dealing with information from samples, the conclusions you draw will not be certain, so use words to show this, such as, 'The data indicates that …' or 'It appears that …'
- ✔ If you want to compare data sets but the data you're given is not in the same format, put the data sets in the same format before you begin the comparison.

ANS
p469

Exercise 7F Comparing data

△ 1, 2(a, b), 3(a, b), 4(a–c), 5, 6, 8, 9, 13

■ 2(c, d), 3(b, c), 4(c, e), 6, 10, 11, 14, 16

⬟ 3(b, c), 4(c, e), 7, 11, 12, 14, 15, 17

1 Use this back-to-back stem-and-leaf plot to answer these questions.

a What is the maximum score:

i in group A? **ii** in group B? **iii** overall?

Leaf	Stem	Leaf
Group A		Group B
9 7 4 3 1	0	7 9
9 8 8 6 4 3 2 1 0	1	3 4 6 8
7 5 3 0	2	0 1 1 4 4 4 5 8 9
8 3	3	2 2 3 4 7 9 9
1	4	0 3 5

Key: 1|3 = 13

b What is the most common score:

i in group A? **ii** in group B? **iii** overall?

c How would you describe the distribution of:

i group A? **ii** group B?

2 Make a brief comparison of the two data sets in each back-to-back stem-and-leaf plot.

a

Leaf	Stem	Leaf
Group A		Group B
1	1	0 1 3 5 8 8
8 7 2	2	2 4 5 6 7 8 9 9
9 7 6 3 1	3	4 6 7 8
9 8 7 7 6 5 3 1 1	4	0 1 2
9 7 6 4 3 1 0	5	1 3

Key: 1|9 = 19

b

Leaf	Stem	Leaf
Group A		Group B
	3	0 0 1 2 4 5 6
9 8 7 6 5 4 4 4	4	1 2 5 5 6 7 8 9 9
8 7 6 6 5 4 3 1 1 0	5	0 3 4 5 5 6 8
8 7 6 5 4 3 3 0	6	
	7	
	8	1

Key: 3|2 = 3.2

c

Leaf	Stem	Leaf
Group A		Group B
	0	2 3 4
9 7 6 5 5 5	1	1 7 8 9 9
9 8 7 7 6 5 4 4 3	2	0 2 3 5 5 7 9
9 7 6 5 4 4 4 3	3	0 4 6 7 8 8
3 2 2 1	4	0 1 5 6
	5	0 6

Key: 1|7 = 170

d

Leaf	Stem	Leaf
Group A		Group B
	5	1
9 8 7 1	6	
7 6 5 4 3 3 3 2 1	7	8 9 9
9 8 7 5 4 2 2 1 0 0	8	1 2 3 4 6 7 8 8
	9	0 0 5 6 7 8 8 8 9

Key: 5|1 = 51

UNDERSTANDING AND FLUENCY

7F.1 **3** For each of the following draw a back-to-back stem-and-leaf plot and use it to make a brief comparison of the two data sets.

a Number of people at a cinema over 6 months of weekends:

Friday: 65, 48, 67, 55, 32, 92, 64, 51, 49, 57, 76, 61, 29, 46, 61, 59, 53, 67, 72, 88, 71, 58, 54, 57, 56, 42, 30

Saturday: 67, 78, 84, 26, 37, 42, 99, 84, 75, 68, 64, 55, 75, 85, 59, 66, 77, 78, 83, 81, 92, 94, 77, 76, 79, 89

b Daily maximum temperature in February:

Darwin: 32.3, 32.1, 32.9, 33.2, 33.5, 33.8, 33.4, 32.8, 32.5, 33.3, 32.5, 29.5, 32.8, 33.4, 33.1, 32.7, 33.4, 31.4, 33.0, 31.3, 29.9, 30.8, 30.0, 32.2, 32.1, 32.6, 31.8, 30.4

Adelaide: 20.6, 23.7, 25.4, 29.4, 34.2, 38.4, 27.0, 28.3, 28.0, 26.3, 28.3, 31.2, 33.8, 34.3, 36.0, 37.1, 39.2, 40.5, 26.6, 28.4, 32.1, 33.8, 35.6, 38.2, 28.7, 33.4, 21.5, 22.7

c Mass of dogs of two breeds (to nearest hundred grams):

Boston Terriers: 5100, 6800, 7800, 10 500, 9200, 5500, 4900, 11 200, 8600, 9200, 10 500, 4900, 5500, 7600, 8400, 6800, 9200, 8600, 7500, 4600, 8300, 10 500, 11 000, 8000, 7600, 5100, 6700, 8300, 9200, 10 000, 4900, 7500, 7900, 6200, 8200, 4600, 7800, 8100, 6400, 9900

French Bulldogs: 9800, 12 900, 8600, 9900, 9800, 12 500, 11 200, 9100, 12 900, 11 200, 9500, 9200, 10 500, 11 900, 12 500, 11 800, 9400, 9800, 10 800, 10 600, 11 500, 9600, 12 300, 13 000, 11 000, 12 000, 10 600, 11 100, 12 700, 10 900, 9500, 11 500, 12 900, 10 100, 11 100, 9200, 11 500, 10 900, 9800, 11 300

7F.2 **4** For each table, use the given statistics to make a comparison of the two data sets.

a

Heights in class	**Mean**	**Median**	**Range**
9A	172 cm	168 cm	46 cm
9B	166 cm	167 cm	22 cm

b

Jeans price	**Mean**	**Median**	**Range**
Store A	\$120	\$122	\$35
Store B	\$124	\$122	\$30

c

Internet usage per week	**Mean**	**Median**	**Range**
Primary school students	4.5 hours	4 hours	8 hours
Secondary school students	11.2 hours	12 hours	17 hours

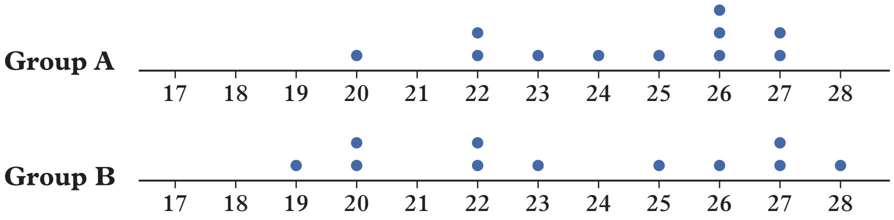
d

SD card capacity	**Mean**	**Median**	**Range**
Store A	20 GB	16 GB	60 GB
Store B	16 GB	16 GB	60 GB

e

Goals per game	**Mean**	**Median**	**Range**
Player A	4.2	4	9
Player B	4.8	4	7

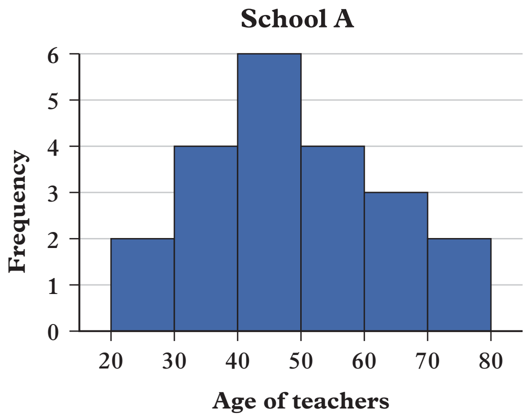
5 Consider the following two dot plots.

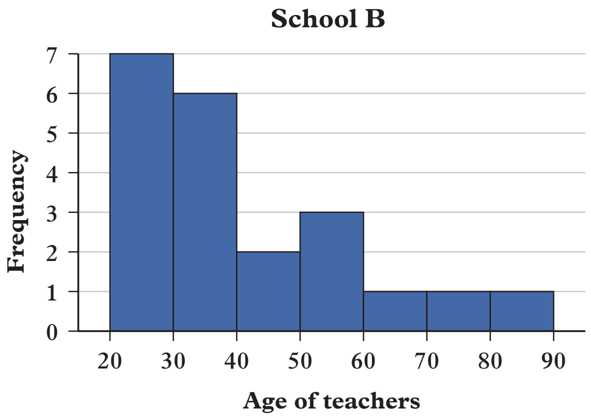


a Which group has the greater range? Justify your answer with calculations.

b Which group has the greater median? Justify your answer.

6 Consider the following two histograms that show the ages of teachers from two different schools.





a Which school has the oldest teacher?

b Which school has the most teachers below the age of 40?

c Make a brief comparison of the two data sets.

7 Consider this back-to-back stem-and-leaf plot detailing the heights of Year 7 and Year 9 students (cm). Note the order of the data in the leaves of this stem-and-leaf plot.

a Make a brief comparison of the two data sets, writing your answer in context.

b Calculate the mean (to two decimal places), median and range for both data sets displayed in the stem-and-leaf plot.

c Use these statistics to make a more detailed comparison of the two data sets.

Leaf **Year 7**	**Stem**	**Leaf** **Year 9**
8	14	
6 7 9 9	15	9
0 0 1 1 2 2 2 3 4 6 7 8	16	9 9 8 7 5 4 3 1
0 1 2 3 4	17	9 8 8 7 6 5 5 4 3 3 3 1
2	18	7 3 2 1
	19	2

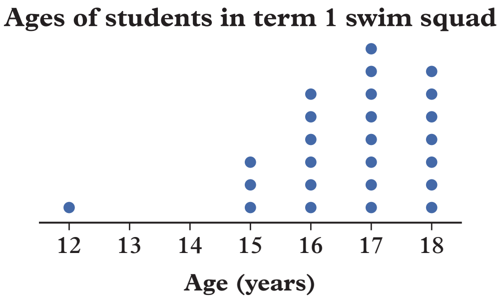
Key: 14|8 = 148

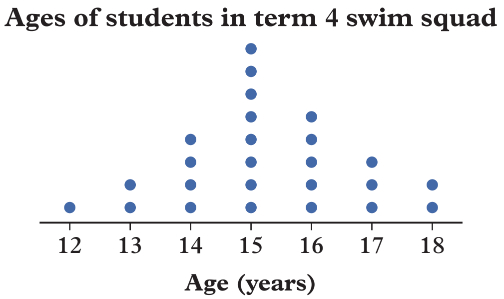
d Does your answer to part **c** support your answer to part **a**? Explain.

8 **a** Compare the shape of the two dot plots below.

b Find the mean, median and range of each dot plot.

c Write a couple of sentences comparing the two data sets with reference to their summary statistics. Be sure to explain the meaning of the range and centre in terms of the context of the data sets.





9 A class of Year 8 students and a class of Year 9 students were each asked to calculate the average number of minutes they spend studying per school night. The data is shown in the stem-and-leaf plots on the right.

- **a** Redraw these separate stem-and-leaf plots as one back-to-back stem-and-leaf plot with the Year 8 group on the left and the Year 9 group on the right.
- **b** Calculate the range of each group.
- **c** Calculate the median of each group.
- **d** Is it accurate to say that, in general, the Year 9 group studies more than the Year 8 group? Justify your answer.

Year 8

Stem	Leaf
0	0 5
1	4
2	3 5 7 7
3	0 0 2 3 8
4	1 4 4 9
5	5 8 9
6	0 5
7	1 4
8	3 6

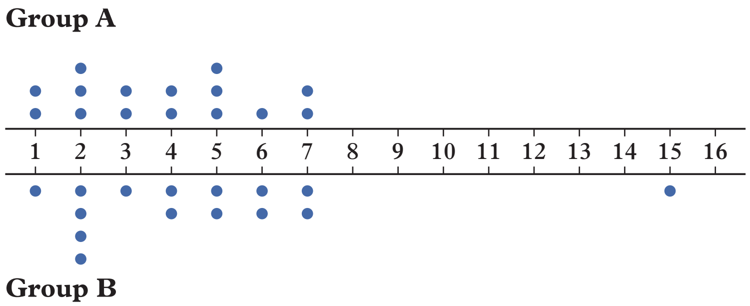
Key: 4 | 1 = 41

Year 9

Stem	Leaf
1	8 9
2	0 6
3	5 7
4	5 8 9
5	1 2 4 5
6	0 5
7	1 4 6 7 7
8	3 6
9	0 2 7

Key: 5 | 2 = 52

10 Two groups of 15 people were asked how many times they go to the hairdressers each year. Their responses are recorded in the back-to-back dot plot below. The data for Group A is above the number line and the data for Group B is below the number line.



- **a** Find the median of each group.
- **b** Complete the table below. Round the mean of each data set to 1 decimal place.

	Mean	Range
A		
B		

- **c** Explain why the summary statistics in part **b** may be misleading to a person who has not seen the back-to-back dot plot.

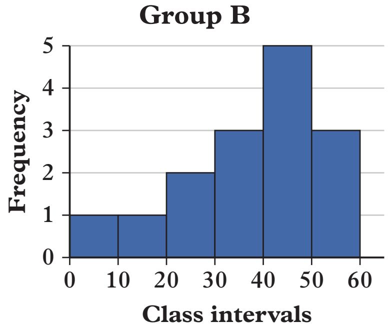
11 Consider the following stem-and-leaf plot and histogram.

- **a** Explain why you cannot redraw the histogram as a stem-and-leaf plot.
- **b** Redraw the Group A data in the stem-and-leaf plot as a histogram with class intervals of 10 starting at 0.
- **c** Use the two histograms to make a brief comparison of the two data sets.

Group A

Stem	Leaf
0	3 8 9
1	2 4 6 7
2	3 7 7
3	1 5
4	6 7
5	1

Key: 3 | 5 = 35

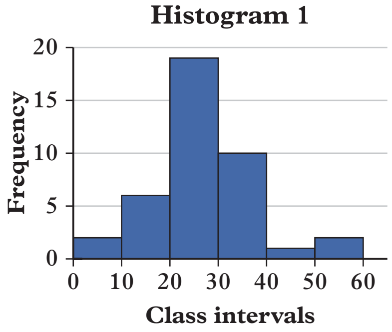


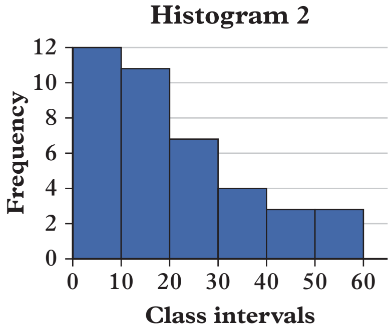
12 Consider the following summary statistics for two different data sets.

These data sets have been drawn as histograms, but they were mistakenly drawn without labels.

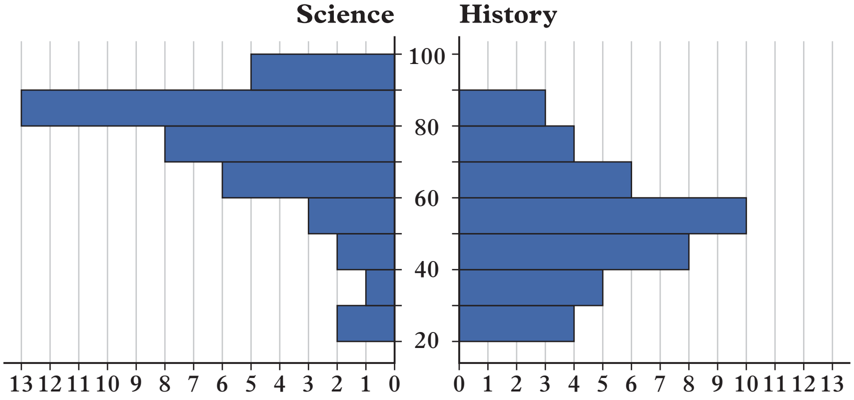
Which data set matches with which histogram? Explain your answer.

	Mean	Median	Range
A	21	17	57
B	27	25	56





13 Two sets of grouped data can be displayed in one back-to-back histogram. The following back-to-back histogram shows the exam scores (out of 100) for the same 40 students in two different exams: Science and History.

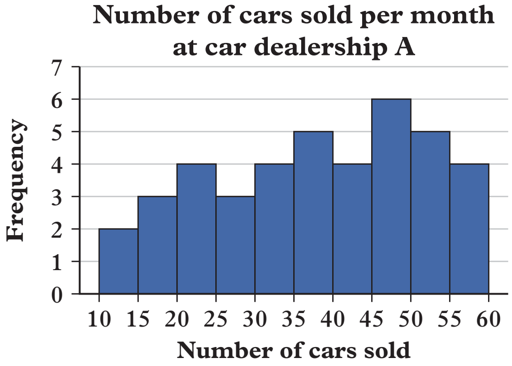


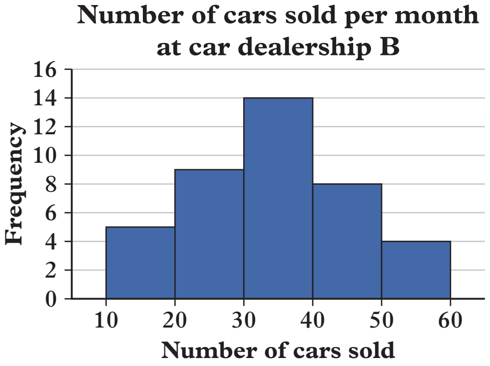
a Describe the distribution of the Science exam scores.

b Describe the distribution of the History exam scores.

c How many exam scores overall were in the interval 80–<90?

14 Consider these two histograms.





a Why is it difficult to make a quick comparison of the two histograms as presented?

b Redraw the first histogram so that its class intervals match those of the second.

c Use your answer to part **b** to make a comparison of the two data sets.

d Why can't you redraw the second histogram to match the first histogram?

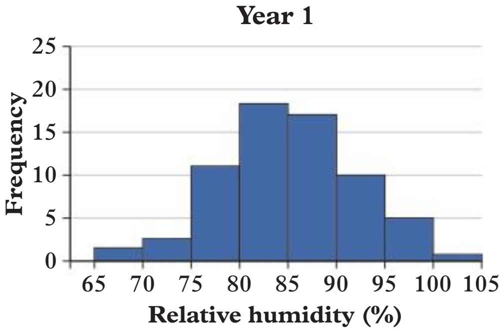
PROBLEM SOLVING AND REASONING

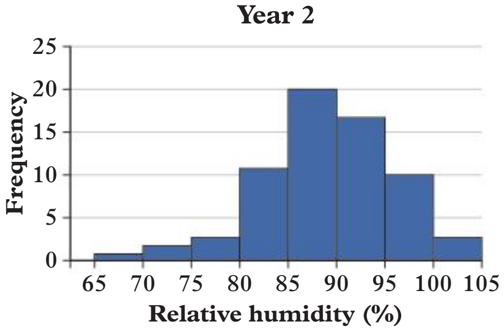
15 Relative humidity is an indicator of how much water vapor is in the air. As part of his research on climate change, Patrick recorded the relative humidity of this hometown every day for three consecutive years. The data he collected has been drawn as histograms shown on the right and below.

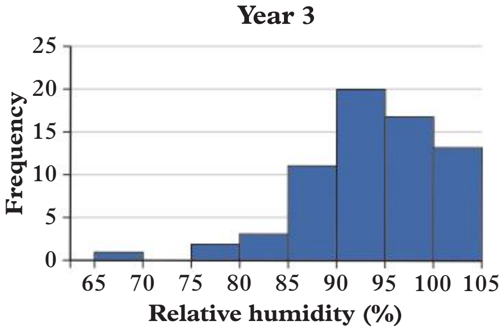
a What is the most common class interval in Year 1, Year 2, and Year 3?

b Which year has the greater median?

c What would you expect the histogram to look like in Year 4, assuming Patrick continues his research?







CHALLENGE

16 A study on the gaming habits of Year 9 students focused on three separate classes, each with exactly 25 students. Each student was asked how many hours they spent per week playing video games. The study found the following results.

	Mean	Median	Range
Class A	5.5	5	12.5
Class B	7	6	28
Class C	6.5	8.5	14.5

a One class had an outlier. Which class had the outlier and how can you tell?

b The median for class C is much higher than the mean. Does this mean the data for class C is positively skewed or negatively skewed? Explain your answer.

c In general, which class plays video games the most? Justify your answer.

17 In a school there are 250 students in Year 8 and 200 students in Year 9. The following statistics have been calculated about the students' heights (in cm).

	Mean	Median	Minimum	Maximum
Year 8	160	161	142	173
Year 9	166	164	144	179

a Find the range for the heights of each year level.

b Find the range for the heights of all students in two year levels combined.

c Find the average height of all students in the two year levels combined, rounded to 1 decimal place. Hint: The answer is not 163 cm.

d Explain why it is not possible to find the median height of all students in the two year levels combined.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Comparing data

Investigation
Closing the Gap

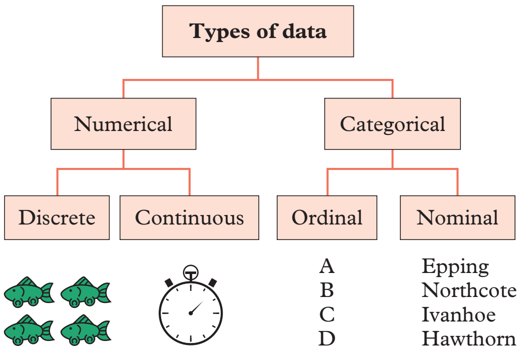
Topic quiz
7F

Chapter summary

Sampling

- Stratified sampling divides the population into strata and takes a random sample from each strata.
- Systematic sampling selects data at fixed intervals.
- Convenience sampling is when a sample that is easily accessible is selected.
- Quota sampling is when the researcher selects the size of each group to sample.

Classifying data



- Bivariate data contains two variables.

Grouping data

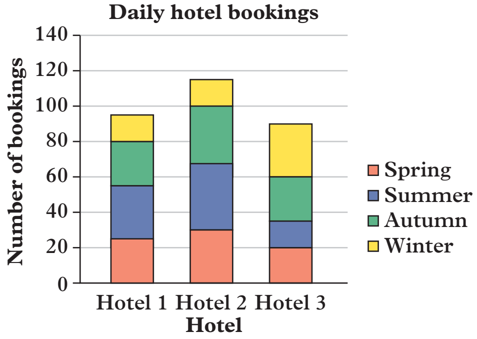
- Frequency tables: 5–10 class intervals
- Discrete data: 0–9
- Continuous data: 0–<10

Histograms

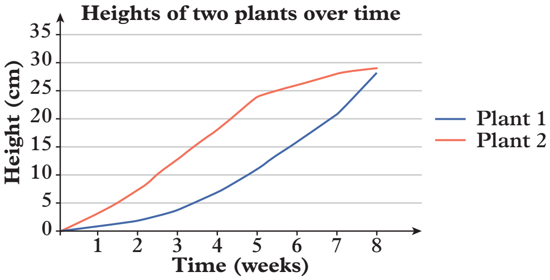
- Lower bound on left of each column
- For continuous data: upper bound on right of each column
- For discrete data: lower bound of next class interval on right of each column

Displaying data

- Bar charts: numerical variable or frequency (vertical axis), categorical data (horizontal axis)
- Stacked bar charts: second categorical variable in legend



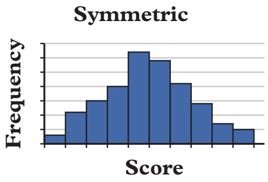
- Line graphs: two numerical variables
- Multiple line graphs/area charts: catagorical variable in legend

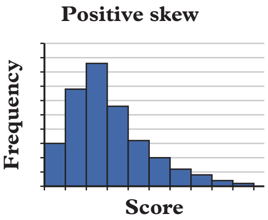


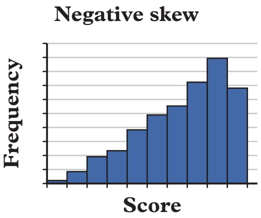
Measures of centre and spread

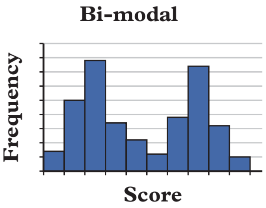
- $\text{Mean} = \dfrac{\text{sum of all data points}}{\text{total number of data points}}$
- Median = middle value when data is listed in order
- Mode = most common value
- Range = maximum value – minimum value
- To calculate the mean and median from a frequency table, add ‘score × frequency’ and ‘cumulative frequency’ columns to the table.

Data distributions









Describing distributions

- Symmetric: use mean or median
- Skewed: use median
- bi-modal distribution: use mode of peaks
- Outliers: use median

Comparing data sets

- Put data sets in same format
- Compare centre and spread

Leaf	Stem	Leaf
9 8 7 7 7 7	1	1 4 5 6 7 9
9 8 7 6 4 4 3 2	2	0 1 1 5 6 8 8
9 7 5 5 4 2 2	3	1 2 2 4 9 9 9
7 6 2 2	4	2 4 5 7

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

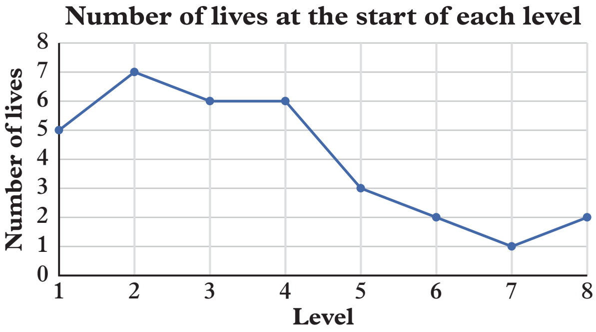
7A **1** Which of the following situations represents stratified sampling?

A Surveying the first 30 people you stop in the street
B Selecting 10 people from each of 5 age groups (20–29, 30–39, etc.) to complete a survey
C Asking your friends and family to complete a survey
D Surveying students from schools in a city in proportion to the number of students at each school
E Selecting every 10th student from the school register to complete a survey

7B **2** Which of the following variables is categorical?

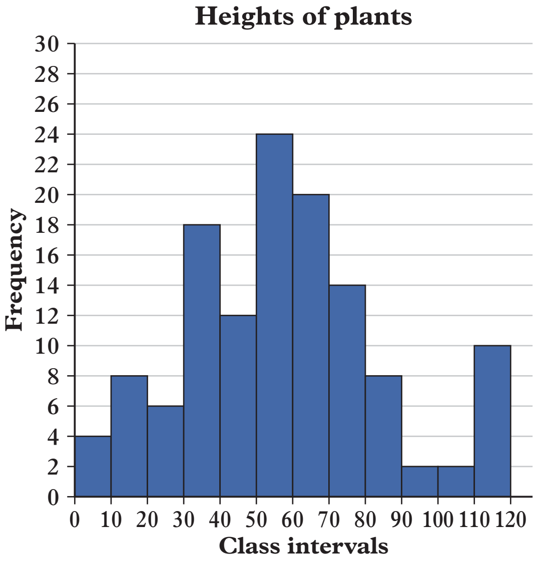
A Number of pets owned
B Percentage score on a test
C Preferred music genre
D Time spent waiting in line at the canteen
E Number of books sold

7B **3** The line graph below tracks the number of lives a gamer has when they begin each level of a video game. The gamer gains two lives during each level. During which level did they lose the most lives?



A 1 **B** 2 **C** 3 **D** 4 **E** 5

7C **4** The histogram below displays the results of research where the heights of plants (in cm) were measured. How many plants are less than 70 cm tall?

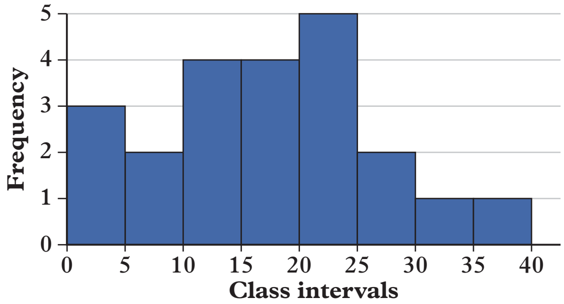


A 14 **B** 20 **C** 36 **D** 44 **E** 92

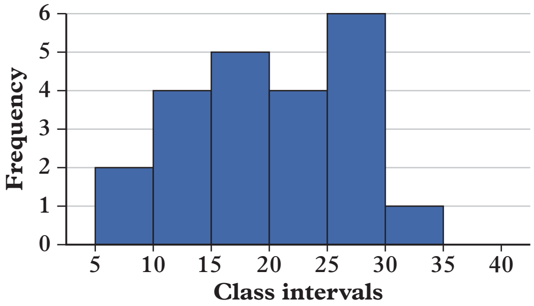
7C **5** The table of data below is represented in which histogram?

Class	0–<5	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	3	2	4	4	5	2	1	1

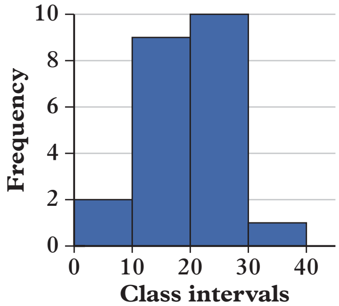
A



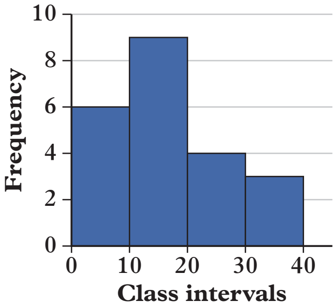
B



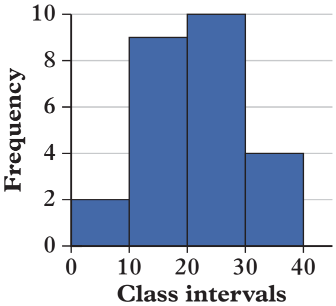
C



D



E



7D **6** In the histogram below, which class interval contains the median?

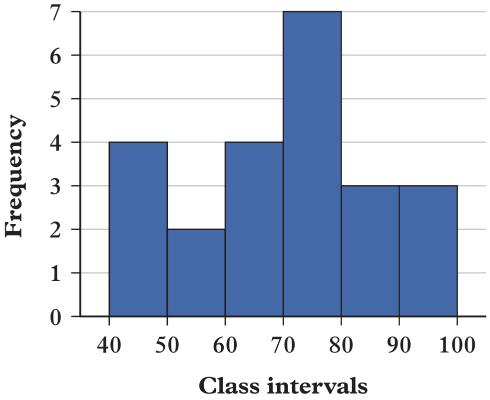
A 40–<50

B 50–<60

C 60–<70

D 70–<80

E 80–<90



7D **7** What is the range of the data shown in the stem-and-leaf plot below?

A 60

B 7.5

C 10.2

D 6

E 102

Stem	**Leaf**
1	4 7 8
2	3 8
3	8 9
4	1
5	6 7
6	0 2 4 5
7	6 6
8	9
9	
10	1 4
11	2 6

Key: 5 | 6 = 5.6

7E **8** Which statement is true if describing distribution of data?

A Symmetric distributions have a centre closer to the right of the distribution, away from the y-axis.

B Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.

C Positively skewed distributions have a middle peak and a roughly even spread on either side of this peak.

D Negatively skewed distributions have a centre closer to the left of the distribution, towards the vertical axis.

E The peaks in a bi-modal distribution must be exactly the same height.

7E **9** Which of the following is an appropriate measure of centre for the dot plot shown?

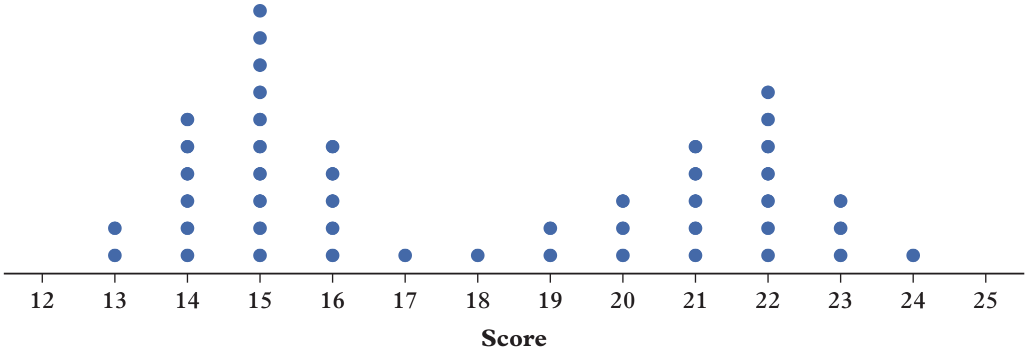
A Mean

B Range

C Median

D Mode(s)

E No centre



7F **10** How is each data set distributed in the back-to-back stem-and-leaf plot on the right?

A Both groups are symmetric.

B Group A is negatively skewed and Group B is positively skewed.

C Group A is positively skewed and Group B is negatively skewed.

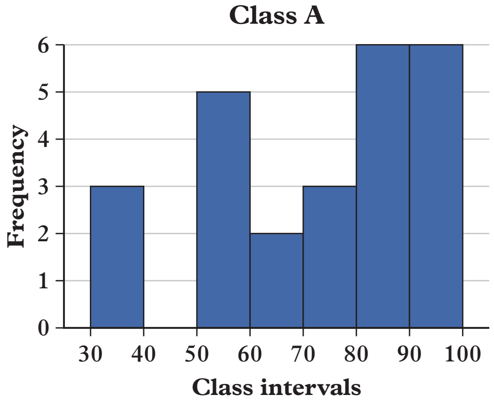
D Both groups are negatively skewed.

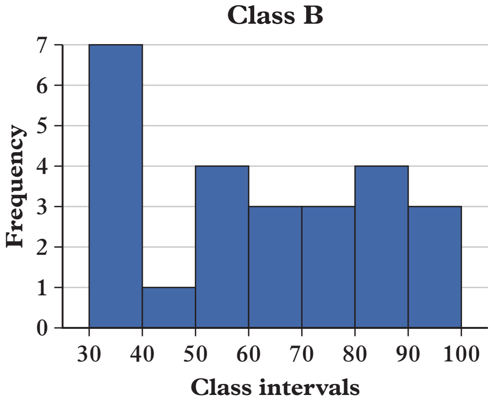
E Both groups are positively skewed.

Leaf Group A	Stem	Leaf Group B
9 8 6 6 4 4 3 2	0	
8 7 5 4 3 1	1	9
7 7 1	2	1
5	3	3 4
4	4	2 4 7 9
1	5	2 3 3 4 6 8 9 9
	6	1 5 6 6 8

Key: 1 | 9 = 19

7F **11** The two histograms below show the Geography exam scores of two different Year 9 classes. Which of the following statements is definitely true?





A The range of each class is at least 60.

B There are more students in Class A than in Class B.

C The range of Class A is greater than the range of Class B.

D The median of Class A is greater than the median of Class B.

E Class B performed better on the exam than Class A.

Short answer

7A **1** Decide whether each survey below will provide fair or biased results, giving reasons for your answers.

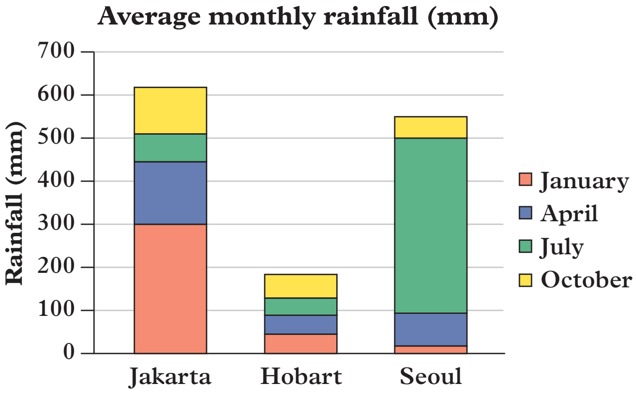
- **a** Asking 20 of your friends their favourite film, in order to determine the favourite films of students at your school.
- **b** Randomly asking 50 people (stratified by age) from your local council area who they want to win most seats at the next election to find the most popular Australian political party.

7A **2** Explain how each of the following headlines might be misrepresenting the data described in brackets.

- **a** 'Swimming is the best form of exercise.' (18 of 20 swimming coaches were surveyed and agreed)
- **b** 'Veganism increasing in popularity.' (75% of people liked a new vegan burger)

7B **3** The stacked bar chart on the right shows the average monthly rainfall in three cities across four different months.

- **a** Identify the three variables shown in the graph and classify each as either numerical or categorical.
- **b** Which city has the most consistent rainfall over the year?
- **c** Of the months shown, when does Seoul have more rain than Jakarta?
- **d** Suggest a reason that the wettest months in Seoul and Jakarta do not occur at the same time.



7B **4** Marina has started a new job selling cars at a car dealership. The number of cars she sold during each of her first 9 months is shown in the table below.

Month	1	2	3	4	5	6	7	8	9
Number of cars sold	2	4	8	14	12	15	19	21	22

- **a** Draw a line graph to represent this data.
- **b** In which month(s) did Marina sell fewer cars than in the previous month?
- **c** At the car dealership, Marina is paid a fixed salary of $4000 per month plus an extra $200 for each car she sells. What was her total income from this job during month 6?

7C **5** Data was collected on the number of hours Year 9 students spent using various forms of social media over the course of a week.

45 21 37 21 20 17 31 32 11 15 17 18 20 31 48 32 21 5 11 7 19 18 27 42
40 21 32 23 24 19 38 37 14 15 19 28 22 35 41 33 27 2 10 5 18 18 25 43

- **a** Construct a frequency table with class intervals of width 10.
- **b** How many people were surveyed?
- **c** What is the most common class interval?
- **d** Draw a histogram to represent this data.
- **e** Which interval is the most common?

7D **6** Find the mean, median, mode and range for this data set, correct to two decimal places where appropriate.

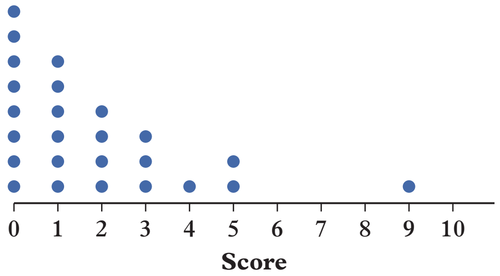
Score	Frequency
3	10
4	12
5	23
6	5

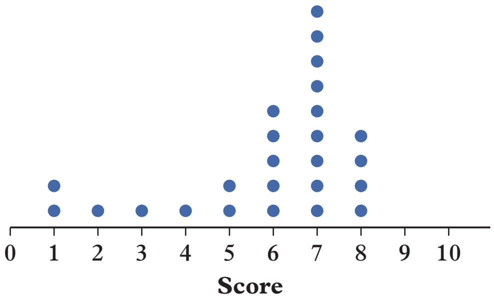
7D **7** Calculate the mean, mode, median and range for the data represented in this stem-and-leaf plot, correct to two decimal places where appropriate.

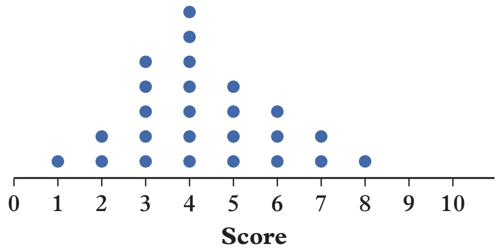
Stem	Leaf
0	2 4 5 7 7 7 7
1	5 8 8 8 9
2	1 1 3 4 7 9
3	7 8 8
4	1 2 5
5	8

Key 2|4 = 24

7E **8** Consider the following three dot plots.


Dot plot 1


Dot plot 2


Dot plot 3

a Which dot plot has the greatest range?

b Which dot plot is positively skewed?

c Which dot plot has a roughly symmetric distribution?

d Is the mean or median the most appropriate measure of centre for dot plot 2? Explain your reasoning.

e One of the dot plots represents the number of pets owned by each student in a Year 9 class. Which dot plot do you think represents this data? Explain your reasoning.

7E **9** The ages of 53 people are shown in the stem-and-leaf plot below.

Stem	Leaf
0	4 4 5 5 7 8
1	0 1 1 1 3 4 4 5 6 6 6 6 7 9 9 9
2	0 0 1 3 4 4 4 4 5 6 6 6 7 7 7 8 9 9
3	0 0 0 0 1 1 1 2 2 3 3 3 3

Key: 1|3 = 13

a Find the median age of the group.

b Find the range of the data set.

c Draw a histogram to represent this data. Use class intervals of 5 with the first interval starting at 0.

d Describe the distribution of ages in the data set.

7F **10** The results achieved in Maths tests by two different classes are recorded in a back-to-back stem-and-leaf plot.

Leaf Class 9A	Stem	Leaf Class 9B
8 5 5 3	4	6 7 8 8 9
9 7 3	5	1 3 6 7
7 6 5 4 3	6	3 4 7
0	7	2 4 8
9 7 5 3 1	8	2 5 6 7
0	9	8 9

Key: 4|6 = 46

a Find the mean, median, mode and range for each class.

b Compare the Maths results of the two classes.

7F **11** The following data comparing the number of hours spent at football training per fortnight were recorded for Year 9 boys from two different secondary schools.

School 1: 35 11 27 11 10 7 21 22 1 5 7 8 10 21 38 22 11 5 1 7 9 8 17 32

School 2: 12 14 16 17 20 11 22 11 11 16 19 14 2 4 10 19 17 6 4 6 2 7 16 31

a Collate the data in a back-to-back stem-and-leaf plot using class intervals of width 5.

b Calculate the mean, median and range for each school, correct to one decimal place.

c Which measure of centre would you use to discuss the results for each school?

Analysis

This stem-and-leaf plot shows the measurements of shrubs taken at different nurseries.

Leaf Brisbane	Stem	Leaf Sydney
7 4 4 4	11	4 4 4 5 7 8 9
7 6 4 3 2 2	12	1 3 6 7
8 7 6 2 1	13	2 3 9
1 0	14	1 4 6 7 8 9
9 8 5 3 2	15	1 2 5 5

Key: 11|4 = 11.4 cm

a Compare the number of shrubs measured at each location.

b Calculate the mean, median, mode and range of shrub heights at each location, correct to two decimal places where necessary.

c Write a short comparison of the height of shrubs at the two locations.

The manager of the nursery company wanted to collate the data for all of his businesses.

d Generate lists of raw data for Brisbane and Sydney. Using this data and the lists below, create a frequency table using suitable class intervals and collate all of the data about shrub height.

Victoria: 12.1, 11.7, 18.3, 11.4, 11.4, 14.5, 15.6, 17.1, 16.5, 18.6, 13.0, 12.6, 12.4, 10.9, 11.4, 14.0, 16.9, 17.1, 16.5, 18.6

Western Australia: 10.2, 10.6, 19.3, 11.4, 11.4, 15.9, 14.7, 18.3, 17.7, 13.4, 10.0, 12.5

e How many plants were measured in total?

f What is the modal class?

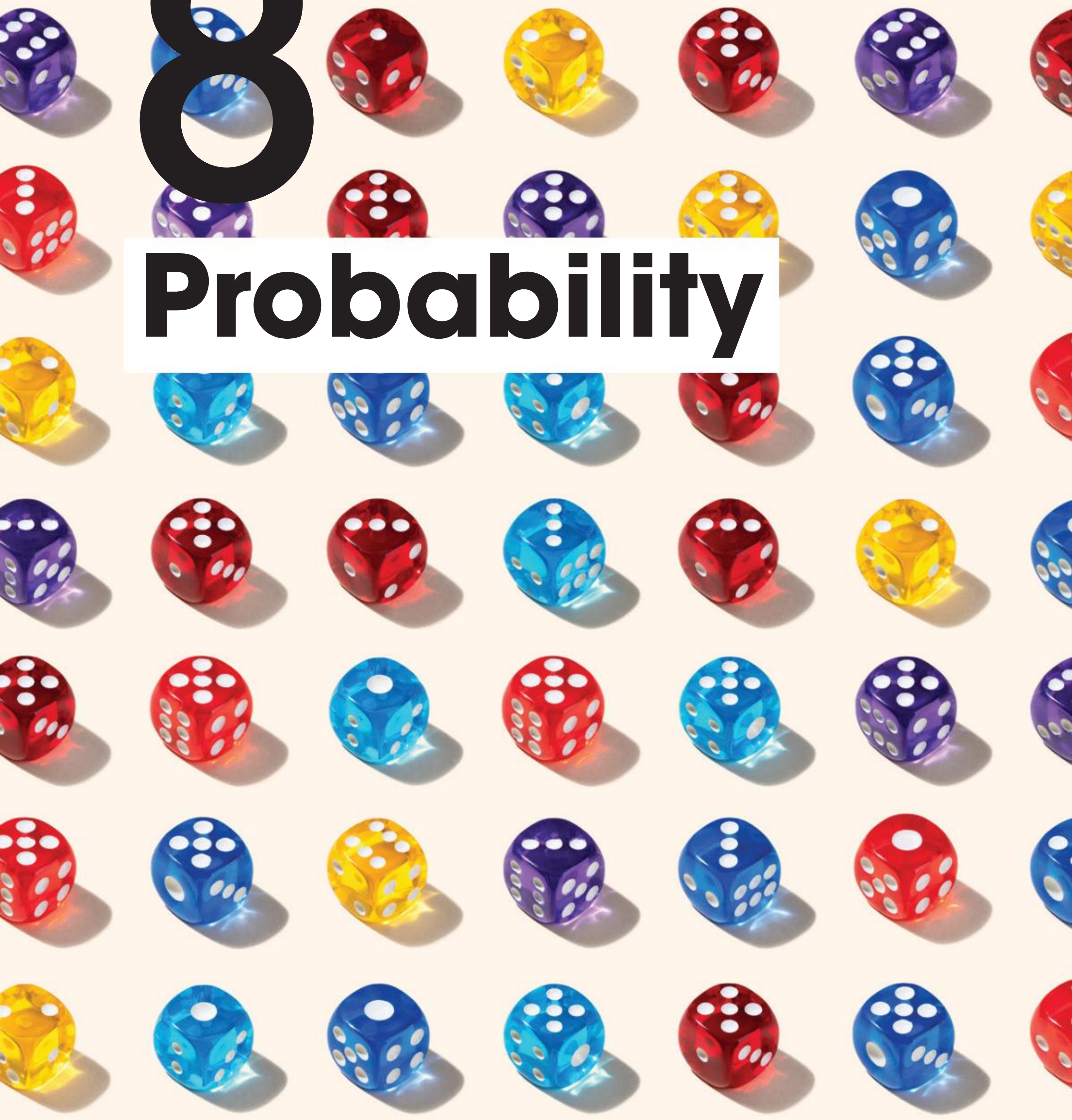
g Represent this data as a histogram and comment on the shape of the distribution.

h Calculate the mean shrub height for Victoria and Western Australia correct to two decimal places.

i Write a statement that could be used in a marketing campaign, which includes information about the smallest and largest shrubs and the average shrub height.

8

Probability



Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Calculating theoretical probability
- ✓ Fractions, decimals and percentages
- ✓ Adding and subtracting fractions
- ✓ Multiplying and dividing fractions

Curriculum links

- List all outcomes for compound events both with and without replacement, using lists, tree diagrams, tables or arrays; assign probabilities to outcomes (AC9M9P01)
- Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and', inclusive 'or' and exclusive 'or' (AC9M9P02)
- Design and conduct repeated chance experiments and simulations, using digital tools to compare probabilities of simple events to related compound events, and describe results (AC9M9P03)

Materials

✓ Calculator

8A Two-step chance experiments

Learning intentions

By the end of this topic you will be able to ...

✔ create tree diagrams and arrays to represent sample spaces

✔ use tree diagrams and arrays to calculate probabilities.

Inter-year links

Support	Understanding probability
Year 7	10E Theoretical probability
Year 8	9D Theoretical probability
Year 10	10A Theoretical probability

Calculating theoretical probability

- The **sample space** of an experiment is a list of all possible outcomes in the **experiment**.
- An **event** is a collection of one or more outcomes from the sample space of an experiment.
- The probability of an event occurring can be written as Pr(A), where A represents all possible outcomes in the event.
- If all the outcomes in an experiment are equally likely to occur, the **theoretical probability** of an event occurring is the ratio of the number of outcomes in the event to the number of outcomes in the sample space. It can be expressed as a fraction, decimal or percentage and calculated using the formula:
  - → $\text{Pr(event)} = \dfrac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$
- The complement of an event A is the event that A does not occur. Events A and ‘not A’ are **complementary events**.
  - → Complementary events are denoted using the symbol prime (′).
    A and A′ are complementary events.
  - → All outcomes in the sample space that are not in A must be in A′.
    $\Pr(A) + \Pr(A') = 1$
- When calculating the probability of an event involving multiple outcomes, the probabilities of each outcome are added together to give the probability of the event.
  - → Make sure you do not include any individual outcome more than once. For example, when a standard six-sided die is rolled, the probability of the event ‘rolling an even or a prime number’ must only count the outcome ‘2’ once.

Two-step chance experiments

- Two-step chance experiments involve two experiments, which may be completed at the same time, or one after the other. For example, rolling a standard six-sided die and tossing a coin is a two-step chance experiment.
- The outcomes for two-step experiments can be recorded in an **array** or in a **tree diagram**.

Arrays

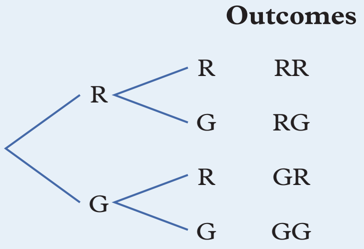
- An array is a list of items arranged in rows and columns.
- The sample space of a two-step chance experiment shown in an array have the outcomes of one event in columns and the outcomes of the other event in rows. For example, this array shows the outcomes from rolling a standard six-sided die and tossing a coin.
  - → There are two outcomes when a coin is tossed: heads (H) or tails (T).
  - → There are six outcomes when a die is rolled: 1, 2, 3, 4, 5, 6.

		Rolling a standard six-sided die					
		1	**2**	**3**	**4**	**5**	**6**
Tossing a coin	**H**	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
	T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

- When calculating probability using an array, all the outcomes are equally likely, and the theoretical probability formula can be used.
- Arrays are best used to display the sample space when there are a large number of outcomes in either step of a two-step experiment.

Tree diagrams

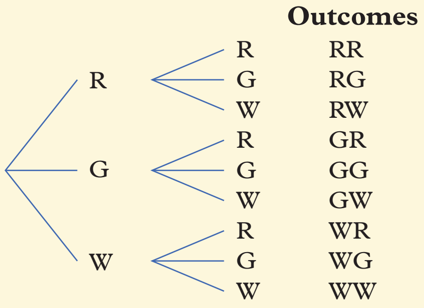
- In a tree diagram, the outcomes of each step of a chance experiment are listed vertically, with the events joined with branches. For example, this tree diagram shows the outcomes for selecting two marbles at random from a bag of red and green marbles.



- If the outcomes in a tree diagram are equally likely, the theoretical probability formula can be used.

Example 8A.1 Understanding tree diagrams

This tree diagram displays the outcomes of selecting two lollies at random from a jar containing red (R), green (G) and white (W) lollies.



Use this tree diagram to find:

a the total number of outcomes

b the number of outcomes containing at least 1 white lolly.

THINK	**WRITE**
a Count the number of final outcomes at the right end of the tree diagram.	**a** There are 9 possible outcomes: RR, RG, RW, GR, GG, GW, WR, WG, WW.
b Trace the branches carefully and count the number that contains at least one white lolly.	**b** 5 outcomes contain a white lolly: RW, GW, WR, WG, WW.

Example 8A.2 Calculating probability using an array

Use an array to calculate the probability of a total sum of 10 or above when rolling two standard six-sided dice.

THINK

1 Draw an outline for the array, with enough columns to list all the outcomes for the first step, and enough rows to list all the outcomes for the second step.

2 Complete the array by showing the result for the first step before the result for the second step in brackets, separated by a comma.

3 Count the outcomes that have a total sum 10 or more (these cells have been highlighted in the answer).

4 Count the total number of outcomes in the array.

5 Use the theoretical probability formula and simplify if possible.

WRITE

		1st dice roll					
		1	**2**	**3**	**4**	**5**	**6**
2nd dice roll	**1**	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

$$\Pr(10 \text{ or above}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

Example 8A.3 Calculating probability using a tree diagram

Use a tree diagram to calculate the probability of exactly one tail when tossing a coin twice.

THINK

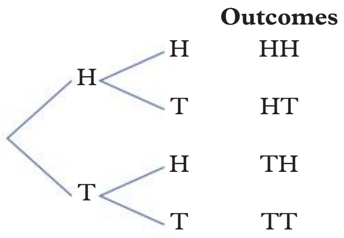
1 Draw the first two branches to represent the first coin toss. Label the ends of these branches with H and T to represent the two different outcomes, heads and tails.

2 From each branch, draw another two branches to represent the next coin toss and label them appropriately.

3 Write the final outcome on each of the four branches to complete the tree diagram.

4 There are four possible outcomes and two outcomes contain exactly one tail (HT, TH). As each of the outcomes are equally likely, the probability formula can be used.

WRITE



$$\Pr(\text{exactly one tail}) = \frac{2}{4}$$

$$= \frac{1}{2}$$

Helpful hints

✔ In other contexts, an event means an occurrence or an occasion, but in probability it means a collection of outcomes from the sample space. Don't get confused!

✔ When creating a tree diagram, make sure you leave plenty of space to draw in all the branches!

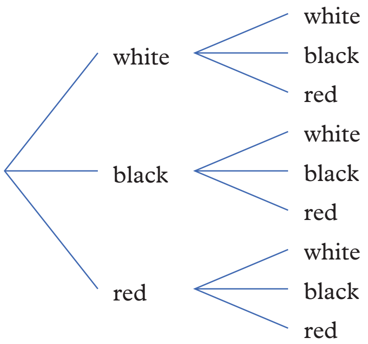
ANS p473

Exercise 8A Two-step chance experiments

△ **1–8, 10, 11** ■ **2–4, 6, 8–12** ⬟ **2–4, 6, 9–13**

UNDERSTANDING AND FLUENCY

8A.1 **1** Consider this tree diagram that shows the results of choosing two coloured balloons at random.



a How many possible outcomes are there?

b How many of these outcomes contain at least one red?

8A.2 **2** Consider this array, which shows the results of two people randomly picking a number between 1 and 3.

a How many possible outcomes are there?

b State the probability of obtaining a sum of:

- **i** 3
- **ii** an odd number
- **iii** a prime number
- **iv** a square number
- **v** a number greater than 6.

		1st number		
		1	**2**	**3**
2nd number	**1**	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

3 Two four-sided dice (numbered 1–4) are rolled and the numbers showing are added together to give the final outcome.

a Create an array that lists all the outcomes.

b How many equally likely outcomes are detailed in the array?

c What is the most likely final outcome? What is the probability of this occurring?

d What are the least likely final outcomes? What is the probability of one of these outcomes occurring?

e State the probability of rolling two four-sided dice (numbered 1–4) and obtaining a sum:

- **i** of 3
- **ii** greater than 6
- **iii** of an odd number
- **iv** less than 5.

4 Two cards are drawn out of a standard deck of cards. Assuming that all outcomes are equally likely, use the tree diagram to find the probability of selecting:

a at least one diamond

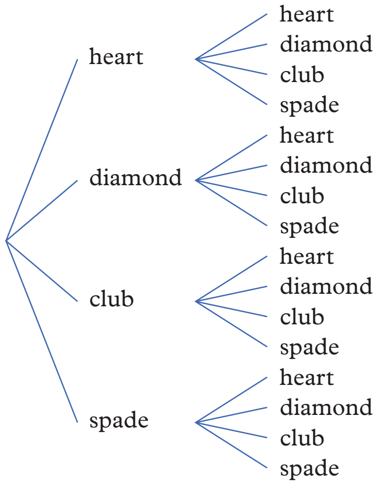
b a club and a heart

c two spades

d not a heart

e a diamond or a spade

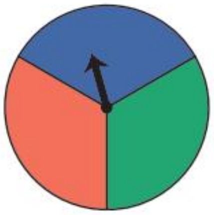
f a spade then a club.



8A.3 **5** A coin is tossed twice. Use a tree diagram to find the probability of:

- **a** two heads
- **b** one head and one tail
- **c** no heads

6 This spinner is spun twice. Use a tree diagram to find the probability of:



- **a** two different results
- **b** spinning blue at least once
- **c** spinning the same colour both times.

PROBLEM SOLVING AND REASONING

7 This array represents a single round of a game of rock (r), paper (p), scissors (s).

		Your choice		
		r	**p**	**s**
Opponent's choice	**r**	(r, r)	(p, r)	(s, r)
	p	(r, p)	(p, p)	(s, p)
	s	(r, s)	(p, s)	(s, s)

rock beats scissors

paper beats rocks

scissors beats paper

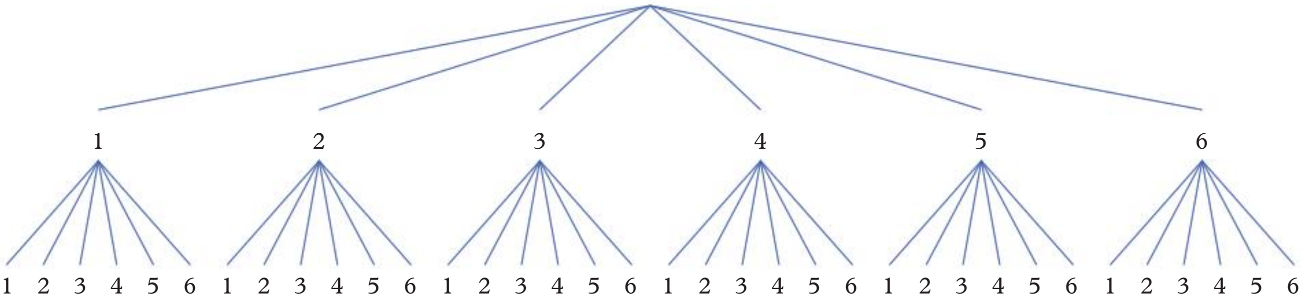
- **a** List the outcomes that result in you:
  - **i** winning
  - **ii** losing
  - **iii** drawing the game.
- **b** Hence calculate the probability that you win a game of rock, paper, scissors.

8 Consider sitting a quiz consisting of multiple-choice questions, with answers A–D.

- **a** Draw a tree diagram to represent the possible different options for the first two questions.
- **b** Draw an array to represent the possible different options for the first two questions.
- **c** Explain why you could use a tree diagram but not an array to represent guessing the answers to the first three questions.

9 The tree diagram below shows the result of rolling two standard six-sided dice.

- **a** How many possible outcomes are there?
- **b** Use the tree diagram to find the probability of rolling:
  - **i** two odd numbers
  - **ii** one even number
  - **iii** a double
  - **iv** a total of 6.
- **c** Why would it be more appropriate to use an array to display the sample space?



PROBLEM SOLVING AND REASONING

10 Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you toss a coin and then roll a standard six-sided die.

a Draw a tree diagram to represent this two-step experiment.

b How many outcomes are there?

c What is the probability that you toss a tail:

i and roll a 6? **ii** and roll a number less than 4?

iii or roll a 6? **iv** or roll a number less than 4?

d Explain the mistake made by someone who answers $\frac{8}{12}$ to part **c iii**.

11 Similar to tree diagrams, arrays are not limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you select a vowel (A, E, I, O, U) at random and roll a standard four-sided die.

a Draw an array to represent this two-step experiment.

b How many outcomes are there?

c What is the probability that:

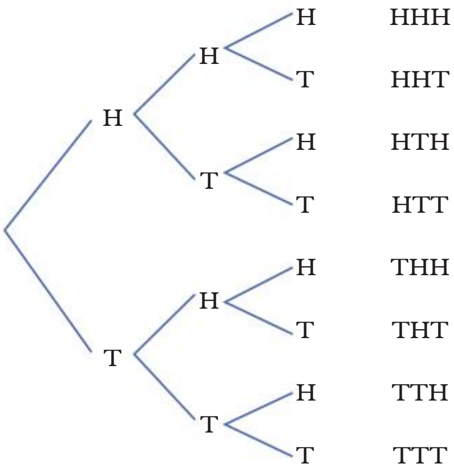
i you select E and roll a 3?

ii you select E or roll a 3?

iii you select a vowel with only straight lines and roll a number less than 4?

iv you select a vowel with only straight lines or roll a number less than 4?

12 Experiments can have more than two steps. When using a tree diagram to represent this, we add more sets of branches to the end of each branch. For example, if a coin is tossed three times (or three coins are tossed), then the tree diagram that represents the sample space would look like this.



a Use the tree diagram shown to determine the probability of tossing a fair coin three times and getting:

i three heads

ii two heads and one tail in any order

iii two tails and one head in any order

iv an alternating sequence of heads and tails in any order.

b Explain why you couldn't use an array to show the sample space for an experiment with three or more steps where each set of outcomes at each step is shown. That is, without using the outcomes of a group of steps against the outcomes of another group.

CHALLENGE

13 Anna is investigating the possibilities when rolling four fair six-sided dice. Use a tree diagram to:

a find the probability of rolling four of the same number

b find the probability of rolling exactly three of the same number

c find the probability of rolling two of the same number and two different numbers

d find the probability of rolling two copies of each of two different numbers (e.g. two 2s and two 3s)

e find the probability of rolling four different numbers.

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Interactive skillsheet
Arrays

Interactive skillsheet
Tree diagrams

Investigation
Dominoes and tree diagrams

Topic quiz
8A

8B Experiments with replacement

Learning intentions

By the end of this topic you will be able to …

✔ calculate probabilities for two-step chance experiments with replacement.

Inter-year links

Year 7	10E Theoretical probability
Year 8	9D Theoretical probability
Year 10	10B Experiments with and without replacement

Calculating probabilities for two-step experiments

- The probabilities for each step of a two-step experiment can be written on the branches of a tree diagram.
- The probabilities for each outcome can be found by multiplying together the probabilities of each branch leading to that outcome.
  - → The probabilities of all the outcomes of an experiment always sum to 1.
- If the event you are looking for consists of more than one outcome, the probabilities of each outcome are added together to find the probability of the event.

Experiments with replacement

- Experiments with replacement involve selecting an item at random, recording the result, and then replacing the item before making another selection.
- When replacing the item before making another selection, the probabilities for each step of the experiment remain the same.

Example 8B.1 Representing experiments with equally likely outcomes

A box contains equal numbers of blue, red and yellow activity cards. A card is selected at random, its colour is recorded and then it is replaced. Another card is then selected at random. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

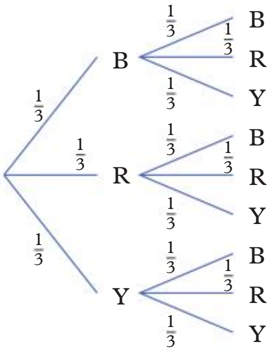
THINK

1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.

2 Write the probability of each branch on your tree diagram. Each colour is equally likely so there is a $\frac{1}{3}$ chance of selecting a blue card, a $\frac{1}{3}$ chance of selecting a red card and a $\frac{1}{3}$ chance of selecting a yellow card.

3 Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE



Outcomes	Probability
BB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
BR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
BY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
RY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YR	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
YY	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

Example 8B.2 Calculating probabilities for experiments with equally likely outcomes

Use the tree diagram from Example 8B.1 to find the probability that:

a two blue cards are selected

b a yellow card is selected first.

THINK	**WRITE**
a Identify the outcome in which both selections are blue cards.	**a** $\Pr(\text{two blue}) = \Pr(\text{BB})$ $= \frac{1}{9}$
b 1 Identify the outcomes in which a yellow activity card is selected first.	**b** $\Pr(\text{yellow first}) = \Pr(\text{YB}, \text{YR}, \text{YY})$
2 Find the sum of the probabilities and simplify.	$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ $= \frac{3}{9}$ $= \frac{1}{3}$

Example 8B.3 Representing experiments with outcomes that are not equally likely

A bag contains 15 red balls and 5 green balls. A ball is selected at random, its colour is recorded and then it is replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each outcome.

THINK

1 Draw a tree diagram to represent this experiment, listing all the final outcomes.

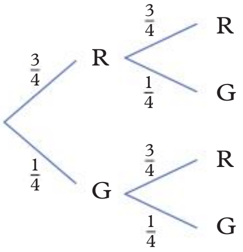
2 Find the probability of a green ball being selected: $\Pr(\text{G}) = \frac{5}{20} = \frac{1}{4}$

Find the probability of a red ball being selected: $\Pr(\text{R}) = \frac{15}{20} = \frac{3}{4}$

3 Write the probability on each branch.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

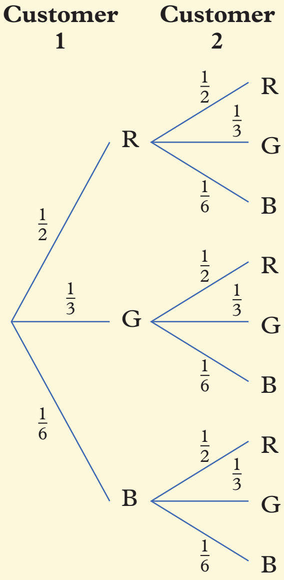
WRITE



Outcomes	**Probability**
RR	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
RG	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$
GR	$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$
GG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Example 8B.4 Calculating probabilities for experiments with outcomes that are not equally likely

This tree diagram shows the possible outcomes when two customers select a ball at random from a box containing 3 red balls, 2 green balls and 1 blue ball. The balls are returned after the first customer's selection.



Outcomes	Probability
RR	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
RG	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
RB	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
GR	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
GG	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
GB	$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
BR	$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
BG	$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
BB	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Calculate the probability of selecting:

a exactly one blue ball

b a blue ball and a green ball.

THINK	**WRITE**
a Identify the outcomes in which exactly one blue ball is selected. Find the sum of the probabilities of the outcomes and simplify.	**a** $\Pr(\text{exactly one blue}) = \Pr(\text{RB}) + \Pr(\text{GB}) + \Pr(\text{BR}) + \Pr(\text{BG})$ $= \frac{1}{12} + \frac{1}{18} + \frac{1}{12} + \frac{1}{18}$ $= \frac{10}{36}$ $= \frac{5}{18}$
b Identify the outcomes in which a blue ball and a green ball are selected (in any order). Find the sum of the probabilities of the outcomes and simplify.	**b** $\Pr(\text{blue and green}) = \Pr(\text{GB}) + \Pr(\text{BG})$ $= \frac{1}{18} + \frac{1}{18}$ $= \frac{2}{18}$ $= \frac{1}{9}$

Helpful hints

✔ The probabilities at each step of a two-step experiment always sum to 1.

✔ After calculating the probabilities of the final outcomes, check that the sum of these probabilities is equal to 1.

ANS p474

Exercise 8B Experiments with replacement

△ 1–9, 13 ■ 2, 4–7, 10, 11, 13–15 ⬠ 2, 4, 6, 7, 11, 12, 15, 16

UNDERSTANDING AND FLUENCY

1 For each chance experiment, state the theoretical probability of the outcome in brackets in any given trial.

a Selecting a marble and recording its colour from a bag with 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)

b Rolling a standard six-sided die and recording the number on top (rolling a 5)

c Drawing a card from a standard deck of 52 playing cards (drawing a red picture card – jack, queen or king)

d Selecting a gift voucher from a lucky dip containing twelve \$5 vouchers, eight \$10 vouchers, and four \$20 vouchers (selecting a \$20 voucher)

8B.1 **2** Draw a tree diagram to represent each of the following situations, complete with probabilities on each branch and for each final outcome.

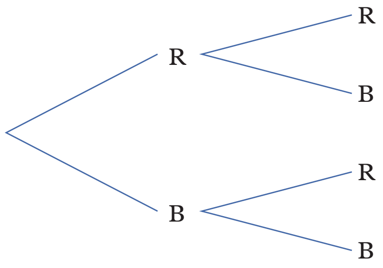
a A pencil case contains equal numbers of red and blue pens. A pen is selected, its colour is recorded and then it is replaced. This is repeated one more time.

b A box contains equal numbers of \$5, \$20 and \$75 vouchers. A voucher is selected, its value is recorded and then it is replaced. This is repeated one more time.

c A box contains 11 cards, numbered 1 to 11. A card is selected, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated one more time.

d A ball-pit contains equal numbers of blue, red, green and yellow balls. A ball is selected, its colour is recorded and then it is replaced. This is repeated one more time.

8B.2 **3** A bag contains six red counters and six black counters. A counter is drawn twice, being replaced after the first draw. This tree diagram shows the outcomes of the two selections.



a In any given trial, what is the probability of selecting a black counter?

b How many final outcomes are there?

c What is the probability of selecting:

i two black counters?

ii exactly one black counter?

iii two red counters?

4 A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected at random from the box, its flavour recorded and then it is replaced. This is repeated once more. What is the probability that:

a both chocolates are white?

b at least one chocolate is dark?

c the first chocolate is white and the second chocolate is milk?

d one chocolate is white and one chocolate is milk?

5 There are 52 cards in a standard pack of playing cards with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is chosen at random from a pack, its suit is recorded and then it is replaced. This is repeated once more. What is the probability of selecting:

a two hearts?

b at least one diamond?

c exactly one club?

d no spades?

e at least one diamond or spade?

f one heart and one club?

UNDERSTANDING AND FLUENCY

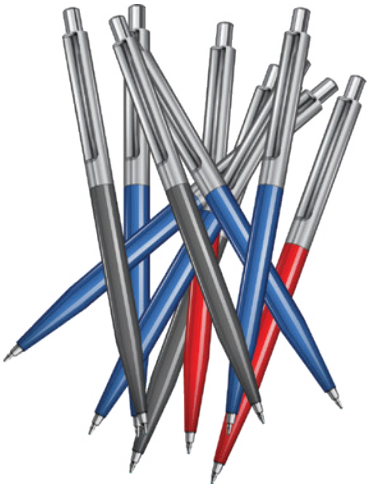
8B.3 **6** Draw a tree diagram with probabilities on the branches for each chance experiment.

- **a** A pencil case contains five blue pens and two red pens. A pen is chosen, its ink colour is recorded and then it is replaced. This is repeated one more time.
- **b** A box contains fifteen \$5 vouchers, ten \$20 vouchers and five \$75 vouchers. A voucher is chosen, its value is recorded and then it is replaced. This is repeated one more time.
- **c** A box contains 15 cards, numbered 1 to 15. A card is chosen, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated a total of two times.
- **d** A ball-pit contains five blue, four red, three green and two yellow balls. A ball is chosen, its colour recorded, and then it is replaced. This is repeated one more time.

8B.4 **7** A lucky dip contains 10 pink gift vouchers for \$100 and 40 green gift vouchers for \$10. A voucher is chosen, its value is recorded and then it is replaced. If this is repeated, find the probability that:

- **a** a \$100 voucher is selected twice
- **b** a \$100 voucher is not selected at all
- **c** a \$100 voucher is selected first, and a \$10 voucher selected second.

8 The contents of a pencil case are shown here. The owner of the pencil case takes out a pen at random for each lesson.



- **a** Use the pens shown to draw a tree diagram to represent the colours of the pens chosen for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.
- **b** Find the probability that the owner chooses:
  - **i** a blue pen each time
  - **ii** a red pen each time
  - **iii** a black pen each time
  - **iv** a blue pen, then a black pen
  - **v** a blue pen, then a red pen
  - **vi** a red pen, then a black pen.

PROBLEM SOLVING AND REASONING

9 The probability of selecting a picture card from a standard 52-card pack is $\frac{3}{13}$.
A card is chosen from a standard pack of 52 cards and it is recorded whether it is a picture card or not, before the card is replaced. This is repeated once more.

- **a** Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome as a decimal number rounded to four decimal places.
- **b** Find the probability of selecting:
  - **i** exactly one picture card
  - **ii** at least one picture card
  - **iii** two picture cards
  - **iv** no picture cards.

10 In Example 8B.4 part **a** there are four favourable outcomes out of nine possible outcomes. Explain why the theoretical probability of selecting exactly one blue ball is $\frac{5}{18}$ instead of $\frac{4}{9}$.

11 A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you choose from a box at random. If you take a red ball you get 10% off, if you take a green ball you get 25% off and if you take a blue ball you get 50% off. There are 10 balls of each colour in the box.

You can have a second chance if you don't take a blue ball out of the box first up, as long as you put the first ball back into the box.

Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of selecting a blue ball from the box.

PROBLEM SOLVING AND REASONING

12 An array can be used to list the outcomes of a two-step experiment with replacement.

a Explain why an array may be a better choice to show the outcomes of a two-step experiment with replacement than a tree diagram when there is a large number of equally likely outcomes for each step (even when like outcomes are combined).

b Explain how you could represent outcomes that are not equally likely in an array.

13 A sock drawer contains 10 socks; some are black and some are white. You need to find out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.

a If you selected with replacement 10 times and selected 3 black socks and 7 white socks, does this mean that there are 3 black and 7 white socks in the drawer? Explain.

b If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?

c If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?

14 A bag contains 12 aqua marbles and some green marbles.

a If the probability of selecting an aqua marble is 0.3, find the number of green marbles.

Three marbles are drawn from the bag and each marble is replaced in the bag before the next is drawn.

b Draw a tree diagram representing this situation. Include the probability of each step on the branches and calculate the probability of each outcome.

c Use your tree diagram to find the probability of selecting:

i three marbles of the same colour

ii exactly two green marbles

iii exactly two marbles of the same colour

iv a green marble first and an aqua marble third.

15 A bag contains some white, black and red tiles. Three tiles are drawn from the bag and their colours noted. Each tile is replaced in the bag before the next tile is drawn.

a If you were to represent this situation with a tree diagram, how many different outcomes would there be?

b Write out all possible outcomes where three different coloured tiles are drawn.

c If Pr (drawing a while tile) = w and Pr (drawing a black tile) = b, find the probability that three different coloured tiles are drawn.

CHALLENGE

16 A bag contains different numbers of tiles labelled A, B, and C. A tile is taken out of the bag at random, the letter is observed, then the tile is replaced into the bag before a second tile is taken out of the bag at random. Let the probability of selecting a tile labelled A on any given selection be $\Pr(A) = a$, and the probability of selecting a tile labelled B on any given selection be $\Pr(B) = b$.

Write an expression for each of the following in terms of a and b.

a The probability of selecting two tiles labelled A

b The probability of selecting a tile labelled A and a tile labelled B in any order

c The probability of selecting a tile labelled C on any given selection

d The probability of selecting a tile labelled A and a tile labelled C in any order in:

i factorised form **ii** expanded form.

e The probability of selecting two tiles labelled C in:

i factorised form **ii** expanded form.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Experiments with replacement

Investigation
Three-step experiments with replacement

Topic quiz
8B

8C Experiments without replacement

Learning intentions

By the end of this topic you will be able to ...

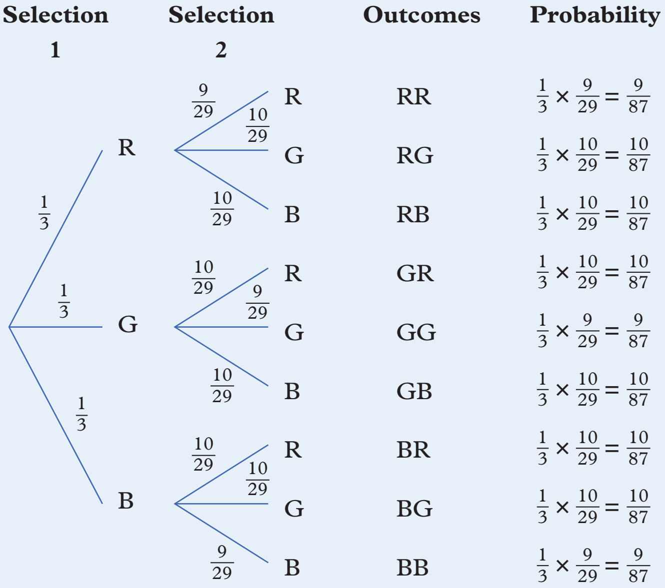
✔ calculate probabilities for two-step chance experiments without replacement.

Inter-year links

Year 7	10E Theoretical probability
Year 8	9D Theoretical probability
Year 10	10B Experiments with and without replacement

Experiments without replacement

- Experiments without replacement involve selecting an item at random, recording the result, and then not replacing the item before making another selection.
- The selected items are not replaced, so the probability of the remaining items being selected changes.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.
- For example, the following tree diagram shows the results of a customer choosing two socks at random from 10 red, 10 green and 10 blue socks. The first sock is not replaced so the probabilities change for subsequent selections.



  → If a pair of red socks is selected, the first selection is from 10 red socks out of 30, and the second selection is from 9 red socks out of 29.

  Selection 1: $\Pr(R) = \frac{10}{30} = \frac{1}{3}$

  Selection 2: $\Pr(R) = \frac{9}{29}$

  → If a red then a green sock is selected, the first selection is from 10 red socks out of 30 and the second selection is from 10 green socks out of 29.

  Selection 1: $\Pr(R) = \frac{10}{30} = \frac{1}{3}$

  Selection 2: $\Pr(G) = \frac{10}{29}$

  → The probabilities of the other outcomes are calculated in a similar manner.

  → The probability of selecting a pair of socks that are the same colour can be calculated:

  $\Pr(RR) + \Pr(GG) + \Pr(BB) = 3 \times \frac{1}{3} \times \frac{9}{29} = \frac{9}{29}$

  → The probability of selecting a pair of socks that are different colours can be calculated:

  $\Pr(RG) + \Pr(RB) + \Pr(GR) + \Pr(GB) + \Pr(BR) + \Pr(BG) = 6 \times \frac{1}{3} \times \frac{10}{29} = \frac{20}{29}$

Example 8C.1 Representing experiments without replacement

A lucky dip contains five red gift vouchers for \$50 and five green gift vouchers for \$5. A gift voucher is selected and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each possible outcome.

THINK

1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.

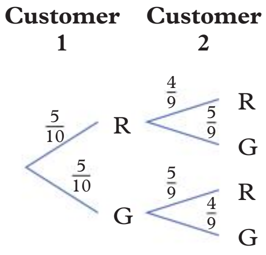
2 Write the probability at the first branches of your tree diagram. Initially there is a $\frac{5}{10}$ chance of selecting a red gift voucher, and a $\frac{5}{10}$ chance of selecting a green gift voucher.

3 Determine the probabilities at the second branches and write on your tree diagram.

- If you select a red gift voucher, 4 red and 5 green vouchers remain.
- If you select a green gift voucher, 5 red and 4 green gift vouchers remain.

4 Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE



Outcomes	Probability
RR	$\frac{5}{10}\times\frac{4}{9}=\frac{20}{90}\approx 0.22$
RG	$\frac{5}{10}\times\frac{5}{9}=\frac{25}{90}\approx 0.28$
GR	$\frac{5}{10}\times\frac{5}{9}=\frac{25}{90}\approx 0.28$
GG	$\frac{5}{10}\times\frac{4}{9}=\frac{20}{90}\approx 0.22$

Example 8C.2 Calculating probabilities for experiments without replacement

Use the tree diagram from Example 8C.1 to find the probability that:

a both customers select a \$50 voucher

b the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.

THINK	WRITE
a Identify the outcome where both customers select a \$50 voucher.	**a** $\Pr(\text{both } \$50) = \Pr(RR) = \frac{20}{90} = \frac{2}{9}$
b Identify the outcome where the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.	**b** $\Pr(\text{first } \$5 \text{ and second } \$50) = \Pr(GR) = \frac{25}{90} = \frac{5}{18}$

Helpful hints

- ✔ The number of total outcomes decreases by 1 after the first step of a two-step chance experiment without replacement.
- ✔ The number of favourable outcomes decreases by 1 only for the outcome occurring in the first step.
- ✔ When calculating probabilities involving multiple final outcomes, it is often easier to not simplify fractions until the end of your working, as it is easier to add fractions when they have a common denominator.

ANS p476

Exercise 8C Experiments without replacement

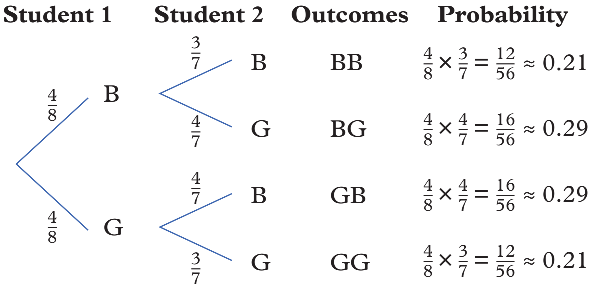
△ 1–4, 6, 7, 9, 10, 14 □ 1(b, c), 2, 4–6, 8, 11, 12, 13(a), 15 ⬠ 1(c), 4–6, 11–13, 15–17

UNDERSTANDING AND FLUENCY

8C.1 **1** Draw a tree diagram with probabilities on the branches for each of these chance experiments without replacement.

- **a** A drawer contains five black socks and five white socks. A sock is selected at random and its colour recorded. This is repeated one more time.
- **b** An esky contains six cans of Coke and six cans of Pepsi. A can is selected at random and its type recorded. This is repeated one more time.
- **c** A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is selected at random and its capacity recorded. This is repeated once more.

8C.2 **2** The following tree diagram shows how two students can be selected at random from a group of four boys and four girls. Find the probability of selecting:



- **a** two boys
- **b** no boys
- **c** a boy and a girl.

3 A lucky dip contains four purple gift vouchers for \$100 and four yellow gift vouchers for \$10. A gift voucher is chosen at random and the customer keeps it. If this was repeated for a second customer, find the probability that:

- **a** both customers select a \$100 voucher
- **b** the first customer selects a \$100 voucher and the second customer selects a \$10 voucher.

4 Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toys: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and a friend each select a bonbon at random from the pack.

- **a** Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.
- **b** Find the probability that:
  - **i** you both select a bonbon with a whistle
  - **ii** the first bonbon has a yo-yo and the second has a bouncy ball.

UNDERSTANDING AND FLUENCY

5 Consider the bonbons from question **4**. Find the probability that:

a you both select a bonbon with a different toy

b at least one of you selects a bonbon with a bouncy ball

c a bonbon with a whistle is not chosen.

6 Experiments without replacement can also start with unequally likely outcomes. A lucky dip contains 15 red, 10 green and 5 blue balls. Customers select one ball at random and they do not replace the ball before the next customer makes a selection.

a Complete the tree diagram.

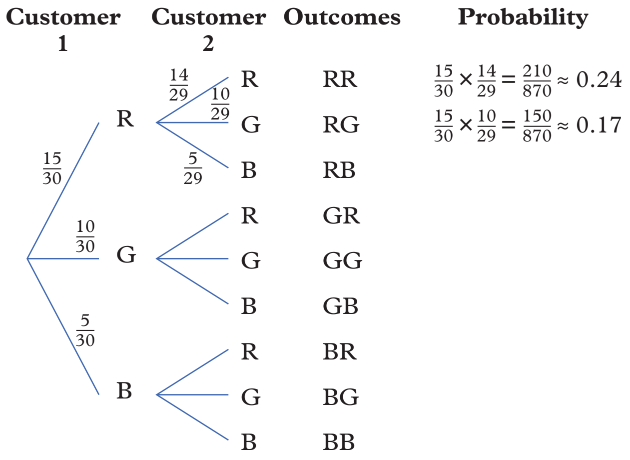
b Use the tree diagram to calculate the probability that:

i both customers select a blue ball

ii both customers select a green ball

iii both customers select a red ball

iv the first customer selects a red ball and the second selects a green ball.



7 Use the tree diagram from question **6** to calculate the probability of selecting:

a at least one green ball
b at least one red ball
c at least one blue ball
d exactly one red ball
e exactly one green ball
f a blue ball and a green ball.

PROBLEM SOLVING AND REASONING

8 There are 52 cards in a standard pack of playing cards, with 13 cards of each suit (clubs, diamonds, hearts and spades).

A card is selected and its suit is recorded. The card is not replaced and another card is selected.

What is the probability of selecting:

a two hearts?
b at least one diamond?
c no spades?
d one heart and one club?

9 A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to calculate the probability that you take out a pair when you select two socks from the drawer at random.

10 Imagine instead that the drawer from question **9** contains six socks of each colour. How does this change the probability of selecting a pair when you select two socks from the drawer at random?

11 If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected at random from the drawer form a pair.

12 A sports team needs to select a captain and a vice-captain. Five people have put their names forward: Adrian, Chantelle, Katie, Guillermo and Sam.

a Draw a tree diagram to represent the selection (start with ‘captain’ branches).

b How does this tree diagram differ from the other ones you have done beforehand? Hint: Does the second set of branches contain the same number as the first set of branches?

c How many different combinations of captains and vice-captains are there? Remember that order is important!

d If each person has an equal chance, then find the probability that:

i Katie is selected captain
ii Sam is selected either captain or vice-captain
iii Adrian is captain and Chantelle is vice-captain
iv Guillermo does not get a position
v Katie and Sam both get a position.

13 An array can be used to show the outcomes for a two-step experiment without replacement. It is important to show the outcomes that cannot occur as they are the repeated outcomes that have been removed after the first step. We show this by placing a cross in that cell. For example, when selecting two marbles from a bag that contains five red marbles and four blue marbles without replacement, the array can be drawn as shown.

First

Second

	R	**R**	**R**	**R**	**R**	**B**	**B**	**B**	**B**
R	×	RR	RR	RR	RR	BR	BR	BR	BR
R	RR	×	RR	RR	RR	BR	BR	BR	BR
R	RR	RR	×	RR	RR	BR	BR	BR	BR
R	RR	RR	RR	×	RR	BR	BR	BR	BR
R	RR	RR	RR	RR	×	BR	BR	BR	BR
B	RB	RB	RB	RB	RB	×	BB	BB	BB
B	RB	RB	RB	RB	RB	BB	×	BB	BB
B	RB	RB	RB	RB	RB	BB	BB	×	BB
B	RB	RB	RB	RB	RB	BB	BB	BB	×

a Determine the probability of selecting the following combinations of marbles from the bag with five red marbles and four blue marbles.

- **i** Two red marbles
- **ii** Two blue marbles
- **iii** A red marble then a blue marble
- **iv** A red and a blue marble in either order

b Another bag contains three green marbles and one purple marble. Two marbles are selected from the bag at random without replacement.

- **i** Draw an array to show the outcomes for this experiment.
- **ii** Determine the probability of selecting one purple marble out of the two marbles.
- **iii** Determine the probability of selecting two purple marbles.

14 A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Two chocolates are selected from the box, without looking. Find the probability of selecting:

a two white chocolates
b no dark chocolates
c two different types of chocolate
d at least one white chocolate.

15 Johan receives a chocolate box like the one in question **14**. He decides to eat three randomly chosen chocolates and share the rest with his friends. Find the probability that he chooses:

a three milk chocolates
b one of each chocolate
c no milk chocolates.

16 A lottery consists of 45 numbered balls, of which six winning balls are chosen at random, with none of the six balls drawn replaced. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery? Express your answer as both a fraction and a decimal correct to three significant figures.

17 An album contains nine songs and the lengths of the songs are shown in the table in minutes and seconds. Find the probability that the lengths of the first two different songs played on shuffle are above the mean length of a track on the album.

4:17	1:08	4:22	1:38	4:16	0:53	4:20	1:44	4:46

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Experiments without replacement

Investigation
Draws for tennis tournaments

Topic quiz
8C

Checkpoint

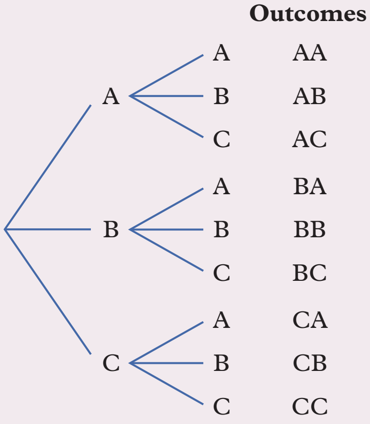
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

8A 1 A fair coin is tossed twice in a row. Draw a tree diagram that shows all outcomes for this experiment.

8A 2 A fair, four-sided die numbered 1 to 4 is rolled twice in a row. Draw an array that shows all outcomes for this experiment.

8A 3 In the tree diagram shown on the right, all of the outcomes are equally likely. Determine the probability of the outcome consisting of:

- **a** two consonants
- **b** a vowel and a consonant
- **c** at least one consonant.



8A 4 Use the array shown to determine the probability of:

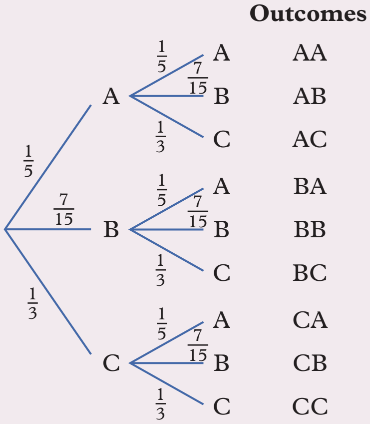
- **a** B and Y
- **b** B or Y or both
- **c** B or Y but not both.

	A	B	C	D
X	(A, X)	(B, X)	(C, X)	(D, X)
Y	(A, Y)	(B, Y)	(C, Y)	(D, Y)
Z	(A, Z)	(B, Z)	(C, Z)	(D, Z)

8B 5 A bag contains three pieces of marble and four pieces of quartz. A piece is selected from the bag at random, returned to the bag, and then another piece is selected. Draw a tree diagram showing the probability of all outcomes, including probabilities on the branches for this experiment.

8B 6 Use the tree diagram shown on the right to determine the probability of:

- **a** the same letter twice in a row
- **b** at least one A
- **c** less than two Cs.



8B 7 Use your tree diagram from question **5** to determine the probability of:

- **a** selecting a piece of marble and a piece of quartz
- **b** selecting a piece of marble second
- **c** selecting at least one piece of quartz.

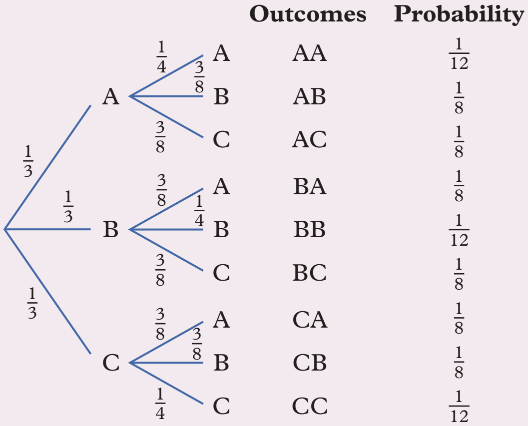
8C 8 A bag contains nine tiles, three tiles labelled A and six labelled B. A tile is drawn at random, the letter observed, then another tile is drawn without replacing the first. Draw a tree diagram with probabilities on the branches for this experiment and calculate the probability of each outcome.

8C 9 Use the tree diagram shown on the right to determine the probability of:

- **a** the same letter twice in a row
- **b** at least one A
- **c** less than two Cs.

8C 10 Use the tree diagram from question **9** to determine the probability of:

- **a** the same letter twice in a row
- **b** a tile labelled A second
- **c** at least one B.



8D Relative frequency

Learning intentions

By the end of this topic you will be able to ...

✔ calculate relative frequencies

✔ calculate the expected number of times an outcome will occur in an experiment.

Inter-year links

Support	Chance experiments
Year 7	10F Experimental probability

Relative frequency

- **Relative frequency** is calculated using the results of experiments or surveys rather than using theoretical probability.
- The relative frequency of a random event is the number of times a favourable outcome occurs divided by the total number of trials.
  - → Relative frequency $= \dfrac{\text{number of occurrences}}{\text{total number of trials}}$
- The **expected number** of each outcome in a chance experiment can be used to investigate whether there is any bias in an experiment. To calculate an expected number, multiply the theoretical probability by the number of trials in the experiment.
  - → Expected number = theoretical probability × number of trials
- If there is bias in a chance experiment, the experiment will favour one outcome over another outcome even when they are supposed to have an equal likelihood of occurring.
  - → When considering if a chance experiment is biased, consider both the expected number of occurrences and the actual number of occurrences. If there is a sizable difference, consider the total number of trials.
  - → With a small number of trials, the results often vary significantly from the expected numbers, but in the long term you can expect the relative frequency of a random event to approach the theoretical probability, due to a powerful concept in probability called the law of large numbers.
  - → Start from the assumption that bias does not exist, and if you suspect bias may exist, carry out more trials to confirm your assumption.
- The theoretical probabilities of the outcomes from surveys, long-run experiments and statistical investigations with large data sets are not usually known. The relative frequency can be used to estimate the theoretical probability of outcomes in real-world events.

Example 8D.1 Calculating relative frequencies

In Brisbane, it rained for 11 days in March. Find the relative frequency of rainy days in March correct to two decimal places.

THINK	**WRITE**
1 Write the relative frequency formula.	Relative frequency $= \dfrac{\text{number of occurrences}}{\text{total number of outcomes}}$
2 Identify the number of occurrences (11) and the total number of outcomes (31). Substitute into the formula and solve, simplifying if possible.	$= \dfrac{11}{31}$
3 Convert to a decimal, correct to two decimal places, and write your answer.	≈ 0.35 The relative frequency of rainy days in Brisbane in March is 0.35.

Example 8D.2 Calculating an expected number

Find the expected number of 5s or 6s if a standard six-sided die is rolled 90 times.

THINK

1 Write the formula for an expected number.

2 Identify the theoretical probability of rolling a 5 or a 6 $\left(\frac{2}{6}\right)$ and the number of trials (90) and substitute into the formula.

WRITE

$$\begin{aligned}\text{Expected number} &= \text{theoretical probability} \times \text{number of trials}\\ &= \frac{2}{6} \times 90\\ &= 30\end{aligned}$$

Example 8D.3 Comparing the relative frequency and expected number

A standard six-sided die is rolled 80 times and a 5 is obtained 17 times.

a Find the expected number of 5s if a standard six-sided die is rolled 80 times.

b Comment on whether you believe this die may be biased.

THINK

a 1 Write the expected number formula.

2 Substitute the theoretical probability $\left(\frac{1}{6}\right)$ and the number of trials (80) into the formula and evaluate.

b Look to see if there is a significant difference between the expected number and the actual number. Consider the number of trials and make a reasonable assessment with the given information.

WRITE

a

$$\begin{aligned}\text{Expected number} &= \text{theoretical probability} \times \text{number of trials}\\ &= \frac{1}{6} \times 80\\ &\approx 13\end{aligned}$$

b Although there is a difference between the expected number (≈13) and the actual number (17), the difference is not significant. To determine whether the die is biased, more trials should be conducted.

Helpful hints

- ✔ The minimum value of a relative frequency is 0. For example, the number of rainy days cannot be −2 days out of 31 days. The minimum number of rainy days is 0 days out of 31 days.
- ✔ The maximum value of a relative frequency is 1. For example, the number of rainy days cannot be 32 days out of 31 days. The maximum number of rainy days is 31 days out of 31 days.
- ✔ The results from an experiment or survey are more reliable when the sample size is larger. This should be considered when drawing conclusions from data.

ANS
p478

Exercise 8D Relative frequency

△ 1–3, 5, 6, 7(a, b), 8, 10   ■ 1, 2, 4–6, 8, 10, 11, 13, 14(a, b)   ⬠ 1–2(d–f), 5, 6, 9, 10, 12–14

UNDERSTANDING AND FLUENCY

8D.1 **1** Find the relative frequency of the outcomes in the following situations. Give your answers as fractions and decimals, rounded to two decimal places where necessary.

a Of 50 families surveyed about pets, 18 had a pet dog.

b Of 300 people surveyed about being right-handed, left-handed or ambidextrous, 3 people were ambidextrous.

c Of 56 people surveyed about playing sport, 21 people did not play a team sport.

d Of 450 people surveyed, 300 people had less than two video game consoles.

e Of 348 people surveyed about pizza or pasta, 87 people preferred pasta.

f Of 1200 cats that had their tails measured, 655 had tails longer than 30 cm.

8D.2 **2** Find the expected number of:

a heads if a coin is tossed 250 times

b 1s or 2s if a standard six-sided die is rolled 120 times

c consonants if a letter is selected randomly from the alphabet 130 times

d 6s if a standard six-sided die is rolled 30 times

e two-digit integers if an integer from 1 to 999 is selected 444 times

f double heads if a pair of coins is tossed 856 times.

3 A standard six-sided die is rolled 150 times and the results are shown in this table.

Outcome	1	2	3	4	5	6
Frequency	38	32	19	24	21	16

a Find the relative frequency of rolling a number greater than 2, based on this data. Give your answer as a fraction and a decimal rounded to two decimal places.

b Assuming the die is not biased, describe how you would expect the relative frequency of rolling a number greater than 2 to change in the long term.

4 A standard pack of 52 cards has four suits (diamonds, clubs, hearts and spades), with each suit consisting of 10 number cards (Ace to 10) and three picture cards (Jack, Queen and King).

A card is chosen at random from a pack and replaced. This is repeated 129 times for a total of 130 selections.

a What is the expected number of picture cards?

b If seven picture cards are obtained, find the relative frequency of selecting a picture card. Give your answer as a fraction and a decimal correct to three decimal places.

c Assuming the deck is not biased, describe how you would expect the relative frequency of selecting a picture card to change in the long term.

8D.3 **5** Consider each of these situations.

a Rolling a 4, if a standard six-sided die is rolled 180 times and a 4 is obtained 27 times

b Tossing a tail, if a coin is tossed 36 times and a head is obtained 16 times

c Rolling an odd number, if a standard six-sided die is rolled 200 times and an odd number is obtained 53 times

i Find the expected number of outcomes. Where necessary, give your answer to the nearest whole number.

ii Comment on whether you believe the experiment may be biased.

UNDERSTANDING AND FLUENCY

6 A number of experiments were performed and their results recorded below. For each experiment, find the number of times each outcome occurred.

a Total number of trials = 40

Outcome	Heads	Tails
Relative frequency	0.625	0.375

b Total number of trials = 120

Outcome	1	2	3	4	5	6
Relative frequency	0.15	0.2	0.175	0.1	0.125	0.25

c Total number of trials = 60

Outcome	Hearts	Diamonds	Clubs	Spades
Relative frequency	0.2	0.3	0.35	0.15

PROBLEM SOLVING AND REASONING

7 A magician uses a number of props in his show to demonstrate magic tricks. He tosses a coin, rolls a six-sided die and selects a card from a pack. To test whether the props are fair (standard) or biased, the results over several shows were recorded and are shown in the tables below.

a

Outcome	Heads	Tails
Frequency	8	2

b

Outcome	1	2	3	4	5	6
Frequency	966	971	1036	994	1031	1002

c

Outcome	Hearts	Diamonds	Clubs	Spades
Frequency	38	31	19	24

For each experiment:

- **i** find the total number of trials
- **ii** state the theoretical probability of each outcome
- **iii** calculate the expected number of each outcome
- **iv** calculate the relative frequency of each outcome based on the data
- **v** state whether you think the prop used is fair, biased or if there are not enough trials to make a firm decision
- **vi** give a reason to support your answer to part **v**.

8 Three coins (5c, 10c and 20c) are tossed one at a time.

a There are 8 possible outcomes; for example, HTH represents a head, tail and head outcome. List all the possible outcomes.

b What is the theoretical probability of tossing a 'triple'?

c How many 'triples' would you expect to get if you performed 40 trials?

The experiment is performed 40 times and 5 triples are recorded.

d Does the actual occurrence of a 'triple' match the expected number?

e Describe how you would expect the relative frequency to change if you performed 4000 trials.

PROBLEM SOLVING AND REASONING

9 A four-sided die (numbered 1–4) is rolled 20 times and the number 2 appears uppermost on 13 occasions. Explain why, considering the law of large numbers, this does not mean that the die is biased, and what should be done to check for any bias.

10 Tulio reads a headline that said '51% of adults worldwide don't get enough sleep'. Concerned for his parents' health, he asks if they both get the recommended 7–8 hours of sleep, which they do. Tulio is now sceptical of the headline.

a Explain why Tulio's sample would not verify or refute the statistics in the headline regardless of his parents' responses.

Later, Tulio finds another article that says 'one in three Australians not getting enough sleep'. He believes this is more accurate because he only asked two people, so the third person he asks won't be getting enough sleep.

b Assuming the statistic is accurate, explain why Tulio would not be guaranteed to have a person that is not getting enough sleep next.

c Give at least two reasons why the statistics from the two headlines are different.

11 Miguel read two similar headlines '39% of cricket fans are female' and '20% of cricket fans are female'. After reading the articles more closely, Miguel found out that the 39% group comprised more than 19 000 interviews, whereas the 20% group was from a sample of 2000. Which percentage is more reliable? Why?

12 Julia read an article that said 'about 10% of the world population is left-handed'. The article also mentions that it was only about 4% in 1920 when left-handedness was stigmatised. Explain why we do not know the exact percentage of the world population that is left-handed.

13 A toothpaste company claims that 'more than 80% of dentists would recommend' their company. Explain how this statistic is misleading.

CHALLENGE

14 Sometimes people fake data, because it can be either time-consuming or costly to collect primary data. Consider the instance of faking data on coin tosses, where people who create fake lists of results often avoid putting in long streaks of heads or tails because they think it would not happen in reality.

a If you toss a coin 7 times, what is the probability you get 7 heads?

b Explain why you would expect to get at least one streak of 7 or more heads if you toss a coin 1000 times.

c If you toss a coin 1000 times, how many streaks of exactly 5 heads in a row do you expect?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Relative frequency	**Investigation** Conducting chance experiments	**Investigation** Conducting probability simulations	**Topic quiz** 8D

8E Two-way tables

Learning intentions

By the end of this topic you will be able to ...

- ✔ create two-way relative frequency tables
- ✔ use two-way tables to solve problems.

Inter-year links

Year 8	9E Two-way tables
Year 10	10C Two-way tables and Venn diagrams

Two-way tables

- **Two-way tables** display the results of a survey or experiment with two different categories. The results can be displayed as raw numbers or as relative frequencies. For example, both of these two-way tables display the same data.

	Dark hair	Light hair	Total
Dark eyes	16	4	20
Pale eyes	8	12	20
Total	24	16	40

	Dark hair	Light hair	Total
Dark eyes	0.4	0.1	0.5
Pale eyes	0.2	0.3	0.5
Total	0.6	0.4	1

Identifying the required outcome

- The language used to describe an outcome can help you to identify which cells of the two-way table are being referred to.
  - → 'and' means both outcomes must be met. For example, 16 people have dark eyes and dark hair.
  - → 'or' can be inclusive, meaning if both outcomes are met the outcome is favourable. For example, 4 + 8 + 12 = 24 have pale eyes or light hair.
  - → 'or' can also be exclusive, meaning if both outcomes are met but the outcome is not favourable. For example, 4 + 8 = 12 people have pale eyes or light hair but not both.

Example 8E.1 Understanding a two-way table

Consider this two-way table, which shows whether Year 8 and Year 9 students prefer jam or Vegemite.

a How many students were surveyed?

b How many students are in Year 9 and prefer Vegemite?

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

THINK	WRITE
a The total number of people surveyed is displayed in the bottom right-hand corner.	**a** 125 students were surveyed.
b Identify the cell that is in the 'Year 9' column and the 'Vegemite' row.	**b** 34 students are in Year 9 and prefer Vegemite.

Example 8E.2 Creating a two-way relative frequency table

This two-way table shows whether or not a group of 50 people from Melbourne and Perth own a pet. Convert it into a two-way relative frequency table.

	Melbourne	Perth	Total
Owns pet	13	19	32
Does not own pet	11	7	18
Total	24	26	50

THINK

Divide the values in each cell by the value in the total cell in the bottom right-hand corner.

The totals should add to 1 in the bottom right-hand corner.

WRITE

	Melbourne	Perth	Total
Owns pet	$\frac{13}{50} = 0.26$	$\frac{19}{50} = 0.38$	$\frac{32}{50} = 0.64$
Does not own pet	$\frac{11}{50} = 0.22$	$\frac{7}{50} = 0.14$	$\frac{18}{50} = 0.36$
Total	$\frac{24}{50} = 0.48$	$\frac{26}{50} = 0.52$	1

Example 8E.3 Creating and completing a relative frequency two-way table

A survey collected information on whether motorists from the country and city drive automatic or manual cars. Of the respondents, 45% are from the city and drive automatic cars, 43% are from the country and 34% drive manual cars.

Use this information to complete a relative frequency two-way table.

THINK

1. Draw the table with one of the categories (car type) placed in the rows and the other category (location) placed in the columns.
2. Convert the percentages to relative frequencies by dividing by 100, e.g. 43% = 0.43.
3. Insert the given information in the relevant cells. Remember that the total in the bottom right-hand corner is always 1.
4. The numbers in each row and column add to give the value in the total cells, so wherever there are two out of three cells in a row or column, the unknown value can be found by using addition or subtraction.

WRITE

	City	Country	Total
Automatic	0.45	0.66 – 0.45 = 0.21	1 – 0.34 = 0.66
Manual	0.57 – 0.45 = 0.12	0.43 – 0.21 = 0.22	0.34
Total	1 – 0.43 = 0.57	0.43	1

Example 8E.4 Calculating probability using a two-way table

Consider the two-way table in example 8E.1, showing whether Year 8 and 9 students prefer jam or Vegemite. What is the probability of a student chosen randomly from the group being in Year 8 or preferring jam (or both)?

	Year 8	Year 9	Total
Jam	28	38	66
Vegemite	25	34	59
Total	53	72	125

THINK	WRITE
1 Identify the relevant cells in the table that represent this outcome (any cell in the Year 8 column or Jam row).	$\frac{28 + 38 + 25}{125} = \frac{91}{125}$
2 Divide the total of these cells by the total in the table. Simplify if possible.	

Helpful hints

✔ When adding the totals in rows and columns, be careful not to double the cells where the rows and columns intersect.

✔ In some problems it may appear that there is insufficient information to answer the question. Start by writing down what you know and then use this information to find the unknown values.

ANS
p479

Exercise 8E Two-way tables

△ 1, 3–10, 12 ■ 2, 4–9, 11, 14, 15(a, b), 16(a–d) ⬟ 4, 6–9, 11, 13, 15(b, c), 16, 17

UNDERSTANDING AND FLUENCY

8E.1
8E.4

1 Consider this two-way table that shows whether students in primary school (PS) and high school (HS) prefer to watch or play sport.

a How many students were surveyed in total?

b How many students in high school prefer to watch sport?

c What is the probability of a student chosen randomly from the group being in high school and preferring to watch sport?

	PS	HS	Total
Watch sport	8	7	15
Play sport	22	18	40
Total	30	25	55

2 Consider this two-way table that shows the hair colours and eye colours of a group of people.

a How many people were surveyed in total?

b How many people with dark hair have blue/green eyes?

c What is the probability of a person chosen randomly from the group having dark hair or blue/green eyes but not both?

	Fair	Dark	Total
Blue/green	23	11	34
Brown	6	35	41
Total	29	46	75

3 This two-way table shows whether Year 8 and Year 9 students prefer alternative or mainstream music. Use this two-way table to find the probability, as a simplified fraction, of a person chosen randomly from the group:

	Year 8	**Year 9**	**Total**
Alternative	14	23	37
Mainstream	29	19	48
Total	43	42	85

a being a Year 8 student who prefers mainstream music

b preferring alternative music

c being a Year 9 student

d being a Year 9 student who prefers alternative music.

4 This two-way table shows whether Year 7, 8 and 9 students prefer tennis, basketball or hockey. Use this table to find the probability, as a simplified fraction, that a person chosen randomly from this group:

	Year 7	**Year 8**	**Year 9**	**Total**
Tennis	23	26	37	86
Basketball	19	42	34	95
Hockey	31	13	25	69
Total	73	81	96	250

a is in Year 9

b plays basketball

c is in Year 8 and plays tennis

d plays hockey or tennis

e does not play hockey

f is in Year 7 and doesn't play basketball.

8E.2 **5** This two-way table shows whether students from two classes prefer sweet or savoury food. Convert the two-way table into a two-way relative frequency table.

	Class A	**Class B**	**Total**
Prefer sweet food	23	32	55
Prefer savoury food	29	16	45
Total	52	48	100

6 This two-way table shows whether a group of introverts and extroverts prefer to read books, play games or play sports. Convert the two-way table into a two-way relative frequency table. Write your answers as fractions in their simplest form.

	Reads books	**Plays games**	**Plays sports**	**Total**
Introvert	56	44	32	132
Extrovert	12	44	68	124
Total	68	88	100	256

8E.3 **7** A survey collected information on whether people with light or dark hair preferred pop music or rock music. Of the respondents, 22% had light hair and preferred pop music, 65% had dark hair and 40% preferred rock music.

Use this information to complete a relative frequency two-way table.

8 A survey collected information on whether people who subscribe to Spotify also subscribe to Netflix. Of the respondents, $\frac{1}{6}$ had a subscription to both Spotify and Netflix, $\frac{1}{4}$ had a subscription to Spotify, and $\frac{1}{3}$ did not have a subscription to Netflix. Use this information to complete a relative frequency two-way table. Write your answers as fractions in their simplest form.

9 **a** Copy and complete this two-way table, which shows whether people prefer to watch films at the cinema or at home and whether they prefer action or comedy films.

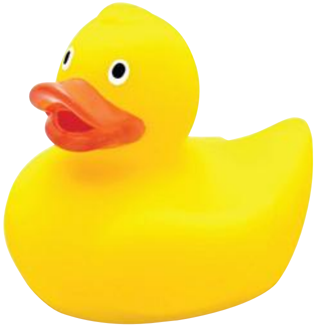
	Cinema	**Home**	**Total**
Action	22		
Comedy		14	33
Total			85

b Use it to calculate the probability, as a simplified fraction, that a person chosen randomly from the group is someone who prefers to:

i watch films at the cinema **ii** watch comedy films at home **iii** watch action films.

10 A group of 200 people, each with a single pet, were surveyed on their pets. Of the 113 who owned cats, 29 had a purebred. This gives a total of 104 pet owners who owned a purebred. Create a two-way table showing this information and use it to calculate the probability, as a simplified fraction, that a person chosen at random is an owner of a purebred that is not a cat.

11 In a group of people who were surveyed on their bathing habits, 60% of women preferred a bath to a shower, whereas 80% of men said they preferred a shower rather than a bath; 55% of the group was female.



a Create a relative frequency two-way table to represent this information using decimals. Hint: The statement ‘60% of women prefer a bath’ refers to 60% of the proportion of women, not the total.

b If a person was randomly selected from a group of 500 people, find the probability that they are:

i a male who prefers to shower

ii someone who prefers a bath

iii a female who prefers to shower.

c Of a group of 500 people, find the number of people who:

i prefer a bath to a shower **ii** are female **iii** are male and prefer a bath.

12 Elsa recorded the make and colour of cars that went past her house over a week and recorded her results as shown. Add totals to her results and then find the probability, as a simplified fraction, that a car going past her house is:

a white
b a Holden
c a white Holden
d blue or red
e not a Toyota
f not black
g neither red nor a Ford
h blue but not a Hyundai
i a Mitsubishi but not silver.

	Ford	**Hyundai**	**Holden**	**Mitsubishi**	**Toyota**	**Other**
Silver	11	9	17	8	6	12
White	15	12	16	11	8	10
Red	12	10	8	6	13	14
Blue	8	15	12	7	9	11
Black	13	8	10	10	7	8
Other	4	3	4	2	1	2

13 $n(\text{A})$ means the number of elements in category A. For the relative frequency two-way table shown, determine the population size if:

a $n(\text{A}) = 30$ **b** $n(\text{B}) = 72$ **c** $n(\text{A and B}) = 98$ **d** $n(\text{A or B}) = 102$

	A	**A'**	**Total**
B	0.1	0.35	0.45
B'	0.4	0.15	0.55
Total	0.5	0.5	1

14 Consider the relative frequency two-way table below. If the total frequency is 120, then draw the frequency two-way table.

	A	**A'**	**Total**
B	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
B'	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{5}{12}$
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

PROBLEM SOLVING AND REASONING

15 Complete each pair of frequency and relative frequency two-way tables.

a

	A	**A'**	**Total**
B	40		75
B'			
Total			

	A	**A'**	**Total**
B	0.2		
B'			
Total	0.4		1

b

	C	**C'**	**Total**
D			9
D'		9	
Total			

	C	**C'**	**Total**
D		0.25	
D'			
Total	0.3		1

c

	E	**E'**	**Total**
F	18		
F'		12	
Total			

	E	**E'**	**Total**
F		$\frac{1}{9}$	$\frac{11}{18}$
F'			
Total			1

CHALLENGE

16 Consider this two-way table, which shows whether a group of Year 8 and Year 9 students prefer Maths or English.

	Year 8	**Year 9**	**Total**
Maths		11	
English	10		
Total	22		50

a Copy and complete the table.

b Use it to find the probability that a student selected randomly from the group:

- **i** prefers Maths
- **ii** is in Year 9
- **iii** is in Year 9 and prefers Maths.

c A student is selected randomly from the group. You know that they are in Year 9.

- **i** How many students are in Year 9?
- **ii** How many students in Year 9 prefer Maths?
- **iii** What is the probability that a student in Year 9 prefers Maths?

d What is the difference between parts **b iii** and **c iii**?

The problem represented in part **c iii** is an example of conditional probability. It looks at the probability of an outcome given certain conditions. In part **c iii**, you are looking for the probability that a student likes Maths, given that they are in Year 9. This means that you consider only the limited group of the condition rather than the entire population.

e Use these steps to calculate the probability of a student being in Year 8, given that they prefer English.

- **i** What is the condition? How many students are in this group?
- **ii** What is the specific group you are after? Hint: It is not just somebody in Year 8.
- **iii** How many students are in this specific group?
- **iv** Use your answers to parts **i** and **iii** to calculate the probability of selecting somebody in Year 8, given that they prefer English.

17 Use the two-way table in question **4** to calculate the probability of randomly selecting a person from the group that:

- **a** plays hockey, given that they are in Year 8
- **b** plays tennis, given that they are in Year 9
- **c** is in Year 7, given that they play basketball
- **d** is in Year 8, given that they play tennis
- **e** is in Year 9, given that they *don't* play hockey
- **f** is not in Year 7, given that they play basketball.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Two-way tables

Topic quiz
8E

8F Venn diagrams

Learning intentions

By the end of this topic you will be able to ...

✔ use set notation to represent sets of data

✔ create Venn diagrams to represent two or more sets of data

✔ calculate the probabilities of outcomes using data displayed in Venn diagrams.

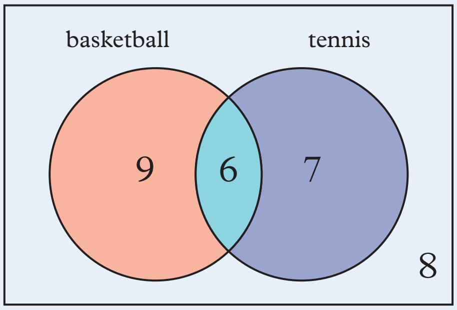
Inter-year links

Year 8	9F Venn diagrams
Year 10	10F Conditional probability

Venn diagrams

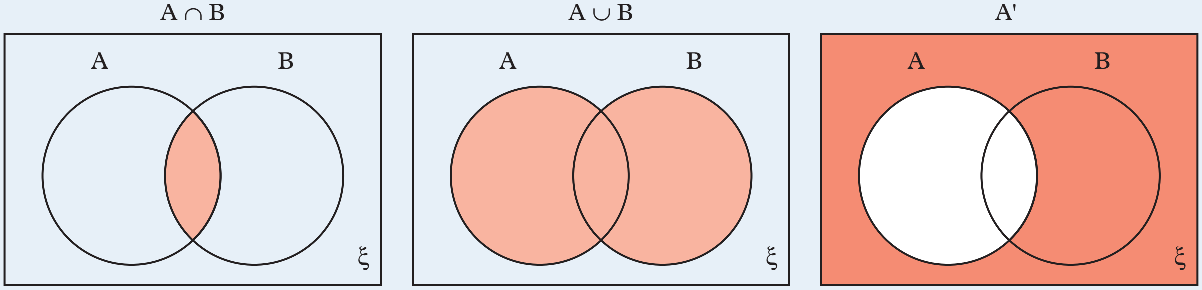
- In mathematics, a **set** is a collection of distinct objects.
- A **Venn diagram** is used to display the relationship between different sets of data.
- In a Venn diagram, each circle represents a different set and the rectangle containing the circles represents the entire data set.
- Frequencies are placed within Venn diagrams to represent the number of data points for each region of the diagram.

  For example, the Venn diagram on the right shows whether Year 9 students play either basketball or tennis. 6 students play both basketball and tennis (the region where these sets overlap).

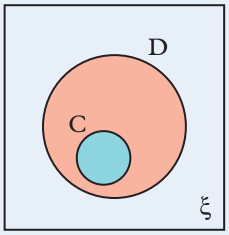


Set notation

- All elements contained in the rectangle of a Venn diagram belong to the **universal set**, which is represented by the Greek letter xi (ξ).
- $A \cap B$ means the **intersection** of sets A and B, which includes only the elements that are in A and are also in B. This can be read as 'A and B'.
- $A \cup B$ means the **union** of sets A and B, which includes all elements in either A or B or both. This can be read as 'A or B'.
  - → The intersection ($A \cap B$) and union ($A \cup B$) of sets A and B are also sets themselves.
- A' means the **complement** of A, which includes all elements not in A.



- $C \subset D$ means C is a **subset** of D. This means that set C is contained within set D and every element in set C is also in set D.



Example 8F.1 Understanding Venn diagrams

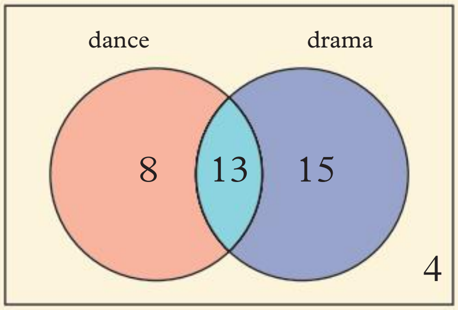
This Venn diagram shows the results of a survey where people were asked if they attend optional dance and drama classes.

a How many people attend both the dance and drama classes?

b How many people attend the drama class?

c How many people do not attend the dance class?

d How many people were surveyed in total?



THINK	WRITE
a Identify the required region, which is the intersection of both sets. This is the middle section where the two circles overlap.	**a** 13 people attend both classes.
b Identify the required region, which is the ‘drama’ circle. Add the numbers in this region.	**b** 13 + 15 = 28 attend the drama class.
c Identify the required region, which is everything outside the ‘dance’ circle. Add the numbers in this region.	**c** 15 + 4 = 19 people do not attend the dance class.
d Find the total of all numbers in the Venn diagram.	**d** 8 + 13 + 15 + 4 = 40 people were surveyed in total.

Example 8F.2 Using Venn diagrams to calculate relative frequencies

Use the Venn diagram from Example 8F.1 to find the relative frequency that a person chosen randomly from the survey group attends the dance class.

THINK	WRITE
1 Identify the required region, which is the ‘dance’ circle. Add the numbers in this region.	8 + 13 = 21 people attended the dance class.
2 Find the total number of people who were surveyed by adding all the numbers in the Venn diagram (don’t forget the number outside the circles).	8 + 13 + 15 + 4 = 40 were surveyed.
3 To determine the relative frequency, divide the number in the required region by the total number.	Relative frequency $= \frac{21}{40}$ $= 0.525$

Example 8F.3 Creating a Venn diagram

In a group of 40 people, 15 have a cat and 22 have a dog. There are 11 people who do not have either a cat or a dog. Draw a Venn diagram to represent this situation.

THINK

1 Draw two circles, one representing cats and the other representing dogs. Draw a rectangle around the two circles.

2 Place the number of people who do not have either a cat or a dog (11) in the rectangle outside the circles.

3 Determine the number of people who have a cat only by subtracting the number of people who have a dog (22) and the number of people who do not have either a cat or a dog (11) from the total (40).

4 Repeat step 3 to determine the number of people who have a dog only.

5 Determine the number of people who have a cat and a dog by subtracting the number of people who have a dog only (14) from the number of people who have a dog (22).

WRITE

Number of people who have a cat only

$= 40 - 22 - 11$

$= 7$

Number of people with a dog only

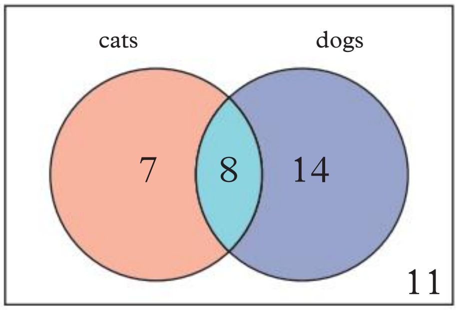
$= 40 - 15 - 11$

$= 14$

Number of people with a cat and a dog

$= 22 - 14$

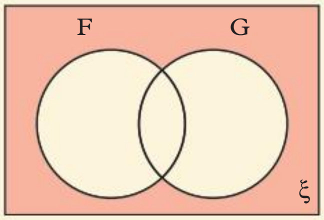
$= 8$



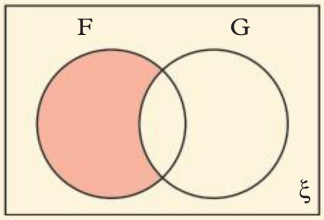
Example 8F.4 Using set notation

Use set notation to describe the following regions in this Venn diagram.

a



b



THINK

a 1 Look at the shaded region. This represents the elements not in F and not in G.

2 Use set notation to represent the region. Use the complement symbol G′ for ‘not in G’ and the symbol ∩ for ‘and’.

b 1 Look at the shaded region. This represents the elements in F and not in G.

2 Use set notation to represent the region.

WRITE

a $F' \cap G'$

b $F \cap G'$

Note: There is sometimes more than one way to describe a region. For example, the region in part **a** could also be described as $(F \cup G)'$.

Helpful hints

✔ When creating Venn diagrams, check that the total in all the individual regions equals the number in the entire data set.

ANS p480

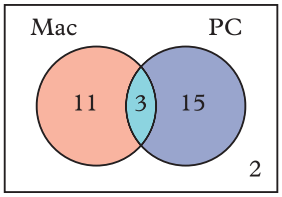
Exercise 8F Venn diagrams

△ 1, 3, 4, 6–8, 11, 12 ■ 2, 4–9, 12, 13, 16 ⬟ 5, 7–10, 12–15, 17

UNDERSTANDING AND FLUENCY

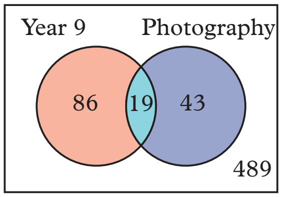
8F.1 1 Consider the Venn diagram on the right that shows whether people own a Mac or a PC.

- **a** How many people own a Mac?
- **b** How many people own a PC but not a Mac?
- **c** How many people don't own either a Mac or a PC?
- **d** How many people were surveyed in total?



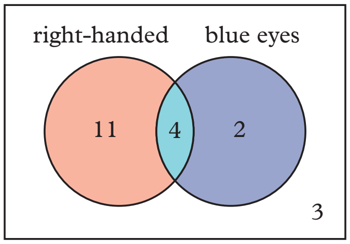
2 Consider the Venn diagram on the right that shows whether people are in Year 9 and whether they take a Photography class.

- **a** How many people take Photography?
- **b** How many people are in Year 9 and take Photography?
- **c** How many people are not in Year 9?
- **d** How many people were surveyed in total?



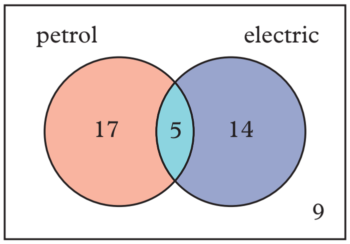
8F.2 3 This Venn diagram shows whether people are right-handed and whether they have blue eyes. Use this diagram to find the relative frequency of a person:

- **a** being right-handed with blue eyes
- **b** having blue eyes
- **c** being right-handed with eyes not blue
- **d** not having blue eyes
- **e** being left-handed with blue eyes
- **f** being left-handed.



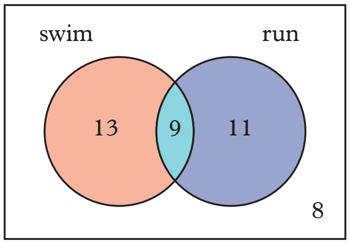
4 This Venn diagram shows whether people drive petrol or electric cars. Use this diagram to find the relative frequency of a person who:

- **a** drives petrol cars only
- **b** drives both petrol and electric cars
- **c** drives electric cars
- **d** drives neither
- **e** does not drive petrol cars
- **f** drives petrol or electric cars (but not both).



5 This Venn diagram shows whether people like to swim or run for exercise. Use this diagram to find the relative frequency of a person who:

- **a** swims and runs
- **b** does not run
- **c** runs but does not swim
- **d** does not swim or run
- **e** swims
- **f** swims or runs.

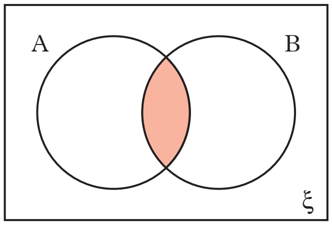


8F.3 6 In a group of 50 students, 24 are in Year 9, 19 walk to school, and 16 are not in Year 9 and do not walk to school. Draw a Venn diagram to represent this situation.

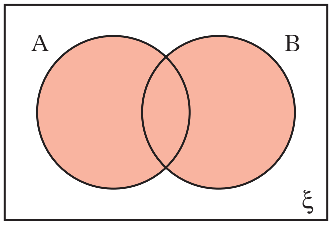
7 In a group of 160 students, 78 take music, 95 take drama, and 46 take music and drama. Draw a Venn diagram to represent this situation.

8F.4 **8** Use set notation to describe the following shaded regions in the Venn diagrams below.

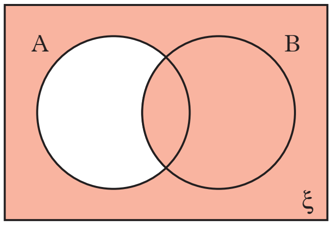
a



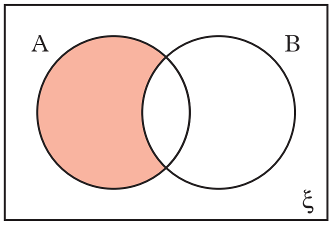
b



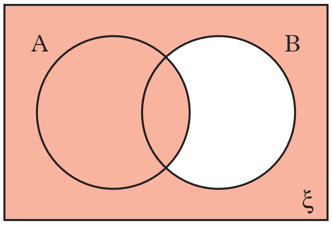
c



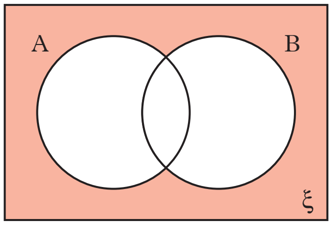
d



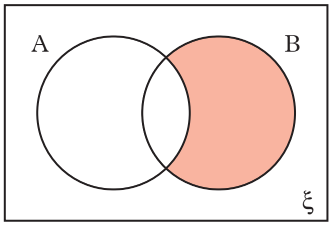
e



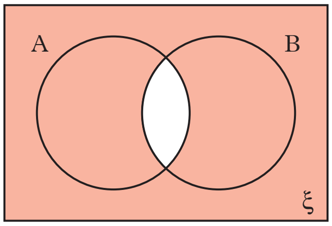
f



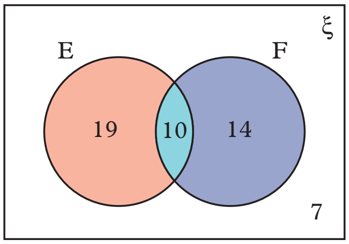
g



h



9 Consider the Venn diagram on the right.



a Given that $n(F)$ means the number of elements in set F, find:

i $n(E)$ **ii** $n(E \cup F)$ **iii** $n(F')$

iv $n(E \cap F)$ **v** $n(\xi)$ **vi** $n(E' \cap F)$.

b Given that $Pr(E)$ means the probability of selecting an element from set E, find:

i $Pr(F)$ **ii** $Pr(E')$ **iii** $Pr(E \cap F)$

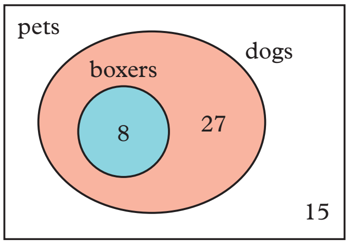
iv $Pr(E \cup F)'$ **v** $Pr(E' \cap F')$ **vi** $Pr(F \cup E')$.

10 Consider the Venn diagrams in question **8**.

a State which pairs are complementary by writing both set equalities using the symbol for the complementary event, ′. For example, $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$.

b Explain the relationship between the complements of intersections and unions. (Note: These relationships are known as De Morgan's laws.)

11 Venn diagrams can be used to show the relationship between three or more sets and also include subsets. Consider these two Venn diagrams, the first of which details the pets of pet owners, and the second of which shows whether students take any of three elective subjects at school.

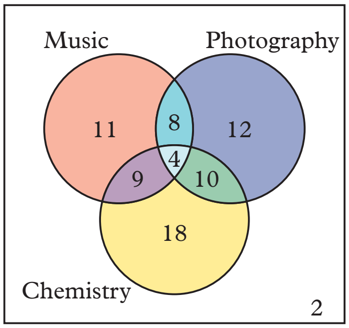


a Which Venn diagram shows subsets of data? Explain the relationship shown in this Venn diagram.

b Copy the first Venn diagram three times and, on separate copies, shade the section that represents:

i pets that are not dogs **ii** dogs that are not boxers

iii all dogs.



c The other Venn diagram shows the relationship between three different sets. Copy this Venn diagram five times and shade the section that represents students:

i taking all three elective subjects **ii** taking only Photography

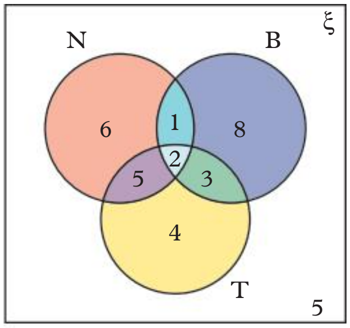
iii taking Music or Chemistry **iv** taking Music

v not taking any of these three electives.

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

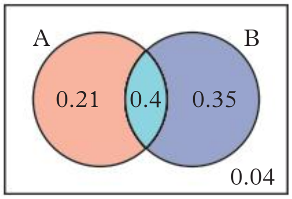


12 This Venn diagram shows the number of students who play netball, basketball and tennis. Find the probability that a student chosen randomly:

a plays netball but not tennis
b plays basketball only
c plays all three sports
d doesn't play any of these sports
e plays basketball or tennis
f plays tennis and netball.

13 Use the following information to draw a Venn diagram that shows the relationship between three different sets of data. In a group of 100 people surveyed:

- 35 liked action films
- 46 liked horror films
- 5 people liked all three genres
- 16 only liked horror
- 45 liked romance
- 55 did not like romance films
- 15 people liked both action and romance
- 18 liked both horror and romance
- 19 people did not like films in any of these three genres.

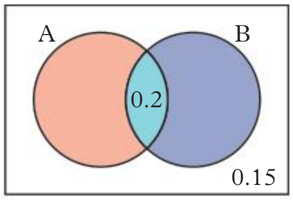


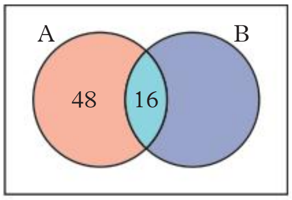
14 The Venn diagram on the right shows the relative frequency of each intersection between the events A, B and their complements. If the sample size was 400, redraw the Venn diagram using frequencies.

15 The following Venn diagrams represent the same sample of people for the events A and B.

a Complete the Venn diagrams.

b Calculate: **i** Pr(A) **ii** Pr(B)

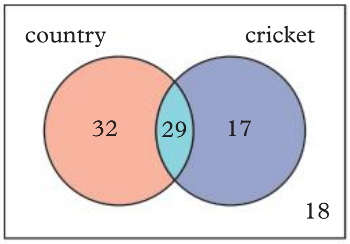




CHALLENGE

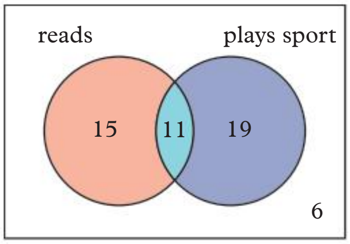
16 The probability of an event can change when new information is obtained. For example, the probability of selecting a diamond from a deck of cards is $\frac{1}{4}$. However, if you are told the card is red, then the probability it is a diamond is now $\frac{1}{2}$, based on this new information. This would usually be asked with wording such as: "What is the probability that a randomly selected card from a deck is a diamond, given that it is red?"

This Venn diagram shows whether people are from the country and if they like cricket or not. Use this diagram to calculate the probability that a person chosen randomly:



a likes cricket, given that they are from the country
b is from the country, given that they like cricket
c does not like cricket, given that they are from the country
d is not from the country, given that they do not like cricket.

17 This Venn diagram shows whether people like to read and/or play sport. Use this diagram to calculate the probability that a person chosen randomly:



a reads given that they play sport
b plays sport given that they read
c does not play sport given that they read
d does not read given that they do not play sport.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet
  Venn diagrams
- Interactive skillsheet
  Using 'and' and 'or' in probability
- Interactive skillsheet
  Set notation
- Investigation
  Venn diagram applications
- Topic quiz
  8F

Chapter summary

Theoretical probability

- Sample space: all possible outcomes
- Event: collection of one or more outcomes from the sample space
- If outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

- A' means not A. A and A' are complementary events. Pr(A) + Pr(A') = 1

Arrays

- List of items arranged in rows and columns.
- Use when there is a large number of outcomes in either step of a two-step experiment.

	1	**2**	**3**	**4**	**5**	**6**
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

Relative frequency

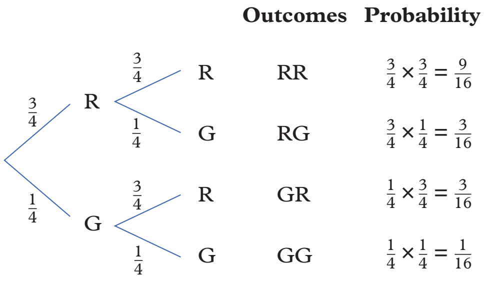
- Relative frequency = $\frac{\text{number of occurrences}}{\text{total number of trials}}$
- Expected number = theoretical probability × number of trials
- With a small number of trials, results may vary significantly from expected.
- In the long term, the relative frequency will approach the theoretical probability.

Experiments with/ without replacement

- With replacement: probabilities for each step remain the same.
- Without replacement: probabilities for all remaining steps are changed.
- Tree diagrams and lists of outcomes can help to find the probabilities of multi-step experiments.

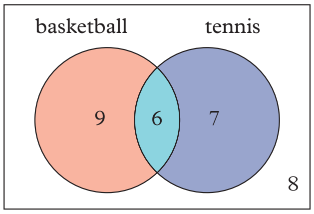
Combining like outcomes

- When grouping like outcomes, the probability of the grouped outcome is the sum of the probabilities in the group.
- The probabilities of final outcomes can be found by multiplying the probabilities leading to that outcome.



Venn diagrams

- Each region represents a different set.
- Rectangle represents the entire data set.
- Frequencies represent number of data points for each region of the diagram.



Two-way tables

- Raw numbers or relative frequencies.

	Dark hair	**Light hair**	**Total**
Dark eyes	0.4	0.1	0.5
Pale eyes	0.2	0.3	0.5
Total	0.6	0.4	1

- ‘and’: both outcomes must be met
- Inclusive ‘or’: if both outcomes are met the outcome is favourable.
- Exclusive ‘or’: if both outcomes are met the outcome is not favourable.

Set notation

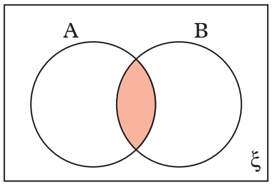
- Everything within the rectangle of a Venn diagram belongs to the universal set, ξ.

A ∩ B is the intersection of sets A and B (A and B)

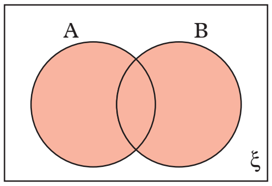
A ∪ B is the union of sets A and B (A or B)

A' is the complement A (not A)

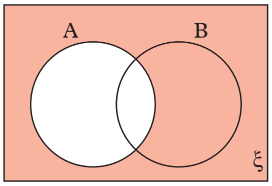
A ∩ B



A ∪ B



A'



Chapter review

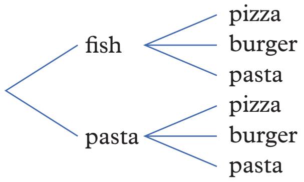
Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Multiple choice

8A **1** If there is an equal probability of choosing every option on the following menu, the probability that a person orders pasta for both entrée and main is:

Meal choices at a local cafe



A $\frac{1}{3}$ **B** $\frac{2}{3}$ **C** $\frac{1}{6}$ **D** $\frac{1}{2}$ **E** $\frac{1}{9}$

8A **2** Two numbers are chosen randomly and their product is recorded. One number is chosen from the set $\{1, 2, 3, 4, 5\}$ and the other is chosen from the set $\{2, 3, 4\}$. The probability of which pair of events is not equal?

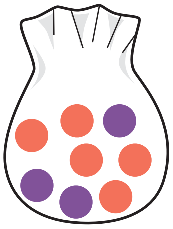
A Pr(4), Pr(8)
B Pr(odd), Pr(multiple of 5)
C Pr(multiple of 3), Pr(multiple of 2)
D Pr(square number), Pr(multiple of 10)
E Pr(prime), Pr(cube number)

8B **3** There is a $\frac{1}{13}$ chance of selecting an Ace from a standard pack of playing cards. A card is selected at random and an Ace is chosen. The card is replaced into the pack. What is the probability of selecting an Ace on the second selection?

A $\frac{1}{13}$ **B** $\frac{1}{169}$ **C** $\frac{12}{13}$ **D** $\frac{144}{169}$ **E** $\frac{3}{676}$

Questions **4** and **5** refer to the diagram on the right.

A marble is selected and not replaced and then a second selection is made.



8C **4** The first marble selected is orange. What is the probability the second marble selected is also orange?

A $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{1}{7}$ **D** $\frac{5}{7}$ **E** $\frac{2}{7}$

8C **5** The probability of selecting two purple marbles is:

A $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$ **E** $\frac{3}{4}$

8D **6** A six-sided die is rolled many times. Consider this table showing the frequencies of the outcomes of this experiment.

Number on die	Frequency
1	45
2	35
3	27
4	30
5	32
6	31

Which one of these statements is true?

A The number of trials performed was 200 rolls.
B The relative frequency of rolling an even number is $\frac{13}{25}$.
C The number rolled the least frequently was 1.
D The relative frequency of rolling a 6 is $\frac{1}{6}$.
E The expected number for rolling a 4 is 30.

8E **7** Consider the following frequency table.

	Under 30s	30 and over	Total
Prefer coffee	176	98	
Prefer tea	101	25	126
Total	277		400

The relative frequency of a person preferring coffee is:

A $\frac{137}{200}$ **B** $\frac{49}{200}$ **C** $\frac{11}{25}$ **D** $\frac{101}{400}$ **E** $\frac{176}{277}$

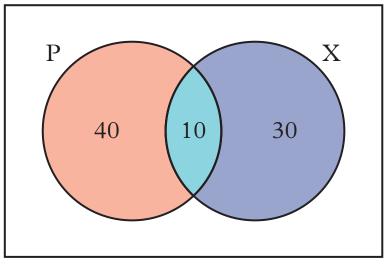
8E **8** This two-way table is to be converted to a relative frequency two-way table. Which relative frequency will not appear?

	A	A'	Total
B	5	5	10
B'	7	8	15
Total	12	13	25

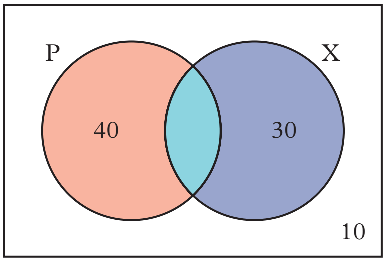
A 0.2 **B** 0.28 **C** 0.52 **D** 0.5 **E** 0.4

8F **9** A group of 80 students were asked if they owned a PlayStation (P) or an Xbox (X). 40 students owned a PlayStation, 30 students owned an Xbox, and 10 said they owned both. Which of the following Venn diagrams correctly displays this information?

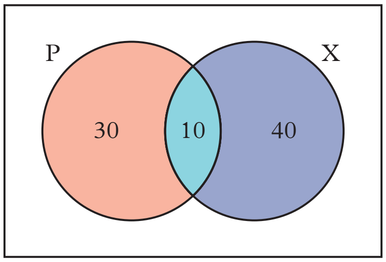
A



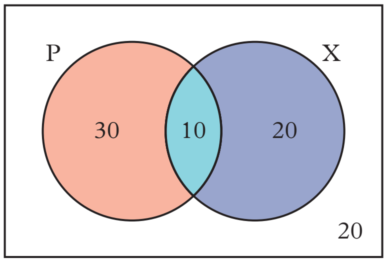
B



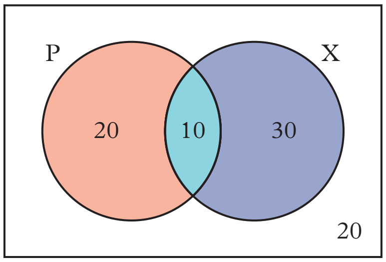
C



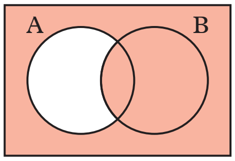
D



E



8F **10** Which of the following is represented by the shaded section in the Venn diagram?



A $A \cap B$ **B** $A \cup B$ **C** $A' \cup B$
D $A' \cap B$ **E** $A \cap B'$

Short answer

8A **1** A company runs a competition where one out of every four purchases contains the winning bar code for an ebook.

a What is the probability of not winning an ebook?

b Complete a tree diagram showing all probabilities on the branches for two trials.

c Calculate the probability of:

i winning twice in a row

ii the complement of winning twice in a row

iii winning on the first try and then not winning on the second try.

8A **2** A coin is tossed and a fair, standard six-sided die is rolled.

a Show all possible outcomes on an array.

b Calculate the probability that:

  i the coin lands on heads and the die lands on 2

  ii the coin lands on heads or the die lands on 2 or both

  iii the coin lands on heads or the die lands on an even number, but not both.

c Calculate the probability that the coin lands on heads and the die lands on a multiple of 3.

d Calculate the probability that the coin lands on heads or the die lands on a factor of 6 or both.

8B **3** A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is chosen at random and the colour is recorded. The chocolate is replaced and then another selected.

a Show all possible outcomes and probabilities on a tree diagram.

b Calculate the probability that:

  i both wrappers are orange **ii** both wrappers are green **iii** both wrappers are pink.

c Calculate the probability that the first wrapper is green.

d Calculate the probability that the second wrapper is pink.

8C **4** There are four suits (clubs, diamonds, hearts and spades) in a standard pack of 52 cards. Two cards are selected from a pack and the suits are noted. Assuming selection without replacement, calculate:

a Pr(two spades) **b** Pr(heart, then spade).

8D **5** Calculate the relative frequency of:

a a 3 being rolled, given that 45 out of 120 rolls of the die landed on 3

b the number 387 being generated when a random integer was generated 1000 times and 387 appeared 5 times

c selecting someone in a shoe store at random wearing boots when 300 people visited the shoe store in a day and 33 were wearing boots

d selecting a blue puzzle piece at random when a 1000-piece puzzle has 650 pieces with similar shades of blue on them.

8D **6** Calculate the expected number of the following.

a Rolling a 1 when a 12-sided die is rolled 420 times

b Generating 676 when a random integer from 1 to 1000 is generated 10 000 times

c *Hungry Like the Wolf* playing from a workout playlist with 73 tracks, when 365 tracks are played on random shuffle

d Rolling a double if a pair of dice is thrown 150 times while playing a board game

8E **7** For the following questions, refer to this two-way table for Years 8, 9 and 10 students and their preferred brands of make-up.

	Year 8	**Year 9**	**Year 10**	**Total**
Napoleon Perdis	32	25	45	
Rimmel	22		22	54
Covergirl	17	31	13	
Maybelline		29		
Total	120		95	310

a Copy and complete the table.

b Calculate the probability that a student chosen at random from the group:

  i prefers Maybelline **ii** is in Year 8 and prefers Rimmel

  iii is in Year 9 and prefers Napoleon Perdis.

c Calculate the probability that a student chosen from the Year 9 students:

  i prefers Napoleon Perdis **ii** prefers Covergirl or Maybelline.

8F **8** A survey of 120 secondary school students was conducted about whether they like to play video games and board games. Of the 20 Year 9 students surveyed, 8 play video games, 10 play board games, and this includes 3 who play both. Of the students in other year levels who were surveyed, 55 play video games, 60 play board games and this includes 35 who play both.

Let A be the set of Year 9 students and let V be the set of students who play video games.

a Use a Venn diagram with circles for the sets A and V to display this information.

b Calculate:

i $\Pr(A \cap V)$ **ii** $\Pr(A \cup V)$ **iii** $\Pr(A' \cap V')$ **iv** $\Pr(V)$

Let B be the set of students who play board games.

c Use a Venn diagram with circles for the sets B and V to display the information.

d Calculate:

i $\Pr(B \cap V)$ **ii** $\Pr(B' \cup V)$ **iii** $\Pr(B \cap V')$ **iv** $\Pr(B)$

Analysis

When moving from Year 9 into Year 10, students undertake core studies such as English and Mathematics, and can select from a range of electives to complete their study program.

A student must select two from these four electives: Health and Human Development (HHD), Art, Italian and Food Technology. The student writes each subject on a card and then chooses two at random from a hat.

a Would this selection be with or without replacement? Explain your reasoning.

b **i** Draw a tree diagram to represent the possible subject combinations.

ii How many combinations are possible?

c Find the probability of selecting:

i Art and Food Technology **ii** Italian and HHD.

d The senior students at the school were surveyed about the Food Technology and Art electives offered. 12% of the respondents were Year 12 students who preferred Art to Food Technology; 28% of the respondents were Year 11 students who preferred Food Technology to Art. Overall, 52% of the respondents were in Year 11.

i Construct a two-way table showing these percentages as relative frequencies.

Based on these survey results, if a person was randomly selected from the 200 students who participated in the survey, find the probability that they are:

ii a Year 12 student who prefers Food Technology

iii a Year 11 student who prefers Art

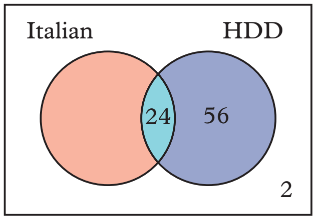
iv a student who prefers Art.

Of these 200 students, find the number of people who:

v are in Year 12 and prefer Food Technology

vi prefer Food Technology.

e An analysis of numbers of students studying Italian and/or HHD was also undertaken and the results collated into this Venn diagram.



i How many of the 150 students study both Italian and HHD?

ii How many students study neither subject?

iii How many students study Italian only?

iv If selecting a student at random from the cohort, what is the probability that they study Italian only?

v If selecting a student at random from the cohort, what is the probability that they study HHD only?

Algorithmic Thinking

Algorithmic thinking: Statistics and probability

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Once the actions in an algorithm have been identified, it may be possible to encode the algorithm in a coding language so it can be automated by a computer or machine.

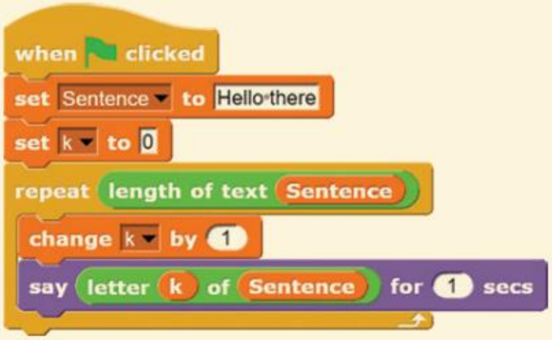
The term iteration refers to repetitive actions, as shown using pseudocode in the algorithms in Examples 1–3. Actions that need to be repeated are grouped together and indented with directives such as Repeat, For and Foreach defining the iteration. Iteration is very useful for working on lists and tables of numbers, and for generating the sample spaces for combinations of events in probability.

Example 1

The Repeat directive can be used for iteration, as shown in this example.

```
Set the variable Sentence = Hello there
Set the variable k = 0
Repeat (length of Sentence) times:
    Add 1 to k
    Report $k^{th}$ letter of Sentence
```

The example can be coded in Snap!, a free block-based coding language, using the defined variables k and Sentence.



```
when ⚑ clicked
set Sentence ▾ to Hello there
set k ▾ to 0
repeat (length of text (Sentence))
    change k ▾ by 1
    say (letter (k) of (Sentence)) for 1 secs
```

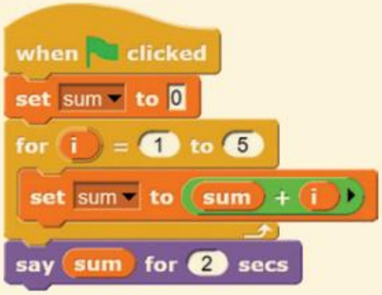
The For directive for iteration repeats a fixed number of times, for example ‘For j = 1 to 4 do’ will repeat the nested actions 4 times, setting the j variable to j = 1, j = 2, j = 3 and j = 4 as it runs.

Example 2

In this example, the For directive is used for iteration.

```
Set the variable sum to 0
For i = 1 to 5 do:
    Add i to sum
Report sum
```

This example can be coded in Snap! using the variables i and sum.



```
when ⚑ clicked
set sum ▾ to 0
for (i) = 1 to 5
    set sum ▾ to ((sum) + (i))
say (sum) for 2 secs
```

The Foreach directive repeats for each element in a list, for example ‘Foreach letter in {A,B,C} do’ will repeat the nested actions 3 times setting the letter variable to letter = A, letter = B and letter = C as it runs.

Task 1

1 Identify the repeated actions in each of the algorithms defined in Examples 1 and 2.

2 What are the outputs reported for the algorithms defined in Examples 1 and 2?

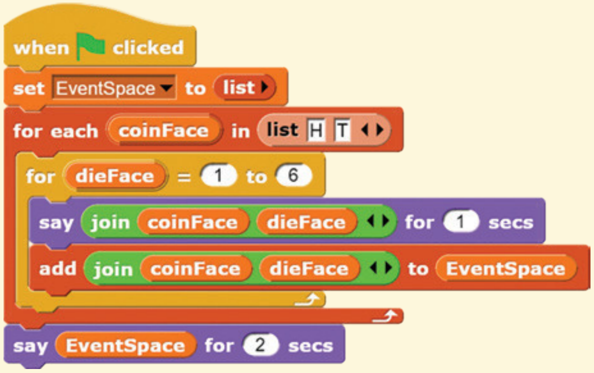
Iterations defined with Repeat, For and Foreach can also be nested within themselves for defining more complicated algorithms in pseudocode.

Example 3

This example shows how you can use nested iteration with the Foreach and For directives.

```
    Create empty list variable SampleSpace
    Foreach coinFace in {H,T} do:
        For dieFace = 1 to 6 do:
                Report coinFace, dieFace
                Add coinFace, dieFace to SampleSpace
    Report SampleSpace
```

This example can be coded in Snap! using the variables coinFace, dieFace and SampleSpace.



```
when ⚑ clicked
set EventSpace to list
for each coinFace in list H T
    for dieFace = 1 to 6
        say join coinFace dieFace for 1 secs
        add join coinFace dieFace to EventSpace
say EventSpace for 2 secs
```

Task 2

1 Identify the nested repeated actions in the algorithm defined in Example 3.

2 What is the output reported for the algorithm in Example 3? Hint: the variable names are describing the reported values.

Task 3

At a fashion show there are 11 tops labelled in uppercase {A, B, C, D, E, F, G, H, I, J, K} and 10 bottoms labelled in lowercase {q, r, s, t, u, v, w, x, y, z}. Any top can be paired with any bottom to make an outfit.

1 How big is the sample space for the outfit combinations in the fashion show?

2 Using ideas from Examples 1–3, create an algorithm in pseudocode to create the sample space for the tops and bottoms in the fashion show.

3 **CHALLENGE:** Code your sample space algorithm in Snap! or another coding language. Check the size of the sample space.

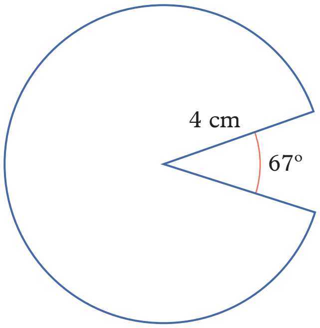
4 What is the theoretical probability that an outfit combination will be picked out at random?

Semester 2 review

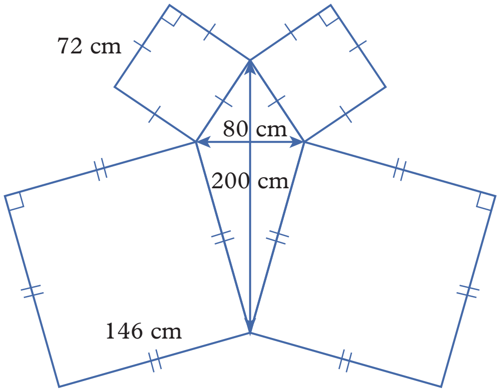
Short answer

1 Calculate the areas of these composite shapes, correct to two decimal places.

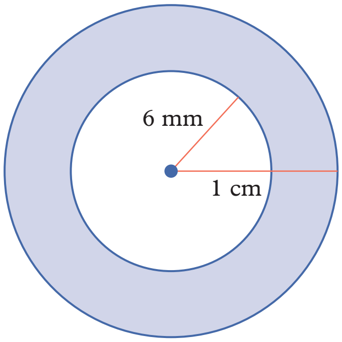
a



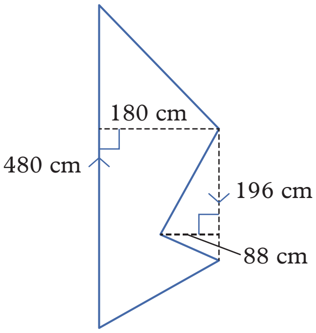
b



c

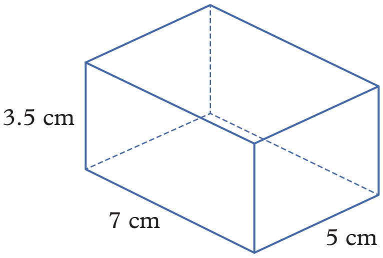


d

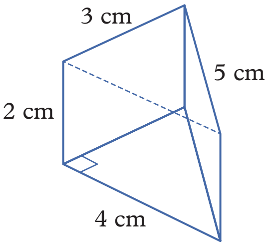


2 Calculate the surface area of the following objects, correct to one decimal place where required.

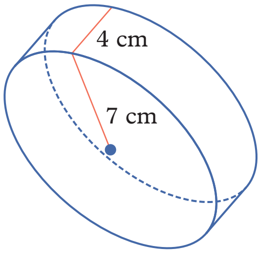
a



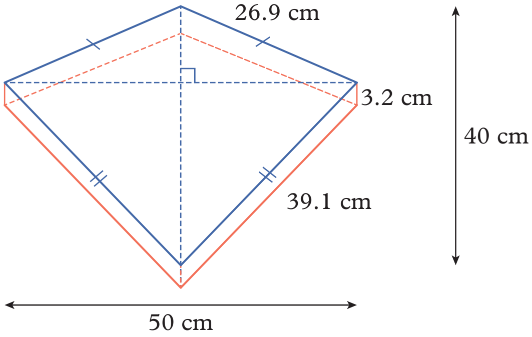
b



c



d



3 Calculate the volume of the objects in question **2**, correct to one decimal place where required.

4 Find the absolute value of the following:

a $|1 - 3|$ **b** $|2.4 - 1|$ **c** $|-1 - 2.4|$

5 Find the actual measurements given the following lower and upper bounds:

a 24.5 mL and 25.5 mL **b** 54.95 g and 55.05 g **c** 4.55 m and 4.65 m

6 A scale drawing is drawn to a scale of 1 : 200. What is the size of each of these rooms given the size in the scale drawing?

a Bathroom (1.6 cm by 1.3 cm)

b Bedroom (2.4 cm by 1.8 cm)

c Kitchen (2.8 cm by 1.6 cm)

7 The exchange rate between Australian dollars (AUD) and United States dollars (USD) is 0.75. How many US dollars are equivalent to the following amounts of Australian dollars?

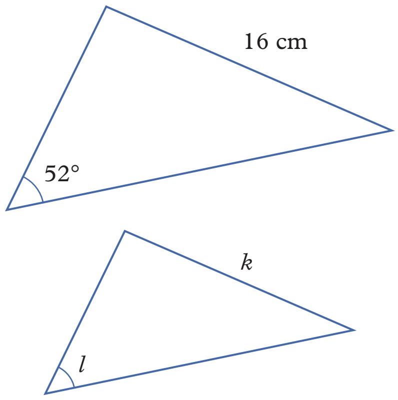
a \$8000 AUD

b \$420 AUD

c \$2250 AUD

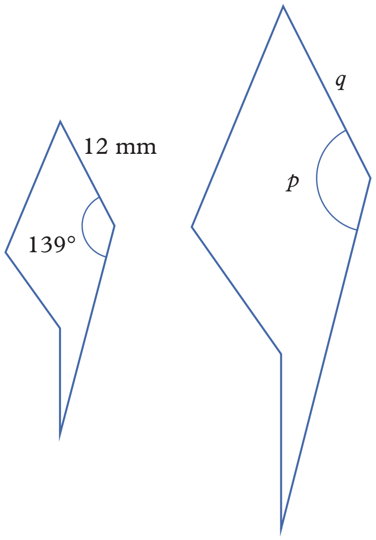
8 The shapes with labelled measurements have been dilated by the given scale factors to create the other shapes. Determine the value of the unknown pronumerals.

a



Scale factor = $\frac{3}{4}$

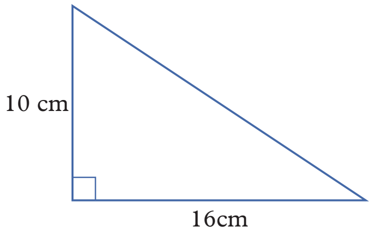
b



Scale factor = 2.5

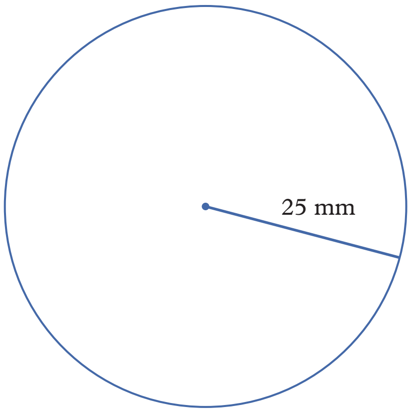
9 For each shape, state the area of the image if the original figure is dilated by the given scale factor. Give your answer to two decimal places where necessary.

a



Scale factor = 2

b

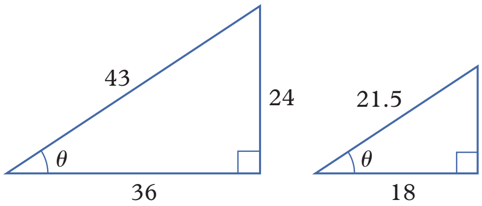


Scale factor = $\frac{1}{4}$

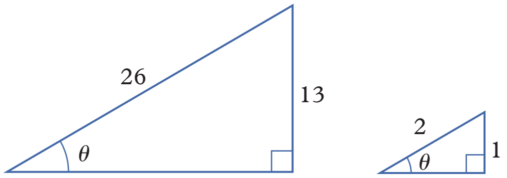
10 For each of the following pairs of similar triangles state the following values. Where appropriate, round to four decimal places.

i the scale factor (original on left) **ii** $\sin(\theta)$ **iii** $\cos(\theta)$ **iv** $\tan(\theta)$

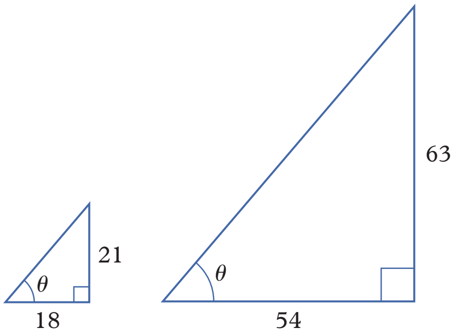
a



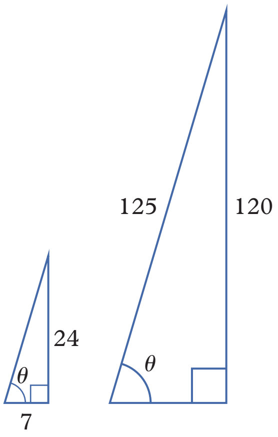
b



c

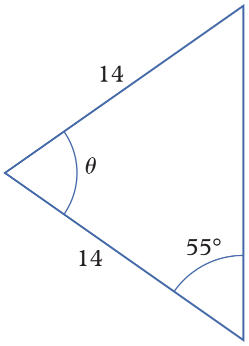


d

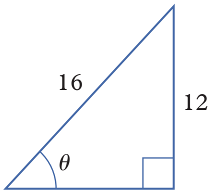


11 Determine the value of θ in each of the following, correct to the nearest degree.

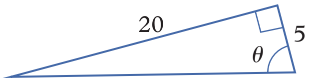
a



b



c



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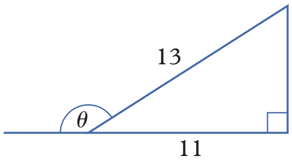
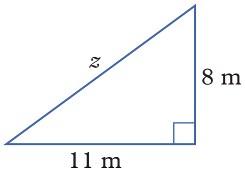
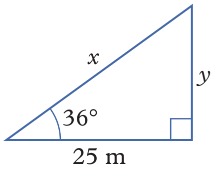


12 Determine the length of the unknown side lengths, correct to three significant figures.

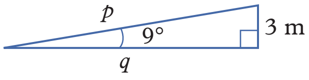
a



b



c



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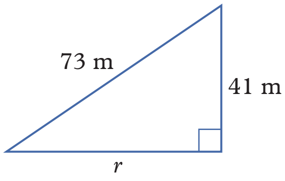


13 Determine whether these side lengths can form right-angled triangles.

a 7.7 cm, 41.7 cm, 42.7 cm

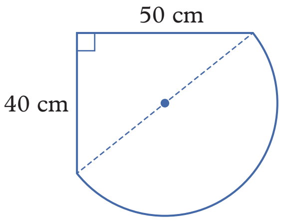
b 29 mm, 128 mm, 131 mm

c 20 cm, 48 cm, 52 cm

d 4 m, 4.2 m, 5.8 m

14 Calculate the area of these shapes, correct to one decimal place.

a



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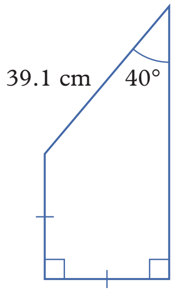
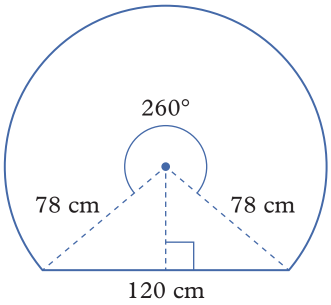


c



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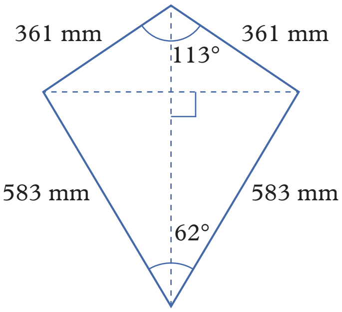
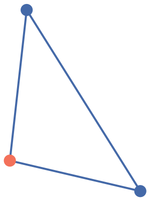


15 Dilate each figure using the orange centre of dilation and the scale factor provided.

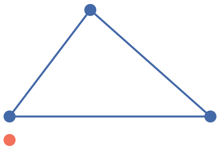
a

Scale factor: $\frac{1}{2}$



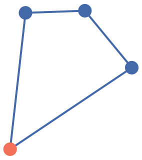
b

Scale factor: 2



c

Scale factor: 3



16 Classify each of the following variables as:

i categorical or numerical

ii ordinal, nominal, discrete or continuous.

a Name of train line

b Distance to Southern Cross Station

c Number of passengers on the train

17 Draw an appropriate frequency table to represent each of these numerical data sets.

a 24.9, 31.1, 26.8, 19.2, 19.1, 29.8, 26,
25.5, 16, 23.7, 18.1, 25.8, 23, 18.3,
7.7, 15.3, 26.7, 27.6, 6.6, 23.8

b 45, 94, 94, 75, 113, 75, 98,
87, 73, 51, 4, 90, 94, 48,
99, 62, 63, 64, 32, 101, 81,
119, 85, 98, 103, 92, 90, 86,
84, 56, 99, 81, 86, 82, 75,
38, 89, 89, 71, 46

18 Construct the specified graph for each of the following data sets.

a Bar chart

Favourite game	Frequency
Board games	10
Card games	45
Video games	50
Physical games	25

b Line graph

Day	1	2	3	4	5	6	7	8	9
Number	12	20	16	18	24	22	24	24	26

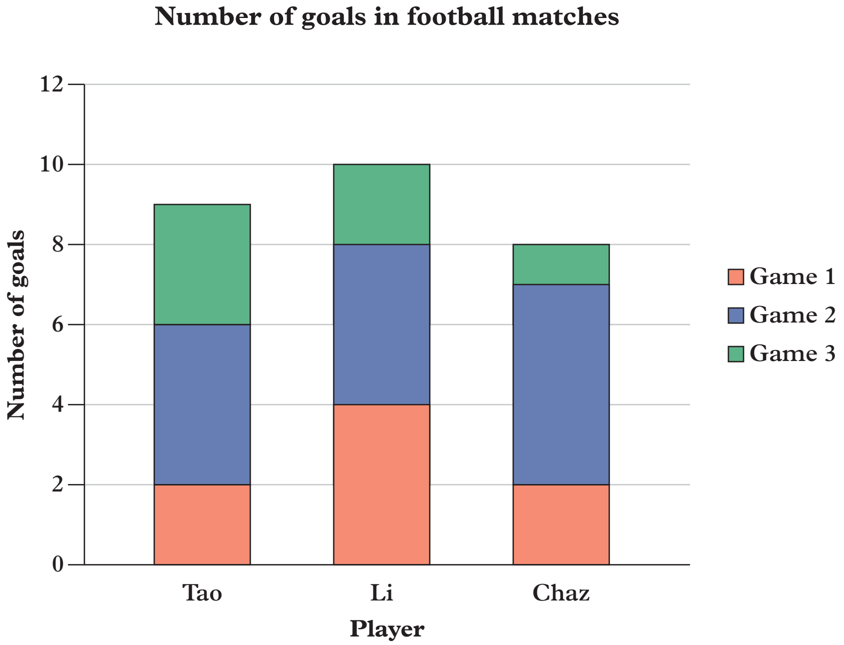
c Bar chart

Value	Frequency
8	10
9	26
10	34
11	38
12	30

d Histogram

Class	Frequency
0–<10	30
10–<20	60
20–<30	10
30–<40	90
40–<50	30

19 Consider the stacked bar chart below.



a Who scored the most goals in total?

b Who scored the most goals in the second game?

c In which game did Tao score more goals than Li?

20 Determine each of the following values. Round to two decimal places if required.

i Mean **ii** Median **iii** Mode **iv** Range

a 21, 19, 19, 19, 17, 16, 18, 26, 24, 19

b

Score (x)	10	11	12	13	14
Frequency (f)	6	12	18	20	14

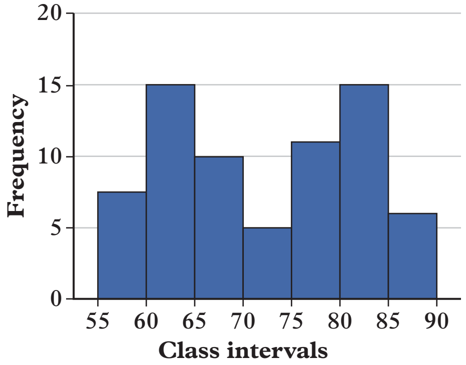
c

Stem	**Leaf**
2	0 3 4 6 7
3	1 2 3 3 4 4 4 5 7
4	1 2 5
5	1 3 3 6 8
6	0 1 1 2 4 6 7

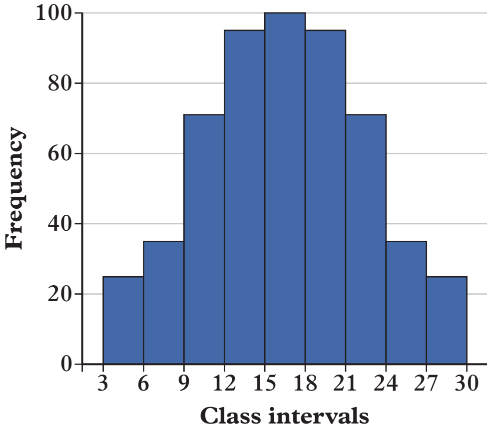
Key 2|1 = 2.1

21 Describe the shape of the distribution in each of the following graphs.

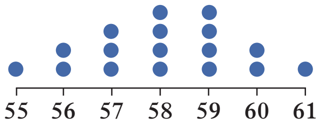
a



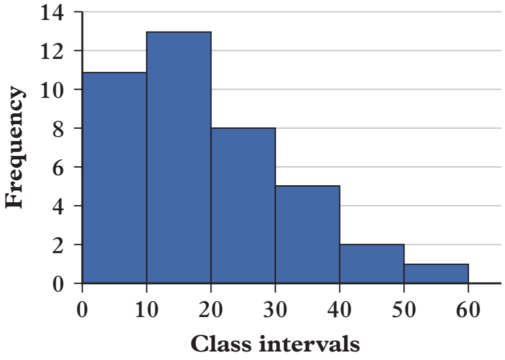
b



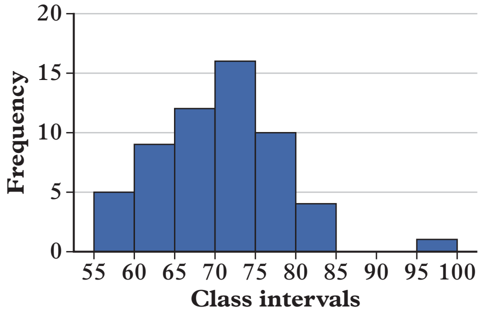
c



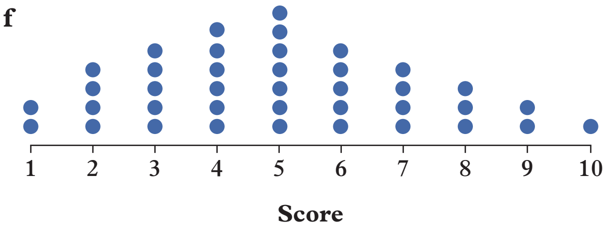
d



e



f



22 State which measures of centre are most appropriate to use to describe each of the distributions in question **21**.

23 Brea plans to play video games on two different consoles this afternoon. She has an Xbox, a PlayStation and a Nintendo Switch to choose from.

- **a** Construct a tree diagram to show all outcomes in the sample space.
- **b** Assuming each outcome is equally likely, calculate the probability that Brea plays:
  - **i** Xbox then PlayStation
  - **ii** Xbox second
  - **iii** Xbox either first or second
  - **iv** Xbox and PlayStation in either order.

24 A fair six-sided die is rolled and the result is noted. Following that a card is drawn from a standard 52-card deck and the suit is noted.

- **a** Construct an array to show all outcomes in the sample space.
- **b** Calculate the probability of:
  - **i** rolling a 6 and drawing a spade
  - **ii** rolling an even number and drawing a red card (heart or diamond)
  - **iii** rolling an even number or drawing a red card or both
  - **iv** rolling an even number or drawing a red card but not both.

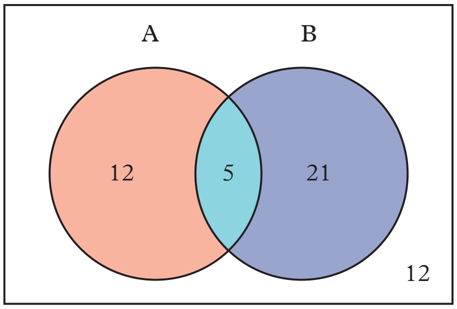
25 A bag contains 21 coloured buttons: 4 red, 8 blue and 9 yellow. Two buttons are selected at random from the bag. Calculate the probability that:

- **a** a yellow and a blue button are selected, if the first button is replaced
- **b** a yellow and a blue button are selected, if the first button is not replaced
- **c** two red buttons are selected, if the first button is not replaced
- **d** two of the same-coloured buttons are selected, if the first button is not replaced.

26 **a** Display the information in this two-way table in a Venn diagram.

	A	**A'**	**Total**
B	11	57	68
B'	47	25	72
Total	58	82	140

b Display the information in this Venn diagram in a two-way table.



27 A school has 212 Year 9 students. Of the 98 students with a smartphone, 56 have a gaming console. 80 students do not have a gaming console.

a Show this information in a two-way table.

b Calculate the relative frequency of students that:

- **i** do not have a smartphone
- **ii** have a gaming console
- **iii** have neither a smartphone nor a gaming console
- **iv** have a smartphone or a gaming console but not both.

28 a Convert this two-way table to a relative frequency two-way table.

	A	**A'**	**Total**
B	75	5	80
B'	35	10	45
Total	110	15	125

b Convert this relative frequency two-way table to a two-way table given that the sample size is 120.

	A	**A'**	**Total**
B	$\frac{17}{40}$	$\frac{29}{120}$	$\frac{2}{3}$
B'	$\frac{1}{8}$	$\frac{5}{24}$	$\frac{1}{3}$
Total	$\frac{11}{20}$	$\frac{9}{20}$	1

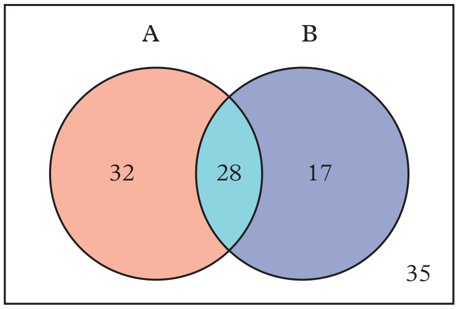
29 A large box contains fluorescent and LED light globes from two companies: Phillips and Osram. A sample of 90 light globes is taken and 46 are Phillips light globes, 39 are fluorescent and 21 are Osram LED light globes.

a Show this information on a Venn diagram.

b From the sample of 90 light globes, calculate the relative frequency of:

- **i** Phillips fluorescent light globes
- **ii** LED light globes
- **iii** Osram light globes
- **iv** fluorescent or not Phillips or both.

30 Consider this Venn diagram.



a Given that $n(\text{A})$ means the number of elements in set A, find:

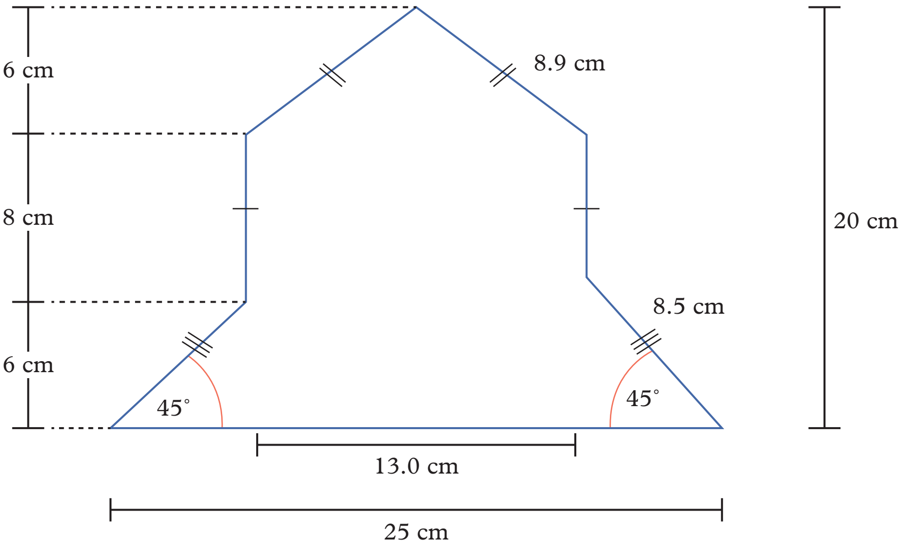
- **i** $n(\text{A})$
- **ii** $n(\text{A} \cup \text{B})$
- **iii** $n(\text{A}')$
- **iv** $n(\text{A} \cap \text{B})$
- **v** $n(\xi)$
- **vi** $n(\text{A}' \cap \text{B})$

b Given that Pr(A) means the probability of selecting an element from set A, find:

- **i** $\Pr(\text{A})$
- **ii** $\Pr(\text{A}')$
- **iii** $\Pr(\text{A} \cap \text{B})$
- **iv** $\Pr((\text{A} \cup \text{B})')$
- **v** $\Pr(\text{A}' \cap \text{B}')$
- **vi** $\Pr(\text{A} \cup \text{B}')$

Analysis

1 Xav is planning to create a flat plywood spaceship to mount on his wall. He draws a rough diagram of the spaceship using a scale factor of 1 : 5. The desired lengths and angles in the spaceship are shown.



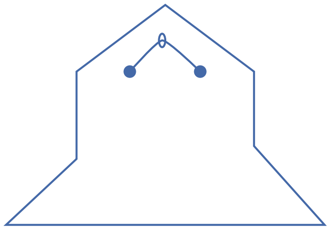
a The angles and integer lengths in the diagram are correct, but Xav measured the other lengths. Use trigonometry or Pythagoras' Theorem to check whether the lengths 8.9 cm, 8.5 cm and 13.0 cm are correct. If not, calculate the correct lengths correct to one decimal place.

b Draw a labelled diagram of the spaceship showing the correct lengths and angles.

c Calculate the area of:

- **i** the diagram of the spaceship
- **ii** the actual plywood spaceship.

To mount the plywood spaceship on the wall, Xav wants to attach an 80 cm string to two points on the back to hang on a hook. The hook will pull the string 20 cm up a vertical distance from where the string is connected, as shown.



d Determine the horizontal distance between the two points where Xav will attach the string, correct to one decimal place.

e Determine the angle the string will make with the line between the two attachment points.

f Determine whether the string is parallel to the edge of the plywood. Explain why or why not.

2 Arielle and Brenna have started to track their sleeping habits. They recorded how many minutes each slept in February, correct to the nearest 10 minutes.

Arielle: 550, 530, 480, 480, 470, 410, 480,
510, 520, 490, 430, 480, 480, 370,
460, 440, 490, 480, 500, 520, 470,
470, 410, 480, 490, 500, 430, 470

Brenna: 570, 420, 560, 460, 530, 480, 510,
410, 450, 510, 430, 540, 600, 570,
600, 440, 530, 460, 630, 370, 580,
500, 500, 550, 510, 400, 510, 460

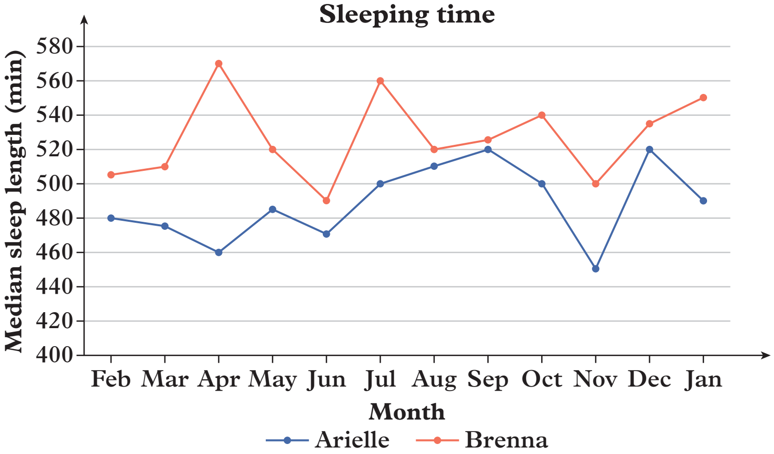
a Construct a back-to-back stem-and-leaf plot using the data above. Use intervals of 50 and the key 1 | 2 = 120 minutes.

b Compare the shape of the two data sets.

c Determine the median and range for Arielle's and Brenna's minutes of sleeping.

d Write two sentences comparing the two data sets with reference to their summary statistics from part **c**.

The median values of Arielle's and Brenna's sleep times are calculated each month for a year and are plotted on the multiple line graph below.



e State the independent and dependent variables.

f Compare the trends in Arielle's and Brenna's median sleep length over the year.

3 An avid puzzle collector has puzzles with 1000, 5000 and 10 000 pieces. Some of the puzzles have irregular-shaped designs, as shown in the table below.

	1000 pieces	5000 pieces	10 000 pieces
Number of puzzles in collection	20	9	1
Number with irregular-shaped designs	5	2	0

a Determine the relative frequency of:

- **i** 1000-piece puzzles out of all puzzles
- **ii** puzzles with irregular designs out of all puzzles
- **iii** 1000-piece puzzles with irregular designs out of all 1000-piece puzzles
- **iv** 1000-piece puzzles with regular designs out of all puzzles with regular designs

The puzzle collector selects two puzzles at random with replacement.

b Determine the probability of choosing:

- **i** an irregular-shaped 1000-piece puzzle and then a regular-shaped puzzle that is not 1000 pieces
- **ii** a 1000-piece puzzle and then an irregular shaped design.

The puzzle collector starts with a rectangular 1000-piece puzzle. The puzzle actually consists of 1026 pieces in a 38-piece by 27-piece grid. The collector separates the corner, edge and centre pieces.

c Calculate the probability of selecting at random:

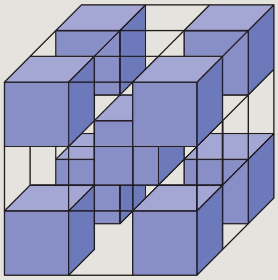
- **i** an edge piece connecting to any corner piece
- **ii** a centre piece connecting to any edge piece.

EXPLORATIONS 2

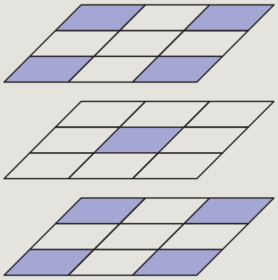
1 Cube stacks

Jali has a large supply of blue and clear cubes of identical size. She arranges 9 blue cubes and 18 clear cubes into a 3 × 3 × 3 stack as shown below. The simpler diagram in the middle shows the position of the blue cubes in each 3 × 3 horizontal layer. Jali notices that this arrangement results in the same pattern, shown on the right, when she views the stack from all six directions: top, bottom, left, right, front and back. In fact, she realises that if two cubes are removed from the top and bottom layers, it is still possible to see the same final pattern from all directions.

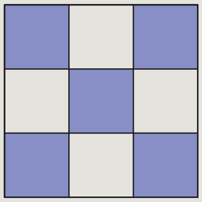
Jali wonders what other symmetrical patterns can be seen from all directions and how many blue cubes are needed to make them.



3D view



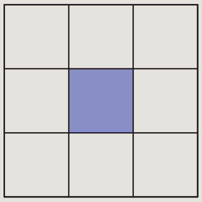
layer view



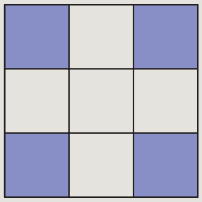
directional view

a For each of the following, determine both the maximum and minimum number of blue cubes that Jali can use in a 3 × 3 × 3 stack so that the resulting pattern can be seen from all six directions. Draw a layer view for each.

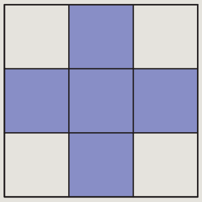
i



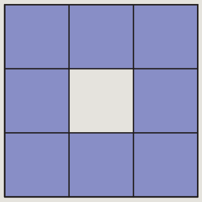
ii



iii

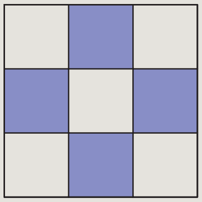


iv



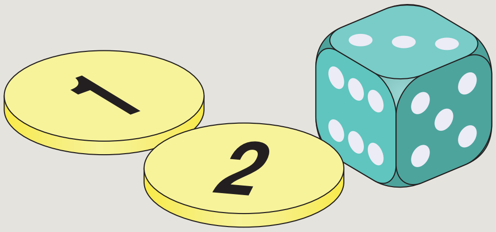
b Jali wants to make the symmetrical pattern shown on the right.

- **i** Describe a stack in which this pattern can be seen from exactly two directions.
- **ii** Describe a stack in which this pattern can be seen from exactly four directions.
- **iii** Explain why it is not possible to see this pattern from all six directions.



2 Counter attack

Connie has a number of counters, each labelled 1 on one side and 2 on the other. She flips the counters and adds up the numbers showing on top. At the same time, Dieter selects a die and rolls it. The game ends in a draw if Connie and Dieter get the same score, otherwise the winner is the player with the higher score.



a Connie flips two counters and Dieter rolls a standard 6-sided die.

- **i** Calculate the probability that both players get a score of 3.
- **ii** Explain why Dieter is more likely to win than Connie.

b Connie flips three counters and Dieter rolls a 6-sided die. Is this version of the game fair? That is, do Connie and Dieter have the same chance of winning?

c Dieter selects an octahedral die numbered 1 to 8. How many counters must Connie flip for the game to be fair?

d Connie flips five counters. Assuming they are numbered 1, 2, 3 … how many sides must there be on Dieter's die for the game to be fair?

e Generalise part **d** as follows: Connie flips n counters and Dieter spins a spinner with k wedges of equal size labelled 1, 2, 3 …, k. Find the relationship between n and k if the game is fair.

f Is there a fair version of the game if Dieter uses a dodecahedral die numbered 1 to 12? What about an icosahedral die numbered 1 to 20?

3 Differagons

A differagon is a polygon in which each side length is a different positive integer.

a For each of the following types of differagon, find the least possible perimeter:

i 3-sided **ii** 3-sided with a right angle

iii 4-sided **iv** 4-sided with one right angle

v 4-sided with two adjacent right angles **vi** 4-sided with two opposite right angles

vii 5-sided with three consecutive right angles **viii** 5-sided with three right angles, not all consecutive.

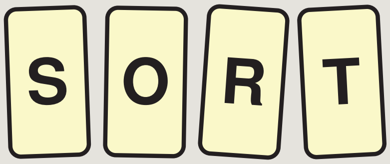
b Which types of differagon listed in part **a** can have an odd perimeter? Explain.

c Diago claims he has found a 6-sided differagon ABCDEF with the following properties:

- angles ABC, ACD, ADE and AFE are all right angles
- BC = 2, CD = 6, DE = 3 and EF = 7.

Show that Diago must have made a mistake.

4 All sorts



Imagine you have a row of cards in front of you, each with a single letter written on it. The goal is to sort them into alphabetical order. Here are three different approaches to this problem. As you work through them, see what shortcuts you can come up with to answer the questions as efficiently as possible.

a The SelectSort algorithm works as follows:

- Find the letter that appears earliest in the alphabet (in the case of repeats, choose the first occurrence).
- Swap it with the letter in the first position.
- Repeat the above steps with the remainder of the list.

For example, consider the cards S O R T in that order. The first step swaps O (the earliest letter) with S (the first letter), producing O S R T. Now repeat this procedure with the remaining list S R T, swapping R and S. The final step applied to S T has no effect, since S is earliest and in the first position. So only two steps are required to arrive at the final order, O R S T.

For each of the following, find the number of steps that SelectSort will take to sort the cards into alphabetical order.

i ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED

b In the SwapThree algorithm, a move consists of reversing the order of three consecutive cards. For example, S O R T can be converted to R O S T by reversing the first three cards, or to S T R O by reversing the last three. For each of the following, decide whether it is possible to sort the cards into alphabetical order using SwapThree and, if so, determine the minimum number of moves required.

i ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED

c Each pass of the iSort algorithm works as follows:

- Compare the first two letters in the list. Whichever is earlier in the alphabet is moved to a new list. The other stays in the original list.
- Repeat the previous step with the remaining letters, moving one letter to the end of the new list at each stage.
- When there is one letter remaining, move it to the end of the new list.

For example, comparing the first two letters of S O R T, the letter O starts the new list and S R T remain in the original list. Comparing the first two letters again, R is moved to the new list, leaving S T. Finally, S and then T are moved to the new list, giving O R S T. This is already in alphabetical order, so the original list was sorted in one pass of the iSort algorithm. But in general, more passes may be needed.

For each of the following, find the number of passes of iSort needed to sort the cards into alphabetical order.

i ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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CHAPTER 1 Exponents

Non-calculator

1 What is $2^3 + 3^2$?

☐ 12 ☐ 17 ☐ 25 ☐ 36

2 Simplify $x^6 \times x^4 \times x^{-11}$ using positive exponents.

3 Simplify $\frac{w^{11}}{w^6}$.

4 Write 2.97×10^4 as a basic numeral.

5 What is 0.000 000 000 000 000 000 000 000 029 in scientific notation?

6 Which symbol makes the following statement true?

4.98×10^3 ___ 0.0498×10^6

☐ $>$ ☐ $<$ ☐ $=$ ☐ $\leq$

7 What does $(5 \times 10^2) \times (4 \times 10^3)$ simplify to in scientific notation?

☐ 9×10^5 ☐ 2×10^6 ☐ 20×10^5 ☐ 9×10^1

Calculator

8 Which expression is equivalent to $5a^2 \times 3a^4$?

☐ $8a^6$ ☐ $8a^8$ ☐ $15a^6$ ☐ $15a^8$

9 Which expression shows $\frac{8r^{-2}p^{-6}}{12p^{-2}}$ in simplified form with positive exponents only?

☐ $\frac{2r^2}{3p^4}$ ☐ $\frac{8r^2}{12p^4}$ ☐ $\frac{2}{3r^2p^4}$ ☐ $\frac{2}{3r^2p^{-8}}$

10 Which expression is equivalent to $\frac{x^5yz^2}{x^3y^4}$?

☐ $\frac{x^2z^2}{y^3}$ ☐ $x^2y^3z^2$ ☐ $x^8y^5z^2$ ☐ $\frac{z^2}{x^2y^3}$

11 Which expression is equivalent to $\left(\frac{2m^6}{n^4}\right)^3$?

☐ $\frac{2m^{18}}{n^4}$ ☐ $\frac{6m^9}{n^7}$ ☐ $\frac{6m^{18}}{n^{12}}$ ☐ $\frac{8m^{18}}{n^{12}}$

12 Which of the following is equivalent to $(2x^2y)^0 \times 3x^0$?

☐ 1 ☐ $6x^2$ ☐ 3 ☐ $3x^2y$

13 Which of the following is equivalent to 4^{-6}?

☐ $-\frac{1}{4^{-6}}$ ☐ -4^6 ☐ $\frac{1}{4^6}$ ☐ $-(-4)^6$

CHAPTER 2 Algebra

Non-calculator

1 Which expression is equivalent to $7(2b + 4)$?

☐ $14b + 4$ ☐ $14b + 28$ ☐ $9b + 4$ ☐ $18b$

2 What is the value of $4x^2$ when $x = -5$?

☐ 100 ☐ −100 ☐ 400 ☐ 220

3 If $a = -3$ and $b = 3$, what is the value of $a^2 + b^2$?

4 What is the missing coefficient value that makes this statement true for all values of x?

$7(x + 6) +$ ____$(x - 2) = 10x + 36$

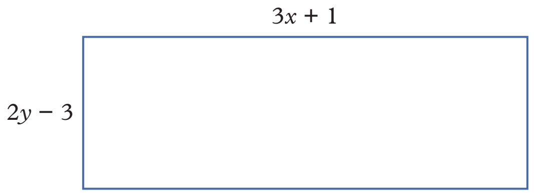
5 If $m = -7$, what is the value of $2m - 11$?

6 Which expression is equivalent to $5a + 6 + 7a - 11$?

☐ $12a - 5$ ☐ $7a$

☐ $12a + 17$ ☐ $11a - 4$

Questions **7–10** refer to this rectangle.



7 Write the perimeter of this shape as an algebraic expression in simplest form.

8 Calculate the perimeter when $x = 5$ cm and $y = 2$ cm.

9 Write an algebraic expression for the area of the shape.

10 Calculate the area when $x = 12$ cm and $y = 8$ cm.

Calculator

11 In a set of three consecutive whole numbers, the value of the lowest number is w. What is the value of the highest number in the set?

☐ $3w$ ☐ $w + 3$

☐ $3 - w$ ☐ $w + 2$

12 What is the highest common factor of $18x^2yz$ and $12xy^2z$?

☐ $6xyz$ ☐ $9x^2yz$

☐ $6xy^2z$ ☐ $6xy$

13 Expand $(a - 3)(a + 5)$.

14 Factorise $b^2 - 13b + 40$.

15 Factorise $m^2 + m - 12$.

16 Factorise $7d^2 - 14d + 9d - 18$.

17 What is the expanded form of $(x + 2y)^2$?

☐ $x^2 + 4y^2$

☐ $x^2 + 2y^2$

☐ $x^2 + 2xy + 2y^2$

☐ $x^2 + 4xy + 4y^2$

18 What is the factorised form of $p^2 - 8p + 16$?

☐ $(p + 4)^2$

☐ $(p - 2)^2$

☐ $(p - 4)^2$

☐ $(p + 2)^2$

CHAPTER 3 Linear relationships

Non-calculator

1 The solution to $\frac{2x - 3}{4} = 3$ is:

☐ $x = 4\frac{1}{2}$

☐ $x = 7\frac{1}{2}$

☐ $x = 12$

☐ $x = 34$

2 Which equation does *not* have the solution $x = 3$?

☐ $\frac{4x - 2}{5} = 2$

☐ $\frac{9 - x}{6} = 1$

☐ $5 - 3x = 4$

☐ $4 - 2x = -2$

3 Which point does *not* lie on the graph of $y = 2x - 4$?

☐ $(1, -2)$ ☐ $(2, 1)$

☐ $(3, 2)$ ☐ $(4, 4)$

4 Which point does *not* lie on the graph of $-x + 2y = 4$?

☐ $(-1, 1.5)$ ☐ $(2, 3)$

☐ $(-2, 1)$ ☐ $(-3, 3.5)$

5 What is the gradient of the line with equation $-4x + 2y = 9$?

Questions **6–7** refer to this information. Consider the linear relationship with the rule $2x - 3y = -6$.

6 What is the x-intercept of the graph?

7 What is the y-intercept of the graph?

Questions **8–10** refer to this information.
Let A be the point $(3, 5)$ and B be the point $(-1, 2)$.

8 What is the gradient of the line connecting A and B?

☐ $-\frac{3}{4}$ ☐ $-\frac{4}{3}$ ☐ $\frac{3}{4}$ ☐ $\frac{4}{3}$

9 What is the midpoint of the line segment AB?

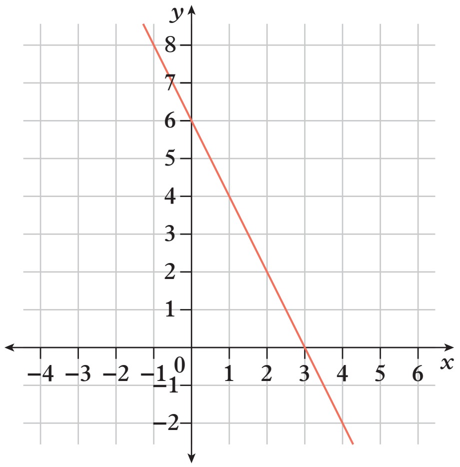
☐ $(2, 3.5)$ ☐ $(1, 3.5)$

☐ $(4, 7)$ ☐ $(2, 3)$

10 What is the distance between the two points A and B?

☐ 2 ☐ 5 ☐ 7 ☐ $\sqrt{7}$

11 What is the equation of the graph shown below?



☐ $y = 3x + 6$ ☐ $y = -2x + 3$

☐ $y = -3x + 6$ ☐ $y = -2x + 6$

12 The solution to $3x + 6 < 12$ is:

☐ $x < 6$

☐ $x > 2$

☐ $x < 2$

☐ $x > 3$

Calculator

13 Answer true or false to this statement: The equation $4x - 3 = 2$ is a linear equation.

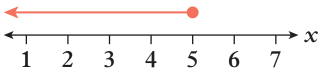
14 If 5 is added to 3 times a number n and the answer is divided by 4, then the result is 6. The equation representing this process is:

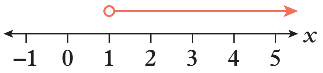
☐ $5 + \frac{3n}{4} = 6$ ☐ $\frac{3(5 + n)}{4} = 6$

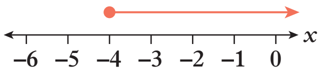
☐ $\frac{5(3 \times n)}{4} = 6$ ☐ $\frac{3n + 5}{4} = 6$

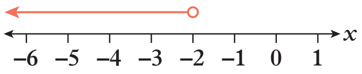
15 A number n is divided into 4, and then 5 is added. The result is 10. Write this as an equation.

16 Which of the following number lines represent the answer to $7x - 2 < 3x - 10$?

☐ 

☐ 

☐ 

☐ 

17 Consider an odd integer represented by n. The next odd integer would be represented by:

☐ $n + 1$ ☐ $n + 2$

☐ $n - 1$ ☐ $n - 2$

18 The sum of three consecutive odd numbers is 33. What is the smallest of the three numbers?

19 Which rule would *not* produce a linear graph?

☐ $y = \frac{x}{2} + 3$ ☐ $y = 3x - \frac{5}{2}$

☐ $y = \frac{4}{x} + 5$ ☐ $y = \frac{1}{2}x - 2$

20 The gradient of the graph of $y = x - 6$ is:

☐ positive ☐ negative

☐ zero ☐ undefined

21 The gradient of the graph of $y = -4$ is:

☐ positive ☐ negative

☐ zero ☐ undefined

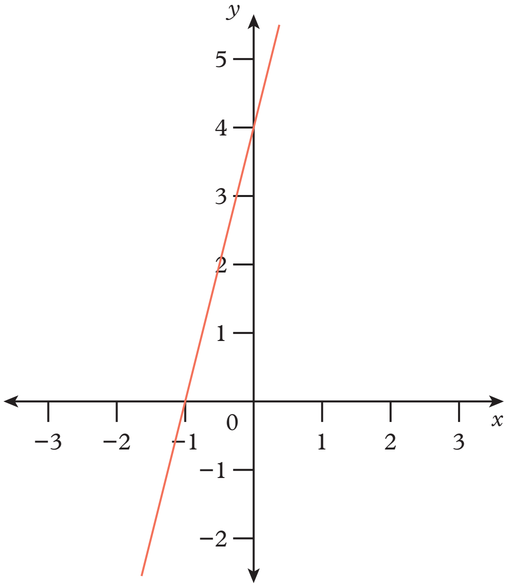
22 The gradient of the graph of $x = 10$ is:

☐ positive ☐ negative

☐ zero ☐ undefined

23 Compare the gradients of the graphs of $y = -x$ and $y = -4x$. Which is the steeper line?

Questions **24–26** refer to the linear graph drawn on this Cartesian plane.



24 What is the x-coordinate of the x-intercept?

☐ −1 ☐ 0

☐ 1 ☐ 4

25 What is the y-coordinate of the y-intercept?

☐ −1 ☐ 0

☐ 1 ☐ 4

26 What is the gradient?

☐ -4

☐ $-\frac{1}{4}$

☐ $\frac{1}{4}$

☐ 4

Questions **27** and **28** refer to this information.
A line segment $\overline{AB}$ has a midpoint $M(2, 4)$.
The coordinates of the point B are $(5, 6)$.

27 What are the coordinates of point A?

28 What is the length of the line segment $\overline{AM}$, correct to one decimal place?

Questions **29–31** refer to a triangle, $\triangle PQR$, with vertices at $P(-1, 3)$, $Q(-1, -2)$ and $R(4, -2)$.

29 Which line segment represents the longest side of the triangle?

☐ $\overline{PQ}$ ☐ $\overline{QR}$ ☐ $\overline{PR}$ ☐ $\overline{QP}$

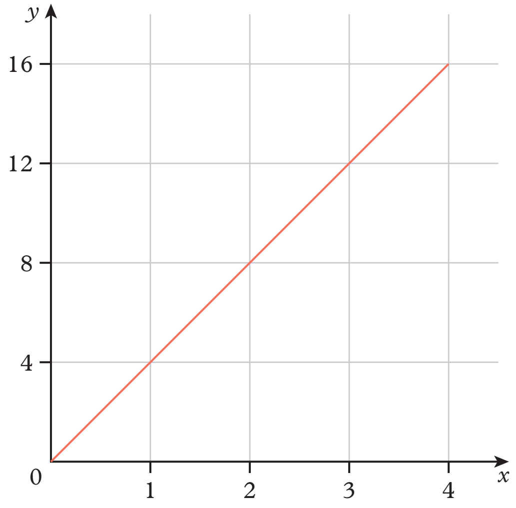
30 Calculate the length of the longest side of the triangle, correct to one decimal place.

31 Find the coordinates of the midpoint of the line segment $\overline{QR}$.

32 Which of these statements is *not* correct? The graph of a linear relationship:

☐ may pass through the origin.
☐ can be sketched using two distinct points.
☐ has a constant gradient.
☐ always has two axis intercepts.

33 Write the linear relationship between x and y shown in this graph.



CHAPTER 4 Non-linear relationships

Non-calculator

1 If $(x - 6)(x + 1) = 0$, the two solutions for x are:

☐ $x = -6$ or $x = 1$ ☐ $x = 6$ or $x = -1$
☐ $x = -6$ or $x = -1$ ☐ $x = 6$ or $x = 1$

2 The expanded form of the equation $(x - 8)(x + 2) = 0$ is:

☐ $x^2 + 6x - 16 = 0$ ☐ $x^2 - 6x - 16 = 0$
☐ $x^2 - 10x - 16 = 0$ ☐ $x^2 + 10x - 16 = 0$

3 What is the solution or solutions to $x^2 - 11x + 10 = 0$?

4 For $y = x^2 + 3x - 2$, what is the value of y when $x = -1$?

5 Write the coordinates of the x-intercepts for the graph of $y = x^2 + x - 20$.

6 The axis of symmetry for the graph of $y = x^2 + 2x - 15$ has the rule:

☐ $x = 1$ ☐ $x = -1$
☐ $y = 1$ ☐ $y = -1$

7 Write the coordinates of the turning point for the graph of $y = x^2 - 8x + 15$.

8 Which of the following relationships is non-linear?

☐ $x + y = 1$
☐ $xy = 1$
☐ $x - y = 1$
☐ $x = y$

Calculator

9 What transformation has been performed on the graph of $y = x^2$ to produce the graph of $y = x^2 - 3$?

☐ Dilated by factor of 3 in the y-direction
☐ Horizontal translation of 3 units
☐ Reflection in the x-axis
☐ Vertical translation of 3 units

10 What are the coordinates of the y-intercept for the graph of $y = -3(x - 2)^2 - 4$?

11 What are the coordinates of the turning point for the graph of $y = -3(x - 2)^2 - 4$?

☐

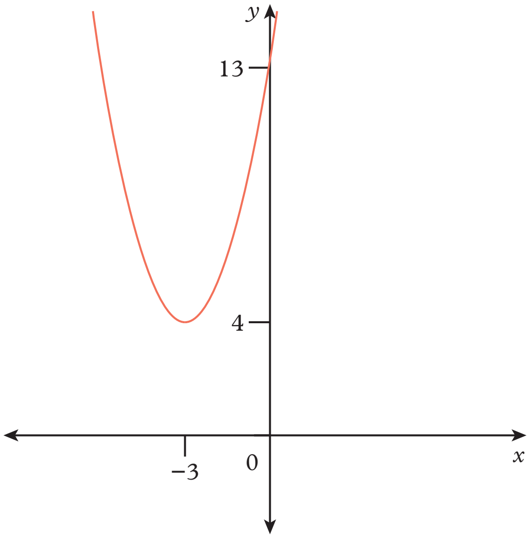
12 Which rule best matches the graph shown?

☐ $y = (x - 3)^2 + 4$

☐ $y = (x - 3)^2 - 4$

☐ $y = (x + 3)^2 + 4$

☐ $y = (x + 3)^2 - 4$



13 How many x-intercepts does the graph of $y = (x - 3)^2$ have?

0	1	2	3
☐	☐	☐	☐

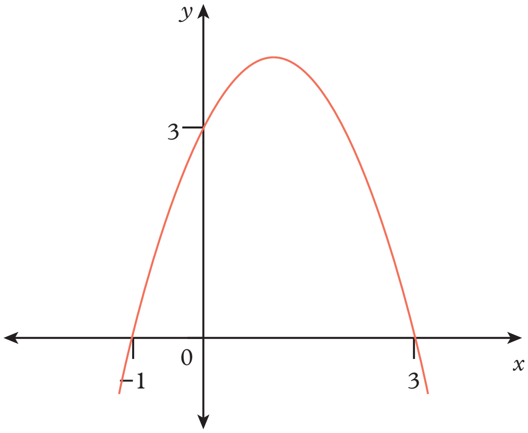
14 Which rule best matches the graph shown?

☐ $y = x^2 + 2x + 3$

☐ $y = -x^2 + 2x + 3$

☐ $y = x^2 - 2x - 3$

☐ $y = -x^2 + 2x - 3$



15 What are the coordinates of the y-intercept of $y = 2 + \sqrt{x}$?

☐

CHAPTER 5 Measurement

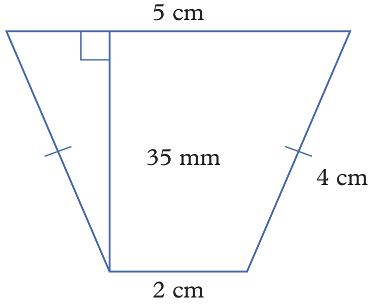
Calculator

1 The area of a rectangle is 96 mm². If the rectangle has a width of 8 mm, calculate the length.

☐ 12 mm ☐ 24 mm

☐ 48 mm ☐ 72 mm

Questions **2** and **3** refer to this shape.



2 The correct formula to calculate the area of this shape is:

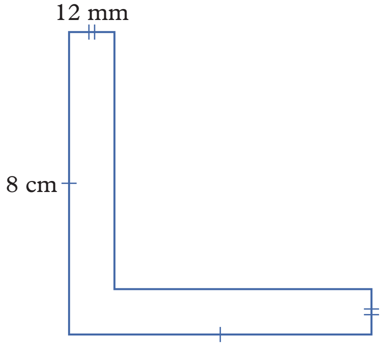
☐ $A = xy$ ☐ $A = \frac{1}{2}bh$

☐ $A = \frac{1}{2}(a + b)h$ ☐ $A = lw$

3 The area of the shape is:

☐

4 The area of this composite shape is:



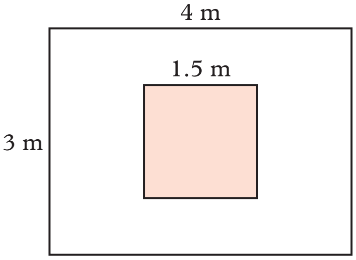
☐ 19.2 cm²

☐ 96 cm²

☐ 46.24 cm²

☐ 17.76 cm²

5 The shaded square represents a garden and the unshaded part represents the paving around the garden. What is the area of paving?

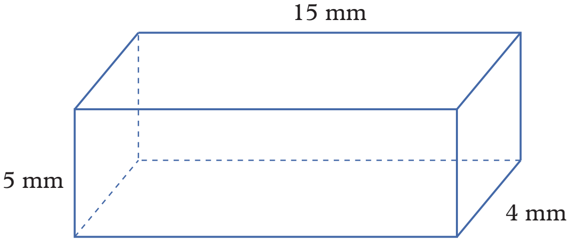


☐ 12 m^2 ☐ 9.75 m^2
☐ 2.25 m^2 ☐ 14.25 m^2

6 The surface area of a cube is 726 cm^2. How long is an edge of the cube?

☐ cm

Questions **7** and **8** refer to this prism.



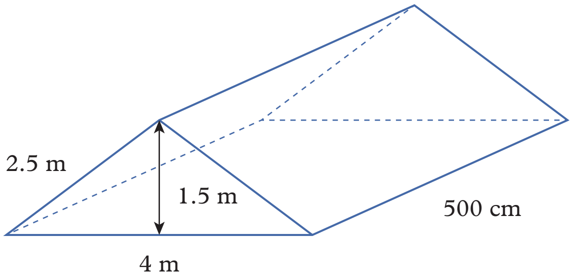
7 The surface area of this rectangular prism in square centimetres is:

☐ cm^2

8 The volume of the rectangular prism, correct to one decimal place, is:

☐ cm^3

Questions **9** and **10** refer to this prism.



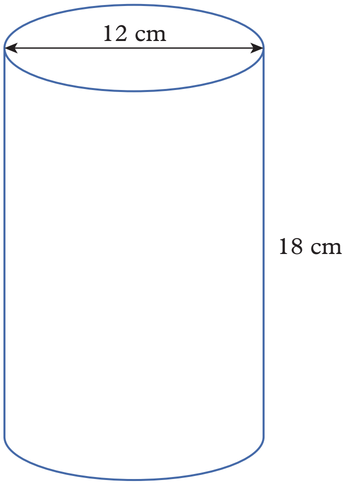
9 This solid triangular prism is to have all surfaces painted. If two coats of paint are required, what is the total area to be painted?

☐ 51 m^2 ☐ 102 m^2
☐ 43.5 m^2 ☐ 48 m^2

10 The volume of the triangular prism, correct to one decimal place, is:

☐ m^3

Questions **11** and **12** refer to this cylinder.



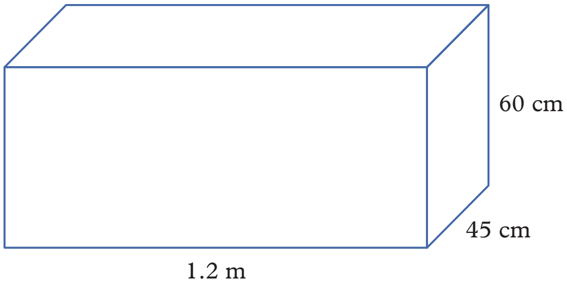
11 The surface area of this cylinder to the nearest centimetre is:

☐ 905 cm^2 ☐ 2262 cm^2
☐ 904 cm^2 ☐ 2261 cm^2

12 The volume of the cylinder, correct to two decimal places, is:

☐ cm^3

13 Given that 1000 cm^3 is 1 L, what is the capacity of the following prism?



☐ 3.24 L ☐ 32.4 L
☐ 324 L ☐ 3240 L

14 Which of the following has the biggest percentage error?

☐ Exact value: 1.02 cm, estimated value: 1 cm
☐ Exact value: 14.7 cm, estimated value: 15 cm
☐ Exact value: 21 cm, estimated value: 20 cm
☐ Exact value: 0.98 cm, estimated value: 1 cm

CHAPTER 6 Geometry

Non-calculator

1 The ratio 5 : 25 expressed in simplest form is:

☐ 1 : 5 ☐ 1 : 21 ☐ 5 : 1 ☐ 1 : 20

2 Juliet is paid an hourly rate of $48. If she earns $1920, how many hours did she work?

☐ 20 ☐ 40 ☐ 45 ☐ 60

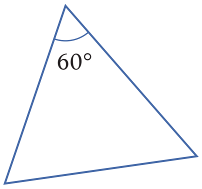
3 If a scale drawing uses a scale of 1 : 2000, how much does 1 cm in the drawing represent in real life?

☐ 2 cm ☐ 20 cm ☐ 2 m ☐ 20 m

4 If a scale drawing uses a scale of 1 : 100, how long should a wall which is 4 metres long in real life be in the drawing?

☐ 4 cm ☐ 40 cm ☐ 40 m ☐ 400 m

5 If the following triangle is dilated by a factor of 2, the size of the angle corresponding to the 60° angle is:



☐ 30° ☐ 60° ☐ 90° ☐ 120°

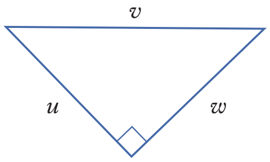
6 If a shape is dilated by a scale factor of 4, the area scale factor is:

☐ 2 ☐ 4 ☐ 16 ☐ 64

7 The two smaller sides of a right-angled triangle are 3 m and 4 m. What is the length of the third side?

Calculator

Questions **8** and **9** refer to this triangle.



8 Which pronumeral represents the hypotenuse in the triangle?

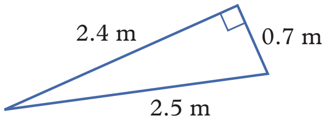
9 Using Pythagoras' theorem, which of these statements shows the relationship between the side lengths of the triangle?

☐ $w^2 = u^2 + v^2$ ☐ $v^2 = u^2 + w^2$

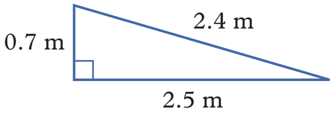
☐ $u^2 = v^2 + w^2$ ☐ $v = u + w$

10 A right-angled triangle contains the side lengths 0.7 m, 2.4 m and 2.5 m. Which of these triangles best matches this description?

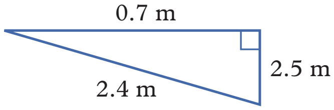
☐ Triangle A



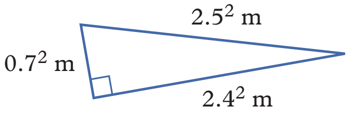
☐ Triangle B



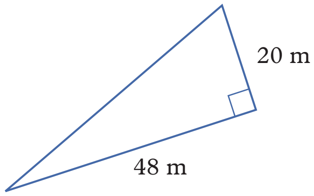
☐ Triangle C



☐ Triangle D



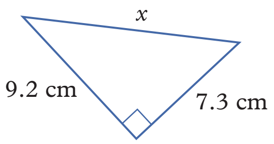
11 What is the length of the hypotenuse in this triangle?



☐ 68 m ☐ 8.24 m

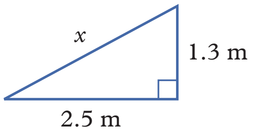
☐ 52 m ☐ 43.63 m

12 What is the value of x in this triangle, correct to two decimal places?

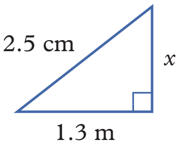


13 A ladder leans against a vertical wall of a building so that the top of the ladder reaches 2.5 m up the wall and the foot is 1.3 m from the base of the wall. Which of these triangles correctly displays this information?

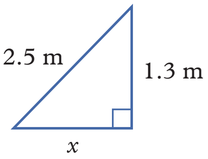
☐ Triangle A



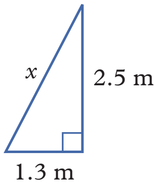
☐ Triangle B



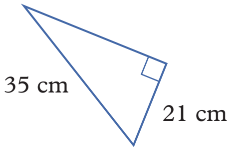
☐ Triangle C



☐ Triangle D



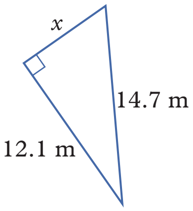
14 What is the length of the unknown side in this triangle, correct to the nearest centimetre?



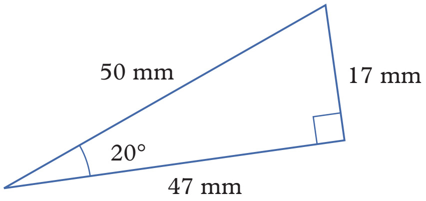
☐ 14 cm ☐ 28 cm

☐ 41 cm ☐ 784 cm

15 What is the value of x in this triangle, correct to two decimal places?



Questions **16** and **17** refer to this triangle.



16 What is the value of $\cos(20°)$ as a fraction?

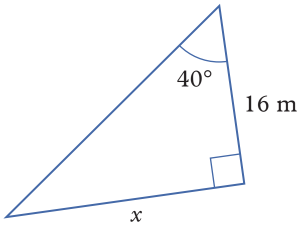
☐ $\frac{47}{50}$ ☐ $\frac{17}{50}$ ☐ $\frac{17}{47}$ ☐ $\frac{47}{17}$

17 What is the value of $\tan(20°)$, correct to two decimal places?

☐ 0.36 ☐ 2.76

☐ 0.94 ☐ 0.34

18 Which equation can be used to find the side length x in the triangle?



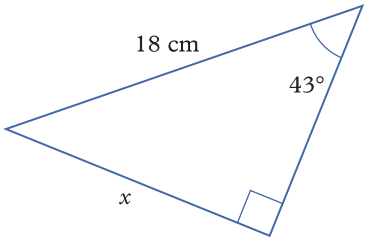
☐ $\tan(40°) = \frac{16}{x}$

☐ $\tan(40°) = \frac{x}{16}$

☐ $\cos(40°) = \frac{x}{16}$

☐ $\sin(40°) = \frac{x}{16}$

19 What is the side length x in this triangle, correct to two decimal places?

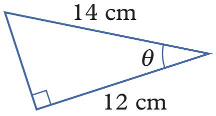


20 If $\tan(\theta) = \frac{22}{25}$, then θ is equal to:

☐ $\tan^{-1}\left(\frac{22}{25}\right)$ ☐ $\tan\left(\frac{22}{25}\right)$

☐ $\tan^{-1}\left(\frac{25}{22}\right)$ ☐ $\tan\left(\frac{22}{25}\right)^{-1}$

21 What is the size of the unknown angle θ in this triangle to the nearest degree?



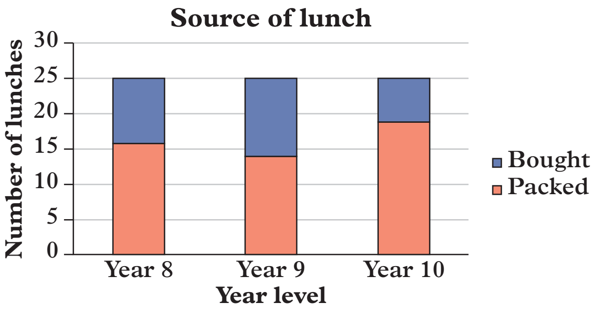
CHAPTER 7 Statistics

Calculator

1 Which of these is classified as ordinal data?

☐ eye colour (hazel, blue, green)
☐ pizza sizes (small, medium, family)
☐ the number of SMS messages sent in a month
☐ the time taken to walk from home to school

2 The students from three different classes were asked after lunch if they brought a packed lunch to school or if they bought their lunch at the canteen. The results are shown in the following stacked bar chart.



How many Year 9 students brought a packed lunch?

Questions **3** and **4** refer to the following data, which is listed below and also displayed in the stem-and-leaf plot.

25, 27, 29, 41, 32, 35, 20, 27, 49

Stem	Leaf
2	5 7 7 9
3	2 5
4	1 9

3 Which score is missing in the stem-and-leaf plot?

4 What is an appropriate key for this stem-and-leaf plot?

Questions **5** and **6** refer to this frequency table.

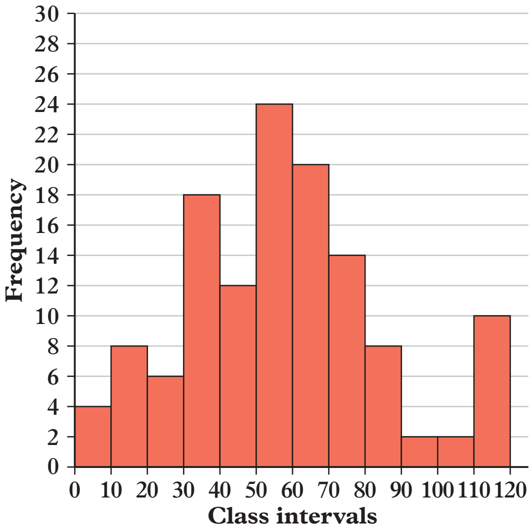
Score	Frequency
24	97
25	111
26	378
27	246
28	
29	301
Total	1325

5 What is the missing value in the frequency table?

6 Find the percentage of scores that are 25 or less, correct to one decimal place.

Questions **7** and **8** refer to the histogram below.

The histogram displays the results of research where the heights of plants (in cm) were measured.



7 How many plants are less than 70 cm tall?

☐ 92 ☐ 36 ☐ 20 ☐ 14

8 What percentage of plants are 100 cm or taller, correct to two decimal places?

☐ 1.56% ☐ 12%
☐ 9.38% ☐ 90.62%

Questions **9** and **10** refer to the data set below.

15, 17, 18, 45, 13, 15, 15, 16, 15,

9 Which value is closest to the mean of this data set?

☐ 18 ☐ 15
☐ 32 ☐ 19

10 A score was recorded incorrectly in the list of data. If the score of 45 should be 15, which statement is true?

☐ The range will be unchanged.
☐ The mean will be unchanged.
☐ The mode will be unchanged.
☐ The median will change.

11 A friend purchased 12 concert tickets for a total of $1188. What is the average price per ticket?

12 The heights of a group of Year 9 students were recorded, as follows:

145 cm, 152 cm, 147 cm, 1.35 m, 165 cm, 170 cm.

Which statement is false?

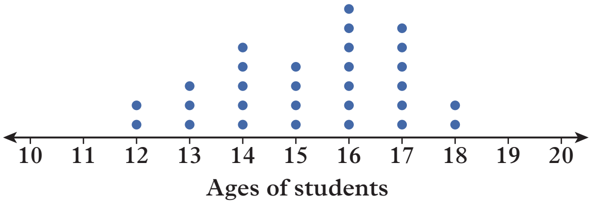
☐ The mean height is 1.52 m.
☐ The standard deviation is large, indicating a significant spread from the mean.
☐ The range is 35 cm.
☐ The median height is 152 cm.

13 Twenty teachers were asked how many cups of coffee they have each day. The results are displayed in the frequency table below.

What is the mean number of cups of coffee consumed per day?

Score	Frequency
0	3
1	8
2	6
3	2
6	1

14 The ages of students in a chess club at a particular high school are shown on the dot plot below. What is the median age of students in the chess club?



☐ 15 ☐ 15.3 ☐ 16 ☐ 17

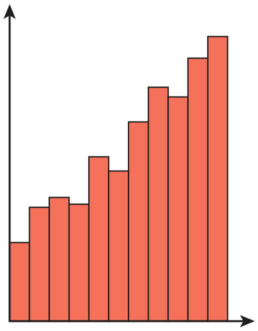
15 What is the range of the data in the following stem-and-leaf plot?

Stem	Leaf
2	3 4 7 9
3	5 6
4	1 3 8
5	2 4 6 6
6	5 7

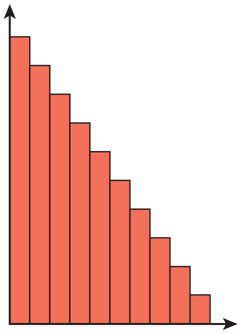
Key: 3|6 = 3.6

16 Which graph shows a positively skewed distribution?

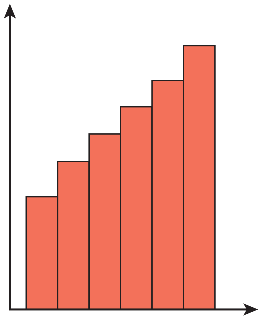
☐ Graph A



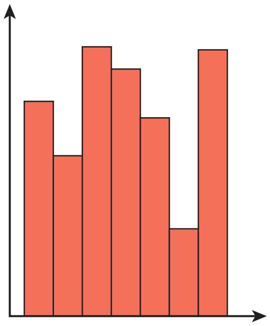
☐ Graph B



☐ Graph C



☐ Graph D



CHAPTER 8 Probability

Non-calculator

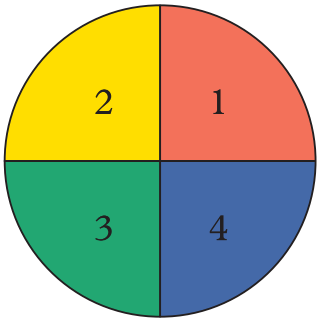
1 The theoretical probability of rolling a number greater than 4 on a standard die is:

$\frac{1}{6}$ ☐ $\frac{1}{3}$ ☐ $\frac{1}{2}$ ☐ 0 ☐

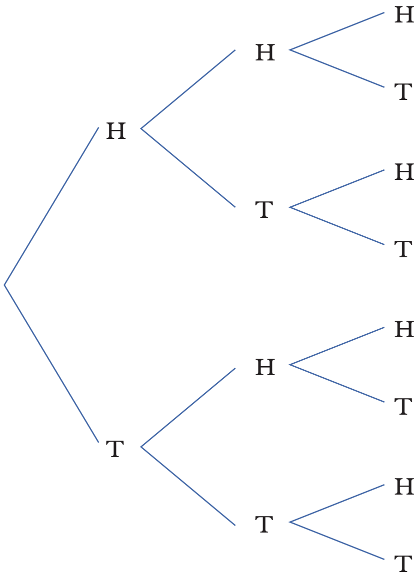
2 Two spinners are spun at the same time. The first spinner has four equally likely outcomes and the second has five equally likely outcomes. The total number of possible outcomes to be represented on a tree diagram is:

4 ☐ 5 ☐ 10 ☐ 20 ☐

3 The following spinner is spun twice. What is the probability that the sum of the two numbers is 6?



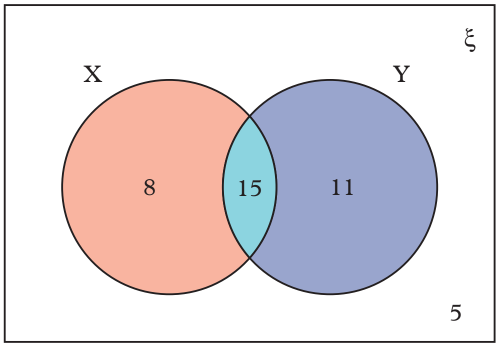
4 The following tree diagram represents the possible outcomes when a coin is tossed three times.



What is the probability that the coin lands on tails at least twice?

$\frac{1}{8}$ ☐ $\frac{1}{4}$ ☐ $\frac{3}{8}$ ☐ $\frac{1}{2}$ ☐

Questions **5–7** refer to the following Venn diagram. The number of elements in sets X and Y is shown in the Venn diagram.



5 $n(\text{Y})$ is:

8 ☐ 11 ☐ 23 ☐ 26 ☐

6 $n(\text{X} \cup \text{Y})$ is:

23 ☐ 26 ☐ 34 ☐ 39 ☐

7 $n(\text{X} \cap \text{Y})$ is:

8 ☐ 15 ☐ 11 ☐ 5 ☐

Calculator

8 The results of an experiment are recorded in this table.

Option	Frequency
A	400
B	250
C	360
D	375
E	198
F	220
G	315
H	

Which of these statements is *not* correct?

☐ If 2500 trials were performed, the frequency for outcome H was 382.

☐ If 2500 trials were performed, the experimental probability for option A is $\frac{4}{25}$.

☐ A spinner with six equal segments may have been used in this simulation.

☐ If an additional 2500 trials were performed and the options were equally likely, the experimental probability for each option would theoretically get closer to $\frac{1}{8}$.

9 The following result was recorded when a fair coin was tossed a number of times.

Outcome	Heads	Tails
Relative frequency		0.465

Which of these statements is false?

☐ The relative frequency of heads for this experiment is 0.535.

☐ If 3000 trials were performed, 1605 heads were recorded.

☐ If 5000 trials were performed, 2500 tails would be expected in theory.

☐ The relative frequency of heads for this experiment is 0.465.

Questions **10** and **11** refer to the information and table below.

A survey of 700 students was conducted relating to student enjoyment in different subjects at primary, secondary and tertiary levels.

	Primary	**Secondary**	**Tertiary**	**Total**
Maths	100		50	
English		69	122	
Sport	75	96		
Total	250	250		700

10 The probability that a student chosen at random from the group was a secondary student who enjoyed Maths is:

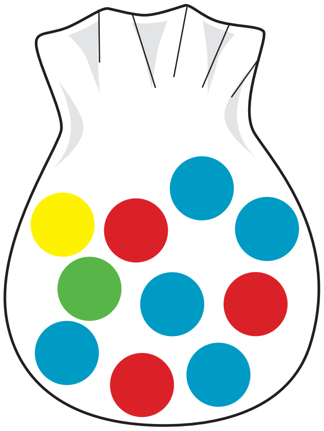
$\frac{1}{7}$	$\frac{1}{14}$	$\frac{17}{140}$	$\frac{47}{140}$
☐	☐	☐	☐

11 The probability that a student chosen from the primary group of the survey group enjoyed English is:

$\frac{2}{5}$	$\frac{3}{10}$	$\frac{69}{250}$	$\frac{3}{28}$
☐	☐	☐	☐

Questions **12–15** refer to the following diagram.

A bag contains a number of coloured marbles: 1 yellow, 1 green, 3 red and 5 blue. A marble is drawn, the colour recorded, the marble is replaced and then another is selected.



12 The probability of selecting two blue marbles is:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{9}{100}$	$\frac{1}{100}$
☐	☐	☐	☐

13 The probability of selecting a red marble first is:

$\frac{3}{20}$	$\frac{3}{100}$	$\frac{9}{100}$	$\frac{3}{10}$
☐	☐	☐	☐

Now assume that marbles are selected without replacement.

14 The probability of selecting a blue marble on the second trial, given that the first marble was not blue, is:

$\frac{5}{9}$	$\frac{1}{2}$	$\frac{3}{9}$	$\frac{1}{9}$
☐	☐	☐	☐

15 The probability of selecting three red marbles, if a third trial is performed, is:

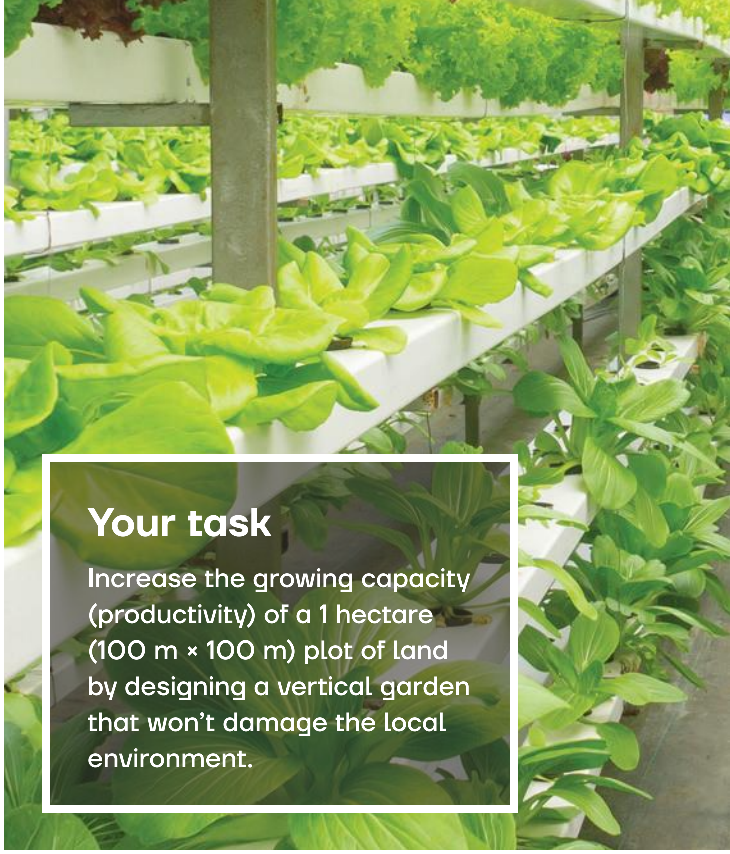
$\frac{2}{9}$	$\frac{1}{120}$	$\frac{2}{72}$	0
☐	☐	☐	☐

[STEAM project 1]

How can we use sustainable farming practices so that no one goes hungry in the future?

The United Nations ranks food shortages and hunger among the most serious issues affecting humankind. It predicts that more than 840 million people will be hungry by 2030. Even in a high-income country such as Australia, 5 per cent of the population are unable to access enough nutritious food. The experience of having inadequate access to food, or having an inadequate supply of food, is known as food insecurity. Food insecurity is linked to poor general health, higher rates of some cancers and higher mortality.

Rapid climate change is increasing threats to Australia's and the world's food security. Changes in the amount of rainfall, longer droughts and an increase in the number of extreme weather events are expected to disrupt the amount and quality of food that Australia can produce. A hotter climate is expected to cause stress in livestock animals such as chickens, sheep and cattle, and to increase the amount of water needed for crop irrigation.



Your task

Increase the growing capacity (productivity) of a 1 hectare (100 m × 100 m) plot of land by designing a vertical garden that won't damage the local environment.

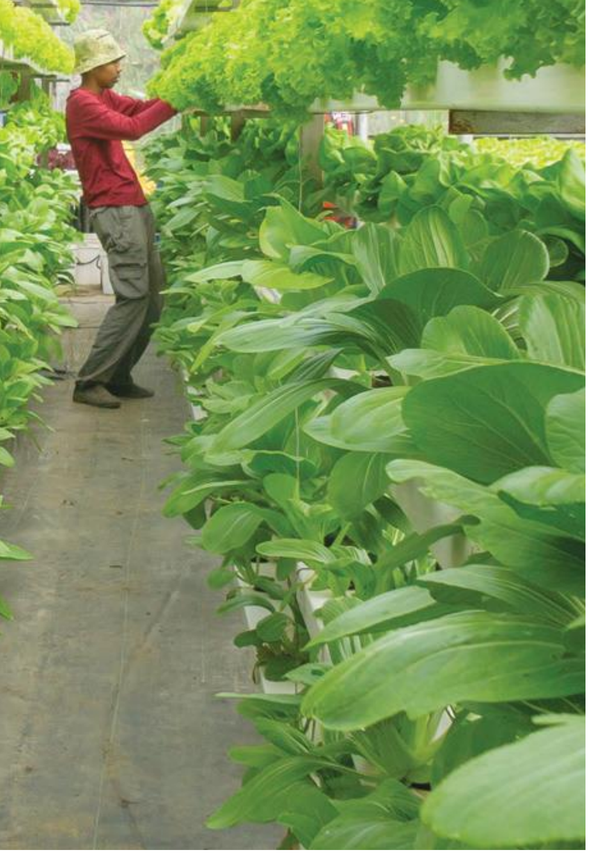
Figure 1 Vertical farming allows people to grow more food in a smaller space.

Figure 2 Drought impacts Australia's production of important crops, such as wheat.

Sustainable farming

Sustainable farming practices use methods that balance the needs of all members of the community. This means that new and old technologies are used to make sure that food production is:

- economically viable – if farmers cannot make enough money to survive, then the farming practice is not sustainable
- socially supportive – if the lifestyle of the farming community is not supported, then people will not want to live in the area
- ecologically sound – if the local environment is not supported, then the land will be unable to support food production. Sustainable



farming also works to maintain the diversity of the local wildlife.

Sustainable farming uses technology to increase the production of fresh, nutritious food while minimising the impact on the local environment.

HUMANITIES

In Geography this year, you will learn about food security around the world and food production in Australia. You will investigate the factors that influence crop yield (such as soil moisture) and how food production can alter a biome. In Economics and Business, you will study the agricultural resources (such as wheat) that form a large part of Australia's trade economy.

To complete this task successfully, you will need to investigate the environmental constraints on agricultural production in Australia, such as climate and distribution of water resources. You will also need to understand the extent to which agricultural innovations have overcome these constraints.

You will find more information on this in Chapter 3 'Food security', Chapter 2 'Biomes' and Chapter 14 'Understanding the economy' of *Oxford Humanities and Social Sciences 9 Australian Curriculum*.

MATHS

In Maths this year, you will build on your knowledge of measurement and geometry to determine areas and volumes of more complicated shapes. You will study right-angled triangles using Pythagoras' theorem and trigonometry. You will also extend your skills in collecting, representing and investigating data.

To complete this task successfully, you will need to perform calculations involving angles, lengths and areas of two-dimensional and three-dimensional shapes. You will need to apply your understanding of scale factors to build a prototype of your designed product. To consider the situation at local, national and international scales, you will need skills in dealing with ratios and proportions. You may also find it helpful to use scientific notation for very large or very small numbers.

You will find help for applying these maths skills in Chapter 5 'Measurement', Chapter 6 'Geometry', and section 1E 'Scientific notation' of *Oxford Maths 9 Australian Curriculum*.

SCIENCE

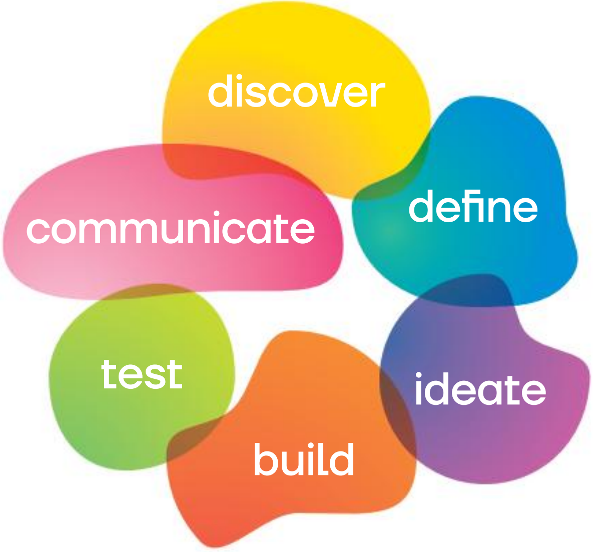
In Science this year, you will learn about the carbon cycle and the ways human activity can disrupt it. You will also consider the consequences of disruption, including the enhanced greenhouse effect. You will also learn about asexual reproduction and investigate vegetative propagation.

To complete this task successfully, you will need to understand the factors required to keep a system, such as a vertical garden, alive. You may need to consider how these factors can be monitored and controlled automatically. You will also need to be familiar with the scientific method, and understand how to conduct a fair test.

You will find more information on this in Chapter 3 'Reproduction' and Chapter 6 'The carbon cycle' of *Oxford Science 9 Victorian Curriculum*.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.



Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, those who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your vertical garden, you need to define the criteria that you will use to test the success of your vertical garden in achieving your goal.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide:

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1 What is the total area of the 100 m × 100 m plot of land? (Remember to use the correct units.)
2 If the plants are planted 25 cm apart in a 100 m row, and the rows are placed 1 m apart, how many plants could be planted in the plot of land? HINT: Draw the plot of land to make sure you reach maximum capacity.
3 What criteria will you use to measure the success of your solution or design? How will you measure how much the end-users have been helped?

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

As a group, brainstorm ways the problem can be solved. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, it is time to sort them by possibility. Select three to five ideas that are possible. Research whether these ideas have already been produced by someone else. If they are already on the market, can you make a better version?

Build

Draw your top two vertical garden designs. Label each part of the designs. Include the materials that will be used for their construction.

Include in the designs:

a the total surface area available for plant growth
b a description of how food production will be increased

c a description of how the design (inputs and waste) will impact the local ecosystem

d a description of how the workers will access all areas of the design to tend the plants

e at least one advantage and disadvantage of each design.

Select one of the designs to take to the building and testing stage.

Build the prototype

You will need to build at least three versions of your vertical garden design prototype. The first prototype garden will be tested for effectiveness. The second prototype will be used to survey the group you are helping. The third prototype will be used for the presentation. The prototype may be full size, or it may be a scale model (10 cm represents 1 m). Use the following questions as a guideline for your prototype:

- What materials will you use?
- What material will you use to represent the plants?
- How will you represent the height, width and angle of the finished prototype?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first vertical garden prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests. What criteria will you use to determine the success of your prototype? Conduct your tests and record your results.

Prototype 2

If your prototype will be used to help market gardeners, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would they consider buying it?)

If your prototype will be used to help another group, or native plants and animals, you will need to consider how you could test the impact it will have. (Will the prototype affect normal behaviours? How will the prototype affect the soil or waterways?)

Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved.

Communicate

Present your vertical garden design to the class as though you are trying to get your peers to invest in it. Describe the criteria and testing used to measure the effectiveness of your vertical garden design.

In your presentation, you will need to:

- explain why we need to be more efficient with food production
- describe the key features of your design and how they improve or solve the problem of food shortages
- show a labelled, to-scale diagram of your prototype
- describe how the ecosystem will be affected by the installation of the prototype
- explain the relevant scientific principles that support your designed solution (e.g. water cycle, photosynthesis, nitrogen/carbon cycle)
- quantify the increase in food production that your design allows; present calculations to justify your claim
- present a calculation for the estimated cost of producing a full-size model of your design
- explain the implications of your design at a state or national level, by comparing the benefits and costs.

Check your student obook pro for the following digital resources to help you with this STEAM project:

Student booklet
This helpful booklet will guide you step-by-step through the project.

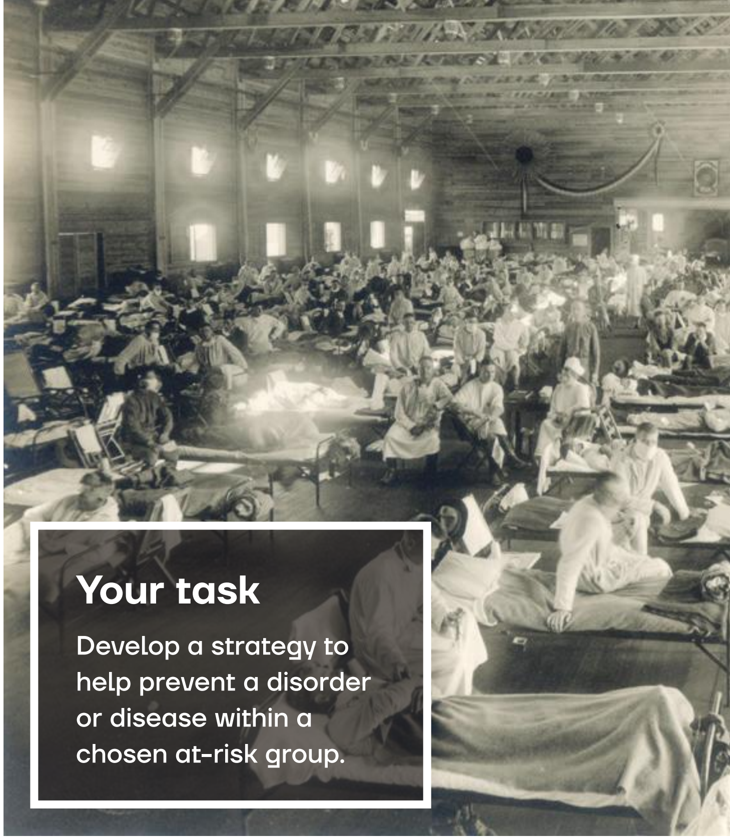
What is the design cycle?
This video will help you to better understand each phase of the design cycle.

How to manage a project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

[STEAM project 2]

How can we harness technology so that we can live healthier lives?



Your task

Develop a strategy to help prevent a disorder or disease within a chosen at-risk group.

Figure 1 During the 1918 flu pandemic (sometimes called the Spanish flu), an estimated 500 million people, or a third of the world's population, were infected with the virus.

Figure 2 Healthcare workers wear personal protective equipment (PPE) to prevent the spread of infectious disease.

A disorder or disease is a condition that affects the normal functioning of the body. Different disorders and diseases can affect many parts of the body. They can be caused by infectious agents such as bacteria or viruses that spread from person to person. Some disorders or diseases are inherited. Environmental factors (such as pollution or diet) can also have an impact on the development of disorders or diseases.

Heart disease, a non-infectious disease, is the leading cause of death globally. Mental health disorders, such as depression, bipolar disorder and dementia, also affect many people around the world.

Disorders and disease affect both high-income and low-income countries, but there are large differences in the ability of different healthcare systems to provide adequate care for people. The need for low-cost health care has led many researchers to investigate how technology can be used to help people live healthier lives.

Prevention of disorders and disease

There are many disorders and diseases that can be prevented through simple, low-cost interventions. Below are a few examples.

- Wearing a helmet or a seat belt has been shown to decrease the risk of brain injury from a road accident. In Vietnam, when wearing a helmet was made mandatory for motorcycle riders, it resulted in a 16 per cent decrease in head injuries.
- The use of mosquito nets can help to prevent malaria, a disease that can lead to life-long neurological impairment, such as epilepsy in children if they have a severe infection.
- Providing vaccinations for viruses such as polio and meningitis can also prevent neurological conditions.
- Promoting a healthy lifestyle and educating the population about the importance of diet can reduce the prevalence of stroke. In Japan, campaigns and treatment for high blood

pressure have reduced the rate of strokes by 70 per cent.

- Personal protective equipment (PPE) is used to protect people from catching infectious diseases, such as Covid-19.

HUMANITIES

In Economics and Business this year, you will learn about how health services are provided in Australia. In Geography, you will study how people are interconnected through travel, technology and trade. These connections affect where and how people access the services they need. In History, you will examine the experiences of different groups during the Industrial Revolution, and the reforms made to improve living standards.

To complete this task successfully, you will need to research the demographics of your local area, and the location and accessibility of health services. You should also consider the economic performance of your area to determine what type of preventative strategy would be most successful for your at-risk group.

You will find more information on this in Chapter 14 ‘Understanding the economy’ Chapter 4 ‘An interconnected world’ and Chapter 9 ‘The Industrial Revolution and the movement of peoples’ of *Oxford Humanities and Social Sciences 9 Australian Curriculum*.

MATHS

In Maths this year, you will extend your skills in representing and interpreting data. You will consider media reports that use statistics and collect secondary data to investigate social issues. You will relate real-world data to probabilities of events, and compare data sets using summary statistics and different graphical displays. You will evaluate and represent data, both with and without digital technology.

To complete this task successfully, you will need to find data to quantify the problem, work out how much your strategy will cost, and calculate a quantitative, evidence-based estimate of the possible benefits of your strategy. You will need skills in dealing with ratios, proportions and percentages to consider the situation at local, national and international scales.

You will find help for applying these maths skills in Chapter 7 ‘Statistics’ of *Oxford Maths 9 Australian Curriculum*.

SCIENCE

In Science this year, you will learn about how the body coordinates and regulates its internal systems so that it can respond to changes. When things change in the environment (such as the emergence of a disease-causing agent), or a part of the body fails, the normal functioning of the body is interrupted. The body needs to respond and attempt to return to a normal homeostatic state before permanent damage is caused.

To complete this task successfully, you will need to identify how the body’s systems work together to maintain a functioning body. You should consider the type of disorder or disease that you will be fighting, and how it may cause changes in the body’s normal function and response mechanisms.

You will find more information on this in Chapter 3 ‘Control and regulation’ of *Oxford Science 9 Australian Curriculum*.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? Is it the people directly affected by the disorder or disease, or do their families and carers need support too?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your solution to the problem, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1 Describe the type of life that the people you are helping lived before their lives were affected by the disorder or disease.
2 Describe how the people affected by the disease have needed to change their lives to cope with the effects of the disorder or disease.
3 Describe how you will know that you have made their lives better as a result of your prototype strategy.

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

Outline the criteria or requirements your design must fulfil (i.e. usability, accessibility, cost).

Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Each group member should draw an individually designed solution. Label each part of the design. Include the material that will be used for its construction.

Include in the individual designs:

a a detailed diagram of the design
b a description of why it is needed by the individual or group

c a description of any similar designs that are already available to buy

d an outline of why your idea or design is better than others that can be purchased.

Present your design to your group.

Build the prototype

Choose one design and build two or three prototypes.

Use the following questions as a guideline for your prototype.

- What materials or technology will you need to build or represent your prototype design?
- What skills will you need to construct your prototype design?
- How will you make sure your prototype design is able to be used by the people who need it?
- How will you describe the way the prototype design will work?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

Prototype 2

If your prototype will be used to help individuals with the disorder or disease, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would it make their life easier or harder? Would they consider buying it? How much would they be willing to pay to access the design?)

Prototype 3

Your last prototype should be adapted using the information gathered from testing the first two versions to make it more effective and usable for the group you are helping. You may want to use the first two prototypes as a demonstration of how the design has been improved over time.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your design.

In your presentation, you will need to:

- outline the relevant disorder or disease and how it affects individuals, as well as society as a whole
- create a working model, or a detailed series of diagrams, with a description of how it will be used by an individual and group
- explain how you changed your design as a result of testing or feedback
- describe how the design will improve the life of an individual or group
- describe how many people need or will use the design
- describe how individuals will be able to access the design (will they need to purchase it or will it be publicly funded?)
- describe how the design will improve an individual's ability to contribute to society as a whole.

Check your Student obook pro for the following digital resources to help you with this STEAM project:

Student booklet
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase of the design cycle.

How to manage a project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

Answers

CHAPTER 1 Exponents

EX 1A Exponents

p7

1 **a** $6 \times 6 \times 6 \times 6 = 1296$

b $8 \times 8 \times 8 = 512$

c $(-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

d $(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) = 729$

e $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64}$

f $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{128}$

g $\left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) = \frac{16}{81}$

h $\left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) = -\frac{243}{3125}$

2 **a** $b \times b \times b \times b \times b \times b$

b $(-n) \times (-n) \times (-n) \times (-n) \times (-n)$

c $(-cd) \times (-cd)$

d $2pq \times 2pq \times 2pq \times 2pq$

e $2 \times p \times q \times q \times q \times q$

f $-4 \times a \times a \times b \times b \times b \times c$

g $3m^2 \times 3m^2 \times 3m^2 \times 3m^2 \times 3m^2$

h $3 \times m^2 \times m^2 \times m^2 \times m^2 \times m^2$

3 **a** 5^4 **b** a^4 **c** $7k^3v^4$

d $(qu)^5$ or q^5u^5 **e** $(-h)^3$ **f** $-h^3$

g $(n^3)^6$ **h** $(5b^3d^4)^2$

4 **a** 2×5^2 **b** $2^3 \times 3^2$ **c** $3^3 \times 5$

d $2 \times 3^3 \times 7$ **e** $2^3 \times 19$ **f** $2^2 \times 7 \times 29$

g $2 \times 5^2 \times 11$ **h** $3^2 \times 5 \times 37$

5 **a** 0.04 **b** 0.04 **c** 0.0004

d 0.008 **e** −0.008 **f** 0.000 008

g 0.0016 **h** 0.0016 **i** 0.000 000 16

6 **a** 5^4 **b** -5^3 **c** a^4b^4

d $5^8x^8y^8$ **e** $5x^8y^8$ **f** $\frac{11^6}{2^6}$

g $-3^5a^5b^5c^5$ **h** $3^8a^8b^8c^8$

7 **a** 343 **b** 6 **c** $-\frac{81}{320}$ **d** 2817

8 **a** $2^3 \times 3^2$ **b** $5^6 \times 6$ **c** $13^4 \times 17^5$

d $101^2 \times 103^5$ **e** $4^3 \times x^4$ **f** $7^2(xy)^7$ or $7^2x^7y^7$

9 **a** $2^3 \times 3 \times 5^2$ **b** $2 \times 3 \times 5^2$

c $2^2 \times 3^2 \times 5^2$ **d** $2^2 \times 3 \times 5^3$

e $2^3 \times 3 \times 5^3$ **f** $2^3 \times 3^2 \times 5^2$

g $2^2 \times 3 \times 5^2 \times 7$ **h** $2^5 \times 3 \times 5^2$

10 **a** x^7 **b** a^3b^2 **c** $3rs^2t^3$ **d** $4e^7f^2$

11 **a** Only k has the exponent 5, so the t should not be repeatedly multiplied. $tk^5 = t \times k \times k \times k \times k \times k$

b All of $2rw$ is raised to the power of 4, so $2rw$ should be repeatedly multiplied.
$(2rw)^4 = 2rw \times 2rw \times 2rw \times 2rw$

c The product was evaluated before the power.
$-3 \times (-2)^4 = -3 \times 16 = -48$

12 **a** 1 **b** 39 **c** −60

d $\frac{63}{4} = 15\frac{3}{4}$

13 **a** **i** 1 **ii** −1 **iii** 1 **iv** −1 **v** 1

vi −1 **vii** 1 **viii** −1 **ix** 1 **x** −1

b **i** When the exponent n, is odd, the basic numeral of $(-1)^n$ is −1.

ii When the exponent n, is even, the basic numeral of $(-1)^n$ is 1.

c **i** negative **ii** negative

iii positive **iv** positive

v positive **vi** negative

vii positive **viii** positive

14 **a** **i** 0.343 and 0.000 343

ii In exponent form, both are the cube of a decimal with the digit 7 in different place values: one is 7 tenths, the other is 7 hundredths. The basic numerals use the same digits 343 also in different place values; 0.000 343 is $10^3 = 1000$ times smaller than 0.343.

b **i** −0.064 and −0.064

ii In exponent form, both are the cube of 0.4 but one has a negative inside the brackets, the other outside. The basic numerals for both are exactly the same.

c **i** −1.728 and 2.0736

ii In exponent form, both raise −1.2 to a power: one is to the power of 3; the other to the power of 4. The basic numeral for the power of 3 is negative while the power of 4 is positive. The two numerals have different values.

d **i** 9.261 and 8.120 601

ii In exponent form, both are cubes of a number that uses the digits 2 and 1. The basic numerals share similar digits where 8.120 601 has its digits spaced out one place more than 9.261 does.

15 **a** 1.2

b 2021: 24, 2022: 29, 2025: 50

16 **a** 3 days: 2^3, 8 days: 2^8, 12 days: 2^{12}

b 3 days: 2^6, 8 days: 2^{16}, 12 days: 2^{24}

c 3 days: 2, 8 days: 2^4, 12 days: 2^6

d A: 24, B: 192, C: 6

17 **a** LCM: $2^8 \times 3^{15} \times 5^2 \times 7^4$ HCF: $2^4 \times 3^5 \times 5^2 \times 7$

b LCM: $a^8b^{15}c^2d^4$ HCF: $a^4b^5c^2d$

c LCM: $pq^5r^{10}s^4$ HCF: $pq^3r^7s^2$

d LCM: $24x^3y^9z^4$ HCF: $4xy^3z^4$

18 **a** 2^{10} **b** 5^{10} **c** $2^{12} \times 5^8$

19 $64^1 = 64$, $8^2 = 64$, $(-8)^2 = 64$, $4^3 = 64$, $2^6 = 64$, $(-2)^6 = 64$

20 **a** $-\frac{1}{12}$ **b** $\frac{243}{32}$ **c** $-\frac{2401}{10} = -240.1$

EX p12

1B Products and quotients of powers

1 **a** 3^9 **b** 7^6 **c** $(-2)^{12}$ **d** 6^3
e $(-8)^7$ **f** 10^{11} **g** 3^1 or 3 **h** 5^2
i 4^2 **j** $(-9)^9$ **k** 13^3 **l** 2^{10}

2 **a** 19 683 **b** 117 649 **c** 4096 **d** 216

3 **a** $3y^9$ **b** $7g^7$ **c** $6b^{11}$
d $-12k^{13}$ **e** $6b^{11}$ **f** $-80g^{11}$
g $27c^{14}$ **h** $15p^{10}$

4 **a** a^2 **b** d **c** g^{10}
d p^3 **e** a^5 **f** n^3
g r^8 **h** $8x^{11}$

5 **a** $12x^{11}$ **b** $10x^7$ **c** $-24x^9$ **d** $54x^{11}$
e $3x^4$ **f** $4x^4$ **g** $\frac{2x}{5}$ **h** $\frac{5x^8}{3}$ or $\frac{5}{3}x^8$
i $-8t^{12}$ **j** $\frac{5r^6}{8}$ or $\frac{5}{8}r^6$ **k** $5c^4$ **l** $\frac{5y^7}{2}$ or $\frac{5}{2}y^7$

6 **a** x^9 **b** $\frac{b}{a}$ **c** n **d** b^8
e x^5 **f** m^4 **g** $6a^6$ **h** n^5
i 5 **j** $-4t^2$ **k** 2 **l** 1

7 **a** a^9b^6 **b** $-18m^{11}n^3$ **c** c^2d^2
d k^2m^3 **e** x^8y^7 **f** $30g^{10}h^3$
g a^8b^6 **h** $15x^8y^{10}$ **i** $54w^4x^{13}y^4$
j $\frac{u^4}{5}$ or $\frac{1}{5}u^4$ **k** $-\frac{3}{4}ef^{10}$ **l** $-108v^{17}y^{10}$

8 **a** x^6 **b** $2k^3$ **c** $6a$
d $\frac{m^2}{x^2}$ **e** b^3 **f** $n^{13}p^3$
g $-2j^5q^6$ **h** $2w^2x^4$

9 **a** False: When multiplying you add, not multiply exponents. $x^3 \times x^4 = x^7$

b False: There is no exponent law for adding terms, but you can add these as they are like terms. $k^3 + k^3 = 2k^3$

c True

d False: There is a power of 1 in the middle term, which must be added. $a^5 \times a \times a^5 = a^{11}$

e False: Add the powers of m and n separately. $m^3n^5 \times m^2n^4 = m^5n^9$

f False: When dividing you subtract, not divide exponents. $100^8 \div 100^2 = 100^6$

g True

h False: The first b^6 has been missed. $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2b^6$

10 **a** **i** $\frac{1}{3^6}$ **ii** $\frac{1}{5^6}$ **iii** $\frac{1}{2^4 \times 3^5}$
iv $\frac{3^5}{2^4}$ **v** $\frac{2^4}{3^5}$

b Copy and complete the following.

i $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2} = \frac{1}{2^{5-3}}$

ii $\frac{2^4}{2^8} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4} = \frac{1}{2^{8-4}}$

iii $\frac{5^6}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5} = \frac{1}{5^1} = \frac{1}{5^{7-6}}$

c Subtracting the exponents on the numerator (larger exponent on the numerator) or denominator (larger exponent on the denominator) gives the exponent on the numerator or denominator when simplified because the common bases are cancelled.

11 **a** 2^{15} **b** 3^{15} **c** $2^6 \times 3^6$ **d** 2^{30}

12 **a** $w = 5, x = 3, y = 2, z = 8$
b $x = 9, y = 6, z = 4$
c $a = 16, b = 9, c = 4, d = 6$

13 No. An exponent represents the number of times the base is repeatedly multiplied. If you write $2^4 \times 3^2$ as $(2 \times 3)^{4+2}$, you are saying that $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is equal to $(2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$, which is not true. Similarly, if you write $y^8 \div x^5$ as $\left(\frac{y}{x}\right)^{8-5}$, you are saying that $\frac{y \times y \times y \times y \times y \times y \times y \times y}{x \times x \times x \times x \times x}$ is equal to $\left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right) \times \left(\frac{y}{x}\right)$, which is not true.

14 **a** $2^3 = 8$ **b** $3^3 = 27$
c $5^2 = 25$ **d** $10^4 = 10\,000$

15 **a** $2^{12} = 8^4$ **b** $3^{15} = 27^5$ **c** $5^{18} = 25^9$
d $10^{12} = 10\,000^3$ **e** $4^{14} = 16^7$ **f** $2^{30} = 32^6$
g $6^6 = 216^2$ **h** $3^{30} = 243^6$

16 **a** $a^{m+n}b^{x+y}$ **b** $a^{m-n}b^{x-y}$

EX p17

1C Raising exponents and the zero exponent

1 **a** 6^{12} **b** 3^4 **c** $3^6 \times 4^2$ **d** 2^{24}
e $5^4 \times 2^{28}$ **f** $\frac{3^2}{4^2}$ **g** $\frac{5^{12}}{2^8}$ **h** $\frac{1}{8^{25}}$
i -3^{28} **j** 3^{20} **k** $7^{28} \times 11^{21}$ **l** $\frac{13^{48}}{17^{24}}$

2 **a** b^{10} **b** m^8 **c** j^{10} **d** j^{10} **e** n^{80} **f** p^{99}

3 **a** x^6y^6 **b** 2^3d^3 **c** $(-5)^7k^7 = -5^7k^7$
d $9^{10}p^{10}$ **e** 3^4m^4 **f** $\frac{8^2}{p^2}$
g $\frac{x^6}{y^6}$ **h** g^2h^2 **i** a^5b^5
j $\frac{k^3}{m^3}$ **k** 2^8x^8 **l** $-\frac{d^5}{3^5}$

4 **a** 3^4x^{24} **b** $5a^{28}b^7$
c $\frac{2^3m^3}{n^3}$ **d** $\frac{a^8}{b^{20}}$
e $-2u^{12}$ **f** $\frac{4}{3}v^{70}w^{30} = \frac{4v^{70}w^{30}}{3}$
g $\frac{7p^7}{9q^{42}}$ **h** $\frac{3^{10}}{2^3}r^{45} = \frac{3^{10}r^{45}}{2^3}$
i $\frac{8}{5^{24}t^{44}}$ **j** $3^2 \times 7 \times i^{34} = 63i^{34}$
k $2^{35}c^{40}$ **l** $\frac{5^4x^{75}}{7^4y^{90}}$

5 **a** 1 **b** 1 **c** 1 **d** 1

6 **a** 2 **b** 1 **c** −7 **d** 1
e −1 **f** 2 **g** 1 **h** 2
i 2 **j** 3 **k** 1 **l** 1
m 1 **n** 1 **o** −1 **p** −1

7 **a** x^{13} **b** x^{22} **c** x^{27} **d** x^{27}
e x^{10} **f** w^6 **g** $\frac{b}{3}$ **h** e^6
i 1 **j** $4a^9$ **k** t^{25} **l** f^{109}

8 **a** 1 **b** -7 **c** 1 **d** 3
e 1 **f** k^2 **g** $10g^4$ **h** 3
i x^{15} **j** 2 **k** $-8b$ **l** $\frac{2}{5}$

9 **a** -2 **b** k^{10} **c** $x^{11}y^9$ **d** $\frac{m^{12}}{2n^5}$ **e** 1

10 **a** x^9y^7 **b** $1568k^7$ **c** $81x^{24}$ **d** $-5a^{28}b^7$
e $\frac{x^{10}}{y^{11}}$ **f** $\frac{8m^3}{n^3}$ **g** $\frac{a^8}{b^{20}}$ **h** $\frac{w^{10}x^6}{y^8}$
i k^7mn^2 **j** $\frac{16t^{13}}{p^{21}}$ **k** ab^{30} **l** $72h^6$

11 **a** True **b** True
c False: Not every power in the bracket has been raised to the sixth power. $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$
d False: The bracket has been expanded incorrectly. $\frac{(k^3)^2 \times k^4}{k^2} = \frac{k^6 \times k^4}{k^2} = k^8$
e True **f** False, $100^9 \div 100^9 = 1$
g True

12 **a** 7 **b** 4 **c** 0 **d** 8
e 3 **f** 5 **g** 7 **h** 5

13 Eden added the 4 and 5 exponents before multiplying the 5 by the 3 exponent. $3^4 \times (3^5)^3 = 3^4 \times 3^{15} = 3^{19}$

14 **a**
$$\begin{aligned}(2^3)^5 &= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3\\ &= 2^{3+3+3+3+3}\\ &= 2^{3\times 5}\end{aligned}$$
b
$$\begin{aligned}(x^7)^4 &= x^7 \times x^7 \times x^7 \times x^7\\ &= x^{7+7+7+7}\\ &= x^{7\times 4}\end{aligned}$$
c
$$\begin{aligned}(2\times 3)^4 &= (2\times 3)\times(2\times 3)\times(2\times 3)\times(2\times 3)\\ &= 2\times 2\times 2\times 2\times 3\times 3\times 3\times 3\\ &= 2^4\times 3^4\end{aligned}$$
d
$$\begin{aligned}\left(\frac{2}{3}\right)^6 &= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\\ &= \frac{2^6}{3^6}\end{aligned}$$

15 **a** 3^{64} **b** 5^{81} **c** 7^{1024} **d** 10^{3125}

16 8^{15} cm^3

17 $\frac{4^5}{3^5}$

18 $(a^m)^n = a^{m\times n} = a^{n\times m} = (a^n)^m$

19 $x = \frac{3}{20}$

Checkpoint

1 **a** $2\times 2\times 2\times 2\times 2\times 2 = 64$
b $(-3)\times(-3)\times(-3)\times(-3) = 81$
c $(-4)\times(-4)\times(-4) = -64$
d $\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{125}{216}$

2 **a** $a\times a\times a\times a\times a\times a$
b $(-b)\times(-b)\times(-b)\times(-b)$
c $3y\times 3y\times 3y\times 3y\times 3y$
d $3\times xy\times xy\times xy\times xy\times xy$

3 **a** 8^7 **b** u^4 **c** $(4b)^5$ **d** $-7k^3h^5$

4 **a** $2^2\times 7$ **b** $2^3\times 3^2$
c $2^2\times 11^2$ **d** $2\times 3^3\times 5$

5 **a** 8^{11} **b** $5^{10}\times 7^{12}$ **c** 6^5 **d** $3^8\times 10^7$

6 **a** a^{12} **b** $-12b^{18}c^{21}$ **c** u^5 **d** $\frac{5}{7}p^{14}q^{17}$

7 **a** 3^{10} **b** k^8
c $\frac{t}{c^{11}}$ **d** $-\frac{15}{4}d^{11}w^7$

8 **a** 1 **b** 1 **c** -1 **d** 8

9 **a** 3^{24} **b** j^{45}
c $(-5)^6a^{18}b^{42} = 5^6a^{18}b^{42}$ **d** $-\frac{3^8p^{40}}{2^8q^{56}}$

10 **a** 1 **b** $\frac{g^{20}}{3^7}$ **c** $5^7m^{66}n^{87}$ **d** $\frac{6}{j^{42}}$

EX p23

1D Negative exponents

1 **a** $\frac{7}{8}$ **b** 2 **c** $-\frac{1}{3}$ **d** $\frac{1}{3w}$
e d **f** $\frac{m}{r}$ **g** $5y$ **h** $\frac{1}{u^2}$

2 **a** $\frac{1}{5}$ **b** $\frac{1}{8}$ **c** $-\frac{1}{2}$
d $\frac{1}{x}$ **e** $\frac{1}{p}$ **f** $\frac{1}{3w}$

3 **a** 5^{-1} **b** $(-13)^{-1}$ **c** $\frac{1}{5^{-1}}$
d $-\frac{1}{8^{-1}}$ **e** m^{-1} **f** $\frac{1}{x^{-1}}$
g $(5y)^{-1}$ **h** $\frac{1}{(3w)^{-1}}$ **i** $\frac{3^{-1}}{4^{-1}} = \left(\frac{3}{4}\right)^{-1}$
j $\frac{6^{-1}}{5^{-1}} = \left(\frac{6}{5}\right)^{-1}$ **k** $\frac{a^{-1}}{w^{-1}} = \left(\frac{a}{w}\right)^{-1}$ **l** $\frac{1}{v^{-3}}$

4 **a** $\frac{1}{4^2}$ **b** $\frac{1}{2^6}$ **c** $\frac{1}{(-9)^3} = -\frac{1}{9^3}$
d $\frac{1}{(-5)^4} = \frac{1}{5^4}$ **e** $-\frac{1}{7^8}$ **f** $\frac{1}{10^5}$
g $\frac{1}{a^4}$ **h** $\frac{1}{x^7}$ **i** $\frac{1}{k^{10}}$
j $\frac{1}{m^2}$ **k** $\frac{1}{u^9}$ **l** $\frac{1}{g^{11}}$

5 **a** 3^{-4} **b** 4^{-7} **c** 6^{-5}
d $(-5)^{-3} = -5^{-3}$ **e** $(-9)^{-2} = 9^{-2}$ **f** -11^{-6}
g n^{-2} **h** g^{-11} **i** x^{-8}
j a^{-9} **k** p^{-4} **l** w^{-7}

6 **a** 2^3 **b** 5^6 **c** $(-8)^4 = 8^4$
d 3^9 **e** $(-7)^5 = -7^5$ **f** -4^2
g x^7 **h** y^3 **i** c^4
j z^{35} **k** $(5t)^4 = 5^4t^4 = 625t^4$
l $(uv)^{15} = u^{15}v^{15}$

7 **a** $\frac{5^2}{4^2}$ **b** $\frac{3}{7}$ **c** $-\frac{4^3}{3^3}$
d $\frac{2^4}{a^4}$ **e** $\frac{m}{6}$ **f** $\frac{u}{144}$

g $\frac{y^6}{x^6}$ **h** $-\frac{7^{11}}{9^{11}}$ **i** $\frac{17^{14}}{13^{14}}$

j $\frac{43^{11}}{500^{11}}$ **k** $\frac{97^{198}}{u^{198}}$ **l** $\frac{g^{654}}{123^{654}}$

8 **a** $\frac{y^3}{x^2}$ **b** $\frac{m^6}{n^4}$ **c** $\frac{c^7}{a}$

d $\frac{2k^5}{p^3}$ **e** $\frac{5b^2}{a^8}$ **f** $-\frac{4}{w^2x^6}$

g $\frac{a^4c^7}{b^5}$ **h** $\frac{m^5}{k^3n^8}$ **i** $\frac{7b^9d}{c^6}$

j $\frac{3}{x^2y^7z^4}$ **k** $\frac{-12p^{17}}{q^{18}u^{21}}$ **l** $\frac{-34}{j^{65}b^{78}}$

9 **a** $\frac{b^3}{a^5}$ **b** $\frac{p^2}{k^6}$ **c** $\frac{m^7}{h^4}$

d d^5e^8 **e** u^3w^8 **f** $3x^2y^4$

g $8g^6h^5$ **h** $-\frac{2c^9}{d^5}$ **i** $\frac{2n^3}{5m^7}$

j $\frac{1}{6e^5y^8}$ **k** $-\frac{1}{2g^{99}h^{11}}$ **l** $\frac{2c^{101}}{5r^{72}}$

10 **a** $\frac{n^4p^6}{m^3}$ **b** $\frac{4c^2e^6}{d^5}$ **c** $\frac{2p^4}{3kn^3}$

d $\frac{a^2b^3}{3c^5d^4}$ **e** $\frac{7k^2n}{m^6}$ **f** $\frac{u^5x^3}{10k^2w^8}$

g $-\frac{5pq^4}{r^4s^3}$ **h** $63b^{14}g^{15}k^{17}u^{10}$ **i** $\frac{1}{5t^4u^3v^7w^{12}}$

j $-\frac{49m^{59}n^{41}}{64l^{88}v}$ **k** $\frac{a^{97}c^{153}d^{167}f^{89}}{b^{105}e^{111}}$ **l** $k^{402}l^{812}m^{999}n^{571}$

11 **a** $\frac{1}{4^3}$ **b** $\frac{1}{7}$ **c** 2^2

d $\frac{1}{(-3)^6} = \frac{1}{3^6}$ **e** 5^4 **f** $\frac{1}{(-2)^7} = -\frac{1}{2^7}$

g $\frac{1}{9^2}$ **h** 3^8 **i** $\frac{1}{4^9}$

j $\frac{1}{10^3}$ **k** $\frac{1}{211^{14}}$ **l** $\frac{1}{13^{100}}$

12 **a** $\frac{1}{5^6}$ **b** $\frac{1}{3^8}$ **c** -2^4

d $\frac{1}{3^2}$ **e** $\frac{1}{6^4}$ **f** $\frac{1}{4}$

g $\frac{1}{9}$ **h** 5^8 **i** 7^3

j $\frac{1}{2^3}$ **k** 99^{127} **l** $\frac{1}{15^{162}}$

13 **a** $\frac{1}{64}$ **b** $\frac{1}{7}$ **c** 4

d $\frac{1}{729}$ **e** 625 **f** $-\frac{1}{128}$

14 **a** $\frac{1}{a^7b^7}$ **b** $\frac{1}{3^5x^5} = \frac{1}{243x^5}$ **c** $\frac{1}{4y}$

d $\frac{1}{7^2k^2} = \frac{1}{49k^2}$ **e** $\frac{1}{2^3p^3} = \frac{1}{8p^3}$ **f** $\frac{1}{m^{11}k^{11}}$

15 **a** −3 **b** −7 **c** −1 **d** 2

e −2 **f** −3 **g** −2 **h** −4

16 **a** $\frac{1}{64}$ mm **b** 0.015 625 mm

17 **a** $\frac{1}{100\,000\,000}$ **b** 0.000 000 01

18 **a** $\frac{1}{15\,625}$ **b** 0.000 064

19 **a**

Exponent form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

b Each term is half of the term before it. The basic numeral of a term with base 2 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c $\frac{1}{64}$ **d** $\frac{1}{1024}$ **e** 128

20 **a**

Exponent form	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

b Each term is one third of the term before it. The basic numeral of a term with base 3 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c $\frac{1}{729}$ **d** $\frac{1}{6561}$ **e** 2187

21 **a**

Exponent form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral	10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

b Each term is one tenth of the term before it. The basic numeral of a term with base 10 and a negative exponent is the reciprocal of the value of the term with the matching positive exponent.

c **i** 100 000 **ii** 1 000 000

iii 10 000 000 **iv** 100 000 000

v 1 000 000 000

d **i** $\frac{1}{100\,000}$ **ii** $\frac{1}{1000\,000}$

iii $\frac{1}{10\,000\,000}$ **iv** $\frac{1}{100\,000\,000}$

v $\frac{1}{1000\,000\,000}$

e **i** $\frac{1}{10}$ **ii** 0.1

f **i** $\frac{1}{100} = 0.01$ **ii** $\frac{1}{1000} = 0.001$

iii $\frac{1}{10\,000} = 0.0001$

g **i** $\frac{1}{100\,000} = 0.000\,01$

ii $\frac{1}{1000\,000} = 0.000\,001$

iii $\frac{1}{10\,000\,000} = 0.000\,000\,1$

iv $\frac{1}{100\,000\,000} = 0.000\,000\,01$

v $\frac{1}{1000\,000\,000} = 0.000\,000\,001$

h The number of decimal places in the decimal matches the number of zeros in the denominator of the fraction, which matches the power of 10.

22 a **i** 20 000 **ii** 7000
iii 300 000 **iv** 400 000 000 000
v 90 000 000

b **i** $\frac{5}{100}$ **ii** $\frac{8}{100\,000}$ **iii** $\frac{2}{1000}$
iv $\frac{7}{10\,000}$ **v** $\frac{6}{1000\,000\,000}$

c **i** 0.05 **ii** 0.000 08 **iii** 0.002
iv 0.0007 **v** 0.000 000 006

d When multiplying a whole number by a negative power of ten, move the decimal point one place to the left for each zero you see.

23 a **i** $2 \times 3^{-1} = 2 \times \frac{1}{3} = \frac{2}{3} = 2 \div 3$

ii $2 \div 3^{-1} = \frac{2}{3^{-1}} = 2 \times \frac{1}{3^{-1}} = 2 \times 3$

b Multiplication by the reciprocal of a number is equivalent to division by that number. Division by the reciprocal of a number is equivalent to multiplication by that number.

24 a **i** 3 **ii** 5 **iii** $\frac{5}{4}$

b Using the power of a power law, $(a^{-1})^{-1} = a^{(-1) \times (-1)} = a^1 = a$.

c The reciprocal of the reciprocal of a number is the original number.

25 a $2a^{-2}b^{-1}$ **b** $-3t^2v^{-3}$ **c** $5^{-1}x^4y^{-4}$ **d** $p^{-1}q^{-5}r^2$

26 a $\frac{1}{x^2}$ **b** $\frac{1}{x^4}$ **c** $8x^3$ **d** $\frac{30}{x^5}$
e 3 **f** x^9 **g** $\frac{1}{x^3}$ **h** $2x^5$
i $\frac{1}{3x^{10}}$ **j** $\frac{4}{7x^4}$ **k** $\frac{1}{x^2}$ **l** x^{11}
m $\frac{4}{x^{13}}$ **n** $\frac{2}{x^8}$ **o** $\frac{1}{x^5}$ **p** $\frac{1}{x^7y^7}$

27 c $\frac{8}{x^{-3}} = \frac{1}{8^{-1}x^{-3}}$ **e** $\frac{1}{3^{-1}}$ **f** $\frac{1}{x^{-9}}$
h $\frac{2}{x^{-5}} = \frac{1}{2^{-1}x^{-5}}$ **l** $\frac{1}{x^{-11}}$

28 a $a^m \times a^{-n} = a^{m + (-n)}$

b $\frac{1}{a^{-m} \times a^n} = \frac{1}{a^{-m + n}}$

c $a^m \div a^n = a^{m - n}$ and $a^{-n} \div a^{-m} = a^{-n - (-m)}$

EX p30

1E Scientific notation

1 a 540 **b** 73 600
c −1800 **d** 405 000
e 2 753 000 **f** 0.61
g 0.000 008 22 **h** −0.000 976
i 0.000 070 03

2 a 10^2 **b** 10^3 **c** 10^4 **d** 10^5 **e** 10^6
f 10^{-1} **g** 10^{-2} **h** 10^{-3} **i** 10^{-4} **j** 10^{-5}

3 a 320 000 **b** 8 140 000 000
c −500 **d** −23 450 000
e 11 000 **f** 0.0064
g 0.000 007 28 **h** 0.000 000 9
i −0.000 030 2 **j** −0.0541
k 450 000 000 000 **l** 0.000 000 006 12
m 0.57 **n** 1306.8
o 0.000 273 16

4 a Check the answers on your calculator with the answers in question **3**.

b Some of the very large and very small numbers are difficult to see on the calculator due to limited screen size.

5 a 4.5×10^3 **b** 7.32×10^6
c 2×10^5 **d** -1.9×10^2
e 3.216×10^3 **f** 6.3×10^{-3}
g 1.8×10^{-7} **h** 5×10^{-2}
i -7.02×10^{-5} **j** 4.27×10^{-1}
k 1.122×10^4 **l** 4×10^{-6}
m -5.682×10^2 **n** 2.49×10^{-4}
o 6.793×10^5 **p** -1.02×10^{-2}

6 a 3 **b** 2 **c** 4 **d** 1
e 3 **f** 2 **g** 1 **h** 4
i 5 **j** 4 **k** 4 **l** 3

7 a 2 **b** 3 **c** 4 **d** 2 **e** 4 **f** 3

8 a 2.6×10^5 **b** -5.04×10^4
c 9.104×10^6 **d** -6.0×10^3
e 460 **f** 73 050
g 1000 **h** 40 000
i −5.14 **j** 0.035
k −42.06 **l** 0.9

9 a 3.3×10^2 **b** 4.87×10^4
c -1.908×10^5 **d** 3×10^3
e 4.03×10^{-1} **f** -5.4×10^{-2}
g 2.072×10^{-4} **h** -8×10^{-3}
i 7.6×10^2 **j** -2.070×10^4
k 4.02×10^1 **l** 5.4008×10^4

10 a A, B, F, H **b** B, F
c A, D, G **d** B, C, E, F, H

11 a 5×10^{-4} m **b** 2.3×10^4 ML
c 4.8×10^{-2} mm **d** 9.3×10^9 people

12 **a** 6400 times in a minute

b 0.000 08 mm

c 149 600 000 km

d 0.000 000 000 000 28 cm

13

	1234.56	**4.0191**
$\times 10^3$	$\underline{1.23456 \times 10^3}$	0.004 0191 $\times 10^3$
$\times 10^2$	12.3456 $\times 10^2$	0.040 191 $\times 10^2$
$\times 10^1$	123.456 $\times 10^1$	0.401 91 $\times 10^1$
$\times 10^0$	1234.56 $\times 10^0$	$\underline{4.0191 \times 10^0}$
$\times 10^{-1}$	12 345.6 $\times 10^{-1}$	40.191 $\times 10^{-1}$
$\times 10^{-2}$	123 456 $\times 10^{-2}$	401.91 $\times 10^{-2}$
$\times 10^{-3}$	1 234 560 $\times 10^{-3}$	4019.1 $\times 10^{-3}$

	0.0492	**0.007 40**
$\times 10^3$	0.000 0492 $\times 10^3$	0.000 007 40 $\times 10^3$
$\times 10^2$	0.000 492 $\times 10^2$	0.000 0740 $\times 10^2$
$\times 10^1$	0.004 92 $\times 10^1$	0.000 740 $\times 10^1$
$\times 10^0$	0.0492 $\times 10^0$	0.007 40 $\times 10^0$
$\times 10^{-1}$	0.492 $\times 10^{-1}$	0.0740 $\times 10^{-1}$
$\times 10^{-2}$	$\underline{4.92 \times 10^{-2}}$	0.740 $\times 10^{-2}$
$\times 10^{-3}$	49.2 $\times 10^{-3}$	$\underline{7.40 \times 10^{-3}}$

14 **a** 6.8×10^7 **b** 2.0×10^2 **c** 4.5×10^4

d -4.0×10^4 **e** -1.23×10^{-1} **f** 3.0×10^5

15 **a** 7.3034×10^5 **b** 8.36×10^4 **c** 6.3×10^{-3}

d -2.68×10^{-2} **e** -1.057×10^4 **f** 9.35×10^4

16 1.08×10^9 km/h

17 **a** 2.4×10^6 km **b** 400 days

18 **a** **i** 86 coins **ii** 774 g

b 0.025 m = 2.5 cm

19 Approximately 507 seconds or about 8 minutes

20 **a** **i** 2 (1 s.f.), 1.9 (2 s.f.), 1.90 (3 s.f.)

ii 2 (1 s.f.), 2.0 (2 s.f.), 1.99 (3 s.f.)

iii 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)

iv 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)

v 2 (1 s.f.), 2.0 (2 s.f.), 2.01 (3 s.f.)

vi 2 (1 s.f.), 2.1 (2 s.f.), 2.10 (3 s.f.)

b If the trailing zeros were not written as significant, then many of these numbers would round to the same number for different significant figures, which means we would be unable to consistently determine the number of significant figures the number was rounded to.

21 **a** 0.4 (1 s.f.), 0.000 000 000 001 (1 s.f.), −0.000 03 (1 s.f.)

b 0, 0, and 0

c If leading zeros are significant, then all values just more than −0.5 and just less than 0.5 would round to 0 for 1 significant figure, which does not provide useful information about very small numbers and is no different to rounding to the nearest integer (or decimal place for more significant figures).

22 **a** Jane rounded to 3 decimal places, not 3 significant figures.

b Kaleb included the leading zeros as significant.

c Lisa did not include the 0 between the 1 and 2 as significant.

d Marius did not keep the place value of the significant figures.

23 **a** **i** 2.82×10^3 seconds

ii 1.2096×10^6 seconds

iii 2.4192×10^7 seconds

iv 3.1536×10^7 seconds

b **i** 2.78 hours **ii** 11.6 days

iii 31.7 years **iv** 31.7 millenia

24 **a** **i** 7.3×10^3 seconds

ii 9.1×10^{-6} seconds

iii 54×10^{-9} seconds

iv 82×10^{12} seconds

v 129×10^6 seconds

vi 974×10^{-12} seconds

b **i** 56.01×10^6 seconds, 56.01 megaseconds (56.01 Ms)

ii 920×10^3 seconds, 920 kiloseconds (920 ks)

iii 43.1×10^{-6} seconds, 43.1 microseconds (43.1 μs)

iv 788×10^{-9} seconds, 788 nanoseconds (788 ns)

v 80×10^{-3} seconds, 80 milliseconds (80 ms)

vi 103.56×10^{12} seconds, 103.56 terraseconds (103.56 Ts)

25 **a** Light is much faster than sound.

b **i** 3.3×10^{-7} seconds

ii about 0.30 seconds

c Watch for the smoke rather than listen for the shot (or listen for the shot and add 0.3 s onto the time).

26 Diameter of one atom = 2.54×10^{-9} cm

The line will be 2.54×10^{-3} cm long.

27 1.8×10^{356}

EX p37

1F Surds

1 **a** **i** 8 **ii** 12 **iii** 20 **iv** 24 **v** 28

b **i** 18 **ii** 27 **iii** 45 **iv** 54 **v** 63

c **i** 50 **ii** 75 **iii** 125 **iv** 150 **v** 175

d **i** 32 **ii** 48 **iii** 80 **iv** 96 **v** 112

e **i** 72 **ii** 108 **iii** 180 **iv** 216 **v** 252

f **i** 128 **ii** 192 **iii** 320 **iv** 384 **v** 448

2 **a** $2\sqrt{5}$ **b** $3\sqrt{3}$ **c** $2\sqrt{7}$ **d** $5\sqrt{2}$

e $6\sqrt{2}$ **f** $20\sqrt{2}$ **g** $7\sqrt{3}$ **h** $8\sqrt{6}$

i $12\sqrt{3}$ **j** $11\sqrt{7}$ **k** $9\sqrt{6}$ **l** $9\sqrt{11}$

3 **a** $\sqrt{45}$ **b** $\sqrt{96}$ **c** $\sqrt{12}$ **d** $\sqrt{44}$

e $\sqrt{250}$ **f** $\sqrt{448}$ **g** $\sqrt{507}$ **h** $\sqrt{720}$

4 **a** $\sqrt{30}$ **b** $\sqrt{96}$ **c** $\sqrt{135}$ **d** $\sqrt{336}$

e $\sqrt{351}$ **f** $\sqrt{600}$ **g** $\sqrt{707}$ **h** $\sqrt{5535}$

5 **a** $\sqrt{3}$ **b** $\sqrt{13}$ **c** $\sqrt{21}$ **d** 3
e $\sqrt{11}$ **f** $\sqrt{17}$ **g** $2\sqrt{6}$ **h** $\sqrt{30}$

6 **a** $4\sqrt{3}$ **b** $10\sqrt{6}$ **c** $6\sqrt{3}$ **d** $5\sqrt{21}$
e $8\sqrt{10}$ **f** $9\sqrt{42}$ **g** $24\sqrt{10}$ **h** $15\sqrt{35}$

7 **a** $8\sqrt{5}$ **b** $2\sqrt{2}$ **c** $3\sqrt{14}$ **d** $6\sqrt{11}$
e $12\sqrt{3}$ **f** $6\sqrt{7}$ **g** $2\sqrt{21}$ **h** $4\sqrt{3}$

8 **a** $\sqrt{9}, \sqrt{6}, \sqrt[3]{6}, \sqrt{\frac{4}{9}}, \sqrt{8}, \sqrt{27}, \sqrt{0.16}, \sqrt[4]{16}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$
b $\sqrt{6}\ \sqrt{8}, \sqrt{27}, \sqrt{\frac{7}{5}}, \sqrt{1.6}$

9 **a** 3 **b** 5 **c** 8 **d** 7
e 1 **f** 50 **g** 1 **h** 1

10 **a** **i** $4\sqrt{3}$ **ii** $3\sqrt{5}$
iii $6\sqrt{6}$ **iv** $2\sqrt{10}$
b **i** $\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$
ii $\sqrt{6} + \sqrt{6}$
iii $\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$
iv $\sqrt{11} + \sqrt{11} + \sqrt{11}$

11 **a** $6\sqrt{10}$ **b** $99\sqrt{35}$ **c** $12\sqrt{105}$ **d** $50\sqrt{30}$

12 **a** **i** $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
ii $2 \times 3 \times 3 \times 3 \times 5$
iii $2 \times 2 \times 5 \times 7 \times 7$
iv $3 \times 3 \times 5 \times 5 \times 5$
b **i** $12\sqrt{2}$ **ii** $3\sqrt{30}$
iii $14\sqrt{5}$ **iv** $15\sqrt{15}$

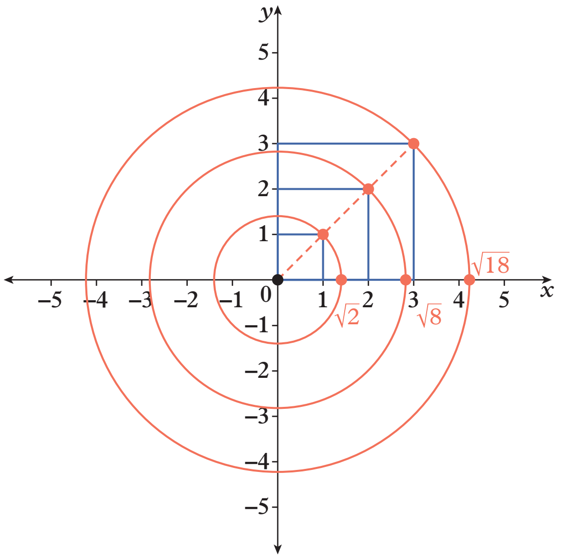
13 **a** $\sqrt{3 \times 7} \times \sqrt{2 \times 7} = 7\sqrt{6}$
b $\sqrt{3^3} \times \sqrt{3 \times 11} = 9\sqrt{11}$
c $\sqrt{2 \times 3 \times 5} \times \sqrt{2 \times 3^2} = 6\sqrt{15}$
d $\sqrt{2 \times 3 \times 7} \times \sqrt{3 \times 7 \times 11} = 21\sqrt{22}$

14 **a** $28\sqrt{6}$ **b** $120\sqrt{10}$
c $4\sqrt{7}$ **d** $\frac{\sqrt{6}}{2}$
e $840\sqrt{35}$ **f** $\frac{3\sqrt{5}}{11} = \frac{3}{11}\sqrt{5}$
g $1872\sqrt{42}$ **h** $\frac{49\sqrt{6}}{4} = \frac{49}{4}\sqrt{6}$

15 **a** $\sqrt{88}$
b 9.38
e $\sqrt{8} \approx 2.83, \sqrt{11} \approx 3.32$
d $940 > 9.38 \approx \sqrt{88}$
e **i** $\sqrt{100}, 10, \sqrt{20} \approx 4.47,$
$\sqrt{5} \approx 2.24, 10.01 > 10 = \sqrt{100}$
ii $6\sqrt{10}, 18.97, \sqrt{30} \approx 5.48,$
$\sqrt{12} \approx 3.46, 18.96 < 18.97 \approx 6\sqrt{10}$
f Rounding during a calculation can make the final answer less accurate compared to using the exact values throughout the calculation and finding an approximation as the final step.

16 **a** 180 **b** 2 **c** 210 **d** 15

17 **a, b, c**



d $\sqrt{8} = 2\sqrt{2}, \sqrt{18} = 3\sqrt{2}$
e They are all multiples of $\sqrt{2}$ so they are evenly spaced along the number line.

18 **a** $7\sqrt{3}$ **b** $3\sqrt{3}$
c $13\sqrt{5}$ **d** $\sqrt{11}$
e $8\sqrt{7} - 11\sqrt{3}$ **f** $13\sqrt{2} + 7\sqrt{3} + 3\sqrt{5}$
g $10\sqrt{2}$ **h** $6\sqrt{3} + 5\sqrt{5}$

19 **a** 2 **b** $2\sqrt{3}$ **c** $\sqrt{3}$

20 **a** 75 **b** 45 **c** 112 **d** 384

21 **a** $\sqrt{2}$ **b** $\sqrt{5}$ **c** $\sqrt{3}$
d $\sqrt{2}$ **e** $\sqrt{3}$ **f** $\sqrt{6}$

Chapter 1 review

Multiple choice

1 B **2** C **3** C **4** E **5** B
6 D **7** A **8** E **9** D **10** B

Short answer

1 **a** 81 **b** −125 **c** −64
d $\frac{243}{32}$ **e** 0.216 **f** 2.0736

2 **a** 17^6 **b** $(-5b^2)^5$ **c** $-10f^3v^7$ **d** $\left(\frac{b^4d^5}{6n^3}\right)^4$

3 **a** a^{16} **b** b **c** $\frac{d^3}{3}$

4 **a** c^{16} **b** e^{47} **c** 9

5 **a** m^5n^8 **b** $54k^{18}l^{12}$

6 **a** $\frac{1}{4}$ **b** $\frac{1}{b}$ **c** $\frac{1}{m^7}$
d $3x^3y^3$ **e** $\frac{g^5}{f^4}$ **f** $\frac{5^2bc^4}{a^3} = \frac{25bc^4}{a^3}$
g $\frac{1}{p^{15}}$ **h** $\frac{l^7}{k^7}$

7 **a** $\frac{1}{65\,536}$ **b** 343

8 **a** 58 760 **b** 0.000 009 02

9 **a** 4 **b** 3

10 a 5.4×10^5 **b** 7.6×10^{-4}

11 a 880 **b** 300 000

12 a 38 600 **b** 39 000

c 3.86×10^4 or 3.9×10^4 (rounded)

13 a $6\sqrt{2}$ **b** 7 **c** $9\sqrt{3}$

d 16 **e** $10\sqrt{10}$ **f** $5\sqrt{14}$

14 a $\sqrt{35}$ **b** $10\sqrt{42}$

c $\sqrt{5}$ **d** $5\sqrt{5}$

15 a $\sqrt{30}$ m **b** $\sqrt{71}$ m

c $2\sqrt{11}$ m **d** $3\sqrt{21}$ m

16 a $12\sqrt{3}$ **b** $108\sqrt{30}$

c $50\sqrt{14}$ **d** $30\sqrt{35}$

Analysis

1 a i Tas, NT, ACT

ii NSW, Vic, QLD, SA, WA

b–e

State	Population	Population (rounded to leading digit)	Population (scientific notation)
NSW	8 166 400	8 000 000	8×10^6
Vic	6 680 600	7 000 000	7×10^6
QLD	5 184 800	5 000 000	5×10^6
SA	1 770 600	2 000 000	2×10^6
WA	2 667 100	3 000 000	3×10^6
Tas	541 400	500 000	5×10^5
NT	246 500	200 000	2×10^5
ACT	431 200	400 000	4×10^5

f i NSW **ii** NT **iii** 7.8×10^6

iv 3.1×10^6 **v** 2.61×10^7

g 4.069×10^5 or 406 900. The difference is due to rounding.

2 a

	Thy	Asha
Round 1	$\frac{2^{10}}{5^3}$	$\frac{3^6}{5^5}$
Round 2	$\frac{5^6}{2^3 \times 3}$	2×3^5
Round 3	$\frac{3^4}{2^3}$	$\frac{2^4 \times 5^3}{3^5}$

b Thy: $2^4 \times 3^3 \times 5^3$, Asha: $\frac{2^5 \times 3^6}{5^2}$

c Thy

d No. Asha had more factors, but the factors of 5 are divided (not multiplied) by the other factors, reducing the value. Thy's value was 54 000, whereas Asha's value was 933.12.

e Thy: $2^{-4} \times 3^{-3} \times 5^{-3} = \frac{1}{2^4 \times 3^3 \times 5^3}$,

Asha: $2^{-5} \times 3^{-6} \times 5^2 = \frac{5^2}{2^5 \times 3^6}$

f Thy: $2^2 \times 3 \times 5^4$, Asha: $2^9 \times 3$

g Thy: $2^{-6} \times 4^1 \times 6^{-4}$, Asha: $1^6 \times 5^3 \times 5^{-4}$

h Asha by two factors

CHAPTER 2 Algebra

EX p49

2A Simplifying

1 a $3x, -x, 20x$

b Like terms have exactly the same pronumerals raised to exactly the same powers. The coefficient can vary.

c 3, 7, −1, 2, 1, 20

2 $\{2ba^2, 3aab, 6a^2b\}$, $\{3a, 6a\}$, $\{2b^2a, 6ab^2, 3abb\}$, $\{6a^3, aaa, 6a^2a\}$

3 a $10a$ **b** $-8k$ **c** $6x^2$

d $4cd - 9cde$ **e** $12x + 6y$ **f** $4a + 6b$

g $9m + 2p$ **h** $1 - k$ **i** $3xy + 5x^2$

j $2d - 4de^2$ **k** $4m^3 + 2$ **l** $abc + 4ab + ac$

4 a $10x + y$ **b** $5ab - 5b + b^2 + a$

c $2km + 4$ **d** $-3x^2 + 3x - 4$

e $2a + a^2 + a^3 - 3$ **f** $6m^2n + 4mn^2 - 2n^2$

5 a $68a^2$ **b** $-27b^3$

c $-3c^2d^2$ **d** $3p^2q^2r + 2pq^2r$

e $8e + 8f + 4$ **f** $7x^2y + \frac{3xy}{2}$

6 a $6abcd$ **b** $-20mpxy$ **c** $9g^2h$

d $-24k^2mn$ **e** $56bjp^2t$ **f** ax^2y^2

g $18a^3bcd$ **h** $-20h^2k^2p$ **i** $-6b^3$

j $4km^2n^3$ **k** $15x^4y^2$ **l** $56a^4b^3c^2$

7 a $75a^{18}z^{36}$ **b** $-432b^{31}y^{16}$

c $-160c^8m^9x^9$ **d** $\frac{44d^7n^{10}}{w^7}$

e $\frac{1}{45e^{11}p^{24}t^{19}}$ **f** $\frac{168q^{33}}{g^{14}u^{28}}$

8 a b **b** $\frac{mn}{p}$ **c** $4c$ **d** $-\frac{2}{x}$

e $-\frac{1}{2}$ **f** $\frac{y}{4}$ **g** $\frac{6b}{5c}$ **h** $\frac{2}{11}$

i $6a$ **j** $-3xy$ **k** $\frac{5n^2}{3}$ **l** $-\frac{ac}{4}$

9 a $\frac{4}{d^5}$ **b** $\frac{1}{c^{24}d^{15}}$ **c** $-\frac{125u^3r^4}{49}$

d $\frac{16p^{38}}{9k^{24}}$ **e** $\frac{2x^9y^6}{z^3}$ **f** $20f$

10 a p^3q **b** $-2d$ **c** $-\frac{3x}{c}$

d $\frac{k}{2p}$ **e** $\frac{7c}{bd^2}$ **f** $\frac{m}{n^4}$

g $-\frac{3}{2xy^2z^3}$ **h** $-16a^5b^7c^9d$ **i** $-\frac{2k^5m^4n^3}{3}$

11 a true **b** false **c** false **d** true

12 a Set B is correct. Set A: all signs changed to + when expression rearranged. Set C: The sign in front of the $2a$ term has changed from + to − and 2 − 7 should be −5, not 9.

b Set A is correct. Set B: −3 × 4 should be −12, not 12, and b should be squared in the last step. Set C: $b \times b$ (or b^2) has been written as $b \times 2$.

c Set C is correct. Set A: The terms are not in their appropriate locations in the fraction in the last step. Set B: error in cancelling the 'a' terms.

13 a $8x + y = 22$ **b** $-6xy = 36$ **c** $15xy = -90$

d $\frac{10}{y} = -5$ **e** $x^2y^3 = -72$ **f** $3x = 9$

g $6y - 2xy = 0$ **h** $x^2 + 4x - y = 23$

i $\frac{y^4}{x^3} = \frac{16}{27}$

14 a $2a + 3b + 2c = 11$ **b** $8ab - a = -18$

c $-2a^2b + ab^2 + 3ac = 40$ **d** $10a^2b^2c^2 = 1000$

e $3ab = -6$ **f** $\frac{4ac}{3} = \frac{40}{3} = 13\frac{1}{3}$

15 a $6x + 5y + 2$ **b** $4x + 2y - 2$ **c** $12x + 20$

16 a 51 cm **b** 24 cm **c** 68 cm

17 a $20xy$ **b** $49x^2$ **c** $9xy$ **d** $2y^2$

18 a 120 m^2 **b** 441 m^2

c 54 m^2 **d** 8 m^2

19 a $2y$ **b** 10 m, 160 m^2

20 a $3x$ **b** 96 cm^2, 16 cm

21 a i $6k$ **ii** $2k^2$

b ii 30 cm and 50 cm^2

22 a i $4x$ m **ii** x^2 m^2

iii $(4x + 8)$ m **iv** $(4x + 4)$ m^2

b i 36 m^2 **ii** 32 m

iii 40 m **iv** 64 m^2

23 a $10xy$ **b** 200 cm^2 **c** $12x + 8y$

d 88 cm **e** 158 cm

24 $2^{-48}x^{24}$

EX p55

2B Expanding

1 a $4a + 12$ **b** $7b + 35$ **c** $3c - 6$

d $5d - 5$ **e** $24 + 6e$ **f** $-2f - 16$

g $-3g - 12$ **h** $-8h + 40$ **i** $-4x + 36$

j $-10 + 5j$ **k** $kp + 6k$ **l** $ab - 4a$

m $18m + 6k$ **n** $2np + 4nq$ **o** $x^2 - 7xy$

2 a $4x + 6$ **b** $2x + 3$ **c** $3 - n$

d $7y + 5z$ **e** $-7b + 10c = 10c - 7b$

f $3t + 7$

3 a $11x + 6$ **b** $5p + 6$ **c** $ab + 2a$

d $11y - 4$ **e** $4h + k + 2$ **f** $-6m$

4 a $5x - 8$ **b** $13k - 4$ **c** $7p + 1$

d $x^2 + 4x + 12$ **e** $m^2 + 5m + 6$ **f** $y^2 - 7y + 10$

5 a $12az + 15a$ **b** $56b + 40by$ **c** $14c - 6cx$

d $-20dw - 25d$ **e** $-48ev + 54e$ **f** $-40et - 30eu$

g $8g^2 - 28gt$ **h** $-30hr + 42h^2$ **i** $12i^2jk + 15ij^2k$

j $12m^5 + 15m^8$ **k** $5n^4 - 3n^3$ **l** $p^6 - p^9q^4$

6 a $ab + 4a + 3b + 12$ **b** $cd + 7c + 2d + 14$

c $mn + m + 5n + 5$ **d** $xy + 3x + 9y + 27$

e $kp - 2k + 6p - 12$ **f** $fg - f + 4g - 4$

g $ac + 3a - 5c - 15$ **h** $wf + 2w - 7f - 14$

i $xy - 8x - 4y + 32$ **j** $jk - 5j - 9k + 45$

k $2ab + 6a + 7b + 21$ **l** $15cd - 20c + 6d - 8$

7 a $a^2 + 5a + 6$ **b** $x^2 + 15x + 50$

c $d^2 - 2d - 24$ **d** $y^2 - 5y - 24$

e $k^2 + 2k - 63$ **f** $m^2 - 3m - 18$

g $5e^2 - 22e + 8$ **h** $21a^2 - 31a + 8$

i $6y^2 - 13y + 5$

8 a $5\mathrm{b} + 20 - ab - 4a$ **b** $-x^2 - 3x + 18$

c $-8c^2 - 2c + 15$ **d** $44d^2 - 102de + 54e^2$

e $-42p^2 + 13pq + 40p^2$ **f** $4x^2 - 9x - 1$

g $x^8 + x^7 + x^4 + x^3$ **h** $-20x^4 + 13x^2 + 84$

i $2x^2 + 11x + 11$

9 a i Area of large rectangle $= 7(3 + 2)$

$= 7 \times 5$

$= 35$

ii Area of large rectangle $= 7 \times 3 + 7 \times 2$

$= 21 + 14$

$= 35$

b i Area of large rectangle $= 5(x + 2)$

ii Area of large rectangle $= 5x + 5 \times 2$

$= 5x + 10$

c The area of the large rectangle is length × width or $a \times (b + c)$.
The total area of the two smaller rectangles is $a \times b + a \times c$.
As the two areas are equal, $a(b + c) = ab + ac$.

10 a i Area of large rectangle $= (6 + 4)(3 + 5)$

$= 10 \times 8$

$= 80$

ii Area of large rectangle $= 6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$

$= 18 + 30 + 12 + 20$

$= 80$

b i Area of large rectangle $= (k + 2)(m + 3)$

ii Area of large rectangle $= km + 3k + 2m + 2 \times 3$

$= km + 3k + 2m + 6$

c The area of the large rectangle is length × width or $(a + b) \times (c + d)$.
The total area of the four smaller rectangles is $a \times c + a \times d + b \times c + b \times d$.
As the two areas are equal,
$(a + b)(c + d) = ac + ad + bc + bd$.

11 a $a^2 - 9$ **b** $b^2 - 4$ **c** $c^2 - 25$

d $9p^2 - 64$ **e** $h^2 - 1$ **f** $k^2 - m^2$

12 The middle two terms always combine to give zero, so you are left with the first term squared minus the second term squared. The two factors are almost the same with one set of brackets containing a sum of two terms and the other containing a difference of the same two terms.

13 a The result is the first term squared minus the second term squared.

b No, as you can multiply two factors in any order.

c **i** $x^2 - 4$ **ii** $y^2 - 81$ **iii** $m^2 - 36$

iv $9d^2 - 100$ **v** $m^2 - n^2$ **vi** $9 - x^2$

14 **a** $(100 + 3)(100 - 3) = 100^2 - 3^2$

b $103 \times 97 = (100 + 3)(100 - 3) = 10\,000 - 9 = 9991$

c **i** 9996 **ii** 9975 **iii** 999 999

iv 999 964

15 **a** $a^2 + 4a + 4$ **b** $b^2 + 14b + 49$

c $c^2 + 8c + 16$ **d** $9p^2 - 30p + 25$

e $h^2 - 2h + 1$ **f** $m^2 + 2mn + n^2$

16 The result is always the first term squared then 2 times the product of both terms + second term squared. The two factors are exactly the same.

17 **a** $(a + b)^2$ is a perfect square.

b **i** $x^2 + 6x + 9$ **ii** $y^2 + 12y + 36$

iii $m^2 + 4m + 4$ **iv** $16b^2 + 88b + 121$

v $m^2 + 2mn + n^2$ **vi** $25 + 10x + x^2$

c $(a - b)^2$ is also a perfect square. The rule or pattern holds true. The negative sign in the factor $(a - b)$ means that the middle term is $-2ab$ rather than $2ab$.

d **i** $a^2 - 4a + 4$ **ii** $b^2 - 8b + 16$

iii $c^2 - 14c + 49$ **iv** $25w^2 - 60w + 36$

vi $k^2 - 2kp + p^2$ **vi** $9 - 6x + x^2$

18 **a** Some possible answers are: $2x + 5$, $9p - 7$, $6c^2 - 14$, and $ab + 4c$.

b A binomial product is where two terms are multiplied by two terms.

c Three terms

d Some possible answers are: $x + 6y + 9$, $a - 2d + 4$ and $81 - 18z + y$.

e A quadratic trinomial is an expression with three terms, where the highest power of a variable is 2. For example: $p^2 - 10p + 25$, $b^2 + 22b + 121$ and $9 - 6xy + x^2$.

19 **a** $12a - 20b + 40$

b $6xw - 10x^2 + 14xy - 8xz$

c $5x^2 + 35x + 60$

d $6ax + 10bx - 12x + 9ay + 15by - 18y$

e $2x^2 + 7xy + 15x + 3y^2 + 15y + 18$

f $x^3 + 8x^2 + 19x + 14$

20 E

21 **a** $8(2x + 5)$ **b** $16x + 40$

c **i** 88 cm² **ii** 88 cm²

d The answers should be the same if you have expanded correctly.

22 **a** **i** $(x + 7)(x + 3)$ **ii** $x^2 + 10x + 21$

iii 96 m²

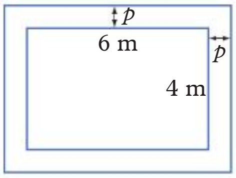
b **i** $(x + 9)(x + 9)$ **ii** $x^2 + 18x + 81$

iii 196 cm²

c **i** $(2x - 1)(x + 2)$ **ii** $2x^2 + 3x - 2$

iii 63 cm²

23 **a**



b **i** $(2p + 6)$ m **ii** $(2p + 4)$ m

c $(2p + 6)(2p + 4) = 4p^2 + 20p + 24$

d $(4p^2 + 20p + 24) - (6 \times 4) = 4p^2 + 20p$

e **i** 24 m² **ii** 80 m² **iii** 56 m²

24 **a** $100y^2 - 140y + 49$ **b** $x^7 - x^6$

c $x^4 - 25$ **d** $y^{12} + 6y^7 + 9y^2$

e $-5a^3$ **f** $x^6y^6 + 2wx^5y^2$

25 $(a - b)^2 + (c - d)^2$

$= (a^2 - 2ab + b^2) + (c^2 - 2cd + d^2)$

$= (b^2 - 2ab + a^2) + (d^2 - 2cd + c^2)$

$= (b - a)^2 + (d - c)^2$

26 **a** Square: length = y, width = y

Rectangle: length = y, width = $x - y$

b Area of the original rectangle = xy

Total area of the two shapes $= (y)(x - y) + y^2$

$= xy - y^2 + y^2$

$= xy$

27 $x = 5$

EX p63

2C Factorising using the HCF

1 **a** 4 **b** 2 **c** 5

d d **e** 2 **f** 3

g 4 **h** $3h$ **i** $12x$

2 **a** c **b** $2y$ **c** $9m$

d bd **e** $4y$ **f** $2k$

g p **h** 5 **i** $3p$

3 **a** $c(b + d)$ **b** $2y(x + 1)$ **c** $9m(2n - 1)$

d $bd(ac + f)$ **e** $4y(2x + 7)$ **f** $2k(3k - 5)$

g $p(1 + 11p)$ **h** $5(9ab - 8cd)$ **i** $3p(q + 2)$

4 **a** a^4 **b** b^2 **c** $2c^3$

d $6d^3z^5$ **e** $7e^6y^5$ **f** $2fx^9$

g $-2g$ **h** $6k^5$ **i** $x^9y^7z^{13}$

5 **a** $5(2x + 1)$ **b** $3(y - 7)$ **c** $4(2k + 3)$

d $3(5 - 2d)$ **e** $7(4x + 3y)$ **f** $10(2n - 5)$

g $xy(x + 3y^3)$ **h** $mn^2(mn - 9)$

6 **a** $5b(4a - 1)$ **b** $8d(1 + ce)$ **c** $5x(3x + 2)$

d $2k(2k - 11)$ **e** $6n(5 - 3n)$ **f** $a(16a + 1)$

g $2h(h - 7)$ **h** $6p(1 + p)$

7 **a** $8m^4(3m^2 + 2)$ **b** $7q^3r^2(3q^2 + 5r^4)$

c $5t^4u^2(3 - t^4u^5)$ **d** $6c^4d^4(2c^5d + 1)$

e $7e^5f^3(e^6 - 1)$ **f** $3ij^2k(i^4k^5 + 9j^5)$

g $b^3c^7(a^7b^9 + c^2\mathrm{d}^4)$ **h** $pq(5m - 3np^9q^4)$

i $w^5z^3(u^4y^2 + v^4x^2)$

8 **a** $-5n(m + 2)$ **b** $-7x(2y + 1)$ **c** $-6c(1 - d)$

d $-a(a + 3)$ **e** $-2k(2k + 1)$ **f** $-8x(x - 1)$

g $-3(4 + xy)$ **h** $-2m(8 + 5m)$ **i** $-9xy(x - 2)$

9 a $(w+4)(x+2)$ **b** $(x-1)(y+7)$

c $(a+6)(a-3)$ **d** $(5-n)(p+8)$

e $(4-k)(3k-5)$ **f** $(3x-4)(2x+9)$

g $(2g+1)(4g+1)$ **h** $(8n-d)(2h-1)$

i $(7x+6y)(3a+2b)$

10 a $(a+5)(b+4)$ **b** $(y-6)(x+7)$

c $(n+4)(m-2)$ **d** $(y+3)(y+5)$

e $(k-7)(k+2)$ **f** $(x+3)(6+x)$

g $(a-7)(a-2)$ **h** $(p+5)(p-2)$

i $(3c-1)(2c+3)$

11 Expanding means to write an expression without brackets. Factorising means to write an expression in factor form and generally involves using brackets. Factorising and expanding are opposite processes.

12 a i $6(x+1)$ cm **ii** $2x(x+4)$ cm

b i 36 cm **ii** 90 cm

13 a $(2x+5)$ m **b** $(m+15)$ cm

14 a $2(2x+9)$ m **b** $2(2m+15)$ cm

15 a $3(9x-3y+5z)$ **b** $5(9p-10q-1)$

c $4(1-5i+10j-15k)$ **d** $a^2(a^3+a+1)$

e $6b^3c^4(3c-6b+4b^5c)$

f $7r^5t^7(12r^7t+1+7r^3t^7+2r^4t)$

16 a $\frac{1}{2}(5x+3y)$ **b** $\frac{3}{5}(3g+h)$

c $\frac{5}{4}(2p+3q)$ **d** $\frac{1}{12}(28m-15n)$

17 a $x(10-x)$ mm^2

b Length and width are x mm and $(10-x)$ mm.

c Sample answers: 6 mm by 4 mm, 7 mm by 3 mm, 9 mm by 1 mm.

18 a Some possible answers are:

Height = $(7-x)$ cm, base length = $6x$ cm

Height = $2(7-x)$ cm, base length = $3x$ cm

Height = 6 cm, base length = $x(7-x)$ cm

b Some possible answers, using the first sample answer in **a**:

Height = 3 cm, base length = 24 cm ($x = 4$)

Height = 6 cm, base length = 6 cm ($x = 1$)

Height = 2 cm, base length = 30 cm ($x = 5$)

19 Length = width = $3x$ cm, Height = $(x+5)$ cm

20 a $(x+3)(x-4)$ **b** $(x-4)(x+3)$

c They are equivalent.

21 a $n+1, n+2$ **b** $3n+3$

c $3(n+1)$; three times middle number

d $n+(n+2)+(n+4) = 3n+6 = 3(n+2)$; also three times middle number

22 For four consecutive numbers, the sum is four times the average of the two middle numbers.

For six consecutive numbers, the sum is six times the average of the two middle numbers.

For 10 consecutive numbers, the sum is 10 times the average of the two middle numbers.

$$10 \times \left[\frac{(n+4)+(n+5)}{2}\right] = 10 \times \left(\frac{2n+9}{2}\right) = 5(2n+9)$$

23 a $(y+5)(x-3)$ **b** $(2q-3)(p-1)$

c $(a-3)(2a-3)$

24 Area of square = $l \times w = 2r \times 2r = 4r^2$

Area of circle = πr^2

Area of shaded section = $4r^2 - \pi r^2 = r^2(4-\pi)$

25 a $5\left(\frac{9x}{5}+\frac{17}{5}\right)$

b $12\left(x^2+\frac{x}{4}+\frac{1}{2}\right)$

c $x\left(3x+5y+\frac{y^2}{x}\right)$

d $(x+2)\left(7(x+2)+5+\frac{9}{x+2}\right)$

Checkpoint

1 a $4a-2b-5$ **b** $4t^2+12t-4$

c $2x^3-7x^2+5x+2$ **d** $4cd^2-10d^2+2c$

2 a $-35ab$ **b** $48a^2bc^2$

c $\frac{9h}{4}$ **d** $\frac{t}{3u}$

3 a $-800a^{16}b^{32}$ **b** $-\frac{729p^6}{8k^3}$

c $\frac{9t^4}{8r^7}$ **d** $\frac{2x^{28}}{3y^4}$

4 a $8a+24$ **b** $-8b+20$

c $21c-3cy$ **d** $18d^2-9d$

5 a $3w+41$ **b** $-x+35$

c $15y-9$ **d** $z^2+5x-15$

6 a $pv+2v+5p+10$ **b** $-qu-3q+4u+12$

c $r^2+4r-12$ **d** $6t^2-29t+35$

7 a $a^7+2a^5+5a^4+10a^2$ **b** $b^3+7b^2-28b+1$

c $c^2-11c-10$ **d** $7d^2-8d+19$

8 a $3(2a+3)$ **b** $12(b-3)$

c $-7(4c+1)$ **d** $4d(2d+9)$

9 a $(3g+2)(2m+5)$ **b** $(h-3)(8-n)$

c $(q+7r)(p-1)$ **d** $(x+3)(2x-5)$

10 a $(b+5)(a+4)$ **b** $(3d-2)(4c-7)$

c $(3f-1)(5e+1)$ **d** $(x+6)(x+5)$

EX p69

2D Factorising the difference of two squares

1 a $9x^2-49$ **b** $9x^2-49$

c Show each term as a square ($(3x)^2$ and 7^2) and then write the base of each term in two pairs of brackets. One pair of brackets will contain the sum of the two bases and the other the difference.

2 a $(x+6)(x-6)$ **b** $(a+10)(a-10)$

c $(3+y)(3-y)$

3 a $(5m+2)(5m-2)$ **b** $(7+8c)(7-8c)$

c $(ab+3)(ab-3)$ **d** $(xy+w)(xy-w)$

e $(p+qr)(p-qr)$ **f** $(bn+dm)(bn-dm)$

g $(5jk + 6mn)(5jk - 6mn)$

h $(9ab + 7cd)(9ab - 7cd)$

i $(12pq + 11rt)\ (12pq - 11rt)$

4 **a** $3(x + 2)(x - 2)$ **b** $2(3p + 5)(3p - 5)$

c $5(10 + 7g)(10 - 7g)$ **d** $7(1 + 2u)(1 - 2u)$

e $4(3r + 5)(3r - 5)$ **f** $100(8 + 11z)(8 - 11z)$

g $6(x + y)(x - y)$ **h** $p(q + r)(q - r)$

i $u^2(t + d)(t - d)$ **j** $4d(3k + 2j)(3k - 2j)$

k $2a^2(9b + 11d)(9b - 11d)$ **l** $9x(1 + 3x)(1 - 3x)$

5 **a** $(k + 1)(k - 7)$ **b** $(8 - m)(6 - m)$

c $(y + 5)(1 - y)$ **d** $-a(a + 4)$

e $(y + 5 + x)(y + 5 - x)$ **f** $(u + t - 9)(u - t + 9)$

g $(3p + 4)(p + 4)$ **h** $(9 - 3m)(9 - 5m)$

i $3(2x + 3)$ **j** $(q + r + 1)(q - r + 11)$

k $(10x - 9)(4x + 1)$ **l** $4z$

6 **a** $(6 + x)(6 - x)$ **b** $(b + 4)(b - 4)$

c $(3h + 5)(3h - 5)$ **d** $(8n + 7d)(8n - 7d)$

e $(1 + rp)(1 - rp)$ **f** $(tu + wx)(tu - wx)$

7 **a** $\left(t + \frac{2}{3}\right)\left(t - \frac{2}{3}\right)$ **b** $\left(r + \frac{5}{7}\right)\left(r - \frac{5}{7}\right)$

c $\left(\frac{9}{4} + u\right)\left(\frac{9}{4} - u\right)$ **d** $\left(\frac{3}{10}d + \frac{11}{12}\right)\left(\frac{3}{10}d - \frac{11}{12}\right)$

e $\left(\frac{1}{8}k + \frac{1}{5}\right)\left(\frac{1}{8}k - \frac{1}{5}\right)$ **f** $\left(\frac{w}{13} + \frac{x}{14}\right)\left(\frac{w}{13} - \frac{x}{14}\right)$

8 **a** $(z + 0.4)(z - 0.4)$ **b** $(y + 1.1)(y - 1.1)$

c $(0.2 + x)(0.2 - x)$ **d** $(c + 0.05)(c - 0.05)$

e $(0.7g + 0.1h)(0.7g - 0.1h)$

f $(0.12 + 0.11k)(0.12 - 0.11k)$

9 **a** $(x^3 + 3)(x^3 - 3)$ **b** $(6 + y^5)(6 - y^5)$

c $(z^{11} + 7)(z^{11} - 7)$ **d** $(2p^7 + 5)(2p^7 - 5)$

e $\left(\frac{q^9}{4} + \frac{9}{11}\right)\left(\frac{q^9}{4} - \frac{9}{11}\right)$ **f** $(1 + r^{15}t^{25})(1 - r^{15}t^{25})$

10 **a** **i** 18 × 10 **ii** 25 × –5 **iii** 197 × 1

iv 12 × –4 **v** 16 × 10 **vi** 29 × 1

b **i** 180 **ii** –125 **iii** 197

iv –48 **v** 160 **vi** 29

c **i** 64 – 25 **ii** 144 – 324 **iii** 900 – 1

iv 81 – 9 **v** 100 – 900 **vi** 196 – 1

d **i** 39 **ii** –180 **iii** 899

iv 72 **v** –800 **vi** 195

11 **a** 2600 **b** 11.4 **c** $\frac{1}{25}$

12 **a** $2n + 1$ **b** odd number

13 $4(x + 1)$

14 **a** $(x^2 + 4)(x + 2)(x - 2)$

b $(x^4 + y^4)\ (x^2 + y^2)\ (x + y)(x - y)$

15 **a** $\pi R^2 - \pi r^2$ **b** $\pi(R + r)(R - r)$

c **i** 50.27 cm² **ii** 100.53 cm² **iii** 59.69 cm²

16 5 cm and 7 cm

17 **a** $(5^2\pi - 4.75^2\pi + 3.75^2\pi - 3.5^2\pi + 2.5^2\pi - 2.25^2\pi + 1.25^2\pi - 1^2\pi)$ m²

b $(5^2\pi - 4.75^2\pi + 3.75^2\pi - 3.5^2\pi + 2.5^2\pi - 2.25^2\pi + 1.25^2\pi - 1^2\pi)$

$= \pi((5^2 - 4.75^2) + (3.75^2 - 3.5^2) + (2.5^2 - 2.25^2) + (1.25^2 - 1^2))$

$= \pi((5 + 4.75)(5 - 4.75) + (3.75 + 3.5)(3.75 - 3.5) + (2.5 + 2.25)(2.5 - 2.25) + (1.25 + 1)(1.25 - 1))$

$= \pi(0.25(5 + 4.75) + 0.25(3.75 + 3.5) + 0.25(2.5 + 2.25) + 0.25(1.25 + 1))$

$= 0.25\pi(5 + 4.75 + 3.75 + 3.5 + 2.5 + 2.25 + 1.25 + 1)$ m²

18 $(x + 1)(x + y)^2 - (x + 1)(x - y)^2$

$= (x + 1)[(x + y)^2 - (x - y)^2]$

$= (x + 1)[(x + y) + (x - y)][(x + y) - (x - y)]$

$= (x + 1)(x + y + x - y)(x + y - x + y)$

$= (x + 1)(2x)(2y)$

$= 4xy(x + 1)$

19 **a** $(\sqrt{a} + \sqrt{5})(\sqrt{a} - \sqrt{5})$ **b** $(b + \sqrt{7})(b - \sqrt{7})$

c $(\sqrt{c} + 2\sqrt{2})(\sqrt{c} - 2\sqrt{2})$ **d** $(\sqrt{3d} + 4)(\sqrt{3d} - 4)$

e $(2\sqrt{e} + 3\sqrt{5})(2\sqrt{e} - 3\sqrt{5})$

f $(f\sqrt{2} + 5\sqrt{3})(f\sqrt{2} - 5\sqrt{3})$

g $(x\sqrt{x} + y^2\sqrt{y})(x\sqrt{x} - y^2\sqrt{y})$

h $4(x\sqrt{2} + y\sqrt{3})(x\sqrt{2} - y\sqrt{3})$

20 **a** 9 m and 1 m **b** 7 m and 3 m

c 10 m and 3 m **d** 10.5 m and 4.5 m

21 9 cm²

22 **a** $x^3 + y^3$ **b** $(x + 2)(x^2 - 2x + 4)$

c $(x - 2)(x^2 + 2x + 4)$

You can use the previous result, replacing 2 with −2.

23 31

EX p76

2E Factorising quadratic expressions

1 **a** 1 and 4 **b** 2 and 4 **c** 2 and 11

d 4 and 5 **e** 4 and 6 **f** 3 and 4

g 6 and 7 **h** 5 and 7 **i** 4 and 4

2 **a** −2 and 4 **b** −3 and 2 **c** −6 and −2

d −2 and 5 **e** −9 and 1 **f** −3 and −2

g −1 and 6 **h** −9 and 3 **i** −11 and −1

3 **a** $(x + 1)(x + 4)$ **b** $(x + 2)(x + 4)$

c $(x + 2)(x + 11)$ **d** $(x + 4)(x + 5)$

e $(x + 4)(x + 6)$ **f** $(x + 3)(x + 4)$

g $(x + 6)(x + 7)$ **h** $(x + 5)(x + 7)$

i $(x + 4)^2$

4 **a** $(a + 3)(a + 1)$ **b** $(b + 2)(b + 7)$

c $(c + 1)(c + 6)$ **d** $(d + 3)(d + 7)$

e $(e + 1)(e + 7)$ **f** $(f + 3)(f + 5)$

g $(g + 4)(g + 7)$ **h** $(h + 4)(h + 9)$

i $(x + 3)(x + 6)$ **j** $(j + 5)(j + 9)$

k $(k + 5)(k + 6)$ **l** $(y + 5)(y + 8)$

5 **a** $(x-2)(x+4)$ **b** $(x-3)(x+2)$
c $(x-6)(x-2)$ **d** $(x-2)(x+5)$
e $(x-9)(x+1)$ **f** $(x-3)(x-2)$
g $(x-1)(x+6)$ **h** $(x-9)(x+3)$
i $(x-11)(x-1)$

6 **a** $(a-1)(a+3)$ **b** $(b-5)(b+3)$
c $(c-4)(c-1)$ **d** $(d-2)(d+7)$
e $(e-6)(e-4)$ **f** $(f-5)(f+2)$
g $(g-3)(g+4)$ **h** $(h-5)(h-3)$
i $(x-8)(x+3)$ **j** $(j-8)(j-2)$
k $(k-3)(k+6)$ **l** $(y-2)(y+1)$

7 **a** $3(x+1)(x+2)$ **b** $2(x+3)(x+5)$
c $5(x-1)(x+4)$ **d** $-4(x+2)(x+3)$
e $-6(x-5)(x-1)$ **f** $-(x-5)(x+7)$

8 **a** $a(x+2)(x+6)$ **b** $b(x-7)(x-4)$
c $10c(x-6)(x+5)$ **d** $-2d(x-5)(x-12)$
e $p^2(x+3)(x+10)$ **f** $x^2(q+11)(q+4)$

9 **a** $(x-3)^2$ **b** $(x-3)(x+3)$
c $(x+0)(x-6) = x(x-6)$

10 **a** $(x-5)(x+5)$ **b** $(x-5)^2$
c $x(x-25)$ **d** $(x+7)^2$
e $x(x-49)$ **f** $(x-7)(x+7)$

11 **a** $(a+3)(a+7)$ **b** $(b-5)(b+7)$
c $(c-6)(c+3)$ **d** $-(d+6)(d-8)$
e $-(e+4)(e+6)$ **f** $-(f-2)(f-8)$
g $(g+8)(g+9)$ **h** $(h-7)(h+2)$
i $-(x+4)(x+9)$

12 **a** $x^2 + 6x + 0 = (x+0)(x+6)$
$x^2 + 6\text{x} + 5 = (x+1)(x+5)$
$x^2 + 6x + 8 = (x+2)(x+4)$
$x^2 + 6x + 9 = (x+3)(x+3)$
b $x^2 + 6x - 7 = (x-1)(x+7)$
$x^2 + 6x - 16 = (x-2)(x+8)$
$x^2 + 6x - 27 = (x-3)(x+9)$
$x^2 + 6x - 40 = (x-4)(x+10)$
c When the coefficient of x is positive, both factors are positive when the constant is positive. When the coefficient of x is positive, one factor is positive and one is negative when the constant is negative.

13 **a** $x^2 - 6x - 7 = (x-1)(x-7)$
$x^2 - 6x - 16 = (x-2)(x-8)$
$x^2 - 6x - 27 = (x-3)(x-9)$
$x^2 - 6x - 40 = (x-4)(x-10)$
b $x^2 - 6x + 0 = (x-0)(x-6)$
$x^2 - 6x + 5 = (x-1)(x-5)$
$x^2 - 6x + 8 = (x-2)(x-4)$
$x^2 - 6x + 9 = (x-3)(x-3)$
c When the coefficient of x is negative, both factors are negative when the constant is positive. When the coefficient of x is negative, one factor is positive and one is negative when the constant is negative.

14 **a** $x^2 + 65x + 64 = (x+1)(x+64)$
$x^2 + 34x + 64 = (x+2)(x+32)$
$x^2 + 20x + 64 = (x+4)(x+16)$
$x^2 + 16x + 64 = (x+8)(x+8)$
b $x^2 - 65x + 64 = (x-1)(x-64)$
$x^2 - 34x + 64 = (x-2)(x-32)$
$x^2 - 20x + 64 = (x-4)(x-16)$
$x^2 - 16x + 64 = (x-8)(x-8)$
c **i** If the coefficient of x is positive, use positive factors when the constant is positive. If the coefficient of x is negative, use negative factors when the constant is positive.
ii Sum

15 **a** $x^2 - 63x - 64 = (x+1)(x-64)$
$x^2 - 30x - 64 = (x+2)(x-32)$
$x^2 - 12x - 64 = (x+4)(x-16)$
$x^2 - 0x - 64 = (x+8)(x-8)$
b $x^2 + 63x - 64 = (x-1)(x+64)$
$x^2 + 30x - 64 = (x-2)(x+32)$
$x^2 + 12x - 64 = (x-4)(x+16)$
$x^2 + 0x - 64 = (x-8)(x+8)$
c **i** If the coefficient of x is positive, the larger factor is positive and the smaller factor is negative when the constant is negative. If the coefficient of x is negative, the larger factor is negative and the smaller factor is positive when the constant is negative.
ii Difference

16 **a** $(x+4)(x+4) = (x+4)^2$
b $(y-5)(y-5) = (y-5)^2$
c $(v+1)(v+1) = (v+1)^2$
d $(2z-3)(2z-3) = (2z-3)^2$
e $(3w+7)(3w+7) = (3w+7)^2$
f $3(h-2)(h-2) = 3(h-2)^2$
g $-(k+9)(k+9) = -(k+9)^2$
h $(f+g)(f+g) = (f+g)^2$
i $(5p-4q)(5p-4q) = (5p-4q)^2$

17 **a** Check for and factorise out any common factors. Halve the coefficient of the linear term and check if it is equal to the product of the square root of the squared terms (or squared term and constant). Both squared terms must be positive.
b **i** Both squared terms are not positive.
ii Both squared terms are not positive.
iii Half of the linear term is not the product of the square root of the squared terms.
iv Half of the linear term is not the product of the square root of the squared terms.

18 **a** $(x+6)$ **b** $(x+3)(x+6)$
c **i** Length = 8 m, width = 5 m, area = $8 \times 5 = 40$ m^2
ii Area = $2^2 + 9 \times 2 + 18 = 40$ m^2

19 a i $(x + 9)$ cm **ii** $(3y + 4)$ m

b 10 **c** 7

d $x = 2, y = \frac{4}{3}$

e While one of the lengths is positive, the other length and area are both negative, which does not make sense for a physical rectangle. That is, the values for the length, width and area for both items must all be positive.

20 a i $(x - 2)(x - 3)$ **ii** $(x - 2)(x - 5)$

iii $(x - 2)(x - 2)$ **iv** $(x - 2)(x - 4)$

b 2 m

c $x^2 - x - 6 = (x - 3)(x + 2)$ and
$x^2 + x - 6 = (x - 2)(x + 3)$
Both are wider than their backyards' width, x m.

d $x^2 - 6x + 8$

e 8 m^2

f 19.25 m^2

g Melissa: 16 m^2, Lena: 22 m^2

21 a $(3x - 2)(3x + 7)$ **b** $(11x + 10)(11x - 8)$

c $(5x - 4)(5x - 8)$ **d** $(x + 6)(x + 1)$

e $(x - 9)(x - 10)$ **f** $x(x - 4)$

g $3(x + 2)(3x - 7)$ **h** $2(x + 5)(2x - 3)$

i $16(2x - 1)(2x + 3)$

22 a $(x - 2)(x + 2)(x - 3)(x + 3)$

b $(x^2 + 4)(x - 4)(x + 4)$

c $(x^2 + 25)(x - 1)(x + 1)$

23 a $(x + 4)(x + 1)$ **b** $(x + 4)(x + 2)$

c $(x + 11)(x + 2)$ **d** $(x + 5)(x - 2)$

e $(x - 9)(x + 1)$ **f** $(x - 3)(x - 2)$

Chapter 2 review

Multiple choice

1	E	**2**	B	**3**	A	**4**	B
5	B	**6**	E	**7**	A	**8**	B
9	A	**10**	A	**11**	C	**12**	D
13	E	**14**	C	**15**	A	**16**	C

Short answer

1 a $16t$ **b** $13a - 18p$ **c** $-2k + km - 15$

d $13m^2n - 5m^2 + 11n^2 - 4mn^2$

2 a $44x^2y^2z$ **b** $72m^4n^3p^2$ **c** $\frac{5e}{6f}$ **d** $-\frac{n}{2m}$

3 a $4x - 28$

b $-40 + 24y$

c $35x - 30wx$

d $ut - 4u + 3t - 12$

e $40r - 15rv + 48 - 18v$

f $63mp - 27np + 77mq - 33nq$

4 a $8a - 11$ **b** $b^2 - 9b - 22$

c $12c^2 - 23c + 10$ **d** $d^2 - w^2$

e $36 + 12e + e^2$ **f** $f^2 - 18f + 81$

5 a $x^7 - x^6$ **b** $y^{12}z^4 - y^3z^6$

c $x^{11} + x^5y^4 - x^6y^3 + y^7$

6 a $4(a - 6)$ **b** $36pq(q + 4)$

c $(8 - d)(7d - 4)$ **d** $(1 + 3f)(5e + 2)$

7 a $z^4(6z - 5)$ **b** $3d(3dy + 5x^2)$

c $-12r^7t^8u^3(3r^3u^3 + 8)$

d $a^4b^2c^7d^5(ad^3 + a^2b^3c^2 + cd^2)$

8 a $8x(x^2 + 2)(3x - 1)$ **b** $(r + q)(t + p + 2)$

c $2(x + 4)(2x - 3)$

9 a $(a + 8)(a - 8)$ **b** $(11 + b)(11 - b)$

c $(6m + 7n)(6m - 7n)$ **d** $(p + 3)(p - 1)$

e $2(e + 4)(e - 4)$ **f** $9(2f - 1)$

10 a $(y^5 - z^5)(y^5 + z^5)$ **b** $-8(q + 2)$

c $(10 + r)(10 - r)$ **d** $(9j - 8)(9j + 8)$

11 a $\left(\frac{14}{5}x + \frac{13}{2}y\right)\left(\frac{14}{5}x - \frac{13}{2}y\right)$ **b** $\left(\frac{12}{c} + \frac{15}{d}\right)\left(\frac{12}{c} - \frac{15}{d}\right)$

c $\left(\frac{6}{7} + \frac{u}{4}\right)\left(\frac{6}{7} - \frac{u}{4}\right)$

d $(0.2v - 1.1)(0.2v + 1.1)$

12 a $(a + 5)(a + 1)$ **b** $(b - 4)(b - 3)$

c $(c - 3)(c + 7)$ **d** $(d - 18)(d + 2)$

13 a $(x + 3)(x + 5)$ **b** $-5(x - 6)(x + 2)$

c $-2(x - 10)(x - 6)$ **d** $-3(x + 10)(x - 8)$

14 a $2(7 - x)(7 + x)$ **b** $16(y^2 + 4x^2)$

c $(x - 5)^2$ **d** $9x(x - 4)$

Analysis

1 a i $(x + 5)(x + 11)$ **ii** $(x - 6)(x - 14)$

iii $(x + 6)(x + 16)$

b The numbers in the brackets in factor form are $a + b$ and $a - b$.

c i $x^2 + 16x + 55$ **ii** $x^2 - 20x + 84$

iii $x^2 + 22x + 96$

d The coefficient of x in the expanded form is $2a$.

e The constant in the expanded form is $a^2 - b^2$.

f i $(x + 3)^2 - 4 = x^2 + 6x + 5 = (x + 1)(x + 5)$

ii $(x - 5)^2 - 16 = x^2 - 10x + 9 = (x - 9)(x - 1)$

iii $(x + 7)^2 - 100 = x^2 + 14x - 51 = (x - 3)(x + 17)$

2 a $\frac{1}{2}n(n + 1) = \frac{1}{2}n^2 + \frac{1}{2}n, \frac{(n + 1)^2 - (n + 1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$

b i 15 **ii** 15

c 5050 **d** $n(n + 1)$ **e** 10 100

f The sum of the first n even numbers is double the sum of the first n counting numbers.

g $\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

h i 55 **ii** 55

i 338 350

CHAPTER 3 Linear relationships

EX p88

3A Solving linear equations

1 a $x = 6$ b $x = 5$ c $x = 36$ d $x = 10$
e $x = -20$ f $x = 4$ g $x = 15$ h $x = -1$
i $x = -3$ j $x = -3$ k $x = 12$ l $x = -6$

2 a $x = 2$ b $x = -8$ c $x = 9$ d $x = 5$
e $x = 20$ f $x = -8$ g $x = 0$ h $x = -4$
i $x = -5$ j $x = 10$ k $x = -20$ l $x = 4$

3 a $x = 6.9$ b $x = -2.6$ c $x = 10.8$ d $x = -3.16$

4 a $x = 6$ b Yes, $x = 6$ c $x = 5.6$ d $x = 5.6$
e Expanding brackets first means you do not have to work with fractions or decimals until the last step.

5 a $x = 3$ b $x = -9$ c $x = \frac{11}{2}$
d $x = -\frac{12}{5}$ e $x = 4$ f $x = \frac{11}{6}$

6 a $x = 4$ b multiply both sides by x
c $x = 4$

7 a $x = 5$ b $x = -3$ c $x = -6$ d $x = 4$
e $x = \frac{11}{2}$ f $x = -\frac{8}{5}$ g $x = 4.8$ h $x = -\frac{1}{3}$

8 a $x = 2$ b $x = 7$ c $x = -1$ d $x = -3$
e $x = -2$ f $x = 7$ g $x = 4$ h $x = -4$

9 a $x = -1$ b $x = 3$ c $x = 7$ d $x = -3$
e $x = 7$ f $x = -3$ g $x = -6$ h $x = 9$

10 a $2x + 5 = x - 4$ b $x = -9$

11 a $x = 5$ b $x = -3$ c $x = 2$ d $x = -3$

12 a $\frac{3(5x-1)}{6} = \frac{2(x+5)}{6}$ b $x = 1$

13 a $x = 6$ b $x = -4$ c $x = 7$ d $x = -27$

14 a Let n = number of jellybeans
b $4n + 2 = 34$
c $n = 8$
d Each person received 8 jellybeans.

15 a Let x = number of months
b $70x + 115 = 395$
c $x = 4$ d 4 months

16 a $185 b 14 goals
c 31 m × 19 m d 84 people

17 a B b $3.50

18 a $P = 3l$
b $w = 10$ m

19 56 sausages

20 100°

21 Numbers are 5, 6 and 7.

22 a i always ii never iii sometimes
iv never v always vi never
b After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are equal. Alternatively, the equation reduces to $0 = 0$.
c After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are not equal. Alternatively, the equation reduces to an equation that is not true, such as $0 = 2$.

23 a $x = \frac{11}{5} = 2.2$ b $x = \frac{19}{13}$ c $x = \frac{13}{7}$

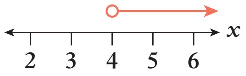
EX p95

3B Solving linear inequalities

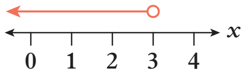
1 a $2 < 3$ b $3 > 2$
c $3 \geq 2$ d $-3 \leq -2$
e $3 \neq -3$ f $-3 \neq 3$

2 a $x \leq 5$ b $x > 1$
c $x \geq -4$ d $x < -2$
e $x < 0$ f $x \neq 3$
g $x \geq 0$ h $x \neq -1$

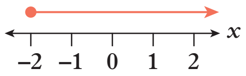
3 a $x > 4$



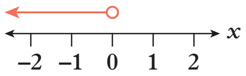
b $x \leq 3$



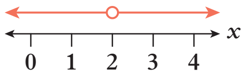
c $x \geq -2$



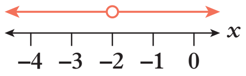
d $x < 0$



e $x \neq 2$



f $x \neq -2$



4 a i $1 < 14$ ii $-13 < 0$
iii $-12 < 27$ iv $-2 < \frac{9}{2}$
v $12 > -27$ vi $2 > -\frac{9}{2}$
vii $-\frac{4}{3} < 3$ viii $\frac{4}{3} > -3$
b i $x + 5 \geq 11$ ii $x - 9 \geq -3$
iii $3x \geq 18$ iv $\frac{x}{2} \geq 3$
v $-3x \leq -18$ vi $-\frac{x}{2} \leq -3$
vii $\frac{x}{3} \geq 2$ viii $-\frac{x}{3} \leq -2$

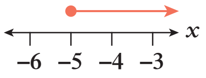
5 a $x > 14$ b $x \leq 300$ c $x < -5$
d $x \leq 5$ e $x < -70$ f $x \leq 5$
g $x \geq 10$ h $x > -3$ i $x \leq -9$

j $x \le 7$ **k** $x > -37$ **l** $x > -\frac{13}{10}$

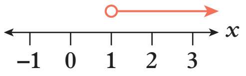
m $x < -\frac{1}{7}$ **n** $x \le \frac{29}{6}$ **o** $x > \frac{11}{6}$

p $x \le -\frac{95}{33}$

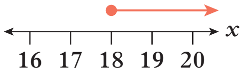
6 a $x \ge -5$



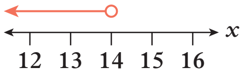
b $x > 1$



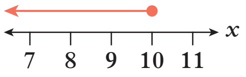
c $x \ge 18$



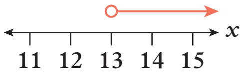
d $x < 14$



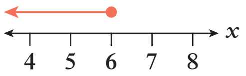
e $x \le 10$



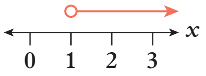
f $x > 13$



g $x \le 6$



h $x > 1$



7 a $x > 4$ **b** $x \le -2$ **c** $x \ge 3$

d $x > 4$ **e** $x < 1$ **f** $x \ge -1$

g $x \le 5$ **h** $x < 2$

8 a $x > 11$ **b** $x \le 13$ **c** $x \ge -2$

d $x > -19$ **e** $x > -15$ **f** $x \le 1$

g $x \le 3$ **h** $x > -\frac{1}{4}$ **i** $x \ge -\frac{13}{2}$

9 a $x \le -7$ **b** $x > 1$ **c** $x \le -3$

d $x < 1$ **e** $x \ge 4$ **f** $x > -5$

10 a $p \ge 750\,000$, where p is the selling price in dollars

b $h < 200$, where h is the height of a person in centimetres

c $s \le 55$, where s is the speed in km/h

d $h < 196$, where h is the height of a person in centimetres

11 a $m = \frac{1}{2}(20 - x)$, where m is the number of watermelons they will each take home

b $\frac{1}{2}(20 - x) \le 3$; $x \ge 14$

c 14 or more watermelons

12 a 4

b i Yes **ii** No

c 5

13 a $3n \le 25$, where n is the number of packs of playing cards Hassan could buy; $n \le \frac{25}{3}$

b Hassan could buy 1, 2, 3, 4, 5, 6, 7 or 8 packs of playing cards.

14 a i $4 < 9$ **ii** $4 < 9$

iii $4 < 9$ **iv** $4 < 9$

v $\frac{1}{4} > \frac{1}{9}$ **vi** $\frac{1}{4} > \frac{1}{9}$

vii $4 = 4$

b The square of a negative and a positive number are both positive, so when two values in an inequality have different signs, then the order of the inequality may change.

c i $\frac{1}{3} < 2$ **ii** $-\frac{1}{3} < 2$

iii $-\frac{1}{3} > -2$ **iv** $\frac{1}{3} > -2$

v $3 > 2$ **vi** $-3 < 2$

d The greater the positive number, the smaller its reciprocal will be. The smaller the negative number, the greater its reciprocal will be. Therefore, when the signs of the values are the same, the order will reverse. However, when the signs of the values are different, the order does not change as the negative value will still be smaller than the positive value.

15 a $4x + 5y \le 100$ **b** $x \ge 0$ and $y \ge 0$

c $x = 25$ **d** $y = 20$

e $x = 15$

16 a $x \ge -11$ **b** $x < 4$ **c** $x \ge 2$

d $x < -8$ **e** $x \le 2$ **f** $x \le -3$

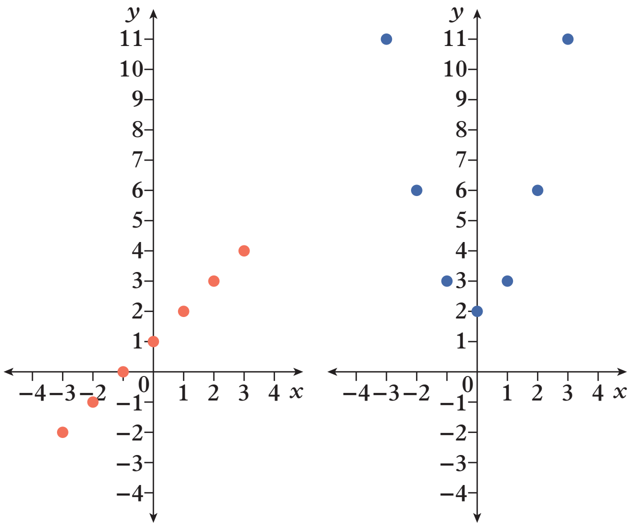
17 a $-1 \le x \le 4$ **b** $-\frac{9}{2} < x < -\frac{5}{2}$

c $-8 < x < 12$ **d** $-\frac{9}{2} < x \le \frac{9}{10}$

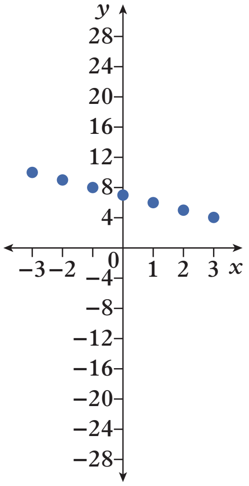
EX p101

3C Plotting linear relationships

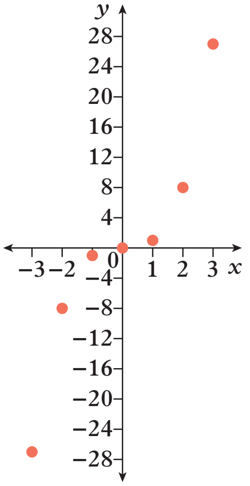
1 a i **ii**



iii



iv



b **i** linear **ii** non-linear **iii** linear

iv non-linear

2 **a** $y = x + 2$

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5

(–3, –1), (–2, 0), (–1, –1), (0, 2), (1, 3), (2, 4), (3, 5)

b $y = x - 4$

x	–3	–2	–1	0	1	2	3
y	–7	–6	–5	–4	–3	–2	–1

(–3, –7), (–2, –6), (–1, –5), (0, –4), (1, –3), (2, –2), (3, –1)

c $y = 3 - x$

x	–3	–2	–1	0	1	2	3
y	6	5	4	3	2	1	0

(–3, 6), (–2, 5), (–1, 4), (0, 3), (1, 2), (2, 1), (3, 0)

d $y = 2 - x$

x	–3	–2	–1	0	1	2	3
y	5	4	3	2	1	0	–1

(–3, 5), (–2, 4), (–1, 3), (0, 2), (1, 1), (2, 0), (3, –1)

e $y = - x - 3$

x	–3	–2	–1	0	1	2	3
y	0	–1	–2	–3	–4	–5	–6

(–3, 0), (–2, –1), (–1, –2), (0, –3), (1, –4), (2, –5), (3, –6)

f $y = 4x$

x	–3	–2	–1	0	1	2	3
y	–12	–8	–4	0	4	8	12

(–3, -12), (–2, –8), (–1, –4), (0, 0), (1, 4), (2, 8), (3, 12)

g $y = 2x + 1$

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

(–3, –5), (–2, –3), (–1, –1), (0, 1), (1, 3), (2, 5), (3, 7)

h $y = 3x - 2$

x	–3	–2	–1	0	1	2	3
y	–11	–8	–5	–2	1	4	7

(–3, –11), (–2, –8), (–1, –5), (0, –2), (1, 1), (2, 4), (3, 7)

i $y = 4 - 2x$

x	–3	–2	–1	0	1	2	3
y	10	8	6	4	2	0	–2

(–3, 10), (–2, 8), (–1, 6), (0, 4), (1, 2), (2, 0), (3, –2)

3 **a** IV: radius

DV: area because the formula for the area of a circle depends on the radius.

b IV: Number sold

DV: Revenue because it depends on the number of items sold.

c IV: Distance

DV: Light intensity because it depends on the distance from the source.

d IV: Number of words

DV: Number of pages because it depends on the number of words.

e IV: Amount of breath

DV: Volume because the volume of the note depends on the amount of breath used to play a note.

f IV: Number of cards

DV: Chance because the chance of drawing an ace of spades depends on the number of cards remaining.

4 **a** **i**

x	0	3	6	9	12	15	18
y	24	21	18	15	12	9	6

ii

x	–12	–6	–2	2	6	12
y	–2	–4	–12	12	4	2

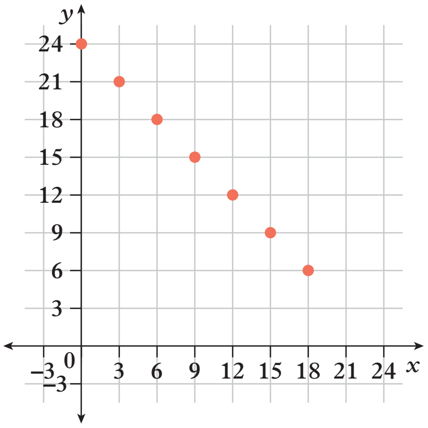
iii

x	–3	–2	–1	0	1	2	3
y	3	0	–1	0	3	8	15

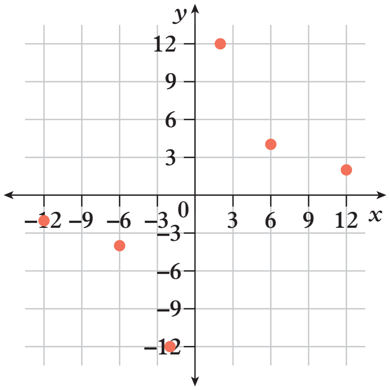
iv

x	–14	–10	–6	–2	2	6	10
y	–3	–2	–1	0	1	2	3

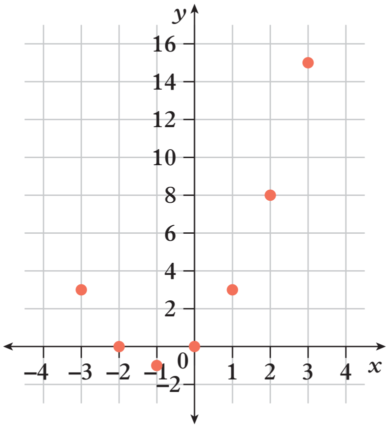
b **i** linear



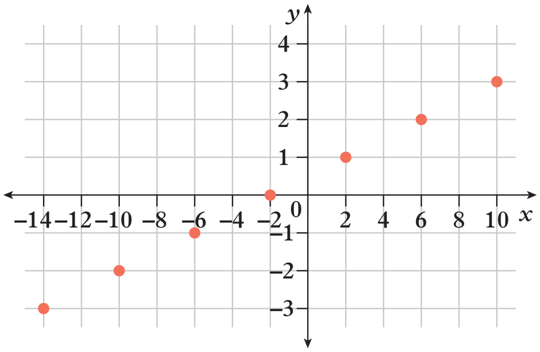
ii non-linear



iii non-linear



iv linear



5 a i $y = x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii $y = 2x$

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

iii $y = 3x$

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

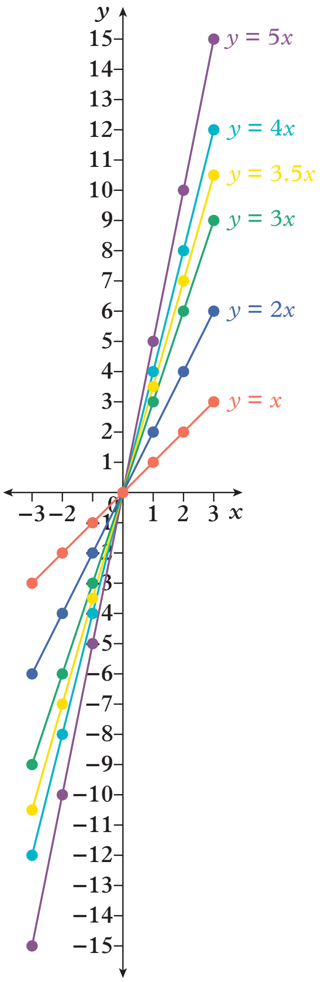
iv $y = 4x$

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12

b All four graphs pass through the origin. However, the steepness of each line is different.

c As the coefficient of x increases, the steepness of the corresponding graph increases.

a, d



6 a i $y = -x$

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

ii $y = -2x$

x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

iii $y = -3x$

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

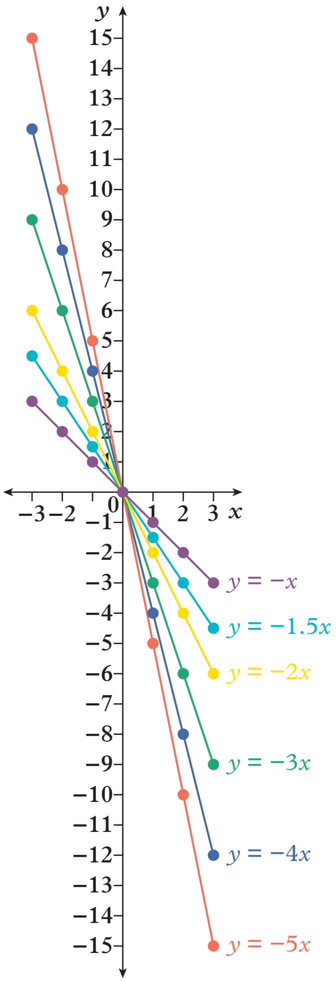
iv $y = -4x$

x	−3	−2	−1	0	1	2	3
y	12	8	4	0	−4	−8	−12

b The values of y decrease from left to right.

c As the coefficient of x decreases, the steepness of the corresponding graph increases.

a, d



7 a i $y = x$

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii $y = x + 1$

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

iii $y = x + 2$

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

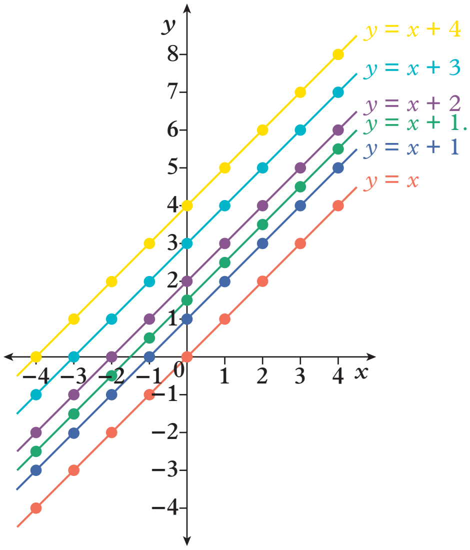
iv $y = x + 3$

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

b All four graphs have the same steepness, but pass through different points on the x- and y-axes.

c As the value added to x increases, the graph moves upwards.

a, d



8 a i

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

ii

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

iii

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1

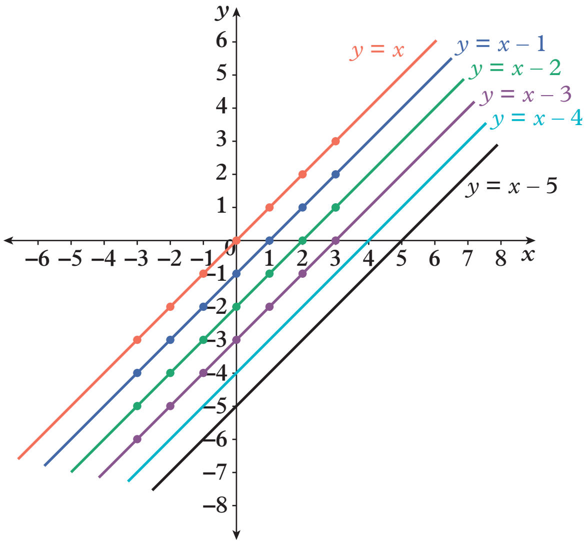
iv

x	−3	−2	−1	0	1	2	3
y	−6	−5	−4	−3	−2	−1	0

b All four graphs have the same steepness, but pass through different points on the x- and y-axes.

c As the value added to x decreases, the graph moves downwards.

a, d



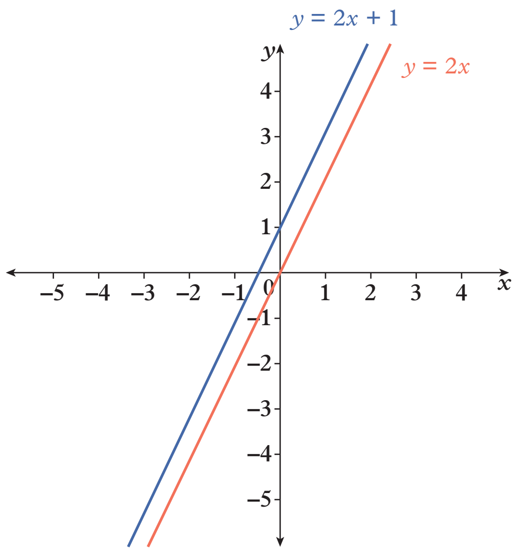
9 a A

b D

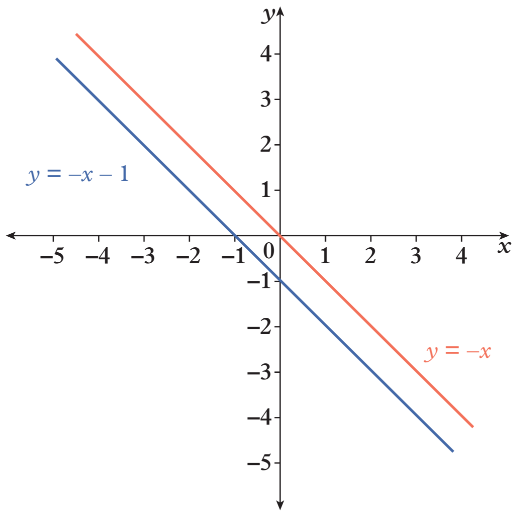
c B

d C

10 a



b



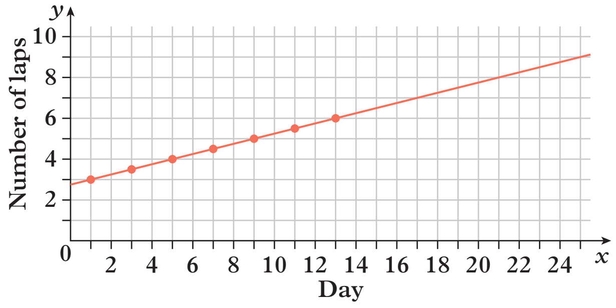
c Both graphs have the same steepness in each case, but the second graph is moved up in **a** and down in **b** in comparison to the first graph.

11 a IV: day number, DV: number of laps

b i The number of laps increases by the same amount every 2 days.

ii 2 points

c



d i 7 laps **ii** day 21

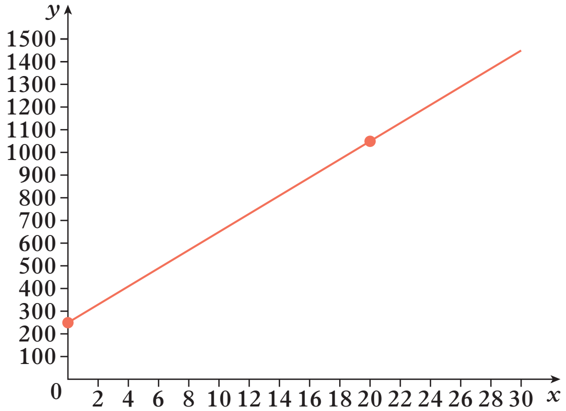
12 a $m = 40n + 250$

b Independent variable: number of students, n

Dependent variable: total amount of money collected for bus hire, m

c i \$250 **ii** \$1050

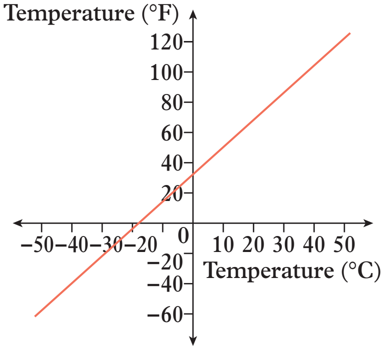
d



e Relationship is linear because it forms a straight line.

f \$1450 **g** 15 students **h** 27 students

13 a



b 86°F **c** −30°C

14 −40°F = −40°C

15 a (1,2) **b** (0,0) **c** (2,0) **d** (4,4)

EX
p107

3D Gradient and intercepts

1 a $\frac{3}{4}$ **b** $-\frac{1}{2}$ **c** -1

2 a $\frac{1}{3}$ **b** $-\frac{3}{2}$ **c** 2

3 a zero **b** positive **c** negative

d undefined **e** zero **f** negative

4 a i 2 **ii** 1 **iii** −2

b i 1 **ii** 4 **iii** −4

c i −2 **ii** −1 **iii** −2

d i 0 **ii** none **iii** 1

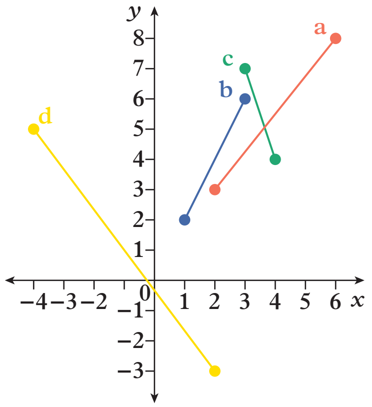
e i $\frac{3}{5}$ **ii** −5 **iii** 3

f i $-\frac{2}{3}$ **ii** −3 **iii** −2

g i undefined **ii** −3 **iii** none

h i −6 **ii** 0 **iii** 0

5 **i**



a **ii** $\frac{5}{4}$

b **ii** 2

c **ii** −3

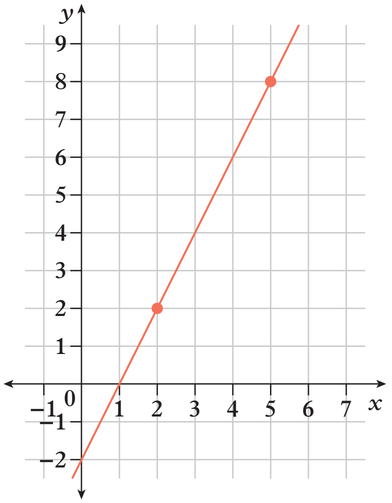
d **ii** $-\frac{4}{3}$

6 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** 2 **d** $-\frac{1}{3}$

e $\frac{2}{5}$ **f** −3 **g** undefined **h** −5

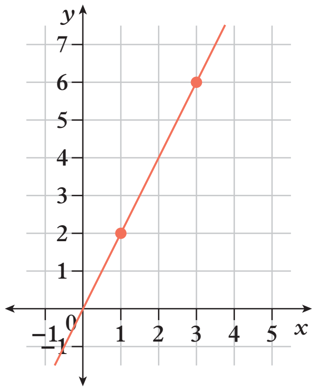
i $\frac{7}{3}$ **j** undefined **k** −2 **l** 0

7 **a** **i**



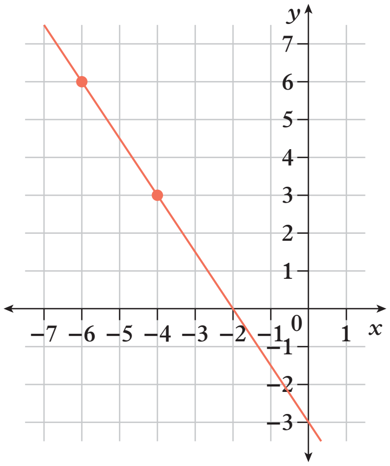
ii (1,0), (0, −2)

b **i**



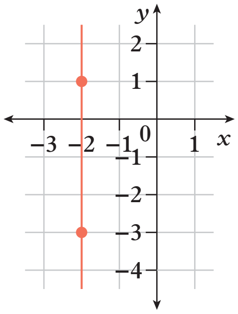
ii (0,0)

c **i**



ii (−2,0), (0, −3)

d **i**



ii (−2, 0)

8 **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** 2 **d** $\frac{3}{2}$ **e** 1 **f** 2

9 **a** **i** $\frac{3}{2}$ **ii** $\frac{3}{2}$

b The answers are the same. The gradient of a line is the same as the gradient of any line segment on that line.

10 **a** **i** 0 **ii** 1 **iii** $\frac{1}{2}$ **iv** −1

b **i** $\frac{1}{2}$ **ii** 0 **iii** 1 **iv** −1

11 Kane subtracted the second y-coordinate from the first, and the first x-coordinate from the second; $m = -\frac{3}{4}$.

12 The order in which the coordinates are substituted into the gradient formula does not affect the value of the gradient.

13

Run	8	−20	−28	40	4	−6
Rise	12	16	−4	−16	28	30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	−5

14 **a**

	Equation	**Gradient**	***y*-intercept**
i	$y = 2x - 3$	2	(0, −3)
ii	$y = -\frac{3}{4}x + 5$	$-\frac{3}{4}$	(0, 5)
iii	$y = x + 4$	1	(0, 4)
iv	$y = -2x - 1$	−2	(0, −1)
v	$y = 4x$	4	(0, 0)
vi	$y = 2$	0	(0, 2)

b gradient is coefficient of x; y-intercept is the constant

c **i** gradient = 6, y-intercept: (0, 4)

ii gradient = 1, y-intercept: (0, −5)

iii gradient = −3, y-intercept: (0, 0)

15 **a** $x = 3$ **b** $y = 0$ **c** $y = 8$

d $y = 10$ **e** $y = 3$ **f** $x = 7$

16 **a** $\frac{y-4}{x} = 2$ **b** $y = 2x + 4$

c $\frac{y-4}{x-1} = 2$ **d** $y = 2x + 2$

Checkpoint

1 **a** $x = -27$ **b** $x = 5$ **c** $x = 22$ **d** $x = 7$

2 **a** $x = 6$ **b** $x = \frac{1}{6}$ **c** $x = 6.25$ **d** $x = 40$

3 **a** $x = -4$ **b** $x = 2$ **c** $x = -5$ **d** $x = -3$

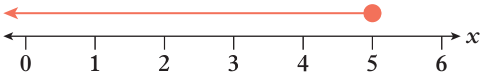
4 **a** Let n be the cost of one video game.

b $5n + 9.95 = 409.7$

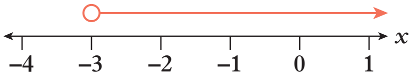
c $n = 79.95$

d $79.95

5 **a**



b



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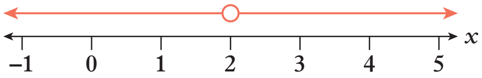

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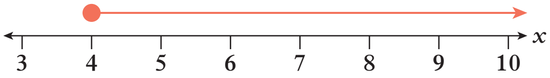


6 **a** $x < 5$ **b** $x \neq 6$

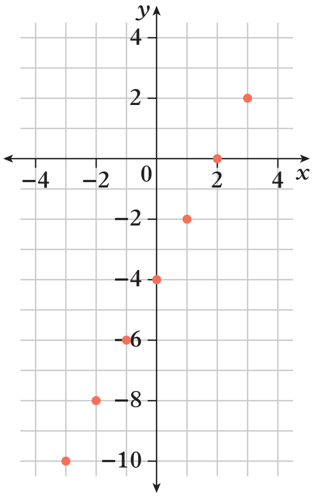
c $x \geq -5$ **d** $x \leq -2$

7 **a** $x > 3$ **b** $x \leq -5$

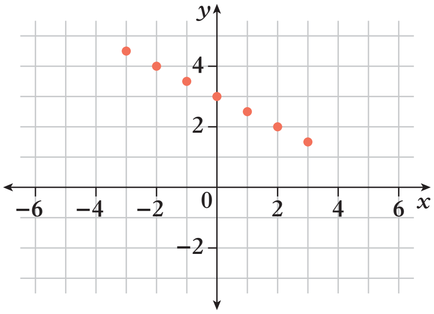
c $x > -25$ **d** $x > 1$

e $x \neq -3$ **f** $x \leq \frac{22}{3}$

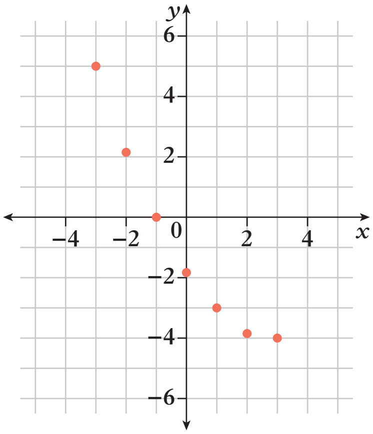
8 **a** **i** **ii** linear



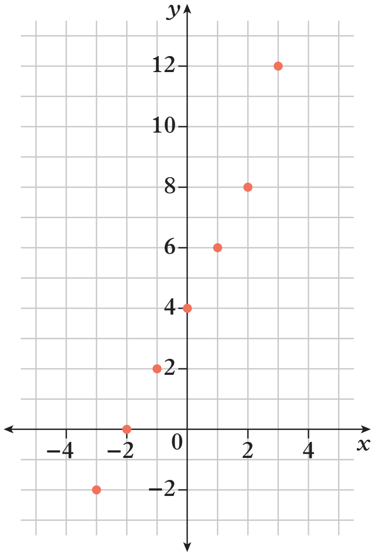
b **i** **ii** linear



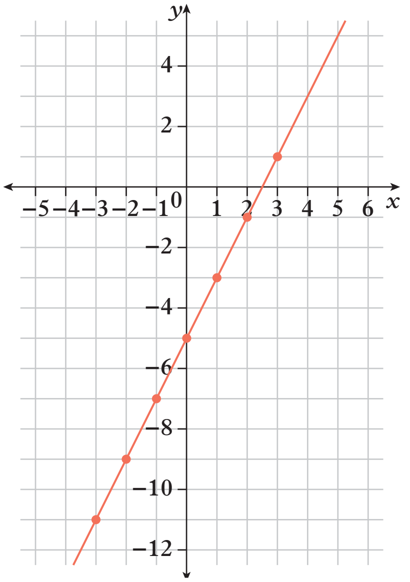
c **i** **ii** non-linear



d **i** **ii** non-linear



9 a



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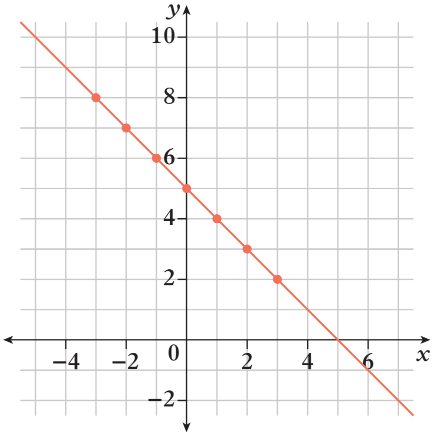

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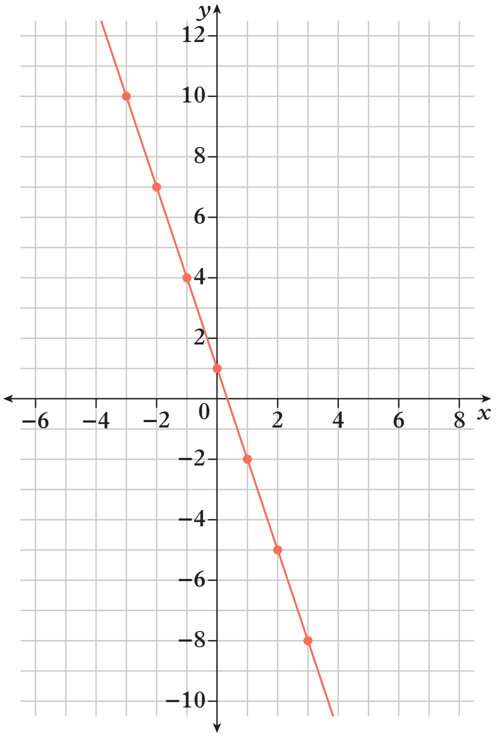

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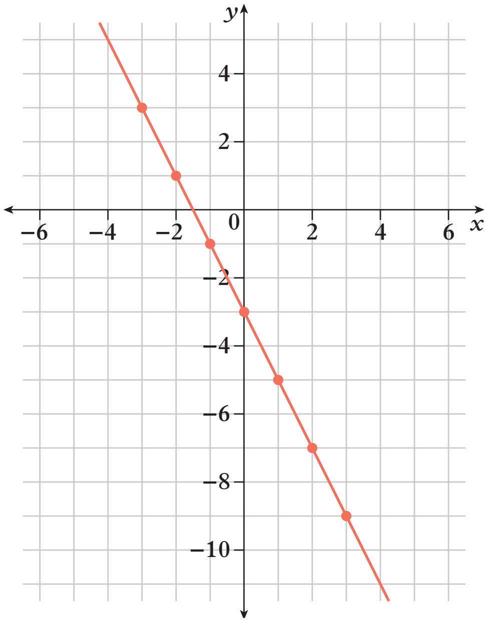


10 **a** IV: length, DV: volume

b IV: number of burgers, DV: calories

c IV: time, DV: number of achievements

11 **a** zero **b** negative **c** undefined **d** positive

12 **a** 3 **b** −3 **c** $-\frac{5}{3}$ **d** $-\frac{6}{5}$

13 **a** (3,0), (0,4) **b** (−2,0), (0,3) **c** (−6,0), (0,−1)

EX p117

3E Sketching linear graphs using intercepts

1 **a** (12, 0); (0, 3)

b (−4, 0); (0, 4)

c (3, 0); (0, 6)

d (5, 0); (0, −5)

e (4, 0); (0, 8)

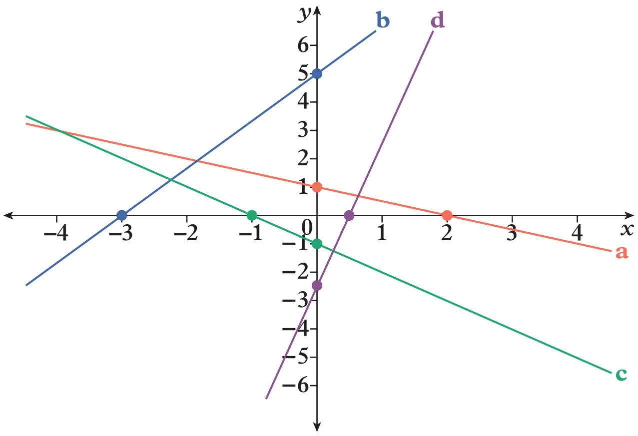
f (2, 0); (0, −6)

g (−2, 0); (0, −10)

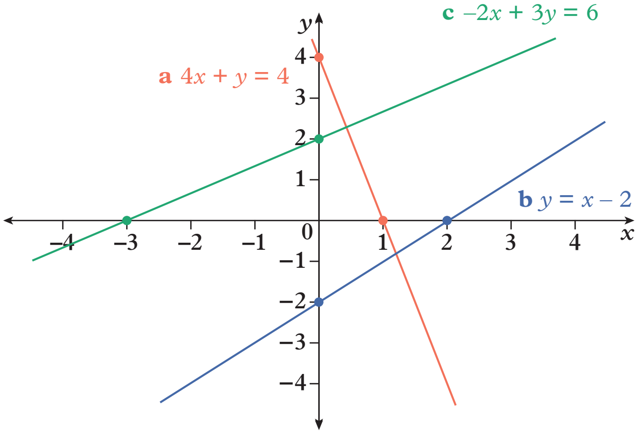
h (7, 0); (0, 7)

i (4, 0); (0, 4)

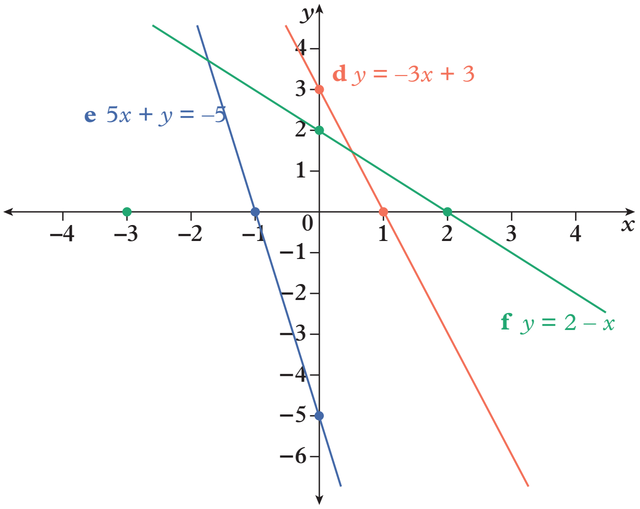
2 **a–d**



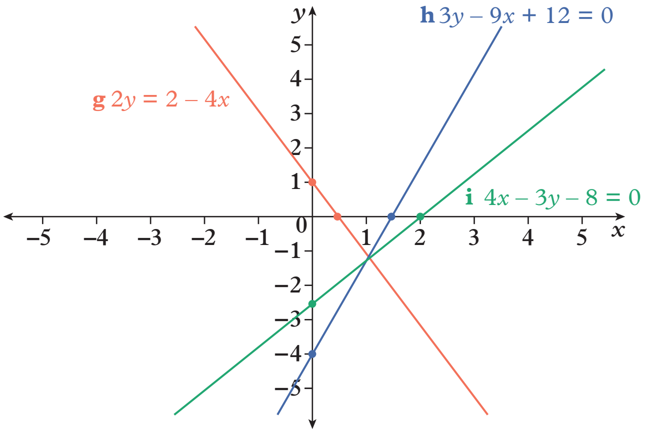
3 a–c



d–f



g–i



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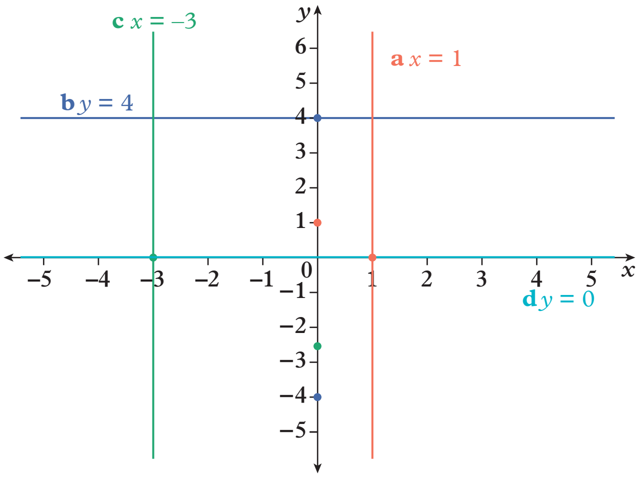
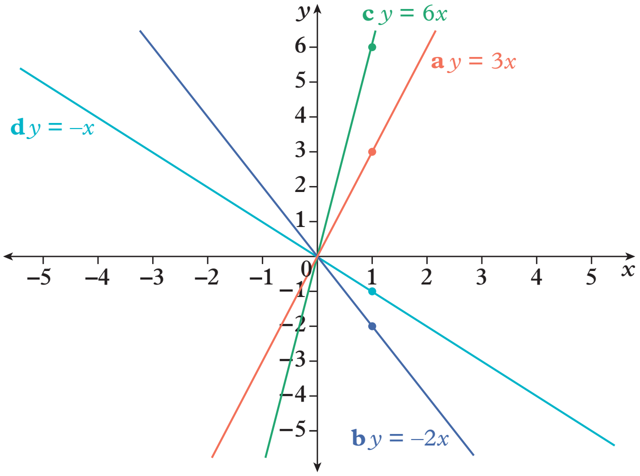
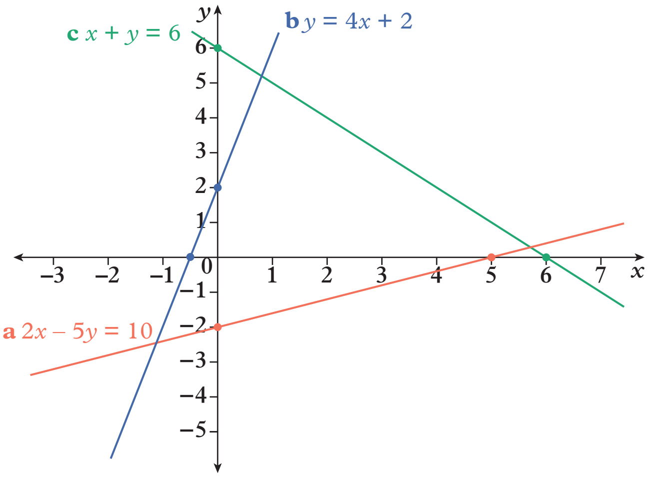


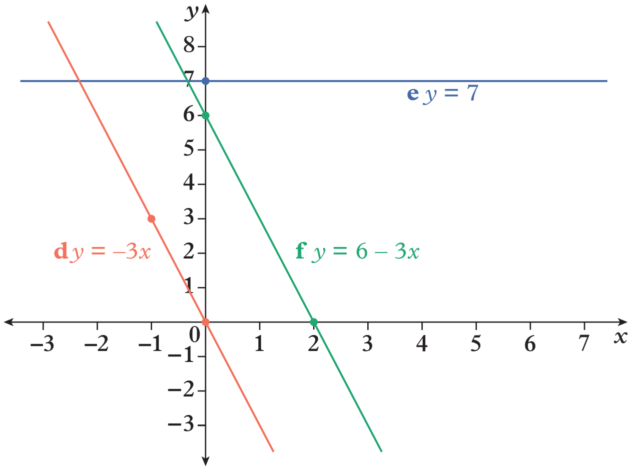
5



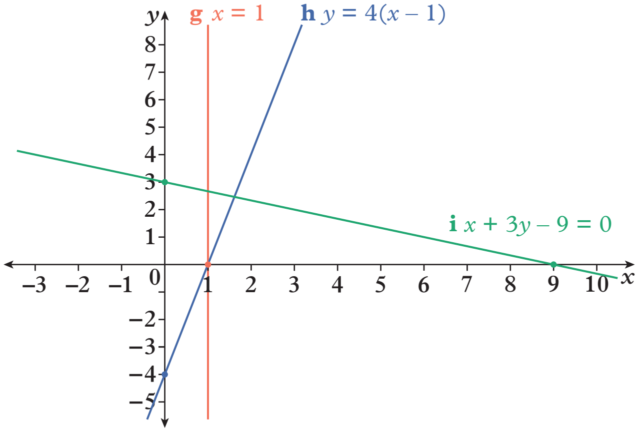
6 a–c



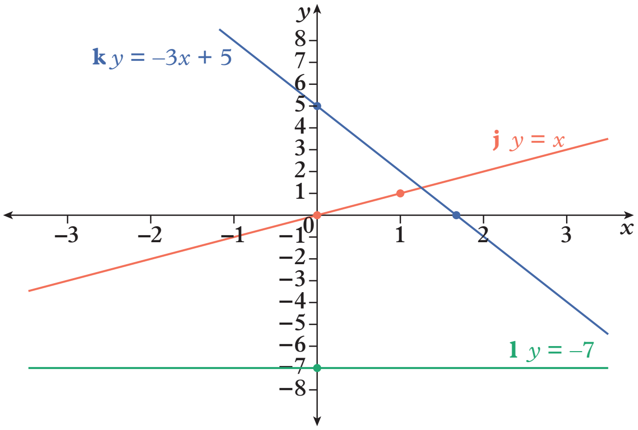
d–f



g–i



j–l



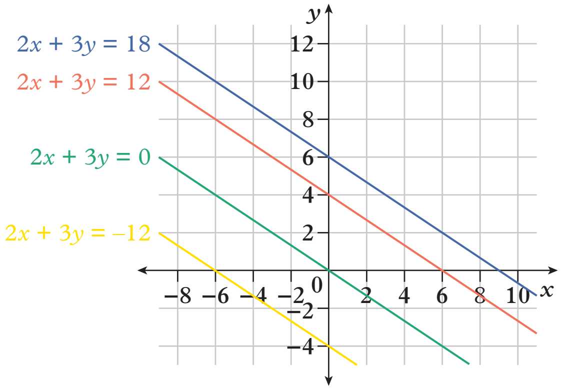
7 **a** 1 **b** 2 **c** 1
d 1 **e** 2 **f** 2
g 1 **h** 1 **i** 1

8 Graph B is correct. In graph A, the coefficients of variables have been used as intercepts. Graph B is a correct sketch of $3x + 2y = 6$. Graph C has an x-intercept at -2 instead of $+2$.

9 **a** true
b false; the x-intercept is -2
c true
d true
e false; the line passes through $(-2, 0)$ and $(0, 6)$
f true

10 **a** false; the graph passes through the origin
b true
c true
d false; the point $(1, -4)$ lies on the line
e true
f false; the gradient of the line is -4

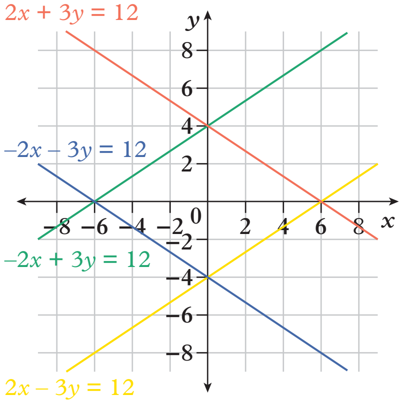
11 **a**



b The equations have the same variable sum, $2x + 3y$, and are parallel (have the same gradient).

c Increasing the constant translates the line up and to the right. Decreasing the constant translates the line down and to the left. If the constant is 0, the line passes through the origin. If the constant is positive, the line has positive intercepts. If the constant is negative, the line has negative intercepts.

12 **a**



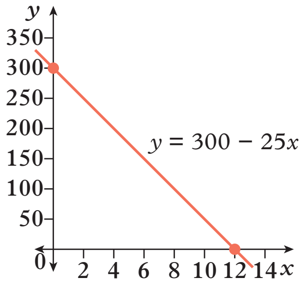
b All equations are equal to the same positive constant and have coefficients with the same unsigned values (but not necessarily the same sign).

c The term(s) with a positive coefficient have positive intercepts. The term(s) with a negative coefficient have negative intercepts. Compared to $2x + 3y = 12$, the line is reflected in the x-axis when the y-term is negative and reflected in the y-axis when the x-term is negative.

13 **a** $\frac{3}{5}$ **b** $-\frac{7}{6}$ **c** $-\frac{1}{2}$ **d** 2

14 a The independent variable is x (number of weeks) so Tony should calculate y (amount still owed in dollars), which depends on the number of payments over x weeks.

b



c original cost of skateboard

d number of weeks needed to pay off skateboard

e Tony is paying \$25 a week towards the skateboard. He will pay it off in 12 weeks.

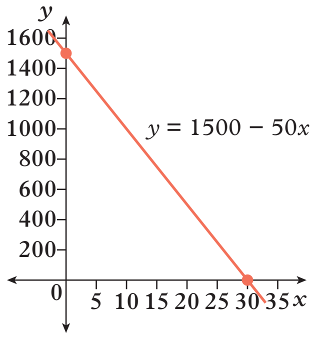
15 a IV: number of days, DV: litres of water – because time passing does not depend on the volume of water in the tank, but the volume of water in the tank depends on the time.

b Yes, the constant rate of water use and no additions to the tank suggest a constant gradient and hence a linear relationship between time, x, and amount of water, y.

c (30, 0)

d (0, 1500)

e



f 1000 L

g end of day 18

16 a When $x = 0$, the equation becomes $by = 1$, which has the solution $y = \frac{1}{b}$ (the y-intercept), the reciprocal of b. Likewise, when $y = 0$, the equation becomes $ax = 1$, which has the solution $x = \frac{1}{a}$ (the x-intercept), the reciprocal of a.

b Divide both sides of the equation by d, then the reciprocals of the coefficients of x and y will be the x- and y-intercepts, respectively.

c $d = ab$

17 a $\left(-\frac{7}{3}, 0\right)$ and $\left(0, \frac{7}{5}\right)$ **b** $\left(\frac{6}{5}, 0\right)$ and $\left(0, \frac{8}{25}\right)$

c $(\sqrt{3}, 0)$ and $(0, -\sqrt{2})$ **d** $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{3}\right)$

EX p123

3F Determining linear equations

1 a $m = 2, c = 5$ **b** $m = 4, c = 1$

c $m = -3, c = 7$ **d** $m = -5, c = -3$

e $m =1, c = -6$ **f** $m = -1, c = 1$

g $m = \frac{4}{3}, c = 2$ **h** $m = \frac{1}{2}, c = -8$

i $m = -\frac{4}{3}, c = \frac{1}{4}$ **j** $m = 0, c = 9$

k $m = -7, c = 0$ **l** $m = -\frac{2}{5}, c = 5$

2 a $y = 3x + 4$ **b** $y = -2x + 10$

c $y = x - 7$ **d** $y = -12x - 1$

e $y = -x + 20$ **f** $y = 5x$

g $y = \frac{1}{3}x + \frac{4}{5}$ **h** $y = -\frac{3}{4}x$

3 a $y = 3x + 6$ **b** $y = -\frac{2}{3}x + 4$

c $y = \frac{1}{2}x - 5$ **d** $y = -\frac{1}{5}x - 2$

e $y = 2x - 3$ **f** $y = -3x + 5$

g $y = -\frac{1}{2}x$ **h** $y = \frac{5}{2}x$

4 a $y = 2x - 22$ **b** $y = -2x + 8$

c $y = -x + 5$ **d** $y = 3x$

e $y = 2x + 1$ **f** $y = 5x - 49$

g $y = -\frac{1}{2}x - \frac{5}{2}$ **h** $y = \frac{4}{3}x - 11$

5 a $x = 5$ **b** $y = -2$

c $y = 60$ **d** $x = -40$

e $y = -\frac{3}{7}$ **f** $x = \frac{2}{3}$

6 a $y = -3x + 12$ **b** $y = 2x + 10$

c $y = 7x - 21$ **d** $y = -4x - 24$

e $y = 10x + 10$ **f** $y = -\frac{x}{3} + 6$

g $y = \frac{2}{5}x - 2$ **h** $y = \frac{3}{2}x + 9$

7 a $y = \frac{1}{2}x + \frac{9}{2}$ **b** $y = \frac{2}{3}x + \frac{14}{3}$

c $y = -\frac{1}{5}x + \frac{18}{5}$ **d** $y = \frac{3}{4}x + \frac{1}{2}$

e $y = -\frac{7}{6}x - \frac{1}{3}$ **f** $y = -\frac{11}{9}x - \frac{5}{3}$

8 a IV: number of years, DV: amount in Azami's account

b $A = 300n + 1900$

9 a $P = 60n + 30$ **b** \$21 930

10 a IV: number of years from 2010, DV: length of stalactite in centimetres

b $g = \frac{1}{5}n + 85$

c The stalactite is 425 years old in 2010, which means it began to form in 1585.

11 a $y = 9x - 5$ **b** $y = \frac{3}{2}x + \frac{5}{2}$

c $y = \frac{3}{10}x + 45$ **d** $y = -2x + 30$

12 a $C = 45n + 125$ **b** $R = 99n + 275$

c $P = 54n + 150$

13 **a** **i** $y = 3(x + 4)$

ii $y = -5(x + 3)$

iii $y = 2(x - 5)$

iv $y = 2(3x - 4)$

v $y = 7(3 - x)$ or $y = -7(x - 3)$

vi $y = 8(2x - 5)$

b **i** $x = -4$ **ii** $x = -3$ **iii** $x = 5$

iv $x = \frac{4}{3}$ **v** $x = 3$ **vi** $x = \frac{5}{2}$

c The x-intercept is the value of x that makes the bracketed factor equal to zero. It is the opposite sign of the constant divided by the coefficient of x.

14 **a** $y = -\frac{21}{10}x + \frac{7}{5}$

b $y = -\frac{5}{3}x + \frac{1}{12}$

c $y = \frac{22}{81}x - \frac{19}{36}$

15 **a** $y = 9x + 11$ and $y = 11x + 9$ **b** (1, 20)

EX p167

3G Midpoint and length of a line segment

1 **a** (2, 1) **b** (4, −2) **c** (−3, 3) **d** (3, 2)

2 **a** (2, 7) **b** (5, 4) **c** (3, 1) **d** (2, 8)

e (4, 7) **f** (5, 1) **g** (4, 3) **h** (1, −4)

i (−3, 0) **j** (3.5, 8.5) **k** (6.5, 5.5) **l** (0,0)

3 **a** 2.2 units **b** 4.5 units **c** 3.2 units

d 4.0 units **e** 6.3 units **f** 5.0 units

g 1.4 units **h** 14.0 units **i** 3.6 units

j 4.1 units **k** 4.5 units **l** 15.6 units

4 **a** **i** (3, 3) **ii** 4.5 units **iii** $-\frac{1}{2}$

b **i** (0.5, 0) **ii** 7.8 units **iii** $\frac{6}{5}$

c **i** (−2.5, 3) **ii** 8.5 units **iii** $\frac{8}{3}$

d **i** (−6, −5) **ii** 8.5 units **iii** −1

5 **a** 1.1 units **b** 2.2 units **c** 1.6 units **d** 2.0 units

e 3.2 units **f** 2.5 units **g** 0.7 units **h** 7.0 units

i 1.8 units **j** 2.1 units **k** 2.2 units **l** 7.8 units

6 (10, 5)

7 (−14, 9)

8 **a** 5 units **b** (5, 7)

9 20.3 units

10 **a** 20.5 units **b** 17.0 units **c** 22.6 units **d** 19.5 units

11 **a** leg 1: 21.5 km; leg 2: 16.5 km; leg 3: 28.8 km

b 66.9 km **c** 24.1 km

12 **a** (−5, −1), (−2, 3), (6, 3) and (3, −1)

b **i** (0.5, 1) **ii** (0.5, 1)

c Midpoints of both diagonals have same coordinates

d 26 cm **e** 5.3 cm

13 **a** **i** 21.6 cm **ii** 10.8 cm

iii Perimeter of orange shape = $\frac{1}{2}$ perimeter of blue shape

b **i** 20.7 cm **ii** 16.7 cm

iii Perimeter of orange shape ≈ $\frac{4}{5}$ perimeter of blue shape

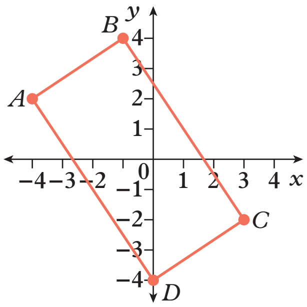
14 **a** AB: 3.6 m, BC: 7.2 m, CD: 3.6 m and AD: 7.2 m

b Midpoint for both diagonals (−0.5, 0)

c A: 4.0 m, B: 4.0 m, C: 4.0 m, D: 4.0 m

d The parallelogram is a rectangle because it has two opposite pairs of sides of equal length, diagonals equal in length and bisect each other, with each vertex the same distance from the midpoint.

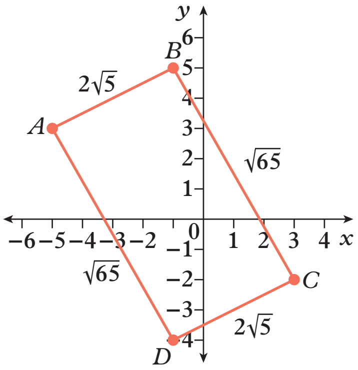
e 26 m^2



15 **a** Two sides are equal in length, $AB = AC = 4\sqrt{5}$ units and $BC = 4\sqrt{2}$ units.

b 24 units^2

16 Two pairs of equal-length sides, $AB = CD = 2\sqrt{5}$ units and $BC = AD = \sqrt{65}$ units; two pairs of parallel sides with gradients $\frac{1}{2}$ and $-\frac{7}{4}$.



17 (0, 1)

18 $(\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, -2\sqrt{5})$

Chapter 3 review

Multiple choice

1 B **2** C **3** D **4** D **5** D **6** C **7** D

8 B **9** E **10** B **11** A **12** C **13** A **14** C

Short answer

1 **a** $x = 2$ **b** $x = \frac{2}{3}$ **c** $x = 10$ **d** $x = 3$

e $x = -1$ **f** $x = -\frac{4}{5}$ **g** $x = 3\frac{3}{4}$ **h** $x = \frac{1}{2}$

2 **a** $x = 4$ **b** $x = -1$

3 **a** Let n be the number of people in the class.

b $\frac{3}{5}n + 1 = 16$ **c** $n = 25$ **d** 25 people

4 **a** $1\frac{5}{8}$, 2.6, 8.5, $\frac{3}{4}$

b −3.9, −8.5, $\frac{3}{4}$

c 2.6, 8.5

5 **a** $x \geq -3$ **b** $x < -10$ **c** $x \leq 2.5$

6 **a** $x > -1$ **b** $x < -2\frac{2}{3}$

7 **a**

x	−2	−1	0	1	2
y	3	4	5	6	7

b

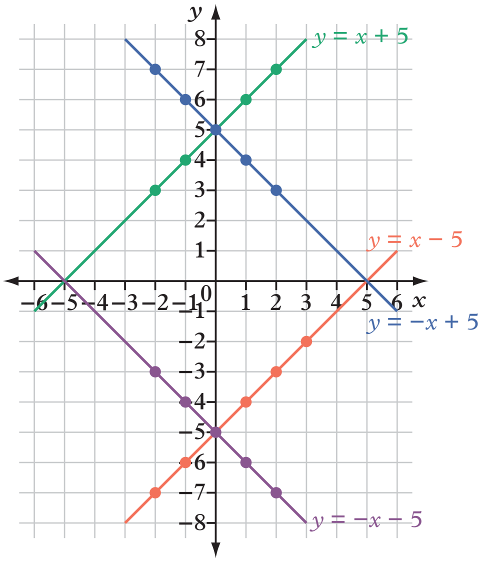
x	−2	−1	0	1	2
y	−7	−6	−5	−4	−3

c

x	−2	−1	0	1	2
y	7	6	5	4	3

d

x	−2	−1	0	1	2
y	−3	−4	−5	−6	−7



8 **a** IV: number of attempts, DV: accuracy

b IV: number of chess matches played, DV: number of victories

9 **a** **i** $\frac{1}{3}$ **ii** (−6, 0) **iii** (0, 2)

b **i** undefined **ii** (3, 0)

iii no y-intercept

c **i** 0 **ii** no x-intercept **iii** (0, −2)

d **i** −2 **ii** (0, 0) **iii** (0, 0)

10 **a** $\frac{3}{2}$ **b** −1

11 **a** **i** (9, 0) **ii** (0, 6)

b **i** (2, 0) **ii** (0, −6)

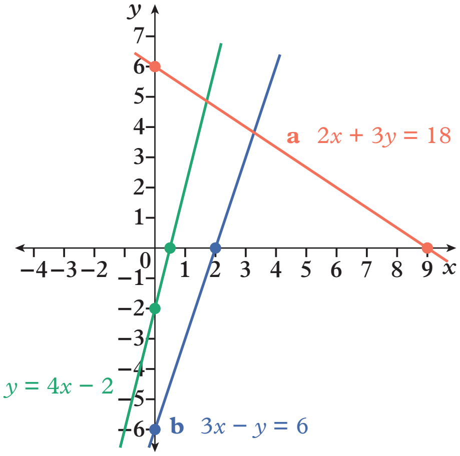
c **i** $\left(\frac{1}{2}, 0\right)$ **ii** (0, −2)

d **i** $\left(\frac{3}{5}, 0\right)$ **ii** $\left(0, -\frac{3}{2}\right)$

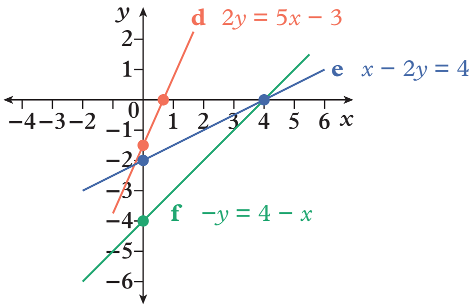
e **i** (4, 0) **ii** (0, −2)

f **i** (4, 0) **ii** (0, −4)

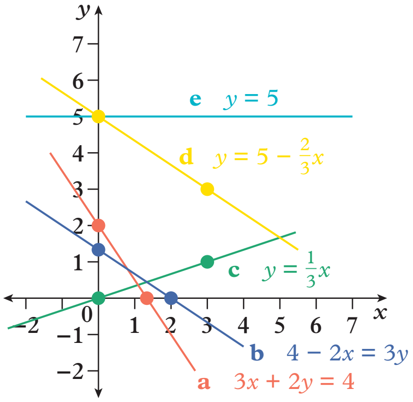
12 **a–c**



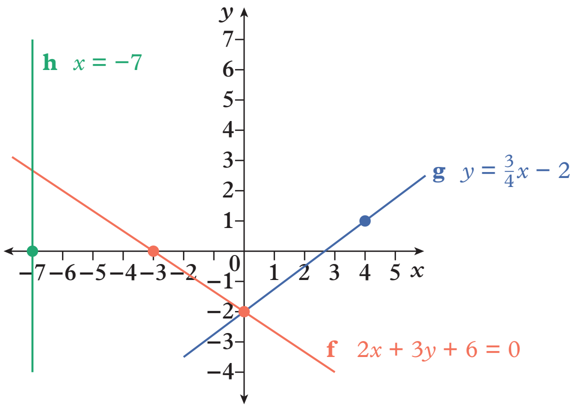
d–f



13 **a–e**

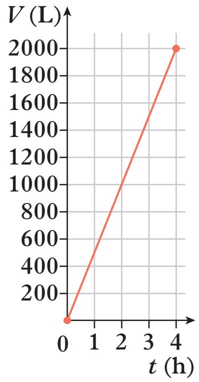


f–h



14 a $y = 4x - 2$ **b** $y = \frac{1}{4}x$ **c** $y = -\frac{1}{2}$

15 a i



ii 500

b 500 L/hr

c $V = 500t$

d 20 hours

16 a $y = -2x - 4$ **b** $y = \frac{3}{2}x$

c $y = 1.25x + 1.25$ **d** $y = -\frac{1}{3}x + 1$

17 a $y = -x - 3$ **b** $y = 39$

c $y = -\frac{4}{3}x$ **d** $y = \frac{4}{5}x + \frac{32}{5}$

18 a i (5, 5) **ii** 7.2 units

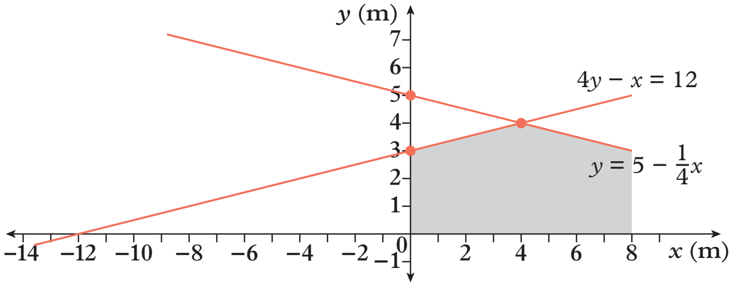
b i (−3.5, 5.5) **ii** 1.4 units

c i (0, 0) **ii** 8.9 units

d i (−0.5, −5) **ii** 9.4 units

Analysis

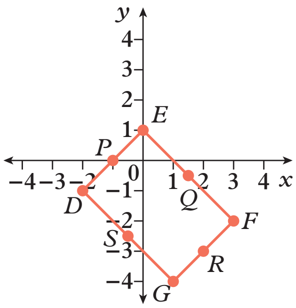
1 a, b



c 4 m **d** 4.1 m **e** $\frac{1}{4}$

f Chimney will be at (2, 3.5).

2 a



b *DE* and *FG* are 2.8 units long, *EF* and *DG* 4.2 units long.

c i (0, 1) **ii** (0, 1) **iii** (0, −3) **iv** none

d i 1 **ii** −1 **iii** −1 **iv** 1

e i (−1, 0) **ii** (1.5, −0.5)

iii (2, −3) **iv** (−0.5, −2.5)

f *PQRS* is a rhombus with all sides equal in length (*PQ*, *RS*, *SP* and *QR*).

g If *DEFG* were square, then *PQRS* would also have been square because the midpoints of each side would have been the same distance from midpoints of diagonals, which cross at 90°.

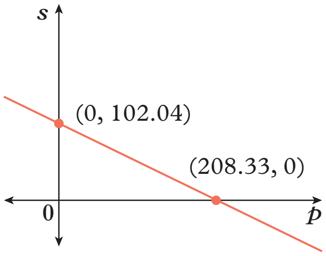
3 a $0.71s + 0.26p \leq 60$

b $p \leq 94$ or $p \leq 94.23$ (2 decimal places)

c $s \leq 29$ or $s \leq 29.58$ (2 decimal places)

d $P = 0.49s + 0.24p$

e



f 208 party pies

g 32 sausage rolls and 143 party pies

CHAPTER 4 Non-linear relationships

4A Solving quadratic equations

EX p143

1 **a**, **c**, **e** and **h**

2 **a** yes **b** no **c** no **d** yes
e no **f** yes **g** yes **h** no

3 **a** $x = -3$ **b** $x = 6$ **c** $x = 2$ **d** $x = -\frac{1}{3}$

4 **a** $x = -2$ or $x = 3$ **b** $x = 1$ or $x = 7$
c $x = -4$ or $x = 4$ **d** $x = 0$ or $x = 6$
e $x = -5$ or $x = -1$ **f** $x = -2$ or $x = 0$
g $x = -8$ or $x = 8$ **h** $x = -1$ or $x = 7$
i $x = 0$ or $x = 11$ **j** $x = -\frac{3}{2}$ or $x = 5$
k $x = \frac{1}{2}$ **l** $x = -\frac{5}{3}$ or $-\frac{3}{2}$

5 **a** $x = -2$ or $x = -5$ **b** $x = 1$ or $x = 2$
c $x = 0$ or $x = -5$ **d** $x = 0$ or $x = 3$
e $x = -6$ or $x = 6$ **f** $x = -7$ or $x = -3$

6 **a** $x = -2$ or $x = 4$ **b** $x = -1$ or $x = 1$
c $x = 0$ or $x = -8$ **d** $x = 1$ or $x = 3$
e $x = -3$ **f** $x = 1$

7 Check answers in questions **5** and **6**.

8 **a** $x = -8$ or $x = 2$ **b** $x = 1$ or $x = 4$
c $x = -6$ or $x = 6$ **d** $x = -9$ or $x = 0$

9 **a** $x = -2$ or $x = -1$ **b** $x = 0$ or $x = -4$
c $x = -2$ or $x = 4$ **d** $x = 2$
e $x = -2$ or $x = 1$ **f** $x = 0$ or $x = 2$
g $x = -7$ or $x = -3$ **h** $x = -4$
i $x = -4$ or $x = 4$

10 **a** $x = -8$ or $x = -6$ **b** $x = 0$ or $x = 4$
c $x = 0$ or $x = 1$ **d** $x = -8$ or $x = 8$
e $x = -3$ or $x = 1$ **f** $x = 4$
g $x = 2$ or $x = 3$ **h** $x = -5$
i $x = -2$ or $x = 2$

11 **a** **i** 2 **ii** 1 **iii** 0
b **i** The equation is factorised into 2 different linear expressions.
ii The equation is factorised into a perfect square.
iii The equation cannot be factorised.

12 **a** 2 **b** 2
c 1 **d** 0
e 2 **f** 2
g 2 **h** 0

13 **a** $w(w + 8)$ cm^2 **b** $w^2 + 8w$ cm^2
c $w^2 + 8w = 560$
d $w = 20$ or $w = -28$. $w = 20$ is the feasible solution and $w = -28$ is not, as it's not possible to have a negative length.
e width = 20 cm and length = 28 cm

14 **a** $w^2 + 2w = 35$ **b** $w = -7$ or $w = 5$
c length = 7 m, width = 5 m

15 $x(x - 12) = 640$, length = 32 cm, width = 20 cm

16 **a** **i** 8 m **ii** 0 m
b 8 m **c** 2 s
d Alex is above ground level when he throws the ball so it is only at ground level when the ball falls down.

17 **a** 2 seconds and 6 seconds
b The first time is when the rocket is ascending, the second time is when the rocket is descending.

18 **a** $x = 1$, $x = 3$ or $x = 5$
b $x = 0$, $x = -6$ or $x = 6$
c $x = -11$ or $x = 4$
d $x = 2$, $x = -\frac{1}{3}$ or $x = \frac{3}{2}$

4B Plotting quadratic relationships

EX p148

1 **a** **i** $(-3, 0)$, $(1, 0)$
ii $(0, -3)$
iii $(-1, -4)$
iv minimum
v $x = -1$

b **i** $(0, 0)$, $(4, 0)$
ii $(0, 0)$
iii $(2, 4)$
iv maximum
v $x = 2$

c **i** $(-2, 0)$, $(2, 0)$
ii $(0, 4)$
iii $(0, 4)$
iv maximum
v $x = 0$

d **i** $(-1, 0)$, $(5, 0)$
ii $(-5, 0)$
iii $(2, -9)$
iv minimum
v $x = 2$

e **i** $(-5, 0)$, $(1, 0)$
ii $(0, 5)$
iii $(-2, 9)$
iv maximum
v $x = -2$

f **i** $(1, 0)$
ii $(0, 1)$
iii $(1, 0)$
iv minimum
v $x = 1$

g **i** $(0, 0)$
ii $(0, 0)$
iii $(0, 0)$

iv minimum

v $x = 0$

h **i** none

ii (0, −7)

iii (−2, −3)

iv maximum

v $x = -2$

i **i** (3, 0)

ii (0, −6)

iii (3, 0)

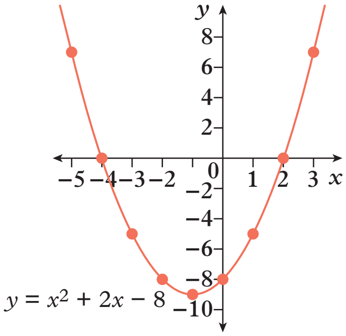
iv maximum

v $x = 3$

2 **a** **i**

x	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7

ii



iii x-intercepts: (−4, 0) and (2, 0)

y-intercept: (0, −8)

turning point: (−1, −9)

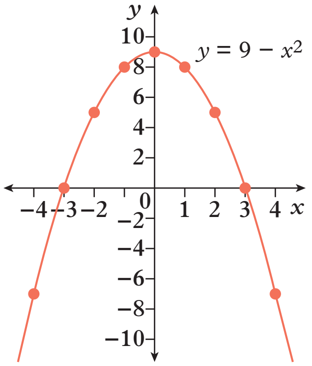
axis of symmetry: $x = -1$

iv minimum

b **i**

x	−4	−3	−2	−1	0	1	2	3	4
y	−7	0	5	8	9	8	5	0	−7

ii



iii x-intercepts: (−3, 0) and (3, 0)

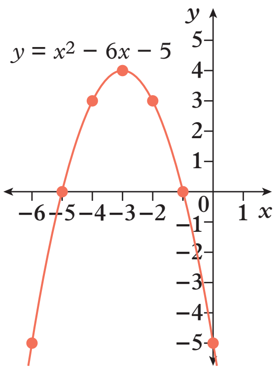
y-intercept: (0, 9)

turning point: (0, 9)

axis of symmetry: $x = 0$

iv maximum

3 **a** **i**



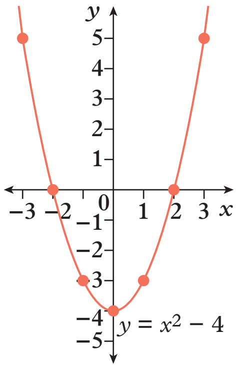
ii x-intercepts: (−5, 0) and (−1, 0)

y-intercept: (0, −5)

turning point: (−3, 4)

axis of symmetry: $x = -3$

b **i**



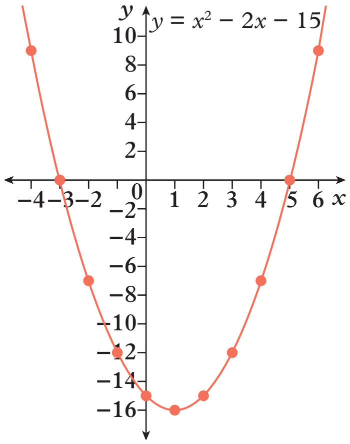
ii x-intercepts: (−2, 0) and (2, 0)

y-intercept: (0, −4)

turning point: (0, −4)

axis of symmetry: $x = 0$

c **i**



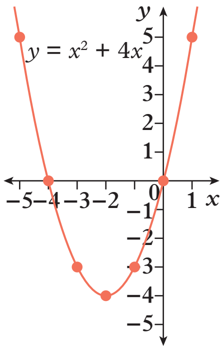
ii x-intercepts: (−3, 0) and (5, 0)

y-intercept: (0, −15)

turning point: (1, −16)

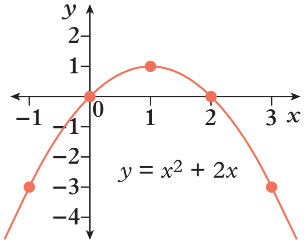
axis of symmetry: $x = 1$

d **i**



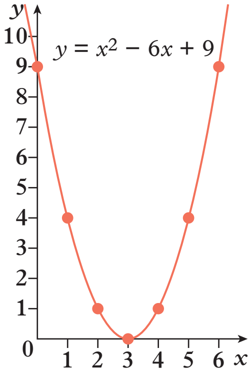
ii x-intercepts: (−4, 0) and (0, 0)
y-intercept: (0, 0)
turning point: (−2, −4)
axis of symmetry: $x = -2$

e **i**



ii x-intercepts: (0, 0) and (2, 0)
y-intercept: (0, 0)
turning point: (1, 1)
axis of symmetry: $x = 1$

f **i**



ii x-intercept: (3, 0)
y-intercept: (0, 9)
turning point: (3, 0)
axis of symmetry: $x = 3$

4 **a** **i** 2 **ii** 1 **iii** 0

b A parabola changes direction only once, so there can only be a maximum of two x-intercepts.

c One

d The y-coordinate of the y-intercept is given by the constant term, c, in the equation.

5 **a** **i** no x-intercepts **ii** 1 y-intercept

b **i** 1 x-intercept **ii** 1 y-intercept

6 **a** $x = 1$

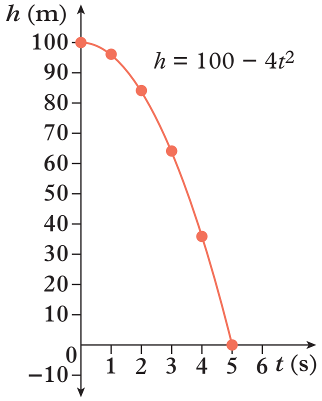
b $x = 5.5$

c $x = -0.5$

d $x = -1.5$

7 **a** 20 m **b** 26 m **c** 5 s

8 **a**



b It's not possible to have negative time values and it is not possible for the hairclip to fall below ground level.

c **i** 84 m **ii** 64 m

d 100 m **e** 5 s

9 **a** (−2, 0) and (5, 0) **b** $x = -2$ or $x = 5$

c They are the same.

d The x-intercepts of a parabola represent the solutions to a quadratic equation.

10 **a** $x = 5$ **b** $x = 0$ **c** $x = -4$

11 **a** $x = 4$ and $x = 5$

b $x = 3$ and $x = 8$

c $x = -3$ and $x = -0.5$

d $x = 3$

12 (0, 0)

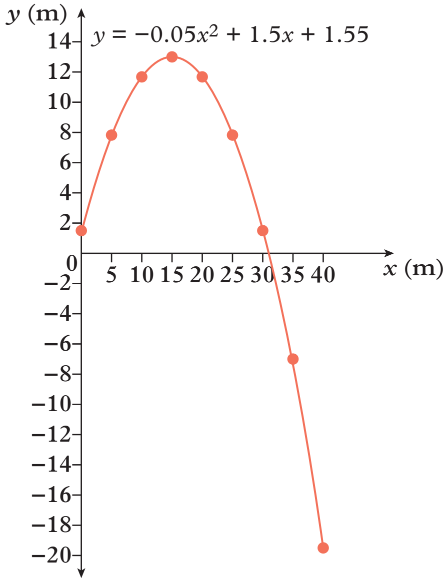
13 **a** x-intercepts: (p, 0) and (q, 0)

b $y = (x + 7)(x - 5)$

c $y = -36$

d $y = -(x + 7)(x - 5)$

14 **a**



b 1.55 m **c** $x = -1$ and $x = 31$

d 31 m **e** 12.8 m

EX p155

4C Sketching parabolas using intercepts

1 **a** **i** $(-7, 0)$ and $(5, 0)$ **ii** $(0, -35)$

b **i** $(2, 0)$ and $(6, 0)$ **ii** $(0, 36)$

c **i** $(-4, 0)$ and $(-2, 0)$ **ii** $(0, 96)$

2 **a** **i** $(0, 0)$ and $(2, 0)$ **ii** $(0, 0)$

b **i** $(-8, 0)$ and $(0, 0)$ **ii** $(0, 0)$

c **i** $(-4, 0)$ and $(-2, 0)$ **ii** $(0, 8)$

d **i** $(2, 0)$ and $(6, 0)$ **ii** $(0, 12)$

e **i** $(-1, 0)$ and $(5, 0)$ **ii** $(0, -5)$

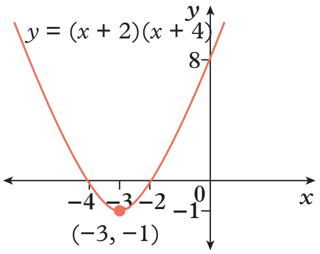
f **i** $(-3, 0)$ and $(3, 0)$ **ii** $(0, -9)$

3 **a** $(2, -4)$ **b** $(4, -1)$ **c** $(1, -25)$ **d** $(-2, 1)$

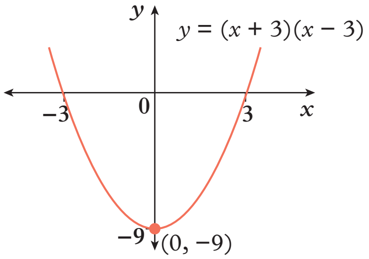
e $(0, -4)$ **f** $(1, 9)$ **g** $\left(-\frac{3}{2}, -\frac{9}{4}\right)$ **h** $\left(-\frac{7}{2}, \frac{49}{4}\right)$

i $(5, -18)$

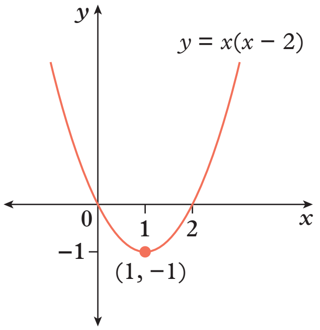
4 **a**



b



c



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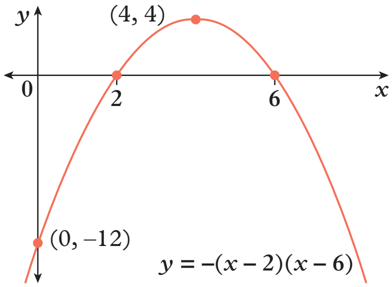

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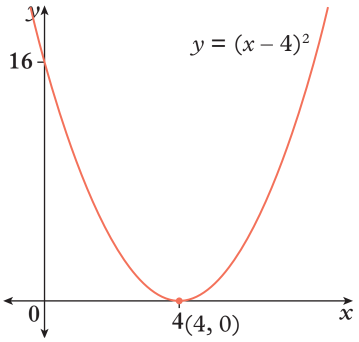

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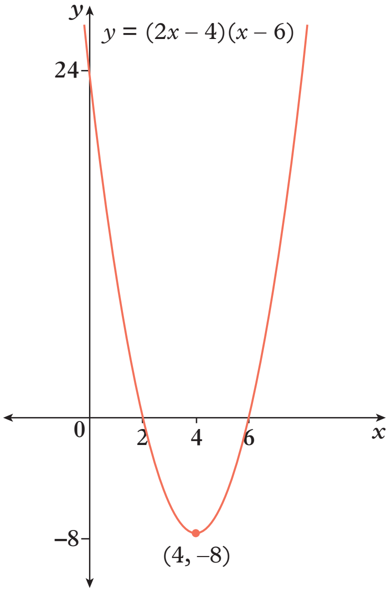
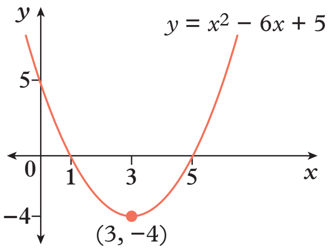


5 **a** Both graphs have x-intercepts at $(-3, 0)$ and $(1, 0)$.

b $y = (x + 3)(x - 1)$ has turning point at $(-4, 0)$ and $y = -(x + 3)(x - 1)$ has turning point at $(4, 0)$.

c Both graphs have the same x-intercepts and the same shape, but they are reflections of each other in the x-axis. So $y = (x + 3)(x - 1)$ has a minimum turning point and $y = -(x + 3)(x - 1)$ has a maximum turning point.

6 **a**



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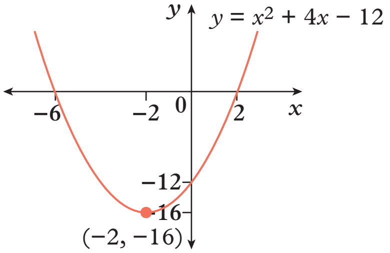

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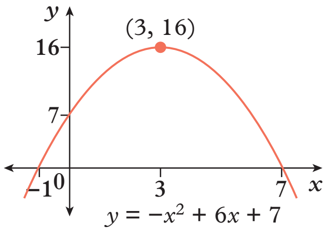

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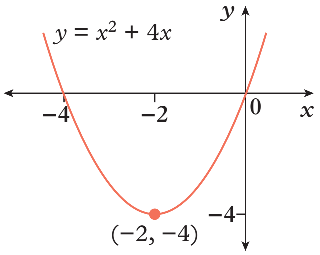

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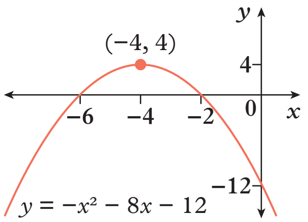

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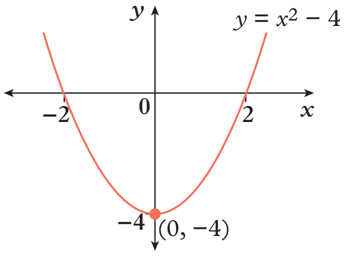

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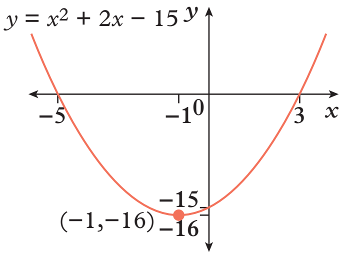

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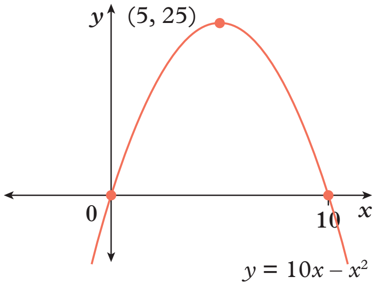
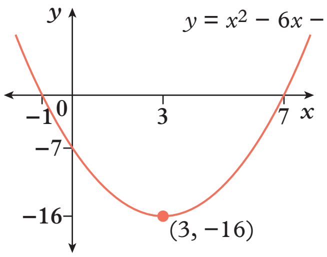


i



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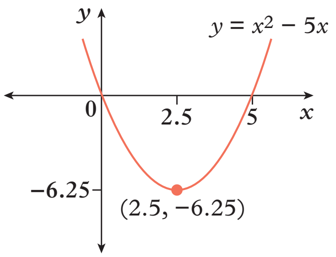
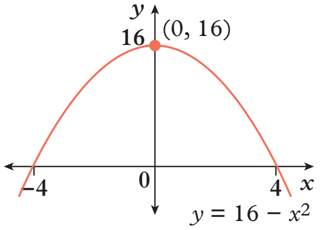
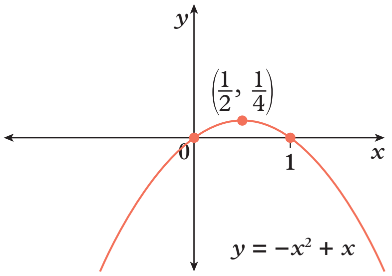


k



l



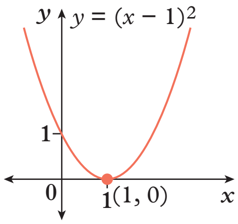
7 **a** C **b** A **c** B

8 Look at coefficient of x^2 term (a). If $a > 0$, parabola is upright, e.g. $y = x^2 + x + 6$ produces an upright parabola. If $a < 0$, parabola is inverted, e.g. $y = -x^2 - x + 6$ produces an inverted parabola.

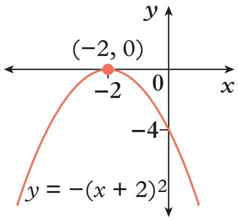
9 **a** upright **b** upright **c** inverted
d inverted **e** upright **f** inverted

10 **a** upright
b one (0, 9)
c one (3, 0)
d (3, 0)
e The turning point is also the x-intercept.
f The negative value of p is the x-coordinate of the turning point. The turning point is also the x-intercept, so the y-coordinate of the turning point will be 0.

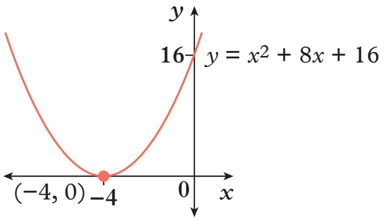
11 **a**



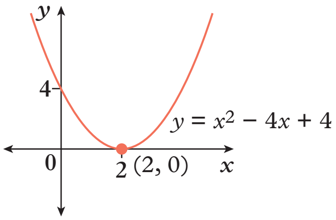
b



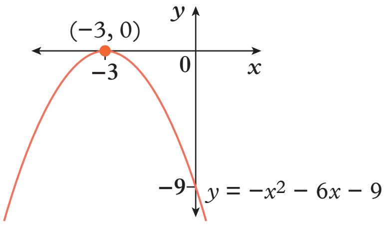
c



d



e

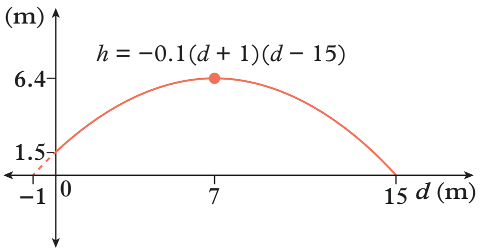


12 a inverted **b** one (0, −1)

c none **d** (0, −1)

e The value of c is the y-coordinate of the turning point. The turning point is also the y-intercept, so the x-coordinate of the turning point will be 0.

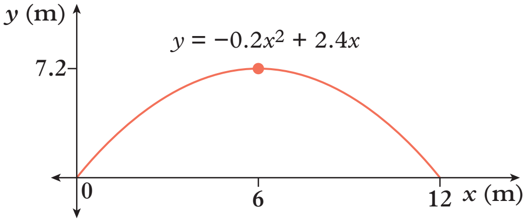
13 a



b The graph represents the path of the arrow from $d = 0$ m to $d = 15$ m.

c 6.4 m **d** 1.5 m **e** 15 m

14 a



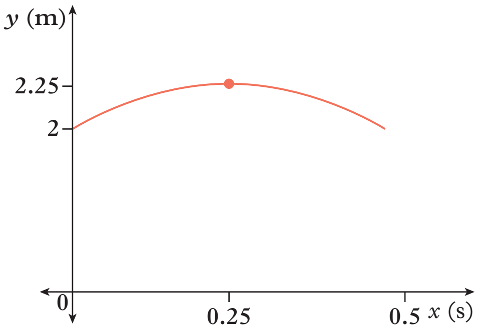
b 7.2 m **c** 6 m **d** 12 m

15 a 2 m

b (0, 2) and (0.5, 2), The ball is thrown and caught at the same height of 2 m.

c (0.25, 2.25)

d



e 2.25 m

16 a $y = \frac{1}{2}(x - 6)(x + 1)$

b $y = -2(x - 6)(x + 2)$

17 $y = \frac{3}{8}(x - 3)^2$

Checkpoint

1 a $x = 3$ or $x = 4$ **b** $x = -2$ or $x = 2$

c $x = -9$ **d** $x = 0$ or $x = -4$

2 a $x = 5$ or $x = 7$ **b** $x = -7$ or $x = 4$

c $x = 6$ **d** $x = -9$ or $x = 9$

3 a $x = -4$ or $x = 4$ **b** $x = -2$ or $x = 4$

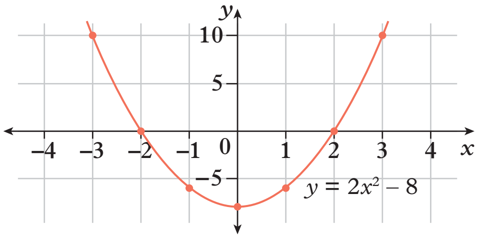
c $x = -5$ or $x = -1$ **d** $x = 0$ or $x = 15$

4 a (2, 0) and (−4, 0) **b** (0, 8)

c (−1, 9) **d** maximum

e $x = -1$

5



6 a $x = 5$ **b** (5, 0)

7 a i (0, 0) and (−3, 0) **ii** (0, 0)

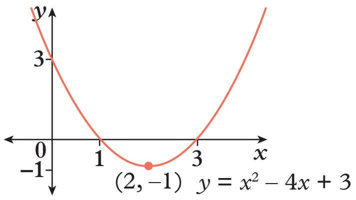
b i (−5, 0) and (5, 0) **ii** (0, −50)

c i (−4, 0) and (2, 0) **ii** (0, −8)

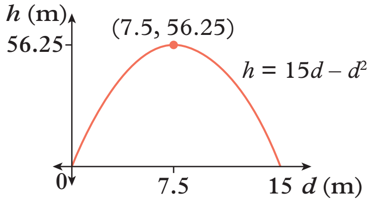
d i (8, 0) **ii** (0, 64)

8 a (−1, −9) **b** (2, −4) **c** (0, −36) **d** (10, 4)

9



10 **a**



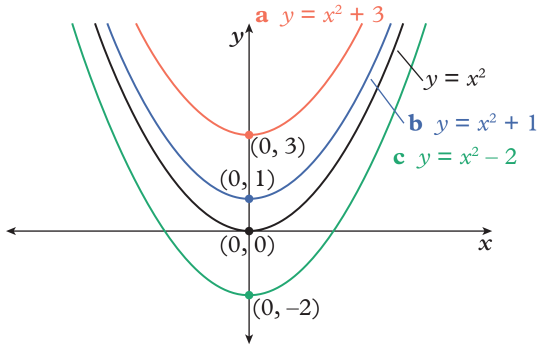
b 15 m **c** 56.25 m

EX p162

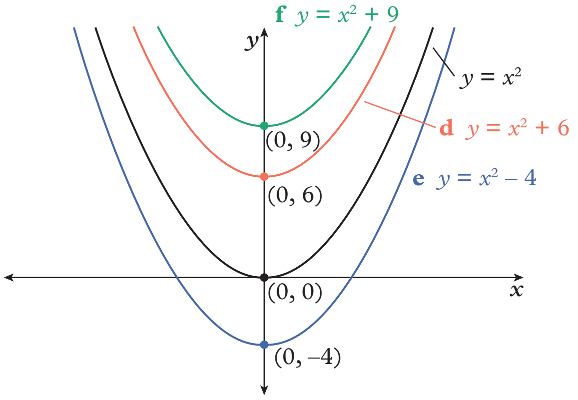
4D Sketching parabolas using transformations

1 **a** 10, down
b 5, right
c 12, left
d 8, up

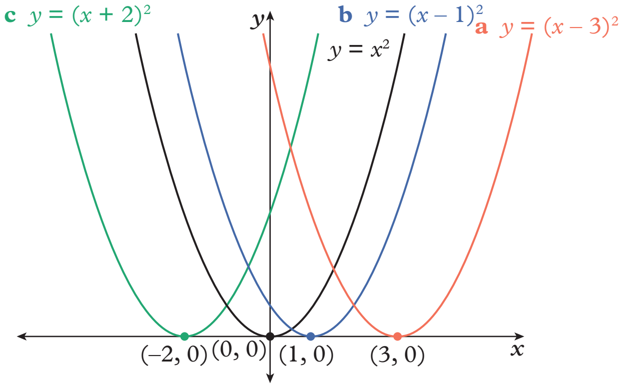
2 **a–c**



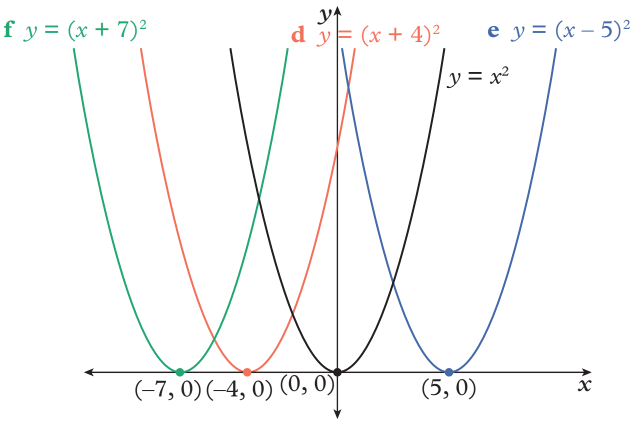
d–f



3 **a–c**

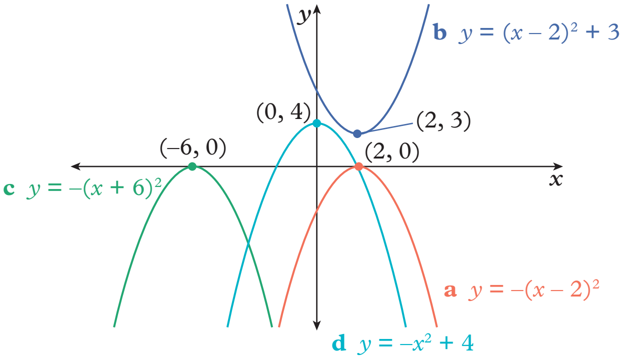


d–f

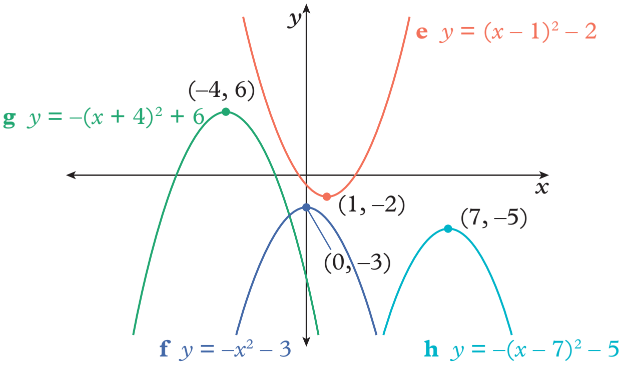


4 **a** 2, left, 3, down **b** reflected, 4, up

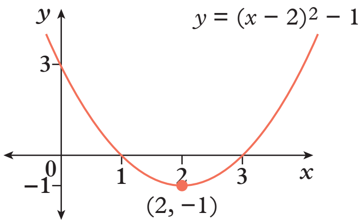
5 **a–d**



e–h



6 **a** **i** upright
ii x-intercepts: (1, 0) and (3, 0); y-intercept: (0, 3)
iii (2, –1)
iv

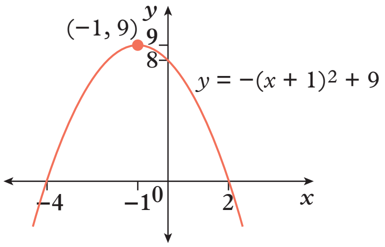


b **i** inverted

ii x-intercepts: $(-4, 0)$ and $(2, 0)$; y-intercept: $(0, 8)$

iii $(-1, 9)$

iv

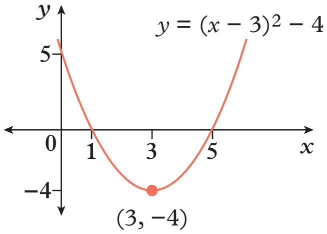


c **i** upright

ii x-intercepts: $(1, 0)$ and $(5, 0)$; y-intercept: $(0, 5)$

iii $(3, -4)$

iv

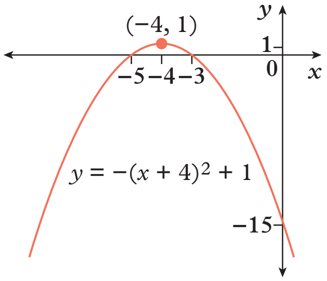


d **i** inverted

ii x-intercepts: $(-5, 0)$ and $(-3, 0)$;
y-intercept: $(0, -15)$

iii $(-4, 1)$

iv



7 **a** **i** upright **ii** $(1, -4)$

iii a is positive **iv** $h = 1, k = -4$

b **i** inverted **ii** $(-3, 1)$

iii a is negative **iv** $h = -3, k = 1$

c **i** upright **ii** $(3, -9)$

iii a is positive **iv** $h = 3, k = -9$

8 **a** E **b** D **c** A **d** F **e** C

f B **g** H **h** I **i** G

9 **a** $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$

c $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$

e $y = (x - 9)^2$ **f** $y = -x^2 + 4$

g $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

10 **a** $(4, 5)$ **b** 5

c There is no maximum value for y because the parabola is upright (it has a minimum turning point).

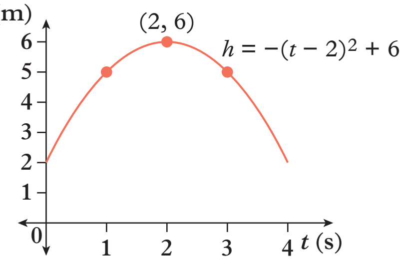
11 Graph is inverted so it has a maximum turning point. The y-coordinate of the turning point is 2, so the maximum y-value is 2.

12 **a** $(2, 6)$

b **i** 2 m

ii 2 m

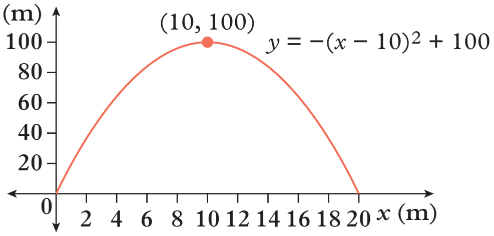
c



d **i** 2 m

ii 6 m

13 **a**



b 100 m **c** 20 m

14 **a** $a = 1$

b $y = 2x^2, a = 2$

c $y = \frac{1}{2}x^2, a = \frac{1}{2}$

d If $a > 1$, the graph will be vertically enlarged.
If $0 < a < 1$, the graph will be vertically compressed.

15 **a** E **b** A **c** D **d** C **e** F **f** B

16 **a** **i** narrower **ii** upright **iii** $(4, -3)$

b **i** wider **ii** upright **iii** $(-2, 6)$

c **i** narrower **ii** upright **iii** $(-2, 0)$

d **i** narrower **ii** inverted **iii** $(0, -4)$

e **i** wider **ii** inverted **iii** $(7, 0)$

f **i** narrower **ii** inverted **iii** $(-1, 5)$

g **i** wider **ii** inverted **iii** $(3, -4)$

h **i** wider **ii** upright **iii** $(0, 3)$

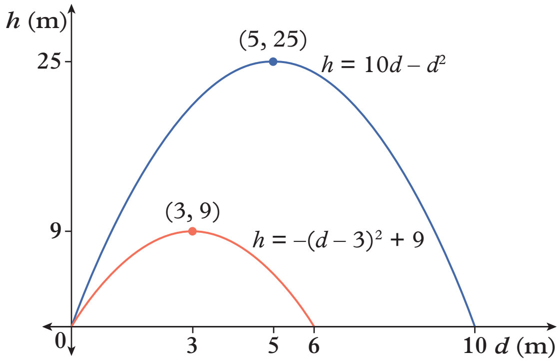
i **i** the same **ii** inverted **iii** $(5, 4)$

j **i** wider **ii** upright **iii** $(-3, -7)$

17 **a** $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$

c $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

18 **a, e**



b 9 m

c 6 m

d $h = -(d - 5)^2 + 25$

f 16 m

EX p170

4E Other non-linear relationships

1 **a** B

b D

c A

d C

2 **a** x-intercepts (0, 0), y-intercepts (0, 0)

b x-intercepts (0, 0), y-intercepts (0, 0)

c x-intercepts (0, 0), y-intercepts (0, 0)

d x-intercepts undefined, y-intercepts undefined

3 **a** **i** (0, 0) **ii** (0, 0)

b **i** (−5, 0) **ii** $(0, \sqrt{5})$

c **i** (0, 0) **ii** (0, 0)

d **i** (0, 0) **ii** (0, 0)

4 **a** **i** (0, 0), (0, 0)

ii (2, 0), (2, 0)

iii (0, −5), (0, −5)

iv (3, 3), (3, 3)

b **i** $(-5, 0), (0, -\sqrt{5})$

ii $(-3, 0), (2, \sqrt{5})$

iii $(-5, -5), (0, \sqrt{5} - 5)$

iv $(-2, 3), (3, 3 + \sqrt{5})$

c **i** (0, 0), (0, 0)

ii (2, 0), (2, 0)

iii (0, −5), (0, −5)

iv (3, 3), (3, 3)

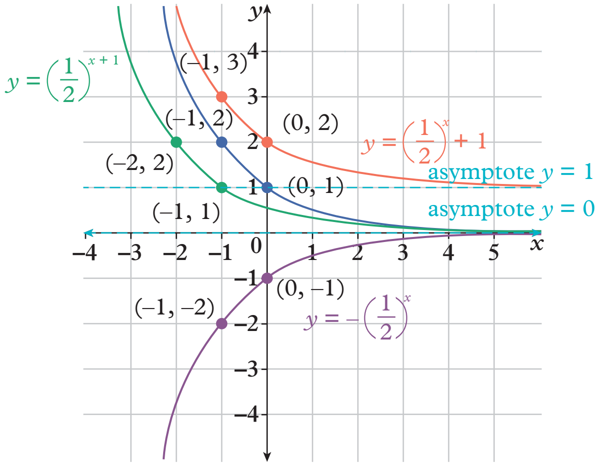
d **i** (0, 0), (0, 0)

ii (2, 0), (2, 0)

iii (0, −5), (0, −5)

iv (3, 3), (3, 3)

5



6 **a** To sketch the graph of $y = \frac{1}{x} + 20$, the graph of $y = \frac{1}{x}$ is translated 20 units up.

b To sketch the graph of $y = \frac{1}{x - 15}$, the graph of $y = \frac{1}{x}$ is translated 15 units right.

c To sketch the graph of $y = -1 + \sqrt[3]{x}$, the graph of $y = \sqrt[3]{x}$ is translated 1 units down.

d To sketch the graph of $y = \sqrt[3]{2 + x}$, the graph of $y = \sqrt[3]{x}$ is translated 2 units left.

7 **a** $y = 3 \times 5^x$

b $y = -5^x$

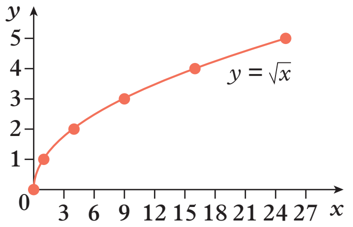
c $y = 5^{x-3}$

d $y = 5^x - 2$

8

x	0	1	4	9	16	25
y	0	1	2	3	4	5

b, d



No; not possible to take the square root of a negative number.

c As x increases, y increases, starting off steep and becoming less steep. The graph starts at the origin and extends into the first quadrant.

e **i** (3, 0) **ii** translate 3 units to right

f **i** translate 2 units up

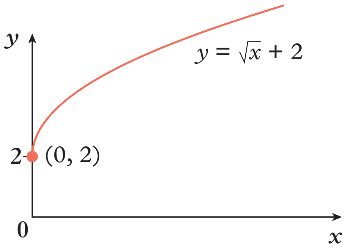
ii translate 1 unit down

iii translate 1 unit right

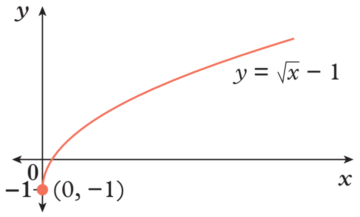
iv translate 4 units left

v translate 2 units right and 3 units up

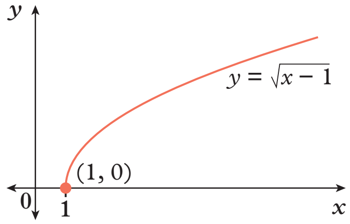
g **i**



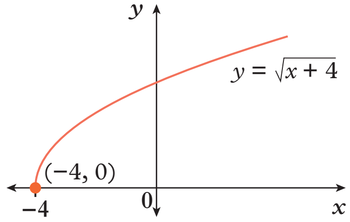
ii



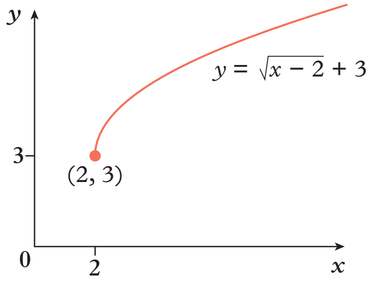
iii



iv



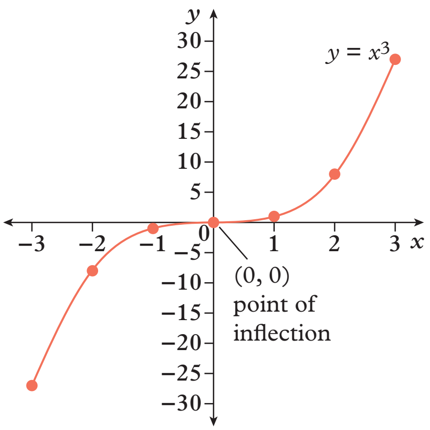
v



9 **a**

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

b



c As x increases, y increases. Graph appears to level out at $x = 0$ before rising more steeply again.

d **i** (0, 2) **ii** translate graph 2 units up

e **i** translate 1 unit up

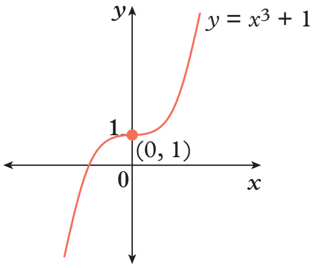
ii translate 3 units down

iii translate 2 units right

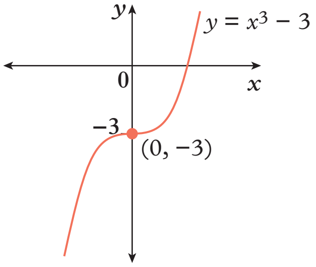
iv translate 4 units left

v translate 1 unit right and 2 units up

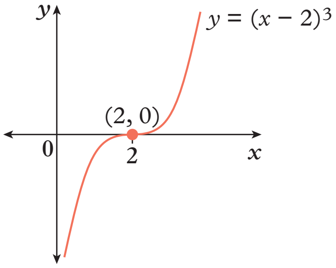
f **i**



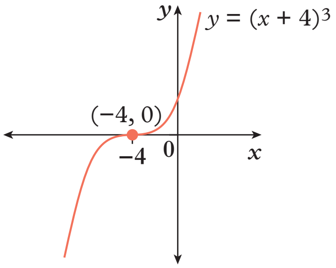
ii



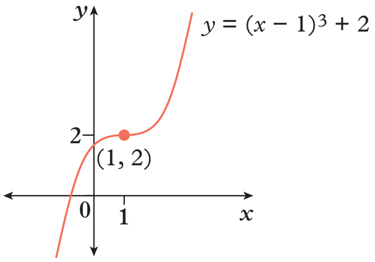
iii



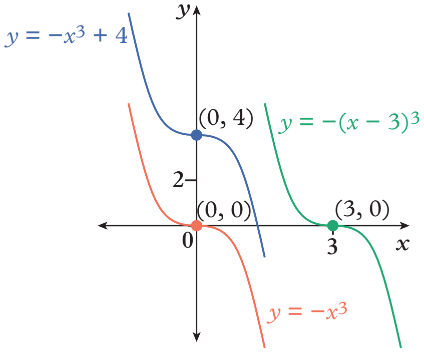
iv



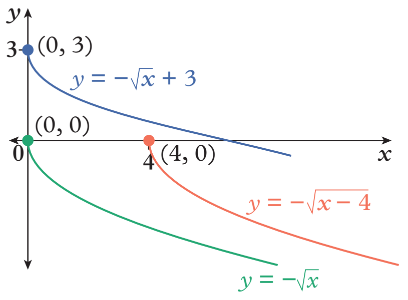
v



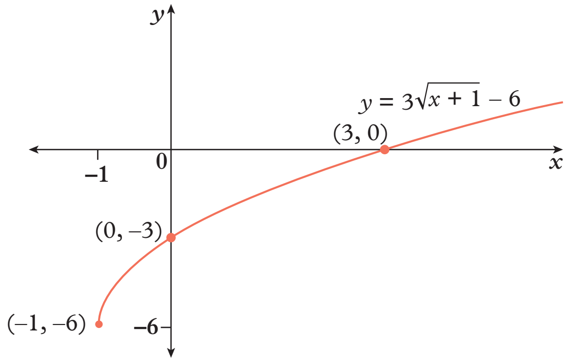
10 **a**



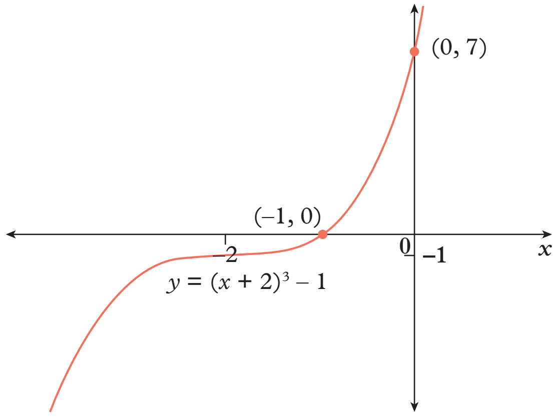
b



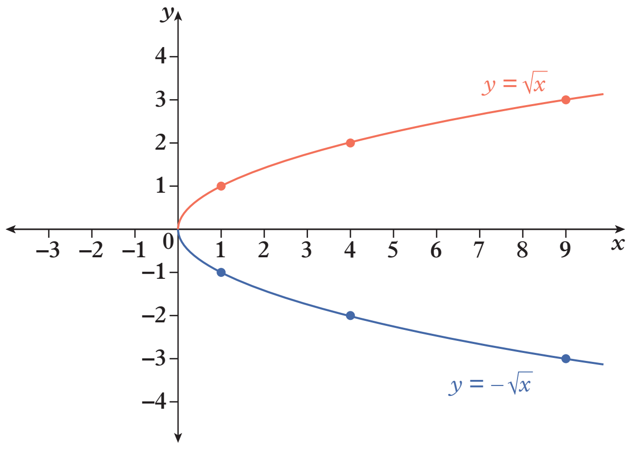
11 a



b

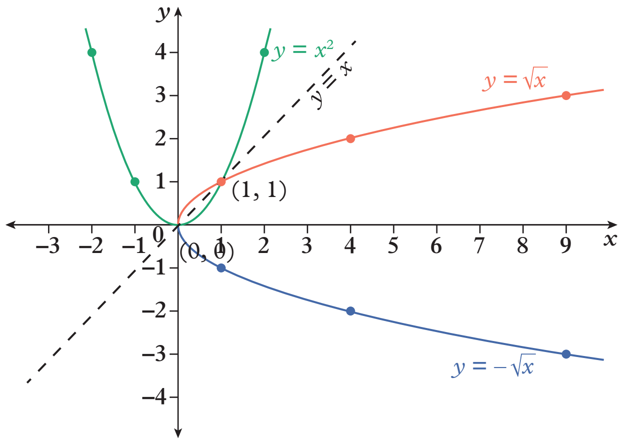


12 a



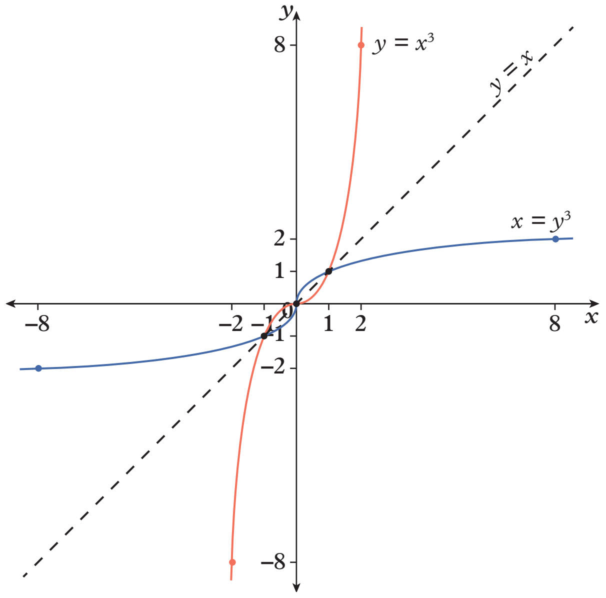
b $y = \pm\sqrt{x}$ or $y^2 = x$

c, d



e The graph of $y^2 = x$ and $y = x^2$ are reflected in the line $y = x$, or the graph of $y = x^2$ is rotated 90 degrees clockwise to make $y^2 = x^2$.

13



Chapter 4 review

Multiple choice

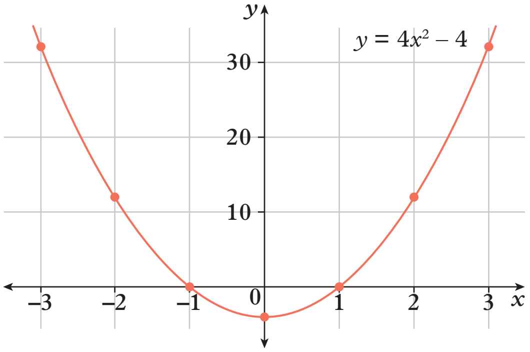
1	D	**2**	A	**3**	A	**4**	D	**5**	C
6	C	**7**	E	**8**	B	**9**	C	**10**	D

Short answer

1 **a** $x = 2$ or $x = 3$ **b** $x = -6$ or $x = 5$

c no solutions **d** $x = 0$ or $x = 12$

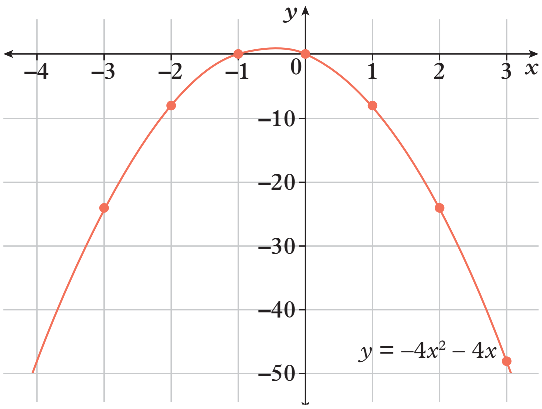
2 **a**



i x-intercepts: $(-1, 0)$ and $(1, 0)$, y-intercept: $(0, -4)$

ii $(0, -4)$ **iii** minimum **iv** $x = 0$

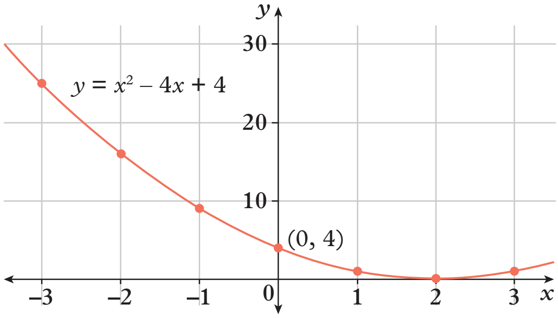
b



i x-intercepts: $(-1, 0)$ and $(0, 0)$, y-intercept: $(0, 0)$

ii $(-0.5, 1)$ **iii** maximum **iv** $x = -0.5$

c



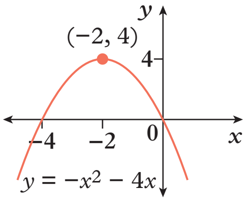
i x-intercept: $(2, 0)$, y-intercept: $(0, 4)$

ii $(2, 0)$ **iii** minimum **iv** $x = 2$

3 **a** **i** x-intercepts: $(-4, 0)$ and $(0, 0)$, y-intercept: $(0, 0)$

ii $(-2, 4)$

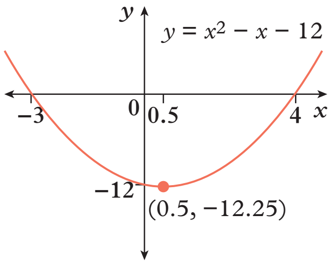
iii



b **i** x-intercepts: $(-3, 0)$ and $(4, 0)$, y-intercept: $(0, -12)$

ii $(0.5, -12.25)$

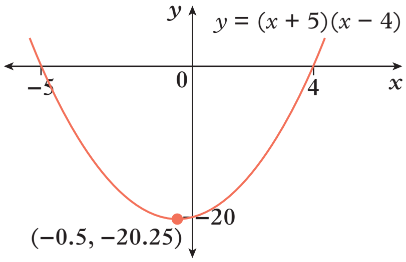
iii



c **i** x-intercepts: $(-5, 0)$ and $(4, 0)$, y-intercept: $(0, -20)$

ii $(-0.5, -20.25)$

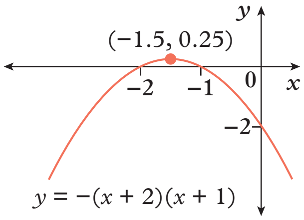
iii



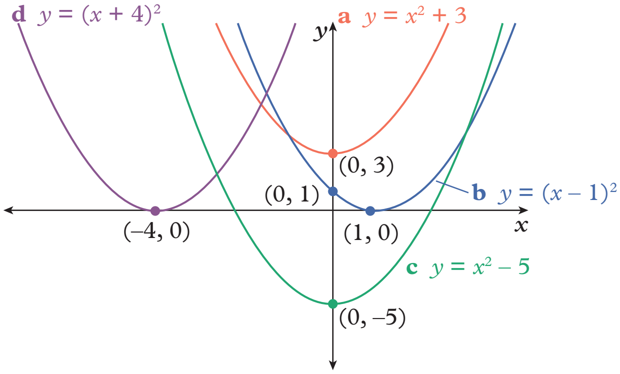
d **i** x-intercepts: $(-2, 0)$ and $(-1, 0)$, y-intercept: $(0, -2)$

ii $(-1.5, 0.25)$

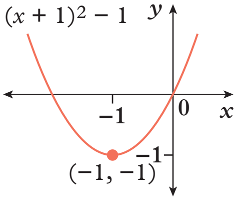
iii



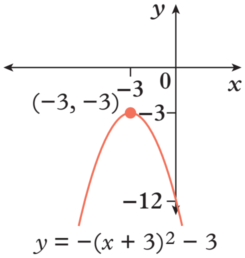
4 **a–d**



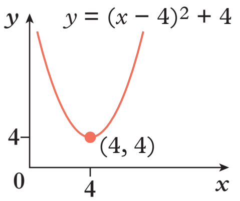
5 **a**



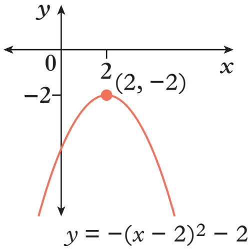
b



c



d

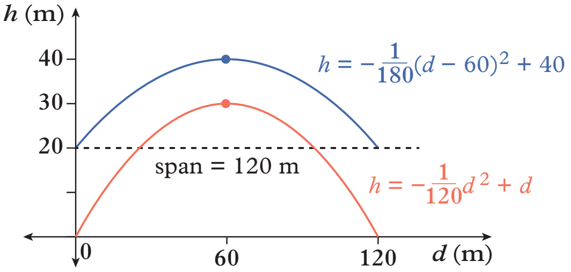


6 **a** Move 3 units up
b Move 2 units right

7 **c** Reflect across the *x*-axis
d Move 2 units up

Analysis

a, d(i)



b **i** upper arch: (0, 20), lower arch: (0, 0)
ii upper arch: (60, 40), lower arch: (60, 30)
iii Both have the same axis of symmetry, $d = 60$.

c 20 m

d **i** 120 m
ii Sample answer: Let $h = 0$

$$-\frac{1}{120}d^2 + d = 0$$
$$-d\left(\frac{1}{120}d - 1\right) = 0$$
$$d = 0 \text{ or } d = 120$$

So, span = 120 m

e **i** 30 m **ii** 40 m

f 10 m

g 12.5 m

h The two arches are 20 m apart at the start. Approaching the highest points on both arches, the distance between the two arches decreases until it is a minimum of 10 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches increases until they are a distance of 20 m apart at the end of the bridge.

EX p176

SEMESTER 1 Review

Short answer

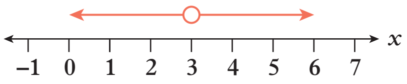
1 **a** 20.040 **b** 20.0
c 0.0008 **d** 7.827×10^{-4}

2 **a** $4\sqrt{2}$ **b** $6\sqrt{5}$ **c** $2\sqrt{21}$ **d** $12\sqrt{7}$

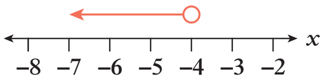
3 **a** $\sqrt{33}$ **b** $\sqrt{3}$ **c** $56\sqrt{3}$ **d** $3\sqrt{10}$

4 **a** $x < 4$ **b** $x \neq 2$
c $x \geq 7$ **d** $-5 \leq x < 2$

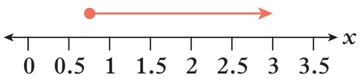
5 **a** $x \neq 3$



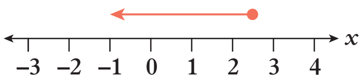
b $x < -4$



c $x \geq \frac{3}{4}$



d $x \leq \frac{5}{2}$



6 **a** 3^4 **b** $2^2 \times 5^3$
c $2^3 \times 3^3$ **d** $2^2 \times 3^3 \times 5^2$

7 **a** 343 **b** 375
c 82 130 **d** 0.010 530
e $\frac{27}{8} = 3.375$ **f** $\frac{49}{25} = 1.96$

8 **a** $2^{30} \times 7^{18}$ **b** $\frac{11^8}{17^9}$
c $\frac{1}{97^4 \times 101^{12}}$ **d** $41^{66} \times 57^{16}$

9 **a** $a^{16}b^{14}$ **b** $\frac{e}{c^{10}}$ **c** $\frac{g^4}{f}$ **d** $\frac{q^{51}}{p^4}$

10 **a** 6 **b** 2 **c** 6 **d** 5

11 **a** 7.514×10^8 **b** 2.5360×10 or 2.5360×10^1
c 8.6×10^{-5} **d** 7.50×10^4

12 **a** $56 - 16x$ **b** $-6x^5 + 16x^3$
c $x^2 - 5x - 14$ **d** $10xy + 15x - 4y - 6$
e $-12x - 23$ **f** $5x^2 + 6x + 6$

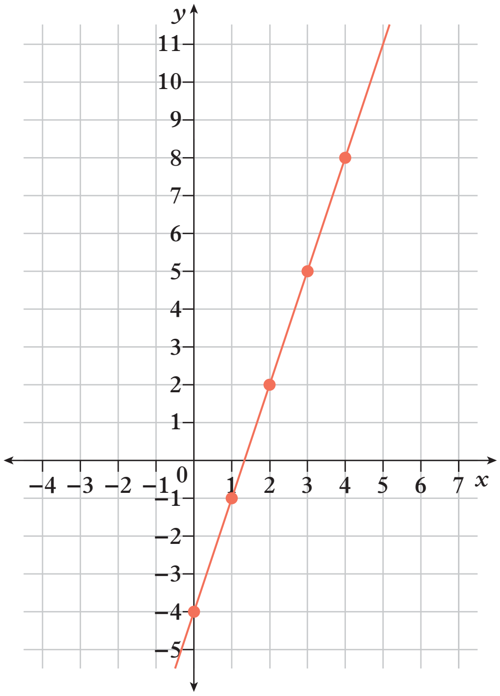
13 **a** $6(2a + 3b^2 - cd)$ **b** $7g(3g - 1)$
c $(a - 5)(a + 5)$ **d** $2(2m - 3n)(2m + 3n)$
e $(t - 5)(t + 9)$ **f** $(r - 4)(r + 9)$

14 **a** $\frac{1}{2}$ **b** 0 **c** -3
d undefined **e** $-\frac{3}{2}$

15 **a** $x = \frac{11}{12}$ **b** $x = -3$
c $x = 8$ **d** $x = 7$
e $x = \frac{4}{5}$ **f** $x = -3, x = 7$
g $x = \frac{1}{9}, x = -\frac{3}{5}$ **h** $x = -5, x = -7$

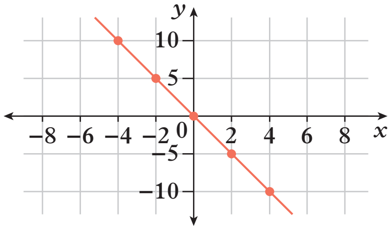
16 a

x	0	1	2	3	4
y	−4	−1	2	5	8



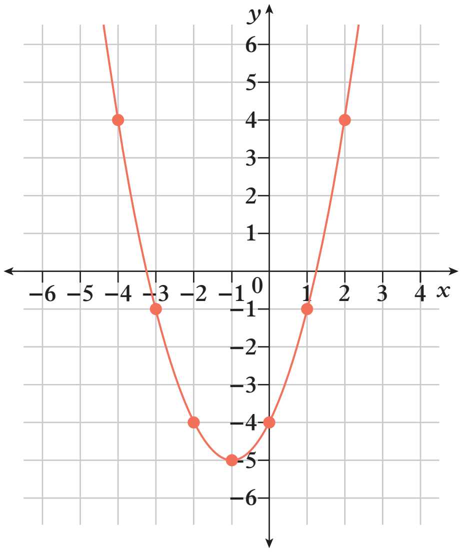
b

x	−4	−2	0	2	4
y	10	5	0	−5	−10



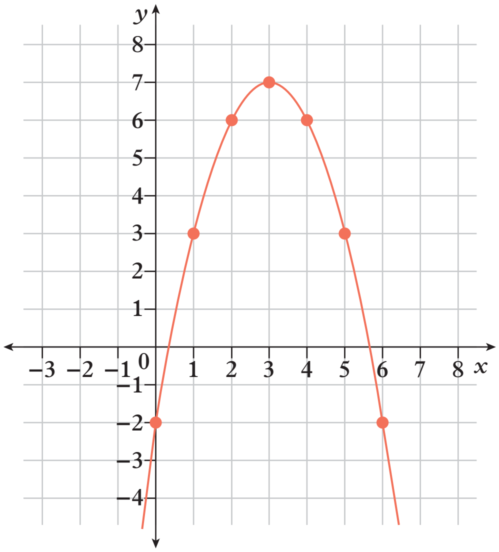
c

x	−4	−3	−2	−1	0	1	2
y	4	−1	−4	−5	−4	−1	4



d

x	0	1	2	3	4	5	6
y	−2	3	6	7	6	3	−2



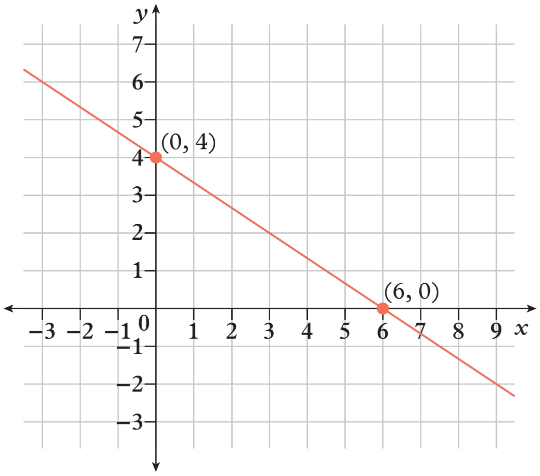
17 a (2, 0), (0, −6)

b (−12, 0), $\left(0, -\frac{15}{2}\right)$

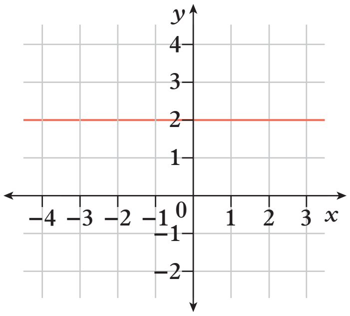
c (5, 0), (−9, 0), (0, −45)

d (10, 0), (−4, 0), (0, −40)

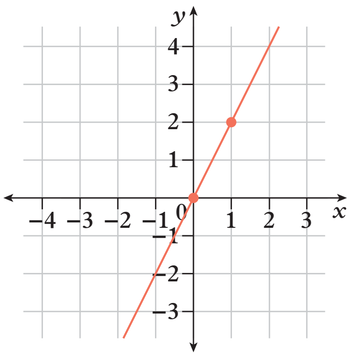
18 a



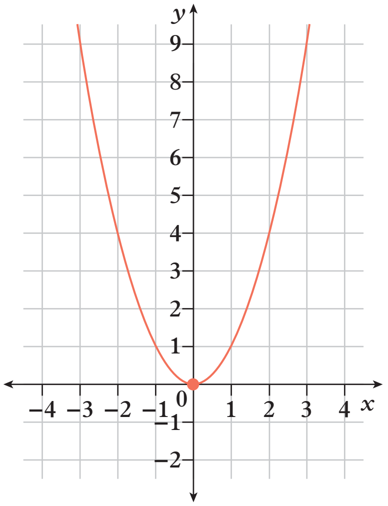
b



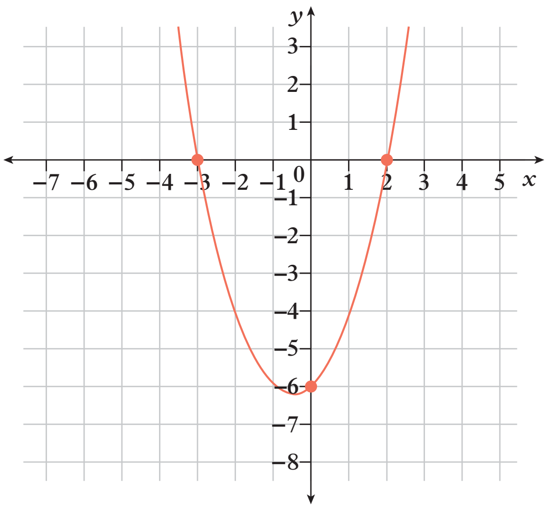
c



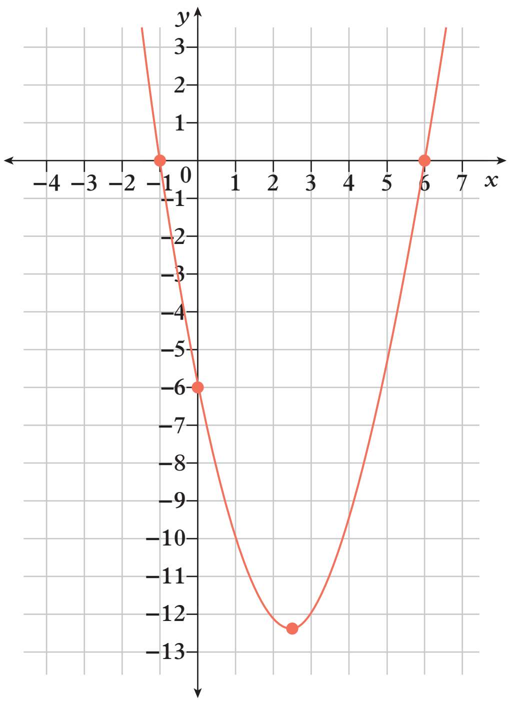
d



e



f



19 **a** $y = -3x + 5$ **b** $x = -3$

20 **a** $(-1, 2)$ **b** $(2, 4.5)$

21 **a** 8.54 units **b** 6.32 units

22 **a** $x^2 - 14x + 52$ **b** $x^2 + 24x + 149$

c $-x^2 + 9x - 13$ **d** $-x^2 + 6x - 11$

23 **a** horizontal translations of 2 units to the right

b horizontal translations of 5 units to the left

c horizontal translations of 3 units to the left, then vertical translation of 1 unit up

Analysis

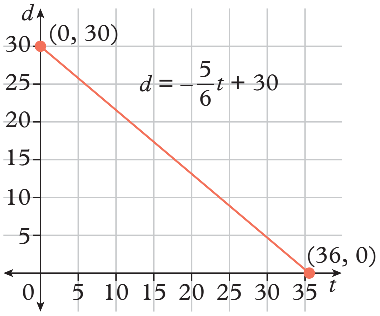
1 **a** d-intercept: $(0, 30)$, t-intercept: $(36, 0)$

b $\frac{5}{6}$ km/min

c From the axis intercepts, the graph must have a negative gradient, so the gradient, m, is the negative of the speed, $m = -\frac{5}{6}$.

d $d = -\frac{5}{6}t + 30$

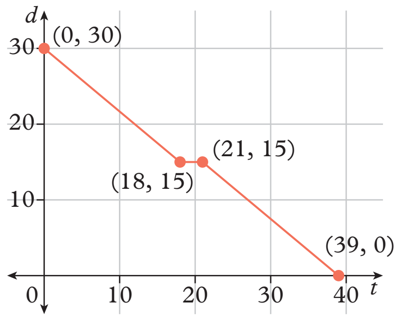
e



f $(18, 15)$

g Add a horizontal line segment at height $d = 15$ for $18 \le t < 21$ since Jane's car isn't moving for those three minutes. The second half of the original graph is translated horizontally right by three units. The t-intercept moves to $(39, 0)$.

h



i $d = -\frac{5}{6}t + 30 \qquad (0 \le t < 18)$

$d = 15 \qquad (18 \le t < 21)$

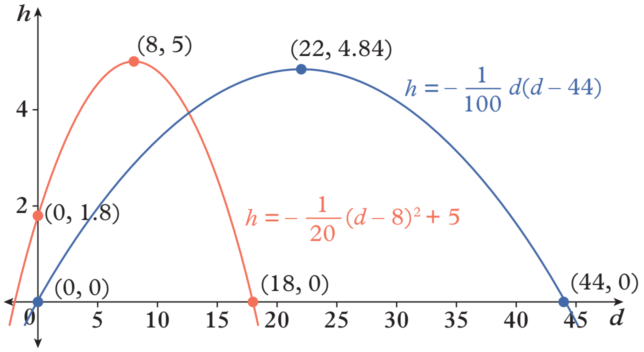
$d = -\frac{5}{6}t + \frac{65}{2} \qquad (21 \le t \le 39)$

2 **a** **i** 0 **ii** 18

b **i** 4.84 **ii** 5

c **i** 44 **ii** 18

d



e

The relationship is non-linear.

EX p180

EXPLORATIONS 1

1 **a** **i** The van's speed is 90 km/h and the bicycle's is 40 km/h. The van overtakes the bicycle at 4:24 pm.

ii 104 km/h **iii** 6:40 pm

b Speed of van = $\frac{1}{4}(c + 3t)$ km/h

Speed of bicycle = $(2t - c)$ km/h

The van overtakes the bicycle at 4:24 pm, which is independent of c and t.

2 **a** **i** The number of border squares is $2x + 2y - 4$ and the number of interior squares is $xy - 2x - 2y$.

ii $xy - 4x - 4y + 4 = 0$

iii 12, so the equation becomes $(x - 4)(y - 4) = 12$

iv $(x, y) = (5, 16), (6, 10), (7, 8), (8, 7), (10, 6)$ or $(16, 5)$

v Switching x and y rotates the design, so we can ignore the last three solutions.

b **i** Three: $(x, y) = (7, 34), (8, 20)$ or $(10, 13)$

ii One: $(x, y) = (4, 10)$

3 **a** To four decimal places: 2.4495, 2.9068, 2.9844, 2.9974, 2.9996, …, which appears to approach 3.

b Solve $x^2 - x - 6 = 0$ to give $x = -2$ or 3 but ignore $x = -2$ as x must be positive.

c **i** 4 and 1

ii $\sqrt{20 + \sqrt{20 + \ldots}}$ and $\sqrt{30 - \sqrt{30 - \ldots}}$

4 **a** $12 \longrightarrow 6 \longrightarrow 3 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1 \longrightarrow 2$

b $13 \longrightarrow \cdots \longrightarrow 14 \longrightarrow 7 \longrightarrow 20 \longrightarrow 10 \longrightarrow 5 \longrightarrow 14$

c This cycle has length 18: There are other solutions.

$17 \longrightarrow 50 \longrightarrow 25 \longrightarrow \cdots \longrightarrow 136 \longrightarrow 68 \longrightarrow 34 \longrightarrow 17$

d Use the fact that a multiple of 3 can only be the result of halving.

e There is only one: the cycle in part **b**.

f One solution is $2^{500} + 1$. There are many others.

5 **a** Clockwise from the top left, the lengths and areas are as follows:

Length (cm)	14	14	12	12	6	6	8	8
Area (cm^2)	80	96	72	54	18	17	30	32

b Dark length = 14 + 12 + 6 + 8 = 40 cm

$= \frac{1}{2}$ of total

Dark area = 80 + 72 + 18 + 30 = 200 cm^2

$= \frac{1}{2}$ of total

c Show that, for any a and b, the total dark length is always $2c$ and the total dark area is always $\frac{1}{2}c^2$.

d Let P be at the centre and choose any three points on the boundary that divide the perimeter into three equal lengths. Show that cutting from P to the three boundary points creates three fair slices.

e It is possible for all positive integers by adapting the strategy in part **d**.

EX p182

Algorithmic Thinking 1 Number and Algebra

Task 1

1 Example 1: 2, 4
Example 2: (−3, 9), (−2, 4), (−1, 1), (0, 0), (1, 1), (2, 4), (3, 9)

Task 2

1 THE CATS AWAY

2 For each integer x in the set {2, 20, 11, 14, 8, 17, 1, 6, 12, 9, 10, 9, 24}:

```
    set the variable $y = x + 1$
    print $(x, y)$
    add $(x, y)$ to the table
```

3 Teacher to check.

CHAPTER 5 Measurement

EX p189

5A Area of composite shapes

1 **a** 20.79 cm^2 **b** 1800 mm^2 **c** 452.39 cm^2
d 14.16 mm^2 **e** 44.2 cm^2 **f** 768 mm^2

2 **a** 7.07 cm^2 **b** 127.41 cm^2 **c** 184.31 mm^2

3 **a** triangle, semicircle
b three rectangles
c triangle, rectangle, semicircle
d triangle, rectangle
e rectangles, semicircles
f parallelogram, triangle, semicircle

4 **a** 122.27 cm^2 **b** 56 cm^2 **c** 31.57 mm^2
d 69 mm^2 **e** 160 cm^2 **f** 174.63 m^2

5 **a** 70.87 cm^2 **b** 47.43 mm^2 **c** 424 cm^2
d 69.73 mm^2 **e** 70.5 cm^2 **f** 60.5 cm^2

6 **a** 141.02 mm^2 **b** 330 cm^2 **c** 557.08 cm^2
d 34 cm^2 **e** 34.28 m^2 **f** 63.26 cm^2

7 **a** An annulus is the shape formed by two different-sized circles with a common centre. It is the area between the two circles.
b area of outer circle – area of inner circle
c 137.44 cm^2

8 **a** 12.57 cm^2 **b** 65.97 cm^2 **c** 28.27 m^2
d 226.19 cm^2 **e** 103.67 cm^2 **f** 47.12 cm^2

9 12 166.55 m^2

10 **a** 35.5 cm^2
b 19 525 cm^2
Sample answers: To cater for wastage between tags when they are cut out; in case any mistakes are made in production of tags.

11 **a** 105.35 m^2 **b** \$4740.75 **c** \$1580.25

12 176.70 cm^2

13 **a** 354.01 cm^2 **b** 336 cm^2 **c** 97.425 cm^2
d 68.57 cm^2 **e** 35 m^2 **f** 78.72 cm^2

14 **a** A, B: 25 cm^2, C, E, G: 12.5 cm^2, D, F: 6.25 cm^2
b **i** 100 cm^2 **ii** 100 cm^2 **iii** 100 cm^2

15 47.75 cm^2

16 15 509.73 cm^2

17 71.78 cm^2

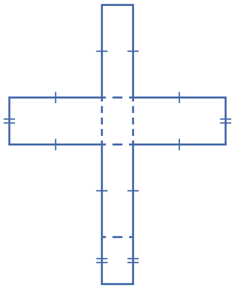
18 **a** 44 m^2 **b** 17.80 cm^2 **c** 15 cm^2

EX p196

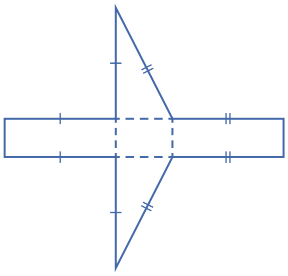
5B Surface area

1 **b**, **c**

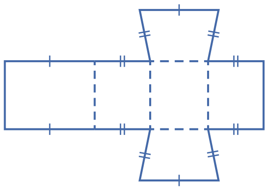
2 **a**



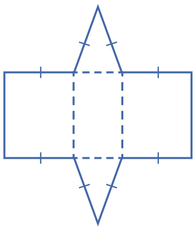
b



c



d



3 **a** 188 cm^2 **b** 136 m^2
c 150 cm^2 **d** 148 m^2

4 **a** 240 cm^2 **b** 120 cm^2
c 976 cm^2 **d** 290 mm^2

5 **a** 54 cm^2 **b** 52 cm^2
c $\frac{25}{6}$ cm^2 **d** $\frac{17}{18}$ cm^2

6 **a** 188 cm^2 **b** 195.52 mm^2 **c** 360 cm^2
d 162.174 m^2 **e** 120 cm^2 **f** 120 cm^2

7 **a** **i** 664 cm^2 **ii** 1328 cm^2
b **i** 396 cm^2 **ii** 792 cm^2

8 **a** 3820 cm^2 **b** 2961.2844 cm^2

9 **a** 500 cm^2
b **i** 292 cm^2 **ii** 584 cm^2
c Answer to **b ii** is 84 cm^2 greater, because two more faces (7 cm by 6 cm) are created when the block of butter is cut in half.
d 4032 cm^2
e Sample answer: Cutting butter into smaller pieces increases the amount of surface exposed to the heat, so butter will melt more quickly.

10 **a** cream: 30 m^2, blue: 53.575 m^2
b cream: 4 L, blue: 7.15 L

11 **a** 25 cm^2 **b** 5 cm **c** $l = \sqrt{\frac{\text{TSA}}{6}}$

12 Sample answer: A cube has six identical square faces; hence all sides are equal in length. Therefore, the length of one face will determine the length of all sides. A rectangular prism could have a different length, width and height.

13 **a** 150 cm^2
b 250 cm^2

c **i** 350 cm^2
ii 450 cm^2
iii 550 cm^2

d $25 \times (4n + 2)$, when $n = 17$, the surface area is 1750 cm^2

e 37.5 cm^2

f 212.5 cm^2

g 3

EX p202

5C Volume

1 **a** 450 cm^3 **b** 15 cm^3 **c** 105 cm^3
2 **a** 48 cm^3 **b** 1080 cm **c** 288 cm^3
3 **a** 210 cm^3 **b** 36 cm^3 **c** 82.5 cm^3
4 **a** 540 cm^3 **b** 2457 cm^3 **c** 140 mm^3
5 **a** 30 cm^2 **b** 4 cm
6 **a** 60 cm^2 **b** 40 mm
7 **a** $6x^2$ **b** x^3 **c** $x, x, x + 4$
d $6x^2 + 16x$ **e** 5 **f** 225 cm^3
8 160 cm^3
9 **a** $6x^2$ **b** x^3 **c** $x, x, x - 2$
d $6x^2 - 8x$ **e** 6 **f** 96 cm^3
10 18 cm^3
11 54 cm^3
12 **a** 33.14 m^3 **b** 33.14 m^3
c 20 **d** 24

e The length of the small container is 5.9 m, the small container can hold 5 layers of cubic boxes with 2 boxes in each row and 2 boxes in each column of the layer. The length of the large container is 11.8 m, which will fit in 11 layers of boxes, with 4 boxes in each layer. As the length of the small container is doubled, the empty space left at the end of the container will be doubled from 0.9 m to 1.8 m, which is enough to fit in one extra layer of boxes of 4.

13 **a** 400 cm^3 **b** 14.33 cm^3 **c** 39 cm^3

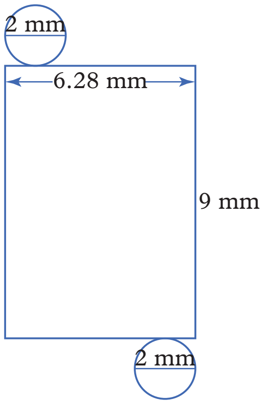
Checkpoint

1 **a** 4.71 cm^2 **b** 41.89 cm^2
2 **a** 37.44 m^2 **b** 284 m^2
3 **a** 195 mm^2 **b** 57.52 mm^2
4 **a** 25.2 cm^3 **b** 1080 m^3
c 125 cm^3 **d** 60 m^3

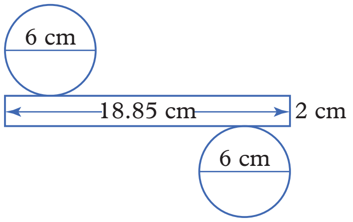
EX p206

5D Surface area of cylinders

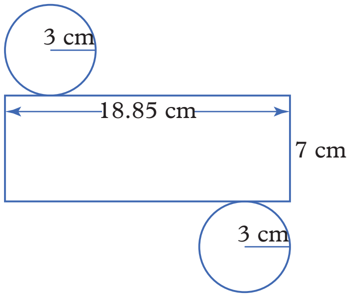
1 **a**



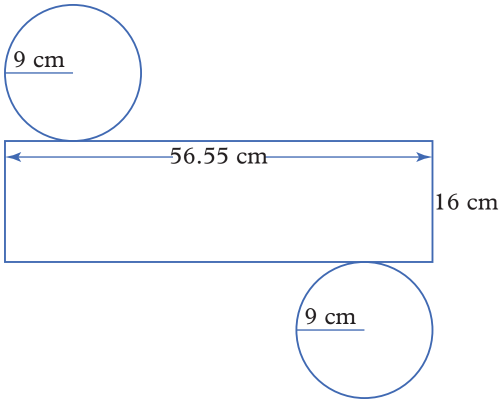
b



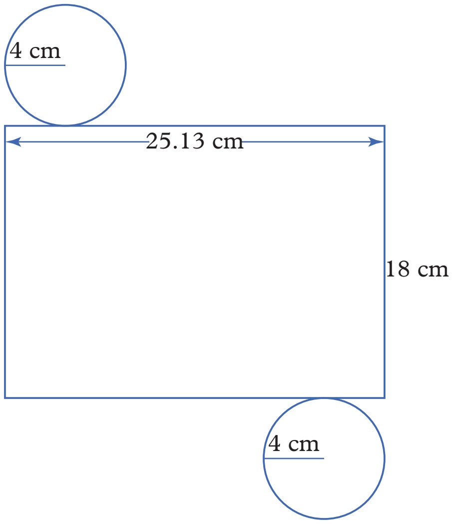
c



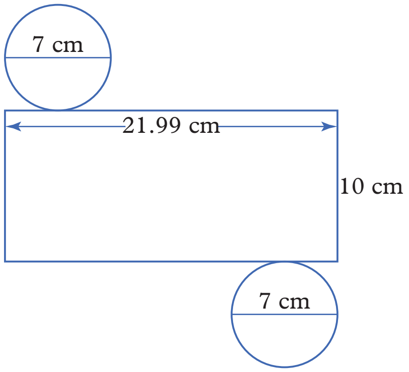
d



e

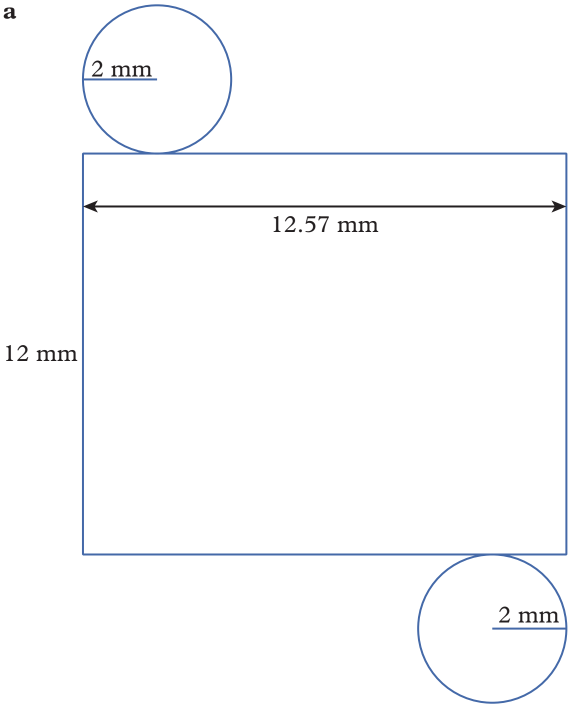


f

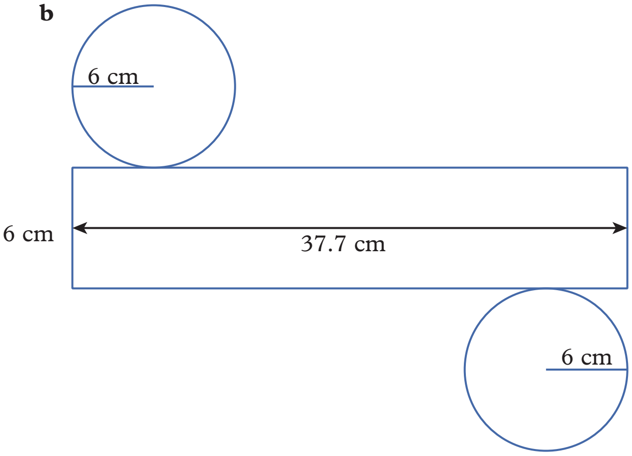


2 **a** 62.83 mm^2 **b** 94.25 cm^2 **c** 188.50 cm^2
d 1413.72 cm^2 **e** 552.92 cm^2 **f** 296.88 cm^2

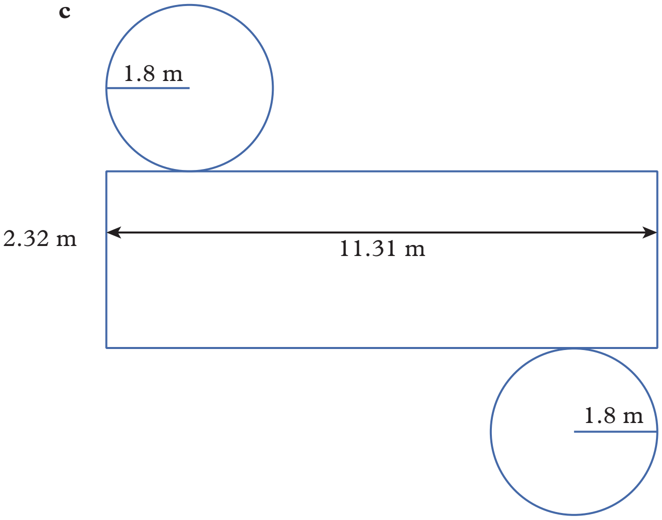
3 **a**



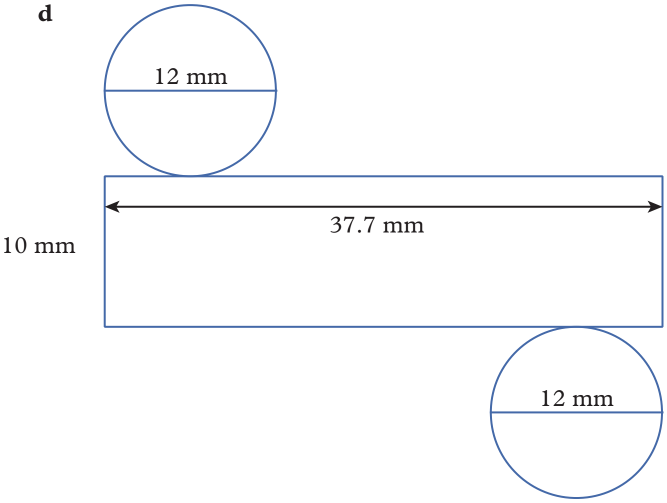
b



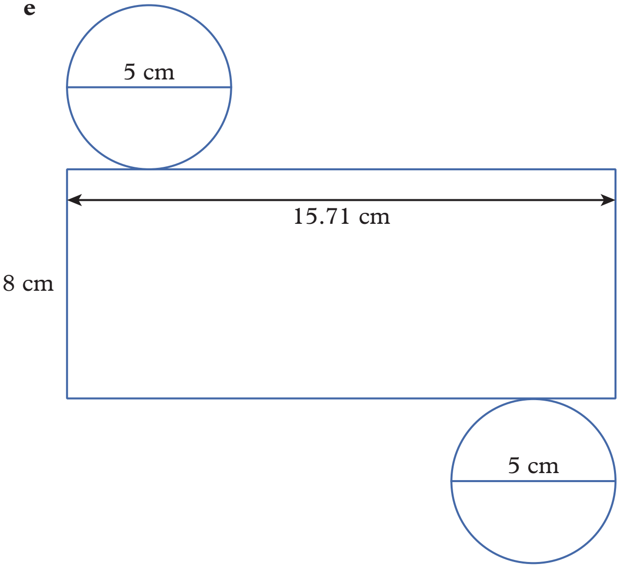
c



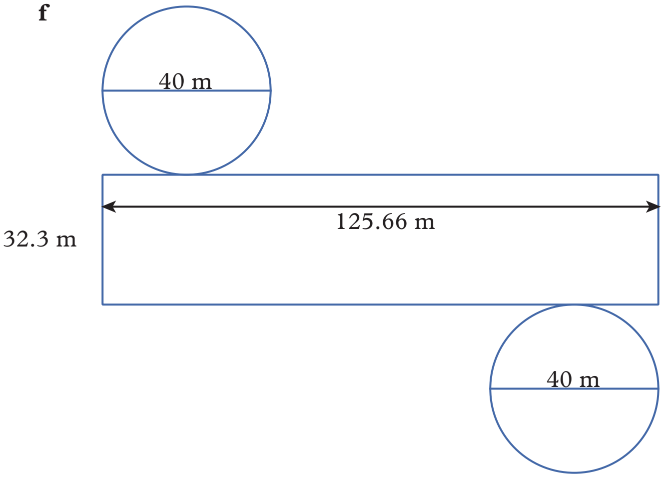
d



e



f



4 **a** 175.93 mm^2 **b** 452.39 cm^2 **c** 46.60 m^2
d 603.19 mm^2 **e** 164.93 cm^2 **f** 6572.21 m^2

5 **a** 100.53 cm^2 **b** 541.92 cm^2 **c** 603.19 cm^2
d 18.85 m^2 **e** 7.85 m^2 **f** 3.39 m^2

6 **a** 1130.97 cm^2 **b** 34.57 cm^2 **c** 96.13 m^2
d 157.10 m^2 **e** 15.36 mm^2 **f** 9.08 mm^2

7 3924.63 cm^2

8 \$1429 (\$1450 if rounded to needing 29 L of paint)

9 first roller with radius 4 cm and length 25 cm

10 **a** 376.99 cm^2

b Lucian has calculated the surface area of the whole cylinder (ignoring the hollowed-out section) and Curtis forgot to include the area of the inside surface.

11 8 cm

12 Sample answer: 2 × area of base calculates the surface area of the matching ends of a prism. The surface area of each of the rectangular faces joining the matching ends can be calculated by multiplying the section of the perimeter of the base that the face occupies by the height, so together the area of all of the rectangular faces joining the matching ends is perimeter of base × height.

13 No, surface area will not double if you double the height of the cylinder. The surface area of the curved surface will double, but the area of the ends will not change.

14 **a** 201.06 cm^2 **b** 615.75 mm^2

c 314.16 cm^2 **d** 127.32 cm^2

15 $h = \frac{SA - 2\pi r^2}{2\pi r}$ or $h = \frac{SA}{2\pi r} - r$

16 48.42

EX p210

5E Volume of cylinders

1 **a** 424.12 cm^3 **b** 942.48 mm^3 **c** 39.27 cm^3

d 904.78 mm^3 **e** 367.57 cm^3 **f** 2290.22 cm^3

2 **a** 150.80 mm^3 **b** 678.58 cm^3 **c** 23.61 m^3

d 1130.97 mm^3 **e** 157.08 cm^3 **f** 40589.38 m^3

3 **a** 2513.27 cm^3 **b** 15.1 cm^3

c 59.38 m^3 **d** 133.93 m^3

e 63 711.50 mm^3 **f** 17 359.66 cm^3

4 **a** 1 570 796 L **b** 1357 L **c** 1 L (618 mL)

5 **a** 3 L **b** 0 L **c** 59 380 L

d 133 930 L **e** 0 L **f** 18 L

6 **a** 1.33 cm **b** 2.82 cm

7 **a** 462 cm^3 **b** 600 g

8 **a** 44 178.65 mm^3 or 44.18 cm^3 **b** 679 coins

9 The cylinder has size and capacity, it's just small and rounds to 0 when measured in litres. Saying that its capacity is 0 L is misleading.

10 Second glass holds 55.37 mL more.

11 6.77 mL

12 In the calculation for the volume of a cylinder the radius is squared, so if the radius is doubled, the volume will be multiplied by 4 ($2^2 = 4$). The height is only used once in the calculation for the volume, so if the height is doubled, the volume will also be doubled.

13 **a** 348 cm^3 **b** 329.87 cm^3 **c** 730 cm^3

14 451.26 cm^3 or 451.26 mL

15 **a** 385.65 m^3 **b** 66.74 mm^3

EX p216

5F Errors

1 **a** 3 **b** $\frac{2}{5}$ **c** 8

d 4.2 **e** 0 **f** $\sqrt{2}$

2 **a** 2 **b** 2 **c** 1.4

d 3.4 **e** 0.068 **f** 0.068

3 **a** 1.65 **b** 2.325

4 **a** 12 **b** 8

5 **a** \$1000 **b** 0.15625 **c** 15.625%

6 **a** 20 **b** 0.125 **c** 12.5%

7 **a** 0.0162 **b** 1.62%

8 **a** 0.05% **b** 0.05%

9 **a** lower bound: 429.5g, upper bound: 431.4g

b lower bound: 427.5, upper bound: 432.4g

10 **a** 8 g **b** 184 g **c** 4.35%

11 42.19%

12 **a** **i** 0.0402%

ii 0.0402%

iii 0.0402%

b The percentage error is the same regardless the value of the radius.

c The approximation of $\frac{22}{7}$ is more accurate, because it has a smaller percentage error.

13 **a** $\sqrt{51}$ cm

b $\sqrt{624.75}$ cm^2

c 7.1 cm

d 24.85 cm^2

e 0.0058

14 **a** 0.2 cm

b lower bound: 13.5 cm; upper bond: 14.5 cm

c lower bound error: 4.93%; upper bound error: 2.11%

15 **a** Left eye: lower bound: 2.45, upper bound: 2.54

Right eye: lower bound: 2.35, upper bound: 2.44

b Left eye: 0.0119; right eye: 0.0127

16 Paul's suggestion is correct if the smallest measuring unit on the ruler is 0.1 cm. Adding a decimal place may suggest that the measurement is more accurate than it really is.

17 **a**

	Andy	Bill	Carol	Daniel	Emily
Exact total (\$)	25.65	99.79	64.77	44.94	12.27
Cash payment (\$)	26.65	99.80	64.75	44.95	12.25

b **i** Andy: \$0, Bill: 0.01, Carol: −\$0.02, Daniel: 0.01, Emily: −\$0.02.

Average: −\$0.004

ii Andy: \$0, Bill: 0.01, Carol: \$0.02, Daniel: 0.01, Emily: \$0.02.

Average: \$0.012

c The signed error is more suitable. Positive error represents the customer paying extra and negative error represents paying less. The average error should be close to zero and the overpayment and underpayment should balance out.

Chapter 5 review

Multiple choice

1 C **2** B **3** C **4** B **5** E
6 C **7** C **8** D **9** B **10** A
11 C **12** D

Short answer

1 118 cm^2
2 167.87 cm^2
3 **a** 276 cm^2 **b** 294 cm^2
4 **a** 216 cm^3 **b** 180 cm^3
5 **a** 1187.52 cm^2 **b** 180.64 cm^2
6 **a** 3078.76 cm^3 **b** 176.71 cm^3
7 **a** 4000 kL or 4 ML **b** 13 m
8 **a** 55.05 cm and 54.95 cm
b 32.75 mm and 32.25 mm
c 30.5 g and 29.5 g
d 108.5 mL and 107.5 mL

Analysis

1 **a** 5 cm
b pink: 36 cm^2, purple: 900 cm^2
c yes
d 0.82 m^2
e 2

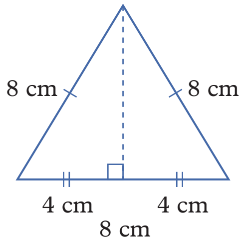
2 **a** Because all measurements are estimates. Errors exist in all measurements, especially when the instrument used is not accurate enough.
b Stack D. 0.084 mm.
c 0.3; 0.25; 0; 0.5. Stack C.
d 17.6%; 5.9%; 0%; 1.2%. Stack B has smaller absolute error than stack D, but a larger percentage error, because it has a smaller exact value. This suggests a small error could be significant when measuring relatively small quantities, but less significant when the quantity is large.

CHAPTER 6 Geometry

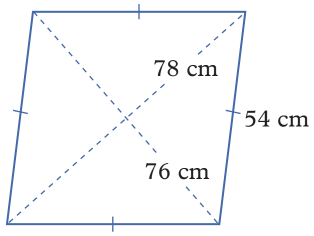
EX p226

6A Pythagoras' theorem

1 **a** right-angled **b** right-angled **c** not right-angled
d right-angled **e** not right-angled
f not right-angled
2 **c** obtuse triangle
e acute triangle
f acute triangle
3 **a** 3.6 cm **b** 7.1 mm **c** 33.8 cm
d 7.3 cm **e** 2.0 cm **f** 75.0 mm
g 8.7 cm **h** 4.2 cm **i** 8.1 cm
4 **a** $10\sqrt{5}$ cm **b** $4\sqrt{13}$ mm **c** $5\sqrt{10}$ cm
d $3\sqrt{2}$ mm **e** $2\sqrt{10}$ cm **f** $4\sqrt{17}$ mm
5 **a** 22.4 cm **b** 14.4 cm **c** 15.8 cm
d 4.2 mm **e** 6.3 cm **f** 16.5 mm
6 **a** $5\sqrt{2}$ cm **b** $20\sqrt{2}$ mm **c** $8\sqrt{2}$ cm
d $25\sqrt{2}$ cm
7 **a** 7.1 cm **b** 28.3 mm **c** 11.3 cm
d 35.4 cm
8 **a**



b **i** $4\sqrt{3}$ cm **ii** 6.9 cm
9 **a** **i** $10\sqrt{17}$ cm **ii** 41.2 cm
b **i** $(3\sqrt{11}+\sqrt{31})$ cm **ii** 15.5 cm
c **i** $(4\sqrt{14}-\sqrt{21})$ cm **ii** 10.4 cm
d **i** $(4\sqrt{21}+\sqrt{17})$ cm **ii** 22.5 cm
e **i** $8\sqrt{21}$ cm **ii** 36.7 cm
f **i** $5\sqrt{13}$ **ii** 18.0 cm
10 **a** $78^2 > 54^2 + 54^2$, so the frame is not truly square. The two sides meet at an obtuse angle.
b



11 **a** 11 cm **b** 2 folds **c** 2 folds
12 **a** (1) Afra used Pythagoras' theorem on the left right-angled triangle. (2) Afra used Pythagoras' theorem on the right right-angled triangle. Afra showed the 6 cm edge is the sum of the lengths of a and b.
b **i** $h^2 = 13 - a^2$ **ii** $b = 6 - a$
c

$$\begin{aligned} b^2 + h^2 &= 25 \\ (6-a)^2 + (13-a^2) &= 25 \\ 36 - 12a + a^2 + 13 - a^2 &= 25 \\ 49 - 12a &= 25 \end{aligned}$$

d $a = 2$ cm
e $b = 4$ cm, $h = 3$ cm
f $h = 1$ cm ($a = 2$, $b = 5$)
13 **a** All three sets of numbers (**i–iii**) satisfy Pythagoras' theorem.
b All three sets of numbers (**i–iii**) satisfy Pythagoras' theorem so all three triangles described are right-angled triangles.

c If the three numbers of a Pythagorean triad are multiplied or divided by the same number, the resulting numbers are also a Pythagorean triad.

d Each side is multiplied by the same scale factor, so the corresponding sides are in the same ratio.

14 a $x^2 + y^2$ will be the hypotenuse; yes, that is always the case because $x^2 + y^2$ will always be greater than $x^2 - y^2$ and $2xy$ (because $x^2 - 2xy + y^2 > 0$ in this case; $x^2 - 2xy + y^2 = 0$ implies $x = y$ and a side of length zero)

b Answers will vary. Any values can be chosen as long as $x > y$, for example $x = 3$, $y = 2$.

c Answers will vary; one possible answer is, if $x = 3$ and $y = 2$:

i 5 **ii** 12 **iii** 13

d Using Pythagoras' theorem to verify the numbers from the answer for part **c**, as an example: $13^2 = 5^2 + 12^2 = 169$.

e The restriction ensures that the number that results from $x^2 - y^2$ is positive.

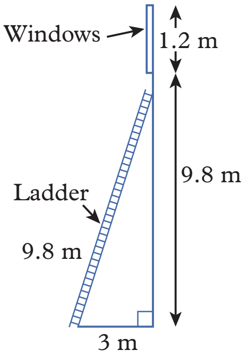
g $x = 2$, $y = 1$

15 a $5\sqrt{5}$ cm, $10\sqrt{5}$ cm

b 11.2 cm, 22.4 cm

16 The only way the length of one side of a triangle could be equal to the sum of the lengths of the other two sides would be if they all were all on the same straight line, but in that case the shape would not be a triangle, it would just be a straight line.

17 a



b 9.3 m

c The top of the window is 9.8 + 1.2 = 11 m above the ground. If Romesh stands on the very top of the ladder he is the same height above the ground as the top of the window (9.3 m + 1.7 m = 11 m). He will be able to clean the windows.

18 16.83 m

19 $x = 4\sqrt{2}$ cm, $y = 9$ cm, $z = 3\sqrt{91}$ cm

EX p232

6B Ratios, rates, direction proportion and scale

1 a 75 km/hour

b 0.2 km/minute

c 55 words/minute

d $2.50/donut

e $1.86/litre

f $5.35/day

2 26

3 $2380 USD

4 $135.20

5 7.84 m by 3.6 m

6 a 3.6 m by 2.76 m

b 4.2 m by 3.36 m

c 4.56 m by 2.64 m

d 5.04 m by 4.86 m

7 7 μm

8 a 1.5

b 40 minutes

9 a Jasmine: 8:10 am, Rajesh: 8:25 am

b 8:05 am

10 a 0.002 seconds

b 8000

11 a 5 Newtons

b 3.75 Newtons

c 24.8 cm

d Sample answer: After a certain amount of force is applied, elastic materials are permanently deformed.

12 a 0.17 m or 170 mm

b 17

c 408 cm

13 a quote 1: $420, quote 2: $390, quote 3: $380

b The cost of quotes 1 and 2 would be $0 for 0 hours, whereas quote 3 would be $80 for zero hours.

14 a As one quantity increases, the other quantity increases in the same ratio.

b 67 inches

c 170.18 cm

15 a original image: 16 : 9, resized image: 32 : 19

b no

16 a yes **b** no **c** yes **d** no

17 a yes **b** no **c** no **d** no

18 a 4 **b** −10 **c** 3.5 **d** $\frac{1}{3}$

19 a 600

b $C = 600A$

c 210 m^2

EX p237

6C Dilations and similar figures

1 a 3 **b** 1.5 **c** 7 **d** 0.5

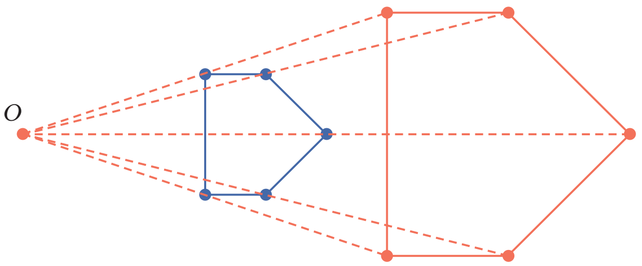
2 a $x = 6$ cm, $y = 38°$

b $m = 71°$, $n = 27$ cm

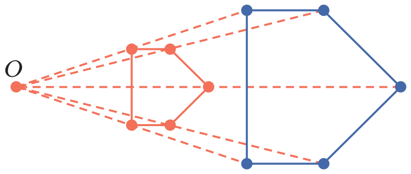
c $p = 49°$, $q = 7$ cm

d $v = 1.9$ cm, $w = 18°$

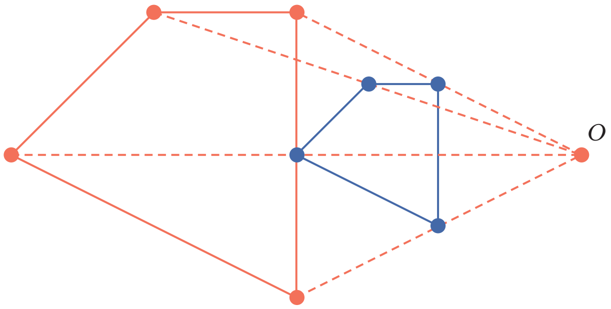
3 a i



ii



b i



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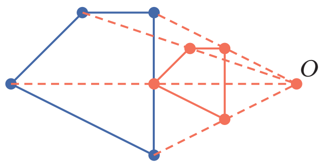
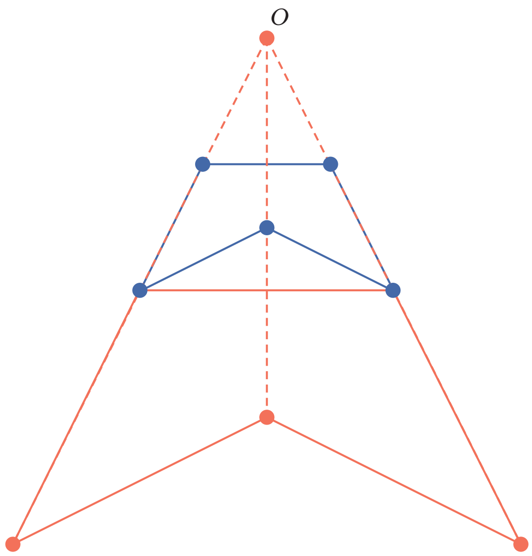
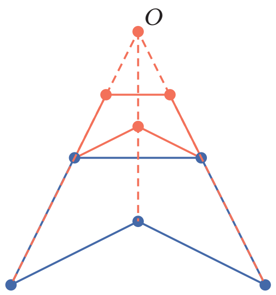


c i



ii



4 a 4 **b** 9 **c** 16

d $\frac{1}{4}$ **e** $\frac{1}{9}$ **f** $\frac{1}{16}$

5

	Original area	Dilation scale factor	New area
a	12 cm^2	2	48 cm^2
b	5 cm^2	3	45 cm^2
c	10 cm^2	5	250 cm^2
d	40 cm^2	$\frac{1}{2}$	10 cm^2
e	27 cm^2	$\frac{1}{3}$	3 cm^2
f	100 cm^2	$\frac{1}{5}$	4 cm^2

6 a 336 cm^2

b 180 cm^2

c 750 cm^2

d 12.57 cm^2

7 To increase the area by a scale factor of 9, you need to increase the length by a scale factor of $\sqrt{9} = 3$.

8 You can find the length scale factor by taking the square root of the area scale factor.

9 a 7 **b** 5 **c** 10

d 8 **e** 2 **f** 20

10 B

11 a 3

b 9

c i 4.52 mm^2

ii 40.72 mm^2

iii 36.19 mm^2

12 2

13 5 km

14 a 2 cm

b 8 cm^3

c i 64 cm^3

ii 216 cm^3

iii 512 cm^3

d The increase in volume is the cube of the increase in the side lengths.

e The length is increased in three dimensions, so the length scale factor is multiplied three times (cubed).

f i 1000 cm^3

ii 1728 cm^3

iii 8000 cm^3

15 a 240 000 cm^3

b 3750 cm^3

c 10

16 a 2 m by 2 m by 60 cm

b 8

c 2.1 m^3

17 a 0.000 006 25

b 62.5 cm by 75 cm

c 172.8 m^2

18 a 2 **b** 2

Checkpoint

1 **a** yes **b** yes

c no **d** yes

2 **a** **i** $\sqrt{41}$ cm **ii** 6.40 cm

b **i** $6\sqrt{14}$ cm **ii** 22.45 cm

3 **a** $(\sqrt{351} - \sqrt{15})$ cm

b $6\sqrt{7}$ cm

4 **a** 21.5 km/hour

b \$0.96/orange

5 £285.71

6 12.5 minutes

7 **a** $x = 20$ cm, $y = 66°$

b $p = 17.5$ cm, $q = 37°$

8 **a** 1.11 cm^2

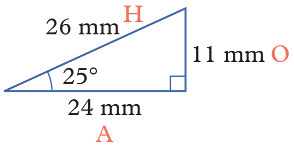
b 113.10 cm^2

9 16 200 cm^2

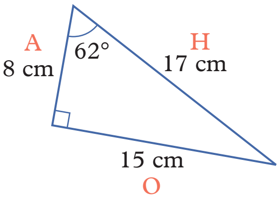
EX p244

6D Trigonometric ratios

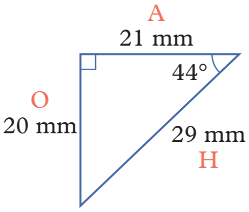
1 **a**



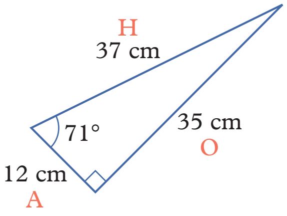
b



c



d



2 **a** $\sin(\theta) = \frac{12}{17}$ **b** $\cos(\theta) = \frac{8}{15}$

c $\tan(\theta) = \frac{24}{33} = \frac{8}{11}$ **d** $\cos(\theta) = \frac{35}{40} = \frac{7}{8}$

e $\sin(\theta) = \frac{15}{24} = \frac{3}{8}$ **f** $\tan(\theta) = \frac{13.1}{6.6} = \frac{131}{6.6}$

3 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

4 **a** $\frac{6}{7}$ **b** $\frac{6}{7}$ **c** $\frac{6}{7}$ **d** $\frac{6}{7}$

5 **a** $\frac{4}{3}$ **b** $\frac{4}{3}$ **c** $\frac{4}{3}$ **d** $\frac{4}{3}$

6 **a** 0.3420 **b** 0.5878 **c** 0.8090

d 0.7265 **e** 1.9626 **f** 0.4540

7 **a** 2.052 **b** 17.54 **c** 0.057 00

d 1 **e** 0.7660 **f** 0.3640

8 **a** They are similar triangles.

b **i, iii, iv, v**

9 **a** **i** $\sin(\theta) = \frac{\text{O}}{\text{H}}, \cos(\theta) = \frac{\text{A}}{\text{H}}, \tan(\theta) = \frac{\text{O}}{\text{A}}$

ii $\sin(\theta) = \frac{\text{O}}{\text{H}}, \cos(\theta) = \frac{\text{A}}{\text{H}}, \tan(\theta) = \frac{\text{O}}{\text{A}}$

iii $\sin(\theta) = \frac{\text{O}}{\text{H}}, \cos(\theta) = \frac{\text{A}}{\text{H}}, \tan(\theta) = \frac{\text{O}}{\text{A}}$

b The values of the sine, cosine and tangent ratios in similar triangles are equal.

10 **a** 100 m **b** 148 m **c** 6.7 m **d** 5 m

11 **a**

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\frac{1}{\tan(\theta)}$
1	0.0175	0.9998	0.0175	57.2900
5	0.0872	0.9962	0.0875	11.4301
15	0.2588	0.9659	0.2679	3.7321
30	0.5000	0.8660	0.5774	1.7321
45	0.7071	0.7071	1.0000	1.0000
60	0.8660	0.5000	1.7321	0.5774
75	0.9659	0.2588	3.7321	0.2679
85	0.9962	0.0872	11.4301	0.0875
89	0.9998	0.0175	57.2900	0.0175

b Sine and tangent increase, and cosine and reciprocal tangent decrease.

c **i** $\sin(27°) = \cos(63°)$

ii $\sin(34°) = \cos(56°)$

iii $\tan(12°) = \frac{1}{\tan(78°)}$

iv $\tan(43°) = \frac{1}{\tan(47°)}$

d The cosine of an angle is the sine of the complementary angle. The sine of an angle is the cosine of the complementary angle.

e The reciprocal of the tangent of an angle is the tangent of the complementary angle.

12 **a** $a = 1, b = 1, c = \sqrt{2}, x = 45°, y = 90°, z = 45°$

b $\sin(45°) = \frac{1}{\sqrt{2}}, \cos(45°) = \frac{1}{\sqrt{2}}, \tan(45°) = 1$

c $a = 1, b = \sqrt{3}, c = 2, x = 30°, y = 90°, z = 60°$

d $\sin(30°) = \frac{1}{2}, \cos(30°) = \frac{\sqrt{3}}{2}, \tan(30°) = \frac{1}{\sqrt{3}}$

$\sin(60°) = \frac{\sqrt{3}}{2}, \cos(60°) = \frac{1}{2}, \tan(60°) = \sqrt{3}$

e **i** $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{3}$ **iv** $\frac{3}{4}$

v $\frac{1}{4}$ **vi** $\frac{1}{4}$ **vii** 1 **viii** $\frac{1}{2}$

f Determine the square of the value of sine/cosine/tangent of 30°/45°/60°, then write the square root of the squared value.

13 a The values of sine and cosine of θ for $0° < \theta < 90°$ are the ratio of a shorter side to the longer hypotenuse. This means all values of sine and cosine can be considered as a smaller number divided by a larger number, which results in a value less than one.

b Tangent is the ratio of the two shorter sides. The two shorter sides have no restriction on which is longer or shorter.

c The tangent ratio is greater than one when the opposite side is longer than the adjacent side, which occurs when $45° < \theta < 90°$.

14 a

θ	$\tan(\theta)$	$\dfrac{\sin(\theta)}{\cos(\theta)}$
1	0.0175	0.0175
5	0.0875	0.0875
15	0.2679	0.2679
30	0.5774	0.5774
45	1	1
60	1.7321	1.7321
75	3.7321	3.7321
85	11.4301	11.4301
89	57.2900	57.2900

b **i** $\dfrac{\sin(27°)}{\cos(27°)} = \tan(27°)$

ii $\dfrac{\sin(56°)}{\cos(56°)} = \tan(56°)$

iii $\dfrac{\sin(12°)}{\cos(12°)} = \tan(12°)$

iv $\dfrac{\sin(47°)}{\cos(47°)} = \tan(47°)$

c $\dfrac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$

15 a **i** $a = 6.40$ m **ii** $b = 6.32$ m

b **i** $\sin(\theta) = 0.6247, \cos(\theta) = 0.7809, \tan(\theta) = 0.8$

ii $\sin(\theta) = 0.4286, \cos(\theta) = 0.9035, \tan(\theta) = 0.4743$

16 a **i** $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

b **i** $\frac{1}{2}$ **ii** $\frac{5}{3}$ **iii** $\frac{6}{5}$ **iv** 7

c The tangent of the angle from the horizontal is equal to the gradient of the hypotenuse.

17 a $x = \cos(31°), y = \sin(31°)$

b $x = \cos(54°), y = \sin(54°)$

c **i** $a = 6.857, b = 4.120$

ii $a = 0.5143, b = 0.3090$

18 a $y = \tan(31°), \frac{1}{z} = \cos(31°)$ or $z = \dfrac{1}{\cos(31°)}$

b **i** $a = 6.009, b = 11.67$

ii $a = 1.863, b = 3.617$

c $\frac{1}{x} = \tan(31°)$ or $x = \dfrac{1}{\tan(31°)}, \frac{1}{z} = \sin(31°)$ or $z = \dfrac{1}{\sin(31°)}$

d **i** $a = 8.321, b = 9.708$

ii $a = 12.32, b = 14.37$

19 a The person has not written the angle, θ. $\cos(\theta) = \frac{10}{11}$

b The person has written the reciprocal of the sine ratio. So, the fraction is upside down.

$$\sin(\theta) = \frac{4}{8} \text{ or } \frac{1}{\sin(\theta)} = \frac{8}{4}$$
$$\sin(\theta) = \frac{1}{2} \quad \frac{1}{\sin(\theta)} = 2$$

c The person has written the cosine ratio instead of the tangent ratio. $\tan(\theta) = \frac{6}{5}$

d The person has written the reciprocal of the tangent ratio. So, the fraction is upside down. $\tan(\theta) = \frac{4}{7}$

20 a **i** $\tan(27°) = \frac{1}{2}$ **ii** $\tan(117°) = -2$

iii $\tan(194°) = \frac{1}{4}$ **iv** $\tan(326°) = -\frac{2}{3}$

b **i** 1 **ii** -1 **iii** 1 **iv** -1

c All four angles (45°, 135°, 225°, and 315°) are 45° rotations from the horizontal, so the gradient of the corresponding line will either be $+1$ or -1 for a line inclined at a 45° angle depending on which way it is rotated.

d **i** The gradient of a horizontal line is zero. Therefore, the tangent of 0° is zero.

ii The gradient of a vertical line is undefined. Therefore, the tangent of 90° is undefined.

21 a **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $-\frac{1}{2}$ **iv** $-\frac{1}{2}$

v $\frac{1}{2}$ **vi** $-\frac{1}{2}$ **vii** $-\frac{1}{2}$ **viii** $\frac{1}{2}$

b **i** sine: positive, cosine: negative

ii sine: negative, cosine: positive

iii sine: negative, cosine: negative

EX p254

6E Using trigonometry to find lengths

1 a $\cos(\theta) = \frac{A}{H}, \cos(64°) = \frac{k}{35}$

b $\sin(\theta) = \frac{O}{H}, \sin(49°) = \frac{k}{12}$

c $\tan(\theta) = \frac{O}{A}, \tan(26°) = \frac{k}{50}$

d $\cos(\theta) = \frac{A}{H}, \cos(51°) = \frac{k}{17}$

e $\sin(\theta) = \frac{O}{H}, \sin(37°) = \frac{k}{24}$

f $\tan(\theta) = \frac{O}{A}, \tan(21°) = \frac{k}{16}$

2 a $x = 2.11$ **b** $x = 11.83$

c $x = 2.11$ **d** $x = 11.83$

e $x = 6.62$ **f** $x = 18.28$

g $x = 3.89$ **h** $x = 3.89$

3 a 15.34 mm **b** 9.06 m **c** 24.39 cm

d 10.70 m **e** 14.44 mm **f** 6.14 m

4 a 31.84 cm **b** 25.86 mm **c** 2.78 m

d 22.65 cm **e** 11.11 m **f** 49.32 mm

g 61.80 cm **h** 129.35 m **i** 5.26 cm

5 a 118.31 cm **b** 3.72 m **c** 38.92 mm

d 11.02 cm **e** 46.27 cm **f** 4.72 m

6 **a** 4.44 cm **b** 8.31 cm **c** 10.39 m
d 78.15 cm

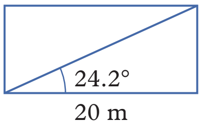
7 **a** 33.64 m **b** 34.64 m

8 4.62 m

9 **a** 9.60 m **b** 16.00 m

10 **a** 1.93 m **b** 2.13 m

11 **a**



b 9 m

c 58 m

12 8.60 m

13 **a** One angle size (other than the right angle) and one side length

b Two side lengths

14 **a** $a = 17.2$ cm **b** $b = 42.2$ mm **c** $c = 52.9$ m
d $d = 32.2$ cm **e** $e = 29.9$ mm **f** $f = 52.0$ cm

15 **a** $a = 3.28$ cm, $b = 3.77$ cm, $x = 90°$, $y = 41°$, $z = 49°$

b $g = 27°$, $h = 63°$, $m = 2.94$ m, $n = 3.30$ m

16 **a** $f = 4.01$ cm, $g = 7.04$ cm

b $a = 4.00$ cm, $b = 7.21$ cm

c $m = 17.69$ cm, $n = 12.00$ cm

17 $r = 8.81$ cm and $t = 10.88$ cm

18 $p = 35.96$ cm, $q = 11.96$ cm

19 **a** $x = 5.01$ cm **b** $a = 7.09$ cm

20 **a** $x = 2.78$ cm **b** $y = 13.22$ cm
c $z = 8.56$ cm **d** $b = 15.75$ cm

EX
p261

6F Using trigonometry to find angles

1 **a** 13° **b** 44° **c** 56° **d** 72°
e 42° **f** 16° **g** 74° **h** 50°

2 **a** 20° **b** 36° **c** 33° **d** 25°
e 8° **f** 1° **g** 72° **h** 45°

3 **a** 53° **b** 38° **c** 21° **d** 62°
e 69° **f** 23° **g** 57° **h** 83°

4 **a** 50° **b** 42° **c** 67° **d** 54° **e** 31° **f** 46°

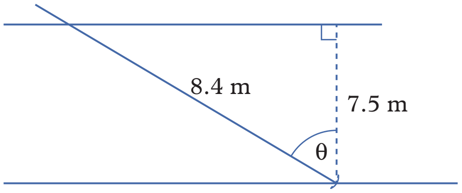
5 **a** $\theta = 138.6°$ **b** $\theta = 13.6°$
c $\theta = 271.1°$ **d** $\theta = 25.8°$

6 **a** 36.9°, 53.1° **b** 26.4°, 63.6°
c 72.9°, 17.1° **d** 25.7°, 64.3°

7 **a** 14° **b** 7.01 m

8 **a** 47° **b** 86°

9 **a**



b 27°

c Larger. If the chain is longer, the angle with the vertical would be greater. If the chain is shorter, the angle is closer to the vertical, hence making a smaller angle size with the vertical.

10 **a** **i** 37° **ii** 37° **iii** 37°

b $\theta = 37°$

c **i** Using trigonometry can be accurate to a required degree of accuracy but is difficult to perform without a calculator.

ii It is an advantage to use a scale diagram to check the results from trigonometric calculations, but it is difficult to record measurements with sufficient accuracy.

11 **a** First, calculate the angle of the Sun's rays at the time of measurement by using the length of a metre ruler's shadow. Then use the length of the tree's shadow as a measurement to calculate the height of the tree.

b 37°

c It refers to the angle of the Sun's rays at the time of measurement, both measurements being taken at the same time.

d 11.71 m

e By forming an equivalent ratio statement using the corresponding sides.

x : 1 m = 15.46 m : 1.32 m

$$x = \frac{1 \times 15.46}{1.32}$$

= 11.71 m

f You could draw the diagram to scale.

12 **a** **i** 28°, 62° **ii** 28°, 62° **iii** 28°, 62°

b The two non-right angles are complementary, so $90° - \theta$.

13 **a** $\theta = 123.7°$ **b** $\theta = 110.4°$ **c** $\theta = 78.6°$

14 **a** 30.0 m, 18.4°, 71.6°

b 20.1 m, 42.6°, 47.4°

15 The corresponding angles in the bottom left of each triangle are approximately 64.707° and 64.652°, so the lines are very nearly parallel.

16 1548.4 cm^2

17 $\theta = 38.4°$

Chapter 6 review

Multiple choice

1 E **2** C **3** A **4** B **5** D
6 B **7** D **8** A **9** A **10** B
11 C

Short answer

1 **a** $a^2 + b^2 = 30^2 + 40^2 = 2500$, $c^2 = 50^2 = 2500$

b Yes, as they are three whole numbers that satisfy Pythagoras' theorem.

2 **a** 13 cm **b** 18.41 cm

3 $x = 19.90$ cm, $y = 15.13$ cm

4 **a** \$1.80/avocado

b 85 beats/minute

c 84 km/hour

5 **a** 2.2 Newtons

b 11.16 cm

6 **a** 1590.43 cm^2

b 15.33 cm^2

7 **a** 1260 mm^3

b 45 mm by 31.5 mm by 24 mm

c 34 020 mm^3

8 $\sin(\theta) = \frac{55}{73}$, $\cos(\theta) = \frac{48}{73}$, $\tan(\theta) = \frac{55}{48}$

9 **a** 5.44 cm **b** 19.43 m

c 28.31 m **d** 4.15 cm

10 **a** 35° **b** 60°

c $\theta = 68°$ **d** $\theta = 29°$

11 $\theta = 47.5°$

Analysis

1 **a** 9.3 cm

b 4.7 cm

c The ratio of the side lengths is directly proportional.

d $\frac{1}{2}$

e $\frac{1}{4}$

f 5 : 3

g 630

h **i** 15 120 cm^2 **ii** 9072 cm^2

i \$27.22

2 **a** 74.8 m **b** 599 m **c** 129.6 m

d 33.75°, 53.19° **e** 19.44°

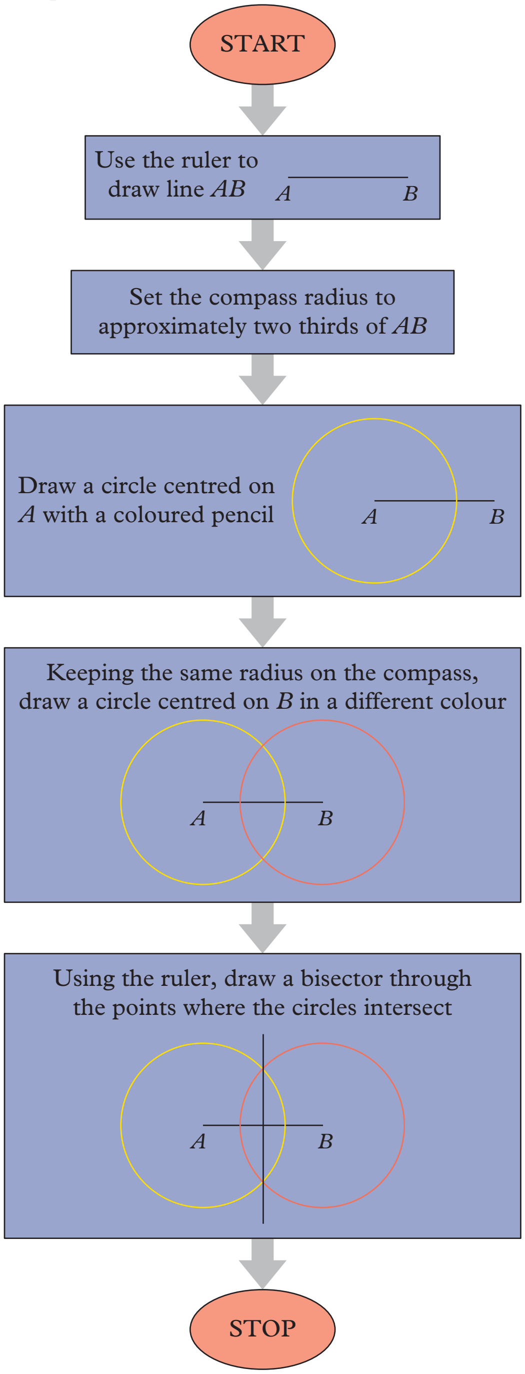
EX p270

Algorithmic Thinking 2 Measurement and Space

Task 1

1 Teacher to check.

2 Sample answer:



START

Use the ruler to draw line AB

Set the compass radius to approximately two thirds of AB

Draw a circle centred on A with a coloured pencil

Keeping the same radius on the compass, draw a circle centred on B in a different colour

Using the ruler, draw a bisector through the points where the circles intersect

STOP

Task 2

1 Teacher to check.

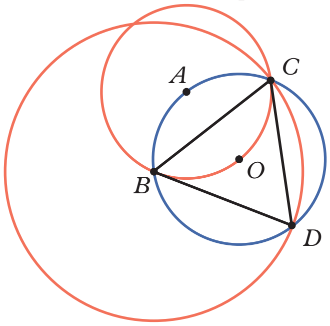
2 Sample answer:

1. Label a point O.
2. Using a compass, draw a circle of any radius centred on point O.
3. Label a point A anywhere on the circumference of the circle.
4. Using the ruler, draw a straight line connecting OA extending to the diameter of the circle.
5. Label the other end of the diameter B.

6 Construct a perpendicular bisector to *AB* through the centre point *O*. Label the points intersecting with the circumference *C* and *D*.

7 Using a ruler, draw the straight lines *AC*, *CB*, *BD*, *DA* in the given order.

3 Three circles and an equilateral triangle.



4 Teacher to check.

Task 3

1 Teacher to check.

2 Teacher to check.

EX
p276

CHAPTER 7 Statistics

7A Surveys and sampling methods

1 **a** systematic sampling

b stratified sampling

c convenience sampling

d quota sampling

2 **a** Biased; people at an AFL football match are not a representative sample of the whole country.

b Fair; this should provide an answer representative of the views of all the people in school.

c Biased; the question is not fair and leads to only responses regarding people's attitude to dogs.

d Biased; a sample of 100 is too small to represent the population.

3 **a** There is no indication of how people were chosen for survey or exactly what they were asked.

b People shopping in an Apple store are probably Apple customers/supporters, so this is a biased sample.

c The sample is too small, and there is no indication of the gender or employment status of people making up the sample or whether it was their opinions or facts about their personal earnings that they were questioned about.

d Shareholders in a company have a vested interest in seeing that company's product succeed.

4 **a** **i** The data collection method is not fair. Tess only surveyed people from her town, when the population needing to be considered is the entire poulation of NSW.

ii The interpretation is biased because the sample is biased.

iii Tess should have surveyed people across all of NSW rather than just in her town.

b **i** The sample is too small for any results to be reliable.

ii Because the sample is so small, the median would be the most reliable result to take.

iii Survey a larger sample, and watch out for outliers.

c **i** The question only asked if the people surveyed like Mac computers, but not if they prefer them to other types of computers. This is not a fair data collection method.

ii Because the question is biased, no assumption can be made.

iii The question could have been, 'Which brand of computer do you prefer?'

d **i** The data collection method is fair.

ii The interpretation of the data is poor. The figure does not need to be above 50% for the soft drink to be the most popular. It just needs to be a greater percentage than any other soft drink.

iii She should conclude Coke is the most popular.

5 **a** Only one person claimed 18 hours pain free. Most of the claims are far less than 18 hours but, because one person said 18 hours, the claim of 'up to 18 hours' has been made.

b mean = 3.1, median = 2, mode = 2

c This claim is not too far from the truth. However, the median or the mode would be a better representation of the data than the mean.

6 How was the data collected? How big was the sample? Does the data represent a sample of the population? How was the questionnaire designed? Is the interpretation fair?

7 **a** If a phone is in stand-by mode, its battery could last up to 7 days. But, in normal use, a phone would not be in stand-by mode for 7 days!

b The 42 people surveyed could have been chosen for the survey because they were purchasing that brand of toilet paper at a supermarket. It is also a very small sample size that is not representative of all of Australia.

c This moisturising lotion was not compared with another moisturising lotion, but with soap.

8 **a** ~54%

b Sample answer: The infographic is visual and highlights the data, but it can be difficult to interpret the exact percentage portrayed in the infographic.

9 **a** To determine the most popular gelato flavour

b The popularity of five different gelato flavours

c The exact number of people who liked each gelato flavour

d Sample answer: The infographic is a good visual aid to get a quick snapshot of the results, however it lacks depth.

10 Sample answer: Both of these factors have made it more difficult to collect data from surveys over the phone and will have had an impact on the random sampling of phone surveys, as some demographics will be less likely to use landline phones and/or more likely to screen phone calls. Other surveying methods, such as online surveys, have become more prevalent during this time.

11 Teacher to check.

12 Sample answer: The operating profits of the mining industry were the most variable, with a big drop in the middle of the decade, followed by a big rise at the end of the decade. Manufacturing followed a similar pattern to mining, with a sharp initial drop in operating profit, followed by a steady rise to a new height at the end of the decade. After an initial drop in 2012–13, construction steadily increased in operating profit through the rest of the decade, while the operating of the retail industry gradually fell, and the profit of the media industry fell significantly after a slight boost in the middle of the decade.

13 **a** 8th

b Because 98 countries were assessed

c This is a sample, because data was not collected from all countries in the world, with China specifically being excluded.

d No, because other countries that were not included may have handled the coronavirus pandemic better than Australia did.

e Look up the Lowy Institute and find the data they used to rank the countries.

14 Teacher to check.

15 Teacher to check.

16 Teacher to check.

EX
p284

7B Classifying and displaying data

1 **a** D **b** A **c** B **d** C

2 **a** 1 and 3 are numerical; 2 and 4 are categorical

b height: continuous; mark: discrete; method of transport: nominal; karate belt: ordinal

3 **a** fuel consumption: numerical; car model: categorical

b number of ice cream sales: numerical; maximum daily temperature: numerical

c exam score: numerical; subject: categorical

4 **a** Temperature over a 24-hour period, measured every 2 hours

b time: continuous; temperature: continuous

c 11°C **d** 2 pm **e** 5°C, 6 am

5 **a** number of people, month, year

b July 2017

c May 2019

6 **a** number of goals, player, game number

b game 4

c game 3

d Hashani

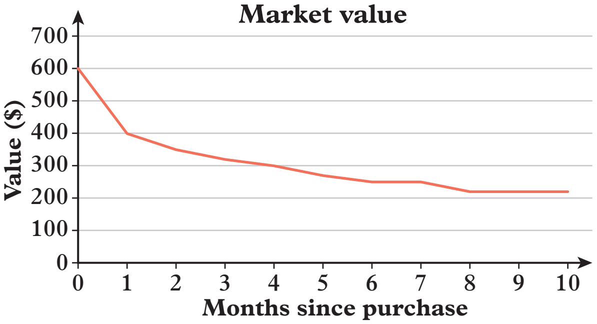
7 **a** portions of vegetables consumed, day, person

b day 2

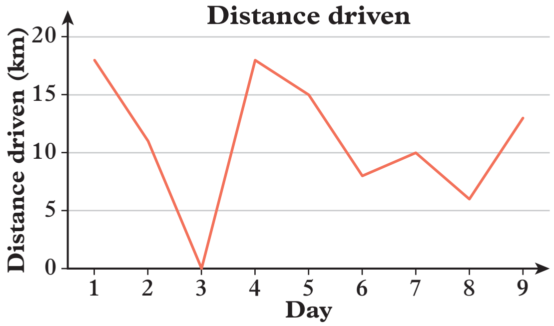
c 3

d 53

8



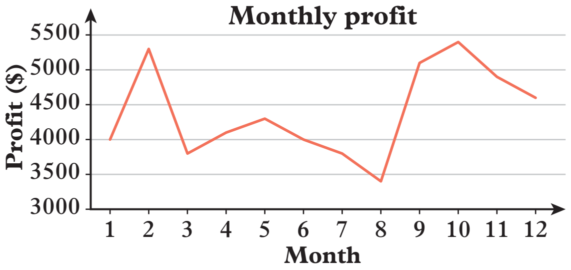
9 **a**



b day 1 and day 4

c 5

10 **a**



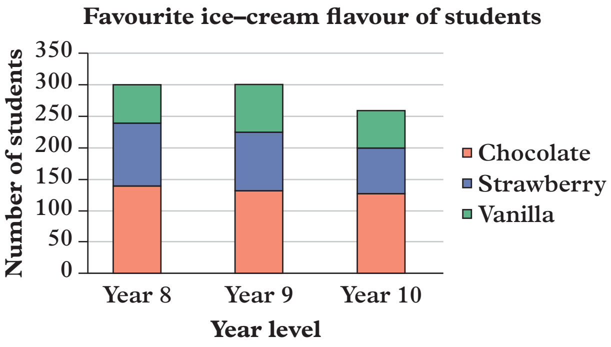
b month 10

c month 8 fit

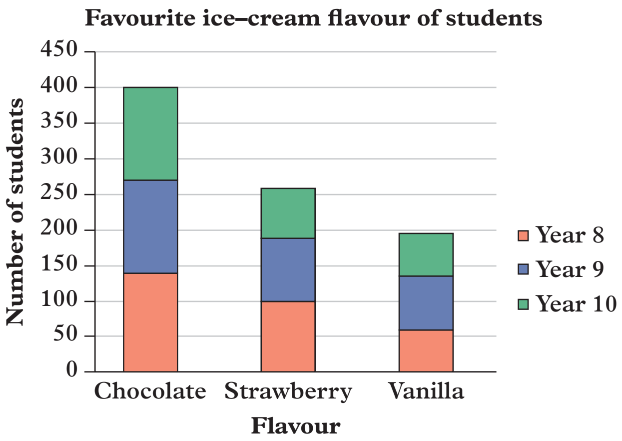
d month 9, $1700

e month 3, $1500

11 **a**



b



c The chart in part **a** with the year level on the horizontal axis

d No. Although slightly more Year 9 students preferred chocolate ice-cream than Year 10 students, there were more Year 9 students in the data.

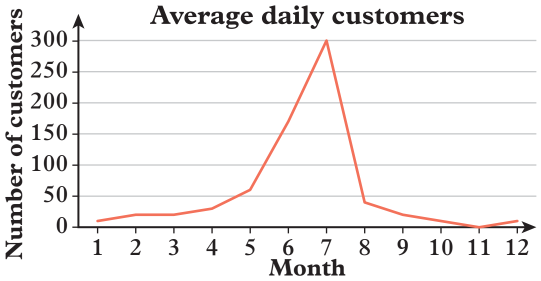
12 **a** 20

b Year 2

c Students can regularly do both: watch films on a streaming service and go to the cinema.

d Sample answer: The columns will stay similar to how they are in Year 5, with a majority of students watching films regularly on streaming services, and some students regularly going to the cinema.

13 **a**



b Month 7. It is the most popular month, and it is followed by a large drop off, which makes sense because January would not be busy.

c No. The data is rounded to the nearest 10 customers and it is the average number of daily customers for the month, not the total number of customers in the month. The average could have been just below 5 customers per day, which would be around 150 customers total for the month.

14 **a** Darwin **b** Adelaide

c Adelaide **d** Melbourne

15 **a** Osaka

b Shanghai

c 16 million

d Shanghai

e 2008

f Sample answer: The population of Osaka stayed relatively level throughout the 30-year period, whereas the population of Shanghai increased quickly, and the population of Mumbai increased slowly.

16 **a** Joey

b Ram

c 58

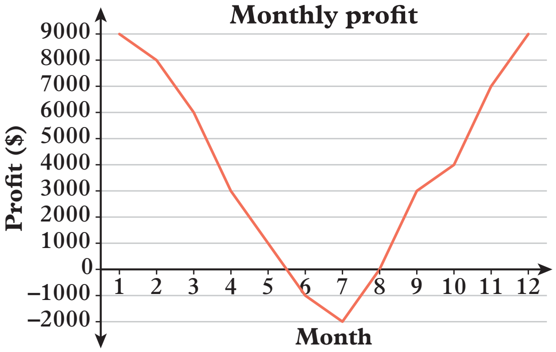
d Ram. He consistently ate either 3 or 4 hotdogs per minute.

17 **a** Area chart; it is useful to visualise the total number of steps taken in the challenge.

b Stacked bar chart; there are two categorical variables, and the favourite topping overall can be easily seen.

c Multiple line graph; the heights can be viewed as progressing over time.

18 **a**



b Negative profit means the store lost money.

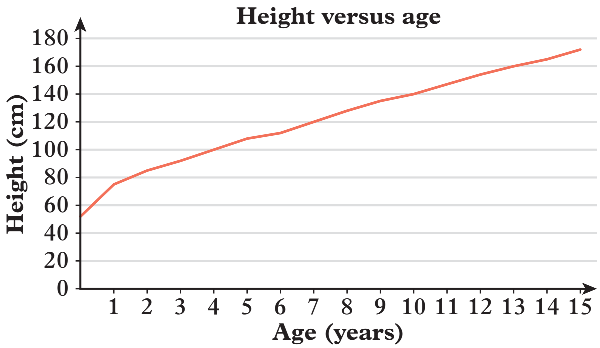
c $0 profit means the store broke even (did not make money or lose money).

d $47 000

e No. The profit is most likely associated with the time of year so it will be back down again in the middle of next year.

f Summer in the northern hemisphere occurs in the middle of the year so the months with the highest profit would be the months in the middle of the year, unlike the Geelong store, which has the highest profits at the start and end of the year.

19



b 8 cm

c 252 cm

d 185 cm (answers will vary)

e Continue increasing until the age of 18 or 19, then remain constant.

EX
p291

7C Grouped data and histograms

1 **a** 5 **b** 16

c 15–<20, 18 **d** 25–<30

e 14 **f** 19

g 4.5%

2 **a** 6 **b** 0–<20, 4 **c** 69

3 **a** 145 **b** 183 **c** 38

d

Class	Frequency
145–<150	1
150–<155	3
155–<160	5
160–<165	3
165–<170	6
170–<175	6
175–<180	0
180–<185	1
Total	25

4 **a**

Class	Frequency
0–<5	1
5–<10	2
10–<15	3
15–<20	3
20–<25	2
25–<30	3
30–<35	3
35–<40	1

b

Class	Frequency
10–<20	3
20–<30	3
30–<40	4
40–<50	4
50–<60	4
60–<70	2
70–<80	2

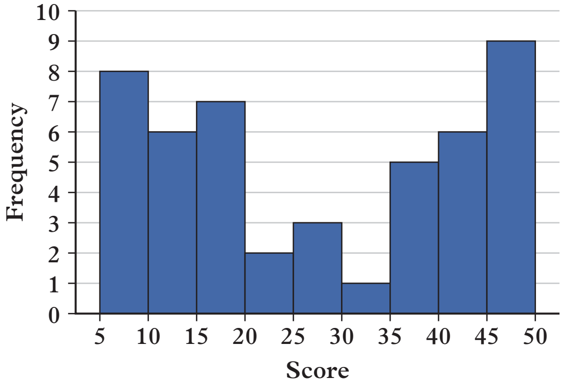
c

Class	Frequency
0–<2	6
2–<4	7
4–<6	6
6–<8	2
8–<10	2
10–<12	4

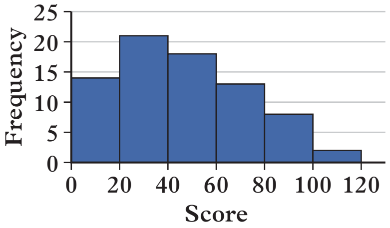
d

Class	Frequency
40–<45	3
45–<50	8
50–<55	7
55–<60	5
60–<65	3
65–<70	3
70–<75	4
75–<80	7

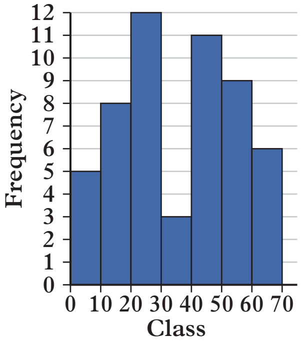
5 **a**



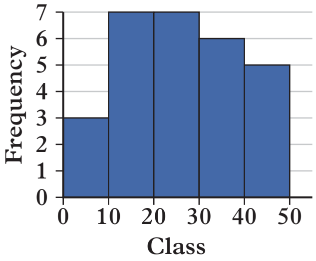
b



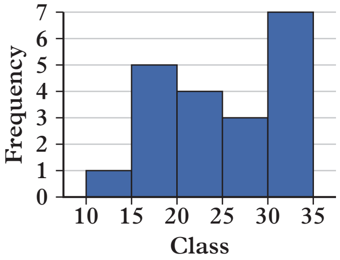
c



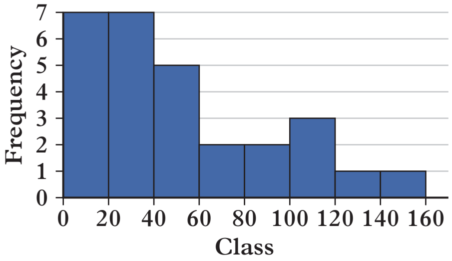
6 **a**



b



c



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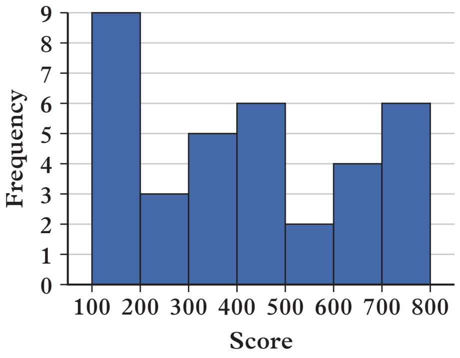
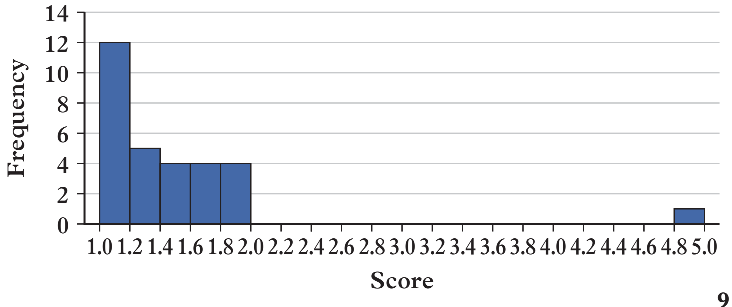
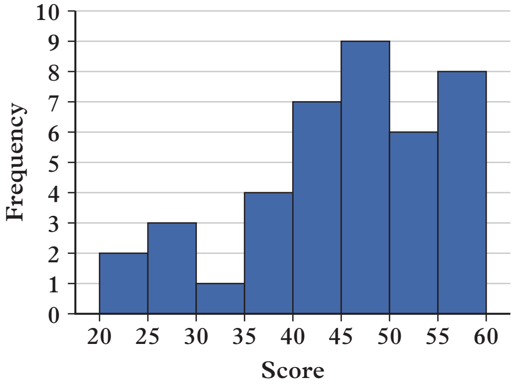


e



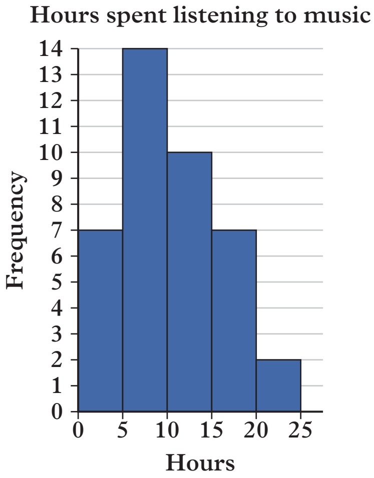
f



7 a

Hours	Frequency
0–<5	7
5–<10	14
10–<15	10
15–<20	7
20–<25	2

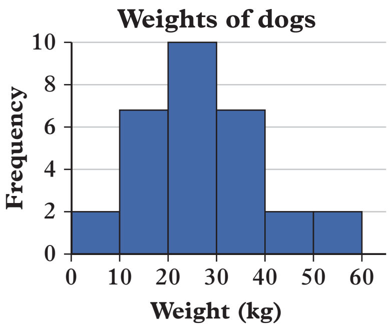
b


Hours spent listening to music

8 a

Class	Frequency
0–<5	2
5–<10	7
10–<15	10
15–<20	7
20–<25	2
25–<30	2

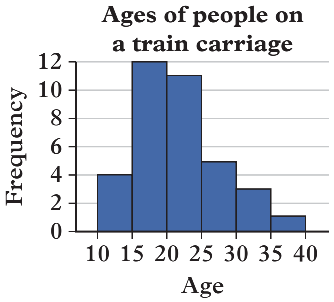
b


Weights of dogs

9 a

Ages of people on a train	Frequency
10–<15	4
15–<20	12
20–<25	11
25–<30	5
30–<35	3
35–<40	1

b


Ages of people on a train carriage

10 a number of Facebook friends

b 50 **c** 350–399; 28

d 100–149

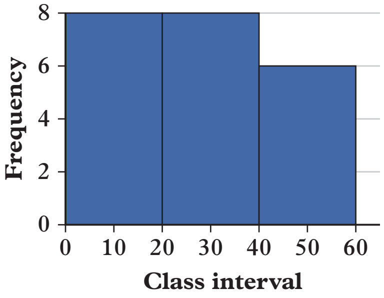
11 a There are too many class intervals, which makes the table long and difficult to read.

b

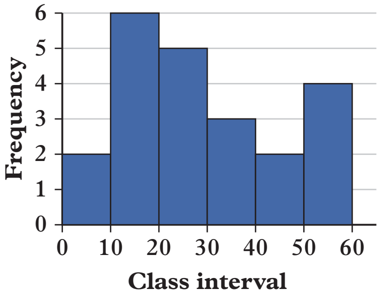
Ages of customers	Frequency
0–<10	2
10–<20	61
20–<30	44
30–<40	20
40–<50	6
≥50	6

12 Without the raw data it is not possible to determine the number in each of the smaller class intervals.

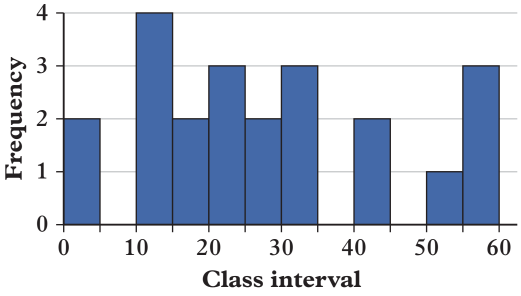
13 a



b

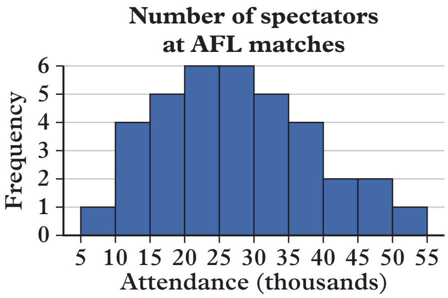


c

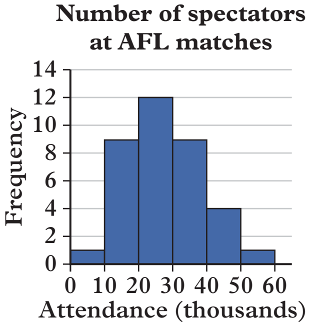


d Sample answer: The histogram in part **b** is most appropriate. The intervals in part **a** are too wide to give an accurate representation of the spread of the data. Both parts **b** and **c** display the data effectively, but the histogram in part **b** looks the neatest as it does not have intervals with frequency of 0 like part **c** does.

14 a


Number of spectators at AFL matches

b

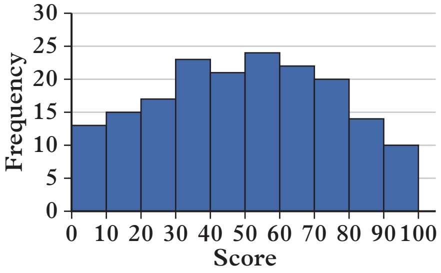

Number of spectators at AFL matches

c Similar shape for each histogram (roughly symmetrical). Using smaller class intervals has resulted in smaller frequency values. That is, largest frequency is 6 compared with 12.

15 a 5

b There appears to be no pattern. Too many intervals make it harder to read.

c



d The distribution is symmetrical, with lower frequencies at either end and higher frequencies in the middle.

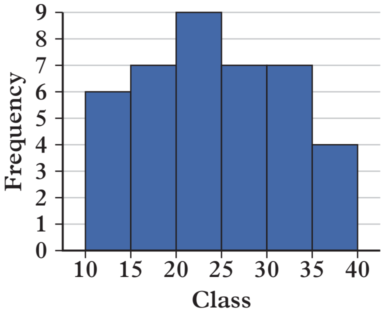
16 a The stems represent the class intervals.

b 10; e.g. 10–19, 20–29 etc.

c 17

d In a stem-and-leaf plot, each individual piece of data can be seen.

17



18 Histograms do not provide specific data within each class interval; they only provide the frequency of data for each class interval.

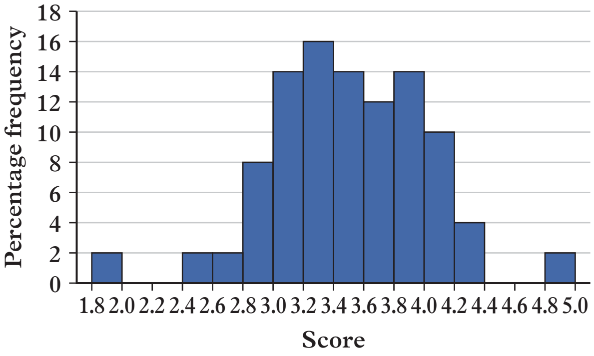
19 a The vertical axis represents percentage frequency rather than frequency.

b 16% **c** 14%

d 100%; because percentages of a whole add to 100%

e i 56 **ii** 196 **iii** 248

20



Checkpoint

1 **a** quota sampling

b convenience sampling

c stratified sampling

d systematic sampling

2 **a** **i** The sample only consisted of people walking through the mall, so this could lead to biased results (especially when the survey is about time spent sitting down). It's also not clear who her population is.

ii A poor method of collection means the interpretation could be biased.

iii Sienna should have specified her population and sampled people in various locations for her survey to be representative of the population. It might also be best to use the median as the measure of centre for this data.

b **i** Kane's survey method is biased because his neighbourhood contains only a small section of the target population and also includes people who are not part of the target population.

ii The interpretation is not fair because the sample is biased.

iii Kane should have surveyed a sample of people who will be attending the school dance.

3 **a** Biased; people outside your community would probably not have any appreciation of particular community issues, so their views could skew the results.

b Biased; a small sample of one section of the population is not likely to be representative of the entire population.

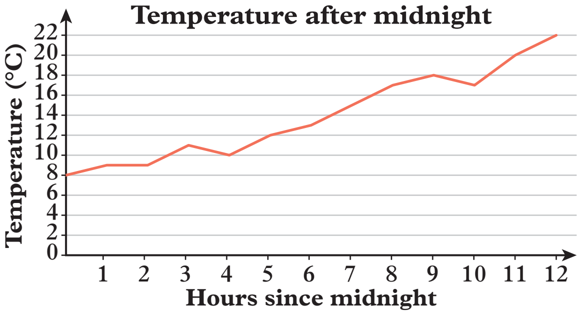
4 **a** numerical

b categorical

c categorical

d numerical

5



6 **a** pop

b Australia

c pop

7 **a** kangaroos

b 2015

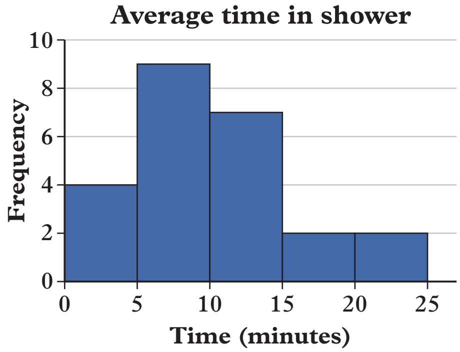
c between 2017 and 2018

8 **a** 90–99 **b** 6 **c** 15

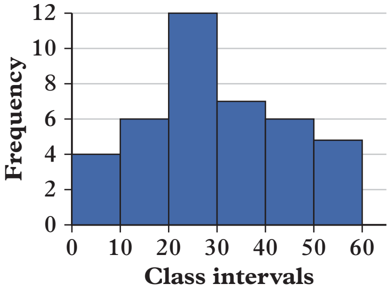
9 **a**

Class	Frequency
0–<5	4
5–<10	9
10–<15	7
15–<20	2
20–<25	2

b



10



EX
p302

7D Summary statistics from tables and displays

1 **a** **i** 2.36 **ii** 2 **iii** 2 **iv** 4

b **i** 21.67 **ii** 20 **iii** 10, 30 **iv** 30

c **i** 15.88 **ii** 16 **iii** 17 **iv** 5

d **i** 1.12 **ii** 1 **iii** 1 **iv** 4

e **i** 24.33 **ii** 25 **iii** 30 **iv** 15

f **i** 3.09 **ii** 2.5 **iii** 2 **iv** 18

2 **a** 30 people **b** mode = 5, range = 9

c

Hours	Frequency
1	3
2	4
3	2
4	6
5	8
6	4
7	2
8	0
9	0
10	1
Total	30

d mean = 4.3, median = 4.5

3 **a** mean = 4.7

b median = 5

c mode = 4

d range = 5

4 **a** four bedrooms **b** 4

c

Number of bedrooms	Frequency
1	2
2	9
3	11
4	14
5	6
Total	42

d mean = 3.31, median = 3

5 **a** mean = 2.3, mode = 2, median = 2, range = 5

b mean = 16.15, mode = 16, median = 16, range = 6

c mean = 1.33, mode = 1, median = 1, range = 4

6 **a** mean = 55.43, median = 59, mode = 42, range = 42

b mean = 2.70, median = 2.3, mode = 1.8, range = 4

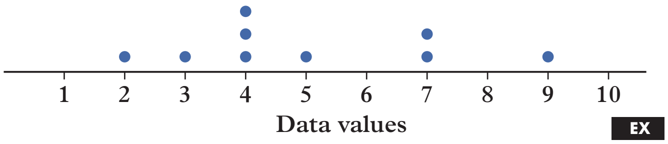
c mean = 280.38, median = 250, mode = 310, range = 670

7 mean = 4.21, median = 4, mode = 3, range = 9

8 Categorical data does not contain any numerical scores. You can only find the most popular/common category that is the mode.

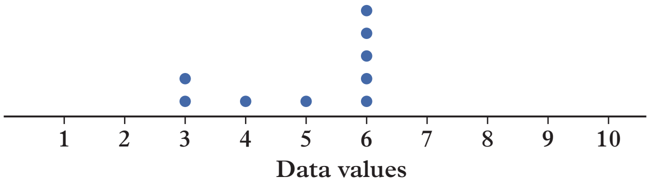
9 **a** B **b** A **c** C

10 **a** Answers will vary.



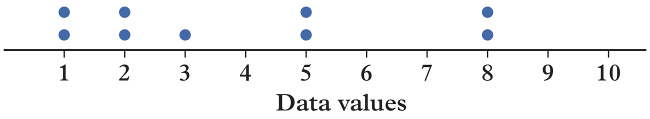
The lowest value must be 2, the highest value must be 9, and the other dots can be anywhere from 2 to 9.

b Answers will vary.



There must be at least five dots on 6 and all other dots must be below 6.

c Answers will vary.



There must be four values with exactly two dots each and one value with exactly one dot.

11 **a** 17 (unique)

b 9 (unique)

c Any number greater than or equal to 13 (not unique)

d 10 (unique)

e 9 or 17 (not unique)

f Any multiple of 9 (not unique)

12 **a** 16th row, 7th column

b 8th row, 19th column

13 **a** 49, because the highest possible value is 69 and the lowest possible value is 20

b 31, because the minimum value of the highest number is 60 and the maximum value of the lowest number is 29.

c The raw data is not available.

d 40–<49

e It is not possible to calculate the mean because the raw data is not available. It is not possible to know which interval contains the mean since, unlike the median, the mean is not found based on position.

14 **a** 39.6 **b** 48.6

c 44.1. Yes, it is reasonable to assume that the values in each interval are evenly spread out rather than all clustered at the top or all clustered at the bottom.

15 **a** 82.4% **b** 62.4% **c** 88%

16 **a** **i** 2.56 **ii** 26.37 **iii** 4.73

b The second set of data has a large spread of data from the mean (standard deviation = 26.37), but the other two data sets have a small spread of data from the mean (standard deviations = 2.56 and 4.73 respectively).

EX
p308

7E Describing data

1 **a** bi-modal **b** negatively skewed

c positively skewed with an outlier

d symmetric

2 **a** symmetric **b** positively skewed

c negatively skewed with an outlier

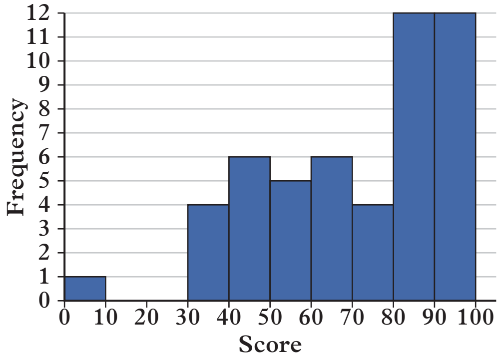
3 **a** mean or median **b** median

c mode of each peak **d** median

4 **a** mode of each peak **b** median

c median **d** mean or median

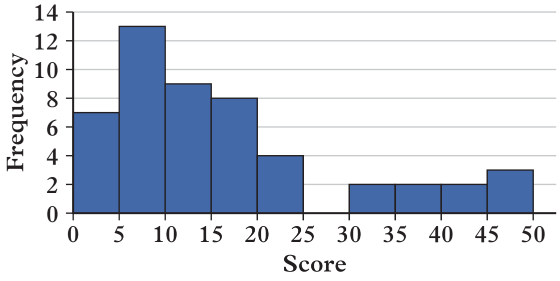
5 **a** **i**



ii negatively skewed

iii Median. The graph is skewed and there is an outlier present, therefore the mean may not be an accurate measure of centre.

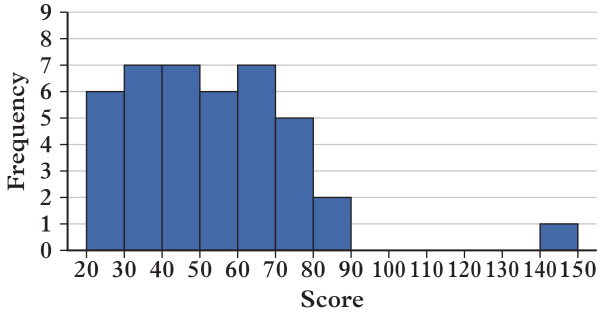
b **i**



ii positively skewed

iii Median. The graph is skewed, therefore the mean may not be an accurate measure of centre.

c **i**



ii symmetric or positively skewed with an outlier

iii Median. An outlier is present, therefore the mean may not be an accurate measure of centre.

6 **a** **i**

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 3 3
1*	5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3
2*	9

Key: 1 | 3 = 13

ii

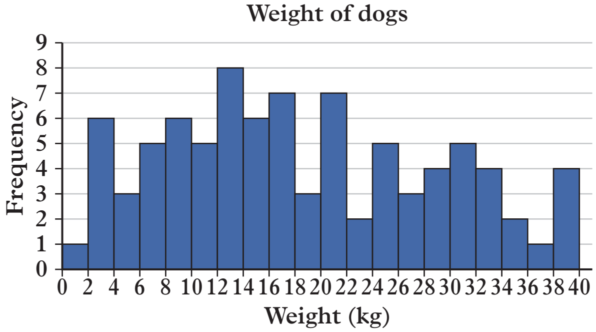
Stem	Leaf
1	0 0 0 1
1*	2 2 2 2 2 3 3
1^	5 5 5 5 5
1#	6 6 7
1+	8 9
2	0 0 1
2*	2 2 2 3 3 3
2^	
2#	
2+	9

Key: 1 | 3 = 13

b **i** positively skewed

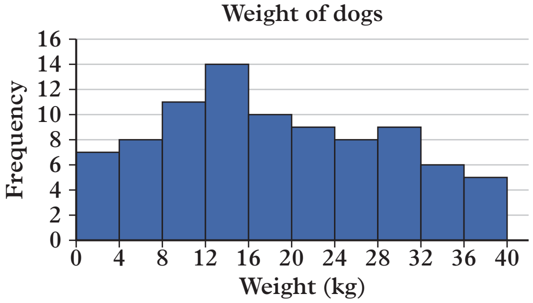
ii bi-modal with an outlier

7 **a**



b no pattern visible

c



d very slightly positively skewed

e Weight of dogs ranges from below 2 kg up to 40 kg. The distribution is fairly even and roughly symmetric with a peak (modal class) around 12–16 kg.

8 **a** symmetric

b mean or median

c

Number of people	Frequency
0–<10	3
10–<20	4
20–<30	9
30–<40	12
40–<50	8
50–<60	6
60–<70	2
Total	44

d 30–<40

e Sample answer: The average number of people in the skate park is between 30 and 40.

9 **a** **i** symmetrical with an outlier

ii median is 35–<40

iii Ages of people at a hairdresser's in a week usually ranges from 15 to 60 years, with the median age being between 35 and 40 years.

b **i** negatively skewed

ii mean = 15.41, median = 15.5

iii Ages of students at a pool ranges from 13 to 18 years, with median age 15.5 years.

c **i** positively skewed **ii** median = 2

iii The majority of homes surveyed have one or two TVs, with the most common number of TVs in each home being two.

d **i** bi-modal

ii mode of each peak = 0–<10 cm and 30–<40 cm

iii Of the Year 9 students surveyed, most have either short hair ranging from 1 to 10 cm in length or longer hair ranging from 30 to 40 cm in length.

10 **a** There were 0 customers on Christmas day because the course was closed.

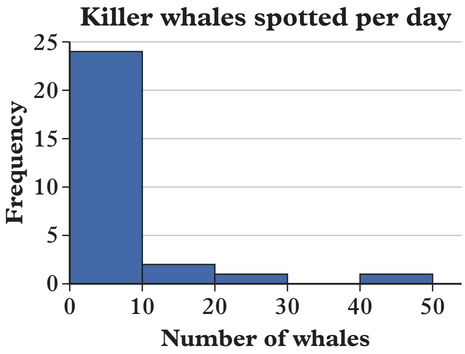
b A mini golf course is likely to have few customers during the week and do the majority of its business on Friday, Saturday, and Sunday.

c Any business that is much busier on weekends: cinemas, golf clubs, paintball, amusement parks, bars, hotel, etc.

11 **a** It could lead people to believe that customers usually see about 4 whales per tour, which is not true.

b The median number of whales spotted is 0, which would not be a statistic worth advertising.

c



d The 24 numbers in this interval are not evenly spread out. 21 of them are 0.

e 25

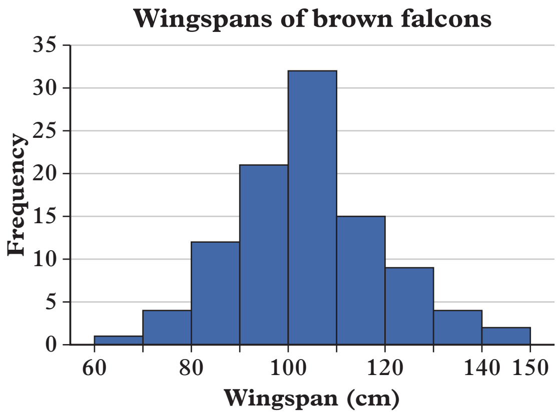
12 **a** mean: 9.4, median: 9, mode: 9

b The mode tells the owner which shoe size is most popular. The median and mean do not tell the owner relevant data for stocktaking.

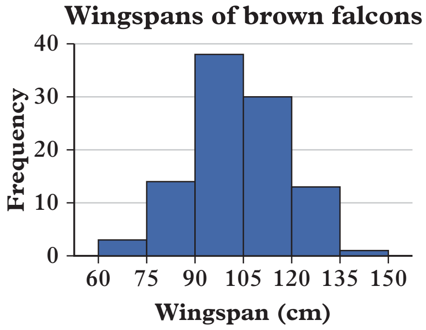
c 70%

d 94%

13 **a**



b



c The histogram with class intervals of size 10, because the smaller size creates intervals that do not follow the pattern properly and the larger size intervals make the pattern less obvious.

d 68%

e 95%

EX p315

7F Comparing data

1 **a** **i** 41 **ii** 45 **iii** 45

b **i** 18 **ii** 24 **iii** 18 and 24

c **i** positively skewed **ii** symmetrical

2 **a** Group A and Group B have a similar spread of data but are skewed in different directions. Group A is negatively skewed, whereas Group B is positively skewed. Group A has a larger centre than group B.

b Both groups A and B are symmetric, but group B has a larger range due to the presence of an outlier. The median should be used as a measure of centre for group B, which has a smaller centre than group A.

c Both groups have a symmetric distribution and have a similar centre, but Group B is spread over a larger range.

d Both groups are negatively skewed, but group A has a smaller centre than Group B. Group B has an outlier present and a larger range.

3 **a**

Leaf Friday	Stem	Leaf Saturday
9	2	6
2 0	3	7
9 8 6 2	4	2
9 8 7 7 6 5 4 3 1	5	5 9
7 7 5 4 1 1	6	4 6 7 8
6 2 1	7	5 5 6 7 7 8 8 9
8	8	1 3 4 4 5 9
2	9	2 4 9

Key: 2|9 = 29

Data sets are spread roughly over same range but data set for Friday night is symmetrical, whereas the data set for Saturday night is negatively skewed. More people seem to attend cinema on Saturday night than on Friday night.

b

Leaf Adelaide	Stem	Leaf Darwin
6	20	
5	21	
7	22	
7	23	
	24	
4	25	
6 3	26	
0	27	
7 4 3 3 0	28	
4	29	5 9
	30	0 4 8
2	31	3 4 8
1	32	1 1 2 3 5 5 6 7 8 8 9
8 8 4	33	0 1 2 3 4 4 4 5 8
3 2	34	
6	35	
0	36	
1	37	
4 2	38	
2	39	
5	40	

Key: 32 | 9 = 32.9

Adelaide has a much larger range of temperatures than Darwin. Adelaide seems to have a roughly symmetric distribution with a lower average temperature than Darwin. Darwin's temperatures cover a small range and appear to be negatively skewed, with a higher average temperature than Adelaide.

c

Leaf Boston terriers	Stem	Leaf French bulldogs
9 9 9 6 6	4	
5 5 1 1	5	
8 8 7 4 2	6	
9 8 8 6 6 5 5	7	
6 6 4 3 3 2 1 0	8	6
9 2 2 2 2	9	1 2 2 4 5 5 6 8 8 8 8 9
5 5 5 0	10	1 5 6 6 8 9 9
2 0	11	0 1 1 2 2 3 5 5 5 8 9
	12	0 3 5 5 7 9 9 9
	13	0

Key: 10 | 9 = 10.9 kg

Both dog breeds appear to have roughly symmetric distributions, with Boston terriers covering a larger range of masses. French bulldogs appear to be heavier on average than Boston terriers.

4 **a** Median heights of students in both classes are similar but 9A has a larger range of heights. The larger range and higher mean values suggests that 9A heights may be affected by presence of an outlier. Median should be used as measure of centre.

b The median and mean price of jeans and range of prices in all stores are similar. Distributions seem to be symmetrical. Mean should be used as measure of centre.

c Both the mean and median indicate that the use of internet in secondary school is greater than its use in primary school. Range of hours is also greater in secondary school. Median should be used as measure of centre.

d Both stores have the same range and median of storage capacity in their cards, however store A has a larger mean than store B (and larger than its median), which suggests that store A has a skewed distribution and the median is a better representation of centre.

e Players A and B have the same median but player B has a larger mean. Combined with the smaller range, it appears that player B is a more consistent goal kicker with a positively skewed distribution, which would make them a better goal kicker.

5 **a** B. The range of B is 9, whereas the range of A is 7.

b A. The median of A is 25, whereas the median of B is 23.

6 **a** School B

b School B (13 at school B and 6 at school A)

c The two data sets are spread out over roughly the same age brackets, but distributed differently. School A has a roughly symmetric distribution, whereas school B is positively skewed.

7 **a** The range of heights is similar for Year 7 and Year 9 but Year 9 heights are slightly skewed in the negative direction. Year 9 students seem to be taller overall.

b

Class heights	Mean (cm)	Median (cm)	Range (cm)
Year 7	164.22	162	34
Year 9	173.54	173.5	33

c Range of heights for Year 7 and Year 9 students is almost identical. Measures of centre are close for each group, but the mean and median for Year 9 is higher, indicating the Year 9 students are taller than the Year 7 students.

d Yes; mean and median indicate that the Year 9 students are slightly taller than the Year 7 students and ranges of heights for Year 7 students and Year 9 students are almost identical.

8 **a** dot plot 1: negatively skewed;
dot plot 2: symmetrical

b dot plot 1: mean = 16.6, median = 17, range = 6
dot plot 2: mean = 15.24, median = 15, range = 6

c Scores in both data sets are spread over same range, 6. The data set in first dot plot has a higher mean, median and mode than the data set in the second dot plot. Therefore, students participating in the swim squad in term 1 tended to be older than students who participated in the school play in term 4.

9 **a**

Leaf Year 8	Stem	Leaf Year 9
5 0	0	
4	1	8 9
7 7 5 3	2	0 6
8 3 2 0 0	3	5 7
9 4 4 1	4	5 8 9
9 8 5	5	1 2 4 5
5 0	6	0 5
4 1	7	1 4 6 7 7
6 3	8	3 6
	9	0 2 7

Key: 5|2 = 52

b Year 8: 86, Year 9: 79

c Year 8: 41, Year 9: 55

d Yes, it is true in general because the median for Year 9 students is much higher.

10 **a** Group A: 4, Group B: 4

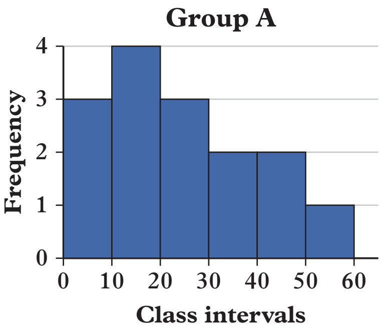
b

	Mean	Range
A	3.8	6
B	4.7	14

c The two data sets are very similar apart from the outlier in Group B.

11 **a** We do not have access to the raw data for the histogram. All that is known is how many numbers are in each interval, but not the actual values of those numbers.

b



c The two data sets are spread out over roughly the same values but distributed differently. Group A is positively skewed, whereas Group B is negatively skewed. Group B would have a higher centre than group A.

12 A matches with histogram 2 and B matches with histogram 1.

Reasons may vary. Any of the following are acceptable:

The median for histogram 2 is in the interval 10–<20 which matches with data set A.

The median for histogram 1 is in the interval 20–<30, which matches with data set B. Histogram 2 will have a lower mean than histogram 1 because so many more values are stacked in the first two bins.

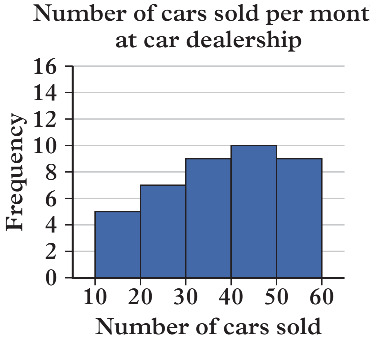
13 **a** negatively skewed

b roughly symmetric

c 16

14 **a** Class intervals used are not same size.

b



c Both histograms display data spread over same range. The first histogram displays data that is negatively skewed. The median class for the number of cars sold each month by car dealership A was 30–40, with the most frequent number sold in any month being 40–50 cars. The second histogram displays data that is symmetrical. Car dealership B had the same median class, which is also the modal class for this data.

d We do not know how the data is spread over the class intervals to enable us to draw the histogram with smaller class intervals.

15 **a** Year: 1 80–85; Year: 2 85–90; Year: 3 90–95

b Year 3

c If the trend continues, in Year 4 the graph will be negatively skewed with a modal class of 95–100.

16 **a** Class B; it has a range of 28.

b Negatively skewed. A higher median than mean indicates that the centre of the distribution is a high value but there is a long tail of lower values to bring the mean down.

c Class C, because its median is the highest by far. Class B has the highest mean but was affected by one outlier, which does not indicate that class B plays video games more in general.

17 **a** Year 8: 31; Year 9: 35

b 37

c 162.7 cm

d Summary statistics cannot help us to find the middle value of all 450 student heights; the raw data is needed.

Chapter 7 review

Multiple choice

1 D **2** C **3** D **4** E **5** A

6 D **7** C **8** B **9** E **10** C

11 D

Short answer

1 **a** biased; your friends are not a representative sample of the whole school.

b biased; people from your local council area are not representative of the population.

2 **a** Swimming coaches will be biased towards swimming as a form of exercise, so this is a biased sample.

b The data is about the popularity of a vegan burger, not about veganism.

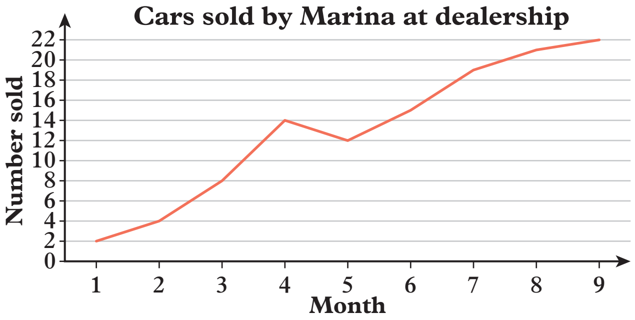
3 **a** average monthly rainfall: numerical; city: categorical; month: categorical

b Hobart

c July

d Jakarta is in the southern hemisphere and Seoul is in the northern hemisphere, so they do not experience summer and winter at the same times.

4 **a**



b month 5

c $7000

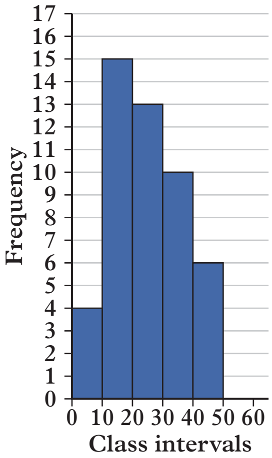
5 **a**

Class interval	Frequency
0–<10	4
10–<20	15
20–<30	13
30–<40	10
40–<50	6
Total	48

b 48

c 10–<20

d



e 10–<20

6 mean = 4.46, median = 5, mode = 5, range = 3

7 mean = 22.84, median = 21, mode = 7, range = 56

8 **a** dot plot 1

b dot plot 1

c dot plot 3

d Median. The mean is not an appropriate measure of centre because the data is skewed.

e Dot plot 1. Reasons may vary. Any of the following are appropriate reasons.

None of the other dot plots have zeros but we would expect some students to have no pets.

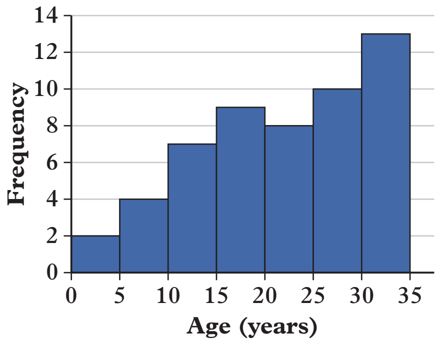
Most values are 0, 1 and 2, which fits the context.

The values in dot plots 2 and 3 are too large.

9 **a** 24

b 29

c Age



d negatively skewed

10 **a** Class 9A: mean = 66.32, median = 65, mode = 45, range = 47

Class 9B: mean = 67.14, median = 64, mode = 48, range = 53

b The means and medians are very similar despite the stem-and-leaf plots looking quite different, but Class 9B has a slightly larger range.

11 **a**

Leaf School 1	Stem	Leaf School 2
1 1	0	2 2 4 4
9 8 8 7 7 7 5 5	0*	6 6 7
1 1 1 0 0	1	0 1 1 1 2 4 4
7	1*	6 6 6 7 7 9 9
2 2 1 1	2	0 2
7	2*	
2	3	1
8 5	3*	

Key: 1 | 1 = 11

b School 1: mean = 14.4, median = 10.5, range = 37

School 2: mean = 12.8, median = 13, range = 29

c In discussing the results, the median will give the best measure of centre, since in school 1 the mean is affected by the positive skew.

Analysis

a Brisbane: 22 shrubs were measured, Sydney: 24 shrubs were measured

b Brisbane: mean = 13.34, median = 13.15, mode = 11.4, range = 4.5

Sydney: mean = 13.33, median = 13.25, mode = 11.4, range = 4.1

c The data for the two locations is very similar: 22 shrubs were measured in Brisbane and 24 were shrubs measured in Sydney, and the mean and mode for both locations is almost identical. The median is slightly higher for Sydney. It could be said that the average shrub height was 13.3 cm.

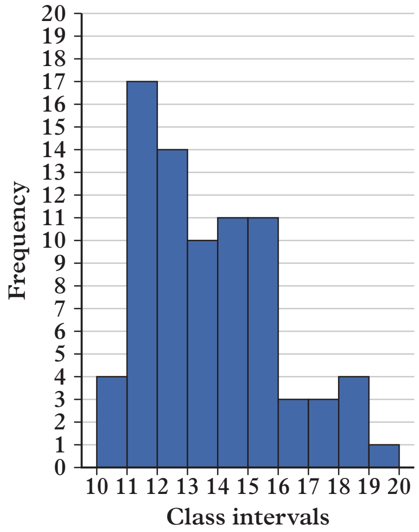
d

Class interval	Frequency
10–<11	4
11–<12	17
12–<13	14
13–<14	10
14–<15	11
15–<16	11
16–<17	3
17–<18	3
18–<19	4
19–<20	1
Total	78

e 78 plants

f 11–<12

g



The data displayed in the histogram is positively skewed.

h Victoria = 14.53, WA = 13.78

i If purchasing a shrub from this nursery chain, it will most likely be in height range of 10–20 cm, with the average shrub height being about 14 cm.

CHAPTER 8 Probability

8A Two-step chance experiments

EX p333

1 **a** 9 **b** 5

2 **a** 9

b **i** $\frac{2}{9}$ **ii** $\frac{4}{9}$ **iii** $\frac{5}{9}$

iv $\frac{1}{3}$ **v** 0

3 **a**

		1st dice roll			
		1	**2**	**3**	**4**
2nd dice roll	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b 16 **c** 5, $\frac{1}{4}$ **d** 2 or 8, $\frac{1}{16}$

e **i** $\frac{1}{8}$ **ii** $\frac{3}{16}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$

4 **a** $\frac{7}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{16}$

d $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{16}$

5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

6 **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{1}{3}$

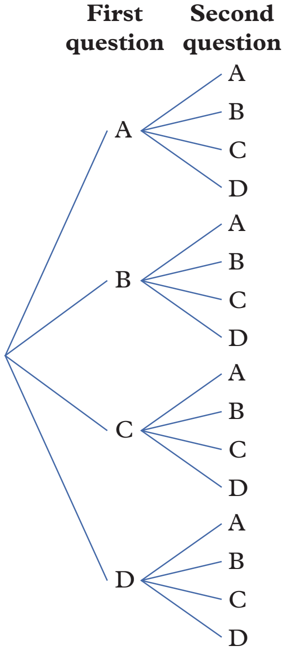
7 **a** **i** (r, s), (p, r), (s, p)

ii (r, p), (p, s), (s, r)

iii (r, r), (p, p), (s, s)

b $\frac{1}{3}$

8 **a**



b

		Second question			
		A	**B**	**C**	**D**
First question	**A**	A, A	A, B	A, C	A, D
	B	B, A	B, B	B, C	B, D
	C	C, A	C, B	C, C	C, D
	D	D, A	D, B	D, C	D, D

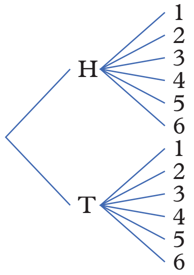
c With a tree diagram you can add a third step to the right of the second step, whereas it is not possible to add another dimension to an array to represent the third step.

9 **a** 36

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{36}$

c When there are a large number of outcomes in each step of a two-step chance experiment, the information is easier to interpret in an array than a tree diagram, and it can be easier to locate the favourable outcomes in an array.

10 **a**



b 12

c **i** $\frac{1}{12}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{12}$ **iv** $\frac{3}{4}$

d Tails on the coin toss and a 6 on the die is counted as one outcome, not two.

11 **a**

		Vowel				
		A	**E**	**I**	**O**	**U**
Die roll	**1**	(A, 1)	(E, 1)	(I, 1)	(O, 1)	(U, 1)
	2	(A, 2)	(E, 2)	(I, 2)	(O, 2)	(U, 2)
	3	(A, 3)	(E, 3)	(I, 3)	(O, 3)	(U, 3)
	4	(A, 4)	(E, 4)	(I, 4)	(O, 4)	(U, 4)

b 20

c **i** $\frac{1}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{9}{20}$

12 **a** **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{4}$

b It is not possible to add another dimension to an array to represent the third or more steps.

13 **a** $\frac{1}{216}$ **b** $\frac{5}{54}$ **c** $\frac{5}{9}$

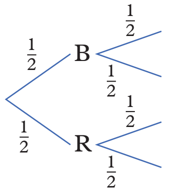
d $\frac{5}{72}$ **e** $\frac{5}{18}$

EX
p339

8B Experiments with replacement

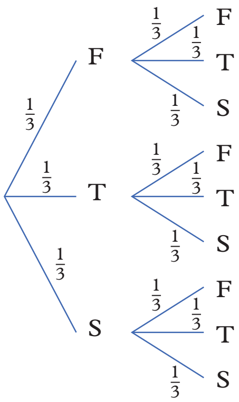
1 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{3}{26}$ **d** $\frac{1}{6}$

2 **a**



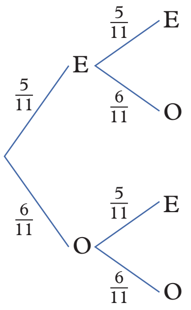
Outcomes	Probability
BB	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
BR	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
RB	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
RR	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

b



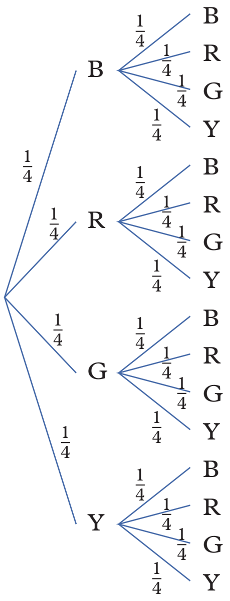
Outcomes	Probability
FF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
FT	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
FS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TT	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
TS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
SF	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
ST	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
SS	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

c



Outcomes	Probability
EE	$\frac{5}{11}\times\frac{5}{11}=\frac{25}{121}$
EO	$\frac{5}{11}\times\frac{6}{11}=\frac{30}{121}$
OE	$\frac{6}{11}\times\frac{5}{11}=\frac{30}{121}$
OO	$\frac{6}{11}\times\frac{6}{11}=\frac{36}{121}$

d



Outcomes	Probability
BB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
BY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
RY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
GY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YB	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YR	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YG	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
YY	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

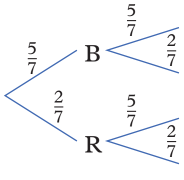
3 **a** $\frac{1}{2}$ **b** 4

c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

4 **a** $\frac{1}{9}$ **b** $\frac{5}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$

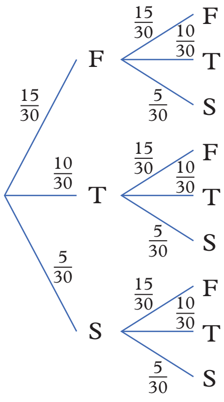
5 **a** $\frac{1}{16}$ **b** $\frac{7}{16}$ **c** $\frac{3}{8}$ **d** $\frac{9}{16}$ **e** $\frac{3}{4}$ **f** $\frac{1}{8}$

6 **a**



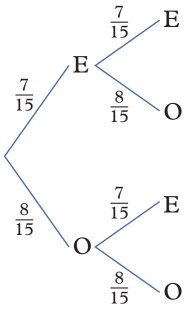
Outcomes	Probability
BB	$\frac{5}{7}\times\frac{5}{7}=\frac{25}{49}$
BR	$\frac{5}{7}\times\frac{2}{7}=\frac{10}{49}$
RB	$\frac{2}{7}\times\frac{5}{7}=\frac{10}{49}$
RR	$\frac{2}{7}\times\frac{2}{7}=\frac{4}{49}$

b



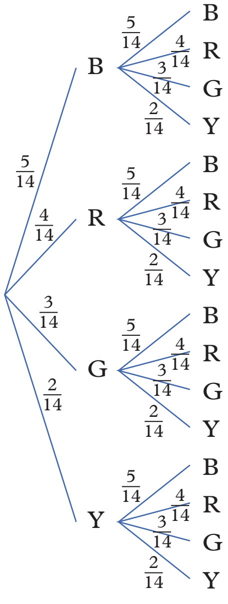
Outcomes	Probability
FF	$\frac{15}{30}\times\frac{15}{30}=\frac{225}{900}$
FT	$\frac{15}{30}\times\frac{10}{30}=\frac{150}{900}$
FS	$\frac{15}{30}\times\frac{5}{30}=\frac{75}{900}$
TF	$\frac{10}{30}\times\frac{15}{30}=\frac{150}{900}$
TT	$\frac{10}{30}\times\frac{10}{30}=\frac{100}{900}$
TS	$\frac{10}{30}\times\frac{5}{30}=\frac{50}{900}$
SF	$\frac{5}{30}\times\frac{15}{30}=\frac{75}{900}$
ST	$\frac{5}{30}\times\frac{10}{30}=\frac{50}{900}$
SS	$\frac{5}{30}\times\frac{5}{30}=\frac{25}{900}$

c



Outcomes	Probability
EE	$\frac{7}{15}\times\frac{7}{15}=\frac{49}{225}$
EO	$\frac{7}{15}\times\frac{8}{15}=\frac{56}{225}$
OE	$\frac{8}{15}\times\frac{7}{15}=\frac{56}{225}$
OO	$\frac{8}{15}\times\frac{8}{15}=\frac{64}{225}$

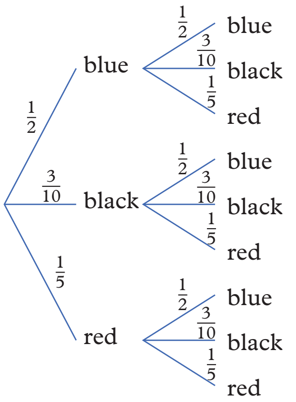
d



Outcomes	Probability
BB	$\frac{5}{14}\times\frac{5}{14}=\frac{25}{196}$
BR	$\frac{5}{14}\times\frac{4}{14}=\frac{20}{196}$
BG	$\frac{5}{14}\times\frac{3}{14}=\frac{15}{196}$
BY	$\frac{5}{14}\times\frac{2}{14}=\frac{10}{196}$
RB	$\frac{4}{14}\times\frac{5}{14}=\frac{20}{196}$
RR	$\frac{4}{14}\times\frac{4}{14}=\frac{16}{196}$
RG	$\frac{4}{14}\times\frac{3}{14}=\frac{12}{196}$
RY	$\frac{4}{14}\times\frac{2}{14}=\frac{8}{196}$
GB	$\frac{3}{14}\times\frac{5}{14}=\frac{15}{196}$
GR	$\frac{3}{14}\times\frac{4}{14}=\frac{12}{196}$
GG	$\frac{3}{14}\times\frac{3}{14}=\frac{9}{196}$
GY	$\frac{3}{14}\times\frac{2}{14}=\frac{6}{196}$
YB	$\frac{2}{14}\times\frac{5}{14}=\frac{10}{196}$
YR	$\frac{2}{14}\times\frac{4}{14}=\frac{8}{196}$
YG	$\frac{2}{14}\times\frac{3}{14}=\frac{6}{196}$
YY	$\frac{2}{14}\times\frac{2}{14}=\frac{4}{196}$

7 **a** $\frac{1}{25}$ **b** $\frac{16}{25}$ **c** $\frac{4}{25}$

8 **a**

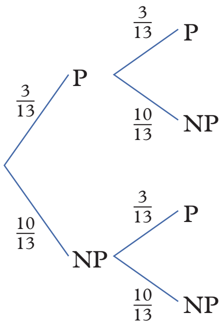


Outcomes	Probability
blue, blue	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
blue, black	$\frac{1}{2}\times\frac{3}{10}=\frac{3}{20}$
blue, red	$\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}$
black, blue	$\frac{3}{10}\times\frac{1}{2}=\frac{3}{20}$
black, black	$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$
black, red	$\frac{3}{10}\times\frac{1}{5}=\frac{3}{50}$
red, blue	$\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$
red, black	$\frac{1}{5}\times\frac{3}{10}=\frac{3}{50}$
red, red	$\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$

b **i** $\frac{1}{4}$ **ii** $\frac{1}{25}$ **iii** $\frac{9}{100}$

iv $\frac{3}{20}$ **v** $\frac{1}{10}$ **vi** $\frac{3}{50}$

9 **a**



Outcomes	Probability
P, P	$\frac{3}{13}\times\frac{3}{13}=\frac{9}{169}\approx 0.0533$
P, NP	$\frac{3}{13}\times\frac{10}{13}=\frac{30}{169}\approx 0.1775$
NP, P	$\frac{10}{13}\times\frac{3}{13}=\frac{30}{169}\approx 0.1775$
NP, NP	$\frac{10}{13}\times\frac{10}{13}=\frac{100}{169}\approx 0.5917$

b **i** $\frac{60}{169}\approx 0.3550$ **ii** $\frac{69}{169}\approx 0.4083$

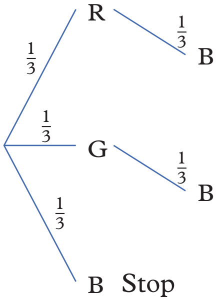
iii $\frac{9}{169}\approx 0.0533$ **iv** $\frac{100}{169}\approx 0.5917$

10 The grouped outcomes of this chance experiment are not equally likely, so the theoretical probability is calculated by adding the probabilities of the successful outcomes instead of using the theoretical probability formula.

11 Pr(blue ball) = Pr(blue ball on first draw) or Pr(not blue, then blue)

$$=\frac{1}{3}+\left(2\times\frac{1}{3}\times\frac{1}{3}\right)$$

$$=\frac{5}{9}$$



Draw 1	Draw 2	Outcomes	Probability
R	B	RB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
G	B	GB	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
B	Stop		$\frac{1}{3}$

12 a If the two-step chance experiment with replacement has lots of outcomes, they may be easier to see in an array than a tree diagram, and it can be easier to locate the favourable outcomes.

b If outcomes are not equally likely, then include the outcomes in the array in the same proportion as the likelihood. For example, if the first event is a ball is drawn at random from 1 blue and 2 red, then the array would have three columns: blue, red, red.

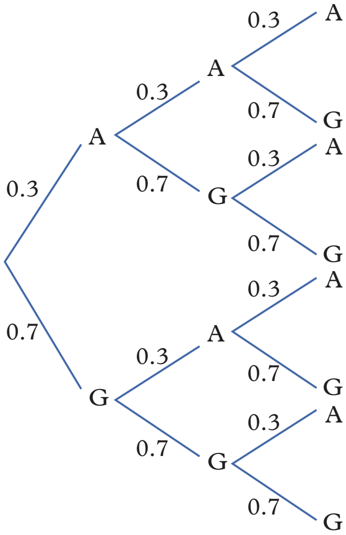
13 a Sample answer: No not necessarily. The socks are replaced each time, so you cannot say that this is the exact number of white and black socks. Some of the same socks may have been chosen multiple times.

b 4 black socks and 6 white socks

c Yes, this supports the previous estimate.

14 a 28 green marbles

b



c **i** 0.37

ii 0.441

iii 0.63

iv 0.21

15 a $3^3 = 27$

b BRW, BWR, RBW, RWB, WBR, WRB

c $6bw(1 - b - w)$

16 a $2a^2$ **b** $2ab$ **c** $1 - a - b$

d **i** $2a(1 - a - b)$ **ii** $2a - 2a^2 - 2ab$

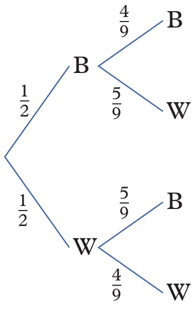
e **i** $(1 - a - b)^2$

ii $1 - 2a - 2b + 2ab + a^2 + b^2$

EX 8C Experiments without replacement

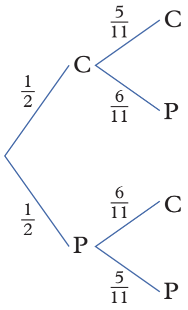
p344

1 a



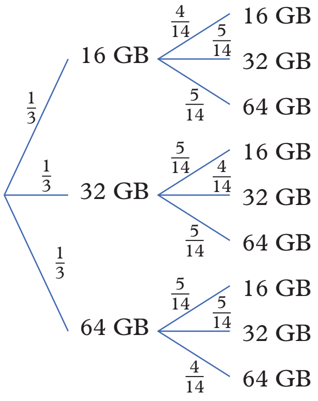
Outcomes	Probability
BB	$\frac{1}{2}\times\frac{4}{9}=\frac{4}{18}$
BW	$\frac{1}{2}\times\frac{5}{9}=\frac{5}{18}$
WB	$\frac{1}{2}\times\frac{5}{9}=\frac{5}{18}$
WW	$\frac{1}{2}\times\frac{4}{9}=\frac{4}{18}$

b



Outcomes	Probability
CC	$\frac{1}{2}\times\frac{5}{11}=\frac{5}{22}$
CP	$\frac{1}{2}\times\frac{6}{11}=\frac{6}{22}$
PC	$\frac{1}{2}\times\frac{6}{11}=\frac{6}{22}$
PP	$\frac{1}{2}\times\frac{5}{11}=\frac{5}{22}$

c

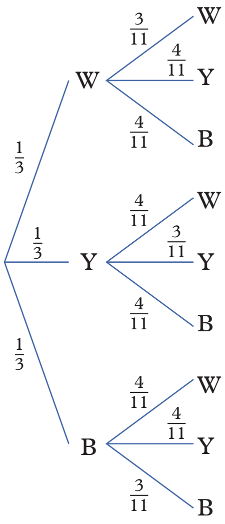


Outcomes	Probability
16, 16	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$
16, 32	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
16, 64	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
32, 16	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
32, 32	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$
32, 64	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 16	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 32	$\frac{1}{3}\times\frac{5}{14}=\frac{5}{42}$
64, 64	$\frac{1}{3}\times\frac{4}{14}=\frac{4}{42}$

2 a 0.21 **b** 0.21 **c** 0.58

3 a $\frac{3}{14}$ **b** $\frac{2}{7}$

4 **a**

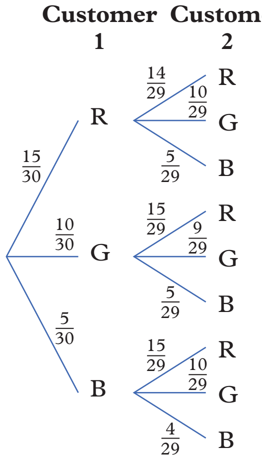


Outcomes	Probability
WW	$\frac{1}{3}\times\frac{3}{11}=\frac{3}{33}$
WY	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
WB	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
YW	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
YY	$\frac{1}{3}\times\frac{3}{11}=\frac{3}{33}$
YB	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
BW	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
BY	$\frac{1}{3}\times\frac{4}{11}=\frac{4}{33}$
BB	$\frac{1}{3}\times\frac{3}{11}=\frac{3}{33}$

b **i** $\frac{1}{11}$ **ii** $\frac{4}{33}$

5 **a** $\frac{8}{11}$ **b** $\frac{19}{33}$ **iii** $\frac{14}{33}$

6 **a**



Outcomes	Probability
RR	$\frac{15}{30}\times\frac{14}{29}=\frac{210}{870}\approx 0.24$
RG	$\frac{15}{30}\times\frac{10}{29}=\frac{150}{870}\approx 0.17$
RB	$\frac{15}{30}\times\frac{5}{29}=\frac{75}{870}\approx 0.09$
GR	$\frac{10}{30}\times\frac{15}{29}=\frac{150}{870}\approx 0.17$
GG	$\frac{10}{30}\times\frac{9}{29}=\frac{90}{870}\approx 0.10$
GB	$\frac{10}{30}\times\frac{5}{29}=\frac{50}{870}\approx 0.06$
BR	$\frac{5}{30}\times\frac{15}{29}=\frac{75}{870}\approx 0.09$
BG	$\frac{5}{30}\times\frac{10}{29}=\frac{50}{870}\approx 0.06$
BB	$\frac{5}{30}\times\frac{4}{29}=\frac{20}{870}\approx 0.02$

b **i** $\frac{20}{870}=\frac{2}{87}\approx 0.0230$

ii $\frac{90}{870}=\frac{3}{29}\approx 0.1034$

iii $\frac{210}{870}=\frac{7}{29}\approx 0.2414$

iv $\frac{150}{870}=\frac{5}{29}\approx 0.1724$

7 **a** $\frac{490}{870}=\frac{49}{87}\approx 0.5632$ **b** $\frac{660}{870}=\frac{22}{29}\approx 0.7586$

c $\frac{270}{870}=\frac{9}{29}\approx 0.3103$ **d** $\frac{450}{870}=\frac{15}{29}\approx 0.5172$

e $\frac{400}{870}=\frac{40}{87}\approx 0.4598$ **f** $\frac{100}{870}=\frac{10}{87}\approx 0.1149$

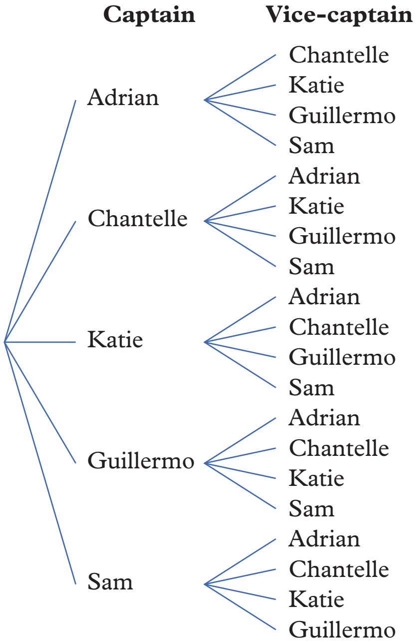
8 **a** $\frac{1}{17}$ **b** $\frac{15}{34}$ **c** $\frac{19}{34}$ **d** $\frac{13}{102}$

9 $\frac{1}{5}=0.2$

10 Pr(pair) $=\frac{5}{17}\approx 0.2941$; probability of a pair increases slightly.

11 $\frac{38}{132}=\frac{19}{66}\approx 0.2879$

12 **a**



Outcomes
AC
AK
AG
AS
CA
CK
CG
CS
KA
KC
KG
KS
GA
GC
GK
GS
SA
SC
SK
SG

b The second set of branches has one fewer branch. Once the captain has been selected there is one fewer person to choose from for vice-captain's position.

c 20

d **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{20}$

iv $\frac{3}{5}$ **v** $\frac{1}{10}$

13 **a** **i** $\frac{5}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{18}$ **iv** $\frac{5}{9}$

b **i**

	G	G	G	P
G	✗	GG	GG	GP
G	GG	✗	GG	GP
G	GG	GG	✗	GP
P	PG	PG	PG	✗

ii $\frac{1}{2}$ **iii** 0

14 **a** $\frac{1}{12}$ **b** $\frac{7}{12}$ **c** $\frac{13}{18}$ **d** $\frac{7}{12}$

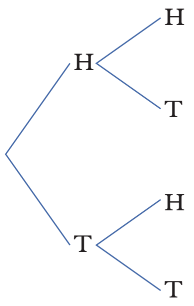
15 **a** $\frac{1}{21}$ **b** $\frac{2}{7}$ **c** $\frac{5}{42}$

16 $\frac{1}{8\,145\,060}\approx 0.000\,000\,123$

17 $\frac{5}{18}$

Checkpoint

1



Outcomes
HH
HT
TH
TT

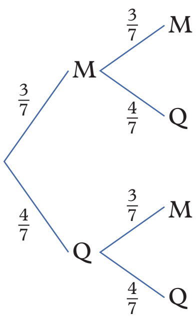
2

	1	2	3	4
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
3	(1, 3)	(2, 2)	(3, 3)	(3, 4)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

3 **a** $\frac{4}{9}$ **b** $\frac{4}{9}$ **c** $\frac{8}{9}$

4 **a** $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{5}{12}$

5

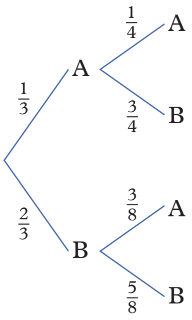


Outcomes	Probability
MM	$\frac{9}{49}$
MQ	$\frac{12}{49}$
QM	$\frac{12}{49}$
QQ	$\frac{16}{49}$

6 **a** $\frac{83}{225}$ **b** $\frac{9}{25}$ **c** $\frac{8}{9}$

7 **a** $\frac{24}{49}$ **b** $\frac{3}{7}$ **c** $\frac{40}{49}$

8



Outcomes	Probability
AA	$\frac{1}{12}$
AB	$\frac{1}{4}$
BA	$\frac{1}{4}$
BB	$\frac{5}{12}$

9 **a** $\frac{1}{4}$ **b** $\frac{7}{12}$ **c** $\frac{11}{12}$

10 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{7}{12}$

EX
p350

8D Relative frequency

1 **a** $\frac{9}{25} \approx 0.36$ **b** $\frac{1}{100} = 0.01$

c $\frac{3}{8} \approx 0.38$ **d** $\frac{2}{3} \approx 0.67$

e $\frac{87}{248} \approx 0.25$ **f** $\frac{131}{240} \approx 0.55$

2 **a** 125 **b** 40 **c** 105 **d** 5

e 40 **f** 214

3 **a** $\frac{8}{15} \approx 0.53$

b In the long term, you would expect the experimental probability of rolling a die and obtaining a number greater than 2 to increase as it approaches the theoretical probability of $\frac{2}{3} \approx 0.67$.

4 **a** 30

b $\frac{7}{130} \approx 0.054$

c In the long term, you would expect the experimental probability of drawing a picture card to increase as it approaches the theoretical probability of $\frac{3}{13} \approx 0.231$.

5 **a** **i** 30

ii There is only a slight difference between the expected number (30) and the actual number (27), so there is no evidence that the experiment is biased.

b **i** 18

ii There is only a slight difference between the expected number (18) and the actual number (16), so there is no evidence that the experiment is biased.

c **i** 100

ii There is a significant difference between the expected number (100) and the actual number (53) and the number of trials is large. It appears that this experiment is biased.

6 **a** heads: 25; tails: 15

b one: 18; two: 24; three: 21; four: 12; five: 15; six: 30

c hearts: 12; diamonds: 18; clubs: 21; spades: 9

7 **a** **i** 10

ii $\Pr(\text{heads}) = \frac{1}{2}, \Pr(\text{tails}) = \frac{1}{2}$

iii heads = 5, tails = 5

iv $\Pr(\text{heads}) = \frac{4}{5} = 0.8, \Pr(\text{tails}) = \frac{1}{5} = 0.2$

v There are not enough trials to make a firm decision.

vi There are only 10 trials; this is too few to tell if it is biased.

b **i** 6000

ii $\Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6)$

$= \frac{1}{6} \approx 0.17$

iii expected number = 1000 for all outcomes

iv $\Pr(1) \approx 0.16, \Pr(2) \approx 0.16, \Pr(3) \approx 0.17,$ $\Pr(4) \approx 0.17, \Pr(5) \approx 0.17, \Pr(6) \approx 0.17$

v The prop is fair.

vi The frequency is very close to the expected number for each outcome. The relative frequency for each outcome is approximately equal to its theoretical probability after a large number of trials.

c **i** 112

ii $\Pr(\text{hearts}) = \Pr(\text{diamonds}) = \Pr(\text{clubs})$ $= \Pr(\text{spades}) = \frac{1}{4} = 0.25$

iii expected number = 28 for all outcomes

iv $\Pr(\text{hearts}) \approx 0.34, \Pr(\text{diamonds}) \approx 0.28,$ $\Pr(\text{clubs}) \approx 0.17, \Pr(\text{spades}) \approx 0.21$

v The prop is biased.

vi Not all frequencies are close to expected numbers of each outcome (0.25). Hearts and diamonds are greater than expected, and clubs and spades are less than expected. Relative frequencies for all outcomes are not approximately equal to their theoretical probabilities after a reasonable number of trials.

8 **a** HHH, HHT, HTH, HTT, THH, TTH, THT, TTT

b $\frac{1}{4}$ **c** 10

d No, the actual occurrences are only half of the expected number.

e The relative frequency should get closer to the theoretical probability after 4000 trials.

9 With a small number of rolls, the actual number (13) can be significantly different from the expected number (5) due to random chance. As the number of rolls increases, the relative frequency will approach the theoretical probability if no bias exists. Therefore, to check for any bias, a higher number of trials should take place.

10 **a** A sample size of two is not a broad enough sample to verify a claim about the worldwide adult population.

b While the expected number would be 1 for a group of 3, 'one in three' does not mean one in any group of three. It means that any given Australian has a one in three chance of being an Australian who does not get enough sleep.

c The first headline is worldwide while the second is Australian only. The samples may be biased. The sample sizes may be too small to draw wide-scale conclusions from.

11 The 39% is likely to be more reliable, as a larger sample has been taken.

12 The number of people who are left-handed changes year to year as more people are born and die.

13 A consumer might believe more than 80% of dentists would recommend this brand over all others, but a dentist could recommend more than one brand of toothpaste (for example).

14 **a** $\frac{1}{128}$

b A streak of 7 of more heads has a 1 in 128 chance of occurring, so it is very likely to occur if you toss a coin 1000 times.

c 31

EX p355

8E Two-way tables

1 **a** 55 **b** 7 **c** $\frac{7}{55}$

2 **a** 75 **b** 11 **c** $\frac{58}{75}$

3 **a** $\frac{29}{85}$ **b** $\frac{37}{85}$ **c** $\frac{42}{85}$ **d** $\frac{23}{85}$

4 **a** $\frac{48}{125}$ **b** $\frac{19}{50}$ **c** $\frac{13}{125}$ **d** $\frac{31}{50}$

e $\frac{181}{250}$ **f** $\frac{27}{125}$

5

	Class A	Class B	Total
Prefer sweet food	0.23	0.32	0.55
Prefer savoury food	0.29	0.16	0.45
Total	0.52	0.48	1

6

	Reads books	Plays games	Plays sports	Total
Introvert	$\frac{7}{32}$	$\frac{11}{64}$	$\frac{1}{8}$	$\frac{33}{64}$
Extrovert	$\frac{3}{64}$	$\frac{11}{64}$	$\frac{17}{64}$	$\frac{31}{64}$
Total	$\frac{17}{64}$	$\frac{11}{32}$	$\frac{25}{64}$	1

7

	Light hair	Dark hair	Total
Prefer pop music	0.22	0.38	0.6
Prefer rock music	0.13	0.27	0.4
Total	0.35	0.65	1

8

	Spotify	No Spotify	Total
Netflix	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$
No Netflix	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$
Total	$\frac{1}{4}$	$\frac{3}{4}$	1

9 **a**

	Cinema	Home	Total
Action	22	30	52
Comedy	19	14	33
Total	41	44	85

b **i** $\frac{41}{85}$ **ii** $\frac{14}{85}$ **iii** $\frac{52}{85}$

10

	Cat	No cat	Total
Purebred	29	75	104
Not purebred	84	12	96
Total	113	87	200

$$\begin{aligned}\Pr(\text{non-cat purebred}) &= \frac{75}{200}\\ &= \frac{3}{8}\end{aligned}$$

11 **a**

	Women	Men	Total
Bath	0.33	0.09	0.42
Shower	0.22	0.36	0.58
Total	0.55	0.45	1

b **i** 0.36 **ii** 0.42 **iii** 0.22

c **i** 210 **ii** 275 **iii** 45

12 **a** $\frac{18}{83}$ **b** $\frac{67}{332}$ **c** $\frac{4}{83}$

d $\frac{125}{332}$ **e** $\frac{72}{83}$ **f** $\frac{69}{83}$

g $\frac{109}{166}$ **h** $\frac{47}{332}$ **i** $\frac{9}{83}$

13 **a** 60 **b** 160 **c** 980 **d** 120

14

	A	A'	Total
B	40	30	70
B'	20	30	50
Total	60	60	120

15 a

	A	A'	Total
B	40	35	75
B'	40	85	125
Total	80	120	200

	A	A'	Total
B	0.2	0.175	0.375
B'	0.2	0.425	0.625
Total	0.4	0.6	1

b

	C	C'	Total
D	4	5	9
D'	2	9	11
Total	6	14	20

	C	C'	Total
D	0.2	0.25	0.45
D'	0.1	0.45	0.55
Total	0.3	0.7	1

c

	E	E'	Total
F	18	4	22
F'	2	12	14
Total	20	16	36

	E	E'	Total
F	$\frac{1}{2}$	$\frac{1}{9}$	$\frac{11}{18}$
F'	$\frac{1}{18}$	$\frac{1}{3}$	$\frac{7}{18}$
Total	$\frac{5}{9}$	$\frac{4}{9}$	1

16 a

	Year 8	Year 9	Total
Maths	12	11	23
English	10	17	27
Total	22	28	50

b **i** $\frac{23}{50}$ **ii** $\frac{14}{25}$ **iii** $\frac{11}{50}$

c **i** 28 **ii** 11 **iii** $\frac{11}{28}$

d Part **c iii** says the student is in Year 9, hence the total number of possible outcomes is restricted to the total number of Year 9 students (28). In part **b iii** you are selecting from the total number of students (50), and of those you need to select a student in Year 9 who likes Maths.

e **i** likes English; 27

ii Year 8 students who like English

iii 10

iv $\frac{10}{27}$

17 a $\frac{13}{81}$ **b** $\frac{37}{96}$ **c** $\frac{1}{5}$

d $\frac{13}{43}$ **e** $\frac{71}{181}$ **f** $\frac{4}{5}$

EX p362

8F Venn diagrams

1 **a** 14 **b** 15 **c** 2 **d** 31

2 **a** 62 **b** 19 **c** 532 **d** 637

3 **a** $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{11}{20}$

d $\frac{7}{10}$ **e** $\frac{1}{10}$ **f** $\frac{1}{4}$

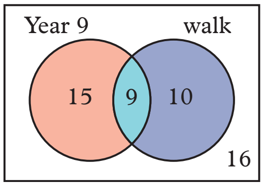
4 **a** $\frac{17}{45}$ **b** $\frac{1}{9}$ **c** $\frac{19}{45}$

d $\frac{1}{5}$ **e** $\frac{23}{45}$ **f** $\frac{31}{45}$

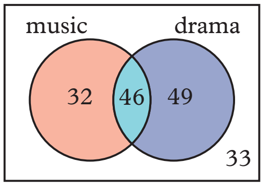
5 **a** $\frac{9}{41}$ **b** $\frac{21}{41}$ **c** $\frac{11}{41}$

d $\frac{8}{41}$ **e** $\frac{22}{41}$ **f** $\frac{33}{41}$

6



7



8 **a** $A \cap B$ **b** $A \cup B$

c $A' \cup B$ **d** $A \cap B'$

e $A \cup B'$ **f** $A' \cap B'$ $(A \cup B)'$

g $A' \cap B$ **h** $A' \cup B'$ or $(A \cap B)'$

9 **a** **i** 29 **ii** 43 **iii** 26

iv 10 **v** 50 **vi** 14

b **i** $\frac{12}{25}$ **ii** $\frac{21}{50}$ **iii** $\frac{1}{5}$

iv $\frac{7}{50}$ **v** $\frac{7}{50}$ **vi** $\frac{31}{50}$

10 **a** $(A \cap B)' = A' \cup B'$ and $(A' \cup B')' = A \cap B$

$(A' \cap B)' = A \cup B'$ and $(A \cup B')' = A' \cap B$

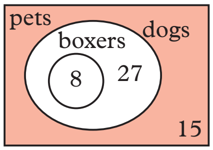
$(A \cap B')' = A' \cup B$ and $(A' \cup B)' = A \cap B'$

$(A' \cap B')' = A \cup B$ and $(A \cup B)' = A' \cap B'$

b The complement of an intersection is the union of the complements of the two events. The complement of a union is the intersection of the complements of the two events.

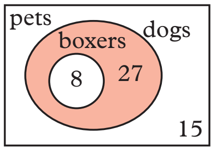
11 **a** The first Venn diagram shows a subset of data. All the set of 'Boxers' belongs to the set of 'Dogs', hence 'Boxers' is a subset of 'Dogs'.

b **i**



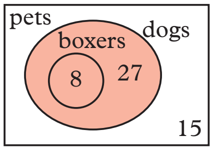
pets that are not dogs

ii



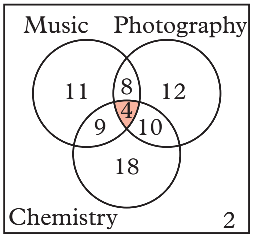
dogs that are not boxers

iii

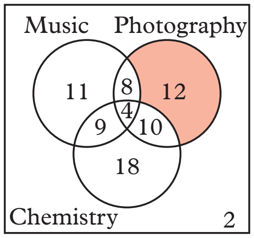


all dogs

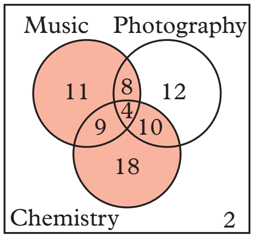
c **i**



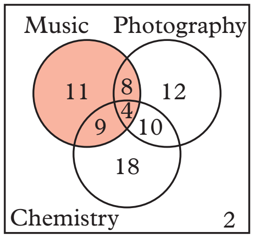
ii



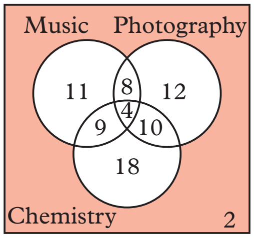
iii



iv



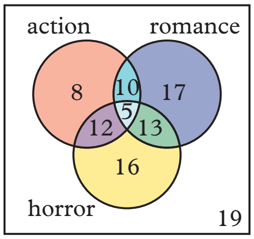
v



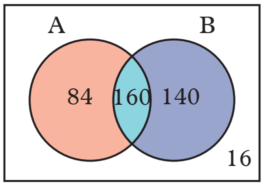
12 **a** $\frac{7}{34}$ **b** $\frac{4}{17}$ **c** $\frac{1}{17}$

d $\frac{5}{34}$ **e** $\frac{23}{34}$ **f** $\frac{7}{34}$

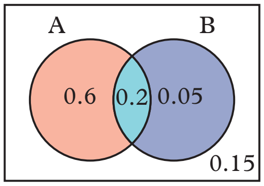
13 Sample answer:

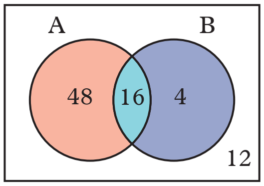


14



15 **a**





b **i** 0.8 **ii** 0.25

16 **a** $\frac{29}{61}$ **b** $\frac{29}{46}$ **c** $\frac{32}{61}$ **d** $\frac{9}{25}$

17 **a** $\frac{11}{30}$ **b** $\frac{11}{26}$ **c** $\frac{15}{26}$ **d** $\frac{2}{7}$

Chapter 8 review

Multiple choice

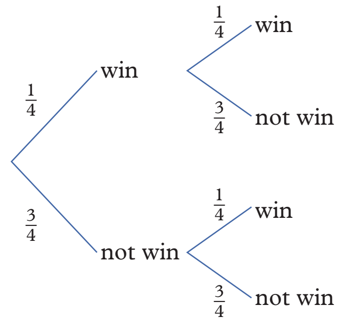
1 C **2** C **3** A **4** A **5** D

6 A **7** A **8** D **9** D **10** C

Short answer

1 **a** $\frac{3}{4}$

b **Chance of winning an eBook**



c **i** $\frac{1}{16}$ **ii** $\frac{15}{16}$ **iii** $\frac{3}{16}$

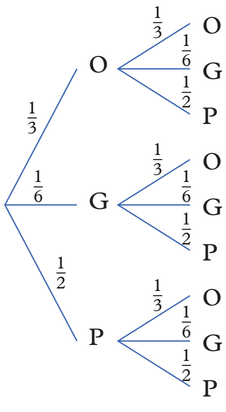
2 **a**

	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

b **i** $\frac{1}{12}$ **ii** $\frac{7}{12}$ **iii** $\frac{1}{2}$

c $\frac{1}{6}$ **d** $\frac{5}{6}$

3 a



Outcomes	Probability
OO	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
OG	$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$
OP	$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$
GO	$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
GG	$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
GP	$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
PO	$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$
PG	$\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$
PP	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

b **i** $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{4}$

c $\frac{1}{6}$ **d** $\frac{1}{2}$

4 a $\frac{1}{17}$ **b** $\frac{13}{204}$

5 a $\frac{3}{8}$ **b** $\frac{1}{200}$ **c** $\frac{11}{100}$ **d** $\frac{13}{20}$

6 a 35 **b** 10 **c** 5 **d** 25

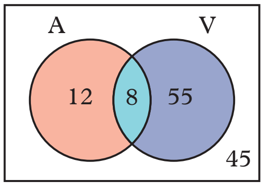
7 a

	Year 8	Year 9	Year 10	Total
Napoleon Perdis	32	25	45	102
Rimmel	22	10	22	54
Covergirl	17	31	13	61
Maybelline	49	29	15	93
Total	120	95	95	310

b **i** $\frac{3}{10}$ **ii** $\frac{11}{155}$ **iii** $\frac{5}{62}$

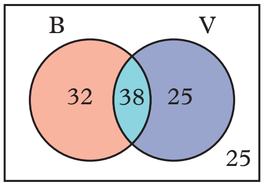
c **i** $\frac{5}{19}$ **ii** $\frac{12}{19}$

8 a



b **i** $\frac{1}{15}$ **ii** $\frac{5}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{21}{40}$

c

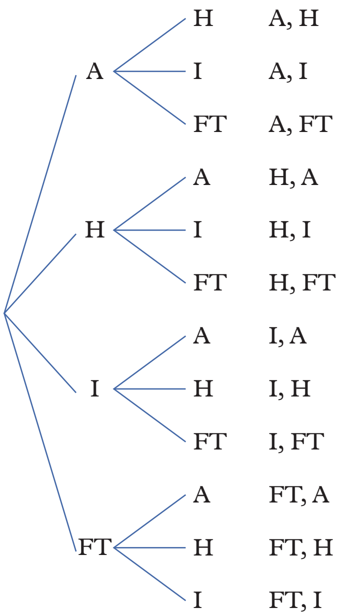


d **i** $\frac{19}{60}$ **ii** $\frac{11}{15}$ **iii** $\frac{4}{15}$ **iv** $\frac{7}{12}$

Analysis

a Without replacement, as the same subject cannot be selected twice

b **i**



Key:
A = Art
H = Health and Human Development
I = Italian
FT = Food Technology

ii 12

c **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$

d **i**

	Year 11	Year 12	Total
Art	0.24	0.12	0.36
Food technology	0.28	0.36	0.64
Total	0.52	0.48	1.00

ii $\frac{9}{25}$ **iii** $\frac{6}{25}$ **iv** $\frac{9}{25}$

v 72 **vi** 128

e **i** 24 **ii** 2 **iii** 68

iv $\frac{34}{75}$ **v** $\frac{28}{75}$

EX p370

Algorithmic Thinking 3 Statistics and Probability

Task 1

1 The repeated actions in Example 1 are adding 1 to k and reporting the k^th^ letter of Sentence.

The repeated action in Example 2 is adding i to sum.

2 Example 1 reports each letter of contents of the variable Sentence.

Example 2 report the sum of the integers 1, 2, 3, 4 and 5, which is computed in the variable sum to be 15.

Task 2

1 The nested repeated actions are reporting coinFace & dieFace and adding coinFace & dieFace to SampleSpace.

2 The nested iteration will report the whole event space for the consecutive random events of tossing a coin and throwing a die in the order H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6.

Task 3

1 110 elements

2 Teacher to check.

3 Teacher to check.

4 $\frac{1}{110}$

EX p372

SEMESTER 2 Review

Short answer

1 **a** 40.91 cm^2 **b** 61 000.00 cm^2
c 201.06 mm^2 or 2.01 cm^2 **d** 52 216.00 cm^2

2 **a** 154 cm^2 or 15 400 mm^2 **b** 36 cm^2
c 483.8 cm^2 **d** 2422.4 cm^2

3 **a** 122.5 cm^3 **b** 12 cm^3
c 615.8 cm^2 **d** 3200 cm^3

4 **a** 2 **b** 1.4 **c** 3.4

5 **a** 25 mL **b** 55.0 g **c** 4.6 m

6 **a** 3.2 m by 2.6 m **b** 4.8 m by 3.6 m
c 5.6 m by 3.2 m

7 **a** \$6000 USD **b** \$315 USD
c \$1687.50 USD

8 **a** $k = 12$ cm, $l = 52°$
b $p = 139°$, $q = 30$ mm

9 **a** 320 cm^2 **b** 122.72 mm^2

10 **a** **i** $\frac{1}{2}$ **ii** 0.5581
iii 0.8372 **iv** 0.6667
b **i** $\frac{1}{13}$ **ii** 0.5
iii 0.8660 **iv** 0.5774
c **i** 3 **ii** 0.7593
iii 0.6508 **iv** 1.1667
d **i** 5 **ii** 0.96
iii 0.28 **iv** 3.4286

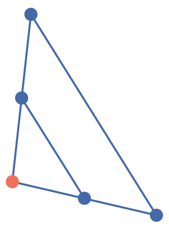
11 **a** 70° **b** 49° **c** 76° **d** 148°

12 **a** $z = 13.6$ m
b $x = 30.9$ m, $y = 18.2$ m
c $p = 19.2$ m, $q = 18.9$ m
d $r = 60.4$ m

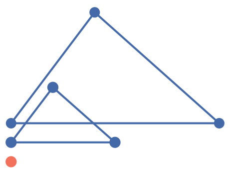
13 **a** no **b** no
c yes **d** yes

14 **a** 2610.1 cm^2 **b** 1008.1 cm^2
c 16 799.9 cm^2 **d** 210 415.2 mm^2

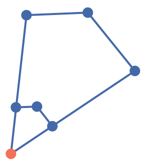
15 **a**



b



c



16 **a** **i** categorical **ii** nominal
b **i** numerical **ii** continuous
c **i** numerical **ii** discrete

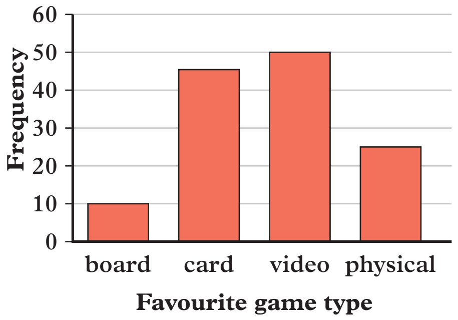
17 **a** Sample answer:

Class	Frequency
5–<10	2
10–<15	0
15–<20	6
20–<25	4
25–<30	7
30–<35	1

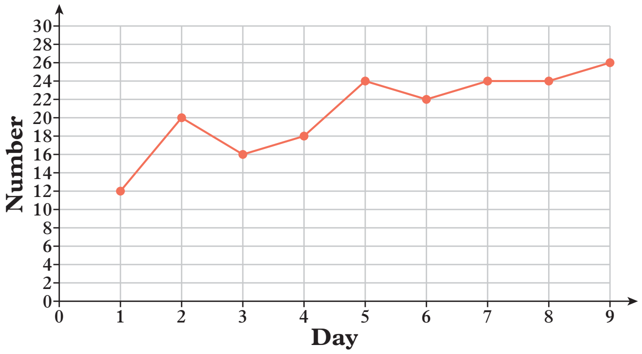
b

Class	Frequency
20–<40	2
40–<60	5
60–<80	8
80–<100	21
100–<120	4

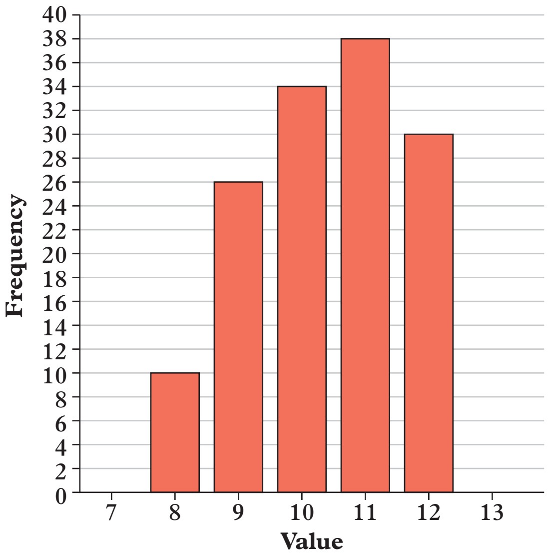
18 **a**



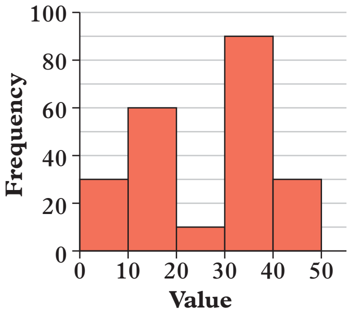
b



c



d



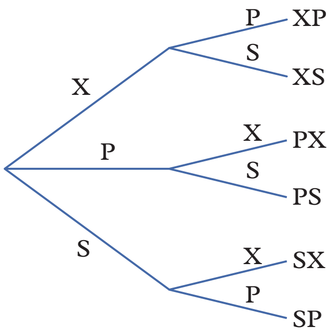
19 a Li **b** Chaz **c** Game 3

20 a i 19.8 **ii** 19
iii 19 **iv** 10
b i 12.34 **ii** 12
iii 13 **iv** 4
c i 4.36 **ii** 4.1
iii 3.4 **iv** 4.7

21 a bi-modal symmetric **b** perfectly symmetric
c symmetric **d** positively skewed
e negatively skewed with outlier
f symmetric

22 a mode of each peak **b** mean, median or mode
c mean, median or mode **d** median
e median or mode **f** mode

23 a



b i $\frac{1}{6}$ **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$

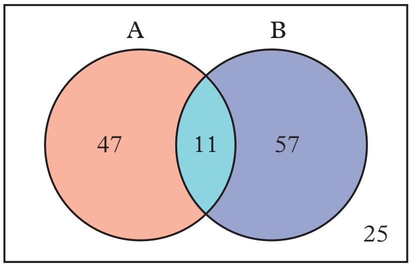
24 a

	1	2	3	4	5	6
Spade	(1, ♠)	(2, ♠)	(3, ♠)	(4, ♠)	(5, ♠)	(6, ♠)
Club	(1, ♣)	(2, ♣)	(3, ♣)	(4, ♣)	(5, ♣)	(6, ♣)
Diamond	(1, ♦)	(2, ♦)	(3, ♦)	(4, ♦)	(5, ♦)	(6, ♦)
Heart	(1, ♥)	(2, ♥)	(3, ♥)	(4, ♥)	(5, ♥)	(6, ♥)

b i $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{2}$

25 a $\frac{16}{49}$ **b** $\frac{12}{35}$ **c** $\frac{1}{35}$ **d** $\frac{1}{3}$

26 a



b

	A	A′	Total
B	5	21	26
B′	12	12	24
Total	17	33	50

27 a

	S	S′	Total
G	56	76	132
G′	42	38	80
Total	98	114	212

b i $\frac{57}{106}$ **ii** $\frac{33}{53}$ **iii** $\frac{19}{106}$ **iv** $\frac{59}{106}$

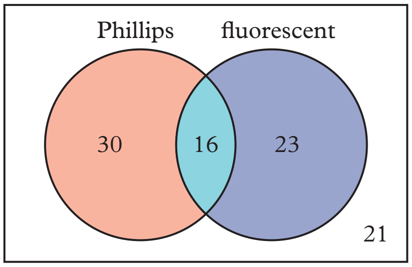
28 a

	A	A′	Total
B	$\frac{3}{5}$	$\frac{1}{25}$	$\frac{16}{25}$
B′	$\frac{7}{25}$	$\frac{2}{25}$	$\frac{9}{25}$
Total	$\frac{22}{25}$	$\frac{3}{25}$	1

b

	A	A′	Total
B	51	29	80
B′	15	25	40
Total	66	54	120

29 a



b i $\frac{8}{45}$ **ii** $\frac{17}{30}$ **iii** $\frac{22}{45}$ **iv** $\frac{2}{3}$

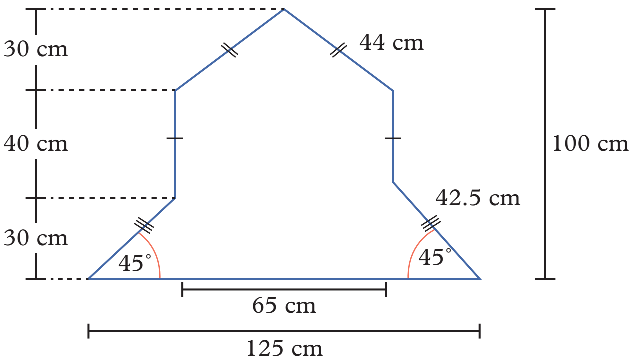
30 a i 60 **ii** 77 **iii** 52
iv 28 **v** 112 **vi** 17
b i $\frac{15}{28}$ **ii** $\frac{13}{28}$ **iii** $\frac{1}{4}$

iv $\frac{5}{16}$ **v** $\frac{5}{16}$ **vi** $\frac{95}{112}$

Analysis

1 a 8.9 cm should be 8.8 cm, correct to one decimal place. 8.5 cm and 13.0 cm are correct.

b



c **i** 257 cm^2 **ii** 6425 cm^2

d 69.3 cm **e** 30°

f They are not parallel as the angle the edge of the plywood makes with the horizontal (≈ 42.7°) is not the same as the angle the string makes with the horizontal (30°), and these are corresponding angles.

2 a

Leaf Arielle	Stem	Leaf Brenna
7	3*	7
4 3 3 1 1	4	0 1 2 3 4
9 9 9 8 8 8 8 8 8 8 7 7 7 7 6	4*	5 6 6 6 8
3 2 2 1 0 0	5	0 0 1 1 1 1 3 3 4
5	5*	5 6 7 7 8
	6	0 0 3

Key: 1 | 2 = 120 minutes

b Arielle's minutes of sleeping is negatively skewed whereas Brenna's minutes of sleeping is approximately symmetric.

c Arielle: median = 480 minutes, range = 180 minutes

Brenna: median = 510 minutes, range = 260 minutes

d Arielle sleeps on average less than Brenna with a median of 480 minutes of sleep compared to 510 minutes of sleep for Brenna. Arielle has a more consistent length of sleep with a range of 180 minutes as compared to 260 minutes for Brenna.

e Independent variable: Month

Dependent variable: Median sleep length

f Arielle and Brenna have valleys in June and November, but their peaks don't coincide. Both Arielle and Brenna's sleep length is, on average, increasing.

3 a i $\frac{2}{3}$ **ii** $\frac{7}{30}$

iii $\frac{1}{4}$ **iv** $\frac{15}{23}$

b i $\frac{2}{45}$ **ii** $\frac{7}{45}$

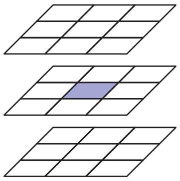
c i $\frac{4}{513}$ **ii** $\frac{61}{513}$

EX p382

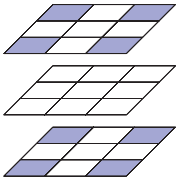
EXPLORATIONS 2

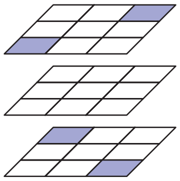
1 a Rotations and reflections of the minimum diagrams in **ii** and **iv** are also possible.

i maximum = minimum = 1

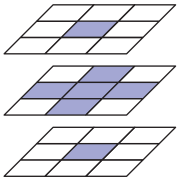


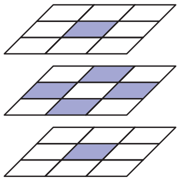
ii maximum = 8 minimum = 4



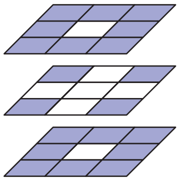


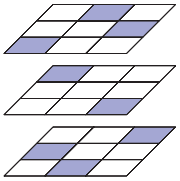
iii maximum = 7 minimum = 6





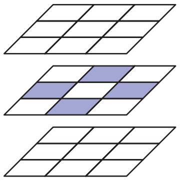
iv maximum = 20 minimum = 8

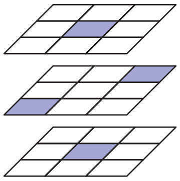




b One possible arrangement is given for **i** and **ii**. There are others.

i Top/bottom views: **ii** Four side views:





iii If the pattern can be viewed from all four sides, deduce that the top and bottom layers must have a cube in the centre, as in **ii** above, so the pattern cannot be seen from above or below.

2 a i $\frac{1}{12}$

ii Dieter has probability $\frac{1}{2}$ of winning but draws are also possible.

b No, Connie has probability $\frac{7}{12}$ of winning.

c 3 coins

d 14 sides

e $k = 3n - 1$

f No for dodecahedral (cannot have $n = 13$ counters).

Yes for icosahedral (with $n = 7$ counters).

3 a i 9 **ii** 12 **iii** 10 **iv** 10

v 16 **vi** 20 **vii** 18 **viii** 22

b Types **i**, **iii** and **iv** only.
c $AB = AF$, so not all side lengths are different.
4 **a** **i** 2 steps **ii** 5 steps
iii 6 steps **iv** 9 steps
b **i** 2 moves **ii** impossible
iii 9 moves **iv** impossible
c **i** 2 passes **ii** 5 passes
iii 5 passes **iv** 9 passes

EX
p452

NAPLAN practice

CHAPTER 1 Exponents

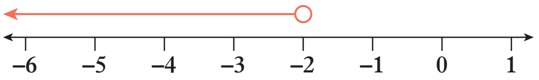
1 17
2 $\frac{1}{x}$
3 w^5
4 29 700
5 2.9×10^{-26}
6 $<$
7 2×10^6
8 $15a^6$
9 $\frac{2}{3r^2p^4}$
10 $\frac{x^2z^2}{y^3}$
11 $\frac{8m^{18}}{n^{12}}$
12 3
13 $\frac{1}{4^6}$

CHAPTER 2 Algebra

1 $14b + 28$
2 100
3 18
4 3
5 -25
6 $12a - 5$
7 $6x + 4y - 4$
8 34 cm
9 $6xy + 2y - 9x - 3$
10 481 cm^2
11 $w + 2$
12 $6xyz$
13 $a^2 + 2a - 15$
14 $(b - 5)(b - 8)$
15 $(m - 3)(m + 4)$
16 $(d - 2)(7d + 9)$
17 $x^2 + 4xy + 4y^2$
18 $(p - 4)^2$

CHAPTER 3 Linear relationships

1 $7\frac{1}{2}$
2 $5 - 3x = 4$
3 (2, 1)
4 (−3, 3.5)
5 2
6 −3
7 2
8 $\frac{3}{4}$
9 (1, 3.5)
10 5 units
11 $y = -2x + 6$
12 $x < 2$
13 true
14 $\frac{3n + 5}{4} = 6$
15 $\frac{4}{n} + 5 = 10$
16



17 $n + 2$
18 9
19 $y = \frac{4}{x} + 5$
20 positive
21 zero
22 undefined
23 $y = -4x$
24 −1
25 4
26 4
27 (−1, 2)
28 3.6 units
29 $\overline{PR}$
30 7.1 units
31 (1.5, −2)
32 always has two axis intercepts
33 $y = 4x$

CHAPTER 4 Non-linear relationships

1 $x = 6$ or $x = -1$
2 $x^2 - 6x - 16 = 0$
3 $x = 1$ or $x = 10$
4 −4
5 (−5, 0) and (4, 0)
6 $x = -1$
7 (4, −1)
8 $xy = 1$
9 vertical translation of 3 units
10 (0, −16)
11 (2, −4)
12 $y = (x + 3)^2 + 4$
13 1
14 $y = -x^2 + 2x + 3$
15 (0, 2)

CHAPTER 5 Measurement

1 12 mm
2 $A = \frac{1}{2}(a + b)h$
3 12.25 cm^2
4 17.76 cm^2
5 9.75 m^2
6 11 cm
7 3.1 cm^2
8 0.3 cm^3
9 102 m^2
10 15 m^3
11 905 cm^2
12 2035.75 cm^3
13 324 L
14 exact value: 21 cm, estimated value: 20 cm

CHAPTER 6 Geometry

1 1 : 5
2 40
3 20 m
4 4 cm
5 60°
6 16
7 5 m
8 v
9 $v^2 = u^2 + w^2$
10 Triangle A
11 52 m
12 11.74 cm
13 Triangle D
14 28 cm
15 8.35 m
16 $\frac{47}{50}$
17 0.36
18 $\tan(40°) = \frac{x}{16}$
19 12.28 cm
20 $\tan^{-1}\left(\frac{22}{25}\right)$
21 31°

CHAPTER 7 Statistics

1 pizza sizes
2 14
3 20
4 Sample answer: Key: 2|7 = 27
5 192
6 15.7%
7 92
8 9.38%
9 19
10 Mode will be unchanged.
11 $99
12 The median height is 152 cm.
13 1.6
14 16
15 4.4
16 Graph B

CHAPTER 8 Probability

1 $\frac{1}{3}$
2 20
3 $\frac{3}{16}$
4 $\frac{1}{2}$
5 26
6 34
7 15
8 A spinner with six equal segments may have been used in this simulation.
9 Relative frequency of heads for this experiment is 0.465.
10 $\frac{17}{140}$
11 $\frac{3}{10}$
12 $\frac{1}{4}$
13 $\frac{3}{10}$
14 $\frac{5}{9}$
15 $\frac{1}{120}$

Glossary

absolute error
the magnitude of an error

absolute value
the distance of a number from zero on the number line

accuracy
the closeness of a measurement to its exact value

adjacent side
(trigonometry) the side, in a right-angled triangle, next to the reference angle that is not the hypotenuse

arc
a portion of the circumference of a circle. Angles are marked using an arc

area chart
a graph created by shading the area between the lines and axes of a multiple line graph to create a sense of quantity over time

area scale factor
the scale factor between the areas of two similar shapes

array
items arranged in rows and columns. Used in probability to display the sample space of a two-step chance experiment

axis of symmetry
imaginary line that divides a symmetrical shape or graph so that one side is a reflection of the other

back-to-back stem-and-leaf plot
stem-and-leaf plot that displays two sets of data using the one set of stems

bar chart
graph where the frequency of categorical data is presented in columns

base
in a value expressed in exponent form, the base is the number that is repeatedly multiplied; for example, 2^4 has a base of 2

bias
an inclination towards one person, group of people, thing or idea over another

BIDMAS
order of operations: brackets, indices (exponents), division and multiplication, addition and subtraction

bi-modal
describes a distribution with two clear peaks

binomial
an expression containing two terms

binomial product
a factorised expression that is the product of two binomial expressions such as $(a + b)(c + d)$

bivariate data
data that shows the relationship between two variables

capacity
amount of a substance that a 3D object (container) can hold (mL, L)

Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described by a pair of coordinates

categorical data
non-numerical data that can be organised into categories

census
a survey of an entire population

centre of dilation
a point in a shape from which a dilation is made; often one of the vertices of the shape

circumference
perimeter of a circle

class intervals
groupings of data into equal-sized classes

coefficient
the number multiplied by a pronumeral

complement
everything in the universal set that is not in the given set; for example, if $\xi = \{1, 2, 3, 4, 5\}$ and set A contains even numbers, $A = \{2, 4\}$, then the complement of A contains odd numbers, $A' = \{1, 3, 5\}$

complementary events
a pair of events A and A′ (not A) where only one of the two events can take place and their probabilities sum to 1

composite shape
figures made up of more than one type of simple shape

conjugates
binomial products with the same terms but one different sign such as $a + b$ and $a - b$

constant
a term without any pronumeral part

constant term
the term in an expression that has no variable or the variable is to power 0; for example, –1 is the constant term in $3x^2 + 4x - 1$

continuous data
numerical data that can be measured; for example, height of a plant

convenience sampling
when a sample that is easily accessible is selected; for example, selecting students from your class to complete a survey

coordinate points (coordinates)
pair of numbers that describe the position of a point on the Cartesian plane

cosine (cos)
in trigonometry, the cosine of an angle in a right-angled triangle equals the ratio of the length of the adjacent side to the length of the hypotenuse

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

cumulative frequency
a running total of frequencies in a frequency table

cylinder
3D object with a uniform circular cross-section

dependent variable
variable that depends on another variable (*see* independent variable) in a relationship of two variables. It is listed second in a table of values and in pairs of coordinates, and shown on the vertical axis of a Cartesian plane

difference of two squares
rule for factorising a difference of two squares, $a^2 - b^2 = (a + b)(a - b)$

dilation
1 transformation that can enlarge or compress the graph of a function in the x- or y-direction

2 transformation that produces an enlargement or a reduction of the original figure

direct proportion
two quantities are in direct proportion if, as they increase or decrease the ratio between the quantities stays the same

discrete data
numerical data that can be counted; for example, number of siblings

distributive law
states that the same result is obtained if you add the numbers inside a pair of brackets first before multiplying by the common factor or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

enlargement
larger image produced after a figure has been dilated

equation
two expressions shown to be equivalent by an equals sign

error
the difference between the exact value and an estimated value of a quantity.
error = exact value – estimated value

event
outcome or group of outcomes in a sample space; for example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or a 4

expand
to convert an expression from factor to expanded form

expanded form
algebraic expression written without brackets, as opposed to factor form; for example, $3(2x + 1)$ expands to $6x + 3$

expected number
number of favourable outcomes expected in a probability experiment:
expected number = theoretical probability × number of trials

experiment
repeatable procedure that has a clearly defined set of possible results

exponent (index)
for a value expressed in exponent form, the exponent indicates the number of times the base is written as a repeated multiplication

exponent form (exponent notation)
shorter form of writing a repeated multiplication, where a number is written with a base and an exponent; for example, 2^4 is written in exponent form

expression
either a single term or the sum or difference of two or more terms

faces
flat surfaces of a polyhedron, where each face is a polygon

factor form
algebraic expression written as the product of its factors, as opposed to expanded form

factor tree
a tree diagram where composite numbers are broken down into factor pairs until prime numbers are found

factorise
to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

gradient *m*
numerical measure of the slope of a graph.
$\text{gradient} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

gradient-intercept form
general form of a linear equation, $y = mx + c$, where m is the gradient and c is the y-coordinate of the y-intercept of the corresponding linear graph

grouped data
numerical data that has been sorted into groups or class intervals

highest common factor (HCF)
greatest factor that is common to two or more given numbers

histogram
type of bar chart that can be used to display grouped continuous or discrete numerical data where the bounds of each class interval are located at the edges of the columns

horizontal translation
transformation where a shape or object is translated (moved) left or right in a straight line without turning or changing size

independent variable
quantity that changes independently in a relationship between two variables. It is listed first in a table of values and in pairs of coordinates, and shown along the horizontal axis on the Cartesian plane

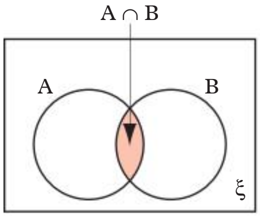
index (plural indices)
see exponent

inequality
a mathematical statement that compares the values of two expressions

inference
a conclusion based on facts and reasoning

intersection

the intersection of two sets ($A \cap B$) is the set of elements that appear in both A and B



inverse cosine ($\cos^{-1}$)

used to obtain an angle from its cosine value

inverse operation

operation that reverses the effect of a previous operation; for example, addition and subtraction are inverse operations

inverse sine ($\sin^{-1}$)

used to obtain an angle from its sine value

inverse tangent ($\tan^{-1}$)

used to obtain an angle from its tangent value

leading coefficient

the number multiplied by the highest power of a variable in an expression; for example, 3 is the leading coefficient in $3x^2 + 4x - 1$

like terms

terms that contain the same pronumerals

line graph

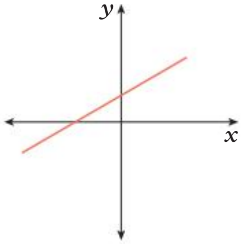
graph that displays the relationship between two variables where points are plotted and joined by a straight line. A continuous variable must be displayed on the horizontal axis; for example, time

linear equation

equation that contains a pronumeral term or terms where the highest power is 1 and no two pronumerals are multiplied together; for example, $2x + 3 = 6$

linear graph

straight line graph of a linear relationship



linear inequality

an inequality containing pronumerals raised only to the power of 1 and no two pronumerals are multiplied together; for example, $2x + 3 > 1$

linear relationship

relationship between two variables that produces a linear graph

linear term

the term with a variable of power 1 in an expression; for example, $4x$ is the linear term in $3x^2 + 4x - 1$

lower bound

1 abbreviation for lower boundary; the lowest value in a class interval

2 the boundary of values that can round up to the given measurement

maximum turning point

a point at which a parabola changes direction and has its maximum y-value

mean

a measure of the centre of a data set (numerical average) calculated by adding all of the data values and then dividing by the number of values

measure of centre

number used to represent the central point in a data set. Measures of centre include the mean, median and mode

measure of spread

number used to indicate how spread out a data set is. The range is a measure of spread

median

a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

midpoint

halfway point or middle point of an interval or line segment

minimum turning point

a point at which a parabola changes direction and has its minimum y-value

mode

a measure of the centre of a data set determined by finding the value, or values, that occur most frequently in the data set

monic quadratic trinomial

a quadratic trinomial in which the leading coefficient is equal to 1

multiple line graph

graph used to display two or more sets of bivariate numerical data. The categorical variable is detailed in the legend

multivariate data

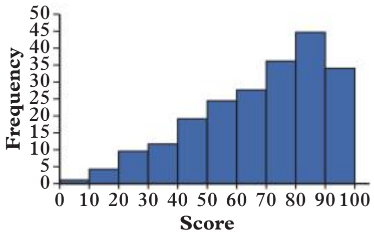
data that shows the relationship between three or more variables

negative exponent

exponent with a negative value indicating repeated division

negatively skewed

describes a distribution that is skewed away from the vertical axis with a tail to the left



net

2D plan that can be folded to form a three-dimensional object

nominal data
categorical data with categories that cannot be ordered; for example, eye colour

non-linear equation
an equation that contains at least one variable with a power other than 1 or a product of variables

non-linear relationship
relationship between two variables that produces a non-linear graph

Null Factor Law
states that if the product of two factors is 0, then one or both of the factors must equal 0

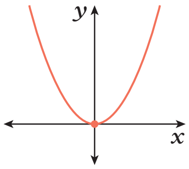
numerical data
data sets of numbers that can be counted or measured

opposite side
(trigonometry) the side opposite the reference angle in a right-angled triangle

ordinal data
categorical data that can be placed in categories in a specific order; for example, a rating system of 1–5

outlier
extreme piece of data that is much higher or lower than the rest of the data set

parabola
the graph of a quadratic relationship



percentage error
the relative error expressed as a percentage

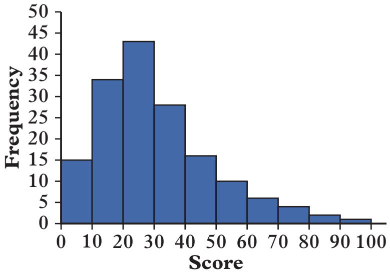
perfect square
the square of an integer; for example 1, 4, 9 and 25 are perfect squares

plot
graphical representation of a relationship consisting of individual coordinate points

plus-minus symbol
± indicates both positive and negative values or both addition and subtraction

population
all potential pieces of data under consideration

positively skewed
describes a distribution that is skewed towards the vertical axis with a tail to the right



prime factorisation
the product of all the prime factors of an integer

prism
3D object with two ends that are identical polygons and joined by straight edges

pronumeral
letter or symbol that takes the place of a number

Pythagoras' theorem
states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the two other sides, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the other side lengths

Pythagorean triple
any set of three positive whole numbers that satisfy Pythagoras' theorem

quadratic equation
an equation that contains a pronumeral term or terms with the highest power of 2; for example, $x^2 + 3x - 2 = 10$. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are constants

quadratic expression
an algebraic expression that contains a squared pronumeral, with no exponents greater than 2 in the expression

quadratic trinomial
expression of the form $ax^2 + bx + c$, where a, b and c are constants

quota sampling
dividing the population into separate categories (such as gender or age) and then taking a specified sample from each category that is chosen by the researcher

range
a measure of spread determined by finding the difference between the highest and lowest values in a data set

rate
comparison between two or more different quantities. It is a measure of how a quantity changes for each unit of another quantity

ratio
an expression of a relationship between two or more quantities, written with a colon between the quantities

rectangular prism
prism with three pairs of identical faces (total of six faces)

reduction
smaller image produced after a figure has been dilated

reflection
transformation where a shape or object is reflected (flipped) across a mirror line to produce its exact mirror image

relative error
the ratio between the absolute error and the exact value of a quantity

relative frequency
the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{Relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

right prism
prism with sides that make right angles with its base

sample
a selection of data that is part of a population

sample space

list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$

scale

the ratio between the drawing length and the actual length in a scale drawing

scale drawing

a drawing that accurately represents a real-life object, but which is drawn to a different scale

scale factor

indicates how many times larger or smaller an image is after a figure has been dilated

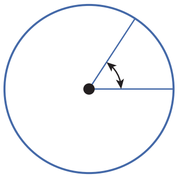
$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

scientific notation (standard form)

a value written in the form $a \times 10^m$, where $1 \leq a < 10$ or $-10 < a \leq -1$ and m is an integer

sector

portion of a circle formed by two radii and part of the circumference



set

a collection of distinct objects or numbers

significant figures

the number of digits in a value that contribute to its accuracy

similar

figures with the same shape but not the same size. Similar figures are produced by a dilation

sine (sin)

the sine of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the hypotenuse

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

sketch

a simple graph in which main features such as intercepts and turning points are labelled

skewed

describes a data set where data points are clustered to the left or right of the distribution

solution

value of the pronumeral that makes an equation a true statement

stacked bar chart

graph used to display frequency of two sets of categorical data. The categorical variable is detailed in the legend

standard form

see scientific notation

stratified sampling

dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size

subset

a set contained within another set

surd

irrational root of an integer that cannot be simplified to remove the root; for example $\sqrt{2}$

survey

a list of questions designed to extract information about a population

symmetrical (symmetric)

describes a data set with a peak in the middle and data points are roughly evenly spread on either side of the peak

systematic sampling

selecting a sample at fixed intervals (such as every third person). The starting point should be random

tangent (tan)

in trigonometry, the tangent of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the adjacent side

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

term

a single pronumeral or number, or the product of a number and one or more pronumerals

theoretical probability

the probability determined by mathematical reasoning, not experiments

total surface area (TSA)

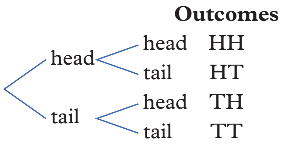
total area of the outer surface of an object. The total surface area of a prism is the sum of the areas of each face of the prism

transformations

general name for translations, reflections, rotations and dilations

tree diagram

a branched display of outcomes for a multi-step experiment; for example, the outcomes when two coins are tossed



triangular prism

3D object with five faces, two identical triangular faces whose respective vertices are joined by parallel lines

trigonometry

the study of relationships between angles and side lengths of a triangle

trinomial

an expression containing three terms

turning point

the point where a parabola changes direction

turning point form

general for of a quadratic equation $y = a(x - h)^2 + k$, where a is the dilation factor and the coordinates of the turning point are (h, k)

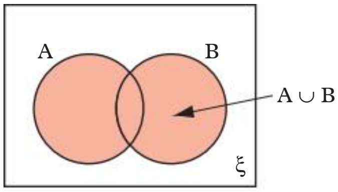
two-way table

display of outcomes of surveys or experiments with two different categories

ungrouped data

raw data that has not been placed into class intervals

union
the union of two sets (A ∪ B) is the set of elements in A or B or both



univariate data
a data set for one variable

universal set
all possible outcomes in a chance experiment or the set of all elements, ξ, inside the rectangle of a Venn diagram

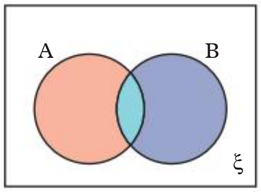
unknown
a quantity or measurement required in a problem that has one possible value only and is usually represented by a pronumeral

upper bound
1 abbreviation for upper boundary; the largest value in a class interval

2 the boundary of values that can round down to the given measurement

variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

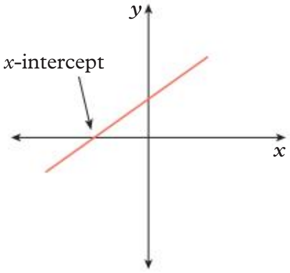
Venn diagram
a display of the relationship between different sets of data. The universal set, ξ, is represented by a rectangle and each set of data is represented by a circle



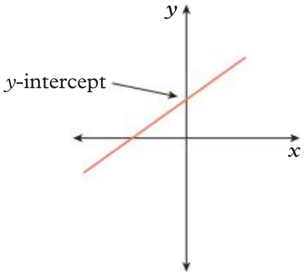
vertical translation
transformation in which a shape or object is translated (moved) up or down in a straight line without turning or changing size

volume
amount of space that a three-dimensional (3D) object occupies (cm^3, m^3)

***x*-intercept**
point where a line crosses the *x*-axis of a Cartesian plane



***y*-intercept**
point where a line crosses the *y*-axis of a Cartesian plane



Index

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Pilots need to apply trigonometry when they are making course corrections due to wind. There is a lot of mental maths to be done in the air!

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