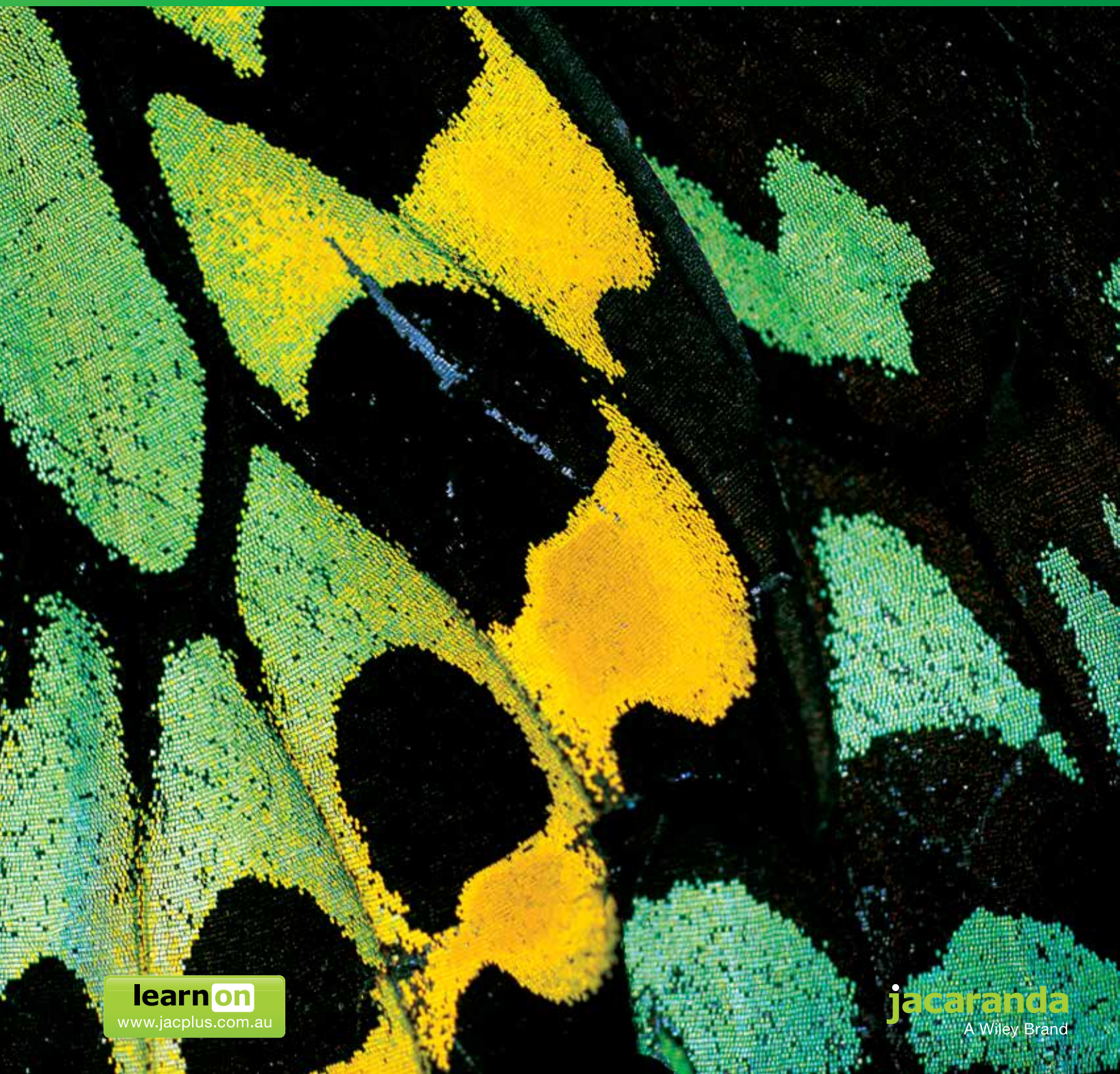


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JACARANDA MATHS QUEST 9

STAGE 5 | NSW AUSTRALIAN CURRICULUM | SECOND EDITION

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OVERVIEW

Maths Quest 9 Stage 5 for the NSW Australian curriculum has been completely revised to help teachers and students discover the NSW Mathematics syllabus. All content has been carefully mapped to Stages 5.1, 5.2 and 5.3 of the syllabus to ensure comprehensive coverage. Individual sub-topics are labelled by stage level to assist teachers with lesson planning and their navigation through the NSW Mathematics syllabus.

Maths Quest is designed to cater for all abilities: no student is left behind and none is held back. Written with the specific purpose of helping students deeply understand mathematical concepts, the content is organised around a number of features, in both print and online through Jacaranda's *learnON* platform, to allow for seamless sequencing through material to scaffold every student's learning.

Each topic mapped to individual stages of the NSW Mathematics Syllabus

Visit your *learnON* title to watch videos which tell the story of mathematics.

Topic introductions put the topic into a real-world context.

Start each topic with an engaging Discussion point.

The learning sequence mapped to stages of the NSW Mathematics Syllabus

NSW Mathematics Syllabus reference linked to contents of the topic

Fully Worked examples throughout the text demonstrate key concepts.

Your FREE online *learnON* resources contain hundreds of videos, interactivities and traditional WorkSHEETs and SkillSHEETs to support and enhance learning.

Carefully graded questions cater for all abilities. Question types are classified according to the Working Mathematically strand of the NSW Mathematics Syllabus.

TOPIC 13 Proportion and rates

[Stages 5.2 and 5.3]

13.1 Overview

Numerous videos and interactivities are embedded just where you need them, at the point of learning, in your *learnON* title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

13.1.1 Why learn this?

Proportion and rates are often used to compare quantities. Kilometres per hour, prices per kilogram, dollars per litre, and pay per hour or per week are all examples of rates. Dividing quantities according to a particular rate requires an understanding of ratios and rates.



DISCUSSION

How has the Golden Ratio been used by different cultures throughout history?

LEARNING SEQUENCE

- 13.2 Stage 5.2 Direct proportion
- 13.3 Stage 5.2 Direct proportion and ratio
- 13.4 Stage 5.2 Inverse proportion
- 13.5 Stage 5.2 Simple rate problems
- 13.6 Stage 5.2 Constant and variable rates
- 13.7 Stage 5.3 Rates of change
- 13.8 Review

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MAS.2-1NM
- interprets mathematical or real-life situations, systematically applying representations to solve problems MAS.2-2NM
- uses algebraic and numerical representations, with some and with increasing levels of complexity MAS.2-3NM
- uses and interprets formal definitions and generalisations when exploring solutions and/or concepts MAS.3-1NM
- uses mathematical ideas and mathematical language to explain, justify and communicate solutions MAS.3-2NM
- uses deductive reasoning in presenting arguments and formal proofs MAS.3-3NM
- draws, interprets and employs graphs of physical phenomena MAS.3-4NM

CONTENT DESCRIPTIONS

Students:

- Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems (ACMSP208)

Source: NSW Syllabus for the Australian curriculum

TOPIC 13 Proportion and rates 595

WORKED EXAMPLE 3

A die is rolled and the number appearing is noted. Determine the probability of each of the following events.

$A = \{1\}$

$B = \{\text{odd numbers}\}$

$C = \{4 \text{ or } 6\}$

There are 6 possible outcomes.

a. A has 1 favourable outcome.

$P(A) = \frac{1}{6}$

b. B has 3 favourable outcomes: 1, 3 and 5.

$P(B) = \frac{3}{6} = \frac{1}{2}$

c. C has 2 favourable outcomes.

$P(C) = \frac{2}{6} = \frac{1}{3}$

learnON RESOURCES — ONLINE ONLY

- Interactivity: Probability scale (in-3824)
- Interactivity: Theoretical probability (in-4081)
- Interactivity: Random number generator (in-4088)
- Digital doc: SASHEET: Probability scale (doc-4051)
- Digital doc: SASHEET: Understanding a die's playing cards (doc-4055)
- Digital doc: SASHEET: Listing the sample space (doc-4059)
- Digital doc: SASHEET: Theoretical probability (doc-4070)

Exercise 9.2 Theoretical probability

Individual pathways

PRACTISE

Questions: 1–16

CONSOLIDATE

Questions: 1–18

MASTER

Questions: 1–20

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your *learnON* title at www.jacplus.com.au. Note: Question numbers may vary slightly. Understanding and Fluency

1. Copy the given probability scale and insert the following events at appropriate points.

- a. The school will have a lunch break on Friday.
- b. Australia will have a swimming team in the Commonwealth Games.
- c. Australia will host two consecutive Olympic Games.
- d. At least one student in a particular class will obtain an A in Mathematics.
- e. Mathematics will be taught in secondary schools.
- f. In the future most cars will run without LPG or petrol.
- g. Water will be cold.
- h. Bean seeds, when sown, will germinate.

TOPIC 9 Probability 491

Fully worked solutions to every question are provided online, and answers are provided at the end of each print topic.

CONTENT DESCRIPTIONS

Students:

- Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP208)
- List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events (ACMSP209)
- Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence (ACMSP240)
- Use the language of 'if... then', 'given', 'or', 'knowing that' to investigate conditional statements and to identify common mistakes in interpreting such language (ACMSP241)

Source: NSW Syllabus for the Australian curriculum

learnON RESOURCES — ONLINE ONLY

Lesson: The story of mathematics: What are the chances? (doc-1700)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

9.2 Theoretical probability

[Stage 5.1]

9.2.1 The language of probability

- The probability of an event is a measure of the likelihood that the event will take place.
- If an event is certain to occur, then it has a probability of 1.
- If an event is impossible, then it has a probability of 0.
- The probability of any other event taking place is given by a number between 0 and 1.
- The higher the probability, the more likely it is for the event to occur.
- Descriptive words such as 'impossible', 'unlikely', 'likely' and 'certain' are commonly used when referring to the chance of an event occurring. Some of these are shown on the probability scale below.



DISCUSSION

List three real-life events for each of the following categories: impossible, unlikely, even chance, likely, certain.

408 Maths Quest 9 Stage 5 NSW Australian curriculum

Individual sub-topics mapped to the NSW Mathematics Syllabus to assist in lesson planning and class development

An extensive glossary of mathematical terms in print and as a hover-over feature in your *learnON* title

Expand the conversation with embedded Discussion points.

Engaging Investigations at the end of each topic to deepen conceptual understanding

Each topic concludes with comprehensive Review questions, both in print and online.

Fully worked solutions are available online; answers are provided at the end of each print topic.

10.7 Review

Investigation | Rich task

Forensic science

Students have been conducted on the relationship between the height of a human and measurements of a variety of body parts. One such study relates the height of a person to the length of the upper arm bone (humerus). The relationship, one different for males and females, and for different races. Let us consider the relationships for white adult Australians.

For males: $h = 7.04a + 75.47$ and for females: $h = 7.74a + 67.47$, where h represents the body height in centimetres and a the length of the humerus in centimetres.

Imagine the following situation.

A decomposed body was found in the bushland. A team of forensic scientists suspects that the body could be the remains of either Alicia Brown or James Kilar. They have been missing for several years. From the description provided by their Missing Persons file, Alicia's height is recorded as '165 cm tall' and James's height is recorded as '171 cm tall'. The forensic scientists hope to identify the body based on the length of the humerus.

14.9 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your *learnON* title at www.jacplus.com.au. Note: Question numbers may vary slightly.

Understanding and Fluency

1. a. $9 + a^2$ b. $9 - a^2$ c. $3 + a^2$ d. $3 - a^2$ e. $9 + 2a^2$ f. $9 - 2a^2$
2. a. $4x^2 + 20x + 25$ b. $4x^2 + 10x + 25$ c. $2x^2 + 20x + 25$ d. $2x^2 + 20x + 5$ e. $4x^2 + 25$
3. a. When factored, $4a^2 + 20a + 25 = (2a + 5)^2$ equals: b. $(6 - x)(6 + 2x)$ c. $(6x + 20 - 2x)$ d. $(6 + x)(6 - 2x)$ e. $2(3 - x)(3 + x)$
4. Expand and simplify these expressions. a. $(x + 4)(x - 4)$ b. $(x + 4)(x + 4)$ c. $(x + 3)(x - 3)$ d. $(1 - 2a)(1 + 2a)$
5. Expand and simplify these expressions. a. $(4x + 3)^2$ b. $(x - 3)^2$ c. $(4x + 1)^2$ d. $(2 - 3y)^2$
6. Expand and simplify the following. a. $(x + 2)^2 + (x + 3)^2$ b. $(x + 2)^2 - (x - 3)^2$ c. $x + 4x^2 - (x - 4)^2$
7. a. $(4x + 9)(4x + 9)$ b. $(4x - 9)(4x - 9)$ c. $(2a - 3)(2a - 3)$ d. $(2a + 3)(2a + 3)$
8. a. $(3x - 12)(x + 5)$ b. $(3x + 15)(x - 4)$ c. $(5x + 6)(x - 2)$
9. a. $(3x - 12)(x + 5)$ b. $(3x + 15)(x - 4)$ c. $(5x + 6)(x - 2)$
10. a. $(3x - 12)(x + 5)$ b. $(3x + 15)(x - 4)$ c. $(5x + 6)(x - 2)$
11. a. $(3x - 12)(x + 5)$ b. $(3x + 15)(x - 4)$ c. $(5x + 6)(x - 2)$
12. Factorise each of the following using the difference of two squares rule. a. $x^2 - 64$ b. $x^2 - 144$ c. $400 - 1$ d. $4x^2 - 9y^2$ e. $16x^2 - 9y^2$ f. $4x^2 - 121y^2$ g. $4x^2 - 121y^2$
13. Factorise each of the following quadratic trinomials. a. $x^2 + 5x + 4$ b. $x^2 + 10x + 24$ c. $x^2 + 30x + 24$

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Maths Quest contains a free activation code for *learnON* (please see instructions on the inside front cover), so students and teachers can take advantage of the benefits of both print and digital, and see how *learnON* enhances their digital learning and teaching journey.

learnon includes:

- Students and teachers connected in a class group
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- Fully worked solutions to every question
- Immediate feedback for students
- Immediate insight into student progress and performance for teachers
- Dashboards to track progress
- Collaboration in real time through class discussions
- Comprehensive summaries for each topic
- Code puzzles and dynamic interactivities to help students engage with and work through challenging concepts
- Formative and summative assessments
- And much more ...



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Faelchle, **315**/Ashley Whitworth, **317** (upper)/TCreativeMedia, **317** (lower)/Igorsky, **318**/Sashkin, **323**/Torderiul, **328**/andrea crisante, **329** (upper)/PinkBlue, **329** (lower)/lucadp, **332**/Kestutis Zitinevicius, **343** (upper)/David H.Seymour, **343** (middle)/Nata-Lia, **343** (lower)/kanvag, **344** (lower)/Vladir09, **345**/James Robey, **359**/Tefi, **362**/Galina Barskaya, **365** (lower)/Mmaxer, **365** (upper)/Divers Vector, **368** (lower)/Ramon grosso dolarea, **368** (upper)/paulaphoto, **369** (upper)/Monkey Business Images, **370** (lower)/Tukaram.Karve, **394**/weknow, **395**/Ed Phillips, **407**/Flat Design, **410** (lower)/Olaf Speier, **410** (upper)/Roma Koshel, **412**/bikeriderlondon, **413**/Jiripravda, **415**/wavebreakmedia, **418**/Africa Studio, **419** (lower)/Val Thoermer, **419** (upper)/Sinisa Botas, **424**/Bo Valentino, **427**/Andresr, **428** (left)/luxorphoto, **428** (right)/Serega K Photo and Video, **429**/Sven Larsen, **430**/Fenton, **431**/Kitch Bain, **432**/Robyn Mackenzie, **436**/chrupka, **437**/Vitalinka, **438** (lower)/Corepics VOF, **438** (upper)/Suzanne Tucker, **444**/Pete Niesen, **445**/Africa Studio, **447**/max blain, **451**/Africa Studio, **452**/Monkey Business Images, **453** (lower)/wavebreakmedia, **453** (upper)/Monkey Business Images, **454** (left)/Alan Poulson Photography, **454** (right)/jessicakirsh, **455** (lower)/Alex James Bramwell, **455** (upper)/Worytko Pawel, **471**/Lee Yiu Tung, **479**/zmkstudio, **482**/Nils Versemann, **485**/Barone Firenze, **486**/foto infot, **487**/e X p o s e, **489**/Luna Vandoorne, **489**/Pavel L Photo and Video, **490**/margouillat photo, **492**/kurhan, **493**/Suzanne Tucker, **494** (lower)/Edw, **494** (upper)/Khakimullin Aleksandr, **495**/Henri et George, **503**/IMR, **508**/StockPhotosLV, **513**/djgis, **514**/ARENA Creative, **520**/kezza, **521**/Tyler Olson, **529**/Andresr, **539** (lower)/ILYA GENKIN, **539** (middle)/Jean-Philippe Menard, **539** (upper)/Gordon Bell, **540** (left)/Taras Vyshnya, **540** (right)/mark higgins, **541**/MidoSemsem, **550**/Mikadun, **571**/Alfio Finocchiaro, **578**/StepStock, **582** (middle)/racorn, **582** (right)/Monkey Business Images, **585**/Karel Gallas, **588** (upper)/Circumnavigation, **588** (lower)/Rosli Othman, **595**/Sergey Nivens, **596**/RenataOs, **597**/Andresr, **598**/ESB Professional, **600**/Robyn Mackenzie, **602** (upper)/Dimedrol68, **602** (lower)/fotohunter, **603** (upper)/cowardlion, **603** (lower)/neoncat, **604**/marco mayer, **605**/nui7711, **606**/Vladyslav Starozhylov, **608**/Mmaxer, **610** (upper)/sunsetman, **610** (lower)/Ashley Whitworth, **614** (lower)/Eric Gevaert, **614** (lower)/Aleksandr Markin, **616**/enlightened101, **616**/Jim Barber, **620**/Maridav, **621** (top left)/Peter Kirillov, **621** (top right)/Eric Gevaert, **621** (lower)/Stefan Schurr, **635**/Vadim Sadovskiy, **648**/Maxim Blinkov, **657**/Olaf Speier, **660** (upper)/J.D.S, **660** (lower)/Tischenko Irina, **664**/Maksim Kabakou, **665**/JUN KAWAGUCHI, **666**/Alvin Travel Junkie, **677**/TechWizard, **688**/Kirschner, **689**/David Stuart Productions, **693**/cassiede alain, **697**/Keith Tarrier, **700**/Shenjun Zhang, **705**/Leah-Anne Thompson, **708**/Radu Bercan, **711**/Lichtmeister, **712** (upper)/dotshock, **712** (lower)/Martina Osmy, **714**/s74, **715**/Pressmaster • Photo: Steven W. 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Text

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TOPIC 1

NAPLAN practice

1.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



1.1.1 Why learn this?

All students in Australia are required to sit NAPLAN tests in May of Years 3, 5, 7 and 9. NAPLAN is an acronym for National Assessment Program — Literacy and Numeracy.

There are two Numeracy tests — one in which students may use a calculator and one in which they may not. Both tests contain 32 questions, most of which are multiple choice. Each test lasts for 40 minutes.

In the exercises, there are NAPLAN practice tests to help you prepare for when you have to sit the actual tests. It is best to complete one practice test per week before you undertake the formal tests.

LEARNING SEQUENCE

- 1.1 Overview
- 1.2 NAPLAN practice 1
- 1.3 NAPLAN practice 2
- 1.4 NAPLAN practice 3
- 1.5 Review

learnon RESOURCES — ONLINE ONLY

eLesson: NAPLAN — Strategies and tips (eles-1688)

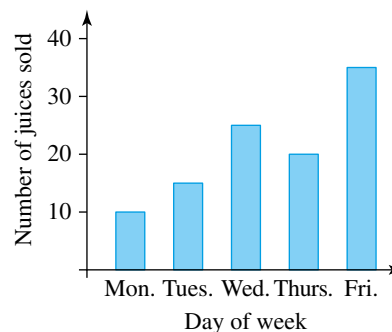
1.2 NAPLAN practice 1

These questions provide the opportunity for students to use their numeracy skills in everyday situations.

1.2.1 Set A Calculator allowed

- Which of the following is the longest time?
A. 95 000 seconds **B.** 22 hours **C.** 1800 minutes **D.** 1 day
- Anne bought 20 pens and Jo bought 30 pens. Each pen costs the same amount, and the pens cost \$75 altogether. How much did Jo pay for her pens?
- When a ball is dropped it bounces on the ground. The maximum height reached after each bounce is half the distance the ball drops. If a ball is dropped from a ladder 6m above the ground, what is the total distance travelled by the time the ball hits the ground the third time?
A. 9m **B.** 18m **C.** 10.5m **D.** 15m
- Last year at Fern Hill High School, there were 985 students. The student population has increased by 20% this year. How many students are at Fern Hill High School this year?
A. 1000 **B.** 1182 **C.** 1218 **D.** 1320
- The length of a rectangle is 9cm and its perimeter is 30cm. What is the area of the rectangle?

- The graph shows the number of juices sold each day of a particular week at a school. Calculate the mean daily number of juices sold at the school that week.



- $(10^2 + 12 \div 3) \div (-8) = ?$

A. 13 **B.** 14
C. -13 **D.** -14

- $\frac{3}{8}$ expressed as a percentage is:

A. 30.8% **B.** 13%
C. 37.5% **D.** 0.375%

- Lena wants to make 2 chocolate cakes. The recipe for a chocolate cake is:

$1\frac{1}{2}$ cups self-raising flour

$\frac{1}{2}$ cup sugar

2 tablespoons cocoa

1 egg

125 grams butter.

How much of each ingredient will she need to use?

___ cups self-raising flour

___ cup sugar

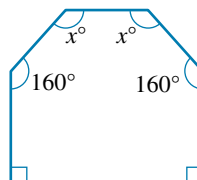
___ tablespoons cocoa

___ eggs

___ grams butter

- In the diagram shown, what is the value of x ?

A. 110° **B.** 105°
C. 120° **D.** 100°



- Express in binary form.

113_{ten}

- Jen bought a car on sale for 15% off the original price, which was \$12500. How much did she pay for the car?

A. \$1875 **B.** \$8505 **C.** \$10625 **D.** \$10 510

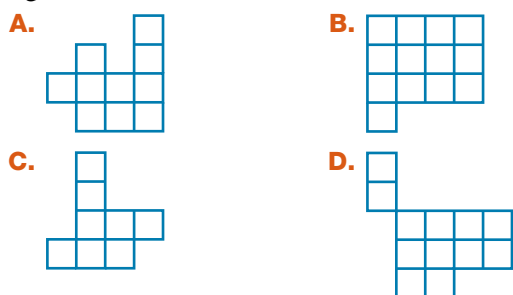
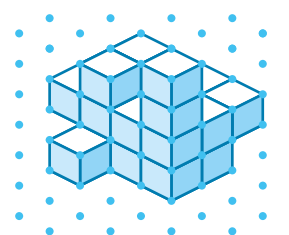
13. Connor climbs to the summit of a mountain 4250 m above sea level. Inga is scuba diving at the bottom of the ocean. The ocean floor is 27 metres below sea level. What is the vertical distance between Connor and Inga?



14. The ratio of cordial to water in Shinji's favourite drink is 2 : 5. How much water does he need to add to 80mL of cordial to make his favourite drink?

A. 100mL B. 200mL C. 300mL D. 400mL

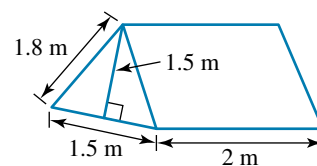
15. Which of the following represents the right side view of the 3-dimensional figure shown?



16. A rectangular paddock is surrounded by a fence. The perimeter of the paddock is 870m. If the length of the paddock is 65m longer than its width, calculate the dimensions of the paddock.

17. Calculate the area of material required to make the tent shown, including the floor.

A. 12m^2 B. 12.5m^2
C. 12.45m^2 D. 24.45m^2



18. Write the rule for the following pattern made from sticks.

Let n = number of sticks.
Let s = number of squares.

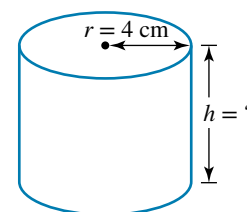


A. $n = 2s + 2$ B. $n = 3s + 1$ C. $n = 4s$ D. $n = s + 3$

19. If 1 teaspoon = 5 mL and 1 tablespoon = 20mL, how many teaspoons are there in a tablespoon?

A. 3 B. 4
C. 5 D. 2

20. The total surface area (TSA) of a cylinder can be found using the formula $\text{TSA} = 2\pi r(r + h)$ where r is the radius and h is the height of the cylinder. Find h when $r = 4\text{cm}$ and the TSA is 377cm^2 .

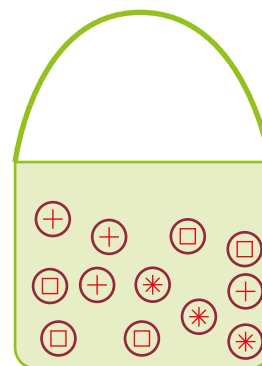


21. A car travelled at 60 kilometres per hour for 2 hours and 15 minutes. How far did it travel?
A. 165 km **B.** 120 km **C.** 75 km **D.** 135 km
22. The table below shows the times for 4 students to run 100m.

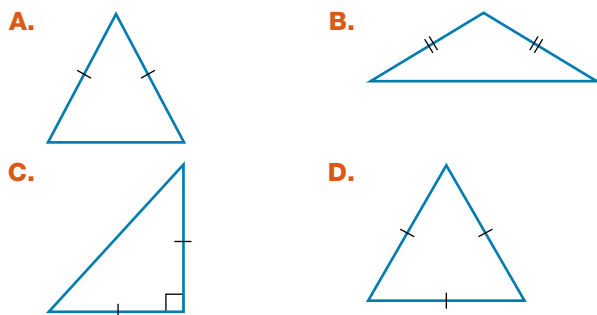
Student	Mitch	Josh	Ben	Liam
Time (s)	14.2	15.4	14.8	14.4

What is the average (mean) time the 4 students ran 100m?

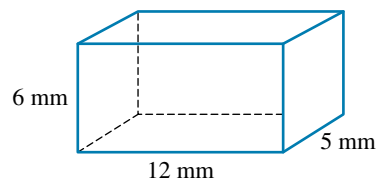
23. Which of the following is the smallest number?
A. $\sqrt[3]{6}$ **B.** 0.18
C. 0.5^2 **D.** $\frac{3}{8}$
24. Complete the following to make this equation true.
 $13 \times (15 - 1) = 180 + ?$
25. The diagram shows a bag of tokens. Aneko chooses a token from the bag without looking. What is the chance of Aneko choosing a star token?



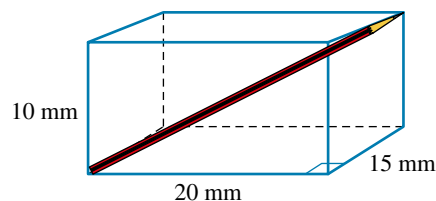
26. Which of the following diagrams shows an equilateral triangle?



27. What is the volume of this rectangular prism?



28. What is the longest pencil that can fit inside the box shown?
 (Round to 1 decimal place.)



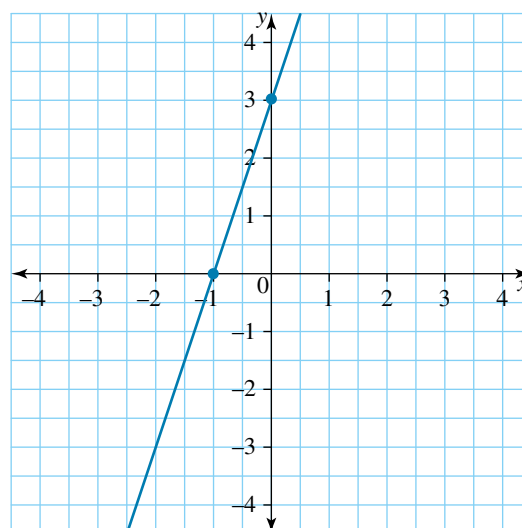
- A.** 26.9 mm
B. 25 mm
C. 22.4 mm
D. 24.6 mm
29. Which coordinates will the graph of $y = -2x + 1$ pass through?
A. (5, 11)
B. (5, -9)
C. (1, 3)
D. (1, -3)
30. If $a = -3$, then $3a^3$ equals:
A. 27
B. -27
C. 81
D. -81

1.2.2 Set B Non-calculator

- How many faces does a rectangular prism have?
A. 4 **B.** 6 **C.** 8 **D.** 12
- Oranges sell for \$2.50 per kilogram. What is the total cost of 3kg of oranges?



- A.** \$2.50 **B.** \$5.00 **C.** \$4.50 **D.** \$7.50
- A fair, six-sided die is rolled. What is the chance of it landing on a 2?
A. $\frac{1}{6}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{2}$ **D.** 2
- $7 + (2 \times 5) - 9 = ?$
A. 36 **B.** 17 **C.** 8 **D.** 7
- Evaluate 1.3×0.6 .
- What is the equation of the graph shown?



- Draw a linear graph using the table of values below.

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	-2	-3	-4

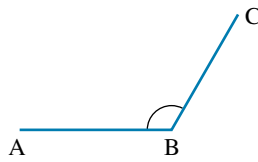
- Sivan recorded the colour of cars travelling along her street between 9am and 10am.

Colour	Blue	White	Red	Silver	Black	Gold
Number	1	6	2	5	3	3

What was the percentage of white cars?

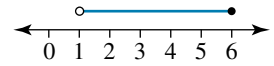
9. What type of angle is this?

- A. Acute
- B. Obtuse
- C. Reflex
- D. Straight



10. What is the inequation that represents the interval shown on the number line?

- A. $1 \leq x \leq 6$
- B. $1 \leq x < 6$
- C. $1 \geq x \geq 6$
- D. $1 < x \leq 6$



11. A store is having a 25% off sale. What would be the reduced price of an item that was originally \$120?

- A. \$40
- B. \$30
- C. \$100
- D. \$90



12. Two friends share 24 lollies. One friend gets twice as many as the other. How many lollies do they each receive?

- A. 10 and 14
- B. 8 and 16
- C. 6 and 18
- D. 4 and 20

13. Calculate the product of $\frac{3}{4}$ and $\frac{5}{6}$.

- A. $1\frac{7}{12}$
- B. $\frac{9}{10}$
- C. $\frac{19}{12}$
- D. $\frac{5}{8}$

14. The sum of 3 consecutive odd numbers is 21. Which equation best represents this situation?

- A. $x + 3 = 21$
- B. $3x = 21$
- C. $3x + 6 = 21$
- D. $6x + 3 = 21$

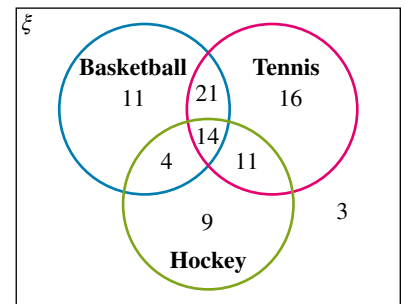
15. Find the mean and the median of the data below.

2, 1, 5, 5, 4, 0, 2, 5, 2, 6

- A. Mean = 3, median = 2
- B. Mean = 3.2, median = 4
- C. Mean = 3, median = 3
- D. Mean = 3.2, median = 3

16. The Venn diagram shows the number of students in Year 9 who play basketball, tennis and hockey. Find n (tennis $\cap$ hockey)'.

- A. 61
- B. 78
- C. 70
- D. 64



17. Evaluate $3^2 + 2^3 + 3^3$.

- A. 40
- B. 44
- C. 36
- D. 38

18. What is the missing number?

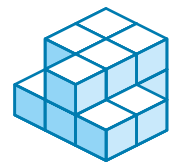
$$5000 + 700 + ? + 6 = 5746$$

19. A piece of string 30cm long forms a rectangle. If the length of the rectangle formed is 10cm, what is its width?

- A. 5 cm
- B. 10 cm
- C. 15 cm
- D. 20 cm

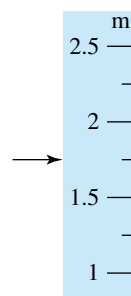
20. Which of the following is the top view of the 3-dimensional figure shown?

- A.
- B.
- C.
- D.



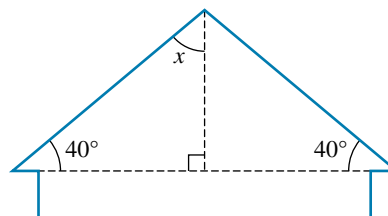
21. This ruler is used to measure the height of a teacher. How tall is the teacher?

A. 1.6m
B. 1.75m
C. 1.075m
D. 175m



22. A roof makes an angle of 40° with the ceiling.
What is the value of x ?

A. 50°
B. 40°
C. 45°
D. 90°



23. The stem-and-leaf plot shows the number of goals scored in each netball match over a season. The range is:

A. 48
B. 50
C. 14
D. 42

Stem	Leaf
1	4 9
2	5 7 9
3	1 2 4 8
4	0 4
5	4 7
6	2

24. A circular spinner with 5 equal-sized sectors is labelled from 1 to 5. The spinner is spun. What is the probability of it landing on an odd number?

A. 3
B. $\frac{1}{2}$
C. $\frac{3}{5}$
D. $\frac{1}{3}$

25. Write an expression to represent the following:

p is added to the square of q and then the sum is divided by y .

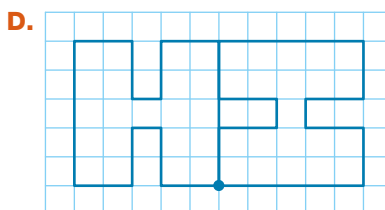
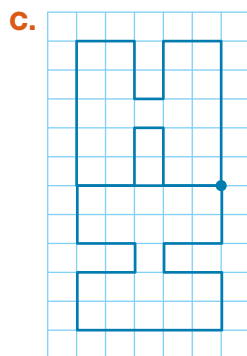
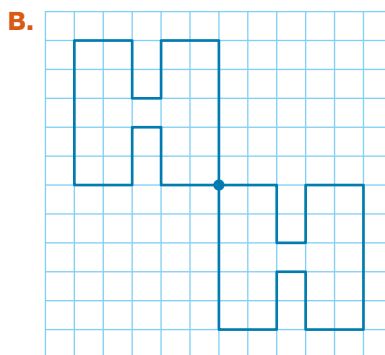
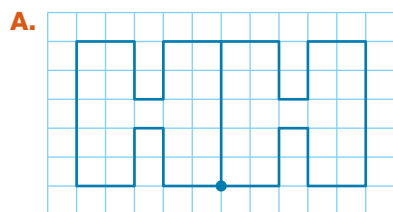
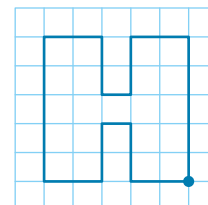
26. If one hour of parking costs \$4.50, how much will it cost to park from 9am until 2pm?

A. \$45
B. \$22.50
C. \$20
D. \$16.50

27. A map has a scale showing that 1 cm on the map represents an actual distance of 2 km. Write the appropriate scale ratio for this map.

A. 1 : 2
B. 1 : 200
C. 1 : 2000
D. 1 : 200 000

28. The shape shown is rotated 90° in a clockwise direction around the black dot.
Which of the following choices represents the result?



29. ABC is an isosceles triangle.

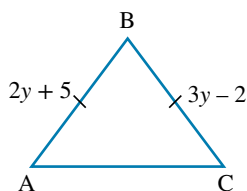
What is the value of y ?

A. 5

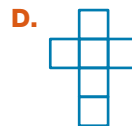
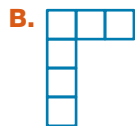
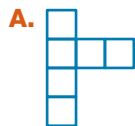
B. 7

C. 9

D. 11



30. Which of the following is the net of a cube?



1.3 NAPLAN practice 2

1.3.1 Set C Calculator allowed

1. The cube root of 64 is between:

A. 1^2 and 1^3

B. 1^2 and 3^2

C. 2^3 and 4^2

D. 3^2 and 4^2

2. A javelin is thrown 3 times. The mean distance of the 3 throws is 55 m. The fourth throw reaches 63 m. What is the mean distance after 4 throws?

A. 52 m

B. 57 m

C. 55 m

D. 60 m

3. The monthly cost (C) of renting a mobile phone is given by the equation $C = 20 + 0.2x$, where x is the call time in minutes. If February's bill was \$120, the call time was:

A. 300 minutes

B. 100 minutes

C. 500 minutes

D. 450 minutes

4. A music store has two promotions running at the same time:

- Buy 2 CDs for \$26.95 each and get the third free.
- 30% off all CDs which normally retail at \$24.95 each.

If Emma wants to buy three CDs, what is the best price she can get?

A. \$53.90

B. \$52.40

C. \$51.90

D. \$54.50

5. A number is multiplied by 3, then 12 is subtracted. The final answer is the square of 6. What is the original number?

6. A painter quotes \$448 to paint the walls of a bedroom with the following dimensions:

length = 4 m

width = 3 m

height = 3.2 m.

How much extra will he charge to paint the ceiling as well (at the same original cost per square metre)?

A. \$100

B. \$110

C. \$120

D. \$150

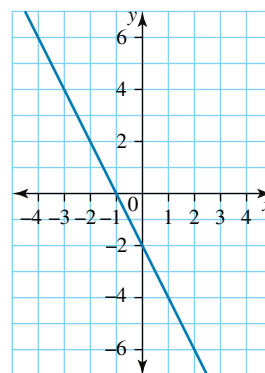


7. Solve the equation for q .

$$\frac{3 - 4q}{2} = \frac{1}{4}$$

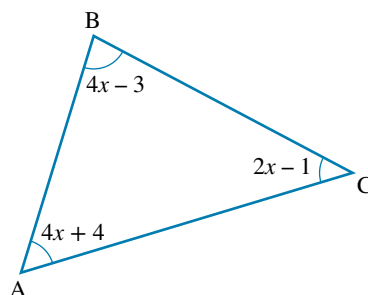
8. What is the equation of the graph shown?

- A. $y = -2x - 2$
 B. $y = 2x - 2$
 C. $y = -2x + 2$
 D. $y = 2x + 2$



9. For the triangle shown, x equals:

- A. 16
 B. 17
 C. 18
 D. 19



10. The scale $200\text{ cm} \Leftrightarrow 4\text{ m}$ simplifies to the ratio:

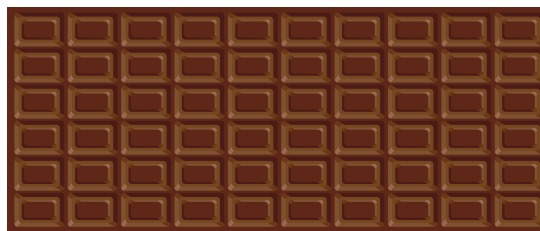
- A. 1 : 2 B. 1 : 4 C. 2 : 4 D. 50 : 1

11. A house is bought for \$450 000 and sold for \$525 000. Express the profit as a percentage of the cost price. (Round to 2 decimal places.)

- A. 14.29% B. 16.67% C. 85.71% D. 16.7%

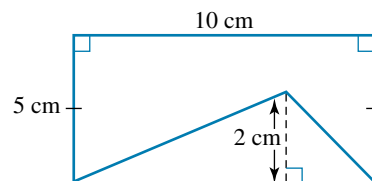
12. A 300-g block of chocolate is to be broken into its individual squares. What is the mass of 4 squares of chocolate?

- A. 10 g
 B. 16 g
 C. 5 g
 D. 20 g



13. The area of the composite shape shown is:

- A. 30 cm^2
 B. 40 cm^2
 C. 50 cm^2
 D. 45 cm^2



14. What is the next term in the following pattern?

1, 4, 9, 16, ...

- A. 20
 B. 25
 C. 29
 D. 36

15. A cyclist travels 15 km in 30 minutes. The speed (in km/h) she is travelling is:

- A. 40 km/h B. 60 km/h
 C. 80 km/h D. 30 km/h

16. The intersection point for $y = 2x + 4$ and $x + y = 1$ is:

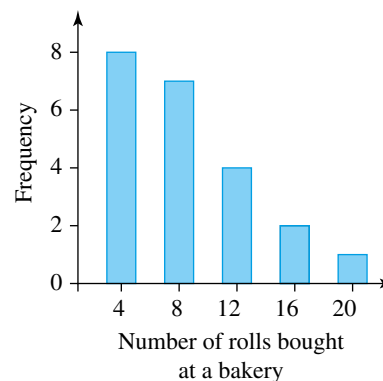
- A. $(-2, 1)$
 B. $(-1, -2)$
 C. $(-1, 2)$
 D. $(2, 1)$



17. The factorised form of $2x^2y^3 - 6xy^2$ is:
- A. $2xy^2(xy - 3)$ B. $2xy(x^2y - 3)$ C. $2xy(xy - 6)$ D. $2x^2y(xy - 3)$

18. The graph shows the frequency of the number of bread rolls bought at a bakery. What is the total number of rolls bought?

- A. 200
B. 188
C. 150
D. 168



19. What are the mean and median of the data shown in the table?

Score (x)	Frequency
1	1
2	2
3	3
4	4

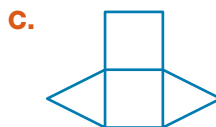
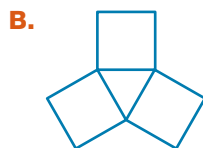
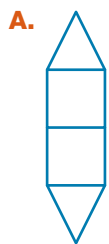
- A. The mean is 3 and median is 4.
B. The mean is 3 and median is 3.
C. The mean is 4 and median is 3.
D. The mean is 4 and median is 4.

20. Which number is exactly halfway between 3.25 and 4.75?

- A. 3.95 B. 4.15 C. 4.25 D. 4

21. Write a fraction equivalent to 0.75.

22. Which diagram is the net of a square-based pyramid?



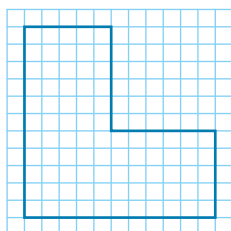
23. The table summarises how much time Lian spent on her Maths project.

What was the average amount of time each day Lian worked on her project?

- A. 25 minutes
B. 30 minutes
C. 43 minutes
D. 1 hour

Time spent on project	
Day	Time
Monday	30 minutes
Tuesday	15 minutes
Wednesday	$1\frac{1}{4}$ hours
Thursday	50 minutes
Friday	45 minutes

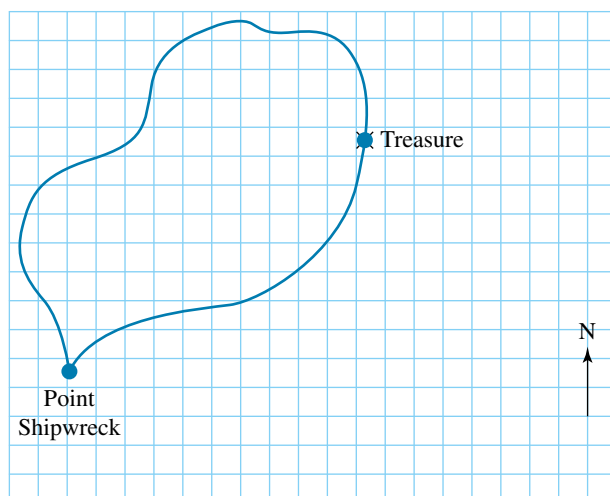
24. Draw a line of symmetry through the diagram.



25. If $x = -2$, what is the value of $\frac{2x}{3x - 2}$?

- A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{2}{3}$ D. $\frac{1}{3}$

26. A map of Fern Island is shown below.

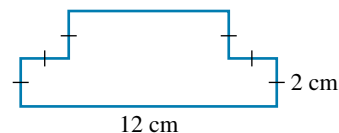


What direction is the treasure from Point Shipwreck?

27. The shape at right is to be enlarged by a factor of 3.

The new area will be:

- A. 3 times the original area B. 6 times the original area
C. 9 times the original area D. 12 times the original area



28. Complete the table of values for the rule $y = 3x^2$.

x	-3	-2	-1	0	1	2	3
y							

29. The population of a city is expected to increase by 2.5% each year for the next 5 years. If this city's current population is 300 000, what is it expected to be in 5 years' time?

- A. 339 422 B. 755 000 C. 3 750 345 D. 355 406

30. Find the area of material (m^2) needed to make a circular cushion with a diameter of 110 cm.

Note: Two pieces of material are cut out and sewn together to make the cushion.

- A. 2.5m^2 B. 2.0m^2 C. 1.9m^2 D. 1.5m^2

1.3.2 Set D Non-calculator

1. What is the equivalent fraction to $3\frac{2}{5}$?

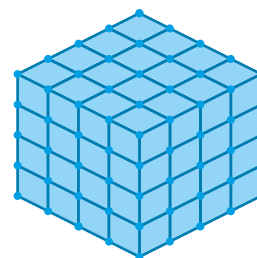
- A. $\frac{17}{3}$ B. $\frac{11}{5}$ C. $\frac{17}{5}$ D. $\frac{11}{2}$

2. The value of $\left[\left(\frac{1}{2}\right)^2 + 2\frac{1}{4}\right] \div \frac{5}{7}$ is:

- A. $\frac{25}{14}$ B. $3\frac{1}{2}$
C. $4\frac{1}{2}$ D. $2\frac{1}{3}$

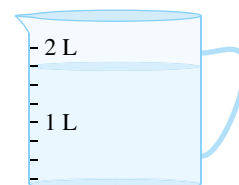
3. A large cube, shown at right, is painted blue. How many cubes, inside this large cube, have no painted faces?

- A. 8 B. 16
C. 36 D. 64

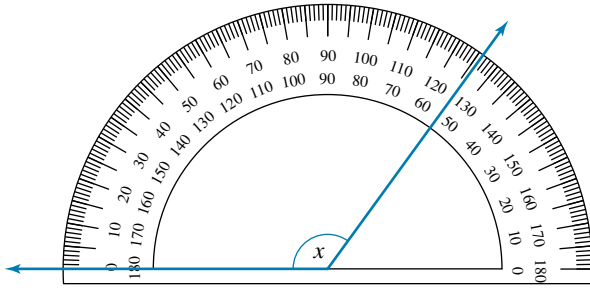


4. A glass of water (250 mL) is poured from the jug. How much water is left in the jug?

- A. 1.75 L B. 1.5 L
C. 1.25 L D. 1 L



5. Measure the size of the unknown angle, x , using the protractor.

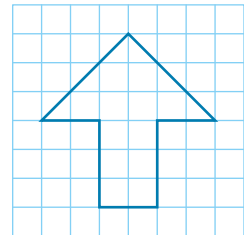
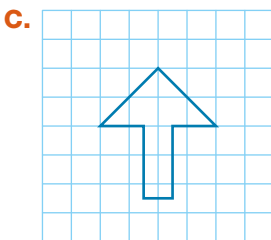
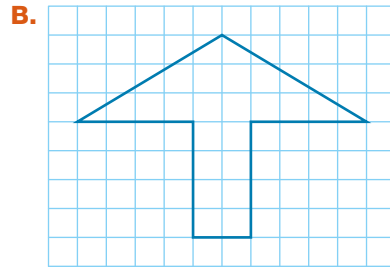
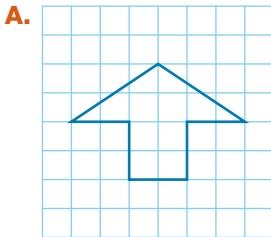


- A. 125° B. 66° C. 54° D. 126°
6. $14.076 - 9.25$ equals:
A. 4.826 B. 4.51 C. 4.926 D. 0.482

7. Jake recorded his classmates' preferred ice-cream flavours in the table below.

Ice-cream flavour	Number of students
Chocolate	14
Vanilla	3
Strawberry	7
Choc-mint	6

- What percentage of students prefers choc-mint flavoured ice-cream?
- A. 60% B. 20% C. 6% D. 25%
8. A car travels at 75 kilometres per hour. How far will it travel in 4 hours?
A. 300km B. 150km C. 100km D. 250km
9. Which of the following diagrams shows the arrow at right multiplied by a scale factor of $\frac{1}{2}$?



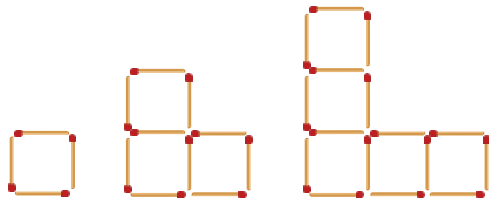
10. $\overline{XZ}$ is 32 cm. $\overline{XY}$ is a quarter of the distance of $\overline{XZ}$. Find $\overline{YZ}$.



- A. 8 cm B. 16 cm C. 20 cm D. 24 cm

11. How many sticks would be needed to make the 5th shape in this pattern?

A. 10
B. 16
C. 28
D. 34



12. To convert from degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$), the following formula is used:

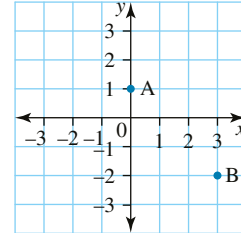
$$C = \frac{5}{9}(F - 32).$$

What is the Fahrenheit temperature equivalent to 35°C ?

A. 86°F B. 95°F C. 70°F D. 100°F

13. The two points A and B are on which line?

A. $y = 1 - x$ B. $y = x + 1$
C. $y = x - 1$ D. $y = 3 - x$



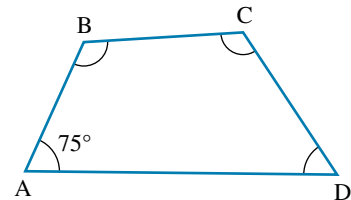
14. Solve the equation $\frac{4m}{5} + 7 = 11$.

A. $m = 5$ B. $m = 4$
C. $m = -5$ D. $m = 2$

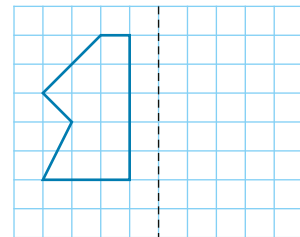
15. A football team played 22 games during a season. The team won 4 more games than it lost. How many games did the team win?

A. 9 B. 11 C. 13 D. 14

16. $\angle\text{BAD}$ and $\angle\text{BCD}$ are supplementary angles. $\angle\text{ABC}$ is 130° . Find the size of $\angle\text{ADC}$.



17. Draw the reflection of the shape shown at right.



18. If $p = 3$ and $q = -5$, evaluate $q^2 - 2p$.

A. -31 B. -19 C. 21 D. 19

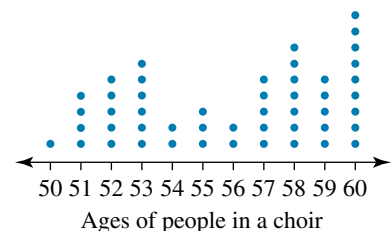
19. The area of a circle is $A = \pi r^2$. Which value best estimates the area of a circle with a radius of 5 cm?

A. 75cm^2 B. 15cm^2 C. 50cm^2 D. 7.5cm^2

20. This dot plot shows the ages of people in a choir.

What is the most common age of the choir members?

A. 50 B. 58
C. 59 D. 60



21. What is the probability of getting Tails when a fair coin is tossed?

A. 0 B. $\frac{1}{2}$
C. 1 D. $\frac{1}{3}$

22. Insert ' $<$ ' or ' $>$ ' to make this statement true.

$$4^3 \underline{\hspace{1cm}} 2^5$$

23. The following scores were recorded in a test: $\frac{18}{20}$ and 88%.

Which is the higher score?

24. The cost of two different books is in the ratio 4 : 5. If the more expensive book costs \$25, what does the cheaper book cost?

A. \$5
B. \$10
C. \$15
D. \$20

25. Calculate the value of $3\text{m}75\text{cm} + 5\text{m}36\text{cm}$.

A. 8m95cm
B. 9m11cm
C. 9m20cm
D. 8m11cm

26. There are 60 squares in a chocolate block. If Stef eats $\frac{2}{3}$ of the block, how many squares are left?

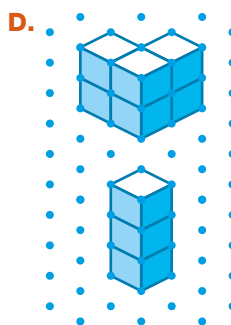
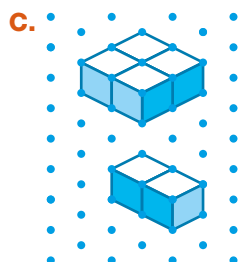
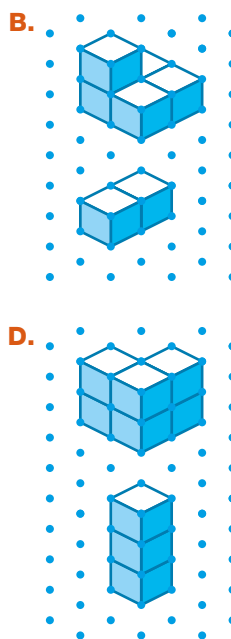
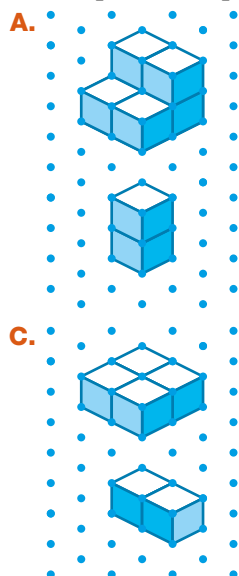
A. 30
B. 25
C. 20
D. 15

27. The length of a rectangular sporting field is four times its width. If the perimeter of the field is 200m, find the length and width.

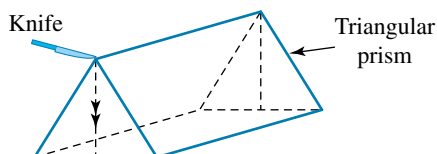
28. A bag of marbles contains 22 red, 20 yellow, 15 green and 18 blue marbles. Darcy chooses one marble without looking. What is the chance it will be green?

A. $\frac{1}{3}$
B. $\frac{1}{4}$
C. $\frac{1}{5}$
D. $\frac{1}{6}$

29. Which pair of shapes fits together to make a cube?



30. What shape is revealed when a triangular prism is cut vertically?



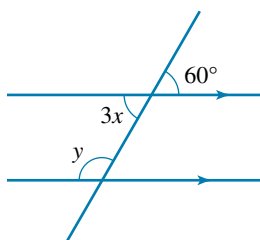
A. Triangle
B. Square
C. Rectangle
D. Trapezium



1.4 NAPLAN practice 3

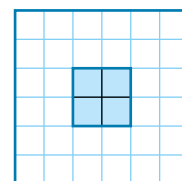
1.4.1 Set E Calculator allowed

- The volume of a cylindrical glass with diameter 7.2cm and height 15.4cm is:
A. 2L **B.** 627cm³ **C.** 550cm³ **D.** 750mL
- Use the diagram below to determine the values of x and y .



- A square window is to be inserted into a square wall. The side length of the window is 70cm. What is the area of the remaining wall?

- A.** 39200m² **B.** 3.92m²
C. 4.41m² **D.** 6.8m²



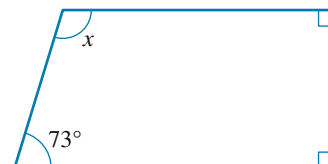
- The formula $y = \frac{3x - 4}{5}$ rearranged to make x the subject is:

- A.** $x = \frac{5y - 3}{4}$ **B.** $x = \frac{4y - 5}{3}$ **C.** $x = \frac{3y + 4}{5}$ **D.** $x = \frac{5y + 4}{3}$

- How many hours are there in 4 weeks and 3 days?

- The value of x in the diagram at right is equal to:

- A.** 107° **B.** 110°
C. 77° **D.** 90°



- An unknown number is multiplied by 6 and then added to 4. The result is 46. Which equation best describes this?

- A.** $6 + 4x = 46$ **B.** $46 + 6x = 4$ **C.** $6x + 4 = 46$ **D.** $6(x + 4) = 46$

- The number line shown can be described by the equation:

- A.** $x < -1$ **B.** $x \geq -1$
C. $x > -1$ **D.** $x < 1$



- Arrange the following in descending order.

2.202, 0.12, $\left(\frac{1}{2}\right)^2$, $\sqrt{\frac{1}{4}}$, 0.01

- A car uses 10L of fuel to travel 80km.

How much fuel is needed to travel 400km?

- A.** 30L **B.** 40L
C. 50L **D.** 60L

- A length of rope is 7.5m. It is to be cut into 2 pieces in the ratio of 2 : 3. What will be the two new lengths?

- A.** 2.5m and 5m
B. 3m and 4.5m
C. 3.5m and 4m
D. 1m and 6.5m

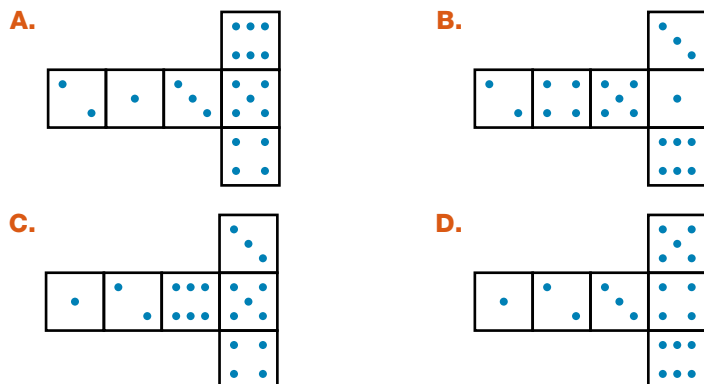


12. A 5-kg bag of potatoes cost \$7.99. The cost of loose potatoes is \$1.55 per kg.

Is it cheaper to buy:

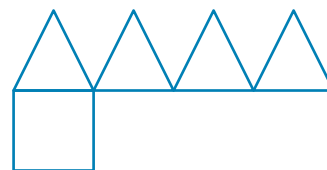
- A.** 5 kg of loose potatoes? **B.** a 5-kg bag of potatoes?

13. Which diagram of the net of a cube has opposite faces that sum to 7?



14. Which of the following shapes is represented by the net shown at right?

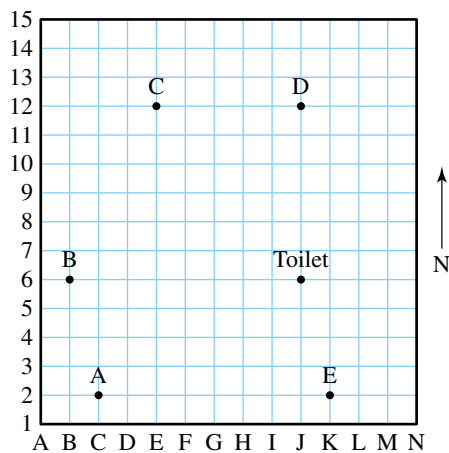
- A.** A triangular prism
B. A triangular pyramid
C. A square pyramid
D. A cube



15. If $A = \{\text{odd numbers between 0 and 10}\}$ and $B = \{3, 6, 9\}$, then $A \cup B$ is equal to:

- A.** $\{1, 3, 5, 7, 9\}$ **B.** $\{3, 9\}$ **C.** $\{2, 4, 8, 10\}$ **D.** $\{1, 3, 5, 6, 7, 9\}$

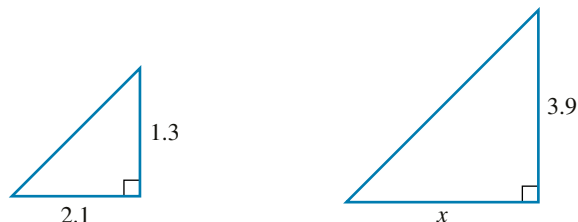
16. The following map shows campsites in a campground.



Jon is camped at grid reference C2. If he is facing north and looks due east, what is he looking at?

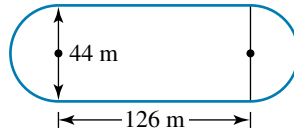
- A.** Toilets **B.** Campsite C **C.** Campsite A **D.** Campsite E

17. Using the similar triangles shown below, find the value of x .



- A.** 1.3 **B.** 2.6 **C.** 6.3 **D.** 9.3

18. The perimeter of the race track shown below is:

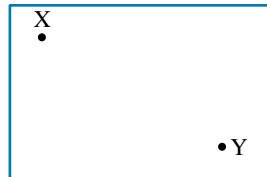


- A. 390.23 m B. 395.2 m C. 405 m D. 309.35 m
19. There are 1500 people in a fun run; 1125 are over 18 years of age. What percentage of people are aged 18 years and under?
- A. 25%
B. 55%
C. 45%
D. 35%
20. The vertices of a triangle are A (1, 1), B (3, 4) and C (4, 2). Give the vertices of the triangle if it is reflected in the y-axis.
21. The data in the table below shows the number of books read by 6 students in Year 9 during one month.

Number of books read	3, 6, 8, 4, 4, 2

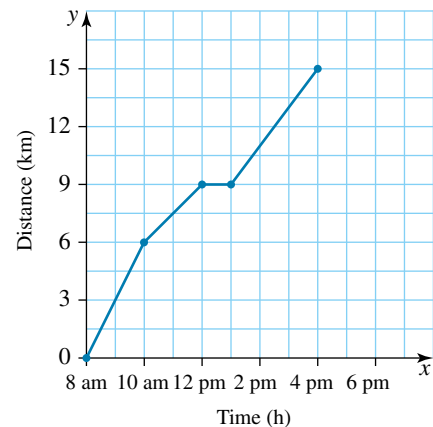
Which of the following is true?

- A. Mean = median B. Median = mode C. Mean = mode D. Mean < mode
22. A bike worth \$420 has been reduced by \$ x . The new value of the bike (in dollars) is:
- A. $420 - x$ B. $420x$
C. $\frac{420}{x}$ D. $420 + x$
23. X and Y are 12.5 cm apart on a map. Calculate the actual distance between X and Y using the given key.

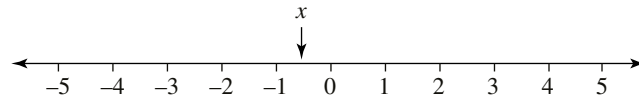


Key: 1 cm $\Leftrightarrow$ 6 km

- A. 12.5 km B. 7.5 km
C. 75 km D. 42 km
24. Pat bought a packet of 100 marbles for \$5.00. The cost of one marble is:
- A. \$0.50 B. \$0.20
C. \$0.02 D. \$0.05
25. A bag contains 6 red, 8 green and 4 blue counters. What is the ratio of red to blue counters?
- A. 3 : 2 B. 2 : 3
C. 3 : 4 D. 2 : 1
26. The graph shows the distance travelled by a hiker. The average speed of the hiker is closest to:
- A. 15 km/h B. 1 km/h
C. 2 km/h D. 8 km/h



27. What is the value of x on the number line shown?



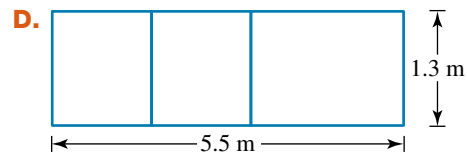
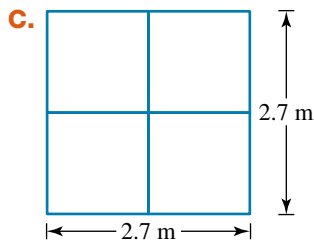
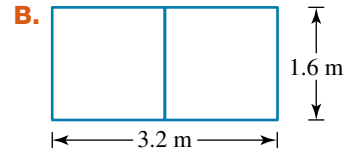
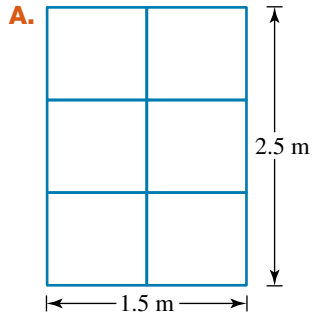
A. -1.5

B. -0.5

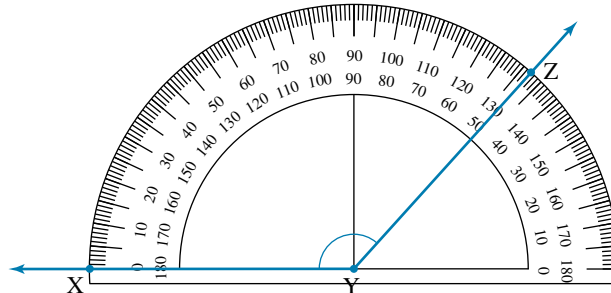
C. 0.5

D. 1.5

28. Which window has the largest area?



29. What is the size of $\angle XYZ$?



30. $24 \times ? = 16$

The missing value is:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

1.4.2 Set F Non-calculator

1. The sum of 11 and a number, r , is 37. What is r ?

A. 27

B. 14

C. 26

D. 8

2. $\sqrt{150}$ is between:

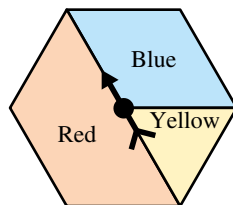
A. 8 and 10

B. 10 and 12

C. 12 and 14

D. 14 and 16

3. The arrow on the spinner is spun 50 times. Which table most likely represents the results?



A.

Colour	Number of spins
Red	25
Blue	17
Yellow	8

B.

Colour	Number of spins
Red	16
Blue	25
Yellow	9

C.

Colour	Number of spins
Red	8
Blue	15
Yellow	27

D.

Colour	Number of spins
Red	25
Blue	12
Yellow	13

4. You are going to share $3\frac{1}{4}$ pizzas with your friends. How many $\frac{1}{6}$ sized slices can you cut from the pizzas?

A. 20

B. $19\frac{1}{2}$

C. 18

D. $16\frac{1}{2}$

5. $2 \times 2 \times 2 \times 2 \times 2$ is equal to:

A. $2^2 \times 2^2$

B. 8×2

C. 2×5

D. $4^2 \times 2$

6. There are 12 boys and 14 girls in a class. What fraction of the class is boys?

A. $\frac{6}{13}$

B. $\frac{6}{7}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

7. Of 100000 newspapers printed, 5% are delivered to homes, with the remaining papers going to the newsagent. How many papers are delivered to the newsagent?

A. 5000

B. 50000

C. 95000

D. 75000

8. A cricketer played 10 innings and scored the following number of runs.

45, 0, 33, 7, 11, 25, 57, 80, 33, 47

What will change if the score of 80 is removed from the set?

A. Mean and range

B. Mean and mode

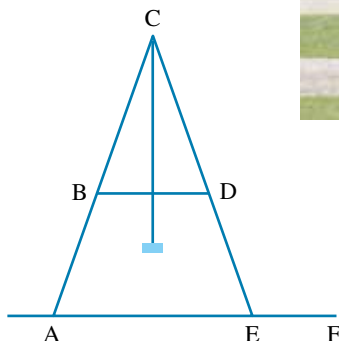
C. Mean and median

D. Mean only

9. The side view of an A-frame swing looks like this.

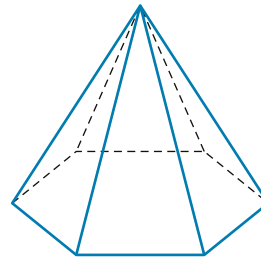
$\angle BCD = 68^\circ$.

Find $\angle BDC$ and $\angle DEF$.

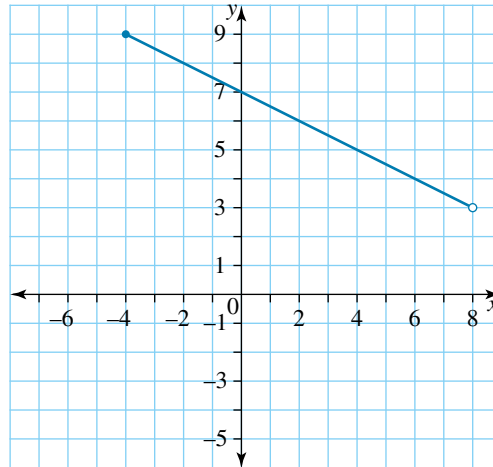


10. How many faces does this pyramid have?

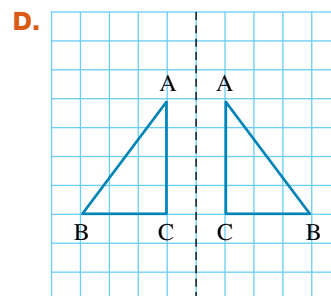
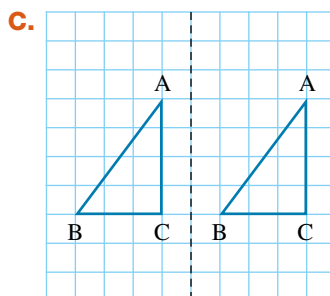
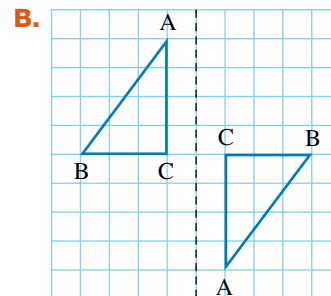
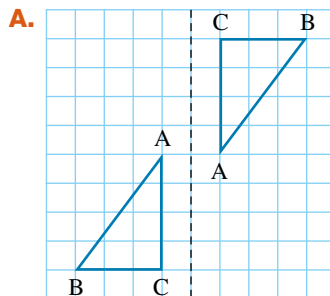
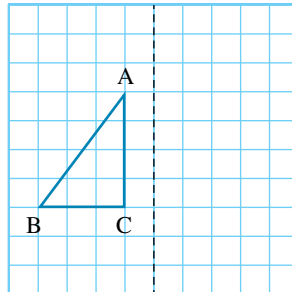
- A. 4
- B. 5
- C. 6
- D. 7



11. For the following graph, state the domain (possible x -values) and range (possible y -values).



12. Which of the following represents the reflection of this triangle in the mirror shown below?



13. Which of the following is the rearranged version of the formula $m = \frac{n - qr}{p}$?

- A. $p = mn - qr$ B. $n = mp + qr$ C. $n = \frac{m}{p} + \frac{q}{r}$ D. $n = \frac{m + qr}{p}$

14. If a number is halved and then increased by 4, it will be equal to 2 less than the original number. What is the original number?

- A. 8 B. 10 C. 12 D. 20

15. When $a = 1$ and $b = -1$, the value of $(a - b)^2$ is:

- A. 0 B. 1 C. 2 D. 4

16. Which linear graph joins the points $(5, -9)$ and $(-3, 7)$?

- A. $y = x + 4$ B. $y = 2x + 1$ C. $y = -x - 4$ D. $y = -2x + 1$

17. A T-shirt worth \$35 is discounted by 15%. Which of the following calculations gives the new price?

- A. 35×0.85 B. 35×0.15 C. $35 + 0.15$ D. $35 - 15$

18. A fair coin is tossed twice. Write down the four possible outcomes.

19. Which of these patterns does not possess a line of symmetry?



20. Arrange the following in ascending order.

$$\frac{1}{2}, 0.2^2, \sqrt{4}, \left(\frac{1}{2}\right)^2$$

21. The solution to $\frac{4 - x}{3} - 5 > 3$ is:

- A. $x > 20$ B. $x > -20$ C. $x < -20$ D. $x < 20$

22. Fill in the missing value to make the following equation true.

$$2(2h - 1) = 3h - 2 + ?$$

23. Evaluate $\sqrt{(-2)^2 + (-4)^2} + 5$.

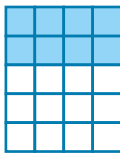
- A. 5 B. 25 C. 10 D. 15

24. What is the probability of choosing a 5 in a regular deck of cards (no jokers)?

- A. $\frac{5}{52}$ B. $\frac{1}{4}$ C. $\frac{1}{13}$ D. $\frac{1}{2}$

25. What percentage of this shape is shaded?

- A. 8%
B. 20%
C. 50%
D. 40%



26. The probability of hiring a certain coloured bicycle is shown in the table below.

Colour	White	Red	Silver	Blue
Probability	0.55	0.25	?	0.05

What is the probability of hiring a silver bicycle if there are no colours other than the ones shown?

- A. 0.15 B. 0.25 C. 0.2 D. 0.5

27. Two people buy a \$5 ticket in a \$200 000 lottery. They each contribute \$2 and \$3. How much would each collect if they won?

28. Andrew rides x km to school. Isabelle rides 3 km more. Together they ride 13 km. Which equation represents this situation?

- A. $x - 3 = 13$ B. $2x + 3 = 13$ C. $x + 3 = 13$ D. $2x - 3 = 13$



29. The maximum temperature recorded for a June day was 14°C . The minimum temperature recorded that day was -2°C . What is the difference between the maximum and minimum temperatures recorded that day?
30. The mean, median and mode for the following set of data is:
0, 1, 1, 2, 3, 3, 3, 3
- A. Mean = 2, median = 3, mode = 1
 - B. Mean = 2, median = 3, mode = 3
 - C. Mean = 2, median = 2.5, mode = 3
 - D. Mean = 3, median = 2.5, mode = 3

1.5 Review

Throughout this topic you have practised completing NAPLAN tests. It is important to be familiar with how the NAPLAN tests are run and what to expect on the day of the tests.

Answers

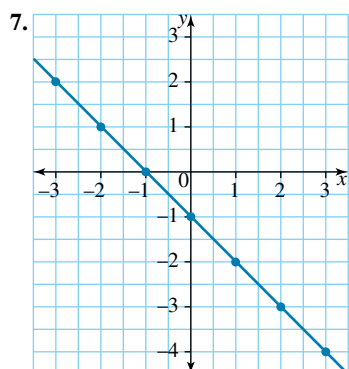
Topic 1 Naplan practice

Exercise 1.2.1 Set A Calculator allowed

1. C	2. \$45	3. D	4. B
5. 54cm^2	6. 21	7. C	8. C
9. 3, 1, 4, 2, 250	10. A	11. 1110001_2	12. C
13. 4277m	14. B	15. C	16. $l = 250\text{ m}$ $w = 185\text{ m}$
17. C	18. B	19. B	20. $h \approx 11\text{ cm}$
21. D	22. 14.7s	23. B	24. 2
25. C	26. D	27. 360mm^3	28. A
29. B	30. D		

Exercise 1.2.2 Set B Non-calculator

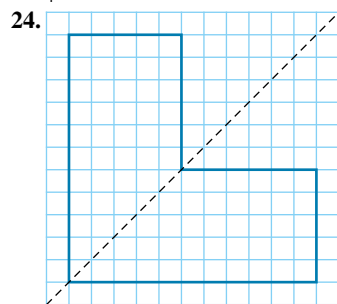
1. B	2. D
3. A	4. C
5. 0.78	6. C



8. 30%	9. B	10. D	11. D	12. B
13. D	14. C	15. D	16. D	17. B
18. 40	19. A	20. C	21. B	22. A
23. A	24. C	25. $\frac{q^2 + p}{y}$	26. B	27. D
28. D	29. B	30. D		

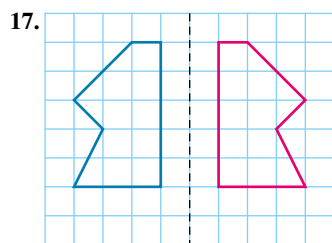
Exercise 1.3.1. Set C Calculator allowed

1. B	2. B	3. C	4. B	5. 16
6. C	7. $q = \frac{5}{8}$	8. A	9. C	10. A
11. B	12. D	13. B	14. B	15. D
16. C	17. A	18. B	19. B	20. D
21. $\frac{3}{4}$	22. D	23. C		



25. A	26. North-east	27. C
28. 27, 12, 3, 0, 3, 12, 27	29. A	30. C

1. C	2. B	3. A	4. B
5. D	6. A	7. B	8. A
9. D	10. D	11. C	12. B
13. A	14. A	15. C	16. 50°



- ### Exercise 1.4.1 Set E Calculator allowed

- ### Exercise 1.4.2 Set F Non-calculator

1. C	2. C	3. A	4. B
5. D	6. A	7. C	8. A
9. $\angle BDC = 56^\circ$ $\angle DEF = 124^\circ$	10. D	11. Domain: $-4 \leq x < 8$ Range: $3 < y \leq 9$	
12. D	13. B	14. C	15. D
16. D	17. A	18. HH, HT, TH, TT	19. B
20. $0.2^2, \left(\frac{1}{2}\right)^2, \frac{1}{2}, \sqrt{4}$	21. C	22. h	23. A
24. C	25. D	26. A	27. \$80000 and \$120000
28. B	29. 16°C	30. C	

TOPIC 2

Financial mathematics

[Stage 5.1]

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

2.1.1 Why learn this?

Unfortunately for most of us, money is not in endless supply. If we monitor our income and expenses we can make our money go further. Understanding budgets and investments can help us to keep track of our money and reach our financial goals.



DISCUSSION

Is it important for individuals and society that every person understands how to manage their money or should it be left to financial experts?

LEARNING SEQUENCE

- 2.1 Overview
- 2.2 [Stage 5.1] Salaries and wages
- 2.3 [Stage 5.1] Special rates
- 2.4 [Stage 5.1] Piecework
- 2.5 [Stage 5.1] Commissions and royalties
- 2.6 [Stage 5.1] Loadings and bonuses
- 2.7 [Stage 5.1] Taxation and net earnings
- 2.8 [Stage 5.1] Simple interest
- 2.9 [Stage 5.1] Compound interest
- 2.10 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- solves financial problems involving earning, spending and investing money MA5.1-4NA

CONTENT DESCRIPTIONS

Students:

Solve problems involving earning money (NSW)

Solve problems involving simple interest (ACMNA211)

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

2.2 Salaries and wages

[Stage 5.1]

2.2.1 Salaries and wages

- Employees may be paid for their work in a variety of ways. Most employees receive either a **wage** or a **salary**.

2.2.2 Salaries

- A salary is a fixed annual (yearly) amount, usually paid fortnightly or monthly. A person who receives a salary is paid to do a job, regardless of the number of hours worked.

WORKED EXAMPLE 1

Susan has an annual salary of \$63 048.92. How much is she paid:

a weekly

b fortnightly

c monthly?

THINK

- a** 1 Annual means per year, so divide the salary by 52 because there are 52 weeks in a year.
2 Write the answer in a sentence.
- b** 1 There are 26 fortnights in a year, so divide the salary by 26.
2 Write the answer in a sentence.
- c** 1 There are 12 months in a year, so divide the salary by 12.
2 Write the answer in a sentence.

WRITE

- a** $63\,048.92 \div 52 \approx 1212.48$
Susan's weekly salary is \$1212.48.
- b** $63\,048.92 \div 26 \approx 2424.96$
Susan's fortnightly salary is \$2424.96.
- c** Monthly salary = $63\,048.92 \div 12 \approx 5254.08$
Susan's monthly salary is \$5254.08.

2.2.3 Wages

- A wage is based on a fixed rate per hour. Hours outside the normal work period are paid at a higher rate.

WORKED EXAMPLE 2

Frisco has casual work at a fast-food store. He is paid \$12.27 per hour Monday to Saturday and \$24.54 per hour on Sunday. Calculate his wage for a week in which he worked from 5.00 pm to 10.00 pm on Friday and from 6.00 pm to 9.00 pm on Sunday.

THINK

- 1 Work out the number of hours Frisco worked each day.
He worked 5 hours on Friday and 3 hours on Sunday.
- 2 Find the total amount earned.
- 3 Write the answer in a sentence.

WRITE

- Friday: $5 \times 12.27 = 61.35$
- $61.35 + 73.62 = 134.97$
- Frisco's wage was \$134.97.

ACTIVITY: COMPARING PAY RATES AND CONDITIONS

Working in small groups, collect a number of job advertisements. They should be obtained from a mixture of print and digital sources. Compare the pay rates and conditions for the different positions and present a report to the class.

learnon RESOURCES — ONLINE ONLY

Interactivity: Salaries (int-6067)

Digital doc: SkillsHEET Converting units of time (doc-10849)

Digital doc: SkillsHEET Multiplying and dividing a quantity (money) by a whole number (doc-10850)

Digital doc: SkillsHEET Multiplying and dividing a quantity (money) by a fraction (doc-10851)

Digital doc: SkillsHEET Increasing a quantity by a percentage (doc-10852)

Digital doc: SkillsHEET Adding periods of time (doc-10853)

Exercise 2.2 Salaries and wages

Individual pathways

PRACTISE

Questions:
1–6, 8, 11, 13

CONSOLIDATE

Questions:
1–6, 7, 9, 10, 12, 14, 15, 17

MASTER

Questions:
1–6, 9, 12–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE1** Johann has an annual salary of \$57 482. How much is he paid:
a. weekly b. fortnightly c. monthly?
- Anna earns \$62 300 per year. How much does she earn:
a. weekly b. fortnightly c. monthly?
- Find the annual salary of workers with the following weekly incomes.
a. \$368 b. \$892.50 c. \$1320.85
- How much is earned per year by those paid fortnightly salaries of:
a. \$995 b. \$1622.46 c. \$3865.31?
- Which of each pair is on the higher salary?
a. \$3890 per month or \$45 700 per year
b. \$3200.58 per fortnight or \$6700 per month
- Find the hourly rate for these workers.
a. Rahni earns \$98.75 for 5 hours.
b. Francisco is paid \$54.75 for $4\frac{1}{2}$ hours.
c. Nhan earns \$977.74 for a 38-hour week.
d. Jessica works $7\frac{1}{2}$ hours a day for 5 days to earn \$1464.75.

7. Henry is a second-year apprentice motor mechanic. He receives the award wage of \$12.08 per hour. Jenny, a fourth-year apprentice, earns \$17.65 per hour.

- a. How much does Henry earn in a 38-hour week?
- b. How much more does Jenny earn in the same period of time?

8. **WE2** Juan has casual work for which he is paid \$13.17 per hour Monday to Saturday and \$26.34 per hour on Sundays. Calculate his total pay for a week in which he worked from 11.00 am to 5.00 pm on Thursday and from 2.00 pm to 7.00 pm on Sunday.

9. Mimi worked the following hours in one week.

Wednesday 5.00 pm to 9.00 pm
Thursday 6.00 pm to 9.00 pm
Friday 7.00 pm to 11.00 pm

If her pay is \$21.79 per hour up to 9.00 pm and \$32.69 per hour after that, what is her total pay?

10. Who earns more money each week: Rhonda, who receives \$38.55 an hour for 38 hours work, or Rob, who receives \$41.87 an hour for 36 hours work?

11. Glenn is a chef and receives \$1076.92 for a week in which he works 35 hours. What is his hourly rate of pay?

12. Julie is considering two job offers for work as a receptionist. Job A pays \$878.56 for a 38-hour working week. Job B pays \$812.16 for a 36-hour working week. Which job has the higher hourly rate of pay?

13. Russell and Gabrielle go to work in different department stores. Russell is paid \$981.77 per week. Gabrielle is paid \$26.36 per hour. How many hours must Gabrielle work to earn more money than Russell?

14. Calculate what pay each of the following salary earners will receive for each of the periods specified.
- a. Annual salary \$83 500, paid each week
 - b. Annual salary \$72 509, paid each fortnight
 - c. Annual salary \$57 200, paid each week
 - d. Annual salary \$105 240, paid each month



15. **MC** When Jack was successful in getting a job as a trainee journalist, he was offered the following choice of four salary packages. Which should Jack choose? Show your working.

-
- A cartoon illustration of a man with a beard and brown hair, wearing a yellow sweater and brown pants, sitting at a wooden desk. He has his arms raised in the air with a joyful expression. On the desk is a laptop, a black desk lamp, and a coffee cup. He is sitting on a grey office chair, and his legs are crossed at the ankles.

[Stage 5.1]

- A normal Australian working week is 38 hours. Wage earners who work extra hours are ‘working overtime’.
- **Overtime** is paid when a wage earner works more than the regular hours each week. When an employee works overtime a higher rate is paid. This higher rate of pay is called a penalty rate. The rate is normally calculated at either time and a half, which means that the person is paid $1\frac{1}{2}$ times the normal rate of pay; double time, which means that the person is paid twice the normal rate of pay; or double time and a half, which means that the person is paid $2\frac{1}{2}$ times the normal rate of pay.
- A person may also be paid these overtime rates for working at unfavourable times, such as at night or during weekends.
- To calculate the hourly rate earned when working overtime, we multiply the normal hourly rate by the overtime factor, which is $1\frac{1}{2}$ for overtime, 2 for double time and $2\frac{1}{2}$ for double time and a half.

WORKED EXAMPLE 3

Ursula works as a waitress and earns \$23.30 per hour. Last week she received the normal rate for 30 hours of work as well as time and a half for 3 hours of overtime and double time for 5 hours of work on Sunday. What was her total wage?



THINK

- 1 Calculate Ursula's normal pay.
- 2 Calculate Ursula's pay for 3 hours at time and a half.
- 3 Calculate Ursula's pay for 5 hours at double time.
- 4 Find the total amount.
- 5 Write the answer in a sentence.

WRITE

Normal pay: $30 \times 23.30 = 699.00$

Overtime: $3 \times 1.5 \times 23.30 = 104.85$

Sunday: $5 \times 2 \times 23.30 = 233.00$

Total = 1036.85

Ursula's total wage was \$1036.85.

DISCUSSION

By how much would Ursula's total wage vary if special rates did not exist?

2.3.2 Time sheets and pay slips

- Employers often use records called time sheets to monitor the number of hours worked by each employee.
- Details of the hours worked and the rate of pay are given to each employee on a pay slip, which they receive with their wages.

WORKED EXAMPLE 4

Fiona works in a department store, and in the week before Christmas she works overtime. Her time sheet is shown below. Fill in the details on her pay slip.

	Start	Finish	Normal hours	O'time 1.5
M	9.00	15.00	6	
T	9.00	17.00	8	
W	9.00	17.00	8	
T	9.00	19.00	8	2
F	9.00	19.00	8	2
S				

Pay slip for: Fiona BLACK	Week ending December 21
Normal hours	
Normal rate	\$17.95
Overtime hours	
Overtime rate	
Total wage	

THINK

- 1 Calculate the number of normal hours worked.
- 2 Calculate the number of overtime hours worked.
- 3 Calculate the overtime rate.
- 4 Calculate the total pay by multiplying the number of normal hours by the normal rate and adding the overtime amount, calculated by multiplying the number of overtime hours by the overtime rate.
- 5 Fill in the amounts on the pay slip.

WRITE

Normal hours: $6 + 8 + 8 + 8 + 8 = 38$

Overtime hours: $2 + 2 = 4$

Overtime rate = 1.5×17.95
 $= 26.93$

Total pay = $38 \times 17.95 + 4 \times 26.93$
 $= 789.82$

Pay slip for: Fiona BLACK	Week ending December 21
Normal hours	38
Normal rate	\$17.95
Overtime hours	4
Overtime rate	\$26.93
Total wage	\$789.82

learnon RESOURCES — ONLINE ONLY

Interactivity: Special rates (int-6068)

Digital doc: SkillsHEET Multiplying a quantity (money) by a decimal (doc-10854)

Exercise 2.3 Special rates

Individual pathways

PRACTISE

Questions:
1–5, 6, 9, 10, 13–15

CONSOLIDATE

Questions:
1–5, 7, 9, 10, 13–16, 18

MASTER

Questions:
1–5, 8, 9, 11–22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Calculate the following special rates.
 - a. Time and a half when the hourly rate is \$15.96
 - b. Double time when the hourly rate is \$23.90
 - c. Double time-and-a-half when the hourly rate is \$17.40
2. Calculate the following total weekly wages.
 - a. 38 hours at \$22.10 per hour, plus 2 hours at time and a half
 - b. 40 hours at \$17.85 per hour, plus 3 hours at time and a half
 - c. 37 hours at \$18.32 per hour, plus 3 hours at time and a half and 2 hours at double time

3. Julio is paid \$956.08 for a regular 38-hour week.
 - a. What is his hourly rate of pay?
 - b. How much is he paid for 3 hours of overtime at time-and-a-half rates?
 - c. What is his wage for a week in which he works 41 hours?
4. **WE3** Geoff is a waiter in a restaurant and works 8 hours most days. Calculate what he earns for 8 hours work on the following days.
 - a. A Monday, when he receives his standard rate of \$21.30 per hour
 - b. A Sunday, when he is paid double time
 - c. A public holiday, when he is paid double time and a half
5. Albert is paid \$870.58 for a 38-hour week. What was his total wage for a week in which he worked 5 extra hours on a public holiday with a double-time-and-a-half penalty rate?
6. Jelessa (aged 16) works at a supermarket on Thursday nights and weekends. The award rate for a 16-year-old is \$7.55 per hour. Calculate what she would earn for:
 - a. 4 hours work on Thursday night
 - b. 6 hours work on Saturday
 - c. 4 hours work on Sunday at double time
 - d. the total of the three days.
7. Jacob works in a pizza shop and is paid \$13.17 per hour.
 - a. Jacob is paid double time and a half for public holiday work. What does he earn per hour on public holidays? (Answer to the nearest cent.)
 - b. What is Jacob's pay for a public holiday where he works 6 hours?
8. If Bronte earns \$7.80 on normal time, how much does she receive per hour:
 - a. at time and a half
 - b. at double time
 - c. at double time and a half?
9. Copy and complete the following time sheet. Calculate the number of hours Susan worked this week.



Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	9.00 am	5.00 pm	
Tuesday	Normal	9.00 am	5.00 pm	
Wednesday	Normal	9.00 am	5.00 pm	
Thursday	Normal	9.00 am	5.00 pm	
Friday	Normal	9.00 am	3.00 pm	

10. **WE4** Copy and complete Susan's pay slip for the week described in question 9.

Pay slip for: Susan WHITE	Week ending 17 August
Normal hours	
Normal pay rate	\$25.60
Overtime hours	0
Overtime pay rate	\$38.40
Total pay	

11. Below is a time sheet for Jason, who works in a department store. Copy and complete the table.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	9.00 am	5.00 pm	
Tuesday	Normal	9.00 am	5.00 pm	
Wednesday	Normal	—		
Thursday	Normal	1.00 pm	9.00 pm	
Friday	Normal	—		
Saturday	Time and a half	8.00 am	12.00 pm	

12. Copy and complete the pay slip for Jason for the week described in question 11.

Pay slip for: Jason RUDD	Week ending 21 December
Normal hours	
Normal pay rate	\$10.90
Overtime hours	
Overtime pay rate	
Total pay	

13. Brett does shift work. Copy and complete his time sheet.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	7.00 am	3.00 pm	
Tuesday	Normal	7.00 am	3.00 pm	
Wednesday				
Thursday				
Friday	Normal	11.00 pm	7.00 am	
Saturday	Time and a half	11.00 pm	7.00 am	
Sunday	Double time	11.00 pm	7.00 am	

14. Copy and complete Brett's pay slip for the week described in question 13.

Pay slip for: Brett SIMPSON	Week ending 15 September
Normal hours	
Normal pay rate	\$16.80
Time-and-a-half hours	
Time-and-a-half pay rate	
Double time hours	
Double time pay rate	
Gross pay	



Communicating, reasoning and problem solving

15. Calculate the following total weekly wages.
- 38 hours at \$18.40 per hour, plus 2 hours at time and a half
 - 32 hours at \$23.70 per hour plus 6 hours on a Sunday at double time
 - 38 hours at \$26.42 per hour, plus 2 hours overtime at one and a half the normal rate and 4 hours on a public holiday that incurred the maximum penalty rate
16. Ruby earns \$979.64 for her normal 38-hour week, but last week she also worked 6 hours overtime at time-and-a-half rates.
- Calculate how much extra she earned and give a possible reason for her getting time-and-a-half rates.
 - What was Ruby's total wage?
17. **MC** A standard working week is 38 hours and a worker puts in 3 hours overtime at the time-and-a-half rate and 2 hours at double time. To how many hours at the standard rate is her total work time equivalent?
- A.** 43 **B.** $46\frac{1}{2}$ **C.** $44\frac{1}{2}$ **D.** 45
18. Glen works 32 hours per week at \$22 per hour and is paid overtime for any time worked over the 32 hours per week. In one week Glen worked 42 hours and was paid \$814. Overtime is paid at 1.5 times the standard wage. Was Glen paid the correct amount (yes or no)? If no, then provide the correct amount.
19. Joshua's basic wage is \$22 per hour. His overtime during the week is paid at time and a half. Over the weekend he is paid double time. Calculate his gross wage in a week when he works his basic 40 hours, together with 1 hour overtime on Monday, 2 hours overtime on Wednesday and 4 hours overtime on Saturday.
20. The table below shows the pay sheet for a small company. If a person works up to 36 hours, the regular pay is \$14.50 per hour.
For hours over 36 and up to 40, the overtime is time and a half.
For hours over 40, the overtime is double time.
Complete the table.

	Hours worked	Regular pay	Overtime pay	Total pay
a	32			
b	38.5			
c	40.5			
d	47.2			

21. Vicki is a supervisor at a local factory. Each fortnight she calculates the wages of the employees. Overtime is paid to any employee who works more than 35 hours each week. The overtime rate is $1\frac{1}{2}$ times the hourly rate. The table below shows the number of hours worked and the hourly rates for three employees for one fortnight.

Employee	Hours worked	Hourly rate
Stewart	72	\$12.75
Helen	56	\$19.80
Amber	x	\$21.50

- a. Determine the total amount, in dollars, in wages for each of Helen and Stewart. Write your answer to the nearest cent.
 - b. Amber worked for x hours including some overtime. Her fortnightly wage was \$1988.75.
 - i. Determine the number of hours she worked.
 - ii. Was it possible for Amber to earn this amount if she did not do any overtime?
 - c. Tax is charged at 45 cents in each dollar earned. Determine the amount of tax, in dollars, that Amber pays for the fortnight. Write your answer correct to the nearest cent.
22. In what situations would being paid according to time sheets be preferable to receiving a wage or salary?

2.4 Piecework

[Stage 5.1]

2.4.1 Non-wage earnings — piecework

- **Piecework** is a system of payment by which a worker is paid a fixed amount for each job or task they complete.

WORKED EXAMPLE 5

Mitchell has a job washing cars in a car yard. He is paid \$5.20 per car washed. Calculate the amount Mitchell earns in an afternoon when he washes 24 cars.

THINK

- 1 Multiply the number of cars Mitchell washes by the amount paid for each car.
- 2 Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Amount earned} &= 24 \times 5.20 \\ &= 124.80\end{aligned}$$

Mitchell earns \$124.80.

- A person may also be paid on a sliding scale where the pay rate increases as the number of completed tasks increases.

WORKED EXAMPLE 6

Angelica is a machinist in a clothing factory. Each week she is paid \$4.28 per garment for the first 180 garments, and \$5.35 per garment thereafter. What will she be paid if she produces 223 garments?



THINK

- 1 Calculate the number of 'extra' garments Angelica makes.
- 2 Calculate her total payment by adding the payment she receives for the first 180 garments to the payment she receives for the extra garments.
- 3 Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Extra garments} &= 223 - 180 \\ &= 43\end{aligned}$$

$$\begin{aligned}\text{Payment} &= 180 \times 4.28 + 43 \times 5.35 \\ &= 1000.45\end{aligned}$$

Angelica earns a total payment of \$1000.45.

- In some cases, piecework is paid for multiple units rather than single units. For example, for letterbox deliveries you may be paid per 1000 deliveries made.

WORKED EXAMPLE 7

Holly is delivering brochures to letterboxes in her local area. She is paid \$43.00 per 1000 brochures delivered. Calculate the amount Holly will earn for a delivery of 3500 brochures.

THINK

- 1 Calculate the number of thousands of brochures Holly will deliver.
- 2 Multiply the number of thousands of brochures delivered by 43 to calculate what Holly will earn.
- 3 Write the answer in a sentence.

WRITE

$$3500 \div 1000 = 3.5$$

So Holly will deliver 3.5 thousand brochures.

$$\begin{aligned}\text{Holly's pay} &= 3.5 \times 43.00 \\ &= 150.50\end{aligned}$$

Holly will earn \$150.50.



RESOURCES — ONLINE ONLY

Interactivity: Piecework (int-6069)

Digital doc: WorkSHEET Financial mathematics (doc-10855)

Exercise 2.4 Piecework**Individual pathways****PRACTISE**

Questions:
1–5, 7

CONSOLIDATE

Questions:
1–5, 6, 7, 9

MASTER

Questions:
1, 2, 4, 6–12

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE5** Hitani is paid 65 cents for each teacup she decorates. How much is she paid for decorating 150 teacups?
2. **WE6** Jack makes leather belts. The piece rate is \$1.25 each for the first 50 belts and \$1.50 thereafter. What is his income for a day in which he produces 68 belts?

3. A production-line worker is paid \$1.50 for each of the first 75 toasters assembled, then \$1.80 per toaster thereafter. How much does she earn on a day in which she assembles 110 toasters?
4. **WE7** Rudolf earns \$42.50 per 1000 leaflets delivered to letterboxes. Calculate what Rudolf will earn for a week in which he delivers 7500 leaflets.
5. Dimitri earns \$7.20 for each box of fruit picked.
 - a. How much does he make for picking 20 boxes?
 - b. How many boxes must he pick to earn at least \$200?
 - c. If he takes 4 hours to pick 12 boxes, what is his hourly rate of pay?
6. Pauline uses her home computer for word processing under contract to an agency. She is paid \$3 per page for the first 50 pages, \$4 per page from 51 to 100 pages, and \$5 per page thereafter. Calculate her total pay for a period in which she prepares:
 - a. 48 pages b. 67 pages c. 123 pages.



Communicating, reasoning and problem solving

7. Rani delivers bills to letterboxes and is paid \$43 per thousand.
 - a. How much does she earn for delivering 2500 items?
 - b. How many thousands must she deliver to earn at least \$1000?
 - c. If she takes 6 hours to deliver each thousand on average, what is her hourly rate of pay?
8. Georgio delivers pizzas. He is paid \$3 per delivery from 5 pm to 9 pm and \$4 per delivery after 9 pm.
 - a. How much does he earn on a night in which he makes 12 deliveries by 9 pm and 4 deliveries between 9 pm and 10.30 pm?
 - b. What are his average earnings per hour if he has worked from 5 pm to 10.30 pm?
9. A shoemaker is paid \$5.95 for each pair of running shoes he can make.
 - a. If the shoemaker made 235 pairs of shoes last week, what was the amount paid?
 - b. The shoemaker is offered a bonus of 5% if he can make more than 250 pairs of shoes in a week. If he makes 251 pairs, what is the total amount earned, including the bonus?
10. A secretarial assistant gets paid \$12 per page that she types. If she manages to type more than 20 pages in a day, she gets a 10% bonus. If she typed 32 pages on Tuesday, how much did she earn?
11. There are both fixed and variable costs associated with some products. Consider the cost of importing a radio from China and selling it in Australia. The costs for a particular company are:
 - import of product \$12.50 per unit
 - transportation costs \$400 per 1000 units
 - warehouse rental space \$1 per unit per month
 - advertising costs \$2000 per month (fixed cost).
 - a. If this company imports and sells 500 units per month, what is the total cost per month?
 - b. At 500 units per month and a selling price of \$25.00, what is the total profit per month?
12. What are the advantages and disadvantages of being paid by piecework?



2.5 Commissions and royalties

[Stage 5.1]

2.5.1 Non-wage earnings — commissions and royalties

- **Commission** is a method of payment used mainly for salespeople. The commission paid is usually calculated as a percentage of the value of goods sold.
- A **royalty** is a payment made to a person who owns a copyright. For example, a musician who writes a piece of music is paid a royalty on CD and online sales. An author who writes a book is also paid a royalty based on the number of books sold. Royalties are calculated as a percentage of sales.

WORKED EXAMPLE 8

Mohamad is a songwriter who is paid a royalty of 12% on all sales of his music. Calculate the royalty that Mohamad earns if a song he writes sells CDs to the value of \$150 000.



THINK

- 1 Find the royalty by calculating 12% of \$150 000.
- 2 Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Royalty} &= 12\% \text{ of } 150\,000 \\ &= 0.12 \times 150\,000 \\ &= 18\,000\end{aligned}$$

Mohamad earns \$18 000 in royalties.

- Sometimes a salesman is paid a small wage, called a retainer, plus a percentage of the value of the goods sold.

WORKED EXAMPLE 9

Gemma, a car salesperson, is paid a retainer of \$350 per week, plus a commission of 8% of the profits made by the company on cars that she sells.

- a How much does Gemma earn in a week when no sales are made?
- b How much does she earn in a week when \$5000 profit was generated by her sales?

THINK

- a If no sales are made, only the retainer is paid.
- b 1 Find the commission paid by calculating 8% of \$5000.
 - 2 Find the total amount paid by adding the retainer and the commission.
 - 3 Write the answer in a sentence.

WRITE

a Gemma earns \$350.

$$\begin{aligned}\text{b Commission} &= 8\% \text{ of } 5000 \\ &= 0.08 \times 5000 \\ &= \$400\end{aligned}$$

$$\begin{aligned}\text{Total earnings} &= 350 + 400 \\ &= \$750\end{aligned}$$

Gemma earns \$750.

- Sometimes the commission is broken into several parts with differing rates.

WORKED EXAMPLE 10

A real estate agency receives 2% commission on the first \$300 000 of a sale and 3% on the remainder. How much commission is received on the sale of a \$380 000 property?

THINK

- 1 Calculate the difference between \$380 000 and \$300 000.
- 2 Calculate 2% of \$300 000.
- 3 Calculate 3% of \$80 000.
- 4 Calculate the total commission by adding the commission earned on \$300 000 and the commission earned on \$80 000.
- 5 Write the answer in a sentence.

WRITE

$$380\,000 - 300\,000 = 80\,000$$

$$2\% \text{ of } 300\,000 = 6000$$

$$3\% \text{ of } 80\,000 = 2400$$

$$6000 + 2400 = 8400$$

The commission received is \$8400.

ACTIVITY: HOW MUCH ROYALTY/COMMISSION?

1. In pairs, collate a list of the commissions and royalties of some common professions. This information can be obtained by contacting record companies, book publishers, art galleries and so on. For commissions, contact car dealers and real-estate companies. Some of this information may be obtained from the internet.
2. Discuss what professions or industries offer the highest commissions as a percentage. Does this mean that they are paid more money?

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Interactivity: Commissions and royalties (int-6070)

Digital doc: SkillSHEET Converting a percentage to a decimal (doc-10856)

Digital doc: SkillSHEET Finding a percentage of a quantity (money) (doc-10857)

Digital doc: SpreadSHEET Converting percentages to fractions or decimals (doc-10905)

Digital doc: SpreadSHEET Finding a percentage of an amount (doc-10906)

Exercise 2.5 Commissions and royalties

Individual pathways

PRACTISE

Questions:
1–5, 7, 11

CONSOLIDATE

Questions:
1–4, 6, 7, 10, 11, 13

MASTER

Questions:
1, 3, 4, 5c, 6d, 7, 8, 9b, 10b, 11–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

-

Communicating, reasoning and problem solving

11. Veronica earns \$400 per week plus 4% on sales, whereas Francis earns 6% commission only.
 - a. How much does each earn on sales of \$8400?
 - b. What level of sales would yield each the same income?
12. Wolfgang, a car salesman, is paid a weekly retainer of \$550, plus 10% of the dealer's profit on each vehicle. Find his total income for weeks in which the dealer's profits on vehicles he sold were:
 - a. \$3500
 - b. \$5980
 - c. \$7036.00

13. Using the commission table for house sales below, calculate the commission on each of the following sales.

Sale price	Commission	Plus
Between \$0 and \$80 000	2% of sale price	0
Between \$80 001 and \$140 000	1.5% of amount over \$80 000	\$1600 (2% of \$80 000)
\$140 000 and over	1.1% of amount over \$140 000	\$2500 (2% of \$80 000 + 1.5% of \$60 000)

- a. \$76 000 b. \$122 500 c. \$145 000 d. \$600 000
14. Mr Hartney is a used car salesman. He receives a basic monthly salary of \$2400 together with 5% commission on all sales. Although his sales for the month amounted to \$48 300, he also had deductions for insurance (\$12.80), association fees (\$25.70) and income tax (\$1100). Calculate the amount, in dollars, he took home that month.
15. A rock musician makes a royalty on all record sales according to the following formula.

Sales from	Sales to	Royalty rate
0	\$100 000	3%
\$100 001	\$500 000	3.5% on amount over \$100 000
\$500 001	1 million	4% on amount over \$500 000
1 million	and above	5% on amount over 1 million

Calculate the musician's royalties for the following years.

- a. 2007 — sales = \$456 000 b. 2008 — sales = \$1 234 500
c. 2009 — sales = \$986 400 d. 2010 — sales = \$2 656 000
16. Four years ago Inka became an employee of TrakRight Tourism, where her starting annual salary was \$55 600. After her first year she received a 2% pay rise. The next year she received a 3% pay rise. Last year she received an $x\%$ pay rise. If her annual salary is now \$61 042, determine the value of x , correct to 1 decimal place.
17. What are the major advantages and disadvantages of getting paid by commission or royalties?

2.6 Loadings and bonuses

[Stage 5.1]

2.6.1 Loadings

- If a wage or salary earner has to work in difficult or hazardous conditions, then the worker may be granted an extra payment or **loading**.
- Most workers are granted a 'holiday loading'. For a 4-week period each year they are paid an extra 17.5% of their usual wage.



WORKED EXAMPLE 11

Rohan works as an electrician and receives \$38.20 per hour for a 36-hour working week. If Rohan works 'at heights' he receives \$2.50 per hour height loading. Calculate Rohan's wage in a week where he works 15 hours at heights.

THINK

- 1 Calculate Rohan's normal weekly wage.
- 2 Calculate Rohan's loading for the time he worked at heights.
- 3 Calculate Rohan's total wage.

WRITE

$$\begin{aligned}\text{Normal wage} &= 36 \times 38.20 \\ &= \$1375.20\end{aligned}$$

$$\begin{aligned}\text{Loading} &= 15 \times 2.5 \\ &= \$37.50\end{aligned}$$

$$\begin{aligned}\text{Total wage} &= 1375.20 + 37.50 \\ &= \$1412.70\end{aligned}$$

WORKED EXAMPLE 12

Jelena works as a hairdresser and is paid a normal rate of \$19.70 per hour for a 38-hour working week.

- a Calculate Jelena's normal weekly wage.
- b For her 4 weeks annual leave, Jelena is paid a loading of 17.5%. Calculate the amount that Jelena receives in holiday loading.
- c Calculate the total amount that Jelena receives for her 4 weeks annual leave.

THINK

- a Calculate Jelena's normal wage by multiplying the hours worked by the hourly rate.
- b1 Find 17.5% of Jelena's normal wage.
2 Multiply this amount by 4 to find the holiday loading.
- c Find the total amount received by multiplying Jelena's normal weekly pay by 4 and adding the holiday loading.

WRITE

$$\begin{aligned}\text{a Normal wage} &= 38 \times 19.70 \\ &= \$748.60\end{aligned}$$

$$\text{b } 17.5\% \text{ of } \$748.60 = \$131.01$$

$$\begin{aligned}\text{Holiday loading} &= 4 \times 131.01 \\ &= \$524.04\end{aligned}$$

$$\begin{aligned}\text{c Holiday pay} &= 4 \times 748.60 + 524.04 \\ &= \$3518.44\end{aligned}$$

ACTIVITY: WHY DO WE HAVE LEAVE LOADING?

Use the internet or other resources to research the reasons for inclusion of leave loading provisions in many awards. Do you think these reasons are valid for today's work force?

2.6.2 Bonuses

- Many people who are employed in managerial positions receive a bonus if the company achieves certain performance targets. The bonus may be a percentage of their annual salary or a percentage of the company's profits.

WORKED EXAMPLE 13

Brooke is the Chief Executive Officer of a fashion company on a salary of \$240 000 per year. Brooke will receive a bonus of 1% of her salary for every percentage point that she increases the company profit. If the company profit grows from \$3.1 million to \$4.4 million in one year, calculate the amount of Brooke's bonus.

THINK

- 1 Calculate the increase in profit.
- 2 Express the increase in profit as a percentage.
- 3 Calculate this percentage of Brooke's annual salary.
- 4 Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Increase in profit} &= \$4.4 \text{ m} - \$3.1 \text{ m} \\ &= \$1.3 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Percentage increase} &= \frac{1.3}{3.1} \times 100\% \\ &= 41.9\%\end{aligned}$$

$$\begin{aligned}\text{Bonus} &= 41.9\% \text{ of } \$240\,000 \\ &= \$100\,560\end{aligned}$$

Brooke's bonus is \$100 560.

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Interactivity: Bonuses (int-6071)

Digital doc: SkillsHEET Expressing one quantity as a percentage of another (doc-10858)

Exercise 2.6 Loadings and bonuses

Individual pathways

PRACTISE

Questions:
1, 3, 4, 7, 9, 10

CONSOLIDATE

Questions:
1, 2, 4, 5, 8–13

MASTER

Questions:
1, 2, 4, 6–16

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE11** Rashid works as an electrician and receives \$35.40 per hour for a 35-hour working week. If Rashid works at heights he receives a height loading of \$0.32 per hour. Calculate Rashid's wage in a week where he works 18 hours at heights.
2. Patrick is a railway linesman. If he works in wet weather he is paid a loading of 43 cents per hour. If he normally works a 38-hour working week at \$21.02 per hour and 16 hours are spent working in wet weather, find Patrick's pay for the week.



3. Saci is an industrial cleaner and is paid at the rate of \$19.82 per hour. If Saci works in a confined space, she is paid a loading of \$0.58 per hour. Calculate Saci's pay for a week in which she worked 38 hours and 19 of those hours were in a confined space.
4. **WE12** Jordan works as the manager of a supermarket and is paid a normal rate of \$37.60 per hour for a 38-hour working week.
 - a. Calculate Jordan's normal weekly wage.
 - b. For her 4 weeks annual leave, Jordan is paid a loading of 17.5%. Calculate the amount that Jordan receives in holiday loading.
 - c. Calculate the total amount that Jordan receives for her 4 weeks annual leave.
5. Charlie earns \$22.80 per hour for a 38-hour week.
 - a. Calculate the amount Charlie will earn in a normal working week.
 - b. Calculate the total amount Charlie will receive for his 4 weeks annual leave if he receives a 17.5% holiday loading.
6. Liam is paid \$15.95 per hour for a 36-hour working week.
 - a. Calculate Liam's weekly wage.
 - b. Liam takes one week's holiday, for which he is given a 17.5% loading. Calculate the holiday loading.
7. Karen receives an annual salary of \$63 212.
 - a. What is her fortnightly pay?
 - b. What is she paid for her annual 4-week holiday, for which she receives an extra 17.5% loading?
8. Brian earns \$956.46 for a standard 38-hour week and a \$27.53 per week allowance for working on scaffolding. Calculate his total pay for a week in which he works on scaffolding and does 4 hours overtime at time and a half.
9. **WE13** Eric is a director of a mining company on a salary of \$380 000 per year. Eric is told that at the end of the year he will receive a bonus of 1% of his salary for every percentage point of increase in the company profit. If the company profit grows from \$4.9m to \$6.4m in one year, calculate the amount of Eric's bonus.



Communicating, reasoning and problem solving

10. Sally is the manager of a small bakery that employs 12 people. As an incentive to her workers she agrees to pay 15% of the business's profits in Christmas bonuses for her employees. The business makes a profit of \$400 000 during the year.
 - a. Find the total amount that Sally pays in bonuses.
 - b. If the bonus is shared equally, what amount does each employee receive as a Christmas bonus?
 - c. If one employee earns \$42 000 per year, calculate the Christmas bonus as a percentage of annual earnings, correct to 2 decimal places. Explain your answer.
11. Shane, the director of an exercise company, earns a salary of \$275 000 a year. Shane gets paid incentives if he is able to increase the company's profit. He gets:
 - 5% if he increases the profit by 0.1–10%
 - 7.5% if he increases the profit by 10.1–20%
 - 10% if he increases the profit by more than 20%.
 If the company's profit grows from \$1.2 million to \$1.4 million in a year:
 - a. explain what percentage incentive Shane will get and why
 - b. calculate his salary for the year.

12. Kevin owns a sports store and has 7 staff working for him. He offers each of them a 5.5% end-of-year bonus on any profits over \$100 000. This year the store made a profit of \$275 000.
- Find the amount each employee earned in bonuses.
 - What is the cost to Kevin in total bonuses for the year?
 - If one employee earned \$64 625 including bonuses for the year, what was their base salary?
13. Jimmy is a high-rise window cleaner. He gets paid \$15 per window for the first five levels. For the next 15 levels he gets an extra 15% per window, and above this he gets 20% extra as danger money. How much does Jimmy earn for cleaning:
- a total of 20 windows on levels 3 to 4
 - 10 windows on levels 4 and a total of 27 windows on levels 10 to 13
 - a total of 30 windows on levels 11 to 14 and a total of 30 windows on levels 21 to 25?
14. Denise works for a real estate agent. She receives a basic wage of \$250 per week plus commission on sales. The rate of commission is variable. For houses up to \$300 000, the commission is 0.5%. For houses over \$300 000, the commission is an additional 0.25% on the amount over \$300 000. How much pay did she receive in the week she sold a house for:
- \$280 000
 - \$428 000?
15. When Jack goes on holidays, he is paid $17\frac{1}{2}\%$ holiday loading in addition to his normal pay. When he went on 2 weeks' leave, his holiday pay was \$1504. What is his normal weekly pay?
16. How are bonuses used to encourage workers?



2.7 Taxation and net earnings

[Stage 5.1]

2.7.1 Taxation

- In Australia, people who earn more than \$18 200 in a financial year must pay a percentage of their earnings as **income tax**.
- The rates of taxation for Australian residents for 2016–17 are shown in the table below.

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

The above rates do not include the Medicare levy of 2.0%.



DISCUSSION

Is the current Australian income tax system fair? Take a look at how it has changed over the years, and how tax systems vary from country to country.

WORKED EXAMPLE 14

Find the amount of tax paid on an annual income of:

a \$22 000

b \$92 000.

THINK

a 1 \$22 000 is in the \$18 201 to \$37 000 bracket.

2 The tax payable is 19c (0.19) for every dollar over \$18 200.

3 Calculate the amount over \$18 200 by subtracting \$18 200 from \$22 000.

4 Apply the rule '19c for every dollar over \$18 200'.

5 Write the answer in a sentence.

b 1 \$92 000 is in the \$87 001 to \$180 000 bracket.

2 Calculate the amount over \$87 000 by subtracting \$87 000 from \$92 000.

3 Apply the rule '\$19 822 plus 37c for each \$1 over \$87 000'.

4 Write the answer in a sentence.

WRITE

a

$$\$22\,000 - \$18\,200 = \$3800$$

$$\begin{aligned}\text{Tax payable} &= 0.19 \times 3800 \\ &= 722\end{aligned}$$

The tax payable on \$22 000 is \$722.

b

$$\$92\,000 - \$87\,000 = \$5\,000$$

$$\begin{aligned}\text{Income tax} &= 19\,822 + 0.37 \times 5000 \\ &= 21\,672\end{aligned}$$

c The tax payable on \$92 000 is \$21 672.

2.7.2 Net earnings

Medicare levy

- Medicare is the scheme that gives Australian residents access to health care.
- Most taxpayers pay 2.0% of their taxable income to pay for this scheme. This is called the Medicare levy.
- People who have private medical insurance can reclaim some of this money.

Pay As You Go (PAYG) taxation

- When you receive your pay, usually as a bank deposit, some of the money has been taken out by the employer to cover your income tax and Medicare levy. This is called 'pay as you go' (PAYG) taxation.
- The initial amount, before tax is taken out, is called your **gross salary** and the amount that you actually receive is called your **net salary**.
- The amount of money to be deducted by the employer each week is published by the Australian Tax Office, as shown in the following table.
- Every tax payer is required to submit a yearly tax return. Usually the tax payer's employer will have deducted the correct weekly amount from the employee's gross earnings. If the employee has received income from another source, such as an interest-bearing bank account, this may mean that the employee needs to pay tax on this extra amount. Alternatively, if the employee has some extra tax deductions, this could mean a tax return to the employee.

PAYG TABLE: Weekly tax withheld (\$)					
Gross wage	With tax-free threshold	Gross wage	With tax-free threshold	Gross wage	With tax-free threshold
450	22	950	165	1450	339
500	37	1000	183	1500	356
550	48	1050	200	1550	373
600	59	1100	217	1600	390
650	69	1150	235	1650	408
700	80	1200	252	1700	426
750	96	1250	270	1750	446
800	113	1300	287	1800	465
850	130	1350	304	1850	485
900	148	1400	321	1900	504

Note: Most Australian citizens qualify for the tax-free threshold. For the purposes of this section, apply the tax-free threshold values.

Deductions

- Often other sums of money, such as union fees and private health insurance, are deducted from gross pay.

Family Tax Benefit

- When a family has young or dependent children, the government may pay an allowance called the 'Family Tax Benefit', which is added to a person's gross salary.



WORKED EXAMPLE 15

Fiona has a gross wage of \$900 per week.

- Use the PAYG table to find the amount of tax that should be deducted.
- What percentage of her gross pay is deducted?
- If Fiona receives \$98 in family allowance but has deductions of \$71 (superannuation) and \$5.50 (union fee), what is her net pay?



THINK

a From the table, PAYG tax payable on a gross wage of \$900 per week is \$148.

b Find 148 as a percentage of 900.

c 1 Fiona receives \$98 in family allowance. Add this to her gross weekly wage to find her total income.

2 Calculate her total deductions.

3 Calculate her net pay by subtracting her total deductions from her total income.

WRITE

a \$148

b $\frac{148}{900} \times 100 = 16.44\%$ deducted

c Total income = $900 + 98$
= \$998

Total deductions = $148 + 71 + 5.50$
= \$224.50

Net pay = $998 - 224.50$
= \$773.50

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eLesson: Small business (eles-0117)

Exercise 2.7 Taxation and net earnings

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–11

MASTER

Questions:
1–12

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE14** Find the amount of tax paid on an annual income of:
 - \$15 000
 - \$22 000
 - \$44 000
 - \$88 000.
- WE15** In the PAYG tax table, look up the amount of tax that must be deducted from the following weekly earnings and find this as a percentage of the gross pay, correct to 2 decimal places.
 - \$650
 - \$1100
 - \$1550
- For each of the following weekly pay values, calculate the net pay.
 - Gross pay \$450.00, tax \$22.00 and union fees \$4.75
 - Gross pay \$550.00, tax \$48.00, private health insurance \$25.85 and superannuation \$53.80
 - Gross pay \$850.00, tax \$130.00, loan repayment \$160.00 and insurance payment \$45.40
- Calculate the net annual salary of a person who has a gross annual salary of \$57 200 and a family allowance of \$4392.20 with deductions of \$17 264 for tax, annual union fees of \$262.75 and social club payments of \$104.00.
- Sergio works as a security guard and receives gross pay of \$950.00 each week. His tax totals \$165 per week. If his other deductions are \$60.10 for superannuation and \$5.05 for union fees, what is his net pay?

6. Lieng works as an interior decorator and earns \$1350 per week.
 - a. How much tax should be deducted from her pay each week?
 - b. What percentage of her gross pay is her tax?
 - c. If Lieng also has deductions of \$105 for superannuation, \$5.20 for union fees, and \$4.00 for a social club, what is her net weekly pay?

Communicating, reasoning and problem solving

7. Yelena works as a chef and is paid \$22.86 per hour and works a 35-hour week.
 - a. Calculate Yelena's gross weekly earnings.
 - b. How much tax should be deducted from Yelena's pay?
 - c. What percentage of her gross pay (correct to 2 decimal places) is deducted in tax?
 - d. If Yelena also has deductions of \$56.20 for superannuation and \$22.50 for her health insurance, and she gets \$60.00 taken out to pay off her car loan, what is her net pay?
 - e. What percentage of her gross pay is her net pay? Give your answer correct to 2 decimal places.
8. Debbie earns \$72 000 per year.
 - a. Explain why she takes home only \$57 053.
 - b. Give reasons why this figure could possibly be different again.
9. Jacko works at an IT firm and earns \$1725 a week.
 - a. How much does he earn a year, gross?
 - b. How much tax will he need to pay per year? (Use the yearly table to calculate the tax payable.)
 - c. If he has no deductions, how much will he need to pay for the Medicare levy?
10. Tamara works as a swimming instructor and earns \$21.50 per hour when working a 38-hour week.
 - a. Using the PAYG table, find the amount of tax that should be deducted from Tamara's salary per week, correct to the nearest dollar.
 - b. What percentage of her gross salary is deducted? Give your answer to 1 decimal place.
 - c. If Tamara receives \$82 per week in family allowance but pays \$50 per week towards her superannuation, what is her net weekly pay?
11. Greg started work as an experienced barista in a café and was paid \$24 an hour when working a 40-hour week. His weekly tax withheld was \$165. After 6 months, he decided to go travelling. If his 6 months of work was all within one financial year, how much money should Greg expect to receive in his tax return? (Assume a compulsory Medicare levy of 2%.)
12. What strategies would you use to remember how to calculate income tax?



2.8 Simple interest

[Stage 5.1]

2.8.1 The simple interest formula

- **Interest** is the fee charged for the use of someone else's money. It is normally a percentage of the amount borrowed.
- Lenders or investors receive interest from banks for lending them money.
- Borrowers pay interest to banks and other financial institutions.

- **Simple interest** or ‘flat rate’ interest can be calculated using a simple formula: $I = PRN$
 where I = the amount of interest to be paid
 P = the principal, which is the amount of money borrowed
 R = the interest rate, usually given as a percentage
 N = the number of times that the interest must be paid.
- The abbreviation p.a. stands for ‘per annum’, which means ‘each year’. For example, an interest rate of 5% p.a. for 4 years means that $R = 5\%$ (or 0.05) and $N = 4$.

WORKED EXAMPLE 16

Zac borrows \$3000 for 2 years at 9% p.a. simple interest.

a How much interest is he charged?

b What total amount must he repay?

THINK

- a 1** Write the simple interest formula and the known values of the variables.
- 2** Substitute the values into the formula to find I .
- 3** Write the answer in a sentence.
- b 1** Repayment = amount borrowed + interest.
- 2** Write the answer in a sentence.

WRITE

- a** $I = PRN$, $P = 3000$, $R = 9\% = 0.09$,
 $N = 2$
- $$I = 3000 \times 0.09 \times 2$$
- $$= \$540$$
- Zac is charged \$540 interest.
- b** $3000 + 540 = 3540$
- Zac must repay \$3540 in total.

- Care needs to be taken with examples where the term of the investment is given in months or even in days. In these examples, the period of the investment needs to be expressed in years.
- The simple interest formula can also be used to find the principal, the interest rate or the term of the investment by substituting the known values into the formula and solving the resulting equation.

WORKED EXAMPLE 17

Anthony invested \$1000 at a simple interest rate of 4.6% p.a. For how long must he invest it in order to earn at least \$100 in interest?

THINK

- 1** Write the formula and the known values of the variables.
- 2** Substitute the given values into the formula.
- 3** Solve the equation.
- 4** Change the decimal part of the years into months.
- 5** Write the answer in a sentence using years and months.

WRITE

- $I = PRN$, where $I = 100$, $P = 1000$, $R = 4.6\% = 0.046$
- $$100 = 1000 \times 0.046 \times N$$
- $$100 = 46N$$
- $$N = \frac{100}{46}$$
- $$\approx 2.1739$$
- $$0.1739 \times 12 = 2.09 \text{ months}$$
- Anthony must invest for 2 years and 2 months.

- The same method is used when R or P are to be found.

WORKED EXAMPLE 18

The Smiths need to buy a new refrigerator at a cost of \$1679. They will pay a deposit of \$200 and borrow the balance at an interest rate of 19.5% p.a. The loan will be paid off with 24 equal monthly payments.

- How much money do the Smiths need to borrow?
- What is the term of the loan?
- How much interest will they pay?
- What will be the total cost of the refrigerator?
- How much is each payment?



THINK

- Subtract the deposit from the cost to find the amount still owing.
 - Write the answer in a sentence.
- The term is 24 months as this is the length of time between borrowing and paying back. The interest rate is per year.
- Identify the principal (P), interest rate (R) and time period (N), and use the formula.
 - Write the answer in a sentence.
- Add the interest to the initial cost.
- Subtract the deposit from the total cost to find the amount to be repaid.
 - Divide the total payment into 24 equal payments.

WRITE

a $1679 - 200 = 1479$

They must borrow \$1479.

- b** 24 months is 2 years. The term is 2 years.

c $P = 1479, R = \frac{19.5}{100} = 0.195, N = 2$
 $I = PRN$
 $= 1479 \times 0.195 \times 2$
 $= 576.81$

The interest will be \$576.81.

- d** $1679.00 + 576.81 = 2255.81$
 The total cost will be \$2255.81.

e $2255.81 - 200 = 2055.81$

$2055.81 \div 24 = 85.66$
 Each repayment will be \$85.66.

- Spreadsheets are often used to make simple interest calculations easier.

2.8.2 Developing a simple interest spreadsheet

- The spreadsheet below calculates the total amount of simple interest for a given number of years.

	A	B	C	D	E	F
1						
2		Principal	1000			
3		Interest rate (per year)	5			
4		Time (years)	6			
5						
6		<i>Year</i>	<i>Principal</i>	<i>Interest</i>	<i>New value</i>	
7		1	1000	50	1050	
8		2	1000	50	1100	
9		3	1000	50	1150	
10		4	1000	50	1200	
11		5	1000	50	1250	
12		6	1000	50	1300	
13						

- Inputs (yellow cells):
 - Cell D2: the amount of principal. Above, the principal is \$1000.
 - Cell D3: the interest rate, as a percentage. Above, the interest rate is 5%.
 - Cell D4: the term. Above, the term is 6 years.
- Outputs (Row 7 and beyond):
 - Column B: shows the years: 1, 2, 3, ... 6
 - Column C: shows the principal each year. Set $C7 = \$D\$2$ and fill down.
 - Column D: shows the interest calculation. Set $D7 = C7 * \$D\$3 / 100$ and fill down.
 - Cell E7: shows the new value after year 1. Set $E7 = C7 + D7$.
 - Cell E8: shows the new value after year 2. Set $E8 = E7 + D8$ and fill down.
- For time periods greater than 6 years, highlight Row 12's cells and fill down.

learnon RESOURCES — ONLINE ONLY

Interactivity: Simple interest (int-6074)

Interactivity: Effects of P , R , I and t (int-0745)

Digital doc: WorkSHEET Simple interest (doc-6246)

Exercise 2.8 Simple interest

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–11

MASTER

Questions:
1–19

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Understanding and fluency

- WE16** Monique borrows \$5000 for 3 years at 8% per annum simple interest.
 - How much interest is she charged?
 - What total amount must she repay?
- Calculate the simple interest earned on an investment of \$15 000 at 5.2% p.a. over 30 months.
- For each loan in the table, calculate:
 - the simple interest
 - the amount repaid.

	Principal (\$)	Interest rate per annum	Time
a	1000	5%	2 years
b	4000	16%	3 years
c	8000	4.5%	48 months
d	2700	3.9%	2 years 6 months
e	15 678	9.2%	42 months

- Find the final value of each of the following investments.
 - \$3000 for 2 years at 5% p.a.
 - \$5000 for 3 years at 4.3% p.a.
- Hasim borrows \$14 950 to buy a used car. The bank charges a 9.8% p.a. flat rate of interest over 60 months.
 - What total amount must he repay?
 - How much is each equal monthly repayment?
- Carla borrows \$5200 for an overseas trip at 8.9% p.a. simple interest over 30 months. If repayment is made in equal monthly instalments, how much is each instalment?
- WE17** Michael invested \$2000 at a simple interest rate of 4% p.a. For how long must he invest it in order to earn \$200 in interest?
- If Jodie can invest her money at 8% p.a., how much does she need to invest to earn \$2000 in 2 years?
- If the simple interest charged on a loan of \$9800 over 3 years is \$2352, what percentage rate of interest was charged?



10. Find the missing quantity in each row of the table.

	Principal	Rate of interest p.a.	Time	Interest earned
a	\$2000	6%		\$240.00
b	\$3760	5.8%		\$545.20
c		7%	3 years	\$126.00
d		4.9%	1 year 9 months	\$385.88
e	\$10 000		$1\frac{1}{2}$ years	\$1200.00
f	\$8500		42 months	\$1041.25

11. **WE18** Mika is buying a used car priced at \$19 998. He has a deposit of \$3000 and will pay the balance in equal monthly payments over 4 years. The simple interest rate will be 12.9% p.a.

- How much money is he borrowing?
- How much interest will he pay?
- What will be the total cost of the car?
- How many payments will he make?
- How much is each payment?



12. A new sound system costs \$3500, but it can be purchased for no deposit, followed by 48 equal monthly payments, at a simple interest rate of 16.2% p.a.

- What will be the total cost of the sound system?
- Under a 'no deposit, no payment for 2 years' scheme, 48 payments are still required, but the first payment isn't made for two years. (This will stretch the loan over 6 years.) How much will the system cost using this scheme?
- What will be the monthly payment under each of the schemes above?

Communicating, reasoning and problem solving

13. A \$269 000 business is purchased on \$89 000 deposit with the balance payable over 5 years at 8.95% p.a. flat rate.

- How much money is borrowed to purchase this business?
- How much interest is charged?
- What total amount must be repaid?
- Find the size of each of the equal monthly repayments and explain two ways in which these payments could be reduced.

14. If a bank offers interest on its savings account of 4.2% p.a. and the investment is invested for 9 months, explain why 4.2 is not substituted into the simple interest formula as the interest rate.

15. A Year 9 girl is paid \$79.50 in interest for an original investment of \$500 for 3 years. What is the annual interest rate?

16. A loan is an investment in reverse: you borrow money from a bank and are charged interest. The value of a loan becomes its total cost.

A worker wishes to borrow \$10 000 from a bank that charges 11.5% interest per year. If the loan is over 2 years:

- calculate the total interest paid
- calculate the total cost of the loan.

17. For the following questions, assume that the interest charged on a home loan is simple interest.

- Tex and Molly purchase their first home and arrange for a home loan of \$375 000. Their home loan interest rate rises 0.25% per annum within the first 6 months of the loan. Determine the monthly increase, in dollars, of their repayments.
- Brad and Angel's interest on their home loan is also increased by 0.25% per annum. Their monthly repayments increase by \$60. Determine the amount of their loan, in dollars.

18. a. Theresa invests \$4500 at 5.72% per annum in an account that attracts simple interest for 6 months. Show that at the end of 6 months she should expect to have \$4628.70.
- b. Barry has \$6273 in his bank account at a simple interest rate of 4.86% per annum. After 39 days he calculates that he will have \$6305.57 in his account. Did Barry calculate his interest correctly? Justify your answer by showing your calculations.
- c. Juanita receives \$10 984 for the sale of her car. She invests $x\%$ of \$10984 in an account at 6.68% per annum simple interest for $1\frac{1}{2}$ years. She spends the remainder of the money from the sale of her car. At the end of the investment she has exactly enough money to purchase a car for \$11 002. Find the value of x , correct to 2 decimal places.
19. How does interest affect the way we live?

2.9 Compound interest

[Stage 5.1]

2.9.1 Compound interest

- Consider \$1000 invested for 3 years at 10% p.a. simple interest.
- Each year the value of the investment increases by \$100, reaching a total value of \$1300.
- The simple interest process can be summarised in the following table.

	Principal	Interest	Total value
Year 1	\$1000	\$100	\$1100
Year 2	\$1000	\$100	\$1200
Year 3	\$1000	\$100	\$1300
Total interest = \$300			

- Under the system called **compound interest**, the interest is added to the principal at the end of each year; in other words, it is compounded annually.
- The compound interest process can be summarised in this table.

	Principal	Interest	Total value
Year 1	\$1000	\$100	\$1100
Year 2	\$1100	\$110	\$1210
Year 3	\$1210	\$121	\$1331
Total interest = \$331			

- The principal grows each year and so does the interest.
- Over many years, the difference between simple interest and compound interest can become enormous.

WORKED EXAMPLE 19

Complete the table to find the interest paid when \$5000 is invested at 11% p.a. compounded annually for 3 years.

	Principal	Interest	Total value
Year 1	\$5000		
Year 2			
Year 3			
Total interest =			

THINK

- Interest for year 1 = 11% of \$5000
Find the principal for year 2 by adding the interest to the year 1 principal.
- Interest for year 2 = 11% of \$5550
Find the total value at the end of year 2.
This is the principal for year 3.
- Interest for year 3 = 11% of \$6160.50
- Calculate the interest earned over 3 years by subtracting the year 1 principal from the final amount.

WRITE

$$11\% = \frac{11}{100} = 0.11$$

$$I = 0.11 \times 5000 = 550$$

$$5000 + 550 = 5550$$

$$0.11 \times 5550 = 610.50$$

$$5550 + 610.50 = 6160.50$$

$$0.11 \times 6160.50 = 677.66$$

$$6160.50 + 677.66 = 6838.16$$

$$6838.16 - 5000 = 1838.16$$

	Principal	Interest	Total value
Year 1	\$5000	\$550	\$5550
Year 2	\$5550	\$610.50	\$6160.50
Year 3	\$6160.50	\$677.66	\$6838.16
Total interest = \$1838.16			

- There is a quicker way of finding the total value of the investment. Look again at Worked example 19.
The investment grows by 11% each year, so its value at the end of the year is 111% $\left(\frac{111}{100} = 1.11\right)$ of its value at the start of the year.

$$\begin{aligned} &111\% \text{ of } 5000 \\ &= 1.11 \times 5000 \\ &= 5550 \end{aligned}$$

- This process is repeated each year for 3 years. The multiplying factor is 1.11.

$$5000 \xrightarrow{\times 1.11} 5550 \xrightarrow{\times 1.11} 6160.50 \xrightarrow{\times 1.11} 6838.16$$

- After 3 years the value of the investment is \$6838.16.

WORKED EXAMPLE 20

Complete the table to find the value, after 4 years, of an investment of \$2000 compounded annually at 8% p.a.

Year	Start of year	End of year
Year 1	\$2000	
Year 2		
Year 3		
Year 4		

THINK

- 1 Interest is compounded at 8%, so at the end of the first year the value is 108% of the initial value.
- 2 For the value at the end of year 2, calculate 108% of the amount accumulated in year 1, so find 108% of **2160**.
- 3 For the value at the end of year 3, calculate 108% of the amount accumulated in year 2, so find 108% of **2332.80**.
- 4 For the value at the end of year 4, calculate 108% of the amount accumulated in year 3, so find 108% of 2519.424.
- 5 Complete the table.

WRITE

$$108\% = \frac{108}{100} = 1.08$$

$$1.08 \times 2000 = 2160$$

$$1.08 \times 2160 = 2332.80$$

$$1.08 \times 2332.80 = 2519.424$$

$$1.08 \times 2519.424 = 2720.98$$

Year	Start of year	End of year
Year 1	\$2000	\$2160
Year 2	\$2160	\$2332.80
Year 3	\$2332.80	\$2519.42
Year 4	\$2519.42	\$2720.98

- In Worked example 20 the principal (\$2000) was multiplied by 108% four times (because there were 4 years). The final amount, A, is \$2720.98.
- If the principal (\$2000) was invested for the same period of time at a flat rate of interest, the final amount would be:

$$A = 2000 + \frac{2000 \times 8 \times 4}{100}$$

$$= \$2640$$

- A compound interest rate over the same period of time returned an extra \$80.98 in interest.
- It is always important when borrowing or investing money to compare terms and hence ensure the most beneficial investment or loan.

WORKED EXAMPLE 21

- a Calculate the amount of interest earned if \$40 000 is invested at 7.5% p.a. compounding annually for 6 years.
- b Calculate the amount of interest earned if \$40 000 is invested at a flat rate of 7.5% p.a. for 6 years.
- c Which is the better investment and by how much?

THINK

- a 1 Interest is compounded at 7.5%, so at the end of the first year the value is 107.5% of the initial value.
The multiplying factor is 1.075.

WRITE

$$\text{a } 107.5\% = \frac{107.5}{100} = 1.075$$

$$1.075 \times 40\,000 = 43\,000$$

2 For the value at the end of year 2, calculate 107.5% of the amount accumulated in year 1, so multiply 1.075 by 43 000.

$$1.075 \times 43\,000 = 46\,225$$

3 For the value at the end of year 6, continue to multiply each successive value by 1.075 (so multiply another 4 times).

$$46\,225 \times 1.075 \times 1.075 \times 1.075 \times 1.075 = \$61\,732.06$$

4 To calculate the interest, subtract the amount invested from the total value.

$$61\,732.06 - 40\,000 = \$21\,732.06$$

5 Write the answer in a sentence.

The compound interest earned is \$21 732.06

b 1 Use the simple interest formula.

$$\begin{aligned} \text{b } SI &= \frac{PRT}{100} \\ &= \frac{40\,000 \times 7.5 \times 6}{100} \\ &= 18\,000 \end{aligned}$$

2 Write the answer in a sentence.

The simple interest earned is \$18 000.

c 1 Compare the two amounts of interest earned

$$\text{c } 21\,732.06 - 18\,000 = 3\,732.06$$

2 Write the answer in a sentence.

The investment in part **a** is the better option. Over the same time period, the interest earned is greater by \$3732.06.

ACTIVITY: CONSIDER YOUR OPTIONS

Nicky has \$15 000 to invest for 5 years.

She considers the following options.

- a. A Bankco term deposit at 5.25% p.a. compounded annually
- b. A Wecare term deposit paying an interest rate of 5.08% p.a. compounded quarterly
- c. A building society paying a return of 5.4% p.a. compounded monthly
- d. A business venture with a guaranteed return of 7.3% p.a. compounded daily
- e. A flat interest rate of 8.2% p.a.

All of the investments are equally secure.

Using a spreadsheet, prepare a report to advise Nicky which would be the best option.

learnON RESOURCES — ONLINE ONLY

Digital doc: SpreadSHEET Simple and compound interest (doc-10907)

Exercise 2.9 Compound interest

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–11

MASTER

Questions:
1–15

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Understanding and fluency

1. **WE19** Complete the tables to find the interest paid when:

- a. \$1000 is invested at 12% p.a. compounded annually for 3 years

	Principal	Interest	Total value
Year 1	\$1000		
Year 2			
Year 3			
Total interest =			

- b. \$100 000 is invested at 9% p.a. compounded annually for 4 years.

	Principal	Interest	Total value
Year 1	\$100 000		
Year 2			
Year 3			
Year 4			
Total interest =			



2. **WE20** Complete the tables to find the final value of each investment.

- a. \$5000 invested at 12% p.a. compounded annually for 3 years

	Start of year	End of year
Year 1	\$5000	
Year 2		
Year 3		

- b. \$200 000 invested at 7% p.a. compounded annually for 3 years

	Start of year	End of year
Year 1	\$200 000	
Year 2		
Year 3		

- c. \$100 000 invested at 8.5% p.a. compounded annually for 5 years

	Start of year	End of year
Year 1	\$100 000	
Year 2		
Year 3		
Year 4		
Year 5		

- d. \$12 000 invested at 15% p.a. compounded annually for 4 years

	Start of year	End of year
Year 1	\$12 000	
Year 2		
Year 3		
Year 4		

3. **WE21** For each of the following investments, use the multiplying factor to find:
- the total value
 - the amount of interest paid.
- \$8000 is invested for 8 years at 15% p.a. interest compounding annually.
 - \$50 000 is invested for 4 years at 6% p.a. interest compounding annually.
 - \$72 000 is invested for 3 years at 7.8% p.a. interest compounding annually.
 - \$150 000 is invested for 7 years at 6.3% p.a. interest compounding annually.
4. Peter invests \$5000 for 3 years at 6% p.a. simple interest, and Maria invests the same amount for 3 years at 5.8% p.a. compounding annually.
- Calculate the value of Peter's investment on maturity.
 - Calculate the value of Maria's investment on maturity.
 - Explain why Maria's investment is worth more, although she received a lower interest rate.
5. Gianni invests \$8000 at 15% p.a. compounded annually, and Dylan invests \$8000 at 15% p.a. flat rate. How much more than Dylan's investment will Gianni's investment be worth after:
- 1 year
 - 2 years
 - 5 years?
6. When her granddaughter was born, Barbara invested \$100 at the rate of 7% p.a. compounding annually. She plans to give it to her granddaughter on her eighteenth birthday. Use a spreadsheet to determine the value of the investment on her eighteenth birthday.
7. Chris and Jenny each invested \$10 000. Chris invested at 6.5% p.a. compounding annually, and Jenny took a flat rate of interest. After 5 years, their investments had equal value.
- Find the value of Chris's investment after 5 years.
 - Find Jenny's interest rate.
 - Find the value of each investment after 6 years.

Communicating, reasoning and problem solving

8. Two investment options are available to invest \$3000.
- Invest for 5 years at 5% p.a. compounding monthly.
 - Invest for 5 years at 5% p.a. compounding weekly.
- Explain which option you would choose and why.
9. There are 3 factors that affect the value of a compound interest investment: the principal, the interest rate and the length of the investment.
- Let the interest rate be 10% p.a. and the length of the investment be 2 years. Calculate the value of an investment of:
 - \$1000
 - \$2000
 - \$4000.
 - Comment on the effect of increasing the principal on the value of the investment.
 - Let the principal be \$1000 and the interest rate be 10% p.a. Calculate the value of an investment of:
 - 2 years
 - 4 years
 - 8 years.



- d. Comment on the effect of increasing the length of the investment on the value of the investment.
- e. Let the principal be \$1000 and the length of the investment be 5 years. Calculate the value of an investment of:
 - i. 6% interest p.a.
 - ii. 8% interest p.a.
 - iii. 10% interest p.a.
- f. Comment on the effect of increasing the interest rate on the value of the investment.
10. Calculate the value of each of the following investments if the principal is \$1000.
 - a. Interest rate = 8% p.a., compounding period = 1 year, time = 2 years
 - b. Interest rate = 8% p.a., compounding period = 6 months, time = 2 years
 - c. Interest rate = 8% p.a., compounding period = 3 months, time = 2 years
11. A bank offers a term deposit for 3 years at an interest rate of 8% p.a. with a compounding period of 6 months. What would be the end value of a \$5000 investment under these conditions?
12. A building society offers term deposits at 9%, compounded annually. A credit union offers term deposits at 10% but with simple interest only.
 - a. After 2 years, which has the larger value?
 - b. After 3 years, which has the larger value?
 - c. How many years does it take for the compound interest offer to have the greater value?
13. One aspect of compound interest is of great importance to investors: how long does it take to double my money? Consider a principal of \$100 and an annual interest rate of 10% (compounding annually).
 - a. How long does it take for this investment to be worth \$200?
 - b. How long would it take for the investment to be worth \$400 (a second doubling)?
14. Which would be better, and by what percentage — a wage rise of 20% or two successive wage rises of 10%?
15. Is compound interest 'fairer' than simple interest?

2.10 Review

Investigation | Rich task

Australian currency

Australia's coins have distinctive features and our notes are unique in colour and texture. Since decimal currency was introduced in Australia in 1966, our notes and coins have undergone many changes. Only our five-, ten- and twenty-cent coins are still minted as they were back then. The one- and two-cent coins are no longer in circulation, the fifty-cent coin is a different shape, the one- and two-dollar notes have been replaced by coins, and our notes have changed from paper to a special type of plastic.

Note: Answer the following questions on a separate sheet of paper.

Coins have two sides: an obverse side and a reverse side. The obverse side of all Australian coins depicts our reigning monarch, Queen Elizabeth II, and the year in which the coin was minted. The reverse side depicts a typical Australian feature and sometimes a special commemorative event.

1. What is depicted on the reverse side of each Australian coin?



The table below includes information on Australia's current coins in circulation. Use the table to answer questions 2 to 4.

Coin	Diameter (mm)	Mass (g)	Composition
Five-cent	19.41	2.83	75% copper, 25% nickel
Ten-cent	23.60	5.65	75% copper, 25% nickel
Twenty-cent	28.52	11.30	75% copper, 25% nickel
Fifty-cent	31.51	15.55	75% copper, 25% nickel
One-dollar	25.00	9.00	92% copper, 6% aluminium, 2% nickel
Two-dollar	20.50	6.60	92% copper, 6% aluminium, 2% nickel

- What are the metal compositions of each of the coins?
- Which is the heaviest coin and which is the lightest? List the coins in order from lightest to heaviest.
- Which has the smaller diameter — the five-cent coin or the two-dollar coin? Indicate the difference in size.

The table below displays information on Australia's current notes in circulation. The column on the far right compares the average life of the previously used paper notes with that of the current plastic notes. Use the table to answer questions 5 to 9.

Note	Date of issue	Size (mm)	Average life of notes (months)	
			Plastic	Paper
Five-dollar	07/07/1992 24/04/1995 01/01/2001 01/09/2016	130 × 65	40	6
Ten-dollar	01/11/1993	137 × 65	40	8
Twenty-dollar	31/10/1994	144 × 65	50	10
Fifty-dollar	04/10/1995	151 × 65	About 100	24
One-hundred-dollar	15/05/1996	158 × 65	About 450	104

- What denominations of notes are available in our Australian currency?
- On what date was Australia's first plastic note issued and what was the denomination of the note?
- Suggest reasons for the four issue dates for the five-dollar note.
- Why do you think each note is of a different size?
- The table clearly shows that the plastic notes last about five times as long as the paper notes we once used. Why do you think the fifty-dollar and one-hundred-dollar notes last longer than the five- and ten-dollar notes?

learn on RESOURCES — ONLINE ONLY

Digital doc: Investigation — Australian currency (doc-11904)

Digital doc: Code puzzle — What are the smallest and largest muscles in the human body? (doc-11905)

Digital doc: Summary Topic 2 (doc-22884)

Interactivity: Crossword Topic 2 (int-2700)

Interactivity: Word search Topic 2 (int-2699)

Interactivity: Sudoku Topic 2 (int-3210)

Exercise 2.10 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Jane earns an annual salary of \$45 650. Calculate her fortnightly pay.
2. Express \$638.96 per week as an annual salary.
3. Frank works as a casual shop assistant and is paid \$8.20 an hour from 3.30 pm to 5.30 pm, Monday to Friday, and \$9.50 an hour from 7 am to 12 noon on Saturday. What is his total pay for the week?
4. Below are the pay details for 4 people. Who receives the most money?
 - a. Billy receives \$18.50 per hour for a 40-hour working week.
 - b. Jasmine is on an annual salary of \$38 400.
 - c. Mladin receives \$1476.90 per fortnight.
 - d. Thuy receives \$3205 per month.
5. Daniel earns \$10 an hour for a regular 38-hour week. If he works overtime, he is paid double time. How much would he earn if he worked 42 hours in one week?
6. Bjorn earns \$468.75 per week award wage for a 38-hour week.
 - a. What is his standard hourly rate?
 - b. If he is paid time and a half for normal overtime, what is his pay for a week in which he works 41 hours?
7. Xana makes gift cards as a hobby and is paid 55 cents for each card.
 - a. How much would she earn if she made 50 cards?
 - b. How many cards would she need to make to earn \$44?
8. Kim sells cakes to the local shop. The shop pays for the ingredients. Kim is paid \$3.20 for each cake that she makes and 50 cents for each slice. If Kim makes 5 cakes and 15 slices one week, how much does she earn?
9. A salesperson earns a \$250 per week retainer, plus 2% commission on the first \$10 000 sale and 1.5% on the remainder. What is their total income for a week in which sales to the value of \$18 000 were made?
10. Jennifer is to start a new job selling mobile phones. She is paid commission only at the rate of 17.5% of sales. What value of sales must she make in order to receive commission of \$600 in one week?

Communicating, reasoning and problem solving

11. Phillipa is paid an annual salary of \$48 800.
 - a. Calculate Phillipa's gross weekly salary.
 - b. Calculate the total amount Phillipa will receive for her 4 weeks annual leave if she is paid a 17.5% holiday loading.
12. The annual cost of operating the rail system in Sydney is \$300 million. The Chief Executive Officer of the rail network is promised a bonus of 0.15% of any saving that she can make to the running of the system. Calculate the bonus paid if the cost is reduced to \$275 million.
13. Geoff works as a waiter and is paid \$12.50 per hour. If he works 40 hours per week, calculate:
 - a. Geoff's gross pay
 - b. the tax that Geoff should pay
 - c. the percentage of gross pay that Geoff pays in tax
 - d. Geoff's net pay if he also has \$5.35 deducted for union fees and \$100.00 for a loan repayment.
14.
 - a. Find the simple interest on an investment of \$4000 for 9 months at 4.9% per annum.
 - b. To what amount will the investment grow by the end of its term?
15. Calculate the monthly instalment needed to repay a loan of \$12 500 over 40 months at 9.75% p.a. simple interest.

16. A simple interest loan of \$5000 over 4 years incurred interest of \$2300. What interest rate was charged?
17. Daniela is going to invest \$16 000 for 2 years at 9% p.a. with interest compounded annually. Calculate:
- the value of the investment
 - the amount of interest earned.
18. Calculate 3.5% of 900.
19. Rearrange the formula $I = PRN$ to make N the subject.
20. Milos works in a supermarket and earns \$11.70 per hour. If Milos works Saturday he is paid at time and a half. State the hourly rate for which Milos works on Saturday.
21. **MC** Frank invests \$1000 at 5% p.a. for three years with interest compounded annually. Which of the following calculations will correctly find the value of Frank's investment?
- $\$1000 \times 5 \times 3$
 - $\$1000 \times 1.05 \times 3$
 - $\$1000 \times 1.05 \times 1.05 \times 1.05$
 - $\$1000 \times 1.05 \times 1.05 \times 1.05 - \$1000$

Answers

Topic 2 Financial mathematics

Exercise 2.2 Salaries and wages

1. a. \$1105.42 b. \$2210.85 c. \$4790.17
2. a. \$1198.08 b. \$2396.15 c. \$5191.67
3. a. \$19136 b. \$46410 c. \$68684.20
4. a. \$25870 b. \$42183.96 c. \$100498.06
5. a. \$3890 per month b. \$3200.68 per fortnight
6. a. \$19.75/h b. \$12.17/h c. \$25.73/h d. \$39.06/h
7. a. \$459.04 b. \$211.66
8. \$210.72
9. \$261.49
10. Rob earns more.
11. \$30.77/h
12. Job A
13. 38 hours
14. a. \$1605.77 b. \$2788.81 c. \$1100 d. \$8770
15. B
16. Minh
17. \$40 per hour
18. \$1735.95, \$1833.75
19. a. \$29.81 b. \$25.64 c. \$72080.58
20. It would depend on the terms and conditions of the position.
21. 12 weeks

Exercise 2.3 Special rates

1. a. \$23.94 b. \$47.80 c. \$43.50
2. a. \$906.10 b. \$794.33 c. \$833.56
3. a. \$25.16 b. \$113.22 c. \$1069.30
4. a. \$170.40 b. \$340.80 c. \$426.00
5. \$1156.96
6. a. \$30.20 b. \$45.30 c. \$60.40 d. \$135.90
7. a. \$32.93 b. \$197.55
8. a. \$11.70 b. \$15.60 c. \$19.50
9. 8, 8, 8, 8, 6

10.

Pay slip for: Susan WHITE	Week ending 17 August
Normal hours	38
Normal pay rate	\$25.60
Overtime hours	0
Overtime pay rate	\$38.40
Total pay	\$972.80

11.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	9.00 am	5.00 pm	8
Tuesday	Normal	9.00 am	5.00 pm	8
Wednesday	Normal	—		
Thursday	Normal	1.00 pm	9.00 pm	8
Friday	Normal	—		
Saturday	Time and a half	8.00 am	12.00 pm	4

12.

Pay slip for: Jason RUDD	Week ending 21 December
Normal hours	24
Normal pay rate	\$10.90
Overtime hours	4
Overtime pay rate	\$16.35
Total pay	\$327.00

13.

Day	Pay rate	Start time	Finish time	Hours worked
Monday	Normal	7.00 am	3.00 pm	8
Tuesday	Normal	7.00 am	3.00 pm	8
Wednesday				
Thursday				
Friday	Normal	11.00 pm	7.00 am	8
Saturday	Time and a half	11.00 pm	7.00 am	8
Sunday	Double time	11.00 pm	7.00 am	8

14.

Pay slip for: Brett SIMPSON	Week ending 15 September
Normal hours	24
Normal pay rate	\$16.80
Time-and-a-half hours	8
Time-and-a-half pay rate	\$25.20
Double time hours	8
Double time pay rate	\$33.60
Gross pay	\$873.60

15. a. \$754.40 b. \$1042.80 c. \$1347.42

16. a. \$232.02 b. \$1211.66

17. B

18. No. Glen should have been paid \$1034.

19. \$1155

20.

	Hours worked	Regular pay	Overtime pay	Total pay
a	32	\$464	\$0	\$464
b	38.5	\$522	\$54.38	\$576.38
c	40.5	\$522	\$101.50	\$623.50
d	47.2	\$522	\$295.80	\$817.80

21. a. Stewart \$930.75, Helen \$1108.80

b. i. 85 hours

ii. No, because she would have worked 92.5 hours, which is 22.5 hours over the required 70 hours per fortnight.

c. \$894.94

22. It would be preferable to be paid using a time sheet if the hourly rate was not very high but the job had much potential for over time.

Exercise 2.4 Piecework

1. \$97.50 2. \$89.50 3. \$175.50 4. \$318.75

5. a. \$144.00 b. 28 c. \$21.60

6. a. \$144 b. \$218 c. \$465

7. a. \$107.50 b. 24 c. \$7.17

8. a. \$52 b. \$9.45

9. a. \$1398.25 b. \$1568.12

10. \$422.40

11. a. \$8950 b. \$3550

12. Answers may vary. Advantages could include flexible hours or working from home; disadvantages could be that time per item is often at a very low hourly rate.

- Figure 1. *Graphical representation of the study design.*

Exercise 2.8 Simple interest

1. a. \$1200	b. \$6200		
2. \$1950			
3. a. i. \$100	ii. \$1100	b. i. \$1920	ii. \$5920
c. i. \$1440	ii. \$9440	d. i. \$263.25	ii. \$2963.25
e. i. \$5048.32	ii. \$20 726.32		
4. a. \$3300	b. \$5645	5. a. \$22 275.50	b. \$371.26
6. \$211.90	7. 2.5 years	8. \$12 500	9. 8%
10. a. 2 years	b. 2.5 years	c. \$600	d. \$4500.06
			e. 8%
11. a. \$16 998	b. \$8770.97	c. \$28 768.97	d. 48
			e. \$536.85
12. a. \$5768	b. \$6902	c. \$120.17, \$143.79	
13. a. \$180 000	b. \$80 550	c. \$260 550	d. \$4342.50
14. The interest rate in the simple interest formula needs to be converted from a percentage into a decimal: 4.2% = 0.042			
15. 5.3%			
16. a. \$2300	b. \$12 300	17. a. \$78.13	b. \$288 000
18. a. Answers will vary.	b. Yes	c. 91.04	
19. Answers may vary but could include that interest can affect the cost of housing and hence standard of living.			

Exercise 2.9 Compound interest

1. a.		Principal	Interest	Total value
	Year 1	\$1000	\$120	\$1120
	Year 2	\$1120	\$134.40	\$1254.40
	Year 3	\$1254.40	\$150.53	\$1404.93
	Total interest = \$404.93			

b.	Principal	Interest	Total value
Year 1	\$100 000	\$9000	\$109 000
Year 2	\$109 000	\$9810	\$118 810
Year 3	\$118 810	\$10 692.90	\$129 502.90
Year 4	\$129 502.90	\$11 655.26	\$141 158.16
Total interest = \$41 158.16			

2. a.	Start of year	End of year
Year 1	\$5000	\$5600
Year 2	\$5600	\$6272
Year 3	\$6272	\$7024.64

b.	Start of year	End of year
Year 1	\$200 000	\$214 000
Year 2	\$214 000	\$228 980
Year 3	\$228 980	\$245 008.60

	Start of year	End of year
Year 1	\$100 000	\$108 500
Year 2	\$108 500	\$117 722.50
Year 3	\$117 722.50	\$127 728.91
Year 4	\$127 728.91	\$138 585.87
Year 5	\$138 585.87	\$150 365.67

d.	Start of year	End of year
Year 1	\$12 000	\$13 800
Year 2	\$13 800	\$15 870
Year 3	\$15 870	\$18 250.50
Year 4	\$18 250.50	\$20 988.08

3. a. i. \$24472.18 ii. \$16472.18 b. i. \$63 123.85 ii. \$13 123.85
 c. i. \$90 196.31 ii. \$18 196.31 d. i. \$230 050.99 ii. \$80 050.99
4. a. \$5900 b. \$5921.44 c. Each year, Maria's principal increases.
5. a. 0 b. \$180 c. \$2090.86
6. \$337.99
7. a. \$13 700.87 b. 7.4% c. Chris, \$14 591.42; Jenny, \$14 440
8. Option B would be the best choice, as the shorter the time between the compounding periods, the greater the interest paid.
9. a. i. \$1210 ii. \$2420 iii. \$4840
 b. Increasing the principal will increase the value of the investment because it will have a higher value of interest.
 c. i. \$1210 ii. \$1464.10 iii. \$2143.59
 d. Increasing the length of the investment will increase the value of the investment because it will have a higher value of interest.
 e. i. \$1338.23 ii. \$1469.33 iii. \$1610.51
 f. Increasing the interest rate will increase the value of the investment because it will have a higher value of interest.
10. a. \$1166.40 b. \$1169.86 c. \$1171.66
11. \$6326.60
12. a. Simple interest b. Simple interest c. 4 years
13. a. 7.27 years b. 14.55 years
14. Two successive wage rises of 10%
15. Compound interest does seem fairer than simple interest because the interest is calculated only on the money in the account. Sometimes the compound interest rate is lower, so a comparison needs to be done on the alternatives.

Investigation | Rich task

- Five-cent coin: echidna
 Ten-cent coin: lyrebird
 Twenty-cent coin: platypus
 Fifty-cent coin: coat of arms
 One-dollar coin: five kangaroos
 Two-dollar coin: Aboriginal elder Gwoya Jungarai
- Refer to the table.
- 5c, 10c, \$2, \$1, 20c, 50c
- The five-cent coin has a smaller diameter, 1.09 mm.
- \$5, \$10, \$20, \$50, \$100
- 7 July 1992; \$5
- Answers will vary.
- The different sizes allow blind people to tell the difference between each note.
- The fifty-dollar and one-hundred-dollar notes are used less frequently.

Exercise 2.10 Review questions

1. \$1755.77	2. \$33 225.92	3. \$129.50	4. a	5. \$460
6. a. \$12.34	b. \$524.28			
7. a. \$27.50	b. 80 cards			
8. \$23.50	9. \$570	10. \$3428.57		
11. a. \$938.46	b. \$4410.76	12. \$37 500		
13. a. \$500	b. \$37.00	c. 7.4%	d. \$357.65	
14. a. \$147	b. \$4147			
15. \$414.06	16. 11.5%			
17. a. \$19 009.60	b. \$3009.60			
18. 31.5	19. $N = \frac{I}{PR}$	20. \$17.55	21. C	

TOPIC 3

Similar figures and scale drawings

[Stages 5.1 and 5.3]

3.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



3.1.1 Why learn this?

Geometry is a study of points, lines and angles and how they combine to make different shapes. Similarity and congruence between two figures are important concepts in geometry. Recognising and using congruent and similar shapes can make calculations and design more effective.

DISCUSSION

Geometry is used in many art forms. Can you think of some examples? Does this use vary between different cultures?

LEARNING SEQUENCE

- 3.1 Overview
- 3.2 [Stage 4] Congruent figures
- 3.3 [Stage 5.1] Scale drawings
- 3.4 [Stage 5.1] Similar figures
- 3.5 [Stage 5.3] Area and volume of similar figures
- 3.6 Review



LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- describes and applies the properties of similar figures and scale drawings MA5.1-11MG
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses formal geometric reasoning to establish properties of triangles and quadrilaterals MA5.3-16MG

CONTENT DESCRIPTIONS

Students:

Use the enlargement transformation to explain similarity (ACMMG220)

Solve problems using ratio and scale factors in similar figures (ACMMG221)

Apply logical reasoning, including the use of congruence and similarity, to numerical exercises involving plane shapes (ACMMG244)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — Mathematics in art (eles-1692)

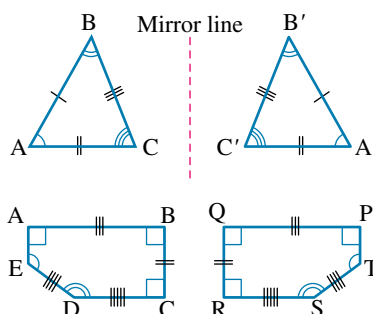
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

3.2 Congruent figures

[Stage 4]

3.2.1 Review of congruent figures

- **Congruent figures** are identical figures; that is, they have exactly the same shape and size.
- They can be superimposed exactly on top of each other, using reflection, rotation and translation.

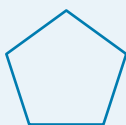


- The symbol for congruence is $\equiv$. This is read as 'is congruent to'.
- For the diagrams shown above, $ABC \equiv A'B'C'$ and $ABCD \equiv PQRS$.
- When writing congruence statements, the vertices of the figures are named in corresponding order.

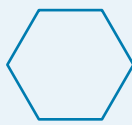
WORKED EXAMPLE 1

Select a pair of congruent shapes from the following set.

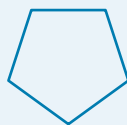
a



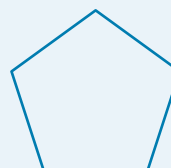
b



c



d



THINK

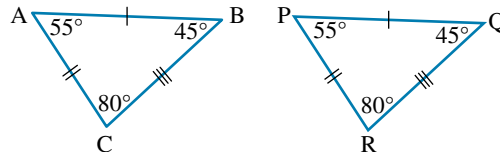
Figures **a** and **c** are identical in shape and size; they just have different orientation.

WRITE

Shape **a** $\equiv$ shape **c**

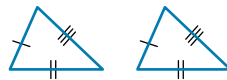
3.2.2 Congruent triangles

- It is not necessary to know that all three sides and all three angles of one triangle are equal to the corresponding sides and angles of another triangle to ensure that the two triangles are congruent. There are certain minimum conditions that will guarantee that this is so.



3.2.3 Side-side-side condition of congruence (SSS)

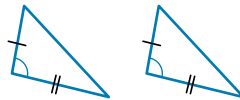
- If two triangles have equal corresponding sides, the angles opposite these corresponding sides will also be equal in size. This means that these two triangles are congruent.



- This is the side-side-side (SSS) condition of congruence.

3.2.4 Side-angle-side condition of congruence (SAS)

- In this situation, two pairs of corresponding sides are equal. If the angles between these sides are equal, then the triangles are congruent.



- This is the side-angle-side (SAS) condition of congruence.

3.2.5 Angle-side-angle condition of congruence (ASA)

- Two pairs of corresponding angles are equal in these triangles. (The third pair of angles will also be equal.)



- If one pair of corresponding sides is equal, then the triangles are congruent.
- This is the angle-side-angle (ASA) condition of congruence.

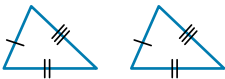
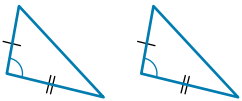
3.2.6 Right angle-hypotenuse-side condition of congruence (RHS)

- In a right-angled triangle, if the hypotenuse and one other side are equal, then the triangles are congruent.



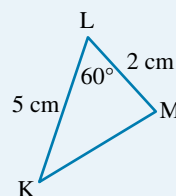
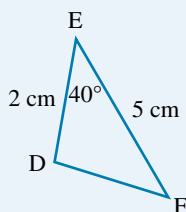
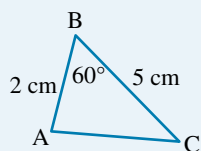
- This is the right angle-hypotenuse-side (RHS) condition of congruence.

3.2.7 Summary of congruence tests

Test	Description	Abbreviation
	All corresponding sides are equal in length.	SSS (side–side–side)
	Two corresponding sides are equal in length and the included angles are equal in size.	SAS (side–angle–side)
	Two angles are equal in size and there is one pair of corresponding sides of equal length.	ASA (angle–side–angle)
	In a right-angled triangle, the hypotenuse and one pair of corresponding sides are equal in length.	RHS (right angle–hypotenuse–side)

WORKED EXAMPLE 2

Which of the following triangles are congruent? Give reasons for your answer.



THINK

In all three triangles, two given sides are of equal length (2 cm and 5 cm). Triangles ABC and KLM also have an included angle of equal size (60°).

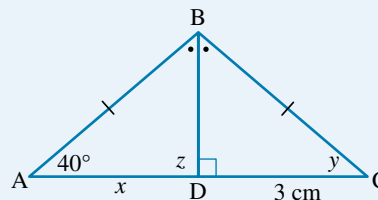
B corresponds to L, and A corresponds to M.

WRITE

$$\triangle ABC \equiv \triangle KLM \text{ (SAS)}$$

WORKED EXAMPLE 3

Given that $\triangle ABD \equiv \triangle CBD$, find the values of the pronumerals in the figure shown.



THINK

- In congruent triangles, corresponding sides are equal in length. Side AD (marked x) corresponds to side CD.
- Since triangles are congruent, corresponding angles are equal.

WRITE

$$\triangle ABD \equiv \triangle CBD$$

$$AD = CD$$

$$x = 3 \text{ cm}$$

$$\angle C = \angle A$$

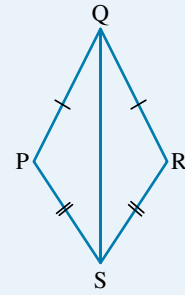
$$y = 40^\circ$$

$$\angle BDA = \angle BDC$$

$$z = 90^\circ$$

WORKED EXAMPLE 4

Prove that $\triangle PQS$ is congruent to $\triangle RQS$.



THINK

- 1 Study the diagram and state which sides and/or angles are equal.
- 2 This fits the SSS test and proves congruence.

WRITE

$QP = QR$ (given)
 $PS = RS$ (given)
 QS is common.
 $\triangle PQS \equiv \triangle RQS$ (SSS)

learnon RESOURCES — ONLINE ONLY

Interactivity: Congruent figures (int-3749)

Interactivity: Congruent triangles (int-3754)

Interactivity: Congruency tests (int-3755)

Digital doc: SkillsHEET Naming angles (doc-6195)

Digital doc: SkillsHEET Complementary and supplementary angles (doc-6196)

Digital doc: SkillsHEET Angles in a triangle (doc-6197)

Digital doc: WorkSHEET Congruent figures (doc-6201)

Exercise 3.2 Congruent figures

Individual pathways

PRACTISE

Questions:
1–7, 10, 11

CONSOLIDATE

Questions:
1–8, 10–13

MASTER

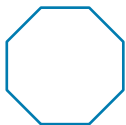
Questions:
1–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE6** Select a pair of congruent shapes from the figures in each part of this question.

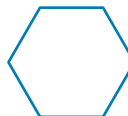
a. i.



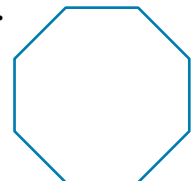
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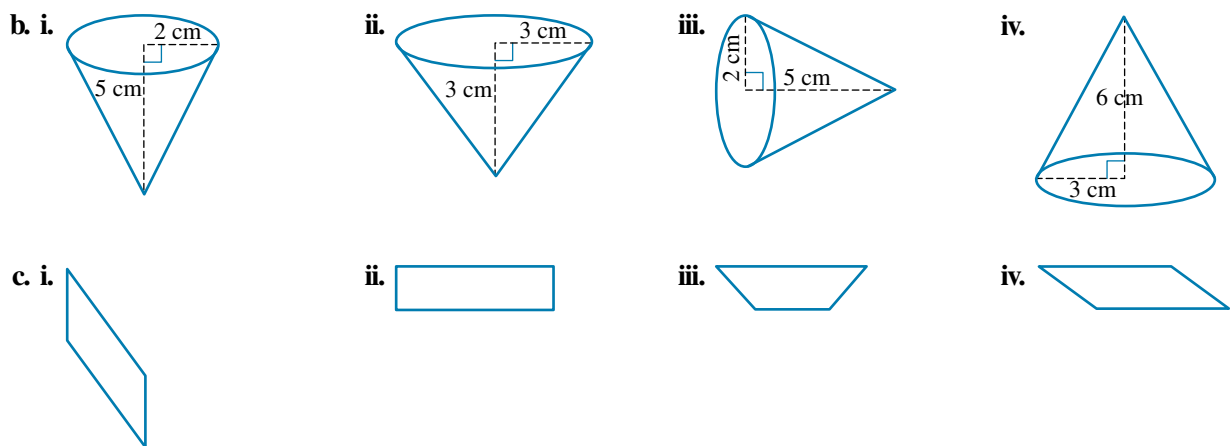


iii.

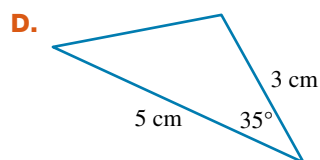
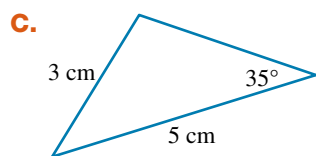
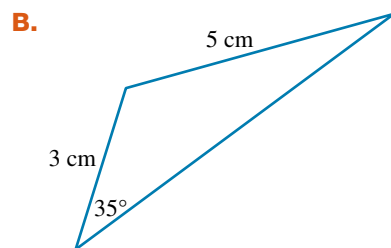
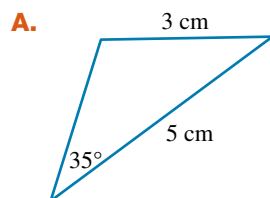
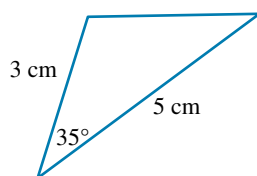


iv.

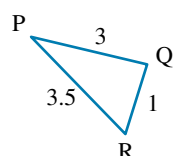
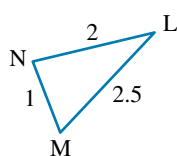
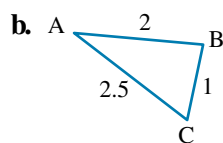
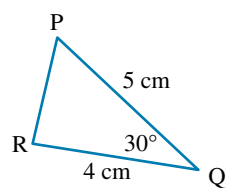
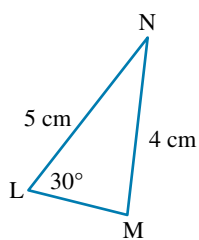
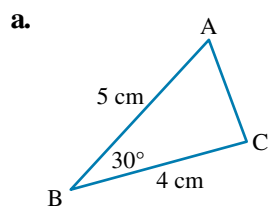


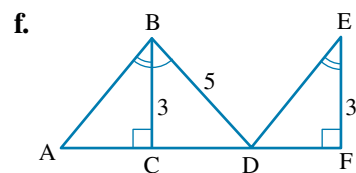
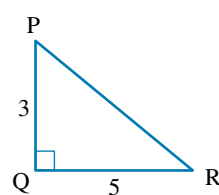
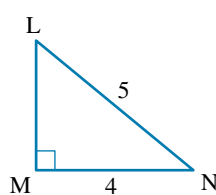
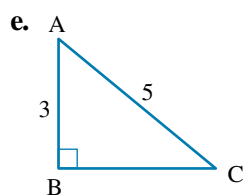
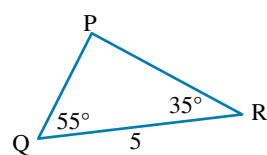
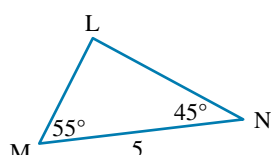
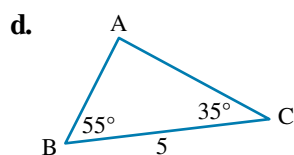
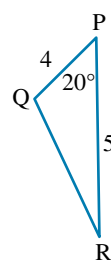
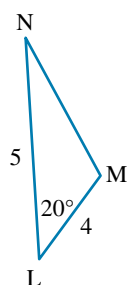
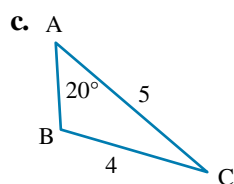


2. **MC** Which of the following triangles is congruent to the triangle shown?

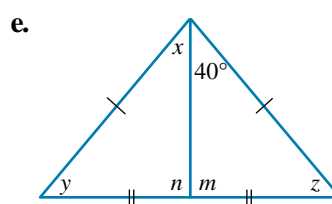
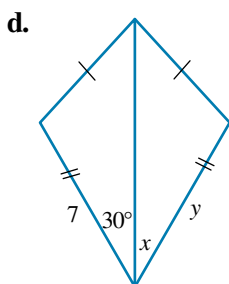
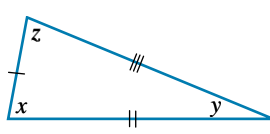
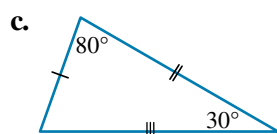
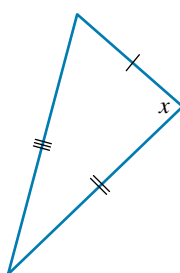
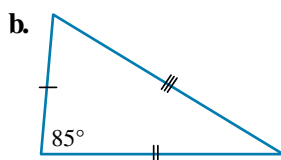
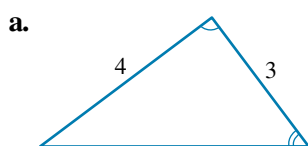


3. **WE7** In each part of the question, which of the triangles are congruent? Give a reason for your answer.



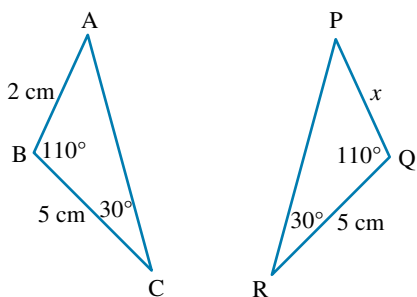


4. **WE8** Find the values of the pronumerals in each of the following pairs of congruent triangles. All side lengths are in centimetres.

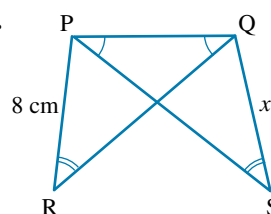


5. Find the length of the side marked with the pronumeral in each of the following pairs of congruent triangles.

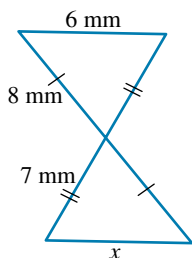
a.



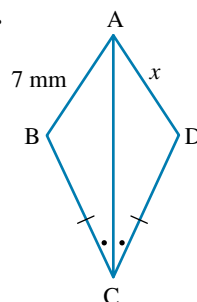
b.



c.



d.

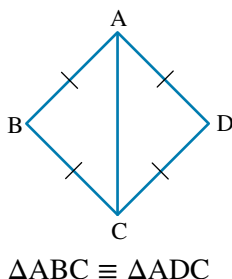


6. Do congruent figures have the same area? Explain.

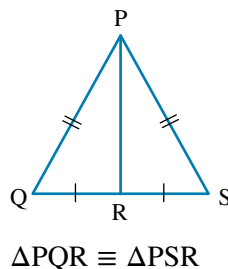
Communicating, reasoning and problem solving

7. **WE9** For each of the following, prove the statement shown.

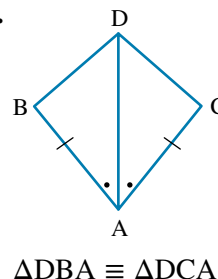
a.



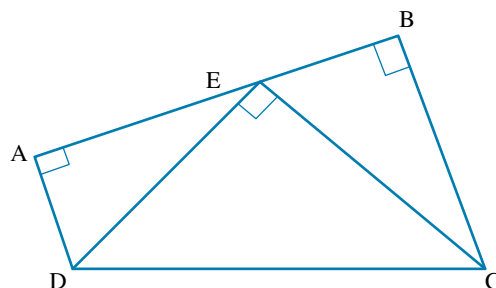
b.



c.

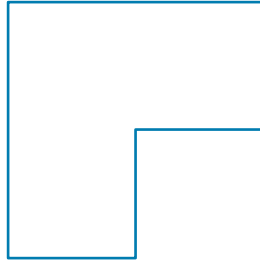


8. Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.
9. ABCD is a trapezium with both AD and BC perpendicular to AB. If a right-angled triangle DEC is constructed with an angle $\angle ECD$ equal to 45° , prove that $\triangle EDA \equiv \triangle ECB$.

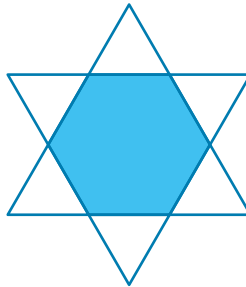


10. If two congruent triangles have a right angle, is the reason always 'right angle, hypotenuse and corresponding side'? Justify your answer.
11. A teacher asked his class to each draw a triangle that has side lengths of 5 cm and 4 cm, and an angle of 45° that is not formed at the point joining the 5-cm and 4-cm sides. Would the triangles drawn by every member of the class be congruent? Explain why.

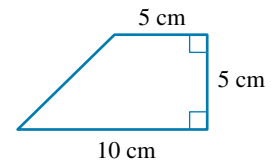
12. Make 5 congruent triangles from 9 matchsticks.
 13. Make 7 congruent triangles from 9 matchsticks.
 14. Show how this figure can be cut into four congruent pieces.



15. Find the ratio of the outer (unshaded) area to the inner (shaded) area of this six-pointed star.



16. What is the easiest way to determine if two figures are congruent?
 17. On a piece of paper, draw a trapezium with the dimensions given in the diagram. Divide your trapezium into four congruent trapeziums that are similar in shape to the original trapezium.



3.3 Scale drawings

[Stage 5.1]

3.3.1 Review of ratio

- **Ratios** are used to compare quantities of the same kind, measured in the same unit.
- The ratio '1 is to 4' can be written in two ways: as $1 : 4$ or as $\frac{1}{4}$.
- The order of the numbers in a ratio is important.
- In simplest form, a ratio is written using the smallest whole numbers possible.

WORKED EXAMPLE 5

Express each of the following ratios in simplest form.

a $24 : 8$

b $3.6 : 8.4$

c $1\frac{4}{9} : 1\frac{2}{3}$

THINK

a Divide both terms by the highest common factor (8).

b 1 Multiply both terms by 10 to obtain whole numbers.

2 Divide both terms by the highest common factor (12).

c 1 Change both mixed numbers into improper fractions.

2 Multiply both terms by the lowest common denominator (9) to obtain whole numbers.

WRITE

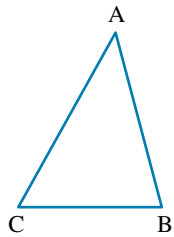
a $24 : 8$
 $= 3 : 1$

b $3.6 : 8.4$
 $= 36 : 84$
 $= 3 : 7$

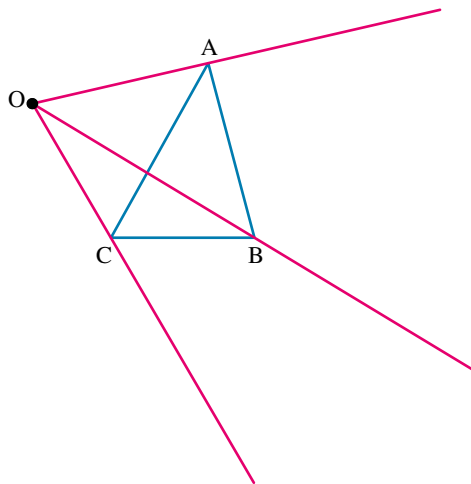
c $1\frac{4}{9} : 1\frac{2}{3}$
 $= \frac{13}{9} : \frac{5}{3}$
 $= 13 : 15$

3.3.2 Enlarging by a scale factor

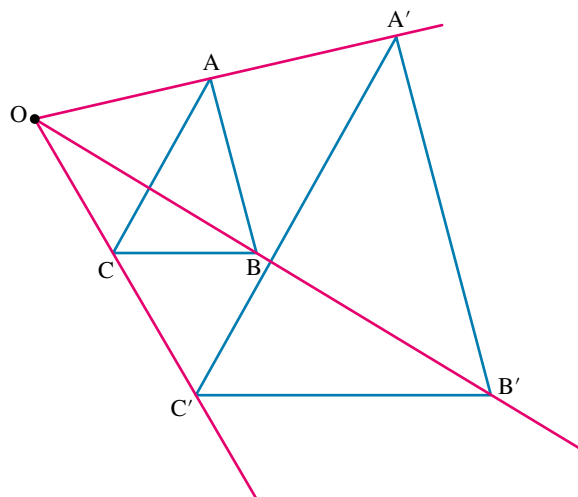
- Consider the case where we want to enlarge triangle ABC (called the **object**) by a **scale factor** of 2, that is, to make it twice its size.



- Here is one method that we can use.
 1. Mark a point O (called the centre of enlargement) somewhere outside the triangle and draw the lines OA, OB and OC as shown.



2. Measure the length of OA and mark in the point A' (called the **image** of A) so that the distance OA' is twice that of OA.
3. In the same way, mark in points B' and C'. ($OB' = 2 \times OB$, and $OC' = 2 \times OC$.)



4. Joining A'B'C' gives a triangle that has side lengths double those of $\triangle ABC$. $\triangle A'B'C'$ is called the image of $\triangle ABC$.

- By definition:

$$\text{scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

DISCUSSION

What is the link between ratios and scale factors?

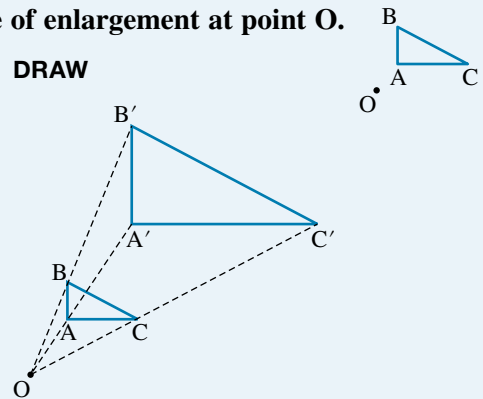
WORKED EXAMPLE 6

Enlarge triangle ABC by a scale factor of 3, with the centre of enlargement at point O.

THINK

- 1 Join each vertex of the triangle to the centre of enlargement O with straight lines and extend them.
- 2 Locate points A', B' and C' along the lines, $OA' = 3OA$, $OB' = 3OB$ and $OC' = 3OC$.
- 3 Join points A', B' and C' to complete the image.

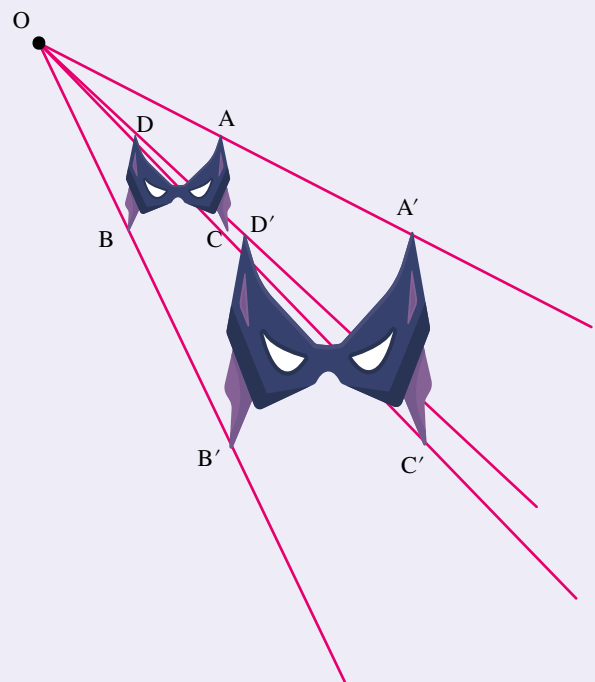
DRAW



- Enlargements have the following properties.
 - The corresponding side lengths of the enlarged figure are changed in a fixed ratio (that is, the same ratio).
 - The corresponding angles are the same.
 - A scale factor greater than 1 produces an enlarged figure.
 - If the scale factor is a positive number less than 1, the image is smaller than the object (reduction has taken place).

ACTIVITY: ENLARGING A CARTOON

- 1 Use the internet to choose a cartoon or a picture to enlarge.
- 2 Copy and paste your picture onto a large sheet of paper.
- 3 Mark the important vertices or points on your cartoon or picture.
- 4 Use the method outlined in section 3.3.2 to enlarge your cartoon or picture by a scale factor of 2.



WORKED EXAMPLE 7

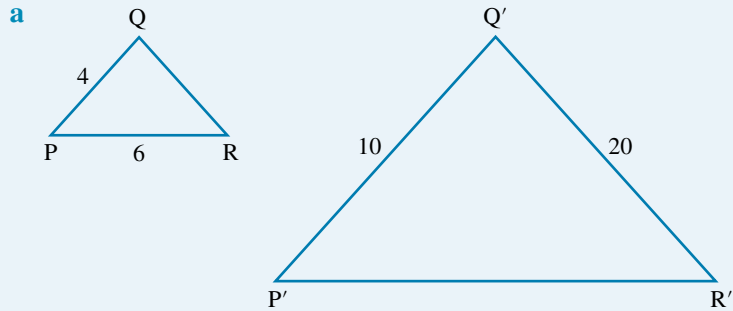
A triangle PQR has been enlarged to triangle P'Q'R'. PQ = 4 cm, PR = 6 cm, P'Q' = 10 cm and Q'R' = 20 cm. Calculate:

- the scale factor for the enlargement
- the length of P'R'
- the length of QR.

THINK

a 1 Draw a diagram.

WRITE/DRAW



- 2** Find two corresponding sides.
P'Q' corresponds to PQ.

$$\begin{aligned}\text{Scale factor} &= \frac{\text{image length}}{\text{object length}} \\ &= \frac{P'Q'}{PQ} \\ &= \frac{10}{4} \\ &= 2.5\end{aligned}$$

- b 1** Apply the scale factor.
 $P'R' = 2.5 \times PR$

b
$$\begin{aligned}P'R' &= 2.5 \times PR \\ &= 2.5 \times 6 \\ &= 15\end{aligned}$$

2 Write the answer.

P'R' is 15 cm long.

- c 1** Apply the scale factor.
 $Q'R' = 2.5 \times QR$

c
$$\begin{aligned}\frac{Q'R'}{QR} &= \frac{20 \text{ cm}}{x \text{ cm}} = 2.5 \\ Q'R' &= 2.5 \times QR \\ 20 &= 2.5 \times QR \\ QR &= \frac{20}{2.5} \\ &= 8\end{aligned}$$

2 Write the answer.

QR is 8 cm long.

3.3.3 Scales in photographs, plans and drawings



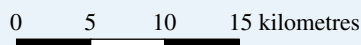
DISCUSSION

The image above is a photo of the largest dinosaur footprint ever found. It was discovered on the north-western coast of Western Australia and is a footprint of a sauropod. In the photograph, the footprint is 6 cm. If the scale factor of the photograph to the real footprint is 28.33, what is the actual length of the footprint to the nearest cm?

- A scale on a photograph, plan or drawing describes the ratio between the distance or size on the photograph, plan or drawing and the actual distance or size.
- A scale may be written several ways.
 - A ratio such as 1 : 100 000 means that 1 cm on the map or plan represents a distance of 100 000 cm on the Earth, 1 mm on the map or plan represents 100 000 mm on the Earth, etc.
 - A statement such as 1 cm $\Leftrightarrow$ 500 km means that 1 cm on the map or plan represents a distance of 500 km on the Earth.
 - A graphical bar scale, such as the one shown in Worked example 8, can be used. In this case, we need to measure the length of the bar to determine the scale ratio.

WORKED EXAMPLE 8

- a** The scale on a house plan is 1 : 2000. How many metres does a length of 1 cm on the map represent?
- b** Write the scale 1 cm $\Leftrightarrow$ 5000 m as a ratio scale.
- c** A map shows a scale of 1 cm $\Leftrightarrow$ 1500 m. What distance is represented by 18 mm on the map?
- d** Use the following graphical scale to determine the distance that 1 cm on the map represents.



THINK

- a 1** Use the ratio to add the same units to both sides. Since the answer asks for cm, use this unit.

WRITE

- a** 1 : 2000 is the same as:
1 cm $\Leftrightarrow$ 2000 cm

2 Bring the length to an appropriate unit. In this case, the answer is required in metres.

$$2000 \text{ cm} = (2000 \div 100) \text{ m} \\ = 20 \text{ m}$$

So, $1 \text{ cm} \Leftrightarrow 20 \text{ m}$.

3 Write the answer.

1 cm on the map represents a length of 20 m in reality.

b 1 Convert the ground units to the same as those of the map.

b $5000 \text{ m} = (5000 \times 100) \text{ cm}$
 $= 500\,000 \text{ cm}$

2 Write the scale as a ratio.

So, $1 \text{ cm} \Leftrightarrow 5000 \text{ m}$ is the same as
 $1 \text{ cm} \Leftrightarrow 500\,000 \text{ cm}$.
Expressed as a ratio, this is $1 : 500\,000$.

c 1 Write a statement explaining this scale.

c $1 \text{ cm} \Leftrightarrow 1500 \text{ m}$ means that 1 cm on the map represents a distance of 1500 m.

2 Change the question distance to the same units as those for the map.

$$18 \text{ mm} = (18 \div 10) \text{ cm} \\ = 1.8 \text{ cm}$$

3 Multiply by this scale factor.

1.8 cm represents $(1.8 \times 1500) \text{ m}$, which is
2700 m or 2.7 km.

d 1 Measure the length of the bars with a ruler.

d The bars measure 3.5 cm.

2 Relate this measurement to the length shown in the scale.

This represents a length of 15 km.

3 Write as a scale.

$$3.5 \text{ cm} \Leftrightarrow 15 \text{ km}$$

4 Calculate a 1 cm map distance and write the answer.

$$\text{So, } 1 \text{ cm} \Leftrightarrow \frac{15}{35} \text{ km.}$$

1 cm on the map represents a distance of 4.3 km.

- Sometimes it is necessary to do the reverse, and convert from a real length to a plan measurement.

WORKED EXAMPLE 9

The length of a wall for a particular house is 12 m. If the scale of a house plan is 1 : 200, what length, on the plan, would represent the actual length of the wall?

THINK

1 Write the scale as a ratio using units of cm.

WRITE

$1 : 200$ means that 1 cm on the plan represents an actual length of 200 cm.

$$1 \text{ cm} \Leftrightarrow 200 \text{ cm}$$

2 Convert the actual length from cm to m units.

$$200 \text{ cm} = (200 \div 100) \text{ m} \\ = 2 \text{ m}$$

So 1 cm on the plan represents an actual length of 2 m.

3 Divide the actual length by 2 to find how many lots of 2 m there are in 12 m.

$$\frac{12 \text{ m}}{2 \text{ m}} = 6$$

4 Multiply both sides of the scale by this scale factor, then simplify.

$$1 \text{ cm} \times 6 \Leftrightarrow 2 \text{ m} \times 6 \\ 6 \text{ cm} \Leftrightarrow 12 \text{ m}$$

5 Answer the question.

The length of the wall on the house plan would be 6 cm.

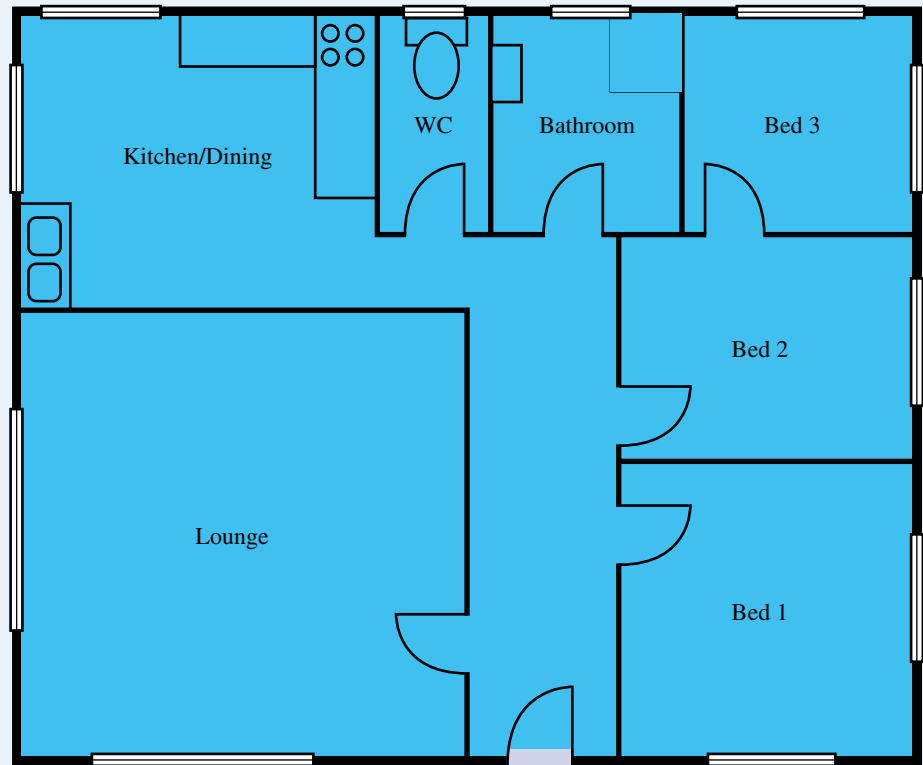
3.3.4 House plans

- Whenever a house is being constructed, floor plans are supplied to the builder so that the footings, walls, roof and other parts of the house are all placed in the correct positions and are the correct length. The scale of the plan is generally given as a ratio. This enables us to calculate all dimensions within the house.

WORKED EXAMPLE 10

A plan for a house is shown.

- Calculate the dimensions of the house.
- Calculate the area of the lounge room.



Scale 1 : 100

THINK

- Measure the length and width of the house on the plan.
 - Multiply each of these measurements by 100.
 - Write your answer.
- Measure the length and width of the lounge room on the plan.
 - Multiply each of these measurements by 100.

WRITE

- $\text{Length of house on plan} = 12 \text{ cm}$
 $\text{Width of house on plan} = 10 \text{ cm}$
 $\text{Actual length of house} = 12 \text{ cm} \times 100$
 $\quad = 1200 \text{ cm}$
 $\quad = 12 \text{ m}$
 $\text{Actual width} = 10 \text{ cm} \times 100$
 $\quad = 1000 \text{ cm}$
 $\quad = 10 \text{ m}$
The dimensions of the house are 12 m by 10 m.
- $\text{Length of lounge room on plan} = 6 \text{ cm}$
 $\text{Width of lounge room on plan} = 6 \text{ cm}$
 $\text{Actual length of lounge room} = 6 \text{ cm} \times 100$
 $\quad = 600 \text{ cm}$
 $\quad = 6 \text{ m}$
The actual width of the lounge room is also 6 m.

3 Calculate the area of the lounge room.

$$\begin{aligned}\text{Area} &= 6^2 \\ &= 36 \text{ m}^2\end{aligned}$$

4 Write your answer.

The area of the lounge room is 36 m^2 .

- In addition to a floor plan, a builder needs plans that show what the house will look like if it is viewed from the front, back and sides. These plans are called *elevations*. These are also drawn to scale.

WORKED EXAMPLE 11

The diagram shows the front elevation of a house.

- Calculate the height of the eaves on the lower side of the house.
- Measure the angle of the pitch of the roof.



THINK

- 1 Measure the height on the plan for the lower side of the house.
2 Multiply the plan measurement by 100.
3 Write your answer.
- 1 Measure the angle that the slope of the roof makes with the horizontal.
2 Write your answer.

WRITE

a Height on the plan = 3.5 cm

$$\begin{aligned}\text{Actual height} &= 3.5 \text{ cm} \times 100 \\ &= 350 \text{ cm} \\ &= 3.5 \text{ m}\end{aligned}$$

The height of the eaves is 3.5 m.

b Angle to horizontal = 45°

The angle of the pitch of the roof is 45° .

ACTIVITY: USING TECHNOLOGY TO CONSTRUCT SCALE DRAWINGS

With an understanding of scale drawings, you should now be able to draw plans of objects familiar to you, such as your home, bedroom or classroom.

Using software such as Google SketchUp, create a 2-D or 3-D model of your classroom or another familiar building.

learn on RESOURCES — ONLINE ONLY

Interactivity: Introduction to ratios (int-3733)

Interactivity: Scale factors (int-6041)

Interactivity: Scales (int-4146)

Interactivity: House plans (int-4147)

Exercise 3.3 Scale drawings

Individual pathways

PRACTISE

Questions:

1, 2, 3a–e, 4a–d, 5, 6, 7, 11, 12

CONSOLIDATE

Questions:

1, 2, 3e–j, 5, 6, 7, 8–13

MASTER

Questions:

2, 3, 8–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE5a** Express each of the following ratios in the simplest form.

a. $12 : 18$

b. $8 : 56$

c. $9 : 27$

d. $14 : 35$

e. $88 : 66$

f. $16 : 60$

g. $200 : 155$

h. $144 : 44$

i. $32 : 100$

j. $800 : 264$

2. **WE5b** Express each of the following ratios in the simplest form.

a. $1.2 : 0.2$

b. $3.9 : 4.5$

c. $9.6 : 2.4$

d. $18 : 3.6$

e. $1.8 : 3.6$

f. $4.4 : 0.66$

g. $0.9 : 5.4$

h. $0.35 : 0.21$

i. $6 : 1.2$

j. $12.1 : 5.5$

k. $8.6 : 4$

l. $0.07 : 14$

3. **WE5c** Write each of the following ratios in the simplest form.

a. $1\frac{1}{2} : 2$

b. $2 : 1\frac{3}{4}$

c. $1\frac{1}{3} : 2$

d. $1\frac{2}{5} : 1\frac{1}{4}$

e. $3 : \frac{2}{3}$

f. $\frac{4}{7} : 2$

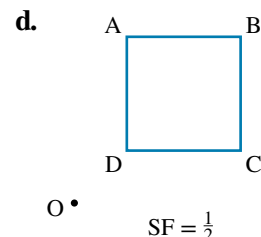
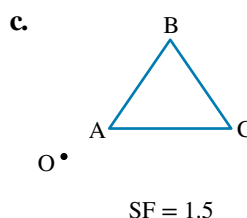
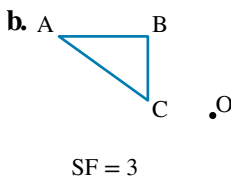
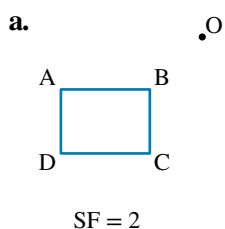
g. $5 : 1\frac{1}{2}$

h. $2\frac{3}{4} : 1\frac{1}{3}$

i. $3\frac{5}{6} : 2\frac{1}{2}$

j. $1\frac{3}{5} : 6\frac{4}{8}$

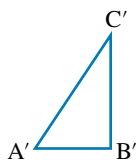
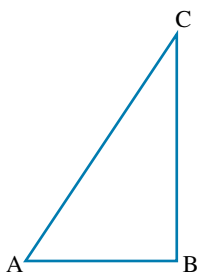
4. **WE6** Enlarge each of the following figures by the given scale factor and the centre of enlargement marked O. Show the image of each figure.



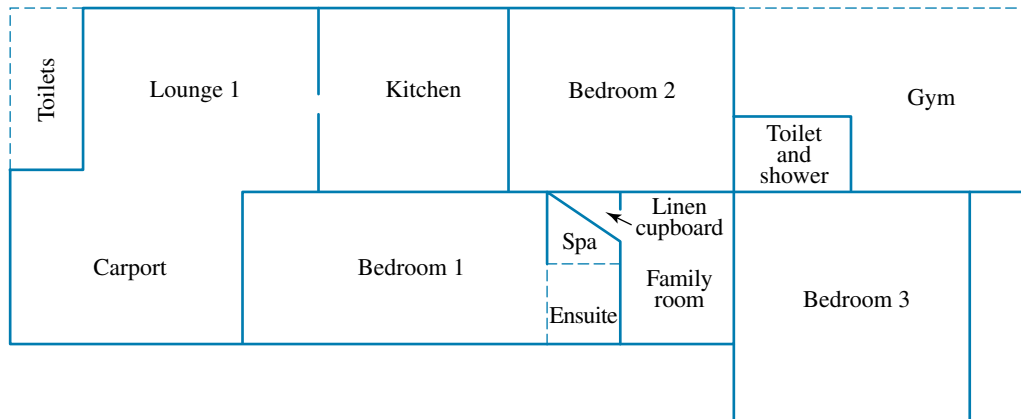
5. **WE7** A quadrilateral ABCD is enlarged to $A'B'C'D'$. $AB = 7$ cm, $AD = 4$ cm, $A'B' = 21$ cm, $B'C' = 10.5$ cm. Find:

- the scale factor for enlargement
- $A'D'$
- BC.

6. $\triangle ABC$ is scaled down to $\triangle A'B'C'$. By measuring the side lengths, determine the scale factor.



7. The diagram shows the ground plan of a house. Bedroom 1 is $8\text{ m} \times 4\text{ m}$.



- a. Using the dimensions given for bedroom 1, find the scale factor when the actual house (object) is built from the plan (image).
- b. Give an estimate of the dimensions of:
 - i. bedroom 3
 - ii. the kitchen.

Communicating, reasoning and problem solving

8. Gordon, a tourist at Kakadu National Park, takes a picture of a two-metre crocodile beside a cliff. When he develops his pictures, the two-metre crocodile is 2.5 cm long and the cliff is 8.5 cm high. What was the actual height of the cliff in cm?

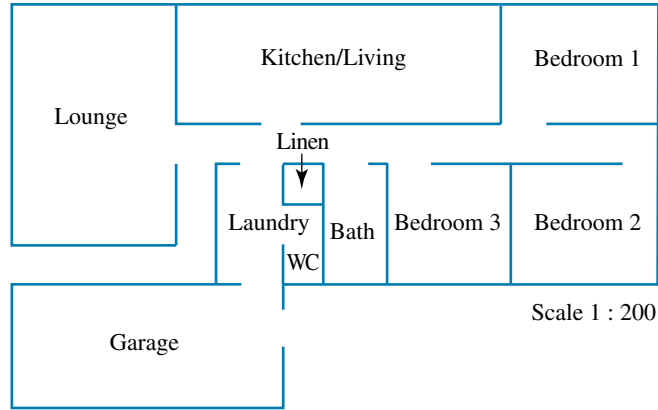


9. If a scale of $1 : 50$ is used to make each of the following drawings:
- a. what would be the width of the drawing of a house which is 25 m wide
 - b. what would be the height of the drawing of a man who is 2 m tall
 - c. what would be the dimensions of the drawing of a rectangular pool 50 m by 20 m
 - d. what would be the dimensions of the drawing of a table 4 m by 1.5 m
 - e. what is the actual height of a tree whose height in the drawing is 10 cm
 - f. what is the actual length of a truck whose length in the drawing is 25 cm
 - g. what are the dimensions of a television set whose dimensions on the drawing are 10 mm by 8 mm
 - h. what are the dimensions of a bed whose dimensions on the drawing are 4.5 cm long by 3 cm wide by 1.2 cm high?

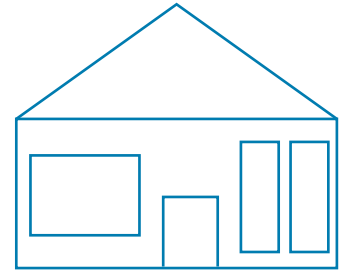
10. Referring to the following plan of a house, find:

- the dimensions of the lounge
- the width of the hallway
- the length of the garage
- the floor area of the kitchen/living room
- the total floor area of the largest bedroom.

(Hint: Use a ruler to find the required lengths on the plan first.)



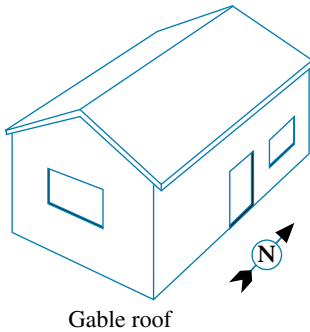
11. The front elevation of a house is shown. Trace the elevation into your book. On your diagram, write all the lengths and angles necessary for the construction of the house.



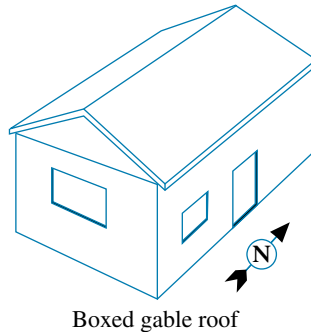
Scale 1 : 200

12. The following diagrams are representations of houses with various roof types. Draw plans of the south and east elevations of each house. The direction of north is given.

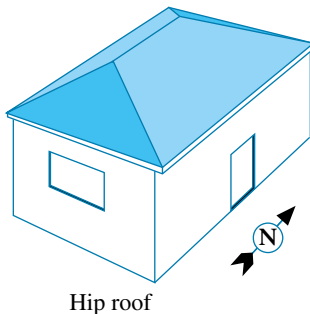
a.



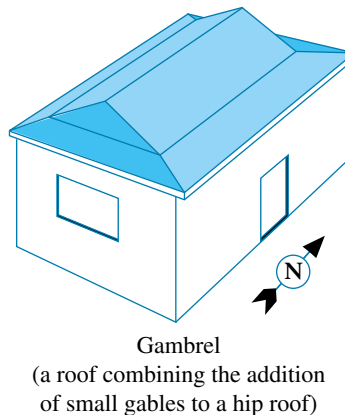
b.



c.



d.



13. Choose an area which is familiar to you, such as your home, bedroom, classroom or school grounds. Draw a plan of your area. Include a scale and a legend. Don't forget to show the northerly direction.
14. A rectangular room has a width of 3 m and a length of 5 m.
- What are the dimensions of the diagram on a scale of 1 : 100?
 - What is the area of the rectangle on the diagram?
 - The diagram has been enlarged so that the dimensions of the new diagram are double the dimensions of the first diagram. What is the area of the rectangle on the new diagram?
 - Calculate the area of a rectangle with dimensions three times bigger than the original diagram.
 - How does enlargement affect the area of a shape?
15. What would happen to the value of an area of a surface if its dimensions were reduced by a factor of n ?
16. Athena the artist paints portraits. For her present subject, Seth, she uses the following measurement relationships.
- Seth's arm span is the same length as his height.
 - The length of Seth's hand is one-tenth of his height.
 - The distance across Seth's shoulders is one-quarter his height.
- What is the relationship between Seth's arm span, the length of his hand and the distance across his shoulders?



17. Mikaela wants to buy a new fridge that is 800 mm deep and 1 m wide. However, she is restricted by the kitchen space. The place for the fridge on her drawing of her flat has a depth of 1.5 cm and a width of 2 cm. If the scale of the drawing is 1 : 50, will this fridge fit in the space given? State the actual dimensions of the fridge space.
18. A map is photocopied and reduced to 70 per cent of its original size. After the reduction, two towns are 3.5 cm apart, and the scale ruler shows that 1.4 cm represents 150 km. Find the actual distance between these two towns and how far apart they were on the original map.
19. It is possible to have a negative scale factor. How would you interpret it?

3.4 Similar figures

[Stage 5.1]

3.4.1 Similar figures

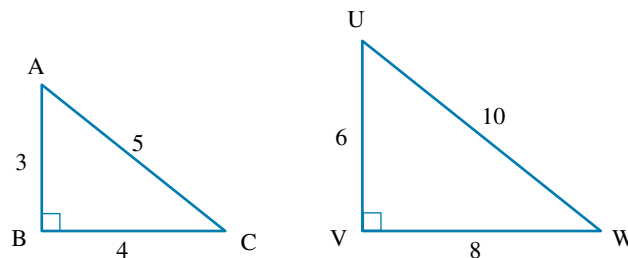
- Similar figures** have identical shape but different size. The corresponding angles in similar figures are equal in size and the corresponding sides are in the same ratio, given by the scale factor.
- The symbol used to denote similarity is $\sim$, which is read as 'is similar to'.
- Similar figures can be obtained as a result of enlargement or reduction transformations.

- If an enlargement (or a reduction) takes place, the original figure can be called the object and the enlarged (or reduced) figure can be called the image.
- Two figures can be described as similar if an enlargement of one is congruent to the other.
- For any two similar figures, the scale factor can be obtained using the following formula:

$$\text{scale factor} = \frac{\text{length of the image}}{\text{length of the object}}$$

Note: The size of the scale factor indicates whether the original object has been enlarged or reduced.

- If the scale factor is greater than 1, an enlargement has occurred.
- If the scale factor is positive but less than 1, a reduction has occurred.
- Consider the pair of similar triangles below.

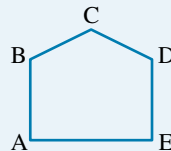


- Triangle UVW is similar to triangle ABC.
That is, $\Delta UVW \parallel \Delta ABC$.
- The corresponding angles of the two triangles are equal in size:
 $\angle CAB = \angle WUV$, $\angle ABC = \angle UVW$ and $\angle ACB = \angle UWV$.
- The corresponding sides of the two triangles are in the same ratio.
 $\frac{UV}{AB} = \frac{VW}{BC} = \frac{UW}{AC} = 2$; that is, the side lengths of ΔUVW are twice as long as the corresponding sides in ΔABC .
- The scale factor is 2.
- The original figure, ΔABC , can be called the object, and ΔUVW , obtained as the result of enlargement, is the image.



WORKED EXAMPLE 12

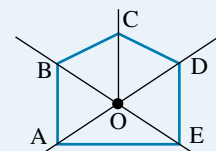
Enlarge the shape shown by a factor of 2.



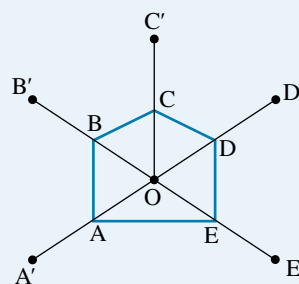
THINK

- 1 Select a point, O, somewhere inside the given shape and join it with straight-line segments to each vertex. Extend the lines beyond the shape.

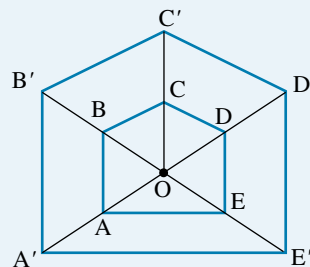
DRAW



- 2 Measure the distance OA and mark in the point A' so that $OA' = 2 \times OA$.
Repeat this for the other vertices.



- 3 Join the image vertices A' B' C' D' E' with straight lines.



DISCUSSION

Referring to Worked example 12, describe the effect of the enlargement transformation on the lengths of the sides and the angles of the image formed.

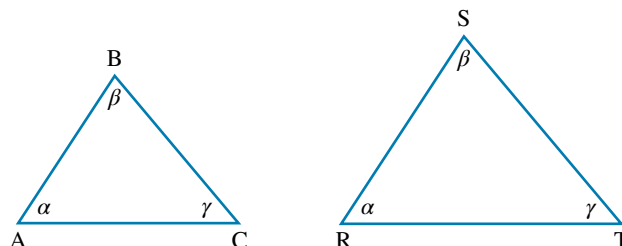
3.4.2 Testing triangles for similarity

- As with congruent triangles, it is not necessary to know that all pairs of corresponding sides are in the same ratio and that all corresponding angles are equal to ensure that two triangles are similar. There are certain minimum conditions which will guarantee that this is so.



3.4.3 Angle-angle-angle condition of similarity (AAA)

- If the angles of one triangle are the same as the angles of a second triangle, then the triangles are similar.



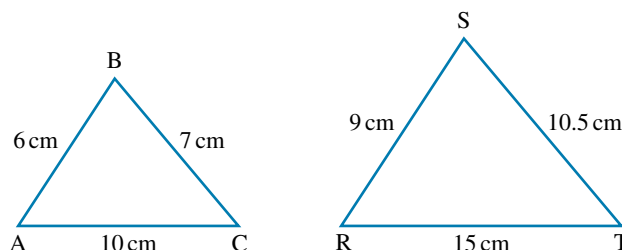
- This is the angle-angle-angle (AAA) condition for similarity.

From the diagram above, $\triangle ABC \sim \triangle RST$ (AAA).

3.4.4 Side-side-side condition of similarity (SSS)

- In the diagram below, the ratio of pairs of corresponding sides is constant.

That is, $\frac{9}{6} = \frac{15}{10} = \frac{10.5}{7} = 1.5$.



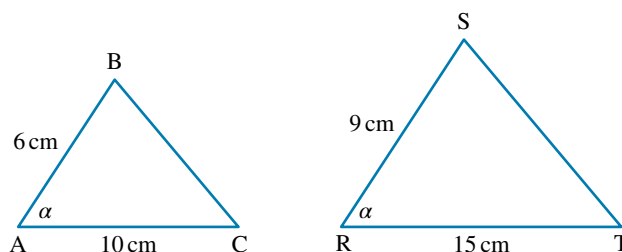
This is enough to show that the triangles are similar.

- This is the side-side-side (SSS) condition for similarity.

In this case, $\triangle ABC \sim \triangle RST$ (SSS).

3.4.5 Side-angle-side condition of similarity (SAS)

- In the diagram below, two pairs of sides are in the same ratio; that is, $\frac{9}{6} = \frac{15}{10} = 1.5$, and the included angles are equal as well.



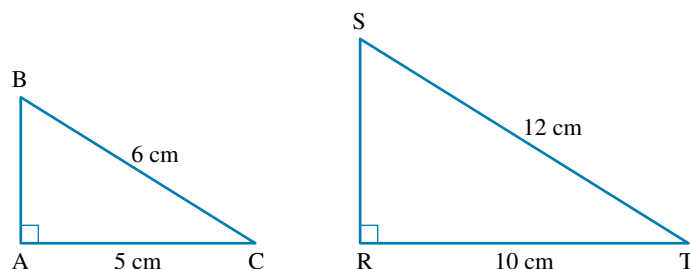
This is enough to show that the triangles are similar.

- This is the side-angle-side (SAS) condition for similarity.

In this case, $\triangle ABC \sim \triangle RST$ (SAS).

3.4.6 Right angle-hypotenuse-side condition for similarity (RHS)

- With right-angled triangles, a special condition can apply.
- If the hypotenuse and one other pair of sides are in the same ratio (e.g. in the diagram below, $\frac{12}{6} = \frac{10}{5}$), then the triangles are similar.



- This is the right angle-hypotenuse-side (RHS) condition for similarity. In this case, $\triangle ABC \sim \triangle RST$ (RHS).

3.4.7 Summary of similarity tests

- Triangles can be checked for similarity using one of the tests described in the table below.

Test description	Abbreviation
All corresponding angles are equal in size.	AAA (angle–angle–angle)
All corresponding sides are in the same ratio.	SSS (side–side–side)
Two pairs of corresponding sides are in the same ratio and the included angles are equal in size.	SAS (side–angle–side)
In right-angled triangles, the hypotenuses and one other pair of sides are in the same ratio.	RHS (right angle–hypotenuse–side)

- *Note:* When using the AAA test, it is sufficient to show that two pairs of corresponding angles are equal. Since the sum of the interior angles in any triangle is 180° , the third pair of angles will automatically be equal.

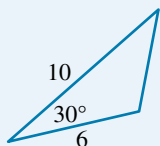
DISCUSSION

What similarities and differences do you find in the tests for similarity and the tests for congruence?

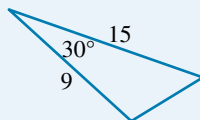
WORKED EXAMPLE 13

Find a pair of similar triangles from those shown. Give a reason for your answer.

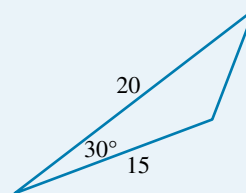
a



b



c



THINK

- 1 In each triangle we know the size of two sides and the included angle, so the SAS test can be applied. Since all included angles are equal (30°), we need to find ratios of corresponding sides, taking two triangles at a time.

- 2 Write the answer.

WRITE

For triangles **a** and **b**:

$$\frac{15}{10} = \frac{9}{6} = 1.5$$

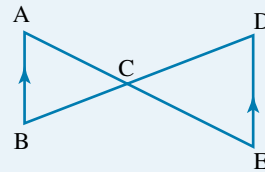
For triangles **a** and **c**:

$$\frac{20}{10} = 2, \frac{15}{6} = 2.5$$

Triangle **a** $\parallel\parallel$ triangle **b** (SAS)

WORKED EXAMPLE 14

Prove that $\triangle ABC$ is similar to $\triangle EDC$.

**THINK**

- 1 AB is parallel to DE. $\angle ABC$ and $\angle EDC$ are alternate angles.
- 2 $\angle BAC$ and $\angle DEC$ are alternate angles.
- 3 The third pair of angles must be equal.
- 4 This proves that the triangles are similar.

WRITE

$$\angle ABC = \angle EDC \text{ (alternate angles)}$$

$$\angle BAC = \angle DEC \text{ (alternate angles)}$$

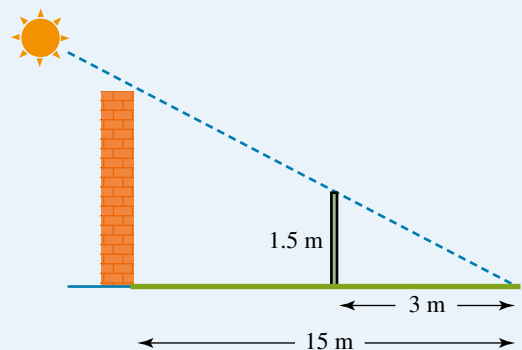
$$\angle BCA = \angle DCE \text{ (vertically opposite angles)}$$

$$\triangle ABC \parallel\parallel \triangle EDC \text{ (AAA)}$$

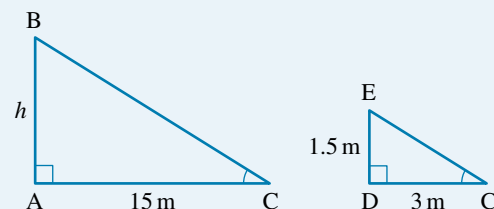
- The ratio of the corresponding sides in similar figures can be used to calculate missing side lengths or angles in these figures.

WORKED EXAMPLE 15

A 1.5-metre pole casts a shadow 3 metres long, as shown. Find the height of a building that casts a shadow 15 metres long at the same time of the day.

**THINK**

- 1 Represent the given information on a diagram.
 $\angle BAC = \angle EDC$; $\angle BCA = \angle ECD$

WRITE/DRAW

2 Triangles ABC and DEC are similar. Therefore, the ratios of corresponding sides are the same. Write the ratios.

$$\triangle ABC \sim \triangle DEC \text{ (AAA)}$$

$$\frac{h}{1.5} = \frac{15}{3}$$

3 Solve the equation for h .

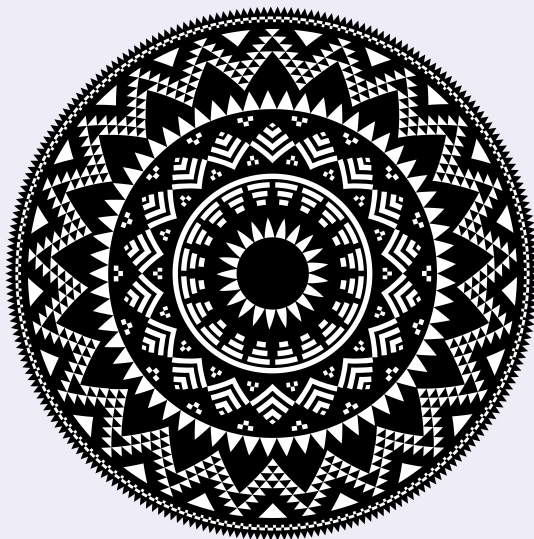
$$\begin{aligned} h &= \frac{15 \times 1.5}{3} \\ &= 7.5 \end{aligned}$$

4 Write the answer in words, including units.

The height of the building is 7.5 m.

ACTIVITY: THE USE OF SIMILAR FIGURES IN DESIGN

Similar figures are frequently used to produce eye-catching designs, such as the concentric circles and similar triangles in the Aztec design below.



Find two examples of similar figures embedded in designs from other cultures or historical periods.

learnon RESOURCES — ONLINE ONLY

eLesson: Similar triangles (eles-1925)

Digital doc: WorkSHEET Similar figures (doc-6202)

Exercise 3.4 Similar figures

Individual pathways

PRACTISE

Questions:

1–4, 5, 8, 11–14, 17, 18

CONSOLIDATE

Questions:

1–4, 6, 9, 11–14, 16, 17, 19, 21, 22

MASTER

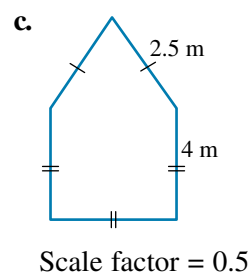
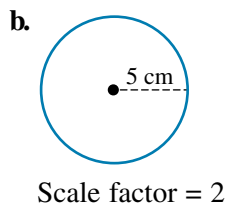
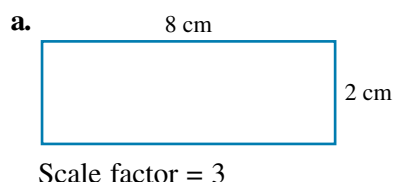
Questions:

1–4, 7, 10–16, 18, 20, 21, 23, 25

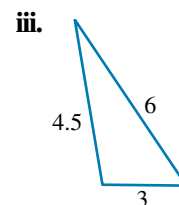
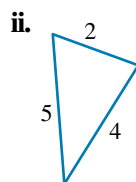
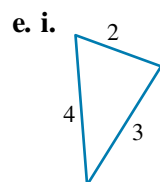
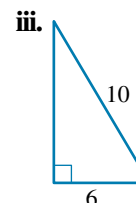
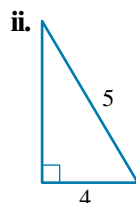
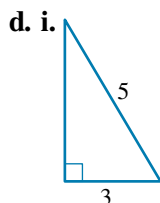
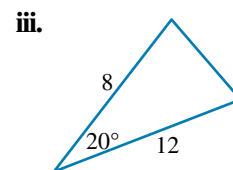
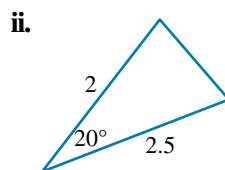
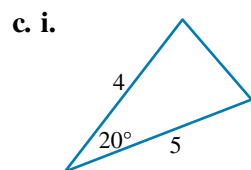
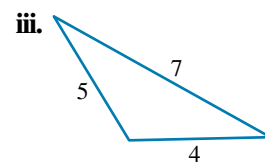
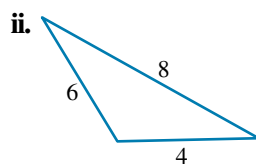
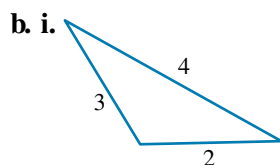
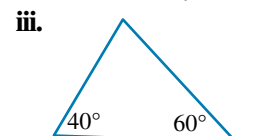
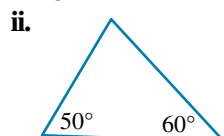
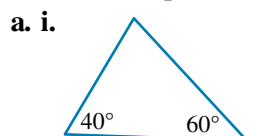
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

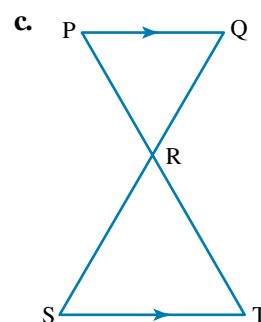
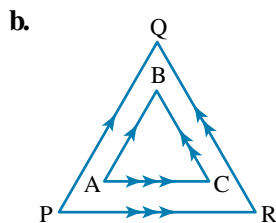
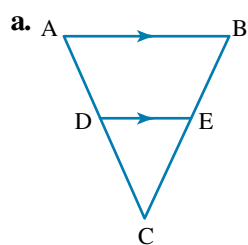
1. **WE10** Enlarge (or reduce) the following shapes by the scale factor given.



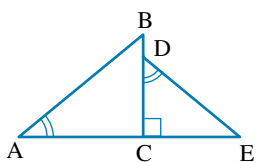
2. **WE11** Find a pair of similar triangles among those shown in each part. Give a reason for your answer.



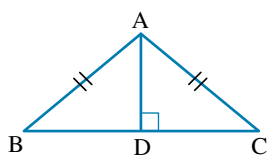
3. Name two similar triangles in each of the following figures, ensuring that vertices are listed in the correct order and using the symbol $\sim$ in the similarity statement.



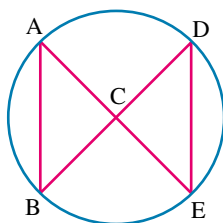
d.



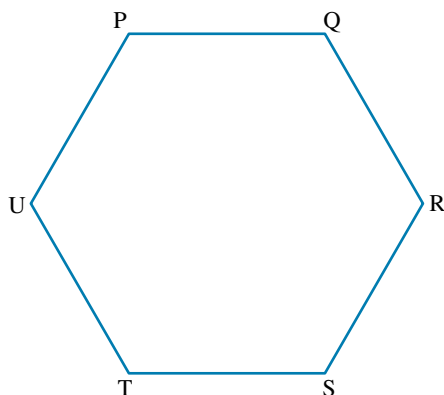
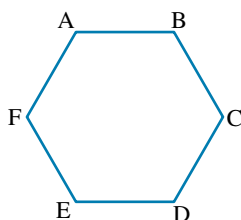
e.



4. In the diagram shown, C is the centre of the circle. Complete this statement: $\triangle ABC$ is similar to ...



5. ABCDEF is a regular hexagon, similar to PQRSTU.



a. What is the length of:

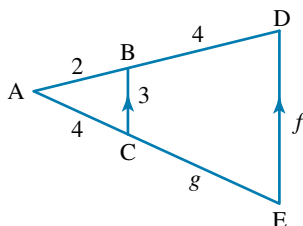
- i. AB ii. RS?

b. What is the scale factor for enlargement?

Note: You need to measure the lengths.

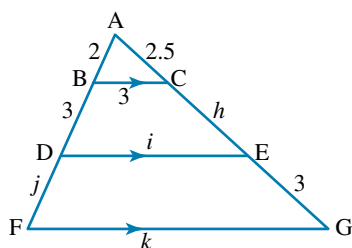
6. a. Complete this statement: $\frac{AB}{AD} = \frac{BC}{AE} = \frac{\quad}{\quad}$.

b. Find the value of the pronumerals.

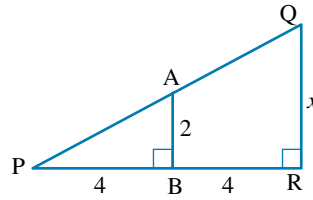


7. a. Find the values of h and i .

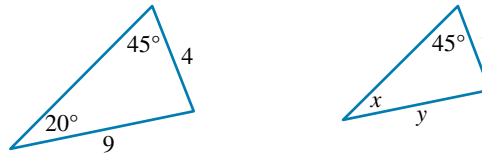
b. Find the values of j and k .



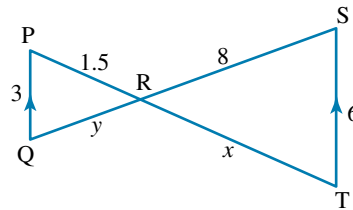
8. Find the value of the pronumeral in the diagram shown.



9. If the triangles shown are similar, find the values of x and y .

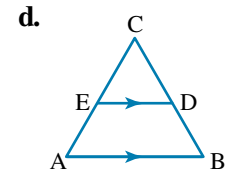
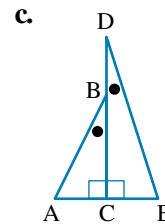
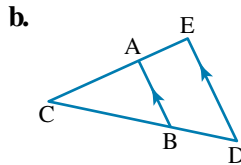
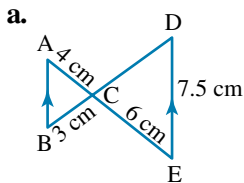


10. Find the values of x and y in the diagram shown.

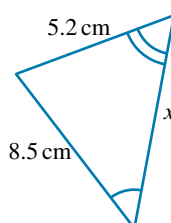
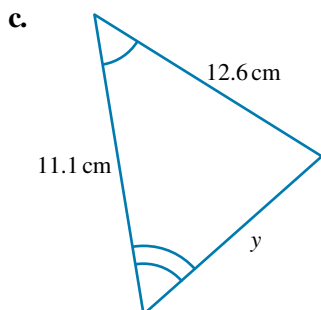
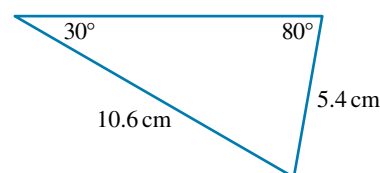
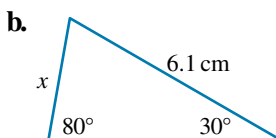
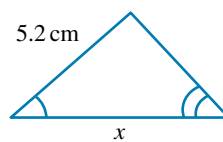
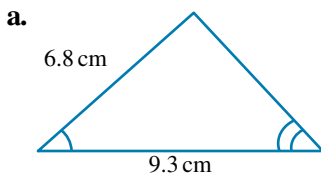


Communicating, reasoning and problem solving

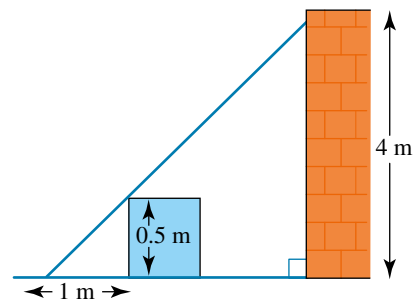
11. **WE12** Prove that $\triangle ABC$ is similar to $\triangle EDC$ in each of the following.



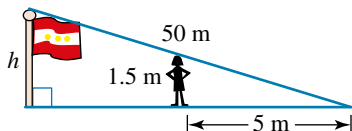
12. Find the value of each pronumeral in these triangles. Show how you arrived at your answers.



13. **WE13** A ladder just touches a bench and leans on a wall 4 metres above the ground, as shown. If the bench is 50 centimetres high and is 1 metre from the base of the ladder, show that the base of the ladder is 8 metres from the wall.



14. Natalie, whose height is 1.5 metres, casts a shadow 2 metres long at a certain time of the day. If Alex is 1.8 metres tall, show that his shadow would be 2.4 metres long.
15. A string 50 metres long is pegged to the ground and tied to the top of a flagpole. It just touches the head of Maureen, who is 5 metres away from the point where the string is held to the ground. If Maureen is 1.5 metres tall, show that the height, h , of the flagpole is 14.37 metres.

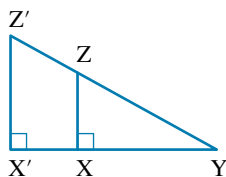


16. Using diagrams or otherwise, explain whether the following statements are true or false.
- Any two equilateral triangles are congruent.
 - Any two equilateral triangles are similar.
 - Any two isosceles triangles are similar.
 - Any two rectangles are similar.
 - Any two squares are congruent.
 - Any two squares are similar.

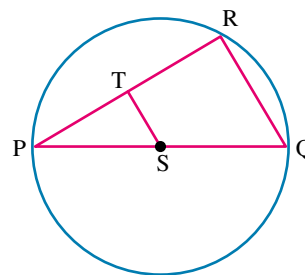
17. Penny and Paul play tennis at night under floodlights. When Penny stands 2.5 m from the base of the floodlight, her shadow is 60 cm long.
- If Penny is 1.3 m tall, how high is the floodlight in metres, correct to 2 decimal places?
 - If Paul, who is 1.6 m tall, stands in the same place, how long will his shadow be in cm?



18. To determine the height of a flagpole, Jenna and Mia decide to measure the shadow cast by the flagpole. They place a 1-m ruler at a distance of 3 m from the base of the flagpole and measure the shadows that both the ruler and flagpole cast. Both shadows finish at the same point. After measuring the shadow of the flagpole, Jenna and Mia calculate that the height of the flagpole is 5 m. Determine the length of the shadow cast by the flagpole, in metres, as measured by Jenna and Mia.
19. Use the diagram to find the value of a if $XZ = 8$ cm, $X'Z' = 12$ cm, $X'X = a$ cm and $XY = (a + 1)$ cm.

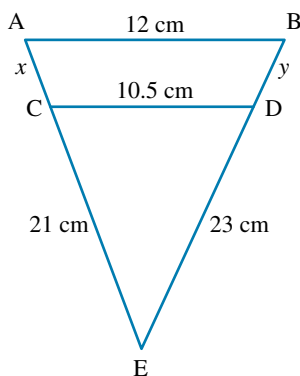


20. PQ is a diameter of this circle with a centre at S. R is any point on the circumference. T is the midpoint of PR.
- Write down everything you know about this figure.
 - Explain why $\triangle PTS$ is similar to $\triangle PRQ$.
 - Find the length of TS if RQ is 8 cm.
 - Find the length of every other side also given that PT is 3 cm and angle PRQ is a right angle.

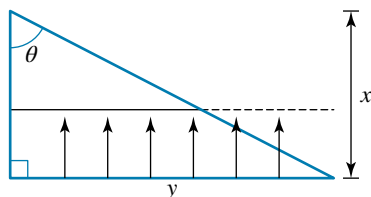


21. AB and CD are parallel lines in the figure shown.

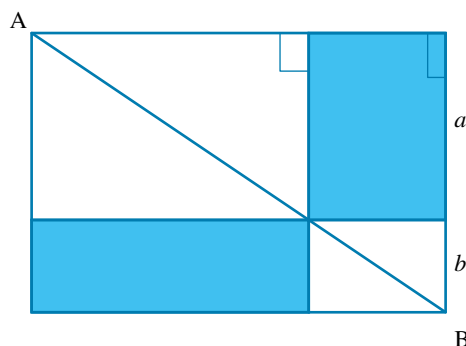
- State the similar triangles.
- Calculate the values of x and y .



22. For the diagram shown, prove that if the base of the triangle is raised to half the height of the triangle, the length of the base of the newly formed triangle will be half of its original length.



23. AB is a straight line. The fraction of the large rectangle that is shaded is $\frac{12}{25}$. Find the ratio $a : b$.



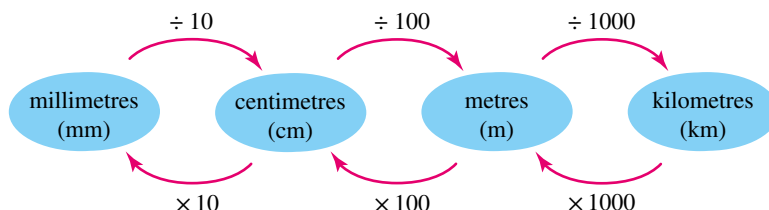
24. Do similar objects have the same perimeters?
25. A tree that is 3.5 m tall casts a 7.5-m shadow at a certain time of the day. At that time of day, Ruby's shadow is 3 m long. How tall is Ruby?

3.5 Area and volume of similar figures

[Stage 5.3]

3.5.1 Units of length

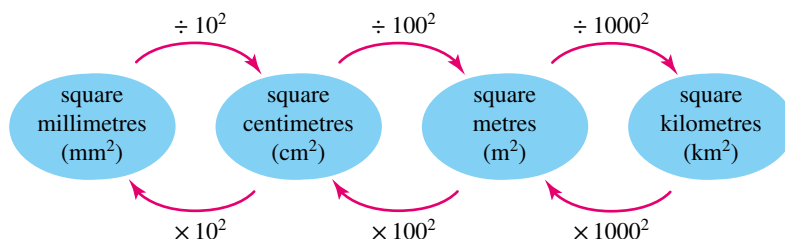
- Metric units of length include millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- To convert between the units of length, we use the conversion chart shown below.



- When converting from a large unit to a smaller unit, multiply by the conversion factor; when converting from a smaller unit to a larger unit, divide by the conversion factor.

3.5.2 Units of area

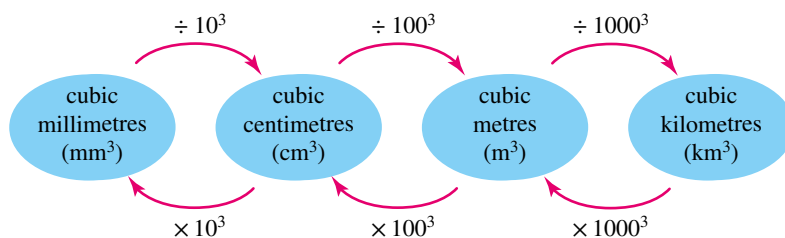
- Area is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).
- Area units can be converted using the chart shown below.



- Area units are the squares of the corresponding length units.

3.5.3 Units of volume

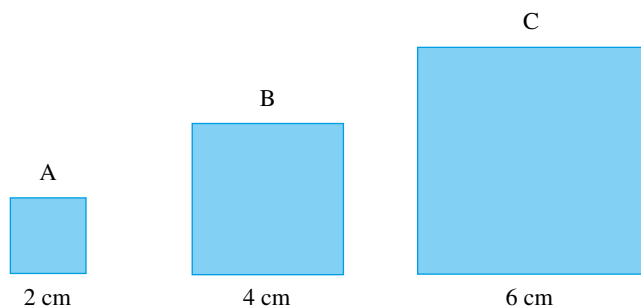
- Volume is measured in cubic units, such as cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).
- Volume units can be converted using the chart shown below.



- Volume units are the cubes of the corresponding length units.

3.5.4 Area and surface area of similar figures

- If the side lengths in any figure are increased by a scale factor of n , then the area of similar figures increases by a scale factor of n^2 .
- For example, consider the squares below.



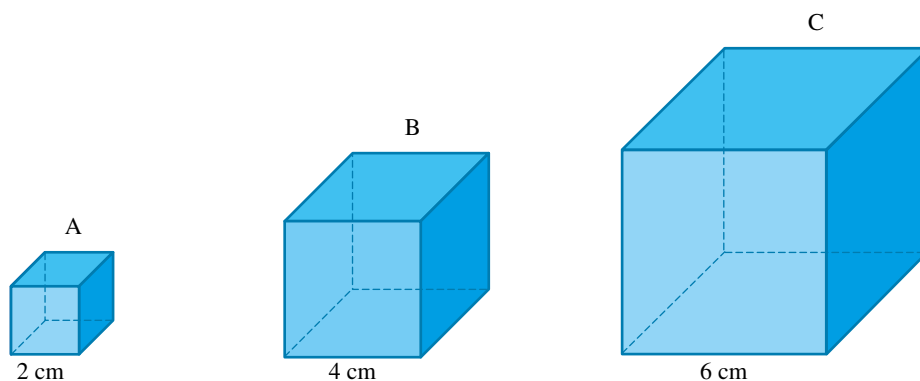
$$\begin{array}{lll} \text{Area} = 2 \times 2 & \text{Area} = 4 \times 4 & \text{Area} = 6 \times 6 \\ = 4 \text{ cm}^2 & = 16 \text{ cm}^2 & = 36 \text{ cm}^2 \end{array}$$

The scale factors for the side lengths and the scale factors for the areas are calculated below.

Squares	Scale factor for side length	Scale factor for area
A and B	$\frac{4}{2} = 2$	$\frac{16}{4} = 4 = 2^2$
A and C	$\frac{6}{2} = 3$	$\frac{36}{4} = 9 = 3^2$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{36}{16} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

- If the side lengths in any figure are increased by a scale factor of n , then the surface area of similar figures increases by a scale factor of n^2 .

Consider the cubes below.



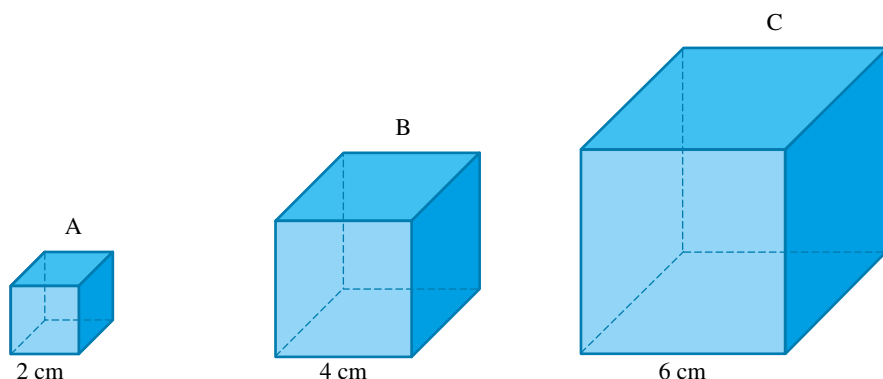
$$\begin{array}{lll} \text{Surface area} = 6 \times 4 & \text{Surface area} = 6 \times 16 & \text{Surface area} = 6 \times 36 \\ = 24 \text{ cm}^2 & = 96 \text{ cm}^2 & = 216 \text{ cm}^2 \end{array}$$

The scale factors for the side lengths and the scale factors for the surface areas are calculated below.

Cubes	Scale factor for side length	Scale factor for surface area
A and B	$\frac{4}{2} = 2$	$\frac{96}{24} = 4 = 2^2$
A and C	$\frac{6}{2} = 3$	$\frac{216}{24} = 9 = 3^2$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{216}{96} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

3.5.5 Volume of similar figures

- If the side lengths in any solid are increased by a scale factor of n , then the volume of similar solids increases by a scale factor of n^3 .
- Once again, consider the cubes shown. The scale factors for the side lengths and the scale factors for the volumes are calculated below.



$$\begin{aligned}\text{Volume} &= 2 \times 2 \times 2 \\ &= 8 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Volume} &= 4 \times 4 \times 4 \\ &= 64 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Volume} &= 6 \times 6 \times 6 \\ &= 216 \text{ cm}^3\end{aligned}$$

Cubes	Scale factor for side length	Scale factor for volume
A and B	$\frac{4}{2} = 2$	$\frac{64}{8} = 8 = 2^3$
A and C	$\frac{6}{2} = 3$	$\frac{216}{8} = 27 = 3^3$
B and C	$\frac{6}{4} = \frac{3}{2}$	$\frac{216}{64} = \frac{27}{8} = \left(\frac{3}{2}\right)^3$

WORKED EXAMPLE 16

The side lengths of a box have been increased by a factor of 3.

a Find the surface area of the new box if the original surface area is 94 cm^2 .

b Find the volume of the new box if the original volume is 60 cm^3 .

THINK

- a** 1 State the scale factor for side length used to produce the new box.
- 2 The scale factor for surface area is the square of the scale factor for length.
- 3 Calculate the surface area of the new box.

WRITE

a Scale factor for side length = 3

$$\begin{aligned}\text{Scale factor for surface area} \\ &= 3^2 \\ &= 9\end{aligned}$$

$$\begin{aligned}\text{Surface area of new box} \\ &= 94 \times 9 \\ &= 846 \text{ cm}^2\end{aligned}$$

b 1 The scale factor for volume is the cube of the scale factor for length.

2 Calculate the volume of the new box.

b Scale factor for volume
 $= 3^3$
 $= 27$

Volume of new box
 $= 60 \times 27$
 $= 1620 \text{ cm}^3$

learnon RESOURCES — ONLINE ONLY

Interactivity: Units of length (int-3779)

Interactivity: Area of similar figures (int-6043)

Interactivity: Volume and surface area of similar figures (int-6044)

Interactivity: Similar figures (int-2768)

Exercise 3.5 Area and volume of similar figures

Individual pathways

PRACTISE

Questions:
1, 2, 4, 6, 8, 13

CONSOLIDATE

Questions:
1–4, 6, 8, 13–16

MASTER

Questions:
1, 3, 5–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- 1. WE14a** The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

	Original surface area	Enlarged surface area
a	100 cm^2	
b	7.5 cm^2	
c	95 mm^2	
d		918 cm^2
e		45 m^2
f		225 mm^2

2. **WE14b** The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

	Original volume	Enlarged volume
a	200 cm^3	
b	12.5 cm^3	
c	67 mm^3	
d		2700 cm^3
e		67.5 m^3
f		27 mm^3

3. A rectangular box has a surface area of 96 cm^2 and volume of 36 cm^3 . Find the volume and surface area of a similar box that has side lengths double the size of the original.
4. The area of a bathroom on a house plan is 5 cm^2 . Find the area of the bathroom if the map has a scale of 1 : 100.
5. The area of a kitchen is 25 m^2 .
- Change 25 m^2 to cm^2 .
 - Find the area of the kitchen on a plan if the scale of the plan is 1 : 120. (Give your answer correct to 1 decimal place.)
6. The volume of a swimming pool from its construction plan is calculated to be 20 cm^3 . Find the volume of the pool if the plan has a scale of 1 : 75.
7. The total surface area of the wings on a 747 aircraft is 120 m^2 .

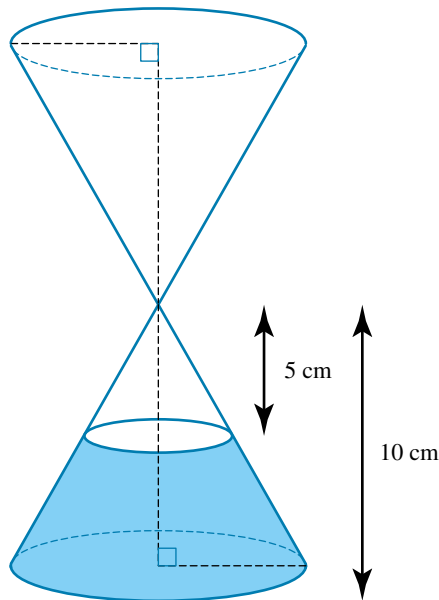
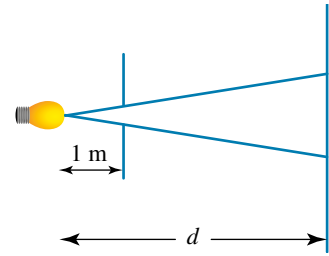


- Change 120 m^2 to cm^2 .
- Find the total surface area of the wings on a scale model built using the scale 1 : 80.

Communicating, reasoning and problem solving

8. A cube has a surface area of 253.5 cm^2 . (Give answers correct to 1 decimal place where appropriate.)
- Show that the side length of the cube is 6.5 cm.
 - Show that the volume of the cube is 274.625 cm^3 .
 - Find the volume of a similar cube that has side lengths twice as long.
 - Find the volume of a similar cube that has side lengths half as long.
 - Find the surface area of a similar cube that has side lengths one-third as long.

9. In the diagram, a light is shining through a hole, resulting in a circular bright spot with a radius of 5 cm on the screen. The hole is 10 mm wide. If the light is 1 m behind the hole, show that the light is 10 m from the screen.
10. A triangle ABC maps to triangle A'B'C' under an enlargement.
 $AB = 7\text{ cm}$, $AC = 5\text{ cm}$, $A'B' = 21\text{ cm}$, $B'C' = 30\text{ cm}$.
 a. Show that the scale factor for the enlargement is 3.
 b. Find BC.
 c. Find A'C'.
 d. If the area of $\triangle ABC$ is 9 cm^2 , show that the area of $\triangle A'B'C'$ is 81 cm^2 .
11. Two rectangles are similar. If the width of one rectangle is twice the width of the other, prove that the ratio of their areas is 4 : 1.
12. A pentagon has an area of 20 cm^2 . If all the side lengths are doubled, show that the area of the enlarged pentagon is 80 cm^2 .
13. a. Calculate the areas of squares with sides 2 cm, 5 cm, 10 cm and 20 cm.
 b. State in words how the ratio of the areas is related to the ratio of the side lengths.
14. The areas of two similar trapeziums are 9 and 25. What is the ratio of a pair of corresponding side lengths?
15. Two cones are similar. The ratio of volumes is 27 : 64. Find the ratio of:
 a. the perpendicular heights
 b. the areas of the bases.
16. Rectangle A has dimensions 5 by 4, rectangle B has dimensions 4 by 3 and rectangle C has dimensions 3 by 2.4.
 a. Which rectangles are similar? Explain.
 b. Find the area scale factor for the similar rectangles.
17. A balloon in the shape of a sphere has an initial volume of 840 cm^3 and is increased to a volume of 430080 cm^3 . What is the increase in the radius of the balloon?
18. Part of an egg timer in the shape of a cone has sand poured into it as shown in the diagram. Find the ratio of the volume of sand in the cone to the volume of empty space in the bottom half of the egg timer.



19. Why does a side length scale factor of 5 result in a volume that is increased by a scale factor of 125?

3.6 Review

Investigation | Rich task

What's this object?

When we use geometrical tools to construct shapes, we have to ensure that the measurements taken are precise. Small errors in each step of the measuring process can result in the creation of an incorrect shape. The object that you have to create for this task is one that has generated a great deal of interest over the years. It is made by combining three congruent shapes. Instructions to make the congruent shapes are as follows.



Part 1: The congruent shape

- Using a ruler, a protractor, a pencil and a pair of compasses, follow the instructions below to construct the first part of the object. Draw the shape in your workbook.
 - Measure a horizontal line AB that is 9.5 cm long. To ensure that there is enough space for the entire object, draw the line close to the bottom of the space.
 - At point B, construct an angle of 120° above the line AB. Extend the line to point C, making BC 2 cm long.
 - At point C, construct an angle of 60° on the same side of the line BC as point A. Measure the line CD to be 7.5 cm long.
 - At point D and above the line DC, construct an angle of 60° . Measure the length DE to be 3.5 cm.
 - On the line DE, at point E, construct an angle of 120° above the line DE. Let the measurement from E to F be 2 cm. Join point F to point A.
- What is the length of the line joining point A to point F?
- What do you notice about the size of the angles FAB and AFE?
- Shade the shape with any colour you wish.

Part 2: The object

You have now constructed the shape that is to be used three times to make the final object. To do this, follow the instructions below.

- Trace the shape onto a piece of tracing paper twice and cut around the edges. Label each copy of the shape with the letters used in its construction.
 - Place the line AF of your first trace against the line CD of your drawn shape. Reproduce this shape on your object. Shade this section using a different colour.
 - Place the line DC of your second trace against the line FA of your drawn shape. Reproduce this shape on your object. Again, shade this section with a different colour.
- Describe the object you have created.
 - Using the internet, library or other references, investigate other impossible objects drawn as two-dimensional shapes. Recreate them on a separate sheet of paper and include reasons why they are termed impossible.



Digital doc: Investigation — What's this object? (doc-15894)

Digital doc: Code puzzle — Why does a giraffe have a long neck? (doc-15895)

Digital doc: Summary Topic 3 (doc-22885)

Interactivity: Crossword Topic 3 (int-2693)

Interactivity: Word search Topic 3 (int-2692)

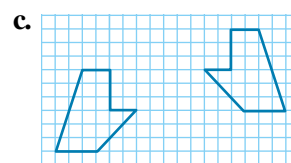
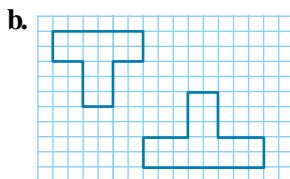
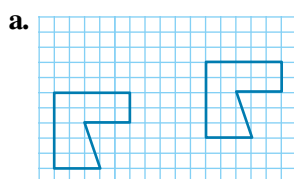
Interactivity: Sudoku Topic 3 (int-3205)

Exercise 3.6 Review questions

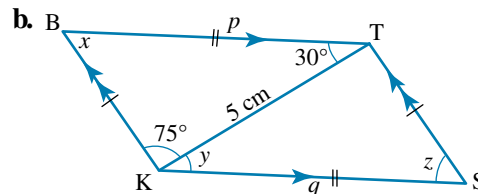
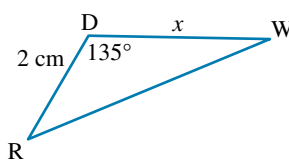
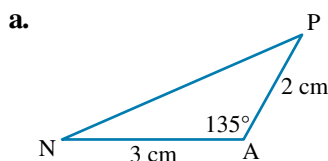
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Which of the following pairs of shapes are congruent?



2. Name the congruent triangles in these figures and find the value of the pronumerals in each case.



3. Express each of the following in simplest form.

a. $8 : 16$

b. $24 : 16$

c. $27 : 18$

d. $56 : 80$

e. $8 : 20$

f. $49 : 35$

4. Express each of the following ratios in simplest form.

a. $4\frac{1}{2} : 1$

b. $6\frac{1}{8} : 9$

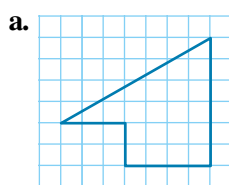
c. $7\frac{1}{4} : 3\frac{1}{2}$

d. $8.4 : 7.2$

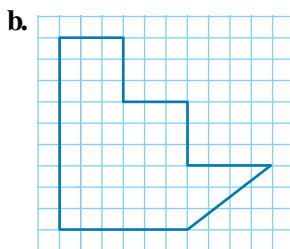
e. $0.2 : 2.48$

f. $6.6 : 0.22$

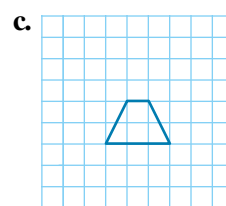
5. Copy each of the following shapes and enlarge (or reduce) them by the given factor.



Enlarge by a factor of 2.

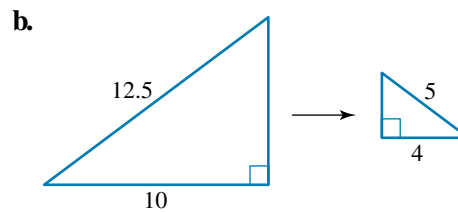
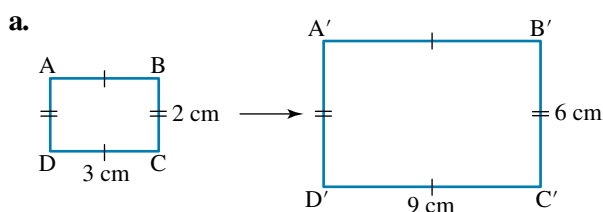


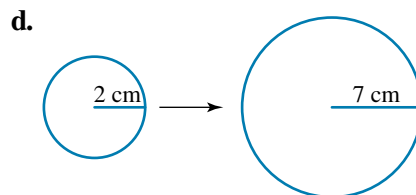
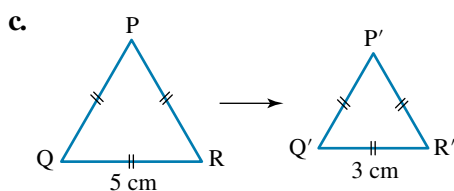
Reduce by a factor of 3.



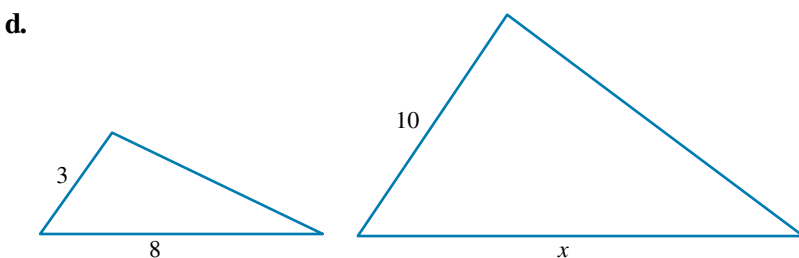
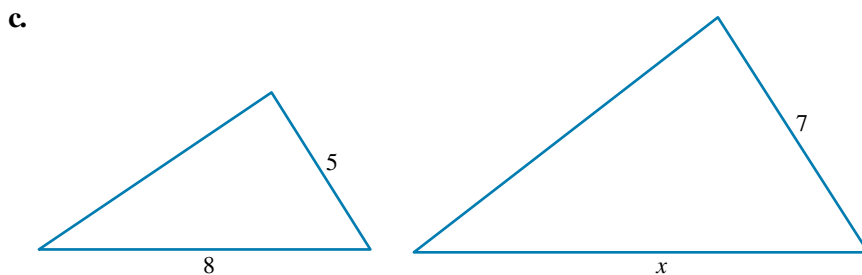
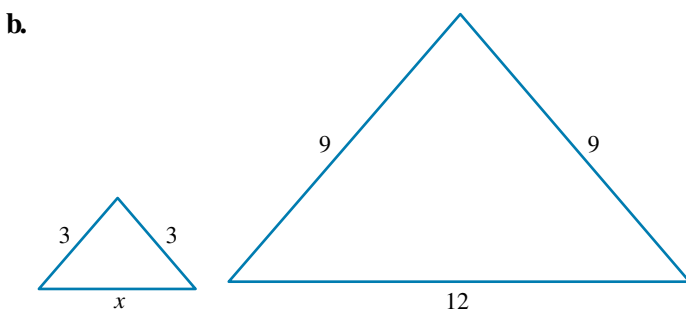
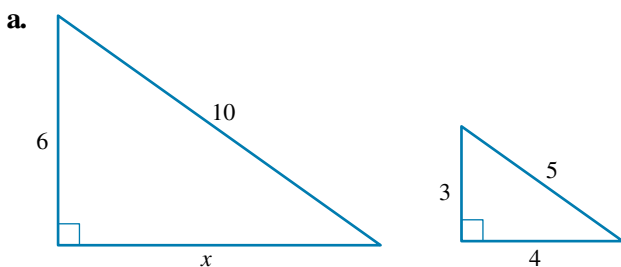
Enlarge by a factor of 4.

6. Determine the enlargement factors that have been used on the following shapes.

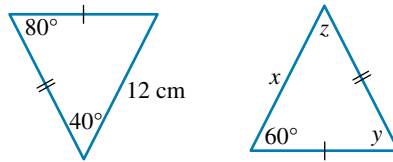




7. The scale on a map is given as 1 : 100000. Find the actual distances between:
- two schools that are 5 cm apart on the map
 - two parks that are 2.5 cm apart on the map
 - two farms that are 4 cm apart on the map.
8. A scale of 1 : 200 is used to make models of a tree and a car.
- What would be the length of the model car if the car is actually 2.5 metres long?
 - What would be the height of the model tree if the tree is actually 6.5 metres tall?
9. $\triangle PQR \sim \triangle DEF$. Determine the length of the missing side in each of the following combinations.
- $PQ = 10$ cm $DE = 5$ cm $PR = 6$ cm Find DF .
 - $PQ = 4$ cm $DE = 12$ cm $QR = 5$ cm Find EF .
 - $DE = 4$ cm $PQ = 6$ cm $EF = 8$ cm Find QR .
 - $DF = 5$ cm $PR = 8$ cm $DE = 6$ cm Find PQ .
 - $QR = 16$ cm $EF = 6$ cm $PQ = 12$ cm Find DE .
10. Each of the diagrams below shows a pair of similar triangles. Find the value of x in each case.



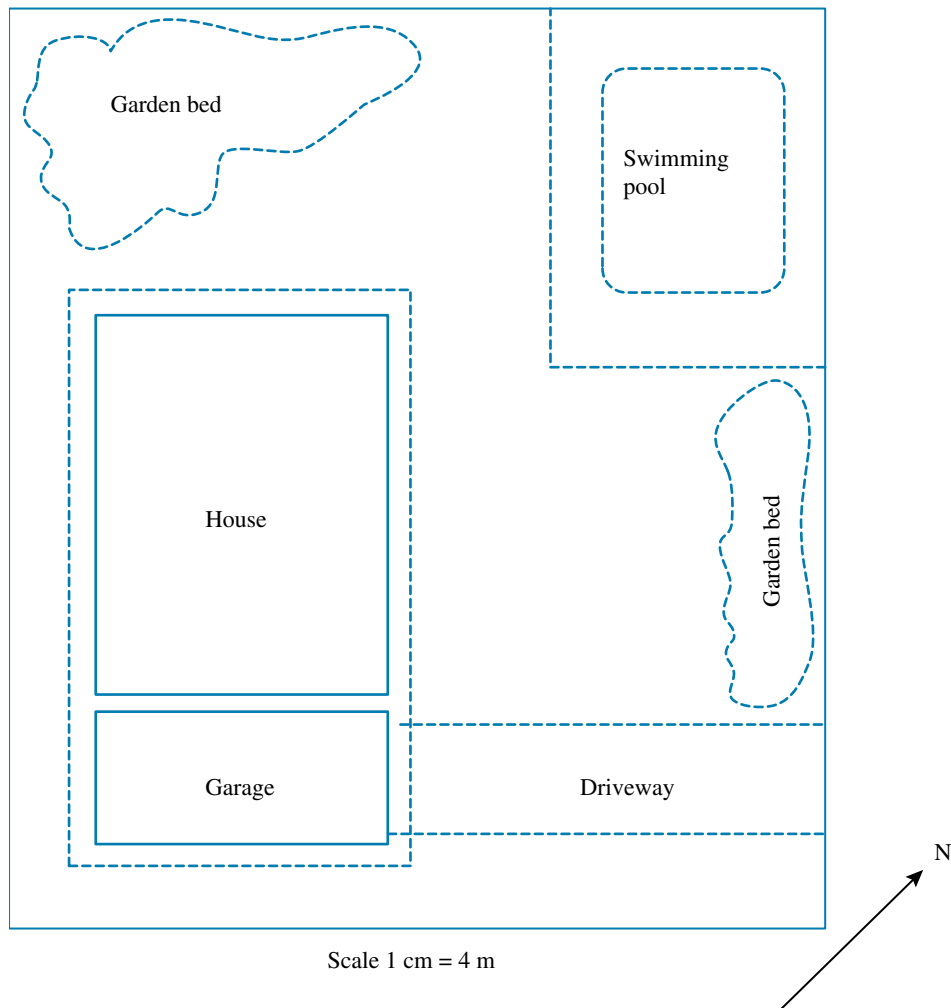
11. Find the values of the pronumerals in the pair of congruent triangles shown.



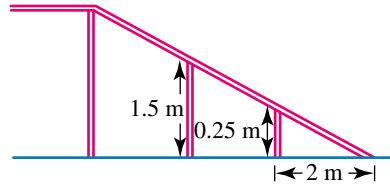
12. The area of a family room is 16 m^2 and the length of the room is 6.4 m . Find the area of the room on a plan that uses a scale of $1 : 20$.

Communicating, reasoning and problem solving

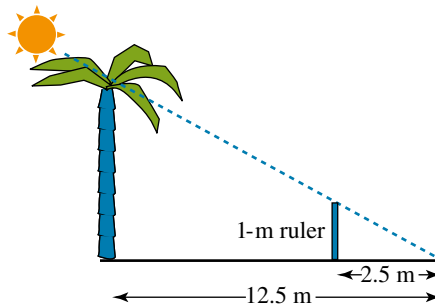
13. Before the start of a yacht race, the judging official must certify that all of the sails are the same. Without unrigging the triangular sails from their masts, which congruency rule can the official most efficiently use to determine if the sails on each of the boats are exactly the same? Explain why this is the most appropriate rule.
14. The scale diagram below is a site plan for a block of land on which a house is to be built. Use this diagram to answer the following questions.
- Calculate the dimensions of the block of land.
 - Calculate the dimensions of the house.
 - Calculate the area of the house in square metres.
 - In which direction would you be travelling as you drive into the garage?



15. The diagram below shows a ramp made by Josef for his automotive class. The first post has a height of 0.25 m and is placed 2 m from the end of the ramp. If the second post is 1.5 m high, how far should it be placed from the first post?



16. Rachel is given a 1-m ruler and asked to estimate the height of a palm tree. She places the ruler vertically so that the ruler's shadow ends at exactly the same point as that of the palm tree. The ruler's shadow is 2.5 m long, and the palm's shadow is 12.5 m long.

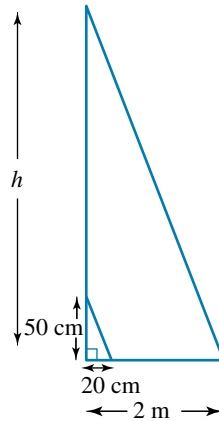


Rachel performs some calculations using similar triangles and calculates the height of the palm tree to be 4 m. Her friend Sandy says that her calculations are incorrect, and that the answer should be 5 m.

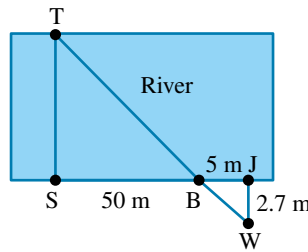
- Which is the correct answer?
 - Explain the error one of the girls has made in her calculations.
17. Calculate the height of the top of the ladder using similar triangles. Give your answer correct to 1 decimal place.



18. A flag stick casts a shadow 2 m long. If a 50-cm ruler is placed in the same upright position and it casts a shadow 20 cm long, what is the height of the flag stick?



19. A student (S) uses a tape measure, backpack (B), water bottle (W), tree (T) and jacket (J) to help calculate the width of a river.

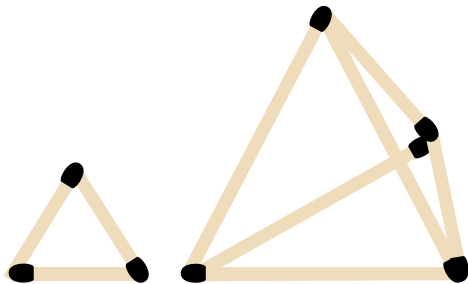


- a. Copy and complete the following:
 - i. $\angle \dots$ and $\angle \dots$ are both right angles $\dots$
 - ii. $\angle SBT$ and $\angle \dots$ are equal because $\dots$
 - iii. $\angle STB$ and $\angle \dots$ are equal because $\dots$
 - b. Rewrite the following statement, selecting the correct alternative from within the brackets.
 $\triangle STB$ and $\triangle JWB$ are (similar/congruent) (SSS/SAS/ASA/AAA/RHS).
 - c. Copy and complete the following equation.

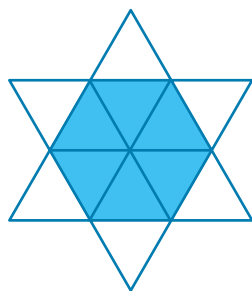
$$\frac{\overline{ST}}{\overline{JW}} = \square$$
 - d. Use your statement from part c to calculate the width of the river.
20. *Slocum* is a yacht whose length is 12 m and beam (width) is 2.50 m. A model is constructed for a maritime museum. If the length and width of the model are one-fifth of the original length and width, how do their volumes compare?

Topic 3 Similar figures and scale drawings

1. a. ii and iii
2. D
3. a. $\triangle ABC$ and $\triangle PQR$, SAS
d. $\triangle ABC$ and $\triangle PQR$, ASA
4. a. $x = 3$ cm
d. $x = 30^\circ$, $y = 7$
5. a. 2 cm
6. Yes, because they are identical.
7. a. SSS
8.
9. Check with your teacher.
10. No; it could be ASA.
11. Because the angle is not between the two given sides, the general shape of the triangle is not set; therefore, many shapes are possible.
12. This can be done with an equilateral triangle and a regular tetrahedron.

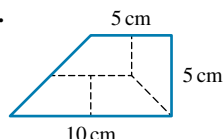


15. 1 : 1



16. If they can be superimposed exactly on top of each other. You may need to use reflection, rotation or translation of the figures.

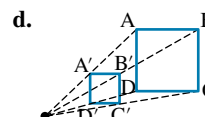
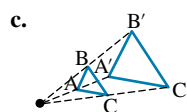
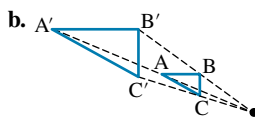
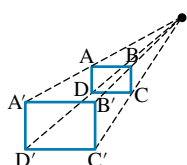
17.



Exercise 3.3 Scale drawings

1. a. 2 : 3	b. 1 : 7	c. 1 : 3	d. 2 : 5	e. 4 : 3	
f. 4 : 15	g. 40 : 31	h. 36 : 11	i. 8 : 25	j. 100 : 33	
2. a. 6 : 1	b. 13 : 15	c. 4 : 1	d. 5 : 1	e. 1 : 2	f. 20 : 3
g. 1 : 6	h. 5 : 3	i. 5 : 1	j. 11 : 5	k. 43 : 20	l. 1 : 200
3. a. 3 : 4	b. 8 : 7	c. 2 : 3	d. 28 : 25	e. 9 : 2	
f. 2 : 7	g. 10 : 3	h. 33 : 16	i. 23 : 15	j. 16 : 65	

4. a.



5. a. 3	b. 12 cm	c. 3.5 cm		
6. $\frac{1}{2}$				
7. a. 200	b. i. $6\text{ m} \times 6\text{ m}$	iii. $5\text{ m} \times 5\text{ m}$		
8. 680 cm				
9. a. 50 cm	b. 4 cm	c. $100\text{ cm} \times 40\text{ cm}$	d. $8\text{ cm} \times 3\text{ cm}$	
e. 5 m	f. 12.5 m	g. $50\text{ cm} \times 40\text{ cm}$	h. $2.25\text{ m} \times 1.5\text{ m} \times 0.6\text{ m}$	
10. a. $6\text{ m} \times 4\text{ m}$	b. 1 m	c. 6.8 m	d. 24 m^2	e. 12 m^2

11. Check with your teacher.

12. Check with your teacher.

13. Check with your teacher.

14. a. 3 cm and 5 cm b. 15 m^2 c. 60 m^2 d. 135 m^2

e. If the dimensions double, the area becomes four times larger. If the dimensions are three times bigger, the area becomes nine times larger. If the dimensions are enlarged by a factor of n , the area is enlarged by a factor of n^2 .

15. If the dimensions were reduced by a factor of n , the area would be reduced by a factor of n^2 .

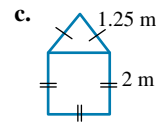
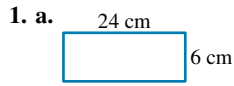
16. Seth's arm span is 10 times the length of his hand and 4 times the distance across his shoulders.

17. No. Depth is 750 cm, width is 1 m.

18. The towns were 5 cm apart on the original map. They are actually 375 km apart.

19. Yes, it is possible to have a negative scale factor. It results in an image which is on the opposite side of the centre of enlargement and is upside down. It can be interpreted by multiplying all the coordinates by the negative scale factor.

Exercise 3.4 Similar figures



2. a. i and iii, AAA
d. i and iii, RHS
3. a. Triangles ABC and DEC
d. Triangles ABC and DEC
4. a. $\triangle EDC$
5. Answers may vary due to inconsistencies in measurement.
a. i. 1.3 cm
ii. 2.6 cm
b. 2
6. a. $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$
b. $f = 9, g = 8$
7. a. $h = 3.75, i = 7.5$
b. $j = 2.4, k = 11.1$
8. $x = 4$
9. $x = 20^\circ, y = 2\frac{1}{4}$
10. $x = 3, y = 4$
11. Check with your teacher.
12. a. $x = 7.1$
b. $x = 3.1$
c. $x = 7.5, y = 7.7$
13. Answers will vary.
14. Answers will vary.
15. Answers will vary.
16. a. T
b. F
c. F
d. T
17. a. 6.72 m
b. 78 cm
18. 3.75 m
19. $a = 1$ cm
20. a. Answers will vary.
b. SAS
d. PR 6 cm, PS 5 cm and PQ 10 cm
- c. 4 cm
21. a. $\triangle EDC$ and $\triangle EBA$
b. $x = 3$ cm, $y \approx 3.29$ cm
22. The triangles are similar (AAA); $l = \frac{y}{2}$
23. 2 : 3 or 3 : 2
24. Similar figures do not have the same perimeter unless they are congruent. The perimeters of similar figures will have the same relationship as the scale factor between the two figures.
25. Ruby is 1.4 m tall.

Exercise 3.5 Area and volume of similar figures

1. a. 900 cm^2
b. 67.5 cm^2
c. 855 mm^2
d. 102 cm^2
e. 5 m^2
f. 25 mm^2
2. a. 5400 cm^3
b. 337.5 cm^3
c. 1809 mm^3
d. 100 cm^3
e. 2.5 m^3
f. 1 mm^3
3. $SA = 384 \text{ cm}^2, V = 288 \text{ cm}^3$
4. 50000 cm^2
5. a. 250000 cm^2
b. 17.4 cm^2
6. 8437500 cm^3
7. a. 1200000 cm^2
b. 187.5 cm^2
8. a. Answers will vary.
b. Answers will vary.
c. 2197 cm^3
d. 34.3 cm^3
e. 28.2 cm^2
9. Answers will vary.
10. a. Answers will vary.
b. 10 cm
c. 15 cm
d. Answers will vary.
11. Answers will vary.

12. Answers will vary.

13. a. 4 cm^2 , 25 cm^2 , 100 cm^2 , 400 cm^2

b. The ratio of the areas is equal to the square of the ratio of the side lengths.

14. 3 : 5

15. a. 3 : 4

b. 9 : 16

16. a. A and C are similar rectangles by the ratio 5 : 3 or scale factor $\frac{5}{3}$.

b. $\frac{25}{9}$

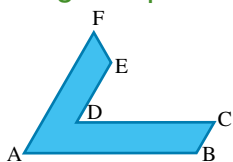
17. The new radius is 8 times the old radius.

18. The ratio is 7 : 1.

19. As the side length is increased by a scale factor of 5, then the volume will increase by a scale factor of 5^3 , which is $5 \times 5 \times 5 = 125$.

Investigation | Rich task

1.



2. 7.5 cm

3. $\angle FAB = 60^\circ$, $\angle AFE = 60^\circ$

4 and 5. The impossible triangle



6. Check with your teacher.

Exercise 3.6 Review questions

1. a. Congruent

b. Not congruent

c. Congruent

2. a. $\triangle ANP \equiv \triangle DWR$, $x = 3 \text{ cm}$

b. $\triangle TKB \equiv \triangle KTS$ or $\triangle TKB \equiv \triangle KST$, $x = 75^\circ$, $y = 30^\circ$, $z = 75^\circ$, $p = 5 \text{ cm}$, $q = 5 \text{ cm}$

3. a. 1 : 2

b. 3 : 2

c. 3 : 2

d. 7 : 10

e. 2 : 5

f. 7 : 5

4. a. 9 : 2

b. 49 : 72

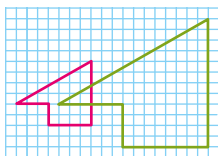
c. 29 : 14

d. 7 : 6

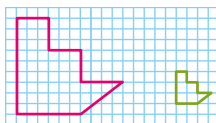
e. 5 : 62

f. 30 : 1

5. a.



b.



c.



6. a. 3

b. 0.4

c. $\frac{3}{5}$

d. 3.5

7. a. 5 km

b. 2.5 km

c. 4 km

8. a. 1.25 cm

b. 3.25 cm

9. a. 3 cm

b. 15 cm

c. 12 cm

d. 9.6 cm

e. 4.5 cm

10. a. $x = 8$

b. $x = 4$

c. $x = 11\frac{1}{5}$

d. $x = 26\frac{2}{3}$

11. $x = 12 \text{ cm}$, $y = 80^\circ$, $z = 40^\circ$

12. 400 cm^2

13. The easiest way to check for congruency is to measure what is within reach. This would be the bottom of each sail and the angles at the bottom of each sail. Hence, ASA would be the most appropriate rule to use.
14. a. $30\text{ m} \times 26\text{ m}$ b. $9.6\text{ m} \times 12\text{ m}$ c. 115.2 m^2 d. SW
15. 10 m
16. a. 5 m
- b. Sandy had the correct answer. Rachel used the distance of 10 m (from the ruler to the tree) in her calculations instead of 12.5 m (the whole length of the tree's shadow).
17. 2.1 m
18. 5 m
19. a. i. $\angle\text{TSB}$ and $\angle\text{WJB}$ are both right angles.
- ii. $\angle\text{SBT}$ and $\angle\text{JBW}$ are equal because they are vertically opposite.
- iii. $\angle\text{STB}$ and $\angle\text{JWB}$ are equal because they are alternate.
- b. $\triangle\text{SBT}$ and $\triangle\text{BJW}$ are similar (AAA).
- c. $\frac{\overline{\text{ST}}}{\overline{\text{JW}}} = 10$
- d. 27 m
20. The model is $\frac{1}{125}$ th the volume of the yacht.

TOPIC 4

Right-angled triangles (trigonometry)

[Stage 5.1]

4.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



4.1.1 Why learn this?

Pythagoras was a great mathematician and philosopher who lived in the 6th century BCE. He is best known for the theorem that bears his name. It concerns the relationship between the lengths of the sides in a right-angled triangle. Geometry and trigonometry are branches of mathematics where Pythagoras' theorem is still widely applied. Trigonometry is a branch of mathematics that allows us to relate the side lengths of triangles to angles. Combining trigonometry with Pythagoras' theorem allows us to solve many problems involving triangles.

DISCUSSION

Pythagoras was a famous Greek mathematician. Who are some of the other Greek mathematicians who helped form the foundations of modern mathematics?

LEARNING SEQUENCE

- 4.1 Overview
- 4.2 [Stage 4] Pythagoras' theorem
- 4.3 [Stage 4] Applications of Pythagoras' theorem
- 4.4 [Stage 5.1] Similarity and the sine, cosine and tangent ratios
- 4.5 [Stage 5.1] Calculating unknown side lengths
- 4.6 [Stage 5.1] Calculating unknown angles
- 4.7 [Stage 5.1] Angles of elevation and depression
- 4.8 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression MA5.1-10MG

CONTENT DESCRIPTIONS

Students:

Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

Solve right-angled triangle problems, including those involving angles of elevation and depression (ACMMG245)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — Secret society (eles-1693)

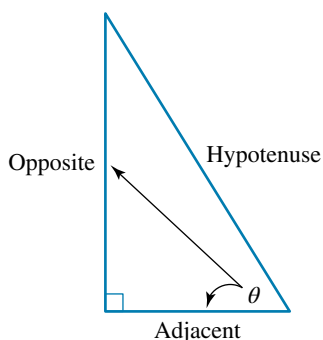
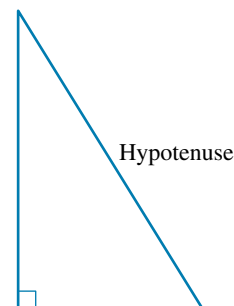
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

4.2 Pythagoras' theorem

[Stage 4]

4.2.1 Right-angled triangles

- Triangles can be classified according to their side lengths or their angles.
- Equilateral, isosceles and scalene triangles are classified by the length of their sides.
- A **right-angled triangle** is classified by the type of angle it contains.
- The longest side of a right-angled triangle is opposite the right angle and is called the **hypotenuse**.
- In all triangles the longest side is opposite the largest angle.
- In order to name the remaining two sides, another angle must be chosen. This angle is called the 'reference angle'. It is labelled θ in the diagram.



- The side that is across from the reference angle, θ , is called the **opposite side**, and the remaining side (the side next to the reference angle) is called the **adjacent side**. This is discussed in more detail in section 4.4.2.

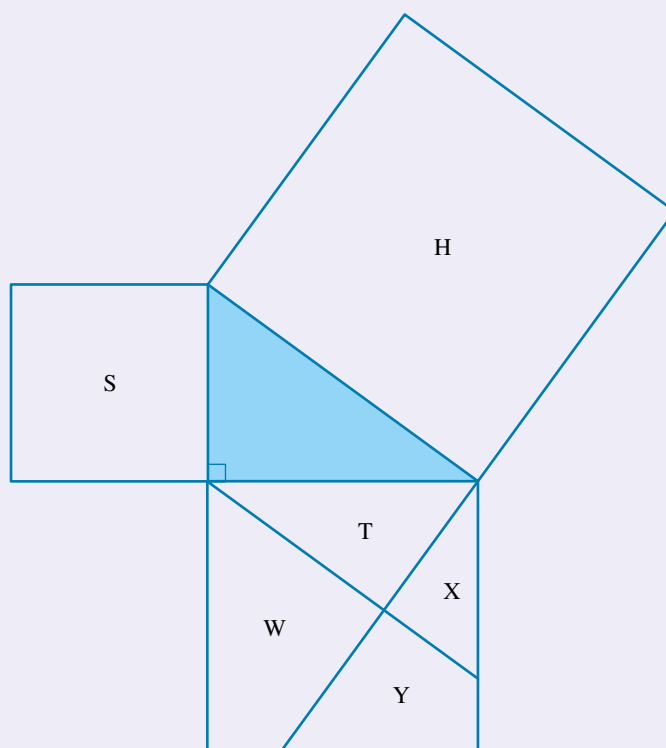
Note: If no reference angle is marked, only the hypotenuse can be named.

- Pythagoras (580–501 BC) was a Greek mathematician who explored the relationship between the lengths of the sides of right-angled triangles.
- The relationship he described, and has been credited with discovering over 2500 years ago, is known as **Pythagoras' theorem**.

4.2.2 Pythagoras' theorem

ACTIVITY: PYTHAGORAS' THEOREM

1. The diagram below shows a right-angled triangle with squares drawn on each of its sides. Copy the figure onto a piece of paper.

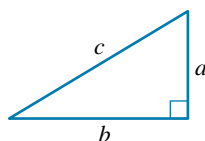


2. Cut around the perimeter of the figure, then cut off the two smaller squares from the base and height of the right-angled triangle. Cut the larger of these two squares into the four pieces shown.
3. You should now have a right-angled triangle with the square H attached to its hypotenuse and five smaller pieces (S, T, W, X and Y). Rearrange these five pieces to fit exactly in the area H.
4. Since the squares from the two shorter sides of the right-angled triangle fit exactly into the square on the hypotenuse, what can you conclude?

- Pythagoras investigated the relationship between the lengths of the sides of a right-angled triangle. This is now known as Pythagoras' theorem.
- The theorem states that:

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is:

$$c^2 = a^2 + b^2$$



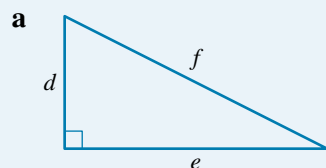
where a and b are the two shorter sides and c is the hypotenuse.

- If the lengths of two sides of a right-angled triangle are given, then Pythagoras' theorem enables the length of the unknown side to be found.

WORKED EXAMPLE 1

For the right-angled triangles shown:

- state which side is the hypotenuse
- write Pythagoras' theorem.

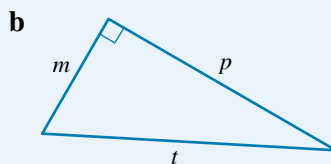


THINK

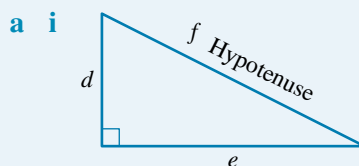
- The hypotenuse is always opposite the right angle.
- If the triangle is labelled as usual with a , b and c , as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).

- b i** The hypotenuse is opposite the right angle

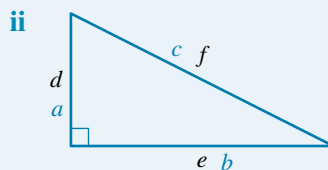
- If the triangle is labelled as usual with a , b and c , as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).



WRITE



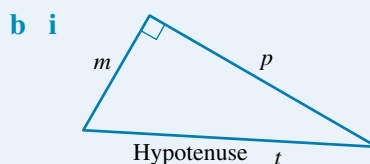
Side f is opposite the right angle.
Therefore, side f is the hypotenuse.



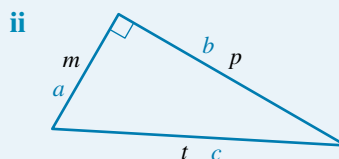
$$c = f; a = d; b = e$$

$$c^2 = a^2 + b^2$$

$$f^2 = d^2 + e^2$$



Side t is opposite the right angle.
Therefore, side t is the hypotenuse.



$$c = t; b = p; a = m$$

$$c^2 = a^2 + b^2$$

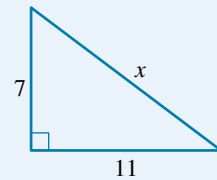
$$t^2 = m^2 + p^2$$

4.2.3 Finding the hypotenuse

- To calculate the length of the hypotenuse given the length of the two other shorter sides, a and b , substitute the values of a and b into the rule $c^2 = a^2 + b^2$ and solve for c .
- Remember that the hypotenuse is the longest side, so the value of c must be *greater than* that of either a or b .

WORKED EXAMPLE 2

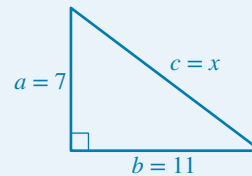
For the triangle shown, calculate the length of the hypotenuse, x , correct to 1 decimal place.



THINK

- 1 Copy the diagram and label the sides a , b and c . Remember to label the hypotenuse as c .
- 2 Write Pythagoras' theorem.
- 3 Substitute the values of a , b and c into this rule and simplify.
- 4 Calculate x by taking the square root of 170. Round the answer correct to 1 decimal place.

WRITE/DRAW



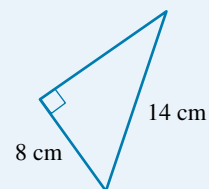
$$\begin{aligned}c^2 &= a^2 + b^2 \\x^2 &= 7^2 + 11^2 \\&= 49 + 121 \\&= 170 \\x &= \sqrt{170} \\x &= 13.0\end{aligned}$$

4.2.4 Finding the length of a shorter side

- When given both the hypotenuse and one other shorter side, either a or b , rearrange Pythagoras' theorem to find the value of the unknown shorter side, for example $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.
- Remember that the value of the unknown shorter side, either a or b , should be *less than* the value of the hypotenuse.

WORKED EXAMPLE 3

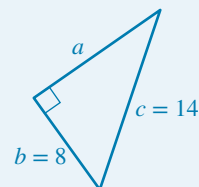
Calculate the length, correct to 1 decimal place, of the unmarked side of the triangle shown.



THINK

- 1 Copy the diagram and label the sides a , b and c . Remember to label the hypotenuse as c .
- 2 Write Pythagoras' theorem for a shorter side.
- 3 Substitute the values of b and c into this rule and simplify.
- 4 Find a by taking the square root of 132. Round to 1 decimal place.

WRITE/DRAW



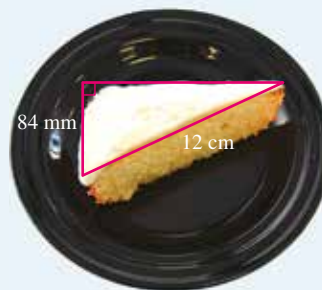
$$\begin{aligned}a^2 &= b^2 - c^2 \\a^2 &= 14^2 - 8^2 \\&= 196 - 64 \\&= 132 \\a &= \sqrt{132} \\&= 11.5\text{ cm}\end{aligned}$$

4.2.5 Practical problems

- When using Pythagoras' theorem to solve practical problems, draw a right-angled triangle to represent the problem.
 - Identify the unknown variable by reading what the question is asking.
 - Identify the known values and substitute these into Pythagoras' theorem.
 - Solve for the unknown value.
 - Use the result to write the answer as a complete sentence.

WORKED EXAMPLE 4

Calculate the value of the missing side length. Give your answer correct to 1 decimal place.



THINK

- Identify the length of the hypotenuse and the two shorter sides.
 - Check that the units for all measurements are the same. Convert 84 mm to centimetres by dividing by 10.
- Substitute the values into the equation and simplify.
- Solve the equation for x by:
 - subtracting 70.56 from both sides, as shown in red
 - taking the square root of both sides.
- Answer the question.
 - Possible values for x are 8.5697 and -8.5697 . It is not possible to have a length of -8.5697 .
 - Round the answer to 1 decimal place.
 - Include the appropriate units in the answer. The calculations were carried out in centimetres. The answer will be in centimetres.

WRITE/DRAW

$c = 12$ cm
 $a = 84$ mm = 8.4 cm
Let x = the length of the missing side.
 $b = x$

$$\begin{aligned}c^2 &= a^2 + b^2 \\12^2 &= (8.4)^2 + x^2 \\144 &= 70.56 + x^2\end{aligned}$$

$$\begin{aligned}144 - 70.56 &= 70.56 - 70.56 + x^2 \\73.44 &= x^2 \\\sqrt{73.44} &= \sqrt{x^2} \\\pm 8.5697 &\approx x\end{aligned}$$

$x \approx 8.5697$ is the only valid solution as x is the length of a side in a triangle. It is not possible to have a side with a negative length.
Therefore, the missing side length is 8.6 cm (correct to 1 decimal place).

4.2.6 Pythagorean triads

- A **Pythagorean triad** is a group of any three whole numbers that satisfy Pythagoras' theorem. For example, $\{5, 12, 13\}$ and $\{7, 24, 25\}$ are Pythagorean triads.
 $13^2 = 5^2 + 12^2$; $25^2 = 7^2 + 24^2$
- Pythagorean triads are useful when solving problems using Pythagoras' theorem. If two known side lengths in a triangle belong to a triad, the length of the third side can be stated without performing any calculations.

- Some well known Pythagorean triads are:
 $\{3, 4, 5\}$; $\{5, 12, 13\}$; $\{8, 15, 17\}$; $\{7, 24, 25\}$.

WORKED EXAMPLE 5

Determine whether the following sets of numbers are Pythagorean triads.

a $\{9, 10, 14\}$

b $\{33, 65, 56\}$

THINK

a 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember c is the longest side.

2 State your conclusion.

b 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember c is the longest side.

2 State your conclusion.

WRITE/DRAW

a $c^2 = a^2 + b^2$

$$\begin{aligned}\text{LHS} &= c^2 \\ &= 14^2 \\ &= 196\end{aligned}$$

$$\begin{aligned}\text{RHS} &= a^2 + b^2 \\ &= 9^2 + 10^2 \\ &= 81 + 100 \\ &= 181\end{aligned}$$

Since $\text{LHS} \neq \text{RHS}$, the set $\{9, 10, 14\}$ is not a Pythagorean triad.

b $c^2 = a^2 + b^2$

$$\begin{aligned}\text{LHS} &= 65^2 \\ &= 4225\end{aligned}$$

$$\begin{aligned}\text{RHS} &= 33^2 + 56^2 \\ &= 1089 + 3136 \\ &= 4225\end{aligned}$$

Since $\text{LHS} = \text{RHS}$, the set $\{33, 65, 56\}$ is a Pythagorean triad.

- If each term in a triad is multiplied by the same number, the result is also a triad. For example, if we multiply each number in $\{5, 12, 13\}$ by 2, the result $\{10, 24, 26\}$ is also a triad.
- Builders and gardeners use multiples of the Pythagorean triad $\{3, 4, 5\}$ to ensure that walls and floors are at right angles.



Interactivity: Pythagorean triples (int-2765)

Interactivity: Finding the hypotenuse (int-3844)

Interactivity: Finding a shorter side (int-3845)

Interactivity: Pythagorean triads (int-3848)

Digital doc: SkillsHEET Rounding to a given number of decimal places (doc-11428)

Digital doc: SkillsHEET Converting units of length (doc-11430)

Exercise 4.2 Pythagoras' theorem

Individual pathways

PRACTISE

Questions:
1–11, 13, 16

CONSOLIDATE

Questions:
1–14, 16

MASTER

Questions:
1–18

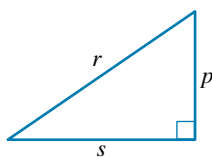
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

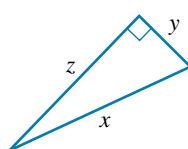
1. **WE1** For each of the following right-angled triangles:

- state which side is the hypotenuse
- write Pythagoras' theorem.

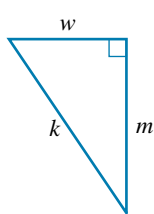
a.



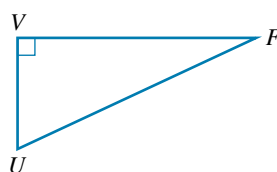
b.



c.

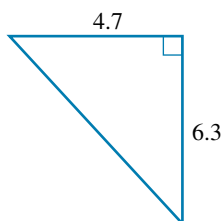


d.

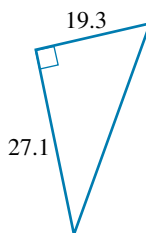


2. **WE2** For each of the following triangles, calculate the length of the hypotenuse, giving answers correct to 2 decimal places.

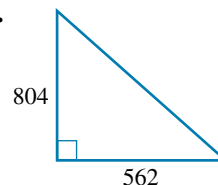
a.



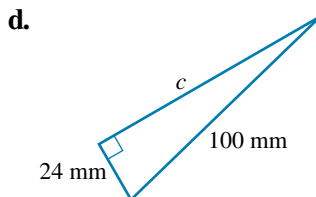
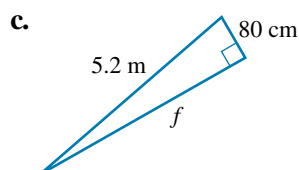
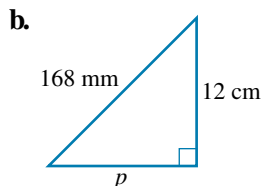
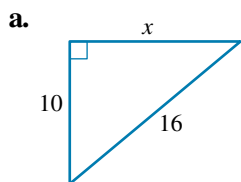
b.



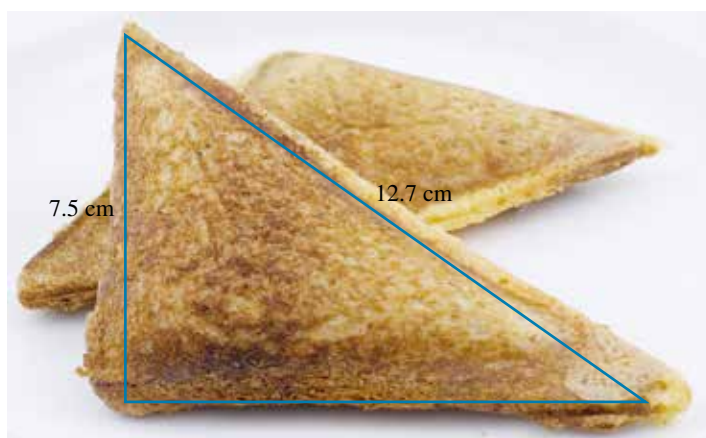
c.



3. **WE3** Calculate the value of the pronumeral in each of the following triangles. Give your answers correct to 2 decimal places.



4. **WE4** Calculate the value of the missing side length. Give your answer correct to 1 decimal place.



5. **WE5** Determine whether the following sets of numbers are Pythagorean triads.

- a. {2, 5, 6} b. {7, 10, 12} c. {18, 24, 30}
d. {72, 78, 30} e. {13, 8, 15} f. {50, 40, 30}

6. In a right-angled triangle, the two shortest sides are 4.2 cm and 3.8 cm.

- a. Draw a sketch of the triangle.
b. Calculate the length of the hypotenuse correct to 2 decimal places.

7. A right-angled triangle has a hypotenuse of 124 mm and another side of 8.5 cm.

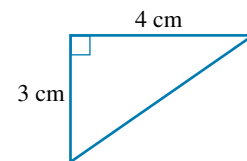
- a. Draw a sketch of the triangle.
b. Calculate the length of the third side. Give your answer in millimetres correct to 2 decimal places.

8. **MC** Which of the following sets is formed from the triad {21, 20, 29}?

- a. {95, 100, 125} b. {105, 145, 100} c. {84, 80, 87}
d. {105, 80, 87} e. {215, 205, 295}

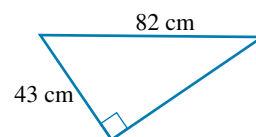
9. **MC** What is the length of the hypotenuse in this triangle?

- A. 25 cm B. 50 cm C. 50 mm
D. 500 mm E. 2500 mm

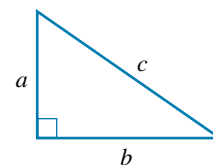


10. **MC** What is the length of the third side in this triangle?

- A. 48.75 cm B. 0.698 m C. 0.926 m
D. 92.6 cm E. 69.8 mm



11. A ladder that is 7 m long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder, correct to 2 decimal places?
12. Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram shows how each triangle is to be labelled.

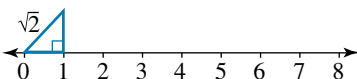


Give your answers correct to 2 decimal places. *Remember: c is always the hypotenuse.*

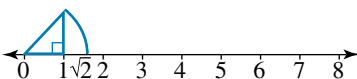
- a. $a = 37$ cm, $c = 180$ cm; find b in cm.
- b. $a = 856$ mm, $b = 1200$ mm; find c in cm.
- c. $b = 4950$ m, $c = 5.6$ km; find a in km.
- d. $a = 125600$ mm, $c = 450$ m; find b in m.

Communicating, reasoning and problem solving

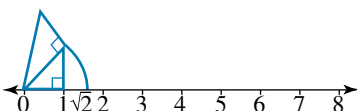
13. An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.
- a. Draw a sketch showing the ladder leaning on the wall.
 - b. How far up the wall does the ladder reach, correct to 2 decimal places?
 - c. The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?
14. A rectangular park is 260 m by 480 m. Danny usually trains by running 5 circuits around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. Show that Danny runs about 970 metres less than his usual training session.
15. An adventure water park has hired Sally to build part of a ramp for a new water slide. She builds a ramp that is 12 m long and rises to a height of 250 cm. To meet the regulations, the ramp must have a gradient between 0.1 and 0.25. Show that the ramp Sally has built is within the regulations.
16. a. The smallest numbers of four Pythagorean triads are given below. For each triad, find the middle number and the third number.
- | | | | |
|------|--------|---------|--------|
| i. 9 | ii. 11 | iii. 13 | iv. 29 |
|------|--------|---------|--------|
- b. What do you notice about the triads formed in part a?
17. We know that it is possible to find the exact square root of some numbers but not others. For example, we can find $\sqrt{4}$ exactly but not $\sqrt{3}$ or 5. Our calculators can find decimal approximations of these, but because they cannot be found exactly they are called irrational numbers. There is a method, however, of showing their exact location on a number line.
- a. Using graph paper, draw a right-angled triangle with two equal sides of length 1 cm as shown below.



- b. Using Pythagoras' theorem, we know that the longest side of this triangle is $\sqrt{2}$ units. Place the compass point at zero and make an arc that will show the location of $\sqrt{2}$ on the number line.



- c. Draw another right-angled triangle using the longest side of the first triangle as one side, and make the other side 1 cm in length.



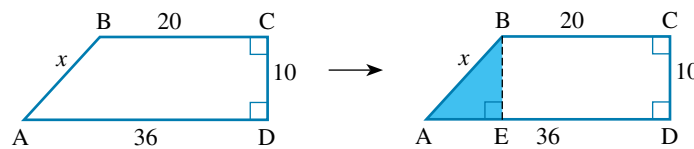
- d. The longest side of this triangle will have a length of $\sqrt{3}$ units. Draw an arc to find the location of $\sqrt{3}$ on the number line.
- e. Repeat steps c and d to draw triangles that will have sides of length $\sqrt{4}$ units, $\sqrt{5}$ units, $\sqrt{6}$ units and so on.
18. What is the quickest way to identify whether a triangle is right-angled or not?

4.3 Applications of Pythagoras' theorem

[Stage 4]

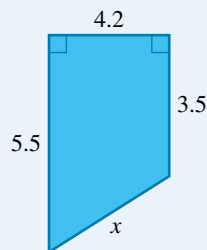
4.3.1 Using composite shapes to solve problems

- Dividing a composite shape into simpler shapes creates shapes that have known properties. For example, to calculate the value of x in the trapezium shown, a vertical line can be added to create a right-angled triangle and a rectangle. The length of x can be found using Pythagoras' theorem.



WORKED EXAMPLE 6

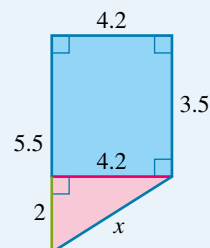
Calculate the value of x in the diagram, correct to 1 decimal place.



THINK

- Divide the shape into smaller shapes that create a right-angled triangle, as shown in red.
- x is the hypotenuse of a right-angled triangle. To use Pythagoras' theorem, the length of the side shown in green must be known. This length can be calculated as the difference between the long and short vertical edges of the trapezium: $5.5 - 3.5 = 2$.

WRITE/DRAW



- 3 Substitute the values of the side lengths into Pythagoras' theorem.

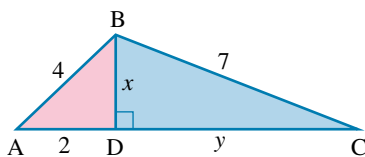
$$\begin{aligned}c^2 &= a^2 + b^2 \\x^2 &= 2^2 + (4.2)^2 \\x^2 &= 4 + 17.64 \\x^2 &= 21.64 \\\sqrt{x^2} &= \sqrt{21.64} \\x &\approx \pm 4.65188\end{aligned}$$

- 4 Answer the question.

- Possible values for x are 4.65188 and -4.65188 . It is not possible for x to have a length of negative value.
- Round the answer to 1 decimal place.
- Include units if appropriate. There are no units.

$x \approx \pm 4.65188$ is the only valid solution, as x is the side length of a triangle. Therefore, $x = 4.7$ (correct to 1 decimal place).

- To find the value of y in the irregular triangle shown, the triangle can be split into two right-angled triangles: $\triangle ABD$ (pink) and $\triangle BDC$ (blue). There is enough information to calculate the missing side length from $\triangle ABD$. This newly calculated length can be used to find the value of y .



WORKED EXAMPLE 7

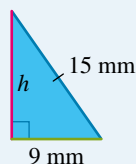
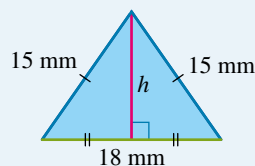
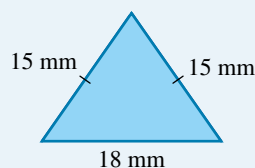
- a Calculate the perpendicular height of the isosceles triangle whose equal sides are each 15 mm long and whose third side is 18 mm long.
- b Calculate the area of the triangle.

THINK

- a 1 Draw the triangle and label all side lengths as described.
- 2 Draw an additional line to represent the height of the triangle, as shown in red, and label it appropriately.
Since the triangle is an isosceles triangle, h bisects the base of 18 mm, as shown in green, and creates 2 right-angled triangles.
- 3 Focus on one right-angled triangle containing the height.

WRITE/DRAW

a



- 4 Substitute the values from this right-angled triangle into Pythagoras' theorem to calculate the value of h .

$$\begin{aligned}c^2 &= a^2 + b^2 \\15^2 &= h^2 + 9^2 \\225 - 81 &= h^2 + 81 - 81 \\144 &= h^2 \\\pm 12 &= h\end{aligned}$$

- 5 Answer the question.

- Possible values for h are 12 and -12 . It is not possible for h to have a length of negative value.
- Include units if appropriate.

$h = 12$ is the only valid solution, since h represents the height of a triangle. The height of the triangle is 12 mm.

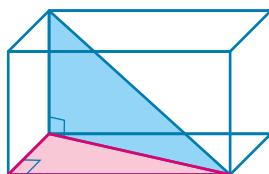
- b 1** Write the formula for the area of a triangle.
In this case the base is 18 mm and the height is 12 mm.

$$\begin{aligned}\mathbf{b} \quad A_{\Delta} &= \frac{1}{2}bh \\&= \frac{1}{2} \times 18 \times 12 \\&= 108\end{aligned}$$

- 2 Answer the question. Include units if appropriate.

The area of the triangle is 108 mm^2 .

4.3.2 Pythagoras' theorem in 3-D



- Pythagoras' theorem can also be used to solve problems in **three dimensions**.
- In 3-D problems, often more than one right-angled triangle exists.
- It is often helpful to redraw sections of the 3-D diagram in 2 dimensions, allowing the right angles to be seen with greater clarity.

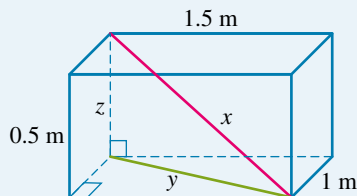
WORKED EXAMPLE 8

Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions $1 \text{ m} \times 1.5 \text{ m} \times 0.5 \text{ m}$.

THINK

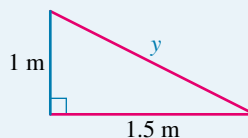
- 1 Draw a diagram of a rectangular box with a rod in it, labelling the dimensions.

WRITE/DRAW



- 2
- Draw in a right-angled triangle that has the metal rod as one of the sides, as shown in red. The length of y in this right-angled triangle is not known.
 - Draw in another right-angled triangle, as shown in green, to calculate the length of y .

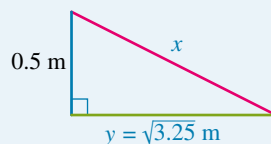
- 3 • Calculate the length of y using Pythagoras' theorem.
• Calculate the exact value of y .



$$\begin{aligned}c^2 &= a^2 + b^2 \\y^2 &= 1.5^2 + 1^2 \\y^2 &= 3.25 \\y &= \sqrt{3.25}\end{aligned}$$

- 4 Draw a right-angled triangle containing the rod and use Pythagoras' theorem to calculate the length of the rod (x).

Note: z = height of the crate = 0.5 m.



$$\begin{aligned}c^2 &= a^2 + b^2 \\x^2 &= (\sqrt{3.25})^2 + 0.5^2 \\x^2 &= 3.25 + 0.25 \\x^2 &= 3.5 \\x &= \sqrt{3.5} \\&\approx 1.87\end{aligned}$$

- 5 Answer the question.

The maximum length of the metal rod is 1.87 m (correct to 2 decimal places).

learnON RESOURCES — ONLINE ONLY

Interactivity: Composite shapes (int-3847)

eLesson: Pythagoras' theorem in 3 dimensions (eles-1913)

Exercise 4.3 Applications of Pythagoras' theorem

Individual Pathways

PRACTISE

Questions:
1–8, 10, 12, 14, 16, 19

CONSOLIDATE

Questions:
1–12, 14–16, 19, 20

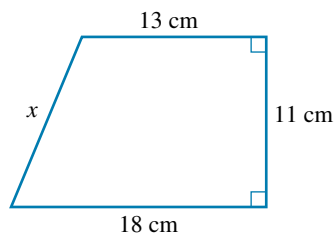
MASTER

Questions:
1–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE6** Calculate the length of the side x in the figure below, correct to 2 decimal places.

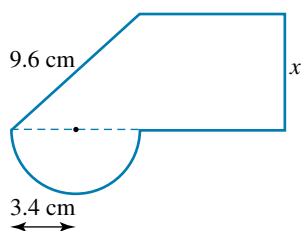


2. Calculate the values of the pronumerals in each of the following. Give your answers correct to 2 decimal places.

a.



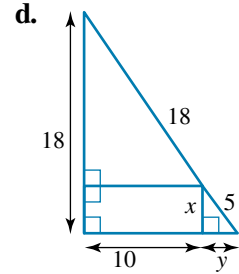
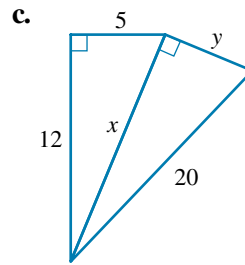
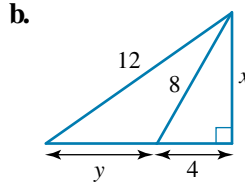
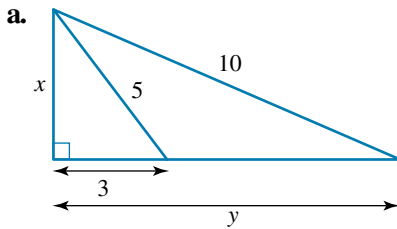
b.



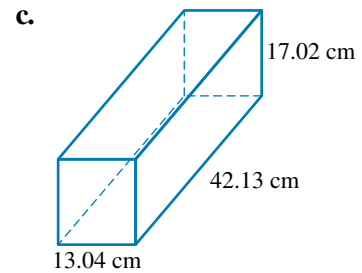
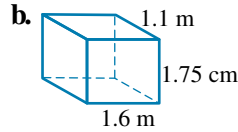
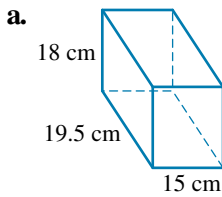
c.



3. For each of the following diagrams, calculate the lengths of the sides marked x and y . Give your answer correct to 2 decimal places where necessary.



4. **WEB** Calculate the length of the longest metal rod that can fit diagonally into each of the boxes shown below. Give your answers correct to 2 decimal places.

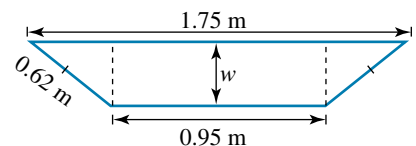


5. **WE7** The height of an isosceles triangle is 3.4 mm and its equal sides are twice as long.
- Sketch the triangle, showing all the information given.
 - Calculate the length of the third side to 2 decimal places.
 - Calculate the area of the triangle to 2 decimal places.
6. The side length of an equilateral triangle is 1 m. Calculate:
- the height of the triangle in metres to 2 decimal places
 - the area of the triangle in m^2 to 3 decimal places.
7. **MC** The longest length object that can fit into a box with dimensions $42 \text{ cm} \times 60 \text{ cm} \times 13 \text{ cm}$ is:
- A.** 74.5 cm **B.** 60 cm **C.** 73.2 cm **D.** 5533 cm **E.** 74 cm
8. A friend wants to pack an umbrella into her suitcase. If the suitcase measures $89 \text{ cm} \times 21 \text{ cm} \times 44 \text{ cm}$, will her 1-m umbrella fit in? Give the length (correct to 3 decimal places) of the longest object that will fit in the suitcase.
9. A friend is packing his lunch into his lunchbox and wants to fit in a 22-cm-long straw. If the lunchbox is $15 \text{ cm} \times 5 \text{ cm} \times 8 \text{ cm}$, will it fit in? What is the length (correct to 2 decimal places) of the longest object that will fit into the lunchbox?
10. A cylindrical pipe has a length of 2.4 m and an internal diameter of 30 cm. What is the largest object that could fit in the pipe? Give your answer correct to the nearest cm.



11. A classroom contains 20 identical desks with dimensions as shown in the diagram at right.

- Calculate the width of each desktop (labelled w) correct to 2 decimal places.
- Calculate the area of each desktop correct to 2 decimal places.
- If the top surface of each desk is to be painted, what is the total area that needs to be painted correct to 2 decimal places?
- What is the cost of the paint needed to finish the job if all desks are to be given two coats of fresh paint and paint is sold in 1-litre containers that cost \$29.95 each and give coverage of 12 m^2 per litre?



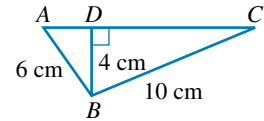
12. A hobby knife has a blade in the shape of a right-angled trapezium with the sloping edge 2 cm long and parallel sides 32 mm and 48 mm long. Calculate the width of the blade and, hence, the area.
13. A cylindrical pen holder has an internal height of 16 cm. If the diameter of the pen holder is 8.5 cm and the width of the pen holder is 2 mm, what is the largest pen that could fit completely inside the holder? Give your answer correct to the nearest mm.



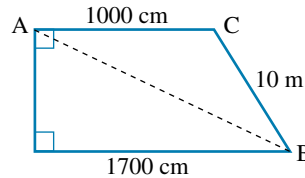
Communicating, reasoning and problem solving

14. Explain in your own words how a 2-D right-angled triangle can be seen in a 3-D figure.

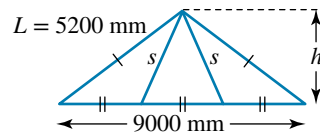
15. Consider the figure shown.



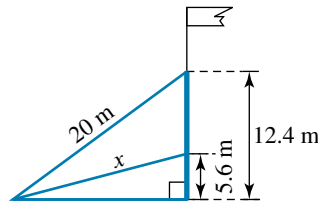
- In which order will you need to calculate the lengths of the following sides: AD, AC and DC?
 - Calculate the lengths of the sides in part a to 2 decimal places.
 - Is the triangle ABC right-angled? Use calculations to justify your answer.
16. Katie goes on a hike and walks 2.5 km north, then 3.1 km east. She then walks 1 km north and 2 km west. She then walks in a straight line back to her starting point. Show that she walks a total distance of 12.27 km.
17. a. Show that the distance AB in the plan of the paddock below is 18.44 metres.



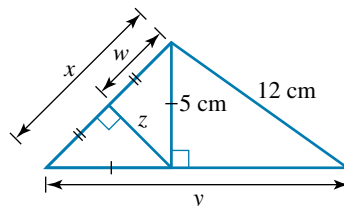
- Prove that the angle $\angle ACB$ is not a right angle.
18. The diagram below shows the cross-section through a roof.



- Calculate the height of the roof, h , to the nearest millimetre.
 - The longer supports, L , are 5200 mm long. Show that the shorter supports, s , are 2193 mm shorter than the longer supports.
19. A flagpole is attached to the ground by two wires as shown in the diagram. Use the information from the diagram to calculate the length of the lower wire, x , to 1 decimal place.



20. Calculate the values of the pronumerals w , x , y and z in the diagram.



21. Which simple shapes should you try to divide composite shapes up into?

4.4 Similarity and the sine, cosine and tangent ratios

[Stage 5.1]

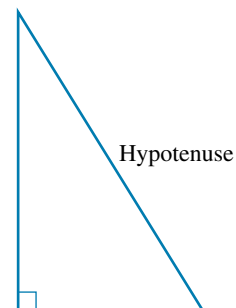
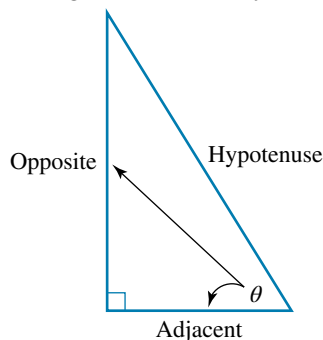
4.4.1 What is trigonometry?

- The word **trigonometry** is derived from the Greek words *trigonon* (triangle) and *metron* (measurement). Thus, it literally means ‘to measure a triangle’.
- Trigonometry deals with the relationship between the sides and the angles of a triangle.
- Modern-day uses of trigonometry include surveying land, architecture, measuring distances and determining heights of inaccessible objects.
- In the remaining sections of this topic, relationships between the sides and angles of a right-angled triangle will be explored.

4.4.2 Naming the sides of a right-angled triangle



- As discussed in section 4.2.1, the longest side of a right-angled triangle (the side opposite the right angle) is called the hypotenuse.
 - The side that is across from the reference angle, θ , is called the opposite side, and the remaining side (the side next to the reference angle) is called the adjacent side.
- Note:* If there is no reference angle marked, only the hypotenuse can be named.



DISCUSSION

Why are there never two equal longest sides in a right-angled triangle?

WORKED EXAMPLE 9

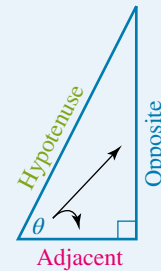
Label the sides of the right-angled triangle shown using the words hypotenuse, adjacent and opposite.



THINK

- 1 The hypotenuse is opposite the right angle.

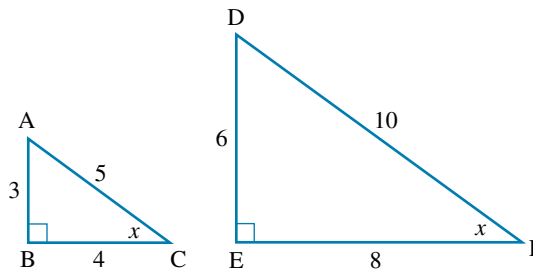
WRITE/DRAW



- 2 Label the side next to angle θ as 'adjacent' and the side opposite angle θ as 'opposite'.

4.4.3 Similar right-angled triangles

- Consider the two right-angled triangles shown below. The second triangle ($\triangle DEF$) is an enlargement of the first, using a scale factor of 2. Therefore, the triangles are similar ($\triangle ABC \sim \triangle DEF$), and $\angle BCA = \angle EFD = x$.



- For $\triangle ABC$, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4}$, and for $\triangle DEF$, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{6}{8} = \frac{3}{4}$.
- Now complete the table below.

	$\triangle ABC$	$\triangle DEF$
$\frac{\text{Opposite side}}{\text{hypotenuse}}$		
$\frac{\text{Adjacent side}}{\text{hypotenuse}}$		

- In similar right-angled triangles, the ratios of corresponding sides are equal.

ACTIVITY: RATIOS OF SIDE LENGTHS IN SIMILAR TRIANGLES

- 1 Using a protractor and ruler, carefully measure and draw a right-angled triangle of base 10 cm and angle of 60° as shown in the diagram. Measure the length of the other two sides to the nearest mm, and mark these lengths on the diagram as well.

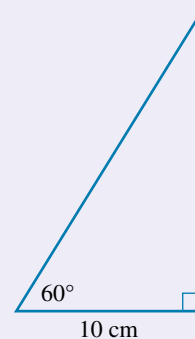
Use your measurements to calculate these ratios correct to 2 decimal places:

$$\frac{\text{opposite}}{\text{adjacent}} = \quad , \quad \frac{\text{opposite}}{\text{hypotenuse}} = \quad , \quad \frac{\text{adjacent}}{\text{hypotenuse}} =$$

- 2 Draw another triangle, similar to the one in part 1 (all angles the same), making the base length anything that you choose, and measuring the length of all the sides. Once again, calculate the three ratios correct to 2 decimal places:

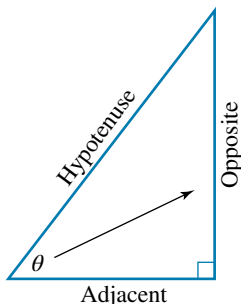
$$\frac{\text{opposite}}{\text{adjacent}} = \quad , \quad \frac{\text{opposite}}{\text{hypotenuse}} = \quad , \quad \frac{\text{adjacent}}{\text{hypotenuse}} =$$

- 3 Compare your results with the rest of the class. What conclusions can you draw?



4.4.4 Trigonometric ratios

- Trigonometry is based upon the ratios between pairs of side lengths, and each one is given a special name as follows.
- In any right-angled triangle:



$$\text{sine}(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine}(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent}(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

- These rules are abbreviated to:

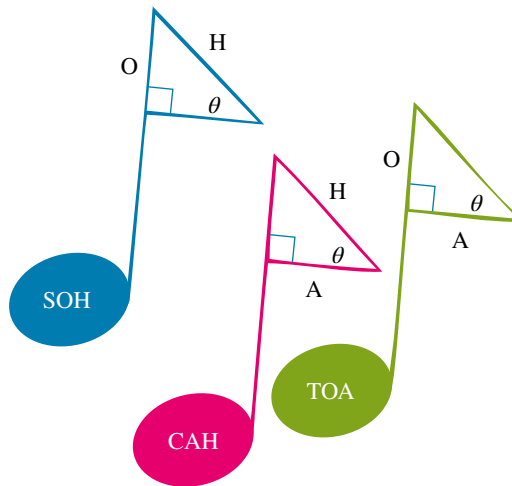
$$\sin(\theta) = \frac{O}{H}, \cos(\theta) = \frac{A}{H} \text{ and } \tan(\theta) = \frac{O}{A}.$$

DISCUSSION

Using your knowledge of similar triangles, explain the meaning and usefulness of the trigonometric ratios sine, cosine and tangent.

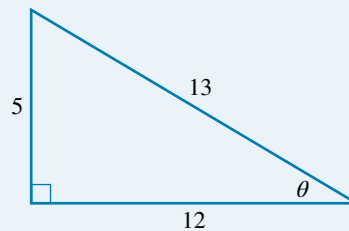
- The following mnemonic can be used to help remember the trigonometric ratios.

SOH – CAH – TOA



WORKED EXAMPLE 10

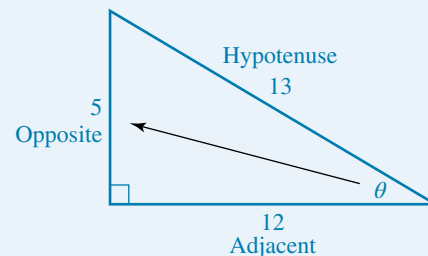
For this triangle, write the equations for the sine, cosine and tangent ratios of the given angle.



THINK

- 1 Label the sides of the triangle.

WRITE/DRAW



- 2 Write the trigonometric ratios.

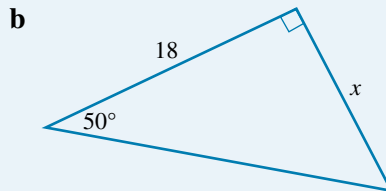
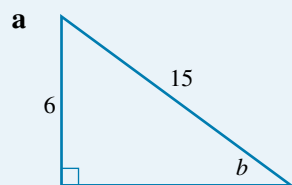
$$\sin(\theta) = \frac{O}{H}, \cos(\theta) = \frac{A}{H}, \tan(\theta) = \frac{O}{A}$$

- 3 Substitute the values of A, O and H into each formula.

$$\sin(\theta) = \frac{5}{13}, \cos(\theta) = \frac{12}{13}, \tan(\theta) = \frac{5}{12}$$

WORKED EXAMPLE 11

Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.



THINK

a 1 Label the given sides.

2 We are given O and H. These are used in SOH. Write the ratio.

3 Substitute the values of the pronumerals into the ratio.

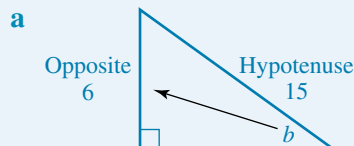
4 Simplify the fraction.

b 1 Label the given sides.

2 We are given A and O. These are used in TOA. Write the ratio.

3 Substitute the values of the angle and the pronumerals into the ratio.

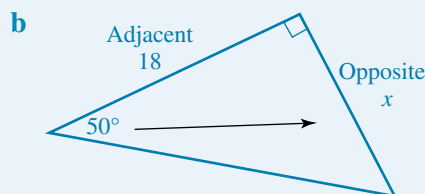
WRITE/DRAW



$$\sin(\theta) = \frac{O}{H}$$

$$\sin(b) = \frac{6}{15}$$

$$\sin(b) = \frac{2}{5}$$



$$\tan(\theta) = \frac{O}{H}$$

$$\tan(50^\circ) = \frac{x}{18}$$

ACTIVITY: SINE, COSINE AND TANGENT RATIOS

Investigate the sine, cosine and tangent ratios for a given fixed angle in similar right-angled triangles. What do you find?

learn on RESOURCES — ONLINE ONLY

Interactivity: Trigonometric ratios (int-2577)

Interactivity: Trigonometric ratios (int-0744)

Interactivity: Right-angled triangles (int-3843)

Digital doc: SkillsSHEET Measuring angles with a protractor (doc-10831)

Digital doc: SkillsSHEET Rounding to a given number of decimal places (doc-11428)

Exercise 4.4 Similarity and the sine, cosine and tangent ratios

Individual Pathways

PRACTISE

Questions:
1–8

CONSOLIDATE

Questions:
1–11

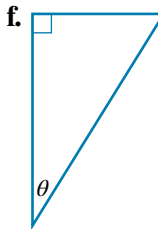
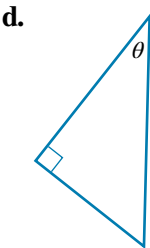
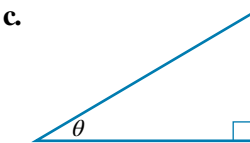
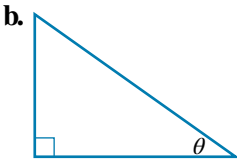
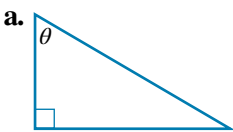
MASTER

Questions:
1–13

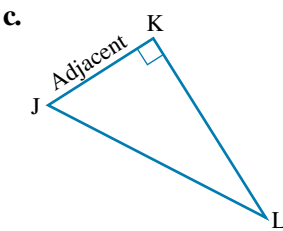
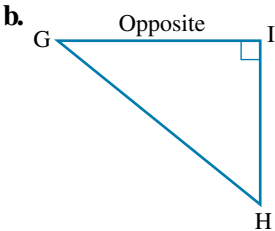
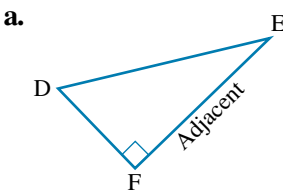
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

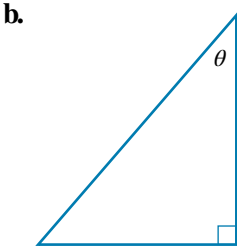
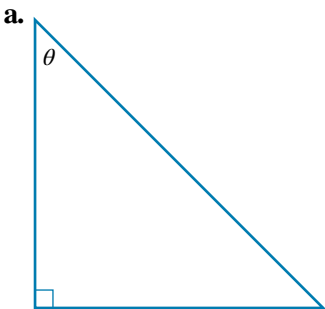
1. **WE9** Label the sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite.



2. Label the hypotenuse, adjacent and opposite sides, and reference angle θ , where appropriate, in each of the following right-angled triangles.

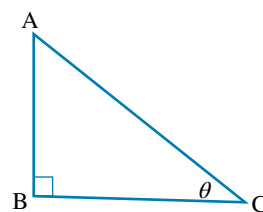


3. For each triangle below, carefully measure θ correct to the nearest degree, then carefully measure each side correct to the nearest mm. Use this information to copy and complete the table below.



	θ	O	A	H	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
a							
b							

4. **MC** Which alternative correctly names the sides and angle of the triangle shown?



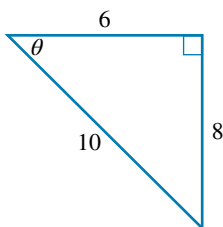
- A.** $\angle C = \theta$, AB = adjacent side, AC = hypotenuse, AC = opposite side
B. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, AC = adjacent side
C. $\angle A = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side
D. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side
5. **WE10** For each of the following triangles, write the expressions for ratios of each of the given angles:

i. sine

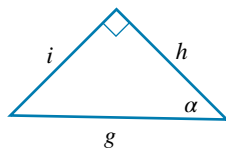
ii. cosine

iii. tangent.

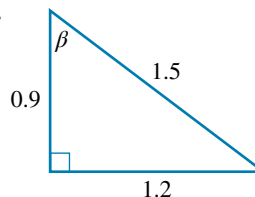
a.



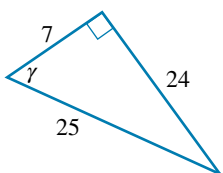
b.



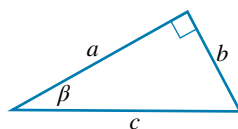
c.



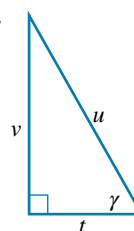
d.



e.

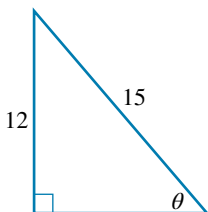


f.

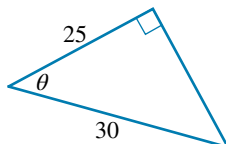


6. **WE11** Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.

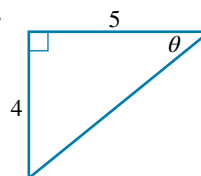
a.



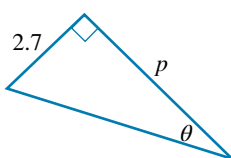
b.



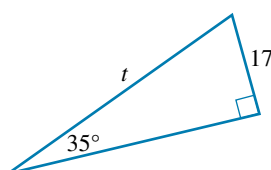
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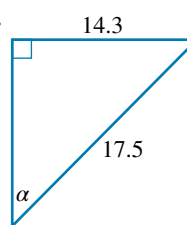
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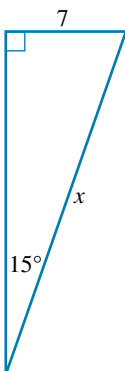
e.



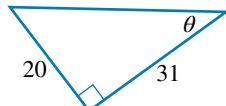
f.



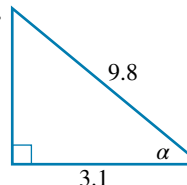
g.



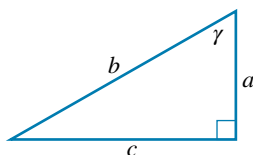
h.



i.

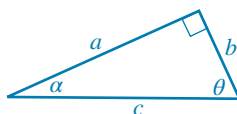


7. **MC** a. What is the correct trigonometric ratio for the triangle shown below?



- A. $\tan(\gamma) = \frac{a}{c}$ B. $\sin(\gamma) = \frac{c}{a}$ C. $\cos(\gamma) = \frac{c}{b}$ D. $\sin(\gamma) = \frac{c}{b}$

- b. Which trigonometric ratio for the triangle shown below is incorrect?

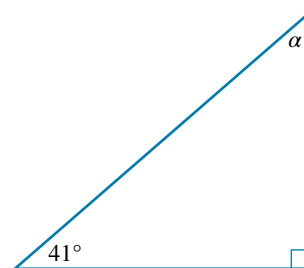


- A. $\sin(\alpha) = \frac{b}{c}$ B. $\sin(\alpha) = \frac{a}{c}$ C. $\cos(\alpha) = \frac{a}{c}$ D. $\tan(\alpha) = \frac{b}{a}$

Communicating, reasoning and problem solving

8. Consider the right-angled triangle shown.

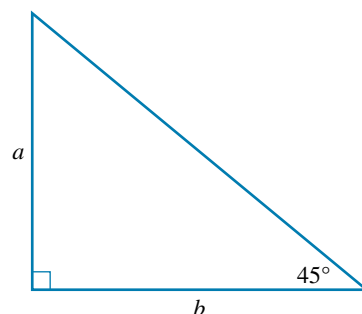
- Label each of the sides using the letters O, A, H with respect to the 41° angle.
- Measure the side lengths (to the nearest millimetre).
- Determine the value of each trigonometric ratio. (Where applicable, answers should be given correct to 2 decimal places.)
 - $\sin(41^\circ)$
 - $\cos(41^\circ)$
 - $\tan(41^\circ)$
- What is the value of the unknown angle, α ?



- Determine the value of each of these trigonometric ratios, correct to 2 decimal places.
 - $\sin(\alpha)$
 - $\cos(\alpha)$
 - $\tan(\alpha)$
 (Hint: First re-label the sides of the triangle with respect to angle α .)
- What do you notice about the relationship between $\sin(41^\circ)$ and $\cos(\alpha)$?
- What do you notice about the relationship between $\sin(\alpha)$ and $\cos(41^\circ)$?
- Make a general statement about the two angles.

9. Given the triangle shown:

- why does $a = b$?
- what would the value of $\tan(45^\circ)$ be?



- If a right-angled triangle has side lengths m , $(m + n)$ and $(m - n)$, which one of the lengths is the hypotenuse? Explain your reasoning.
- A ladder leans on a wall as shown. Use the information from the diagram to answer the following questions. In relation to the angle given, what part of the image represents:
 - the adjacent side
 - the hypotenuse
 - the opposite side?
- Use sketches of right-angled triangles to investigate the following.
 - As the acute angle increases in size, what happens to the ratio of the length of the opposite side to the length of the hypotenuse in any right-angled triangle?



- b. As the acute angle increases in size, what happens to the other two ratios (i.e. the ratio of the length of the adjacent side to the length of the hypotenuse and that of the opposite side to the adjacent)?
- c. What is the largest possible value for:
- i. $\sin(\theta)$ ii. $\cos(\theta)$ iii. $\tan(\theta)$?

13. Why does $\sin(30^\circ) = \cos(60^\circ)$?

4.5 Calculating unknown side lengths

[Stage 5.1]

4.5.1 Values of trigonometric ratios

- The values of trigonometric ratios can be found using a calculator.
- Each calculator has several modes. For the following calculations, your calculator must be in degree mode.

WORKED EXAMPLE 12

Evaluate each of the following, giving answers correct to 4 decimal places.

a $\sin(53^\circ)$

b $\cos(31^\circ)$

c $\tan(79^\circ)$

THINK

- a 1 Set the calculator to degree mode. Write the first 5 decimal places.
2 Round correct to 4 decimal places.
- b 1 Write the first 5 decimal places.
2 Round correct to 4 decimal places.
- c 1 Write the first 5 decimal places.
2 Round correct to 4 decimal places.

WRITE

a $\sin(53^\circ) = 0.79863$
 ≈ 0.7986

b $\cos(31^\circ) = 0.85716$
 ≈ 0.8572

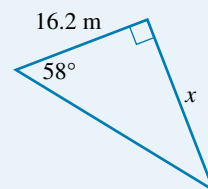
c $\tan(79^\circ) = 5.14455$
 ≈ 5.1446

4.5.2 Finding side lengths

- If a reference angle and any side length of a right-angled triangle are known, it is possible to find the other sides using trigonometry.

WORKED EXAMPLE 13

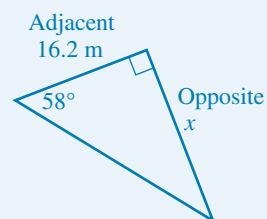
Use the appropriate trigonometric ratio to find the length of the unknown side in the triangle shown. Give your answer correct to 2 decimal places.



THINK

- 1 Label the given sides.

WRITE/DRAW



- 2 These sides are used in TOA. Write the ratio.

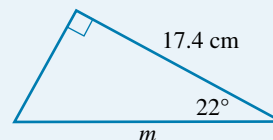
$$\tan(\theta) = \frac{O}{A}$$

- 3 Substitute the values of θ , O and A into the tangent ratio.
- 4 Solve the equation for x .
- 5 Calculate the value of x to 3 decimal places, then round the answer to 2 decimal places.

$$\begin{aligned}\tan(58^\circ) &= \frac{x}{16.2} \\ 16.2 \times \tan(58^\circ) &= x \\ x &= 16.2 \tan(58^\circ) \\ x &= 25.925 \\ x &= 25.93 \text{ m}\end{aligned}$$

WORKED EXAMPLE 14

Find the length of the side marked m in the triangle shown. Give your answer correct to 2 decimal places.



THINK

- 1 Label the given sides.

- 2 These sides are used in CAH. Write the ratio.

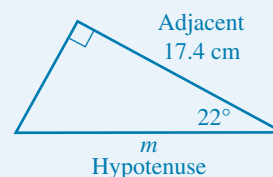
- 3 Substitute the values of θ , A and H into the cosine ratio.

- 4 Solve for m :

- Multiply both sides by m .
- Divide both sides by $\cos(22^\circ)$.

- 5 Calculate the value of m to 3 decimal places, then round the answer to 2 decimal places.

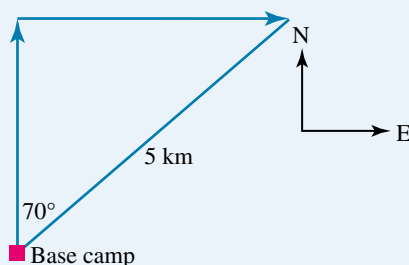
WRITE/DRAW



$$\begin{aligned}\cos(\theta) &= \frac{A}{H} \\ \cos(22^\circ) &= \frac{17.4}{m} \\ m \cos(22^\circ) &= 17.4 \\ m &= \frac{17.4}{\cos(22^\circ)} \\ m &= 18.766 \\ m &= 18.77 \text{ cm}\end{aligned}$$

WORKED EXAMPLE 15

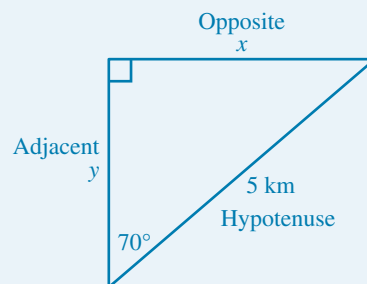
Benjamin set out on a bushwalking expedition. Using a compass, he set off on a course N 70° E (or 070° T) and travelled a distance of 5 km from his base camp.



- a How far east has he travelled?
 - b How far north has he travelled from the base camp?
- Give answers correct to 2 decimal places.

THINK

- a 1** Label the easterly distance x .
 Label the northerly distance y .
 Label the sides of the triangle: Hypotenuse, Opposite, Adjacent.
- 2** To calculate the value of x , use the sides of the triangle:
 $x = O$, $5 = H$.
 These are used in SOH. Write the ratio.
- 3** Substitute the values of the angle and the pronumerals into the sine ratio.
- 4** Make x the subject of the equation.
- 5** Evaluate x to 3 decimal places, using a calculator.
- 6** Round to 2 decimal places.
- 7** Answer the question in sentence form.
- b 1** To calculate the value of y , use the sides:
 $y = A$, $5 = H$.
 These are used in CAH. Write the ratio.
- 2** Substitute the values of the angle and the pronumerals into the cosine ratio.
- 3** Make y the subject of the equation.
- 4** Evaluate y using a calculator.
- 5** Round the answer to 2 decimal places.
- 6** Answer the question in sentence form.

WRITE/DRAW**a**

$$\sin(\theta) = \frac{O}{H}$$

$$\sin(70^\circ) = \frac{x}{5}$$

$$x = 5 \sin(70^\circ)$$

$$= 4.698$$

$$= 4.70 \text{ km}$$

Benjamin has travelled 4.70 km east of the base camp.

b

$$\cos(\theta) = \frac{A}{H}$$

$$\cos(70^\circ) = \frac{y}{5}$$

$$y = 5 \cos(70^\circ)$$

$$= 1.710$$

$$= 1.71 \text{ km}$$

Benjamin has travelled 1.71 km north of the base camp.

learnon

RESOURCES — ONLINE ONLY

eLesson: Using an inclinometer (eles-0116)

Digital doc: SkillsHEET Solving equations of the type $a = \frac{x}{b}$ to find x (doc-10832)

Digital doc: SkillsHEET Solving equations of the type $a = \frac{b}{x}$ to find x (doc-10833)

Digital doc: SkillsHEET Rearranging formulas (doc-10834)

Digital doc: WorkSHEET Trigonometry (doc-10835)

Exercise 4.5 Calculating unknown side lengths

Individual pathways

PRACTISE

Questions:

1–3, 4a–c, 5a–c, 6a–f, 7–9, 11, 12

CONSOLIDATE

Questions:

1–3, 4b–d, 5b–d, 6d–i, 7–10, 12–15

MASTER

Questions:

1–3, 4d–f, 5d–f, 6g–i, 7–10, 11, 13–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE12** a. Evaluate the following correct to 4 decimal places.

i. $\sin(55^\circ)$

ii. $\sin(11.6^\circ)$

- b. Copy and complete the table below. Use your calculator to find each value of $\sin(\theta)$ correct to 2 decimal places.

θ	0°	15°	30°	45°	60°	75°	90°
$\sin(\theta)$							

- c. Summarise the trend in these values.

2. a. Evaluate the following correct to 4 decimal places.

i. $\cos(38^\circ)$

ii. $\cos(53.71^\circ)$

- b. Copy and complete the table below. Use your calculator to find each value of $\cos(\theta)$ correct to 2 decimal places.

θ	0°	15°	30°	45°	60°	75°	90°
$\cos(\theta)$							

3. a. Evaluate the following correct to 4 decimal places.

i. $\tan(18^\circ)$

ii. $\tan(51.9^\circ)$

- b. Copy and complete the table below. Use your calculator to find each value of $\tan(\theta)$ correct to 2 decimal places.

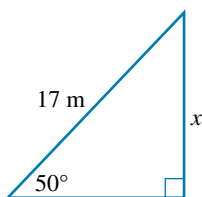
θ	0°	15°	30°	45°	60°	75°	90°
$\tan(\theta)$							

- c. Find the value of $\tan(89^\circ)$ and $\tan(89.9^\circ)$.

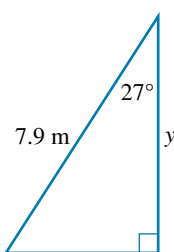
- d. What do you notice about these results?

4. **WE13** Use the appropriate trigonometric ratios to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.

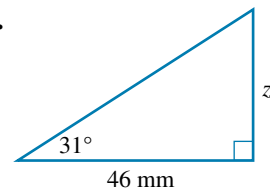
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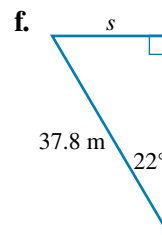
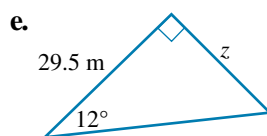
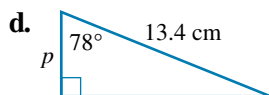


b.

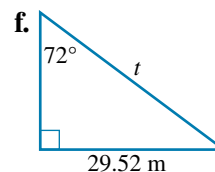
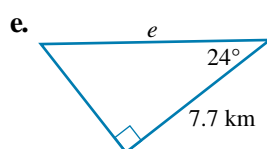
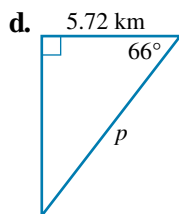
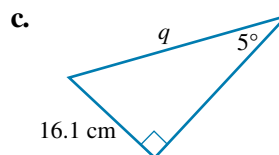
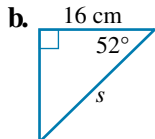
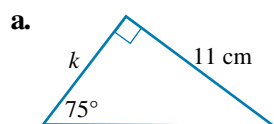


c.

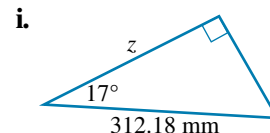
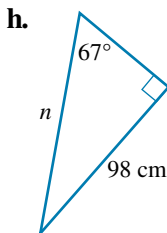
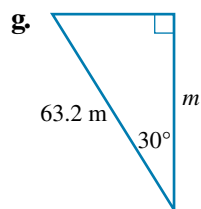
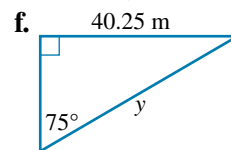
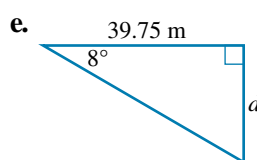
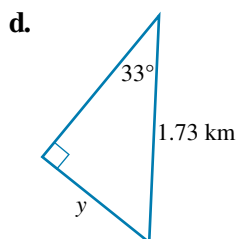
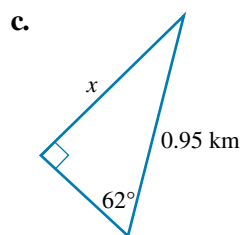
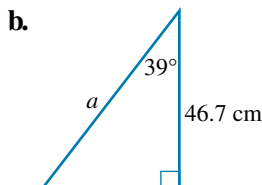
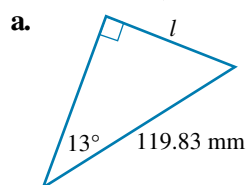




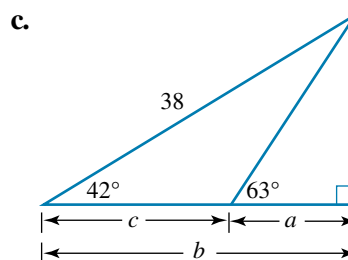
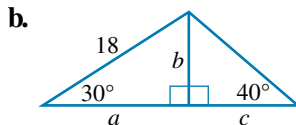
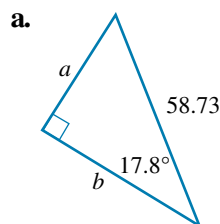
5. **WE14** Use the appropriate trigonometric ratio to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.



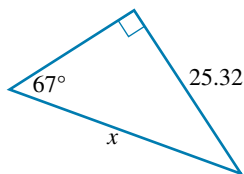
6. Find the length of the unknown side in each of the following triangles, correct to 2 decimal places. (Note: In some cases the unknown will be in the numerator and in other cases it will be in the denominator.)



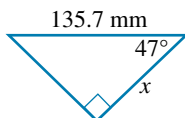
7. Find the lengths of the unknown sides in the triangles shown, correct to 2 decimal places.



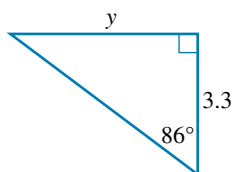
8. **MC** a. The value of x correct to 2 decimal places is:



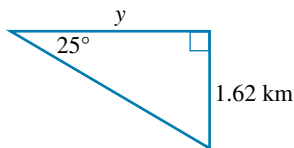
- A.** 59.65 **B.** 23.31 **C.** 64.80 **D.** 27.51
- b. The value of x correct to 2 decimal places is:



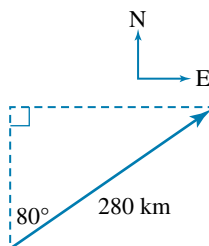
- A.** 99.24 mm **B.** 92.55 mm **C.** 185.55 mm **D.** 198.97 mm
- c. The value of y correct to 2 decimal places is:



- A.** 47.19 **B.** 7.94 **C.** 1.37 **D.** 0.23
- d. The value of y correct to 2 decimal places is:



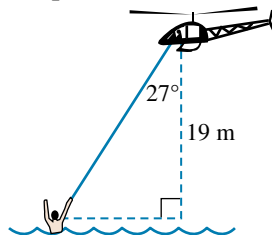
- A.** 0.76 km **B.** 1.79 km **C.** 3.83 km **D.** 3.47 km
9. **WE15** A ship that was to travel due north veered off course and travelled $N80^\circ E$ (or $080^\circ T$) for a distance of 280 km, as shown in the diagram.



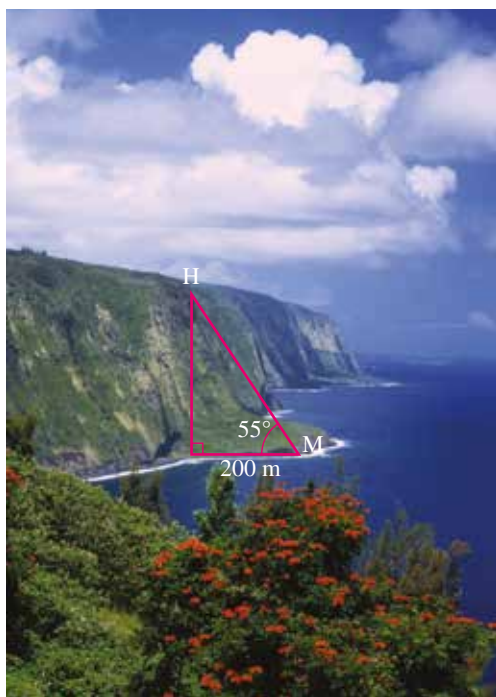
- a. How far east had the ship travelled?
- b. How far north had the ship travelled?



10. A rescue helicopter spots a missing surfer drifting out to sea on his damaged board. The helicopter descends vertically to a height of 19 m above sea level and drops down an emergency rope, which the surfer grips. Due to the wind the rope swings at an angle of 27° to the vertical, as shown in the diagram. What is the length of the rope?

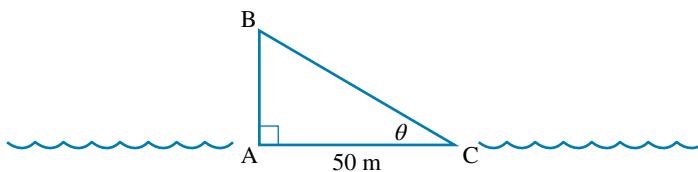


11. Walking along the coastline, Michelle (M) looks up through an angle of 55° and sees her friend Helen (H) on top of the cliff at the lookout point. How high is the cliff if Michelle is 200 m from its base? (Assume both girls are the same height.)



Communicating, reasoning and problem solving

12. One method for determining the distance across a body of water is shown in the diagram below.



The required distance is AB. A surveyor moves at right angles 50 m to point C and uses a tool called a transit to measure the angle θ ($\angle ACB$).

- If $\theta = 12.3^\circ$, show that the length AB is 10.90 m.
 - Show that a value of $\theta = 63.44^\circ$ gives a length of $AB = 100$ m.
 - Find a rule that can be used to calculate the length AC.
13. Using a diagram, explain why $\sin(70^\circ) = \cos(20^\circ)$ and $\cos(70^\circ) = \sin(20^\circ)$. In general, $\sin(\theta)$ will be equal to which cosine?

14. Calculate the value of the pronumeral in each of the following triangles.

a.



b.



c.

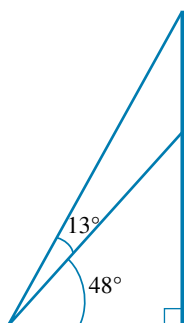
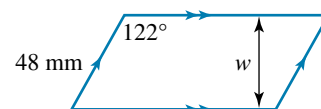


15. A tile is in the shape of a parallelogram with measurements as shown. Calculate the width of the tile, w , to the nearest mm.

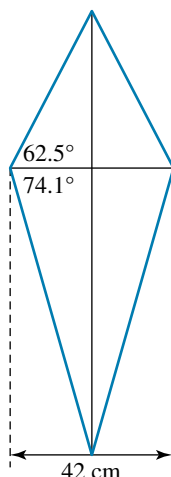
16. A pole is supported by two wires as shown. If the length of the lower wire is 4.3 m, calculate to 1 decimal place:

a. the length of the top wire

b. the height of the pole.



17. The frame of a kite is built from 6 wooden rods as shown. Calculate the total length of wood used to make the frame of the kite to the nearest metre.



18. What does $\sin(60^\circ)$ actually mean?
19. A track in a national park has four legs from its starting point to its return. The first leg travels due east. The second, third and fourth legs follow paths southwest, due north and finally due east back to the starting point. If the distances of the first two legs are 1.5 km and 3.5 km respectively, what distances are the third and fourth legs?

4.6 Calculating unknown angles

[Stage 5.1]

4.6.1 What is trigonometry?

- We have seen that $\sin(30^\circ) = 0.5$; therefore we say, 30° is the inverse sine of 0.5. This is written as $\sin^{-1}(0.5) = 30^\circ$.
- The expression $\sin^{-1}(x)$ is read as ‘the inverse sine of x ’.
- The expression $\cos^{-1}(x)$ is read as ‘the inverse cosine of x ’.
- The expression $\tan^{-1}(x)$ is read as ‘the inverse tangent of x ’.

ACTIVITY: INVERSE SINE, COSINE AND TANGENT

- Use your calculator to find $\sin(30^\circ)$, then find the inverse sine of the answer. Choose another angle and do the same thing.
- Now find $\cos(30^\circ)$ and then find the inverse cosine($\cos^{-1}$) of the answer. Choose another angle and do the same thing.
- Lastly, find $\tan(30^\circ)$ and then find the inverse tangent($\tan^{-1}$) of the answer. Try this with other angles.

- The fact that $\sin$ and $\sin^{-1}$ cancel each other out is useful in solving equations such as:

$$\begin{aligned}\sin(\theta) &= 0.3 \text{ (Take the inverse sine of both sides.)} \\ \sin^{-1}(\sin(\theta)) &= \sin^{-1}(0.3) \\ \theta &= \sin^{-1}(0.3)\end{aligned}$$

$$\begin{aligned}\sin^{-1}(x) &= 15^\circ \text{ (Take the sine of both sides)} \\ \sin(\sin^{-1}(x)) &= \sin(15^\circ) \\ x &= \sin(15^\circ)\end{aligned}$$

Similarly, $\cos(\theta) = 0.522$ means that

$$\theta = \cos^{-1}(0.522)$$

and $\tan(\theta) = 1.25$ means that

$$\theta = \tan^{-1}(1.25).$$

WORKED EXAMPLE 16

Evaluate $\cos^{-1}(0.3678)$, correct to the nearest degree.

THINK

- 1 Set your calculator to degree mode.
- 2 Round the answer to the nearest whole number and include the degree symbol.

WRITE

$$\begin{aligned}\cos^{-1}(0.3678) &= 68.4 \\ &\approx 68^\circ\end{aligned}$$

WORKED EXAMPLE 17

Determine the size of angle θ in each of the following. Give answers correct to the nearest degree.

a $\sin(\theta) = 0.6543$ b $\tan(\theta) = 1.745$

THINK

- a
- 1 θ is the inverse sine of 0.6543.
 - 2 Calculate and record the answer.
 - 3 Round the answer to the nearest degree.
- b
- 1 θ is the inverse tangent of 1.745.
 - 2 Use the inverse tangent function on a calculator.
Record the number shown.
 - 3 Round the answer to the nearest degree.

WRITE

a

$$\begin{aligned}\sin(\theta) &= 0.6543 \\ \theta &= \sin^{-1}(0.6543) \\ &= 40.8 \\ &\approx 41^\circ\end{aligned}$$

b

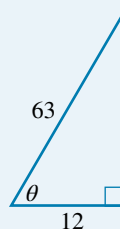
$$\begin{aligned}\tan(\theta) &= 1.745 \\ \theta &= \tan^{-1}(1.745) \\ &= 60.18 \\ &\approx 60^\circ\end{aligned}$$

4.6.2 Finding the angle when 2 sides are known

- If the lengths of any 2 sides of a right-angled triangle are known, it is possible to find an angle using inverse sine, inverse cosine or inverse tangent.

WORKED EXAMPLE 18

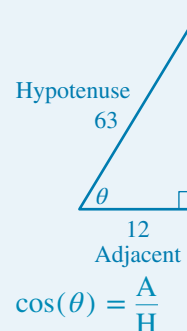
Determine the value of θ in the triangle shown. Give your answer correct to the nearest degree.



THINK

- 1 Label the given sides. These are used in CAH.
Write the ratio.

WRITE/DRAW



2 Substitute the given values into the cosine ratio.

$$\cos(\theta) = \frac{12}{63}$$

3 θ is the inverse cosine of $\frac{12}{63}$.

$$\theta = \cos^{-1}\left(\frac{12}{63}\right)$$

4 Evaluate.

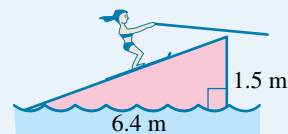
$$= 79.0$$

5 Round the answer to the nearest degree.

$$\approx 79^\circ$$

WORKED EXAMPLE 19

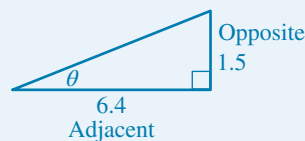
Roberta enjoys water skiing and is about to try a new ramp on the Hawkesbury River. The inclined ramp rises 1.5 m above the water level and spans a horizontal distance of 6.4 m. What is the magnitude (size) of the angle that the ramp makes with the water? Give the answer correct to the nearest degree.



THINK

1 Draw a simple diagram, showing the known lengths and the angle to be found.

WRITE/DRAW



2 Label the given sides. These are used in TOA.
Write the ratio.

$$\tan(\theta) = \frac{O}{A}$$

3 Substitute the values of the pronumerals into the tangent ratio.

$$\tan(\theta) = \frac{1.5}{6.4}$$

4 θ is the inverse tangent of $\frac{1.5}{6.4}$.

$$\theta = \tan^{-1}\left(\frac{1.5}{6.4}\right)$$

5 Evaluate.

$$= 13.19$$

6 Round the answer to the nearest degree.

$$\approx 13^\circ$$

7 Write the answer in words.

The ramp makes an angle of 13° with the water.

learnon RESOURCES — ONLINE ONLY

Interactivity: Finding the angle when two sides are known (int-6046)

Digital doc: SkillsHEET Rounding angles to the nearest degree (doc-10836)

Exercise 4.6 Calculating unknown angles

Individual pathways

PRACTISE

Questions:
1a–c, 2, 3a–f, 4–10

CONSOLIDATE

Questions:
1d–f, 2, 3d–h, 4–11

MASTER

Questions:
1g–i, 2, 3e–i, 4–14

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE16** Evaluate each of the following, correct to the nearest degree.

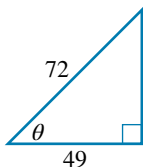
a. $\sin^{-1}(0.6294)$	b. $\cos^{-1}(0.3110)$	c. $\tan^{-1}(0.7409)$
d. $\tan^{-1}(1.3061)$	e. $\sin^{-1}(0.9357)$	f. $\cos^{-1}(0.3275)$
g. $\cos^{-1}(0.1928)$	h. $\tan^{-1}(4.1966)$	i. $\sin^{-1}(0.2554)$

2. **WE17** Determine the size of the angle in each of the following. Give answers correct to the nearest degree.

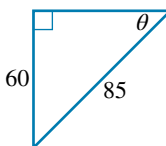
a. $\sin(\theta) = 0.3214$	b. $\sin(\theta) = 0.6752$	c. $\sin(\beta) = 0.8235$	d. $\cos(\beta) = 0.9351$
e. $\cos(\alpha) = 0.6529$	f. $\cos(\alpha) = 0.1722$	g. $\tan(\theta) = 0.7065$	h. $\tan(\alpha) = 1$
i. $\tan(b) = 0.876$	j. $\sin(c) = 0.3936$	k. $\cos(\theta) = 0.5241$	l. $\tan(\alpha) = 5.6214$

3. **WE18** Determine the value of θ in each of the following triangles. Give answers correct to the nearest degree.

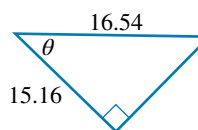
a.



b.



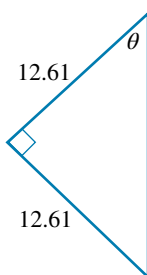
c.



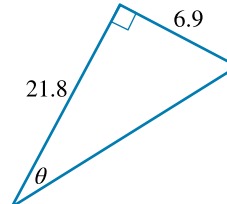
d.



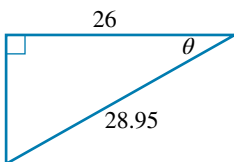
e.



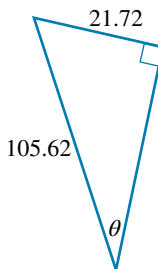
f.



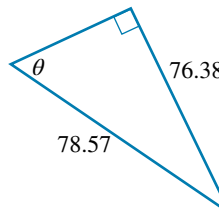
g.



h.



i.



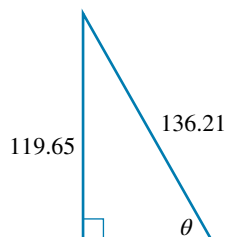
4. **MC** a. If $\cos(\theta) = 0.8752$, the value of θ correct to 2 decimal places is:

A. 61.07°	B. 41.19°	C. 25.84°	D. 28.93°

- b. If $\sin(\theta) = 0.5530$, the value of θ correct to 2 decimal places is:

A. 56.43°	B. 33.57°	C. 28.94°	D. 36.87°

c. The value of θ in the triangle shown, correct to 2 decimal places, is:



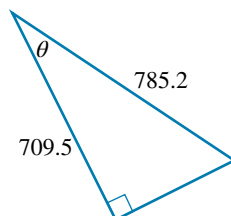
A. 41.30°

B. 28.55°

C. 48.70°

D. 61.45°

d. The value of θ in the triangle shown, correct to 2 decimal places, is:



A. 42.10°

B. 64.63°

C. 25.37°

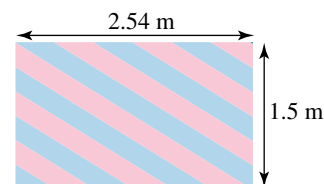
D. 47.90°

5. a. Copy and fill in the table below.

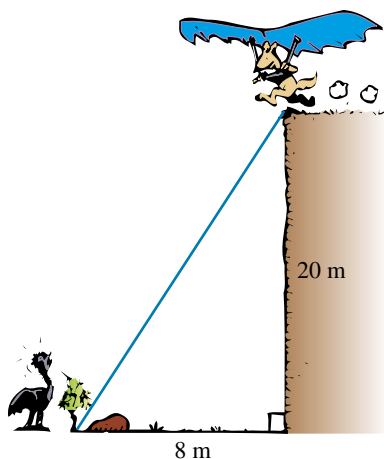
x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y = \cos^{-1}(x)$	90°					60°					0°

b. Plot the data from the table on graph paper or with a spreadsheet or suitable calculator.

6. A piece of fabric measuring 2.54 m by 1.5 m has a design consisting of parallel diagonal stripes. What angle does each diagonal make with the length of the fabric? Give your answer correct to 2 decimal places.



7. **WE19** Danny Dingo is perched on top of a cliff 20 m high watching an emu feeding 8 m from the base of the cliff. Danny has purchased a flying contraption, which he hopes will help him capture the emu. At what angle to the cliff must he swoop to catch his prey? Give your answer correct to 2 decimal places.

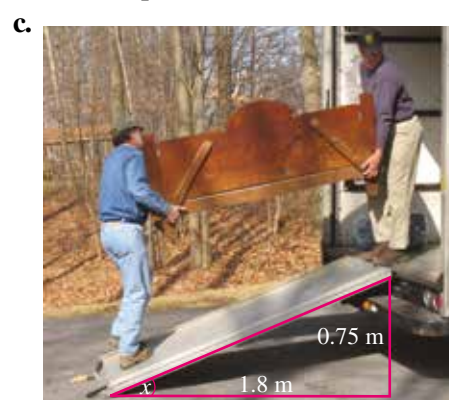
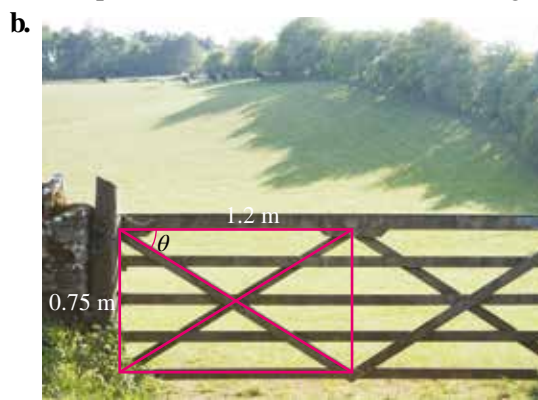


Communicating, reasoning and problem solving

8. Jenny and Les are camping with friends Mark and Susie. Both couples have a 2-m-high tent. The top of each 2-m tent pole is to be tied with a piece of rope that will be used to keep the pole upright.

So that the rope doesn't trip a passerby, Jenny and Les decide that the angle between the rope and the ground should be 60° . Answer the following questions, correct to 2 decimal places.

- a. Find the length of the rope needed from the top of the tent pole to the ground to support their tent pole.
 - b. Further down the camping ground, Mark and Susie also set up their tent. However, they want to use a piece of rope that they know is in the range of 2 to 3 m in length.
 - i. Explain why the rope will have to be greater than 2 m in length.
 - ii. Show that the minimum angle the rope will make with the ground will be 41.8° .
9. Safety guidelines for wheelchair access ramps used to state that the gradient had to be in the ratio 1 : 20.
- a. Using this ratio, show that the angle that the ramp had to make with the horizontal is closest to 3° .
 - b. New regulations have changed the ratio of the gradient, so the angle the ramp must make with the horizontal is now closest to 6° . Explain why, using this angle size, the new ratio could be 1 to 9.5.
10. Calculate the value of the pronumeral in each of the following to 2 decimal places.



11. A family is building a patio extension to their house. One section of the patio will have a gable roof. A similar structure is pictured with the planned post heights and span shown. To allow more light in, the family wants the peak (highest point) of the gable roof to be at least 5 m above deck level. According to building regulations, the slope of the roof (i.e. the angle that the sloping edge makes with the horizontal) must be 22° .

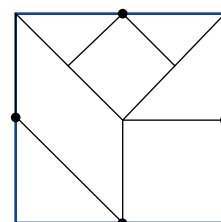


- a. Use trigonometry to calculate whether the roof would be high enough if the angle was 22° .
- b. Use trigonometry to calculate the size of the obtuse angle formed at the peak of the roof.

12. Use the formulas $\sin(\theta) = \frac{o}{h}$ and $\cos(\theta) = \frac{a}{h}$ to prove that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$.

13. Why does $\cos(0^\circ) = 1$?

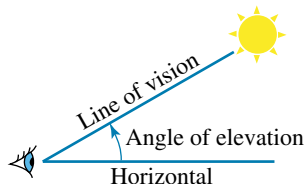
14. a. A square-based prism has a height twice its base length. What angle does the diagonal of the prism make with the diagonal of the base?
- b. Seven smaller shapes are created inside a square with side length 10 cm as shown in the diagram. If the dots represent the midpoints of the square's sides, find the dimensions of each shape.



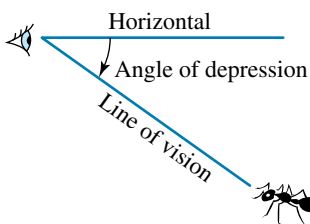
4.7 Angles of elevation and depression [Stage 5.1]

4.7.1 Angles of elevation and depression

- When looking up towards an object, an **angle of elevation** is the angle between the horizontal line and the line of vision.



- When looking down at an object, an **angle of depression** is the angle between the horizontal line and the line of vision.



- Angles of elevation and depression are measured from horizontal lines.

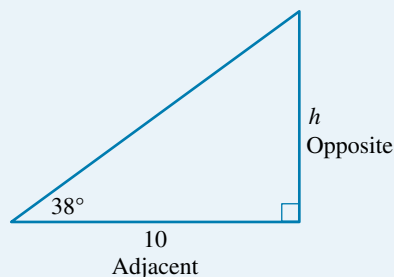
WORKED EXAMPLE 20

At a point 10 m from the base of a tree, the angle of elevation of the treetop is 38° . How tall is the tree to the nearest centimetre?

THINK

- 1 Draw a simple diagram. The angle of elevation is 38° from the horizontal.

WRITE/DRAW



- 2 Label the given sides of the triangle. These sides are used in TOA. Write the ratio.
- 3 Multiply both sides by 10.
- 4 Calculate h correct to 3 decimal places.
- 5 Round to 2 decimal places.
- 6 Write the answer in words.

$$\tan(38^\circ) = \frac{h}{10}$$

$$10 \tan(38^\circ) = h$$

$$h = 7.812$$

$$\approx 7.81$$

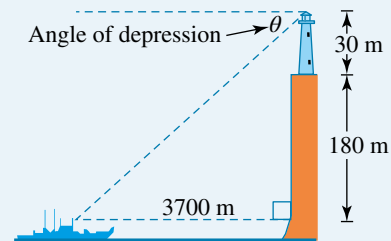
The tree is 7.81 m tall.

DISCUSSION

What are the benefits of drawing a simple diagram to help you answer this question?

WORKED EXAMPLE 21

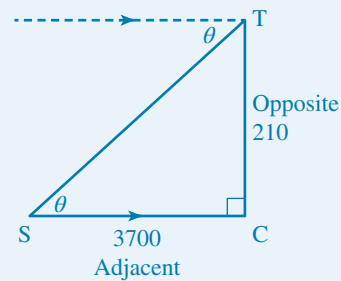
A lighthouse, 30 m tall, is built on top of a cliff that is 180 m high. Find the angle of depression (θ) of a ship from the top of the lighthouse if the ship is 3700 m from the bottom of the cliff.



THINK

- 1 Draw a simple diagram to represent the situation.
The height of the triangle is $180 + 30 = 210$ m.
Draw a horizontal line from the top of the triangle and mark the angle of depression, θ .
Also mark the alternate angle.
- 2 Label the triangle. These sides are used in TOA. Write the ratio.
- 3 Substitute the given values into the ratio.
- 4 θ is the inverse tangent of $\frac{210}{3700}$.
- 5 Evaluate.
- 6 Round the answer to the nearest degree.
- 7 Write the answer in words.

WRITE/DRAW



$$\tan(\theta) = \frac{O}{A}$$

$$\tan(\theta) = \frac{210}{3700}$$

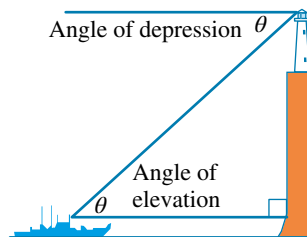
$$\theta = \tan^{-1}\left(\frac{210}{3700}\right)$$

$$= 3.2$$

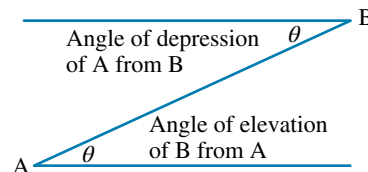
$$\approx 3^\circ$$

The angle of depression of the ship from the top of the lighthouse is 3° .

- *Note:* In Worked example 21, the angle of depression from the top of the lighthouse to the ship is equal to the angle of elevation from the ship to the top of the lighthouse. This is because the angle of depression and the angle of elevation are alternate (or 'Z') angles.



- This can be generalised as follows:
- **For any two objects, A and B, the angle of elevation of B, as seen from A, is equal to the angle of depression of A, as seen from B.**



Interactivity: Angles of elevation and depression (int-6047)

Digital doc: SkillSHEET Drawing a diagram from given directions (doc-10837)

Digital doc: WorkSHEET Trigonometry using elevation and depression (doc-10838)

Exercise 4.7 Angles of elevation and depression

Individual pathways

PRACTISE

Questions:
1–6, 8, 10, 12

CONSOLIDATE

Questions:
1–6, 8, 11–14

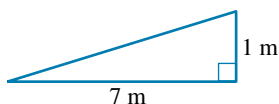
MASTER

Questions:
1–3, 6, 7, 9–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

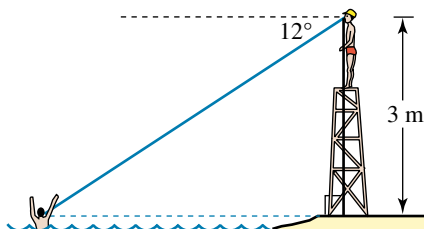
Understanding and fluency

- WE20** Building specifications require the angle of elevation of any ramp constructed for public use to be less than 3° .

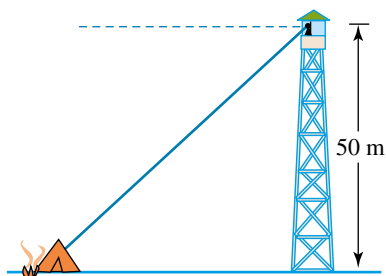


Ramps being constructed at a new shopping centre are each made in the ratio 7 m horizontal length to 1 m vertical height. Find the angle of elevation of these ramps and, hence, decide whether they meet building specifications.

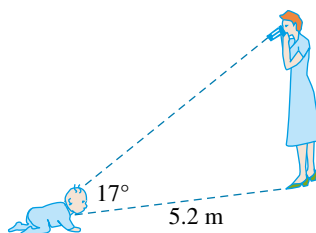
- A lifesaver standing on his tower 3 m above the ground spots a swimmer experiencing difficulty. The angle of depression of the swimmer from the lifesaver is 12° . How far is the swimmer from the lifesaver's tower? (Give your answer correct to 2 decimal places.)



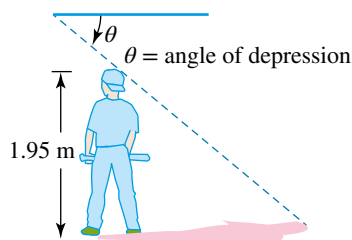
3. From the top of a lookout 50 m above the ground, the angle of depression of a camp site that is level with the base of the lookout is 37° . How far is the camp site from the base of the lookout?



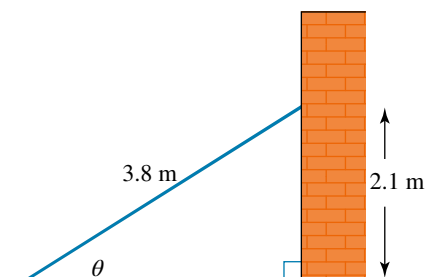
4. From a rescue helicopter 80 m above the ocean, the angles of depression of two shipwreck survivors are 40° and 60° respectively. If the two sailors and the helicopter are in line with each other:
- draw a labelled diagram to represent the situation
 - calculate the distance between the two sailors, to the nearest metre.
5. The angle of elevation of the top of a tree from a point on the ground, 60 m from the tree, is 35° .
- Draw a labelled diagram to represent the situation.
 - Find the height of the tree to the nearest metre.
6. Miriam, an avid camerawoman from Perth, wants to record her daughter Alexandra's first attempts at crawling. As Alexandra lies on the floor and looks up at her mother, the angle of elevation is 17° . If Alexandra is 5.2 m away from her mother, how tall is Miriam? Give your answer correct to 1 decimal place.



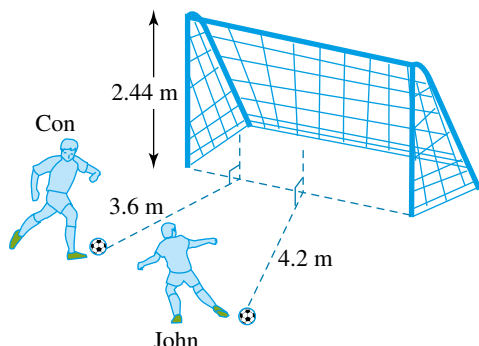
7. **WE21** Stan, who is 1.95 m tall, measures the length of the shadow he casts along the ground as 0.98 m. Find the angle of depression of the sun's rays to the nearest degree.



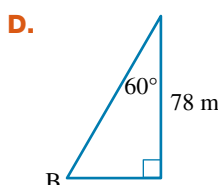
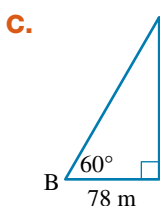
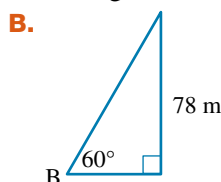
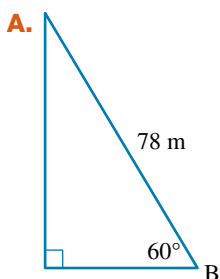
8. What angle does a 3.8-m ladder make with the ground if it reaches 2.1 m up the wall? How far is the foot of the ladder from the wall? (Give your answers to the nearest degree and the nearest metre.)



9. Con and John are practising shots on goal. Con is 3.6 m away from the goal and John is 4.2 m away, as shown in the diagram. If the height of the goal post is 2.44 m, what is the maximum angle of elevation, to the nearest degree, that each can kick the ball in order to score a goal?

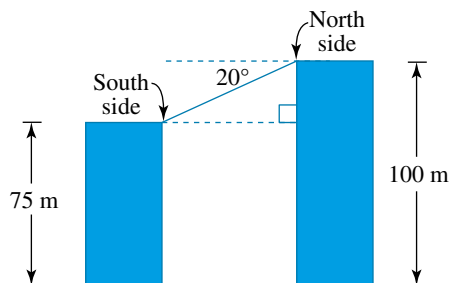


10. **MC** The angle of elevation of the top of a lighthouse tower 78 m tall, from a point B on the same level as the base of the tower, is 60° . The correct diagram for this information is:



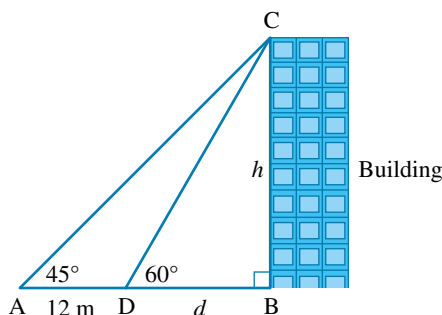
Communicating, reasoning and problem solving

11. Lifesaver Sami spots some dolphins playing near a marker at sea directly in front of him. He is sitting in a tower that is situated 10 m from the water's edge and he is 4 m above sea level. The marker is 20 m from the water's edge.
- Draw a diagram to represent this information.
 - Show that the angle of depression of Sami's view of the dolphins, correct to 1 decimal place, is 7.6° .
 - As the dolphins swim towards Sami, would the angle of depression increase or decrease? Justify your answer in terms of the tangent ratios.
12. Two buildings are 100 m and 75 m high. From the top of the north side of the taller building, the angle of depression to the top of the south side of the smaller building is 20° , as shown at right. Show that the horizontal distance between the north side of the taller building and the south side of the smaller building is closest to 69 m.
13. Rouka was hiking in the mountains when she spotted an eagle sitting up in a tree. The angle of elevation of her view of the

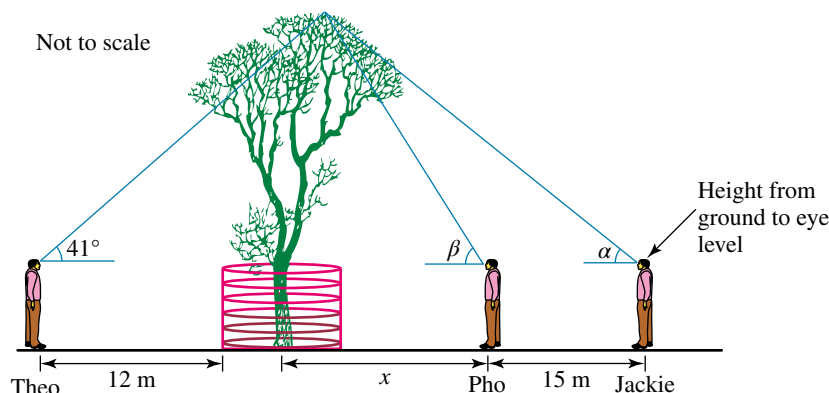


eagle was 35° . She then walked 20 metres towards the tree and her angle of elevation was 50° . The height of the eagle from the ground was 35.5 metres.

- Draw a labelled diagram to represent this information.
 - Determine how tall Rouka is if her eyes are 9 cm from the top of her head. Write your answer in metres, correct to the nearest centimetre.
- 14.** A lookout in a lighthouse tower can see two ships approaching the coast. Their angles of depression are 25° and 30° . If the ships are 100 m apart, show that the height of the lighthouse, to the nearest metre, is 242 m.
- 15.** At a certain distance away, the angle of elevation to the top of a building is 60° . From 12 m further back, the angle of elevation is 45° as shown in the diagram below. Show that the height of the building is 28.4 metres.



- 16.** A tall gum tree stands in a courtyard in the middle of some office buildings. Three Year 9 students, Jackie, Pho and Theo, measure the angle of elevation from three different positions. They are unable to measure the distance to the base of the tree because of the steel tree guard around the base. The diagram below shows the angles of elevation and the distances measured.



- Show that $x = \frac{15 \tan \alpha}{\tan \beta - \tan \alpha}$, where x is the distance, in metres, from the base of the tree to Pho's position.
 - The girls estimate the tree to be 15 m taller than them. Pho measured the angle of elevation to be 72° . What should Jackie have measured her angle of elevation to be, if these measurements are assumed to be correct? Write your answer to the nearest degree.
 - Theo did some calculations and determined that the tree was only about 10.4 m taller than them. Jackie claims that Theo's calculation of 10.4 m is incorrect.
 - Is Jackie's claim correct? Show how Theo calculated a height of 10.4 m.
 - If the height of the tree was actually 15 metres above the height of the students, determine the horizontal distance Theo should have used in his calculations. Write your answer to the nearest centimetre.
- 17.** Why does the angle of elevation have the same value as the angle of depression?

4.8 Review

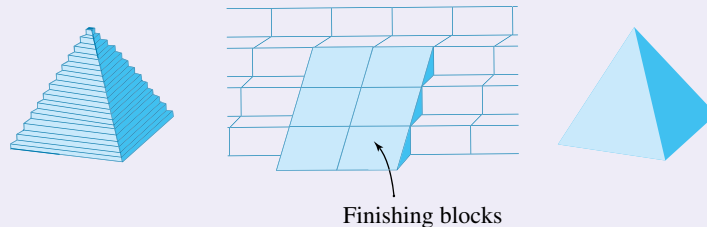
Investigation | Rich task

The Great Pyramid of Giza



The Great Pyramid of Giza was built over four and a half thousand years ago. It was constructed using approximately 2 300 000 rectangular granite blocks and took over 20 years to complete. When built, its dimensions measured 230 m at the base and its vertical height was 146.5 m.

1. Each side of the pyramid has a triangular face. Use the dimensions given and Pythagoras' theorem to calculate the height of each triangle. Give your answer correct to 2 decimal places.
2. Special finishing blocks were added to the ends of each row of the pyramid to give each triangular face a smooth and flat finish. Calculate the area of each face of the pyramid.



Finishing blocks

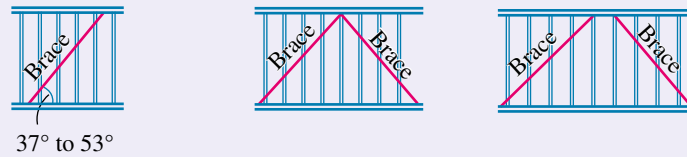
3. The edge of the pyramid joins two faces from the ground to the tip of the pyramid. Use Pythagoras' theorem to calculate the length of the edge.

Wall braces

In the building industry, wall frames are strengthened with the use of braces. These braces run between the top and bottom horizontal sections of the frame.



Industry standards stipulate that the acute angle the brace makes with the horizontal sections lies in the range 37° to 53° . Sometimes, more than one brace may be required if the frame is a long one.



1. Cut thin strips of cardboard and arrange them in the shape of a rectangle to represent a rectangular frame. Pin the corners to hold them together. Notice that the frame moves out of shape easily. Attach a brace according to the angle stipulation of the building industry. Write a brief comment to describe what effect the brace had on the frame.
2. Investigate what happens to the length of the brace required as the acute angle with the base increases from 37° to 53° .
3. Use your finding from question 2 to state which angle requires the shortest brace and which angle requires the longest brace.

Most contemporary houses are constructed with a ceiling height of 2.4 metres. (This is the height of the walls from the floor to the ceiling.) Use this fact to assist in your calculations for the following questions.

4. Assume you have a section of a wall that is 3.5 metres long. What would be the length of the longest brace possible? Draw a diagram and show your working to support your answer in the space below.
5. What would be the minimum wall length in which two braces were required? Show your working, along with a diagram, in the space provided.
6. Some older houses have ceilings over 2.4 metres. Repeat questions 4 and 5 for a frame with a height of 3 metres. Draw diagrams and show your workings to support your answers in the space below.
7. Take the measurements of a wall without windows in your school or at home. Draw a scale drawing of the frame on a separate sheet of paper and show the positions in which a brace or braces might lie. Calculate the length and angle of each brace.

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Digital doc: Investigation — The Great Pyramid of Giza (doc-15896)

Digital doc: Code puzzle — What does it mean? (doc-15897)

Digital doc: Summary Topic 4 (doc-22886)

Interactivity: Crossword Topic 4 (int-0703)

Interactivity: Word search Topic 4 (int-0889)

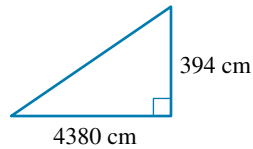
Interactivity: Sudoku Topic 4 (int-3206)

Exercise 4.8 Review questions

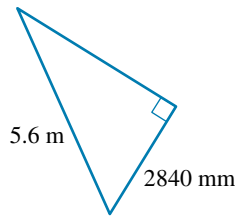
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

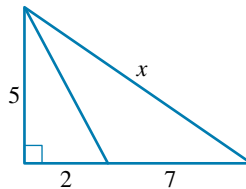
1. **MC** What is the length of the third side in this triangle?



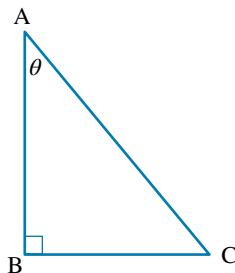
- A. 34.71 m B. 2.96 m C. 5.89 m D. 1722 cm
2. **MC** The most accurate measure for the length of the third side in the triangle below is:



- A. 4.83 m B. 23.3 cm C. 3.94 m D. 4826 mm
3. **MC** What is the value of x in this figure?

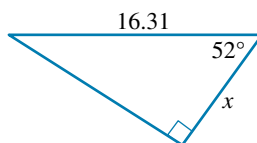


- A. 5.4 B. 7.5 C. 10.1 D. 10.3
4. **MC** Which of the following triads is not a Pythagorean triad?
- A. 3, 4, 5 B. 6, 8, 10 C. 5, 12, 13 D. 2, 3, 4
5. **MC** Which of the following correctly names the sides and angle of the triangle shown?

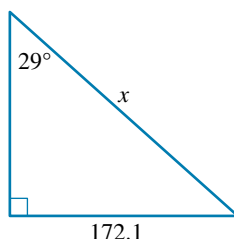


- A. $\angle C = \theta$, AB = adjacent side, AC = hypotenuse, BC = opposite side
B. $\angle C = \theta$, AB = opposite side, AC = hypotenuse, AC = adjacent side
C. $\angle A = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side
D. $\angle A = \theta$, AB = adjacent side, AC = hypotenuse, BC = opposite side
6. **MC** Which one of the following statements is correct?
- A. $\sin(60^\circ) = \cos(60^\circ)$ B. $\cos(25^\circ) = \cos(65^\circ)$
C. $\cos(60^\circ) = \sin(30^\circ)$ D. $\sin(70^\circ) = \cos(70^\circ)$

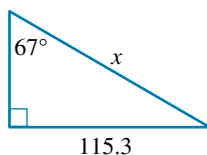
7. **MC** What is the value of x in the triangle shown, correct to 2 decimal places?



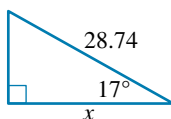
- A. 26.49 B. 10.04 C. 12.85 D. 20.70
8. **MC** Which one of the following could be used to find the value of x in the triangle shown?



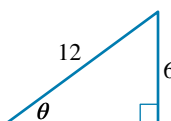
- A. $x = \frac{172.1}{\cos(29^\circ)}$ B. $x = \frac{172.1}{\sin(29^\circ)}$
- C. $x = 172.1 \times \sin(29^\circ)$ D. $x = 172.1 \times \cos(29^\circ)$
9. **MC** Which of the following could be used to find the value of x in the triangle shown?



- A. $x = \frac{115.3}{\sin(23^\circ)}$ B. $x = \frac{115.3}{\cos(67^\circ)}$
- C. $x = 115.3 \times \sin(67^\circ)$ D. $x = \frac{115.3}{\cos(23^\circ)}$
10. **MC** Which of the following could be used to find the value of x in the triangle shown?

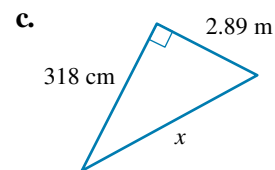
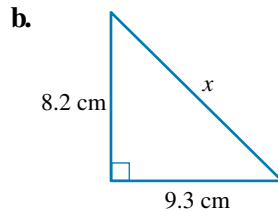
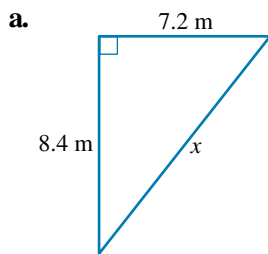


- A. $x = \frac{28.74}{\cos(17^\circ)}$ B. $x = 28.74 \times \sin(17^\circ)$
- C. $x = 28.74 \times \cos(17^\circ)$ D. $x = 28.74 \times \cos(73^\circ)$
11. **MC** The value of $\tan^{-1}(1.8931)$ is closest to:
- A. 62° B. 0.0331° C. 1.08° D. 69°
12. **MC** What is the value of θ in the triangle shown, correct to 2 decimal places?

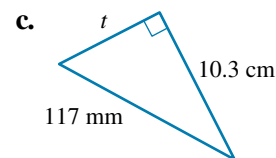
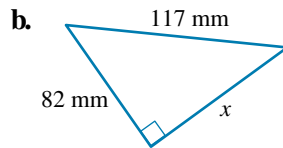
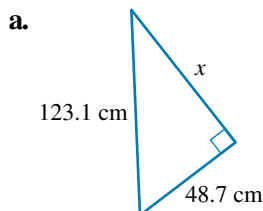


- A. 40.89° B. 60.00° C. 35.27° D. 30.00°

13. Calculate x to 2 decimal places.



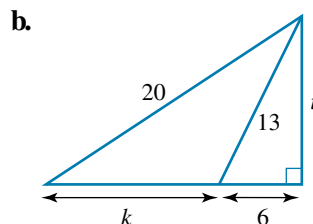
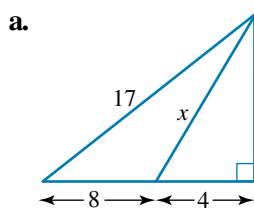
14. Calculate x , correct to 2 decimal places.



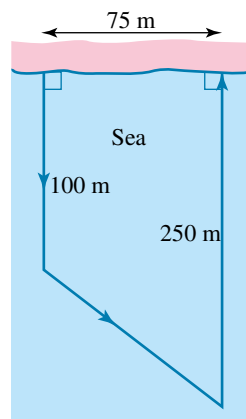
15. The top of a kitchen table measures 160 cm by 90 cm. A beetle walks diagonally across the table. How far does it walk correct to 2 decimal places?

16. A broomstick leans against a wall. The stick is 1.5 m long and reaches 1.2 m up the wall. How far from the base of the wall is the bottom of the broom?

17. Calculate the unknown values in the figures below. Give your answers correct to 2 decimal places.



18. An ironwoman race involves 3 swim legs and a beach run back to the start as shown in the figure. What is the total distance covered in the race?



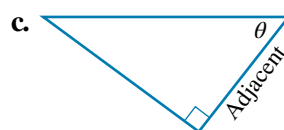
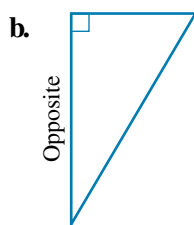
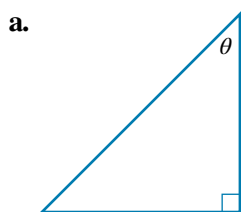
19. Which of the following are Pythagorean triads?

a. 15, 36, 39

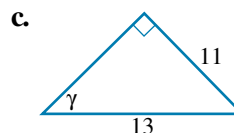
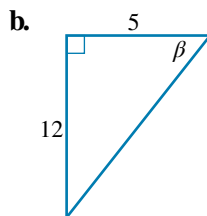
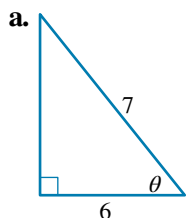
b. 50, 51, 10

c. 50, 48, 14

20. Label the unlabelled sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite, and the symbol θ , where appropriate.



21. Write trigonometric ratios that connect the lengths of the given sides and the size of the given angle in each of the following triangles. Use your ratio to then calculate the size of the angle.



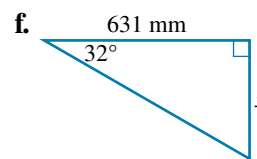
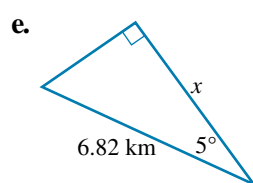
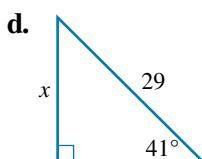
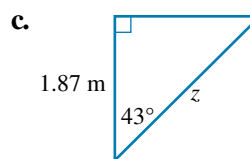
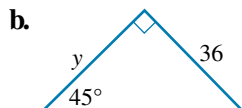
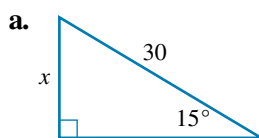
22. Use a calculator to find the values of the following trigonometric ratios, correct to 4 decimal places.

a. $\sin(54^\circ)$

b. $\cos(39^\circ)$

c. $\tan(12^\circ)$

23. Find the values of the pronumerals in each of the following triangles. Give answers correct to 2 decimal places.



24. Evaluate each of the following, correct to the nearest degree.

a. $\sin^{-1}(0.1572)$

b. $\cos^{-1}(0.8361)$

c. $\tan^{-1}(0.5237)$

25. Find the size of the angle in each of the following. Give answers correct to the nearest degree.

a. $\sin(\theta) = 0.5321$

b. $\cos(\theta) = 0.7071$

c. $\tan(\theta) = 0.8235$

d. $\sin(\alpha) = 0.7891$

e. $\cos(\alpha) = 0.3729$

f. $\tan(\alpha) = 0.5774$

g. $\sin(\beta) = 0.8660$

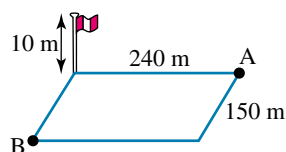
h. $\cos(\beta) = 0.5050$

i. $\tan(\beta) = 8.3791$

26. At a certain time of the day a 6.7-m-tall tree casts a shadow 1.87 m long. Find the angle of depression of the rays of the sun at that time. Round to 2 decimal places.

Communicating, reasoning and problem solving

27. A 10-m high flagpole is in the corner of a rectangular park that measures 240 m by 150 m.

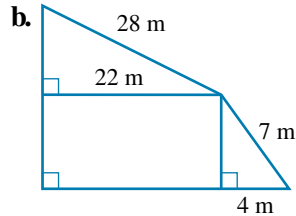
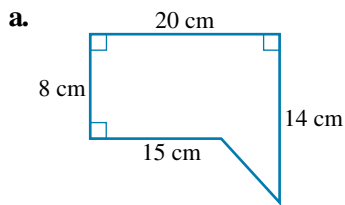


a. Find, correct to 2 decimal places:

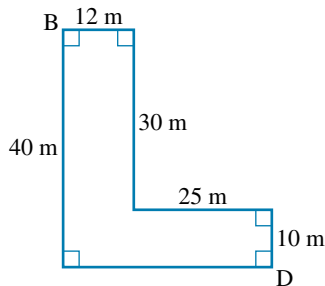
- i. the length of the diagonal of the park
- ii. the distance from A to the top of the pole
- iii. the distance from B to the top of the pole.

b. A bird flies from the top of the pole to the centre of the park. How far does it fly?

28. Find the perimeters of the shapes below.



29. Calculate the length of the shortest distance between points B and D inside the figure.



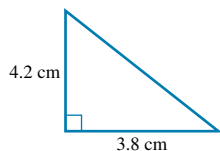
30. Two towers are 30 m apart. From the top of tower A, the angle of depression of the base of tower B is 60° , and the angle of depression of the top of tower B is 30° . What is the height of tower B? Round to the nearest metre.
31. Determine the angles of a triangle whose sides are given by the Pythagorean triad 3, 4, 5.
32. A stack of chairs reaches 2 m in height. They must fit into a doorway which is 1.8 m high. The janitor can safely lean them at an angle of 25° to the vertical. Can the janitor move the stack through the doorway?
33. What is the side-length of the largest square that can be drawn within a circle of radius r ? Give your answer as a surd.

Answers

Topic 4 Right-angled triangles (trigonometry)

Exercise 4.2 Pythagoras' theorem

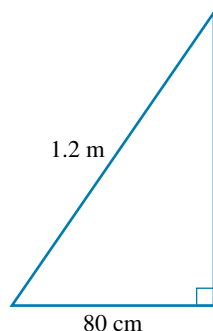
1. a. i. r ii. $r^2 = p^2 + s^2$
 b. i. x ii. $x^2 = y^2 + z^2$
 c. i. k ii. $k^2 = m^2 + w^2$
 d. i. FU ii. $(FU)^2 = (VU)^2 + (VF)^2$
2. a. 7.86 b. 33.27 c. 980.95
 3. a. $x = 12.49$ b. $p = 11.76$ cm c. $f = 5.14$ m d. $c = 97.08$ mm
 4. 10.2 cm
 5. a. No b. No c. Yes
 d. Yes e. No f. Yes
 6. a. b. 5.66 cm



7. a. b. 90.28 mm
-

8. B 9. C 10. B 11. 2.60 m
 12. a. 176.16 cm b. 147.40 cm c. 2.62 km d. 432.12 m

13. a.

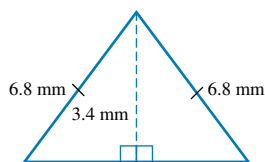


- b. 89.44 cm
 c. Yes, she will reach the hook from the top step.
 14. Answers will vary.
 15. The horizontal distance is 11.74 m, so the gradient is 0.21, which is within the limits.
 16. a. i. {9, 40, 41} ii. {11, 60, 61} iii. {13, 84, 85} iv. {29, 420, 421}
 b. The middle number and the large number are one number apart.
 17. Check with your teacher.
 18. A triangle is right-angled if it has an angle of 90° .

Exercise 4.3 Applications of Pythagoras' theorem

1. a. 12.08 cm
 2. a. $k = 16.40$ m b. $x = 6.78$ cm c. $g = 4.10$ km
 3. a. $x = 4$, $y = 9.17$ b. $x = 6.93$, $y = 5.80$ c. $x = 13$, $y = 15.20$ d. $x = 2.52$, $y = 4.32$
 4. a. 30.48 cm b. 2.61 cm c. 47.27 cm

5. a.



b. 11.78 mm

c. 20.02 mm²

6. a. 0.87 m

b. 0.433 m²

7. E

8. Yes, 1.015 m

9. No, 17.72 cm

10. 2.42 m

11. a. $w = 0.47$ m

b. 0.64 m²

c. 12.79 m²

d. \$89.85

12. 12 mm; 480 mm²

13. 17.9 cm

14. Even though a problem may be represented in 3-D, right-angled triangles in 2-D can often be found within the problem. This can be done by drawing a cross-section of the shape or by looking at individual faces of the shape.

15. a. AD, DC, AC or DC, AD, DC

b. AD = 4.47 cm, DC = 9.17 cm, AC = 13.64 cm

c. The triangle ABC is not right-angled because $(AB)^2 + (BC)^2 \neq (AC)^2$.

16. Answers will vary.

17. Answers will vary.

18. a. 2606 mm

b. Answers will vary.

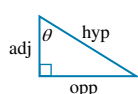
19. 16.7 m

20. $w = 3.536$ m, $x = 7.071$ cm, $y = 15.909$ cm, $z = 3.536$ cm

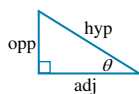
21. Composite shapes should be divided into simple shapes that have known properties. For example, two vertical lines in a trapezium will create two right-angled triangles and a rectangle.

Exercise 4.4 Similarity and the sine, cosine and tangent ratios

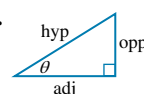
1. a.



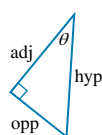
b.



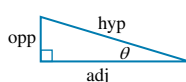
c.



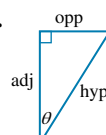
d.



e.



f.



2. a. DE = hyp, DF = opp, $\angle E = \theta$

b. GH = hyp, IH = adj, $\angle H = \theta$

c. JL = hyp, KL = opp, $\angle J = \theta$

3.

	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
a.	45°	0.71	0.71	1.00
b.	35°	0.57	0.82	0.70

4. D

5. a. i. $\sin(\theta) = \frac{4}{5}$

ii. $\cos(\theta) = \frac{3}{5}$

iii. $\tan(\theta) = \frac{4}{3}$

b. i. $\sin(\alpha) = \frac{i}{g}$

ii. $\cos(\alpha) = \frac{h}{g}$

iii. $\tan(\alpha) = \frac{i}{h}$

c. i. $\sin(\beta) = 0.8$

ii. $\cos(\beta) = 0.6$

iii. $\tan(\beta) = 1.3$

d. i. $\sin(\gamma) = \frac{24}{25}$

ii. $\cos(\gamma) = \frac{7}{25}$

iii. $\tan(\gamma) = \frac{24}{7}$

e. i. $\sin(\beta) = \frac{b}{c}$

ii. $\cos(\beta) = \frac{a}{c}$

iii. $\tan(\beta) = \frac{b}{a}$

f. i. $\sin(\gamma) = \frac{v}{u}$

ii. $\cos(\gamma) = \frac{t}{u}$

iii. $\tan(\gamma) = \frac{v}{t}$

6. a. $\sin(\theta) = \frac{12}{15} = \frac{4}{5}$

b. $\cos(\theta) = \frac{25}{30} = \frac{5}{6}$

c. $\tan(\theta) = \frac{4}{5}$

d. $\tan(\theta) = \frac{2.7}{p}$

e. $\sin(35^\circ) = \frac{17}{t}$

f. $\sin(\alpha) = \frac{14.3}{17.5}$

$$\text{g. } \sin(15^\circ) = \frac{7}{x}$$

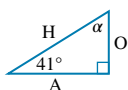
$$\text{h. } \tan(\theta) = \frac{20}{31}$$

$$\text{i. } \cos(\alpha) = \frac{3.1}{9.8}$$

7. a. D

b. B

8. a.



b. $O = 34$ mm, $A = 39$ mm, $H = 52$ mm

c. i. $\sin(41^\circ) = 0.66$

ii. $\cos(41^\circ) = 0.75$

iii. $\tan(41^\circ) = 0.87$

d. $\alpha = 49^\circ$

e. i. $\sin(49^\circ) = 0.75$

ii. $\cos(49^\circ) = 0.66$

iii. $\tan(49^\circ) = 1.15$

f. They are equal.

g. They are equal.

h. The sine of an angle is equal to the cosine of its complement.

9. a. The missing angle is also 45° , so the triangle is an isosceles triangle, therefore $a = b$.

b. 1

10. Provided n is a positive value, $(m + n)$ would be the hypotenuse, as it has a greater value than both m and $(m - n)$.

11. a. Ground

b. Ladder

c. Brick wall

12. a. The ratio of the length of the opposite side to the length of the hypotenuse will increase.

b. The ratio of the length of the adjacent side will decrease, and the ratio of the opposite side to the adjacent will increase.

c. i. 1

ii. 1

iii. ∞

13. In a right-angled triangle, we know that one angle is 90° . If one of the remaining angles is 30° , the opposite angle must be 60° ($90^\circ + 30^\circ + 60^\circ = 180^\circ$). When determining $\sin(30^\circ)$ and $\cos(60^\circ)$, both calculations will use the same side measurement, which is divided by the length of the hypotenuse.

Exercise 4.5 Calculating unknown side lengths

1. a. i. 0.8192

ii. 0.2011

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\sin(\theta)$	0	0.26	0.50	0.71	0.87	0.97	1.00

c. As θ increases, so does $\sin(\theta)$, starting at 0 and increasing to 1.

2. a. i. 0.7880

ii. 0.5919

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\cos(\theta)$	1.00	0.97	0.87	0.71	0.50	0.26	0

c. As θ increases, $\cos(\theta)$ decreases, starting at 1 and decreasing to 0.

3. a. i. 0.3249

ii. 1.2753

b.

θ	0°	15°	30°	45°	60°	75°	90°
$\tan(\theta)$	0	0.27	0.58	1.00	1.73	3.73	Undefined

c. $\tan(89^\circ) = 57.29$, $\tan(89.9^\circ) = 572.96$

d. As θ increases, $\tan(\theta)$ increases, starting at 0 and becoming very large. There is no value for $\tan(90^\circ)$.

4. a. 13.02 m

b. 7.04 m

c. 27.64 mm

d. 2.79 cm

e. 6.27 m

f. 14.16 m

5. a. 2.95 cm

b. 25.99 cm

c. 184.73 cm

d. 14.06 km

e. 8.43 km

f. 31.04 m

6. a. 26.96 mm

b. 60.09 cm

c. 0.84 km

d. 0.94 km

e. 5.59 m

f. 41.67 m

g. 54.73 m

h. 106.46 cm

i. 298.54 mm

7. a. $a = 17.95$, $b = 55.92$

b. $a = 15.59$, $b = 9.00$, $c = 10.73$

c. $a = 12.96$, $b = 28.24$, $c = 15.28$

8. a. D

b. B

c. A

d. D

9. a. 275.75 km b. 48.62 km

10. 21.32 m

11. 285.63 m

12. a, b Answers will vary.

c. $AC = \frac{AB}{\tan(\theta)}$

13. Answers will vary.

14. a. $x = 12.87$ m

b. $h = 3.00$ m

c. $x = 2.60$ m

15. $w = 41$ mm

16. a. 5.9 m

b. 5.2 m

17. 4 m

18. In a given triangle with a given angle of 60° , $\sin(60^\circ)$ is the ratio of the opposite side length to the length of the hypotenuse.

19. 2.47 km; 0.97 km

Exercise 4.6 Calculating unknown angles

1. a. 39°

b. 72°

c. 37°

d. 53°

e. 69°

f. 71°

g. 79°

h. 77°

i. 15°

2. a. 19°

b. 42°

c. 55°

d. 21°

e. 49°

f. 80°

g. 35°

h. 45°

i. 41°

j. 23°

k. 58°

l. 80°

3. a. 47°

b. 45°

c. 24°

d. 43°

e. 45°

f. 18°

g. 26°

h. 12°

i. 76°

4. a. D

b. B

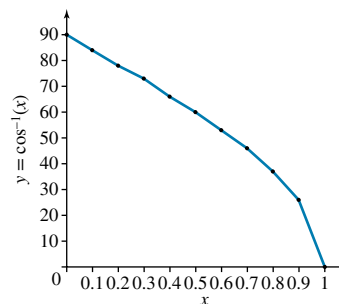
c. D

d. C

5. a.

x	$y = \cos^{-1}(x)$
0.0	90°
0.1	84°
0.2	78°
0.3	73°
0.4	66°
0.5	60°
0.6	53°
0.7	46°
0.8	37°
0.9	26°
1.0	0°

b.



6. 30.56°

7. 21.80°

8. a. 2.31 m

b. Answers will vary.

9. Answers will vary.

10. a. $\theta = 26^\circ$

b. $\theta = 32.01^\circ$

c. $x = 22.62^\circ$

11. a. The roof would not be high enough.

b. 136°

12. Answers will vary.

13. When the angle is 0, to calculate $\cos(0)$ the adjacent side is the same length as the hypotenuse. Hence, $\cos(0) = \frac{1}{1} = 1$.

14. a. 54.74°

b. Large square: $5 \text{ cm} \times 5 \text{ cm}$

Large triangles: $5 \text{ cm} \times 5 \text{ cm}$

Small square: $\frac{5\sqrt{2}}{2} \text{ cm} \times \frac{5\sqrt{2}}{2} \text{ cm}$

Small triangles: $\frac{5\sqrt{2}}{2} \text{ cm} \times \frac{5\sqrt{2}}{2} \text{ cm}$

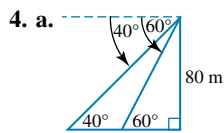
Parallelogram: $5\sqrt{2} \text{ cm} \times 5 \text{ cm}$

Exercise 4.7 Angles of elevation and depression

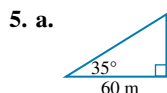
1. 8.13° . No, the ramps do not meet specifications.

2. 14.11 m

3. 66.35 m



b. 49 m



b. 42 m

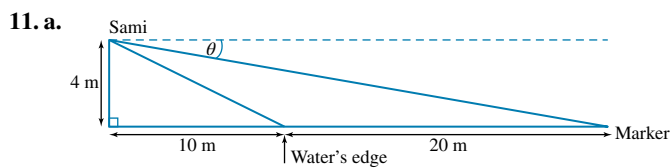
6. 1.6 m

7. 63°

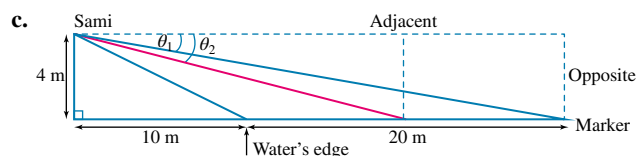
8. 34° , 3 m

9. Con: 34° , John: 30°

10. B



b. $\tan(\theta) = \frac{4}{30}$
 $\theta = \tan^{-1}\left(\frac{4}{30}\right)$
 $\theta \approx 7.595$
 $\approx 7.6^\circ$



As the dolphins swim towards Sami, the adjacent length decreases and the opposite remains unchanged.

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Therefore, θ will increase as the adjacent length decreases.

If the dolphins are at the water's edge,

$$\begin{aligned}\tan(\theta) &= 10 \\ \theta &= \tan^{-1}\left(\frac{4}{10}\right) \\ \theta &\approx 21.801 \\ &\approx 21.8^\circ\end{aligned}$$

12. Answers will vary.

13. a. Answers will vary. b. 1.64 m

14. Answers will vary.

15. Answers will vary.

16. a. Answers will vary. b. 37°

c. i. Yes ii. 17.26 m

17. The angle of elevation is the same as the angle of depression because they are alternate angles (or 'Z' angles).

Investigation | Rich task

The Great Pyramid of Giza

- 186.25 m
- 21 418.75 m²
- 218.89 m

Wall braces

- Answers will vary.
- Answers will vary.
- 53° requires the shortest brace and 37° requires the longest brace.
- 4.24 m
- 3.62 m
- 4.61 m; 4.52 m
- Answers will vary.

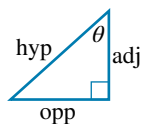
Exercise 4.8 Review questions

1. C	2. D	3. D
4. D	5. D	6. C
7. B	8. B	9. D
10. C	11. A	12. D
13. a. 11.06 m	b. 12.40 cm	c. 429.70 cm or 4.30 m
14. a. 113.06 cm	b. 83.46 mm	c. 55.50 mm or 5.55 cm
15. 183.58 cm		
16. 0.9 m		
17. a. $x = 12.69$	b. $l = 11.53, k = 10.34$	

18. 593 m

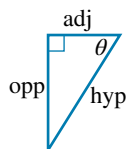
19. a. True

20. a.



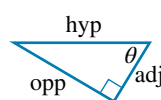
b. False

b.



c. True

c.



21. a. $\cos(\theta) = \frac{6}{7}, \theta = 31^\circ$

b. $\tan(\beta) = \frac{12}{5}, \beta = 67^\circ$

c. $\sin(\gamma) = \frac{11}{13}, \gamma = 58^\circ$

22. a. 0.8090

b. 0.7771

c. 0.2126

23. a. 7.76

b. 36.00

c. 2.56 m

d. 19.03

e. 6.79 km

f. 394.29 mm

24. a. 9°

b. 33°

c. 28°

25. a. 32°

b. 45°

c. 39°

d. 52°

e. 68°

f. 30°

g. 60°

h. 60°

i. 83°

26. 74.41°

27. a. i. 283.02 m

ii. 240.21 m

iii. 150.33 m

b. 141.86 m

28. a. 64.81 cm

b. 84.06 m

29. 59.24 m

30. 35 m

31. $90^\circ, 53^\circ$ and 37°

32. No. The stack of chairs must lean 25.84° to fit through the doorway, which is more than the safe lean angle of 25° .

33. $\frac{2r}{\sqrt{2}} = \sqrt{2}r$

TOPIC 5

Single variable data analysis

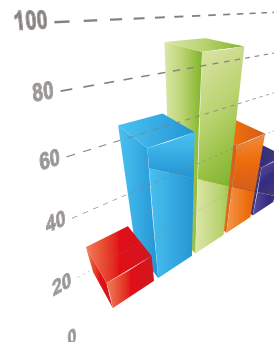
[Stages 5.1 and 5.2]

5.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

5.1.1 Why learn this?

Data gathered from observation and experimentation can be interpreted and analysed using statistics. Statistics allow us to understand real-world issues such as the popularity of politicians, the effectiveness of drugs and the reliability of weather predictions.



DISCUSSION

In what ways might the migration of humans to megacities have an impact on our culture and environment?

LEARNING SEQUENCE

- 5.1 Overview
- 5.2 [Stage 4] Collecting data
- 5.3 [Stage 5.1] Stem-and-leaf plots and dot plots
- 5.4 [Stage 5.1] Comparing and displaying data
- 5.5 [Stage 5.1] Mean, median and mode
- 5.6 [Stage 5.2] Quartiles, interquartile range and distributions
- 5.7 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- uses statistical displays to compare sets of data MA5.1-12SP
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- uses quartiles and box plots to compare sets of data MA5.2-15SP

CONTENT DESCRIPTIONS

Students:

Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi-modal' (ACMSP282)

Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)

Determine quartiles and interquartile range (ACMSP248)

Source: NSW Syllabus for the Australian curriculum

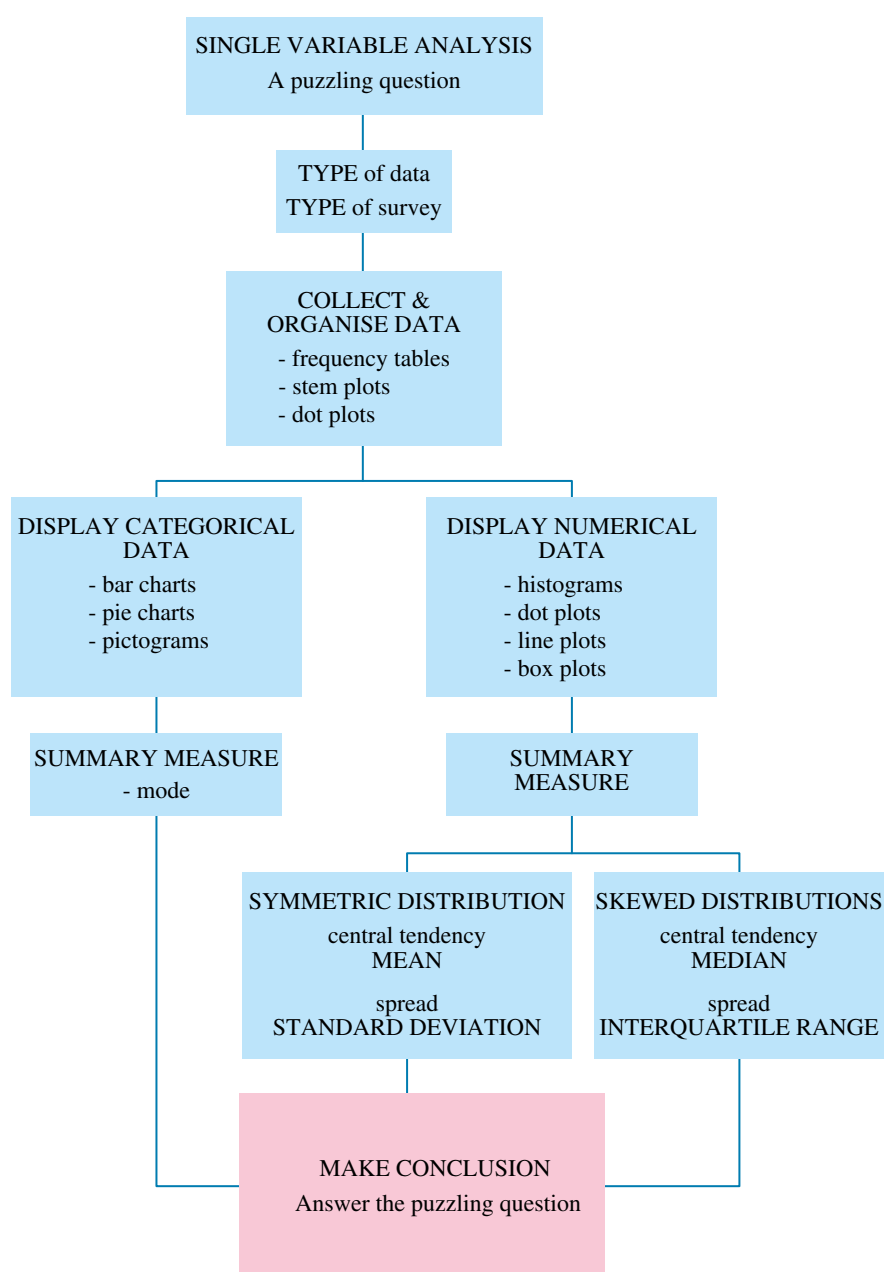
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

5.2 Collecting data

[Stage 4]

5.2.1 Introduction

- **Single variable data analysis** uses statistical methods to collect and analyse data related to one variable in order to respond to a question. Data have to be collected, recorded, organised, presented and analysed in a manner appropriate to the question.
- The process involves many stages as shown in the flow chart below.



5.2.2 Types of data

- When we perform statistical analyses, the type of data being collected is important.
- The different types of data determine the choice of displays, tables and evaluations that are best used.
- The types of data can be classified as follows.

Categorical data		Numerical data	
Data recorded are non-numerical and use labels or categories. For example, categories can be hair colour or year level.		Data are recorded in numerical form. For example, values can be the number of family members or the heights of students in Year 9.	
Nominal	Ordinal	Discrete	Continuous
Categories that are named for each of the possible responses, such as types of pets: cat, dog, bird, fish, and others	Categories that are named with some notion of order, such as quality of work: excellent, good, satisfactory, not satisfactory, poor	Numerical values that can only be expressed in integer form or whole numbers, such as counting the number of stamps in a stamp collection, or counting the number of goals kicked each game by a full forward	Numerical values that can be expressed as fractions, decimals, as well as rounded off integer values, such as the measured heights of basketball players, or the change in temperature during the day
<i>Examples of some types of statistical tools that can be used for each type of data include:</i>			
frequency tables, bar charts, pie charts, percentages and mode	frequency tables, bar charts, pie charts, percentages and mode	frequency tables, histograms, box plots, mean, median and range	frequency tables (grouped), histograms (grouped), box plots, mean, median and range

WORKED EXAMPLE 1

Classify each of the following data sets as categorical data or numerical data.

- Favourite sport watched on TV
- Quantity of books carried in a school bag by 20 Year 9 students
- Cars passing your home each hour of a day
- Eye colours of 100 pre-school children

THINK

Decide how the data would initially be recorded, as either numbers or labels.

WRITE

- Favourite sport watched on TV
Possible data could include football, netball, soccer or tennis. These are labels, so they are categorical data.
- Quantity of books carried in a school bag by 20 Year 9 students
The recorded data would be gathered by counting the number of books in a school bag. This is numerical data.
- Cars passing your home each hour of a day
The recorded data would be the number of cars. This is numerical data.
- Eye colours of 100 pre-school children
The records kept would be blue, brown, green or hazel. These are labels, so this is categorical data.

WORKED EXAMPLE 2

Classify each of the following data sets into the correct groups, stating if the data set is categorical and nominal or ordinal, or numerical and discrete or continuous.

- a** Incomes of the Manly League players
- b** Type of transport used to go to work
- c** Attendance at a zoo each day
- d** Hair colours of 100 Year 9 students
- e** Mass of an individual Easter egg being manufactured
- f** A student's behavioural report

THINK

- 1** For each, determine whether the data collected is made up of numerical values or categories.
- 2** If the data is numerical data, decide if it is collected by counting (numerical discrete data) or collected by measuring (numerical continuous data).
- 3** If the data is categorical, decide the types of labels possible, and if the labels can be ranked in a logical order (categorical ordinal data) or if they cannot be logically ranked (categorical nominal data).

WRITE

- a** Incomes of the Manly League players
As there is a wide range of incomes and they might not be stated precisely, they will be grouped in categories such as \$50 000—<\$100 000, \$100 000—<\$150 000 and so on. As these are labels and have order, it is more appropriate to classify this data set as categorical ordinal data.
- b** Type of transport used to go to work
A worker's transport mode could include car, bus or train. As these are labels with no logical order, it is classified as categorical nominal data.
- c** Attendance at a zoo each day
You would count the number of zoo patrons and record them as whole numbers (integers). Therefore, this is classified as numerical discrete data.
- d** Hair colours of 100 Year 9 students.
The students' hair colours would include blonde, brunette, redhead and so on. These are labels with no logical order, so this is categorical nominal data.
- e** Mass of an individual Easter egg being manufactured.
The mass of each egg would be measured on a scale. The mass of each egg can then be recorded as a whole number (e.g. 55 grams), or more accurately as a decimal number (e.g. 55.23 grams). Therefore, this data is numerical continuous data.
- f** A student's behavioural report
The student's behaviour could be stated as: excellent, good, satisfactory or improvement needed. These are labels that have a logical order; therefore, this is categorical ordinal data.

5.2.3 Size of data sets



- The size of the data collected can be a **population** or a **sample**.
- In statistics, ‘population’ refers to every element or identity that relates to the investigation. A population can be as small as 10 or as large as 7 billion. It depends on what the outcome relates to. For instance, to find the average age of Year 9 students for your school, the population contains only Year 9 students from your school. If the average age of Year 9 students in Australia was needed, then the population includes all Year 9 students in Australia.
- A sample is a representative portion of the population. For example, a sample of 20 Year 9 students could be taken to represent all Year 9 students at your school.
- A single piece of data is often referred to as a score or a value.

5.2.4 Types of data collection

- There are three basic ways to collect data:
 1. **Survey** — data collected from a portion of the population or sample. This type of data collection is most commonly used in market research. It is important that surveys are conducted correctly, as the conclusions drawn from the data should represent those of the population from which they were drawn.
 2. **Census** — data collected from an entire population. The most common census in Australia is the one conducted by the Australian Bureau of Statistics (ABS). Every five years the entire nation takes part in a census.
 3. Experimental — data collected by measuring or counting or assigning labels.

DISCUSSION

What are the benefits and shortcomings of doing a survey as compared to a census?

5.2.5 Question design and data collection methods

- It is important to keep in mind that the questions in a survey or census need to be clear and unambiguous.
- Questions should be unbiased, not too personal, and able to be answered by most respondents.
- The design of the questionnaire should enable the collected data to be analysed easily.
- It is important to consider the methods used to collect data, as they can bias the results of the analysis, leading to incorrect conclusions.
- The sample for the data set can be collected by either a random or a stratified technique.
 - **Random selection** ensures that all elements of a population have an equal chance of being selected.
 - **Stratified selection** ensures that groups within a population have a similar representation in a sample.

ACTIVITY: THE AGE OF MARRIAGE

Use the Australian Bureau of Statistics (ABS) website to investigate the median age of people getting married in Australia. Are you able to find similar statistics for the mean age of marriage? Compare these figures to the median age for marriage in the United States and the United Kingdom.

WORKED EXAMPLE 3

For the following data collection activities, define whether the amount of data collected is a sample or population and what type of data collection was used: census, survey or experimental; and random, stratified or biased.

- a** Sixty Year 7 students collected data on the amount they contributed to a charity, then used this data to find the average donation from Bolan College's 1523 students.
- b** A data set of the force, measured in newtons, needed to break wooden ice block sticks
- c** The average age of the members of your own family

THINK

Know the key terminology used by statisticians.

Decide if it is a:

- sample or population
- census, survey or experimental
- random, stratified or biased.

WRITE

- a** It is a sample because only 60 of Bolan College's 1523 students were surveyed. The data are biased as only Year 7 students were surveyed.
- b** It is a sample as there is an unknown number of wooden ice block sticks. It is impossible to collect data on the entire population. The data are experimental as the sticks are randomly supplied or collected.
- c** A survey of all members of a family about the family is a population.

WORKED EXAMPLE 4

A health club is planning to conduct a survey to research the needs of its members. Fifty members of the club will be asked to participate in the survey. The club has 120 male adult, 90 male adolescent, 60 female adult and 30 female adolescent members. Determine the number of members required from each group to perform a stratified sample survey that is an accurate approximation of the needs of the whole population.



THINK

- 1 Determine what fraction of the population is to be used in the sample.
- 2 Use this fraction to calculate the number to be surveyed from each group within the population.
- 3 Summarise the sampling strata.

WRITE

The total population is $120 + 90 + 60 + 30 = 300$ members.

$$\text{Fraction of members surveyed} = \frac{50}{300} = \frac{1}{6}$$

$$\text{Number of male adults} = \frac{1}{6} \times \frac{120}{1} = 20$$

$$\text{Number of male adolescents} = \frac{1}{6} \times \frac{90}{1} = 15$$

$$\text{Number of female adults} = \frac{1}{6} \times \frac{60}{1} = 10$$

$$\text{Number of female adolescents} = \frac{1}{6} \times \frac{30}{1} = 5$$

50 members surveyed will be:
20 male adults
15 male adolescents
10 female adults
5 female adolescents.

5.2.6 Random number generators

- Random numbers can be obtained from number generators built into most calculators and computers.
- Computer programs and the scientific calculator function RAND randomly select a number from 0 to less than 1. This value multiplied by the size of the sample (plus 1 to avoid choosing the value zero) will ensure any sample size selected is unbiased. Below is an example of randomly selecting a whole number (integer) from 1 to 15.

Button	Possible outcome	Example 1	Example 2	Example 3
RAND	0.000 to 0.999	0.456	0.056	0.932
$\times 15$ (multiply)	0.000 to 14.985	6.84	0.84	13.98
+1	1.000 to 15.985	7.84	1.84	14.98
INT (ignore decimal part)	1 to 15	7	1	14

WORKED EXAMPLE 5

Use a random number generator on a scientific calculator to randomly select 15 months from January 2013 to December 2017.

THINK

1 Using a scientific calculator

Use the random number button to generate a number from 0 to less than 1, then multiply the generated number by 60. Finally, add 1 to the generated number (only use the integers, ignore the decimals).

Random number = integer(random number $\times$ 60 + 1)

2 Repeat until 15 months have been selected.

WRITE/DISPLAY

Label the months numerically as:

1. January 2013

2. February 2013

...

60. December 2017.

29.632 becomes the 29th month — May 2015.

13.775 becomes the 13th month — January 2014.

(Repeat this 15 times, ignoring any doubling up of months.)

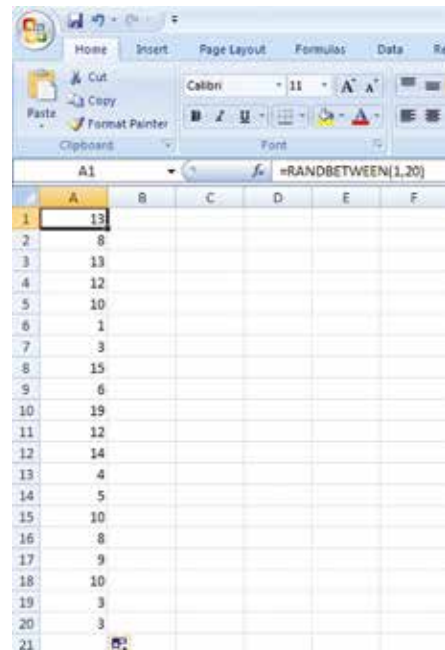
5.2.7 Generating random numbers using an Excel spreadsheet

- The command **RANDBETWEEN** in an Excel spreadsheet generates a random number within the values that you specify.
- If your spreadsheet does not have this function, press the **Office Button** and choose **Excel options**. Choose **Add-Ins** from the menu on the right, click on **Analysis Toolpak**, and press **Go** at the bottom of the page. Tick **Analysis ToolPak**, and press **OK**.

- In order to enter 20 random numbers, enter the function **=RANDBETWEEN(1,20)** into cell **A1**. A random number in the range 1 to 20 will appear in cell **A1**.
- Use the **Fill Down** function to fill this formula down to cell **A20**. You should now have 20 random numbers, as shown in the screen shot.

Note that each time you perform an action on this spreadsheet, the random numbers will automatically change. In order to switch to manual mode:

- choose the **Calculator Options** menu
- select **Manual**
- press **F9** to obtain a recalculation of your random numbers.



learn on RESOURCES — ONLINE ONLY

Interactivity: Types of data (int-6086)

Interactivity: Selecting samples (int-3811)

Interactivity: Random number generator (int-0089)

Exercise 5.2 Collecting data

Individual pathways

PRACTISE

Questions:
1–7, 9, 11–16, 19

CONSOLIDATE

Questions:
1–3, 5, 7, 8, 10, 13–17, 19, 20

MASTER

Questions:
1–4, 6, 8, 10, 13–22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** Classify each of the following data sets according to whether they are numerical or categorical data.
 - a. Favourite TV programs of each student in your year level
 - b. Shoe size of the top ten international models
 - c. Hours of computer games played at home each week by each student in your class
 - d. Birth weight of each baby in a young mothers' group
 - e. The favourite colours for cars sold in Canberra in the last month
2. **WE2** Classify each of the following numerical data sets according to whether they are discrete or continuous.
 - a. Length of a stride of each golfer in a local golf club
 - b. Distance travelled on one full tank from the top ten most popular vehicles
 - c. Rainfall recorded for each day during spring
 - d. Shoe size of each student in your class
 - e. Membership size of the 16 AFL teams
3. Classify each of the following categorical data sets as nominal or ordinal.
 - a. Favourite magazine of each student in your year level
 - b. Sales ranking of the top ten magazines
 - c. Age group of the readers of the top ten magazines
 - d. Gender of 100 randomly chosen readers of each of the top ten magazines
 - e. Type of magazine of the top ten magazines
4. **WE3** Classify each of the following data collections according to whether they are samples or populations.
 - a. Favourite TV programs of secondary school students, determined by surveying students in Year 8
 - b. Size of shoes to be stocked in a department store, determined by measuring the feet of the top ten international models
 - c. Number of hours computer games are played by your class at home each week, determined by surveying each student in your class
 - d. Survey of first 40 customers at the opening of a new store to gauge customer satisfaction
 - e. The most popular duco colours for cars sold in the last month in Victoria, taken from the registration board database of all 2310 cars registered last month



5. For each of the surveys from question 4 that involve sampling, classify the sample as random, stratified and/or biased.
6. **WE4** Determine the number of people that need to be surveyed from each of the following groups if stratified surveying is to be conducted.
 - a. Survey of 40 members from a club of 1500 members, with 1200 males members and 300 female members
 - b. Survey of 10% of all clients from a database of 10 000 clients where an appropriate representation of the 70% female customers and 30% male customers is maintained
 - c. Survey of 100 students from a school with a population of 900 students. There are 90 Year 12 students, 135 Year 11 students, 153 Year 10 students, 180 Year 9 students, and 180 Year 8 students, with the remainder of the population being Year 7 students.
 - d. Survey 8 class members such that there is an unbiased gender representation of the 20 girls and 12 boys.
7. **WE5** Devise a strategy for randomly selecting samples for the following scenarios.
 - a. Select 4 AFL clubs from the 18 clubs available to represent the league at an international sporting convention.
 - b. Select 5 students from your class.
 - c. Select 50 students from your school.
 - d. Select 65 consecutive days from a non-leap year for analysis of the daily maximum temperatures.
8. **MC** A tennis club has 400 members; 280 of the members are male. If the club wants to conduct a sample survey of 80 members, the number of males needed for an unbiased survey is:

A. 32 **B.** 24 **C.** 40 **D.** 56
9. **MC** Which of the following statements is false?
 - A.** A good sample size is 50 or more.
 - B.** Results from samples give an estimate.
 - C.** Results from a population give an estimate.
 - D.** The use of random numbers or a stratified selection process ensures unbiased results.
10. **MC** If a random number generator is used and the first two numbers are 0.796 and 0.059, the first two random numbers, respectively, for a selected sample of size 50 are:

A. 2.95 and 39.8 **B.** 2 and 39 **C.** 4 and 41 **D.** 3 and 40
11. Randomly select 7 days of a non-leap year.

Communicating, reasoning and problem solving

12. The following are 2011 statistics for motorcycle and car fatalities in the United States.

Motorcycle fatalities: 4612

Car fatalities: 11 981

Registered motorcycles: 8 437 502

Registered cars: 134 534 655

Vehicle miles travelled motorcycles (millions): 18 500

Vehicle miles travelled cars (millions): 1 495 303

Population of USA: 311 591 917

Source: National Highway Traffic Safety Administration, <http://www.nhtsa.gov>, 2011.

- a. According to the data, which is the safer mode of transport, a motorcycle or car? Explain.
- b. What ratios or percentages could be extracted from this data to reinforce your conclusion?
- c. If you wanted to advocate riding a motorcycle, which information would you include? Which information would you omit?
- d. Write a sentence that could convince your readers that a motorcycle is a safer mode of transport.
- e. Write a sentence that could convince your readers that a car is a safer mode of transport.

- 13.** A stratified random sample was taken from two high schools, a 200-student high school and a larger high school. The sample assessed how many students watched more than four hours of television per day. The small school represented 12.5% of the total students surveyed, and a total of 208 students were randomly surveyed.
- Show that 26 students were surveyed in the small school.
 - How many students were surveyed in the large school?
 - How many students were in the large school?
 - Show that 1600 students were in both schools.
- 14.** Explain how data are classified into different groups, giving examples for each group.
- 15.** Identify and justify two sets of information a person might want to collect from each of the following sources.
- A maternity hospital
 - An airport
 - Shoppers at a supermarket
- 16.** Develop a question to ask about the issue 'A new Australian flag'. Decide whether a census or survey of a sample population should be conducted. Give reasons for your decision.
- 17.** A TV station requires viewers to vote online in ranking 4 couples in a home renovating show. Is this a random sample? Is it a biased survey? Explain your reasoning.
- 18.** Explain how a random number generator can obtain seven numbers from the number range 0 to 45.
- 19.** Questions are being written for a survey about students' views on sports. Which of the following questions would you use and why?
- What do you think about sport?
 - List these sports in order from your most favourite to least favourite: football; tennis; netball; golf.
 - How often do you watch sports on television? Once a week; once a day; once a month; once a year; never
 - Why do you like sports?
 - How many live sporting events do you attend each year?
 - Can you play a sport?
 - Which sports do you watch on the television? Football; tennis; netball; golf; all of these; none of these
- 20.** Businesses collect and analyse all sorts of data.
- What sort of data do businesses collect?
 - Who collects the data for the businesses?
 - What is the data used for?
- 21.** A company involved in constructing the desalination plant in the Wonthaggi region wishes to gather some data on public opinion about the presence of the desalination plant in the area and its possible effects on the environment. The company has decided to conduct a survey and has asked you to plan and implement the survey and then analyse the data and interpret the results.
- What questions could you include in the survey?
 - What are you trying to find out by asking these questions?
 - What type(s) of data will you collect?
 - How would you display this data visually?
 - What could the company use this data for?
- 22.** What is the chance that any five students chosen in your class are born on the same day of the month?

5.3 Stem-and-leaf plots and dot plots

[Stage 5.1]

5.3.1 Recording and organising data

- Data need to be recorded and organised so that graphs and tables can be drawn and analysis conducted. Data are usually presented in a frequency table to begin with.



WORKED EXAMPLE 6

The following is a result of a survey of 30 students showing the number of people in their family living at home. Draw a frequency table to summarise the data.

2 4 5 6 4 3 4 3 2 5 6 6 5 4 5 3 3 4 5 8 7 4 3 4 2 5 4 6 4 5

THINK

- Identify the data as numerical discrete data.
- There are only 7 different scores, so the data can be left ungrouped.
- Use a frequency table of two (or three) columns. The first column is for the possible scores (x) and can be labelled as 'Number of members in the family'. The last column is for the frequency (f) and can be labelled 'Number of families'. An optional column between these two can be used as a tally for very large sets of data.

WRITE/DRAW

Number of members in the family (x)	Tally	Number of families (f)
2	III	3
3	IIII	5
4	IIII IIII	9
5	IIII II	7
6	IIII	4
7	I	1
8	I	1
		$\Sigma f = 30$

- Add up the frequency (Σf) to confirm scores from 30 students have been recorded.

WORKED EXAMPLE 7

A survey was conducted, asking students the type of vehicle driven by their parents. The responses were collected and recorded as shown below. Display the data as a frequency table and as a dot plot.

Sedan	4WD	Sedan	Sedan
4WD	SUV	SUV	Station wagon
SUV	Convertible	Station wagon	4WD
Sedan	Sports car	Convertible	Station wagon
Sedan	Station wagon	SUV	Station wagon
SUV	SUV	Sedan	Sedan

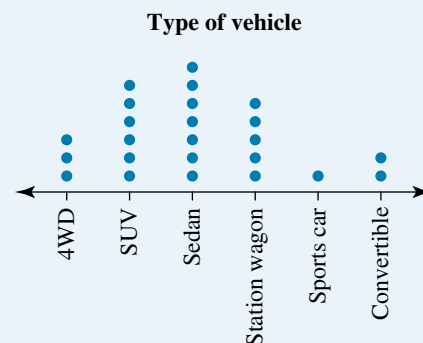
THINK

- 1 Identify this data as categorical ordinal data.
- 2 There are 6 different scores or categorical labels.
- 3 Use a frequency table of two (or three) columns. The first column is for the possible scores (x) and can be labelled as 'Type of vehicle'. The last column is for the frequency (f) and can be labelled 'Number of vehicles'. An optional middle column can be used as a tally for very large sets of data.

WRITE/DRAW

Type of vehicle (x)	Tally	Number of vehicles (f)
4WD	III	3
SUV	IIII I	6
Sedan	IIII II	7
Station wagon	IIII	5
Sports car	I	1
Convertible	II	2
		$\Sigma f = 24$

- 4 Add up the frequency (Σf) to confirm 24 scores have been recorded.
- 5 Place a dot for each vehicle type in the appropriate column.



5.3.2 Stem-and-leaf plots

- A **stem-and-leaf plot** or stem plot is a graph that displays numerical data.
- All data values are shown in stem plots.
- Stem plots have two columns: the stem holds the group value and the leaf holds the final digit in the data value.

- The following stem plot shows the sizes of 20 students' DVD collections.

DVD collection size of 20 students

Key: 2 | 5 = 25 DVDs

Stem	Leaf
0	4
1	4 6
2	2 3 5 8
3	0 4 4 6 7 7 9
4	1 3 5 9
5	4 6

- From the stem plot shown, the scores collected were 4, 14, 16, 22, 23, 25, 28, 30, 34, 34, 36, 37, 37, 39, 41, 43, 45, 49, 54 and 56.
- The distribution of the data is evident in a stem plot. In this example, it is a normal or symmetrical distribution.

WORKED EXAMPLE 8

The following scores represent the distance (in km) travelled by a group of people over a particular weekend. Summarise the data in a stem plot.

48 67 87 2 34 105 34 45 63 98 12 23 35 54 65 41 34 23 12 38 18 58 53 44 39 29

Comment on what the shape of the distribution tells us.

THINK

- Identify the data as numerical discrete data. Write a title and a key.
- There are scores from 2 to 105, so they can be grouped in 10s. The stem will hold the tens place value and the leaf will hold the last digit (the unit/ones place value) for each score.
- Transfer each score by recording its last digit in the leaf alongside the row corresponding to the stem with its tens place digit.
- Reorder each row in the leaf so the digits are in ascending order from the stem.

WRITE/DRAW

Distance travelled over one weekend

Key: 4 | 7 = 47 km

Stem	Leaf
0	2
1	2 2 8
2	3 3 9
3	4 4 4 5 8 9
4	1 4 5 8
5	3 4 8
6	3 5 7
7	
8	7
9	8
10	5

- Comment on the shape of the distribution.

The distribution is not symmetrical. Most people travelled between 20 km and 50 km over the weekend, with only 3 people travelling more than 70 km.

- Sometimes the leaves in the rows of a stem plot become too long. This can be overcome by breaking the stems into smaller intervals. The second part of the interval is identified with an asterisk (*).

WORKED EXAMPLE 9

The heights of 30 students are measured and recorded as follows.

125, 143, 119, 136, 127, 131, 139, 122, 140, 118, 120, 123, 132, 134, 127,
129, 124, 131, 138, 133, 122, 128, 130, 135, 141, 139, 121, 138, 131, 126

Represent the data in a stem plot and comment on the shape of the distribution.

THINK

- 1 Write a title and key, then draw up the stem plot with the numbers in each row of the leaf column in ascending order from the stem.

- 2 The leaves of the two middle stem values are long. They would be easier to interpret with the stem broken up into smaller intervals, i.e. intervals of 5. The stem of 12 would then include the numbers from 120 to 124 inclusive, while the stem of 12* would include the numbers from 125 to 129 inclusive.
Reconstruct the stem plot.

- 3 Comment on the shape of the distribution.

WRITE/DRAW

Heights of 30 students (in cm)

Key: 11 | 8 = 118 cm

Stem	Leaf
11	89
12	012234567789
13	0111234568899
14	013

Heights of 30 students (in cm)

Key: 11* | 8 = 118 cm

Stem	Leaf
11*	89
12	012234
12*	567789
13	0111234
13*	568899
14	013

The distribution is reasonably symmetrical, with most heights falling between 120 cm and 130 cm.



RESOURCES — ONLINE ONLY

Interactivity: Collecting data (int-3807)

Interactivity: Stem plots (int-6242)

Digital doc: SkillsHEET Presenting data in a frequency table (doc-6317)

Digital doc: SkillsHEET Arranging a set of data in ascending order (doc-10954)

Digital doc: WorkSHEET Statistics 1 (doc-6323)

Exercise 5.3 Stem-and-leaf plots and dot plots

Individual pathways

PRACTISE

Questions:

1, 3–5, 7, 9–11, 14

CONSOLIDATE

Questions:

1–3, 5, 7, 9–12, 14, 15

MASTER

Questions:

1, 4, 6, 8, 10–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE6, 7** The following set of data represents the scores a class of students achieved in a multiple choice Maths test.

4, 6, 8, 3, 6, 9, 1, 3, 5, 6, 4, 7, 5, 9, 3, 2, 7, 8, 9, 6, 5, 4, 6, 5, 3, 5, 7, 6, 7, 7

- Present the data as a dot plot.
 - Present the data in a frequency table.
- The following set of data shows the types of pets kept by 20 people.
cat, dog, fish, cat, cat, cat, dog, dog, bird, turtle, dog, cat, snake, cat, dog, dog, frog, bird
- Present the data as a dot plot.
 - Present the data in a frequency table.
 - Explain whether this data can be presented in a stem plot.
- A sample of students surveyed by the Australian Bureau of Statistics were asked: 'What is the colour of your eyes?' The results were as follows:

Brown	Hazel	Blue	Brown	Brown	Green	Green	Brown	Brown	Hazel
Brown	Brown	Brown	Brown	Blue	Brown	Green	Blue	Blue	Blue
Brown	Brown	Brown	Brown	Hazel	Blue	Hazel	Blue	Brown	Blue
Brown	Blue	Green	Hazel	Hazel	Hazel	Green	Blue	Blue	Brown

- Construct a dot plot for the data.
 - Set up a frequency table for the data.
 - Comment on the proportion of people who have brown eyes.
- WE8** Draw a stem-and-leaf plot for each of the following sets of data. Comment on each distribution.
 - 18, 22, 20, 19, 20, 21, 19, 20, 21
 - 24, 19, 31, 43, 20, 36, 26, 19, 27, 24, 31, 42, 29, 25, 38
 - 346, 353, 349, 368, 371, 336, 346, 350, 359, 362
 - 49, 43, 88, 81, 52, 67, 70, 85, 83, 44, 47, 82, 55, 77, 84, 48, 46, 83
 - 2.8, 3.6, 1.5, 1.3, 1.9, 2.4, 2.7, 3.3, 4.1, 2.9, 1.1, 4.6, 4.2, 3.9
 - WE9** Redraw the stem plots in question 4 so that the stems are intervals of 5 rather than intervals of 10.
 - The following times (to the nearest minute) were achieved by 40 students during a school outdoor running activity.
23 45 25 48 21 56 33 34 63 43 42 41 26 44
45 41 40 39 37 53 26 55 48 39 29 52 57 33
31 32 71 60 49 52 32 28 47 42 37 33
- Construct a stem plot with groupings of 5 minutes.
 - Construct a frequency table. (Use groupings of 5 minutes.)
 - If the top 10 runners were chosen for the representative team, what was the qualifying time as given by:
 - the stem plot
 - the frequency table?
 - How many times was the time of 33 minutes recorded? Explain which summary (stem plot or frequency table) was able to give this information.
- The ages of participants in a pump class at a gym are listed below:
17, 21, 36, 38, 23, 45, 32, 53, 18, 25, 14, 29, 42, 26, 18, 27, 37, 19, 34, 20, 35.
Display the data as a stem plot using an interval of 5.



8. **MC** Consider the stem plot shown.

$$7 \mid 6 = 7.6$$

Stem	Leaf
7	8
8	0 8 9
9	1 6 7 8
10	3 5 8
11	2

Which of the following data sets matches the above stem plot?

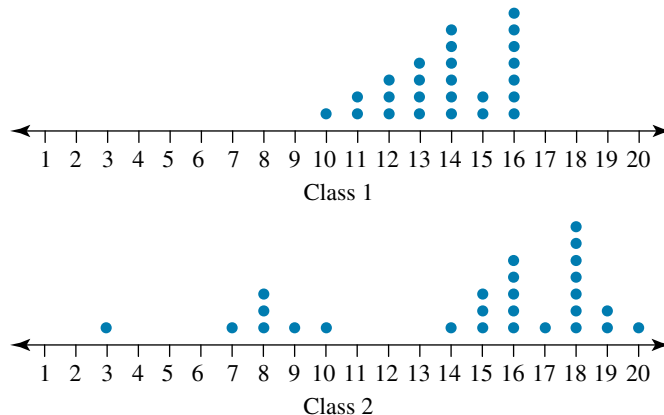
- A.** 78 80 88 89 91 96 97 98 103 105 108 112
B. 8 0 8 9 1 6 7 8 3 5 8 2
C. 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 1.03 1.05 1.08 1.12
D. 7.8 8.0 8.8 8.9 9.1 9.6 9.7 9.8 10.3 10.5 10.8 11.2
9. The following data represent the life expectancy (in years) of Australians in 40 different age groups.
- | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| 83.7 | 84.5 | 84.7 | 85.2 | 84.6 | 84.5 | 85.9 | 84.9 | 88.3 | 86.5 |
| 84.1 | 84.6 | 84.8 | 84.4 | 84.7 | 84.5 | 86.0 | 85.0 | 88.5 | 86.6 |
| 84.2 | 84.6 | 84.8 | 84.4 | 84.7 | 85.7 | 87.3 | 85.0 | 88.8 | 86.8 |
| 84.3 | 84.6 | 84.9 | 84.5 | 87.1 | 85.8 | 87.6 | 85.1 | 89.0 | 86.9 |
- a.** Present the data as a frequency table.
b. Present the data as a stem plot.
c. Comment on the distribution of life expectancy for the 40 different age groups.

Communicating, reasoning and problem solving

10. The dots below are arranged in a particular pattern, even though they may appear to be randomly coloured. Each row is generated by the row immediately above it.

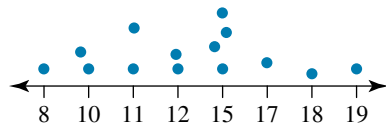


- a.** Can you work out the pattern and add the next 3 rows?
b. Do you think a row could ever have all red dots?
c. Do you think a row could ever have all black dots?
d. Does a row ever repeat itself?
11. The two dot plots below display the latest Mathematics test results for two Year 9 classes. The results show the marks out of 20.



- a.** How many students are in each class?
b. For each class, how many students scored 15 out of 20 for the test?
c. For each class, how many students scored more than 10 for the test?
d. Use the dot plots to compare the performance of the two classes on the test.

12. The following diagram is the result of a student's attempt to draw a dot plot to display the values 8, 10, 10, 11, 11, 12, 12, 15, 15, 15, 15, 17, 18, 19.



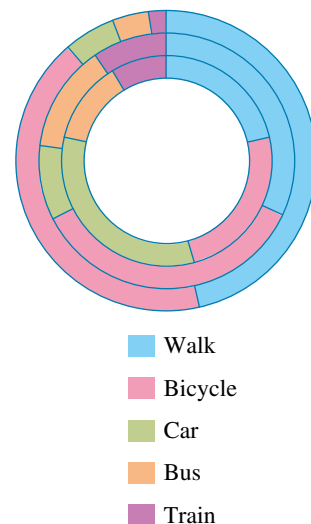
- List the mistakes that the student made in drawing the dot plot.
 - Draw the correct dot plot.
13. The masses of mice (measured in grams) recorded by a Year 9 Science class are shown below.
25.1, 24.8, 25.1, 27.3, 25.3, 29.5, 24.5, 26.7, 24.0, 26.3, 25.4, 26.3, 23.9, 25.8, 25.4, 24.6, 25.1, 23.9, 33.2, 24.5, 28.1, 27.3
- Draw an ordered stem-and-leaf plot for this data.
 - Calculate the range.
 - Draw an ordered stem-and-leaf plot for the masses of the mice rounded to the nearest gram. Discuss whether a stem-and-leaf plot is suitable for the rounded data. Compare the stem-and-leaf plot with the original masses of mice. What can you conclude?
14. The following data show the speeds of 30 cars recorded by a roadside camera outside a school, where the speed limit is 40 km/h.
20, 27, 30, 36, 45, 39, 15, 22, 29, 30, 30, 38, 40, 40, 40, 40, 42, 44, 20, 45, 29, 30, 34, 37, 40, 45, 60, 38, 35, 32
- Present the data as an ordered stem-and-leaf plot.
 - Write a paragraph for the school newsletter about the speed of cars outside the school.



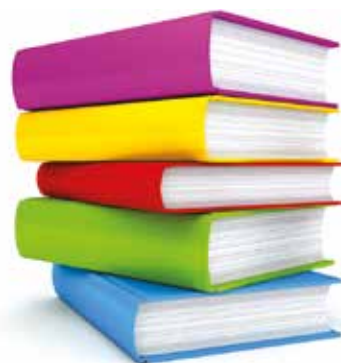
15. A donut graph can be used to compare two or more sets of data using the basic idea of a pie graph. A donut graph comparing students' methods of travel to school on Monday, Tuesday and Wednesday is shown.
- What similarities and differences does a donut graph have to a pie graph?
 - Write a series of instructions for drawing a donut graph.
 - Trial your instructions on the following data by creating a donut graph to represent the data.

Method of travel	Monday	Tuesday	Wednesday
Walk	5	7	25
Ride	6	8	23
Car	8	2	3
Bus	3	3	2
Train	2	2	1

Method of travel to school
Monday (inner ring)
Tuesday (middle ring)
Wednesday (outer ring)



- d. Compare your donut chart with the one shown on the previous page.
 - e. What are the advantages and disadvantages of using a donut graph instead of a multiple column graph to compare two or more sets of data?
16. What are two things that need to be considered when deciding on the most suitable method to collect data?
 17. Analysis of written English prose reveals the proportion of vowels and consonants to be roughly the same in every large sample — approximately 7 : 11. The letter ‘e’ accounts for one-third of the occurrence of vowels. In a passage consisting of 10 000 letters, approximately how many times would you expect the letter ‘e’ to occur?



5.4 Comparing and displaying data

[Stage 5.1]

5.4.1 Displaying data

- Displaying data using graphs reveals many important features at a glance.
- There are many representations available to a statistician, but they must be correctly matched with the type of data. These are summarised in the table below.



	Categorical data	Numerical discrete data	Numerical continuous data
Types of graphs	Dot plots	Dot plots	Histograms
	Line plots	Line plots	Frequency polygons
	Pictographs	Histograms — ungrouped	Box plots
	Bar charts	Histograms — grouped	
	Pie charts	Frequency polygons	
		Stem plots	

5.4.2 Bar charts and pie charts

- **Bar charts** are used to display categorical data only.
- The height of each bar represents **frequency, relative frequency** or percentage frequency.
- The width of the bars and the spaces between the bars must be kept uniform.
Note: Bars cannot touch one another.
- **Pie charts** (circle graphs or sector graphs) use pieces of the pie or sectors to represent a category.
- The size of the piece of pie is in proportion to the frequency (as a percentage) compared with the total (100%).
- The size of each sector is measured as a proportion of the 360 degrees in a circle.

WORKED EXAMPLE 10

The Kelly Clan is having a major family reunion. The age groups of the 200 family members that are attending have been recorded, as shown below.

Age group	Number
Baby	20
Toddler	24
Child	34
Teenager	36
Adult	50
Pensioner	36

Display the frequency table as a:

a percentage bar chart

b pie chart.

THINK

- a 1 Add two extra columns to the frequency table to calculate the percentage frequency and the size of the sector.

For percentage use the rule $\% = \frac{f}{\Sigma f} \times \frac{100}{1}$.

For size of sector use the rule degrees°

$$= \frac{f}{\Sigma f} \times 360^\circ.$$

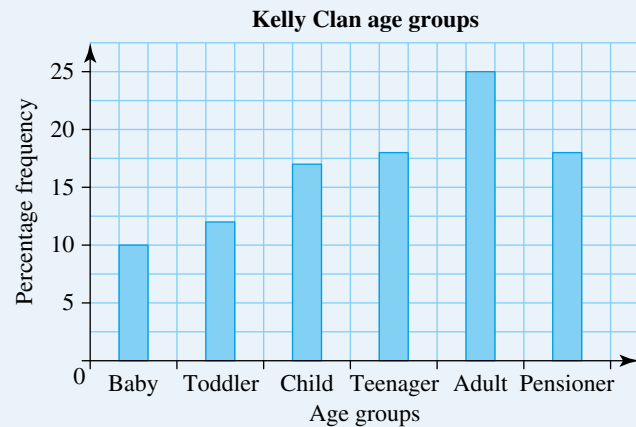
- 2 Give the bar chart a suitable title.

WRITE/DRAW

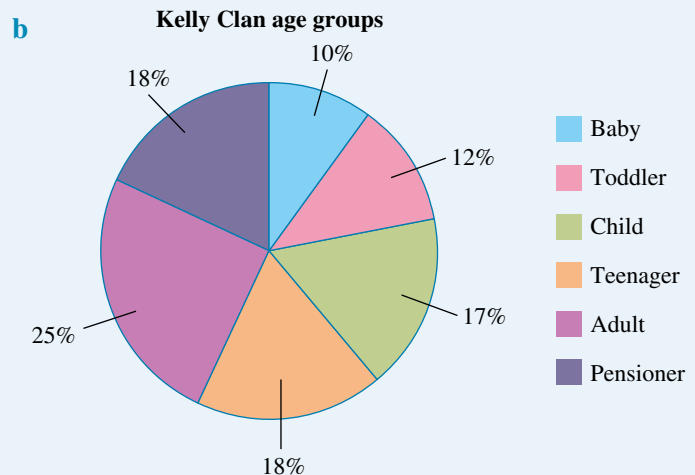
Age group	Number (f)	Percentage frequency	Size of sector
Baby	20	$\frac{20}{200} \times \frac{100}{1} = 10\%$	$\frac{20}{200} \times 360^\circ = 36^\circ$
Toddler	24	$\frac{24}{200} \times \frac{100}{1} = 12\%$	$\frac{24}{200} \times 360^\circ = 43.2^\circ \approx 43^\circ$
Child	34	$\frac{34}{200} \times \frac{100}{1} = 17\%$	$\frac{34}{200} \times 360^\circ = 61.2^\circ \approx 61^\circ$
Teenager	36	$\frac{36}{200} \times \frac{100}{1} = 18\%$	$\frac{36}{200} \times 360^\circ = 64.8^\circ \approx 65^\circ$
Adult	50	$\frac{50}{200} \times \frac{100}{1} = 25\%$	$\frac{50}{200} \times 360^\circ = 90^\circ$
Pensioner	36	$\frac{36}{200} \times \frac{100}{1} = 18\%$	$\frac{36}{200} \times 360^\circ = 64.8^\circ \approx 65^\circ$
Σ	$\Sigma f = 200$	100%	360°

Kelly Clan age group categories include baby, toddler, child, teenager, adult and pensioner.

- 3 Identify the category names and label the horizontal axis with the names of the categories of people. A space between each category is the convention for bar charts.
- 4 Label the vertical axis with a suitable scale and title.
- 5 Draw the bars to the correct percentage frequency height for each category.

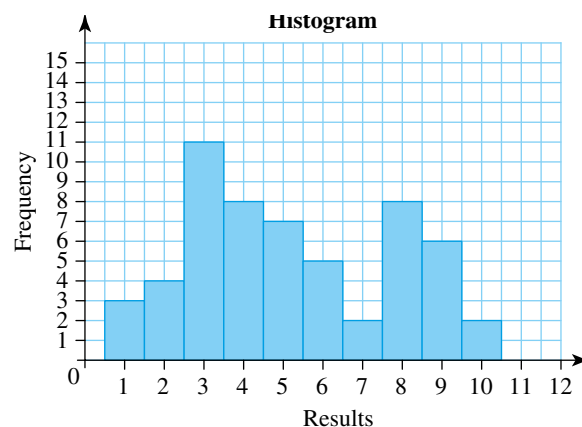


- b 1 Use the information calculated in the extended frequency table from a1.
- 2 Give the pie chart a suitable title.
- 3 Create a legend with a suitable colour for each category.
- 4 Draw a circle.
- 5 Using a protractor, measure and draw the correct angle of each sector.
- 6 Colour the sectors using the legend.



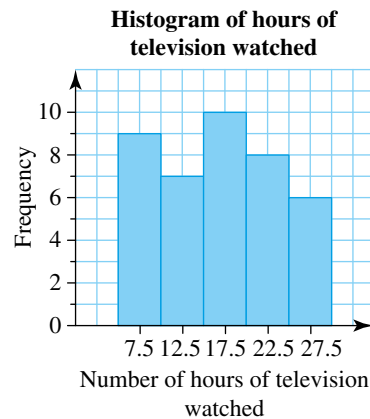
5.4.3 Histograms and frequency polygons

- Histograms** are similar to bar charts except that the columns have no gaps between them.

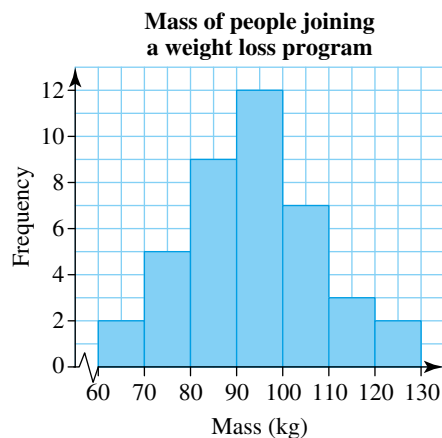


- Histograms can display discrete and continuous numerical data.

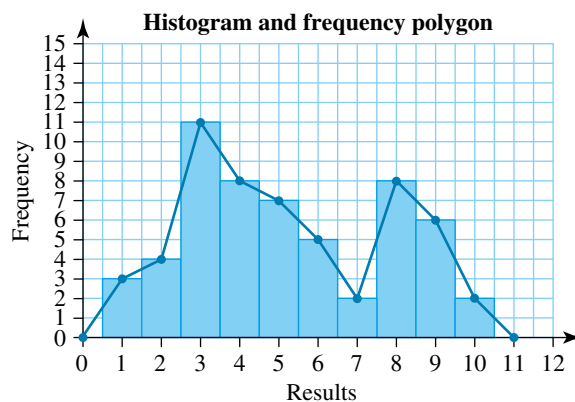
- Histograms should include:
 - a title
 - clearly labelled axes
 - separate axes scaled evenly
 - columns of equal width, with no gaps between
 - a half-interval gap at each end of the graph.
- Each column of the histogram has the range of values shown on either edge of the column.



- Alternatively, the centre score (midpoint) of each class interval can be marked.
- The midpoint is calculated by taking the average of the two extreme values of that class interval.



- Frequency polygons** are constructed by connecting the midpoints of the tops of the columns in the histograms with straight lines.
- The lines close the frequency polygon on either side of the histogram on the horizontal axis.
- The ends of the frequency polygon should be a half-interval either side of the first and last columns of the histogram.



WORKED EXAMPLE 11

A sample of 40 people was surveyed regarding the number of hours per week they spent watching television. The results are listed below.

12.9, 18.3, 9.9, 17.1, 20, 7.8, 24.2, 16.7, 9.1, 27, 7.2, 16, 26.5, 15, 7.4, 28, 11.4, 20, 9, 11, 23.7, 19.8, 29, 12.6, 19, 12.5, 16, 21, 8.3, 5, 16.4, 20.1, 17.5, 10, 24, 21, 5.9, 13.8, 29, 25

- What is the range for the data?
- Organise the data into 5 class intervals and use this to create a frequency distribution table that displays the class intervals, midpoints and frequencies.
- Construct a histogram and frequency polygon to represent the data.

THINK

- Determine the range of the number of hours of television watched.
Note: The range is the difference between the largest and smallest values.

- Determine the size of the class intervals.
Class intervals have been recorded as 5–<10, 10–<15 and so on to accommodate the continuous data.

- Rule a table with three columns, headed ‘Hours of television watched’ (class interval), ‘Midpoint’ (class centre) and ‘Frequency’.

Note: The midpoint of a class interval is calculated by taking the average of the two extreme values of that class interval. For example, the midpoint of the 5–10 class interval is $\frac{5 + 10}{2} = 7.5$.

- Tally the scores and enter the information into the frequency column.

- Calculate the total of the frequency column.

- Rule a set of axes on graph paper. Label the horizontal axis ‘Number of hours of television watched’ and the vertical axis ‘Frequency’.

- Leaving a $\frac{1}{2}$ -unit or interval space, draw in the first column so that it starts and finishes halfway between class intervals and reaches a vertical height of 9 people.

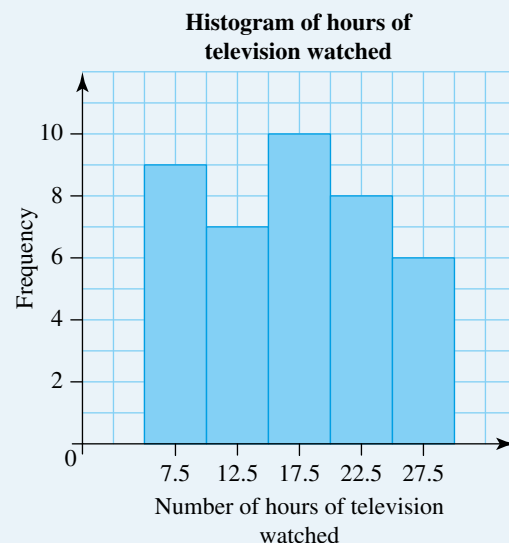
- Draw the columns for each of the other scores.

WRITE

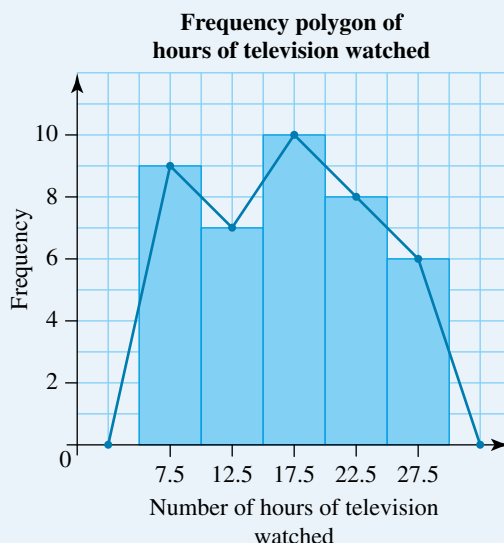
- Range = largest value – smallest value
= 29 – 5
= 24

- Class intervals of 5 hours will create 5 groups and cover a range of 25 numbers.

Hours of television watched	Midpoint	Frequency
5–<10	7.5	9
10–<15	12.5	7
15–<20	17.5	10
20–<25	22.5	8
25–<30	27.5	6
	Total	40



- 4 To draw the frequency polygon, mark the midpoints of the tops of the columns obtained in the histogram.
- 5 Join the midpoints by straight line intervals.
- 6 Close the frequency polygon by drawing lines to points placed a half-interval either side of the first and last columns of the histogram.



WORKED EXAMPLE 12

A stem plot was used to display the weight of airline luggage for flights to the Gold Coast.

- a Convert the stem plot into a histogram.
- b Show the frequency polygon without the histogram superimposed.

Weights of airline luggage

Key: 21 | 3 = 21.3 kg

Stem	Leaf
18	5 8 9
19	2 3 6 7
20	5 7 8 8 9 9
21	0 2 6 9
22	0 1 9
23	2



THINK

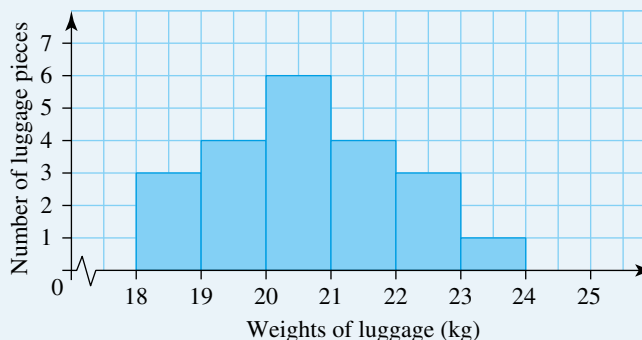
a Histogram

- 1 Identify the data as numerical continuous data with six groups or class intervals. Label the x-axis as 'Weight of airline luggage (kg)'. Leave a half-interval gap at the beginning and use an appropriate scale for the x-axis.
- 2 Label the y-axis as 'Frequency' (or 'Number of luggage pieces').

WRITE

a

Histogram of weights of airline luggage



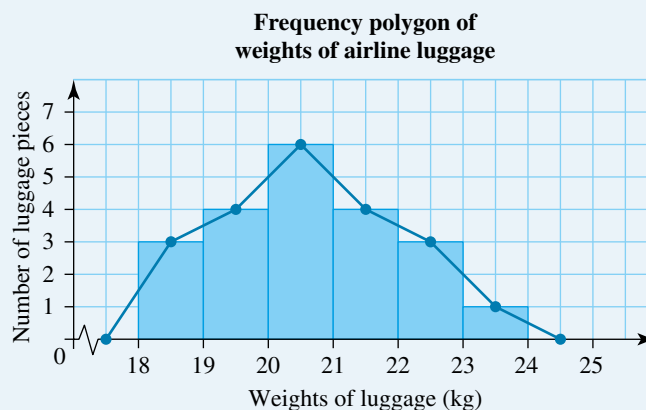
3 Draw the first column on the x -axis to a height of its frequency.

4 Repeat for all class intervals.

b Frequency (histogram) polygon

Use the same procedure as for the histogram, except the columns are replaced with lines with end points lined up with the middle of the class intervals shown on the x -axis and to the height of the frequency on the y -axis.

b



5.4.4 Back-to-back stem-and-leaf plots

- It is important to be able to compare data sets when the data are related.
- When two sets of data are related, we can present them as **back-to-back stem-and-leaf plots**.

WORKED EXAMPLE 13

The ages of male and female groups using a ten-pin bowling centre are listed.

Males: 65, 15, 50, 15, 54, 16, 57, 16, 16, 21, 17, 28, 17, 27, 17, 22, 35, 18, 19, 22, 30, 34, 22, 31, 43, 23, 48, 23, 46, 25, 30, 21.

Females: 16, 60, 16, 52, 17, 38, 38, 43, 20, 17, 45, 18, 45, 36, 21, 34, 19, 32, 29, 21, 23, 32, 23, 22, 23, 31, 25, 28.

Display the data as a back-to-back stem-and-leaf plot and comment on the distribution.



THINK

- 1 Rule up three columns headed 'Leaf (female)', 'Stem', and 'Leaf (male)'.
- 2 Make a note of the smallest and largest values of both sets of data (15 and 65). List the stems in ascending order in the middle column.

WRITE

Key: 1 | 5 = 15

Leaf (female)	Stem	Leaf (male)
987766	1	5566677789
8532331910	2	1872223351
1224688	3	50410
553	4	386
2	5	047
0	6	5

- 3 Beginning with the males, work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.
- 4 Repeat step 3 for the set of data for females.
- 5 Include a key to the plot that informs the reader of the meaning of each entry.
- 6 Redraw the stem-and-leaf plot so that the numbers in each row of the leaf columns are in ascending order.
Note: The smallest values are closest to the stem column and increase as they move away from the stem.

Key: 1 | 5 = 15

Leaf (female)	Stem	Leaf (male)
987766	1	5566677789
9853332110	2	1122233578
8864221	3	00145
553	4	368
2	5	047
0	6	5

- 7 Comment on any interesting features.

The youngest male attending the ten-pin bowling centre is 15 and the oldest 65; the youngest and oldest females attending the ten-pin bowling centre are 16 and 60 respectively. Ten-pin bowling is most popular for men in their teens and 20s, and for females in their 20s and 30s.

learnON RESOURCES — ONLINE ONLY

Interactivity: Frequency polygons (int-6088)

Interactivity: Back-to-back stem plots (int-2773)

Exercise 5.4 Comparing and displaying data

Individual pathways

PRACTISE

Questions:
1, 2, 4–6, 9–11, 13

CONSOLIDATE

Questions:
1, 3, 6, 7, 9, 11, 12, 14, 17

MASTER

Questions:
1, 4, 6, 8–10, 15, 16–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE10** The number of participants in various gym and fitness classes for the local fitness club is given as:
 Aerobics — 120 members Boxing — 15 members Pilates — 60 members
 Spin — 45 members Aqua aerobics — 30 members Step — 90 members
 Represent this information as a:
 a. bar chart b. pie chart.
- WE11** In a survey, 40 people were asked about the number of hours a week they spent watching television. The results are listed below.
 10.3, 13, 7.1, 12.4, 16, 11, 6, 14, 6, 11.1, 5.8, 14, 12.9, 8.5, 27.5, 17.3,
 13.5, 8, 14, 10.2, 13, 7.9, 15, 10.7, 16, 8.4, 18.4, 14, 21, 28, 9, 12, 11.5, 13, 9, 13, 29, 5, 24, 11
 a. Are the data continuous or discrete?
 b. What is the range of the data?
 c. Organise the data into class intervals and create a frequency distribution table that displays the class intervals, midpoints and frequencies.
 d. Construct a histogram and frequency polygon.
- The number of phone calls made per week in a sample of 56 people is listed below.
 21, 50, 8, 64, 33, 58, 35, 61, 3, 51, 5, 62, 16, 44, 56, 17, 59, 23, 34, 57, 49, 2, 24, 50, 27, 33, 55, 7, 52,
 17, 54, 78, 69, 53, 2, 42, 52, 28, 67, 25, 48, 63, 12, 72, 36, 66, 15, 28, 67, 13, 23, 10, 72, 72, 89, 80
 a. Organise the data into a grouped frequency distribution table using a suitable class interval.
 b. Display the data as a combined histogram and frequency polygon.
- For the following data, construct:
 a. a frequency distribution table b. a histogram and frequency polygon.

Class sizes at Jacaranda Secondary College									
38	24	20	23	27	27	22	17	30	26
25	16	29	26	15	26	19	22	13	25
21	19	23	18	30	20	23	16	24	18
12	26	22	25	14	21	25	21	31	25

- WE12** The number of bananas sold during each recess at a high school canteen for the past five weeks was recorded as shown.
 4 6 3 1 3 4 5 7 2 1 6 2 3 5 7 3 4 2 2 5 8 4 2 1 4
 Represent this information as:
 a. a histogram
 b. a frequency polygon without the histogram superimposed.
- The distances 300 students have to travel to attend a primary school are summarised in the following table.

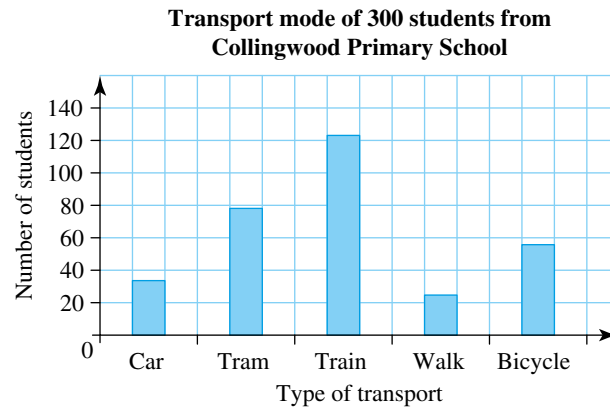


Distance km	Number of students
0–<2	112
2–<4	65
4–<6	56
6–<8	44
8–<10	15
10–<12	8

Represent this information as:

- a. a histogram
- b. a frequency polygon without the histogram superimposed.

7. **MC** From the bar chart, the number of students who catch public transport to school is:
A. 80 **B.** 100 **C.** 120 **D.** 200



8. An investigation into transport needs for an outer suburb community recorded the number of passengers boarding a bus during each of its journeys.

12 43 76 24 46 24 21 46 54 109 87 23 78 37 22 139
 65 78 89 52 23 30 54 56 32 66 49

Construct a histogram using class intervals of 20.

9. A drama teacher is organising his DVD collection into categories of 'Type of show'. He recorded the following for the first 20 shows nominated.

musical, musical, drama, pantomime, horror, musical, pantomime, action, drama, pantomime, drama, drama, musical, drama, pantomime, romance, drama, musical, romance, drama

- a. Show the data set as a bar chart.

He continued with the task, and the classification for his collection of 200 DVDs is summarised in the frequency table shown below.

Type of DVD	Number of DVDs	Size of sector for a pie chart
Musical	52	
Drama	98	
Pantomime	30	
Horror	10	
Action	5	
Romance	5	
Total	200	360°

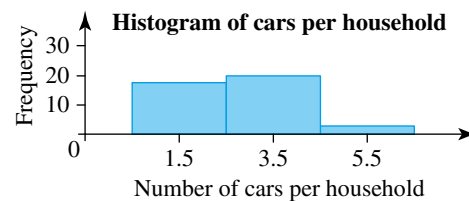
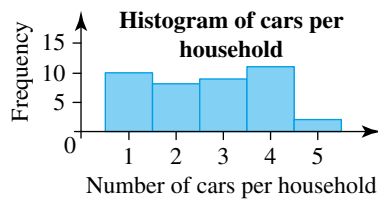
- b. Copy and complete the frequency table.
- c. Construct a pie chart for the 200-DVD collection.

10. The heights, in centimetres, of 30 students in Year 9 were recorded as follows.

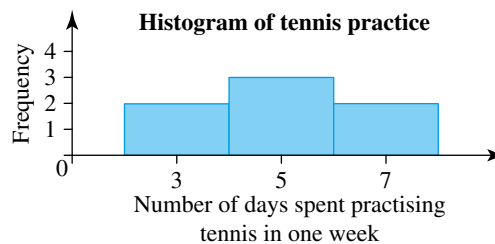
146.5, 163, 156.1, 168, 159.4, 170.1, 152.1, 174.5, 156,
 163, 157.8, 161, 178, 151, 148, 167, 162.9, 157, 166.7,
 154.6, 150.3, 166, 160, 155.9, 164, 157, 171.1, 168, 158, 162

- a. Is the data continuous or discrete?
- b. Organise the data into 7 classes, starting at 145 cm, and draw up a frequency distribution table.

- c. Display the data as a histogram.
- d. Is it possible to obtain the range of heights from the frequency distribution table, without any other information? Explain.
- e. **MC** How many students stood *at least* 160 cm tall?
A. 15 **B.** 16 **C.** 14 **D.** 7
- f. **MC** What percentage were under the minimum height of 165 cm for the basketball team?
A. 21% **B.** 30% **C.** 87% **D.** 70%
- g. Reorganise the data into class intervals of 4 cm; that is, 145–<148 cm.
- h. Draw a new histogram and compare it to the previous one. Discuss any advantages or disadvantages of having a smaller class interval.
11. Could the following histograms be produced from the same data sets? Explain.



12. The following histogram illustrates the number of days that tennis team members practised in one week.



- a. What does the frequency represent in this problem?
- b. What is the average number of days the team members practised in one week?

Communicating, reasoning and problem solving

13. **WE13** The number of goals scored in football matches by Mitch and Yani were recorded as follows:

Mitch	0	3	1	0	1	2	1	0	0	1
Yani	1	2	0	1	0	1	2	2	1	1

Display the data as a back-to-back stem-and-leaf plot and comment on the distribution.

14. Answer the following questions for the back-to-back stem-and-leaf plot in question 13.
- a. How many times did each player score more than 1 goal?
- b. Who scored the greatest number of goals in any one match?
- c. Who scored the greatest number of goals overall?
- d. Who is the more consistent performer? Explain.

15. Percentages in a Mathematics examination for two classes were as follows:

9A	32	65	60	54	85	73	67	65	49	96	57	68
9B	46	74	62	78	55	73	60	75	73	77	68	81

- a. Construct a back-to-back stem-and-leaf plot of the data.
- b. What percentage of each group scored above 50? Give your answers to the nearest whole number.

- c. Which group had more scores over 80?
 d. Compare the clustering for each group.
 e. Comment on extreme values.
 f. Calculate the average percentage for each group.
16. **MC** The back-to-back stem-and-leaf plot displays the heights of a group of Year 9 students.
 Key: 13 | 7 = 137 cm

<i>Leaf (boys)</i>	<i>Stem</i>	<i>Leaf (girls)</i>
98	13	78
98876	14	356
988	15	1237
7665	16	356
876	17	1

- a. The total number of Year 9 students is:
A. 13 **B.** 17 **C.** 30 **D.** 36
- b. The tallest male and shortest female heights respectively are:
A. 186 cm and 137 cm
B. 171 cm and 148 cm
C. 137 cm and 188 cm
D. 178 cm and 137 cm
- c. Compare the heights of boys and girls.
17. When reading the menu at the local Chinese restaurant, you notice that the dishes are divided into sections. The sections are labelled chicken, beef, duck, vegetarian and seafood.
 a. What type of data is this?
 b. What is the best way to represent this data?
18. You are working in the marketing department of a company that makes sunscreen. Your boss wants to know if there are any unusual results from some market research you completed recently. You completed a survey of people who use your sunscreen, asking them how many times a week they used it. What will you say to your boss? Analyse the data set below to help you and try to explain any unusual values.
 7, 2, 5, 4, 7, 5, 7, 2, 5, 4, 3, 5, 7, 7, 4, 5, 21, 5, 5, 2
19. Give two reasons why stem plots are preferred by statisticians rather than frequency tables.

5.5 Mean, median and mode

[Stage 5.1]

- The mean, median and mode are known as **measures of central tendency**.
- The **mean** is a measure of the centre of a set of data. The mean is more commonly known as the average.
 - To calculate the mean from a set of raw scores, use the formula:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}}$$

- To calculate the mean of data organised into a frequency table, use the formula:

$$\bar{x} = \frac{\sum xf}{\sum f} = \frac{\text{sum of (scores} \times \text{frequency)}}{\text{sum of frequencies}}$$

- The **median** is the middle score (symbol M or Q_2) of a set of data organised from lowest to highest.
 - The median score = $\frac{n+1}{2}$ th score, where n is the number of pieces of data or scores.
 - If n is odd, then the median is the middle score.
 - If n is even, then the median is the average of the two middle scores.
- The **mode** is the score (or scores) that occurs most frequently.

DISCUSSION

What are the benefits of having more than one measure of central tendency?

WORKED EXAMPLE 14

For the data below, calculate:

a the median

b the mean

c the mode.

3 7 4 5 6 2 8 3 3 5

THINK

a Median

- 1 Organise the data or scores from smallest to largest.
- 2 The median or middle score is found by first counting how many scores there are. If there is an even number of scores, then the median is the average of the two middle scores. Use the rule median score = $\frac{n+1}{2}$ th score where n is the number of scores.

b Mean

- 1 Add up all the scores.
- 2 Divide by the number of scores.

c The mode is the score that occurs the most frequently in the list.

WRITE

a Median

2 3 3 3 4 5 5 6 7 8

$$\begin{aligned}\text{Median score} &= \frac{n+1}{2}\text{th score} \\ &= \frac{10+1}{2} = 5.5\text{th score}\end{aligned}$$

The median score is between the 5th and 6th scores, which is between 4 and 5.
 $\therefore$ median = 4.5.

b Mean

$$\begin{aligned}\text{Mean} &= \bar{x} \\ &= \frac{\sum x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}} \\ \bar{x} &= \frac{3+7+4+5+6+2+8+3+3+5}{10} \\ \bar{x} &= \frac{46}{10} = 4.6\end{aligned}$$

c Mode = 3

WORKED EXAMPLE 15

The combined number of goals scored in each of the 48 first round games of the 2010 World Cup is summarised in the frequency table below.

Total combined goals scored	Number of matches
0	6
1	13
2	12
3	9
4	5
5	2
6	0
7	1
Σf	48



Find:

- the median number of goals scored
- the mean number of goals scored
- the modal number of goals scored.

THINK

- a Median**

The sum of the frequencies is the number of scores, n . If there is an even number of scores, then the median is the average of two middle scores.
Find where the 24th and 25th scores lie.

- b Mean**

Set up a third column on the frequency table and multiply the score and its frequency.
Sum the products of xf .
Divide by the sum of the frequencies, Σf .

WRITE/DRAW

- a Median**

$$\begin{aligned}\text{Median score} &= \frac{n+1}{2} \text{th match} \\ &= \frac{48+1}{2} = 24.5 \text{th match}\end{aligned}$$

This score is between the 24th match and the 25th match.
The 24th score is 2 combined goals.
The 25th score is 2 combined goals.
Median = 2 combined goals

- b Mean**

Combined goals (x)	Number of matches (f)	$x \times f$
0	6	0
1	13	13
2	12	24
3	9	27
4	5	20
5	2	10
6	0	0
7	1	7
Σf	48	$\Sigma xf = 101$

$$\begin{aligned}\bar{x} &= \frac{\Sigma xf}{\Sigma f} = \frac{\text{sum of (scores} \times \text{frequency)}}{\text{sum of frequencies}} \\ &= \frac{101}{48} \\ &= 2.1\end{aligned}$$

c Mode

Look for the score with the highest frequency.

c Mode = 1 combined goal scored

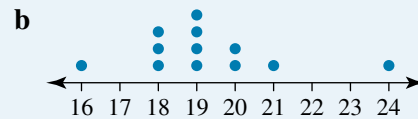
- Sometimes the mean, median and mode are calculated from graphs.

WORKED EXAMPLE 16

Find the median of each data set.

a

Stem	Leaf
1	89
2	225778
3	01467
4	05



THINK

- a 1** Check that the scores are arranged in ascending order.
- 2** Locate the position of the median using the rule $\frac{n+1}{2}$ where $n = 15$.
Note: The position of the median is $\frac{15+1}{2} = 8$ (the eighth score).

- 3** Answer the question.

- b 1** Observe the data in the given dot plot.

- 2** Locate the position of the median using the rule $\frac{n+1}{2}$ where $n = 12$.
Note: The position of the median is $\frac{12+1}{2} = 6.5$; that is, between the sixth and seventh scores.

- 3** Obtain the average of the two middle scores.

- 4** Answer the question.

WRITE/DISPLAY

- a** The scores are arranged in ascending order.

Key: 1 | 8 = 18

Stem	Leaf
1	89
2	225778
3	01467
4	05

The median of the set of data is 28.

- b** The 6th score is 19.
 The 7th score is 19.

$$\text{Median} = \frac{19 + 19}{2}$$

$$\begin{aligned}&= \frac{38}{2} \\ &= 19\end{aligned}$$

The median of the set of data is 19.

Interactivity: Mean (int-3818)

Interactivity: Median (int-3819)

Interactivity: Mode (int-3820)

eLesson: Mean and median (eles-1905)

Digital doc: SkillsHEET Finding the mean (doc-10955)

Digital doc: SkillsHEET Finding the middle score (doc-10956)

Digital doc: SkillsHEET Finding the middle score for data arranged in a dot plot (doc-10957)

Digital doc: SkillsHEET Finding the most frequent score (doc-10958)

Exercise 5.5 Mean, median and mode

Individual pathways

PRACTISE

Questions:

1, 2, 3a, 4a, 5–7, 9–13, 16, 17

CONSOLIDATE

Questions:

1, 2, 3b, 4b, 5, 6, 8, 9–13, 16–22

MASTER

Questions:

1, 2, 3c, 4c, 5, 6, 9, 11, 13, 16–22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE14** Calculate the median, mean and mode for the small raw data sets given. Confirm your answers for the median and mean using a graphics calculator. Where necessary, give your answers to 2 decimal places.

a. 27 32 45 48 53 55 55 57 59 61 75 81

b. 8 8 2 8 8 8 8 9 9 1 9 5 9 7 1 0 0

- WE15** Calculate the median, mean (to 2 decimal places) and mode for this ungrouped frequency table.

Score	Frequency
0	2
1	5
2	12
3	10
4	7
5	2

- WE16a** Find the median of each of the following data sets.

a. Key: 1|0 = 10

Stem	Leaf
1	02
2	1335
3	
4	4

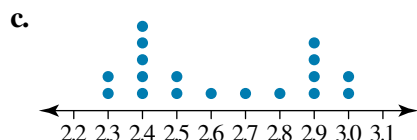
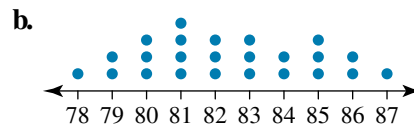
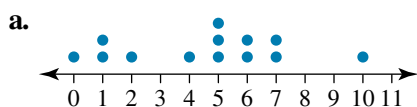
b. Key: 10|0 = 100

Stem	Leaf
10	0
11	0222
12	0466
13	3

c. Key: 6.1|8 = 6.18

Stem	Leaf
6.1	889
6.2	0568
6.3	012444

4. **WE16b** Find the median of each of the following data sets.



5. Calculate the median, mean and mode for each of the following grouped frequency tables. Where necessary, give your answers to 1 decimal place.

a.

Score x	Frequency f
120–129	2
130–139	8
140–149	10
150–159	5
160–169	1

b.

Score	Frequency
120–129	2
130–139	8
140–149	10
150–159	16
160–169	5
170–179	1

Hint: The tables above show grouped discrete data, where the midpoint of the first class interval is $\frac{120 + 129}{2} = 124.5$.

6. For the stem plot shown, calculate:

- a. the median
b. the mean (to 2 decimal places)
c. the mode.

Key: 3* | 7 = 37

Stem	Leaf
1	34
1*	5679
2	02334
2*	5556678
3	1234
3*	57
4	1

7. **MC** For the following set of data:
1, 5, 2, 8, 7, 3, 5, 7, 2, 8, 7, 6, 4, 5, 5
the median, mean and mode is best given by:

A. 5, 6, 5 **B.** 6, 3, 5 **C.** 5, 5, 5 **D.** 5, 5, 7

8. **MC** The median number of members per family surveyed and presented in this frequency table is:

Number of members per family	Frequency
3	19
4	53
5	21
6	4
7	2
8	1

A. 3 **B.** 4 **C.** 4.5 **D.** 4.6

9. A survey of the number of people living in each house on a residential street produced the following data.
- 2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4
- Prepare a frequency distribution table with an $f \times x$ column and use it to find the average (mean) number of people per household. Give your answer to 1 decimal place.
 - Draw a dot plot of the data and use it to find the median number per household.
 - Find the modal number per household.
 - Which of the measures would be most useful to:
 - a real estate agent renting out houses
 - a government population survey
 - an ice-cream mobile vendor?

Communicating, reasoning and problem solving

10. A small business pays these wages (in thousands of dollars) to its employees:
- 18, 18, 18, 18, 26, 26, 26, 35, 80 (boss).
- MC** What is the wage earned by most workers?
 - \$29 400
 - \$18 000
 - \$29 000
 - \$26 000
 - What is the average wage?
 - MC** Find the median of the distribution.
 - \$26 000
 - \$26 500
 - \$18 000
 - \$18 500
 - Which measure might be used in wage negotiations by:
 - the union, representing the employees (other than the boss)
 - the boss?

Explain each answer.
11. A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.
- Explain how this is possible.
 - Explain how many must have scored below 54.
12. A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. Explain how they scored 2 goals in that latest match.
13. A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the table.

Distance in km ('000)	82	78	56	52	50	46
Number of tyres	2	4	10	16	12	4

- Calculate the mean, median and mode.
- Which measure best describes average tyre life? Explain.
- Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered? Give your answer to the nearest km.
- If you selected a tyre at random, what tyre life has the highest probability of being selected?

- e. In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?
- f. As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?
14. For the set of numbers given below, the mean = 11, mode = 4, and the median = 9. All of the numbers in the set are positive integers.
4, __, 19, __, 21, 4
- Find two values for the missing numbers.
 - Find two different values that meet the stated criteria.
 - Can you substitute 9 for both missing values? Explain.
15. a. Create a data set of 8 numbers that meet the following criteria.
- Data values are positive integers
 - Mean = 9.5
 - Median = 7
 - Mode = 6
 - Maximum value in data set = 21
 - Minimum value in data set = 1
- b. Is there only one data set that can meet this criteria? Explain.
16. Calculate the mean for this data set.

Score	4	8	12	16	20	24
Frequency	$4a$	$3a$	$2a$	a	$\frac{a}{2}$	$\frac{a}{4}$

17. a. Create a set of five values with a mean of 4 and a median of 5. Demonstrate that this is true.
b. Change only one of your numbers in the set so that the mean is now 5 and the median remains 5. Demonstrate that this is true.
18. Seven students did a very difficult Science test. The test was out of 15. Six of the students got the following marks: 4, 4, 5, 5, 5, 6. Determine the mark that the 7th student obtained if:
- the mean is the same as the median
 - the mean is greater than the median
 - the mean is equal to the mode
 - the mean is greater than 6.
19. On a piece of paper is a list of six 2-digit prime numbers. They have a mean and median of 39. Their mode is 31, and the smallest number is 13. What are the six numbers? Explain how you deduced your answer.
20. What is the unique feature of the data collected when all three measures of central tendency are the same or very similar?
21. I have chosen 5 different whole numbers. Their mean is 5. The product of the 5 numbers is 945. Two of the numbers are 1 and 9. What are the remaining 3 numbers?
22. The data sets below show the heights (in cm) of 12 boys and 12 girls in a Year 9 class.
Boys: 154, 156, 156, 158, 159, 160, 162, 164, 168, 171, 172, 180
Girls: 145, 148, 152, 153, 155, 156, 160, 161, 161, 163, 165, 168
- Display this data in a back-to-back stem-and-leaf plot.
 - Calculate the mean and median heights for the boys and the girls.
 - Compare the heights of the boys and girls in this data set using the measures of centre.

5.6 Quartiles, interquartile range and distributions

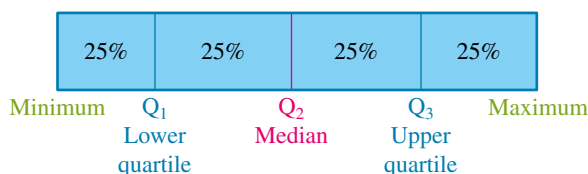
[Stage 5.2]

5.6.1 Measures of spread

- **Measures of spread** are used to describe a distribution. There are three common measures of spread: the range, the interquartile range and the standard deviation. You will learn about the standard deviation in Year 10.
- The **range** is the difference between the highest and lowest scores.

$$\begin{aligned}\text{Range} &= \text{highest score} - \text{lowest score} \\ &= x_H - x_L\end{aligned}$$

- The **interquartile range (IQR)** is the range of the middle 50% of all the scores in an ordered set. When the interquartile range is calculated, the data is first organised into quartiles, each containing 25% of the data. The word ‘quartiles’ comes from the word ‘quarter’.



$$\text{Interquartile range} = \text{upper quartile} - \text{lower quartile}$$

This can be written as:

$$\text{IQR} = Q_{\text{upper}} - Q_{\text{lower}}$$

or

$$\text{IQR} = Q_3 - Q_1$$

WORKED EXAMPLE 17

Find the range and interquartile range of the following data set.

5 6 8 4 12 5 7 8 4 10 9 6 5

THINK

- 1 Arrange data from lowest to highest score.
- 2 Calculate the range.

- 3 Find the median by using the rule:

$$\text{median score} = \frac{n+1}{2}\text{th score.}$$

- 4 Find the lower quartile (Q_{lower}) from the 6 scores in the lower half of the data set.

WRITE

4 4 5 5 5 6 6 7 8 8 9 10 12

$$\begin{aligned}\text{Range} &= \text{highest score} - \text{lowest score} \\ &= x_H - x_L \\ &= 12 - 4 \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{Median score} &= \frac{n+1}{2}\text{th score} \\ &= \frac{13+1}{2} = 7\text{th score}\end{aligned}$$

The median is the 7th score.

$$\therefore \text{median} = 6$$

4 4 5 5 5 6 6 7 8 8 9 10 12

Q_{lower} is between the 3rd and 4th scores

$$Q_{\text{lower}} = 5$$

- 5 Find the upper quartile (Q_{upper}) from the 6 scores in the upper half of the data set.

Q_{upper} is between the 3rd last and 4th last scores.

$$Q_{\text{upper}} = \frac{8 + 9}{2} = 8.5$$

- 6 Subtract the lower quartile from the upper quartile to find the interquartile range.

$$\begin{aligned} \text{IQR} &= Q_{\text{upper}} - Q_{\text{lower}} \\ &= 8.5 - 5 \\ &= 3.5 \end{aligned}$$

WORKED EXAMPLE 18

For the stem plot shown, determine the range and the interquartile range.

Key: 4 | 7 = 4.7 kg

Stem	Leaf
1	9
2	2 7
3	0 1 3 7
4	2 4 5 7 8
5	1 2 8
6	0

THINK

- 1 Calculate the range by finding the difference between the lowest and highest scores.

- 2 Count the number of scores, n .

- 3 Find the median by using the rule:
median score = $\frac{n+1}{2}$ th score.

- 4 Find the lower quartile (Q_{lower}) from the 8 scores in the lower half of the data set.

WRITE

$$\begin{aligned} \text{Range} &= \text{highest score} - \text{lowest score} \\ &= x_H - x_L \\ &= 6.0 - 1.9 \\ &= 4.1 \text{ kg} \end{aligned}$$

$$n = 16$$

$$\begin{aligned} \text{Median score} &= \frac{n+1}{2} \text{th score} \\ &= \frac{16+1}{2} = 8.5 \text{th score} \end{aligned}$$

The median is between the 8th and 9th scores, i.e. between 4.2 and 4.4.

$\therefore$ median = 4.3 kg.

Stem	Leaf
1	9
2	2 7
3	0 1 3 7
4	2 4 5 7 8
5	1 2 8
6	0

Q_{lower} is between the 4th score (3.0) and the 5th score (3.1).

$$\therefore Q_{\text{lower}} = 3.05 \text{ kg}$$

- 5 Find the upper quartile (Q_{upper}) from the 8 scores in the upper half of the data set.

Q_{upper} is between the 4th last score (5.1) and the 5th last score (4.8), that is 4.95 kg.

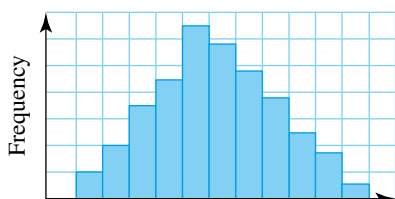
$$\therefore Q_{\text{upper}} = 4.95 \text{ kg}$$

- 6 Determine the interquartile range by subtracting the lower quartile from the upper quartile.

$$\begin{aligned} \text{IQR} &= Q_{\text{upper}} - Q_{\text{lower}} \\ &= 4.95 - 3.05 \\ &= 1.9 \text{ kg} \end{aligned}$$

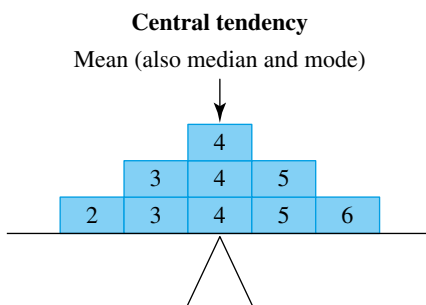
5.6.2 Types of distribution

- Distribution refers to the way data are spread out.
- Depending on how data are distributed or spread out, particular measures of centre and spread may provide different results for the data.
- Symmetrical distributions**, also referred to as bell-shaped or normal distributions, have a clear centre of the distribution and an even spread on either side.

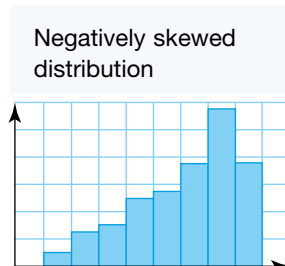
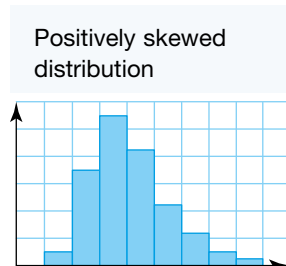


Stem	Leaf
0	7
1	2 3
2	2 4 5 7 9
3	0 2 3 6 8 8
4	4 7 8 9 9
5	2 7 8
6	1 3

- For symmetrical distributions, all three measures of central tendency (the mean, the median and the mode) give similar results.



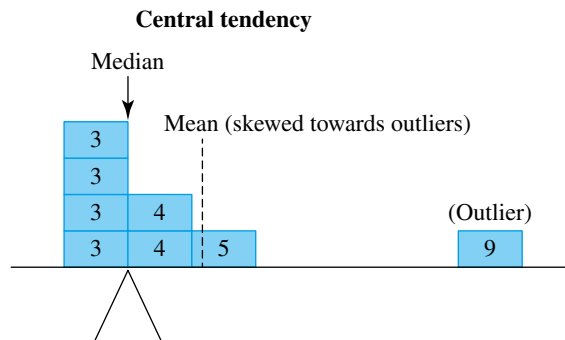
- A skewed distribution is a non-symmetrical distribution.
 - A right or **positively skewed distribution** has few high scores that are spread over a large range to the right. The most common scores are located to the left.
 - A left or **negatively skewed distribution** has few low scores that are spread over a large range to the left. The most common scores are located to the right.



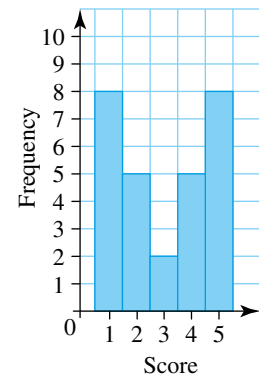
DISCUSSION

Can you think of examples of distributions which may be symmetrical, positively skewed or negatively skewed?

- For skewed distributions, the mean is affected by outlier(s) or unusual scores.

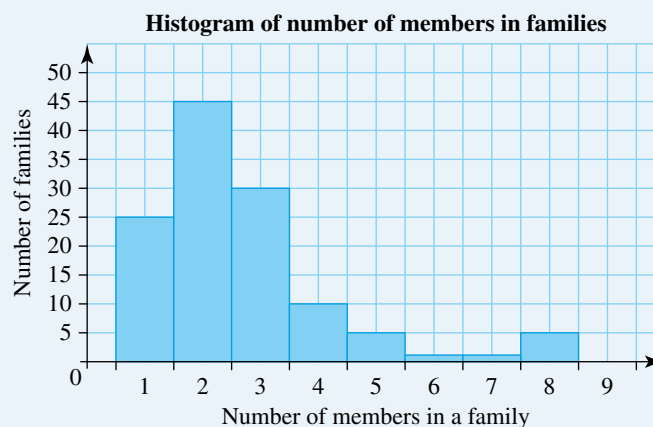


- The mean is shifted towards the outlier(s).
- The median is not affected by outliers and is thus the preferred measure of central tendency.
- Interquartile range is the preferred measure of spread.
- **Bi-modal distributions** have two peaks. When this occurs it indicates that the measurement or observation includes two different groups. An example would be measuring the heights of 100 adults. We could find two modes, one for the heights of adult males and the other mode for adult females. Such distributions would then suggest that the data analysis be modified to separate the two unique groups.
- For skewed distributions, the mean is affected by outliers or unusual scores. The median is not affected and is thus the preferred measure of central tendency. The mean is affected by outliers and is shifted towards the outlier.
- The mode is used primarily for describing the centre of categorical data.



WORKED EXAMPLE 19

For the distribution shown, describe the type of the distribution and determine its centre and spread using the most appropriate measures.



THINK

- 1 Describe the shape of the distribution.

WRITE

The graph is positively skewed.

2 Select appropriate measures of centre and spread.

3 Determine the median and interquartile range.

4 State the answer.

Measure of centre: median

Measure of spread: interquartile range.

The median is the average of the 61st and 62nd scores.

$$\text{Median} = \frac{2 + 2}{2} = 2$$

Q_{lower} is the 31st score.

$$Q_{\text{lower}} = 2$$

Q_{upper} is the 92nd score.

$$Q_{\text{upper}} = 3$$

$$\begin{aligned} \text{IQR} &= Q_{\text{upper}} - Q_{\text{lower}} \\ &= 3 - 2 = 1 \end{aligned}$$

The median number of members in a family is 2 and the interquartile range is 1.

WORKED EXAMPLE 20

Determine the best measures of central tendency and spread for the heights of 25 people shown in the stem plot. Give your answer to the nearest centimetre.

Key: 17 | 8 = 178 cm

Stem	Leaf
14	8
15	3 6
16	1 4 7
17	0 0 7 8 9
18	1 2 2 4 6 8 8
19	3 4 7 7
20	0 2
21	2

THINK

- 1 Describe the distribution.
- 2 State the most appropriate measure of centre.
- 3 Calculate the mean.

WRITE/DISPLAY

A symmetrical or normal distribution.

Measure of centre: mean

$$\begin{aligned} \text{Mean} = \bar{x} &= \frac{\sum x}{n} = \frac{\text{sum of all scores}}{\text{number of scores}} \\ &= \frac{4509}{25} \\ &= 180.36 \end{aligned}$$

4 State the answer.

The mean height of the 25 people is 180 cm.

learn on RESOURCES — ONLINE ONLY

Interactivity: Range (int-3822)

Interactivity: Interquartile range (int-4813)

Interactivity: Skewness (int-3823)

Digital doc: WorkSHEET Statistics 2 (doc-6324)

Exercise 5.6 Quartiles, interquartile range and distributions

Individual pathways

PRACTISE

Questions:
1–12, 16

CONSOLIDATE

Questions:
1, 3, 5, 6, 8, 10, 11–14, 16

MASTER

Questions:
1–3, 5, 7, 9, 11–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE17** Find the range and interquartile range for the following raw data sets.
 - 120 124 125 135 135 137 141 145 151
 - 18 22 25 27 29 31 35 37 40 42 44
- WE18** This stem plot gives the spread of 25 cars caught by a roadside speed camera.

Key: 8 | 2 = 82 km/h

8* | 6 = 86 km/h

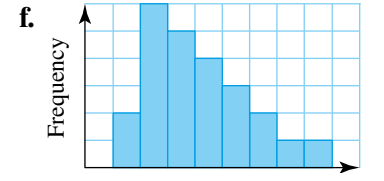
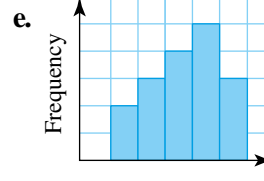
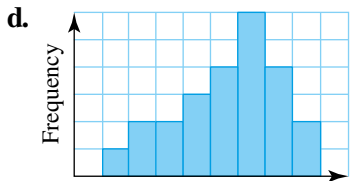
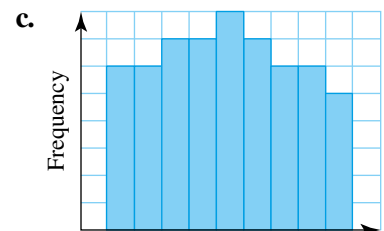
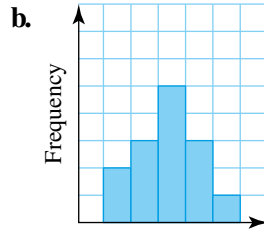
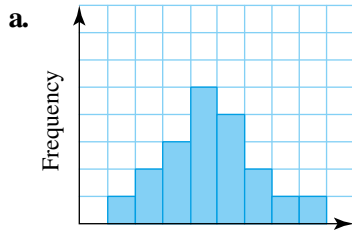
Stem	Leaf
8	2 2 4 4 4 4
8*	5 5 6 6 7 9 9 9
9	0 1 1 2 4
9*	5 6 9
10	0 2
10*	
11	4

Determine:

- the range
 - the interquartile range.
- For each of the following raw data sets, find the range and interquartile range
 - 20 45 73 65 85 127 101
 - 1.22 1.28 1.48 1.52 1.62 1.72 1.85 1.95 2.03
 - For the ungrouped frequency table shown, calculate:
 - the spread — the range and interquartile range
 - the central tendency — the mode, the median and mean (correct to 2 decimal places).

Score	Frequency
0	24
1	17
2	12
3	7
4	2
5	1

5. For each of the following histograms, describe the shape of the distribution of the data.



6. For each of the following stem plots, state the type of distribution.

a. Key: $14 \mid 6 = 14.6$

Stem	Leaf
12	0
13	8 9
14	0 8 9 9
15	1 5 6 6 6 7 8
16	3 5 7 8
17	0 2
18	3 5

b. Key: $14 \mid 6 = 146$

Stem	Leaf
13	1 2 4 6 7 8 9
14	0 4 5 6 8 9 9
15	1 5 6 6 7 8
16	3 7 8
17	0 2
18	3

c. Key: $14 \mid 6 = 1.46$

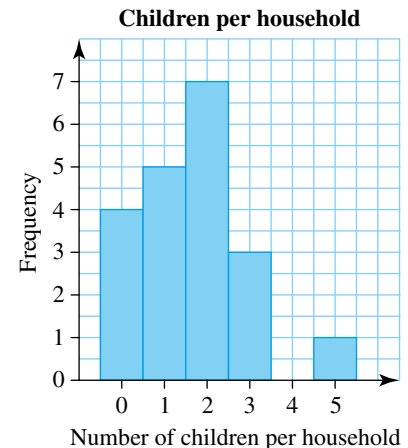
Stem	Leaf
12	7 8
13	6 7 8 9
14	0 9
15	1 5 6
16	3 7 8 9
17	0 2
18	3

d. Key: $14 \mid 6 = 1.46$

Stem	Leaf
1	1
2	
3	6 9
4	0 4 9
5	1 3 5 6 8
6	2 3 7 8 9 9
7	0 4 6 7 7 8

7. **WE19** The histogram at right represents the results of data collected from households in a suburban street. Describe the type of distribution, determining its centre and spread using the most appropriate measure.

8. **WE20** The number of customers served each hour for the past 24-hour period is summarised in the following stem plot.
Key: $2 \mid 4 = 24$ customers

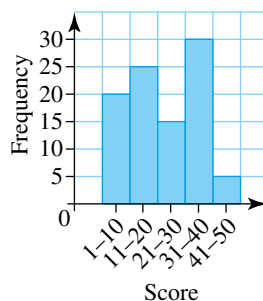


Stem	Leaf
0	0 0 1 4
0*	5 5 8 8 9 9 9
1	1 2 3 4 4 4
1*	5 5 5 7
2	0 2
2*	
3	4

- State the type of data and type of distribution.
 - Choose the most appropriate measure of central tendency and spread. Give a reason for your choice.
 - Calculate the centre and spread chosen in b.
9. **MC** Which of the following statements is false?
- Median and interquartile range are not affected by outliers.
 - Mean and range are affected by outliers.
 - Interquartile range is a good measure of spread for skewed distributions.
 - Range and interquartile range can be used on categorical data.
10. **MC** For the frequency table below, the most likely type of distribution is a:
- negatively skewed distribution
 - positively skewed distribution
 - bimodal distribution
 - normal distribution

Score	0	1	2	3	4	5	6	7
Frequency	3	4	5	7	10	22	34	45

11. Consider the distribution shown.



- Are the data symmetrical?
 - What is the modal class?
 - Can the mean and median be seen from the graph and, if so, what are their values?
- Communicating, reasoning and problem solving**
12. A Geography teacher with a class of 20 students commented that test results for his class out of a score of 50 were negatively skewed. Show and explain a data set for the possible results obtained for the 20 students.
13. Show a data set where the range and interquartile range are the same. Explain what factors cause this.
14. Explain the effect on the range when a set of scores has:
- 5 added to each score
 - each score doubled
 - each score increased by 25%.

15. A symmetric distribution is one where the mean and median are equal. A uniform distribution (one where the elements are all the same) is symmetric. Give an example of a non-uniform distribution which is symmetric.
16. This back-to-back stem-and-leaf plot displays the speeds of 25 cars caught by a speed camera on a major highway on Easter Sunday and Christmas Day.

<i>Easter Sunday</i>		<i>Christmas Day</i>
863	6	23699
8741	7	12245578
665421	8	003467
9877521	9	1233
77521	10	24



The police commissioner was heard to comment that he was surprised to find drivers behaved better on Christmas Day than they did on Easter Sunday. His department had done an analysis of the data and found that both the mean and the interquartile range of speeds on Easter Sunday were greater than those for Christmas Day. Discuss his statement.

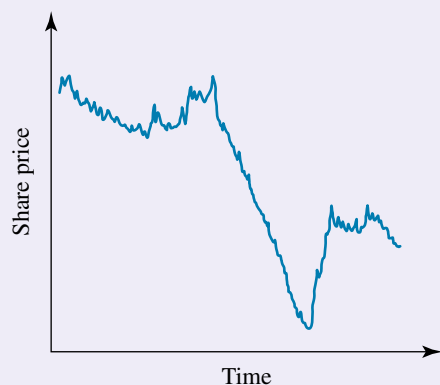
17. A famous phrase possibly first used by Mark Twain was 'Lies, damned lies and statistics'. This phrase suggests statistics can be used to twist the truth. Suggest a golden rule to avoid statistics being used inappropriately.

5.7 Review

Investigation | Rich task

Graphical displays of data

Statistical data can be communicated in the form of written prose, tables and graphs. It is difficult to compare and see trends in written prose. To gain a full understanding of trends and comparisons of data presented in table form, we often need to perform additional calculations. The visual impact of graphs is immediate; however, often the impression we receive is a misleading one if we fail to look carefully at the graph. Graphs can be manipulated to produce the desired effect. Consider the two graphs below.



These graphs both display the same data (the value of shares over the same period of time), yet they look quite different. If the objective of the graph is to create the impression that the value of the shares has dropped quite dramatically, the first graph would serve this purpose. On the other hand, if we are to be convinced that the shares have not changed very much in value, the second graph reflects this point of view.

1. What technique is used to create two quite opposing impressions in the two graphs?

The following table shows the times for the men's and women's 100-m sprints in the Summer Olympic Games. A sports journalist wants to convince readers that, since 1928, the women have been more successful in reducing their time than the men.

Year	100 m men	Men's time (seconds)	100 m women	Women's time (seconds)
1928	Percy Williams, CAN	10.80	Betty Robinson, USA	12.20
1932	Eddie Tolan, USA	10.30	Stella Walsh, POL	11.90
1936	Jesse Owens, USA	10.30	Helen Stephens, USA	11.50
1948	Harrison Dillard, USA	10.30	Fanny Blankers-Koen, HOL	11.90
1952	Lindy Remigino, USA	10.40	Marjorie Jackson, AUS	11.50
1956	Bobby Morrow, USA	10.50	Betty Cuthbert, AUS	11.50
1960	Armin Hary, GER	10.20	Wilma Rudolph, USA	11.00
1964	Bob Hayes, USA	10.00	Wyomia Tyus, USA	11.40
1968	Jim Hines, USA	9.95	Wyomia Tyus, USA	11.08
1972	Valery Borzov, URS	10.14	Renate Stecher, GDR	11.07
1976	Hasely Crawford, TRI	10.06	Annegret Richter, FRG	11.08
1980	Allan Wells, GBR	10.25	Lyudmila Kondratyeva, URS	11.06
1984	Carl Lewis, USA	9.99	Evelyn Ashford, USA	10.97
1988	Carl Lewis, USA	9.92	Florence Griffith Joyner, USA	10.54
1992	Linford Christie, GBR	9.96	Gail Devers, USA	10.82
1996	Donovan Bailey, CAN	9.84	Gail Devers, USA	10.94
2000	Maurice Greene, USA	9.87	Marion Jones, USA	10.75
2004	Justin Gatlin, USA	9.85	Yuliya Nesterenko, BLR	10.93
2008	Usain Bolt, JAM	9.69	Shelley-Ann Fraser, JAM	10.78
2012	Usain Bolt, JAM	9.63	Shelley-Ann Fraser, JAM	10.75
2016	Usain Bolt, JAM	9.81	Tori Bowie, USA	10.90

Note: The Summer Olympic Games were not held in 1940 and 1944.

2. On a separate sheet of graph paper, draw two graphs to support the sports journalist's view.
3. Describe features of your graphs that helped you present the journalist's view effectively.
4. Imagine you are the sports journalist and you are about to publish the graphs in the local newspaper. Write a short journal article to accompany the graphs.

Digital doc: Investigation — Graphical displays of data (doc-15910)

Digital doc: Code puzzle — The five smallest countries in the world (in area) (doc-15911)

Digital doc: Summary Topic 5 (doc-22887)

Interactivity: Crossword Topic 5 (int-2715)

Interactivity: Word search Topic 5 (int-2714)

Interactivity: Sudoku Topic 5 (int-3213)

Exercise 5.7 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **MC** Which one of the following is an example of numerical discrete data?

- A. Your favourite weeknight television show
- B. The speed of a car recorded on a speed camera
- C. The number of home runs by a baseball player
- D. Placing in a field of eight swimmers of a 100 m freestyle race

The following information relates to questions 2 and 3.

Number of bedrooms	1	2	3	4	5	6
Number of homes	4	9	32	36	7	2

2. **MC** For the data summarised, the most likely type of distribution is:
- A. a negatively skewed distribution
 - B. a positively skewed distribution
 - C. skewed to the left
 - D. a symmetric distribution.
3. **MC** The mode for the data collected is:
- A. 36
 - B. 5
 - C. 4
 - D. 3
4. **MC** The results of a Science test marked out of 60 are represented by the stem plot shown below.

Key: 2 | 3 = 23

Stem	Leaf
0	8
1	8
2	3 5
3	0 3 6
4	2 4 5 5 5 8
5	4 7 8 8 9
6	0 0 0 0

The median, mean and mode, respectively, are:

- A. 60, 42, 52
 - B. 40, 45, 60
 - C. 45, 60, 45
 - D. 45, 44, 60
5. **MC** The range and IQR for the following set of data is:
- 46, 46, 49, 53, 61, 63, 67, 72, 81, 84, 93
- A. 46 and 34
 - B. 46 and 65
 - C. 47 and 63
 - D. 47 and 32
6. For the following surveys or studies conducted, decide what type of data would be collected.
- a. The blood type of all workers at a shoe factory
 - b. A survey of 500 Australians' views (strongly agree, agree, disagree, strongly disagree) on bringing in corporal punishment

- c. Mobile phones owned in the households of students in your class
 - d. The top 20 fastest recorded times for the women's 100 metres butterfly
7. Classify each of the following data collections according to whether they are samples or populations.
- a. A survey of 1000 travellers on the quality of train service provided
 - b. A survey of all the students at a secondary college about the state government's rules on under-age alcohol laws
 - c. Local schools' Mathematics performance in this year's Year 9 cohort of all this year's students' assessed examination results
8. Determine the number from each group to be surveyed in the following, if stratified sampling was to be conducted.
- a. Survey of 60 members from a tennis club of 600 members with 200 male adult members, 100 female adult members and 300 teenage members.
 - b. Survey of 100 students from a school population of 1250 students where there are 100 Year 12s, 150 Year 11s, 250 Year 10s, 250 Year 9s, 250 Year 8s and 250 Year 7s.
9. The following set of data shows the types of mobiles owned by the first 30 shoppers surveyed.
Nokia, Samsung, LG, Nokia, Nokia, Nokia, Samsung, Samsung, Blackberry, LG, Samsung, Nokia, LG, Nokia, Samsung, Samsung, LG, Nokia, Samsung, Blackberry, Samsung, LG, Nokia, Nokia, Nokia, Nokia, Samsung, Samsung, Blackberry, LG
- a. Present the data as a dot plot.
 - b. Present the data in a frequency table.
10. The following data represent the number of mp3 songs found on rented laptops. An initial analysis of 25 laptops resulted in the following data being collected.
47, 103, 45, 111, 67, 54, 59, 44, 31, 89, 34, 2, 43, 96, 70, 90, 66, 48, 67, 82, 74, 56, 35, 9, 77
- a. Present the data as stem plot.
 - b. Comment on the shape of distribution.
 - c. Present the data as a frequency table grouped in intervals of 20.
 - d. State the type of data, type of distribution and give the most appropriate measures of central tendency and spread.
11. The following table shows the incomes for a group of professional golfers in 2017.

Golfers' income	Number of golfers
0–\$100 000	146
\$100 001–\$200 000	188
\$200 001–\$300 000	107
\$300 001–\$400 000	57
\$400 001–\$500 000	24
\$500 001–\$600 000	12
\$600 001–\$700 000	4
\$700 001–\$800 000	0
\$800 001–\$900 000	4
Total number of golfers	542



- a. Draw a histogram to display the data.
- b. Describe the type of distribution the histogram displays.
- c. Calculate the median income of these golfers (to the nearest thousand).
- d. Find the modal income for these professional golfers in 2017.

Communicating, reasoning and problem solving

12. The workers in an office are trying to obtain a wage rise. In the previous year, the ten people who work in the office each received a 2% wage rise, while the company CEO received a 42% wage rise.
- What was the mean wage rise received in the office last year? Give your answer to 1 decimal place.
 - What was the median percentage wage rise received?
 - What was the modal percentage wage rise received?
 - The company is trying to avoid paying the rise. Explain the statistics you think they would quote about last year's rise. Give your reason.
 - What statistics do you think the trade union would quote about wage rises? Explain why.
 - Which statistic do you think is the most 'honest' reflection of last year's wage rises? Explain why.
13. A stratified random sample was taken from two high schools: a 312-student high school and a larger high school. The sample would assess how many students watched more than four hours of television per day. The small school represented 12.5% of the total students surveyed, and 200 total students were randomly surveyed.
- How many students were surveyed in the small school?
 - How many students were surveyed in the large school?
 - How many students were in the large high school?
 - How many students were in both schools?
14. If the mean = 15, mode = 8 and the median = 14 for the following set (and the missing values are positive integers):
8, __, 25, __, 21, 8
- find two values for the missing numbers
 - find two different values that meet the stated criteria.
15. Create a data set of 9 numbers that meet the following criteria.
- Data values are positive integers
 - Mean = 8
 - Median = 7
 - Mode = 10
 - Maximum value in data set = 14
 - Minimum value in data set = 3
16. Life expectancies in 2015 for females and males in G20 countries are shown in the table below.
(Note: The G20 is made up of 19 countries plus the European Union.)

Country	Life expectancy (years)	
	Females	Males
Australia	84.8	80.9
Canada	84.1	80.2
Saudi Arabia	76.0	73.2
United States	81.6	76.9
India	69.9	66.9
Russia	76.3	64.7
South Africa	66.2	59.3
Turkey	78.9	72.6
Argentina	79.9	72.7
Brazil	78.7	71.4

Mexico	79.5	73.9
France	85.4	79.4
Germany	83.4	78.7
Italy	84.8	80.5
United Kingdom	83.0	79.4
China	77.6	74.6
Indonesia	71.2	67.1
Japan	86.8	80.5
South Korea	85.5	78.8

- a.** Calculate the mean, median, interquartile range and range for each of the two data sets. Give your answers to the nearest whole number.
- b.** By referring to the means, medians and ranges, compare the life expectancies of females and males in these 19 countries.

Answers

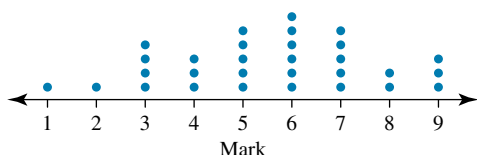
Topic 5 Single variable data analysis

Exercise 5.2 Collecting data

1. a. Categorical b. Numerical c. Numerical
d. Numerical e. Categorical
2. a. Continuous b. Continuous c. Continuous
d. Discrete e. Discrete
3. a. Nominal b. Ordinal c. Ordinal
d. Nominal e. Nominal
4. a. Sample b. Sample c. Population
d. Sample e. Population
5. a. Biased b. Biased c. Biased
6. a. 32 males, 8 females
b. 700 females, 300 males
c. Y12s — 10, Y11s — 15, Y10s — 17, Y9s — 20, Y8s — 20, Y7s — 18
d. 5 girls, 3 boys
7. a. 1–18, randint (1, 18, 4) b. 1– x , randint (1, x , 5)
c. 1– x , randint (1, x , 50) d. 1–365, randint (1, 365, 65)
8. D 9. C 10. D 11. randint (1, 365, 7)
12. a. Car b–e. Answers will vary.
13. a. Answers will vary. b. 182 c. 1400 d. Answers will vary.
14. Answers will vary but should include the following groups.
Categorical data can be:
 - nominal — arranged in categories, for example by colour or age
 - ordinal — arranged in categories that have an order; for example, karate ability ranges from white belt (beginner) to black belt (accomplished).
 Numerical data can be:
 - discrete — data can have only particular values, for example amounts of money or shoe sizes
 - continuous — data can have any value within a range, for example the time it takes a person to run around an oval.
- 15–21. Answers will vary.
22. Check with your teacher as the answer depends on the number of people in your class.

Exercise 5.3 Stem-and-leaf plots and dot plots

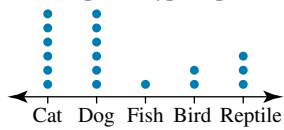
1. a. Multiple choice Maths test results



b.

Score (x)	Frequency (f)
1	1
2	1
3	4
4	3
5	5
6	6
7	5
8	2
9	3

2. a. Dot plot of type of pet

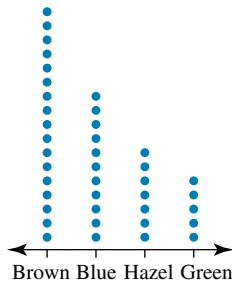


b.

Type of pet (x)	Frequency (f)
Cat	6
Dog	6
Fish	1
Bird	2
Reptile	3

- c. No, only numeric data can be presented in a stem plot.

3. a. Dot plot of eye color of 40 people



b.

Score (x)	Frequency (f)
Brown	17
Blue	11
Hazel	7
Green	5
$\Sigma f =$	40

- c. 17 out of 40 people or 42.5% of the people have brown eyes.

4. a. Key: 118 = 18

Stem	Leaf
1	899
2	000112

Not symmetrical; most scores in low 20s

- b. Key: 119 = 19

Stem	Leaf
1	99
2	0445679
3	1168
4	23

Not symmetrical

- d. Key: 413 = 43

Stem	Leaf
4	346789
5	25
6	7
7	07
8	1233458

Almost symmetrical

5. a. Key: 1*18 = 18 210 = 20

Stem	Leaf
1*	899
2	000112

- c. Key: 3316 = 336

Stem	Leaf
33	6
34	669
35	039
36	28
37	1

Almost symmetrical

- e. Key: 111 = 1.1

Stem	Leaf
1	1359
2	4789
3	369
4	126

Not symmetrical

b. Key: $1 \mid 9 = 19$ $2 \mid 0 = 20$

Stem	Leaf
1*	9 9
2	0 4 4
2*	5 6 7 9
3	1 1
3*	6 8
4	2 3

c. Key: $33 \mid 6 = 336$ $35 \mid 0 = 350$

Stem	Leaf
33*	6
34	
34*	6 6 9
35	0 3
35*	9
36	2
36*	8
37	1

d. Key: $4 \mid 3 = 43$ $4 \mid 6 = 46$

Stem	Leaf
4	3 4
4*	6 7 8 9
5	2
5*	5
6	
6*	7
7	0
7*	7
8	1 2 3 3 4
8*	5 8

e. Key: $1 \mid 1 = 1.1$ $1 \mid 5 = 1.5$

Stem	Leaf
1	1 3
1*	5 9
2	4
2*	7 8 9
3	3
3*	6 9
4	1 2
4*	6

6. a. Running times for 40 students

Key: $4 \mid 3 = 43$ minutes, $4 \mid 8 = 48$ minutes

Stem	Leaf
2	1 3
2*	5 6 6 8 9
3	1 2 2 3 3 3 4
3*	7 7 9 9
4	0 1 1 2 2 3 4
4*	5 5 7 8 8 9
5	2 2 3
5*	5 6 7
6	0 3
6*	
7	1

c. i. 32 min

ii. 30 min to less than 35 min

d. 3, stem plot

7. Key: $1 \mid 4 = 14$

$1 \mid 7 = 17$

Stem	Leaf
1	4
1*	7 8 8
	9
2	0 1 3
2*	5 6 7
	9
3	2 4
3*	5 6 7
	8
4	2
4*	5
5	3

b.

Run times, x (minutes)	Number of runners, f
$20 - < 25$	2
$25 - < 30$	5
$30 - < 35$	7
$35 - < 40$	4
$40 - < 45$	7
$45 - < 50$	6
$50 - < 55$	3
$55 - < 60$	3
$60 - < 65$	2
$65 - < 70$	0
$70 - < 75$	1
$\Sigma f =$	40

8. D

9. a.

Life expectancy, x (years)	Number of age groups, f
83–<84	1
84–<85	20
85–<86	7
86–<87	5
87–<88	3
88–<89	3
89–<90	1
$\Sigma f =$	40

b. Life expectancy of Australians across 40 different age groups

Key: 86 | 1 = 86.1 years

Stem	Leaf
83	7
84	1 2 3 4 4 5 5 5 5 6 6 6 6 7 7 7 8 8 9 9
85	0 0 1 2 7 8 9
86	0 5 6 8 9
87	1 3 6
88	3 5 8
89	0

c. Answers will vary. Half the people will live to 84, with others living longer.

10. a.  Row 7
Row 8
Row 9

The pattern is: two dots of the same colour produce a red dot in the row below; two dots of different colour produce a black dot in the row below. The last dot in the row is always black.

b. No, there must be a black dot at the end of each row.

c. No, as the pattern starts to repeat in Row 9 and no rows have all black dots.

d. Yes

11. a. Class 1 : 25; class 2 : 27

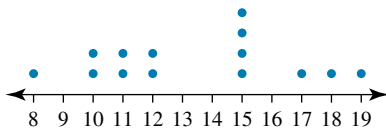
b. Class 1 : 2; class 2 : 3

c. Class 1 : 24; class 2 : 20

d. Eleven students in class 2 scored better than class 1 but 6 students scored worse than class 1. Class 2's results are more widely spread than class 1's results.

12. a. Dots are not aligned vertically. The horizontal scale is not equally spaced.

b.



13. a. Key: 25 | 1 = 25.1 g

Stem	Leaf
23	9 9
24	0 5 5 6 8
25	1 1 1 3 4 4 8
26	3 3 7
27	3 3
28	1
29	5
30	
31	
32	
33	2

b. The range is $33.2 - 23.9 = 9.3$.

c. Answers will vary but could include that the plot for rounded values provides less detail.

Key: $2 \mid 4 = 24 \text{ g}$

$2^* \mid 6 = 26 \text{ g}$

Stem	Leaf
2	4 4
2*	5 5 5 5 5 5 5 5 5 6 6 6 7 7
	7 8
3	0 3

14. a. Key: $3 \mid 2 = 32 \text{ km/h}$

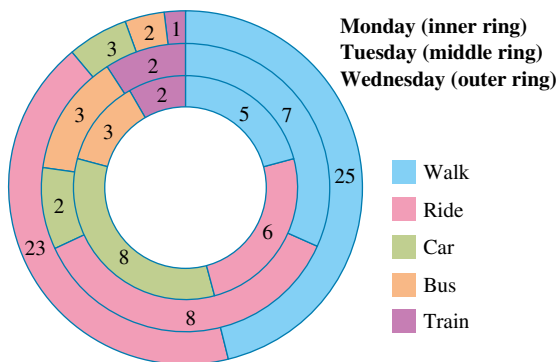
Stem	Leaf
1	5
2	0 0 2 7 9 9
3	0 0 0 0 2 4 5 6 7 8
	8 9
4	0 0 0 0 0 2 4 5 5 5
5	
6	0

b. Answers will vary. A possible answer could include that there were a high number of speeds ending in 0 (possibly indicating a machine malfunction), and most of the speeds (24 out of 30) were within the legal speed limit.

15. a. Answers will vary.

b. Draw circles of different size with the same centre. Then use the same instructions as for creating a pie chart but mark only the lines *between* the two circles.

c. **Methods of travel to school**



d. They are the same graph.

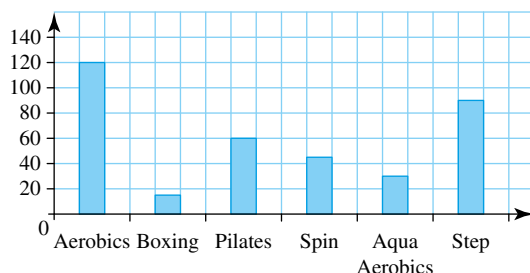
e. Answers will vary.

16. When collecting data it is important to consider: 1. question design, to ensure questions are clear, unambiguous, unbiased, and able to be answered by most respondents; and 2. data collection methods, which can be either random selection (ensures all elements of the population have an equal chance of being selected) or stratified selection (ensures that groups within a populations have a similar representation in the sample).

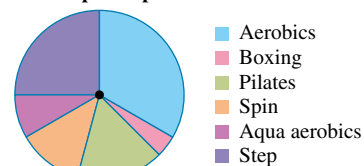
17. 1296

Exercise 5.4 Comparing and displaying data

1. a. **Bar chart of gym class participation**



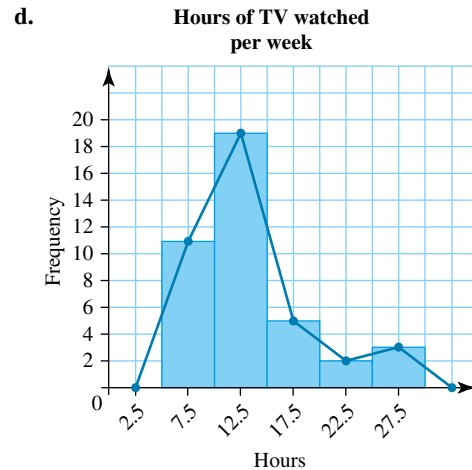
b. **Pie chart of gym class participation**



2. a. Continuous b. 24

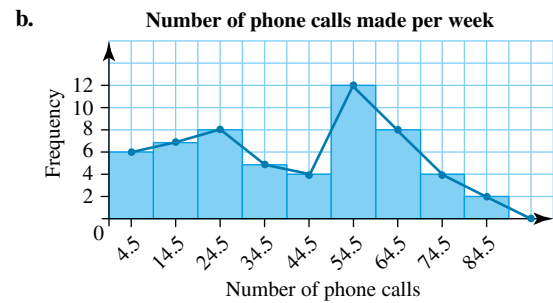
c.

Class interval	Midpoint (class centre)	Tally	Frequency
5–<10	7.5	IIII	11
10–<15	12.5	IIIIIIIIII	19
15–<20	17.5	III	5
20–<25	22.5	II	2
25–<30	27.5	III	3
Total			40



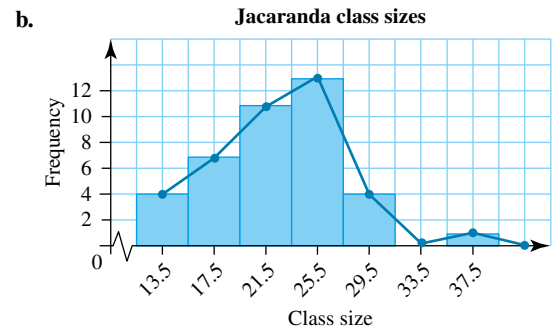
3. a.

Class interval	Midpoint (class centre)	Tally	Frequency
0–9	4.5	IIII	6
10–19	14.5	IIII	7
20–29	24.5	IIIIII	8
30–39	34.5	III	5
40–49	44.5	IIII	4
50–59	54.5	IIIIIIII	12
60–69	64.5	IIIIII	8
70–79	74.5	IIII	4
80–89	84.5	II	2
Total			56

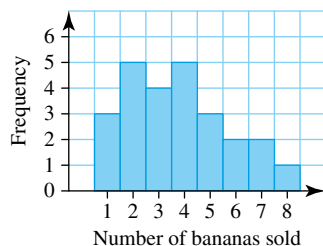


4. a.

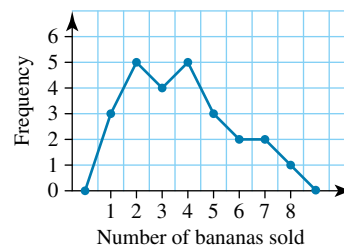
Class interval	Midpoint (class centre)	Tally	Frequency
12–15	13.5	IIII	4
16–19	17.5	IIII	7
20–23	21.5	IIIIII	11
24–27	25.5	IIIIIIII	13
28–31	29.5	IIII	4
32–35	33.5	—	0
36–39	37.5	I	1
Total			40



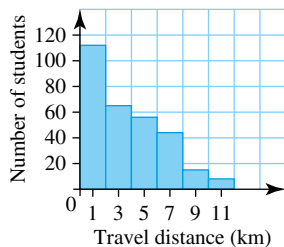
5. a. **Histogram of number of bananas sold at recess**



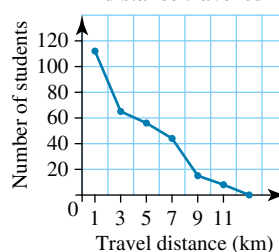
- b. **Frequency polygon of number of bananas sold at recess**



6. a. Histogram of distance travelled

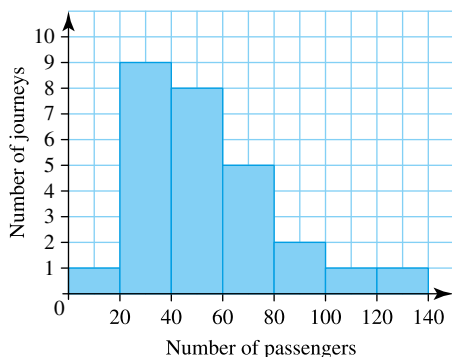


b. Frequency polygon of distance travelled

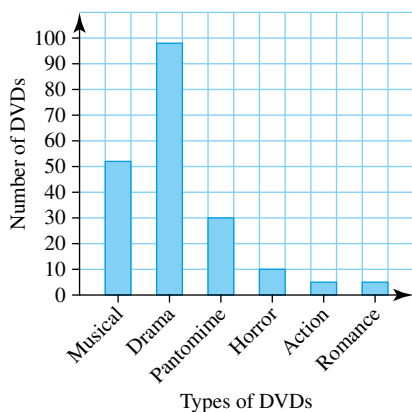


7. D

8. Histogram of number of passengers on bus journeys



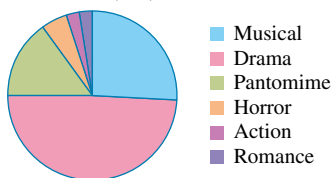
9. a. Bar chart of types of DVDs (20)



b.

Type of DVD	Number of DVDs	Size of sector for a pie chart
Musical	52	93.6°
Drama	98	176.4°
Pantomime	30	54°
Horror	10	18°
Action	5	9°
Romance	5	9°
Total	200	360°

c. Pie chart of types of DVDs (200)



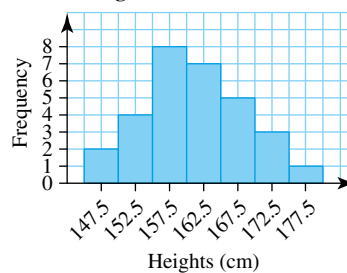
10. a. Continuous

b.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<150	147.5	II	2
150–<155	152.5	IIII	4
155–<160	157.5	IIII III	8
160–<165	162.5	IIII II	7
165–<170	167.5	IIII	5
170–<175	172.5	III	3
175–<180	177.5	I	1
Total			30

c.

Heights of Year 9 students



d. No. Individual scores are lost when data are grouped into class intervals.

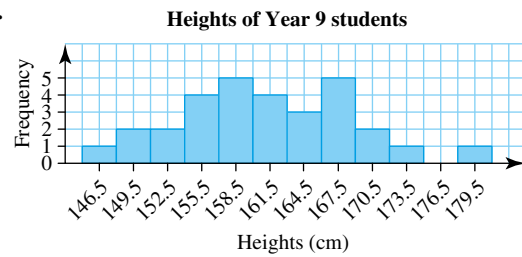
e. B

f. D

g.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<148	146.5	I	1
148–<151	149.5	II	2
151–<154	152.5	II	2
154–<157	155.5	IIII	4
157–<160	158.5	IIII	5
160–<163	161.5	IIII	4
163–<166	164.5	IIII	3
166–<169	167.5	IIII	5
169–<172	170.5	II	2
172–<175	173.5	I	1
175–<178	176.5	—	0
178–<181	179.5	I	1
Total			30

h.



Discuss in class.

11. Yes — change the class intervals.

12. a. Number of players

b. 5

13. Key: 0|0 = 0

Leaf	Stem	Leaf
(Mitch)		(Yani)
3 2 1 1 1 0 0 0 0	0	0 0 1 1 1 1 1 2 2 2

Mitch scored between 0 and 3 goals inclusive.

Yani scored between 0 and 2 goals inclusive.

14. a. Mitch: 2, Yani: 3

b. Mitch

c. Yani: 11

d. Yani, as he scored goals in more games.

15. a. Key: 3|2 = 32

Leaf	Stem	Leaf
(9A)		(9B)
2	3	
9	4	6
7 4	5	5
8 7 5 5 0	6	0 2 8
3	7	3 3 4 5 7
		8
5	8	1
6	9	

b. 9A: 83%, 9B: 92%

c. 9A: 2

d, e. Discuss in class.

f. 9A: 64.25%; 9B: 68.5%

16. a. C

b. D

c. In general the boys are slightly taller than the girls, with the tallest 3 students all being boys.

17. a. Categorical

b. A column graph or a pie chart

18. The most common number of times per week people used the sunscreen was 5. The minimum number of times was 2 and the maximum number was 21, although this value was so much higher than the rest of the data that it should be ignored. Fifty per cent of the people surveyed used the sunscreen between 4 and 7 times per week.

19. Stem plots show all data values, with the stem showing the group value and the leaf the final digit in the data value. This allows the statistician to easily see all the data which make up the data set and a visual representation of their distribution. While frequency tables also show the distribution of data, it is presented in ranges, which does not allow the statistician to see individual data values.

Exercise 5.5 Mean, median and mode

1. a. 55, 54, 55 b. 8, 6.88, 8
2. 2.5, 2.55, 2
3. a. 23 b. 116 c. 6.28
4. a. 5 b. 82 c. 2.55
5. a. 143.5, 142.6, 140–149 b. 150.9, 148.5, 150–159
6. a. 25 b. 25.24 c. 25
7. C
8. B
9. a.

Score (x)	Frequency (f)	Freq. $\times$ score ($f \times x$)
0	1	0
1	3	3
2	6	12
3	3	9
4	3	12
5	1	5
6	1	6
	$n = 18$	$\Sigma f = 47$

$$\bar{x} = 2.6$$

c. 2

d. i. Median ii. Mean iii. Mode

10. a. B b. \$29444 c. A d. Answers will vary.

11. a. The median was calculated by taking the average of the 2 middle scores.

b. 13

12. Answers will vary.

13. a. 55250 km, 52000 km, 52000 km

c. 51810 km; it is reduced by 3440 km.

e. 3333

b. Discuss in class.

d. 52000 km

f. 50000 km; 92% last that distance or more.

14. a, b. Answers will vary; options include 7, 11 and 8, 10.

c. No; 4 is the mode.

15. a. Answers will vary. b. No

16. 8.93

17. Example:

a. 1, 2, 5, 6, 6

$$\frac{1 + 2 + 5 + 6 + 6}{5} = \frac{20}{5} = 4$$

b. 1, 2, 5, 6, 11

$$\frac{1 + 2 + 5 + 6 + 11}{5} = \frac{25}{5} = 5$$

18. a. 6

b. > 6

c. 6

d. > 13

19. 13, 31, 31, 47, 53 and 59

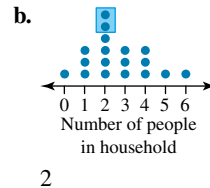
20. When all three measures of central tendency are the same or very similar, the data has a normal (or symmetrical) distribution.

21. 3, 5 and 7

22. a. Key: 15 | 6 = 156

Leaf (boys)	Stem	Leaf (girls)
	14	5 8
9 8 6 6 4	15	2 3 5 6
8 4 2 0	16	0 1 1 3 5 8
2 1	17	
0	18	

b. Mean (boys) = 163.3 cm; median (boys) = 161 cm; mean (girls) = 157.25 cm; median (girls) = 158 cm



- c. The mean and the median are very similar for the girls' heights, so either value could be used to describe the centre of the data. There is a difference of almost 2 cm between the mean and median for the boys' heights.

A study of the stem-and-leaf-plot shows a large value of 180 cm, which is much larger than the next value of 172 cm. This large value has a significant effect on the mean but not as much effect on the median. The median is therefore a better measure of the centre of the data for the boys.

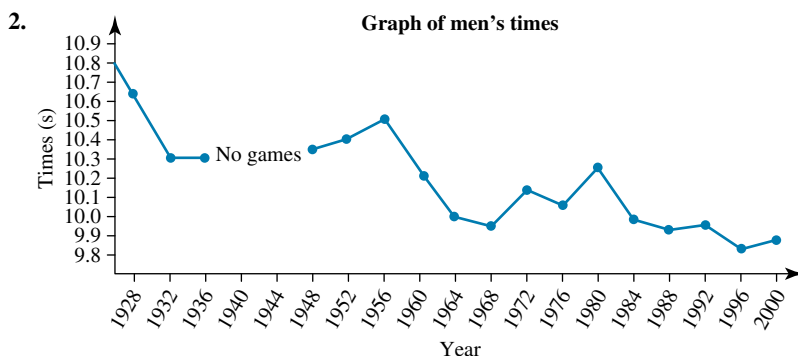
The boys are on average taller than the girls, with a median of 161 cm compared to the girls' median of 158. There are two girls under 150 cm tall and no boys under that height. There are three boys taller than 170 cm and no girls taller than 170 cm.

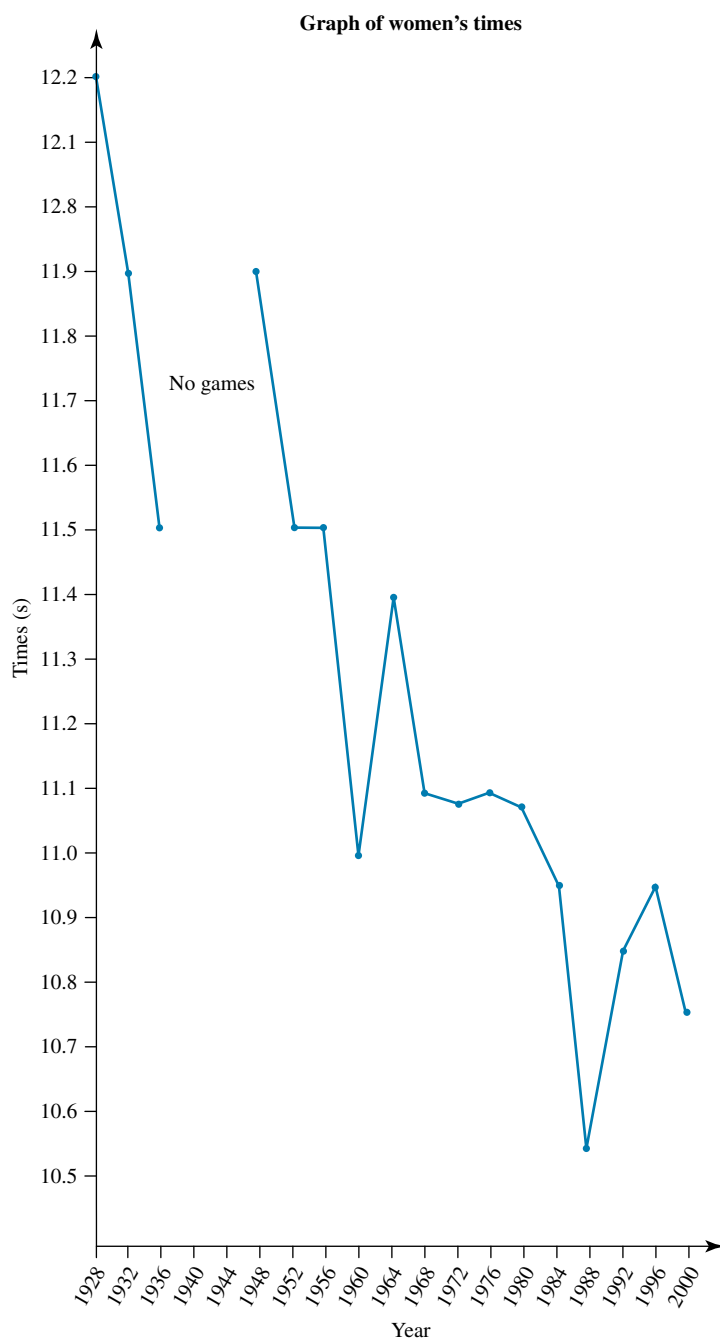
Exercise 5.6 Quartiles, interquartile range and distributions

1. a. 31, 18.5 b. 26, 15
2. a. 32 km/h b. 10 km/h
3. a. 107, 56 b. 0.81, 0.52
4. a. 5, 2 b. 0, 1
5. a. Symmetric b. Symmetric c. Symmetric
d. Negatively skewed e. Negatively skewed f. Positively skewed
6. a. Symmetric b. Positively skewed
c. Bimodal d. Negatively skewed with an outlier
7. The data is positively skewed, with a median of 2, and an interquartile range of 1. The mean is not an appropriate measure of centre as it is affected by the outlier.
8. a. Numerical discrete data with a positively skewed distribution
b. Median and interquartile range are best measure of central tendency and spread for a skewed distribution as they are not affected by outliers.
c. Median = 11.5 customers, IQR = 8.5 customers
9. D
10. A
11. a. No b. 31–40 c. No
12. Answers will vary.
13. Answers will vary. Possible data sets will have the highest score = Q_3 and the lowest score = Q_1 .
14. a. The same
b. Double
c. Increase by 25%
15. For example, { 1, 2, 2, 3, 3, 3, 4, 4, 5 }
16. Easter Sunday: mean = 87.7, IQR = 21
Christmas Day: mean = 79.9, IQR = 17.5
17. Answers will vary, but statistics should not be used to mislead a person into believing something which is not true. It is important to remember that statistics usually produce probabilities, the conclusions drawn from these have errors and error rates and statistical methods and data collection are based on ideal assumptions which may not be met in the real world.

Investigation | Rich task

1. Stretching the axes vertically gives the impression of a fall in the value of the shares. Stretching the axes horizontally gives the impression the value of the shares has not changed very much.





3. For the women's graph, stretching the vertical axis and reducing the distance between the values on the horizontal axis gives the impression women's times have fallen dramatically. To show that the men's times have not fallen greatly, you can reduce the distance between the values on the vertical axis and stretch the values on the horizontal axis.

4. Check with your teacher.

Exercise 5.7 Review questions

1. C

2. D

3. C

4. D

5. D

6. a. Categorical nominal

b. Categorical ordinal

c. Numerical discrete

d. Numerical continuous

7. a. Sample

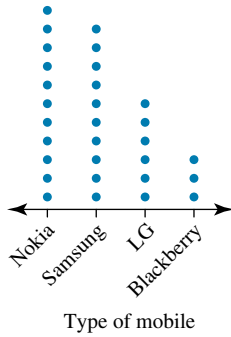
b. Sample

c. Population

8. a. 20 male adults, 10 female adults and 30 teenager members

b. 8 Year 12s, 12 Year 11s and 20 from Years 7, 8, 9 and 10.

9. a. Mobiles owned by 30 shoppers



b. Frequency table of mobile brands

Brand of mobile	Number of mobiles
Nokia	11
Samsung	10
LG	6
Blackberry	3

10. a. Key: 1 | 2 = 12 songs

Stem	Leaf
0	2 9
1	
2	
3	1 4 5
4	3 4 5 7 8
5	4 6 9
6	6 7 7
7	0 4 7
8	2 9
9	0 6
10	3
11	1

b. Positively skewed with outliers

c.

Number of songs	Number of laptops
0–19	2
20–39	3
40–59	8
60–79	6
80–99	4
100–119	2

d. Numerical discrete, symmetric distribution, mean = 60 (rounded to the nearest whole number) and standard deviation = 27.3

11. a.



b. Positively skewed

c. The median is \$166 755.

d. The modal class is \$100 001 to \$200 000.

12. a. 5.6% b. 2% c. 2%
- d. In order to give the impression that the pay rise is generous, the company would quote the mean.
- e. The trade union would quote the median or the mode, because they would be arguing that the wage rise is too low.
- f. The mode is probably the most 'honest' reflection of last year's wage rise, as it represents the situation of most of the workers.

13. a. 25 b. 175 c. 2184 d. 2496

14. a. 8, 20 b. 9, 19

15. 3, 4, 5, 6, 7, 10, 10, 13, 14

16. a.

	Females	Males
Mean	80	74
Median	80	75
Range	21	22
Interquartile range	9	8

- b. The mean and median for the females are the same, and for the males they are very similar. This implies that the data for both males and females is symmetrical in shape and not distorted by outliers.

When the data for females and males are compared, we can see that the range and interquartile range for both sets of data are very similar: 21 and 22, and 9 and 8. Therefore, the spread of the data is very similar, and both the mean and median values are reasonable representations of the average life expectancies for males and females.

Given that the mean life expectancy for females is 6 years more than that for males, and the median life expectancy for females is 5 years more than that for males, it could be said that on average, in the nineteen G20 countries, females live 5–6 years longer than males.

TOPIC 6

Indices, scientific notation and surds

[Stages 5.1, 5.2 and 5.3]

6.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

6.1.1 Why learn this?

Indices (the plural of index) give us a way of abbreviating multiplication, division and so on. They are most useful when working with very large or very small numbers. For calculations involving such numbers, we can use indices to simplify the process.



DISCUSSION

Do you think we should try to limit population growth?

LEARNING SEQUENCE

- 6.1 Overview
- 6.2 [Stage 5.3] Rational and irrational numbers
- 6.3 [Stage 5.3] Simple operations with surds
- 6.4 [Stage 5.1] Rounding and significant figures
- 6.5 [Stage 5.1] Scientific notation
- 6.6 [Stage 5.1] Applying the index laws to variables
- 6.7 [Stage 5.1] Using index laws to simplify algebraic products and quotients
- 6.8 [Stage 5.2] Applying index laws to algebraic expressions involving integer indices
- 6.9 [Stage 5.3] Introduction to fractional indices
- 6.10 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases MA5.1-5NA
- uses scientific notation, and rounds to significant figures MA5.1-9MG
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- applies index laws to operate with algebraic expressions involving integer indices MA5.2-7NA
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM

- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- performs operations with surds and indices MA5.3-6NA

CONTENT DESCRIPTIONS

Students:

Investigate the concept of irrational numbers, including π (ACMNA186)
 Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)
 Investigate very small and very large time scales and intervals (ACMMG219)
 Express numbers in scientific notation (ACMNA210)
 Extend and apply the index laws to variables, using positive-integer indices and the zero index (ACMNA212)
 Simplify algebraic products and quotients using index laws (ACMNA231)
 Apply index laws to numerical expressions with integer indices (ACMNA209)
 Apply index laws to algebraic expressions involving integer indices (NSW)
 Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — The population boom (eles-1697)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

6.2 Rational and irrational numbers

[Stage 5.3]

6.2.1 Natural numbers

- The numbers 1, 2, 3, 4 ... are called **natural numbers**, and the set of natural numbers is called N . That is, positive whole numbers are called natural numbers (N).
- Natural numbers can be used to answer questions such as:
 $5 + 43 = ?$
 $8 \times 6 = ?$



6.2.2 Integers

- What is the value of $3 - 8$?
- The answer to this question is not a natural number, so zero and the negative numbers were defined and a name was given to this new list of numbers.
- The numbers ... $-3, -2, -1, 0, 1, 2, 3$... are called whole numbers or **integers**, and the set of integers is called Z . When we perform statistical analyses, the type of data being collected is important.

DISCUSSION

Can you suggest any reasons why we need negative numbers?

6.2.3 Rational numbers

- What is the value of $4 \div 7$?
- The answer to this question is not an integer, but a fraction (or ratio) written as $\frac{4}{7}$.
- Fractions such as $\frac{4}{7}$ or $-\frac{11}{3}$ in which the denominator and numerator are both integers are called **rational numbers**. A rational number is of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. The set of rational numbers is called Q .



DISCUSSION

Can you suggest reasons why we need fractions?

WORKED EXAMPLE 1

By writing each of the following in fraction form, show that the numbers are rational.

a 7

b -11

c 0

d $-4\frac{2}{5}$

e 1.2

THINK

Each number must be written as a fraction using integers.

- a** The number 7 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.
- b** The number -11 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.
- c** The number 0 has to be written in fraction form. To write a number as a fraction, it must be written with a numerator and denominator. For a whole number, the denominator is 1.
- d** Change $-4\frac{2}{5}$ to an improper fraction.
- e** The number 1.2 can be expressed as $1 + 0.2$. This can then be expressed as $1 + \frac{2}{10}$. Write this as an improper fraction.

WRITE

a $7 = \frac{7}{1}$ is rational.

b $-11 = \frac{-11}{1}$ is rational.

c $0 = \frac{0}{1}$ is rational.

d $-4\frac{2}{5} = \frac{-22}{5}$ is rational.

e $1.2 = \frac{12}{10}$ is rational.

6.2.4 Rational numbers written as decimals

- When a rational number is written as a decimal there are two possibilities:
 - The decimal terminates, e.g. $\frac{5}{4} = 1.25$.
 - The decimal repeats, or recurs, e.g. $\frac{7}{6} = 1.1666\dots$
- For $1.1666\dots$, the 6 is a repeating digit. This number, $\frac{7}{6}$, is called a **recurring decimal**, and it can also be written as $1.1\dot{6}$.
- $1.\dot{6}1\dot{5}$ or $1.\overline{615}$ means $1.615\ 615\ 615\dots$ (The digits 615 in the decimal repeat.)
- $1.6\dot{1}5$ or $1.6\overline{15}$ means $1.615\ 151\ 515\dots$ (The digits 15 in the decimal repeat.)
- $1.61\dot{5}$ means $1.615\ 555\dots$ (Only the digit 5 in the decimal repeats.)

WORKED EXAMPLE 2

Write the first 8 digits of each of the following recurring decimals.

a $3.\dot{0}2$

b $47.\dot{1}$

c $11.5\dot{4}9$

THINK

a In $3.\dot{0}2$, 02 recurs.

b In $47.\dot{1}$, 1 recurs.

c In $11.5\dot{4}9$, 49 recurs.

WRITE

a 3.0202020

b 47.111111

c 11.549494

- 1.237 is rational, because it is a **terminating decimal**. It is easy to show that $1.237 = \frac{1237}{1000}$.
- $0.\dot{8}6$, or $0.868\ 686\dots$, is rational because it is a recurring decimal. It can be shown that $0.\dot{8}6 = \frac{86}{99}$.

WORKED EXAMPLE 3

Write each fraction as a recurring decimal using dot notation. (Use a calculator for the division.)

a $\frac{5}{6}$

b $\frac{57}{99}$

c $\frac{25}{11}$

d $\frac{4}{7}$

THINK

a $5 \div 6 = 0.833\ 333\ 3$
3 recurs — the dot goes above the 3.

b $57 \div 99 = 0.575\ 757\ 575\ 7$
57 recurs — dots go above the 5 and the 7.

c $25 \div 11 = 2.272\ 727\ 27$
27 recurs — dots go above the 2 and the 7.

d $4 \div 7 = 0.571\ 428\ 571$
571 428 is recurring — dots go above the 5 and the 8.

WRITE

a $\frac{5}{6} = 0.8\dot{3}$

b $\frac{57}{99} = 0.\dot{5}\dot{7}$

c $\frac{25}{11} = 2.\dot{2}\dot{7}$

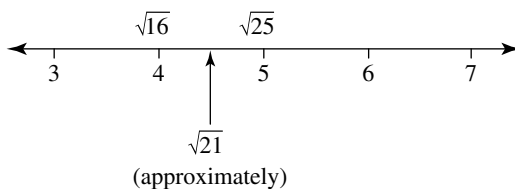
d $\frac{4}{7} = 0.\dot{5}7142\dot{8}$

6.2.5 Irrational numbers

- Ancient mathematicians were shocked to find that rational numbers could not be used to label every point on the number line. In other words they discovered lengths that could not be expressed as fractions. These numbers are called **irrational numbers**.

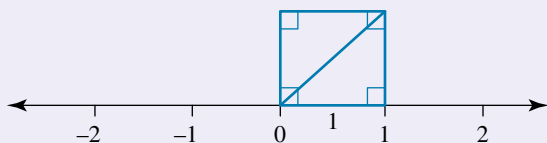


- A number is irrational if it is not rational; that is, if it cannot be written as a fraction, nor as a terminating or recurring decimal.
- Irrational numbers are denoted by the letter I .
- When the square root of a number is an irrational number, it is called a **surd**. For example, $\sqrt{10}$ cannot be written as a fraction, or as a recurring or terminating decimal. It is therefore irrational and is called a surd.
- $\sqrt{10} \approx 3.162\ 277\ 660\ 17\dots$
- One method of approximating the value of a surd is by using a number line. For example, $\sqrt{21}$ will lie between 4 and 5, because $\sqrt{16} = 4$ and $\sqrt{25} = 5$. This can be shown on a number line.



ACTIVITY: REPRESENTING IRRATIONAL NUMBERS ON A NUMBER LINE

- Draw a number line on a page with numbers between -2 and 2 .
- Using a compass, construct a square with the base between 0 and 1 on the number line, as shown in the diagram below.



- Draw the diagonal and calculate the length of the diagonal. (Leave the length in exact form.) Write this length on the diagonal.
- Place the point of the compass at 0 on the number line and draw an arc through the point at the other end of the diagonal.
- Draw a point on the number line equidistant to the arc at the top of the diagonal. Label this point on the number line.
- Can you think of another irrational number you could illustrate in the same way on your number line?

WORKED EXAMPLE 4

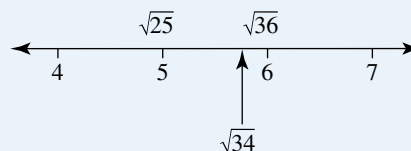
Place $\sqrt{34}$ on a number line.

THINK

- 1 The square number that is smaller than 34 is 25.
The square number that is larger than 34 is 36.
Write the numbers just below and just above $\sqrt{34}$.
- 2 Draw a number line to show the approximate position of $\sqrt{34}$.

WRITE

$\sqrt{34}$ lies between $\sqrt{25}$ and $\sqrt{36}$.



WORKED EXAMPLE 5

Which of the following are surds?

a $\sqrt{0}$

b $\sqrt{20}$

c $-\sqrt{9}$

d $\sqrt[3]{6}$

THINK

- a $\sqrt{0} = 0$. This is a rational number and therefore not a surd.
- b $\sqrt{20} \approx 4.472\ 135\ 955\dots$ It is an irrational number and therefore a surd.
- c $-\sqrt{9} = -3$. This is a rational number and therefore not a surd.
- d $\sqrt[3]{6} \approx 1.817\ 120\ 592\ 83\dots$ It is an irrational number and therefore a surd.

WRITE

- a $\sqrt{0} = 0$, which is not a surd.
- b $\sqrt{20}$ is a surd.
- c $-\sqrt{9} = -3$, which is not a surd.
- d $\sqrt[3]{6}$ is a surd.

learn on RESOURCES — ONLINE ONLY

Interactivity: Surds on the number line (int-6029)

Lesson: Surds (eles-1906)

Digital doc: SkillsHEET Operations with directed numbers (doc-6100)

Digital doc: SkillsHEET Calculating the square root of a number (doc-6101)

Exercise 6.2 Rational and irrational numbers

Individual pathways

PRACTISE

Questions:

1a–l, 2a–f, 3a–e, 4–8

CONSOLIDATE

Questions:

1d–l, 2c–j, 3c–g, 4–11

MASTER

Questions:

1e–l, 2f–j, 3e–j, 4–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** Show that the following numbers are rational by writing each of them in fraction form.

a. 15	b. -8	c. $2\frac{2}{3}$	d. $-5\frac{1}{8}$
e. $\sqrt{16}$	f. $7\frac{3}{10}$	g. 0.002	h. 87.2
i. 0	j. 1.56	k. 3.612	l. -0.08

2. **WE2** Write the first 8 digits of each of the following recurring decimals.

a. $0.\dot{5}$	b. $0.\dot{5}\dot{1}$	c. $0.5\dot{1}$	d. $6.0\dot{3}\dot{1}$
e. $5.\dot{1}8\dot{3}$	f. $-7.02\dot{4}$	g. $8.\dot{9}12\dot{4}$	h. $5.12\dot{3}\dot{4}$
i. $5.\dot{1}23\dot{4}$	j. $3.00\dot{2}$		

3. **WE3** Write the following fractions as recurring decimals using dot notation. (Use a calculator.)

a. $\frac{5}{9}$	b. $\frac{3}{11}$	c. $2\frac{2}{11}$	d. $\frac{3}{7}$
e. $\frac{-173}{99}$	f. $\frac{73}{990}$	g. $-3\frac{5}{6}$	h. $\frac{7}{15}$
i. $\frac{46}{99}$	j. $\frac{46}{990}$		

4. From the following list of numbers: -3 , $-\frac{3}{7}$, 0, 2.3, $2.\dot{3}$, $2\frac{3}{5}$, 15

- write down the natural numbers
- write down the integers
- write down the rational numbers.

5. Write these numbers in order from smallest to largest.

$2.\dot{1}$, $2.\dot{1}\dot{2}$, $2.\dot{1}2\dot{1}$, $2.1\dot{2}\dot{1}$, $2.1\dot{2}$

6. Write down the positive square roots of each of the following.

a. 1	b. 4	c. 0	d. $\frac{1}{9}$
e. $1\frac{9}{16}$	f. 0.16	g. 400	h. 10000
i. $\frac{4}{25}$	j. 1.44	k. 20.25	l. 1000000
m. 0.0009	n. 256		

7. Write down the value of each of the following.

a. $\sqrt{81}$	b. $-\sqrt{81}$	c. $\sqrt{121}$	d. $-\sqrt{441}$
e. $\sqrt[3]{8}$	f. $\sqrt[3]{64}$	g. $\sqrt[3]{343}$	h. $\sqrt[4]{81}$
i. $\sqrt[5]{1024}$	j. $\sqrt[3]{125}$	k. $2\sqrt{49}$	l. $\sqrt[3]{-27}$

8. **WE5** Which of the following are surds?

a. $\sqrt[3]{0}$	b. $\sqrt{10}$	c. $-\sqrt{36}$	d. $\sqrt[3]{9}$
e. $-\sqrt[3]{216}$	f. $\sqrt[3]{-216}$	g. $-\sqrt{2}$	h. $1 + \sqrt{2}$
i. 1.32	j. $1.\dot{3}\dot{2}$	k. $\sqrt[4]{64}$	l. 1.752 16
m. $\sqrt{7} + \sqrt{2}$	n. $\sqrt{9} + \sqrt{49}$	o. $\sqrt{4} - \sqrt{3}$	p. $\sqrt{3^2}$

Communicating, reasoning and problem solving

- Explain why all integers are rational numbers.
- Are all fractions in which both the numerator and denominator are integers rational? Explain why or why not.
- Using the fraction $\frac{a}{b}$, where a and b are natural numbers, write 3 recurring decimals in fractional form using the smallest natural numbers possible.
 - What was the largest natural number you used?
- Write 2 fractions that have the following number of repeating digits in their decimal forms.
 - 1 repeating digit
 - 2 repeating digits
 - 3 repeating digits
 - 4 repeating digits

13. If a \$2 coin weighs 6g, a \$1 coin weighs 9g, a 50c coin weighs 15g, a 20c coin weighs 12g, a 10c coin weighs 5g and a 5c coin weighs 3g, explain what the maximum value of the coins would be if a bundle of them weighed exactly 10kg.
14. A Year 9 student consumes 2 bottles of water every day, except on every 5th day when she only has 1. Calculate her *annual* bottled water consumption.
15. How many recurring digits will there be for $\frac{1}{13}$?

6.3 Simple operations with surds

[Stage 5.3]

6.3.1 Multiplying and dividing surds

- Consider that $3 = \sqrt{9}$, $2 = \sqrt{4}$ and $6 = \sqrt{36}$.
The multiplication $3 \times 2 = 6$ could be written as $\sqrt{9} \times \sqrt{4} = \sqrt{36}$.
- In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.
For example, $\sqrt{7} \times \sqrt{3} = \sqrt{21}$.
- Similarly, $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.
For example, $\sqrt{18} \div \sqrt{3} = \sqrt{6}$.

WORKED EXAMPLE 6

Evaluate the following, leaving your answer in surd form.

a $\sqrt{7} \times \sqrt{2}$ b $5 \times \sqrt{3}$ c $\sqrt{5} \times \sqrt{5}$ d $-2\sqrt{3} \times 4\sqrt{5}$

THINK

- a Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.
- b Only $\sqrt{3}$ is a surd. It is multiplied by 5, which is not a surd.
- c Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.
- d Multiply the whole numbers by each other.
Multiply the surds by each other.

WRITE

a $\sqrt{7} \times \sqrt{2} = \sqrt{14}$

b $5 \times \sqrt{3} = 5\sqrt{3}$

c $\sqrt{5} \times \sqrt{5} = \sqrt{25}$
 $= 5$

d $-2\sqrt{3} \times 4\sqrt{5} = -2 \times 4 \times \sqrt{3} \times \sqrt{5}$
 $= -8 \times \sqrt{15}$
 $= -8\sqrt{15}$

WORKED EXAMPLE 7

Evaluate the following, leaving your answer in surd form.

a $\frac{\sqrt{10}}{\sqrt{5}}$ b $\sqrt{\frac{10}{5}}$ c $\frac{-6\sqrt{8}}{4\sqrt{4}}$ d $\frac{\sqrt{20}}{\sqrt{5}}$ e $\frac{5}{\sqrt{5}}$

THINK

- a Apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.

WRITE

a $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}}$
 $= \sqrt{2}$

b Simplify the fraction first.

$$\mathbf{b} \quad \sqrt{\frac{10}{5}} = \sqrt{2}$$

c Simplify the whole numbers. Then apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.

$$\mathbf{c} \quad \frac{-6\sqrt{8}}{4\sqrt{4}} = \frac{-3\sqrt{2}}{2}$$

d Apply the rule $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$.

$$\mathbf{d} \quad \frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$$

e Rewrite the numerator as the product of two surds and then simplify.

$$\mathbf{e} \quad \frac{5}{\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

6.3.2 Simplifying surds

- Just as a rational number can be written many different ways (e.g. $\frac{1}{2} = \frac{5}{10} = \frac{7}{14}$), so can a surd, and it is expected that surds should normally be written in simplest form.
- A surd is in simplest form when the number inside the radical sign has the smallest possible value.
- Note that $\sqrt{24}$ can be factorised several ways. For example:
 $\sqrt{24} = \sqrt{2} \times \sqrt{12}$
 $\sqrt{24} = \sqrt{3} \times \sqrt{8}$
 $\sqrt{24} = \sqrt{4} \times \sqrt{6}$
In the last case, $\sqrt{4} = 2$:
 $\sqrt{24} = 2 \times \sqrt{6}$
 $\quad = 2\sqrt{6}$
- $2\sqrt{6}$ is equal to $\sqrt{24}$, and $2\sqrt{6}$ is written in simplest form.
- To simplify a surd you must find a factor that is also a perfect square: 4, 9, 16, 25, 36, 49 and so on.

DISCUSSION

How would you explain how best to simplify surds to other learners?

WORKED EXAMPLE 8

Simplify the following surds.

a $\sqrt{18}$

b $6\sqrt{20}$

THINK

a 1 Rewrite 18 as the product of two numbers, one of which is square.

2 Simplify.

b 1 Rewrite 20 as the product of two numbers, one of which is square.

2 Simplify.

WRITE

a $\sqrt{18} = \sqrt{9} \times \sqrt{2}$

$$= 3 \times \sqrt{2} \\ = 3\sqrt{2}$$

b $6\sqrt{20} = 6 \times \sqrt{4} \times \sqrt{5}$

$$= 6 \times 2 \times \sqrt{5} \\ = 12\sqrt{5}$$

- The surd $\sqrt{22}$ cannot be simplified because 22 has no perfect square factors.
- Surds can be simplified in more than one step.

$$\begin{aligned}\sqrt{72} &= \sqrt{4} \times \sqrt{18} \\ &= 2\sqrt{18} \\ &= 2 \times \sqrt{9} \times \sqrt{2} \\ &= 2 \times 3\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

6.3.3 Entire surds

- The surd $\sqrt{45}$, when simplified, is written as $3\sqrt{5}$.
- The surd $\sqrt{45}$ is called an entire surd, because it is written entirely inside the radical sign, whereas $3\sqrt{5}$ is not.
- Writing a surd as an entire surd reverses the process of simplification.

WORKED EXAMPLE 9

Write $3\sqrt{7}$ as an entire surd.

THINK

- 1 In order to place the 3 inside the radical sign, it has to be written as $\sqrt{9}$.
- 2 Apply the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

WRITE

$$\begin{aligned}3\sqrt{7} &= \sqrt{9} \times \sqrt{7} \\ &= \sqrt{63}\end{aligned}$$

WORKED EXAMPLE 10

Which number is larger, $3\sqrt{5}$ or $5\sqrt{3}$?

THINK

- 1 Write $3\sqrt{5}$ as an entire surd.
- 2 Write $5\sqrt{3}$ as an entire surd.
- 3 Compare the values.
- 4 Write your answer.

WRITE

$$\begin{aligned}3\sqrt{5} &= \sqrt{9} \times \sqrt{5} \\ &= \sqrt{45} \\ 5\sqrt{3} &= \sqrt{25} \times \sqrt{3} \\ &= \sqrt{75} \\ \sqrt{75} &> \sqrt{45} \\ 5\sqrt{3} &\text{ is the larger number.}\end{aligned}$$

DISCUSSION

Is there a quicker way to determine which of these two numbers are larger?

6.3.4 Addition and subtraction of surds

- Surds can be added or subtracted if they are like terms.

WORKED EXAMPLE 11

Simplify each of the following.

a $6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5}$

b $3\sqrt{2} - 5 + 4\sqrt{2} + 9$

THINK

- a Collect the $\sqrt{3}$ s and the $\sqrt{5}$ s.
- b Collect the like terms and simplify.

WRITE

$$\begin{aligned} \text{a } 6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5} \\ = 8\sqrt{3} - \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{b } 3\sqrt{2} - 5 + 4\sqrt{2} + 9 \\ = 3\sqrt{2} + 4\sqrt{2} - 5 + 9 \\ = 7\sqrt{2} + 4 \end{aligned}$$

- Surds should be simplified before adding or subtracting like terms.

WORKED EXAMPLE 12

Simplify $5\sqrt{75} - 6\sqrt{12} + \sqrt{8} - 4\sqrt{3}$.

THINK

- 1 Simplify $5\sqrt{75}$.
- 2 Simplify $6\sqrt{12}$.
- 3 Simplify $\sqrt{8}$.
- 4 Rewrite the original expression and simplify by adding like terms.

WRITE

$$\begin{aligned} 5\sqrt{75} &= 5 \times \sqrt{25} \times \sqrt{3} \\ &= 5 \times 5 \times \sqrt{3} \\ &= 25\sqrt{3} \end{aligned}$$

$$\begin{aligned} 6\sqrt{12} &= 6 \times \sqrt{4} \times \sqrt{3} \\ &= 6 \times 2 \times \sqrt{3} \\ &= 12\sqrt{3} \end{aligned}$$

$$\begin{aligned} \sqrt{8} &= \sqrt{4} \times \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} 5\sqrt{75} - 6\sqrt{12} + \sqrt{8} - 4\sqrt{3} \\ = 25\sqrt{3} - 12\sqrt{3} + 2\sqrt{2} - 4\sqrt{3} \\ = 9\sqrt{3} + 2\sqrt{2} \end{aligned}$$

learnON

RESOURCES — ONLINE ONLY

Interactivity: Balancing surds (int-2762)
Interactivity: Simplifying surds (int-6028)
Digital doc: SkillsHEET Simplifying surds (doc-10813)
Digital doc: WorkSHEET Number skills (doc-6119)

Exercise 6.3 Simple operations with surds**Individual pathways****PRACTISE**

Questions:
1–6, 8a–f, 9a–f, 11

CONSOLIDATE

Questions:
1–7, 8e–j, 9e–h, 10–12

MASTER

Questions:
1m–p, 2l–p, 3–7, 8g–l, 9e–j, 10,
12–20

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE6** Simplify each of the following.

a. $\sqrt{3} \times \sqrt{7}$	b. $-\sqrt{3} \times \sqrt{7}$	c. $2 \times \sqrt{6}$	d. $2 \times 3\sqrt{7}$
e. $2\sqrt{7} \times 5\sqrt{2}$	f. $-3\sqrt{2} \times -5\sqrt{5}$	g. $3\sqrt{7} \times 4$	h. $\sqrt{7} \times 9$
i. $-2\sqrt{5} \times -11\sqrt{2}$	j. $2\sqrt{3} \times 11$	k. $\sqrt{3} \times \sqrt{3}$	l. $2\sqrt{3} \times 4\sqrt{3}$
m. $\sqrt{6} \times \sqrt{6}$	n. $\sqrt{11} \times \sqrt{11}$	o. $\sqrt{51} \times \sqrt{51}$	p. $\sqrt{15} \times 2\sqrt{15}$

2. **WE7** Simplify each of the following.

a. $\sqrt{\frac{12}{4}}$	b. $-\sqrt{\frac{10}{5}}$	c. $\frac{\sqrt{18}}{\sqrt{3}}$	d. $\frac{-\sqrt{15}}{-\sqrt{3}}$
e. $\frac{15\sqrt{6}}{5\sqrt{2}}$	f. $\frac{15\sqrt{6}}{10}$	g. $\frac{15\sqrt{6}}{\sqrt{3}}$	h. $\frac{5}{3}\sqrt{\frac{15}{3}}$
i. $\frac{-10\sqrt{10}}{5\sqrt{2}}$	j. $\frac{\sqrt{9}}{\sqrt{3}}$	k. $\frac{3}{\sqrt{3}}$	l. $\frac{7}{\sqrt{7}}$
m. $\frac{35}{\sqrt{35}}$	n. $\frac{\sqrt{28}}{\sqrt{7}}$	o. $\frac{2\sqrt{18}}{\sqrt{2}}$	p. $\frac{\sqrt{45}}{\sqrt{5}}$

3. **WE8** Simplify each of the following.

a. $\sqrt{20}$	b. $\sqrt{8}$	c. $\sqrt{18}$	d. $\sqrt{49}$
e. $\sqrt{30}$	f. $\sqrt{50}$	g. $\sqrt{28}$	h. $\sqrt{108}$
i. $\sqrt{288}$	j. $\sqrt{48}$	k. $\sqrt{500}$	l. $\sqrt{162}$

4. Simplify each of the following.

a. $2\sqrt{8}$	b. $5\sqrt{27}$	c. $6\sqrt{64}$	d. $7\sqrt{50}$
e. $10\sqrt{24}$	f. $5\sqrt{12}$	g. $4\sqrt{42}$	h. $12\sqrt{72}$
i. $9\sqrt{45}$	j. $12\sqrt{242}$		

5. **WE9** Write each of the following in the form $\sqrt{a}$; that is, as an entire surd.

a. $2\sqrt{3}$	b. $5\sqrt{7}$	c. $6\sqrt{3}$	d. $4\sqrt{5}$
e. $8\sqrt{6}$	f. $3\sqrt{10}$	g. $4\sqrt{2}$	h. $12\sqrt{5}$
i. $10\sqrt{6}$	j. $13\sqrt{2}$		

6. **MC** a. $\sqrt{1000}$ is equal to:

A. 31.6228	B. $50\sqrt{2}$	C. $50\sqrt{10}$	D. $10\sqrt{10}$
b. $\sqrt{80}$ in simplest form is equal to:			
A. $4\sqrt{5}$	B. $2\sqrt{20}$	C. $8\sqrt{10}$	D. $5\sqrt{16}$
c. Which of the following surds is in simplest form?			
A. $\sqrt{60}$	B. $\sqrt{147}$	C. $\sqrt{105}$	D. $\sqrt{117}$
d. Which of the following surds is not in simplest form?			
A. $\sqrt{102}$	B. $\sqrt{110}$	C. $\sqrt{116}$	D. $\sqrt{118}$
e. $6\sqrt{5}$ is equal to:			
A. $\sqrt{900}$	B. $\sqrt{30}$	C. $\sqrt{150}$	D. $\sqrt{180}$
f. Which one of the following is not equal to the rest?			
A. $\sqrt{128}$	B. $2\sqrt{32}$	C. $8\sqrt{2}$	D. $64\sqrt{2}$
g. Which one of the following is not equal to the rest?			
A. $4\sqrt{4}$	B. $2\sqrt{16}$	C. 8	D. 16
h. $5\sqrt{48}$ is equal to:			
A. $80\sqrt{3}$	B. $20\sqrt{3}$	C. $9\sqrt{3}$	D. $21\sqrt{3}$

7. Challenge: Reduce each of the following to simplest form.

a. $\sqrt{675}$	b. $\sqrt{1805}$	c. $\sqrt{1792}$	d. $\sqrt{578}$
e. $\sqrt{a^2c}$	f. $\sqrt{bd^4}$	g. $\sqrt{h^2jk^2}$	h. $\sqrt{f^3}$

8. **WE11** Simplify each of the following.

a. $6\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

c. $-3\sqrt{3} - 7\sqrt{3} + 4\sqrt{3}$

e. $10\sqrt{11} - 6\sqrt{11} + \sqrt{11}$

g. $4\sqrt{2} + 6\sqrt{2} + 5\sqrt{3} + 2\sqrt{3}$

i. $5\sqrt{10} + 2\sqrt{3} + 3\sqrt{10} + 5\sqrt{3}$

k. $6\sqrt{6} + \sqrt{2} - 4\sqrt{6} - \sqrt{2}$

b. $4\sqrt{5} - 6\sqrt{5} - 2\sqrt{5}$

d. $-9\sqrt{6} + 6\sqrt{6} + 3\sqrt{6}$

f. $\sqrt{7} + \sqrt{7}$

h. $10\sqrt{5} - 2\sqrt{5} + 8\sqrt{6} - 7\sqrt{6}$

j. $12\sqrt{2} - 3\sqrt{5} + 4\sqrt{2} - 8\sqrt{5}$

l. $16\sqrt{5} + 8 + 7 - 11\sqrt{5}$

9. **WE12** Simplify each of the following.

a. $\sqrt{8} + \sqrt{18} - \sqrt{32}$

c. $-\sqrt{12} + \sqrt{75} - \sqrt{192}$

e. $\sqrt{24} + \sqrt{180} + \sqrt{54}$

g. $2\sqrt{24} + 3\sqrt{20} - 7\sqrt{8}$

i. $6\sqrt{44} + 4\sqrt{120} - \sqrt{99} - 3\sqrt{270}$

b. $\sqrt{45} - \sqrt{80} + \sqrt{5}$

d. $\sqrt{7} + \sqrt{28} - \sqrt{343}$

f. $\sqrt{12} + \sqrt{20} - \sqrt{125}$

h. $3\sqrt{45} + 2\sqrt{12} + 5\sqrt{80} + 3\sqrt{108}$

j. $2\sqrt{32} - 5\sqrt{45} - 4\sqrt{180} + 10\sqrt{8}$

10. **MC** a. $\sqrt{2} + 6\sqrt{3} - 5\sqrt{2} - 4\sqrt{3}$ is equal to:

A. $-5\sqrt{2} + 2\sqrt{3}$

B. $-3\sqrt{2} + 2\sqrt{3}$

b. $6 - 5\sqrt{6} + 4\sqrt{6} - 8$ is equal to:

A. $-2 - \sqrt{6}$

B. $14 - \sqrt{6}$

c. $4\sqrt{8} - 6\sqrt{12} - 7\sqrt{18} + 2\sqrt{27}$ is equal to:

A. $-7\sqrt{5}$

B. $29\sqrt{2} - 18\sqrt{3}$

d. $2\sqrt{20} + 5\sqrt{24} - \sqrt{54} + 5\sqrt{45}$ is equal to:

A. $19\sqrt{5} + 7\sqrt{6}$

B. $9\sqrt{5} - 7\sqrt{6}$

C. $6\sqrt{2} + 2\sqrt{3}$

D. $-4\sqrt{2} + 2\sqrt{3}$

C. $-2 + \sqrt{6}$

D. $-2 - 9\sqrt{6}$

C. $-13\sqrt{2} - 6\sqrt{3}$

D. $-13\sqrt{2} + 6\sqrt{3}$

C. $-11\sqrt{5} + 7\sqrt{6}$

D. $-11\sqrt{5} - 7\sqrt{6}$

11. **WE10** Which number is larger?

a. $\sqrt{10}$ or $2\sqrt{3}$

b. $3\sqrt{5}$ or $5\sqrt{2}$

c. $10\sqrt{2}$ or $4\sqrt{5}$

d. $2\sqrt{10}$ or $\sqrt{20}$

12. Write these numbers from smallest to largest.

a. $6\sqrt{2}$, 8, $2\sqrt{7}$, $3\sqrt{6}$, $4\sqrt{2}$, $\sqrt{60}$

b. $\sqrt{6}$, $2\sqrt{2}$, $\sqrt{2}$, 3, $\sqrt{3}$, 2, $2\sqrt{3}$

Communicating, reasoning and problem solving

13. A man wants to divide his vegetable garden into 10 equal squares, each of area exactly 2 m^2 .

a. What will be the exact side length of each square? Explain your reasoning.

b. What will the side length of each square be, correct to 2 decimal places?

c. The man wishes to group the squares next to each other so as to have the smallest perimeter for his vegetable garden.

i. How should he arrange the squares?

ii. Explain why the exact perimeter of the vegetable patch is $14\sqrt{2}$ m.

iii. What will be the perimeter correct to 2 decimal places?

14. Explain why $\sqrt{a^3b^2}$ can be simplified to $ab\sqrt{a}$.

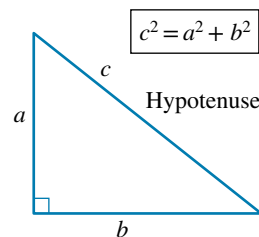
15. Kyle wanted a basketball court in his backyard; however, he could not fit a full-size court into his yard. He had a rectangular court laid with a width of $6\sqrt{2}$ m and length of $3\sqrt{10}$ m. Calculate the area of the basketball court and represent it in its simplest surd form.

16. A netball team went on a pre-season training run.

They completed 10 laps of a triangular course with side lengths $(200\sqrt{3} + 50)$ m, $(50\sqrt{2} + 75\sqrt{3})$ m and $(125\sqrt{2} - 18)$ m. What was the distance they ran in simplest surd form?



17. The area of a square is $x \text{ cm}^2$. Would the side length of the square be a rational number? Explain your answer.
18. To calculate the length of the hypotenuse of a right-angled triangle, use the formula $c^2 = a^2 + b^2$.
- a. Calculate the length of the hypotenuse for triangles with other side lengths of:
- $\sqrt{5}, \sqrt{8}$
 - $\sqrt{2}, \sqrt{7}$
 - $\sqrt{15}, \sqrt{23}$.
- b. Describe any pattern in your answers to part a.
- c. State the length of the hypotenuse (without calculations) for triangles with side lengths of:
- $\sqrt{1000}, \sqrt{500}$
 - $\sqrt{423}, \sqrt{33}$
 - $\sqrt{124}, \sqrt{63}$
- d. Your friend wrote down the following explanation.
- $$\sqrt{b} + \sqrt{c} = \sqrt{a}$$



Explain why this is not correct and where your friend made an error.

19. Are all square root numbers surds?
20. The integer 5 has a square of 25. If we add 1 to 5, we get the integer 6. The square of 6 is 36. If we increase each digit (2 and 5) in 25 by 1, we get the number 36. Therefore, the square of 6 (5 increased by 1) is the number formed by adding 1 to each digit in the square of 5. Find a pair of 2-digit numbers with the same property.

6.4 Rounding and significant figures

[Stage 5.1]

6.4.1 Real numbers

- A **real number** is any number that lies on the number line.
- Together, the rational numbers and the irrational numbers make up the set of real numbers, which is called R .
- Therefore, every point on the number line represents some number, rational or irrational.

6.4.2 The irrational number π

- Surds are not the only irrational numbers. Most irrational numbers cannot be written as surds. A few of these numbers are so important in mathematics and science that they are given special names.
- The most well known of these numbers is π (pi). It is also called a transcendental number.
- π is irrational; therefore, it cannot be written as a fraction. If you tried to write π as a decimal, you would be writing forever, as the digits never recur and the decimal does not terminate. In the 20th century, computers were used to find the value of π to 1 trillion decimal places.
- The value of π is very close to (but not equal to) 3.14 or $\frac{22}{7}$. Most calculators store an approximate value for π .

3.141592653589
79323846264338
32795028841971
69399375105820
97494459230781
64062862089986
28034825342117
06798214808651
32823066470938
446095505822...



6.4.3 Rounding to a number of decimal places

- Because numbers such as $\sqrt{3}$ or π cannot be written exactly as decimals, approximate values are often used. These values can be found using a calculator and then rounded off to the desired level of accuracy.
- When we round numbers, we write them to a certain number of decimal places or significant figures.
- The rules for rounding to a certain number of decimal places are as follows.

For example, let us round 3.456 734 correct to 3 decimal places.

- Identify the rounding digit in the third decimal place. In this case it is 6.

3.456734

- Look at the digit that comes after the rounding digit.

3.456734

If this digit is 0, 1, 2, 3 or 4, leave the rounding digit as it is.

If this digit is 5, 6, 7, 8 or 9, then increase the rounding digit by 1.

In this case the next digit is 7, so the rounding digit is increased by 1, from 6 to 7.

- Leave out all digits that come after the rounding digit.

So $3.456734 \approx 3.457$ correct to 3 decimal places (3 d.p.).

WORKED EXAMPLE 13

Write these numbers correct to 3 decimal places (3 d.p.).

a $\sqrt{3}$

b π

c $5.1\dot{9}$

d $\frac{2}{3}$

e 7.123456

THINK

a $\sqrt{3} = 1.7320\dots$ The rounding digit is 2. The next digit is 0, so leave 2 as is.

b $\pi = 3.1415\dots$ The rounding digit is 1. The next digit is 5, so add 1.

c $5.1\dot{9} = 5.1999\dots$ The rounding digit is 9. The next digit is 9, so add 1.

d $\frac{2}{3} = 0.6666\dots$ The rounding digit is 6. The next digit is 6, so add 1.

e 7.123456. The rounding digit is 3. The next digit is 4, so leave 3 as is.

WRITE

a $\sqrt{3} \approx 1.73$

b $\pi \approx 3.142$

c $5.1\dot{9} \approx 5.200$

d $\frac{2}{3} \approx 0.667$

e $7.1234 \approx 7.123$

6.4.4 Rounding to significant figures

- Another method of rounding decimals is to write them correct to a certain number of significant figures. In a decimal number, the significant figures start with the first non-zero digit.
- Consider the approximate value of $\sqrt{2}$.
- $\sqrt{2} \approx 1.414$. This approximation is written correct to 3 decimal places, but it has 4 significant figures. When we count significant figures, the non-zero digits before the decimal point are included.
- The decimal 0.0302 is written to 4 decimal places, but it has only 3 significant figures. This is because significant figures start with the first non-zero digit. In this case the first significant figure is 3, and the significant figures are 302.

WORKED EXAMPLE 14

How many significant figures are there in each of the following numbers?

- a 25 b 0.04 c 3.02 d 0.100

THINK

- a 25: The first significant figure is 2.
b 0.04: The first significant figure is 4.
c 3.02: The first significant figure is 3.
d 0.100: The first significant figure is 1.

WRITE

- a 2 significant figures
b 1 significant figure
c 3 significant figures
d 3 significant figures

WORKED EXAMPLE 15

Round these numbers correct to 5 significant figures.

- a π b $\sqrt{200}$ c $0.\dot{0}\dot{3}$ d 2530.166

THINK

- a $\pi \approx 3.14159\dots$ The first significant figure is 3. Write 4 more digits.
b $\sqrt{200} \approx 14.1421\dots$ The first significant figure is 1. Write 4 more digits.
c $0.\dot{0}\dot{3} = 0.0303030\dots$ The first significant figure is 3. Write 4 more digits.
d 2530.16... The first significant figure is 2. Write 4 more digits.

WRITE

- a 3.1416
b 14.142
c 0.030303
d 2530.2

DISCUSSION

Peta and Shannon are adventurers and have decided to circumnavigate the earth with a combination of trekking and sailing.

They have done some preliminary research and know that the diameter of the earth is 12,742 km. Peta estimates that the circumference of the earth is 40,009 km but Shannon estimates that the circumference is 40,030 km. Are these two values different enough that the two adventurers need to investigate these values further? Why or why not? Can you suggest a reason why their calculations are different? Use mathematics to explain your answer.



learn on

RESOURCES — ONLINE ONLY

Interactivity: Rounding (int-3730)

Interactivity: Rounding to significant figures (int-6030)

Digital doc: SkillsSHEET Rounding to a given number of significant figures (doc-10814)

Exercise 6.4 Rounding and significant figures

Individual pathways

PRACTISE

Questions:
1–3, 4a–k, 5–11

CONSOLIDATE

Questions:
1–3, 4e–p, 5–13

MASTER

Questions:
1–3, 4i–t, 5–14

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE13** Write each of the following correct to 3 decimal places.

a. $\frac{\pi}{2}$	b. $\sqrt{5}$	c. $\sqrt{15}$	d. 5.12×3.21
e. $5.\dot{1}$	f. $5.1\dot{5}$	g. $5.\dot{1}5$	h. e
i. 11.722	j. $\frac{3}{7}$	k. $\frac{1}{13}$	l. $2\frac{3}{7}$
m. 0.999999	n. 6.58129	o. 4.00001	p. $2.79 \div 11$
q. 0.0254	r. 0.0009136	s. 5.00001	t. 2342.156

2. Write the value of π correct to 4, 5, 6 and 7 decimal places.

3. **WE14** How many significant figures are there in each of the following numbers?

a. 36	b. 207	c. 1631	d. 5.04
e. 176.2	f. 95.00	g. 0.21	h. 0.01
i. 0.000316	j. 0.1007	k. 0.010	l. 0.0512

4. **WE15** Write each number correct to 5 significant figures.

a. $\frac{\pi}{2}$	b. $\sqrt{5}$	c. $\sqrt{15}$	d. 5.12×3.21
e. $5.\dot{1}$	f. $5.1\dot{5}$	g. $5.\dot{1}5$	h. e
i. 11.722	j. $\frac{3}{7}$	k. $\frac{1}{13}$	l. $2\frac{3}{7}$
m. 0.999999	n. 6.58129	o. 4.00001	p. $2.79 \div 11$
q. 0.0254	r. 0.0009136	s. 5.00001	t. 2342.156

5. List the following numbers in order from smallest to largest.

a. $\sqrt{7}$, 3.5, $\sqrt{18}$, 4, $\sqrt{15}$ b. 3.14, π , $\frac{22}{7}$, $\sqrt{10}$, $3.\dot{1}\dot{4}$

6. Explain whether each statement is true or false.

- a. Every surd is a rational number.
b. Every surd is an irrational number.
c. Every irrational number is a surd.
d. Every surd is a real number.

7. Explain whether each statement is true or false.

- a. π is a rational number. b. π is an irrational number.
c. π is a surd. d. π is a real number.

8. Explain whether each statement is true or false.

- a. $1.\dot{3}\dot{1}$ is a rational number. b. $1.\dot{3}\dot{1}$ is an irrational number.
c. $1.\dot{3}\dot{1}$ is a surd. d. $1.\dot{3}\dot{1}$ is a real number.

Communicating, reasoning and problem solving

9. Explain why $\sqrt{25}$ and $-\sqrt{25}$ are defined, but $\sqrt{-25}$ is not.
10. Explain why the number 12.9954123 when rounded to 2 decimal places becomes 13.00.

11. The area of a circle is calculated using the formula $A = \pi \times r^2$, where r is the radius of the circle. Pi (π) is sometimes rounded to 2 decimal places (3.14). A particular circle has a radius of 7 cm.
- Use $\pi = 3.14$ to calculate the area of the circle to 2 decimal places.
 - Use the π key on your calculator to calculate the area of the circle to 4 decimal places.
 - Round your answer for part **b** to 2 decimal places.
 - Are your answers for parts **a** and **c** different? Why or why not?
12. The volume of a sphere (a ball shape) is calculated using the formula $V = \frac{4}{3} \times \pi \times r^3$, where r is the radius of the sphere. A beach ball with a radius of 25 cm is bouncing around the crowd at the MCG during the Boxing Day Test.
- Calculate the volume of the beach ball to 4 decimal places.
 - When the volume is calculated to 4 decimal places, how many significant figures does it have?
 - Is the calculated volume a rational number? Why or why not?
13. In a large sample of written English prose there are about 7 vowels to every 11 consonants. The letter 'e' accounts for about one-third of the occurrence of vowels. How many times would you expect the letter 'e' to occur in a passage of 100 000 letters? Round your answer to the nearest 100.
14. Is there an equal number of rational numbers and irrational numbers?



6.5 Scientific notation

[Stage 5.1]

6.5.1 Scientific notation

- Did you know that Jupiter is approximately 778 547 200 km from the sun? Astronomers and other scientists frequently use large numbers that are difficult to read and manipulate. To make them easier to use and compare, they can be written in a special way called **scientific notation** (or **standard form**).
- A number written in scientific notation looks like this:

$$5.316 \times 10^2$$

A number between 1 and 9.999...
×
A power of 10



WORKED EXAMPLE 16

Here are some numbers written in scientific notation. Write them in decimal notation.

a 7.136×10^2

b 5.017×10^5

c 8×10^6

THINK

a Move the decimal point 2 places to the right, since $10^2 = 100$.

b Move the decimal point 5 places to the right, since $10^5 = 100\,000$.

c Move the decimal point 6 places to the right, since $10^6 = 1\,000\,000$.

WRITE

a $7.136 \times 10^2 = 713.6$

b $5.017 \times 10^5 = 501\,700$

c $8 \times 10^6 = 8\,000\,000$

6.5.2 Using a calculator

- Numbers written in scientific notation can be entered into your calculator using a special button. Find out how this works on your calculator.
- Some calculators have unusual ways of displaying scientific notation. For example, some show 5.71×10^4 as 5.71E4. In writing you should always write this as 5.71×10^4 .

WORKED EXAMPLE 17

a Write each of these numbers in scientific notation.

i 827.2

ii 53681

iii 51900000000

b Round each answer from part a to 3 significant figures.

THINK

WRITE

a i The first part needs to be a number between 1 and 10, so 8.272. The decimal point must be moved 2 places ($\times 10^2$).

a i $827.2 = 8.272 \times 10^2$

ii The first part needs to be a number between 1 and 10, so 5.3681. The decimal point must be moved 4 places ($\times 10^4$).

ii $53681 = 5.3681 \times 10^4$

iii The first part needs to be a number between 1 and 10, so 5.190. The decimal point must be moved 10 places ($\times 10^{10}$).

iii $51900000000 = 5.190 \times 10^{10}$

b i $8.272 \approx 8.27$

b i 8.27×10^2

ii $5.368 \approx 5.37$

ii 5.37×10^4

iii $5.190 \approx 5.19$

iii 5.19×10^{10}

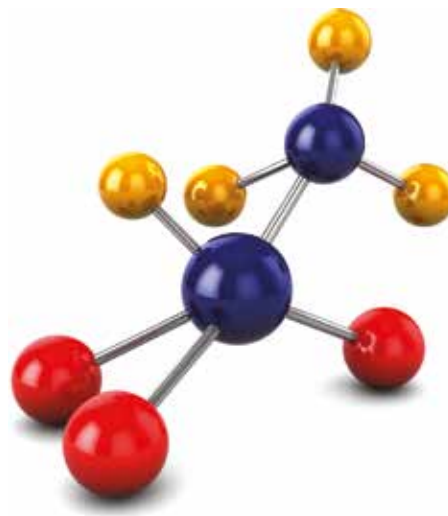
6.5.3 Small numbers in scientific notation

- Very small numbers, such as the length or the mass of a molecule, can also be expressed in scientific notation.

$$\begin{aligned} 0.0412 &= 4.12 \div 10^2 \\ &= 4.12 \times 10^{-2} \end{aligned}$$

It will be shown later that $10^{-2} = \frac{1}{10^2}$.

- Just as
$$6.285 \times 10^2 = 628.5$$
(the decimal point is moved 2 places to the right),
$$6.285 \times 10^{-2} = 0.06285$$
(the decimal point is moved 2 places to the left). Similarly,
$$6.285 \times 10^{-1} = 0.6285$$
and
$$6.285 \times 10^{-3} = 0.006285.$$



WORKED EXAMPLE 18

Write these numbers in decimal notation.

a 9.12×10^{-1}

b 7.385×10^{-2}

c 6.32×10^{-7}

THINK

- a** Move the decimal point 1 place to the left.
- b** Move the decimal point 2 places to the left.
(Insert a zero.)
- c** Move the decimal point 7 places to the left.
(Insert zeros.)

WRITE

a $9.12 \times 10^{-1} = 0.912$

b $7.385 \times 10^{-2} = 0.07385$

c $6.32 \times 10^{-7} = 0.000000632$

WORKED EXAMPLE 19

Write these numbers in scientific notation.

a 0.0051

b 0.1321

c 0.0000000007

THINK

- a** The first part needs to be a number between 1 and 10, so 5.1. The decimal point must be moved 3 places ($\times 10^{-3}$).
- b** The first part needs to be a number between 1 and 10, so 1.321. The decimal point must be moved 1 place ($\times 10^{-1}$).
- c** The first part needs to be a number between 1 and 10, so 7.0. The decimal point must be moved 10 places ($\times 10^{-10}$).

WRITE

a $0.0051 = 5.1 \times 10^{-3}$

b $0.1321 = 1.321 \times 10^{-1}$

c $0.0000000007 = 7.0 \times 10^{-10}$

ACTIVITY: LARGER OR SMALLER

Equipment: A4 paper, marker pens

1. Using a marker pen, write a number using scientific notation on your piece of A4 paper.
2. Compare your number with another person's number and decide which is larger and which is smaller.
3. Join your pair with another pair and arrange the numbers in ascending order.
4. As a group, discuss how you can easily order the size of numbers expressed using scientific notation.

learn^{on} RESOURCES — ONLINE ONLY

Interactivity: Scientific notation (int-6031)

Digital doc: SkillsHEET Multiplying and dividing by powers of 10 (doc-10815)

Digital doc: WorkSHEET Scientific notation (doc-10816)

Exercise 6.5 Scientific notation

Individual pathways

PRACTISE

Questions:
1–7, 9, 11, 13–15

CONSOLIDATE

Questions:
1–8, 10, 12–15, 18

MASTER

Questions:
1–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE16** Write these numbers in decimal notation.

a. 6.14×10^2	b. 6.14×10^3	c. 6.14×10^4	d. 3.518×10^2
e. 1×10^9	f. 3.92673×10^2	g. 5.911×10^2	h. 5.1×10^3
i. 7.34×10^5	j. 7.1414×10^6	k. 3.51×10	l. 8.05×10^4
- WE17a** Write these numbers in scientific notation.

a. 5000	b. 431	c. 38	d. 350 000
e. 72.5	f. 725	g. 7250	h. 725 000 000
- WE17b** Write these numbers in scientific notation, correct to 4 significant figures.

a. 43.792	b. 5317	c. 258.95	d. 110.11
e. 1632 000	f. 1 million	g. 123 456 789	h. 249.9
- WE18** Write these numbers in decimal notation.

a. 2×10^{-1}	b. 4×10^{-3}	c. 7×10^{-4}	d. 3×10^{-2}
e. 8.273×10^{-2}	f. 7.295×10^{-2}	g. 2.9142×10^{-3}	h. 3.753×10^{-5}
i. 5.29×10^{-4}	j. 3.3333×10^{-5}	k. 2.625×10^{-9}	l. 1.273×10^{-15}
- WE19** Write these numbers in scientific notation.

a. 0.7	b. 0.005	c. 0.000 000 3	d. 0.000 000 000 01
e. 0.231	f. 0.003 62	g. 0.000 731	h. 0.063
- Write these numbers in scientific notation, correct to 3 significant figures.

a. 0.006 731	b. 0.142 57	c. 0.000 068 3	d. 0.000 000 005 12
e. 0.0509	f. 0.012 46	g. 0.000 731	h. 0.063
- Write these numbers in ascending order.

a. 8.31×10^2 , 3.27×10^3 , 9.718×10^2 , 5.27×10^2
b. 7.95×10^2 , 4.09×10^2 , 7.943×10^2 , 4.37×10^2
c. 5.31×10^{-2} , 9.29×10^{-3} , 5.251×10^{-2} , 5.27×10^{-1}
d. 8.31×10^2 , 3.27×10^3 , 7.13×10^{-2} , 2.7×10^{-3}
- One carbon atom weighs 1.994×10^{-23} g.
 - Write the weight as a decimal.
 - How much will 1 million carbon atoms weigh?
 - How many carbon atoms are there in 10 g of carbon? Give your answer correct to 4 significant figures.
- The distance from Earth to the Moon is approximately 3.844×10^5 km. If you could drive there without breaking the speed limit (100 km/h), how long would it take? What is this in days (correct to 2 decimal places)?
- The Earth weighs 5.97×10^{24} kg, but the Sun weighs 1.99×10^{30} kg. How many Earths would it take to balance the Sun's weight? Give your answer correct to 2 decimal places.



- 11.** Inside the nucleus of an atom, a proton weighs 1.6726×10^{-27} kg and a neutron weighs 1.6749×10^{-27} kg. Which one is heavier and by how much?
- 12.** The Earth's orbit has a radius of 7.5×10^7 km; the orbit of Venus is equal to 5.4×10^7 km. How far apart are the planets when:
- they are closest to each other
 - they are farthest apart from each other?

Communicating, reasoning and problem solving

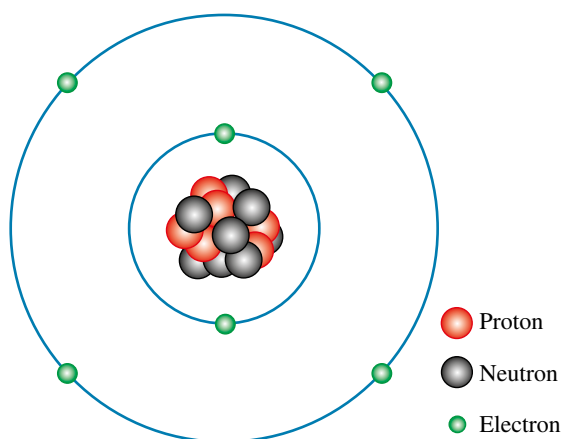
- 13. a.** State the power(s) of 10 that you believe equations **i** and **ii** will have when solved.
- $5.36 \times 10^7 + 2.95 \times 10^3$
 - $5.36 \times 10^7 - 2.95 \times 10^3$
- b.** Evaluate equations **i** and **ii**, correct to 3 significant figures.
- c.** Were your answers to part **a** correct? Why or why not?
- 14.** Explain why $2.39 \times 10^{-3} + 8.75 \times 10^{-7} = 2.39 \times 10^{-3}$, correct to 3 significant figures.
- 15.** Write the following numbers in ascending order.
- 14%, 0.753 , $\frac{5}{8}$, $\sqrt{2}$, 0.52 , $\sqrt[3]{2}$, 1.5×10^{-2}
- 16.** Distance is equal to speed multiplied by time. If we travelled at 100 km/h it would take us approximately 0.44 years to reach the Moon, 89.6 years to reach Mars, 1460 years to reach Saturn and 6590 years to reach Pluto.
- a.** Assuming that there are 365 days in a year, calculate the distance (as a basic numeral) between Earth and:
- the Moon
 - Mars
 - Saturn
 - Pluto.
- b.** Write your answers to part **a** correct to 3 significant figures.
- c.** Write your answers to part **a** using scientific notation correct to 3 significant figures.
- 17.** A light-year is the distance that light travels in one year. Light travels at approximately 300 000 km/s.
- a.**
- Calculate the number of seconds in a year (assuming 1 year = 365 days).
 - Write your answer to part **i** using scientific notation.
- b.** Calculate the distance travelled by light in one year. Express your answer:
- as a basic numeral
 - using scientific notation.
- c.** The closest 'star' to Earth (other than our sun) is the star system Alpha Centauri, which is 4.3 light-years away.
- How far is this in kilometres, correct to 4 significant figures?
 - Travelling at 100 km/h, how long in years would it take to reach Alpha Centauri?



18. Scientists used Earth's gravitational pull on nearby celestial bodies (such as the Moon) to calculate the mass of the Earth at approximately 5.972 sextillion metric tonnes.
- Write 5.972 sextillion using scientific notation.
 - How many significant figures does this number have?



19. An atom consists of smaller particles called protons, neutrons and electrons. Electrons have a mass of $9.10938188 \times 10^{-31}$ kilograms, correct to 9 significant figures.
- Write the mass of an electron correct to 5 significant figures.
 - Protons and neutrons are the same size. They are both 1836 times the size of an electron. Use the mass of an electron (correct to 9 significant figures) and your calculator to find the mass of a proton correct to 5 significant figures.
 - Use the mass of an electron correct to 3 significant figures to calculate the mass of a proton correct to 5 significant figures.
 - Why is it important to work with the original amounts and then round to the specified number of significant figures at the end of a calculation?



20. What is the advantage of converting numbers into scientific notation?
21. The diameter of the Sun is approximately 8.656×10^5 miles. Since there are 1.61 km in a mile, convert this to kilometres using scientific notation and give your answer correct to 4 significant figures. If the Sun's diameter is 10^9 times that of Earth, what is Earth's diameter? Give your answer correct to 3 significant figures.

6.6 Applying the index laws to variables

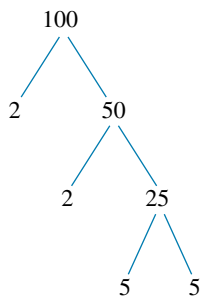
[Stage 5.1]

6.6.1 Review of index notation

- The product of factors can be written in a shorter form called **index notation**.

Index, exponent
Base $\rightarrow 6^4 = 6 \times 6 \times 6 \times 6 \leftarrow$ Factor form
 $= 1296$

- Any composite number can be written as a product of powers of prime factors using a factor tree, or by other methods, such as repeated division.



$$100 = 2 \times 2 \times 5 \times 5$$
$$= 2^2 \times 5^2$$

WORKED EXAMPLE 20

Express 360 as a product of powers of prime factors using index notation.

THINK

- Express 360 as a product of a factor pair.
- Further factorise 6 and 60.
- Further factorise 4 and 15.
- There are no more composite numbers.
- Write the answer using index notation.
Note: The factors are generally expressed with bases in ascending order.

WRITE

$$360 = 6 \times 60$$
$$= 2 \times 3 \times 4 \times 15$$
$$= 2 \times 3 \times 2 \times 2 \times 3 \times 5$$
$$= 2 \times 2 \times 2 \times 3 \times 3 \times 5$$
$$360 = 2^3 \times 3^2 \times 5$$

DISCUSSION

Can negative whole numbers be expressed as a product of powers of prime factors? Why or why not?

6.6.2 Multiplication using indices

- The **First Index Law** states: $a^m \times a^n = a^{m+n}$.
- That is, when multiplying terms with the same bases, add the indices.

WORKED EXAMPLE 21

Simplify $5e^{10} \times 2e^3$.

THINK

- 1 The order is not important when multiplying, so place the coefficients first.
- 2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).

WRITE

$$\begin{aligned} 5e^{10} \times 2e^3 \\ &= 5 \times 2 \times e^{10} \times e^3 \\ &= 10e^{13} \end{aligned}$$

- When more than one base is involved, apply the First Index Law to each base separately.

WORKED EXAMPLE 22

Simplify $7m^3 \times 3n^5 \times 2m^8n^4$.

THINK

- 1 The order is not important when multiplying, so place the coefficients first and group the same pronumerals together.
- 2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).

WRITE

$$\begin{aligned} 7m^3 \times 3n^5 \times 2m^8n^4 \\ &= 7 \times 3 \times 2 \times m^3 \times m^8 \times n^5 \times n^4 \\ &= 42m^{11}n^9 \end{aligned}$$

6.6.3 Division using indices

- The **Second Index Law** states: $a^m \div a^n = a^{m-n}$.
- That is, when dividing terms with the same bases, subtract the indices.

WORKED EXAMPLE 23

Simplify $\frac{25v^6 \times 8w^9}{10v^4 \times 4w^5}$.

THINK

- 1 Simplify the numerator and the denominator by multiplying the coefficients.
- 2 Simplify further by dividing the coefficients and applying the Second Index Law (subtract the indices).

WRITE

$$\begin{aligned} \frac{25v^6 \times 8w^9}{10v^4 \times 4w^5} \\ &= \frac{200v^6w^9}{40v^4w^5} \\ &= \frac{5\cancel{200}}{\cancel{140}} \times \frac{v^6}{v^4} \times \frac{w^9}{w^5} \\ &= 5v^2w^4 \end{aligned}$$

- When the coefficients do not divide evenly, simplify by cancelling.

WORKED EXAMPLE 24

Simplify $\frac{7t^3 \times 4t^8}{12t^4}$.

THINK

- 1 Simplify the numerator by multiplying the coefficients.
- 2 Simplify the fraction by dividing the coefficients by the highest common factor. Then apply the Second Index Law.

WRITE

$$\begin{aligned} & \frac{7t^3 \times 4t^8}{12t^4} \\ &= \frac{28t^{11}}{12t^4} \\ &= \frac{28}{12} \times \frac{t^{11}}{t^4} \\ &= \frac{7t^7}{3} \end{aligned}$$

6.6.4 Zero index

- Any number divided by itself (except zero) is equal to 1.
- Therefore, $\frac{10}{10} = \frac{2.14}{2.14} = \frac{\pi}{\pi} = \frac{592^3}{592^3} = 1$.
- Similarly, $\frac{x^3}{x^3} = 1$. But using the Second Index Law, $\frac{x^3}{x^3} = x^0$. It follows that $x^0 = 1$.
- In the same way, $\frac{n^{10}}{n^{10}} = 1$, and $\frac{n^{10}}{n^{10}} = n^0$, so $n^0 = 1$.
- In general, any number (except zero) to the power zero is equal to 1.
- This is the **Third Index Law: $a^0 = 1$, where $a \neq 0$** .

WORKED EXAMPLE 25

Evaluate the following.

- a** t^0 **b** $(xy)^0$ **c** 17^0 **d** $5x^0$ **e** $(5x)^0 + 2$ **f** $5^0 + 3^0$

THINK

- a** Apply the Third Index Law.
b Apply the Third Index Law.
c Apply the Third Index Law.
d Apply the Third Index Law.
e Apply the Third Index Law.
f Apply the Third Index Law.

WRITE

$$\begin{aligned} \mathbf{a} \quad & t^0 = 1 \\ \mathbf{b} \quad & (xy)^0 = 1 \\ \mathbf{c} \quad & 17^0 = 1 \\ \mathbf{d} \quad & 5x^0 = 5 \times x^0 \\ & \quad = 5 \times 1 \\ & \quad = 5 \\ \mathbf{e} \quad & (5x)^0 + 2 = 1 + 2 \\ & \quad = 3 \\ \mathbf{f} \quad & 5^0 + 3^0 = 1 + 1 \\ & \quad = 2 \end{aligned}$$

WORKED EXAMPLE 26

Simplify $\frac{9g^7 \times 4g^4}{6g^3 \times 2g^8}$.

THINK

- 1 Simplify the numerator and the denominator by applying the First Index Law.
- 2 Simplify the fraction further by applying the Second Index Law.
- 3 Simplify by applying the Third Index Law.

WRITE

$$\begin{aligned}
 & \frac{9g^7 \times 4g^4}{6g^3 \times 2g^8} \\
 &= \frac{36g^{11}}{12g^{11}} \\
 &= \frac{3\cancel{6}g^{11}}{1\cancel{2}g^{11}} \\
 &= 3g^0 \\
 &= 3 \times 1 \\
 &= 3
 \end{aligned}$$

6.6.5 Cancelling fractions

- Consider the fraction $\frac{x^3}{x^7}$. This fraction can be cancelled by dividing the denominator and the numerator by the highest common factor (HCF), x^3 , so $\frac{x^3}{x^7} = \frac{1}{x^4}$.

Note: $\frac{x^3}{x^7} = x^{-4}$ by applying the Second Index Law. We will study negative indices in a later section.

WORKED EXAMPLE 27

Simplify these fractions by cancelling.

a $\frac{x^5}{x^7}$

b $\frac{6x}{12x^8}$

c $\frac{30x^5y^6}{10x^7y^3}$

THINK

- Divide the numerator and denominator by the HCF, x^5 .
- Divide the numerator and denominator by the HCF, $6x$.
- Divide the numerator and denominator by the HCF, $10x^5y^3$.

WRITE

a $\frac{x^5}{x^7} = \frac{1}{x^2}$

$$\begin{aligned}
 \text{b } \frac{6x}{12x^8} &= \frac{6}{12} \times \frac{x}{x^8} \\
 &= \frac{1}{2} \times \frac{1}{x^7} \\
 &= \frac{1}{2x^7}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \frac{30x^5y^6}{10x^7y^3} &= \frac{30}{10} \times \frac{x^5}{x^7} \times \frac{y^6}{y^3} \\
 &= \frac{3}{1} \times \frac{1}{x^2} \times \frac{y^3}{1} \\
 &= \frac{3y^3}{x^2}
 \end{aligned}$$

Interactivity: Review of index form (int-3708)
 Interactivity: First Index Law (int-3709)
 Interactivity: Second Index Law (int-3711)
 Interactivity: Third Index Law (int-3713)
 Interactivity: Index laws (int-2769)
 eLesson: Index laws (eles-1903)
 Digital doc: SkillsHEET Index form (doc-6225)
 Digital doc: SkillsHEET Using a calculator to evaluate numbers in index form (doc-6226)

Exercise 6.6 Applying the index laws to variables

Individual pathways

PRACTISE

Questions:
 1–4, 5a–e, 6, 7a–e, 8–11, 13–18

CONSOLIDATE

Questions:
 1–3, 4a–c, 5d–g, 6, 7d–g, 8–19

MASTER

Questions:
 1, 2, 3e–j, 4d–f, 5f–i, 6, 7f–j, 8–22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE20** Express each of the following as a product of powers of prime factors using index notation.
 a. 12 b. 72 c. 75 d. 240 e. 640 f. 9800
- WE21** Simplify each of the following.
 a. $4p^7 \times 5p^4$ b. $2x^2 \times 3x^6$ c. $8y^6 \times 7y^4$
 d. $3p \times 7p^7$ e. $12t^3 \times t^2 \times 7t$ f. $6q^2 \times q^5 \times 5q^8$
- WE22** Simplify each of the following.
 a. $2a^2 \times 3a^4 \times e^3 \times e^4$ b. $4p^3 \times 2h^7 \times h^5 \times p^3$
 c. $2m^3 \times 5m^2 \times 8m^4$ d. $2gh \times 3g^2h^5$
 e. $5p^4q^2 \times 6p^2q^7$ f. $8u^3w \times 3uw^2 \times 2u^5w^4$
 g. $9y^8d \times y^5d^3 \times 3y^4d^7$ h. $7b^3c^2 \times 2b^6c^4 \times 3b^5c^3$
 i. $4r^2s^2 \times 3r^6s^{12} \times 2r^8s^4$ j. $10h^{10}v^2 \times 2h^8v^6 \times 3h^{20}v^{12}$
- WE23** Simplify each of the following.
 a. $\frac{15p^{12}}{5p^8}$ b. $\frac{18r^6}{5r^2}$ c. $\frac{45a^5}{5a^2}$
 d. $\frac{60b^7}{20b}$ e. $\frac{100r^{10}}{5r^6}$ f. $\frac{9q^2}{q}$
- WE24** Simplify each of the following.
 a. $\frac{8p^6 \times 3p^4}{16p^5}$ b. $\frac{12b^5 \times 4b^2}{18b^2}$ c. $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$
 d. $\frac{27x^9y^3}{12xy^2}$ e. $\frac{16h^7k^4}{12h^6k}$ f. $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$
 g. $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ h. $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ i. $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$

6. **WE25** Evaluate the following.

a. m^0	b. $6m^0$	c. $(6m)^0$	d. $(ab)^0$
e. $5(ab)^0$	f. w^0x^0	g. 85^0	h. $85^0 + 15^0$
i. $x^0 + 1$	j. $5x^0 - 2$	k. $\frac{x^0}{y^0}$	l. $x^0 + y^0$
m. $x^0 - y^0$	n. $3x^0 + 11$	o. $3a^0 + 3b^0$	p. $3(a^0 + b^0)$

7. **WE26** Simplify each of the following.

a. $\frac{2a^3 \times 6a^2}{12a^5}$	b. $\frac{3c^6 \times 6c^3}{9c^9}$	c. $\frac{5b^7 \times 10b^5}{25b^{12}}$	d. $\frac{8f^3 \times 3f^7}{4f^5 \times 3f^5}$
e. $\frac{9k^{12} \times 4k^{10}}{18k^4 \times k^{18}}$	f. $\frac{2h^4 \times 5k^2}{20h^2 \times k^2}$	g. $\frac{p^3 \times q^4}{5p^3}$	h. $\frac{m^7 \times n^3}{5m^3 \times m^4}$
i. $\frac{8u^9 \times v^2}{2u^5 \times 4u^4}$	j. $\frac{9x^6 \times 2y^{12}}{3y^{10} \times 3y^2}$		

8. **WE27** Simplify the following by cancelling.

a. $\frac{x^7}{x^{10}}$	b. $\frac{m}{m^9}$	c. $\frac{m^3}{4m^9}$	d. $\frac{12x^6}{6x^8}$
e. $\frac{12x^8}{6x^6}$	f. $\frac{24t^{10}}{t^4}$	g. $\frac{5y^5}{10y^{10}}$	h. $\frac{35x^2y^{10}}{20x^7y^7}$
i. $\frac{12m^2n^4}{30m^5n^8}$	j. $\frac{16m^5n^{10}}{8m^5n^{12}}$	k. $\frac{20x^4y^5}{10x^5y^4}$	l. $\frac{a^2b^4c^6}{a^6b^4c^2}$

9. Find the value of each of the following expressions if $a = 3$.

a. $2a$	b. a^2	c. $2a^2$	d. $a^2 + 2$	e. $a^2 + 2a$

Communicating, reasoning and problem solving

10. Explain why x^2 and $2x$ are not the same number. Include an example to illustrate your reasoning.

11. **MC** a. $12a^8b^2c^4(de)^0f$ when simplified is equal to:

A. $12a^8b^2c^4$	B. $12a^8b^2c^4f$	C. $12a^8b^2f$	D. $12a^8b^2$

b. $\left(\frac{6}{11}a^2b^7\right)^0 \times -(3a^2b^{11})^0 + 7a^0b$ when simplified is equal to:

A. $7b$	B. $1 + 7b$	C. $-1 + 7ab$	D. $-1 + 7b$

c. You are told that there is an error in the statement $3p^7q^3r^5s^6 = 3p^7s^6$. To make the statement correct, what should the left-hand side be?

A. $(3p^7q^3r^5s^6)^0$	B. $(3p^7)^0q^3r^5s^6$	C. $3p^7(q^3r^5s^6)^0$	D. $3p^7(q^3r^5)^0s^6$

d. You are told that there is an error in the statement $\frac{8f^6g^7h^3}{6f^4g^2h} = \frac{8f^2}{g^2}$. In order to make the statement correct, the left-hand side should be:

A. $\frac{8f^6(g^7h^3)^0}{(6)^0f^4g^2(h)^0}$	B. $\frac{8(f^6g^7h^3)^0}{(6f^4g^2h)^0}$	C. $\frac{8(f^6g^7)^0h^3}{(6f^4)^0g^2h}$	D. $\frac{8f^6g^7h^3}{(6f^4g^2h)}$

e. What does $\frac{6k^7m^2n^8}{4k^7(m^6n)^0}$ equal?

A. $\frac{6}{4}$	B. $\frac{3}{2}$	C. $\frac{3n^8}{2}$	D. $\frac{3m^2n^8}{2}$

12. Explain why $5x^5 \times 3x^3$ is not equal to $15x^{15}$.

13. A multiple choice question requires a student to multiply 5^6 by 5^3 . The student is having trouble deciding which of these four answers is correct: 5^{18} , 5^9 , 25^{18} or 25^9 .

- Which is the correct answer?
- Explain your answer by using another example to explain the First Index Law.

14. A multiple choice question requires a student to divide 5^{24} by 5^8 . The student is having trouble deciding which of these four answers is correct: 5^{16} , 5^3 , 1^{16} or 1^3 .
- Which is the correct answer?
 - Explain your answer by using another example to explain the Second Index Law.
15. a. What is the value of $\frac{5^7}{5^7}$?
- What is the value of any number divided by itself?
 - Applying the Second Index Law dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what index?
 - Explain the Third Index Law using an example.
16. a. For $x^2x^\Delta = x^{16}$ to be an identity, what number must replace the triangle?
- For $x^\Delta x^O x^\Diamond = x^{12}$ to be an identity, there are 55 ways of assigning positive whole numbers to the triangle, circle, and diamond. Give at least four of these.
17. a. Can you find a pattern in the units digit for powers of 3?
- The units digit of 3^6 is 9. What is the units digit of 3^{2001} ?
18. a. Can you find a pattern in the units digit for powers of 4?
- What is the units digit of 4^{105} ?
19. a. Investigate the patterns in the units digit for powers of 2 to 9.
- Predict the units digit for:
 - 2^{35}
 - 3^{16}
 - 8^{51} .
20. Write $4^{n+1} + 4^{n+1}$ as a single power of 2.
21. How do the index laws aid calculations?
22. Find the value of each of the following.
- $2^2 - 1^2$
 - $3^2 - 2^2$
 - $4^2 - 3^2$
- Use your results to investigate a pattern and use it to determine the answer to $87^2 - 86^2$ without actually performing the calculation.

6.7 Using index laws to simplify algebraic products and quotients

[Stage 5.1]

6.7.1 Raising a power to another power

- $(7^2)^3 = 7^2 \times 7^2 \times 7^2$
 $= 7^{2+2+2}$ (using the First Index Law)
 $= 7^{2 \times 3}$
 $= 7^6$
- The indices are multiplied when a power is raised to another power. This is the **Fourth Index Law**: $(a^m)^n = a^{m \times n}$.
- The Fifth and Sixth Index Laws are extensions of the Fourth Index Law.
Fifth Index Law: $(a \times b)^m = a^m \times b^m$
Sixth Index Law: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

WORKED EXAMPLE 28

Simplify the following.

- a $(7^4)^8$ b $(3a^2b^5)^3$

THINK

- a** Simplify by applying the Fourth Index Law (multiply the indices).
- b** **1** Write the expression.
2 Simplify by applying the Fifth Index Law for each term inside the brackets (multiply the indices).
3 Write the answer.

WRITE

$$\begin{aligned}\mathbf{a} \quad & (7^4)^8 \\ &= 7^{4 \times 8} \\ &= 7^{32}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad & (3^1 a^2 b^5)^3 \\ &= 3^{1 \times 3} a^{2 \times 3} b^{5 \times 3} \\ &= 3^3 a^6 b^{15} \\ &= 27 a^6 b^{15}\end{aligned}$$

WORKED EXAMPLE 29

Simplify $(2b^5)^2 \times (5b)^3$.

THINK

- 1** Write the expression, including all indices.
2 Simplify by applying the Fifth Index Law.
3 Simplify further by applying the First Index Law.

WRITE

$$\begin{aligned}& (2^1 b^5)^2 \times (5^1 b^1)^3 \\ &= 2^2 b^{10} \times 5^3 b^3 \\ &= 4 \times 125 \times b^{10} \times b^3 \\ &= 500 b^{13}\end{aligned}$$

WORKED EXAMPLE 30

Simplify $\left(\frac{2a^5}{d^2}\right)^3$.

THINK

- 1** Write the expression, including all indices.
2 Simplify by applying the Sixth Index Law for each term inside the brackets.
3 Write the answer.

WRITE

$$\begin{aligned}& \left(\frac{2^1 a^5}{d^2}\right)^3 \\ &= \frac{2^3 a^{15}}{d^6} \\ &= \frac{8a^{15}}{d^6}\end{aligned}$$

DISCUSSION

When allocating your pocket money, your parent asked if you would like \$20 a month for the next year or 20 cents for the first month, with the money doubled each month for a year. Which of the options would you choose and why?



RESOURCES — ONLINE ONLY

Interactivity: Fourth Index Law (int-3715)

Interactivity: Fourth Index Law 2 (int-3716)

Interactivity: Fifth and Sixth Index Laws (int-6063)

Digital doc: WorkSHEET Indices 1 (doc-6233)

Exercise 6.7 Using index laws to simplify algebraic products and quotients

Individual pathways

PRACTISE

Questions:

1a–f, 2a–f, 3a–d, 4–12, 14, 15

CONSOLIDATE

Questions:

1d–i, 2d–i, 3b–e, 4–12, 14–16

MASTER

Questions:

1g–i, 2g–i, 3e–h, 4–16

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE28** Simplify each of the following.

a. $(e^2)^3$

b. $(f^8)^{10}$

c. $(p^{25})^4$

d. $(r^{12})^{12}$

e. $(a^2b^3)^4$

f. $(pq^3)^5$

g. $(g^3h^2)^{10}$

h. $(3w^9q^2)^4$

i. $(7e^5r^2q^4)^2$

2. **WE29** Simplify each of the following.

a. $(p^4)^2 \times (q^3)^2$

b. $(r^5)^3 \times (w^3)^3$

c. $(b^5)^2 \times (n^3)^6$

d. $(j^6)^3 \times (g^4)^3$

e. $(q^2)^2 \times (r^4)^5$

f. $(h^3)^8 \times (j^2)^8$

g. $(f^4)^4 \times (a^7)^3$

h. $(t^5)^2 \times (u^4)^2$

i. $(i^3)^5 \times (j^2)^6$

3. **WE30** Simplify each of the following.

a. $\left(\frac{3b^4}{d^3}\right)^2$

b. $\left(\frac{5h^{10}}{2j^2}\right)^2$

c. $\left(\frac{2k^5}{3t^8}\right)^3$

d. $\left(\frac{7p^9}{8q^{22}}\right)^2$

e. $\left(\frac{5y^7}{3z^{13}}\right)^3$

f. $\left(\frac{4a^3}{7c^5}\right)^4$

g. $\left(\frac{-4k^2}{7m^6}\right)^3$

h. $\left(\frac{-2g^7}{3h^{11}}\right)^4$

4. Simplify each of the following.

a. $(2^3)^4 \times (2^4)^2$

b. $(t^7)^3 \times (t^3)^4$

c. $(a^4)^0 \times (a^3)^7$

d. $(b^6)^2 \times (b^4)^3$

e. $(e^7)^8 \times (e^5)^2$

f. $(g^7)^3 \times (g^9)^2$

g. $(3a^2)^4 \times (2a^6)^2$

h. $(2d^7)^3 \times (3d^2)^3$

i. $(10r^{12})^4 \times (2r^3)^2$

5. **MC** What does $(p^7)^2 \div p^2$ equal?

A. p^7

B. p^{12}

C. p^{16}

D. $p^{4.5}$

6. **MC** What does $\frac{(w^5)^2 \times (p^7)^3}{(w^2)^2 \times (p^3)^5}$ equal?

A. w^2p^6

B. $(wp)^6$

C. $w^{14}p^{36}$

D. w^2p^2

7. **MC** What does $(r^6)^3 \div r^4$ equal?

A. r^3

B. r^4

C. r^8

D. r^{10}

8. Simplify each of the following.

a. $(a^3)^4 \div (a^2)^3$

b. $(m^8)^2 \div (m^3)^4$

c. $(n^5)^3 \div (n^6)^2$

d. $(b^4)^5 \div (b^6)^2$

e. $(f^7)^3 \div (f^2)^2$

f. $(g^8)^2 \div (g^5)^2$

g. $(p^9)^3 \div (p^6)^3$

h. $(y^4)^4 \div (y^7)^2$

i. $\frac{(c^6)^5}{(c^5)^2}$

j. $\frac{(f^5)^3}{(f^2)^4}$

k. $\frac{(k^3)^{10}}{(k^2)^8}$

l. $\frac{(p^{12})^3}{(p^{10})^2}$

Communicating, reasoning and problem solving

9. a. Simplify each of the following.

i. $(-1)^{10}$

ii. $(-1)^7$

iii. $(-1)^{15}$

iv. $(-1)^6$

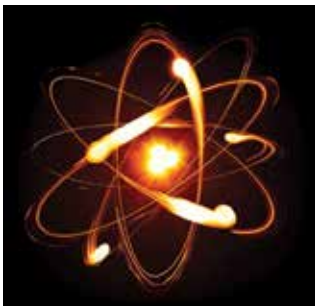
- b. Write a general rule for the result obtained when -1 is raised to a positive power. Justify your solution.

- 10. a.** Replace the triangle with the correct index for $4^7 \times 4^7 \times 4^7 \times 4^7 \times 4^7 = (4^7)^\Delta$.
b. The expression $(p^5)^6$ means to write p^5 as a factor how many times?
c. If you rewrote the expression from part **b** without any exponents, as $p \times p \times p \dots$, how many factors would you need?
d. Explain the Fourth Index Law.
 - 11.** A multiple choice question requires a student to calculate $(5^4)^3$. The student is having trouble deciding which of these three answers is correct: 5^{64} , 5^{12} or 5^7 .
a. Which is the correct answer?
b. Explain your answer by using another example to explain the Fourth Index Law.
 - 12.** Jo and Danni are having an algebra argument. Jo is sure that $-x^2$ is equivalent to $(-x)^2$, but Danni thinks otherwise. Explain who is correct and justify your answer.
 - 13. a.** Without using your calculator, simplify each side to the same base and solve each of the following equations.

i. $8^x = 32$

ii. $27^x = 243$

iii. $1000^x = 100000$

b. Explain why all three equations have the same solution.
 - 14.** Consider the expression 4^{3^2} . Explain how you could get two different answers.
 - 15.** The diameter of a typical atom is so small that it would take about 10^8 of them, arranged in a line, to reach just one centimetre. Estimate how many atoms are contained in a cubic centimetre. Write this number as a power of 10.
 - 16.** What difference, if any, is there between the operation of the index laws on numeric terms compared with similar operations on algebraic terms?
- 

[Stage 5.2]

- As previously stated, $\frac{x^4}{x^6} = \frac{1}{x^2}$ if the numerator and denominator are both divided by the highest common factor, x^4 .
- However, $\frac{x^4}{x^6} = x^{4-6} = x^{-2}$ if the Second Index Law is applied.
- It follows that $a^{-n} = \frac{1}{a^n}$. This is the **Seventh Index Law**.

What is the difference between $-x$ and x^{-1} ? Use some examples to assist your explanation.

$$\mathbf{c} \left(\frac{3}{5} \right)^{-1}$$

THINK

a 1 Apply the rule $a^{-n} = \frac{1}{a^n}$.

2 Simplify.

b Apply the rule $a^{-n} = \frac{1}{a^n}$.

c 1 Apply the Sixth Index Law, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

2 Apply the rule $a^{-n} = \frac{1}{a^n}$ to the numerator and denominator.

3 Simplify and write the answer.

WRITE

$$\begin{aligned}\mathbf{a} \quad 5^{-2} &= \frac{1}{5^2} \\ &= \frac{1}{25}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 7^{-1} &= \frac{1}{7^1} \\ &= \frac{1}{7}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad \left(\frac{3^1}{5^1}\right)^{-1} &= \frac{3^{-1}}{5^{-1}} \\ &= \frac{1}{3} \div \frac{1}{5} \\ &= \frac{1}{3} \times \frac{5}{1} \\ &= \frac{5}{3}\end{aligned}$$

WORKED EXAMPLE 32

Write the following with positive indices.

a x^{-3}

b $5x^{-6}$

c $\frac{x^{-3}}{y^{-2}}$

THINK

a Apply the rule $a^{-n} = \frac{1}{a^n}$.

b 1 Write in expanded form and apply the rule $a^{-n} = \frac{1}{a^n}$.

2 Simplify.

c 1 Write the fraction using division.

2 Apply the Seventh Index Law, $a^{-n} = \frac{1}{a^n}$.

3 Simplify.

WRITE

$$\mathbf{a} \quad x^{-3} = \frac{1}{x^3}$$

$$\begin{aligned}\mathbf{b} \quad 5x^{-6} &= 5 \times x^{-6} \\ &= 5 \times \frac{1}{x^6} \\ &= \frac{5}{x^6}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad \frac{x^{-3}}{y^{-2}} &= x^{-3} \div y^{-2} \\ &= \frac{1}{x^3} \div \frac{1}{y^2} \\ &= \frac{1}{x^3} \times \frac{y^2}{1} \\ &= \frac{y^2}{x^3}\end{aligned}$$

WORKED EXAMPLE 33

Simplify the following expressions, writing your answers with positive indices.

a $x^3 \times x^{-8}$

b $3x^{-2}y^{-3} \times 5xy^{-4}$

THINK

a 1 Apply the First Index Law,

$$a^n \times a^m = a^{m+n}.$$

2 Write the answer with a positive index.

b 1 Write in expanded form and apply the First Index Law.

2 Apply the rule $a^{-n} = \frac{1}{a^n}$.

3 Simplify.

WRITE

$$\begin{aligned} \text{a} \quad x^3 \times x^{-8} &= x^{3+(-8)} \\ &= x^{-5} \\ &= \frac{1}{x^5} \end{aligned}$$

$$\begin{aligned} \text{b} \quad 3x^{-2}y^{-3} \times 5xy^{-4} &= 3 \times 5 \times x^{-2} \times x^1 \times y^{-3} \times y^{-4} \\ &= 15x^{-1}y^{-7} \\ &= \frac{15}{1} \times \frac{1}{x} \times \frac{1}{y^7} \\ &= \frac{15}{xy^7} \end{aligned}$$

WORKED EXAMPLE 34

Simplify the following expressions, writing your answers with positive indices.

a $\frac{t^2}{t^{-5}}$

b $\frac{15m^{-5}}{10m^{-2}}$

THINK

a Apply the Second Index Law, $\frac{a^n}{a^m} = a^{n-m}$.

b 1 Apply the Second Index Law and simplify.

2 Write the answer with positive indices.

WRITE

$$\begin{aligned} \text{a} \quad \frac{t^2}{t^{-5}} &= t^{2-(-5)} \\ &= t^{2+5} \\ &= t^7 \end{aligned}$$

$$\begin{aligned} \text{b} \quad \frac{15m^{-5}}{10m^{-2}} &= \frac{15}{10} \times \frac{m^{-5}}{m^{-2}} \\ &= \frac{3}{2} \times m^{-5-(-2)} \\ &= \frac{3}{2} \times m^{-3} \\ &= \frac{3}{2} \times \frac{1}{m^3} \\ &= \frac{3}{2m^3} \end{aligned}$$

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RESOURCES — ONLINE ONLY

Interactivity: Negative indices (int-6064)

eLesson: Negative indices (eles-1910)

Digital doc: SkillsHEET Operations with directed numbers (doc-6100)

Exercise 6.8 Applying index laws to algebraic expressions involving integer indices

Individual pathways

PRACTISE

Questions:
1–10, 13, 14

CONSOLIDATE

Questions:
1–11, 13–16

MASTER

Questions:
1–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Copy and complete the patterns below.

a. $3^5 = 243$

$3^4 = 81$

$3^3 = 27$

$3^2 =$

$3^1 =$

$3^0 =$

$3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{9}$

$3^{-3} =$

$3^{-4} =$

$3^{-5} =$

c. $10^4 = 10000$

$10^3 =$

$10^2 =$

$10^1 =$

$10^0 =$

$10^{-1} =$

$10^{-2} =$

$10^{-3} =$

$10^{-4} =$

b. $5^4 = 625$

$5^3 =$

$5^2 =$

$5^1 =$

$5^0 =$

$5^{-1} =$

$5^{-2} =$

$5^{-3} =$

$5^{-4} =$

2. **WE31** Evaluate each of the following expressions.

a. 2^{-5}

b. 3^{-3}

c. 4^{-1}

d. 10^{-2}

e. 5^{-3}

f. $\left(\frac{1}{7}\right)^{-1}$

g. $\left(\frac{3}{4}\right)^{-1}$

h. $\left(\frac{3}{4}\right)^{-2}$

i. $\left(\frac{1}{3}\right)^{-3}$

j. $\left(\frac{3}{2}\right)^{-1}$

k. $\left(2\frac{1}{4}\right)^{-2}$

l. $\left(\frac{2}{7}\right)^{-2}$

3. **WE32** Write each expression with positive indices.

a. x^{-4}

b. y^{-5}

c. z^{-1}

d. a^2b^{-3}

e. $7m^{-2}$

f. $m^{-2}n^{-3}$

g. $(m^2n^3)^{-1}$

h. $\frac{x^2}{y^{-2}}$

i. $\frac{5}{x^{-3}}$

j. $\frac{x^{-2}}{w^{-5}}$

k. $\frac{1}{x^{-2}y^{-2}}$

l. $a^2b^{-3}cd^{-4}$

m. $\frac{a^2b^{-2}}{c^2d^{-3}}$

n. $10x^{-2}y$

o. $3^{-1}x$

p. $\frac{m^{-3}}{x^2}$

4. **WE33** Simplify the following expressions, writing your answers with positive indices.

a. $a^3 \times a^{-8}$	b. $m^7 \times m^{-2}$	c. $m^{-3} \times m^{-4}$	d. $2x^{-2} \times 7x$
e. $x^5 \times x^{-8}$	f. $3x^2y^{-4} \times 2x^{-7}y$	g. $10x^5 \times 5x^{-2}$	h. $x^5 \times x^{-5}$
i. $10a^2 \times 5a^{-7}$	j. $10a^{10} \times a^{-6}$	k. $16w^2 \times -2w^{-5}$	l. $4m^{-2} \times 4m^{-2}$
m. $(3m^2n^{-4})^3$	n. $(a^2b^5)^{-3}$	o. $(a^{-1}b^{-3})^{-2}$	p. $(5a^{-1})^2$

5. **WE34** Simplify the following expressions, writing your answers with positive indices.

a. $\frac{x^3}{x^8}$	b. $\frac{x^{-3}}{x^8}$	c. $\frac{x^3}{x^{-8}}$	d. $\frac{x^{-3}}{x^{-8}}$
e. $\frac{10a^4}{5a^5}$	f. $\frac{6a^2c^5}{a^4c}$	g. $10a^2 \div 5a^8$	h. $5m^7 \div m^8$
i. $\frac{a^5b^6}{a^5b^7}$	j. $\frac{a^2b^8}{a^5b^{10}}$	k. $\frac{a^{-3}bc^3}{abc}$	l. $\frac{4^{-2}ab}{a^2b}$
m. $\frac{m^{-3} \times m^{-5}}{m^{-5}}$	n. $\frac{2t^2 \times 3t^{-5}}{4t^6}$	o. $\frac{t^3 \times t^{-5}}{t^{-2} \times t^{-3}}$	p. $\frac{(m^2n^{-3})^{-1}}{(m^{-2}n^3)^2}$

6. Write the following numbers as powers of 2.

a. 1	b. 8	c. 32	d. 64	e. $\frac{1}{8}$	f. $\frac{1}{32}$

7. Write the following numbers as powers of 4.

a. 1	b. 4	c. 64	d. $\frac{1}{4}$	e. $\frac{1}{16}$	f. $\frac{1}{64}$

8. Write the following numbers as powers of 10.

a. 1	b. 10	c. 10000	d. 0.1	e. 0.01	f. 0.00001

Communicating, reasoning and problem solving

9. a. The result of dividing 3^7 by 3^3 is 3^4 . What is the result of dividing 3^3 by 3^7 ?
 b. Explain what it means to have a negative index.
 c. Explain how you write a negative index as a positive index.
10. Indices are encountered in science, where they help to deal with very small and large numbers. The diameter of a proton is 0.0000000000003 cm. Explain why it is logical to express this number in scientific notation as 3×10^{-13} .
11. a. When asked to find an expression equivalent to x^{-2} , a student responded $-2x$. Is this answer correct? Explain why or why not.
 b. When asked to find an expression that is equivalent to $x^3 + x^{-3}$, a student responded x^0 . Is this answer correct? Explain why or why not.
 c. When asked to find an expression that is equivalent to $(x^{-1} + y^{-1})^{-2}$, a student responded $x^2 + y^2$. Is this answer correct? Explain why or why not.
12. a. When asked to find an expression that is equivalent to $x^8 - x^{-5}$, a student responded x^3 . Is this answer correct? Explain why or why not.
 b. Another student said that $\frac{x^2}{x^8 - x^5}$ is equivalent to $\frac{1}{x^6} - \frac{1}{x^3}$. Is this answer correct? Explain why or why not.
13. What is the value of n in the following expressions?
 a. $4793 = 4.793 \times 10^n$
 b. $0.631 = 6.31 \times 10^n$
 c. $134 = 1.34 \times 10^n$
 d. $0.00056 = 5.6 \times 10^n$
14. Write the following numbers as basic numerals.
 a. 4.8×10^{-2}
 b. 7.6×10^3
 c. 2.9×10^{-4}
 d. 8.1×10^0
15. Find one-half of 2^{20} .
16. Find one-third of 3^{21} .
17. Simplify the following expressions.
 a. $(2^{-1} + 3^{-1})^{-1}$
 b. $\frac{3^{400}}{6^{200}}$
 c. $\sqrt{16x^{16}}$
18. What strategy will you use to remember the index laws?

6.9 Introduction to fractional indices

[Stage 5.3]

6.9.1 Square root

- The symbol $\sqrt{\quad}$ means ‘square root’ — a number that multiplies by itself to give the original number.
- Each number actually has a positive and negative square root. For example, $(2)^2 = 4$ and $(-2)^2 = 4$. Therefore the square root of 4 is $+2$ or -2 .
- $\sqrt{x}$ is defined as the positive square root of x , for $x \geq 0$. $\sqrt{x}$ is undefined for $x < 0$.
- The square root is the inverse of squaring (power 2).
- For this reason, a square root is equivalent to an index of $\frac{1}{2}$.
- In general, $\sqrt{a} = a^{\frac{1}{2}}$.

WORKED EXAMPLE 35

Evaluate $\sqrt{16p^2}$.

THINK

- 1 We need to obtain the square root of both 16 and p^2 .
- 2 Which number is multiplied by itself to give 16?
It is 4. Replace the square root sign with a power of $\frac{1}{2}$.
- 3 Use the Fourth Index Law.
- 4 Simplify.

WRITE

$$\begin{aligned}\sqrt{16p^2} &= \sqrt{16} \times \sqrt{p^2} \\ &= 4 \times (p^2)^{\frac{1}{2}} \\ &= 4 \times p^{2 \times \frac{1}{2}} \\ &= 4 \times p^1 \\ &= 4p\end{aligned}$$

6.9.2 Cube root

- The symbol $\sqrt[3]{\quad}$ means ‘cube root’ — a number that multiplies by itself three times to give the original number.
- The cube root is the inverse of cubing (power 3).
- For this reason, a cube root is equivalent to an index of $\frac{1}{3}$.
- In general, $\sqrt[3]{a} = a^{\frac{1}{3}}$.

DISCUSSION

Why does the cube root of a number always have the same sign (positive or negative) as the number?

WORKED EXAMPLE 36

Evaluate $\sqrt[3]{8j^6}$.

THINK

- 1 We need to obtain the cube root of both 8 and j^6 .
- 2 Which number, written 3 times and multiplied gives 8? It is 2.
Replace the cube root sign with a power of $\frac{1}{3}$.

WRITE

$$\begin{aligned}\sqrt[3]{8j^6} &= \sqrt[3]{8} \times \sqrt[3]{j^6} \\ &= 2 \times (j^6)^{\frac{1}{3}}\end{aligned}$$

3 Use the Fourth Index Law.

4 Simplify.

$$\begin{aligned} &= 2 \times j^{6 \times \frac{1}{3}} \\ &= 2 \times j^2 \\ &= 2j^2 \end{aligned}$$

- In general terms, $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.

learnon RESOURCES — ONLINE ONLY

Interactivity: Square roots (int-6066)

Interactivity: Cube roots (int-6065)

Digital doc: WorkSHEET Indices 2 (doc-6234)

Exercise 6.9 Introduction to fractional indices

Individual pathways

PRACTISE

Questions:
1–10, 13, 14

CONSOLIDATE

Questions:
1–11, 13–15

MASTER

Questions:
1–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Write the following in surd form.

a. $x^{\frac{1}{2}}$

b. $y^{\frac{1}{5}}$

c. $z^{\frac{1}{4}}$

d. $(2w)^{\frac{1}{3}}$

e. $7^{\frac{1}{2}}$

2. Write the following in index form.

a. $\sqrt{15}$

b. $\sqrt{m}$

c. $\sqrt[3]{t}$

d. $\sqrt[3]{w^2}$

e. $\sqrt[5]{n}$

3. **WE35** Evaluate the following.

a. $49^{\frac{1}{2}}$

b. $4^{\frac{1}{2}}$

c. $27^{\frac{1}{3}}$

d. $125^{\frac{1}{3}}$

e. $1000^{\frac{1}{3}}$

f. $64^{\frac{1}{2}}$

g. $64^{\frac{1}{3}}$

h. $128^{\frac{1}{7}}$

i. $243^{\frac{1}{5}}$

j. $1000000^{\frac{1}{2}}$

k. $1000000^{\frac{1}{3}}$

l. $\left(27^{\frac{1}{3}}\right)^2$

4. Writing a base as a power itself can be used to simplify an expression. Copy and complete the following calculations.

a. $16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} = \dots\dots\dots$

b. $343^{\frac{2}{3}} = (7^3)^{\frac{2}{3}} = \dots\dots\dots$

5. Simplify the following using index laws.

a. $8^{\frac{1}{3}}$

b. $27^{\frac{4}{3}}$

c. $125^{\frac{2}{3}}$

d. $512^{\frac{2}{9}}$

e. $16^{-\frac{1}{2}}$

f. $4^{-\frac{1}{2}}$

g. $32^{-\frac{1}{5}}$

h. $49^{-\frac{1}{2}}$

6. a. Use the index laws to simplify the following.

i. $(3^2)^{\frac{1}{2}}$

ii. $(4^2)^{\frac{1}{2}}$

iii. $(8^2)^{\frac{1}{2}}$

iv. $(11^2)^{\frac{1}{2}}$

b. Use your answers from part **a** to calculate the values of the following.

i. $9^{\frac{1}{2}}$

ii. $16^{\frac{1}{2}}$

iii. $64^{\frac{1}{2}}$

iv. $121^{\frac{1}{2}}$

c. Use your answers to parts **a** and **b** to write a sentence describing what raising a number to a power of one-half does.

7. **WE36** Simplify the following expressions.

a. $\sqrt{m^2}$

b. $\sqrt[3]{b^3}$

c. $\sqrt{36t^4}$

d. $\sqrt[3]{m^3n^6}$

e. $\sqrt[3]{125t^6}$

f. $\sqrt[5]{x^5y^{10}}$

g. $\sqrt[4]{a^8m^{40}}$

h. $\sqrt[3]{216y^6}$

i. $\sqrt[3]{64x^6y^6}$

j. $\sqrt{25a^2b^4c^6}$

k. $\sqrt[7]{b^{49}}$

l. $\sqrt[3]{b^3} \times \sqrt{b^4}$

8. **MC** a. What does $\sqrt[3]{8000m^6n^3p^3q^6}$ equal?

A. $2666.6m^2npq^2$

B. $20m^2npq^2$

C. $20m^3n^0p^0q^3$

D. $7997m^2npq^2$

b. What does $\sqrt[3]{3375a^9b^6c^3}$ equal?

A. $1125a^3b^2c$

B. $1125a^6b^3c^0$

C. $1123a^6b^3$

D. $15a^3b^2c$

c. What does $\sqrt[3]{15625f^3g^6h^9}$ equal?

A. $25fg^2h^3$

B. $25f^0g^3h^6$

C. $25g^3h^6$

D. $5208.3fg^2h^3$

Communicating, reasoning and problem solving

9. a. Using the First Index Law, explain how $3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3$.

b. What is another way that $3^{\frac{1}{2}}$ can be written?

c. Find $\sqrt{3} \times \sqrt{3}$.

d. How can $\sqrt[n]{a}$ be written in index form?

e. Without a calculator, solve:

i. $8^{\frac{1}{3}}$

ii. $32^{\frac{2}{5}}$

10. a. Explain why calculating $z^{2.5}$ is a square root problem.

b. Is $z^{0.3}$ a cube root problem? Justify your reasoning.

11. Mark and Christina are having an algebra argument. Mark is sure that $\sqrt{x^2}$ is equivalent to x , but Christina thinks otherwise. Who is correct? Explain how you would resolve this disagreement.

12. Verify that $(-8)^{\frac{1}{3}}$ can be evaluated and explain why $(-8)^{\frac{1}{4}}$ cannot be evaluated.

13. If $n^{\frac{3}{4}} = \frac{8}{27}$, what is the value of n ?

14. The mathematician Augustus de Morgan enjoyed telling his friends that he was x years old in the year x^2 . Find the year of Augustus de Morgan's birth, given that he died in 1871.

15. a. Investigate Johannes Kepler.

b. Kepler's Third Law describes the relationship between the distance of planets from the Sun and their orbital periods. It is represented by the equation $d^{\frac{1}{2}} = t^{\frac{1}{3}}$. Solve for:

i. d in terms of t

ii. t in terms of d .



Johannes Kepler.

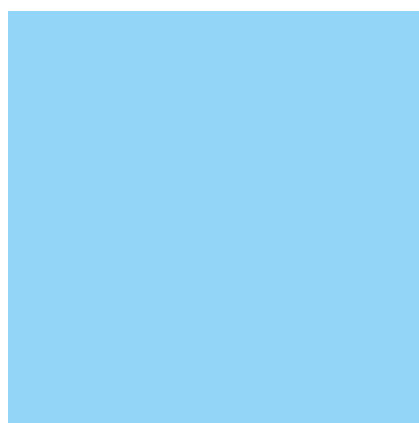
16. An unknown number is multiplied by 4 and then has 5 subtracted from it. It is now equal to the square root of the original unknown number squared.
- Is this a linear algebra problem? Justify your answer.
 - How many solutions are possible? Explain why.
 - Find all possible values for the number.
17. How would $\sqrt[n]{a^b}$ be written in index form?
18. The pronumeral i is used to represent $\sqrt{-1}$. This is the basis for imaginary numbers. It follows that the value of i^2 is -1 . What is the value of i^{40} ?

6.10 Review

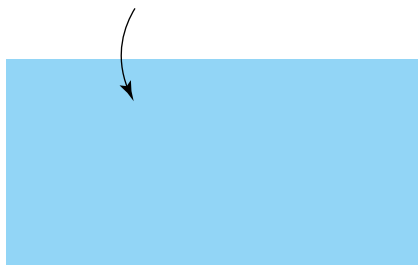
Investigation | Rich task

Paper folds

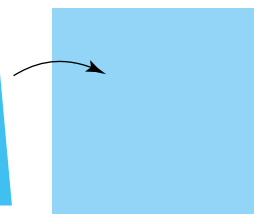
If you take a piece of paper and fold it a few times, you will soon notice that the area of the upper surface of the paper decreases with each fold. In this task, consider the relationships between the number of folds, the resulting thickness of paper and the upper surface area after each fold has been made.



One square piece of paper



One fold, two sheets thick



Two folds, four sheets thick

Take a square piece of paper with a side length of 8 cm. The upper surface of this paper has an area of 64 cm^2 . Fold the paper in half. The paper now shows an area of 32 cm^2 , has 1 fold and is 2 sheets thick. Make another fold. The diagram above displays the changes that take place for 2 folds.

Continue with the folding process for up to 5 folds. The thickness of the paper and the surface area of the upper face change with each fold.

- Write the dimensions of each upper surface after each fold.
- Calculate the area (in cm^2) of each upper surface after each fold.

3. Complete the following table to show the change in the upper surface area and the thickness after each fold.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2				
Area of upper surface after each fold (cm ²)	64	32				

4. Study the values recorded in the table in question 3. Explain whether there is a linear relationship between the number of folds and the thickness of the paper, or between the number of folds and the area after each fold.

Let f represent the number of folds, t represent the thickness of the paper after each fold and a represent the area of the upper face after each fold. A relationship between the pronumerals may be more obvious if the values in the table are presented in a different form.

5. Complete the table below, presenting your values in index form with a base of 2.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1				
Area of upper surface after each fold (cm ²) (a)	2^6	2^5				

6. Consider the values in the table above to write a relationship between the following pronumerals.

- t and f
- a and t
- a and f

7. What difference, if any, would it make to these relationships if the original paper size had been a square with side length of 16 cm? Draw a table to show the change in area of each face and the thickness of the paper with each fold. Write formulas to describe these relationships.



8. Investigate these relationships with squares of different side lengths. Describe whether the relationship between the three features studied during this task can always be represented in index form.

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Digital doc: Investigation — Paper folds (doc-15902)

Digital doc: Code puzzle — What is the name given to a boat with one sculler and two oarsmen? (doc-15903)

Digital doc: Summary Topic 6 (doc-22888)

Interactivities: Crossword Topic 6 (int-2697)

Interactivities: Word search Topic 6 (int-2696)

Interactivities: Sudoku Topic 6 (int-3209)

Exercise 6.10 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- MC** Which of the given numbers, $\sqrt{\frac{6}{12}}$, $\sqrt{0.81}$, 5 , -3.26 , $0.\dot{5}$, $\frac{\pi}{5}$, $\sqrt{\frac{3}{12}}$, are rational?
 - $\sqrt{0.81}$, 5 , -3.26 , $0.\dot{5}$ and $\sqrt{\frac{3}{12}}$
 - $\sqrt{\frac{6}{12}}$ and $\frac{\pi}{5}$
 - $\sqrt{\frac{6}{12}}$, $\sqrt{0.81}$ and $\sqrt{\frac{3}{12}}$
 - 5 , -3.26 and $\sqrt{\frac{6}{12}}$
- For each of the following, state whether the number is rational or irrational and give the reason for your answer.
 - $\sqrt{12}$
 - $\sqrt{121}$
 - $\frac{2}{9}$
 - $0.\dot{6}$
 - $\sqrt[3]{0.08}$
- MC** Which numbers in the set $\{3\sqrt{2}, 5\sqrt{7}, 9\sqrt{4}, 6\sqrt{10}, 7\sqrt{12}, 12\sqrt{64}\}$ are surds?
 - $9\sqrt{4}$, $12\sqrt{64}$
 - $3\sqrt{2}$ and $7\sqrt{12}$ only
 - $3\sqrt{2}$, $5\sqrt{7}$ and $6\sqrt{10}$ only
 - $3\sqrt{2}$, $5\sqrt{7}$, $6\sqrt{10}$ and $7\sqrt{12}$
- Which of the following are not irrational numbers?
 - $\sqrt{7}$
 - $\sqrt{8}$
 - $\sqrt{81}$
 - $\sqrt{361}$
- MC** The number 0.00067 written using scientific notation is:
 - 67×10^{-5}
 - 0.67×10^{-3}
 - 6.7×10^{-4}
 - 6.7×10^5
- MC** The expression $\sqrt{250}$ may be simplified to:
 - $25\sqrt{10}$
 - $5\sqrt{10}$
 - $10\sqrt{5}$
 - $5\sqrt{50}$
- State the base for each of the following.
 - 4^8
 - 5^9
 - a^4
 - x^7
- State the power or index for each of the following.
 - 10^3
 - 6^{12}
 - g^{89}
 - 4^0
- Write each of the following as a basic numeral.
 - 5^2
 - 4^2
 - 2^5
 - 10^3
 - 3^4
 - 12^8
- Express each of the following as a product of powers of primes.
 - 100
 - 121
 - 104
 - 225
 - 588
 - 10080
- Simplify each of the following.
 - $b^7 \times b^3$
 - $m^9 \times m^2$
 - $k^3 \times k^5$
 - $f^2 \times f^8 \times f^4$
 - $h^4 \times h^5 \times h$
 - $2q^5 \times 3q^2 \times q^{10}$
 - $5w^3 \times 7w^{12} \times w^{14}$
 - $2e^2p^3 \times 6e^3p^5$
 - $5a^2b^4 \times 3a^8b^5 \times 7a^6b^8$
- Simplify each of the following.
 - $a^5 \div a^2$
 - $t^5 \div t$
 - $r^{19} \div r^{12}$
 - $p^8 \div p^5$
 - $\frac{f^{17}}{f^{12}}$
 - $\frac{y^{100}}{y^{10}}$
 - $\frac{m^{24}}{m^{14}}$
 - $\frac{g^4 \times g^5}{g^2}$
 - $\frac{x^6 \times x^2 \times x}{x^8}$
 - $\frac{d^6 \times d^7 \times d^2}{d^8}$
 - $\frac{t^7 \times t \times t^3}{t^2 \times t^4}$
 - $\frac{p^5 \times p^3 \times p \times p^4}{p^2 \times p^4 \times p^2}$
 - $\frac{16k^{13}}{21} \div \frac{8k^9}{42}$
 - $\frac{22b^{15}}{c} \div \frac{2b^8}{c^6}$
 - $\frac{9d^8}{16e^{10}} \div \frac{2d^{10}}{e^{16}}$

13. Simplify the following.

a. 5^0	b. 12^0	c. 345^0	d. q^0
e. r^0	f. ab^0	g. $3w^0$	h. $5q^0 - 2q^0$
i. $100s^0 + 99t^0$	j. a^7b^0	k. $v^{10}w^0$	l. prr^0
m. $a^9b^4c^0$	n. $j^8k^0m^3$	o. $4e^2f^0 - 36(a^2b^3)^0$	p. $-8(18x^2y^4z^6)^0$
q. $15 - 12x\left(\frac{3x}{8}\right)^0$			
r. $-4p^0 \times 6(q^2r^3)^0 \div 8(-12q^2)^0$			
s. $3(6w^0)^2 \div 2(5w^5)^0$			

14. Raise each of the following to the given power.

a. $(b^4)^2$	b. $(a^8)^3$	c. $(k^7)^{10}$	d. $(j^{100})^2$
e. $(a^5b^2)^3$	f. $(m^7n^{12})^2$	g. $(st^6)^3$	h. $(qp^{30})^{10}$

15. What does $\left(\frac{4b^4}{d^2}\right)^3$ equal?

A. $\frac{4b^3}{d^3}$	B. $\frac{12b^{12}}{d^6}$	C. $\frac{64b^{12}}{d^6}$	D. $\frac{64b^7}{d^5}$

16. Write each of the following with positive indices.

a. a^{-1}	b. k^{-4}	c. $4m^3 \div 2m^7$	d. $7x^3y^{-4} \times 6x^{-3}y^{-1}$

17. Write each of the following using a negative index.

a. $\frac{1}{x}$	b. $\frac{2}{y^4}$	c. $z \div z^4$	d. $45p^2q^{-4} \times 3p^{-5}q$

18. Simplify each of the following.

a. $\sqrt{100}$	b. $\sqrt{36}$	c. $\sqrt{a^2}$	d. $\sqrt{b^2}$
e. $\sqrt{49f^4}$	f. $\sqrt[3]{27}$	g. $\sqrt[3]{1000}$	h. $\sqrt[3]{x^3}$
i. $\sqrt[3]{8d^3}$	j. $\sqrt[3]{64f^6g^3}$		

Reasoning and problem solving

- If your body produces about 2.0×10^{11} red blood cells each day, how many red blood cells has your body produced after one week?
- If your body produces about 1.0×10^{10} white blood cells each day, how many white blood cells does your body produce each hour?
- If the planet Earth is approximately 1.496×10^8 km from the sun and the planet Mercury is about 5.8×10^7 km from the sun, what is the closest distance from Mercury to Earth?
- Determine the closest distance between Mars and Saturn if Mars is 2.279×10^8 km from the sun and Saturn is 1.472×10^9 km from the sun.
- Consider the numbers 6×10^9 , 3.4×10^7 and 1.5×10^9 .
 - Place the numbers in order from smallest to largest.
 - If the population of Oceania is 3.4×10^7 and the world population is 6×10^9 , what percentage is the Oceania population of the world population? Give your answer to 2 decimal places.
 - What must you multiply 1.5×10^9 by to get 6×10^9 ?
 - Assume the surface area of the Earth is approximately 1.5×10^9 km² and the world population is about 6×10^9 . If we divided all the Earth up into equal portions, how much would each person get?
- A newly discovered super colony of bees contains 2.05×10^8 bees. If 0.4% of these were estimated to be queen bees, how many queen bees live in the super colony?
- The area of Victoria is 2.27×10^5 km². If the total area of Australia is 7.69×10^6 km², what percentage of Australia does the state of Victoria occupy? Write your answer correct to two decimal places.

26. Third-generation fibre optics that can carry up to 10 trillion (1×10^{12}) bits per second down a single strand of fibre have been recently tested. Currently the amount that one fibre can carry is tripling every 6 months and is predicted to do so for the next 20 years. How many years will it take for the amount of bits per second being carried down a single strand of fibre to exceed 6×10^{16} bits per second? Explain, in detail, your solution process.
27. Mach speed refers to speeds at or above the speed of sound. Mach 1 is the speed of sound, which is approximately 343 metres per second (m/s). Mach 2 is double the speed of sound; Mach 3 is triple the speed of sound, and so on. The Earth has a circumference of about 4.0×10^4 km.
- The fastest aircraft recorded travelled at 11 270 kilometres per hour (km/h). Express this as a Mach speed correct to 1 decimal place.
 - Create and complete a table for the four different speeds with columns 'Mach number', 'Speed in m/s' and 'Speed in km/h'.
 - Calculate how long it would take something travelling at Mach 1, 2 and 3 to circle the Earth. Give your answer in hours correct to 2 decimal places.
 - How long does it take the world's fastest aircraft to circle the Earth? Give your answer in hours correct to 2 decimal places.
 - Calculate how many times the world's fastest aircraft could circle the Earth in the time it takes an aircraft going at Mach 1, 2 and 3 to circle the Earth once. Give your answers correct to 2 decimal places.
 - Explain the relationship between your answers in the previous question and their corresponding Mach value.

28. The chessboard problem

Legend has it that an ancient Chinese king challenged the people around him to invent a game that would keep him amused for hours on end and would challenge him mentally. Days later the king was presented with the game of chess.

The king thought that this was a marvellous game and wanted to reward the inventor. The inventor said, *'I do not ask much for my king's pleasure. All I ask for is one grain of rice for the first square, two grains of rice for the second square, four grains of rice for the third square and so on.'*

The king was amazed that one could ask for so little for such a wonderful game. He sent his mathematician away to calculate exactly what the inventor was to be paid.

The mathematician came back and said that the inventor had asked for more than all the rice in the Kingdom of China!



- The inventor asked for one grain of rice for the first square, two for the second, four for the third and so on. For the first row of the board, write down the number of grains of rice for each square.
- We can write 4 as 2×2 , 8 as $2 \times 2 \times 2$ and so on. Write each of the amounts of grain for the first row, in index form.
- For each square on the board, write the number of grains of rice to be paid, in index notation.

d. Complete the table below for the first row.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	
5		
6		
7		
8		

e. Can you see a pattern between the numbers in the total paid column and in the grains paid column?

f. Calculate the total number of rice grains to be paid.

Answers

Topic 6 Indices, scientific notation and surds

Exercise 6.2 Rational and irrational numbers

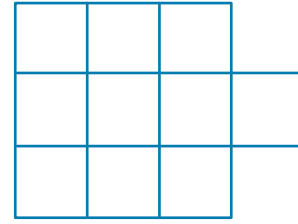
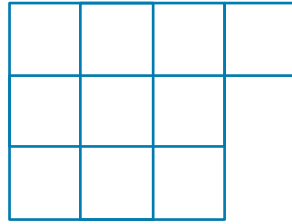
1. a. $\frac{15}{1}$ b. $\frac{-8}{1}$ c. $\frac{8}{3}$ d. $\frac{-41}{8}$ e. $\frac{4}{1}$ f. $\frac{73}{10}$
 g. $\frac{2}{1000}$ h. $\frac{872}{10}$ i. $\frac{0}{1}$ j. $\frac{156}{100}$ k. $\frac{3612}{1000}$ l. $\frac{-8}{100}$
2. a. 0.55555555 b. 0.51515151 c. 0.51111111 d. 6.0313131 e. 5.1831831
 f. -7.0244444 g. 8.9124912 h. 5.1234343 i. 5.1234123 j. 3.0020202
3. a. $0.\dot{5}$ b. $0.2\dot{7}$ c. $2.\dot{1}\dot{8}$ d. $0.42857\dot{1}$ e. $-1.\dot{7}\dot{4}$
 f. $0.\dot{7}\dot{3}$ g. $-3.8\dot{3}$ h. $0.4\dot{6}$ i. $0.4\dot{6}$ j. $0.04\dot{6}$
4. a. 15 b. -3, 0, 15 c. $-3, \frac{-3}{7}, 0, 2.3, 2.\dot{3}, 2\frac{3}{5}, 15$
5. 2.i, 2.i2i, 2.12i, 2.i2, 2.12
6. a. 1 b. 2 c. 0 d. $\frac{1}{3}$ e. $\frac{5}{4}$
 f. 0.4 g. 20 h. 100 i. $\frac{2}{5}$ j. 1.2
 k. 4.5 l. 1000 m. 0.03 n. 16
7. a. 9 b. -9 c. 11 d. -21 e. 2 f. 4
 g. 7 h. 3 i. 4 j. 5 k. -7 l. -3
8. b, d, g, h, k, m, o
9. Every integer a can be written as $\frac{a}{1}$ and is therefore rational.
10. Yes, because the division of an integer by another integer will always result in a rational number.
11. a. $\frac{1}{3}, \frac{2}{3}, \frac{1}{6}$ b. 6
12. Answers will vary. Example answers are shown.
 a. $\frac{1}{9}$ b. $\frac{10}{99}$ c. $\frac{100}{999}$ d. $\frac{1000}{9999}$
13. \$3330.20
14. 657
15. $\frac{1}{13} = 0.07692\dot{3}$, so there are 6 recurring digits.

Exercise 6.3 Simple operations with surds

1. a. $\sqrt{21}$ b. $-\sqrt{21}$ c. $2\sqrt{6}$ d. $6\sqrt{7}$ e. $10\sqrt{14}$ f. $15\sqrt{10}$
 g. $12\sqrt{7}$ h. $9\sqrt{7}$ i. $22\sqrt{10}$ j. $22\sqrt{3}$ k. 3 l. 24
 m. 6 n. 11 o. 51 p. 30
2. a. $\sqrt{3}$ b. $-\sqrt{2}$ c. $\sqrt{6}$ d. $\sqrt{5}$ e. $3\sqrt{3}$ f. $\frac{3\sqrt{6}}{2}$
 g. $15\sqrt{2}$ h. $\frac{5\sqrt{5}}{3}$ i. $-2\sqrt{5}$ j. $\sqrt{3}$ k. $\sqrt{3}$ l. $\sqrt{7}$
 m. $\sqrt{35}$ n. 2 o. 6 p. 3
3. a. $2\sqrt{5}$ b. $2\sqrt{2}$ c. $3\sqrt{2}$ d. 7 e. $\sqrt{30}$ f. $5\sqrt{2}$
 g. $2\sqrt{7}$ h. $6\sqrt{3}$ i. $12\sqrt{2}$ j. $4\sqrt{3}$ k. $10\sqrt{5}$ l. $9\sqrt{2}$
4. a. $4\sqrt{2}$ b. $15\sqrt{3}$ c. 48 d. $35\sqrt{2}$ e. $20\sqrt{6}$ f. $132\sqrt{2}$
 f. $10\sqrt{3}$ g. $4\sqrt{42}$ h. $72\sqrt{2}$ i. $27\sqrt{5}$ j. $\sqrt{384}$
5. a. $\sqrt{12}$ b. $\sqrt{175}$ c. $\sqrt{108}$ d. $\sqrt{80}$ e. $\sqrt{384}$ f. $\sqrt{338}$
 f. $\sqrt{90}$ g. $\sqrt{32}$ h. $\sqrt{720}$ i. $\sqrt{600}$ j. $\sqrt{338}$
6. a. D b. A c. C d. C
 e. D f. D g. D h. B
7. a. $15\sqrt{3}$ b. $19\sqrt{5}$ c. $16\sqrt{7}$ d. $17\sqrt{2}$
 e. $a\sqrt{c}$ f. $d^2\sqrt{b}$ g. $hk\sqrt{j}$ h. $f\sqrt{f}$
8. a. $2\sqrt{2}$ b. $-4\sqrt{5}$ c. $-6\sqrt{3}$ d. 0 e. $5\sqrt{11}$ f. $2\sqrt{7}$
 g. $10\sqrt{2} + 7\sqrt{3}$ h. $8\sqrt{5} + \sqrt{6}$ i. $8\sqrt{10} + 7\sqrt{3}$ j. $16\sqrt{2} - 11\sqrt{5}$ k. $2\sqrt{6}$ l. $5\sqrt{5} + 15$
9. a. $\sqrt{2}$ b. 0 c. $-5\sqrt{3}$ d. $-4\sqrt{7}$ e. $5\sqrt{6} + 6\sqrt{5}$
 f. $2\sqrt{3} - 3\sqrt{5}$ g. $4\sqrt{6} + 6\sqrt{5} - 14\sqrt{2}$ h. $29\sqrt{5} + 22\sqrt{3}$ i. $9\sqrt{11} - \sqrt{30}$ j. $28\sqrt{2} - 39\sqrt{5}$

10. a. D b. A c. C d. A
 11. a. $2\sqrt{3}$ b. $5\sqrt{2}$ c. $10\sqrt{2}$ d. $2\sqrt{10}$
 12. a. $2\sqrt{7}$, $4\sqrt{2}$, $3\sqrt{6}$, $\sqrt{60}$, 8, $6\sqrt{2}$ b. $\sqrt{2}$, $\sqrt{3}$, 2, $\sqrt{6}$, $2\sqrt{2}$, 3, $2\sqrt{3}$
 13. a. $\sqrt{2}\text{m}$ b. 1.41m

c. i. Any of these:



ii. $14\sqrt{2}\text{m}$ iii. 19.80m

14. $\sqrt{a^3b^2} = \sqrt{a^3} \times \sqrt{b^2}$
 $= \sqrt{a^2} \times \sqrt{a} \times \sqrt{b^2}$
 $= a \times \sqrt{a} \times b$
 $= ab\sqrt{a}$

15. $36\sqrt{5}\text{m}^2$

16. $(1750\sqrt{2} + 2750\sqrt{3} + 320)\text{m}$

17. The side length will be rational if x is a perfect square; otherwise, the side length will be a surd.

18. a. i. $\sqrt{13}$ ii. 3 iii. $\sqrt{38}$
 b. For side lengths $\sqrt{a}$ and $\sqrt{b}$, the length of the hypotenuse is $\sqrt{a+b}$.
 c. i. $\sqrt{1500} = 10\sqrt{15}$ ii. $\sqrt{456} = 2\sqrt{114}$ iii. $\sqrt{187}$
 d. The answer should be $\sqrt{b+a} = \sqrt{c}$.

19. No, not all square root numbers are surds. A surd is when the square root of a number is an irrational number, i.e. it cannot be expressed as a fraction or as a recurring or terminating decimal.

20. $45^2 = 2025$; $56^2 = 3136$

Exercise 6.4 Rounding and significant figures

1. a. 1.571 b. 2.236 c. 3.873 d. 16.435 e. 5.111
 f. 5.156 g. 5.152 h. 2.718 i. 137.358 j. 0.429
 k. 0.077 l. 2.429 m. 1.000 n. 6.581 o. 4.000
 p. 0.254 q. 0.025 r. 0.001 s. 5.000 t. 2342.156

2. 3.1416, 3.14159, 3.141593, 3.1415927

3. a. 2 b. 3 c. 4 d. 3 e. 4 f. 4
 g. 2 h. 1 i. 3 j. 4 k. 2 l. 3

4. a. 1.5708 b. 2.2361 c. 3.8730 d. 16.435 e. 5.1111
 f. 5.1556 g. 5.1515 h. 2.7183 i. 137.36 j. 0.42857
 k. 0.076923 l. 2.4286 m. 1.0000 n. 6.5813 o. 4.0000
 p. 0.25364 q. 0.025400 r. 0.00091360 s. 5.0000 t. 2342.2

5. a. $\sqrt{7}$, 3.5, $\sqrt{15}$, 4, $\sqrt{18}$ b. 3.14, 3.14, π , $\frac{22}{7}$, $\sqrt{10}$

6. a. F b. T c. F d. T
 7. a. F b. T c. F d. T
 8. a. T b. F c. F d. T

9. $\sqrt{25}$ means the positive square root of 25, i.e. 5, because $5^2 = 25$. Similarly, $-\sqrt{25}$ means the negative square root of 25, which is -5 because $(-5)^2 = 25$. But -25 , like all other negative numbers, has no real square root, so $\sqrt{-25}$ is 'undefined'.

10. The 5th digit (5) causes the 4th digit to round up from 9 to 10, which causes the 3rd digit to round up from 9 to 10, which causes the 2nd digit to round up from 2 to 3.

11. a. 153.86 cm^2 b. 153.9380 cm^2 c. 153.94 cm^2

d. Yes, because 3.14 is used as an estimate and is not the value of π as used on the calculator.

12. a. 65449.8470 cm^3 b. 9

c. Yes, any number with a finite number of decimal places is a rational number.

13. 13000

14. There is an infinite number of both rational and irrational numbers. It is impossible to determine which is more common.

Exercise 6.5 Scientific notation

1. a. 614 b. 6140 c. 61400 d. 351.8 e. 1000000000 f. 392.673
g. 591.1 h. 5100 i. 734000 j. 7141400 k. 35.1 l. 80500
2. a. 5.00×10^3 b. 4.31×10^2 c. 3.8×10^1 d. 3.5×10^5
e. 7.25×10^1 f. 7.25×10^2 g. 7.25×10^3 h. 7.25×10^8
3. a. 4.379×10^1 b. 5.317×10^3 c. 2.590×10^2 d. 1.101×10^2
e. 1.632×10^6 f. 1.000×10^6 g. 1.235×10^8 h. 2.499×10^2
4. a. 0.2 b. 0.004 c. 0.0007 d. 0.03
e. 0.08273 f. 0.07295 g. 0.0029142 h. 0.00003753
i. 0.000529 j. 0.000033333 k. 0.000000002625 l. 0.000000000000001273
5. a. 7×10^{-1} b. 5×10^{-3} c. 3×10^{-7} d. 1×10^{-11}
e. 2.31×10^{-1} f. 3.62×10^{-3} g. 7.31×10^{-4} h. 6.3×10^{-2}
6. a. 6.73×10^{-3} b. 1.43×10^{-1} c. 6.83×10^{-5} d. 5.12×10^{-9}
e. 5.09×10^{-2} f. 1.25×10^{-2} g. 7.31×10^{-4} h. 6.30×10^{-2}
7. a. 5.27×10^2 , 8.31×10^2 , 9.718×10^2 , 3.27×10^3 b. 4.09×10^2 , 4.37×10^2 , 7.943×10^2 , 7.95×10^2
c. 9.29×10^{-3} , 5.251×10^{-2} , 5.31×10^{-2} , 5.27×10^{-1} d. 2.7×10^{-3} , 7.13×10^{-2} , 8.31×10^2 , 3.27×10^3
8. a. 0.0000000000000000000000001994g
b. 1.994×10^{-17} g c. 5.015×10^{23} atoms
9. 3844hours $\approx$ 160.17days
10. 333333.33
11. The neutron is heavier by 2.3×10^{-30} kg.
12. a. 2.1×10^7 b. 1.29×10^8
13. a. Check with your teacher.
b. i. 5.36×10^7 ii. 5.36×10^7
c. The answers are the same because 2.95×10^3 is very small compared with 5.36×10^7 .
14. 8.75×10^{-7} is such a small amount (0.000000875) that when it is added to 2.39×10^{-3} , it doesn't affect the value when given to 3 significant figures.
15. 1.5×10^{-2} , 14%, 0.5^2 , $\frac{5}{8}$, 0.753, $\sqrt[3]{2}$, $\sqrt{2}$
16. a. i. 385440km ii. 78489600km iii. 1278960000km iv. 5772840000km
b. i. 385000km ii. 78500000km iii. 1280000000km iv. 5770000000km
c. i. 3.85×10^5 km ii. 7.85×10^7 km iii. 1.28×10^9 km iv. 5.77×10^9 km
17. a. i. 31536000s ii. 3.1536×10^7 s
b. i. 9460800000000km ii. 9.4608×10^{12} km
c. i. 4.068×10^{13} km ii. 46440000years
18. a. 5.972×10^{21} b. 4
19. a. 9.1094×10^{-31} b. 1.6725×10^{-27} c. 1.6726×10^{-27}
d. It is important to work with the original amounts and leave rounding until the end of a calculation so that the answer is accurate.
20. Scientific notation is an easier way of representing very large and very small numbers.
21. 1.394×10^6 km; 12800 or 1.28×10^4 km

Exercise 6.6 Applying the index laws to variables

1. a. $2^2 \times 3$ b. $2^3 \times 3^2$ c. 3×5^2
d. $2^4 \times 3 \times 5$ e. $2^7 \times 5$ f. $2^3 \times 5^2 \times 7^2$
2. a. $20p^{11}$ b. $6x^8$ c. $56y^{10}$
d. $21p^8$ e. $84t^6$ f. $30q^{15}$

3. a. $6a^6e^7$
e. $30p^6q^9$
i. $24r^{16}s^{18}$
4. a. $3p^4$
d. $3b^6$
5. a. $\frac{3p^5}{2}$
d. $\frac{9x^8y}{4}$
g. $\frac{4p^2rs}{3}$
6. a. 1
e. 5
i. 2
m. 0
7. a. 1
e. 2
i. v^2
8. a. $\frac{1}{x^3}$
e. $2x^2$
i. $\frac{2}{5m^3n^4}$
9. a. 6
d. 11
10. Answers will vary.
11. a. B
d. A
12. Answers will vary.
13. a. 5^9
b. Answers will vary.
14. a. 5^{16}
b. Answers will vary.
15. a. 1
b. 1
c. Zero
d. Answers will vary.
16. a. $\Delta=14$
b. Answers will vary, but $\Delta + O + \Diamond$ must sum to 12. Possible answers include:
 $\Delta = 3, O = 2, \Diamond = 7$; $\Delta = 1, O = 3, \Diamond = 8$; $\Delta = 4, O = 4, \Diamond = 4$; $\Delta = 5, O = 1, \Diamond = 6$.
17. a. The repeating pattern is 1, 3, 9, 7.
b. 3
18. a. The repeating pattern is 4, 6.
b. 4
19. a. Answers will vary.
b. i. 8
ii. 1
iii. 2
20. 2^{2n+3}
21. Index laws provide short cuts for many calculations.
22. a. 3
b. 5
c. 7
d. $87^2 - 86^2 = 173$

Exercise 6.7 Using index laws to simplify algebraic products and quotients

1. a. e^6
d. r^{144}
g. $g^{30}h^{20}$
- b. f^{80}
e. a^8b^{12}
h. $81w^{36}q^8$
- c. p^{100}
f. p^5q^{15}
i. $49e^{10}r^4q^8$

2. a. p^8q^6
 d. $j^{18}g^{12}$
 g. $a^{21}f^{16}$
3. a. $\frac{9b^8}{d^6}$
 d. $\frac{49p^{18}}{64q^{44}}$
 g. $\frac{-64k^6}{343m^{18}}$
4. a. 2^{20}
 d. b^{24}
 g. $324a^{20}$
- b. $r^{15}w^9$
 e. q^4r^{20}
 h. $t^{10}u^8$
- b. $\frac{25h^{20}}{4j^4}$
 e. $\frac{125y^{21}}{27z^{39}}$
 h. $\frac{16g^{28}}{81h^{44}}$
- b. r^{33}
 e. e^{66}
 h. $216d^{27}$
- c. $b^{10}n^{18}$
 f. $h^{24}j^{16}$
 i. $i^{15}j^{12}$
- c. $\frac{8k^{15}}{27t^{24}}$
 f. $\frac{256a^{12}}{2401c^{20}}$
- c. a^{21}
 f. g^{39}
 i. $40000r^{54}$

5. B

6. B

7. D

8. a. a^6
 e. f^{17}
 i. c^{20}
- b. m^4
 f. g^6
 j. f^7
- c. n^3
 g. p^9
 k. k^{14}
- d. b^8
 h. y^2
 l. p^{16}
9. a. i. 1
 b. $(-1)^{\text{even}} = 1(-1)^{\text{odd}} = -1$
- ii. -1
- iii. -1
- iv. 1

10. a. 5

b. 6

c. 30

d. Answers will vary.

11. a. 5^{12}

b. Answers will vary.

12. Danni is correct. Explanations will vary but should involve $(-x)(-x) = (-x)^2 = x^2$ and $-x^2 = -1 \times x^2 = -x^2$.

13. a. i. $x = \frac{5}{3}$

ii. $x = \frac{5}{3}$

iii. $x = \frac{5}{3}$

b. When equating the powers, $3x = 5$.

14. Answers will vary. Possible answers are 4096 and 262144.

15. $10^8 \times 10^8 \times 10^8 = (10^8)^3$ atoms

16. Index laws operate in exactly the same way with numeric bases as with algebraic bases, the only difference is that a number raised to a power can be written as a basic numeral, an algebraic expression cannot.

Exercise 6.8 Applying index laws to algebraic expressions involving integer indices

1. a. $3^5 = 243, 3^4 = 81, 3^3 = 27, 3^2 = 9, 3^1 = 3, 3^0 = 1, 3^{-1} = \frac{1}{3}, 3^{-2} = \frac{1}{9}, 3^{-3} = \frac{1}{27}, 3^{-4} = \frac{1}{81}, 3^{-5} = \frac{1}{243}$
 b. $5^4 = 625, 5^3 = 125, 5^2 = 25, 5^1 = 5, 5^0 = 1, 5^{-1} = \frac{1}{5}, 5^{-2} = \frac{1}{25}, 5^{-3} = \frac{1}{125}, 5^{-4} = \frac{1}{625}$
 c. $10^4 = 10000, 10^3 = 1000, 10^2 = 100, 10^1 = 10, 10^0 = 1, 10^{-1} = \frac{1}{10}, 10^{-2} = \frac{1}{100}, 10^{-3} = \frac{1}{1000}, 10^{-4} = \frac{1}{10000}$
2. a. $\frac{1}{32}$
 e. $\frac{1}{125}$
 i. 27
- b. $\frac{1}{27}$
 f. 7
 j. $\frac{2}{3}$
- c. $\frac{1}{4}$
 g. $\frac{4}{3}$
 k. $\frac{16}{81}$
- d. $\frac{1}{100}$
 h. $\frac{16}{9}$
 l. $\frac{49}{4}$
3. a. $\frac{1}{x^4}$
 e. $\frac{7}{m^2}$
 i. $5x^3$
- b. $\frac{1}{y^5}$
 f. $\frac{1}{m^2n^3}$
 j. $\frac{w^5}{x^2}$
- c. $\frac{1}{z}$
 g. $\frac{1}{m^2n^3}$
 k. x^2y^2
- d. $\frac{a^2}{b^3}$
 h. x^2y^2
 l. $\frac{a^2c}{b^3d^4}$
- m. $\frac{a^2d^3}{b^2c^2}$
 n. $\frac{10y}{x^2}$
 o. $\frac{x}{3}$
- p. $\frac{1}{m^3x^2}$
4. a. $\frac{1}{a^5}$
 e. $\frac{1}{x^3}$
- b. m^5
 f. $\frac{6}{x^5y^3}$
- c. $\frac{1}{m^7}$
 g. $50x^3$
- d. $\frac{14}{x}$
 h. 1

- i. $\frac{50}{a^5}$ j. $10a^4$ k. $\frac{-32}{w^3}$ l. $\frac{16}{m^4}$
 m. $\frac{27m^6}{n^{12}}$ n. $\frac{1}{a^6b^{15}}$ o. a^2b^6 p. $\frac{25}{a^2}$
 5. a. $\frac{1}{x^5}$ b. $\frac{1}{x^{11}}$ c. x^{11} d. x^5
 e. $\frac{2}{a}$ f. $\frac{6c^4}{a^2}$ g. $\frac{2}{a^6}$ h. $\frac{5}{m}$
 i. $\frac{1}{b}$ j. $\frac{1}{a^3b^2}$ k. $\frac{c^2}{a^4}$ l. $\frac{1}{16a}$
 m. $\frac{1}{m^3}$ n. $\frac{3}{2t^9}$ o. t^3 p. $\frac{m^2}{n^3}$
 6. a. 2^0 b. 2^3 c. 2^5
 d. 2^6 e. 2^{-3} f. 2^{-5}
 7. a. 4^0 b. 4^1 c. 4^3
 d. 4^{-1} e. 4^{-2} f. 4^{-3}
 8. a. 10^0 b. 10^1 c. 10^4
 d. 10^{-1} e. 10^{-2} f. 10^{-5}
 9. a. $3^{-4} = \frac{1}{3^4}$

b. Answers will vary but should convey that if dividing, the power in the numerator is lower than that in the denominator.

c. Answers will vary.

10. Answers will vary but should convey that the negative 13 means the decimal point is moved 13 places to the left of 3. Using scientific notation allows the number to be expressed more concisely.

11. a. No. The equivalent expression is $\frac{1}{x^2}$.

b. No. The equivalent expression with positive indices is $\frac{x^6 + 1}{x^3}$.

c. No. The equivalent expression with positive indices is $\frac{(xy)^2}{(x + y)^2}$.

12. a. No. The equivalent expression with positive indices is $\frac{x^{13} - 1}{x^5}$.

b. No. The correct equivalent expression is $\frac{1}{x^6 - x^3}$.

13. a. 3 b. -1 c. 2 d. -4

14. a. 0.048 b. 7600 c. 0.00029 d. 8.1

15. 2^{19}

16. 3^{20}

17. a. $\frac{6}{5}$ b. $\left(\frac{3}{2}\right)^{200}$ c. $4x^8$

18. Answers will vary. However, you can think of the index laws like this:

- Multiplication using indices means add the indices.
- Division using indices means subtract the indices.
- A number with a zero index equals 1.
- When a power is raised to another power, the indices are multiplied.
- When a product of two numbers is jointly raised to a power, it is equal to the product of both numbers each raised to that power.
- When a fraction is raised to a power, both the numerator and denominator are raised to that same power.

Exercise 6.9 Introduction to fractional indices

1. a. $\sqrt{x}$ b. $\sqrt[5]{y}$ c. $\sqrt[4]{z}$
 d. $\sqrt[3]{2w}$ e. $\sqrt{7}$
 2. a. $15^{\frac{1}{2}}$ b. $m^{\frac{1}{2}}$ c. $t^{\frac{1}{3}}$
 d. $(w^2)^{\frac{1}{3}}$ e. $n^{\frac{1}{5}}$

3. a. 7 b. 2 c. 3 d. 5
 e. 10 f. 8 g. 4 h. 2
 i. 3 j. 1000 k. 100 l. 9
4. a. 4^1 b. 7^2
5. a. 2^1 b. 3^4 c. $\frac{1}{5^2}$ d. 2^2
 e. $\frac{1}{2^2}$ f. $\frac{1}{2}$ g. $\frac{1}{2}$ h. $\frac{1}{7}$
6. a. i. 3 ii. 4 iii. 8 iv. 11
 b. i. 3 ii. 4 iii. 8 iv. 11
- c. Raising a number to a power of one-half is the same as finding the square root of that number.
7. a. m b. b c. $6t^2$ d. mn^2
 e. $5t^2$ f. xy^2 g. a^2m^{10} h. $6y^2$
 i. $4x^2y^2$ j. $5ab^2c^3$ k. b^7 l. b^3
8. a. B b. D c. A
9. a. $3^{\frac{1}{2}+\frac{1}{2}} = 3^1$ b. $\sqrt{3}$ c. 3 d. $a^{\frac{1}{n}}$
 e. i. 2 ii. 4

10. a. $z^{2.5} = z^{\frac{5}{2}} = (z^5)^{\frac{1}{2}} = \sqrt{z^5}$

b. No, it is the tenth root: $z^{0.3} = z^{\frac{3}{10}} = (z^3)^{\frac{1}{10}} = (\sqrt[10]{z^3})^{10}$.

11. Mark is correct: $\sqrt{x^2} = x^{\frac{1}{2} \times 2} = x^1 = x$; x can be a positive or negative number.

12. $(-2^3)^{\frac{1}{3}} = -2$; answers will vary but should include that we cannot take the fourth root of a negative number.

13. $\frac{16}{81}$

14. $\sqrt{1871} \approx 43.25$

$42^2 = 1764$

$43^2 = 1849$

He was 43 years old in 1849. Therefore, he was born in $1849 - 43 = 1806$.

15. a. Answers will vary.

b. i. $d = t^{\frac{2}{3}}$

ii. $t = d^{\frac{3}{2}}$

16. a. No, since it has x^2 and $\sqrt{}$.

b. 2, because the square root of a number has a positive and a negative answer

c. $\frac{5}{3}, 1$

17. $a^{\frac{b}{n}}$

18. $i^{40} = (i^2)^{20} = (-1)^{20} = 1$

Investigation | Rich task

1. Fold 1, $8 \text{ cm} \times 4 \text{ cm}$; fold 2, $4 \text{ cm} \times 4 \text{ cm}$; fold 3, $4 \text{ cm} \times 2 \text{ cm}$; fold 4, $2 \text{ cm} \times 2 \text{ cm}$; fold 5, $2 \text{ cm} \times 1 \text{ cm}$

2. Fold 1, 32 cm^2 ; fold 2, 16 cm^2 ; fold 3, 8 cm^2 ; fold 4, 4 cm^2 ; fold 5, 2 cm^2

3.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2	4	8	16	32
Area of surface after each fold (cm^2)	64	32	16	8	4	2

4. There is no linear relationship.

5.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1	2^2	2^3	2^4	2^5
Area of surface after each fold (cm^2) (a)	2^6	2^5	2^4	2^3	2^2	2^1

6. $t = 2^f$, $at = 2^6$, $a = 2^{6-f}$
 7. $t = 2^f$, $at = 2^8$, $a = 2^{8-f}$
 8. Check with your teacher.

Exercise 6.10 Review questions

1. A
2. a. Irrational, since equal to non-recurring and non-terminating decimal
 b. Rational, since can be expressed as a whole number
 c. Rational, since given in a rational form
 d. Rational, since it is a recurring decimal
 e. Irrational, since equal to non-recurring and non-terminating decimal
3. D
4. c, d
5. C
6. B
7. a. 4 b. 5 c. a d. x
 8. a. 3 b. 12 c. 89 d. 0
 9. a. 25 b. 16 c. 32
 d. 1000 e. 81 f. 429 981 696
 10. a. $2^2 \times 5^2$ b. 11^2 c. $2^3 \times 13$
 d. $3^2 \times 5^2$ e. $2^2 \times 3 \times 7^2$ f. $2^5 \times 3^2 \times 5 \times 7$
 11. a. b^{10} b. m^{11} c. k^8
 d. f^{14} e. h^{10} f. $6q^{17}$
 g. $35w^{29}$ h. $12e^5p^8$ i. $105a^{16}b^{17}$
 12. a. a^3 b. t^4 c. r^7 d. p^3
 e. f^5 f. y^{90} g. m^{10} h. g^7
 i. x j. d^7 k. t^5 l. p^5
 m. $4k^4$ n. $11b^7c^5$ o. $\frac{9e^6}{32d^2}$
13. a. 1 b. 1 c. 1 d. 1
 e. 1 f. a g. 3 h. 3
 i. 199 j. a^7 k. v^{10} l. pr
 m. a^9b^4 n. j^8m^3 o. $4e^2 - 36$ p. -8
 q. $15 - 12x$ r. -3 s. 54
14. a. b^8 b. a^{24} c. k^{70} d. j^{200}
 e. $a^{15}b^6$ f. $m^{14}n^{24}$ g. s^3t^{18} h. $q^{10}p^{300}$
15. C
16. a. $\frac{1}{a}$ b. $\frac{1}{k^4}$ c. $\frac{2}{m^4}$ d. $\frac{42}{y^5}$
 17. a. x^{-1} b. $2y^{-4}$ c. z^{-3} d. $135p^{-3}q^{-3}$
 18. a. 10 b. 6 c. a d. b
 e. $7f^2$ f. 3 g. 10 h. x
 i. $2d$ j. $4f^2g$
19. 1.4×10^{12}
20. 4.167×10^8
21. 9.16×10^7 km
22. 1.2441×10^9 km
23. a. 3.4×10^7 , 1.5×10^9 , 6×10^9
 b. 0.57% c. 4 d. 0.25 km^2

24. 820000 queen bees

25. 2.95%

26. During the 5th year or 5.5 years

27. a. Mach 9.1

b.

Mach number	Speed in m/s	Speed in km/h
1	343	1234.8
2	686	2469.6
3	1029	3704.4
9.1	3130.6	11270

c. Travelling at Mach 1 would take 32.39 hours.

Travelling at Mach 2 would take 16.20 hours.

Travelling at Mach 3 would take 10.79 hours.

d. 3.55 hours

e. The fastest aircraft would travel the Earth 9.12 times in the time an aircraft travelling at Mach 1 would circle the Earth once.

The fastest aircraft would travel the Earth 4.56 times in the time an aircraft travelling at Mach 2 would circle the Earth once.

The fastest aircraft would travel the Earth 3.04 times in the time an aircraft travelling at Mach 3 would circle the Earth once.

f. (Number of times to circle Earth) $\times$ (Mach value) is approximately equal to 9.1.

28. a.

1	2	4	8	16	32	64	128

b.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7

c.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}
2^{24}	2^{25}	2^{26}	2^{27}	2^{28}	2^{29}	2^{30}	2^{31}
2^{32}	2^{33}	2^{34}	2^{35}	2^{36}	2^{37}	2^{38}	2^{39}
2^{40}	2^{41}	2^{42}	2^{43}	2^{44}	2^{45}	2^{46}	2^{47}
2^{48}	2^{49}	2^{50}	2^{51}	2^{52}	2^{53}	2^{54}	2^{55}
2^{56}	2^{57}	2^{58}	2^{59}	2^{60}	2^{61}	2^{62}	2^{63}

d.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

e. The total paid is 1 less than twice the number of grains paid.

$$\text{Total paid} = 2(\text{grains paid}) - 1$$

f. $2(2^{63}) - 1 = 18446744073709551615$ or $1.844674407 \times 10^{19}$

TOPIC 7

Area, surface area and volume

[Stages 5.1 and 5.2]

7.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

7.1.1 Why learn this?

Measurement skills are required in many real-world situations. Measurements may be in one, two or three dimensions. We use measurements to describe such things as length, temperature, time, area and capacity. Measurement concepts have significant connections to other areas of mathematics, such as geometry and statistics.



DISCUSSION

Can you think of more than one potentially fatal error that could be caused by minor errors in measurement?

LEARNING SEQUENCE

- 7.1 Overview
- 7.2 [Stage 5.1] Time scales and units of measurement
- 7.3 [Stage 5.1] Area of composite shapes
- 7.4 [Stage 5.1] Area and perimeter of semicircles and sectors
- 7.5 [Stage 5.1] Surface area of rectangular and triangular prisms
- 7.6 [Stage 5.2] Surface area of cylinders and composite solids
- 7.7 [Stage 5.2] Volume of prisms, cylinders and composite solids
- 7.8 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms MA5.1-8MG
- interprets very small and very large units of measurement MA5.1-9MG
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- calculates the surface areas of right prisms, cylinders and related composite solids MA5.2-11MG
- applies formulas to calculate the volumes of composite solids composed of right prisms and cylinders MA5.2-12MG

CONTENT DESCRIPTIONS

Students:

Investigate very small and very large time scales and intervals (ACMMG219)

Calculate the areas of composite shapes (ACMMG216)

Calculate the surface areas of cylinders and solve related problems (ACMMG217)

Solve problems involving the surface areas and volumes of right prisms (ACMMG218)

Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — Measurement matters (eles-1699)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

7.2 Time scales and units of measurement [Stage 5.1]

7.2.1 Time scales

- Scientists sometimes need to work with very large or very small time scales.
- Very large or very small numbers are best written in scientific notation (as described in Topic 6).

WORKED EXAMPLE 1

Alpha Centauri is the closest star system to Earth at a distance of 4.3 light-years. Light travels at 300 000 km/s. Correct to 4 significant figures:

- write the speed of light in scientific notation**
- determine the distance travelled by light in 1 minute**
- determine the distance travelled by light in 1 day**
- determine the length of a light-year in kilometres**
- determine the distance, in km, of Alpha Centauri from Earth.**

THINK

a Light travels at 300 000 km/s.

b Light travels at the rate of 3.000×10^5 km per second.

1 minute = 60 seconds, so multiply the answer to part **a** by 60.

c Light travels at the rate of 1.8×10^7 km per minute.

1 hour = 60 minutes,

1 day = 24 hours, so multiply the answer to part **b** by 60 and by 24.

WRITE

a $300\,000 = 3.000 \times 10^5$ km/s

b $3.0 \times 10^5 \times 60 = 180 \times 10^5$
 $= 1.800 \times 10^7$ km/min

c $1.800 \times 10^7 \times 60 \times 24 = 2592 \times 10^7$
 $= 2.592 \times 10^{10}$ km/day

- d** Light travels at the rate of 2.592×10^{10} km per day.
1 year = 365.25 days, so multiply the answer to part **c** by 365.25.
- d** $2.592 \times 10^{10} \times 365.25 = 946.728 \times 10^{10}$
 $\approx 9.467 \times 10^{12}$ km/year
A light-year is 9.467×10^{12} km.
- e 1** Alpha Centauri is 4.3 light-years from Earth. Multiply the previous answer by 4.3.
- e** $9.46728 \times 10^{12} \times 4.3 = 40.7093 \times 10^{12}$
 $\approx 4.071 \times 10^{13}$ km
- 2** Express the answer in scientific notation.
- Alpha Centauri is 4.071×10^{13} km from Earth.

7.2.2 Small time scales

- Slow-motion cameras have made it possible to watch events in ultra slow motion, often 1000 times slower than normal. For example, a 1-second recording could take 15 minutes to watch at ultra slow speed.
- There are many examples of slow-motion recordings on the internet. Possible examples you could search for include skateboard tricks, a leopard running, a bubble bursting or a bullet travelling through water.



WORKED EXAMPLE 2

Mary bought a new camera that could record at the rate of 16000 frames per second.

- a** Write this rate in scientific notation.
b She records a balloon bursting that takes 2 seconds. How many frames is this?
c She replays the video and slows it down so that it takes 15 minutes to play. How many frames per second is this?

THINK

- a** Write the speed in the form $a \times 10^n$.
b Multiply the answer to part **a** by 2.
c 1 Calculate the number of seconds in 15 minutes.

2 Frames per second = $\frac{\text{number of frames}}{\text{number of seconds}}$.

- 3** Write the answer in words.

WRITE

a $16000 = 1.6 \times 10^4$ frames/s
b $1.6 \times 10^4 \times 2 = 3.2 \times 10^4$ frames
c $15 \times 60 = 900$ seconds

$$\frac{3.2 \times 10^4}{900} = 0.003556 \times 10^4$$

$$= 3.556 \times 10$$

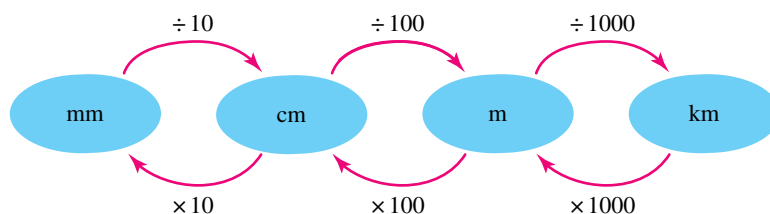
$$= 35.56$$

The rate is 35.56 frames per second.

7.2.3 Common prefixes for units of measurement

- In the **metric system**, the base unit of length is the metre (m).
- Milli-, centi- and kilo- are called prefixes and are placed in front of the base length, the metre, to provide divisions or multiples of this base length.

- The following units are commonly used.
 - millimetre** (mm) one-thousandth of a metre
 - centimetre** (cm) one-hundredth of a metre
 - kilometre** (km) one thousand metres
- The chart below is useful when converting from one unit of length to another.



For example, $36000 \text{ mm} = 36000 \div 10 \div 100 \text{ m}$
 $= 36 \text{ m}$

- In the metric system, the base unit of weight is the gram (gm).
- The following units are commonly used.
 - milligram** (mg) one-thousandth of a gram
 - kilogram** (kg) one thousand grams

WORKED EXAMPLE 3

Convert the following lengths into cm.

a 37 mm

b 2.54 km

THINK

a There will be fewer cm, so divide by 10.

b There will be more cm, so multiply.

km $\rightarrow$ m: $\times 1000$

m $\rightarrow$ cm: $\times 100$

WRITE

a $37 \div 10 = 3.7 \text{ cm}$

b $2.54 \times 1000 = 2540 \text{ m}$
 $2540 \times 100 = 254000 \text{ cm}$

WORKED EXAMPLE 4

Convert the following weights into grams.

a 5.25 kg

b 42 mg

THINK

a There will be more grams, so multiply by 10^3 .

b There will be fewer grams, so divide by 10^3 .

WRITE

a $5.25 \times 10^3 = 5250 \text{ g}$

b $42 \div 10^3 = 0.042 \text{ g}$

7.2.4 Prefixes for very small and very large units of measurement

- Many areas in science, finance and technology use very large and very small numbers. There are a number of standard prefixes used in the metric system for these large and small quantities.

- Some of these prefixes include:
 - pico- (10^{-12})
 - nano- (10^{-9})
 - micro- (10^{-6})
 - mega- (10^6)
 - giga- (10^9)
 - tera- (10^{12})
- These prefixes can be used for very small and very large time scales.
- The following units of time are based on the second.
 - One **pico**second (ps) = 1×10^{-12} s
 - One **nano**second (ns) = 1×10^{-9} s
 - One **micro**second (μ s) = 1×10^{-6} s
 - One **milli**second (ms) = 1×10^{-3} s
 - One **kilo**second (ks) = 1×10^3 s
 - One **mega**second (Ms) = 1×10^6 s
 - One **giga**second (Gs) = 1×10^9 s
- The base unit for measuring digital information storage is the **byte**. (The smallest unit of measurement of data storage is the bit: 8bits = 1 byte.)
- As our digital capacity has grown, so too has the need for larger and larger units of measurement.
- The following units are commonly used.
 - One **kilobyte** (kB) one thousand bytes (10^3)
 - One **megabyte**(MB) one million bytes (10^6) = one thousand kilobytes
 - One **gigabyte** (GB) one billion bytes (10^9) = one thousand megabytes
 - One **terabyte** (TB) one trillion bytes (10^{12}) = one thousand gigabytes

WORKED EXAMPLE 5

a Convert the following measurements into gigabytes.

- i 5 TB** **ii 4000 MB**

b Convert the following measurements into milliseconds.

- i 5 ns** **ii 8 Gs**

THINK

- a i** There will be more gigabytes, so multiply by 10^3 .
ii There will be fewer gigabytes, so divide by 10^3 .
b i There will fewer milliseconds, so divide by 10^6 .
ii There will be more milliseconds, so multiply by 10^{12} .

WRITE

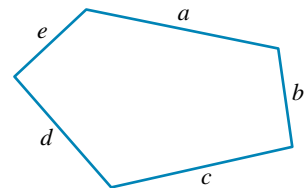
- a i** $5 \times 10^3 = 5000$ GB
ii $4000 \div 10^3 = 4$ GB
b i $5 \div 10^6 = 5 \times 10^{-6}$ ms
ii $8 \times 10^{12} = 8 \times 10^{12}$ ms

DISCUSSION

Are kB, MB, GB and TB the only abbreviations for the units of measurement for data storage?

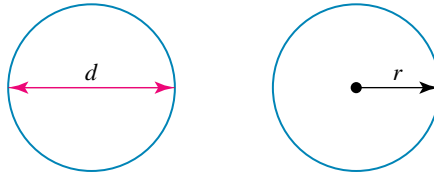
7.2.5 Perimeter

- The **perimeter** of a plane (flat) figure is the distance around the outside of the figure.
- If the figure has straight edges, then the perimeter can be found by simply adding all the side lengths.
- Ensure that all lengths are in the same unit.
- Perimeter = $a + b + c + d + e$



7.2.6 Circumference

- **Circumference** is a special name given to the perimeter of a circle.
- Circumference is calculated using the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius of the circle.
- Use your calculator's value for π unless otherwise directed.

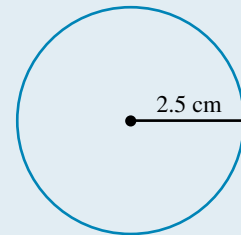


DISCUSSION

How would the answers to some questions change if a rounded value for π (3.14) was used instead of the exact value?

WORKED EXAMPLE 6

Determine the circumference of the circle shown. Give your answer correct to 2 decimal places.



THINK

- 1 The radius is known, so apply the formula $C = 2\pi r$.
- 2 Substitute $r = 2.5$ into $C = 2\pi r$.
- 3 Calculate the circumference to 3 decimal places and then round correct to 2 decimal places.

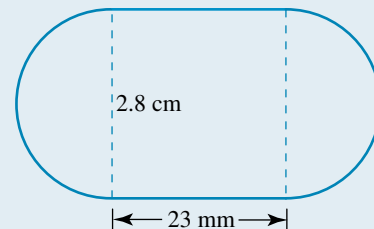
WRITE

$$\begin{aligned}C &= 2\pi r \\C &= 2 \times \pi \times 2.5 \\C &\approx 15.707 \\C &\approx 15.71 \text{ cm}\end{aligned}$$

- Sometimes a standard formula will only form part of the calculation of perimeter.

WORKED EXAMPLE 7

Determine the perimeter of the shape shown. Give your answer in cm correct to 2 decimal places.



THINK

- 1 There are two straight sections and two semicircles. The two semicircles make up a full circle, the diameter of which is known, so apply the formula $C = \pi d$.

WRITE

$$C = \pi d$$

2 Substitute $d = 2.8$ into $C = \pi d$.	$= \pi \times 2.8$ $\approx 8.796 \text{ cm}$
3 Convert 23 mm to cm (23 mm = 2.3 cm). The perimeter is the sum of all the outside lengths.	$P = 8.796 + 2 \times 2.3$ $= 13.396$
4 Round to 2 decimal places.	$\approx 13.40 \text{ cm}$

learnon RESOURCES — ONLINE ONLY

Interactivity: Converting units of length (int-4355)

Interactivity: Perimeter (int-4357)

Interactivity: Circumference (int-3782)

Digital doc: SkillsHEET Converting units of length (doc-6294)

Digital doc: SkillsHEET Substitution into perimeter formulas (doc-10872)

Digital doc: SkillsHEET Perimeter of squares, rectangles, triangles and circles (doc-10873)

Exercise 7.2 Time scales and units of measurement

Individual pathways

PRACTISE

Questions:

1–3, 4a, c, e, g, 5–7, 8a–c, 9a–c, 10, 11, 13a–e, 14, 15, 18, 19

CONSOLIDATE

Questions:

1–3, 4b, d, f, h, 6, 8b–d, 9b–d, 10, 12, 13d–h, 14, 16, 18, 20, 21

MASTER

Questions:

1–3, 4 b, d, f, h, 6, 8d–f, 9d–f, 10, 12, 13g–i, 14, 17–24

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE1** NGC 6782 is a relatively nearby galaxy, residing about 183 million light-years from Earth. Given that light travels at approximately 300 000 km/s:
 - write the speed of light in scientific notation
 - determine the distance travelled by light in 1 hour
 - determine the distance travelled by light in 1 week
 - determine the length of a light-year in kilometres
 - determine the distance, in km, between NGC 6782 and Earth.
- WE2** John bought a new camera that could record at the rate of 12 000 frames per second.
 - Write this rate in scientific notation.
 - He records his friend surfing a wave that takes 3 minutes. How many frames is this?
 - He replays the video and slows it down so that it takes 25 minutes to play. How many frames per second is this?
- WE3, WE4** Fill in the gaps for each of the following.

a. 5 cm = _____ mm	b. 1.52 m = _____ cm
c. 12.5 mm = _____ m	d. 0.0322 m = _____ mm
e. 6.57 g = _____ kg	f. 64 g = _____ kg
g. 0.00001435 kg = _____ g	h. 18.35 g = _____ kg

4. **WE5** Fill in the gaps for each of the following.

a. 5 000 000 kB = _____ MB

c. 70 000 GB = _____ TB

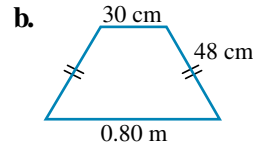
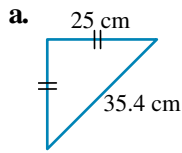
e. 15 Ms = _____ ks

b. 4500 MB = _____ GB

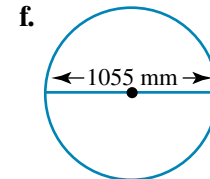
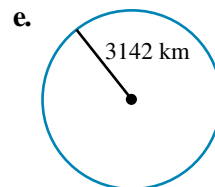
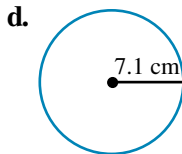
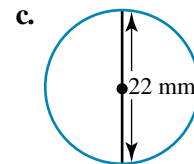
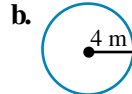
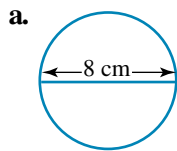
d. 1000 GB = _____ MB

f. 200 000 000 μ s = _____ ks

5. Find the perimeter of each of the figures below. Give your answers in centimetres (cm).



6. **WE6** Find the circumference of each of the following circles. Give your answers correct to 2 decimal places.



7. **MC** A circle has a radius of 34 cm. Its circumference, to the nearest centimetre, is:

A. 102 cm

B. 214 cm

C. 107 cm

D. 3630 cm

E. 68 cm

8. Timber is sold in standard lengths that increase in 300-mm intervals from the smallest available length of 900 mm. (The next two standard lengths available are therefore 1200 mm and 1500 mm.)

a. Write the next four standard lengths (after 1500 mm) in mm, cm and m.

b. How many pieces of length 600 mm could be cut from a 2.4-m length of timber?

c. If I need to cut eight pieces of timber, each 41 cm long, what is the smallest standard length I should buy?

Note: Ignore any timber lost due to the cuts.



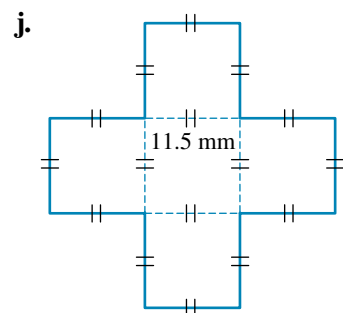
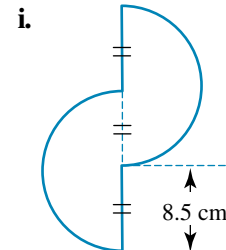
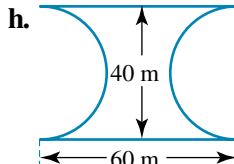
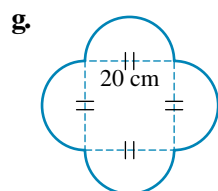
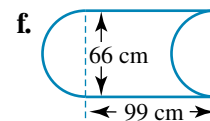
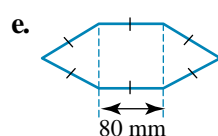
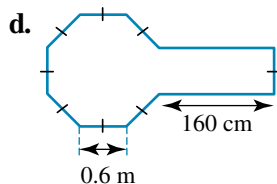
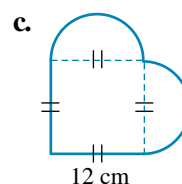
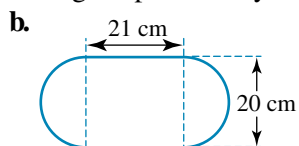
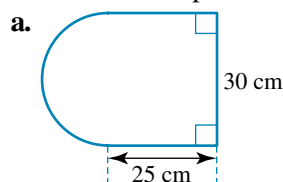
9. The world's longest bridge is the Akashi-Kaikyo Bridge, which links the islands of Honshu and Shikoku in Japan. Its central span covers 1.990 km.

a. How long is the central span, in metres?

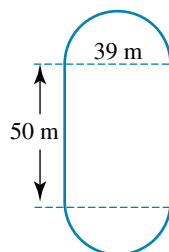
b. How much longer is the span of the Akashi-Kaikyo Bridge than the Sydney Harbour Bridge, which spans 1149 m?



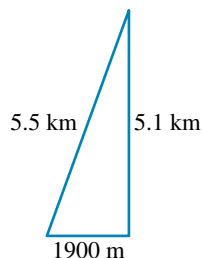
10. **WE7** Find the perimeters of the following shapes. Give your answer correct to 2 decimal places.



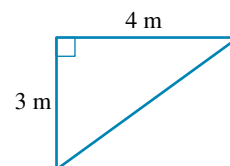
11. Find the perimeter of the racetrack shown in the plan.



12. Yacht races are often run over a triangular course as shown. What distance would the yachts cover if they completed 3 laps of the shown course?



13. Use Pythagoras' theorem to find the length of the missing side and, hence, find the perimeter of the triangular frame shown.



Communicating, reasoning and problem solving

14. Use the following list of time intervals to complete the sentences below.

1 nanosecond, 17 microseconds, 3.6 kiloseconds, 1 gigasecond, 6 teraseconds

- One hour is equal to _____.
- Every year the Earth's days are getting longer by approximately _____.
- The time taken for a person to almost reach their 32nd birthday is _____.
- 190 000 years is approximately _____.
- The time it takes for light to travel 30 centimetres is _____.

15. The Hubble Space Telescope is over 13 m in length. It orbits the Earth at a height of 559 km, where it can take extremely sharp images outside the distortion of the Earth's atmosphere.

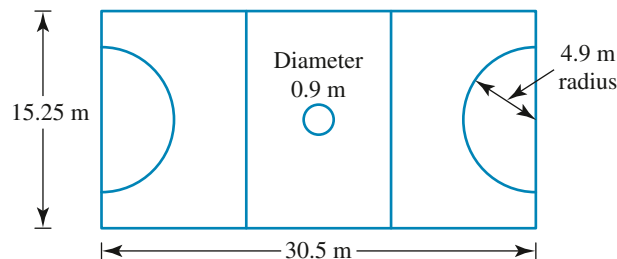


- If the radius of the Earth is 6371 km, show that the distance travelled by the Hubble Space Telescope in one orbit, to the nearest km, is 43 542 km.
- If the telescope completes one orbit in 96 minutes, show that its speed is approximately 7559 m/s.

16. A bullet can travel in air at 500 m/s.

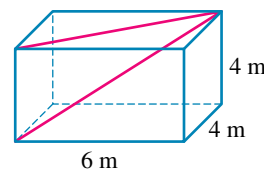
- Show how the bullet travels 50 000 cm in 1 second.
- How long does it take for the bullet to travel 1 centimetre?
- If a super-slow-motion camera can take 100 000 pictures each second, how many shots would be taken by this camera to show the bullet travelling 1 cm?

17. Edward is repainting all the lines of a netball court at the local sports stadium. The dimensions of the netball court are shown below.

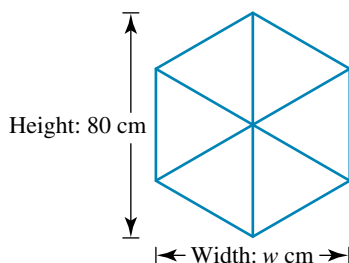


- Calculate the total length of lines that need to be repainted.
 - Edward starts painting at 8 pm when the centre is closing, and it takes him $1\frac{1}{2}$ minutes on average to paint each metre of line. Show that it will take him 233 minutes to complete the job.
18. The radius of the Earth is accepted to be roughly 6400 km.
- How far, to the nearest km, do you travel in one complete rotation of the Earth?
 - As the Earth spins on its axis once every 24 hours, what speed are you moving at?
 - If the Earth is 150 000 000 km from the sun, and it takes 365.25 days to circle around the sun, show that the speed of the Earth's orbit around the sun is 107 515 km/h. Give your answer to the nearest whole number.
19. One-fifth of an 80-cm length of jewellery wire is cut off. A further 22-cm length is then removed. Is there enough wire remaining to make a 40-cm necklace?
20. You are a cook in a restaurant where the clock has just broken. You have a four-minute timer, a seven-minute timer and a pot of boiling water. A very famous fussy food critic enters the restaurant and orders a pasta dish. You remember from a TV show on which she appeared that she likes her pasta cooked for nine minutes exactly. How will you measure nine minutes using the timers?

21. A spider is sitting in one top corner of a room that has dimensions 6 m by 4 m by 4 m. It needs to get to the corner of the floor that is diagonally opposite. The spider must crawl along the ceiling, then down a wall, until it reaches its destination.

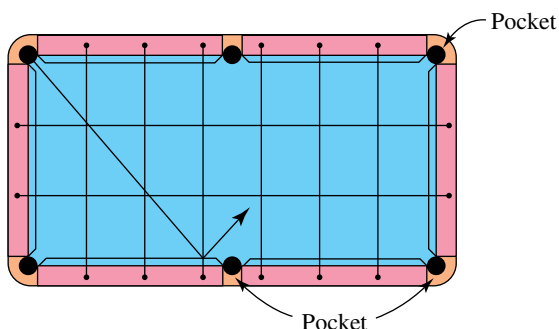


- If the spider crawls first to the diagonally opposite corner of the ceiling, then down the wall to its destination, what distance would it crawl?
 - What is the shortest distance from the top back corner to the lower left corner?
22. A church needs to repair one of its regular hexagonal stained glass windows. Use the information given in the diagram to find the width of the window.



23. Imagine this is an electronic game of billiards in a games arcade.

The ball is ejected from the top left-hand corner at 45° to the side of the table. It follows the path as indicated, always rebounding at the same angle, until it reaches the pocket in the lower right-hand corner. The grid squares are 5 cm square. What distance does the ball travel on its journey to the pocket? Give an exact answer.



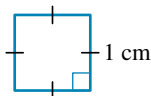
24. What are some ways to remember how to convert between the various metric units?

7.3 Area of composite shapes

[Stage 5.1]

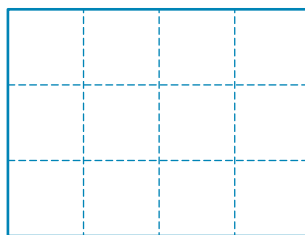
7.3.1 Review of area of quadrilaterals

- The diagram shows a square of side length 1 cm. By definition it has an area of 1 cm^2 (1 square centimetre).



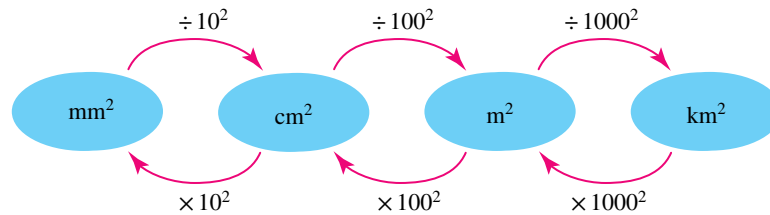
Note: This is a 'square centimetre', not a 'centimetre squared'.

- Area tells us how many squares it takes to cover a figure, so the area of the rectangle shown is 12 cm^2 .



- Area is commonly measured in square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), or square kilometres (km^2).

- The chart below is useful when converting from one unit of area to another.



For example, $54 \text{ km}^2 = 54 \times 1000^2 \times 100^2$
 $= 540\,000\,000\,000 \text{ cm}^2$

- Another common unit is the **hectare** (ha), a $100 \text{ m} \times 100 \text{ m}$ square equal to $10\,000 \text{ m}^2$, which is used to measure small areas of land. A hectare does not need to be square; for example, it could be $1000 \text{ m} \times 10 \text{ m}$.

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

WORKED EXAMPLE 8

Convert 1.3 km^2 into:

- a** square metres **b** hectares.

THINK

- a** There will be more m^2 , so multiply by 1000^2 .
b Divide the result of part **a** by $10\,000$, as $1 \text{ ha} = 10\,000 \text{ m}^2$.

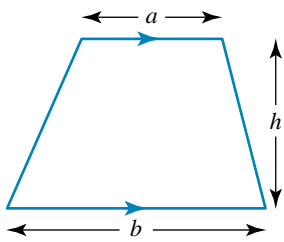
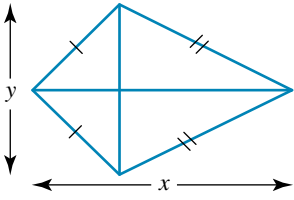
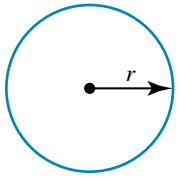
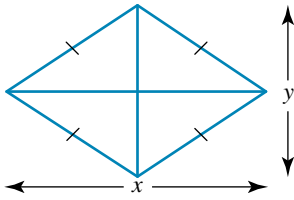
WRITE

- a** $1.3 \times 1000^2 = 1\,300\,000 \text{ m}^2$
b $1\,300\,000 \div 10\,000 = 130 \text{ ha}$

7.3.2 Using formulas to calculate area

- There are many useful **formulas** to find the area of simple shapes. Some common ones are summarised here.

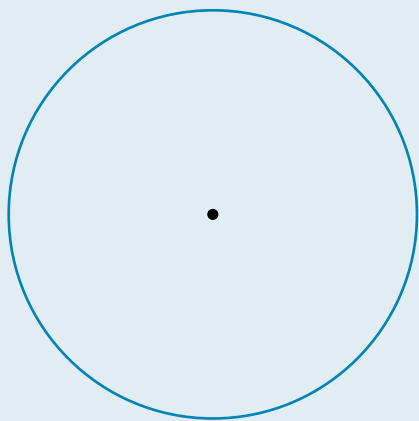
Square $A = l^2$	Rectangle $A = lw$
Parallelogram $A = bh$	Triangle $A = \frac{1}{2}bh$

Trapezium  $A = \frac{1}{2}(a + b)h$	Kite  $A = \frac{1}{2}xy$
Circle  $A = \pi r^2$	Rhombus  $A = \frac{1}{2}xy$

WORKED EXAMPLE 9

By making the appropriate measurements, calculate the area of each of the following figures in cm^2 , correct to 1 decimal place.

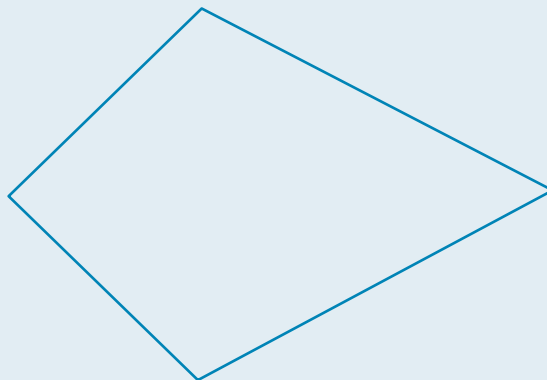
a



THINK

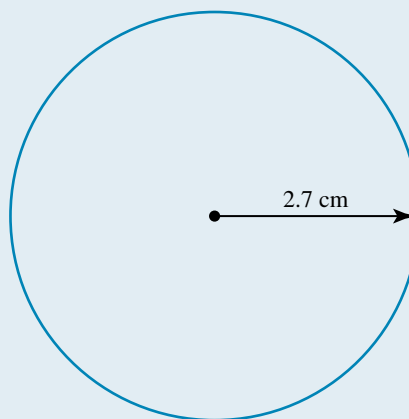
a 1 The figure is a circle. Use a ruler to measure the radius.

b



WRITE/DRAW

a $r = 2.7 \text{ cm}$



- 2 Apply the formula for area of a circle:
 $A = \pi r^2$.

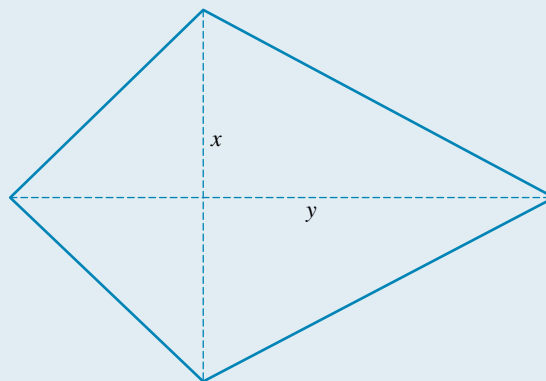
$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 2.7^2 \\ &\approx 22.90 \end{aligned}$$

- 3 Round the answer to 1 decimal place.

$$A \approx 22.9 \text{ cm}^2$$

- b 1** The figure is a kite. Measure the diagonals.

b



$$x \approx 4.9 \text{ cm}, y \approx 7.2 \text{ cm}$$

- 2 Apply the formula for area of a kite:
 $A = \frac{1}{2}xy$.

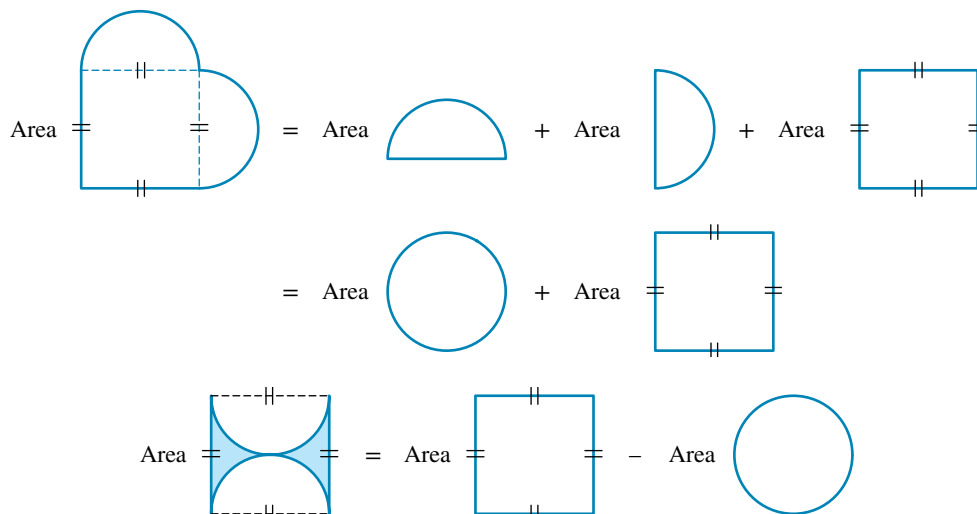
$$\begin{aligned} A &= \frac{1}{2}xy \\ &= \frac{1}{2} \times 4.9 \times 7.2 \\ &= 17.64 \end{aligned}$$

- 3 Round the answer to 1 decimal place.

$$A \approx 17.6 \text{ cm}^2$$

7.3.3 Composite shapes

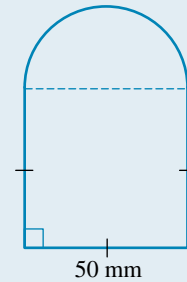
- A composite shape is made up of smaller, simpler shapes. Here are two examples.



- Observe that in the second example the two semicircles are subtracted from the square to obtain the shaded area on the left.

WORKED EXAMPLE 10

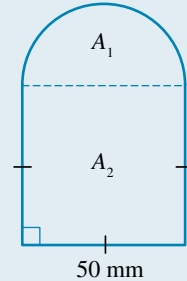
Calculate the area of the figure shown, giving your answer correct to 1 decimal place.



THINK

- 1 Draw a diagram, divided into basic shapes.

WRITE/DRAW



- 2 A_1 is a semicircle of radius 25 mm.
The area of a semicircle is half the area of a complete circle.
- 3 Substitute $r = 25$ into the formula and evaluate, correct to 4 decimal places.
- 4 A_2 is a square of side length 50 mm. Write the formula.
- 5 Substitute $l = 50$ into the formula and evaluate A_2 .
- 6 Sum to find the total area.
- 7 Round the final answer correct to 1 decimal place.

$$A_1 = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2} \times \pi \times 25^2 \\ \approx 981.7477 \text{ mm}^2$$

$$A_2 = l^2$$

$$= 50^2 \\ = 2500 \text{ mm}^2$$

$$\begin{aligned} \text{Total area} &= A_1 + A_2 \\ &= 981.7477 + 2500 \\ &= 3481.7477 \text{ mm}^2 \\ &\approx 3481.7 \text{ mm}^2 \end{aligned}$$

ACTIVITY: COUNTING SQUARES

Equipment: grid paper, ruler, protractor

1. On the grid paper, draw an example of each of the following shapes: square, rectangle, triangle, trapezium and circle.
2. Estimate the area of each of your shapes (in cm^2) by counting the squares.
3. Share your drawings with a partner and get them to calculate the area of each shape by using the appropriate formula.
4. Compare your results.

Interactivity: Conversion chart for area (int-3783)

Interactivity: Area of rectangles (int-3784)

Interactivity: Area of parallelograms (int-3786)

Interactivity: Area of trapeziums 1 (int-3789)

Interactivity: Area of trapeziums 2 (int-3790)

Interactivity: Area of circles (int-3788)

Interactivity: Area of rhombuses (int-3787)

eLesson: Composite area (eles-1886)

Digital doc: SkillSHEET Substitution into area formulas (doc-10874)

Digital doc: SkillSHEET Area of squares, rectangles, triangles and circles (doc-10875)

Digital doc: WorkSHEET Length and area (doc-6303)

Exercise 7.3 Area of composite shapes

Individual pathways

PRACTISE

Questions:

1, 2, 3a–k, 4, 5a–e, 6, 7, 9a–b,
10–12, 15, 18, 21–23

CONSOLIDATE

Questions:

1, 2, 3c–k, 5d–i, 7, 9b–c, 10–13,
16–19, 21–23, 25

MASTER

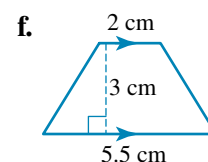
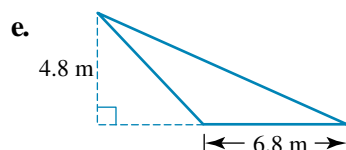
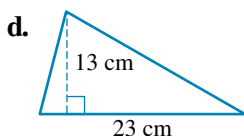
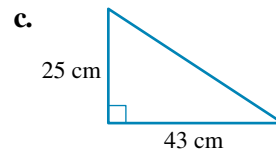
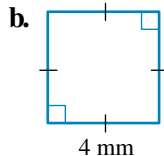
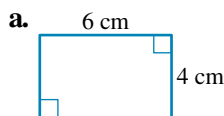
Questions:

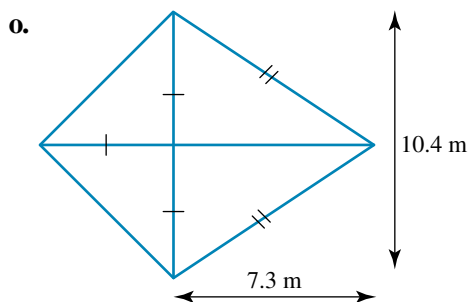
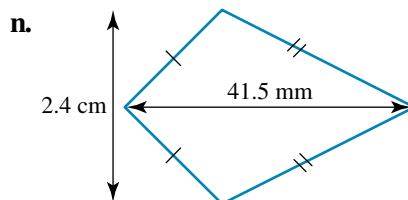
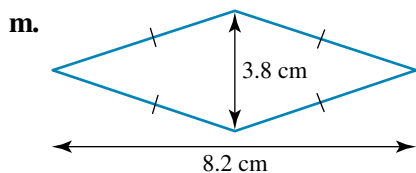
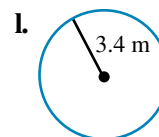
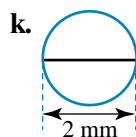
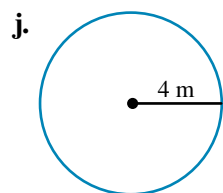
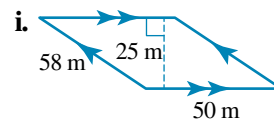
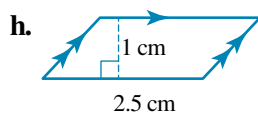
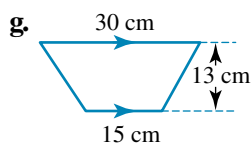
1, 2, 3d–o, 5f–j, 8, 9c–d, 10,
11, 13, 14, 16–26

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

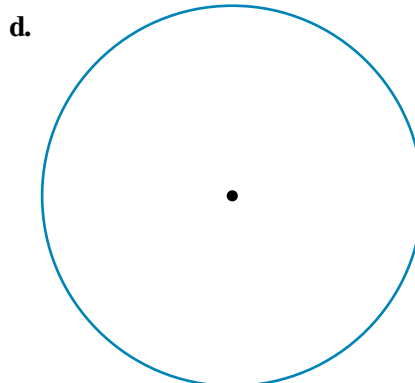
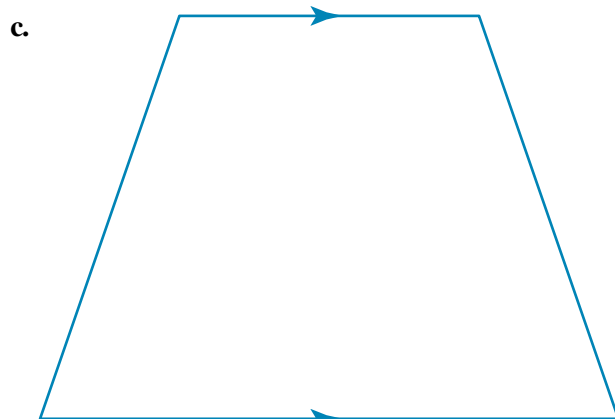
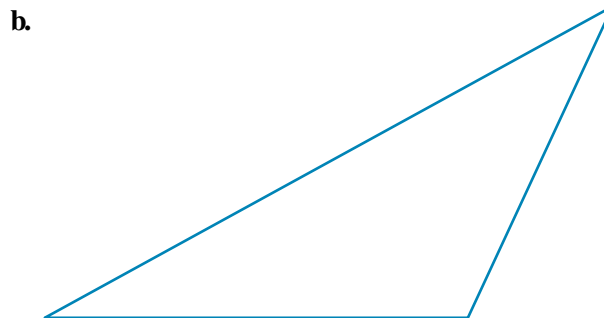
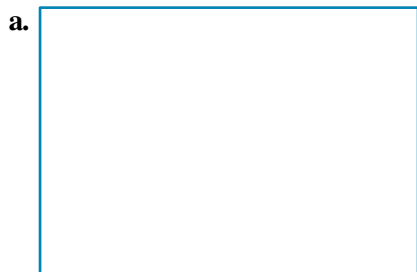
Understanding and fluency

- MC** To convert an area measurement from square kilometres to square metres:
 - divide by 1000
 - multiply by 1000
 - divide by 1 000 000
 - multiply by 1 000 000
 - divide by 1 000 000 000
- WE8** Fill in the gaps:
 - $13\,400\text{ m}^2 = \underline{\hspace{2cm}}\text{ km}^2$
 - $0.04\text{ cm}^2 = \underline{\hspace{2cm}}\text{ mm}^2$
 - $3\,500\,000\text{ cm}^2 = \underline{\hspace{2cm}}\text{ m}^2$
 - $0.005\text{ m}^2 = \underline{\hspace{2cm}}\text{ cm}^2$
 - $0.043\text{ km}^2 = \underline{\hspace{2cm}}\text{ m}^2$
 - $200\text{ mm}^2 = \underline{\hspace{2cm}}\text{ cm}^2$
 - $1.41\text{ km}^2 = \underline{\hspace{2cm}}\text{ ha}$
 - $3800\text{ m}^2 = \underline{\hspace{2cm}}\text{ ha}$
- Calculate the area of each of the following shapes. (Where appropriate, give your answer correct to 2 decimal places.)

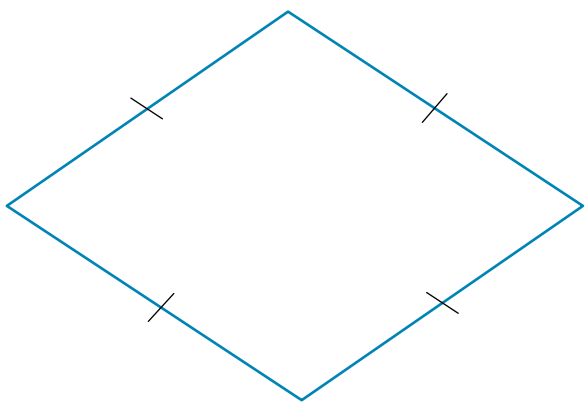




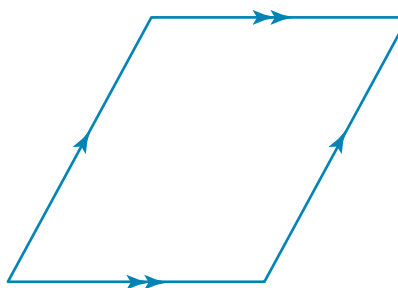
4. **WE9** By making appropriate measurements, calculate the area of each of the following figures in cm^2 , correct to 1 decimal place.



e.

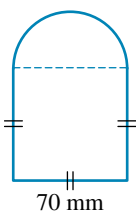


f.

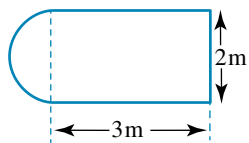


5. **WE10** Calculate the areas of the composite shapes shown. Where appropriate, express your answers correct to 1 decimal place.

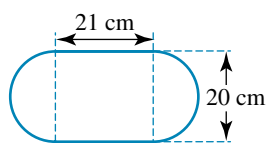
a.



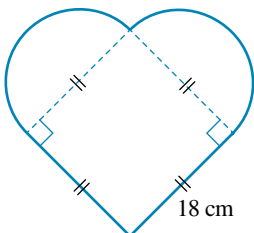
b.



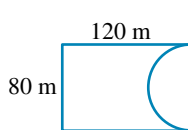
c.



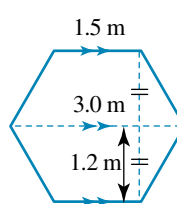
d.



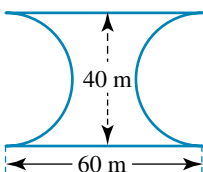
e.



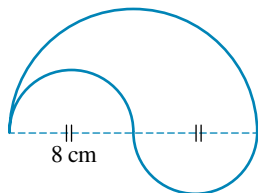
f.



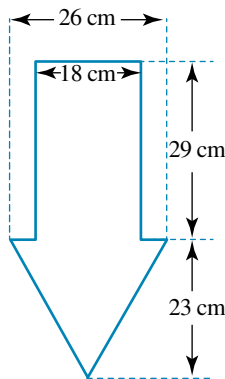
g.



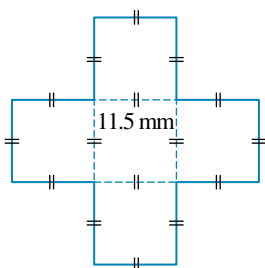
h.



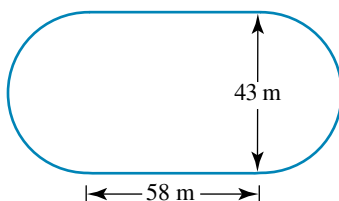
i.



j.



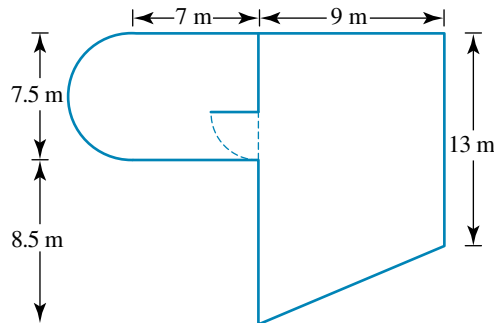
6. What would be the cost of covering the sportsground shown with turf if the turf costs \$7.50 per square metre?



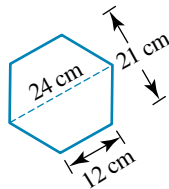
7. The Murray–Darling River Basin is Australia’s largest catchment. Irrigation of farms in the Murray–Darling Basin has caused soil degradation due to rising salt levels. Studies indicate that about 500 000 hectares of the basin could be affected in the next 50 years.
- Convert the possible affected area to square kilometres. ($1 \text{ km}^2 = 100$ hectares.)
 - The total area of the Murray–Darling Basin is about 1 million square kilometres, about one-seventh of the continent. What percentage of this total area may be affected by salinity?



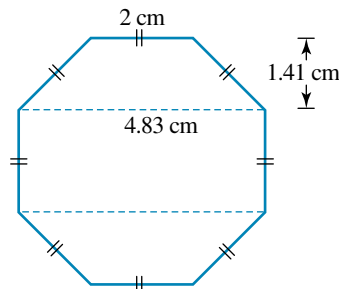
8. The plan shows two rooms that are to be refloored. Calculate the cost if the flooring costs \$45 per square metre. Allow 10% more for wastage and round to the nearest \$10.



9. Calculate the area of the regular hexagon shown by dividing it into two trapeziums.

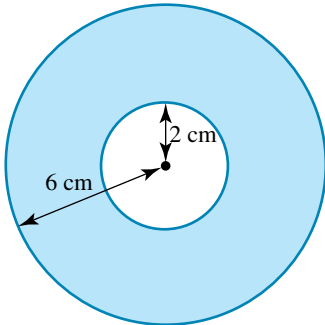


10. Calculate the area of the regular octagon by dividing it into two trapeziums and a rectangle, as shown in the figure.

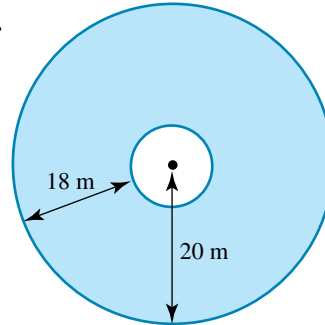


11. An annulus is a shape formed by two concentric circles (two circles with a common centre). Calculate the area of each of the annuli shown below by subtracting the area of the smaller circle from the area of the larger circle. Give answers correct to 2 decimal places.

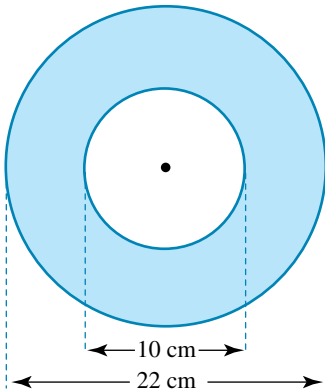
a.



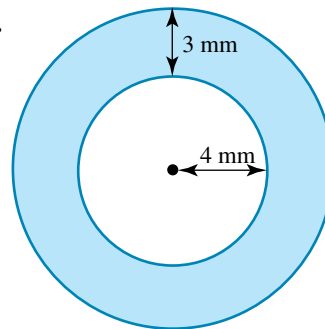
b.



c.



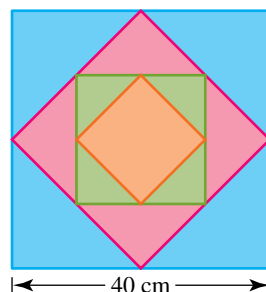
d.



12. **MC** A pizza has a diameter of 30 cm. If your sister eats one-quarter, what is the remaining area of the pizza?
A. 168.8 cm^2 **B.** 176.7 cm^2 **C.** 530.1 cm^2 **D.** 675 cm^2 **E.** 706.9 cm^2
13. A circle has an area of 4500 cm^2 . Calculate its diameter to the nearest mm.

Communicating, reasoning and problem solving

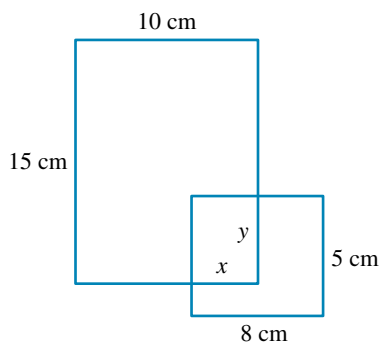
14. A sheet of paper measures 29.5 cm by 21.0 cm.
- What is the area of the sheet of paper?
 - What is the radius of the largest circle that can be drawn on this sheet?
 - What is the area of this circle?
 - If the interior of the circle is shaded red, show that 56% of the paper is red.
15. a. Find the area of a square with side lengths 40 cm.
- If the midpoints of each side of the previous square are joined by straight lines to make another square, find the area of the smaller square.
 - Now the midpoints of the previous square are also joined with straight lines to make another square. Find the area of this even smaller square.
 - This process is repeated again to make an even smaller square. What is the area of this smallest square?
 - What pattern do you observe? Justify your answer.
 - What percentage of the original square's area does the smallest square take up?
 - Show that the area of the combined figure that is coloured red is 1000 cm^2 .



- 16.** A chessboard is made up of 8 rows and 8 columns of squares. Each small square is 42cm^2 in area. Show that the shortest distance from the upper right corner to the lower left corner of the chessboard is 73.32 cm.



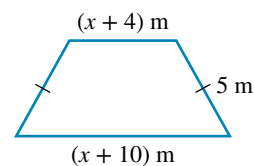
- 17.** Two rectangles of sides 15 cm by 10 cm and 8 cm by 5 cm overlap as shown. Show that the difference in area between the two non-overlapping sections of the rectangles is 110 cm^2 .



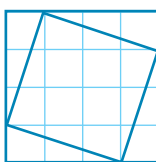
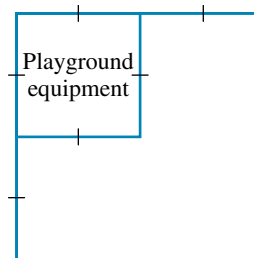
- 18.** The area of a square is $x\text{ cm}^2$. Would the side length of the square be a rational number? Explain your answer.
- 19.** Show that the triangle with the largest area for a given perimeter is an equilateral triangle.
- 20.** Show that a square of perimeter $4x + 20$ has an area of $x^2 + 10x + 25$.
- 21.** The area of a children's square playground is 50 m^2 .
- What is the exact length of the playground?
 - Pine logs 3 m long are to be laid around the playground. How many logs will need to be bought? (The logs can be cut into smaller pieces if required.)



- 22.** A sandpit is designed in the shape of a trapezium, with the dimensions shown. If the area of the sandpit is 14 m^2 , what will be its perimeter?



23. The area of a room must be determined so that floor tiles can be laid. The room measures 2.31 m by 4.48 m. This was rounded off to 2 m by 4 m to calculate the area.
- What problems might arise?
 - If each tile is 0.5 m by 0.5 m, how many tiles are actually required?
24. The playground equipment is half the length and half the width of the square kindergarten yard it is in.
- What fraction of the kindergarten yard is occupied by the play equipment?
 - During a working bee, the playground equipment area is extended 2 m in length and 1 m in width. If x represents the length of the kindergarten yard, write an expression for the area of the play equipment.
 - Write an expression for the area of the kindergarten yard *not* taken up by the playground equipment.
 - The kindergarten yard that is not taken up by the playground equipment is divided into 3 equal-sized sections:
 - a grassed area
 - a sandpit
 - a concrete area.
 - Write an expression for the area of the concrete area.
 - The children usually spend their time on the play equipment or in the sandpit. Write a simplified expression for the area of the yard where the children usually play.
25. A rectangular classroom has a perimeter of 28 m and its length is 4 m shorter than its width. What is the area of the classroom?
26. If you know the conversion factor between two units of length, for example mm and m, how can you quickly work out the conversion factor between the corresponding units of area?
27. The diagram shows one smaller square drawn inside a larger square on grid paper. Express the area of the smaller square as a fraction of the larger square.

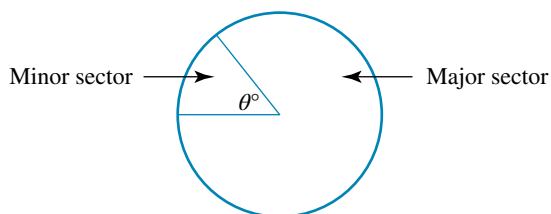


7.4 Area and perimeter of semicircles and sectors

[Stage 5.1]

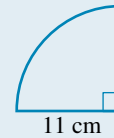
7.4.1 Sectors

- If you draw two **radii** inside a circle, they divide the circle into two regions called sectors.
- A circle, like a pizza, can be cut into many sectors.
- Two important sectors that have special names are the **semicircle** (half circle) and the **quadrant** (quarter circle).



WORKED EXAMPLE 11

Calculate the area enclosed by the figure shown, correct to 1 decimal place.



THINK

- 1 The figure is a quadrant or quarter-circle. Write the formula for its area.
- 2 Substitute $r = 11$ into the formula.
- 3 Evaluate and round the answer correct to 1 decimal place. Include the units.

WRITE

$$A = \frac{1}{4} \times \pi r^2$$

$$A = \frac{1}{4} \times \pi \times 11^2$$

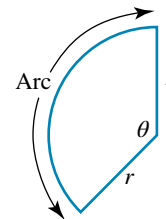
$$\approx 95.03$$

$$\approx 95.0 \text{ cm}^2$$

7.4.2 The formula for the area of a sector

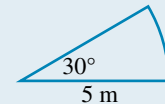
- Usually sectors are specified by the angle (θ) between the two radii.
- For example, in a quadrant, $\theta = 90^\circ$, so a quadrant is $\frac{90}{360}$ or $\frac{1}{4}$ of a circle.
- For any value of θ , the area of the sector is given by:

$$\text{Area}_{\text{sector}} = \frac{\theta}{360} \times \pi r^2$$



WORKED EXAMPLE 12

Calculate the area of the sector shown, correct to 1 decimal place.



THINK

- 1 This sector is $\frac{30}{360}$ of a circle. Write the formula for its area.
- 2 Substitute $r = 5$ into the formula.
- 3 Evaluate and round the answer correct to 1 decimal place. Include the units.

WRITE

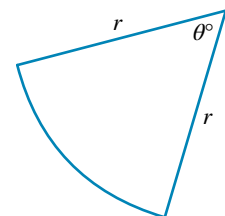
$$A = \frac{30}{360} \pi r^2$$

$$A = \frac{30}{360} \times \pi \times 5^2$$
$$\approx 6.54$$

$$\approx 6.5 \text{ m}^2$$

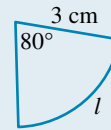
- The perimeter of a sector consists of 2 radii and a curved section, which is the arc of a circle.
- The length of the arc will be $\frac{\theta}{360}$ of the circumference of the circle.
- If the circumference of the circle = C , then the perimeter of the sector, P , will be given by:

$$P = 2r + \frac{\theta}{360} \times C$$
$$= 2r + \frac{\theta}{360} \times 2\pi r$$



WORKED EXAMPLE 13

Calculate the perimeter of the sector shown, correct to 1 decimal place.



THINK

- 1 The sector is $\frac{80}{360}$ of a circle. Write the formula for the length of the curved side.
- 2 Substitute $r = 3$ and evaluate l . Don't round off until the end.
- 3 Add all the sides together to calculate the perimeter.
- 4 Round the answer to 1 decimal place.

WRITE

$$l = \frac{80}{360} \times 2\pi r$$

$$l = \frac{80}{360} \times 2 \times \pi \times 3$$
$$\approx 4.189 \text{ cm}$$

$$P = 4.189 + 3 + 3$$
$$= 10.189 \text{ cm}$$

$$\approx 10.2 \text{ cm}$$

learnon

RESOURCES — ONLINE ONLY

Interactivity: Area of a sector (int-6076)

Interactivity: Perimeter of a sector (int-6077)

Exercise 7.4 Area and perimeter of semicircles and sectors

Individual pathways

PRACTISE

Questions:
1–12

CONSOLIDATE

Questions:
1–12, 15, 16

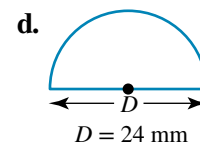
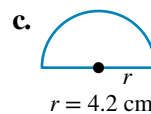
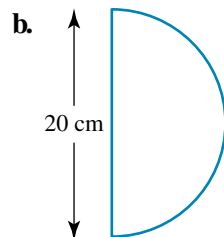
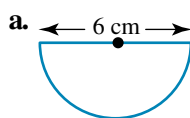
MASTER

Questions:
1d, 2c, 3–8, 10–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

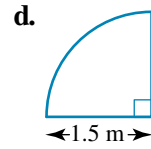
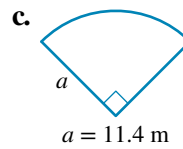
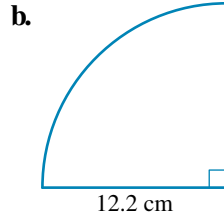
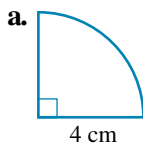
1. Calculate the area of each of the semicircles shown, correct to 2 decimal places.



2. **WE11** For each of the quadrants shown, calculate to 1 decimal place:

i. the perimeter

ii. the area enclosed.



3. **MC** Which is the correct formula for calculating the area of this sector?

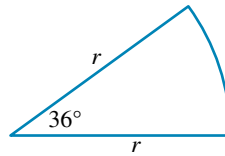
A. $A = \frac{1}{4}\pi r^2$

B. $A = \frac{3}{4}\pi r^2$

C. $A = \frac{1}{2}\pi r^2$

D. $A = 36\pi r^2$

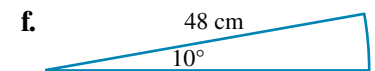
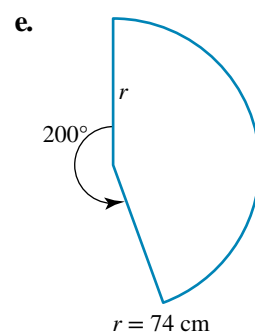
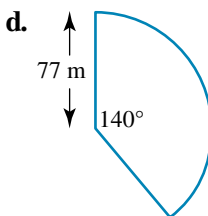
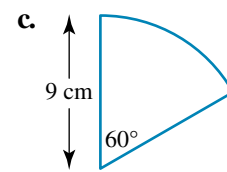
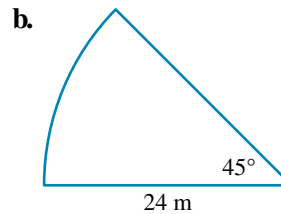
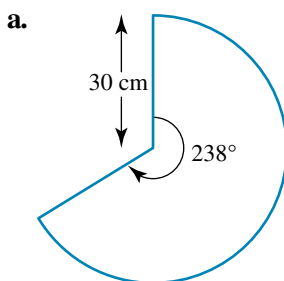
E. $A = \frac{1}{10}\pi r^2$



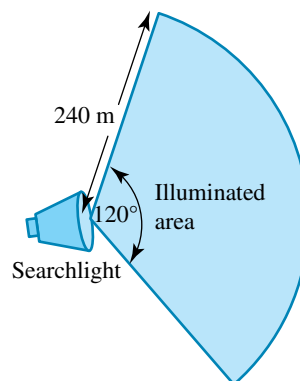
4. **WE12, WE13** For each of the sectors shown, calculate to 1 decimal place:

i. the perimeter

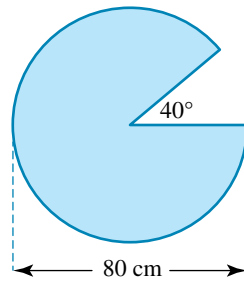
ii. the area.



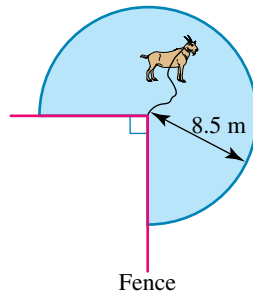
5. A searchlight lights up the ground to a distance of 240 m. What area does the searchlight illuminate if it can swing through an angle of 120° , as shown in the diagram below? (Give your answer correct to 1 decimal place.)



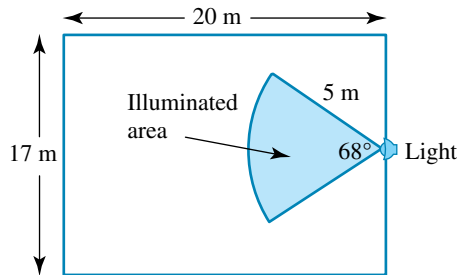
6. Calculate the perimeter, correct to 1 decimal place, of the figure shown.



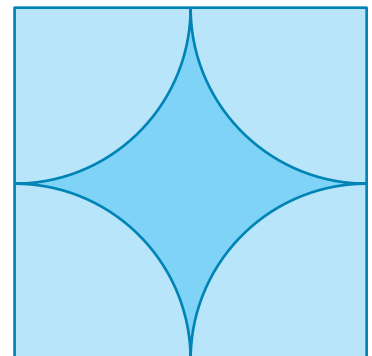
7. A goat is tethered by an 8.5 m rope, to the outside of a corner post in a paddock, as shown in the diagram. Calculate the area of grass (shaded) on which the goat is able to graze. (Give your answer correct to 1 decimal place.)



8. A beam of light is projected onto a theatre stage as shown in the diagram.



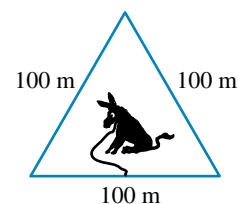
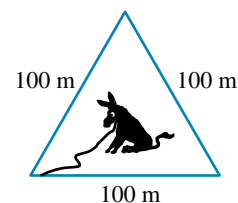
- Calculate the illuminated area (correct to 1 decimal place) by finding the area of the sector.
 - Calculate the percentage of the total stage area that is illuminated by the light beam.
9. **MC** A sector has an angle of 80° and a radius of 8 cm; another sector has an angle of 160° and a radius of 4 cm. The ratio of the area of the first sector to the area of the second sector is:
- 1 : 2
 - 2 : 1
 - 1 : 1
 - 1 : 4
 - 2 : 3
10. Calculate the radii of the following sectors, correct to 1 decimal place.
- Area = 100 m^2 , angle = 13°
 - Area = 100 m^2 , curved arc = 12 m
 - Perimeter = 100 m, angle = 11°
11. Four baseball fields are to be constructed inside a rectangular piece of land. Each field is in the shape of a sector of a circle, as shown in light blue. The radius of each sector is 80 m.
- Calculate the area of one baseball field, correct to the nearest whole number.
 - What percentage, correct to 1 decimal place, of the total area is occupied by the four fields?
 - The cost of the land is \$24 000 per hectare. What is the total purchase price of the land?



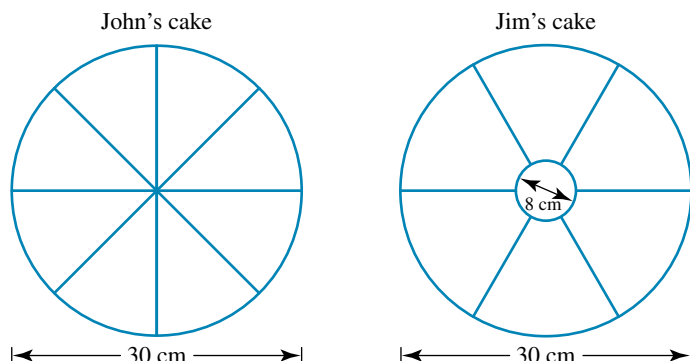
Communicating, reasoning and problem solving

- 12.** Answer the following questions. Where appropriate, give all answers to the nearest whole number.

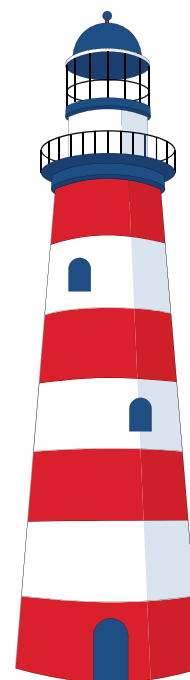
- A donkey inside a square enclosure is tethered to a post at one of the corners, as shown in the diagram at right. Show that the length of the rope required so that the donkey eats only half of the grass in the enclosure is 120 m.
- Suppose two donkeys are tethered at opposite corners of the square region shown. How long should the rope be so that the donkeys together can graze half of the area?
- This time four donkeys are tethered, one at each corner of the square region. How long should the rope be so that all the donkeys can graze only half of the area?
- Another donkey is tethered to a post inside an enclosure in the shape of an equilateral triangle. The post is at one of the vertices. Show that a rope of length 64 m is required so that the donkey eats only half of the grass in the enclosure.
- This time the donkey is tethered halfway along one side of the equilateral triangular region shown. How long should the rope be so that the donkey can graze half of the area?



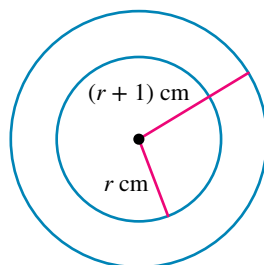
- 13.** John and Jim are twins, and on their birthday they have identical birthday cakes, each cake of diameter 30 cm. Grandma Maureen cuts John's cake into 8 equal sectors. Grandma Mary cuts Jim's cake with a circle in the centre and then 6 equal portions from the rest.



- Show that each sector of John's cake makes an angle of 45° at the centre of the cake.
 - What area of cake, correct to 1 decimal place, is a slice of John's cake?
 - What area of cake, correct to 1 decimal place, is the small central circular part of Jim's cake?
 - What area of cake, correct to 1 decimal place, is a larger portion of Jim's cake?
 - If each boy eats one slice of the largest part of their own cake, does John eat more cake? Justify your answer.
- 14.** A lighthouse has a light beam in the shape of a sector of a circle that rotates at 10 revolutions per minute and covers an angle of 40° . A person stands 200 m from the lighthouse and observes the beam. Show that the time between the end of one flash and the start of the next is approximately 5.33 seconds.



15. A metal washer has an inner radius of r cm and an outer radius of $(r + 1)$ cm, as shown in the diagram.



- State, in terms of r , the area of the circular piece of metal that was cut out of the washer.
 - State, in terms of r , the area of the larger circle.
 - Show that the area of the metal washer in terms of r is $\pi(2r + 1)$ cm².
 - If r is 2 cm, what is the exact area of the washer?
 - If the area of the washer is 15π cm², show that the radius would be 7 cm.
16. The area of a sector of a circle is π cm², and the length of its arc is 2 cm. What is the radius of the circle (in terms of π)?
17. An arbelos is a shape enclosed by three semicircles. The word, in Greek, means ‘shoemaker’s knife’ as it resembles the blade of a knife used by cobblers. Investigate to determine a relationship between the lengths of the three semicircular arcs.
18. What is the relationship between the curved arc of a sector and the area of the sector?

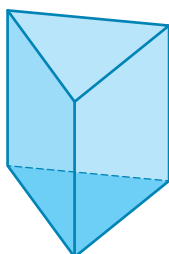


7.5 Surface area of rectangular and triangular prisms

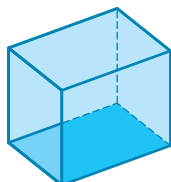
[Stage 5.1]

7.5.1 Right prisms

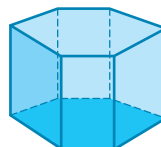
- A **right prism** is a solid object with a uniform (unchanging) **cross-section** and all sides flat. It is called a right prism because the angles between the base and sides are right angles.



Triangular prism

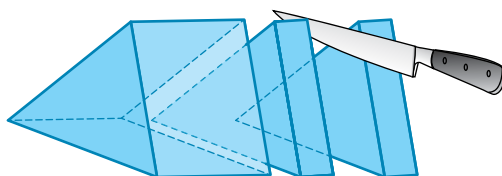


Rectangular prism (cuboid)



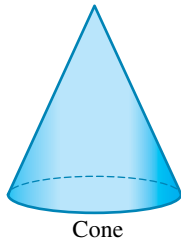
Hexagonal prism

- A prism can be sliced (cross-sectioned) in such a way that each ‘slice’ has an identical base.



‘Slicing’ a prism into pieces produces congruent cross-sections.

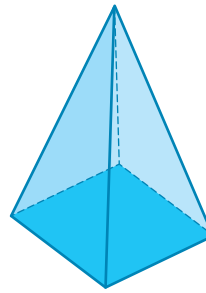
- The following objects are not prisms because they do not have uniform cross-sections.



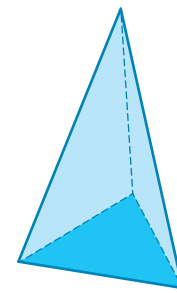
Cone



Sphere



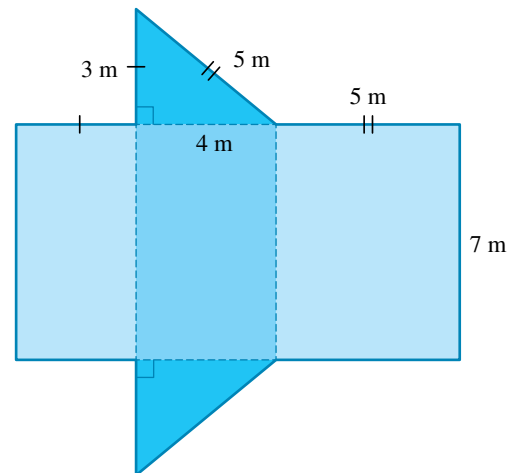
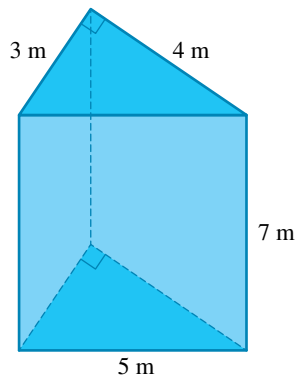
Square pyramid



Triangular pyramid

7.5.2 Surface area of a right prism

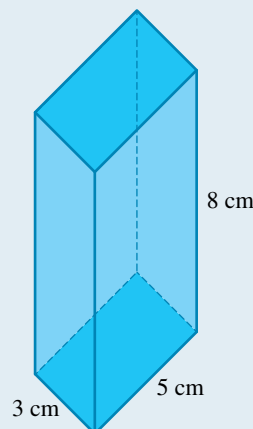
- Consider the **triangular prism** shown.
- It has 2 bases, which are right-angled triangles, and 3 rectangular sides. In all, there are 5 faces. The net of the prism is drawn below right.



- The area of the net is the same as the total surface area (SA) of the prism.
- The angles between the base and sides of this triangular prism are right angles; hence, it is also a right prism.

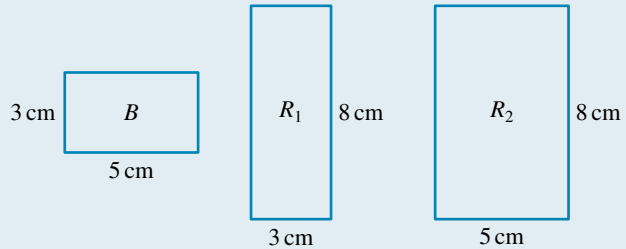
WORKED EXAMPLE 14

Find the surface area of the rectangular prism (cuboid) shown.



THINK

- 1 There are 6 faces: 2 rectangular bases and 4 rectangular sides. Draw diagrams for each pair of faces and label each region.

WRITE/DRAW

- 2 Find the area of each rectangle by applying the formula $A = lw$.
- 3 The total surface area is the sum of the area of 2 of each shape.
Write the answer.

$$B = 3 \times 5 \quad R_1 = 3 \times 8 \quad R_2 = 5 \times 8$$

$$= 15 \quad = 24 \quad = 40$$

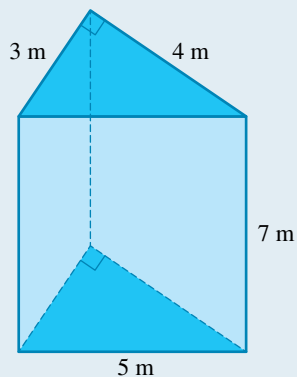
$$SA = 2B + 2 \times R_1 + 2 \times R_2$$

$$= 30 + 48 + 80$$

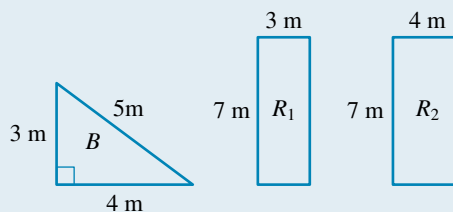
$$= 158 \text{ cm}^2$$

WORKED EXAMPLE 15

Find the surface area of the right-angled triangular prism shown.

**THINK**

- 1 There are 5 faces: 2 triangles and 3 rectangles.
Draw diagrams for each face and label each region.

WRITE/DRAW

- 2 Find the area of the triangular base by applying the formula $A = \frac{1}{2}bh$.
- 3 Find the area of each rectangular face by applying the formula $A = lw$.
- 4 The total surface area is the sum of all the areas of the faces, including 2 bases.

$$B = \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ m}^2$$

$$R_1 = 3 \times 7 \quad R_2 = 4 \times 7 \quad R_3 = 5 \times 7$$

$$= 21 \quad = 28 \quad = 35$$

$$SA = 2B + R_1 + R_2 + R_3$$

$$= 12 + 21 + 28 + 35$$

$$= 96 \text{ m}^2$$

Interactivity: Cross-sections (int-6078)

Interactivity: Surface area of a prism (int-6079)

Interactivity: Surface area of prisms (int-2771)

eLesson: Total surface area of prisms (eles-1909)

Digital doc: SkillsHEET Surface area of cubes and rectangular prisms (doc-10876)

Digital doc: SkillsHEET Surface area of triangular prisms (doc-10877)

Digital doc: WorkSHEET Surface area (doc-6304)

Exercise 7.5 Surface area of rectangular and triangular prisms

Individual pathways

PRACTISE

Questions:
1a–c, 2–15, 17

CONSOLIDATE

Questions:
1b–d, 2b, 3b, 4–17

MASTER

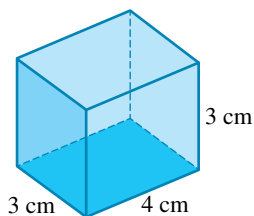
Questions:
1d–f, 2c, 3c, 4–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

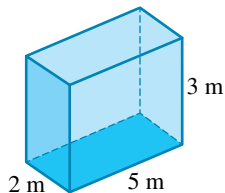
Understanding and fluency

1. **WE14** Find the surface area (SA) of each of the following rectangular prisms (cuboids).

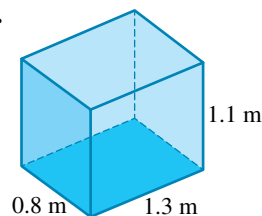
a.



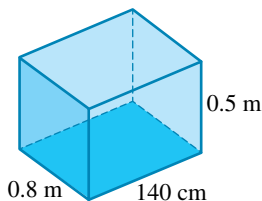
b.



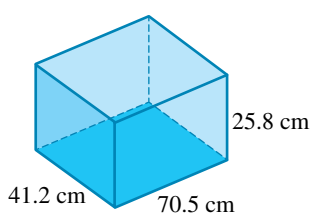
c.



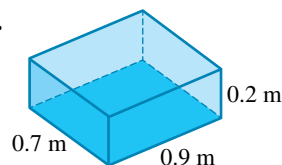
d.



e.

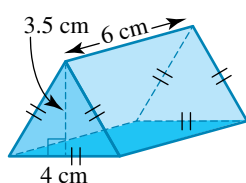


f.

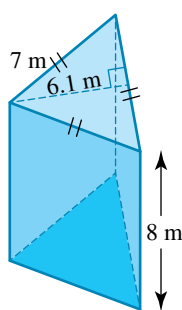


2. **WE15** Find the surface area (SA) of each of the triangular prisms below.

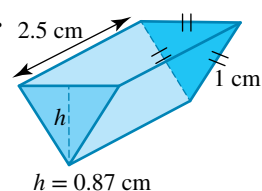
a.



b.

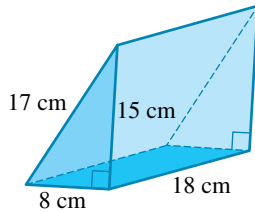


c.

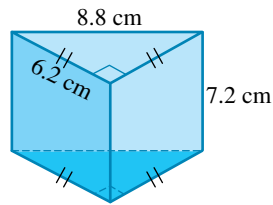


3. Find the surface area (SA) of each of the triangular prisms below.

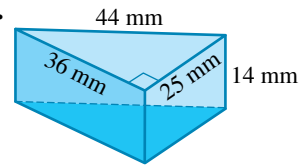
a.



b.



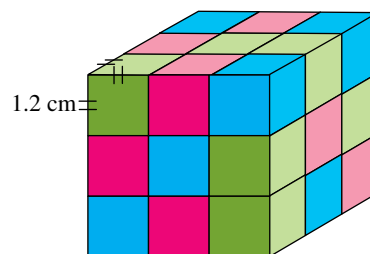
c.



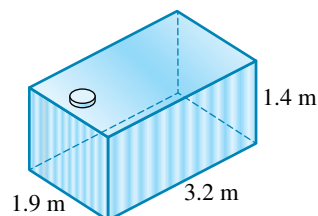
4. Maya is planning to buy and paint the outside surface of one of these shipping containers. How many cans of paint should Maya buy, if the base of the container is not painted, and each can of paint covers about 40 m^2 ?



5. The aim of the Rubik's cube puzzle is to make each face of the cube one colour. Find the surface area of the Rubik's cube if each small coloured square is 1.2 cm in length. Assume that there are no gaps between the squares.

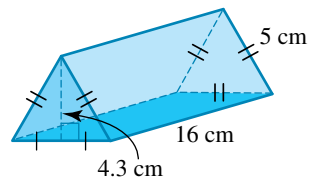


6. How many square metres of iron sheet are needed to construct the water tank shown?

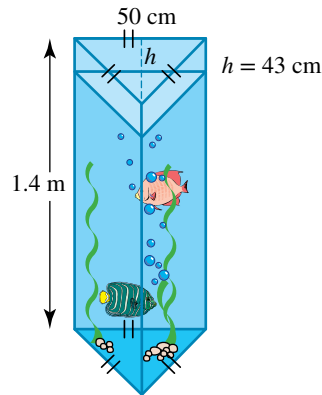


7. What is the surface area of the tank in the previous question if no top is made?

8. What area of cardboard would be needed to construct a box to pack this prism, assuming that no overlap occurs?



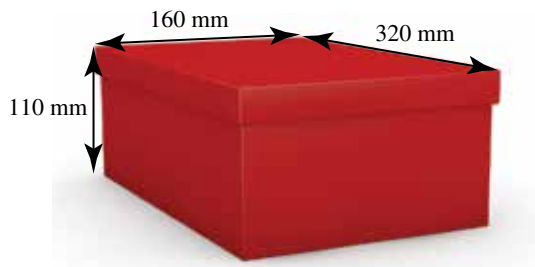
9. An aquarium is a triangular prism with the dimensions shown. The top of the tank is open. What area of glass was required to construct the tank? Give your answer correct to 2 decimal places.



10. A tent is constructed as shown. What area of canvas is needed to make the tent, if a floor is included?



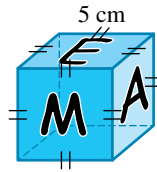
11. How many square centimetres of cardboard are needed to construct the shoebox shown, assuming no overlap and ignoring the overlap on the top? Draw a sketch of a net that could be used to make the box.



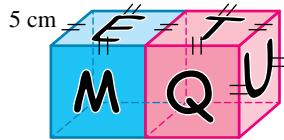
12. Find the surface area of a square-based prism of height 4 cm, given that the side length of its base is 6 cm.
13. A prism has an equilateral triangular base with a perimeter of 12 cm. If the length of the prism is 24 cm, determine the total surface area of the prism. (*Hint:* What is the area of 1 triangle?)

Communicating, reasoning and problem solving

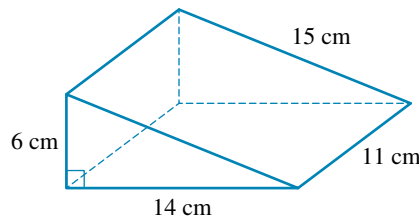
14. a. Find the surface area of the toy block shown.



- b. If two of the blocks are placed together as shown, what is the surface area of the prism which is formed?



- c. What is the surface area of the prism formed by three blocks?
- d. Use the pattern to determine the surface area of a prism formed by eight blocks arranged in a line. Explain your reasoning.
15. A cube has a side length of 2 cm. Show that the least surface area of a solid formed by joining eight such cubes is 96 cm^2 .
16. Ken wants to paint his son's bedroom walls blue and the ceiling white. The room measures 3 m by 4 m with a ceiling height of 2.6 m. There is one 1 m by 2 m door and one 1.8 m by 0.9 m window. Each surface takes two coats of paint and 1 L of paint covers 16 m^2 on the walls and 12 m^2 on the ceiling. Cans of wall paint cost \$33.95 for 1 L, \$63.90 for 4 L, \$147 for 10 L and \$174 for 15 L. Ceiling paint costs \$24 for 1 L and \$60 for 4 L. What is the least it would cost Ken to paint the room?
17. A wedge in the shape of a triangular prism, as shown, is to be painted.



- a. Draw a net of the wedge so that it is easier to calculate the area to be painted.
- b. What area is to be painted? (Do not include the base.)
18. A swimming pool has a length of 50 m and a width of 28 m. The shallow end of the pool has a depth of 0.80 m, which increases steadily to 3.8 m at the deep end.
- a. Calculate how much paint would be needed to paint the floor of the pool.
- b. If the pool is to be filled to the top, how much water will be needed?
19. What is the quickest method of calculating the total surface area of a six-sided box?

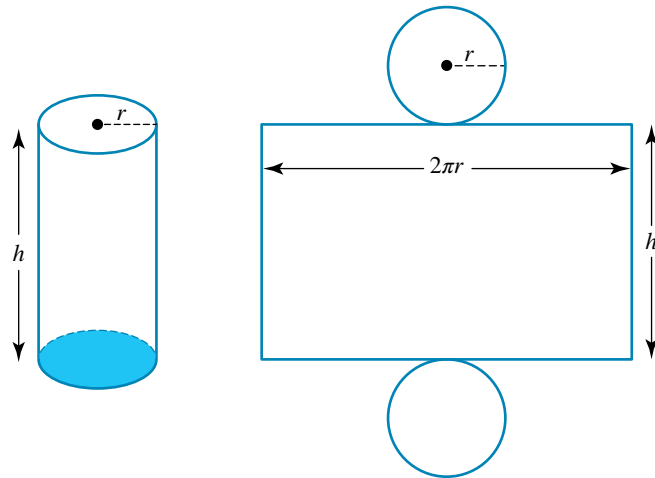
7.6 Surface area of cylinders and composite solids

[Stage 5.2]

7.6.1 Surface area of a cylinder

- A **cylinder** is a solid object with two identical flat circular ends and one curved side. It has a uniform cross-section.

- The net of a cylinder has two circular bases and one rectangular face. The rectangular face is the curved surface of the cylinder.

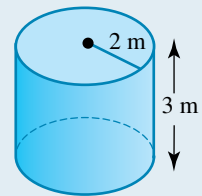


- Because the rectangle wraps around the circular base, the width of the rectangle is the same as the circumference of the circle. Therefore, the width is equal to $2\pi r$.
- The area of each base is πr^2 , and the area of the rectangle is $2\pi rh$. Because there are two bases, the surface area of the cylinder is given by:

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \end{aligned}$$

WORKED EXAMPLE 16

- a** Use the formula $A = 2\pi rh$ to calculate the area of the curved surface of the cylinder shown, correct to 1 decimal place.
- b** Use the formula $SA = 2\pi rh + 2\pi r^2$ to calculate the surface area of the cylinder, correct to 1 decimal place.



THINK

- a 1** Write the formula for the curved surface area.
- 2** Identify the values of the pronumerals.
- 3** Substitute $r = 2$ and $h = 3$.
- 4** Evaluate, round to 1 decimal place and include units.
- b 1** Write the formula for the surface area of a cylinder.
- 2** Identify the values of the pronumerals.
- 3** Substitute $r = 2$ and $h = 3$.
- 4** Evaluate, round to 1 decimal place and include units.

WRITE

a $A = 2\pi rh$

$$r = 2, h = 3$$

$$A = 2 \times \pi \times 2 \times 3$$

$$\approx 37.69$$

$$\approx 37.7 \text{ m}^2$$

b $SA = 2\pi rh + 2\pi r^2$

$$r = 2, h = 3$$

$$SA = (2 \times \pi \times 2 \times 3) + (2 \times \pi \times 2^2)$$

$$\approx 62.83$$

$$\approx 62.8 \text{ m}^2$$

Exercise 7.6 Surface area of cylinders and composite solids

Individual pathways

PRACTISE

Questions:
1–6, 9, 11, 13

CONSOLIDATE

Questions:
1–7, 10, 12–14, 16

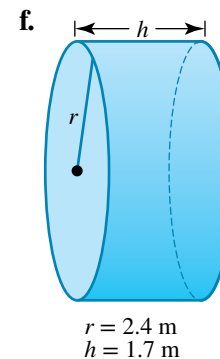
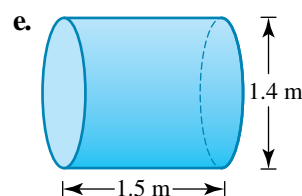
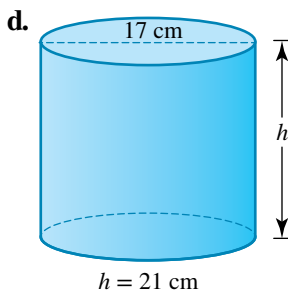
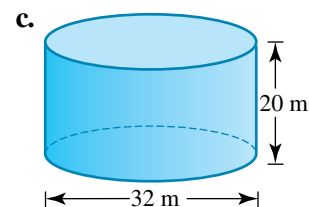
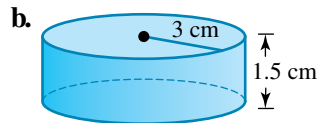
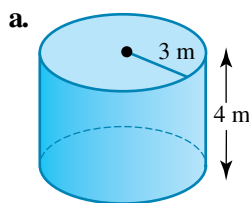
MASTER

Questions:
1–6, 8–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

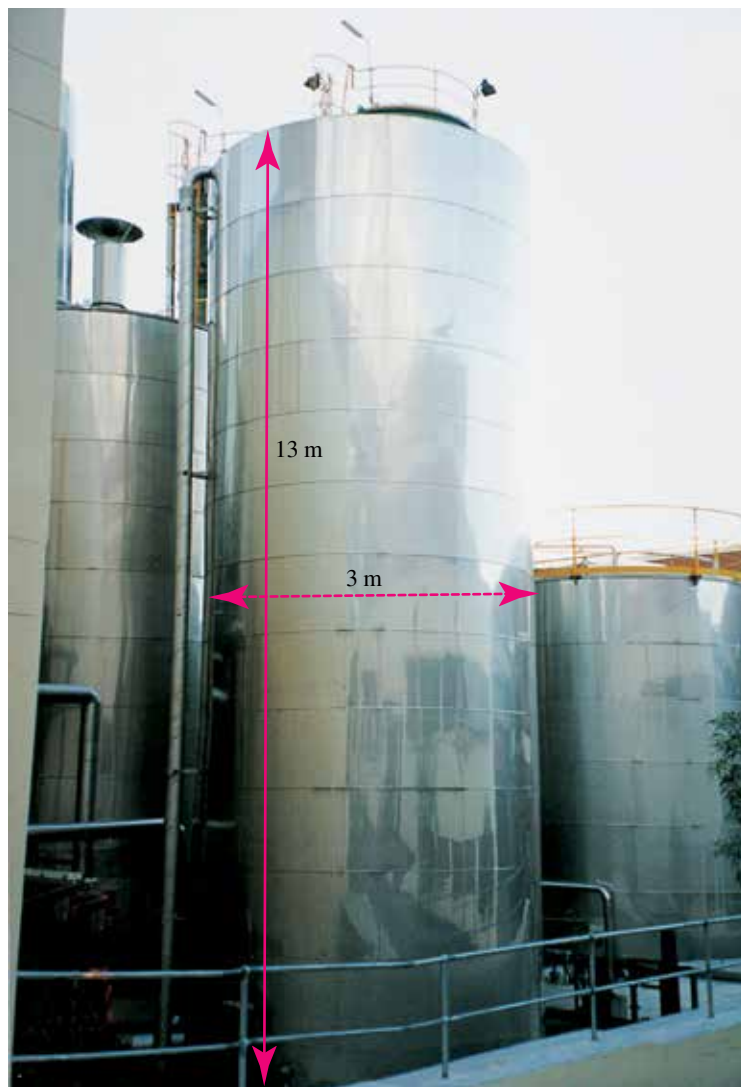
1. **WE16a** Use the formula $A = 2\pi rh$ to find the area of the curved surface of each of the cylinders below. (Express your answers correct to 1 decimal place.)



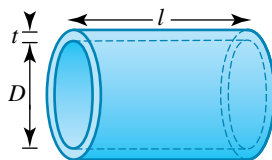
2. **WE16b** Use the formula $SA = 2\pi rh + 2\pi r^2$ to find the total surface area of each of the cylinders in question 1.
3. A can containing an energy drink has a height of 130 mm and a radius of 24 mm. Draw the net of the can. (*Hint:* Plan ahead.)
4. A can of asparagus spears is 137 mm tall and has a diameter of 66 mm, a can of tomatoes is 102 mm tall and has a diameter of 71 mm, and a can of beetroot is 47 mm tall with a diameter of 84 mm.
- Which can has the largest surface area? Which can has the smallest surface area?
 - What is the difference between the largest and smallest surface areas, correct to the nearest cm^2 ?
5. A cylinder has a radius of 15 cm and a height of 45 mm. Determine its surface area correct to 1 decimal place.
6. If the radius of a cylinder is twice its height, write a formula for the surface area in terms of its height only.



7. A cylinder has a surface area of 2000 cm^2 and a radius of 8 cm. Determine the cylinder's height correct to 2 decimal places.
8. A 13 m-high storage tank was constructed from stainless steel (including the lid and the base). The diameter is 3 metres as shown.
 - a. What is the surface area of the tank? Give your answer correct to 2 decimal places.
 - b. How much did the steel cost for the side of the tank if it comes in sheets 1 m wide that cost \$60 a metre?



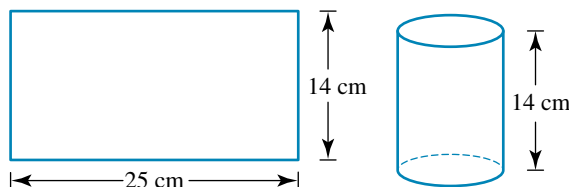
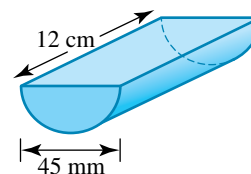
9. The concrete pipe shown in the diagram has the following measurements:
 $t = 30 \text{ mm}$, $D = 18 \text{ cm}$, $l = 27 \text{ cm}$.



Find the following areas. Give your answers correct to 2 decimal places.

- a. Calculate the outer curved surface area.
- b. Calculate the inner curved surface area.
- c. Calculate the total surface area of both ends.
- d. Hence calculate the surface area for the entire shape.

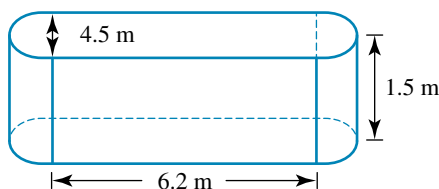
10. Wooden mouldings are made by cutting cylindrical dowels in half as shown at right. Calculate the surface area of the moulding.
11. Kiara has a rectangular sheet of cardboard with dimensions 25 cm by 14 cm. She rolls the cardboard to form a cylinder so that the shorter side, 14 cm, is its height, and glues the edges together with a 1-cm overlap.



- What is the radius of the circle Kiara needs to construct to put at the top of her cylinder? Give your answer correct to 2 decimal places.
- What is the total surface area of her cylinder if she also makes the top and bottom of her cylinder out of cardboard? Give your answer correct to 1 decimal place.

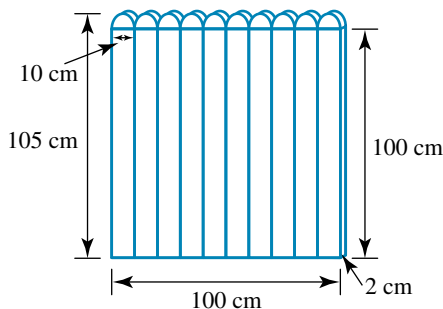
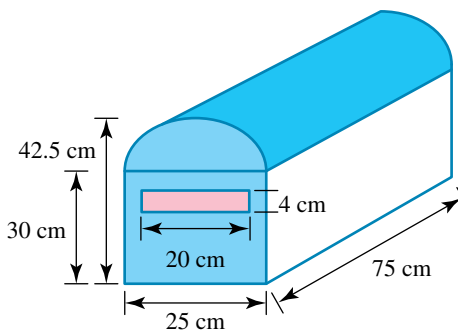
Communicating, reasoning and problem solving

12. Cylinder A has a 10% greater radius and a 10% greater height compared with Cylinder B. Show that the ratio of their surface areas is 121 : 100.
13. An above-ground swimming pool has the following shape, with semicircular ends.



How much plastic would be needed to line the base and sides of the pool?

14. A over-sized solid wooden die is constructed for a children's playground. The side dimensions of the die are 50 cm. The number on each side of the die will be represented by cylindrical holes which will be drilled out of each side. Each hole will have a diameter of 10 cm and depth of 2 cm. All surfaces on the die will be painted (including the die holes). Show that the total area required to be painted is 1.63 m^2 .
15. The following letterbox is to be spray-painted on the outside. What is the total area to be spray-painted? Assume that the end of the letterbox is a semicircle above a rectangle. The letter slot is open and does not require painting.
16. A timber fence is designed as shown. How many square metres of paint are required to completely paint the fence front, back, sides and top with 2 coats of paint? Assume each paling is 2 cm in thickness and that the top of each paling is a semicircle.



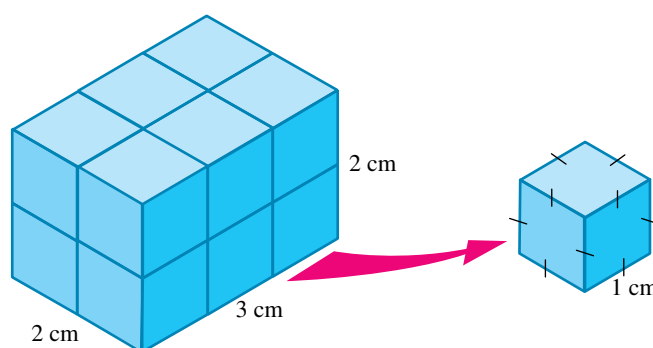
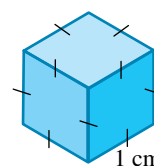
17. Devise an easy way to remember the formula for the surface area of a cylinder.

7.7 Volume of prisms, cylinders and composite solids

[Stage 5.2]

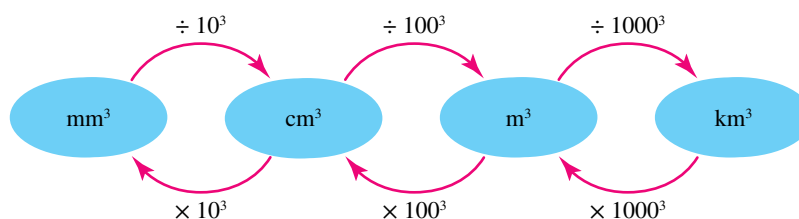
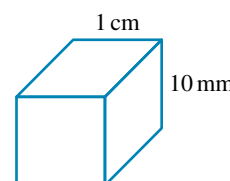
7.7.1 Volume

- The diagram shows a cube of side length 1 cm.
By definition the cube has a **volume** of 1 cm^3 (1 cubic centimetre).
Note: This is a 'cubic centimetre', not a 'centimetre cubed'.
- The volume of a solid is the amount of space it fills or occupies.
- The volume of some solids can be found by dividing them into cubes with 1-cm sides.



This solid fills the same amount of space as 12 cubic centimetres.
The volume of this cuboid is 12 cm^3 .

- Volume is commonly measured in cubic millimetres (mm^3), cubic centimetres (cm^3), cubic metres (m^3) or cubic kilometres (km^3).
- The volume of the cube shown is equal to 1 cm^3 or 1000 mm^3 , so
 $1 \text{ cm}^3 = 1000 \text{ mm}^3$,
or $1 \text{ cm}^3 = 10^3 \text{ mm}^3$.
Similarly $1 \text{ m}^3 = 100^3 \text{ cm}^3$
and $1 \text{ km}^3 = 1000^3 \text{ m}^3$.
- The chart below is useful when converting from one unit of volume to another.



- For example, $3 \text{ m}^3 = 3 \times 100^3 \times 10^3 \text{ mm}^3$.
 $= 3\,000\,000\,000 \text{ mm}^3$

7.7.2 Capacity

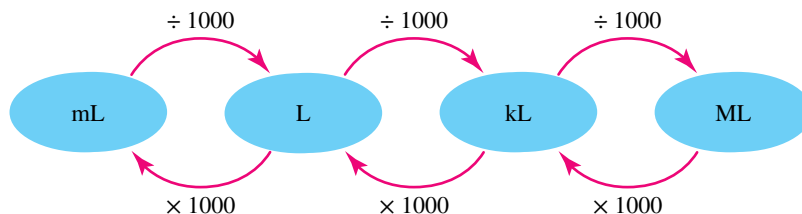
- Capacity** is a term usually applied to the measurement of liquids and containers.
- The capacity of a container is the volume of liquid that it can hold.
- The standard measurement for capacity is the litre (L).

Other common units are the millilitre (mL), kilolitre (kL), and megalitre (ML), where

$$1 \text{ L} = 1000 \text{ mL}$$

$$1 \text{ kL} = 1000 \text{ L}$$

$$1 \text{ ML} = 1\,000\,000 \text{ L}$$



- The units of capacity and volume are related as follows:

$$1 \text{ cm}^3 = 1 \text{ mL and}$$

$$1 \text{ m}^3 = 1000 \text{ L}.$$

WORKED EXAMPLE 17

Convert:

a 13.2 L into cm^3

b 3.1 m^3 into litres

c 0.13 cm^3 into mm^3

d 3.8 kL into m^3 .

THINK

a 1 L = 1000 mL, so multiply by 1000.

2 1 mL = 1 cm^3 . Convert to cm^3 .

b 1 m^3 = 1000 L, so multiply by 1000.

c There will be more mm^3 , so multiply by 10^3 .

d 1 kL = 1 m^3 . Convert to m^3 .

WRITE

$$\begin{aligned} \text{a } 13.2 \times 1000 &= 13\,200 \text{ mL} \\ &= 13\,200 \text{ cm}^3 \end{aligned}$$

$$\text{b } 3.1 \times 1000 = 3100 \text{ L}$$

$$\text{c } 0.13 \times 1000 = 130 \text{ mm}^3$$

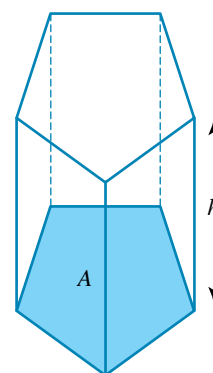
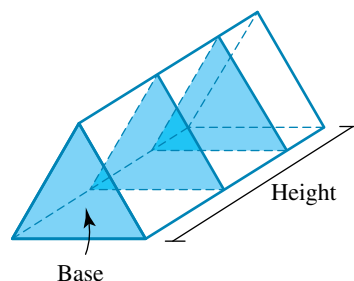
$$\text{d } 3.8 \text{ kL} = 3.8 \text{ m}^3$$

7.7.3 Volume of a prism

- The volume of a prism can be found by multiplying its cross-sectional area (A) by its height (h).

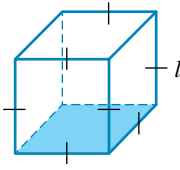
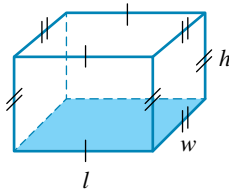
$$\text{Volume} = A \times h$$

- The cross-section (A) of a prism is often referred to as the base, even if it is not at the bottom of the prism.
- The height (h) is always measured perpendicular to the base, as shown in the diagram below.



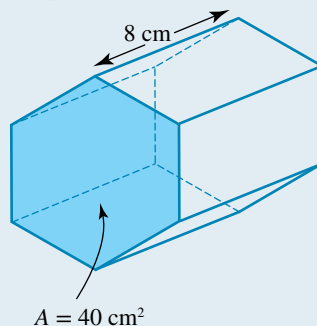
7.7.4 Volume of a cube or a cuboid

- A specific formula can be developed for the volumes of cubes and cuboids.

Cube  Volume = base area $\times$ height $= l^2 \times l$ $= l^3$	Cuboid (rectangular prism)  Volume = base area $\times$ height $= lw \times h$ $= lwh$

WORKED EXAMPLE 18

Calculate the volume of the hexagonal prism.



THINK

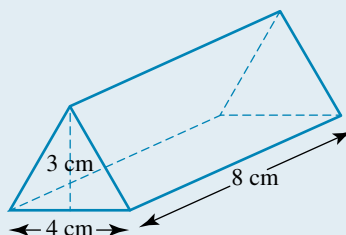
- Write the formula for the volume of a prism.
- Identify the values of the pronumerals.
- Substitute $A = 40$ and $h = 8$ into the formula and evaluate. Include the units.

WRITE

$$\begin{aligned}
 V &= Ah \\
 A &= 40, h = 8 \\
 V &= 40 \times 8 \\
 &= 320 \text{ cm}^3
 \end{aligned}$$

WORKED EXAMPLE 19

Calculate the volume of the prism.



THINK

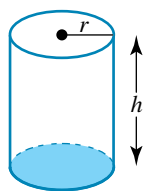
- 1 The base of the prism is a triangle. Write the formula for the area of the triangle.
- 2 Substitute $b = 4$, $h = 3$ into the formula and evaluate.
- 3 Write the formula for volume of a prism.
- 4 State the values of A and h .
- 5 Substitute $A = 6$ and $h = 8$ into the formula and evaluate. Include the units.

WRITE

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 4 \times 3 \\
 &= 6 \text{ cm}^2 \\
 V &= A \times h \\
 A &= 6, h = 8 \\
 V &= 6 \times 8 \\
 &= 48 \text{ cm}^3
 \end{aligned}$$

7.7.5 Volume of cylinders and composite solids

- A cylinder has a circular base and uniform cross-section.
- A formula for the volume of a cylinder is shown.

Cylinder

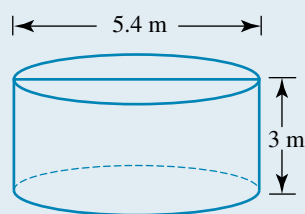
$$\begin{aligned}
 \text{Volume} &= \text{base area} \times \text{height} \\
 &= \text{area of circle} \times \text{height} \\
 &= \pi r^2 \times h \\
 &= \pi r^2 h
 \end{aligned}$$

WORKED EXAMPLE 20

Calculate the capacity, in litres, of a cylindrical water tank that has a diameter of 5.4 m and a height of 3 m. (Give your answer correct to the nearest litre.)

THINK

- 1 Draw a labelled diagram of the tank.
- 2 The base is a circle, so $A = \pi r^2$. Write the formula for the volume of a cylinder.
- 3 Write the values of r and h .
- 4 Substitute $r = 2.7$, $h = 3$ into the formula and find the volume.
- 5 Convert this volume to litres (multiply by 1000).

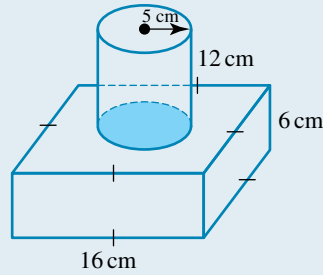
WRITE/DRAW

$$\begin{aligned}
 V &= \pi r^2 h \\
 r &= 2.7, h = 3 \\
 V &= \pi \times 2.7^2 \times 3 \\
 &\approx 68.706631 \text{ m}^3 \\
 V &= 68.706631 \times 1000 \\
 &= 68706.631 \\
 &\approx 68.707 \text{ L}
 \end{aligned}$$

- A composite solid is made up of two or more prisms.

WORKED EXAMPLE 21

Find the total volume of this solid.



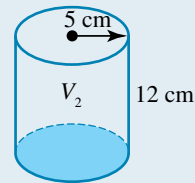
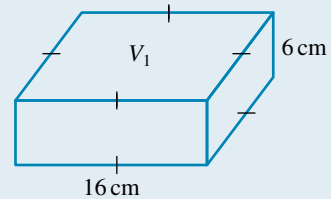
THINK

- 1 The solid is made from two objects. Draw and label each object.

Let V_1 = volume of the square prism.

Let V_2 = volume of the cylinder.

WRITE/DRAW



- 2 Find V_1 (square prism).

$$\begin{aligned} V_1 &= Ah \\ &= 16^2 \times 6 \\ &= 1536 \text{ cm}^3 \end{aligned}$$

- 3 Find V_2 (cylinder).

$$\begin{aligned} V_2 &= \pi r^2 h \\ &= \pi \times 5^2 \times 12 \\ &\approx 942.478 \text{ cm}^3 \end{aligned}$$

- 4 To find the total volume, add the volumes found above.

$$\begin{aligned} V &= V_1 + V_2 \\ &= 1536 + 942.478 \\ &= 2478.478 \\ &\approx 2478 \text{ cm}^3 \end{aligned}$$

DISCUSSION

How does rounding during your working as opposed to using unrounded values affect the answers to questions?

learn on RESOURCES — ONLINE ONLY

Interactivity: Conversion chart for volume (int-3791)

Interactivity: Prisms? (int-3792)

Digital doc: SkillsHEET Volume of cubes and rectangular prisms (doc-10878)

Digital doc: SkillsHEET Volume of triangular prisms (doc-10879)

Digital doc: SkillsHEET Volume of cylinders (doc-10880)

Digital doc: WorkSHEET Volume (doc-6305)

Exercise 7.7 Volume of prisms, cylinders and composite solids

Individual pathways

PRACTISE

Questions:

1, 2, 3a–e, 4a, 5a, b, 6a, b, 7, 9a, b, 10, 13, 14, 20–22

CONSOLIDATE

Questions:

1, 3b–f, 4b, 5b, 6b–e, 8, 9c, d, 11, 13–16, 20–23

MASTER

Questions:

1, 3e–h, 4c, 5c, d, 6d–f, 8, 9e, f, 12–25

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE17** a. Convert the following units into mL.

i. 325 cm^3

ii. 2.6 m^3

iii. 5.1 L

iv. 0.63 kL

b. Convert the following units into cm^3 .

i. 5.8 mL

ii. 6.1 L

iii. 3.2 m^3

iv. 59.3 mm^3

c. Convert the following units into kL.

i. 358 L

ii. 55.8 m^3

iii. 8752 L

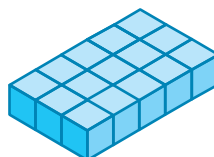
iv. 5.3 ML

2. Calculate the volumes of the cuboids below. Assume that each small cube has sides of 1 cm .

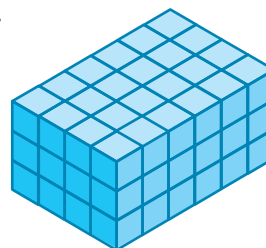
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b.

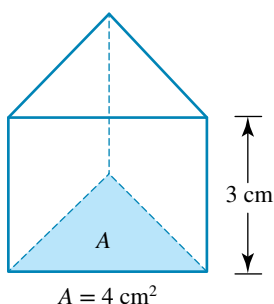


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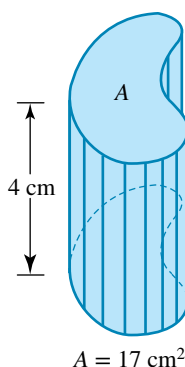


3. **WE18** Calculate the volumes of these objects.

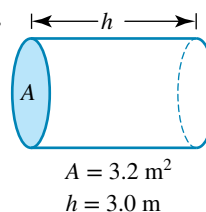
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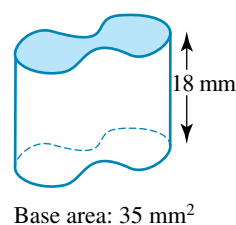
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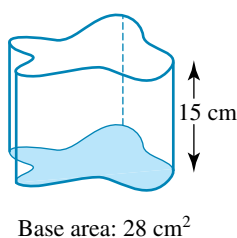
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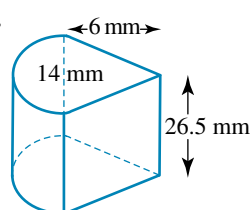
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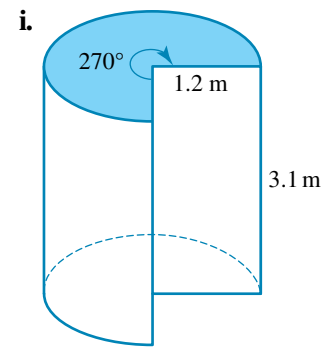
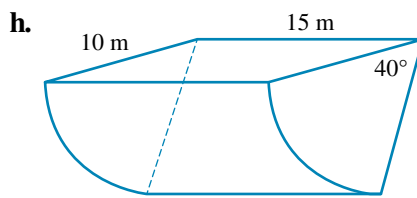
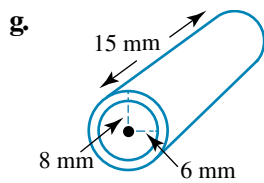


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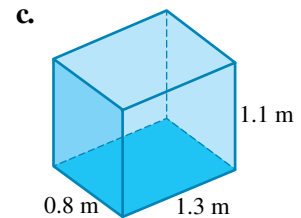
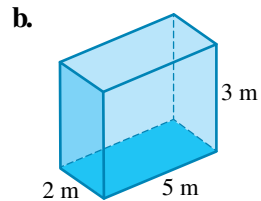
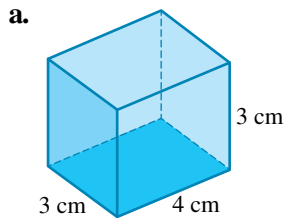


f.

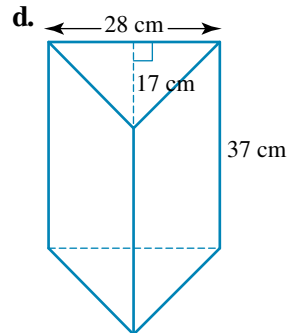
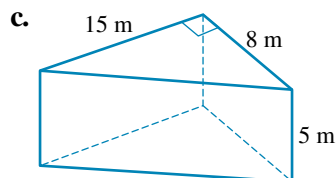
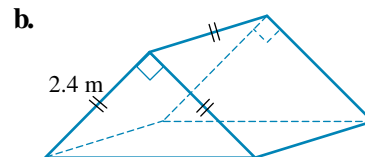
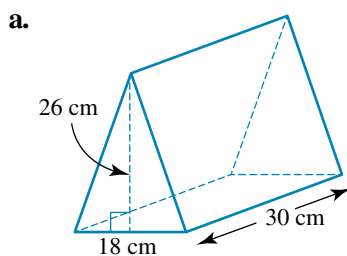




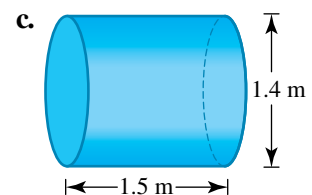
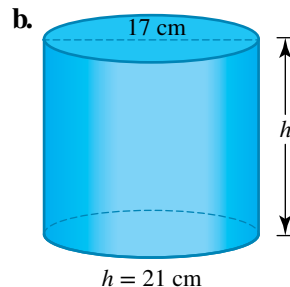
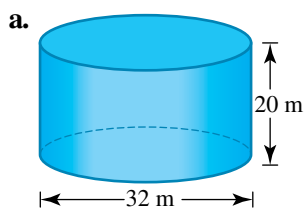
4. Calculate the volume of these rectangular prisms.

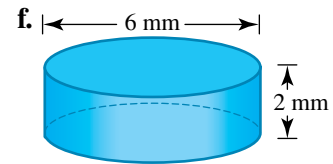
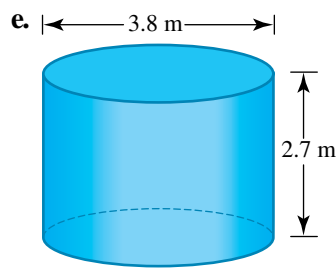
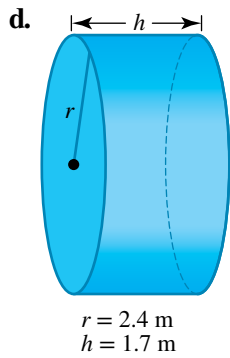


5. **WE19** Calculate the volumes of the prisms shown below. Give your answers correct to 2 decimal places.



6. Calculate the volumes of the following cylinders. Give your answers correct to 1 decimal place.

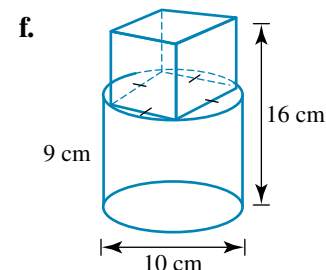
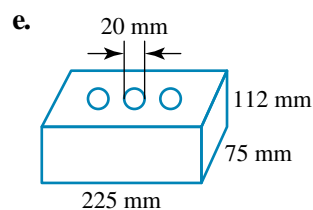
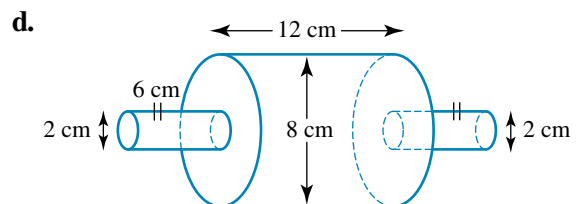
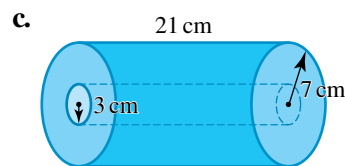
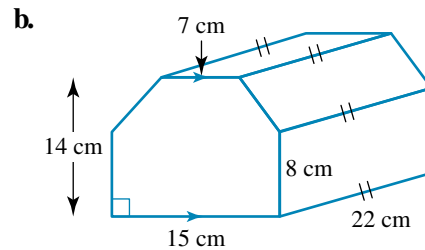
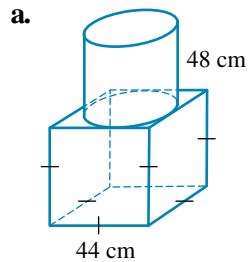




7. **WE20** Calculate the volume of water, in litres (L), that can fill a cylindrical water tank that has a diameter of 3.2 m and a height of 1.8 m.
8. What is the capacity in litres of the Esky shown?



9. **WE21** Find the volume of each solid to the nearest cm^3 .



Note: In the diagram above, the diameter of each of the small holes is 20 mm, and each hole runs through to the other side of the block.

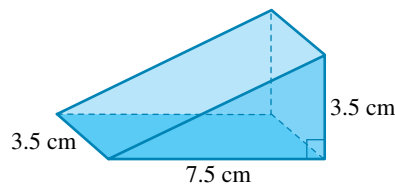
10. Calculate the capacity, in litres, of the cylindrical storage tank shown. Give your answer correct to 1 decimal place.



11. What is the capacity (in mL) of this cylindrical coffee plunger when it is filled to the level shown? Give your answer correct to 1 decimal place.
12. Until its closure in 2001, Fresh Kills on Staten Island outside New York City was one of the world's biggest landfill garbage dumps. (New Yorkers throw out about 100 000 tonnes of refuse weekly.) Calculate the approximate volume (m^3) of the Fresh Kills landfill if it covers an area of 1215 hectares and is about 240 m high. (Note: 1 hectare = 10 000 m^2 .)

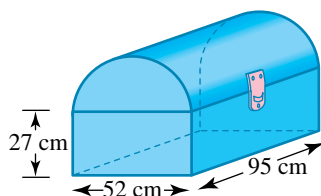


13. Sudhira is installing a rectangular pond in a garden. The pond is 1.5 m wide and 2.2 m long, and has a uniform depth of 1.5 m.
- Calculate the volume of soil (m^3) that Sudhira must remove to make the hole before installing the pond.
 - What is the capacity of the pond in litres? (Ignore the thickness of the walls in this calculation.)
14. a. Calculate the volume of plastic needed to make the door wedge shown. Give your answer correct to 2 decimal places.



- The wedges can be packed snugly into cartons $45 \text{ cm} \times 70 \text{ cm} \times 35 \text{ cm}$. How many wedges fit into each carton?

15. Calculate the internal volume (in cm^3) of the wooden chest shown, correct to 2 decimal places.
(Ignore the thickness of the walls.)

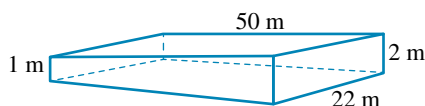


16. An internal combustion engine consists of 4 cylinders. In each cylinder a piston moves up and down. The diameter of each cylinder is called the bore and the height that the piston moves up and down within the cylinder is called the stroke (stroke = height).
- If the bore of a cylinder is 84 mm and the stroke is 72 mm, calculate the volume (in litres) of 4 such cylinders correct to 2 decimal places.
 - When an engine gets old, the cylinders have to be 're-bored', that is, the bore is increased by a small amount (and new pistons put in them). If the re-boring increases the diameter by 1.1 mm, what is the increase (in litres) of the volume of the 4 cylinders? Give your answer correct to 3 decimal places.
17. a. Calculate the volume, in m^3 , of the refrigerator shown.
b. What is the capacity of the refrigerator if the walls are all 5 cm thick?



Communicating, reasoning and problem solving

18. A cylindrical glass is designed to hold 1.25 L. Show that its height is 132 mm if it has a diameter of 110 mm.
19. Mark is responsible for the maintenance of the Olympic (50 m) pool at an aquatic centre. The figure below shows the dimensions of an Olympic pool.



- a. What is the shape of the pool?
 - b. Draw the cross-section of the prism and calculate its area.
 - c. Show that the capacity of the pool is 1 650 000 L.
 - d. Mark needs to replace the water in the pool every 6 months. If the pool is drained at 45 000 L per hour and refilled at 35 000 L per hour, how long will it take to:
 - i. drain
 - ii. refill (in hours and minutes)?
20. Use examples to show that the volume of a cone is equal to $\frac{1}{3}$ the volume of the cylinder it can be taken from.
 21. Use examples to show that the volume of a pyramid is equal to $\frac{1}{3}$ the volume of the rectangular prism it can be taken from.
 22. A cylindrical container of water has a diameter of 16 cm and is 40 cm tall. How many full cylindrical glasses can be filled from the container if the glasses have a diameter of 6 cm and are 12 cm high?
 23. a. A square sheet of metal with dimensions 15 cm by 15 cm has a 1-cm square cut out of each corner. The remainder of the square is folded to form an open box. Calculate the volume of the box.
b. Write a general formula for calculating the volume of a box created from a metal sheet of any size with any size square cut out of the corners.
 24. Why do some people think that a cylinder is a prism?
 25. A foam cube has a side length of 10 cm. What increase in side length would be necessary to double the volume of the cube?



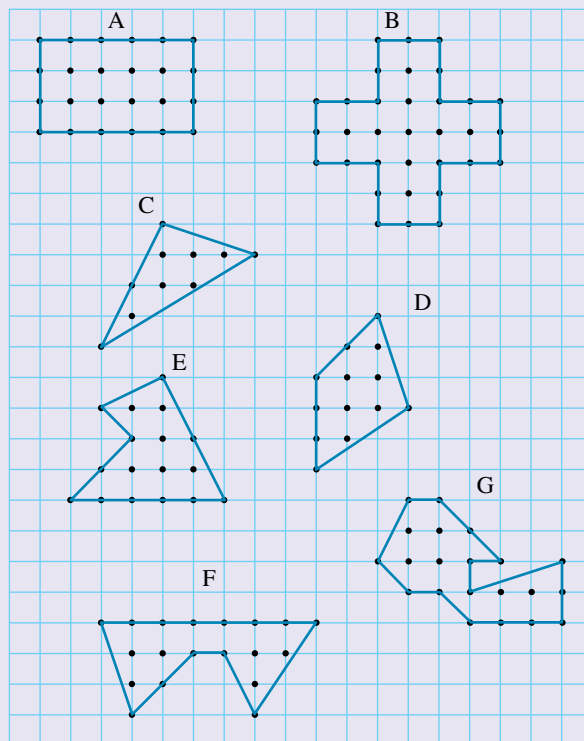
7.8 Review

Investigation | Rich task

Areas of polygons

The area of plane figures can be found using a formula. For example, if the figure is a rectangle, its area is found by multiplying its length by its width. If the figure is a triangle, the area is found by multiplying half the base length by its perpendicular height. If the figures are complex, break them down into simple shapes and find the area of these shapes. The area of a complex shape is the sum of the area of its simple shapes. Drawing polygons onto grid paper is one method that can be used to determine their area.

Consider the following seven polygons drawn on 1-cm grid paper.



1. By counting the squares or half-squares, determine the area of each polygon in cm^2 . In some cases, it may be necessary to divide some sections into half-rectangles in order to determine the exact area.

Formulas define a relationship between dimensions of figures. In order to search for a formula to find the area of polygons drawn on the grid paper, consider the next question.

2. For each of the polygons provided, complete the table on the next page. Count the number of dots on the perimeter of each polygon and count the number of dots that are within the perimeter of each polygon.

Polygon	Dots on perimeter (b)	Dots within perimeter (i)	Area of polygon (A)
A			
B			
C			
D			
E			
F			
G			

- Choose a pronumeral to represent the headings in the table. Investigate and determine a relationship between the area of each polygon and the dots on and within the perimeter. Test that the relationship determined works for each polygon. Write the relationship as a formula.
- Draw some polygons on a sheet of grid paper. Use your formula to determine the area of each shape. Confirm that your formula works by counting the squares in each polygon. The results of both methods (formula and method) should be the same.

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Digital doc: Investigation — Areas of polygons (doc-15906)

Digital doc: Code puzzle — When is a wall absolutely plumb? (doc-15907)

Digital doc: Summary Topic 7 (doc-22889)

Interactivity: Crossword Topic 7 (int-2709)

Interactivity: Word search Topic 7 (int-2708)

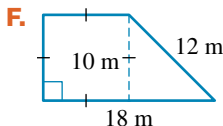
Interactivity: Sudoku Topic 7 (int-3211)

Exercise 7.8 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

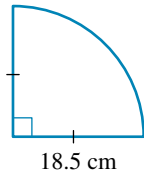
- MC** Which of the following is true?
 - 5 kg is 100 times as big as 5 g.
 - 5 metres is 100 times as big as 5 cm.
 - 5 km is 1000 times as small as 5 m.
 - 5 MB is 1000 times as small as 5 kB.
 - 5 MB is 1000 times as big as 5 TB.
- MC** The circumference of a circle with a diameter of 12.25 cm is:
 - 471.44 cm
 - 384.85 mm
 - 76.97 cm
 - 117.86 cm
 - 471.44 m
- MC** The area of the following shape is:
 - 216 m²
 - 150 m²
 - 140 m²
 - 90 m²
 - 80 m²



- MC** The area of a circle with diameter 7.5 cm is:
 - 176.7 cm²
 - 47.1 cm²
 - 23.6 cm²
 - 44.2 cm²
 - 128.2 cm²

5. **MC** The perimeter of the shape shown is:

- A. 1075.2 cm B. 55.5 cm C. 153.2 cm D. 49.3 cm
E. 66.1 cm F.



6. **MC** The surface area and volume of a cube with side length 7 m are respectively:

- A. 294 m^2 , 343 m^3 B. 49 m^2 , 343 m^3 C. 147 m^2 , 49 m^3 D. 28 m^2 , 84 m^3 E. 44 m^2 , 70 m^3

7. **MC** The surface area of a rectangular box with dimensions 7 m, 3 m and 2 m is:

- A. 41 m^2 B. 42 m^2 C. 72 m^2 D. 82 m^2 E. 53 m^2

8. **MC** The surface area and volume of a cylinder with radius 35 cm and height 40 cm are:

- A. 16493.36 cm^2 and 153938 cm^3 B. 8796.5 cm^2 and 11246.5 cm^3
C. 153938 cm^2 and 11246.5 cm^3 D. 8796.5 cm^2 and 153938 cm^3
E. 8796.5 cm^2 and 16493.36 cm^3

9. Fill in the gaps.

- a. $26 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$ b. $1385 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$
c. $1.63 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$ d. $1.5 \text{ kg} = \underline{\hspace{2cm}} \text{ g}$
e. $200000 \text{ kB} = \underline{\hspace{2cm}} \text{ MB}$ f. $2850 \text{ TB} = \underline{\hspace{2cm}} \text{ GB}$

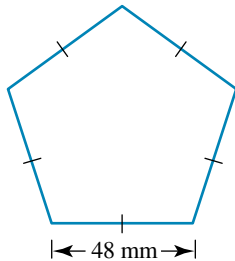
10. Calculate the cost of 1.785 km of cable if the cable costs \$4.20 per metre.

11. Calculate the circumferences of circles with the following dimensions (correct to 1 decimal place).

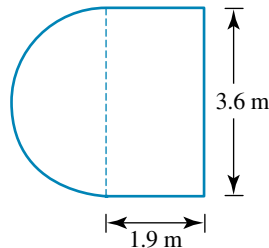
- a. radius 4 cm b. radius 5.6 m c. diameter 12 cm

12. Calculate the perimeters of the following shapes (correct to 1 decimal place).

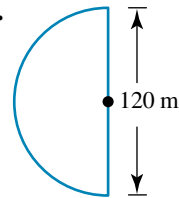
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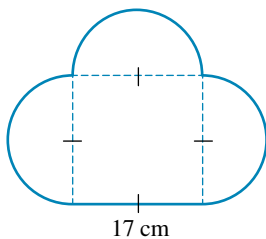
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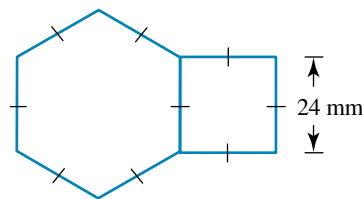
c.



d.

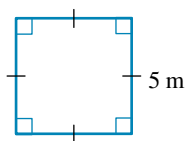


e.

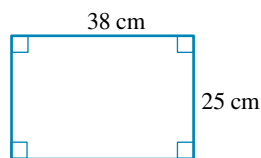


13. Calculate the areas of the following shapes.

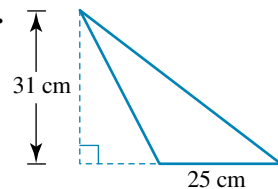
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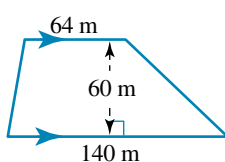
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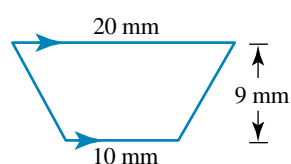
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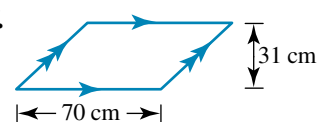
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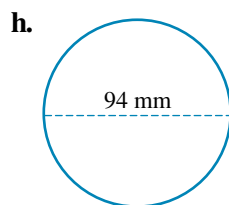
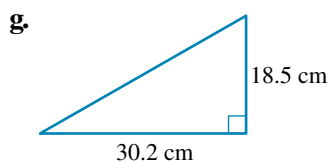


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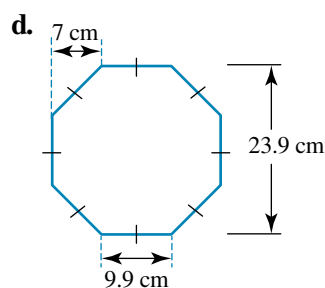
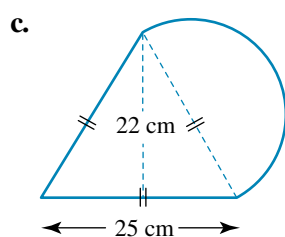
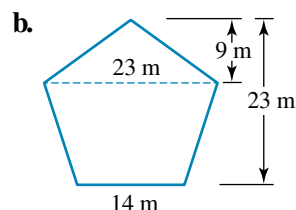
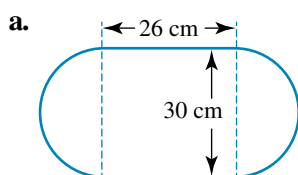


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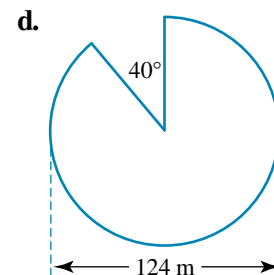
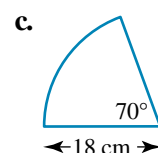
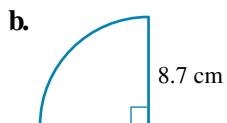
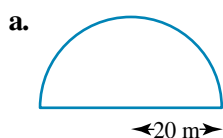




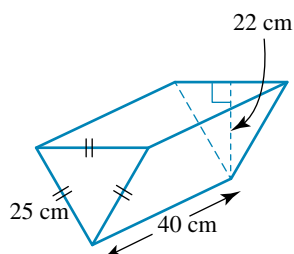
- 14.** Calculate the areas of the following 2-dimensional shapes by dividing them into simpler shapes.
(Where necessary, express your answer correct to 1 decimal place.)



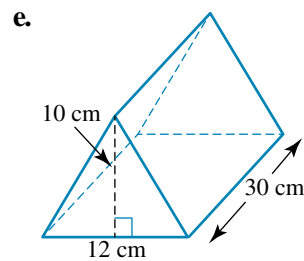
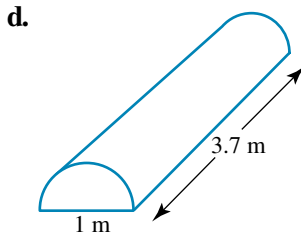
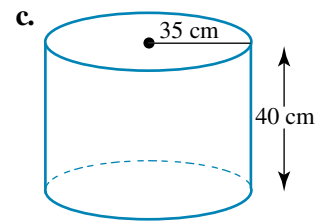
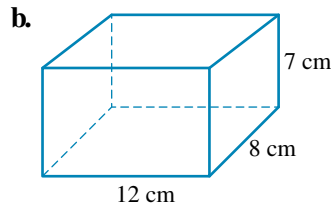
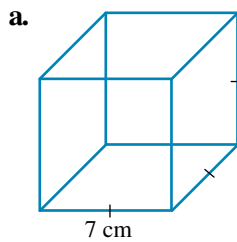
- 15.** Calculate the areas of the sectors below (correct to 2 decimal places).



- 16.** Calculate the perimeters of the sectors in question 15, correct to 2 decimal places.
- 17.** Calculate the inner surface area of the grape-collecting vat using the dimensions for length, width and depth shown.
- 18.** Calculate the surface area of the triangular prism shown.

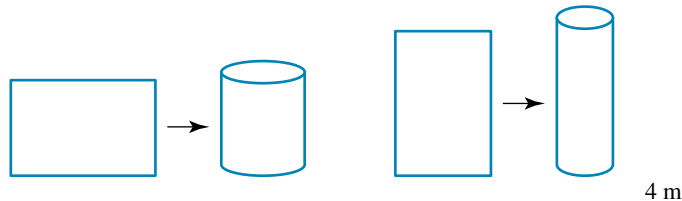


19. Calculate the volume of each of the following, giving your answers to 2 decimal places where necessary.



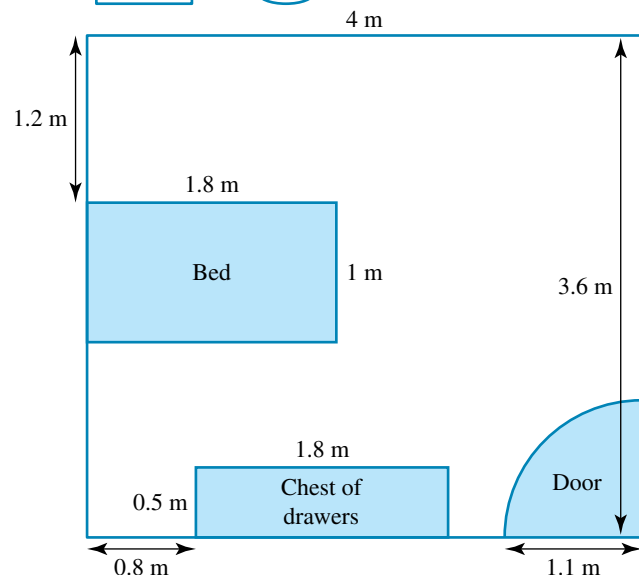
Communicating, reasoning and problem solving

20. A cylindrical water tank has a radius of 2.2 m and a height of 1.6 m. Calculate the cost of painting the outer surface (including the lid) of the tank if the paint costs \$1.90 per square metre.
21. A harvester travels at 11 km/h. The comb (harvesting section) is 8.7 m wide. How many hectares per hour can be harvested with this comb?
22. A block of chocolate is in the shape of a triangular prism. The face of the prism is an equilateral triangle of 5-cm sides, and the length is 22 cm.
- Determine the area of one of the triangular faces.
 - Calculate the total surface area of the block.
 - Calculate the volume of the block.
 - If it cost \$0.025 per cm^3 to produce the block, what is the total production cost?
23. An A4 sheet of paper has dimensions 210 mm $\times$ 297 mm and can be rolled two different ways (by rotating the paper) to make baseless cylinders as illustrated below. Which cylinder has the greater volume?

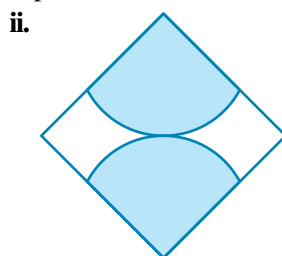
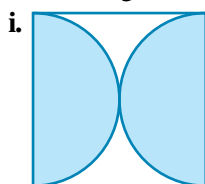


24. Nina decides to invite some friends for a sleepover. She plans to have her guests sleep on inflatable mattresses with dimensions 60 cm by 170 cm. A plan of Nina's bedroom is shown below.

- What area of floor space is available?
- The area of an inflatable mattress is 1.02 m^2 . Nina's best friend suggests that by dividing the available floor space by the area of an inflatable mattress the number of mattresses that can fit into the bedroom can be calculated. Is the friend correct?



- c. How many self-inflatable mattresses can fit into the room as it is?
 - d. How would this change if the bed could be moved within the bedroom?
25. These congruent squares have shaded parts of circles inside them.



Compare the shaded area in figure **i** with that in figure **ii**.

26. An isosceles triangle has its base on the side of a square. The areas of the triangle and the square are equal. What is the length of one of the equal sides of the isosceles triangle in terms of the length of one side of the square? Give an exact answer.
27. The widespread manufacture of soft drinks in cans only really started in the 1960s. Today cans of soft drink are very common. They can be purchased in a variety of sizes. You can also purchase them as single cans or bulk-buy them in larger quantities. Does the arrangement of the cans in a multi-pack affect the amount of packaging used to wrap them?

For this investigation we will look at the packaging of 375-mL cans. This can has a radius of approximately 3.2 cm and a height of approximately 12 cm. We will consider packaging these cans in cardboard in the shape of a prism, ignoring any overlap for gluing.

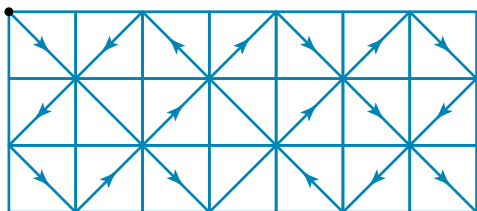
- a. A 6-can pack could be packed in a single row of cans.
 - i. Draw a diagram to show what this pack would look like.
 - ii. Calculate the amount of cardboard required for the package.
- b. The cans could also be packaged in a 3×2 arrangement.
 - i. Show the shape of this pack.
 - ii. Calculate the amount of cardboard required for this pack.
- c. Which of the two 6-can pack arrangements is more economical in terms of its packaging?
- d. These cans can be packaged as a single layer of 4×3 . How much cardboard would this require?
- e. They could also have a two-level arrangement of 3×2 each layer. What packaging would this require?
- f. It is also possible to have a three-level arrangement of 2×2 each layer. How much packaging would be needed in this case?
- g. Comment on the best way to package a 12-can pack so that it uses the minimum amount of wrapping. Provide diagrams and mathematical evidence to support your conclusion.

Answers

Topic 7 Area, surface area and volume

Exercise 7.2 Time scales and units of measurement

1. a. $300\,000 = 3.0 \times 10^5$ km/s
d. 9.4349×10^{12} km
2. a. 1.2×10^4
3. a. 50 mm
d. 32.2 mm
g. 0.01435 g
4. a. $5\,000\,000$ kB = 5000 MB
d. $1\,000$ GB = $1\,000\,000$ MB
5. a. 85.4 cm
6. a. 25.13 cm
d. 44.61 cm
7. B
8. a. 1800 mm, 2100 mm, 2400 mm, 2700 mm, 180 cm, 210 cm, 240 cm, 270 cm, 1.8 m, 2.1 m, 2.4 m, 2.7 m
b. 4
c. 3300 mm
9. a. 1990 m
b. 841 m
10. a. 127.12 cm
d. 8 m
g. 125.66 cm
j. 138 mm
11. 222.5 m
12. 37.5 km
13. 12 m
14. a. 3.6 kiloseconds
d. 6 teraseconds
15. Answers will vary.
16. a. Answers will vary.
17. a. 155.62 m
18. a. 40212 km
19. Yes, 42 cm of wire remains.
20. Several solutions are possible. An example is shown below.
To start, flip both timers over and put the pasta in the water.
When the four-minute timer runs out, flip it back over immediately. (Total time: 4 minutes)
When the seven-minute timer runs out, flip that back over immediately too. (Total time: 7 minutes)
The four-minute timer will run out again. Flip the *seven*-minute timer back over. (Total time: 8 minutes)
The seven-minute timer had only been running for a minute, so it will only run for a minute more before running out. (Total time: 9 minutes)
21. a. $(2\sqrt{13} + 4)$ m
b. $2\sqrt{17}$ m
22. 69.3 cm
23. This is the journey of the ball.



The ball travels along the diagonal of each of the squares. The length of each diagonal is $5\sqrt{2}$ cm.
Total length travelled = $21 \times 5\sqrt{2}$ cm = $105\sqrt{2}$ cm

24. Because the metric system has a base of 10, conversions between units involve multiplying or dividing by factors of 10. The prefix of the unit is also a clue. A **centimetre** is one-hundredth of a metre and a **millimetre** is one thousandth of a metre, while a **kilometre** is one thousand metres.

Exercise 7.3 Area of composite shapes

1. D
2. a. $13\,400\text{ m}^2 = 0.0134\text{ km}^2$ b. $0.04\text{ cm}^2 = 4\text{ mm}^2$ c. $3\,500\,000\text{ cm}^2 = 350\text{ m}^2$
 d. $0.005\text{ m}^2 = 50\text{ cm}^2$ e. $0.043\text{ km}^2 = 43\,000\text{ m}^2$ f. $200\text{ mm}^2 = 2\text{ cm}^2$
 g. $1.41\text{ km}^2 = 141\text{ ha}$ h. $3800\text{ m}^2 = 0.38\text{ ha}$
3. a. 24 cm^2 b. 16 mm^2 c. 537.5 cm^2
 d. 149.5 cm^2 e. 16.32 m^2 f. 11.25 cm^2
 g. 292.5 cm^2 h. 2.5 cm^2 i. 1250 m^2
 j. 50.27 m^2 k. 3.14 mm^2 l. 36.32 m^2
 m. 15.58 cm^2 n. 4.98 cm^2 o. 65 m^2
4. *Note: Answers may vary due to screen resolution.*
 a. 17.5 cm^2 b. 11.5 cm^2 c. 30.7 cm^2
 d. 19.6 cm^2 e. 19.4 cm^2 f. 25.0 cm^2
5. a. 6824.2 mm^2 b. 7.6 m^2 c. 734.2 cm^2
 d. 578.5 cm^2 e. 7086.7 m^2 f. 5.4 m^2
 g. 1143.4 m^2 h. 100.5 cm^2 i. 821 cm^2
 j. 661.3 mm^2
6. $\$29\,596.51$
7. a. 5000 km^2 b. 0.5%
8. $\$10\,150$
9. 378 cm^2
10. $\approx 19.3\text{ cm}^2$
11. a. 100.53 cm^2 b. 1244.07 m^2 c. 301.59 cm^2 d. 103.67 mm^2
12. C
13. 75.7 cm or 757 mm
14. a. 619.5 cm^2 b. 10.5 cm c. 346.36 cm^2 d. Answers will vary.
15. a. 1600 cm^2 b. 800 cm^2 c. 400 cm^2 d. 200 cm^2
 e. The area halves each time. f. 12.5% g. Answers will vary.
16. Answers will vary.
17. Answers will vary.
18. Answers will vary. The side length may be a rational number or a surd. It will be rational if x is a perfect square.
19. Answers will vary.
20. Check with your teacher.
21. a. $\sqrt{50} = 5\sqrt{2}\text{ mm}$ b. 10 logs
22. 17 m
23. a. Answers will vary. b. 45
24. a. $\frac{1}{4}$ b. $\frac{1}{4}(x^2 + 6x + 8)$ c. $\frac{1}{4}(3x^2 - 6x - 8)$
 d. i. $\frac{1}{12}(3x^2 - 6x - 8)$ ii. $\frac{1}{6}(3x^2 + 6x + 8)$
25. 45 m^2
26. It will be the length conversion factor squared (i.e. multiplied by itself).
27. $\frac{5}{8}$

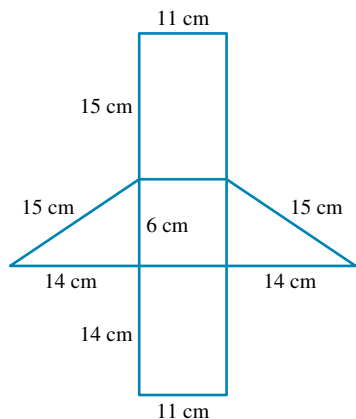
Exercise 7.4 Area and perimeter of semicircles and sectors

1. a. 14.14 cm^2 b. 157.08 cm^2 c. 27.71 cm^2 d. 226.19 mm^2
2. a. i. 14.3 cm ii. 12.6 cm^2
 b. i. 43.6 cm ii. 116.9 cm^2

- c. i. 40.7 m ii. 102.1 m²
 d. i. 5.4 m ii. 1.8 m²
3. E
4. a. i. 184.6 cm ii. 1869.2 cm²
 b. i. 66.8 m ii. 226.2 m²
 c. i. 27.4 cm ii. 42.4 cm²
 d. i. 342.1 m ii. 7243.6 m²
 e. i. 354.6 cm ii. 7645.9 cm²
 f. i. 104.4 cm ii. 201.1 cm²
5. 60 318.6 m²
 6. 303.4 cm
 7. 170.2 m²
 8. a. 14.8 m² b. 4.4%
 9. B
 10. a. 29.7 m b. 16.7 m c. 45.6 m
 11. a. 5027 m² b. 78.5% c. \$61 440
 12. a. Answers will vary. b. 85 m each c. 60 m each
 d. Answers will vary. e. 37 m
 13. a. Answers will vary. b. 88.4 cm² c. 50.3 cm²
 d. 109.4 cm² e. Jim
 14. Answers will vary.
 15. a. πr^2 cm² b. $\pi(r + 1)^2$ cm² c. Answers will vary.
 d. 5π cm² e. Answers will vary.
 16. Radius = π cm
 17. The length of the large arc equals the sum of the lengths of the two smaller arcs.
 18. They are the same fraction of the total circle circumference and circle area respectively, in a ratio of the sector angle: 360°

Exercise 7.5 Surface area of rectangular and triangular prisms

1. a. 66 cm² b. 62 m² c. 6.7 m²
 d. 4.44 m² e. 11 572.92 cm² f. 1.9 m²
2. a. 86 cm² b. 210.7 m² c. 8.37 cm²
3. a. 840 cm² b. 191.08 cm² c. 2370 mm²
4. 2 cans of paint
5. 77.76 cm² 6. 26.44 m² 7. 20.36 m²
 8. 261.5 cm² 9. 2.21 m² 10. 15.2 m²
 11. 2080 cm² 12. 168 cm² 13. 301.86 cm²
 14. a. 150 cm² b. 250 cm² c. 350 cm² d. 850 cm²
 15. Answers will vary. The solid formed is a cube with side length 4 cm.
 16. \$145.85 (2L of ceiling paint, 1 L + 4 L for walls)
 17. a. b. 315 cm²



18. a. 1402.52 m^2 b. 3220 m^3

19. Assuming all six sides are the same size, find the area of one side and multiply by 6.

Exercise 7.6 Surface area of cylinders and composite solids

1. a. 75.4 m^2 b. 28.3 cm^2 c. 2010.6 m^2
d. 1121.5 cm^2 e. 6.6 m^2 f. 25.6 m^2
2. a. 131.9 m^2 b. 84.8 cm^2 c. 3619.1 m^2
d. 1575.5 cm^2 e. 9.7 m^2 f. 61.8 m^2
3. Check with your teacher.
4. a. Asparagus is largest, beetroot is smallest. b. 118 cm^2
5. $\approx 1837.8 \text{ cm}^2$
6. $12\pi h^2$
7. $\approx 31.79 \text{ cm}$
8. a. $\approx 136.66 \text{ m}^2$ b. $\$7351.33$
9. a. $\approx 2035.75 \text{ cm}^2$ b. $\approx 1526.81 \text{ cm}^2$ c. $\approx 395.84 \text{ cm}^2$ d. $\approx 3958.40 \text{ cm}^2$
10. 154.73 cm^2
11. a. 3.82 cm b. 427.7 cm^2
12. Answers will vary.
13. 83.6 m^2
14. The area to be painted is 1.63 m^2 .
15. 11231.12 cm^2
16. 4.30 m^2
17. Answers will vary. However, you need to find the surface area of the cylindrical base/top, multiply it by 2 and add it to the area of the side, which will be the base/top circumference multiplied by the height of the cylinder.

Exercise 7.7 Volume of prisms, cylinders and composite solids

1. a. i. 325 mL ii. 2600000 mL iii. 5100 mL iv. 630000 mL
b. i. 5.8 cm^3 ii. 6100 cm^3 iii. 3200000 cm^3 iv. 0.0593 cm^3
c. i. 0.358 kL ii. 55.8 kL iii. 8.752 kL iv. 5300 kL
2. a. 36 cm^3 b. 15 cm^3 c. 72 cm^3
3. a. 12 cm^3 b. 68 cm^3 c. 9.6 m^3 d. 630 mm^3
e. 420 cm^3 f. $\approx 3152.7 \text{ mm}^3$ g. $\approx 1319.5 \text{ mm}^3$ h. $\approx 523.6 \text{ m}^3$
i. $\approx 10.5 \text{ m}^3$
4. a. 36 cm^3 b. 30 m^3 c. 1.144 m^3
5. a. 7020 cm^3 b. 6.91 m^3 c. 300 m^3 d. 8806 cm^3
6. a. $16\,085.0 \text{ m}^3$ b. 4766.6 cm^3 c. 2.3 m^3 d. 30.8 m^3
e. 30.6 m^3 f. 56.5 mm^3
7. $14\,476.5 \text{ L}$
8. 176.4 L
9. a. $158\,169 \text{ cm}^3$ b. 4092 cm^3 c. 2639 cm^3 d. 641 cm^3
e. 1784 cm^3 f. 1057 cm^3
10. $75\,322.8 \text{ L}$
11. 1145.1 mL
12. $2.916 \times 10^9 \text{ m}^3$
13. a. 4.95 m^3 b. 4950 L
14. a. 45.94 cm^3 b. 2400 (to the nearest whole number)
15. $234\,256.54 \text{ cm}^3$
16. a. 1.60 L b. 0.042 L
17. a. 1.1475 m^3 b. 840 L
18. Answers will vary.

19. a. Prism
d. i. 36 h 40 min
20. Answers will vary.
21. Answers will vary.
22. 23
23. a. 169 cm^3
b. $V = x(l-2x)(w-2x)$
24. Some people think that a cylinder is a prism because it has a uniform cross-section. However, prisms have flat faces, so a cylinder is not a prism.
25. Increase side length to 12.6 cm.

Investigation | Rich task

1. A. 15 cm^2
E. 10.5 cm^2
B. 20 cm^2
F. 11.5 cm^2
C. 6 cm^2
G. 12.5 cm^2
D. 8.5 cm^2

2.

Polygon	Dots on perimeter (b)	Dots within perimeter (i)	Area of polygon (A)
A	16	8	15
B	24	9	20
C	4	6	6
D	7	6	8.5
E	11	6	10.5
F	13	6	11.5
G	15	6	12.5

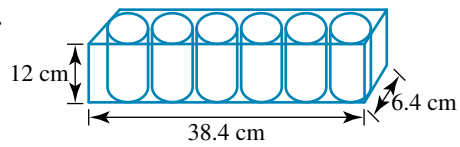
3. Answers will vary.
4. Answers will vary.

Exercise 7.8 Review questions

1. B
5. E
9. a. $26 \text{ mm} = 2.6 \text{ cm}$
d. $1.5 \text{ kg} = 1500 \text{ g}$
10. \$7497
11. a. 25.1 cm
12. a. 240 mm or 24 cm
d. 97.1 cm
13. a. 25 m^2
e. 135 mm^2
14. a. 1486.9 cm^2
15. a. 628.32 m^2
16. a. 102.83 m
17. $48\,920 \text{ cm}^2$ or 4.892 m^2
18. 3550 cm^2
19. a. 343 cm^3
d. 1.45 m^3
20. \$99.80
21. 9.57 hectares
22. a. 10.83 cm^2
23. The cylinder of height 210 mm has the greater volume.
24. a. 10.75 m^2
d. 9 mattresses could fit into the room.
25. The shaded areas are the same area.
2. B
6. A
b. $1385 \text{ mm} = 138.5 \text{ cm}$
e. $200\,000 \text{ kB} = 200 \text{ MB}$
b. 35.2 m
b. 13.1 m
e. 192 mm or 19.2 cm
b. 950 cm^2
f. 2170 cm^2
b. 362.5 m^2
b. 59.45 cm^2
b. 31.07 cm
b. 672 cm^3
e. 1800 cm^3
3. C
7. D
c. $1.63 \text{ cm} = 16.3 \text{ mm}$
f. $2850 \text{ TB} = 2\,850\,000$
c. 37.7 cm
c. 308.5 m
c. 387.5 cm^2
g. 279.4 cm^2
c. 520.4 cm^2
c. 197.92 cm^2
c. 57.99 cm
c. $153\,938.04 \text{ cm}^3$
4. D
8. A
d. 6120 m^2
h. 6939.8 mm^2
d. 473.2 cm^2
d. $10\,734.47 \text{ m}^2$
d. 470.27 m
d. 6120 m^2
h. 6939.8 mm^2
d. 473.2 cm^2
d. $10\,734.47 \text{ m}^2$
d. 470.27 m
d. \$5.96
c. 8

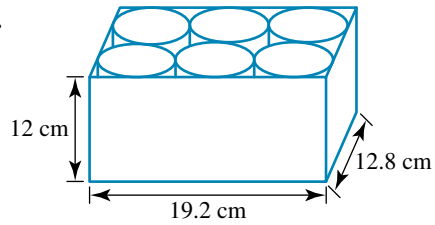
26. $\frac{\sqrt{17}}{2}$ times the side length of the square

27. a. i.



ii. 1566.72 cm²

b. i.



ii. 1259.52 cm²

c. 3×2 arrangement

d. 2058.24 cm²

e. 2027.52 cm²

f. Yes, 2170.88 cm²

g. The smallest amount of packaging required is 3×2 , with two levels of packaging.

TOPIC 8

Algebraic techniques 1

[Stage 5.2]

8.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

8.1.1 Why learn this?

Algebra relates to you and the world around you. It is part of everyday life and you will be using it without knowing it. If you want to be a scientist or engineer, you will certainly need algebra. If you want to do well in Mathematics, you will need to study algebra. When you learn algebra you learn a different way of thinking. It helps with problem solving, decision making, reasoning and creative thinking.



DISCUSSION

Can you think of any technological advancement that didn't need the input of mathematicians?

LEARNING SEQUENCE

- 8.1 Overview
- 8.2 [Stage 4] Variables
- 8.3 [Stage 4] Algebra in worded problems
- 8.4 [Stage 4] Simplifying algebraic expressions
- 8.5 [Stage 5.2] Simplifying algebraic fractions 1
- 8.6 [Stage 5.2] Expanding algebraic expressions
- 8.7 [Stage 5.2] The highest common factor
- 8.8 [Stage 5.2] Factorising monic quadratics
- 8.9 Review

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2 1WM
- constructs arguments to prove and justify results MA5.2 3WM
- simplifies algebraic fractions, and expands and factorises quadratic expressions MA5.2 6NA

CONTENT DESCRIPTIONS

Students:

Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)

Apply the four operations to algebraic fractions with pronumerals in the denominator (NSW)

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)

Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)

Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

8.2 Variables

[Stage 4]

8.2.1 Expressions

- A **pronumeral** is a letter or symbol which represents a number. Algebra is the branch of mathematics where we use and manipulate pronumerals.
- In algebra an **expression** is a group of **terms** in which each term is separated by a + sign or a – sign.
- Here are some important things to note about the expression $5x - 3y + 2$:
 - The expression has 3 terms and is called a **trinomial**.
 - The pronumerals x and y may take different values and are called **variables**.
 - The term 2 has only one value and is called a **constant**.
 - The **coefficient** (number in front) of x is 5, and the coefficient of y is -3 .
- An expression with 2 terms is called a binomial, and an expression with 1 term is called a monomial. A polynomial is an expression with many terms.

WORKED EXAMPLE 1

For the expression $6x - 3xy + z + 2 + x^2z + \frac{y^2}{7}$, determine:

- the number of terms
- the coefficient of the second term
- the coefficient of the last term
- the constant term
- the term with the lowest coefficient
- the coefficient of x^2z .

THINK

- Count the number of terms.
- The second term is $-3xy$. The number part is the coefficient.
- The last term is $\frac{y^2}{7}$. This can be rewritten as $\frac{1}{7} \times y^2$.
- The constant is the term with no pronumeral.
- Identify the lowest coefficient and write the whole term to which it belongs.
- x^2z can be written as $1x^2z$.

WRITE

- There are 6 terms.
- The coefficient of the second term is -3 .
- The coefficient of the last term is $\frac{1}{7}$.
- The constant term is 2.
- The term with the lowest coefficient is $-3xy$.
- The coefficient of x^2z is 1.

8.2.2 Substitution of integers

- We can evaluate (find the value of) an algebraic expression if we replace the pronumerals with their known values.
- This process is called substitution.
- Consider the expression $4x + 3y$. If we substitute the values $x = 2$ and $y = 5$, then the expression takes the value

$$4 \times 2 + 3 \times 5 = 8 + 15 \\ = 23$$

- Rather than showing the multiplication signs, it is common in mathematics to write the substituted values in brackets.
- We would write the example above as:

$$4x + 3y = 4(2) + 3(5) \\ = 8 + 15 \\ = 23$$

WORKED EXAMPLE 2

Evaluate the following terms for the values $x = 4$ and $x = -4$.

a $3x^2$

b $-x^2$

c $\sqrt{5-x}$

THINK

- a** 1 Substitute 4 for x .
 2 Only the x term is squared.
 3 Evaluate.
 4 Repeat for $x = -4$.

WRITE

a If $x = 4$: $3x^2 = 3(4)^2$
 $= 3 \times 16$
 $= 48$

If $x = -4$: $3x^2 = 3(-4)^2$
 $= 3 \times 16$
 $= 48$

- b** 1 Substitute 4 for x .
 2 Evaluate.
 3 Repeat for $x = -4$.

b If $x = 4$: $-x^2 = -(4)^2$
 $= -16$

If $x = -4$: $-x^2 = -(-4)^2$
 $= -16$

- c** 1 Substitute 4 for x .
 2 Evaluate.

c If $x = 4$: $\sqrt{5-x} = \sqrt{5-4}$
 $= \sqrt{1}$
 $= 1$

- 3 Repeat for $x = -4$.

If $x = -4$: $\sqrt{5-x} = \sqrt{5-(-4)}$
 $= \sqrt{5+4}$
 $= \sqrt{9}$
 $= 3$

WORKED EXAMPLE 3

If $x = 3$ and $y = -2$, evaluate the following expressions.

a $3x + 2y$

b $5xy - 3x + 1$

c $2x^2 + y^2$



THINK

- a** 1 Write the expression.
 2 Substitute $x = 3$ and $y = -2$.
 3 Evaluate.
- b** 1 Write the expression.
 2 Substitute $x = 3$ and $y = -2$.
 3 Evaluate.
- c** 1 Write the expression.
 2 Substitute $x = 3$ and $y = -2$.
 3 Evaluate.

WRITE

a $3x + 2y$
 $= 3(3) + 2(-2)$
 $= 9 - 4$
 $= 5$

b $5xy - 3x + 1$
 $= 5(3)(-2) - 3(3) + 1$
 $= -30 - 9 + 1$
 $= -38$

c $2x^2 + y^2$
 $= 2(3)^2 + (-2)^2$
 $= 18 + 4$
 $= 22$

8.2.3 Substitution into formulas

- A **formula** is a mathematical rule, usually written using variables.
- We know that the formula for the area of a rectangle is given by:

$$A = lw$$

where A represents the area of the rectangle, l represents the length and w represents the width.

- If a rectangular kitchen tile has length $l = 20$ cm and width $w = 15$ cm, we can substitute these values into the formula to find its area.

$$\begin{aligned} A &= lw \\ &= 20 \times 15 \\ &= 300 \text{ cm}^2 \end{aligned}$$

- If we are given the area of the tile to be 400 cm^2 and the width to be 55 cm, then we can calculate its length.

$$\begin{aligned} A &= lw \\ 400 &= l \times 55 \\ l &= \frac{400}{55} \\ &= \frac{80}{11} \\ &\approx 7.3 \text{ cm (to 1 d.p.)} \end{aligned}$$



WORKED EXAMPLE 4

The formula for the voltage in an electrical circuit can be found using the formula known as Ohm's Law:

$$V = IR$$

where I = current in amperes
 R = resistance in ohms
 V = voltage in volts.

a Calculate V when:

i $I = 2$ amperes, $R = 10$ ohms

ii $I = 20$ amperes, $R = 10$ ohms.

b Calculate I when $V = 300$ volts and $R = 600$ ohms.

THINK

a i 1 Write the formula.

2 Substitute $I = 2$ and $R = 10$.

3 Evaluate and express the answer in the correct units.

ii 1 Write the formula.

2 Substitute $I = 20$ and $R = 10$.

3 Evaluate and express the answer in the correct units.

b 1 Write the formula.

2 Substitute $V = 300$ and $R = 600$.

3 Evaluate and express the answer in the correct units.

WRITE

a i $V = IR$

$$= (2)(10)$$

$$= 20$$

The voltage is 20 volts.

ii $V = IR$

$$= (20)(10)$$

$$= 200$$

The voltage is 200 volts.

b $V = IR$

$$300 = I(600)$$

$$I = \frac{300}{600}$$
$$= 0.5$$

The current is 0.5 amperes.

Note: Even if we know nothing about volts, amperes and ohms, we can still do the calculation.

learnON RESOURCES — ONLINE ONLY

Interactivity: Substituting positive and negative numbers (int-3765)

eLesson: Substitution (eles-1892)

Digital doc: SkillsHEET Alternative expressions used for the four operations (doc-6122)

Exercise 8.2 Variables

Individual pathways

PRACTISE

Questions:

1, 2a–h, 3a–f, 4–6, 8, 9a–g,
10–12, 20

CONSOLIDATE

Questions:

1, 2e–l, 3c–h, 4–8, 9d–k, 13–16,
18, 20

MASTER

Questions:

1, 2i–o, 3e–j, 4–6, 8, 9j–o, 15–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Find the coefficient of each of the following terms.

a. $3x$

b. $7a$

c. $-2m$

d. $-8q$

e. w

f. $-n$

g. $\frac{x}{3}$

h. $\frac{y}{2}$

i. $-\frac{at}{4}$

j. $-\frac{r^2}{9}$

2. State the coefficient of the x term in each of the following expressions.

a. $6x - 3y$

b. $5 + 7x$

c. $5x^2 + 3x - 2$

d. $-7x^2 - 2x + 4$

e. $3x - 2x^2$

f. $-9x^2 - 2x$

g. $5x^2 + 3 - 7x$

h. $-11x + 5 - 2x^2$

i. $2ax^3 - \frac{x}{2} + x^2$

j. $1 - \frac{2x}{3} - bx^2$

k. $\frac{x^2}{6} + \frac{5}{7} + \frac{x}{4}$

l. $x^3 + x + 4$

m. $-3x - 4bx + 6$

n. $4cx^2 - 2x + 4a$

o. $2x^2 - 5$

3. **WE1** For each expression below, write:

i. the number of terms

ii. the coefficient of the first term

iii. the constant term

iv. the term with the smallest coefficient.

a. $5x^2 + 7x + 8$

b. $-9m^2 + 8m - 6$

c. $5x^2y - 7x^2 + 8xy + 5$

d. $9ab^2 - 8a - 9b^2 + 4$

e. $11p^2q^2 - 4 + 5p - 7q - p^2$

f. $-9p + 5 - 7q^2 + 5p^2q + q$

g. $4a - 2 + 9a^2b^2 - 3ac$

h. $5s + s^2t + 9 + 12t - 3u$

i. $-m + 8 + 5n^2m + m^2 + 2n$

j. $7c^2d + 5d^2 + 14 - 3cd^2 - 2e$

4. **WE2** Evaluate the following expressions for $x = 1$, $x = 0$ and $x = -2$.

a. $5x + 2$

b. $x^2 + 5x + 2$

c. $x(x + 2)$

d. $x^2 + x - 2$

e. $\sqrt{x^2}$

f. $\frac{2}{x}$

5. **MC** a. If $x = -3$, then the value of $-5x - 3$ is:

A. -18

B. 12

C. 30

D. 18

E. 15

b. An expression that is a trinomial is:

A. $3x^2$

B. $3x$

C. $x + 5 + 2$

D. $x^2 - 7$

E. $x^2 + x + 2$

c. The expression $\sqrt{x - 6}$ cannot be evaluated if x is equal to:

A. 2

B. 6

C. 7

D. 10

E. 36

d. In the expression $5x^2 - 3xy + 0.5x - 0.3y - 5$, the smallest coefficient is:

A. 5

B. -3

C. 0.5

D. -0.3

E. -5

6. **WE3** Find the value of the following expressions if $x = 2$, $y = -1$ and $z = 3$.

a. $2x$

b. $3xy$

c. $2y^2z$

d. $\frac{14}{x}$

e. $6(2x + 3y - z)$

f. $x^2 - y^2 + xyz$

7. If $x = 4$ and $y = -3$, evaluate the following expressions.

a. $4x + 3y$

b. $3xy - 2x + 4$

c. $x^2 - y^2$

8. **WE4** The change in the voltage in an electrical circuit can be found using the formula known as Ohm's Law: $V = IR$, where I = current in amperes, R = resistance in ohms and V = voltage in volts.

a. Calculate V when:

i. $I = 4$, $R = 8$

ii. $I = 25$, $R = 10$.

b. Calculate R when:

i. $V = 100$, $I = 25$

ii. $V = 90$, $I = 30$.

9. Evaluate each of the following by substituting the given values into each formula.

a. If $A = bh$, find A when $b = 5$ and $h = 3$.

b. If $d = \frac{m}{v}$, find d when $m = 30$ and $v = 3$.

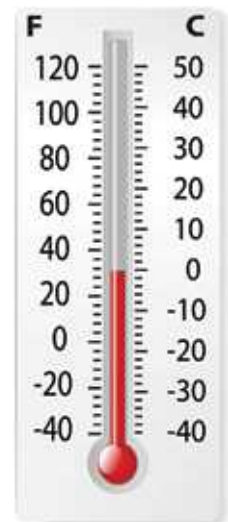
c. If $A = \frac{1}{2}xy$, find A when $x = 18$ and $y = 2$.

d. If $A = \frac{1}{2}(a + b)h$, find A when $h = 10$, $a = 7$ and $b = 2$.

- e. If $V = \frac{AH}{3}$, find V when $A = 9$ and $H = 10$.
- f. If $v = u + at$, find v when $u = 4$, $a = 3.2$ and $t = 2.1$.
- g. If $t = a + (n - 1)d$, find t when $a = 3$, $n = 10$ and $d = 2$.
- h. If $A = \frac{1}{2}(x + y)h$, find A when $x = 5$, $y = 9$ and $h = 3.2$.
- i. If $A = 2b^2$, find A when $b = 5$.
- j. If $y = 5x^2 - 9$, find y when $x = 6$.
- k. If $y = x^2 - 2x + 4$, find y when $x = 2$.
- l. If $a = -3b^2 + 5b - 2$, find a when $b = 4$.
- m. If $s = ut + \frac{1}{2}at^2$, find s when $u = 0.8$, $t = 5$ and $a = 2.3$.
- n. If $F = \frac{mp}{r^2}$, find F correct to 2 decimal places when $m = 6.9$, $p = 8$ and $r = 1.2$.
- o. If $C = \pi d$, find C correct to 2 decimal places when $d = 11$.

Communicating, reasoning and problem solving

10. The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the length of the base and h is the perpendicular height of the triangle.
- a. Show that the area is 12 cm^2 when $b = 6 \text{ cm}$ and $h = 4 \text{ cm}$.
- b. What is h if $A = 24 \text{ cm}^2$ and $b = 4 \text{ cm}$?
11. The formula to convert degrees Fahrenheit (F) to degrees Celsius (C) is $C = \frac{5}{9}(F - 32)$.
- a. Find C when $F = 59$.
- b. Show that when Celsius (C) is 15, Fahrenheit (F) is 59.
12. The length of the hypotenuse of a right-angled triangle (c) can be found using the formula $c = \sqrt{a^2 + b^2}$ where a and b are the lengths of the other two sides.
- a. Find c when $a = 3$ and $b = 4$.
- b. Show that when $a = 5$ and $c = 13$, $b = 12$.
13. If the volume of a prism (V) is given by the formula $V = AH$ where A is the area of the cross-section and H is the height of the prism, determine:
- a. V when $A = 7 \text{ cm}^2$ and $H = 9 \text{ cm}$
- b. H when $V = 120 \text{ cm}^3$ and $A = 30 \text{ cm}^2$.
14. Using $E = F + V - 2$ where F is the number of faces on a prism, E is the number of edges and V is the number of vertices, calculate:
- a. E if $F = 5$ and $V = 7$
- b. F if $E = 10$ and $V = 2$.



15. The kinetic energy (E) of an object is found by using the formula $E = \frac{1}{2}mv^2$ where m is the mass and v is the velocity of the object.
- a. Determine E when $m = 3$ and $v = 3.6$.
- b. Show that when $E = 25$ and $v = 5$, $m = 2$.

16. The volume of a cylinder (v) is given by $v = \pi r^2 h$ where r is the radius in centimetres and h is the height of the cylinder in centimetres.
- Determine v correct to 2 decimal places if $r = 7$ and $h = 3$.
 - Determine h correct to 2 decimal places if $v = 120$ and $r = 2$.
17. The surface area of a cylinder (S) is given by $S = 2\pi r(r + h)$ where r is the radius of the circular end and h is the height of the cylinder.
- Calculate S (to 2 decimal places) if $r = 14$ and $h = 5$.
 - Show that for a cylinder of surface area 240 units^2 and radius 5 units, the height is 2.64 units, correct to 2 decimal places.
18. a. In a magic square, every row, column and diagonal adds to the same number. This number is called the magic number for that square. Complete the unfinished magic square by first finding an algebraic expression for the magic number.

	$2d + 2g$	
	$d + g$	
	0	$d + 2g$

- Make three different magic squares by substituting different values for g and d in the completed magic square from part a.
 - Calculate the magic number for each magic square.
19. A flat, rectangular board is built by gluing together a number of square pieces of the same size. The board is m squares wide and n squares long. In terms of m and n , write expressions for:
- the total number of squares
 - the number of completely surrounded squares
 - the perimeter of the figure, given that each square is 1 unit by 1 unit.
20. Which one of the following eight expressions is not equivalent to the others?
- $x - y + z$
 - $z - y + x$
 - $z - (y - x)$
 - $-y + x + z$
 - $z + x - y$
 - $x - (y - z)$
 - $y - (z - x)$
 - $x + z - y$
21. Which letters should you avoid using when constructing algebraic expressions?

8.3 Algebra in worded problems

[Stage 4]

8.3.1 Algebra in worded problems

- An important skill in algebra is to be able to convert worded expressions and sentences into algebraic expressions.
- The first step in converting a worded statement into an algebraic expression is to assign pronumerals to the unknown quantities.

WORKED EXAMPLE 5

Write an algebraic expression for each of the following, choosing an appropriate pronumeral if necessary.

- a The number that is 6 more than Ben's age
- b The total value of some \$10 notes
- c The total cost of 8 adults' and 3 children's train tickets
- d The product of a and w
- e w less than a

THINK

- a 1 Ben's age is the unknown quantity.
Note: You must not say 'Let Ben = b '; b is a number.
- 2 Write an expression for Ben's age plus 6.
- b 1 The number of notes is the unknown quantity.
- 2 The value is 10 times the number of notes.
- c 1 The cost of tickets is the unknown quantity.
- 2 The adults' tickets cost $8 \times a$ and the children's tickets cost $3 \times c$.
- d 'Product' means to multiply.
- e w is subtracted from a .

WRITE

- a Let Ben's age = b years.
 $b + 6$
- b Let n = the number of \$10 notes.
Total value = $10n$ dollars
- c Let an adult's ticket cost a dollars and a child's ticket cost c dollars.
Total cost = $8a + 3c$ dollars.
- d $a \times w$
- e $a - w$

learnON RESOURCES — ONLINE ONLY

Digital doc: SkillsHEET Algebraic expressions (doc-6123)

Digital doc: SkillsHEET Substitution into algebraic expressions (doc-6124)

Exercise 8.3 Algebra in worded problems

Individual pathways

PRACTISE

Questions:
1–5, 7, 12, 14

CONSOLIDATE

Questions:
3–8, 10, 12, 14, 15

MASTER

Questions:
4–11, 13, 15–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. Note: Question numbers may vary slightly.

Understanding and fluency

- 1. Jacqueline studies 5 more subjects than Helena. How many subjects does Jacqueline study if:
 - a. Helena studies 6 subjects
 - b. Helena studies x subjects
 - c. Helena studies y subjects?

2. Dianne and Angela walk home from school together. Dianne's home is 2 km further from school than Angela's home. How far does Dianne walk if Angela's home is:

- a. 1.5 km from school
- b. x km from school?

3. Lisa watched television for 2.5 hours today. How many hours will she watch tomorrow if she watches:

- a. 1.5 hours more than she watched today
- b. t hours more than she watched today
- c. y hours fewer than she watched today?

4. **WE5** Write an algebraic expression for each of the following questions.

- a. If it takes 10 minutes to iron a single shirt, how long would it take to iron all of Anthony's shirts?
- b. Ross has 30 dollars more than Nick. If Nick has N dollars, how much money does Ross have?
- c. In a game of Aussie Rules football, Luciano kicked 4 more goals than he kicked behinds. (*Remember*: 1 goal scores 6 points, 1 behind scores 1 point.)
 - i. How many behinds did Luciano score if g is the number of goals kicked?
 - ii. How many points did Luciano score?

5. Jeff and Chris play Aussie Rules football for opposing teams, and Jeff's team won when the two teams played one another.

- a. How many points did Jeff's team score if they kicked:
 - i. 14 goals and 10 behinds
 - ii. x goals and y behinds?
- b. How many points did Chris's team score if his team kicked:
 - i. 10 goals and 6 behinds
 - ii. p goals and q behinds?
- c. How many points did Jeff's team win by if:
 - i. Chris's team scored 10 goals and 6 behinds, and Jeff's team scored 14 goals and 10 behinds
 - ii. Chris's team scored p goals and q behinds, and Jeff's team scored x goals and y behinds?

6. Yvonne's mother gives her x dollars for each school subject she passes. If she passes y subjects, how much money does she receive?

7. Brian buys a bag containing x Smarties.

- a. If he divides them equally among n people, how many does each person receive?
- b. If he keeps half the Smarties for himself and divides the remaining Smarties equally among n people, how many does each person receive?

8. A piece of licorice is 30cm long.

- a. If David cuts d cm off, how much licorice remains?
- b. If David cuts off $\frac{1}{4}$ of the remaining licorice, how much licorice has been cut off?
- c. How much licorice remains now?



Communicating, reasoning and problem solving

9. One-quarter of a class of x students plays tennis on the weekend. One-sixth of the class plays tennis and also swims on the weekend.

- a. Write an expression to represent the number of students playing tennis on the weekend.
- b. Write an expression to represent the number of students playing tennis and also swimming on the weekend.
- c. Show that the number of students playing only tennis on the weekend is $\frac{x}{12}$.



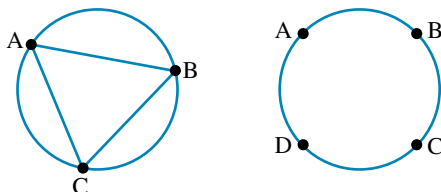
10. During a 24-hour period, Vanessa uses her computer for c hours. Her brother Darren uses it for $\frac{1}{7}$ of the remaining time.
 - a. For how long does Darren use the computer?
 - b. Show that the total number of hours that Vanessa and Darren use the computer during a 24-hour period can be expressed as $\frac{6c + 24}{7}$.
11. Marty had a birthday party last weekend and invited n friends. The table below indicates the number of friends at Marty's party at certain times during the evening. Everybody arrived by 8.30 pm and had all left the party by 11 pm.

Time	Number of friends
7.00 pm	$n - 24$
7.30 pm	$n - 23$
8.00 pm	$n - 8$
8.30 pm	$n - 5$
9.00 pm	$n - 5$
9.30 pm	$n - 7$
10.00 pm	$n - 12$
10.30 pm	$n - 18$
11.00 pm	$n - 24$

- a. Show that 1 person arrived between 7.00 pm and 7.30 pm.
 - b. Between which times were the most friends present at the party?
 - c. How many friends were invited but did not arrive?
 - d. Prove that 24 friends were invited in total.
 - e. Between which times did the most friends arrive?
12. A child builds a pyramid as shown.
 - a. Find a rule that gives the number of blocks on the bottom layer, b , for a tower that is h blocks high.
 - b. If the child wishes the tower to be 10 blocks high, how many blocks should she begin with on the bottom layer?

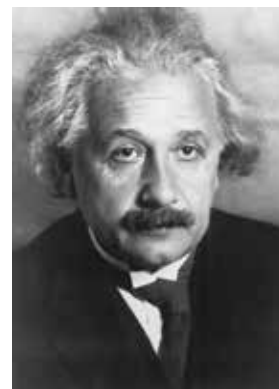


13. If a circle has three equally spaced dots (A, B and C) on its circumference, only 1 triangle can be formed by joining the dots. Investigate to determine the number of triangles that could be formed by 4, 5, ... equally spaced dots on the circumference.



From your investigation, can you see a pattern in the number of triangles (t) formed for a circle with d equally spaced dots on its circumference? Is it possible to express this as a formula?

14. A shop owner is ordering vases from a stock catalogue. Red vases cost \$4.20 and clear vases cost \$5.70. If 25 vases cost \$120, how many of each type of vase did she buy?
15. a. Investigate what the variables in the formula $E = mc^2$ mean.
b. Prepare a presentation about Albert Einstein and present this to the class.
16. a. Transpose the equation $T = \frac{x-3}{x-1}$ to make x the subject.
b. What is the value of T when $x = 1$? Explain.
c. What is the value of x when $T = 1$? Explain.
17. Why is it important to define pronumerals or variables in terms of what they represent?
18. A father is 4 times the age of his son. In 4 years time, the father will be 3 times the age of his son. How old is the father now?



8.4 Simplifying algebraic expressions

[Stage 4]

8.4.1 Addition and subtraction of like terms

- When we simplify an expression, we write it with the smallest number of terms, without using $\times$ or $\div$ signs.
- Like terms** have identical pronumeral parts.
For example: $3x$ and x are like terms, but $3x^2$ and x are not.
 $5a^2b$ and $-2ba^2$ are like terms, but $5a^2b$ and $-2b^2a$ are not.
 $6x + 5x$ can be simplified (to $11x$), but $6x + 5$ cannot.
- When there is more than one pronumeral in a term, it is preferable to write them in alphabetical order.
For example, $3xy$ is preferable to $3yx$, although these are identical terms. This makes it easy to recognise like terms.

WORKED EXAMPLE 6

Simplify the following expressions.

a $x + 2x + 3x - 5$

b $9a^2b + 5ba^2 + ab^2$

c $-12 - 4c^2 + 10 + 5c^2$

THINK

a 1 Write the expression.

2 Collect the like terms: $x, 2x, 3x$.

b 1 Write the expression.

2 Collect the like terms: $9a^2b, 5ba^2$.

c 1 Write the expression.

2 Collect the like terms.

3 Simplify.

WRITE

a $x + 2x + 3x - 5$

$$= 6x - 5$$

b $9a^2b + 5ba^2 + ab^2$

$$= 14a^2b + ab^2$$

c $-12 - 4c^2 + 10 + 5c^2$

$$= -12 + 10 - 4c^2 + 5c^2$$

$$= -2 + c^2 \text{ or } c^2 - 2$$

DISCUSSION

Why is it preferable to write variables in alphabetical order? How might this help with simplifying expressions?

8.4.2 Multiplication and division

Multiplication

- Any two algebraic terms can be multiplied together.
- Terms can be multiplied together in any order, so it is easier to deal with the coefficients first and then each pronumeral in turn.

$$\begin{aligned} 3xy \times 5y &= 3 \times x \times y \times 5 \times y \\ &= 3 \times 5 \times x \times y \times y \\ &= 15 \times x \times y^2 \\ &= 15xy^2 \end{aligned}$$

WORKED EXAMPLE 7

Simplify the following terms.

a $4a \times 2b \times a$

b $7ax \times -6bx \times -2abx$

THINK

a 1 Write the term.

2 Rearrange, writing the coefficients first.

3 Multiply the coefficients and pronumerals separately.

b 1 Write the term.

2 Rearrange, writing the coefficients first.

3 Multiply the coefficients and pronumeral separately.

WRITE

a $4a \times 2b \times a$

$$= 4 \times 2 \times a \times a \times b$$

$$= 8a^2b$$

b $7ax \times -6bx \times -2abx$

$$= 7 \times -6 \times -2 \times a \times a \times b \times b \times x \times x \times x$$

$$= 84a^2b^2x^3$$

Division

- Division is performed by writing the terms in fraction form and then cancelling to simplify.

$$\begin{aligned}\text{For example, } 10x \div 5x &= \frac{10x}{5x} \\ &= \frac{\overset{2}{\cancel{10}}\overset{1}{x}}{\underset{1}{\cancel{5}}\overset{1}{x}} \\ &= 2\end{aligned}$$

WORKED EXAMPLE 8

Simplify the following terms.

a $\frac{12xy}{4xz}$

b $-8ab^2 \div -16a^2b$

THINK

a 1 Write the term.

2 Cancel the common factors.

3 Write the answer.

b 1 Write the term as a fraction.

2 Cancel the common factors.

3 Simplify and write the answer.

WRITE

a $\frac{12xy}{4xz}$

$$= \frac{\overset{3}{\cancel{12}} \times \overset{1}{x} \times y}{\underset{1}{\cancel{4}} \times \overset{1}{x} \times z}$$

$$= \frac{3y}{z}$$

b $\frac{-8ab^2}{-16a^2b}$

$$= \frac{\overset{1}{\cancel{8}} \times \overset{1}{a} \times \overset{2}{b^2}}{\underset{2}{\cancel{16}} \times \underset{a}{a^2} \times \underset{1}{b}}$$

$$= \frac{1}{2} \times \frac{1}{a} \times \frac{b}{1}$$

$$= \frac{b}{2a}$$

learn on RESOURCES — ONLINE ONLY

Interactivity: Simplifying expressions (int-3770)

Interactivity: Multiplying variables (int-3772)

Interactivity: Dividing expressions with variables (int-3773)

eLesson: Simplification of expressions (eles-1884)

Digital doc: SkillsSHEET Like terms (doc-6125)

Digital doc: SkillsSHEET Collecting like terms (doc-6126)

Digital doc: SkillsSHEET Multiplying algebraic terms (doc-6127)

Digital doc: SkillsSHEET Dividing algebraic terms (doc-6128)

Digital doc: SkillsSHEET Adding and subtracting integers (doc-10817)

Digital doc: SkillsSHEET Multiplying and dividing integers (doc-10818)

Digital doc: WorkSHEET Introductory algebra (doc-6138)

Exercise 8.4 Simplifying algebraic expressions

Individual pathways

PRACTISE

Questions:

1, 2a–l, 3, 4a–i, 5a–l, 6, 7, 10, 12

CONSOLIDATE

Questions:

1, 2g–r, 3, 4g–o, 5d–o, 6–13

MASTER

Questions:

1, 2p–z, 3, 4k–r, 5h–r, 6, 8–11, 13, 14

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. For each of the following terms, select those terms listed in brackets that are like terms.

- a. $6ab$ ($7a$, $8b$, $9ab$, $-ab$, $4a^2b^2$)
 b. $-x$ ($3xy$, $-xy$, $4x$, $4y$, $-yx$)
 c. $3az$ ($3ay$, $-3za$, $-az$, $3z^2a$, $3a^2z$)
 d. x^2 ($2x$, $2x^2$, $2x^3$, $-2x$, $-x^2$)
 e. $-2x^2y$ (xy , $-2xy$, $-2xy^2$, $-2x^2y$, $-2x^2y^2$)
 f. $3x^2y^5$ ($3xy$, $3x^5y^2$, $3x^4y^3$, $-x^2y^5$, $-3x^2y^5$)
 g. $5x^2p^3w^5$ ($-5x^3w^5p^3$, $p^3x^2w^5$, $5xp^3w^5$, $-5x^2p^3w^5$, $w^5p^2x^3$)
 h. $-x^2y^5z^4$ ($-xy^5$, $-y^2z^5x^4$, $-x + y + z$, $4y^5z^4x^2$, $-2x^2z^4y^5$)

2. **WE6** Simplify the following expressions.

a. $5x + 2x$	b. $3y + 8y$	c. $7m + 12m$
d. $13q - 2q$	e. $17r - 9r$	f. $-x + 4x$
g. $5a + 2a + a$	h. $9y + 2y - 3y$	i. $7x - 2x + 8x$
j. $14p - 3p + 5p$	k. $2q^2 + 7q^2$	l. $5x^2 - 2x^2$
m. $6x^2 + 2x^2 - 3y$	n. $3m^2 + 2n - m^2$	o. $9x^2 + x - 2x^2$
p. $9h^2 - 2h + 3h + 9$	q. $-2g^2 - 4g + 5g - 12$	r. $-5m^2 + 5m - 4m + 15$
s. $12a^2 + 3b + 4b^2 - 2b$	t. $6m + 2n^2 - 3m + 5n^2$	u. $3xy + 2y^2 + 9yx$
v. $3ab + 3a^2b + 2a^2b - ab$	w. $9x^2y - 3xy + 7yx^2$	x. $4m^2n + 3n - 3m^2n + 8n$
y. $-3x^2 - 4yx^2 - 4x^2 + 6x^2y$	z. $4 - 2a^2b - ba^2 + 5b - 9a^2$	

3. **MC** For each of the following, choose the correct simplified expression.

- a. $18p - 19p$
 A. p B. $-p$ C. p^2 D. -1 E. $37p$
- b. $5x^2 - 8x + 6x - 9$
 A. $3x - 9$ B. $3x^2 - 9$ C. $5x^2 + 2x - 9$ D. $5x^2 - 2x - 9$ E. $5x^2 - 14x - 9$
- c. $12a - a + 15b - 14b$
 A. $11a + b$ B. 12 C. $11a - b$ D. $13a + b$ E. $a + b$
- d. $-7m^2n + 5m^2 + 3 - m^2 + 2m^2n$
 A. $-9m^2n + 4m^2 + 3$ B. $-9m^2n + 8$ C. $-5m^2n - 4m^2 + 3$
 D. $-5m^2n + 4m^2 + 3$ E. $9m^2n + 4m^2 + 3$

4. **WE7** Simplify the following.

a. $3m \times 2n$	b. $4x \times 5y$	c. $2p \times 4q$
d. $5x \times -2y$	e. $3y \times -4x$	f. $-3m \times -5n$
g. $5a \times 2a$	h. $4y \times 5y$	i. $5p \times p$
j. $m \times 7m$	k. $3mn \times 2p$	l. $-6ab \times b$
m. $-5m \times -2mn$	n. $-6a \times 3ab$	o. $-3xy \times -5xy \times 2x$
p. $4pq \times -p \times 3q^2$	q. $4c \times -7cd \times 2c$	r. $-3a^2 \times -5ab^3 \times 2ab^4$

5. **WE8** Simplify the following.

a. $\frac{6x}{2}$

b. $\frac{9m}{3}$

c. $\frac{12y}{6}$

d. $\frac{8m}{2}$

e. $12m \div 3$

f. $14x \div 7$

g. $-21x \div 3$

h. $-32m \div 8$

i. $\frac{4m}{8}$

j. $\frac{6x}{18}$

k. $\frac{8mn}{18n}$

l. $\frac{16xy}{12y}$

m. $\frac{2}{10m}$

n. $\frac{6ab}{12a^2b}$

o. $\frac{28xyz}{14x}$

p. $\frac{70ab^2}{4b}$

q. $\frac{2x^2yz}{8xz}$

r. $-7xy^2z^2 \div 11xyz$

6. Simplify the following.

a. $5x \times 4y \times 2xy$

b. $7xy \times 4ax \times 2y$

c. $x \times 4xy \times 3yx$

d. $\frac{6x^2y}{12y^2}$

e. $\frac{-15x^2ab}{12b^2x^2}$

f. $\frac{2p^3q^2}{p^3q^2}$

g. $-4a \times -5ab^2 \times 2a$

h. $-a \times 4ab \times 2ba \times b$

i. $2a \times 2a \times 2a \times 2a$

7. Jim buys m pens at p cents each and n books at q dollars each.

a. How much does he spend in:

i. dollars

ii. cents?

b. What is his change from \$20?

8. At a local discount clothing store, 4 shirts and 3 pairs of trousers cost \$138. If a pair of trousers cost 2.5 times as much as a shirt, determine the cost of each.

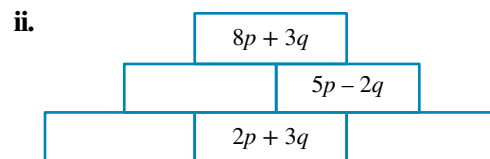
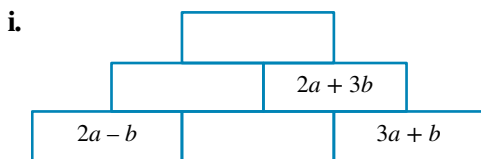
Communicating, reasoning and problem solving

9. Class 9A were given an algebra test. One of the questions is shown below.

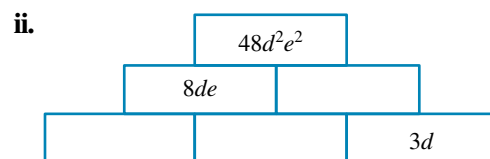
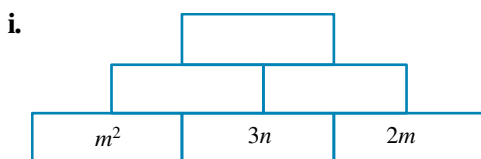
Simplify the following expression: $\frac{3ab}{2} \times \frac{4ac}{6b} \times 7c$.

Sean, who is a student in class 9A, wrote his answer as $\frac{12aabc}{12b} \times 7c$. Explain why Sean's answer is incorrect, and write the correct answer.

10. a. Fill in the empty bricks in the following pyramids. The expression in each brick is obtained by adding the expressions in the two bricks below it.



b. Fill in the empty bricks in the following pyramids. The expression in each brick is obtained by multiplying the expressions in the two bricks below it.



11. A rectangular chocolate block has dimensions x and $(4x - 7)$ as shown.

a. Write an expression for the perimeter (P) in the form

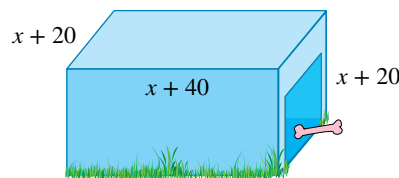
$$P = \_\_x + \_\_(4x - 7).$$

b. Expand and simplify this equation.

c. Calculate the perimeter of the chocolate block when $x = 3$.

d. Explain why x cannot equal 1.

12. A dog house in the shape of a rectangular prism is to be reinforced with steel edging along its outer edges. The cost of the steel edging is \$1.50 per metre. All measurements are in centimetres.

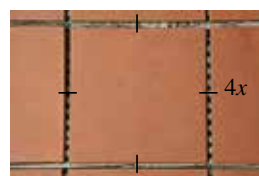
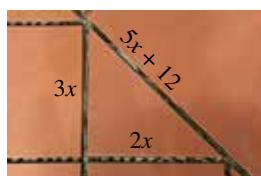


a. Write an expression for the total length of the straight edges of the frame.

b. Expand the expression.

c. Calculate the cost of steel needed when $x = 80$ cm.

13. A tile manufacturer produces tiles that have the following side lengths. All measurements are in centimetres.



a. Write an expression for the perimeter of each shape.

b. Calculate the value of x for which the perimeter of the triangular tile is the same as the perimeter of the square tile.

14. Is the expression ab the same as ba ? Explain.



8.5 Simplifying algebraic fractions 1

[Stage 5.2]

8.5.1 Adding and subtracting algebraic fractions

- Algebraic fractions contain pronumerals that may represent particular numbers or changing values.
- Simplifying algebraic fractions follows the same principles as that used for simplifying numerical fraction expressions such as $\frac{2}{5} + \frac{5}{7}$ or $\frac{3}{4} \div \frac{4}{5}$. That is, find a common denominator for addition and subtraction and use reciprocals for division.
- To add or subtract algebraic fractions we perform the following steps.
 - Step 1 Find the lowest common denominator (LCD) by finding the lowest common multiple (LCM) of the denominators.
 - Step 2 Rewrite each fraction as an equivalent fraction with this common denominator.
 - Step 3 Add (or subtract) the new numerators.
 - Step 4 Simplify.

WORKED EXAMPLE 9

Simplify each of the following expressions.

a $\frac{x}{2} + \frac{x}{5}$

b $\frac{y}{3} - \frac{2y}{7}$

c $\frac{y+1}{5} + \frac{x+y}{6}$

THINK

- a 1 Write the expression.
- 2 Find the lowest common denominator (LCD).
The lowest common multiple (LCM) of 2 and 5 is 10.
- 3 Add the numerators.
- b 1 Write the expression.
- 2 Find the LCD. The LCM of 3 and 7 is 21.
- 3 Subtract the numerators.
- c 1 Write the expression.
- 2 Find the LCD. The LCM of 5 and 6 is 30.
- 3 Add the numerators.
- 4 Expand the grouping symbols in the numerator.
- 5 Simplify the numerator by collecting like terms.

WRITE

a $\frac{x}{2} + \frac{x}{5}$

$$= \frac{5x}{10} + \frac{2x}{10}$$

$$= \frac{7x}{10}$$

b $\frac{y}{3} - \frac{2y}{7}$

$$= \frac{7y}{21} - \frac{6y}{21}$$

$$= \frac{y}{21}$$

c $\frac{y+1}{5} + \frac{x+y}{6}$

$$= \frac{6(y+1)}{30} + \frac{5(x+y)}{30}$$

$$= \frac{6(y+1) + 5(x+y)}{30}$$

$$= \frac{6y + 6 + 5x + 5y}{30}$$

$$= \frac{5x + 11y + 6}{30}$$

- If pronumerals appear in the denominator we can treat these separately to their coefficients (numbers).
- In such a case the lowest common denominator (LCD) is found by finding the lowest common multiple (LCM) of the coefficients, then including in the LCD every pronumeral used.

WORKED EXAMPLE 10

Simplify each of the following expressions.

a $\frac{1}{x} - \frac{3}{2x}$

b $\frac{2x}{3y} + \frac{3}{5y}$

c $\frac{6}{5z} - \frac{3x}{10}$

THINK

- a 1 Write the expression.
- 2 Find the LCD. The LCM of 1 and 2 is 2. The only pronumeral is x , so include in the LCD. The LCD is $2x$.

WRITE

a $\frac{1}{x} - \frac{3}{2x}$

$$= \frac{2}{2x} - \frac{3}{2x}$$

3 Subtract the numerators.

$$= -\frac{1}{2x}$$

b 1 Write the expression.

$$\frac{2x}{3y} + \frac{3}{5y}$$

2 Find the LCD. The LCM of 3 and 5 is 15. The only pronumeral is y so include it in the LCD. The LCD is $15y$

$$= \frac{10x}{15y} + \frac{9}{15y}$$

3 Add the numerators.

$$= \frac{10x + 9}{15y}$$

c 1 Write the expression.

$$\frac{6}{5z} - \frac{3x}{10}$$

2 Find the LCD. The LCM of 5 and 10 is 10. The only pronumeral is z , so include it in the LCD. The LCD is $10z$.

$$= \frac{12}{10z} - \frac{3xz}{10z}$$

3 Subtract the numerators.

$$= \frac{12 - 3xz}{10z}$$

8.5.2 Multiplying and dividing algebraic fractions

- The rules for multiplication and division are the same as for numerical fractions. When multiplying algebraic fractions, multiply the numerators and multiply the denominators, then cancel any common factors in the numerator and denominator.

WORKED EXAMPLE 11

Simplify each of the following.

a $\frac{x}{2} \times \frac{6}{y}$

b $\frac{y}{2x} \times \frac{2z}{3y}$

THINK

a 1 Write the algebraic fractions.

2 Multiply the numerators and multiply the denominators.

3 Check for common factors in the numerator and denominator and simplify. The numbers 6 and 2 have a common factor of 2.

b 1 Write the algebraic fractions.

2 Multiply the numerators and multiply the denominators

3 Check for common factors in the numerator and denominator and simplify. The numbers 2 and 6 have a common factor of 2. Cancel y

WRITE

a $\frac{x}{2} \times \frac{6}{y}$

$$= \frac{6x}{2y}$$

$$= \frac{3x}{y}$$

b $\frac{y}{2x} \times \frac{2z}{3y}$

$$= \frac{2yz}{6xy}$$

$$= \frac{z}{3x}$$

- When dividing algebraic fractions, change the division sign to a multiplication sign and write the following fraction as its reciprocal.

WORKED EXAMPLE 12

Simplify each of the following.

a $\frac{1}{x} \div \frac{4}{x}$

b $\frac{3xy}{2} \div \frac{4x}{9y}$

THINK

- a**
- 1 Write the algebraic fractions.
 - 2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.
 - 3 Multiply the numerators and multiply the denominators.
 - 4 Check for common factors in the numerator and denominator. Cancel x .
- b**
- 1 Write the algebraic fractions.
 - 2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.
 - 3 Multiply the numerators and multiply the denominators.
 - 4 Check for common factors in the numerator and denominator. Cancel x .

WRITE

$$\begin{aligned} \text{a } \frac{1}{x} \div \frac{4}{x} &= \frac{1}{x} \times \frac{x}{4} \\ &= \frac{x}{4x} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{3xy}{2} \div \frac{4x}{9y} &= \frac{3xy}{2} \times \frac{9y}{4x} \\ &= \frac{27xy^2}{8x} \\ &= \frac{27y^2}{8} \end{aligned}$$

DISCUSSION

Using numerical values for a , explain the difference between $\frac{3a}{9}$ and $\frac{9}{3a}$.

learnON RESOURCES — ONLINE ONLY

Digital doc: SkillSHEET Adding and subtracting fractions (doc-6857)

Digital doc: SkillSHEET Multiplying and dividing fractions (doc-6858)

Exercise 8.5 Simplifying algebraic fractions 1

Individual pathways

PRACTISE

Questions:
1a–f, 2a–c, 3a–g, 4a–g

CONSOLIDATE

Questions:
1a–j, 2a–f, 3a–o, 4a–i, 5–7

MASTER

Questions:
1a–j, 2a–f, 3i–r, 4i–o, 5–9

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE9** Simplify each of the following expressions.

a. $\frac{x}{3} + \frac{x}{4}$

b. $\frac{y}{2} - \frac{y}{3}$

c. $\frac{m}{8} - \frac{m}{4}$

d. $\frac{x}{6} + \frac{x}{12}$

e. $\frac{m}{2} + \frac{m}{7}$

f. $\frac{t}{3} - \frac{t}{5}$

g. $\frac{3a}{2} - \frac{a}{5}$

h. $\frac{2p}{3} - \frac{5p}{6}$

i. $\frac{4q}{5} + \frac{q}{3}$

j. $\frac{5x}{6} - \frac{2x}{3}$

2. **WE10** Simplify each of the following expressions.

a. $\frac{3}{2p} - \frac{2}{p}$

b. $\frac{1}{2x} + \frac{3}{5x}$

c. $\frac{3}{4m} - \frac{7}{2m}$

d. $\frac{8}{5b} - \frac{4}{4b}$

e. $\frac{11}{6c} - \frac{4}{9c}$

f. $\frac{2}{3y} + \frac{4}{5y}$

3. **WE11** Simplify each of the following.

a. $\frac{2}{3} \times \frac{9}{2}$

b. $\frac{3}{5} \times \frac{10}{3}$

c. $\frac{5}{12} \times \frac{3}{5}$

d. $\frac{4}{15} \times \frac{3}{4}$

e. $\frac{x}{3} \times \frac{9}{x}$

f. $\frac{4}{y} \times \frac{y}{12}$

g. $\frac{4}{3} \times \frac{m}{16}$

h. $\frac{n}{9} \times \frac{3}{2}$

i. $\frac{7m}{5} \times \frac{10}{m}$

j. $\frac{5}{3x} \times \frac{x}{15}$

k. $\frac{20}{3y} \times \frac{6}{5}$

l. $\frac{2x}{3} \times \frac{15}{6}$

m. $\frac{4m}{27} \times \frac{9}{7m}$

n. $\frac{2}{15} \times \frac{7}{p} \times \frac{p}{21}$

o. $\frac{x}{22} \times \frac{11}{12} \times \frac{6}{x}$

p. $\frac{10a^4}{5a^2}$

q. $\frac{9a^2b}{3ab}$

r. $\frac{3ab}{9a^2b}$

4. **WE12** Simplify each of the following.

a. $\frac{2}{5} \div \frac{2}{15}$

b. $\frac{3}{4} \div \frac{3}{8}$

c. $\frac{5}{6} \div \frac{15}{6}$

d. $\frac{9}{10} \div \frac{36}{10}$

e. $\frac{x}{3} \div \frac{x}{9}$

f. $\frac{4}{m} \div \frac{12}{m}$

g. $\frac{a}{5} \div \frac{a}{20}$

h. $\frac{6}{b} \div \frac{20}{b}$

i. $\frac{3a}{14} \div \frac{a}{7}$

j. $\frac{21}{4} \div \frac{3}{b}$

k. $\frac{6m}{15} \div \frac{2}{3}$

l. $\frac{ab}{9} \div \frac{a}{24}$

m. $\frac{2m}{3p} \div \frac{10m}{9pq}$

n. $\frac{3}{5} \times \frac{10}{m} \div \frac{12}{m}$

o. $\frac{3x}{8} \times \frac{2y}{15} \div \frac{y}{4}$

Communicating, reasoning and problem solving

5. Simplify each of the following

a. $\frac{3x^2}{8y^5} \div \frac{15x^3}{4y}$

b. $\frac{a^2b^4}{6} \times \frac{9}{a^2b^2}$

c. $\frac{3}{x} - \frac{1}{2x}$

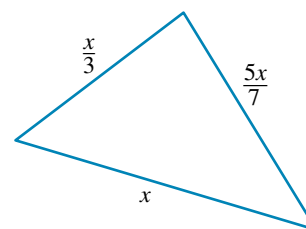
d. $\frac{21b^2}{4} \div \frac{3}{b^3}$

e. $\frac{x}{22y} \times \frac{11y^2}{12z} \times \frac{6z}{xy}$

f. $\frac{3x^3}{8y} \times \frac{2y^2}{15} \div \frac{y^2}{4}$

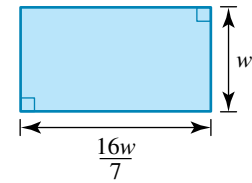
6. For the triangle shown:

- write an expression for the perimeter of the triangle
- show that the perimeter can be simplified to $\frac{43}{21}x$
- if x is 2 metres, calculate the cost (to the nearest dollar) to frame the triangle using timber that costs \$4.50 per metre.



7. For the rectangle shown:

- write an expansion for the perimeter of the rectangle
- show that the perimeter can be simplified to $6\frac{4}{7}w$
- if $w = 7$ metres, calculate the cost (to the nearest cent) to create a wire frame for the rectangle if the wire frame costs \$1.57 per metre.



- A student's work is shown at right. What errors has the student made?
 - What should the student have written as the solutions?

i $\frac{5a+4}{2} = 5a+2$

ii $\frac{3p+r}{3p+s} = \frac{r}{s}$

9. Simplify the following expressions.

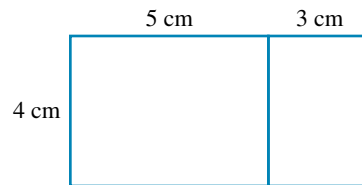
- $\frac{2}{3b} \times \frac{b^2}{4} \div \frac{7b}{8}$
- $\frac{3}{x-2} \times \frac{2x-4}{4x+1} \div \frac{9}{20x+5}$

8.6 Expanding algebraic expressions

[Stage 5.2]

8.6.1 The Distributive Law

- What is the area of the large rectangle drawn below? There are two ways of finding out.



1. The rectangle can be treated as one large shape.

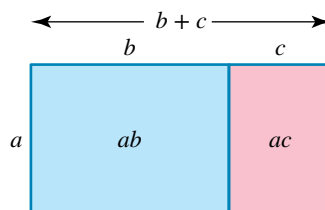
The length of the rectangle is 4 cm, and its width is $5 + 3$ cm.

$$\begin{aligned} A &= l \times w \\ &= 4(5 + 3) \\ &= 4 \times 8 \\ &= 32 \text{ cm}^2 \end{aligned}$$

2. The areas of the two small rectangles can be added together.

$$\begin{aligned} A &= 4 \times 5 + 4 \times 3 \\ &= 20 + 12 \\ &= 32 \text{ cm}^2 \end{aligned}$$

- The length of this rectangle is a , and its width is $b + c$.



- The area can be found in two different ways.

$$\begin{aligned} 1. A &= l \times w \\ &= a(b + c) \end{aligned}$$

This expression is **factorised** because it is one number (or factor) multiplied by another. The two factors are a and $b + c$.

2. The areas of the two small rectangles can be added together.

$$A = ab + ac$$

This expression is described as expanded or written without brackets.

- $a(b + c) = ab + ac$ is called the **Distributive Law**.

WORKED EXAMPLE 13

Use two different methods to find the value of the following.

a $7(5 + 15)$

b $10(9 - 1)$

THINK

a 1 Method 1: Work out the brackets first, then evaluate.

2 Method 2: Expand the brackets first, then evaluate.

b 1 Method 1: Work out the brackets first, then evaluate.

2 Method 2: Expand the brackets first, then evaluate.

WRITE

a $7(5 + 15)$
 $= 7 \times 20$
 $= 140$

$$\begin{aligned} 7(5 + 15) &= 7 \times 5 + 7 \times 15 \\ &= 35 + 105 \\ &= 140 \end{aligned}$$

b $10(9 - 1)$
 $= 10 \times 8$
 $= 80$

$$\begin{aligned} 10(9 - 1) &= 10 \times 9 + 10 \times -1 \\ &= 90 - 10 \\ &= 80 \end{aligned}$$

DISCUSSION

In Worked example 13, which of these methods is your preferred method? Is there a reason why?

- A good way to expand brackets is to draw arrows as you work out each multiplication.

$$x(x - y) = x^2 \dots$$

$$x(x - y) = x^2 - xy$$

WORKED EXAMPLE 14

Expand the following.

a $5(x + 3)$

b $-4y(2x - w)$

THINK

a 1 Draw arrows to assist in expanding.

2 Simplify.

b 1 Draw arrows to assist in expanding.

2 Simplify.

WRITE

a $5(x + 3)$
 $= 5 \times x + 5 \times 3$
 $= 5x + 15$

b $-4y(2x - w)$
 $= -4y \times 2x - 4y \times -w$
 $= -8xy + 4wy$

8.6.2 Expanding and collecting like terms

- With more complicated expansions, like terms may need to be collected after the expansion of the brackets.
- First expand the brackets, then collect the like terms.

WORKED EXAMPLE 15

Expand and simplify by collecting like terms.

a $4(x - 4) + 5$

b $x(y - 2) + 5x$

c $-x(y - z) + 5x$

d $7x - 6(y - 2x)$

THINK

a 1 Expand the brackets.

2 Simplify.

b 1 Expand the brackets.

2 Simplify.

c 1 Expand the brackets.

2 Simplify. (There are no like terms.)

d 1 Expand the brackets.

2 Simplify.

WRITE

a $4(x - 4) + 5$

$$= 4x - 16 + 5$$

$$= 4x - 11$$

b $x(y - 2) + 5x$

$$= xy - 2x + 5x$$

$$= xy + 3x$$

c $-x(y - z) + 5x$

$$= -xy + xz + 5x$$

d $7x - 6(y - 2x)$

$$= 7x - 6y + 12x$$

$$= 19x - 6y$$

8.6.3 Expanding two brackets

- When simplifying an expression containing two (or more) brackets, the steps are the same.
- Expand each bracket, and then simplify by collecting like terms.

WORKED EXAMPLE 16

Expand and simplify the following expressions.

a $5(x + 2y) + 6(x - 3y)$

b $5x(y - 2) - y(x + 3)$

THINK

a 1 Expand each bracket.

2 Simplify.

b 1 Expand each bracket.

2 Simplify.

WRITE

a $5(x + 2y) + 6(x - 3y)$

$$= 5x + 10y + 6x - 18y$$

$$= 11x - 8y$$

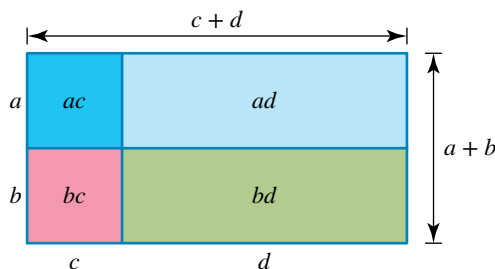
b $5x(y - 2) - y(x + 3)$

$$= 5xy - 10x - xy - 3y$$

$$= 4xy - 10x - 3y$$

8.6.4 Expanding binomial factors

- Remember, a **binomial** is an expression containing two terms, such as $x + 3$ or $2y - z^2$. In this section two binomials are multiplied together.
- The rectangle below has length $a + b$ and width $c + d$.



- There are two ways of finding the area of the large rectangle.

$$\begin{aligned} 1. A &= L \times W \\ &= (a + b) \times (c + d) \\ &= (a + b)(c + d) \end{aligned}$$

This is a factorised expression, where the two factors are $(a + b)$ and $(c + d)$.

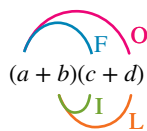
- The areas of the four small rectangles can be added together.

$$A = ac + ad + bc + bd$$

$$\text{So } (a + b)(c + d) = ac + ad + bc + bd.$$

- There are several methods that can be helpful in remembering how to expand binomial factors. One commonly used method is the FOIL method.
- This method is given the name **FOIL** because the letters stand for:
First — multiply the first term in each bracket.
Outer — multiply the 2 outer terms of each bracket.
Inner — multiply the 2 inner terms of each bracket.
Last — multiply the last term of each bracket.

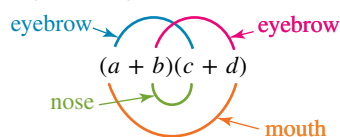
‘FOIL’ – First Outer Inner Last



$$(a + b)(c + d) = ac + ad + bc + bd$$

OR

‘eyebrow, eyebrow, nose and mouth’



$$(a + b)(c + d) = ac + bd + bc + ad$$

WORKED EXAMPLE 17

Expand and simplify each of the following expressions.

a $(x - 5)(x + 3)$

b $(x + 2)(x + 3)$

c $(2x + 2)(2x + 3)$

THINK

- Expand using the FOIL method.
 - Simplify the expression by collecting like terms.
- Expand the brackets using FOIL.
 - Simplify the expression by collecting like terms.
- Expand the brackets using FOIL.
 - Simplify the expression by collecting like terms.

WRITE

$$\begin{aligned} \mathbf{a} \quad (x - 5)(x + 3) \\ &= x^2 + 3x - 5x - 15 \\ &= x^2 - 2x - 15 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad (x + 2)(x + 3) \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad (2x + 2)(2x + 3) \\ &= 4x^2 + 6x + 4x + 6 \\ &= 4x^2 + 10x + 6 \end{aligned}$$

DISCUSSION

Use mathematical reasoning to explain why $(a + b)(c + d) = (c + d)(a + b)$.



RESOURCES — ONLINE ONLY

Interactivity: Expanding brackets: Distributive Law (int-3774)

Interactivity: Expanding brackets (int-6034)

Interactivity: Like terms (int-6035)

Interactivity: Expanding binomial factors (int-6033)

eLesson: Expansion of brackets (eles-1888)

eLesson: Expansion of binomial expressions (eles-1908)

Digital doc: SkillsSHEET Expanding brackets (doc-10819)

Digital doc: SkillsSHEET Expanding a pair of brackets (doc-10820)

Exercise 8.6 Expanding algebraic expressions

Individual pathways

PRACTISE

Questions:

1, 2a–d, 3a–d, 4a, c, m, o, 5a, c, g, k, m, 6a, e, i, m, o, 7, 8a, d, g, j, m, 9, 10a, d, g, j, m, 11, 12, 19, 20

CONSOLIDATE

Questions:

1, 2c–f, 3c–f, 4b, f–j, 5i–p, 6b, d, j, l, p, 7, 8b, e, h, k, n, 9d–f, 10b, e, h, k, n, 13, 14, 19, 20, 22

MASTER

Questions:

1, 2e–h, 3e–h, 4i–p, 5i–p, 6i–p, 7c, 8c, f, i, l, o, 9g–i, 10c, f, i, l, o, 13–18, 21, 22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE13** Use two different methods to find the value of these expressions.
a. $8(10 - 2)$ b. $11(99 + 1)$ c. $-5(3 + 1)$ d. $7(100 - 1)$
- WE14** Expand the following expressions. For the first two examples, draw a diagram to represent the expression.
a. $3(x + 2)$ b. $4(x + 3)$ c. $4(x + 1)$ d. $7(x - 1)$
e. $-3(p - 2)$ f. $-(x - 1)$ g. $3(2b - 4)$ h. $8(3m - 2)$
- WE14** Expand each of the following. For the first two examples, draw a diagram to represent the expression.
a. $x(x + 2)$ b. $a(a + 5)$ c. $x(4 + x)$ d. $m(7 - m)$
e. $2x^2(y + 2x)$ f. $-3y^2(y - 4x)$ g. $b^3(3 - b^2)$ h. $-3x^2(5x^2 + 2x^4y)$
- WE15** Expand and simplify by collecting like terms.
a. $2(p - 3) + 4$ b. $5(x - 5) + 8$ c. $-7(p + 2) - 3$
d. $-4(3p - 1) - 1$ e. $6x(x - 3) - 2x$ f. $2m(m + 5) - 3m$
g. $3x(p + 2) - 5$ h. $4y(y - 1) + 7$ i. $-4p(p - 2) + 5p$
j. $5(x - 2y) - 3y - x$ k. $2m(m - 5) + 2m - 4$ l. $-3p(p - 2q) + 4pq - 1$
m. $-7a(5 - 2b) + 5a - 4ab$ n. $4c(2d - 3c) - cd - 5c$ o. $6p + 3 - 4(2p + 5)$
p. $5 - 9m + 2(3m - 1)$

5. **WE16** Expand and simplify the following expressions.

a. $2(x + 2y) + 3(2x - y)$	c. $7(2a + 3b) + 4(a + 2b)$	e. $-4(m + 2n) + 3(2m - n)$
b. $4(2p + 3q) + 2(p - 2q)$	d. $5(3c + 4d) + 2(2c + d)$	f. $-3(2x + y) + 4(3x - 2y)$
g. $-2(3x + 2y) + 3(5x + 3y)$	i. $6(a - 2b) - 5(2a - 3b)$	k. $4(2p - 4q) - 3(p - 2q)$
h. $-5(4p + 2q) + 2(3p + q)$	j. $5(2x - y) - 2(3x - 2y)$	l. $2(c - 3d) - 5(2c - 3d)$
m. $7(2x - 3y) - (x - 2y)$	n. $-5(p - 2q) - (2p - q)$	o. $-3(a - 2b) - (2a + 3b)$
p. $4(3c + d) - (4c + 3d)$		

6. **WE16** Expand and simplify the following expressions.

a. $a(b + 2) + b(a - 3)$	b. $x(y + 4) + y(x - 2)$	c. $c(d - 2) + c(d + 5)$
d. $p(q - 5) + p(q + 3)$	e. $3c(d - 2) + c(2d - 5)$	f. $7a(b - 3) - b(2a + 3)$
g. $2m(n + 3) - m(2n + 1)$	h. $4c(d - 5) + 2c(d - 8)$	i. $3m(2m + 4) - 2(3m + 5)$
j. $5c(2d - 1) - (3c + cd)$	k. $-3a(5a + b) + 2b(b - 3a)$	l. $-4c(2c - 6d) + d(3d - 2c)$
m. $6m(2m - 3) - (2m + 4)$	n. $2p(p - 4) + 3(5p - 2)$	o. $7x(5 - x) + 6(x - 1)$
p. $-2y(5y - 1) - 4(2y + 3)$		

7. **MC** a. What is the equivalent of $3(a + 2b) + 2(2a - b)$?

A. $5a + 6b$	B. $7a + 4b$	C. $5(3a + b)$	D. $7a + 8b$	E. $5(a + b)$
- b. What is the equivalent of $-3(x - 2y) - (x - 5y)$?
- | | | | | |
|-----------------------|-----------------------|----------------------|---------------------|-----------------|
| A. $-4x + 11y$ | B. $-4x - 11y$ | C. $4x + 11y$ | D. $4x + 7y$ | E. $-9y$ |
|-----------------------|-----------------------|----------------------|---------------------|-----------------|
- c. What is the equivalent of $2m(n + 4) + m(3n - 2)$?
- | | | | | |
|-------------------------|----------------------|-----------------------|----------------------|--------------------------|
| A. $3m + 4n - 8$ | B. $5mn + 4m$ | C. $5mn + 10m$ | D. $5mn + 6m$ | E. $5mn + 8m - 2$ |
|-------------------------|----------------------|-----------------------|----------------------|--------------------------|

8. **WE17** Expand and simplify each of the following expressions.

a. $(a + 2)(a + 3)$	b. $(x + 4)(x + 3)$	c. $(y + 3)(y + 2)$	d. $(m + 4)(m + 5)$
e. $(b + 2)(b + 1)$	f. $(p + 1)(p + 4)$	g. $(a - 2)(a + 3)$	h. $(x - 4)(x + 5)$
i. $(m + 3)(m - 4)$	j. $(y + 5)(y - 3)$	k. $(y - 6)(y + 2)$	l. $(x - 3)(x + 1)$
m. $(x - 3)(x - 4)$	n. $(p - 2)(p - 3)$	o. $(x - 3)(x - 1)$	

9. Expand and simplify each of the following.

a. $(2a + 3)(a + 2)$	b. $(3m + 1)(m + 2)$	c. $(6x + 4)(x + 1)$	d. $(c - 6)(4c - 7)$
e. $(7 - 2t)(5 - t)$	f. $(1 - x)(9 - 2x)$	g. $(2 + 3t)(5 - 2t)$	h. $(7 - 5x)(2 - 3x)$
i. $(5x - 2)(5x - 2)$			

10. Expand and simplify each of the following.

a. $(x + y)(z + 1)$	b. $(p + q)(r + 3)$	c. $(2x + y)(z + 4)$	d. $(3p + q)(r + 1)$
e. $(a + 2b)(a + b)$	f. $(2c + d)(c - 3d)$	g. $(x + y)(2x - 3y)$	h. $(4p - 3q)(p + q)$
i. $(3y + z)(x + z)$	j. $(a + 2b)(b + c)$	k. $(3p - 2q)(1 - 3r)$	l. $(7c - 2d)(d - 5)$
m. $(4x - y)(3x - y)$	n. $(p - q)(2p - r)$	o. $(5 - 2j)(3k + 1)$	

11. **MC** a. The equivalent of $(x + 7)(x - 2)$ is:

A. $x^2 + 5x - 14$	B. $2x + 5$	C. $x^2 - 5x - 14$	D. $x^2 + 5x + 14$	E. $x^2 - 14$

b. What is the equivalent of $(4 - y)(7 + y)$?

A. $28 - y^2$	B. $28 - 3y + y^2$	C. $28 + 3y - y^2$	D. $11 - 2y$	E. $28 - 3y - y^2$

c. The equivalent of $(2p + 1)(p - 5)$ is:

A. $2p^2 - 5$	B. $2p^2 - 11p - 5$	C. $2p^2 - 9p - 5$	D. $2p^2 - 6p - 5$	E. $2p^2 - 11p + 5$

12. Expand the following expressions using the FOIL method, then simplify.

a. $(x + 3)(x - 3)$	b. $(x + 5)(x - 5)$	c. $(x + 7)(x - 7)$
d. $(x - 1)(x + 1)$	e. $(x - 2)(x + 2)$	f. $(2x - 1)(2x + 1)$

Can you see a pattern? If so, explain.

13. Expand the following expressions using the FOIL method, then simplify.

a. $(x + 1)(x + 1)$	b. $(x + 2)(x + 2)$	c. $(x + 8)(x + 8)$
d. $(x - 3)(x - 3)$	e. $(x - 5)(x - 5)$	f. $(x - 9)(x - 9)$

Can you see a pattern? If so, explain.

14. Simplify the following expressions.

a. $2.1x(3x + 4.7y) - 3.1y(1.4x + y)$

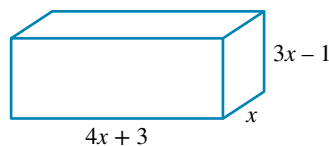
c. $(3.4x + 5.1y)^2$

15. For the box shown, find in expanded form:

a. the total surface area

b. the volume.

b. $(2.1x - 3.2y)(2.1x + 3.2y)$



Communicating, reasoning and problem solving

16. For each of the following shapes:

i. write down the area in factor form

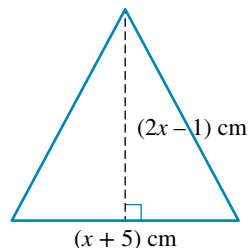
iii. discuss any limitations on the value of x .

a. $(x + 3y)$ m



ii. expand and simplify the expression

b.



17. Shown below are three students' attempts at expanding $(3x + 4)(2x + 5)$.

STUDENT A

$$\begin{aligned} & (3x + 4)(2x + 5) \\ &= 3x \times 2x + 3x \times 5 + 4 \times 2x + 4 \times 5 \\ &= 6x + 15x + 8x + 20 \\ &= 29x + 20 \\ &= 49x \end{aligned}$$

STUDENT B

$$\begin{aligned} & (3x + 4)(2x + 5) \\ &= 3x \times 2x + 4 \times 2x + 4 \times 5 \\ &= 6x^2 + 8x + 20 \end{aligned}$$

STUDENT C

$$\begin{aligned} & (3x + 4)(2x + 5) \\ &= 3x \times 2x + 3x \times 5 + 4 \times 2x + 4 \times 5 \\ &= 6x^2 + 15x + 8x + 20 \\ &= 6x^2 + 23x + 20 \end{aligned}$$

a. Which student's work was correct?

b. Copy each of the incorrect answers into your workbook and correct the mistakes in each as though you were the teacher of these students.

18. Show that $(a - x)(a + x) - 2(a - x)(a - x) - 2x(a - x) = -(a - x)^2$.

19. Three students gave the following incorrect answers when expanding $-5(3x - 20)$.

i. $-5x - 20$

ii. $-8x + 25$

iii. $15x - 100$

a. Explain the errors made by each student.

b. What is the correct answer?

20. In a test, a student expanded brackets and obtained the following answers. Identify and correct the student's errors and write the correct expansions.

a. $-2(a - 5) = -2a - 10$

b. $2b(3b - 1) = 6b^2 - 1$

c. $-2(c - 4) = 2c + 8$

21. Three incorrect expansions of $(x + 8)(x - 3)$ are shown below. For each incorrect expansion, explain what mistake has been made.
- a. $x^2 + 11x + 24$
 - b. $x^2 + 5x + 24$
 - c. $x^2 - 3x$
22. Explain why, when expanded, $(x + y)(2x + y)$ gives the same result as $(2x + y)(x + y)$.

8.7 The highest common factor

[Stage 5.2]

8.7.1 Factorising expressions

- **Factorising** a number or an expression means writing it as a product. For example, 35 is factorised by writing it as 7×5 , the product of 7 and 5.

The factors of 35 are 7 and 5:

$$35 \div 7 = 5$$

$$35 \div 5 = 7$$

- $5(x + 2)$ is also a factorised expression. (Remember that it means $5 \times (x + 2)$.)

The factors of $5(x + 2)$ are 5 and $(x + 2)$:

$$5(x + 2) \div 5 = (x + 2)$$

$$5(x + 2) \div (x + 2) = 5$$

- In section 8.6, factorised expressions were expanded as

$$5(x + 2) = 5x + 10$$

This section examines the opposite process. We will start with expanded expressions and try to factorise them.

WORKED EXAMPLE 18

Complete the following factorisations.

a $15x = 5x \times \dots$

b $15x^2 = 3x \times \dots$

c $-18ab^2 = 6a \times \dots$

THINK

a Divide $15x$ by $5x$.

b Divide $15x^2$ by $3x$.

c Divide $-18ab^2$ by $6a$.

WRITE

a $\frac{15x}{5x} = 3$

So $15x = 5x \times 3$.

b $\frac{15x^2}{3x} = 5x$

So $15x^2 = 3x \times 5x$.

c $\frac{-18ab^2}{6a} = -3b^2$

So $-18ab^2 = 6a \times -3b^2$.

- The **highest common factor (HCF)** of two or more numbers is the largest factor that divides into all the numbers with no remainder. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. The factors of 16 are 1, 2, 4, 8 and 16. The HCF of 12 and 16 is 4.
- The simplest way to factorise an expression is to take out the highest common factor or HCF.

WORKED EXAMPLE 19

Determine the highest common factor for each of the following pairs of terms.

a $25a^2b$ and $10ab$

b $3xy$ and $-3xz$

THINK

a 1 Write $25a^2b$ in expanded form.

2 Write $10ab$ in expanded form.

3 Write the HCF.

b 1 Write $3xy$ in expanded form.

2 Write $-3xz$ in expanded form.

3 Write the HCF.

WRITE

a $25a^2b = 5 \times 5 \times a \times a \times b$

$$10ab = 2 \times 5 \times a \times b$$

$$\text{HCF} = 5ab$$

b $3xy = 3 \times x \times y$

$$-3xz = -1 \times 3 \times x \times z$$

$$\text{HCF} = 3x$$

8.7.2 Factorising expressions by finding the highest common factor

- To factorise an expression such as $4xy + 12x$, find the highest common factor of both terms. The HCF for $4xy$ and $12x$ is $4x$, so $4xy + 12x = 4x \times (\dots)$ where the brackets contain a binomial.

$$\frac{4xy}{4x} = y \text{ and } \frac{12x}{4x} = 3$$

$$\begin{aligned} \text{So } 4xy + 12x &= 4x \times (y + 3) \\ &= 4x(y + 3) \end{aligned}$$

The answer can be checked by expanding the brackets.

WORKED EXAMPLE 20

Factorise each expression by first finding the HCF.

a $5x + 15y$

b $-14xy - 7y$

c $6x^2y + 9xy^2$

THINK

a 1 HCF = 5. Write 5 multiplied by brackets.

2 Divide each term by 5 to get the binomial.

$$\frac{5x}{5} = x, \frac{15y}{5} = 3y$$

3 Check the answer by expanding.

b 1 HCF = $7y$ or $-7y$, but $-7y$ works neatly.

$$\frac{-14xy}{-7y} = 2x, \frac{-7y}{-7y} = 1$$

3 Check the answer by expanding.

c 1 HCF = $3xy$

$$\frac{6x^2y}{3xy} = 2x, \frac{9xy^2}{3xy} = 3y$$

3 Check the answer by expanding.

WRITE

a $5x + 15y = 5(\quad)$

$$= 5(x + 3y)$$

$$5(x + 3y) = 5x + 15y \text{ (correct)}$$

b $-14xy - 7y$

$$= -7y(2x + 1)$$

$$-7y(2x + 1) = -14xy - 7y \text{ (correct)}$$

c $6x^2y + 9xy^2$

$$= 3xy(2x + 3y)$$

$$3xy(2x + 3y) = 6x^2y + 9xy^2 \text{ (correct)}$$

Interactivity: Highest common factor (int-6037)

eLesson: Factorisation (eles-1887)

Digital doc: SkillsSHEET Finding the highest common factor (doc-10822)

Digital doc: SkillsSHEET Factorising by finding the HCF (doc-10823)

Exercise 8.7 The highest common factor

Individual pathways

PRACTISE

Questions:

1a-f, 2a-i, 3, 4a-i, 5a-i, 6a-i,
7a-i, 8a-i, 9

CONSOLIDATE

Questions:

1d-i, 2e-k, 3, 4g-n, 5g-n, 6g-o,
7g-o, 8g-o, 9, 11

MASTER

Questions:

1g-l, 2i-p, 3, 4i-r, 5i-o, 6i-r, 7j-o,
8j-o, 9-12

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE18** Complete each of the following factorisations by writing in the missing factor.

a. $8a = 4 \times \dots$	b. $8a = 2a \times \dots$	c. $12x^2 = 4x \times \dots$
d. $-12x^2 = 3x^2 \times \dots$	e. $3x^2 = x \times \dots$	f. $15a^2b = ab \times \dots$
g. $12x = -4 \times \dots$	h. $10mn = 10n \times \dots$	i. $10mn = -10 \times \dots$
j. $a^2b^2 = ab \times \dots$	k. $30x^2 = 10x \times \dots$	l. $-15mn^2 = -3m \times \dots$
- WE19** Find the highest common factor (HCF) of each of the following.

a. 4 and 12	b. 6 and 15	c. 10 and 25
d. 24 and 32	e. 12, 15 and 21	f. 25, 50 and 200
g. 17 and 23	h. $6a$ and $12ab$	i. $14xy$ and $21xz$
j. $60pq$ and $30q$	k. $50cde$ and $70fgh$	l. $6x^2$ and $15x$
m. $6a$ and $9c$	n. $5ab$ and 25	o. $3x^2y$ and $4x^2z$
p. $4k$ and 6		
- MC** What is $5m$ the highest common factor of?

A. $2m$ and $5m$	B. $5m$ and m	C. $25mn$ and $15lm$
D. $20m$ and $40m$	E. 5 and m	
- WE20** Factorise each of the following expressions.

a. $4x + 12y$	b. $5m + 15n$	c. $7a + 14b$	d. $7m - 21n$
e. $-8a - 24b$	f. $8x - 4y$	g. $-12p - 2q$	h. $6p + 12pq + 18q$
i. $32x + 8y + 16z$	j. $16m - 4n + 24p$	k. $72x - 8y + 64pq$	l. $15x^2 - 3y$
m. $5p^2 - 20q$	n. $5x + 5$	o. $56q + 8p^2$	p. $7p - 42x^2y$
q. $16p^2 + 20q + 4$	r. $12 + 36a^2b - 24b^2$		
- Factorise each expression.

a. $9a + 21b$	b. $4c + 18d^2$	c. $12p^2 + 20q^2$	d. $35 - 14m^2n$
e. $25y^2 - 15x$	f. $16a^2 + 20b$	g. $42m^2 + 12n$	h. $63p^2 + 81 - 27y$
i. $121a^2 - 55b + 110c$	j. $10 - 22x^2y^3 + 14xy$	k. $18a^2bc - 27ab - 90c$	l. $144p + 36q^2 - 84pq$
m. $63a^2b^2 - 49 + 56ab^2$	n. $22 + 99p^3q^2 - 44p^2r$	o. $36 - 24ab^2 + 18b^2c$	

6. Factorise the following expressions.

a. $-x + 5$	b. $-a + 7$	c. $-b + 9$	d. $-2m - 6$
e. $-6p - 12$	f. $-4a - 8$	g. $-3n^2 + 15m$	h. $-7x^2y^2 + 21$
i. $-7y^2 - 49z$	j. $-12p^2 - 18q$	k. $-63m + 56$	l. $-12m^3 - 50x^3$
m. $-9a^2b + 30$	n. $-15p - 12q$	o. $-18x^2 + 4y^2$	p. $-3ab + 18m - 21$
q. $-10 - 25p^2 - 45q$	r. $-90m^2 + 27n + 54p^3$		

7. Factorise each of the following expressions.

a. $a^2 + 5a$	b. $m^2 + 3m$	c. $x^2 - 6x$	d. $14q - q^2$
e. $18m + 5m^2$	f. $6p + 7p^2$	g. $7n^2 - 2n$	h. $a^2 - ab + 5a$
i. $7p - p^2q + pq$	j. $xy + 9y - 3y^2$	k. $5c + 3c^2d - cd$	l. $3ab + a^2b + 4ab^2$
m. $2x^2y + xy + 5xy^2$	n. $5p^2q^2 - 4pq + 3p^2q$	o. $6x^2y^2 - 5xy + x^2y$	

8. Factorise each of the following expressions.

a. $5x^2 + 15x$	b. $10y^2 + 2y$	c. $12p^2 + 4p$	d. $24m^2 - 6m$	e. $32a^2 - 4a$
f. $-2m^2 + 8m$	g. $-5x^2 + 25x$	h. $-7y^2 + 14y$	i. $-3a^2 + 9a$	j. $-12p^2 - 2p$
k. $-15b^2 - 5b$	l. $-26y^2 - 13y$	m. $4m - 18m^2$	n. $-6t + 36t^2$	o. $-8p - 24p^2$

Communicating, reasoning and problem solving

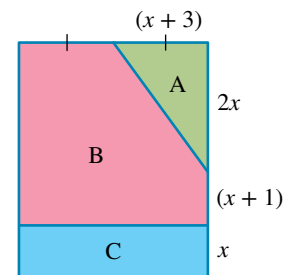
9. A large billboard display is in the shape of a rectangle. There are 3 regions (A, B, C) with dimensions in terms of x as shown.

- Determine the total area of the rectangle. Give your answer in factorised form.
- Determine the area of each region in simplest form.

10. On her recent Algebra test, Marcia wrote her answer of $(a + b)(a - b)^2$ to the question 'Using factorisation, simplify the following expression $a^2(a - b) - b^2(a - b)$.' Is Marcia's answer correct? Use mathematical reasoning to explain.

11. It has been said that, for any two numbers, the product of their LCM and HCF is the same as the product of the two numbers themselves. Show whether this is true.

12. Why would it be said that $24x^2y + 16xy^2 = 4xy(6x + 4y)$ is only partially factorised? Factorise the expression $24x^2y + 16xy^2$ fully.



8.8 Factorising monic quadratics

[Stage 5.2]

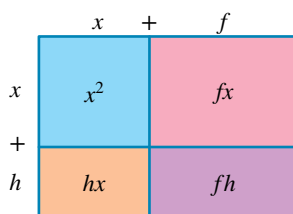
8.8.1 Quadratic trinomials

- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where a , b and c are constants.
- If the expression contains $1x^2$, that is if $a = 1$, then it is called a monic quadratic trinomial.

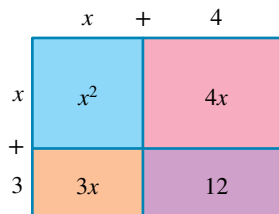
8.8.2 Factorising monic quadratics

- The area model of the binomial expansion can be reversed to find a pattern for factorising a general quadratic expression. For example:

$$\begin{aligned}(x + f)(x + h) &= x^2 + fx + hx + fh \\ &= x^2 + (f + h)x + fh\end{aligned}$$



$$\begin{aligned}(x + 4)(x + 3) &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12\end{aligned}$$



To factorise a general quadratic, look for factors of c that sum to b .

$$x^2 + bx + c = (x + f)(x + h)$$

Factors of c that sum to b

For example, $x^2 + 7x + 12 = (x + 3)(x + 4)$

$$3 + 4 = 7$$

$$3 \times 4 = 12$$

DISCUSSION

Explain why the factorisation $x^2 - 6x - 8 = (x - 4)(x - 2)$ is incorrect.

WORKED EXAMPLE 21

Factorise the following quadratic expressions.

a $x^2 + 5x + 6$

b $x^2 + 10x + 24$

THINK

a 1 The general quadratic expression has the pattern $x^2 + 5x + 6 = (x + f)(x + h)$, where f and h are a factor pair of 6 that add to 5.

Calculate the sums of factor pairs of 6. The factors of 6 that add to 5 are 2 and 3, as shown in blue.

2 Substitute the values of f and h into the expression in its factorised form.

b 1 The general quadratic expression has the pattern $x^2 + 10x + 24 = (x + f)(x + h)$, where f and h are a factor pair of 24 that add to 10.

Calculate the sums of factor pairs of 24. The factors of 24 that add to 10 are 4 and 6, as shown in blue.

2 Substitute the values of f and h into the expression in its factorised form.

WRITE

a

Factors of 6	Sum of factors
1 and 6	7
2 and 3	5

$$(x^2 + 5x + 6) = (x + 2)(x + 3)$$

b

Factors of 24	Sum of factors
1 and 24	25
2 and 12	14
3 and 8	11
4 and 6	10

$$x^2 + 10x + 24 = (x + 4)(x + 6)$$

WORKED EXAMPLE 22

Factorise the following expressions.

a $x^2 - 9x + 18$

b $x^2 + 6x - 16$

THINK

a 1 The general quadratic expression has the pattern $x^2 - 9x + 18 = (x + f)(x + h)$, where f and h are a factor pair of 18 that add to -9 .

Calculate the sums of factor pairs of 18. As shown in blue, -3 and -6 are factors of 18 that add to -9 .

WRITE

a

Factors of 18	Sum of factors
1 and 18	19
-1 and -18	-19
2 and 9	11
-2 and -9	-11
3 and 6	9
-3 and -6	-9

2 Substitute the values of f and h into the expression in its factorised form.

$$x^2 - 9x + 18 = (x - 3)(x - 6)$$

- b 1** The general quadratic expression has the pattern $x^2 + 6x - 16 = (x + f)(x + h)$, where f and h are a factor pair of -16 that add to 6.

Calculate the sums of factor pairs of -16 . As shown in blue, -2 and 8 are factors of -16 that add to 6.

b

Factors of -16	Sum of factors
1 and -16	-15
-1 and -16	15
2 and -8	-6
-2 and 8	6

2 Substitute the values of f and h into the expression in its factorised form.

$$x^2 - 6x + 16 = (x - 2)(x + 8)$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Factorising monic quadratics (int-6092)

Interactivity: Factorise trinomials (int-0748)

Digital doc: SkillsHEET Factorising quadratic trinomials (doc-10981)

Digital doc: SkillsHEET Finding the factor pair that adds to a given number (doc-10982)

Exercise 8.8 Factorising monic quadratics

Individual pathways

PRACTISE

Questions:

1a–k, 2, 3, 4a–i, 5, 6a–i, 7–9, 11

CONSOLIDATE

Questions:

1g–o, 4g–o, 5, 6g–o, 7–10, 12–14

MASTER

Questions:

1m–k, 4i–o, 6i–o, 7–11, 12–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Expand each of the following.

a. $(x + 4)(x + 2)$

d. $(x + 4)(x + 5)$

g. $(t + 8)(t + 11)$

j. $(x + 3)(x - 2)$

m. $(x + 7)(x - 2)$

p. $(a + 3)(a - 6)$

s. $(n - 3)(n - 7)$

b. $(x - 2)(x - 4)$

e. $(m + 1)(m + 5)$

h. $(t - 10)(t - 20)$

k. $(v + 5)(v - 8)$

n. $(t + 1)(t - 12)$

q. $(z + 4)(z + 4)$

t. $(v - 1)(v + 2)$

c. $(x - 4)(x - 5)$

f. $(m - 1)(m - 5)$

i. $(x + 2)(x - 3)$

l. $(v - 5)(v + 8)$

o. $(n + 15)(n - 2)$

r. $(z + 11)(z - 8)$

2. Make a complete systematic list of pairs of positive numbers that add up to:

a. 5

b. 6

c. 12

d. 20.

3. Make a complete systematic list of pairs of negative numbers that add up to:

a. -5

b. -7

c. -8

d. -11 .

4. **WE21** Factorise each of the following.

a. $x^2 + 4x + 3$

d. $x^2 - 12x + 11$

b. $x^2 - 4x + 3$

e. $a^2 + 6a + 5$

c. $x^2 + 12x + 11$

f. $a^2 - 6a + 5$

- g. $x^2 - 7x + 12$ h. $x^2 - 7x + 10$ i. $n^2 + 8n + 16$
 j. $n^2 + 10n + 16$ k. $y^2 - 12y + 27$ l. $x^2 - 13x + 42$
 m. $t^2 - 8t + 12$ n. $t^2 + 11t + 18$ o. $u^2 + 5u + 6$
5. Make a systematic list of 5 pairs of numbers with opposite signs whose sum is:
 a. 6 b. -6 c. 3
 d. -3 e. -8 f. 14
 g. -5 h. 7.
6. **WE22** Factorise each of the following.
 a. $x^2 + 3x - 18$ b. $x^2 - 3x - 18$ c. $x^2 - 2x - 15$
 d. $x^2 + 2x - 15$ e. $n^2 - 13n - 14$ f. $n^2 + 2n - 35$
 g. $v^2 + 5v - 6$ h. $v^2 - 5v - 6$ i. $t^2 + 4t - 12$
 j. $t^2 - 5t - 14$ k. $x^2 - x - 20$ l. $x^2 + x - 20$
 m. $n^2 + n - 90$ n. $n^2 - 3n - 70$ o. $x^2 - 4x - 5$
7. Consider the quadratic trinomial $x^2 + 7x + c$ where c is a positive integer.
 a. Factorise the expression if $c = 6$.
 b. What other positive whole number values can c take if the expression is to be factorised? Factorise the expression for each of these values.
8. Consider the quadratic trinomial $x^2 + 3x + c$ where c is a negative integer.
 a. Factorise the expression if $c = -4$.
 b. Find 3 more values of c for which the expression can be factorised, and factorise each one.
9. Factorise the following trinomials, by first removing a common factor.
 a. $2x^2 - 10x - 28$ b. $4x^2 + 28x + 40$ c. $-2x^2 - 2x + 24$ d. $5x^2 - 40x + 75$
10. **MC** When factorised, $x^2 - 3x - 18$ is equal to:
 A. $(x - 3)(x - 6)$ B. $(x - 3)(x + 6)$ C. $(x + 3)(x + 6)$
 D. $(x + 3)(x - 6)$ E. $(x + 2)(x - 9)$

Communicating, reasoning and problem solving

11. Show that $x^2 + 8x + 10$ has no factors, if only whole numbers can be used.
12. To factorise the quadratic trinomial $6x^2 + 9x + 3$, the expression is rewritten as an equivalent form consisting of four terms, $6x^2 + \square x + \square x + 3$, that are then grouped using the 'two and two' method, $6x^2 + \square x + \square x + 3$. Investigate all the possible combinations for $\square x + \square x = 9x$ and show that there is only one combination that will factorise the trinomial appropriately.
13. Rectangular floor mats have an area of $x^2 + 2x - 15$.
 a. If the length of the mat is $(x + 5)$ cm, find an expression for the width.
 b. If the length of the mat is 70 cm, what is the width?
 c. If the width of the mat is 1 m, what is the length?
14. A particular rectangle has an area of 120 m^2 .
 a. Generate a list of possible whole number dimensions for the rectangle.
 b. The area of the rectangle can also be expressed as $A = x^2 + 2x - 48$. Determine algebraic expressions for the length and the width of the rectangle in terms of x .
 c. Use your list of possible dimensions and your algebraic expressions for width and length to determine the dimensions of the rectangle. *Hint:* The value of x in the length and width must be the same.
15. How will you memorise the steps involved in factorising quadratic trinomials when $a = 1$?

8.9 Review

Investigation | Rich task

Quilt squares

Quilts are decorative blankets that are also used to keep us warm. For many generations, people all over the world have been interested in quilt making, which involves stitching pieces of fabric together and inserting stuffing between the sewn fabric. The fabric can be arranged and sewn in a variety of ways to create attractive geometric designs. Quilts can also be used as decorative objects.

Medini is designing a special quilt. She is sewing pieces of different coloured fabrics together to make a block, and then copying the block and sewing the blocks together repeatedly.

A scaled diagram of Medini's basic block is shown below. The letters indicate the colours of the fabric that make up the block: yellow, black and white. The yellow and white pieces are square; the black pieces are rectangular. The blocks are sewn together in rows and columns.



Trace or copy the basic block onto a sheet of paper. Repeat this process until you have nine blocks. Colour in each section and cut out all nine blocks.

Diagram of basic block used in the quilt

y	b	y
b	w	b
y	b	y

1. Place all nine blocks together to form a 3×3 square. Draw and colour a scaled diagram of your result.
2. Describe the feature created by arranging the blocks in the manner described in question 1. Observe the shapes created by the different colours.

Medini sold her design to a company that now manufactures quilts made from 100 of these blocks. Each quilt covers an area of 1.44 m^2 . Each row and column has the same number of blocks. Answer the following, ignoring any seam allowances.

3. Calculate the side length of each square block.
4. If the entire quilt has an area of 1.44 m^2 , what is the area of each block?
5. Determine the dimensions of the yellow, black and white pieces of fabric of each block.
6. Calculate the area of the yellow, black and white pieces of fabric of each block.
7. Determine the total area of each of the three different colours required to construct this quilt.

8. Due to popular demand, the company that manufactures these quilts now makes them in different sizes. All that the customer is required to do is to specify the approximate quilt area, the three colours required and the number of blocks in the quilt (they must conform to a square or rectangular shape). Derive a general formula that would allow the company to quickly determine the areas of the three coloured fabrics in each block. Give an example and a diagram on a separate sheet of paper to illustrate your formula.



learnon RESOURCES — ONLINE ONLY

Digital doc: Investigation — Quilt squares (doc-15890)

Digital doc: Code puzzle — Why does a giraffe have a long neck? (doc-15891)

Digital doc: Summary Topic 8 (doc-22890)

Interactivity: Crossword Topic 8 (int-0685)

Interactivity: Word search Topic 8 (int-0699)

Interactivity: Sudoku Topic 8 (int-3203)

Exercise 8.9 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- MC** For the expression $-5xy^2 + 2y + 8y^2 + 6$, the coefficient of the second term is:
A. -5 **B.** 2 **C.** 8 **D.** 6 **E.** $2y$
- MC** Jodie had \$55 in her purse. She spent \$ x . The amount left is:
A. $x - 55$ **B.** $x + 55$ **C.** $55x$ **D.** $55 - x$ **E.** x
- MC** What is $\frac{3x}{5} - \frac{x}{4}$ simplified?
A. $\frac{7x}{20}$ **B.** $\frac{2x}{20}$ **C.** $\frac{2x}{1}$ **D.** $\frac{4x}{20}$ **E.** $\frac{7x}{5}$
- MC** What does $\frac{3}{p} \div \frac{6}{p}$ equal?
A. 2 **B.** $\frac{1}{2}$ **C.** $12p$ **D.** $12p^2$ **E.** 18

5. **MC** The volume, V , of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere. If $r = 3$, the volume is:
A. 25.12 **B.** 38.37 **C.** 64.05 **D.** 113.10 **E.** 36
6. **MC** What is the equivalent of $6 - 4x(x + 2) + 3x$?
A. $6 - 4x^2 - 5x$ **B.** $6 + 4x^2 + 5x$ **C.** $6 - 4x^2 + 5x$ **D.** $6 + 4x^2 - 5x$ **E.** $6 - 4x^2 + 11x$
7. **a.** For the expression $-8xy^2 + 2x + 8y^2 - 5$:
i. state the number of terms **ii.** state the coefficient of the first term
iii. state the constant term **iv.** state the term with the smallest coefficient.
- b.** Write expressions for the following, where x and y represent numbers:
i. a number 8 more than y
ii. the difference between x and y
iii. the sum of x and y
iv. 7 times the product of x and y
v. 2 times x is subtracted from 5 times y .
8. **a.** Leo receives x dollars for each car he washes. If he washes y cars, how much money does he earn?
b. A piece of rope is 24m long.
i. If George cuts k m off, how much is left?
ii. After George has cut k m off, he divides the rest into three pieces of equal length. How long is each piece?
9. Simplify the following expressions by collecting like terms.
a. $8p + 9p$
b. $5y^2 + 2y - 4y^2$
c. $9s^2t - 12s^2t$
d. $11c^2d - 2cd + 5dc^2$
e. $n^2 - p^2q - 3p^2q + 6$
f. $8ab + 2a^2b^2 - 5a^2b^2 + 7ab$
10. Simplify the following expressions.
a. $6a \times 2b$ **b.** $2ab \times b$ **c.** $2xy \times 4yx$ **d.** $\frac{4x}{12}$ **e.** $18 \div 4b$
11. Expand these expressions.
a. $5(x + 3)$ **b.** $-(y + 5)$ **c.** $-x(3 - 2x)$ **d.** $-4m(2m + 1)$
12. Expand and simplify these expressions by collecting like terms.
a. $3(x - 2) + 9$
b. $-2(5m - 1) - 3$
c. $4m(m - 3) + 3m - 5$
d. $7p - 2 - (3p + 4)$
13. Expand and simplify the following expressions.
a. $3(a + 2b) + 2(3a + b)$
b. $-4(2x + 3y) + 3(x - 2y)$
c. $2m(n + 6) - m(3n + 1)$
d. $-2x(3 - 2x) - (4x - 3)$
14. Expand and simplify these expressions.
a. $(x + 4)(x + 5)$ **b.** $(m - 2)(m + 1)$
c. $(3m - 2)(m - 5)$ **d.** $(2a + b)(a - 3b)$
15. Expand and simplify these expressions.
a. $(x + 2)(x + 1) + (x + 3)(x + 2)$ **b.** $(m + 7)(m - 2) + (m + 3)^2$
c. $(x + 6)(x + 2) - (x + 3)(x - 1)$ **d.** $(b - 7)^2 - (b - 3)(b - 4)$
16. If $y = 5x^2 + 2x - 1$, find y when:
a. $x = 2$ **b.** $x = 5$.

17. The volume (V) of each of the following paint tins is given by the formula $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Find V (to 2 decimal places).

a. $r = 7$ cm and $h = 2$ cm

b. $r = 9.3$ cm and $h = 19.8$ cm

18. Factorise each of the following by finding common factors.

a. $6x + 12$

b. $6x^2 + 12x^3y$

c. $8a^2 - 4b$

d. $16x^2 - 24xy$

e. $-2x - 4$

f. $b^2 - 3b + 4bc$

19. Simplify the following expressions.

a. $\frac{x}{2} + \frac{x}{5}$

b. $\frac{x}{6} - \frac{x}{12}$

c. $\frac{3x}{4} - \frac{5x}{8}$

d. $\frac{5}{12m} + \frac{7}{8m}$

e. $\frac{a}{6} \times \frac{12}{a}$

f. $\frac{2}{y} \times \frac{y}{8}$

g. $\frac{7}{2x} \times \frac{x}{14}$

h. $\frac{1}{3p} \times \frac{6p}{5}$

i. $\frac{a}{12} \times \frac{4}{5} \times \frac{10}{a}$

j. $\frac{a}{2} \div \frac{a}{4}$

k. $\frac{5}{m} \div \frac{30}{m}$

l. $\frac{24}{7y} \div \frac{8}{y}$

m. $\frac{5m}{6y} \div \frac{15m}{3xy}$

n. $\frac{20a}{14c} \div \frac{4ab}{7c}$

20. a. $c^2 + 5c + 4$

b. $x^2 - 6x - 7$

c. $x^2 - 4x + 4$

d. $p^2 + 10p - 24$

e. $q^2 + q - 42$

f. $x^2 - 2x - 24$

g. $y^2 - 10y + 24$

h. $x^2 + 3x + 2$

i. $c^2 - 11c - 26$

j. $m^2 - 7m + 10$

k. $x^2 + 6x - 27$

l. $m^2 + 24m + 44$

Communicating, reasoning and problem solving

21. A rectangular rug has a length of $3x$ cm and a width of x cm.

a. Write an expression for its perimeter.

b. Write an expression for its area.

c. i. If its side length is increased by y cm, write an expression for its new side length.

ii. Write an expression for its new perimeter and expand.

iii. Calculate the perimeter when $x = 90$ cm and $y = 30$ cm.

iv. Write an expression for its new area and expand.

v. Calculate the area when $x = 90$ cm and $y = 30$ cm.

22. A rectangular garden bed has a length of 15 m and a width of 8 m. It is surrounded by a path of width p m.

a. Write the total area of the garden bed and path in factorised form.

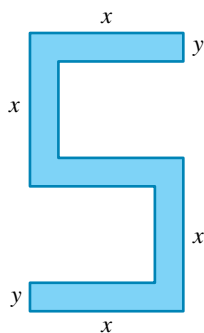
b. Expand the expression you found in part a.

c. Find the area of the path in terms of p .

d. Write an equation that could be solved to find the width of the path if the area of the path is 200 m^2 .

23. Find the circumference of a circle whose area is $\pi \left(\frac{x}{y}\right)^2 \text{ cm}^2$. Give your answer in terms of π .

24. This large sign appears in a parking lot at the entrance to car park 5. It has a uniform width, with the dimensions as shown. Write an algebraic expression for the area of the front of the sign.



- 25.** At the end of a lesson using boxes of Smarties, the remaining Smarties are shared equally among the members of each group. In one group there are 6 boxes of Smarties left over and 42 Smarties that are no longer in boxes.
- a.** Write an expression for the total number of Smarties left over. (*Hint:* Let n = the number of Smarties in a box.)
 - b.** Write the expression from part **a** in factorised form.
 - c.** If there are 3 people in the group, write an expression for the number of Smarties each person receives.
 - d.** If there are x people in the group, write an expression for the number of Smarties each person receives.

Answers

Topic 8 Algebraic techniques 1

Exercise 8.2 Variables

1. a. 3 b. 7 c. -2 d. -8 e. 1
 f. -1 g. $\frac{1}{3}$ h. $\frac{1}{2}$ i. $-\frac{1}{4}$ j. $-\frac{1}{9}$
 2. a. 6 b. 7 c. 3 d. -2 e. 3
 f. -2 g. -7 h. -11 i. $-\frac{1}{2}$ j. $-\frac{2}{3}$
 k. $\frac{1}{4}$ l. 1 m. -3 n. -2 o. 0
 3. a. i. 3 ii. 5 iii. 8 iv. $5x^2$
 b. i. 3 ii. -9 iii. -6 iv. $-9m^2$
 c. i. 4 ii. 5 iii. 5 iv. $-7x^2$
 d. i. 4 ii. 9 iii. 4 iv. $-9b^2$
 e. i. 5 ii. 11 iii. -4 iv. $-7q$
 f. i. 5 ii. -9 iii. 5 iv. $-9p$
 g. i. 4 ii. 4 iii. -2 iv. $-3ac$
 h. i. 5 ii. 5 iii. 9 iv. $-3u$
 i. i. 5 ii. -1 iii. 8 iv. $-m$
 j. i. 5 ii. 7 iii. 14 iv. $-3cd^2$
 4. a. 7, 2, -8 b. 8, 2, -4 c. 3, 0, 0 f. 2, cannot be done, -1
 d. 0, -2, 0 e. 1, 0, 2 c. A d. B
 5. a. B b. E c. 6 f. -3
 6. a. 4 b. -6 c. 7
 d. 7 e. -12
 7. a. 7 b. -40
 8. a. i. 32 ii. 250
 b. i. 4 ii. 3
 9. a. 15 b. 10 c. 18 d. 45 e. 30
 f. 10.72 g. 21 h. 22.4 i. 50 j. 171
 k. 4 l. -30 m. 32.75 n. 38.33 o. 34.56
 10. a. 12 cm^2 b. 12 cm
 11. a. 15 b. Answers will vary.
 12. a. 5 b. Answers will vary.
 13. a. 63 cm^3 b. 4 cm
 14. a. 10 b. 10
 15. a. 19.44 b. Answers will vary.
 16. a. 461.81 b. 9.55
 17. a. 1671.33 b. 2.64

18. a.

d	$2d + 2g$	g
$2g$	$d + g$	$2d$
$2d + g$	0	$d + 2g$

Magic number = $3d + 3g$

- b, c. Answers will vary.
 19. a. mn b. $(m - 2)(n - 2)$ c. $(2m + 2n)$ units
 20. $y - (z - x)$
 21. You should avoid using any letter which could easily be mistaken for a number, such as 'o'.

Exercise 8.3 Algebra in worded problems

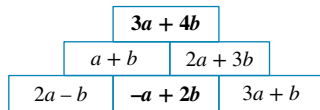
1. a. 11 b. $x + 5$ c. $y + 5$
2. a. 3.5 km b. $(x + 2)$ km
3. a. 4 b. $2.5 + t$ c. $2.5 - y$
4. a. $10n$, where n = number of shirts
b. $N + 30$, where N = the number of Nick's dollars
c. i. $g - 4$ ii. $7g - 4$
5. a. i. 94 ii. $6x + y$
b. i. 66 ii. $6p + q$
c. i. 28 ii. $6x + y - 6p - q$
6. xy dollars
7. a. $\frac{x}{n}$ b. $\frac{x}{2n}$
8. a. $(30 - d)$ cm b. $\frac{30 - d}{4}$ cm c. $\frac{3(30 - d)}{4}$ cm
9. a. $\frac{x}{4}$ b. $\frac{x}{6}$ c. $\frac{x}{4} - \frac{x}{6} = \frac{x}{12}$
10. a. $\frac{24 - c}{7}$ hours b. $c + \frac{24 - c}{7} = \frac{6c + 24}{7}$
11. a. 1 b. 8.30 pm and 9.00 pm c. 5
d. 24 e. 7.30 pm and 8.00 pm
12. a. $b = 2h - 1$ b. 19 blocks
13. 4 dots = 4 triangles
5 dots = 10 triangles
6 dots = 20 triangles
7 dots = 35 triangles
 $t = \frac{d!}{6(d - 3)!}$ where $d!$ is the product of all positive integers less than or equal to d .
14. 15 red vases and 10 clear vases
15. Answers will vary.
16. a. $x = \frac{T - 3}{T - 1}$
b, c. Undefined; answers will vary.
17. We define pronumerals or variables in terms of what they represent to avoid confusion as to what we are trying to determine.
18. The father is 32 years old.

Exercise 8.4 Simplifying algebraic expressions

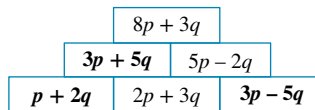
1. a. $9ab, -ab$ b. $4x$ c. $-3za, -az$ d. $2x^2, -x^2$
e. $-2x^2y$ f. $-x^2y^5, -3x^2y^5$ g. $p^3x^2w^5, -5x^2p^3w^5$ h. $4y^5z^4x^2, -2x^2z^4y^5$
2. a. $7x$ b. $11y$ c. $19m$ d. $11q$
e. $8r$ f. $3x$ g. $8a$ h. $8y$
i. $13x$ j. $16p$ k. $9q^2$ l. $3x^2$
m. $8x^2 - 3y$ n. $2m^2 + 2n$ o. $7x^2 + x$ p. $9h^2 + h + 9$
q. $-2g^2 + g - 12$ r. $-5m^2 + m + 15$ s. $12a^2 + b + 4b^2$ t. $3m + 7n^2$
u. $12xy + 2y^2$ v. $2ab + 5a^2b$ w. $16x^2y - 3xy$ x. $m^2n + 11n$
y. $-7x^2 + 2x^2y$ z. $-3a^2b - 9a^2 + 5b + 4$
3. a. B b. D c. A d. D
4. a. $6mn$ b. $20xy$ c. $8pq$ d. $-10xy$
e. $-12xy$ f. $15mn$ g. $10a^2$ h. $20y^2$
i. $5p^2$ j. $7m^2$ k. $6mnp$ l. $-6ab^2$
m. $10m^2n$ n. $-18a^2b$ o. $30x^3y^2$ p. $-12p^2q^3$
q. $-56c^3d$ r. $30a^4b^7$

5. a. $3x$ b. $3m$ c. $2y$ d. $4m$
 e. $4m$ f. $2x$ g. $-7x$ h. $-4m$
 i. $\frac{m}{2}$ j. $\frac{x}{3}$ k. $\frac{4m}{9}$ l. $\frac{4x}{3}$
 m. $\frac{1}{5m}$ n. $\frac{1}{2a}$ o. $2yz$ p. $\frac{35ab}{2}$
 q. $\frac{xy}{4}$ r. $\frac{-7yz}{11}$
 6. a. $40x^2y^2$ b. $56ax^2y^2$ c. $12x^3y^2$
 d. $\frac{x^2}{2y}$ e. $\frac{-5a}{4b}$ f. 2
 g. $40a^3b^2$ h. $-8a^3b^3$ i. $16a^4$
7. a. i. $(0.01mp + nq)$ dollars ii. $(mp + 100nq)$ cents
 b. $20 - (0.01mp + nq)$ dollars
8. Shirt = \$12 each
 Trousers = \$30 each
9. Explanations will vary. The correct answer is $7a^2c^2$.

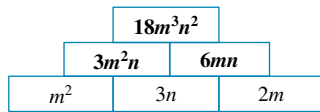
10. a. i.



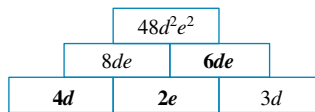
ii.



b. i.



ii.



11. a. $P = 2x + 2(4x - 7)$
 b. $P = 10x - 14$
 c. 16
 d. If $x = 1$ the rectangle will have a negative value perimeter, which is not possible.
12. a. $l = 4(x + 20) + 4(x + 40) + 4(x + 20)$
 b. $l = 12x + 320$
 c. \$19.20
13. a. $P_1 = 10x + 12 \text{ cm}$
 $P_2 = 16x \text{ cm}$
 b. $x = 2$
14. The expression ab is the same as ba because the order in which we multiply does not matter. However, it is convention to write pronumerals in alphabetical order, so ab is preferred over ba .

Exercise 8.5 Simplifying algebraic fractions 1

1. a. $\frac{7x}{12}$ b. $\frac{y}{6}$ c. $-\frac{m}{8}$ d. $\frac{x}{4}$ e. $\frac{9m}{14}$
 f. $\frac{2t}{15}$ g. $\frac{13a}{10}$ h. $-\frac{p}{6}$ i. $\frac{17q}{15}$ j. $\frac{x}{6}$
2. a. $-\frac{1}{2p}$ b. $\frac{11}{10x}$ c. $-\frac{11}{4m}$
 d. $\frac{3}{5b}$ e. $\frac{25}{18c}$ f. $\frac{22}{15y}$
3. a. 3 b. 2 c. $\frac{1}{4}$ d. $\frac{1}{5}$ e. 3
 f. $\frac{1}{3}$ g. $\frac{m}{12}$ h. $\frac{n}{6}$ i. 14 j. $\frac{1}{9}$
 k. $\frac{8}{y}$ l. $\frac{5x}{3}$ m. $\frac{4}{21}$ n. $\frac{2}{45}$ o. $\frac{1}{4}$
 p. $2a^2$ q. $3a$ r. $\frac{1}{3a}$

4. a. 3 b. 2 c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. 3
 f. $\frac{1}{3}$ g. 4 h. $\frac{3}{10}$ i. $\frac{3}{2}$ j. $\frac{7b}{4}$
 k. $\frac{3m}{5}$ l. $\frac{8b}{3}$ m. $\frac{3q}{5}$ n. $\frac{1}{2}$ o. $\frac{x}{5}$
 5. a. $\frac{1}{10xy^4}$ b. $\frac{3b^2}{2}$ c. $\frac{5}{2x}$ d. $\frac{7b^5}{4}$ e. $\frac{1}{4}$ f. $\frac{x^3}{5y}$
 6. a. $P = \frac{x}{3} + x + \frac{5x}{7}$ b. Answers will vary. c. \$18
 7. a. $P = 2w + \frac{32w}{7}$ b. Answers will vary. c. \$72.22
 8. a. The student has cancelled both fractions incorrectly. Only common factors can be cancelled. A term cannot be cancelled if it is part of an addition.
 b. The student should have left the answers as they are shown on the left-hand side, because there are no common factors to factorise the expressions.
 9. a. $\frac{4}{21}$ b. $\frac{10}{3}$

Exercise 8.6 Expanding algebraic expressions

1. a. 64 b. 1100 c. -20 d. 693
 2. a. $3x + 6$ b. $4x + 12$
 3

$3x$	6

 4

$4x$	12

 x 2 x 3
 c. $4x + 4$ d. $7x - 7$ e. $-3p + 6$ f. $-x + 1$
 g. $6b - 12$ h. $24m - 16$
 3. a. $x^2 + 2x$ b. $a^2 + 5a$
 x

x^2	$2x$

 a

a^2	$5a$

 x 2 a 5
 c. $4x + x^2$ d. $7m - m^2$ e. $2x^2y + 4x^3$ f. $-3y^3 + 12xy^2$
 g. $3b^3 - b^5$ h. $-15x^4 - 6x^6y$
 4. a. $2p - 2$ b. $5x - 17$ c. $-7p - 17$ d. $-12p + 3$
 e. $6x^2 - 20x$ f. $2m^2 + 7m$ g. $3px + 6x - 5$ h. $4y^2 - 4y + 7$
 i. $-4p^2 + 13p$ j. $4x - 13y$ k. $2m^2 - 8m - 4$ l. $-3p^2 + 10pq - 1$
 m. $-30a + 10ab$ n. $7cd - 12c^2 - 5c$ o. $-2p - 17$ p. $3 - 3m$
 5. a. $8x + y$ b. $10p + 8q$ c. $18a + 29b$ d. $19c + 22d$
 e. $2m - 11n$ f. $6x - 11y$ g. $9x + 5y$ h. $-14p - 8q$
 i. $-4a + 3b$ j. $4x - y$ k. $5p - 10q$ l. $-8c + 9d$
 m. $13x - 19y$ n. $-7p + 11q$ o. $-5a + 3b$ p. $8c + d$
 6. a. $2ab + 2a - 3b$ b. $2xy + 4x - 2y$ c. $2cd + 3c$ d. $2pq - 2p$
 e. $5cd - 11c$ f. $5ab - 21a - 3b$ g. $5m$ h. $6cd - 36c$
 i. $6m^2 + 6m - 10$ j. $9cd - 8c$ k. $-15a^2 + 2b^2 - 9ab$ l. $-8c^2 + 3d^2 + 22cd$
 m. $12m^2 - 20m - 4$ n. $2p^2 + 7p - 6$ o. $-7x^2 + 41x - 6$ p. $-10y^2 - 6y - 12$
 7. a. B b. A c. D
 8. a. $a^2 + 5a + 6$ b. $x^2 + 7x + 12$ c. $y^2 + 5y + 6$ d. $m^2 + 9m + 20$
 e. $b^2 + 3b + 2$ f. $p^2 + 5p + 4$ g. $a^2 + a - 6$ h. $x^2 + x - 20$
 i. $m^2 - m - 12$ j. $y^2 + 2y - 15$ k. $y^2 - 4y - 12$ l. $x^2 - 2x - 3$
 m. $x^2 - 7x + 12$ n. $p^2 - 5p + 6$ o. $x^2 - 4x + 3$
 9. a. $2a^2 + 7a + 6$ b. $3m^2 + 7m + 2$ c. $6x^2 + 10x + 4$
 d. $4c^2 - 31c + 42$ e. $2t^2 - 17t + 35$ f. $2x^2 - 11x + 9$
 g. $-6t^2 + 11t + 10$ h. $15x^2 - 31x + 14$ i. $25x^2 - 20x + 4$

10. a. $xz + x + yz + y$
 e. $a^2 + 3ab + 2b^2$
 i. $3xy + 3yz + xz + z^2$
 m. $12x^2 - 7xy + y^2$
- b. $pr + 3p + qr + 3q$
 f. $2c^2 - 5cd - 3d^2$
 j. $ab + ac + 2b^2 + 2bc$
 n. $2p^2 - pr - 2pq + qr$
- c. $2xz + 8x + yz + 4y$
 g. $2x^2 - xy - 3y^2$
 k. $3p - 9pr - 2q + 6qr$
 o. $15k + 5 - 6jk - 2j$
- d. $3pr + 3p + qr + q$
 h. $4p^2 + pq - 3q^2$
 l. $7cd - 35c - 2d^2 + 10d$
11. a. A
 b. E
 c. C
12. a. $x^2 - 9$
 d. $x^2 - 1$
 b. $x^2 - 25$
 e. $x^2 - 4$
 c. $x^2 - 49$
13. a. $x^2 + 2x + 1$
 d. $x^2 - 6x + 9$
 b. $x^2 + 4x + 4$
 e. $x^2 - 10x + 25$
 c. $x^2 + 16x + 64$
14. a. $6.3x^2 + 5.53xy - 3.1y^2$
 b. $4.41x^2 - 10.24y^2$
 c. $11.56x^2 + 34.68xy + 26.01y^2$
15. a. Surface area = $38x^2 + 14x - 6$
 b. Volume = $12x^3 + 5x^2 - 3x$
16. a. i. $(x + 3y)(4x - y)$
 b. i. $\frac{(2x - 1)(x + 5)}{2}$
 ii. $4x^2 + 11xy - 3y^2$
 ii. $\frac{2x^2 + 9x - 5}{2}$
 iii. $x > -3y, x > \frac{y}{4}$
 iii. $x > \frac{1}{2}$

17. a. Student C

b. Corrections to the students' answers are shown in bold.

Student A:

$$\begin{aligned} (3x + 4)(2x + 5) \\ = 3x \times 2x + 3x \times 5 + 4 \times 2x + 4 \times 5 \\ = \mathbf{6x^2} + 23x + 20 \end{aligned}$$

(Also, $29x + 20$ does not equal $49x$.)

Student B:

$$\begin{aligned} (3x + 4)(2x + 5) \\ = 3x \times 2x + \mathbf{3x \times 5} + 4 \times 2x + 4 \times 5 \\ = 6x^2 + 15x + 8x + 20 \\ = 6x^2 + 23x + 20 \end{aligned}$$

18. Answers may vary.

19. a. i. The student did not multiply both terms.

ii. The student used addition instead of multiplication.

iii. The student did not change negative and positive signs.

b. $100 - 15x$

20. a. $-2(a - 5) = 2a - 10$ is incorrect because the student did not change the multiplied negative signs for -2×-5 to a positive sign. The correct answer is $-2a + 10$.

b. $2b(3b - 1) = 6b^2 - 1$ is incorrect because the student did not multiply -1 and $2b$ together. The correct answer is $6b^2 - 2b$.

c. $-2(c - 4) = 2c + 8$ is incorrect because the student has left out the negative sign when multiplying -2 and c . The correct answer is $-2c + 8$.

21. a. Negative sign ignored

b. Negative sign ignored

c. Distributive law not used

22. The algebraic expressions $(2x + y)(x + y)$ and $(x + y)(2x + y)$ are equal because they are commutative under multiplication; that is, the order in which $(2x + y)$ and $(x + y)$ are multiplied together makes no difference to the result.

Exercise 8.7 The highest common factor

1. a. $2a$
 e. $3x$
 i. $-mn$
 2. a. 4
 e. 3
 i. $7x$
 m. 3
 3. C
 4. a. $4(x + 3y)$
- b. 4
 f. $15a$
 j. ab
 b. 3
 f. 25
 j. $30q$
 n. 5
 b. $5(m + 3n)$
- c. $3x$
 g. $-3x$
 k. $3x$
 c. 5
 g. 1
 k. 10
 o. x^2
 c. $7(a + 2b)$
- d. -4
 h. m
 l. $5n^2$
 d. 8
 h. $6a$
 l. $3x$
 p. 2
 d. $7(m - 3n)$

- e. $-8(a + 3b)$
 i. $8(4x + y + 2z)$
 m. $5(p^2 - 4q)$
 q. $4(4p^2 + 5q + 1)$
5. a. $3(3a + 7b)$
 e. $5(5y^2 - 3x)$
 i. $11(11a^2 - 5b + 10c)$
 m. $7(9a^2b^2 - 7 + 8ab^2)$
6. a. $-(x - 5)$
 e. $-6(p + 2)$
 i. $-7(y^2 + 7z)$
 m. $-3(3a^2b - 10)$
 q. $-5(2 + 5p^2 + 9q)$
7. a. $a(a + 5)$
 e. $m(18 + 5m)$
 i. $p(7 - pq + q)$
 m. $xy(2x + 1 + 5y)$
8. a. $5x(x + 3)$
 e. $4a(8a - 1)$
 i. $-3a(a - 3)$
 m. $2m(2 - 9m)$
9. a. $2(x + 3)(4x + 1)$
 b. $A = x(x + 3)$
 $C = 2(x + 3)x$
 $B = 5x^2 + 17x + 6$
- f. $4(2x - y)$
 j. $4(4m - n + 6p)$
 n. $5(x + 1)$
 r. $12(1 + 3a^2b - 2b^2)$
- b. $2(2c + 9d^2)$
 f. $4(4a^2 + 5b)$
 j. $2(5 - 11x^2y^3 + 7xy)$
 n. $11(2 + 9p^3q^2 - 4p^2r)$
- b. $-(a - 7)$
 f. $-4(a + 2)$
 j. $-6(2p^2 + 3q)$
 n. $-3(5p + 4q)$
 r. $-9(10m^2 - 3n - 6p^3)$
- b. $m(m + 3)$
 f. $p(6 + 7p)$
 j. $y(x + 9 - 3y)$
 n. $pq(5pq - 4 + 3p)$
- b. $2y(5y + 1)$
 f. $-2m(m - 4)$
 j. $-2p(6p + 1)$
 n. $-6t(1 - 6t)$
- g. $-2(6p + q)$
 k. $8(9x - y + 8pq)$
 o. $8(7q + p^2)$
- c. $4(3p^2 + 5q^2)$
 g. $6(7m^2 + 2n)$
 k. $9(2a^2bc - 3ab - 10c)$
 o. $6(6 - 4ab^2 + 3b^2c)$
- c. $-(b - 9)$
 g. $-3(n^2 - 5m)$
 k. $-7(9m - 8)$
 o. $-2(9x^2 - 2y^2)$
- c. $x(x - 6)$
 g. $n(7n - 2)$
 k. $c(5 + 3cd - d)$
 o. $xy(6xy - 5 + x)$
- c. $4p(3p + 1)$
 g. $-5x(x - 5)$
 k. $-5b(3b + 1)$
 o. $-8p(1 + 3p)$
- h. $6(p + 2pq + 3q)$
 l. $3(5x^2 - y)$
 p. $7(p - 6x^2y)$
- d. $7(5 - 2m^2n)$
 h. $9(7p^2 + 9 - 3y)$
 l. $12(12p + 3q^2 - 7pq)$
- d. $-2(m + 3)$
 h. $-7(x^2y^2 - 3)$
 l. $-2(6m^3 + 25x^3)$
 p. $-3(ab - 6m + 7)$
- d. $q(14 - q)$
 h. $a(a - b + 5)$
 l. $ab(3 + a + 4b)$
- d. $6m(4m - 1)$
 h. $-7y(y - 2)$
 l. $-13y(2y + 1)$

10. Yes, the two expressions are equivalent.

11. True

12. The answer $4xy(6x + 4y)$ is only partially factorised, as $6x + 4y$ has a factor of 2. Therefore, the fully factorised expression for $24x^2y + 16xy^2$ is $8xy(3x + 2y)$.

Exercise 8.8 Factorising monic quadratics

1. a. $x^2 + 6x + 8$
 e. $m^2 + 6m + 5$
 i. $x^2 - x - 6$
 m. $x^2 + 5x - 14$
 q. $z^2 + 8z + 16$
2. a. 1, 4; 2, 3
 3. a. -1, -4; -2, -3
 b. -1, -6; -2, -5; -3, -4
 c. -1, -7; -2, -6; -3, -5; -4, -4
 d. -1, -10; -2, -9 ... -5, -6
4. a. $(x + 1)(x + 3)$
 e. $(a + 5)(a + 1)$
 i. $(n + 4)^2$
 m. $(t - 6)(t - 2)$
- b. $(x - 1)(x - 3)$
 f. $(a - 5)(a - 1)$
 j. $(n + 8)(n + 2)$
 n. $(t + 9)(t + 2)$
5. a. 7 and -1; 8 and -2; 9 and -3; 10 and -4; 11 and -5
 b. 1 and -7; 2 and -8; 3 and -9; 3 and -10; 5 and -11
 c. 4 and -1; 5 and -2; 6 and -3; 7 and -4; 8 and -5
 d. 1 and -4; 2 and -5; 3 and -6; 4 and -7; 5 and -8
 e. 1 and -9; 2 and -10; 3 and -11; 4 and -12; 5 and -13
- c. $x^2 - 9x + 20$
 g. $t^2 + 19t + 88$
 k. $v^2 - 3v - 40$
 o. $n^2 + 13n - 30$
 s. $n^2 - 10n + 21$
 c. 1, 11; 2, 10 ... 6, 6
- d. $x^2 + 9x + 20$
 h. $t^2 - 30t + 200$
 l. $v^2 + 3v - 40$
 p. $a^2 - 3a - 18$
 t. $v^2 + v - 2$
 d. 1, 19; 2, 18 ... 10, 10
- d. $(x - 1)(x - 11)$
 h. $(x - 5)(x - 2)$
 l. $(x - 7)(x - 6)$
- o. $(u + 2)(u + 3)$

- f. 15 and -1 ; 16 and -2 ; 17 and -3 ; 18 and -4 ; 19 and -5
 g. 1 and -6 ; 2 and -7 ; 3 and -8 ; 4 and -9 ; 5 and -10
 h. 8 and -1 ; 9 and -2 ; 10 and -3 ; 11 and -4 ; 12 and -5
6. a. $(x + 6)(x - 3)$ b. $(x - 6)(x + 3)$ c. $(x - 5)(x + 3)$ d. $(x + 5)(x - 3)$
 e. $(n - 14)(n + 1)$ f. $(n + 7)(n - 5)$ g. $(v + 6)(v - 1)$ h. $(v - 6)(v + 1)$
 i. $(t + 6)(t - 2)$ j. $(t - 7)(t + 2)$ k. $(x - 5)(x + 4)$ l. $(x - 4)(x + 5)$
 m. $(n + 10)(n - 9)$ n. $(n - 10)(n + 7)$ o. $(x - 5)(x + 1)$
7. a. $(x + 1)(x + 6)$
 b. $x^2 + 7x + 10 = (x + 2)(x + 5)$, $x^2 + 7x + 12 = (x + 3)(x + 4)$
8. a. $(x - 1)(x + 4)$ b. Answers will vary.
9. a. $2(x + 2)(x - 7)$ b. $4(x + 2)(x + 5)$ c. $-2(x + 4)(x - 3)$ d. $5(x - 5)(x - 3)$
10. D
11. Answers will vary.
12. $3x + 6x = 9x$
 $6x^2 + 3x + 6x + 3 = 3x(2x + 1) + 3(2x + 1)$
 $= (3x + 3)(2x + 1)$
 $= 3(x + 1)(2x + 1)$
13. a. $w = \frac{x^2 + 2x - 15}{x + 5}$
 b. $w = 62$ cm
 c. $l = 103$ cm
14. a. 1×120 m
 2×60 m
 3×40 m
 4×30 m
 5×24 m
 6×20 m
 8×15 m
 10×12 m
 b. $l = x + 8$, $w = x - 6$
 c. 6×20 m
15. Answers will vary. However, if we consider the expression $x^2 + bx + c$, you could look for factors of the constant (c) that sum to b . Alternatively you could reverse the area model of the binomial expansion to find a pattern for factorising.

Investigation | Rich task

- Solution not required
- Answers will vary.
- 0.12 m
- 0.0144 m^2
- Yellow: $0.03 \text{ m} \times 0.03 \text{ m}$
 Black: $0.03 \text{ m} \times 0.06 \text{ m}$
 White: $0.06 \text{ m} \times 0.06 \text{ m}$
- Yellow: 0.0009 m^2
 Black: 0.0018 m^2
 White: 0.0036 m^2
- Yellow: 0.0036 m^2
 Black: 0.0072 m^2
 White: 0.0036 m^2
- Answers will vary.

Exercise 8.9 Review questions

1. B
2. D
3. A
4. B
5. D
6. A
7. a. i. 4
b. i. $y + 8$
- ii. -8
iii. $x - y$
- iv. $-8xy^2$
v. $5y - 2x$
8. a. xy dollars
b. i. $(24 - k)m$
ii. $\frac{24 - k}{3}m$
9. a. $17p$
d. $16c^2d - 2cd$
- b. $y^2 + 2y$
e. $n^2 - 4p^2q + 6$
- c. $-3s^2t$
f. $15ab - 3a^2b^2$
10. a. $12ab$
b. $2ab^2$
c. $8x^2y^2$
d. $\frac{x}{3}$
e. $\frac{9}{2b}$
11. a. $5x + 15$
b. $-y - 5$
c. $-3x + 2x^2$
d. $-8m^2 - 4m$
12. a. $3x + 3$
b. $-10m - 1$
c. $4m^2 - 9m - 5$
d. $4p - 6$
13. a. $9a + 8b$
b. $-5x - 18y$
c. $-mn + 11m$
d. $4x^2 - 10x + 3$
14. a. $x^2 + 9x + 20$
b. $m^2 - m - 2$
c. $3m^2 - 17m + 10$
d. $2a^2 - 5ab - 3b^2$
15. a. $2x^2 + 8x + 8$
b. $2m^2 + 11m - 5$
c. $6x + 15$
d. $-7b + 37$
16. a. 23
b. 134
17. a. 307.88 cm^3
b. 5379.98 cm^3
18. a. $6(x + 2)$
d. $8x(2x - 3y)$
b. $6x^2(1 + 2xy)$
e. $-2(x + 2)$
c. $4(2a^2 - b)$
f. $b(b - 3 + 4c)$
19. a. $\frac{7x}{10}$
e. 2
i. $\frac{2}{3}$
m. $\frac{x}{6}$
b. $\frac{x}{12}$
f. $\frac{1}{4}$
j. 2
n. $\frac{5}{2b}$
c. $\frac{x}{8}$
g. $\frac{1}{4}$
k. $\frac{1}{6}$
d. $\frac{31}{24m}$
h. $\frac{2}{5}$
l. $\frac{3}{7}$
20. a. $(c + 1)(c + 4)$
e. $(q + 7)(q - 6)$
i. $(c - 13)(c + 2)$
b. $(x - 7)(x + 1)$
f. $(x - 6)(x + 4)$
j. $(m - 2)(m - 5)$
c. $(x - 2)(x - 2) = (x - 2)^2$
g. $(y - 6)(y - 4)$
k. $(x + 9)(x - 3)$
d. $(p - 2)(p + 12)$
h. $(x + 1)(x + 2)$
l. $(m + 22)(m + 2)$
21. a. $8x \text{ cm}$
c. i. $(3x + y) \text{ cm}$
ii. $(8x + 2y) \text{ cm}$
iii. 780 cm
iv. $(3x^2 + xy) \text{ cm}^2$
v. $27\,000 \text{ cm}^2$
b. $3x^2 \text{ cm}^2$
22. a. $(8 + 2p)(15 + 2p) \text{ m}^2$
c. $(4p^2 + 46p) \text{ m}^2$
b. $(4p^2 + 46p + 120) \text{ m}^2$
d. $4p^2 + 46p = 200$
23. $\frac{2\pi x}{y} \text{ cm}$
24. $5xy - 4y^2$
25. a. $6n + 42$
b. $6(n + 7)$
c. $2(n + 7)$
d. $\frac{6(n + 7)}{x}$

TOPIC 9

Probability

[Stages 5.1 and 5.2]

9.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



9.1.1 Why learn this?

Probability lies at the heart of nature. Think about all the events that had to happen for you to be born, for example ... the odds are extraordinary.

Probability is that part of mathematics that gives meaning to the idea of uncertainty, of not fully knowing or understanding the occurrence of some event.

We often hear that there is a good chance of rain; people bet with different odds that a favourite horse will win at Caulfield, and so on. In each case, we are making a guess as to what will be the outcome of some event.

It is important to learn about probability so that you can understand that chance is involved in many decisions that you will have to make in your life and in everyday events.

DISCUSSION

Can all events in life be given an estimated probability of happening?

LEARNING SEQUENCE

- 9.1 Overview
- 9.2 [Stage 5.1] Theoretical probability
- 9.3 [Stage 5.1] Experimental probability
- 9.4 [Stage 5.1] Venn diagrams and two-way tables
- 9.5 [Stage 5.2] Two-step experiments
- 9.6 [Stage 5.2] Mutually exclusive and independent events
- 9.7 [Stage 5.2] Conditional probability
- 9.8 Review

LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- selects and uses appropriate strategies to solve problems MA5.1-2WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- calculates relative frequencies to estimate probabilities of simple and compound events MA5.1-13SP
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- describes and calculates probabilities in multi-step chance experiments MA5.2-17SP

CONTENT DESCRIPTIONS

Students:

Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP226)

List all outcomes for two-step chance experiments, with and without replacement, using tree diagrams or arrays; assign probabilities to outcomes and determine probabilities for events (ACMSP225)

Describe the results of two- and three-step chance experiments, with and without replacement, assign probabilities to outcomes, and determine probabilities of events; investigate the concept of independence (ACMSP246)

Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and to identify common mistakes in interpreting such language (ACMSP247)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — What are the chances? (eles-1700)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

9.2 Theoretical probability

[Stage 5.1]

9.2.1 The language of probability

- The **probability** of an event is a measure of the likelihood that the event will take place.
- If an event is certain to occur, then it has a probability of 1.
- If an event is impossible, then it has a probability of 0.
- The probability of any other event taking place is given by a number between 0 and 1.
- The higher the probability, the more likely it is for the event to occur.
- Descriptive words such as 'impossible', 'unlikely', 'likely' and 'certain' are commonly used when referring to the chance of an event occurring. Some of these are shown on the **probability scale** below.



DISCUSSION

List three real-life events for each of the following categories: impossible, unlikely, even chance, likely, certain.

WORKED EXAMPLE 1

Copy the probability scale shown and insert the following events at appropriate points.



- You will sleep tonight.
- You will come to school the next Monday during a school term.
- It will snow in Victoria this year.

THINK

a 1 Carefully read the given statement and assign a position on the probability scale.

2 Provide reasoning.

b 1 Carefully read the given statement and assign a position on the probability scale.

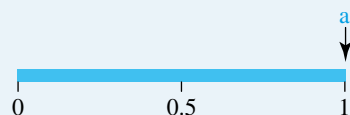
2 Provide reasoning.

c 1 Carefully read the given statement and assign a position on the probability scale.

2 Provide reasoning.

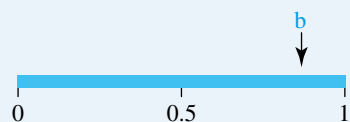
WRITE/DRAW

a



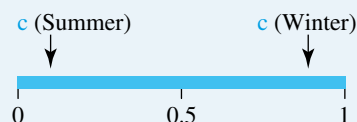
Under normal circumstances, I will certainly sleep tonight.

b



It is very likely but not certain that I will come to school on a Monday during term. Circumstances such as illness or public holidays may prevent me from coming to school on a specific Monday during a school term.

c



It is highly likely but not certain that it will snow in Victoria during winter. The chance of snow falling in Victoria in summer is highly unlikely but not impossible.

9.2.2 Key terms

- The study of probability uses many special terms that must be clearly understood. Here are explanations of some of the more common terms.

Chance experiment: A chance experiment is a process, such as rolling a die, that can be repeated many times.

Trial: A trial is one performance of an experiment to get a result. For example, each roll of the die is called a trial.

Outcome: The outcome is the result obtained when the experiment is conducted. For example, when a normal six-sided die is rolled, the outcome can be 1, 2, 3, 4, 5 or 6.

Sample space: The set of all possible outcomes is called the sample space and is given the symbol ξ (or S). For the example of rolling a die, $\xi = \{1, 2, 3, 4, 5, 6\}$.

Event: An event is the favourable outcome of a trial and is often represented by a capital letter. For example, when a die is rolled, A could be the event of getting an even number; $A = \{2, 4, 6\}$.

Favourable outcome: A favourable outcome for an event is any outcome that belongs to the event. For event A above (rolling an even number), the favourable outcomes are 2, 4 and 6.

WORKED EXAMPLE 2

For the chance experiment of rolling a die:

- a list the sample space
- b list the events:
 - i rolling a 4
 - ii rolling an even number
 - iii rolling at least 5
 - iv rolling at most 2
- c list the favourable outcomes for:
 - i {4, 5, 6}
 - ii not rolling 5
 - iii rolling 3 or 4
 - iv rolling 3 and 4.



THINK

- a The outcomes are the numbers 1 to 6.
- b
 - i This describes only 1 outcome.
 - ii The possible even numbers are 2, 4 and 6.
 - iii 'At least 5' means 5 is the smallest.
 - iv 'At most 2' means 2 is the largest.
- c
 - i The outcomes are shown inside the brackets.
 - ii 'Not 5' means everything except 5.
 - iii The event is {3, 4}.
 - iv There is no number that is both 3 and 4.

WRITE

- a $\xi = \{1, 2, 3, 4, 5, 6\}$
- b
 - i {4}
 - ii {2, 4, 6}
 - iii {5, 6}
 - iv {1, 2}
- c
 - i 4, 5, 6
 - ii 1, 2, 3, 4, 6
 - iii 3, 4
 - iv There are no favourable outcomes.

9.2.3 Theoretical probability

- When a coin is tossed, there are two possible outcomes, Heads or Tails. That is, $\xi = \{H, T\}$.
- In ideal circumstances, the two outcomes have the same likelihood of occurring, so they are allocated the same probability.

For example, $P(\text{Heads}) = \frac{1}{2}$ (this says the probability of Heads = $\frac{1}{2}$) and $P(\text{Tails}) = \frac{1}{2}$.

- The total of the probabilities equals 1, as there are no other possible outcomes.
- In general, if all outcomes are **equally likely** to occur (ideal circumstances), then the probability of event A occurring is given by:

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}.$$



WORKED EXAMPLE 3

A die is rolled and the number uppermost is noted. Determine the probability of each of the following events.

a $A = \{1\}$

b $B = \{\text{odd numbers}\}$

c $C = \{4 \text{ or } 6\}$

THINK

There are 6 possible outcomes.

a A has 1 favourable outcome.

b B has 3 favourable outcomes: 1, 3 and 5.

c C has 2 favourable outcomes.

WRITE

a $P(A) = \frac{1}{6}$

b $P(B) = \frac{3}{6}$
 $= \frac{1}{2}$

c $P(C) = \frac{2}{6}$
 $= \frac{1}{3}$

learn^{on} RESOURCES — ONLINE ONLY

Interactivity: Probability scale (int-3824)

Interactivity: Theoretical probability (int-6081)

Interactivity: Random number generator (int-0089)

Digital doc: SkillsHEET Probability scale (doc-6307)

Digital doc: SkillsHEET Understanding a deck of playing cards (doc-6308)

Digital doc: SkillsHEET Listing the sample space (doc-6309)

Digital doc: SkillsHEET Theoretical probability (doc-6310)

Exercise 9.2 Theoretical probability

Individual pathways

PRACTISE

Questions:
1–16

CONSOLIDATE

Questions:
1–18

MASTER

Questions:
1–20

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and Fluency

1. **WE1** Copy the given probability scale and insert the following events at appropriate points.

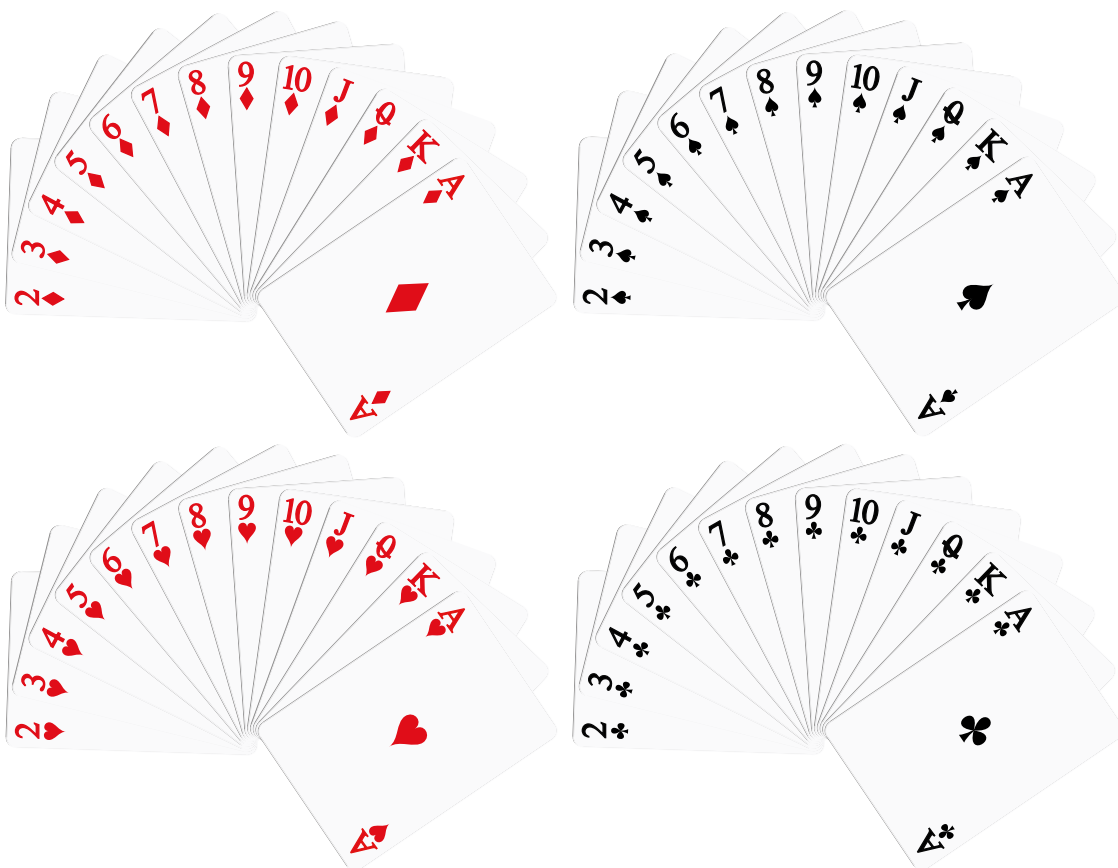


- a. The school will have a lunch break on Friday.
- b. Australia will have a swimming team in the Commonwealth Games.
- c. Australia will host two consecutive Olympic Games.
- d. At least one student in a particular class will obtain an A for Mathematics.
- e. Mathematics will be taught in secondary schools.
- f. In the future most cars will run without LPG or petrol.
- g. Winter will be cold.
- h. Bean seeds, when sown, will germinate.

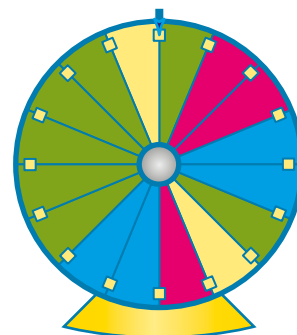
2. Indicate the chance of each event listed in question 1 using one of the following terms: certain, likely, unlikely, impossible.
3. **WE2a** For each chance experiment below, list the sample space.
 - a. Rolling a die
 - b. Tossing a coin
 - c. Testing a light bulb to see whether it is defective or not
 - d. Choosing a card from a normal deck and noting its colour
 - e. Choosing a card from a normal deck and noting its suit



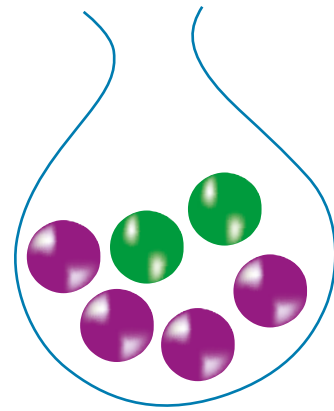
4. A normal 6-sided die is rolled. List the sample space for each of the following events.
 - a. Rolling a number less than or equal to 3
 - b. Rolling an odd number
 - c. Rolling an even number or 1
 - d. Not rolling a 1 or 2
 - e. Rolling at most a 4
 - f. Rolling at least a 5
5. **WE2c** A normal 6-sided die is rolled. List the favourable outcomes for each of the following events.
 - a. $A = \{3, 5\}$
 - b. $B = \{1, 2\}$
 - c. $C =$ 'rolling a number greater than 5'
 - d. $D =$ 'not rolling a 3 or a 4'
 - e. $E =$ 'rolling an odd number or a 2'
 - f. $F =$ 'rolling an odd number and a 2'
 - g. $G =$ 'rolling an odd number and a 3'
6. **WE2a, b** A card is selected from a normal deck of 52 cards and its suit is noted.
 - a. List the sample space.
 - b. List each of the following events.
 - i. 'Drawing a black card'
 - ii. 'Drawing a red card'
 - iii. 'Not drawing a heart'
 - iv. 'Drawing a black or a red card'
7. How many outcomes are there for:
 - a. rolling a die
 - b. tossing a coin
 - c. drawing a card from a standard deck
 - d. drawing a card and noting its suit
 - e. noting the remainder when a number is divided by 5?



8. A card is drawn at random from a standard deck of 52 cards.
Note: 'At random' means that every card has the same chance of being selected.
 Find the probability of selecting:
- an ace
 - a king
 - the 2 of spades
 - a diamond.
9. **WE3** A card is drawn at random from a deck of 52. Find the probability of each event below.
- | | |
|--|-----------------------------------|
| a. $A = \{5 \text{ of clubs}\}$ | b. $B = \{\text{black card}\}$ |
| c. $C = \{5 \text{ of clubs or queen of diamonds}\}$ | d. $D = \{\text{hearts}\}$ |
| e. $E = \{\text{hearts or clubs}\}$ | f. $F = \{\text{hearts and 5}\}$ |
| g. $G = \{\text{hearts or 5}\}$ | h. $H = \{\text{aces or kings}\}$ |
| i. $I = \{\text{aces and kings}\}$ | j. $J = \{\text{not a 7}\}$ |
10. A letter is chosen at random from the letters in the word PROBABILITY. What is the probability that the letter is:
- | | | | |
|------|----------|------------|-----------------|
| a. B | b. not B | c. a vowel | d. not a vowel? |
|------|----------|------------|-----------------|
11. The following coloured spinner is spun and the colour is noted. What is the probability of each of the events given below?
- $A = \{\text{blue}\}$
 - $B = \{\text{yellow}\}$
 - $C = \{\text{yellow or red}\}$
 - $D = \{\text{yellow and red}\}$
 - $E = \{\text{not blue}\}$

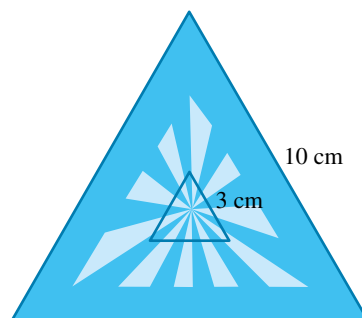
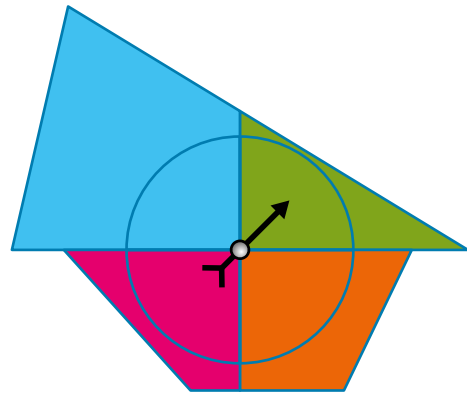


12. a. A bag contains 4 purple balls and 2 green balls. If a ball is drawn at random, then what is the probability that it will be:
 i. purple ii. green?
 b. Design an experiment like the one in part a but where the probability of drawing a purple ball is 3 times that of drawing a green ball.
13. Design a spinner using red, white and blue sections to meet each of the following conditions. (You will need to design three spinners in total.)
 a. Each colour has the same probability of being spun.
 b. Red is twice as likely to be spun as either of the other 2 colours.
 c. Red is twice as likely to be spun as white and 3 times as likely to be spun as blue.



Communicating, reasoning and problem solving

14. Do you think that the probability of tossing Heads is the same as the probability of tossing Tails if your friend tosses the coin? What are some reasons that it might not be?
15. If the following four probabilities were given to you, which two would you say were not correct? Give reasons why.
 0.725, -0.5, 0.005, 1.05
16. Consider this spinner. Discuss whether the spinner has equal chance of falling on each of the colours.
17. A box contains two coins. One is a double-headed coin, and the other is a normal coin with Heads on one side and Tails on the other. You draw one of the coins from a box and look at one of the sides. It is Heads. What is the probability that the other side shows Heads also?
18. 'Unders and Overs' is a game played with two normal six-sided dice. The two dice are rolled, and the numbers uppermost are added to give a total. Players bet on the outcome being 'under 7', 'equal to 7' or 'over 7'. If you had to choose one of these outcomes, which would you choose? Explain why.
19. Justine and Mary have designed a new darts game for their Year 9 Fete Day. Instead of a circular dart board, their dart board is in the shape of two equilateral triangles. The inner triangle (bullseye) has a side length of 3 cm, and the outer triangle has side length 10 cm.



Given that a player's dart falls in one of the triangles, what is the probability that it lands in the bullseye? Write your answer correct to 2 decimal places.

20. Write a sentence using the word 'probability' and showing its meaning.

9.3 Experimental probability

[Stage 5.1]

9.3.1 Relative frequency

- A die is rolled 12 times and the outcomes are recorded in the table below.

Outcome	1	2	3	4	5	6
Frequency	3	1	1	2	2	3

In this chance experiment there were 12 trials.

The table shows that the number 1 was rolled 3 times out of 12.

- So the **relative frequency** of 1 is 3 out of 12, or $\frac{3}{12} = \frac{1}{4}$.

As a decimal, the relative frequency of 1 is equal to 0.25.

- In general, the relative frequency of an outcome = $\frac{\text{the frequency of the outcome}}{\text{total number of trials}}$.

If the number of trials is very large, then the relative frequency of each outcome becomes very close to the theoretical probability.

WORKED EXAMPLE 4

For the chance experiment of rolling a die, the following outcomes were noted.

Outcome	1	2	3	4	5	6
Frequency	3	1	4	6	3	3



- How many trials were there?
- How many threes were rolled?
- What was the relative frequency for each number written as decimals?

THINK

WRITE

- Adding the frequencies will give the number of trials.

a $1 + 3 + 4 + 6 + 3 + 3 = 20$ trials

- The frequency of 3 is 4.

b 4 threes were rolled.

- Add a relative frequency row to the table and complete it.

c

Outcome	1	2	3	4	5	6
Frequency	3	1	4	6	3	3
Relative frequency	$\frac{3}{20} = 0.15$	$\frac{1}{20} = 0.05$	$\frac{4}{20} = 0.2$	$\frac{6}{20} = 0.3$	$\frac{3}{20} = 0.15$	$\frac{3}{20} = 0.15$

ACTIVITY: A CHANCE EXPERIMENT

Organise for the class to toss a coin at least 500 times. For example, if the class has 20 students, each one should record 25 outcomes and enter their information into a grid as shown below.

Group	H	T	Total T	Total H	Total outcomes	Relative frequency (Heads)	Relative frequency (Tails)
A	17	8	17	8	25	$\frac{17}{25} = 0.68$	$\frac{8}{25} = 0.32$
B	15	10	32	18	50	$\frac{32}{50} = 0.64$	$\frac{18}{50} = 0.36$
C					75		
etc.							

Questions:

1. In an ideal situation, what would you expect the relative frequencies to be? Has this occurred?
 2. As more information was added to the table, what happened to the relative frequencies?
 3. What do you think might happen if the experiment was continued for another 500 tosses?
- A rule called the law of large numbers indicates that as the number of trials increases, then the relative frequencies will tend to get closer to the expected value (in this case 0.5).

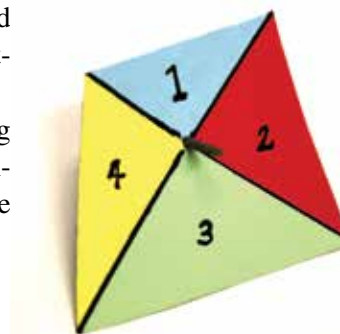
9.3.2 Experimental probability

- Sometimes it is not possible to calculate theoretical probabilities and in such cases experiments, sometimes called simulations, are conducted to determine the **experimental probability**.
- The relative frequency is equal to the experimental probability.

$$\text{Experimental probability} = \frac{\text{the frequency of the outcome}}{\text{total number of trials}}$$

- For example, the spinner shown below (made from light cardboard and a toothpick) is not symmetrical, and the probability of each outcome cannot be determined theoretically.

However, the probability of each outcome can be found by using the spinner many times and recording the outcomes. If a large number of trials is conducted, the relative frequency of each outcome will be very close to its theoretical probability.



DISCUSSION

How would you explain the difference between theoretical probability and experimental probability to other learners?

WORKED EXAMPLE 5

The spinner shown above was spun 100 times and the following results were achieved.

Outcome	1	2	3	4
Frequency	7	26	9	58

- a How many trials were there?
- b What is the experimental probability of each outcome?
- c What is the sum of the 4 probabilities?

THINK

- a Adding the frequencies will determine the number of trials.
- b The experimental probability equals the relative frequency.

WRITE

a $7 + 26 + 9 + 58 = 100$ trials

b $P(1) = \frac{7}{100}$
 $= 0.07$

$P(2) = \frac{26}{100}$
 $= 0.26$

$P(3) = \frac{9}{100}$
 $= 0.09$

$P(4) = \frac{58}{100}$
 $= 0.58$

- c Add the probabilities (they should equal 1).

c $0.07 + 0.26 + 0.09 + 0.58 = 1$

ACTIVITY: MAKING A SPINNER

Construct an irregular spinner using cardboard and a toothpick. By carrying out a number of trials, estimate the probability of each outcome.

learnON RESOURCES — ONLINE ONLY

Interactivity: Experimental probability (int-3825)

Digital doc: WorkSHEET Probability 1 (doc-6313)

Exercise 9.3 Experimental probability

Individual pathways

PRACTISE

Questions:
1–4, 6–13, 18, 19, 25

CONSOLIDATE

Questions:
1–4, 6–11, 15, 16, 18–20, 25

MASTER

Questions:
1–3, 5–8, 12–14, 16–25

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE4** Each of the two tables at the top of the next page contains the results of a chance experiment (rolling a die). For each table, find:
 - i. the number of trials held
 - ii. the number of fives rolled
 - iii. the relative frequency for each outcome, correct to 2 decimal places
 - iv. the sum of the relative frequencies.

a.

Number	1	2	3	4	5	6
Frequency	3	1	5	2	4	1

b.

Number	1	2	3	4	5	6
Frequency	52	38	45	49	40	46

2. A coin is tossed in two chance experiments. The outcomes are recorded in the tables below. For each experiment, find:

i. the relative frequency of both outcomes

ii. the sum of the relative frequencies.

a.

Outcome	H	T
Frequency	22	28

b.

Outcome	H	T
Frequency	31	19

3. **WE5** An unbalanced die was rolled 200 times and the following outcomes were recorded.

Number	1	2	3	4	5	6
Frequency	18	32	25	29	23	73

Using these results, find:

a. $P(6)$

b. $P(\text{odd number})$

c. $P(\text{at most } 2)$

d. $P(\text{not } 3)$.

4. A box of matches claims on its cover to contain 100 matches. A survey of 200 boxes established the following results.

Number of matches	95	96	97	98	99	100	101	102	103	104
Frequency	1	13	14	17	27	55	30	16	13	14

If you were to purchase a box of these matches, what is the probability that:

- the box would contain 100 matches
- the box would contain at least 100 matches
- the box would contain more than 100 matches
- the box would contain no more than 100 matches?



5. Here is a series of statements based on experimental probability. If a statement is not reasonable, give a reason why.
- I tossed a coin 5 times and there were 4 Heads, so $P(H) = 0.8$.
 - Sydney Roosters have won 1064 matches out of the 2045 that they have played, so $P(\text{Sydney will win their next game}) = 0.54$.
 - $P(\text{the sun will rise tomorrow}) = 1$.
 - At a factory, a test of 10 000 light globes showed that 7 were faulty. Therefore, $P(\text{faulty light globe}) = 0.0007$.
 - In Sydney it rains an average of 143.7 days each year, so $P(\text{it will rain in Sydney on the 17th of next month}) = 0.39$.
6. At a birthday party, some cans of soft drink were put in a container of ice. There were 16 cans of Coke, 20 cans of Sprite, 13 cans of Fanta, 8 cans of Sunkist and 15 cans of Pepsi. If a can was picked at random, what is the probability that it was:
- a can of Pepsi
 - not a can of Fanta?
7. **MC** In Ozlotto, 6 numbers are drawn from the numbers 1, 2, 3, ... 45. The number of different combinations of 6 numbers is 8 145 060. If you buy 1 ticket, what is the probability that you will win the draw?

A. $\frac{1}{8\,145\,060}$

B. $\frac{1}{45}$

C. $\frac{45}{8\,145\,060}$

D. $\frac{1}{6}$

E. $\frac{6}{8\,145\,060}$



8. **WE8** If a fair coin is tossed 400 times, how many Tails are expected?
9. If a fair die is rolled 120 times, how many threes are expected?
10. **WE9 MC** A survey of high school students asked 'Should Saturday be a normal school day?' Three hundred and fifty students voted yes, and 450 voted no. What is the probability that a student chosen at random said no?
- A.** $\frac{7}{16}$ **B.** $\frac{9}{16}$ **C.** $\frac{7}{9}$ **D.** $\frac{9}{14}$ **E.** $\frac{1}{350}$
11. In a poll of 200 people, 110 supported party M, 60 supported party N and 30 were undecided. If a person is chosen at random from this group of people, what is the probability that he or she:
- a.** supports party M **b.** supports party N
c. supports a party **d.** is not sure what party to support?
12. A random number is picked from $N = \{1, 2, 3, \dots, 100\}$. What is the probability of picking a number that is:
- a.** a multiple of 3 **b.** a multiple of 4 or 5
c. a multiple of 5 and 6?
13. The numbers 3, 5 and 6 are combined to form a three-digit number such that no digit may be repeated.
- a. i.** How many numbers can be formed? **ii.** List them.
b. Find $P(\text{the number is odd})$. **c.** Find $P(\text{the number is even})$.
d. Find $P(\text{the number is a multiple of 5})$.
14. **MC** In a batch of batteries, 2 out of every 10 in a large sample were faulty. At this rate, how many batteries are expected to be faulty in a batch of 1500?
- A.** 2 **B.** 150 **C.** 200 **D.** 300 **E.** 750
15. Svetlana, Sarah, Leonie and Trang are volleyball players. The probabilities that they will score a point on serve are 0.6, 0.4, 0.3 and 0.2 respectively. How many points on serve are expected from each player if they serve 10 times each?



16. **MC** A survey of the favourite leisure activity of 200 Year 9 students produced the following results.

Activity	Playing sport	Fishing	Watching TV	Video games	Surfing
Number of students	58	26	28	38	50

The probability (given as a percentage) that a student selected at random from this group will have surfing as their favourite leisure activity is:

- A.** 50% **B.** 100% **C.** 25% **D.** 0% **E.** 29%
17. The numbers 1, 2 and 5 are combined to form a three-digit number, allowing for any digit to be repeated up to three times.
- How many different numbers can be formed?
 - List the numbers.
 - Determine $P(\text{the number is even})$.
 - Determine $P(\text{the number is odd})$.
 - Determine $P(\text{the number is a multiple of 3})$.

Communicating, reasoning and problem solving

18. John has a 12-sided die numbered 1 to 12 and Lisa has a 20-sided die numbered 1 to 20. They are playing a game where the first person to get the number 10 wins. They are rolling their dice individually.
- Find $P(\text{John gets a 10})$.
 - Find $P(\text{Lisa gets a 10})$.
 - Is this game fair? Explain.
19. At a supermarket checkout, the scanners have temporarily broken down and the cashiers must enter the bar codes manually. One particular cashier overcharged 7 of the last 10 customers she served by entering the incorrect bar code.
- Based on the cashier's record, what is the probability of making a mistake with the next customer?
 - Should another customer have any objections with being served by this cashier?
 - Justify your answer to part **b**.
20. A survey of the favourite foods of Year 9 students is recorded, with the following results.

Meal	Tally
Hamburger	45
Fish and chips	31
Macaroni and cheese	30
Lamb souvlaki	25
BBQ pork ribs	21
Cornflakes	17
T-bone steak	14
Banana split	12
Corn-on-the-cob	9
Hot dogs	8
Garden salad	8
Veggie burger	7
Smoked salmon	6
Muesli	5
Fruit salad	3

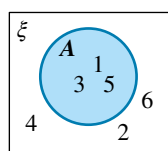
- Estimate the probability that macaroni and cheese is the favourite food of a randomly selected Year 9 student.
- Estimate the probability that a vegetarian dish is a randomly selected student's favourite food.
- Estimate the probability that a beef dish is a randomly selected student's favourite food.

21. If you flip a coin 6 times, how many of the possible outcomes could include a Tail on the second toss?
22. In a jar, there are 600 red balls, 400 green balls, and an unknown number of yellow balls. If the probability of selecting a green ball is $\frac{1}{5}$, how many yellow balls are in the jar?
23. In another jar there are an unknown number of balls, N , with 20 of them green. The other colours contained in the jar are red, yellow and blue, with $P(\text{red or yellow}) = \frac{1}{2}$, $P(\text{red or green}) = \frac{1}{4}$ and $P(\text{blue}) = \frac{1}{3}$. Determine the number of red, yellow and blue balls in the jar.
24. The gender of babies in a set of triplets is simulated by flipping 3 coins. If a coin lands Tails up, the baby is a boy. If a coin lands Heads up, the baby is a girl. In the simulation, the trial is repeated 40 times. The following results show the number of Heads obtained in each trial:
 0, 3, 2, 1, 1, 0, 1, 2, 1, 0, 1, 0, 2, 0, 1, 0, 1, 2, 3, 2, 1, 3, 0, 2, 1, 2, 0, 3,
 1, 3, 0, 1, 0, 1, 3, 2, 2, 1, 2, 1.
 - a. Calculate the probability that exactly one of the babies in a set of triplets is female.
 - b. Calculate the probability that more than one of the babies in the set of triplets is female.
25. What are the most important similarities between theoretical and experimental probability calculations?

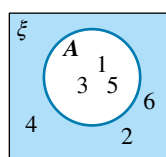
9.4 Venn diagrams and two-way tables [Stage 5.1]

9.4.1 The complement of an event

- Suppose that a die is rolled: $\xi = \{1, 2, 3, 4, 5, 6\}$.
 If A is the event 'rolling an odd number', then $A = \{1, 3, 5\}$.
- There is another event called 'the **complement** of A ', or 'not A '. This event contains all the outcomes that do not belong to A . It is given the symbol A' .
- In this case $A' = \{2, 4, 6\}$.
- A and A' can be shown on a **Venn diagram**.



A is coloured.



A' (not A) is coloured.

WORKED EXAMPLE 6

For the sample space $\xi = \{1, 2, 3, 4, 5\}$, list the complement of each of the following events.

- a $A = \{\text{multiples of 3}\}$
- b $B = \{\text{square numbers}\}$
- c $C = \{1, 2, 3, 5\}$

THINK

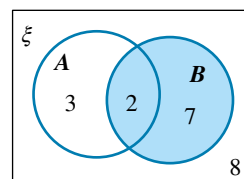
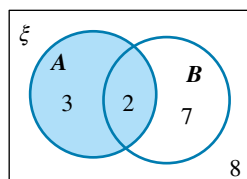
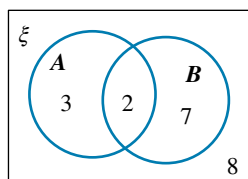
- a The only multiple of 3 in the set is 3.
 Therefore, $A = \{3\}$. A' is every other element of the set.
- b The only square numbers are 1 and 4.
 Therefore, $B = \{1, 4\}$. B' is every other element of the set.
- c $C = \{1, 2, 3, 5\}$. C' is every other element of the set.

WRITE

- a $A' = \{1, 2, 4, 5\}$
- b $B' = \{2, 3, 5\}$
- c $C' = \{4\}$

9.4.2 Venn diagrams

- Venn diagrams convey information in a concise manner and are often used to illustrate sample spaces and events. Here is an example.
 - In a class of 20 students, 5 study Art, 9 study Biology, and 2 students study both subjects.
 - This information is shown on the diagram below, where $A = \{\text{students who study Art}\}$ and $B = \{\text{students who study Biology}\}$.



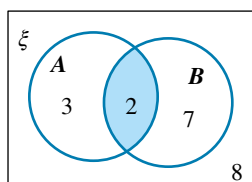
A contains 5 students.

B contains 9 students.

Note: In the case shown above, 8 students in the class study neither Art nor Biology.

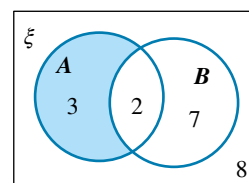
- The Venn diagram has 4 regions, each with its own name.

$A \cap B$



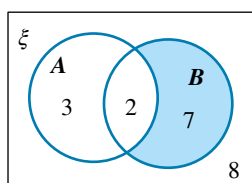
There are 2 students who study Art and Biology. They occupy the region called 'A and B' or $A \cap B$.

$A \cap B'$



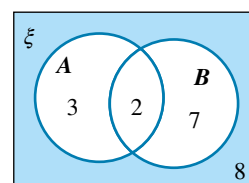
There are 3 students who study Art but not Biology. They occupy the region called 'A and not B' or $A \cap B'$.

$A' \cap B$



There are 7 students who study Biology but not Art. They occupy the region called 'not A and B' or $A' \cap B$.

$A' \cap B'$



The remaining 8 students study neither subject. They occupy the region called 'not A and not B' or $A' \cap B'$.

9.4.3 Two-way tables

- The information from the previous section can also be summarised in a **two-way table**.

	Biology	Not Biology	Total
Art	2	3	5
Not Art	7	8	15
Total	9	11	20

Note: Nine students in total study Biology and 11 do not. Five students in total study Art and 15 do not.

9.4.4 Number of outcomes

- If event A contains 7 outcomes or members, this is written as $n(A) = 7$.
- So $n(A \cap B') = 3$ means that the event 'A and not B' has 3 outcomes.

WORKED EXAMPLE 7

For the Venn diagram shown, write down the number of outcomes in each of the following.

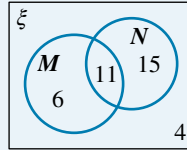
a M

b M'

c $M \cap N$

d $M \cap N'$

e $M' \cap N'$



THINK

a Identify the regions showing M and add the outcomes.

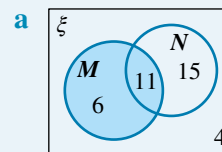
b Identify the regions showing M' and add the outcomes.

c $M \cap N$ means ' M and N '. Identify the region.

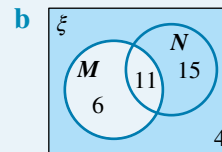
d $M \cap N'$ means ' M and not N '. Identify the region.

e $M' \cap N'$ means 'not M and not N '. Identify the regions.

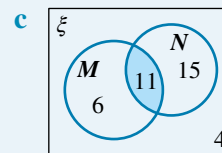
WRITE/DRAW



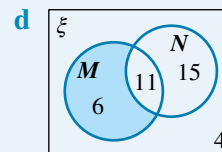
$$n(M) = 6 + 11 = 17$$



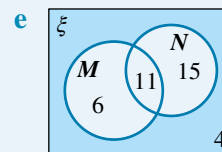
$$n(M') = 4 + 15 = 19$$



$$n(M \cap N) = 11$$



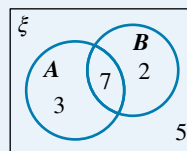
$$n(M \cap N') = 6$$



$$n(M' \cap N') = 4$$

WORKED EXAMPLE 8

Show the information from the Venn diagram on a two-way table.



THINK

1 Draw a 2×2 table and add the labels A, A', B and B' .

2 There are 7 elements in A and B .

There are 3 elements in A and 'not B '.

There are 2 elements in 'not A ' and B .

There are 5 elements in 'not A ' and 'not B '.

3 Add in a column and a row to show the totals.

WRITE

	A	A'
B		
B'		

	A	A'
B	7	2
B'	3	5

	A	A'	Total
B	7	2	9
B'	3	5	8
Total	10	7	17

WORKED EXAMPLE 9

Show the information from the two-way table on a Venn diagram.

	Left-handed	Right-handed
Blue eyes	7	20
Not blue eyes	17	48

**THINK**

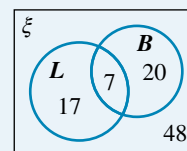
1 Draw a Venn diagram that includes a sample space and events L for left-handedness and B for blue eyes. (Right-handedness = L')

2 $n(L \cap B) = 7$

$n(L \cap B') = 17$

$n(L' \cap B) = 20$

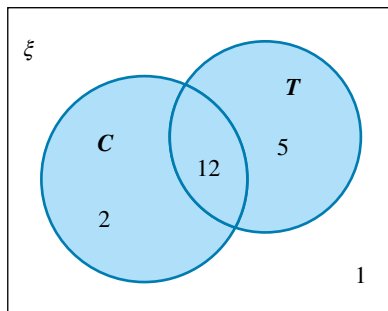
$n(L' \cap B') = 48$

DRAW**9.4.5 Event A or B**

- This Venn diagram illustrates the results of a survey of 20 people, showing whether they drink tea and whether they drink coffee.

In all, there are 19 people who drink tea or coffee. They are found in the shaded region of the diagram. This large group of people is written as $C \cup T$ and called ' C or T '. Note that the people who drink both tea and coffee, $C \cap T$, are included in this group.

$$n(C \cup T) = 19$$



A number of people drink tea or coffee, but not both. This group contains the 2 people who drink only tea and the 5 people who drink only coffee.

$$n(\text{people who drink tea or coffee, but not both}) = 7$$

WORKED EXAMPLE 10

In a class of 24 students, 11 students play basketball, 7 play tennis, and 4 play both sports.

a Show the information on a Venn diagram.

b If one student is selected at random, then find the probability that:

i the student plays basketball

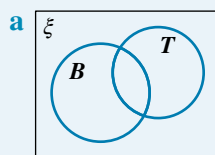
ii the student plays tennis or basketball

iii the student plays tennis or basketball but not both.

THINK

a 1 Draw a sample space with events B and T .

WRITE/DRAW



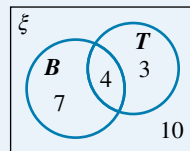
$$2 \quad n(B \cap T) = 4$$

$$n(B \cap T') = 11 - 4 = 7$$

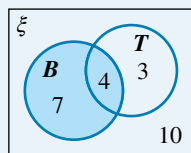
$$n(T \cap B') = 7 - 4 = 3$$

So far, 14 students out of 24 have been placed.

$$n(B' \cap T') = 24 - 14 = 10$$

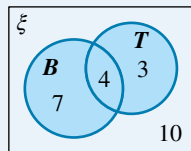


b i Identify the number of students who play basketball.



$$P(B) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

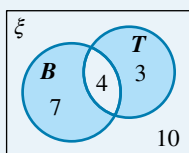
ii Identify the number of students who play tennis or basketball.



$$\begin{aligned} \mathbf{b \ i} \quad P(B) &= \frac{\text{number of students who play basketball}}{\text{total number of students}} \\ &= \frac{n(B)}{24} \\ &= \frac{11}{24} \end{aligned}$$

$$\begin{aligned} \mathbf{ii} \quad P(T \cup B) &= \frac{n(T \cup B)}{24} \\ &= \frac{14}{24} \\ &= \frac{7}{12} \end{aligned}$$

- iii Identify the number of students who play tennis or basketball but not both.



$$\text{iii } n(B \cap T') + n(B' \cap T) = 3 + 7 = 10$$

$$\begin{aligned} P(\text{tennis or basketball but not both}) &= \frac{10}{24} \\ &= \frac{5}{12} \end{aligned}$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Venn diagrams (int-3828)

Interactivity: Two-way tables (int-6082)

eLesson: Venn diagrams (eles-1934)

Digital doc: SkillsHEET Determining complementary events (doc-6311)

Digital doc: SkillsHEET Calculating the probability of a complementary event (doc-6312)

Exercise 9.4 Venn diagrams and two-way tables

Individual pathways

PRACTISE

Questions:
1–5, 7, 9, 11, 13, 15–17

CONSOLIDATE

Questions:
1–4, 6, 8, 10, 13–15, 17–19

MASTER

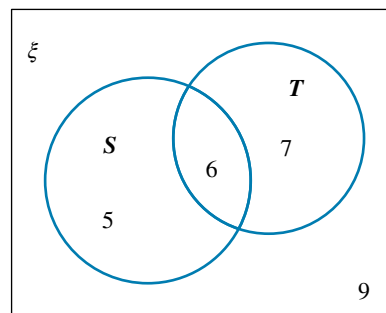
Questions:
1, 3, 4, 6, 8, 10, 12, 14, 16–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

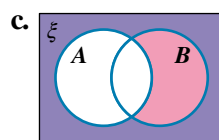
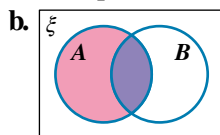
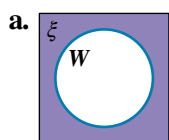
- WE6** If $\xi = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, list the complement of each of the following events.
 - $A = \{\text{multiples of 3}\}$
 - $B = \{\text{numbers less than 20}\}$
 - $C = \{\text{prime numbers}\}$
 - $D = \{\text{odd numbers and numbers greater than 16}\}$
- WE7** For the Venn diagram shown, write down the number of outcomes in:

a. ξ	b. S	c. T	d. $S \cap T$
e. $T \cap S$	f. $T \cap S'$	g. $S' \cap T'$	
- WE8** Show the information from question 2 on a two-way table.
- WE9** Show the information from this two-way table on a Venn diagram.



	W	W'
V	21	7
V'	2	10

5. For each of the following Venn diagrams, use set notation to write the name of the region coloured in:
- purple
 - pink.



6. The membership of a tennis club consists of 55 men and 45 women. There are 27 left-handed people, including 15 men.

- Show the information on a two-way table.
- Show the information on a Venn diagram.
- If one member is chosen at random, find the probability that the person is:
 - right-handed
 - a right-handed man
 - a left-handed woman.

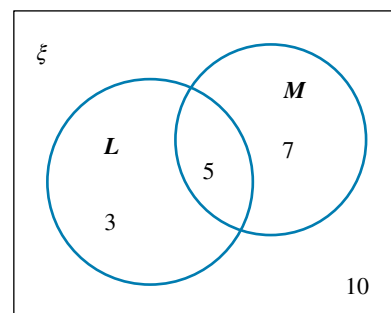


7. Using the information given in the Venn diagram, if one outcome is chosen at random, find:

- $P(L)$
- $P(L')$
- $P(L \cap M)$
- $P(L \cap M')$.

8. Using the information given in the table, if one family is chosen at random, find the probability that they own:

- a cat
- a cat and a dog
- a cat or a dog or both
- a cat or a dog but not both
- neither a cat nor a dog.

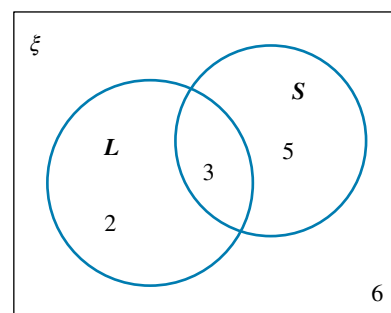


Pets owned by families		
	Cat	No cat
Dog	4	11
No dog	16	9

9. A group of athletes was surveyed and the results were shown on a Venn diagram.

$S = \{\text{sprinters}\}$ and $L = \{\text{long jumpers}\}$.

- How many athletes were included in the survey?
- If one of the athletes is chosen at random, what is the probability that the athlete competes in:
 - long jump
 - long jump and sprints
 - long jump or sprints
 - long jump or sprints but not both?



10. If $\xi = \{\text{children}\}$, $S = \{\text{swimmers}\}$ and $R = \{\text{runners}\}$, describe in words each of the following.
- a. S' b. $S \cap R$ c. $R' \cap S'$ d. $R \cup S$
11. A group of 12 students was asked whether they liked hip hop (H) and whether they liked classical music (C). The results are shown in the table below.

	C	H
Ali	✓	✓
Anu		
Chris		✓
George		✓
Imogen		✓
Jen	✓	✓
Luke	✓	✓
Pam	✓	
Petra		
Roger	✓	
Seedevi		✓
Tomas		

- a. Show the results on:
- i. a Venn diagram ii. a two-way table.
- b. If one student is selected at random, find:
- i. $P(H)$
- ii. $P(H \cup C)$
- iii. $P(H \cap C)$
- iv. $P(\text{student likes classical or hip hop but not both}).$



12. Place the elements of the following sets of numbers in their correct position in a single Venn diagram.
- $A = \{\text{prime numbers from 1 to 20}\}$
- $B = \{\text{even numbers from 1 to 20}\}$
- $C = \{\text{multiples of 5 from 1 to 20}\}$
- $\xi = \{\text{numbers between 1 and 20 inclusive}\}$

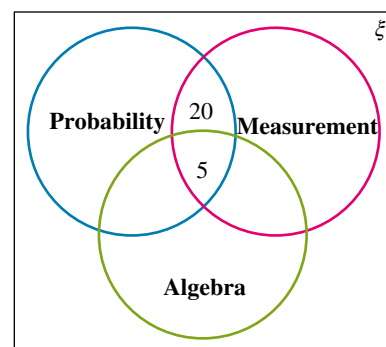
Communicating, reasoning and problem solving

- 13.** One hundred Year 9 Maths students were asked to indicate their favourite topic in mathematics. Sixty chose Probability, 50 chose Measurement and 43 chose Algebra. Some students chose two topics: 15 chose Probability and Algebra, 18 chose Measurement and Algebra, and 25 chose Probability and Measurement. Five students chose all three topics.

- Copy and complete the Venn diagram.
- How many students chose Probability only?
- How many students chose Algebra only?
- How many students chose Measurement only?
- How many students chose any two of the three topics?

A student is selected at random from this group. Find the probability that this student has chosen:

- Probability
- Algebra
- Algebra and Measurement
- Algebra and Measurement but not Probability
- all of the topics.



- 14.** Create a Venn diagram using two circles to accurately describe the relationships between the following quadrilaterals: rectangle, square and rhombus.
- 15.** Use the Venn diagram shown to write the numbers of the correct regions for each of the following problems.

- $A \cup (B' \cap C)$
- $A \cap (B \cap C')$
- $A' \cap (B' \cup C')$
- $(A \cup B \cup C)'$

- 16.** A recent survey taken at a cinema asked 90 teenagers what they thought about three different movies. In total, 47 liked 'Hairy Potter', 25 liked 'Stuporman' and 52 liked 'There's Something About Fred'.

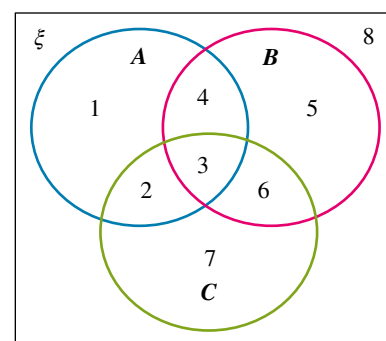
16 liked 'Hairy Potter' only.

4 liked 'Stuporman' only.

27 liked 'There's Something About Fred' only.

There were 11 who liked all three films and 10 who liked none of them.

- Construct a Venn diagram showing the results of the survey.
- What is the probability that a teenager chosen at random liked 'Hairy Potter' and 'Stuporman' but not 'There's Something About Fred'?



- 17.** 120 children attended a school holiday program during September. They were asked to select their favourite board game from Cluedo, Monopoly and Scrabble. They all selected at least one game, and 4 children chose all three games.

In total, 70 chose Monopoly and 55 chose Scrabble.

Some children selected exactly two games — 12 chose Cluedo and Scrabble, 15 chose Monopoly and Scrabble, and 20 chose Cluedo and Monopoly.

- Draw a Venn diagram to represent the children's selections.
- What is the probability that a child selected at random did not choose Cluedo as a favourite game?



18. Valleyview High School offers three sports at Year 9: baseball, volleyball and soccer. There are 65 students in Year 9.
 2 have been given permission not to play sport due to injuries and medical conditions.
 30 students play soccer.
 9 students play both soccer and volleyball but not baseball.
 9 students play both baseball and soccer (including those who do and don't play volleyball).
 4 students play all three sports.
 12 students play both baseball and volleyball (including those who do and don't play soccer).
 The total number of players who play baseball is 1 more than the total of students who play volleyball.
- Determine the number of students who play volleyball.
 - If a student was selected at random, what is the probability that this student plays soccer and baseball only?
19. Consider the numbers 156 and 520. Show how a Venn diagram could be used to determine their:
- HCF
 - LCM.
20. A group of 200 shoppers was asked which type of fruit they had bought in the last week. The results are shown in the table.

Fruit	Number of shoppers
Apples (A) only	45
Bananas (B) only	34
Cherries (C) only	12
A and B	32
A and C	15
B and C	26
A and B and C	11

- Display this information in a Venn diagram.
 - Calculate $n(A \cap B' \cap C)$.
 - How many shoppers purchased apples and bananas but not cherries?
 - Calculate the relative frequency of shoppers who purchased:
 - apples
 - bananas or cherries.
 - Estimate the probability that a shopper purchased cherries only.
21. How will you remember the difference between when one event *and* another occurs and when one event *or* another occurs?
22. Bar codes are used to identify items. One particular system uses bars arranged in groups of five, each group containing two long bars and three short bars. In how many different ways can two long bars and three short bars be arranged in a group?

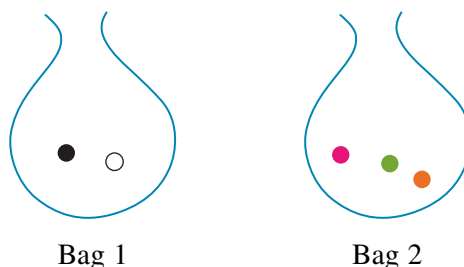


9.5 Two-step experiments

[Stage 5.2]

9.5.1 The sample space

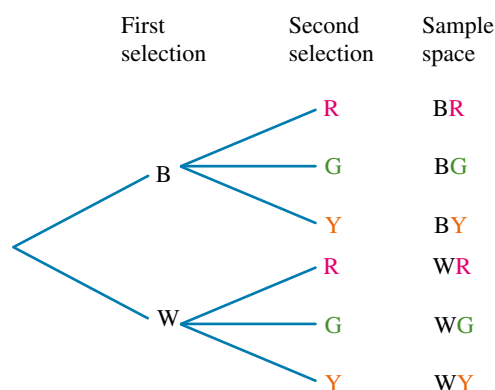
- Imagine two bags (that are not transparent) that contain coloured counters. The first bag has a mixture of black and white counters, and the second bag holds red, green and yellow counters. In a probability experiment, one counter is to be selected at random from each bag and its colour noted.
- The sample space for this experiment can be found using a table called an **array** that systematically displays all the outcomes.



		Bag 2		
		R	G	Y
Bag 1	B	B R	B G	B Y
	W	W R	W G	W Y

The sample space, $\xi = \{BR, BG, BY, WR, WG, WY\}$.

- The sample space can also be found using a **tree diagram**.



WORKED EXAMPLE 11

Two dice are rolled and the numbers uppermost are noted. List the sample space in an array.

- How many outcomes are there?
- How many outcomes contain at least one 5?
- What is P (at least one 5)?



THINK

Draw an array (a table) showing all the possible outcomes.

WRITE/DRAW

		Second die					
		1	2	3	4	5	6
First die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

- a The table shows 36 outcomes.
- b Count the outcomes that contain 5.
The cells are shaded in the table.
- c There are 11 favourable outcomes and 36 in total.

- a There are 36 outcomes.
- b Eleven outcomes include 5.
- c $P(\text{at least one } 5) = \frac{11}{36}$

WORKED EXAMPLE 12

Two coins are tossed and the outcomes are noted.
Show the sample space on a tree diagram.

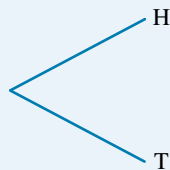
- a How many outcomes are there?
- b Find the probability of tossing at least one Head.

**THINK**

- 1 Draw a tree representing the outcomes for the toss of the first coin

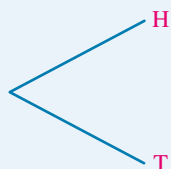
WRITE/DRAW

First coin



- 2 For the second coin the tree looks like this:
Add this tree to both ends of the first tree.

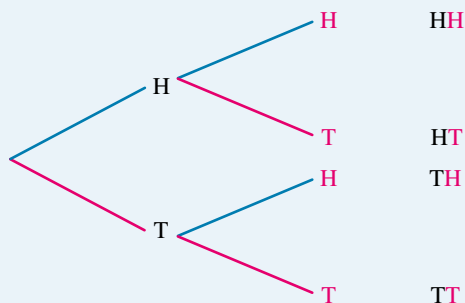
Second coin



First coin

Second coin

Sample space



3 List the outcomes.

a Count the outcomes in the sample space.

a There are 4 outcomes (HH, HT, TH, TT).

b Three outcomes have at least one Head.

b $P(\text{at least one Head}) = \frac{3}{4}$

9.5.2 Two-step experiments

- When a coin is tossed, $P(H) = \frac{1}{2}$, and when a die is rolled, $P(3) = \frac{1}{6}$.

If a coin is tossed and a die is rolled, what is the probability of getting a Head *and* a 3?

- Consider the sample space.

	1	2	3	4	5	6
H	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

There are 12 outcomes, and $P(\text{Head and } 3) = \frac{1}{12}$.

- In this case, $P(\text{Head and } 3) = P(H) \times P(3)$; that is, $\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$.
- In general, if A is the outcome of one event and B is the outcome of a separate event, then

$$P(A \cap B) = P(A) \times P(B).$$



WORKED EXAMPLE 13

In one cupboard Joe has 2 black t-shirts and 1 yellow one. In his drawer there are 3 pairs of white socks and 1 black pair. If he selects his clothes at random, what is the probability that his socks and t-shirt will be the same colour?

THINK

If they are the same colour, they must be black.

$$P(\text{black t-shirt}) = P(B_t) = \frac{2}{3}$$

$$P(\text{black socks}) = P(B_s) = \frac{1}{4}$$

WRITE

$$\begin{aligned} P(B_t \cap B_s) &= P(B_t) \times P(B_s) \\ &= \frac{2}{3} \times \frac{1}{4} \\ &= \frac{1}{6} \end{aligned}$$

9.5.3 Choosing with replacement

- Consider what happens when replacement is allowed in an experiment. Worked example 14 illustrates this situation.

WORKED EXAMPLE 14

A bag contains 3 red and 2 blue counters. A counter is taken at random from the bag, its colour is noted, then it is returned to the bag and a second counter is chosen.

- Show the outcomes on a tree diagram.
- Find the probability of each outcome.
- Find the sum of the probabilities.



THINK

- a 1** Draw a tree for the first trial. Write the probability on the branch.

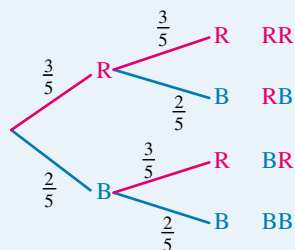
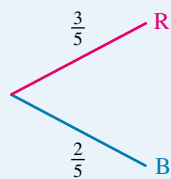
Note: The probabilities should sum to 1.

- 2** For the second trial the tree is the same. Add this tree to both ends of the first tree.

- 3** List the outcomes.

- b** For both draws, $P(R) = \frac{3}{5}$ and $P(B) = \frac{2}{5}$. Use the rule $P(A \cap B) = P(A) \times P(B)$ to determine the probabilities.

- c** Add the probabilities.

WRITE/DRAW

$$\begin{aligned} \mathbf{b} \quad P(R \cap R) &= P(R) \times P(R) \\ &= \frac{3}{5} \times \frac{3}{5} \\ &= \frac{9}{25} \end{aligned}$$

$$\begin{aligned} P(R \cap B) &= P(R) \times P(B) \\ &= \frac{3}{5} \times \frac{2}{5} \\ &= \frac{6}{25} \end{aligned}$$

$$\begin{aligned} P(B \cap R) &= P(B) \times P(R) \\ &= \frac{2}{5} \times \frac{3}{5} \\ &= \frac{6}{25} \end{aligned}$$

$$\begin{aligned} P(B \cap B) &= P(B) \times P(B) \\ &= \frac{2}{5} \times \frac{2}{5} \\ &= \frac{4}{25} \end{aligned}$$

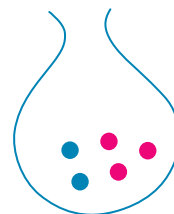
$$\mathbf{c} \quad \frac{9}{25} + \frac{6}{25} + \frac{6}{25} + \frac{4}{25} = 1$$

- In Worked example 14, $P(R) = \frac{3}{5}$ and $P(B) = \frac{2}{5}$ for both trials. This would not be so if a counter is selected *but not replaced*.

9.5.4 Choosing without replacement

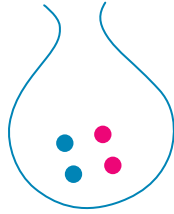
- Let us consider again the situation described in Worked example 14, and consider what happens if the first marble is not replaced.
- Initially the bag contains 3 red and 2 blue counters, and either a red counter or a blue counter will be chosen.

$$P(R) = \frac{3}{5} \text{ and } P(B) = \frac{2}{5}.$$



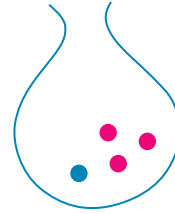
- If the counter is not replaced, then the sample space is affected as follows:

If the first counter randomly selected is red, then the sample space for the second draw looks like this:



$$\text{So } P(R) = \frac{2}{4} \text{ and } P(B) = \frac{2}{4}.$$

If the first counter randomly selected is blue, then the sample space for the second draw looks like this:



$$\text{So } P(R) = \frac{3}{4} \text{ and } P(B) = \frac{1}{4}.$$

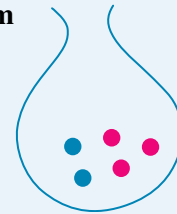
WORKED EXAMPLE 15

A bag contains 3 red and 2 blue counters. A counter is taken at random from the bag and its colour is noted, then a second counter is drawn, without replacing the first one.

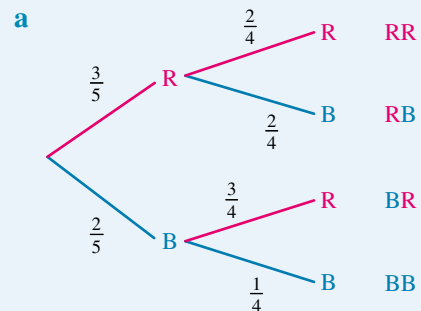
- Show the outcomes on a tree diagram.
- Find the probability of each outcome.
- Find the sum of the probabilities.

THINK

- Draw a tree diagram listing the probabilities.



WRITE/DRAW



- Use the rule $P(A \cap B) = P(A) \times P(B)$ to determine the probabilities.

b

$$\begin{aligned}
 P(R \cap R) &= P(R) \times P(R) \\
 &= \frac{3}{5} \times \frac{2}{4} \\
 &= \frac{6}{20} \\
 &= \frac{3}{10} \\
 P(R \cap B) &= P(R) \times P(B) \\
 &= \frac{3}{5} \times \frac{2}{4} \\
 &= \frac{6}{20} \\
 &= \frac{3}{10} \\
 P(B \cap R) &= P(B) \times P(R) \\
 &= \frac{2}{5} \times \frac{3}{4} \\
 &= \frac{6}{20} \\
 &= \frac{3}{10}
 \end{aligned}$$

$$\begin{aligned}
 P(B \cap B) &= P(B) \times P(B) \\
 &= \frac{2}{5} \times \frac{1}{4} \\
 &= \frac{2}{20} \\
 &= \frac{1}{10}
 \end{aligned}$$

c Add the probabilities.

$$c \quad \frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{1}{10} = 1$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Two-step experiments (int-6083)

Interactivity: Two-step chance (int-2772)

eLesson: Tree diagrams (eles-1894)

Digital doc: WorkSHEET Probability 2 (doc-6314)

Exercise 9.5 Two-step experiments

Individual pathways

PRACTISE

Questions:
1–10, 12

CONSOLIDATE

Questions:
1–15

MASTER

Questions:
1–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

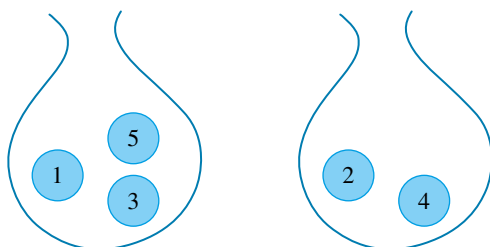
- In her cupboard Rosa has 3 scarves (red, blue and pink) and 2 beanies (brown and purple). If she randomly chooses 1 scarf and 1 beanie, show the sample space in an array.
- WE11** If two dice are rolled and their *sum* is noted, complete the array below to show the sample space.

		Die 1					
		1	2	3	4	5	6
Die 1	1	2					
	2					7	
	3						
	4						
	5						
	6			9			

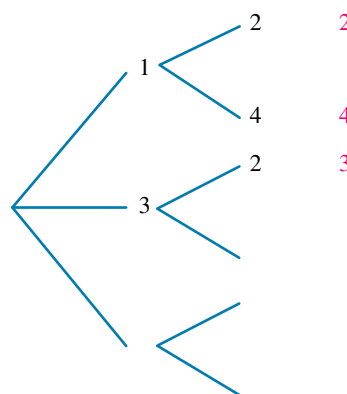
- What is P (rolling a total of 5)?
- What is P (rolling a total of 1)?
- What is the most probable outcome?



3. One box contains red and blue pencils, and a second box contains red, blue and green pencils. If one pencil is chosen at random from each box and the colours are noted, draw a tree diagram to show the sample space.
4. **WE12** A bag contains 3 discs labelled 1, 3 and 5, and another bag contains two discs, labelled 2 and 4, as shown below. A disc is taken from each bag and the larger number is recorded.



- a. Complete the tree diagram at right to list the sample space.
- b. What is:
- $P(2)$
 - $P(1)$
 - $P(\text{odd number})$?
5. Two dice are rolled and the difference between the two numbers is found.
- a. Use an array to find all the outcomes.
- b. Find:
- $P(\text{odd number})$
 - $P(0)$
 - $P(\text{a number more than } 2)$
 - $P(\text{a number no more than } 2)$.
6. **WE13** A die is rolled twice. What is the probability of rolling:
- a 6 on the first roll
 - a double 6
 - an even number on both dice
 - a total of 12?
7. A coin is tossed twice.
- a. Show the outcomes on a tree diagram.
- b. What is:
- $P(2 \text{ Tails})$
 - $P(\text{at least } 1 \text{ Tail})$?
8. **WE14** A bag contains 3 red counters and 1 blue counter. A counter is chosen at random. A second counter is drawn with replacement.
- a. Show the outcomes and probabilities on a tree diagram.
- b. Find the probability of choosing:
- a red counter then a blue counter
 - two blue counters.
9. **WE15** A bag contains 3 black balls and 2 red balls. If two balls are selected, randomly, without replacement:
- show the outcomes and their probabilities on a tree diagram
 - find $P(2 \text{ red balls})$.
10. The kings and queens from a deck of cards are shuffled, then 2 cards are chosen. Find the probability that 2 kings are chosen:
- if the first card is replaced
 - if the first card is not replaced.

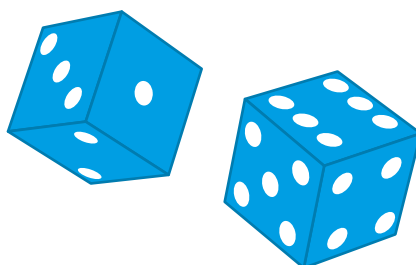


11. Each week John and Paul play 2 sets of tennis against each other. They each have an equal chance of winning the first set. When John wins the first set, his probability of winning the second set rises to 0.6, but if he loses the first set, he has only a 0.3 chance of winning the second set.
- Show the possible outcomes on a tree diagram.
 - What is:
 - $P(\text{John wins both sets})$
 - $P(\text{Paul wins both sets})$
 - $P(\text{they win 1 set each})$



Communicating, reasoning and problem solving

12. A bag contains 4 red and 6 yellow balls. If the first ball drawn is yellow, explain the difference in the probability of drawing the second ball if the first ball was replaced compared to not being replaced.
13. Three dice are tossed and the total is recorded.
- What are the smallest and largest possible totals?
 - Calculate the probabilities for all possible totals.
14. You draw two cards, one after the other without replacement, from a deck of 52 cards.
- What is the probability of drawing two aces?
 - What is the probability of drawing two face cards (J, Q, K)?
 - What is the probability of getting a 'pair' (22, 33, 44 ... QQ, KK, AA)?
15. A chance experiment involves flipping a coin and rolling two dice. Determine the probability of obtaining Tails and two numbers whose sum is greater than 4.
16. In a jar there are 10 red balls and 6 green balls. Jacob takes out two balls, one at a time, without replacing them. What is the probability that both balls are the same colour?
17. In the game of 'Texas Hold'Em' poker, 5 cards are progressively placed face up in the centre of the table for all players to use. At one point in the game there are 3 face-up cards (two hearts and one diamond). You have 2 diamonds in your hand for a total of 3 diamonds. Five diamonds make a flush. Given that there are 47 cards left, what is the probability that the next two face-up cards are both diamonds?
18. How does replacement affect the probability of an event occurring?
19. Professional gambling cheats use a variety of techniques to help their cause. One such method is to re-number the dice with only three numbers on each die. Consider two dice. The first die is numbered 1, 1, 2, 2, 3 and 3. The second die is numbered 4, 4, 5, 5, 6 and 6. When rolling two fair dice, the chance of getting a total of 7 is $\frac{1}{6}$. Would you be advantaged or disadvantaged by using the two re-numbered dice? Explain.

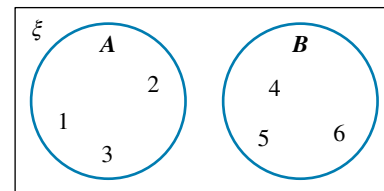


9.6 Mutually exclusive and independent events

[Stage 5.2]

9.6.1 Mutually exclusive events

- If two events cannot both occur at the same time, then it is said the two events are **mutually exclusive**. For example, when rolling a die, the events ‘getting a 1’ and ‘getting a 5’ are mutually exclusive.
- If two sets are disjoint (have no elements in common), then the sets are mutually exclusive. For example, if $A = \{\text{prime numbers} > 10\}$ and $B = \{\text{even numbers}\}$, then A and B are mutually exclusive.
- If A and B are two mutually exclusive events (or sets), then $P(A \cap B) = 0$.
- Consider the Venn diagram shown. Since A and B are disjoint (do not overlap), then A and B are mutually exclusive sets.
- If two events A and B are mutually exclusive, then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$.



9.6.2 Examples of mutually exclusive events

- Draw a card from a standard deck: the drawn card is a heart or a club.
 - Reason: it is impossible to get both a heart and a club at the same time.
- Record the time of arrival of overseas flights: a flight is late, on time or it is early.
 - Reason: it is impossible for the flight to arrive late, on time or early all at the same time.

9.6.3 Examples of non-mutually exclusive events

- Draw a card from a standard deck: the drawn card is a heart or a king.
 - Reason: it is possible to draw the king of hearts.
- Record the mode of transport of school students: count students walking or going by bus.
 - Reason: a student can walk (to the bus stop) and take a bus.

DISCUSSION

Can you think of more instances of mutually and non-mutually exclusive events?

WORKED EXAMPLE 16

A card is drawn from a pack of 52 cards. What is the probability that the card is a diamond or a spade?

THINK

- 1 The events are mutually exclusive because diamonds and spades cannot be drawn at the same time.
- 2 Determine the probability of drawing a diamond and the probability of drawing a spade.

WRITE

The two events are mutually exclusive as $P(A \cap B) = 0$.

Number of diamonds, $n(E_1) = 13$

Number of spades, $n(E_2) = 13$

Number of cards, $n(S) = 52$

$$P(\text{diamond}) = \frac{13}{52} \\ = \frac{1}{4}$$

$$P(\text{spade}) = \frac{13}{52} \\ = \frac{1}{4}$$

3 Write the probability.

$$P(A \cup B) = P(A) + P(B)$$

4 Evaluate and simplify.

$$\begin{aligned} P(\text{diamond or spade}) &= P(\text{diamond}) + P(\text{spade}) \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$

9.6.4 Independent events

- Two events are considered independent if the outcome of one event is not dependent on the outcome of the other event.
- For example, if $E_1 = \{\text{rolling a 4 on a first die}\}$ and $E_2 = \{\text{rolling a 2 on a second die}\}$, the outcome of event E_1 is not influenced by the outcome of event E_2 , so the events are independent.

WORKED EXAMPLE 17

Three coins are flipped simultaneously. Draw a tree diagram for the experiment. Calculate the following probabilities.

a P(3 Heads)

b P(2 Heads)

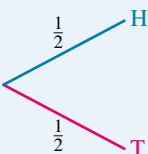
c P(at least 1 Head)

THINK

- Use branches to show the individual outcomes for the first part of the experiment (flipping the first coin).
- Link each outcome of the first flip with the outcomes of the second part of the experiment (flipping the second coin).
- Link each outcome from the second flip with the outcomes of the third part of the experiment (flipping the third coin).

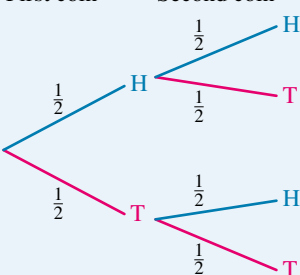
WRITE/DRAW

First coin



First coin

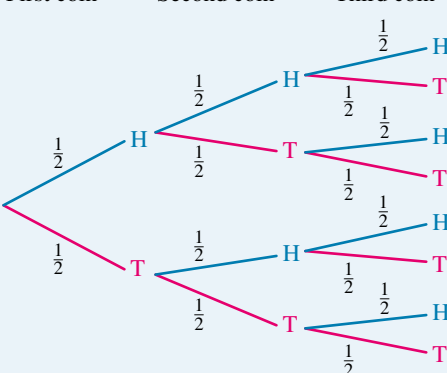
Second coin



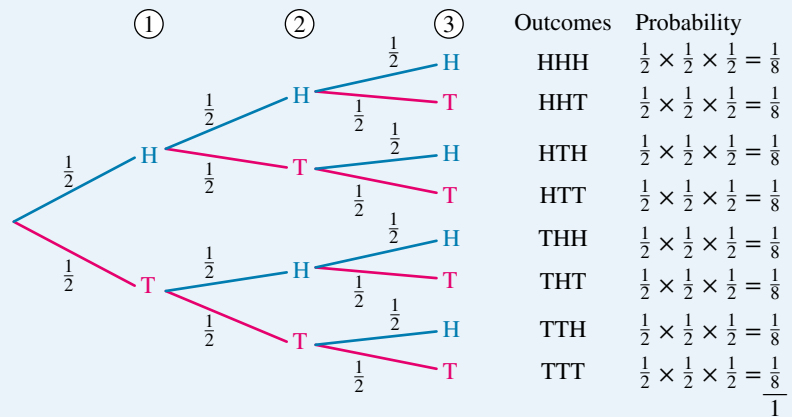
First coin

Second coin

Third coin



- 4 Determine the probability of each outcome. *Note:* The probability of each result is found by multiplying along the branches and in each case this will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.



- a The probability of three heads is $P(H, H, H)$

a $P(3 \text{ Heads}) = \frac{1}{8}$.

- b 1 { 2 Heads } has 3 satisfactory outcomes: (H, H, T), (H, T, H) and (T, H, H), which are mutually exclusive.

b $P(2 \text{ Heads})$
 $= P(H, H, T) + P(H, T, H) + P(T, H, H)$
 $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
 $= \frac{3}{8}$

- 2 Write your answer.

The probability of obtaining exactly 2 Heads is $\frac{3}{8}$.

- c 1 At least 1 Head means any outcome that contains one or more Head. This is every outcome except three Tails. That is, it is the complementary event to obtaining 3 Tails.

c $P(\text{at least 1 Head})$
 $= 1 - P(T, T, T)$
 $= 1 - \frac{1}{8}$
 $= \frac{7}{8}$

- 2 Write your answer.

The probability of obtaining at least 1 Head is $\frac{7}{8}$.

Note: The probabilities of all outcomes add to 1.

9.6.5 Dependent events

- Many real-life events have some dependence upon each other, and their probabilities are likewise affected.
 Examples include:
 - the chance of rain today and the chance of a person taking an umbrella to work
 - the chance of growing healthy vegetables and the availability of good soil
 - the chance of Victory Soccer Club winning this week and winning next week
 - drawing a card at random, not replacing it, and drawing another card.
- It is important to be able to recognise the difference between *dependent* events and *independent* events.

WORKED EXAMPLE 18

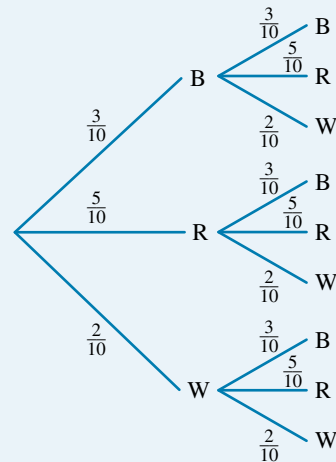
A jar contains three black marbles, five red marbles, and two white marbles. Find the probability of choosing a black marble (with replacement), then choosing another black marble.

THINK

- 1 The events, draw 1 and draw 2, are independent because the result of the first draw is not dependent on the result of the second draw.
- 2 Demonstrate using a tree diagram.

WRITE/DRAW

E_1 and E_2 are independent events.



- 3 Determine the probability.
Evaluate and simplify.

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

$$P(\text{black and black}) = P(\text{black}) \times P(\text{black})$$

$$\begin{aligned} P(\text{black and black}) &= \frac{3}{10} \times \frac{3}{10} \\ &= \frac{9}{100} \end{aligned}$$

- If the first marble had not been replaced in the previous worked example, the second draw would be dependent on the outcome of the first draw, and so it follows that the sample space for the second draw is different to that for the first draw.

WORKED EXAMPLE 19

Repeat Worked example 18 without replacing the first marble before the second one is drawn.

THINK

- 1 The words 'without replacing' indicate that the two events are dependent.
Write the sample space and state the probability of choosing a black marble on the first selection.
- 2 Assume that a black marble was chosen in the first selection. Determine how many black ones remain, and the total number of remaining marbles.
Write the sample space and state the probability of choosing a black marble on the second selection.

WRITE/DRAW

There are 10 marbles and 3 of these are black.

The sample space is

$\{B, B, B, R, R, R, R, R, W, W\}$.

$$P(E_1) = P(\text{black}) = \frac{3}{10}$$

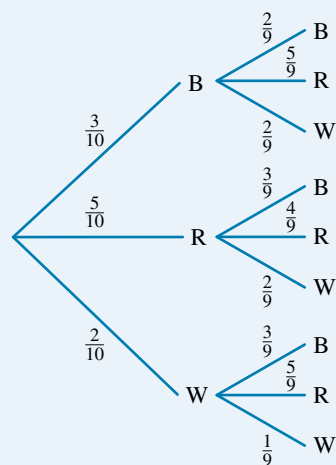
A black one was chosen, leaving 2 black ones and a total of 9 marbles.

The sample space is

$\{B, B, R, R, R, R, R, W, W\}$.

$$P(E_2) = P(\text{black}) = \frac{2}{9}$$

3 Demonstrate using a tree diagram.



4 Multiply the probabilities.

$$\begin{aligned} P(E_1 \text{ and } E_2) &= P(E_1) \times P(E_2) \\ &= \frac{3}{10} \times \frac{2}{9} \\ &= \frac{1}{15} \end{aligned}$$

5 Answer the question.

The probability of choosing two black marbles without replacing the first marble is $\frac{1}{15}$.

DISCUSSION

Your friend has just tossed a coin 4 times in a row and heads has been uppermost those 4 times. What is the probability of your friend tossing a head with her fifth toss?

learnON RESOURCES — ONLINE ONLY

Interactivity: Mutually exclusive events (int-6084)

Exercise 9.6 Mutually exclusive and independent events

Individual pathways

PRACTISE

Questions:

1–3, 5, 7, 9, 11, 13–16, 18, 20, 22, 24, 26, 29

CONSOLIDATE

Questions:

1, 3–6, 8, 10, 12–15, 18, 20, 24–26, 28, 29

MASTER

Questions:

1, 5–8, 11, 13, 14, 16, 17, 19–23, 25, 27, 28, 30, 31

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **MC** If a card is drawn from a pack of 52 cards, what is the probability that the card is not a queen?

A. $\frac{4}{52}$

B. $\frac{4}{48}$

C. $\frac{13}{12}$

D. $\frac{48}{52}$

E. $\frac{1}{52}$

2. **MC** Which events are not mutually exclusive?
- A. Drawing a queen and drawing a jack from 52 playing cards
 - B. Drawing a red card and drawing a black card from 52 playing cards
 - C. Drawing a vowel and drawing a consonant from cards representing the 26 letters of the alphabet
 - D. Obtaining a total of 8 and rolling doubles (when rolling two dice)
3. When a six-sided die is rolled 3 times, the probability of getting 3 sixes is $\frac{1}{216}$. What is the probability of not getting 3 sixes?
4. **MC** Eight athletes compete in a 100-m race. The probability that the athlete in lane 1 will win is $\frac{1}{5}$. What is the probability that one of the other athletes wins? (Assume that there are no dead heats.)
- A. $\frac{1}{5}$
 - B. $\frac{5}{8}$
 - C. $\frac{8}{5}$
 - D. $\frac{4}{5}$
 - E. $\frac{2}{5}$



5. A pencil case has 4 red pens, 3 blue pens and 5 black pens. If a pen is randomly drawn from the pencil case, find:
- a. P(drawing a blue pen)
 - b. P(not drawing a blue pen).
6. Seventy Year 9 students were surveyed. Their ages ranged from 13 years to 15 years, as shown in the table below.

Age	13	14	15	Total
Boys	10	20	9	39
Girls	7	15	9	31
Total	17	35	18	70

- A student from the group is selected at random. Find:
- a. P(selecting a student of the age of 13 years)
 - b. P(not selecting a student of the age of 13 years)
 - c. P(selecting a 15-year-old boy)
 - d. P(not selecting a 15-year-old boy).
7. **WE16** A card is drawn from a pack of 52 cards. What is the probability that the card is a king or an ace?



8. **MC** A die is rolled. Find the probability of getting an even number or a 3.

A. $\frac{3}{6}$

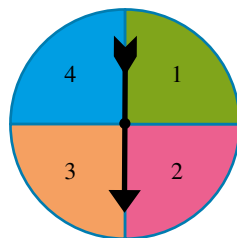
B. $\frac{4}{6}$

C. $\frac{1}{6}$

D. $\frac{5}{6}$

E. $\frac{2}{6}$

9. If you spin the following spinner, what is the probability of obtaining:



a. a 1 or a 3

b. an even number or an odd number?

10. The probabilities of Dale placing 1st, 2nd, 3rd or 4th in the local surf competition are:

1st = $\frac{1}{6}$

2nd = $\frac{1}{5}$

3rd = $\frac{2}{5}$

4th = $\frac{7}{30}$.

Find the probability that Dale places:

a. 1st or 2nd

b. 3rd or 4th

c. 1st, 2nd or 3rd

d. not 1st.

11. **WE17** A circular spinner that is divided into two equal halves, coloured red and blue, is spun 3 times.

a. Draw a tree diagram for the experiment.

b. Calculate the following probabilities.

i. P(3 red sectors)

ii. P(2 red sectors)

iii. P(1 red sector)

iv. P(0 red sectors)

v. P(at least 1 red sector)

vi. P(at least 2 red sectors)

12. **WE18** There are two yellow tickets, three green tickets, and four black tickets in a jar. Choose one ticket, replace it, then choose another ticket. Find the probability that a yellow ticket is drawn first, then a black ticket.



13. **WE19** Repeat question 12 with the first ticket not being replaced before the second ticket is drawn.

14. A coin is tossed two times. Determine P(a Head and a Tail in any order).

15. A coin is tossed three times. Determine P(H, H, T) (in that order).

16. A coin and a die are tossed. What is the probability of a Heads–2 outcome?

17. Holty is tossing two coins. He claims that flipping two Heads and flipping zero Heads are complementary events. Is he correct? Explain your answer.

18. Each of the numbers 1, 2, 3, ... 20 is written on a card and placed in a bag. If a card is drawn from the bag, find:

a. P(drawing a multiple of 3 or a multiple of 10)

b. P(drawing an odd number or a multiple of 4)

c. P(drawing a card with a 5 or a 7)

d. P(drawing a card with a number less than 5 or more than 16).

19. From a shuffled pack of 52 cards, a card is drawn. Find:

a. P(hearts or the jack of spades)

b. P(a queen or a jack)

c. P(a 7, a queen or an ace)

d. P(neither a club nor the king of spades).

20. **MC** Which of the following are not mutually exclusive?
- A. Obtaining an odd number on a die and obtaining a 4 on a die
 - B. Obtaining a Head on a coin and obtaining a Tail on a coin
 - C. Obtaining a red card and obtaining a black card from a pack of 52 playing cards
 - D. Obtaining a diamond and obtaining a king from a pack of 52 playing cards
 - E. None of the above
21. Greg has a 30% chance of scoring an A on an examination, Carly has 70% chance of scoring an A on the examination, and Chilee has a 90% chance of scoring an A on the examination. What is the probability that all three can score an A on the examination?
22. From a deck of playing cards, a card is drawn at random, noted and replaced, then another card is drawn at random. Find the probability that:
- a. both cards are spades
 - b. neither card is a spade
 - c. both cards are aces
 - d. both cards are the ace of spades
 - e. neither card is the ace of spades.
23. Repeat question 22 with the first drawn card not being replaced before the second card is drawn.
24. Assuming that it is equally likely that a boy or a girl will be born, answer the following.
- a. Show the gender possibilities of a 3-child family on a tree diagram.
 - b. In how many ways is it possible to have exactly 2 boys in the family?
 - c. What is the probability of getting exactly 2 boys in the family?
 - d. Which is more likely, 3 boys or 3 girls in the family?
 - e. What is the probability of having at least 1 girl in the family?

Communicating, reasoning and problem solving

25. Give an example of mutually exclusive events that are not complementary events using:
- a. sets
 - b. a Venn diagram.
26. Explain why all complementary events are mutually exclusive but not all mutually exclusive events are complementary.
27. A married couple plans to have four children.
- a. List the possible outcomes in terms of boys and girls.
 - b. What is the probability of them having exactly two boys?
 - c. Another couple plans to have two children. What is the probability that they have exactly one boy?
28. A bag contains 6 marbles, 2 of which are red, 1 is green and 3 are blue. A marble is drawn, the colour is noted, the marble is replaced and another marble is drawn.



- a. Show the possible outcomes on a tree diagram.
- b. List the outcomes of the event 'the first marble is red'.
- c. Calculate $P(\text{the first marble is red})$.
- d. Calculate $P(2 \text{ marbles of the same colour are drawn})$.

29. A tetrahedron (prism with 4 identical triangular faces) is numbered 1, 1, 2, 3 on its 4 faces. It is rolled twice. The outcome is the number facing downwards.
- Show the results on a tree diagram.
 - Are the outcomes 1, 2 and 3 equally likely?
 - Find the following probabilities.
 - $P(1, 1)$
 - $P(1 \text{ is first number})$
 - $P(\text{both numbers the same})$
 - $P(\text{both numbers are odd})$
30. Robyn is planning to watch 3 league games on one weekend. She has a choice of two games on Friday night: (A) Broncos vs Eels and (B) Bulldogs vs Cowboys. On Saturday, she can watch one of three games: (C) Knights vs Storm, (D) Titans vs Sharks and (E) Rabbitohs vs Raiders. On Sunday, she also has a choice of three games: (F) Roosters vs Tigers, (G) Dragons vs Panthers and (H) Warriors vs Sea Eagles. She plans to watch one game each day and will choose a game at random.
- To determine the different combinations of games she can watch, Robyn draws a tree diagram using the codes A, B, ... H. List the sample space for Robyn's selections.
 - Robyn's favourite team is the Broncos. What is the probability that one of the games Robyn watches involves the Broncos?
 - Robyn has a good friend who plays for the Roosters. What is the probability that Robyn watches both the matches involving the Broncos and the Roosters?
31. What is the difference between independent events and mutually exclusive events?



9.7 Conditional probability

[Stage 5.2]

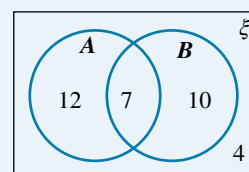
- The probability that an event occurs given that another event has already occurred is called **conditional probability**.
- The probability that event B occurs, given that event A has already occurred is denoted by $P(B|A)$. The symbol 'I' stands for 'given'.
- The formula for conditional probability is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0.$$

WORKED EXAMPLE 20

This Venn diagram at right shows the results of a survey where students were asked to indicate whether they liked apples or bananas. If one student is selected at random:

- What is the probability that the student likes bananas?
- What is the probability that the student likes bananas, given that they also like apples?
- Comment on any differences between the answers for parts a and b.



THINK

- a 1** Find the total number of students.
- 2** Find the total number of students who like bananas.
- 3** Determine the probability using the correct formula.
- 4** Write the answer.

- b 1** Determine the number of students who like apples.
- 2** Find the probability that a student likes apples.
- 3** Note the number liking both apples and bananas. This is the overlapping region of the two sets.
- 4** Determine the probability a student likes both apples and bananas.
- 5** Apply the formula to determine the conditional probability.

6 Write the answer.

- c** Why aren't the answers for parts **a** and **b** the same?

WRITE

$$\begin{aligned} \mathbf{a} \text{ Total number of students in survey} \\ &= 12 + 7 + 10 + 4 \\ &= 33 \end{aligned}$$

$$\begin{aligned} \text{Total number who like bananas} \\ &= 7 + 10 \\ &= 17 \end{aligned}$$

$$\begin{aligned} P(\text{bananas}) &= P(B) \\ &= \frac{\text{Total number who like bananas}}{\text{Total number of students}} \\ &= \frac{17}{33} \end{aligned}$$

The probability that a student likes bananas is $\frac{17}{33}$.

$$\begin{aligned} \mathbf{b} \text{ Number of students who like apples} \\ &= 12 + 7 \\ &= 19 \end{aligned}$$

$$\begin{aligned} P(\text{apples}) &= P(A) \\ &= \frac{\text{Number of students who like apples}}{\text{Total number of students}} \\ &= \frac{19}{33} \end{aligned}$$

$$\begin{aligned} \text{Number who like both apples and bananas} \\ &= n(A \cap B) \\ &= 7 \end{aligned}$$

$$P(A \cap B) = \frac{7}{33}$$

$$\begin{aligned} P(B | A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{\frac{7}{33}}{\frac{19}{33}} \\ &= \frac{7}{19} \end{aligned}$$

The probability that a student likes bananas, given that they also like apples is $\frac{7}{19}$.

This answer is also supported by the figures in the Venn diagram.

- c** The answer for part **a** determines the proportion of students who like bananas out of the whole group of students. The part **b** answer gives the proportion of students who like bananas out of those who like apples.

Note: These probabilities could also be expressed as decimals or percentages.
It is possible to transpose the conditional probability formula to determine $P(A \cap B)$.

$$P(A \cap B) = P(A) \times P(B | A)$$

WORKED EXAMPLE 21

In a student survey, the probability that a student likes apples is $\frac{19}{33}$. The probability that a student likes bananas, given that they also like apples, is $\frac{7}{19}$. What is the probability that a student selected at random likes both apples and bananas?

THINK

1 Write the given information.

WRITE

$$P(A) = \frac{19}{33}$$

$$P(B | A) = \frac{7}{19}$$

2 Apply the rearranged conditional probability formula.

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B | A) \\ &= \frac{19}{33} \times \frac{7}{19} \\ &= \frac{7}{33} \end{aligned}$$

3 Answer the question.

The probability that a student selected at random likes both apples and bananas is $\frac{7}{33}$.

- Conditional probability can also be determined by examining outcomes from a tree diagram.

WORKED EXAMPLE 22

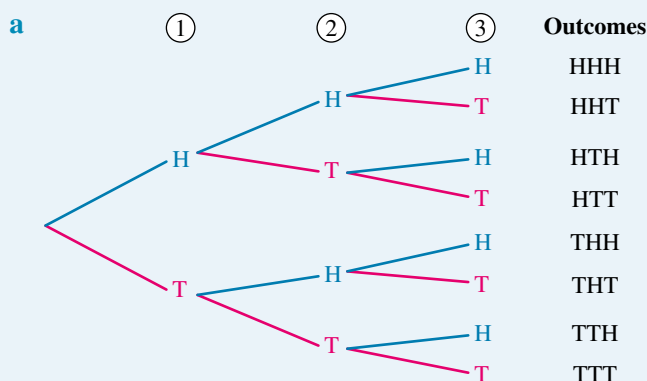
Three coins are flipped simultaneously.

- Display the outcomes as a tree diagram.
- Determine the probability that a Head will result from the third coin, given that the first two coins resulted in a Head (H) and a Tail (T).

THINK

a Draw a tree diagram to display the flipping of three coins. Write the individual outcomes.

WRITE/DRAW



b 1 Look for the outcomes where the first two flips resulted in a Head and a Tail.

b There are four outcomes where the first two flips are a Head and a Tail — HTH, HTT, THH and THT.

2 How many of these outcomes have a Head for the third flip?

3 Calculate the probability.

There are two of these outcomes where the third flip resulted in a Head — HTH and THH.

From four possible outcomes, two satisfy the conditions.

$$\begin{aligned} P(\text{H on third flip} \mid \text{H and T on first two flips}) \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

- A two-way table can also be used to determine conditional probability.

WORKED EXAMPLE 23

Two dice are rolled and the numbers are added.

a Show the results in a two-way table.

b Determine the probability that the sum of the two dice is 7, given that their total is greater than 6.

THINK

a Show the results of rolling two dice in a two-way table.

WRITE/DISPLAY

a

		Die 2					
		1	2	3	4	5	6
Die 1	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b 1 Which outcomes have a total greater than 6?

2 Which of these outcomes have a total equal to 7?

3 Write the probability.

b There are 36 outcomes. 21 of these have a total greater than 6: (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (6, 2) ... etc.

There are 6 of these outcomes that have a total of 7: (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6).

$$P(\text{total of 7} \mid \text{total greater than 6}) = \frac{6}{21} = \frac{2}{7}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Conditional probability (int-6085)

eLesson: Conditional probability (eles-1928)

Digital doc: WorkSHEET Probability 3 (doc-6315)

Individual pathways

Questions:
1-3, 5, 7, 9-11

Questions:
1-4, 6-13

Questions:
1-16

Understanding and fluency

-
- A Venn diagram with two overlapping circles, labeled A and B . The universal set is labeled with the Greek letter ξ in the top right corner. The number of elements in each region is as follows:
- Region A only: 23
 - Intersection of A and B : 16
 - Region B only: 15
 - Outside both sets (in ξ): 6

- $$P(A) = \frac{7}{10} \text{ and } P(B | A) = \frac{3}{7}.$$

Determine $P(A \cap B)$.



4. **WE22** Two fair coins are tossed.
- Display the outcomes as a tree diagram.
 - Determine the probability that a Head results on the second coin, given that the first coin also resulted in a Head.
5. **WE23** Two standard dice are rolled and the numbers are added together.
- Show the results in a two-way table.
 - Determine the probability that the sum of the two dice is even, given that their total is greater than 7.
6. A group of 40 people was surveyed regarding the types of movies, comedy or drama, that they enjoyed. The results are shown below.
- 28 enjoyed comedy.
17 enjoyed drama.
11 liked both comedy and drama.
6 did not like either type.
- Draw a Venn diagram to display the results of the survey.
 - Determine the probability that a person selected at random:
 - likes comedy
 - likes drama
 - likes both comedy and drama
 - likes drama, given that they also like comedy
 - likes comedy, given that they also like drama.
 - Arrange the probabilities in part **b** in order from least probable to most probable.



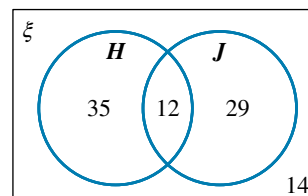
7. A teacher gave her class two tests. Only 25% of the class passed both tests, but 40% of the class passed the first test. What percentage of those who passed the first test also passed the second test?

Communicating, reasoning and problem solving

8. If $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$, calculate:
- $P(A \cap B)$
 - $P(B | A)$
 - $P(A | B)$.
9. A group of 80 boys is auditioning for the school musical. They are all able to either sing, play a musical instrument, or both. Of the group, 54 can play a musical instrument and 35 are singers. What is the chance that if a randomly selected student is a singer he can also play a musical instrument?
10. A white die and a black die are rolled. The dice are 6-sided and unbiased. Consider the following events.
- Event A: the white die shows a 6.
Event B: the black die shows a 2.
Event C: the sum of the two dice is 4.
Determine the following probabilities.
- $P(A | B)$
 - $P(B | A)$
 - $P(C | A)$
 - $P(C | B)$

11. A die is rolled and the probability of rolling a 6 is $\frac{1}{6}$. However, with the condition that the number rolled was an even number, its probability is $\frac{1}{3}$. Explain why the probabilities are different, using conditional probability.

12. A group of students was asked to nominate their favourite form of dance, hip hop (H) or jazz (J). The results are illustrated in the Venn diagram. Use the Venn diagram given to calculate the following probabilities relating to a student's favourite form of dance.



- a. What is the probability that a randomly selected student likes jazz?
b. What is the probability that a randomly selected student enjoys hip hop, given that they like jazz?
13. At a school classified as a 'Music school for excellence', the probability that a student elects to study Music and Physics is 0.2. The probability that a student takes Music is 0.92. What is the probability, correct to 2 decimal places, that a student takes Physics, given that the student is taking Music?



14. A medical degree requires applicants to participate in two tests: an aptitude test and an emotional maturity test. This year, 52% passed the aptitude test and 30% passed both tests. Use the conditional probability formula to calculate the probability, correct to 2 decimal places, that a student who passed the aptitude test also passed the emotional maturity test.



15. The probability that a student is well and misses a work shift the night before an examination is 0.045, and the probability that a student misses a work shift is 0.05. What is the probability that a student is well, given they miss a work shift the night before an examination?
16. How can you determine when a probability question is a conditional one?

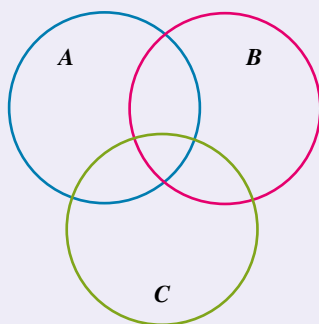
9.8 Review

Investigation | Rich task

Sand-rings

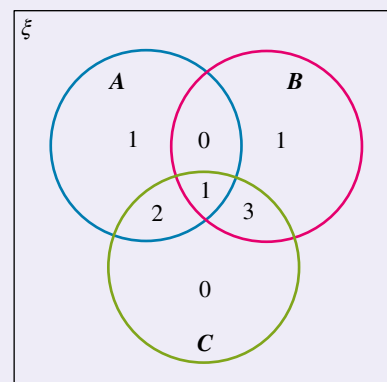


A class of students and their teacher spent a day at the beach as part of a school excursion. Part of the day was devoted to activities involving puzzles in the sand. One of the popular — and most challenging — puzzles was ‘sand-rings’. Sand-rings involve drawing three rings in the sand as shown in the diagram.



The first sand-rings puzzle requires 8 shells to be arranged inside the circles, so that 4 shells appear inside circle A, 5 shells appear inside circle B and 6 shells appear inside circle C. The overlapping of the circles shows that the shells can be counted in 2 or 3 circles. One possible arrangement is shown at right. Use this diagram to answer questions 1 to 4.

1. How many shells appear inside circle A but not circle B?
2. How many shells appear in circles B and C but not circle A?



A shell is selected at random from the sand.

3. What is the probability it came from circle A?
4. What is the probability it was not in circle C?
5. The class was challenged to find the rest of the arrangements of the 8 shells. (*Remember:* 4 shells need to appear in circle A, 5 in circle B and 6 in circle C.)

After completing the first puzzle, the students are given new rules. The number of shells to be arranged in the circles is reduced from 8 to 6. However, the number of shells to be in each circle remains the same; that is, 4 shells in circle A, 5 shells in circle B and 6 shells in circle C.

6. Using 6 shells, in how many ways can the shells be arranged so that there are 4, 5 and 6 shells in the three circles?
7. Explain the system or method you used to determine your answer to question 6 above. Draw diagrams to show the different arrangements.
8. Using 7 shells, in how many ways can the shells be arranged so that there are 4, 5 and 6 shells in the three circles?
9. Again, explain the system or method you used to determine your answer to question 8. Draw diagrams to show the different arrangements.
10. What would be the minimum number of shells required to play sand-rings so that there are 4, 5 and 6 shells in the three circles?
11. Modify the rules of this game so that different totals are required for the 3 circles. Challenge your classmates to find all possible solutions to your modified game.



Digital doc: Investigation — Sand-rings (doc-15908)

Digital doc: Code puzzle — Roy G. Biv is a mnemonic (memory aid) for what purpose? (doc-15909)

Digital doc: Summary Topic 9 (doc-22891)

Interactivity: Crossword Topic 9 (int-2712)

Interactivity: Word search Topic 9 (int-2711)

Interactivity: Sudoku Topic 9 (int-3212)

Exercise 9.8 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- MC** In a trial, it was found that a drug cures $\frac{2}{5}$ of those treated by it. If 700 sufferers are treated with the drug, how many of them are not expected to be cured?
A. 280 **B.** 420 **C.** 140 **D.** 350 **E.** 100
- MC** Two dice are rolled and their numbers added. The probability that their total is odd, given that the total is less than 5 is:
A. $\frac{2}{5}$ **B.** $\frac{1}{2}$ **C.** $\frac{3}{5}$ **D.** $\frac{1}{3}$ **E.** $\frac{2}{3}$
- MC** If a die is rolled and a coin tossed, what is the probability of a 6–Heads result?
A. $\frac{1}{6}$ **B.** $\frac{1}{2}$ **C.** $\frac{1}{8}$ **D.** $\frac{1}{12}$ **E.** $\frac{1}{3}$
- MC** Which one of the following does not represent independent events?
A. Flipping a coin, then rolling a die
B. The colour of your hair, and your marks in school
C. Choosing a card from a standard deck of cards without replacing it, then choosing another card from the same deck
D. Flipping a coin ten times
E. None of the above
- MC** Twelve nuts are taken from a jar containing macadamias and cashews. If 3 macadamias are obtained, the experimental probability of obtaining a cashew is:
A. $\frac{1}{12}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{3}$ **D.** $\frac{3}{4}$ **E.** $\frac{2}{3}$
- MC** From a normal pack of 52 playing cards, one card is randomly drawn and replaced. If this is done 208 times, the number of red or picture cards expected to turn up is:
A. 150 **B.** 130 **C.** 128 **D.** 120 **E.** 104
- MC** A cubic die with faces numbered 2, 3, 4, 5, 6 and 6 is rolled. The probability of rolling an even number is:
A. $\frac{1}{3}$ **B.** $\frac{2}{3}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{2}$ **E.** $\frac{1}{5}$
- MC** The probability of rolling an odd number or a multiple of 2 using the die in question 7 is:
A. 1 **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{3}{4}$ **E.** $\frac{2}{3}$

The following information applies to questions 9 and 10.

Students in a Year 9 class chose the following activities for a recreation day.

Activity	Tennis	Fishing	Golf	Bushwalking
Number of students	8	15	5	7

- MC** If a student is selected at random from the class, the probability that the student chose fishing is:
A. $\frac{1}{7}$ **B.** $\frac{2}{7}$ **C.** $\frac{3}{7}$ **D.** $\frac{4}{7}$ **E.** $\frac{5}{7}$

10. **MC** If a student is selected at random, the probability that the student did not choose bushwalking is:
- A. $\frac{1}{35}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$ E. $\frac{5}{7}$

11. The mass of 40 students in a Year 9 Maths class was recorded in a table.

Mass (kg)	Less than 50	50–<55	55–<60	60–<65	65 and over
Number of students	4	6	10	15	5

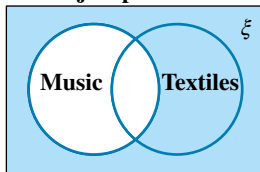
What is the experimental probability of selecting a student who has:

- a mass of 55 kg or more, but less than 60 kg?
 - a mass less than 50 kg?
 - a mass of 65 kg or greater?
12. Calculate the following expected values.
- The number of Heads in 80 tosses of a coin
 - The number of sixes in 200 rolls of a die
 - The number of hearts if a card is picked from a reshuffled pack and replaced 100 times
13. Consider a family of three children. Given that the first child is a girl, what is the probability that the other two children are both boys?
14. Insert each of the letters **a** to **d** to represent the following events at appropriate places on the probability scale shown.

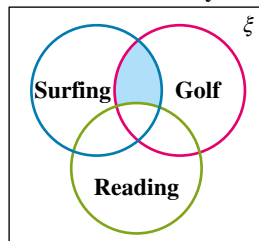


- You will go to school on Christmas Day.
 - All Year 9 students can go to university without doing Year 10.
 - Year 9 students will study Mathematics.
 - An Australian TV channel will telecast the news at 6.00pm.
15. Indicate the set that each of the shaded regions represents.

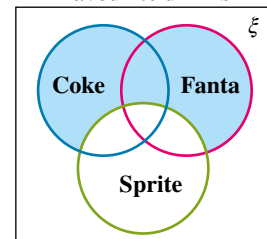
a. Subject preference



b. Leisure activity



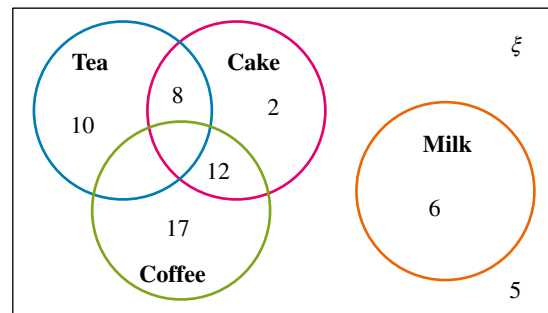
c. Favourite drinks



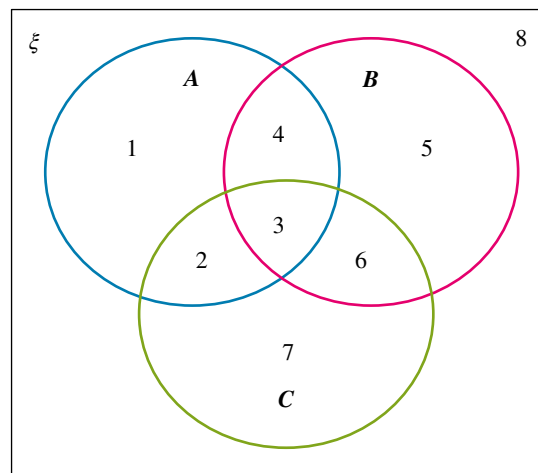
16. A normal six-sided die is rolled. What is the probability of getting an odd number or a multiple of 4?
17. A card is drawn from a pack of 52 cards. What is the probability that the card is a heart or a club?
18. An equal number of red (R), black (B) and yellow (Y) counters makes up a total of 30 in a bag.
- List the sample space for picking a counter from the bag.
 - Event A is 'the first counter is yellow' if two counters are selected. List the sample space.
 - Is 'choosing a green counter' an outcome? Explain your answer.

Communicating, reasoning and problem solving

19. Teachers at a school opted for the choice of morning recess refreshments shown in the Venn diagram at right.
- How many teachers are in the set 'cake $\cap$ coffee'?
 - What is the total number of teachers surveyed?



- c. If a teacher is selected at random, find the probability that the teacher:
- chose tea
 - chose coffee only
 - chose milk
 - did not choose tea, coffee, cake or milk
 - did not choose coffee.
20. Thirty-two students ordered fried rice, chicken wings and dim sims for lunch. Four students ordered all three, 2 ordered fried rice and chicken wings only, 3 ordered fried rice and dim sims only, and 3 ordered chicken wings and dim sims only. When the waiter organised the orders, he found that 16 students ordered fried rice and 18 students ordered dim sims.
- Show this information on a Venn diagram.
 - How many students ordered chicken wings only?
 - If a student is picked at random, find the probability that the student has:
 - ordered chicken wings and dim sims
 - ordered fried rice
 - not ordered dim sims.
21. The following are options for dorm rooms at a university. You are required to choose one option from each of the four categories. How many different combinations of rooms are there to choose from?
- Upstairs or downstairs rooms
 - Single or double rooms
 - Male or female roommates
 - A choice of 10 different locations on campus
22. Use the Venn diagram to calculate each of the following.
- $A' \cup (B' \cap C')$
 - $A' \cap (B \cap C')$
 - $A \cap (B' \cup C')$
 - $(A \cup B \cup C)' \cap (A \cup B \cup C)$
23. A witness described a getaway car as having a NSW registration plate starting with TLK. The witness could not remember the three digits that followed, but recalled that all three digits were different. How many cars in NSW could have a registration plate with these letters and numbers?
24. There are 12 people trying out for a badminton team. Five of them are girls. What is the probability that a team chosen at random to play is a mixed doubles team?
25. An ace is chosen from a deck of standard cards and not replaced. A king is then chosen from the deck. What is the probability of choosing an ace and a king in this order?
26. If you randomly select one number from 1 to 500 (inclusive), what is the probability that the selected number will have at least one four in the digits?
27. If you flip a coin 8 times, how many of the possible outcomes would you expect to have a Head on the second toss?
28. Any future king is the oldest male child. The future king of Mainland has two siblings. What is the probability he has an older sister?

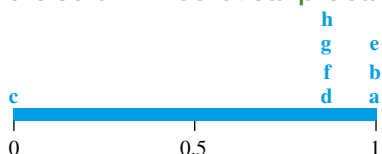


Answers

Topic 9 Probability

Exercise 9.2 Theoretical probability

1.



2. a. Certain

e. Certain

3. a. {1, 2, 3, 4, 5, 6}

d. {red, black}

4. a. {1, 2, 3}

d. {3, 4, 5, 6}

5. a. 3, 5

e. 1, 2, 3, 5

6. a. {hearts, clubs, diamonds, spades}

b. i. {clubs, spades}

iii. {clubs, diamonds, spades}

7. a. 6

d. 4

8. a. $\frac{1}{13}$

9. a. $\frac{1}{52}$

e. $\frac{1}{2}$

i. 0

10. a. $\frac{2}{11}$

11. a. $\frac{1}{4}$

d. 0

12. a. i. $\frac{2}{3}$

b. Answers will vary.

b. Certain

f. Likely

b. {H, T}

e. {hearts, clubs, diamonds, spades}

b. {1, 3, 5}

e. {1, 2, 3, 4}

b. 1, 2

f. no favourable outcomes

c. Unlikely

g. Likely

c. {1, 2, 4, 6}

f. {5, 6}

c. 6

g. 3

d. Likely

h. Likely

c. {defective, not defective}

ii. {hearts, diamonds}

iv. {hearts, clubs, diamonds, spades}

c. 52

c. $\frac{1}{52}$

c. $\frac{1}{26}$

g. $\frac{4}{13}$

c. $\frac{4}{11}$

c. $\frac{5}{16}$

d. $\frac{1}{4}$

d. $\frac{1}{4}$

h. $\frac{2}{13}$

d. $\frac{7}{11}$

13, 14. Check with your teacher.

15. Probabilities must be between 0 and 1, so -0.5 and 1.05 can't be probabilities.

16. The coloured portions outside the arc of the spinner shown are of no consequence. The four colours within the arc of the spinner are of equal area (each $\frac{1}{4}$ circle), so there is equal chance of falling on each of the colours.

17. $\frac{2}{3}$

18. There are 36 outcomes, 15 under 7, 6 equal to 7 and 15 over 7. So, you would have a greater chance of winning if you chose 'under 7' or 'over 7' rather than 'equal to 7'.

19. 0.09

20. Answers will vary. For example: 'If you have read the text, there is a likely probability that you will get this answer correct, however it is not certain.'

Exercise 9.3 Experimental probability

1. a. i. 16

ii. 4

iii.

Outcome	1	2	3	4	5	6
Relative frequency	0.19	0.06	0.31	0.13	0.25	0.06

iv. 1

b. i. 270

ii. 40

iii.	Outcome	1	2	3	4	5	6
	Relative frequency	0.19	0.14	0.17	0.18	0.15	0.17

iv. 1

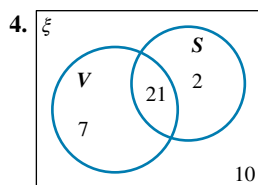
2. a. i. $r.f.(H) = 0.44$, $r.f.(T) = 0.56$ ii. 1
b. i. $r.f.(H) = 0.62$, $r.f.(T) = 0.38$ ii. 1
3. a. 0.365 b. 0.33 c. 0.25 d. 0.875
4. a. 0.275 b. 0.64 c. 0.365 d. 0.635
5. a. Not reasonable; not enough trials were held.
b. Not reasonable; the conditions are different under each trial.
c. Not reasonable; there are seasonal influences.
d. Reasonable; enough trials were performed under the same conditions.
e. Not reasonable; monthly rainfall in Sydney is not consistent throughout the year.
6. a. $\frac{5}{24}$ b. $\frac{59}{72}$
7. A
8. 200
9. 20
10. B
11. a. $\frac{11}{20}$ b. $\frac{3}{10}$ c. $\frac{17}{20}$ d. $\frac{3}{20}$
12. a. $\frac{33}{100}$ b. $\frac{40}{100} = \frac{2}{5}$ c. $\frac{3}{100}$
13. a. i. 6 ii. {356, 365, 536, 563, 635, 653} d. $\frac{1}{3}$
b. $\frac{2}{3}$ c. $\frac{1}{3}$
14. D
15. Svetlana 6, Sarah 4, Leonie 3, Trang 2
16. C
17. a. 27
b. {111, 112, 115, 121, 122, 125, 151, 152, 155, 211, 212, 215, 221, 222, 225, 251, 252, 255, 511, 512, 515, 521, 522, 525, 551, 552, 555}
c. $\frac{1}{3}$ d. $\frac{2}{3}$ e. $\frac{1}{3}$
18. a. $\frac{1}{12}$
b. $\frac{1}{20}$
c. No, because John has a higher probability of winning.
19. a. $\frac{7}{10}$
b, c. Yes, far too many mistakes
20. a. $\frac{30}{241}$ b. $\frac{91}{241}$ c. $\frac{59}{241}$
21. 32
22. 1000 balls
23. Red = 10, yellow = 50, blue = 40
24. a. $\frac{7}{20}$ b. $\frac{2}{5}$
25. Both theoretical and experimental probability calculations aim to give the probability of each possible outcome.

Exercise 9.4 Venn diagrams and two-way tables

1. a. $A' = \{11, 13, 14, 16, 17, 19, 20\}$ b. $B' = \{20\}$
c. $C' = \{12, 14, 15, 16, 18, 20\}$ d. $D' = \{12, 14, 16\}$
2. a. 27 b. 11 c. 13 d. 6
e. 6 f. 7 g. 9

3.

	T	T'
S	6	5
S'	7	9



5. a. i. W'

b. i. $A \cap B$

ii. $A \cap B'$

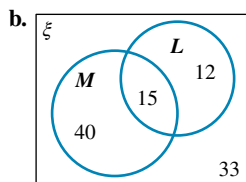
c. i. $A' \cap B'$

ii. $A' \cap B$

6. a.

	Left-handed	Right-handed
Male	15	40
Female	12	33

a.



c. i. $\frac{73}{100}$

ii. $\frac{2}{5}$

iii. $\frac{3}{25}$

7. a. $\frac{8}{25}$

b. $\frac{17}{25}$

c. $\frac{1}{5}$
c. $\frac{31}{40}$

d. $\frac{3}{25}$
d. $\frac{27}{40}$

e. $\frac{9}{40}$

8. a. $\frac{1}{2}$

b. $\frac{1}{10}$

iii. $\frac{5}{8}$

iv. $\frac{7}{16}$

9. a. 16

b. i. $\frac{5}{16}$

ii. $\frac{3}{16}$

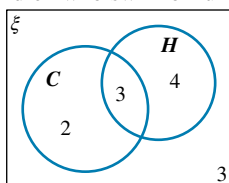
10. a. Children who are not swimmers

b. Children who are swimmers and runners

c. Children who neither swim nor run

d. Children who swim or run or both

11. a. i.



ii.

	H	H'
C	3	2
C'	4	3

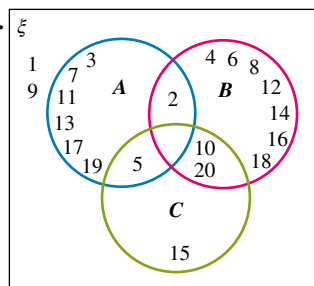
b. i. $\frac{7}{12}$

ii. $\frac{3}{4}$

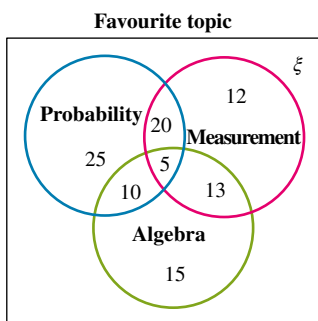
iii. $\frac{1}{4}$

iv. $\frac{1}{2}$

12.



13. a.



b. 25

c. 15

d. 12

e. 43

f. $\frac{3}{5}$

g. $\frac{43}{100}$

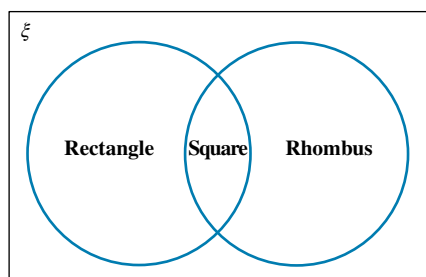
h. $\frac{9}{50}$

i. $\frac{13}{100}$

j. $\frac{1}{20}$

14.

Quadrilaterals



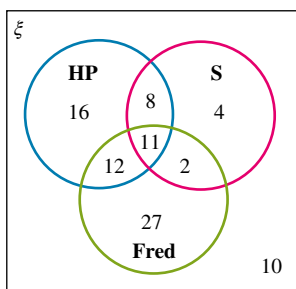
15. a. 2, 5, 6, 7, 8

b. 4

c. 5, 7, 8

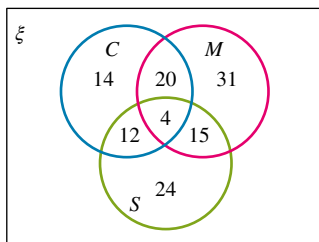
d. 8

16. a.



b. $\frac{8}{90} = \frac{4}{45}$

17. a.

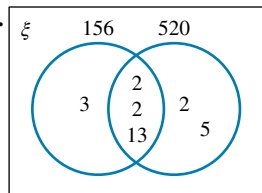


b. $\frac{7}{12}$

18. a. 31 students

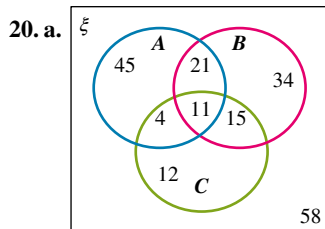
b. $\frac{1}{13}$

19.



a. $\text{HCF} = 2 \times 2 \times 13 = 52$

b. $\text{LCM} = 3 \times 2 \times 2 \times 13 \times 2 \times 5 = 1560$



b. 4

c. 21

d. i. $\frac{81}{200}$

ii. $\frac{97}{200}$

e. $\frac{3}{50}$

21. When one event *and* another event occurs, it is expressed $A \cap B$, but when one event *or* another event occurs, it is expressed $A \cup B$. However, it is important to remember that $A \cap B$ is included in the group $A \cup B$.

22. 120

Exercise 9.5 Two-step experiments

1.

		Scarves		
		R	Bl	Pi
Beanies	Br	Br, R	Br, Bl	Br, Pi
	Pu	Pu, R	Pu, Bl	Pu, Pi

2.

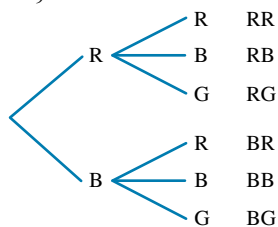
		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a. $\frac{1}{9}$

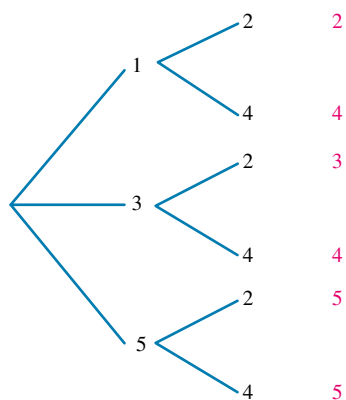
b. 0

c. A total of 7

3.



4. a.



- b. i. $\frac{1}{6}$
 ii. 0
 iii. $\frac{1}{2}$

5. a.

		Die 1					
		1	2	3	4	5	6
Die 2	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

- b. i. $\frac{1}{2}$

- ii. $\frac{1}{6}$

- iii. $\frac{1}{3}$

- iv. $\frac{2}{3}$

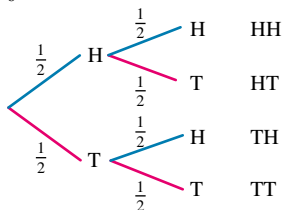
6. a. $\frac{1}{6}$

b. $\frac{1}{36}$

c. $\frac{1}{4}$

d. $\frac{1}{36}$

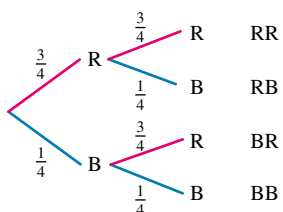
7. a.



- b. i. $\frac{1}{4}$

- ii. $\frac{3}{4}$

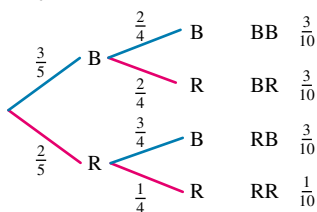
8. a.



- b. i. $\frac{3}{16}$

- ii. $\frac{1}{16}$

9. a.

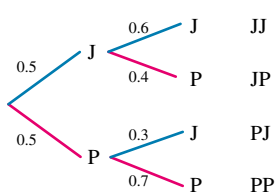


- b. $\frac{1}{10}$

10. a. $\frac{1}{4}$

- b. $\frac{3}{14}$

11. a.



- b. i. 0.3

- ii. 0.35

- iii. 0.35

12. If the first ball is replaced, the probability of drawing a yellow ball stays the same on the second draw, i.e. $\left(\frac{3}{5}\right)$.

If the first ball isn't replaced, the probability of drawing a yellow ball on the second draw decreases, i.e. $\left(\frac{5}{9}\right)$.

13. a. Smallest total: 3, largest total: 18

b.

Total	Probability
3	$\frac{1}{216}$
4	$\frac{3}{216}$
5	$\frac{6}{216}$
6	$\frac{10}{216}$
7	$\frac{15}{216}$
8	$\frac{21}{216}$
9	$\frac{25}{216}$
10	$\frac{27}{216}$
11	$\frac{27}{216}$
12	$\frac{25}{216}$
13	$\frac{21}{216}$
14	$\frac{15}{216}$
15	$\frac{10}{216}$
16	$\frac{6}{216}$
17	$\frac{3}{216}$
18	$\frac{1}{216}$

14. a. $\frac{1}{221}$

b. $\frac{11}{221}$

c. $\frac{1}{17}$

15. $\frac{5}{12}$

16. $\frac{1}{2}$

17. $\frac{45}{1081}$

18. If replacement does not occur before a second draw of an event, then the sample space is reduced by 1, which changes the probability of these second events.

19. Advantaged. The chance of getting a total of 7 would be $\frac{1}{3}$.

Exercise 9.6 Mutually exclusive and independent events

1. D

2. A

3. $\frac{215}{216}$

4. D

5. a. $\frac{1}{4}$
b. $\frac{17}{70}$

b. $\frac{3}{4}$
b. $\frac{53}{70}$

c. $\frac{9}{70}$

d. $\frac{61}{70}$

7. $\frac{2}{13}$

8. B

9. a. $\frac{1}{2}$

b. 1

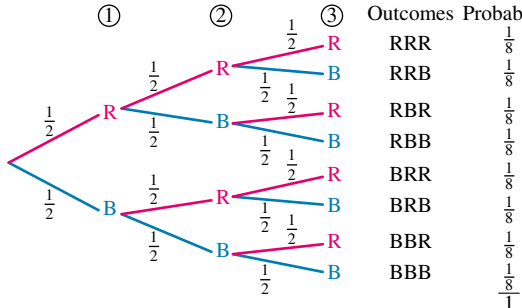
10. a. $\frac{11}{30}$

b. $\frac{19}{30}$

c. $\frac{23}{30}$

d. $\frac{5}{6}$

11. a.



b. i. $\frac{1}{8}$
iv. $\frac{1}{8}$

ii. $\frac{3}{8}$
v. $\frac{1}{8}$

iii. $\frac{3}{8}$
vi. $\frac{1}{2}$

12. $\frac{8}{81}$

13. $\frac{1}{9}$

14. $\frac{1}{2}$

15. $\frac{1}{8}$

16. $\frac{1}{12}$

17. No, because there is also the possibility of 1 Head (HT or TH).

18. a. $\frac{8}{20}$

b. $\frac{15}{20}$

c. $\frac{4}{20}$

d. $\frac{8}{20}$

19. a. $\frac{14}{52}$

b. $\frac{2}{13}$

c. $\frac{3}{13}$

d. $\frac{38}{52}$

20. D

21. $\frac{189}{1000}$ or 0.189

22. a. $\frac{1}{16}$

b. $\frac{9}{16}$

c. $\frac{1}{169}$

d. $\frac{1}{2704}$

e. $\frac{2601}{2704}$

23. a. $\frac{1}{17}$

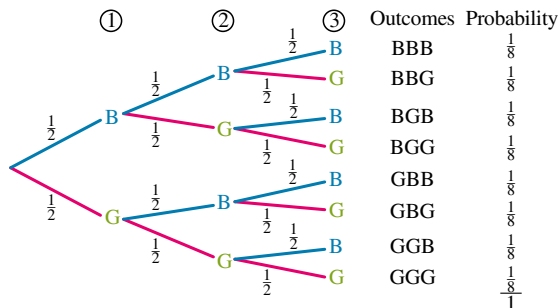
b. $\frac{19}{34}$

c. $\frac{1}{221}$

d. 0

e. $\frac{25}{26}$

24. a.



b. 3

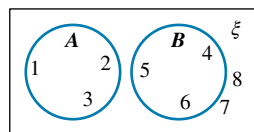
c. $\frac{3}{8}$

d. They are equally likely.

e. $\frac{7}{8}$

25. a. $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

b.

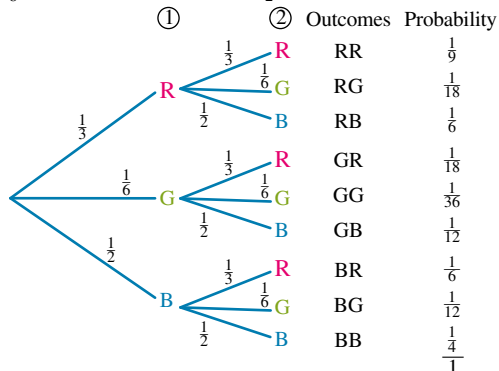
26. If two events are complementary, they cannot occur at the same time, thus their intersection is $\emptyset$, the same as mutually exclusive sets. However, if events are mutually exclusive, they do not need to have a sum equal to 1.

27. a. {BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG}

b. $\frac{3}{8}$

c. $\frac{1}{2}$

28. a.

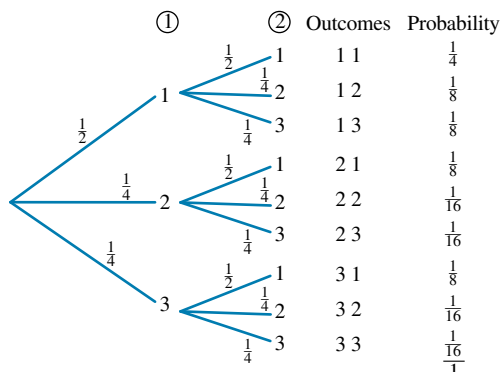


b. {(R, R), (R, G), (R, B)}

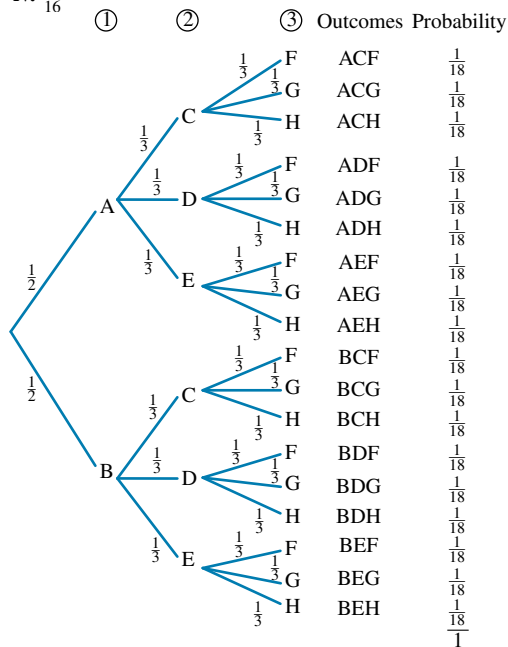
c. $\frac{1}{3}$

d. $\frac{7}{18}$

29. a.



30. a.



Sample space = { ACF, ACG, ..., BEG, BEH }

b. $\frac{1}{2}$
c. $\frac{1}{6}$

31. Events are independent if the outcome of one event is not dependent on the outcome of the other event. Mutually exclusive events cannot occur at the same time or have no elements in common.

Exercise 9.7 Conditional probability

1. a. 60

b. $\frac{4}{15}$
c. **i.** $\frac{13}{20}$
ii. $\frac{16}{39}$
d. **i.** $\frac{31}{60}$
ii. $\frac{16}{31}$

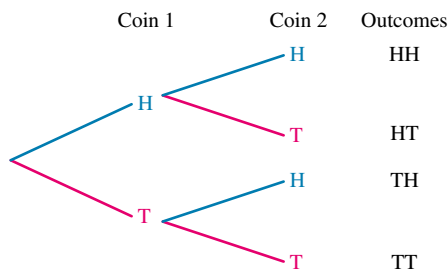
2. a. $\frac{4}{15}$

b. $\frac{4}{15}$

c. They are the same, and equal to the probability calculated in question 1 part b.

3. $\frac{3}{10}$

4. a.



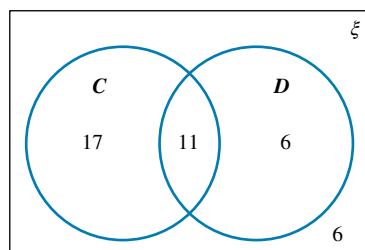
b. $\frac{1}{2}$

5. a.

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b. $\frac{3}{5}$

6. a.



b. i. $\frac{7}{10}$

ii. $\frac{17}{40}$

iii. $\frac{11}{40}$

iv. $\frac{11}{28}$

v. $\frac{11}{17}$

c. Order from least to most probable: iii, iv, ii, v, i.

7. 62.5%

8. a. 0.2

b. $\frac{2}{3}$

c. $\frac{2}{5}$

9. $\frac{9}{35}$

10. a. $\frac{1}{6}$

b. $\frac{1}{6}$

c. 0

d. $\frac{1}{6}$

11. Conditional probability reduces the sample space that the probability is calculated from. In this instance the sample space is reduced from 6 numbers (1, 2, 3, 4, 5, 6) to 3 numbers (2, 4, 6).

12. a. $P(J) = \frac{41}{90}$

b. $P(H|J) = \frac{12}{41}$

13. 0.22

14. 0.58

15. 0.9

16. Conditional probability is the probability of one event occurring given that another event has already occurred. Conditional probability questions are expressed $P(B|A)$.

Investigation | Rich task

1. 3

2. 4

3. $\frac{1}{2}$

4. $\frac{1}{4}$

5. 18

6. 2

7. Answers will vary.

8. 8

9. Answers will vary.

10. 6

11. Answers will vary.

Exercise 9.8 Review questions

1. B

2. D

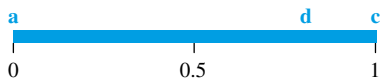
3. D

4. C

5. D
6. C
7. B
8. A
9. C
10. D

11. a. $\frac{1}{4}$ b. $\frac{1}{10}$ c. $\frac{1}{8}$
12. a. 40 b. 33 c. 25
13. $\frac{1}{4}$

14. b



15. a. Students who do not like music
b. People who like surfing and golf as leisure activities, but not reading
c. People who like Coke or Fanta or both but not Sprite

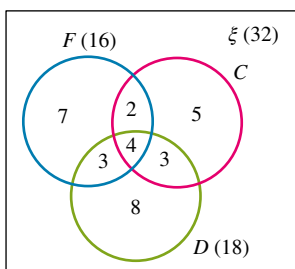
16. $\frac{2}{3}$
17. $\frac{1}{2}$

18. a. {R, B, Y}
b. {(YR), (YY), (YB)}
c. No, there is no green counter.

19. a. 12
b. 60

- c. i. $\frac{3}{10}$ ii. $\frac{17}{60}$ iii. $\frac{1}{10}$ iv. $\frac{1}{12}$ v. $\frac{31}{60}$

20. a.



b. 5

- c. i. $\frac{7}{32}$ ii. $\frac{1}{2}$ iii. $\frac{7}{16}$

21. 80

22. a. {1, 5, 6, 7, 8} b. {5} c. {1, 2, 4} d. $\emptyset$

23. 720

24. $\frac{35}{66}$

25. $\frac{4}{663}$

26. $\frac{176}{500}$

27. 128

28. $\frac{3}{7}$

TOPIC 10

Linear equations

[Stage 5.2]

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

10.1.1 Why learn this?

Looking for patterns in numbers, relationships and measurements helps us to understand the world around us. A mathematical model is a mathematical representation of a situation. If we can see a pattern in a table of values or a graph that shows ordered pairs following an approximately straight line, the model is called a linear model.



DISCUSSION

The Romans used mathematics to determine the numbers of men and equipment as well as strategic formations to build a formidable army. Are there any other ways in which the Romans used mathematics to build the dominant empire of their time?

LEARNING SEQUENCE

- 10.1** Overview
- 10.2** [Stage 5.2] Solving linear equations
- 10.3** [Stage 5.2] Solving linear equations with brackets
- 10.4** [Stage 5.2] Solving linear equations with pronumerals on both sides
- 10.5** [Stage 5.2] Solving problems with linear equations
- 10.6** [Stage 5.2] Substituting values into formulas
- 10.7** Review

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- solves linear equations MA5.2-8NA

CONTENT DESCRIPTIONS

Students:

Solve linear equations (ACMNA215)

Solve linear equations involving simple algebraic fractions (ACMNA240)

Substitute values into formulas to determine an unknown (ACMNA234)

Solve problems involving linear equations, including those derived from formulas (ACMNA235)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

10.2 Solving linear equations

[Stage 5.2]

10.2.1 What is a linear equation?

- An **equation** is a mathematical statement that contains an equals sign (=).
- For an equation, the **expression** on the left-hand side of the equals sign has the same value as the expression on the right-hand side.
- Solving a **linear equation** means finding a value for the pronumeral that makes the statement true.
- ‘Doing the same thing’ to both sides of the equation ensures that the two expressions remain equal.

WORKED EXAMPLE 1

For each of the following equations, determine whether $x = 10$ is a solution.

a $\frac{x+2}{3} = 6$

b $2x + 3 = 3x - 7$

c $x^2 - 2x = 9x - 10$

THINK

a 1 Substitute 10 for x in the left-hand side of the equation.

2 Write the right-hand side.

3 Is the equation true? That is, does the left-hand side equal the right-hand side?

4 State whether $x = 10$ is a solution.

b 1 Substitute 10 for x in the left-hand side.

2 Substitute 10 for x in the right-hand side.

3 Is the equation true?

4 State whether $x = 10$ is a solution.

c 1 Substitute 10 for x in the left-hand side.

WRITE

$$\begin{aligned} \text{a LHS} &= \frac{x+2}{3} \\ &= \frac{10+2}{3} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

$$\text{RHS} = 6$$

$$\text{LHS} \neq \text{RHS}$$

$x = 10$ is not a solution.

$$\begin{aligned} \text{b LHS} &= 2x + 3 \\ &= 2(10) + 3 \\ &= 23 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 3x - 7 \\ &= 3(10) - 7 \\ &= 23 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

$x = 10$ is a solution.

$$\begin{aligned} \text{c LHS} &= x^2 - 2x \\ &= 10^2 - 2(10) \\ &= 100 - 20 \\ &= 80 \end{aligned}$$

2 Substitute 10 for x in the right-hand side.

$$\begin{aligned}\text{RHS} &= 9x - 10 \\ &= 9(10) - 10 \\ &= 90 - 10 \\ &= 80\end{aligned}$$

3 Is the equation true?

$$\text{LHS} = \text{RHS}$$

4 State whether $x = 10$ is a solution.

$$x = 10 \text{ is a solution.}$$

10.2.2 Solving one-step equations

- If only one operation has been performed on a pronumeral, it is known as a **one-step equation**.
- Simple equations can be solved by performing the **inverse operation**.
- The inverse operation has the effect of undoing the original operation.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

WORKED EXAMPLE 2

Solve each of the following linear equations.

a $x - 7 = 153$

b $x + 46 = 82$

c $6x = 100$

d $\frac{x}{7} = 19$

THINK

a 1 7 is subtracted from x to give 153.

2 Apply the inverse operation by adding 7 to both sides of the equation.

3 Write the value of x .

b 1 46 is added to x to give 82.

2 Apply the inverse operation by subtracting 46 from both sides of the equation.

3 Write the value of x .

c 1 6 is multiplied by x to give 100.

2 Perform the inverse operation by dividing both sides of the equation by 6.

3 Write the value of x .

d 1 x is divided by 7 to give 19.

2 Perform the inverse operation by multiplying both sides of the equation by 7.

3 Write the value of x .

WRITE

a $x - 7 = 153$

$$x = 153 + 7$$

$$x = 160$$

b $x + 46 = 82$

$$x = 82 - 46$$

$$x = 36$$

c $6x = 100$

$$x = \frac{100}{6}$$

$$x = 16\frac{2}{3}$$

d $\frac{x}{7} = 19$

$$x = 19 \times 7$$

$$= 133$$

Note: In each case the result can be checked by substituting the value obtained for x into the original equation and confirming that it will make the equation a true statement.

10.2.3 Solving two-step equations

- If two operations have been performed on the pronumeral, it is known as a **two-step equation**.
- To solve two-step equations, determine the order in which the operations were performed.
- Perform inverse operations in the reverse order to both sides of the equation.
- Each inverse operation must be performed one step at a time.
- This principle will apply to any equation with two or more steps, as shown in the examples that follow.

WORKED EXAMPLE 3

Solve the following linear equations.

a $2y + 4 = 12$

b $6 - x = 8$

c $\frac{x}{3} - 4 = 2$

d $\frac{3x}{5} = 6$

THINK

a 1 First subtract 4 from both sides.

2 Divide both sides by 2.

3 Write the value of y .

b 1 $6 - x$ is the same as $-x + 6$. Rewrite the equation.

2 Subtract 6 from both sides.

3 Divide both sides by -1 .

4 Write the value of x .

c 1 Add 4 to both sides.

2 Multiply both sides by 3.

3 Write the value of x .

d 1 Multiply both sides by 5.

2 Divide both sides by 3.

3 Write the value of x .

WRITE

a $2y + 4 = 12$
 $2y + 4 - 4 = 12 - 4$

$$\begin{aligned} 2y &= 8 \\ \frac{2y}{2} &= \frac{8}{2} \\ y &= 4 \end{aligned}$$

b $6 - x = 8$
 $-x + 6 = 8$
 $-x + 6 - 6 = 8 - 6$
 $-x = 2$
 $\frac{-x}{-1} = \frac{2}{-1}$
 $x = -2$

c $\frac{x}{3} - 4 = 2$
 $\frac{x}{3} - 4 + 4 = 2 + 4$
 $\frac{x}{3} = 6$
 $\frac{x}{3} \times 3 = 6 \times 3$
 $x = 18$

d $\frac{3x}{5} = 6$
 $\frac{3x}{5} \times 5 = 6 \times 5$
 $3x = 30$
 $\frac{3x}{3} = \frac{30}{3}$
 $x = 10$

DISCUSSION

What happens if you perform the inverse operations in the wrong order? Use an example to explain your response.

WORKED EXAMPLE 4

Solve the following linear equations.

a $\frac{x+1}{2} = 11$

b $\frac{7-x}{5} = -6.3$

THINK

a 1 All of $x + 1$ has been divided by 2.

2 Multiply both sides by 2.

3 Subtract 1 from both sides.

b 1 All of $7 - x$ has been divided by 5.

2 Multiply both sides by 5.

3 $7 - x$ is the same as $-x + 7$.

4 Subtract 7 from both sides.

5 Divide both sides by -1 .

WRITE

a $\frac{x+1}{2} = 11$

$$\frac{x+1}{2} \times 2 = 11 \times 2$$

$$x+1 = 22$$

$$x = 21$$

b $\frac{7-x}{5} = -6.3$

$$\frac{7-x}{5} \times 5 = -6.3 \times 5$$

$$7-x = -31.5$$

$$7-x-7 = -31.5-7$$

$$-x = -38.5$$

$$x = 38.5$$

10.2.4 Algebraic fractions with the pronumeral in the denominator

- If a pronumeral is in the denominator, there is an extra step involved in finding the solution.
Consider the following example:

$$\frac{4}{x} = \frac{3}{2}$$

- In order to solve this equation, we first multiply both sides of the equation by x .

$$\frac{4}{x} \times x = \frac{3}{2} \times x$$

$$4 = \frac{3x}{2}$$

$$\text{or } \frac{3x}{2} = 4$$

- The pronumeral is now in the numerator, and the equation is easy to solve.

$$\frac{3x}{2} = 4$$

$$3x = 8$$

$$x = \frac{8}{3}$$

WORKED EXAMPLE 5

Solve each of the following linear equations.

a $\frac{3}{a} = \frac{4}{5}$

b $\frac{5}{b} = -2$

THINK

a 1 Multiply both sides by a .

2 Multiply both sides by 5.

3 Divide both sides by 4.

b 1 Write the equation.

2 Multiply both sides by b .

3 Divide both sides of the equation by -2 .

WRITE

a $\frac{3}{a} = \frac{4}{5}$
 $3 = \frac{4a}{5}$

$$15 = 4a$$

$$a = \frac{15}{4}$$

or $a = 3\frac{3}{4}$

b $\frac{5}{b} = -2$

$$5 = -2b$$

$$\frac{5}{-2} = b$$

$$b = -2\frac{1}{2}$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Using algebra to solve problems (int-3805)

eLesson: Solving linear equations (eles-1895)

Digital doc: SkillSHEET Solving one-step equations (doc-6150)

Digital doc: SkillSHEET Checking solutions to equations (doc-6151)

Digital doc: SkillSHEET Solving equations (doc-6152)

Digital doc: SkillSHEET Writing equations from worded statements (doc-10826)

Digital doc: WorkSHEET Solving linear equations (doc-6156)

Exercise 10.2 Solving linear equations

Individual pathways

PRACTISE

Questions:

1a-f, 2a-l, 3a-h, 4, 5, 6a-f, 7a-f,
8a-f, 9a-f, 10-12, 17

CONSOLIDATE

Questions:

1d-i, 2g-r, 3d-i, 4, 5, 6d-i, 7d-i,
8d-i, 9d-l, 10-13, 17-19

MASTER

Questions:

1g-l, 2i-u, 3g-l, 4, 5, 6g-l, 7g-l,
8g-l, 9g-l, 10-12, 14-22

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** For each of the following equations, determine whether $x = 6$ is a solution.

a. $x + 3 = 7$	b. $2x - 5 = 7$	c. $x^2 - 2 = 38$
d. $\frac{6}{x} + x = 7$	e. $\frac{2(x+1)}{7} = 2$	f. $3 - x = 9$
g. $x^2 + 3x = 39$	h. $3(x+2) = 5(x-4)$	i. $x^2 + 2x = 9x - 6$
j. $x^2 = (x+1)^2 - 14$	k. $(x-1)^2 = 4x + 1$	l. $5x + 2 = x^2 + 4$

2. **WE2** Solve each of the following linear equations. Check your answers by substitution.

a. $x - 43 = 167$	b. $x - 17 = 35$	c. $x + 286 = 516$
d. $58 + x = 81$	e. $x - 78 = 64$	f. $209 - x = 305$
g. $5x = 185$	h. $60x = 1200$	i. $5x = 250$
j. $\frac{x}{23} = 6$	k. $\frac{x}{17} = 26$	l. $\frac{x}{9} = 27$
m. $y - 16 = -31$	n. $5.5 + y = 7.3$	o. $y - 7.3 = 5.5$
p. $6y = 14$	q. $0.2y = 4.8$	r. $0.9y = -0.05$
s. $\frac{y}{5} = 4.3$	t. $\frac{y}{7.5} = 23$	u. $\frac{y}{8} = 21.04$

3. **WE3a** Solve each of the following linear equations.

a. $2y - 3 = 7$	b. $2y + 7 = 3$	c. $5y - 1 = 0$
d. $6y + 2 = 8$	e. $7 + 3y = 10$	f. $8 + 2y = 12$
g. $15 = 3y - 1$	h. $-6 = 3y - 1$	i. $6y - 7 = 140$
j. $4.5y + 2.3 = 7.7$	k. $0.4y - 2.7 = 6.2$	l. $600y - 240 = 143$

4. **WE3b** Solve each of the following linear equations.

a. $3 - 2x = 1$	b. $3x - 1 = 5$	c. $-4x - 7 = -19$
d. $1 - 3x = 19$	e. $-5 - 7x = 2$	f. $-8 - 2x = -9$
g. $9 - 6x = -1$	h. $-5x - 4.2 = 7.4$	i. $2 = 11 - 3x$
j. $-3 = -6x - 8$	k. $-1 = 4 - 4x$	l. $35 - 13x = -5$

5. Solve each of the following linear equations.

a. $7 - x = 8$	b. $8 - x = 7$	c. $5 - x = 5$
d. $5 - x = 0$	e. $15.3 = 6.7 - x$	f. $5.1 = 4.2 - x$
g. $9 - x = 0.1$	h. $140 - x = 121$	i. $-30 - x = -4$
j. $-5 = -6 - x$	k. $-x + 1 = 2$	l. $-2x - 1 = 0$

6. **WE3c,d** Solve each of the following linear equations.

a. $\frac{x}{4} + 1 = 3$	b. $\frac{x}{3} - 2 = -1$	c. $\frac{x}{8} = \frac{1}{2}$
d. $-\frac{x}{3} = 5$	e. $5 - \frac{x}{2} = -8$	f. $4 - \frac{x}{6} = 11$
g. $\frac{2x}{3} = 6$	h. $\frac{5x}{2} = -3$	i. $-\frac{3x}{4} = -7$
j. $-\frac{8x}{3} = 6$	k. $\frac{2x}{7} = -2$	l. $-\frac{3x}{10} = -\frac{1}{5}$

7. **WE4** Solve each of the following linear equations.

a. $\frac{z-1}{3} = 5$	b. $\frac{z+1}{4} = 8$	c. $\frac{z-4}{2} = -4$
d. $\frac{6-z}{7} = 0$	e. $\frac{3-z}{2} = 6$	f. $\frac{-z-50}{22} = -2$
g. $\frac{z-4.4}{2.1} = -3$	h. $\frac{z+2}{7.4} = 1.2$	i. $\frac{140-z}{150} = 0$
j. $\frac{-z-0.4}{2} = -0.5$	k. $\frac{z-6}{9} = -4.6$	l. $\frac{z+65}{73} = 1$

8. Solve each of the following linear equations.

a. $\frac{5x + 1}{3} = 2$

b. $\frac{2x - 5}{7} = 3$

c. $\frac{3x + 4}{2} = -1$

d. $\frac{4x - 13}{9} = -5$

e. $\frac{4 - 3x}{2} = 8$

f. $\frac{1 - 2x}{6} = -10$

g. $\frac{-5x - 3}{9} = 3$

h. $\frac{-10x - 4}{3} = 1$

i. $\frac{4x + 2.6}{5} = 8.8$

j. $\frac{5x - 0.7}{-0.3} = -3.1$

k. $\frac{1 - 0.5x}{4} = -2.5$

l. $\frac{-3x - 8}{14} = \frac{1}{2}$

9. **WE5** Solve each of the following linear equations.

a. $\frac{2}{x} = \frac{1}{2}$

b. $\frac{3}{x} = 7$

c. $\frac{-4}{x} = \frac{7}{2}$

d. $\frac{5}{x} = \frac{-3}{4}$

e. $\frac{0.4}{x} = \frac{9}{2}$

f. $\frac{8}{x} = 1$

g. $\frac{-4}{x} = \frac{2}{3}$

h. $\frac{-6}{x} = \frac{-4}{5}$

i. $\frac{1.7}{x} = \frac{1}{3}$

j. $\frac{6}{x} = -1$

k. $\frac{4}{x} = \frac{-15}{22}$

l. $\frac{50}{x} = \frac{-35}{43}$

10. **MC** a. The solution to the equation $82 - x = 44$ is:

A. $x = 126$

B. $x = -126$

C. $x = 122$

D. $x = 38$

E. $x = 44$

b. What is the solution to the equation $5x - 12 = -62$?

A. $x = -14.8$

B. $x = 14.8$

C. $x = 10$

D. $x = -10$

E. $x = -50$

c. What is the solution to the equation $\frac{x - 1}{2} = 5.3$?

A. $x = 2$

B. $x = 3.65$

C. $x = 9.6$

D. $x = 10.6$

E. $x = 11.6$

11. Solve each of the following linear equations.

a. $3a + 7 = 4$

b. $5 - b = -5$

c. $4c - 4.4 = 44$

d. $\frac{d - 4}{67} = 0$

e. $5 - 3e = -10$

f. $\frac{2f}{3} = 8$

g. $100 = 6g + 4.2$

h. $\frac{h + 2}{6} = 5.5$

i. $452i - 124 = -98$

j. $\frac{6j - 1}{17} = 0$

k. $\frac{12 - k}{5} = 4$

l. $\frac{l - 5.2}{3.4} = 1.5$

12. Write each of the following worded statements as a mathematical sentence and then solve for the unknown.

a. Seven is added to the product of x and 3, which gives the result of 4.

b. Four is divided by x and this result is equivalent to $\frac{2}{3}$.

c. Three is subtracted from x and this result is divided by 12 to give 25.

13. Driving lessons are usually quite expensive, but a discount of \$15 per lesson is given if a family member belongs to the automobile club. If 10 lessons cost \$760 (after the discount), find the cost of each lesson before the discount.

14. Anton lives in Australia and his pen pal, Utan, lives in the USA. Anton's home town of Horsham experienced one of the hottest days on record with a temperature of 46.7°C . Utan said that his home town had experienced a day hotter than that, with the temperature reaching 113°F . The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Was Utan correct?

Communicating, reasoning and problem solving

15. Santo solved the linear equation $9 = 5 - x$. His second step was to divide both sides by -1 . Trudy, his mathematics buddy, said that she multiplied both sides by -1 . Explain why they are both correct.

16. Find the mistake in the following working and explain what is wrong.

$$\begin{aligned}\frac{x}{5} - 1 &= 2 \\ x - 1 &= 10 \\ x &= 11\end{aligned}$$

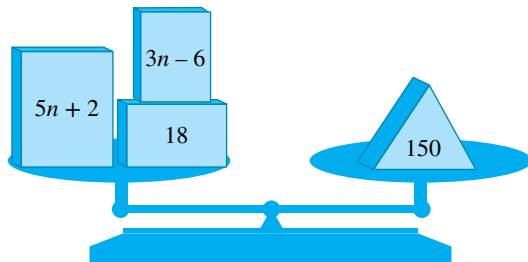
17. Sweet-tooth Sammy goes to the corner store and buys an equal number of 25-cent and 30-cent lollies for a total of \$16.50. How many lollies did he buy?

18. In a cannery, cans are filled by two machines that together produce 16000 cans during an 8-hour shift. If the newer machine produces 340 more cans per hour than the older machine, how many cans does each machine produce in an 8-hour shift?

19. Tickets to an exhibition cost \$55 for an adult, \$27 for a child and \$130 for a family of two adults and two children.

- How much is saved by buying a family ticket instead of buying two adult and two child tickets?
- Is it worthwhile buying a family ticket if the family has only one child?

20. A teacher comes across a clue shown below in a cryptic mathematics cross-number. What is the value of n that the teacher is looking for?



21. How are linear equations defined?

22. The value of the expression $\frac{12}{x-4}$ is an integer. What are the possible values for x ; given that x is also an integer?

10.3 Solving linear equations with brackets

[Stage 5.2]

10.3.1 Linear equations with brackets

Consider the equation $3(x + 5) = 18$.

There are two appropriate methods for solving this equation.

Method 1:

First divide both sides by 3.

$$\begin{aligned}\frac{3(x+5)}{3} &= \frac{18}{3} \\ x+5 &= 6 \\ x &= 1\end{aligned}$$

Method 2:

First expand the brackets.

$$\begin{aligned}3(x+5) &= 18 \\ 3x+15 &= 18 \\ 3x &= 3 \\ x &= 1\end{aligned}$$

In this case, method 1 works well because 3 divides exactly into 18.

Now try the equation $7(x + 2) = 10$.

Method 1:

First divide both sides by 7.

$$\begin{aligned}\frac{7(x + 2)}{7} &= \frac{10}{7} \\ x + 2 &= \frac{10}{7} \\ x &= -\frac{4}{7}\end{aligned}$$

Method 2:

First expand the brackets.

$$\begin{aligned}7(x + 2) &= 10 \\ 7x + 14 &= 10 \\ 7x &= -4 \\ x &= -\frac{4}{7}\end{aligned}$$

In this case, method 2 works well because it avoids fraction addition or subtraction.

Try both methods and choose the one that works best for each question.

DISCUSSION

There are different algebraic techniques for solving linear equations. Explain and justify your choice of technique for solving $\frac{x}{3} + \frac{x}{2} = 5$.

WORKED EXAMPLE 6

Solve each of the following linear equations.

a $7(x - 5) = 28$

b $6(x + 3) = 7$

THINK

a 1 7 is a factor of 28, so divide both sides by 7.

2 Add 5 to both sides.

3 Write the value of x .

b 1 6 is not a factor of 7, so it will be easier to expand the brackets first.

2 Subtract 18 from both sides.

3 Divide both sides by 6.

WRITE

a $7(x - 5) = 28$

$$\frac{7(x - 5)}{7} = \frac{28}{7}$$

$$x - 5 = 4$$

$$x = 9$$

b $6(x + 3) = 7$

$$6x + 18 = 7$$

$$6x + 18 = 7 - 18$$

$$6x = -11$$

$$x = -\frac{11}{6} \left(\text{or } -1\frac{5}{6} \right)$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Linear equations with brackets (int-6039)

Exercise 10.3 Solving linear equations with brackets

Individual pathways

PRACTISE

Questions:

1a–f, 2a–h, 3a–f, 4a–f, 5, 6, 8, 10

CONSOLIDATE

Questions:

1d–i, 2d–i, 3d–i, 4d–i, 5, 7–11

MASTER

Questions:

1g–l, 2g–l, 3g–l, 4g–l, 5, 7–13

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE6** Solve each of the following linear equations.

a. $5(x - 2) = 20$

b. $4(x + 5) = 8$

c. $6(x + 3) = 18$

d. $5(x - 41) = 75$

e. $8(x + 2) = 24$

f. $3(x + 5) = 15$

g. $5(x + 4) = 15$

h. $3(x - 2) = -12$

i. $7(x - 6) = 0$

j. $-6(x - 2) = 12$

k. $4(x + 2) = 4.8$

l. $16(x - 3) = 48$

2. **WE6** Solve each of the following equations.

a. $6(b - 1) = 1$

b. $2(m - 3) = 3$

c. $2(a + 5) = 7$

d. $3(m + 2) = 2$

e. $5(p - 2) = -7$

f. $6(m - 4) = -8$

g. $-10(a + 1) = 5$

h. $-12(p - 2) = 6$

i. $-9(a - 3) = -3$

j. $-2(m + 3) = -1$

k. $3(2a + 1) = 2$

l. $4(3m + 2) = 5$

3. Solve each of the following equations.

a. $9(x - 7) = 82$

b. $2(x + 5) = 14$

c. $7(a - 1) = 28$

d. $4(b - 6) = 4$

e. $3(y - 7) = 0$

f. $-3(x + 1) = 7$

g. $-6(m + 1) = -30$

h. $-4(y + 2) = -12$

i. $-3(a - 6) = 3$

j. $-2(p + 9) = -14$

k. $3(2m - 7) = -3$

l. $2(4p + 5) = 18$

4. Solve the following linear equations. Round the answers to 3 decimal places where appropriate.

a. $2(y + 4) = -7$

b. $0.3(y + 8) = 1$

c. $4(y + 19) = -29$

d. $7(y - 5) = 25$

e. $6(y + 3.4) = 3$

f. $7(y - 2) = 8.7$

g. $1.5(y + 3) = 10$

h. $2.4(y - 2) = 1.8$

i. $1.7(y + 2.2) = 7.1$

j. $-7(y + 2) = 0$

k. $-6(y + 5) = -11$

l. $-5(y - 2.3) = 1.6$

5. **MC** a. The best first step in solving the equation $7(x - 6) = 23$ would be to:

A. add 6 to both sides

B. subtract 7 from both sides

C. divide both sides by 23

D. divide both sides by 7

E. expand the brackets

b. The solution to the equation $84(x - 21) = 782$ is closest to:

A. $x = 9.06$

B. $x = 9.31$

C. $x = 9.56$

D. $x = 30.31$

E. $x = -11.69$

6. In 1974 a mother was 6 times as old as her daughter. If the mother turned 50 in the year 2000, in what year was the mother double her daughter's age?

7. New edging is to be placed around a rectangular children's playground. The width of the playground is x m and the length is 7 metres longer than the width.

- a. Write down an expression for the perimeter of the playground. Write your answer in factorised form.
b. If the amount of edging required is 54 m, determine the dimensions of the playground.

Communicating, reasoning and problem solving

8. Juanita is solving the following equation: $2(x - 8) = 10$. She performs the following operations to both sides of the equation in order: $+8, \div 2$. Explain why Juanita will not find the correct value of x using her order of inverse operations, then solve the equation.

9. As your first step to solve the equation $3(2x - 7) = 18$, you are given three options:

- Expand the brackets on the left-hand side.
- Add 7 to both sides.
- Divide both sides by 3.

Which of the options is your least preferred and why?

10. Five times the sum of 4 and a number is equal to 35. What is the number?

11. Kyle earns \$55 more than Noah each week, but Callum earns three times as much as Kyle. If Callum earns \$270 a week, how much do Kyle and Noah earn each week?

12. A school wishes to hire a bus to travel to a football game. The bus will take 28 passengers, and the school will contribute \$48 towards the cost of the trip. The price of each ticket is \$10. If the hiring of the bus is $\$300 + 10\%$ of the cost of all the tickets, what should be the cost per person?



13. Explain the two possible methods for solving equations in factorised form.

10.4 Solving linear equations with pronumerals on both sides

[Stage 5.2]

10.4.1 Linear equations with pronumerals on both sides

When we solve equations, it is important to remember that whatever we do to one side of an equation we must do to the other.

If the pronumeral occurs on both sides of the equation, first remove it from one side, as shown in the example below.

WORKED EXAMPLE 7

Solve each of the following linear equations.

a $5y = 3y + 3$

b $7x + 5 = 2 - 4x$

c $3(x + 1) = 14 - 2x$

d $2(x + 3) = 3(x + 7)$

THINK

a 1 $3y$ is smaller than $5y$. Subtract $3y$ from both sides.

2 Divide both sides by 2.

WRITE

a $5y = 3y + 3$
 $5y - 3y = 3y + 3 - 3y$
 $2y = 3$

$y = \frac{3}{2}$ (or $1\frac{1}{2}$)

b 1 $-4x$ is smaller than $7x$. Add $4x$ to both sides.

2 Subtract 5 from both sides.

3 Divide both sides by 11.

c 1 Expand the bracket.

2 $-2x$ is smaller than $3x$. Add $2x$ to both sides.

3 Subtract 3 from both sides.

4 Divide both sides by 5.

d 1 Expand the brackets.

2 $2x$ is smaller than $3x$.

Subtract $2x$ from both sides.

3 Subtract 21 from both sides.

4 Write the answer with the pronumeral written on the left-hand side.

$$\begin{aligned}\mathbf{b} \quad & 7x + 5 = 2 - 4x \\ & 7x + 5 + 4x = 2 - 4x + 4x \\ & 11x + 5 = 2 \\ & 11x + 5 - 5 = 2 - 5 \\ & 11x = -3 \\ & x = -\frac{3}{11}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad & 3(x + 1) = 14 - 2x \\ & 3x + 3 = 14 - 2x \\ & 3x + 3 + 2x = 14 - 2x + 2x \\ & 5x + 3 = 14 \\ & 5x + 3 - 3 = 14 - 3 \\ & 5x = 11 \\ & x = \frac{11}{5}\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad & 2(x + 3) = 3(x + 7) \\ & 2x + 6 = 3x + 21 \\ & 2x + 6 - 2x = 3x + 21 - 2x \\ & 6 = x + 21 \\ & 6 - 21 = x + 21 - 21 \\ & -15 = x \\ & x = -15\end{aligned}$$

ACTIVITY: DICE TRICKS

Equipment: pair of dice

1. Roll a pair of dice and follow the instructions listed below.

- Multiply the number on top of one die by 2.
- Add 3 to this.
- Multiply by 5.
- To this result, add the number on the second die.
- Multiply this sum by 10.
- Finally, subtract 150 from your answer.
- Note your answer and repeat this experiment several times.

2. Discuss your results and show algebraically what you have discovered.

3. Use the idea of inverse operations to write a list of instructions for the result of the die roll which will give a predetermined result. For example, write a set of instructions that will always result in an answer of 2.

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Interactivity: Solving equations (int-2764)

eLesson: Solving linear equations with pronumerals on both sides (eles-1901)

Digital doc: SkillsSHEET Simplifying like terms (doc-10828)

Digital doc: WorkSHEET Solving equations with the pronumerals on both sides (doc-6159)

Exercise 10.4 Solving linear equations with pronumerals on both sides

Individual pathways

PRACTISE

Questions:

1a-f, 2, 3a-f, 4, 5, 6a-f, 7, 8, 11

CONSOLIDATE

Questions:

1d-i, 2, 3d-i, 4, 5, 6d-i, 7-12

MASTER

Questions:

1g-l, 2, 3g-l, 4, 5, 6g-l, 7-15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE7a** Solve each of the following linear equations.

a. $5y = 3y - 2$

b. $6y = -y + 7$

c. $10y = 5y - 15$

d. $25 + 2y = -3y$

e. $8y = 7y - 45$

f. $15y - 8 = -12y$

g. $7y = -3y - 20$

h. $23y = 13y + 200$

i. $5y - 3 = 2y$

j. $6 - 2y = -7y$

k. $24 - y = 5y$

l. $6y = 5y - 2$

2. **MC** a. To solve the equation $3x + 5 = -4 - 2x$, the first step is to:

A. add $3x$ to both sides

B. add 5 to both sides

C. add $2x$ to both sides

D. subtract $2x$ from both sides

E. divide both sides by 3

b. To solve the equation $6x - 4 = 4x + 5$, the first step is to:

A. subtract $4x$ from both sides

B. add $4x$ to both sides

C. subtract 4 from both sides

D. add 5 to both sides

E. divide both sides by 4

3. **WE7b** Solve each of the following linear equations.

a. $2x + 3 = 8 - 3x$

b. $4x + 11 = 1 - x$

c. $x - 3 = 6 - 2x$

d. $4x - 5 = 2x + 3$

e. $3x - 2 = 2x + 7$

f. $7x + 1 = 4x + 10$

g. $5x + 3 = x - 5$

h. $6x + 2 = 3x + 14$

i. $2x - 5 = x - 9$

j. $10x - 1 = -2x + 5$

k. $7x + 2 = -5x + 2$

l. $15x + 3 = 7x - 3$

4. Solve each of the following linear equations.

a. $x - 4 = 3x + 8$

b. $3x + 12 = 4x + 5$

c. $2x + 9 = 7x - 1$

d. $-2x + 7 = 4x + 19$

e. $-3x + 2 = -2x - 11$

f. $11 - 6x = 18 - 5x$

g. $6 - 9x = 4 + 3x$

h. $x - 3 = 18x - 1$

i. $5x + 13 = 15x + 3$

5. **MC** a. The solution to $5x + 2 = 2x + 23$ is:

A. $x = 3$

B. $x = -3$

C. $x = 5$

D. $x = 7$

E. $x = 8\frac{1}{3}$

b. The solution to $3x - 4 = 11 - 2x$ is:

A. $x = 15$

B. $x = 7$

C. $x = 3$

D. $x = 5$

E. $x = 1.4$

6. **WE7c,d** Solve each of the following.

a. $5(x - 2) = 2x + 5$

b. $7(x + 1) = x - 11$

c. $2(x - 8) = 4x$

d. $3(x + 5) = x$

e. $6(x - 3) = 14 - 2x$

f. $9x - 4 = 2(3 - x)$

g. $4(x + 3) = 3(x - 2)$

h. $5(x - 1) = 2(x + 3)$

i. $8(x - 4) = 5(x - 6)$

j. $3(x + 6) = 4(2 - x)$

k. $2(x - 12) = 3(x - 8)$

l. $4(x + 11) = 2(x + 7)$

7. Aamir's teacher gave him an algebra problem and told him to solve it.

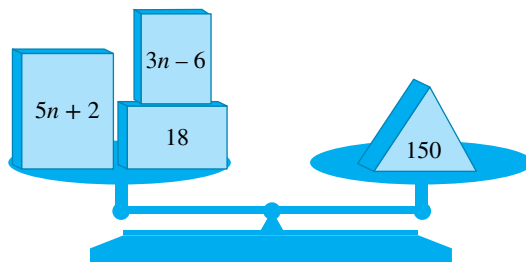
$$3x + 7 = x^2 + k = 7x + 15$$

Can you help him find the value of k ?

8. A classroom contained an equal number of boys and girls. Six girls left to play hockey, leaving twice as many boys as girls in the classroom. What was the original number of students present?

Communicating, reasoning and problem solving

9. Express the following information as an equation, then show that $n = 29$ is the solution.



10. Explain the difficulty of trying to solve the equation $4(3x - 5) = 6(4x + 2)$ without expanding the brackets first.
11. This year Tom is 4 times as old as his daughter. In 5 years' time he will be only 3 times as old as his daughter. Find the ages of Tom and his daughter now.
12. If you multiply an unknown number by 6 and then add 5, the result is 7 less than the unknown number plus 1 multiplied by 3. Find the unknown number.
13. You are investigating having business cards printed for your new game store. A local printing company charges \$250 for the cardboard used and an hourly rate for labour of \$40.



- a. If h is the number of hours of labour required to print the cards, construct an equation for the cost of the cards, C .
- b. You have budgeted \$1000 for the printing job. How many hours of labour can you afford? Give your answer to the nearest minute.
- c. The company estimates that it can print 1000 cards per hour of labour. How many cards will you get printed with your current budget?
- d. An alternative to printing is photocopying. The company charges 15 cents per side for the first 10000 cards and then 10 cents per side for the remaining cards. Which is the cheaper option for 18750 single-sided cards and by how much?
14. A local pinball arcade offers its regular customers the following deal. For a monthly fee of \$40 players get 25 \$2 pinball games. Additional games cost \$2 each. After a player has played 50 games in a month, all further games are \$1.
- a. If Tom has \$105 to spend in a month, how many games can he play if he takes up the special deal?
- b. How much did Tom save by taking up the special deal, compared to playing the same number of games at \$2 a game?
15. Draw a diagram that could represent $2x + 4 = 3x + 1$.

10.5 Solving problems with linear equations

[Stage 5.2]

10.5.1 Converting worded sentences to algebraic equations

- An important skill in mathematics is the ability to translate written problems into **algebraic equations** in order to solve problems.

WORKED EXAMPLE 8

Write linear equations for each of the following statements, using x to represent the unknown. (Do not attempt to solve the equations.)

- a When 6 is subtracted from a certain number, the result is 15.
b Three more than seven times a certain number is zero.
c When 2 is divided by a certain number, the answer is 4 more than the number.

THINK

- a 1 Let x be the number.
2 Write x and subtract 6. This expression equals 15.
b 1 Let x be the number.
2 7 times the number is $7x$. Three more than $7x$ equals $7x + 3$. This expression equals 0.
c 1 Let x be the number.
2 Write the term for 2 divided by a certain number.
Write the expression for 4 more than the number.
3 Write the equation.

WRITE

- a $x = \text{unknown number}$
 $x - 6 = 15$
b $x = \text{unknown number}$
 $7x + 3 = 0$
c $x = \text{unknown number}$
 $\frac{2}{x}$
 $x + 4$
 $\frac{2}{x} = x + 4$

WORKED EXAMPLE 9

In a basketball game, Hao scored 5 more points than Seve. If they scored a total of 27 points between them, how many points did each of them score?



THINK

- 1 Define a pronumeral.
- 2 Hao scored 5 more than Seve.
- 3 Between them they scored a total of 27 points.
- 4 Solve the equation.
- 5 Since $x = 11$, this is Seve's score.
Write Hao's score.
- 6 Write the answer in words.

WRITE

Let Seve's score be x .

Hao's score is $x + 5$.

$$x + (x + 5) = 27$$

$$2x + 5 = 27$$

$$2x = 22$$

$$x = 11$$

$$\text{Hao's score} = x + 5$$

$$= 11 + 5$$

$$= 16$$

Seve scored 11 points and Hao scored 16 points.

WORKED EXAMPLE 10

Taxi charges are \$3.60 plus \$1.38 per kilometre for any trip in Sydney. If Elena's taxi fare was \$38.10, how far did she travel?

**THINK**

- 1 The distance travelled by Elena has to be found. Define the pronumeral.
- 2 It costs 1.38 to travel 1 kilometre, so the cost to travel x kilometres $= 1.38x$. The fixed cost is \$3.60. Write an expression for the total cost.
- 3 Let the total cost $= 38.10$.
- 4 Solve the equation.
- 5 State the solution in words.

WRITE

Let x = distance travelled
(in kilometres).

$$\text{Total cost} = 3.60 + 1.38x$$

$$3.60 + 1.38x = 38.10$$

$$3.60 + 1.38x = 38.10$$

$$1.38x = 34.50$$

$$x = \frac{34.50}{1.38}$$

$$= 25$$

Elena travelled 25 kilometres.

Exercise 10.5 Solving problems with linear equations

Individual pathways

PRACTISE

Questions:

1–4, 7, 9, 11–14

CONSOLIDATE

Questions:

1–5, 7–10, 12–15

MASTER

Questions:

1–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE8** Write linear equations for each of the following statements, using x to represent the unknown. (Do not attempt to solve the equations.)
 - When 3 is added to a certain number, the answer is 5.
 - Subtracting 9 from a certain number gives a result of 7.
 - Seven times a certain number is 24.
 - A certain number divided by 5 gives a result of 11.
 - Dividing a certain number by 2 equals -9 .
 - Three subtracted from five times a certain number gives a result of -7 .
 - When a certain number is subtracted from 14 and this result is then multiplied by 2, the result is -3 .
 - When 5 is added to three times a certain number, the answer is 8.
 - When 12 is subtracted from two times a certain number, the result is 15.
 - The sum of 3 times a certain number and 4 is divided by 2, which gives a result of 5.
- MC** Which equation matches each of the following statements?
 - A certain number, when divided by 2, gives a result of -12 .

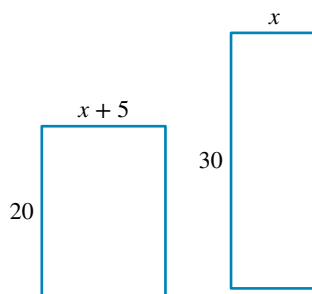
A. $x = \frac{-12}{2}$	B. $2x = -12$	C. $\frac{x}{2} = -12$
D. $\frac{x}{12} = -2$	E. $\frac{x}{2} = 12$	
 - Dividing 7 times a certain number by -4 equals 9.

A. $\frac{x}{-4} = 9$	B. $\frac{-4x}{7} = 9$	C. $\frac{7+x}{-4} = 9$
D. $\frac{7x}{-4} = 9$	E. $\frac{7}{-4x} = 9$	
 - Subtracting twice a certain number from 8 gives 12.

A. $2x - 8 = 12$	B. $8 - 2x = 12$	C. $2 - 8x = 12$
D. $8 - (x + 2) = 12$	E. $12 - 2x = 8$	
 - When 15 is added to a quarter of a number, the answer is 10.

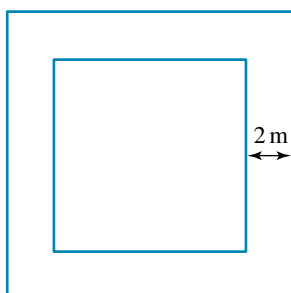
A. $15 + 4x = 10$	B. $10 = \frac{x}{4} + 15$	C. $\frac{x + 15}{4} = 10$
D. $15 + \frac{4}{x} = 10$	E. $\frac{15 + x}{4x} = 10$	
- When a certain number is added to 3 and the result is multiplied by 4, the answer is the same as when the same number is added to 4 and the result is multiplied by 3. Find the number.
- WE9** John is three times as old as his son Jack, and the sum of their ages is 48. How old is John?
- In one afternoon's shopping Seedeви spent half as much money as Georgia, but \$6 more than Amy. If the three of them spent a total of \$258, how much did Seedeви spend?

6. These rectangular blocks of land have the same area. Find the dimensions of each block, and the area.



Communicating, reasoning and problem solving

7. A square pool is surrounded by a paved area that is 2 metres wide. If the area of the paving is 72 m^2 , what is the length of the pool?



8. Maria is paid \$11.50 per hour plus \$7 for each jacket that she sews. If she earned \$176 for one 8-hour shift, how many jackets did she sew?
9. Mai hired a car for a fee of \$120 plus \$30 per day. Casey's rate for his car hire was \$180 plus \$26 per day. If their final cost and rental period were the same, how long was the rental period?
10. **WE10** The cost of producing music CDs is quoted as \$1200 plus \$0.95 per disk. If Maya's recording studio has a budget of \$2100, how many CDs can she have made?



11. Joseph wishes to have some flyers delivered for his grocery business. Post Quick quotes a price of \$200 plus 50 cents per flyer, while Fast Box quotes \$100 plus 80 cents per flyer.
- If Joseph needs to order 1000 flyers, which distributor would be cheaper to use?
 - For what number of fliers will the cost be the same for either distributor?
 - Why is the solution to the linear equation generated in part **b** not a solution to the problem?

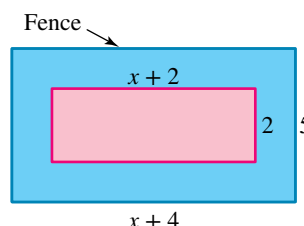
12. A number is multiplied by 8, then 16 is subtracted. The result is the same as 4 times the original number minus 8. What is the number?

13. Carmel sells three different types of healthy drinks: herbal, vegetable and citrus fizz. One hour she sells 4 herbal, 3 vegetable and 6 citrus fizz for \$60.50. The next hour she sells 2 herbal, 4 vegetable and 3 citrus fizz. The third hour she sells 1 herbal, 2 vegetable and 4 citrus fizz. The total amount in cash sales for the three hours is \$136.50. Carmel made \$7 less in the third hour than she did in the second hour of sales.



Determine Carmel's sales in the fourth hour if she sells 2 herbal, 3 vegetable and 4 citrus fizz drinks.

14. A rectangular swimming pool is surrounded by a path which is enclosed by a pool fence. All measurements are in metres and are not to scale in the diagram shown.



- Write an expression for the area of the entire fenced-off section.
 - Write an expression for the area of the path surrounding the pool.
 - If the area of the path surrounding the pool is 34 m^2 , find the dimensions of the swimming pool.
 - What fraction of the fenced-off area is taken up by the pool?
15. Why is it important to define the pronumeral used when forming a linear equation to solve a problem?

10.6 Substituting values into formulas [Stage 5.2]

10.6.1 Rearranging (transposing) formulas

- Formulas are generally written in terms of two or more pronumerals or variables.
- One pronumeral is usually written on one side of the equal sign.
- When rearranging formulas, use the same methods as for solving linear equations (use inverse operations in reverse order).
- The difference between rearranging formulas and solving linear equations is that rearranging formulas does not require a value for the pronumeral(s) to be found.
- The **subject** of the formula is the pronumeral or variable that is written by itself. It is usually written on the left-hand side of the equation.
- A formula is simply an equation that is used for some specific purpose. By now you will be familiar with many mathematical or scientific formulas.
- For example, $C = 2\pi r$ relates the circumference of a circle to its radius. When the formula is shown in this order, C is called the subject of the formula. The formula can be **transposed** (rearranged) to make r the subject.

$$C = 2\pi r \quad \text{Divide both sides by } 2\pi.$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = r$$

$$\text{or } r = \frac{C}{2\pi} \quad \text{Now } r \text{ is the subject.}$$

WORKED EXAMPLE 11

Rearrange each formula to make x the subject.

a $y = kx + m$

THINK

a 1 Subtract m from both sides.

2 Divide both sides by k .

3 Rewrite the equation so that x is on the left-hand side.

b 1 Expand the brackets.

2 Add 14 to both sides.

3 Divide both sides by 7.

4 Rewrite the equation so that x is on the left-hand side.

b $6(y + 1) = 7(x - 2)$

WRITE

a $y = kx + m$

$$y - m = kx$$

$$\frac{y - m}{k} = \frac{kx}{k}$$

$$\frac{y - m}{k} = x$$

$$x = \frac{y - m}{k}$$

b $6(y + 1) = 7(x - 2)$

$$6y + 6 = 7x - 14$$

$$6y + 20 = 7x$$

$$\frac{6y + 20}{7} = x$$

$$x = \frac{6y + 20}{7}$$

WORKED EXAMPLE 12

For each of the following, make the variable shown in brackets the subject of the formula.

a $g = 6d - 3$ (d)

b $a = \frac{v - u}{t}$ (v)

THINK

a 1 Add 3 to both sides.

2 Divide both sides by 6.

3 Rewrite the equation so that d is on the left-hand side.

b 1 Multiply both sides by t .

2 Add u to both sides.

3 Rewrite the equation so that v is on the left-hand side.

WRITE

a $g = 6d - 3$

$$g + 3 = 6d$$

$$\frac{g + 3}{6} = d$$

$$d = \frac{g + 3}{6}$$

b $a = \frac{v - u}{t}$

$$at = v - u$$

$$at + u = v$$

$$v = at + u$$

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Interactivity: Rearranging formulas (int-6040)

eLesson: Formulas in the real world (eles-0113)

Digital doc: SkillsSHEET Transposing and substituting into a formula (doc-10829)

Exercise 10.6 Substituting values into formulas

Individual pathways

PRACTISE

Questions:

1a–f, 2a–f, 3, 6

CONSOLIDATE

Questions:

1e–h, 2e–h, 3–6, 8

MASTER

Questions:

1g–l, 2g–n, 3–13

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE11** Rearrange each formula to make x the subject.

a. $y = ax$

c. $y = 2ax - b$

e. $6(y + 2) = 5(4 - x)$

g. $x(y - 2) = y + 1$

i. $6(x + 2) = 5(x - y)$

k. $5(a - 2x) = 9(x + 1)$

b. $y = ax + b$

d. $y + 4 = 2x - 3$

f. $x(y - 2) = 1$

h. $5x - 4y = 1$

j. $7(x - a) = 6x + 5a$

l. $8(9x - 2) + 3 = 7(2a - 3x)$

2. **WE12** For each of the following, make the variable shown in brackets the subject of the formula.

a. $g = 4P - 3$ (P)

c. $f = \frac{9c}{5} + 32$ (c)

e. $v = u + at$ (t)

g. $m = \frac{y - k}{h}$ (y)

i. $m = \frac{y - a}{x - b}$ (a)

k. $C = \frac{2\pi}{r}$ (r)

m. $s = ut + \frac{1}{2}at^2$ (a)

b. $f = \frac{9c}{5}$ (c)

d. $V = IR$ (I)

f. $d = b^2 - 4ac$ (c)

h. $m = \frac{y - a}{x - b}$ (y)

j. $m = \frac{y - a}{x - b}$ (x)

l. $f = ax + by$ (x)

n. $F = \frac{GMm}{r^2}$ (G)

3. The cost to rent a car is given by the formula $C = 50d + 0.2k$, where d = the number of days rented and k = the number of kilometres driven. Lin has \$300 to spend on car rental for her 4-day holiday. How far can she travel on this holiday?



4. The formula for perimeter of a rectangle is $P = 2l + 2b$, where P is the perimeter, l is the length and b is the base. If the perimeter of a rectangle is 20 cm and its length is 6 cm, what is the base?
5. The formula connecting degrees Celsius and degrees Fahrenheit is $C = \frac{5}{9}(F - 32)$. If, on a particular day the maximum temperature recorded is 35°C , what is the equivalent temperature in degrees Fahrenheit?
6. A cyclist pumps up a bike tyre that has a slow leak. The volume of air (in cm^3) after t minutes is given by the formula $V = 24000 - 300t$.



- a. What is the volume of air in the tyre when it is first filled?
- b. Write an equation and solve it to work out how long it takes the tyre to go completely flat.

Communicating, reasoning and problem solving

7. The total surface area of a cylinder is given by the formula $T = 2\pi r^2 + 2\pi rh$, where r = radius and h = height. A car manufacturer wants the engine's cylinders to have a radius of 4 cm and a total surface area of 400 cm^2 . Show that the height of the cylinder is approximately 11.92 cm, correct to 2 decimal places. (*Hint: Express h in terms of T and r .*)
8. If $B = 3x - 6xy$, write x as the subject. Explain the process by showing all working.
9. Use algebra to show that $\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$ can also be written as $u = \frac{fv}{v + f}$.
10. Consider the formula $d = \sqrt{b^2 - 4ac}$. Rearrange the formula to make a the subject.
11. Find values for a and b such that:

$$\frac{4}{x+1} - \frac{3}{x+2} = \frac{ax+b}{(x+1)(x+2)}$$
12. How does rearranging formulas differ to solving linear equations?
13. The volume, V , of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. What is the radius of a spherical ball that has the capacity to hold 5 litres of water?

10.7 Review

Investigation | Rich task

Forensic science



Studies have been conducted on the relationship between the height of a human and measurements of a variety of body parts. One such study relates the height of a person to the length of the upper arm bone (humerus). The relationships are different for males and females, and for different races. Let us consider the relationships for white adult Australians.

For males, $h = 3.08l + 70.45$, and for females, $h = 3.36l + 57.97$, where h represents the body height in centimetres and l the length of the humerus in centimetres.

Imagine the following situation.

A decomposed body was found in the bushland. A team of forensic scientists suspects that the body could be the remains of either Alice Brown or James King; they have been missing for several years. From the description provided by their Missing Persons file, Alice's height is recorded as 162 cm, and James's height is recorded as 172 cm. The forensic scientists hope to identify the body based on the length of the body's humerus.



1. Use the equations above to complete the following tables. Calculate body heights to the nearest centimetre.

Table for males

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)					

Table for females

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)					

2. On a piece of graph paper, draw the first quadrant of a Cartesian plane. Since the length of the humerus is the independent variable, place it on the x -axis. Place the dependent variable, body height, on the y -axis.



3. Plot the points from the two tables from question 1 onto the set of axes drawn in question 2. Join the points with straight lines, using different colours to represent males and females.
4. Describe the shape of the two graphs.
5. Measure the length of your humerus. Use your graph to predict your height. How accurate is the measurement?
6. The two lines of your graph will intersect if extended. At what point does this occur? Comment on this value.
7. The forensic scientists measured the length of the humerus of the bone remains and found it to be 33 cm. Using methods covered in this activity, identify the body, justifying your decision with mathematical evidence.

learn**on** RESOURCES — ONLINE ONLY

Digital doc: Investigation — Forensic science (doc-15892)

Digital doc: Code puzzle — The driest place (doc-15893)

Digital doc: Summary Topic 10 (doc-22892)

Interactivity: Crossword Topic 10 (int-0700)

Interactivity: Word search Topic 10 (int-0686)

Interactivity: Sudoku Topic 10 (int-3204)

Exercise 10.7 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **MC** The linear equation represented by the sentence 'When a certain number is multiplied by 3, the result is 5 times the certain number plus 7' is:
A. $3x + 7 = 5x$ **B.** $5(x + 7) = 3x$ **C.** $5x + 7 = 3x$
D. $5x = 3x + 7$ **E.** $3x = 5 + 7x$
2. **MC** The solution to the equation $\frac{x}{3} = 5$ is:
A. $x = -15$ **B.** $x = 15$ **C.** $x = 1\frac{2}{3}$
D. $x = 3$ **E.** $x = 2$
3. **MC** What is the solution to the equation $7 = 21 + x$?
A. $x = 28$ **B.** $x = -28$ **C.** $x = -14$
D. $x = 14$ **E.** $x = 3$
4. **MC** What is the solution to the equation $5x + 3 = 37$?
A. $x = 8$ **B.** $x = -8$ **C.** $x = 6.8$
D. $x = 106$ **E.** $x = 200$
5. **MC** The solution to the equation $8 - 2x = 22$ is:
A. $x = 11$ **B.** $x = 15$ **C.** $x = -15$
D. $x = 7$ **E.** $x = -7$
6. **MC** The solution to the equation $4x + 3 = 7x - 33$ is:
A. $x = -12$ **B.** $x = 12$ **C.** $x = \frac{36}{11}$
D. $x = \frac{30}{11}$ **E.** $x = -\frac{36}{11}$
7. **MC** The solution to the equation $7(x - 15) = 28$ is:
A. $x = 11$ **B.** $x = 19$ **C.** $x = 20$
D. $x = 6.14$ **E.** $x = -19$
8. **MC** When rearranging $y = ax + b$ in terms of x , we obtain:
A. $x = \frac{y - a}{b}$ **B.** $x = \frac{y - b}{a}$ **C.** $x = \frac{b - y}{a}$
D. $x = \frac{y + b}{a}$ **E.** $x = \frac{y}{a} - b$
9. Which of the following are linear equations?
a. $5x + y^2 = 0$ **b.** $2x + 3 = x - 2$ **c.** $\frac{x}{2} = 3$
d. $x^2 = 1$ **e.** $\frac{1}{x} + 1 = 3x$ **f.** $8 = 5x - 2$
g. $5(x + 2) = 0$ **h.** $x^2 + y = -9$ **i.** $r = 7 - 5(4 - r)$
10. Solve each of the following linear equations.
a. $3a = 8.4$ **b.** $a + 2.3 = 1.7$ **c.** $\frac{b}{21} = -0.12$
d. $b - 1.45 = 1.65$ **e.** $b + 3.45 = 0$ **f.** $7.53b = 5.64$
11. Solve each of the following linear equations.
a. $\frac{2x - 3}{7} = 5$ **b.** $\frac{5 - x}{2} = -4$ **c.** $\frac{-3x - 4}{5} = 3$
d. $\frac{6}{x} = 5$ **e.** $\frac{4}{x} = \frac{3}{5}$ **f.** $\frac{x + 1.7}{2.3} = -4.1$

12. Solve each of the following linear equations.

a. $5(x - 2) = 6$

b. $7(x + 3) = 40$

c. $4(5 - x) = 15$

d. $6(2x + 3) = 1$

e. $4(x + 5) = 2x - 5$

f. $3(x - 2) = 7(x + 4)$

13. Liz has a packet of 45 Easter eggs. She saves 21 to eat tomorrow but rations the remainder so that she can eat 8 eggs each hour.

a. Write a linear equation in terms of the number of hours, h , to represent this situation.

b. Work out how many hours it will take to eat today's share.

14. Solve each of the following linear equations.

a. $11x = 15x - 2$

b. $3x + 4 = 16 - x$

c. $5x + 2 = 3x + 8$

d. $8x - 9 = 7x - 4$

e. $2x + 5 = 8x - 7$

f. $3 - 4x = 6 - x$

15. Translate these sentences into algebraic equations. Use x for the certain number.

a. Twice a certain number is equal to 3 minus that certain number.

b. When 8 is added to 3 times a certain number, the result is 19.

c. Multiplying a certain number by 6 equals 4.

d. Dividing 10 by a certain number is one more than dividing that number by 6.

e. Multiply a certain number by 2, then add 5. Multiply this result by 7. This expression equals 0.

f. Twice the distance travelled is 100 metres more than the distance travelled plus 50 metres.

16. Samuel decides to go on a holiday. He travels a certain distance on the first day, twice that distance on the second day, three times that distance on the third day and four times that distance on the fourth day. If his total journey is 2000km, how far did he travel on the third day?

17. For each of the following, make the variable shown in brackets the subject of the formula.

a. $y = 6x - 4$ (x)

b. $y = mx + c$ (x)

c. $q = 2(P - 1) + 2r$ (P)

d. $P = 2l + 2w$ (w)

e. $v = u + at$ (a)

f. $s = \left(\frac{u + v}{2}\right)t$ (t)

g. $v^2 = u^2 + 2as$ (a)

h. $2A = h(a + b)$ (b)

Communicating, reasoning and problem solving

18. John is comparing two car rental companies, Golden Ace Rental Company and Silver Diamond Rental Company. Golden Ace Rental Company charges a flat rate of \$38 per day and \$0.20 per kilometre. The Silver Diamond Rental Company charges a flat rate of \$30 per day plus \$0.32 per kilometre. John plans to rent a car for three days.

a. Write an algebraic equation for the cost of renting a car for three days from the Golden Ace Rental Company in terms of the number of kilometres travelled, k .

b. Write an algebraic equation for the cost of renting a car for three days from the Silver Diamond Rental Company in terms of the number of kilometres travelled, k .

c. How many kilometres would John have to travel so that the cost of hiring from each company is the same?

19. Frederika has \$24000 saved for a holiday and a new stereo. Her travel expenses are \$5400 and her daily expenses are \$260.

a. Write an equation for the cost of her holiday if she stays for d days.

b. Upon her return from holidays, Frederika wants to purchase a new stereo system that will cost her \$2500. How many days can she spend on her holiday if she wishes to purchase a new stereo upon her return?

- 20.** A maker of an orange drink purchases her raw materials from two sources. The first source provides liquid with 6% orange juice, while the second source provides liquid with 3% orange juice. She wishes to make 1 litre of drink with 5% orange juice. Let x = amount of liquid (in litres) purchased from the first source.
- Write an expression for the amount of orange juice from the first supplier, given that x is the amount of liquid.
 - Write an expression for the amount of liquid from the second supplier, given that x is the amount of liquid used from the first supplier.
 - Write an expression for the amount of orange juice from the second supplier.
 - Write an equation for the total amount of orange juice in the mixture of the 2 supplies, given that 1 litre of drink is mixed to contain 5% orange juice.
 - How much of the first supplier's liquid should she use?
- 21.** Rachel, a bushwalker, goes on a 4-day journey. She travels a certain distance on the first day, half that distance on the second day, a third that distance on the third day and a fourth of that distance on the fourth day. If the total journey is 50 km, how far did she walk on the first day?
- 22.** Svetlana, another bushwalker, goes on a 5-day journey using the same pattern as Rachel in the previous question (a certain amount, then half that amount, then one third, one fourth and one fifth). If her journey is also 50 km, how far did she travel on the first day?
- 23.** Nile.com, the internet bookstore, advertises its shipping cost to Australia as a flat rate of \$20 for up to 10 books; while Sheds & Meager, their competitor, offers a rate of \$12 plus \$1.60 per book. For how many books (6, 7, 8, 9 or 10) is Nile.com's cost a better deal?

Answers

Topic 10 Linear equations

Exercise 10.2 Solving linear equations

1. a. No	b. Yes	c. No	d. Yes
e. Yes	f. No	g. No	h. No
i. Yes	j. No	k. Yes	l. No
2. a. $x = 210$	b. $x = 52$	c. $x = 230$	d. $x = 23$
e. $x = 142$	f. $x = -96$	g. $x = 37$	h. $x = 20$
i. $x = 50$	j. $x = 138$	k. $x = 442$	l. $x = 243$
m. $y = -15$	n. $y = 1.8$	o. $y = 12.8$	p. $y = 2\frac{1}{3}$
q. $y = 24$	r. $y = -\frac{1}{18}$	s. $y = 21.5$	t. $y = 172.5$
u. $y = 168.32$			
3. a. $y = 5$	b. $y = -2$	c. $y = 0.2$	d. $y = 1$
e. $y = 1$	f. $y = 2$	g. $y = 5\frac{1}{3}$	h. $y = -1\frac{2}{3}$
i. $y = 24.5$	j. $y = 1.2$	k. $y = 22.25$	l. $y = \frac{383}{600}$
4. a. $x = 1$	b. $x = 2$	c. $x = 3$	d. $x = -6$
e. $x = -1$	f. $x = \frac{1}{2}$	g. $x = 1\frac{2}{3}$	h. $x = -2.32$
i. $x = 3$	j. $x = -\frac{5}{6}$	k. $x = 1\frac{1}{4}$	l. $x = 3\frac{1}{13}$
5. a. $x = -1$	b. $x = 1$	c. $x = 0$	d. $x = 5$
e. $x = -8.6$	f. $x = -0.9$	g. $x = 8.9$	h. $x = 19$
i. $x = -26$	j. $x = -1$	k. $x = -1$	l. $x = -\frac{1}{2}$
6. a. $x = 8$	b. $x = 3$	c. $x = 4$	d. $x = -15$
e. $x = 26$	f. $x = -42$	g. $x = 9$	h. $x = -1\frac{1}{5}$
i. $x = 9\frac{1}{3}$	j. $x = -2\frac{1}{4}$	k. $x = -7$	l. $x = \frac{2}{3}$
7. a. $z = 16$	b. $z = 31$	c. $z = -4$	d. $z = 6$
e. $z = -9$	f. $z = -6$	g. $z = -1.9$	h. $z = 6.88$
i. $z = 140$	j. $z = 0.6$	k. $z = -35.4$	l. $z = 8$
8. a. $x = 1$	b. $x = 13$	c. $x = -2$	d. $x = -8$
e. $x = -4$	f. $x = 30\frac{1}{2}$	g. $x = -6$	h. $x = -\frac{7}{10}$
i. $x = 10.35$	j. $x = 0.326$	k. $x = 22$	l. $x = -5$
9. a. $x = 4$	b. $x = \frac{3}{7}$	c. $x = -1\frac{1}{7}$	d. $x = -6\frac{2}{3}$
e. $x = \frac{4}{45}$	f. $x = 8$	g. $x = -6$	h. $x = 7.5$
i. $x = 5.1$	j. $x = -6$	k. $x = -5\frac{13}{15}$	l. $x = -61\frac{3}{7}$
10. a. D	b. D	c. E	
11. a. $a = -1$	b. $b = 10$	c. $c = 12.1$	
d. $d = 4$	e. $e = 5$	f. $f = 12$	
g. $g = 15\frac{29}{30}$	h. $h = 31$	i. $i = \frac{13}{226}$	
j. $j = \frac{1}{6}$	k. $k = -8$	l. $l = 10.3$	
12. a. -1	b. 6	c. 303	
13. \$91			
14. No. $46.7^{\circ}\text{C} \approx 116.1^{\circ}\text{F}$.			
15. Answers will vary.			
16. The mistake is in the second line: the -1 should have been multiplied by 5.			
17. 60 lollies			
18. Old machine: 6640 cans; new machine: 9360 cans			
19. a. \$34	b. Yes, a saving of \$7		

20. 17

21. Linear equations are equations that contain a pronumeral to the power of 1. Solving a linear equation means finding the value of this pronumeral to make the equation true.

22. $x = -8, -2, 0, 1, 2, 3, 5, 6, 7, 8, 10, 16$

Exercise 10.3 Solving linear equations with brackets

1. a. $x = 6$	b. $x = -3$	c. $x = 0$	d. $x = 56$
e. $x = 1$	f. $x = 0$	g. $x = -1$	h. $x = -2$
i. $x = 6$	j. $x = 0$	k. $x = -0.8$	l. $x = 6$
2. a. $b = 1\frac{1}{6}$	b. $m = 4\frac{1}{2}$	c. $a = -1\frac{1}{2}$	d. $m = -1\frac{1}{3}$
e. $p = \frac{3}{5}$	f. $m = 2\frac{2}{3}$	g. $a = -1\frac{1}{2}$	h. $p = 1\frac{1}{2}$
i. $a = 3\frac{1}{3}$	j. $m = -2\frac{1}{2}$	k. $a = -\frac{1}{6}$	l. $m = -\frac{1}{4}$
3. a. $x = 16\frac{1}{9}$	b. $x = 2$	c. $a = 5$	d. $b = 7$
e. $y = 7$	f. $x = -3\frac{1}{3}$	g. $m = 4$	h. $y = 1$
i. $a = 5$	j. $p = -2$	k. $m = 3$	l. $p = 1$
4. a. $y = -7.5$	b. $y = -4.667$	c. $y = -26.25$	d. $y = 8.571$
e. $y = -2.9$	f. $y = 3.243$	g. $y = 3.667$	h. $y = 2.75$
i. $y = 1.976$	j. $y = -2$	k. $y = -3.167$	l. $y = 1.98$
5. a. E	b. D		
6. 1990			
7. a. $[2(2x + 7)]$ m			
b. Width 10 m, length 17 m			
8. Answers will vary; $x = 3$.			
9. Adding 7 to both sides is the least preferred option, as it does not resolve the subtraction of 7 within the brackets.			

10. 3

11. Kyle: \$90, Noah: \$35

12. \$20

13. The first method is to divide both sides of the equation by the factor outside the brackets, and then solving for the pronumeral. The second method is to expand the brackets by multiplying the bracket side by the factor, and then solving for the pronumeral.

Exercise 10.4 Solving linear equations with pronumerals on both sides

1. a. $y = -1$	b. $y = 1$	c. $y = -3$	d. $y = -5$
e. $y = -45$	f. $y = \frac{8}{27}$	g. $y = -2$	h. $y = 20$
i. $y = 1$	j. $y = -1\frac{1}{5}$	k. $y = 4$	l. $y = -2$
2. a. C	b. A		
3. a. $x = 1$	b. $x = -2$	c. $x = 3$	d. $x = 4$
e. $x = 9$	f. $x = 3$	g. $x = -2$	h. $x = 4$
i. $x = -4$	j. $x = \frac{1}{2}$	k. $x = 0$	l. $x = -\frac{3}{4}$
4. a. $x = -6$	b. $x = 7$	c. $x = 2$	d. $x = -2$
e. $x = 13$	f. $x = -7$	g. $x = \frac{1}{6}$	h. $x = -\frac{2}{17}$
i. $x = 1$			
5. a. D	b. C		
6. a. $x = 5$	b. $x = -3$	c. $x = -8$	d. $x = -7\frac{1}{2}$
e. $x = 4$	f. $x = \frac{10}{11}$	g. $x = -18$	h. $x = 3\frac{2}{3}$
i. $x = \frac{2}{3}$	j. $x = -1\frac{3}{7}$	k. $x = 0$	l. $x = -15$
7. -3			
8. 24			
9. $3(n - 36) - 98 = -11n + 200$			
10. You cannot easily divide the left-hand side by 6 or the right-hand side by 4.			
11. Daughter = 10 years, Tom = 40 years			

12. The unknown number is -3 .

13. a. $C = 40h + 250$

c. 18750

14. a. 65 games

15. Check with your teacher.

b. 18 hours, 45 minutes

d. The printing is cheaper by \$1375.

b. \$25

Exercise 10.5 Solving problems with linear equations

1. a. $x + 3 = 5$

b. $x - 9 = 7$

c. $7x = 24$

d. $\frac{x}{5} = 11$

e. $\frac{x}{2} = -9$

f. $5x - 3 = -7$

g. $2(14 - x) = -3$

h. $3x + 5 = 8$

i. $2x - 12 = 15$

j. $\frac{3x + 4}{2} = 5$

2. a. C

b. D

c. B

d. B

3. 0

4. 36 years

5. \$66

6. 20×15 ; 30×10 ; Area = 300

7. 7 m

8. 12 jackets

9. 15 days

10. 947 CDs

11. a. Post Quick (cost = \$700)

b. The cost is nearly the same for 333 flyers (\$366.50 and \$366.40).

c. The solution to the equation in part b is not a solution to the problem, because the answer is a fraction and the problem refers to the number of flyers.

12. 2

13. \$42.50

14. a. $A_{\text{fenced}} = (5x + 20) \text{ m}^2$

b. $A_{\text{path}} = (3x + 16) \text{ m}^2$

c. $l = 8 \text{ m}$, $w = 2 \text{ m}$

d. $\frac{8}{25}$

15. It is important to define the pronumeral by translating written problems into algebraic problems so they can be solved.

Exercise 10.6 Substituting values into formulas

1. a. $x = \frac{y}{a}$

b. $x = \frac{y - b}{a}$

c. $x = \frac{y + b}{2a}$

d. $x = \frac{y + 7}{2}$

e. $x = \frac{8 - 6y}{5}$

f. $x = \frac{1}{y - 2}$

g. $x = \frac{y + 1}{y - 2}$

h. $x = \frac{4y + 1}{5}$

i. $x = -5y - 12$

j. $x = 12a$

k. $x = \frac{5a - 9}{19}$

l. $x = \frac{14a + 13}{93}$

2. a. $P = \frac{g + 3}{4}$

b. $c = \frac{5f}{9}$

c. $c = \frac{5(f - 32)}{9}$

d. $I = \frac{V}{R}$

e. $t = \frac{v - u}{a}$

f. $c = \frac{b^2 - d}{4a}$

g. $y = hm + k$

h. $y = m(x - b) + a$

i. $a = y - m(x - b)$

j. $x = \frac{y - a + mb}{m}$

k. $r = \frac{2\pi}{C}$

l. $x = \frac{f - by}{a}$

m. $a = \frac{2(s - ut)}{t^2}$

n. $G = \frac{Fr^2}{Mm}$

3. 500 km

4. $b = 4 \text{ cm}$

5. 95°F

6. a. $24\,000 \text{ cm}^3$

b. $t = 80 \text{ min} = 1 \text{ h } 20 \text{ min}$

7. $h = \frac{T - 2\pi r^2}{2\pi r} = \frac{(400 - 2 \times \pi \times 4^2)}{(2 \times \pi \times 4)} = 11.9155 \text{ cm}$

8. $\frac{B}{3(1 - 2y)} = x$

9. Answers will vary.

10. $a = \frac{b^2 - d^2}{4c}$

11. $a = 1$ and $b = 5$

12. Linear equations contain only one pronumeral, which can be found by solving the equation. As a formula contains more than 1 pronumeral, rearranging a formula does not require the value of the pronumerals to be found.

13. $r = 10.608 \text{ cm}$

Investigation | Rich task

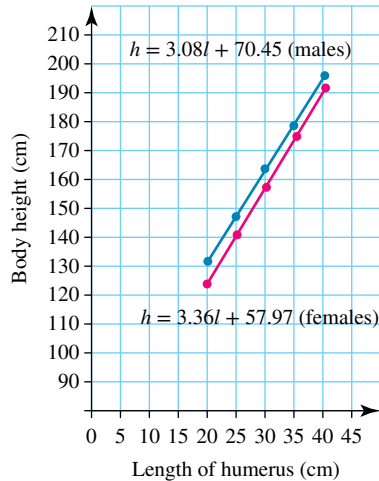
1. Table for males

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)	132	147	163	178	194

Table for females

Length of humerus l (cm)	20	25	30	35	40
Body height h (cm)	125	142	159	176	192

2, 3



4. Linear

5. Answers will vary.

6. (44.6, 207.8)

7. James King

Exercise 10.7 Review questions

1. C

2. B

3. C

4. C

5. E

6. B

7. B

8. B

9. b, c, f, g, i

10. a. $a = 2.8$

d. $b = 3.1$

11. a. $x = 19$

d. $x = 1\frac{1}{5}$

12. a. $x = 3\frac{1}{5}$

d. $x = -1\frac{5}{12}$

13. a. $8h + 21 = 45$

14. a. $x = \frac{1}{2}$

d. $x = 5$

15. a. $2x = 3 - x$

d. $\frac{10}{x} - 1 = \frac{x}{6}$

16. 600 km

17. a. $x = \frac{y+4}{6}$

e. $a = \frac{v-u}{t}$

18. a. $C_G = 114 + 0.20k$

19. a. $5400 + 260d = C_H$

20. a. $0.06x$

d. $0.06x + 0.03(1-x) = 0.05$

21. 24 km

b. $a = -0.6$

e. $b = -3.45$

b. $x = 13$

e. $x = 6\frac{2}{3}$

b. $x = 2\frac{5}{7}$

e. $x = -12\frac{1}{2}$

b. 3 hours

b. $x = 3$

e. $x = 2$

b. $3x + 8 = 19$

e. $7(2x + 5) = 0$

b. $x = \frac{y-c}{m}$

f. $t = \frac{2s}{u+v}$

b. $C_S = 90 + 0.32k$

b. 61 days

b. $(1-x)$

e. 0.667 or 66.7%

22. $21\frac{123}{137} \approx 21.9$ km

c. $b = -2.52$

f. $b = 0.749$

c. $x = -6\frac{1}{3}$

f. $x = -11.13$

c. $x = 1\frac{1}{4}$

f. $x = -8\frac{1}{2}$

c. $x = 3$

f. $x = -1$

c. $6x = 4$

f. $2x - 100 = x + 50$

c. $P = \frac{q-2r}{2} + 1$

g. $a = \frac{v^2 - u^2}{2s}$

c. 200 km

c. $0.03(1-x)$

23. 6, 7, 8, 9 or 10 books

d. $w = \frac{P-2l}{2}$

h. $b = \frac{2A-ah}{h}$

TOPIC 11

Linear and non-linear relationships

[Stages 5.1 and 5.2]

11.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

11.1.1 Why learn this?

It is very common that one quantity depends on another quantity, and we can often use mathematical modelling to gain an understanding of the situation. If we know the equation for a function, we can draw a graph and use the graph to analyse, interpret and explain the relationship between the variables, and to make predictions about the future. Science, engineering, health and finance all rely heavily on using mathematical equations to model real-life situations and solve problems. Slope, gradient and distance between points are important concepts to understand.



DISCUSSION

How might the impact of performance-enhancing drugs have affected the progression of the 100-metre world record?

LEARNING SEQUENCE

- 11.1** Overview
- 11.2** [Stage 5.1] Sketching linear graphs using the coordinates of two points
- 11.3** [Stage 5.1] Midpoint of a line segment and distance between two points
- 11.4** [Stage 5.1] The gradient of a straight line
- 11.5** [Stage 5.2] Using the gradient–intercept form to graph linear relationships
- 11.6** [Stages 5.1 and 5.2] Parallel and perpendicular lines
- 11.7** [Stage 5.2] Finding the equation of a straight line
- 11.8** [Going further] Practical applications of linear graphs
- 11.9** [Stages 5.1 and 5.2] Non-linear relationships (parabolas, exponentials, circles)
- 11.10** Review



LEARNING OUTCOMES

A student:

- uses appropriate terminology, diagrams and symbols in mathematical contexts MA5.1-1WM
- provides reasoning to support conclusions that are appropriate to the context MA5.1-3WM
- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- constructs arguments to prove and justify results MA5.2-3WM
- determines the midpoint, gradient and length of an interval, and graphs linear relationships MA5.1-6NA
- graphs simple non-linear relationships MA5.1-7NA
- uses the gradient-intercept form to interpret and graph linear relationships MA5.2-9NA
- connects algebraic and graphical representations of simple non-linear relationships MA5.2-10NA

CONTENT DESCRIPTIONS

Students:

- Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)
- Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software (ACMNA214)
- Sketch linear graphs using the coordinates of two points (ACMNA215)
- Solve problems involving parallel and perpendicular lines (ACMNA238)
- Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line (NSW)
- Graph simple non-linear relationships, with and without the use of digital technologies, and solve simple related equations (ACMNA296)
- Explore the connection between algebraic and graphical representations of relationships such as simple quadratics, circles and exponentials using digital technologies as appropriate (ACMNA239)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — How fast can humans go? (eles-1694)

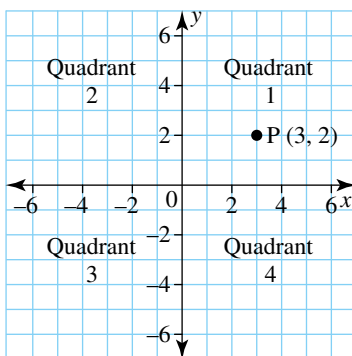
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

11.2 Sketching linear graphs using the coordinates of two points

[Stage 5.1]

11.2.1 The Cartesian plane

- The **Cartesian plane** is divided into 4 regions (quadrants) by the x - and y -axes, as shown in the diagram.
- Every point in the plane is described exactly by a pair of **Cartesian coordinates**, (x, y) . The point $P(3, 2)$ is marked on the diagram.



11.2.2 Plotting from a rule

- A graph can be drawn by plotting a series of points on a Cartesian plane. To do this requires:
 - a set of x -values
 - a rule.

WORKED EXAMPLE 1

Plot the graph specified by the rule $y = x + 2$ for the x -values $-3, -2, -1, 0, 1, 2, 3$.

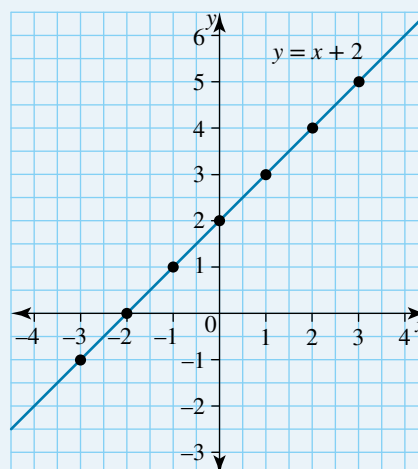
THINK

- Draw a table and write in the required x -values.
- Substitute each x -value into the rule $y = x + 2$ to obtain the corresponding y -value.
When $x = -3$, $y = -3 + 2 = -1$.
When $x = -2$, $y = -2 + 2 = 0$ etc.
Write the y -values into the table.
- Plot the points from the table: $(-3, -1)$ etc.
- Join the points with a straight line and label the graph with its equation, $y = x + 2$.

WRITE/DRAW

x	-3	-2	-1	0	1	2	3
y							

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5



- A straight line graph is called a **linear graph** and its rule is called a linear relation. The rule for a linear graph can always be written in the form $y = mx + c$, for example $y = 4x - 5$ or $y = x + 1.2$.
- If a graph is linear, then a minimum of two points need be plotted to locate the straight line. It is sensible to choose points that are some distance apart and to use a third point to check an error has not been made.

WORKED EXAMPLE 2

Plot two points and hence draw the linear graph $y = 2x - 1$.

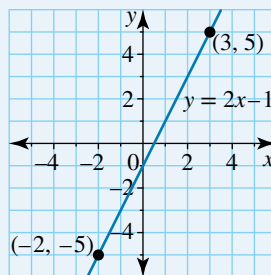
THINK

- Choose any two x -values, for example $x = -2$ and $x = 3$
- Calculate y by substituting each x -value into $y = 2x - 1$.
When $x = -2$, $y = 2 \times -2 - 1 = -5$.
When $x = 3$, $y = 2 \times 3 - 1 = 5$.

WRITE/DRAW

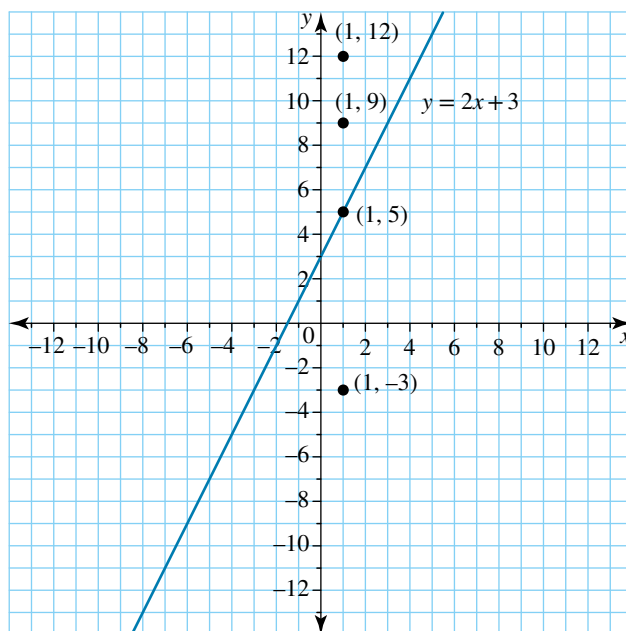
x	-2	3
y	-5	5

- 3 Plot the points $(-2, -5)$ and $(3, 5)$.
- 4 Draw a line through the points and add a label.



11.2.3 Points on a line

- Consider the line that has the rule $y = 2x + 3$ as shown in the graph.
- If $x = 1$, then $y = 2(1) + 3$
 $\quad\quad\quad = 5$
 So the point $(1, 5)$ lies on the line $y = 2x + 3$.



- The points $(1, 0)$, $(1, -3)$, $(1, 9)$, $(1, 12)$... are not on the line; they lie above or below it.

WORKED EXAMPLE 3

Does the point $(2, 4)$ lie on the line given by:

a $y = 3x - 2$

b $x + y = 5$?

THINK

- a 1 Substitute $x = 2$ into the equation $y = 3x - 2$ and find y .

- 2 When $x = 2$, $y = 4$, so the point $(2, 4)$ lies on the line. Write the answer.

WRITE

a $y = 3x - 2$
 $x = 2:$ $y = 3(2) - 2$
 $\quad\quad\quad = 6 - 2$
 $\quad\quad\quad = 4$

The point $(2, 4)$ lies on the line $y = 3x - 2$.

b 1 Substitute $x = 2$ into the equation $x + y = 5$ and find y .

2 The point $(2, 3)$ lies on the line, but the point $(2, 4)$ does not. Write the answer.

b $x + y = 5$

$$x = 2: \quad 2 + y = 5$$

$$y = 3$$

The point $(2, 4)$ does not lie on the line $x + y = 5$.

learnon RESOURCES — ONLINE ONLY

Interactivity: Plotting linear graphs (int-3834)

Interactivity: Drawing a graph (int-1020)

Digital doc: SkillsHEET Plotting coordinate points (doc-6161)

Digital doc: SkillsHEET Substituting into a rule (doc-6162)

Digital doc: SkillsHEET Completing a table of values (doc-6163)

Digital doc: SkillsHEET Plotting a line from a table of values (doc-6164)

Exercise 11.2 Sketching linear graphs using the coordinates of two points

Individual pathways

PRACTISE

Questions:
1–8, 12

CONSOLIDATE

Questions:
1–10, 12, 13

MASTER

Questions:
1–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. MC a. The point with coordinates $(-2, 3)$ is:

A. in quadrant 1

B. in quadrant 2

C. in quadrant 3

D. in quadrant 4

b. The point with coordinates $(-1, -5)$ is:

A. in the first quadrant

B. in the second quadrant

C. in the third quadrant

D. in the fourth quadrant

c. The point with coordinates $(0, -2)$ is:

A. in the third quadrant

B. in the fourth quadrant

C. on the x -axis

D. on the y -axis

2. WE1 For each of the following rules, complete the table below and plot the linear graph.

x	-3	-2	-1	0	1	2	3
y							

a. $y = x$

b. $y = 2x + 2$

c. $y = 3x - 1$

d. $y = -2x$

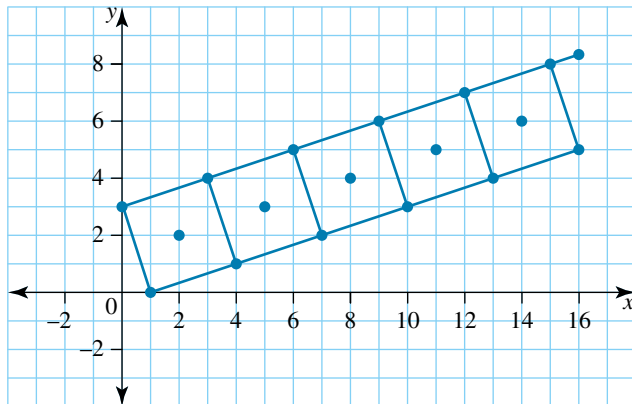
e. $y = 2$

f. $y = -3$

3. **WE2** By first plotting 2 points, draw the linear graph given by each of the following.
- a. $y = -x$ b. $y = \frac{1}{2}x + 4$ c. $y = -2x + 3$ d. $y = x - 3$
e. $x = 3$ f. $x = -1$
4. **WE3** Do these points lie on the graph of $y = 2x - 5$?
a. (3, 1) b. (-1, 3) c. (0, 5) d. (5, 5)
5. Does the given point lie on the given line?
a. $y = -x - 7$, (1, -8) b. $y = 3x + 5$, (0, 5) c. $y = x + 6$, (-1, 5)
d. $y = 5 - x$, (8, 3) e. $y = -2x + 11$, (5, -1) f. $y = x - 4$, (-4, 0)
g. $y = 7x - 11$, (1, -4) h. $2x + y = 10$, (3, 4)
6. **MC** The line that passes through the point (2, -1) is:
A. $y = -2x + 5$ **B.** $y = 2x - 1$ **C.** $y = -2x + 1$
D. $x + y = 1$ **E.** $y = x - \frac{1}{2}$
7. Match each point with a line passing through that point.
a. (1, 1) b. (1, 3) c. (1, 6) d. (1, -4)
A. $x + y = 4$ **B.** $2x - y = 1$ **C.** $y = 3x - 7$ **D.** $y = 7 - x$

Communicating, reasoning and problem solving

8. The line through (1, 3) and (0, 4) passes through every quadrant except one. Which one? Explain your answer.
9. a. Which quadrant(s) does the line $y = x + 1$ pass through?
b. Show that the point (1, 3) does not lie on the line $y = x + 1$.
10. Explain the process of how to check whether a point lies on a given line.
11. Using the coordinates (-1, -3), (0, -1) and (2, 3), show that a rule for the linear graph is $y = 2x - 1$.
12. Consider the pattern of squares shown on the grid.



What would be the coordinates of the centre of the 20th square?

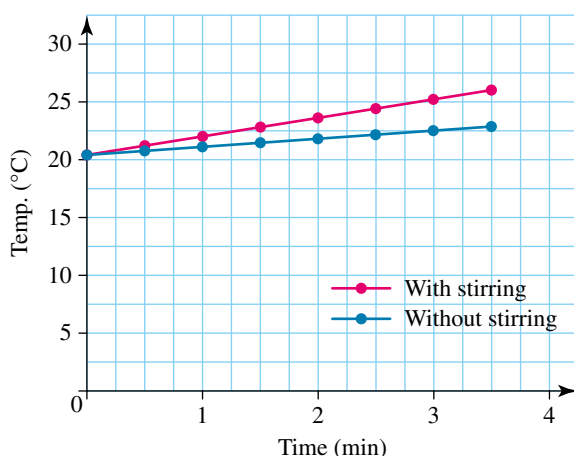
13. It is known that the mass of a certain kind of genetically modified tomato increases linearly over time. The following results were recorded.

Time, t (weeks)	1	4	6	9	16
Mass, m (grams)	6	21	31	46	81

- a. Plot the above points on a Cartesian plane.
b. Determine the rule connecting mass with time.
c. Show that the mass after 20 weeks is 101 grams.



14. As a particular chemical reaction proceeds, the temperature increases at a constant rate. The graph represents the same chemical reaction with and without stirring. How does stirring affect the reaction?



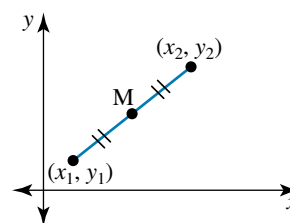
15. What do the lines with equations $x = 0$ and $y = 0$ represent?

11.3 Midpoint of a line segment and distance between two points

[Stage 5.1]

11.3.1 Finding the midpoint of a line segment

- The x - and y -coordinates of the **midpoint** are halfway between those of the end points of a line segment or between any two given points.
- The coordinates of the midpoint, M , of a line can be found by averaging the x - and y -coordinates of the end points.
- The coordinates of the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) are:



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- This method is similar to finding the mean of the x -values and the mean of the y values.

WORKED EXAMPLE 4

Find the midpoint of the segment joining $(5, 9)$ and $(-3, 11)$.

THINK

1 Average the x -values: $\frac{x_1 + x_2}{2}$.

2 Average the y -values: $\frac{y_1 + y_2}{2}$.

3 Write the answer.

WRITE

$$x = \frac{5 - 3}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

$$y = \frac{9 + 11}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

The midpoint is $(1, 10)$.

WORKED EXAMPLE 5

M (7, 2) is the midpoint of the line segment AB. If the coordinates of A are (1, -4), find the coordinates of B.

THINK

- 1 Let B have the coordinates (x, y) .
- 2 The midpoint is $(7, 2)$, so the average of the x -values is 7. Solve for x .
- 3 The average of the y -values is 2. Solve for y .
- 4 Write the answer.

WRITE

A $(1, -4)$, B (x, y) , M $(7, 2)$

$$\frac{1 + x}{2} = 7$$

$$1 + x = 14$$
$$x = 13$$

$$\frac{-4 + y}{2} = 2$$

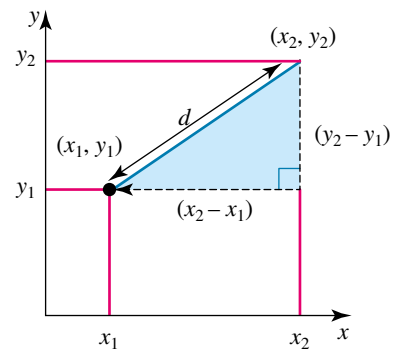
$$-4 + y = 4$$
$$y = 8$$

Hence, the coordinates of point B are $(13, 8)$.

11.3.2 The distance between two points

- The distance between two points on the Cartesian plane is calculated using Pythagoras' theorem applied to a right-angled triangle.
- The distance between the points (x_1, y_1) and (x_2, y_2) is calculated using the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



WORKED EXAMPLE 6

Find the distance between the points $(-1, 3)$ and $(4, 5)$:

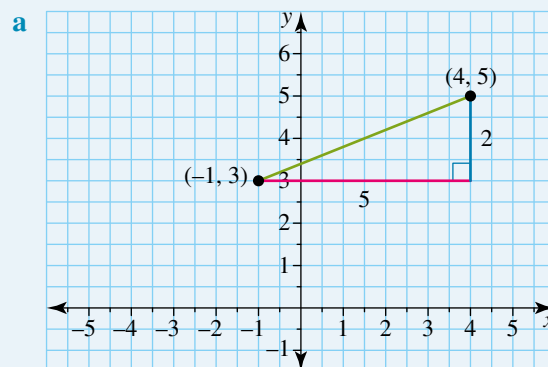
a exactly

b correct to 3 decimal places.

THINK

- a 1** Draw a diagram showing the right-angled triangle (optional).

WRITE/DRAW



2 Write the formula for the distance between two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3 Let $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, 5)$. Substitute the x - and y -values into the equation.

$$d = \sqrt{(4 - -1)^2 + (5 - 3)^2}$$

4 Simplify.

$$d = \sqrt{5^2 + 2^2}$$

$$d = \sqrt{25 + 4}$$

$$d = \sqrt{29}$$

b 1 Write $\sqrt{29}$ as a decimal to 4 decimal places.

$$\text{b } \sqrt{29} = 5.3851$$

2 Write the answer correct to 3 decimal places.

$$d \approx 5.385$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Midpoints (int-6052)

Interactivity: Distance between two points (int-6051)

Interactivity: Distance between two points (int-2766)

Exercise 11.3 Midpoint of a line segment and distance between two points

Individual pathways

PRACTISE

Questions:

1a–f, 2–5, 6a–f, 7, 8, 11, 12

CONSOLIDATE

Questions:

1c–h, 2–5, 6c–h, 7, 8, 10–14, 16

MASTER

Questions:

1e–j, 2–5, 6e–j, 7–9, 11–17

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Understanding and fluency

1. **WE4** Find the midpoint of the segment joining each of the following pairs of points.

a. (1, 3) and (3, 5)

b. (6, 4) and (4, -2)

c. (2, 3) and (12, 1)

d. (6, 3) and (10, 15)

e. (4, 2) and (-4, 8)

f. (0, -5) and (-2, 9)

g. (8, 2) and (-18, -6)

h. (-3, -5) and (7, 11)

i. (-8, -3) and (8, 27)

j. (-4, 1) and (-2, 13)

2. Find the midpoint of the segment joining each of the following pairs of points. (Graphing software may be used to calculate the midpoint of the segments.)

a. (7, -2) and (-4, 13)

b. (0, 22) and (-6, -29)

c. (-15, 8) and (-4, 11)

d. (-3, 40) and (0, -27)

3. **WE5** Find the value of a in each set of points below so that the point M is the midpoint of the segment joining points A and B .

a. $A(-2, a)$, $B(-6, 5)$, $M(-4, 5)$

b. $A(a, 0)$, $B(7, 3)$, $M\left(8, \frac{3}{2}\right)$

c. $A(3, 3)$, $B(4, a)$, $M\left(3\frac{1}{2}, -6\frac{1}{2}\right)$

d. $A(-4, 4)$, $B(a, 0)$, $M(-2, 2)$

4. M is the midpoint of the line interval AB . Find the coordinates of B if:

a. $A = (0, 0)$ and $M = (2, 3)$

b. $A = (2, 3)$ and $M = (0, 0)$

c. $A = (-3, 2)$ and $M = (4, 2)$

d. $A = (3, -1)$ and $M = (-2, -2)$.

5. **WE6** Find the distance between each of the following pairs of points.

a. $(4, 5)$ and $(1, 1)$

b. $(7, 14)$ and $(15, 8)$

c. $(2, 4)$ and $(2, 3)$

d. $(12, 8)$ and $(10, 8)$

e. $(14, 9)$ and $(2, 14)$

f. $(5, -13)$ and $(-3, -7)$

g. $(-14, -9)$ and $(-10, -6)$

h. $(0, 1)$ and $(-15, 9)$

i. $(-4, -8)$ and $(1, 4)$

j. $(12, 9)$ and $(-4, -3)$

6. Calculate the distance between each of the following pairs of points, correct to 3 decimal places. (Graphing software may be used to calculate the distance between points.)

a. $(-14, 10)$ and $(-8, 14)$

b. $(6, -7)$ and $(13, 6)$

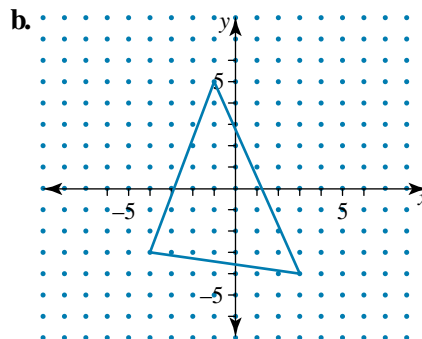
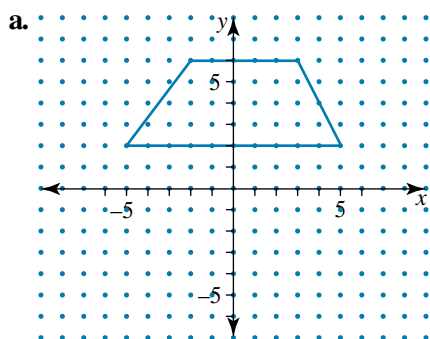
c. $(-11, 1)$ and $(2, 2)$

d. $(9, 0)$ and $(5, -8)$

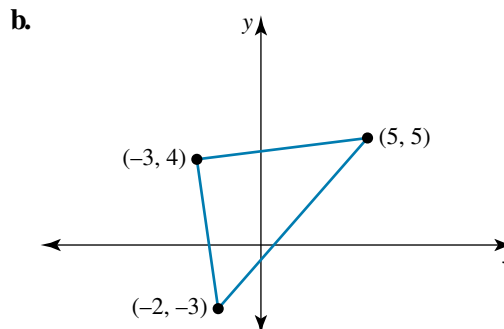
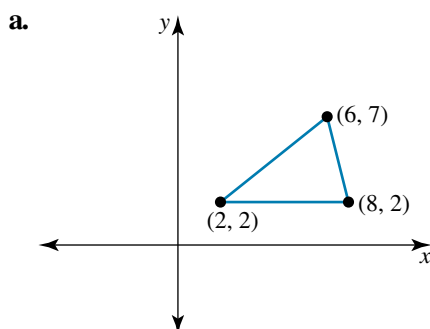
e. $(2, -7)$ and $(-2, 12)$

f. $(9, 4)$ and $(-10, 0)$

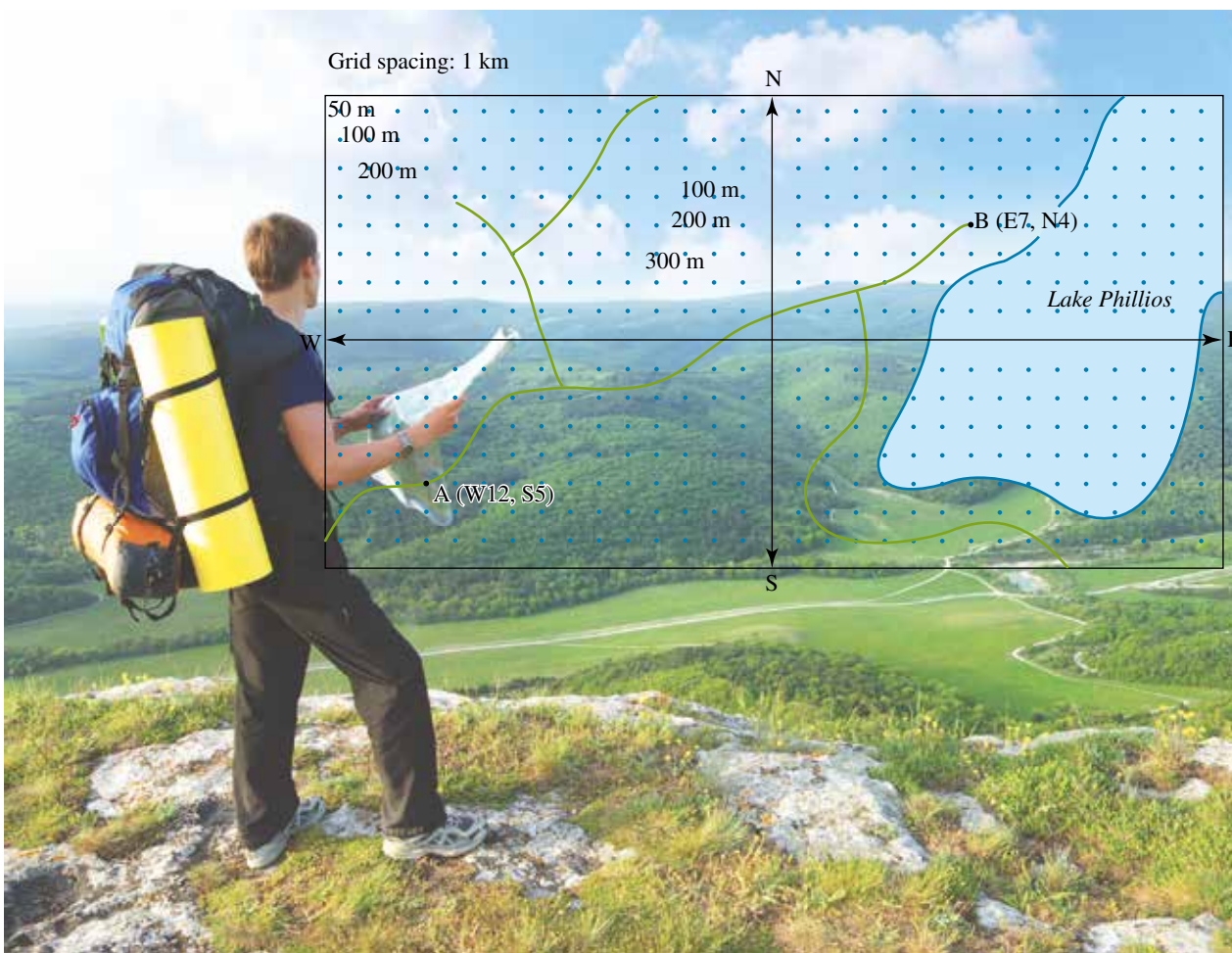
7. Find the perimeter of each figure below, giving your answers correct to 3 decimal places.



8. Find the perimeter of each triangle, giving your answers correct to 3 decimal places.



9. Two hikers are about to hike from A to B (shown on the map below). How far is it from A to B in a straight line?



Communicating, reasoning and problem solving

10. a. Plot the following points on a Cartesian plane: A $(-1, -4)$, B $(2, 3)$, C $(-3, 8)$ and D $(4, -5)$.
 b. Show that the midpoint of the interval AC is $(-2, 2)$.
 c. Find the exact distance between the points A and C.
 d. If B is the midpoint of an interval CM, find the point M.
11. The point M $(-2, -4)$ is the midpoint of the interval AB. Show that the point B is $(-9, -2)$, given A is $(5, -6)$.
12. Show that the distance between the points A $(2, 2)$ and B $(6, -1)$ is 5.
13. Show that the point B $(6, -10)$ is equidistant from the points A $(15, 3)$ and C $(-7, -1)$.
14. A triangle has vertices A $(-4, 1)$, B $(2, 3)$ and C $(0, -3)$. Show that $\triangle ABC$ is isosceles.
15. A map of a town drawn on a Cartesian plane shows the main street extending from $(-4, 5)$ to $(0, -7)$. Five street lights are positioned in the street. There is one at either end and three spaced evenly down the street. Give the position of the five lights in the street.
16. If the two end points of a line segment both have negative x - and y -coordinates, the distance formula still produces a positive answer. Why is this so?
17. Describe how the distance between two points can be calculated using Pythagoras' theorem.

11.4 The gradient of a straight line

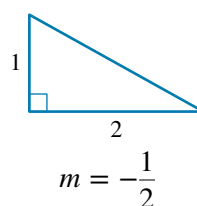
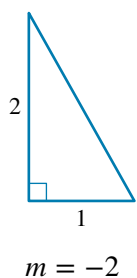
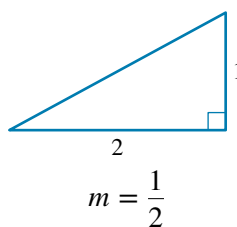
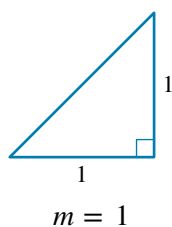
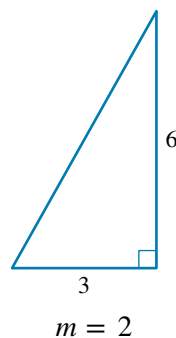
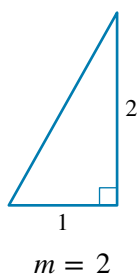
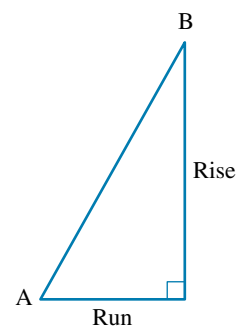
[Stage 5.1]

11.4.1 The gradient (m)

- A straight line goes on forever, and has constant steepness or **gradient**.



- The gradient of an interval (portion of a line) is equal to the gradient of the entire line.
- The gradient of an interval AB is defined as the distance up (rise) divided by the distance across (run), and is usually given the symbol m .
- So $m = \frac{\text{rise}}{\text{run}}$.
- Compare these intervals and their gradients.



- Note that if the line is sloping downwards (from left to right), the gradient has a negative value.

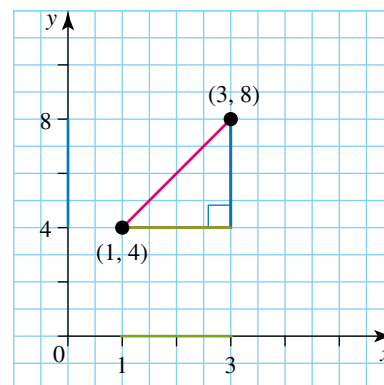
11.4.2 Finding the gradient of a line passing through two points

- Suppose a line passes through the points (1, 4) and (3, 8), as shown in the graph.
- By completing a right-angled triangle, it can be seen that the rise = $8 - 4$ (the difference in y -values), and the run = $3 - 1 = 2$ (the difference in x -values). So

$$\begin{aligned} m &= \frac{8 - 4}{3 - 1} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

- In general, if the line passes through the points (x_1, y_1) and (x_2, y_2) , then

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$



WORKED EXAMPLE 7

Find the gradient of the line passing through the points $(-2, 5)$ and $(1, 14)$.

THINK

- Let the two points be (x_1, y_1) and (x_2, y_2) .
- Write the formula for gradient.
- Substitute the coordinates of the given points into the formula and evaluate.
- Write the answer.

WRITE

$$(-2, 5) = (x_1, y_1), (1, 14) = (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{14 - 5}{1 - (-2)}$$

$$m = \frac{9}{1 + 2}$$

$$m = \frac{9}{3}$$

$$= 3$$

The gradient of the line passing through $(-2, 5)$ and $(1, 14)$ is 3.

- Note:* In Worked example 7, let $(x_1, y_1) = (1, 14)$ and $(x_2, y_2) = (-2, 5)$.

$$\begin{aligned} \text{The calculation becomes } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 14}{-2 - 1} \\ &= \frac{-9}{-3} \\ &= 3 \end{aligned}$$

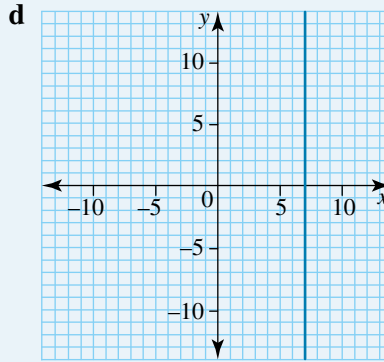
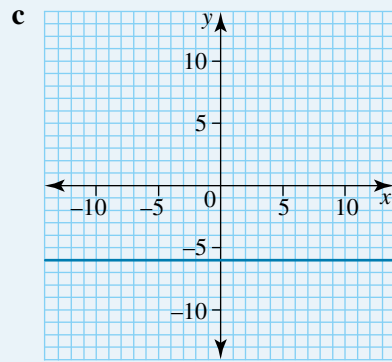
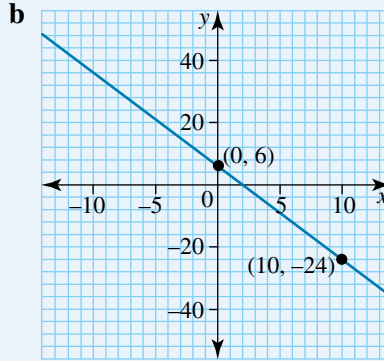
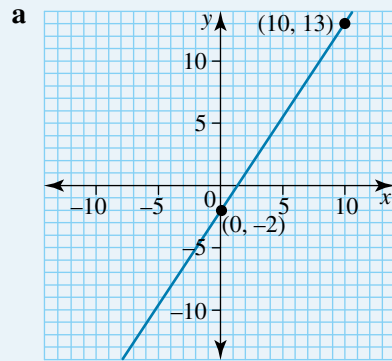
The result is the same.

DISCUSSION

When calculating the gradient, why is the same result obtained regardless of which point is matched to which pair of coordinates in the equation?

WORKED EXAMPLE 8

Calculate the gradients of the lines shown.



THINK

a 1 Write down two points that lie on the line.

2 Calculate the gradient by finding the ratio $\frac{\text{rise}}{\text{run}}$.

b 1 Write down two points that lie on the line.

2 Calculate the gradient.

c 1 Write down two points that lie on the line.

2 There is no rise between the two points.

WRITE

a Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (10, 13)$.
 Rise $= y_2 - y_1 = 13 - (-2) = 15$
 Run $= x_2 - x_1 = 10 - 0 = 10$

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{15}{10} \\ &= \frac{3}{2} \text{ or } 1.5 \end{aligned}$$

b Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (10, -24)$.
 Rise $= y_2 - y_1 = -24 - 6 = -30$
 Run $= x_2 - x_1 = 10 - 0 = 10$

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-30}{10} \\ &= -3 \end{aligned}$$

c Let $(x_1, y_1) = (5, -6)$ and $(x_2, y_2) = (10, -6)$.

$$\begin{aligned} \text{Rise} &= y_2 - y_1 = -6 - (-6) = 0 \\ \text{Run} &= x_2 - x_1 = 10 - 5 = 5 \end{aligned}$$

3 Calculate the gradient.

Note that the gradient of a horizontal line is always zero. The line has no slope.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{0}{5} \\ &= 0 \end{aligned}$$

d 1 Write down two points that lie on the line.

2 The vertical distance between the selected points is 13 units. There is no run between the two points.

3 Calculate the gradient.

Note: The gradient of a vertical line is always undefined.

d Let $(x_1, y_1) = (7, 10)$ and $(x_2, y_2) = (7, -3)$.

$$\text{Rise} = y_2 - y_1 = -3 - 10 = 13$$

$$\text{Run} = x_2 - x_1 = 7 - 7 = 0$$

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{13}{0} \text{ undefined} \end{aligned}$$

11.4.3 Finding the gradient of a straight line from its rule

- When an equation is written in the form $y = mx + b$, m is the value of the gradient. For example, consider the line with equation $y = 3x + 1$. The gradient is 3.
- To confirm this, find the gradient using the formula. Two points that lie on the line $y = 3x + 1$ are $(0, 1)$ and $(5, 16)$.

$$\begin{aligned} \text{Gradient} &= \frac{16 - 1}{5 - 0} \\ &= \frac{15}{5} \\ &= 3 \end{aligned}$$

WORKED EXAMPLE 9

Find the gradients of the straight lines whose rules are given.

a $y = -2x + 3$

b $2y - 3x = 6$

c $y = 4$

THINK

a The equation is the form $y = mx + b$, so the gradient is the coefficient of x .

b 1 First rearrange the given rule so that it is in the form $y = mx + b$. (Add $3x$ to both sides, then divide both sides by 2.)

2 Write the value of the gradient.

c 1 Rewrite the equation in the form $y = mx + b$.

2 Write the value of the gradient.

WRITE

a $y = -2x + 3$
 $m = -2$

b $2y - 3x = 6$
 $2y = 6 + 3x$
 $y = \frac{6}{2} + \frac{3}{2}x$
 $y = \frac{3}{2}x + 3$

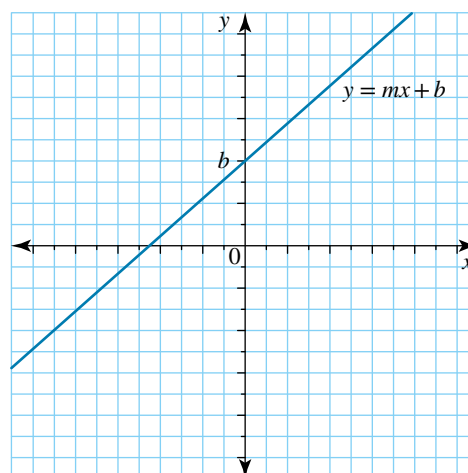
$$m = \frac{3}{2}$$

c $y = 4$

$$m = 0$$

11.4.4 The y-intercept

- For the line given by $y = mx + b$, when $x = 0$, $y = b$.
- The line passes through the point $(0, b)$. This is the point where the graph cuts the y-axis.
- The point where the graph cuts the y-axis is called the **y-intercept**.
- In this case the y-intercept is $(0, b)$, often simply called b .
- The y-intercept of any line is easily found by substituting 0 for x and calculating the y-value.
- $y = mx + b$ is called the 'gradient-intercept form' of the equation of a line, because it plainly displays the gradient (m) and the y-intercept (b).



WORKED EXAMPLE 10

Find the y-intercepts of the lines whose linear rules are given, and hence state the coordinates of each y-intercept.

a $y = -4x + 7$

b $5y + 2x = 10$

c $y = 2x$

d $y = -8$

THINK

- a** The rule is in the gradient-intercept form, $y = mx + b$.
The y-intercept is the value of b . State the coordinates.
- b** **1** To find the y-intercept, substitute $x = 0$ into the equation.
2 Solve for y .
3 Write the coordinates of the y-intercept.
- c** The rule is in the gradient-intercept form, $y = mx + b$.
The y-intercept is the value of b . State the coordinates.
- d** The rule is in the form $y = mx + b$.
State the coordinates.

WRITE

a $y = -4x + 7$
 $b = 7$
y-intercept: $(0, 7)$

b $5y + 2x = 10$
 $5y + 2(0) = 10$
 $5y = 10$
 $y = 2$
y-intercept: $(0, 2)$

c $y = 2x$
 $b = 0$
y-intercept: $(0, 0)$

d $y = -8$
 $y = 0x - 8$
 $b = -8$
y-intercept: $(0, -8)$

learn on

RESOURCES — ONLINE ONLY

Interactivity: The gradient (int-3836)

Interactivity: Linear graphs (int-6484)

eLesson: Gradient (eles-1889)

Digital doc: SkillSHEET Transposing a linear equation to general form (doc-6165)

Digital doc: SkillSHEET Finding the gradient given two points (doc-10839)

Digital doc: SkillSHEET Measuring the rise and the run (doc-10840)

Digital doc: SkillSHEET Finding the gradient of a line from its equation (doc-10841)

Digital doc: WorkSHEET Linear graphs (doc-6170)

Exercise 11.4 The gradient of a straight line

Individual pathways

PRACTISE

Questions:

1a–f, 2a–e, 3a–f, 4, 5a–f, 6–11, 15–16

CONSOLIDATE

Questions:

1d–i, 2c–f, 3e–j, 4, 5c–j, 6, 7b–e, 8, 9–12, 15–17

MASTER

Questions:

1g–l, 2e–i, 3g–l, 4, 5f–l, 6, 7c–e, 8–20

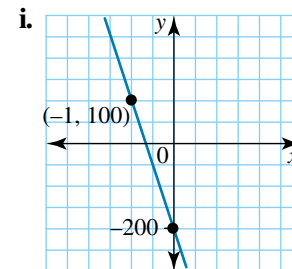
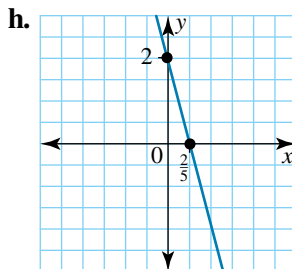
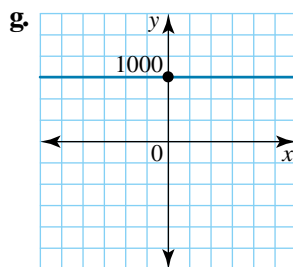
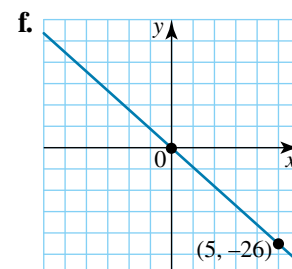
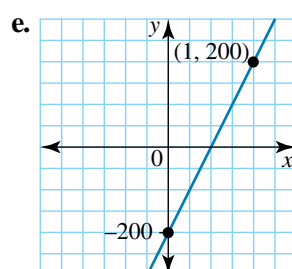
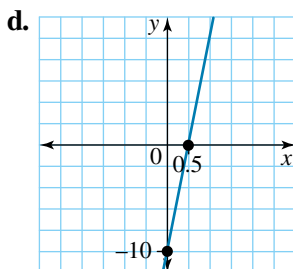
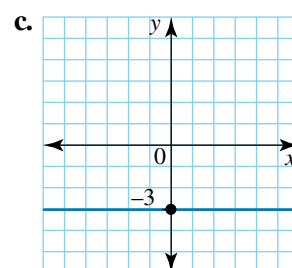
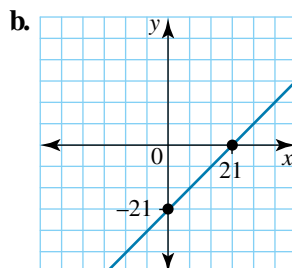
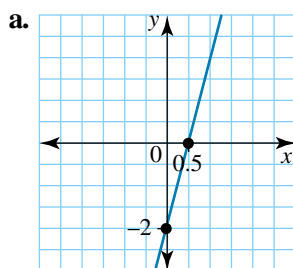
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

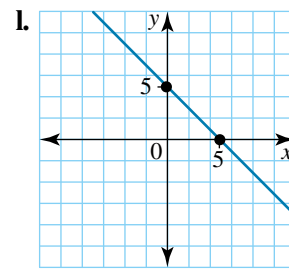
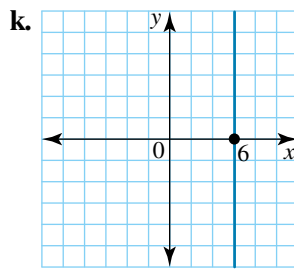
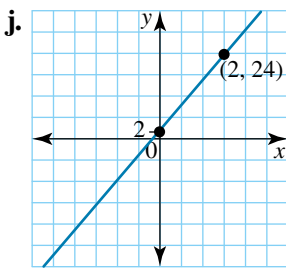
Understanding and fluency

1. **WE7** Find the gradients of the lines passing through the following pairs of points.

a. (2, 10) and (4, 22)	b. (1, -2) and (3, -10)
c. (-3, 0) and (7, 0)	d. (-4, -7) and (1, -1)
e. (0, 4) and (4, -4.8)	f. (-2, 122) and (1, -13)
g. (2, 3) and (17, 3)	h. (-2, 2) and (2, 2.4)
i. (1, -5) and (5, -15.4)	j. (-12, -7) and (8.4, -7)
k. (-2, -17.7) and (0, 0.3)	l. (-3, 3.4) and (5, 2.6)

2. **WE8** Calculate the gradients of the lines shown.





3. **WE9** Find the gradients of the straight lines whose rules are given below.
- | | | | |
|-----------------------------|----------------------------|-------------------|---------------------|
| a. $y = 5x + 23$ | b. $y = 54 - 3x$ | c. $y = 3(x - 2)$ | d. $y = 70 - 2x$ |
| e. $y = \frac{1}{2}(x + 2)$ | f. $y = \frac{x}{2} + 5.2$ | g. $y = 100 - x$ | h. $y = 100$ |
| i. $y = -87 + 23x$ | j. $2y - 4x - 5 = 0$ | k. $2y + 4x = -5$ | l. $6y - 21x = -19$ |

4. **WE10** Find the y -intercepts of the lines whose linear rules are given in question 3.

5. Write each equation in gradient–intercept form, then find the gradient and y -intercept of the lines defined by the following rules.

a. $y = 4x + 8$	b. $y = -4x + 8$	c. $y = -2x + 7$	d. $y = 12x$
e. $y = 0.5x + 2.5$	f. $y = -40x + 83$	g. $y + 4x = -18$	h. $5y - 6x = -18$
i. $y = \frac{x}{2} + \frac{3}{2}$	j. $15y - x = 0$	k. $3y - 9x = 15$	l. $8x - 2y = 16$

6. **MC** Which of the following statements about linear graphs is *false*?

- A. A gradient of zero means the graph is a horizontal line.
- B. Gradient can be any real number.
- C. A linear graph can have two y -intercepts.
- D. In the form $y = mx + b$, the y -intercept equals b .
- E. A linear graph is the graph of a straight line.

7. In the form $y = mx + b$, the y -intercept is b and the gradient is m . Find a formula for the x -intercept in terms of m and b . (*Hint*: The x -intercept is the point where the graph crosses the x -axis. At such a point $y = 0$.)

8. Find the coordinates of the y -intercepts of the lines with the following rules.

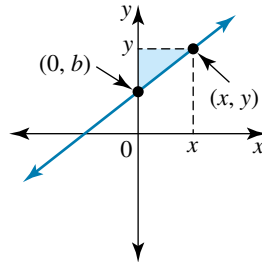
a. $y = -6x - 10$	b. $3y + 3x = -12$	c. $7x - 5y + 15 = 0$
d. $y = 7$	e. $x = 9$	

Communicating, reasoning and problem solving

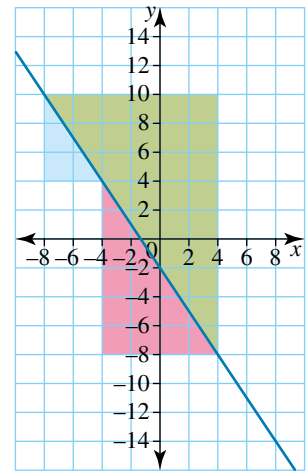
9. Explain why the gradient of a vertical line is undefined.
10. Explain why the gradient of a horizontal line is zero.
11. Show the gradient of the line passing through the points (a, b) and (c, d) is $\frac{d - b}{c - a}$.
12. a. Sketch a line for the rule $y = -x + b$, if the value of b can be any value.
b. Explain the effect of the change in b for the line $y = -x + b$.
13. Find how many points (x, y) satisfy the equation $x + y = 25$, assuming that the values of both x and y must be:
a. equal
b. positive integers.
14. Water is leaking out of a water tank such that the amount of water remaining in the tank is given by the formula $V = 3000 - 48t$, where V is the volume of water in the tank in litres and t is the time it takes to drain in minutes.
a. How much water was in the tank before it started leaking?
b. How much water is left in the tank after leaking for 20 minutes?



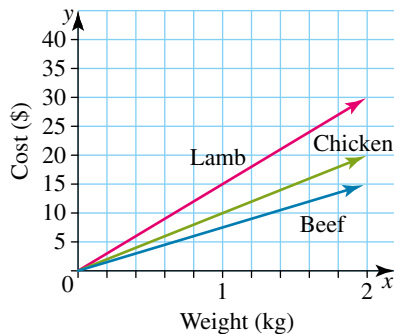
- c. Show that if the tank is checked after 65 minutes there will be no water left.
 d. Use this information to plot the graph of V against t and state the intercepts.
 e. In the context of this problem, explain what the gradient of -48 means.
15. When using the gradient to draw a line, does it matter if you rise before you run or run before you rise? Explain.
16. a. Using the graph shown, write a general formula for the gradient m in terms of x , y and b .
 b. Transpose your formula to make y the subject. What do you notice?



17. Three right-angled triangles have been superimposed on the graph shown.
 a. Use each of these to determine the gradient of the line.
 b. Does it matter which points are chosen to determine the gradient of a line? Explain.
 c. Describe the shape of the graph.
18. Lilly has entered the competition for the Readers' Award, which awards prizes for the students who read the most books from 1 February to 31 May. First prize is awarded to a student who has read more than 50 books over the four months, second prize is awarded to a student who has read from 30 to 50 books, and third prize is awarded to a student who has read between 15 and 30 books. At the beginning of the n th week, Lilly had read 5 books.



- Lilly then read an average of b books each week until the end of the competition and qualified for second prize. Explain why the least value of b must be 1.56.
19. The price per kilogram for 3 different types of meat is illustrated in the graph.



- a. Calculate the gradient (using units) for each graph.
 b. What is the cost of 1 kg of each type of meat?
 c. What is the cost of purchasing:
 i. 1 kg of lamb
 ii. 0.5 kg of chicken
 iii. 2 kg of beef?
 d. What is the total cost of the order in part c?



e. Copy and complete the table below to confirm your answer from part d.

Meat type	Cost per kilogram (\$/kg)	Weight required (kg)	Cost = \$/kg $\times$ kg
Lamb		1	
Chicken		0.5	
Beef		2	
		Total cost	

20. Why is the y-intercept of a graph found by substituting $x = 0$ into the equation?

11.5 Using the gradient–intercept form to graph linear relationships

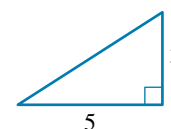
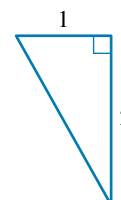
[Stage 5.2]

11.5.1 The gradient–intercept method

- To use this method, the gradient and the y-intercept must be known.
- The line is drawn by plotting the y-intercept, then drawing a line with the correct gradient through that point.

Note:

- A line interval of gradient $3 \left(= \frac{3}{1} \right)$ can be drawn with a rise of 3 and a run of 1.
- Similarly, a line interval with a gradient of $-2 \left(= \frac{-2}{1} \right)$ can be shown as an interval sloping downwards.
- A line interval with a gradient of $\frac{3}{5}$ can be shown with rise = 3 and run = 5.



WORKED EXAMPLE 11

Using the gradient–intercept method, sketch the graphs of:

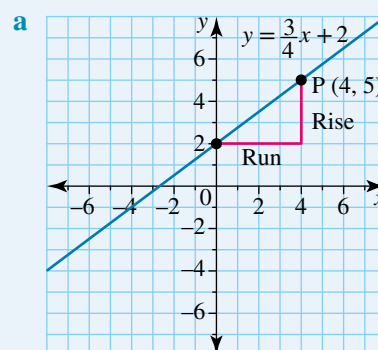
a $y = \frac{3}{4}x + 2$

b $4x + 2y = 3$.

THINK

- a 1 From the equation, the y-intercept is 2.
Plot the point (0, 2).

WRITE/DRAW



2 From the equation, the gradient is $\frac{3}{4}$, so $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$.

From $(0, 2)$, run 4 units and rise 3 units.

Mark the point P $(4, 5)$.

3 Draw a line through $(0, 2)$ and P $(4, 5)$.

Label the graph.

b 1 Write the rule in gradient–intercept form: $y = mx + b$.

From the equation, $m = -2$, $b = \frac{3}{2}$.

Plot the point $(0, \frac{3}{2})$.

2 The gradient is -2 , so $\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$.

From $(0, \frac{3}{2})$, run 1 unit and rise -2 units

(i.e. go down 2 units). Mark the point P $(1, -\frac{1}{2})$.

3 Draw a line through $(0, \frac{3}{2})$ and P $(1, -\frac{1}{2})$.

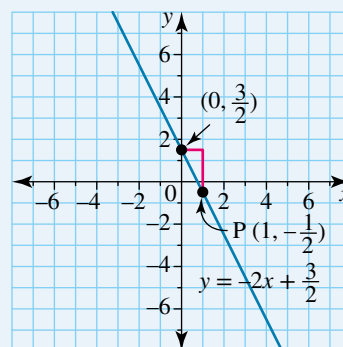
Label the graph.

$$b \quad 4x + 2y = 3$$

$$2y = 3 - 4x$$

$$y = \frac{3}{2} - 2x$$

$$y = -2x + \frac{3}{2}$$

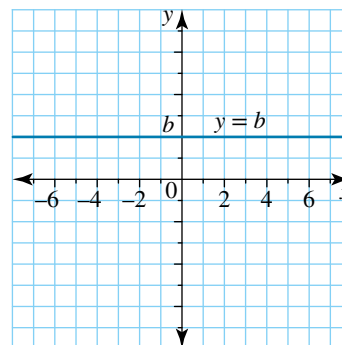


11.5.2 Horizontal and vertical lines

Horizontal lines: $y = b$

- $y = b$ is the same as $y = 0x + b$.
- This is a line with gradient 0 and y-intercept b .
- As a fraction, $0 = \frac{0}{3}, \frac{0}{4}$ and so on. Therefore, a line with gradient of 0 has a rise of 0 and a run of any length except 0. This is a horizontal line.
- Using a table to find points on the line $y = c$ gives:

x	-2	0	2	4
y	b	b	b	b

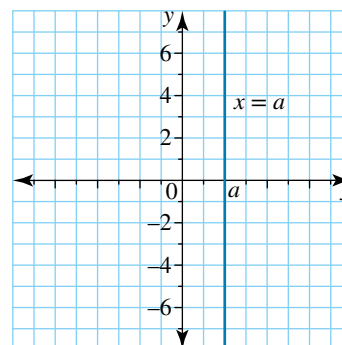


Vertical lines: $x = a$

- The equation $x = a$ states that x has the same value as a , regardless of what value y may take.
- A table of values looks like this:

x	a	a	a	a
y	-2	0	2	4

- Plotting these points gives a vertical line as shown.
- The run of the graph is 0, so using the formula $m = \frac{\text{rise}}{\text{run}}$ involves dividing by zero, which cannot be done. The gradient is said to be **undefined**.



DISCUSSION

Why is it impossible to divide a value by zero?

WORKED EXAMPLE 12

a On a pair of axes, sketch the graphs of:

- i $x = -3$ ii $y = 4$.

b Label the point of intersection of the two lines.

THINK

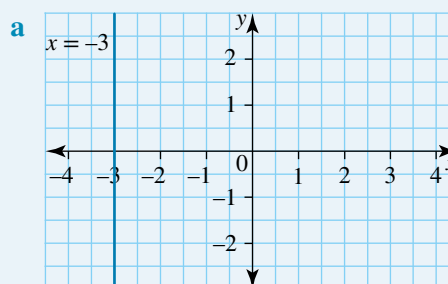
a i 1 The line $x = -3$ is in the form $x = a$.

This is a vertical line.

2 Rule the vertical line where $x = -3$.

Label the graph.

WRITE/DRAW



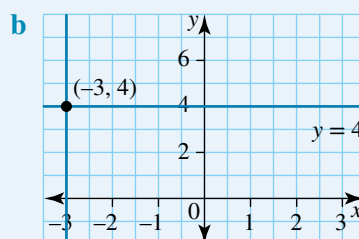
ii 1 The line $y = 4$ is in the form $y = b$.

This is a horizontal line.

2 Rule the horizontal line where $y = 4$.

Label the graph.

b The lines intersect at $(-3, 4)$.



learn on RESOURCES — ONLINE ONLY

Interactivity: Drawing a graph (int-1020)

Interactivity: The gradient–intercept method (int-3840)

Interactivity: Vertical and horizontal lines (int-6049)

eLesson: Sketching linear graphs using the gradient–intercept method (eles-1920)

Exercise 11.5 Using the gradient–intercept form to graph linear relationships

Individual pathways

PRACTISE

Questions:
1–9, 13

CONSOLIDATE

Questions:
1–11, 13, 14

MASTER

Questions:
1–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE11** Sketch the graphs of the following using the gradient–intercept method.

a. $y = x - 7$	b. $y = 2x + 1$	c. $y = 2x + 2$	d. $y = -2x + 2$
e. $y = \frac{1}{2}x - 1$	f. $y = 4 - x$	g. $y = \frac{5}{4}x + 5$	h. $y = -x - 10$
2. **WE12** Sketch the graphs of the following.

a. $y = 4$	b. $y = -3$	c. $y = -12.5$	d. $y = \frac{4}{5}$
3. Sketch the graphs of the following.

a. $x = 2$	b. $x = -6$	c. $x = -2.5$	d. $x = \frac{3}{4}$
4. Sketch the graphs of the following.

a. $y = 3x$	b. $y = -2x$	c. $y = \frac{3}{4}x$	d. $y = -\frac{1}{3}x$
5. **MC** a. Which of the following statements about the rule $y = 4$ is not true?
 - A. The gradient $m = 0$.
 - B. The y -intercept is at $(0, 4)$.
 - C. The graph is parallel to the x -axis.
 - D. The point $(4, 2)$ lies on this graph.
 - E. The graph has no x -intercept.
 b. Which of the following statements is not true about the rule $y = -\frac{3}{5}x$?
 - A. The graph passes through the origin.
 - B. The gradient $m = -\frac{3}{5}$.
 - C. The x -intercept is at $x = 0$.
 - D. The graph cannot be sketched using the gradient–intercept method.
 - E. The gradient is negative.
6. a. $2x + 5y = 20$ is a linear equation in the form $px + qy = c$.
 - i. Rearrange this equation into the form $y = mx + b$.
 - ii. What is the gradient?
 - iii. State the x - and y -intercepts.
 - iv. Sketch this straight line.
 b. If the x -intercept of a straight line is -3 and the y -intercept is 5 :
 - i. state the gradient
 - ii. draw the graph
 - iii. write the equation in the form $y = mx + b$ and the form $px + qy = c$.

Communicating, reasoning and problem solving

7. Consider the relationship $4x - 3y = 24$.
 - a. Rewrite this relationship, making y the subject.
 - b. Show that the x - and y -intercepts are $(6, 0)$ and $(0, -8)$ respectively.
 - c. Sketch a graph of this relationship.
8. a. Rewrite the relationship $px + qy = c$, making y the subject.
 b. If p , q and c are positive integer values, explain how the gradient is negative.
9. Josie accidentally spilled a drink on her work. Part of her calculations were smudged. The line $y = \frac{1}{2}x + \frac{3}{4}$ was written in the form $ax + 4y = 3$. Show that the value of $a = -2$.
10. Explain why the descriptions ‘right 3 up 2’, ‘right 6 up 4’, ‘left 3 down 2’, ‘right $\frac{3}{2}$ up 1’ and ‘left 1 down $\frac{2}{3}$ ’ all describe the same gradient for a straight line.
11. Find the slope of the line containing the points $(3, 5)$ and $(6, 11)$. Find coordinates for another point that lies on the same line. Explain the method used to find the coordinate and gradient.

12. a. Match the descriptions given below with their corresponding lines.

- Straight line with a y-intercept of (0, 1) and a positive gradient
- Straight line with a gradient of $1\frac{1}{2}$
- Straight line with a gradient of -1

b. Write a description for the unmatched graph.

13. a. Sketch the linear equation $y = -\frac{5}{7}x - \frac{3}{4}$:

- using the y-intercept and the gradient
- using the x- and y-intercepts
- using two other points.

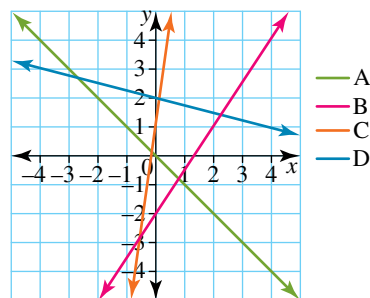
b. Compare and contrast the methods. Generate a list of advantages and disadvantages for each method. Which method do you think is best? Why?

14. Consider these two linear graphs:

$$y - ax = b \text{ and } y - cx = d.$$

Show that if these two graphs intersect where both x and y are positive, then $a > c$ when $d > b$.

15. Why are gradients of vertical lines undefined?



11.6 Parallel and perpendicular lines [Stages 5.1 and 5.2]

11.6.1 Graphing with technology

- There are many digital technologies that can be used to graph linear relationships. Digital technologies can be very useful when you want to draw multiple graphs in order to investigate important features.

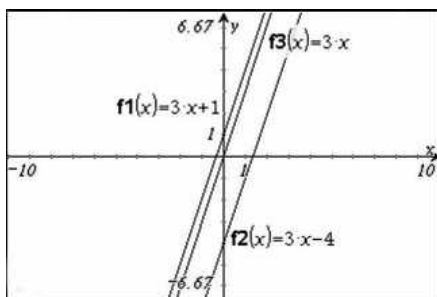
11.6.2 Parallel lines

- Lines with the same gradient are called **parallel lines**.

That is, $m_1 = m_2$.

For example, $y = 3x + 1$, $y = 3x - 4$ and $y = 3x$ are all parallel lines, because $m = 3$.

Note: This technology uses $f1(x)$, $f2(x)$ etc. in place of y .

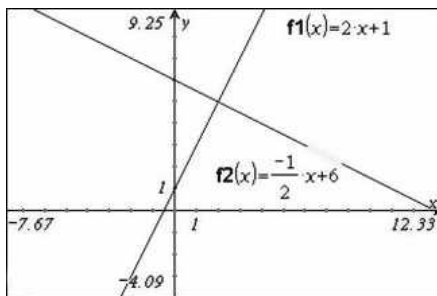


11.6.3 Perpendicular lines

- Lines that meet at right angles are called **perpendicular lines**.
- The product of the gradients of two perpendicular lines is equal to -1 .

That is, $m_1 \times m_2 = -1$.

For example, $y = 2x + 1$ and $y = -\frac{1}{2}x + 6$ are perpendicular, because $2 \times -\frac{1}{2} = -1$.



Exercise 11.6 Parallel and perpendicular lines

Individual pathways

PRACTISE

Questions:
1–11, 15

CONSOLIDATE

Questions:
1–12, 14, 15

MASTER

Questions:
1–16

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

Use technology wherever possible to answer the following questions.

1. On the same Cartesian plane, sketch the following graphs.

a. $y = x$ **b.** $y = 2x$ **c.** $y = 3x$

The steepness of a graph is called the gradient of the line.

- What happens to the steepness of the graph as the coefficient of x increases in value?
- Where does each graph cut the x -axis? (That is, what is the x -intercept?)
- Where does each graph cut the y -axis? (That is, what is the y -intercept?)

2. On another Cartesian plane, sketch the following graphs.

a. $y = -x$ **b.** $y = -2x$ **c.** $y = -3x$

- What happens to the steepness of the graph as the magnitude of the coefficient of x decreases in value? (That is, the coefficient becomes more negative.)

- Where does each graph cut the x -axis? (That is, what is the x -intercept?)

- Where does each graph cut the y -axis? (That is, what is the y -intercept?)

3. Copy the following sentences, deleting the incorrect responses.

- If the coefficient of x is **(positive/negative)**, then the graph will have an upward slope to the right. That is, the gradient of the graph is **(positive/negative)**.
- If the coefficient of x is negative, then the graph will have a **(downward/upward)** slope to the right. That is, the gradient of the graph is **(positive/negative)**.
- The bigger the magnitude of the coefficient of x (more positive or more negative), the **(bigger/smaller)** the steepness of the graph.
- If there is no constant term in the equation, the graph **(will/will not)** pass through the origin.

4. On another Cartesian plane, sketch the following graphs.

a. $y = x$ **b.** $y = x + 2$ **c.** $y = x - 2$

- Is the coefficient of x the same for each graph? If so, what is it?
- Does the steepness (or gradient) of each graph differ?
- Where does each graph cut the x -axis? (That is, what is the x -intercept?)
- Where does each graph cut the y -axis? (That is, what is the y -intercept?)

5. On another Cartesian plane, sketch the following graphs.

a. $y = -x$ **b.** $y = -x + 2$ **c.** $y = -x - 2$

- Is the coefficient of x the same for each graph? If so, what is it?
- Does the steepness (or gradient) of each graph differ?

- iii. Where does each graph cut the x -axis? (That is, what is the x -intercept?)
 - iv. Where does each graph cut the y -axis? (That is, what is the y -intercept?)
6. Copy the following sentences, deleting the incorrect responses.
- a. For a given set of linear graphs, if the coefficient of x is the **(same/different)**, the graphs will be parallel.
 - b. The constant term in the equation is the **(y-intercept/x-intercept)** or where the graph cuts the **(y-axis/x-axis)**.
7. On another Cartesian plane, sketch the following graphs.
- a. $y = x + 5$
 - b. $y = -x + 5$
- i. Is the coefficient of x the same for each graph?
 - ii. What do you notice about the intersection of the graphs?
 - iii. Write down the gradient of each linear graph.
 - iv. What is the product of the gradients of the two graphs?
8. On another Cartesian plane, sketch the following graphs.
- a. $y = 3x + 5$
 - b. $y = -\frac{1}{3}x + 5$
- i. Is the coefficient of x the same for each graph?
 - ii. What do you notice about the intersection of the graphs?
 - iii. Write down the gradient of each linear graph.
 - iv. What is the product of the gradients of the two graphs?
9. Copy the following sentences, deleting the incorrect responses.
- One of the general forms of the equation of a linear graph is $y = mx + c$, where m is the **(steepness/x-coordinate)** of the graph. We call the steepness of the graph the gradient.
- When the **(sum/product)** of the gradients of two linear graphs is -1 , then the graphs are **(parallel/perpendicular)**.
10. For each of the following lines, write:
- i. the gradient
 - ii. the y -intercept.
- a. $y = 2x$
 - b. $y = x + 1$
 - c. $y = -3x + 5$
 - d. $y = \frac{2}{3}x - 7$
- Communicating, reasoning and problem solving**
11. Using graphing software, graph the lines $y = 3(x - 1) + 5$, $y = 2(x - 1) + 5$ and $y = \frac{1}{2}(x - 1) + 5$.
What do they all have in common? Explain how they differ from each other.
12. Shirley walks dogs after school for extra pocket money. She finds that she can use the equation $P = -15 + 10N$ to calculate her profit (in dollars) each week.
- a. Explain the real-world meaning of the numbers -15 and 10 and the variable N .
 - b. Explain what the equation means.
13. Plot the points $(6, 3.5)$ and $(-1, -10.5)$ using graphing software and:
- a. find the equation of the line joining them
 - b. sketch the graph, showing x - and y -intercepts
 - c. find the value of y when $x = 8$
 - d. find the value of x when $y = 12$.
14. A school investigating the price of a site licence for their computer network found that it would cost \$1750 for 30 computers and \$2500 for 60 computers.
- a. Find a linear equation that represents the cost of a site licence in terms of the number of computers in the school.
 - b. What is the y -intercept of the linear equation and how does it relate to the cost of a site licence?
 - c. How much would it cost for 200 computers?
 - d. How many computers could you connect for \$3000?

15. Dylan starts his exercise routine by jogging to the gym, which burns 325 calories. He then pedals a stationary bike burning 3.8 calories a minute.
- Graph the data using graphing software.
 - After 15 minutes of pedalling, how many calories has Dylan burned?
 - How long did it take for Dylan to burn a total of 450 calories?
16. If two lines look like they intersect at right angles, can you assume that they are perpendicular?



11.7 Finding the equation of a straight line

[Stage 5.2]

11.7.1 Finding the equation of a straight line given the gradient and the y-intercept

- For a linear graph, if the gradient (m) is known and the y-intercept (b) is known, then the equation ($y = mx + b$) can be determined.

WORKED EXAMPLE 13

Determine the rule of the line whose gradient is -2 and y-intercept is 3 .

THINK

- Write the equation of a straight line.
- Substitute the values $m = -2$, $b = 3$.

WRITE

$$y = mx + b$$

$$m = -2, b = 3$$

$$y = -2x + 3$$

11.7.2 Finding the rule given the gradient and the coordinates of one point

- If the gradient (m) and any one point are known, then the y-intercept can be calculated.

WORKED EXAMPLE 14

Determine the rule of a straight line that goes through the point $(1, -3)$ if its gradient is -2 .

THINK

- Write the equation of a straight line.
- Substitute the value $m = -2$.
- Since the line passes through the point $(1, -3)$, substitute $x = 1$ and $y = -3$ into $y = -2x + b$ to find the value of b .

WRITE

$$y = mx + b$$

$$y = -2x + b$$

$$\text{When } x = 1, y = -3.$$

$$-3 = -2 \times 1 + b$$

4 Solve for b .

$$\begin{aligned}-3 &= -2 + b \\ -3 + 2 &= b \\ b &= -1\end{aligned}$$

5 Write the rule.

$$y = -2x - 1$$

11.7.3 Finding the rule given the coordinates of two points

- If two points on the line are known, then the gradient (m) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ or } m = \frac{\text{rise}}{\text{run}}.$$

- Using the gradient and one of the points, the equation can be found as above.

WORKED EXAMPLE 15

Find the rule of the straight line passing through the points $(-1, 6)$ and $(3, -2)$.

THINK

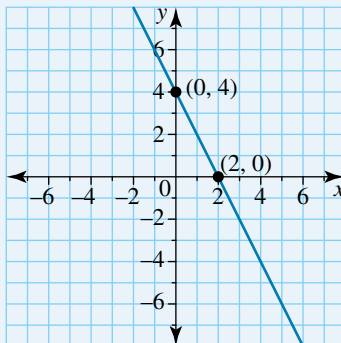
- 1 Write the equation of a straight line.
- 2 Write the formula for finding the gradient, m .
- 3 Let $(x_1, y_1) = (-1, 6)$ and $(x_2, y_2) = (3, -2)$.
Substitute the values into the formula and determine the value of m .
- 4 Substitute the value of m into the equation.
- 5 Select either of the two points, say $(3, -2)$, and substitute into $y = -2x + b$.
- 6 Solve for b .
- 7 Write the rule using the values $b = 4$, $m = -2$.

WRITE

$$\begin{aligned}y &= mx + b \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{-2 - 6}{3 - -1} \\ &= \frac{-2 - 6}{3 + 1} \\ &= -\frac{8}{4} \\ &= -2 \\ y &= -2x + b \\ \text{Point } (3, -2): & -2 = -2(3) + b \\ & -2 = -6 + b \\ & b = 4 \\ y &= -2x + 4\end{aligned}$$

WORKED EXAMPLE 16

Find the equation of the linear graph shown.



THINK

- 1 Read the important information from the graph. The y-intercept is 4 and the x-intercept is 2. Write the coordinates of each point.
- 2 Find the gradient using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (0, 4)$.

- 3 From the graph, the y-intercept is 4.
- 4 Write the equation.

WRITE

The graph passes through $(0, 4)$ and $(2, 0)$.

$$m = \frac{4 - 0}{0 - 2}$$

$$= \frac{4}{-2}$$

$$y = -2x + b$$

$$b = 4$$

$$y = -2x + 4$$

learnon

RESOURCES — ONLINE ONLY

eLesson: The equation of a straight line (eles-2313)

Digital doc: WorkSHEET Linear rules (doc-6172)

Exercise 11.7 Finding the equation of a straight line

Individual pathways

PRACTISE

Questions:

1a–c, 2, 3a–d, 4, 5a–f, 6a–d,
7a–c, 8a–c, 9–13, 16, 17

CONSOLIDATE

Questions:

1c–e, 2, 3e–h, 4, 5d–i, 6b–e,
7c–e, 8c–e, 9–14, 16–18

MASTER

Questions:

1d–f, 2, 3g–j, 4, 5g–i, 6d–f, 7d–f,
8d–f, 9, 10, 12–15, 18–21

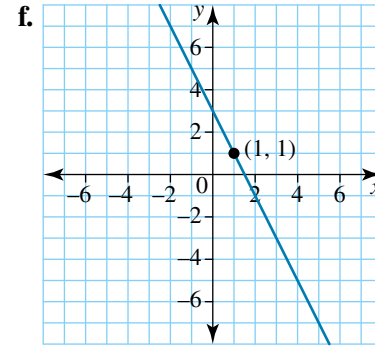
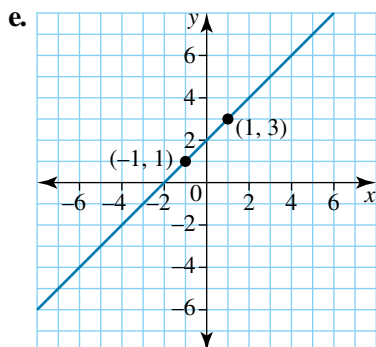
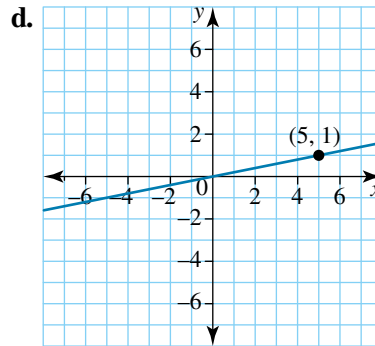
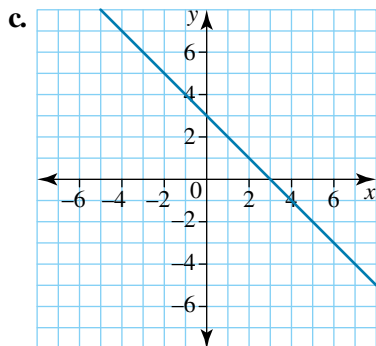
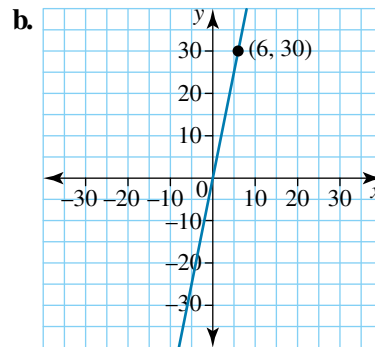
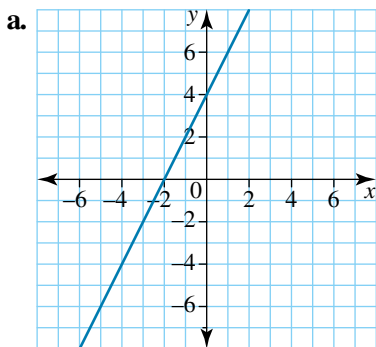
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

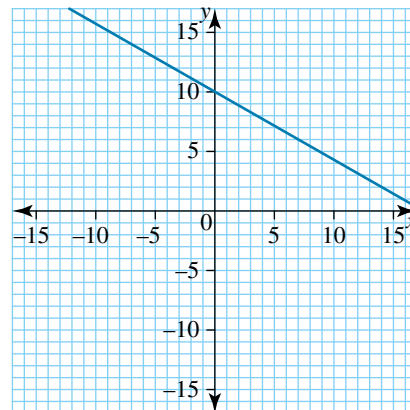
1. **WE13** Determine the rules of the straight lines with the gradients and y-intercepts given below.
 - a. Gradient = 4, y-intercept = 2
 - b. Gradient = -4, y-intercept = 1
 - c. Gradient = 4, y-intercept = 8
 - d. Gradient = 6, y-intercept = 7
 - e. Gradient = -2.5, y-intercept = 6
 - f. Gradient = 45, y-intercept = 135
2. Find the rule for each straight line passing through the origin and with the gradient given.
 - a. Gradient = -2
 - b. Gradient of 4
 - c. Gradient = 10.5
 - d. Gradient of -20
 - e. Gradient = 1.07
 - f. Gradient of 32
3. **WE14** Determine the rule for each straight line with the gradient and passing through the point given below.
 - a. Gradient = 1, point = (3, 5)
 - b. Gradient = -1, point = (3, 5)
 - c. Gradient = -4, point = (-3, 4)
 - d. Gradient = 2, point = (5, -3)
 - e. Gradient = -5, point = (13, 5)
 - f. Gradient = 2, point = (10, -3)
 - g. Gradient = -6, point = (2, -1)
 - h. Gradient = -1, point = (-2, 0.5)
 - i. Gradient = 6, point = (-6, -6)
 - j. Gradient = -3.5, point = (3, 5)
 - k. Gradient = 1.2, point = (2.4, -1.2)
 - l. Gradient = 0.2, point = (1.3, -1.5)
4. Determine the rules when:
 - a. gradient = -4 and x-intercept = -6
 - b. gradient = 2 and x-intercept = 3

- c. gradient = -2 and x -intercept = 2
 e. gradient = 1.5 and x -intercept = 2.5
5. **WE15** Find the rule for each straight line passing through the given points.
- a. $(-6, 11)$ and $(6, 23)$
 c. $(4, 11)$ and $(6, 11)$
 e. $(1.5, 2)$ and $(6, -2.5)$
 g. $(25, -60)$ and $(10, 30)$
 i. $(1, 3)$ and $(3, 1)$
 k. $(9, -2)$ and $(2, -4)$
6. Determine the rules for the linear graphs that have the following x - and y -intercepts.
- a. x -intercept = -3 , y -intercept = 3
 c. x -intercept = 1 , y -intercept = 6
 e. x -intercept = -8 , y -intercept = 8
 g. x -intercept = -7 , y -intercept = -3
7. Find the rule for the straight line passing through the origin and:
- a. the point $(4, 7)$
 c. the point $(-4, 8)$
 e. the point $(-22, 48)$
- b. gradient = 5 and x -intercept = -7
 f. gradient = 0.4 and x -intercept = 2.4 .
- b. $(1, 2)$ and $(-5, 8)$
 d. $(3, 6.5)$ and $(6.5, 10)$
 f. $(-7, 3)$ and $(2, 4)$
 h. $(5, 100)$ and $(25, 500)$
 j. $(2, 5)$ and $(-2, 6)$
 l. $(1, 4)$ and $(-0.5, 3)$
- b. x -intercept = 4 , y -intercept = 5
 d. x -intercept = -40 , y -intercept = 35
 f. x -intercept = 3 , y -intercept = 6
 h. x -intercept = -200 , y -intercept = 50
- b. the point $(5, 5)$
 d. the point $(-1.2, 3.6)$
 f. the point $(-105, 35)$.

8. **WE16** Find the equation of the line shown on each of the following graphs.

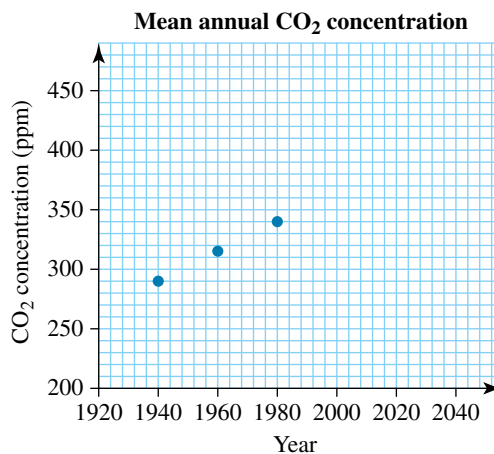


9. **MC** a. The gradient of the straight line that passes through (3, 5) and (5, 3) is:
A. -2 **B.** -1 **C.** 0 **D.** 1 **E.** 2
- b. A straight line with an x -intercept of 10 and a y -intercept of 20 has a gradient of:
A. -2 **B.** -1 **C.** -0.5 **D.** 0 **E.** 5
- c. The rule $2y - 3x = 20$ has an x -intercept of:
A. $-\frac{3}{2}$ **B.** $-\frac{2}{3}$ **C.** 0
D. 10 **E.** None of the above
10. Given that the x -intercept of a straight line graph is $(-5, 0)$ and the y -intercept is $(0, -12)$:
a. determine the equation of the straight line
b. find the value of y when $x = 19.3$.
11. a. Determine the equation of the straight line shown in blue in the graph at right. Use the fact that when $x = 5$, $y = 7$.
b. Determine the x -intercept.

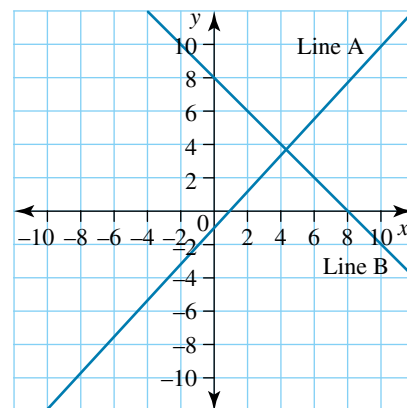


Communicating, reasoning and problem solving

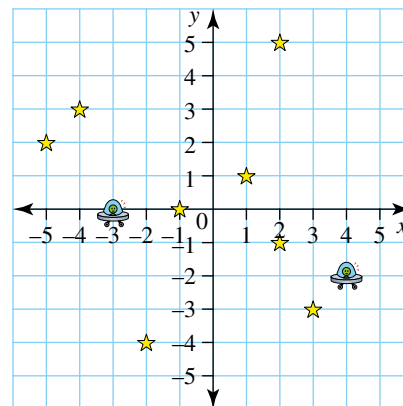
12. The graph shows the carbon dioxide (CO_2) concentration in the atmosphere, measured in parts per million (ppm).
a. If the trend follows a linear pattern, find the equation for the line.
b. Explain why b cannot be read directly from the graph.
c. What might be the concentration of CO_2 in 2020? What is the assumption that is made when finding this value?



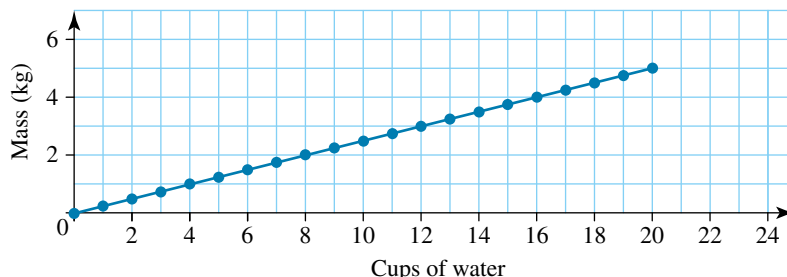
13. Show that the equation for the line that passes through the point (3, 6) parallel to the line through the points (0, -7) and (4, -15) is $y = -2x + 12$.
14. a. Determine the equations for line A and line B as shown.
b. Write the point of intersection between line A and line B and mark it on the Cartesian plane.
c. Show that the equation of the line that is perpendicular to line B and passes through the point $(-4, 6)$ is $y = x + 10$.



15. A student plays the game Space Galaxy. She has the following Stars and Spaceships on her screen.
- Copy the diagram into your workbook. On the diagram, draw a straight line that will hit 3 stars.
 - What is the equation of the straight line that will hit 3 stars?
 - The student types in the equation $y = \frac{1}{2}x + \frac{1}{2}$ and manages to hit 2 stars. Draw the straight line on the screen.
 - If the student was to type in the equation from part **b** and the equation from part **c**, what is the coordinate of the star that both lines would hit?
 - If a student types $y = 2$, how many stars will they hit?
 - What is another equation of a straight line that will hit 2 stars?

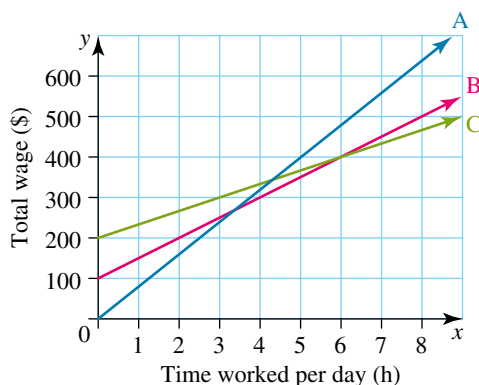


16. The graph shown describes the mass in kilograms of metric cups of water.



Write a rule to describe the mass of water relative to the number of cups.

17. The temperature of water in a kettle is 15°C before the temperature increases at a constant rate for 20 seconds to reach boiling point (100°C). A classmate argues that $T = 5t + 15$ describes the water temperature, citing the starting temperature of 15°C and that to reach 100°C in 20 seconds an increase of 5°C for every second is required. Explain why the equation is incorrect and devise another equation that correctly describes the temperature of the water.
18. A father wants to administer Children's Panadol[®] to his child. The recommended dosage is a range, 7.5–9 mL for an average weight of 12–14 kg. The child weighs 12.8 kg. The father uses a linear relationship to calculate an exact dosage. What dosage does the father calculate?
19. The graph shows the wages earned in three different remote locations.



- What is the set allowance for each location?
- Determine the hourly rates for each location.
- Using your answers from parts **a** and **b**, determine linear equations that describe the wages at each location.

- d. Match each working lifestyle below to the most appropriate location.
- Working lifestyle 1: Earn the most money possible while working at most 4 hours in a day
 - Working lifestyle 2: Earn the most money possible while working an 8-hour day
- e. If a person works on average 8 hours a day, what is the advantage of location C?
- f. How much money is earned at each location for:
- a 2-hour day
 - a 6-hour day?
20. Why are only 2 points needed to find the rule for the line that passes through the points?
21. A certain linear pattern has its Cartesian coordinates in fraction form as (x, y) , $(\frac{1}{6}, \frac{1}{4})$, $(\frac{1}{3}, \frac{1}{6})$, $(\frac{1}{2}, \frac{1}{12})$, $(\frac{2}{3}, 0)$. Find the rule that satisfies these Cartesian coordinates.

11.8 Practical applications of linear graphs

[Going further]

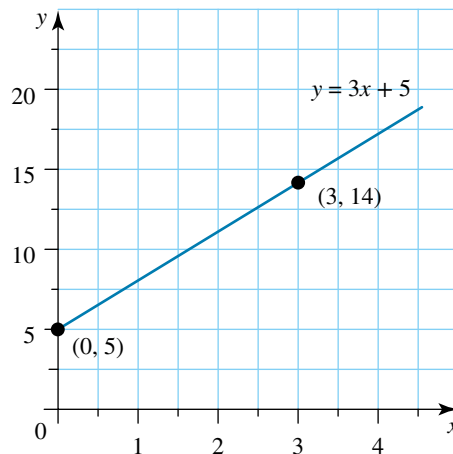
11.8.1 Finding a linear rule from a table of values

- If two variables are linked by a linear rule, then as one variable increases, the other increases (or decreases) at a steady rate.
- Here is an example of a linear relationship.

x	0	1	2	3
y	5	8	11	14

Each time x increases by 1, y increases by 3.

The linear rule connecting x and y is $y = 3x + 5$.

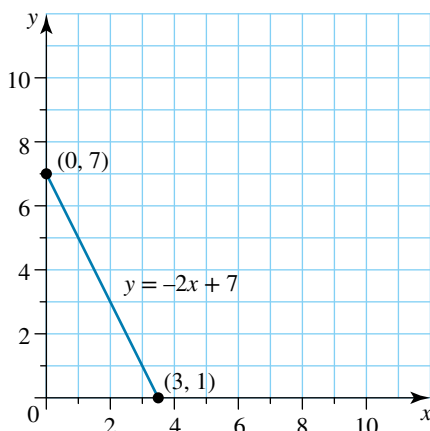


- Consider this relationship.

x	0	1	2	3
y	7	5	3	1

- Each time x increases by 1, y decreases by 2.

- The linear rule in this case is $y = -2x + 7$.



WORKED EXAMPLE 17

Find the rule connecting x and y in each of the following value tables.

a

x	0	1	2	3
y	-3	2	7	12

b

x	3	4	5	6
y	12	11	10	9

THINK

a 1 y increases at a steady rate, so this is a linear relationship.
Write the rule.

2 Find m : y increases by 5 each time x increases by 1.
Write the value of the gradient.

3 From the table, when $x = 0$, $y = -3$.
Write the value of the y -intercept.

4 Write the rule.

b 1 y decreases at a steady rate, so this is a linear relationship.

2 y decreases by 1 each time x increases by 1. Write the value of the gradient.

3 Find b : $y = 12$ when $x = 3$.
To find the y -intercept, substitute the x - and y -values of one of the points, and solve for b .

4 Write the rule.

WRITE

a $y = mx + b$

$$m = 5$$

$$b = -3$$

$$y = 5x - 3$$

b $y = mx + b$

$$m = -1$$

$$\begin{aligned} y &= -x + b \\ (3, 12): 12 &= -(3) + b \\ 12 &= -3 + b \\ b &= 15 \end{aligned}$$

$$y = -x + 15$$

11.8.2 Modelling linear relationships

- Relationships between real-life variables are often modelled (described) by a mathematical equation. In other words, an equation or formula is used to link the two variables.
- For example,
 - $A = l^2$ represents the relationship between the area and the side length of a square;
 - $C = \pi d$ represents the relationship between the circumference and the diameter of a circle.
- If one variable changes at a constant rate compared to the other, then the two variables have a linear relationship.
- Many relationships are linear, which means they can be modelled by the equation $y = mx + b$.

WORKED EXAMPLE 18

An online bookstore sells a certain book for \$21 and charges \$10 for the delivery of any number of books.

- Find the rule connecting the cost (\$ C) with the number of books delivered (n).
- Use the rule to find the cost of delivering 35 books.
- How many books can be delivered for \$1000?

THINK

- Set up a table.

$$\text{Cost for 1 book} = 21 + 10$$

$$\begin{aligned}\text{Cost for 2 books} &= 2(21) + 10 \\ &= 52\end{aligned}$$

- The cost rises steadily, so there is a linear relationship. Write the rule.

- To find the gradient, use the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ with the points } (1, 31) \text{ and } (2, 52).$$

- To find the value of b , substitute $C = 31$ and $n = 1$.

- Write the rule.

- Substitute $n = 35$ and find C .

- Write the answer.

- Substitute $C = 1000$ and find n .

- You cannot buy 47.14 books, so round down. Write the answer.

WRITE

n	1	2	3
C	31	52	73

$$C = mn + b$$

$$\begin{aligned}m &= \frac{52 - 31}{2 - 1} \\ &= 21\end{aligned}$$

$$C = 21n + b$$

$$\begin{aligned}(1, 31): \quad 31 &= 21(1) + b \\ b &= 10\end{aligned}$$

$$C = 21n + 10$$

- $$\begin{aligned}C &= 21(35) + 10 \\ &= 735 + 10 \\ &= 745\end{aligned}$$

The cost including delivery for 35 books is \$745.

- $$\begin{aligned}1000 &= 21n + 10 \\ 21n &= 990 \\ n &= 47.14\end{aligned}$$

For \$1000, 47 books can be bought and delivered.

- In Worked example 18, compare the rule $C = 21n + 10$ with the original question. It is clear that the $21n$ refers to the cost of the books (a variable amount, depending on the number of books) and that 10 refers to the fixed (constant) delivery charge.
- In this case C is called the **dependent variable**, because it depends on the number of books (n).
- The variable n is called the **independent variable**.

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Interactivity: Dependent and independent variables (int-6050)

Exercise 11.8 Practical applications of linear graphs

Individual pathways

PRACTISE

Questions:

1a–d, 2–4, 7, 8, 10, 11

CONSOLIDATE

Questions:

1e–h, 2, 4, 5, 7, 9–12, 14

MASTER

Questions:

1g–i, 2, 4, 6, 8–16

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE17** Find the linear rule linking the variables in each of the following tables.

a.

x	0	1	2	3
y	–5	1	7	13

c.

x	0	1	2	3
y	4	6	8	10

e.

x	2	3	4	5
y	7	10	13	16

g.

n	0	1	2	3
C	5	9	13	17

i.

t	3	4	5	6
v	18	15	12	9

b.

x	0	1	2	3
y	8	5	2	–1

d.

x	0	1	2	3
y	1.1	2.0	2.9	3.8

f.

x	5	6	7	8
y	12	11	10	9

h.

t	0	1	2	3
n	0	5	10	15

j.

d	1	2	3	4
C	11	14	17	20

2. **MC** Greg and Tanya hire a car. They are charged a fixed fee of \$150 for hiring the car and then \$25 per day. They hire the car for d days. Which one of the following could describe the rule that describes the number of days the car is hired and the total cost, C , they are charged?

A. $C = 25d$

B. $C = 150d$

C. $C = 175d$

D. $C = 25d + 150$

E. $C = 150d + 25$

3. **WE18** Nathan's bank balance has increased in a linear manner since he started his part-time job. If after 20 weeks of work his bank balance was at \$560 and after 21 weeks of work it was at \$585, find:

- the rule that relates the size of his bank balance, A , and the time (in weeks) worked, t
- the amount in his account after 200 weeks
- the initial amount.

4. The cost of producing a shoe increases as the size of the shoe increases. It costs \$5.30 to produce a size 6 shoe, and \$6.40 to produce a size 8 shoe. Assuming that a linear relationship exists:

- determine the rule relating cost (C) to shoe size (s).
- how much does it cost to produce a size 12 shoe?

5. The number of publications in a library (N) increased steadily with time (t). After 10 years there were 7200 publications in the library, and after 12 years there were 8000 publications.



- a. Determine the rule predicting the number of publications in the library.
 - b. How many publications were there after 5.5 years?
 - c. How many publications will there be after 25 years?
6. The Robinsons' water tank sprang a leak and has been losing water at a steady rate. Four days after the leak occurred, the tank contained 552 L of water, and ten days later it held only 312 L.
- a. Find the rule linking the amount of water in the tank (w) and the number of days (t) since the leak occurred.
 - b. How much water was in the tank initially?
 - c. If water loss continues at the same rate, when will the tank be empty?



Communicating, reasoning and problem solving

7. A software company claims that its staff can fix 22 bugs per month. They are working to fix a program that initially had 164 bugs.
- a. Determine the linear rule connecting the number of bugs left, N , and the time in months, t , from the beginning of eradication.
 - b. How many bugs will there be left after 2 months?
 - c. After how many months will there be 54 bugs left?
 - d. How long will it take this company to eradicate all bugs? Justify your answer.
8. A skyscraper can be built at a rate of 4.5 storeys per month.
- a. How many storeys will be built after 6 months? Justify your answer.
 - b. How many storeys will be built after 24 months? Justify your answer.
9. The cost of a taxi ride is \$3.50 flag fall plus \$2.14 for each kilometre travelled.
- a. Determine the linear rule connecting the cost, C , and the distance travelled, d .
 - b. How much will an 11.5-km trip cost?
 - c. How much will a 23.1-km trip cost? Justify your answer.
10. After 11 pm the taxi company charges \$3.50 flag fall plus \$2.57 for each kilometre travelled.
- a. Determine the linear rule connecting the cost, C , and the distance travelled, d .
 - b. How much will an 11.5-km trip cost with this taxi?
 - c. If I have \$22 in my pocket, how far can I travel, correct to 1 decimal place? Justify your answer.
11. A certain kind of eucalyptus tree grows at a linear rate for its first 2 years of growth. If the growth rate is 5 cm per month, show that the tree will be 1.07 m tall after 21.4 months.



12. The pressure inside a boiler increases steadily as the temperature increases. For each 1°C , the pressure increases by 10 units, and at a temperature of 100°C the pressure is 1200 units. If the maximum pressure allowed is 2000 units, show that the temperature cannot exceed 180°C .
13. Michael produces and sells prints of his art at a local gallery. For each print run his profit (P) is given by the equation $P = 200n - 800$, where n is the number of prints sold.
 - a. Sketch the graph of this rule.
 - b. What is the y -intercept? What does this mean in this example?
 - c. What is the x -intercept? What does this mean in this example?
 - d. What is the gradient of the graph? What does this mean in this example?
14. Cheng lives in Australia and he is going on holiday to Japan. One yen (¥) buys 0.0127 Australian dollars (A\$).
 - a. Write an equation that converts Australian dollars to Japanese yen, where A represents amount of Australian dollars and Y represents amount of yen.
 - b. Using the equation from part a, how many yen (¥) will Cheng receive if he has A\$2500?
 - c. There is a commission to be paid on exchanging currency. Cheng needs to pay 2.8% for each Australian dollar that is exchanged into yen. Write down an equation that calculates the total amount of yen Cheng will receive. Write your equation in terms of Y_T , total amount of yen, and Australian dollars.
15. Sam and Cody need to make a journey to the other branch of their store across town. The traffic is very busy at this time of the day, so Sam catches the train that travels halfway and then walks the rest of the way. Cody travels by bike the whole way. The bike path travels along the train line and then along the roadway to the other branch of their store. The bike's speed was twice walking speed and the train's speed was four times the bike's speed. Who arrives at the destination first?
16. How are the dependent and independent variables determined?

11.9 Non-linear relationships (parabolas, exponentials, circles) [Stages 5.1 and 5.2]

11.9.1 Non-linear relationships

- There are many examples of non-linear relationships in mathematics. Some of them are the parabola, exponentials and the circle.
- Note:* This section introduces these non-linear relationships; parabolas will be explored in more depth in Topic 15.

11.9.2 The parabola

- The **parabola** is a curve that is found in many phenomena such as those shown in the following images.
- These images reveal a number of common features of parabolic shapes.



- They are all symmetrical. Specifically, a line of symmetry can be drawn in the middle of the parabola, such that each half is an exact reflection of the other.
- They have either a maximum (highest) point or a minimum (lowest point) — both are known as **vertices** (singular **vertex**) or turning points.
- All parabolas have the same basic shape; however, they can be wider or narrower depending on the equation.
- The most basic parabolic graph is produced by the equation:

$$y = x^2$$
- The graph of $y = x^2$ can be plotted using a table of values as shown in Worked example 19.



WORKED EXAMPLE 19

Plot the graph of $y = x^2$ for values of x from -3 to 3 . State the equation of the axis of symmetry and the coordinates of the vertex.

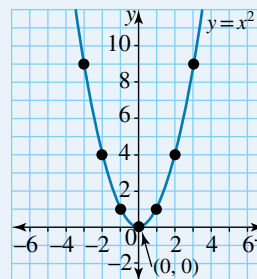
THINK

- 1 Write the equation.
- 2 Produce a table of values using x -values from -3 to 3 .
- 3 Draw a set of clearly labelled axes, plot the points and join them with a smooth curve. The scale on the y -axis would be from -2 to 10 and from -4 to 4 on the x -axis.
- 4 Write the equation of the line that divides the parabola exactly in half.
- 5 Write the coordinates of the vertex.

WRITE/DRAW

$$y = x^2$$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



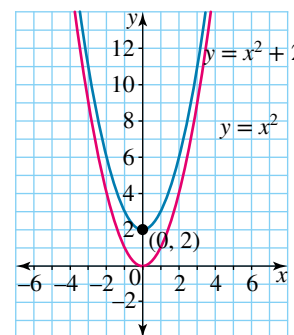
The equation of the axis of symmetry is $x = 0$.

The vertex is $(0, 0)$.

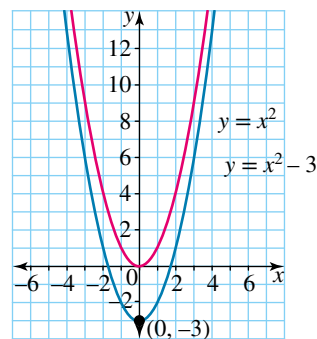
- A parabola can be sketched using its key features, such as its general shape, its axis of symmetry and its turning point or vertex.
- A parabola can be seen to undergo transformations by making changes to the equation $y = x^2$.
- Some transformations include:
 - vertical translations
 - horizontal translations
 - reflections.

11.9.3 Vertical translation

- Compare the graph of $y = x^2 + 2$ with that of $y = x^2$.
 - The whole graph has been moved or translated 2 units upwards.
 - The vertex has become $(0, 2)$.
 - The y -coordinate of the vertex has increased by 2 units to show that the graph has been moved up 2 units.



- Compare the graph of $y = x^2 - 3$ with that of $y = x^2$.
 - The whole graph has been moved or translated 3 units downwards.
 - The vertex has become $(0, -3)$.
 - The y -coordinate of the vertex has decreased by 3 units to show that the graph has been moved down 3 units.



WORKED EXAMPLE 20

State the vertical translation (when compared with the graph of $y = x^2$) and the coordinates of the vertex for the graphs of each of the following equations.

a $y = x^2 + 5$

b $y = x^2 - 4$

THINK

a 1 Write the equation.

2 $+5$ means the graph is translated upwards 5 units.

3 Translate the vertex of $y = x^2$, which is $(0, 0)$.
The x -coordinate of the vertex remains 0, and the y -coordinate has 5 added to it.

b 1 Write the equation.

2 -4 means the graph is translated downwards 4 units.

3 Translate the vertex of $y = x^2$, which is $(0, 0)$.
The x -coordinate of the vertex remains 0, and the y -coordinate has 4 subtracted from it.

WRITE

a $y = x^2 + 5$

Vertical translation of 5 units up

The vertex becomes $(0, 5)$.

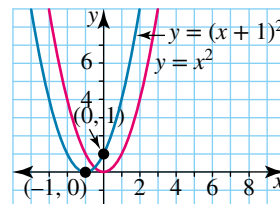
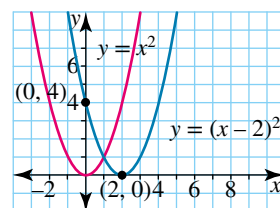
b $y = x^2 - 4$

Vertical translation of 4 units down

The vertex becomes $(0, -4)$.

11.9.4 Horizontal translation

- Compare the graph of $y = (x - 2)^2$ with that of $y = x^2$.
 - The whole graph has been moved or translated 2 units to the right.
 - The vertex has become $(2, 0)$.
 - The x -coordinate of the vertex has increased by 2 units to show that the graph has been moved 2 units to the right.
- Compare the graph of $y = (x + 1)^2$ with that of $y = x^2$.
 - The whole graph has been moved or translated 1 unit left.
 - The vertex has become $(-1, 0)$.
 - The x -coordinate of the vertex has decreased by 1 unit to show that the graph has been moved left 1 unit.



WORKED EXAMPLE 21

State the horizontal translation (when compared to the graph of $y = x^2$) and the coordinates of the vertex for the graphs of each of the following equations.

a $y = (x - 3)^2$

b $y = (x + 2)^2$

THINK

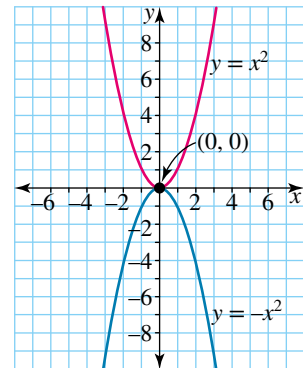
- a** 1 Write the equation.
- 2 -3 means the graph is translated to the right 3 units.
- 3 Translate the vertex of $y = x^2$, which is $(0, 0)$.
The y -coordinate of the vertex remains 0, and the x -coordinate has 3 added to it.
- b** 1 Write the equation.
- 2 $+2$ means the graph is translated to the left 2 units.
- 3 Translate the vertex of $y = x^2$, which is $(0, 0)$.
The y -coordinate of the vertex remains 0, and the x -coordinate has 2 subtracted from it.

WRITE

- a** $y = (x - 3)^2$
Horizontal translation of 3 units to the right
The vertex becomes $(3, 0)$.
- b** $y = (x + 2)^2$
Horizontal translation of 2 units to the left
The vertex becomes $(-2, 0)$.

11.9.5 Reflection

- Compare the graph of $y = -x^2$ with that of $y = x^2$.
 - In each case the axis of symmetry is the line $x = 0$ and the vertex is $(0, 0)$.
 - The only difference between the equations is the $-$ sign in $y = -x^2$. The difference between the graphs is that $y = x^2$ 'sits' on the x -axis and $y = -x^2$ 'hangs' from the x -axis. (One is a reflection or mirror image of the other).
 - $y = x^2$ has a minimum vertex and $y = -x^2$ has a maximum vertex.
- Any quadratic graph where x^2 is positive has a $\cup$ shape and is said to be concave up. Conversely, if x^2 is negative the graph has a $\cap$ shape and is said to be concave down.



11.9.6 The exponential

- An exponential is a non-linear graph whose equation is $y = a^x$, where a is a real number not equal to 1 and x is the power or exponent.

WORKED EXAMPLE 22

Complete the table of values below and use it to plot the graph of $y = 2^x$.

x	-3	-2	-1	0	1	2	3
y							

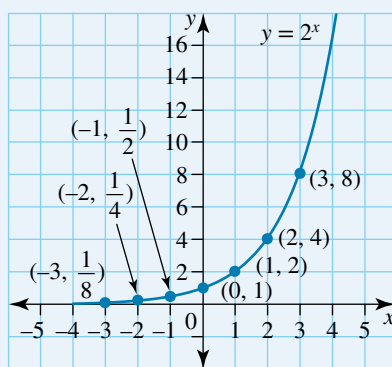
THINK

- 1 Substitute each x -value into the function $y = 2^x$ to obtain the corresponding y -value.

WRITE/DRAW

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

- 2 Draw a set of axes and plot the points from the table. Join them with a smooth curve.



- The graph in Worked example 22 has several important features.
 - The y -intercept is 1.
 - The value of y is always greater than zero.
 - As x decreases, y gets closer to but never reaches zero. So the graph gets closer to but never touches the x -axis. The x -axis (or the line $y = 0$) is called a horizontal **asymptote**.
 - As x increases, y becomes very large.

WORKED EXAMPLE 23

Sketch and describe similarities and differences between the following pairs of graphs.

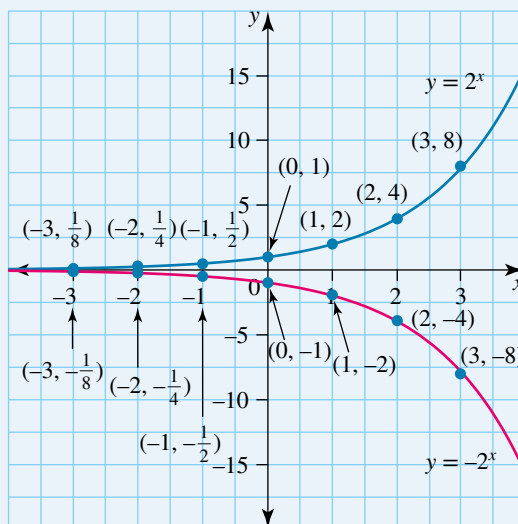
- a $y = 2^x$ and $y = -2^x$
 b $y = 2^x$ and $y = 2^{-x}$
 c $y = 2^x$ and $y = 2^x + 1$

THINK

- a 1 Substitute each x -value into the function $y = 2^x$ to obtain the corresponding y -value.
 Substitute each x -value into the function $y = -2^x$ to obtain the corresponding y -value.
- 2 Draw a set of axes and plot the points from the table. Join them with a smooth curve.

WRITE/DRAW

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = -2^x$	$-\frac{1}{8}$	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	-4	-8



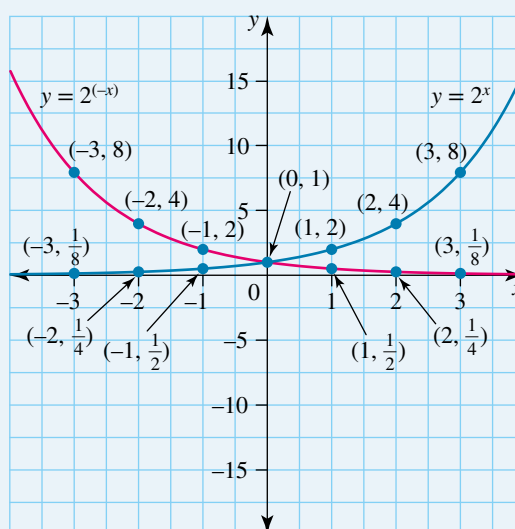
3 Compare the graphs.

- b 1** Substitute each x -value into the function $y = 2^x$ to obtain the corresponding y -value. Substitute each x -value into the function $y = 2^{-x}$ to obtain the corresponding y -value.

The graphs of $y = 2^x$ and $y = -2^x$ are the same shape. The y -intercept of $y = 2^x$ is $(0, 1)$, and the y -intercept of $y = -2^x$ is $(0, -1)$. The graph of $y = -2^x$ is the graph of $y = 2^x$ reflected in the x -axis.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = 2^{-x}$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

- 2** Draw a set of axes and plot the points from the table. Join them with a smooth curve.



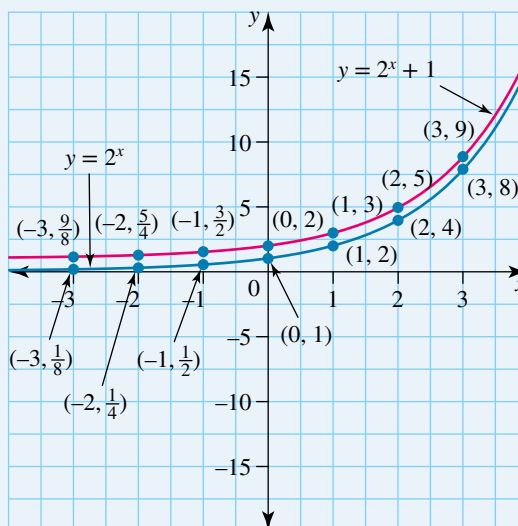
3 Compare the graphs.

- c 1** Substitute each x -value into the function $y = 2^x$ to obtain the corresponding y -value. Substitute each x -value into the function $y = 2^x + 1$ to obtain the corresponding y -value.

The graphs of $y = 2^x$ and $y = 2^{-x}$ are the same shape. The y -intercept for both graphs is $(0, 1)$. The graph of $y = 2^{-x}$ is the graph of $y = 2^x$ reflected in the y -axis.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = 2^x + 1$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{3}{2}$	2	3	5	9

- 2 Draw a set of axes and plot the points from the table. Join them with a smooth curve.



- 3 Compare the graphs.

The graphs of $y = 2^x$ and $y = 2^x + 1$ are the same shape.

The y -intercept of $y = 2^x$ is $(0, 1)$, and the y -intercept of $y = 2^x + 1$ is $(0, 2)$.

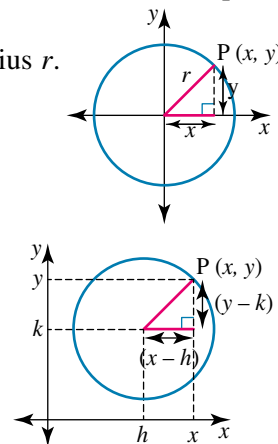
The graph of $y = 2^x + 1$ is the graph of $y = 2^x$ moved up one unit.

11.9.7 The circle

- A circle is the path traced out by a point at a constant distance (the radius) from a fixed point (the centre).
- Consider the circles shown. The first circle has its centre at the origin and radius r .
 - Let $P(x, y)$ be a point on the circle.
 - By Pythagoras' theorem, $x^2 + y^2 = r^2$.
 - This relationship is true for all points, P , on the circle.
 - The equation of a circle with centre $(0, 0)$ and radius r is:
- If the circle is translated h units to the right, parallel to the x -axis, and k units upwards, parallel to the y -axis, then the equation of a circle with centre (h, k) and radius r is:

$$x^2 + y^2 = r^2$$

$$(x - h)^2 + (y - k)^2 = r^2$$



WORKED EXAMPLE 24

Sketch the graph of $x^2 + y^2 = 49$, stating the centre and radius.

THINK

- Write the equation.
- State the coordinates of the centre.
- Find the length of the radius by taking the square root of both sides. (Ignore the negative results.)

WRITE/DRAW

$$x^2 + y^2 = 49$$

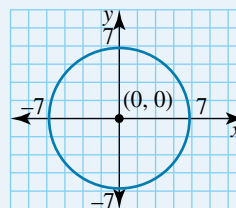
Centre $(0, 0)$

$$r^2 = 49$$

$$r = 7$$

Radius = 7 units

4 Sketch the graph.



WORKED EXAMPLE 25

Sketch the graph of $(x - 2)^2 + (y + 3)^2 = 16$, clearly showing the centre and radius.

THINK

- 1 Express the equation in general form.
- 2 State the coordinates of the centre.
- 3 State the length of the radius.
- 4 Sketch the graph.

WRITE/DRAW

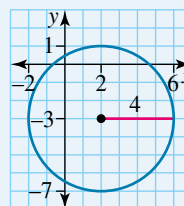
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 2)^2 + (y + 3)^2 = 4^2$$

Centre $(2, -3)$

$$r^2 = 16$$

$$r = 4$$

Radius = 4 units



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Interactivity: Vertical translations of parabolas (int-6055)

Interactivity: Horizontal translations of parabolas (int-6054)

Interactivity: Equations of circles (int-6053)

Exercise 11.9 Non-linear relationships (parabolas, exponentials, circles)

Individual pathways

PRACTISE

Questions:
1–12, 16

CONSOLIDATE

Questions:
1–13, 16, 17

MASTER

Questions:
1–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE20** State the vertical translation and the coordinates of the vertex for the graph of each of the following equations.

a. $y = x^2 + 3$

b. $y = x^2 - 1$

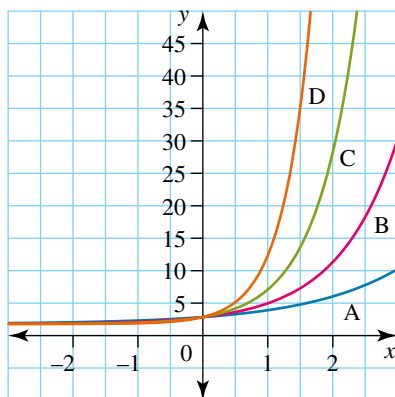
c. $y = x^2 - 7$

d. $y = x^2 + \frac{1}{4}$

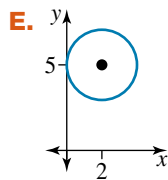
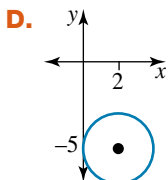
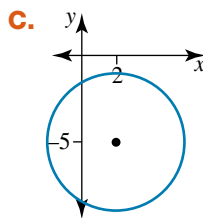
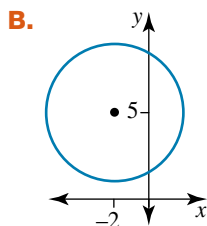
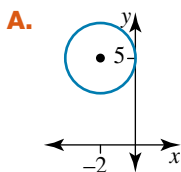
2. **WE21** State the horizontal translation and the coordinates of the vertex for the graph of each of the following equations.
- a. $y = (x - 1)^2$ b. $y = (x - 2)^2$ c. $y = (x + 10)^2$ d. $y = (x + 4)^2$
3. For each of the following graphs, give the coordinates of the vertex and state whether it is a maximum or a minimum.
- a. $y = -x^2 + 1$ b. $y = x^2 - 3$ c. $y = -(x + 2)^2$
4. **WE22** a. Complete the table of values below.

x	-3	-2	-1	0	1	2	3
y							

- a. Use the table of values to plot the graph of $y = 3^x$.
5. **WE23** Sketch and describe similarities and differences between the following pairs of graphs. (Graphing software may be used to draw the graphs.)
- a. $y = 3^x$ and $y = -3^x$ b. $y = 3^x$ and $y = -3^{-x}$ c. $y = 3^x$ and $y = 3^x - 1$
6. Match the following graphs A, B, C and D with their correct equations a, b, c and d. (Graphing software may be of assistance.)



- a. $y = 3^x$ b. $y = 10^x$ c. $y = 2^x$ d. $y = 5^x$
7. Use your answer to question 6 to describe the effect of increasing the value of a on the graph of $y = a^x$.
8. **WE24** Sketch the graphs of the following, stating the centre and radius of each.
- a. $x^2 + y^2 = 49$ b. $x^2 + y^2 = 4^2$ c. $x^2 + y^2 = 36$
d. $x^2 + y^2 = 81$ e. $x^2 + y^2 = 25$ f. $x^2 + y^2 = 100$
9. **WE25** Sketch the graphs of the following, clearly showing the centre and the radius.
- a. $(x - 1)^2 + (y - 2)^2 = 5^2$ b. $(x + 2)^2 + (y + 3)^2 = 6^2$
c. $(x + 3)^2 + (y - 1)^2 = 49$ d. $(x - 4)^2 + (y + 5)^2 = 64$
e. $x^2 + (y + 3)^2 = 4$ f. $(x - 5)^2 + y^2 = 100$
10. **MC** The graph of $(x - 2)^2 + (y + 5)^2 = 4$ is:



Communicating, reasoning and problem solving

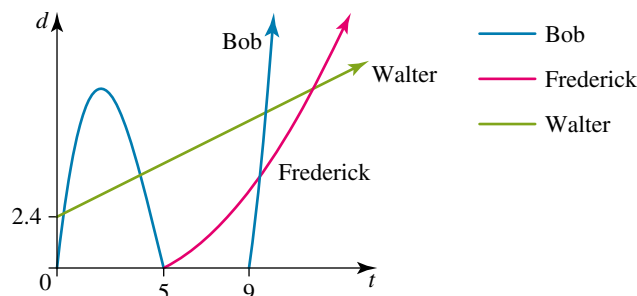
11. Show that $y = x^2$ is the same as $y = (-x)^2$.
12. Explain the similarities and differences between $y = a^x$ and $y = -a^x$.
13. Show that the vertex for $y = (x + 3)^2 - 1$ is $(-3, -1)$. Explain whether this is a maximum or minimum vertex.
14. A house in a Sydney suburb was sold for \$680 000 at the beginning of 2016. The purchaser had the house valued at the beginning of 2017 and it had increased by 11%. The purchaser expects this increase to be maintained at this level for the foreseeable future.
 - a. Express this information in the form of an exponential equation with V as the value of the house and x as the number of years after 2016.
 - b. According to this model, what will the house be worth by the end of 2020?
15. Three kindergarten children, Walter, Frederick and Bob, ran a 20-metre race. Each child ran according to the following equations, where d is the distance, in metres, from the starting line and t is the time, in seconds.

Walter: $d = 2.4 + 0.5t$

Frederick: $d = 0.1t(t - 5)$

Bob: $d = 0.2t(t - 5)(t - 9)$

This sketch shows their running paths.



- a. Describe the race style of Bob.
 - b. How long did it take Frederick to start running?
 - c. At the start of the race, how far was Walter from the starting line?
 - d. If the winner of the 20-metre race won the race in 10.66 seconds, determine whether Frederick or Bob won the race. Justify your answer using calculations.
16. A circle with centre $(0, 2)$ passes through the point $(3, 6)$. What are the coordinates of the points where the circle crosses the y -axis?
17. Consider the relationship $\frac{1}{y} = \frac{1}{x} + 1$. It appears that both variables are being raised to the power of 1. The relationship can be transposed as shown below.

$$\begin{aligned}\frac{1}{y} &= \frac{1}{x} + 1 \\ \frac{1}{y} &= \frac{1}{x} = \frac{x}{x} \\ \frac{1}{y} &= \frac{1+x}{x} \\ y &= \frac{x}{1+x}\end{aligned}$$

- a. Generate a table of values for x versus y using the transposed relationship.
 - b. Plot a graph to show the points contained in the table of values.
 - c. Use your plot to confirm whether this relationship is linear.
18. How can you tell whether the equation of a non-linear relationship represents a parabola, an exponential or a circle?

11.10 Review

Investigation | Rich task

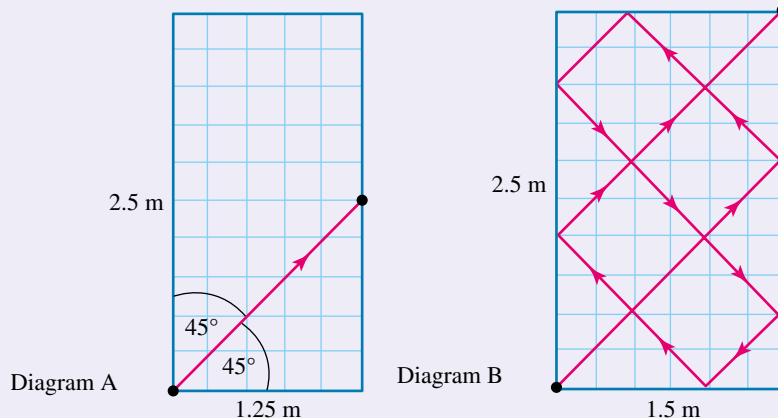
Path of a billiard ball



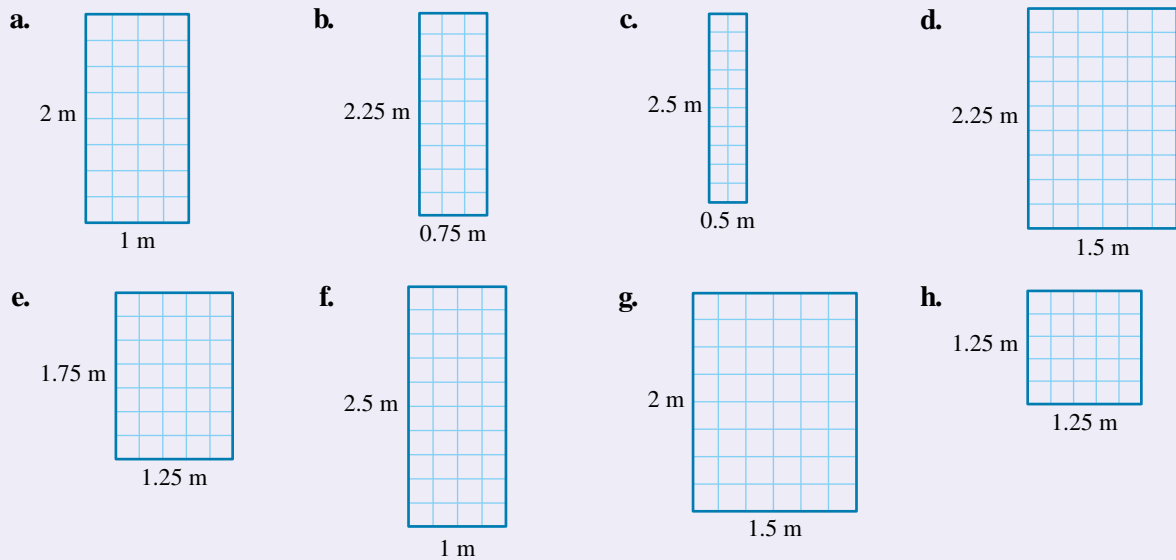
The path of a billiard ball can be mapped by using mathematics. The billiard table can be represented by a rectangle, the ball by a point and its path by line segments. In this investigation, we will look at the trajectory of a single ball, unobstructed by other balls. A billiard table has a pocket at each of its corners and one in the middle of each of its long sides.

Consider the path of a ball that is hit on its side, from the lower left-hand corner of the table, so that it travels at a 45° angle from the corner of the table. Assume that the ball continues to move, rebounding from the sides and stopping only when it comes to a pocket. Diagrams of the table drawn on grid paper are shown at right. Each grid square has a side length of 0.25 metres.

Diagram A shows the trajectory of a ball that has been hit on its side at a 45° angle, from the lower left-hand corner of a table that is 2.5 m long and 1.25 m wide. Note that, because the ball has been hit on its side at a 45° angle, it travels diagonally through each square in its path, from corner to corner. Diagram B shows the trajectory of a ball on a 2.5 m by 1.5 m table.



1. With reference to Diagram B, how many times does the ball rebound off the sides before going into a pocket?
2. Eight tables of different sizes are drawn on grid paper in the following diagrams. For each table, determine the trajectory of a ball hit at 45° on its side, from the lower left-hand corner of the table. Draw the path each ball travels until it reaches a pocket.



- Which tables show the ball travelling through the simplest path? What is special about the shape of the tables?
- Which table shows the ball travelling through the most complicated path? What is special about the path? Draw another table (and path of the ball) with the same feature.
- Which tables show the ball travelling through a path that does not cross itself? Draw another table (and path of the ball) with the same feature.
- Consider the variety of table shapes. Will a ball hit on its side from the lower left corner of a table at 45° always end up in a pocket (assuming it does not run out of energy)?

Simplify matters a little and consider a billiard table with no pockets in the middle of the long sides. Look, in a systematic way, for patterns for tables whose dimensions are related in a special way.

- On paper, draw a series of billiard tables of length 3 m. Increase the width from an initial value of 0.25 m in increments of 0.25 m. Investigate the destination pocket of a ball hit from the lower left corner. Complete the table below.

	Length of table (m)	Width of table (m)	Destination pocket
a	3	0.25	
b	3	0.5	
c	3	0.75	
d	3	1	
e	3	1.25	
f	3	1.5	
g	3	1.75	
h	3	2	

- How can you predict (without drawing a diagram) the destination pocket of a ball hit from the lower left-hand corner of a table that is 3 m long? Provide an illustration with your answer to verify your prediction.

Digital doc: Investigation — Path of a billard ball (doc-15898)

Digital doc: Code puzzle — Christmas day, 1859 (doc-15899)

Digital doc: Summary Topic 11 (doc-22893)

Interactivity: Crossword Topic 11 (int-2688)

Interactivity: Word search Topic 11 (int-2687)

Interactivity: Sudoku Topic 11 (int-3207)

Exercise 11.10 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- MC** For the rule $y = 3x - 1$, what is y when $x = 2$?
 A. -1 B. 1 C. 2
 D. 5 E. 6
- MC** What is the gradient of the linear rule $y = 4 - 6x$?
 A. 6 B. -6 C. 4
 D. -4 E. $-1\frac{1}{2}$
- MC** The graph with the rule $2y - x + 6 = 0$ has an x -intercept of:
 A. -2 B. 0 C. 2
 D. 6 E. -1
- MC** The graph with the rule $2y - x + 6 = 0$ has a y -intercept of:
 A. -6 B. -3 C. 0
 D. 3 E. 6
- MC** Consider a linear graph that goes through the points $(6, -1)$ and $(0, 5)$.
 The gradient of this line is:
 A. 5 B. -5 C. 1
 D. -1 E. 0
- MC** A straight line passes through the points $(2, 1)$ and $(5, 4)$. Its rule is:
 A. $y = x - 1$ B. $y = x + 1$ C. $y = 2x$
 D. $4y = 5x$ E. $y = 2.5x + 3$
- MC** The rule for a line whose gradient is -4 and y -intercept $= 8$ is:
 A. $y = -4x + 32$ B. $y = -4x + 8$ C. $y = 4x - 32$
 D. $y = 4x - 8$ E. $y = -4x - 8$
- MC** Which of the following linear rules will *not* intersect with the straight line defined by $y = 3x$?
 A. $y = 3x + 2$ B. $y = -3x + 1$ C. $y = -3x + 2$
 D. $y = -2x + 1$ E. $y = x$
- MC** If $y = 2x + 1$, then a point that could not be on the line is:
 A. $(3, 7)$ B. $(-3, -5)$ C. $(0, 1)$
 D. $(2, 5)$ E. $(-3, 0)$
- MC** The solution to $y = 3x + 1$ and $y = -3x + 1$ is:
 A. $(0, 1)$ B. $(1, 0)$ C. $(0, -\frac{1}{3})$
 D. $(-\frac{1}{3}, 0)$ E. $(-1, 1)$

11. Write down **i** the gradient and **ii** the y-intercept of each of the following linear graphs. You may need to first rearrange the equations.

a. $y = 8x - 3$

b. $y = 5 - 9x$

c. $2x + y - 6 = 0$

d. $4x - 2y = 0$

e. $y = \frac{2x - 1}{3}$

12. Which of the following lines are parallel to the line with the equation $y = 6 - x$?

a. $y + x = 4$

b. $y = 13 - x$

c. $2y - 2x = 1$

d. $x + 2y - 4 = 0$

13. Determine which of the following rules will yield a linear graph.

a. $3y = -5x - \frac{1}{2}$

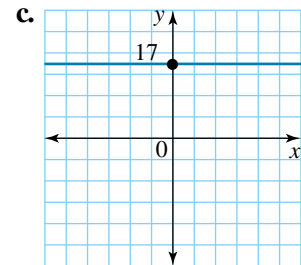
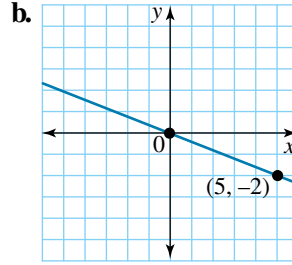
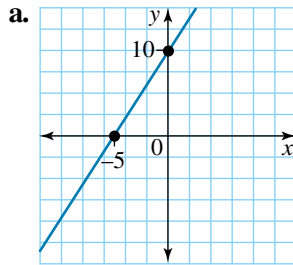
b. $y = 3x^2 - 2\frac{1}{2}$

c. $3x + 4y + 6 = 0$

d. $x + y - 2xy = 0$

e. $2y = 4^2x + 9^2$

14. Find the gradients of the lines shown.



15. Find the gradients of the lines passing through the following pairs of points:

a. $(2, -3)$ and $(4, 1)$

b. $(0, -5)$ and $(4, 0)$.

16. For each of the following rules, state the gradient and the y-intercept.

a. $y = -3x + 7$

b. $2y - 3x = 6$

c. $y = -\frac{2}{5}x$

d. $y = 4$

17. For the following rules, use the gradient-intercept method to sketch linear graphs.

a. $y = -x + 5$

b. $y = 4x - 2.5$

c. $y = \frac{2}{3}x - 1$

d. $y = 3 - \frac{5}{4}x$

18. For the following rules, sketch the linear graphs.

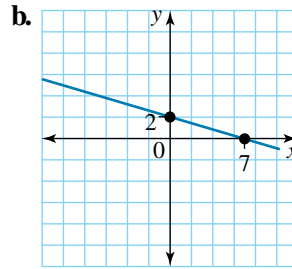
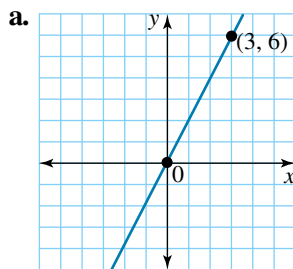
a. $y = -3x$

b. $y = \frac{1}{4}x$

c. $y = -2$

d. $x = 3$

19. Find the rules of the lines shown.



20. Determine the linear rules given the following pieces of information.

a. Gradient = 2, y-intercept = -7

b. Gradient = 2, x-intercept = 7

c. Gradient = 2, passing through $(7, 9)$

d. Gradient = -5, passing through the origin

e. y-intercept = -2, passing through $(1, -3)$

f. Passing through $(1, 5)$ and $(5, -6)$

g. x-intercept = 3, y-intercept = -3

h. y-intercept = 5, passing through $(-4, 13)$

21. Find the midpoint of the line interval joining the points $(-2, 3)$ and $(4, -1)$.

22. Find the distance between the points $(1, 1)$ and $(4, 5)$.

23. Sketch each of the following, comparing it with the graph of $y = x^2$.

a. $y = x^2 - 3$

b. $y = (x + 2)^2$

24. a. Plot the graph of $y = 4^x$.
 b. What sort of graph is this?
25. Sketch the following circle. Clearly show the centre and radius.

$$x^2 + y^2 = 16$$

Communicating, reasoning and problem solving

26. Louise owes her friend Sula \$400 and agrees to pay her back \$15 per week.
 a. State a linear rule that demonstrates this reducing debt schedule and sketch the graph.
 b. How many weeks does it take her to repay the debt?
 c. How much does she owe after 15 weeks?
 d. After how many repayments does she owe \$85?
27. A bushwalker is 40 km from his base camp when he decides to head back.
 He is able to walk 3.5 km each hour.
 a. Determine the linear rule that describes this situation and sketch its graph.
 b. How long, correct to 1 decimal place, will it take him to reach base camp?
 c. How far will he have walked in 6.5 hours?
28. Sue is writing test questions. She has already written 25 questions and can write a further 5 questions per hour.
 a. Represent this information as a linear equation where t hours is the time spent writing test questions and n is the number of questions written.
 b. Predict the total number of questions written after a further 8 hours, assuming the same linear rule.
 c. How long, to the nearest minute, will it take Sue to have 53 written questions?
29. Catherine earns a daily rate of \$200 for working in her mother's store. She receives \$5 for each necklace that she sells.
 a. Write an equation to show how much money (m) Catherine earned for the day after selling (n) necklaces.
 b. Graph the equation that you created in part a, showing the two intercepts.
 c. Which part of the line applies to her earnings? Explain.
 d. Which part of the line does not apply to her earnings? Explain.
30. Calculate the gradient of the line passing through the points $(2, 3)$ and $(6 + 4t, 5 + 2t)$. Write your answer in simplest form.
31. What is the point on the line $y = 2x + 7$ that is also 5 units above the x -axis?
32. An experiment was conducted, and data were collected for two variables p and t .

p	$-\frac{1}{2}$		$\frac{1}{2}$	3.6
t	$2\frac{1}{4}$	1.75		-5.95

It is known that the relationship between p and t is a linear one. What are the two missing values?

33. The distance from the origin to the y -intercept of a linear graph is three times the distance from the origin to the x -intercept. The area of the triangle formed by the line and the axes is 3.375 units^2 . The line has a negative gradient and a negative y -intercept. What is the equation of the line?

Answers

Topic 11 Linear and non-linear relationships

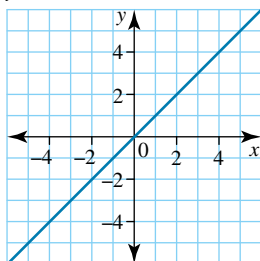
Exercise 11.2 Sketching linear graphs using the coordinates of two points

1. a. B

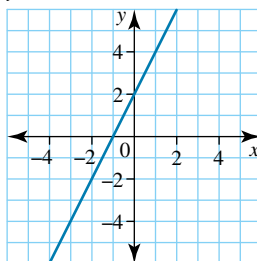
b. C

c. D

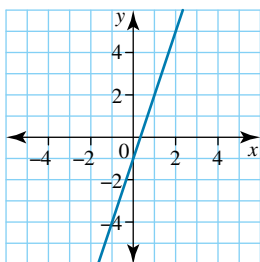
2. a. $y = x$



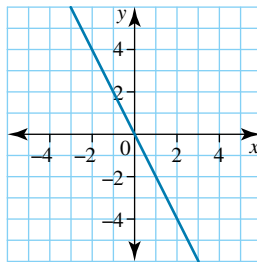
b. $y = 2x + 2$



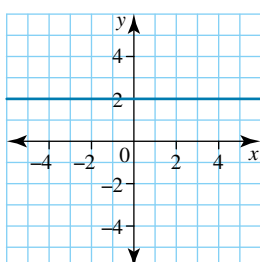
c. $y = 3x - 1$



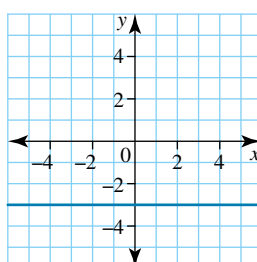
d. $y = -2x$



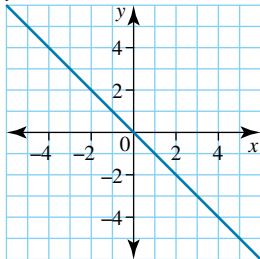
e. $y = 2$



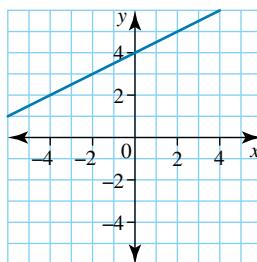
f. $y = -3$



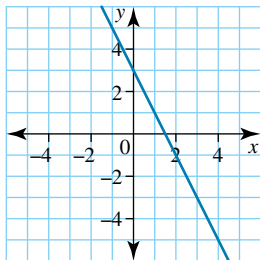
3. a. $y = -x$



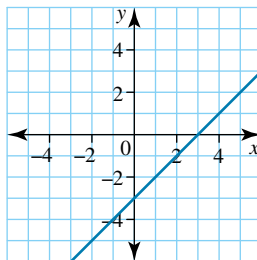
b. $y = \frac{1}{2}x + 4$



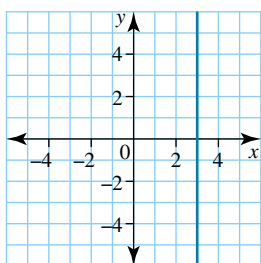
c. $y = -2x + 3$



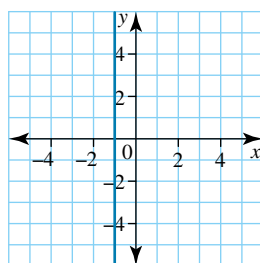
d. $y = x - 3$



e. $x = 3$



f. $x = -1$



4. a. Yes

b. No

c. No

d. Yes

5. a. Yes

b. Yes

c. Yes

d. No

e. No

f. No

g. Yes

h. Yes

6. D

7. a. B

b. A

c. D

d. C

8. Third quadrant. By plotting the points on the Cartesian plane and joining them to make a line it can be seen that it does not pass through the third quadrant.

9. a. The first, second and third quadrants.

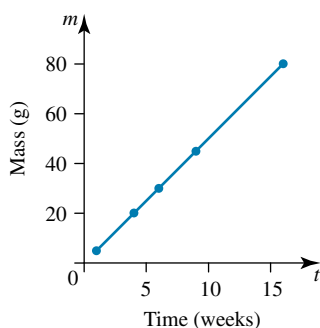
b. Answers will vary. Substitute point (1, 3) into line equation. $3 \neq 1 + 1$

10. Answers will vary. Substitute the point into the line equation. If the LHS equals the RHS of the equation, then the point lies on the line. Alternatively, graph the line and check the point on the Cartesian plane.

11. Answers will vary. Plot the points on the Cartesian plane and draw a line through the points. Find the equation of the line. Check all points lie on the line by substituting into the line equation.

12. (59, 21)

13. a.



b. $m = 5t + 1$

c. Answers will vary.

14. Stirring increases the rate of reaction.

15. The equation $x = 0$ is the equation of the y-axis and the equation $y = 0$ is the equation of the x-axis.

Exercise 11.3 Midpoint of a line segment and distance between two points

1. a. (2, 4)

b. (5, 1)

c. (7, 2)

d. (8, 9)

e. (0, 5)

f. (-1, 2)

g. (-5, -2)

h. (2, 3)

i. (0, 12)

j. (-3, 7)

2. a. $(1\frac{1}{2}, 5\frac{1}{2})$

b. $(-3, -3\frac{1}{2})$

c. $(-9\frac{1}{2}, 9\frac{1}{2})$

d. $(-1\frac{1}{2}, 6\frac{1}{2})$

3. a. 5

b. 9

c. -16

d. 0

4. a. (4, 6)

b. (-2, -3)

c. (11, 2)

d. (-7, -3)

5. a. 5

b. 10

c. 1

d. 2

e. 13

f. 10

g. 5

h. 17

i. 13

j. 20

6. a. 7.211

b. 14.765

c. 13.038

d. 8.944

e. 19.416

f. 19.416

7. a. 24.472

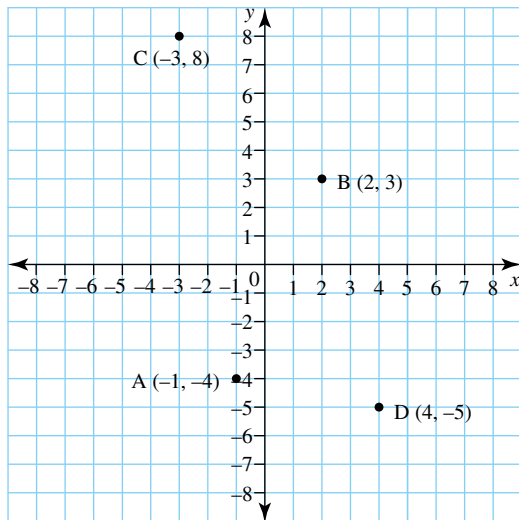
b. 25.464

8. a. 17.788

b. 25.763

9. 21.024 km

10. a.



c. $2\sqrt{37}$

$$11. -2 = \frac{x+5}{2} \Rightarrow x = -9$$

$$-4 = \frac{-6+y}{2} \Rightarrow y = -2$$

The required point is $(-9, -2)$.

$$12. d = \sqrt{(6-2)^2 + (-1-2)^2} \\ = \sqrt{16+9} \\ = 5$$

$$13. d_{AB} = \sqrt{(3-(-10))^2 + (15-6)^2} \\ = \sqrt{169+81} \\ = \sqrt{250} \\ = 5\sqrt{10}$$

$$d_{BC} = \sqrt{(-1-(-10))^2 + (-7-6)^2} \\ = \sqrt{81+169} \\ = \sqrt{250} \\ = 5\sqrt{10}$$

$$d_{AB} = d_{BC}$$

Therefore, B is equidistant from A and C.

14. Answers will vary; a sample answer is shown.

$$d_{AB} = \sqrt{(3-1)^2 + (2+4)^2} \\ = \sqrt{4+36} \\ = \sqrt{40} \\ = 2\sqrt{10}$$

$$d_{BC} = \sqrt{(3+3)^2 + (0+2)^2} \\ = \sqrt{36+4} \\ = \sqrt{40} \\ = 2\sqrt{10}$$

$$d_{AC} = \sqrt{(-3-1)^2 + (0+4)^2} \\ = \sqrt{16+16} \\ = \sqrt{32} \\ = 4\sqrt{2}$$

Side length AB is equal to side length BC but not equal to side length AC. Therefore, $\triangle ABC$ is an isosceles triangle.

$$b. M = \left(\frac{-1+(-3)}{2}, \frac{-4+8}{2} \right) \\ = (-2, 2)$$

d. $(7, -2)$

15. $(-4, 5)$, $(-3, 2)$, $(-2, -1)$, $(-1, -4)$, $(0, -7)$

16. The distance formula determines the distance between two points, which cannot be negative as it is a measurement.

17. Draw a right-angled triangle using the line segment as the hypotenuse and hence use Pythagoras's theorem to calculate the length of the hypotenuse.

Exercise 11.4 The gradient of a straight line

1. a. 6

e. -2.2

i. -2.6

2. a. 4

e. 400

i. -300

3. a. 5

e. 0.5

i. 23

4. a. 23

e. 1

i. -87

b. -4

f. -45

j. 0

b. 1

f. -5.2

j. 11

b. -3

f. 0.5

j. 2

b. 54

f. 5.2

j. $\frac{5}{2}$

c. 0

g. 0

k. 9

c. 0

g. 0

k. Undefined

c. 3

g. -1

k. -2

c. -6

g. 100

k. $\frac{-5}{2}$

d. $\frac{6}{5}$

h. 0.1

l. -0.1

d. 20

h. -5

l. -1

d. -2

h. 0

l. $\frac{7}{2}$ or 3.5

d. 70

h. 100

l. $\frac{-19}{6}$

5. a. Gradient = 4, y-intercept = 8

c. Gradient = -2 , y-intercept = 7

e. Gradient = 0.5, y-intercept = 2.5

g. Gradient = -4 , y-intercept = -18

i. Gradient = 0.5, y-intercept = 1.5

k. Gradient = 3, y-intercept = 5

b. Gradient = -4 , y-intercept = 8

d. Gradient = 12, y-intercept = 0

f. Gradient = -40 , y-intercept = 83

h. Gradient = 1.2, y-intercept = -3.6

j. Gradient = $\frac{1}{15}$, y-intercept = 0

l. Gradient = 4, y-intercept = -8

6. C

7. x-intercept = $\frac{-c}{m}$

8. a. $(0, -10)$

d. $(0, 7)$

b. $(0, -4)$

e. No y-intercept

c. $(0, 3)$

9. Answers will vary. There is a rise and no run; $\frac{\text{rise}}{0} = \text{undefined}$

10. Answers will vary. There is no rise and a run; $\frac{0}{\text{run}} = 0$.

11. Answers will vary.

12. a. Answers will vary.

b. The change in b changes the y-intercept and moves the line along the y-axis.

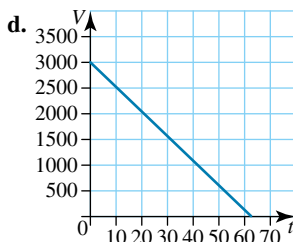
13. a. 1 point $(12.5, 12.5)$

b. 24 points

14. a. 3000 L

b. 2040 L

c. Answers will vary.



e. Water is leaking out at a rate of 48 litres per minute.

Intercepts: $(0, 3000)$ and $(62.5, 0)$

15. It does not matter if you rise before you run or run before you rise, as long as you take into account whether the rise or run is negative.

16. a. $m = \frac{y - b}{x}$

b. $y = mx + b$

17. a. $m = -\frac{3}{2}$

b. It does not matter which points are chosen to determine the gradient of the graph because the gradient will always remain the same.

c. Straight line with a y-intercept of $(0, -2)$ and a slope of $-\frac{3}{2}$

18. Answers will vary.

19. a. Lamb: $m = \$15/\text{kg}$
 Chicken: $m = \$10/\text{kg}$
 Beef: $m = \$7.50/\text{kg}$

b. Lamb: \$15
 Chicken: \$10
 Beef: \$7.50

c. i. \$15

ii. \$5

iii. \$15

d. \$35

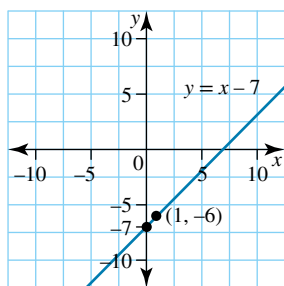
e.

Meat type	Cost per kilogram (\$/kg)	Weight required (kg)	Cost = \$/kg $\times$ kg
Lamb	15	1	15
Chicken	10	0.5	5
Beef	7.50	2	15
Total cost			35

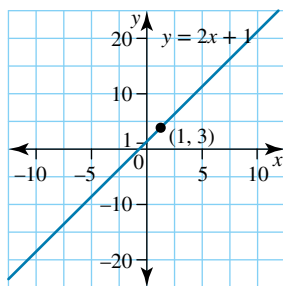
20. The line crosses the y-axis when $x = 0$, so to determine the y-intercept, we simply substitute $x = 0$ into the equation.

Exercise 11.5 Using the gradient–intercept form to graph linear relationships

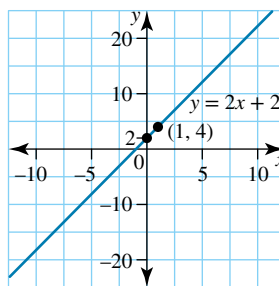
1. a.



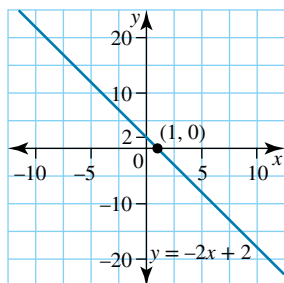
b.



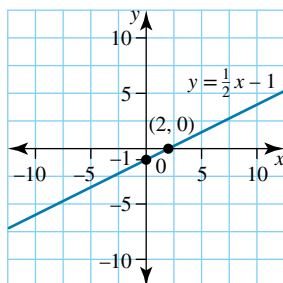
c.



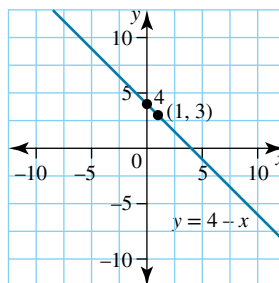
d.



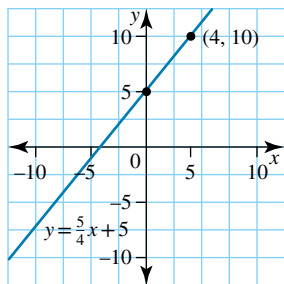
e.



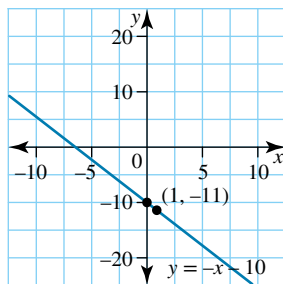
f.



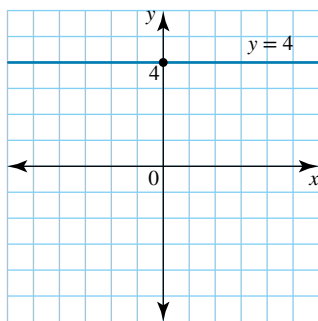
g.



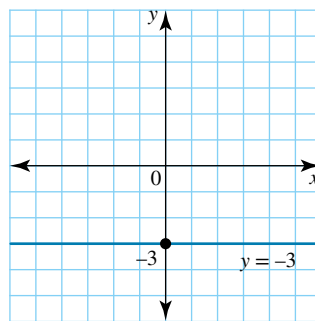
h.



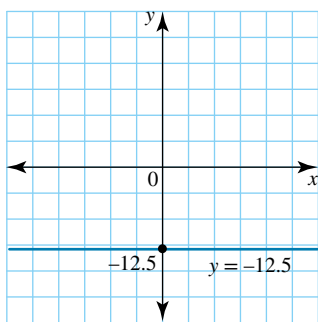
2. a.



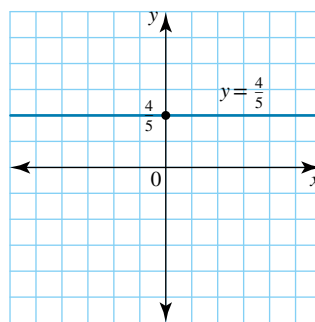
b.



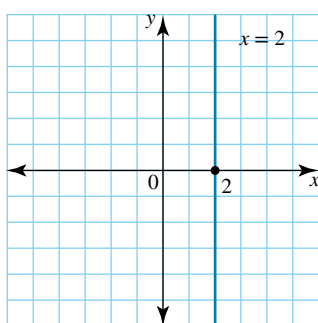
c.



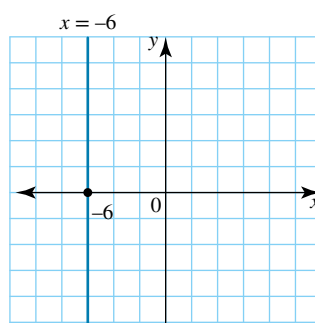
d.



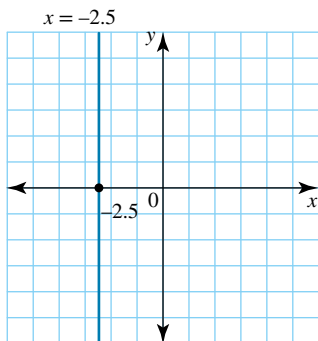
3. a.



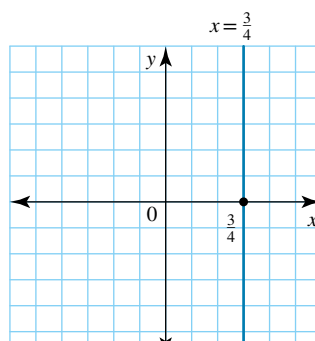
b.



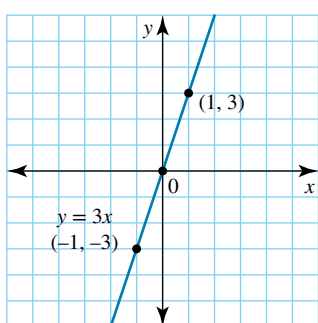
c.



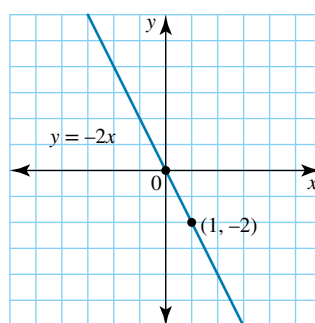
d.

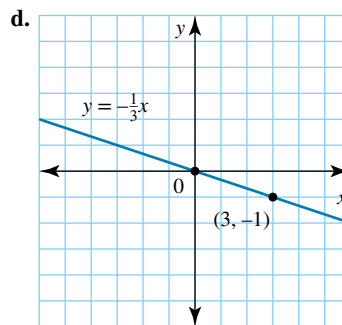
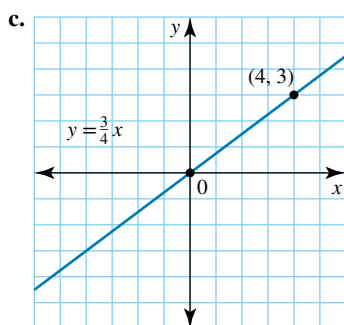


4. a.



b.





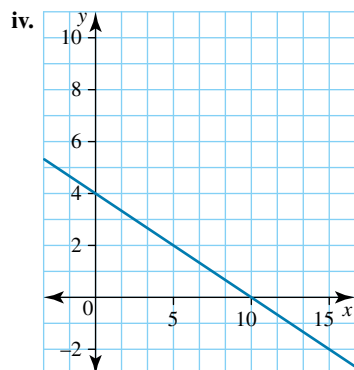
5. a. D

b. D

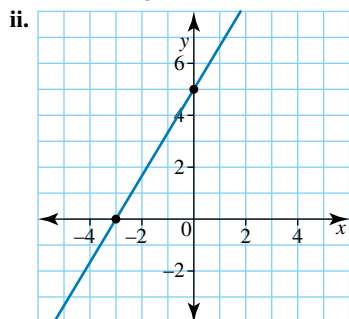
6. a. i. $y = \frac{-2x}{5} + 4$

ii. $-\frac{2}{5}$

iii. The x -intercept is 10, the y -intercept is 4.



b. i. $m = \frac{\text{rise}}{\text{run}} = \frac{5}{3}$

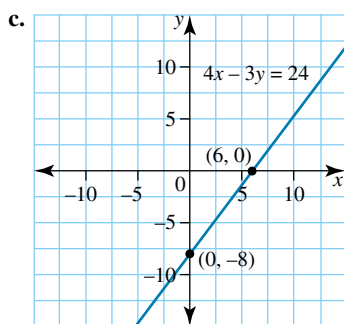


iii. $y = \frac{5x}{3} + 5$

$-5x + 3y = 15$

7. a. $y = \frac{4}{3}x - 8$

b. $y = 0$, $4x = 24$, $x = 6$; $x = 0$, $-3y = 24$, $y = -8$



8. a. $y = \frac{-px + c}{q}$

b. Answers will vary. The gradient is $\frac{-p}{q}$; substituting positive values always results in a negative gradient.

9. Answers will vary; a possible answer is shown.

$$4y = \frac{4}{2}x + 3$$

$$4y = 2x + 3$$

$$-2x + 4y = 3$$

10. Answers will vary. All descriptions use the idea that a gradient is equal to $\frac{\text{rise}}{\text{run}}$, which equals $\frac{2}{3}$ in all of these cases.

11. Answers will vary. The gradient is 2. The equation of the line is $y = 2x - 1$. One point is (1, 1).

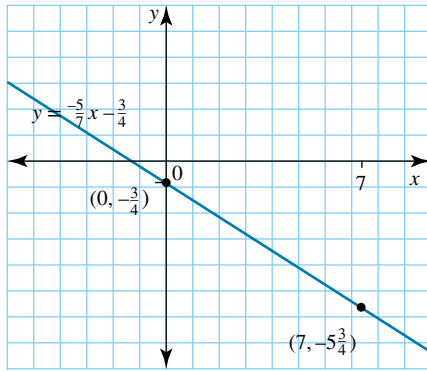
12. a. i. C

ii. B

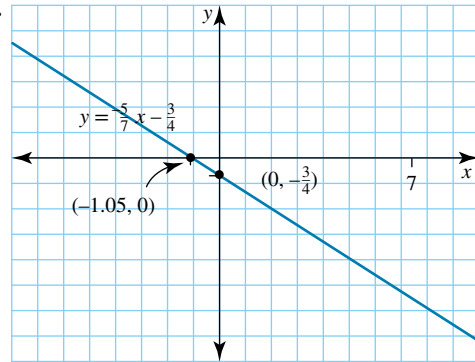
iii. A

b. Straight line y-intercept of (0, 2) with a slope of $-\frac{1}{4}$

13. a. i.



ii.



iii. Answers will vary.

b. Answers will vary.

14. If these two graphs intersect, then $x = \frac{d-b}{a-c}$. We need $x > 0$, so when $d > b$, the denominator is positive, so the numerator must also be positive so $a > c$.

15. Gradients are calculated by the ratio $\frac{\text{rise}}{\text{run}}$. For a vertical line, there is no change in the x -value, so the ratio is $\frac{\text{rise}}{0}$. Any number divided by zero is undefined.

Exercise 11.6 Parallel and perpendicular lines

1. i. As the size of the coefficient increases, the steepness of the graph increases.

ii. Each graph cuts the x -axis at (0, 0).

iii. Each graph cuts the y -axis at (0, 0).

2. i. As the magnitude of the coefficient decreases, the steepness of the graph increases.

ii. Each graph cuts the x -axis at (0, 0).

iii. Each graph cuts the y -axis at (0, 0).

3. a. positive, positive

b. downward, negative

c. bigger

d. will

4. i. Yes, 1

ii. No, the lines are parallel.

iii. a. (0, 0), i.e. $x = 0$

b. (-2, 0), i.e. $x = -2$

c. (2, 0), i.e. $x = 2$

iv. a. (0, 0), i.e. $y = 0$

b. (0, 2), i.e. $y = 2$

c. (0, -2), i.e. $y = -2$

5. i. Yes, -1

ii. No, the lines are parallel.

iii. a. (0, 0), i.e. $x = 0$

b. (2, 0), i.e. $x = 2$

c. (-2, 0), i.e. $x = -2$

iv. a. (0, 0), i.e. $y = 0$

b. (0, 2), i.e. $y = 2$

c. (0, -2), i.e. $y = -2$

6. i. same

ii. y-intercept, y-axis

7. i. No

ii. They intersect at right angles.

iii. a. 1, -1

iv. -1

8. i. No

ii. They intersect at right angles.

iii. a. 3, $-\frac{1}{3}$

iv. -1

9. One of the general forms of the equation of a linear graph is $y = mx + b$, where m is the **steepness** of the graph. We call the steepness of the graph the gradient. When the **product** of the gradients of two linear graphs is -1 , then the graphs are **perpendicular**.

10. a. i. 2

ii. (0, 0), i.e. $y = 0$

b. i. 1

ii. (0, 1), i.e. $y = 1$

c. i. -3

ii. (0, 5), i.e. $y = 5$

d. i. $\frac{2}{3}$

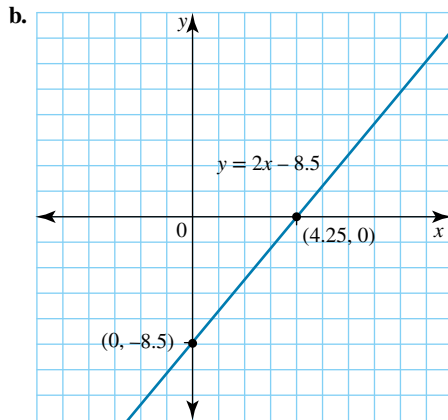
ii. (0, -7), i.e. $y = -7$

11. Answers will vary.

12. a. N is the number of dogs walked; $-\$15$ is Shirley's starting cost and out-of-pocket expense before she walks a dog, and she earns $\$10$ for every dog walked.

b. She needs to walk at least 2 dogs a week before she can make a profit.

13. a. $y = 2x - 8.5$



c. $y = 7.5$

d. $x = 10.25$

14. a. $C = 25n + 1000$

b. $(0, 1000)$. This is the initial cost of the site licence.

c. $\$6000$

d. 80

15. a. The method will vary: possible methods include plotting the points $(0, 325)$ and $(1, 328.8)$ or using the equation $c = 3.8m + 325$.

b. 382 calories

c. 32.9 minutes

16. You cannot assume they are perpendicular as they must intersect at exactly 90° , not a number very close to 90° .

Exercise 11.7 Finding the equation of a straight line

1. a. $y = 4x + 2$

d. $y = 6x + 7$

2. a. $y = -2x$

d. $y = -20x$

3. a. $y = x + 2$

d. $y = 2x - 13$

g. $y = -6x + 11$

j. $y = -3.5x + 15.5$

4. a. $y = -4x - 24$

d. $y = 5x + 35$

5. a. $y = x + 17$

d. $y = x + 3.5$

g. $y = -6x + 90$

j. $y = -0.25x + 5.5$

6. a. $y = x + 3$

d. $y = \frac{7}{8}x + 35$

g. $y = \frac{-3}{7}x - 3$

7. a. $y = \frac{7}{4}x$

d. $y = -3x$

b. $y = -4x + 1$

e. $y = -2.5x + 6$

b. $y = 4x$

e. $y = 1.07x$

b. $y = -x + 8$

e. $y = -5x + 70$

h. $y = -x - 1.5$

k. $y = 1.2x - 4.08$

b. $y = 2x - 6$

e. $y = 1.5x - 3.75$

b. $y = -x + 3$

e. $y = -x + 3.5$

h. $y = 20x$

k. $y = \frac{2}{7}x = \frac{32}{7}$ (or $7y = 2x - 32$)

b. $y = \frac{-5}{4}x + 5$

e. $y = x + 8$

h. $y = \frac{1}{4}x + 50$

b. $y = x$

e. $y = \frac{-24}{11}x$

c. $y = 4x + 8$

f. $y = 45x + 135$

c. $y = 10.5x$

f. $y = 32x$

c. $y = -4x - 8$

f. $y = 2x - 23$

i. $y = 6x + 30$

l. $y = 0.2x - 1.76$

c. $y = -2x + 4$

f. $y = 0.4x - 0.96$

c. $y = 11$

f. $y = \frac{1}{9}x + \frac{34}{9}$ (or $9y = x + 34$)

i. $y = -x + 4$

l. $y = \frac{2}{3}x + \frac{10}{3}$ (or $3y = 2x + 10$)

c. $y = -6x + 6$

f. $y = -2x + 6$

c. $y = -2x$

f. $y = \frac{-x}{3}$

8. a. $y = 2x + 4$

d. $y = \frac{1}{5}x$

9. a. B

10. a. $y = -2.4x - 12$

11. a. $y = -0.6x + 10$

12. a. The equation is $y = \frac{5}{4}x - 2135$.

b. Values of c cannot be read directly from the graph because the graph doesn't contain the origin, and since c is found at $x = 0$, we need to use the equation.

c. In 2020, the concentration of CO_2 will be 390 ppm. The assumption is that the concentration of CO_2 will continue to follow this linear pattern.

13. $m = \frac{-15 + 7}{4 - 0} = -2$

$y = -2x + b$

$6 = -6 + b$

$b = 12$

The equation is $y = -2x + 12$.

14. a. Line A: $y = x - 1$; Line B: $y = -x + 8$

b. (4.5, 3.5)

c. Answers will vary. Possible methods include graphing on a Cartesian plane or using algebra.

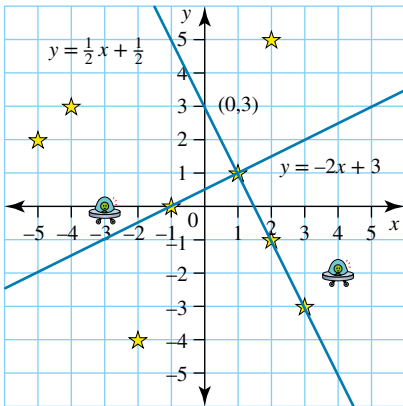
Line B: $y = -x + 8$, $m = -1$, $m_{\perp} = 1$

$y = x + b$

$6 = -4 + b$

$b = 10$

15. a, c



b. $y = -2x + 3$

d. (1, 1)

e. 1 star

f. Answers will vary but could include the following.

$y = -x - 1$, $x = 2$

16. $m = \frac{1}{4}C$

17. Answers will vary but should include the correct equation: $T = 4.25t + 15$.

18. 8.1 mL

19. a. A: \$0

B: \$100

C: \$200

b. A: \$80

B: \$50

C: \$33.33

c. A: $w = 80h$

B: $w = 50h + 100$

C: $w = 33.\bar{3}h + 200$

d. i. C

ii. A

e. The advantage of location C is that it has the highest minimum pay, so you would be guaranteed at least \$200 per day, regardless of how many hours you work.

f. i. A: \$160

B: \$200

C: \$266.66

ii. A: \$480

B: \$400

C: \$400

20. Only 2 points are needed to find the rule for the line, because a line is defined by 2 points.

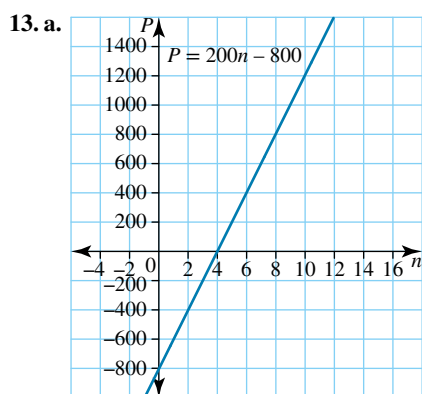
21. $y = \frac{1}{2}x - \frac{1}{3}$

Exercise 11.8 Practical applications of linear graphs

1. a. $y = 6x - 5$ b. $y = -3x + 8$ c. $y = 2x + 4$ d. $y = 0.9x + 1.1$
 e. $y = 3x + 1$ f. $y = -x + 17$ g. $C = 4n + 5$ h. $n = 5t$
 i. $v = -3t + 27$ j. $C = 3d + 8$
2. D
3. a. $A = 25t + 60$ b. \$5060 c. \$60
4. a. $C = 0.55s + 2.0$ b. \$8.60
5. a. $N = 400t + 3200$ b. 5400 c. 13 200
6. a. $W = -40t + 712$ b. 712 L c. 18 days
7. a. $N = -22t + 164$ b. 120 c. 5 d. 7.5 months
8. a. 27 b. 108
9. a. $C = 2.14d + 3.50$ b. \$28.11 c. \$52.93
10. a. $C = 2.57d + 3.50$ b. \$33.06 c. 7.20 km

11. $5 \times 21.4 = 107 \text{ cm} = 1.07 \text{ m}$

12. Answers will vary but should include the correct equation: $P = 10t + 200$.



- b. $(0, -800)$, i.e. $y = -800$; fixed costs are \$800
 - c. $(4, 0)$, i.e. $x = 4$; break-even amount
 - d. 200; sale price per print that contributes to profit
14. a. $Y = 78.7A$ b. ¥196750 c. $Y_T = 76.5A$

15. Cody arrives first.

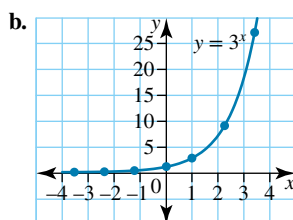
16. The value of a dependent variable depends on the value of the independent variable. The value of the independent variable is constant.

Exercise 11.9 Non-linear relationships (parabolas, exponentials, circles)

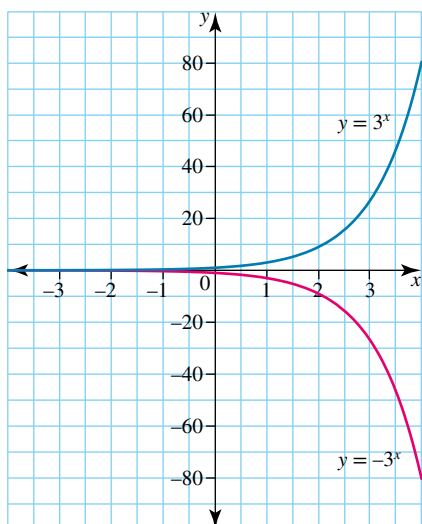
1. a. Vertical 3 up, vertex $(0, 3)$ b. Vertical 1 down, vertex $(0, -1)$
 c. Vertical 7 down, vertex $(0, -7)$ d. Vertical $\frac{1}{4}$ up, vertex $(0, \frac{1}{4})$
2. a. Horizontal 1 right, $(1, 0)$ b. Horizontal 2 right, $(2, 0)$
 c. Horizontal 10 left, $(-10, 0)$ d. Horizontal 4 left, $(-4, 0)$
3. a. $(0, 1)$, max b. $(0, -3)$, min c. $(-2, 0)$, max

4. a.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



5. a.

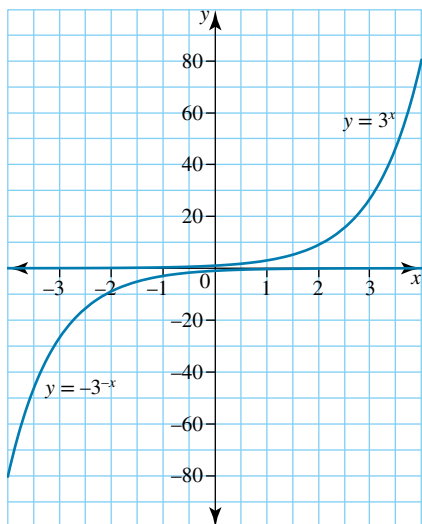


The graphs of $y = 3^x$ and $y = -3^x$ are the same shape.

The y -intercept of $y = 3^x$ is $(0, 1)$, the y -intercept of $y = -3^x$ is $(0, -1)$.

The graph of $y = -3^x$ is the graph of $y = 3^x$ reflected in the x -axis.

b.

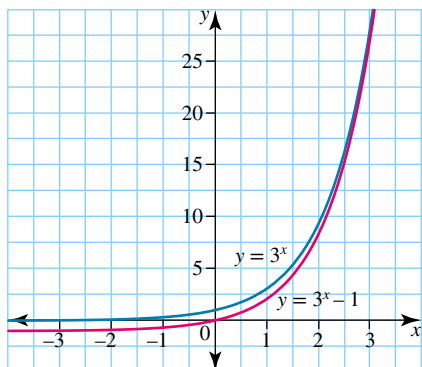


The graphs of $y = 3^x$ and $y = -3^{-x}$ are the same shape.

The y -intercept of $y = 3^x$ is $(0, 1)$, the y -intercept of $y = -3^{-x}$ is $(0, -1)$.

The graph of $y = -3^{-x}$ is the graph of $y = 3^x$ reflected in both the x and y -axes.

c.



The graphs of $y = 3^x$ and $y = 3^x + 1$ are the same shape.

The y -intercept of $y = 3^x$ is $(0, 1)$, the y -intercept of $y = 3^x - 1$ is $(0, 0)$.

The graph of $y = 3^x - 1$ is the graph of $y = 3^x$ moved down one unit.

6. B: **a** $y = 3^x$

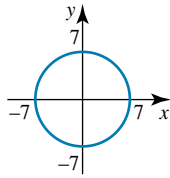
D: **b** $y = 10^x$

A: **c** $y = 2^x$

C: **d** $y = 5^x$

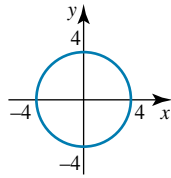
7. As the value of a gets larger, the graph becomes steeper.

8. a.



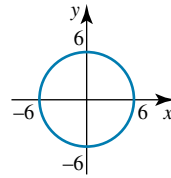
Centre (0, 0), radius 7

b.



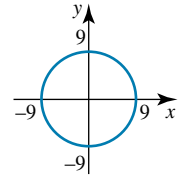
Centre (0, 0), radius 4

c.



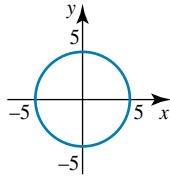
Centre (0, 0), radius 6

d.



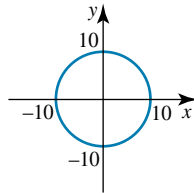
Centre (0, 0), radius 9

e.



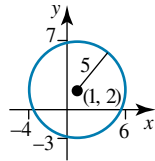
Centre (0, 0), radius 5

f.

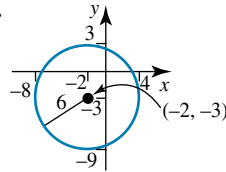


Centre (0, 0), radius 10

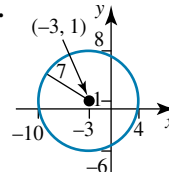
9. a.



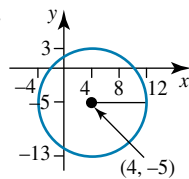
b.



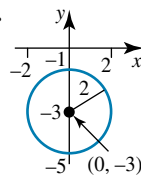
c.



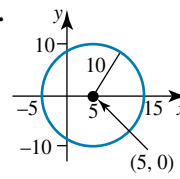
d.



e.



f.



10. D

11. Answers will vary. Possible methods include using a table of values or graphing both equations on a Cartesian plane.

12. Similarity: they both have the same asymptote, $y = 0$.

Difference: one is a reflection of the other about the x -axis.

13. Answers will vary; use algebra or graph the equation. The vertex is a minimum.

14. a. $V = 680\,000 \times 1.11^x$

b. \$1145839.55

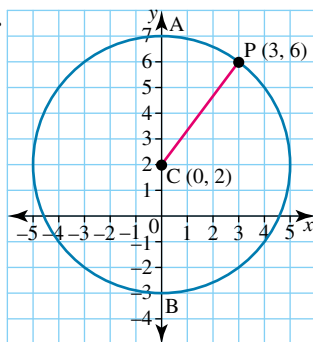
15. a. Bob started well and raced ahead of Frederick and Walter. He then turned around and went back to the starting line. He stayed at the starting line for an amount of time before turning around and sprinting to the finishing line, passing Frederick and Walter.

b. From the graph, Frederick took 5 seconds to begin running.

c. From the graph, at $t = 0$, Walter is 2.4 metres from the starting line.

d. Bob won the race.

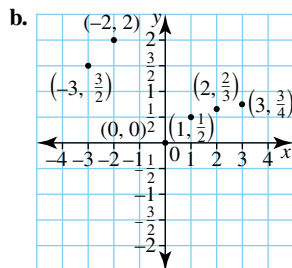
16.



At (0, 7) and (0, -3)

17. a.

x	-3	-2	-1	0	1	2	3
y	$\frac{3}{2}$	2	Undefined	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$



c. Not linear

18. Parabolas are symmetrical about a vertex and the simplest parabola is expressed as $y = x^2$. An exponential has a horizontal asymptote. Its most simple form is $y = a^x$, in this form there will never be a value for x when $y = 0$ and the y -intercept is 1. A circle is a path traced an equal distance from a centre point and its most simple form is $x^2 + y^2 = r^2$, where r is the radius or distance from the centre point.

Investigation | Rich task

1. 6
2. Teacher to check
3. a and h
4. e
5. a, b, c and h
6. Yes

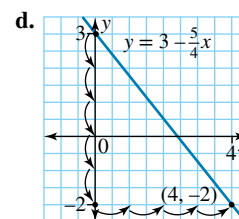
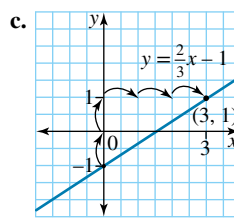
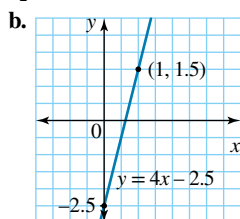
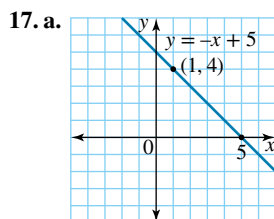
7.

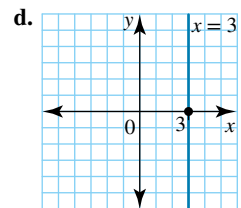
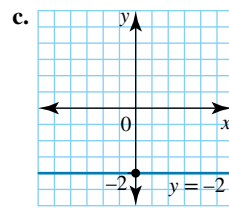
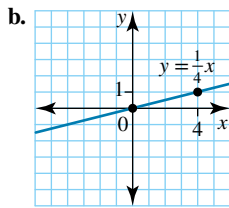
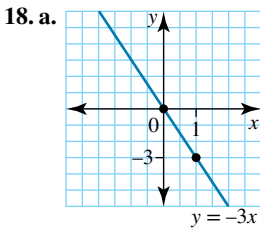
Length of table (m)	Width of table (m)	Destination pocket
3	0.25	Far left
3	0.5	Far left
3	0.75	Far left
3	1	Far right
3	1.25	Far left
3	1.5	Far left
3	1.75	Far right
3	2	Close right

8. Answers will vary.

Exercise 11.10 Review questions

1. D
2. B
3. D
4. B
5. D
6. A
7. B
8. A
9. E
10. A
11. a. i. 8
ii. -3
b. i. -2
ii. 6
c. i. $\frac{2}{3}$
ii. $-\frac{1}{3}$
12. a, b
13. a, c, e
14. a. 2
b. $-\frac{2}{5}$
c. 0
15. a. 2
b. $\frac{5}{4}$
16. a. $m = -3$, $b = 7$
b. $m = \frac{3}{2}$, $b = 3$
c. $m = -\frac{2}{5}$, $b = 0$
d. $m = 0$, $b = 4$





19. a. $y = 2x$

b. $y = 2 - \frac{2}{7}x$

20. a. $y = 2x - 7$

b. $y = 2x - 14$

c. $y = 2x - 5$

d. $y = -5x$

e. $y = -x - 2$

f. $y = -2.75x + 7.75$

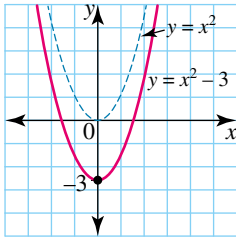
g. $y = x - 3$

h. $y = -2x + 5$

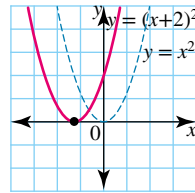
21. (1.1)

22. 5 units

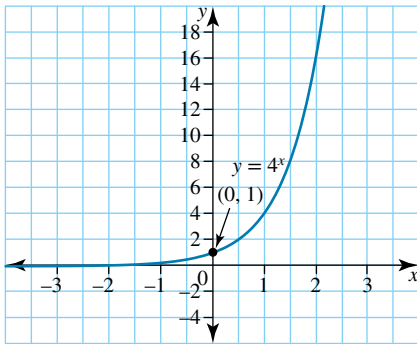
23. a. Vertical translation 3 units down; vertex = (0, -3)



b. Horizontal translation 2 units to the left; vertex = (2, 0)

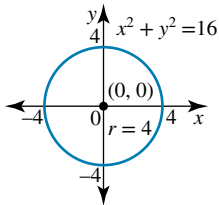


24. a.



b. Exponential

25.

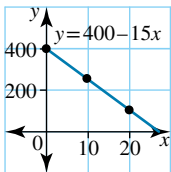


26. a. $y = 400 - 15x$

b. 26.7 or 27 weeks

c. \$175

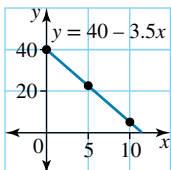
d. 21 repayments



27. a. $y = 40 - 3.5x$

b. 11.4 hours

c. 22.75 km

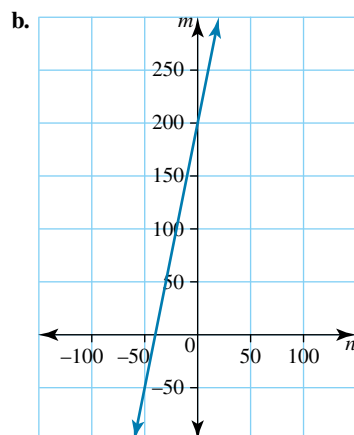


28. a. $n = 5t + 25$

b. 65

c. 336 minutes

29. a. $m = 5n + 200$



- c. Everything to the right of the vertical axis and including the vertical axis applies to her earnings because she can only sell 0, 1, 2, etc. necklaces.
- d. Everything to the left of the vertical axis does not apply to her earnings, because she cannot sell a negative number of necklaces and because she cannot make less than her base salary of \$200.

30. $\frac{1}{2}$

31. $(-1, 5)$

32. $p = -0.25, t = \frac{1}{4}$

33. $y = -3x - 4.5$

TOPIC 12

Quadratic equations

[Stage 5.2]

12.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

12.1.1 Why learn this?

Algebraic techniques help us to understand many things in the everyday world. What we learn in mathematics can be applied to many other subjects; for example, physics is often described as using mathematics as a tool for studying the world around us. The concepts studied in quadratic algebra can be applied to solving quadratic equations. Many physical phenomena can be described using quadratic equations, and in this chapter you will learn the techniques necessary to solve quadratic equations.



DISCUSSION

What benefits are there to knowing the movements of planetary bodies? How might this help the human race in the future?

LEARNING SEQUENCE

- 12.1** Overview
- 12.2** [Stage 5.2] Quadratic equations
- 12.3** [Stage 5.2] The Null Factor Law
- 12.4** [Stage 5.2] Solving the quadratic equation $ax^2 + bx + c = 0$
- 12.5** [Stage 5.2] Solving quadratic equations with two terms
- 12.6** [Stage 5.2] Applications of quadratic equations
- 12.7** Review

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- solves simple quadratic equations MA5.2-8NA

Related life skills outcome: MALS-19NA

CONTENT DESCRIPTIONS

Students:

Solve simple quadratic equations using a range of strategies (ACMNA241)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

12.2 Quadratic equations

[Stage 5.2]

12.2.1 General form of a quadratic equation

- A **quadratic equation** is an equation of the general form $ax^2 + bx + c = 0$.
- For example, $x^2 + 2x - 7 = 0$ is a quadratic equation, and so are $2x^2 = 8$ and $x^2 = 5x$.

WORKED EXAMPLE 1

Rearrange the following quadratic equations so that they are in general form and state the values of a , b and c .

a $5x^2 - 2x + 3 = 2x^2 + 4x - 12$ **b** $\frac{x^2}{2} - \frac{1}{6} = x\left(\frac{x}{3}\right) - 4$ **c** $x(3 - 2x) = 4(x - 6)$

THINK

- a**
- Write the equation.
 - Subtract $2x^2$ from both sides of the equation.
 - Subtract $4x$ from both sides of the equation.
 - Add 12 to both sides of the equation. The equation is now in the general form $ax^2 + bx + c = 0$.
 - Write the values of a , b and c .
- b**
- Write the equation.
 - Expand the bracket.
Lowest common denominator (LCD) = 6.
 - Multiply through by 6.
 - Simplify each fraction.
 - Collect like terms on the left-hand side. The equation is now in the general form $ax^2 + bx + c = 0$.
 - Write the values of a , b and c .
- c**
- Write the equation.
 - Expand the brackets.
 - To collect like terms on the left-hand side of the equation, subtract $4x$ from both sides.
 - Add 24 to both sides.

WRITE

a $5x^2 - 2x + 3 = 2x^2 + 4x - 12$

$$3x^2 - 2x + 3 = 4x - 12$$

$$3x^2 - 6x + 3 = -12$$

$$3x^2 - 6x + 15 = 0$$

$$a = 3, b = 6, c = 15$$

b $\frac{x^2}{2} - \frac{1}{6} = x\left(\frac{x}{3}\right) - 4$

$$\frac{x^2}{2} - \frac{1}{6} = \frac{x^2}{3} - 4$$

$$\frac{x^2}{2} \times \frac{6}{1} - \frac{1}{6} \times \frac{6}{1} = \frac{x^2}{3} \times \frac{6}{1} - 4 \times 6$$

$$3x^2 - 1 = 2x^2 - 24$$

$$x^2 - 1 = -24$$

$$x^2 + 23 = 0$$

$$a = 1, b = 0, c = 23$$

c $x(3 - 2x) = 4(x - 6)$

$$3x - 2x^2 = 4x - 24$$

$$-2x^2 - x = -24$$

$$-2x^2 - x + 24 = 0$$

5 Multiply all terms by -1 to make the x^2 term positive.

The equation is now in the general form

$$ax^2 + bx + c = 0.$$

6 Write the values of a , b and c .

$$2x^2 + x - 24 = 0$$

$$a = 2, b = 1, c = -24$$

12.2.2 Solving the equation $ax^2 + c = 0$

- Solving a quadratic equation means finding the values of x that satisfy the equation.
- Some quadratic equations have two solutions, some have only one solution, and some have no solutions. The solutions may be rational or irrational numbers. If the solutions are irrational, they can be expressed as exact values or decimal approximations.

WORKED EXAMPLE 2

Solve the following equations.

a $2x^2 - 18 = 0$

b $x^2 + 9 = 0$

c $3x^2 + 4 = 4$

d $3x^2 = 6$

THINK

a 1 Write the equation. The aim is to make x the subject.

2 Add 18 to both sides.

3 Divide both sides by 2.

4 Take the square root of both sides.

There are two square roots of 9.

b 1 Write the equation. The aim is to make x the subject.

2 Take 9 from both sides.

3 Take the square root of both sides.

c 1 Write the equation. The aim is to make x the subject.

2 Take 4 from both sides.

3 Divide both sides by 3.

4 Take the square root of both sides. Zero has only one square root.

d 1 Write the equation. The aim is to make x the subject.

2 Divide both sides by 3.

3 Take the square root of both sides. This gives two solutions.

4 Write the answers as decimal approximations.

WRITE

a $2x^2 - 18 = 0$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = 3, -3$$

Two solutions: $x = 3, x = -3$

b $x^2 + 9 = 0$

$$x^2 = -9$$

-9 has no square root.

The equation has no solution.

c $3x^2 + 4 = 4$

$$3x^2 = 0$$

$$x^2 = 0$$

$$x = 0$$

One solution: $x = 0$

d $3x^2 = 6$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$x = -1.414 \text{ or } x = 1.414$$

DISCUSSION

Explain why quadratic equations could be expected to have two solutions.

12.2.3 Confirming the solutions to quadratic equations

- Solutions to quadratic equations are confirmed by substituting the values into the equation.

WORKED EXAMPLE 3

Determine whether any of the following are solutions to the quadratic equation $x^2 = 4x - 3$.

a $x = 1$

b $x = 2$

c $x = 3$

THINK

- a**
- 1 Substitute $x = 1$ into both sides of the equation.
 - 2 Evaluate the left-hand side.
 - 3 Evaluate the right-hand side.
 - 4 Since the left-hand side equals the right-hand side, $x = 1$ is a solution. Write the answer.
- b**
- 1 Substitute $x = 2$ into both sides of the equation.
 - 2 Evaluate the left-hand side.
 - 3 Evaluate the right-hand side.
 - 4 The left-hand side does not equal the right-hand side, so $x = 2$ is not a solution. Write the answer.
- c**
- 1 Substitute $x = 3$ into both sides of the equation.
 - 2 Evaluate the left-hand side.
 - 3 Evaluate the right-hand side.
 - 4 The left-hand side equals the right-hand side, so $x = 3$ is a solution. Write the answer.

WRITE

- a** $x^2 = 4x - 3$
- $$\begin{aligned}\text{LHS} &= (1)^2 \\ &= 1 \\ \text{RHS} &= 4(1) - 3 \\ &= 4 - 3 \\ &= 1\end{aligned}$$
- $x = 1$ is a solution to the equation.
- b** $x^2 = 4x - 3$
- $$\begin{aligned}\text{LHS} &= (2)^2 \\ &= 4 \\ \text{RHS} &= 4(2) - 3 \\ &= 8 - 3 \\ &= 5\end{aligned}$$
- $x = 2$ is not a solution to the equation.
- c** $x^2 = 4x - 3$
- $$\begin{aligned}\text{LHS} &= (3)^2 \\ &= 9 \\ \text{RHS} &= 4(3) - 3 \\ &= 12 - 3 \\ &= 9\end{aligned}$$
- $x = 3$ is a solution to the equation.

Exercise 12.2 Quadratic equations

Individual pathways

PRACTISE

Questions:
1–3, 5, 7, 9, 10

CONSOLIDATE

Questions:
1, 3, 5, 7–12

MASTER

Questions:
2, 4–12

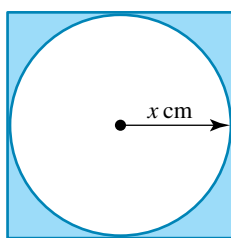
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE1** Rearrange each of the following quadratic equations so that they are in the form $ax^2 + bx + c = 0$.
 - $3x - x^2 + 1 = 5x$
 - $5(x - 2) = x(4 - x)$
 - $x(5 - 2x) = 6(5 - x^2)$
 - $4x^2 - 5x + 2 = 6x - 4x^2$
 - $5(x - 12) = x(4 - 2x)$
 - $x^2 - 4x - 16 = 1 - 4x$
- MC** a. Which of the following is a quadratic equation?
 - $2x - 1 = 0$
 - $2^x - 1 = 0$
 - $x^2 - x = 1 + x^2$
 - $x^2 - x = 1 - x^2$
 - $\frac{1}{x} = 4$
 b. **MC** Which of the following is not a quadratic equation?
 - $x(x - 1) = 2x - 1$
 - $-3x^2 + 2x = 1$
 - $3(x + 2) + 5(x + 3) = 2(x + 1)$
 - $2(x - 1) + 3x = x(2x - 3)$
 - $x^2 = 4$
- WE2a, b, c** Solve the following quadratic equations.
 - $x^2 - 16 = 0$
 - $2x^2 + 18 = 0$
 - $3x^2 + 2x = x(x + 2)$
 - $2(x^2 + 7) = 16$
 - $x^2 + 17 = 13$
 - $-3x^2 + 17 = 5$
- WE2** Solve the following quadratic equations.
 - $4(x + 2)^2 - 4 = 0$
 - $4(x + 2)^2 + 4 = 0$
 - $2(x + 3)^2 - 32 = 0$
 - $2(x - 4)^2 - 32 = 0$
 - $(x + 2)^2 - 9 = 0$
 - $-(x - 2)^2 - 9 = 0$
 - $3(x + 5)^2 - 10 = 2$
 - $3(x - 4)^2 + 48 = 36$
 - $2(x - 8)^2 + 4 = 4$
- WE3** Determine whether $x = 4$ is a solution to the following equations.
 - $x^2 = x + 12$
 - $x^2 = 3x + 1$
 - $x^2 = 4x$
- Determine whether $x = -3$ is a solution to the following equations.
 - $x^2 = x + 12$
 - $x^2 = 3x + 1$
 - $x^2 = 4x$
- Determine whether $x = 0$ is a solution to the following equations.
 - $x^2 = x + 12$
 - $x^2 = 3x + 1$
 - $x^2 = 4x$
- Determine whether either of the following are solutions to the equation $5(x - 1)^2 + 7 = 27$.
 - $x = -1$
 - $x = 1$

Communicating, reasoning and problem solving

- a. Show that the shaded area can be represented by $A = x^2(4 - \pi)$.
 b. If the shaded area = 10 cm^2 , find the value of x correct to 3 significant figures.
 c. Why can't x be equal to -3.41 ?



- Explain why $x^2 + 7x + 4 = 7x$ has no solutions.
- The sum of 5 and the square of a number is 41.
 - What is the number?
 - Is there more than one possible number? If so, what is the other number and why is there more than one?
- Two less than a number is squared and its result is tripled. The difference between this and 7 is 41.
 - Write the algebraic equation.
 - What is the number? Give both possible answers.

12.3 The Null Factor Law

[Stage 5.2]

- The **Null Factor Law** states that if the product of two or more factors is zero, then *at least one* of the factors must be zero.

$$\text{If } a \times b = 0,$$

then:

$$a = 0 \text{ or}$$

$$b = 0 \text{ or}$$

$$a = b = 0.$$

For example, if $(x - 5)(x - 2) = 0$, then:

$$(x - 5) = 0 \text{ or } (x - 2) = 0 \text{ or both } x - 5 = x - 2 = 0.$$

$$\text{If } x - 5 = 0 \quad \text{If } x - 2 = 0$$

$$x = 5 \quad x = 2$$

Both $x = 5$ and $x = 2$ make the equation true.

- This can be checked by substituting the values into the original equation.

For example:

if $x = 5$, then:

$$(x - 5)(x - 2) = (5 - 5)(x - 2)$$

$$\text{if } x = 2, \text{ then: } = 0(x - 2)$$

$$= 0$$

$$(x - 5)(x - 2) = (x - 5)(2 - 2)$$

$$= (x - 5)0$$

$$= 0$$

WORKED EXAMPLE 4

Solve each of the following quadratic equations.

a $(x - 2)(2x + 1) = 0$

c $x(x - 3) = 0$

THINK

- a 1** The product of 2 factors is 0, so apply the Null Factor Law.

- 2** One of the factors must equal zero.

- 3** Solve the equations.

- 4** Write the solutions.

- b 1** The product of 2 factors is 0, so apply the Null Factor Law.

- 2** One of the factors must equal zero.

- 3** Solve the equations.

- 4** Write the solutions.

b $(4 - 3x)(6 + 11x) = 0$

d $(x - 1)^2 = 0$

WRITE

a $(x - 2)(2x + 1) = 0$

Either $x - 2 = 0$ or $2x + 1 = 0$.

$$x = 2 \quad 2x = -1$$

$$x = \frac{-1}{2}$$

$$x = -\frac{1}{2}, x = 2$$

b $(4 - 3x)(6 + 11x) = 0$

Either $4 - 3x = 0$ or $6 + 11x = 0$.

$$-3x = -4 \quad 11x = -6$$

$$-3x = -4 \quad 11x = -6$$

$$x = \frac{4}{3} \quad x = \frac{-6}{11}$$

$$x = \frac{4}{3}, -\frac{6}{11}$$

c 1 The product of 2 factors is 0, so apply the Null Factor Law. 2 One of the factors must equal zero. 3 Solve the equations. 4 Write the solutions.	c $x(x - 3) = 0$ Either $x = 0$ or $x - 3 = 0$. $x = 3$ $x = 0, 3$
d 1 Write the equation. 2 Rewrite the squared factors as the product of two factors. 3 Apply the Null Factor Law. 4 Solve the equations. The two solutions are the same. 5 Write the solution.	d $(x - 1)^2 = 0$ $(x - 1)(x - 1) = 0$ Either $x - 1 = 0$ or $x - 1 = 0$. $x = 1$ $x = 1$ $x = 1$

Note: Quadratic equations can have a maximum of two solutions. (This is known as the **Fundamental Theorem of Algebra**.)

learnon

RESOURCES — ONLINE ONLY

Interactivity: The Null Factor Law (int-6095)

eLesson: The Null Factor Law (eles-2312)

Digital doc: SkillsHEET Solving linear equations (doc-10984)

Exercise 12.3 The Null Factor Law

Individual pathways

PRACTISE

Questions:
1a–i, 2a–d, 3–6

CONSOLIDATE

Questions:
1d–l, 2c–f, 3–7

MASTER

Questions:
1h–o, 2e–h, 3–10

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Understanding and fluency

- 1. WE14** Solve each of the following quadratic equations.

a. $(x - 2)(x + 3) = 0$	b. $(2x + 4)(x - 3) = 0$	c. $(x + 2)(x - 3) = 0$
d. $(2x + 5)(4x + 3) = 0$	e. $(x + 4)(2x + 1) = 0$	f. $(2x - 1)(x + 30) = 0$
g. $(2x + 1)(3 - x) = 0$	h. $(1 - x)(3x - 1) = 0$	i. $x(x - 2) = 0$
j. $\left(x + \frac{1}{3}\right)\left(2x - \frac{1}{2}\right) = 0$	k. $(5x - 1.5)(x + 2.3) = 0$	l. $\left(2x + \frac{1}{3}\right)\left(2x - \frac{1}{3}\right)$
m. $(x - 2)^2 = 0$	n. $x(4x - 15) = 0$	o. $(x + 4)^2 = 0$

2. The Null Factor Law can be extended to products of more than 2 factors. Use this to find all the solutions to the following equations.

a. $(x - 2)(x + 2)(x + 3) = 0$

b. $(x + 2)(x + 2)(2x - 5) = 0$

c. $(x + 2)(x + 2)(x + 4) = 0$

d. $x(x + 2)(3x + 12) = 0$

e. $(2x - 2.2)(x + 2.4)(x + 2.6) = 0$

f. $(2x + 6)\left(x + \frac{1}{2}\right)(9x - 15) = 0$

g. $3(x - 3)^2 = 0$

h. $(x + 1)(x - 2)^2 = 0$

3. **MC** a. The solutions to $(2x - 4)(x + 7) = 0$ are:

A. $x = 4, x = 7$

B. $x = 4, x = -7$

C. $x = 2, x = 7$

D. $x = 2, x = -7$

E. $x = -4, x = -7$

- b. The Null Factor Law cannot be applied to the equation $x\left(x + \frac{1}{2}\right)\left(\frac{x}{2} - 1\right) = 1$ because:

A. there are more than 2 factors

B. the right-hand side equals 1

C. the first factor is a simple x -term

D. the third term has x in a fraction

E. none of the above

4. Rewrite the following equations so that the Null Factor Law can be used. Then solve the resulting equation.

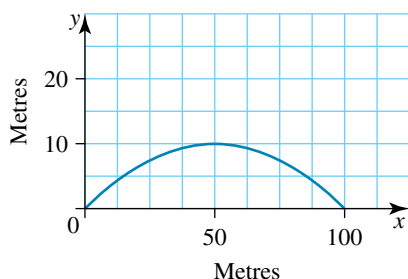
a. $x^2 + 10x = 0$

b. $2x^2 - 14x = 0$

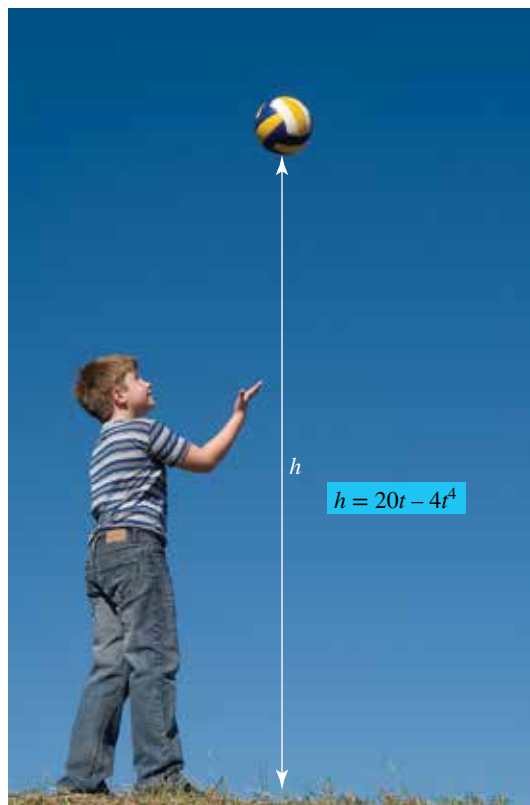
c. $25x^2 - 40x = 0$

Communicating, reasoning and problem solving

5. a. Explain why $x = 2$ is not a solution of the equation $(2x - 2)(x + 2) = 0$.
b. What are the solutions to the equation?
6. What is the maximum number of solutions that a quadratic such as $x^2 - x - 56 = 0$ can have? Explain why.
7. A bridge is constructed with a supporting structure in the shape of a parabola as shown in the diagram. The origin $(0, 0)$ is at the left-hand edge of the bridge, which is 100 m long.



- a. What is the maximum height of the bridge support?
- b. If the equation of the support is $y = ax(b - x)$, determine the values of a and b . (Hint: Let $y = 0$.)
- c. What is the height of the support when $x = 62$?
8. Consider a ball thrown upwards so that it reaches a height of h metres after t seconds. The expression $20t - 4t^2$ represents the height of the ball, in metres, after t seconds.
- a. Factorise the expression for height.
- b. How high is the ball after:
- 1 second
 - 5 seconds?
- c. How did factorising the expression make it easier to evaluate?



9. The product of 7 more than a number and 14 more than that same number is 330. What are the possible values of the number?
10. When can the Null Factor Law be used, and when is it not able to be used?

12.4 Solving the quadratic equation

$$ax^2 + bx + c = 0$$

[Stage 5.2]

- If $ax^2 + bx + c$ can be factorised, then the quadratic equation $ax^2 + bx + c = 0$ can be solved using the Null Factor Law.

WORKED EXAMPLE 5

Solve the following quadratic equations.

a $x^2 - 5x - 6 = 0$

b $x^2 + 14x = 15$

c $2x^2 + 7x + 3 = 0$

THINK

a 1 Write the equation.

2 Factorise $x^2 - 5x - 6$.

3 Apply the Null Factor Law.

4 Solve the equations.

5 Write the solutions.

b 1 Write the equation and transpose so that the right-hand side equals 0.

2 Factorise $x^2 - 14x - 15$.

3 Apply the Null Factor Law.

4 Solve the linear equations.

5 Write the solutions.

c 1 Write the equation, making sure that the right-hand side is equal to zero. Check for common factors.

2 Rewrite the expression on the left-hand side by breaking the middle term into two terms.

3 Factorise by grouping terms.

4 Apply the Null Factor Law.

5 Solve the linear equations.

6 Write the solutions.

WRITE

a $x^2 - 5x - 6 = 0$

$$(x - 6)(x + 1) = 0$$

$$\begin{array}{l|l} x - 6 = 0 & x + 1 = 0 \\ x = 6 & x = -1 \end{array}$$

$$x = -1, x = 6$$

b $x^2 + 14x = 15$

$$x^2 + 14x - 15 = 0$$

$$(x + 15)(x - 1) = 0$$

$$\begin{array}{l|l} x + 15 = 0 & x - 1 = 0 \\ x = -15 & x = 1 \end{array}$$

$$x = -15, x = 1$$

c $2x^2 + 7x + 3 = 0$

$$2x^2 + 6x + x + 3 = 0$$

$$2x(x + 3) + 1(x + 3) = 0$$

$$(2x + 1)(x + 3) = 0$$

$$\begin{array}{l|l} 2x + 1 = 0 & x + 3 = 0 \\ x = -\frac{1}{2} & x = -3 \end{array}$$

$$x = -\frac{1}{2}, x = -3$$

Exercise 12.4 Solving the quadratic equation $ax^2 + bx + c = 0$

Individual pathways

PRACTISE

Questions:

1a, c, g, k, o, 2–4, 6

CONSOLIDATE

Questions:

1b, d, f, j, n, 2–4, 6–8

MASTER

Questions:

1b, d, f, h, j, l, n, 2–10

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Understanding and fluency

1. **WE15** Solve each of the following quadratic equations.

a. $x^2 - 6x + 8 = 0$

b. $x^2 + 6x + 8 = 0$

c. $x^2 + 6x + 5 = 0$

d. $x^2 + x - 6 = 0$

e. $x^2 + 2x - 15 = 0$

f. $x^2 + 4x + 4 = 0$

g. $x^2 + 2x - 24 = 0$

h. $x^2 - 5x - 24 = 0$

i. $x^2 - x - 12 = 0$

j. $x^2 + 13x + 12 = 0$

k. $x^2 - 10x = 11$

l. $x^2 + x = 20$

m. $x^2 + 29x = -100$

n. $x^2 - 15x = -50$

o. $0 = x^2 - 2x - 8$

2. **MC** a. The quadratic equation $x^2 + 2x - 8 = 0$ has solutions:

A. $x = 4$ and $x = -2$

B. $x = -4$ and $x = 2$

C. $x = -4$ and $x = 4$

D. $x = -2$ and $x = 2$

E. $x = -8$ and $x = 0$

- b. The quadratic equation $x^2 - 7x - 8 = 0$ has solutions:

A. $x = 1$ and $x = 8$

B. $x = -1$ and $x = -8$

C. $x = -8$ and $x = 1$

D. $x = -1$ and $x = 8$

E. $x = -8$ and $x = -7$

- c. The quadratic equation $4a^2 + 13a + 3 = 0$ has solutions:

A. $a = 3$ and $a = 1$

B. $a = -3$ and $a = -1$

C. $a = \frac{1}{4}$ and $a = 3$

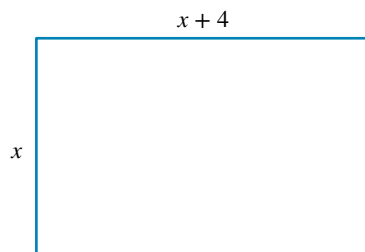
D. $a = -3$ and $a = -\frac{1}{4}$

E. $x = -3$ and $x = 4$

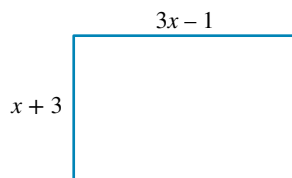
3. Solve the following for x .

$$2x^2 + 4x + 5 = 3x^2 + 9x - 1$$

4. The rectangle shown has an area of 45 cm^2 . By solving a quadratic equation, find the dimensions of the rectangle.

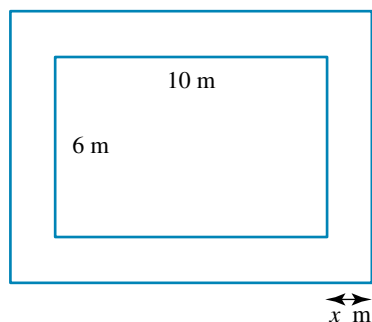


5. Find the dimensions of this rectangle, which has an area of 48 m^2 .

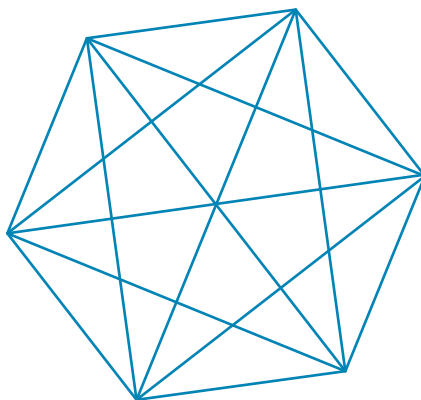


Communicating, reasoning and problem solving

6. A small rectangular pool, 6 m by 10 m, is surrounded by a paved pathway of uniform width as shown. How wide is the pavement if it covers an area of 57 m^2 ?



7. The number of diagonals in a polygon is given by the formula $D = \frac{n}{2}(n - 3)$, where n is the number of sides in the polygon.
- How many diagonals are there in:
 - a triangle
 - a square
 - a decagon?
 - What type of polygon has:
 - 20 diagonals
 - 170 diagonals?



8. a. Write algebraic expressions for each of the parts below. Your friend picks a number, n .
- He adds 1 to it.
 - He adds 7 to it.
 - He multiplies these two new numbers together.
 - He then divides the product in part **iii** by the product of 2 more than the original number, n , and 1 more than the original number, n .
- b. The result of the division is 2. What number, n , did your friend begin with?
9. Calculate the value(s) of x for which the expressions $4x^2 + 13x - 16$ and $3x^2 + 8x - 2$ are equal by following the steps outlined below:
- Equate the expressions.
 - Rewrite the resulting equation by collecting like terms so that the equation is equal to zero.

- c. Factorise the quadratic equation.
 d. Apply the Null Factor Law and solve.
10. A girl, her brother and her teacher are shown below. The product of the teacher's and brother's ages is 256.
- Write an algebraic expression in terms of g to represent the teacher's age.
 - Use the algebraic expressions to write an expression for the product of the brother's and teacher's ages and expand this product.
 - Write an equation using your answer to part **b** for the product of their ages. *Hint:* An equation has an 'is equal to' part to it.
 - Rearrange your answer to part **c** by collecting like terms and making it equal to zero.
 - Find the value of g by solving the equation in part **d** using the Null Factor Law. *Note:* Discard any value that does not make sense.
 - State the ages of the girl, her brother and the teacher.

I am 6 years younger than my sister.



I am 18 years older than this student.



If I am g years old, my brother is $g - 6$.



12.5 Solving quadratic equations with two terms

[Stage 5.2]

12.5.1 Solving quadratic equations of the form $ax^2 + c = 0$

- As this form of quadratic only has two terms, it can be solved by rewriting the equation with only the x^2 term on the left-hand side.

WORKED EXAMPLE 6

Solve each of the following quadratic equations.

a $x^2 - 1 = 0$

b $2x^2 - 16 = 0$

THINK

- Write the equation.
- Rewrite the equation with x^2 only on the left-hand side

WRITE

a $x^2 - 1 = 0$
 $x^2 = 1$

3 Take the square root of both sides.

4 State the two solutions.

$$x = \pm\sqrt{1}$$

$$x = 1, x = -1$$

(This can be abbreviated to $x = \pm 1$.)

b 1 Write the equation.

2 Take out the common factor.

3 Divide both sides of the equation by 2.

4 Rewrite the equation with x^2 only on the left-hand side.

5 Take the square root of both sides.

6 Simplify the surd.

7 State the two solutions as exact values and as approximate values.

$$\mathbf{b} \quad 2x^2 - 16 = 0$$

$$2(x^2 - 8) = 0$$

$$x^2 - 8 = 0$$

$$x^2 = 8$$

$$x = \pm\sqrt{8}$$

$$x = \pm 2\sqrt{2}$$

$$x = \pm 2\sqrt{2}$$

$$x = \pm 2.82$$

12.5.2 Solving quadratic equations of the form $ax^2 + bx = 0$

- The equation $ax^2 + bx = 0$ can be solved easily using the Null Factor Law.

WORKED EXAMPLE 7

Solve each of the following equations.

a $x^2 + 4x = 0$

b $-2x^2 + 4x = 0$

THINK

a 1 Write the equation.

2 Factorise by taking out a common factor of x .

3 Apply the Null Factor Law.

4 Solve the equations.

5 Write the solutions.

b 1 Write the equation.

2 Factorise by taking out the common factor of $-2x$.

3 Apply the Null Factor Law.

4 Solve the equations.

5 Write the solutions.

WRITE

a $x^2 + 4x = 0$

$$x(x + 4) = 0$$

$$x = 0 \quad x + 4 = 0$$

$$x = 0 \quad x = -4$$

$$x = -4, x = 0$$

b $-2x^2 + 4x = 0$

$$-2x(x + 2) = 0$$

$$-2x = 0 \text{ or } x + 2 = 0$$

$$x = 0 \quad x = -2$$

$$x = -2, x = 0$$

WORKED EXAMPLE 8

If the square of a number is multiplied by 5, the answer is 45. Find the number.

THINK

1 Define the number.

2 Write an equation that can be used to find the number.

3 Transpose to make the right-hand side equal to zero.

WRITE

Let x be the number.

$$5x^2 = 45$$

$$5x^2 - 45 = 0$$

Solve the equation.

$$5(x^2 - 9) = 0$$

$$x^2 - 9 = 0$$

$$x^2 - 3^2 = 0$$

$$(x + 3)(x - 3) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -3 \quad \text{or} \quad x = 3$$

4 Write the answer in a sentence.

The number is either 3 or -3 .

learnON RESOURCES — ONLINE ONLY

Digital doc: SkillsHEET Factorising expressions of the type $ax^2 + bx$ (doc-10985)

Exercise 12.5 Solving quadratic equations with two terms

Individual pathways

PRACTISE

Questions:
1–7

CONSOLIDATE

Questions:
1–9, 11, 12

MASTER

Questions:
1–13

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Understanding and fluency

1. **WE16** Solve each of the following quadratic equations.

a. $x^2 - 9 = 0$

b. $x^2 - 16 = 0$

c. $2x^2 - 18 = 0$

d. $2x^2 - 50 = 0$

e. $100 - x^2 = 0$

f. $49 - x^2 = 0$

g. $3x^2 - 27 = 0$

h. $5x^2 - 20 = 0$

i. $x^2 + 6 = 0$

j. $2x^2 + 18 = 0$

k. $-x^2 + 9 = 0$

l. $-3x^2 + 48 = 0$

m. $-4x^2 + 100 = 0$

n. $x^2 = 0$

o. $-x^2 = 0$

2. **WE17** Solve each of the following equations.

a. $x^2 + 6x = 0$

b. $x^2 - 8x = 0$

c. $x^2 + 9x = 0$

d. $x^2 - 11x = 0$

e. $2x^2 - 12x = 0$

f. $2x^2 - 15x = 0$

g. $3x^2 - 2x = 0$

h. $4x^2 + 7x = 0$

i. $2x^2 - 5x = 0$

j. $x^2 + x = 0$

k. $4x^2 - x = 0$

l. $-x^2 - 5x = 0$

m. $-2x^2 - 24x = 0$

n. $-x^2 + 18x = 0$

o. $x^2 - 2.5x = 0$

3. **MC** a. The solutions to $4x^2 - 36 = 0$ are:

A. $x = 3$ and $x = -3$

B. $x = 9$ and $x = -9$

C. $x = 1$ and $x = -1$

D. $x = 2$ and $x = -2$

E. $x = 4$ and $x = 0$

b. What are the solutions to $x^2 - 5x = 0$?

A. $x = 1$ and $x = 5$

B. $x = 0$ and $x = -5$

C. $x = 0$ and $x = 5$

D. $x = -1$ and $x = 5$

E. $x = 0$

4. **WE18** If the square of a number is multiplied by 2, the answer is 32. Find the number.

5. A garden has two vegetable plots. One plot is a square; the other plot is a rectangle with one side 3 m shorter than the side of the square and the other side 4 m longer than the side of the square. Both plots have the same area. Sketch a diagram and find the dimensions of each plot.
6. The square of a number is equal to 10 times the same number. What is the number?

Communicating, reasoning and problem solving

7. Explain why the equation $x^2 + 9 = 0$ cannot be solved.
8. Solve the equation $m^2x^2 - n^2 = 0$ by using inverse operations.
9. Explain why the equation $x^2 + bx = 0$ cannot be solved by using inverse operations.
10. A quadratic expression may be written as $ax^2 + bx + c$. Using examples, explain why b and c may take any values, but a cannot equal zero.
11. The square of a number tripled is equal to 24 times the same number. What is the number?
12. The sum of 6 times the square of a number and 72 times the same number is equal to zero. What is the number?
13. A rectangular horse paddock has a width of x and a length that is 30 metres longer than its width. The area of the paddock is $50x$ square metres.
 - a. Write the algebraic equation for the area described.
 - b. Factorise the equation.
 - c. Solve the equation.
 - d. Can you use all your solutions? Explain why or why not.



12.6 Applications of quadratic equations

[Stage 5.2]

- There are many situations in science, engineering, economics and other fields where quadratic equations can be applied to finding solutions to problems.

WORKED EXAMPLE 9

The distance travelled by an accelerating skier is given by the formula $d = 3t + t^2$, where t is the time in seconds and d is the distance in metres. If the distance travelled was 130 m, for how long was the skier travelling?



THINK

- 1 Write the equation.
- 2 Substitute the given value, $d = 130$, into the equation.
- 3 Rearrange the equation so that it is in the form $ax^2 + bx + c = 0$.

WRITE

$$\begin{aligned}
 d &= 3t + t^2 \\
 130 &= 3t + t^2 \\
 0 &= 3t + t^2 - 130 \\
 t^2 + 3t - 130 &= 0
 \end{aligned}$$

4 Factorise the left-hand side of the equation, using the factors of -130 that add up to 3 .

$$\begin{aligned} -130: -10 + 13 &= 3 \\ (t - 10)(t + 13) &= 0 \end{aligned}$$

5 Solve the equation by using the Null Factor Law.

$$\begin{aligned} \text{Either } t - 10 &= 0 \text{ or } t + 13 = 0 \\ t &= 10 \qquad t = -13 \end{aligned}$$

6 Evaluate the result.

The only feasible answer is $t = 10$, because time is always positive.

7 Answer the question in a sentence.

The skier was travelling for 10 seconds.

WORKED EXAMPLE 10

When 10 is added to the square of a positive number, the result is equal to 3 times the number subtracted from twice its square. Find the number.

THINK

- 1 Define the unknown quantity.
- 2 Translate each part of the sentence into an algebraic expression or term.
- 3 Form a quadratic equation and rearrange it so that it is in the form $ax^2 + bx + c = 0$.
- 4 Solve the equation by factorising.
- 5 Evaluate the result.
- 6 Answer the question in a sentence.

WRITE

Let x be the positive number.

The square of a positive number: x^2

Add 10 to the (positive number)²: $x^2 + 10$

Three times the number: $3x$

Three times the number is subtracted from twice its square: $2x^2 - 3x$

$$\begin{aligned} x^2 + 10 &= 2x^2 - 3x \\ 10 &= x^2 - 3x \\ x^2 - 3x - 10 &= 0 \end{aligned}$$

$$(x - 5)(x + 2) = 0$$

$$x - 5 = 0 \text{ or } x + 2 = 0$$

$$x = 5 \qquad x = -2$$

The number is positive, so $x = 5$ is the only valid solution.

The number is 5.

Exercise 12.6 Applications of quadratic equations

Individual pathways

PRACTISE

Questions:
1, 3, 5, 7, 9

CONSOLIDATE

Questions:
1, 3–5, 7, 8, 10

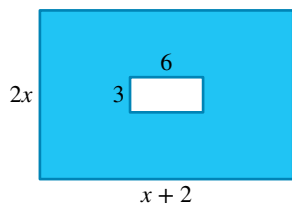
MASTER

Questions:
2, 4–11

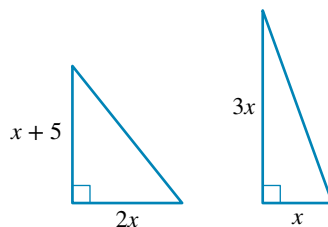
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. The product of two consecutive numbers is 42. What are the numbers?
2. The product of two consecutive odd numbers is 143. What are the numbers?
3. If the shaded area is 180 square units, find the value of x .

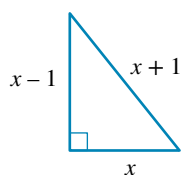


4. If these triangles have the same area, what is the value of x ?

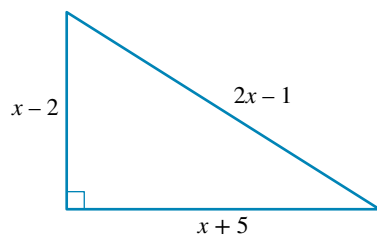


Communicating, reasoning and problem solving

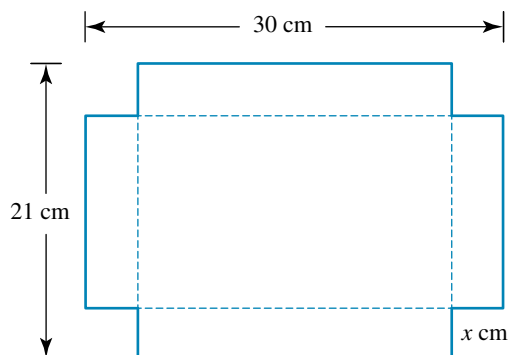
5. Find the side lengths of this right-angled triangle. Show your working.



6. Find the side lengths of this right-angled triangle. Show your working.



7. A sheet of A4 paper is approximately 30 cm by 21 cm. Four corners are cut from a sheet as shown, and the remaining paper is folded to make a box with a base area of 286 cm^2 . What are the dimensions of the box?



8. A gardener maintains a large rectangular rose garden of length 15 m and width 12 m. He plans to lay a wide pathway around the garden that will require 160 m^2 of paving. What will be the width of the pathway?



9. A cricket ball is struck high into the air. After t seconds its height, h , is given by the formula $h = 20t - 5t^2$.
- What is its height after 1 second?
 - When will the ball strike the ground?
10. Each term in the sequence $-3, -1, 1, 3, 5, \dots$ is 2 more than the previous term. The sum S of the first n terms in the sequence is given by the formula $S = n^2 - 4n$.
- What is the sum of the first 4 terms?
 - How many terms are needed to make a sum of 77?
11. Describe a situation that might require the application of quadratic equations.



12.7 Review

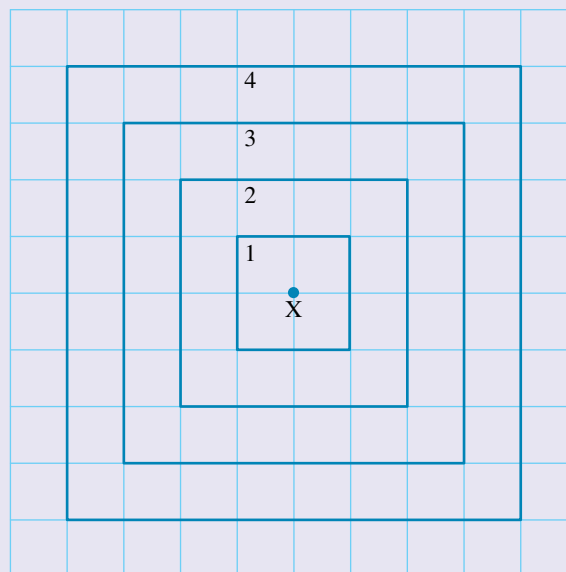
Investigation | Rich task

Concentric squares

Consider a set of squares drawn around a central point on a grid as shown in this diagram,

These squares can be regarded as concentric squares, because the central point of each square is the point X. The diagram shows four labelled squares drawn on 1-centimetre grid paper.

1. Use the diagram at right to complete the following table, leaving your answers in simplest surd form, if necessary. The first square has been completed for you.



Square	1	2	3	4
Side length (cm)	2			
Diagonal length (cm)	$2\sqrt{2}$			
Perimeter (cm)	8			
Area (cm ²)	4			

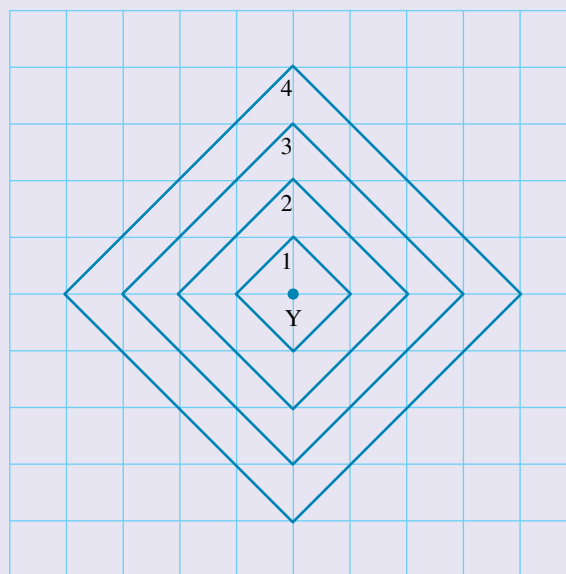
Observe the patterns in the table and answer questions 2 and 3.

2. What would be the side length of the tenth square of this pattern?
3. What would be the length of the diagonal of this tenth square?

Consider, now, a different arrangement of the squares on 1-centimetre grid paper, as shown in the diagram.

These squares all still have a central point, labelled Y.

4. Use the diagram to complete the following table for these squares, leaving your answers in simplest surd form if necessary.

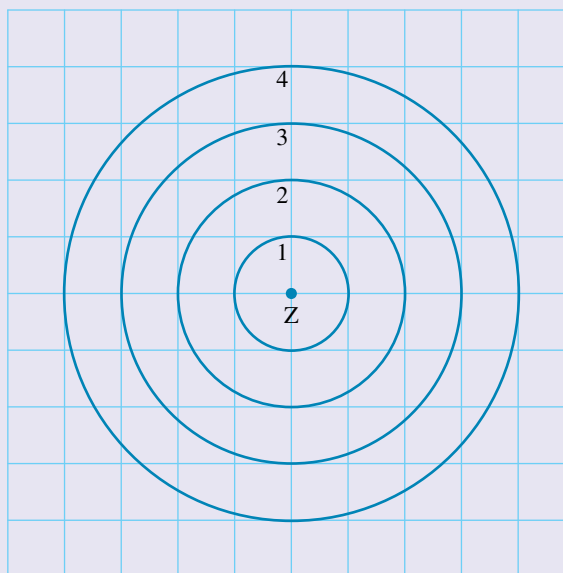


Square	1	2	3	4
Side length (cm)	$\sqrt{2}$			
Diagonal length (cm)	2			
Perimeter (cm)	$4\sqrt{2}$			
Area (cm ²)	2			

5. What would be the side length of the tenth square in this pattern of squares?
6. What would be the length of the diagonal of the tenth square?

Use the two diagrams above to answer questions 7 to 11.

7. In which arrangement does the square have the greater side length?
8. Which arrangement shows the squares with the greater diagonal length?
9. Compare the area of a square in the first diagram with the area of the corresponding square in the second diagram.
10. Compare the perimeter of corresponding squares in the two diagrams.
11. Examine the increase in area from one square to the next in the set. What increase in area would you expect from square 7 to square 8 in each of the patterns?



Consider a set of concentric circles around a centre Z, drawn on 1-centimetre grid paper.

12. Investigate the change in circumference of the circles, moving from one circle to the next (smallest to largest). Write a general formula for this increase in circumference in terms of π .
13. Write a general formula in terms of π and r that can be used to calculate the increase in area from one circle to the next (smallest to largest).

learnON RESOURCES — ONLINE ONLY

Digital doc: Investigation — Concentric squares (doc-15888)

Digital doc: Code puzzle — Inventions and their inventors (doc-15913)

Digital doc: Summary Topic 12 (doc-22894)

Interactivity: Crossword Topic 12 (int-0707)

Interactivity: Word search Topic 12 (int-0693)

Interactivity: Sudoku Topic 12 (int-3214)

Exercise 12.7 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **MC** Which of the following is *not* a quadratic equation?
 - A. $x^2 - 1 = 0$
 - B. $x^2 - 1 + 2x = 0$
 - C. $x^2 - \frac{1}{2} = 0$
 - D. $x^2 - \frac{1}{x} = 0$
2. **MC** Which of the following is in general form?
 - A. $x^2 + 1 = 0$
 - B. $x^2 - 1 = 2$
 - C. $2x^2 = x^2 + 3$
 - D. $x^2 = x^2 + 1$
3. **MC** The Null Factor Law cannot be applied to the equation $x(x + 3)(x - 2) = 1$ because:
 - A. the first factor is a simple x term
 - B. there are more than two factors
 - C. the right hand side equals 1
 - D. the second term is positive
 - E. it is a cubic

4. **MC** If the solutions to the quadratic equation $(x - 3)(x - b) = 0$ are 3 and -5 , then b is equal to:
A. 5 **B.** -5 **C.** 3 **D.** -3 **E.** 0
5. **MC** The solutions to $3x^2 - 27 = 0$ are:
A. $x = 3$ and $x = -3$ **B.** $x = 9$ and $x = -9$ **C.** $x = 1$ and $x = -1$
D. $x = 2$ and $x = -2$ **E.** $x = -3$ and $x = 9$
6. **MC** The quadratic equation $2x^2 + 7x + 3 = 0$ has solutions at:
A. $x = 3$ and $x = 1$ **B.** $x = -3$ and $x = -1$ **C.** $x = \frac{1}{2}$ and $x = 3$
D. $x = -3$ and $x = -\frac{1}{2}$ **E.** $x = 2$ and $x = 3$
7. **MC** The quadratic equation $9(x + 9)^2 - 9 = 0$ has:
A. the solutions 9 and -9 **B.** the solutions $x = 1$ and -1 **C.** the solutions 8 and 10
D. the solutions -8 and -10 **E.** $x = -9$ and $x = 0$
8. Find the solutions to the quadratic equation $(x - 3)(2x + 8) = 0$.
9. Determine the solutions to the equation $(4 - x)(2x - 7) = 0$.
10. Find the solutions to the following quadratic equations.
a. $2x^2 - 98 = 0$ **b.** $5x^2 - 125 = 0$ **c.** $2x^2 + 14x = 0$
d. $x^2 - 4x - 5 = 0$ **e.** $x^2 - 11x + 10 = 0$ **f.** $2x^2 - 9x - 5 = 0$
11. Solve the following equations, identifying those with no real solutions.
a. $x^2 + 11x + 10 = 0$ **b.** $3x^2 + 6x = 0$ **c.** $-2x^2 - 1 = 0$
12. Determine the solutions to the following equations.
a. $(x + 2)^2 - 16 = 0$ **b.** $4(x - 3)^2 - 36 = 0$ **c.** $(x + 1)^2 = 25$

Communicating, reasoning and problem solving

13. The distance travelled by a motorbike is given by the formula $d = 18t + 2t^2$, where t is the time in seconds and d is the distance in metres. How long would it take the motorbike to travel a distance of 180 m?
14. When 20 is subtracted from the square of a certain number, the result is 8 times that number. Find the number, which is negative.
15. A ball is thrown from the balcony of an apartment building. The ball is h metres above the ground when it is a horizontal distance of x metres from the building. The path of the ball follows the rule $h = -x^2 + 3x + 28$. How far from the building will the ball land?
16. An astronaut needs to find a positive number such that twice the number plus its square gives 35.
a. Write this as an equation.
b. Solve the equation to find the answer for him.
17. A window cleaner was 3 metres off the ground, cleaning the windows of a high-rise building, when he dropped his bucket. The height, h m, of the bucket above the ground t seconds after it is dropped is given by the equation $h = 3 - t^2 - 2t$.
a. What is the value of h when the bucket hits the ground?
b. Rewrite the equation, replacing h with the value it takes when the bucket hits the ground.
c. Solve the equation.
d. Find the time it takes for the bucket to reach the ground.

Answers

Topic 12 Quadratic equations

Exercise 12.2 Quadratic equations

1. **a.** $a = 1, b = 2, c = -1$
d. $a = 8, b = -11, c = 2$
2. **a.** D
3. **a.** $x = 4, x = -4$
d. $x = 1, x = -1$
4. **a.** $x = -1, x = -3$
d. $x = 0, x = 8$
g. $x = -3, x = -7$
5. **a.** Y
6. **a.** Y
7. **a.** N
8. **a.** Y
9. **a.** Check with your teacher.
b. $x = 3.41$
c. A negative number is not appropriate for a length.
10. Check with your teacher.
11. **a.** 6 or -6
b. There are two possible numbers, as the square root of a positive number, in this case 36, has two possible solutions, one positive and one negative.
12. **a.** $3(x - 2)^2 - 7 = 41$
 $7 - 3(x - 2)^2 = 41$ has no solution.
b. -2 and 6

Exercise 12.3 The Null Factor Law

1. **a.** $x = -3, x = 2$
d. $x = -2\frac{1}{2}, x = -\frac{3}{4}$
g. $x = -\frac{1}{2}, x = 3$
j. $x = -\frac{1}{3}, x = \frac{1}{4}$
m. $x = 2$
2. **a.** $x = 2, x = -2, x = -3$
c. $x = -2, x = -4$
e. $x = 1.1, x = -2.4, x = -2.6$
g. $x = 3$
3. **a.** D
4. **a.** $x(x + 10) = 0; x = 0$ or $x = -10$
c. $5x(5x - 8) = 0; x = 0$ or $x = \frac{8}{5}$
5. **a.** When $x = 2$, the first bracket equals 2 and the second bracket equals 4; therefore, the product is 8.
b. -2 and 1
6. A quadratic can have a maximum of two solutions, because a quadratic can at most be factorised into two separate pairs of brackets, each of which represent one solution.
7. **a.** 10 metres
c. 9.424 m
8. **a.** $h = -4t(t - 5)$
b. i. 16 m
c. Answers will vary.
9. -29 or 8
- b.** $x = 1, b = 1, c = -10$
e. $a = 2, b = 1, c = -60$
- b.** C
b. There is no solution.
e. There is no solution.
b. There is no solution.
e. $x = 1, x = -5$
h. There is no solution.
b. N
b. N
b. N
b. N
- c.** $a = 4, b = 5, c = -30$
f. $a = 1, b = 0, c = -17$
c. $x = 0$
f. $x = 2, x = -2$
c. $x = 1, x = -7$
f. There is no solution.
i. $x = 8$
c. Y
c. N
c. Y
- c.** $x = -2, x = 3$
f. $x = \frac{1}{2}, x = -30$
i. $x = 0, x = 2$
l. $x = -\frac{1}{6}, x = \frac{1}{6}$
o. $x = -4$
- b.** $x = -2, x = 3$
e. $x = -4, x = -\frac{1}{2}$
h. $x = 1, x = \frac{1}{3}$
k. $x = -2.3, x = 0.3$
n. $x = 0, x = \frac{15}{4}$
- b.** $x = -2, x = 2.5$
d. $x = 0, x = -2, x = -4$
f. $x = -3, x = -\frac{1}{2}, x = 1\frac{2}{3}$
h. $x = -1, x = 2$
b. B
- b.** $2x(x - 7) = 0; x = 0$ or $x = 7$
- b.** $y = \frac{1}{250}x(100 - x); a = \frac{1}{250}, b = 100$
- ii.** 0 m

Exercise 12.4 Solving the quadratic equation $ax^2 + bx + c = 0$

1. a. $x = 2, x = 4$ b. $x = -2, x = -4$ c. $x = -1, x = -5$ d. $x = 2, x = -3$
 e. $x = 3, x = -5$ f. $x = -2$ g. $x = 4, x = -6$ h. $x = 8, x = -3$
 i. $x = -3, x = 4$ j. $x = -12, x = -1$ k. $x = 11, x = -1$ l. $x = 4, x = -5$
 m. $x = -25, x = -4$ n. $x = 5, x = 10$ o. $x = 4, x = -2$
2. a. B b. D c. D
3. $x = 1, -6$
4. 9 m by 5 m
5. 8 m by 6 m
6. 1.5 m
7. a. i. 0 ii. 2 iii. 35
 b. i. Octagon ii. Icosahedron
8. a. i. $n + 1$ ii. $n + 7$ iii. $(n + 1)(n + 7)$ iv. $\frac{(n + 7)}{(n + 2)}$
 b. $n = 3$
9. a. $4x^2 + 13x - 16 = 3x^2 + 8x - 2$ b. $x^2 + 5x - 14 = 0$
 c. $(x - 2)(x + 7) = 0$ d. $x = 2$ or -7
10. a. $g + 18$
 b. $g^2 + 12g - 108$
 c. $g^2 + 12g - 108 = 256$
 d. $g^2 + 12g - 364 = 0$
 e. $g = 14$
 f. The girl is 14 years old, her brother is 8 years old and her teacher is 32 years old.

Exercise 12.5 Solving quadratic equations with two terms

1. a. $x = -3, x = 3$ b. $x = -4, x = 4$ c. $x = -3, x = 3$ d. $x = -5, x = 5$
 e. $x = -10, x = 10$ f. $x = -7, x = 7$ g. $x = -3, x = 3$ h. $x = -2, x = 2$
 i. No real solutions j. No real solutions k. $x = -3, x = 3$ l. $x = -4, x = 4$
 m. $x = -5, x = 5$ n. $x = 0$ o. $x = 0$
2. a. $x = 0, x = -6$ b. $x = 0, x = 8$ c. $x = 0, x = -9$ d. $x = 0, x = 11$
 e. $x = 0, x = 6$ f. $x = 0, x = 7.5$ g. $x = 0, x = \frac{2}{3}$ h. $x = 0, x = -1\frac{3}{4}$
 i. $x = 0, x = 2\frac{1}{2}$ j. $x = 0, x = -1$ k. $x = 0, x = \frac{1}{4}$ l. $x = 0, x = -5$
 m. $x = 0, x = -12$ n. $x = 0, x = 18$ o. $x = 0, x = 2.5$
3. a. A b. C
4. 4 or -4
5. The square plot is $12 \text{ m} \times 12 \text{ m}$; the rectangular plot is $16 \text{ m} \times 9 \text{ m}$.
6. The number is 0 or 10.
7. $x^2 + 9$ cannot be factorised.
8. $x = \pm \frac{n}{m}$
9. x cannot be isolated.
10. If $a = 0$, then the expression is not quadratic.
11. 8
12. -12 or 0
13. a. $x(x + 30) = 50x$
 b. $x(x - 20) = 0$
 c. $x = 0$ or $x = 20$
 d. No, x cannot be 0, because the width has to have a positive value.

Exercise 12.6 Applications of quadratic equations

1. 6 and 7 or -7 and -6 2. 11 and 13 or -13 and -11
3. $x = 9$ 4. $x = 10$

5. 3, 4, 5

6. 5, 12, 13

7. Length = 22 cm, width = 13 cm, height = 4 cm

8. 2.5 m

9. a. 15 m

b. 4 seconds

10. a. 0

b. 11 terms

11. Answers will vary, but quadratic equations can be used to describe many situations where there can be a rise or fall in a value of what you are measuring. This might be the amount of time taken to do something, the profit of a business activity, or the distance covered in a particular time.

Investigation | Rich task

1.

Square	1	2	3	4
Side length (cm)	2	4	6	8
Diagonal length (cm)	$2\sqrt{2}$	$4\sqrt{2}$	$6\sqrt{2}$	$8\sqrt{2}$
Perimeter (cm)	8	16	24	32
Area (cm²)	4	16	36	64

2. 20 cm

3. $20\sqrt{2}$ cm

4.

Square	1	2	3	4
Side length (cm)	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}$	$4\sqrt{2}$
Diagonal length (cm)	2	4	6	8
Perimeter (cm)	$4\sqrt{2}$	$8\sqrt{2}$	$12\sqrt{2}$	$16\sqrt{2}$
Area (cm²)	2	8	18	32

5. $10\sqrt{2}$ cm

6. 20 cm

7. First diagram

8. First diagram

9. The squares in the first diagram are twice the area of the corresponding square in second diagram.

10. The squares in the first diagram have a perimeter that is $\sqrt{2}$ times the perimeter of the corresponding square in the second diagram.

11. An increase of 60 cm² in the first diagram and an increase of 30 cm² for the second diagram

12. 2π

13. $\pi(2r + 1)$

Exercise 12.7 Review questions

1. D

2. A

3. C

4. B

5. A

6. D

7. D

8. $x = -4, x = 3$

9. $x = 3.5, x = 4$

10. a. $x = -7, x = 7$

b. $x = -5, x = 5$

c. $x = -7, x = 0$

d. $x = -1, x = 5$

e. $x = 1, x = 10$

f. $x = -\frac{1}{2}, x = 5$

11. a. $x = -10, x = -1$

b. $x = -2, x = 0$

c. No real solutions

12. a. $x = -6, x = 2$

b. $x = 0, x = 6$

c. $x = -6, x = 4$

13. 6s

14. $x = -2$

15. 7 metres

16. a. $x^2 + 2x = 35$

b. 5

17. a. $h = 0$

b. $0 = 3 - t^2 - 2t$

c. $t = -3$ or $t = 1$

d. 1s

TOPIC 13

Proportion and rates

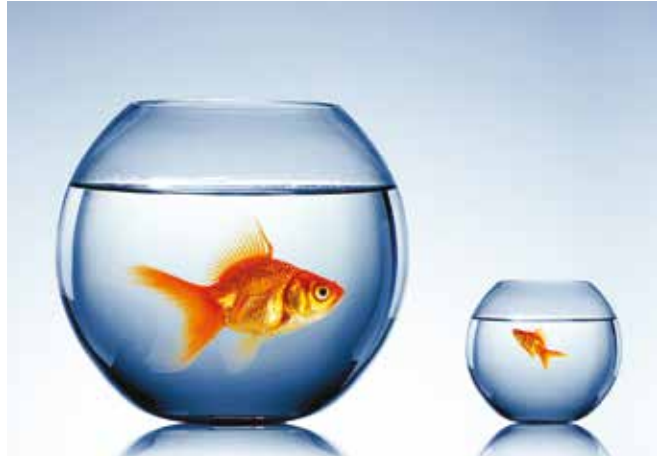
[Stages 5.2 and 5.3]

13.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

13.1.1 Why learn this?

Proportion and rates are often used to compare quantities. Kilometres per hour, prices per kilogram, dollars per litre, and pay per hour or per week are all examples of rates. Dividing quantities according to a particular rule requires an understanding of ratios and rates.



DISCUSSION

How has the Golden Ratio been used by different cultures throughout history?

LEARNING SEQUENCE

- 13.1 Overview
- 13.2 [Stage 5.2] Direct proportion
- 13.3 [Stage 5.2] Direct proportion and ratio
- 13.4 [Stage 5.2] Inverse proportion
- 13.5 [Stage 5.2] Simple rate problems
- 13.6 [Stage 5.3] Constant and variable rates
- 13.7 [Stage 5.3] Rates of change
- 13.8 Review

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- recognises direct and indirect proportion, and solves problems involving direct proportion MA5.2-5NA
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- draws, interprets and analyses graphs of physical phenomena MA5.3-4NA

CONTENT DESCRIPTIONS

Students:

Solve problems involving direct proportion; explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

13.2 Direct proportion

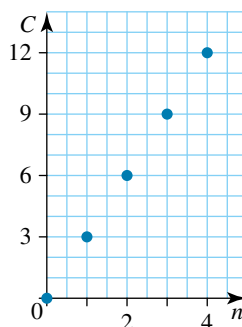
[Stage 5.2]

- Suppose that ice-creams cost \$3 each and that you are to buy some for your friends.
- There is a relationship between the cost of the ice-creams (C) and the number of the ice-creams that you buy (n).



- The relationship can be illustrated in a table or a graph.

n	0	1	2	3	4
C (\$)	0	3	6	9	12



- This relationship has some important characteristics:
 - As n increases, so does C .
 - When $n = 0$, $C = 0$.
 - The graph of the relationship is a straight line passing through the origin.
- When all of these characteristics apply, the relationship is called **direct proportion**.
- We say that ' C is directly proportional to n ' or ' C varies directly as n .' This is written as $C \propto n$.

WORKED EXAMPLE 1

Does direct proportion exist between these variables?

- a** The height of a stack of photocopy paper (h) and the number of sheets (n) in the stack
- b** Your Mathematics mark (m) and the number of hours of Mathematics homework you have completed (n)

THINK

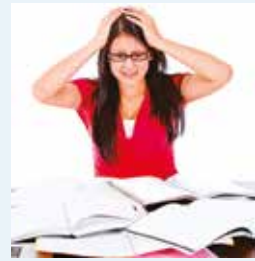
- a** When n increases, so does h .
When $n = 0$, $h = 0$.
If graphed, the relationship would be linear.

- b** As n increases, so does m .
When $n = 0$, I may get a low mark at least, so $n \neq 0$.

WRITE

a $h \propto n$

b m is not directly proportional to n .



WORKED EXAMPLE 2

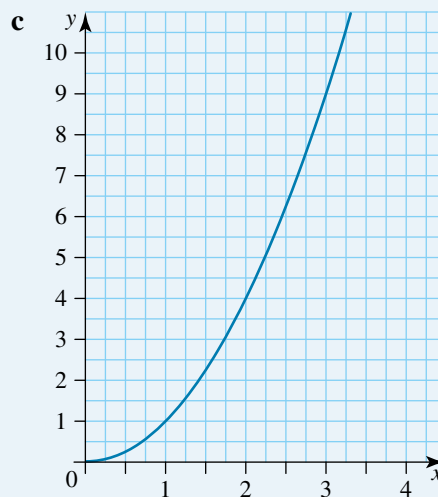
Do the following relationships show direct proportion?

a

t	0	1	2	3	4
y	0	1	3	7	15

b

n	1	2	3	4
C	5	10	15	20



THINK

- a** From the table, when t increases, so does y .
When $t = 0$, $y = 0$. The t -values increase by a constant amount but the y -values do not, so the relationship is not linear.

When t is doubled, y is not.

- b** From the table, as n increases, so does m .
Extending the pattern gives $n = 0$, $C = 0$. The n -values and C -values increase by constant amounts, so the relationship is linear.

- c** When x increases, so does y .
When $x = 0$, $y = 0$.
The graph is not a straight line.

WRITE

a y is not directly proportional to t .

b $C \propto n$

c y is not directly proportional to x .

Interactivity: Direct proportion (int-6056)

Interactivity: Direct proportion (int-2767)

Digital doc: SkillsHEET Measuring the rise and the run (doc-6174)

Exercise 13.2 Direct proportion

Individual pathways

PRACTISE

Questions:
1, 2

CONSOLIDATE

Questions:
1, 2, 4, 5

MASTER

Questions:
1–8

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** For each of the following pairs of variables, state whether direct proportion exists. If it does not exist, give a reason.
 - a. The distance (d) travelled in a car travelling at 60km/h and the time taken (t)
 - b. The speed of a swimmer (s) and the time the swimmer takes to complete one lap of the pool (t)
 - c. The cost of a bus ticket (c) and the distance travelled (d)
 - d. The perimeter (p) of a square and the side length (l)
 - e. The area of a square (A) and the side length (l)
 - f. The total cost (C) of buying n boxes of pencils
 - g. The weight of an object in kilograms (k) and the weight in pounds (p)
 - h. The distance (d) travelled in a taxi and the cost (c)
 - i. A person's height (h) and their age (a)



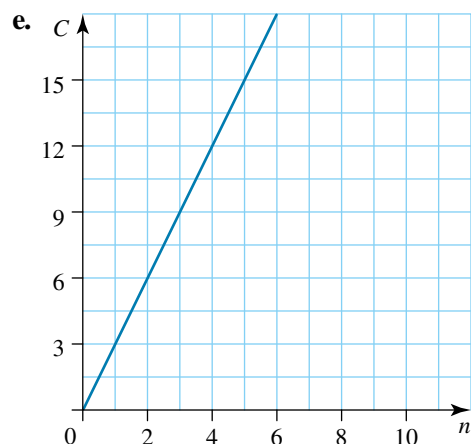
2. **WE2** For each of the following, determine whether direct proportion exists between the variables. If it does not, give a reason.

a.

x	0	1	2	3	4
y	0	1	3	8	15

c.

t	0	1	2	4	8
d	0	3	6	9	12

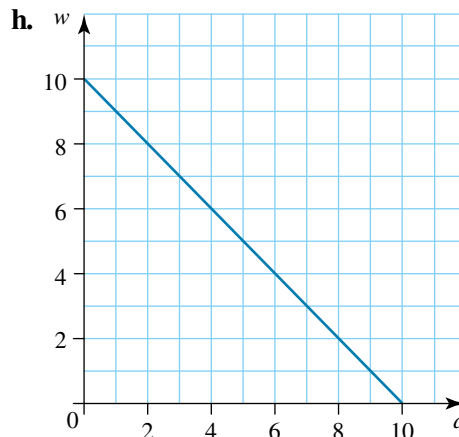
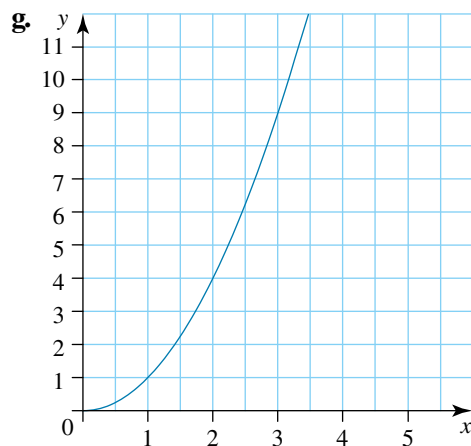
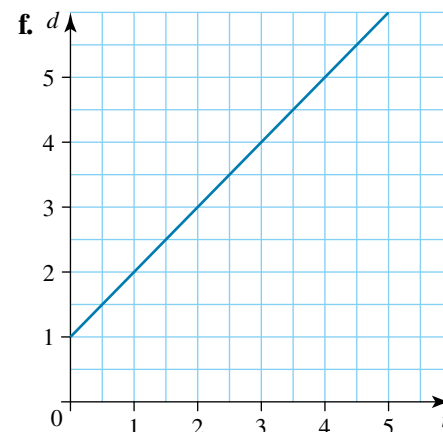


b.

a	0	1	2	3	6
M	0	8	16	24	48

d.

n	0	1	2	3	4
C	10	20	30	40	50



3. List five pairs of real-life variables that exhibit direct proportion.

Communicating, reasoning and problem solving

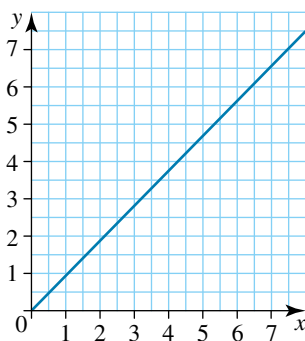
4. Which point must always exist in a table of values if the two variables exhibit direct proportionality?
5. If direct proportion exists between two variables, m and n , fill out the table and explain your reasoning.
- | | | | |
|-----|---|---|----|
| m | 0 | 2 | 5 |
| n | 0 | | 20 |
6. Mobile phone calls are charged at 17 cents per 30 seconds
- Does direct proportion exist between the cost of a phone bill and the number of 30-second time periods?
 - If a call went for 7.5 minutes, how much would the call cost?
7. Bruce is building a pergola and needs to buy treated pine timber. He wants 4.2-metre and 5.4-metre lengths of timber. If it costs \$23.10 for the 4.2-metre length and \$29.16 for the 5.4-metre length, does direct proportion exist between the cost of the timber and the length of the timber per metre? Explain.
8. How do you know when two quantities are directly proportional?

13.3 Direct proportion and ratio

[Stage 5.2]

13.3.1 Constant of proportionality

- A direct proportion relationship ($y \propto x$) is shown by the graph.
- Because the graph passes through the origin, the y -intercept is equal to zero and the graph has the equation $y = kx$, where k is the gradient of the graph.



- In the equation $y = kx$:
 - k is a constant and is called the **constant of proportionality** (or the constant of variation)
 - k can be a negative number
 - $y \propto x$ is equivalent to the equation $y = kx$
 - y is called the dependent variable and is normally placed in the bottom row of a table
 - x is called the independent variable and is normally placed in the top row of a table.

WORKED EXAMPLE 3

Given that $y \propto x$ and $y = 12$ when $x = 3$, find the constant of proportionality and state the rule linking y and x .

THINK

- 1 $y \propto x$, so write the linear rule.
- 2 Substitute $y = 12$, $x = 3$ into $y = kx$.
- 3 Find the constant of proportionality by solving for k .
- 4 State the rule.

WRITE

$$y = kx$$

$$12 = 3k$$

$$k = 4$$

$$y = 4x$$

WORKED EXAMPLE 4

The weight (W) of \$1 coins in a bag varies directly as the number of coins (n). Twenty coins weigh 180 g.

- a Find the relationship between W and n .
- b How much will 57 coins weigh?
- c How many coins weigh 252 g?



THINK

Summarise the information given in a table.

WRITE

$$W = kn$$

n	20	57	
W	180		252

a 1 Substitute $n = 20$, $W = 180$ into $W = kn$.

2 Solve for k .

3 State the relationship between W and n .

b 1 State the rule.

2 Substitute 57 for n to find W .

3 State the answer.

c 1 State the rule.

2 Substitute 252 for W .

3 Solve for n .

4 State the answer.

$$180 = 20k$$

$$k = 9$$

$$W = 9n$$

$$W = 9n$$

$$W = 9 \times 57$$

$$= 513$$

57 coins will weigh 513 g.

$$W = 9n$$

$$252 = 9n$$

$$n = \frac{252}{9}$$

$$= 28$$

28 coins weigh 252 g.

13.3.2 Ratio

- If $y \propto x$, then $y = kx$, where k is constant.

Transposing this formula gives $\frac{y}{x} = k$.

In other words, for any pair of values (x, y) , the ratio $\frac{y}{x}$ is constant and equals the constant of proportionality.

- For example, this table shows that $v \propto t$.

t	1	2	3	4
v	5	10	15	20

It is clear that $\frac{20}{4} = \frac{15}{3} = \frac{10}{2} = \frac{5}{1} = 5$.

WORKED EXAMPLE 5

Sharon works part time and is paid at a fixed rate per hour. If she earns \$135 for 6 hours work, how much will she earn for 11 hours?

THINK

- Sharon's payment is directly proportional to the number of hours worked. Write the rule.
- Summarise the information given. The value of x is to be found.

- Since $P = kn$, $\frac{P}{n}$ is constant.

Solve for x .

- State the answer.

WRITE

$$P = kn$$

n	6	11
P	135	x

$$\begin{aligned}\frac{135}{6} &= \frac{x}{11} \\ x &= \frac{135 \times 11}{6} \\ x &= 247.5\end{aligned}$$

Sharon earns \$247.50 for working 11 hours.

Interactivity: Constants of proportionality (int-6057)

Digital doc: SkillSHEET Rounding to a given number of decimal places (doc-6175)

Digital doc: WorkSHEET Direct proportion (doc-6181)

Exercise 13.3 Direct proportion and ratio

Individual pathways

PRACTISE

Questions:

1–6, 9–11, 14

CONSOLIDATE

Questions:

1–5, 7, 9, 10, 12, 14, 15

MASTER

Questions:

1–5, 8–10, 12–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE3** If a is directly proportional to b and $a = 30$ when $b = 5$, find the constant of proportionality and state the rule linking a and b .
- If $a \propto b$ and $a = 2.5$ when $b = 5$, find the rule linking a and b .
- If $C \propto t$ and $C = 100$ when $t = 8$, find the rule linking C and t .
- If $v \propto t$ and $t = 20$ when $v = 10$, find the rule linking v and t .
- If $F \propto a$ and $a = 40$ when $F = 100$, find the rule linking a and F .
- WE4** Springs are often used to weigh objects, because the extension of a spring (E) is directly proportional to the weight (W) of the object hanging from the spring. A 4-kg load stretches a spring by 2.5 cm.
 - Find the relationship between E and W .
 - What load will stretch the spring by 12 cm?
 - How much will 7 kg extend the spring?
- Han finds that 40 shelled almonds weigh 52 g.
 - Find the relationship between the weight (W) and the number of almonds (n).
 - How many almonds would there be in a 500-g bag?
 - How much would 250 almonds weigh?
- Petra knows that her bicycle wheel turns 40 times when she travels 100 m.
 - Find the relationship between the distance travelled (d) and the number of turns of the wheel (n).
 - How far does she go if her wheel turns 807 times?
 - How many times does her wheel turn if she travels 5 km?
- Fiona, who operates a plant nursery, uses large quantities of potting mix. Last week she used 96 kg of potting mix to place 800 seedlings in medium-sized pots.
 - Find the relationship between the mass of potting mix (M) and the number of seedlings (n).
 - How many seedlings can she pot with her remaining 54 kg of potting mix?
 - How much potting mix will she need to pot 3000 more seedlings?



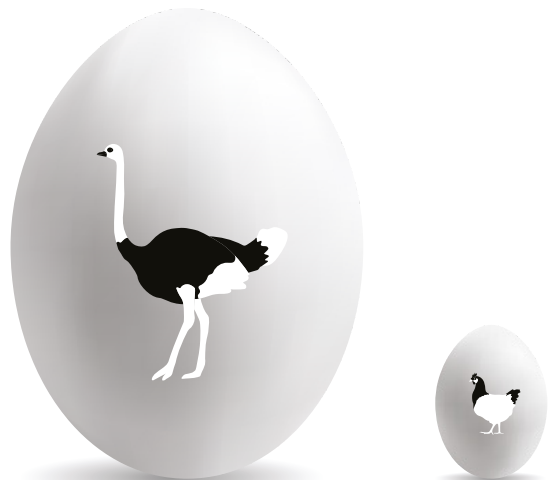
10. **WES** Tamara is paid at a fixed rate per hour. If she earns \$136 for 5 hours work, how much will she earn for working 8 hours?



11. It costs \$158 to buy 40 packets of cards. How much will 55 packets cost?
12. If 2.5 L of lawn fertiliser will cover an area of 150 m^2 , how much fertiliser is needed to cover an area of 800 m^2 ?

Communicating, reasoning and problem solving

13. Paul paid \$68.13 for 45 L of fuel. At the same rate, how much would he pay for 70 L? Justify your answer.
14. Rose gold is an alloy of gold and copper that is used to make high-quality musical instruments. If it takes 45 g of gold to produce 60 g of rose gold, how much gold would be needed to make 500 g of rose gold? Justify your answer.
15. If Noah takes a group of friends to the movies for his birthday and it would cost \$62.50 for five tickets, how much would it cost if there were 12 people (including Noah) in the group?
16. Sharyn enjoys quality chocolate, so she makes a trip to her favourite chocolate shop. She is able to select her favourite chocolates for \$7.50 per 150 grams. Since Sharyn loves her chocolate, she decides to purchase 675 grams. How much did she spend?
17. Anthony drives to Mildura, covering an average of 75 kilometres in 45 minutes. He has to travel 610 kilometres to get to Mildura.
a. Find the relationship between the distance travelled in kilometres, D , and his driving time in hours, T .
b. How long will it take Anthony to complete his trip?
c. If he stops at Bendigo, 220 kilometres from his starting position, how long does it take him to reach Bendigo?
d. How much longer will it take him to arrive at Mildura after leaving Bendigo?
18. What is ratio?
19. The volume of a bird's egg can be determined by the formula $V = kl^3$, where V is the volume in cm^3 , l is the length of the egg in cm and k is a constant. A typical ostrich egg is 15 cm long and has a volume of 7425 cm^3 . What is the volume of a typical 5 cm long chicken egg?



13.4 Inverse proportion

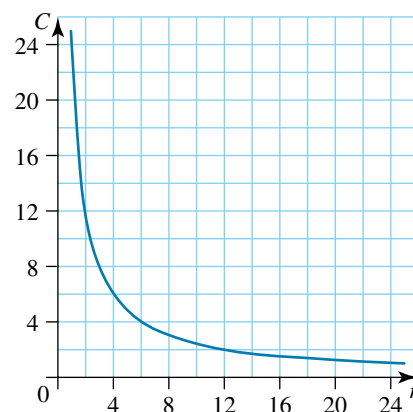
[Stage 5.2]

- If 24 sweets are shared between 4 children, then each child will receive 6 sweets. If the sweets are shared by 3 children, then each will receive 8 sweets.
- The relationship between the number of children (C) and the number of sweets for each child (n) can be given in a table.

C	1	2	3	4	6	8	12
n	24	12	8	6	4	3	2



- As the number of children (C) increases, the number of sweets for each child (n) decreases. This is an example of **inverse proportion** or inverse variation.
- We say that ' n is inversely proportional to C ' or ' n varies inversely as C '.
This is written as $C \propto \frac{1}{n}$, or $C = \frac{k}{n}$, where k is a constant (the constant of proportionality). This formula can be rearranged to $Cn = k$. Note that multiplying any pair of values in the table (3×8 , 12×2) gives the same result.
- The relationship has some important characteristics:
 - As C increases, n decreases, and vice versa.
 - The graph of the relationship is a **hyperbola**.



WORKED EXAMPLE 6

y is inversely proportional to x and $y = 10$ when $x = 2$.

- Calculate the constant of proportionality, k , and hence the rule relating x and y .
- Plot a graph of the relationship between x and y for values of x from 2 to 10.

THINK

- Write the relationship between the variables.
 - Rewrite as an equation using k , the constant of proportionality.
 - Substitute $y = 10$, $x = 2$ into $y = \frac{k}{x}$ and solve for k .
 - Write the rule by substituting $k = 20$ into $y = \frac{k}{x}$.
- b 1** Use the rule $y = \frac{20}{x}$ to set up a table of values for x and y , taking values for x which are positive factors of k so that only whole number values of y are obtained.

For example, $x = 4$, $y = \frac{20}{4} = 5$.

WRITE/DRAW

a $y \propto \frac{1}{x}$

$$y = \frac{k}{x}$$

$$10 = \frac{k}{2}$$

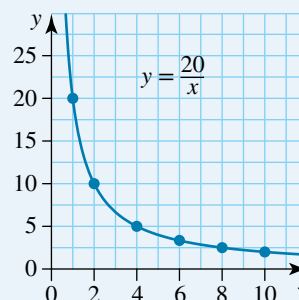
$$k = 20$$

$$y = \frac{20}{x}$$

b

x	2	4	6	8	10
y	10	5	3.3	2.5	2

- 2 Plot the points on a clearly labelled set of axes and join the points with a smooth curve. Label the graph.



WORKED EXAMPLE 7

When a wire is connected to a power source, the amount of electrical current (I) passing through the wire is inversely proportional to the resistance (R) of the wire. If a current of 0.2 amperes flows through a wire of resistance 60 ohms:

- find the constant of proportionality
- determine the rule relating R and I
- find the resistance if the current equals 5 amperes
- find the current that will flow through a wire of resistance 20 ohms.



THINK

Summarise the information in a table.

$$I \propto \frac{1}{R}$$

Write the rule.

a 1 Substitute $R = 60$, $I = 0.2$ into $I = \frac{k}{R}$.

2 Solve for k .

b Write the rule using $k = 12$.

c 1 Substitute $I = 5$ into $I = \frac{12}{R}$.

2 Solve for R .

3 Write the answer.

d 1 Substitute $R = 20$ into $I = \frac{12}{R}$.

2 Write the answer.

WRITE

R	60		20
I	0.2	5	

$$I = \frac{k}{R}$$

a $0.2 = \frac{k}{60}$

$$0.2 \times 60 = k$$

$$k = 12$$

b $I = \frac{12}{R}$

c $5 = \frac{12}{R}$

$$5R = 12$$

$$R = \frac{12}{5}$$

$$= 2.4$$

The resistance equals 2.4 ohms.

d $I = \frac{12}{20}$

$$= 0.6$$

The current will be 0.6 amperes.

Exercise 13.4 Inverse proportion

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–13

MASTER

Questions:
1–14

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Decide whether inverse proportion exists between each pair of variables. If it does exist, write an equation to describe the relationship.
 - The speed of a car (s) and the time (t) it takes to complete one lap of a race circuit
 - The amount of money (D) that I have and the number (n) of cards that I can buy
 - The time (t) that it takes to make a pair of jeans and the number of pairs (p) that can be made in one day
 - The price (P) of petrol and the amount (L) that can be bought for \$80
 - The price (P) of petrol and the cost (C) of buying 80 L
 - The number of questions (n) in a test and the amount of time (t) available to answer each one
- List three examples of inverse proportion.
- WE6** y varies inversely as x and $y = 100$ when $x = 10$.
 - Calculate the constant of proportionality, k , and hence the rule relating x and y .
 - Plot a graph of the relationship between x and y for values of x that are positive factors of k less than 21.
- p is inversely proportional to q and $p = 12$ when $q = 4$.
 - Calculate the constant of proportionality, k , and hence the rule relating p and q .
 - Plot a graph of the relationship between q and p , for values of q that are positive factors of k less than 11.
- y varies inversely as x and $y = 42$ when $x = 1$.
 - Calculate the constant of proportionality, k , and hence the rule relating x and y .
 - Plot a graph of the relationship between x and y for values of x from 1 to 10.
- WE7** When a constant force is applied to an object, its acceleration is inversely proportional to its mass. When the acceleration of an object is 40 m/s^2 , the corresponding mass is 100 kg.
 - Calculate the constant of proportionality.
 - Determine the rule relating mass and acceleration.
 - Determine the acceleration of a 200-kg object.
 - Determine the acceleration of a 1000-kg object.
- The number of colouring pencils sold is inversely proportional to the price of each pencil. Two thousand pencils are sold when the price is \$0.25 each.
 - Calculate the constant of proportionality.
 - Determine the number of pencils that could be sold for \$0.20 each.
 - Determine the number of pencils that could be sold for \$0.50 each.



8. The time taken to complete a journey is inversely proportional to the speed travelled. A trip is completed in 4.5 hours travelling at 75 km per hour.
 - a. Calculate the constant of proportionality.
 - b. Determine how long, to the nearest minute, the trip would take if the speed was 85 km per hour.
 - c. Determine the speed required to complete the journey in 3.5 hours, correct to 1 decimal place.
 - d. Determine the distance travelled in each case.
9. The cost per person travelling in a charter plane is inversely proportional to the number of people in the charter group. It costs \$350 per person when 50 people are travelling.
 - a. Calculate the constant of variation.
 - b. Determine the cost per person, to the nearest cent, if there are 75 people travelling.
 - c. Determine how many people are required to reduce the cost to \$250 per person.
 - d. Determine the total cost of hiring the charter plane.

Communicating, reasoning and problem solving

10. The electrical current in a wire is inversely proportional to the resistance of the wire to that current. There is a current of 10 amperes when the resistance of the wire is 20 ohms.
 - a. Calculate the constant of proportionality.
 - b. Determine the current possible when the resistance is 200 ohms.
 - c. Determine the resistance of the wire when the current is 15 amperes.
 - d. Justify your answer to parts **b** and **c** using a graph.
11. Two equations relating the time of a trip, T , and the speed at which they travel, S , are given. For both cases the time is inversely proportional to the speed: $T_1 = \frac{5}{S_1}$ and $T_2 = \frac{7}{S_2}$. Explain what impact the different constants of proportionality have on the time of the trip.
12. The time it takes to pick a field of strawberries is inversely proportional to the number of pickers. It takes 2 people 5 hours to pick all of the strawberries in a field.
 - a. Calculate the constant of proportionality.
 - b. Determine the rule relating time (T) and the number of pickers (P).
 - c. Determine the time spent if there are 6 pickers.
13. For a constant distance covered by a sprinter, the sprinter's speed is inversely proportional to their time. If a sprinter runs at a speed of 10.4 m/s, the corresponding time is 9.62 seconds.
 - a. Calculate the constant of variation.
 - b. Determine the rule relating speed (V) and time (T).
 - c. Determine the time, correct to 2 decimal places, if they ran at a speed of 10.44 m/s.
 - d. Determine the time, correct to 2 decimal places, if they ran at a speed of 6.67 m/s.
14. Explain what is meant by inverse proportion.



13.5 Simple rate problems

[Stage 5.2]

13.5.1 Rates

- The word **rate** occurs commonly in news reports and conversation.
 - ‘Home ownership rates are falling.’
 - ‘People work at different rates.’
 - ‘What is the current rate of inflation?’

‘Do you offer a student rate?’

‘The crime rate seems to be increasing.’

- Rate is a word often used when referring to a specific ratio.



DISCUSSION

Can you think of other examples where the word ‘rate’ might be used in conversation?

13.5.2 Average speed

- If you travel 160 km in 4 hours, your average speed is 40 km per hour.
Speed is an example of a rate, and its unit of measurement (kilometres per hour) contains a formula.
The word ‘per’ can be replaced with ‘divided by’, so $\text{speed} = \frac{\text{distance (in km)}}{\text{time (in hours)}}$.
- It is important to note the units involved. For example, an athlete’s speed is often measured in metres per second (m/s) rather than km/h.

13.5.3 Finding rates

- In general, to find a rate (or special ratio), one quantity is divided by another.

WORKED EXAMPLE 8

Calculate the rates suggested by these statements.

a A shearer shears 1110 sheep in 5 days.

b Eight litres of fuel costs \$12.56

c A cricket team scored 152 runs in 20 overs.

THINK

WRITE

a The rate suggested is ‘sheep per day’.
Write the ratio (rate).

$$\text{a Rate} = \frac{\text{number of sheep}}{\text{number of days}} = \frac{1110}{5} = 222 \text{ sheep/day}$$

b The rate suggested is ‘dollars per litre’. Write the ratio (rate).

$$\text{b Rate} = \frac{\text{number of dollars}}{\text{number of litres}} = \frac{12.56}{8} = \$1.57 \text{ per litre}$$

c The rate suggested is ‘runs per over’.
Write the ratio (rate).

$$\text{c Rate} = \frac{\text{number of runs}}{\text{number of overs}} = \frac{152}{20} = 7.6 \text{ runs per over}$$

WORKED EXAMPLE 9

The concentration of a solution is measured in g/L (grams per litre). What is the concentration of the solution when 10 g of salt is dissolved in 750 mL of water?

THINK

Concentration is measured in g/L, which means that concentration = number of grams (mass) $\div$ number of litres (volume). Write the ratio (rate).

WRITE

$$\begin{aligned}\text{Concentration} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{10}{0.75} \\ &= 13.3 \text{ g/L}\end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Speed (int-6457)

Digital doc: WorkSHEET Rates (doc-6182)

Exercise 13.5 Simple rate problems

Individual pathways

PRACTISE

Questions:
1–5

CONSOLIDATE

Questions:
1–11

MASTER

Questions:
1–12

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Hayden drove from Hay to Bee. He covered a total distance of 96 km and took 1.5 hours for the trip. What was Hayden's average speed for the journey?
- Find the average speed in km/h for each of the following.
 - 90 km in 45 min
 - 5500 km in 3 h 15 min (correct to 1 decimal place)
 - 8 m/s
- WE8** Calculate the rates suggested by these statements, giving your answers to 2 decimal places where necessary.
 - It costs \$736 for 8 theatre tickets.
 - Penelope decorated 72 small cakes in 3 hours.
 - Usain Bolt has a 100-m world sprint record of 9.58 seconds.
 - It takes 30 hours to fill the swimming pool to a depth of 90 cm.
 - Peter received \$260 for 15 hours work.
 - Yan received \$300 for assembling 6 air conditioners.
- A metal bolt of volume 25 cm^3 has a mass of 100 g. Find its density (mass per unit of volume).
- WE9** One hundred and twenty grams of sugar is dissolved in 200 mL of water. What is the concentration of this solution in g/L? Explain your answer.

Communicating, reasoning and problem solving

6. In the race between the tortoise and the hare, the hare ran at 72 km per hour while the giant tortoise moved at 240 cm per minute. Compare and explain the difference in the speed of the two animals.
7. The average speed of a car is determined by the distance of the journey and the time the journey takes. Explain the two ways in which the speed can be increased.
8. In the 2000 Sydney Olympics, Cathy Freeman won gold in the 400-metre race. Her time was 49.11 seconds. In the 2008 Beijing Olympics, Usain Bolt set a new world record for the men's 100-metre race. His time was 9.69 seconds. Calculate the average speed of the winner for each race in kilometres per hour.
9. A school had 300 students in 2013 and 450 students in 2015. What was the average rate of growth in students per year?
10. Mt Feathertop is Victoria's second-highest peak. To walk to the top involves an increase in height of 1500 m over a horizontal distance of 10 km (10000 m). Calculate the average gradient of the track.
11. Beaches are sometimes unfit for swimming if heavy rain has washed pollution into the water. A beach is declared unsafe for swimming if the concentration of bacteria is more than 5000 organisms per litre. A sample of 20 millilitres was tested and found to contain 55 organisms. Calculate the concentration in the sample (in organisms/litre) and state whether or not the beach should be closed.
12. What is an example of a rate that is not listed in this section?



13.6 Constant and variable rates

[Stage 5.3]

13.6.1 Constant rates

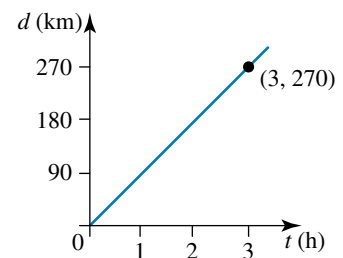
- Consider a car travelling along a highway at a constant speed (rate) of 90 km/h. After 1 hour it will have travelled 90 km, as shown in the table below.

Time (h)	0	1	2	3
Distance (km)	0	90	180	270

- The distance–time graph is shown.

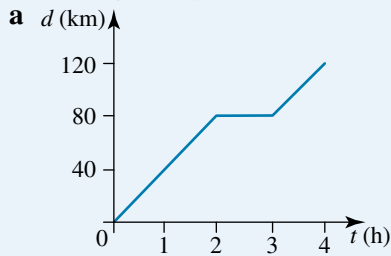
The gradient of the graph is $\frac{270 - 0}{3 - 0} = 90$.

The equation of the graph is $d = 90t$, and the gradient is equal to the speed, or rate of progress.



WORKED EXAMPLE 10

Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.



THINK

a There are three distinct sections.

1 In the first 2 hours, the car travels 80 km:

$$\frac{80}{2} = 40 \text{ km/h.}$$

2 In the next hour, the car does not move.

3 In the fourth hour, the car travels 40 km.

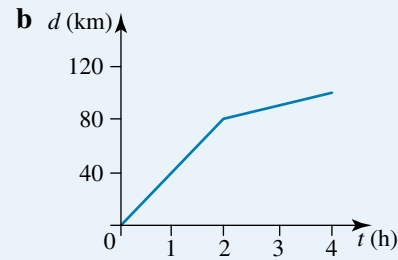
b There are two distinct sections.

1 In the first 2 hours, the car travels 80 km:

$$\frac{80}{2} = 40 \text{ km/h.}$$

2 In the next 2 hours, the car travels 20 km:

$$\frac{20}{2} = 10 \text{ km/h.}$$



WRITE

a The car travels at a speed of 40 km/h for 2 hours, and then stops for 1 hour. After this it travels for 1 hour at 40 km/h.

b The car travels at a speed of 40 km/h for 2 hours, and then travels at 10 km/h for a further 2 hours.

WORKED EXAMPLE 11

Draw a distance–time graph to illustrate the following journey.

A cyclist travels for 1 hour at a constant speed of 10 km/h and then stops for a 30-minute break before riding a further 6 km for half an hour at a constant speed.

THINK

There are three phases to the journey.

The graph starts at (0, 0).

1 In the first hour, the cyclist travels 10 km.

Draw a line segment from (0, 0) to (1, 10).

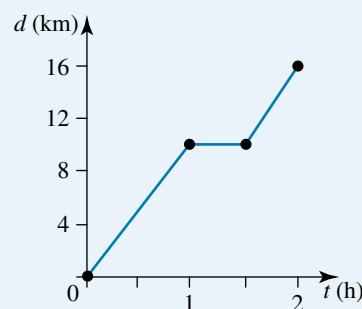
2 For the next half-hour, the cyclist is stationary,

so draw a horizontal line segment from (1, 10) to (1.5, 10).

3 In the next half-hour, the cyclist travels 6 km.

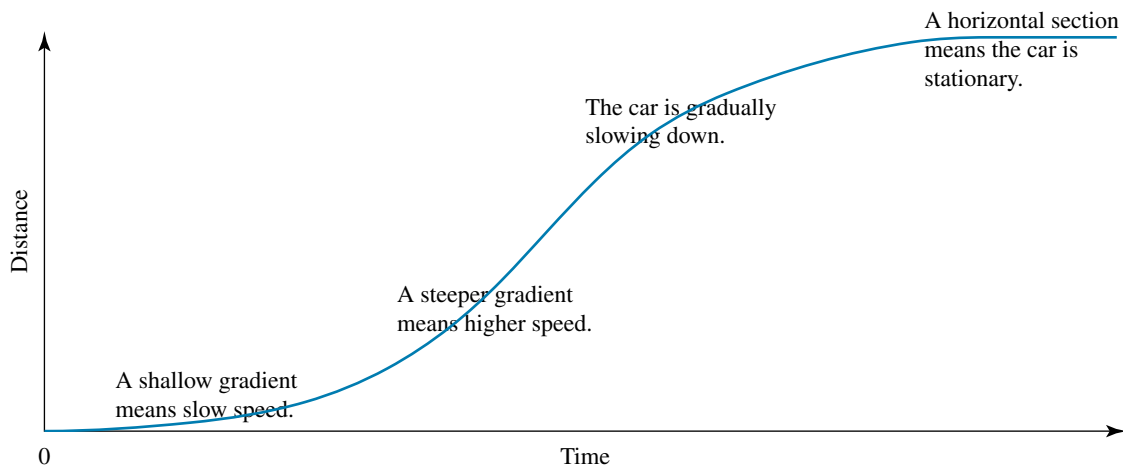
Draw a line segment from (1.5, 10) to (2, 16).

DRAW



13.6.2 Variable rates

- In reality, a car tends not to travel at constant speed. It starts from rest and gradually picks up speed.
- When the speed is low, the distance–time graph will have a small gradient, and when the speed is high, the gradient will be steep.

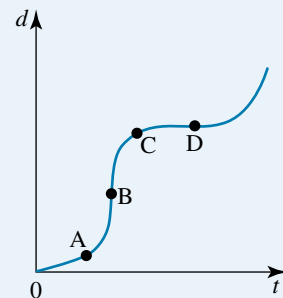


DISCUSSION

Would it be possible to have a section of a distance–time graph with a vertical gradient? What would this mean?

WORKED EXAMPLE 12

The diagram illustrates the distance travelled by a car over time. Describe what is happening, in terms of speed, at each of the marked points.



THINK

From the graph:

At A the gradient is small, but becoming steeper.

At B the gradient is at its steepest and is not changing.

At C the graph is becoming flatter.

At D the graph is horizontal.

WRITE

At A the car is travelling slowly but accelerating.

At B the car is at its greatest speed.

At C the car is slowing down.

At D the car is stationary.

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RESOURCES — ONLINE ONLY

Interactivity: Constant rates (int-6060)

Interactivity: Variable rates (int-6061)

Digital doc: SkillsHEET Converting time in minutes and seconds into seconds only (doc-10844)

Digital doc: SkillsHEET Equivalent rates (doc-10845)

Exercise 13.6 Constant and variable rates

Individual pathways

PRACTISE

Questions:
1–7

CONSOLIDATE

Questions:
1–13

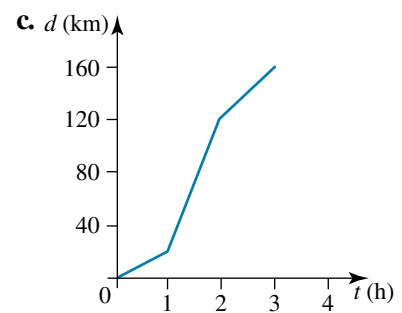
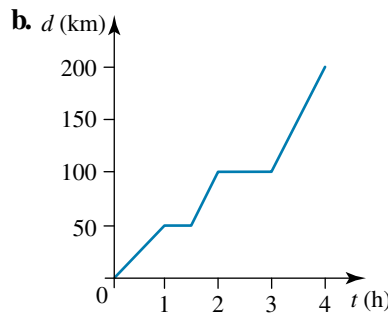
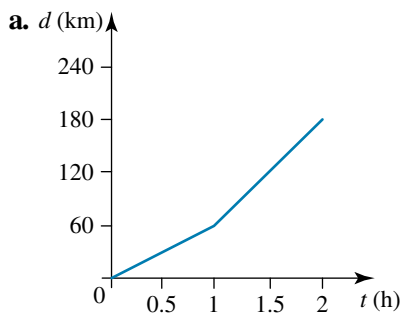
MASTER

Questions:
1–10, 13–15

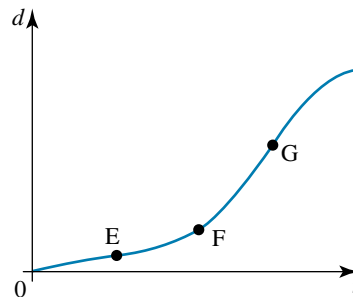
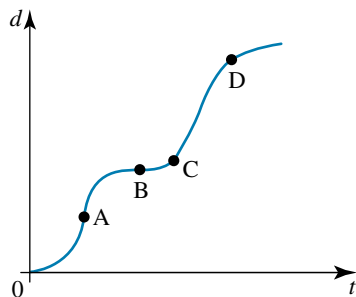
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Understanding and fluency

- Two friends take part in a 24-kilometre mini-marathon. They run at constant speed. Ali takes 2 hours and Beth takes 3 hours to complete the journey.
 - On the same diagram, draw a distance–time graph for each runner.
 - What is the equation for each graph?
 - What is the difference between the two graphs?
- WE10** Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.



- WE11** Draw a distance–time graph to illustrate each of the following journeys.
 - A cyclist rides at 40 km/h for 30 minutes, stops for a 30-minute break, and then travels another 30 km at a speed of 15 km/h.
 - Zelko jogs at a speed of 10 km/h for one hour, and then at half the speed for another hour.
- WE12** The diagrams below illustrate the distances travelled by two cars over time.



- Describe what is happening in terms of speed at each of the marked points.
- For each diagram:
 - at which point is the speed the greatest?
 - at which point is the speed the least?
 - at which point is the car stationary?

5. The table below shows the distance travelled, D , as a person runs for R minutes.

R (minutes)	10	20	50
D (km)	2	4	10

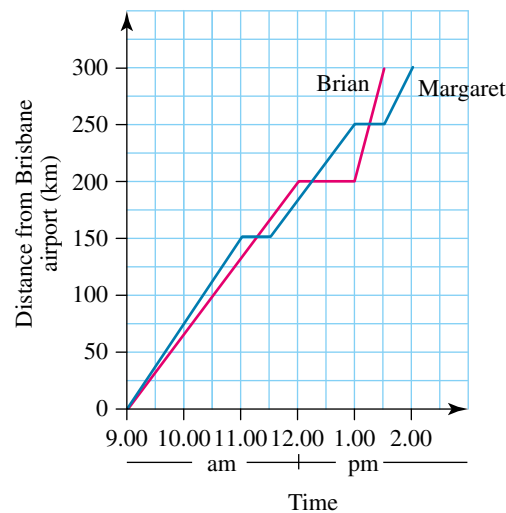
- Find the rate in km/min between the times 10 minutes and 20 minutes.
 - Find the rate in km/min between the distances 4 km and 10 km.
 - Is the person's speed constant? Why?
6. The table below shows the water used, W , after the start of the shower, where T is the time after the shower was started.

T (minutes)	0	1	2	4
W (litres)	0	20	30	100

- Find the rate of water usage in L/min for the 4 stages of the shower.
- Was the rate of water usage constant?

Communicating, reasoning and problem solving

7. Margaret and Brian left Brisbane airport at 9.00 am. They travelled separately but on the same road and in the same direction. Their journeys are represented by the travel graph below.



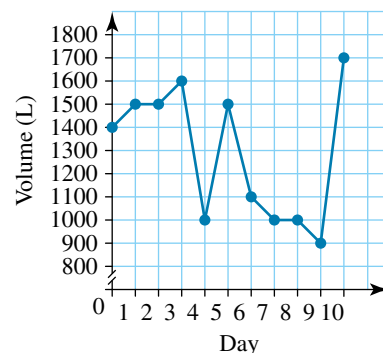
- At what distance from the airport did their paths cross?
 - How far apart were they at 1.00 pm?
 - For how long did each person stop on the way?
 - What was the total time spent driving and the total distance for each person?
 - Calculate the average speed while driving for each person.
8. Hannah rode her bike along the bay one morning. She left home at 7.30 am and covered 12 km in the first hour. She felt tired and rested for half an hour. After resting she completed another 8 km in the next hour to reach her destination.
- How long did Hannah take for the entire journey?
 - What is the total distance for which she actually rode her bike?
 - Draw a travel graph for Hannah's journey.



9. Rebecca and Joanne set off at the same time to jog 12 kilometres. Joanne ran the entire journey at constant speed and finished at the same time as Rebecca. Rebecca set out at 12 km/h, stopping after 30 minutes to let Joanne catch up, then she ran at a steady rate to complete the distance in 2 hours.

- Show the progress of the two runners on a distance–time graph.
- How long did Rebecca wait for Joanne to catch up?

10. Use the graph showing the volume of water in a rainwater tank to answer the following questions.



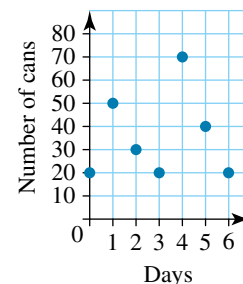
- During which day(s) was the rate of change positive?
- During which day(s) was the rate of change negative?
- During which day(s) was the rate of change zero?
- On which day did the volume of water increase at the fastest rate?
- On which day did the volume of water decrease at the fastest rate?

11. An internet service provider charges \$30 per month plus \$0.10 per megabyte downloaded. The table of monthly cost versus download amount is shown.

Download (MB)	0	100	200	300	400	500
Cost (\$)	30	40	50	60	70	80

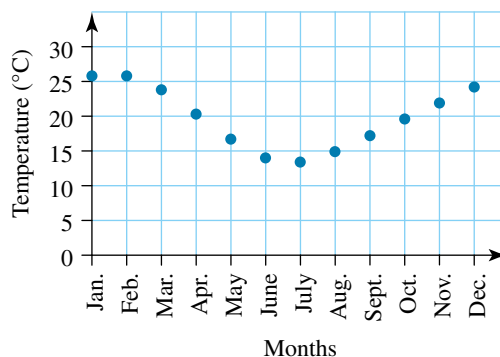
- By how much does the cost increase when the download amount increases from:
 - 0 to 100 MB
 - 100 to 200 MB
 - 200 to 300 MB
- Is the cost increasing at a constant rate?

12. The graph shows the number of soft-drink cans in a vending machine at the end of each day.



- By how much did the number of soft-drink cans change in the first day?
- By how much did the number of soft-drink cans change in the fifth day?
- Is the number of soft-drink cans changing at a constant or variable rate?

13. The table and graph show the average daily maximum temperature over the year for a particular city.



	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Mean maximum (°C)	25.8	25.8	23.8	20.3	16.7	14.0	13.4	14.9	17.2	19.6	21.9	24.2

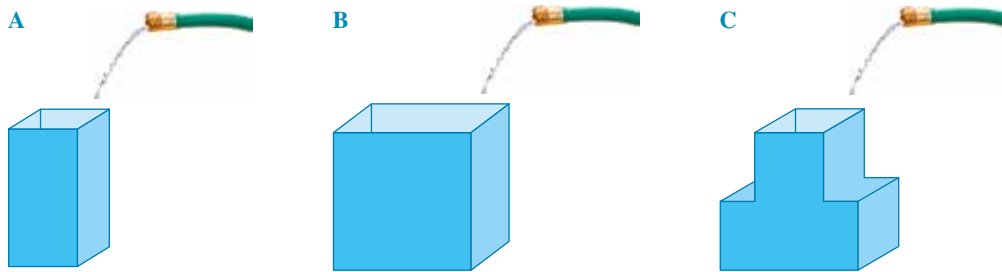
- a. What is the average maximum temperature in:
 - i. February
 - ii. June?
 - b. What is the change in temperature from:
 - i. January to August
 - ii. November to December?
 - c. Is the temperature changing at a constant rate?
14. How can you tell the difference between constant and variable rates?
15. An early method of catching speeding motorists was the amphometer. It consisted of two airtight tubes placed across the road at a separation of 15 m. The tubes were connected to a timing device that detected the change in pressure as a car crossed them. In a 60 km/h zone, what was the minimum time for the car to travel across the device without being booked for speeding?



13.7 Rates of change

[Stage 5.3]

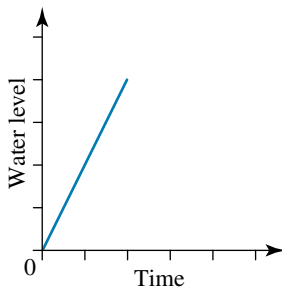
- Imagine that water is flowing out of a hose at a steady rate and will be used to fill several containers.



Because they are of different widths, they will fill at different rates. The narrow container will fill at a faster rate than the other two.

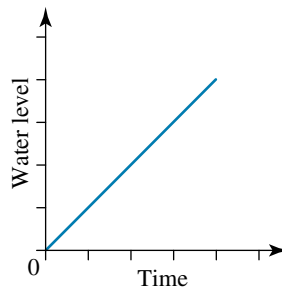
- Consider a graph of the water level against time for the three containers.

Container *A* is narrow, so the water level will rise quickly.



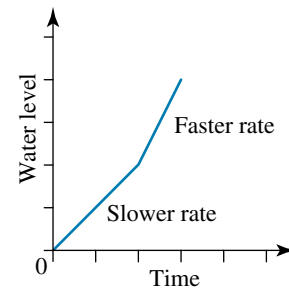
The water level changes at a constant rate.

Container *B* is wide and will fill at a slower rate.



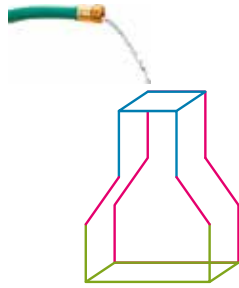
The water level changes at a constant rate.

Container *C* is wide at the bottom, so the water will rise slowly at first, then quickly when it reaches the narrow part.

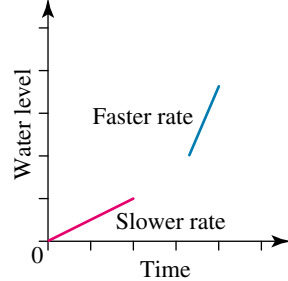


The water level changes at two different but constant rates.

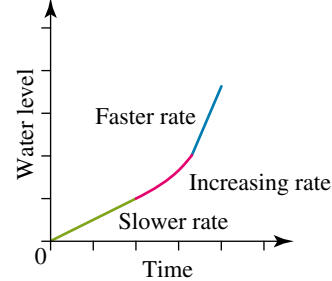
- Here is a more complex container, with three distinct sections.



The bottom section fills slowly and the top section fills quickly.

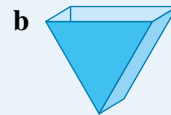
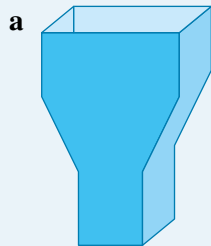


In between, the rate changes steadily from slow to fast. In this section, the rate is increasing.



WORKED EXAMPLE 13

Each of these containers is being filled with water at a steady rate. For each container, sketch a graph of the water level against time.

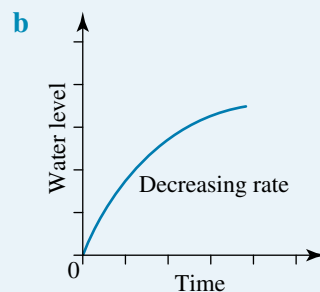
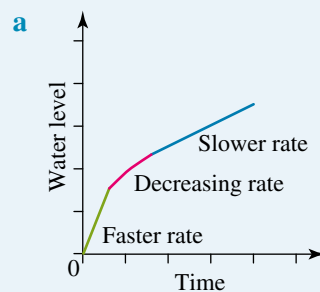


THINK

- a** The container has three distinct sections. The bottom section will fill quickly. The top section will fill slowly. In the middle section, the rate will increase gradually from slow to quick.

- b** At the bottom of the container the water will rise rapidly, slowing down as the container becomes wider.

DRAW



Interactivity: Rates of change (int-6062)

Digital doc: SkillSHEET Completing a table of values (doc-10846)

Digital doc: SkillSHEET Plotting from a table of values (doc-10847)

Digital doc: WorkSHEET Problems using rates (doc-6187)

Exercise 13.7 Rates of change

Individual pathways

PRACTISE

Questions:
1–6

CONSOLIDATE

Questions:
1–8

MASTER

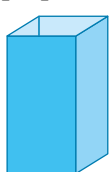
Questions:
1–9

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

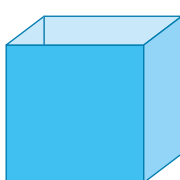
Understanding and fluency

1. These containers are filled with water at a steady rate. Match each container with the appropriate graph.

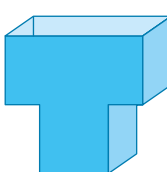
a.



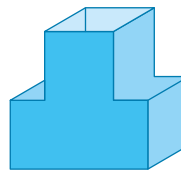
b.



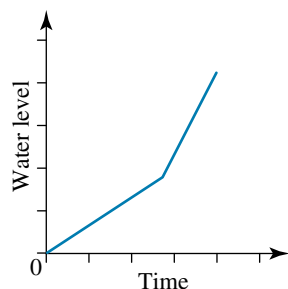
c.



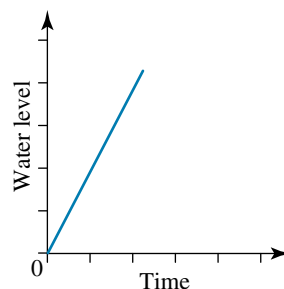
d.



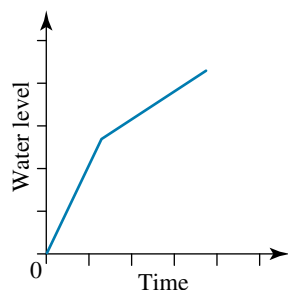
A.



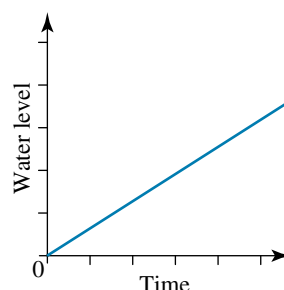
B.



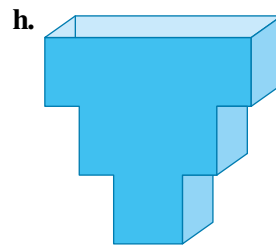
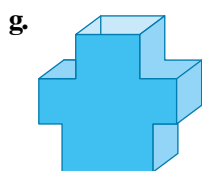
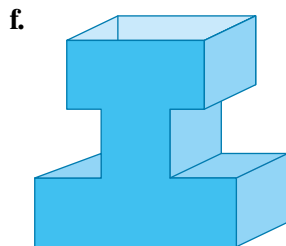
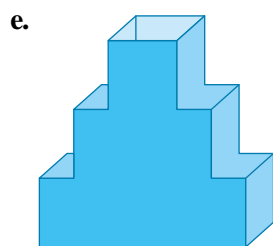
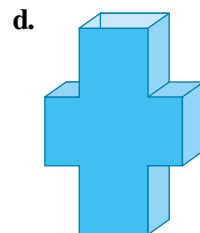
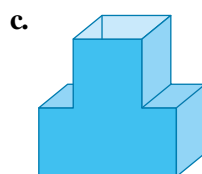
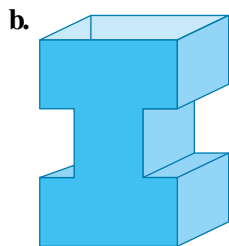
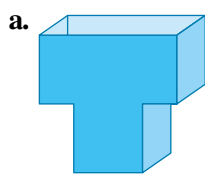
C.



D.



2. These containers are being filled with water at a steady rate. Sketch a graph of the water level against time.

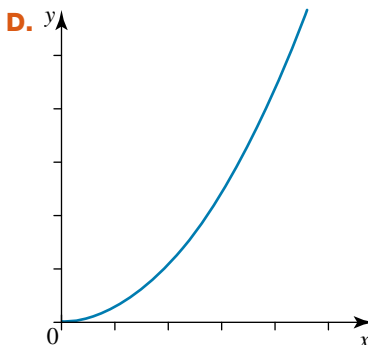
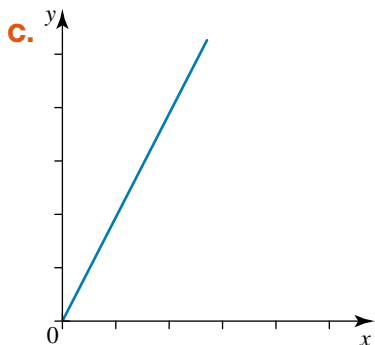
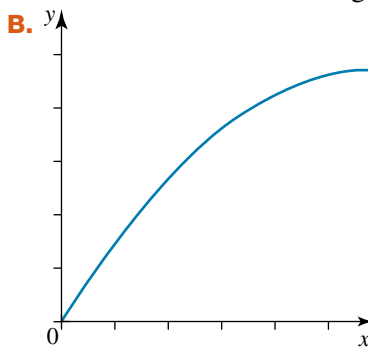
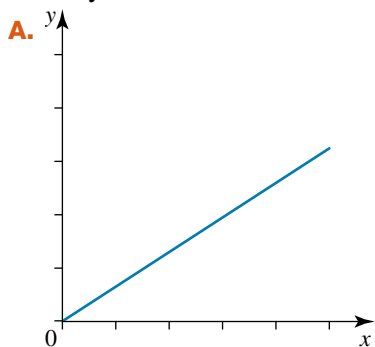


3. Which of these graphs show:

a. a steady rate

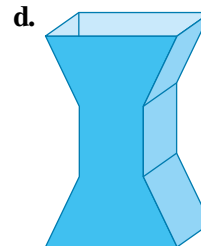
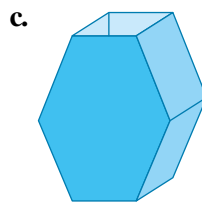
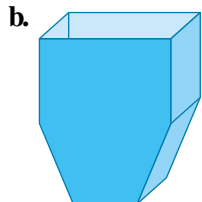
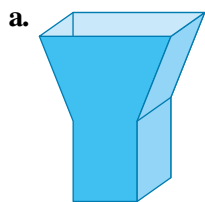
b. an increasing rate

c. a decreasing rate?

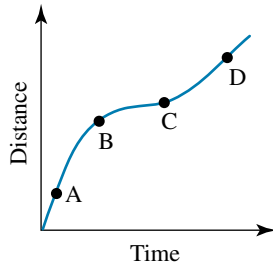


Communicating, reasoning and problem solving

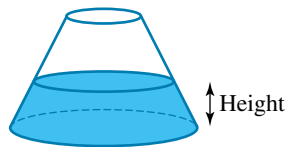
4. **WE13** These containers are being filled with water at a steady rate. For each container, sketch a graph of water level against time and explain your reasons.



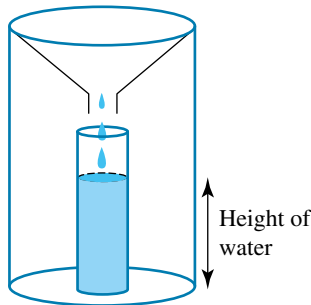
5. The distance versus time graph for a runner is shown below. Four times during the run are marked as A, B, C and D on the graph. State, with reasoning, at which of the points the runner is:



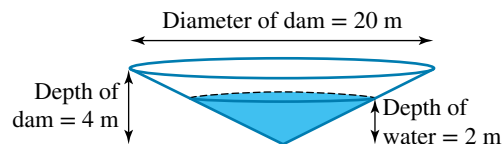
- travelling fastest
 - travelling slowest
 - slowing down
 - speeding up
 - travelling at constant speed.
6. Water is poured at a constant rate into the container shown. Sketch the graph of height of water against time.



7. A rain gauge is made up of an inner cylinder and an outer cylinder, as shown in the diagram. When the inside cylinder fills and overflows, then the outside cylinder fills, so there are two scales on the device. The inner cylinder measures smaller quantities accurately and the outer cylinder measures larger quantities. For a day when the rain falls steadily, sketch a graph of the height of the water versus time.



8. A dam can be thought of as an inverted cone with a large radius and small height, as shown in the diagram. If the depth of water is half of the depth of the dam, at what percentage of its capacity is the dam? The formula for the volume of a cone is $V = \frac{1}{3} \pi r^2 h$.



9. How can you tell from a graph whether the rate of change is constant or variable?

13.8 Review

Investigation | Rich task

Fastest speeds



To calculate the average speed of a journey, we can use the speed formula:

$$\text{speed} = \frac{\text{distance}}{\text{time}}.$$

Speed is a *rate*, because it compares two quantities of different units. Speed can be expressed in units such as km/h, km/min and m/s. The units of the quantities substituted into the numerator and denominator determine the final units of speed. In order to compare the speeds of different events, it is useful to convert them to the same units.

In 2008, the summer and winter Olympics were held. At the summer Olympics, sprinter Usain Bolt ran 100 m in 9.69 seconds. At the winter Olympics, cross-country skier Petter Northug covered 50 km in 2 hours and 5 minutes, and speed skater Mika Poutala covered 0.5 km in 34.86 seconds. Which competitor was the fastest? In order to answer this question, we need to determine the speed of each competitor. Since the information is quoted in a variety of units, we need to decide on a common unit for speed.



1. Calculate the speed of each athlete in m/s, correct to 1 decimal place.
2. Which athlete was the fastest?



Consider the speed of objects in the world around you. This task requires you to order the following objects from fastest to slowest, assuming that each object is travelling at its fastest speed possible:

electric car, submarine, diesel train, car ferry, skateboard, bowled cricket ball, windsurfer, served tennis ball, solar-powered car, motorcycle, aircraft carrier, Jaguar motorcar, helicopter, airliner, rocket-powered car, the Concorde supersonic airliner.

3. From your personal understanding and experience, order the above list from fastest to slowest.

We can make a more informed judgement if we have facts available about the movement of each of these objects. Consider the following facts:

When travelling at its fastest speed, the Concorde can cover a distance of about 7000 km in 3 hours. The fastest airliner can cover a distance of 5174 km in 2 hours and a helicopter 600 km in 1.5 hours. It would take 9.5 hours for a rocket-powered car to travel 9652 km, 3.25 hours for an electric car to travel 1280 km and 1.5 hours for a Jaguar car to travel 525 km. On a sunny day, a solar-powered car can travel 39 km in half an hour. In 15 minutes, a car ferry can travel 26.75 km, an aircraft carrier 14 km and a submarine 18.5 km. It takes the fastest diesel train 0.42 hours to travel 100 km and a motorcyclist just 12 minutes to travel the same distance. In perfect weather conditions, a skateboarder can travel 30 km in 20 minutes and a windsurfer can travel 42 km in 30 minutes. A bowled cricket ball can travel 20 m in about 0.45 seconds, whereas a served tennis ball can travel 25 m in about 0.4 seconds.

4. Using the information above, decide on a common unit of speed and determine the speed of each of the objects.

5. Order the above objects from fastest to slowest.

6. Compare the order from question 5 with the list you made in question 3.

7. Conduct an experiment with your classmates and record times that objects take to cover a certain distance. For example, record the time it takes to run 100 m or throw a ball 20 m. Compare the speeds with the speeds of the objects calculated during this investigation.

learnON RESOURCES — ONLINE ONLY

Digital doc: Investigation — Fastest speeds (doc-15900)

Digital doc: Code puzzle — Who created the fictional character Sherlock Holmes? (doc-15901)

Digital doc: Summary Topic 13 (doc-22895)

Interactivity: Crossword Topic 13 (int-2690)

Interactivity: Word search Topic 13 (int-6188)

Interactivity: Sudoku Topic 13 (int-3208)

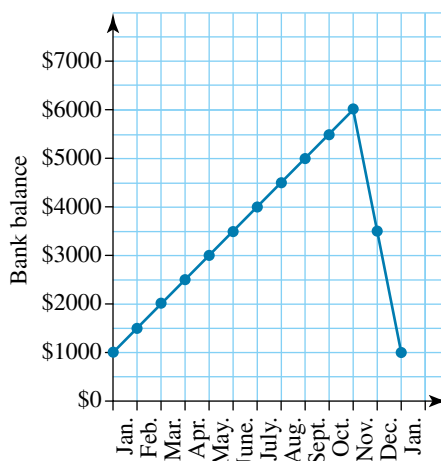
Exercise 13.8 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- MC** t is directly proportional to s and the gradient $m = 1.5$. When $s = 2$, t is equal to:
A. 1.5 **B.** 0.66 **C.** 0.75 **D.** 3 **E.** 5
- MC** y is directly proportional to x and $y = 450$ when $x = 15$. The rule relating x and y is:
A. $y = 0.0333x$ **B.** $y = 30x$ **C.** $y = 60x$ **D.** $y = 6750x$ **E.** $y = 60750x$
- MC** If $y \propto x$ and $y = 10$ when $x = 50$, the constant of proportionality is:
A. 10 **B.** 5 **C.** 1 **D.** 0.2 **E.** 0.1

4. **MC** If $y \propto x$ and $y = 10$ when $x = 50$, the value of x when $y = 12$ is:
A. 6 **B.** 60 **C.** 40 **D.** 2.4 **E.** 4
5. **MC** If y is inversely proportional to x , then which of the following statements is true?
A. $x + y$ is a constant value
B. $y \div x$ is a constant value
C. $y \times x$ is a constant value
D. $y - x$ is a constant value
E. $x \div y$ is a constant value
6. **MC** The number of calculators a company sells is inversely proportional to the selling price. If a company can sell 1000 calculators when the price is \$22, how many could they sell if they reduced the price to \$16?
A. 2000 **B.** 727 **C.** 6000 **D.** 1375 **E.** 4500
7. **MC** Maya is sharing her collection of 60 fuchsia plants among three members of the family in the ratio of 2 : 3 : 5. The difference between the largest and smallest share of fuchsias is:
A. 12 **B.** 18 **C.** 36 **D.** 42 **E.** 54
8. **MC** A speed of 60 km/h is equivalent to approximately:
A. 17 m/min **B.** 1 km/s **C.** 1 m/s **D.** 17 m/s **E.** 1.7 m/s
9. **MC** A metal part has a density of 37 g/mm³. If its volume is 6 mm³, it has a mass of:
A. 6.1 g **B.** 6.1 kg **C.** 222 kg **D.** 222 g **E.** 22.2 g
10. **MC** Which of the following is not a rate?
A. 50 km/h **B.** 70 beats/min **C.** 40 kg
D. Gradient **E.** 13 cows/ha
11. **MC** How long does it take to travel 240 km at an average of 60 km/hour?
A. 0.25 hours **B.** 240 minutes **C.** 180 hours
D. 14400 hours **E.** 140 hours
12. **MC** A plane takes 2 hours to travel 1600 km. How long does it take to travel 1000 km?
A. 150 minutes **B.** 48 minutes **C.** 2 hours
D. 75 minutes **E.** 1 hour
13. **MC** The graph below is of Sandy's bank balance. How many months during the year has she been saving?

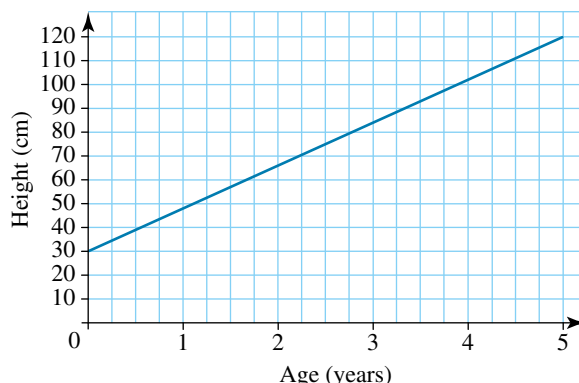


- A.** 2 months **B.** 5 months **C.** 10 months **D.** 12 months **E.** 18 months
14. If $w \propto v$, and $w = 7.5$ when $v = 5$, calculate k , the constant of proportionality.
15. The gear ratio for front and back sprockets of a bicycle is 10 : 3. If the front (large) sprocket has 40 teeth, how many teeth does the back sprocket have?

16. Calculate the missing quantities in the table below.

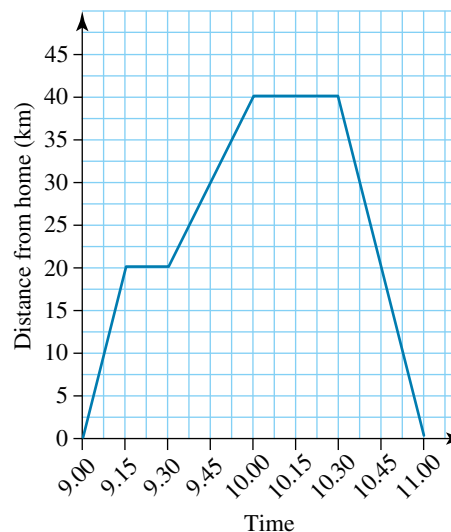
Mass	Volume	Density
500 g	20 cm ³	
1500 g		50 g/cm ³
	120 cm ³	17 g/cm ³

17. a. A 2000-litre water tank takes 2 days to fill. Express this rate in litres per hour with 2 decimal places.
 b. How far do you have to travel vertically to travel 600 metres horizontally if the gradient of a track is 0.3?
 c. How long does it take for a 60-watt (60 joules/second) light globe to use 100 kilojoules?
 d. Find the cost of 2.3 m³ of sand at \$40 per m³.
18. The graph shows the partial variation between the height of a child over time. What is the rate of change of height with respect to age in cm/year?



Communicating, reasoning and problem solving

19. The authors of a Physics textbook are going to share royalties from sales of the book in the ratio proportional to the number of chapters each has written. Miss Alan wrote 4 chapters, Mr Bradley wrote 3 chapters, Mrs Cato wrote 7 chapters and Ms Dawn wrote 6 chapters. If the expected amount to be shared is \$28000, how much money will each author get?
20. Lisa drove to the city from her school. She covered a distance of 180 km in 2 hours.
 a. What is Lisa's average speed?
 b. Lisa travelled back at an average speed of 60 km/h. How long did she take?
21. Seventy grams of ammonium sulfate crystals are dissolved in 0.5 L of water.
 a. What is the concentration of the solution in g/mL?
 b. Another 500 mL of water is added. What is the concentration of the solution now?
22. Karina left home at 9.00 am. She spent some time at a friend's house then travelled to the airport to pick up her sister. She then travelled straight back home. Her journey is shown by the travel graph at right.
- How far from Karina's house is her friend's house?
 - How much time did Karina spend at her friend's place?
 - How far is the airport from Karina's house?
 - How much time did Karina spend at the airport?
 - How much time did Karina take to drive home?
 - Find the average speed of Karina's journey:
 - from her home to the friend's place
 - from her friend's place to the airport
 - from the airport to her home.



23. A skyscraper can be built at a rate of 4.5 storeys per month.
- How many storeys will be built after 6 months?
 - How many storeys will be built after 24 months?
24. A certain kind of eucalyptus tree grows at a linear rate for its first 2 years of growth. If the growth rate is 5 cm per month, how long will it take to grow to be 1.07 m tall?
25. The pressure inside a boiler increases as the temperature increases. For each 1°C , the pressure increases by 10 units. At a temperature of 100°C the pressure is 600 units. If the boiler can withstand a pressure of 2000 units, at what temperature does this occur?
26. Hector has a part-time job as a waiter at a local café and is paid \$8.50 per hour. Complete the table of values relating the amount of money received to the number of hours worked.

Number of hours	0	2	4	6	8	10
Pay						

27. A fun park charges a \$10 entry fee and an additional \$3 per ride. Complete the following table of values relating the total cost to the number of rides.

Rides	0	2	4	6	8	10
Cost						

28. Speed is a measure of the distance covered in a period of time. Distance is a length measurement and the units can be kilometres, metres, centimetres, millimetres or other metric units. The length measurements could also be in imperial units, such as miles, yards, feet or inches. Time can be measured in years, days, hours, minutes or seconds, just to name a few possible units. This means that speed can be quoted in units such as kilometres per hour, metres per minute, miles per hour or feet per second.

In Australia, we use the metric measurement of length (kilometres, metres, centimetres, millimetres). This system is used in most parts of the world. However, the United States uses imperial units of length (miles, yards, feet, inches), and the United Kingdom uses a combination of imperial and metric. How can you compare, for instance, the speed of a car travelling at 100 km/h in Australia with one travelling at 100 miles/h (or mph) in the United States?

To help compare speed in different units, a length conversion chart is useful. Time is measured in the same units throughout the world, so a time conversion chart is not necessary.

Length conversion chart

Imperial unit	Conversion factor	Metric unit
inches (ins)	25.4	mm
feet (ft)	30.5	cm
yards (yds)	0.915	m
miles	1.61	km

To convert an imperial length unit into its equivalent metric unit, multiply by the conversion factor. Divide by the conversion factor when converting from metric to imperial units.

- Use the table to complete each of the following.
 - 12 ins = ____ mm
 - 3 ft = ____ mm
 - 1 m = ____ yd
 - 1 km = ____ miles
 - One car is travelling at 100 km/h, and another car is travelling at 100 miles/h. Which car is faster?
29. The speed limit of 60 km/h in Australia would be equivalent to a speed limit of ____ miles/h in USA. Give your answer correct to 3 decimal places.

30. The first supersonic land speed record for wheeled vehicles on ground was set by Andy Green of the United States in 1997. He reached a speed of 763.035 miles/h. How fast is this in km/h? Give your answer correct to 3 decimal places.
31. A commercial aircraft covers a distance of 1700 km in 2 hours 5 minutes. How many metres does the aircraft travel each second?
32. A launched rocket covers a distance of 17 miles in 10 seconds. Calculate its speed in km/h.
33. Here are some record speeds for moving objects.

Motorcycle	149 m/sec
Train	302 miles/h
Human skiing	244 km/h
Bullet from 38-calibre revolver	4000 ft/sec

Convert these speeds to the same unit. Then place them in order from fastest to slowest.

Answers

Topic 13 Proportion and rates

Exercise 13.2 Direct proportion

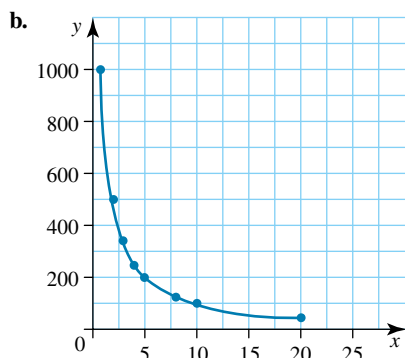
1. a. Yes
b. No; as speed increases, time decreases.
c. No; doubling distance doesn't double cost.
d. Yes
e. No; doubling the side length doesn't double the area.
f. Yes
g. Yes
h. No; doubling distance doesn't double cost (due to the initial fee).
i. No; doubling age doesn't double height.
2. a. No; as x doubles, y does not.
b. Yes
c. No; as t doubles, d does not.
d. No; when $n = 0$, d does not.
e. Yes
f. No; when $t = 0$, d does not.
g. No; as x doubles, y does not.
h. No; when $d = 0$, w does not.
3. Answers will vary.
4. The point (0, 0) must always exist in a table of values of two variables exhibiting direct proportionality.
5. The missing value is 8. The relationship between n and m can be calculated by the values 5 and 20 in the table ($n = 4m$).
6. a. Yes, direct proportion does exist.
b. \$2.55
7. Direct proportion does not exist. The price per metre for the 4.2-metre length is \$5.50, and the price per metre for the 5.4-metre length is \$5.40.
8. If two quantities are directly proportional, when one quantity increases, so does the other; when one quantity equals zero, so does the other; and the graph of the relationship between the two quantities is a straight line passing through the origin.

Exercise 13.3 Direct proportion and ratio

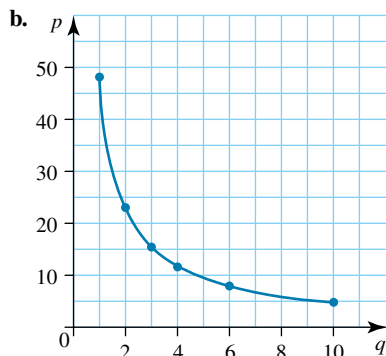
1. $k = 6$, $a = 6b$	2. $a = 0.5b$	3. $C = 12.5t$	
4. $v = 0.5t$	5. $F = 2.5a$		
6. a. $E = 0.625W$	b. 19.2 kg	c. 4.375 cm	
7. a. $W = 1.3n$	b. ≈ 385 almonds	c. 325 g	
8. a. $d = 2.5n$	b. 2017.5 km	c. 2000 turns	
9. a. $M = 0.12n$	b. 450 seedlings	c. 360 kg	
10. \$217.60	11. \$217.25	12. 13.33 L	13. \$105.98
14. 375 g	15. \$150	16. \$33.75	
17. a. $D = 100T$	b. 6 hours 6 minutes	c. 2 hours 12 minutes	d. 3 hours 54 minutes
18. If two values (x, y) are directly proportional ($y \propto x$), then $y = kx$, where k is a constant. This constant k is equal to the ratio $\frac{y}{x}$.
19. 275 cm^3

Exercise 13.4 Inverse proportion

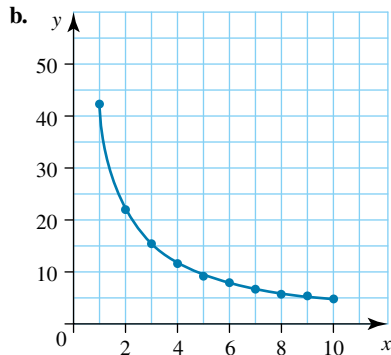
1. a. $s = \frac{k}{t}$	b. No	c. $t = \frac{k}{p}$ or $p = \frac{k}{t}$	d. $L = \frac{k}{p}$ or $p = \frac{k}{L}$
e. No	f. $t = \frac{k}{n}$ or $n = \frac{k}{t}$		
2. Answers will vary.
3. a. $k = 1000$, $y = \frac{1000}{x}$



4. a. $k = 48$, $p = \frac{48}{q}$



5. a. $k = 42$, $y = \frac{42}{x}$

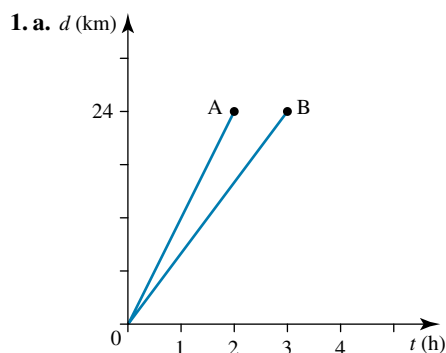


6. a. 4000	b. $a = \frac{4000}{m}$	c. 20 m/s ²	d. 4 m/s ²
7. a. 500	b. 2500 pencils	c. 1000 pencils	
8. a. 337.5	b. 3.97 hours = 3 h 58 min	c. 96.4 km/h	d. 337.5 km
9. a. 17500	b. \$233.33	c. 70 people	d. \$17500
10. a. 200	b. 1 ampere	c. 13.3 ohms	d. Check with your teacher.
11. The constant of proportionality represents the distance of the trip. Therefore, when this value is smaller, the time taken to complete the trip at the same speed is also smaller.
- | | | |
|----------------|----------------------------|----------------------|
| 12. a. 10 | b. $T = \frac{10}{P}$ | c. 1 hour 40 minutes |
| 13. a. 100.048 | b. $T = \frac{100.048}{V}$ | c. 9.58 seconds |
| | | d. 15.00 seconds |
14. Inverse proportion is when the number of one variable increases, the number of the other variable decreases. It is expressed as $x \propto \frac{1}{y}$ or $x = \frac{k}{y}$, where k is the constant of proportionality.

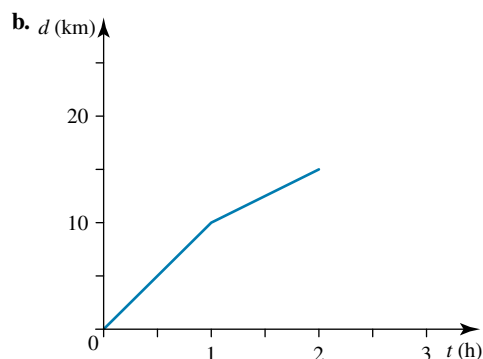
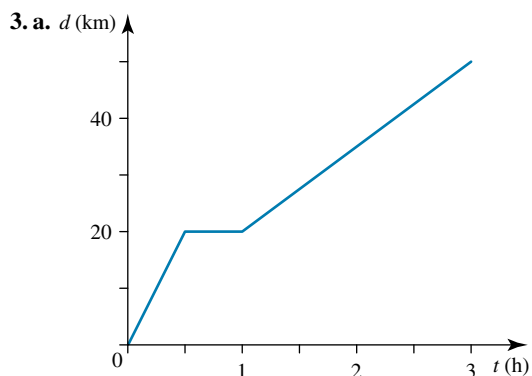
Exercise 13.5 Simple rate problems

1. 64 km/h
2. a. 120 km/h b. 1692.3 km/h c. 28.8 km/h
3. a. \$92 per ticket b. 24 cakes per hour c. 10.44 m/s d. 3 cm/h
- e. \$17.33/h f. \$50 per air conditioner
4. 4 g/cm³
5. 600 g/L
6. The hare runs at 120000 cm per minute; the tortoise runs at 0.144 km per hour. The hare runs 500 times faster than the tortoise.
7. Either the distance of the journey decreases and the time remains constant, or the distance of the journey remains constant and the time decreases.
8. Cathy Freeman: 29.32 km/h; Usain Bolt: 37.15 km/h
9. 75 students/year
10. 0.15
11. 2750 organisms/litre. The beach should not be closed.
12. A currency exchange rate

Exercise 13.6 Constant and variable rates

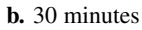


- b. i. A: $d = 12t$, B: $d = 8t$ ii. A has a steeper gradient than B.
2. a. 1 hour at 60 km/h, then 1 hour at 120 km/h
- b. 1 hour at 50 km/h, a 30-minute stop, 30 minutes at 100 km/h, a 1-hour stop, then 1 hour at 100 km/h
- c. 1 hour at 20 km/h, 1 hour at 100 km/h, 1 hour at 40 km/h

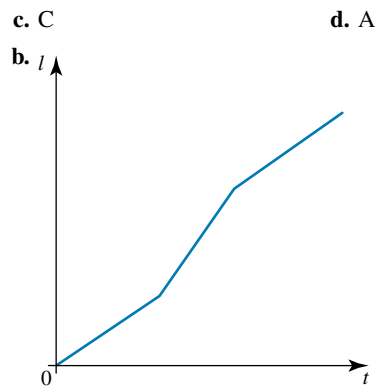


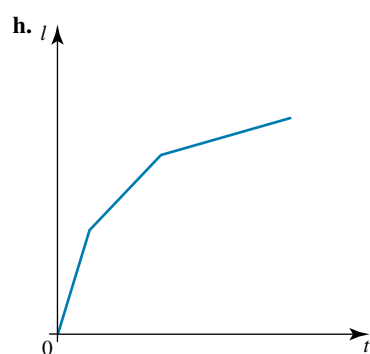
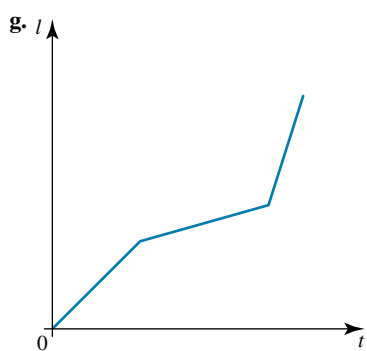
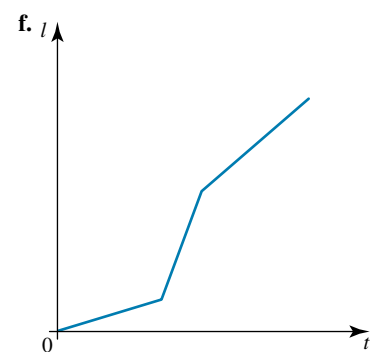
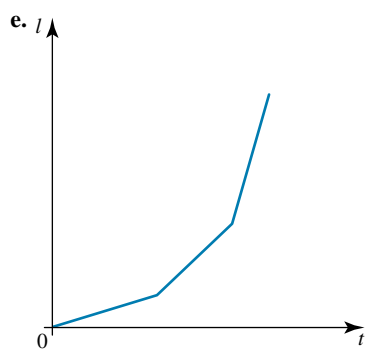
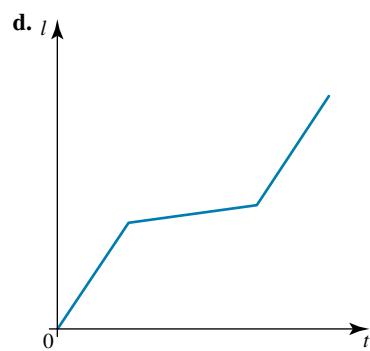
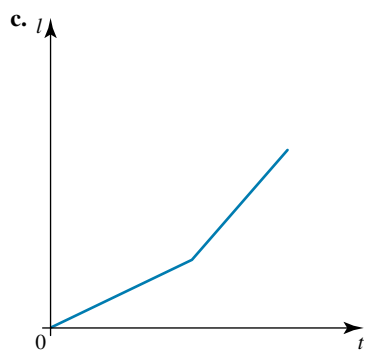
4. a. A The car is moving with steady speed.
 B The car is momentarily stationary.
 C The speed is increasing.
 D The car is slowing down.
 E The car is moving at a slow steady speed.
 F The speed is increasing.
 G The car is moving at a faster steady speed.
- b. i. A, G ii. B, E iii. B

- c.
-
- A distance-time graph showing the journey from home to school. The vertical axis is labeled 'Distance from home (km)' and ranges from 0 to 20 in increments of 5. The horizontal axis is labeled 'Time (am)' and ranges from 7.30 to 10.00 in increments of 0.30. The graph consists of three segments: a straight line from (7.30, 0) to (8.30, 12), a horizontal line from (8.30, 12) to (9.00, 12), and a straight line from (9.00, 12) to (10.00, 20).
- | Time (am) | Distance from home (km) |
|-----------|-------------------------|
| 7.30 | 0 |
| 8.30 | 12 |
| 9.00 | 12 |
| 10.00 | 20 |



- ### Exercise 13.7 Rates of change



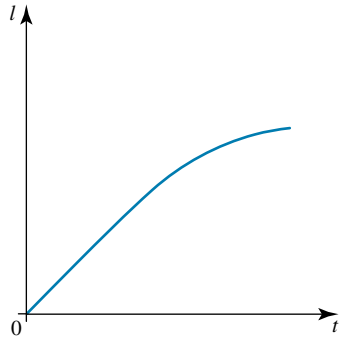


3. a. A, C

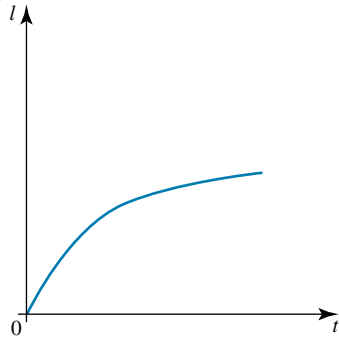
b. D

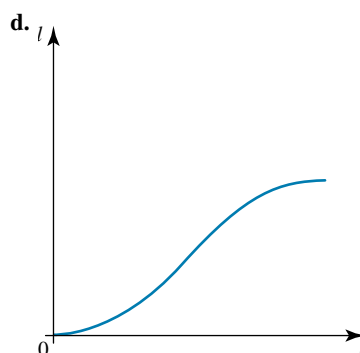
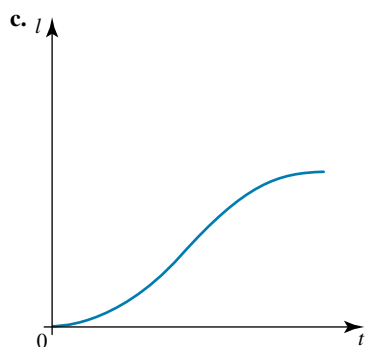
c. B

4. a.



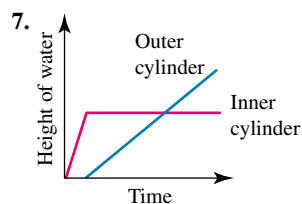
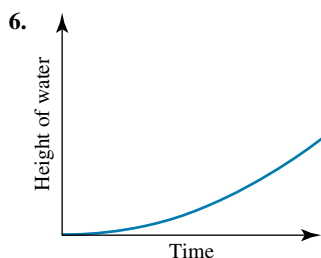
b.





5. a. Point A — the gradient is greatest.
 c. Point B — the gradient is decreasing.
 e. Points A and D — the gradient is constant.

- b. Point C — the gradient is least.
 d. Point C — the gradient is increasing.



8. 12.5%

9. If the rate of change is constant, the graph will be a straight line. If the rate of change is variable, the graph line will show changes in gradient.

Investigation | Rich task

1. Usain Bolt = 10.3 m/s

Petter Northug = 6.7 m/s

Mika Poutala = 14.3 m/s

2. Mika Poutala

3. Answers will vary.

4. Concorde: 2333.3 km/h

Airliner: 2587 km/h

Helicopter: 400 km/h

Rocket-powered car: 1016 km/h

Electric car: 393.8 km/h

Jaguar car: 350 km/h

Solar-powered car: 78 km/h

Car ferry: 107 km/h

Aircraft carrier: 56 km/h

Submarine: 74 km/h

Diesel train: 238.1 km/h

Motorcyclist: 500 km/h

Skateboarder: 90 km/h

Windsurfer: 84 km/h

Bowled cricket ball: 160 km/h

Served tennis ball: 225 km/h

5. Airliner, Concorde, rocket-powered car, motorcyclist, helicopter, electric car, Jaguar car, diesel train, served tennis ball, bowled cricket ball, oar ferry, skateboarder, windsurfer, solar-powered car, submarine, aircraft carrier

6. Answers will vary.

7. Answers will vary.

Exercise 13.8 Review questions

1. D 2. B 3. D 4. B 5. C
 6. D 7. B 8. D 9. D 10. C
 11. B 12. D 13. C 14. 1.5 15. 12 teeth

16.

Mass	Volume	Density
500 g	20 cm ³	25 g/cm ³
1500 g	30 cm ³	50 g/cm ³
2040 g	120 cm ³	17 g/cm ³

17. a. 41.67 L/h b. 180 m c. 28 minutes d. \$92

18. 18 cm/year

19. Miss Alan \$5600, Mr Bradley \$4200, Mrs Cato \$9800, Ms Dawn \$8400

20. a. 90 km/h b. 3 h

21. a. 0.14 g/mL b. 0.07 g/mL

22. a. 20 km b. 15 min c. 40 km d. 30 min e. 30 min

f. i. 80 km/h ii. 40 km/h iii. 80 km/h

23. a. 27 b. 108

24. 21.4 months

25. 240 °C

26.

Number of hours	0	2	4	6	8	10
Pay	\$0	\$17	\$34	\$51	\$68	\$85

27.

Rides	0	2	4	6	8	10
Cost	\$10	\$16	\$22	\$28	\$34	\$40

28. a. i. 304.8 mm ii. 915 mm iii. 1.09 yd iv. 0.62 miles

b. The one travelling at 100 miles/h

29. 37.267 miles/h

30. 1228.486 km/h

31. $226\frac{2}{3}$ m/sec

32. 9853.2 km/h

33. The conversions may vary, but the order from fastest to slowest is:

- bullet from 38-calibre revolver
- motor cycle
- train
- human skiing.

TOPIC 14

Algebraic techniques 2

[Stage 5.3]

14.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

14.1.1 Why learn this?

Algebra enables us to move from a specific example to a generalisation. We can then create formulas that are powerful because they can be used in every situation. Formulas impact on us every day as they can be used in very many diverse situations, from modelling weather patterns or landing space modules on Mars to repaying mortgages. Algebra helps us understand numbers better and to think logically. Many physical phenomena can be described using quadratic functions, and in this topic you will learn more algebraic techniques to help you to graph quadratic functions.



DISCUSSION

What is the role of algebra in the real world?

LEARNING SEQUENCE

- 14.1 Overview
- 14.2 [Stage 5.3] Special binomial products
- 14.3 [Stage 5.3] Further expansions
- 14.4 [Stage 5.3] The highest common binomial factor
- 14.5 [Stage 5.3] Factorisation patterns
- 14.6 [Stage 5.3] Factorising non-monic quadratics
- 14.7 [Stage 5.3] Simplifying algebraic fractions 2
- 14.8 [Stage 5.3] Applications of binomial products
- 14.9 Review

LEARNING OUTCOMES

A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- selects and applies appropriate algebraic techniques to operate with algebraic expressions MA5.3-5NA



CONTENT DESCRIPTIONS

Students:

Add and subtract algebraic fractions with numerical denominators, including those with binomial numerators (NSW)

Expand binomial products using a variety of strategies (ACMNA233)

Factorise monic and non-monic quadratic expressions (ACMNA269)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — The Arabic scholar Al-Khwarizmi (eles-1841)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

14.2 Special binomial products

[Stage 5.3]

14.2.1 Difference of two squares

- Look at the expansion of $(x + 3)(x - 3)$.

$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 9 \\ &= x^2 - 9\end{aligned}$$

Compare it with

$$\begin{aligned}(x + 5)(x - 5) &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25\end{aligned}$$

In both cases the middle terms cancel each other out, leaving just two terms. There seems to be a pattern, so we can take the short cut:

$$\begin{aligned}(x + 10)(x - 10) &= x^2 - 10^2 \\ &= x^2 - 100\end{aligned}$$

The terms x^2 and 100 are both squares, so an expression such as $x^2 - 100$ is called a **difference of two squares** (or DOTS). (Remember, difference means subtraction.) It is important to recognise this type of expression and remember its name.

$$\text{Difference of two squares: } (a + b)(a - b) = a^2 - b^2$$

- Does this rule work if $a = 7$ and $b = 3$? Let's check.

$$\begin{aligned}(7 + 3)(7 - 3) &= 10 \times 4 \\ &= 40 \\ 7^2 - 3^2 &= 49 - 9 \\ &= 40\end{aligned}$$

The rule works. Try some more examples yourself.

WORKED EXAMPLE 1

Use the difference of two squares rule to expand and simplify each of the following.

a $(x + 8)(x - 8)$

b $(6 - x)(6 + x)$

c $(2x - 3)(2x + 3)$

d $(3x + 5)(5 - 3x)$

THINK

a 1 Write the factorised expression.

2 This expression is in the form $(a + b)(a - b)$, so apply the rule $a^2 - b^2$.

b 1 Write the factorised expression.

2 The difference of two squares rule can be used. Expand using the formula.

Note: $36 - x^2$ is not the same as $x^2 - 36$.

c 1 Write the factorised expression.

2 The difference of two squares rule can be used. Expand using the formula.

d 1 Write the factorised expression.

2 The difference of two squares rule can be used by rearranging the terms, since $3x + 5 = 5 + 3x$. Expand using the formula.

WRITE

a $(x + 8)(x - 8)$

$$= x^2 - 8^2$$

$$= x^2 - 64$$

b $(6 - x)(6 + x)$

$$= 6^2 - x^2$$

$$= 36 - x^2$$

c $(2x - 3)(2x + 3)$

$$= (2x)^2 - 3^2$$

$$= 4x^2 - 9$$

d $(3x + 5)(5 - 3x)$

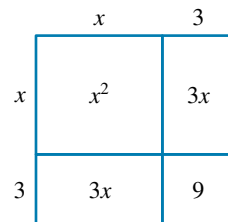
$$(5 + 3x)(5 - 3x)$$

$$= 5^2 - (3x)^2$$

$$= 25 - 9x^2$$

14.2.2 The perfect square of a binomial

- A **perfect square** is the square of a whole number. For example, 1, 4, 9, 16 ... and so on are all perfect squares, since $1 = 1 \times 1 = 1^2$, $4 = 2 \times 2 = 2^2$, $9 = 3 \times 3 = 3^2$, $16 = 4 \times 4 = 4^2$ and so on.
- Similarly, $(x + 3)^2$ is a perfect square since it is equivalent to $(x + 3)(x + 3)$, or $x + 3$ multiplied by itself. It is called the 'perfect square of a binomial.'
- Consider a picture of $(x + 3)^2$. What shape is it?



The area is given by

$$x^2 + 3x + 3x + 9 = x^2 + 6x + 9$$

The large square is made up of two smaller squares and two rectangles, and because the rectangles are the same size ($x \times 3$), there is a short cut to the answer.

The first term is x^2 ,

the third term is 3^2 and

the middle term is $x \times 3 \times 2$.

- Once again there is a pattern to remember. It is called the 'square of a binomial'.

$$\text{Square of a binomial: } (a + b)^2 = a^2 + 2ab + b^2$$

$$\text{So } (x + 5)^2 = x^2 + 2 \times x \times 5 + 5^2$$

$$= x^2 + 10x + 25$$

- Does this rule work if $a = 7$ and $b = 3$? Let's check.

$$\begin{aligned}(7 + 3)^2 &= 10^2 \\ &= 100 \\ (7 + 3)^2 &= 7^2 + 2 \times 7 \times 3 + 3^2 \\ &= 49 + 42 + 9 \\ &= 100\end{aligned}$$

The rule works.

- Try to remember this rule in words:
'The square of a binomial equals the square of the first term plus the square of the second term plus twice the product of the two terms.'
- Note that $(a - b)^2 = a^2 - 2ab + b^2$
since $a^2 = a^2$
 $(-b)^2 = b^2$
and $2 \times a \times -b = -2ab$.

$$\text{Square of a binomial: } (a \pm b)^2 = a^2 \pm 2ab + b^2$$

WORKED EXAMPLE 2

Use $(a \pm b)^2 = a^2 \pm 2ab + b^2$ to expand and simplify the following.

a $(x + 1)(x + 1)$

b $(x - 2)^2$

c $(2x + 5)^2$

d $(4x - 5y)^2$

THINK

a 1 This is the square of a binomial.

2 Apply the formula for perfect squares:
 $(a + b)^2 = a^2 + 2ab + b^2$.

b 1 This is the square of a binomial.

2 Apply the formula for perfect squares:
 $(a - b)^2 = a^2 - 2ab + b^2$.

c 1 This is the square of a binomial.

2 Apply the formula for perfect squares:
 $(a + b)^2 = a^2 + 2ab + b^2$.

d 1 This is the square of a binomial.

2 Apply the formula for perfect squares:
 $(a - b)^2 = a^2 - 2ab + b^2$.

WRITE

a $(x + 1)(x + 1)$

$$\begin{aligned}&= x^2 + 2 \times x \times 1 + 1^2 \\ &= x^2 + 2x + 1\end{aligned}$$

b $(x - 2)^2$

$$\begin{aligned}&= (x - 2)(x - 2) \\ &= x^2 - 2 \times x \times 2 + 2^2 \\ &= x^2 - 4x + 4\end{aligned}$$

c $(2x + 5)^2$

$$\begin{aligned}&= (2x)^2 + 2 \times 2x \times 5 + 5^2 \\ &= 4x^2 + 20x + 25\end{aligned}$$

d $(4x - 5y)^2$

$$\begin{aligned}&= (4x)^2 - 2 \times 4x \times 5y + (5y)^2 \\ &= 16x^2 - 40xy + 25y^2\end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Difference of two squares (int-6036)

Digital doc: SkillSHEET Recognising expansion patterns (doc-10821)

Exercise 14.2 Special binomial products

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–12

MASTER

Questions:
1–15

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

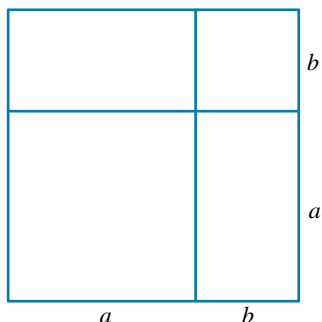
Understanding and fluency

1. **WE1** Use the difference of two squares rule to expand and simplify each of the following.
 - a. $(x + 2)(x - 2)$
 - b. $(y + 3)(y - 3)$
 - c. $(m + 5)(m - 5)$
 - d. $(a + 7)(a - 7)$
 - e. $(x + 6)(x - 6)$
 - f. $(p - 12)(p + 12)$
 - g. $(a + 10)(a - 10)$
 - h. $(m - 11)(m + 11)$
2. Use the difference of two squares rule to expand and simplify each of the following.
 - a. $(2x + 3)(2x - 3)$
 - b. $(3y - 1)(3y + 1)$
 - c. $(5d - 2)(5d + 2)$
 - d. $(7c + 3)(7c - 3)$
 - e. $(2 + 3p)(2 - 3p)$
 - f. $(d - 9x)(d + 9x)$
 - g. $(5 - 12a)(5 + 12a)$
 - h. $(3x + 10y)(3x - 10y)$
 - i. $(2b - 5c)(2b + 5c)$
 - j. $(10 - 2x)(2x + 10)$
3. **WE2** Use the square of a binomial rule to expand and simplify each of the following.
 - a. $(x + 2)(x + 2)$
 - b. $(a + 3)(a + 3)$
 - c. $(b + 7)(b + 7)$
 - d. $(c + 9)(c + 9)$
 - e. $(m + 12)^2$
 - f. $(n + 10)^2$
 - g. $(x - 6)^2$
 - h. $(y - 5)^2$
 - i. $(9 - c)^2$
 - j. $(8 + e)^2$
 - k. $2(x + y)^2$
 - l. $(u - v)^2$
4. Use the perfect squares rule to expand and simplify each of the following.
 - a. $(2a + 3)^2$
 - b. $(3x + 1)^2$
 - c. $(2m - 5)^2$
 - d. $(4x - 3)^2$
 - e. $(5a - 1)^2$
 - f. $(7p + 4)^2$
 - g. $(9x + 2)^2$
 - h. $(4c - 6)^2$
 - i. $(3 + 2a)^2$
 - j. $(5 + 3p)^2$
 - k. $(2 - 5x)^2$
 - l. $(7 - 3a)^2$
 - m. $(9x - 4y)^2$
 - n. $(8x - 3y)^2$
5. A square has a perimeter of $4x + 12$. What is its area?
6. Frances has fenced off a square in her paddock for spring lambs. The area of the paddock is $9x^2 + 6x + 1 \text{ m}^2$. By using pattern recognition, find the side length of the paddock in terms of x .

Communicating, reasoning and problem solving

7. Show that $a^2 - b^2 = (a + b)(a - b)$ is true for each of the following.
 - a. $a = 5, b = 4$
 - b. $a = 9, b = 1$
 - c. $a = 2, b = 7$
 - d. $a = -10, b = -3$
8. Anne has a square bedroom. Her sister Beth has a room that is 1 m shorter in length than Anne's room, but 1 m wider.
 - a. Show that Anne has the larger bedroom.
 - b. How much bigger is Anne's bedroom compared to Beth's bedroom?

9. A large square has been subdivided into two squares and two rectangles.



- a. Write formulas for the areas of these four pieces using the dimensions a and b marked on the diagram.
 - b. Write an equation that states that the area of the large square is equal to the combined area of its four pieces. Do you recognise this equation?
10. Expand each of the following pairs of expressions.
- a. i. $(x - 3)^2$ and $(3 - x)^2$
 ii. $(x - 15)^2$ and $(15 - x)^2$
 iii. $(3x - 7)^2$ and $(7 - 3x)^2$
 - b. What do you notice about the answers to the pairs of expansions above?
 - c. Explain how this is possible.
11. Expand each of the following pairs of expressions.
- a. i. $(x - 4)(x + 4)$ and $(4 - x)(4 + x)$
 ii. $(x - 11)(x + 11)$ and $(11 - x)(11 + x)$
 iii. $(2x - 9)(2x + 9)$ and $(9 - 2x)(9 + 2x)$
 - b. What do you notice about the answers to the pairs of expansions above?
 - c. Explain how this is possible.
12. a. Expand $(10k + 5)^2$.
 b. Show that $(10k + 5)^2 = 100k(k + 1) + 25$
 c. Using part b, evaluate 25^2 and 85^2 .
13. The expansions of perfect squares $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$ can be used to simplify some arithmetic calculations.
 For example:

$$97^2 = (100 - 3)^2$$

$$= 100^2 - 2 \times 100 \times 3 + 3^2$$

$$= 9409$$
 Use this method to calculate the following.
- a. 103^2
 - b. 62^2
 - c. 997^2
 - d. 1012^2
 - e. 53^2
 - f. 98^2
14. How could you represent $(x - 3)^2$ on a diagram?
15. Complete this algebraic magic square, so that all rows, columns and diagonals add up to the same amount.

$-3(x - 1)^2$		
	$3x$	
$(x + 1)^2$		$3(x^2 + 1)$

14.3 Further expansions

[Stage 5.3]

14.3.1 Expanding several sets of brackets

- We need to take care with signs when one expression is subtracted from another.

For example,

$$\begin{aligned}x^2 + x + 2 - (x - 2)(x + 2) \\&= x^2 + x + 2 - (x^2 - 4) \quad (\text{Remember — all of } (x - 2)(x + 2) \text{ is being subtracted.}) \\&= x^2 + x + 2 - x^2 + 4 \\&= x + 6\end{aligned}$$

WORKED EXAMPLE 3

Expand and simplify each of the following expressions.

a $(x + 3)(x + 4) + 4(x - 2)$ **b** $(x - 2)(x + 3) - (x - 1)(x + 2)$

THINK

a 1 Expand each set of brackets.

2 Simplify by collecting like terms.

b 1 Expand and simplify each pair of brackets.
Keep the second expression in a bracket.

2 Subtract all of the second result from the first result. Remember that $-(x^2 + x - 2) = -1(x^2 + x - 2)$.

3 Simplify by collecting like terms.

WRITE

a $(x + 3)(x + 4) + 4(x - 2)$

$$\begin{aligned}&= x^2 + 4x + 3x + 12 + 4x - 8 \\&= x^2 + 11x + 4\end{aligned}$$

b $(x - 2)(x + 3) - (x - 1)(x + 2)$

$$= x^2 + 3x - 2x - 6 - (x^2 + 2x - x - 2)$$

$$\begin{aligned}&= x^2 + x - 6 - (x^2 + x - 2) \\&= x^2 + x - 6 - x^2 - x + 2\end{aligned}$$

$$= -4$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Expanding brackets (int-2763)

Exercise 14.3 Further expansions

Individual pathways

PRACTISE

Questions:
1–16, 31, 34

CONSOLIDATE

Questions:
10–22, 32–34

MASTER

Questions:
18–36

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Understanding and fluency

WE3 Expand and simplify each of the following expressions.

1. $(x + 3)(x + 5) + (x + 2)(x + 3)$

2. $(x + 4)(x + 2) + (x + 3)(x + 4)$

3. $(x + 5)(x + 4) + (x + 3)(x + 2)$

4. $(x + 1)(x + 3) + (x + 2)(x + 4)$

5. $(p - 3)(p + 5) + (p + 1)(p - 6)$

6. $(a + 4)(a - 2) + (a - 3)(a - 4)$

7. $(p - 2)(p + 2) + (p + 4)(p - 5)$
9. $(y - 1)(y + 3) + (y - 2)(y + 2)$
11. $(x + 2)(x + 3) + (x - 4)(x - 1)$
13. $(x + 2)^2 + (x - 5)(x - 3)$
15. $(p + 2)(p + 7) + (p - 3)^2$
17. $(x + 3)(x + 5) - (x + 2)(x + 5)$
19. $(x + 3)(x + 2) - (x + 4)(x + 3)$
21. $(b + 4)(b - 6) - (b - 1)(b + 2)$
23. $(p - 1)(p + 4) - (p - 2)(p - 3)$
25. $(c - 2)(c - 1) - (c + 6)(c + 7)$
27. $(m + 3)^2 - (m + 4)(m - 2)$
29. $(p - 3)(p + 1) - (p + 2)^2$
8. $(x - 4)(x + 4) + (x - 1)(x + 20)$
10. $(d + 7)(d + 1) + (d + 3)(d - 3)$
12. $(y + 6)(y - 1) + (y - 2)(y - 3)$
14. $(y - 1)^2 + (y + 2)(y - 4)$
16. $(m - 6)(m - 1) + (m + 5)^2$
18. $(x + 5)(x + 2) - (x + 1)(x + 2)$
20. $(m - 2)(m + 3) - (m + 2)(m - 4)$
22. $(y - 2)(y - 5) - (y + 2)(y + 6)$
24. $(x + 7)(x + 2) - (x - 3)(x - 4)$
26. $(f - 7)(f + 2) - (f + 4)(f + 5)$
28. $(a - 6)^2 - (a - 2)(a - 3)$
30. $(x + 5)(x - 4) - (x - 1)^2$

Communicating, reasoning and problem solving

31. Show that $(p - 1)(p + 2) + (p - 3)(p + 1) = 2p^2 - p - 5$.
32. Show that $(x + 2)(x - 3) - (x + 1)^2 = -3x - 7$.
33. a. Show that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.
b. Using part a, write $(2^2 + 1^2)(3^2 + 4^2)$ as the sum of two squares and evaluate.
34. a. Expand $(x^2 + x - 1)^2$.
b. Show that $(x^2 + x - 1)^2 = (x - 1)x(x + 1)(x + 2) + 1$.
c. i. Evaluate $4 \times 3 \times 2 \times 1 + 1$.
ii. What is the value of x if $4 \times 3 \times 2 \times 1 + 1 = (x - 1)x(x + 1)(x + 2) + 1$?
35. a. Expand $(a + b)(d + e)$
b. Expand $(a + b + c)(d + e + f)$. Draw a diagram to illustrate your answer.
36. How would you show $(m + 2)(m + 3) - (m + 2)(m + 4)$ on a diagram?

14.4 The highest common binomial factor [Stage 5.3]

14.4.1 The common binomial factor

- When factorising, always look for the highest common factor(s) first.
- Common factors can be expressions.
- Consider the following expression: $5(x + y) + 6b(x + y)$.
The factors of $5(x + y)$ are 5 and $x + y$, and the factors of $6b(x + y)$ are $6b$ and $x + y$.

So $x + y$ is a common factor.

$$\frac{5(x + y)}{x + y} = 5 \quad \text{and} \quad \frac{6b(x + y)}{x + y} = 6b$$

$$5(x + y) + 6b(x + y) = (x + y)(5 + 6b)$$

WORKED EXAMPLE 4

Factorise each of the following expressions.

a $3(m + n) + 2p(m + n)$ b $2b(a - 3b) - (a - 3b)$

In both these expressions it can be seen that the HCF is a binomial factor.

THINK

a 1 The HCF = $(m + n)$.

2 $\frac{3(m + n)}{m + n} = 3, \frac{2p(m + n)}{m + n} = 2p$

WRITE

a $3(m + n) + 2p(m + n)$
 $= (m + n)(3 + 2p)$

b 1 The HCF = $(a - 3b)$.

$$2 \frac{2b(a - 3b)}{a - 3b} = 2b, \frac{-1(a - 3b)}{a - 3b} = -1$$

$$\mathbf{b} \quad 2b(a - 3b) - (a - 3b)$$

$$= 2b(a - 3b) - 1(a - 3b) \\ = (a - 3b)(2b - 1)$$

14.4.2 Factorising by grouping terms

- If an algebraic expression has 4 terms and no common factor in all the terms, it may be possible to group the terms in pairs and find a common factor in each pair.

WORKED EXAMPLE 5

Factorise each of the following expressions by grouping the terms in pairs.

a $5a + 10b + ac + 2bc$

b $x - 3y + ax - 3ay$

c $5p + 6q + 15pq + 2$

THINK

a 1 $5a + 10b = 5(a + 2b)$
 $ac + 2bc = c(a + 2b)$

2 $a + 2b$ is a common factor.

b 1 $x - 3y = 1(x - 3y)$, $ax - 3ay = a(x - 3y)$

2 Take out the common factor $x - 3y$.

c 1 There are no simple common factors.
Write the terms in a different order.

2 $5p + 15pq = 5p(1 + 3q)$
 $6q + 2 = 2(3q + 1)$

3 $1 + 3q = 3q + 1$

4 Factorise by taking out a binomial common factor.

WRITE

a $5a + 10b + ac + 2bc$

$$= 5(a + 2b) + c(a + 2b) \\ = (a + 2b)(5 + c)$$

b $x - 3y + ax - 3ay$

$$= 1(x - 3y) + a(x - 3y) \\ = (x - 3y)(1 + a)$$

c $5p + 6q + 15pq + 2$

$$= 5p + 15pq + 6q + 2$$

$$= 5p(1 + 3q) + 2(3q + 1) \\ = 5p(1 + 3q) + 2(1 + 3q)$$

$$= (1 + 3q)(5p + 2)$$

- The answers can be checked by expanding the brackets.

learnON RESOURCES — ONLINE ONLY

Interactivity: Common binomial factor (int-6038)

Digital doc: WorkSHEET Factorising (doc-10824)

Exercise 14.4 The highest common binomial factor

Individual pathways

PRACTISE

Questions:
1–3, 5

CONSOLIDATE

Questions:
1–5

MASTER

Questions:
1–7

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Understanding and fluency

1. **WE4** Factorise each of the following expressions.

a. $2(a + b) + 3c(a + b)$

c. $7x(2m + 1) - y(2m + 1)$

e. $z(x + 2y) - 3(x + 2y)$

g. $3p^2(x - y) + 2q(x - y)$

i. $p^2(q + 2p) - 5(q + 2p)$

b. $4(m + n) + p(m + n)$

d. $4a(3b + 2) - b(3b + 2)$

f. $12p(6 - q) - 5(6 - q)$

h. $4a^2(b - 3) + 3b(b - 3)$

j. $6(5m + 1) + n^2(5m + 1)$

2. **WE5** Factorise each of the following expressions by grouping the terms in pairs.

a. $xy + 2x + 2y + 4$

c. $xy - 4y + 3x - 12$

e. $3ab + a + 12b + 4$

g. $m - 2n + am - 2an$

i. $15mn - 5n - 6m + 2$

k. $6x - 2 - 3xy + y$

m. $10xy + 5x - 4y - 2$

o. $5ab - 10ac - 3b + 6c$

q. $5pr + 10qr - 3p - 6q$

b. $ab + 3a + 3b + 9$

d. $2xy + x + 6y + 3$

f. $ab - 2a + 5b - 10$

h. $5 + 3p + 15a + 9ap$

j. $10pq - q - 20p + 2$

l. $16p - 4 - 12pq + 3q$

n. $6ab + 9b - 4a - 6$

p. $4x + 12y - xz - 3yz$

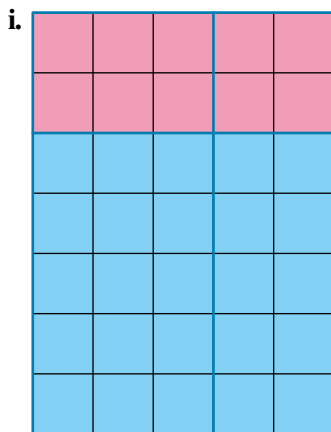
r. $ac - 5bc - 2a + 10b$

Communicating, reasoning and problem solving

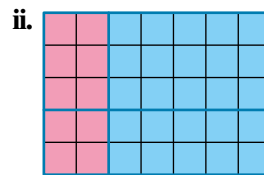
3. Using the method of rectangles to expand, show how $a(m + n) + 3(m + n)$ equals $(a + 3)(m + n)$.

4. a. Expand the expression $(3x + 2)(2x - 5)$.

- b. Diagrams I and II below represent the expansion of $(3x + 2)(2x - 5)$. Examine the diagrams and explain how they work to expand the expression.



I



II

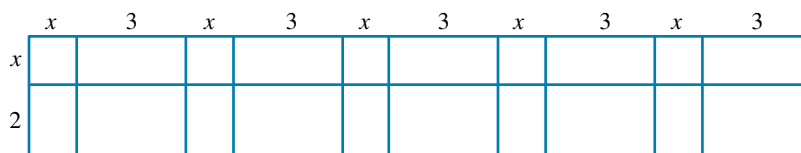
- c. For the products below, use grid paper to construct and appropriately colour representations for their expansions. Use your diagrams to write the expanded form of each.

i. $(2x - 3)(x - 4)$

ii. $(x + 5)(-2x + 1)$

- d. Expand each product in part c and compare with your diagrams.

5. a. Write out the product $5(x + 2)(x + 3)$ and show that it also corresponds to the diagram below.



- b. Explain why $5(x + 2)(x + 3)$ is equivalent to $(5x + 10)(x + 3)$. Use bracket expansion and a labelled diagram to support your answer.
- c. Explain why $5(x + 2)(x + 3)$ is equivalent to $(x + 2)(5x + 15)$. Use bracket expansion and a labelled diagram to support your answer.

6. The area formulas below relate to either squares or rectangles.
- i. $9s^2 + 48s + 64$
 - ii. $25s^2 - 4$
 - iii. $s^2 + 4s + 3$
 - iv. $4s^2 - 28s - 32$
- a. Without completing any algebraic operations, examine the formulas and determine which belong to squares and which to rectangles. Explain your answer.
- b. Factorise each formula and classify it as a square or rectangle. Check this against your answer to part a.
7. How do you factorise expressions with 4 terms?

14.5 Factorisation patterns

[Stage 5.3]

14.5.1 Difference of two squares

- Recall the rule for the difference of two squares from Section 14.2.1:

$$(a + b)(a - b) = a^2 - b^2.$$

This rule can be used in reverse to factorise the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

- Factorise by taking out any common factor first.

$$\begin{aligned}\text{For example: } 5x^2 - 80 \\ &= 5(x^2 - 16) \\ &= 5(x - 4)(x + 4)\end{aligned}$$

WORKED EXAMPLE 6

Factorise each of the following expressions.

a $x^2 - 9$

b $64m^2 - 25n^2$

c $(c + 7)^2 - 16$

d $3x^2 - 48$

THINK

- a 1 The expression is a difference of two squares.
- 2 Rewrite, showing the two squares.
- 3 Factorise using the formula $a^2 - b^2 = (a + b)(a - b)$ where $a = x$ and $b = 3$.

- b 1 The expression is a difference of two squares.

2 Rewrite, showing the two squares.

3 Factorise.

- c 1 The expression is a difference of perfect squares.

2 Rewrite, showing the two squares.

3 Put $c + 7$ in each bracket and complete the factors.

4 Simplify.

- d 1 There is a common factor.

2 Take out the common factor.

3 $x^2 - 16$ is a difference of two squares.

4 Factorise.

WRITE

a $x^2 - 9$

$$= x^2 - 3^2$$

$$= (x + 3)(x - 3)$$

b $64m^2 - 25n^2$

$$= (8m)^2 - (5n)^2$$

$$= (8m + 5n)(8m - 5n)$$

c $(c + 7)^2 - 16$

$$= (c + 7)^2 - 4^2$$

$$= (c + 7 + 4)(c + 7 - 4)$$

$$= (c + 11)(c + 3)$$

d $3x^2 - 48$

$$= 3(x^2 - 16)$$

$$= 3(x^2 - 4^2)$$

$$= 3(x + 4)(x - 4)$$

14.5.2 Perfect squares

- Recall from section 14.2.2 that a perfect square is an expression such as $(x + 2)^2$ or $(t - 2)^2$. The expansion of perfect squares produces the following patterns or identities.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

- The identities can also be used in reverse to factorise the quadratic expressions.

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

- Is $x^2 + 8x + 16$ a perfect square?

The constant term must be the square of half the coefficient of the x -term if the expression is a perfect square. Because half of 8 is 4 and $4^2 = 16$, $x^2 + 8x + 16$ is a perfect square.

$$x^2 + 8x + 16 = (x + 4)^2$$

DISCUSSION

Does this technique work to determine if any quadratic expression is a perfect square?

WORKED EXAMPLE 7

Factorise each of the following expressions.

a $x^2 - 6x + 9$

b $64m^2 + 80mn + 25n^2$

c $3x^2 - 24x + 48$

THINK

a 1 x^2 and 9 are perfect squares, so $x^2 - 6x + 9$ might be $(x - 3)^2$. Check as follows.

2 Halve the coefficient of the x -term and then square the result.

3 The middle term fits the pattern, so factorise using the formula $a^2 - 2ab + b^2 = (a - b)^2$, where $a = x$ and $b = 3$.

b 1 $64m^2$ and $25n^2$ are perfect squares, so $64m^2 + 80mn + 25n^2$ might be $(8m + 5n)^2$.

2 Check the middle term.

3 The middle term fits the pattern, so write down the factors.

c 1 There is a common factor.

2 Factorise.

3 Factorise $x^2 - 8x + 16$.

WRITE

a $x^2 - 6x + 9$

$$\frac{-6}{2} = -3, (-3)^2 = 9$$

$$x^2 - 6x + 9 = (x - 3)^2$$

b $64m^2 + 80mn + 25n^2$

$$8m \times 5n \times 2 = 80mn$$

$$64m^2 + 80mn + 25n^2 = (8m + 5n)^2$$

c $3x^2 - 24x + 48$

$$= 3(x^2 - 8x + 16)$$

$$= 3(x - 4)^2$$

learn**on**

RESOURCES — ONLINE ONLY

Interactivity: Difference of two squares (int-6091)

Digital doc: SkillSHEET Factorising difference of two squares expressions (doc-10980)

Exercise 14.5 Factorisation patterns

Individual pathways

PRACTISE

Questions:

1a–e, 2a–e, 3a–f, 4a–f,
5–8, 10, 11

CONSOLIDATE

Questions:

1d–g, 2d–g, 3d–i, 4d–i, 5–9,
11, 12–14

MASTER

Questions:

1e–i, 2e–i, 3g–l, 4g–l, 5–18

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Understanding and fluency

1. **WE6a** Factorise each of the following expressions using the difference of two squares rule.

a. $x^2 - 25$

b. $x^2 - 81$

c. $a^2 - 16$

d. $25 - p^2$

e. $121 - a^2$

f. $36 - y^2$

g. $4b^2 - 25$

h. $9a^2 - 16$

i. $25d^2 - 1$

2. **WE6b** Factorise each of the following expressions.

a. $x^2 - y^2$

b. $a^2 - b^2$

c. $p^2 - q^2$

d. $25m^2 - n^2$

e. $81x^2 - y^2$

f. $p^2 - 36q^2$

g. $36m^2 - 25n^2$

h. $16q^2 - 9p^2$

i. $4m^2 - 49n^2$

3. **WE6c** Factorise each of the following expressions.

a. $(x + 9)^2 - 16$

b. $(p + 8)^2 - 25$

c. $(p - 2)^2 - q^2$

d. $(c - 6)^2 - d^2$

e. $(x + 7)^2 - y^2$

f. $(p + 5)^2 - q^2$

g. $(a + 3)^2 - b^2$

h. $(a - 3)^2 - 1$

i. $(b - 1)^2 - 36$

4. **WE6d** Factorise each of the following expressions completely after removing any common factors.

a. $2m^2 - 32$

b. $5y^2 - 45$

c. $6p^2 - 24$

d. $4m^2 - 100$

e. $288 - 2x^2$

f. $80 - 5a^2$

g. $100x^2 - 25y^2$

h. $144p^2 - 4q^2$

i. $32y^2 - 2x^2$

j. $ax^2 - 9a$

k. $8b - 2bx^2$

l. $3(b + 5)^2 - 48$

5. **WE7** Factorise each of the following expressions by recognising the perfect square rule.

a. $x^2 + 10x + 25$

b. $p^2 - 24p + 144$

c. $n^2 + 20n + 100$

d. $x^2 + 2xy + y^2$

e. $u^2 - 2uv + v^2$

f. $64 + 16e + e^2$

g. $4m^2 - 20m + 25$

h. $49 + 42a + 9a^2$

i. $81x^2 + 72xy + 16y^2$

j. $2x^2 + 24x + 72$

k. $3x^2 - 24xy + 48y^2$

l. $18a^2 + 24ab + 8b^2$

6. a. Which of the expressions below are the difference of two squares?

- b. Which of the expressions below are perfect squares?

i. $x^2 - 36$

ii. $x^2 + 36$

iii. $x^2 + 6x + 36$

iv. $x^2 - 12x + 36$

v. $x^2 + 12x - 36$

vi. $x^2 + 12x + 36$

7. **MC** a. The expression $x^2 - 121$ factorises to:

A. $(x - 11)^2$

B. $(x - 11)(x + 12)$

C. $(x + 11)(x - 11)$

D. $(x + 11)^2$

E. $(x + 121)(x - 1)$

- b. The expression $36m^2 - 12m + 1$ factorises to:

A. $(6m - 1)(6m - 1)$

B. $36(m + 1)(m - 1)$

C. $(6m + 1)^2$

D. $6(6m - 1)(6m - 1)$

E. $6(m - 1)^2$

- c. The expression $16a^2 - 25b^2$ factorises to:

A. $16(a + 5b)(a - 5b)$

B. $4(4a - 5b)^2$

C. $(16a + 25b)(16a - 25b)$

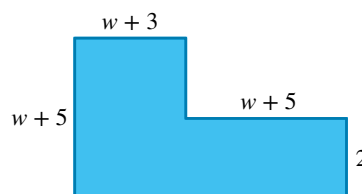
D. $(4a + 5b)(4a - 5b)$

E. $(4a - 5b)^2$

- d. The expression $5c^2 - 20c + 20$ factorises to:
- A. $5(c - 2)(c - 2)$ B. $5(c + 4)(c - 4)$ C. $5(c - 4)(c - 4)$
D. $(5c + 20)(5c - 20)$ E. $5(c - 2)(c + 2)$
- e. The expression $(x + 4)^2 - 9$ factorises to:
- A. $9(x + 4)(x - 4)$ B. $(x + 13)(x - 5)$ C. $(x + 7)(x + 1)$
D. $(x - 1)(x + 5)$ E. $(x + 2)(x - 3)$
8. A circular pool with a radius of r metres is surrounded by a circular path 1 m wide.
- Find the area of the surface of the pool.
 - Find, in terms of r , the distance from the centre of the pool to the outer edge of the path.
 - Find the area of the circle that includes the path and the pool. (Do not expand the expression.)
 - Write an expression for the area of the path.
 - Simplify this expression.
 - If the pool had a radius of 5 m, what would be the area of the path to the nearest square metre?
 - If the pool had a radius of 7 m, what would be the area of the path to the nearest square metre?



9. An L-shaped piece of farm land is to be planted with wheat.
- Write an expression for the area of the wheat in simplest form.
 - Calculate how much wheat is to be planted if w is:
 - 5
 - 10
 - 20.
 - Invent your own L-shaped area using the pronumeral w that has an area that can be expressed as a perfect square.

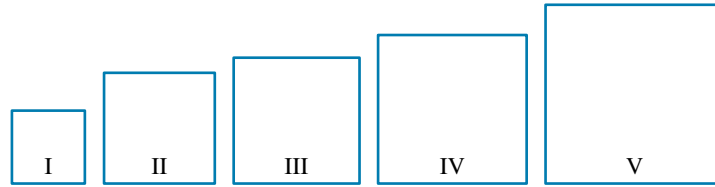


10. Factorise the following using the difference of two squares rule.
- $x^2 - 10$
 - $4x^2 - 32$
 - $x^4 - y^2$
 - $(x + 2)^2 - (x - 3)^2$
 - $y^2 - 4x^4$
 - $10^4 - x^4$

Communicating, reasoning and problem solving

- Is $(x + 4)^2$ equal to $x^2 + 16$? Explain using a numerical example.
- Show that $(a + b)(c + d) = (c + d)(a + b)$.
- Show that $(a + b)(a - b) = (a - b)(a + b)$.
- The area of a rectangle is represented by $x^2 - 49 \text{ mm}^2$.
 - Factorise the expression.
 - Using the factorised expression, find a possible length and width for the rectangle.
 - If $x = 25 \text{ mm}$, find the dimensions for the rectangle.
 - If $x = 50 \text{ mm}$, how much bigger is the area of the rectangle than in part c?

15. Five squares of increasing size and five area formulas are given below.



$$A_a = x^2 + 6x + 9$$

$$A_b = x^2 + 10x + 25$$

$$A_c = x^2 + 16x + 64$$

$$A_d = x^2 - 6x + 9$$

$$A_e = x^2 + 12x + 36$$

- Use factorisation to find the side length that correlates to each area formula.
 - Using the areas given and side lengths found, match the squares with the appropriate algebraic expressions for their areas.
 - If $x = 5$ cm, use the formula given to calculate the area of each square.
16. The expansion of perfect squares
 $(a + b)^2 = a^2 + 2ab + b^2$
 and $(a - b)^2 = a^2 - 2ab + b^2$
 can be used to simplify some arithmetic calculations. For example:
 $97^2 = (100 - 3)^2$
 $= 100^2 - 2 \times 100 \times 3 + 3^2$
 $= 9409$
 Use this method to calculate the following.
- 103^2
 - 62^2
 - 997^2
 - 1012^2
 - 53^2
 - 98^2
17. What is the best way to remember all three expansion patterns from this section?
18. Use the difference of two squares to completely factorise $x^8 - 1$.

14.6 Factorising non-monic quadratics [Stage 5.3]

14.6.1 Factorising quadratic trinomials where $a \neq 1$

- When a quadratic trinomial in the form $ax^2 + bx + c$ is written as $ax^2 + mx + nx + c$, where $m + n = b$, the four terms can be factorised by grouping.

$$2 \times 12 = 24$$

Numbers that multiply to give 24

and add to 11

therefore (8) and (3).

$$\begin{aligned} 2x^2 + 11x + 12 &= 2x^2 + (8x) + (3x) + 12 \\ &\quad \uparrow \quad \times \quad \uparrow \\ &= 2x^2 + 8x + 3x + 12 \\ &= 2x(x + 4) + 3(x + 4) \\ &= 2x(x + 4) + 3(x + 4) \\ &= (x + 4)(2x + 3) \end{aligned}$$

- There are many combinations of numbers that satisfy $m + n = b$, however only one particular combination can be grouped and factorised.

For example,

$$\begin{aligned}
 2x^2 + 11x + 12 &= 2x^2 + 7x + 4x + 12 \quad \text{or} \quad 2x^2 + 11x + 12 = 2x^2 + 8x + 3x + 12 \\
 &= 2x^2 + 7x + 4x + 12 \quad \quad \quad = 2x^2 + 8x + 3x + 12 \\
 &= x(2x + 7) + 4(x + 3) \quad \quad \quad = 2x(x + 4) + 3(x + 4) \\
 &\text{cannot be factorised further} \quad \quad \quad = (x + 4)(2x + 3)
 \end{aligned}$$

- In examining the general binomial expansion, a pattern emerges that can be used to help identify which combination to use for $m + n = b$.

$$\begin{aligned}
 (dx + e)(fx + g) &= dfx^2 + dgx + efx + eg \\
 &= dfx^2 + (dg + ef)x + eg
 \end{aligned}$$

$$\begin{aligned}
 m + n &= dg + ef & \text{and} & & m \times n &= dg \times ef \\
 &= b & & & &= dgef \\
 & & & & &= dfeg \\
 & & & & &= ac
 \end{aligned}$$

- Therefore, m and n are factors of ac that sum to b .
- To factorise a general quadratic where $a \neq 1$, look for factors of ac that sum to b . Then rewrite the quadratic trinomial with four terms that can then be grouped and factorised.

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

Factors of ac that sum to b

WORKED EXAMPLE 8

Factorise $6x^2 - 11x - 10$.

THINK

- 1 Write the expression and look for common factors and special patterns. The expression is a general quadratic with $a = 6$, $b = -11$ and $c = -10$.

- 2 Since $a \neq 1$, rewrite $ax^2 + bx + c$ as $ax^2 + mx + nx + c$, where m and n are factors of $ac(6 \times -10)$ that sum to $b(-11)$. Calculate the sums of factor pairs of -60 . As shown in blue, 4 and -15 are factors of -60 that add to -11 .

WRITE

$$6x^2 - 11x - 10 = 6x^2 + -11x + -10$$

Factors of -60	Sum of factors
$-60, 1$	-59
$-20, 3$	-17
$-30, 2$	-28
$15, -4$	11
$-15, 4$	-11

- 3 Rewrite the quadratic expression:
 $ax^2 + bx + c = ax^2 + mx + nx + c$ with $m = 4$ and $n = -15$.

$$6x^2 - 11x - 10 = 6x^2 + 4x + -15x - 10$$

- 4 Factorise using the grouping method:
 $6x^2 + 4x = 2x(3x + 2)$ and $-15x - 10 = -5(3x + 2)$.
 Write the answer.

$$\begin{aligned}
 6x^2 - 11x - 10 &= 2x(3x + 2) + -5(3x + 2) \\
 &= (3x + 2)(2x - 5)
 \end{aligned}$$

learn on RESOURCES — ONLINE ONLY

Interactivity: Factorising quadratic trinomials where $y = ax^2 + bx + c$, $a > 1$ (int-6093)

eLesson: Factorisation of trinomials (eles-1921)

Digital doc: WorkSHEET Quadratics 1 (doc-10986)

Exercise 14.6 Factorising non-monic quadratics

Individual pathways

PRACTISE

Questions:
1a–e, 2a–e, 3–8

CONSOLIDATE

Questions:
1d–h, 2d–h, 3–10

MASTER

Questions:
1g–i, 2g–i, 3–12

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE8** Factorise each of the following quadratic trinomials.

a. $2x^2 + 10x + 12$	b. $3x^2 + 15x + 12$	c. $2x^2 - 12x + 16$
d. $3x^2 - 12x + 9$	e. $4x^2 - 4x - 24$	f. $3x^2 + 9x - 30$
g. $2x^2 + 8x - 42$	h. $3x^2 - 9x - 54$	i. $5x^2 + 20x - 60$

2. Factorise each of the following quadratic trinomials.

a. $2x^2 + 7x + 3$	b. $2x^2 + 7x + 6$	c. $3x^2 + 7x + 2$
d. $2x^2 - 7x + 3$	e. $3x^2 - x - 2$	f. $5x^2 + 3x - 2$
g. $7x^2 - 17x + 6$	h. $10x^2 - 11x - 6$	i. $2x^2 + 5x - 12$

3. **MC** a. The expression $3x^2 + 21x + 36$ is equal to:

A. $3(x + 6)(x + 2)$	B. $3(x + 4)(x + 3)$	C. $(3x + 9)(x + 4)$
D. $(3x + 2)(x + 18)$	E. $(3x + 4)(x + 3)$	

- b. The expression $2x^2 - 16x + 14$ is equal to:

A. $2(x - 1)(x - 7)$	B. $2(x + 1)(x + 7)$	C. $(2x - 7)(x - 2)$
D. $(2x - 2)(x - 7)$	E. $2(x - 1)(x + 7)$	

- c. The expression $12x^2 + 58x - 10$ is equal to:

A. $(12x - 2)(x - 5)$	B. $2(6x + 1)(x - 5)$	C. $2(6x + 5)(x - 1)$
D. $2(6x - 1)(x + 5)$	E. $(6x - 1)(2x + 5)$	

4. Factorise the following trinomials.

a. $2x^2 + 3x - 2$	b. $8x^2 + 2x - 3$
c. $6x^2 + x - 2$	d. $2x^2 + 9x + 10$
e. $3x^2 - x - 4$	f. $6x^2 + 5x - 21$

5. Factorise the following.

a. $6x^2 - 5x - 6$	b. $4x^2 + 13x + 10$	c. $4x^2 - 19x + 15$	d. $6x^2 + 5x - 4$

6. Factorise $3x^2 + 15x + 12$:

- a. by first taking out a common factor b. by first separating the middle term, then grouping.

7. Solve each of the following quadratic equations.

a. $2x^2 + 7x + 3 = 0$	b. $2x^2 + x - 1 = 0$	c. $2x^2 - x - 15 = 0$	d. $2x^2 + 8x + 6 = 0$
e. $3x^2 + 13x + 14 = 0$	f. $3x^2 + 5x + 2 = 0$	g. $5x^2 - 22x + 21 = 0$	h. $5x^2 - 17x + 6 = 0$
i. $7x^2 - 2x - 5 = 0$	j. $7x^2 - 33x + 20 = 0$	k. $4x^2 + 4x - 3 = 0$	l. $6x^2 + 17x + 5 = 0$

Communicating, reasoning and problem solving

8. Complete the statements below by writing in the second factor. Check your answer by expanding.

a. $4x^2 - 17x - 15 = (4x + 3)(\quad)$	b. $3x^2 + 10x + 3 = (x + 3)(\quad)$
c. $10x^2 - 9x - 9 = (5x + 3)(\quad)$	d. $15x^2 + x - 2 = (3x - 1)(\quad)$

9. a. Explain why the quadratic $4x^2 - 9x - 9$ can be written as $4x^2 + 3x - 12x - 9$.

- b. Factorise the quadratic $4x^2 - 9x - 9$.

10. a. Factorise $12x^2 - 4x - 5$.
 b. Assume that the factors represent the length and width of a rectangle.
 i. What is the area of the rectangle?
 ii. For what value of x is the rectangle a square?
 iii. For what values of x is one side twice as long as the other? Give both possible answers.
11. The area formulas below relate to either squares or rectangles.
 i. $9s^2 + 48s + 64$ ii. $25s^2 - 4$ iii. $s^2 + 4s + 3$ iv. $4s^2 - 24s - 36$
 a. Without completing any algebraic operations, examine the formulas and determine which belong to squares and which to rectangles. Explain your answer.
 b. Factorise each formula and classify it as a square or rectangle. Check this against your answer to part a.
12. Factorise the following expressions.
 a. $9b^2 + 48b + 64$ b. $(ac)^2 - d^2$ c. $a^2 + ab - 182b^2$ d. $b^2 - 2abc + a^2c^2$

14.7 Simplifying algebraic fractions 2 [Stage 5.3]

14.7.1 Cancelling common factors

- Algebraic fractions** are fractions that have algebraic expressions in the numerator and/or denominator, for example $\frac{a}{2}$, $\frac{b+3}{4}$ and $\frac{x^2+3x}{4x+2}$.
- Just as fractions can be simplified by cancelling common factors, so too can algebraic fractions.
- A fraction can only be simplified if the numerator and denominator are both products of the same term.
- A factor can be cancelled if it is a factor of the whole of the numerator and the whole of the denominator. For example:

$$\frac{\cancel{2a}}{\cancel{a}b} = \frac{2}{b} \quad \frac{\cancel{2(x+3)}}{5a\cancel{(x+3)}} = \frac{2}{5a}$$

WORKED EXAMPLE 9

Simplify each of the following.

a $\frac{4}{x+3} \times \frac{x+3}{5}$

b $\frac{(z-3)(z-4)}{(z-5)(z+1)} \times \frac{(z+1)(z-1)}{(z+3)(z-3)}$

THINK

a 1 Write the expression.

2 Cancel the common factors.

3 Simplify the answer.

b 1 Write the expression.

2 Cancel the common factors.

3 Simplify the resultant fraction.

WRITE

$$\begin{aligned} \text{a } \frac{4}{x+3} \times \frac{x+3}{5} \\ &= \frac{4}{\cancel{(x+3)}} \times \frac{\cancel{(x+3)}^1}{5} \\ &= \frac{4}{5} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{(z-3)(z-4)}{(z-5)(z+1)} \times \frac{(z+1)(z-1)}{(z+3)(z-3)} \\ &= \frac{\cancel{(z-3)}^1(z-4)}{(z-5)\cancel{(z+1)}_1} \times \frac{\cancel{(z+1)}^1(z-1)}{(z+3)\cancel{(z-3)}_1} \\ &= \frac{(z-4)(z-1)}{(z-5)(z+3)} \end{aligned}$$

WORKED EXAMPLE 10

Simplify the following.

a $\frac{1}{x-3} \div \frac{3}{x-3}$

b $\frac{(z+2)(z-3)}{(z-4)} \div \frac{z-3}{z-4}$

THINK

a 1 Write the expression.

2 Change to a multiplication problem by changing the $\div$ to a $\times$ and turning the second fraction upside down.

3 Cancel the common factors.

4 Simplify the fraction.

b 1 Write the expression.

2 Change to multiplication.

3 Cancel the common factors.

4 Simplify the resultant fraction.

WRITE

$$\begin{aligned} \text{a } \frac{1}{x-3} \div \frac{3}{x-3} &= \frac{1}{x-3} \times \frac{x-3}{3} \\ &= \frac{1}{\cancel{(x-3)}_1} \times \frac{\cancel{(x-3)}^1}{3} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{(z+2)(z-3)}{(z-4)} \div \frac{z-3}{z-4} &= \frac{(z+2)(z-3)}{(z-4)} \times \frac{(z-4)}{(z-3)} \\ &= \frac{(z+2)\cancel{(z-3)}^1}{\cancel{(z-4)}_1} \times \frac{\cancel{(z-4)}^1}{\cancel{(z-3)}_1} \\ &= \frac{z+2}{1} \\ &= z+2 \end{aligned}$$

WORKED EXAMPLE 11

Simplify each of the following fractions by factorising the numerator and the denominator, and then cancelling as appropriate.

a $\frac{3x+9}{15}$

b $\frac{5}{10x+20}$

c $\frac{10x+15}{6x+9}$

d $\frac{x^2+4x}{x^2-5x}$

THINK

a 1 Write the fraction.

2 Factorise both the numerator and the denominator.

3 Cancel the common factors.

4 Simplify the resulting fraction.

WRITE

$$\begin{aligned} \text{a } \frac{3x+9}{15} &= \frac{3(x+3)}{15} \\ &= \frac{\cancel{3}^1(x+3)}{\cancel{3}_1 \times 5} \\ &= \frac{(x+3)}{5} \end{aligned}$$

b 1 Write the fraction.

2 Factorise both the numerator and the denominator.

3 Cancel the common factors.

4 Simplify the resulting fraction.

$$\begin{aligned}\mathbf{b} \quad & \frac{5}{10x + 20} \\ &= \frac{5 \times 1}{10(x + 2)} \\ &= \frac{5^1 \times 1}{10_2(x + 2)} \\ &= \frac{1}{2(x + 2)}\end{aligned}$$

c 1 Write the fraction.

2 Factorise both the numerator and the denominator.

3 Cancel the common factors.

$$\begin{aligned}\mathbf{c} \quad & \frac{10x + 15}{6x + 9} \\ &= \frac{5(2x + 3)^1}{3(2x + 3)^1} \\ &= \frac{5}{3}\end{aligned}$$

d 1 Write the fraction.

2 Factorise both the numerator and the denominator.

3 Cancel the common factors.

$$\begin{aligned}\mathbf{d} \quad & \frac{x^2 + 4x}{x^2 - 5x} \\ &= \frac{x(x + 4)}{x(x - 5)} \\ &= \frac{x + 4}{x - 5}\end{aligned}$$

WORKED EXAMPLE 12

Simplify each of the following algebraic fractions.

a $\frac{x^2 + 3x - 4}{x - 1}$

b $\frac{x^2 - 7x - 8}{x^2 + 3x + 2}$

c $\frac{x^2 - 6x + 8}{2x^2 - 16x + 30}$

THINK

a 1 Write the expression.

2 Factorise the numerator and denominator and cancel the common factors.

3 Simplify.

b 1 Write the original fraction.

2 Factorise the numerator and denominator.

3 Cancel any common factors and simplify.

WRITE

$$\begin{aligned}\mathbf{a} \quad & \frac{x^2 + 3x - 4}{x - 1} \\ &= \frac{(x - 1)(x + 4)}{x - 1} \\ &= x + 4\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad & \frac{x^2 - 7x - 8}{x^2 + 3x + 2} \\ &= \frac{(x + 1)(x - 8)}{(x + 1)(x + 2)} \\ &= \frac{x - 8}{x + 2}\end{aligned}$$

c 1 Write the original fraction.

$$\text{c } \frac{x^2 - 6x + 5}{2x^2 - 16x + 30}$$

2 Factorise the numerator and denominator.

$$= \frac{(x-1)(x-5)^1}{2(x-3)(x-5)^1}$$

3 Cancel any common factors and simplify.

$$= \frac{(x-1)}{2(x-3)}$$

- *Note:* A fraction such as $\frac{x-1}{x-3}$ cannot be simplified any further. It may be tempting to cancel the x in the numerator and denominator, but x is neither a factor of $(x-1)$ nor $(x-3)$, so it cannot be cancelled.

WORKED EXAMPLE 13

Simplify $\frac{2x+4}{3} \times \frac{3x-3}{x+2}$ **by factorising the numerator and denominator and cancelling as appropriate.**

THINK

1 Write the expression.

2 Factorise each numerator and denominator and cancel common factors.

3 Simplify.

WRITE

$$\begin{aligned} & \frac{2x+4}{3} \times \frac{3x-3}{x+2} \\ &= \frac{2(x+2)^1}{1 \cdot 3} \times \frac{1 \cdot 3(x-1)}{x+2_1} \\ &= \frac{2(x-1)}{1} \\ &= 2(x-1) \end{aligned}$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Cancelling common factors (int-6094)

Digital doc: SkillsHEET Simplifying algebraic fractions (doc-10983)

Exercise 14.7 Simplifying algebraic fractions 2

Individual pathways

PRACTISE

Questions:

1a, c, e, k, 2a, c, e, k, 3a–h,
4a–h, 5a–h, 6a, c, e, g, 7

CONSOLIDATE

Questions:

1a, c, g, i, k, 2a–k, 3e–i, 4e–i,
5e–i, 6a–i, 7–9

MASTER

Questions:

1b–l, 2b–l, 3i–p, 4i–p, 5i–p, 6b–j,
7–11

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE9** Simplify the following.

a. $\frac{x+1}{2} \times \frac{1}{x+1}$

b. $\frac{y-1}{3} \times \frac{2}{y-1}$

c. $\frac{p-2}{4} \times \frac{4}{p-2}$

d. $\frac{b+5}{3} \times \frac{9}{b+5}$

e. $\frac{m+4}{m} \times \frac{m}{m+4}$

f. $\frac{3p}{p-6} \times \frac{p-6}{p}$

g. $\frac{y-7}{4y} \times \frac{y}{y-7}$

h. $\frac{ha}{a-2} \times \frac{a-2}{2a}$

i. $\frac{p-3}{p+3} \times \frac{5(p+3)}{p-2}$

j. $\frac{(x+2)(x-7)}{x+1} \times \frac{x+1}{x-7}$

k. $\frac{(b-2)(b+3)}{(b+5)(b-4)} \times \frac{b-4}{b-2}$

l. $\frac{x-3}{x+2} \times \frac{(x+2)(x+1)}{(x-6)(x-3)}$

2. **WE10** Simplify the following.

a. $\frac{x+2}{3} \div \frac{x+2}{x}$

b. $\frac{m-3}{4} \div \frac{m-3}{8}$

c. $\frac{5}{x-3} \div \frac{10}{x-3}$

d. $\frac{12}{a+2} \div \frac{4}{a+2}$

e. $\frac{m}{m-6} \div \frac{m}{m-6}$

f. $\frac{p+2}{p-1} \div \frac{p-2}{p-1}$

g. $\frac{s-6}{3(s+1)} \div \frac{s}{s+1}$

h. $\frac{(a+1)(a-3)}{a+2} \div \frac{a+1}{a+2}$

i. $\frac{8m}{(m+2)(m-5)} \div \frac{2m}{m-5}$

j. $\frac{(a+1)(a-3)}{(a-5)(a+4)} \div \frac{a+1}{a+4}$

k. $\frac{m(m-2)}{(m+1)(m+5)} \div \frac{m}{m+1}$

l. $\frac{p}{p+7} \div \frac{p(p-4)}{(p+2)(p+7)}$

3. **WE11** Simplify each of the following by factorising the numerator and the denominator, and then cancelling as appropriate.

a. $\frac{2a+2}{2}$

b. $\frac{3a+6}{9}$

c. $\frac{4a-4}{4}$

d. $\frac{7p-21}{7}$

e. $\frac{4x+8}{8}$

f. $\frac{3x-12}{6}$

g. $\frac{9x+54}{3}$

h. $\frac{12x-96}{4}$

i. $\frac{12x-24}{18}$

j. $\frac{9x-45}{15}$

k. $\frac{2}{2x+10}$

l. $\frac{-6}{6x-24}$

m. $\frac{2x+2}{3x+3}$

n. $\frac{3x-6}{4x-8}$

o. $\frac{5x-15}{6x-18}$

p. $\frac{9x+18}{3x+6}$

4. **WE11** Simplify each of the following fractions by factorising the numerator and the denominator, and then cancelling as appropriate.

a. $\frac{x^2+3x}{x}$

b. $\frac{p^2-5p}{p}$

c. $\frac{y^2-4y}{y}$

d. $\frac{a^2+10a}{a}$

e. $\frac{7x}{5x-x^2}$

f. $\frac{5b}{5b-b^2}$

g. $\frac{9y}{y^2-9y}$

h. $\frac{4m}{m^2-4m}$

i. $\frac{7x}{x-2x^2}$

j. $\frac{3m}{2m^2-m}$

k. $\frac{a-2a^2}{a^2+3a}$

l. $\frac{2b^2+b}{5b-8b^2}$

m. $\frac{3x^2+x}{9x-x^2}$

n. $\frac{7b-2b^2}{b^2+5b}$

o. $\frac{9m^2+18m}{12m^2-3m}$

p. $\frac{32a-16a^2}{24a+16a^2}$

5. **WE12** Simplify each of the following algebraic fractions by first factorising the numerator, then the denominator.

a. $\frac{x^2+5x+6}{x+3}$

b. $\frac{x^2+7x+12}{x+4}$

c. $\frac{x^2-9x+20}{x-5}$

d. $\frac{b^2-8b+7}{b-1}$

e. $\frac{a^2-18a+81}{a-9}$

f. $\frac{a^2-22a+121}{a-11}$

g. $\frac{x^2-49}{x-7}$

h. $\frac{m^2-64}{m-8}$

i. $\frac{a^2-7a+12}{a^2-16}$

j. $\frac{p^2-4p-5}{p^2-25}$

k. $\frac{x^2+6x+9}{x^2+2x-3}$

l. $\frac{m^2-2m+1}{m^2+5m-6}$

m. $\frac{y^2-4y-12}{y^2-36}$

n. $\frac{x^2-4x+4}{x^2-4}$

o. $\frac{x^2+3x-40}{x^2+6x-16}$

p. $\frac{x^2+3x-18}{x^2-6x+9}$

6. **WE13** Simplify each of the following by factorising the numerator and denominator and cancelling as appropriate.

a. $\frac{20x + 40}{2x + 8} \times \frac{x + 4}{x + 2}$

b. $\frac{9a + 27}{3a - 3} \times \frac{a - 1}{a + 3}$

c. $\frac{m + 5}{m - 2} \times \frac{8m - 16}{2m + 10}$

d. $\frac{q - 3}{q + 1} \times \frac{15q + 15}{5q - 15}$

e. $\frac{3x - 6}{x + 6} \times \frac{x + 6}{4x - 8}$

f. $\frac{3a + 6}{2a - 6} \div \frac{a + 3}{8a - 24}$

g. $\frac{10x - 5}{4x + 28} \div \frac{20x - 10}{6x + 1}$

h. $\frac{m^2 + 8m + 15}{m^2 + 7m + 10} \times \frac{m^2 + 6m + 8}{m^2 + 10m + 21}$

i. $\frac{x^2 + 6x + 8}{x^2 + 5x + 6} \times \frac{x^2 + 8x + 15}{x^2 + 7x + 12}$

j. $\frac{x^2 + x - 6}{x^2 + 5x - 14} \times \frac{x^2 + 9x + 14}{x^2 - x - 12}$

Communicating, reasoning and problem solving

7. Explain why $\frac{x + 5}{x + 1}$ does not equal $\frac{5}{1}$. Illustrate your answer with an example.

8. Show that $\frac{3x + 11}{x + 2} = 3 + \frac{5}{x + 2}$.

9. Simplify the following fractions.

a. $\frac{c^2 + 14c + 24}{c^2 - 16c - 36}$

b. $\frac{m^2 + 2m + 1}{m^2 + 9m + 8}$

c. $\frac{f^2 - 81}{f^2 - 3f - 54}$

d. $\frac{r^2 - 28r + 196}{r^2 + 28r + 196}$

e. $\frac{w^2 + 10w + 25}{w^2 + 25w + 100}$

f. $\frac{p^2 - 49}{p^2 - 64}$

10. What is the sequence of steps involved in simplifying algebraic expressions that are being multiplied or divided?

11. Simplify the expression $1 + \frac{\frac{1}{x}}{1 + \frac{1}{x}}$.

14.8 Applications of binomial products

[Stage 5.3]

14.8.1 Converting worded problems

- A very important skill is to be able to convert a worded or real-life problem into an algebra sentence or expression.

WORKED EXAMPLE 14

A rectangular swimming pool measures 30 m by 20 m. A path around the edge of the pool is x m wide on each side.

- Determine the area of the pool.
- Write an expression for the area of the pool plus the path.
- Write an expression for the area of the path.
- If the path is 1.5 m wide, calculate the area of the path.



THINK

a 1 Construct a drawing of the pool.

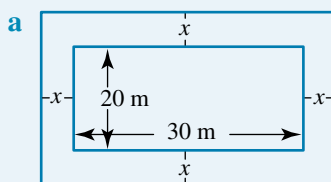
2 Calculate the area of the pool.

b 1 Write expressions for the total length and width.

2 Area = length $\times$ width

c Find an expression for the area of the path by subtracting the area of the pool from the total area.

d Substitute 1.5 for x in the expression found for the area of the path.

WRITE

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 20 \times 30 \\ &= 600 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{b Length} &= 30 + x + x \\ &= 30 + 2x \\ \text{Width} &= 20 + x + x \\ &= 20 + 2x\end{aligned}$$

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= (30 + 2x)(20 + 2x) \\ &= 600 + 60x + 40x + 4x^2 \\ &= (600 + 100x + 4x^2) \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{c Area of path} &= \text{total area} - \text{area of pool} \\ &= 600 + 100x + 4x^2 - 600 \\ &= (100x + 4x^2) \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{d When } x &= 1.5, \\ \text{Area of path} &= 100(1.5) + 4(1.5)^2 \\ &= 159 \text{ m}^2\end{aligned}$$

- The algebraic expression found in part c of Worked example 14 allows us to calculate the area of the path for any given width, x .

WORKED EXAMPLE 15

Suppose that the page of a typical textbook is 24 cm high by 16 cm wide. The page has margins at the top and bottom of x cm and on the left and right of y cm.

- Write an expression for the height of the page that is inside the margins.
- Write an expression for the width of the page that is inside the margins.
- Write an expression for the area of the page that is inside the margins.
- Show that if the margins on the left and right are doubled, then the area available to be printed is reduced by $(48y - 4xy) \text{ cm}^2$.
- If the margins at the top and the bottom of the page are 1.5 cm and the margins on the left and right of the page are 1 cm, calculate the size of the area that is available to be printed.



THINK

a 1 Construct a drawing, showing the key dimensions.

2 The real height (24 cm) is effectively reduced by x cm at the top and x cm at the bottom.

b The width (16 cm) is reduced by y cm on the left and y cm on the right.

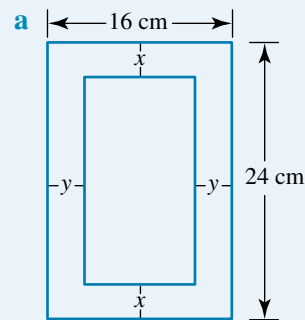
c The area of the page is the product of the width and height.

d 1 If the left and right margins are doubled, they become $2y$ and $2y$, respectively. Determine the new expression for the width of the page.

2 Determine the new expression for the reduced area.

3 Determine the difference in area by subtracting the reduced area from the area obtained in part **c**.

e Using the area found in part **c**, substitute the values for x and y , where $x = 1.5$ and $y = 1$.

WRITE/DRAW

$$\begin{aligned}\text{Height} &= 24 - x - x \\ &= (24 - 2x) \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{b Width} &= 16 - y - y \\ &= (16 - 2y) \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{c Area}_1 &= (24 - 2x)(16 - 2y) \\ &= (384 - 48y - 32x + 4xy) \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{d Width} &= 16 - 2y - 2y \\ &= (16 - 4y) \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Area}_2 &= (24 - 2x)(16 - 4y) \\ &= 24 \times 16 + 24 \times -4y - 2x \times 16 \\ &\quad - 2x \times -2y \\ &= (384 - 96y - 32x + 8xy) \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Difference in area} &= \text{Area}_1 - \text{Area}_2 \\ &= 384 - 48y - 32x + 4xy \\ &\quad - (384 - 96y - 32x + 8xy) \\ &= 384 - 48y - 32x + 4xy \\ &\quad - 384 + 96y + 32x - 8xy \\ &= 48y - 4xy\end{aligned}$$

So the amount by which the area is reduced is $(48y - 4xy) \text{ cm}^2$.

$$\begin{aligned}\text{e Area}_1 &= (384 - 48y - 32x + 4xy) \text{ cm}^2 \\ &= 384 - 48(1) - 32(1.5) + 4(1.5)(1) \\ &= 294 \text{ cm}^2\end{aligned}$$

Exercise 14.8 Applications of binomial products

Individual pathways

■ PRACTISE

Questions:
1–4, 8, 13, 14

■ CONSOLIDATE

Questions:
1–3, 5, 7–10, 13, 14

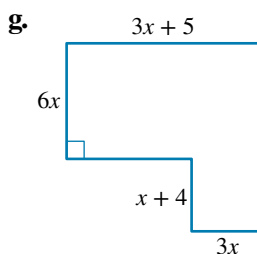
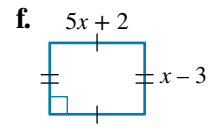
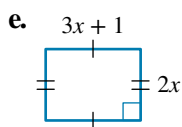
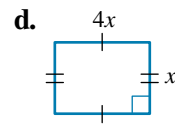
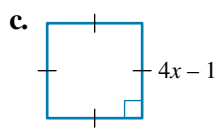
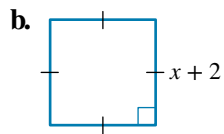
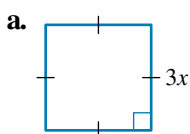
■ MASTER

Questions:
1–3, 6, 7, 9–16

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Answer the following for each shape.
 - i. Determine an expression for the perimeter.
 - ii. Determine the perimeter when $x = 5$.
 - iii. Determine an expression for the area and simplify by expanding if necessary.
 - iv. Determine the area when $x = 5$.



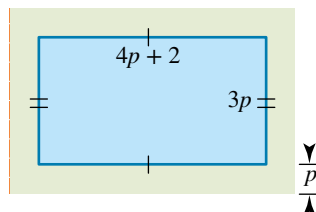
2. **WE14** A rectangular swimming pool measures 50 m by 25 m. A path around the edge of the pool is x m wide on each side.
 - a. Determine the area of the pool.
 - b. Write an expression for the area of the pool plus the path.
 - c. Write an expression for the area of the path.
 - d. If the path is 2.3 m wide, calculate the area of the path.
 - e. If the area of the path is 200 m^2 , write an equation that could be solved to find the width of the path.



3. **WE15** The page of a book is 20 cm high by 15 cm wide. The page has margins at the top and bottom of x cm and on the left and right of y cm.
 - a. Write an expression for the height of the page that is inside the margins.
 - b. Write an expression for the width of the page that is inside the margins.



- c. Write an expression for the area of the page that is inside the margins.
- d. Show that if the margins on the left and right are doubled, then the area available to be printed on is reduced by $(40y - 4xy)$ cm.
4. A rectangular book cover is 8 cm long and 5 cm wide.
- Calculate the area of the book cover.
 - If the length of the book cover is increased by v cm, write an expression for its new length.
 - If the width of the book cover is increased by v cm, write an expression for its new width.
 - Write an expression for the new area of the book cover and expand.
 - Calculate the area of the book cover if $v = 2$ cm.
 - If the length of the book cover is decreased by d cm, write an expression for its new length.
 - If the width of the book cover is decreased by d cm, write an expression for its new width.
 - Write an expression for the new area of the rectangle and expand.
 - Calculate the area of the book cover if $d = 2$ cm.
 - If the length of the book cover is made x times as long, write an expression for its new length.
 - If the width of the book cover is increased by x m, write an expression for its new width.
 - Write an expression for the new area of the book cover and expand.
 - Calculate the area of the book cover if $x = 5$ cm.
5. A square has dimensions of $5x$ m.
- Write an expression for its perimeter.
 - Write an expression for its area.
 - If its length is decreased by 2 m, write an expression for its new length.
 - If its width is decreased by 3 m, write an expression for its new width.
 - Write an expression for its new area and expand.
 - Calculate its area when $x = 6$ m.
6. A rectangular sign has a length of $2x$ cm and a width of x m.
- Write an expression for its perimeter.
 - Write an expression for its area.
 - If its length is increased by y cm, write an expression for its new length.
 - If its width is decreased by y cm, write an expression for its new width.
 - Write an expression for its new area and expand.
 - Calculate its area when $x = 4$ cm and $y = 3$ cm using your expression.
7. A square has a side length of x cm.
- Write an expression for its perimeter.
 - Write an expression for its area.
 - If its side length is increased by y cm, write an expression for its new side length.
 - Write an expression for its new perimeter and expand.
 - Calculate the perimeter when $x = 5$ cm and $y = 9$ cm.
 - Write an expression for its new area and expand.
 - Calculate the area when $x = 3.2$ cm and $y = 4.6$ cm.
8. A swimming pool with length $(4p + 2)$ m and width $3p$ m is surrounded by a path of width p m. Obtain the following in expanded form.

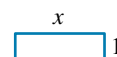


- An expression for the perimeter of the pool
- An expression for the area of the pool

- c. An expression for the length of the pool and path
- d. An expression for the width of the pool and path
- e. An expression for the perimeter of the pool and path
- f. An expression for the area of the pool and path
- g. An expression for the area of the path
- h. The area of the path when $p = 2$ m

Communicating, reasoning and problem solving

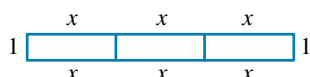
9. The figure at right is a rectangle. For the purpose of this activity we are going to call this rectangle an ‘algebra rectangle’.



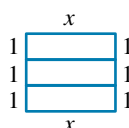
The length of each algebra rectangle is x cm and the width is 1 cm.

These algebra rectangles are put together to form larger rectangles in one of two ways.

Long algebra rectangle (3-long)



Tall algebra rectangle (3-tall)



Let's find the perimeter of each algebra rectangle.

Perimeter of 3-long algebra rectangle; $P = (x + x + x + x + x + x) + (1 + 1)$
 $= 6x + 2$

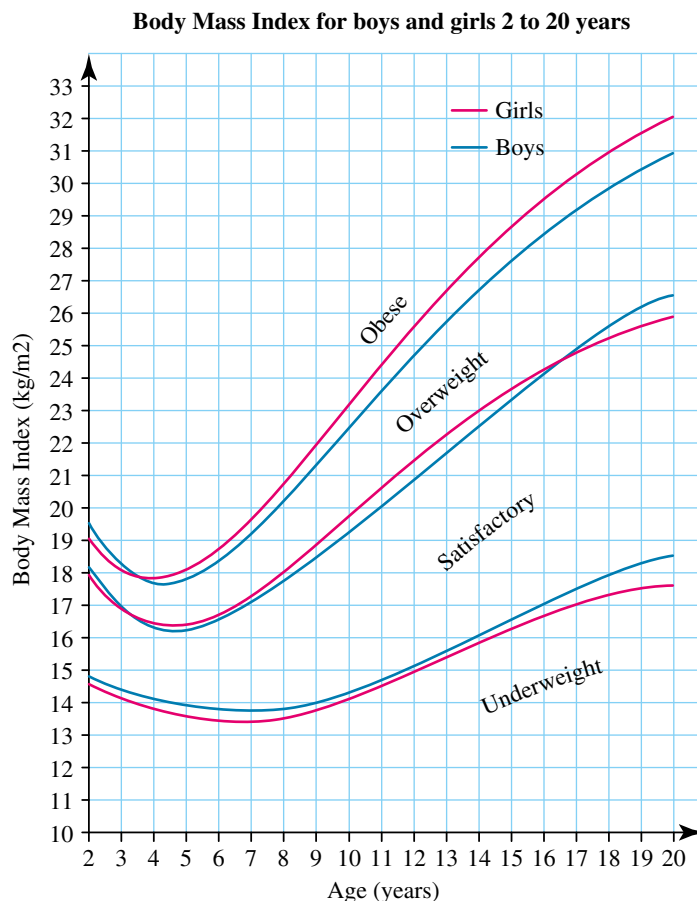
Perimeter of 3-tall algebra rectangle; $P = (x + x) + (1 + 1 + 1 + 1 + 1 + 1)$
 $= 2x + 6$

- a. Find the perimeter of each of the algebra rectangles. Copy and complete the table below to show your results.

Type of algebra rectangle	Perimeter	Type of algebra rectangle	Perimeter
1-long		1-tall	
2-long		2-tall	
3-long	$6x + 2$	3-tall	$2x + 6$
4-long		4-tall	
5-long		5-tall	

- b. Can you see a pattern? What would be the perimeter of a 20-long algebra rectangle and a 20-tall algebra rectangle?
- c. For what values of x will the tall rectangle have a larger perimeter than the long rectangle?
10. The Body Mass Index (B) is used as an indicator of whether or not a person is in a healthy weight range for their height. It can be found using the formula $B = \frac{m}{h^2}$, where m is the person's mass in kilograms and h is the person's height in metres. Use the formula to answer the following questions, correct to 1 decimal place.
- a. Calculate Richard's Body Mass Index if he weighs 85 kg and is 1.75 m tall.
 - b. A person is considered to be in a healthy weight range if his or her Body Mass Index lies in the range of 21 to 25 inclusive. Comment on Richard's weight for a person of his height.
 - c. Calculate the Body Mass Index, correct to 1 decimal place, for each of the following people:
 - i. Judy, who is 1.65 m tall and has a mass of 52 kg
 - ii. Karen, who is 1.78 m tall and has a mass of 79 kg
 - iii. Manuel, who is 1.72 m tall and has a mass of 65 kg.

- d. The Body Mass Index (BMI) range for children changes until adulthood. This graph shows the Body Mass Index for boys and girls aged 2 to 20 years. Study the graph carefully and decide on possible age ranges (between 2 and 20) for Judy, Karen and Manuel if their BMI was in the Satisfactory range. State the age ranges in half-year intervals.



11. Can you evaluate 997^2 without a calculator and in less than 90 seconds? We would be able to evaluate this using long multiplication, but it would take a fair amount of time and effort. Mathematicians are always looking for quick and simple ways of solving problems. What if we consider expanding the formula that produces the difference of two squares?

$$(a + b)(a - b) = a^2 - b^2$$

Adding b^2 to both sides gives $(a + b)(a - b) + b^2 = a^2 - b^2 + b^2$.

Simplifying and swapping sides gives $a^2 = (a + b)(a - b) + b^2$.

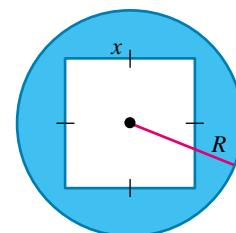
We can use this new formula and the fact that multiplying by 1000 is an easy operation to evaluate 997^2 .

- If $a = 997$, what should we make the value of b so that $(a + b)$ equals 1000?
- Substitute these a and b values into the formula to evaluate 997^2 .
- Try this method to evaluate the following.
 - 995^2
 - 990^2

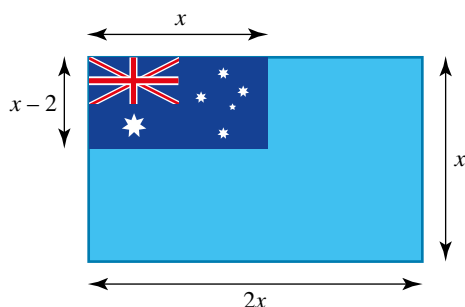
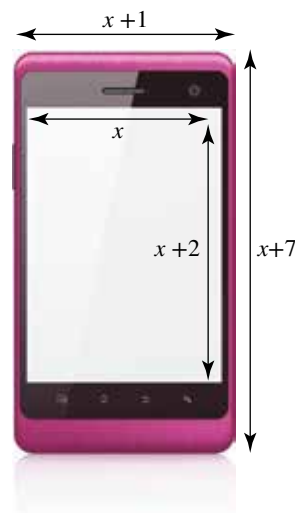
12. A new game has been created by students for the school fair. To win the game you need to hit the target with 5 darts in the shaded region.

- Write an expression for the area of the game board (the shaded region).
- If $R = 7.5$ cm and $x = 4$ cm, find the area of the shaded region.

- Show that $R = \sqrt{\frac{A + x^2}{\pi}}$ by transposing the formula found in part a.



- d. If $A = 80 \text{ cm}^2$ and $x = 3 \text{ cm}$, calculate the value of R .
- e. The students found that the best size for the game board is when $R = 10$ and $x = 5$. What is the percentage of the total board that is shaded?
13. A picture of a mobile phone is shown (dimensions are in cm).
- Use the information in the picture to write an expression for the area of:
 - the viewing screen
 - the front face of the phone.
 - A friend has a phone that is 4 cm longer but 1 cm narrower. Write expressions, in expanded form, for:
 - the length and width of your friend's phone
 - the area of your friend's phone.
 - If the phone pictured is 5 cm wide, use your answer to part **b** to find the area of the front face of your friend's phone.
14. A proposed new flag for Australian schools will have the Australian flag in the top left-hand corner. The dimensions given are in metres.



- Write an expression, in factorised form, for the area of:
 - the Australian flag
 - the proposed flag.
 - Use the answers from part **a** to write the area of the Australian flag as a *fraction* of the school flag and then simplify this fraction.
 - Use the fraction from part **b** to express the area of the Australian flag as a *percentage* of the proposed school flag.
 - Using the formula for the percentage of area taken up by the Australian flag, find the percentages for the following school flag widths.
 - 4 m
 - 4.5 m
 - 4.8 m
 - If the percentage of the school flag taken up by the Australian flag measures the importance a school places on Australia, what can be said for the three flags?
15. Cubic expressions are the expanded form of expressions with three linear factors. The expansion process for three linear factors is called trinomial expansion.
- Explain how the area model can be altered to show that binomial expansion can also be altered to become a model for trinomial expansion.
 - Investigate the powers in cubic expressions by expanding the following expressions.
 - $(3x + 2)(-x + 1)(x + 1)$
 - $(x + 5)(2x - 2)(4x - 8)$
 - $(3 - x)(x + 8)(5 - x)$
 - Describe the patterns of the powers in a cubic expression.
16. How can you use algebraic skills in real-life situations?

14.9 Review

Investigation | Rich task

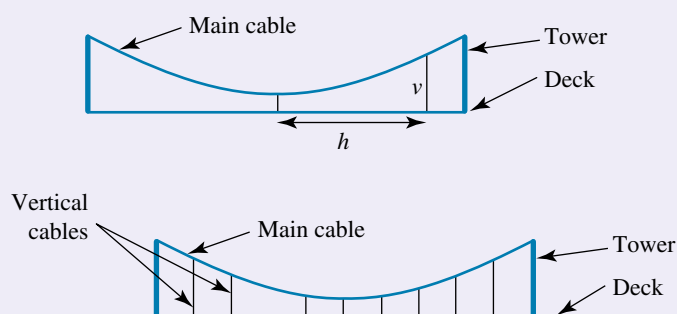
Suspension bridges



When you hold up a chain at both ends, it forms a curve. Its shape is actually a *catenary* — a trigonometric function with a shape similar to that of a quadratic equation whose shape is a parabola. When constructing a suspension bridge, the main cables are attached to a tower at both ends and the curve is a catenary. However, when vertical cables are attached from the main cable to the deck, the curve takes the shape of a parabola. A catenary curves under its own weight; the bridge's main cables are curving not just under their own weight, but also curving from holding up the weight of the deck.

There are many well-known suspension bridges around the world, for example the Golden Gate Bridge in San Francisco.

A sketch of one side of the bridge can be represented as follows (we will consider just one side of the bridge in our calculations). Note that there are no vertical cables at the ends where the main cable is attached to the towers.



The shape of the cable can be represented by the equation $v = 0.000832h^2 + 3$ where v represents the vertical distance between the main cable and the deck (in metres) and h represents the horizontal distance from the centre point of the main cable (in metres). The curve is symmetrical around this central point.

Let us consider a suspension bridge with 55-m high towers at the ends and vertical cables evenly spaced every 50 m.

1. Substitute into the equation to calculate the horizontal distance of the towers from the centre of the main cable.
2. Determine the length of the deck.
3. Draw a diagram displaying the spacing of all the vertical cables on the bridge.
4. How many vertical cables are there in the structure?
5. Use the equation to calculate the length (to the nearest metre) of each of these vertical cables.
6. What is the total length of cable required for these vertical cables?



The Golden Gate Bridge consists of a main suspension span in the centre with side suspension spans at both ends.

Here are some facts about the structure of the bridge.

- Length of suspension span including main span and side spans: 1966 m
 - Horizontal distance between towers: 1280 m
 - Clearance above mean high water: 67 m
 - Height of tower above roadway: 152 m
 - The bridge has two main cables that pass over the tops of the two main towers and are secured at either end. Length of each main cable: 2332 m
 - Total length of wire used in main cables and vertical cables: 129000 km
7. Draw a sketch of the bridge showing as much detail as possible.

learnon RESOURCES — ONLINE ONLY

Digital doc: Investigation — Suspension bridges (doc-15912)

Digital doc: Summary Topic 14 (doc-22896)

Interactivity: Crossword Topic 14 (int-0707)

Interactivity: Word search Topic 14 (int-0693)

Interactivity: Sudoku Topic 14 (int-3214)

Exercise 14.9 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

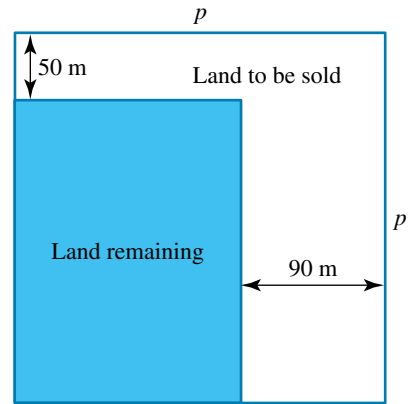
Understanding and fluency

- MC** The equivalent of $(3 - a)(3 + a)$ is:
A. $9 + a^2$ **B.** $9 - a^2$ **C.** $3 + a^2$ **D.** $3 - a^2$ **E.** $9 - 2a + a^2$
- MC** The equivalent of $(2y + 5)^2$ is:
A. $4y^2 + 20y + 25$ **B.** $4y^2 + 10y + 25$ **C.** $2y^2 + 20y + 25$ **D.** $2y^2 + 20y + 5$ **E.** $4y^2 + 25$
- MC** When factorised, $6(a + 2b) - x(a + 2b)$ equals:
A. $6 - x(a + 2b)$ **B.** $(6 - x)(a + 2b)$ **C.** $6(a + 2b - x)$ **D.** $(6 + x)(a - 2b)$ **E.** $2(3 - x)(a + b)$
- Expand and simplify these expressions.
a. $(x + 4)(x - 4)$ **b.** $(9 - m)(9 + m)$ **c.** $(x + y)(x - y)$ **d.** $(1 - 2a)(1 + 2a)$
- Expand and simplify these expressions.
a. $(x + 5)^2$ **b.** $(m - 3)^2$ **c.** $(4x + 1)^2$ **d.** $(2 - 3y)^2$
- Expand and simplify the following.
a. $(x + 2)^2 + (x + 3)^2$ **b.** $(x - 2)^2 - (x - 3)^2$ **c.** $(x + 4)^2 - (x - 4)^2$
- MC** The expression $4a^2 - 12a + 9$ factorises to:
A. $(4a + 9)(4a + 9)$ **B.** $(4a - 9)(4a - 9)$ **C.** $(2a - 3)(2a - 3)$
D. $(2a - 3)(2a + 3)$ **E.** $(2a - 9)(2a - 1)$
- MC** What does the expression $9m^2 - 16n^2$ factorise to?
A. $9(m - 4n)(m + 4n)$ **B.** $3(3m - 4n)^2$ **C.** $(9m - 16n)(9m + 16n)$
D. $(3m - 4n)(3m + 4n)$ **E.** $(3m - 4n)^2$
- MC** What does the expression $5x^2 + 20x - 60$ factorise to?
A. $(5x - 12)(x + 5)$ **B.** $(5x + 15)(x - 4)$ **C.** $5(x + 6)(x - 2)$
D. $5(x - 6)(x + 2)$ **E.** $(5x + 10)(x - 6)$
- MC** What does the expression $3x^2 - 13x + 4$ factorise to?
A. $(3x - 1)(x - 4)$ **B.** $(3x - 2)(x + 2)$ **C.** $(3x - 2)^2$
D. $(3x - 4)(x - 1)$ **E.** $(3x - 1)(x + 4)$
- MC** $\frac{x + 3}{x + 2} \div \frac{(x + 3)^2}{x + 2}$ is equivalent to:
A. $\frac{(x + 3)^2}{(x + 2)^2}$ **B.** $\frac{1}{x + 3}$ **C.** $\frac{x + 2}{x + 3}$ **D.** $\frac{-1}{2(x + 2)}$ **E.** $x + 3$
- Factorise each of the following using the difference of two squares rule.
a. $x^2 - 64$ **b.** $a^2 - 144$ **c.** $49b^2 - 1$
d. $4f^2 - 9g^2$ **e.** $(n + 1)^2 - m^2$ **f.** $(r - 1)^2 - 4s^2$
- Factorise each of the following quadratic trinomials.
a. $c^2 + 5c + 4$ **b.** $p^2 + 10p - 24$ **c.** $y^2 - 10y + 24$
d. $x^2 + 3x + 2$ **e.** $m^2 - 7m + 10$ **f.** $m^2 + 24m + 44$
- Factorise each of the following.
a. $2a^2 + 16a + 24$ **b.** $3b^2 - 24b + 36$ **c.** $4c^2 - 16c - 48$
d. $2x^2 + 3x + 1$ **e.** $3x^2 - x - 2$ **f.** $6x^2 + 13x + 6$
- Simplify each of the following.
a. $\frac{(x - 3)(x + 4)}{x + 1} \times \frac{x + 1}{x - 3}$ **b.** $\frac{x(x + 5)}{x - 5} \div \frac{x + 5}{x(x - 5)}$ **c.** $\frac{4x - 12}{2x + 2} \times \frac{x + 1}{x - 3}$
d. $\frac{10x + 20}{3x - 3} \div \frac{5x + 10}{6x + 6}$ **e.** $\frac{x^2 + 7x + 12}{x^2 + 4x + 3} \times \frac{x^2 - 6x - 7}{x^2 + 2x - 8}$ **f.** $\frac{x^2 - 25}{x^2 + 4x - 5} \div \frac{x^2 - 6x + 5}{x^2 + 5x - 6}$

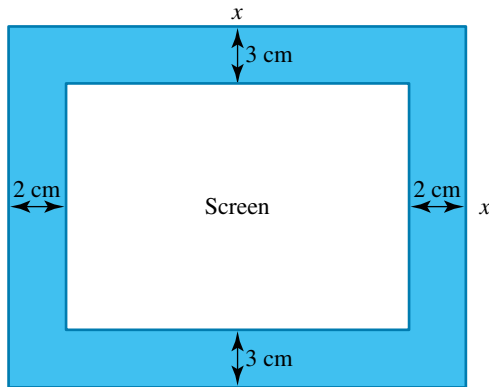
Communicating, reasoning and problem solving

- 16.** David owned land in the shape of a square with side length p . He decided to sell part of this land by reducing it by 50 metres in one direction and 90 metres in the other.

- Find an expression in terms of p for the area of the original land.
- Find an expression in terms of p , in its simplest form, for the area of land David has remaining after he sells the section of land shown.
- Find an expression in terms of p for the area of land he sold.



- 17.** A new computer monitor is made up of a rectangular screen surrounded by a hard plastic frame in which speakers can be inserted.



- Write expressions for the length and width of the screen in terms of x .
- Write an expression for the area of the screen, using expanded form.
- Find the area of the screen if $x = 30$ cm.

Answers

Topic 14 Algebraic techniques 2

Exercise 14.2 Special binomial products

1. a. $x^2 - 4$ b. $y^2 - 9$ c. $m^2 - 25$ d. $a^2 - 49$
 e. $x^2 - 36$ f. $p^2 - 144$ g. $a^2 - 100$ h. $m^2 - 121$
2. a. $4x^2 - 9$ b. $9y^2 - 1$ c. $25d^2 - 4$ d. $49c^2 - 9$
 e. $4 - 9p^2$ f. $d^2 - 81x^2$ g. $25 - 144a^2$ h. $9x^2 - 100y^2$
 i. $4b^2 - 25c^2$ j. $100 - 4x^2$
3. a. $x^2 + 4x + 4$ b. $a^2 + 6a + 9$ c. $b^2 + 14b + 49$ d. $c^2 + 18c + 81$
 e. $m^2 + 24m + 144$ f. $n^2 + 20n + 100$ g. $x^2 - 12x + 36$ h. $y^2 - 10y + 25$
 i. $81 - 18c + c^2$ j. $64 + 16e + e^2$ k. $2x^2 + 4xy + 2y^2$ l. $u^2 - 2uv + v^2$
4. a. $4a^2 + 12a + 9$ b. $9x^2 + 6x + 1$ c. $4m^2 - 20m + 25$ d. $16x^2 - 24x + 9$
 e. $25a^2 - 10a + 1$ f. $49p^2 + 56p + 16$ g. $81x^2 + 36x + 4$ h. $16c^2 - 48c + 36$
 i. $9 + 12a + 4a^2$ j. $25 + 30p + 9p^2$ k. $4 - 20x + 25x^2$ l. $49 - 42a + 9a^2$
 m. $81x^2 - 72xy + 16y^2$ n. $64x^2 - 48xy + 9y^2$

5. $(x^2 + 6x + 9)$ units²

6. $(3x + 1)$ m

7. Answers will vary.

8. Anne's bedroom is larger by 1 m².

9. a. $a^2 + ab + ab + b^2$ b. $A = (a + b) \times (a + b)$
 $= (a + b)^2$
 $= a^2 + ab + ab + b^2$
 $= a^2 + 2ab + b^2$
 Perfect squares

10. a. i. $x^2 - 6x + 9$ and $9 - 6x + x^2$

ii. $x^2 - 30x + 225$ and $225 - 30x + x^2$

iii. $9x^2 - 42x + 49$ and $49 - 42x + 9x^2$

b. The answers to the pairs of expansions are the same.

c. This is possible because when a negative number is multiplied by itself, it becomes positive; and when expanding a perfect square, where the two expressions are the same, the negative signs cancel out and result in the same answer.

11. a. i. $x^2 - 16$ and $16 - x^2$

ii. $x^2 - 121$ and $121 - x^2$

iii. $4x^2 - 81$ and $81 - 4x^2$

b. The answers to the pairs of expansions are the same, except that the negative and positive signs are reversed.

c. This is possible because when a negative number is multiplied by a positive number, it becomes negative; and when expanding a DOTS, where the two expressions have different signs, the signs will be reversed.

12. a. $(10k + 5)^2 = 100k^2 + 100k + 25$

b. $(10k + 5)^2 = 100 \times k \times k + 100 \times k + 25$
 $= 100k(k + 1) + 25$

c. $25^2 = (10 \times 2 + 5)^2$

Let $k = 2$.

$25^2 = 100k(k + 1) + 25$

$= 100 \times 2 \times (2 + 1) + 25$

$= 625$

$85^2 = (10 \times 8 + 5)^2$

Let $k = 8$.

$85^2 = 100k(k + 1) + 25$

$= 100 \times 8 \times (8 + 1) + 25$

$= 7225$

13. a. 10609

b. 3844

c. 994009

d. 1024144

e. 2809

f. 9604

14. $(x - 3)^2$ is a perfect square. As a diagram it would be a square, each with a side length of $(x - 3)$.

15.

$-3(x - 1)^2$	$4x^2 - x + 4$	$-x^2 + 4x - 1$
$2x^2 + x + 2$	$3x$	$-2x^2 + 5x - 2$
$(x + 1)^2$	$-4x^2 + 7x - 4$	$3(x^2 + 1)$

Exercise 14.3 Further expansions

1. $2x^2 + 13x + 21$
2. $2x^2 + 13x + 20$
3. $2x^2 + 14x + 26$
4. $2x^2 + 10x + 11$
5. $2p^2 - 3p - 21$
6. $2a^2 - 5a + 4$
7. $2p^2 - p - 24$
8. $2x^2 + 19x - 36$
9. $2y^2 + 2y - 7$
10. $2d^2 + 8d - 2$
11. $2x^2 + 10$
12. $2y^2$
13. $2x^2 - 4x + 19$
14. $2y^2 - 4y - 7$
15. $2p^2 + 3p + 23$
16. $2m^2 + 3m + 31$
17. $x + 5$
18. $4x + 8$
19. $-2x - 6$
20. $3m + 2$
21. $-3b - 22$
22. $-15y - 2$
23. $8p - 10$
24. $16x + 2$
25. $-16c - 40$
26. $-14f - 34$
27. $4m + 17$
28. $-7a + 30$
29. $-6p - 7$
30. $3x - 21$

$$31. (p-1)(p+2) + (p-3)(p+1) = (p^2 + p - 2) + (p^2 - 2p - 3) \\ = p^2 + p^2 + p - 2p - 2 - 3 \\ = 2p^2 - p - 5$$

$$32. (x+2)(x-3) - (x+1)^2 = (x^2 - x - 6) - (x^2 + 2x + 1) \\ = -3x - 7$$

$$33. \text{ a. } (a^2 + b^2)(c^2 + d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \\ = (a^2c^2 + b^2d^2) + (a^2d^2 + b^2c^2) \\ = ((ac)^2 - 2abcd + (bd)^2) + ((ad)^2 + 2abcd + (bc)^2) \\ = (ac - bd)^2 + (ad + bc)^2$$

$$\text{ b. } a = 2, b = 1, c = 3, d = 4$$

$$(2^2 + 1^2)(3^2 + 4^2) = (2 \times 3 - 1 \times 4)^2 + (2 \times 4 + 1 \times 3)^2 \\ = 4 + 121 \\ = 125$$

$$34. \text{ a. } (x^2 + x - 1)^2 = (x^2 + x - 1)(x^2 + x - 1) \\ = x^4 + x^3 - x^2 + x^3 + x^2 - x - x^2 - x + 1 \\ = x^4 + 2x^3 - x^2 - 2x + 1$$

$$\text{ b. } x^4 + 2x^3 - x^2 - 2x + 1 \\ = x(x^3 + 2x^2 - x - 2) + 1 \\ = x(x+1)(x+2)(x-1) + 1$$

$$\text{ c. i. } 4 \times 3 \times 2 \times 1 + 1 = 25$$

$$\text{ ii. } x = 2$$

$$35. \text{ a. } (a+b)(d+e) = ad + ae + bd + be$$

$$\text{ b. } (a+b+c)(d+e+f) = ad + ae + bd + be + cd + ce + af + bf + cf$$

	a	b	c
d	ad	bd	cd
e	ae	be	ce
f	af	bf	cf

36. To show $(m+2)(m+3) - (m+2)(m+4)$ on a diagram, we need to expand each set of brackets and simplify.

$$(m+2)(m+3) - (m+2)(m+4) \\ = (m^2 + 2m + 3m + 6) - (m^2 + 2m + 4m + 8) \\ = m^2 + 5m + 6 - m^2 - 6m - 8 \\ = -m - 2$$

Exercise 14.4 The highest common binomial factor

1. a. $(a+b)(2+3c)$
2. a. $(y+2)(x+2)$
3. a. $(2m+1)(7x-y)$
4. a. $(3b+2)(4a-b)$
5. a. $(x+2y)(z-3)$
6. a. $(6-q)(12p-5)$
7. a. $(x-y)(3p^2+2q)$
8. a. $(b-3)(4a^2+3b)$
9. a. $(q+2p)(p^2-5)$
10. a. $(5m+1)(6+n^2)$
11. a. $(3m-1)(5n-2)$
12. a. $(10p-1)(q-2)$
13. a. $(3x-1)(2-y)$
14. a. $(4p-1)(4-3q)$
15. a. $(2y+1)(x+3)$
16. a. $(5+3p)(1+3a)$
17. a. $(x-4)(y+3)$
18. a. $(m-2n)(1+a)$
19. a. $(b-2)(a+5)$
20. a. $(3x-1)(2-y)$
21. a. $(4p-1)(4-3q)$
22. a. $(2y+1)(5x-2)$
23. a. $(2a+3)(3b-2)$
24. a. $(b-2c)(5a-3)$
25. a. $(x+3y)(4-z)$
26. a. $(p+2q)(5r-3)$
27. a. $(a-5b)(c-2)$

3. Answers will vary.

4. a. $6x^2 - 11x - 10$

b. Answers will vary.

c. Answers will vary.

d. i. $2x^2 - 11x + 12$

ii. $-2x^2 - 9x + 5$

5. a. $5(x+2)(x+3) = 5(x^2 + 5x + 6) = 5x^2 + 15 + 30$

	x	3	x	3	x	3	x	3	x	3
3	x^2	$3x$	x^2	$3x$	x^2	$3x$	x^2	$3x$	x^2	$3x$
2	$2x$	6	$2x$	6	$2x$	6	$2x$	6	$2x$	6

b. $5(x+2)(x+3) = (5 \times x + 2 \times 5)(x+3) = (5x+10)(x+3)$

	x	2	x	2	x	2	x	2	x	2
x	x^2	$2x$	x^2	$2x$	x^2	$2x$	x^2	$2x$	x^2	$2x$
3	$3x$	6	$3x$	6	$3x$	6	$3x$	6	$3x$	6

c. $5(x+2)(x+3) = (5 \times x + 3 \times 5)(x+2) = (5x+15)(x+2)$

	$5x$	15
x	$5x^2$	$15x$
2	$10x$	30

6. a. i. Square, because it is a perfect square.

ii. Rectangle, because it is a DOTS.

iii. Rectangle, because it is a trinomial.

iv. Rectangle, because it is a trinomial.

b. i. $(3s+8)^2$

ii. $(5s+2)(5s-2)$

iii. $(s+1)(s+3)$

iv. $4(s^2-7s-8)$

7. If an expression has 4 terms and no common factor in all the terms, it may be possible to group terms in pairs and find a common factor in each pair.

Exercise 14.5 Factorisation patterns

1. a. $(x+5)(x-5)$

b. $(x+9)(x-9)$

c. $(a+4)(a-4)$

d. $(5+p)(5-p)$

e. $(11+a)(11-a)$

f. $(6+y)(6-y)$

g. $(2b+5)(2b-5)$

h. $(3a+4)(3a-4)$

i. $(5d+1)(5d-1)$

2. a. $(x+y)(x-y)$

b. $(a+b)(a-b)$

c. $(p+q)(p-q)$

d. $(5m+n)(5m-n)$

e. $(9x+y)(9x-y)$

f. $(p+6q)(p-6q)$

g. $(6m+5n)(6m-5n)$

h. $(4q+3p)(4q-3p)$

i. $(2m+7n)(2m-7n)$

3. a. $(x-1)(x+7)$

b. $(p+13)(p+3)$

c. $(p-2+q)(p-2-q)$

d. $(c-6+d)(c-6-d)$

e. $(x+7+y)(x+7-y)$

f. $(p+5+q)(p+5-q)$

g. $(a+3+b)(a+3-b)$

h. $(a-2)(a-4)$

i. $(b+5)(b-7)$

4. a. $2(m+4)(m-4)$

b. $5(y+3)(y-3)$

c. $6(p+2)(p-2)$

d. $4(m+5)(m-5)$

e. $2(12+x)(12-x)$

f. $5(4+a)(4-a)$

g. $25(2x+y)(2x-y)$

h. $4(6p+q)(6p-q)$

i. $2(4y+x)(4y-x)$

j. $a(x+3)(x-3)$

k. $2b(2+x)(2-x)$

l. $3(b+9)(b+1)$

5. a. $(x+5)^2$

b. $(p-12)^2$

c. $(n+10)^2$

d. $(x+y)^2$

e. $(u-v)^2$

f. $(8+e)^2$

g. $(2m-5)^2$

h. $(7+3a)^2$

i. $(9x+4y)^2$

j. $2(x+6)^2$

k. $3(x-4y)^2$

l. $2(3a+2b)^2$

6. a. i

b. iv, vi

7. a. C

b. A

c. D

d. A

e. C

8. a. $\pi r^2 \text{ m}^2$

b. $(r+1) \text{ m}$

c. $\pi(r+1)^2 \text{ m}^2$

d. $(\pi(r+1)^2 - \pi r^2) \text{ m}^2$

e. $\pi(2r+1) \text{ m}^2$

f. 35 m^2

g. 47 m^2

9. a. $w^2 + 10w + 25$
 b. i. 100 ii. 225 iii. 625
 c. Check with your teacher.
10. a. $(x + \sqrt{10})(x - \sqrt{10})$ b. $4(x + \sqrt{8})(x - \sqrt{8})$ c. $(x^2 - y)(x^2 + y)$ d. $5(2x - 1)$
 e. $(y - 2x^2)(y + 2x^2)$ f. $(10^2 - x^2)(10^2 + x^2) = (10 - x)(10 + x)(100 + x^2)$
- 11, 12, 13 Check with your teacher.
14. a. $(x - 7)(x + 7)$ b. Length = $x + 7$; $x - 7$
 c. Length = 32 mm, width = 18 mm d. Area = 2451 mm^2 ; 1875 mm^2 larger
15. a. Side length_a = $x + 3$ b. $A_a = \Pi = (x + 3)^2$ c. $A_a = \Pi = 64 \text{ cm}$
 Side length_b = $x + 5$ $A_b = \text{III} = (x + 5)^2$ $A_b = \text{III} = 100 \text{ cm}$
 Side length_c = $x + 8$ $A_c = \text{V} = (x + 8)^2$ $A_c = \text{V} = 169 \text{ cm}$
 Side length_d = $x - 3$ $A_d = \text{I} = (x - 3)^2$ $A_d = \text{I} = 4 \text{ cm}$
 Side length_e = $x + 6$ $A_e = \text{IV} = (x + 6)^2$ $A_e = \text{IV} = 121 \text{ cm}$
16. a. 10609 b. 3844 c. 994009 d. 1024144 e. 2809 f. 9604
17. Answers will vary. The difference of two squares can be factorised: $a^2 - b^2 = (a + b)(a - b)$. Quadratic expressions of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ can be factorised as $(a + b)^2$ or $(a - b)^2$ respectively.
18. $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

Exercise 14.6 Factorising non-monic quadratics

1. a. $2(x + 2)(x + 3)$ b. $3(x + 4)(x + 1)$ c. $2(x - 2)(x - 4)$
 d. $3(x - 3)(x - 1)$ e. $4(x + 2)(x - 3)$ f. $3(x + 5)(x - 2)$
 g. $2(x + 7)(x - 3)$ h. $3(x - 6)(x + 3)$ i. $5(x + 6)(x - 2)$
2. a. $(x + 3)(2x + 1)$ b. $(x + 2)(2x + 3)$ c. $(x + 2)(3x + 1)$
 d. $(x - 3)(2x - 1)$ e. $(x - 1)(3x + 2)$ f. $(x + 1)(5x - 2)$
 g. $(x - 2)(7x - 3)$ h. $(2x - 3)(5x + 2)$ i. $(x + 4)(2x - 3)$
3. a. B b. A c. D
4. a. $(2x - 1)(x + 2)$ b. $(4x + 3)(2x - 1)$ c. $(3x + 2)(2x - 1)$
 d. $(2x + 5)(x + 2)$ e. $(3x - 4)(x + 1)$ f. $(2x - 3)(3x + 7)$
5. a. $(2x - 3)(3x + 2)$ b. $(4x + 5)(x + 2)$ c. $(x - 1)(4x - 15)$ d. $(3x + 4)(2x - 1)$
6. $3(x + 1)(x + 4)$
7. a. $-3, -\frac{1}{2}$ b. $-1, \frac{1}{2}$ c. $-2\frac{1}{2}, 3$ d. $-3, -1$
 e. $-2\frac{1}{3}, -2$ f. $-1, -\frac{2}{3}$ g. $1\frac{2}{5}, 3$ h. $\frac{2}{5}, 3$
 i. $-\frac{5}{7}, 1$ j. $\frac{5}{7}, 4$ k. $-1\frac{1}{2}, \frac{1}{2}$ l. $-2\frac{1}{2}, -\frac{1}{3}$
8. a. $(4x + 3)(x - 5)$ b. $(x + 3)(3x + 1)$ c. $(5x + 3)(2x - 3)$ d. $(3x - 1)(5x + 2)$
9. a. The middle term, $-9x$, can be split as $3x - 12x$.
 b. $(4x + 3)(x - 3)$
10. a. $(6x - 5)(2x + 1)$ b. i. $12x^2 - 4x - 5$ ii. 1.5 iii. 1.1 or 3.5
11. a. i. Square, because it is a perfect square.
 iii. Rectangle, because it is a trinomial.
 b. i. $(3s + 8)^2$ ii. $(5s + 2)(5s - 2)$ iii. $(s + 1)(s + 3)$ iv. $4(s^2 - 6s - 9)$
12. a. $(3b + 8)^2$ b. $(ac + d)(ac - d)$ c. $(a - 13b)(a + 14b)$ d. $(b - ac)^2$

Exercise 14.7 Simplifying algebraic fractions 2

1. a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. 1 d. 3 e. 1 f. 3
 g. $\frac{1}{4}$ h. $\frac{h}{2}$ i. $\frac{5(p - 3)}{p - 2}$ j. $x + 2$ k. $\frac{b + 3}{b + 5}$ l. $\frac{x + 1}{x - 6}$
2. a. $\frac{x}{3}$ b. 2 c. $\frac{1}{2}$ d. 3 e. 1 f. $\frac{p + 2}{p - 2}$
 g. $\frac{s - 6}{3s}$ h. $a - 3$ i. $\frac{4}{m + 2}$ j. $\frac{a - 3}{a - 5}$ k. $\frac{m - 2}{m + 5}$ l. $\frac{p + 2}{p - 4}$

3. a. $a + 1$ b. $\frac{a+2}{3}$ c. $a - 1$ d. $p - 3$ e. $\frac{x+2}{2}$ f. $\frac{x-4}{2}$
 g. $3(x+6)$ h. $3(x-8)$ i. $\frac{2(x-2)}{3}$ j. $\frac{3(x-5)}{5}$ k. $\frac{1}{x+5}$ l. $-\frac{1}{x-4}$
 m. $\frac{2}{3}$ n. $\frac{3}{4}$ o. $\frac{5}{6}$ p. 3
 4. a. $x + 3$ b. $p - 5$ c. $y - 4$ d. $a + 10$ e. $\frac{7}{5-x}$ f. $\frac{5}{5-b}$
 g. $\frac{9}{y-9}$ h. $\frac{4}{m-4}$ i. $\frac{7}{1-2x}$ j. $\frac{3}{2m-1}$ k. $\frac{1-2a}{a+3}$ l. $\frac{2b+1}{5-8b}$
 m. $\frac{3x+1}{9-x}$ n. $\frac{7-2b}{b+5}$ o. $\frac{3(m+2)}{4m-1}$ p. $\frac{2(2-a)}{3+2a}$
 5. a. $x + 2$ b. $x + 3$ c. $x - 4$ d. $b - 7$ e. $a - 9$ f. $a - 11$
 g. $x + 7$ h. $m + 8$ i. $\frac{a-3}{a+4}$ j. $\frac{p+1}{p+5}$ k. $\frac{x+3}{x-1}$ l. $\frac{m-1}{m+6}$
 m. $\frac{y+2}{y+6}$ n. $\frac{x-2}{x+2}$ o. $\frac{x-5}{x-2}$ p. $\frac{x+6}{x-3}$
 6. a. 10 b. 3 c. 4 d. 3 e. $\frac{3}{4}$ f. $\frac{12(a+2)}{a+3}$
 g. $\frac{6x+1}{8(x+7)}$ h. $\frac{m+4}{m+7}$ i. $\frac{x+5}{x+3}$ j. $\frac{x+2}{x-4}$
 7. Check with your teacher. 8. Check with your teacher.
 9. a. $\frac{(c+12)}{(c-18)}$ b. $\frac{(m+1)}{(m+8)}$ c. $\frac{(f+9)}{(f+6)}$
 d. Cannot be simplified e. $\frac{(w+5)}{(w+20)}$ f. Cannot be simplified

10. After writing out the expression, the common factors are cancelled and then the expression can be simplified.

11. $\frac{x+1}{x(x+2)} + 1$

Exercise 14.8 Applications of binomial products

1. a. i. $12x$ ii. 60 iii. $9x^2$ iv. 225
 b. i. $4x + 8$ ii. 28 iii. $x^2 + 4x + 4$ iv. 49
 c. i. $16x - 4$ ii. 76 iii. $16x^2 - 8x + 1$ iv. 361
 d. i. $10x$ ii. 50 iii. $4x^2$ iv. 100
 e. i. $10x + 2$ ii. 52 iii. $6x^2 + 2x$ iv. 160
 f. i. $12x - 2$ ii. 58 iii. $5x^2 - 13x - 6$ iv. 54
 g. i. $20x + 18$ ii. 118 iii. $21x^2 + 42x$ iv. 735
 2. a. 1250 m^2 b. $(1250 + 150x + 4x^2) \text{ m}^2$ c. $(150x + 4x^2) \text{ m}^2$
 d. 366.16 m^2 e. $150x + 4x^2 = 200$
 3. a. $(20 - 2x) \text{ cm}$ b. $(15 - 2y) \text{ cm}$
 c. $(300 - 40y - 30x + 4xy) \text{ cm}^2$ d. Check with your teacher.
 4. a. 40 cm^2
 b. i. $(8 + v) \text{ cm}$ ii. $(5 + v) \text{ cm}$ iii. $(v^2 + 13v + 40) \text{ cm}^2$ iv. 70 cm^2
 c. i. $(8 - d) \text{ cm}$ ii. $(5 - d) \text{ cm}$ iii. $(40 - 13d + d^2) \text{ cm}^2$ iv. 18 cm^2
 d. i. $8x \text{ cm}$ ii. $(5 + x) \text{ cm}$ iii. $(8x^2 + 40x) \text{ cm}^2$ iv. 400 cm^2
 5. a. $20x \text{ m}$ b. $25x^2 \text{ m}^2$
 c. i. $(5x - 2) \text{ m}$ ii. $(5x - 3) \text{ m}$ iii. $(25x^2 - 25x + 6) \text{ m}^2$ iv. 756 m^2
 6. a. $6x \text{ cm}$ b. $2x^2 \text{ cm}^2$
 c. i. $(2x + y) \text{ cm}$ ii. $(x - y) \text{ cm}$ iii. $(2x^2 - xy - y^2) \text{ cm}^2$ iv. 11 cm^2
 7. a. $4x \text{ cm}$ b. $x^2 \text{ cm}^2$
 c. i. $(x + y) \text{ cm}$ ii. $(4x + 4y) \text{ cm}$ iii. 56 cm iv. $(x^2 + 2xy + y^2) \text{ cm}^2$
 d. 60.84 cm^2

8. a. $(14p + 4)$ m b. $(12p^2 + 6p)$ m² c. $(6p + 2)$ m d. $5p$ m
 e. $(22p + 4)$ m f. $(30p^2 + 10p)$ m² g. $(18p^2 + 4p)$ m² h. 80 m²

9. a.

Type of 'algebra' rectangle	Perimeter	Type of 'algebra' rectangle	Perimeter
1-long	$2x + 2$	1-tall	$2x + 2$
2-long	$4x + 2$	2-tall	$2x + 4$
3-long	$6x + 2$	3-tall	$2x + 6$
4-long	$8x + 2$	4-tall	$2x + 8$
5-long	$10x + 2$	5-tall	$2x + 10$

b. Perimeter: long $40x + 2$, tall $2x + 40$

c. $0 < x < 1$

10. a. 27.8

b. Richard is not within the healthy weight range.

c. i. 19.1

ii. 24.9

iii. 22.0

d. Judy: 9 to 20 years of age; Karen: 17.5 to 20 years of age; Manuel: 13.5 to 20 years of age

11. a. 3

b. 994009

c. i. 990025

ii. 980100

12. a. $A = \pi R^2 - x^2$

b. $A = 160.71$ cm²

c. Answers will vary.

d. $R = 5.32$ cm

e. 92.0%

13. a. i. $A_{\text{screen}} = x(x + 2) = x^2 + 2x$

ii. $A_{\text{phone 1}} = (x + 1)(x + 7) = x^2 + 8x + 7$

b. i. $l = x + 11$, $w = x$

ii. $A_{\text{phone 2}} = x(x + 11) = x^2 + 11x$

c. $x = 4$ cm, $A_{\text{phone 2}} = 60$ cm²

14. a. i. $x(x - 2)$ m

ii. $x(2x)$ m²

b. $\frac{x - 2}{2x}$

c. $\frac{50(x - 2)}{x}\%$

d. i. 25%

ii. 27.8%

iii. 29.17%

e. The third flag places the most importance on Australia.

15. a. Answers will vary.

b. i. $-3x^3 - 2x^2 + 3x + 2$

ii. $8x^3 + 16x^2 - 104x + 80$

iii. $x^3 - 49x + 120$

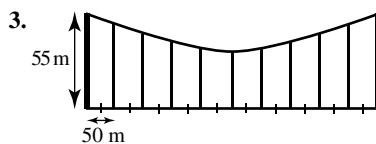
c. In a cubic expression there are four terms with descending powers of x and ascending values of its pronumeral. For example, $(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$, where the powers of x are descending and the values of a are ascending.

16. Answers will vary, but examples include construction and building, adjusting recipes to cater for larger or fewer numbers of people, or when you are shopping. Algebra also teaches you how to think logically and organise information – something that is useful in all aspects of life.

Investigation | Rich task

1. $h = 250$ m

2. 500 m

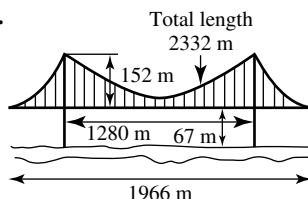


4. 18

5. 3 m; 5 m; 11 m; 22 m; 36 m

6. 305 m

7.



Exercise 14.9 Review questions

1. B

2. A

3. B

4. a. $x^2 - 16$

b. $81 - m^2$

c. $x^2 - y^2$

d. $1 - 4a^2$

5. a. $x^2 + 10x + 25$

b. $m^2 - 6m + 9$

c. $16x^2 + 8x + 1$

d. $4 - 12y + 9y^2$

6. a. $2x^2 + 10x + 13$

b. $2x - 5$

c. $16x$

7. C

8. D

9. C

10. A

11. B

12. a. $(x + 8)(x - 8)$

b. $(a + 12)(a - 12)$

c. $(7b + 1)(7b - 1)$

d. $(2f + 3g)(2f - 3g)$

e. $(n + 1 + m)(n + 1 - m)$

f. $(r - 1 + 2s)(r - 1 - 2s)$

13. a. $(c + 1)(c + 4)$

b. $(p - 2)(p + 12)$

c. $(y - 6)(y - 4)$

d. $(x + 1)(x + 2)$

e. $(m - 2)(m - 5)$

f. $(m + 22)(m + 2)$

14. a. $2(a + 6)(a + 2)$

b. $3(b - 6)(b - 2)$

c. $4(c - 6)(c + 2)$

d. $(x + 1)(2x + 1)$

e. $(x - 1)(3x + 2)$

f. $(2x + 3)(3x + 2)$

15. a. $x + 4$

b. x^2

c. 2

d. $\frac{4(x + 1)}{x - 1}$

e. $\frac{x - 7}{x - 2}$

f. $\frac{x + 6}{x - 1}$

16. a. p^2

b. $p^2 - 140p + 4500$

c. $140p - 4500$

17. a. Length = $(x - 4)$ cm, width = $(x - 6)$ cm

b. $(x^2 - 10x + 24)$ cm²

c. 624 cm²

TOPIC 15

Quadratic relationships

[Stage 5.3]

15.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

15.1.1 Why learn this?

A quadratic function can be used to describe a physical thing such as the arch of a bridge, the path of a batted ball, water rising from a fountain, or a whale's spout. Over the centuries the quadratic equation has played a major role in the whole of human civilisation as we know it. We would not be able to watch satellite television or engage in other modern activities without the developments that have come about through the application of quadratic functions.



DISCUSSION

Why is the cross-section of a satellite dish parabolic in shape?

LEARNING SEQUENCE

15.1 Overview

15.2 [Stage 5.3] Graphs of quadratic functions

15.3 [Stage 5.3] Plotting points to graph quadratic functions

15.4 [Stage 5.3] Sketching parabolas of the form $y = ax^2$

15.5 [Stage 5.3] Sketching parabolas of the form $y = ax^2 + k$

15.6 [Stage 5.3] Sketching parabolas of the form $y = (x - h)^2$

15.7 [Stage 5.3] Sketching parabolas of the form $y = (x - h)^2 + k$

15.8 [Stage 5.3] Sketching parabolas of the form $y = (x + a)(x + b)$

15.9 [Stage 5.3] Applications

15.10 Review

LEARNING OUTCOMES

A student:

- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- sketches and interprets a variety of non-linear relationships MA5.3-9NA

CONTENT DESCRIPTIONS

Students:

Describe, interpret and sketch parabolas and their transformations (ACMNA267)

Source: NSW Syllabus for the Australian curriculum

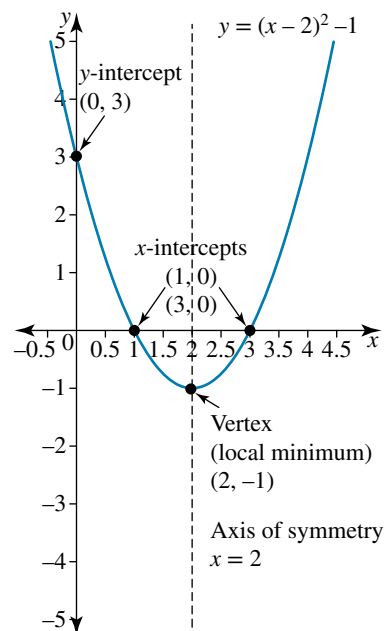
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

15.2 Graphs of quadratic functions

[Stage 5.3]

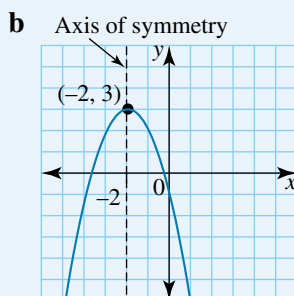
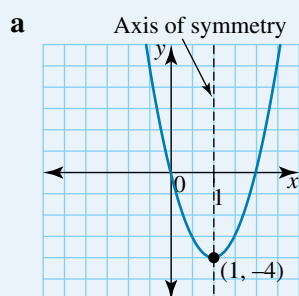
15.2.1 The parabola

- The graph shown is a typical parabola with the following features.
 - The dotted line is the **axis of symmetry**; the parabola is the same on either side of this line.
 - The **vertex** or turning point is the lowest point on the graph; the point where the graph changes direction. It is also called the local minimum.
 - The parabola is upside down, or inverted, if the vertex is the highest point on the graph.
 - The x -intercept(s) is where the graph crosses (or sometimes just touches) the x -axis. Not all parabolas have x -intercepts.
 - The y -intercept is where the graph crosses the y -axis. All parabolas have one y -intercept.



WORKED EXAMPLE 1

For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the vertex and whether it is a maximum or a minimum.



THINK

- State the equation of the vertical line that cuts the parabola in half.
 - State the vertex.
 - Determine the nature of the vertex by observing whether it is the highest or lowest point of the graph.
- b 1** State the equation of the vertical line that cuts the parabola in half.

WRITE

- a** The axis of symmetry is $x = 1$.

The vertex is at (1, -4).

Minimum vertex

- b** The axis of symmetry is $x = -2$.

2 State the vertex.

The vertex is at $(-2, 3)$.

3 Determine the nature of the vertex by observing whether it is the highest or lowest point of the graph.

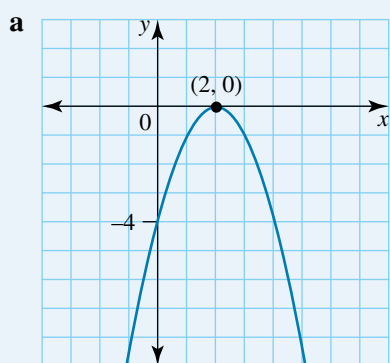
Maximum vertex

15.2.2 The x - and y -intercepts

- The **x -intercept** is where the graph crosses (or just touches) the x -axis.
- The **y -intercept** is where the graph crosses the y -axis. All parabolas have one y -intercept.
- When sketching a parabola, the x -intercepts (if any) and the y -intercept should always be marked on the graph, with their respective coordinates.

WORKED EXAMPLE 2

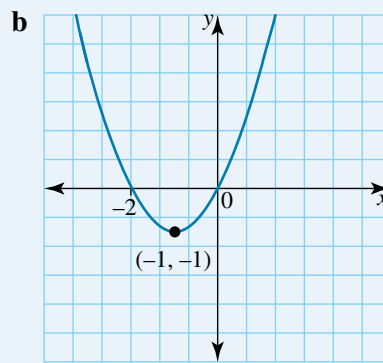
For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the vertex, whether the vertex is a maximum or a minimum, and the x - and y -intercepts.



THINK

- a 1** State the equation of the vertical line that cuts the parabola in half.
- 2** State the vertex and its nature; that is, determine whether it is the highest or lowest point of the graph.
- 3** Observe where the parabola crosses the x -axis. In this case, the graph touches the x -axis when $x = 2$, so there is only one x -intercept.
- 4** Observe where the parabola crosses the y -axis.

- b 1** State the equation of the vertical line that cuts the parabola in half.
- 2** State the vertex and its nature; that is, determine whether it is the highest or lowest point of the graph.
- 3** Observe where the parabola crosses the x -axis.
- 4** Observe where the parabola crosses the y -axis.



WRITE

a The axis of symmetry is $x = 2$

The maximum vertex is at $(2, 0)$.

The x -intercept is 2. It occurs at the point $(2, 0)$.

The y -intercept is -4 . It occurs at the point $(0, -4)$.

b The axis of symmetry is $x = -1$.

The minimum vertex is at $(-1, -1)$.

The x -intercepts are -2 and 0 . They occur at the points $(-2, 0)$ and $(0, 0)$.

The y -intercept is 0 . It occurs at the point $(0, 0)$.

Exercise 15.2 Graphs of quadratic functions

Individual pathways

PRACTISE

Questions:

1–8, 10, 16, 17

CONSOLIDATE

Questions:

1a, c, e, 2a, c, e, 3, 4, 6, 7, 9–11, 13, 15–17

MASTER

Questions:

1a, c, f, 2a, f, 3, 4, 7, 9–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

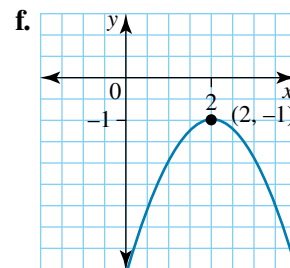
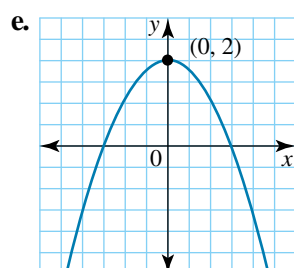
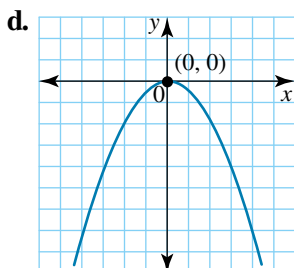
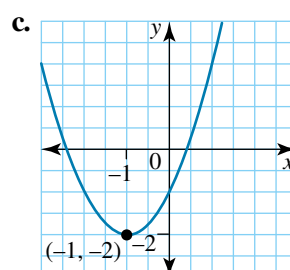
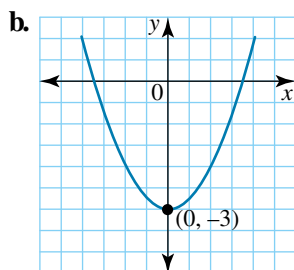
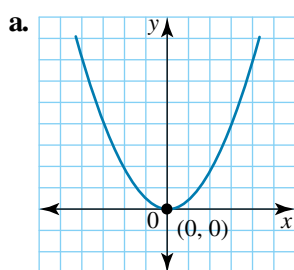
Understanding and fluency

1. **WE1** For each of the following graphs:

i. state the equation of the axis of symmetry

ii. give the coordinates of the vertex

iii. indicate whether it is a minimum or maximum vertex.

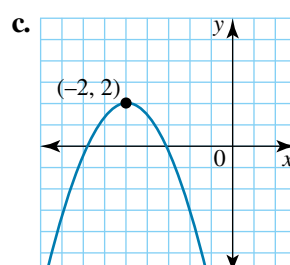
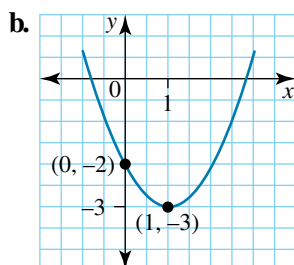
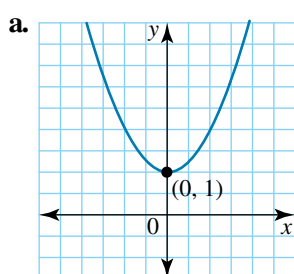


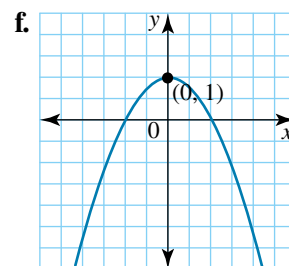
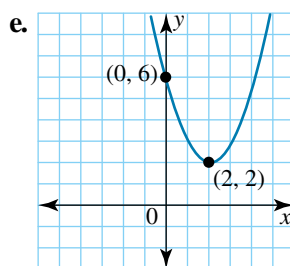
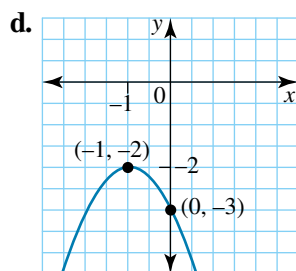
2. For each of the following graphs, state:

i. the equation of the axis of symmetry

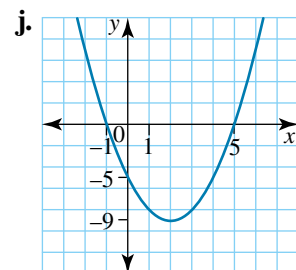
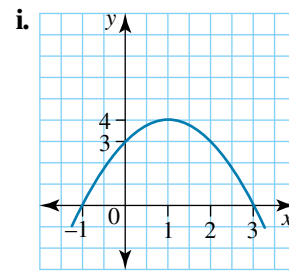
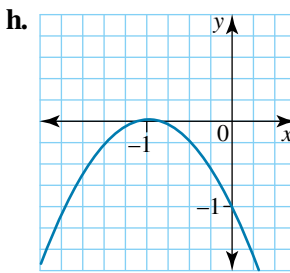
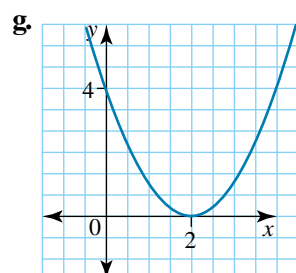
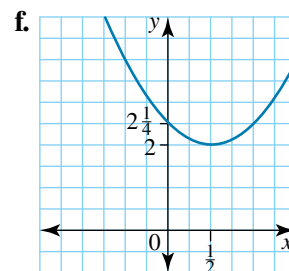
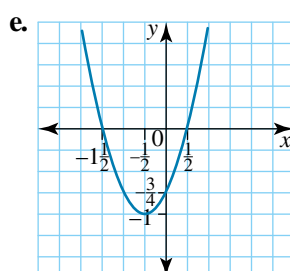
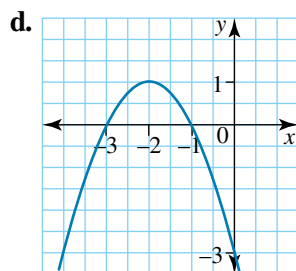
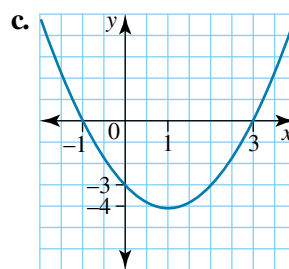
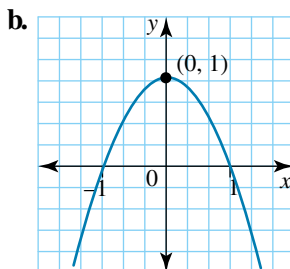
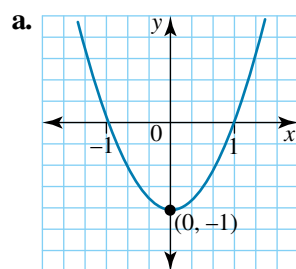
ii. the coordinates of the vertex

iii. whether the vertex is a maximum or minimum.





3. **WE2** For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the vertex and whether it is a maximum or a minimum, and the x - and y -intercepts.



4. **MC** a. The axis of symmetry for the graph shown is:

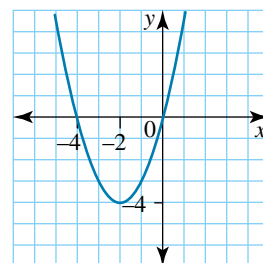
A. $x = 0$ **B.** $x = -2$ **C.** $x = -4$

D. the y -axis **E.** the x -axis

- b. The coordinates of the vertex for the graph are:

A. $(0, 0)$ **B.** $(-2, 4)$ **C.** $(-2, -4)$

D. $(2, -4)$ **E.** $(-4, -2)$



c. The y-intercept is:

- A.** 0 **B.** -2 **C.** -4 **D.** 2 **E.** 4

d. The x-intercepts are:

- A.** 0 and 4 **B.** 0 and 2 **C.** 0 and -2 **D.** 0 and -4 **E.** -2 and -4

5. Consider the table of values below.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

a. Plot these points on graph paper. What shape is the graph?

b. Locate the axis of symmetry.

c. Locate the y-intercept.

d. Locate the x-intercept(s).

6. Consider the function $y = x^2 + x$. Complete this table of values for the function.

x	-6	-4	-2	0	2	4	6	8
y				0				

7. **MC** Which of the following rules is not a parabola?

- A.** $y = -2x^2$ **B.** $y = 2x^2 - x$ **C.** $y = -2 \div x^2$ **D.** $y = -2 + x^2$ **E.** $y = x^2$

8. Consider the graph of $y = -x^2$.

a. State the vertex of this graph.

b. State whether the vertex is a maximum or a minimum.

9. Given the following information, make a sketch of the graph involved.

a. Maximum vertex = $(-2, -2)$, y-intercept = $(0, -6)$

b. Minimum vertex = $(-3, -2)$, x-intercepts $(1, 0)$ and $(-7, 0)$

Communicating, reasoning and problem solving

10. Consider the parabola given by the rule $y = x^2$ and the straight line given by $y = x$. Show that the two graphs meet at $(0, 0)$ and $(1, 1)$.

11. A window-cleaning company varies its charges as the square of the height of the building. Let the heights of the buildings be 10, 15, 20, ... 50 m. The cost of window washing for a 10-m building is \$100.

a. Determine the window washing costs for the buildings listed above. By plotting a graph, what shape is the graph?

b. What would be the cost for a 45-m building?

c. If the cost is \$729, show that the height of the building is 27 m.

12. Another window-cleaning company also varies its charges as the square of the height of the building. The cost for a 20-m building is \$600.

a. Show that the cost for a 40-m building is \$2400.

b. If the cost was \$2053.50, what was the height of the building?

13. If the axis of symmetry of a parabola is $x = -4$ and one of the x-intercepts is $(10, 0)$, show the other x-intercept is $(-18, 0)$.

14. If the x-intercepts of a parabola are $(-2, 0)$ and $(5, 0)$, show that the axis of symmetry is $x = 1.5$.

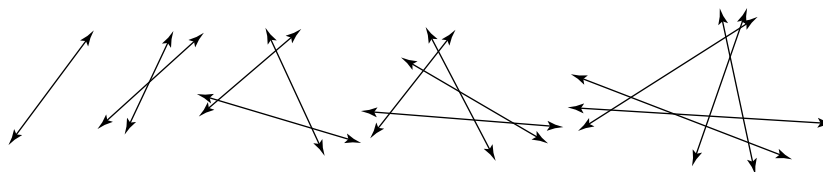
15. On a set of axes, sketch a parabola that has no x-intercepts and has an axis of symmetry $x = -2$. Can the parabola have a maximum or minimum vertex or both? Explain your reasoning.

16. Sketch a graph where the vertex and the x-intercept are the same. Suggest a possible equation.

17. Sketch a graph where the vertex and the y-intercept are the same. Suggest a possible equation.

18. a. Complete the table to show how many intersections there are for 3, 4, 5, and 6 lines.

n (number of lines)	1	2	3	4	5	6
N (number of intersections)	0	1				



- b. Find the rule connecting the number of lines, n , with the number of intersections, N , and hence show that the relationship is quadratic.
- c. If you could draw 100 lines, how many intersections would there be?
19. What are the major features of all parabolas?

15.3 Plotting points to graph quadratic functions

[Stage 5.3]

15.3.1 Graphing quadratic functions

- If there is a rule connecting y and x , a table of values can be used to determine actual coordinates.
- When drawing straight line graphs, a minimum of two points is required. For parabolas there is *no minimum* number of points, but between 6 and 12 points is a reasonable number.
- When more points are used, the parabola will appear 'smoother'. The points should be joined with a smooth curve, not ruled.
- Ensure that points plotted include (or are near) the main features of the parabola, namely the axis of symmetry, the vertex and the x - and y -intercepts.

WORKED EXAMPLE 3

Copy and complete the table of values for each of the following equations, then list the coordinates of each of the points.

a $y = x^2$

x	-3	-2	-1	0	1	2	3
$y = x^2$	9						

b $y = -x^2 + 2$

x	-3	-2	-1	0	1	2	3
$y = -x^2 + 2$							

c $y = 2x^2 - 5x + 1$

x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x + 1$							

THINK

a 1 Write the equation.

2 Substitute the x -values into the equation to obtain the corresponding y -values.

WRITE

a $y = x^2$

When $x = -3$, $y = (-3)^2 = 9$.

When $x = -2$, $y = (-2)^2 = 4$.

...

When $x = 3$, $y = (3)^2 = 9$.

- 3 Complete the table of values.

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

- 4 List the coordinates of each of the points.

$(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)$

- b** 1 Write the equation.

b $y = -x^2 + 2$

- 2 Substitute the x -values into the equation to obtain the corresponding y -values.

When $x = -3$, $y = -(-3)^2 + 2 = -7$.

When $x = -2$, $y = -(-2)^2 + 2 = -2$.

...

When $x = 3$, $y = -(3)^2 + 2 = -7$.

- 3 Complete the table of values.

x	-3	-2	-1	0	1	2	3
$y = -x^2 + 2$	-7	-2	1	2	1	-2	-7

- 4 List the coordinates of each of the points.

$(-3, -7), (-2, -2), (-1, 1), (0, 2), (1, 1), (2, -2), (3, -7)$

- c** 1 Write the equation.

c $y = 2x^2 - 5x + 1$

- 2 Substitute the x -values into the equation to obtain the corresponding y -values.

When $x = -2$, $y = 2(-2)^2 - 5(-2) + 1 = 19$.

When $x = -1$, $y = 2(-1)^2 - 5(-1) + 1 = 8$.

...

When $x = 4$, $y = 2(4)^2 - 5(4) + 1 = 13$.

- 3 Complete the table of values.

x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x + 1$	19	8	1	-2	-1	4	13

- 4 List the coordinates of each of the points.

$(-2, 19), (-1, 8), (0, 1), (1, -2), (2, -1), (3, 4), (4, 13)$

- Occasionally a list of x -values will be provided and the corresponding y -values can be calculated. In the following example, the set of x -values is specified as $-4 \leq x \leq 2$.

WORKED EXAMPLE 4

Plot the graph of $y = x^2 + 2x - 3$, $-4 \leq x \leq 2$ and hence state:

- a** the equation of the axis of symmetry
b the coordinates of the vertex and whether it is a maximum or a minimum
c the x - and y -intercepts.

THINK

- 1 Write the equation.

WRITE/DRAW

$y = x^2 + 2x - 3$

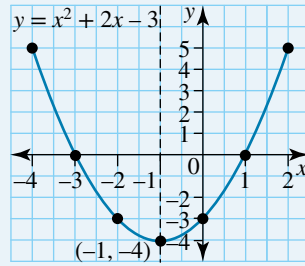
- 2 Complete a table of values by substituting into the equation each integer value of x from -4 to 2 . For example, when $x = -4$, $y = (-4)^2 + 2 \times (-4) - 3 = 5$.

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0	5

3 List the coordinates of the points.

$(-4, 5), (-3, 0), (-2, -3), (-1, -4),$
 $(0, -3), (1, 0), (2, 5)$

4 Draw and label a set of axes, plot the points listed and join the points to form a smooth curve.



a Find the equation of the line that divides the parabola exactly into two halves.

a The axis of symmetry is $x = -1$.

b Find the point where the graph turns or changes direction, and decide whether it is the highest or lowest point of the graph. State the coordinates of this point.

b The minimum vertex is at $(-1, -4)$.

c 1 State the x -coordinates of the points where the graph crosses the x -axis.

c The x -intercepts are at -3 and 1 . They occur at the points $(-3, 0)$ and $(1, 0)$.

2 State the y -coordinate of the point where the graph crosses the y -axis.

The y -intercept is at -3 . It occurs at the point $(0, -3)$.

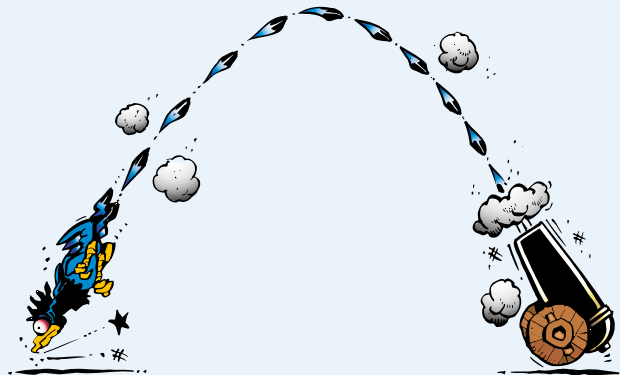
- Occasionally a rule connecting x and y will be provided. From this rule, pairs of x - and y -values can be calculated.

In the following example, the rule is given as $h = -5x^2 + 25x$.

- Graphs can be drawn using a graphing program on your computer or by hand.

WORKED EXAMPLE 5

Rudie, the cannonball chicken, was fired out of a cannon. His path could be traced by the equation $h = -5x^2 + 25x$, where h is Rudie's height, in metres, above the ground and x is the horizontal distance, in metres, from the cannon. Plot the graph for $0 \leq x \leq 5$ and use it to find the maximum height of Rudie's path.



THINK

- 1 Write the equation.
- 2 Complete a table of values by substituting into the equation each integer value of x from 0 to 5. For example, when $x = 0$, $h = -5 \times 0^2 + 25 \times 0 = 0$.
- 3 List the coordinates of the points.
- 4 As a parabola is symmetrical, the greatest value of h must be greater than 30 and occurs when x lies between 2 and 3, so find the value of h when $x = 2.5$.
- 5 Draw and label a set of axes, plot the points from the table and join the points to form a smooth curve.
- 6 The maximum height is the value of h at the highest point of the graph.
- 7 Answer the question in a sentence.

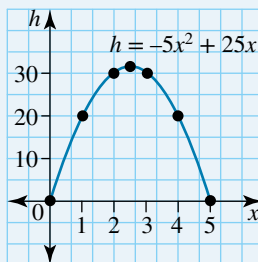
WRITE

$$h = -5x^2 + 25x$$

x	0	1	2	3	4	5
h	0	20	30	30	20	0

$(0, 0), (1, 20), (2, 30), (3, 30), (4, 20), (5, 0)$

$$\text{When } x = 2.5, \quad h = -5 \times (2.5)^2 + 25 \times 2.5 = 31.25$$



$$h = 31.25$$

The maximum height of Rudie's path is 31.25 metres.



RESOURCES — ONLINE ONLY

Digital doc: SkillSHEET Substitution into quadratic equations (doc-10990)

Digital doc: SkillSHEET Plotting coordinate points (doc-10991)

Exercise 15.3 Plotting points to graph quadratic functions

Individual pathways

PRACTISE

Questions:

1–3, 4a, c, e, g, 5–7, 10, 14

CONSOLIDATE

Questions:

1–3, 4b, d, f, h, 5, 6, 8, 10, 12, 14, 15

MASTER

Questions:

1–3, 4a, f, g, 5, 6, 9–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE3** Copy and complete the table of values for each of the following equations, then list the coordinates of each of the points.

a. $y = 2x^2$

x	-3	-2	-1	0	1	2	3
$y = 2x^2$							

b. $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 4$							

c. $y = -x^2 + 4x + 5$

x	-2	-1	0	1	2	3	4	5	6
$y = -x^2 + 4x + 5$									

2. Using the equations in question 1:
- plot the points and join with a smooth curve
 - identify the axis of symmetry and state its equation.
3. Complete the following table of values, plot the points, and then join the points with a smooth curve.

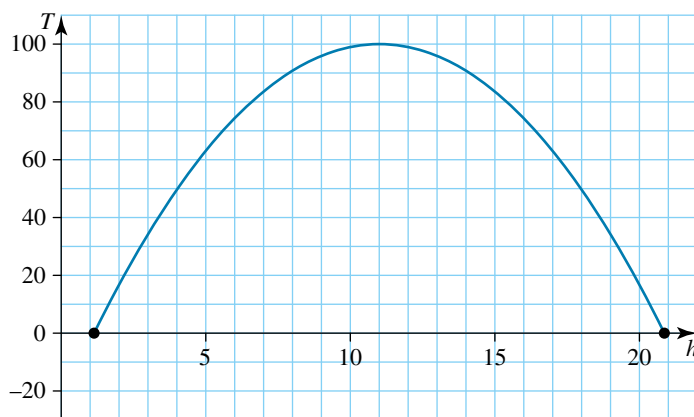
x	-2	-1	0	1	2	3	4	5
$y = -x^2 + x$								

4. **WE4** Plot the graph of each of the following and hence state:
- the equation of the axis of symmetry
 - the coordinates of the vertex and whether it is a maximum or a minimum
 - the x - and y -intercepts.

Remember: $-7 \leq x \leq 0$ means the graph is drawn from $x = -7$ to $x = 0$.

- $y = x^2 + 8x + 15$, $-7 \leq x \leq 0$
 - $y = x^2 - 1$, $-3 \leq x \leq 3$
 - $y = x^2 - 4x$, $-1 \leq x \leq 5$
 - $y = x^2 - 2x + 3$, $-2 \leq x \leq 4$
 - $y = x^2 + 12x + 35$, $-9 \leq x \leq 0$
 - $y = -x^2 + 4x + 5$, $-2 \leq x \leq 6$
 - $y = 2x^2 - 16x - 18$, $-1 \leq x \leq 9$
 - $y = -x^2 - 4x - 3$, $-4 \leq x \leq 2$
5. Consider the following equations for $-3 \leq x \leq 3$.
- $y = x^2 + 2$
 - $y = x^2 + 3$
- Make a table of values for both equations and plot the points on the same set of axes.
 - State the equation of the axis of symmetry for each equation.
 - State the x -intercepts for each equation.
6. **WE5** A missile was fired from a boat during a test. The missile's path could be traced by the equation $h = -\frac{1}{2}x^2 + x$, where h was the missile's height above the ground, in kilometres, and x was the horizontal distance from the boat, in kilometres.
- Plot the graph for $0 \leq x \leq 2$ and use it to find the maximum height of the missile's path, in metres.
7. The speed versus distance graph of a car braking efficiently has the equation $s = \frac{v^2}{260}$, where v is the speed, in km/h, and s is the stopping distance, in metres.
- Use graph paper to plot this graph for $0 \leq v \leq 120$ (choose an appropriate scale). From the graph, find the stopping distance, to the nearest metre, for a car at:
 - 60 km/h
 - 100 km/h
 - 120 km/h.
 - What is the maximum speed a car can travel if it must stop within 25 m? Round answers to the nearest whole number. Compare your answers from the graph and formula.

8. SpaceCorp sent a lander to Mars to measure the temperature change over a period of time. The results were plotted on a set of axes as shown below. From the graph it can be seen that the temperature change follows the quadratic rule $T = -h^2 + 22h - 21$, where T is the temperature in degrees Celsius and h is the time elapsed in hours.



- a. What was the initial temperature on Mars?
 - b. When was the temperature measured as 0°C ?
 - c. When was the highest temperature recorded?
 - d. What was the highest temperature recorded?
9. A ball is thrown from the top of a building. Its height above the ground, h , is given by $h = -4.9t^2 + 5t + 12.5$, where h is in metres and t is the time, in seconds, since the ball was thrown.
- a. What is the height of the building? (*Hint: Where was the ball at $t = 0$?*)
 - b. Sketch a graph of h versus t .
 - c. How long does it take for the ball to reach the ground?

Communicating, reasoning and problem solving

10. The Grand Old Duke of York marched his men up a hill that followed the path of the equation $y = -x^2 + 16x$, where y is the vertical distance travelled and x is the horizontal distance. Both measurements are in metres. Plot the graph for $0 \leq x \leq 16$, and use it to find out whether the Duke and his men were halfway across when they were halfway up. If not, explain how far across they were on the upward journey. Round answers to 1 decimal place.
11. When a golfer hits the ball from the tee with a 7 iron, the ball follows the path of the parabola $h_1 = -0.01x^2 + 0.95x$. When hit with a 9 iron, the ball follows the path of the parabola $h_2 = -0.02x^2 + 1.6x$. In both cases, h is the height above ground, in metres, and x is the horizontal distance from the golfer, in metres. The green is 80 metres away from the tee. Plot both graphs to determine which club the golfer should use in order to land the ball on the front of the green.
12. On a basketball court, Sam threw a basketball such that it followed the path defined by the equation $y = -0.1(x - 13)(x + 1)$, where y is the height of the ball in metres and x is the horizontal distance from Sam in metres.
- a. How far off the ground was the ball when Sam threw it?
 - b. How far from Sam did the ball bounce?
 - c. If Saney, another player, is 10 metres away from Sam, explain how high he will need to jump in order to catch the ball directly above his head. (Saney is 1.85 m tall and his arm length is 70 cm.)

13. In order for Ruby to participate in a trip to the snow this year, she will need to fund-raise. By selling mini Easter eggs at school for different prices each day, Ruby found that the relationship between the cost (c) of a sachet of mini Easter eggs and the total profit (P) was modelled by the equation $P = (c + 2)(6 - c)$. One day she did not sell any Easter eggs but a teacher kindly gave her \$12 towards her trip.



- Draw a graph of $P = (c + 2)(6 - c)$.
 - Explain why the graph should not appear to the left of the P axis.
 - What will the mini Easter eggs cost if the maximum profit is to be made? How much will this profit be?
 - Up to what amount could Ruby charge before she makes a loss?
14. a. The axis of symmetry of a parabola is $x = -4$. If one x -intercept is -10 , what is the other x -intercept?
b. Suggest a possible equation for the parabola.
15. a. Compare the vertices of the parabolas $y = 2x^2 - 6x$ and $y = 2x^2 - 6x - 3$.
b. Find an equation for a quadratic curve that has the same axis of symmetry as $y = 2x^2 - 6x$ and a vertex $(1.5, -2.5)$.
16. Water comes out of a garden hose in the shape of a parabola and can be modelled by the equation $y = -0.016x^2 + 0.5x + 4.5$, where x is the horizontal distance from the hose's nozzle and y is the height of the water (both in metres). Investigate whether the water can be jetted over a 2-metre fence that is located 35.5 metres from the nozzle.
17. The data below models the equation of $y = ax^2 + bx + c$. Use the table to show that $c = 0$ and find the values of a and b .

x	0	1	2	3	4
y	0	2	6	12	20

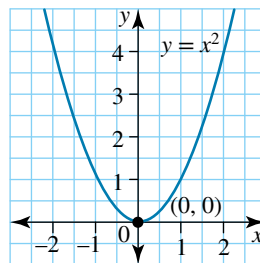
18. If given a choice, what is the best way to choose the points to be plotted on a graph?

15.4 Sketching parabolas of the form $y = ax^2$

[Stage 5.3]

15.4.1 The graph of the quadratic function $y = x^2$

- The graph shows the simplest parabola, $y = x^2$.



- Both the x - and y -axes are clearly indicated, along with their scales.
- The vertex $(0, 0)$ is indicated.
- The x - and y -intercepts are indicated. For this graph they are all $(0, 0)$.
- This is an example of a parabola that just touches (does not cross) the x -axis at $(0, 0)$.

15.4.2 Parabolas of the form $y = ax^2$, where $a > 0$

- A coefficient in front of the x^2 term affects the **dilation** of the graph, making it wider or narrower than the graph of $y = x^2$.
- If $a > 1$, then the graph becomes narrower, whereas if $0 < a < 1$, the graph becomes wider.
- The following features of the parabola remain *unchanged*, for $a > 0$, regardless of the value of a :
 - the axis of symmetry is $x = 0$
 - the vertex is $(0, 0)$
 - the x -intercept is $(0, 0)$
 - the y -intercept is $(0, 0)$
 - the shape of the parabola is always upright or a U shape.

WORKED EXAMPLE 6

On the same set of axes sketch the graph of $y = x^2$ and $y = 3x^2$, marking the coordinates of the vertex and the intercepts. State which graph is narrower.

THINK

- 1 Write the equation of the first graph.
- 2 State its axis of symmetry.
- 3 State the coordinates of the vertex.
- 4 State the intercepts.
- 5 Find the coordinates of one other point.
- 6 Write the equation of the second graph.
- 7 State its axis of symmetry.
- 8 State the coordinates of the vertex.
- 9 State the intercepts.
- 10 Find the coordinates of one other point.
- 11 Sketch the graphs, labelling the vertex.

WRITE/DRAW

$$y = x^2$$

The axis of symmetry is $x = 0$.

The vertex is $(0, 0)$.

The x -intercept is 0 and the y -intercept is 0.

When $x = 1$, $y = 1$. $(1, 1)$

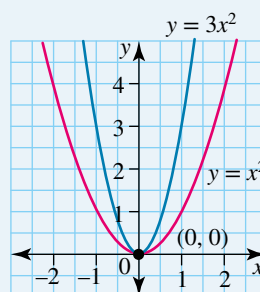
$$y = 3x^2$$

The axis of symmetry is $x = 0$.

The vertex is $(0, 0)$.

The x -intercept is 0 and the y -intercept is 0.

When $x = 1$, $y = 3$. $(1, 3)$



- 12 State which graph is narrower.

The graph of $y = 3x^2$ is narrower.

15.4.3 Parabolas of the form $y = ax^2$, where $a < 0$

- When $a < 0$, the graph is inverted; that is, it is $\cap$ shaped.
- A coefficient in front of the x^2 term affects the dilation of the graph, making it wider or narrower than the graph of $y = x^2$.

- If $-1 < a < 0$, the graph is wider than $y = -x^2$.
- If $a < -1$, the graph is narrower than $y = -x^2$.
- The following features of the parabola remain unchanged for $a < 0$, regardless of the value of a :
 - the axis of symmetry is $x = 0$
 - the vertex is $(0, 0)$
 - the x -intercept is $(0, 0)$
 - the y -intercept is $(0, 0)$
 - the shape of the parabola is always inverted or an upside-down U shape ($\cap$).

WORKED EXAMPLE 7

On the same set of axes sketch the graphs of $y = -x^2$ and $y = -2x^2$, marking the coordinates of the vertex and the intercept. State which graph is narrower.

THINK

- 1 Write the equation of the first graph.
- 2 State its axis of symmetry.
- 3 State the coordinates of the vertex.
- 4 State the intercepts.
- 5 Calculate the coordinates of one other point.
- 6 Write the equation of the second graph.
- 7 State its axis of symmetry.
- 8 State the coordinates of the vertex.
- 9 State the intercepts.
- 10 Calculate the coordinates of one other point.
- 11 Sketch the two graphs on a single set of axes, labelling the vertex (as well as intercepts and maximum).

WRITE

$$y = -x^2$$

The axis of symmetry is $x = 0$.

The vertex is $(0, 0)$.

The x -intercept is $(0, 0)$ and the y -intercept is also $(0, 0)$.

When $x = 1$, $y = -1$, $(1, -1)$.

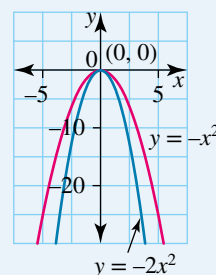
$$y = -2x^2$$

The axis of symmetry is $x = 0$.

The vertex is $(0, 0)$.

The x -intercept is $(0, 0)$ and the y -intercept is also $(0, 0)$.

When $x = 1$, $y = -2$, $(1, -2)$.



The graph of $y = -2x^2$ is narrower.

learn on RESOURCES — ONLINE ONLY

Interactivity: Dilation of parabolas (int-6096)

Exercise 15.4 Sketching parabolas of the form $y = ax^2$

Individual pathways

PRACTISE

Questions:

1–7, 8, 10, 13, 16

CONSOLIDATE

Questions:

1–7, 9, 10, 12, 13, 16

MASTER

Questions:

1–7, 10, 12–18

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE6** On the same set of axes, sketch the graph of $y = x^2$ and $y = 4x^2$, marking the coordinates of the vertex and the intercepts. State which graph is narrower.
- On the same set of axes, sketch the graph of $y = x^2$ and $y = \frac{1}{2}x^2$, marking the coordinates of the vertex and the intercepts. State which graph is narrower.
- Sketch the graph of the following table. State the equation of the graph.

x	-4	-3	-2	-1	0	1	2	3	4
y	4	2.25	1	0.25	0	0.25	1	2.25	4

- WE7** Using the same set of axes, sketch the graphs of $y = -x^2$ and $y = -0.5x^2$, marking the coordinates of the vertex and the intercepts. State which graph is narrower.
- Using the same set of axes from question 4, sketch the graph of $y = -5x^2$, marking the coordinates of the vertex and the intercepts. State which graph is the narrowest.
- MC** a. The graph of $y = -3x^2$ is:

A. wider than $y = x^2$

C. the same width as $y = x^2$

B. narrower than $y = x^2$

D. a reflection of $y = x^2$ in the x -axis

b. The graph of $y = \frac{1}{3}x^2$ is:

A. wider than $y = x^2$

C. the same width as $y = x^2$

B. narrower than $y = x^2$

D. a reflection of $y = x^2$ in the x -axis

c. The graph of $y = \frac{1}{2}x^2$ is:

A. wider than $y = \frac{1}{4}x^2$

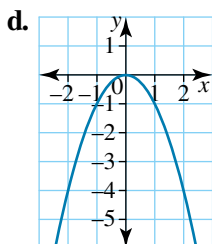
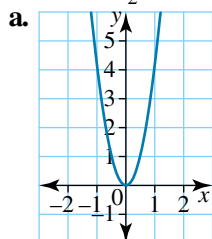
C. the same width as $y = \frac{1}{4}x^2$

B. narrower than $y = \frac{1}{4}x^2$

D. a reflection of $y = \frac{1}{4}x^2$ in the x -axis
- Match each of the following parabolas with the appropriate equation from the list below.

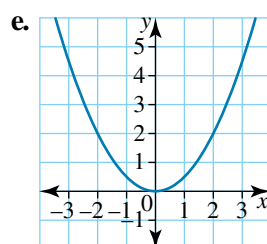
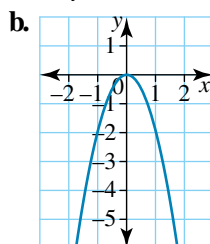
i. $y = 3x^2$

iv. $y = \frac{1}{2}x^2$



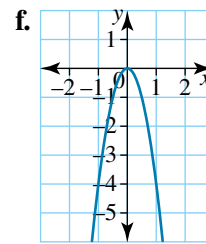
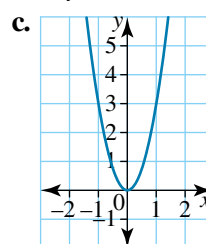
ii. $y = -x^2$

v. $y = -4x^2$



iii. $y = 4x^2$

vi. $y = -2x^2$



8. Write an equation for a parabola that has a minimum vertex and is narrower than $y = x^2$.
9. Write an equation for a parabola that has a maximum vertex and is wider than $y = x^2$.
10. Find the equation of a quadratic relation if it has an equation of the form $y = ax^2$ and passes through:
 - a. (1, 3) b. (-1, -1).
11. Consider the equation $y = -3.5x^2$. Calculate the values of y when x is:
 - a. 10 b. -10 c. -3 d. 1.5 e. -2.2.
12. The amount of power (watts) in an electric circuit varies as the square of the current (amperes). If the power is 100 watts when the current is 2 amperes, calculate:
 - a. the power when the current is 4 amperes
 - b. the power when the current is 5 amperes.

Communicating, reasoning and problem solving

13. a. Sketch the following graphs on the same axes: $y = x^2$, $y = 2x^2$ and $y = -3x^2$. Shade the area between the two graphs above the x -axis and the area inside the graph below the x -axis. Describe the shape that has been shaded.
 b. Sketch the following graphs on the same axes: $y = x^2$, $y = \frac{1}{3}x^2$ and $y = -4x^2$. Shade the area inside the graphs of $y = x^2$ and $y = -4x^2$. Also shade the area between the graph of $y = \frac{1}{3}x^2$ and the x -axis. Describe the shape that you have drawn.
14. This figure shows the parabolic shape of a skate ramp. The rule of the form $y = ax^2$ describes the shape of the ramp. If the top of the ramp has coordinates (3, 6), find a possible equation that describes the shape. Justify your answer.
15. The total sales of a fast food franchise vary as the square of the number of franchises in a given city. Let S be the total sales (in millions of dollars per month) and f be the number of franchises. If sales = \$25 million when $f = 4$, then:
 - a. show that the equation relating S and f is $S = 1562500f^2$
 - b. determine the number of franchises needed to (at least) double the sales from \$25 000 000.
16. Xanthe and Carly were comparing parabolas on their CAS calculators. Xanthe graphed $y = 0.001x^2$ with window settings $-1000 \leq x \leq 1000$ and $0 \leq y \leq 1000$. Carly graphed $y = x^2$ with window settings $-k \leq x \leq k$ and $0 \leq y \leq k$. Except for the scale markings, the graphs looked exactly the same. What is the value of k ?
17. The parabola $y = x^2$ is rotated 90° clockwise about the origin. Find the equation.
18. List the features of a parabola that remain unchanged when a changes from positive to negative in $y = ax^2$.



15.5 Sketching parabolas of the form $y = ax^2 + k$

[Stage 5.3]

15.5.1 Sketching $y = ax^2 + k$

- Adding a constant, k , to the rule $y = ax^2$ **translates** the graph vertically.
- If $k > 0$, the value of y increases, therefore the graph translates vertically upwards.
- If $k < 0$, the value of y decreases, therefore the graph translates vertically downwards.

WORKED EXAMPLE 8

For each part of the question, sketch the graph of $y = x^2$, then, on the same axes, sketch the given graph, clearly labelling the vertex.

a $y = x^2 + 2$

b $y = -x^2 - 3$

THINK

a 1 Sketch the graph of $y = x^2$ by drawing a set of labelled axes, marking the vertex $(0, 0)$ and noting that it is symmetrical about the y -axis.

2 Find the vertex of $y = x^2 + 2$ by adding 2 to the y -coordinate of the vertex of $y = x^2$.

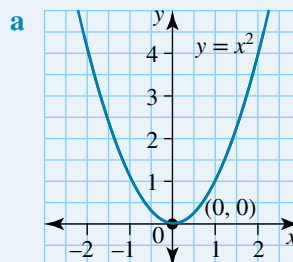
3 Using the same axes as for the graph of $y = x^2$, sketch the graph of $y = x^2 + 2$, marking the vertex and making sure that it is the same width as the graph of $y = x^2$. (The coefficient of x^2 is the same for both graphs.)

b 1 Sketch the graph of $y = x^2$ by drawing a set of labelled axes, marking the vertex $(0, 0)$ and noting that it is symmetrical about the y -axis.

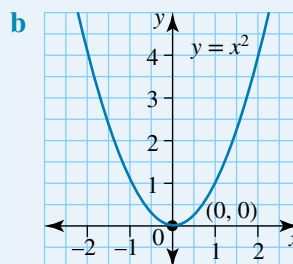
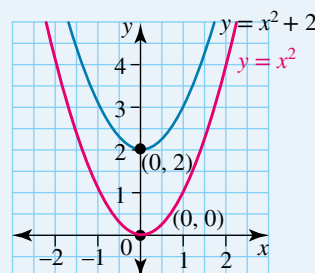
2 Find the vertex of $y = -x^2 - 3$ by subtracting 3 from the y -coordinate of the vertex of $y = x^2$.

3 Using the same axes as for the graph of $y = x^2$, sketch the graph of $y = -x^2 - 3$, marking the vertex, inverting the graph and making sure that the graph is the same width as the graph of $y = x^2$.

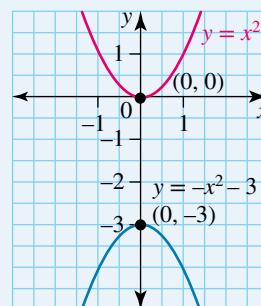
WRITE/DRAW



The vertex of $y = x^2 + 2$ is $(0, 2)$.



The vertex of $y = -x^2 - 3$ is $(0, -3)$.



- If the graph is of the form $y = -x^2 + k$ and $k > 0$, the graph translates vertically upwards. If $k < 0$, the graph translates vertically downwards.

WORKED EXAMPLE 9

Sketch the graph of $y = -x^2 + 4$, drawing clearly labelled axes and marking the axis of symmetry, vertex and y -intercept. State whether the vertex is a maximum or minimum.

THINK

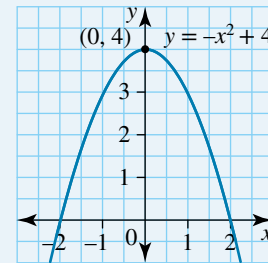
- 1 The equation of the axis of symmetry is the same as for $y = x^2$.
- 2 The vertex has been moved up k units and is $(0, k)$.
- 3 The coefficient of x^2 is negative so the graph is inverted.
- 4 Draw clearly labelled axes, mark the vertex and draw the graph.

WRITE/DRAW

The axis of symmetry is $x = 0$.

The vertex is $(0, 4)$.

Maximum vertex



learnON RESOURCES — ONLINE ONLY

Interactivity: Vertical translation of parabolas (int-6097)

Interactivity: Vertical translation: $y = x^2 + c$ (int-1192)

Digital doc: WorkSHEET Quadratic functions 1 (doc-11009)

Exercise 15.5 Sketching parabolas of the form $y = ax^2 + k$

Individual pathways

PRACTISE

Questions:
1–8, 10, 13, 15

CONSOLIDATE

Questions:
1–10, 12, 13, 15–17

MASTER

Questions:
1–8, 10–20

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE8** For each part of the question, sketch the graph of $y = x^2$, then, on the same axes, sketch the given graph, clearly labelling the vertex.
a. $y = x^2 + 1$ b. $y = x^2 + 4$ c. $y = x^2 - 1$ d. $y = x^2 - 4$ e. $y = -x^2 + 1$ f. $y = -x^2 - 1$
2. How does a positive number at the end of the equation affect the graph?
3. How does a negative number at the end of the equation affect the graph?
4. **WE9** Sketch each of the following graphs on clearly labelled axes, marking the axis of symmetry, vertex and y -intercept of each one. State whether the vertex is a maximum or a minimum.
a. $y = x^2 + 2$ b. $y = x^2 - 5$ c. $y = -x^2 + 3$ d. $y = -x^2 + 4$ e. $y = -x^2 - 3$ f. $y = x^2 - \frac{1}{2}$
5. Sketch the following graphs, indicating the vertex and estimating the x -intercepts for each graph.
a. $y = 4 - x^2$ b. $y = -4 - x^2$ c. $y = 1 - x^2$

6. a. Does the vertex change if there is a negative number in front of the x^2 term in the equation $y = x^2$?
 b. How does a negative coefficient of x^2 affect the graph?
 c. What is the axis of symmetry for all the graphs in this exercise?
7. **MC** a. The vertex for the graph of the equation $y = -x^2 + 8$ is:
 A. (0, 0) B. (-1, 8) C. (0, 8) D. (0, -8) E. (-8, 0)
 b. The vertex of the graph of the equation $y = x^2 - 16$ is:
 A. (1, -16) B. (-16, 1) C. (0, 0) D. (0, -16) E. (-16, 0)
 c. The graph of $y = x^2 - 7$ moves the graph of $y = x^2$ in the following way:
 A. up 1 B. down 1 C. up 7 D. down 7 E. up 49
 d. The y-intercept of the graph of $y = -x^2 - 6$ is:
 A. 1 B. -1 C. 6 D. -6 E. 0

8. Match each of the following parabolas with the appropriate equation from the list below.

i. $y = x^2 - 3$

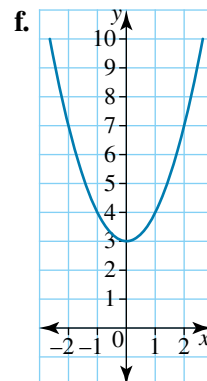
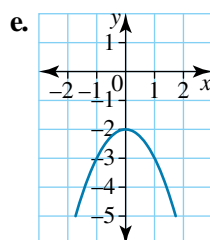
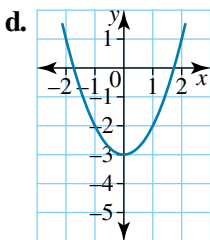
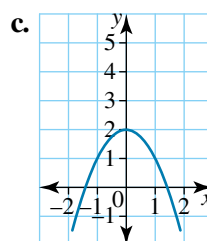
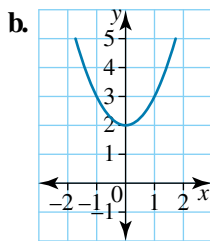
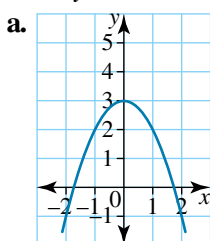
ii. $y = x^2 + 3$

iii. $y = 3 - x^2$

iv. $y = x^2 + 2$

v. $y = -x^2 + 2$

vi. $y = -x^2 - 2$



9. The vertical cross-section through the top of the mountain called the Devil's Tower can be approximated by the graph $y = -x^2 + 5$. Sketch the graph. If the x -axis represents sea level, and both x and y are in kilometres, find the maximum height of the mountain.
10. The cross-section of a large bowl can be given by the rule $y = 3x^2 - 243$, where both x (measured across the bowl) and y (the depth of the bowl) are measured in centimetres.
- By factorising the rule, find the points where $y = 0$. What are these points called?
 - If the bowl's rim occurs at the point where $y = 0$, find the greatest depth of the bowl.
 - What is the width of the bowl at its rim?
11. The photo shows an imaginary line drawn across the surface of a lake. A vertical cross-section of the lake is taken at the line, such that the depth of the lake can be approximated by the graph $y = x^2 - 12$, where x and y are in metres.
- What would be a suitable 'domain' (set of possible x -values) for this graph?



- b. Sketch the graph over this domain.
- c. What is the greatest depth of the lake?
- d. What is the width of the lake along the white line shown in the photo?

Communicating, reasoning and problem solving

12. Show that the equation of the parabola that is of the form $y = x^2 + k$ and passes through:
 - a. $(2, 1)$ is $y = x^2 - 3$
 - b. $(-3, -1)$ is $y = x^2 - 10$.
13. A ball is dropped from a height and its descent follows the downward path of the parabola given by $h = at^2 + k$, where h is the height of the ball in metres and t is the time of flight in seconds. If the ball was dropped from a height of 12 m and took 2 seconds to reach the ground, show that the rule for the path of the ball is $h = -3t^2 + 12$.
14. Sketch $y = x^2 - 2$ and $y = -x^2 + 2$ on the same set of axes. Use algebra to explain where they intersect.
15. The path of a ball rolling off the end of a table follows a parabolic curve and can be modelled by the equation $y = ax^2 + k$. A student rolls a ball off a tabletop that is 128 cm above the floor, and the ball lands 80 cm horizontally away from the desk. If the student sets a cup 78 cm above the floor to catch the ball during its fall, where should the cup be placed?
16. On a set of axes, plot the following points.

x	0	1	-3	-2
y	2	4	20	10

On the same set of axes, reflect the points about the y -axis and plot the points. Join all the points together and find the equation.

17. Find the intersection points between the graphs with equations $y = -2.5x^2 + 277.5$ and $y = -16x^2 + 555$. Describe the points.
18. For the equation $y = x^2 - 5$, find the exact value of the y -coordinate when the x -coordinate is $2 + \sqrt{5}$.
19. Describe the effect of changing a and k , separately, in the rule $y = ax^2 + k$.
20. A rocket is released from a rocket launcher on the ground and travels in an upward parabolic path to a maximum height of 125 metres before falling and landing on the ground 10 metres from where it was launched. Determine the equation of the path taken by the rocket.



15.6 Sketching parabolas of the form $y = (x - h)^2$

[Stage 5.3]

15.6.1 Sketching $y = (x - h)^2$

- Subtracting a constant value h from $y = x^2$ translates the parabola h units to the right; otherwise, the parabola's shape is unchanged.
 - The equation is $y = (x - h)^2$, where h is a positive quantity.
 - The x -intercept occurs when $x = h$.
- Adding a constant value h to x in $y = x^2$ translates the parabola h units to the left; otherwise, the parabola's shape is unchanged.
 - The equation is $y = (x + h)^2$, where h is a positive quantity.
 - The x -intercept occurs when $x = -h$.
- The y -intercept occurs when $x = 0$, and is always at $y = h^2$.

WORKED EXAMPLE 10

On clearly labelled axes, sketch the graph of $y = (x - 2)^2$, marking the vertex and y-intercept. State whether the vertex is a maximum or a minimum.

THINK

- 1 Write the equation.
- 2 Find the axis of symmetry ($x = h$ where h is 2).
- 3 Find the vertex, which has been moved to the right.
- 4 The sign in front of the bracket is positive so the parabola is upright.
- 5 Substitute 0 for x in the equation to find the y-intercept.
- 6 Draw a clearly labelled set of axes, mark the vertex and y-intercept and draw the graph of $y = (x-2)^2$. Note that the sign in the brackets is negative, so the graph moves 2 units to the right.

WRITE/DRAW

$$y = (x - 2)^2$$

The axis of symmetry is $x = 2$.

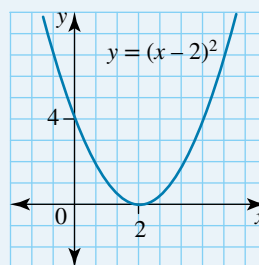
The vertex is $(2, 0)$.

Minimum vertex

y-intercept: when $x = 0$,

$$\begin{aligned} y &= (0 - 2)^2 \\ &= 4 \end{aligned}$$

The y-intercept is 4.



- Observe that the vertex also shifts by the same amount and direction as the x -intercept.

WORKED EXAMPLE 11

Sketch the graph of $y = -(x + 1)^2$, labelling the vertex, stating whether it is a maximum or minimum, finding the y-intercept and the equation of the axis of symmetry.

THINK

- 1 Write the equation.
- 2 Find the vertex, which has been moved to the left.
- 3 The sign in front of the bracket is negative, so the parabola is inverted.
- 4 The y-intercept is where $x = 0$, so substitute 0 for x in the equation.
- 5 Find the axis of symmetry ($x = h$, where h is -1).

WRITE/DRAW

$$y = -(x + 1)^2$$

The vertex is $(-1, 0)$.

Maximum vertex

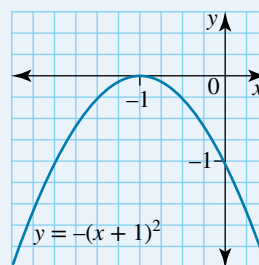
y-intercept: when $x = 0$,

$$\begin{aligned} y &= -(0 + 1)^2 \\ &= -1 \end{aligned}$$

The y-intercept is -1 .

The axis of symmetry is $x = -1$.

- 6 Draw a clearly labelled set of axes, mark the vertex and y-intercept, and draw the graph.



learnon RESOURCES — ONLINE ONLY

Interactivity: Horizontal translation of parabolas (int-6098)

Interactivity: Horizontal translation: $y = (x - h)^2$ (int-1193)

Exercise 15.6 Sketching parabolas of the form $y = (x - h)^2$

Individual pathways

PRACTISE

Questions:
1–8, 10, 12, 15

CONSOLIDATE

Questions:
1–10, 12, 13, 15, 16

MASTER

Questions:
1–8, 9a, 10a, c, 11–19

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE10** On clearly labelled axes, sketch the graph of each of the following, marking the vertex and y-intercept. State whether the vertex is a maximum or a minimum.

a. $y = (x - 1)^2$	b. $y = (x - 3)^2$	c. $y = (x + 4)^2$
d. $y = (x + 2)^2$	e. $y = (x - 5)^2$	f. $y = (x + 6)^2$
- State the axis of symmetry for each of the graphs in question 1.
- How does a positive number for h in $y = (x - h)^2$ affect the graph of $y = x^2$?
- How does a negative number for h in $y = (x - h)^2$ affect the graph of $y = x^2$?
- WE11** On clearly labelled axes, sketch the graph of each of the following, marking the vertex and y-intercept. State whether the vertex is a maximum or a minimum.

a. $y = -(x - 1)^2$	b. $y = -(x - 3)^2$
c. $y = -(x + 4)^2$	d. $y = -(x + 2)^2$
e. $y = -(x - 5)^2$	f. $y = -(x + 6)^2$
- Do the answers to questions 3 and 4 change if there is a negative sign in front of the bracket? How does this negative sign affect the graph?
- MC** a. The axis of symmetry for the graph $y = (x + 3)^2$ is:

A. $y = 0$	B. $x = 3$	C. $x = -3$	D. $x = 2$	E. $x = 0$

 b. The vertex of the graph $y = (x + 3)^2$ is:

A. (0, 0)	B. (1, 3)	C. (1, -3)	D. (-3, 0)	E. (-3, 1)

8. Match each of the following parabolas with the appropriate equation from the list below.

i. $y = (x - 2)^2$

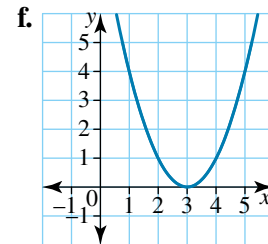
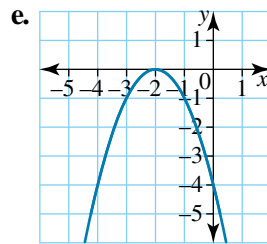
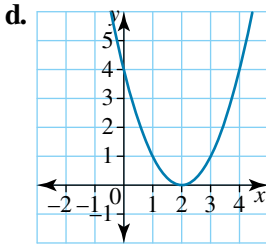
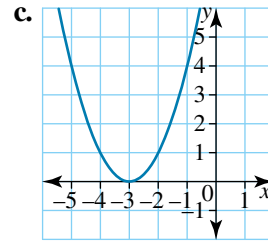
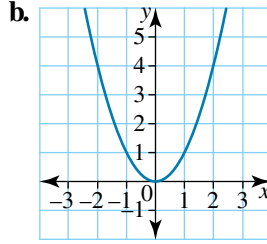
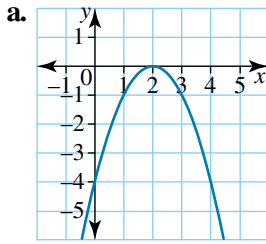
ii. $y = x^2$

iii. $y = (x + 3)^2$

iv. $y = -(x + 2)^2$

v. $y = x - 3^2$

vi. $y = -(x - 2)^2$



9. Find the equation of the parabola that is of the form $y = (x - h)^2$ and passes through:

a. (3, 1)

b. (-1, 9).

10. State the differences, if any, between the following pairs of parabolas.

a. $y = (1 - x)^2$ and $y = (x - 1)^2$

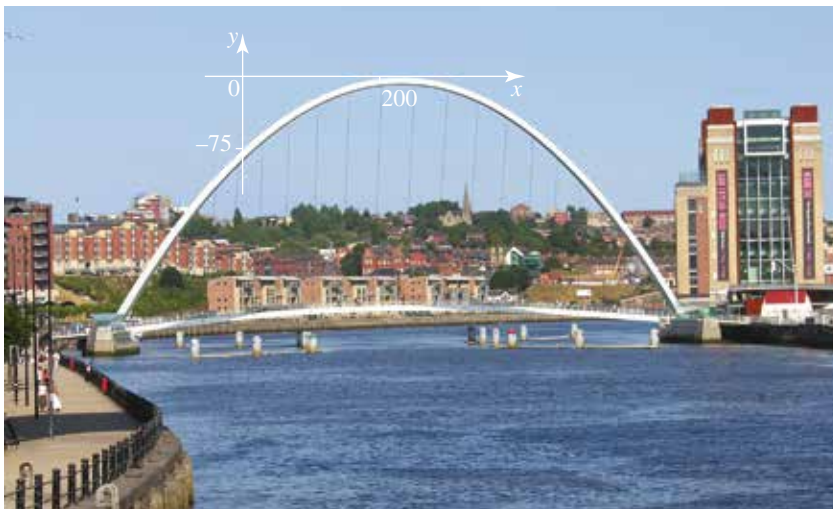
b. $y = (x - 2)^2$ and $y = (x + 2)^2$

c. $y = -(x - 4)^2$ and $y = (4 - x)^2$

11. Write the rule for the graph in the form $y = a(x - h)^2$ such that the y-intercept is $= -12$ and the x-intercept is at $x = 2$.

Communicating, reasoning and problem solving

12. The photo shows the span of the Gateshead Millennium Bridge in England. A set of axes has been superimposed onto the photo. Use the coordinates (200, 0) and (0, -75) to show that a possible equation is $y = -0.001875(x - 200)^2$.



13. a. By sketching the graph or using another method, determine the x- and y-intercepts of $y = 4(x - 3)^2$.
 b. Using the result from part a, show that the x-intercept and y-intercept for the equation $y = a(x - h)^2$ are $(h, 0)$ and $(0, ah^2)$ respectively.

14. The amount of profit made by a new company each month can be modelled by the rule $y = a(t - h)^2$, where y is the monthly profit and t is the integer value of the month ($0 = \text{January}$). If the January profit was \$9000 and the March profit was \$0:
- determine the values of a and h
 - determine the month when the profit is \$20250
 - explain why normally three points are required to uniquely determine a parabola, but in this example only two points are required.
15. The graph of a quadratic function touches the x -axis and has its vertex at $(3, 0)$. Graph two parabolas that apply to this description and find equations for them. How many examples are possible? Explain.
16. Sketch the graphs of $y = (x - 2)^2$ and $y = (2 - x)^2$. What do you notice about the graphs? Explain why this is true.
17. Find the equation of a parabola that has the vertex $(-2, 0)$ and has the same dilation factor as the graph $y = 2(x - 3)^2$.
18. What changes and what remains the same when a changes from a positive to a negative number in $y = a(x - h)^2$?

15.7 Sketching parabolas of the form $y = (x - h)^2 + k$

[Stage 5.3]

15.7.1 Sketching $y = (x - h)^2 + k$

- The equation $y = (x - h)^2 + k$ combines a vertical translation of k and a horizontal translation of h together.
- The equation $y = (x - h)^2 + k$ is called **vertex form** (or turning point form) because the vertex is given by the coordinates (h, k) .

WORKED EXAMPLE 12

Sketch the graph of $y = (x + 2)^2 - 1$, marking the vertex and the y -intercept, and indicate the type of vertex.

THINK

- Write the equation.
- State the vertex. As the equation is in the form $y = (x - h)^2 + k$, the vertex is (h, k) .
- There is no sign outside the brackets, so the parabola is upright.
- Find the y -intercept by substituting $x = 0$ into the equation.
- Draw clearly labelled axes, mark the coordinates of the vertex, the y -intercept and draw a symmetrical graph.

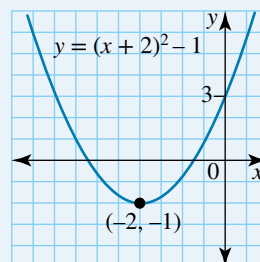
WRITE/DRAW

$$y = (x + 2)^2 - 1$$

The vertex is $(-2, -1)$.

Minimum vertex

$$\begin{aligned} \text{y-intercept: when } x = 0, \\ y &= (0 + 2)^2 - 1 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$



- Note that in the previous example the graph has shifted 2 units left ($h = 2$) and 1 unit down ($k = -1$).
- The graph has the same shape as $y = x^2$.

WORKED EXAMPLE 13

On the same set of axes, sketch the graphs of each of the following, clearly marking the coordinates of the vertex and the y-intercept:

a $y = x^2$ b $y = (x - 2)^2$ c $y = (x - 2)^2 + 1$.

State the changes that are made from part a to part b and from part a to part c.

THINK

- a Sketch the graph of $y = x^2$, marking the coordinates of the vertex and the y-intercept.

- b 1 Write the equation.

2 Find the coordinates of the vertex.

3 Find the y-intercept.

- 4 On the same set of axes, sketch the graph of $y = (x - 2)^2$, marking the coordinates of the vertex and the y-intercept.

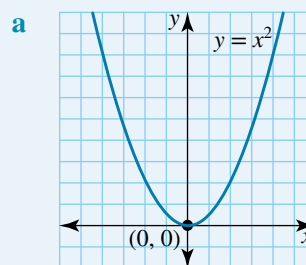
- c 1 Write the equation.

2 Find the coordinates of the vertex.

3 Find the y-intercept.

- 4 On the same set of axes, sketch the graph of $y = (x - 2)^2 + 1$, marking the coordinates of the vertex and the y-intercept.

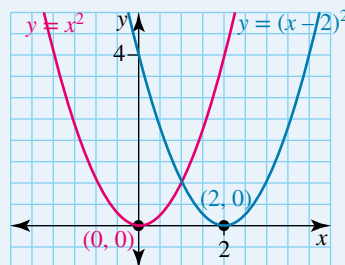
WRITE/DRAW



- b $y = (x - 2)^2$
The vertex is $(2, 0)$.

y-intercept: when $x = 0$,

$$y = (0 - 2)^2 \\ = 4$$

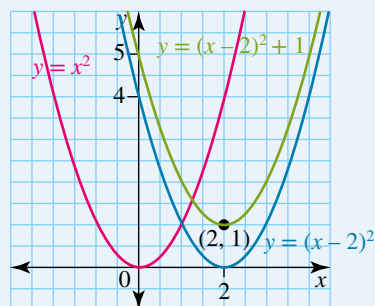


- c $y = (x - 2)^2 + 1$

The vertex is $(2, 1)$.

y-intercept: when $x = 0$,

$$y = (0 - 2)^2 + 1 \\ = 4 + 1 \\ = 5$$



- 5 State how $y = x^2$ is changed to form $y = (x - 2)^2$.
State how $y = x^2$ is changed to form $y = (x - 2)^2 + 1$.

If $y = x^2$ is moved 2 units to the right, the resulting graph is $y = (x - 2)^2$.

If $y = x^2$ is moved 2 units to the right and 1 unit up, the resulting graph is $y = (x - 2)^2 + 1$.

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Interactivity: Turning point form (int-6099)

Digital doc: WorkSHEET Quadratic functions 2 (doc-11010)

Exercise 15.7 Sketching parabolas of the form $y = (x - h)^2 + k$

Individual pathways

PRACTISE

Questions:
1–8, 10, 12, 13, 16

CONSOLIDATE

Questions:
1, 2, 3a, c, e, 4–8, 10–12, 14,
16–18

MASTER

Questions:
1, 2, 3a, e, f, 4–10, 12–21

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE12** Sketch the graph of each of the following, marking the vertex, the type of vertex and the y-intercept.

a. $y = (x - 1)^2 + 1$	b. $y = (x + 2)^2 - 1$
c. $y = (x - 3)^2 + 2$	d. $y = (x + 3)^2 - 2$
e. $y = -(x + 2)^2 - 1$	f. $y = -(x - 1)^2 + 2$
- Explain why the equation presented in the form $y = (x - h)^2 + k$ is known as ‘vertex form’.
- WE13** Using the same set of axes, sketch each (a–f) of the following sets (i, ii, iii) of graphs, clearly marking the coordinates of the vertex and the y-intercept.
State the changes that are made from i to ii and from i to iii.

a. i. $y = x^2$	ii. $y = (x - 1)^2$	iii. $y = (x - 1)^2 + 3$
b. i. $y = x^2$	ii. $y = (x - 2)^2$	iii. $y = (x - 2)^2 - 1$
c. i. $y = x^2$	ii. $y = (x + 3)^2$	iii. $y = (x + 3)^2 + 2$
d. i. $y = x^2$	ii. $y = (x - 4)^2$	iii. $y = (x - 4)^2 - 1$
e. i. $y = x^2$	ii. $y = -(x + 1)^2$	iii. $y = -(x + 1)^2 - 2$
f. i. $y = x^2$	ii. $y = -(x - 2)^2$	iii. $y = -(x - 2)^2 - 2$
- MC** a. For the graph of $y = (x + 5)^2 - 2$, the coordinates of the vertex are:

A. (5, -2)	B. (-2, 5)	C. (-2, -5)	D. (-5, -2)	E. (25, -2)

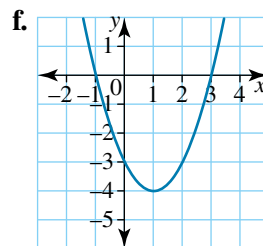
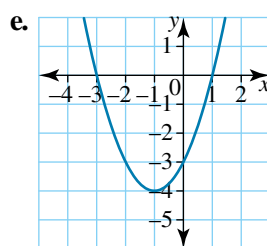
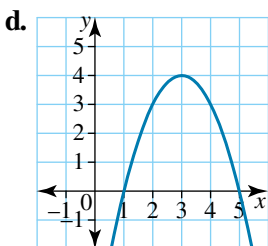
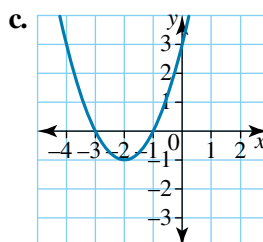
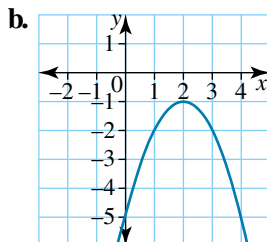
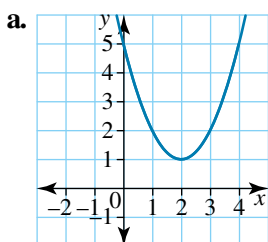
 b. For the graph of $y = -(x - 3)^2 + 7$, the axis of symmetry is:

A. $x = 3$	B. $x = 7$	C. $x = -3$	D. $y = 3$	E. $y = -3$

 c. For the graph of $y = (x - 1)^2 - 4$, the y-intercept is:

A. 1	B. -1	C. -3	D. 4	E. 3

5. The graph of $y = x^2$ is translated 2 units to the left and 3 units up.
- State the equation of the translated graph.
 - State the location of the vertex.
6. The vertex (maximum) of a graph is $(-3, -4)$.
- State the equation of the graph in vertex form.
 - Calculate the y -intercept.
7. Sketch the graph that is the 'mirror' image (reflected vertically) of $y = (x - 2)^2 + 3$.
8. State the equation of each of the following given it is of the form $y = (x - h)^2 + k$ or $y = -(x - h)^2 + k$.



9. Aravind threw a ball into the air. It followed the path of a parabola defined by the equation $h = -(x - 2)^2 + 5$, where h metres represents the height of the ball above the ground x metres horizontally from Aravind's hand.
- Sketch the graph showing the path of the ball during its flight from Aravind's hand until it reaches the ground. Mark the vertex and the y -intercept on the graph.
 - Use the graph to find the height of the ball when it leaves Aravind's hand.
 - Use the graph to find the maximum height that the ball reaches.
10. Write the equations for the following transformations on $y = x^2$.
- Reflection in the x -axis and translation of 2 units to the left
 - Translation of 1 unit down and 3 units to the right
 - Translation of 1 unit left and 2 units up
 - Reflection in the x -axis, translation of 4 units to the right and 6 units up
11. A rocket is shot in the air from a given point $(0, 0)$. The rocket follows a parabolic trajectory. It reaches a maximum height of 400m and lands 300m away from the launching point.
- How far horizontally from the launching point does the rocket reach its maximum height?
 - State the equation of the path of the rocket.
12. **MC** A parabola has an equation of $y = x^2 - 6x + 14$. Its vertex form is:
- $y = (x - 6)^2 + 14$
 - $y = (x - 14)^2 - 6$
 - $y = (x - 3)^2 + 5$
 - $y = (x - 5)^2 + 3$
 - $y = (x - 3)^2$

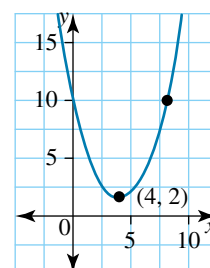
Communicating, reasoning and problem solving

13. Nikki wanted to keep a carnivorous plant, so after school she recorded the temperature on her windowsill for 8 hours every day for several months. One summer evening the temperature followed the relationship $t = (h - 5)^2 + 15$, where t was the temperature in degrees Celsius, h hours after 4 pm.

- Find the temperature on the windowsill at 4 pm.
- Show that the minimum temperature reached during the 8-hour period was 15°C .
- Find the number of hours it took for the windowsill to reach the minimum temperature.
- Sketch a graph of the relationship between the temperature and the number of hours after recording began. Mark the vertex and the t -intercept on the graph.



14. A graph has an axis of symmetry at $x = 1$ and a y -intercept of -4 . If the graph's minimum value is -10 , determine its equation in the form $y = a(x - h)^2 + k$. Demonstrate the correctness of your result.
15. Students were asked to choose values for a , h and k and substitute them into the general equation $y = a(x - h)^2 + k$ to form a quadratic equation. The graph shown was generated using the values 0.5, 2 and 4 for the three variables. Match the numerical values with the appropriate variable. Justify your answer.
16. a. Describe how to transform the parabola $y = (x - 3)^2$ to obtain the parabolas $y = (x - 3)^2 - 2$ and $y = (x - 3)^2 + 1$.
 b. Another parabola is created by moving $y = (x - 3)^2$ so that its vertex is $(5, -4)$. Write an equation for this parabola.
17. Find an equation for the parabola that has its vertex at $(-1, 4)$ and passes through the point $(1, 3)$.
18. Find the x -intercepts of $y = a(x - 2a)^2 - 9a^3$ in terms of a .
19. Use the three points $(-2, 1)$, $(3, 1)$ and $(0, 7)$ to determine a , h and k in the equation $y = a(x - h)^2 + k$.
20. Find the x -intercepts of $y = a(x - h)^2 - k$ in terms of a , h and k .
21. What are the advantages of having the equation of a parabola in vertex form?



15.8 Sketching parabolas of the form $y = (x + a)(x + b)$

[Stage 5.3]

15.8.1 Sketching $y = (x + a)(x + b)$

- The equation $y = (x + a)(x + b)$ consists of a pair of linear factors $(x + a)$ and $(x + b)$ multiplied together.
- The x -intercepts are found by setting each factor to 0, namely:
 $(x + a) = 0$, or $x = -a$
 $(x + b) = 0$, or $x = -b$.
- The y -intercept can be found by letting $x = 0$ in the original equation. That is,
 $y = (0 + a)(0 + b)$, or $y = ab$.
- The axis of symmetry is half-way between the x -intercepts $-a$ and $-b$. That is, $x = \frac{-(a + b)}{2}$.
- Substitute the x -value of the axis of symmetry into $y = (x + a)(x + b)$ to find the y -coordinate of the vertex.

DISCUSSION

If a quadratic function is given in the form $y = (x + a)(x + b)$, explain the information you can determine from the equation in order to sketch the parabolic shape of the function.

WORKED EXAMPLE 14

Sketch the graph of $y = (x - 4)(x + 2)$ by finding first the x - and y -intercepts and then the vertex.

THINK

- 1 Write the equation.
- 2 Find the x -intercepts.
- 3 Find the y -intercept.
- 4 Find the x -value of the vertex by averaging the values of the two x -intercepts.
- 5 Find the y -value of the vertex by substituting the x -value of the vertex into the equation of the graph.
- 6 State the coordinates of the vertex.
- 7 Sketch the graph.

WRITE/DRAW

$$y = (x - 4)(x + 2)$$

x -intercepts: when $y = 0$,

$$(x - 4)(x + 2) = 0$$

$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 4 \qquad \qquad x = -2$$

The x -intercepts are -2 and 4 .

y -intercept: when $x = 0$,

$$y = (0 - 4)(0 + 2)$$

$$= -4 \times 2$$

$$= -8$$

The y -intercept is -8 .

$$\text{At the vertex, } x = \frac{4 + (-2)}{2} = 1.$$

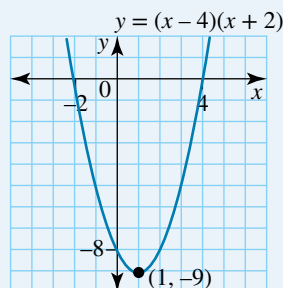
When $x = 1$,

$$y = (1 - 4)(1 + 2)$$

$$= -3 \times 3$$

$$= -9$$

The vertex is $(1, -9)$.



Quadratic equations written in the form $y = ax^2 + bx + c$ can sometimes be factorised into intercept form.

WORKED EXAMPLE 15

Sketch the graph of $y = x^2 + 6x + 8$.

THINK

- 1 Write the equation.
- 2 Factorise the expression on the right-hand side of the equation, $x^2 + 6x + 8$.

WRITE/DRAW

$$y = x^2 + 6x + 8$$

$$= (x + 2)(x + 4)$$

3 Find the two x -intercepts.

x -intercepts: when $y = 0$,

$$(x + 2)(x + 4) = 0$$

$$x + 2 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = -2 \quad \quad \quad x = -4$$

The x -intercepts are -4 and -2 .

4 Find the y -intercept.

y -intercept: when $x = 0$,

$$y = 0 + 0 + 8$$

$$= 8$$

The y -intercept is 8 .

5 Find the x -value of the vertex.

$$\text{At the vertex, } x = \frac{-2 + -4}{2} \\ = -3$$

6 Find the y -value of the vertex.

When $x = -3$,

$$y = (-3 + 2)(-3 + 4)$$

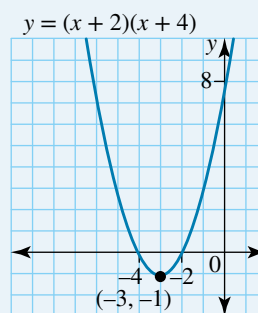
$$= -1 \times 1$$

$$= -1$$

The vertex is $(-3, -1)$.

7 State the vertex.

8 Sketch the graph.



learnon RESOURCES — ONLINE ONLY

Interactivity: Parabola of the form $y = (x + a)(x + b)$ (int-6100)

Interactivity: Sketching parabolas (int-2776)

Digital doc: SkillsHEET Solving quadratic equations of the form $(x + a)(x + b) = 0$ (doc-10992)

Digital doc: SkillsHEET Factorising quadratic trinomials of the form $ax^2 + bx + c$ where $a = 1$ (doc-11011)

Digital doc: SkillsHEET Solving quadratic trinomials of the type $ax^2 + bx + c = 0$ where $a = 1$ (doc-11012)

Digital doc: WorkSHEET Quadratic functions 3 (doc-11013)

Exercise 15.8 Sketching parabolas of the form $y = (x + a)(x + b)$

Individual pathways

PRACTISE

Questions:

1–6, 10, 12

CONSOLIDATE

Questions:

1a, b, f, 2b, e, h, k, 3, 4, 6, 9, 10,
12, 13

MASTER

Questions:

1a, b, d, 2c, f, i, l, 3–14

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE14** Sketch the graph of each of the following by finding first the x - and y -intercepts and then the vertex.

a. $y = (x + 2)(x + 6)$

b. $y = (x - 3)(x - 5)$

c. $y = (x - 3)(x + 1)$

d. $y = (x + 4)(x - 6)$

e. $y = (x - 5)(x + 1)$

f. $y = (x + 1)(x - 2)$

2. **WE15** Sketch the graph of each of the following.

a. $y = x^2 + 8x + 12$

b. $y = x^2 + 8x + 15$

c. $y = x^2 + 6x + 5$

d. $y = x^2 + 10x + 16$

e. $y = x^2 + 2x - 8$

f. $y = x^2 + 4x - 5$

g. $y = x^2 - 4x - 12$

h. $y = x^2 - 6x - 7$

i. $y = x^2 - 10x + 24$

j. $y = x^2 - 6x + 5$

k. $y = x^2 - 5x + 6$

l. $y = x^2 - 3x - 10$

3. Sketch the graph of a parabola whose x -intercepts are at $x = -4$ and $x = 6$ and whose y -intercept is at $y = 2$.

4. State a possible equation of a graph with an axis of symmetry at $x = 4$ and one intercept at $x = 6$.

5. Out on the cricket field, Michael Clarke chases the ball. He picks it up, runs 4 metres towards the stumps (which are 25 metres away from where he picks up the ball), then throws the ball, which follows the path described by the quadratic equation $y = -0.1(x - m)(x - n)$, where m and n are positive integers. The ball lands 1 metre from the stumps, where another player quickly scoops it up and removes the bails. Take the origin as the point where Michael picked up the ball initially.

- Find how far the ball has travelled horizontally while in flight.
- Find the values of m and n .
- Find the x -intercepts.
- Find the axis of symmetry.
- Find the vertex.
- Find the highest point reached by the cricket ball.
- Find how far horizontally the ball has travelled when it reaches its highest point.
- Sketch the flight of the ball, showing all of the relevant details on the graph.

6. McDonald's golden arches were designed by Jim Schindler in 1962. The two arches both approximate the shape of a parabola. A large McDonald's sign stands on the roof of a shopping centre. The shape of one of the parabolas can be modelled by the quadratic function $y = -x^2 + 8x - 7$, where both x and y are measured in metres. The complete sign is supported by a beam underneath the arches. What is the minimum length required for this beam?



7. For the equation $y = (x - 3)(x - 9)$:

- determine the vertex
- rewrite the equation in vertex form
- expand both forms of the equation, showing that they are equivalent
- sketch the equation showing the x -intercepts and vertex.

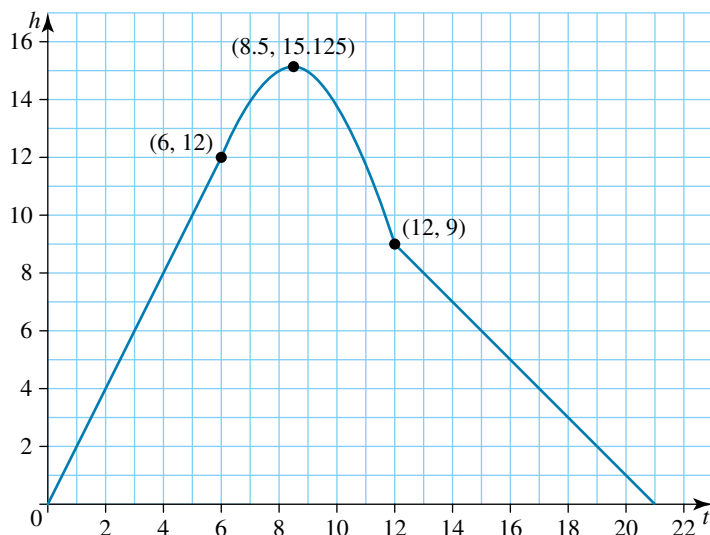
8. The dimensions of a rectangular backyard can be given by $(x + 2)$ m and $(x - 4)$ m. Find the value of x if the yard's area is 91 m^2 .

Communicating, reasoning and problem solving

9. The height, $h(t)$, that an object reaches when it is thrown into the air is determined by the formula $h(t) = -8t^2 + 128t$, where t is time in seconds and h is height in metres.

- Does the graph of the formula have a maximum or a minimum vertex? Explain or show your reasoning.
- What is the maximum height that the object reaches and at what time is this height reached?

10. Daniel is in a car on a roller-coaster ride as shown in the following graph, where the height, h , is in metres above the ground and time for the ride, t , is in minutes.



- a. The ride can be represented by three separate equations. Show that the first section of the ride is $h = 2t$ from $t = 0$ to $t = 6$. The middle section is a parabola and the final section is a straight line. Find the other two equations for the ride.
- b. What is the required domain (set of possible t -values) for each section of the ride?
11. On a set of axes, sketch several parabolas of the form $y = -x^2 + bx$. What do you notice about the vertices of each graph sketched?
12. Find a quadratic equation given the x -intercepts of the parabola are $-\frac{1}{2}$ and $\frac{3}{4}$. Is there more than one equation possible?
13. Find the equation of the parabola that passes through the points $(-1, -2)$, $(1, -4)$ and $(3, 10)$.
14. What are the advantages of having the equation of a parabola in intercept form?

15.9 Applications

[Stage 5.3]

- Quadratic graphs and equations can be used to solve practical problems in science and engineering.
- Quadratic graphs and equations can be used to solve problems where a maximum or minimum needs to be found.
- When working with physical phenomena, ensure that the solution achieved satisfies any physical constraints of the problem. For example, if measurements of length or time are involved, there can be no negative solutions.

DISCUSSION

Can you think of any other examples where the solution achieved might not satisfy the physical constraints of the problem?

WORKED EXAMPLE 16

A flare is fired from a yacht in distress off the coast of Brisbane. The flare's height, h metres above the horizon t seconds after firing, is given by $h = -2t^2 + 18t + 20$.

- a When will the flare fall into the ocean?
- b How high is the flare after 2 seconds?
- c At which other time will the flare be at the same height?
- d For how long is the flare above the 'lowest visible height' of 56 m?

THINK

- a Substitute $h = 0$ into the equation and solve for t using the Null Factor Law. Since t cannot equal -1 seconds, answer the question with the appropriate solution.
- b Substitute $t = 2$ into the equation and solve for h . Answer the question.
- c Substitute $h = 48$ into the equation, rearrange, factorise and solve for t .
- d Substitute $h = 56$ into the equation, rearrange, factorise and solve for t . Answer the question.

WRITE

- a $0 = -2t^2 + 18t + 20$
 $= -2(t^2 - 9t - 10)$
 $= -2(t - 10)(t + 1)$
 $t = 10$ or -1
 $t = 10$ seconds
- b $h = -2(2)^2 + 18(2) + 20$
 $= -8 + 36 + 20$
 $= 48$
The flare is 48 metres high after 2 seconds.
- c $48 = -2t^2 + 18t + 20$
 $0 = -2t^2 + 18t - 28$
 $= -2(t^2 - 9t + 14)$
 $= -2(t - 7)(t - 2)$
 $t = 7$ seconds
- d $56 = -2t^2 + 18t + 20$
 $= -2t^2 + 18t - 36$
 $= -2(t^2 - 9t + 18)$
 $= -2(t - 6)(t - 3)$
 $t = 3$ seconds and 6 seconds
The flare is above 56m for 3 seconds (between 3 and 6 seconds).

Exercise 15.9 Applications

Individual pathways

PRACTISE

Questions:
1–5, 9, 12

CONSOLIDATE

Questions:
1–4, 6, 7, 10, 12, 13

MASTER

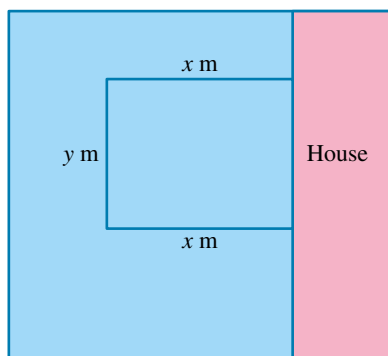
Questions:
1–4, 7–17

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. A spurt of water emerging from an outlet just below the surface of an ornamental fountain follows a parabolic path described by the equation $h = -x^2 + 8x$, where h is the height of the water and x is the horizontal distance from the outlet in metres.

8. A woman wished to build a fence around part of her backyard as shown in the diagram. The fence will have one side abutting the wall of the house. She has enough fencing material for 55 m of fence.



- Show that $y = 55 - 2x$.
- Write an expression for the area enclosed by the fence in terms of x alone.
- Determine the dimensions of the fence such that the area is a maximum and calculate that area.

Communicating, reasoning and problem solving

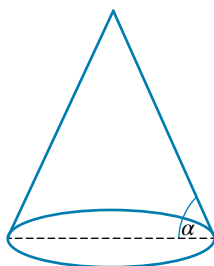
9. A basketball thrown from the edge of the court to the goal shooter is described by the formula $h = -t^2 + 6t + 1$, where h is the height of the basketball in metres after t seconds.
- What was the height of the ball when it was first thrown?
 - What was the height of the basketball after 2 s?
 - Show that the ball was first at a height of 6 m above the ground at 1 second.
 - During which time interval was the ball above a height of 9 m?
 - Plot the path of the basketball.
 - What was the maximum height of the basketball during its flight?
 - How long was the ball in flight if it was caught at a height of 1 m above the ground on its downward path?



10. Jack and his dog were playing outside. Jack was throwing a stick in the air for his dog to catch. The height of the stick (in metres) followed the equation $h = 20t - 5t^2$.
- The graph of h is a parabola. Is the parabola upright or inverted?
 - Factorise the expression on the right-hand side of the equation.
 - Find the t -intercepts. These will be the two points at which the stick is on the ground, once at take-off and once at landing. How long does the stick remain in the air? (For simplicity, assume that Jack throws the stick from ground level.)
 - Justify whether the parabola has a maximum or minimum vertex, and find its coordinates.
 - What will be the maximum height reached by the stick? Explain.
 - Use the information you have found to produce a sketch of the path of the stick.



11. An engineer wishes to build a footbridge in the shape of an inverted parabola across a 40-m wide river. The bridge will be symmetrical and the greatest difference between the lowest part and highest part will be 4 m. Taking the origin as one side, show that the equation that models this footbridge is $y = -0.01x(x - 40)$. Explain what x and y are, and state the domain (the possible values that x can take).
12. A car engine spark plug produces a spark of electricity. The size of the spark depends on how far apart the terminals are. The percentage performance, Z , of a certain brand is thought to be $Z = -400(g - 0.5)^2 + 100$, where g is the distance between the terminals.
 - a. Sketch the graph of Z .
 - b. When is the performance greatest?
 - c. From your graph, find the values of g for which the percentage performance is greater than 50%.
13. The monthly profit or loss, p (in thousands of dollars), for a new brand of soft drink is given by $p = -2(x - 7.5)^2 + 40.5$, where x is the number (integer) of months after its introduction (when $x = 0$).
 - a. In which month was the greatest profit made?
 - b. Between which months did the company make a profit?
14. NASA uses a parabolic flight path to simulate zero gravity and the gravity experienced on the moon. Use the internet to investigate the flight path and create a mathematical model to represent the flight path.
15. An arch bridge is modelled in the shape of a parabolic arch. The arch span is 50 m wide and the maximum height of the arch above water level is 5.5 m. A floating platform 35 m wide is towed under the bridge. What is the greatest height of the deck above water level if the platform is to be towed under the bridge with at least a 35 cm clearance on either side?
16. What are some important things to remember when solving practical problems with quadratic graphs and equations?
17. Take a solid cone standing on its circular base. Let the angle that the slant height makes with the base be α . Cross-sections of the cone can reveal shapes of a parabola, an ellipse and a circle. Describe how these shapes can be obtained. Make reference to the angle α in each case.



15.10 Review

Investigation | Rich task

Constructing a parabola

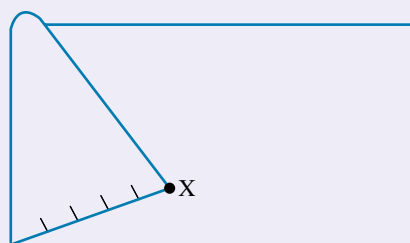


The word *parabola* comes from the Ancient Greek language. It originally meant ‘thrown’, referring to the path followed by a projectile in flight. Notice that the water streams shown in the photo above are moving in the path of a parabola.

This investigation explores the technique of folding paper to display the shape of a parabola. Take care with each step to ensure your finished product is a well-constructed parabola that can be used in later parts of this investigation.

Forming a parabola by folding paper

- Take a sheet of A4 paper. Cut it into two pieces by dividing the longer side into two. Only one of the halves is required for this investigation.
- Along one of the longer sides of your piece of paper, mark points that are equally spaced 1 cm apart. Start with the first point being on the very edge of the paper.
- Turn over the piece of paper and mark a point, X, 3 cm above the centre of the edge that has the markings on the reverse side.
- Fold the paper so that the first point you marked on the edge touches point X. Make a sharp crease and open the paper flat.
- Fold the paper again so that the second mark touches the point X. Crease and unfold again.
- Repeat this process until all the marks have been folded to touch point X.
- With the paper flat and the point X facing up, you should notice the shape of a parabola appearing in the creases.



1. Trace the curve with a pencil.

The point X is called the focus of the parabola. Consider the parabola to represent a mirror. Rays of light from the focus would hit the mirror (parabola) and be reflected. The angle at which each ray hits the mirror is the same size as the angle at which it is reflected.

2. Using your curve traced from your folding activity, accurately draw a series of lines to represent rays of light from the point X to the parabola (mirror). Use a protractor to carefully measure the angle each line makes with the mirror and draw the path of these rays after reflection in the mirror.
3. Draw a diagram to describe your finding from question 2 above. Provide a brief comment on your description.
4. Retrace your parabola onto another sheet of paper. Take a point other than the focus and repeat the process of reflection of rays of light from this point by the parabolic mirror.
5. Draw a diagram to describe your finding from question 4. Provide a brief comment on your description.
6. Give examples where these systems could be used in society.



learnon RESOURCES — ONLINE ONLY

Digital doc: Investigation — Constructing a parabola (doc-15914)

Digital doc: Code puzzle — Whatever happened to Australia's first 50-cent coin of 1966? (doc-15915)

Digital doc: Summary Topic 15 (doc-22897)

Interactivity: Crossword Topic 15 (int-0708)

Interactivity: Word search Topic 15 (int-0694)

Interactivity: Sudoku Topic 15 (int-3215)

Exercise 15.10 Review questions

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

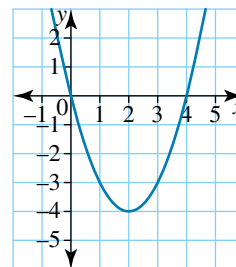
1. **MC** a. The axis of symmetry for the graph shown is:

A. $x = 0$	B. $x = 2$	C. $x = 4$
D. the y-axis	E. $x = -4$	
- b. The coordinates of the vertex for the graph are:

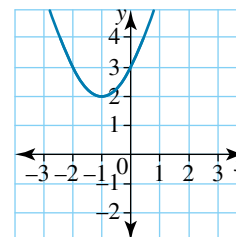
A. (0, 0)	B. (2, 4)	C. (2, -4)
D. (-2, -4)	E. (2, 0)	
- c. The y-intercept is:

A. 0	B. -2	C. -4
D. 2	E. 4	
- d. The x-intercepts are:

A. 2 and -4	B. 0 and 2	C. 0 and -2
D. 0 and -4	E. 0 and 4	



2. **MC** a. The graph of $y = -4x^2$ is:
- A. wider than $y = x^2$
 - B. narrower than $y = x^2$
 - C. the same width as $y = x^2$
 - D. a reflection of $y = x^2$ in the x -axis
 - E. moved 4 units left
- b. The graph of $y = \frac{1}{2}x^2$ is:
- A. wider than $y = x^2$
 - B. narrower than $y = x^2$
 - C. the same width as $y = x^2$
 - D. a reflection of $y = x^2$ in the x -axis
 - E. moved $\frac{1}{2}$ units right
- c. Compared to the graph of $y = x^2$, the graph of $y = -2x^2$ would be:
- A. half as wide
 - B. twice as wide
 - C. moved 2 units to the right
 - D. moved 2 units up
 - E. moved 2 units down
3. **MC** a. The vertex for the graph of the equation $y = -x^2 + 6$ is:
- A. (0, 0)
 - B. (-1, 6)
 - C. (0, 6)
 - D. (0, -6)
 - E. (-6, 0)
- b. The vertex of the graph of the equation $y = x^2 - 12$ is:
- A. (1, -12)
 - B. (-12, 1)
 - C. (0, 12)
 - D. (0, -12)
 - E. (-12, 0)
- c. The graph of $y = x^2 - 4$ moves the graph of $y = x^2$ in the following way:
- A. up 4
 - B. down 4
 - C. no change
 - D. up 2
 - E. down 2
- d. The y -intercept of the graph of $y = -x^2 - 3$ is:
- A. 1
 - B. -1
 - C. 3
 - D. -3
 - E. 9
4. **MC** a. The axis of symmetry for the graph $y = (x + 1)^2$ is:
- A. $y = 0$
 - B. $x = 1$
 - C. $x = -1$
 - D. $x = 2$
 - E. $y = -1$
- b. The vertex of the graph $y = (x + 1)^2$ is:
- A. (0, 0)
 - B. (1, 1)
 - C. (1, -1)
 - D. (-1, 0)
 - E. (0, -1)
- c. Compared to the graph of $y = x^2$, the graph of $y = (x - 2)^2$ would be:
- A. twice as wide
 - B. moved 2 units to the left
 - C. moved 2 units to the right
 - D. moved 2 units up
 - E. moved 2 units down
5. **MC** a. For the graph of $y = (x + 6)^2 - 1$, the coordinates of the vertex are:
- A. (6, -1)
 - B. (-1, 6)
 - C. (-1, -6)
 - D. (-6, -1)
 - E. (6, 1)
- b. For the graph of $y = -(x - 4)^2 + 5$, the axis of symmetry is:
- A. $x = 4$
 - B. $x = 5$
 - C. $x = -4$
 - D. $y = -5$
 - E. $y = 4$
- c. For the graph of $y = (x - 2)^2 - 3$, the y -intercept is:
- A. -5
 - B. 1
 - C. -4
 - D. -3
 - E. -1
6. **MC** To change the graph of $y = x^2$ to make the graph of $y = 2(x - 1)^2 - 3$, we need to:
- A. move it 1 unit to the right and 3 units down
 - B. make it narrower by a factor of 3 and move it 1 unit to the left and 3 units down
 - C. make it wider by a factor of 2 and move it 1 unit to the right and 3 units down
 - D. make it narrower by a factor of 2 and move it 1 unit to the right and 3 units down
 - E. make it narrower by a factor of 2 and move it 3 units to the right and 1 unit down
7. **MC** Human cannonball Stephanie Smith was fired from a cannon. Her flight path is a parabola. If c is her height above the net and b is her horizontal distance from the cannon, which rule could describe her flight?
- A. $y = x^2 + c$
 - B. $y = bx^2 + c$
 - C. $y = (x - b)^2 + c$
 - D. $y = (x - b)^2 - c$
 - E. $y = -(x - b)^2 + c$
8. **MC** The curve shown could have the rule:
- A. $y = -(x + 1)^2 - 2$
 - B. $y = (x + 1)^2 + 2$
 - C. $y = (x + 1)^2 - 2$
 - D. $y = -(x + 2)^2 - 1$
 - E. $y = (x - 1)^2 - 2$



9. **MC** If $y = x^2 - x - 6$, then the x -intercepts would be:

A. -2 and 3

B. -3 and 2

C. 2 and 3

D. -2 and -3

E. 1 and 6

10. **MC** The sketch in the diagram shown is a graph of:

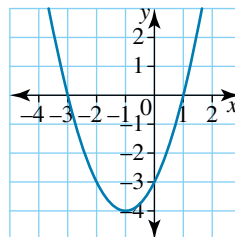
A. $y = (x - 1)(x - 3)$

B. $y = (x + 1)(x + 3)$

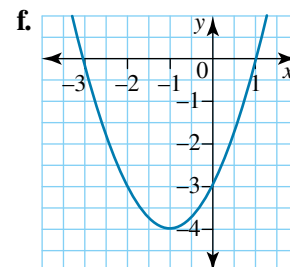
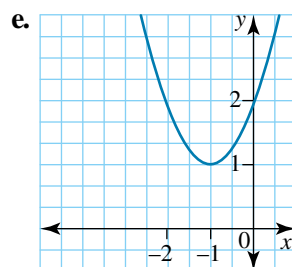
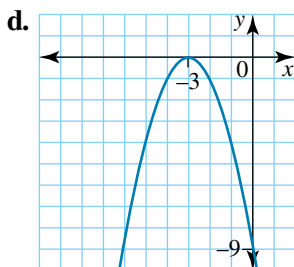
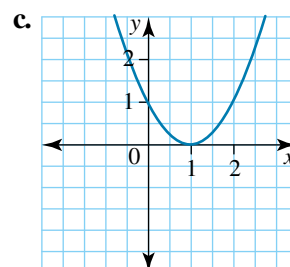
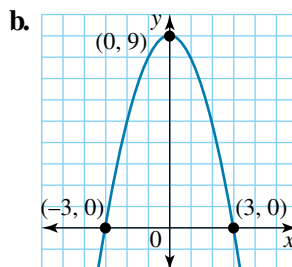
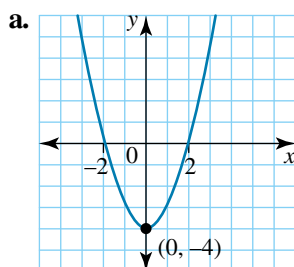
C. $y = (x - 1)(x + 3)$

D. $y = (x + 1)(x - 3)$

E. $y = (x + 1)(x - 4)$



11. For each of the following graphs, state the equation of the axis of symmetry, the coordinates of the vertex, whether the point is a maximum or a minimum, and the x - and y -intercepts.



12. For each of the following equations, draw a table of values, use it to plot the graph, and hence state the equation of the axis of symmetry, the coordinates of the vertex and whether it is a maximum or a minimum, and the x - and y -intercepts.

a. $y = x^2 - 4x$, $-2 \leq x \leq 6$

b. $y = -x^2 - 2x + 8$, $-5 \leq x \leq 3$

c. $y = 2x^2 - 4x + 4$, $-2 \leq x \leq 3$

d. $y = -x^2 + 6x - 5$, $0 \leq x \leq 6$

13. Sketch each of the following graphs, labelling the axis of symmetry, the vertex and the intercepts, and stating the type of vertex.

a. $y = 2x^2$

b. $y = \frac{1}{2}x^2$

c. $y = -4x^2$

d. $y = -\frac{1}{3}x^2$

14. Sketch each of the following graphs, labelling the axis of symmetry, the vertex and the y -intercept, and stating the nature of the vertex.

a. $y = x^2 + 2$

b. $y = x^2 - 4$

c. $y = x^2 + 5$

d. $y = x^2 - 3$

15. Sketch each of the following graphs, labelling the vertex and the y -intercept, and stating the type of vertex.

a. $y = (x + 1)^2$

b. $y = -(x - 2)^2$

c. $y = (x - 5)^2$

d. $y = -(x + 2)^2$

16. Sketch each of the following graphs, labelling the vertex and the intercepts, and stating the nature of the vertex.

a. $y = (x - 1)^2 + 1$

b. $y = (x + 2)^2 + 3$

c. $y = (x - 3)^2 - 2$

d. $y = -(x + 1)^2 - 4$

Communicating, reasoning and problem solving

17. A railway bridge has an arch below it that can be modelled by the equation $H = -(x + 1)^2 + 16$, where H is the height of the bridge, in metres. Sketch a graph of the bridge and hence find its maximum height.
18. Sketch the graph of each of the following by finding the x - and y -intercepts and the vertex.
a. $y = (x + 2)(x - 2)$ b. $y = (x + 3)(x - 7)$ c. $y = x^2 - 2x - 15$ d. $y = x^2 + 3x + 2$
19. A valley in the countryside has a river running through the centre of it. A vertical cross-section of the valley can be modelled by the equation $H = \frac{1}{10}x^2 - 6x + 100$, where H is the height above sea level, in metres, and x is the horizontal distance, in metres. If two villages are situated in the valley, one at a horizontal distance of 10 metres from the river and the other 20 metres from the river, what are their heights, respectively, above sea level?
20. Children playing on a cricket pitch throw a ball that follows the path $y = \frac{1}{120}(x - 16)^2$, where y is the height, in yards, above the ground and x is the horizontal distance from the stumps at the end from which the ball is thrown. If the stumps are 22 yards away and 0.5 yards high, will the ball hit the stumps? Find out by sketching the graph. (As a cricket pitch is 22 yards long, a yard is a convenient unit to use in this example. A yard is about 91.5 cm.)
21. A toy rocket is thrown up into the air from a balcony. The path the rocket takes is given by the equation $h = 49 - (t - 2)^2$, where h is the height above the ground in metres and t is the time in seconds.
a. How high is the balcony?
b. When does the rocket land on the ground?
c. What is the highest point the rocket reaches?
d. Sketch a graph of the rockets path for $0 \leq t \leq 9$.
22. A piece of wire 1 m long is bent into the shape of a rectangle.
a. If w cm is the width and l cm is the length, write a rule connecting w and l .
b. If the area of the rectangle is A cm², write an equation connecting l in terms of A .
c. Sketch the graph of A versus l .
d. What will be the maximum area enclosed by the wire?
23. The organising committee of the Greensborough Tennis Club found that the profit, $\$P$, from a Saturday morning barbeque depended on the number of sausages sold, n . The estimated profit is given by $P = -n^2 + 50n - 400$.
a. What is the profit if the number of sausages sold is 35?
b. Sketch the graph of P versus n for $0 \leq n \leq 50$.
c. Find the maximum profit possible and the number of sausages that need to be sold to make this amount.
d. What would be the minimum number of sausages that would need to be sold to avoid a loss?
24. In a demonstration using the width of a street, a skateboarder performs an ollie over a distance of 2 m. He remains on the ground for $\frac{1}{4}$ m before he begins the ollie, the path of which can be described by the quadratic equation $y = -2(x - p)(x - q)$ (where all dimensions are in metres), and lands with 35 cm to spare before hitting the opposite edge of the curb. Taking the origin as the start of his run:
a. How far has he travelled horizontally?
b. Find the values of p and q .
c. State the axis of symmetry of the ollie.
d. Find the highest point reached by the skateboarder.
e. Sketch the path of the skateboarder, showing all of the relevant details on the graph.

Answers

Topic 15 Quadratic relationships

Exercise 15.2 Graphs of quadratic functions

1. a. Axis of symmetry: $x = 0$, vertex $(0, 0)$, minimum
 c. Axis of symmetry: $x = -1$, vertex $(-1, -2)$, minimum
 e. Axis of symmetry: $x = 0$, vertex $(0, 2)$, maximum
2. a. $x = 0$, $(0, 1)$, minimum
 c. $x = -2$, $(-2, 2)$, maximum
 e. $x = 2$, $(2, 2)$, minimum
3. a. $x = 0$; vertex $(0, -1)$, minimum; x -intercepts are -1 and 1 , y -intercept is -1 .
 b. $x = 0$; vertex $(0, 1)$, maximum; x -intercepts are -1 and 1 , y -intercept is 1 .
 c. $x = 1$; vertex $(1, -4)$, minimum; x -intercepts are -1 and 3 , y -intercept is -3 .
 d. $x = -2$; vertex $(-2, 1)$, maximum; x -intercepts are -3 and -1 , y -intercept is -3 .
 e. $x = -\frac{1}{2}$; vertex $(-\frac{1}{2}, 1)$, minimum; x -intercepts are $-1\frac{1}{2}$ and $\frac{1}{2}$, y -intercept is $-\frac{3}{4}$.
 f. $x = \frac{1}{2}$; vertex $(\frac{1}{2}, 2)$, minimum; no x -intercepts, y -intercept is $2\frac{1}{4}$.
 g. $x = 2$; vertex $(2, 0)$, minimum; x -intercept is 2 , y -intercept is 4 .
 h. $x = -1$; vertex $(-1, 0)$, maximum; x -intercept is -1 , y -intercept is -1 .
 i. $x = 1$; vertex $(1, 4)$, maximum; x -intercepts are -1 and 3 , y -intercept is 3 .
 j. $x = 2$; vertex $(2, -9)$, minimum; x -intercepts are -1 and 5 , y -intercept is -5 .

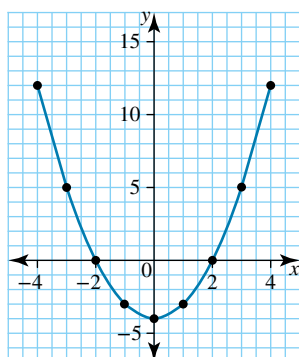
4. a. B

b. C

c. A

d. D

5. a.



The shape is a parabola.

b. $x = 0$

c. $y = -4$

d. $x = -2$, $x = 2$

6.

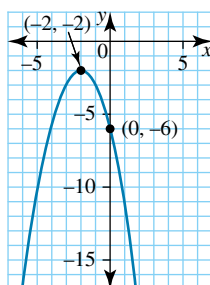
x	-6	-4	-2	0	2	4	6	8
y	30	12	2	0	6	20	42	72

7. C

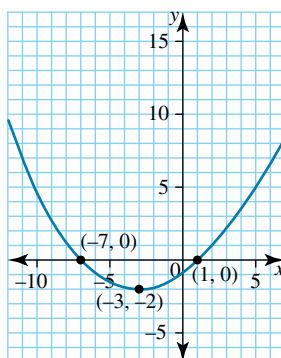
8. a. $(0, 0)$

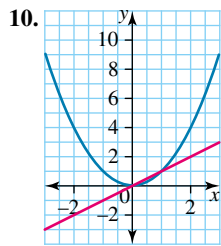
b. Maximum

9. a.



b.





By inspection or algebra, the graphs meet at (0, 0) and again at (1, 1).

11. a. The graph has the shape of a parabola.

Height	10	15	20	25	30	35	40	45	50
Cost	100	225	400	625	900	1225	1600	2025	2500

b. \$2025

c. Answers will vary.

12. a. Answers will vary.

b. 37 metres

13. Answers will vary.

14. Answers will vary.

15. Answers will vary. If the parabola is upright, it has a minimum vertex. If the parabola is inverted, it has a maximum vertex.

16. Answers will vary but must be of the form $y = a(x - h)^2$.

17. Answers will vary but must be of the form $y = ax^2 + c$.

18. a.

n (number of lines)	1	2	3	4	5	6
N (number of intersections)	0	1	3	6	10	15

b. $N = \frac{n(n-1)}{2}$

c. $N = \frac{100(100-1)}{2}$
= 4950 intersections

19. All parabolas have an axis of symmetry and a vertex.

Exercise 15.3 Plotting points to graph quadratic functions

1. a. $y = 2x^2$

x	-3	-2	-1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18

$(-3, 18), (-2, 8), (-1, 2), (0, 0), (1, 2), (2, 8), (3, 18)$

b. $y = x^2 - 4$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 4$	5	0	-3	-4	-3	0	5

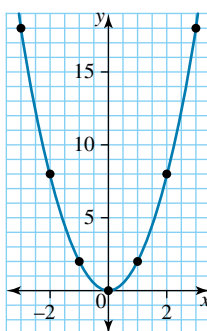
$(-3, 5), (-2, 0), (-1, -3), (0, -4), (1, -3), (2, 0), (3, 5)$

c. $y = -x^2 + 4x + 5$

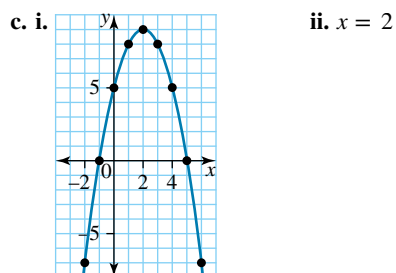
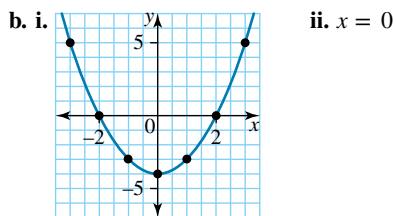
x	-2	-1	0	1	2	3	4	5	6
y	-7	0	5	8	9	8	5	0	-7

$(-2, -7), (-1, 0), (0, 5), (1, 8), (2, 9), (3, 8), (4, 5), (5, 0), (6, -7)$

2. a. i.

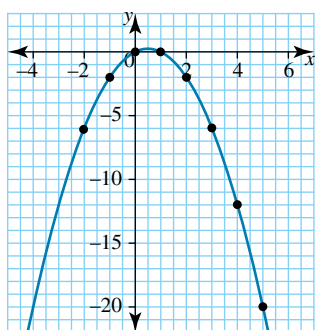


ii. $x = 0$

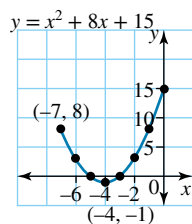


3.

x	-2	-1	0	1	2	3	4	5
$y = -x^2 + x$	-6	-2	0	0	-2	-6	-12	-20



4. a. $y = x^2 + 8x + 15$, $-7 \leq x \leq 0$

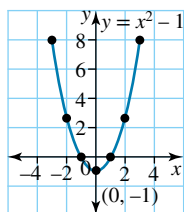


i. $x = -4$

ii. $(-4, -1)$, minimum

iii. x -intercepts are -5 and -3 , y -intercept is 15 .

b. $y = x^2 - 1$, $-3 \leq x \leq 3$

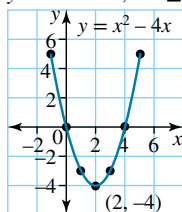


i. $x = 0$

ii. $(0, -1)$, minimum

iii. x -intercepts are -1 and 1 , y -intercept is -1 .

c. $y = x^2 - 4x, -1 \leq x \leq 5$

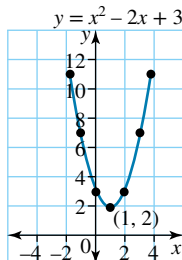


i. $x = 2$

ii. $(2, -4)$, minimum

iii. x -intercepts are 0 and 4, y -intercept is 0.

d. $y = x^2 - 2x + 3, -2 \leq x \leq 4$

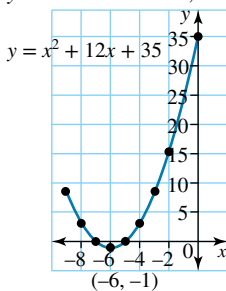


i. $x = 1$

ii. $(1, 2)$, minimum

iii. No x -intercepts, y -intercept is 3.

e. $y = x^2 + 12x + 35, -9 \leq x \leq 0$

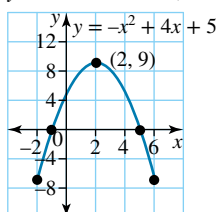


i. $x = -6$

ii. $(-6, -1)$, minimum

iii. x -intercepts are -7 and -5 , y -intercept is 35.

f. $y = -x^2 + 4x + 5, -2 \leq x \leq 6$

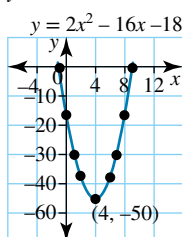


i. $x = 2$

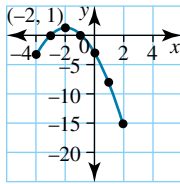
ii. $(2, 9)$, maximum

iii. x -intercepts are -1 and 5 , y -intercept is 5.

g. $y = 2x^2 - 16x - 18, -1 \leq x \leq 9$



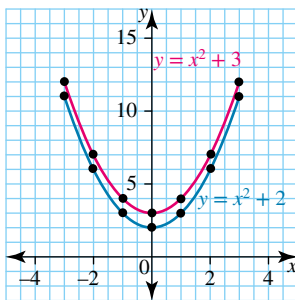
- i. $x = 4$
 - ii. $(4, -50)$, minimum
 - iii. x -intercepts are -1 and 9 , y -intercept is -18 .
- h. $y = -x^2 - 4x - 3$, $-4 \leq x \leq 2$



- i. $x = -2$
- ii. $(-2, 1)$, maximum
- iii. x -intercepts are -3 and -1 , y -intercept is -3 .

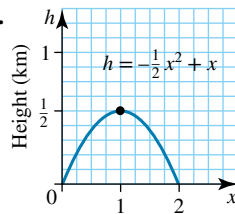
5. a.

x	-3	-2	-1	0	1	2	3
$y = x^2 + 2$	11	6	3	2	3	6	11
$y = x^2 + 3$	12	7	4	3	4	7	12



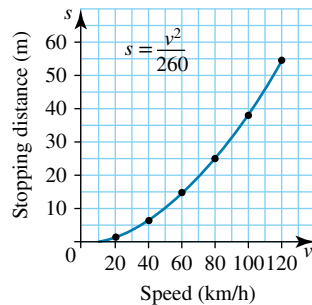
- b. Axis of symmetry: $x = 0$ for both equations.
- c. No x -intercepts for either equation.

6.



The maximum height is 500 m.

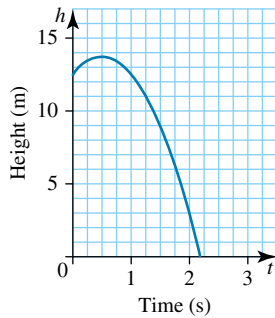
7. a. Stopping distances



- i. 14 m
 - ii. 38 m
 - iii. 55 m
- b. 81 km/h
- 8. a. -21°C
 - b. 1 hour and at 21 hours
 - c. 100°C

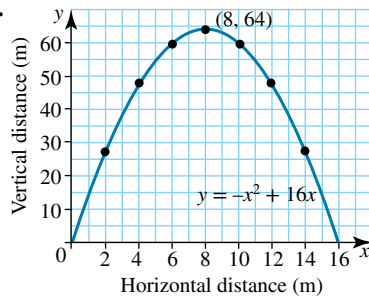
9. a. 12.5 metres

b.



c. 2.2 seconds

10.

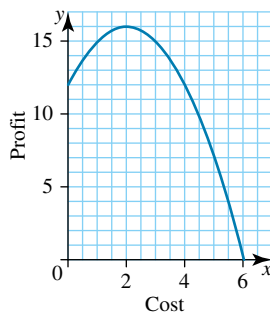


No. They were 2.3 metres across.

12. a. 1.3 m

b. 13 m

13. a.



b. The graph should not be to the left of the P axis because Ruby would be selling the mini Easter eggs at a negative cost there.

c. The cost of the Easter eggs should be \$2 to make a profit of \$16.

d. Ruby could charge up to \$6 before she makes a loss.

14. a. $\frac{x_2 + 10}{2} = -4$
 $x_2 = 2$

b. Answers will vary but must be of the form $y = a(x - 2)(x + 10)$.

15. a. The vertex of $y = 2x^2 - 6x$ is $\left(\frac{3}{2}, -\frac{9}{2}\right)$.

The vertex of $y = 2x^2 - 6x - 3$ is $\left(\frac{3}{2}, -\frac{15}{2}\right)$.

b. Both graphs have the same x -coordinate of the vertex.

$$y = \left(x - \frac{3}{2}\right)^2 - 2.5$$

16. $y = -0.016x^2 + 0.5x + 4.5$

$$y = -0.016 \times 35.5^2 + 0.5 \times 35.5 + 4.5$$

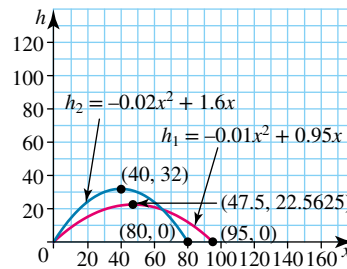
$$y = 2.086$$

The water just makes it over the fence.

17. $y = x^2 + x$

18. For a parabola, it is best to use between 6 and 12 points to plot a graph. These points should be near the main features of the parabola: the axis of symmetry, the vertex and the x - and y -intercepts.

11.

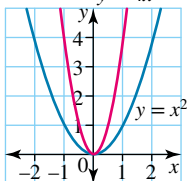


The 9 iron

c. 75 cm

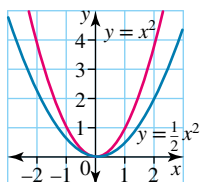
Exercise 15.4 Sketching parabolas of the form $y = ax^2$

1. $y = 4x^2$ is narrower.



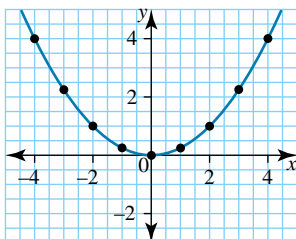
Vertex for each is at $(0, 0)$ x -int. and y -int. is 0 for both.

2. $y = x^2$ is narrower.

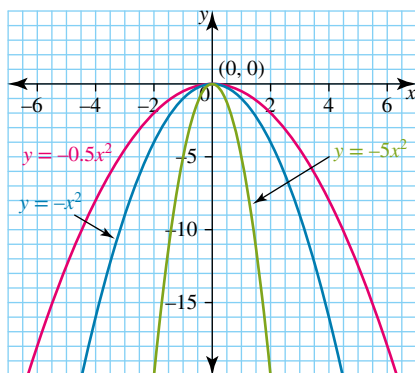


Vertex for each is at $(0, 0)$ x -int. and y -int. is 0 for both.

3. $y = 0.25x^2$



4, 5



$y = -x^2$ is narrower than $y = -0.5x^2$.

$y = -5x^2$ is the narrowest.

6. a. B b. A c. B

7. a. iii b. vi c. i d. ii e. iv f. v

8. Answers will vary; for example, $y = 2x^2$.

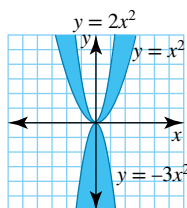
9. Answers will vary; for example, $y = -0.5x^2$.

10. a. $y = 3x^2$ b. $y = -x^2$

11. a. -350 b. -350 c. -31.5 d. -7.875 e. -16.94

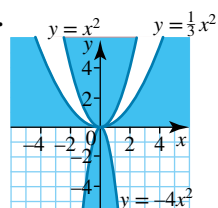
12. a. 400 watts b. 625 watts

13. a.



A tie

b.



A bow

14. $y = \frac{2}{3}x^2$

15. a. Answers will vary. b. 6

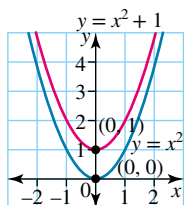
16. $k = 1$

17. $x = y^2$

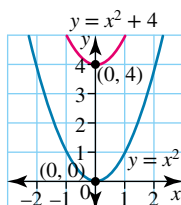
18. The axis of symmetry and the vertex location remain the same, although the vertex changes from a minimum to a maximum.

Exercise 15.5 Sketching parabolas of the form $y = ax^2 + k$

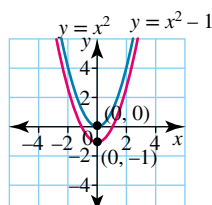
1. a.



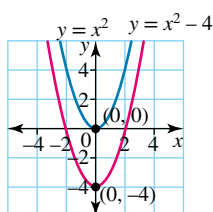
b.



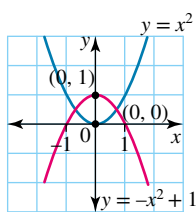
c.



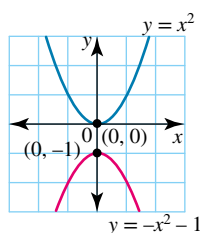
d.



e.



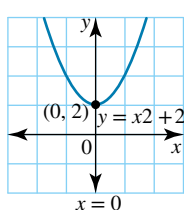
f.



2. A positive number moves the graph up.

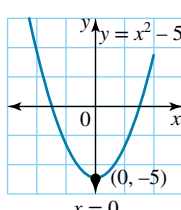
3. A negative number moves the graph down.

4. a.



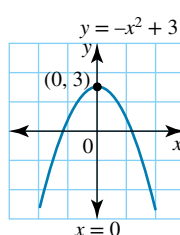
Minimum

b.



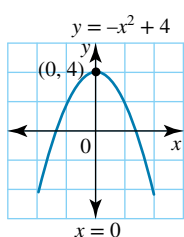
Minimum

c.



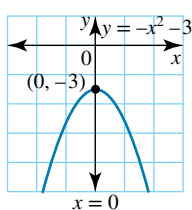
Maximum

d.



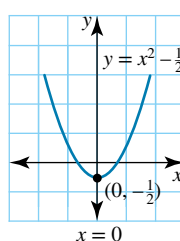
Maximum

e.



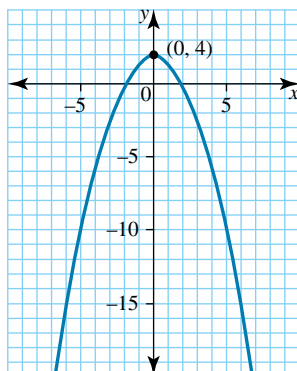
Maximum

f.

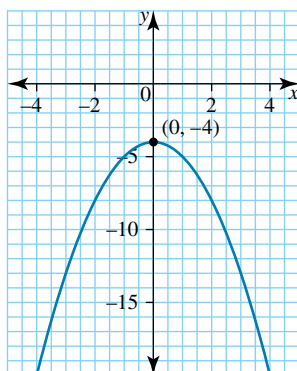


Minimum

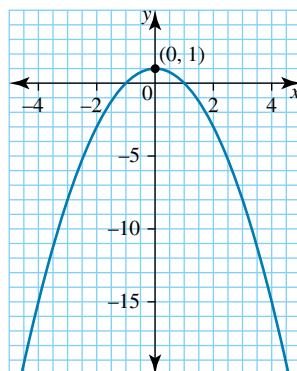
5. a. x -intercepts: $(-2, 0)$ $(2, 0)$



b. No x -intercepts



c. x -intercepts: $(-1, 0)$ $(1, 0)$



6. a. No

7. a. C

b. D

8. a. iii

b. iv

b. A negative sign inverts the graph.

c. D

d. D

c. v

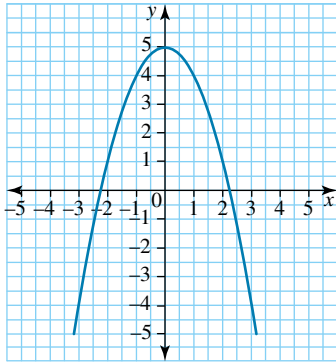
d. i

c. $x = 0$ (the y-axis)

e. vi

f. ii

9. 5 km above sea level



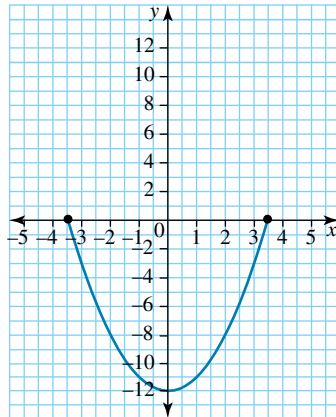
10. a. x-intercepts: $(-9, 0)$, $(9, 0)$

b. 243 cm

c. 18 cm

11. a. $-2\sqrt{3} \leq x \leq 2\sqrt{3}$

b.



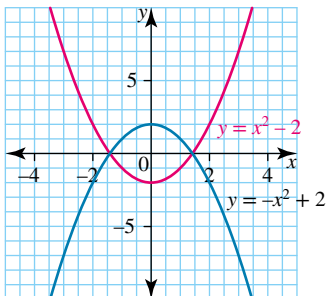
c. 12 m

d. 6.9 metres

12. a, b Answers will vary.

13. Answers will vary.

14. Intersections at $(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$



15. The cup should be placed 50 cm horizontally away from the desk.

16. $y = 2x^2 + 2$

17. The parabolas intersect symmetrically at $(-4.53, 226.11)$ and $(4.53, 226.11)$.

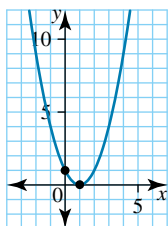
18. $y = 4 + 4\sqrt{5}$

19. If a becomes a larger number, the graph is narrower. Changing k changes the value of the y-intercept.

20. $y = -5x^2 + 50x$

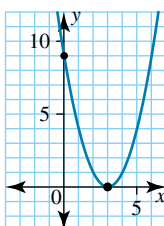
Exercise 15.6 Sketching parabolas of the form $y = (x - h)^2$

1. a. Minimum vertex (1, 0)



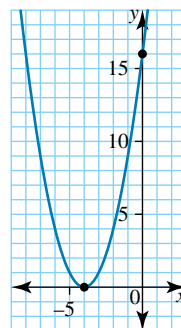
y-intercept (0, 1)

- b. Minimum vertex (3, 0)



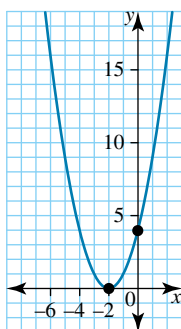
y-intercept (0, 9)

- c. Minimum vertex (-4, 0)



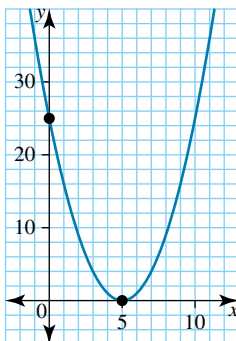
y-intercept (0, 16)

- d. Minimum vertex (-2, 0)



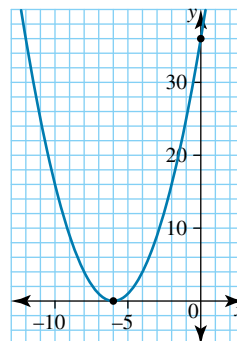
y-intercept (0, 4)

- e. Minimum vertex (5, 0)



y-intercept (0, 25)

- f. Minimum vertex (-6, 0)



y-intercept (0, 36)

2. a. $x = 1$

- b. $x = 3$

- c. $x = -4$

- d. $x = -2$

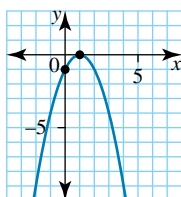
- e. $x = 5$

- f. $x = -6$

3. The graph is translated h units to the right.

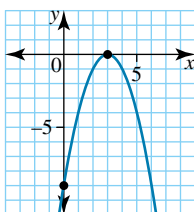
4. The graph is translated h units to the left.

5. a. Maximum vertex (1, 0)



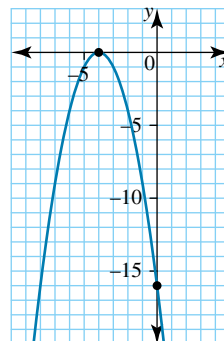
y-intercept (0, -1)

- b. Maximum vertex (3, 0)



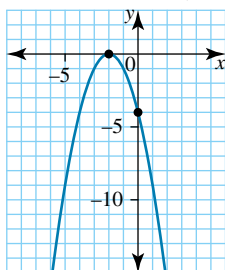
y-intercept (0, -9)

- c. Maximum vertex (-4, 0)



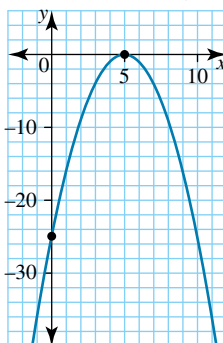
y-intercept (0, -16)

- d. Maximum vertex (-2, 0)



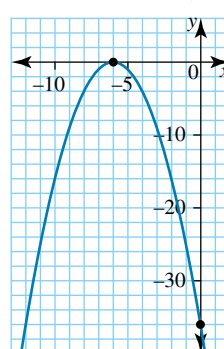
y-intercept (0, -4)

- e. Maximum vertex (5, 0)



y-intercept (0, -25)

- f. Maximum vertex (-6, 0)



y-intercept (0, -36)

6. No, the graph is inverted, but with the same axis of symmetry and vertex. The y -intercept changes.

7. a. C

b. D

8. a. vi

b. ii

c. iii

d. i

e. iv

f. v

9. a. $y = (x - 2)^2$ or $y = (x - 4)^2$

b. $y = (x - 2)^2$ or $y = (x + 4)^2$

10. a. No difference

b. 2nd graph is translated 4 units to the left of the 1st graph.

c. 2nd graph is 'inverted' and is the mirror image of the 1st graph in the x -axis.

11. $y = -3(x - 2)^2$

12. Answers will vary.

13. a. x -intercept (3, 0), y -intercept (0, 36)

b. Answers will vary.

14. a. $a = 2250$, $h = 2$

b. $t = 5$ (June)

c. There are only two unknown quantities (a and h). In this case there is only one x -intercept while most parabolas have two x -intercepts.

15. Answers will vary. The graphs must have vertices at (3, 0) and be of the form $y = a(x - 3)^2$.

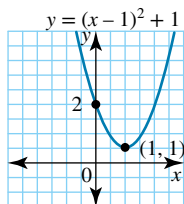
16. They are the same graph.

17. $y = 2(x + 2)^2$

18. The axis of symmetry and the location of the vertex remains the same, but the vertex changes from a minimum to a maximum.

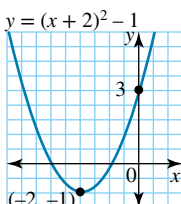
Exercise 15.7 Sketching parabolas of the form $y = (x - h)^2 + k$

1. a.



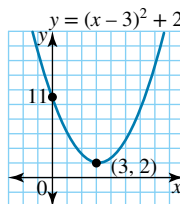
Minimum

b.



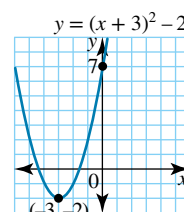
Minimum

c.



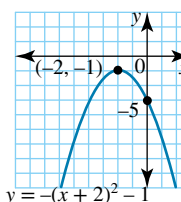
Minimum

d.



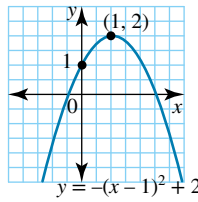
Minimum

e.



Maximum

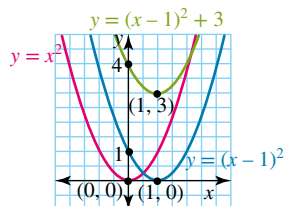
f.



Maximum

2. It is easy to find the vertex (h , k) from the equation.

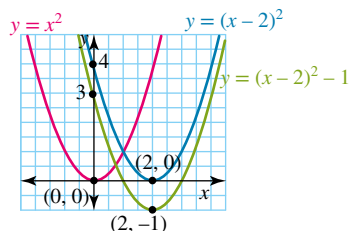
3. a. i.



ii. 1 right

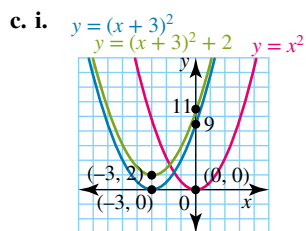
iii. 1 right, 3 up

b. i.



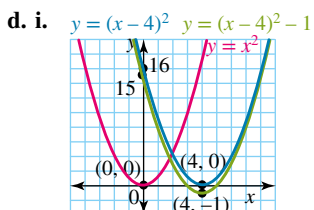
ii. 2 right

iii. 2 right, 1 down



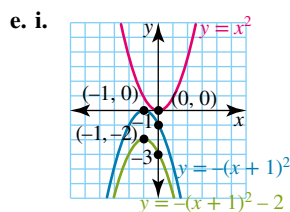
ii. 3 left

iii. 3 left, 2 up



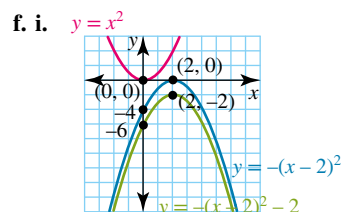
ii. 4 right

iii. 4 right, 1 down



ii. Reflect in the x -axis, 1 left

iii. Reflect in the x -axis, 1 left, 2 down



ii. Reflect in the x -axis, 2 right

iii. Reflect in the x -axis, 2 right, 2 down

4. a. D

b. A

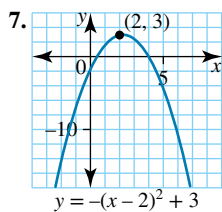
c. C

5. a. $y = (x+2)^2 + 3$

b. $(-2, 3)$

6. a. Answers may vary, for example $y = (x+3)^2 - 4$ is the simplest.

b. 5



8. a. $y = (x-2)^2 + 1$

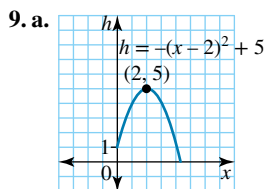
b. $y = -(x-2)^2 - 1$

c. $y = (x+2)^2 - 1$

d. $y = -(x-3)^2 + 4$

e. $y = (x+1)^2 - 4$

f. $y = (x-1)^2 - 4$



b. 1 metre

c. 5 metres

10. a. $y = -(x+2)^2$

b. $y = (x-3)^2 - 1$

c. $y = (x+1)^2 + 2$

d. $y = -(x-4)^2 + 6$

11. a. 150 metres

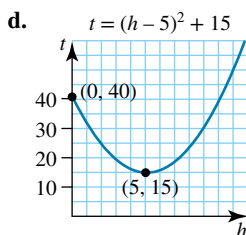
b. $y = \frac{-4}{225}(x-150)^2 + 400$

12. C

13. a. 40°C

b. Answers will vary.

c. 5 hours



14. $y = 6(x - 1)^2 - 10$

Axis of symmetry $x = 1$

Therefore, $h = 1$.

$k = -10$ (minimum)

y -intercept $= -4$ implies $-4 = a(0 - 1)^2 - 10$, or $a = 6$.

15. Minimum value at $y = 2$, so $k = 2$.

Axis of symmetry at $x = 4$, so $h = 4$.

Therefore, $a = 0.5$.

$y = 0.5(x - 4)^2 + 2$

Check using the y -intercept.

$y = 0.5(-4)^2 + 2 = 10$, therefore confirmed.

16. a. $y = (x - 3)^2 - 2$ is obtained by translating $y = (x - 3)^2$ vertically 2 units down.

$y = (x - 3)^2 + 1$ is obtained by translating $y = (x - 3)^2$ vertically 1 unit up.

b. $y = (x - 5)^2 - 4$

17. $y = -\frac{1}{4}(x + 1)^2 + 4$

18. $x = 5a$ and $x = -a$

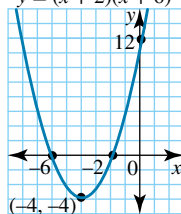
19. $y = -\left(x - \frac{1}{2}\right)^2 + \frac{29}{4}$

20. $x = h \pm \sqrt{\frac{k}{a}}$

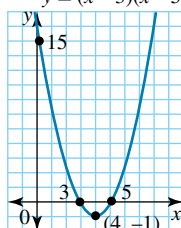
21. When the equation is in vertex form $y = (x - h)^2 + k$, it is easy to determine the location of the vertex (h, k) and the location of the axis of symmetry $x = h$.

Exercise 15.8 Sketching parabolas of the form $y = (x + a)(x + b)$

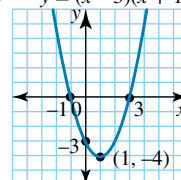
1. a. $y = (x + 2)(x + 6)$



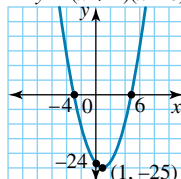
b. $y = (x - 3)(x - 5)$



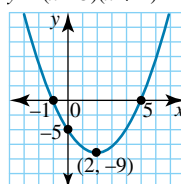
c. $y = (x - 3)(x + 1)$



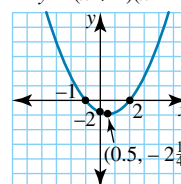
d. $y = (x + 4)(x - 6)$



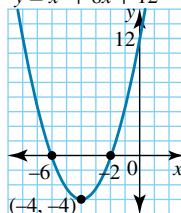
e. $y = (x - 5)(x + 1)$



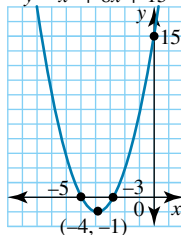
f. $y = (x + 1)(x - 2)$



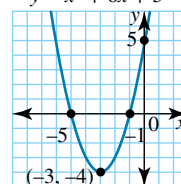
2. a. $y = x^2 + 8x + 12$

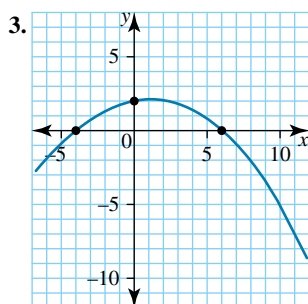
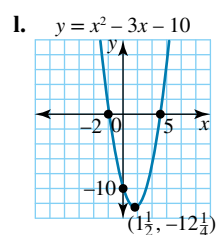
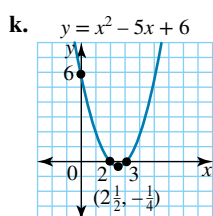
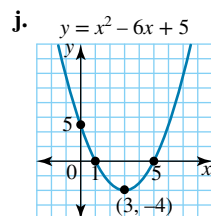
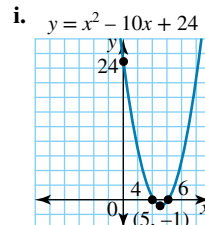
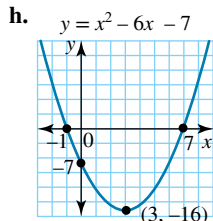
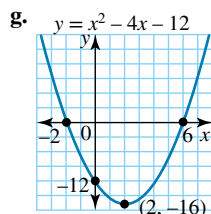
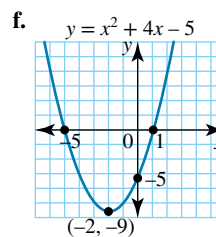
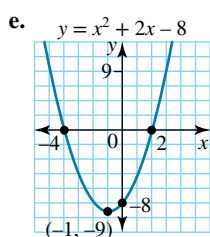
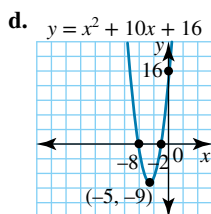


b. $y = x^2 + 8x + 15$



c. $y = x^2 + 6x + 5$





4. $y = (x - 2)(x - 6)$

5. a. 20 m b. 4, 24

c. (4, 0), (24, 0)

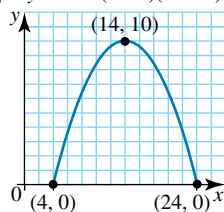
d. $x = 14$

e. (14, 10)

f. 10 m

g. 10 m

h. $y = -0.1(x - 4)(x - 24)$

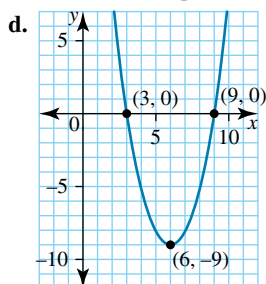


6. 12 m

7. a. (6, -9) b. $y = (x - 6)^2 - 9$

c. Intercept form expanded: $y = x^2 - 12x + 27$

Vertex form expanded: $y = x^2 - 12x + 36 - 9 = x^2 - 12x + 27$



8. $x = 11$

9. a. Since the coefficient of t^2 is negative, the vertex is a maximum.

b. Maximum height = 512 m at $t = 8$ seconds.

10. a. From $t = 6$ to $t = 12$; $h = -0.5(t - 8.5)^2 + 15.125$

From $t = 12$ to $t = 21$; $h = -t + 21$

b. $0 \leq t \leq 6$, $6 \leq t \leq 12$, $12 \leq t \leq 21$

11. The general vertex is $\left(\frac{b}{2}, \frac{b^2}{4}\right)$.

12. Answers will vary but must be of the general form $y = a\left(x + \frac{1}{2}\right)\left(x - \frac{3}{4}\right)$.

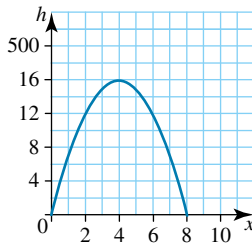
13. $y = 2x^2 - x - 5$

14. The intercept form of the parabola equation is $y = (x + a)(x + b)$. In this form it is easy to determine the intercepts. The x -intercepts are $x = -a$ and $x = -b$. The y -intercept is $y = ab$. The axis of symmetry is midway between the x -intercepts,

$$x = \frac{-(a + b)}{2}.$$

Exercise 15.9 Applications

1. a.

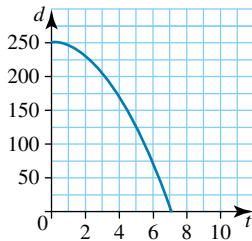


b. 16 m

2. a. i. 19.6 m

ii. 240.1 m

b.



3. a. 120 metres

b. 2 minutes

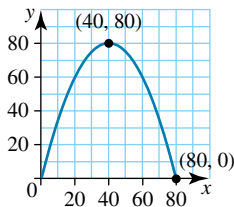
4. a. i. 75 km

ii. 60 km

b. 80 km

c. 80 km high after travelling 40 km horizontally

d.



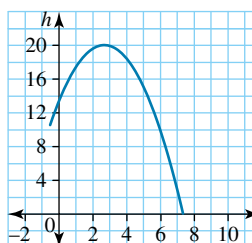
5. a. 14 m

b. 20 m

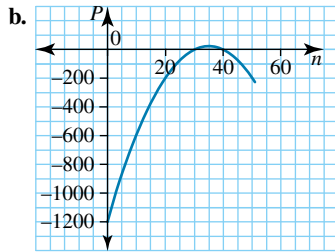
c. 7 s

d. 20.25 m

e.



6. a. No profit



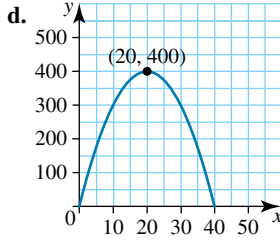
c. 30 bears

d. \$25; 35 bears

7. a. $2(x + y) = 80$

b. Answers will vary.

c. $A = x(40 - x)$



e. 400 m^2

f. $x = 20 \text{ m}; y = 20 \text{ m}$

8. a. Perimeter $= 2x + y = 55 \text{ m}$, so $y = 55 - 2x$

b. Area $= xy = x(55 - 2x) = 55x - 2x^2$

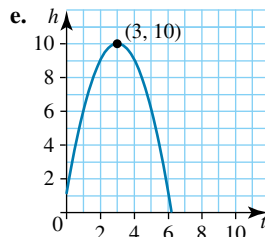
c. $x = 13.75 \text{ m}, y = 27.5 \text{ m}$, area $= 378.125 \text{ m}^2$

9. a. 1 m

b. 9 m

c. Answers will vary.

d. From 2 to 4 seconds



f. 10 m

g. 6 s

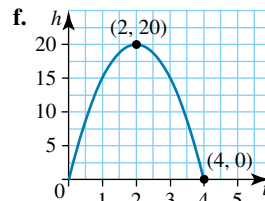
10. a. Inverted

b. $h = 5t(4 - t)$

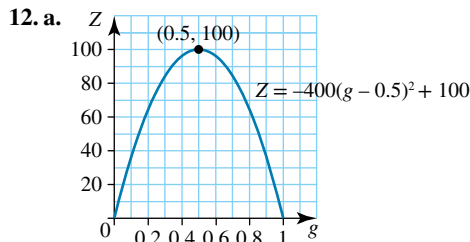
c. $t = 0, t = 4$; 4 seconds

d. Maximum vertex at (2, 20)

e. 20 m



11. Let x = horizontal distance, y = vertical distance across the bridge ($0 \leq x \leq 40$) $y = -0.01x(x - 40)$



b. $g = 0.5$

c. $0.15 \leq g \leq 0.85$

13. a. 7th to 8th month

b. 3rd and 12th months

14. Answers will vary.

15. The height of the platform deck above the water is 2.7 m.

16. When applying your knowledge to practical problems it is important to ensure that the solution satisfies any physical constraints of the problem. For example, there cannot be negative solutions to measurements of length or time.

17. Answers will vary.

Investigation | Rich task

1–5 Check with your teacher.

6 Answers will vary.

Exercise 15.10 Review questions

1. a. B b. C c. A d. E
2. a. B b. A c. A
3. a. C b. D c. B d. D
4. a. C b. D c. C
5. a. D b. A c. B
6. D
7. E
8. B
9. A

10. C

11. a. $x = 0$, vertex $(0, -4)$, minimum, x -intercepts are -2 and 2 , y -intercept is -4 .

b. $x = 0$, vertex $(0, 9)$, maximum, x -intercepts are 3 and -3 , y -intercept is 9 .

c. $x = 1$, vertex $(1, 0)$, minimum, x -intercept is 1 , y -intercept is 1 .

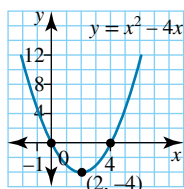
d. $x = -3$, vertex $(-3, 0)$, maximum, x -intercept is -3 , y -intercept is -9 .

e. $x = -1$, vertex $(-1, 1)$, minimum, no real x -intercepts, y -intercept is 2 .

f. $x = -1$, vertex $(-1, -4)$, minimum, x -intercepts are 1 and -3 , y -intercept is -3 .

12. a. $y = x^2 - 4x$, $-2 \leq x \leq 6$

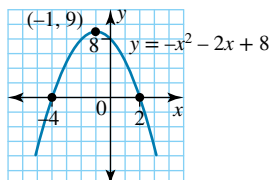
x	-2	-1	0	1	2	3	4	5	6
y	12	5	0	-3	-4	-3	0	5	12



$x = 2$, vertex $(2, -4)$, minimum, x -intercepts are 0 and 4 , y -intercept is 0 .

b. $y = -x^2 - 2x + 8$, $-5 \leq x \leq 3$

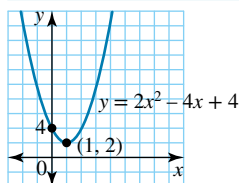
x	-5	-4	-3	-2	-1	0	1	2	3
y	-7	0	5	8	9	8	5	0	-7



$x = -1$, vertex $(-1, 9)$, maximum, x -intercepts are -4 and 2 , y -intercept is 8 .

c. $y = 2x^2 - 4x + 4$, $-2 \leq x \leq 3$

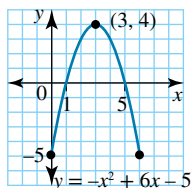
x	-2	-1	0	1	2	3
y	20	10	4	2	4	10



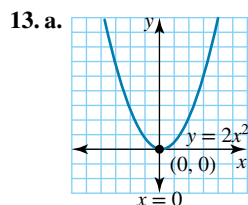
$x = 1$, vertex $(1, 2)$, minimum, no x -intercepts, y -intercept is 4 .

d. $y = -x^2 + 6x - 5, 0 \leq x \leq 6$

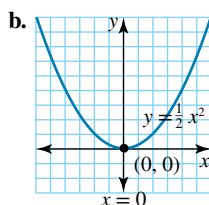
x	0	1	2	3	4	5	6
y	-5	0	3	4	3	0	-5



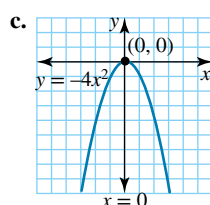
$x = 3$, vertex $(3, 4)$, maximum, x -intercepts are 1 and 5, y -intercept is -5 .



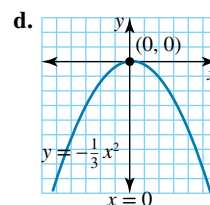
Minimum



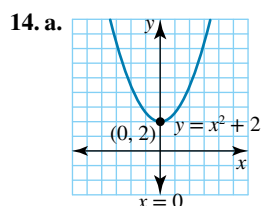
Minimum



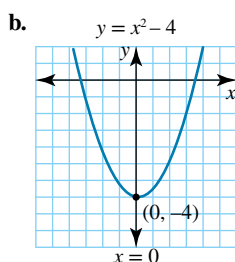
Maximum



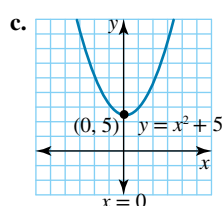
Maximum



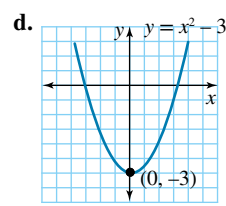
Minimum



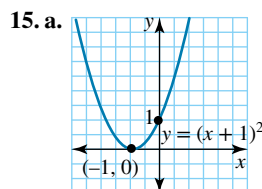
Minimum



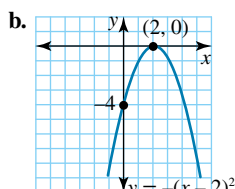
Minimum



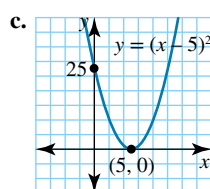
Minimum



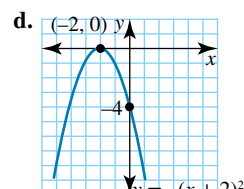
Minimum



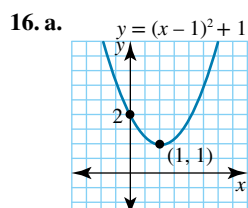
Maximum



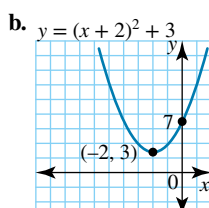
Minimum



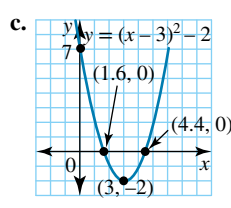
Maximum



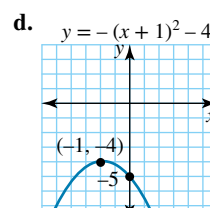
Minimum



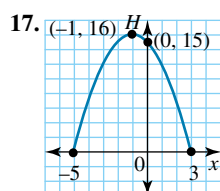
Minimum



Minimum



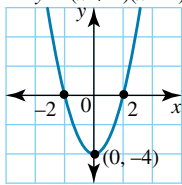
Maximum



$H = -(x + 1)^2 + 16$

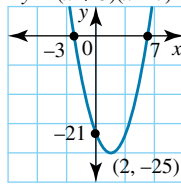
16 metres

18. a. $y = (x + 2)(x - 2)$



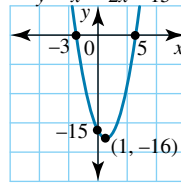
Minimum

b. $y = (x + 3)(x - 7)$



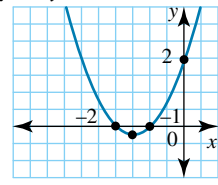
Minimum

c. $y = x^2 - 2x - 15$



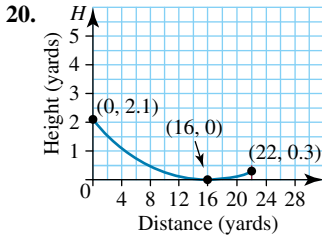
Minimum

d. $y = x^2 + 3x + 2$



Minimum

19. 50 metres, 20 metres

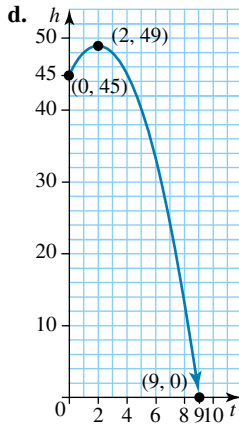


Yes

21. a. 45 m

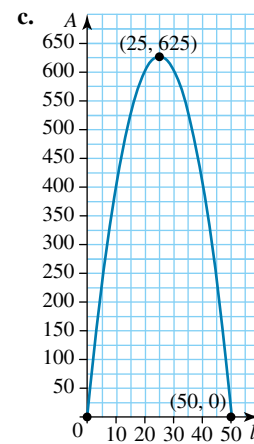
b. 9 s

c. 49 m



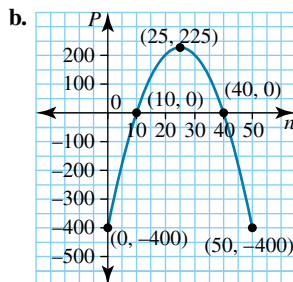
22. a. $w + l = 50$

b. $A = l(50 - l)$



d. 625 cm²

23. a. \$125



c. \$225 for 25 sausages

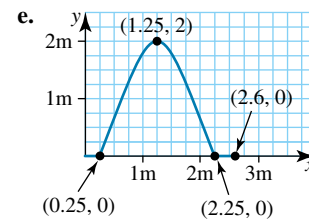
d. 10 or 40 sausages

24. a. 2.6 m

b. $p = 0.25, q = 2.25$

c. $x = 1.25$

d. 2 m



GLOSSARY

Acute angle: angle greater than 0° but less than 90°

Adjacent side [trigonometry]: the side of a right-angled triangle that is adjacent to the reference angle

Algebraic equations: equations that contain pronumerals (letters)

Algebraic fractions: fractions that contain pronumerals (letters)

Angle of depression: angle measured down from a horizontal line (through the observation point) to the line of vision

Angle of elevation: angle measured up from the horizontal line (through the observation point) to the line of vision

Arc: a portion of the circumference of a circle

Area: amount of surface enclosed by the shape. Area is measured in square units; such as square metres, m^2 , and square kilometres, km^2 .

Array: an arrangement of numbers

Asymptote: a line that a graph approaches but never meets

Average speed: the total distance travelled during a journey divided by the total time taken; given by the formula: $\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$

Axis of symmetry: vertical line dividing a parabola into two equal halves, one the mirror image of the other

Back-to-back stem-and-leaf plot: a stem-and-leaf plot that compares two related sets of data

Bar charts: charts that use bars to display categorical data; the height of each bar represents the frequency of that category

Base: digit at the bottom of numbers written in index form; for example, in 6^4 , the base is 6. This tells us that 6 is multiplied by itself four times.

Basic numeral: answer to a number written in index form; for example, the basic numeral of 6^2 is 36

Bias: opinion or process that prevents a fair consideration of an issue

Bi-modal distributions: distributions of data that have two modes

Binomial factor: factor containing two terms; for example, in the expression $5(x + y) + 6b(x + y)$, $(x + y)$ is a binomial factor. It contains two terms and is common to both terms of the expression.

Boxplot: graphical representation of the 5-number summary; that is, the lowest score, lower quartile, median, upper quartile and highest score for a particular set of data

Cancelling: dividing the numerator and denominator of a fraction by the highest common factor of both

Capacity: maximum amount of fluid contained in an object. It is usually applied to the measurement of liquids and measured in units; such as, millilitres (mL), litres (L) and kilolitres (kL).

Cartesian coordinates (x, y) : pairs of numbers that give the position of a point on the 2-dimensional flat plane, the Cartesian plane. The first number is the x -coordinate and the second number is the y -coordinate.

Cartesian plane: an area that consists of two number lines at right angles to each other and includes all the space between the two number lines

Census: process of collecting data from the entire target population

Chance experiment: a process, such as rolling a die, that can be repeated many times

Circumference: distance around the outside of a circle. It is given by the rule $2\pi r$ or πD , where r is the radius and D is the diameter of the circle.

Class centre: *see* Midpoint

Class intervals: groups of equal size into which grouped data is divided

Coefficient: the number part of a term

Commission: method of payment usually calculated as a percentage of the value of goods sold

Complement (of a set): complement of a set, A , written A' is the set of elements that are in ξ but not in A

Composite shape: figure made up of more than one basic shape

Compound interest: interest that is paid on the sum of the principal plus preceding interest over time;

$$A = P(1 + R)^n$$

Compute: calculate

Conditional probability: where the probability of an event is conditional (depends) on another event occurring first. For two events A and B , the conditional probability of event B , given that event A occurs, is denoted by $P(B|A)$ and can be calculated using the formula: $P(B|A) = \frac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$.

Cone: pyramid with a circular base

Congruent figures: identical figures with exactly the same shape and size

Constant: term without a pronumeral; that is, a number

Constant of proportionality, k [direct proportion]: the number multiplying x in the equation $y = kx$; equal to the gradient of the corresponding graph; also called the constant of variation

Continuous: describes numerical data in which the information can take values that are not whole numbers

Cosine (cos) ratio: ratio of the adjacent side to the hypotenuse in a right-angled triangle;

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ or } \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

Counting numbers: *see* Natural numbers

Cross-section: identical 'slice' produced when cuts are made across a prism parallel to its ends

Cube: polyhedron with 6 faces. All faces are squares of the same size.

Cube root ($\sqrt[3]{}$): one of three equal factors of a number. Finding a cube root is the opposite of cubing a number.

Cuboid (or rectangular prism): polyhedron with 6 faces, at least two of which are rectangles opposite each other

Cylinder: solid with circular ends

Denominator: bottom term of a fraction. It shows the total number of parts into which the whole has been divided.

Dependent: describes successive events in which one event affects the occurrence of the next

Dependent variable: y -axis (or vertical) variable

Difference of two squares: type of expansion of a pair of brackets $(a + b)(a - b) = a^2 - b^2$

Dilation: non-isometric transformation, also known as stretching, enlargement or reduction

Direct proportion: describes a particular relationship between two variables (or quantities); that is, as one variable increases, so does the other variable. The graph of the relationship is a straight line, passing through the origin and the rule used to relate the two variables is $y = kx$.

Discount: reduction in the price of an article usually calculated as a percentage of the marked price

Discrete: describes numerical data in which the information can only take certain exact values, usually whole numbers

Disjoint sets: sets with no elements in common with each other

Distributive Law: the product of one number with the sum of two others equals the sum of the products of the first number with each of the others; for example $4(6 + 2) = 4 \times 6 + 4 \times 2$. It is also applicable to algebra; for example, $3x(x + 4) = 3x^2 + 12x$.

Double time: paid twice the normal hourly rate

Empty set: a set that contains no elements (also known as a null set); represented by $\emptyset$

Equally likely outcomes: outcomes in a probability experiment with the same chance of occurring

Equation: a mathematical statement that shows two equal expressions

Equidistant: equal in distance

Equivalent fractions: different fractions representing the same amount; formed by multiplying (or dividing) both the numerator and denominator by the same (non-zero) value

Error: difference between a measurement and the actual value

Estimation: approximate answer based on information given

Event: set of favourable outcomes in each trial of a probability experiment

Expanding (algebra): multiplying everything inside the brackets by what is directly outside the brackets.

Expanding is the opposite of factorising.

Expected value: number of times an event is expected to occur from a certain number of trials.

The expected value $E(x) = n \times p$, where n represents the number of trials and p is the probability of success or the probability of that event occurring in one trial.

Experiment: process of performing repeated trials of a probability activity to obtain data that predict the chances of certain things happening

Experimental probability: probability determined by observing an experiment and gathering data;

$$P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

Expression: group of terms separated by + or – signs. Expressions do not contain equals signs.

Factor form: displays all the multiplications of a number written in index form; for example,

$$5^3 = 5 \times 5 \times 5$$

Factor pairs: pairs of numbers that multiply together to equal another number. They are factors of the same number.

Factorising: breaking down a number or expression into smaller factors

Factors: whole numbers that divide exactly into another whole number, without any remainder

Favourable outcome: A favourable outcome for an event is any outcome that belongs to the event. For event A (rolling an even number on a die), the favourable outcomes are 2, 4 and 6.

Fifth Index Law: to remove brackets containing a product, raise every part of the product to the index outside the brackets. So, $(ab)^m = a^m b^m$.

Finite: fixed number or amount; for example the decimal 0.25 has a fixed number of decimal places

First Index Law: when terms with the same base are multiplied, the indices are added. So,

$$a^m \times a^n = a^{m+n}.$$

Fixed (costs): describes costs that occur at regular intervals and at the same amount

FOIL: diagrammatic method of expanding a pair of brackets where the letters in **FOIL** represent the order of the expansion; **F**irst, **O**uter, **I**nnner and **L**ast

Formula: an equation or rule that describes the relationship between different quantities

Fourth Index Law: to remove brackets, multiply the indices inside the brackets by the index outside the brackets. Where no index is shown, assume that it is 1. So, $(a^m)^n = a^{mn}$.

Frequency: number of times a particular piece of data appears

Frequency polygon: special type of line graph with the same scaled axes as the histogram. Midpoints on top of the histogram columns are joined by straight lines. The polygon is closed by drawing lines at each end down to the score (x)-axis.

Function: a process that takes a set of x -values and produces a related set of y -values. For each distinct x -value, there is only one related y -value. A function is usually defined by a formula for $f(x)$ in terms of x ; for example, $f(x) = x^2$.

Fundamental Theorem of Algebra: The Fundamental Theorem of Algebra states that quadratic functions can have a maximum of two solutions.

Gradient (slope), m : measure of how steep something is. The gradient of a straight line is given by

$$m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{y_2 - y_1}{x_2 - x_1} \text{ and is constant anywhere along that line.}$$

Gross salary: the salary a person receives before deductions

Gross wage: amount paid before tax is taken out

Hectare (ha): a unit of area measurement where the space enclosed is equal to the space enclosed by a square with side lengths of 100 m. Hectares are often used to measure land area ($1 \text{ ha} = 10000 \text{ m}^2$). There are 100 hectares in 1 square kilometre.

Higher order roots ($\sqrt[n]{}$): include the cube root, the fourth root and so on; for example, the cube root of 27 is written as ($\sqrt[3]{27}$) and equals 3. The fourth root of 16 is written as ($\sqrt[4]{16}$) and equals 2, since $2 \times 2 \times 2 \times 2 = 16$.

Highest common factor (HCF): largest factor common to all the numbers or expressions given

Histogram: column graph with no gaps between the columns and each column ‘straddles’ an x -axis score, starting and finishing halfway between scores. The x -axis scale is continuous and usually a half-interval is left before the first column and after the last column.

Hyperbola: the shape of the graph formed by an inverse proportion relationship

Hypotenuse: longest side of a right-angled triangle, always opposite the right angle

Image: an enlarged or reduced copy of a shape (the original shape is called the object)

Improper fraction: fraction with numerator larger than its denominator, for example $\frac{5}{4}$

Income tax: a tax levied on the financial income of people

Independent variable: x -axis (or horizontal) variable

Index (power or exponent): indicates the number of times a base is to be multiplied, when written in index form

Index form: form of writing the product of prime factors. It has a base and an index or power. The index indicates how many times the base is being multiplied by itself.

Index notation: the short way of writing a number or pronumeral if it is multiplied by itself

$$8^6 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 262144$$

Index form Expanded form Basic numeral

Inequation: expression containing an inequality sign; that is, $<$, $>$, $\geq$ or $\leq$

Infinite: never ending; for example the decimal $0.\dot{3}$ is non-terminating and therefore its number of decimal places cannot be counted

Integers (Z): positive whole numbers, negative whole numbers and zero

Interest: percentage charged for the use of someone else’s money

Interquartile range (IQR): difference between the upper quartile, Q_3 and the lower quartile, Q_1 ; that is, $IQR = Q_3 - Q_1$. It is the range of the middle half of the data.

Intersection (of sets): represents the common elements of two or more sets. $A \cap B$ denotes the intersection of sets A and B .

Inverse operation: opposite operations such as $+$ and $-$, or $\div$ and $\times$

Inverse proportion: a relationship in which increasing one variable decreases the other

Irrational numbers (I): cannot be written as fractions. Examples of irrational numbers include surds, π and non-terminating, non-recurring decimals.

Like terms: terms containing exactly the same pronumeral (letter) part. For example, $3ab$ and $7ab$ are like terms, but $5a$ is not.

Linear equations: equations in which the variable has an index (power) of 1

Linear graph: a relationship between two variables where a straight line is formed when the points are plotted

Loading: an additional payment granted to a worker for holidays or for working in hazardous or difficult conditions

Loss: difference between the total of a retailer’s costs (cost price) and the price at which the goods are sold (selling price). It occurs when the costs exceed the selling price and is calculated by the rule:

$$\text{Loss} = \text{cost} - \text{selling price}.$$

Lower quartile: score marking the end of the first quarter in an ordered set of data. It is denoted by Q_1 and is calculated by finding the median of the lower half of the data.

Lowest common multiple (LCM): lowest multiple that two or more numbers have in common

Maximum error [measurement]: half the degree of accuracy

Maximum vertex: highest point on an inverted parabola

Mean: measure of the centre (average) of a set of data, given by $\text{Mean} = \frac{\text{sum of all scores}}{\text{number of scores}}$ or

$$\bar{x} = \frac{\Sigma x}{n}. \text{ When data are presented in a frequency distribution table, } \bar{x} = \frac{\Sigma(f \times x)}{n}.$$

Measure of central tendency: a single value that represents a data set; *mean*, *median* and *mode* are all measures of central tendency

Measures of spread: measures of how far data values are spread from the centre or from each other; range and *interquartile range (IQR)* are both measures of spread

Median: measure of the centre (middle) of a set of data. It is the middle score for an odd number of scores arranged in numerical order (ordered). For an even number of scores, the median is the average of the two middle scores when they are ordered. Its location is determined by the rule: $\frac{n+1}{2}$.

Mediator: perpendicular bisector of a line segment

Metric system: the system of measurement based on the metre

Midpoint [coordinates]: point halfway between two given points

Midpoint [data]: centre score of each class interval in grouped data

Minimum vertex: lowest point on an upright parabola

Mode: measure of the centre of a set of data. It is the score that occurs most often. There may be no mode (all scores occur once), one mode or more than one mode (two or more scores occur equally frequently).

Multi-step equations: require more than one arithmetic operation to find the value of the variable

Mutually exclusive: cannot occur together. On a Venn diagram, two mutually exclusive events appear as disjoint sets.

Natural numbers (N): positive whole numbers; 1, 2, 3, 4,...

Negatively skewed (distribution): describes a distribution that has larger amounts of data as the values of the data increase

Net: 2-dimensional plan that can be cut out and folded to form the 3-dimensional shape

Net salary: the salary a person receives after deductions

Null Factor Law (NFL): used to solve quadratic equations; states that if $a \times b = 0$, then either $a = 0$ or $b = 0$ or both $a = 0$ and $b = 0$

Null set: a set that contains no elements (also known as an empty set); represented by $\emptyset$

Numerator: top term of a fraction. It shows how many parts there are.

Object [drawing]: the original figure

Obtuse angle: angle greater than 90° but less than 180°

Odds: probabilities in gambling

One-step equation: requires only one arithmetic operation to find the value of the variable

Opposite side [trigonometry]: the side of a right-angled triangle that is opposite the reference angle

Origin: the point on the Cartesian plane where the x - and y -axes intersect

Outcome: result obtained from a probability experiment

Outliers: extreme values that distort a set of data

Overtime: time worked beyond normal hours

Pair of compasses: instrument with two hinged legs used to make geometrical constructions

Parabola: graph of a quadratic function

Parallel lines: lines that run in the same direction and never meet

Partial variation: describes a relationship between two variables in which the dependent variable is the sum of two components — a direct variation (kx) and a constant (c)

Percentage: out of 100 or per hundred; for example, 81% means 81 out of 100 or $\frac{81}{100}$; used to represent part of a quantity

Perfect square: an algebraic expression or a whole number that is multiplied by itself

Perimeter: distance around the outside (border) of a shape

Perpendicular lines: lines that intersect at right angles to each other

Pi (π): the irrational number that is the ratio of a circle's circumference to its diameter; 3.14159...

Pie charts: charts that use sectors of a circle (pieces of a pie) to represent categories

Piecework: payment for each 'piece' or article completed

Plot: place points on the Cartesian plane using the x - and y -coordinates to draw a graph of an equation

Plotting graphs: placing points on the Cartesian plane using the x - and y -coordinates to draw graphs

Polygon: 2-dimensional, closed shape with 3 or more sides, each a straight line

Polyhedron: 3-dimensional shape in which each flat surface (face) is a polygon. The plural is polyhedra.

Population: the whole group from which a sample is drawn

Positively skewed (distribution): describes a distribution that has larger amounts of data as the values of the data decrease

Prime numbers: counting numbers with exactly two factors, itself and 1

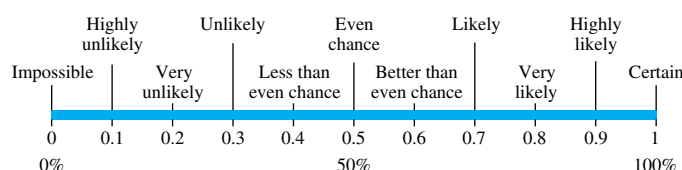
Prisms: solid shapes with identical opposite ends joined by straight edges. They are 3-dimensional figures with uniform cross-sections.

Probability: likelihood or chance of a particular event (result) occurring, given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}.$$

The probability of an event occurring ranges from 0 (impossible/will not occur) to 1 (certainty/will definitely occur) inclusive.

Probability scale:



Profit: difference between the total of a retailer's costs (cost price) and the price at which the goods are sold (selling price); given by the rule: Profit = selling price – cost

Pronumeral: letter used in place of a number

Proportion: corresponding elements are in proportion if there is a constant ratio; for example,
$$\frac{\text{circumference}}{\text{diameter}} = \pi \text{ for all circles}$$

Pythagoras' theorem: in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is written as $c^2 = a^2 + b^2$ where c is the hypotenuse and a and b are the two shorter sides.

Pythagorean triad: a set of three whole numbers that satisfy Pythagoras' theorem

Quadrant (sector): a quarter circle

Quadrants [Cartesian plane]: four regions of the Cartesian plane produced by the intersection of the x - and y -axes

Quadratic binomial: algebraic expression containing two terms, where the pronumeral with the highest power is a squared term

Quadratic equation: equation where the term with the highest power is a squared term

Quadratic trinomial: algebraic expression containing three terms, where the pronumeral with the highest power is a squared term

Quadrilateral: 2-dimensional closed shape formed by four straight sides

Quotient: the answer you obtain when you divide one number by another

Radius [plural 'radii']: the distance from the centre of a circle to its circumference

Random: governed by chance; cannot be predicted

Random selection (or random sampling): survey that ensures all subjects have an equal chance of inclusion in the sample

Range: difference between the highest and lowest scores in a set of data:

Range = highest score – lowest score

Rate: a ratio that compares the size of one quantity with that of another quantity; a method of measuring the change in one quantity in response to another

Ratio: comparison of two or more quantities of the same kind

Rational numbers (Q): can be written as fractions, where the denominator is not zero

Real numbers (R): set of all rational and irrational numbers

Recurring decimal: a decimal with one or more digits repeated continuously; for example 0.999...

They can be expressed exactly by placing a dot or horizontal line over the repeating digits; as in $8.343434 \dots = 8.\dot{3}4$ or $8.\overline{34}$.

Relative frequency [data analysis]: frequency of a particular score divided by the total sum of the frequencies; given by the rule: $\text{Relative frequency of a score} = \frac{\text{frequency of the score}}{\text{total sum of frequencies}}$

Relative frequency [probability]: the number of successful events for a finite sample of trials divided by the total number of trials

Retainer: fixed amount paid to a sales representative

Revolution: *see* Perigon

Right angle: angle that equals 90°

Right-angled triangle: triangle containing a right (90°) angle

Right prism: a solid object with a uniform cross-section and flat sides at right angles to the base

Rounding: technique used to give an approximate answer. If rounding to the first digit when the second digit is 0, 1, 2, 3 or 4, the first digit stays the same. If the second digit is 5, 6, 7, 8 or 9, then the first digit is rounded up to the next value. This also applies to decimals; for example 2.456 rounded to 2 decimal places is 2.46.

Royalty: payment made to people who own copyright, calculated as a percentage of sales; for example, a musician who writes a piece of music is paid a royalty on CD sales

Salary: money earned by people at a fixed annual (yearly) amount, usually paid fortnightly or monthly. There is no extra pay for hours outside the normal work period.

Sample: part of a population chosen so as to give information about the population as a whole

Sample space: lists all the possible outcomes from a probability experiment; written as ξ or S , and the list is enclosed in a pair of curled brackets { }

Scale factor: the factor by which an object is enlarged or reduced

Scientific notation (or standard form): used to express very large or very small numbers. To express a number in standard form, write it as a number between 1 and 10 multiplied by a power of 10; for example 64350000 can be written as 6.435×10^7 in standard form.

Second Index Law: when terms with the same base are divided, the indices are subtracted.

So, $a^m \div a^n = a^{m-n}$.

Sector: portion of a circle formed by two radii and the arc between them

Semicircle: a half circle

Set: collection of similar elements

Seventh Index Law: a term with a negative index can be expressed with a positive index using this law.

So, $a^{-n} = \frac{1}{a^n}$.

SI: stands for the International System of Units

Significant figures: the number of digits in a whole number or decimal that are reliably known. They are counted from the first non-zero digit; for example 0.0075 has two significant figures (7 and 5). Any zeros after a decimal point are considered to be significant; for example 1.20 has three significant figures (1, 2 and 0). Trailing zeros at the end of a whole number are not considered significant; 1900 has only two significant figures (1 and 9).

Similar figures: identical shapes with different sizes. The corresponding angles are equal in size and the corresponding sides are in the same ratio, called a scale factor.

Simple interest: the interest accumulated when the interest payment in each period is a fixed fraction of the principal. The formula used is $I = PRN$, where I is the interest earned (in \$) when a principal of \$ P is invested at an interest rate of R p.a. for a period of N years.

Simulation: simple probability experiment; such as rolling a die, tossing a coin or drawing a card, that may model or imitate real-life situations

Simultaneous equations (linear): two (or more) linear graphs with the same solution; that is, a common point

Sine (sin) ratio: ratio of the opposite side to the hypotenuse in a right-angled triangle;

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \text{ or } \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

Single variable data analysis: use of statistical methods to collect and analyse data related to one variable

Sixth Index Law: to remove brackets containing a fraction, multiply the indices of both the numerator and denominator by the index outside the brackets. So, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Sketching: drawing the graph of an equation using special features. For a straight line, two points usually the x - and y -intercepts are used. For a parabola, the y -intercept, vertex and axis of symmetry are used.

Speed: a rate that describes how quickly distance changes over a period of time; given by the formula:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Sphere: 3-dimensional object with a round shape and a fixed radius

Square root ($\sqrt{\quad}$): one of two equal factors of a number. Finding a square root is the opposite of squaring a number; for example $\sqrt{49} = 7$.

Statistics: the branch of mathematics that deals with the collection, organisation, analysis and interpretation of data

Stem-and-leaf plots: display organised data. Each piece of data in a stem-and-leaf plot is made up of two components, a stem and a leaf.

Straight angle: angle equal to 180°

Stratified selection: describes a method of random sampling of a population consisting of identifiable groups or strata, so that each group receives fair representation

Subject: pronumeral left by itself, usually on the left-hand side of a formula; for example, in the formula $V = IR$, V is the subject

Subset: smaller set within another set. It is denoted by the symbol $\subset$.

Substitute: a number that replaces a pronumeral when evaluating a formula or algebraic expression

Surd: any n th root of a number that results in an infinite non-recurring decimal pattern

Surface area (Total surface area): area of the outside surface of a 3-dimensional figure

Survey: process of collecting data from a representative sample of the population

Symmetrical distributions: distributions that have a clear centre and an even spread on either side; also referred to as *bell-shaped* or *normal distributions*

Tangent (tan) ratio: ratio of the opposite side to the adjacent side in a right-angled triangle;

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \text{ or } \tan \theta = \frac{\text{opp}}{\text{adj}}$$

Terms: parts of an equation or expression. Terms can be expressed as one or more than one pronumeral or a number only; for example $5x$, $3ab$, xyz , 8 and so on.

Terminating decimal: a decimal with a fixed number of decimal places; for example 0.6 or 2.54

Theoretical probability: probability determined by predicting outcomes rather than by gathering data;

$$P(\text{event}) = \frac{\text{number of times the event occurred}}{\text{total number of trials}}$$

Third Index Law: any term (excluding 0) with an index of 0 is equal to 1. So, $a^0 = 1$.

3-dimensional: shape that occupies space with dimensions; such as length, width and height in three directions

Three dimensions: length, width and height; the spatial dimensions formed by three lines at right angles to each other

Time and a half: payment $1\frac{1}{2}$ times the normal hourly rate

Tolerance: amount by which a quantity may vary from its normal value. A measurement is accepted if it is within the range: normal value $\pm$ tolerance.

Total surface area (TSA): *see* Surface area

Translation: horizontal (left/right) or vertical (up/down) movement

Transpose: rearrange a formula to make a particular pronumeral the subject

Transversal: line intersecting a pair (a set) of parallel lines

Travel graph: a graph representing the relationship between distance and time. The distance covered is shown on the vertical axis and the time taken on the horizontal axis.

Tree diagram: diagram used to list all possible outcomes of two or more events; the branches show the possible links between one outcome and the next outcome

Trial: number of times a probability experiment is conducted

Triangular prism: prism with triangular ends

Trigonometric ratios: three different ratios of one side of a right-angle triangle to another; sine, cosine and tangent ratios

Trigonometry: deals with the relationship between the sides and the angles of a triangle.

Trinomial: an algebraic expression containing three terms

Turning point: *see* Vertex

Turning point form: for a quadratic equation, $a(x - h)^2 + k = 0$

2-dimensional: plane shape with dimensions, such as length and width or length and height in two directions

Two-step equation: an equation that requires two arithmetic operations to find the value of the variable

Two-way table: lists all the possible outcomes of a probability experiment in a logical manner

Union (of sets): represents the combination of elements of two or more sets: $A \cup B$ denotes the union of sets A and B

Universal set (ξ): largest set that contains all possible elements

Upper quartile: score marking the end of the third quarter in an ordered set of data and denoted by Q_3 . It is calculated by finding the median of the upper half of the data.

Variable [algebra]: pronumeral in an equation or expression that may take on different values

Variable (quantity): a quantity that can change or take on different values, such as the temperature during the day or the number of people in a shop. When a pronumeral represents a variable quantity, it is called a variable.

Venn diagram: series of circles, representing sets, within a rectangle, representing the universal set. They show the relationships between the sets.

Vertex: point at which the graph of a quadratic function (parabola) changes direction (either up or down)

Vertex form: the equation of a parabola in the form $y = (x - h)^2 + k$ gives the vertex, (h, k)

Volume: amount of space a 3-dimensional object occupies. The units used are cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

Wage: money earned by people based on a fixed rate per hour. Hours outside the normal work period are paid at a higher rate.

x-intercept: point where a graph cuts the x -axis

y-intercept: point where a graph cuts the y -axis. In the equation of a straight line, $y = mx + c$, the constant term, c , represents the y -intercept of that line.

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