SACE TWO – AUSTRALIAN CURRICULUM

PHYSICS

WORKBOOK
THIRD EDITION

MARIA CARUSO

Essentials education

ADELAIDE TUITION CENTRE

THE AUTHOR

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Preface

This workbook is designed to cover the core topics of the new Stage 2 SACE Subject Outline in order from topic 1 through to topic 3. However, the topics can be taught in any order, as each topic is treated individually.

The worked examples are designed to help model the use and application of the theoretical concepts. The exercises are designed to help consolidate ideas. There are a variety of question types, which range in difficulty. These question types include recalling facts, describing, graphing and explaining. Clear and detailed solutions are provided. This workbook can be used in class or as a supplement at home.

The workbook also addresses *Science as a human endeavour*, providing examples and ideas for further investigation taken from the Subject Outline. Similarly, Student Practical Investigations are referred to in the body of the workbook and in some of the exercises.

I have tried to cover all aspects of the Subject Outline. I trust that you will find this workbook valuable to your teaching of this new course.

All students and teachers should refer to the SACE website – **www.sace.sa.edu.au** – for the most up-to-date version of the Subject Outline.

Maria Caruso

November 2024

Contents

Topic 1: Motion and relativity

1.1 Projectile motion

Science understanding

1. When the acceleration is constant, motion is described in terms of relationships between measurable scalar and vector quantities, including displacement, speed, velocity, and acceleration.

   Motion under constant acceleration can be described quantitatively using the following formulae:

$$v = v_0 + at$$

$$s = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2as$$

2. Projectile motion can be analysed quantitatively by treating the horizontal and vertical components of the motion independently.
   - Construct, identify, and label displacement, velocity, and acceleration vectors.
   - Resolve velocity into vertical and horizontal components, using $v_H = v\cos\theta$ and $v_V = v\sin\theta$ for the horizontal and vertical components respectively.
   - Solve problems using the constant acceleration formulae.
   - Use vector addition and trigonometric calculations to determine the magnitude and direction of the velocity of a projectile at any moment of time.
3. An object experiences a constant gravitational force near the surface of the Earth, which causes it to undergo uniform acceleration.
   - Explain that, in the absence of air resistance, the horizontal component of the velocity is constant.
4. The motion formulae are used to calculate measurable quantities for objects undergoing projectile motion.
   - Calculate the time of flight when a projectile is launched horizontally.
   - Calculate the time of flight and maximum height for a projectile when the launch height is the same as the landing height.
   - Calculate the horizontal range of a projectile when it is launched horizontally or when the launch height is the same as the landing height (or the flight time is given).
   - Determine the velocity of a projectile at any time using trigonometric calculations or vector addition.
   - Explain qualitatively that the maximum range occurs at a launch angle of 45° for projectiles that land at the same height from which they were launched.
   - Describe the relationship between launch angles that result in the same range.
   - Describe and explain the effect of launch height, speed, and angle on the time of flight and the maximum range of a projectile.
   - Analyse multi-image representations of projectile paths.
5. When a body moves through a medium (such as air), the body experiences a drag force that opposes the motion of the body.
   - Explain the effects of speed, cross-sectional area of the body, and density of the medium on the drag force on a moving body.
   - Explain that terminal velocity occurs when the magnitude of the drag force results in zero net force on the moving body.
   - Describe situations (such as skydiving and the maximum speed of racing cars) where terminal velocity is achieved.
   - Describe and explain the effects of air resistance on the vertical and horizontal components of the velocity, maximum height, and range of a projectile.
   - Describe and explain the effects of air resistance on the time for a projectile to reach the maximum height or to fall from the maximum height.

In the stage 1 **Subtopic 1.1, Motion Under Constant Acceleration,** uniformly accelerated motion was described in terms of relationships between measurable scalar and vector quantities, including displacement, speed, velocity, and acceleration.

Recall that quantities that have size or ***magnitude only*** are called ***scalar*** quantities and quantities that have both a ***magnitude and direction*** are called ***vector*** quantities.

Motion under constant acceleration was described quantitatively using the following equations:

$$v = v_0 + at$$
$$s = v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2as$$

where
v_0 = *initial velocity* (ms^{-1})
v = *final velocity* (ms^{-1})
t = *time* (s)
a = *acceleration* (ms^{-2})
s = *displacement* (m)

Velocity is defined as displacement per unit time and the equation $\vec{v} = \frac{\vec{s}}{t}$ was used when dealing with constant velocity($\vec{v}$).

An object projected horizontally above the surface of the Earth follows a parabolic path towards the surface of the Earth. Similarly the motion of an object that is projected at an angle (θ) above the surface of the Earth follows a parabolic path as it rises and then falls towards the surface of the Earth. The object follows a parabolic path in both examples due to the constant, downwards gravitational force that acts near the surface of the Earth. Motion under the action of the gravitational force near the surface of the Earth is referred to as **projectile motion**.

A multi-image photograph such as that pictured in Figure 1.1.1 can capture the motion of a projectile. Figure 1.1.1 illustrates that the tennis ball has both a horizontal and vertical component to its motion.

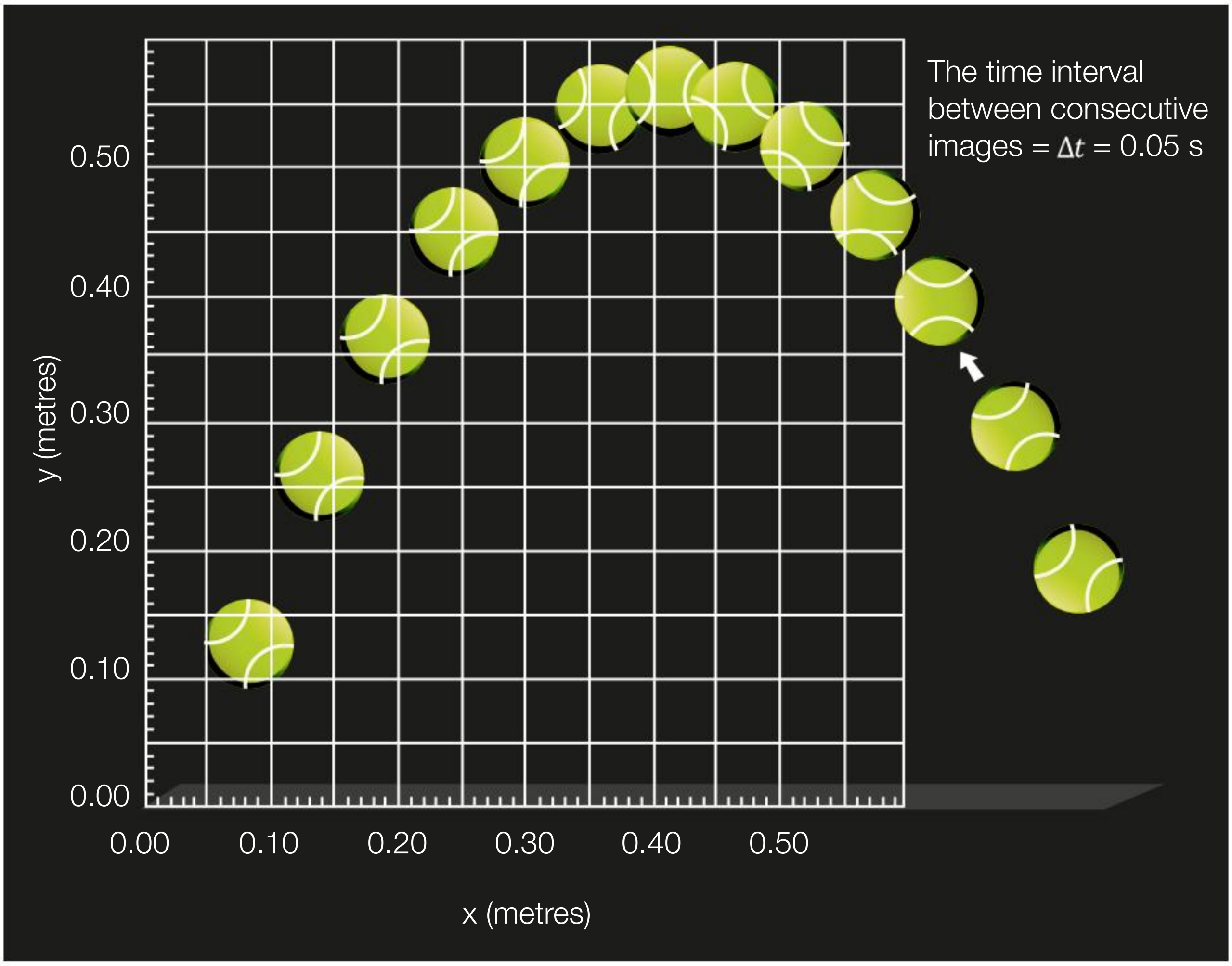


Figure 1.1.1

1

Horizontal and vertical components of motion can be treated independently

In Figure 1.1.1, the time interval (Δt) between consecutive images is constant. The displacement of the tennis ball in the horizontal direction (x) is also constant between consecutive images. This means that the **horizontal component** of the ball's velocity is constant, and indicates that there are no unbalanced forces acting in the horizontal direction (Newton's First Law).

Figure 1.1.1 indicates that the tennis ball experiences an acceleration in the vertical direction. The vertical displacement between consecutive images decreases as the ball rises and increases as the ball falls. This is because all objects experience a constant, downwards gravitational force near the surface of the Earth. The acceleration of the ball is therefore constant and has a magnitude of **9.80 ms^{-2}**. The acceleration is directed downwards towards the ground and is independent of the object's mass.

Analysis of projectile motion qualitatively

Consider the three cases below.

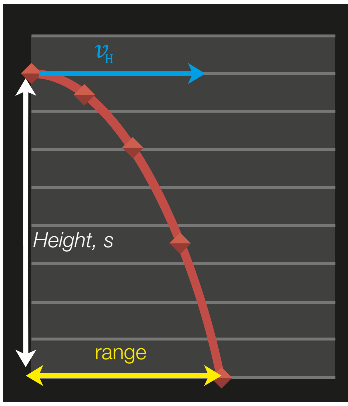


Figure 1.1.2

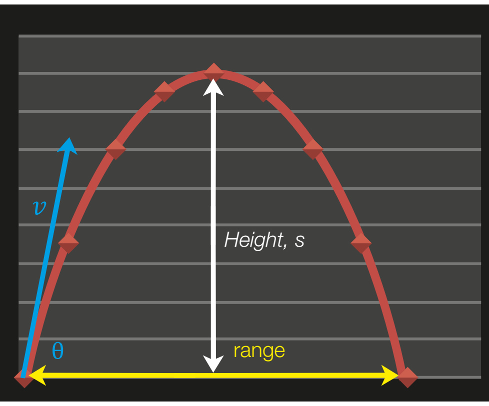


Figure 1.1.3

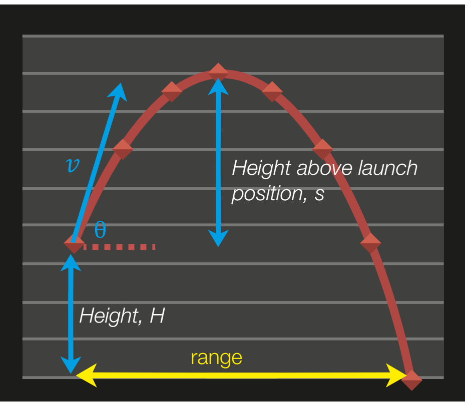
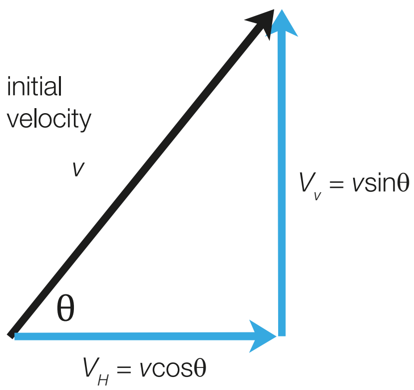


Figure 1.1.4

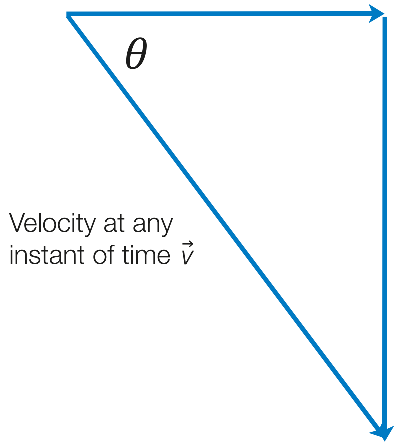
The table below summarises the quantitative methods for determining the most common measurable quantities.

Figure 1.1.2 **Object launched horizontally with a speed v_H**	**Figure 1.1.3** **Object launched at an angle θ above the horizontal with a speed v** **(uni-level projection)**	**Figure 1.1.4** **Object launched at a height H and an angle θ above the horizontal with a speed v**
The vertical component of the initial velocity is zero when a projectile is launched horizontally above the ground. The three equations of motion that describe the vertical motion of the projectile are: $v = at$ $s = \frac{1}{2}at^2$ $v^2 = 2as$	In this case, the initial or launch velocity has a horizontal and a vertical component as shown below. That is, the initial velocity can be resolved into its horizontal (v_H) and vertical (v_v) components.  	The analysis in this case is similar to an object launched at an angle above the horizontal. The initial or launch velocity has a horizontal and vertical component given by $v_H = v\cos\theta$ and $v_v = v\sin\theta$ respectively. .

Time of flight		
Most commonly, $$s=\frac{1}{2}at^2$$ is rearranged to give $$t=\sqrt{\frac{2s}{a}}$$ and is used to find the time of flight (*t*) of the projectile.	Most commonly, the equation $$v = v_0 + at$$ is rearranged to give $$t=\frac{v-v_0}{a}=\frac{-v_0}{-9.8}=\frac{v_0}{9.8}$$ and is used to calculate the time taken for the projectile to reach its maximum height (or vertical displacement). In this case, *v* is the final velocity in the vertical direction (zero), *a* is the acceleration (-9.80 ms^{-2}) and v_o is the initial vertical velocity ($v_v = v\sin\theta$). Since the motion is symmetrical, this value is doubled to give the total time of flight.	Calculating the time of flight for this situation is outside the scope of the course. It will be discussed for completeness. In this case, the time taken for the projectile to reach its maximum height above its launch position is found in the same way it is calculated for an object launched at an angle above the horizontal. The time taken to fall from maximum height can be found by rearranging $$s=\frac{1}{2}at^2$$ so that $$t=\sqrt{\frac{2s}{a}}$$ The total time of flight is the sum of these two times.
Height (or vertical displacement)		
Most commonly, $$s=\frac{1}{2}at^2$$ is used to find the height (*s*) from which the projectile was launched given the time of flight (*t*).	Most commonly, the equation $v^2 = v_0^2 + 2as$ is rearranged to give $$s=\frac{v^2-v_0^2}{2a}=\frac{-v_0^2}{-19.6}=\frac{v_0^2}{19.6}$$ and is used to calculate the maximum height reached (*s*) by the projectile above its launch position (usually the ground). In this case, *v* is the final velocity in the vertical direction (zero), *a* is the acceleration (-9.80 ms^{-2}) and v_o is the initial vertical velocity ($v_v = v\sin\theta$). Alternatively, if the time to maximum height (*t*) is known, then the equation $$s=v_0t+\frac{1}{2}at^2$$ can be used to calculate the height reached by the projectile.	Calculating the maximum height for this situation is outside the scope of the course. It will be discussed for completeness. In this case, the height reached by the projectile above its launch position (*s*) is found in the same way it is calculated for an object launched at an angle above the horizontal but the launch height (*H*) is added to determine the height above the ground.
Range s_H (or horizontal displacement)		
$s_H = v_H t$ is used to calculate the horizontal range.	$s_H = v_H t$ is used to calculate the horizontal range.	$s_H = v_H t$ is used to calculate the horizontal range.

Velocity at any point along the path

The horizontal and vertical components of velocity at any instant of time can be added vectorially to give the velocity vector.



Vertical component of velocity = $v_o + at$

Here v_o is the initial vertical velocity ($v\sin\theta$) where v is the initial or launch velocity.

The acceleration is a =-9.80 ms^{-2}.

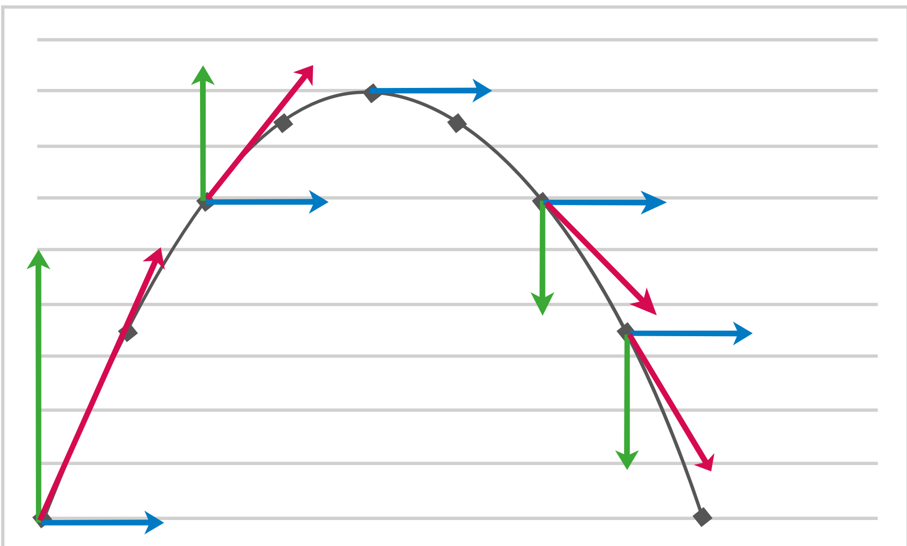
If the value of $v_o + at$ is negative this means that the vertical component of velocity is directed downwards as shown in the adjacent vector triangle. If the value is positive the vertical component of velocity is directed upwards.

Scale diagrams

A scale diagram can be used to determine the components of a projectile's initial or launch velocity or the velocity of the projectile at any instant of time after it is launched. This is not a common method. Opportunities to use scale diagrams to solve problems will be provided in the examples and exercises.

Velocity and acceleration at different points along a projectile's path

Earlier in the chapter we saw that the horizontal component of a projectile's velocity is constant. We also discussed that the vertical component of velocity decreases as the object rises until it is zero at maximum height and then increases as the object falls. The velocity vector at any instant of time is at a tangent to the parabolic path and a vector sum of the horizontal and vertical components of velocity. Figure 1.1.5 illustrates the horizontal and vertical components of a projectile's velocity as well as the velocity vector at various points along a projectile's path.



Key:
Horizontal component of velocity
Vertical component of velocity
Velocity

Figure 1.1.5

Figure 1.1.6 illustrates the acceleration of a projectile at various points along its path. Acceleration is represented by vector arrows of equal length at each point along the path as it is constant in magnitude and direction.

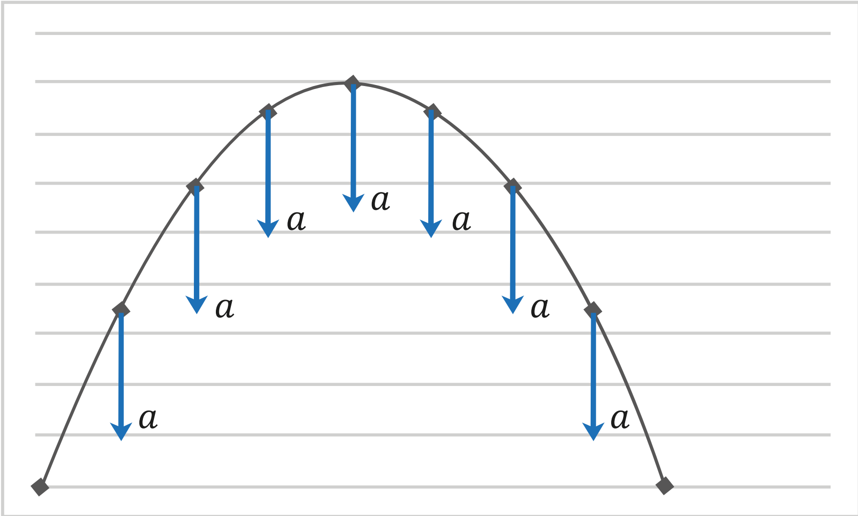


Figure 1.1.6

Range

The vertical component of the initial or launch velocity determines the height reached by a projectile. The larger the vertical component, the higher the projectile rises. This results in the projectile spending more time in the air. The product of the horizontal component of the launch velocity and the time of flight determines the range. The affect of launch speed, angle and height can be investigated using a spreadsheet or simulations such as: *https://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html*

Effect of launch speed on range

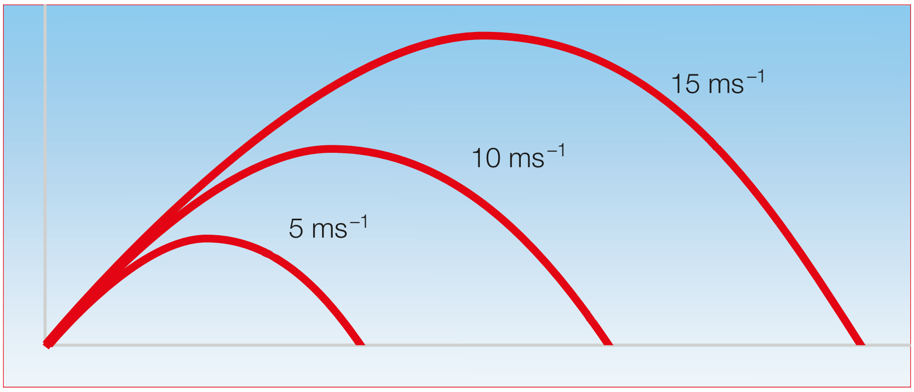


Figure 1.1.7

Figure 1.1.7 illustrates the range that results from three different launch speeds (5 ms^{-1}, 10 ms^{-1} and 15 ms^{-1}) at the same launch angle.

A greater launch speed results in both the vertical and horizontal components of the launch velocity being larger. A larger vertical component of velocity results in a greater height and a greater time of flight. Since the product of the horizontal component of the launch velocity and the time of flight determines the range, the range increases with launch speed.

Effect of launch angle on range

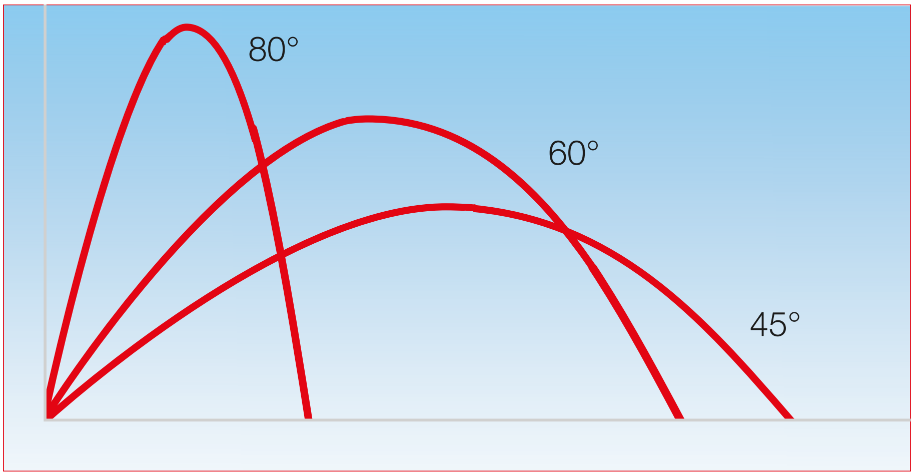


Figure 1.1.8

Figure 1.1.8 illustrates the range that results from the same launch speed at three different angles (45°, 60° and 80°).

Figure 1.1.9 shows that a greater launch angle (θ_1) for a given launch velocity ($\vec{v}$) results in the vertical component of the launch velocity being larger and the horizontal component being smaller.

Figure 1.1.9

A larger vertical component of velocity results in a greater height and therefore a greater time of flight. However, reducing the horizontal component of the launch velocity results in the product of the horizontal component of the launch velocity and the time of flight decreasing as the angle increases. It follows that the range decreases.

Launch angles that result in the same range

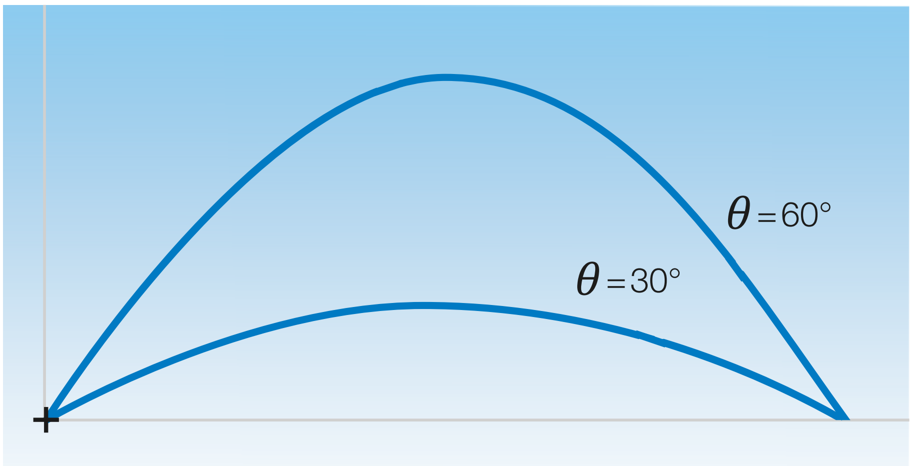


Figure 1.1.10

Figure 1.1.10 illustrates that complementary angles (θ and 90-θ) produce the same range if a projectile lands at the same height that it is launched.

Maximum range

(i) A projectile launched at an angle above the horizontal and landing at the same height that it is launched (uni-level projection)

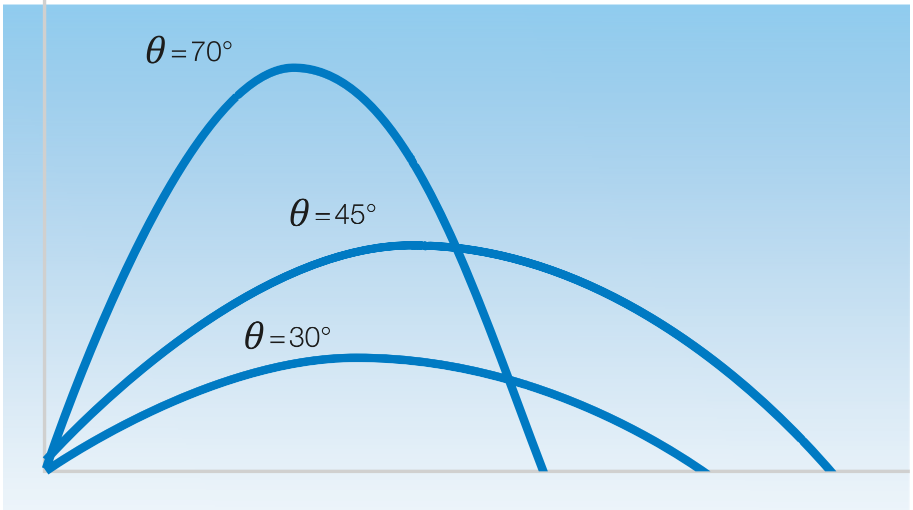


Figure 1.1.11

Figure 1.1.11 illustrates a projectile launched with the same speed at three different angles. A launch angle of 45° produces a maximum range for any given value of launch speed. This is because the horizontal and vertical components of the launch velocity are equal (*cos45 = sin45 = 0.707*). The vertical component causes the projectile to rise high enough to produce a time of flight that maximises the product between the horizontal component of the launch velocity and the time of flight. Thus, an angle of 45° produces the largest range.

(ii) Effect of launch height and launch angle on range

For a projectile launched at an angle from a given height (e.g. a shot put), maximum range will depend on the launch angle and the launch height.

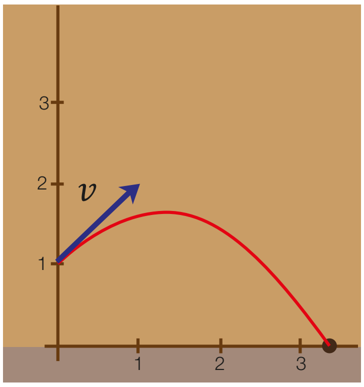


Figure 1.1.12

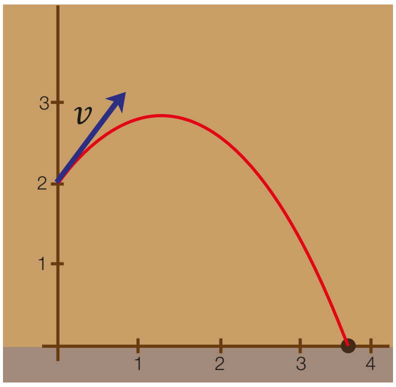


Figure 1.1.13

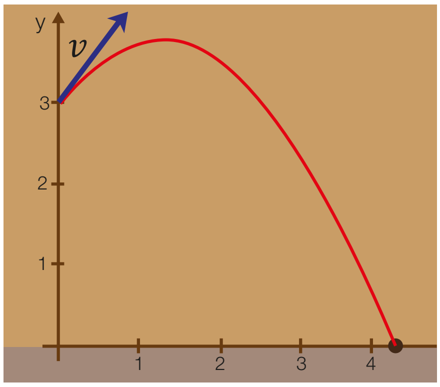


Figure 1.1.14

The three diagrams above illustrate a projectile being launched from three different heights with the same velocity. The greatest range is achieved in Figure 1.1.14 i.e. for the greatest launch height of 3 m.

This is because the projectile is in the air for a greater length of time as it falls the extra vertical height. Since range is given by the product of the time of flight and the horizontal component of velocity then the range is greater because the time of flight is greater while the horizontal component of velocity remains constant.

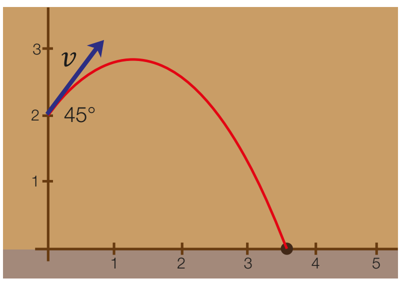


Figure 1.1.15

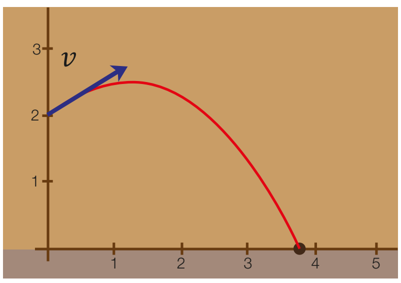


Figure 1.1.16

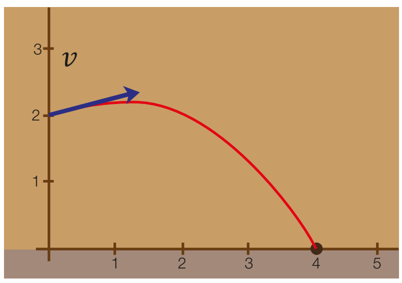


Figure 1.1.17

The above diagrams illustrate a projectile launched from the same height with the same speed at three different angles. The launch angle progressively decreases from 45° (Figure 1.1.15). Figure 1.1.17 depicts the projectile being launched from the smallest angle but resulting in the greatest range. It can be concluded that a launch angle less than 45° produces a maximum range in this case.

This is because a smaller launch angle will result in the horizontal component of the launch velocity being larger than the vertical component. While the projectile does not rise as high and the time of flight is reduced, the increase in the horizontal component of velocity results in the product of the horizontal component of the launch velocity and the time of flight (i.e. range) increasing as the launch angle decreases.

The greater the launch height, the smaller the launch angle that produces a maximum range.

? Science inquiry activity

The following are some possible ideas for science investigations.

1. Use a projectile launcher to investigate the effect of launch angle or launch height on range.
2. Set up a spreadsheet to investigate the effect of launch height and angle on range.
3. Investigate the 'monkey and the hunter' problem both quantitatively and qualitatively.
4. Use video footage to analyse projectile motion in a variety of contexts.
5. Analyse the constant horizontal component of the velocity qualitatively and quantitatively, using various recording technologies.
6. Model and demonstrate that the maximum range occurs at an angle other than 45° when the launch height is different to the landing height.
7. In terms of projectile motion, analyse footage of students undertaking a sport like shot put.
8. Use concepts from projectile motion to analyse sporting activities such as aerial skiing, golf, javelin, shot put, and various ball sports.

Worked Examples

1. A tennis player at the Australian Tennis Open serves a ball horizontally with a speed of 212 kmh^{-1} straight down the centre line of the tennis court. The ball takes 0.721 seconds to hit the ground on the other side of the net.

(a) Calculate the height at which the ball was struck.

$$s = v_o t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.8 \times (0.721)^2 = 2.55\ m$$

(b) The court has a length of 40.2 m. Calculate the range of the ball and comment on why the serve was called a fault.

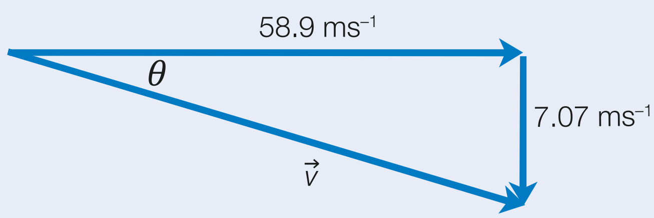
$$s = vt = \frac{212}{3.6} \times 0.721 = 42.5\ m$$

The range is greater than the length of the court. It would land outside the court and therefore be called a fault.

(c) Determine the velocity with which the ball strikes the ground.

$$v_H = \frac{212}{3.6} = 58.9 ms^{-1} \rightarrow$$

$$v_v = v_0 + at$$
$$= 0 + 9.8 \times 0.721$$
$$= 7.07 \downarrow\ ms^{-1}$$



$$v = \sqrt{58.9^2 + 7.07^2} = 59.3\ ms^{-1}$$

$$\tan\theta = \frac{7.07}{58.9} \Rightarrow \theta = 6.84°$$

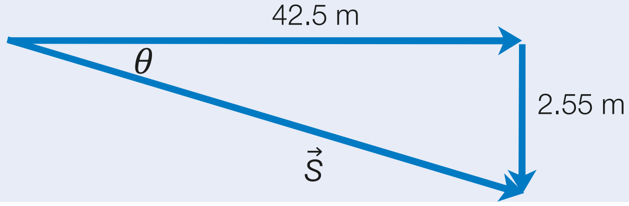
$$\vec{v} = 59.3 ms^{-1}\ 6.84°\ \textit{below the horizontal}$$

(d) Determine the displacement of the tennis ball on impact with the ground.

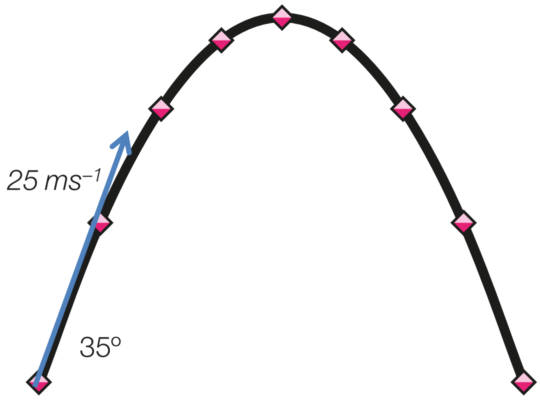
$s=\sqrt{42.5^2+2.55^2}=42.6m$

$\tan\theta=\frac{2.55}{42.5}\Rightarrow\theta=3.43°$

$\vec{s}=42.6m\ \ 3.43°$ *below the horizontal*



2. A golf ball is struck from the ground with a velocity of 25 ms^{-1} and at an angle of 35° above the horizontal.



(a) Calculate the magnitude of the horizontal and vertical components of the initial velocity of the golf ball.

$v_H = v\cos\theta = 25\cos35 = 20\ ms^{-1}$

$v_V = v\sin\theta = 25\sin35 = 14\ ms^{-1}$

(b) Calculate the maximum height reached by the golf ball.

$v^2=v_o^2+2as$

$0=14^2-2\times9.8s$

$s=\frac{14^2}{2\times9.8}=10\ m$

(c) Calculate the time of flight of the golf ball.

$v=v_o+at\ \ \therefore\ t=\frac{v-v_o}{a}=\frac{0-14}{-9.8}=1.43s$

Total time in the air = 1.43 × 2 = 2.9s

(d) Calculate the range of the golf ball.

$s = vt = 20 \times 2.9 = 58m$

(e) State two ways that the golf player can hit the golf ball so that it lands further than the value calculated in part (d).

The golf player could strike the ball harder so that it has a greater launch velocity or they could hit the golf ball with the same launch speed but at an angle of 45° above the ground.

(f) Determine the velocity of the golf ball after 2.0 s.

$v_H=20ms^{-1}\rightarrow$

$v_v=v_0+at$

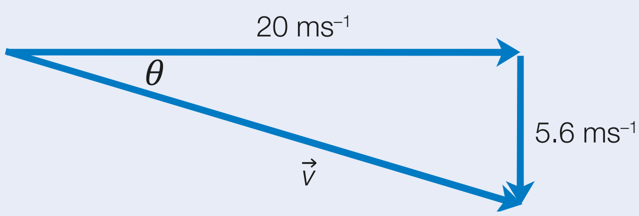
$=14-9.8\times2$

$=-5.6\ \ ms^{-1}\ =\ 5.6\ ms^{-1}\downarrow$

$v=\sqrt{20^2+5.6^2}=21ms^{-1}$

$\tan\theta=\frac{5.6}{20}\Rightarrow\theta=16°$

$\vec{v}=21ms^{-1}\ \ 16°$ *below the horizontal*



3. Jürgen Schult of Germany broke the world record in 1986 by throwing a discus 74.08 metres.

Consider an athlete that releases a discus from a height of 1.60 m, with a speed of 20.0 ms^{-1} at an angle of 40.0° above the horizontal.

(a) Resolve the release velocity of the discus into its component vectors.

$v_H = v\cos\theta = 20\cos40 = 15.3\ ms^{-1}$

$v_V = v\sin\theta = 20\sin 40 = 12.9\ ms^{-1}$

(b) Explain why the discus' velocity is not zero at maximum height.

The discus has two components of velocity. While the vertical component of velocity is zero at maximum height, the horizontal component is constant. The velocity at maximum height is therefore 15.3 ms^{-1} horizontally and in the same direction that it was thrown.

(c) Calculate the height reached by the discus as measured from the ground.

$v^2 = v_o^2 + 2as$

$0 = 12.9^2 - 2\times 9.8s$

$s = \frac{12.9^2}{2\times 9.8} = 8.49\ m$

Total height above the ground = 1.60 + 8.49 = 10.1 m

(d) Calculate the discus' time of flight.

time to maximum height $v = v_o + at \ \therefore \ t = \frac{v - v_o}{a} = \frac{0-12.9}{-9.8} = 1.32\ s$

Time to fall from maximum height $s = \frac{1}{2}at^2 \ \therefore \ t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 10.1}{9.8}} = 1.44s$

total time = 1.32 + 1.44 = 2.76 s

(e) Determine whether the world record is broken.

$s = vt = 15.3 \times 2.76 = 42.2\ m$

The discus falls well short (31.9 m) of the world record. The world record was not broken by this athlete.

(f) Calculate the displacement of the discus at its maximum height.

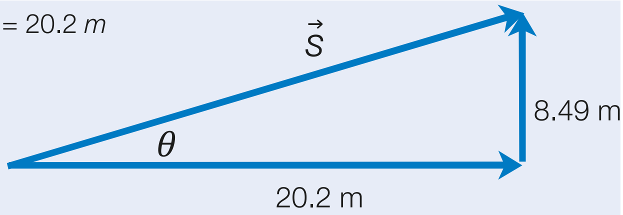
horizontal displacement $s = vt = 15.3 \times 1.32 = 20.2\ m$

vertical displacement = 8.49 m

$s = \sqrt{20.2^2 + 8.49^2} = 21.9\ \ m$

$\tan\theta = \dfrac{8.49}{20.2} \Rightarrow \theta = 22.8°$

$\vec{s} = 21.9\ \ m\ \ \ 22.8°$ *above the horizontal*



(g) Determine the velocity of the javelin after 2.00 s using

(i) a scale diagram

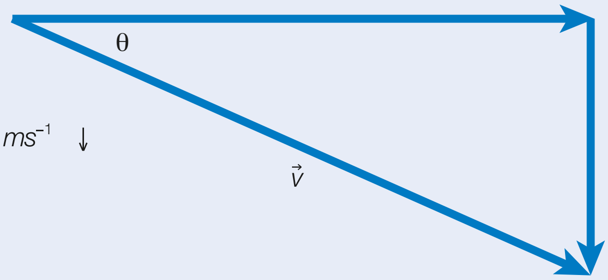
$v_H = 15.3\ \ ms^{-1}\ \ \rightarrow$

$v_v = v_o + at = 12.9 - 9.8 \times 2 = -6.7\ \ ms^{-1}\ = \ 6.70\ \ ms^{-1}\ \downarrow$

scale: 1 cm = 2 ms^{-1}

By measurement

$\vec{v} = 16.7\ \ ms^{-1}\ \ 24°$ *below the horizontal*

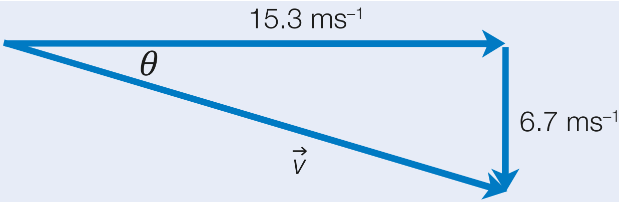


(ii) trigonometry

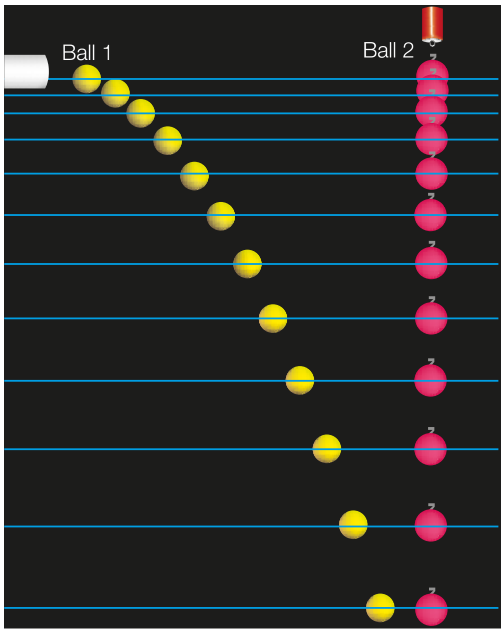
$v = \sqrt{15.3^2 + 6.7^2} = 16.7\ \ ms^{-1}$

$\tan\theta = \dfrac{6.7}{15.3} \Rightarrow \theta = 23.6°$

$\vec{v} = 16.7\ \ ms^{-1}\ 23.6°$ *below the horizontal*



4. A multi-image photograph is taken of two identical balls falling under the action of gravity. Ball 2 is dropped from rest while Ball 1 is given an initial horizontal velocity. Both balls fall a vertical distance of 24 cm and the time interval between successive images is $\frac{1}{50}s$.



(a) Explain how the multi-image photograph illustrates that the horizontal motion of Ball 1 is uniform.

The horizontal distance between successive images is the same. This means that the ball travels the same distance every $\frac{1}{50}s$. By definition, the ball is travelling with a constant velocity in the horizontal direction.

(b) Calculate the magnitude of the acceleration due to gravity for Ball 2.

The total time taken for the ball to fall 24cm = 0.24 m is $\frac{11}{50}s$ = 0.22 s.

$s = \frac{1}{2}at^2\ \ \therefore\ \ a = \dfrac{2s}{t^2} = \dfrac{2 \times 0.24}{0.22^2} = 9.9\ \ ms^{-2}$

(c) Comment on the accuracy of the acceleration calculated.

$\%error = \dfrac{9.9 - 9.80}{9.8} \times 100 = 1.0\%$

The accepted value for the acceleration of an object in free-fall is 9.80 ms^{-2}. The experimental value for the acceleration is very close to the accepted value with only 1.0% error. The value is accurate.

(d) Use your answer to part (b) and evidence from the photograph to state and justify the magnitude of the acceleration of Ball 1.

The vertical positions of Ball 1 are identical to the vertical positions of Ball 2 as it falls. It can be concluded that the acceleration of Ball 1 is identical to Ball 2 i.e. 9.9 ms^{-2}

Drag forces

A drag force is the unbalanced force that acts to oppose the motion of an object as it moves through a medium such as air or water.

Drag forces occurs due to collisions between the object and the particles making up the medium. The object exerts a force on the particles of the medium and by Newton's Third Law the particles of the medium exert an equal and opposite force on the object.

Factors that affect drag forces

The drag formula is given by $Drag = \frac{1}{2}C_dA\rho v^2$. In this formula, A, represents the cross-sectional area of the object, ρ represents the density of the medium and v represents the speed of the object. C_d represents the coefficient of drag. This is a factor representing the drag acting on an object due to its shape and inclination.

1 Speed

The faster the object moves, the greater the drag force. This is because there are more collisions with the particles of the medium per unit time.

The drag force is directly proportional to the square of the speed. This means that drag increases significantly with speed. If the speed of the object doubles, the drag force increases by a factor of four, if the speed triples, the drag force increases by a factor of 9 and so on.

2 Cross-sectional area

The drag force is directly proportional to the cross-sectional area of the object. This means that the larger the area of cross-section of the object, the greater the drag force. This is because the object will collide with more particles of the medium.

3 Density of the medium

The drag force is directly proportional to the density of the medium. This means that a medium with a greater density produces a greater drag force. This is because there are more particles of the medium packed into a given volume. The object will therefore experience more collisions with the particles of the medium. It follows that the drag force is greater than for an object moving through a less dense medium.

Extra understanding

Shape

Figure 1.1.18

The particles of a medium can flow over and around a streamline object. Drag forces are therefore smaller than for a non-aerodynamic shape with the same area of cross-section (*A*).

The greater the drag coefficient, the greater the drag force. The objects illustrated in Figure 1.1.18 all have the same cross-sectional area but experience a different amount of drag due to their shape.

Terminal velocity

We have already discussed that drag forces experienced by an object moving through a medium increase as the object speeds up or accelerates. If the drag forces increase to a stage that they equal to the force accelerating the object, the forces become balanced and the object no longer accelerates but travels with a constant velocity.

Terminal velocity occurs when the magnitude of the drag forces results in zero net force on the moving object.

Two such examples are discussed below.

1 Skydiving

Air resistance is the unbalanced force that acts to oppose the motion of an object moving through the air. Air resistance is an example of a drag force. Air resistance occurs because a moving object will collide with air particles.

As a skydiver jumps from a plane, they are immediately acted on by the gravitational force or their weight (*W*=*mg*) which acts vertically downwards. The skydiver accelerates towards the ground. As the skydiver falls the upward force due to air resistance increases as the velocity of the diver increases. Eventually the force due to air resistance will balance the downward force due to the skydiver's weight and terminal velocity is reached. This is illustrated in Figure 1.1.19.

A skydiver falling from a height of 5000 m without a parachute would have a terminal velocity of around 60 ms^{-1}. This speed would cause death on impact with the ground. This is why a parachute is used. A parachute increases the surface area in contact with the air and greatly increases the force due to air resistance. This will decreases the terminal velocity of the skydiver and ensures a slower and safer descent to the ground.

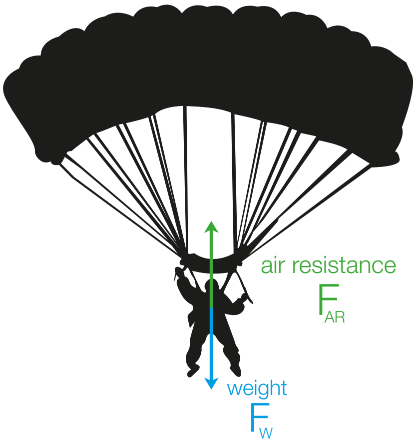


Figure 1.1.19

2 Racing cars

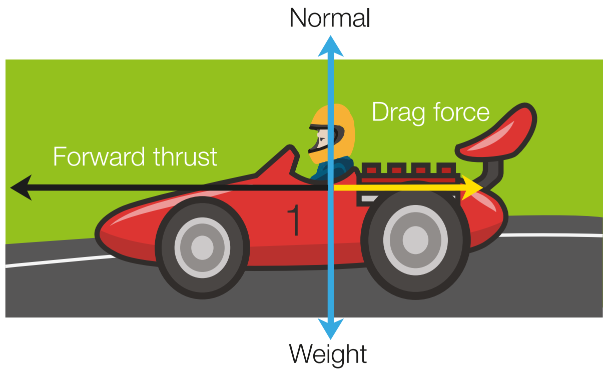


Figure 1.1.20

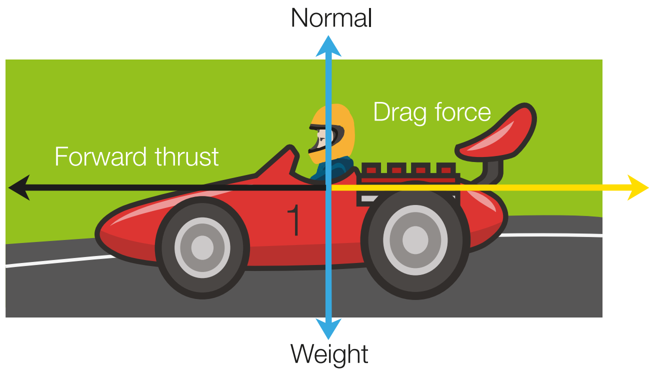


Figure 1.1.21

Figures 1.1.20 and 1.1.21 illustrate the four forces acting on a racing car. In the vertical direction, the weight of the car acts downwards and is balanced by the normal force acting upwards (Newton's Third Law). It follows that these two forces do not have an effect on the motion of the racing car. In the horizontal direction, the thrust of the engine turns the wheels and because friction is present, the car accelerates forwards. Drag forces act in the opposite direction that the car is moving. As the car speeds up, the drag force increases. When the drag force becomes equal in magnitude to the forward thrust, the racing car reaches terminal velocity.

The car in Figure 1.1.20 is travelling at a lower speed than the car in Figure 1.1.21. The drag force opposing the motion of the car in Figure 20 is smaller than the forward thrust force. The car can accelerate forward. The drag force opposing the motion of the car in Figure 1.1.21 is equal in magnitude to the forward thrust force. The car travels with constant velocity.

In subtopic 1.3 we revisit the forces acting on a vehicle travelling with constant velocity and discuss the force acting when the vehicle travels along a circular path with constant speed.

Worked Example

A light aircraft of mass of 4200 kg starts from rest and accelerates along a straight horizontal runway. The aircraft engine produces a constant thrust of 6400 N.

(a) Describe the change in the magnitude of the acceleration as the aircraft's speed increases.

As the aircraft increases its speed, the magnitude of the drag force increases. This means that the net forward force acting on the aircraft decreases which means that the acceleration of the aircraft decreases.

(b) State the magnitude of the drag force acting when the aircraft reaches terminal velocity.

6400 N

Effects of air resistance on the motion of a projectile

Figure 1.1.22 illustrates the effect of air resistance on the path of a projectile. The path is no longer parabolic nor symmetrical. The projectile does not rise as high and the range is reduced.

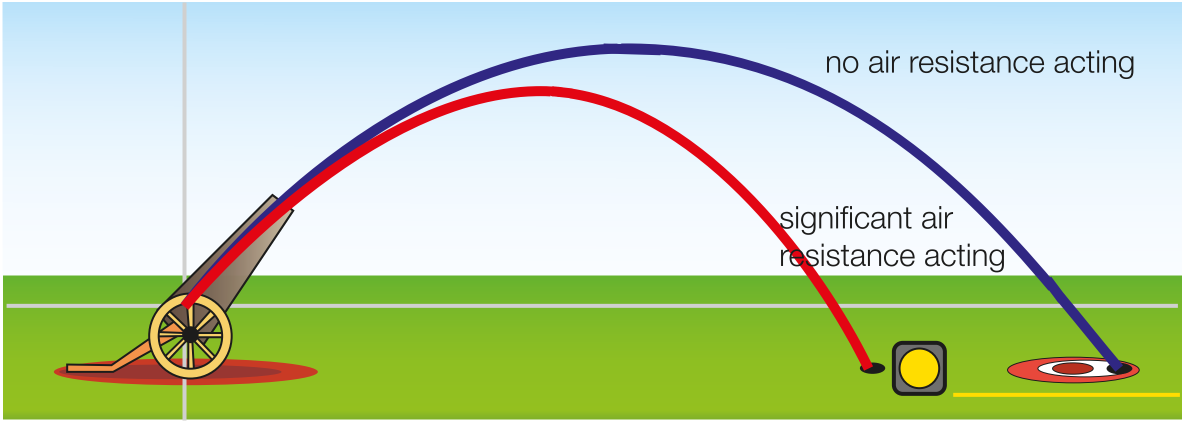


Figure 1.1.22

1 Effect of air resistance on the components of velocity

Air resistance reduces both the horizontal and vertical components of velocity as a projectile moves through the air.

2 Effect of air resistance on maximum height reached

The vertical height (s) reached by a projectile depends on the initial vertical velocity (v_o) and the acceleration (a) experienced as it rises. The vertical height reached can be calculated using the equation

$$s = \frac{v^2 - v_o^2}{2a} = \frac{-v_o^2}{2a}.$$

When air resistance acts to oppose the ascending motion of the projectile, the vertical height reached is smaller. This is because the initial vertical component of velocity (v_o) does not change but the object decelerates more quickly than if gravity was acting alone. That is the value of acceleration (a) becomes greater than 9.80 ms^{-2}.

3 Effect of air resistance on the time taken to reach maximum height and to fall from maximum height

When air resistance acts, the projectile decelerates more quickly as it rises (greater than if gravity was acting alone i.e. greater than 9.80 ms^{-2}). The projectile therefore reaches maximum height in a short time. As it returns to the ground it accelerates at a smaller rate (smaller than if gravity was acting alone i.e. smaller than 9.80 ms^{-2}). The two effects almost cancel. The time of flight is generally reduced by a small amount as both the range and maximum height reached are reduced.

4 Effect of air resistance on range

The product of the horizontal component of a projectile's velocity and it's time of flight is used to calculate the range. While the time of flight may not be reduced significantly when air resistance opposes the motion of a projectile, the horizontal component of the projectile's velocity is. It follows that the range is also reduced.

? Science inquiry activity

The following are some possible ideas for science investigations.

1. Determine the terminal velocity of a spherical object by dropping it into a viscous liquid.
2. Determine the drag coefficients by dropping coffee filters or cupcake holders. By manipulating the mass and recording the time taken to reach the ground, use the air resistance formula to calculate the drag coefficient.
3. Discuss the conclusions of experiments comparing swimming in syrup with swimming in water: http://www.nature.com/news/2004/040920/full/news040920-2.html.

Science as a human endeavour

The following is a possible idea for an investigation.

Explore examples of the way that scientists have been able to develop solutions affecting aerodynamics (such as shape, texture, and spin) of different objects like balls, planes and cars.

Exercises

1. A baseball pitcher throws a ball horizontally with a speed of 40.0 ms^{-1}. The ball leaves the pitcher's hand at a height of 1.40 m above the ground. The batter stands 18.4 m away. The batter swings but misses the ball. This is called a strike.

(a) Calculate the time taken for the ball to reach the batter.

(b) Determine the height above the ground at which the ball passes the batter.

(c) Calculate the velocity with which the ball passes the batter.

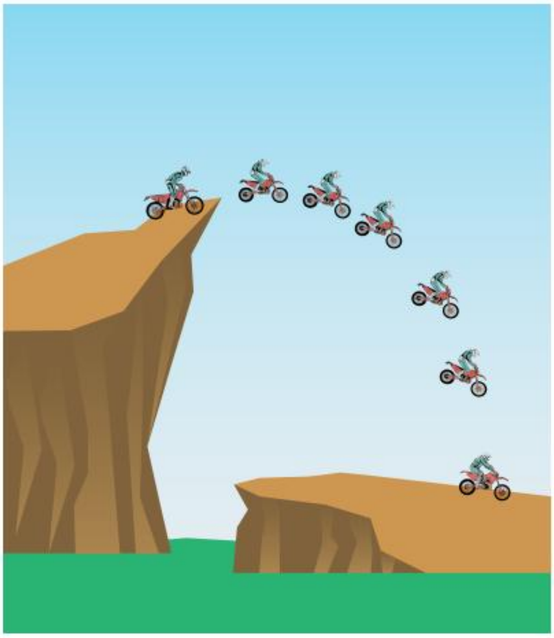
Space for vector diagram

(d) Calculate the displacement of the ball as it passes the batter.

Space for vector diagram

(e) The pitcher throws the ball harder and therefore with a greater horizontal velocity. Explain how this affects the height at which the ball passes the batter.

2. A thrill-seeking motorcyclist drives his bike horizontally off the top of a hill 10.0 m above the ground with a velocity of 80.0 kmh^{-1}.



(a) Calculate the time taken for the motorcyclist to reach the ground.

(b) Calculate the range of the motorcyclist.

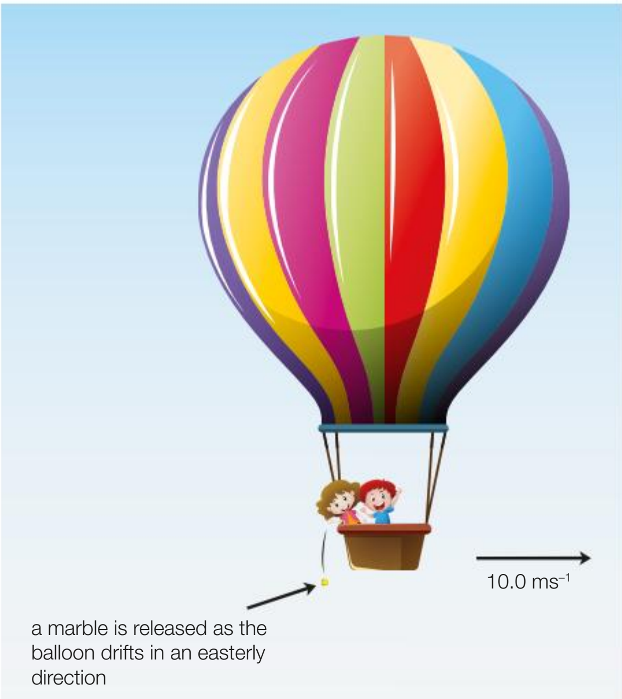
(c) Determine the velocity of the motorcyclist on impact with the ground.

Space for vector diagram

(d) Being a thrill seeker, the motorcyclist rides off the top of the hill several times in an attempt to increase his range. The greatest range he achieves is 52.0 m. Calculate the speed with which the motorcyclist left the top of the hill in order to achieve this range.

(e) Explain why the range of the motorcyclist increases if the motorcycle is driven off a higher hill with the same speed.

3. A balloon is drifting across the countryside with a velocity of 10.0 ms^{-1} in an easterly direction. A marble is dropped over the side of the balloon and takes 11.2 s to hit the ground.



(a) Calculate the height above the Earth's surface at which the balloon is drifting.

(b) Calculate the magnitude of the marble's horizontal displacement.

(c) Calculate the horizontal and vertical components of the marble's velocity on impact with the ground.

(d) Draw a vector triangle which represents the velocity of the marble on impact with the ground. Use it to determine the velocity with which the marble hits the ground.

Space for vector diagram

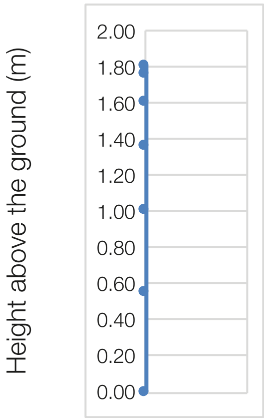
(e) Describe the path of the marble as seen by

(i) the balloonist

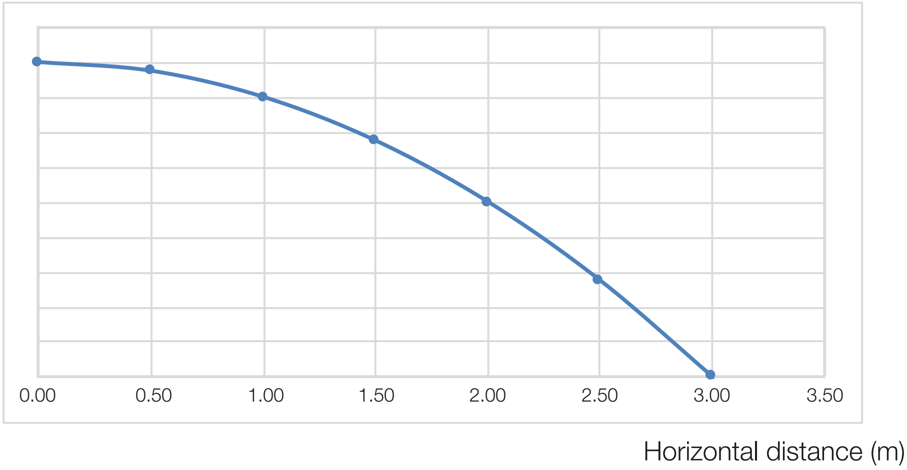
(ii) a person standing on the ground

4. A multi-image photograph is taken of two identical balls falling 1.80 m under the action of gravity. Ball 1 is dropped from rest while Ball 2 is given an initial horizontal velocity. The time between successive images is 0.100 s.

Ball 1



Ball 2



(a) Show that Ball 1 accelerates vertically downwards at 10.0 ms^{-2}.

(b) Explain how the multi-image photograph shows that the horizontal motion of Ball 2 is constant.

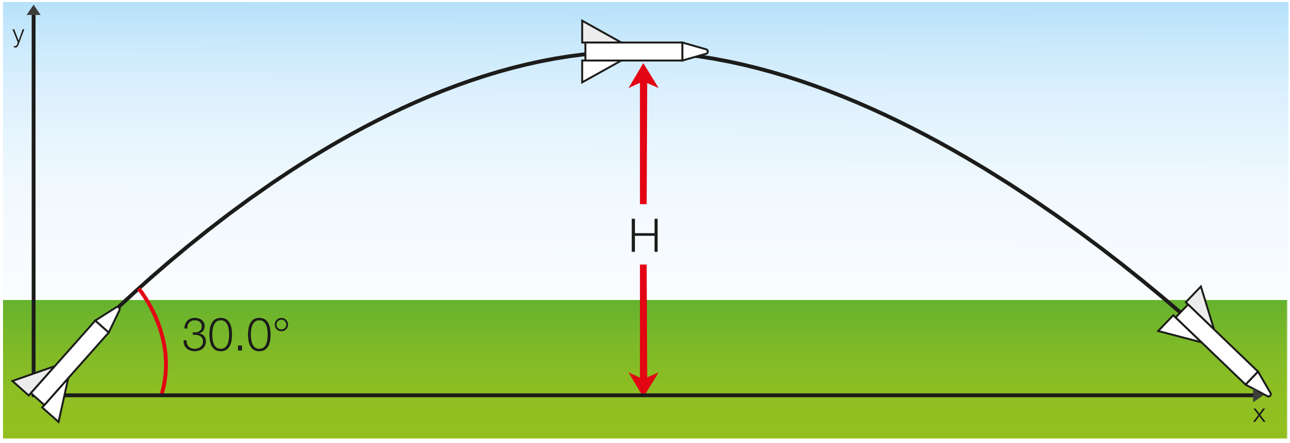
(c) Explain how the multi-image photograph shows that Ball 2 is accelerating vertically downwards at 10.0 ms^{-2}.

(d) Calculate the initial velocity of Ball 2.

(e) Determine the velocity of Ball 2 after 0.300 s.

Space for vector diagram

5. A model rocket is fired at 40.0 ms^{-1}, 30.0° to the ground.



Calculate the

(a) magnitude of the initial components of the model rocket's velocity.

(b) maximum height H, reached by the model rocket.

(c) time taken for the model rocket to reach its maximum height.

(d) displacement of the model rocket at its maximum height.

Space for vector diagram

(e) range of the model rocket and state one other launch angle that produces the same range.

(f) velocity of the model rocket after 1.50 s.

Space for vector diagram

6. A golf ball is struck with a velocity of 25 ms^{-1} and at an angle of 45° above the ground.

(a) Calculate the magnitude of the components of the golf ball's initial velocity.

(b) Calculate the height reached by the golf ball.

(c) Determine the time of flight for the golf ball.

(d) Calculate the range of the golf ball.

(e) Use a scale diagram to determine the velocity of the golf ball after 1.5 s.

Space for vector diagram

(f) The golfer now strikes another golf ball with the same speed but at an angle of 60.0° above the ground. State whether each of the following quantities is smaller, larger or unaffected when compared to the golf ball struck at 45°.

(i) range

(ii) height reached

(iii) time of flight

7. A hockey ball is struck with a velocity $\vec{v}$, at an angle θ above the ground. The ball reaches a height of 0.82 m above the ground and travels a horizontal distance of 4.0 m. Air resistance can be considered to be negligible.

(a) State the change, if any at all, in the magnitude of the

(i) horizontal component of velocity during the hockey ball's time in the air.

(ii) vertical component of velocity as the hockey ball rises.

(iii) vertical component of velocity as the hockey ball falls.

(b) Calculate the

(i) magnitude of the initial vertical component of the hockey ball's velocity.

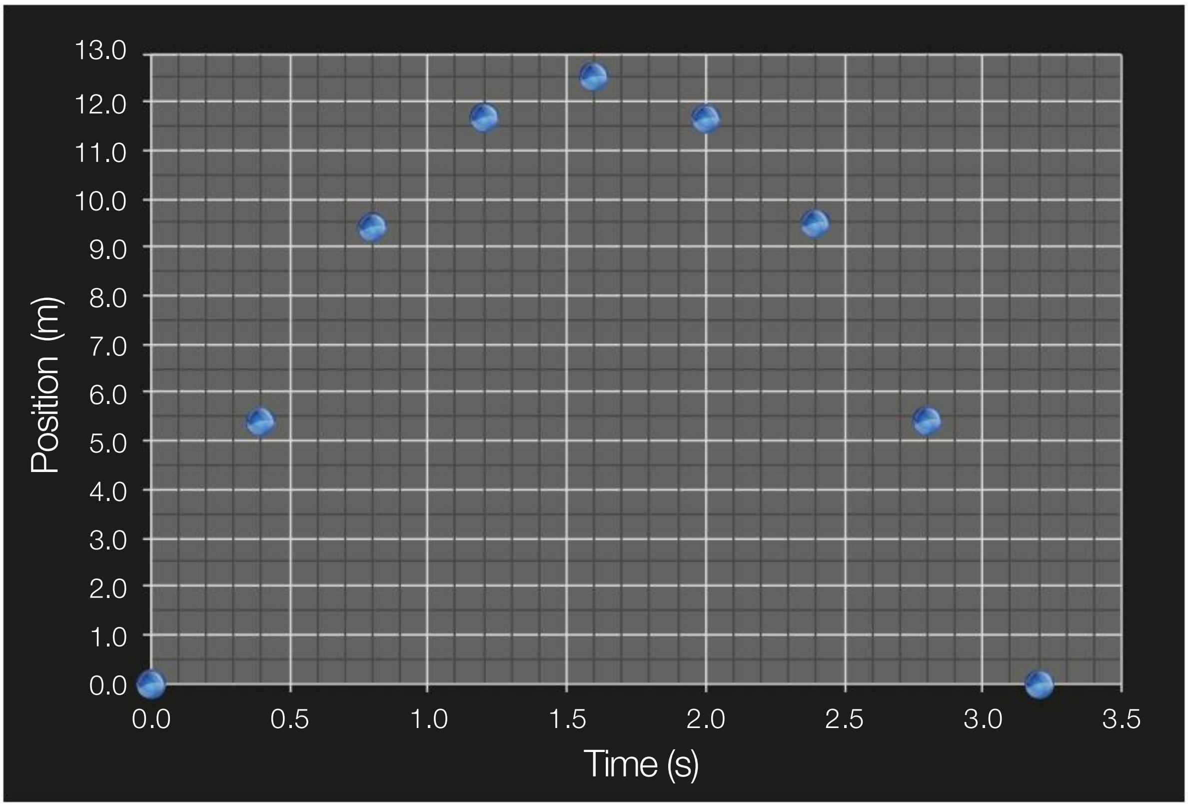
(ii) hockey ball's time of flight.

(iii) velocity with which the hockey ball was struck.

Space for vector diagram

(c) Describe and explain the effect that a significant amount of air resistance would have on the range and height reached by the hockey ball.

8. Consider the multi-image diagram shown below.



(a) Describe, with reason what the multi-image diagram indicates about the horizontal motion of the object.

....

....

....

(b) Calculate the acceleration of the object in the vertical direction.

....

....

....

(c) On the diagram, draw vector arrows that represent the velocity of the object at 0.4 s and 2.0 s.

9. An athlete releases a javelin from a height of 2.00 m, with a velocity of 23.0 ms^{-1} 40.0° above the horizontal.

(a) Resolve the initial velocity of the javelin into it's component vectors.

(b) State, with reason, the javelin's velocity at maximum height.

(c) Calculate the height reached by the javelin.

(d) The javelin is thrown 55.4 m. Calculate the time the javelin spends in the air.

(e) Another taller athlete throws the javelin with the same velocity. Explain why the taller athlete will throw the javelin further.

10. A snowboarder makes a jump from a hill made of snow with a velocity of 9.0 ms^{-1}, 45° above the horizontal.

(a) Calculate the height achieved by the snowboarder relative to his launch position.

(b) The snow boarder lands 1.5 m below the launch position. Calculate the time that the snowboarder spends in the air.

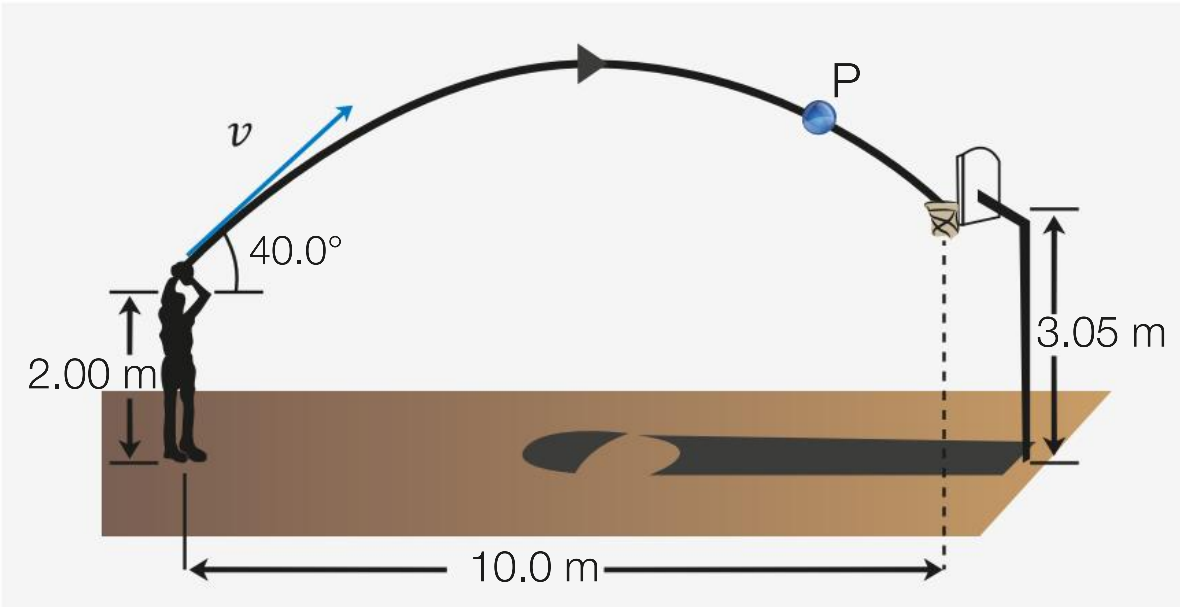
(c) Calculate the magnitude of the horizontal displacement achieved by the snow boarder.

(d) Suggest two ways the snowboarder could achieve a greater horizontal displacement.

(e) State the magnitude and direction of the snow boarder's acceleration at maximum height.

(f) Draw a vector arrow to represent the velocity of the snow boarder at maximum height.

11. A basketball player shoots and scores a goal. The vertical component of the ball's velocity as it leaves the player's hand is 7.00 ms^{-1}. The ball is thrown at an angle of 40.0° above the horizontal. The ball is released at a height of 2.00 m and the goal hoop is 3.05 m above the ground and 10.0 m from the player.



(a) Calculate the magnitude of the velocity *v*, with which the ball is thrown.

(b) Calculate the horizontal component of the ball's initial velocity and hence calculate the time taken for the ball to reach the goal hoop.

(c) Show that the ball takes 0.714 s to reach its maximum height.

(d) Calculate the magnitude and direction of the ball's velocity as it passes through the goal hoop.

Space for vector diagram

(e) Determine the displacement of the ball as it passes through the goal hoop.

Space for vector diagram

12. Two athletes can exert the same force on a ball when they throw it, to produce the same launch speed. One athlete is considerably taller than the other. Assuming they release the ball from the same point relative to their body;

(a) State which of the two athletes should throw the ball at a larger angle above the horizontal in order to achieve a maximum range.

(b) If the ball is launched at 10.0 ms^{-1} and at and angle of 40.0° above the horizontal, state the magnitude of each of the following quantities at the maximum height:

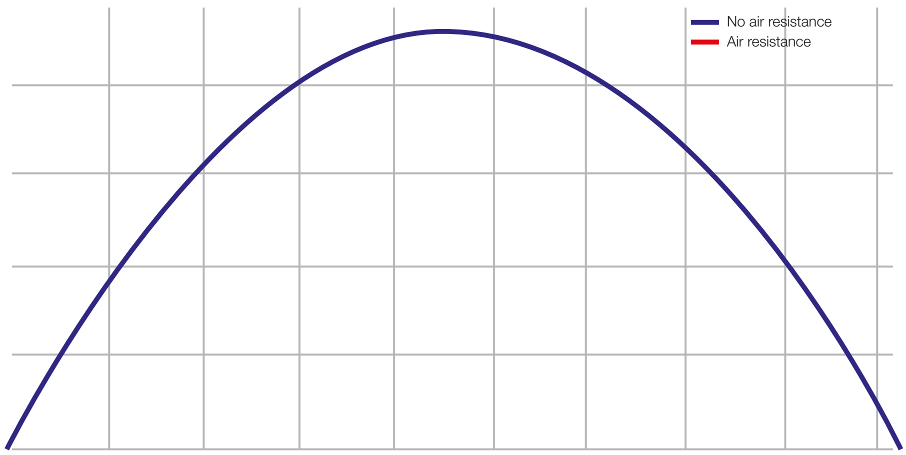
(i) acceleration

(ii) horizontal component of velocity

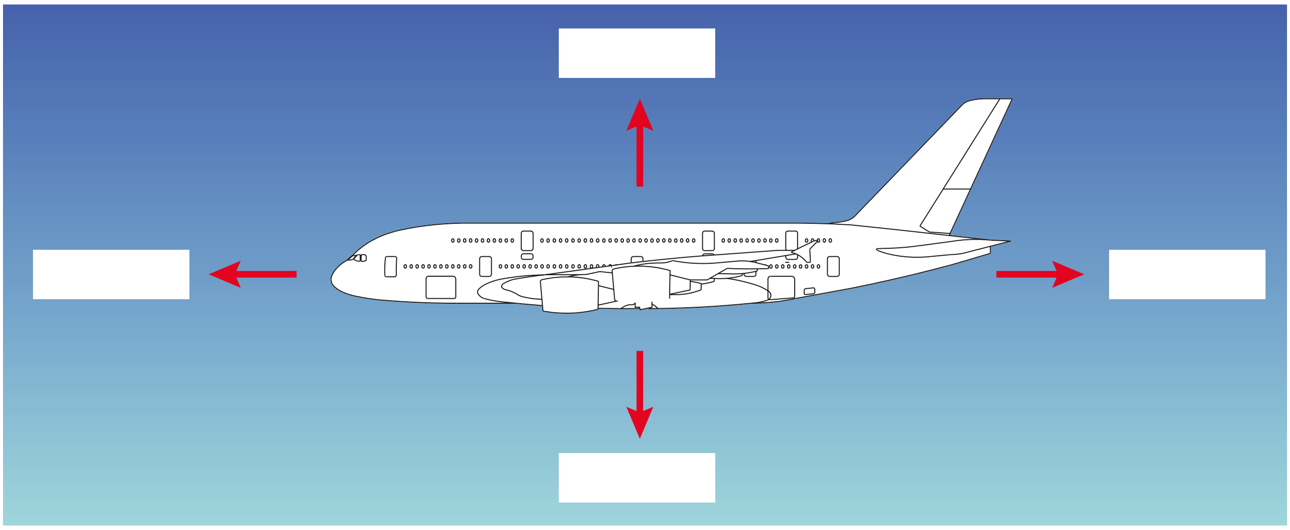
(iii) vertical component of velocity

(iv) velocity

13. The line drawn in blue below represents the path taken by a projectile in the absence of air resistance. On the same diagram, draw a red line to represent the path when a significant amount of air resistance acts on the projectile.



14. The aeroplane shown below is flying with a constant velocity. Place labels in each of the four boxes to name the force acting in each direction indicated by the red arrows.



15. A 55.0 kg student exerts a force of 58.0 N on a bicycle of mass 20.0 kg in order to accelerate forwards. A total drag force of 16.0 N acts.

(a) Draw labelled vector arrows on the diagram below to represent the forces acting on the bicycle.

(b) State the magnitude and direction of the resultant force on the bicycle.

....

(c) Calculate the magnitude and direction of the acceleration of the bicycle.

....

....

(d) Explain why terminal velocity will be reached as the student increases their speed.

....

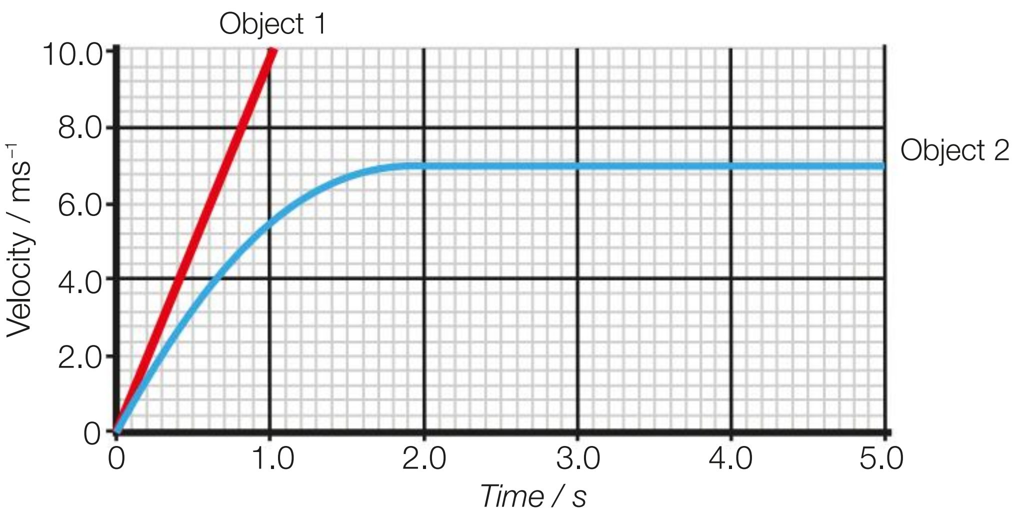
....

16. A 200 g ball falls vertically through the air with a constant velocity.

(a) Draw labelled vector arrows on the diagram below to represent the forces acting on the ball.

(b) Calculate the magnitude of the drag force acting on the ball. Justify your answer.

17. The following graph represents how the speed of two falling objects changes with time. One object is acted upon by air resistance while the other is not.



(a) State which of the two objects is being acted upon by air resistance.

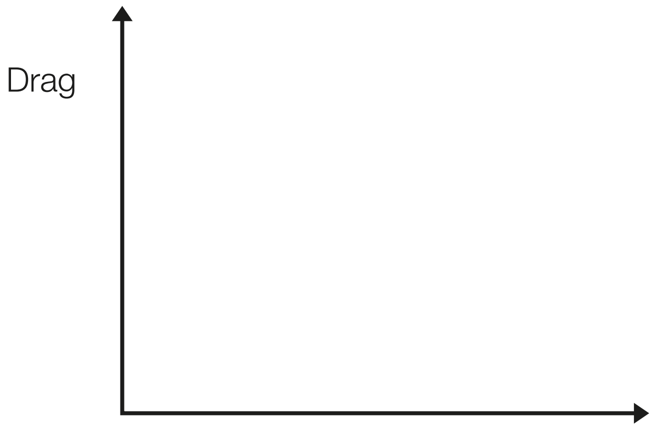
(b) Describe and explain how the motion of the two objects differ.

18. The drag formula is given by $Drag = \frac{1}{2}C_d A\rho v^2$. In this formula, A, represents the area of cross-sectional area of the object, ρ represents the density of the medium and v represents the speed of the object. *Cd* represents the coefficient of drag. This is a factor representing the drag acting on an object due to its shape and inclination.

A student is planning to investigate the relationship between the drag forces experienced by a toy car and its speed in a wind tunnel.

(a) State in words, the expected relationship between the drag force experienced by the toy car and the speed of the toy car.

(b) The student plans to plot a graph in order to establish this relationship. The student plans to plot the drag force acting on the vertical axis. Add the quantity that should be plotted on the horizontal axis.



(c) Describe the appearance of the line of best fit that the student would expect if the relationship between the drag force and speed is successfully established.

1.2 Forces and momentum

Science understanding

1. Momentum is a property of moving objects and is defined as the product of the mass and the velocity of the object. It is conserved in an isolated system and may be transferred from one object to other objects when a force acts over a time interval.

   Kinetic energy is a property of moving objects, and is given by the formula $E_K = \frac{1}{2}mv^2$.

   Newton's Second Law of Motion can be expressed as two formulae, $\vec{F} = m\vec{a}$ and $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, where $\vec{p} = m\vec{v}$ is the momentum of the object.

   - Derive $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ by substituting the defining formula for acceleration ($\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$) into Newton's Second Law of Motion, $\vec{F} = m\vec{a}$, for particles of fixed mass. (The net force $\vec{F}$, and hence the acceleration $\vec{a}$ are assumed to be constant. Otherwise, average or instantaneous quantities apply.)
   - Draw vector diagrams in one dimension or in two dimensions (with right-angled or equilateral triangles) in which the initial momentum is subtracted from the final momentum, giving the change in momentum, $\Delta\vec{p}$.
   - Solve problems (in both one dimension and in two dimensions) using the formulae $\vec{F} = m\vec{a}$, $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ and $E_K = \frac{1}{2}mv^2$.

2. Newton's Third Law of Motion states, $\vec{F}_1 = -\vec{F}_2$. Momentum is conserved in an isolated system of particles. In such a system, the particles are subject only to the forces that they exert on each other.

   - Derive a formula expressing the conservation of momentum for two interacting particles by substituting $\vec{F}_1 = \frac{\Delta\vec{p}_1}{\Delta t}$ and $\vec{F}_2 = \frac{\Delta\vec{p}_2}{\Delta t}$ into $\vec{F}_1 = -\vec{F}_2$.
   - Use the law of the conservation of momentum to solve problems in one and two dimensions.
   - Use vector addition or subtraction in one dimension or in two dimensions (with right-angled or equilateral triangles) to solve problems using the law of conservation of momentum.
   - Analyse multi-image representations to solve conservation of momentum problems, using only situations in which the mass of one object is an integral multiple of the mass of the other object(s). The scale of the representations and the flash rate can be ignored.

3. The conservation of momentum can be used to explain the propulsion of spacecraft, ion thrusters, and solar sails.

   - Use the conservation of momentum to describe and explain the change in momentum and acceleration of spacecraft due to the emission of gas particles or ionised particles.
   - Use the conservation of momentum to describe and explain how the reflection of particles of light (photons) can be used to accelerate a solar sail.
   - Use vector diagrams to compare the acceleration of a spacecraft, using a solar sail where photons are reflected with the acceleration of a spacecraft, and using a solar sail where photons are absorbed.

Many of these ideas have been introduced in Stage 1 through one-dimensional situations. The focus here should be on two-dimensional situations.

This topic uses the concepts of acceleration and force developed in the Stage 1, Subtopics 1.1: Motion under Constant Acceleration and 1.2 Forces and Momentum.

Momentum

The concepts of momentum and impulse were introduced in the Stage 1 Workbook and applied to one-dimensional situations. The Stage 2 Subject Outline requires students to solve problems in both one dimension and two dimensions.

Momentum $\vec{p}$ is defined as the product of an object's mass and velocity.

$$\vec{p} = m\vec{v}$$

where m is the mass in kilograms (kg) and v is the velocity in ms^{-1}.

Momentum is a vector quantity which means that it has both magnitude and direction.

The unit of momentum can be written as $kgms^{-1}$ or sN (this unit is explained below).

1

Newton's Second Law in terms of momentum

Newton's Second Law $\vec{F} = m\vec{a}$, can be expressed as $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ where $\Delta\vec{p}$ is the change in momentum.

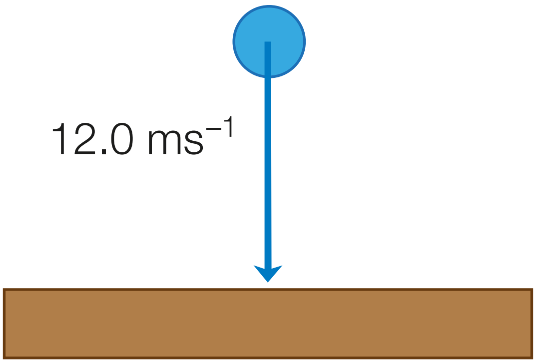
Derivation

$$\vec{F} = m\vec{a} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

This equation means that Newton's Second Law can be expressed as the time rate of change in momentum. The force exerted on an object is directly proportional to the change in momentum experienced by the object and inversely proportional to the time over which the force acts. Rearranging the formula gives $\Delta\vec{p} = \Delta t\vec{F}$. This leads to the idea that the unit of momentum can be expressed as sN.

Worked Examples

1. A 50.0 g super ball strikes the ground and bounces without a loss in speed.



(a) Calculate the change in velocity $\Delta\vec{v}$, of the super ball.

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 12.0\uparrow - 12.0\downarrow = 12.0\uparrow + 12.0\uparrow = 24.0\ ms^{-1}\uparrow$ (i.e. 90° away from the surface of the ground)

(b) Calculate the change in momentum $\Delta\vec{p}$ of the super ball.

$\Delta\vec{p} = m\Delta\vec{v} = 0.0500 \times 24.0 = 1.20\ kgms^{-1}\ or\ sN\ \uparrow$ (*90° away from the ground*)

(c) The super ball is in contact with the ground for 0.0100 s. Calculate the magnitude and direction of the force that the super ball exerts on the ground.

$\vec{F}_{ball} = \frac{\Delta\vec{p}_{ball}}{\Delta t} = \frac{1.20}{0.0100} = 120N\uparrow$

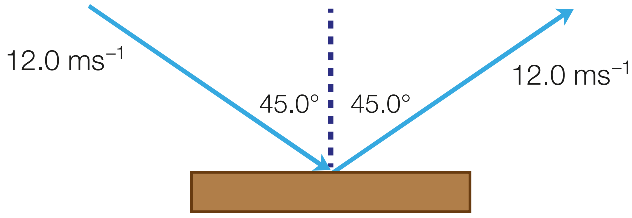
Using Newton's Third Law, $\vec{F}_{ground} = -\vec{F}_{ball} = 120N\downarrow$ (i.e. 90° *towards the ground*)

(d) Calculate the acceleration $\vec{a}$ experienced by the super ball over the period of 0.0100 s.

$\vec{a} = \frac{\vec{F}}{m}$ *or* $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ can be used

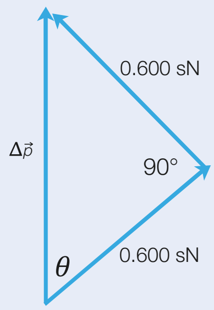
$\vec{a} = \frac{\vec{F}}{m} = \frac{120}{0.0500} = 2.40 \times 10^3\ ms^{-2}\uparrow$

2. Consider the same super ball from question 1. This time, the super ball collides with the ground at an angle as shown in the diagram below.



(a) Calculate the magnitude and direction of the change in momentum of the super ball.

You could follow the process in question 1(a) and (b) or find the change in momentum directly as shown below.



$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = 0.0500 \times 12.0 \nearrow - 0.0500 \times 12.0 \searrow$

$\Delta\vec{p} = 0.600 \nearrow + 0.600 \nwarrow$

$\Delta\vec{p} = \sqrt{(0.600)^2 + (0.600)^2} = 0.849 \; sN$

$\tan\theta = \frac{0.600}{0.600} \; \therefore \theta = 45.0°$

$\Delta\vec{p} = 0.849 \; sN$ $90°$ *away from the ground*

(b) Calculate the force that the super ball exerts on the ground in this situation.

$\vec{F}_{ball} = \frac{\Delta\vec{p}_{ball}}{\Delta t} = \frac{0.849}{0.0100} = 84.9N \uparrow$

Using Newton's Third Law, $\vec{F}_{ground} = -\vec{F}_{ball} = 84.9N \downarrow$ (i.e. 90° towards the ground)

Law of conservation of momentum

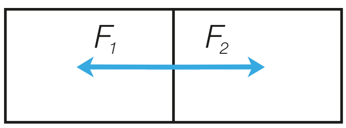
The law of conservation of momentum states that in an isolated system where no external forces act, the total momentum of the system before an interaction is equal to the total momentum of the system after an interaction.

The Stage 2 Subject Outline specifically refers to a system of two interacting particles but the law extends to any number of particles.

Derivation

The law of conservation of momentum can be derived by considering two interacting particles which are subject only to the force of each other.

Consider the two particles colliding head-on.



According to Newton's Third Law of Motion: $\vec{F}_1 = -\vec{F}_2$

According to Newton's Second Law of Motion: $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

It follows that:

$$\frac{\Delta\vec{p}_1}{\Delta t} = -\frac{\Delta\vec{p}_2}{\Delta t}$$

$$m_1v_{1f} - m_1v_{1i} = -(m_2v_{2f} - m_2v_{2i})$$

$$m_1v_{1f} - m_1v_{1i} = -m_2v_{2f} + m_2v_{2i}$$

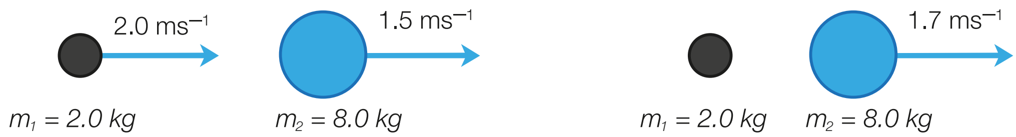
$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$$

i.e. The total initial momentum is equal to the total final momentum.

1

Worked Examples

1. Two masses m_1 = 2.0 kg and m_2 = 8.0 kg are moving East on a smooth horizontal surface with speeds 2.0 ms^{-1} and 1.5 ms^{-1} respectively. The two masses collide and after the collision m_2 = 8.0 kg continues to move in an easterly direction with a speed of 1.7 ms^{-1}.



Before the collision *After the collision*

(a) Determine the magnitude and direction of the velocity of the 2.0 kg mass after the collision.

Using the law of conservation of momentum $\vec{p}_i = \vec{p}_f$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$2.0\,x\,2.0 \rightarrow +8.0x1.5 \rightarrow = 2xv + 8.0x1.7 \rightarrow$$

$$4.0 \rightarrow + 12 \rightarrow = 2.0xv \ +13.6 \rightarrow$$

$$16 \rightarrow = 2.0xv \ +13.6 \rightarrow$$

$$16 \rightarrow \ -13.6 \rightarrow = 2.0xv$$

$$16 \rightarrow \ +13.6 \leftarrow = 2.0xv$$

$$2.4 \rightarrow = 2.0xv$$

$$v = \frac{2.4 \rightarrow}{2.0} = 1.2\ ms^{-1} \rightarrow$$

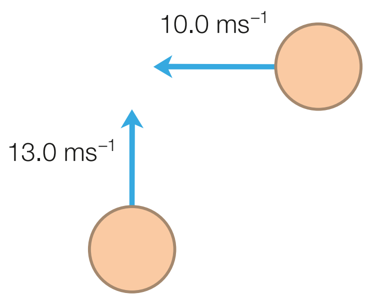
i.e. *1.2 ms^{-1}* East

Note: Direction can be defined as positive to the right and negative to the left as discussed in the Stage 1 Workbook. If this is the case, the value for *v* will be negative and indicates the velocity is to the West.

(b) State any assumption that you have made in calculating your answer.

The system is isolated i.e. no external forces are acting.

2. A 166 g ice hockey puck travelling with a velocity of 10.0 ms^{-1} West, collides with an identical puck travelling with a velocity of 13.0 ms^{-1} North. After the collision the ice hockey pucks stick together and move off as one mass. Assume any frictional forces to be negligible.



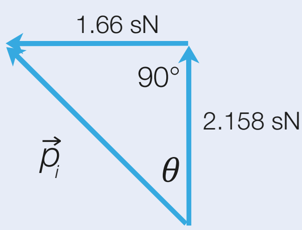
(a) Calculate the total initial momentum of the ice hockey pucks.

$$p_i = m_1v_{1i} + m_2v_{2i} = 0.166 \times 13.0 \uparrow + 0.166 \times 10.0 \leftarrow = 2.158 \uparrow + 1.66 \leftarrow$$

$$p_i = \sqrt{(1.66)^2 + (2.158)^2} = 2.72\ sN$$

$$\tan\theta = \frac{1.66}{2.158} \quad \therefore \quad \theta = 37.6^\circ$$

$$\vec{p}_i = 2.72\ \ sN \quad N37.6^\circ W$$



(b) State the total final momentum of the ice hockey pucks and hence calculate the velocity of the pucks after the collision has occurred.

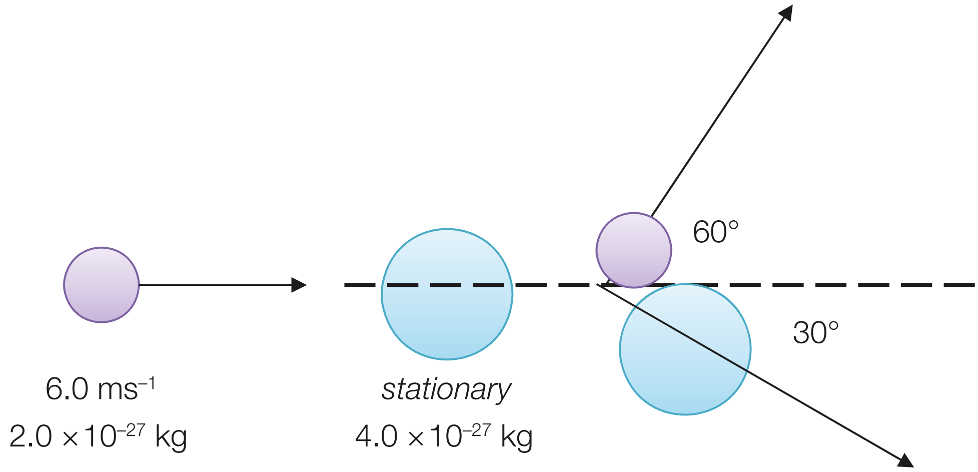
$\vec{p}_f = 2.72 \ sN \quad N37.6°W$

Total mass = 332 g

$p_f = m_{total} v_f = 2.72$

$\therefore \ v_f = \frac{2.72}{0.332} = 8.19 \ ms^{-1} \ N37.6°W$

3. Two gas particles collide as shown below.



(a) Calculate the total initial momentum of the gas particles before the collision occurred.

$p_i = m_1 v_{1i} + m_2 v_{2i} = 6.0 \times 2.0 \times 10^{-27} \rightarrow = 1.2 \times 10^{-26} \ sN \ \rightarrow$

(b) State, with reason, the total final momentum of the gas particles.

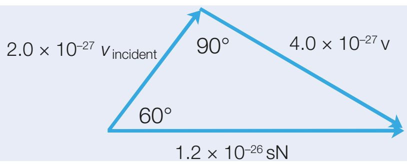
Assuming the system is isolated, then using the law of conservation of momentum, the total initial momentum is equal to the total final momentum.

$p_f = 1.2 \times 10^{-26} \ sN \ \rightarrow$

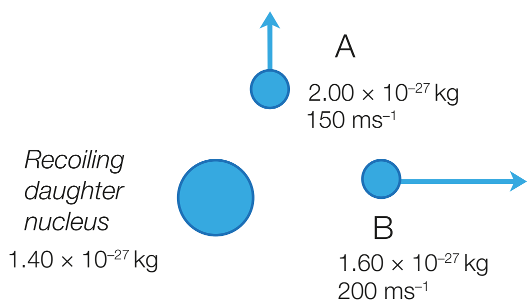
(c) Determine the velocity $\vec{V}$ of the initially stationary gas particle after the collision has occurred.

$\sin 60 = \frac{4.0 \times 10^{-27} v}{1.2 \times 10^{-26}} \ \therefore \ v_{blue} = 2.6 \ ms^{-1}$

$\vec{v}_{blue} = 2.6 \ ms^{-1} \ 30° \ below \ the \ horizontal$



4. A nucleus of mass 5.00×10^{-27} kg decays by ejecting two particles A and B. Particle A has a mass of 2.00×10^{-27} kg and is ejected with a velocity of 150 ms^{-1} North. Particle B has a mass of 1.60×10^{-27} kg and is ejected with a velocity of 200 ms^{-1} in an easterly direction.



(a) State the total initial and final momentum of the system. Justify your answer.

The original nucleus was stationary so the initial momentum is 0 sN.

Assuming the system is isolated and using the law of conservation of momentum, the total initial momentum is equal to the total final momentum. The final momentum is also 0 sN

(b) Determine the 'recoil' momentum of the remaining (daughter) nucleus.

$\vec{p}_i = \vec{p}_f$

$0 = m_A v_A + m_B v_B + p_{DN}$

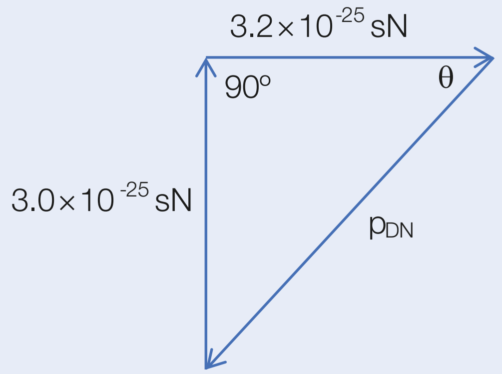
$0 = 2.00 \times 10^{-27} \times 150 \uparrow + 1.60 \times 10^{-27} \times 200 \rightarrow + P_{DN}$

$0 = 3.0 \times 10^{-25} \uparrow + 3.2 \times 10^{-25} \rightarrow + P_{DN}$

$p_{DN} = \sqrt{(3.0 \times 10^{-25})^2 + (3.2 \times 10^{-25})^2} = 4.4 \times 10^{-25}\ sN$

$\tan\theta = \dfrac{3.0 \times 10^{-25}}{3.2 \times 10^{-25}} \quad \therefore \quad \theta = 43.2°$

$\vec{p}_{DN} = 4.4 \times 10^{-25}\ sN\ \ W43\ S$



(c) Calculate the recoil velocity of the daughter nucleus.

$p_{DN} = m_{DN} v_{DN}$

$4.4 \times 10^{-25} = 1.4 \times 10^{-27} v_{DN}$

$v_{DN} = 310\ ms^{-1}\ W43\ S$

Multi-image representations

Multi-image representations such as Figure 1.2.1 below can be analysed to determine whether momentum is conserved during a collision involving two or more particles. The term 'particle' can refer to any object or body.

Any flash rate involved in taking the image can be ignored because the time (Δt) between successive images is constant. Figure 1.2.1 therefore indicates that the objects are travelling with constant velocity both before and after they collide. This is because $v = \frac{s}{\Delta t}$ and the distance (*s*) between successive images is constant. This indicates that the system is isolated.

Since $v = \frac{s}{\Delta t}$, it follows that the velocity is proportional to the distance travelled between successive images. Since $p = mv$, the momentum is proportional to mass and the distance between successive images. This means that both the time interval between the images and the actual mass of the object can be ignored. The momentum of an object of mass *m* can be representation by a vector arrow connecting two successive images. An arrow twice the length (i.e. across two time intervals) is drawn for an object with mass *2m*, an arrow three times the length (i.e. across three time intervals) is drawn for an object with mass *3m* etc.

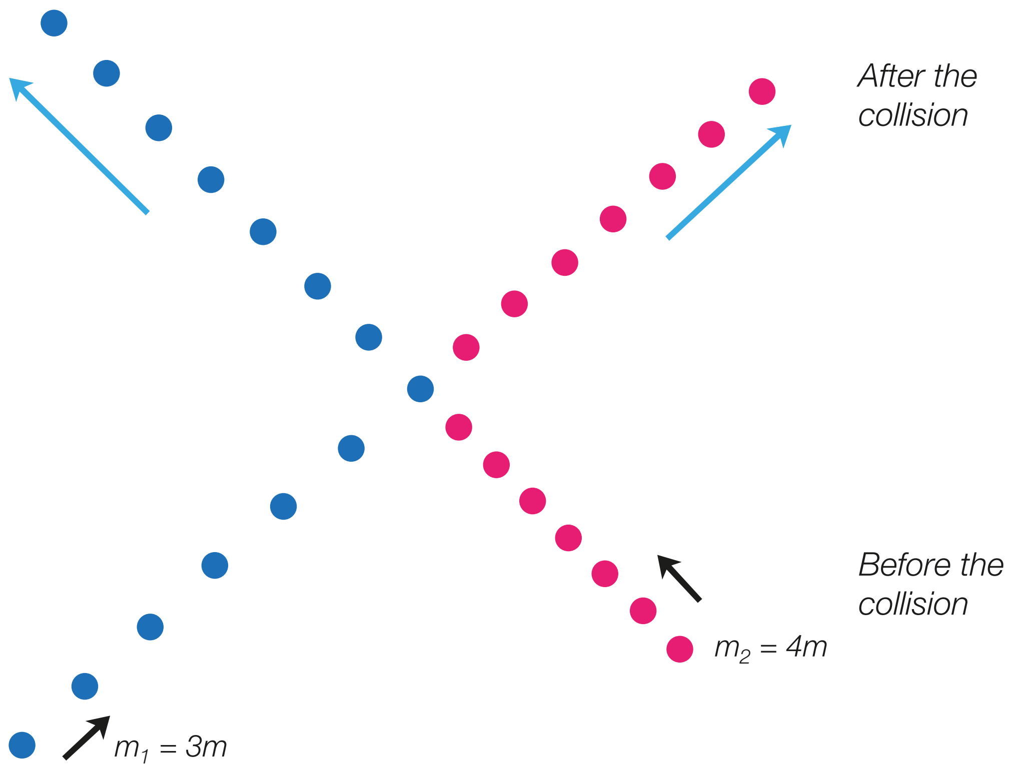
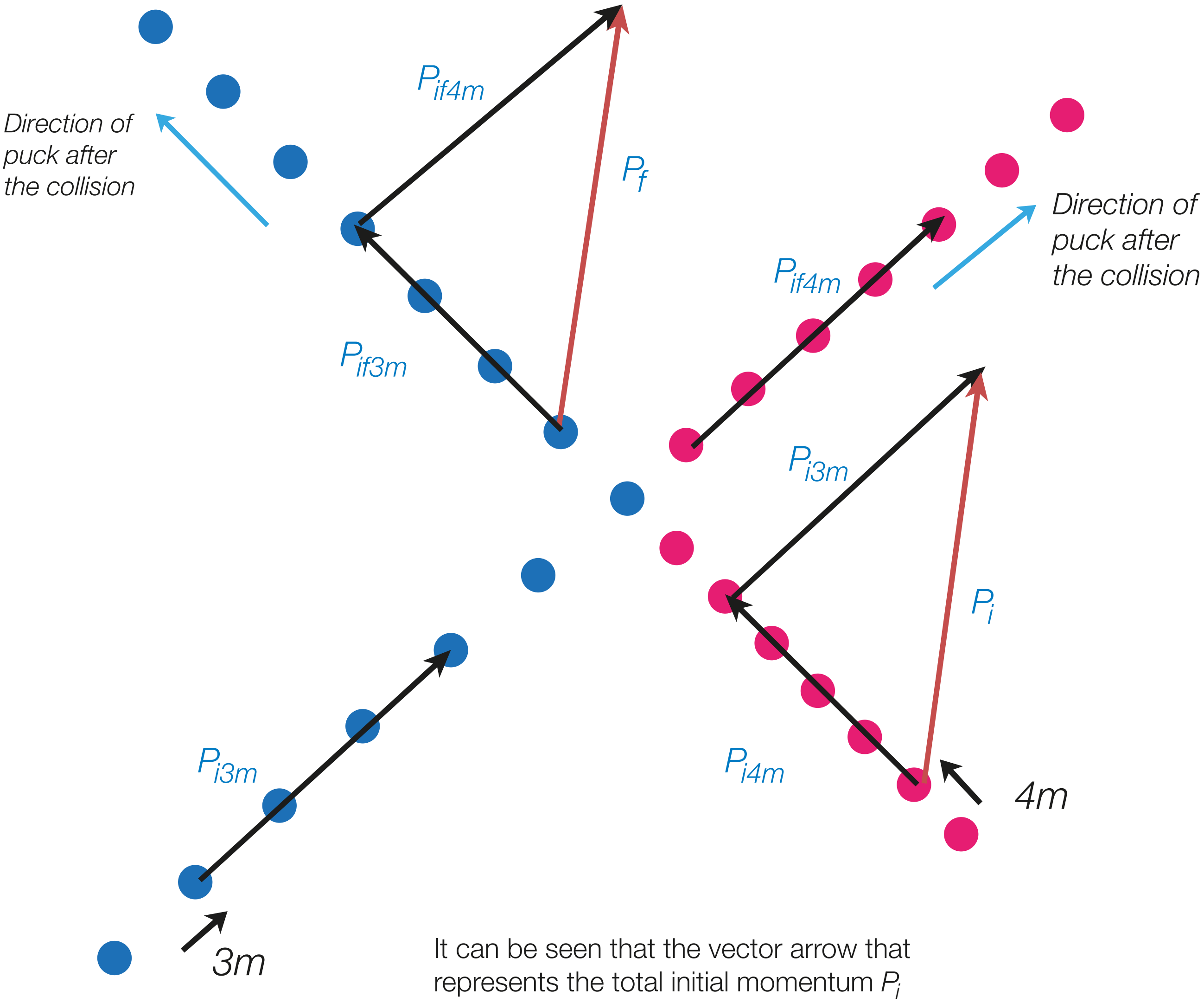


Figure 1.2.1

Worked Example

The multi-image representation below captures the collision between two pucks on an air table. Draw appropriate vector arrows to show that momentum is conserved during this collision.



It can be seen that the vector arrow that represents the total initial momentum P_i is equal in length and parallel to the vector arrow that represents the total final momentum P_f. Momentum is conserved in this collision.

Spacecraft propulsion

Change in momentum and acceleration of a spacecraft due to the emission of gas particles or ionised particles.

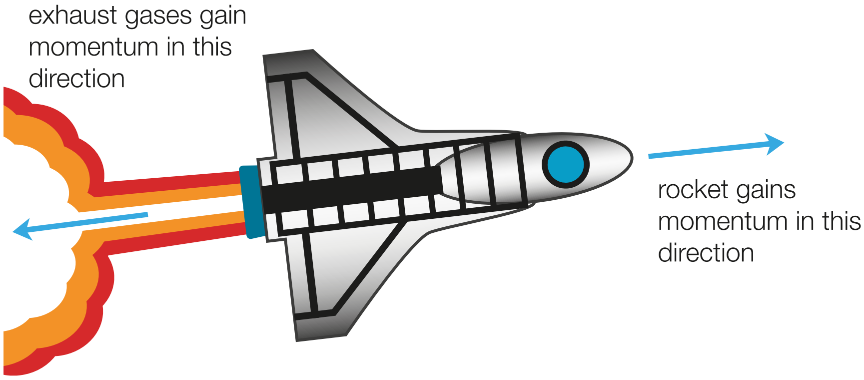


Figure 1.2.2

A spacecraft can accelerate forward by emitting gas particles or ionised particles such as xenon ions created by bombarding xenon gas particles with electrons. The incident electrons collide with electrons in the xenon atoms removing them from the xenon atoms. The xenon atoms now have a net positive charge. They are referred to as xenon ions. The xenon ions are then accelerated to the rear of the spacecraft.

The spacecraft and its onboard propulsion gas can be considered an isolated system. When the spacecraft ejects charged particles with a large velocity from its rear it will gain momentum and accelerate in the opposite direction to the ejected charged particles. This is because the charged particles gain momentum towards the rear of the spacecraft. Using the law of conservation of momentum, the total momentum of the system must remain constant. As a consequence, the spacecraft gains an equal magnitude of momentum but in the opposite direction to the charged particles. A gain in momentum leads to a gain in velocity and by definition an acceleration.

Figure 1.2.2 illustrates how a spacecraft can gain momentum by emitting charged particles or ions. Figures 1.2.3 and 1.2.4 illustrate xenon ion discharge from ion thrusters on spacecraft.

Figure 1.2.3

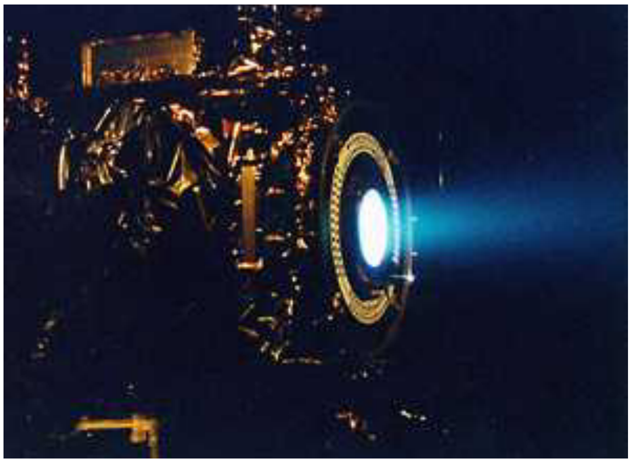


Figure 1.2.4

Xenon ion discharge from the NSTAR ion thruster of Deep Space 1. Credit: NASA

Acceleration of solar sails by the reflection of light particles (called photons)

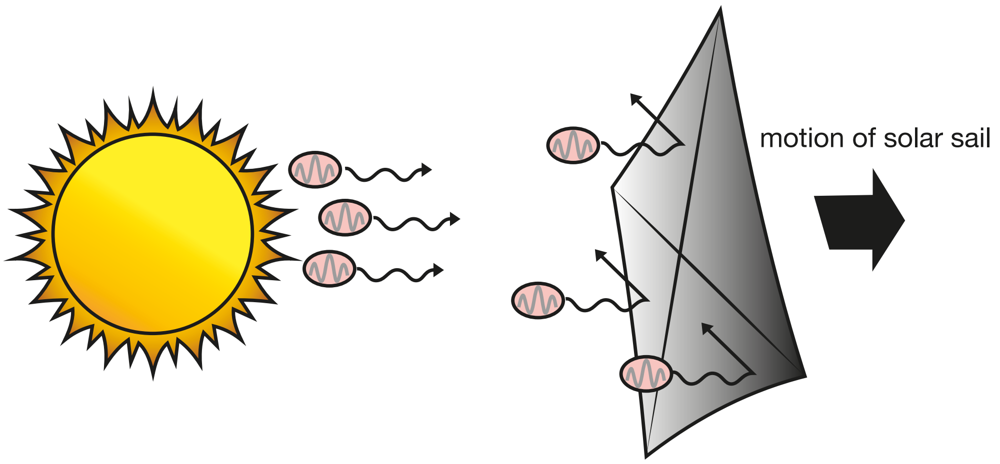


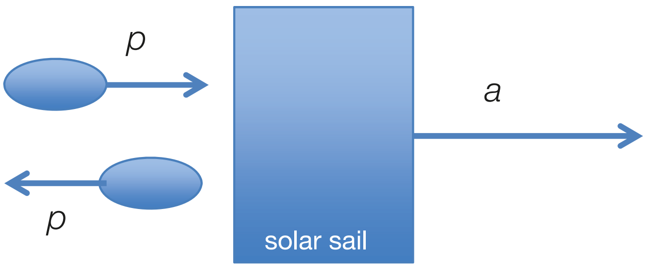
Figure 1.2.5

Figure 1.2.5 illustrates a light particle (or photon) striking a solar sail. When it is reflected it will experience a change in momentum because its direction of motion has changed. In accordance with the law of conservation of momentum, the total momentum of the system remains unchanged. The solar sail will experience the same change in momentum (Δp_{sail}) as the incident light particle but in the opposite direction.

The force exerted on the solar sail by the incident photon is given by $F_{sail} = \frac{\Delta p_{sail}}{\Delta t}$ where Δt is the time over which the reflection occurs. In accordance with Newton's Second Law, the solar sail will experience an acceleration given by $a_{sail} = \frac{F_{sail}}{m_{sail}}$.

When many light particles (say n) are reflected the resultant acceleration of the solar sail is given by $a_{sail} = \frac{nF_{sail}}{m_{sail}}$.

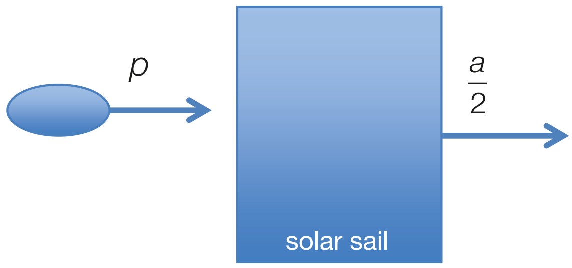
Comparing the acceleration of a spacecraft using a solar sail where photons are reflected from the solar sail to the acceleration of a spacecraft using a solar sail where photons are absorbed by the solar sail.



$\Delta\vec{p} = p_f - p_i = p\leftarrow - p\rightarrow = 2p\leftarrow$

Photon being reflected

Figure 1.2.6



$\Delta\vec{p} = p_f - p_i = 0 - p\rightarrow = p\leftarrow$

Photon being absorbed

Figure 1.2.7

Figure 1.2.6 illustrates a photon of momentum p to the right being reflected from a solar sail. The photon experiences a change in momentum of $2p$ directed to the left.

Figure 1.2.7 illustrates the same photon being absorbed. This time the photon only experiences a change in momentum of magnitude p directed to the left.

In accordance with the law of conservation of momentum, the change in momentum experienced by the solar sail is equal in magnitude and opposite in direction to the collective change in momentum experienced by the many incident photons that strike the sail. It follows that the solar sail gains twice the momentum when the incident photons are reflected. Since $F_{sail} = \frac{\Delta p_{sail}}{\Delta t}$ and $a_{sail} = \frac{F_{sail}}{m_{sail}}$, this means that the force and hence acceleration experienced by the solar sail when photons are reflected rather than absorbed from its surface is twice as large.

Worked Example

A satellite has an initial mass of 750 kg. It emits 50 kg of particle emissions with a velocity of 210 ms^{-1}.

Calculate the gain in velocity of the satellite.

Using the law of conservation of momentum

$0 = m_{satellite-ejectedparticles}\vec{v}_{satellite-ejectedparticles} + m_{particles}\vec{v}_{particles}$

$0 = 700v + 50 \times 210 \leftarrow$

$0 = 700v + 10500 \leftarrow$

$v = \frac{10500}{700} = 15ms^{-1} \rightarrow$

i.e. 15 ms^{-1} in the opposite direction to the ejected particles

? Science inquiry practical

Use the conservation of momentum to determine the speed of a projectile by firing it into a trolley.

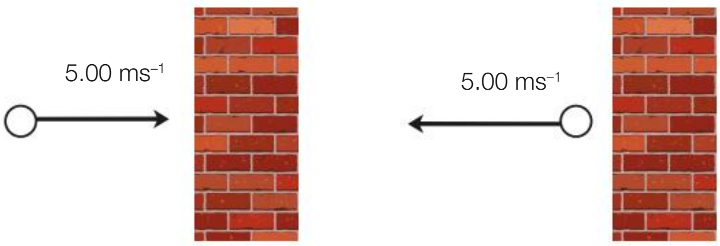
Science as a human endeavour

Some ideas:

1. Investigate how the use of the law of conservation of momentum was used to predict the existence of neutrinos.
2. Explore perspectives in the public debate about economics of space exploration. Is government funding likely to be maintained?
3. Research the most appropriate types of spacecraft propulsion for journeys to different destinations, considering technical challenges and speculative technologies.

Exercises

1. A particle of mass 10.0 g collides a wall as shown below.



Before colliding with the wall   After colliding with the wall

(a) Calculate the change in velocity $\Delta\vec{v}$ of the particle.

....

....

....

(b) Calculate the change in momentum $\Delta\vec{p}$ of the particle.

....

....

(c) Given the collision occurs in a time of 0.0500 s, calculate the force $\vec{F}$ that the wall exerts on the particle.

....

....

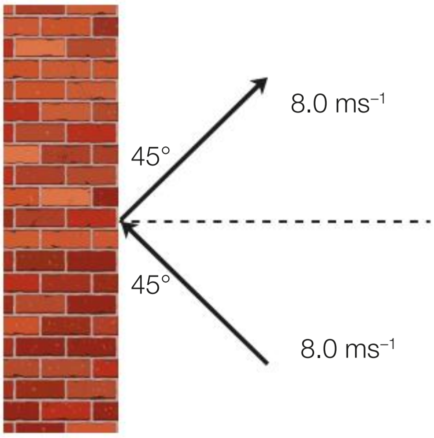
....

(d) State, with reason the force $\vec{F}$ that the particle exerts on the wall.

....

....

2. A 200 g particle collides with a wall as shown below.



(a) Calculate the magnitude and direction of the change in momentum of the particle.

....

....

....

....

....

....

....

Space for vector diagram

(b) Determine the force that the particle exerts on the wall if the collision occurs in a time of 1.00×10^{-2} s.

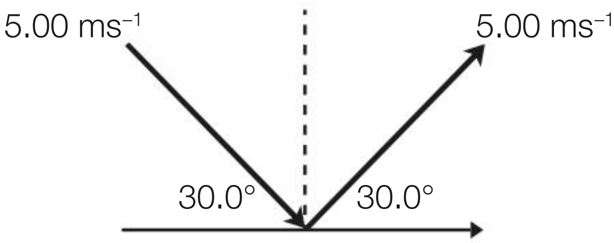
....

....

....

....

3. A 65.0 g tennis ball collides with the ground as shown below. The tennis ball is in contact with the ground for a time of 3.00×10^{-2} s



(a) State the change in speed of the tennis ball.

....

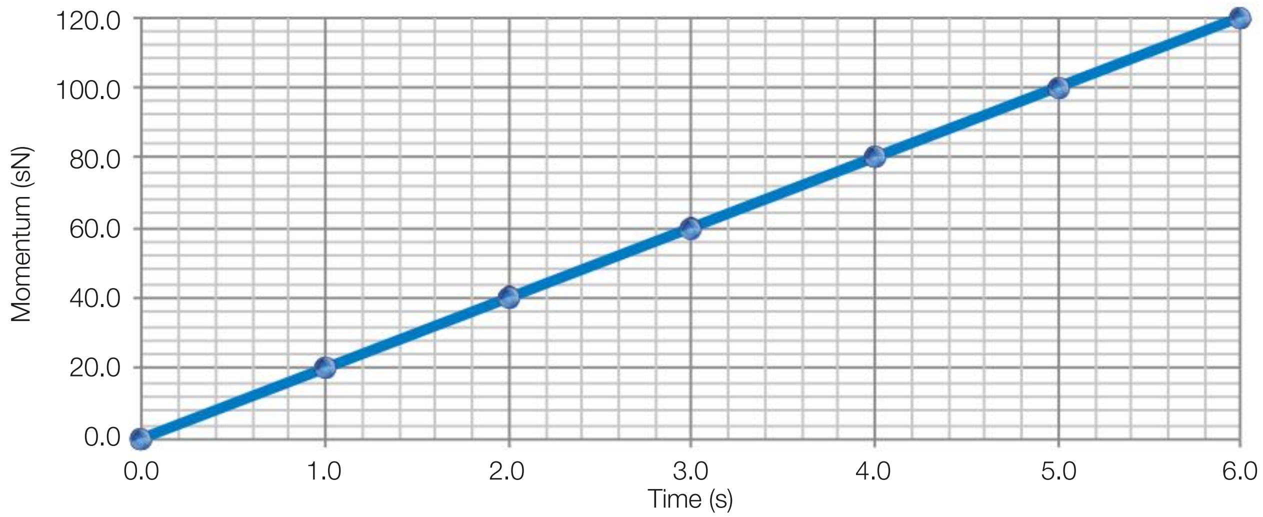
(b) Calculate the change in velocity $\Delta\vec{V}$ of the tennis ball.

Space for vector diagram

(c) Calculate the force $\vec{F}$ experienced by the tennis ball.

(d) State the force $\vec{F}$ that the tennis ball exerts on the ground.

4. The graph shown below illustrates how the momentum of an object varies with time *t*.



(a) Explain why the gradient of the line represents the force acting on the object.

(b) Hence, calculate the magnitude of the force acting on the object.

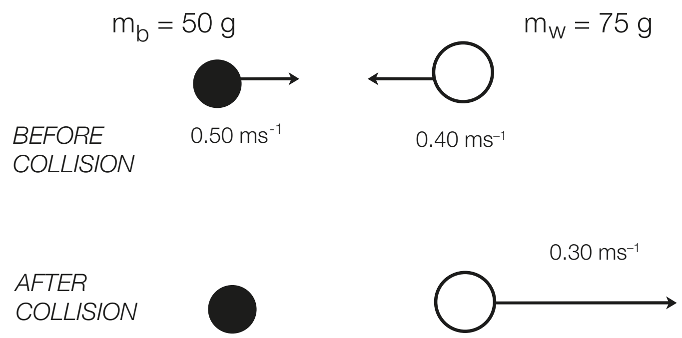
5. (a) State the law of conservation of momentum.

(b) Describe what is meant by an isolated system.

6. Derive the law of conservation of momentum by considering a two particle interaction where the particles are subject only to the force between each other.

Space for diagram

7. Consider the collision below between a black ball and a white ball billiard ball. Assume that the collision occurs in an isolated system. The balls are in contact for a time of 0.040 s during the collision.



(a) Determine the magnitude and direction of the final velocity of the black ball after the collision.

(b) Calculate the magnitude and direction of the change in momentum $\Delta\vec{p}$ of the black ball.

(c) Determine the force $\vec{F}$ that the black ball exerts on the white ball.

8. A particle of mass m, travelling with a velocity $\boldsymbol{v}$ to the right collides with another stationary object that has three times the mass of the first particle. The initially stationary object moves off with a velocity of $\frac{1}{5}\boldsymbol{v}$ to the right.

Before the collision *After the collision*

(a) Write an expression for the total initial momentum of the system.

(b) State the total final momentum of the system. What assumption have you made?

(c) Write an *expression* in terms of v for the magnitude and direction of the velocity of the object of mass m after the collision.

9. A car of mass 1.00×10^3 kg is initially moving due South with a velocity of 72.0 km h^{-1}. Another car of mass 1.40×10^3 kg is travelling due West with a velocity of 60.0 km h^{-1}.

The cars collide on a wet intersection and stick together after the collision.

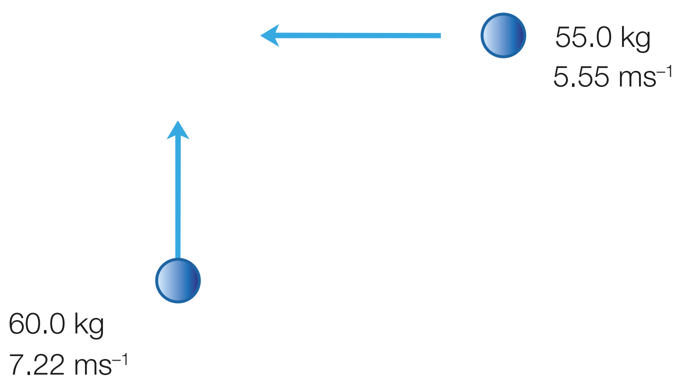
(a) Determine the magnitude and direction of the total initial momentum of the two cars.

Space for vector diagram

(b) State, with reason, the total final momentum of the two cars.

(c) Calculate the combined velocity $\vec{V}$ of the cars after the collision.

10. An ice skater of mass 55.0 kg is skating in a westerly direction with a speed of 5.55 ms^{-1} while another ice skater of mass 60.0 kg is skating with a speed of 7.22 ms^{-1} in a northerly direction. The skaters meet and hold hands so that they move off as one mass.

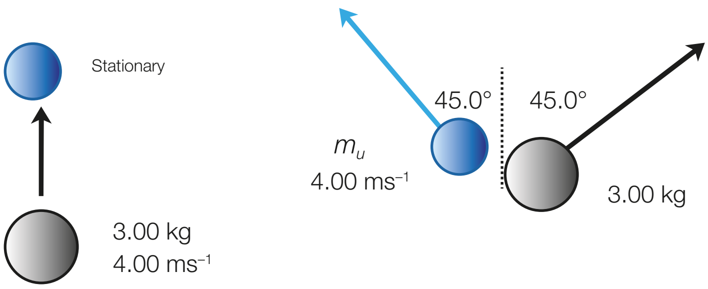


(a) Calculate the total initial momentum of the ice skaters.

Space for vector diagram

(b) Calculate the final velocity of the ice skaters after the collision as they move as one combined mass.

11. A 3.00 kg mass moving with a velocity of 4.00 ms^{-1} North on a frictionless surface, collides with an unknown mass which is initially stationary. The 3.00 kg mass moves off at an angle of 45.0° to the right of its initial path, while the unknown mass moves off at 4.00 ms^{-1} at an angle of 45.0° to the left of the 3.00 kg masses' initial path.



Before the collision *After the collision*

Calculate the

(a) total initial momentum of the system.

(b) speed of the 3.00 kg mass after the collision.

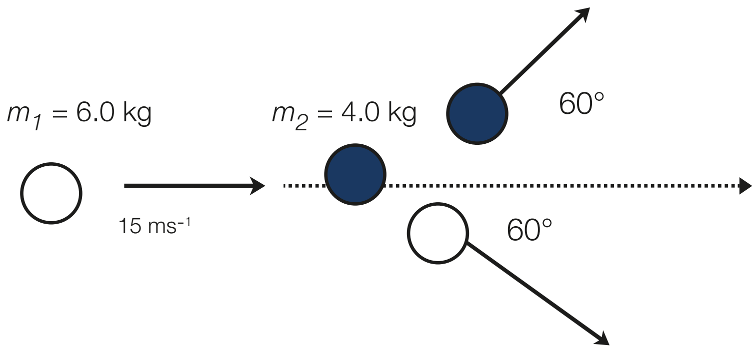
Space for vector diagram

(c) magnitude m_u of the unknown mass.

(d) magnitude of the change in momentum of the unknown mass.

(e) magnitude of the force acting on each mass given that the collision lasts for 0.0500 s.

12. The diagram below illustrates the collision between two masses m_1 and m_2. You can assume that m_2 was originally at rest and that the collision takes place in an isolated system.

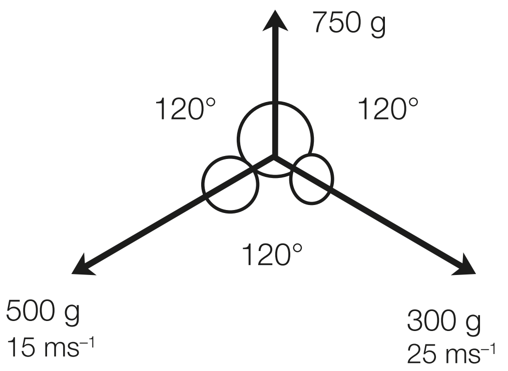


(a) Determine the speed of m_1 after the collision.

Space for vector diagram

(b) Determine the speed of m_2 after the collision.

13. An object explodes into three pieces as shown in the diagram below.

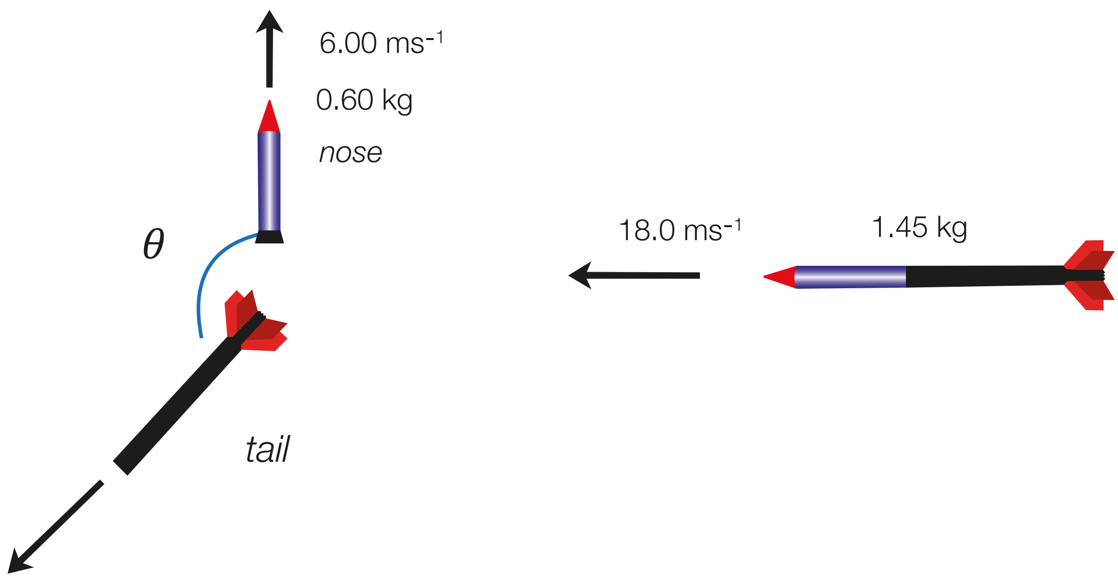


(a) State the total initial and final momentum for the explosion.

(b) Calculate the speed of the 750 g mass after the explosion.

Space for vector diagram

14. A toy rocket of mass 1.45 kg is travelling with a speed of 18.0 ms^{-1} when it explodes into two pieces; the nose section (m_n = 0.60 kg) and the tail section. The nose section moves off as shown with a speed of 6.00 ms^{-1}.



(a) Calculate the momentum of the rocket before it exploded.

....

....

(b) Show that the angle θ is approximately 98°.

(Space for vector diagram)

(c) Calculate the magnitude of the velocity of the tail piece after the explosion.

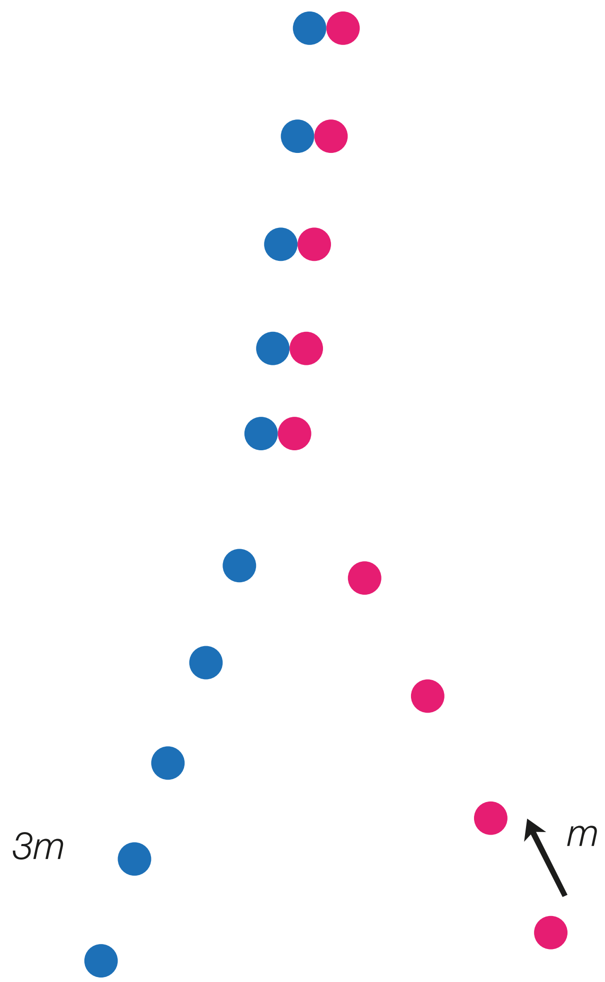
....

....

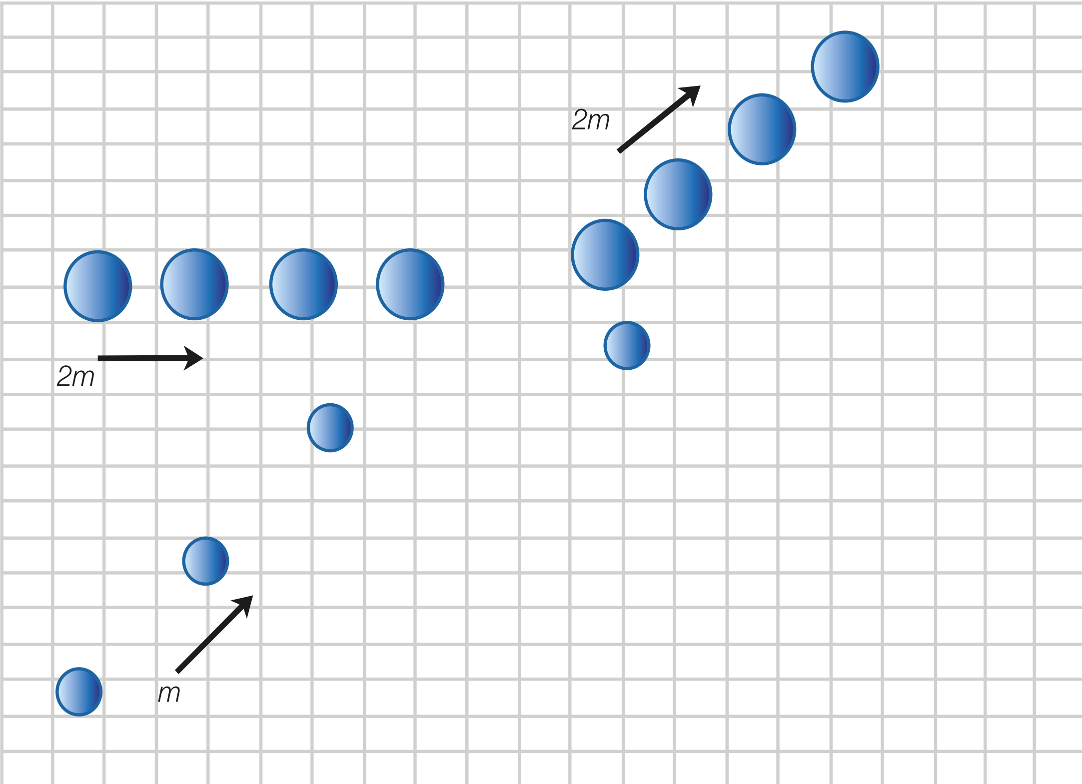
....

15. Consider the multi-image representation shown below.

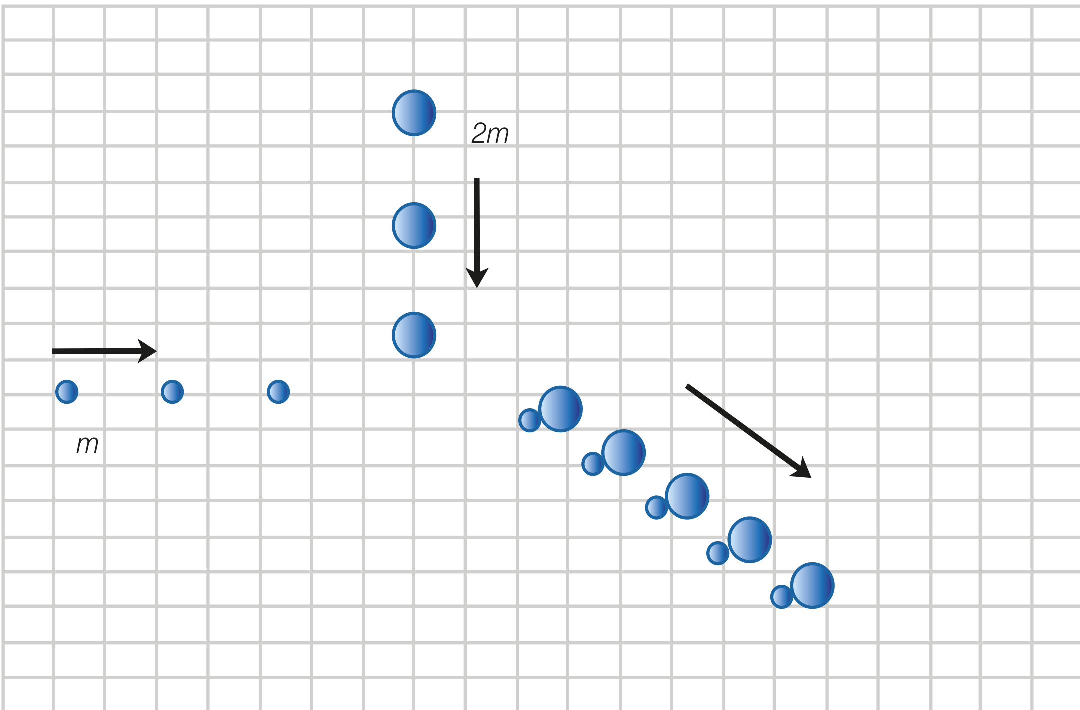
Show that momentum is conserved both in its magnitude and direction during the collision.



16. Two ice hockey pucks collide. The path taken after the collision by the puck with mass 2m is shown in the diagram. One position of the puck with mass *m* is shown after the collision. Draw the next two successive positions for the puck with mass *m* after the collision.



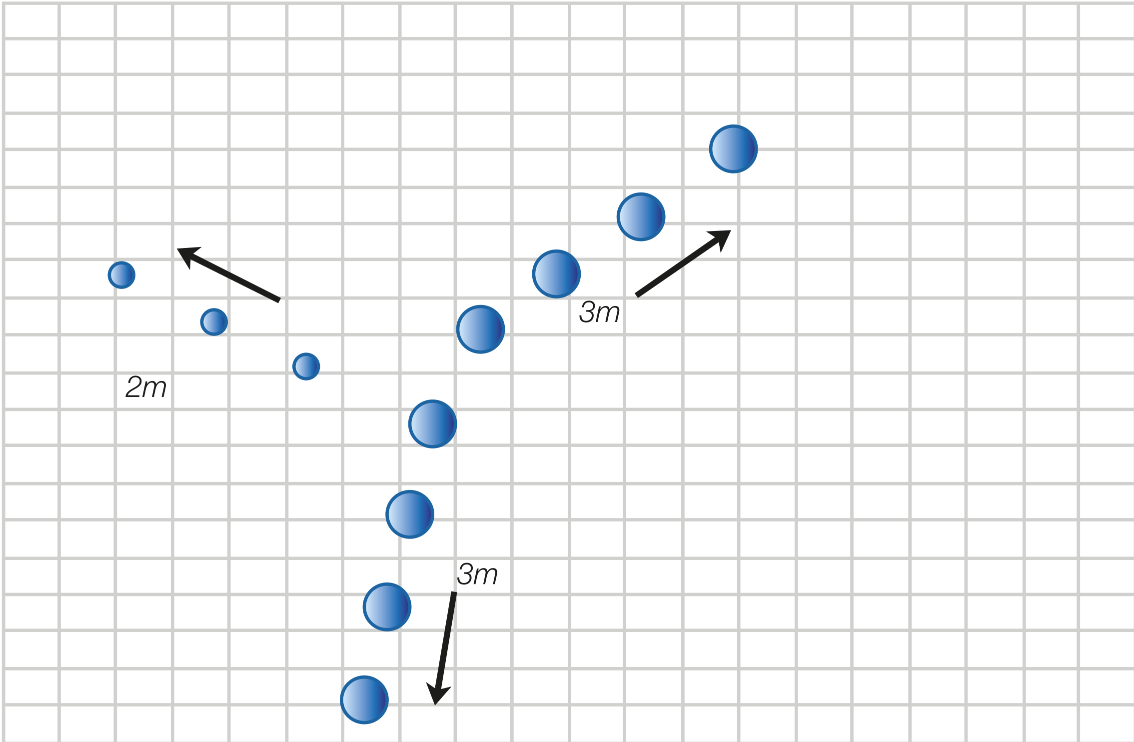
17. (a) Consider the multi-image diagram below. Show that the law of conservation of momentum is not obeyed during this collision.



(b) Suggest a possible reason for the law of conservation of momentum not being obeyed during this collision.

18. The multi-image representation below depicts an explosion. Two fragments of mass *3m* and one of mass *2m* fly off in the directions shown.

Show that momentum is conserved during this explosion.



19. (a) Use the law of conservation of momentum to explain how a spacecraft can gain speed in a given direction by emitting ionised particles.

(b) A spacecraft of mass 5.00×10^4 kg, increases it speed from rest to 5.00 ms^{-1} relative to a second spacecraft by emitting ionised particles with a speed of 50.0 ms^{-1}. Calculate the mass of ionised particles emitted in order to achieve this gain in speed.

20. A solar sail is fitted to a spacecraft of mass 850 kg. Photons with a momentum of 2.0×10^{-27} sN are reflected from the solar sail.

(a) Determine the change in momentum of the photons as they are reflected from the solar sail.

(b) If 5.0×10^{20} photons are reflected per second, calculate the total change in momentum of the photons per second.

(c) Calculate the magnitude of the force exerted on the solar sail each second.

(d) Calculate the acceleration of the solar sail.

(e) How would your answer to part (d) differ if the photons were absorbed instead of being reflected?

21. (a) A given solar sail has a highly reflective surface. Explain why photons of light reflected from the solar sail can result in the acceleration of the solar sail.

(b) A solar sail made from an absorbent material will not accelerate at the same rate as a highly reflective solar sail. Compare with reason, the acceleration of the reflective and black sail solar sails.

1.3 Circular motion and gravitation

Science understanding

1. Centripetal acceleration occurs when the acceleration of an object is perpendicular to the velocity of the of the object. An object that experiences centripetal acceleration undergoes uniform circular motion. The centripetal acceleration is directed towards the centre of the circular path.
2. The magnitude of the centripetal acceleration is constant for a given speed and radius and is given by $a = \frac{v^2}{r}$.
3. The formula $v = \frac{2\pi r}{T}$ relates the speed, v, to the period, T, for an object undergoing circular motion with a radius, r.
   - Solve problems involving the use of the formulae $a = \frac{v^2}{r}$, and $v = \frac{2\pi r}{T}$ and $\vec{F} = m\vec{a}$.
   - Use vector subtraction to show that the change in the velocity $\Delta\vec{v}$, and hence the acceleration, of an object over a very small time interval is directed towards the centre of the circular path.
4. On a flat curve, the friction force between the tyres and the road causes the centripetal acceleration. To improve safety, some roads are banked at an angle above the horizontal.
   - Draw a diagram showing the force vectors (and their components) for a vehicle travelling around a flat curve and around a banked curve.
   - Explain how a banked curve reduces the reliance on friction to provide centripetal acceleration.
5. Objects with mass produce a gravitational field in the space that surrounds them.
6. An object with mass experiences a gravitational force when it is within the gravitational field of another mass.
7. Gravitational field strength, *g,* is defined as the net force per unit mass at a particular point in the field.
8. This definition is expressed quantitatively as $\vec{g} = \frac{\vec{F}}{m}$, hence it is equal to the acceleration due to gravity. The magnitude of the acceleration due to gravity at the surface of the Earth is 9.80 ms^{-2}.
9. All objects with mass attract one another with a gravitational force; the magnitude of this force can be calculated using Newton's Law of Universal Gravitation.

   Every particle in the universe attracts every other particle with a force that is directly proportional to the product of the two masses and inversely proportional to the square of the distance between their centres.

   The force between two masses, m_1 and m_2, separated by distance, r, is given by: $F = \frac{Gm_1m_2}{r^2}$
   - Solve problems using Newton's Universal Law of Gravitation.
   - Use proportionality to discuss changes in the magnitude of the gravitational force on each of the masses as a result of a change in one or both of the masses and/or a change in the distance between them.
   - Explain that the gravitational forces are consistent with Newton's Third Law.
   - Use Newton's Law of Universal Gravitation and Second Law of Motion to calculate the value of the acceleration due to gravity, g, on a planet or moon.
10. Many satellites orbit the Earth in circular orbits.
    - Explain why the centres of the circular orbits of Earth satellites must coincide with the centre of the Earth.
    - Explain that the speed, and hence the period, of a satellite moving in a circular orbit depends only on the radius of the orbit and the mass of the central body (m_2) about which the satellite is orbiting and not on the mass of the satellite.
    - Derive the formula $v = \sqrt{\frac{GM}{r}}$ for the speed, v, of a satellite moving in a circular orbit of radius, r, about a spherically symmetric mass, M, given that its gravitational effects are the same as if all its mass were located at its centre.
11. Kepler's Laws of Planetary Motion describe the motion of planets, their moons, and other satellites.
12. Kepler's First Law of planetary motion: All planets move in elliptical orbits with the Sun at one focus.
13. Kepler's Second Law of Planetary Motion: The radius vector drawn from the Sun to a planet sweeps equal areas in equal time intervals.
    - Use Kepler's first two laws to solve problems involving the motion of comets, planets, moons, and other satellites.
14. Kepler's Third Law of Planetary Motion shows that the period of any satellite depends upon the radius of its orbit.

15. For circular orbits Kepler's Third Law can be expressed as: $T^2 = \frac{4\pi^2}{GM}r^3$.
- Derive: $T^2 = \frac{4\pi^2}{GM}r^3$.
- Solve problems involving the use of the formulae $v = \sqrt{\frac{GM}{r}}$, $v = \frac{2\pi r}{T}$ and $T^2 = \frac{4\pi^2}{GM}r^3$.
- Explain why a satellite in a geostationary orbit must have an orbit in the Earth's equatorial plane, with a relatively large radius and in the same direction as the Earth's rotation.
- Explain the differences between polar, geostationary, and equatorial orbits. Justify the use of each orbit for different applications.
- Perform calculations involving orbital periods, radii, altitudes above the surface, and speeds of satellites, including examples that involve the orbits of geostationary satellites.

This topic uses the concepts of acceleration and force developed in the Stage 1, Subtopics 1.1: Motion under Constant Acceleration and 1.2 Forces.

Uniform circular motion

Uniform circular motion involves an object moving with **constant speed in a circular path**. The **velocity** at any point is at a **tangent to the circular path**.

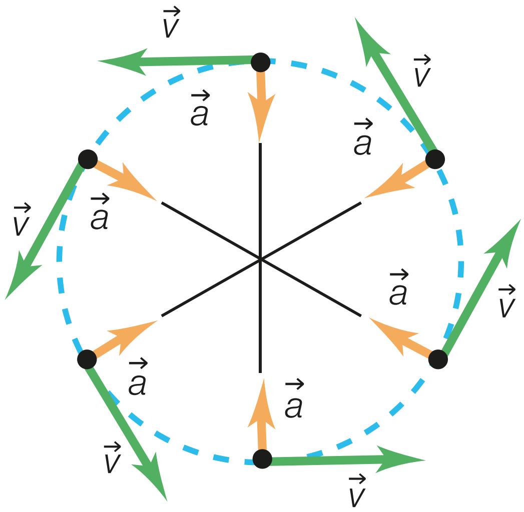


Figure 1.3.1

Figure 1.3.1 shows that the object is constantly changing direction. This means that the velocity is constantly changing. Since $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$, an object undergoing uniform circular motion is accelerating.

The name given to this acceleration is **centripetal acceleration ($\vec{a}$)** and it can be shown using a vector subtraction that this acceleration is directed towards the centre of the circular path.

Centripetal acceleration

The acceleration experienced by an object undergoing uniform circular motion always act perpendicularly to the object's velocity and towards the centre of the circular path.

A vector subtraction can be used to show that the change in velocity and hence acceleration, over a short time interval is directed towards the centre of the circular path.

Consider an object moving in a circular path with constant speed v as shown in Figure 1.3.2. Its velocity is at a tangent to the circular path at any instant of time. The magnitude of the velocity does not change, but its direction changes instantaneously as it moves around the circle. Consider the object moving from point A to B over a small time interval Δt.

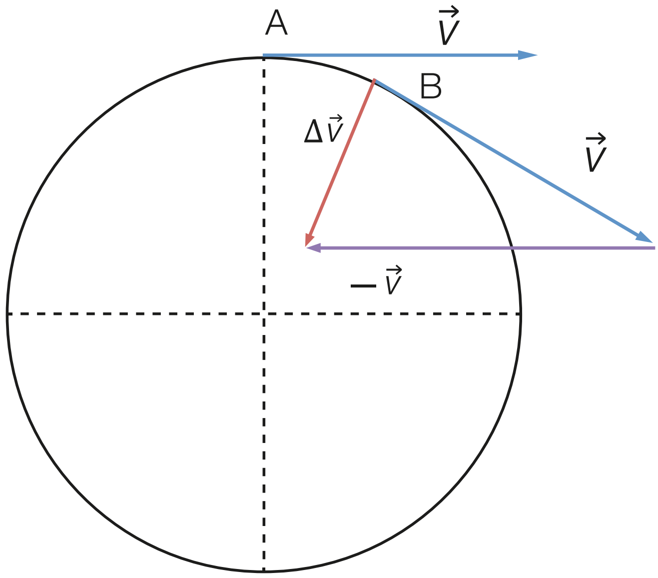


Figure 1.3.2

The change in velocity is given by $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. From the vector subtraction shown on the diagram, it can be seen that the change in velocity is directed towards the centre of the circular path.

Since $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, it follows that acceleration is directed towards the centre of the circular path.

The magnitude of the centripetal acceleration is given by $a = \frac{v^2}{r}$

where v is the speed of the object undergoing uniform circular motion and r is the radius of the circular path.

The direction of the centripetal acceleration is always changing but is always directed towards the centre of the circular path.

Worked Example

A wind turbine has blades that rotate about a central axis. A typical 2.5 MW turbine has blades with a length of 50.0 m from the central axis and a blade tip speed of 84.0 ms^{-1}. Calculate the centripetal acceleration at the tip of the turbine blades.

$a = \frac{v^2}{r} = \frac{84.0^2}{50.0} = 141\ ms^{-2}$ towards the central axis

Force

Since $a = \frac{v^2}{r}$, then using Newton's Second Law, the magnitude of the force producing this centripetal acceleration is given by

$F = ma = \frac{mv^2}{r}$ where m is the mass of the object undergoing uniform circular motion.

Speed

Since speed is defined as the distance travelled per unit time, then for one complete circle

$v = \frac{s}{t} = \frac{2\pi r}{T}$ where T is the time taken for one revolution (called the period)

and $2\pi r$ is the distance travelled in one complete circle.

Different forces can cause a centripetal acceleration

The centripetal acceleration can be caused by a

- tension force in the case of an object being whirled in a horizontal circular path on a string.
- frictional force in the case of a car turning a corner.
- gravitational force in the case of an artificial satellite circling the Earth or a natural satellite such as the moon circling the Earth or a planet circling the Sun.

Each of these is discussed further in the section that follows.

Tension force

Figure 1.3.3 shows an object being whirled in a circular path that is in a horizontal plane.

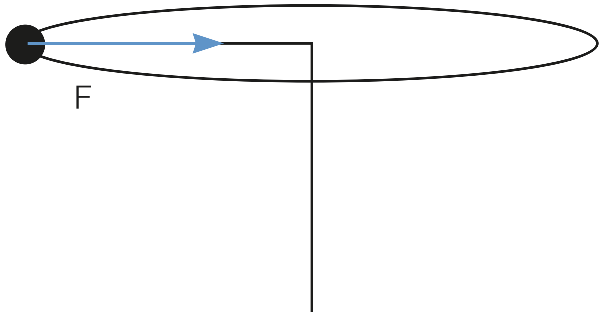


Figure 1.3.3

The tension force provides the centripetal acceleration. Since $F = ma = \frac{mv^2}{r}$, the tension in the string depends on three factors:

(i) *The speed of motion: $F \alpha v^2$*

The tension force providing the centripetal acceleration is proportional to the square of the speed of motion providing the mass (m) of the object and the radius of the circular path (r) does not change. If the speed doubles, the tension force required to keep the object in a circular path is four times larger. If the speed increases by a factor of ten, then the tension force required to keep the object in a circular path is one hundred times larger. Similarly, if the speed decreases by a factor of three, then the tension force decreases by a factor of nine.

Since increasing the speed of the object being whirled increases the tension force required to keep the object in a circular path ($F \alpha v^2$), this means there is a limit to the speed with which the object can be whirled because all strings have a maximum tension that they can withstand before they snap. If the object is whirled too fast, the maximum tension force that the string can provide is exceeded and the string snaps. If the force providing the centripetal acceleration of the object were suddenly removed, the object will travel in a straight line that is tangential to the circular path.

(ii) The radius of the circular path: $F \alpha \frac{1}{r}$

The tension force and the radius of the circular path are inversely proportional providing the mass (m) and speed (v) of the object do not change. If the radius of the circular path is doubled then the tension force required to keep the object in a circular path is halved. If the radius of the circular path is increased by a factor of ten, then the tension force required to keep the object in a circular path is ten times smaller. Similarly, if the radius of the circular path is three times smaller, the tension force becomes is three times larger.

(iii) The mass of the object being whirled: $F \alpha m$

The tension force is proportional to the mass (m) of the object undergoing uniform circular motion providing the speed (v) and radius of the circular path (r) do not change. If the object being whirled is replaced with one having twice the mass, the tension required to keep the mass in a circular path is doubled. If the object being whirled is replaced with one having 10 times the mass, the tension required to keep the mass in a circular path is 10 times larger. Alternatively, if the object being whirled is replaced with one with a mass that is three times smaller, the tension force becomes three times smaller.

Frictional force

Consider a **vehicle travelling with constant velocity on a flat road**.

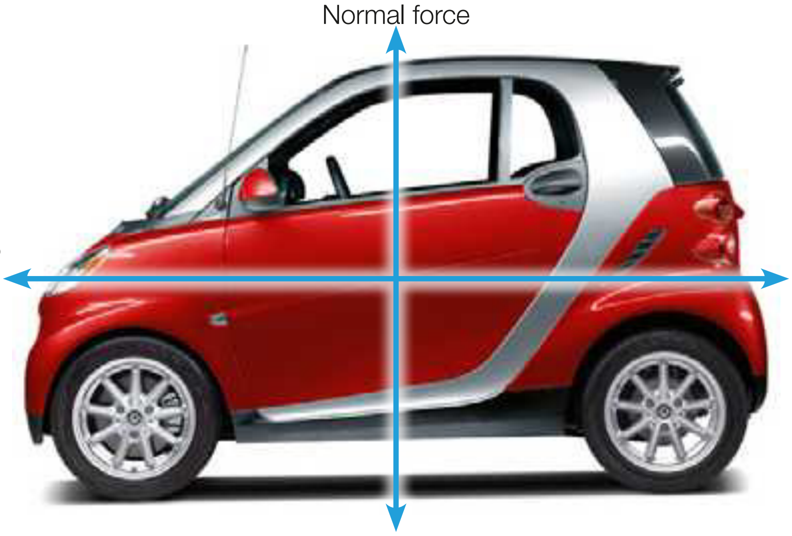


Figure 1.3.4

According to Newton's First Law of Motion, a vehicle can only travel with constant velocity if the net force acting on the vehicle is zero i.e. There are no unbalanced forces acting.

Figure 1.3.4 identifies the forces acting on the vehicle. The weight of the vehicle acts down towards the ground and is balanced by the normal force acting up (Newton's Third Law). In addition the forward force provided by the thrust of the engine is balanced with the total drag forces directed towards the rear of the vehicle while it travels with constant velocity.

Now consider a **vehicle turning a corner on a flat road** as shown in Figure 1.3.5. The vehicle experiences a centripetal acceleration towards the centre of the circular path even though it is travelling with constant speed. The sideways frictional force between the tyres and the road provides this centripetal acceleration.

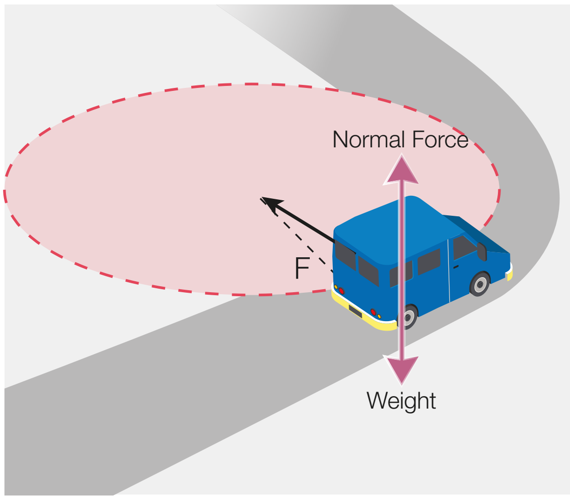


Figure 1.3.5

Just like the object being whirled on a string, the magnitude of the frictional force depends on the mass of the vehicle, the speed of the vehicle and the radius of curvature of the circular path.

Since $F = ma = \frac{mv^2}{r}$, the greater the speed of the vehicle as it turns the corner, the greater the frictional force required to turn the corner ($F \alpha v^2$). There is a limit to the frictional force that can be provided by the tyres and therefore a limit to the speed with which a vehicle can safely round a corner of a given radius. If the maximum frictional force is exceeded, the vehicle slides off at a tangent to the circular path (from the point at which the frictional force is exceeded). As the tyres wear, the maximum amount of frictional force that the tyres can provide decreases and turning the corner at high speed can become dangerous.

Gravitational force

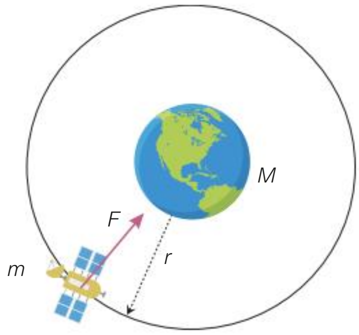
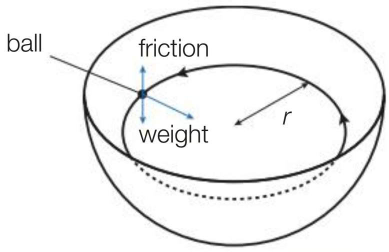


Figure 1.3.6

In the case of an artificial satellite circling the Earth or a planet (including Earth) circling the Sun, the gravitational force provides the centripetal acceleration. The magnitude of the force is given by $F = ma = \frac{mv^2}{r}$ where m is the mass of the satellite, r is the radius of orbit and v is the speed of the satellite. This force is equivalent to $F = \frac{GmM}{r^2}$ as described later in this chapter.

Normal force

A ball can be made to move in a circular path inside a bowl. The wall of the bowl provides a force that is directed towards the centre of the circular path. This force is called a normal force and provides the centripetal acceleration for uniform circular motion.



The normal force provides the centripetal acceleration when a railway carriage or our local O-Bahn bus moves around a circular section of a track.

Figure 1.3.7

A train moving around a circular section of track

Figure 1.3.8

The O-Bahn bus moving around a circular section of track

Worked Examples

1. A 225 g mass is attached to a wire and swung in a horizontal circle so that it completes 6 revolutions in 2.00 seconds. The radius of the circular path is 15.0 cm

   (a) Calculate the period of motion.

   $$T = \frac{2.00}{6} = 0.333\ s$$

   (b) Calculate the speed of the mass.

   $$v = \frac{2\pi r}{T} = \frac{2\pi \times 0.150}{(0.333)} = 2.83\ ms^{-1}$$

   (c) Calculate the centripetal acceleration experienced by the mass.

   $$a = \frac{v^2}{r} = \frac{2.83^2}{0.150} = 53.4\ ms^{-2} \text{ towards the centre of motion}$$

(d) Calculate the tension in the wire.

F = *ma* = 0.225 × 53.4 =12.0 *N towards the centre of motion*

(e) Describe the path followed by the mass if the wire were to snap.

The mass would travel in a straight line at a tangent to the circular path and with a speed of 2.83 ms^{-1} .

2. A car with a mass of 1.50×10^3 kg enters a roundabout at an intersection. The road is flat and the radius of curvature of the roundabout is 6.00 m.

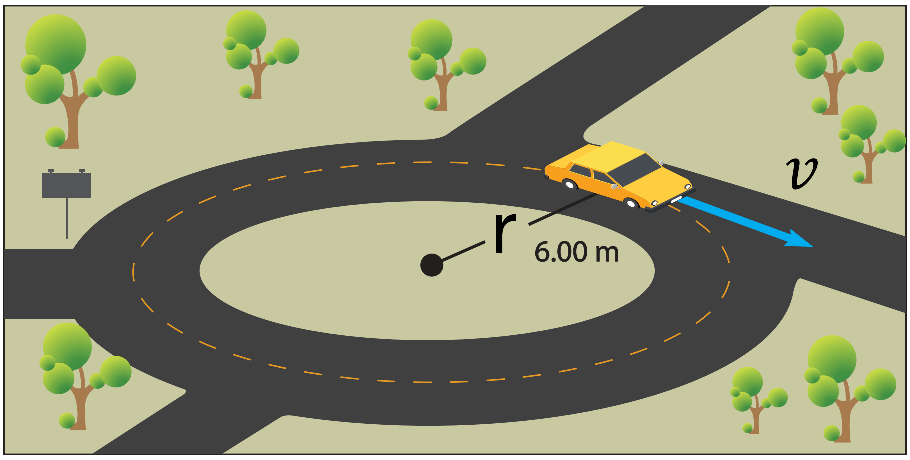


Figure 1.3.9

Calculate the maximum speed with which the car can safely circle this roundabout if the maximum frictional force the tyres can provide is 5.10×10^4 N.

$$F = ma = \frac{mv^2}{r} \quad \therefore \quad v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{5.10 \times 10^4 \times 6.00}{1.50 \times 10^3}} = 14.3 \ ms^{-1}$$

Banked curves

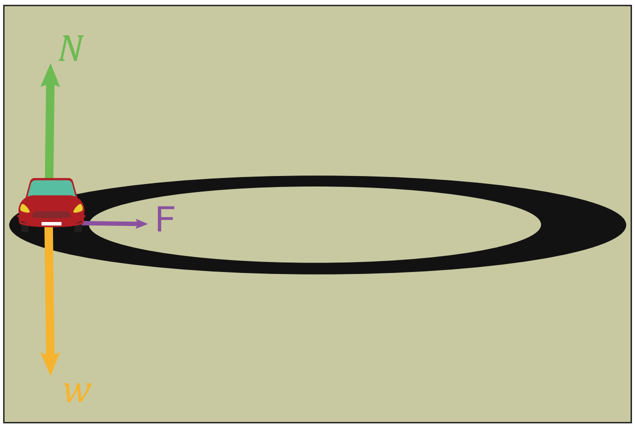


Figure 1.3.10

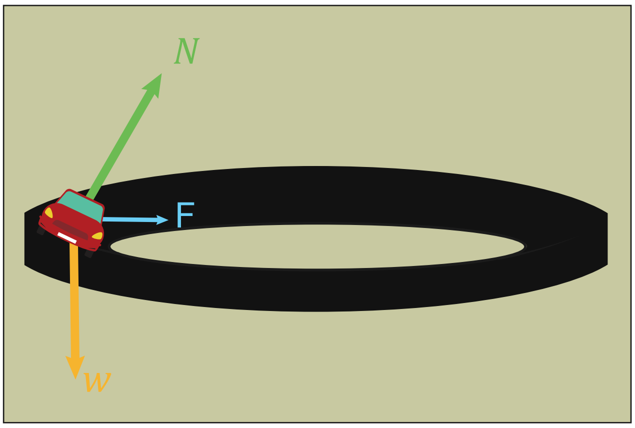


Figure 1.3.11

Earlier in the chapter the frictional force that provides the centripetal acceleration when a car undergoes uniform circular motion on a flat road was discussed. The amount of frictional force needed to round a curve increases with speed. If the car is travelling too fast, the tyres may not be able to provide all of the friction needed to make the turn safely. This situation is shown in figure 1.3.10.

To improve safety, a road can be banked at an angle above the horizontal as shown in Figure 1.3.11. If a road is banked, it is possible to achieve a situation where friction is not required in providing the centripetal acceleration. This eliminates dangers involved with slippery roads and allows cars to move around a corner at higher speeds.

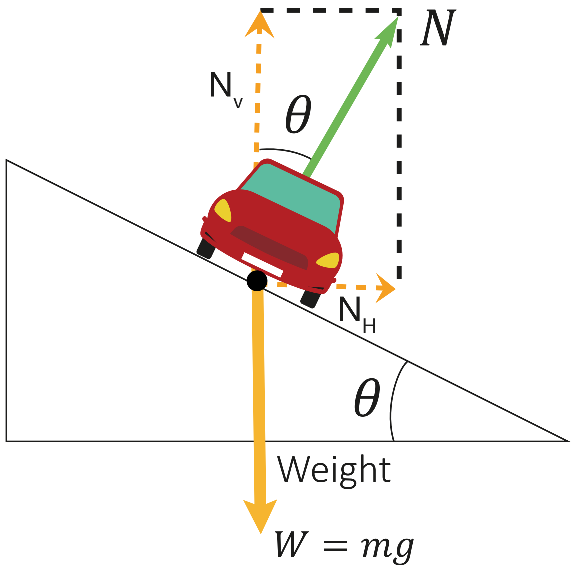


Figure 1.3.12

Figure 1.3.12 illustrates the forces acting on a vehicle when the road is banked at an angle above the horizontal.

The normal force (N) provided by the road can be resolved into two perpendicular vectors. The **vertical component** (N_V) is equal in magnitude and opposite in direction to the weight (*mg*) of the vehicle.

The **horizontal component** (N_H) points towards the centre of the circular path and can provide part or all of the centripetal acceleration. If the horizontal component provides all of the centripetal acceleration, no frictional force is required.

Extra understanding

The subject outline does not require you to be able to calculate the banking angle but it may be of interest.

The banking angle (θ) needed to achieve a situation in which friction is not needed to round a banked curve can be calculated using the equation $Tan\theta = \frac{v^2}{rg}$ where *v* is the speed of the vehicle, *g* is the gravitational acceleration (9.80 ms^{-2}) and *r* is the radius of the circular path.

Derivation

Using Figure 1.3.12, the vertical component (N_V) of the normal has a magnitude of *mg*. If the horizontal component (N_H) points towards the centre of the circular path and provides all of the centripetal acceleration,

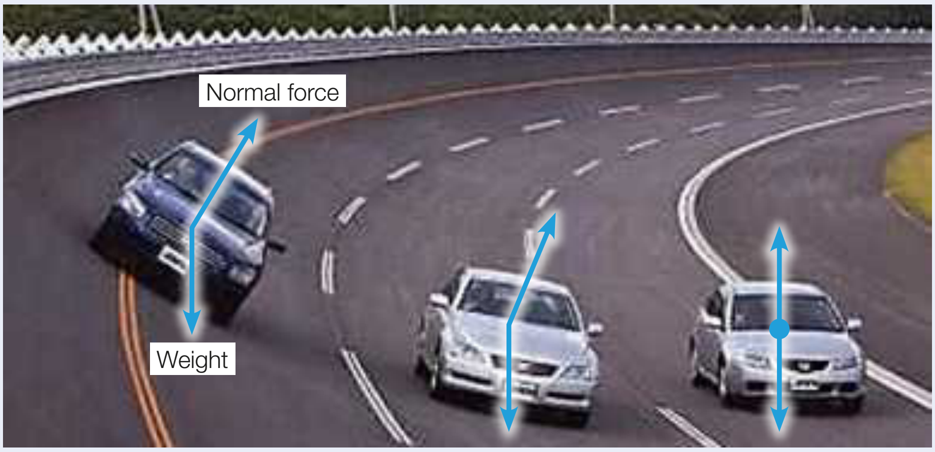
then $N_H = ma = \frac{mv^2}{r}$. It follows that $\tan\theta = \frac{opp}{adj} = \frac{N_H}{N_V} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$

This means that when a vehicle travels around a banked curve at the correct speed for the banking angle, the horizontal component of the normal (not the frictional force) causes the centripetal acceleration.

If the banking angle is less than that calculated using $\tan\theta = \frac{v^2}{rg}$, then friction between the tyres and the road provides some of the force required to cause the centripetal acceleration of the car. Similarly if the vehicle travels faster that the correct speed for the banking angle, then friction between the tyres and the road provides some of the force required to cause the centripetal acceleration of the car.

Worked Examples

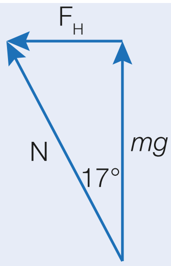
1. Draw vector arrows to represent the forces acting on each of the three cars shown in the diagram below.



2. A section of a road of radius r = 300.0 m, is banked at an angle of 17.0° so that a car may make a turn at a speed v = 108 kmh^{-1} without depending on friction.

The car has a mass of 1450 kg. Calculate the magnitude of the normal force acting on the car.

$$\cos 17 = \frac{mg}{N} \quad \therefore \quad N = \frac{mg}{\cos 17} = \frac{1450 \times 9.80}{\cos 17} = 1.49 \times 10^4\ N$$



? Science inquiry practical

1. Investigate the force causing centripetal acceleration, using a tube, stopper, washers or weights and connecting string. A possible arrangement is shown below.

   Ideas include investigating the relationship between the force causing centripetal acceleration and the speed of the stopper or the relationship between the force causing centripetal acceleration and the period of the stopper.

   Write a report that includes

   - relevant physics concepts
   - a hypothesis
   - variables (independent, dependent and factors held constant)
   - a clear and detailed procedure
   - safely considerations
   - results represented in table and in graphical form
   - a summary of any trends in the results
   - an evaluation of the procedures and results
   - a discussion of the sources of error
   - improvements to the method
   - a conclusion that is consistent with the results

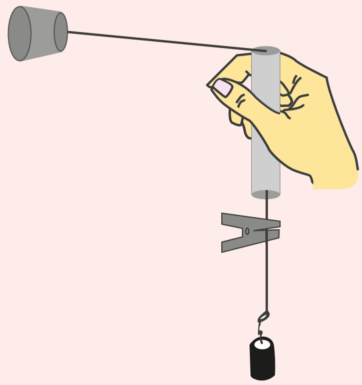


Figure 1.3.13

2. Find and test the speed required for a marble to 'loop the loop', using flexible railing or a slot-car set.

Science as a human endeavour

Explore the benefits and limitations in the design and use of banked curves, such as velodromes, motor racing circuits, amusement park rides, and high-speed train tracks.

Gravitation

Gravitational field

Objects with mass have a gravitational field in the space that surrounds them.

The strength of the gravitational field decreases with distance *r* from the centre of the object. Lines of force are used to represent the gravitational field in the space around the mass. The field acts **radially towards the centre of the mass.** Gravitational field is a vector quantity. The **spacing** of the field lines indicates the **strength** of the field. A greater spacing between the field lines represents a weaker field. Figures 1.3.14 and 1.3.15 show the gravitational field for two masses. The field shown in figure 1.3.15 is stronger.

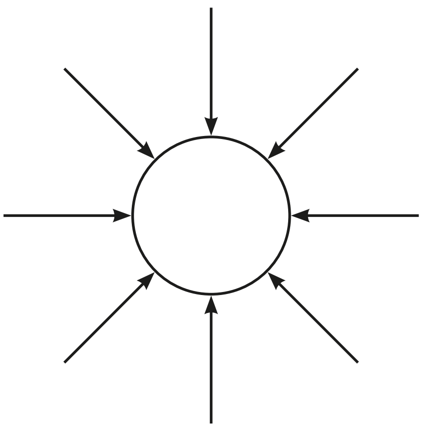


Figure 1.3.14

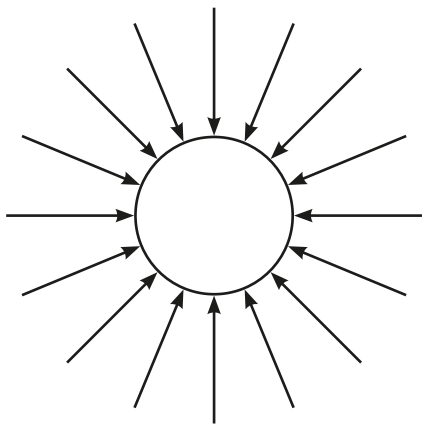


Figure 1.3.15

Any object with mass experiences a gravitational force when it is within the gravitational field of another mass.

The gravitational field strength, *g*, at a point is defined as the net force per unit mass at a particular point in the field.

$$\vec{g} = \frac{\vec{F}}{m}$$

Unit: Nkg^{-1} or ms^{-2}

NOTES:

1 Since $a = \frac{F}{m}$ then it follows that the gravitational field strength is equivalent to the **gravitational acceleration.**

2 Gravitational field strength is a vector quantity.

The principle of **superposition** applies to gravitational field. If more than two masses are present, the gravitational field strength at any point is a vector sum of the gravitational field due to each of the other masses present.

A section later in the chapter uses the concept of superposition to calculate the gravitational field strength at a point between two masses.

Worked Example

A 22.0 kg mass experiences a gravitational force of 36.8 N on the surface of the Earth's moon. Calculate the gravitational field strength on the surface of the moon.

$$a = \frac{F}{m} = \frac{36.8}{22.0} = 1.67\ Nkg^{-1}$$

Gravitational force

Any two objects with mass will attract each other due to a gravitational force.

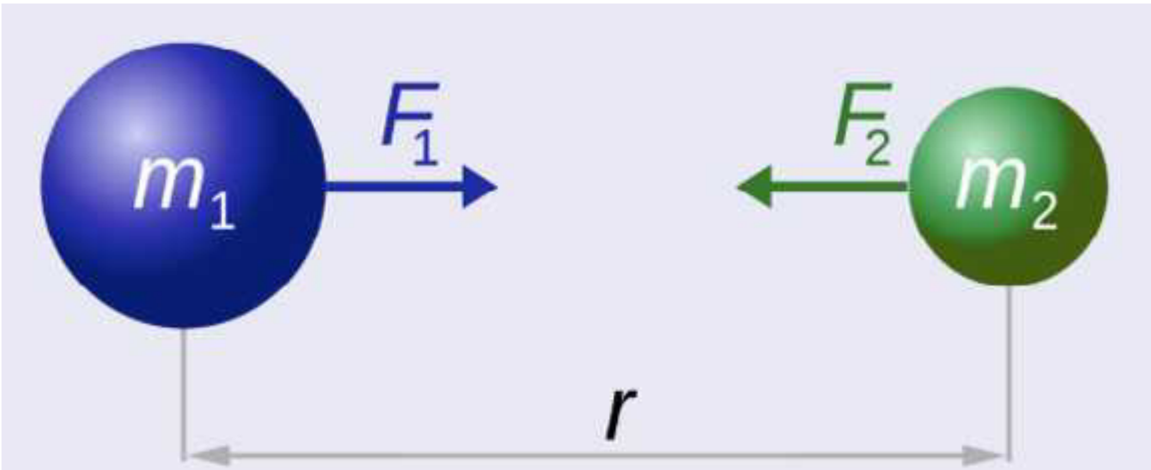


Figure 1.3.16

Newton's Law of Universal Gravitation

The gravitational force of *attraction* between any two masses is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres.

$$F = \frac{Gm_1m_2}{r^2}$$

where m_1 and m_2 are the masses in kilograms (kg).

G = Gravitational constant = $6.67 \times 10^{-11}\ Nm^2kg^2$

r is the distance between the centre of the two masses

The gravitational force acts along the line joining the centres of the masses as though the mass were concentrated at a point.

Principle of Superposition: If more than two masses are present, the force on any one of the masses is a vector sum of the gravitational force due to each of the other masses present.

Applying the concept of proportionality

(i) The gravitational force is proportional to one mass, providing the other mass and the distance between the centre of the two masses does not change.

$$F \alpha m_1 \quad m_2, r \quad \text{constant}$$

This concept is illustrated by example. If one mass is replaced with one having twice the mass, the gravitational force between the two masses doubles. Similarly if one mass is replaced with one having a mass that is ten times smaller, then the gravitational force between the two masses decreases by a factor of ten.

(ii) The gravitational force is proportional to the product of the two masses, providing the distance between the centre of the two masses does not change.

$$F \alpha m_1 m_2 \quad r \text{ constant}$$

An example to illustrate this concept is that if one mass is replaced with one having double its mass and the other is replaced with a mass having five times its mass, then the gravitational force becomes ten times larger.

(iii) The gravitational force is inversely proportional to the square of the distance between the centre of the two masses, providing the masses do not change.

$$F \alpha \frac{1}{r^2} \quad m_1, m_2 \quad \text{constant}$$

An example to illustrate this concept is that if the distance between the centre of the two masses is increased by a factor of four, then the gravitational force becomes sixteen times smaller.

Worked Examples

1. (a) Calculate the gravitational force acting between two astronauts A and B, separated by a distance of 50.0 m, if their masses (including their space suits) are 105 kg and 110 kg respectively.

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times105\times110}{50.0^2} = 3.08\times10^{-10}\ N \ \textit{attraction}$$

(b) Calculate the magnitude and direction of the acceleration experienced by astronaut B.

$$a = \frac{F}{m} = \frac{3.08\times10^{-10}}{110} = 2.80\times10^{-12}\,ms^{-2} \quad \textit{towards astronaut A}$$

2. The magnitude of the gravitational force between two 50.0 kg masses is 2.20×10^{-6} N.

(a) Calculate the distance between the centre of the two masses.

$$F = \frac{Gm_1m_2}{r^2} \quad \therefore \quad r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{6.67\times10^{-11}\times50.0^2}{2.20\times10^{-6}}} = 0.275m$$

(b) The distance calculated in part (a) is doubled. Use proportionality to describe the effect this would have on the magnitude of the gravitational force acting between the masses.

$$F \alpha \frac{1}{r^2} \quad m_1, m_2 \quad \text{constant}$$

If the distance is doubled, the gravitational force becomes four times smaller.

(c) One of the masses is replaced with one having a mass of 150 kg. Use proportionality to calculate the magnitude of the gravitational force acting between the masses.

$$F \alpha m_1 \quad m_2, r \quad \text{constant}$$

If one of the masses is tripled, so is the gravitational force.

F' = 6.60×10^{-6} N

(d) The distance between the centre of the two masses is doubled and one mass is replaced with one having a mass of 150 kg. Determine the magnitude of the gravitational force acting between the two masses.

$F \alpha \frac{m_1}{r^2}\ m_2$ constant

The gravitational force changes by a factor of $\frac{1}{4} \times 3 = \frac{3}{4}$

The magnitude of the gravitational force becomes $\frac{3}{4} \times 2.2 \times 10^{-6} = 1.65 \times 10^{-6}\ N$

Gravitational forces are consistent with Newton's Third Law

Figure 1.3.16 illustrates the gravitational force acting between two masses. Both masses experience a force which has the same magnitude but the force on each mass acts in opposite directions. This means that gravitational forces are consistent with Newton's Third Law which states that if object A exerts a force on object B, then object B exerts and equal and opposite force on object A.

The acceleration due to gravity *g* on a planet or moon.

Consider a particle of small mass *m*, placed at the surface of a planet or moon of mass *M*. Assume that the radius of the planet or moon is *r*.

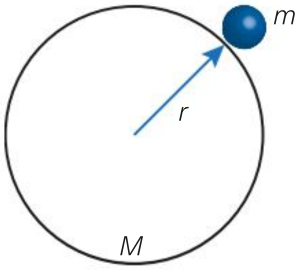


Figure 1.3.17

The gravitational force between the mass and the moon or planet is given by $F = \frac{GmM}{r^2}$.

Using Newton's Second Law, the gravitational acceleration is given by $g = \frac{F}{m}$.

It follows that $g = \frac{F}{m} = \frac{\frac{GmM}{r^2}}{m} = \frac{GM}{r^2}$.

The acceleration due to gravity on the surface of a planet or moon has a magnitude given by:

$$g = \frac{GM}{r^2}$$

where *M* is the mass of the planet or moon and *r* is the radius of the planet or moon.

At a height *h* above the surface of the planet or moon, the acceleration due to gravity is given by $g = \frac{GM}{(r+h)^2}$

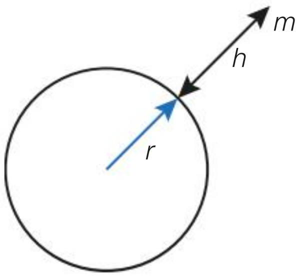


Figure 1.3.18

It can be seen that the magnitude of the acceleration due to gravity decreases with height above the surface of the planet or moon.

Worked Examples

1. Saturn is a planet often referred to as a gas giant as it is made up of mostly hydrogen and helium gas. It does not have a defined surface but still has a gravitational field surrounding it. Calculate the magnitude of the acceleration due to gravity at the perceived surface of Saturn which has a mass of 5.683×10^{26} kg and an average radius of 58 232 km.

$$g = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times5.683\times10^{26}}{(58232\times10^3)^2} = 11.2\ ms^{-2}$$

(Note: This formula is not on the Formula Sheet but can easily be derived using Newton's Law of Universal Gravitation and Second Law of Motion – see page 66)

2. Calculate the height above the Earth's surface at which the magnitude of the acceleration due to gravity has a value of one quarter the value on the Earth's surface. Take the mass of the Earth to be 5.97×10^{24} kg and its radius to be 6.37×10^6 m. (Note: The values for the mass and mean radius of the Earth can be found on the Formula Sheet)

$$g = \frac{GM}{(r+h)^2} \quad \therefore\ (r+h)^2 = \frac{GM}{g} \quad \therefore\ r+h = \sqrt{\frac{GM}{g}}$$

$$h = \sqrt{\frac{GM}{g}} - r = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{\frac{1}{4}\times9.8}} - 6.37\times10^6 = 6.38\times10^6 m$$

? Possible teaching or learning strategy

Compare the acceleration due to gravity at various points on the surface of the Earth e.g. at sea level and at the top of Mount Everest (h = 8848 m).

The point of weightlessness between the Earth and the Moon

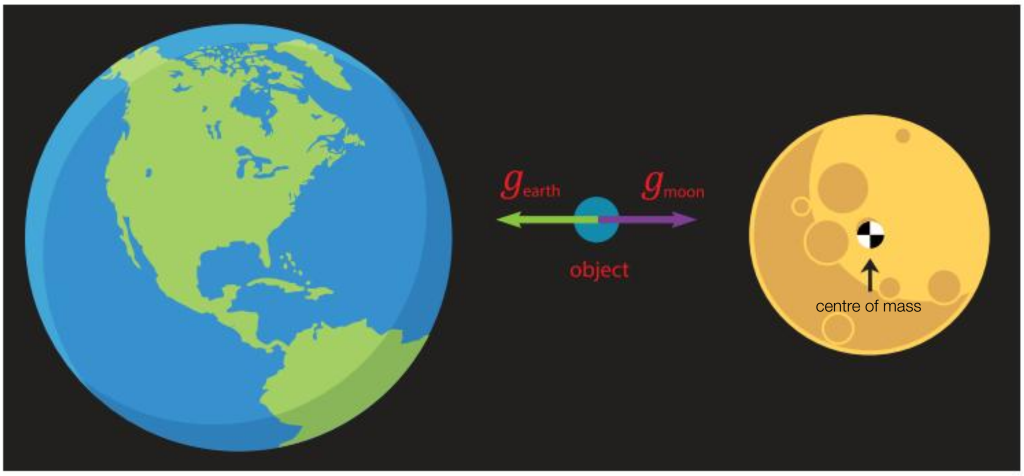


Figure 1.3.19

An object placed between the Earth and the Moon experiences a gravitational pull from the Earth and the Moon. These forces act in opposite directions.

Since weight $W = mg$ and g decreases with height above the Earth's surface, so too does the weight of an object. The gravitational pull from the Earth decreases but the gravitational pull from the moon increases. There is a point between the Earth and the Moon where the object is pulled equally in both directions as shown in Figure 1.3.19. At this point the net gravitational force, acceleration due to gravity and the weight of the object are all zero. There is a question about this in the end of chapter exercises.

Satellite motion

Stable orbits

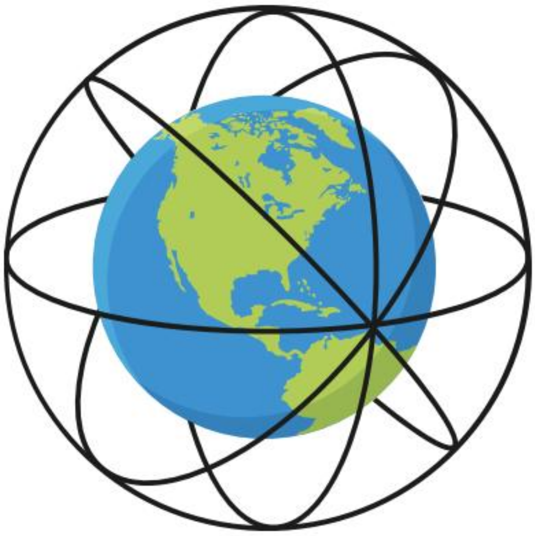


Figure 1.3.20

The centres of the circular orbits of Earth satellites must coincide with the centre of the Earth in order for the satellite orbit to be stable. Figure 1.3.20 illustrates examples of such orbits. The centres must coincide because the gravitational force provides the centripetal acceleration. The gravitational force must be directed towards the centre of the circular orbit in order to provide the centripetal acceleration for uniform circular motion and by definition towards the centre of the Earth because the gravitational force acts between the centres of the two masses. As a result, the centre of the circular orbit must coincide with the centre of the Earth.

Speed of a satellite

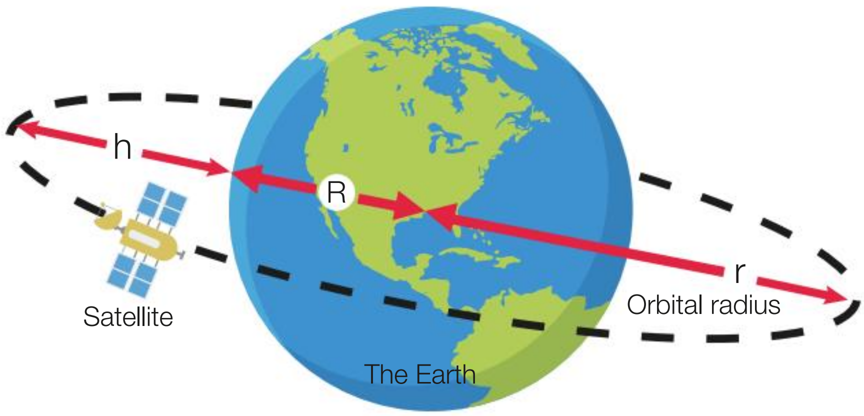


Figure 1.3.21

The speed of a satellite moving in a circular orbit is given by

$$v = \sqrt{\frac{GM}{r}}$$

where G is the gravitational constant $6.67 \times 10^{-11} Nm^2kg^{-2}$, M is the mass of the central body being orbited and r is the radius of orbit of the satellite.

N.B. The central body may be the Earth in the case of a satellite circling the Earth, the Sun in the case of a planet circling the Sun or it may be a planet in the case of a moon circling a planet.

Derivation

The gravitational force between the satellite of mass m and the central body of mass M provides the centripetal acceleration required for a satellite to undergo uniform circular motion.

$$F = \frac{GmM}{r^2} = \frac{mv^2}{r} \quad \therefore \quad \frac{GM}{r} = v^2 \quad v = \sqrt{\frac{GM}{r}}$$

Kepler's Laws of Planetary Motion

In the early 1600s Johannes Kepler explained the motion of planets around the Sun. Kepler's Laws of Planetary Motion describe the motion of planets, their moons, and other satellites.

Kepler's three Laws of Planetary Motion are described below:

1 Kepler's First Law - all planets move in elliptical orbits with the Sun at one focus.

An elliptical path has the shape of an elongated circle. The sum of the distances from every point on the elliptical path to the points referred to as foci of the ellipse is constant. The closer together the foci are, the closer the path is to a circular path. In fact a circle is an ellipse in which the two foci are at the same point. Figure 1.3.22 shows the elliptical path of the Earth around the Sun.

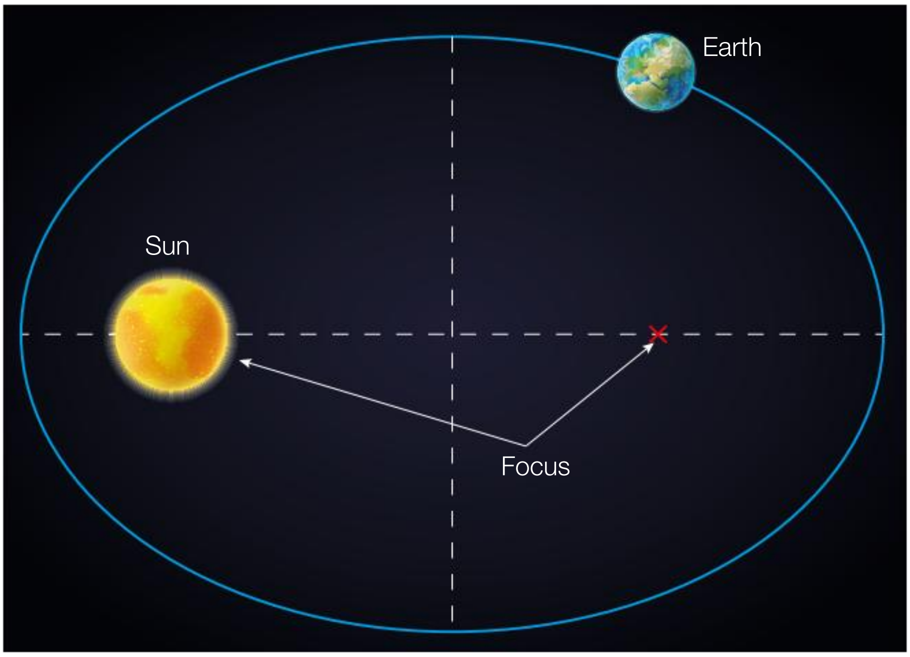


Figure 1.3.22

Kepler's First Law is sometimes called the Law of Ellipses. It simply states that the planets orbit the Sun in an elliptical orbit with the Sun located at one of the foci of that ellipse.

2 Kepler's Second Law - The radius vector drawn from the Sun to a planet sweeps equal areas in equal time intervals.

Kepler's Second Law is sometimes called the Law of Equal Areas. It describes the speed of a planet moving in its orbit around the Sun. This is because a planet is moving faster when it is closer to the Sun and slower when it is further away from the Sun.

This principle is illustrated in Figure 1.3.23. The radius vector is drawn from the planet to the Sun. In a given time interval, for instance one month, the area swept out by the line is always the same. The triangles formed change dimension but their area is the same.

This principle extends to comets, moons, satellites and other bodies orbiting in elliptical paths.

3 Kepler's Third Law - The period of any satellite depends upon the radius of its orbit. For circular orbits Kepler's Third Law can be expressed as:

$T^2 = \frac{4\pi^2}{GM} r^3$ or $T^2 \propto r^3$

Kepler's Third Law can be stated in words: the period of revolution of a satellite squared is proportional to the radius of orbit cubed.

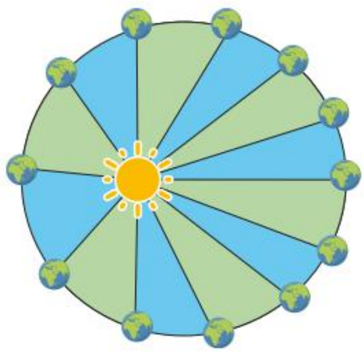


Figure 1.3.23

Derivation

The speed of a satellite is given by $v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$

Squaring both sides of the equation gives: $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$

It follows that $GMT^2 = 4\pi^2 r^3 \therefore T^2 = \frac{4\pi^2}{GM} r^3$

Additional notes

It can be seen from the equations $v = \sqrt{\frac{GM}{r}}$ and $T^2 = \frac{4\pi^2}{GM} r^3$ that the speed and period of a satellite in a circular orbit depends only on the radius of orbit (r) and the mass of the central body (M) not the mass of the satellite (m).

Worked Examples

Useful data – see Formula Sheet: *(M_{earth} = 5.97 × 10^{24} kg r_{earth} = 6.37 × 10^6 m)*

1. A satellite circles the Earth over its equator once every 12.0 h. Calculate the

   (a) period of the satellite in seconds.

   12.0 h = 12.0 × 60 × 60 = 4.32 × 10^4 s

   (b) satellite's height above the Earth.

   $T^2 = \frac{4\pi^2}{GM} r^3 \quad \therefore \quad r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11}\times5.97\times10^{24}\times(4.32\times10^4)^2}{4\pi^2}} = 2.66\times10^7 m$

   Height = $r - r_E$ = 2.66 × 10^7 – 6.37 × 10^6 = 2.02 × 10^7 m

   (c) speed of the satellite.

   $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{2.66\times10^7}} = 3.87\times10^3 ms^{-1}$

2. A large Earth satellite of mass 2.00 × 10^3 kg orbits the Earth in a circular path with a speed of 5.90 × 10^3 ms^{-1}. Calculate the

   (a) radius of orbit for this satellite.

   $v = \sqrt{\frac{GM}{r}} \quad \therefore \quad r = \frac{GM}{v^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(5.90\times10^3)^2} = 1.14\times10^7 m$

   (b) period of orbit for this satellite.

   $v = \frac{2\pi r}{T} \quad \therefore \quad T = \frac{2\pi r}{v} = \frac{2\pi(1.14\times10^7)}{5.90\times10^3} = 1.21\times10^4 s$

   (c) magnitude and direction of the force acting on the satellite.

   $F = \frac{mv^2}{r} = \frac{2.00\times10^3\times(5.90\times10^3)^2}{1.14\times10^7} = 6.11\times10^3 N$ *towards the centre of the Earth*

   (d) Name the force that provides the centripetal acceleration in this example.

   The gravitational force

Geostationary satellites

A geostationary satellite is a satellite that remains fixed above one point of the Earth's surface. Figure 1.3.24 illustrates a geostationary satellite in orbit.

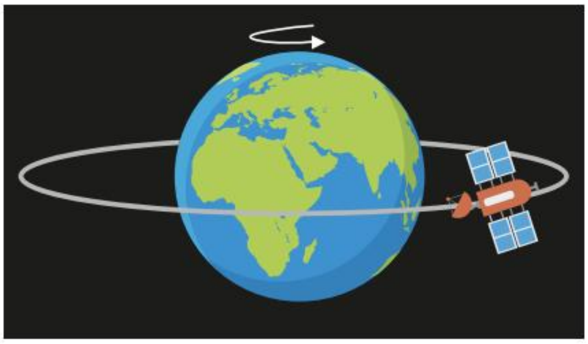


Figure 1.3.24

The circular orbit of a geostationary satellite has the following characteristics:

1. The satellite travels in the same direction as the Earth's rotation

   The Earth rotates from west to east. The only way that the satellite can remain fixed above one position of the Earth's surface is to move in the same direction as the Earth's rotation.

2. The period of the satellite is 24 h

   The period of the Earth's rotation is 24 h so it follows that to remain in a fixed position above the Earth's surface the orbital period of a geostationary satellite must also be 24 h.

3. The orbit is equatorial

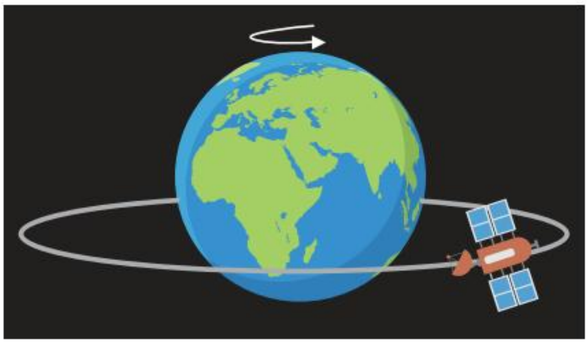


Figure 1.3.25

   A satellite in a geostationary orbit must have an orbit in the Earth's equatorial plane if it is to move in west to east direction with the Earth's rotation and remain in a stable orbit. This is because the centre of the orbit must coincide with the centre of the Earth. Figure 1.3.25 shows a satellite circling the Earth in a plane below the equator. The orbit is not stable. A geostationary satellite orbits the Earth directly above the equator as shown in Figure 1.3.24.

4. The radius of orbit is relatively large

   According to Kepler's Third Law, $T^2 \alpha r^3$. This means that a relatively large period of 24 hours will correspond to a relatively large radius of orbit. One of the questions in the exercises at the end of this chapter is to calculate the radius of orbit and altitude of a geostationary Earth satellite. Such a satellite is positioned at a height of around 36 000 km above the surface of the Earth.

Types of orbits

1. **Equatorial orbit** (Figure 1.3.26) – the satellite is in an orbit such that it moves directly above the equator as it circles the Earth.

Figure 1.3.26

2. **Polar orbit** (Figure 1.3.27) – the satellite is in an orbit such that it moves over the North and South poles as it orbits the Earth.

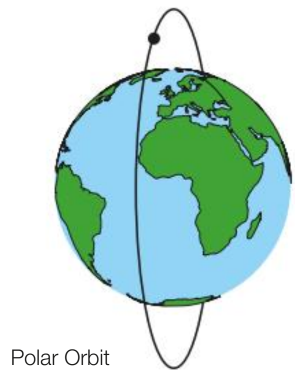


Figure 1.3.27

3. **Geostationary orbit** – the satellite remains fixed over one point of the Earth's surface. The characteristics of such an orbit were explained earlier in the chapter.

Applications

Equatorial orbit/Geostationary

Geostationary orbits allow for constant communication between two ground stations. This is because the satellite is always in the same position and communication is not interrupted by the motion of the satellite or the rotational motion of the Earth. Geostationary satellites also allow continuous monitoring of a particular region of the Earth's surface since they are in a fixed position. It should be noted that the radius of orbit being large, positions the satellite at a large altitude above the surface of the Earth. This has the advantage of a large coverage Their large altitude make them useful for meterology, but the disadvantage of producing images with low resolution.

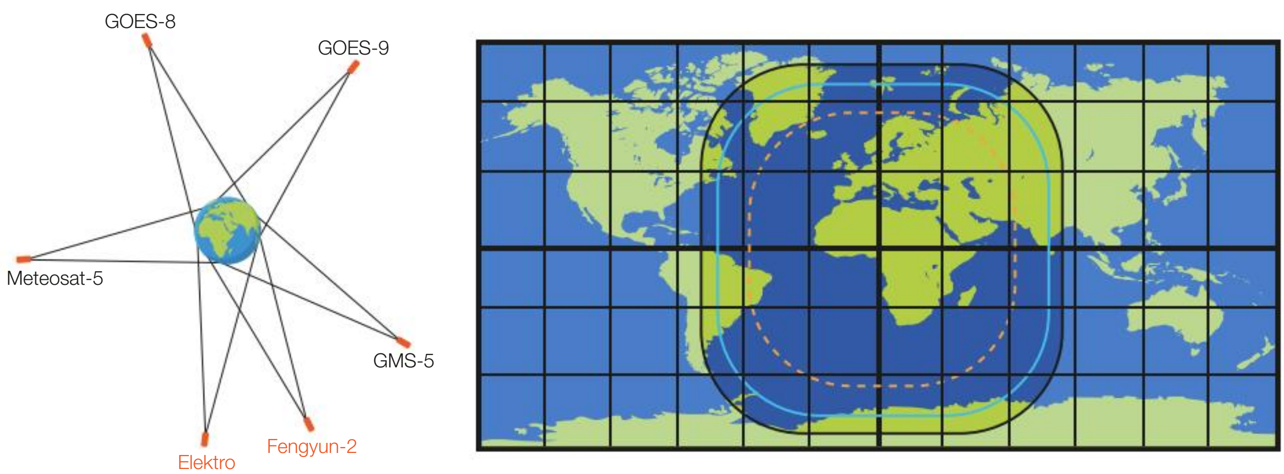


Figure 1.3.28

Figure 1.3.28 shows the coverage of a geostationary satellite.

Polar orbit

Geostationary satellites are not always the best choice for surveillance and meteorology. Low-altitude polar-orbit satellites are often used. The low altitude produces images with a higher resolution as the satellite is closer to the ground. In addition, these satellites take images directly beneath their path. This means that distortion of the images produced is low. For these reasons, low-altitude polar-orbit satellites are useful for surveillance.

A low-altitude polar-orbit satellite is also useful for meteorology. Such a satellite passes over the North and South poles of the Earth many times a day and scans a full picture of the Earth's surface, one strip at a time as the Earth rotates beneath it. This is shown in figure 1.3.29. Bad weather conditions can be detected and precautions taken.

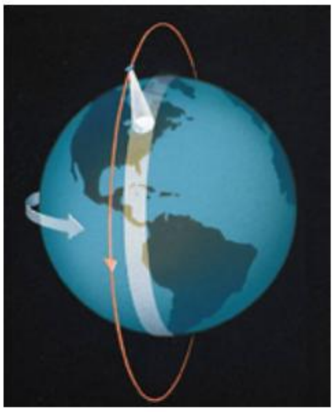


Figure 1.3.29

Figure 1.3.30 illustrates the relative altitude of a geostationary and polar orbiting satellite.

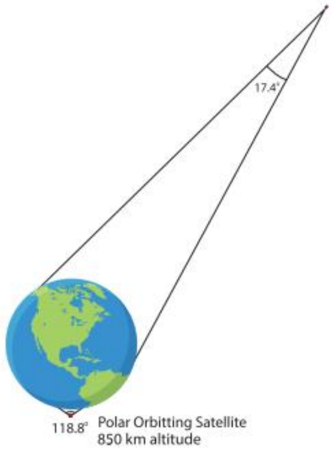


Figure 1.3.30

For further discussion in class

Below are some ideas for further discussion with your teacher:

- How can we use Kepler's Laws to explain the motion of comets and predict times when they may be seen?
- How can data giving the orbital radii and periods of the natural satellites of a planet be used to determine the mass of a planet (e.g. for Saturn)? Is there a technique to determine the mass of the Sun?
- What is the geometric definition of an ellipse and its relation to planetary and satellite motion.
- Explore the eccentricities of planets within the solar system to explore how Kepler's Laws may be modelled as uniform circular motion.

? Science inquiry practical

Track satellites in real time at:

http://www.n2yo.com/?s=00050

Science as a human endeavour

Some suggestions for possible investigations

- Analyse how the models for the motion of planets, stars, and other bodies were modified in the light of new evidence.
- Research the benefits, limitations, and/or unexpected consequences of the uses of satellites. Examples include: the Hubble Space Telescope, the International Space Station, GPS satellites, and decommissioned satellites.
- Use Kepler's Laws to analyse highly elliptical orbits, such as HD 80606 b and HD 20782. Consider the effect of these orbits on the composition and temperature changes on these exoplanets.
- Investigate how Kepler's Laws can be used to estimate the mass of black holes, including Sagittarius A* — the black hole hypothesised to exist within the Milky Way Galaxy.

Helpful online resources

http://curious.astro.cornell.edu/about-us/95-the-universe/galaxies/general-questions/512-do-stars-orbits-in-galaxies-obey-kepler-s-laws-intermediate

http://io9.gizmodo.com/the-video-that-revealed-the-black-hole-at-the-center-of-1114918644

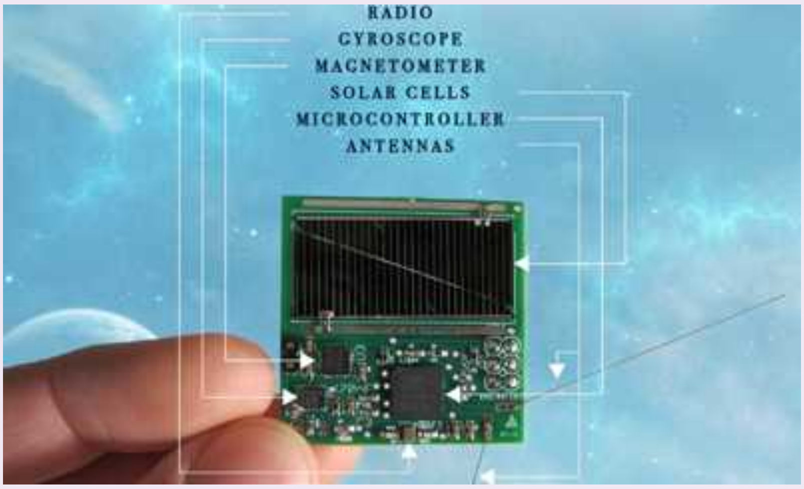
Science as a human endeavour activity – Gravitation 1

The text below forms part of an article, 'Breakthrough Starshot successfully launch world's smallest spacecraft'.

Date: July 29, 2017

Source: Guardian, Nicola Davis

https://www.theguardian.com/science/2017/jul/28/breakthrough-starshot-successfully-launch-worlds-smallest-spacecraft



The smallest spacecraft ever launched are successfully travelling in low Earth orbit and communicating with systems on Earth, scientists have announced.

Known as "Sprites", the miniature satellites are just 3.5cm × 3.5cm and carry radios, sensors and computers, with each device powered by sunlight and weighing just four grams.

While nanosatellites known as CubeSats have previously been sent into space, such systems have a mass thousands of times that of the Sprites, weighing more than 1kg. Scientists say the latest development is an important a precursor to an ambitious attempt to send space probes to planets beyond our solar system, dubbed Breakthrough Starshot.

"This is a new frontier of tiny, gram-scale spacecraft" said Professor Avi Loeb of Harvard University, chair of the advisory committee for the Breakthrough Starshot Initiative.

The Sprites, Loeb adds, are also cheap. "Each of them is only tens of dollars in cost," he said.

Describe an example of how the use of 'Sprites' demonstrates science as a human endeavour.

Suggested solution

While nanosatellites have previously been sent into space, Sprites indicate a major development because they have a mass that is around one thousand times smaller. This makes them the 'smallest spacecraft ever launched'. The Sprites travel in low Earth orbit and successfully communicate with systems on Earth. Space exploration is traditionally very expensive, but this new technology is cheap at around ten dollars per spacecraft and Spikes are 'an important a precursor to an ambitious attempt to send space probes to planets beyond our solar system' (Starshot). They will therefore help improve the efficiency of scientific procedures, data collection and analysis and enable scientists to collect data they have not been able to collect in the past. This may reveal new information and help further develop scientist's understanding of space.

1

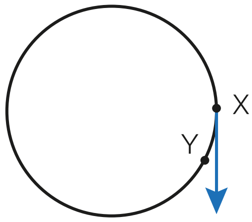
Exercises

Uniform circular motion

1. Describe the meaning of the following terms
   (a) uniform circular motion

   (b) centripetal acceleration

2. The vector arrow drawn at point X, represents the velocity *v*, of an object undergoing uniform circular motion at the point X.



   (a) Draw a vector arrow at point Y to represent the velocity of the object at point Y.
   (b) Annotate the diagram and show that the object experiences an acceleration towards the centre of the circular path as it travels from point X to point Y.

3. A 20.0 g marble circles the rim of a bowl of diameter 38.0 cm. It completes 10 revolutions in 8.50 s.
   (a) State the name of the force that provides the centripetal acceleration of the marble.

   (b) (i) Define the term period as it relates to the motion of the marble.

   (ii) Calculate the period of motion for the marble.

   (c) Calculate the speed of the marble as it circles the rim of the bowl.

(d) Calculate the magnitude and direction of the force acting on the marble.

4. A 26 g rubber stopper is whirled horizontally in a circular path of radius 54 cm with a speed of 5.0 ms^{-1}.

(a) Calculate the period of revolution of the rubber stopper.

(b) Calculate the centripetal acceleration experienced by the rubber stopper.

(c) Calculate the magnitude of the tension in the string.

(d) The rubber stopper is progressively whirled faster. Explain why the string will eventually break.

5. Consider the amusement ride shown in the photograph below. When it is spinning at its maximum speed of 15 ms^{-1} the carriages and their occupants move out into a horizontal circular path. The radius of the circular path traced by the carriages is 8.0 m.

Assuming the average mass of a carriage and its occupant is 74 kg, calculate the

(a) centripetal acceleration $\vec{a}$ experienced by each carriage.

(b) magnitude of the average tension in the carriage support wires.

(c) time taken for the carriages to complete ten revolutions.

6. A centripetal acceleration of 6.0 ms^{-2} acts on a mass undergoing uniform circular motion.

Describe the affect on the magnitude of the centripetal acceleration and state its new value if the

(a) speed v of the mass is increased to $5v$.

(b) radius of curvature r is changed to $\frac{r}{4}$.

7. A 1250 kg car rounds a curve of radius 85.0 m with a speed of 17.0 ms^{-1} on a level road. Calculate the magnitude of the frictional force that the tyres provide during the turn.

8. A young boy hangs an increasing number of weights to the end of piece of fishing wire. He can hang 3.75 kg of mass before the fishing wire snaps.

(a) Calculate the maximum tension that the fishing wire can withstand.

(b) The young boy whirls a 50.0 g mass on the end of a 40.0 cm length of the fishing wire in a horizontal circular path.

Calculate the maximum speed achievable before the fishing wire breaks.

(c) Calculate the minimum period of revolution of the 50.0 g mass.

9. A force of magnitude ***F*** Newtons acts on an object moving in a circular path at constant speed.

Use proportionality to describe the affect on the magnitude of the force acting if each of the following changes occur. State the new force in terms of ***F***.

(a) The object is replaced with one that has three times the mass.

(b) The speed of the object is reduced by a factor of four.

(c) The radius of the circular path is halved.

10. A 1500 kg satellite circles the Earth. The satellite has a period of revolution of 3.0 hours.

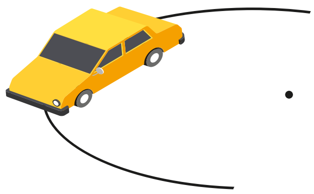
(a) Determine the altitude at which the satellite circles the Earth.

(b) Calculate the magnitude of the force acting on the satellite.

(c) Name the force that provides the centripetal acceleration in this situation.

11. A 1.6 tonne car rounds a corner with a radius of curvature of 250 m.

    (a) Draw vector arrows on the car to represent the velocity and the force acting on the car as it turns the corner illustrated below.



    (b) Calculate the magnitude and direction of the frictional force acting if the car makes the turn at a speed of 60.0 kmh^{-1}.

    (c) The tyres can provide a maximum frictional force of 4.0×10^3 N. Calculate the maximum speed with which the car can turn this corner safely.

    (d) Explain why the tyres don't need to provide as much frictional force, if any at all, when the road is banked.

12. A geostationary satellite is one that remains fixed over a certain point of the Earth's surface. It has a period of 24.0 hours and is positioned approximately 3.70×10^4 km above the surface of the Earth.

    (a) Calculate the speed of the satellite.

    (b) Calculate the centripetal acceleration $\vec{a}$ experienced by the satellite.

    (c) Calculate the force $\vec{F}$ acting on the satellite given it has a mass of 1.00×10^3 kg.

(d) Name the force acting on the satellite to provide the centripetal acceleration for uniform circular motion.

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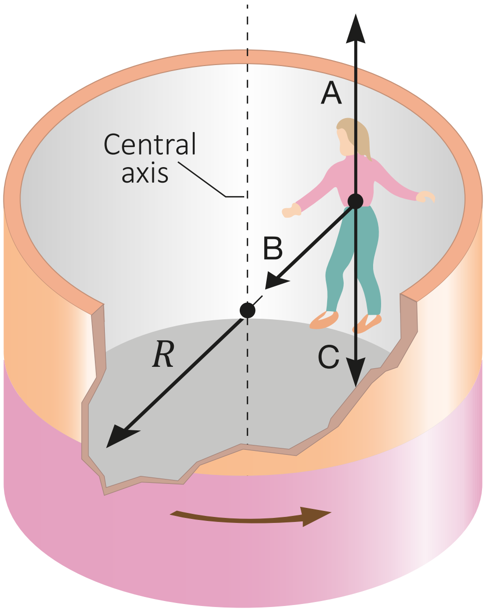
13. The amusement park ride shown below rotates at a high speed of 24 revolutions per minute before the floor drops and the occupants remain suspended against the outside wall of the ride. The ride has a radius *R* of 6.6 m.



(a) Name each of the forces acting on the person enjoying the ride. The forces are represented by the vector arrows A, B and C.

A..

B..

C..

(b) Calculate the period of rotation of this amusement ride.

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(c) The person shown enjoying the ride has a mass of 67 kg. Determine the magnitude of each of the forces A, B and C.

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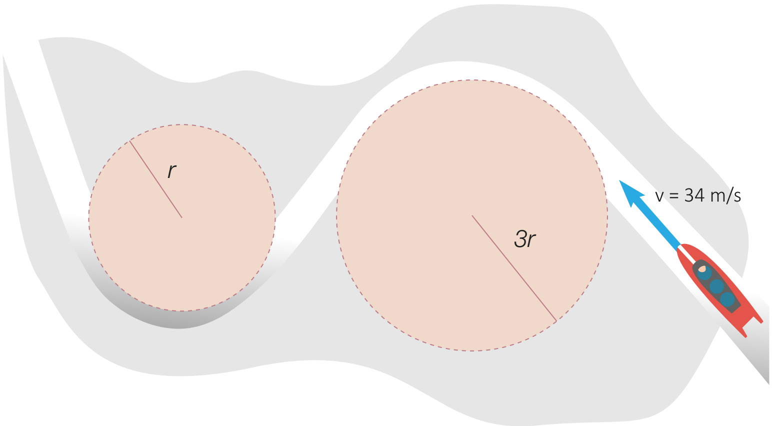
14. Consider the bob sled track shown below.



The bob sled maintains a constant speed along the track. Calculate the ratio of

(a) the magnitude of the centripetal acceleration experienced by the bob sled as it rounds the first turn to the centripetal acceleration experienced by the bob sled as it rounds the second turn.

(b) the period of motion of the bob sled as it rounds the first turn to the period of motion of the bob sled as it rounds the second turn.

15. (a) A vehicle turns a banked road of radius r at a constant speed v.
With the aid of a labelled diagram, show that the road should be banked at an angle given by $\theta = \tan^{-1}(\frac{v^2}{rg})$ in order for the vehicle to turn the road without any frictional force contributing to the centripetal acceleration.

(b) Hence, calculate the banking angle for a road with a radius of curvature of 120 m such that a vehicle can make the turn with a speed 25 ms^{-1} without friction providing any of the centripetal acceleration.

16. A section of a road of radius r = 300 m, is banked so that a car can make a turn at a speed of v = 100 kmh^{-1} without relying on friction.

(a) On the diagram above, draw a vector arrow, showing the direction of the normal force acting on the car. Label this vector N.

(b) Explain how the horizontal and vertical components of the normal force affect the motion of the car.

Gravitation

1. (a) Sketch the gravitational field of the Earth on the diagram below.

(b) Define the term gravitational field strength.

(c) A mass of 3.40 kg experiences a gravitational force of 37.4 N on the surface of Uranus. Calculate the magnitude of the gravitational field strength at the surface of Uranus.

2. The gravitational field strength on Mars is 3.7 Nkg^{-1}. Calculate the magnitude of the gravitational force that a 10.0 kg object would experience on the surface of Mars.

3. (a) State Newton's Universal Law of Gravitation in words.

(b) Calculate the magnitude and direction of the gravitational force acting between two point masses, m_1 = 20.0 kg and m_2 = 55.0 kg placed 1.80 m apart in a vacuum.

(c) Explain why gravitational forces are consistent with Newton's Third Law of Motion.

4. Two dancers are standing 1.00 m apart. One dancer has a mass of 55.0 kg and the other a mass of 62.0 kg.

(a) Calculate the gravitational force $\vec{F}$ acting between the two dancers.

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(b) Add vector arrows to represent the gravitational force acting on each dancer depicted above.

(c) Calculate the magnitude and direction of the acceleration experienced by the 62.0 kg dancer.

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5. Two identical masses separated by a distance of 5.0 cm, experience a gravitational force of attraction of magnitude 5.0 N.

(a) Calculate the magnitude of the masses.

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(b) Without performing a calculation, describe the affect on the magnitude of the gravitational force between the masses if

(i) one mass is replaced with another half the mass.

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(ii) one mass is replaced with another that has half its mass and the other mass is replaced with a mass six times heavier.

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(iii) the distance between the centre of the masses is quartered.

6. When placed a fixed distance d apart in air, two masses exert a gravitational force of magnitude $\boldsymbol{F}$ on each other. State the magnitude of the new force in terms of $\boldsymbol{F}$ if

(a) both masses are replaced with masses that are twice as heavy.

(b) the distance between the centre of the two masses becomes $\frac{d}{2}$.

7. (a) Calculate the distance between the centre of two identical objects with a mass of 5.0×10^2 kg if they experience a gravitational force of attraction of magnitude 3.7×10^{-9} N.

(b) The distance calculated in part (a) is changed. The force becomes four times larger. State, with reason the change made to the distance between the centres of the two objects.

8. (a) Show that the acceleration due to gravity g at the surface of the Earth, mass M_E and radius r_E is given by $g = \frac{GM_E}{r_E^2}$.

(b) Calculate the magnitude of the acceleration due to gravity at the surface of the Earth.

(c) Determine the magnitude of the acceleration due to gravity experienced by a geostationary satellite that is situated at an altitude of 3.6×10^4 km above the surface of the Earth.

9. (a) Explain why gravitational field strength and acceleration due to gravity are the same quantity.

(b) The planet Venus has a mass of 4.90×10^{24} kg. Calculate the radius of Venus if the gravitational field strength at its surface is $8.90\ Nkg^{-1}$.

10. Determine the position between the Earth and Moon at which the gravitational field strength (or acceleration due to gravity) is zero. You may label the diagram below and use the data listed to help you.

Useful data: $m_{Earth} = 5.97 \times 10^{24}$ kg

$m_{moon} = 7.35 \times 10^{22}$ kg

$r_{moon's\ orbit} = 3.84 \times 10^{8}$ m

11. The gravitational force between the Earth and a space vehicle will allow the space vehicle to move with uniform circular motion around the Earth.

(a) Calculate the magnitude of the gravitational force acting on a space vehicle of mass $m = 1.30 \times 10^{3}$ kg if it circles the Earth in an orbit of radius $r = 5.64 \times 10^{7}$ m.

(b) Show that the speed of the space vehicle is given by $v = \sqrt{\frac{GM}{r}}$.

(c) Use the expression from part (b) to calculate the speed of the space vehicle.

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(d) Show that an expression for the period of the satellite is $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$.

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(e) Determine the factor by which the period of motion of the space vehicle is changed if the radius of orbit is doubled.

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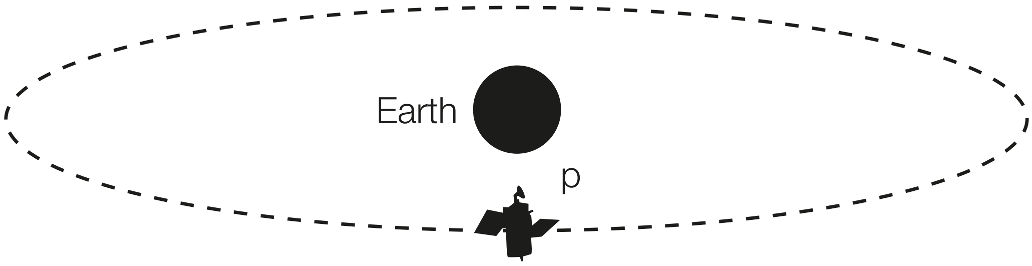
12. The orbit, A, of a satellite is shown in the diagram below.



This diagram is not to scale

(a) Draw and label vector arrows on the diagram to represent the velocity and the acceleration of the satellite at position P.

(b) Calculate the speed of the satellite given it is positioned at an altitude of 8000 km above the surface of the Earth.

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(c) The satellite is moved to a different orbit, B, which has half the radius of orbit of A.

Calculate the ratio of the speed of the satellite in orbit A to the speed of the satellite in orbit B.

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(d) Calculate the period of the satellite's motion in orbit A.

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13. A artificial satellite of mass 850 kg travels with a speed of 1.6×10^3 ms^{-1} around the planet Mercury with a radius of orbit of 8.44×10^6 m.

(a) Calculate the mass of Mercury.

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(b) Calculate the gravitational force $\vec{F}$ experienced by the satellite.

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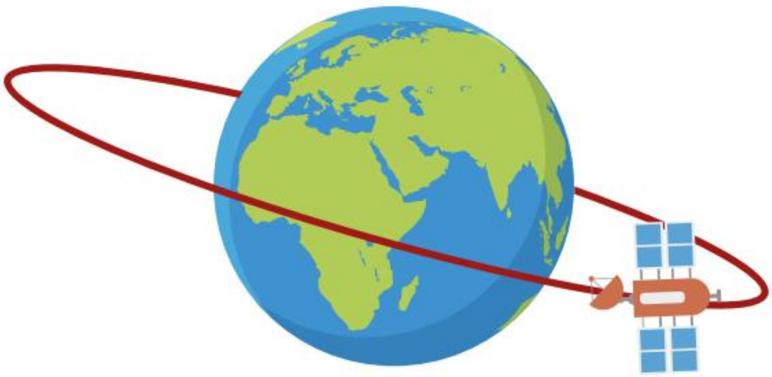
14. The satellite shown in the diagram below is in a stable orbit around the Earth.



(a) Explain why the centre of the circular orbit must coincide with the centre of the Earth.

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(b) On the diagram shown, draw one orbit that would not be possible.

(c) Describe one use of low-altitude polar orbits in meteorology.

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15. (a) Explain why a geostationary satellite must orbit in the same direction as the Earth rotates.

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(b) Describe why the orbit of a geostationary satellite must be equatorial.

(c) State the period of orbit for a geostationary satellite and hence calculate the radius of orbit for a geostationary satellite.

(d) Calculate the altitude of a geostationary satellite.

16. (a) State Kepler's three laws of planetary motion.

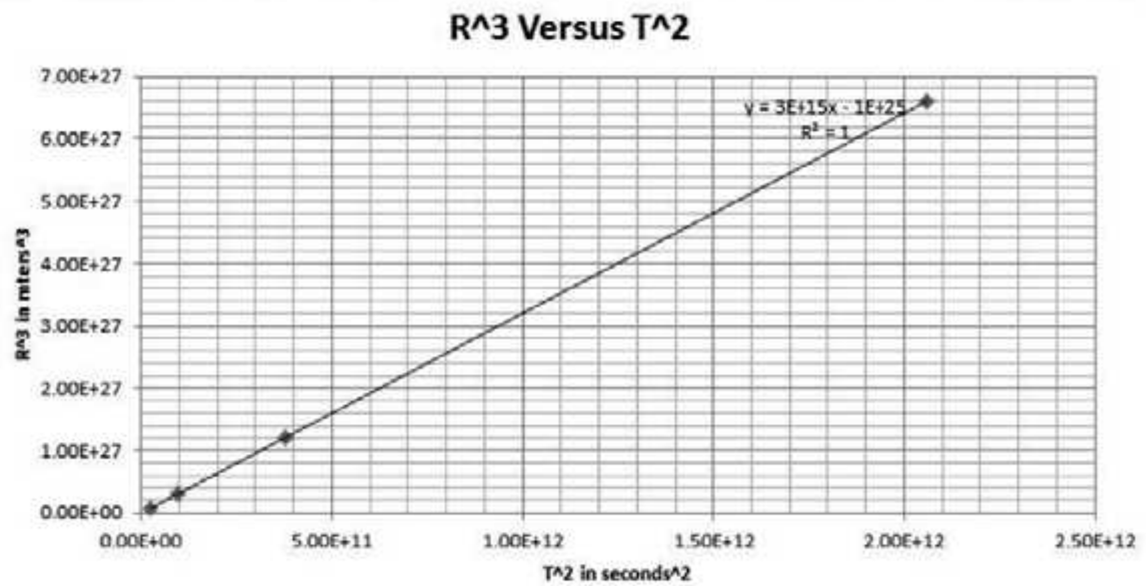
(i) First Law

(ii) Second Law

(iii) Third Law

(b) Describe how Kepler's first and second laws can be used to explain the motion of the planets in their orbit around the Sun.

17. A student plots a graph of the radius of orbit of Jupiter's moons cubed against their period squared.



(a) The equation of the line shows that the value of the gradient of the line is 3×10^{15}.

Use the graph to calculate the gradient of the line to the appropriate number of significant figures. Include units for the gradient.

(b) Use the graph to calculate the mass of Jupiter.

(c) The accepted value for the mass of Jupiter is 1.90×10^{27} kg.

Comment on the accuracy of the experimental value for the mass of Jupiter.

(d) Comment on the precision of the data.

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18. Science as a human endeavour activity – Gravitation 2

The following text forms part of an article 'Tiny Neptune moon spotted by Hubble may have broken from larger moon'.

Date: February 21, 2019

Source: NASA/Goddard Space Flight Center

https://www.sciencedaily.com/releases/2019/02/190221095033.htm

After several years of analysis, a team of planetary scientists using NASA's Hubble Space Telescope has at last come up with an explanation for a mysterious moon around Neptune that they discovered with Hubble in 2013.

The tiny moon, named Hippocamp, is unusually close to a much larger Neptunian moon called Proteus. Normally, a moon like Proteus should have gravitationally swept aside or swallowed the smaller moon while clearing out its orbital path.

So why does the tiny moon exist? Hippocamp is likely a chipped-off piece of the larger moon that resulted from a collision with a comet billions of years ago. The diminutive moon, only 20 miles (about 34 kilometers) across, is 1/1000th the mass of Proteus (which is 260 miles [about 418 kilometers] across).

This scenario is supported by Voyager 2 images from 1989 that show a large impact crater on Proteus, almost large enough to have shattered the moon. "In 1989, we thought the crater was the end of the story," said Showalter. "With Hubble, now we know that a little piece of Proteus got left behind and we see it today as Hippocamp." The orbits of the two moons are now 7,500 miles (about 12,070 kilometers) apart.

"Based on estimates of comet populations, we know that other moons in the outer solar system have been hit by comets, smashed apart, and re-accreted multiple times," noted Jack Lissauer of NASA's Ames Research Center in California's Silicon Valley, a coauthor on the new research. "This pair of satellites provides a dramatic illustration that moons are sometimes broken apart by comets."

The team of astronomers in this study consists of M. Showalter (SETI Institute, Mountain View, California), I. de Pater (University of California, Berkeley, California), J. Lissauer (NASA Ames Research Center, Silicon Valley, California), and R. French (SETI Institute, Mountain View, California).

Describe two examples of how this study using NASA's Hubble Space Telescope demonstrates science as a human endeavour.

1.4 Relativity

Science understanding

1. Motion can only be measured relative to an observer; length and time are relative quantities that depend on the observer's frame of reference.
2. Some measured quantities of objects travelling at very high speeds cannot be explained by Newtonian physics. Einstein's Theory of Special Relativity predicts significantly different results to those of Newtonian physics for velocities approaching the speed of light.
3. The Theory of Special Relativity is based on two postulates:
    - that the laws of physics are the same in all inertial reference frames.
    - that the speed of light in a vacuum is an absolute constant.
4. In relativistic mechanics, there is no absolute length or time interval.
5. At relativistic speeds, time intervals in moving frames of reference are dilated when observed from a stationary reference frame according to $t = \gamma t_0$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor, t_0, is the time interval in the moving frame of reference and t is the time interval in the stationary observer's frame of reference.
    - Solve problems using $t = \gamma t_0$ and the Lorentz factor formula.
    - Explain the effects of time dilation on objects moving at relativistic speeds.
6. Some subatomic particles exist in the laboratory for very short time periods before decaying. These same particles are detected as part of cosmic ray showers in the atmosphere, travelling at relativistic speeds close to the speed of light. Time dilation effects allow these particles to travel significant distances without decay.
    - Calculate and compare lifetimes and therefore distances travelled by subatomic particles in stationary and moving reference frames.
    - Solve problems involving subatomic particles moving at relativistic speeds.
7. An object moving at relativistic speeds is shorter to an observer in a stationary frame of reference, and the length is given by: $l = \frac{l_0}{\gamma}$, where l_0 is the length in the moving object's frame of reference and l is the length in the stationary observer's frame of reference.
    - Solve problems using $l = \frac{l_0}{\gamma}$.
    - Explain the effects of length contraction on objects moving at relativistic speeds.
8. The magnitude of the relativistic momentum of a moving object is given by $p = \gamma m_0 v$, where m_0 is the mass of the object in the frame of reference where the object is stationary and v is the speed of the object.
    - Solve problems using $p = \gamma m_0 v$.
    - Explain why masses moving at relativistic speeds are unable to reach the speed of light.

Introduction

In classical physics (pre-relativity), time, length and mass are absolute quantities. At low speeds, classical physics could explain how two observers view the same event when one observer was moving. However, at very high speeds, classical physics failed to explain the observations made. Einstein thought of space and time as one entity. He did not consider time, length and mass as absolute quantities.

The following YouTube clip summarises the theory of relativity in simple terms. It may be worth watching before starting this subtopic. Relative motion, time dilation and length contraction are discussed.

Helpful online resources

The following YouTube clip summarises the theory of relativity in simple terms.

https://www.youtube.com/watch?v=ttZCKAMpcAo

Frames of reference

A **reference frame or frame of reference** is a set of coordinates used to define position. In most cases the Cartesian reference is used. That is, an origin with a set of axes. A reference frame enables us to refer to the position of an object and to make measurements such as length and time. A reference frame gives us a perspective or way of viewing a situation.

Figure 1.4.1 shows a student in a laboratory. The student is stationary in the reference frame of the laboratory. The clock indicates that the student is measuring the time it takes for an event to occur. In this case the event is the time taken for a candle to burn out. We say that the student is stationary relative to the event.

Figure 1.4.2 shows the same student, timing the same event on a plane that is moving with a constant velocity. This time, the frame of reference is the plane. Now pretend that you are watching the plane from the ground. You may be referred to as a ground-based observer. The plane is now a moving reference frame and the student timing the burning of the candle is stationary within the moving frame of reference.

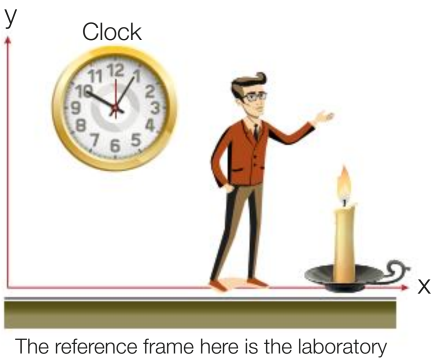


Figure 1.4.1

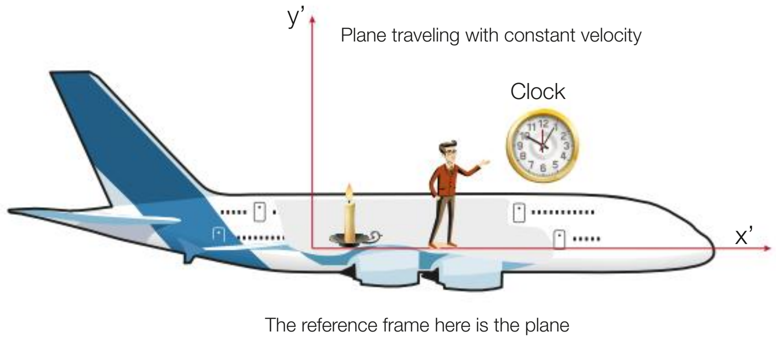


Figure 1.4.2

An **inertial frame of reference** is one in which Newton's laws of motion apply. An inertial frame is therefore either stationary or moving with a constant velocity.

An accelerated frame of reference is not inertial neither is one which involves a change in direction. A playground merry-go-round is not inertial neither is an accelerating vehicle.

Suppose you were doing an experiment in class. Your reference frame would be the laboratory. You are stationary relative to the experiment and you are in an inertial reference frame.

Viewing the same event from different frames of reference

Galileo Galilei proposed that motion is relative. In 1632 he wrote a book with a title of 'Dialogues Concerning the Two Chief World Systems'. In the book, Galileo described a thought experiment sometimes referred to as 'Galileo's ship experiment'.

The thought experiment involved dropping an object while on a ship that was moving with constant velocity. Galileo concluded that the movement of the ship did not affect the motion of the object but the frame of reference did.

To explain this, consider a similar situation as shown in Figure 1.4.3. An object is dropped from the top of a boat's mast.

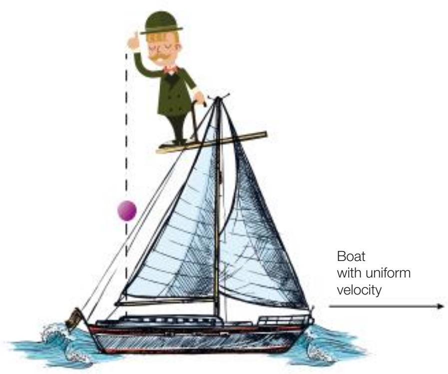


Figure 1.4.3

Figure 1.4.4

The person dropping the object sees it fall straight down towards the deck of the boat and Newton's laws of motion apply. Just like Galileo, we can conclude that the object would fall in the same way it would if the boat was stationary. That is, there is no difference between a stationary boat and one that is moving with constant velocity.

Figure 1.4.4 shows a stationary observer on the beach, who sees the stone follow a parabolic path. They observe the object as having a horizontal component of velocity. In addition, the object has a final displacement to the right. Recall that Galileo concluded that the movement of the boat does not affected the motion, but the frame of reference does.

Whose observation is correct? Galileo's explanation was simple, both observers are correct because motion is relative.

The same event may be observed differently in a different frame of reference.

At this point it may be worth revisiting question three in the Subtopic 1.1 exercises.

The question was about a hot air balloon drifting across the countryside when a marble was dropped over the side. Part (e) asked you to describe the path of the marble as seen by the balloonist and a person standing on the ground. This is essentially the same as Galileo's thought experiment.

Figure 1.4.5 shows a person throwing a ball vertically into the air while moving forward in a vehicle.

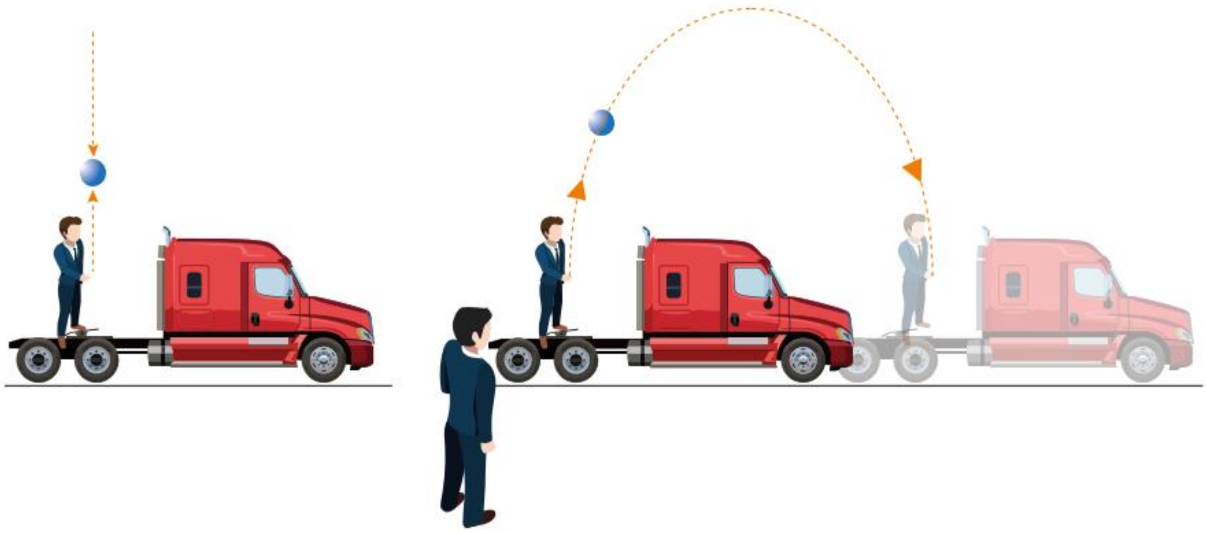


Figure 1.4.5

From the vehicle's frame of reference (the moving frame of reference), the person standing in the car observes the ball moving upwards and then falling back into their hand. A person standing on the side of the road (stationary frame of reference) observes the ball moving in a parabolic path. To this observer the ball has a horizontal component of velocity. The final position of the ball is also different for this observer.

Relative velocity

We have already seen that motion is relative to the frame of reference. Figure 1.4.6 shows a ball being thrown horizontally from a vehicle travelling with constant velocity.

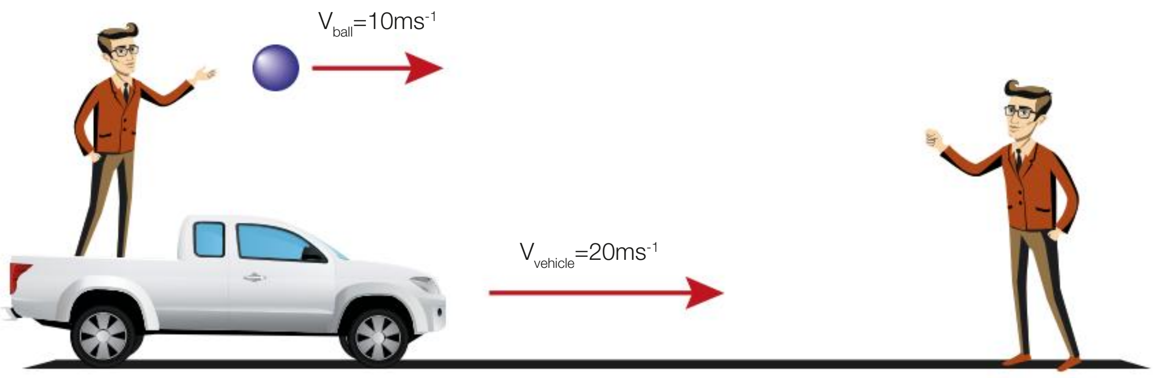


Figure 1.4.6

The person throwing the ball is stationary within the frame of reference of the vehicle. The magnitude of the velocity of the ball is 10.0 ms^{-1}. To the ground-based observer the magnitude of the velocity of the ball is 30.0 ms^{-1}.

Figure 1.4.7 shows a dog chasing a postman.

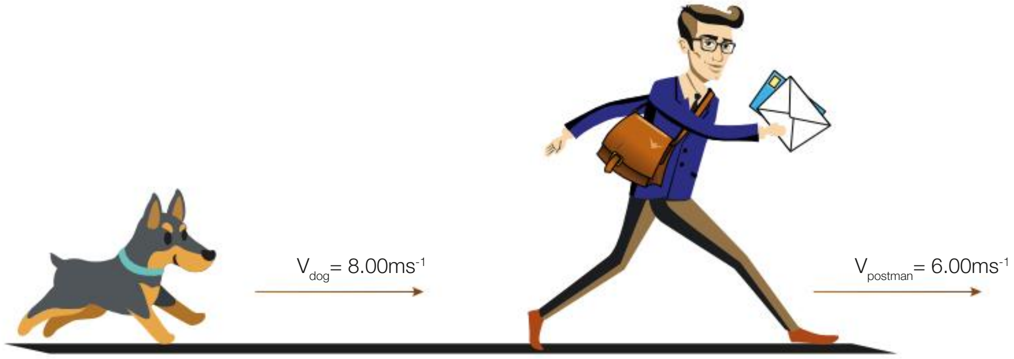


Figure 1.4.7

The postman observes the dog to be running with a velocity of 2.00 ms^{-1} to the right (or +2.00 ms^{-1}). The dog observes the postman's velocity to be 2.00 ms^{-1} to the left (or –2.00 ms^{-1}).

Galilean transformations and Maxwell – the constancy of the speed of light

Key idea

Observations of objects travelling at very high speeds cannot be explained by Newtonian physics.

Although not part of this course, the Galilean transformations are a set of equations that represent the relationship between space and time in two different frames of reference that are moving at a constant velocity relative to one another. Newtonian physics is applied, and this assumes that space and time are absolute quantities and that length, time, and mass are independent of the relative motion of the observer.

The Galilean transformations are of interest because, they are able to describe motion at speeds much smaller than the speed of light. The examples discussed when examining relative velocity essentially applied the concepts underpinning the Galilean transformations.

Consider the situation illustrated in Figure 1.4.8. Using the concepts of motion that we know, the speed of light would depend upon the relative motion of the observer.

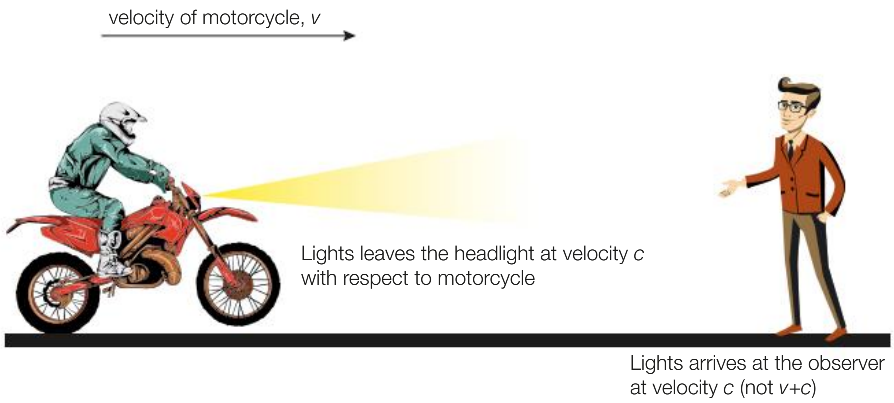


Figure 1.4.8

James Clerk Maxwell had already theorised that the speed of light is the same in all inertial reference frames. This means that the Galilean transformation equations cannot work for light as the equations predict the speed of light to be different depending of the frame of reference.

In fact the equations could not explain observations of objects made a very high speeds.

Consider Figure 1.4.6 again. It showed a ball being thrown horizontally from a vehicle travelling with constant velocity. If the person throwing the ball could through it at a speed close to the speed of light (say 0.4c), and the vehicle was travelling close to the speed of light (say 0.8c), then the magnitude of the velocity of the ball as observed by the ground-based observer would be greater than the speed of light. Figure 1.4.9 shows that the ground-based observer would observe the ball with a velocity of 1.2c. This is not possible.

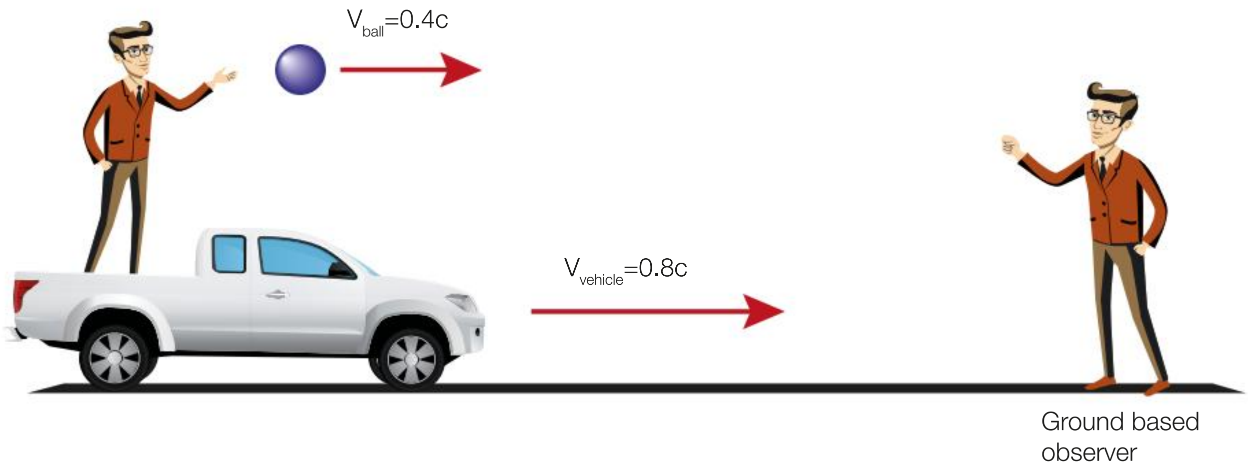


Figure 1.4.9

Helpful online resources

1. This video explains frames of reference and explains clear examples
   https://www.youtube.com/watch?v=ClPH7b_LAQk

2. The following online resources will help you learn more about relative motion and the Galilean transformations.
   https://www.youtube.com/watch?v=mkQYSkioO98
   https://www.youtube.com/watch?v=QRKtQKCbEIl
   https://www.youtube.com/watch?v=NH3_llkSB9s

Einstein's Theory of Special Relativity

Key ideas

1. Motion can only be measured relative to an observer; length and time are relative quantities that depend on the observer's frame of reference.
2. Einstein's Theory of Special Relativity predicts significantly different results to those of Newtonian physics for velocities approaching the speed of light.

Two postulates of the Theory of Special Relativity

Recall that an inertial frame of reference is one in which Newton's laws apply. An inertial reference frame is either stationary or moving with a constant velocity. An accelerated frame of reference is not inertial neither is one which involves a change in direction.

The Theory of Special Relativity is based on two postulates or assumptions.

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light in a vacuum is an absolute constant. That is, it is the same for all inertial frames of reference.

Concept of simultaneity

Key idea

In relativistic mechanics, there is no absolute length or time interval.

Two events are said to occur **simultaneously** if they happen at the same instant of time. Simultaneous events are not necessarily simultaneous to all observers.

Figure 1.4.10 shows a student positioned in the middle of a train carriage that is moving with constant velocity to the right.

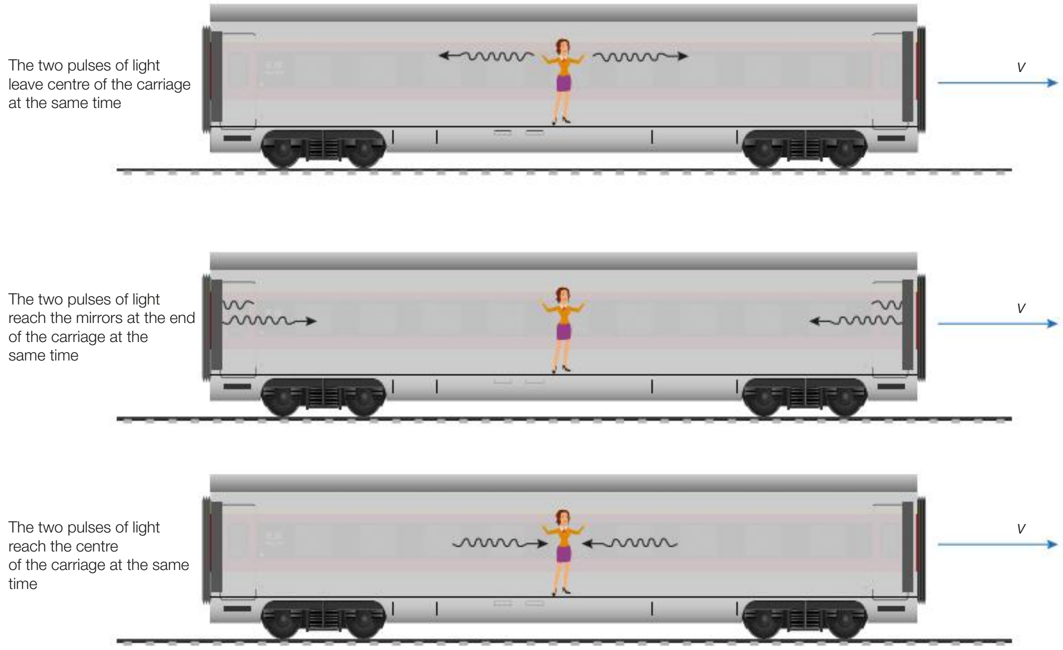


Figure 1.4.10

The student sends two pulses of light that travel in opposite directions to each end of the train carriage. Mirrors at each end of the train carriage reflect the pulses back towards the middle of the train carriage.

The student is at rest with respect to the train carriage. The light pulses were sent out simultaneously, they hit the mirrors simultaneously and return to the centre of the carriage simultaneously.

Figure 1.4.11 illustrates the situation for a stationary ground-based observer.

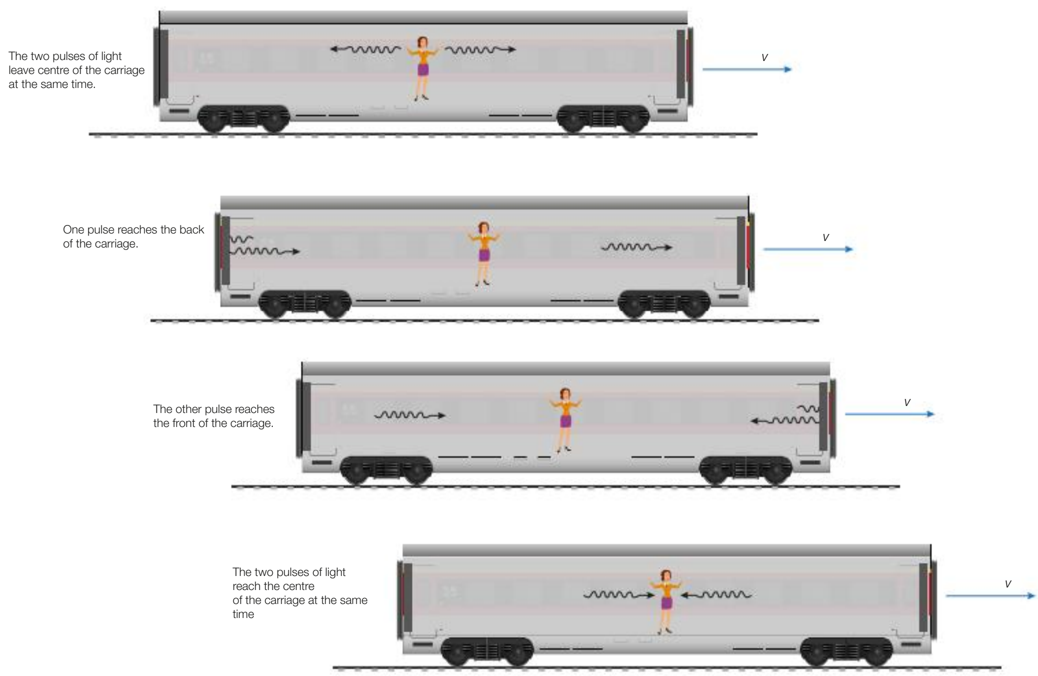


Figure 1.4.11

The ground-based observer sees one of the light pulses reaching the back of the train carriage first. This is because the left hand side of the carriage is moving towards the pulse and the right hand side is moving away from the pulse. The light pulses travel with the same velocity (c) but the pulse traveling to the back of the carriage does not travel as far before reaching the mirror. The reflection will therefore happen from the back of the train carriage first. Time in relativity is not an absolute quantity.

The time interval between two events and the determination of whether two events are simultaneous depends on the observer's frame of reference.

Two events that are simultaneous in one frame of reference will not be simultaneous in a frame of reference which is moving with respect to the other.

The ground-based observer will, however, see the pulses arrive at the centre of the carriage simultaneously.

Helpful online resources

The following YouTube clip explains the concept of Simultaneity

https://www.youtube.com/watch?v=wteiuxyqtoM

Time dilation

A light clock is an imaginary device. Figure 1.4.12 shows a light clock consisting of a mirror on its bottom surface and one on the top surface. The mirrors are separated by a distance l. A beam of light is reflected between the two mirrors.

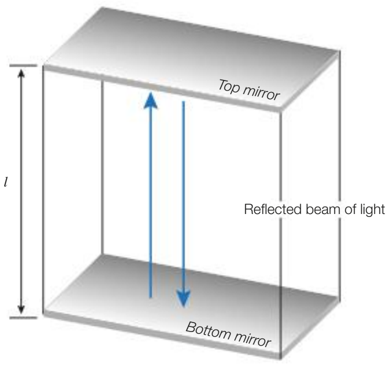


Figure 1.4.12

Consider a simple event, the time taken for the light to leave the bottom mirror and reach the top mirror (t_0). The time taken for this event for an observer in the light box is $t_0 = \frac{l}{c}$. This is because light travels with a constant velocity, so $v = \frac{s}{t}$ becomes $t_0 = \frac{s}{v} = \frac{l}{c}$.

This is an expression for the **proper time**.

Proper time is the time interval between two events when measured in the reference frame in which the event is at rest (or its rest frame).

Proper time is the shortest possible time that any observer could record for the event.

Figure 1.4.13 shows the same light clock moving with a constant velocity v to the right.

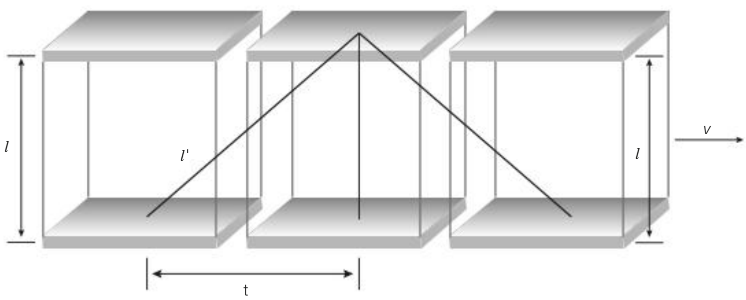


Figure 1.4.13

The same event is measured by a stationary ground-based observer outside of the light clock. The time taken for the light to reach the top mirror from the bottom mirror will be taken to be t. In a time t the clock has moved a distance vt. The distance travelled by the light l', can be calculated using Pythagoras' theorem.

$$l' = \sqrt{(vt)^2 + l^2}$$

$$\text{but} \qquad t = \frac{l'}{c}$$

$$\therefore \quad t^2 = \frac{v^2t^2 + l^2}{c^2}$$

$$\therefore \quad t^2c^2 = v^2t^2 + l^2$$

$$\therefore \quad t^2c^2 - v^2t^2 = l^2$$

$$\therefore \quad t^2(c^2 - v^2) = l^2$$

$$\therefore \quad \frac{t^2(c^2 - v^2)}{c^2} = \frac{l^2}{c^2}$$

$$\therefore \quad t^2\left(1 - \frac{v^2}{c^2}\right) = \frac{l^2}{c^2} = t_0^2$$

$$\therefore \quad t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This means that to the stationary observer outside the light clock, the time taken for the light to cross between the two mirrors (t) is larger than the time measured by the stationary observer within the time clock (t_0). This is because the denominator of the above expression has a value less than one for all values of velocity v. A greater value of v, produces a larger time for an event in the moving frame of reference.

This phenomenon is referred to as time dilation. Another way of thinking about this is that the time taken for an event is always greater in the moving frame of reference than the frame of reference that is stationary relative to the event.

Time dilation refers to the slowing down of clocks that are in motion relative to an observer.

That is, clocks appear to run slow when they are in motion.

Time in a moving reference frame is dilated when observed from a stationary reference frame according to the formula:

$$t = \gamma t_0$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the **Lorentz factor** and t_0 is the time in the moving frame of reference (i.e. the reference frame in which the event is at rest).

At very low velocities, the effect of time dilation is insignificant. For time dilation to be noticeable, speeds close to the speed of light are required so that the Lorentz factor is large. Figure 1.4.14 illustrates the concept of time dilation.

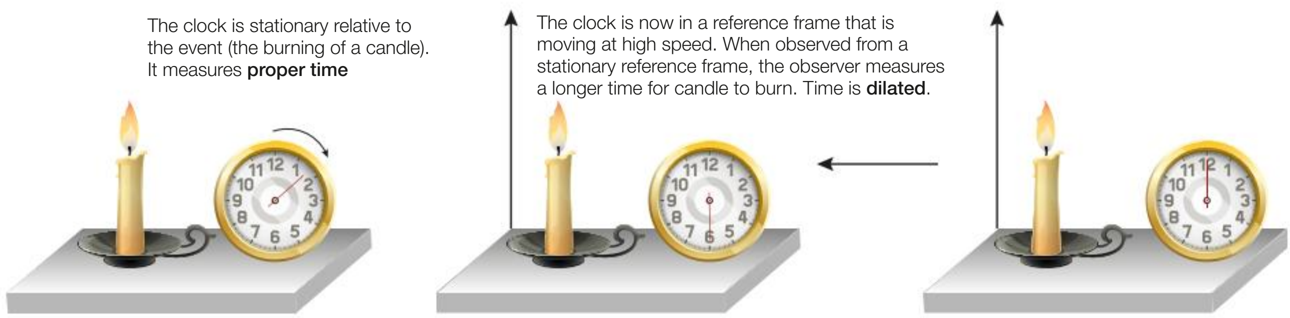


Figure 1.4.14

Helpful online resources

The following YouTube clip discusses the concepts discussed in the section on time dilation.

https://www.youtube.com/watch?v=HHRK6ojWdtU

The Lorentz factor

Although not part of this course, the Lorentz transformations are a set of equations that relate the space and time coordinates of two systems moving at a constant velocity relative to each other. They deal with situations that involve motion close to the speed of light. They work on the assumption that space and time are not absolute and that length, time and mass depend on the relative motion of the observer. They also assume that the speed of light in a vacuum is constant and independent of the motion of the observer or the source of light.

The **Lorentz factor (γ)** is used as the relative velocity increases between two inertial frames of reference. The Lorentz factor is a ratio that indicates the factor by which length, time and mass will change in a moving reference frame when compared to the stationary value.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where v is the velocity of the moving reference frame or the relative velocity between two reference frames and c is the speed of light.

Figure 1.4.15 shows how the Lorentz factor varies as the relative velocity between two frames of reference increases. At low velocity the Lorentz factor is approximately equal to 1, but it approaches infinity when the relative velocity between two reference frames approaches the speed of light ($v \rightarrow c$).

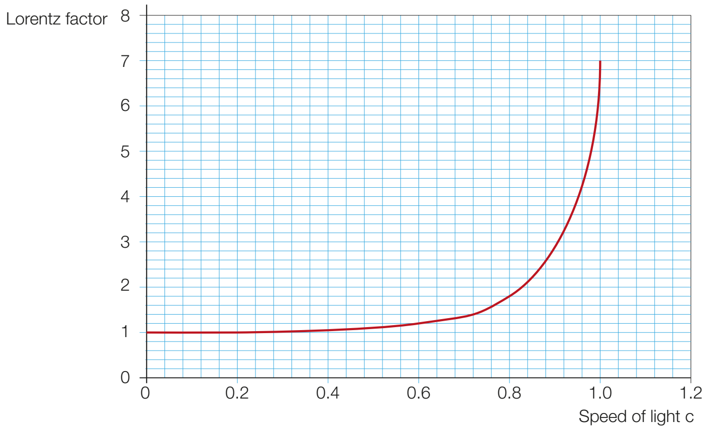


Figure 1.4.15

For convenience, the relative velocity between the two reference frames is often expressed as a fraction of the speed of light c.

We can see that for a velocity of v = 0.1c, the denominator of the Lorentz factor ($\sqrt{1-\frac{v^2}{c^2}}$) is close to one.

It follows that the Lorentz factor ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$) is close to one.

For a velocity of v = 0.999c, the denominator of the Lorentz factor is closer to zero. The Lorentz factor increases to approximately 71. The closer v gets to c, the larger the Lorentz value.

Worked examples

1. Calculate the Lorentz factor for an object travelling at a speed of 0.400c to three significant figures.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.400c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.400^2}} = 1.09$$

2. Calculate the Lorentz factor for an object travelling at a speed of 2.4×10^8 ms^{-1}.

$$\frac{2.4 \times 10^8}{3.00 \times 10^8} = 0.80 \quad \therefore \quad 2.4 \times 10^8 \text{ ms}^{-1} = 0.80c$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.80c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.80^2}} = 1.7$$

3. Show that at a speed of 0.98c, a time interval of 1.0 second as measured by a stationary observer is measured as approximately 5.0 seconds by a moving observer.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.98c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.98^2}} = 5.03$$

$$t = \gamma t_0 = 5.03 \times 1 = 5.03 = 5.0 \text{ s}$$

4. An unstable particle has a lifetime of 3.8×10^{-8} s as measured on a spacecraft travelling with a velocity of 95% the speed of light. Calculate the lifetime of the unstable particle as measured by a stationary observer on a near by space station.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.95^2}} = 3.2$$

$$t = \gamma t_0 = 3.2 \times 3.8 \times 10^{-8} = 1.2 \times 10^{-7}\ s$$

5. An observer sets up an experiment in a laboratory and records the period of a pendulum to be 2.20 s. A second observer moving at a high speed, records the period of the pendulum to be 3.30s.

(a) State, with reason, which of the two observers records the proper time for the period of the pendulum.

Proper time is the time interval between two events measured in the reference frame in which it is at rest. The pendulum is at rest in the laboratory. The proper time is 2.20 s.

(b) Determine the relative velocity between the two observers.

$$t = \gamma t_0 \quad \therefore \quad \gamma = \frac{t}{t_0} = \frac{3.30}{2.20} = 1.50$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \therefore \quad \gamma^2 = \frac{1}{1-\frac{v^2}{c^2}} \qquad 1-\frac{v^2}{c^2} = \frac{1}{\gamma^2} \quad \therefore \quad \frac{v^2}{c^2} = 1-\frac{1}{\gamma^2} = 1-\frac{1}{1.50^2} = 0.556$$

Since $\frac{v^2}{c^2} = 0.556$ it follows that $v = 0.746c$ or $2.24 \times 10^8\ ms^{-1}$

Extra understanding

Twin Paradox–time dilation and the implications for long-distance space travel.

The Twin Paradox is a thought experiment in which two identical twins compare their views of time. One twin remains on Earth while the other twin travels to a distant star and back again at a very high speed. The twin on Earth observes the travelling twin's clock running slow. When the travelling twin returns to Earth, they will have aged less than the twin who remained on Earth.

We have already seen that for a speed of 0.98c, one second for the travelling twin is about the same as five seconds to the twin on Earth (worked example 3). Consider the following example. The twins are 30 years old at the beginning of the thought experiment and ten years have elapsed as measured by the twin in the spacecraft. The twin on Earth observes the travelling twin's clock running slow and measuring 50 years. The twin on Earth is now 80 years old but the travelling twin is 40 years old. The situation is illustrated in Figure 1.4.16.

Figure 1.4.16

So why is there a paradox? The travelling twin observes the twin on Earth to have aged less as the twin on Earth is moving relative to the spacecraft. Which of the two twins is correct? The twin on Earth is in an inertial reference frame. The twin that took the journey had to turn around and return to Earth. This is not an inertial reference frame. Einstein's Theory of Special Relativity applies to two reference frames that are in constant relative motion. It does not deal with accelerating frames of reference. Non-inertial reference frames are dealt with by Einstein's Theory of General Relativity but is not part of this course.

Helpful online resources

The following YouTube clip summarises the theory of relativity, time dilation and the twin paradox.

https://www.youtube.com/watch?v=ERgwVm9qWKA

Investigate how relativistic effects are taken into consideration in the Global Positioning System (GPS):

http://www.brighthub.com/science/space/articles/32969.aspx

http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit5/gps.html

Length contraction

We have already seen that measurements of time are dependent on the relative motion of the observer. Proper time (t_0) was defined as the time interval between two events when measured in the reference frame in which the event is at rest (or its rest frame).

In a similar fashion, measurements of length are also dependent on the relative motion of the observer.

Proper length (l_0) of an object is the length recorded in a reference frame where the object is at rest.

Proper length is the longest possible length that any observer could record.

The separation of two points or the length of an object travelling at very high speed appears shorter (or contracted) in a moving reference frame. The contraction is in the same direction as the relative motion. An expression can be derived, just as the time dilation formula was derived.

Length contraction refers to the decrease in length of an object that is in motion relative to an observer.

Length in a moving reference frame is contracted when observed from a stationary frame of reference according to the formula: $l = \frac{l_0}{\gamma}$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the **Lorentz factor** and l_0 is the length in the frame of reference where the object is stationary.

At very low velocities, the effect of length contraction is insignificant. For length contraction to be noticeable, speeds close to the speed of light are required so that the Lorentz factor is large. Figure 1.4.17 illustrates the concept of length contraction.

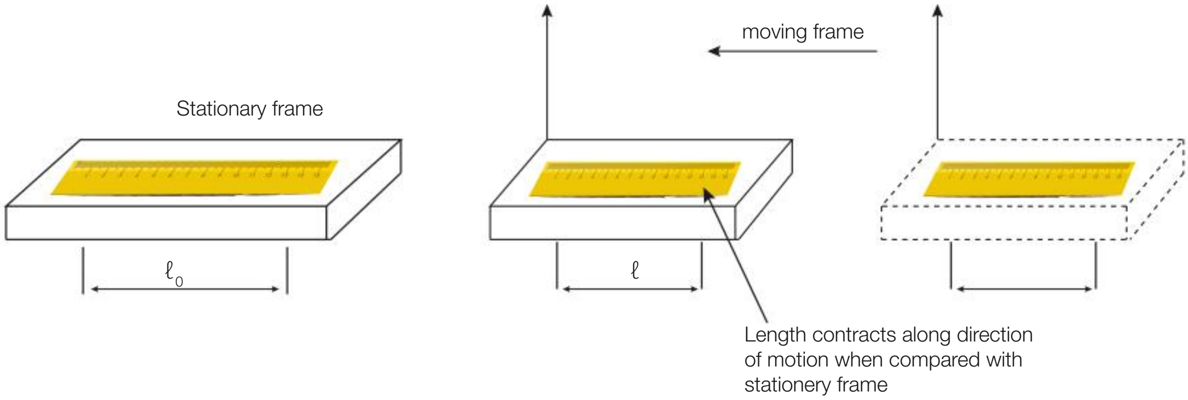


Figure 1.4.17

Worked examples

1. A physicist measures the length of a motion trolley to be 32.0 cm. The motion trolley is placed on a spacecraft travelling with a velocity of 1.50×10^8 ms^{-1}. The physicist remains on Earth.

   (a) Calculate the Lorentz factor for the spacecraft travelling with a velocity of 1.50×10^8 ms^{-1}.

   $$\frac{1.50 \times 10^8}{3.00 \times 10^8} = 0.500 \quad \therefore \quad 1.50 \times 10^8 \text{ ms}^{-1} = 0.500c$$

   $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.500c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.500^2}} = 1.15$$

   (b) Calculate the length of the motion trolley as observed by the scientist while it is on the spacecraft moving with a velocity of 1.50×10^8 ms^{-1}.

   $$l = \frac{l_0}{\gamma} = \frac{32.0}{1.15} = 27.8 \textit{ cm}$$

   (c) Calculate the speed with which the spacecraft must travel in order for the motion trolley's length to measure 25.0 cm.

   $$l = \frac{l_0}{\gamma} \quad \therefore \quad \gamma = \frac{l_0}{l} = \frac{32.0}{25.0} = 1.28$$

   $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \therefore \quad \gamma^2 = \frac{1}{1-\frac{v^2}{c^2}} \qquad 1-\frac{v^2}{c^2} = \frac{1}{\gamma^2} \quad \therefore \quad \frac{v^2}{c^2} = 1-\frac{1}{\gamma^2} = 1-\frac{1}{1.28^2} = 0.390$$

   Since $\frac{v^2}{c^2} = 0.390$ it follows that $v = 0.624c$ or 1.87×10^8 *ms*$^{-1}$

2. Proxima Centuri is one of the stars in the Alpha Centauri triple-star system. This is the star closest to Earth at a distance of 4.24 light years away as measured from Earth. One light year is the distance travelled by light in one year.

   Calculate the distance to Proxima Centuri as measured by an astronaut in a rocket travelling towards Proxima Centuri at a speed of 0.780c.

   $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.780c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.780^2}} = 1.60$$

   $$l = \frac{l_0}{\gamma} = \frac{4.24}{1.60} = 2.65 \textit{ light years}$$

3. Two astronauts Alex and Sam are flying identical spacecraft which have a length of 150 m. The spacecrafts are travelling with a velocity of 0.200c relative to one another.

   (a) Determine the length of Alex's spacecraft relative to Sam.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.200c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.200^2}} = 1.02$$

$$l = \frac{l_0}{\gamma} = \frac{150}{1.02} = 147\ m$$

   (b) State with reason, the length of Sam's spacecraft relative to Alex.

   The situation is symmetrical. Alex with observe Sam's spacecraft to be 147 m long.

Helpful online resources

The following YouTube clip discusses the difficulties surrounding providing experimental evidence of length contraction. There is an indirect way to measure the relativistic effects of length contraction in relation to magnetism:

https://www.youtube.com/watch?v=1TKSfAkWWN0

Muon decay experiment

Muons are subatomic as particles called leptons. Such particles will be discussed in Subtopic 3.4. Muons can be produced in a laboratory and have a short average lifetime of 2.2×10^{-6} s.

Muons are also created when cosmic rays in the Earth's upper atmosphere collide with nitrogen and oxygen nuclei. This occurs at a distance of around 10 km above the Earth's surface. Muons have a large velocity close to the speed of light (0.99c). As they travel through the atmosphere they will quickly decay. In the time that they decay, and without taking relativistic effects into account, muons are not expected to reach the Earth's surface. A simple calculation ($s = vt = 0.99 \times 3.00 \times 10^8 \times 2.2 \times 10^{-6} = 650$m) shows that the muons would travel approximately 650 m before decaying. This is far smaller that the 10 km they would need to travel to reach the Earth's surface.

Another way of looking at the situation is to calculate the time it would take a muon to reach the Earth surface, 10 km away at a speed of 0.99c. A simple calculation shows that it would take $t = \frac{s}{v} = \frac{10\times10^3}{0.99 \times 3.00 \times 10^8} = 3.4\times10^{-5} s$. This is equivalent to about 15 half lives ($\frac{3.4\times10^{-5}}{2.2\times10^{-6}} = 15$).

This means that the number of muons expected to reach the Earth's surface, if any at all, would be very low. The question is, 'why then, are a significant number of muons detected reaching the Earth's surface as shown in Figure 1.4.18.

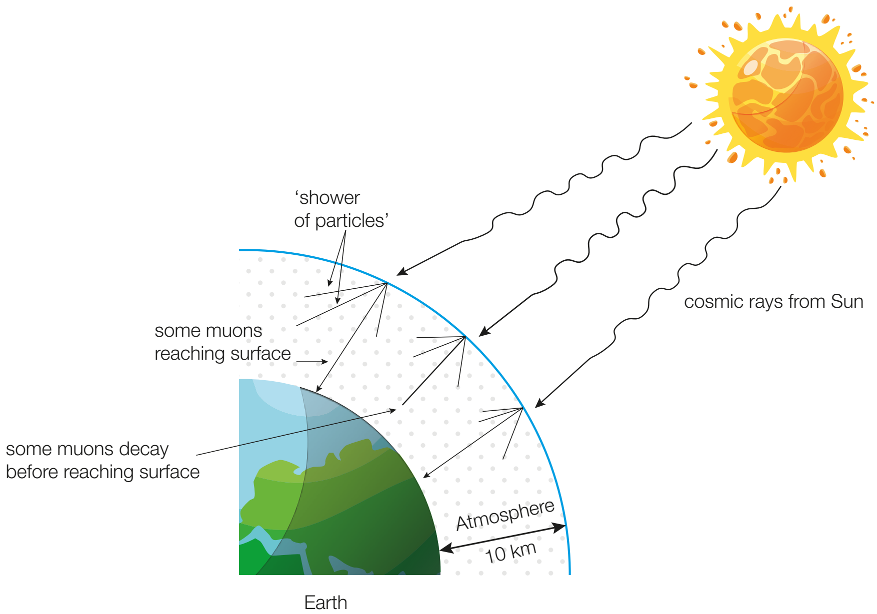


Figure 1.4.18

If relativistic effects are taken into account then a lifetime of 2.2×10^{-6}s in the muon's reference frame will be time dilated to a stationary observer on Earth.

The Lorentz factor for a speed of 0.99c is 7.1 ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.99c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.99^2}} = 7.1$)

To an observer on Earth, the average lifetime of a muon is 7.1 times longer than 2.2×10^{-6} s (or 15.6×10^{-6} s). It can be expected that muons will reach the Earth's surface as the time taken to reach the Earth's surface will be just over 2 half lives ($\frac{3.4\times10^{-5}}{15.6\times10^{-6}} = 2.2$).

In the muon's frame of reference they exist for an average time of 2.2×10^{-6} s. They make it down to the Earth's surface because the atmosphere (and Earth) is moving with respect to the muon. This means that the atmosphere is length contracted.

The 10 km distance as measured by an observer on the Earth will only be $\frac{10}{7.1} = 1.4$ km.

There will be a significant number of the muons that last long enough for the Earth to travel this distance towards them.

The muon decay experiment gives experimental evidence of both time dilation and length contraction.

Relativistic mass

Mass, just like time and length is affected by relative motion. A frame that is moving with a speed v with respect to the object will record a larger mass called the **relativistic mass.** This is also the mass of the object moving with a velocity v relative to a stationary observer.

Rest mass m_0, is defined as the mass of an object when it is at rest relative to the observer.

As the speed of the object increases, so too does its mass. We already know that energy is required, that is, work must be done to increase an object's speed. Einstein believed that mass and energy are interchangeable. The gain in energy at very high speeds is due to a gain in mass.

Mass in a moving reference frame is increased when observed from a stationary reference frame according to: $m = \gamma m_0$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor and m_0 is the mass in the frame of reference where the object is stationary.

Objects moving at relativistic speeds are unable to reach the speed of light

As an object's speed approaches the speed of light, c, it's mass will increase.

At the speed of light, the Lorentz factor is infinitely large ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{0} = \infty$).

The mass of the object, $m = \gamma m_0$ will also become infinitely large. According to Newton's Second Law, $F = ma$, the force required to accelerate the object would be infinite. This would require an infinite amount of energy which is not possible. This means that an object can never travel at a speed equal to, or greater than, the speed of light.

Momentum

Momentum is defined as the product of an object's mass and velocity. The magnitude of the relativistic momentum of a moving object is given by

$$p = mv = \gamma m_0 v$$

Worked examples

1. (a) Calculate the relativistic mass of an electron moving at 95% the speed of light.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.95^2}} = 3.2$$

$$m = \gamma m_0 = 3.2 \times 9.11 \times 10^{-31} = 2.9 \times 10^{-30}\ kg$$

(b) Calculate the relativistic momentum of the electron moving at 95% the speed of light.

$$p = mv = 2.9 \times 10^{-30} \times 0.95 \times 3.00 \times 10^8 = 8.3 \times 10^{-22}\ sN$$

2. An object with a mass of 3.5 kg is accelerated to a velocity of 2.0×10^8 ms^{-1}.

Determine the relativistic momentum of the object

$$\frac{2.0 \times 10^8}{3.00 \times 10^8} = 0.67 \quad \therefore \quad 2.0 \times 10^8\ ms^{-1} = 0.67c$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.67c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.67^2}} = 1.347$$

$$p = mv = \gamma m_0 v = 1.347 \times 3.5 \times 2.0 \times 10^8 = 9.4 \times 10^8\ sN$$

Extra understanding

Relativity and an upper limit for velocity

Consider the Lorentz factor. As discussed earlier in the chapter, when the relative velocity between two observers is equal to the speed of light c, then the Lorentz factor is infinite ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{0} = \infty$). It follows that $t = \gamma t_0$ becomes infinite.

This would mean that a clock in a moving frame of reference would measure an infinite time. In other words, time would stand still. Similarly, $l = \frac{l_0}{\gamma} = 0$. This would mean that the effect of length contraction would be that an object has zero length in the direction of travel. Additionally, $m = \gamma m_0$ indicates that mass would be infinite. The Theory of Special Relativity therefore sets an upper limit for the speed of an object to be the speed of light. It also indicates that this speed cannot be achieved.

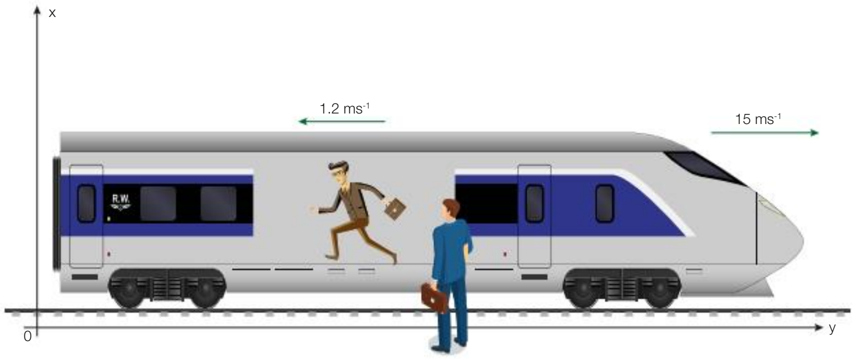
Science as a human endeavour

Some possible ideas for investigation are:

1. Explore how new evidence led scientists to modify models to account for high-speed particles which exhibited properties that were inconsistent with Newtonian physics.
2. In what way has the evidence from different sources, such as X-rays from binary star systems and other experiments on moving gamma radiation sources, supported Einstein's second postulate of relativity?
3. What happens if you try to apply the equation for the relativistic momentum of a moving object to a photon? Explain in terms of the postulates of special relativity.

Exercises

1. The diagram below shows a passenger on a train walking at a velocity of 1.2 ms^{-1} to the left. The train is moving with a velocity of 15 ms^{-1} to the right.



State the velocity of the passenger in the

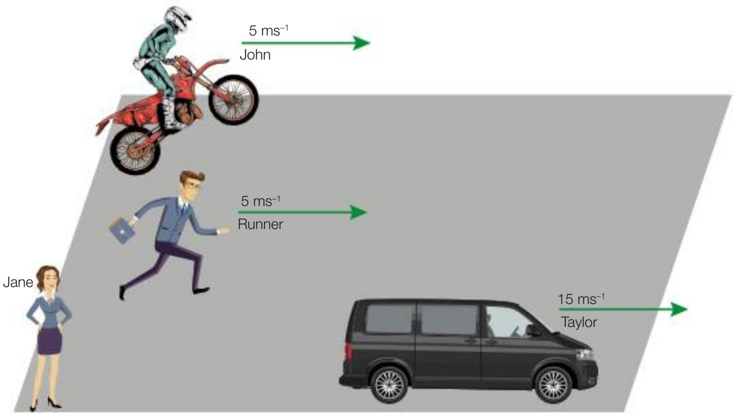
(a) frame of reference of the train.

....

(b) ground-observer's frame of reference.

....

2. The diagram below shows a runner and three other observers.



State the velocity of the runner as observed in each of the three other reference frames.

(a) John's reference frame.

....

(b) Jane's reference frame.

....

(c) Taylor's reference frame.

....

3. Define the term inertial reference frame

....

....

....

4. State the two postulates of the Theory of Special Relativity.

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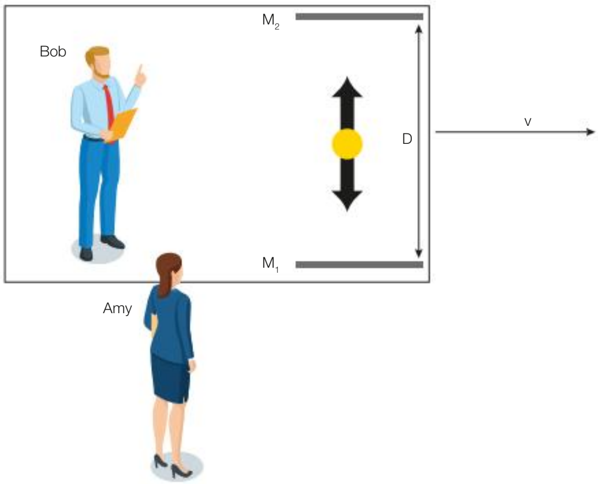
5. The diagram below shows Bob reflecting a pulse of light between two mirrors M_1 and M_2. Bob is in a reference frame that is moving to the right relative to Amy.



Consider a pulse of light leaving M_1 and reaching M_2.

(a) On the diagram, draw the path of the pulse of light as seen by Amy.

(b) Explain why Amy sees the pulse of light reach M_2 after Bob.

....

....

....

....

(c) Bob is travelling with a speed of 0.500c and records the time taken for the pulse of light to reach to M_2 as t.

In terms of t, determine the time taken for the pulse of light to reach to M_2 as observed by Amy.

....

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....

6. Show that at a speed of 0.85c, a time interval of half a minute measured by a stationary observer is measured as approximately 57 seconds by a moving observer.

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7. A space probe travelling with a velocity of 0.600c emits a light signal that lasts 10.0 s towards a nearby space station. An observer on the space station observes the signal for a time interval greater than 10.0 s.

(a) Explain why the observer on the space station sees the light signal for a time interval greater than 10.0 s.

....

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....

(b) Calculate the time interval for which the observer on the space station observes the light signal.

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8. The diagram below shows a red beacon located on the top of an aircraft.

A pilot flying the plane at 2.7×10^8 ms^{-1} turns the red flashing light on. The light flashes every 3.0 s.

(a) Calculate the Lorentz factor for the aircraft flying at 2.7×10^8 ms^{-1}.

....

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....

(b) Calculate the time interval between the flashes as measured by a ground-based observer.

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9. The Twin Paradox is an imaginary situation involving two twins. One twin stays on Earth while the another travels a large distance at a speed close to the speed of light before returning to Earth. At the end of the journey, the twin that stayed on Earth thinks that the twin that took the journey is younger.

Explain why the twin that stayed on Earth thinks a longer period of time has passed compared to the twin that took the journey.

....

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10. Two astronauts Peter and Jane are flying identical spacecraft which have a length of 20.0 m. The spacecraft are traveling with a velocity of 0.400c relative to one another.

(a) Determine the length of Peter's spacecraft relative to Jane.

....

....

....

....

(b) State with reason, the length of Jane's spacecraft relative to Peter.

....

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....

11. A spacecraft leaving the Earth's surface at a constant speed of 0.600c flashes a beam of light back towards the Earth. The spacecraft carries several metal rods of length L.

(a) Calculate the Lorentz factor for the spacecraft travelling at 0.600c.

(b) Explain, with reason, the speed of the light beam recorded by an observer on Earth.

(c) State, in terms of L, the length of the metal rods as measured by an

(i) astronaut on the spacecraft.

(ii) observer on Earth.

12. The diagram below shows an athlete preparing to throw a javelin. The athlete is running while holding the javelin horizontally above his shoulder. The javelin has a length of 2.6 m.

Imagine a situation in which the athlete could run at a speed of 1.6×10^8 ms^{-1}.

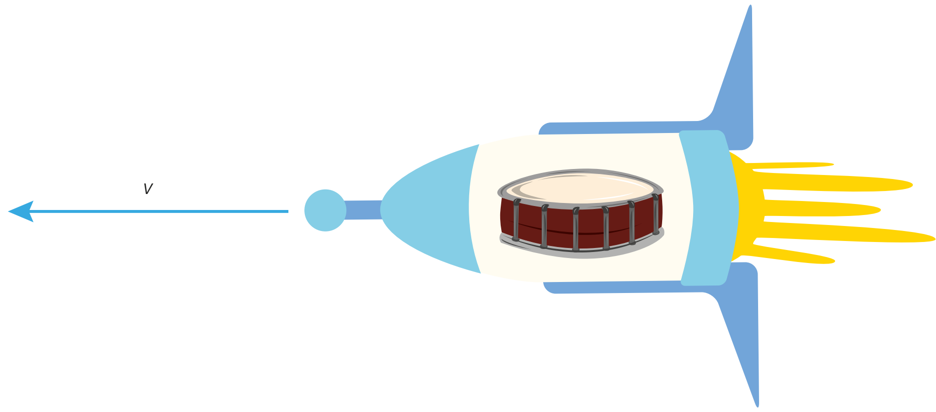
(a) Express the speed of the runner in terms of the speed of light *c*.

(b) Calculate the length of the javelin as measured by an observer in the grandstand watching the athlete.

(c) The athlete runs faster and the observer in the grandstand observes the javelin to have a length of 2.0 m.

Calculate the velocity of the athlete relative to the observer.

13. The diagram below shows a rocket travelling to the left at a speed close to the speed of light. It is carrying a cylindrical drum of supplies for a space station.



Redraw the cylindrical drum as seen by an observer on Earth. Justify your answer.

14. The Lorentz factor for a speed of 0.95c is 3.2.

An alpha particle is travelling at a constant speed of 0.95c in a vacuum. A laboratory observer measures the distance that the alpha particle penetrates the air to be 18 cm.

Calculate the distance that the observer moves as measured in the alpha particle's frame of reference. Explain your answer.

15. Calculate, to three significant figures, the relative velocity between two inertial observers that reduces the proper length by 50%.

16. Muons are created in the upper atmosphere at a height of 10.0 km above the surface of the Earth. The muons have an average lifespan of 2.20×10^{-6} s before decaying and travel at a speed of 0.980c.

(a) Show that the muons travel an average distance of 647 m before decaying and are therefore unlikely to reach the Earth's surface.

(b) Calculate the time it would take the muons to reach the Earth's surface at a speed of 0.980c.

(c) With the aid of a calculation, explain why the muons as observed from Earth have an average lifespan of 1.11×10^{-5} s.

(d) Explain how the Theory of Special Relativity helps explain why a significant number of muons are detected at the surface of the Earth.

17. An observer in a reference frame A measures the mass of a ball, its width and the time taken for the ball to fall vertically, bounce and return to its original position. The observer is at rest relative to the ball.

A second observer in a reference frame B is moving with a velocity of 2.50×10^{8} ms^{-1} in a horizontal direction relative to the observer in reference frame A. The observer in reference frame B measures the mass of the same ball, its width and the time taken for the ball to fall vertically, bounce and return to its original position.

(a) Calculate the factor by which the measurements made by the observer in reference frame B differ from the measurements made by the observer in reference frame A.

(b) Complete the table below by stating whether the observer in reference frame B measures the mass, width and the time taken for a ball to fall vertically and bounce and return to its original position as greater, smaller or the same as the observer in reference frame A.

Quantity being measured	Effect of relative motion on measurement
mass	
width	
time taken to bounce	

18. The Lorentz factor for an object travelling at 0.500c is 1.15.

A proton of mass 1.67×10^{-27} kg is accelerated to a speed of 0.500c.

(a) Calculate the mass of the proton travelling at 0.500c.

(b) Calculate the momentum of the proton at a speed of 0.500c.

19. An object with a mass of 200 g is accelerated to a velocity of 2.8×10^8 ms^{-1}.

Determine the momentum of the object when the effects of relativity are taken into account.

20. Define each of the following terms in the context of relativity.

(a) Time dilation

(b) Length contraction

(c) The Lorentz factor

(d) Rest mass

21. (a) Explain why an increase in mass is only significant at relativistic speeds.

(b) Explain why a speed equal to the speed of light can never be achieved.

22. Science as a human endeavour activity – Relativity

Date: May 29, 2019

Source: Breakthrough Needed? Starshot Interstellar-Flight Project Faces Challenges

https://www.space.com/interstellar-flight-breakthrough-starshot-challenges.html

In 2016 the Starshot Probes project was announced. A large array of lasers on the Earth will form one single powerful beam of around 100 gigawatts. The beam is directed towards a highly reflective solar sail, which is already in orbit. The solar sail is attached to a small Starshot probe. The probe would be about the size of a postage stamp, with a mass of around one gram. The powerful laser beam will propel the probes to relativistic speeds of 0.2c.

The fast moving Starshot probes could reach Pluto in as little as 3.5 days, and reach nearby exoplanets such a Proxima b in just over 20 years. Proxima b is a potentially life-supporting world which is about 4.2 light-years away. It would take a traditional spacecraft several tens of thousands of years to reach.

The project is not without its challenges. Scientists know the signal returning to Earth will be weak and being so small, it will be difficult to place the necessary electronics on board. Additionally, the probe will gain mass at relativistic speeds and will require a more powerful laser beam.

Describe two ways in which the Starshot Probes project demonstrate science as a human endeavour.

Topic 2: Electricity and magnetism

2.1 Electric fields

Science understanding

1. Electrostatically charged objects exert forces upon one another; the magnitude of these forces can be calculated using Coulomb's Law.
   - Solve problems involving the use of: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
   - Using proportionality, discuss changes in the magnitude of the force on each of the charges as a result of a change in one or both of the charges and/or a change in the distance between them.
   - Describe how the electric forces are consistent with Newton's Third Law.
2. When more than two point charges are present, the force on any one of them is equal to the vector sum of the forces due to each of the other point charges.
   - Use vector addition in one dimension or two dimensions (with right–angled, or equilateral triangles) to calculate the magnitude and direction of the force on a point charge due to two other point charges.
3. Point charges and charged objects produce electric fields in the space that surrounds them. A charged object in an electric field experiences an electric force.

   The direction and number of electric field lines per unit area represent the direction and magnitude of the electric field.
   - Sketch the electric field lines:

     (i) for an isolated positive or negative point charge and for two point charges

     (ii) between and near the edges of two finite oppositely charged parallel plates.
4. A positively charged body placed in an electric field will experience a force in the direction of the field; the strength of the electric field is defined as the force per unit charge
   - Solve problems involving the use of: $\vec{E} = \frac{\vec{F}}{q}$.
   - Using Coulomb's Law, derive the formula: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$.
   - Solve problems using: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, for one isolated point charge.
   - Use vector addition in one one dimension or two dimensions (with right-angled, or equilateral triangles) to determine the magnitude and direction of an electric field due to two point charges.
5. There is no electric field inside a hollow conductor of any shape, provided that there is no charge in the cavity.
   - Sketch the electric field produced by a hollow spherical charged conductor.
6. Electric fields are strongest near sharp points on conductors. These fields may be large enough to ionise molecules in the air near of the sharp points, resulting in charge movement away from the conductor. This is called a 'corona discharge'.
   - Sketch the electric field produced by a charged conductor of an irregular shape.
   - Explain how the electric field near sharp points may ionise the air.

Coulomb's Law was introduced in subtopic 2.1 **Potential difference and electric current of the Stage 1 Workbook.**

Electric force

In the Stage 1 Workbook we saw that electrostatically charged objects exert a force upon one another.

Like charges repel and unlike charges attract.

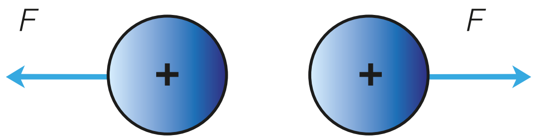


Figure 2.1.1

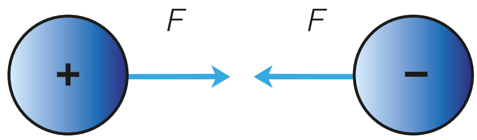


Figure 2.1.2

Electric forces are non–contact forces. A positively charged object will exert a repulsive force upon a second positively charged object that will push the two objects apart as shown in Figure 2.1.1. The same concept applies for two negatively charged objects.

A positively charged object will exert an attractive force upon a negatively charged object that will draw the two objects together as shown in Figure 2.1.2.

Coulomb's Law

The electric force of attraction or repulsion between two charged objects is directly proportional to the product of the two charges and inversely proportional to the square of the distance between their centres.

The electric force acts along the line joining the centres of the charges as though the charge were concentrated at a point. In physics we often refer to such charges as point charges.

The magnitude of the electric force between two charges in a vacuum or air can be calculated using Coulomb's Law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 represent the magnitude of the charges in Coulombs

r is the distance between the centre of the charges in metres

$\frac{1}{4\pi\varepsilon_0}$ is Coulomb's Law constant = $8.99 \times 10^9\ Nm^2C^{-2}$

F is the force in Newtons

Worked Example

Two charged spheres are placed 40.0 cm apart in a vacuum. Charge q_1 = +3.00 mC and q_2 = –5.00 mC. Calculate the magnitude and direction of the force acting between the two spheres.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times 3.00 \times 10^{-3} \times 5.00 \times 10^{-3}}{0.400^2} = 8.43\times10^5\ N\ attraction$$

Protons and electrons

It will be assumed that you know that the magnitude of the charge of an electron and a proton are the same and that a proton has a positive charge while an electron has a negative charge. The symbol for the charge of an electron is e and the magnitude of the charge is provided on the formula sheet. For an electron $e = -1.60 \times 10^{-19}$ C.

You may also require the mass of the electron and proton in order to solve problems. Both of these are found on the formula sheet.

Applying the concept of proportionality

Proportionality can be used to determine the change in the magnitude of the electric force between two charges as a result of a change in one or both of the charges and/or a change in the distance between them. The concept of proportionality was introduced and applied to gravitational forces in subtopic 1.2.

(i) The electric force is proportional to one charge, providing the other charge and the distance between the charges does not change.

$$F \propto q_1 \quad q_2, r \text{ constant}$$

This concept is illustrated by example. If one charge is replaced with one having twice the charge, the electric force between the two charges will double. Similarly if one charge is replaced with one having a charge that is ten times smaller, then the electric force between the two charges will decrease by a factor of ten.

(ii) The electric force is proportional to the product of the two charges, providing the distance between the centre of the two charges does not change.

$$F \propto q_1 q_2 \quad r \text{ constant}$$

An example to illustrate this concept is that if one charge is replaced with one having double its charge and the other charge is replaced with a charge having five times its charge, then the electric force between the two charges becomes ten times larger.

(iii) The electric force is inversely proportional to the square of the distance between the centre of the two charges, providing the magnitude of the charges does not change.

$$F \propto \frac{1}{r^2} \quad q_1, q_2 \text{ constant}$$

An example to illustrate this concept is that if the distance between the centre of the two charges is increased by a factor of five, then the electric force becomes 25 times smaller.

Note: An attractive force becomes a repulsive force and vice versa if the sign of one of the charges is reversed.

Worked Example

A force of magnitude 35.0 N acts between two charges. Determine the magnitude of the force acting between the charges if

(a) one charge is replaced with one with triple its charge.

$F \propto q_1 \quad q_2, r$ constant

If one charge is replaced with one that has three times its charge, the magnitude of the force triples to 105 N.

(b) both charges are replaced with charges that have triple their charge.

$F \propto q_1 q_2 \quad r$ constant

If both charges are replaced with charges that have triple their charge, the magnitude of the force increases by a factor of 9 to 315 N.

(c) one charge is replaced with one with triple its charge and the distance is doubled.

If one charge is replaced with one that has three times its charge, the magnitude of the force triples. In addition, if the distance between the charges doubles, the magnitude of the force between the charges is reduced by a factor of 4. The combined effect is that the magnitude of the force changes by a factor of $\frac{3}{4}$ to become $\frac{3}{4} \times 35.0 = 26.3N$.

2

Electric forces are consistent with Newton's Third Law

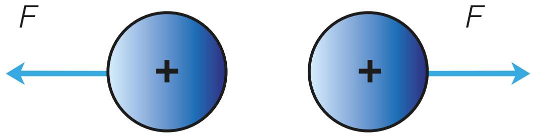


Figure 2.1.3

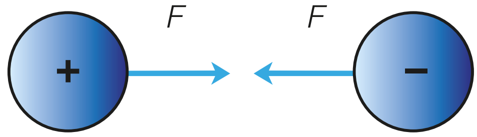


Figure 2.1.4

Consider the force acting between two charges as shown in Figures 2.1.3 and 2.1.4. The magnitude of the force is given by $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ where q_1 and q_2 represent the magnitude of the charges. This is a mutual force. This means that the magnitude of the force on each charge is the same. The diagrams clearly show that the direction of the force on each charge acts in the opposite direction. Newton's Third Law states that if object A exerts a force on another object B, then object B exerts an equal and opposite force on object A. Electric forces are therefore consistent with Newton's Third Law.

Science as a human endeavour

Explore an example of the development of complex models using evidence from many sources using the video: *Coulomb's Law:* https://youtu.be/B5LVoU_a08c

Electric forces and the principle of superposition

Coulomb's Law can be used to calculate the force between two charges but when there are more than two charges the force acting on any one of them can be calculated by adding the force acting on it due to each of the other charges present. Since force is a vector quantity, this will involve a vector addition of the forces acting.

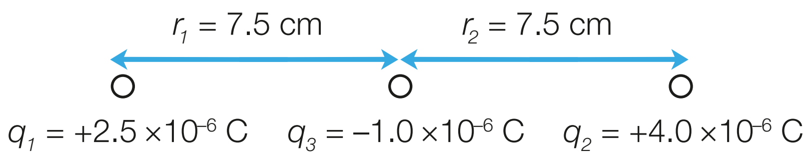
This leads to the principle of superposition for electric forces which states:

> When more than two point charges are present, the force on any one of them is a vector sum of the electric forces acting due to each of the other point charges present.

This course only requires that you consider the case when there are three point charges present. You will need to be able to solve one dimensional and two dimensional problems which involve right–angled or equilateral triangles. Two examples are outlined below. It should be noted that while the symbols q_1 and q_2 are the symbols most often used in questions, there will be times when other symbols are used to represent charges e.g. q_A, *Q or* Q_1. You should be able to apply Coulomb's Law in these situations by adjusting the symbols accordingly.

Worked Examples

1. Two point charges $q_1 = +2.5 \times 10^{-6}$ C and $q_2 = +4.0 \times 10^{-6}$ C are placed 15 cm apart in a vacuum. A third charge $q_3 = -1.0 \times 10^{-6}$ C is placed midway between them.



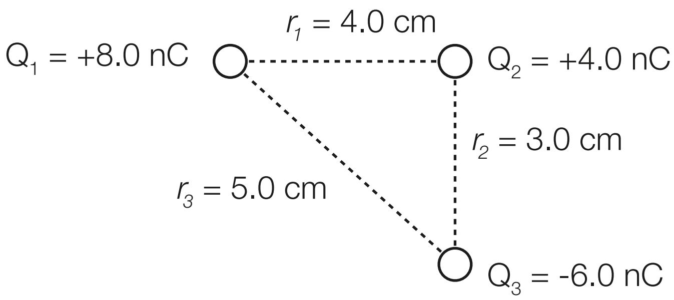
Calculate the magnitude and direction of the total force acting on q_3.

Force due to q_1: $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_1^2} = \frac{8.99 \times 10^9 \times 2.5 \times 10^{-6} \times 1.0 \times 10^{-6}}{0.075^2} = 4.0\ N \leftarrow$

Force due to q_2: $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_2^2} = \frac{8.99 \times 10^9 \times 4.0 \times 10^{-6} \times 1.0 \times 10^{-6}}{0.075^2} = 6.4\ N \rightarrow$

Total force: $\vec{F} = \vec{F}_1 + \vec{F}_2 = 4.0 \leftarrow + 6.4 \rightarrow = 2.4\ N \rightarrow$

2. Consider three point charges positioned in a vacuum as shown in the diagram below.



(a) Calculate the magnitude and direction of the force acting on Q_2 due to Q_1.

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_1^2} = \frac{8.99\times10^9\times8.0\times10^{-9}\times4.0\times10^{-9}}{0.040^2} = 1.8\times10^{-4}\ N \ \rightarrow$$

(b) Calculate the magnitude and direction of the force acting on Q_2 due to Q_3.

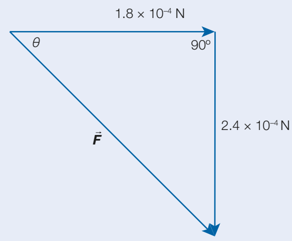
$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_2^2} = \frac{8.99\times10^9\times4.0\times10^{-9}\times6.0\times10^{-9}}{0.040^2} = 2.4\times10^{-4}\ N \ \downarrow$$

(c) Calculate the total or resultant force $\vec{F}$ on Q_2.

$$F = \sqrt{(1.8\times10^{-4})^2+(2.4\times10^{-4})^2} = 3.0\times10^{-4}\ N$$

$$\theta = \tan^{-1}\left(\frac{2.4\times10^{-4}}{1.8\times10^{-4}}\right) = 53^\circ$$

$\vec{F}$ = 3.0 × 10^{-4} N 53° clockwise from the line joining Q_1 and Q_2



Electric fields

In accordance with Coulomb's Law, two charges exert a force on each other even though the charges are not in contact. This is because there is a field around each charge and we call this field an electric field. It is the electric field around each charge that produces a force on the other charge. Instead of saying that the two charges exert a force on each other we can say that the field of one charge exerts a force on the other charge.

In general, all electric charges establish an electric field ($\vec{E}$) in the space that surrounds them. A charged object in an electric field experiences an electric force.

Representing electric fields

Helpful online resource

The following computer interactive: 'Electric Fields and Charges' allows you to explore electric fields surrounding charges.

https://phet.colorado.edu/en/simulations/charges-and-fields

Since a charge in an electric field experiences a force, we can represent the electric field with lines of force called electric field lines. By definition, the direction of an electric field is taken as the direction a positive test charge would move. A test charge is simply a very small charge with a weak electric field in the space surrounding it. It will not interfere with the electric field created by a charge.

Consider the electric field surrounding an isolated positive charge as shown in Figure 2.1.5. At any point, a positive test charge will experience a force away from the positive charge. Using Coulomb's Law, the force on the positive test charge will decreases with distance ($F \alpha \frac{1}{r^2}$). It follows that the electric field is weaker further from the isolated charge.

The electric field surrounding an isolated positive charge is directed radially outwards. This is because the direction of the electric field is determined by the direction of the force on a positive test charge. The electric field is also non–uniform. A positive test charge will experience a greater force closer to the positive charge creating the electric field. There are more electric field lines per unit area closer to the charge creating the field. This indicates that the electric field is stronger.

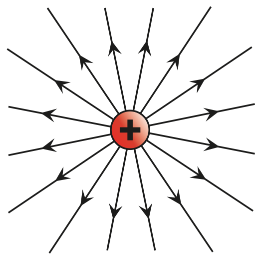


Figure 2.1.5

The arrows in electric field diagrams represent the direction a positive test charge would move.

The number of electric field lines per unit area represents the magnitude of the electric field. The greater the number of electric field lines, the stronger the field.

Electric field lines leave and enter the surface of a conductor at right angles. Although it is no longer part of this course to explain why, your teacher may explain why this is the case.

The electric field surrounding an isolated negative point charge is shown in Figure 2.1.6. The electric field surrounding two point charges of unlike and like sign are shown in Figures 2.1.7 and 2.1.8 respectively and the electric field in the region between two finite charged parallel conducting plates is illustrated in Figure 2.1.9.

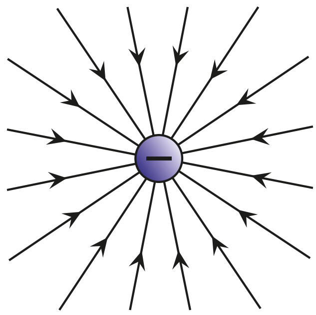


Figure 2.1.6: The electric field for an isolated negative charge

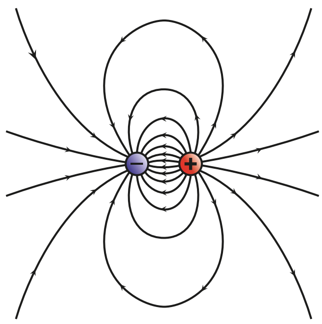


Figure 2.1.7: Two point charges with unlike sign

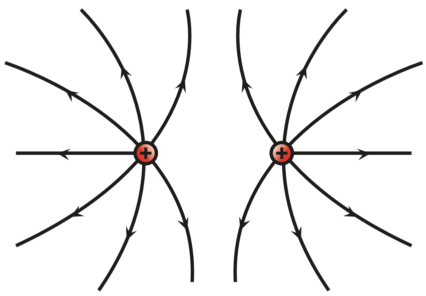


Figure 2.1.8: Two point charges with like sign

Figure 2.1.9 shows the electric field between two finite parallel conducting plates with equal and opposite charges per unit area is uniform between the plates. This is represented by evenly spaced electric field lines and means that a positive test charge will experience the same force no matter where it is placed within the electric field.

Near and beyond the edges of the plates the electric field is non–uniform. This is commonly referred to as the 'end effect'. The end effect is a result of the field lines leaving and arrive on the surface of the plates at right angles.

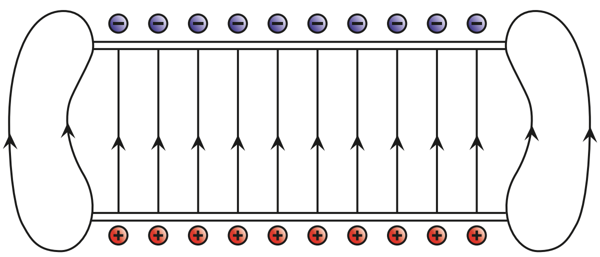


Figure 2.1.9

Science as a human endeavour activity – Electric fields

Figure 2.1.10

Shark Shields

A typical Shark Shield is shown in Figure 2.1.10. It consists of two electrodes in a long cord that trails behind the user. The device can be worn around the ankle and some models can be fitted to a surf board. When a small current passes through the electrodes, an electric field is emitted and surrounds the user.

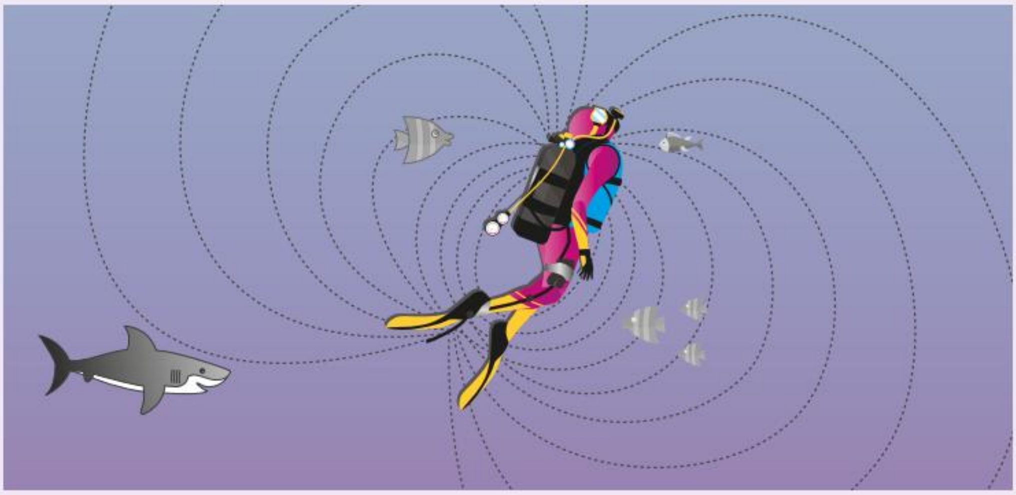


Figure 2.1.11

Figure 2.1.11 shows the Shark Shield in action. Sharks have an electrical receptor in their snouts called the 'Ampullae of Lorenzini' which contains tiny gel filled sacs. This organ can sense electric fields created by nearby prey (usually about one metre away). The electric fields are created by any muscle activations such as the heartbeat of a fish. When the shark approaches the electric field created by the Shark Shield that surrounds the user, the electric field is much stronger than that created by prey and causes the gel filled sacs to spasm. This is believed to create an uncomfortable sensation for the shark and it quickly turns away.

The idea that sharks may be affected by electric fields was discovered in 1995. The Shark Shield took over 20 years to develop. Its effectiveness is still being research, including here in South Australia. Research has discovered that the effect is not uniform across all sharks including the Great White Shark.

The device is important in not only protecting the user but in removing the need for culling sharks. There is no known long term effect on sharks or other sea animals.

Describe how application and limitation, one of the key concepts of science as a human endeavour applies to Shark Shields.

Suggested solution

Scientific knowledge, understanding, and inquiry has enabled scientists to develop a solution to shark attacks which often result in human injury or death. The electric field that is created by a current passing through an electrode surrounds the user and irritates the shark causing it to turn away from the user. This application not only has the benefit of saving lives, but the shark is not harmed in any way. The main limitation is that the effect of the Shark Shield is not uniform across all sharks which means that there is a risk that the user may still be attacked. Further monitoring and assessment will help scientists better understand how this technology can be improved.

? Science inquiry activity

The following is a possible idea for a science investigation.

Use electric field sensors to map electric fields and explore the relationship between electric field strength and distance from charged conductors.

Electric field strength

We have already discussed that a charged object experiences a force in an electric field.

The strength of an electric field ($\vec{E}$) at a point is defined as the electric force $\vec{F}$ per unit charge (q) experienced by a small positive test charge when placed at that point in the field.

As an equation, we write

$$\vec{E} = \frac{\vec{F}}{q}$$

The SI unit of electric field is Newton per Coulomb (NC^{-1}).

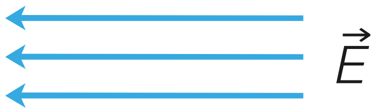
The direction of the electric field is the same as the direction of the force experienced by a positive test charge. This means that the direction of the electric force on a charge is parallel to the field if the charge is positive and antiparallel if the charge is negative.

Worked Examples

1. An electron experiences a force of magnitude 2.8×10^{-13} N when it is placed at a point in an electric field. Calculate the magnitude of the electric field at the point where the electron is placed.

$$E = \frac{F}{q} = \frac{2.8 \times 10^{-13}}{1.60 \times 10^{-19}} = 1.8 \times 10^{6}\, NC^{-1}$$

2. The uniform electric field $\vec{E}$, in a given region has a magnitude of 2.00×10^{3} NC^{-1} and acts to the left of the page as shown below.



(a) Calculate the magnitude and direction of the force acting on a +2.0 μC charge.

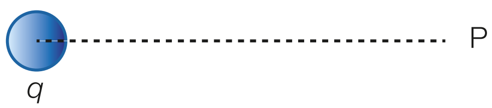
$$\vec{F} = Eq = 2.00 \times 10^{3} \times 2.0 \times 10^{-6} = 4.0 \times 10^{-3}\, N \ \leftarrow$$

(b) The +2.0 μC charge is replaced with another charge of magnitude –2.0 μC. Describe how the magnitude and direction of the force acting on the –2.0 μC charge would differ, if at all.

The magnitude of the force will not differ as the magnitude of the charge has not changed. Since the charge is negative, it will move anti–parallel to the electric field i.e. to the right.

The magnitude of an electric field at a distance (r) from an isolated point charge q

Consider a point P, at a distance r from an isolated point charge q creating an electric field.



The magnitude of the electric field is a measure of the force per unit charge on a small positive test charge. If a small positive test charge (q_+) were placed at P, then the force acting between the positive test charge and the charge q would be given by: $F = \frac{1}{4\pi\varepsilon_0}\frac{qq_+}{r^2}$

The magnitude of the electric field would be $E = \frac{F}{q_+} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{qq_+}{r^2}}{q_+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

The magnitude of the electric field ($\vec{E}$) at a distance r from an isolated point charge q is given by:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Note:

The direction of the electric field is given by the direction of the force on a positive test charge.

1 If $\boldsymbol{q}$ is positive then the direction of the electric field is radially away from q.

2 If $\boldsymbol{q}$ is negative then the direction of the electric field is radially towards q.

2

Worked Example

(a) Calculate the magnitude and direction of the electric field 1.50 cm from a +30.0 nC charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{8.99 \times 10^9 \times 30.0 \times 10^{-9}}{0.0150^2} = 1.20\times10^6 \ NC^{-1} \ \textit{radially outwards}$$

(b) Calculate the force, $\vec{F}$ that an electron would experience 1.50 cm from the +30.0 nC charge.

$$F = Eq = 1.20 \times 10^6 \times 1.60 \times 10^{-19} = 1.92 \times 10^{-13}\,N \ \textit{radially inwards}$$

(c) Use proportionality to determine the magnitude of the electric field at a point 15.0 cm from the +30.0 nC charge.

Since $E \alpha \frac{1}{r^2}$ it follows that if the distance from the charge creating the electric field is increased by a factor of ten, then the electric field becomes 100 times smaller i.e. 1.20×10^4 NC^{-1}

Electric fields and the principle of superposition

The magnitude of the electric field $\vec{E}$ at a distance from an isolated point charge can be calculated using the formula $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$. If there is more than one charge present, the electric field at any point can be determined by calculating the electric field at that point due to each of the charges present and performing a vector sum. This is because electric field is a vector quantity. This principle is similar to the principle of superposition applied to electric forces.

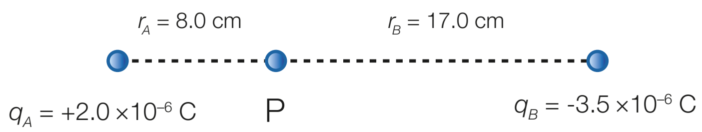
This leads to the idea of the principle of superposition for electric fields which states:

The electric field acting at a point when more than one point charge is present is a vector sum of the individual electric fields at that point due to each of the point charges present.

This course only requires that you consider the case when there are two point charges present. Two examples are outlined below. It should be noted that while the symbols q_1 and q_2 are the symbols most often used in questions, there will be times when other symbols are used to represent charges e.g. q_A, Q or Q_1. You should be able to apply the electric field formula in these situations by adjusting the symbols accordingly.

Worked Examples

1. Two point charges q_A = +2.0 × 10^{-6} C and q_B = –3.5 × 10^{-6} C are separated by a distance of 25 cm in a vacuum. Calculate the electric field at a point P which is located between the charges at a distance of 8.0 cm from A and 17.0 cm from B as shown below.



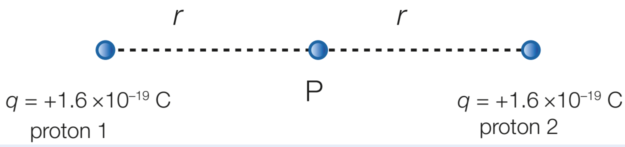
E field due to q_A $\vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_A^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6}}{0.080^2} = 2.8\times10^6 \ NC^{-1} \rightarrow$

E field due to q_B $\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{r_B^2} = \frac{8.99 \times 10^9 \times 3.5 \times 10^{-6}}{0.170^2} = 1.1\times10^6 \ NC^{-1} \rightarrow$

$\vec{E}_p = 2.8\times10^6 \rightarrow + 1.1\times10^6 \rightarrow = 3.9\times10^6 \ NC^{-1} \rightarrow$

***Hint**: The direction is determined by the direction of the force on a positive test charge*

2. Two protons are separated by a small distance in a vacuum. Explain why the magnitude of the electric field at a point midway between the two protons is zero.



The electric field at the midpoint P due to proton 1 has a magnitude given by $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ and acts to the right. The electric field at the midpoint P due to proton 2 has a magnitude given by $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ and acts to the left. Since the magnitude of the field at P due to each proton is the same but the electric field acts in the opposite direction, then the two electric fields cancel to zero.

The electric field inside a hollow conductor

In the Stage 1 Workbook, the idea of charging objects negatively or positively was discussed. A neutral object has negative and positive charge in equal magnitude in the form of electrons and protons. If negative charge in the form of electrons is introduced to an object, it becomes negatively charged due to having excess negative charge. If electrons or negative charge is removed from the object it becomes positively charged due to having excess positive charge.

Consider a hollow spherical charged conductor. The excess charged particles will undergo electrostatic repulsion, causing them to move as far apart as possible. To do this, they occupy the largest surface area possible i.e. they reside on the outside surface of the conductor.

This means there is no electric field inside the conductor as there are no charges to provide the field. In addition, if a positive test charge is used to detect the presence of an electric field inside the hollow charged conductor, the sum of the electric fields due to each charge on the surface is zero because the fields cancel. Figure 2.1.12 illustrates this idea by using a simplified model of only eight charges distributed evenly on the surface of the conductor. Using the Principal of Superposition, the electric field at the centre of the hollow conductor is zero because the individual electric fields due to each charge on the surface cancel. Although quite complex, it can be shown that the net force on a positive test charge placed anywhere inside the hollow conductor is zero. This means that the electric field anywhere inside the hollow charged spherical conductor is zero providing there is no charge inside the cavity. This concept is extended to a charged conductor of any shape.

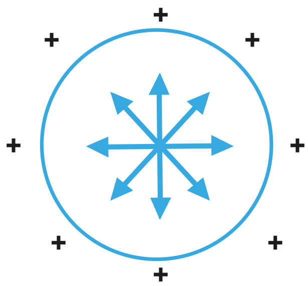


Figure 2.1.12

There is no electric field inside a hollow conductor of any shape, provided that there is no charge in the cavity.

Figure 2.1.13 illustrates the electric field surrounding a negatively charged hollow sphere.

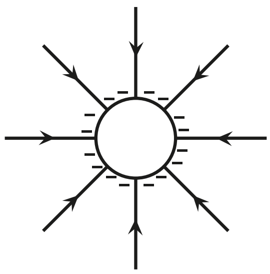


Figure 2.1.13

Science as a human endeavour

Electrostatic shielding

Michael Faraday was an English scientist who invented the Faraday cage in 1836. Faraday demonstrated the idea that there is no electric field inside a charged hollow conductor. To do this he constructed a small room inside a larger room and covered the smaller room with metal foil. Faraday reportedly sat inside the smaller room with an electroscope which could detect the presence of charge. He used an electrostatic generator to charge both rooms and watched while sparks were created between the two rooms. The electroscope did not detect any charge in the smaller room. The smaller room covered in foil protected Faraday from the static charge because it remained on the outside surface of the inner room without penetrating the room. That is, Faraday was shielded from the electric field. The room was called a Faraday cage. This term is still used today to describe any closed conducting surface. Faraday cages can also be made from metal mesh.

Figure 2.1.14

Figure 2.1.14 shows a person being shielded from an electrical discharge by a Faraday cage. Faraday cages can also shield objects from electromagnetic radiation. They will distribute charges or electromagnetic radiation on the outside of the cage and are used to protect electrical components including computer chips.

Figure 2.1.15 illustrates a Faraday suit. This type of suit is used to protect electricians or anyone working on live wires from unexpected electrical discharge. Figure 2.1.16 shows the metal casing and mesh in the door of a microwave oven. They act as a Faraday cage and do not allow electromagnetic radiation to escape from the oven.

Faraday cages are also used in MRI imaging rooms to prevent external interference from electromagnetic radiation when scanning a patient. The metal shell of a vehicle or aeroplane also acts as a Faraday cage protecting the passengers from lightning storms and any other electrical discharges that may occur.

A bar owner in Hove, East Sussex in the South East of England recently built a Faraday cage into the walls and ceiling of the building. His reasoning is that customers are distracted from interacting because of their constant use of mobile phones. The cage blocks all electromagnetic radiation meaning there is no mobile phone signal inside the bar. Is it possible that this tactic will encourage customers to socialise by having to talk to each other?

Figure 2.1.15

Figure 2.1.16

Read more: https://www.bbc.com/news/uk-england-sussex-36943686#

Science as a human endeavour

Assess the benefits and limitations of applications of electrical shielding such as:

- Faraday cages
- Microwave ovens
- Nuclear Magnetic Resonance imaging rooms
- Coaxial and USB cables
- Difficulties with mobile phone reception

Electric fields near sharp points

Earlier in the chapter we discussed the fact that charges distribute themselves evenly over the surface of a spherical hollow charge conductor. When the conductor is irregular in shape, the charges do not distribute themselves evenly. In fact charges tend to concentrate at points or regions on a conductor where the radius of curvature is small. This means that the electric field is strongest at sharp points.

The electric charges will still distribute themselves in a fashion to reduce the electric force between them. Figure 2.1.17 shows a positive charge q at the end of positively charged the conductor with a smaller radius of curvature. It lies between two positive charges q_1 and q_2 that are equidistant from q. Using the principle of superposition, the force on q is a vector sum of the forces acting on it due to q_1 and q_2.

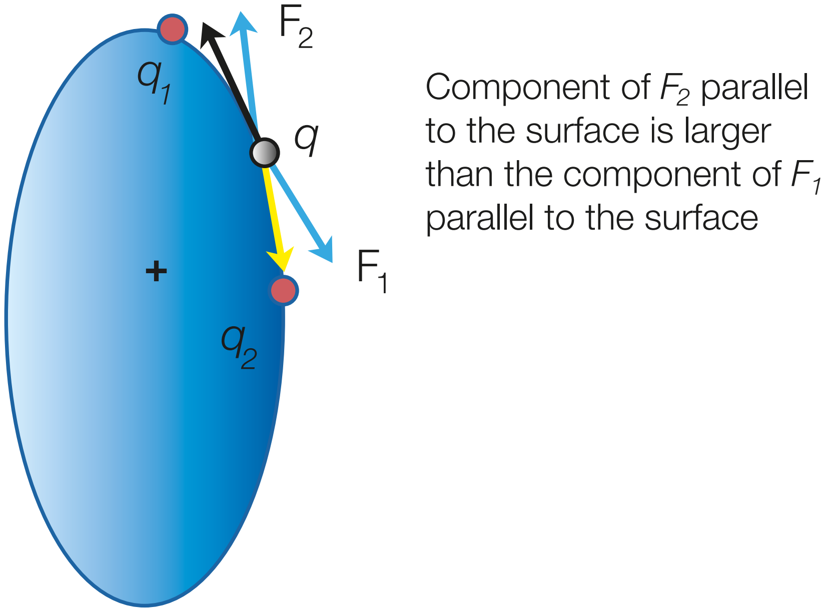


Figure 2.1.17

While the magnitude of the forces F_1 and F_2 are the same, the component of F_2 parallel to the surface (illustrated by a black arrow) is larger than the component of F_1 parallel to the surface (illustrated in yellow). This means that the charge q is forced closer to the sharper end. In general, charges are forced towards the end of the conductor with a smaller radius of curvature.

Figure 2.1.18 illustrates the electric field near a positively charged pear–shaped conductor.

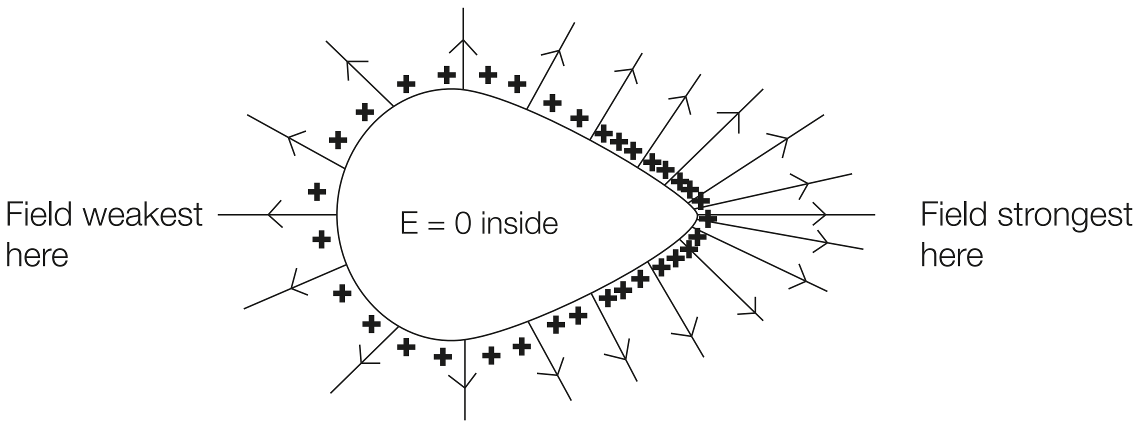


Figure 2.1.18

Corona discharge

Electric fields are strongest near sharp points on charged conductors. These fields may be large enough to ionise molecules in the air near the sharp points, resulting in charge movement away from the conductor. This movement of charge away from the conductor by the ionisation of molecules in the air is called a 'corona discharge'.

Figure 2.1.19 below illustrates the concept of corona discharge.

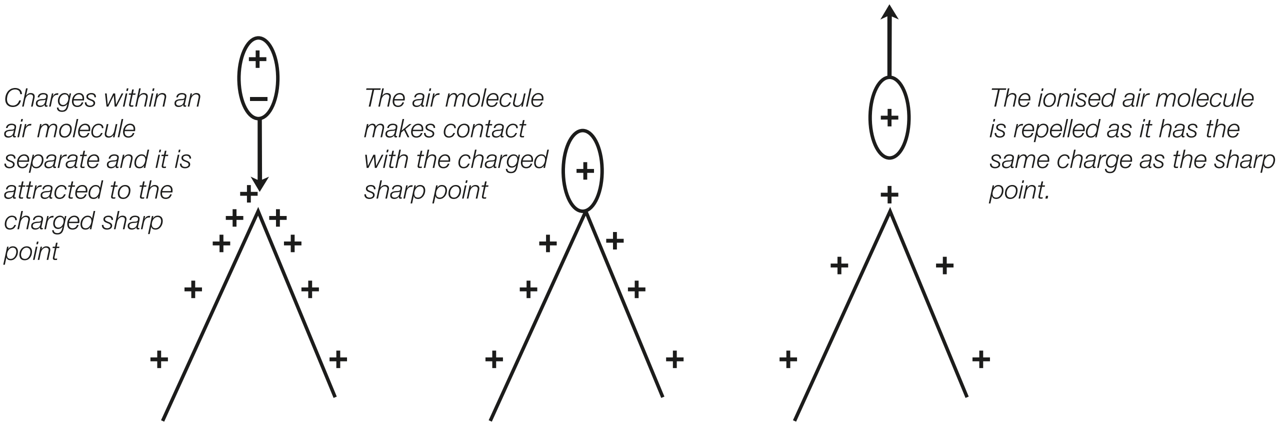


Figure 2.1.19

We have already discussed that charges tend to accumulate at sharp points on a conductor to produce a strong electric field. Consider a positively charged sharp point. If the electric field is strong enough, it can cause a separation of charges in nearby air molecules. This is because negative charges are attracted to the sharp point while positive charges are repelled. We can see this in Figure 2.1.19. Since electric forces are inversely proportional to the square of the distance between the charges, we can see that the force of attraction between negative charges in the air molecule and the positively charged sharp point is greater than the repulsive force between the positive charges which are further away. We say the air molecules are charged by induction and as a consequence they will be attracted to the charged sharp point.

When the air molecules come into contact with the sharp point of the conductor, they will gain the same charge as the conductor (positive in the case above). The air molecules are now charged or ionised.

The ionised air molecules are then repelled away from the sharp point due to the repulsive electric force between like charges. When this happens to many air molecules, a stream of the charge moves away from the conductor. This is called a corona discharge.

A similar situation would occur if the sharp point were negatively charged but there would be a net movement of negative charge away from the conductor.

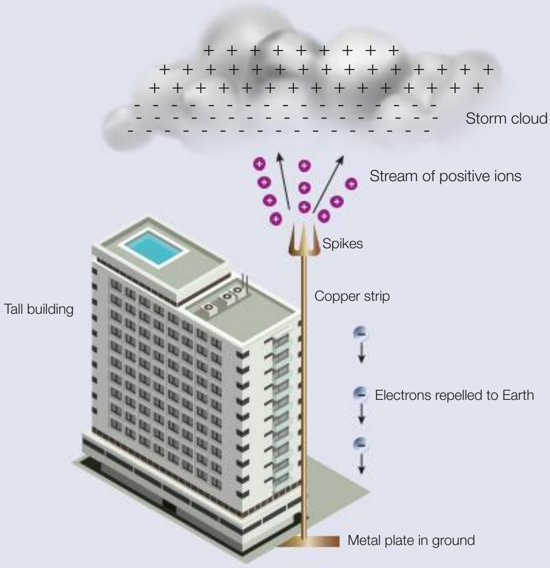
Science as a human endeavour

Applications of corona discharge

The lightning rod

A lightning rod is a conductive rod or a series of spikes mounted at the top of buildings and structures. The lightning rod is connected to a rod or plate inserted into the ground by a copper wire.

Lightning rods are designed to protect buildings or structures by providing a pathway for lightning. They can also help prevent lightning from striking via the mechanism of corona discharge. The Earth's surface becomes positively charged by induction as the large negative charge carried on the bottom surface of clouds in an electrical storm force electrons in the Earth's surface deeper into the ground. As a consequence the lightning rod becomes positively charged. The strong electric field around its tip ionises air particles to produce positive charge that is repelled away from the lightning rod and onto the cloud via corona discharge. This neutralises some of the negative charge on the cloud and helps prevent a lightning strike.



However, there are times when the rate at which negative charges accumulate on a cloud is greater than the rate at which a lightning rod can ionise air particles. When this occurs, ionised air particles provide a conductive pathway for the lightning to discharge to the lightning rod which is situated high above the ground.

Science as a human endeavour

Explore problems for which scientists have developed practical solutions by making use of strong electric fields.

Examples include:

- Photocopier (charging drum /discharging paper)
- Lightning rods
- Electrostatic precipitator
- Spark plugs

Extra understanding

The photocopier

A photocopier makes use of corona discharge.

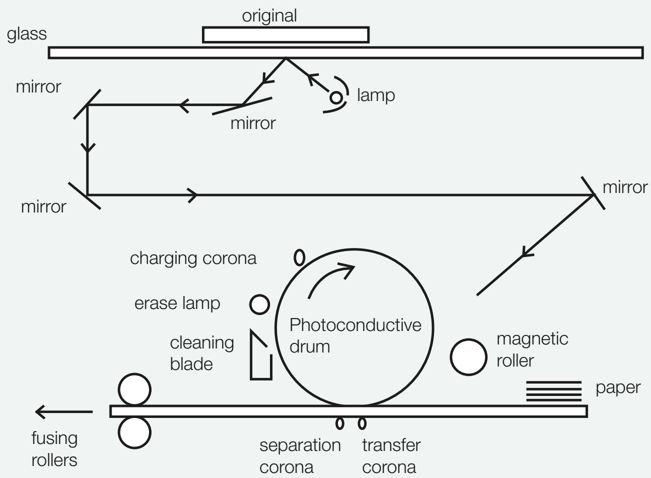


Figure 2.1.20

There are five main steps involved in producing an image with a photocopier.

1. The photoconductive drum is charged positively
2. A latent image is produced on the drum in the form of charges
3. Toner is applied to the drum
4. Toner is transferred to the paper
5. The image is fixed to the paper and the drum is cleared, ready for the next copy to be produced

The drum is coated with a photoconductive material (e.g. selenium) which only conducts charge when it is exposed to light. The region surrounding the drum is dark.

A thin wire (called the charging corona) extends along the length of the drum. This wire is positively charged. The strong electric field surrounding it will ionise air molecules in the gap between the wire and the drum. This produces positive charges that are repelled away from the wire and onto the drum. The drum rotates under the wire and becomes evenly charged.

Next light is reflected from the original onto the drum. The parts of the original that are light/white will reflect light onto the drum whereas the parts of the original which are dark/black will not reflect light onto the drum. Therefore the parts that are light in colour cause the drum to conduct charge and these parts of the drum are earthed. The corresponding dark parts of the original do not cause the drum to conduct charge and the charges remain on the drum. We say a 'latent image' of the original remains on the drum in the form of positive charge.

The next step is to apply toner to the drum. Toner is a fine powdery substance consisting of carbon and resins. Toner is attracted towards the positively charged parts of the drum. It can be sprayed on or transferred to the drum by a magnetic roller.

The toner needs to be transferred from the drum to the paper. To do this, a positive charge is applied to the bottom surface of the paper by a different thin wire (called the transfer corona). When the paper comes into contact with the drum, the toner particles are attracted from the drum onto the paper. This is because the paper has a greater concentration of positive charge (when compared to the drum).

Once the toner is on the paper, a third thin negatively charged wire (called the separation corona) neutralises most of the charge on the paper as it passes over the wire so that the paper does not cling to the drum.

Finally the paper is passed through a pair of hot rollers that melts the toner and presses it into the fibres of the paper.

The drum now needs to be cleared of any remaining charge so that it is ready for a new photocopy. This is achieved by scraping any excess toner off the drum with a cleaning blade and exposing it to light that will cause any remaining charge on the drum to be earthed.

Exercises

1. Two conducting spheres are placed 40.0 cm apart. One sphere carries a charge of +1.00 C and the other carries a charge of +2.00 C.

   Calculate the magnitude and direction of the force acting between the spheres.

2. (a) State Coulomb's Law.

   (b) Calculate the magnitude and direction of the electric force between two charges q_1 = –3.4 μC and q_2 = +4.5 μC separated by a distance of 8.0 cm.

   (c) Use proportionality to discuss the effect on the magnitude of the force between the two charges if the distance between them is reduced to 4.0 cm.

3. The magnitude of the force experienced between two point charges in a vacuum is *F*. State, in terms of *F*, the magnitude of the force if

   (a) One charge is replaced with one that has four times its charge.

   (b) One charge is replaced with one that has twice its charge and the other charge is replaced with one that has a charge three times larger.

   (c) One charge is replaced with one that has twice its charge and the distance between the charges is doubled.

4. Two charged conducting spheres q_A =+500 mC and q_B = –150 mC are placed 30.0 cm apart in a vacuum.

   (a) Calculate the force $\vec{F}$, acting between the two charged spheres.

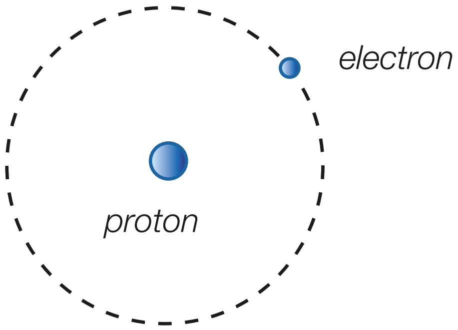
   (b) State the force $\vec{F}$, that q_A exerts on q_B.

(c) State the force $\vec{F}$, that q_B exerts on q_A.

(d) The charged spheres are now placed 5.0 cm apart in a vacuum, determine the force $\vec{F}$, that q_A exerts on q_B.

(e) The charged spheres are now brought momentarily into contact and then separated by 30.0 cm. Calculate the new force $\vec{F}$, that q_B exerts on q_A.

5. In a hydrogen atom an electron circles a proton in a circular path of radius 5.92×10^{-11} m.



(a) Calculate the magnitude of the electric force between the proton and the electron in the hydrogen atom.

(b) On the diagram above, draw a vector arrow that indicates the direction of the electric force on the electron due to the proton.

(c) Explain why the electron undergoes uniform circular motion.

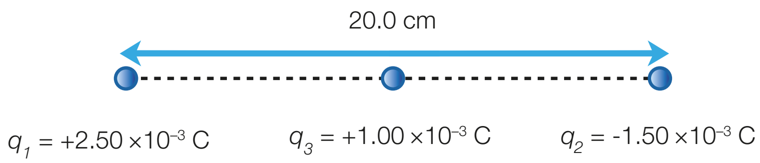
(d) Calculate the speed of the electron in its orbit.

6. (a) State the principal of superposition for electric forces.

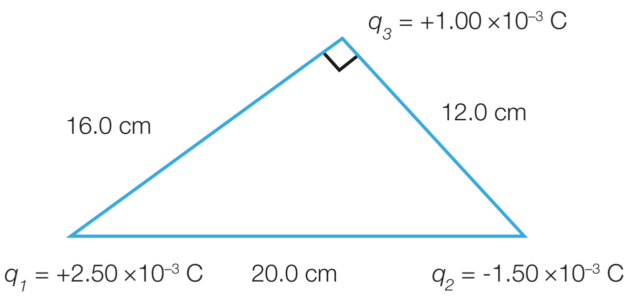
(b) Two point charges $q_1 = +2.50 \times 10^{-3}$ C and $q_2 = -1.50 \times 10^{-3}$ C are placed 20.0 cm apart in a vacuum.

Determine the magnitude and direction of the electric force experienced by a point charge $q_3 = +1.00 \times 10^{-3}$ C placed:

(i) at a point midway between q_1 and q_2 and along the line joining their centres.

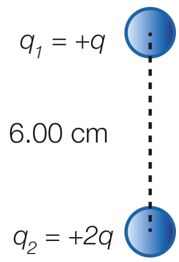


(ii) at a point which is 16.0 cm from q_1 and 12.0 cm from q_2.



Space for vector diagram

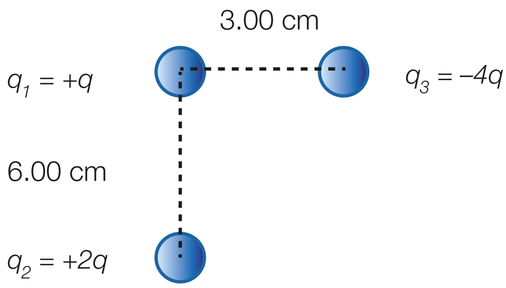
7. Two point charges q_1 and q_2 are placed 6.00 cm apart in a vacuum.



The magnitude of the force between q_1 and q_2 is 25.0 N.

(a) Show that the magnitude of the charge on q_1 is 2.24×10^{-6} C.

(b) A charge $q_3 = -4q$ is placed 3.00 cm to the right of q_1.

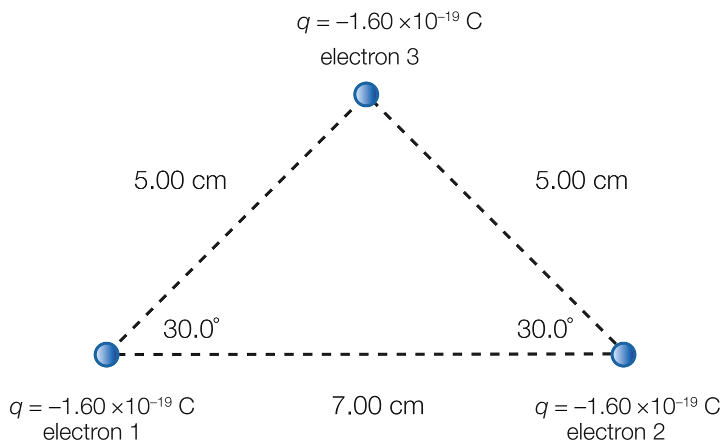


(i) Use proportionality to determine the magnitude of the force on q_1 due to q_3.

(ii) Calculate the total force on q_1 due to q_2 and q_3.

Space for vector diagram

8. Two electrons are placed 7.00 cm apart in a vacuum. A third electron is positioned so that it is 5.0 cm from each of the other two electrons as shown in the diagram below.



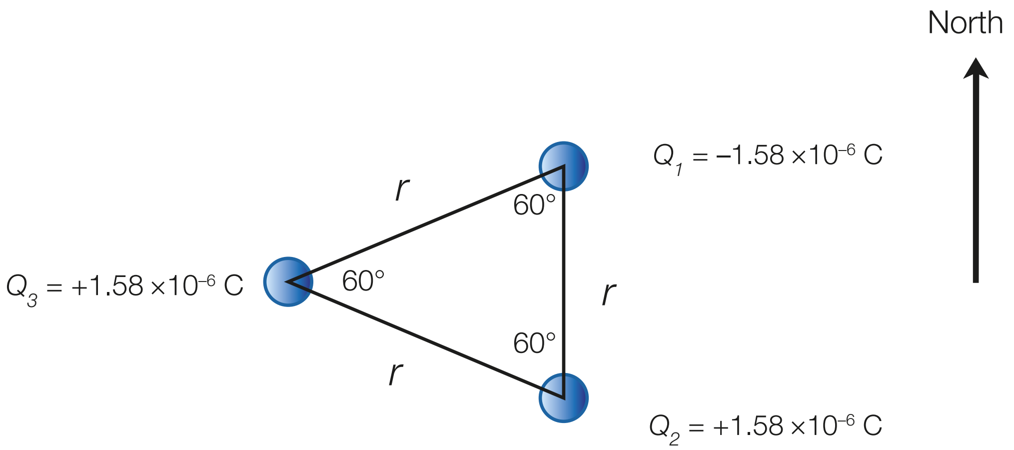
(a) Calculate the magnitude and direction of the force on electron 3 due to electron 1.

(b) State the magnitude and direction of the force on electron 3 due to electron 2.

(c) Calculate the total force on electron 3 due to electron 1 and 2.

Space for vector diagram

9. Three point charges are positioned at the corners of an equilateral triangle as shown. The magnitude of the force on Q_1 due to Q_2 is 36.0 N. The direction of north is shown on the diagram.



(a) Show that the distance r is 2.50×10^{-2} m.

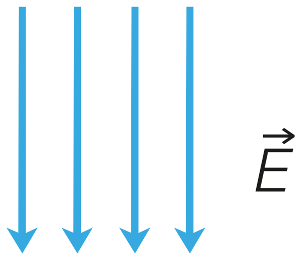
(b) Calculate the magnitude and direction of the total force on Q_3.

Space for vector diagram

10. (a) Define the term 'electric field'.

(b) A point charge $q = +5.00 \times 10^{-6}$ C experiences a force of 2.8 N South when placed at a point in an electric field. Calculate the magnitude and direction of the electric field at the point where q is placed.

11. An alpha particle has a charge of $+2e = +3.2 \times 10^{-19}$ C and a mass of 6.645×10^{-27} kg. It is placed in a uniform electric field $\vec{E}$ of magnitude 1800 NC^{-1} acting down the plane of the page.



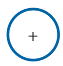
(a) Calculate the magnitude and direction of the force acting on the alpha particle.

(b) Calculate the acceleration $\vec{a}$, of the alpha particle.

(c) Explain without a calculation why the force $\vec{F}$, experienced by an electron placed in the same electric field would be half that calculated in part (a) and in the opposite direction.

12. A $+4.50 \times 10^{-6}$ C isolated charge is placed in a vacuum.

(a) Sketch the electric field surrounding a positive isolated charge.



(b) Calculate the electric field $\vec{E}$, at a point 10.0 cm from the charge.

(c) State, with reason, the magnitude of the electric field at a point 40.0 cm from the charge.

13. A Shark shield consists of two electrodes in a cord that trails behind the surfer. A large current flows between the electrodes to produce an electric field.

The magnitude of the electric field produced 80.0 cm below the stationary surfboard of a surfer wearing a Shark Shield and 40.0 cm from the electrode is 70.0 NC^{-1}.

(a) Calculate the charge produced by the electrode at this point.

(b) Sharks experience discomfort as they enter the electric field produced by a Shark Shield. Suggest why the discomfort increases as the shark approaches the surfboard until it becomes unbearable and the shark turns away.

14. (a) Calculate the magnitude and direction of the electric field at a distance of 20.0 cm from the dome of a Van de Graaff generator which holds a charge of -6.00×10^{-4} C.

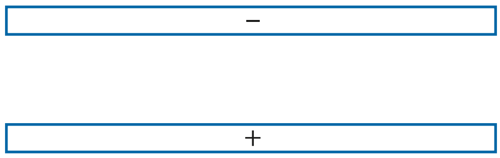
(b) Calculate the magnitude of the electric field at a distance of 60.0 cm from the dome of the same Van de Graaff generator.

(c) Calculate the number of electrons distributed on the dome of this an de Graaff generator.

15. Sketch the electric field in the vicinity of:

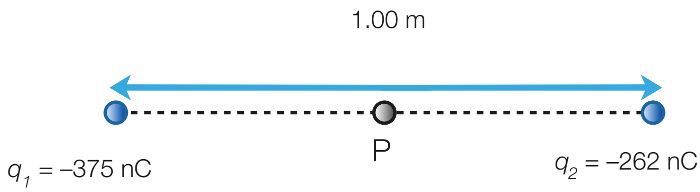
(a) two identical charges with opposite signs.

(b) two finite parallel conducting plates



16. (a) Two point charges q_1 = –375 nC and q_2=–262 nC are placed 1.00 m apart in a vacuum.

Determine the magnitude and direction of the electric field $\vec{E}$, at a point P which is located midway between the charges and along the line joining their centres.



(b) There is a point, a distance *r* from q_1 and between the two charges where the total electric field is zero. Determine the value of the distance *r*.

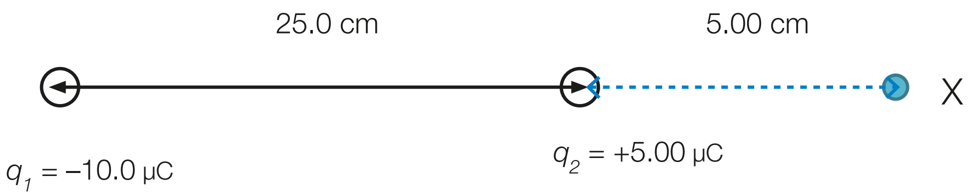
17. Two point charges Q_1 = +50.0 μC and Q_2 = +75.0 μC are separated by a distance of *r cm* in a vacuum.

Explain why the total electric field can never be zero at a point A directly to the right of Q_2.

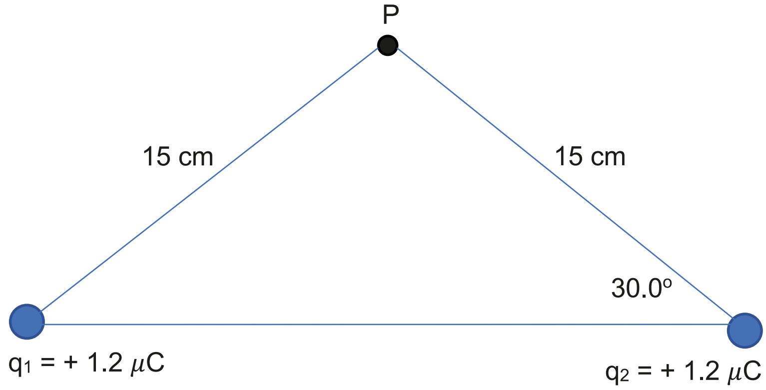
18. (a) Show that the magnitude of the electric field 10.0 cm from an isolated charge q_1 = −10.0 μC is 8.99 × 10^6 NC^{-1}.

(b) State, with reason, the distance from the charge at which the electric field would have a magnitude of 3.60 × 10^7 NC^{-1}.

(c) A second charge q_2 is placed 25.0 cm to the right of q_1 as shown in the diagram below. Calculate the electric field $\vec{E}$, at the position marked X.



19. Two charges q_1 and q_2 are separated as shown in the diagram below.

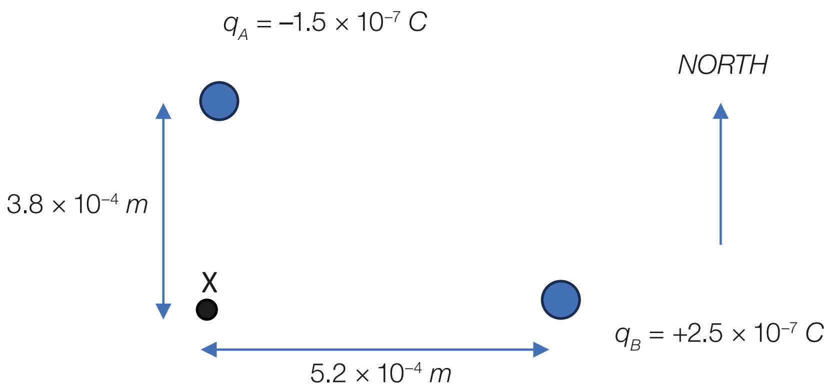


(a) Calculate the magnitude of the electric field at position P due to q_1.

(b) State the magnitude of the electric field at P due to q_2.

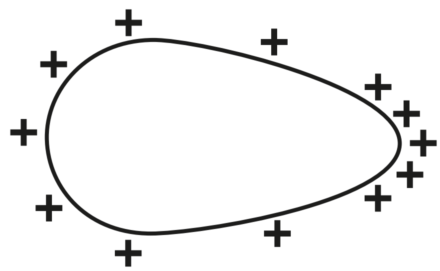
(c) Determine the total magnitude and direction of the electric field at P.

20. Two point charges q_A and q_B are arranged at right angles to point X as shown in the diagram below.



Calculate the magnitude and direction of the *net electric field* at X due to the charges q_A and q_B.

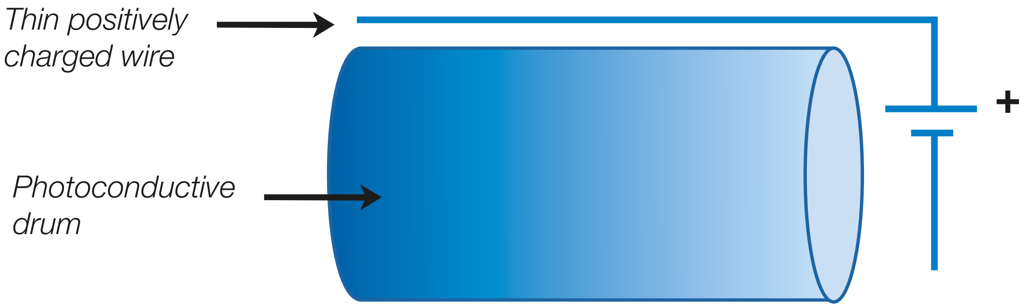
21. (a) Sketch the electric field in the region surrounding the positively charged pear–shaped conductor.



(b) Explain how the large electric field in the vicinity of a sharp point may ionise nearby air molecules.

22. A photocopier consists a drum coated with a photoconductive material. A thin positively charged wire extends along the length of the drum as shown in the diagram below. There is a small gap of air between the wire and the drum.

Part of the process involved in photocopying requires the photoconductive drum to be charged positively.



Explain how the photoconductive drum can be charged positively as it rotates beneath the wire.

2.2 Motion of charged particles in electric fields

Science understanding

1. When a charged body moves or is moved from one point to another in an electric field, work is done on or by the field.
2. The electric potential difference, ΔV, between two points is the work done per unit charge on a small positive test charge moved between the points, provided that all other charges remain undisturbed.
3. The electronvolt (eV) is a unit of measurement which describes the energy carried by a particle. It is the work done when an electron moves through a potential difference of 1 volt.
   - Solve problems involving the use of $W = q\Delta V$.
   - Convert energy values between joules and electronvolts.
4. The magnitude of the electric field (away from the edges) between two oppositely charged parallel plates a distance d apart, where ΔV is the potential difference between the plates, is given by the formula: $E = \frac{\Delta V}{d}$.
   - Solve problems involving the use of $E = \frac{\Delta V}{d}$.
5. The force on a charged particle moving in a uniform electric field is constant in magnitude and direction, thus producing a constant acceleration.
   - Derive the formula $\vec{a} = \frac{q\vec{E}}{m}$ for the acceleration of a charged particle in an electric field.
   - Solve problems using $\vec{a} = \frac{q\vec{E}}{m}$ and the constant acceleration formulae of charged particles moving parallel or antiparallel to a uniform electric field.
   - Describe the motion of charged particles parallel or antiparallel to a uniform electric field.
6. In a cyclotron, the electric field in the gap between the dees increases the speed of the charged particles.
   - Describe how an electric field between the dees can transfer energy to a charged particle passing between them.
   - Describe how charged particles could be accelerated to high energies if they could be made to repeatedly move across an electric field.
   - Calculate the energy transferred to a charged particle each time it passes between the dees.
   - Explain why charged particles do not gain kinetic energy when inside the dees.
7. A charged particle moving at an angle to a uniform electric field experiences a force which effects both components of its velocity differently. The component of the velocity parallel to the electric field changes due to the electric force and the component perpendicular to the field remains constant.
   - Compare the motion of a projectile in the absence of air resistance with the motion of a charged particle in a uniform electric field.
   - Solve problems for the motion of charged particles that enter a uniform electric field perpendicular to the field.
   - Solve problems for the motion of charged particles that enter a uniform electric field at an angle to the field where the displacement of the charged particle parallel to the field is zero.

This chapter uses the concepts of force developed in Stage 1, Subtopic 1.2: Forces and energy in Stage 1, Subtopic 4.1: Energy.

2

Electric potential difference

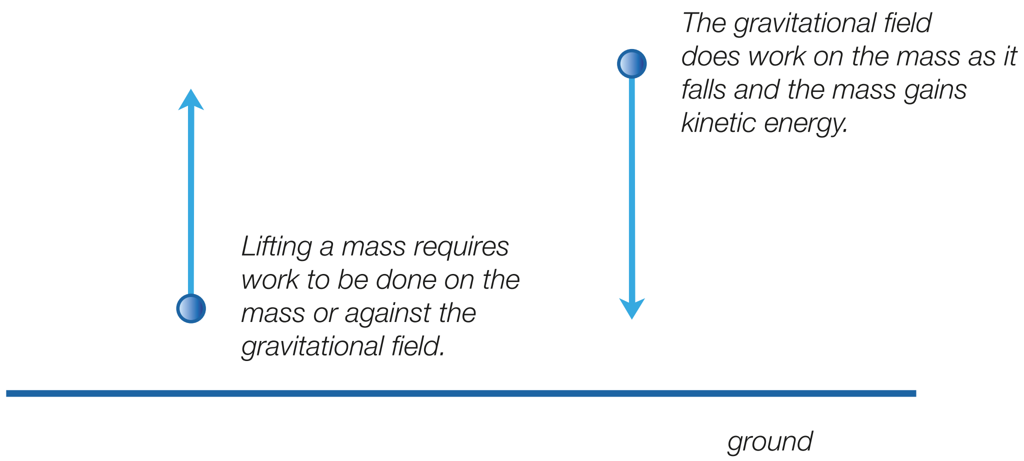


Figure 2.2.1

Figure 2.2.1 illustrates an object of mass *m* being raised in a gravitational field. Work is done against the gravitational field and the mass gains gravitational potential energy. In accordance with the law of conservation of energy, the work done (*W*) on the mass is equal to the gravitational potential energy gained. In Stage 1 you learnt that the energy involved is given by W = Fs = mgh where *h* is the vertical height through which the mass is raised. When the same mass falls, the gravitational field does work on the mass and the mass gains kinetic energy (E_k) and hence speed (*v*). In this case $W = \Delta E_k = \frac{1}{2}mv^2$. We can say that gravitational fields store gravitational potential energy.

In a similar way, we can say that electric fields store electric potential energy. When a charged body moves or is moved from one point to another in an electric field and its potential energy changes, work is done on or by the field.

Figure 2.2.2 illustrates a positive test charge (q_+) being moved from point A to point B in the electric field created by a positive charge Q.

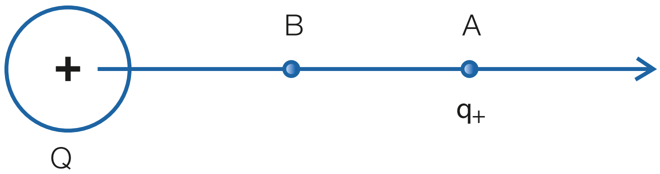


Figure 2.2.2

In moving the positive test charge from point A to point B, work must be done against the electric field to overcome the electrostatic force of repulsion and the positive test charge gains electrical potential energy.

The **electric potential difference,** ΔV, between two points in an electric field is defined as the work done (*W*) per unit charge (*q*) on a small positive test charge moved between two points, provided that all other charges remain undisturbed.

$$\Delta V = \frac{W}{q}$$

The **unit of potential difference** is the volt (*V*) and is equivalent to one Joule per Coulomb (JC^{-1}).

Rearranging the equation $\Delta V = \frac{W}{q}$ gives the expression for work done in moving the positive test charge from point A to point B: $W = q\Delta V$.

If the positive test charge moves from point B to point A, in the same direction as the electric field, then the electric field does work on the positive test charge and it gains kinetic energy. Using the law of conservation of energy, it follows that $W = \Delta E_k = \frac{1}{2}mv^2$.

Worked Example

A +2.0 mC charge is accelerated through a potential difference of 4.0×10^3 V.

(a) Calculate the work done on the +2.0 mC charge in moving it through the potential difference.

$W = q\Delta V = 2.0 \times 10^{-3} \times 4.0 \times 10^3 = 8.0\ J$

(b) State the kinetic energy gained by the +2.0 mC charge as it moves through the potential difference.

8.0 J

(c) Calculate the speed gained by the +2.0 mC charge given it has a mass $m = 1.4 \times 10^{-15}$ kg.

$$\Delta E_k = W = q\Delta V = \frac{1}{2}mv^2 \ \therefore\ v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 8.0}{1.4 \times 10^{-15}}} = 1.1 \times 10^8 ms^{-1}$$

The electronvolt (eV)

2

The electronvolt is a unit of measurement that describes the energy carried by a charged particle.

One **electronvolt** (eV) is the work done when an electron moves through a potential difference of one volt.

Applying $W = q\Delta V$ one electronvolt is equivalent to $W = q\Delta V = 1.60 \times 10^{-19} \times 1 = 1.60 \times 10^{-19}\ J$

1eV = 1.60 × 10^{-19} J

To convert energy from electronvolts to Joules: multiply by *1.60 × 10^{-19} J*

To convert energy from Joules to electronvolts: divide by *1.60 × 10^{-19} J*

Worked Examples

1. A charged particle has 2.5 eV of kinetic energy. Convert the kinetic energy to Joules.

$2.5 \times 1.60 \times 10^{-19} = 4.0 \times 10^{-19}\ J$

2 A charged particle gains 7.20×10^{-16} J of kinetic energy in an electric field. Convert this energy to eV.

$$\frac{7.2 \times 10^{-16}}{1.60 \times 10^{-19}} = 4.50 \times 10^3 eV$$

3 An electron is accelerated through a potential difference of 6.0×10^3 V.

Calculate the kinetic energy gained by the electron in J and eV as it accelerates through this potential difference.

$\Delta E_k = W = q\Delta V = 1.6 \times 10^{-19} \times 6 \times 10^3 = 9.6 \times 10^{-16}\ J = 6.0 \times 10^3\ eV$

Note: If an electron accelerates through a potential difference $\Delta V = x$, then we already have the kinetic energy that it gains in eV i.e. the electron gains x eV of kinetic energy.

Uniform electric fields

The electric field between two oppositely charged parallel conducting plates

A uniform electric field can be set up in the laboratory by applying a constant potential difference ΔV between two oppositely charged parallel conducting plates separated by a small distance d (Figure 2.2.3).

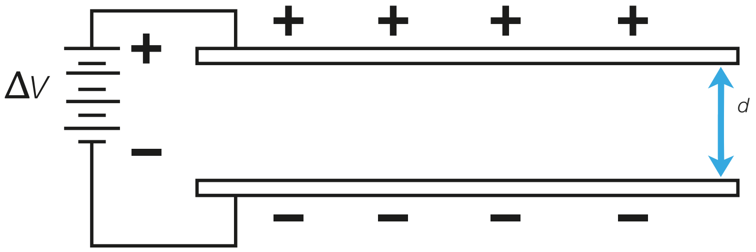


Figure 2.2.3

In subtopic 2.1 we defined electric field $\vec{E}$ at a point as the electric force $\vec{F}$ per unit charge (q) experienced by a small positive test charge when placed at that point in the electric field.

The electric field between two oppositely charged parallel conducting plates is illustrated in Figure 2.2.4. The electric field between the plates and away from the edges is uniform. A uniform electric field is represented by evenly spaced electric field lines. This means that a positive test charge will experience the same force no matter where it is placed within the electric field.

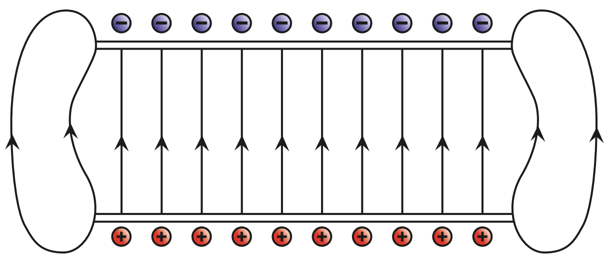


Figure 2.2.4

Near and beyond the edges of the plates the electric field is non–uniform. This is commonly referred to as the 'end effect'. The end effect is a result of the electric field lines still having to leave and arrive on the surface of the plates at right angles.

A charge in a uniform electric field experiences the same force both in magnitude and direction at all points within the electric field.

Now consider a small positive test charge being moved from the lower plate to the top plate in Figure 2.2.3. Work needs to be done on the charge, where $W = q\Delta V = Fd$.

Rearranging yields: $\frac{F}{q} = \frac{\Delta V}{d}$. Since $E = \frac{F}{q}$ it follows that $E = \frac{\Delta V}{d}$.

The electric field E between two oppositely charged parallel conducting plates separated by a distance d is given by

$$E = \frac{\Delta V}{d}$$

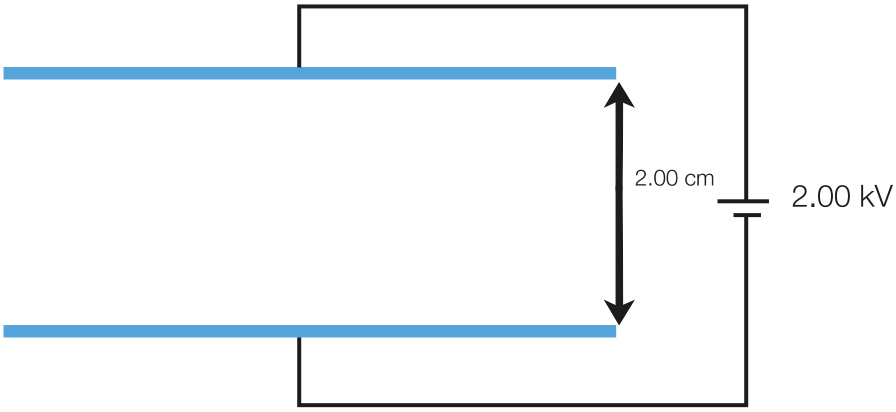
where ΔV is the potential difference between the plates.

The **unit of electric field** in this context is the volt per metre (Vm^{-1}) and is equivalent to the Newton per Coulomb (NC^{-1}).

The **direction** of the electric field is from the positive plate to the negative plate.

Worked Example

Consider the uniform electric field created between the two parallel conducting plates pictured below.



Calculate the magnitude and direction of the electric field between the plates.

$$E = \frac{\Delta V}{d} = \frac{2.00 \times 10^3}{0.0200} = 1.00 \times 10^5\ Vm^{-1} \text{ towards the lower plate}$$

The motion of charged particles parallel and antiparallel to a uniform electric field

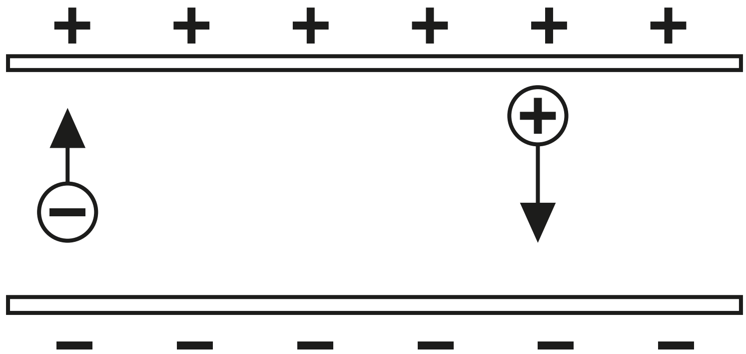


Figure 2.2.5

Since the force on a charged particle in a uniform electric field is constant in magnitude and direction, then it follows that the **acceleration** of a charged particle in a uniform electric field is **constant** in magnitude and direction.

Consider the two charges illustrated in Figure 2.2.5. The positive charge experiences a downward force due to the electric field and accelerates towards the negative plate. Its motion is parallel to the electric field. The negative charge experiences an upward force due to the electric field and accelerates towards the positive plate. Its motion is antiparallel to the electric field.

Derivation for the magnitude of acceleration

Using Newton's Second Law, the acceleration of a charge in an electric field is given by: $\vec{a} = \frac{\vec{F}}{m}$

Since $\vec{E} = \frac{\vec{F}}{q}$ *then* $\vec{F} = q\vec{E}$.

It follows that $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

The magnitude of the acceleration of a charge q of mass m in an electric field $\vec{E}$ is given by

$$\vec{a} = \frac{q\vec{E}}{m}$$

Worked Example

An electron is released from rest in a uniform electric field $\vec{E} = 2.4 \times 10^4$ Vm^{-1} acting to the right of the page as shown below.

(a) Describe the motion of the electron.

The electron accelerates at a constant rate antiparallel to the electric field i.e. to the left.

(b) Calculate the magnitude of the acceleration of the electron in this electric field.

$$a = \frac{qE}{m} = \frac{1.60\times10^{-19}\times2.4\times10^{4}}{9.11\times10^{-31}} = 4.2\times10^{15}ms^{-2}$$

Analysing the motion of a charged particle parallel and antiparallel to a uniform electric field

The equations of motion can be applied to the motion of a charge in a uniform electric field. These equations have already been discussed in the Stage 1 Workbook and in Subtopic 1.1.

$$v = v_0 + at$$

$$s = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2as$$

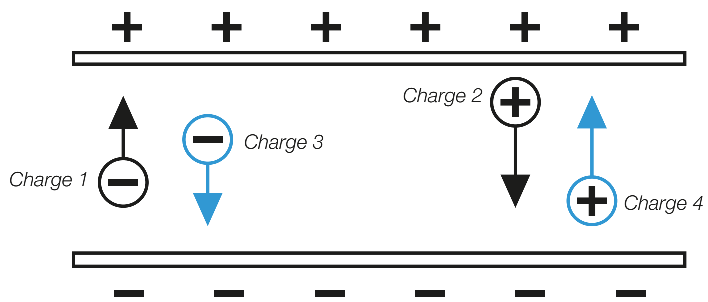


Figure 2.2.6

Consider the four charges illustrated in Figure 2.2.6.

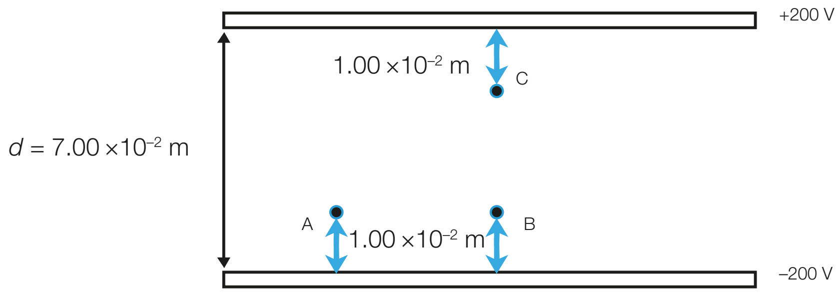
If the charges are released into the electric field from rest then the initial velocity v_o is zero.

Charges 1 and 2 experience a force due to the electric field that cause them to speed up. The value of the acceleration used in the equations of motion is positive when analysing their motion.

Charges 3 and 4 have an initial velocity and experience an opposing force due to the electric field. The value of the acceleration used in the equations of motion is negative.

Worked Examples

1. Two oppositely charged parallel conducting plates are placed 7.00 cm apart as shown in the diagram below.



This diagram is not to scale

(a) Calculate the magnitude and direction of the electric field between the plates.

$E = \frac{\Delta V}{d} = \frac{400}{7.00 \times 10^{-2}} = 5.71 \times 10^{3} Vm^{-1}$ *towards the lower plate* ↓

(b) Show that the magnitude of the acceleration of a proton in this electric field is 5.46×10^{11} ms^{-2}.

$a = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 5.71 \times 10^{3}}{1.67 \times 10^{-27}} = 5.47 \times 10^{11} ms^{-2}$

(c) Calculate the force $\vec{F}$, acting on a proton in this electric field at each of the points A, B and C.

$F = qE = 1.60 \times 10^{-19} \times 5.71 \times 10^{3} = 9.14 \times 10^{-16} N$ ↓

The electric field is uniform. The force on a proton is the same, both in magnitude and direction at any position with in the electric field.

Note: Newton's Second Law gives the same value for the force (allowing for some rounding off differences).

$F = ma = 1.67 \times 10^{-27} \times 5.47 \times 10^{11} = 9.13 \times 10^{-16} N$

(d) Calculate the time taken for a proton released from the top plate to cross the distance between the plates.

$s = v_0 t + \frac{1}{2} a t^2 \therefore t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 7.00 \times 10^{-2}}{5.47 \times 10^{11}}} = 5.06 \times 10^{-7} s$

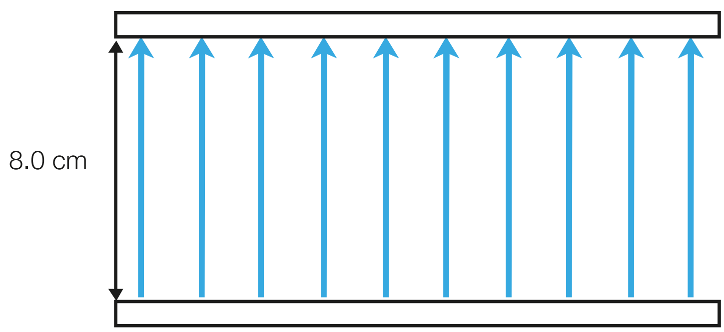
(e) Calculate the work done by the electric field on the proton as it crosses the distance between the plates.

$W = q\Delta V = 1.60 \times 10^{-19} \times 400 = 6.40 \times 10^{-17} J$

(f) Calculate the gain in speed of the proton in crossing the distance between the plates.

$\Delta Ek = W = q\Delta V = \frac{1}{2} m v^2 \therefore v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 6.40 \times 10^{-17}}{1.67 \times 10^{-27}}} = 2.77 \times 10^{5} ms^{-1}$

2. Consider the electric field created between the oppositely charged parallel conducting plates shown below. The plates are 8.0 cm apart and the electric field has a magnitude of 3.0×10^{4} Vm^{-1} towards the top plate.



(a) Calculate the potential difference that created the electric field between the plates.

$E = \frac{\Delta V}{d} \therefore \Delta V = Ed = 3.00 \times 10^{4} \times 0.080 = 2400 V$

(b) State the sign of the top plate.

Negative

(c) Calculate the magnitude and direction of the acceleration of a –1.8 μC charge in this electric field given it has a mass of 7.2 × 10⁻¹⁶ kg.

$$a=\frac{qE}{m}=\frac{1.8\times10^{-6}\times3.0\times10^{4}}{7.2\times10^{16}}=7.5\times10^{13}ms^{-2}\ \downarrow(\textit{towards the lower plate})$$

(d) The –1.8 μC charge is released from rest in this electric field. Calculate the time it would take the charge to move 5.0 cm.

$$s=v_0t+\frac{1}{2}at^2\ \therefore\ t=\sqrt{\frac{2s}{a}}=\sqrt{\frac{2\times0.050}{7.5\times10^{13}}}=3.7\times10^{-8}s$$

(e) Calculate the energy gained by the –1.8 μC in moving 5.0 cm in this electric field.

$W = q\Delta V = qEd = 1.8 \times 10^{-6} \times 3.0 \times 10^{4} \times 0.050 = 2.7 \times 10^{-3}\ J$

Note: The charge does not move through a potential difference of 2400V. An alternative method is to use a ratio to calculate the potential difference crossed by the charge i.e. $\frac{5}{8}x2400=1500V$.

The calculation for the work done: $W = q\Delta V = 1.8 \times 10^{-6} \times 1500 = 2.7 \times 10^{-3}\ J$, yields the same answer.

(f) Calculate the gain in speed of the –1.8 μC charge after it has moved 5.0 cm in this electric field.

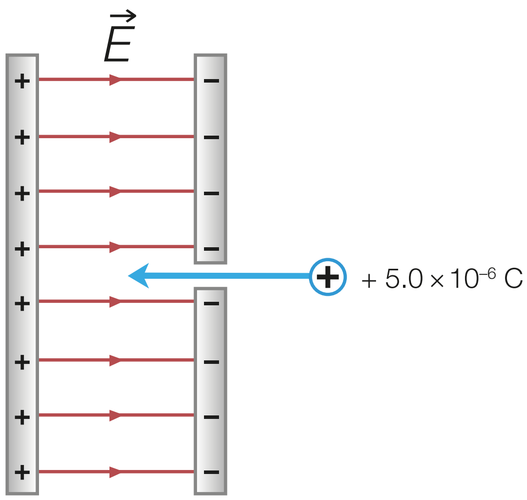
$v = v_0 + at = 0 + 7.5 \times 10^{13} \times 3.7 \times 10^{-8} = 2.8 \times 10^{6}\ ms^{-1}$

Please note: An alternative method is to use the law of conservation of energy. The work done by the electric is equal to the kinetic energy gained by the charge. The calculation for speed:

$$W=q\Delta V=\frac{1}{2}mv^2\ \therefore\ v=\sqrt{\frac{2W}{m}}=\sqrt{\frac{2\times2.7\times10^{-3}}{7.2\times10^{-16}}}=2.7\times10^{6}ms^{-1},$$

yields the same answer (allowing for some rounding off differences).

3. Consider the + 5.0 × 10⁻⁶ C charge entering the uniform electric field between the two parallel conducting plates shown in the diagram below. The charge enters the electric field of magnitude 2.0 × 10⁵ Vm⁻¹ with a speed of 6.0 × 10⁶ ms⁻¹. The mass of the charge is 3.5 × 10⁻¹⁵ kg.



(a) Calculate the magnitude and direction of the acceleration of the charge in this electric field.

$$a=\frac{qE}{m}=\frac{5.0\times10^{-6}\times2.0\times10^{5}}{3.5\times10^{-15}}=2.9\times10^{14}ms^{-2}\rightarrow$$

(b) Calculate the distance that the charge penetrates the electric field before stopping.

$$v^2=v_0^2+2as\ \therefore\ s=\frac{v^2-v_0^2}{2a}=\frac{0^2-(6.0\times10^{6})^2}{2\times-2.9\times10^{14}}=0.062m$$

The cyclotron

The first cyclotron was constructed by Ernest Lawrence in 1931. Lawrence did not work in isolation and had an assistant, Milton Stanley Livingston. Figure 2.2.7 pictures Lawrence standing next to an early cyclotron (the 17–inch cyclotron) built around 1932. Figure 2.2.8 illustrates a modern cyclotron. It is used for medical research and is located in Camperdown, Sydney.

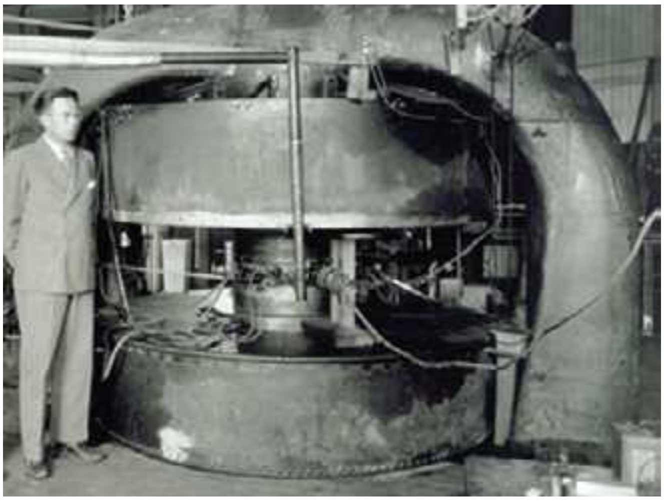


Figure 2.2.7

Figure 2.2.8

The cyclotron makes use of electric and magnetic fields. The cyclotron will be discussed further in Subtopic 2.4 when the motion of charges in magnetic fields is explored. A cyclotron is a **particle accelerator** used to accelerate charged particles (often protons) to high speeds. These charged particles are then used to bombard stable atomic nuclei to produce short–lived radioactive isotopes that are used in medical diagnosis or the treatment of cancer.

The main components of a cyclotron are shown in Figure 2.2.9. An electromagnet is placed above and below two hollow D–shaped copper conductors. This produces a uniform magnetic field inside the dees. Charged particles are **accelerated by a uniform electric field** in the gap between the dees. The uniform **magnetic field** causes the charged particles to move in a **semi–circular path** so that they return to the electric field. The electric field is reversed when the charges return to the gap so that the charges once again accelerate across the electric field. The process repeats many times and the charges eventually exit the cyclotron having **accumulated a large amount of kinetic energy and hence speed**.

Note: The dees are open at the diameter so that charges can pass from one dee into the other easily without colliding with the material making up the dees.

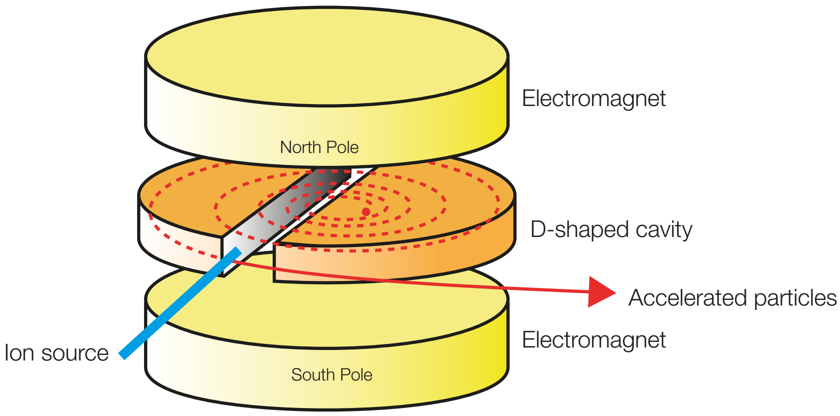


Figure 2.2.9

The ion source

The discussion that follows will assume that protons are accelerated by the cyclotron. An **ion source** injects a small amount of hydrogen gas into the apparatus. An electric current heats a filament that releases electrons in the gap between the dees. The electrons collide with the hydrogen gas atoms, ionising them by knocking out atomic electrons. This produces protons or positive hydrogen ions often symbolised using H^+.

The uniform electric field in the gap between the dees

The potential difference in the gap between the dees produces a uniform electric field between them. When the charged particles enters the electric field they are accelerated by the field because the electric field exerts a force on the charges. The magnitude of the force is given by $F = qE$. According the Newton's Second Law the magnitude of the acceleration is be given by $a = \frac{F}{m}$ and by definition, the charges gain speed.

Alternatively energy concepts can be used to explain why the charges gain speed. The electric field does work on the charges. Using the law of conservation of energy, the work done by the electric field is equal to the kinetic energy gained by the charges. That is, the electric field transfers kinetic energy to the charged particles.

$$\Delta W = q\Delta V = \Delta E_k = \frac{1}{2}mv^2$$

Since the charges are made to repeatedly cross the electric field, they are accelerated to high kinetic energies. The gain in kinetic energy is $q\Delta V$ every time the charges cross the electric field. It follows that the total kinetic energy gained by the charges as they exit the cyclotron and cross the electric field N times is given by $Nq\Delta V$.

The charges do not gain energy when inside the dees

There is no electric field inside the dees, as there is no electric field inside a hollow conductor (refer to Subtopic 2.1 for an explanation). The magnetic field inside the dees is designed to force the charged particles in a semi–circular path within the dees so that they repeatedly return to the electric field. We will discuss why the charges move in a circular path in Subtopic 2.3.

Worked Example

An early cyclotron had a diameter of 12.5 cm. The protons it accelerated completed 2000 complete revolutions inside the cyclotron when the potential difference applied between the dees was 5000 V. Calculate the

(a) kinetic energy of the protons as they left the cyclotron in J and MeV.

$\Delta E_k = W = Nq\Delta V = 4000 \times 1.60 \times 10^{-19} \times 5000 = 3.20 \times 10^{-12}\ J = 20\ MeV$

(b) speed of the protons as they left the cyclotron.

$\Delta E_k = W = q\Delta V = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 3.2 \times 10^{-12}}{1.67 \times 10^{-27}}} = 6.19 \times 10^{7}\ ms^{-1}$

Science as a human endeavour

We have already seen that Ernest Lawrence developed the first cyclotron along with Livingston. Since then, it has been developed further and can accelerate charged particles to very high kinetic energies. There are around 1200 cyclotrons used world wide for medical treatment and research. However, there are other types of particle accelerators, each useful in their own way.

Explore solutions to scientific problems developed using the motion of charges parallel or antiparallel to electric fields, such as:

- linear accelerators
- electron guns (e.g. in electron microscopes, oscilloscopes)
- ion thrusters (e.g. in spacecraft propulsion)
- X–ray tubes (e.g. in medicine).

The motion of charged particles at an angle to a uniform electric field

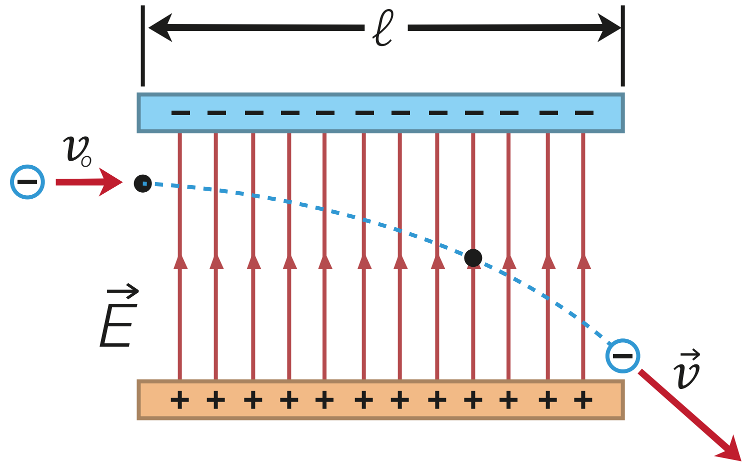


Figure 2.2.10

Consider a negative charge entering the uniform electric field between two parallel conducting plates as shown in Figure 2.2.10. The charge enters in a direction that is perpendicular to the electric field. The electric field exerts a constant force ($F=qE$) towards the lower plate. That is, in the vertical direction. The electric field does not exert a force on the charge in the horizontal direction, so it follows that the component of velocity perpendicular to the electric field (horizontal in Figure 2.2.10) remains constant. The result is that the charge follows a parabolic path towards the lower plate.

The path taken by the charged particle inside the electric field can be treated the same way that a projectile is treated in a gravitational field (covered in Subtopic 1.1). The magnitude of the acceleration of the charge is constant just as it is for a projectile in the absence of air resistance near the surface of the Earth but will not be 9.80 ms^{-2}. The magnitude of the acceleration of the charge needs to be calculated using $a = \frac{qE}{m}$.

The direction of the acceleration and hence path of the charge will depend on the sign of the charge moving in the electric field as well as the sign of the charge on the parallel conducting plates. For instance if the charge in Figure 2.2.10 was positive, it would deflect upwards. For a projectile near the surface of the Earth, the acceleration is always directed towards the ground resulting in a parabolic path towards the ground.

Similarly if the negative charge enters the uniform electric field at an angle other than 90°, its path will be parabolic and similar to that shown in Figure 2.2.11.

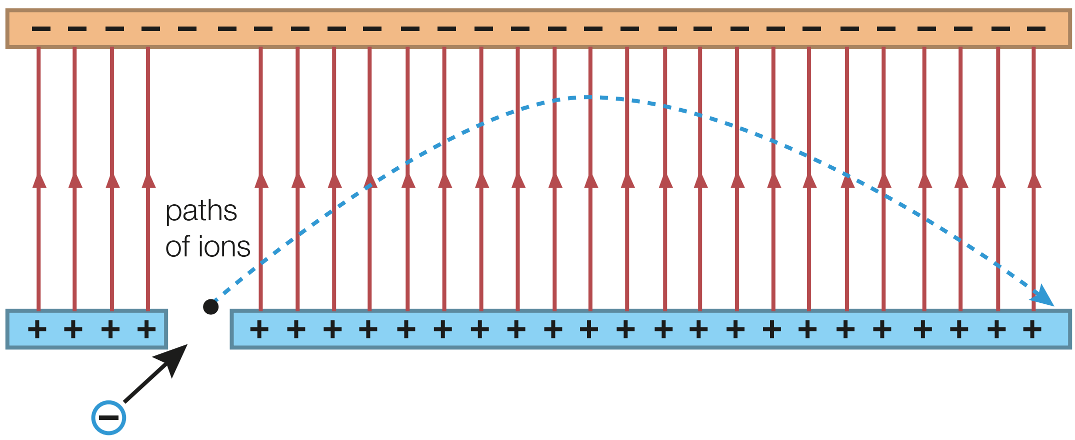
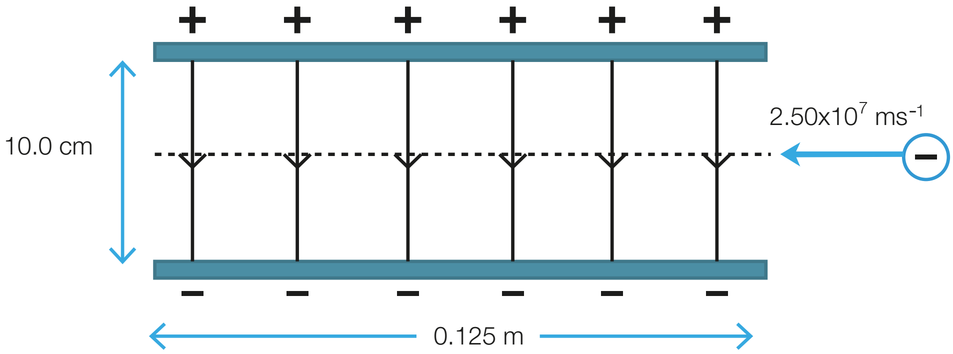


Figure 2.2.11

The motion of the charge within the electric field can be analysed using the equations of motion in the same way as a projectile near the surface of the Earth.

Worked Examples

1. A uniform electric field of magnitude 2.10×10^4 Vm^{-1} is created by applying a constant potential difference across two parallel conducting plates. The plates are separated by a distance of 10.0 cm as shown below.



An electron is fired horizontally and enters the electric field halfway between the plates with a speed of 2.50×10^7 ms^{-1}. The plates have a length of 0.125 m.

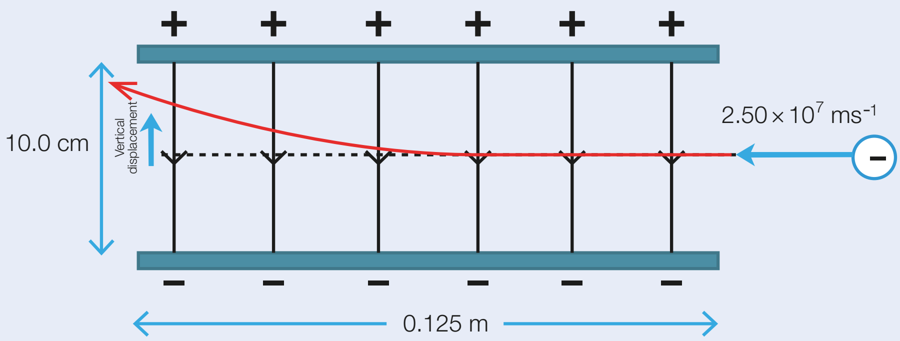
(a) Calculate the time of flight of the electron through the uniform electric field.

$$v = \frac{s}{t} \quad \therefore t = \frac{s}{v} = \frac{0.125}{2.50\times10^7} = 5.00\times10^{-9}s$$

(b) Calculate the magnitude and direction of the acceleration of the electron.

$$a = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 2.10 \times 10^4}{9.11\times10^{-31}} = 3.69\times10^{15}ms^{-2} \text{ towards the top plate}$$

(c) Draw the path of the electron in this electric field and label the vertical displacement as it leaves the field.



(d) Calculate the magnitude of the vertical displacement of the electron as it leaves the electric field.

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times3.69\times10^{15}\times(5.00\times10^{-9})^2 = 4.61\times10^{-2}m$$

(e) Calculate the velocity of the electron as it leaves the electric field.

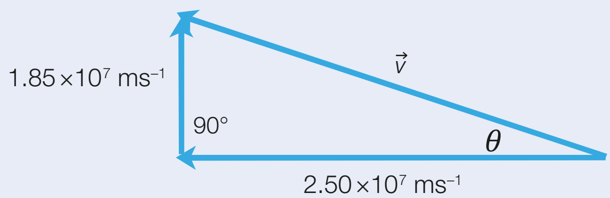
$$v_H = 2.50\times10^7\ ms^{-1} \leftarrow$$

$$v_v = v_0 + at$$
$$= 0 + 3.69\times10^{15}\times5\times10^{-9}$$
$$= 1.85\times10^7\ ms^{-1} \ \uparrow$$

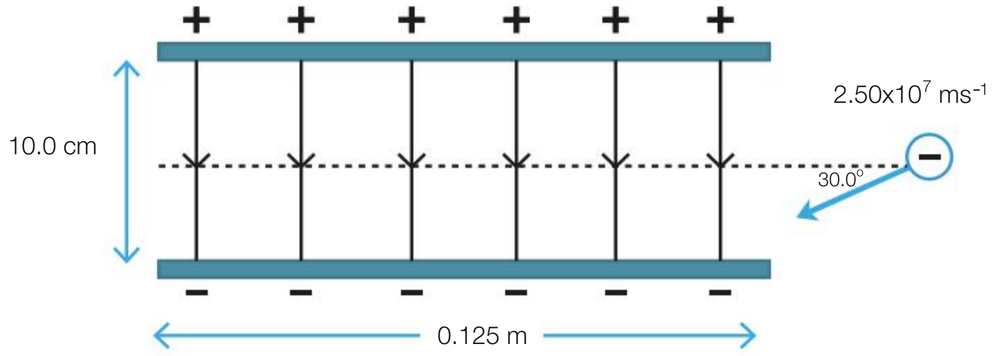
$$v = \sqrt{(2.5\times10^7)^2 + (1.85\times10^7)^2} = 3.11\times10^7 ms^{-1}$$

$$\tan\theta = \frac{1.85\times10^7}{2.5\times10^7} \Rightarrow \theta = 36.5°$$

$$\vec{v} = 3.11\times10^7 ms^{-1} \ 36.5° \text{ above the horizontal}$$



2. Now consider an electron entering the uniform electric field described in question 1 at a point midway between the plates with the same speed of 2.50×10^7 ms^{-1} but at an angle of 30.0° as shown in the diagram below.



This diagram is not to scale

(a) Calculate the magnitude of the horizontal component of the initial velocity of the electron.

$v_H = v\cos\theta = 2.5 \times 10^7 \cos 30.0 = 2.17 \times 10^7$ ms^{-1}

(b) Calculate the magnitude of the vertical component of the initial velocity of the electron.

$v_v = v\sin\theta = 2.5 \times 10^7 \sin 30.0 = 1.25 \times 10^7$ ms^{-1}

(c) Show that the electron does not strike the bottom plate.

$$v^2 = v_0^2 + 2as \quad \therefore \quad s = \frac{v^2 - v_0^2}{2a} = \frac{0^2 - (1.25\times10^7)^2}{2\times -3.69\times10^{15}} = 0.0212m$$

The distance between the midway point of the plates and the bottom plate is 5.00 cm. The electron undergoes a maximum vertical displacement of 2.12 cm and therefore does not strike the bottom plate.

(d) Show that the electron spends 5.76×10^{-9} s in the electric field.

$$v_H = \frac{s_H}{t} \quad \therefore \quad t = \frac{s_H}{v_H} = \frac{0.125}{2.17\times10^7} = 5.76\times10^{-9}s$$

? Science inquiry activity

A Teltron tube is pictured in Figure 2.1.12 and can be used to investigate the motion of electrons in a uniform electric field.

A small current heats the filament which in turn releases electrons. The electrons then pass through a pair of accelerating plates and gain speed. The filament and the accelerating plates are called an **electron gun**.

Next, the electrons enter a uniform electric field created between two parallel conducting plates that have a constant potential difference applied across them. A grid within the tube can be used to measure the deflection of the electron beam.

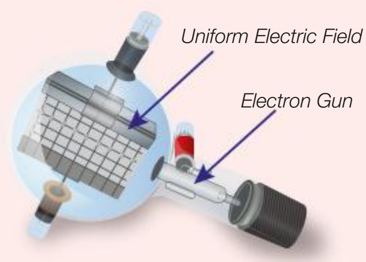


Figure 2.1.12

A possible investigation is to determine the relationship between the electric field and the deflection of the electrons. This can be done by varying the potential difference that creates the uniform electric field ($E = \frac{\Delta V}{d}$) and measuring the deflection of the electron beam directly from the grid within the tube.

Exercises

1. An electron gun produces a stream of electrons that are accelerated across a potential difference of 3.8×10^3 V.

   Calculate the energy supplied to the electrons by the accelerating potential.

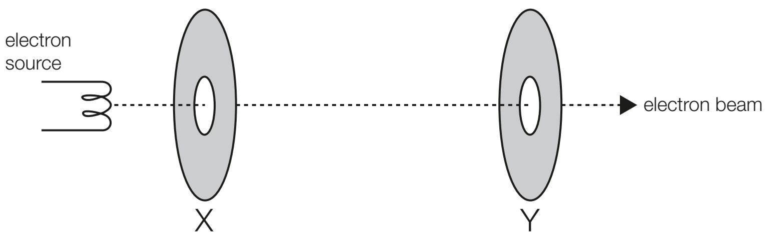
2. An alpha particle has a charge of $q = +3.20 \times 10^{-19}$ C and a mass of 6.645×10^{-27} kg. It is released from rest in a uniform electric field and accelerates across a potential difference of 50.0 KV.

   Calculate the

   (a) work done on the alpha particle by the electric field in J and eV.

   (b) gain in speed of the alpha particle accelerated by this potential difference.

3. A synchrotron is a device used to accelerate charged particles. The diagram below shows the injector gun of a synchrotron.



   An accelerating potential is used to accelerate electrons across XY. The electrons, initially at rest, enter the injector gun at X and leave at Y with a speed of 2.68×10^7 ms^{-1}.

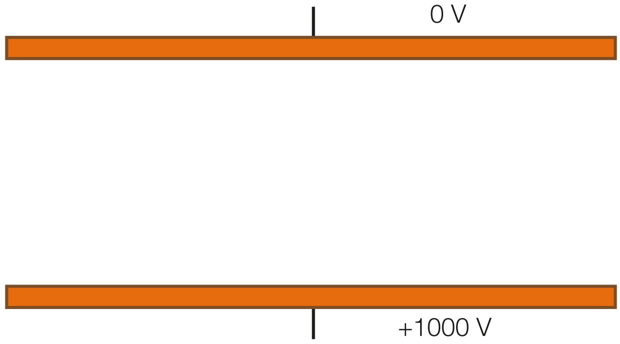
   (a) Show that the potential difference between XY is given by:

$$\Delta V = \frac{mv^2}{2q}$$

   (b) Hence, calculate the potential difference across XY.

   (c) The accelerating potential between X and Y is doubled. Use proportionality to discuss the effect on the speed of the electrons leaving Y.

4. Consider the two parallel conducting plates pictured below. The plates are separated by a distance $d = 2.0 \times 10^{-2}$ m and a constant potential difference ΔV of 1000 V is applied across the plates.



(a) Draw the electric field between the plates (ignore any end effects).

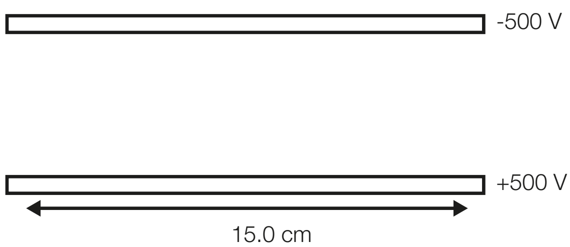
(b) Calculate the magnitude of the electric field between the plates.

(c) An electron is released from the top plate. Calculate the work done by the electric field in accelerating the electron to the lower plate.

(d) Show that the magnitude of the acceleration of the electron is given by $a = \frac{qE}{m}$.

(e) Hence calculate the magnitude and direction of the acceleration of the electron in this electric field.

5. Consider the parallel conducting plates pictured below.



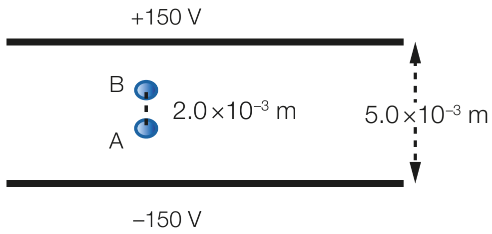
(a) Describe and explain the nature of the electric field between the plates.

(b) A uniform electric field of magnitude $E = 4.50 \times 10^3$ Vm^{-1} is created by the potential difference. Calculate the distance between the plates.

(c) A proton is released from the lower plate and accelerates across the electric field. Calculate the speed of the proton as it strikes the top plate.

(d) Calculate the time taken for the proton to traverse the field.

6. The diagram below shows two parallel conducting plates 5.0×10^{-3} m apart.



(a) Calculate the magnitude and direction of the electric field between the plates.

(b) A charge of magnitude –3.4 μC and mass 2.6×10^{-15} kg is released from rest at point A in the electric field.

(i) Calculate the magnitude and direction of the acceleration of the –3.4 μC charge in this electric field.

(ii) Calculate the work done by the electric field in accelerating the –3.4 μC charge between the two points A and B which are 2.0×10^{-3} m apart.

(iii) Calculate the time taken for the –3.4 μC charge to move from point A to point B in this electric field.

7. (a) Calculate the magnitude of the electric field that is created when a potential difference of 2.00×10^3 V is applied across two parallel conducting plates that are placed 10.0 cm apart.

(b) A proton is placed in the middle of the electric field as shown in the diagram below.

(i) Calculate the magnitude of the force on the proton.

(ii) Explain whether the magnitude of the force calculated in part (i) depends on the position of the proton.

8. The ion thruster of the Dawn spacecraft is used to accelerate xenon ions (Xe^+) with a single positive charge of $+1.60 \times 10^{-19}$ C and mass 2.20×10^{-25} kg across a potential difference of 1280 V. The xenon ions are initially at rest.

Calculate the

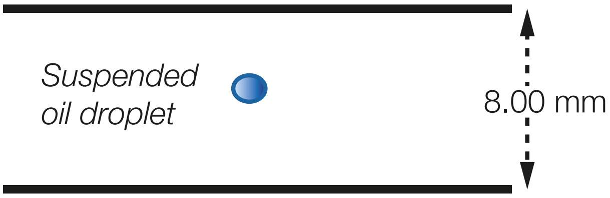
(a) kinetic energy gained by the xenon ions as they are accelerated through the ion thruster.

(b) gain in speed of the xenon ions as they leave the ion thruster.

(c) Calculate the magnitude of the acceleration of the xenon ions as they pass through the ion thruster in 3.00×10^{-5} s.

(d) distance the xenon ions need to be accelerated in order to exit the ion thruster in 3.00×10^{-5} s.

9. A negatively charged oil droplet with a mass of 4.90×10^{-16} kg remains suspended between two parallel conducting plates separated by distance of 8.00 mm when an electric field of 1.50×10^{4} NC^{-1} is created between the plates. The top plate is held at a positive potential.



(a) Draw labelled vector arrows to represent the forces acting on the oil drop.

(b) Determine the charge and hence the number of electrons on the oil drop.

(c) Calculate the potential difference applied between the parallel conducting plates.

(d) The separation of the plates is increased to 24.0 mm without changing the potential difference between the plates. Discuss the effect this would have on the electric field between the plates.

10. An X–ray tube is used to accelerate electrons across a potential difference of 120 KV before striking a metal target and producing X–rays for medical imaging.

Calculate the speed of the electrons as they strike the metal target.

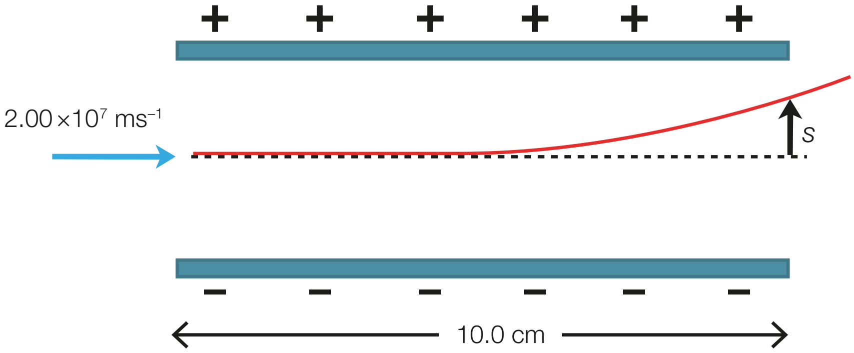
11. A cyclotron has a potential difference of 5.5×10^{3} V applied between the dees. Protons within the cyclotron complete 8000 complete revolutions before leaving the cyclotron.

(a) Describe how the electric field can transfer energy to a proton passing between the dees of a cyclotron.

(b) Calculate the kinetic energy of the protons as they leave the cyclotron in J and MeV.

12. Consider the parallel conducting plates shown below. The magnitude of the electric field between the plates is 3.60×10^3 Vm^{-1}.

An electron is fired horizontally and mid–way between the plates with a speed of 2.00×10^7 ms^{-1}. The electron traces a parabolic path towards the upper plate.



(a) Explain why the path of the electron is parabolic.

(b) Calculate the magnitude and direction of the acceleration of the electron.

(c) Calculate the time taken for the electron to cross the uniform field.

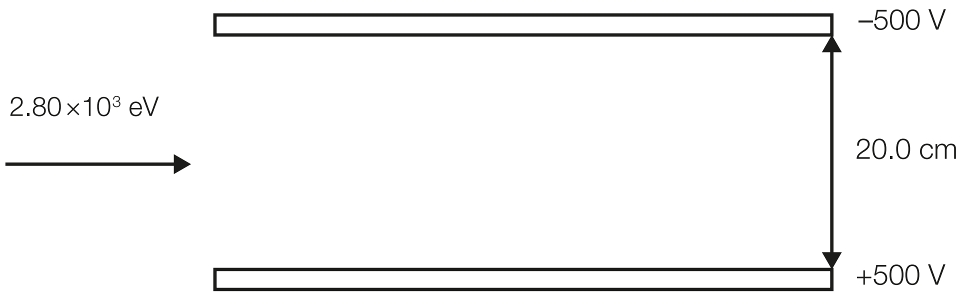
(d) Show that the vertical deflection of the electron as they leave the electric field *s*, is 7.90×10^{-3} m.

(e) Calculate the magnitude and direction of the velocity of the electron as it leaves the region between the plates.

Space for vector diagram

(f) Determine the work done by the electric field in deflecting the electron.

13. A beam of protons enters the region between two parallel conducting plates as shown below. The protons takes 66.0 ns to cross the length of the plates.



(a) Calculate the speed of the protons as they entered the electric field.

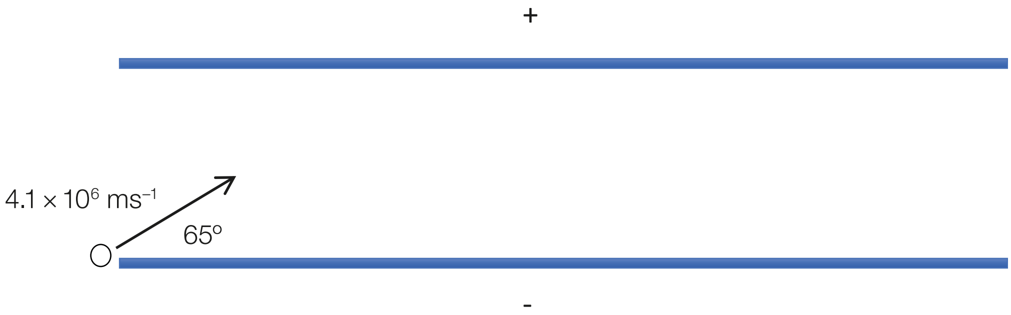
(b) Calculate the magnitude and direction of the electric field between the plates.

(c) Calculate the length of the parallel conducting plates.

(d) Calculate the vertical distance through which the protons are deflected.

(e) Calculate the work done by the electric field in deflecting the protons.

14. A charged particle $q = +4.6$ nC with mass $m = 1.8 \times 10^{-25}$ kg enters a uniform electric field between two parallel conducting plates with a speed of 4.1×10^{6} ms^{-1} at an angle of 65°. The charge enters so that its initial position coincides with the edge of the lower plate.



(a) Calculate, in J and eV, the kinetic energy of the +4.6 nC charge as it enters the electric field.

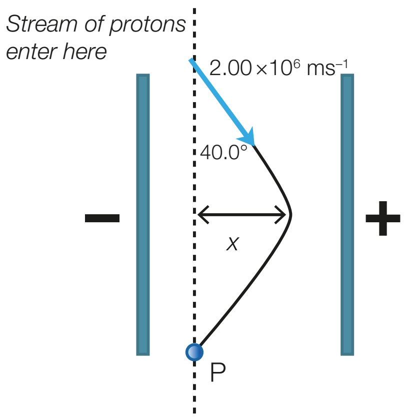
(b) Calculate the magnitude of the horizontal component of the initial velocity of the charge.

(c) Calculate the magnitude of the vertical component of the initial velocity of the charge.

The charged particle reaches a maximum vertical displacement of 4.0×10^{-3} m within the electric field.

(d) Calculate the magnitude and direction of the acceleration of the charge in this electric field.

15.



A stream of protons enter an electric field at an angle of 40.0° as shown in the above diagram. The protons experience an acceleration of magnitude 1.54×10^{14} ms^{-2}.

(a) Calculate the magnitude of the component of initial velocity that is parallel to the electric field.

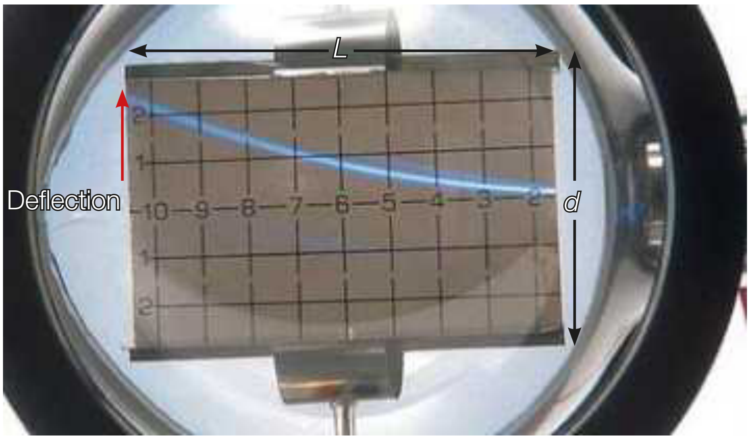
(b) Calculate the magnitude of the component of initial velocity that is perpendicular to the electric field.

(c) Show that the value of the distance x shown in the diagram is 5.40×10^{-3} m.

(d) Calculate the time of flight of the protons in this electric field if they leave the electric field at the point marked P.

(e) Use your answer to part (d) to calculate the length of the parallel conducting plates.

16. The image below shows a beam of electrons being deflected by a uniform electric field in a Teltron tube. The electrons enter the tube with a speed *v* that is perpendicular to the electric field. The plates are a distance *d* = 6.0 cm apart and have a length *L* = 12 cm.



A student varied the potential difference ΔV between the parallel conducting plates and measured the vertical deflection *s* of the electrons as shown. The student repeated the experiment three times and averaged the vertical deflection measured for each potential difference applied across the plates.

(a) State with reason, the independent variable in this experiment.

(b) Explain why repeating the experiment and averaging three values for the vertical deflection measured for each potential difference is a good experimental technique.

(c) Show that the deflection of the electrons is given by:

$$s = \frac{e\Delta VL^2}{2mdv^2}$$

where *e* is the charge of an electron.

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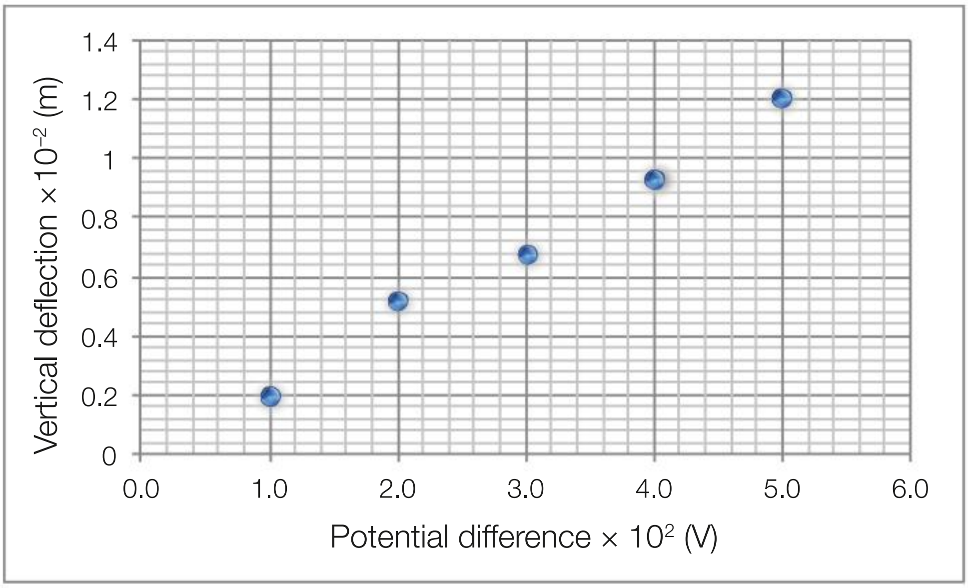
The student plotted a graph of the average deflection versus potential difference across the plates. The graph is shown below.



(d) (i) Draw a line of best fit for the plotted data.

(ii) Calculate the gradient of the line of best fit.

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(iii) Use your answer to part d (ii) to calculate the speed with which the electrons enter the electric field.

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2.3 Magnetic fields

Science understanding

1. Magnetic fields are associated with permanent magnets and moving charges, such as charges in an electric current.
2. Current–carrying conductors produce magnetic fields; these fields are utilised in solenoids.
3. Magnetic field lines can be used to represent the magnetic field. The direction of the magnetic field depends on the direction of the moving charge that is producing the magnetic field.
4. The direction and number of magnetic field lines per unit area represent the direction and magnitude of the magnetic field.
   - Sketch and/or interpret the magnetic field lines produced by a bar magnet, and an electric current flowing in a straight conductor, a loop, and a solenoid.
5. The magnitude of the magnetic field strength, B, at a radial distance, r, from the centre of a current-carrying conductor is given by $B = \frac{\mu_0}{2\pi}\frac{I}{r}$.
   - Solve problems involving the use of $B = \frac{\mu_0}{2\pi}\frac{I}{r}$
   - Use vector addition in one dimension or in two dimensions (with right-angled or equilateral triangles) to calculate the magnitude and direction of the magnetic field due to two current-carrying conductors)

This chapter uses the concepts of electric current developed in the Stage 1, Subtopic 2.1 Potential Difference and Electric Current.

The magnetic field of a bar magnet

? Science inquiry activity

This activity will enable you to plot the magnetic field associated with bar magnets and better understand the concepts discussed later in the chapter.

A bar magnet has a north and south pole at opposite ends and produces a magnetic field in the space surrounding it. The magnetic field surrounding bar magnets can be mapped by sprinkling iron filings in the vicinity of the magnet.

1. Place a bar magnet on a bench.
2. Cover the magnet with a clear sheet of plastic.
3. Sprinkle iron filings over the plastic.
4. Clear the iron filings from the plastic sheet and try two bar magnets in different orientations i.e. two north poles facing one another and a north and south pole facing one another.

Figure 2.3.1: Bar magnets

Result

Each iron filing becomes magnetised by induction and acts like a tiny magnet. The end of the iron filing closest to the north pole of the magnet becomes a south pole and the other end a north pole. The iron filings move so that the induced north pole aligns itself closer to the south pole of the bar magnet. Each iron filing behaves in a similar fashion. The force experienced by the iron filings cause them to line up with the magnetic field. This reveals the magnetic field of the bar magnet as shown in Figure 2.3.2.

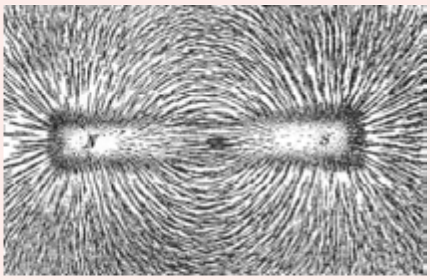


Figure 2.3.2

? Science inquiry activity

A magnetic field will cause a compass needle to deflect. The direction of a magnetic field at a point is defined as the direction that the north pole of a compass points.

1. Place a bar magnet on a bench.
2. Place a compass at the north pole of the bar magnet and note the direction that it points.
3. Place a compass at the south pole of the bar magnet and note the direction that it points.
4. Place the compass at the north pole of the bar magnet and slowly move the compass around the bar magnet from the north pole to the south pole. What do you notice about the deflection of the compass?
5. Repeat step 4 but this time place the compass further from the magnet. What do you notice about the deflection of the compass?

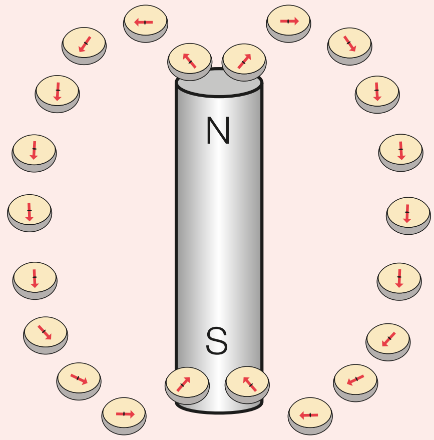


Figure 2.3.3

Result

The compass will point away from the north pole of the bar magnet and towards the south pole as shown in Figure 2.3.3. The deflection of the compass is weaker further from the bar magnet. This indicates a weaker magnetic field.

The magnetic field for a single bar magnet is shown in Figure 2.3.4.

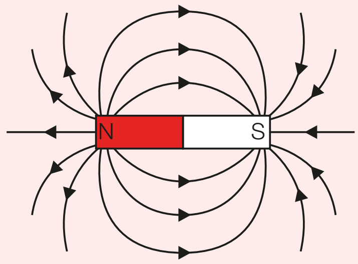


Figure 2.3.4

Explore the plotting of magnetic field lines and their properties.

https://www.youtube.com/watch?v=NWUgK8W-4JM

Key ideas

The magnetic field surrounding the bar magnet can be represented using magnetic field lines.

The **direction** of the magnetic field is given by the direction that the north pole of a small compass needle points. The **direction** of the magnetic field **at a given point** is at a tangent to the magnetic field lines.

Magnetic field lines are directed from the north seeking pole to the south seeking pole.

The **magnitude** of the magnetic field is represented by the number of magnetic field lines crossing a unit area perpendicular to the field in the vicinity of the point. The greater the number of field lines crossing a unit area, the stronger the magnetic field.

The **unit of magnetic field** is the **Tesla (T)**.

The **symbol** for magnetic field is $\vec{B}$.

Magnetic field is a vector quantity.

For interest, the magnetic fields produced by two bar magnets in different orientations are shown in Figures 2.3.5 to 2.3.8.

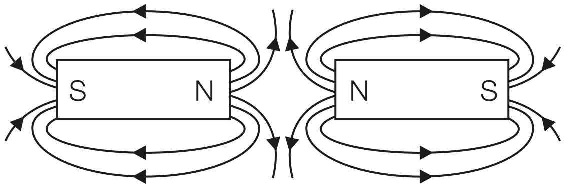


Figure 2.3.5: Like poles facing each other

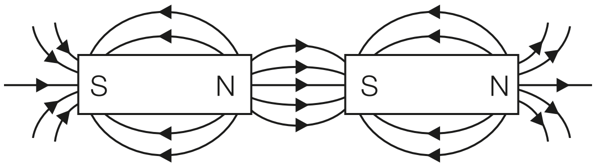


Figure 2.3.6: Unlike poles facing each other

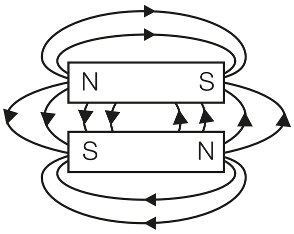


Figure 2.3.7: Unlike poles adjacent

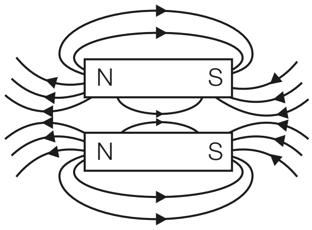


Figure 2.3.8: Like poles adjacent

Magnetic fields and moving charges

We have already seen in Subtopic 2.1 that stationary charges produce an electric field in the space surrounding them. Moving charges will produce both electric and magnetic fields.

In 1819 Hans Christian Oersted, a Danish physicist showed that a current–carrying conductor produced a magnetic field. Activity 3 recreates how he did this.

? Science inquiry activity

The magnetic field for a straight current–carrying conductor

Moving charges, such as the charges in a current flowing through a conductor will produce a magnetic field in the space surrounding the conductor. This activity will allow you to investigate the magnetic field associated with a straight current–carrying conductor.

1. Form a horizontal plane using a piece of stiff cardboard with a hole in its centre.
2. Pass a long wire through the hole and straighten the wire by supporting it vertically using two retort stands.
3. Connect the wire in series to a switch, resistor and a power supply so that a current of about 5 Amperes flows through the wire. Connect the top of the wire to the negative terminal of a power supply and the bottom of the wire to the positive terminal as shown in Figure 2.3.9.
4. Place a compass on the horizontal card.
5. Complete the circuit by closing the switch. What happens to the compass?

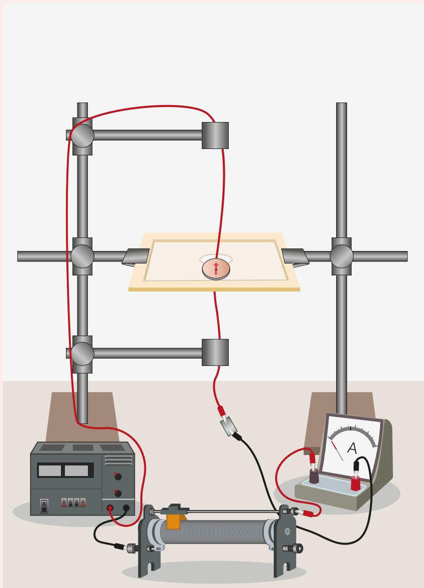


Figure 2.3.9

6. Move the compass around the wire at a constant radial distance. What do you notice about the deflection of the compass? Note the direction that the north pole of the compass points.
7. Move the compass to a position that is at a greater radial distance from the wire. What do you notice about the deflection of the compass?
8. Reverse the direction of the current flowing through the wire by swapping the connecting leads at the top and bottom of the wire.

Results

When the switch is closed, a current (I) will flow up through the straight current–carrying conductor and you will notice that the needle of the compass will deflect. This indicates the presence of a magnetic field. As you move the compass around the wire, you will find that it forms tangents to a circle. The direction that the north pole of the compass points at any given position indicates the direction of the magnetic field at that point. This is illustrated in Figure 2.3.10.

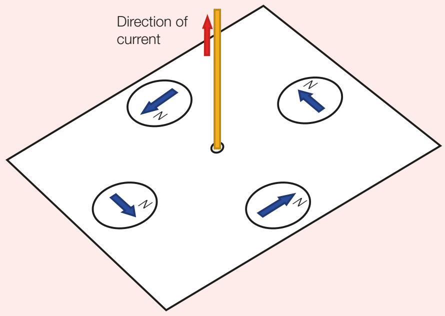


Figure 2.3.10

Recall: The **direction** of the magnetic field is given by the direction that the north pole of a small compass needle points. The **direction** of the magnetic field **at a given point** is at a tangent to the magnetic field lines.

The deflection of the compass decreases further from the wire. This indicates that the magnetic field becomes weaker further from the wire.

The magnetic field surrounding a straight current–carrying conductor can be represented using magnetic field lines that form concentric circles around the conductor. The magnetic field lines become further apart as the distance from the current–carrying conductor increases.

Figure 2.3.11 below illustrates the magnetic field around a straight current–carrying conductor with the current flowing up the plane of the page.

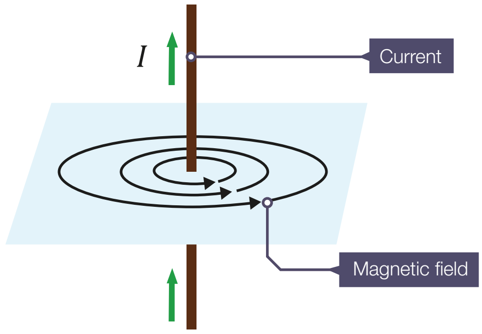


Figure 2.3.11: The magnetic field for a straight current–carrying conductor

Reversing the current in the wire reverses the direction of the magnetic field. This is illustrated in Figure 2.3.12.

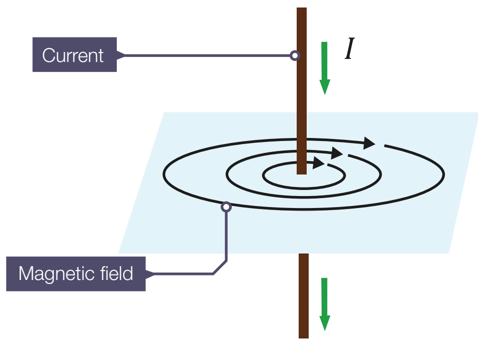


Figure 2.3.12

The right hand rule for determining the magnetic field for a conductor

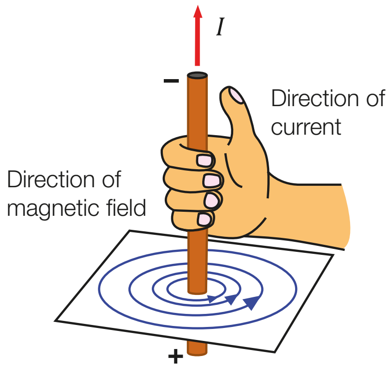


Figure 2.3.13

Figure 2.3.13 illustrates the **right hand rule**. This rule determines the direction of the magnetic field surrounding a current carrying conductor. The thumb points in the direction of conventional current and the curl of the fingers indicates the direction of the magnetic field. The magnetic field extends along the length of the wire.

Other representations

A current carrying conductor with a current flowing into the plane of the page is represented by a circle with a cross in it. ⊗

A current carrying conductor with a current flowing out of the plane of the page is represented by a circle with a dot in it. ⊙

The magnetic field surrounding a straight current–carrying conductor with the current flowing out of the plane of the page is shown in Figure 2.3.14 and the magnetic field surrounding a straight current–carrying conductor with the current flowing into the plane of the page is shown in Figure 2.3.15 below.

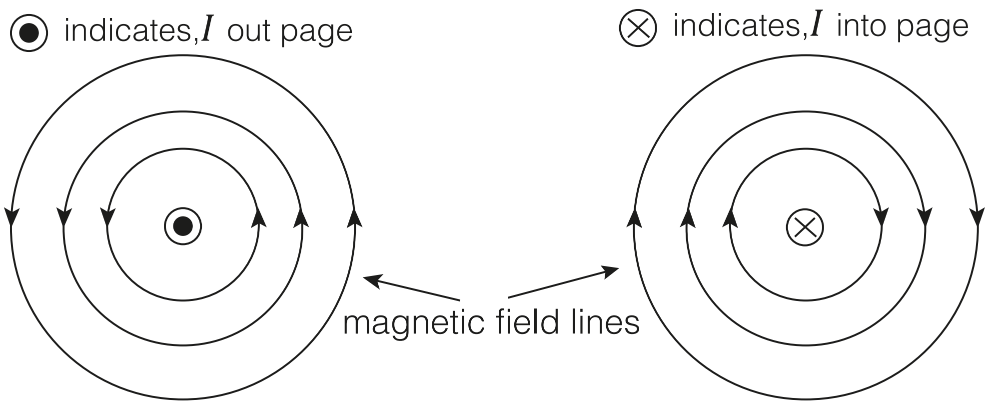


Figure 2.3.14 *Figure 2.3.15*

The magnetic field for a single loop

If a straight current carrying conductor is used to form a loop, the magnetic field inside the loop is stronger than the magnetic field outside the loop. The right hand rule can be used to determine the direction of the magnetic field both inside and outside the loop.

The magnetic field surrounding a single loop is illustrated in different ways in Figures 2.3.16 to 2.3.19. Figure 2.3.16 shows the magnetic field all the way around a loop which rests in the plane of the page. The magnetic field lines inside the loop are closer together than the magnetic field lines outside the loop. This indicates that the magnetic field inside the loop is stronger than the magnetic field outside the loop. Figure 2.3.17 illustrates the magnetic field in a plane perpendicular to the loop while Figure 2.3.18 illustrates the magnetic field when the current–carrying conductor is viewed from the side. Figure 2.3.19 illustrates the magnetic field using the convention of crosses without a circle around them (magnetic field into the page) and dots without a circle around them (magnetic field out of the page).

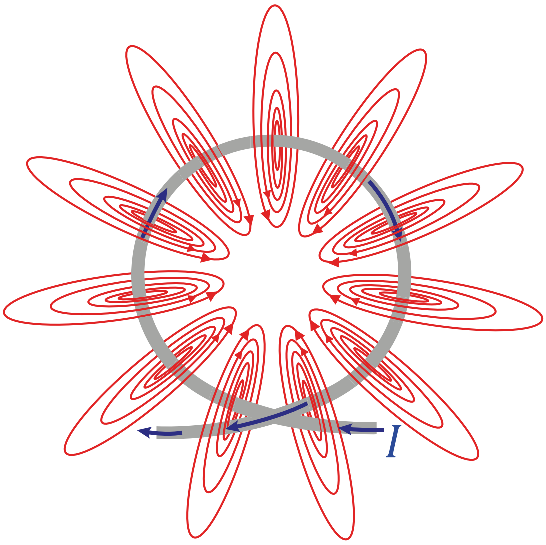


Figure 2.3.16

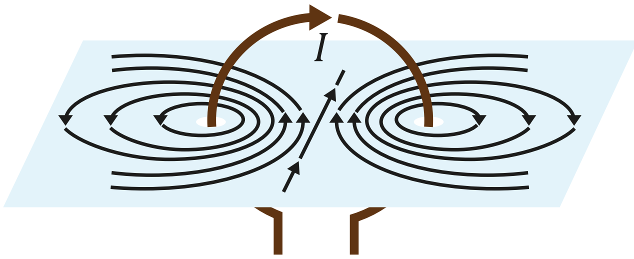


Figure 2.3.17

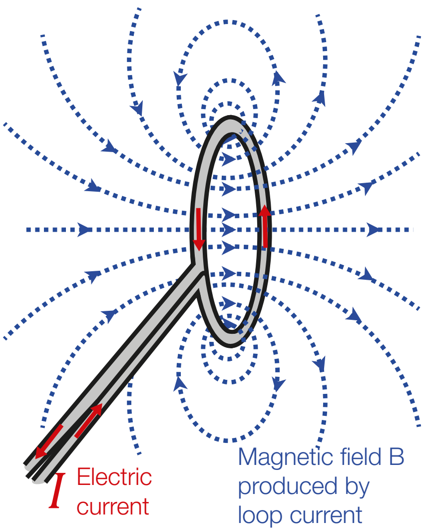


Figure 2.3.18

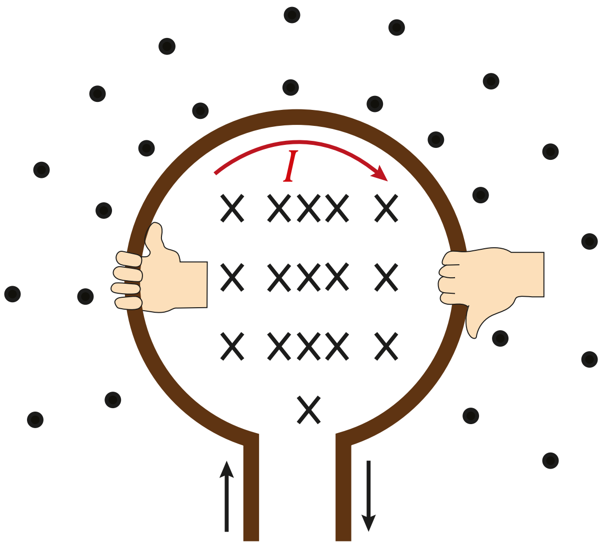


Figure 2.3.19

Solenoids

We have already seen that current–carrying conductors produce magnetic fields. These magnetic fields can be utilised in solenoids.

A **solenoid** consists of many coils of wire. The coils are often wrapped around a plastic cylinder.

When a current flows through a solenoid, a magnetic field is produced. The magnetic field of a solenoid can be investigated by replacing the straight current–carrying conductor in Activity 3 with a solenoid. A compass will deflect and the field surrounding the solenoid can be mapped.

Figure 2.3.20 illustrates a solenoid with a current flowing clockwise through the coils. It can be seen that there is a magnetic field both inside and surrounding the solenoid. The magnetic field resembles the magnetic field of a bar magnet. The north pole of the compass needle is pointing away from the open end shown in the diagram. This indicates that a north pole is produced at this end. The other end of the solenoid produces a south pole.

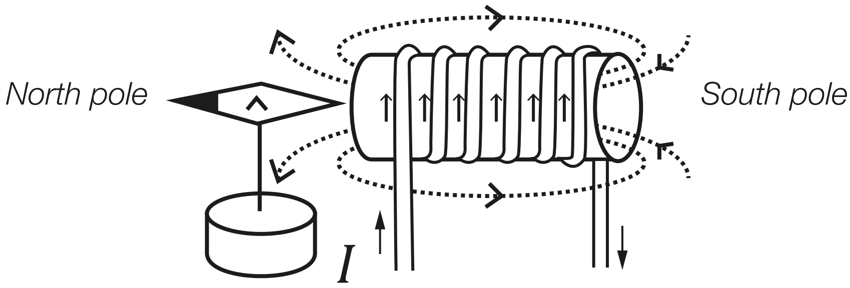


Figure 2.3.20

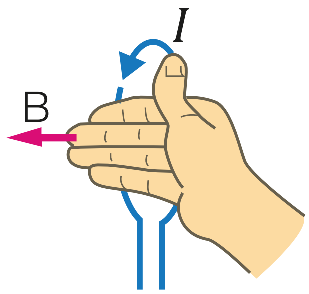


Figure 2.3.21

The **right hand rule** can also be used to determine which end of the solenoid is a north pole if you know the direction of the current. In Figure 2.3.20, your thumb would point in the direction of the current and the curl of your fingers would point into the solenoid on the right hand side and out on the left hand side. This is illustrated in Figure 2.3.21 by considering a single loop of the solenoid.

The **strength** of the magnetic field produced by a current–carrying solenoid can be increased by:

1. increasing the number of coils
2. increasing the current passing through the coils
3. placing an iron core inside the solenoid.

Figures 2.3.22 and 2.3.23 show two ways to illustrate the magnetic field of a solenoid. Figure 2.3.23 show the magnetic field for a cross sectional view of a solenoid.

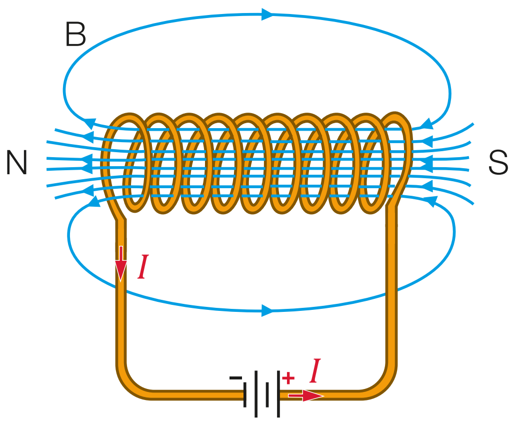


Figure 2.3.22

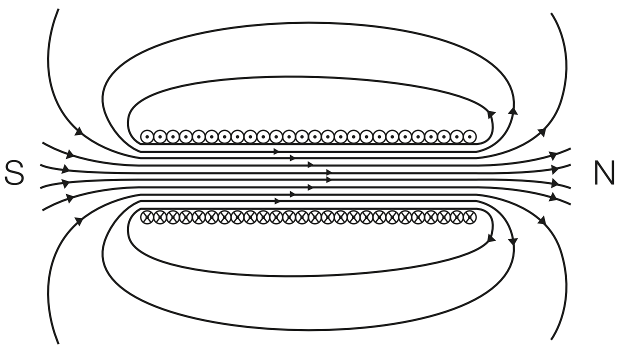


Figure 2.3.23

? Science inquiry activity

There are various data logging probes that can be used to measure the magnetic field. One such probe is the Pasco **PASPORT Magnetic Field Sensor–**PS–2112.

A possible investigation is to investigate the factors that affect the magnetic field strength near a solenoid e.g. the relationship between the current flowing through the solenoid and the magnetic field strength.

Extra Understanding - Electromagnets and their uses

A solenoid with a current flowing through its coils is referred to as an electromagnet.

An electromagnet can easily be made in a laboratory. Figure 2.3.24 illustrates the apparatus needed to do this.

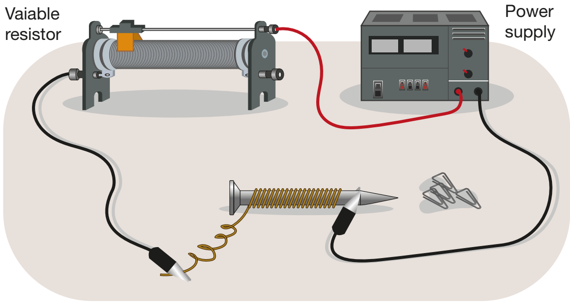


Figure 2.3.24

1. Wrap a long piece of insulated copper wire around an iron nail to form a solenoid.
2. Sand the ends of the wire to remove the insulation.
3. Connect the solenoid to a variable resistor and a power supply (8 V).
4. When the power supply is turned on, the solenoid becomes an electromagnet. That is magnetism is induced when the current flows.
5. Its strength can be investigated by varying the number of turns of wire on the nail or varying the current (via the variable resistor or power supply) and seeing how many paper clips can be picked up.

Uses of electromagnets

Electromagnets are useful because they behave like a magnet when current flows. When the current no longer flows, the electromagnet no longer acts as a magnet. In addition, the strength of the magnetic field can be controlled by adjusting the number of coils or the current flowing through the coils. If either or both of these increase, the magnetic field will be stronger.

This is useful in car wrecking yards. Cars are picked up by large electromagnets, moved and then released (Figure 2.3.25). Other common uses include electric bells, ticker timers used to record the motion of objects, motors, loud speakers, cyclotrons and MRI scanners.

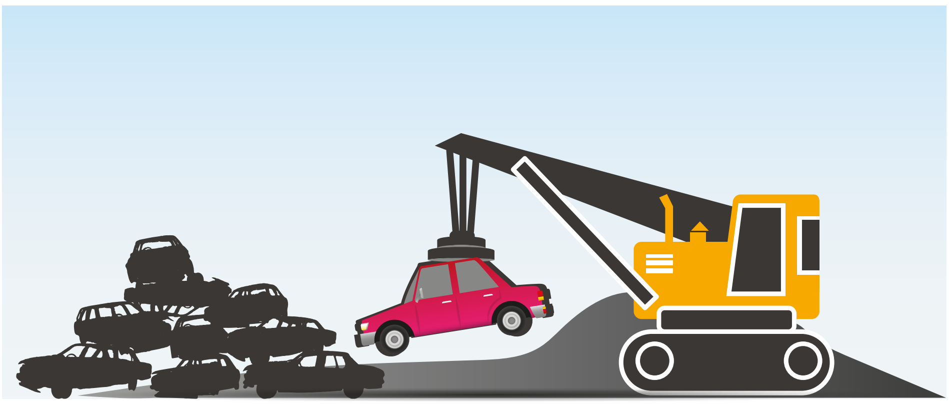
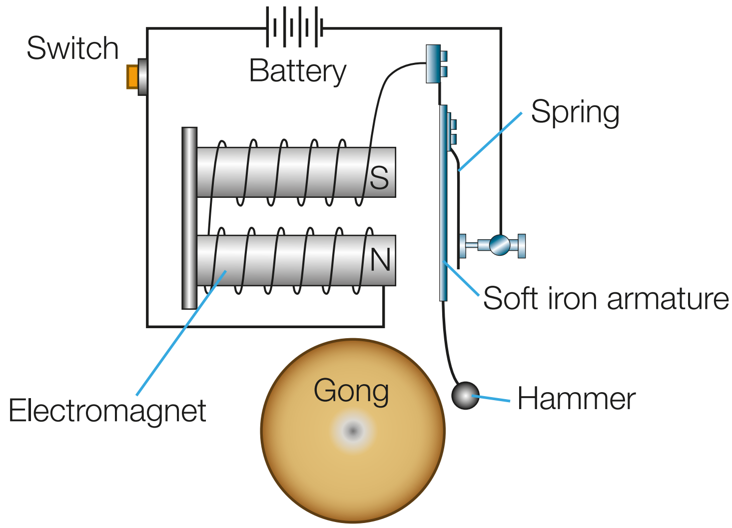


Figure 2.3.25

Worked Example

Consider the diagram below for an electric bell.



Use the labels on the diagram to explain why the bell rings continuously when the switch is pushed.

Pushing the switch completes the circuit and a current flows. The electromagnet creates a magnetic field that attracts the soft iron armature. The hammer hits the gong. The spring moves to the left and the circuit is broken.

The electromagnet releases the armature and hammer and the circuit is once again complete. The process repeats.

The magnitude of the magnetic field strength

We have already seen that the magnetic field in the vicinity of a straight current–carrying conductor consists of concentric circles and that the strength of the magnetic field deceases with distance from the conductor.

The magnetic field strength is directly proportional to the current flowing through the conductor ($B \alpha I$), just as gravitational field strength is directly proportional to the magnitude of the mass creating the gravitational field and electric field strength is directly proportional to the charge creating the electric field.

The magnetic field strength is inversely proportional to the radial distance from a conductor ($B \alpha \frac{1}{r}$).

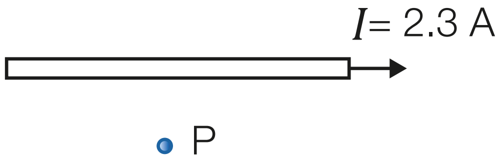
The magnitude of the magnetic field strength in the vicinity of a current-carrying conductor is given by

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

where $\frac{\mu_0}{2\pi}$ is the constant for the magnetic field around a conductor and equal to 2.00×10^{-7} T m A^{-1} and r is the radial distance from the centre of the current-carrying conductor in metres. The unit of magnetic field is the Tesla (T).

Worked Examples

1. Consider the straight current–carrying conductor shown below.



(a) State the direction of the magnetic field at the point P located 12 cm directly below the centre of the conductor.

Into the page

(b) Calculate the magnitude of the magnetic field at the point P.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 2.3}{0.12} = 3.8 \times 10^{-6}\ T$$

(c) Use proportionality to discuss the effect on the magnitude of the magnetic field strength at P if the current through the conductor is tripled.

Since $B \alpha I$, the magnitude of the magnetic field strength at P will be three times larger if the current is increases by a factor of three.

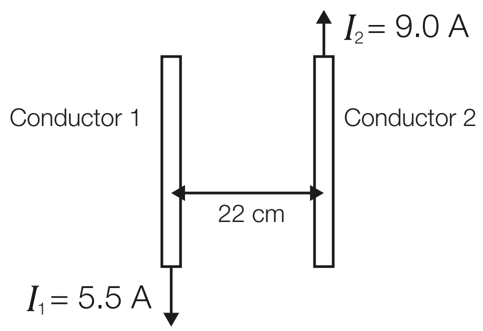
2. The magnitude of the magnetic field strength is 5.44×10^{-5} T at a radial distance of 5.00 cm from the centre of a conductor.

Calculate the current flowing through the conductor.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times I}{r} = 5.44 \times 10^{-5}\ T$$

$$\therefore\ I = \frac{Br}{2.00 \times 10^{-7}} = \frac{5.44 \times 10^{-5} \times 0.0500}{2.00 \times 10^{-7}} = 13.6\ A$$

3. Calculate the magnetic field midway between the two straight current–carrying conductors shown below.



The magnitude of the magnetic field midway between the conductors due to conductor 1

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 5.5}{0.11} = 1.0 \times 10^{-5}\ T\ \textit{out of the page}$$

The magnitude of the magnetic field midway between the conductors due to conductor 2

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 9.0}{0.11} = 1.6 \times 10^{-5}\ T\ \textit{out of the page}$$

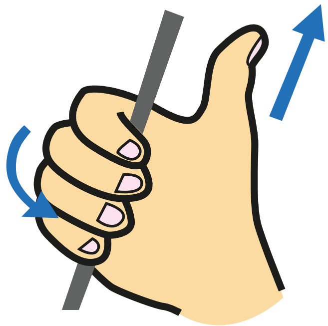
Magnetic field is a vector quantity, therefore the total magnetic field midway between the two conductors is given by the vector sum of the individual magnetic fields:

1.0×10^{-5} T out of the page + 1.6×10^{-5} T out of the page = 2.6×10^{-5} T out of the page

Exercises

1. The right hand rule can be used to determine the direction of a magnetic field due to a current flowing through a conductor.

   Consider the diagram below of a right hand with the thumb pointing upwards and the fingers curled around.

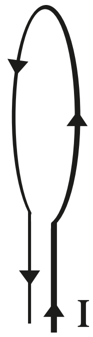


   Clearly label each arrow with the quantity it represents.

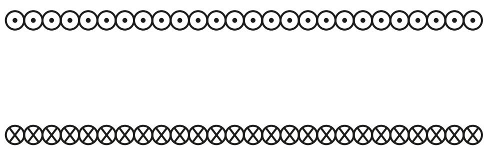
2. Sketch magnetic field lines in the vicinity of each of the following:

   (a) a single straight conductor carrying a current out of the plane of the page.

   (b) a circular loop of wire carrying a current.



   (c) a solenoid carrying a current



3. Explain how magnetic field lines can be used to represent the
   (a) direction of the magnetic field at a ***point***.

   ....

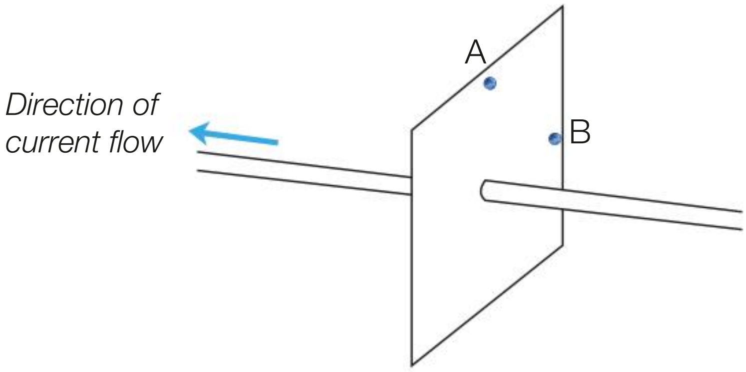
   ....

   (b) magnitude of the magnetic field at a ***point***.

   ....

   ....

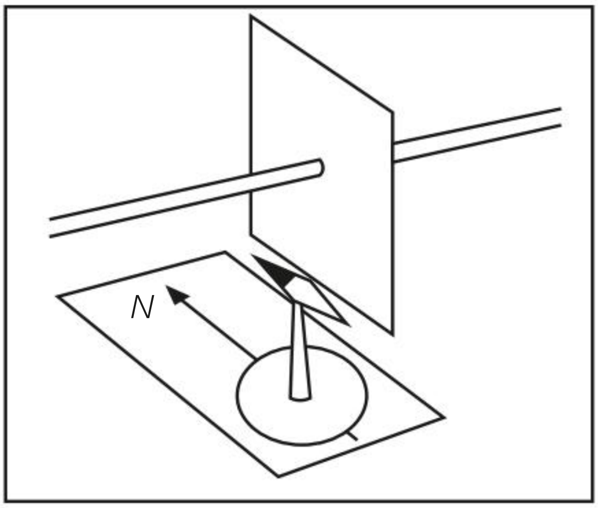
4. (a) Sketch the magnetic field due to the electric current flowing through the conductor shown below. The current flows to the left of the page.



   (b) Consider the two points A and B as shown on the diagram. Point A lies directly above the conductor while point B directly to the right of the conductor.

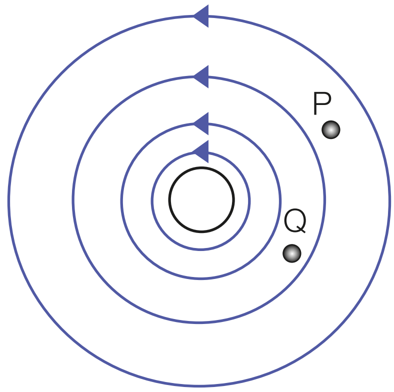
   Draw an arrow at each point A and B to indicate the direction of the magnetic field at each of these points.

5. The diagram below illustrates the deflection of a compass when it is placed near a straight current–carrying conductor.



   (a) Draw three magnetic field lines in the vertical plane shown that represent the magnetic field of the conductor.
   (b) Use a labelled arrow to clearly indicate the direction of the current *I*, through the conductor.

6. Consider the magnetic field surrounding the straight current–carrying conductor below.



(a) State the direction of the current flowing through the conductor.

....

(b) State the effect on the spacing of the magnetic field lines if the current flowing through the conductor is doubled.

....

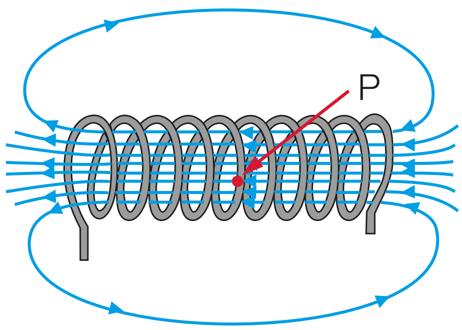
(c) Consider the points P and Q. Point P is positioned at a radial distance of 10 cm from the centre of the conductor while point Q is positioned at a radial distance of 7 cm from the centre of the conductor. State the ratio of the magnetic field strength at the point P to the magnetic field strength at the point Q.

....

....

....

7. The magnetic field surrounding a solenoid is pictured below.



(a) Clearly label the end of the solenoid that produces a south pole.

(b) Clearly indicate the direction of the current at the point P located at the front and centre of the solenoid.

8. (a) A young child stands 1.50 m from the centre of a conductor carrying a current of 45.0 A. Calculate the magnitude of the magnetic field strength at this point.

....

....

....

(b) Use proportionality to determine the new value for the magnitude of the magnetic field if the

(i) child stands 3.00 m from the centre of the conductor

....

....

....

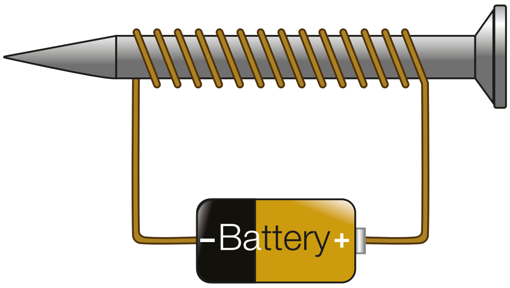
(ii) current in the conductor is reduced to 15.0 A.

9. The magnitude of the magnetic field strength a radial distance *r* from the centre of a straight current–carrying conductor carrying a current of *I* is *B*. State the magnitude of the magnetic field strength in terms of B

(a) at a distance 10*r* from the centre of the conductor.

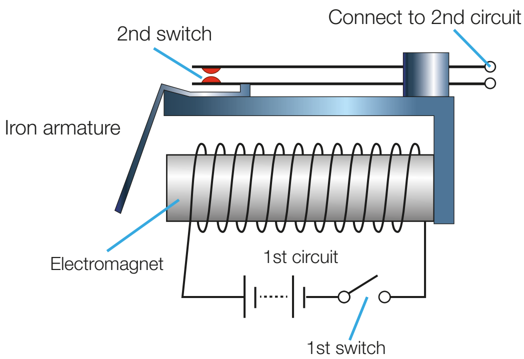
(b) when the current flowing through the conductor is increased to 4*I*.

10. An electromagnet is formed by passing a current through a coil of wire wrapped around an iron nail.



Label the pole (north or south) produce at the pointy end of the nail.

11. A relay switch can control (open or close) one circuit by opening and closing contacts in another circuit. A relay switch makes use of an electromagnet.



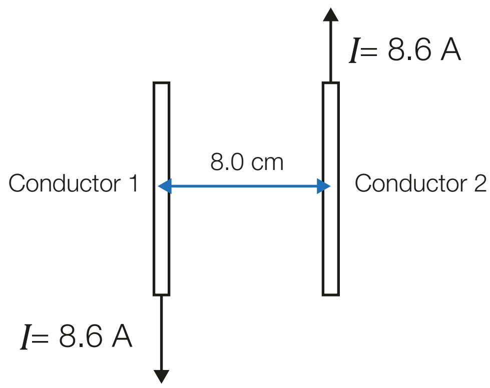
Use the labels on the diagram to help you explain how closing the switch in circuit 1 will allow current to flow through circuit 2 and opening the switch will stop current flowing in circuit 2.

12. Calculate the current flowing through a conductor when the magnitude of the magnetic field strength is 2.5×10^{-4} T at a radial distance of 6.0 cm from the centre of the conductor.

13. A straight conductor (conductor 1) carries a current I = 8.6 A down the plane of the page as shown.

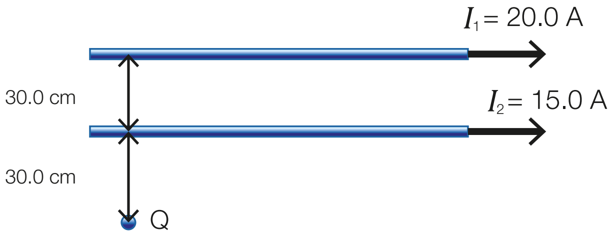
(a) Calculate the magnitude and direction of the magnetic field 3.0 cm to the right of the centre of conductor 1.

(b) A second identical straight conductor (conductor 2), carrying the same current as conductor 1, is placed 8.0 cm to the right of the centre of conductor 1. The current in conductor 2 flows in the opposite direction to the current flowing in conductor 1 as shown in the diagram below.



Calculate the magnitude and direction of the magnetic field 3.0 cm to the right of the centre of conductor 1.

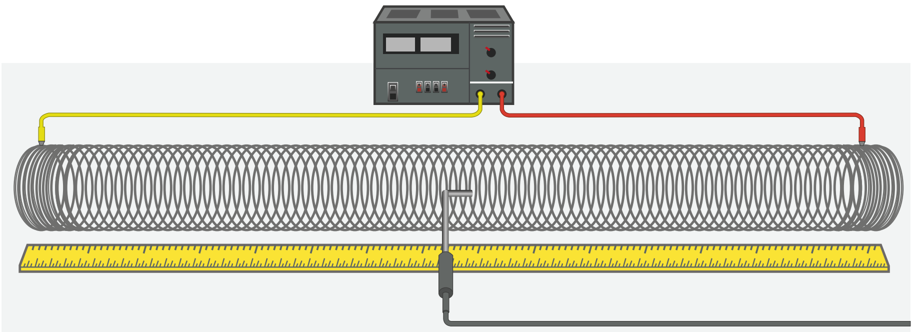
14. Two straight current–carrying conductors are separated by a distance of 30.0 cm. A current I_1 = 20.0 A flows through conductor 1 and a current I_2 = 15.0 A flows through conductor 2 as shown below.



Calculate the magnitude and direction of the magnetic field at point Q, which is located 30.0 cm below the centre of the conductor carrying the current I_2 = 15.0 A.

15. A slinky spring can be used to investigate concepts that relate to a solenoid. When a current is passed through the slinky spring, a magnetic field forms inside the slinky spring.

A student uses the arrangement illustrated below to investigate the relationship between the current (I) passed through a solenoid and the magnetic field strength (B) inside the solenoid by inserting a magnetic field sensor between the coils of the slinky spring.

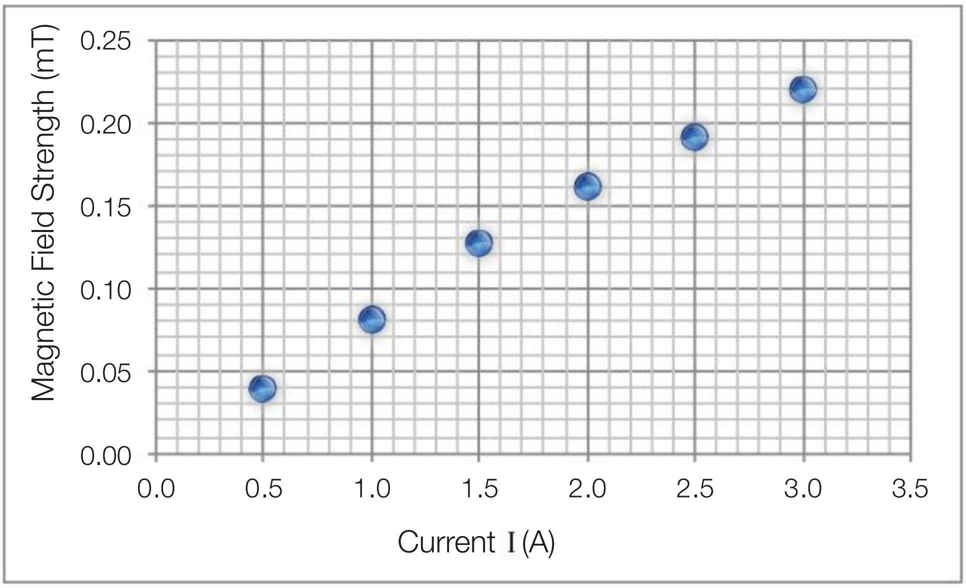


The results collected by the student are summarised in the table below.

Current (A)	Magnetic field strength (mT)			
	Trial 1	Trial 2	Trial 3	Average
0.5	0.03	0.05	0.04	
1.0	0.09	0.09	0.07	
1.5	0.12	0.11	0.15	
2.0	0.16	0.17	0.15	
2.5	0.19	0.19	0.18	
3.0	0.22	0.22	0.22	

(a) Complete the column of data that represents the average magnetic field strength.

(b) The student plots a graph that illustrates how the magnetic field strength varies with the current passing through the solenoid. The graph is shown below.



(i) Draw a line of best fit for the data that has been plotted.

(ii) State the relationship between the magnetic field strength inside the solenoid and the current passing through the solenoid for the data that has been plotted.

....

....

(iii) Calculate the gradient of the line of best fit.

....

....

....

(iv) The expression for the magnetic field strength inside a solenoid is given by $B = \mu_0 nI$ where n represents the number of turns per unit length and μ_0 represents the constant of permeability of free space.

The slinky spring used is 50.0 cm long and contains 35 turns of wire. Use your value for the gradient of the line of best fit to determine the magnitude of μ_0.

....

....

....

(c) (i) The accepted value of μ_0 has a magnitude of 1.3×10^{-6}.

Comment on the accuracy of the value of μ_0 obtained from this set of data.

....

....

....

(ii) Comment on the precision of the plotted data.

....

....

....

2

16. Science as a human endeavour activity – Magnetic fields

Date: March 3, 2020

Source: University of Leeds (Information sourced from:
Hope for a new permanent magnet that's cheap and sustainable)

https://www.sciencedaily.com/releases/2020/03/200303140142.htm

The demand for permanent magnets is increases as they have many uses including renewable energy, electronics and electric-powered vehicles. Permanent magnets are made from alloys of rare earth metals. These metals need to be mined, and mining causes environmental issues. Toxic by-products also cause ecological problems.

At the University of Leeds, a team of scientists are working on a hybrid film of cobalt covered with molecules of a form of carbon. Cobalt is already magnetic, but the presence of the film makes the magnetic strength of cobalt five times greater. However, this effect is only evident at temperature of –195ºC. Scientist's, are working on a way to manipulate the carbon chemically in the hope that the same effect will occur at room temperature.

Hybrid films of magnetic materials will eventually replace rare earth permanent magnets. This will reduce the impact of the environment.

The research was funded by the Engineering and Physical Sciences Research Council, Horizon 2020 European Research Infrastructure project OpenDreamKit, Taibah University and the Science and Technology Facilities Council.

Describe three ways in which the research on hybrid magnets demonstrates the key concepts of science as a human endeavour.

2.4 Motion of charged particles in magnetic fields

Science understanding

1. Magnets, magnetic materials, moving charges, and current-carrying conductors experience a force in a magnetic field.
2. The magnetic force on a moving charged particle depends on the velocity of the particle, its charge, the magnetic field, and the angle between the velocity and magnetic field.
3. The force on a current-carrying conductor within a uniform magnetic field depends on the current in the conductor, the length of the conductor within the magnetic field, the magnetic field strength, and the angle between the conductor and magnetic field.
4. Determine the direction of one of:
   - force
   - magnetic field
   - charge movement

   given the direction of the other two.
5. Solve problems involving the use of $F = IlB\sin\theta$ for a current-carrying conductor and $F = qvB\sin\theta$ for a moving charged particle.
6. A charged particle moving at right angles to a uniform magnetic field experiences a force of constant magnitude at right angles to the velocity. The force changes the direction but not the speed of the charged particle, therefore causes centripetal acceleration.
   - Explain how the velocity dependence of the magnetic force on a charged particle causes the particle to move with uniform circular motion when it enters a uniform magnetic field at right angles.
   - Derive $r = \frac{mv}{qB}$ for the radius r of the circular path of an ion of charge q and mass m that is moving with speed v at right angles to a uniform magnetic field of magnitude B.
   - Solve problems involving the use of $r = \frac{mv}{qB}$.
7. Cyclotrons are used to accelerate ions to high speed. Radioisotopes used in medicine and industry may be produced from collisions between high speed ions and nuclei.
8. The magnetic field within the dees of a cyclotron causes the charged particles to travel in a circular path, so that they repeatedly pass through the electric field.
   - Describe the nature and direction of the magnetic field needed to deflect ions into a circular path in the dees of a cyclotron.
   - Derive the formula $T = \frac{2\pi m}{qB}$ for the period T of the circular motion of an ion, and hence show that the period is independent of the speed of the ion.
   - Use $f = \frac{1}{T}$ to relate the period to the frequency of the alternating potential differences.
   - Derive the formula $E_K = \frac{q^2B^2r^2}{2m}$ for the kinetic energy E_K of the ions emerging at radius r from a cyclotron.
   - Explain why E_K is independent of the potential difference across the dees

     and, for given ions, depends only on the magnetic field and the radius of the cyclotron.
   - Solve problems involving the use of $T = \frac{2\pi m}{qB}$ and $E_K = \frac{q^2B^2r^2}{2m}$

This chapter uses the concept of force developed in the Stage 1, Subtopic 1.2: Forces and the concept of circular motion in Stage 2, Subtopic 1.3: Circular motion and gravitation.

Magnets, magnetic materials, moving charges and current-carrying conductors in magnetic fields

In subtopic 2.3 we saw that bar magnets and moving charges, such as charges in an electric current produce magnetic fields in the space surrounding them. Two bar magnets with like poles facing one another will repel one another. This is because the magnetic field of one magnet interacts with the magnetic field of the other magnet producing a repulsive force. We also saw that a compass, which is essentially a small magnetised needle, deflected in the magnetic field produced by a straight current-carrying conductor. This is because the magnetic field of the straight current-carrying conductor exerted a force on the compass and vice versa.

Magnets, magnetic materials, moving charges and current-carrying conductors experience a force in a magnetic field.

2

Figure 2.4.1 below illustrates the force on a bar magnet in a uniform magnetic field. Recall, that the direction of a magnetic field is given by the direction that the north pole of a compass points. The north pole of the magnet experiences a force parallel to the magnetic field (right) while the south pole experiences a force that is antiparallel to the magnetic field (left).

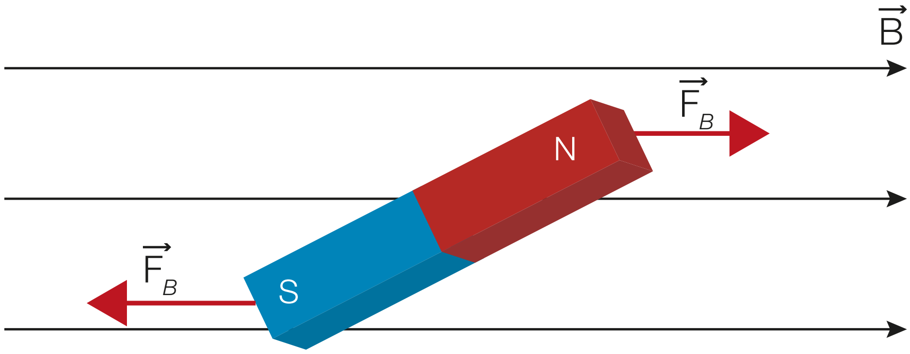


Figure 2.4.1

Figures 2.4.2 to 2.4.4 illustrate this principle for a straight current-carrying conductor placed in a uniform magnetic field created between the opposite poles of two magnets.

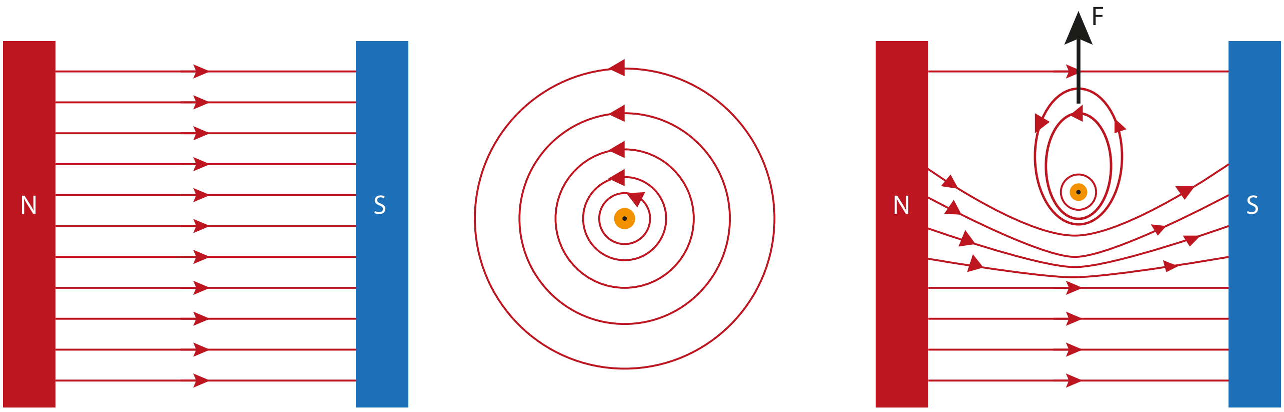


Figure 2.4.2 *Figure 2.4.3* *Figure 2.4.4*

Figures 2.4.2 and 2.4.3 illustrate the uniform magnetic field between the two bar magnets and the magnetic field surrounding a current-carrying conductor respectively. Figure 2.4.4 shows the individual magnetic fields interacting and the resultant force (F) on the straight current-carrying conductor.

The force F on the straight current-carrying conductor shown in Figure 2.4.4 can be shown to be proportional to:

1. the current (I) flowing through the conductor
2. the length (l) of the conductor
3. the magnetic field strength (B)
4. sine of the angle (θ) that the conductor makes with the magnetic field

The magnetic force on a current-carrying conductor in a magnetic field

As discussed above, whenever a current-carrying conductor is placed in a magnetic field, it experiences a force.

A current element is the product of the current (I) and the length (l) of a small current-carrying conductor i.e. Il

The magnitude of the magnetic field (B) is defined as the force per unit current element placed at right angles to the magnetic field.

$$B = \frac{F}{Il}$$

We have already seen that the unit of magnetic field is the Tesla. An alternative unit in $NA^{-1}m^{-1}$.

Calculating the force on a current-carrying conductor in a uniform magnetic field

Figure 2.4.5 shows a conductor of length l carrying a current I placed in a uniform magnetic field B at an angle θ to the magnetic field.

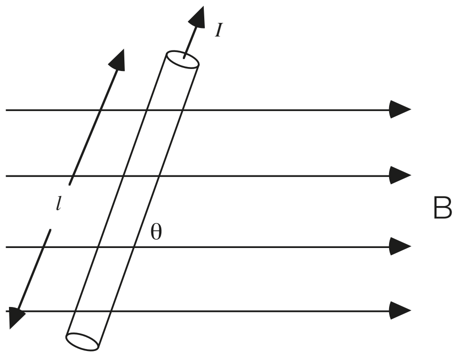


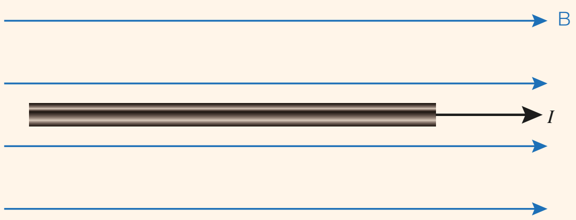
Figure 2.4.5

The magnitude of the force on a straight current-carrying conductor in a uniform magnetic field is given by:

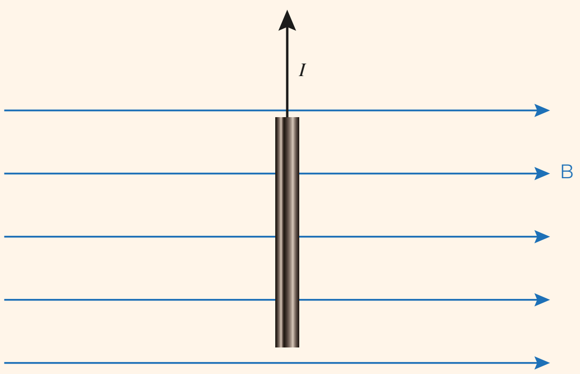
$$F = IlB\sin\theta$$

Key ideas

1. A current-carrying conductor placed parallel to a magnetic field (i.e. $\theta = 0°$) does not experience a force due to the magnetic field ($\sin 0 = 0$ therefore $F = 0$).



2. A current-carrying conductor placed perpendicular to a magnetic field (i.e. $\theta = 90°$) experiences a maximum force given by IlB (since sin90 = 1).



The direction of the force on a current-carrying conductor

The direction of the force, conventional current and the magnetic field are at right angles to each other.

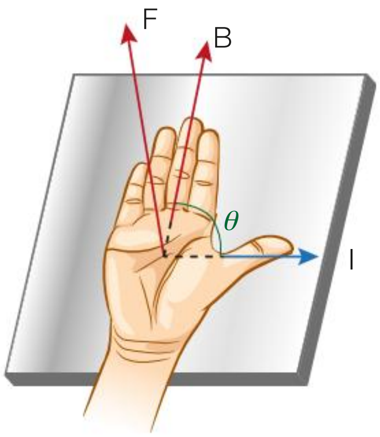


Figure 2.4.6

Figure 2.4.6 illustrates how the direction of the uniform magnetic field and current through the conductor are related to the direction of the force on the conductor. This is referred to as the **right hand rule**.

The **thumb** points in the direction of the conventional **current**, the **outstretched fingers** point in the direction of the **magnetic field** and the **force** on the conductor acts perpendicularly away from the palm of the hand.

The right hand rule can be used to determine the direction of one of:

- force
- magnetic field
- current

given the direction of the other two.

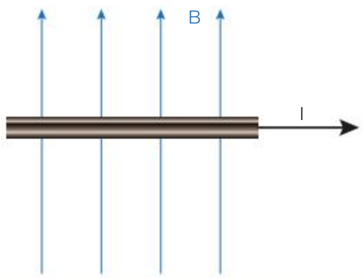
2

- For a straight current-carrying conductor:
  - ⊗ Current into the page
  - ⊙ Current out of the page
- For magnetic field:
  - × Field into the page
  - • Field out of the page

Worked Examples

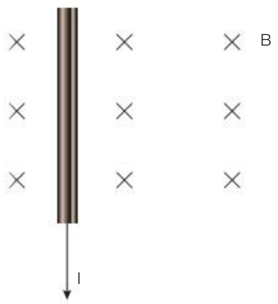
1. State the direction of the magnetic force on each of the conductors below.

(a)



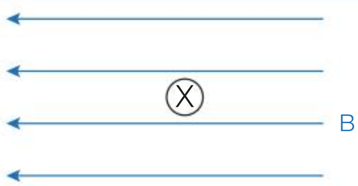
The force on the conductor is directed out of the page.

(b)



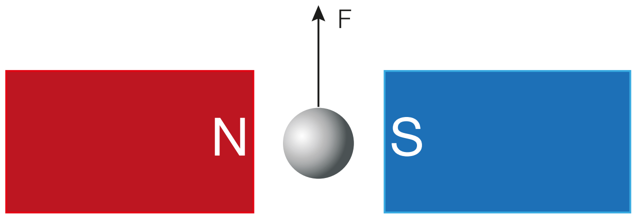
The force on the conductor is directed to the right of the page.

(c)



The force on the conductor is directed up the plane of the page.

2. The diagram below shows a straight current-carrying conductor lying between two magnets. The magnitude of the magnetic field is 0.55 T and a current of magnitude 32 A flows through the conductor. The conductor experiences a magnetic force directed up the plane of the page.



(a) Calculate the magnitude of the magnetic force acting on a 1.5 cm length of the conductor.

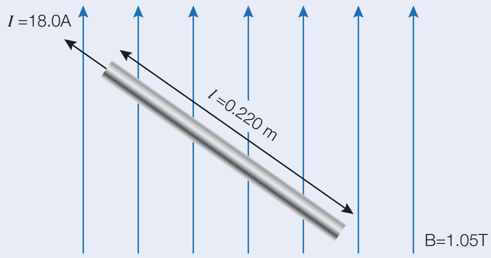
$F = IlB\sin\theta = 32 \times 0.015 \times 0.55 \sin 90 = 0.26 N$

(b) State the direction of the current flowing through the conductor.

Perpendicularly out of the page

3. A straight current-carrying conductor of length l = 0.220 m carries a current I = 18.0 A and lies in a uniform magnetic field of magnitude B = 1.05 T. The magnitude of the magnetic force on the conductor is 2.90 N.

Calculate the angle θ between the current-carrying conductor and the magnetic field.



$$F = IlB\sin\theta$$

$$\theta = \sin^{-1}\left(\frac{F}{IlB}\right)$$

$$\theta = \sin^{-1}\left(\frac{2.90}{18.0 \times 0.220 \times 1.05}\right) = 44.2°$$

Calculating the force on an individual charged particle moving at an angle θ to a uniform magnetic field

We have already seen that a current-carrying conductor in a uniform magnetic field experiences a force. A current is a flow of charge. Individual charges create their own magnetic field as they move. It follows that individual charged particles will also experience a force when moving in a uniform magnetic field.

Figure 2.4.7 shows an individual charged particle q moving at an angle θ and speed v in a uniform magnetic field of magnitude B .

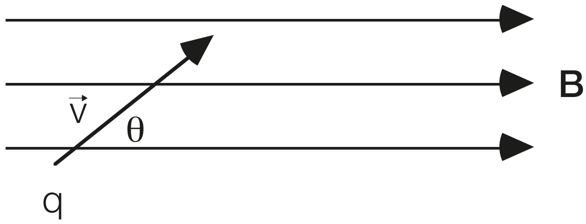


Figure 2.4.7

The magnitude of the force on an individual charged particle in a uniform magnetic field is given by:

$$F = qvB\sin\theta$$

Worked Example

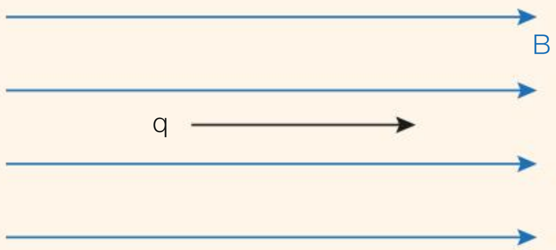
A proton enters a uniform magnetic field of magnitude B = 3.20 T with a speed v = 4.40 × 10^6 ms^{-1} at an angle θ = 35.0° to the magnetic field.

Calculate the magnitude of the force acting on the proton.

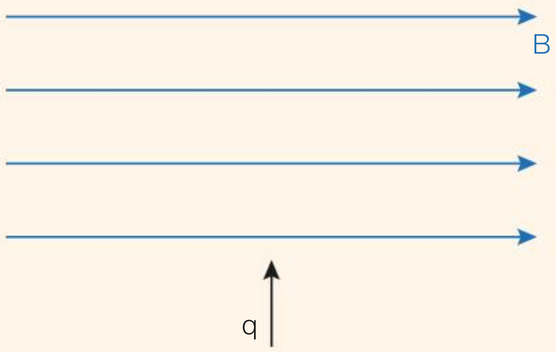
$F = qvB\sin\theta = 1.60 \times 10^{-19} \times 4.40 \times 10^6 \times 3.20\sin 35 = 1.29 \times 10^{-12} N$

Key ideas

1. A stationary charge (i.e. $v = 0$) does not experience a force in a magnetic field.
2. The force on an individual charged particle that is moving parallel or antiparallel to a uniform magnetic field is zero (i.e. $\theta = 0°$ therefore $\sin 0 = 0$ and $F = 0$).



3. An individual charged particle that is moving at right angles to a uniform magnetic (i.e. $\theta = 90°$) experiences a maximum force given by qvB (since $\sin 90 = 1$).



The direction of the force on an individual charged particle moving at right angles to a uniform magnetic field

The direction of the force, charge movement and the magnetic field are at right angles to each other when an individual charged particle moves through a uniform magnetic field.

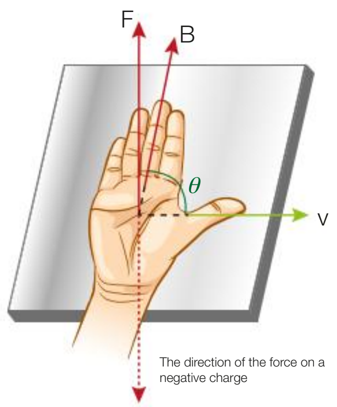


Figure 2.4.8

Figure 2.4.8 illustrates how the direction of the uniform magnetic field and the direction of charge movement are related to the direction of the force on the charge. This is referred to as the **right hand rule**.

The **thumb** points in the direction of **charge movement**, the **outstretched fingers** point in the direction of the **magnetic field** and the **force** on a **positive charge** acts perpendicularly **away from the palm** of the hand. Negative charges would experience a force in the opposite direction.

The right hand rule can be used to determine the direction of one of:

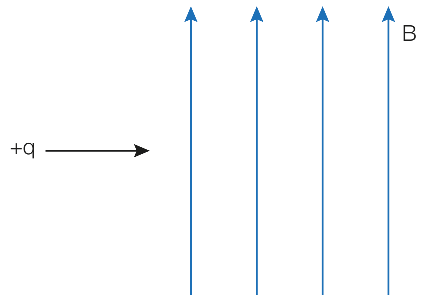
- force
- magnetic field
- charge movement

given the direction of the other two.

Worked Example

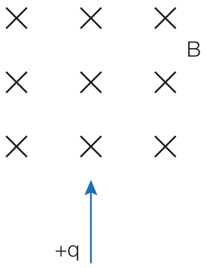
State the direction of the force on each of the charges as they enter the uniform magnetic field.

(a)



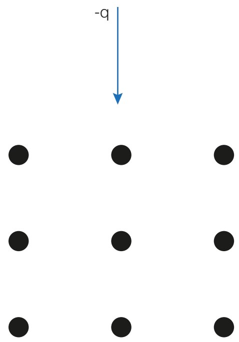
The force on the positive charge is directed out of the page.

(b)



The force on the positive charge is directed to the left of the page.

(c)



The force on the negative charge is directed to the right of the page.

Key ideas

Since a charged particle moving at right angles to a uniform magnetic field experiences a force of constant magnitude at right angles to its velocity, the charge changes direction without a change in speed. The magnetic force provides the centripetal acceleration required for uniform circular motion.

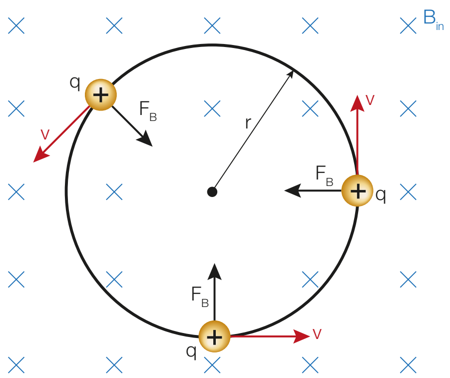


Figure 2.4.9

Figure 2.4.9 illustrates the circular path traced by a positive charge moving at right angles to a uniform magnetic field directed into the plane of the page.

Derivation for the radius *r* of the circular path of a charged particle moving at right angles to a uniform magnetic field.

Consider a charge q with mass m that is moving with a speed v at right angles to a uniform magnetic field of magnitude B.

The magnetic force provides the centripetal acceleration for uniform circular motion.

$$F_{centripetal} = F_{magnetic}$$

$$\frac{mv^2}{r} = qvB$$

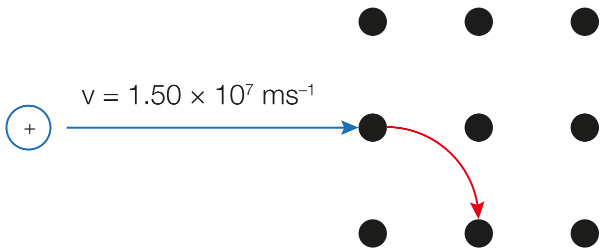
$$r = \frac{mv}{qB}$$

The radius of the circular path is given by $r = \frac{mv}{qB}$

2

Worked Examples

1. The diagram below shows a charge q = +7.00 µC with mass $m = 3.00 \times 10^{-15}$ kg entering a uniform magnetic field $\vec{B}$ = 2.40 T as shown below. The charge is moving with a speed $v = 1.50 \times 10^7$ ms^{-1}.



(a) On the diagram above, sketch the possible path of the charge.

See diagram

(b) Explain the path traced by the charge in this magnetic field.

Since the charged particle is moving at right angles to the uniform magnetic field, it experiences a force of constant magnitude at right angles to its velocity. The charge experiences a change in direction without a change in speed. The magnetic force therefore provides the centripetal acceleration required for uniform circular motion.

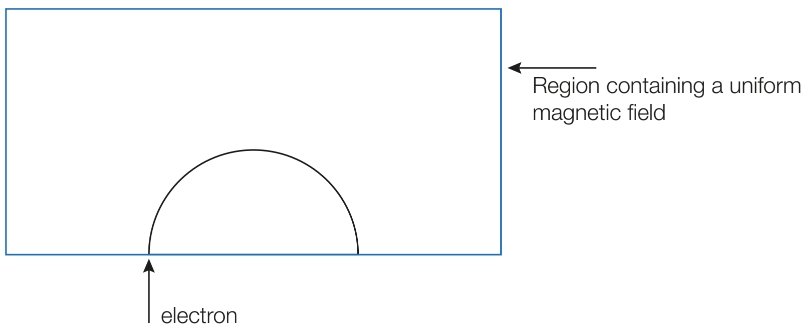
(c) Calculate the magnitude of the force acting on the charge.

$F = qvB\sin\theta = 7.00 \times 10^{-6} \times 1.50 \times 10^{7} \times 2.40\sin 90 = 252\ N$

(d) Calculate the radius of the circular path traced by the charge.

$r = \frac{mv}{qB} = \frac{3.00 \times 10^{-15} \times 1.50 \times 10^{7}}{7.00 \times 10^{-6} \times 2.40} = 2.68 \times 10^{-3}\ m$

2. The diagram below represents the path of an electron as it enters a uniform magnetic field of magnitude $B = 8.50 \times 10^{-3}$ T at a right angle to the field. The electron is deflected to the right.



(a) State the direction of the magnetic field. Justify your answer.

Using the right hand rule, the direction of charge movement is up the page, the force acts to the right, the magnetic field must act into the plane of the page.

(b) The electron travels in a circular path of radius 14.0 mm. Determine the speed with which the electron entered the magnetic field.

$$r = \frac{mv}{qB} \therefore v = \frac{rqB}{m} = \frac{14.0 \times 10^{-3} \times 1.60 \times 10^{-19} \times 8.50 \times 10^{-3}}{9.11 \times 10^{-31}} = 2.09 \times 10^{7} ms^{-1}$$

(c) Describe how the radius of the circular path of the electron will alter if the

(i) electron enters the magnetic field at twice the speed calculated in part (b).

$$r \alpha v$$

The radius of the circular path will double if the electron enters the magnetic field at twice the speed.

(ii) magnitude of the magnetic field is tripled.

$$r \alpha \frac{1}{B}$$

If the magnitude of the magnetic field is tripled, the radius of the circular path will be three times smaller.

? Science inquiry activity

1. A possible investigation is to use a current balance shown in Figure 2.4.10 to investigate factors that affect the force on a current-carrying conductor due to an external magnetic field.

   A permanent magnet assembly is placed on an electronic balance and the balance is zeroed. A circuit board containing a wire loop (conductor) is placed into the permanent magnet assembly so that the wire loop is at right angles to the magnetic field but is not touching the magnets. Figure 2.4.11 shows circuit boards with conductors of various lengths.

   When a current flows through the conductor, it will experience a force due to the magnetic field. The force on the conductor can be determined by recording the reading on the electronic scales. The magnetic force acts on the permanent magnet assembly and will cause its weight ($W = mg$) to either increase or decrease depending on the direction of the current and the direction of the magnetic field. The change in the magnet assembly's weight will allow you to determine the magnetic force ($W = F = mg = IlB$).

   An example of a possible investigation would be to design a method to determine the relationship between the length of the conductor and the magnetic force experienced by the conductor.

Figure 2.4.10

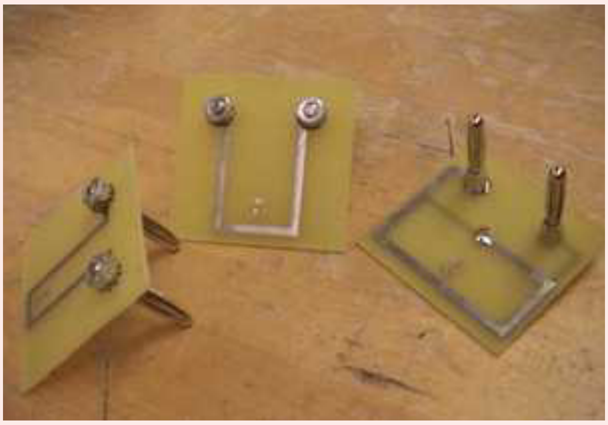


Figure 2.4.11 Circuit boards with conductors of different length.

? Science inquiry activity

2. The Teltron tube was introduced in Subtopic 2.2. It can be used to investigate the motion of charged particles in magnetic fields as well as electric fields.

   Figure 2.4.12 shows a Teltron tube with a pair of Helmholtz coils situated on either side of the glass tube. Figure 2.4.13 shows the experimental set up for investigating the motion of charged particles in magnetic fields. When a current is passed through the coils, the magnetic field created between the coils is almost uniform. The magnitude of the magnetic field can be increased by increasing the current flowing through the coils. When electrons released by the electron gun are accelerated by a potential difference, they gain kinetic energy and enter the magnetic field at right angles. The electrons undergo uniform circular motion.

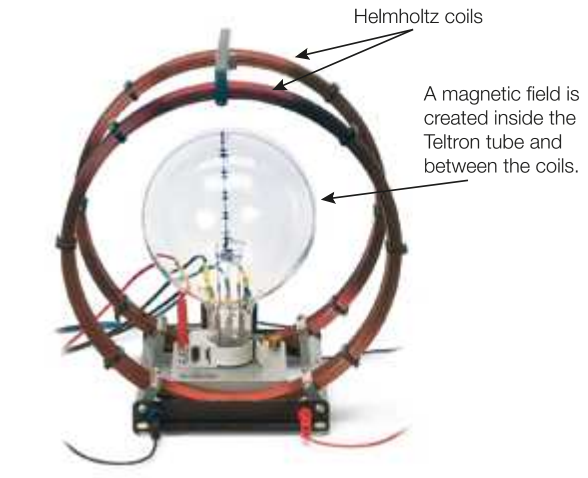


Figure 2.4.12

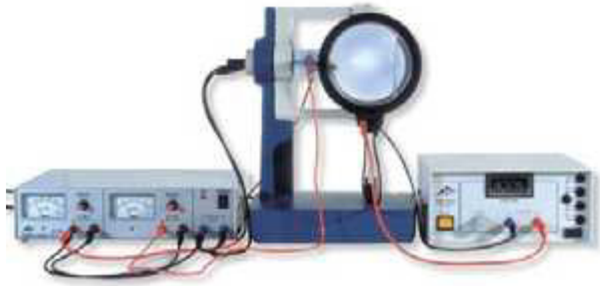


Figure 2.4.13

   A possible investigation is to investigate one factor that affects the circular path of charges in a magnetic field. An example would be to design an investigation to determine the relationship between the magnetic field strength and the radius of the circular path traced by the electrons in the magnetic field.

3. The Teltron tube can also be set up so that it produces an electric and magnetic field at right angles to each other. This arrangement of fields can be used to measure the charge-to-mass ratio of electrons. The experimental set up is shown in Figure 2.4.14.

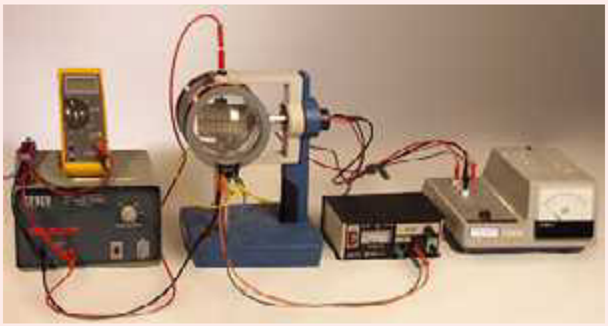


Figure 2.4.14

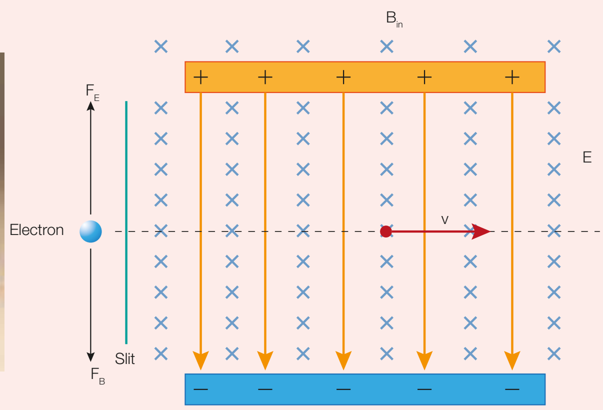


Figure 2.4.15

Figure 2.4.15 illustrates the electric and magnetic fields at right angles to each other. If the magnitude of the force due to the electric field is equal to the magnitude of the force due to the magnetic field and acts in the opposite direction as shown in Figure 2.4.15, then the electron's path is not deviated. The speed of the electron can be derived as shown below:

$$F = qE = qvB \text{ therefore } v = \frac{E}{B}$$

Science as a human endeavour

Possible investigations and discussions could include to evaluate the economic, social, and environmental impacts of some applications of charges moving within magnetic fields, such as:

- moving-coil loudspeaker
- synchrotron
- mass spectrometer
- electric motors
- use of magnetic fields in electron microscopes
- maglev trains.

The cyclotron

The cyclotron was introduced in Subtopic 2.2. Cyclotrons are used to accelerate charged particles (usually protons) or ions to high speed. The high-speed ions are forced to collide with other nuclei to cause nuclear reactions. These nuclear reactions produce radioisotopes (or radioactive isotopes) that can be used in medicine and industry. The radioisotopes often have a short half-life. This is the time taken for half the number of nuclei to decay. For this reason, cyclotrons are kept in hospitals so that the radioactive materials they produce can be made when they are needed.

Please note: Concepts relating to radioactivity and half-life can be reviewed in Chapter 6 of the Stage 1 Workbook.

Fluorine-18 is an example of a radioisotope that is produced using a cyclotron. Fluorine-18 is produced when protons collide with Oxygen-18 nuclei. The protons used to do this are accelerated to large kinetic energies by a cyclotron. This is because the large repulsive electrostatic force between the positive protons and the positive Fluorine-18 nuclei needs to be overcome in order for the collisions to occur. As a result of the collisions, a neutron is ejected from the Oxygen-18 nuclei and the radioisotope Fluorine-18 is produced. The nuclear equation shown below represents the formation of Fluorine-18.

$$^{1}_{1}H + ^{18}_{8}O \rightarrow ^{18}_{9}F + ^{1}_{0}n$$

Fluorine-18 can be used directly for bone scans but is most commonly used in PET scans to monitor organ function (e.g. the brain or kidney) or to detect cancer. Fluorine-18 is used to make the radiopharmaceutical FDG (Fluoro deoxy glucose). The Fluorine-18 attaches itself to glucose to make the FDG. The FDG moves through the body and is accumulated by the organ being examined. Fluorine-18 decays by emitting positrons (beta plus decay). When the positrons collide with nearby electrons they annihilate and produce two gamma rays that travel in opposite directions in order to conserve momentum. A PET scanner has a ring of photon detectors and uses a computer to create a three dimensional image of the organ being examined. Figure 2.4.16 shows a PET scanner in action while Figure 2.4.17 is a PET scan of the cross-section of a patient's abdomen.

Figure 2.4.16

Figure 2.4.17 Cancer is identified in this PET scan (a cross-section of a patients abdomen) as the bright yellow areas

Areas of high activity in an organ and cancers appear brighter on a scan. This is because the cells need energy and consume the radioactive glucose. Cancers in particular, use more energy and therefore use a lot more of the radioactive glucose. More photons are produced in the area of the activity and this appears as a bright area on a scan. This enables diseases and cancers to be quickly and accurately diagnosed.

A medical production facility located in Lucas Heights Australia is the twin PETNET cyclotrons. These are small cyclotrons used to produce Fluorine-18 needed for FDG.

The half-life of Fluorine-18 is short at 110 minutes. This only allows sites within one or two hours of the cyclotron to provide PET scans effectively. The old Royal Adelaide Hospital sourced its Fluorine-18 daily from Melbourne. Time was needed to prepare the isotope, send it to the airport, fly it to Adelaide and then transport it to the hospital. This meant that more than the required amount of the isotope needed to be made to allow for the radioactive decay that occurred. With the new hospital upgrade this is no longer necessary.

Having a cyclotron and radiopharmacy on-site reduces the cost and increases the number of scans available to patients on a daily basis. In addition, some isotopes have a very short half-life and this makes them impractical to ship in from other locations.

The main components of a cyclotron

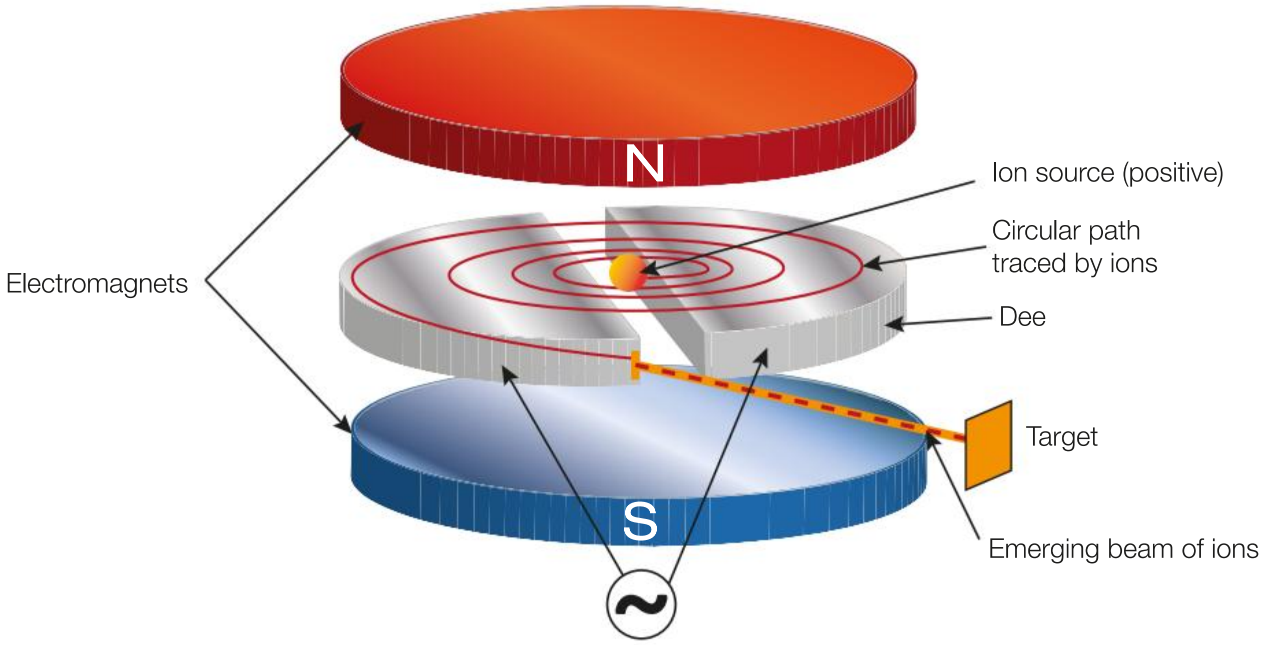


Figure 2.4.18

Figures 2.4.18 illustrates the main components of a cyclotron. Recall that an electromagnet is placed above and below two hollow D-shaped copper conductors. This produces a uniform magnetic field inside the dees. Charged particles are **accelerated by a uniform electric field** in the gap between the dees. This is because the electric field does work on the ions and transfers kinetic energy to the ions ($W = q\Delta V = \Delta E_k = \frac{1}{2}mv^2$). The uniform **magnetic field** causes the charged particles to move in a **circular path** so that they return to the electric field. This is because a magnetic force of constant magnitude acts at right angles to the velocity of the ions. The magnetic force provides the centripetal acceleration for uniform circular motion. By the time the ions return to the gap between the dees, the high frequency alternating potential difference **reverses the electric field**. This causes the ions to once again accelerate across the electric field. The ions enter the second dee with a greater speed. Since $r = \frac{mv}{qB}$, the ions travel in a circular path of **greater radius**. The ions return to the gap between the dees and the electric field is once again reversed. The ions accelerate and enter the first dee again. The ions trace a circular path of greater radius and the **process repeats many times**. By the time the ions exit the cyclotron they have **accumulated a large amount of kinetic energy and hence speed**.

Note: The dees are open at the diameter so that protons and other ions can easily pass from one dee into the other without colliding with the material making up the dees.

Figure 2.4.19 illustrates a top view of the circular path traced by positive ions in figure 2.4.18 above. The magnetic field acts into the plane of the page. Using the right hand rule, the magnetic force on the ions at any point along their path can be determined. For example, consider position A. The direction of movement of the charge is up the plane of the page (↑) and the magnetic field acts into the plane of the page (X). The force on a positive charge at position A acts to the left (←).

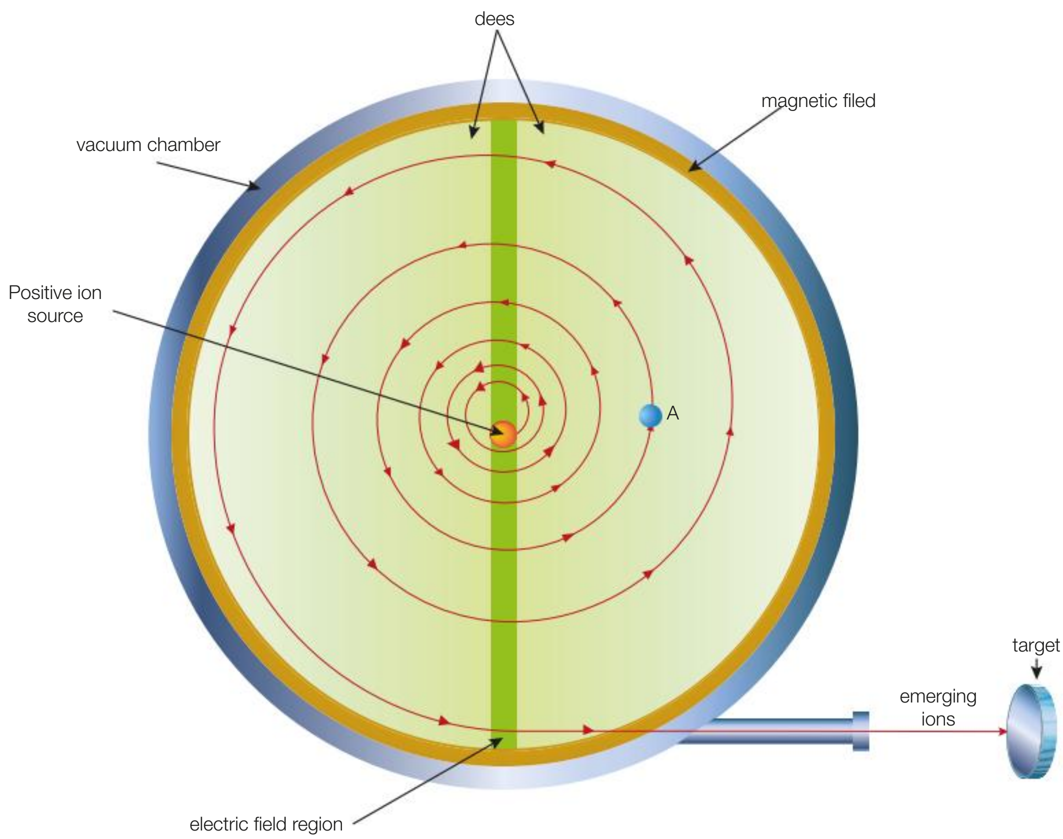


Figure 2.4.19

Key ideas

1. The potential difference between the dees produces a uniform electric field. This causes the ions to gain kinetic energy and accelerate.
2. The uniform magnetic field inside the dees causes the ions to move in a circular path so that they repeatedly pass through the uniform electric field.
3. The uniform electric field reverses direction every time the alternating potential difference reverses. The time taken for this is half the period (T) of motion of the charges. The expression for this is derived later in the chapter and is given by $T = \frac{2\pi m}{qB}$.
4. The total kinetic energy (E_k) of the ions as they exit the cyclotron and cross the uniform electric field N times is given by $Nq\Delta V$.
5. The total kinetic energy of the ions as they emerge from a cyclotron of radius r is given by $E_K = \frac{q^2B^2r^2}{2m}$. This expression is derived later in the chapter.

Additional notes:

1. As discussed in Subtopic 2.2, the ion source is often ionised hydrogen. This produces protons.
2. The dees are hollow copper electrodes shaped like the letter D. There is no electric field inside the dees.
3. The dees are placed inside an evacuated chamber. This prevents the ions from colliding with air particles and losing kinetic energy or from being scattered.

The period of circular motion of an ion in a cyclotron

The period (T) of the circular path of ions in a cyclotron is given by: $T=\frac{2\pi m}{qB}$

where m is the mass of the ions in kg, q is the charge of the ions in C and B is the magnitude of the magnetic field in T.

Derivation

$$T=\frac{2\pi r}{v} \quad \text{but } r=\frac{mv}{qB}$$

$$T=\frac{2\pi \frac{mv}{qB}}{v}=\frac{2\pi mv}{vqB}=\frac{2\pi m}{qB}$$

The **period does not depend on the speed of the ions**. This means that the period of all the ions in the cyclotron is the same and they all reach the gap between the dees and hence the uniform electric field at the same time. Ions closer to the centre of the cyclotron travel in a circular path with a smaller radius and therefore travel a smaller distance at a slower speed. Ions closer to the outside of the cyclotron travel in a circular path with a larger radius and therefore travel further at a greater speed. Since $t=\frac{s}{v}$ all the ions reach the electric field at the same time. The potential difference can be reversed on a regular basis so that the charges continue to move through the cyclotron in one direction.

The relationship between period and frequency

The formula $f=\frac{1}{T}$ relates the period to the frequency of the alternating potential difference.

The potential difference applied across the dees is reversed every half revolution i.e. every $\frac{\pi m}{qB}$ seconds.

The kinetic energy of ions emerging from the cyclotron

The kinetic energy (E_K) of the ions emerging from a cyclotron at a radius r is given by:

$$E_K=\frac{q^2B^2r^2}{2m}$$

Derivation

$$E_k=\frac{1}{2}mv^2 \quad \text{using } r=\frac{mv}{qB} \quad \text{it follows that } v=\frac{rqB}{m}$$

$$E_k=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{rqB}{m}\right)^2=\frac{1}{2}m\frac{r^2q^2B^2}{m^2}=\frac{r^2q^2B^2}{2m}$$

The kinetic energy of the emerging ions is **independent of the potential difference** across the dees. A larger potential difference across the dees transfers more kinetic energy to the ions, they travel faster and in circular paths of greater radii. The ions cross the electric field fewer times before they emerge from the cyclotron. A smaller potential difference across the dees transfers less kinetic energy to the ions, they travel slower and in circular paths of smaller radii. The ions cross the electric field more times before they emerge from the cyclotron. The result is that the ions have the same kinetic energy by the time they emerge from the cyclotron.

The kinetic energy of the emerging ions **depends only on the magnetic field** and the radius of the final circular path i.e. the **radius of the cyclotron**.

Worked Example

One of the earliest cyclotrons was constructed with a diameter of 12.5 cm. The magnitude of the magnetic field acting in the dees was 3.00 T. The cyclotron was used to accelerate protons.

(a) Calculate the period of the protons in this cyclotron.

$$T=\frac{2\pi m}{qB}=\frac{2\pi\times 1.67\times 10^{-27}}{1.60\times 10^{-19}\times 3.00}=2.19\times 10^{-8}\,s$$

(b) Calculate the frequency of the alternating potential difference.

$$f=\frac{1}{T}=\frac{1}{2.19\times 10^{-8}}=4.57\times 10^{7}\,Hz$$

(c) Calculate kinetic energy of the protons as they emerged from this cyclotron in J and MeV.

$$E_k=\frac{q^2B^2r^2}{2m}=\frac{(1.60\times 10^{-19})^2\times 3.00^2\times(6.25\times 10^{-2})^2}{2\times 1.67\times 10^{-27}}=2.69\times 10^{-13}\,J=1.68\,MeV$$

? Science inquiry activity

Some ideas for possible investigations include:

- Discuss the advantages and disadvantages of generating radioisotopes in a cyclotron compared to a nuclear reactor. Make recommendations for particular contexts.
- Debate the need for both cyclotrons and nuclear reactors in the production of radioisotopes, including the relationship between public debate and science.
- Discuss the importance of the cyclotron in the South Australian Health and Medical Research Institute (SAHMRI) facility.
- Investigate medical uses and disadvantages of radioisotopes for diagnostic and therapeutic purposes (e.g. PET scanners, boron neutron capture therapy).
- Investigate benefits and limitations of using radioisotopes in industry (e.g. in quality assurance processes).
- Discuss the safe storage and disposal of radioactive materials.
- Study the production and use of radioisotopes, for medical or industrial use.
- Explore the limitation on the energy of a charged particle emerging from a cyclotron due to relativistic effects.

Exercises

1. A straight current-carrying conductor of length l and carrying a current of magnitude I, is placed at right angles to a uniform magnetic field of magnitude B.

   (a) Write an expression for the force experienced by the conductor.

   (b) Write a new expression for the force on the conductor if the

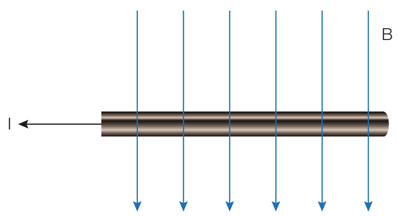
   (i) current flowing through the conductor is doubled

   (ii) magnitude of the magnetic field is reduced by a factor of five.

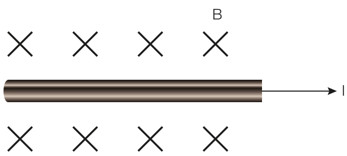
   (iii) current is doubled and the magnitude of the magnetic field is reduced by a factor of five.

2. State the direction of the force on the current-carrying conductor in each case below.

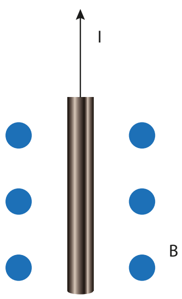
   (a)



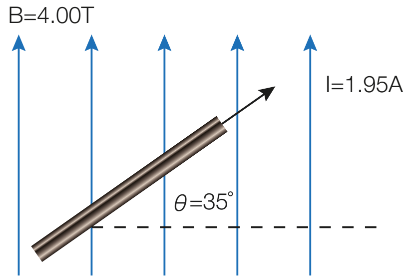
   (b)



   (c)



3. The diagram below shows a conductor of length 50.0 cm carrying a current of 1.95 A in a uniform magnetic field of magnitude 4.00 T.



Calculate the magnitude of the force acting on the conductor.

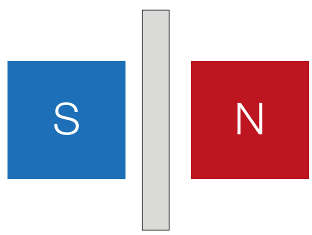
4. A current of 2.0×10^2 A flows through a conductor to the starter motor of a car. The conductor is 1.0 m long and is in the Earth's magnetic field of magnitude 1.8×10^{-5} T.

(a) State the angle between the conductor and the Earth's magnetic field when the conductor experiences a maximum magnetic force.

(b) Calculate the magnitude of the maximum force that the conductor would experience due to the Earth's magnetic field.

(c) The position of the conductor is changed so that it experiences one quarter the maximum force. Calculate the angle between the conductor and the Earth's magnetic field at this new position.

5. The diagram below shows a current-carrying conductor placed at right angles to a uniform magnetic field of magnitude 0.360 T produced between the opposite poles of two magnets.

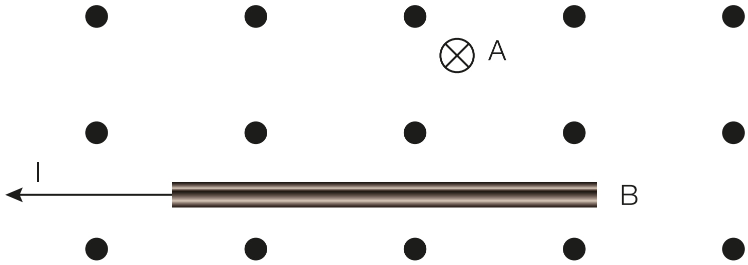


A current of 2.00 A flows through the conductor and it experiences a force $\vec{F} = 5.00 \times 10^{-2} N$ into the plane of the page.

(a) State, with reason, the direction of the current flowing through the conductor.

(b) Calculate the length of conductor in the magnetic field.

6. The diagram below shows two current-carrying conductors A and B in a uniform magnetic field.



State the conductor, A or B for which each of the following is true.

(a) The force is zero. ....

(b) The force is BIl and directed up the plane of the page. ....

7. A straight current-carrying conductor experiences a force of magnitude F when it is placed in a uniform magnetic field.

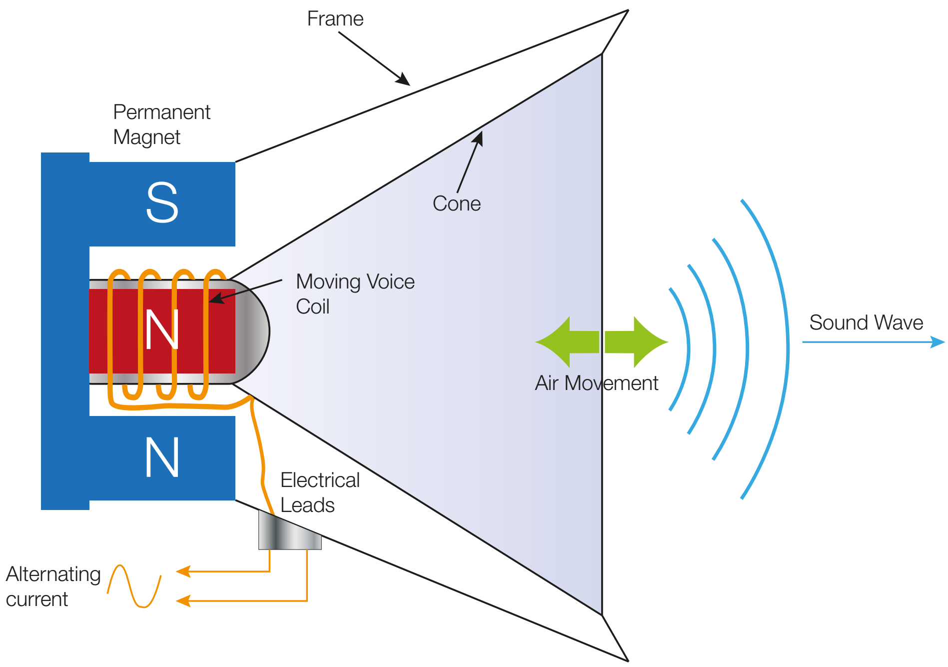
Use proportionality to discuss the effect on the force experienced by the conductor if the length of the conductor in the magnetic field is halved and the magnitude of the magnetic field is tripled.

....

....

....

8. The diagram below illustrates a typical moving-coil loud speaker. Such a loudspeaker consists of a coil of wire called the voice coil that is placed at right angles to a uniform magnetic field. The voice coil is attached to a cone.



When an alternating current flows through the voice coil, the voice coil vibrates and since it is attached to the cone, the cone also vibrates. The vibrating cone vibrates air particles to produce sound waves.

(a) Explain why the voice coil vibrates when an alternating current flows through the voice coil.

....

....

....

....

....

....

(b) The voice coil of a loud speaker has a radius of 3.0 cm and is situated in a uniform magnetic field of magnitude 0.80 T . The coil consists of 10 turns of wire.

(i) Calculate the length of the voice coil.

....

....

....

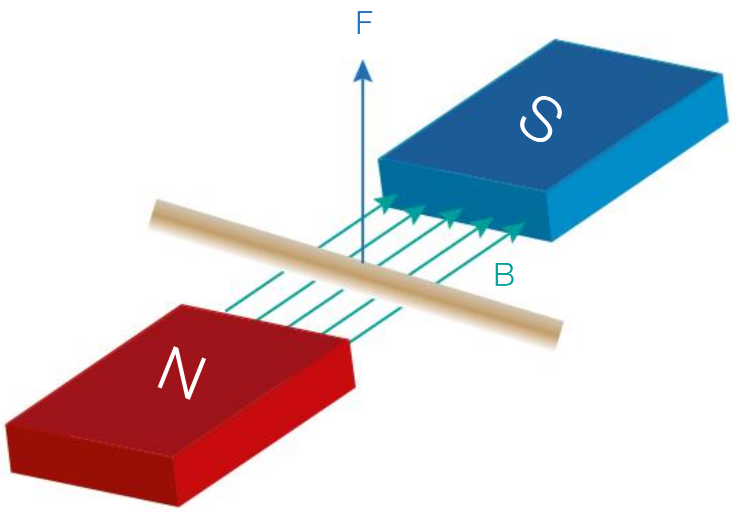
(ii) Calculate the magnitude of the force on the voice coil when a current of 0.95 A flows through the voice coil.

....

....

....

9. The diagram below shows a conductor that remains suspended in a uniform magnetic field created between the opposite poles of two magnets.



The conductor has a mass of 7.50×10^{-2} kg per unit length and is lying in a horizontal plane that contains the uniform magnetic field of magnitude 2.50×10^{-2} T perpendicular to the conductor. The conductor experiences a magnetic force in the direction shown while it remains suspended in the magnetic field.

(a) Determine the magnitude of the current flowing through the conductor.

....

....

....

(b) Draw an arrow on the diagram that indicates the direction of the current flowing through the conductor.

10. Figure A shows a simple motor consisting of a rectangular coil of wire containing many loops (called an armature) mounted on an axle.

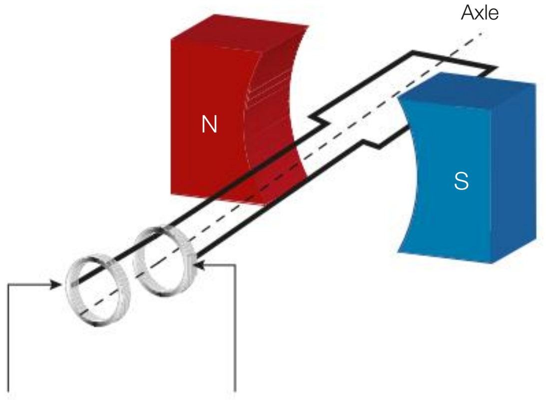


Figure A

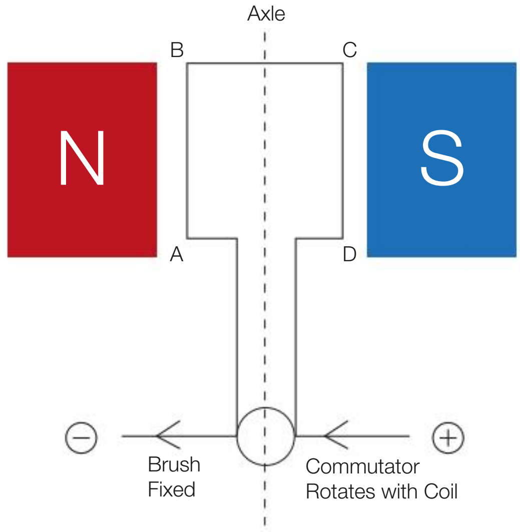


Figure B

Each end of the rectangular coil is connected to a ring of copper, called a commutator. Two brushes are lightly pressed against the commutator by springs. The brushes are connected to an electrical supply. This is illustrated in Figure B. When a current flows through the armature, each side of the armature experiences a force.

(a) State and explain how the direction of the magnetic force on side AB in Figure B can be determined.

....

....

....

(b) State the direction of the force on side CD in Figure B.

....

(c) State the magnitude of the force on sides BC and AD.

....

(d) Describe the motion of the armature.

....

....

(e) State, with reason, the effect that each of the following would have on the motion of the armature.

(i) The number of loops of wire used to make the armature is increased.

....

....

....

(ii) The existing magnets are replaced with stronger magnets.

....

....

....

(iii) A larger potential difference is applied across the armature.

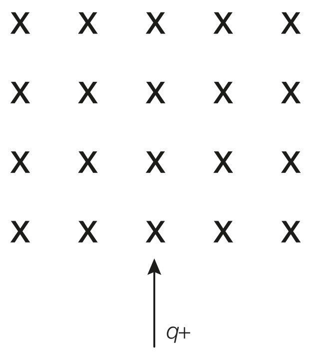
....

....

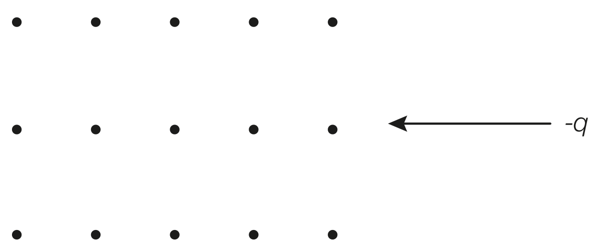
....

11. Represent the circular path traced by each charge in the uniform magnetic fields shown below.

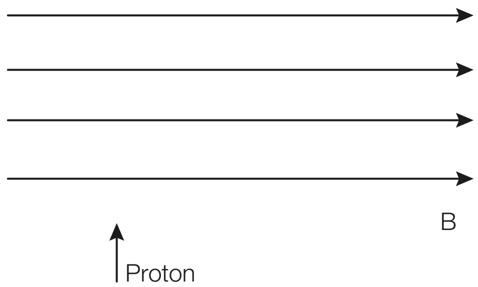
(a)



(b)



12. The diagram below shows a proton as it enters a uniform magnetic field of magnitude 1.80 T with a speed of 2.50×10^6 ms^{-1}.



(a) Calculate the magnitude and direction of the force acting on the proton as it enters the magnetic field.

(b) Describe and explain the path of the proton.

(c) Calculate the radius of curvature of the path followed.

13. An electron and a proton travelling with the same velocity enter a uniform magnetic field at right angles to the magnetic field direction.

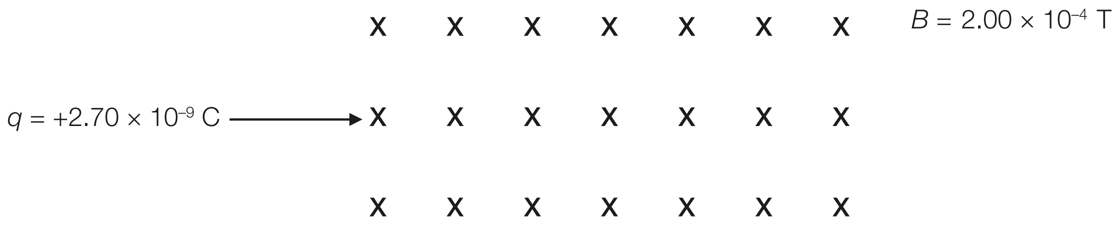
(a) Compare the magnitude of the force on the electron and the proton as they enter the magnetic field.

(b) Explain whether the electron or the proton travels through the magnetic field in a circular path with a smaller radius.

14. A charged particle of mass m and charge q, enters a uniform magnetic field of magnitude B with a speed v at right angles to the magnetic field.

    (a) Show that the radius of the circular path is given by $r = \frac{mv}{qB}$.

    (b) The diagram below shows a charge of magnitude $q = +2.70 \times 10^{-9}$ C and mass $m = 1.72 \times 10^{-18}$ kg entering a uniform magnetic field of magnitude $B = 2.00 \times 10^{-4}$ T.



    (i) Sketch a possible path for the charged particle.

    (ii) The charged particle traces a circular path with a radius of 5.0 mm. Determine the kinetic energy of the charge in eV as it enters the magnetic field.

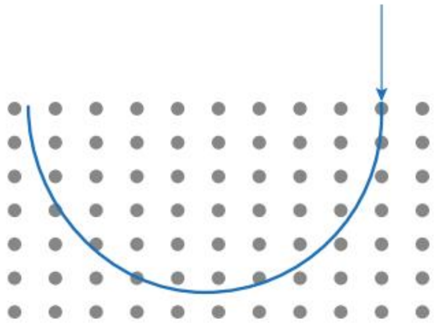
15. A bubble chamber is a tank of superheated liquid hydrogen that is just kept on the verge of boiling. The tank is placed in a strong magnetic field. When a charged particle moves through the bubble chamber, it will collide with the hydrogen atoms. This ionises the hydrogen atoms and the path of the charged particle becomes visible.

    The diagram below shows the path of an electron as it moves through a bubble chamber under the influence of a magnetic field perpendicular to the plane of the diagram.

    (a) The electron moves in a circular path that spirals inwards. Explain why the radius of the circular path decreases as the electron moves through the bubble chamber.

    (b) State the direction of the magnetic field in the bubble chamber.

16. The diagram below shows a charged particle entering a region containing a uniform magnetic field. The charged particle travels in a circular arc.



(a) State the sign of the charge.

....

(b) If the charged particle enters the magnetic field with a greater speed, state with reason, the affect this would have on the

(i) magnetic force experienced by the charge.

....

....

(ii) radius of the circular arc.

....

....

17. A charge particle enters a uniform magnetic field B_1 at right angles to the field. The charged particle moves in a circular path of radius *r*. When the same charged particle enters a second uniform magnetic field B_2 with the same speed, it moves in a circular path of radius *3r*.

Determine the ratio B_2 to B_1.

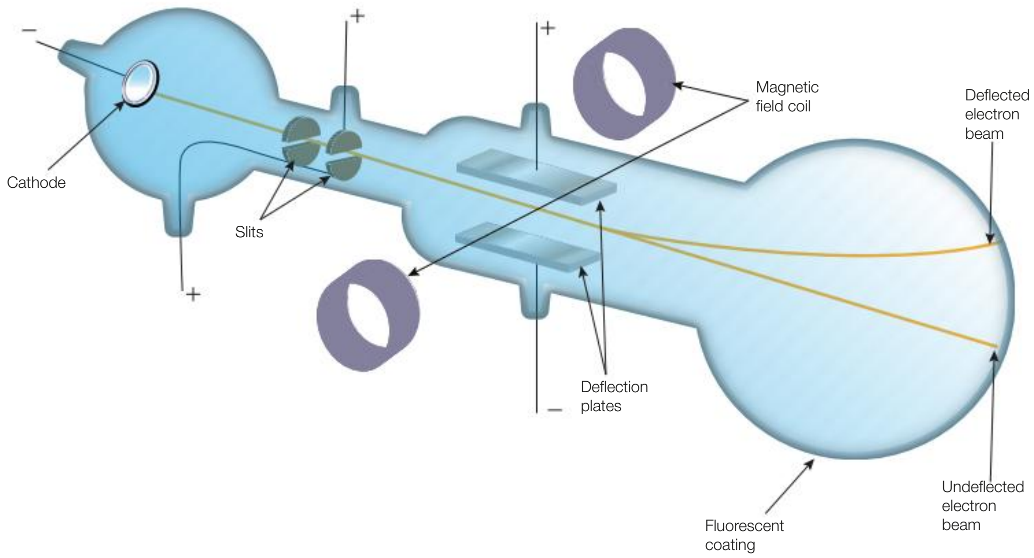
....

....

....

....

18. The diagram below shows the arrangement for a velocity selector.



Electrons released from the cathode pass through an electric and magnetic field that are orientated at right angles to one another. The force exerted on the electrons by the electric field acts in the opposite direction to the force due to the magnetic field. The magnitude of the magnetic field is 2.5 T and the undeflected beam of electrons has a speed of 140 ms^{-1}.

(a) Draw an arrow on the diagram that indicates the direction of the magnetic field through the coils.

(b) State whether the magnitude of the electric or magnetic force acting on the electrons is greater for the deflected electron beam shown in the diagram.

....

(c) Show that the speed of the undeflected electron beam shown on the diagram is given by $v = \frac{E}{B}$.

....

....

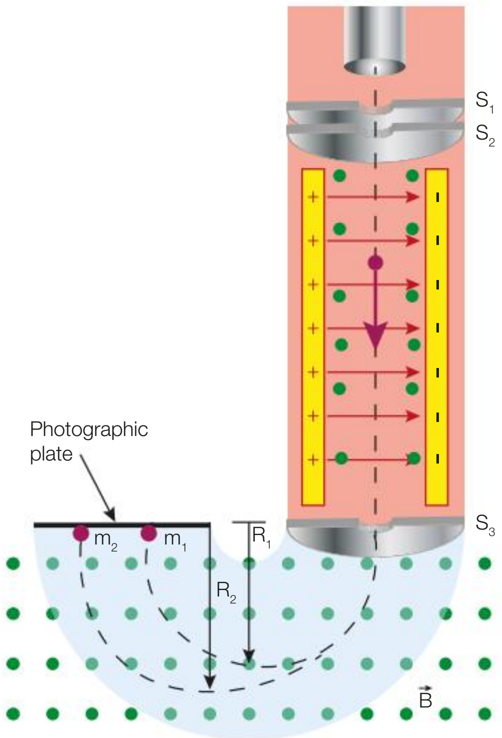
....

(d) Hence show that the magnitude of the electric field is 350 Vm^{-1}.

....

....

19. The diagram below represents a mass spectrometer.



Ions pass through two slits S_1 and S_2 before entering a velocity filter that selects ions of speed $v = 2.00 \times 10^6$ ms^{-1}. These ions then enter a uniform magnetic field $\vec{B} = 0.860T$. Ions with the same charge but of different mass are separated by the magnetic field and are detected on a photographic plate. Ions of mass m_1 and m_2 are detected. The radius of the circular path traced by ions of mass m_1 is 25.0 cm and the radius of the circular path of the ions of mass m_2 is 25.6 cm.

(a) State the sign of the ions in this spectrometer.

....

(b) Explain whether the ions with mass m_1 or m_2 have a larger mass.

....

....

....

(c) Calculate the charge to mass ratio of the ions of mass m_1.

20. A magnetic field of magnitude 1.50 T is used in the operation of a cyclotron that is designed to accelerate singly charged deuterons (H_2^+). The deuterons are extracted at a radius of 25.0 cm and have a mass of $H_2^+ = 3.34 \times 10^{-27}$ kg.

(a) Calculate the period of the deuterons in the cyclotron.

(b) Calculate the frequency of the alternating potential difference.

(c) Determine the kinetic energy of the deuterons as they are extracted from the cyclotron (in J and MeV).

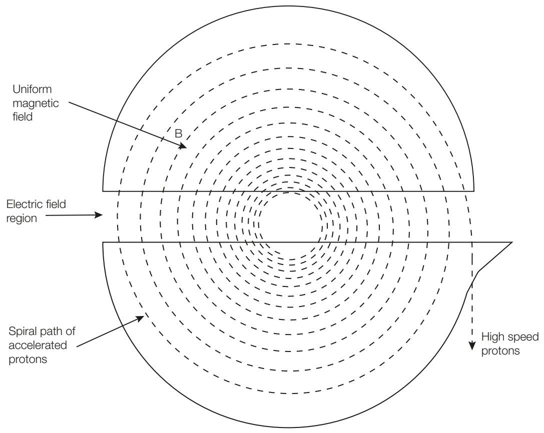
21. The first prototype cyclotron was built with a dee diameter of 9.0 cm and could fit in the palm of a hand. The cyclotron could accelerate protons to a kinetic energy of 81 KeV.

(a) Determine the magnitude of the magnetic field used in this cyclotron.

(b) Calculate the period of the protons in this cyclotron.

(c) Calculate the frequency of the alternating potential difference.

22. The diagram below represents a typical cyclotron that is used to accelerate protons to high speeds.



(a) Describe the function of the uniform electric field.

(b) Describe the function of the uniform magnetic field.

(c) State the direction of the magnetic field in this cyclotron.

A modern medical cyclotron uses a magnetic field of 1.70 T and accelerates protons to a maximum kinetic energy of 30.0 MeV.

(d) Show that the kinetic energy of the protons as they emerge from the cyclotron is given by $E_K = \dfrac{q^2B^2r^2}{2m}$.

(e) Calculate the radius of the cyclotron required to achieve this kinetic energy.

23. (a) Show that the period of charges accelerated in a cyclotron is given by $T = \dfrac{2\pi m}{qB}$.

(b) Explain why the period of the charges accelerated by a cyclotron is independent of their speed within the cyclotron.

24. Science as a human endeavour activity – Magnetic fields

The following text forms part of an article 'Electrical Components of Maglev Systems: Emerging Trends'.

Date: 29 May 2019

Source: Nisha Prasad, Shailendra Jain and Sushma Gupta

https://www.novuslight.com/real-time-tissue-diagnostics-with-miniature-photonics-spectrometers_N8051.html#atop

Rapidly increasing urbanization has given rise to an aggravating transport crisis and deteriorating environmental conditions [1,2,3]. According to recent studies, road transport dominates the global transport industry with a percentage share of 35.1%. Road transport not only contributes around 72.6% of total CO_2 emission resulting from the transportation, but it also consumes 75.3% of total transport energy demand [3]. These facts emphasize the need for clean and efficient mass transit systems. Rail transportation industry has the capability to cope with the expanding transport network. However, on-wheel railways worldwide fulfil 60% of their total energy demand through petroleum products. This not only indicates the necessity of the electrification of railways, but it also emphasizes on the need of improvement in their technological performance [3]. This has motivated the researchers and manufacturers to foster the development of magnetic levitation (maglev)-based rail technology worldwide. Being a fully electrified system, a maglev system can assure future passenger transport. Electrification makes it fully congruous with the renewable energy resources without any technological modifications, which provides sustainability to the system [1,2,3].

Explain how influence, one of the key concepts of science as a human endeavour applies to magnetic levitation (maglev)-based rail technology worldwide.

2.5 Electromagnetic induction

Science understanding

1. Magnetic flux (Φ) is defined as the product of magnetic field strength (B) and the area perpendicular to the magnetic field ($A_\perp$). Hence: $\Phi = BA_\perp$.
   - Solve problems involving the use of $\Phi = BA_\perp$.
2. Electromagnetic induction is the process in which a changing magnetic flux induces a potential difference in a conductor.
3. The induced potential difference is referred to as an electromotive force (ε).
4. The changing magnetic flux is due to relative movement of the conductor or variation of the magnetic field strength.
5. Faraday's Law states that the induced *emf* is equal to the rate of change of the magnetic flux.
6. For N conducting loops the induced ε is given by $\varepsilon = \frac{N\Delta\Phi}{\Delta t}$
7. Lenz's Law states that the direction of a current created by an induced ε is such that it opposes the change in magnetic flux producing the ε.
   - Solve problems involving the induction of an ε in a straight conductor.
   - Solve problems involving the induction of an ε in N conducting loops.
   - Use the law of conservation of energy to explain Lenz's Law.
   - Use Lenz's Law to determine the direction of the current produced by the induced ε.
   - Use Lenz's Law to explain the production of eddy currents.
8. Some generators use a fixed magnet to generate *emf*s in rotating conducting loops for electricity production.
   - Explain how generators can be used to produce an alternating electric current.
9. Transformers allow generated voltage to be either increased or decreased before it is used. A transformer consists of a input coil (with N_{input} turns) with a potential difference V_{input} and a output coil (with N_{output} turns) with a potential difference V_{output}. The relationship between the potential differences is given by the formula:
   $$\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}}$$
   - Explain how a transformer increases or decreases an alternating potential difference.
   - Describe the purpose of transformers in electrical circuits.
   - Compare step-up and step-down transformers.
   - Solve problems involving the use of: $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}}$

This chapter uses the concept of electric current developed in the Stage 1, Subtopic 2.1: Potential difference and electric current.

Magnetic flux (Φ)

Magnetic flux Φ is a measure of the number of magnetic field lines passing through an area.

Figure 2.5.1 shows a uniform magnetic field passing through three equal areas in different orientations to the magnetic field.

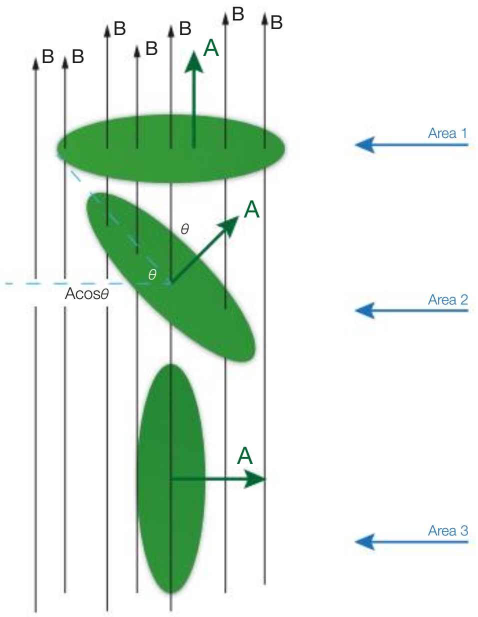


Figure 2.5.1

The angle between the area being considered and the magnetic field affects the magnetic flux. Area 1 is perpendicular to the magnetic field. This situation creates the greatest magnetic flux. Six magnetic field lines can be counted passing through Area 1. The magnetic flux is given by the product of the magnetic field (*B*) and the area (*A*) i.e. $\Phi = BA$.

Area 2 represents Area 1 rotated clockwise through an angle θ. The magnetic flux is less than it is for Area 1. Only four magnetic field lines can be counted passing through Area 2. The component of area perpendicular to the magnetic field is used to calculate the magnetic flux i.e. $\Phi = BA_\perp = BA\cos\theta$.

Area 3 has been rotated through an angle of ninety degrees and is parallel to the magnetic field ($\theta = 90°$). The magnetic flux is zero as there are no magnetic field lines passing through the area. It follows that $\Phi = BA_\perp = BA\cos\theta = BA\cos 90 = 0$.

Magnetic flux Φ through an area A, is defined as the product of the magnetic field strength *B* and the area perpendicular to the magnetic field $A_\perp$.

$$\Phi = BA_\perp = BA\cos\theta$$

where θ is the angle between the area and a line normal to the magnetic field.

The SI unit of magnetic flux is the Weber (Wb).

Converting units of area

The SI unit of area is m^2. To convert cm^2 to m^2 divide by 100^2 or 10 000. This is equivalent to multiplying by 10^{-4}. To convert mm^2 to m^2 divide by 1000^2 or one million. This is equivalent to multiplying by 10^{-6}.

$$cm^2 \xrightarrow{\times 10^{-4}} m^2$$

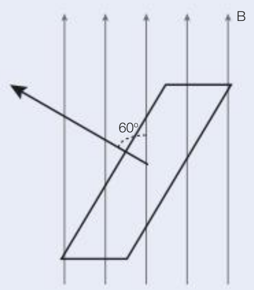
$$mm^2 \xrightarrow{\times 10^{-6}} m^2$$

Worked Example

1. Calculate the magnetic flux through an area of 2.3×10^{-2} m^2 placed at right angles to a magnetic field of magnitude 1.6 T.

$$\Phi = BA_{\perp} = 2.3 \times 10^{-2} \times 1.6 = 3.7 \times 10^{-2}\ Wb$$

2. Calculate the magnetic flux for an area of 16.0 cm^2 placed at an angle of 60.0° to a uniform magnetic field of magnitude 3.00 T.



$$\Phi = BA_{\perp} = BA\cos\theta = 3.00 \times 16.0 \times 10^{-4} \cos 60.0 = 2.40 \times 10^{-3}\ Wb$$

Electromagnetic induction and electromotive force (ε)

Michael Faraday was the first person to show that a current could be induced in a conductor by moving it through a magnetic field. He did this using a straight conductor which he placed in the uniform magnetic field produced by a U-shaped magnet. The conductor was connected to a very sensitive ammeter or galvanometer and moved within the magnetic field. The arrangement is shown in Figure 2.5.2.

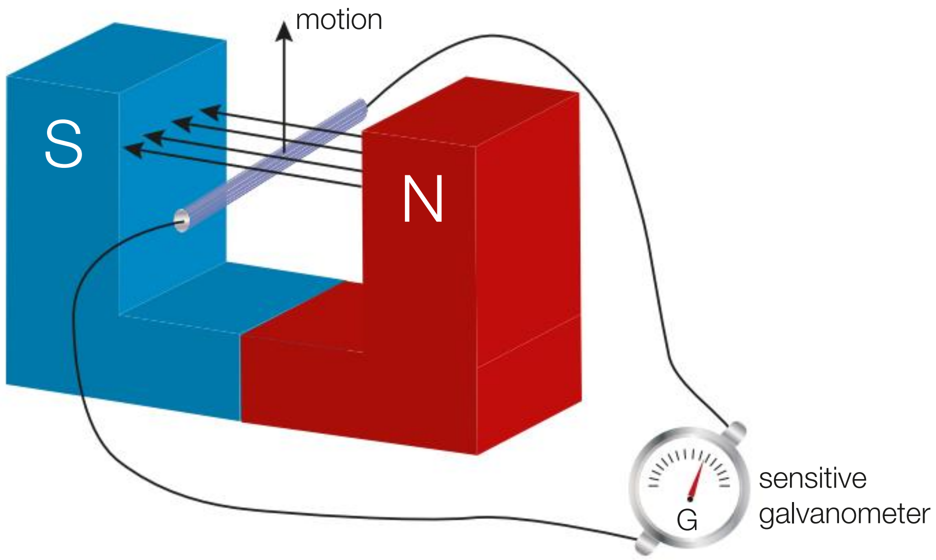


Figure 2.5.2

While the conductor was stationary, no current was detected by the galvanometer. As soon as the conductor was moved, a small current was detected. When the conductor was moved in the opposite direction, the needle deflected in the opposite direction. This indicated that a current flowed when the conductor moved and that the direction of the current reversed when the direction of movement of the conductor was reversed.

When a conductor is moved in a magnetic field, a potential difference is induced in the conductor. This induced potential difference is referred to as an induced *emf* and causes a current to flow.

A changing magnetic flux due to the relative movement of a conductor

Figure 2.5.3 shows a straight conductor of length l, moving with a speed v at right angles to a uniform magnetic field acting into the plane of the page.

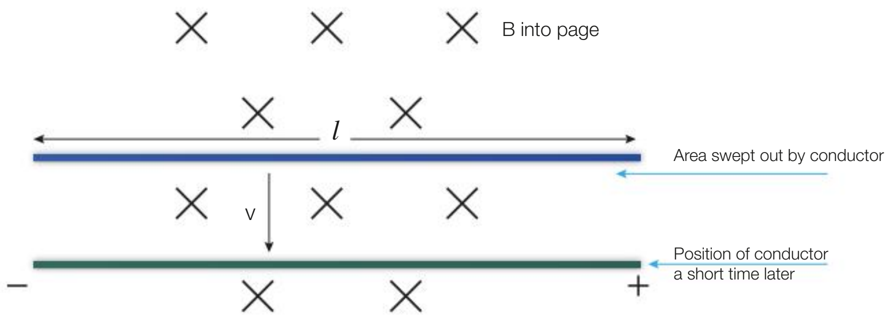


Figure 2.5.3

Electrons in the conductor are forced to move with the same speed as the conductor and experience a force given by $F = qvB\sin\theta = qvB$ since $\theta = 90°$. Using the right hand rule, electrons flow to the left end of the conductor. This means that one end of the conductor has excess electrons and the other end lacks them (i.e. is positively charged). Figure 2.5.3 shows that the left end of the conductor is negative relative to the right end. This creates an induced potential difference or *emf* between the two ends of the conductor. The *emf* increases until it balances the magnetic force acting on the electrons and stops them from moving.

Figure 2.5.3 also illustrates that the conductor sweeps out an area as it moves.

Although not part of this course, it can be shown that the induced *emf* = B × rate at which the area is swept out $= B\frac{\Delta A}{\Delta t} = \frac{\Delta\Phi}{\Delta t}$. That is, the induced *emf* is given by the rate of change in the magnetic flux.

Faraday's Law

Faraday's Law states that the induced ε in a circuit is equal to the rate of change of the magnetic flux.

$$\varepsilon = \frac{\Delta\Phi}{\Delta t}$$

The SI unit of ε is the Volt (V).

Faraday's Law can be extended to a situation involving n conducting loops. An example of this would be a solenoid consisting of ***N* loops** of wire.

$$emf = N\frac{\Delta\Phi}{\Delta t}$$

Notes

1. A change in magnetic flux can occur when a straight conductor moves in a magnetic field. The conductor sweeps out an area while the magnetic field remains constant.
2. A change in magnetic flux can also occur when the area is constant but the magnitude of the magnetic field changes.

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} = NA\frac{(B_2 - B_1)}{\Delta t}$$

Key ideas

1. Electromagnetic induction is the process in which a changing magnetic flux induces a potential difference in a conductor.
2. The induced potential difference is referred to as an electromotive force (ε).
3. The changing magnetic flux is due to relative movement of the conductor or variation of the magnetic field strength.

Helpful online resources

The following computer interactives may be useful in further understanding Faraday's Law: 'Faraday's Law' from https://phet.colorado.edu/en/simulation/faradays-law

'Faraday's Electromagnetic Lab' from https://phet.colorado.edu/en/simulations/faradays-law

Worked Example

1. A circular solenoid has a radius of 5.0 cm and consists of 40 turns. It is placed at right angles to a uniform magnetic field of magnitude 1.4 T. The magnetic field strength is doubled in a time of 0.75 s. Calculate the

   (a) change in magnetic flux for the solenoid.

   $\Delta\Phi = N\Delta BA = N(B_2 - B_1)A = N(B_2 - B_1)\pi r^2 = 40 \times (2.8 - 1.4) \times \pi\,(0.050)^2 = 0.44\ Wb$

   (b) *emf* induced through the coil.

   $emf = \frac{\Delta\Phi}{\Delta t} = \frac{0.44}{0.75} = 0.59V$

Lenz's Law

Heinrich Lenz was a German physicist who conducted experiments in the late 1800s in order to determine the direction of the current induced by a change in magnetic flux.

Figure 2.5.4 shows a straight conductor moving to the right of the page in a uniform magnetic field acting into the plane of the page.

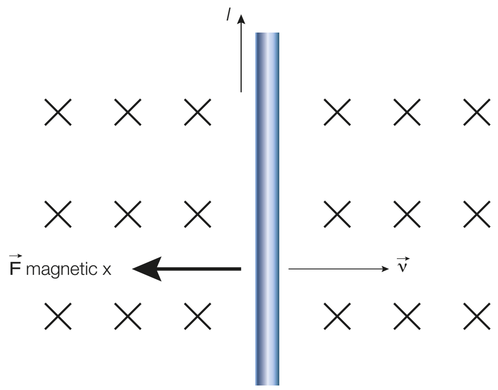


Figure 2.5.4

We have already seen that the movement of the conductor through the magnetic field produces an induced *emf* and therefore current through the conductor. If an ammeter is connected to the conductor, it would show that the current flows upwards through the conductor. Using the right hand rule, this means that a force, directed to the left, acts on the wire. **The induced *emf* produces a current that flows to produce a magnetic force that opposes the movement that created change in magnetic flux.**

A change in magnetic flux can also be caused when the strength of the magnetic field changes. Figure 2.5.5 shows a magnet being moved towards the open end of a solenoid. Figure 2.5.6 shows the magnet being withdrawn. An ammeter connected to the solenoid can be used to detect the direction of the current flowing through the solenoid.

When the north pole of the magnet is brought close to the solenoid (Figure 2.5.5), a current is induced in the solenoid. This is because the strength of the magnetic field closer to the north pole of the magnet is stronger than the magnetic field further from the end of the magnet. A change in magnetic flux due to the changing magnetic field strength induces an *emf* and therefore current across the solenoid. The direction of the current is indicated by the ammeter (from the right end of the solenoid to the left end of the solenoid). This means that the **current flows through the solenoid in a direction that creates a magnetic field that opposes the change in magnetic flux created by the motion of the magnet.** That is a north pole is created at the end closest to the magnet.

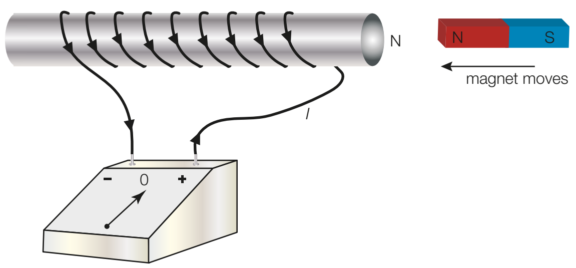


Figure 2.5.5

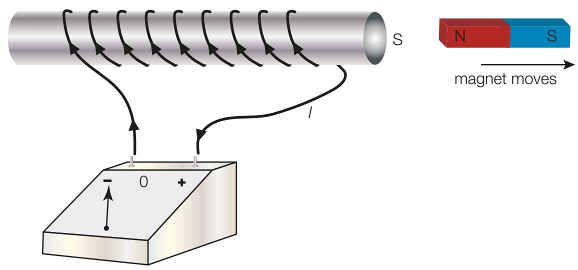


Figure 2.5.6

When the magnet is withdrawn from the solenoid (Figure 2.5.6), the ammeter indicates that the current changes direction. Again the **current flows in a direction that produces a magnetic field that opposes the change in magnetic flux created by the motion of the magnet.** That is a south pole is created at the end of the solenoid closest to the magnet.

It should be noted that the same effect occurs if the solenoid is moved towards a stationary magnet instead of the magnet being moved towards the solenoid.

Lenz's Law states that the induced *emf* creates a current in a direction that opposes the change in magnetic flux producing the *emf*.

Helpful online resources

The following YouTube clip is a lecture that discusses and demonstrates the concepts associated with electromagnetic induction, Faraday's Law and Lenz's Law.

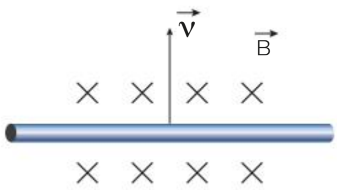
https://www.youtube.com/watch?v=nGQbA2jwkWI

Lenz's Law in terms of the conservation of energy

Figure 2.5.5 showed the north pole of a magnet being moved towards the open end of a solenoid. This resulted in a north pole being induced at the open end closest to the magnet. If a south pole were induced instead of a north pole, the magnetic force of attraction between the two unlike poles would cause the magnet to accelerate towards the solenoid. This would induce a greater current in the solenoid and the magnet would further accelerate. This would continue and the speed and hence kinetic energy of the magnet would theoretically increase indefinitely. This means that energy is created and violates the law of conservation of energy. The induced current must therefore flow in a direction to oppose the change in magnetic flux that produced the induced *emf*.

Worked Examples

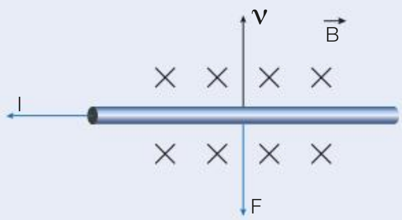
1. The diagram below shows a straight conductor moving upwards in a uniform magnetic field.



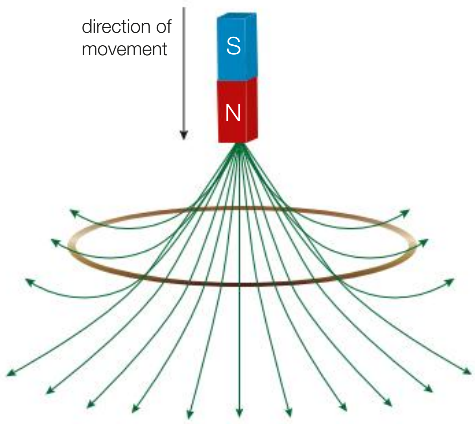
Determine the direction of the current induced in the conductor. Explain your answer.

According to Lenz's Law, the induced current will flow in a direction that opposes the change in magnetic flux producing the *emf*. This means that a force acts on the conductor that is directed down the plane of the page as shown in the diagram.

The force acts down the plane of the page and the magnetic field acts into the plane of the page. Using the right hand rule, the current must flow towards the left of the page.



2. The diagram below shows the north pole of a bar magnet being moved towards a single loop of wire.



(a) Explain why a current is induced in the loop of wire as the magnet approaches the loop.

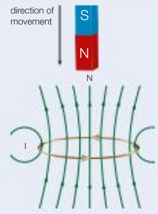
As the magnet moves towards the loop of wire, there is a change in magnetic flux through the loop of wire. A change in magnetic flux through the wire loop will induce an *emf* and hence current through the loop of wire.

(b) Explain the effect that moving the magnet towards the loop of wire with a greater speed would have on the magnitude of the current induced through the wire loop.

Using Faraday's Law, the induced *emf* is equal to the rate of change in magnetic flux through the coil ($\varepsilon = \frac{\Delta\Phi}{\Delta t}$). If the magnet is moved faster, the change in magnetic flux occurs over a smaller time interval. The induced *emf* will increase ($\varepsilon \,\alpha\, \frac{1}{\Delta t}$) which means that the current through the wire loop will also increase.

(c) Use Lenz's Law to determine the direction of the induced current flowing through the loop of wire.

As the magnet moves towards the wire loop, there is a change in magnetic flux (the magnetic flux increases). According to Lenz's Law, the induced current will flow to oppose the change in magnetic flux. Since the magnetic field of the magnet acts downwards. The current flows to produce a magnetic field that acts upwards through the loop. Using the right hand rule, the current flows anticlockwise as shown in the adjacent diagram.



? Science inquiry activity

The following are some ideas for a possible investigation:

- Investigate induced *emf* and currents using data loggers.
- Investigate the output of a hand-turned generator.
- Compare the structure and function of a generator to an electric motor.

The production of eddy currents in terms of Lenz's Law

It would be useful to watch the following YouTube clips before continuing to the notes on eddy currents in this chapter.

Helpful online resources

1. The YouTube clip below demonstrates an eddy current cushioning the fall of a large industrial magnet falling towards a copper plate.

   https://www.youtube.com/watch?v=Yu1uRvErM80

2 The YouTube clip below demonstrates a swinging magnet stopping when it moves over a copper plate. This phenomenon, referred to as 'magnetic braking' is explained in terms of eddy currents.

   https://www.youtube.com/watch?v=ezXVzc64qRE

3 A magnet falling vertically down a metal tube takes a lot longer to fall that if they were to fall down a plastic tube. Magnetic attraction cannot explain this phenomenon as copper is not magnetic. The magnet does not make contact with the side of the copper tube, so friction cannot explain why it takes longer to fall. The following YouTube clip demonstrates this phenomenon using an Eddy Tube. It also explains magnetic braking.

   https://www.youtube.com/watch?v=otu-KV3iH_I

Earlier in the chapter we discussed that a current is induced in a conductor such as a coil (or solenoid) whenever there is a change in magnetic flux produced by a changing magnetic field. A current can also be induced in a solid conductor such as a flat steel sheet, an aluminium tube or a copper plate. In this case we refer to eddy currents. This is because the current that is induced flows in circular swirls much like 'eddies' in water.

An eddy current is the current induced in little swirls ("eddies") on a large solid conductor due to a changing magnetic field.

Figure 2.5.7 shows a large metal plate swinging into a magnetic field. As the metal sheet swings into the magnetic field, a changing magnetic flux induces an *emf* which forces electrons to flow in the metal plate. This creates eddy currents within the metal plate.

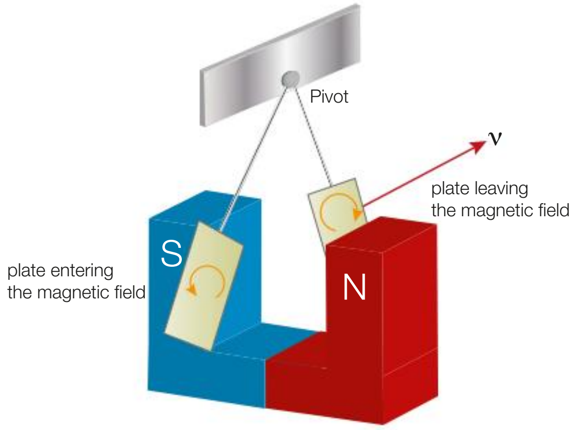


Figure 2.5.7

Figure 2.5.8 below shows the metal plate entering the magnetic field from a different point of view. As the plate enters the magnetic field there is a change in magnetic flux through the plate (the magnetic flux increases). Since the magnetic field produced by the magnet acts into the page, in accordance with Lenz's Law, the eddy currents flow in a direction that opposes the change in magnetic flux. That is, the eddy currents flow in a direction that produces an opposing magnetic field out of the page. This magnetic field will act to reduce the magnetic flux through the plate. Using the right hand rule, the eddy currents flow in an anticlockwise direction.

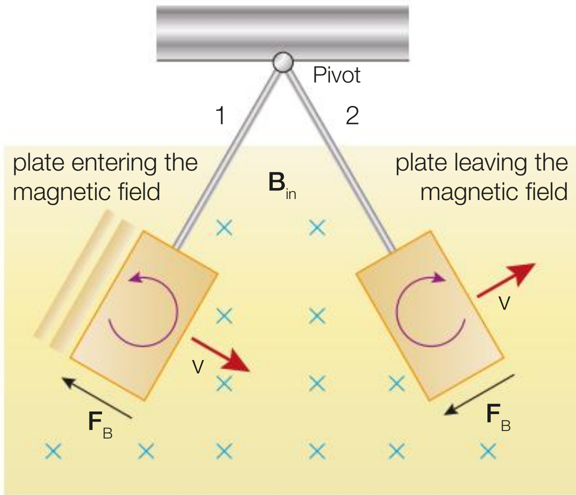


Figure 2.5.8

As the metal plate leaves the magnetic field in Figure 2.5.8, there is another change in magnetic flux through the plate (the magnetic flux decreases). The eddy currents flow in a direction that opposes this change; that is, the eddy currents flow to produce a magnetic field into of the plane of the page. This acts to increase the magnetic flux through the plate. Using the right hand rule, the eddy currents flow in a clockwise direction.

The result is an opposing force on the metal plate and it slows down or even stops. This is because the kinetic energy of the plate is used to force the movement of electrons (i.e. the eddy currents) inside the metal plate and some thermal energy is produced.

Figure 2.5.9 shows a current-carrying solenoid falling towards a conductive plate. The magnetic field produced by the solenoid acts down as it approaches the plate. A change in magnetic flux induces an *emf* in the plate and causes electrons to flow creating eddy currents in the plate. In accordance with Lenz's Law, the eddy currents flow in a direction to oppose the change in magnetic flux. The eddy currents produce a magnetic field that acts upwards. Using the right hand rule, the eddy current shown flows in an anticlockwise direction and the solenoid is repelled upwards.

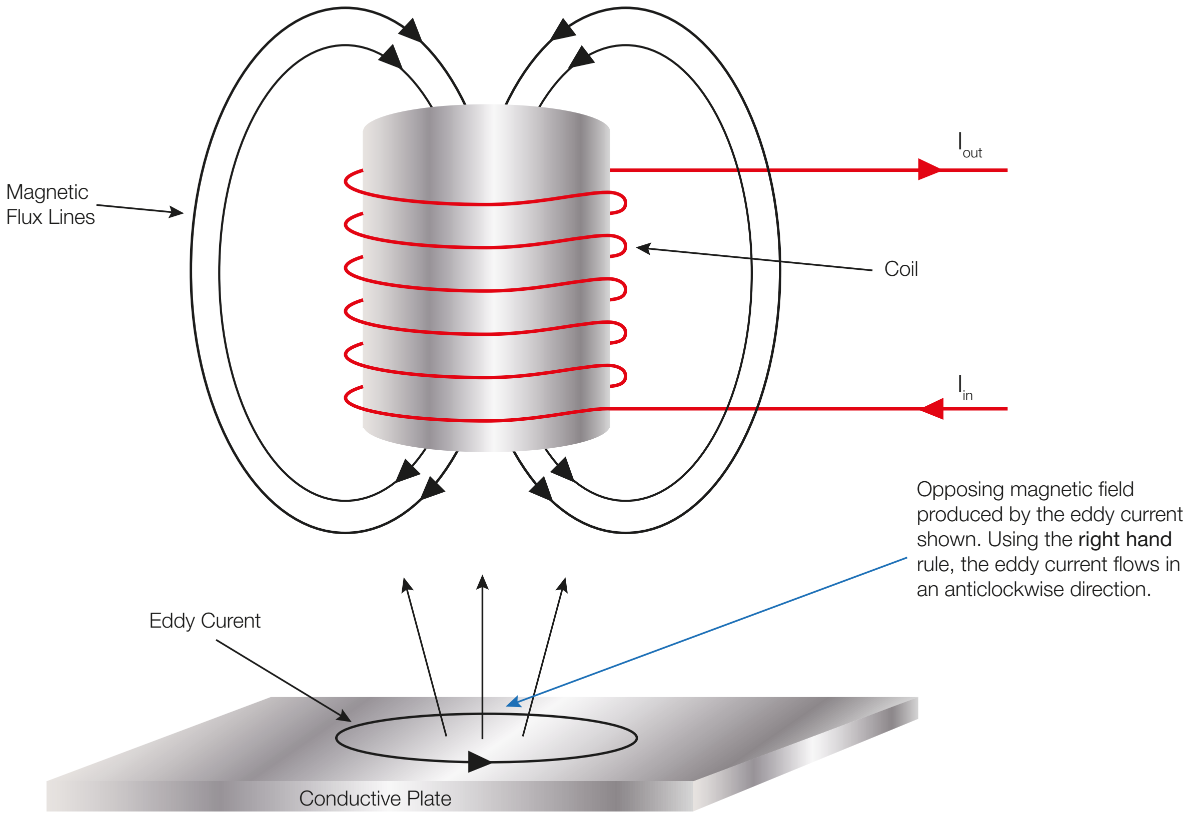


Figure 2.5.9

Magnetic brakes work on this principle. For example, eddy currents can be used to quickly stop rotating power tools when they are turned off. Adding a strong magnetic field around a spinning piece of metal will create eddy currents in the metal. The eddy currents create magnetic fields which oppose any change in magnetic flux. This in turn, produces an opposing force on the spinning metal and it slows down.

The braking system on some subways and trains also use electromagnetic induction and eddy currents. An electromagnet is attached to the train and placed near the steel tracks. When a large current flows through the electromagnet, the relative motion of the magnet and the rails induces eddy currents in the rails. The eddy currents flow in a direction that opposes the change in magnetic flux that created them. This creates a force that opposes the motion of the train. The magnitude of the eddy currents decreases as the train begins to slow down. As a consequence the retarding force on the train reduces and the braking system is smooth.

To avoid large eddy currents being produced in a solid conductor, slits can be cut into the metal as shown in Figure 2.5.10. The braking effect is smaller.

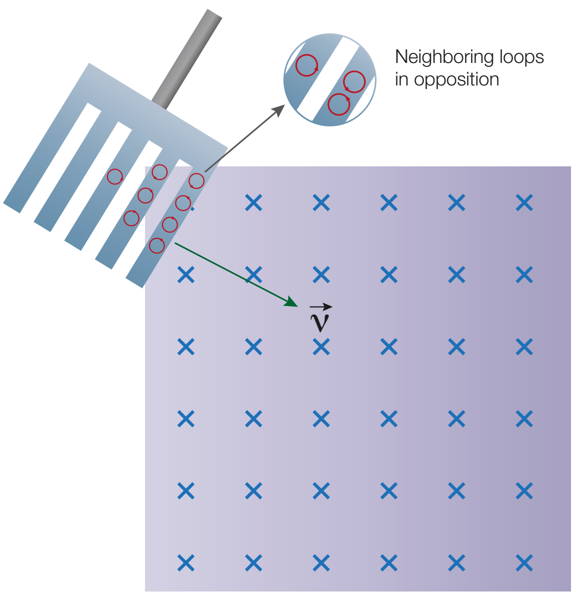


Figure 2.5.10

Generators

A generator uses a fixed magnet to generate *emfs* in rotating conducting loops for electricity production. A generator therefore converts rotational mechanical energy into electrical energy.

Extra Understanding

Main components of a generator

There are two types of generators: direct current (DC) and alternating current (AC).

DC and AC generators

Figure 2.5.11 below shows the main components of a single loop DC generator while Figure 2.5.12 shows the main components of an AC generator.

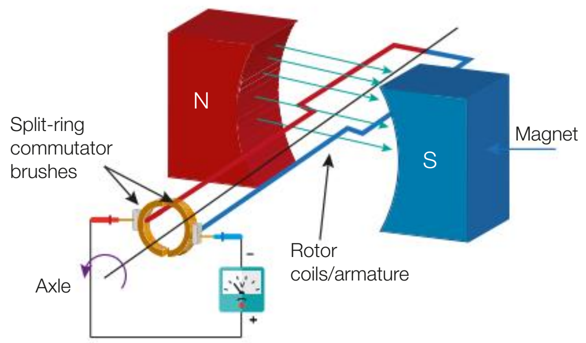


Figure 2.5.11 DC generator

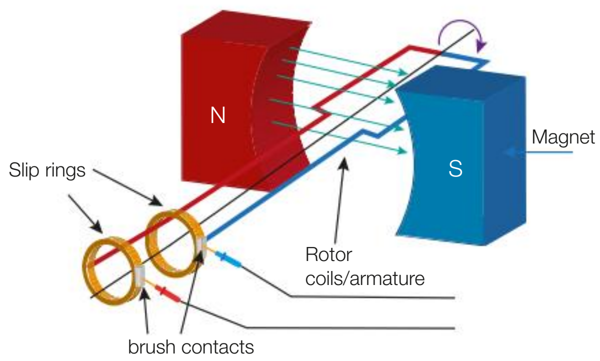


Figure 2.5.12 AC generator

Main Component	Description/Function
Magnets	Usually curved permanent magnets or electromagnets that produce a uniform magnetic field.
Iron core	A soft iron that serves to maximise the magnetic field. The iron core is laminated to reduce the effect of eddy currents.
Rotor coils or armature	Several loops of wire wound around the iron core. A current is induced in the rotor coils as they rotate in the magnetic field.
Split-ring commutator (DC generator)	Two metal half-rings that connect either end of the rotor coils to the brushes. The split-rings provide points of electrical contact so that the induced current can flow to an external circuit through the brushes. The induced current is reversed every half rotation of the coil. This produces current in one direction.
Slip-ring commutator (AC generator)	Two full-rings that connect either end of the rotor coils to the brushes. The slip-rings provide points of electrical contact so that the induced current can flow to an external circuit through the brushes. The induced current is not reversed every half rotation of the coils. This produces AC current.
Brushes	Two graphite blocks on either side of the commutator. The brushes provide points of electrical contact with the split or slip-rings so that the induced current can flow to an external circuit.
Axle	A rod that passes through the iron core and rotor coils and provides an axis of rotation for the rotor coils.

Using generators to produce an alternating electric current

In an AC generator, both ends of a coil are connected to separate slip-rings as shown in Figure 2.5.13. The slip-rings rotate with the coil.

Every half turn or 180°, the direction of the current reverses and an alternating current is generated. Alternating current is supplied to homes and industry.

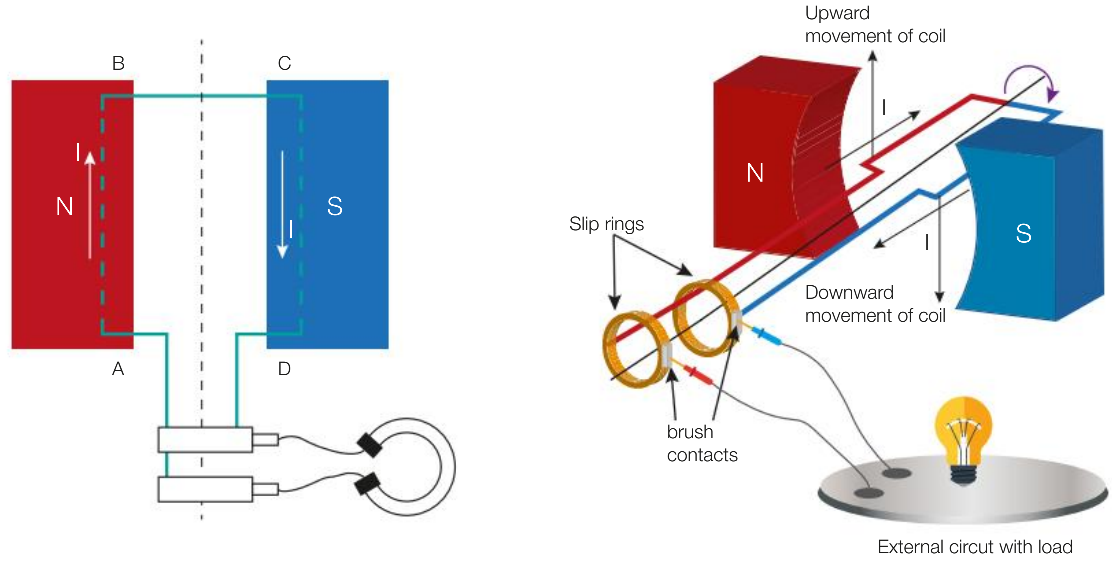


Figure 2.5.13

Figure 2.5.13 shows the coil of an AC generator in a horizontal position. The coil is rotating clockwise. The *emf* and hence current is greatest when there is the greatest change in magnetic flux. This occurs as it begins to rotate from the horizontal position. Once in the vertical position the current flowing is zero. As it continues to rotate back towards the horizontal position the change in magnetic flux gets gradually larger and so does the induced current. The result is a sinusoidal change in *emf* and current through the coil.

Figure 2.5.14 below illustrates the change in current as the coil rotates.

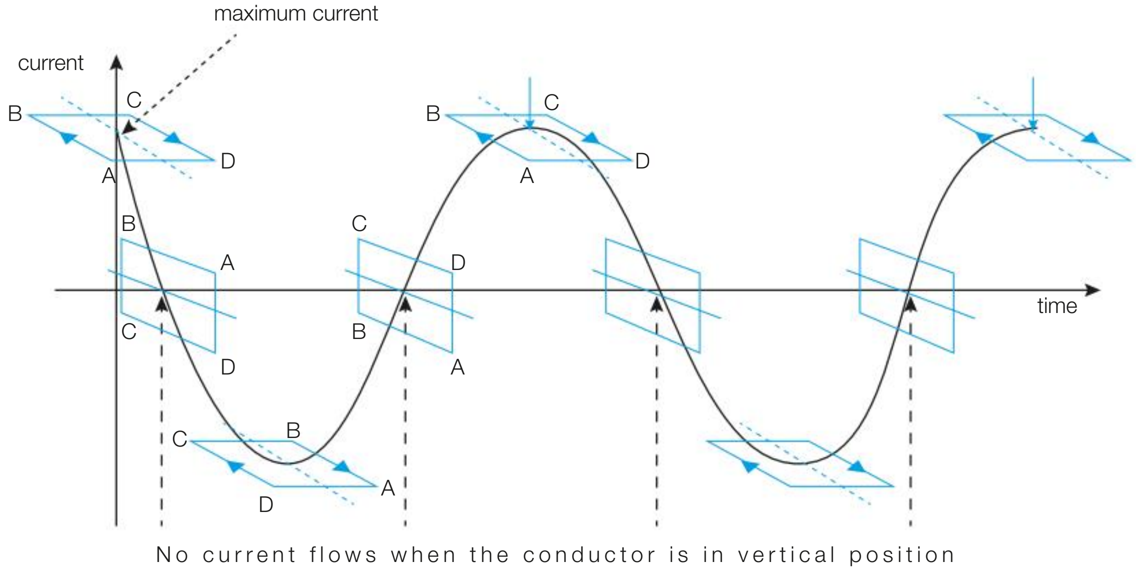


Figure 2.5.14

Transformers

Transformers allow generated voltages to be either increased or decreased before they are used.

Main components

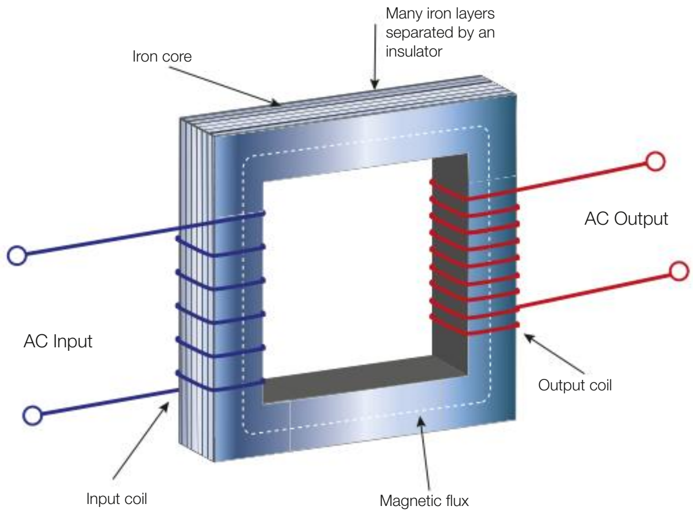


Figure 2.5.15

A transformer consists of two coils of wire called the **input coil** and the **output coil**. The input coil consists of N_{input} **turns** of wire while the output coil consists of N_{output} **turns** of wire. The input and output coils are wrapped around a soft iron core that increases the strength of the magnetic field. A potential difference V_{input} exists across the input coil and a potential difference of V_{output} exists across the output coil.

The relationship between the potential differences across the primary and secondary coils is given by: $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}}$

Step-up and step-down transformers

When an alternating potential difference V_{input} is applied to the input coil, a current flows through the input coil. A magnetic field is produced around the input coil. It quickly grows and cuts the output coil, producing a change in magnetic flux through the output coil. This induces an emf (ε) and hence current in the output coil.

The current in the input coil will drop to zero as the potential difference reverses. This means that the magnetic field in the input coil decreases and once again cuts the output coil. This produces another change in magnetic flux through the output coil. A current flows through the output coil in the opposite direction. An alternating potential difference of the same frequency is therefore induced in the output coil. This is a frequency of 50 Hz in Australia and in many other countries. Some countries use a frequency of 60 Hz.

1. If the number of turns in the output coil is greater than the number of turns in the input coil, then the transformer is referred to as a ***step-up transformer***. This is because the potential difference in the input coil is less than that in the output coil. The output current is lower than the input current.
2. If the number of turns in the output coil is smaller than the number of turns in the input coil, then the transformer is referred to as a ***step-down transformer***. This is because the potential difference in the input coil is greater than that in the output coil. The output current is higher than the input current.

Worked Examples

1. A step-up transformer has an input coil with 85 turns of wire and an output coil with 1200 turns. A potential difference of 240 V is applied across the input coil.

   (a) Calculate the potential difference in the output coil.

   $$\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore V_{output} = \frac{V_{input} \times N_{output}}{N_{input}} = \frac{240 \times 1200}{85} = 3400\ V$$

   (b) The transformer needs to be modified if it is to produce a potential difference of 5500 V in the output coil. Determine the increase in the number of turns of wire in the output coil needed to achieve this.

   $$\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore N_{output} = \frac{V_{output} \times N_{input}}{V_{input}} = \frac{5500 \times 85}{240} = 1948$$

   The original transformer has 1200 turns in the output coil. A further 748 (approximately 750) turns are needed.

2. A step-down transformer has a potential difference of 600 V applied across its input coil and produces a potential difference of 12 V across its output coil.

   Calculate the ratio of the number of turns in the input coil to the number of turns in the output coil.

   $$\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore \frac{N_{input}}{N_{output}} = \frac{600}{12} = 50$$

Science as a human endeavour

The following are some possible ideas for investigation:

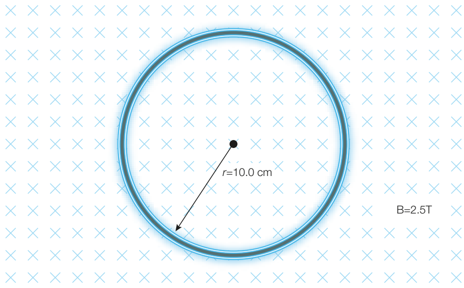
1. Explore the benefits and limitations of applications of electromagnetic induction, such as:
   - reading data from computer hard drives
   - induction cooktops
   - electromagnetic (eddy-current) braking
   - maglev trains
   - security systems
   - vehicle detection at traffic lights
   - metal detectors
   - minesweepers.
2. Assess the economic, social, and environmental impacts of power generation by:
   - mechanically powered torches
   - domestic and industrial electricity power stations
   - alternators in vehicles.
3. Analyse changes that have resulted from the use of transformers in contexts such as:
   - step-up and step-down transformers in electrical power transmission
   - step-down transformers in home appliances
   - induction coils in vehicles.

Exercises

1. (a) Define the term magnetic flux.

   (b) Calculate the magnetic flux through a conducting loop with an area of $4.0 \times 10^{-2}\ m^2$ which is placed at right angles to a magnetic field of magnitude 1.2 T.

2. The diagram below shows a circular loop of wire with a radius 10.0 cm in a uniform magnetic field of magnitude 2.50 T.



(a) Calculate the magnetic flux through the circular loop.

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(b) Discuss the effect that doubling the magnetic field strength would have on the magnetic flux through the circular loop.

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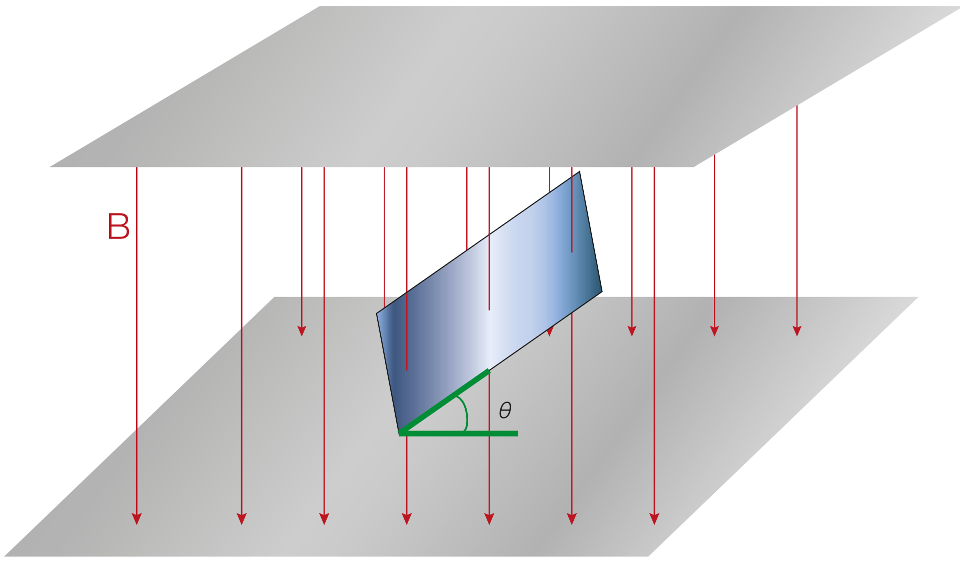
3. The diagram below shows a square conducting loop with dimensions 12.0 cm x 12.0 cm in a uniform magnetic field of strength 0.980 T. The angle $\theta = 35.0°$.



(a) Calculate the area of the rectangular loop in m^2.

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(b) Calculate the magnetic flux through the rectangular conductor.

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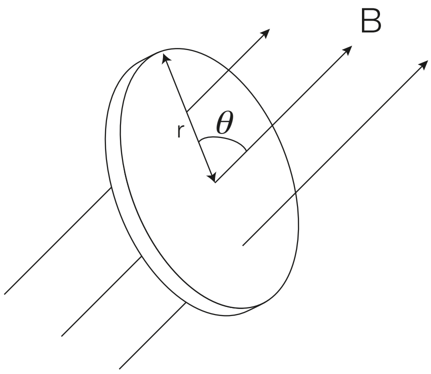
(c) Determine the new value of θ if the magnetic flux through the rectangular loop is half its maximum possible value.

4. The diagram below shows a circular coil of wire of radius r placed at an angle to a uniform magnetic field of strength B.



Show that the magnetic flux through the circular loop is given by $\pi r^2 B \sin\theta$.

5. (a) State Faraday's Law of electromagnetic induction.

(b) A circular loop with an area of 0.15 m^2 is placed in a magnetic field of magnitude 2.2 T. The magnetic field strength is increased to 2.8 T in 3.0 ms.

Calculate the induced *emf* across the circular loop.

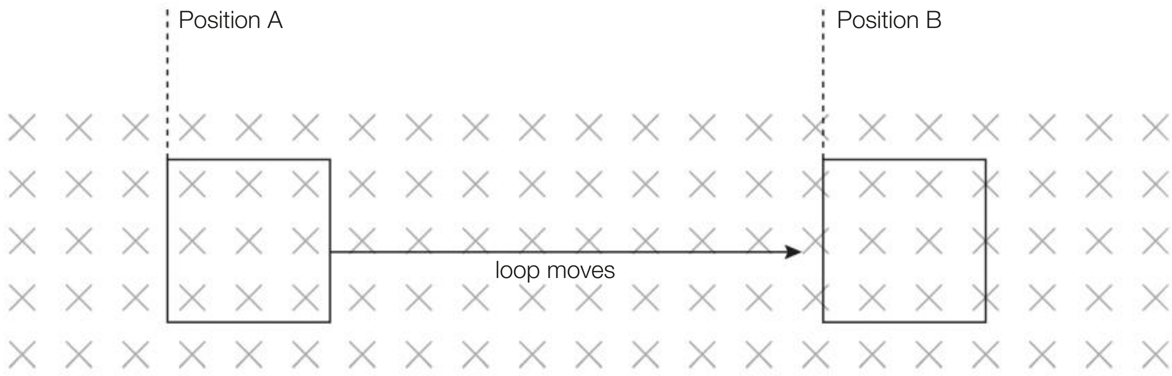
6. An emf ε is induced across a conducting loop of area A that is placed in a changing magnetic field.

Use proportionality to discuss the effect that each of the following changes would have on the *emf* induced across the conductor. State the new *emf* in terms of ε.

(a) The time over which the magnetic flux through the loop changes is tripled.

(b) The area of the conducting loop is halved.

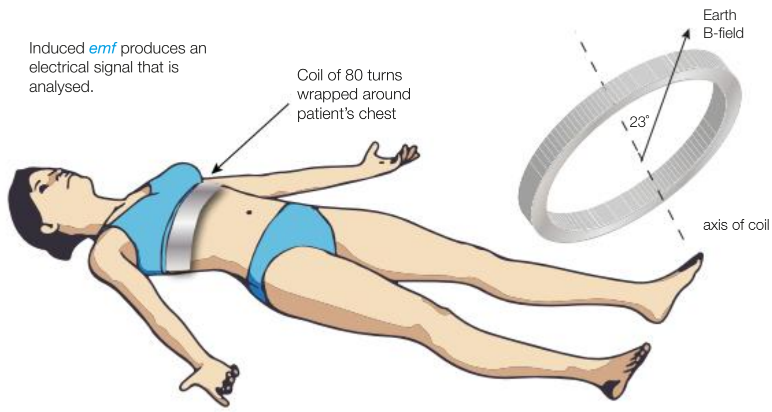
7. A square loop of wire is moved from position A to position B in a time *t*.



Explain why there is no *emf* induced in the loop.

8. The magnetic flux through a conducting coil with 25 loops is reduced from 60.0 Wb to zero in 5.20 seconds. Calculate the *emf* induced in the conducting coil.

9. The diagram below shows a patient being monitored for breathing problems while they sleep (sleep apnea). The device used consists of a coil with 80 turns of wire wrapped around the patient's chest.



The Earth's magnetic field has a magnitude of 5.0×10^{-6} T and makes an angle of 23° with the axis of the coil.

As the patient takes a breath, the area of the coil increases from 0.089 m^2 to 0.12 m^2 in a time of 0.82 s.

(a) Calculate the average change in magnetic flux through the coil.

(b) Calculate the average ε induced in the coil.

(c) The patient becomes anxious and starts breathing at twice the rate. State the effect on the magnitude of the average induced *emf* through the coil.

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10. (a) State Lenz's Law.

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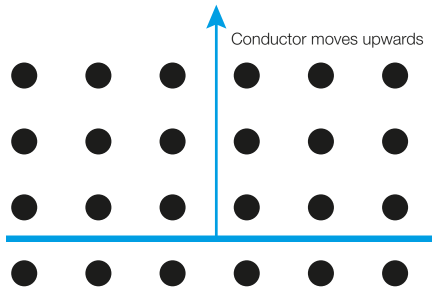
(b) The diagram below shows a straight conductor being moved through a uniform magnetic field.



(i) Explain why an *emf* is induced across the conductor as it is moved through the magnetic field.

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(ii) Draw an arrow on the diagram that represents the direction of the induced current that flows through the conductor.

(iii) Use Lenz's Law and the right hand rule to explain your reasoning for the direction of the induced current.

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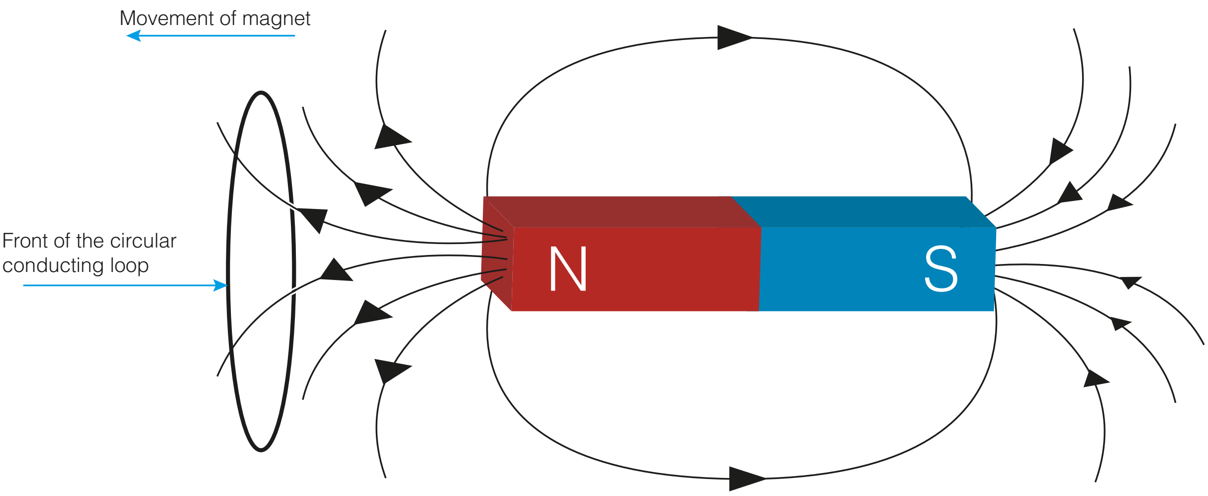
11. The diagram below illustrates a bar magnet moving towards a circular conducting loop.



(a) Describe the reason for an *emf* being induced across the circular loop.

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(b) Draw an arrow on the conducting loop that represents the direction of the current induced in the circular loop as the magnet is moved towards it.

(c) Use Lenz's Law to explain your reasoning for the direction of the induced current.

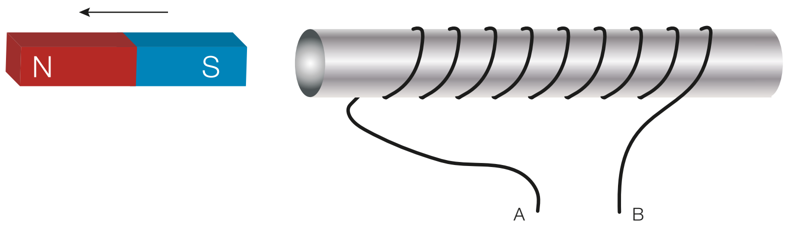
(d) State, with reason, the effect that each of the following would have on the magnitude and/or direction of the current induced in the circular loop.

(i) The bar magnet stops moving.

(ii) The bar magnet is moved towards the circular loop at a higher speed.

(iii) The bar magnet is withdrawn from the circular loop.

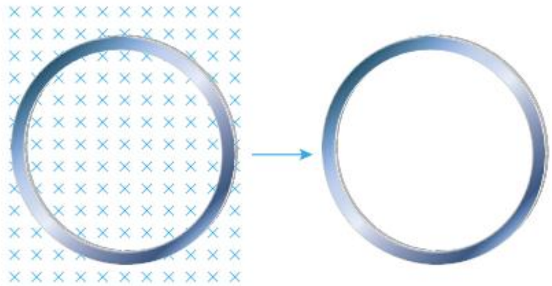
12. The diagram below illustrates a bar magnet being moved away from a solenoid.



(a) Use Lenz's Law to determine the direction (from A to B or B to A) of the induced current through the solenoid.

(b) Use this situation to explain Lenz's Law in terms of the conservation of energy.

13. The diagram below shows a conducting coil in a magnetic field that is directed into the plane of the page. The coil is moved to the right so that it is no longer in the magnetic field.



The loop has a radius of 6.0 cm and the magnitude of the magnetic field is 4.0 T.

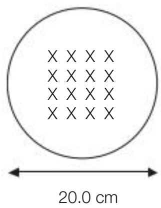
(a) Calculate the change in magnetic flux through the coil due to it being moved out of the magnetic field.

(b) Calculate the *emf* induced across the coil given the movement of the coil takes 1.1×10^{-2} s.

(c) State, with reason, the direction of the induced current through the coil.

(d) At what point in time does the induced current in the coil stop flowing?

14. The diagram below shows the north pole of a bar magnet being inserted into a single coil of wire of diameter 20.0 cm.

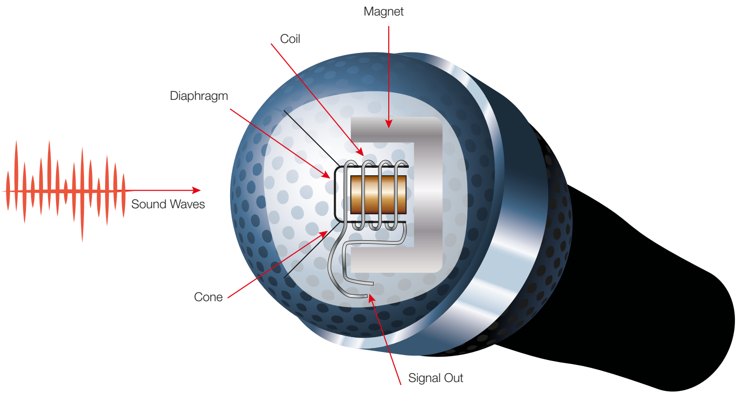


(a) Determine, with reasons, the direction of the induced current in the coil.

(b) An *emf* of 3.00 V is induced in the coil of wire in a time of 0.450 s. Calculate the change in magnetic flux through the coil.

(c) Calculate the change in magnetic field strength that created the induced current.

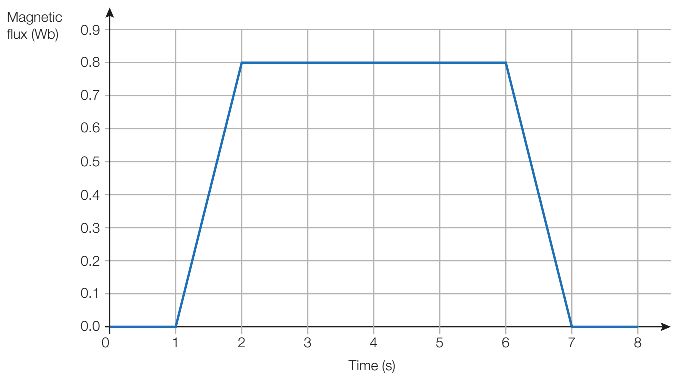
15. The diagram below shows the structure of a dynamic microphone. The diaphragm is attached to a coil of wire that is free to move in a uniform magnetic field.



(a) Explain how the sound waves are transformed into an oscillating electrical signal by the microphone.

(b) Explain how the frequency of the sound waves affects the *emf* induced across the coil.

16. The graph below represents how the magnetic flux through a conducting coil changes as the coil moves into a uniform magnetic field and is then withdrawn.



(a) Explain why the gradient of the graph represents the *emf* induced in the conducting coil.

(b) (i) State the time at which the coil just enters the magnetic field.

....

(ii) State the time interval during which the conducting coil is stationary within the magnetic field.

....

(iii) State the time at which the coil has just been completely removed from within the magnetic field.

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(c) Calculate the *emf* induced in the coil.

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17. Figure 1 below shows a metal plate moving into the magnetic field produced by a horse-shoe magnet. The metal plate moves at right angles to the magnetic field.

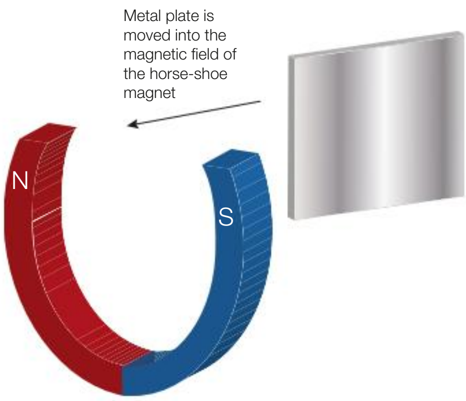

Figure 1

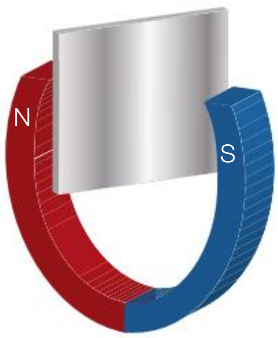

Figure 2

(a) On Figure 2, draw one eddy current that may be produced in the plate as it is being moved into the magnetic field.

(b) Explain the direction of the eddy currents produced in the metal plate.

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(c) The metal plate would be difficult to push into the magnetic field. State the reason for this.

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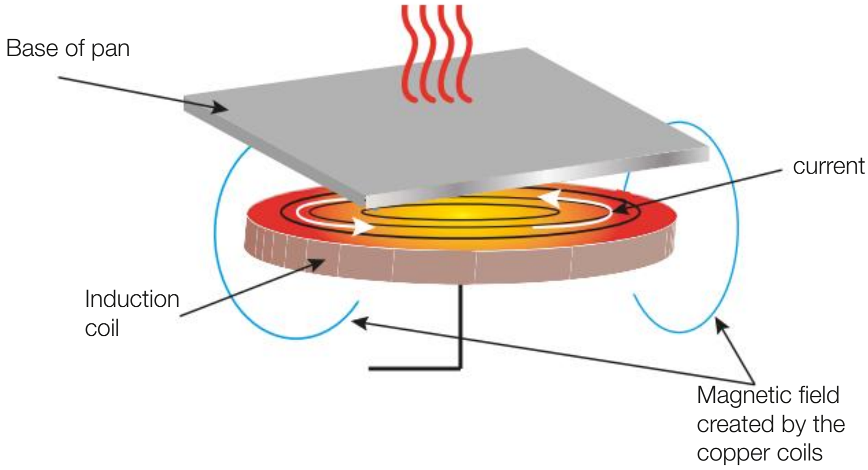
18. An induction cooktop contains tightly wound copper coils called the induction coil. The induction coil is located under a ceramic cooktop. The pan used to cook food is placed on top of the ceramic cooktop and over the induction coil.



When an alternating current flows through the induction coil, eddy currents are set up in the base of the pan. Electrons collide with atoms in the metal base of the pan. Thermal energy is released which heats the contents of the pan. The direction of the current through the induction coil is shown by the white arrows:

(a) On the diagram, show the direction of the magnetic field created by the induction coil by adding a clear arrow on each of the two lines already drawn to represent the magnetic field of the induction coil.

(b) Draw one large circular arrow in the base of the pan that represents the direction of the eddy currents produced in the base of the pan.

(c) The photograph below shows a fry pan cut in half and placed over the heating element of an induction cooktop. A block of chocolate is positioned so that half is in the pan and the other half lies directly on the cooktop.

(i) Explain why the chocolate that lies directly on the cooktop does not melt.

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(ii) Suggest two possible advantages of an induction cooker compared to cooking with gas.

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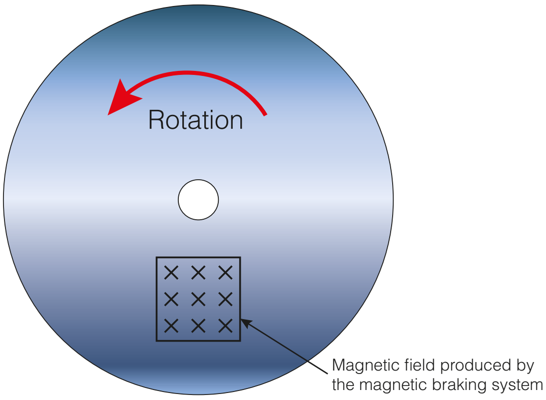
19. The photograph below shows an angle grinder in use. This is a power tool that consists of a rotating metal disc. Angle grinders have many uses, including metal fabrication. A magnetic braking system is used to quickly stop the rotating metal disc when the grinder is turned off.

The diagram below shows the rotating disc and the magnetic field produced by the magnetic braking system.



(a) On the diagram of the rotating disc, draw several eddy currents that could be produced by the magnetic braking system. Clearly indicate the direction of the eddy currents.

(b) Briefly describe why the rotating disc stops when the eddy currents are produced in the disc.

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20. (a) Describe the purpose of a transformer in an electrical circuit.

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(b) A transformer has an input coil with 50 turns and an output coil with 250 turns.

(i) State whether the transformer is a step-up or step-down transformer.

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(ii) State the ratio of the potential difference across the input coil to the potential difference across the output coil.

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21. A transformer has 1850 turns in its input coil with a potential difference of 1150 V. The output coil has 650 turns.

Calculate potential difference across the output coil.

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22. Compare each of the following features of a step-up and step-down transformer.

(a) The number of turns in the input and output coils.

(b) The voltage across the input and output coils.

(c) The output current in the output coil.

23. A transformer is used to change 240 V to 12 V.

(a) State whether the transformer is a step-up or step-down transformer.

(b) Calculate the number of turns in the output coil, if the input coil of the transformer has 2000 turns.

24. The ratio of potential difference in the output coil of a transformer to the potential difference in the input coil of a transformer is 20.

(a) State whether the transformer is a step-up or step-down transformer.

(b) Calculate the number of turns in the input coil of the transformer if the output coil has 860 turns.

25. A soldering iron is a tool commonly used to connect electrical wires or other electrical connections. The soldering iron generates heat to melt solder. The image below shows a soldering iron in use.

A soldering iron is an example of a step-down transformer with a large step ratio. Describe what a large step ratio would indicate in the case of a soldering iron.

Topic 3: Light and atoms

3.1 Wave behaviour of light

Science understanding

1. Oscillating charges produce electromagnetic waves of the same frequency as the oscillation; electromagnetic waves cause charges to oscillate at the frequency of the wave.
   - Use the frequency of oscillation of the electrons in the transmitting and receiving antennae to explain the transmission and reception of electromagnetic signals.
2. Electromagnetic waves are transverse waves made up of mutually perpendicular, oscillating electric and magnetic fields.
   - Relate the orientation of the receiving antenna to the plane of polarisation of electromagnetic waves.
3. The speed of a wave, its frequency, and its wavelength are related through the formula $v = f\lambda$.
   - Solve problems using $v = f\lambda$.
4. Most light sources emit waves that radiate in all directions away from the source.
5. Monochromatic light is light composed of a single frequency.
6. Coherent waves maintain a constant phase relationship with each other.
   - Describe what is meant by two wave sources being in phase or out of phase.
   - Explain why light from an incandescent source is neither coherent nor monochromatic.
7. When two or more electromagnetic waves overlap, the resultant electric and magnetic fields at a point can be determined using the principle of superposition.
8. When the waves at a point are in phase, 'constructive interference' occurs.
9. When the waves at a point are out of phase, 'destructive interference' occurs.
   - Use the principle of superposition to describe and represent constructive and destructive interference.
10. For two monochromatic sources in phase, the waves at a point some distance away in a vacuum:
    - constructively interfere when the path difference from the sources to the point is $m\lambda$
    - destructively interfere when the path difference from the sources to the point is $\left(m+\frac{1}{2}\right)\lambda$

    where m is an integer and λ is the wavelength.
    - Use the principle of superposition to determine points of maximum or minimum amplitude resulting from the interference of light from two wave sources of the same frequency.
    - Use constructive and destructive interference to explain the maximum and minimum amplitudes.
11. Young's double-slit experiment can be used to demonstrate the wave behaviour of light.
12. The formulae $d\sin\theta = m\lambda$ and $\Delta y = \frac{\lambda L}{d}$ can be used to analyse the interference pattern, where d is the distance between the slits, θ is the angular position of the maximum, Δy is the distance between adjacent minima or maxima on the screen, and L is the slit-to-screen distance.
    - Describe how two-slit interference is produced in the laboratory using a coherent light source.
    - Describe how diffraction of the light by the slits in a two-slit interference apparatus allows the light to overlap and hence interfere.
    - Sketch a graph of the intensity distribution for two-slit interference of monochromatic light. (Consider only cases where the slit separation is much greater than the width of the slits.)
    - Explain the bright fringes of a two-slit interference pattern using constructive interference, and the dark fringes using destructive interference.
    - Solve problems involving the use of $d\sin\theta = m\lambda$ and $\Delta y = \frac{\lambda L}{d}$.
    - Determine the wavelength of monochromatic light from measurements of the two-slit interference pattern.
13. The interference pattern produced by light passing through a transmission diffraction grating demonstrates the wave behaviour of light.
14. Transmission diffraction gratings can be used to analyse the spectra of various light sources.

15. The formula $d\sin\theta = m\lambda$ can be used to analyse the interference pattern.
    - Describe how diffraction by the very thin slits in a transmission diffraction grating allows the light from the slits to overlap and hence interfere to produce significant intensity maxima at large angles.
    - Derive $d\sin\theta = m\lambda$ for the intensity maxima in the pattern produced by a transmission diffraction grating, where d is the distance between the slits in the grating and θ is the angular position of the m^{th} maximum (m specifies the order of the maximum).
    - Solve problems involving the use of $d\sin\theta = m\lambda$.
    - Sketch a graph of the intensity distribution of the maxima produced by a transmission diffraction grating, for monochromatic light.
    - Determine, from the distance between the slits in the transmission diffraction grating, the maximum number of orders possible for a given transmission diffraction grating and wavelength.
    - Describe how a transmission diffraction grating can be used to experimentally determine the wavelength of light from a monochromatic source.
    - Describe and explain the white-light pattern produced by a transmission diffraction grating.
    - Identify the properties of a transmission diffraction grating that make it useful in spectroscopy.

This chapter uses the concept of waves developed in the Stage 1, Subtopic 5.1: Wave model and 5.3 Light.

Oscillating charges and electromagnetic waves

In subtopic 2.1 we saw that stationary charges produce a stationary electric field. An oscillating charge will therefore produce an oscillating electric field.

In subtopic 2.3 we saw that a moving charge produces a magnetic field that is perpendicular to the movement of the charge. This is in addition to the electric field associated with the charge. If the charge is moving with constant velocity, the magnetic field is constant. If the charge is oscillating, the magnetic field will also oscillate. In subtopic 2.5 we saw that a changing or oscillating magnetic field induced an oscillating current in a conductor.

James Clerk Maxwell was a Scottish mathematician and physicist. In 1864 he introduced his theory for electromagnetic radiation. This theory is sometimes referred to as the wave model for light and is based on the idea that an oscillating electric field produces an oscillating magnetic field and an oscillating magnetic field produces an oscillating electric field.

When charges are forced to oscillate, they produce electromagnetic waves that consist of oscillating electric and magnetic fields. The two fields oscillate at right angles to each other and to the direction of wave propagation (travel). By definition, electromagnetic waves are transverse waves and travel with a constant speed of 3.00×10^8 ms^{-1} in a vacuum.

Figure 3.1.1 below, illustrates an electromagnetic wave.

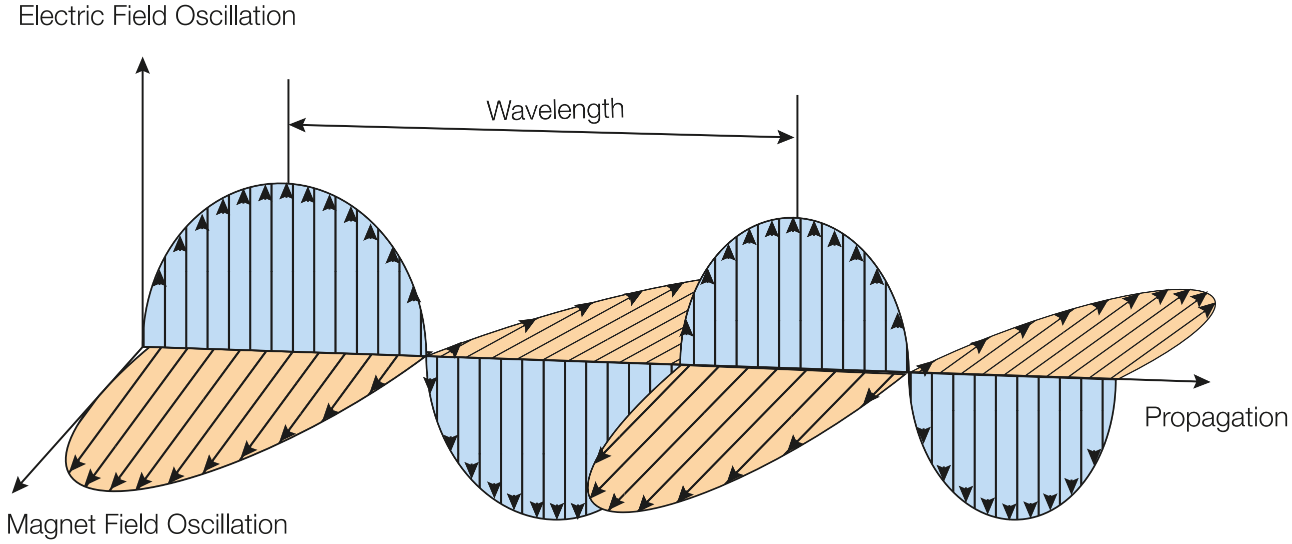


Figure 3.1.1

The concept of electromagnetic waves and the electromagnetic spectrum, which consists of all forms of light, were introduced in the Stage 1 Workbook: Subtopic 5.3. The frequency of an electromagnetic wave is the same as the frequency of the oscillating charges that produce the wave. The frequency of the oscillation will therefore determine the type of electromagnetic wave that is produced.

The wave equation relates the speed v of a wave to the frequency f and wavelength and is given by $v = f\lambda$.

Figure 3.1.2 illustrates the electromagnetic spectrum and gives an indication of the magnitude for the wavelength and frequency of different types of electromagnetic waves.

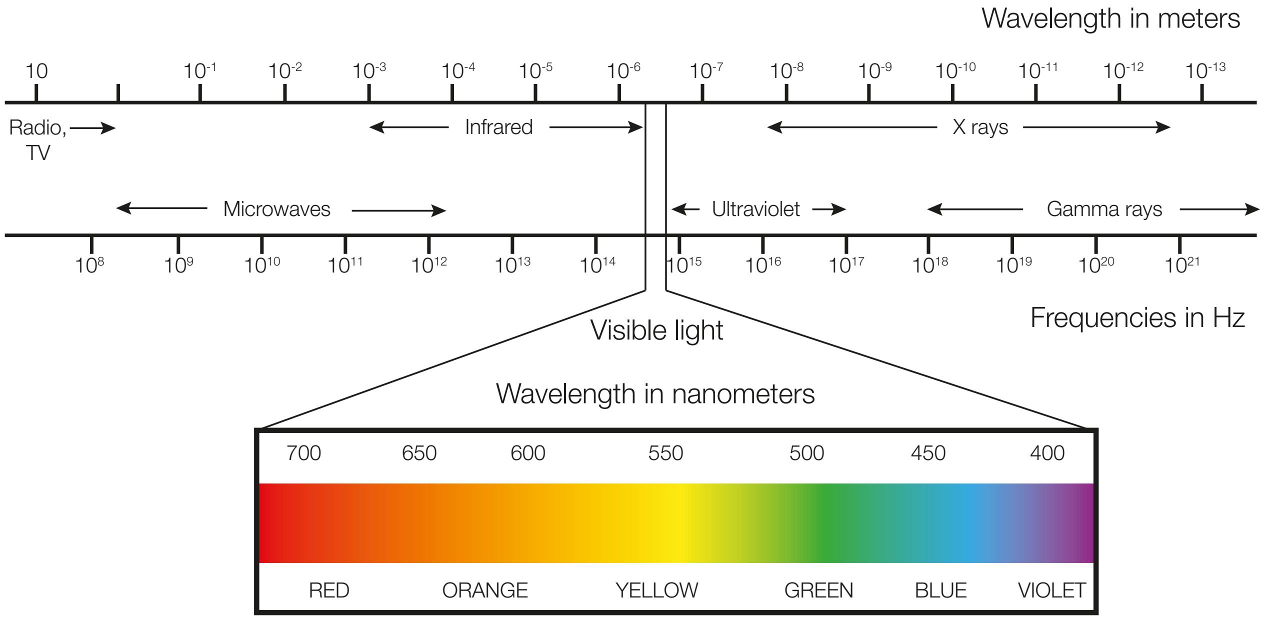


Figure 3.1.2

Key ideas

1. Oscillating charges produce electromagnetic waves of the same frequency as the oscillation; electromagnetic waves cause charges to oscillate at the frequency of the wave.
2. Electromagnetic waves are transverse waves made up of mutually perpendicular, oscillating electric and magnetic fields.
3. Electromagnetic waves are transverse waves and travel with a constant speed of 3.00×10^8 ms^{-1} in a vacuum.
4. The wave equation for the speed v of a wave applies to all forms of electromagnetic radiation.

Helpful online resources

1. The following YouTube clip shows the electric field of an electron when it is stationary and while it vibrates.

   https://www.youtube.com/watch?v=DOBNo654pwQ

2. The following YouTube clip is an animation of an oscillating charge that produces an electromagnetic wave. The electric field is produced in the same plane that the charge oscillates.

   https://www.youtube.com/watch?v=hXSXp3QNTjQ

Worked Example

Calculate the frequency of X-rays that have a wavelength of 2.00×10^{-12}m.

$v = f\lambda$

Therefore $f = \frac{v}{\lambda} = \frac{3.00 \times 10^8}{2.00 \times 10^{-12}} = 1.50 \times 10^{20}$ Hz

Radio and television signals

Antennae are usually made of aluminium rods and convert an electric current into electromagnetic radiation and vice versa. Antennae are designed to transmit and/or to receive radio or television signals within a certain frequency range according to their size and form.

The simplest type of antenna consists of two metal rods, and is known as a dipole antenna. The monopole antenna, consists of a single rod placed vertically on a large metal plate. The antenna mounted on vehicles is usually a monopole antenna, with the metal roof of the vehicle serving as a ground plane. In general antennae all work on the same principle.

Figure 3.1.3 shows two of the three tall broadcasting transmission towers located in Mount Lofty about 15 km away from the city centre of Adelaide. The three broadcast towers are Broadcast Australia, TX Australia and TX Australia NWS-9 towers.

Figure 3.1.3

The plane of polarisation of radio and television waves

The polarisation of waves was discussed in Subtopic 5.3 of the Stage 1 Workbook. Plane-polarisation involves the restriction of vibrations to a single plane. Radio and television waves are produced when electrons are forced to oscillate in a transmitting antenna. Since the electrons can only oscillate in one plane parallel to the antenna, the emitted waves are said to be plane-polarised. In other words, the electromagnetic wave is plane-polarised in the direction of the vibrating charge. Figure 3.1.4 shows a vertically plane-polarised electromagnetic wave and a horizontally plane-polarised electromagnetic wave.

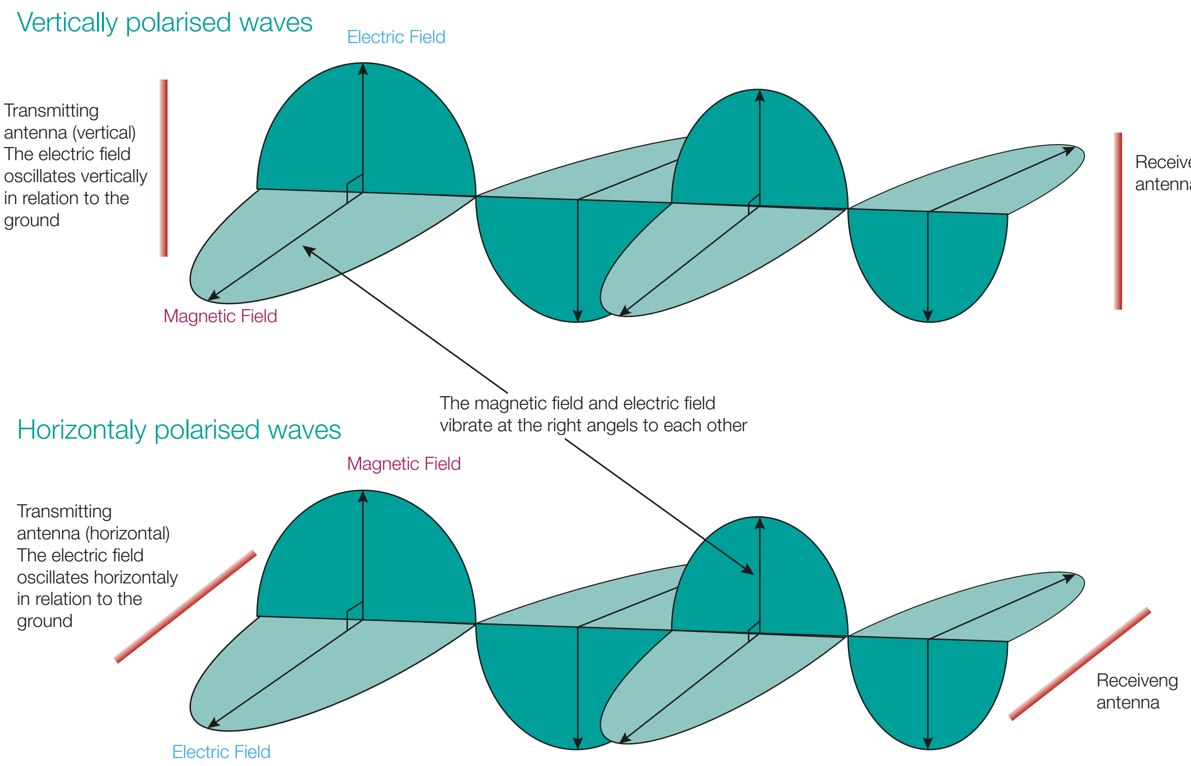


Figure 3.1.4

The plane of polarisation of an electromagnetic wave is defined as the plane of the oscillating electric field.

? Science inquiry activity

1 Your teacher may demonstrate the polarisation of microwaves.

2 Utilise this computer interactive showing the production and transmission of electromagnetic waves: https://phet.colorado.edu/en.simulation/legacy/radio-waves.

3

Transmitting and receiving an electromagnetic signal using antennae

Electrons in the transmitting antenna are forced to oscillate along the length of the antenna by an alternating potential difference (sometimes referred to as a radio frequency RF) attached to the antenna. This creates an oscillating electric field. The oscillating electric field produces an oscillating magnetic field at right angles to the electric field. Both fields are perpendicular to one another and the direction that that the wave travels. Since an oscillating electric field can create an oscillating magnetic field and vice versa, these two fields continually reproduce one another and a plane-polarised electromagnetic signal (e.g. radio or television), radiates away from the antenna in all directions. Figure 3.1.5 shows a vertically plane-polarised radio wave being emitted from an antenna. The frequency of the radio wave is the same as the frequency of oscillation of the electrons in the transmitting antenna.

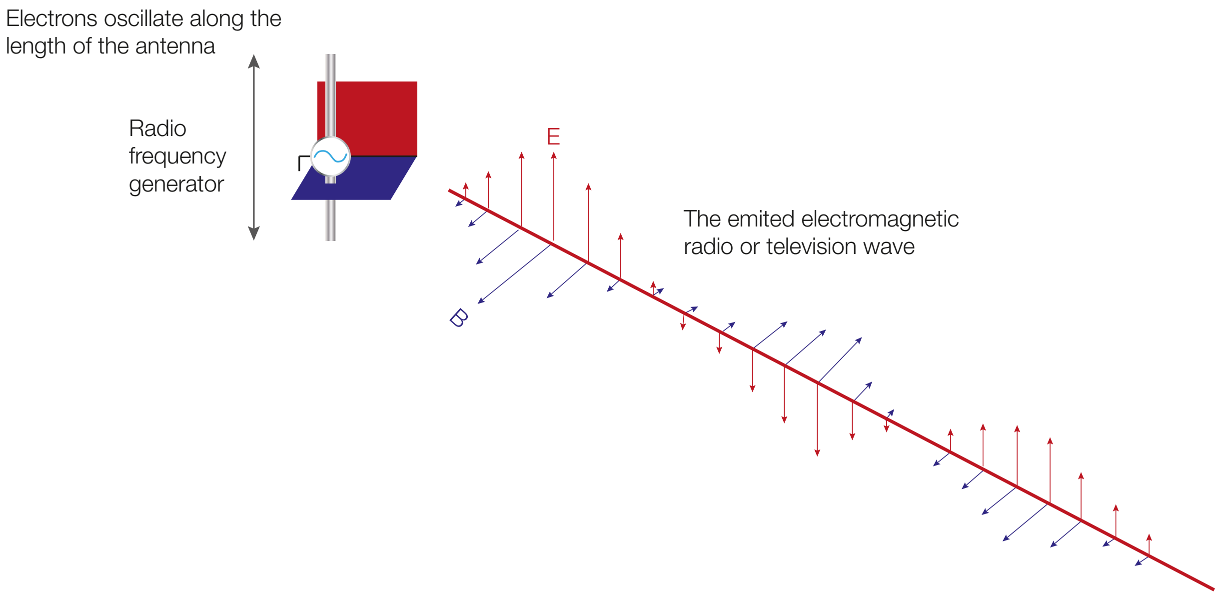


Figure 3.1.5

As the electromagnetic wave transmitted from an antenna approaches a receiving antenna, the oscillating electric field exerts a force ($F = Eq$) on the stationary electrons in the receiving antenna. The electrons will oscillate at the same frequency as the electric field. This produces an alternating potential difference between the two rods of the dipole. The frequency of the alternating potential difference is the same as the frequency of oscillation of the electrons. In this way, the electromagnetic signal that was originally emitted is now received.

Figure 3.1.6 shows an electromagnetic wave approaching a receiving antenna.

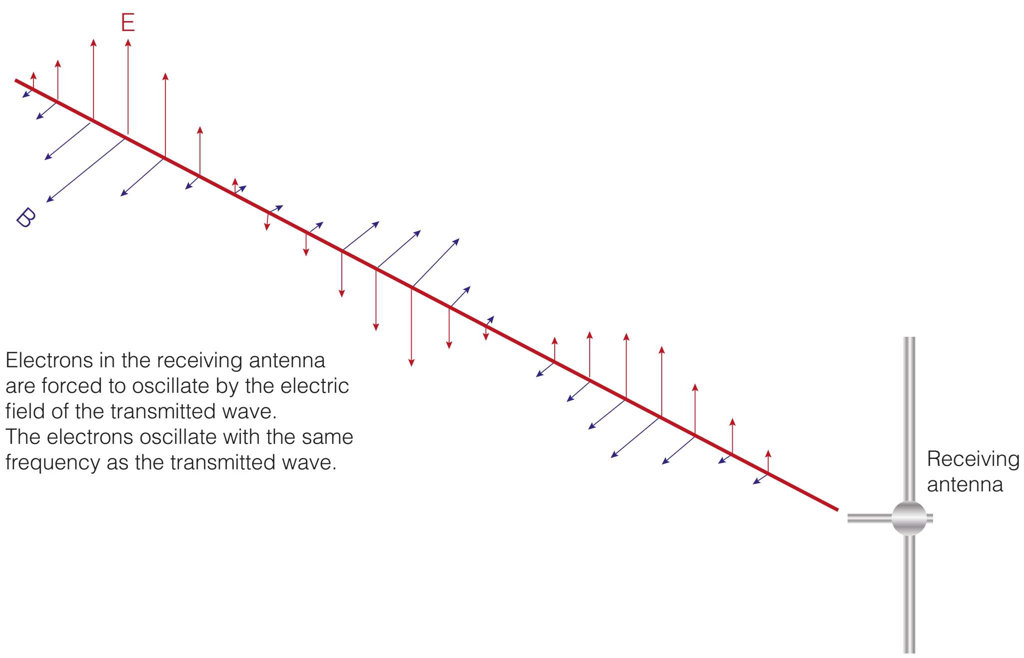


Figure 3.1.6

The orientation of the receiving antenna and the plane of polarisation of electromagnetic waves

A strong signal will be received by a receiving antenna if it is orientated parallel the antenna transmitting the signal. That is, the receiving antenna is orientated in the same plane of polarisation as the transmitted electromagnetic wave. This is because the electrons in the receiving antenna are made to oscillate by the electric field not the magnetic field which does not exert a force on stationary electrons in the receiving antenna ($F = qvB\sin\theta$). The electric field is emitted in the same plane that the electrons are forced to vibrate in the transmitting antenna so it follows that the transmitting antenna and the receiving antenna need to be parallel to receive a strong signal.

If the receiving antenna is orientated at right angles to the antenna transmitting the radio or television wave, the electric field of the transmitted signal cannot oscillate the electrons in the receiving antenna so that they oscillate to produce a large oscillating potential difference across the rods of the antenna. In this case, a very weak signal is received.

Extra understanding

Figure 3.1.7 below shows a typical city antenna, while Figure 3.1.8 shows a typical country antenna. These antennae are mounted on the roof of a home and are used to receive television and radio signals.

Figure 3.1.7

Figure 3.1.8

City and country radio and television signals are polarised in planes perpendicular to each other. City signals are horizontally plane-polarised (antennae are orientated horizontally) while country signals are vertically-plane polarised (antennae are orientated vertically). Strong signals require a receiving antenna to be orientated in the same plane of polarisation as the transmitted signal. This means that city antennae can't receive a strong country signal and vice versa. This reduces the possibility of the two signals interfering.

Science as a human endeavour

The following are suggestions that may lead to an investigation.

Explore examples of new technologies enabled by an understanding of electromagnetic waves:

- data recording, storage, transmission and reproduction
- AM and FM radio/TV
- mobile phone transmission, Bluetooth, Wi-Fi
- orientation of transmitting and receiving antennae
- synchrotron radiation.

3

Wave terminology

Monochromatic light

The word monochromatic comes from two Greek words: *mono*, meaning single and *chroma*, meaning colour. Monochromatic visible light is therefore light of one colour. Recall that the electromagnetic spectrum contains all forms of light. Oscillating charges produce electromagnetic waves and the frequency of the oscillations will determine the frequency and hence type of electromagnetic wave produced. This means that monochromatic light consists of a single frequency and therefore wavelength of light.

Monochromatic light is light composed of a single frequency.

Wave sources that are in phase or out of phase

Two wave sources that are in phase emit waves that are in step. This means that if one wave source emits a crest so does the other, alternatively if one source emits a trough so does the other. This concept can be extended to any number of wave sources.

Transverse waves, such as light, are said to **be in phase** if the crest of one wave coincides with the crest of another. The trough of one wave therefore coincides with the trough of the other. Figure 3.1.9 illustrates the waves produced by two wave sources that are in phase.

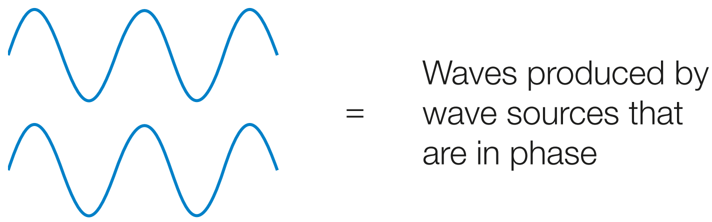


Figure 3.1.9

Two wave sources that are **out of phase** emit waves that are out of step. For the purpose of this course two wave sources being out of phase will refer to the waves being half a wavelength ($\frac{\lambda}{2}$) out of step. This means that if one wave source emits a crest, the other wave source emits a trough. Figure 3.1.10 illustrates the waves produced by two wave sources that are out of phase.

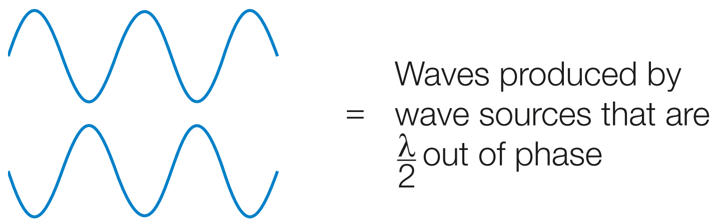


Figure 3.1.10

Coherent wave sources

Coherent wave sources produce waves that remain in phase as they radiate in all directions away from a source. Wave sources can be coherent and in phase or coherent and out of phase. To remain in step, the waves must have the same frequency.

Coherent waves maintain a constant phase relationship with each other.

Light from an incandescent source

An incandescent light source is a source of light produced by heating a material until it glows. A common example is a light bulb. A light bulb is an artificial source of incandescent light produced by heating a wire filament with electricity. This process is called incandescence. The resistance of the filament causes electrons to vibrate randomly and with a range of frequencies as they move through the filament. These oscillations produce electromagnetic radiation or light.

The light produced by an incandescent source consists of a range of frequencies. In addition, random oscillations mean that the emitted light does not maintain a constant phase relationship. Light from an incandescent light source is neither monochromatic nor coherent.

Figure 3.1.11 shows glowing hot coals and Figure 3.1.12 shows a common light bulb. Both are examples of incandescent sources of light.

Figure 3.1.11

Figure 3.1.12

Principle of superposition

We have already seen the principle of superposition being applied to electric forces and electric fields in Subtopic 2.1. Superposition also applies to waves. In general, the resultant amplitude of a wave produced when two or more waves meet at a point in space is a vector sum of the individual amplitudes of the waves that meet.

The principle of superposition for electromagnetic waves states that when two or more electromagnetic waves overlap, the resultant electric and magnetic fields at a point are the vector sum of their separate fields.

Constructive and destructive interference

Figure 3.1.13 shows two identical waves that are in phase. When waves at a point are **in phase,** the principle of **superposition** states that the amplitude of the resultant wave is the vector sum of the individual amplitudes i.e. twice the amplitude of the original waves.

This is referred to as **constructive interference.**

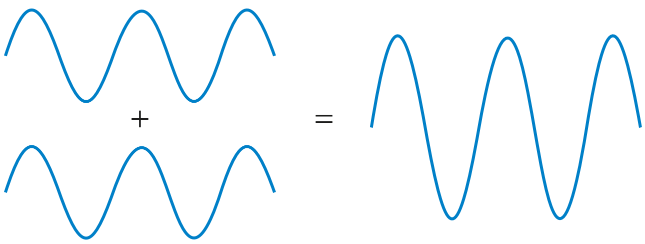


Figure 3.1.13

Figure 3.1.14 shows two identical waves that are out of phase. When waves at a point are half a wavelength **out of phase,** the principle of **superposition** states that the amplitude of the resultant wave is the vector sum of the individual amplitudes i.e. the amplitude of the original waves cancels to zero.

This is referred to as **destructive interference.**

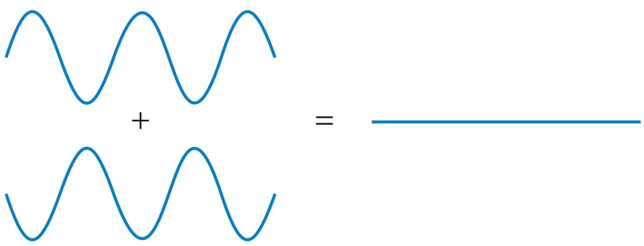


Figure 3.1.14

Two-source interference

In Subtopic 5.3 of the Stage 1 Workbook , the interference pattern that resulted when two coherent wave sources overlap was discussed in terms of path difference. Your teacher may demonstrate a two-source interference pattern using a ripple tank or two-source simulation application such as

https://phet.colorado.edu/en/simulations/wave-interference

Figure 3.1.15 below maps the crests and troughs of two coherent wave sources that overlap. For the purpose of this course, only sources that are in phase will be considered. The concepts described can be extended to other forms of waves but of particular interest is light.

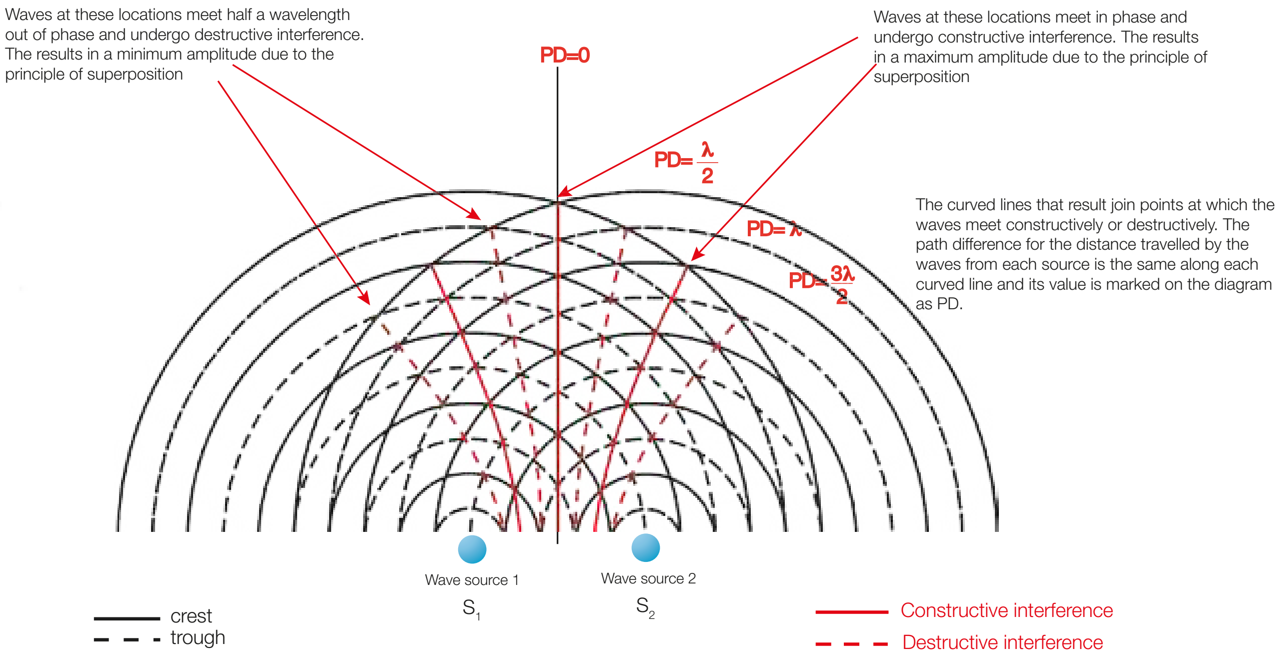


Figure 3.1.15

Waves from each of the coherent sources travel different distances to the point were they meet and undergo superposition. This difference in path is called the **path difference** and is expressed in terms of wavelength (λ).

For two monochromatic light sources in phase, the waves at a point some distance away in a vacuum:

- constructively interfere when the path difference from the sources to the point is $m\lambda$ (m = 0, 1, 2, 3…)
- destructively interfere when the path difference from the sources to the point is $(m + \frac{1}{2})\lambda$ (m = 0, 1, 2, 3…)

In other words, constructive interference occurs when the path difference is equal to a whole number of wavelengths, while destructive interference occurs when the path difference is equal to an odd number of half wavelengths.

Young's double slit experiment

In 1801 Thomas Young demonstrated that light displayed interference effects using his two-slit experiment. This provided experimental evidence for the wave behaviour of light.

The two-slit interference pattern for light consists of alternating bright and dark fringes or bands of equal width. The colour of the bright fringes corresponds to the colour of the monochromatic light used. Figure 3.1.16 shows the two-slit interference pattern for sodium light.

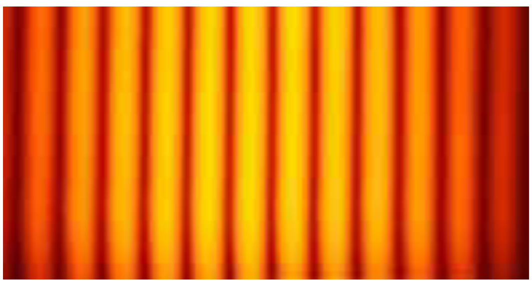


Figure 3.1.16

Producing two-slit interference in the laboratory

Using a light source that is not coherent

Two-slit interference can be produced in a laboratory using a Young's double slit interferometer. Figure 3.1.17 shows an interferometer while Figure 3.1.18 is a diagrammatical representation of an interferometer.

The interferometer is constructed so that monochromatic light passes through a single slit (S). The single slit causes the light to diffract producing circular waves that travel the same distance to a pair of double slits S_1 and S_2. The waves produced at the single slit S, reach the double slits S_1 and S_2 simultaneously. The waves are therefore in phase and the double slits act as two coherent light sources. Further diffraction occurs at each of the double slits, S_1 and S_2 to produce circular waves that overlap and allow an interference pattern to form. The interference pattern can be viewed on a screen through a telescopic eyepiece.

Figure 3.1.17

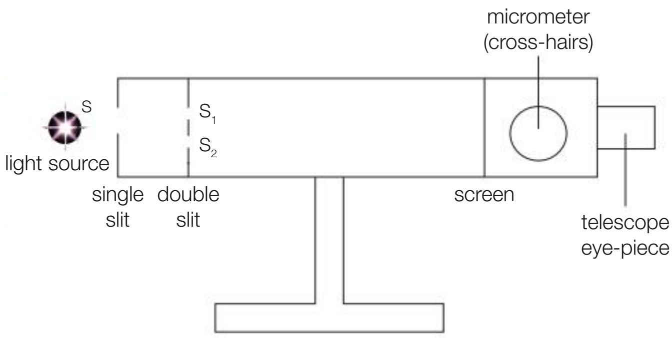


Figure 3.1.18

Single slit diffraction

The diffraction of waves was discussed in Subtopic 5.3 of the Stage 1 Workbook.

Diffraction is the bending (and hence spreading) of a wave as it passes through an opening or around an obstacle.

For observable diffraction effects, the wavelength of the waves must be comparable to the slit size.

Figure 3.1.19 shows waves diffracting though various slits and around an obstacle. Diffraction is significant when the slit width is comparable to the wavelength of the waves.

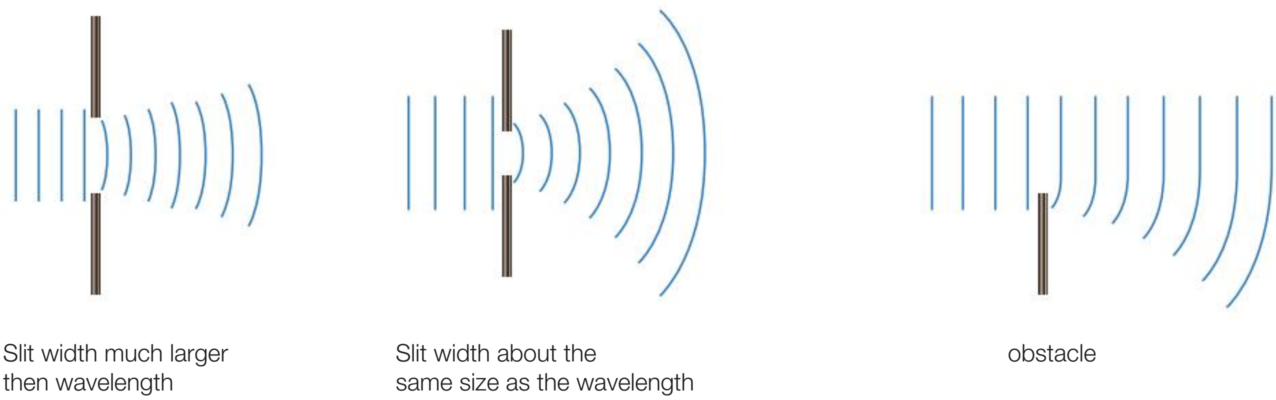


Figure 3.1.19

Using a light source that is coherent

If a coherent light source such as a laser is used, the single slit is not required. The arrangement is illustrated in Figure 3.1.20. Dots are produced on the screen instead of fringes or bands because a laser produces a beam of light.

Double slit slides can be purchased commercially. Figure 3.1.20 shows a double slit slide with a slit separation of 0.5 mm. The screen can be placed about a metre away from the double slits. This arrangement will easily allow you to experimentally observe the effect of different slit separations (*d*) on the fringe separation (Δy) by passing the laser through a double slits slide with a different slit separation. It also allows you to change the distance between the double slits and the screen (*L*) and determine the effect this has on the fringe separation (Δy).

A commercial slide with double slits separated by 0.5mm

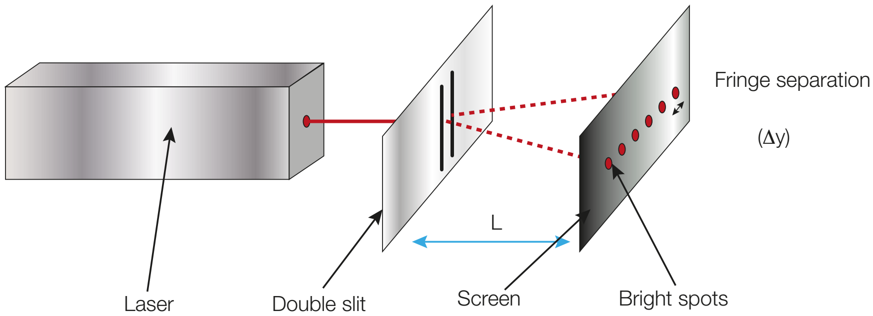


Figure 3.1.20

The intensity distribution graph for two-slit interference

The two-slit interference pattern for monochromatic light can be represented graphically as shown in Figure 3.1.21. The peaks on the graph represent the maxima or bright fringes while the troughs represent the minima or dark fringes. The path difference is also indicated on the graph.

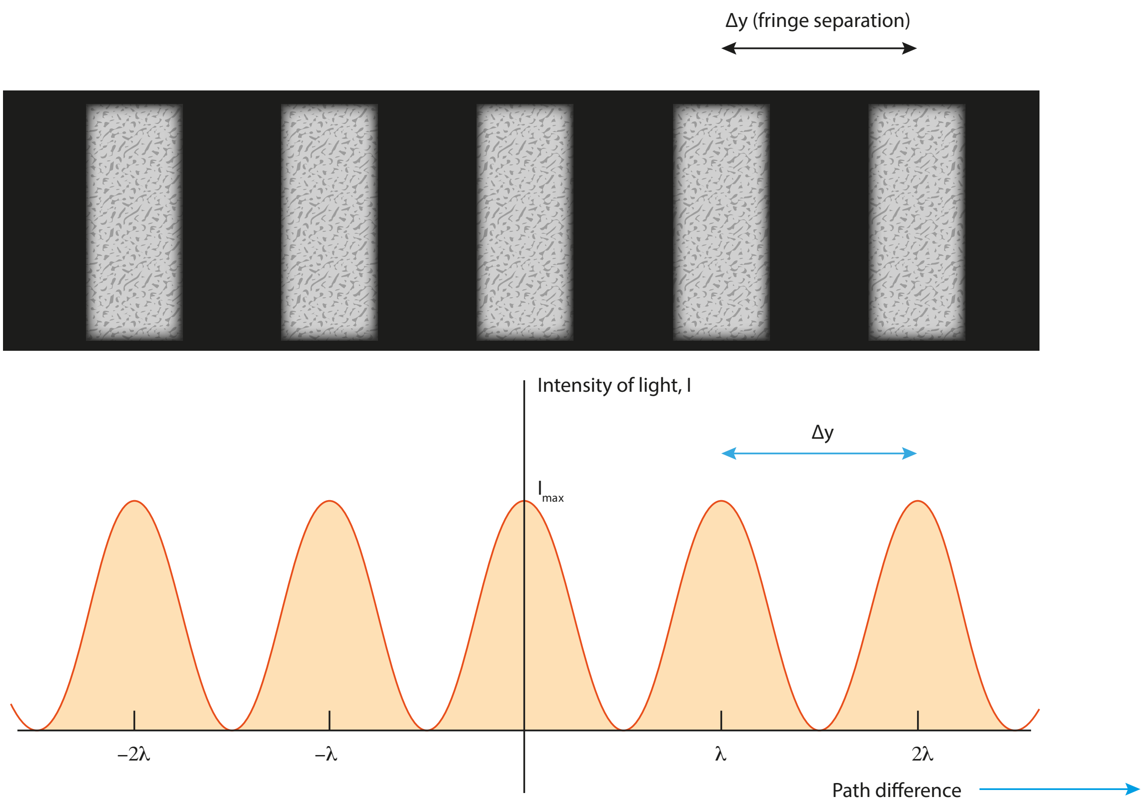


Figure 3.1.21

Explaining the bright and dark fringes

Figure 3.1.22 shows two coherent wave sources overlapping and producing an interference pattern for monochromatic light on a screen. The bright and dark fringes can be explained in terms of constructive and destructive interference.

Positions of maximum light intensity occur when the waves from each of the double slits reach the screen in phase. This occurs at positions where the path difference is $m\lambda$ (m is an integer). The light undergoes constructive interference and a bright fringe results.

Positions of minimum light intensity, occur when the waves from each of the double slits reach the screen half a wavelength out of phase. This occurs at positions where the path difference is $(m + \frac{1}{2})\lambda$ (m is an integer). The light undergoes destructive interference and a dark fringe results.

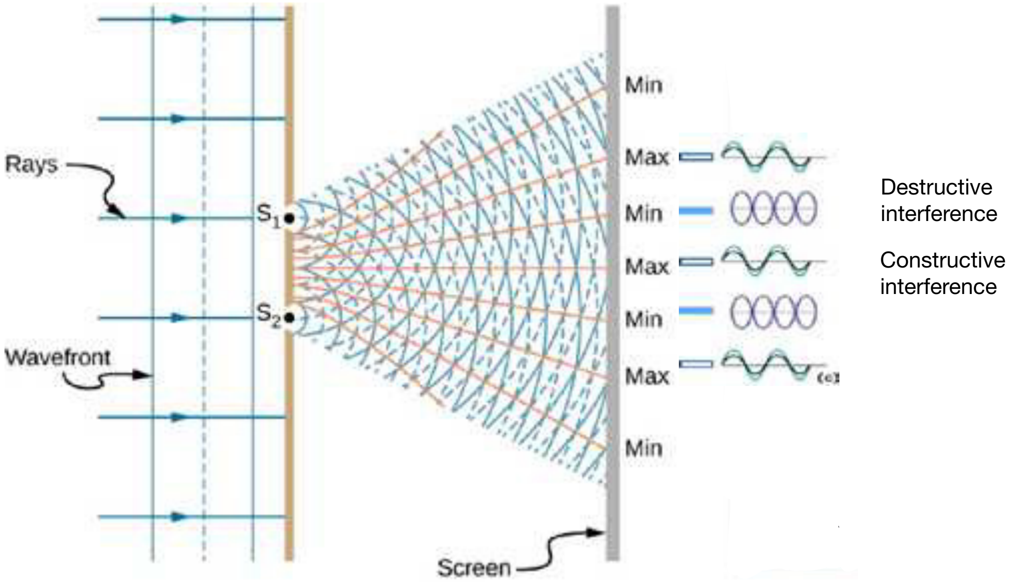


Figure 3.1.22

Analysing the interference pattern

Position of the maxima

Figure 3.1.23 below is the geometrical arrangement for the two slit-interference of monochromatic light. Although not part of this course, it can be shown that the **position of the maxima can be predicted using dsinθ = mλ.** In this formula, d is the distance between the double slits, θ is the angular position of the mth maximum from the centre of the screen and λ is the wavelength of the monochromatic light used to illuminate the double slits.

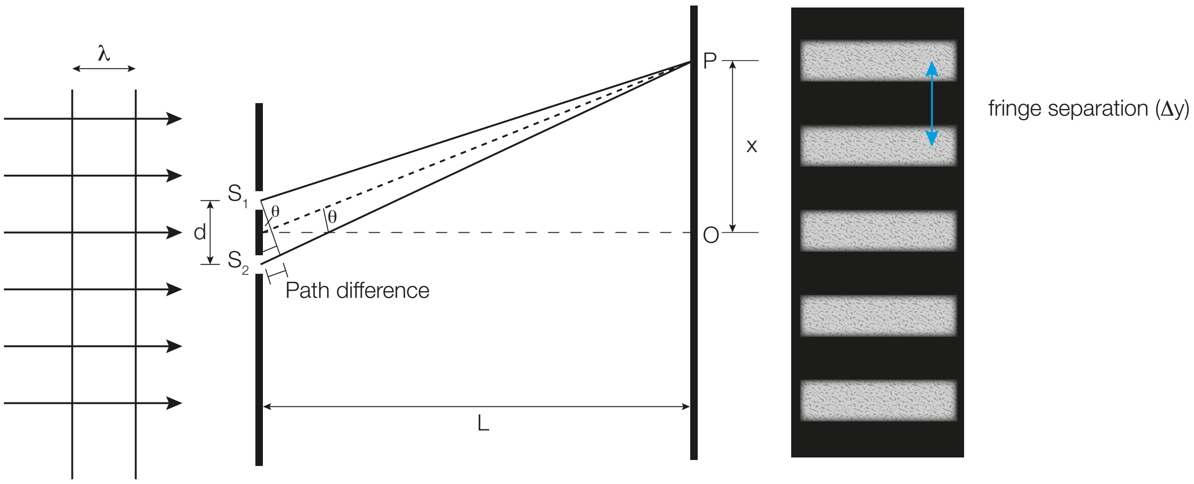


Figure 3.1.23

The distance between adjacent minima or maxima (Δy)

The distance between adjacent minima or maxima (Δy) is marked on Figure 3.1.21 as well as Figure 3.1.23. It can be referred to as the fringe separation. The distance between the double slits is allocated the symbol *d*, while the slit-to-screen distance is allocated the symbol *L*.

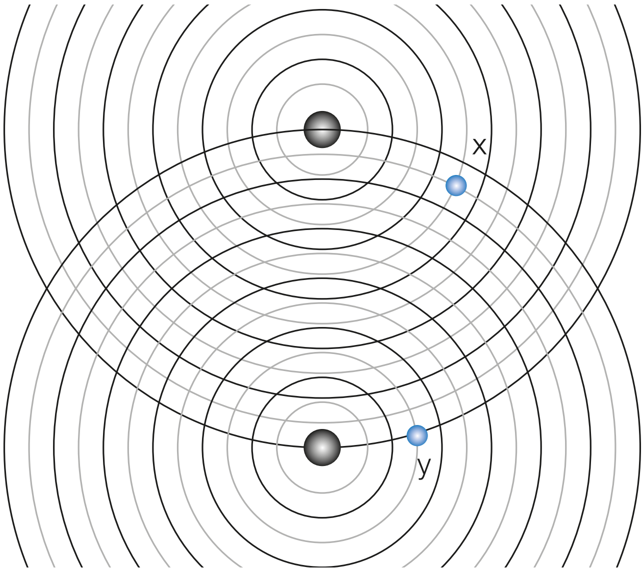
The distance between adjacent minima or maxima of an interference pattern can be calculated using the formula:

$$\Delta y = \frac{\lambda L}{d}$$

where λ is the wavelength of the monochromatic light used.

Worked Examples

1. The diagram below represents the waves produced by two coherent wave sources that are in phase. The solid lines represent crests while the grey lines represent troughs. The amplitude of the waves is *A*.



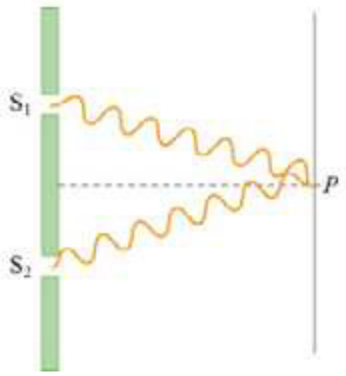
Describe the resultant amplitude at each point X and Y in terms of the principle of superposition.

The waves at X are in phase. They undergo constructive superposition. This result is an amplitude of 2*A*.

The waves at Y are half a wavelength out of phase. They undergo destructive superposition. This result is an amplitude of zero.

2. A two-slit interferometer has a slit separation $d = 5.00 \times 10^{-5}$ m and a slit-to-screen distance $L = 40.0$ cm. Light of wavelength 485 nm is used to illuminate the apparatus. An interference pattern consisting of equally spaced bright and dark fringes is observed on the screen.

(a) Explain why a fringe of maximum intensity is viewed at the centre of the screen. Use a diagram of the geometrical arrangement to help you explain your answer.



Coherent light from the double slits travels the same distance to the centre of the screen. The light arrives in phase and undergoes constructive interference. This results in a fringe of maximum light intensity i.e. a bright fringe.

(b) Calculate the angle at which the third-order maxima occurs on the screen.

$d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3 \times 485 \times 10^{-9}}{5.00 \times 10^{-5}}\right) = 1.67°$

(c) Calculate the distance between adjacent maxima for this interference pattern.

$\Delta y = \frac{\lambda L}{d} = \frac{485 \times 10^{-9} \times 0.400}{5.00 \times 10^{-5}} = 3.88 \times 10^{-3}$ m

(d) Calculate the distance between the centre of the screen and the third-order maxima.

$3\Delta y = 3 \times 3.88 \times 10^{-3} = 1.16 \times 10^{-2}$ m

(e) The slit-to-screen distance can easily be changed in the interferometer. This distance is adjusted so that the screen is moved further away from the double slits. Explain the change in the separation of the maxima.

$\Delta y \alpha L$

Increasing the distance between the double slits and the screen will increase the separation of the maxima observed on the screen.

3. A red laser of wavelength 628 nm is used to illuminate two narrow parallel slits. The interference pattern that is produced is observed on a screen 3.00 m from the slits. The distance between adjacent maxima is 1.50×10^{-2} m.

(a) Calculate the distance between the parallel slits used to form this interference pattern.

$\Delta y = \frac{\lambda L}{d} = \therefore d = \frac{\lambda L}{\Delta y} = \frac{628 \times 10^{-9} \times 3.00}{1.50 \times 10^{-2}} = 1.26 \times 10^{-4}$ m

(b) The red laser is replaced with a blue laser. State with reason, how the distance between adjacent maxima will change.

Blue laser has a smaller wavelength than red laser. Since $\Delta y \alpha \lambda$, the distance between adjacent maxima will decrease.

(c) When the blue laser is used, the angular position of the fifth-order maxima is 1.04°. Calculate the wavelength of the blue laser.

$d\sin\theta = m\lambda \therefore \lambda = \frac{d\sin\theta}{m} = \frac{1.26 \times 10^{-4} \times \sin 1.04}{5} = 4.57 \times 10^{-7}$ m

Determining the wavelength of monochromatic light from measurements of the two-slit interference pattern

The wavelength of monochromatic light can be determined from measurements taken from a two-slit interference pattern produced on a screen using one of the methods described earlier in the chapter. Figure 3.1.24 shows the arrangement for a laser. The slit-to-screen distance *L*, needs to be measured using a ruler or tape measure and the distance between the double slits *d* will be specified by the manufacturer of the commercial double slit slide.

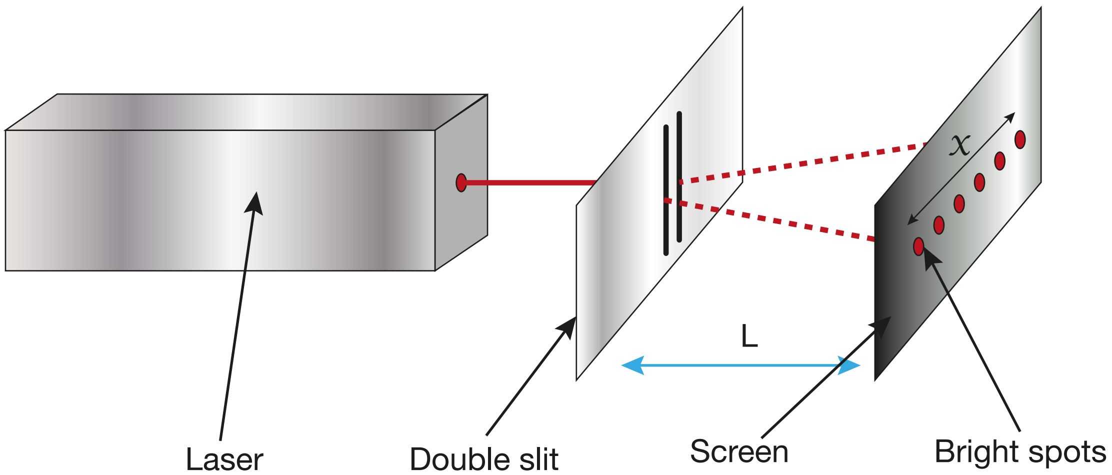


Figure 3.1.24

The distance between adjacent fringes can be measured from the screen. However, the fringes often have 'fuzzy' edges. To minimise the effect of random error, the distance between several maxima, say n, should be measured. Let's call this distance x.

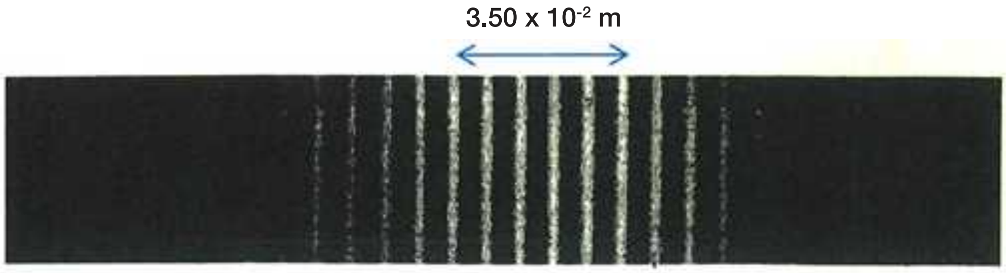
The average distance between adjacent maxima (Δy) can then be calculated using $\frac{x}{n}$.

The formula $\Delta y = \frac{\lambda L}{d}$ can then be rearranged to calculate the wavelength of the monochromatic light used to illuminate the double slits i.e. $\frac{d\Delta y}{L}$

Worked Example

The photograph below shows the two-slit interference pattern produced using monochromatic light.

The distance between the double slits is 2.45×10^{-4} m and the slit-to-screen distance is 2.80 m.



(a) A student measures the distance between several maxima as shown on the photograph. Calculate the average distance between adjacent maxima.

$5\Delta y = 3.50 \times 10^{-2}$ m

$\Delta y = 7.00 \times 10^{-3}$ m

(b) Calculate the wavelength of the monochromatic light used to produce this interference pattern.

$$\Delta y = \frac{\lambda L}{d} \therefore \lambda = \frac{d\Delta y}{L} = \frac{2.45 \times 10^{-4} \times 7.00 \times 10^{-3}}{2.80} = 6.13 \times 10^{-7} \text{ m}$$

Key ideas

The formulae $d\sin\theta = m\lambda$ and $\Delta y = \frac{\lambda L}{d}$ can be used to analyse a two-slit interference pattern, where d is the distance between the slits, θ is the angular position of the maximum, Δy is the distance between adjacent minima or maxima on the screen, and L is the slit-to-screen distance.

? Science inquiry activity

Some ideas for possible investigations are:

- Investigate interference patterns produced by light using, e.g. thin films, multiple wave-sources, and multiple layers of nanoparticles.
- Explore the effect of frequency/ wavelength on a two-source interference pattern.
- Investigate the effect of slit separation on a two-slit interference pattern.
- Determine the wavelength from a two-source interference pattern.

Science as a human endeavour

An idea for a possible investigation is to explore the opportunities for innovation provided by applying the interference of electromagnetic waves in applications such as Blu-ray players and anti-reflective surfaces.

Transmission diffraction gratings

A transmission diffraction grating is a piece of plastic or glass that consists of a large number of thin, parallel, equally spaced slits. These slits are much closer together than the double slits used in two-slit interference.

Figure 3.1.25 shows a commercial diffraction grating with three windows. The diffraction grating is made by ruling parallel lines on plastic or glass. This is why the windows are marked with the number of lines per millimetre. The lines are made by an extremely precise machine called a ruling engine. A ruling engine uses a diamond-tipped tool to press thousands of very narrow, shallow lines onto the plastic or glass. Lines can also be ruled using a laser. The lines that are ruled make the plastic or glass opaque. Light will not pass through the opaque lines but will pass through the transparent plastic or glass between them. Thus the lines create the same effect as having many slits.

The first window consists of 100 lines per millimetre. This means that the distance between the slits is $\frac{1\ mm}{100} = \frac{1 \times 10^{-3}}{100} = 1 \times 10^{-5} m$. The third window consists of 600 lines per millimetre. This means that the distance between the slits is $\frac{1\ mm}{600} = \frac{1 \times 10^{-3}}{600} = 1.67 \times 10^{-6} m$.

Figure 3.1.25

? Science inquiry activity

Much like two-slit interference, a transmission diffraction grating forms an interference pattern when it is illuminated with light. The interference pattern produced by light passing through a transmission diffraction grating demonstrates the wave behaviour of light.

Your teacher may demonstrate an interference pattern produced by a transmission diffraction grating using coherent monochromatic light and/or white light.

You can investigate spectra of light from various sources (e.g. incandescent globe, fluorescent globe, vapour lamp, LED) through a spectroscope.

The interference pattern produced by a transmission diffraction grating

When a parallel beam of monochromatic light is directed at right angles to a transmission diffraction grating, light is transmitted to produce very narrow, intense (bright) lines or maxima which are separated by large regions of negligible intensity or darkness. That is the maxima occur at large angles.

Figure 3.1.26 shows the interference pattern produced by a red laser passing through a diffraction grating while Figure 3.1.27 shows the interference pattern produced by a green light source passing through a transmission diffraction grating. In both cases the maxima are narrow, intense and occur at large angles.

Figure 3.1.26

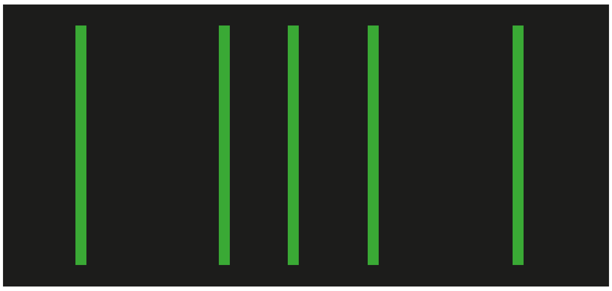


Figure 3.1.27

The role of single slit diffraction in producing the interference pattern produced by a transmission diffraction grating

When parallel beams of light are used to illuminate a transmission diffraction grating at right angles, the light is in phase as it passes through the slits. Since the slits are very thin and the width is comparable to the wavelength of the incident light, the light bends or diffracts at each slit producing circular wavelets. This is shown in Figure 3.1.28. The smaller the slit width, the more the light diffracts. The circular wavelets overlap and interfere producing maxima at positions on a screen where the path difference between the light from adjacent slits is a whole number of wavelengths.

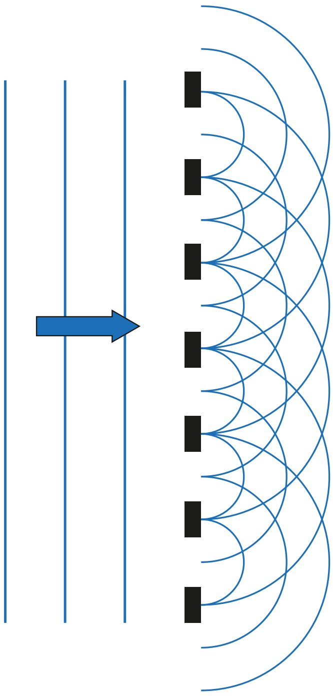


Figure 3.1.28

Why maxima occur at large angles

Light passing through a diffraction grating passes through many slits. Light reaching the centre of a screen produces a maximum because much like two-slit interference, the light reaches the centre of the screen in phase and undergoes constructive interference. As you move away from the centre of the screen, the path difference increases. The light from adjacent slits becomes out of phase. Since there are so many slits on the grating, there are always two slits on the grating for which the light is half a wavelength out of phase. For instance, if the light from adjacent slits is only slightly out of phase (say one hundredth of a wavelength), then light from the first slit will be half a wavelength out of phase with light from the fifty first slit and light from the second slit will be half a wavelength out of phase with light from the fifty second slit and so on. The light will undergo destructive interference and the light intensity will be low or negligible. The intensity remains negligible until the path difference increases to one wavelength. At this point, the light from adjacent slits interferes constructively and a maximum is produced on the screen. Similarly as the path difference increases beyond one wavelength the intensity is once again negligible until it is exactly two wavelengths.

The intensity distribution graph of maxima produced by diffraction grating for monochromatic light

Figure 3.1.29 shows an intensity distribution graph for the maxima produced by a diffraction grating illuminated with monochromatic light. The maxima are symmetrical about the central maxima and get further apart. Figure 3.1.30 is another representation, instead of peaks, it is acceptable to draw thin lines to represent the narrow intense maxima.

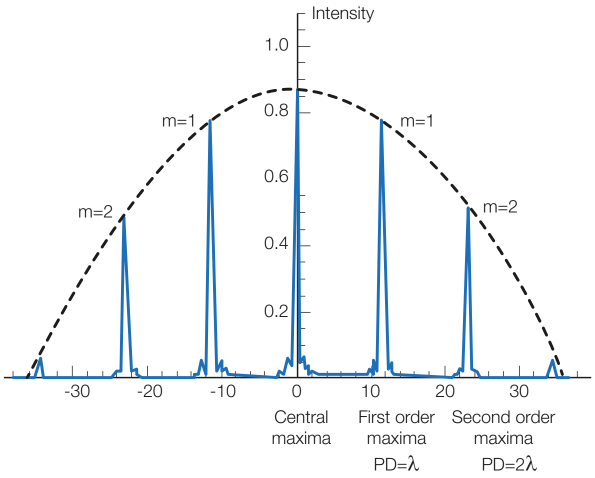


Figure 3.1.29

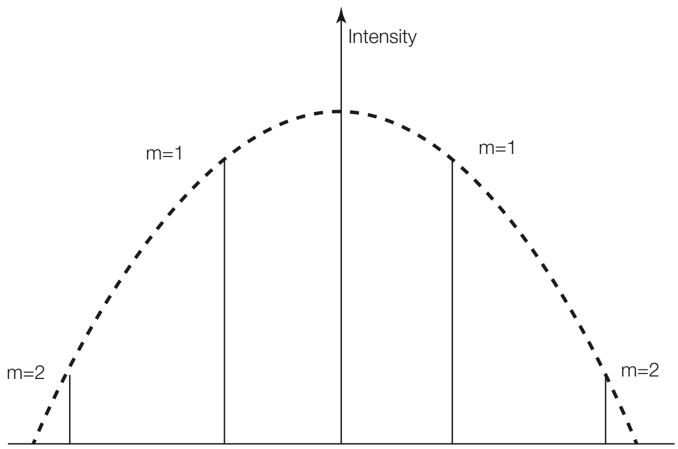


Figure 3.1.30

Analysing the interference pattern produced by a diffraction grating $d\sin\theta = m\lambda$

Just like two-slit interference, the position of the maxima can be predicted using $d\sin\theta = m\lambda$, where d is the distance between the slits or diffraction grating lines, θ is the angular position of the mth maximum (referred to as the order) and λ is the wavelength of the monochromatic light used to illuminate the diffraction grating.

The formula $d\sin\theta = m\lambda$ can be used to analyse the interference pattern produced by a diffraction grating.

Derivation of $d\sin\theta = m\lambda$

Figure 3.1.31 shows the geometrical arrangement for a diffraction grating. The rays of light emerging from the diffraction grating are parallel. A convex lens is used to focus the light rays onto a screen.

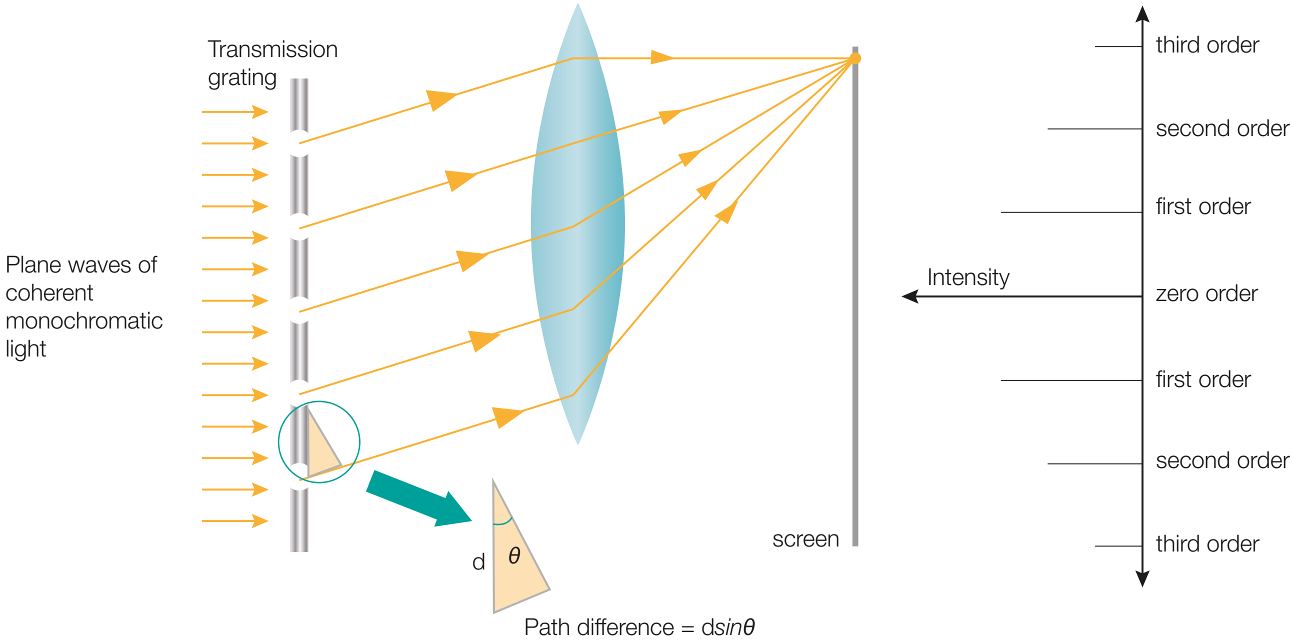


Figure 3.1.31

The path difference between rays of light from consecutive slits is given by $d\sin\theta$ as shown in Figure 3.1.31. For the *mth* order maximum, the path difference is $m\lambda$. It follows that $d\sin\theta = m\lambda$ where $m = 0, 1, 2\ldots$

Worked Examples

1. (a) A transmission diffraction grating consists of 3000 lines per centimetre. Calculate the distance between the grating lines to two significant figures.

$$d = \frac{1 \times 10^{-2}}{3000} = 3.3 \times 10^{-6} \text{ m}$$

(b) The diffraction grating is illuminated with monochromatic red light of wavelength 680 nm. Calculate the angle of the third order maximum.

$$d\sin\theta = m\lambda \quad \therefore \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3 \times 680 \times 10^{-9}}{3.3 \times 10^{-6}}\right) = 38°$$

2 A transmission grating is illuminated with monochromatic light with a wavelength of 5.56×10^{-7} m. The second order maximum is observed at an angle of 48.0°.

Determine the number of lines per metre on the diffraction grating.

$$d\sin\theta = m\lambda \quad \therefore d = \frac{m\lambda}{\sin\theta} = \frac{2 \times 5.56 \times 10^{-7}}{\sin 48.0} = 1.50 \times 10^{-6} \text{ m}$$

d is the distance between the lines. If N represents the number of lines per metre, then $d = \frac{1}{N}$.

$$N = \frac{1}{d} = \frac{1}{1.50 \times 10^{-6}} = 6.66 \times 10^{5}$$

Determining the maximum number of orders for diffraction grating

It is possible to determine the maximum number of orders for a given grating and wavelength of light.

Since $d\sin\theta = m\lambda$, it follows that $\sin\theta = \frac{m\lambda}{d}$.

For a given diffraction grating, the angular position of the maxima is larger for larger wavelengths of light. In addition, the smaller the distance between the grating lines, the greater the angle at which a particular maximum will occur for a given wavelength.

To determine the maximum number of orders, an angle of 90° is substituted for θ.

That is the maximum number of orders is given by $m = \frac{d\sin 90}{\lambda} = \frac{d}{\lambda}$.

Worked Example

In a previous worked example, a transmission grating consisted of 3000 lines per centimetre. The distance between the grating lines was found to be 3.33×10^{-6} m.

Calculate the maximum number of orders possible for this diffraction grating when it is illuminated with monochromatic red light of wavelength 680 nm.

$$d\sin\theta = m\lambda \quad \therefore m = \frac{d\sin 90}{\lambda} = \frac{3.33 \times 10^{-6}}{680 \times 10^{-9}} = 4.9$$

A maximum of four orders is possible.

Using a diffraction grating to measure the wavelength of light from a monochromatic light source

Figure 3.1.32 shows a grating spectrometer set up to view the interference pattern for sodium light using a transmission grating. When an electrical discharge is applied to sodium gas atoms, they emit a characteristic yellow light. This is referred to as the atomic spectrum for sodium. A grating spectrometer can be used to accurately determine the wavelength of the emitted light. Figure 3.1.33 shows the arrangement shown in Figure 3.1.32 diagrammatically.

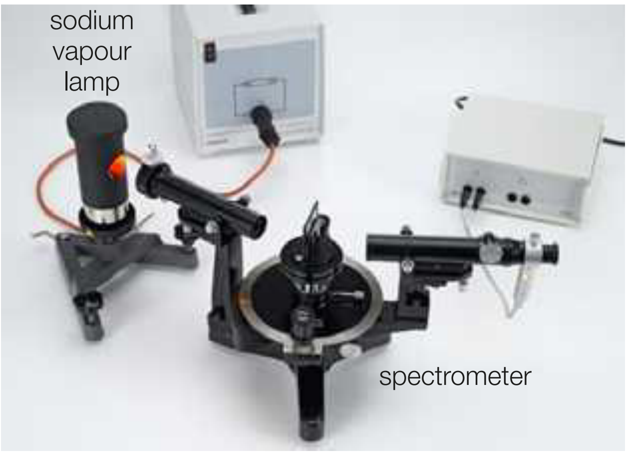


Figure 3.1.32

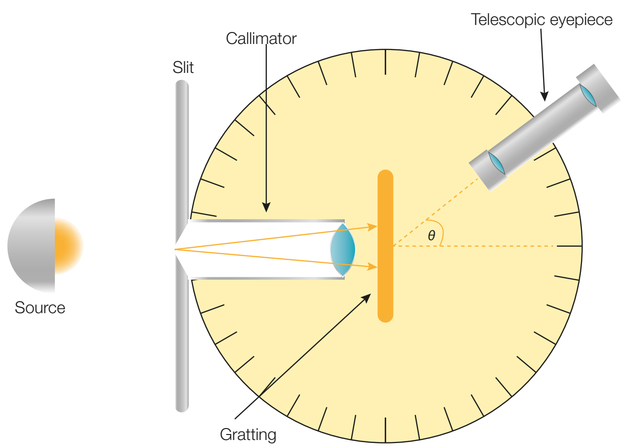


Figure 3.1.33

Light enters the spectrometer and is collimated into a thin beam. This produces parallel beams of light that pass through the diffraction grating.

The deviated beams are focused and viewed by a rotating telescopic eyepiece fitted with a lens which focuses the parallel beams of light emerging from the transmission diffraction grating. The eyepiece rotates on a calibrated turntable so that the angle at which the maxima occur can be measured.

As the eyepiece is rotated from the central maximum, the order *m* of the maximum being viewed is noted, the angular position of the maximum is measured from the calibrated turntable and the distance between the slits *d* is calculated from the number of lines marked on the transmission diffraction grating.

The wavelength of the light can be found by rearranging the relationship $d\sin\theta = m\lambda$ for wavelength i.e. $\lambda = \frac{d\sin\theta}{m}$.

One way to reduce the effect of random errors is to calculate the wavelength of the light for several maxima on either side of the central maximum and averaging the results.

Worked Examples

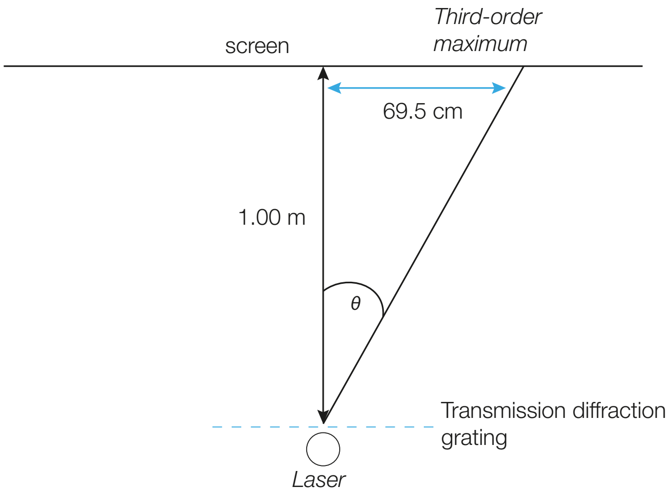
1. A grating spectrometer is used to view sodium light using a transmission grating consisting of 5.00×10^5 lines per metre. The first-order maximum is viewed at an angle of 17.1°. Calculate the wavelength of sodium light.

$$d = \frac{1}{5.00 \times 10^5} = 2.00 \times 10^{-6} \text{ m}$$

$$d\sin\theta = m\lambda \quad \therefore \; \lambda = \frac{d\sin\theta}{m} = \frac{2.00 \times 10^{-6} \times \sin 17.1}{1} = 5.88 \times 10^{-7} \text{ m}$$

2. The example below outlines an alternative method for determining the wavelength of laser light.

A transmission diffraction grating consisting of 300 lines per millimetre is illuminated with a helium–neon laser. The laser is placed 1.00 m from a screen. A third-order maximum is observed at a distance of 69.5 cm from the centre of the screen.



(a) Calculate the angle of the third-order maximum.

$$\tan\theta = \frac{0.695}{1.00} \quad \therefore \; \theta = 34.8°$$

(b) Calculate the wavelength of the laser light used to illuminate the diffraction grating.

$$d = \frac{1 \times 10^{-3}}{300} = 3.33 \times 10^{-6} \text{ m}$$

$$d\sin\theta = m\lambda \quad \therefore \; \lambda = \frac{d\sin\theta}{m} = \frac{3.33 \times 10^{-6} \times \sin 34.8}{3} = 6.33 \times 10^{-7} \text{ m}$$

The white-light pattern produced by a diffraction grating

As discussed earlier in the chapter, white light is composed of a continuous range of wavelengths from violet (approximately 400 nm) through to red (approximately 700 nm). When white light is passed through a diffraction grating the interference pattern will have the appearance shown in Figure 3.1.34.

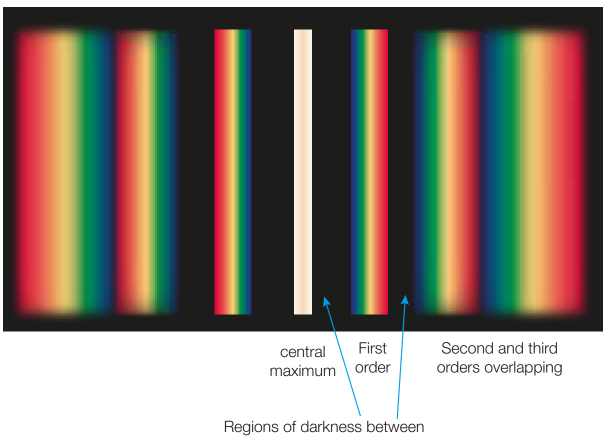


Figure 3.1.34

A white line is seen in the centre of the screen because the path difference between light from each slit is zero for all wavelengths. Every wavelength meets in phase and undergoes constructive interference. A region of darkness is seen followed by the first-order. Since $\sin\theta \alpha \lambda$, violet is seen in the first-order followed by all the other colours through to red. This is because the wavelength of violet light is smaller than the wavelength of red light and therefore produces a maximum at a smaller angle than red light. A second region of darkness is seen followed by the second-order. In theory this pattern would continue but higher orders are less intense and tend to spread. The third-order often overlaps with the second and is difficult to view clearly.

The properties of a diffraction grating that make it useful in spectroscopy

Spectroscopy refers to the viewing of atomic spectra. Atomic spectra will be discussed in Subtopic 3.3: Structure of the atom. In addition to viewing atomic spectra, the wavelength of light can be determined accurately using a diffraction grating. The method for this was discussed earlier in the chapter.

The following properties of a diffraction grating make it useful for spectroscopy.

1. The grating lines are close together. This means that the maxima are viewed at larger angles. The angular position of the maxima can be measured precisely as the experimental error in their measurement is small compared to their spacing.
2. The maxima are thin, intense lines rather than bands. The error involved with judging the centre of the maxima and hence their position is small.

These properties, when combined, allow the wavelength of light to be determined accurately (i.e. close to the accepted or known value).

3

? Science inquiry activity

Some ideas for possible investigations are:

- To determine the wavelength of a monochromatic source.
- To investigate the spectra of different light sources.

Science as a human endeavour

An idea for a possible investigation is to explore the emerging technologies which use optical data storage; consider the interplay with technology and engineering.

Exercises

1. Explain how a radio wave of a given frequency can be transmitted using a transmitting antenna.

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2. (a) Define the term 'plane polarisation' of electromagnetic waves.

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(b) Explain why transmitted television waves are plane polarised and define the plane of polarisation in terms of the oscillating electrons.

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(c) A television wave is horizontally plane polarised when it is transmitted. State the orientation of the receiving antenna for a strong signal is to be received.

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3. The photograph below shows a typical antenna found on most suburban Adelaide roofs. The antenna is used to receive television waves.

(a) State the plane of polarisation of the television waves being received by this antenna.

(b) State the direction of the oscillating magnetic field of the television waves being received by this antenna.

(c) Television antenna in country areas are positioned vertically. Suggest, with explanation, a reason for this.

4. An antenna transmits an electromagnetic waves such that the magnetic field oscillates horizontally. Explain the orientation of a receiving antenna if a strong signal is to be received by the receiving antenna.

5. *SAFM* is an Adelaide radio-station that broadcasts radio signals at a frequency of 107.1 MHz.

(a) In terms of the oscillation of the electrons in the transmitting antenna, describe how a radio-signal of this frequency is transmitted.

(b) Explain how a radio-signal of this frequency is received.

(c) Calculate the wavelength of the radio-waves transmitted by *SAFM*.

6. (a) Define the following terms:

(i) monochromatic light

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(ii) coherent wave sources

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(b) Explain why light from an incandescent source is neither coherent nor monochromatic.

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7. (a) Describe constructive interference in terms of the principle of superposition.

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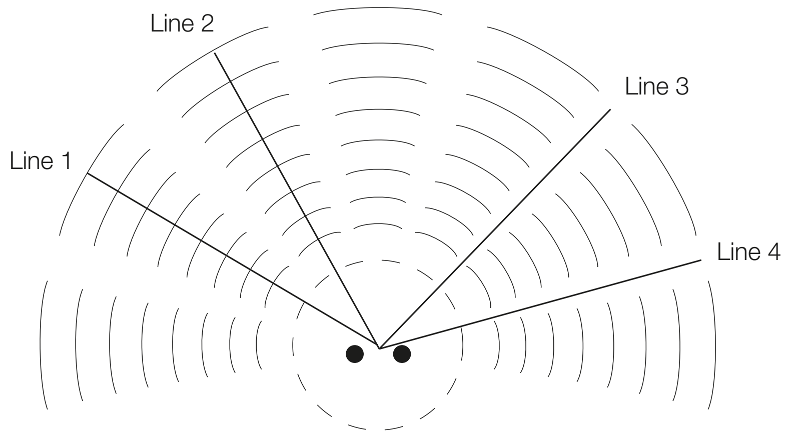
(b) The diagram below represents the interference pattern produced by two coherent wave sources of wavelength λ.



(i) State the line(s) that represent constructive interference.

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(ii) State the path difference between the two wave sources at any point along line 4.

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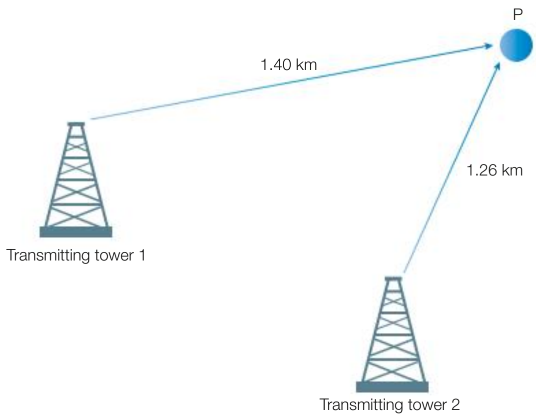
8. Identical radio waves are emitted from two transmitting towers with a wavelength of 2.80 m and an amplitude *A*. Position P is 1.40 km away from transmitting tower 1 and 1.26 km away from transmitting tower 2.



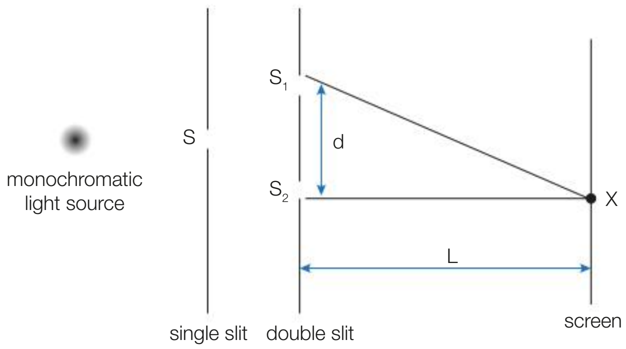
(a) Calculate the path difference, in metres, between the radio waves transmitted by tower 1 and tower 2 at position P.

(b) Determine whether the radio waves interfere constructively or destructively at position P.

(c) State the amplitude of the radio waves received at position P.

(d) The amplitude to the radio waves emitted by transmitting tower 2 is increased to *3A* while the amplitude of the radio waves emitted by transmitting tower 1 remain unchanged. State the new amplitude of the radio waves received at position P.

9. In Young's double slit experiment, the double slits are separated by a distance $d = 2.50 \times 10^{-4}$ m and the slit-to-screen distance $L = 35.0$ cm.



The single slit is illuminated with a monochromatic light source. This produces two coherent light sources at the double slits.

(a) Describe the pattern that is observed on the screen.

(b) The wavelength of the monochromatic light used to illuminate the apparatus is $\lambda = 5.00 \times 10^{-7}$ m. Calculate the

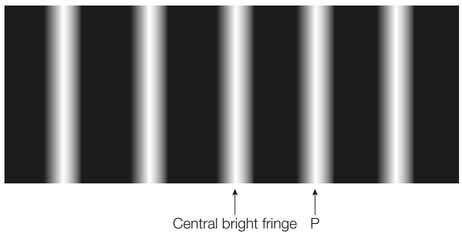
(i) angle of the fifth maximum.

(ii) distance between adjacent maxima, Δy.

(c) The path difference between the light from S_1 and S_2 is $\frac{\lambda}{2}$ at the position X shown on the screen.

Explain why a dark fringe is observed at X.

10. The diagram below shows the two-slit interference pattern produced by a monochromatic light source. The distance between the two slits is 1.80×10^{-4} m and the screen is 40.0 cm from the two slits.



(a) State the path difference between the two coherent light sources at the position P shown on the screen and explain why a bright fringe is seen.

(b) The distance between the centre of the screen and P is 1.29×10^{-3} m.

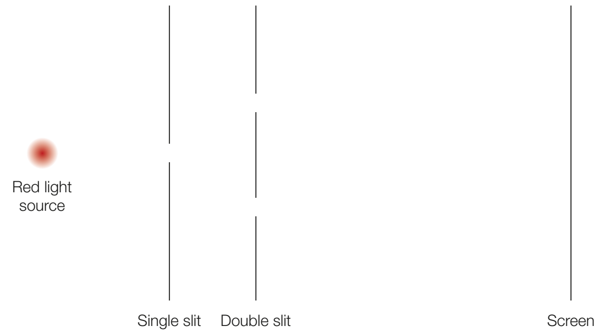
Calculate the wavelength of the monochromatic light source.

(c) State, with reason, the affect on the distance between the centre of the screen and P if the

(i) screen is placed further away from the two-slits.

(ii) source of light used to illuminate the two-slits is changed to one with a smaller wavelength.

11. The diagram below shows a two-slit experiment for investigating the interference of light. Monochromatic yellow light of wavelength $\lambda = 590$ nm is used to illuminate a single slit placed between the light source and the double slits. This produces two coherent light sources at the double slits. The separation of the double slits is 6.0×10^{-4} m and the screen is 0.45 m from the double slits.



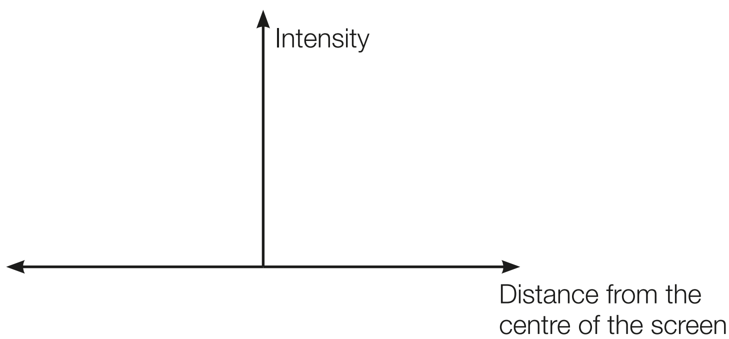
(a) Describe the role of diffraction in producing the interference pattern on the screen.

(b) Calculate the angle of the third-order maximum.

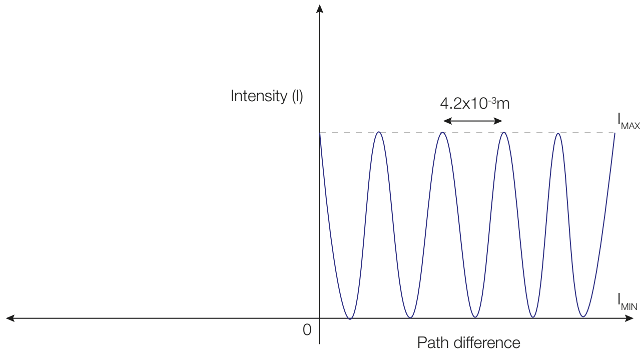
(c) Calculate the distance between adjacent bright fringes.

(d) Calculate the distance between the two third-order fringes on either side of the central maximum.

(e) Sketch the intensity distribution graph for the interference pattern on the diagram below.



12. The graph below represents the intensity distribution for the two-slit interference pattern of a coherent monochromatic light source.



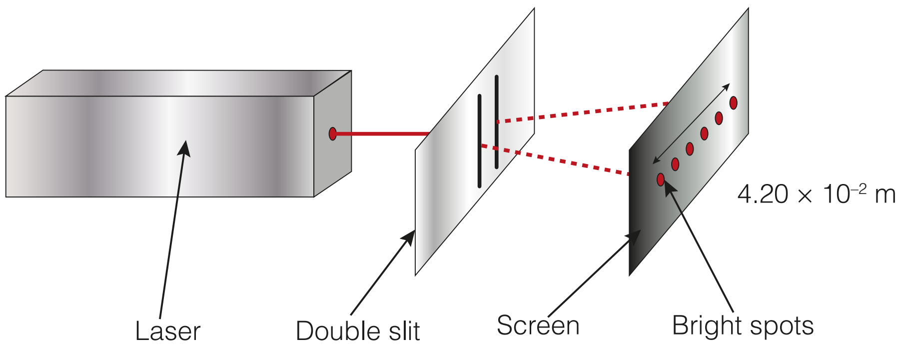
The distance between the double slits is 0.27 mm and the slit-to-screen distance is 2.5 m. The distance between the second and third bright fringes is 4.2×10^{-3} m.

(a) Calculate the wavelength of the light used to produce the interference pattern.

(b) State the path difference between the two light sources for the second bright fringe.

(c) Calculate the angular position of the second bright fringe.

13. The diagram below shows a red laser beam of wavelength 633 nm illuminating two narrow slits. The diagram also shows the interference pattern that is produced on a screen placed 2.00 m from the double slits. The distance between several maxima is measured to be 7.00×10^{-2} m. The diagram is not drawn to scale.



(a) Calculate the average distance between adjacent maxima on the screen.

(b) Calculate the slit separation.

(c) Describe and explain the change in the distance between adjacent maxima if the

(i) slit-to-screen distance is reduced.

(ii) red laser is replaced with a blue laser.

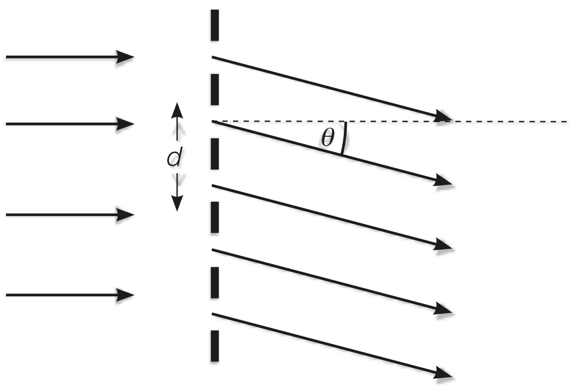
14. A diffraction grating has 6.00×10^5 lines per metre ruled on its surface. The grating is illuminated with normally incident monochromatic light of wavelength 5.40×10^{-7} m. Calculate the

(a) distance between the grating lines.

(b) angle of the first-order maximum.

(c) maximum number of possible orders.

15. The diagram below shows some of the slits of a transmission diffraction grating. The slits are separated by a distance *d*. Parallel beams of monochromatic light are incident on the diffraction grating at 90°. The light diffracted by the slits at an angle θ is shown.



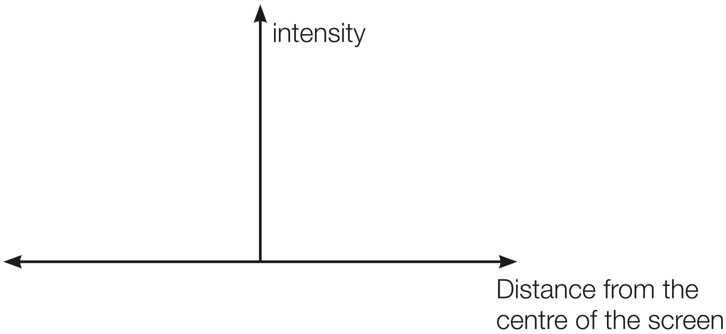
A convex lens is placed between the diffraction grating and a screen which is used to view the intensity pattern produced.

(a) Clearly indicate the path difference between two adjacent rays on the diagram.

(b) Show that $d\sin\theta = m\lambda$ for the intensity maxima in the pattern produced.

(c) There are 800 lines per mm on the diffraction grating. Show that there are only two possible orders on either side of the central maximum for this grating when it is illuminated with monochromatic light with a wavelength of 5.00×10^{-7} m.

(d) On the axes below, sketch a graph of the intensity distribution of the maxima produced by this grating.



(e) The diffraction grating in this arrangement is replaced with one that has fewer lines per millimetre. Explain whether the number of orders seen on either side of the central maximum increases or decreases.

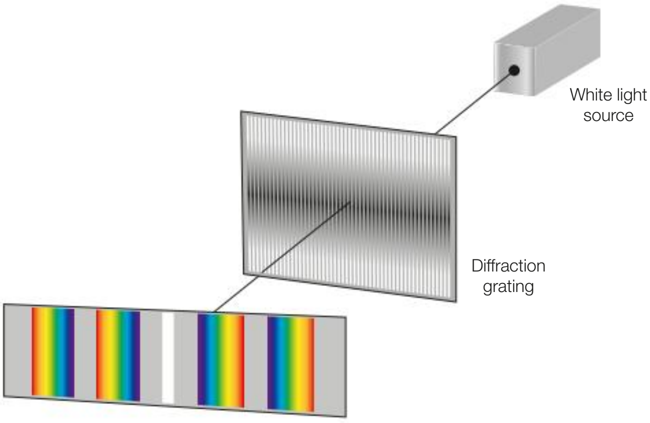
16. A diffraction grating has 360 lines per millimetre ruled on its surface. The grating is illuminated with normally incident monochromatic light of wavelength 6.0×10^{-7} m. Show that the angle of the third-order maximum is approximately 40°.

17. A diffraction grating is illuminated with monochromatic light with a wavelength of 4.50×10^{-7} m. The fourth-order maxima is observed at an angle of 76.0°.

(a) Calculate the distance between the grating lines on the diffraction grating.

(b) Calculate the number of lines per metre that exist on the diffraction grating.

18. The diagram below shows the diffraction pattern produced by white light.



(a) Explain why the central maximum is white.

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(b) Explain why the first order maximum consists of a continuous spread of frequencies as shown in the diagram.

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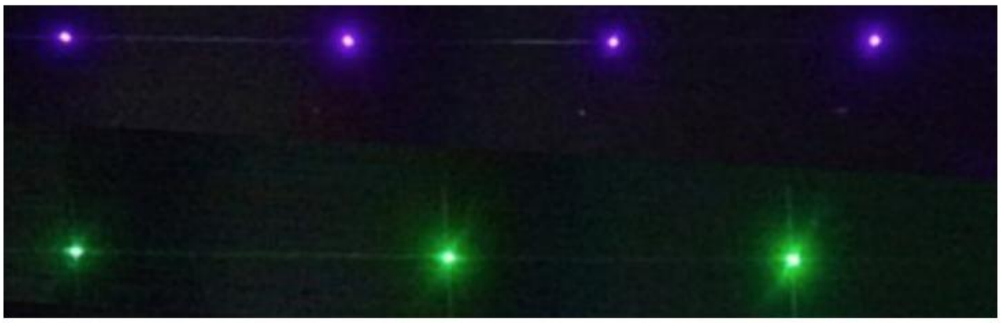
19. The photograph below shows the interference pattern produced when a violet laser and a green laser are passed through the same transmission diffraction grating.



(a) Explain how the high intensity maxima are produced.

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(b) Explain why the maxima of the violet light are closer together.

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20. White light is incident normally on a diffraction grating that consists of 2000 lines per centimetre.

Show that the second-order red ($\lambda_{red} = 7.00 \times 10^{-7}$ m) overlaps with the third-order violet ($\lambda_{violet} = 4.00 \times 10^{-7}$ m).

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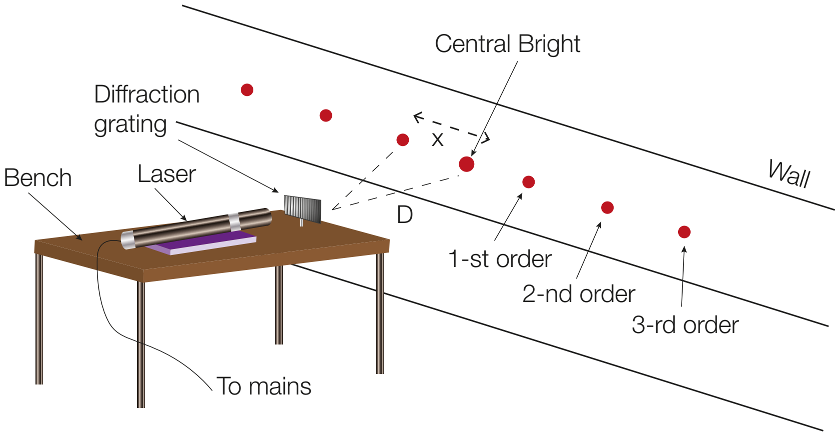
21. A student plans to confirm the relationship between the angle of the maxima and the order of the maxima when monochromatic light is passed through a transmission diffraction grating.

The student sets up the experimental arrangement shown in the diagram below.



(a) Describe the expected relationship between the angle of the maxima and the order of the maxima.

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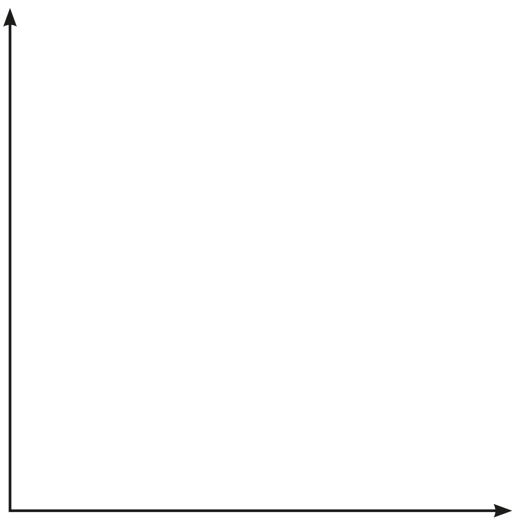
(b) Place appropriate labels on the axes below that show the graph that the student should plot in order to establish this relationship.



(c) Draw a line on the graph in part (b) that represents the expected results.

(d) Given the wavelength of a red laser is 6.33×10^{-7} m, explain how the student could use the graph to calculate the number of lines per metre on the diffraction grating.

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22. Science as a human endeavour activity – Light magnetometers

Source: https://www.novuslight.com/fiber-optic-sensor-measures-tiny-magnetic-fields_N8449.html

Magnetic resonance imaging (MRI) systems are currently used to map brain activity. MRI machines measure the very weak magnetic fields produced by neurons in the brain. However, MRI machines require expensive cooling systems and electromagnetic shielding.

Researchers have been able to produce light magnetometers that use fibre optics. The magnetometer is a compact sensor that uses light to measure very weak magnetic fields. These sensors are inexpensive, and do not require shielding.

"A portable, low-cost brain imaging system that can operate at room temperature in unshielded environments would allow real-time brain activity mapping after potential concussions on the sports field and in conflict zones where the effect of explosives on the brain can be catastrophic," says researcher Babak Amirsolaimani of the University of Arizona, Tucson (US).

These new sensors can also help scientists better understand dementia and Alzheimer's (diseases of the brain). Additionally, the sensors may prove useful in predicting volcanic eruptions and earthquakes by measuring magnetic fields and oil and minerals deposits can be more easily found.

(a) Identify one key concept of science as a human endeavour that is evident in the quote from Babak Amirsolaimani. Justify your answer.

(b) Describe one other example of how light magnetometers demonstrate science as a human endeavour.

3.2 Wave–particle duality

Science understanding

1. In interacting with matter, light behaves like particles (called 'photons'), with energy given by $E = hf$ and momentum given by $p = \frac{h}{\lambda}$ where h is Planck's constant, f is the frequency of the light, and λ is its wavelength.
   - Solve problems using $E = hf$ and $p = \frac{h}{\lambda}$
2. Electrons may be emitted from a metal surface when light of sufficiently high frequency is incident on the metal surface. This process is called the 'photoelectric effect'.
3. If monochromatic light is used the intensity of the incident light affects the number, but not the energy, of emitted electrons.
4. The minimum frequency, f_0, at which electrons are emitted varies with the type of material and is called the 'threshold frequency'.
5. The work function, W, of a surface is the minimum energy required to remove an electron from it.
6. The work function is related to the threshold frequency by $W = hf_0$.
   - Describe an experimental method for investigating the relationship between the maximum kinetic energy of the emitted electrons, calculated from the measured stopping voltage using $E_{K_{max}} = eV_s$ and the frequency of the light incident on a metal surface.
   - Describe how Einstein used the concept of photons and the conservation of energy to explain the experimental observations of the photoelectric effect.
   - Deduce the formula $E_{K_{max}} = hf - W$ where $E_{K_{max}}$ is the maximum kinetic energy of the emitted electrons.
   - Plot experimental values of maximum kinetic energy vs frequency, and relate the slope and axes intercepts to the formula: $E_{K_{max}} = hf - W$.
   - Solve problems that require the use of $E_{K_{max}} = hf - W$.
7. X-ray photons can be produced when electrons that have been accelerated to high speed interact with a target. This is done in an X-ray tube.
   - Describe the purpose of the following features of a simple X-ray tube: filament, target, potential difference across the tube, evacuated tube, and a means of cooling the target.
   - Describe the energy changes that occur during the production of X-rays, including the heat produced.
8. The three main features of the spectrum of the X-rays produced in this way are:
   – a continuous range of frequencies (bremsstrahlung)
   – a maximum frequency given by $f_{max} = \frac{e\Delta V}{h}$ where ΔV is the potential difference across the X-ray tube
   – high-intensity peaks at particular frequencies (known as characteristic X-rays).
   - Sketch a graph of the spectrum from an X-ray tube, showing the three main features of the spectrum.
   - Explain the continuous range of frequencies and the maximum frequency in the spectrum of the X-rays.
   - Explain the effect of manipulating the filament current or potential difference across the X-ray tube on an X-ray spectrum.
   - Derive the formula for the maximum frequency, $f_{max} = \frac{e\Delta V}{h}$.
   - Solve problems involving the use of $f_{max} = \frac{e\Delta V}{h}$.
9. X-rays are attenuated (reduce in intensity) as they pass through matter by scattering and absorption.
   - Explain the effect of the filament current on the intensity of X-rays produced by an X-ray tube.
   - Relate the attenuation of X-rays to the types of tissue through which they pass (e.g. soft tissue, bone, metals).
   - Relate the penetrating power (hardness) of X-rays required to pass through a particular type of material to the energy and frequency of the X-rays.
   - Relate the minimum exposure time for X-ray photographs of a given hardness to the intensity of the X-rays.
10. Particles exhibit wave behaviour with a wavelength (called the 'de Broglie wavelength') that depends on the momentum of the particle. The de Broglie wavelength is given by the formula $\lambda = \frac{h}{p}$, where h is Planck's constant and p is the momentum of the particles.
11. The wave behaviour of particles can be demonstrated using a double-slit experiment and the Davisson–Germer experiment.
    - Solve problems involving the use of the formula $\lambda = \frac{h}{p}$, for electrons and other particles.
    - Describe two-slit interference pattern produced by electrons in double-slit experiments.
    - Describe the Davisson–Germer experiment, in which the diffraction of electrons by the surface layers of a crystal lattice was observed.
    - Compare the de Broglie wavelength of electrons with the wavelength required to produce the observations of the Davisson–Germer experiment and in two-slit interference experiments.

This chapter uses the concept of energy developed in Stage 1, Subtopic 4.1: Energy and Stage 2, Subtopic 2.2: Motion of charged particles in electric fields, momentum in Stage 1, Subtopic 4.2:Momentum and waves in Stage 1, Subtopics 5.1: Wave model and 5.3: Light.

3

Photons

The energy and momentum of photons

In subtopic 3.1 we discussed Young's double slit experiment and the interference pattern produced by light passing through a transmission diffraction grating demonstrated the wave like nature of light.

When light interacts with matter, the wave theory for light cannot explain the observations. Instead, the observations are explained in terms of the particle behaviour of light. That is, light behaves like particles which we call ***photons***.

Key ideas

1. **Photons** are considered discrete packets of energy.
2. Each photon has an **energy** given by $E = hf$ and **momentum** given by $p = \frac{h}{\lambda}$ where h is Planck's constant (6.63×10^{-34} Js), f is the frequency of the light in Hertz and λ is its wavelength in metres.
3. The equation $v = f\lambda$ applies, where $v = 3.00 \times 10^8$ ms^{-1}. This equation can be used to find the wavelength when the frequency of light is known and vice versa.

Extra understanding

An image can be shown to build up over time in light of low intensity. Individual photons can be recorded by a single photon imaging camera. These cameras have a CCD (charge coupled device) photon detector. Microscopic observations show that localised bundles of energy (photons) reach the screen. At first the bundles of light seem to arrive on the screen at random, but after a while, an image builds up. A two-slit interference pattern can also be shown to build up over time in light of very low intensity.

Figure 3.2.1 below shows a photograph of a woman building up over time. At first it is difficult to see any image but there are clear bursts of light on the screen. Over a large exposure time, more bust of light (photons) reach the screen and a clear image results.

Figure 3.2.1

Figure 3.2.2 shows a two-slit interference pattern building up over time in low intensity light.

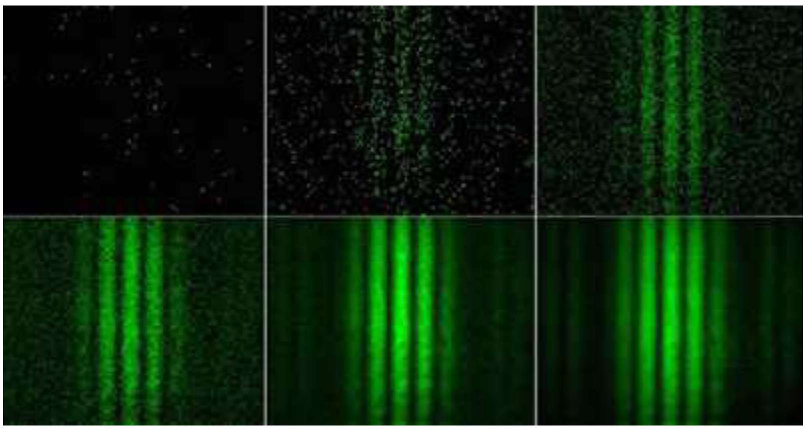


Figure 3.2.2

Helpful online resources

Investigate photon by photon build-up of an interference pattern, using a simulation:

https://phet.colorado.edu/en/simulation/quantum-wave-interference

Science as a human endeavour

A possible investigation may be to explore ways in which engineers use an understanding of photons to design devices.

Examples include:

- charge-coupled devices in digital cameras
- photomultiplier tubes for neutrino detection inside huge underground water tanks.

Worked examples

1. (a) Red light lies in the visible region of the electromagnetic spectrum. Calculate the energy of the photons in red light of frequency 4.29×10^{14} Hz.

$$E = hf = 6.63 \times 10^{-34} \times 4.29 \times 10^{14} = 2.84 \times 10^{-19}\ J$$

(b) Calculate the wavelength of the photons in red light.

$$v = f\lambda \quad \therefore\ \lambda = \frac{v}{f} = \frac{3.00 \times 10^{8}}{4.29 \times 10^{14}} = 6.99 \times 10^{-7}\ m$$

(c) Calculate the momentum of the photons in red light.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.99 \times 10^{-7}} = 9.48 \times 10^{-28}\ sN$$

2. Photons in the X-ray part of the electromagnetic spectrum have an energy of 9.56×10^{4} eV.

Determine the frequency of these X-ray photons.

$$E = 9.56 \times 10^{4}\ eV = 9.56 \times 10^{4} \times (1.6 \times 10^{-19}) = 1.53 \times 10^{-14}\ J$$

$$E = hf \quad \therefore\ f = \frac{E}{h} = \frac{1.53 \times 10^{-14}}{6.63 \times 10^{-34}} = 2.31 \times 10^{19}\ Hz$$

The Photoelectric effect

Figure 3.2.3 shows the emission of electrons from a metal surface when it is illuminated with light of sufficiently high frequency. This phenomenon is called the **photoelectric effect.**

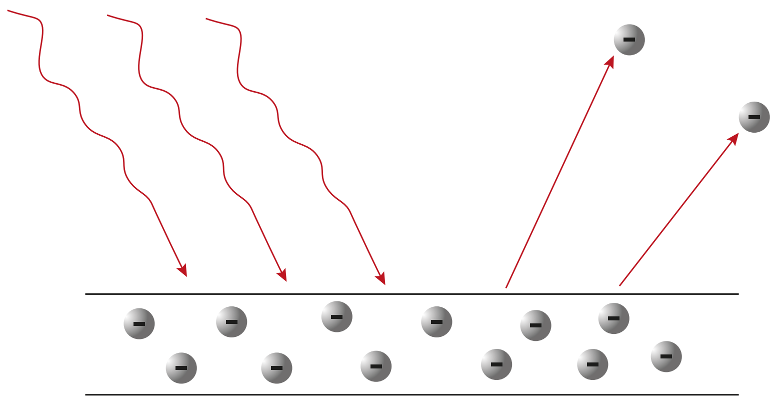


Figure 3.2.3

The photoelectric effect was first observed by Heinrich Hertz in 1887. The photoelectric effect is an important phenomenon because the wave model for light failed to explain the experimental observations. In 1905 Albert Einstein published a paper that explained the photoelectric effect and in 1921 he was awarded the Nobel Prize in Physics for his services to theoretical physics and in particular his explanation of the photoelectric effect.

Helpful online resources

Try this simulation for the photoelectric effect:

https://phet.colorado.edu/en/simulation/photoelectric

The experimental observations of the photoelectric effect

A photoelectric cell is used to observe the photoelectric effect. It consists of a curved metal cathode (or emitter) and an anode (or collector) in an evacuated glass envelope or tube.

Figure 3.2.4 shows a photoelectric cell while Figure 3.2.5 is a diagrammatic representation of a photoelectric cell.

Figure 3.2.4

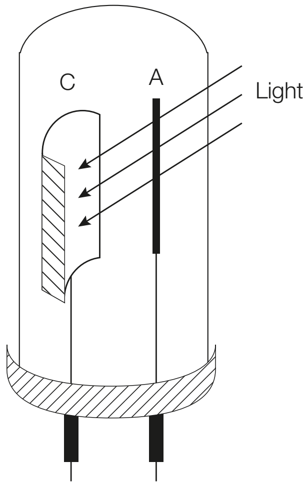


Figure 3.2.5

Key ideas

1. When light of sufficiently high frequency is incident on matter, it may be absorbed by the matter, from which electrons are then emitted. This process is called the 'photoelectric effect'.
2. The intensity of the incident light affects the number, but not the energy, of emitted electrons.
3. The minimum frequency, f_0 at which electrons are emitted varies with the type of material and is called the 'threshold frequency'.
4. The work function, *W*, of a surface is the minimum energy required to remove an electron from it.
5. The work function is related to the threshold frequency by $W = hf_0$.

In order to detect electrons that are ejected from the cathode (or emitter), the photoelectric cell is placed in a circuit as shown in Figure 3.2.6.

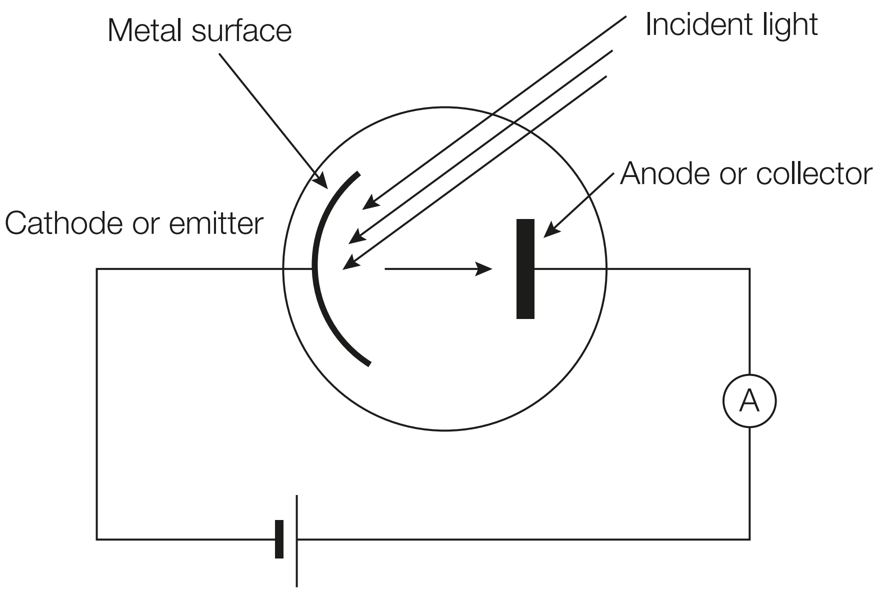


Figure 3.2.6

When light strikes the metal surface (cathode), electrons are emitted. The evacuated glass envelope allows the electrons to travel freely towards the positive anode without colliding with air particles. A sensitive ammeter records the size of the current. The current is proportional to the number of electrons reaching the anode.

The experimental observations of the photoelectric effect are stated below:

1. Electrons are emitted instantaneously.
2. There is a minimum frequency, f_0 below which no electrons are ejected. This frequency is called the 'threshold frequency' and varies with the type of material.

   Figure 3.2.7 shows red, green and violet light striking a metal surface. The violet and green light both emit electrons from the metal. As the frequency of the light decreases, electrons are not emitted. In this case, red light has the smallest frequency and does not eject electrons.

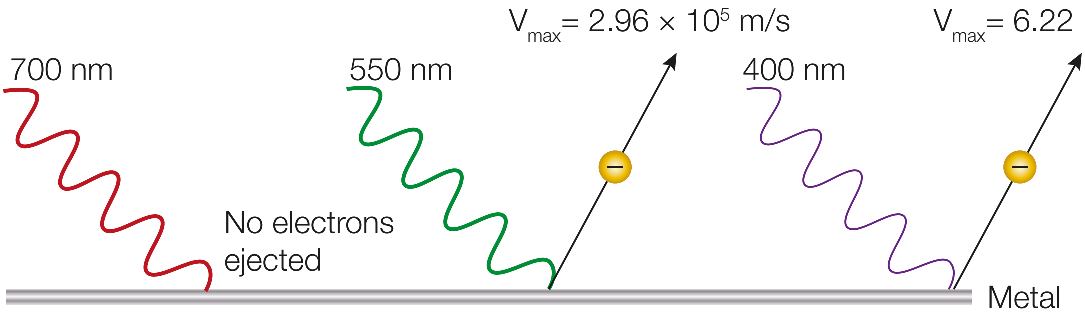


Figure 3.2.7

3. Increasing the intensity (brightness) of the light, increases the number of electrons ejected. These two factors are proportional. A larger current is detected when the intensity of the light is increased. The frequency of the light does not affect the number of electrons emitted.
4. Electrons are ejected with a range of kinetic energies up to a maximum. Increasing the frequency of the incident light increases the maximum kinetic energy of the emitted electrons. This is illustrated in Figure 3.2.7. Violet light has a larger frequency than green light. The electrons are emitted with a greater speed and hence kinetic energy. The intensity of light does not affect the kinetic energy of the emitted electrons.

Helpful online resources

The following YouTube clip describes demonstrates the photoelectric effect with different frequencies.

https://www.youtube.com/watch?v=kcSYV8bJox8

The following YouTube clip shows an experiment that can only be described in terms of the photoelectric effect.

https://www.youtube.com/watch?v=puT36rd9dkQ

The following YouTube clip describes the observations of the photoelectric effect.

https://www.youtube.com/watch?v=92-SI5Uf0rs

The following YouTube clip describes the experimental arrangement for the photoelectric effect.

https://www.youtube.com/watch?v=maFUYiQgwUU

? Science inquiry activity

A possible idea is to investigate the photoelectric effect experimentally.

Extra understanding

The failure of the wave theory in explaining the photoelectric effect.

If light is a continuous wave, then there is a continuous transfer of energy to the electrons in the surface of the metal. This means that any frequency of light should be able to emit electrons. The electrons absorb energy from the light and when they have built up sufficient energy they will escape from the metal. This doesn't happen, there is the threshold frequency below which no electrons are ejected at all.

In addition, if the electrons absorb energy continuously from light, then there may be a time delay in their emission. This does not happen. The electrons are emitted instantly or not at all.

The intensity or brightness of light is proportional to the amplitude and the hence energy of the radiation. Light with a greater intensity should emit electrons with a greater kinetic energy. This is not the case, more electrons are emitted and their kinetic energy depends on the frequency of the light.

According to the wave theory, increasing the frequency of light increases the number of waves striking the metal surface per second. More electrons should be ejected. This is not the case, the electrons are ejected with greater kinetic energy.

Albert Einstein was awarded the Nobel Prize in Physics in 1921 for his contribution to theoretical physics and in particular his explanation of the photoelectric effect.

Einstein's explanation of the photoelectric effect

Einstein was able to use the concept of photons and the law of conservation of energy to explain the experimental observations of the photoelectric effect. His explanation was based on the assumption that only ***one electron*** in the metal surface can absorb the energy of ***one photon***.

The electrons in a metal surface are bound in the metal by differing amounts depending on how deep within the metal they were found.

The **work function**, W, is defined as the minimum amount of energy needed to emit an electron from the surface of a material.

1. Photons behave like a particle. When an electron in the metal absorbs the energy of a photon, the transfer of energy is immediate. The electrons are therefore **emitted instantaneously** as long as the photons have an energy equivalent to or greater than the work function.
2. If the frequency of the light incident on a metal is reduced, the energy of the photons is reduced ($E = hf$). When this energy drops below the work function, the incident photons do not have sufficient energy to release the least bound or surface electrons i.e. no electrons are emitted from the metal. This explained the **threshold frequency** f_0.

The work function is given by: $W = hf_0$

3. The intensity or brightness of light is directly proportional to the number of photons in the light. Increasing the intensity of the light used to illuminate the metal surface, increases the number of photons incident on the metal without changing their energy. This means that more electrons can absorb the energy of one photon and be emitted. This explains why **more electrons are emitted with more intense light**.
4. Using the law of conservation of energy, part of the energy of the incident photons is used to release an electron, the rest is transformed into the kinetic energy of the electron. Since electrons deeper in the metal require more energy to be released, the amount of kinetic energy available varies. Electrons close to the surface are emitted with more kinetic energy than those deeper in the metal. This means that **electrons are emitted with a range of kinetic energies** up to a maximum value. Those electrons bound with an energy greater than that of the incident photons are not emitted from the metal. **Increasing the frequency** of the incident light **increases** the energy of the incident photons and the **maximum kinetic energy** of the electrons as shown in Figure 3.2.7.

The maximum kinetic energy ($E_{K_{max}}$) of electrons emitted during the photoelectric effect

The least bound electrons are released with the greatest kinetic energy i.e. $E_{K_{max}}$. As explained above, we can apply the law of conservation of energy. The energy of the photon is used to release an electron and the remaining energy is transformed into the kinetic energy of the emitted electrons. The energy needed to release the least bound electrons is the work function W.

$$E_{photon} = E_{release} + E_{K_{max}}$$
$$hf = W + E_{K_{max}}$$
$$E_{K_{max}} = hf - W$$

Worked examples

1. The photoelectric effect is observed in a metal using visible light. The minimum frequency that emits electrons from the metal surface is 6.2×10^{14} Hz.

   (a) State the threshold frequency of the metal.

   6.2×10^{14} Hz

   (b) Calculate the work function of the metal.

   $W = hf_0 = 6.63 \times 10^{-34} \times 6.2 \times 10^{14} = 4.1 \times 10^{-19}\ J$

   (c) The metal is illuminated with light of wavelength $\lambda = 200$ nm. Calculate the maximum kinetic energy of the emitted electrons.

   $v = f\lambda \quad \therefore\ f = \frac{v}{\lambda} = \frac{3.00 \times 10^{8}}{200 \times 10^{-9}} = 1.5 \times 10^{15}\ Hz$

   $E_{K_{max}} = hf - W = 6.63 \times 10^{-34} \times 1.5 \times 10^{15} - 4.1 \times 10^{-19} = 5.8 \times 10^{-19}\ J$

2. Silver has a work function of 4.73 eV.

   (a) Calculate the threshold frequency for silver.

   $W = 4.73 \times 1.6 \times 10^{-19} = 7.57 \times 10^{-19}\ J$

   $W = hf_0 \quad \therefore\ f_0 = \frac{W}{h} = \frac{7.57 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.14 \times 10^{15}\ Hz$

   (b) Light with a frequency 3.56×10^{15} Hz illuminates silver. Calculate the maximum kinetic energy of the emitted electrons.

   $E_{K_{max}} = hf - W = 6.63 \times 10^{-34} \times 3.56 \times 10^{15} - 7.57 \times 10^{-19} = 1.60 \times 10^{-18}\ J$

   (c) Calculate the speed of the electrons emitted with maximum kinetic energy.

   $E_K = \frac{1}{2}mv^2 \quad \therefore\ v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-18}}{9.11 \times 10^{-31}}} = 1.87 \times 10^{6}\ ms^{-1}$

3. Sodium has a work function of 2.46 eV. Show that light of frequency 3.55×10^{14} Hz will not eject electrons from the surface of sodium metal.

   $W = hf_0 \quad \therefore\ f_0 = \frac{W}{h} = \frac{2.46 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.94 \times 10^{14}\ Hz$

   The frequency of the incident light is lower than the threshold frequency. The threshold frequency is the frequency below which no electrons are emitted. It follows that light with a frequency of 3.55×10^{14} Hz will not eject electrons from the surface of sodium metal.

3

Experimental method for investigating the relationship between $E_{K_{max}}$ and the frequency of light

To experimentally determine the relationship between the maximum kinetic energy of the emitted electrons and the frequency of light, a stopping voltage is applied.

A photoelectric cell is placed in a circuit as shown in Figure 3.2.8. White light is passed through a coloured filter so that only one frequency of light (monochromatic) illuminates the metal (cathode).

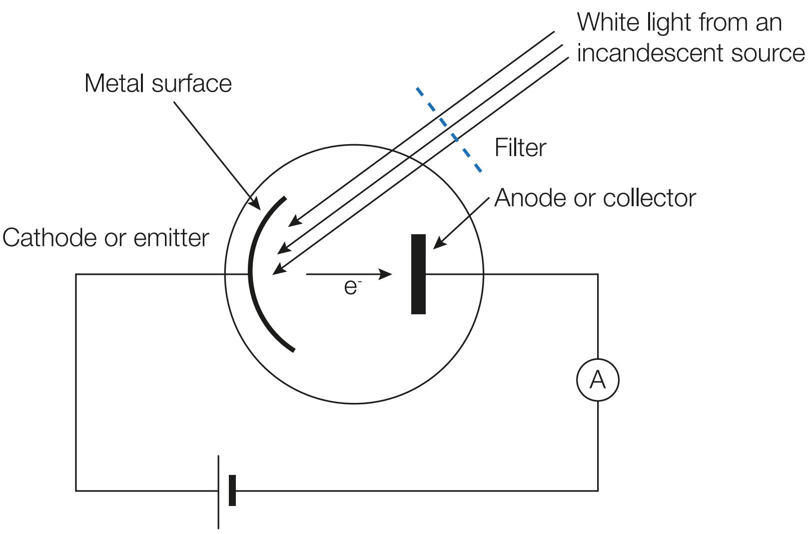


Figure 3.2.8

A negative potential is applied to the anode. This produces an electric field which repels the electrons and stops them from reaching the anode.

The potential difference is increased making the anode increasingly more negative. Fewer electrons have enough kinetic energy to overcome the repulsive force and reach the anode. When no more electrons reach the anode the current registered by the sensitive ammeter drops to zero. This potential difference is referred to as the **stopping voltage (V_S).** When the stopping voltage is achieved, the work done by the electric field matches the maximum kinetic energy of the emitted electrons.

$$W = q\Delta V = eV_s = E_{K_{max}}$$

The maximum kinetic energy of the emitted electrons from a metal surface is given by:

$$E_{K_{max}} = eV_s$$

where e is the charge of an electron and V_s is the stopping voltage.

The colour of the filter is changed and the corresponding stopping voltage is recorded for a different frequency of light. This is repeated for other coloured filters. The maximum kinetic energy of the emitted electrons for each frequency of light can be calculated using $E_{K_{max}} = eV_s$

A graph of $E_{K_{max}}$ against frequency is plotted. Since $E_{K_{max}} = hf - W$, a linear relationship is expected. A straight line graph should result but it will not pass through the origin. The expected relationship is illustrated in Figure 3.2.9.

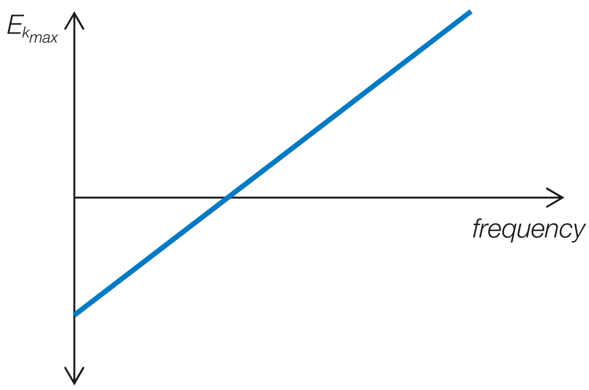


Figure 3.2.9

Analysing a graph of $E_{K_{max}}$ against frequency

When $E_{K_{max}} = hf - W$ is compared to the equation of a straight line, $y = mx + c$, it can be seen that the gradient of the graph gives the value for Planck's constant and the vertical intercept gives the value for the work function. The horizontal axis intercept gives the value for the threshold frequency. These quantities are illustrated on Figure 3.2.10.

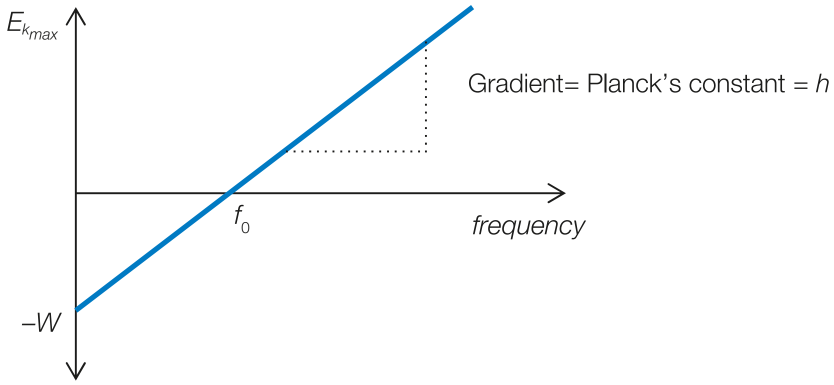


Figure 3.2.10

If a different metal with a different threshold frequency is used a similar straight-line graph results. Figure 3.2.11 represents the data for two different metals. Metal 2 has a larger threshold frequency and therefore crosses the horizontal axis at f_{0_2} which is higher than f_{0_1}. The gradient of both lines is the same because it represents Planck's constant.

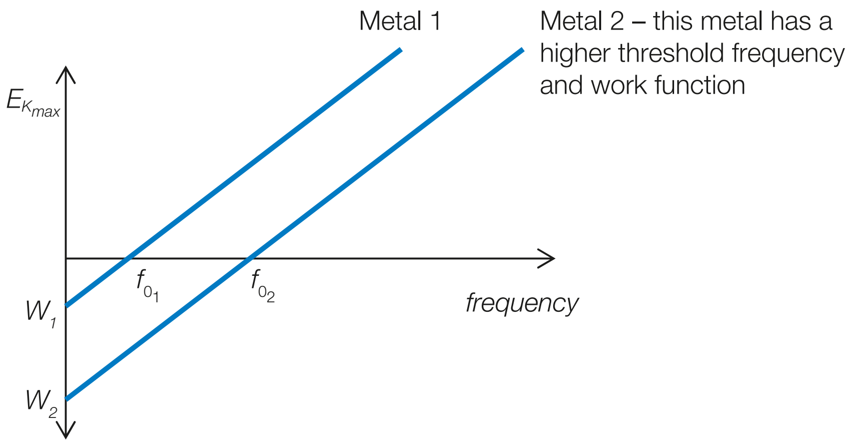


Figure 3.2.11

3

Worked examples

1. Most school laboratories have equipment that can establish the relationship between the maximum kinetic energy of the emitted electrons and the frequency of the incident light in the photoelectric effect.

   A traditional experiment is to use a photoelectric cell, a set of high quality coloured filters and a cathode ray oscilloscope to determine the stopping voltage for various frequencies. However, your school may have other digital technology that enables data to be collected and Planck's constant to be determined.

   Figure 3.2.12 shows the data obtained in a school laboratory for the photoelectric effect.

Frequency (*Hz*) × 10^{14}	**Stopping potential (V_s)**	**Maximum kinetic energy of emitted electrons × 10^{-19} (*J*)**
5.20	0.5	
5.50	0.7	
6.90	1.2	
7.40	1.4	
8.20	1.8	

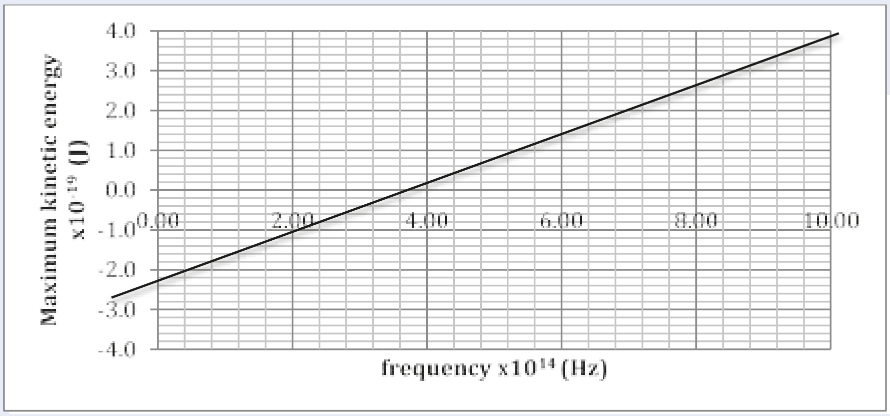
Figure 3.2.12

(a) Complete the third column of the table by calculating the maximum kinetic energy of the electrons emitted with each frequency of light.

Use the equation $E_{K_{max}} = eV_s$

Frequency (*Hz*) × 10^{14}	Stopping potential (V_s)	Maximum kinetic energy of emitted electrons × 10^{-19} (*J*)
5.20	0.5	0.8
5.50	0.7	1.1
6.90	1.2	1.9
7.40	1.4	2.2
8.20	1.8	2.9

(b) Plot a graph of maximum kinetic energy versus frequency of light. Draw a line of best fit for the plotted data.



(c) Use the graph to state the

(i) threshold frequency for the metal.

The frequency/horizontal axis intercept = f_0 = 3.80 × 10^{14} Hz

(ii) Work function of the metal.

The vertical/maximum kinetic energy axis intercept = W = 2.4 × 10^{-19} J

(d) Calculate the gradient of the graph and justify why it represents Planck's constant.

$$gradient = \frac{2.6 \times 10^{-19}}{8.00 \times 10^{14} - 3.80 \times 10^{14}} = 6.2 \times 10^{-34}\ Js$$

When the formula $E_{K_{max}} = hf - W$ is compared to the equation of a straight line, $y = mx + c$, it can be seen that the gradient of the graph gives the values for Planck's constant.

(e) Calculate the percentage error in the value of Planck's constant and comment on its accuracy.

$$percentage\ error = \frac{6.63 \times 10^{-34} - 6.2 \times 10^{-34}}{6.63 \times 10^{-34}} \times 100 = 6.5\%$$

The experimental value of Planck's constant is close to the accepted value. The accuracy is good.

2. A metal is illuminated with electromagnetic radiation. Electrons are emitted from the metal surface with a maximum kinetic energy of 1.88 eV. Show that the stopping voltage is 1.88 V.

$$E_{K_{max}} = eV_s \therefore V_s = \frac{E_{K_{max}}}{e} = \frac{1.88 \times 1.60 \times 10^{-19}}{1.60 \times 10^{-19}} = 1.88\ V$$

3. In a photoelectric experiment, a metal with a work function of 2.4 eV is illuminated with light of frequency 6.7×10^{14} Hz.

   Calculate the stopping voltage for the emitted electrons.

$$E_{K_{max}} = hf - W \therefore V_s = \frac{hf}{e} - \frac{W}{e} = \frac{6.63 \times 10^{-34} \times 6.7 \times 10^{14}}{1.60 \times 10^{-19}} - \frac{2.4 \times 1.60 \times 10^{-19}}{1.60 \times 10^{-19}}$$

$V_s = 0.38$ V

Extra Understanding

It is possible to determine the threshold frequency, Planck's constant and the work function from a graph of stopping voltage versus frequency.

When $E_{K_{max}} = eV_s = hf - W$ is rearranged for V_s we have $V_s = \frac{hf}{e} - \frac{W}{e}$. When this equation is compared to the equation of a straight line, $y = mx + c$, it can be seen that the *gradient* $= \frac{h}{e}$. Planck's constant is calculated by multiplying the gradient by the charge of an electron (h = *gradient* $\times\ e$). The vertical axis intercept $= \frac{-W}{e}$. The work function is calculated by multiplying the vertical axis intercept by the charge of an electron ($W = e \times$ *vertical intercept*). The horizontal axis intercept gives the value for the threshold frequency. Figure 3.2.13 illustrates these values on a sketch of a V_s versus frequency graph.

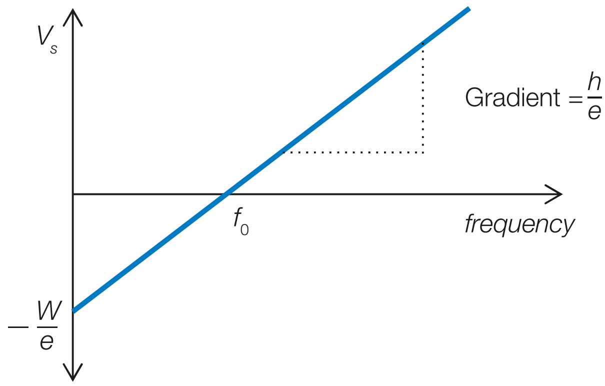


Figure 3.2.13

Science as a human endeavour

Explore innovations which utilise the photoelectric effect, for example:

- solar cells
- photocells (used as light sensors in cameras and many other automated electronic and security systems)
- photomultiplier tubes used in many scientific instruments that monitor light and other electromagnetic radiation
- the production of sound tracks on movie films
- some smoke detectors.

X-rays

X-rays are produced in an X-ray tube (Figure 3.2.15) when electrons are accelerated across a large potential difference and strike a metal target. They were discovered in 1895 by Wilhelm Röntgen, a German physicist. He used a tube much like the X-ray tubes we use today. He noticed that the rays that were emitted caused crystals to glow. He later discovered that the rays could penetrate living tissue but not bones. Figure 3.2.14 shows an early X-ray taken by Röntgen. The X-ray is of his wife's hand. X-rays are commonly used in medicine but they have many industrial uses too. These include sterilisation, thickness gauging, airport security (baggage scanning) and detecting stress fractures in materials used to build large structures such as bridges and aeroplanes.

Figure 3.2.14

Figure 3.2.15

Key ideas

1. X-ray photons can be produced when electrons that have been accelerated to high speed interact with a target.
2. The three main features of the spectrum of the X-rays produced in this way are:
   - a continuous range of frequencies (bremsstrahlung)
   - a maximum frequency given by $f_{max} = \frac{e\Delta V}{h}$ where ΔV is the potential difference across the X-ray tube
   - high-intensity peaks at particular frequencies (known as characteristic Xrays).
3. The intensity of X-rays is decreased (i.e. attenuated) as they pass through matter by scattering and absorption.

The production of X-rays using an X-ray tube

Figure 3.2.16 illustrates a typical X-ray tube.

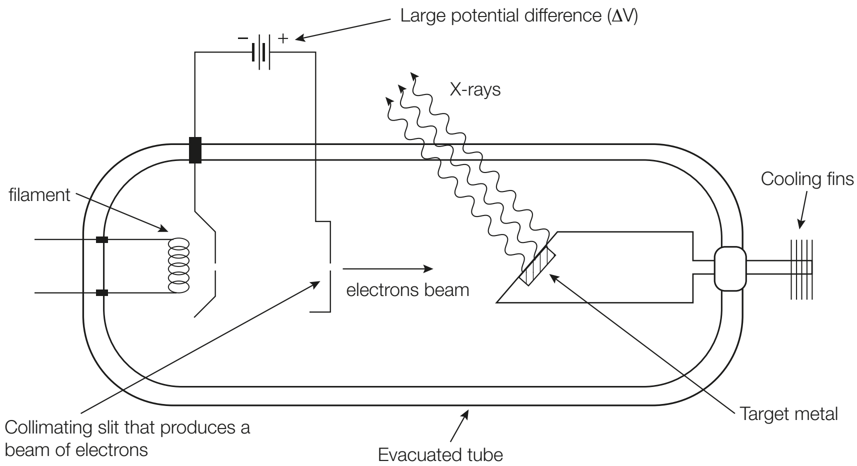


Figure 3.2.16

A current is passed through a wire filament. This heats the filament and electrons are released. A high voltage supply provides a potential difference that accelerates the electrons towards a target metal (tungsten). The large potential difference (ΔV) creates a strong electric field which does work on the electrons. The work done is transferred to the electrons as kinetic energy. X-rays are emitted as the electrons strike the target metal. The X-rays are focused through the quartz window (not shown on the diagram).

Approximately 99% of the incident electron's kinetic energy is converted to heat energy and the rest to X-ray photons. Water or oil circulates through the cooling fins and allows them to draw heat away from the target metal. This prevents it from melting. Tungsten is often chosen as the target metal because it is hard and can withstand the collisions from the electrons and it has a high melting point.

The X-ray tube is evacuated so that electrons do not lose kinetic energy or scatter from their intended path as they collide with air particles.

In solving problems, the equations $W = \Delta E_k = q\Delta V = e\Delta V = \frac{1}{2}mv^2$ can be used to calculate the kinetic energy (E_k) and hence speed (v) of the electrons as they strike the target metal. ΔV is the potential difference across the X-ray tube, e is the charge of an electron and m is the mass of an electron.

The X-ray spectrum

Figure 3.2.17 shows a typical X-ray spectrum from an X-ray tube. Intensity represents the number of X-ray photons. This means that the area under the curve represents the total number of X-ray photons emitted. The three main features of the spectrum are labelled on the graph.

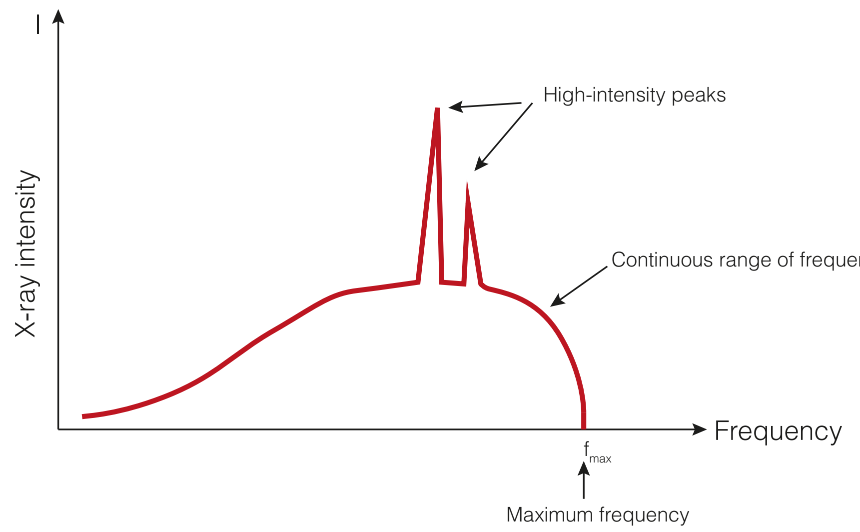


Figure 3.2.17

It should be noted that it is also acceptable to draw the high-intensity peaks as vertical lines as shown in Figure 3.2.18 below.

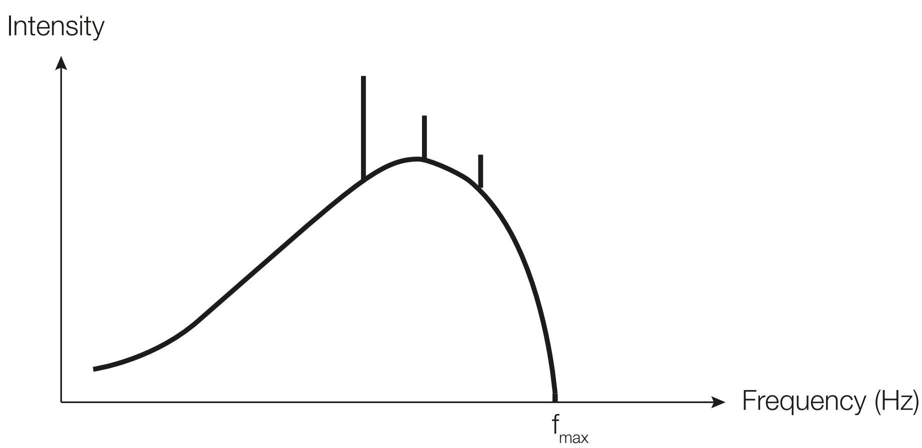


Figure 3.2.18

Explaining the continuous range of frequencies and the maximum frequency in the X-ray spectrum

This course requires that you understand how the continuous range of frequencies and the maximum frequency in the X-ray spectrum are produced. An explanation of the high-intensity peaks is provided in Subtopic 3.3.

As electrons with fixed energy collide with the target metal, their path is deviated by the electrostatic force between the electron and the nucleus of the target atoms. The electrons slow down and their kinetic energy decreases. Using the law of conservation of energy, the difference in the kinetic energy of the electron before the collision and after the collision with the nucleus is transformed into and released as an X-ray photon.

$$\Delta E_k = E_{X\text{-}ray} = hf_{X\text{-}ray}$$

The difference in the kinetic energy of the electron before and after the collision varies depending on how close the electron collides with the nucleus. It follows that the energy transformed into and released as an X-ray photon also varies. This results in a **continuous range of frequencies** for the X-rays and is represented by the curved part of the intensity against frequency graph (bremsstrahlung).

Figure 3.2.19 shows two electrons colliding with the nucleus of a target atom. Electron 2 collides more closely with the nucleus and 'loses' more kinetic energy. This results in a higher energy and hence frequency X-ray photon.

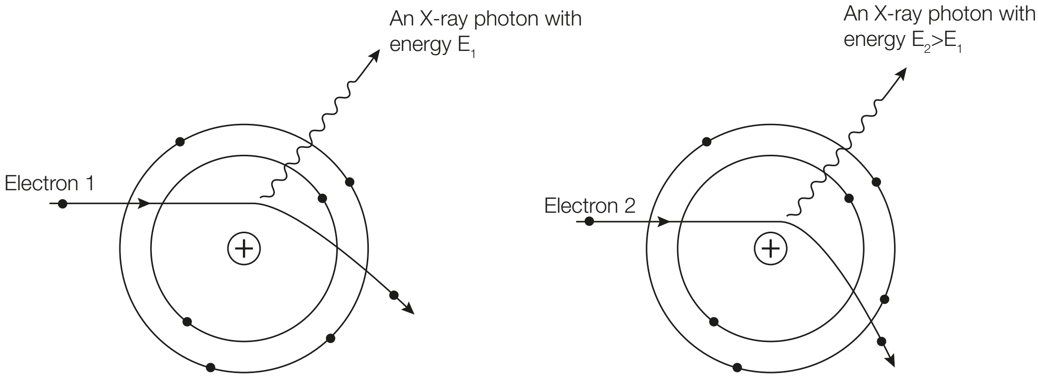


Figure 3.2.19

When an electron collides **head on** with the nucleus of a target atom, all of its initial kinetic energy is transferred to an X-ray photon. The X-ray photon has a **maximum** energy and hence **frequency** (f_{max}).

The derivation of f_{max}

The potential difference (ΔV) does work (W) on the electrons which is transformed into kinetic energy. In a head on collision with the nucleus of the target atoms all of this kinetic energy is transformed into the energy of an X-ray photon.

$$W = \Delta E_k = q\Delta V = e\Delta V = hf_{max}$$

Rearranging yields $f_{max} = \frac{e\Delta V}{h}$

The formula for calculating the maximum frequency (f_{max}) of X-rays produced in an X-ray tube is given by $f_{max} = \frac{e\Delta V}{h}$ where (ΔV) is the potential difference across the X-ray tube, e is the charge of an electron and h is Planck's constant.

Increasing the filament current

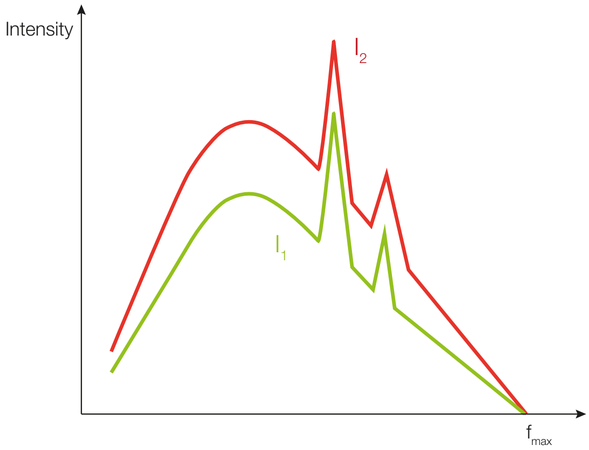


Figure 3.2.20

Figure 3.2.20 shows the graph of the X-ray spectrum for two different filament currents. I_2 is larger than I_1 and produces more X-ray photons.

Increasing the filament current will increase the number of electrons released from the filament. More electrons collide with the target metal to produce more X-ray photons. The intensity curve therefore shifts upwards. There is no change in the maximum frequency.

Increasing the accelerating potential

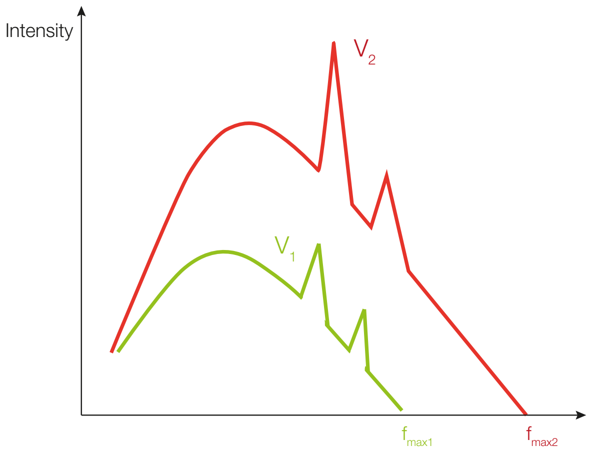


Figure 3.2.21

Figure 3.2.21 shows the graph of the X-ray spectrum for two potential differences across the X-ray tube. V_2 is larger than V_1 and produces a larger maximum frequency.

Increasing the accelerating potential will increase the maximum frequency of the X-rays produced. This is because the electrons acquire more kinetic energy as they are accelerated across the potential difference. The electrons can therefore transfer more energy to an X-ray photon. This increases the maximum frequency of the X-ray photons produced ($E=hf_{max}$). The intensity curve shifts to the right. The intensity can also increase because the electrons colliding with the nuclei of the target atoms often have enough kinetic energy to undergo multiple collisions.

Extra Understanding

Changing the target metal

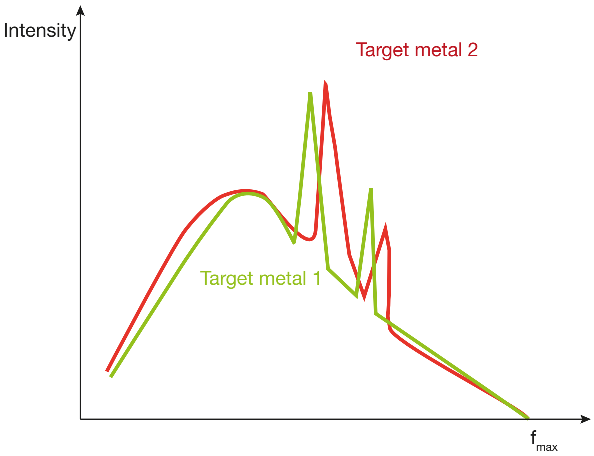


Figure 3.2.22

Figure 3.2.22 shows the graph of the X-ray spectrum for two different target metals.

A different target metal will produce X-rays with the same range of frequencies but the high-intensity peaks appear at different frequencies. These peaks are therefore characteristic of the target atoms and produce what are known as characteristic X-rays.

3

Worked examples

1. A potential difference of 4.00×10^4 V is applied across an X-ray tube.

   (a) Calculate the kinetic energy of the electrons just before they strike the target metal.

   $W = \Delta E_k = q\Delta V = 1.60 \times 10^{-19} \times 4.00 \times 10^4 = 6.40 \times 10^{-15}\ J$

   (b) Calculate the maximum frequency of the X-ray produced.

   $$f_{max} = \frac{e\Delta V}{h} = \frac{6.40 \times 10^{-15}}{6.63 \times 10^{-34}} = 9.65 \times 10^{18}\ Hz$$

2. An X-ray tube produces X-rays with a maximum frequency of 1.80×10^{19} Hz by accelerating electrons through a large potential difference towards a metal target.

   (a) Calculate the potential difference that is applied across the X-ray tube in order to produce these X-rays.

   $$f_{max} = \frac{e\Delta V}{h} = \therefore\ \Delta V = \frac{hf_{max}}{e} = \frac{6.63 \times 10^{-34} \times 1.80 \times 10^{19}}{1.60 \times 10^{-19}} = 7.46 \times 10^4\ V$$

   (b) Calculate the speed of the electrons as they strike the target metal.

   $E_k = q\Delta V = 1.60 \times 10^{-19} \times 7.46 \times 10^4 = 1.19 \times 10^{-14}\ J$

   $$E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.19 \times 10^{-14}}{9.11 \times 10^{-31}}} = 1.62 \times 10^8\ ms^{-1}$$

The attenuation of X-rays by different materials

X-rays are extremely useful as a diagnostic tool in detecting problems such as fractures and break in bones. They can also be used as a therapeutic tool in the treatment of cancers and have many uses in industry.

The attenuation of X-rays is the reduction in intensity of X-rays as they pass through a material due to the absorption and scattering of the X-ray photons.

The degree of attenuation depends on the physical properties of the material through which the X-rays pass. If all materials attenuated X-rays equally, an X-ray photograph could not result. That is, the differing amount of attenuation by different materials creates contrast. Figure 3.2.23 shows an X-ray photograph of a broken leg. The whiter the region on the X-ray photograph, the greater the attenuation of X-rays. This is because fewer X-rays penetrate the tissue to expose the photographic film. Photographic film turns black when exposed to X-rays.

Figure 3.2.23

Material thickness

When comparing the same type of material, the thicker the material, the greater the attenuation. For instance, the attenuation of X-rays passing through an arm bone is less than for a thicker leg bone.

Material density

When comparing materials of different density, the greater the density, the greater the attenuation. For instance, the attenuation of X-rays passing through a compact bone such as a leg bone is greater than the attenuation of X-rays passing through less dense muscle or fatty tissue such as breast tissue.

The penetrating power (hardness) of X-rays

The penetrating power of X-rays is referred to as their hardness**.**

Hard or highly penetrating X-rays are needed for X-rays such as chest and leg X-ray. This is due to the greater attenuation of the X-rays. Hard X-rays have **high photon energies** and **frequencies**. Hard X-rays result from **large potential differences** across the X-ray tube. Recall that a larger potential difference will do more work on the electrons. The kinetic energy gained by the electrons is greater and converted into higher energy X-ray photons.

Soft or less penetrating X-rays are needed for X-rays such as mammograms. This is due to the lower attenuation of the X-rays. Soft X-rays have **low photon energies** and **frequencies**. Soft X-rays result from **lower potential differences** across the X-ray tube.

Exposure time for X-rays

Just like a traditional photograph, a short exposure time will minimise blurring due to movement. This requires a more intense beam of X-rays for a given hardness. The **greater** the **intensity** of the beam, the greater the number of photons present. The number of photons is determined by the number of electrons striking the target metal per second. More electrons are released from the filament by a **larger filament current**.

Science as a human endeavour

Some possible ideas for investigation include:

1. Investigate how the uses of X-rays are monitored and assessed, and how the risks are evaluated.
   - Examples in medicine include diagnostic medicine, such as CAT or CT scans.
   - Examples in industry include Xray diffraction used in crystallography and the non-destructive analysis of art objects, Xray microscopy of biological materials, security screening, and quality control on production lines.
2. Explore the uses of alternative techniques for producing or detecting X-rays, including:
   - synchrotron
   - X-ray fluorescence
   - X-ray lasers
   - X-ray astronomy.

   Assess the benefits and disadvantages of these in different contexts.

The Wave Behaviour of Particles

If electromagnetic radiation previously thought to be a wave could behave like a particle (photon), then perhaps particles under the right conditions could exhibit wave properties.

Key ideas

1. Particles exhibit wave behaviour with a wavelength (called the de Broglie wavelength) that depends on the momentum of the particle. The de Broglie wavelength is given by the formula $\lambda = \frac{h}{p}$ where h is Planck's constant and p is the momentum of the particles.
2. The wave behaviour of particles can be demonstrated using a double-slit experiment and the Davisson–Germer experiment.

The de Broglie wavelength

In 1923, a French physicist called Victor de Broglie proposed that moving particles had a wavelength that depended on their momentum. In 1929 de Broglie was awarded the Nobel Prize for Physics.

The formula for calculating the de Broglie wavelength (λ) of a particle is given by

$$\lambda = \frac{h}{p}$$

where p is the momentum of the particle.

Since $p = mv$ it follows that $\lambda = \frac{h}{p} = \frac{h}{mv}$ where m is the mass and v is the speed of the particle.

It can be seen that the larger the momentum of the particle, the shorter the de Broglie wavelength. Most objects such as a ball being thrown through the air, have such a small wavelength that it was impossible to detect. However, calculation shows that the de Broglie wavelength of fast moving electrons is larger and in the order of the wavelength of X-rays ($\times 10^{-11} - \times 10^{-10}$ m).

Worked examples

1. Calculate the de Broglie wavelength of a 145 g ball travelling with a speed of 50.0 kmh^{-1}.

   50.0 kmh^{-1} = 13.9 ms^{-1}

   $$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{0.145 \times 13.9} = 3.29 \times 10^{-34}\ m$$

2. Calculate the wavelength of electrons travelling with a speed of 3.0×10^{7} ms^{-1}.

   $$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.0 \times 10^{7}} = 2.4 \times 10^{-11}\ m$$

The two-slit interference pattern produced by electrons

The wave behaviour of electrons can be demonstrated using a double slit experiment. Figure 3.2.24 shows a beam of electrons released from an electron gun passing through two narrow slits such that they are incident normally on a fluorescent screen. The screen will glow wherever the electrons strike it. The result is an interference pattern similar to that of light in two-slit interference (Figure 3.2.25). This infers that the electrons are behaving in the same manner as light. Even if the number of electrons per unit time (or their intensity) is reduced, the interference pattern builds up over time just like it did with light of low intensity.

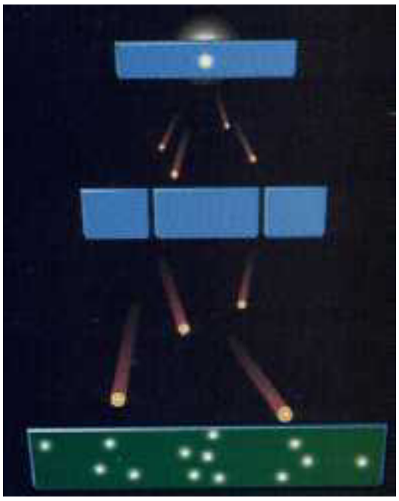


Figure 3.2.24

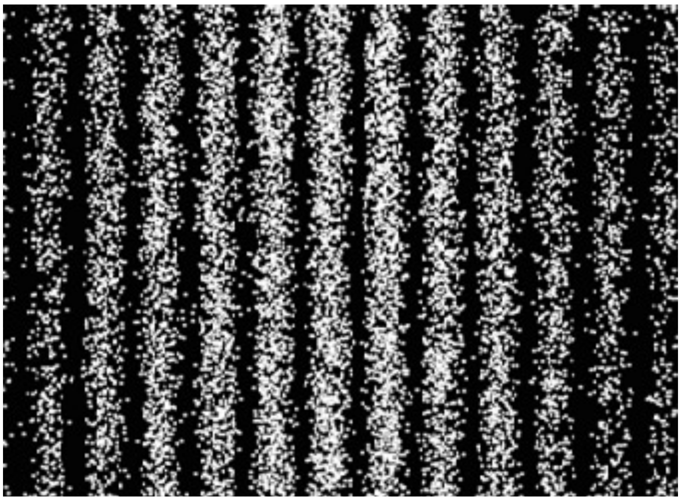


Figure 3.2.25

Helpful online resources

A simulation of two-slit interference of electrons.

https://phet.colorado.edu/en/simulation/quantum-wave-interference

The Davisson Germer experiment

In 1927 Clinton Davisson and Lester Germer provided experimental evidence for the wave nature of electrons. The two reasoned that if electrons behave like waves, then they should undergo diffraction in the same way that visible light undergoes diffraction. Figure 3.2.26 shows the experimental arrangement and Figure 3.2.27 is a graphical representation of the results.

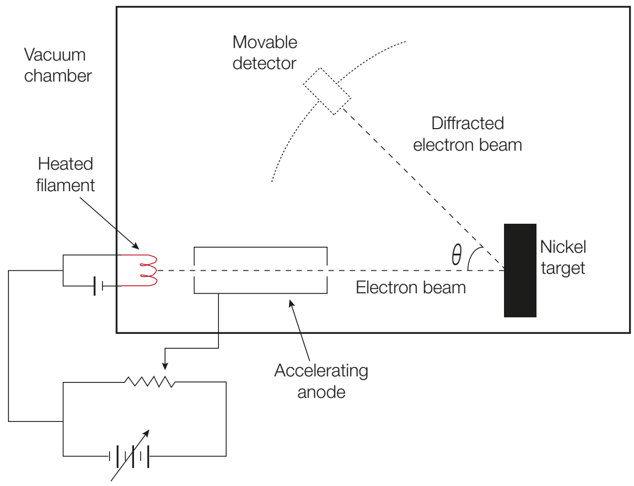


Figure 3.2.26

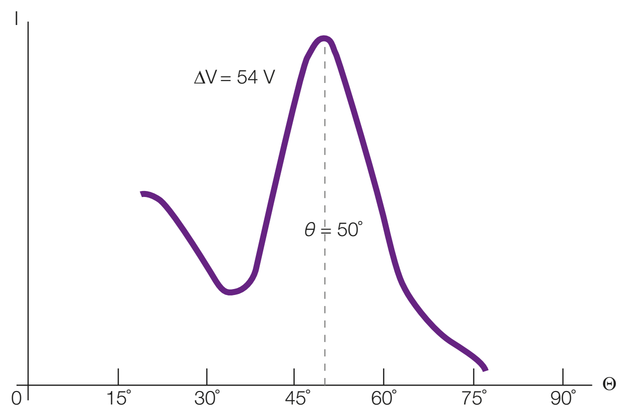


Figure 3.2.27

Electrons were released from a heated filament and accelerated through a small potential difference so that they had a low energy. They were passed through a slit to produce a thin beam and fired towards a nickel crystal. An electron detector was used to measure the intensity of the scattered electrons at various angles. Instead of the electrons reflecting from the crystal and scatter in all directions evenly, their intensity peaked at certain angles. That is, the electrons were diffracted by the surface layers of the crystal at preferred angles in a similar way to light. The equation $d\sin\theta = m\lambda$ was able to predict the angles at which the electrons were diffracted.

The crystal spacing d was known and the angle θ of the diffracted electrons was measured for a given order m.

Davisson and Germer compared the wavelength found experimentally using the wave relationship $\lambda = \frac{d\sin\theta}{m}$ to the theoretical wavelength calculated using the de Broglie relationship $\lambda = \frac{h}{p}$. The results agreed, confirming the de Broglie relationship.

A crystal lattice was used because the spacing of the crystal planes was of the order of 10^{-10} m. This was comparable to the de Broglie wavelength calculated for electrons and it was already known that for observable diffraction effects, the spacing needed to be of the same order as the wavelength.

Helpful online resources

1. A summary of the experiment and results of the Davisson-Germer experiment.

   https://www.youtube.com/watch?v=Ho7K27B_Uu8

2. A simulation of the Davisson–Germer experiment

   https://phet.colorado.edu/en/simulation/davisson-germer

Worked example

In the Davisson-Germer experiment, the incident electrons had a kinetic energy of 54.0 eV and the first order maximum was detected at an angle of 50.0°.

(a) Given that the inter-atomic spacing of the nickel crystal was 2.15×10^{-10} m. Calculate the experimental wavelength of the electrons.

$$d\sin\theta = m\lambda \quad \therefore \quad \lambda = \frac{d\sin\theta}{m} = \frac{2.15 \times 10^{-10}\sin 50.0}{1} = 1.65 \times 10^{-10}\ m$$

(b) Determine the theoretical or de Broglie wavelength of the electrons. Compare this to the value obtained in part (a).

$$E_k = 1.60 \times 10^{-19} \times 54.0 = 8.64 \times 10^{-18}\ J$$

$$E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 8.64 \times 10^{-18}}{9.11 \times 10^{-31}}} = 4.36 \times 10^6\ ms^{-1}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.36 \times 10^6} = 1.67 \times 10^{-10}\ m$$

Taking experimental errors into account, the experimental and theoretical values for wavelength agree.

Science as a human endeavour

Explore how the wave nature of electrons has led to a diverse range of contemporary applications. Examples include: electron microscope, materials research, forensics, pharmaceutical quality control.

Exercises

1. The microwaves produced in a microwave oven have a frequency of 3.2×10^9 Hz.

   (a) Calculate the energy of a microwave photon.

   (b) Calculate the momentum of a microwave photon.

2. Photons in green light have an energy of 3.62×10^{-19} J.

   (a) Calculate the frequency of the photons in green light.

   (b) Calculate the momentum of the photons in green light.

3. Light from a neon sign is produced by exciting neon atoms with an electrical discharge. The wavelength of the emitted photons is 640 nm.

   Calculate the energy of the photons in neon light.

4. Determine the momentum of photons which have an energy of 20.0 eV.

5. The power rating of a red light bulb is 60.0 W. This means that the light globe releases 60.0 J of energy per second.

   In converting electrical energy to light energy the light bulb produces a great deal of heat energy and operates at an efficiency of 22.0 %.

   Assuming that all the light released is red light with a wavelength of 7.00×10^{-7} m, determine the number of photons that are emitted by the red light globe every second.

6. The threshold frequency of iron is 1.1×10^{14} Hz. When iron metal is illuminated with light of frequency 7.5×10^{15} Hz, electrons are emitted from its surface.

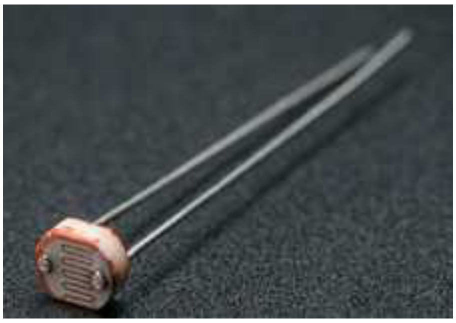
   (a) Describe how Einstein explained the ejection of electrons from such a metal surface.

   (b) Calculate the work function for iron.

   (c) Calculate the maximum kinetic energy of the emitted electrons.

7. The diagram below shows a photocell. The photoelectric effect is the basis by which a photocell works. A photocell consists of a vacuum tube with two electrodes. One electrode is photosensitive and emits electrons when exposed to light. The other electrode is held at a positive potential. The electrons that are emitted are attracted to the positive electrode.

This creates a current that can be used to operate many electrical devices including toys, the shutter of a camera, the motor that opens a door as you approach it and water taps that turn on when you wave your hand under the faucet.



A particular photocell peaks in its operation when light of wavelength 520 nm illuminates its photosensitive surface. The work function of the photocell is 2.1 eV.

(a) Calculate the threshold frequency for the photocell.

(b) Calculate the maximum kinetic energy of the electrons emitted from the photocell.

(c) When the intensity of the light incident on a photocell increases, more electrons are emitted from the photosensitive surface. Describe how Einstein used the concept of photons to explain this observation.

8. (a) Describe what is meant by the photoelectric effect.

Ultra-violet light of frequency 1.00×10^{15} Hz is used to illuminate a metal plate whose work function is 3.50 eV.

(b) Calculate the

(i) energy of the incident photons.

(ii) threshold frequency of the metal plate.

(iii) maximum kinetic energy of the electrons emitted from the metal plate.

(iv) speed of the fastest electrons emitted from the metal plate.

(v) stopping voltage for the electrons emitted from the metal plate.

(c) The intensity of the ultra-violet light is doubled. Explain how this would affect the value of the stopping voltage.

(d) The ultra-violet light used to illuminate the metal plate is replaced with visible light which has a wavelength of 600 nm. Determine whether or not light with a wavelength of 600 nm would emit electrons from the same metal plate.

9. A student illuminates a photoelectric cell with light of frequency 6.25×10^{15} Hz and measures a stopping voltage of 3.13 V.

(a) Calculate the maximum kinetic energy of the electrons emitted from the photoelectric cell.

(b) Calculate the work function of the photoelectric cell.

10. The photoelectric effect is the emission of electrons from the surface of a material when it is illuminated with light of sufficiently high frequency.

    One of the experimental observations of the photoelectric effect is that electrons are ejected with a range of kinetic energies up to a maximum value.

    Describe Einstein's explanation of this experimental observation.

11. Silver has a threshold frequency of 1.02×10^{14} Hz. When illuminated with electromagnetic radiation of frequency 6.4×10^{14} Hz, electrons are emitted from the surface of silver.

    Determine the stopping voltage for the ejected electrons.

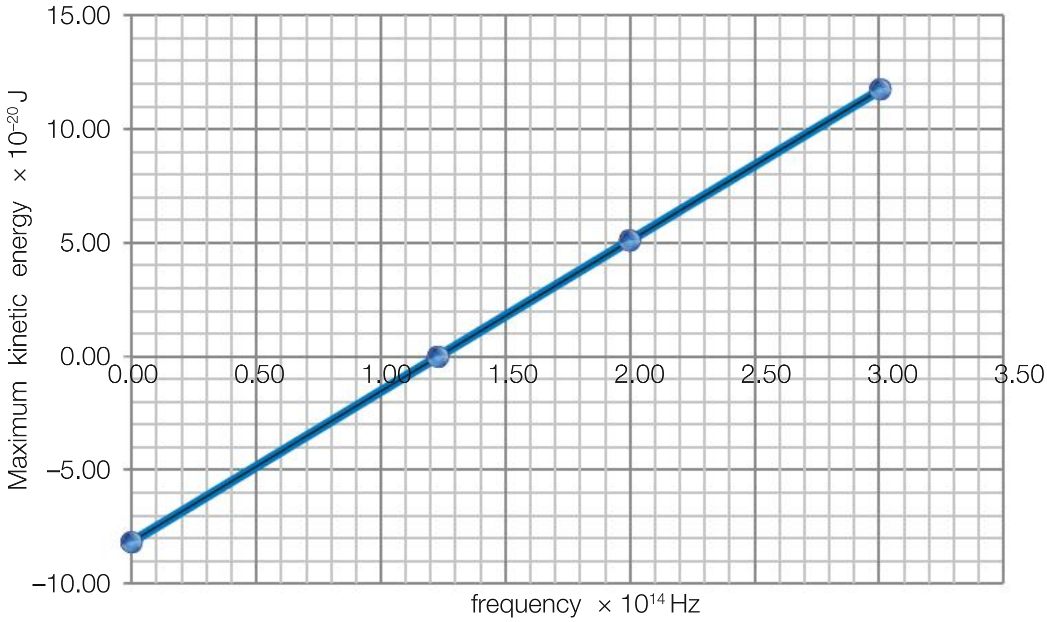
12. The table below lists the threshold frequencies for three different metals.

Metal	Threshold frequency (Hz) $\times 10^{14}$
Aluminium	9.85
Zinc	1.04
Gold	1.23

(a) Calculate the work function for gold.

(b) All three metals are illuminated with light of frequency *f* to produce the photoelectric effect. Explain which of the three metals listed in the table will emit electrons with the greatest kinetic energy.

(c) The frequency of the light used to illuminate gold is varied and a graph of the maximum kinetic energy of the emitted electrons is plotted against frequency.



(i) State the quantity represented by the vertical axis intercept of the graph.

....

(ii) State the quantity represented by the horizontal axis intercept of the graph.

....

(iii) State the quantity represented by the gradient of the graph.

....

(iv) Draw a new line on the graph that would represent the results for the maximum kinetic energy of the emitted electrons versus frequency graph for zinc.

13. A student carries out an experiment to investigate how the maximum kinetic energy of the emitted electrons in the photoelectric effect varies with frequency.

The table below summarises the student's results.

$E_{K\ max}$ (eV)	$E_{K\ max}$ (J)	Frequency (×10^{14} Hz)
0.6	....	7.0
1.1	....	8.0
1.5	....	9.0
1.8	....	10.0
2.1	....	10.5

(a) Complete the middle column of the table by calculating the maximum kinetic energy of the emitted electrons in Joules.

(b) Plot a graph of $E_{K_{max}}$ *(J) vs frequency (Hz)*. Draw a line of best fit for the plotted points.

3

(c) Use your graph to determine

(i) Planck's constant.

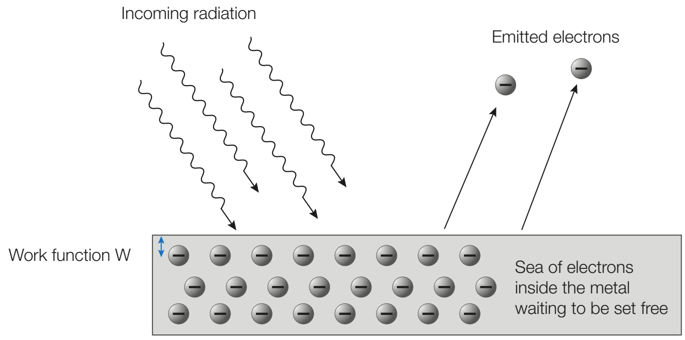
(ii) the threshold frequency of the metal.

.. ..

(iii) the work function of the metal cathode used.

.. ..

14. (a) The diagram below represents the emission of electrons from the surface of a metal when it is illuminated with monochromatic electromagnetic radiation which has a frequency *f*.



The work function *W* is marked on the diagram.

(i) Define the term work function.

.. ..

.. ..

.. ..

(ii) Show that the maximum kinetic energy $E_{K_{max}}$ of the emitted electrons is given by $E_{K_{max}} = hf - W$.

.. ..

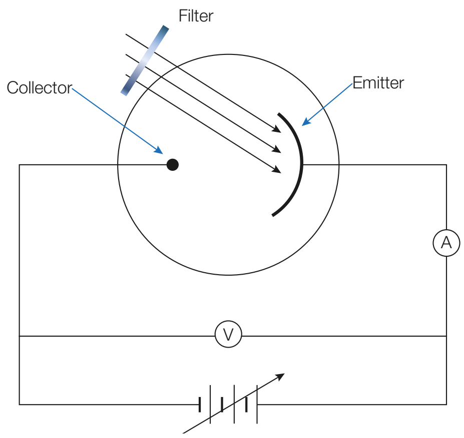
.. ..

.. ..

.. ..

.. ..

(b) The diagram below shows an experimental arrangement that can be used to investigate the relationship between the maximum kinetic energy of the emitted electrons in the photoelectric effect and the frequency of visible light. That is, $E_{K_{max}} = hf - W$.



Describe how this arrangement can be used to investigate this relationship.

.. ..

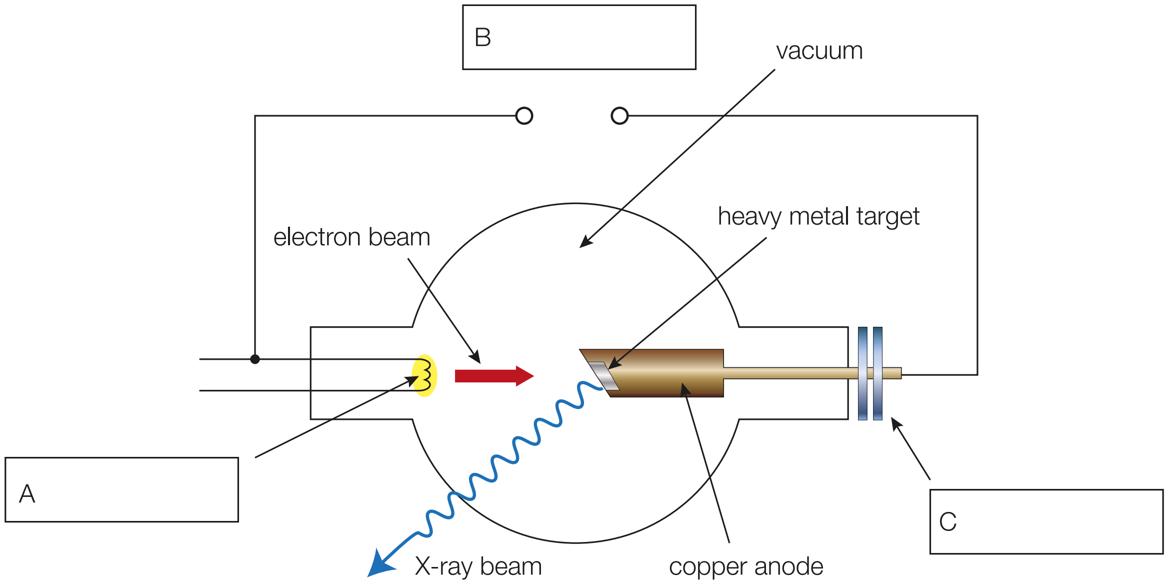
.. ..

.. ..

.. ..

.. ..

15. An X-ray tube produces X-ray photons. The diagram below shows a typical X-ray tube.



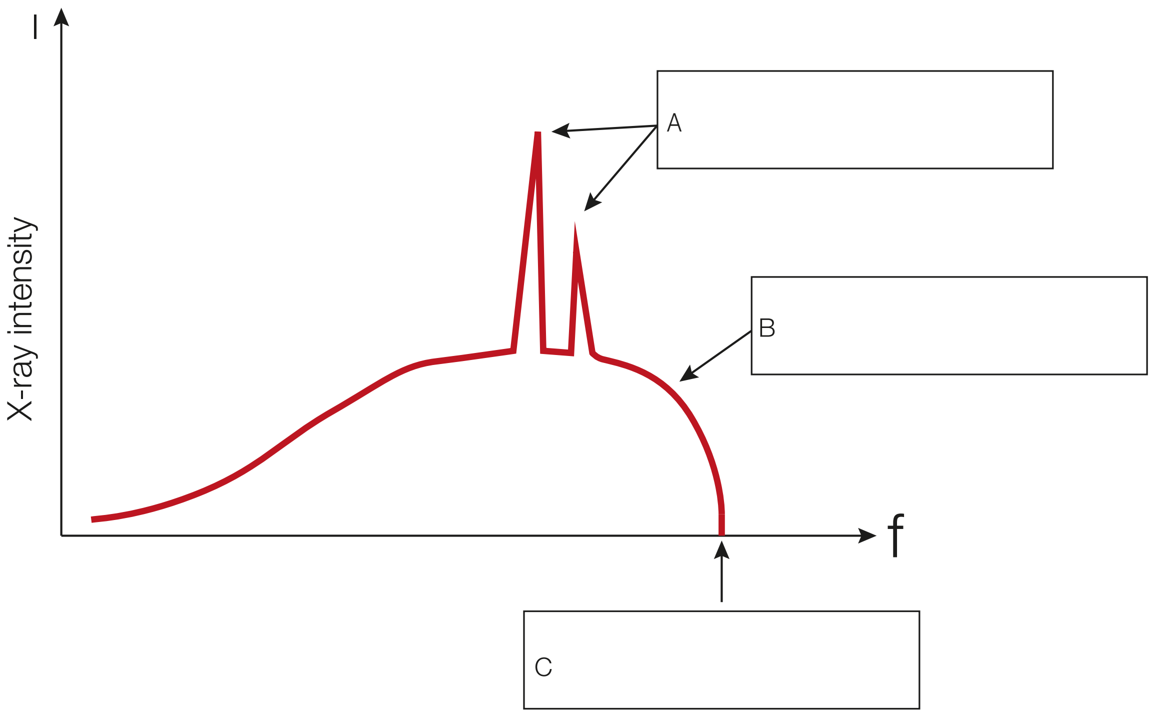
(a) Fill in the empty boxes labelled A to C with the missing features of the X-ray tube.

(b) Complete the following table by describing the purpose of each of the features A to C.

Feature	Purpose
A	.. ..
B	.. ..
C	.. ..

16. The diagram below represents the spectrum from an X-ray tube.

On the diagram, label the three main features of the X-ray spectrum.

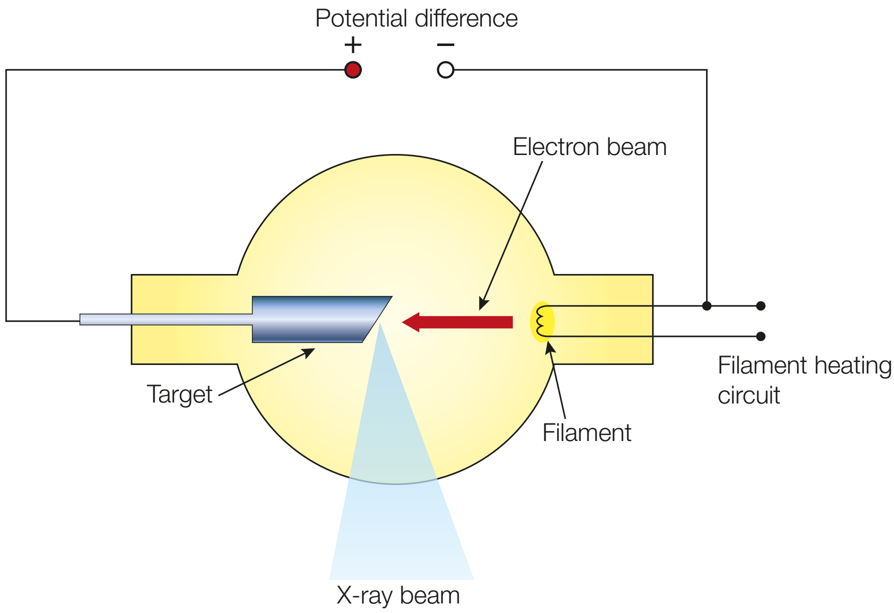


17. Electrons are accelerated across a potential difference of 5.00×10^4 V in an X-ray tube.

(a) Show that the maximum frequency of the X-ray is 1.21×10^{19} Hz.

(b) The accelerating potential across the X-ray tube is increased to 1.00×10^5 V. State with reason, the change this would have on the maximum frequency of the X-rays.

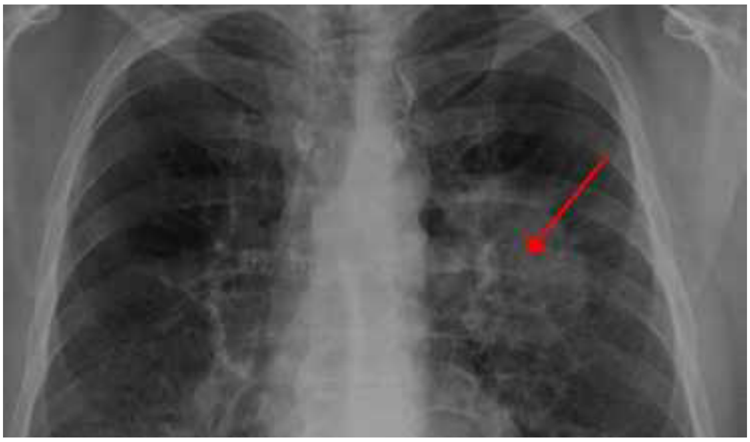
18. An X-ray tube is shown in the diagram below.



(a) Explain why X-rays with a continuous range of frequencies up to a maximum frequency are produced by the X-ray tube.

(b) Show that the maximum frequency of the X-ray photons is given by $f_{max} = \frac{e\Delta V}{h}$, where ΔV is the potential difference applied across the X-ray tube.

19. An X-ray photograph of a patient's chest is shown below. The patient is a smoker and the X-ray photograph reveals a mass in the lungs. The mass is indicated by the red arrow. The patient was diagnosed with lung cancer. The lung cancer tumour is more dense than the surrounding tissue.



Explain in terms of attenuation why the lung cancer tumour is seen as a white mass on the X-ray photograph.

20. (a) X-ray photons are produced in an X-ray tube. Calculate the potential difference that needs to be applied across the tube to produce X-rays with a maximum frequency of 2.88×10^{19} Hz.

(b) X-ray tubes generally operate at potential differences between 10 kV and 140 KV.

Consider X-rays with a maximum frequency of 2.88×10^{19} Hz. With reason classify them as soft or hard X-rays and describe their penetrating power.

(c) Suggest one type of tissue that could be successfully examined with X-rays that have a maximum frequency of 2.38×10^{19} Hz.

(d) The filament current is increased while the potential difference across the tube remains unchanged. Describe and explain the effect this would have on the maximum frequency of the X-rays produced.

21. A potential difference of 100 KV is applied across an X-ray tube.

(a) Calculate the speed of the electrons as they strike the target metal.

(b) Determine the minimum wavelength of the X-rays produced.

22. Image 1 shows a full body low-level X-ray scanner commonly used in airports. Travellers pass between two walls of the scanner to have their body scanned. The X-ray image that results is a virtual three-dimensional naked scan that can easily reveal concealed weapons (Image 2).

Image 1

Image 2

(a) Travellers are exposed to low intensity beam of X-rays. Explain how this is achieved in the X-ray tube.

(b) List one disadvantage of a low intensity beam.

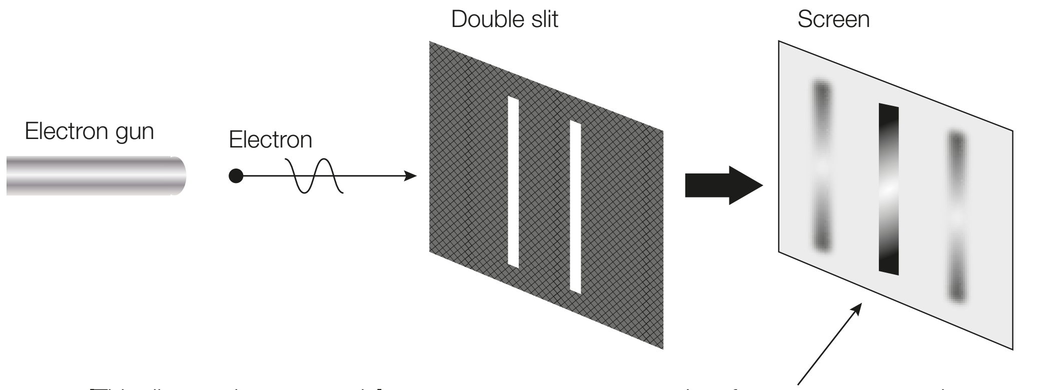
23. Calculate the de Broglie wavelength of a proton travelling with a momentum of 1.12×10^{-14} sN.

24. An electron microscope uses the wavelike properties of electrons to achieve images of high resolution. In a particular electron microscope, the electrons are accelerated to a speed of 2.9×10^7 ms^{-1} by an electron gun.

Calculate the wavelength of the electrons as they emerge from the electron gun.

25. A potential difference of 6.50×10^4 V is applied across an X-ray tube. Determine the de Broglie wavelength of the electrons as they strike the target.

26. The diagram below shows electrons released from an electron gun passing through two narrow slits placed 1.6×10^{-10} m apart.

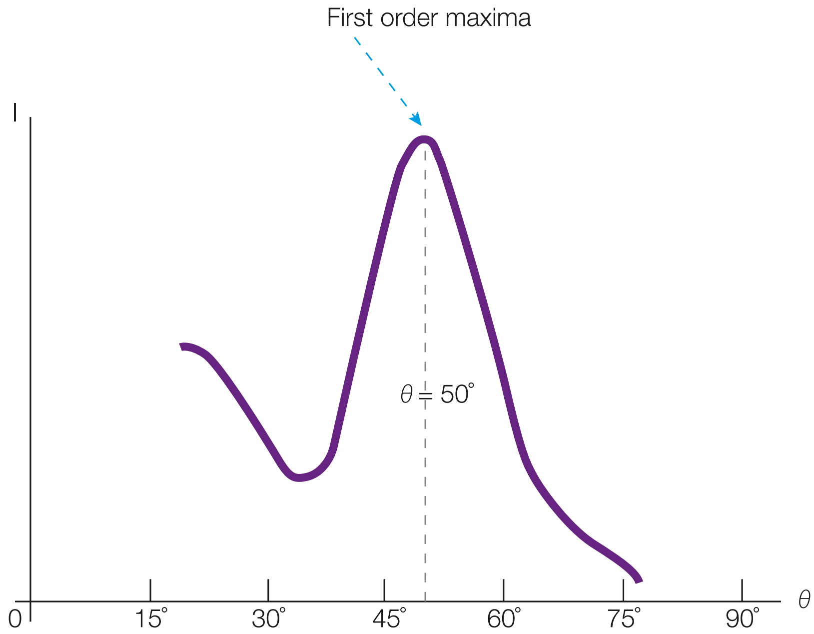


[This diagram is not to scale]

(a) Describe the significance of the interference pattern that appears on the screen.

(b) The interference pattern produces a first order maxima at an angle of 22°. Calculate the wavelength of the electrons.

27. Davisson and Germer used the diffraction of electrons from a crystal to show that electrons had a de Broglie wavelength. The diagram below shows the results of the Davisson-Germer experiment.



(a) Describe the Davisson-Germer experiment.

(b) The wavelength of the electrons was found to be 1.6×10^{-10} m. Calculate the spacing between the planes of the crystal used by Davisson and Germer.

(c) Calculate the potential difference through which Davisson and Germer accelerated the electrons.

28. Show that in an X-ray tube, the wavelength of the electrons just before they strike the target is given by $\lambda = \frac{h}{\sqrt{2qm\Delta V}}$.

3.3 Structure of the atom

Science understanding

1. A continuous spectrum contains a continuous range of frequencies.
2. Solid, liquid, or dense gaseous objects radiate a continuous spectrum, which may extend into or beyond the visible region. The process is known as incandescence. The frequency distribution, and hence the dominant colour, depends on the temperature of the object.
   - Describe the changes in the spectrum of incandescent source as the temperature of the incandescent increases.
3. Atoms can be raised to excited states by heating or by bombardment with light or particles such as electrons. An atom is in an excited state when an electron has been raised to a higher energy level.
4. The heated vapour of a pure element emits light of discrete frequencies, resulting in a line emission spectrum when the light is viewed with a spectrometer.
   - Describe the general characteristics of the line emission spectra of elements.
   - Explain how the uniqueness of the spectra of elements can be used to identify the presence of an element.
   - Explain the production of characteristic X-rays in an X-ray tube.
   - Solve problems that require comparing spectra of different elements
5. The presence of discrete frequencies in the line emission spectra of atoms is evidence for the existence of discrete electron energy-levels in atoms. The different electron energy-levels can be represented on an energy-level diagram.
6. When an electron makes a transition from a higher-electron energy level to a lower-electron energy level in an atom, the energy of the atom decreases and a photon is emitted.
7. The energy of the emitted photon is given by the difference in the electron energy-levels of the atom. An atom is in its ground state when its electrons are in their lowest possible electron energy-level in the atoms.
   - Explain how the presence of discrete frequencies in line emission spectra provides evidence for the existence of states with discrete electron energy-levels in atoms.
   - Solve problems involving emitted photons and electron energy-levels.
   - Draw arrows on an electron energy-level diagram showing transitions between electron energy-levels in atoms.
   - Draw electron energy-level diagrams to represent the energies of different states in an atom.
   - Given an electron energy-level diagram, calculate the frequencies and wavelengths of lines corresponding to specified transitions.
8. The line emission spectrum of atomic hydrogen consists of several series of lines.
   - Draw, on an electron energy-level diagram of hydrogen, transitions corresponding to each of the series terminating at the three lowest-energy levels.
   - Relate the magnitude of the transitions on an electron energy-level diagram to the region in the electromagnetic spectrum of the emitted photons (ultraviolet, visible, or infrared).
9. The ionisation energy of an atom is the minimum energy required to remove the electron from the atom in its ground state.
   - Determine the ionisation energy (in either joules or electron volts) of atoms using an electron energy-level diagram.
10. When light with a continuous spectrum is incident on a gas of an element, discrete frequencies of light are absorbed, resulting in a line absorption spectrum.
11. The frequencies of the absorption lines are a subset of those in the line emission spectrum of the same element.
    - Describe the line absorption spectrum of an atom.
    - On an energy-level diagram, draw transitions corresponding to the line absorption spectrum of atoms.
    - Explain why there are no absorption lines in the visible region for hydrogen at room temperature.
    - Account for the presence of absorption lines (Fraunhofer lines) in the Sun's spectrum.
12. One type of fluorescence is when an electron in an atom absorbs a photon to reach a higher electron energy-level, but then reverts to its previous state by emitting two or more photons with lower energy and longer wavelength.
    - Explain, using an electron energy-level diagram, the production of multiple photons via fluorescence.

13. When an electron in an atom absorbs a photon and reaches a higher electron energy-level the atom is said to be in an excited state. Excited states are generally short-lived and the electron returns spontaneously to its previous electron energy-level often by emitting a series of lower-energy photons. This is known as 'spontaneous emission'.
14. When a photon is incident on an electron that has been raised to a higher electron energy-level, and the energy of the photon corresponds to a transition to a lower electron energy-level, then the photon can stimulate an electron to transition to the lower electron energy-level. This results in two identical photons; the original photon and a second photon that results from the transition. This is known as 'stimulated emission'.
    - Compare the process of stimulated emission with that of spontaneous emission.
15. Stimulated emission in gas lasers produces laser light. The photon emitted in stimulated emission is identical (in energy, direction, and phase) to the incident photon.
    - Explain how stimulated emission can produce coherent light in a laser.
16. A population inversion is produced in a set of atoms whenever there are more atoms in a higher-energy state than in a lower-energy state. For practical systems, the higher-energy state must be metastable if a population inversion is to be produced.
    - Explain the conditions required for stimulated emission to predominate over absorption when light is incident on a set of atoms.
17. The energy carried by a laser beam is concentrated in a small area and can travel efficiently over large distances, giving laser radiation a far greater potential to cause injury than light from other sources.
    - Describe the useful properties of laser light (i.e. it is coherent and monochromatic, and may be of high intensity).
    - Discuss the requirements for the safe handling of lasers.

This chapter uses the concept of energy developed in the Stage 1, Subtopic 4.1: Energy and Stage 2, Subtopic 2.2: Motion of Charged Particles in Electric Fields, momentum in Stage 1, Subtopic 4.2:Momentum and waves in Stage 1, Subtopics 5.1: Wave Model and 5.3 : Light.

Atomic spectra

An atomic spectrum is the range of frequencies of electromagnetic radiation emitted or absorbed by matter.

There are three types of atomic spectra: Continuous, line emission and line absorption.

Each type is pictured in Figure 3.3.1.

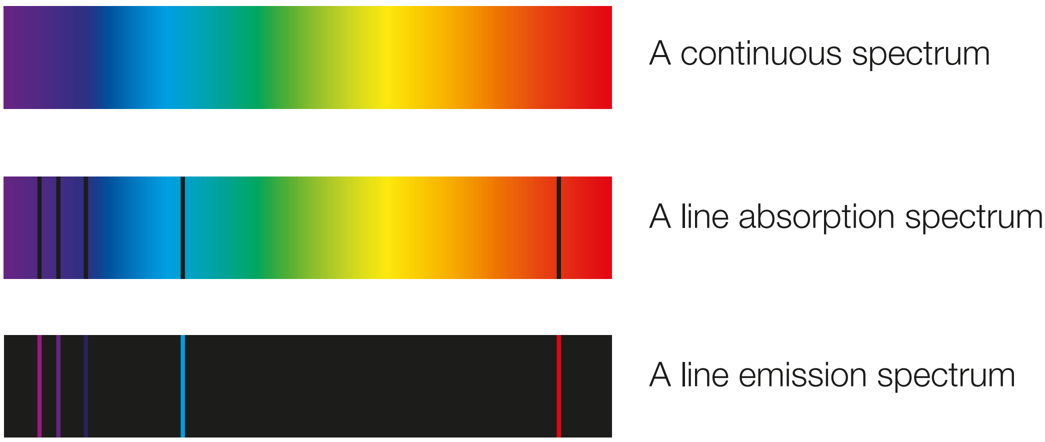


Figure 3.3.1

The continuous spectrum contains of a range of frequencies of light emitted by matter. A line emission spectrum consists of discrete frequencies and a line absorption spectrum consists of a series of dark lines superimposed on a continuous spectrum. These dark lines correspond to a subset of the coloured lines produced in the line emission spectrum of the same element.

The production and features of each type of spectrum will be discussed in this chapter. Atomic spectra are viewed using an instrument called a spectroscope or grating spectrometer. A grating spectrometer contains a diffraction grating that separates the light and allows the different frequencies to be viewed. An instrument that photographs spectra is called a spectrograph.

The continuous spectra

Key ideas

1. A continuous spectrum contains a continuous range of frequencies.
2. Solid, liquid, or dense gaseous objects radiate a continuous spectrum, which may extend into or beyond the visible region. The process is known as incandescence. The frequency distribution, and hence the dominant colour, depends on the temperature of the object.

When light radiated from a solid, liquid or dense gas is viewed through a spectrometer a continuous spectrum consisting of a range of frequencies is observed. This is because all matter consists of atoms and all atoms contain charges. The kinetic theory suggests that the particles of matter are in constant random motion. This was explained in the Stage 1 Workbook, Subtopic 3.1. The particles of the material at any given temperature have a range of kinetic energies as shown in Figure 3.3.2. The average kinetic energy of the particles increases with temperature.

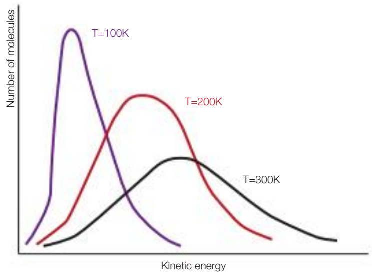


Figure 3.3.2

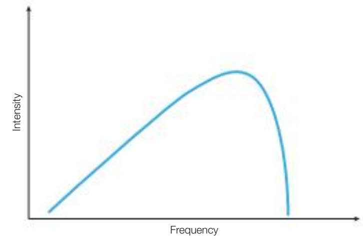


Figure 3.3.3

This means that the particles and hence charges in an object at a given temperature are vibrating with a range of frequencies. Vibrating charges emit electromagnetic radiation and the frequency of the emitted radiation is the same as the frequency with which the charges vibrate. It therefore follows that electromagnetic radiation with a range of frequencies is emitted from a hot object as shown in Figure 3.3.3. The frequency distribution may extend into or beyond the visible region of electromagnetic spectrum.

If the material is heated, the average vibrational kinetic energy of the particles increases. This corresponds to an increase in the frequency of the electromagnetic radiation emitted.

Figure 3.3.4 shows the frequency distribution of the emitted light for an object at three different temperatures. The frequency distribution and hence dominant colour depends on the temperature of the object. A temperature of 600 K (327°C) produces light completely in the Infra-red region. In fact, any object at low temperature (including room temperature) emits light in the infra-red region. The dominant frequency (peak) is therefore also in the infra-red region. No visible light is observed.

As an object is heated, it starts to glow a red colour. This is because it emits a range of frequencies with the dominant being red. Figure 3.3.5 shows three stars. The star Betelgeuse has a temperature of about 3000 K and has a red appearance.

As heating continues, the dominant colour shifts towards the blue end of the visible spectrum. Figure 3.3.4 illustrates that a temperature of 6000 K produces a range of frequencies that cross the infra-red, visible and ultra-violet regions of the electromagnetic spectrum. Although the dominant or peak frequency is in the visible region (green light), there is a fairly even spread of frequencies. Such an object would appear red-white. Figure 3.3.5 shows an image of our Sun which has a temperature of around 6000 K.

A temperature of 60 000 K produces a range of frequencies that cross the infra-red, visible and ultra-violet regions of the electromagnetic spectrum. The dominant (peak) frequency is in the ultra-violet region. The object would appear a violet-white colour. The star Rigel shown in Figure 3.3.5 has this appearance.

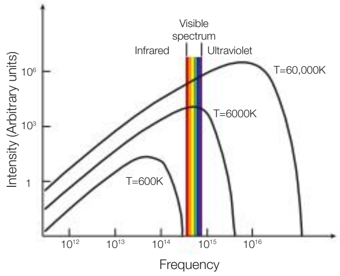


Figure 3.3.4

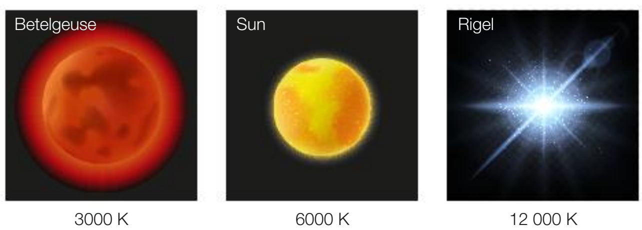


Figure 3.3.5

The changes in the spectrum of an incandescent source as the temperature of the incandescent source increases

In Subtopic 3.1 we defined incandescent light as light produced by heating a material until it glows. A filament globe is an example of an incandescent light source. It produces light when a wire filament is heated with electricity. Charges in the filament vibrate randomly and with a range of frequencies. These oscillations produce electromagnetic radiation with a range of frequencies.

If the light emitted from a filament globe is observed through a spectrometer, a continuous spectrum is seen. As the temperature of the filament increases, the frequency of the vibrations increases and light is emitted with a greater frequency. Since the frequency and wavelength of light are inversely proportional ($\lambda = \frac{v}{f}$ where v is the speed of light 3.00×10^8 ms^{-1}), higher frequencies produce smaller wavelengths. The wavelength of violet/blue light is smaller than the wavelength of red/orange light. Thus, as the temperature increases, there is a shift towards the blue/violet end of the visible spectrum for light i.e. more blue light is seen.

In general, if the temperature of the incandescent source increases, a larger range of frequencies are produced. The intensity of each frequency increases as does the peak frequency. If the incandescent source is cool the spectrum contains a greater proportion of red and orange in the spectrum but as the incandescent source heats up the spectrum shifts and there is more violet and blue in the spectrum.

? Science inquiry activity

Your teacher may demonstrate this by building a simple circuit which includes an incandescent filament globe and a variable power supply such as a power pack. View the continuous spectrum of the globe through a spectrometer; start with a low voltage (e.g. 2 V) and then progressively increase the voltage.

Helpful online resources

A possible idea for investigation is to investigate the relationship between temperature and frequency distribution, using a simulation.

https://phet.colorado.edu/en/simulation/blackbody-spectrum

Science as a human endeavour

Explore examples of the application of incandescence, such as:

- red hot vs white hot
- white fireworks
- filament light bulbs.

Propose contexts for which the use of each is appropriate.

The Line emission spectra for elements

Key ideas

The heated vapour of a pure element emits light of discrete frequencies, resulting in a line emission spectrum when the light is viewed with a spectrometer.

The general characteristics of the line emission spectra of elements

A line emission spectrum is produced when the atoms of a pure gas are heated to high temperatures or subjected to a large potential difference. The emitted light is viewed through a spectrometer or a diffraction grating.

Figure 3.3.6 below shows a discharge tube containing hydrogen gas. When a large potential difference (about 5000 V) is applied across the discharge tube, the light emitted by the hydrogen is viewed through a spectrometer or diffraction grating which separates the different wavelength of emitted light to reveal the line emission spectrum for hydrogen.

Figure 3.3.6

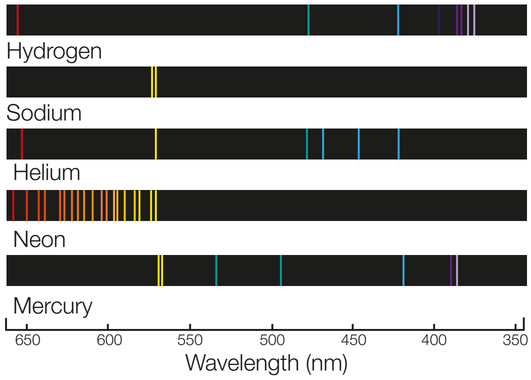


Figure 3.3.7

The spectrum consists of thin discrete coloured lines on a dark background.

That is the heated vapour emits light of discrete frequencies. Figure 3.3.7 shows the line emission spectra for several elements.

The uniqueness of the spectra of elements can be used to identify the presence of elements

It can be seen that the line emission spectrum for each element shown in Figure 3.3.7 are all different. The spectrum is **unique** to the element. That is the frequencies of light emitted by hydrogen are different to those emitted by sodium, helium and all other elements. This means that the coloured lines of the spectrum are different for each gas. A line emission spectrum can therefore be used to **identify an element** by matching the position and therefore wavelength (or frequency) of the emitted light in the line emission spectrum of an unknown gas to that of a known gas.

? Science inquiry activity

Some possible ideas for investigation:

- Use flame tests to identify various metal atoms.
- Use spectroscopes to identify gases in a fluorescent light globe.

Science as a human endeavour

Some possible ideas for investigation are:

1. Explore advantages and disadvantage of using vapour lamps (e.g. neon lights and sodium-vapour street lamps). When is the use of one type more appropriate than the other?
2. Explore how the temperature of stars is determined from the spectrum of emitted light. How confident of their accuracy can scientists be?

Electron energy-levels and electron energy-level diagrams

Key ideas

1. The existence of discrete electron energy-levels in an atom.
2. The different electron energy-levels can be represented on an energy-level diagram.
3. Atoms can be raised to excited states by heating or by bombardment with light or particles such as electrons. An atom is in an excited state when an electron has been raised to a higher energy level.
4. When an electron makes a transition from a higher electron energy-level to a lower electron energy-level in an atom, the energy of the atom decreases and a photon is emitted.
5. The energy of the emitted photon is given by the difference in the electron energy-levels of the atom. An atom is in its ground state when its electrons are in their lowest possible electron energy-level.
6. The ionisation energy of an atom is the minimum energy required to remove a single electron from the atom in its ground state.

At the start of the 20th century, a basic model of the atom had already been established. The model described a tiny dense nucleus consisting of protons and neutrons with electrons orbiting the nucleus. In 1913, a Danish physicist, Niels Bohr proposed that electrons inside an atom could only occupy certain orbitals with a specific energy. Figure 3.3.8 illustrates the Bohr model for the hydrogen atom. The atom consists of a single proton (nucleus) with one electron orbiting the nucleus. It is the simplest of all atoms.

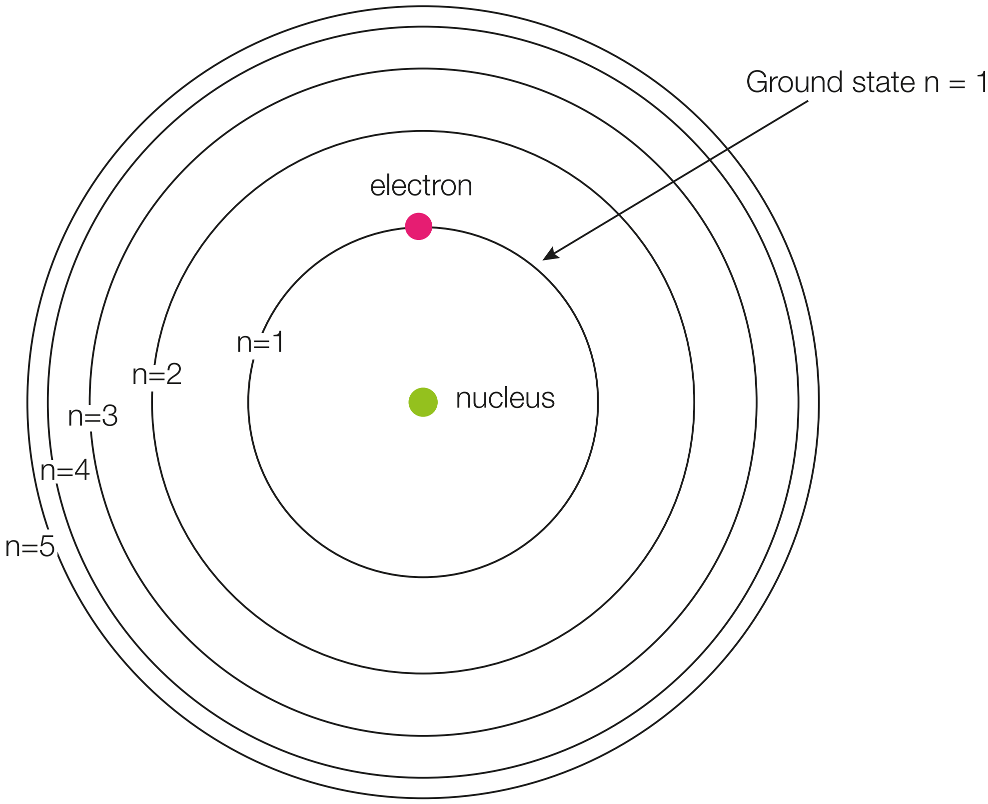


Figure 3.3.8

Each electron energy-level is labelled *n=1, n = 2, n = 3 etc* and the electron in each energy level has a specific energy E_1, E_2, E_3 etc. For hydrogen the possible energies of an electron are given by the equation $E_n = \frac{-13.6}{n^2}\ eV$. The ground state is the lowest energy available for the electrons and is labelled $n = 1$. The energy of the electron in the ground state is assigned an energy E_1, given by $E_1 = \frac{-13.6}{1^2} = -13.6\ eV$. Similarly an electron in the second energy level would have an energy $E_2 = \frac{-13.6}{2^2} = -3.40\ eV$. This can be repeated for all other energy levels.

Figure 3.3.9 shows some of the electron energy-levels for hydrogen.

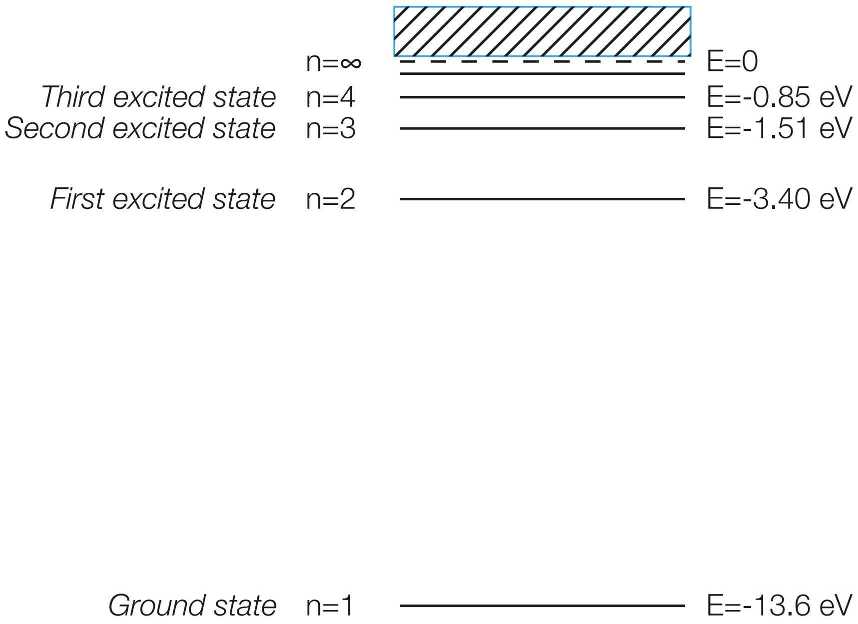


Figure 3.3.9

Further from the nucleus, the energy levels get closer until they converge at infinity ($n = \infty$). The energy of an electron is negative because it is bound within an energy level.

For hydrogen at room temperature the electron is found in the ground state. If hydrogen atoms are heated or bombarded with light or electrons, the electron can absorb this energy and jump or make a transition to a higher-energy level. We say the atom is excited or has been raised to an excited state. An electron that has been raised to $n = 2$ is said to be in the first excited state. An electron that has been raised to $n = 3$ is said to be in the second excited state and so on.

The ionisation energy of an atom is the minimum energy required to remove the electron from the ground state.

Using Figure 3.3.9 we can see that the ionisation energy for hydrogen is 13.6 eV.

If an electron in an excited state makes a transition to a lower electron energy-level, the energy of the atom decreases and a photon of energy exactly equal to the difference between the electron energy-levels will be released. Figure 3.3.10 illustrates this principle.

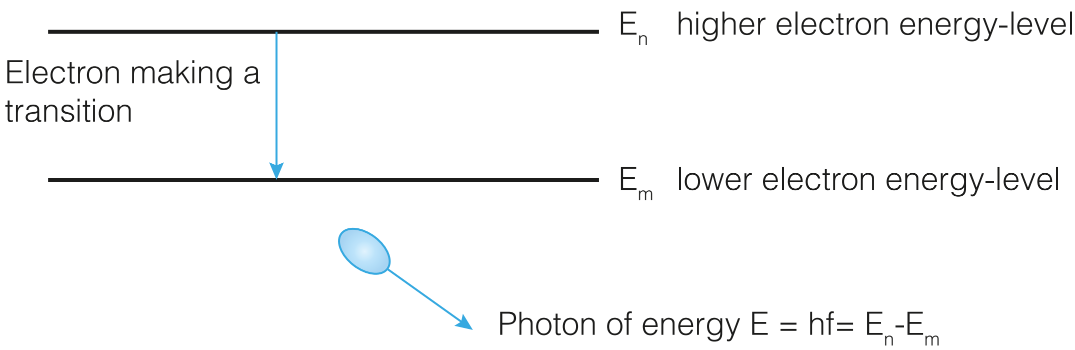


Figure 3.3.10

The concepts discussed for hydrogen can be extend to all atoms but the electron energy-levels are different and unique to the atom.

Extra understanding

Figure 3.3.11 shows the atom of a gas being excited by a photon. The energy of the incident photon must exactly match the energy difference between a lower and a higher electron-energy level. This is because the energy of a photon is discrete. If a photon is absorbed by an electron in an atom, all of its energy will be transferred to the electron. If the photon is not absorbed, it will be scattered elastically (without loss in energy).

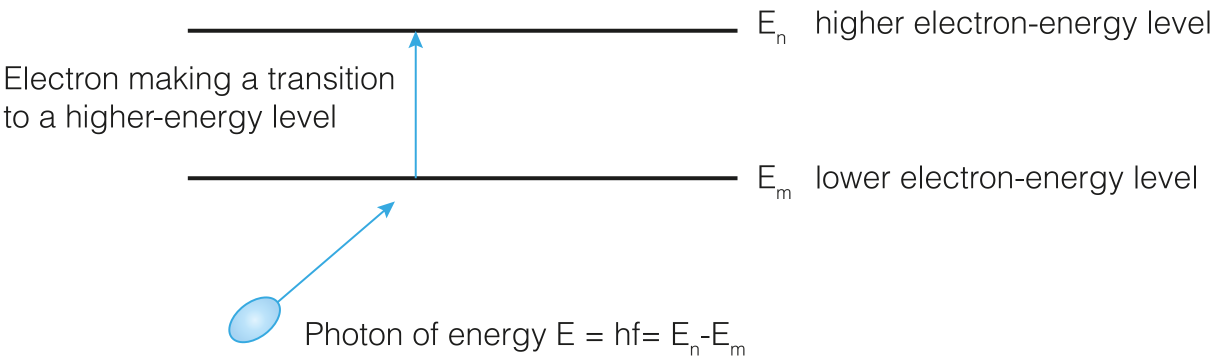


Figure 3.3.11

Figure 3.3.12 shows the atom of a gas being excited by an electron. The kinetic energy of the incident electron does not need to exactly match the energy difference between a lower and a higher electron-energy level. An electron is a particle and can share its energy. Providing the kinetic energy of the incident electron is equal to or greater than the energy difference between a lower and a higher electron-energy level, the electron in the atom can absorb all or part of the incident electron's kinetic energy. If the kinetic energy of the incident electron is greater than the energy needed to elevate the electron in the atom to a higher electron-energy level, the incident electron will be scattered with some kinetic energy. Using the law of conservation of energy, the kinetic energy of the scattered electron will be given by: $E_{k_{scattered}} = E_{k_{incident}} - (E_n - E_m)$

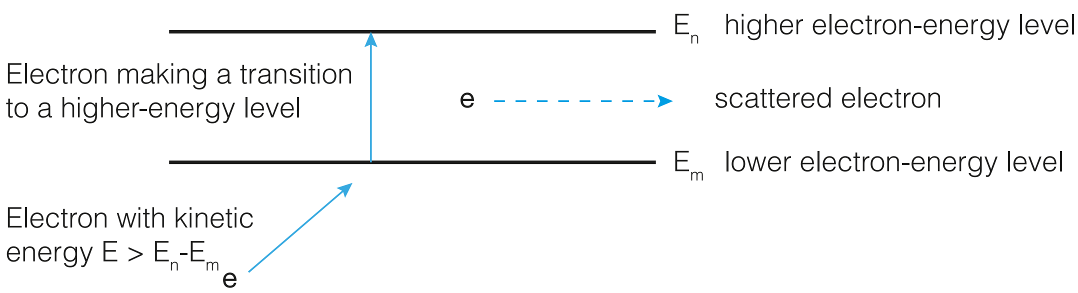


Figure 3.3.12

Key idea

The presence of discrete frequencies in the line emission spectra of atoms is evidence for the existence of discrete electron-energy levels.

When a large number of atoms in a sample of a gas are excited by heating the gas or bombarding the gas with light or electrons, the electrons in the gas atoms absorb this energy and are elevated to higher electron-energy levels. These excited states are short-lived and the electrons in the atoms make transitions (jumps) from higher-energy levels to lower-energy levels within the atom. The energy of the atom decreases and can be released as a photon or a series of photons. A descending electron emits a **discrete energy photon** of **energy exactly equal to the difference in energy between the two energy levels involved.** Since there are a number of different pathways back to the lower electron-energy levels, several photons may be emitted. Since photons have a discrete energy given by $E = hf$ it follows that the frequency of the emitted photons are also discrete and will appear as discrete spectral lines in the spectrum of the atom. That is, discrete frequencies in the spectra of atoms is evidence for the existence of discrete electron-energy levels in atoms.

Figure 3.3.13 shows some of the energy levels of an atom. Consider an electron making a transition from the third energy level to the second energy level in many of these atoms. This would release a photon with energy equal to the difference in energy between $n = 3$ and $n = 2$ and appear as a discrete frequency in the line emission spectrum for the atom. In this case this transition accounts for the red spectral line. Different transitions would result in the other spectral lines shown.

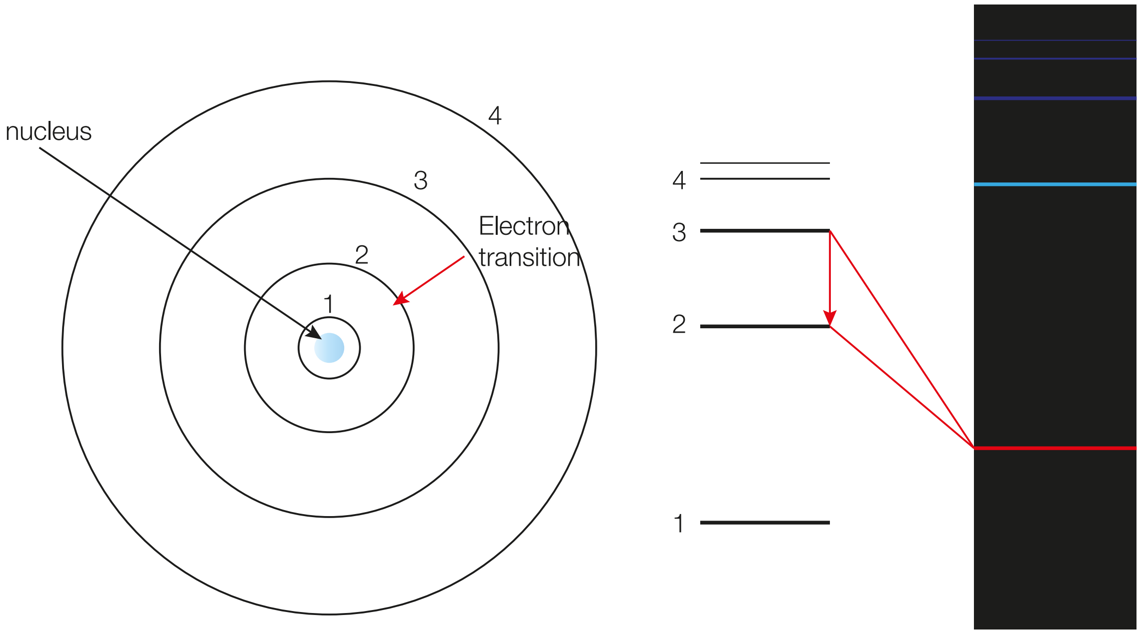
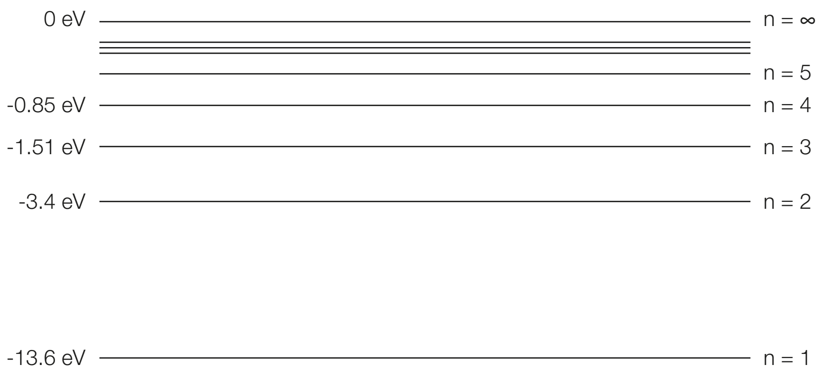


Figure 3.3.13

Worked examples

1. The diagram below shows the some of the electron energy-levels for a hydrogen atom. Assume the atoms are in their ground state.



(a) Calculate the ionisation energy for hydrogen in Joules.

$13.6 \times 1.60 \times 10^{-19} = 2.18 \times 10^{-18}\ J$

(b) A large number of hydrogen atoms are heated and excited to n = 4.

(i) State the number of spectral lines with discrete frequency that could be produced by these atoms as they return to the ground state.

6

(ii) State the energy of the photons released when electrons return to the ground state from $n = 4$ in a single transition.

$E_4 - E_1 = -0.85 - (-13.6) = 12.75 = 12.8\ eV$

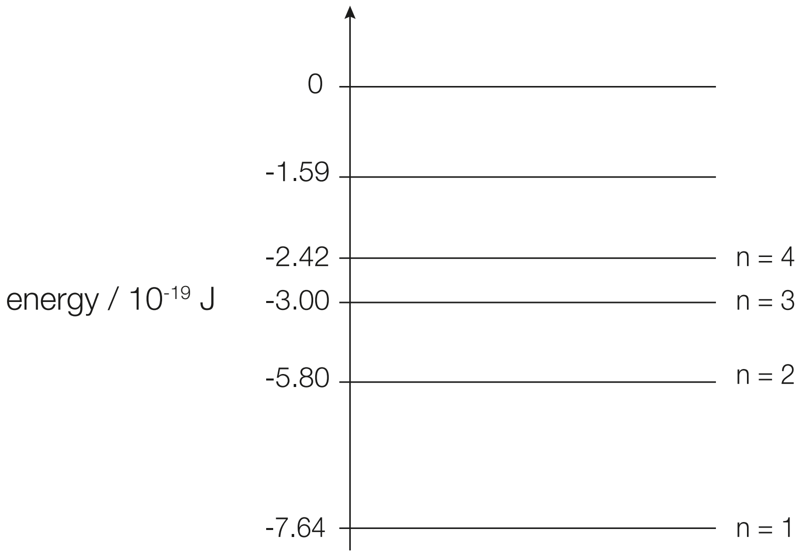
(iii) Calculate the frequency of the photons released when electrons return to the ground state from $n = 4$ in a single transition.

$$E = hf \therefore f = \frac{E}{h} = \frac{12.8 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 3.09 \times 10^{15}\ Hz$$

(iv) Calculate the wavelength of the photons released when electrons return to the ground state from $n = 4$ in a single transition.

$$v = f\lambda \therefore \lambda = \frac{v}{f} = \frac{3.00 \times 10^{8}}{3.09 \times 10^{15}} = 9.71 \times 10^{-8}\ m$$

2. Some of the electron energy-levels of the helium atom are shown below.



Singly ionised helium is sometimes compared to hydrogen as it only contains one electron.

(a) A large number of singly ionised helium atoms are bombarded with photons that have a frequency of 4.52×10^{14} Hz. Determine whether or not these photons will excite the singly ionised helium atoms in the ground state.

$E = hf = 6.63 \times 10^{-34} \times 4.52 \times 10^{14} = 3.00 \times 10^{-19}\ J$

The energy of the incident photons does not match the difference in energy between the ground state and a higher electron energy-level. The incident photons will not be absorbed by electrons in the helium atoms and they will not be excited.

(b) The photons in part (a) are replaced with electrons. The kinetic energy of the electrons is equal to the energy of the photons in part (a). When the electrons are used to bombard helium atoms, excitation occurs.

(i) Determine the energy transition that could occur when helium atoms are bombarded with these electrons.

The energy difference between $n = 1$ and $n = 2$ is 1.84×10^{-19} J and the energy difference between $n = 1$ and $n = 3$ is 4.64×10^{-19} J. Incident electrons with a kinetic energy of 3.00×10^{-19} J can elevate electrons in the ground state to the first excited state ($n = 2$). The incident electrons do not have enough energy to elevate the ground state electrons to the second excited state ($n = 3$).

The energy transition that could occur is $n = 1$ to $n = 2$.

(ii) Calculate the kinetic energy of the incident electrons as they emerge from the singly ionised helium atoms.

$E_k = 3.00 \times 10^{-19} - 1.84 \times 10^{-19} = 1.16 \times 10^{-19}\ J$

The Line emission spectra of atomic hydrogen

Recall that hydrogen is the simplest atom with only one electron. When excited electrons make transitions to lower electron energy-levels, several series of spectral lines are produced. This course requires that you are familiar with the series that involve electron transitions terminating at the three lowest-energy levels. These transitions produce photons in different regions of the electromagnetic spectrum.

Figure 3.3.14 illustrates the transitions that terminate at the three lowest-energy levels.

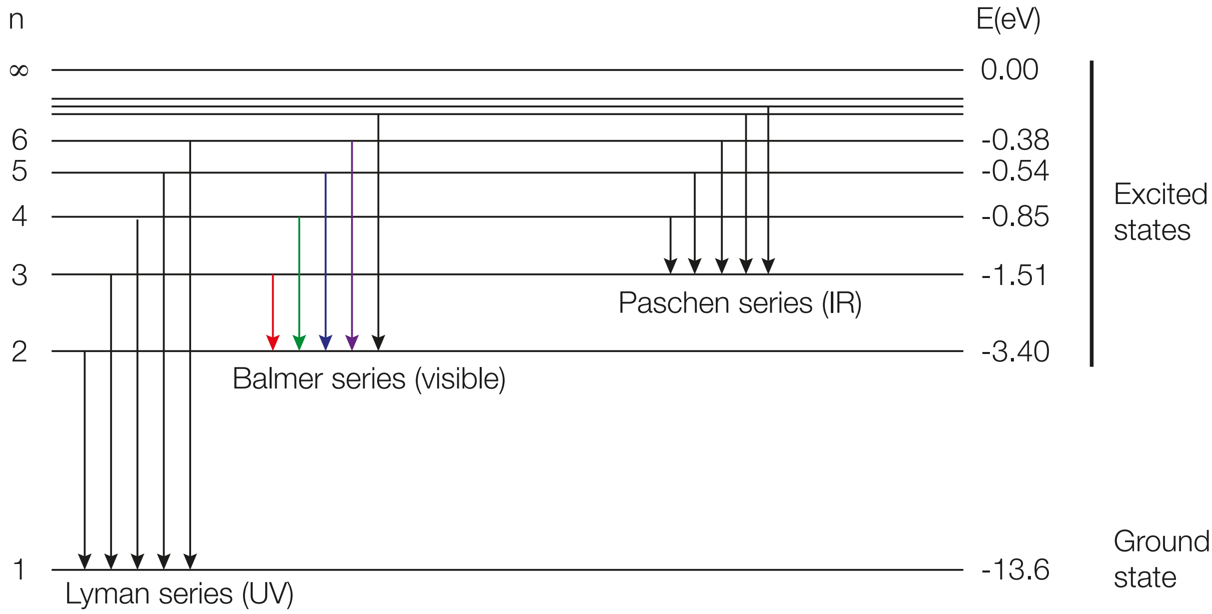
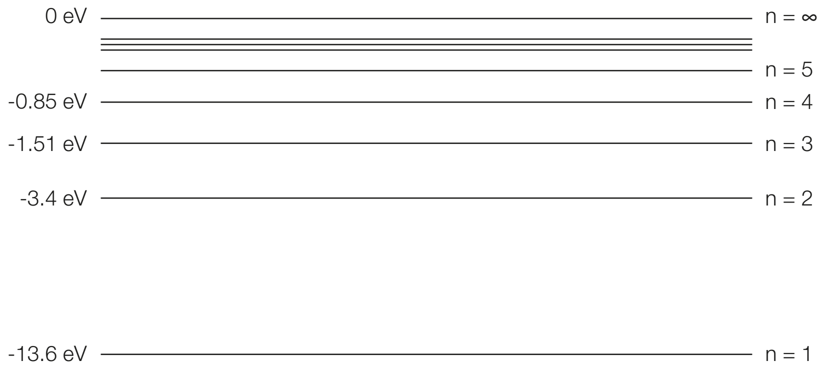


Figure 3.3.14

Transitions which end in the ground state ($n = 1$) produce photons with the greatest energy. This results in the highest frequency or lowest wavelength ($E = hf = \frac{hc}{\lambda}$). These transitions result in spectral lines in the **ultra-violet region** of the electromagnetic spectrum and constitute the Lyman series.

Transitions which end in the first excited state ($n = 2$), result in spectral lines in the **visible region** of the electromagnetic spectrum and constitute the Balmer series. Transitions which end in the second excited state ($n = 3$) produce the lowest energy photons and result in spectral lines in the **infra-red region**. This is called the Paschen series.

Worked example



Use the electron energy-level diagram for hydrogen to show that

(a) any transition that ends in the ground state produces photons with a wavelength in the ultra-violet region of the electromagnetic spectrum.

Consider the transition $n = 2$ to $n = 1$.

The energy of the emitted photon is 10.2 eV.

$$E = hf = \frac{hc}{\lambda} \therefore \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{10.2 \times 1.60 \times 10^{-19}} = 1.22 \times 10^{-7}\ m$$

This photon has a wavelength in the ultra-violet region of the electromagnetic spectrum. All other transitions that end at the ground state have a larger energy and therefore a smaller wavelength which will also be in the ultra-violet region of the electromagnetic spectrum.

(b) Calculate the largest frequency photon produced in the infra-red region of the electromagnetic spectrum.

The largest frequency photon in the infra-red region would result from the transition $n = \infty$ to $n = 3$.

The energy of the emitted photon is 1.51 eV.

$$E = hf \therefore f = \frac{E}{h} = \frac{1.51 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} 3.64 \times 10^{14}\ Hz$$

Production of characteristic X-rays in an X-ray tube.

Recall that X-rays are produced when electrons are accelerated to high kinetic energies and forced to collide with a target metal. Figure 3.3.15 illustrates a typical X-ray spectrum. The production of the continuous range of frequencies and the maximum frequency were discussed in Subtopic 3.2. The production of the characteristic peaks can now be discussed in terms of electron transitions within the atoms of the target metal.

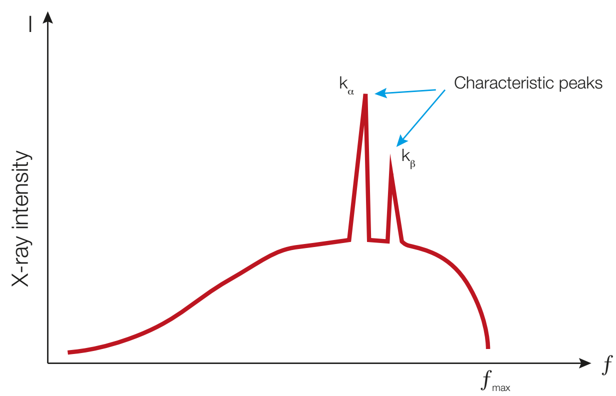


Figure 3.3.15

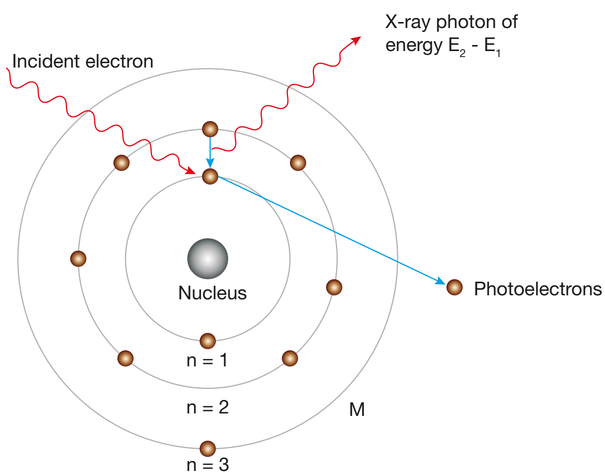


Figure 3.3.16

When incident electrons collide with the electrons in the lower electron energy-levels (inner shell electrons) of the atoms of the target metal, they can transfer energy to these inner shell electrons and knock them out of the atom. Electrons from higher electron energy-levels ‘drop down’ or make descending transitions to fill their place. This causes X-ray photons of energy equal to the energy difference between the electron energy-levels to be emitted. Since the energy difference is discrete, this results in X-ray photons of discrete energy and hence frequency. The intensity (number) of these discrete X-ray photons is greater than those of all other frequencies and produces the peaks in the X-ray spectrum graph.

Figure 3.3.16 shows an electron in the lowest energy level or ground state (*n* = *1*) being knocked out of a target metal atom by an incident electron. An electron in the second energy level has descended to take its place. The result is the emission of an X-ray photon of energy $E_2 - E_1$.

Extra understanding

The lowest energy state or level (*n* = *1*) is often referred to as the K-shell. The second and third energy levels (*n* = *2* and *n* = *3*) are referred to as the L and M-shells respectively as shown in Figure 3.3.16.

If a K-shell electron is knocked out by an incident electron, and an electron from the L-shell descends to take its place, we say that a K-alpha ($K\alpha$) X-ray photon is released.

If a K-shell electron is knocked out by an incident electron, and an electron from the M-shell descends to take its place, we say that a K-beta ($K\beta$) X-ray photon is released.

The intensity of the $K\alpha$ characteristic X-ray is typically greater than the $K\beta$ characteristic X-ray. This is because the electrons in the L-shell are closer to the K-shell, and more likely to fill the position of an electron ejected from the K-shell.

It should be noted that electrons can be knocked out of other energy levels. For example if an electron is knocked out of the second energy level (*n* = *2*), transitions made to *n* = *2* are referred to as L X-rays. A transition from *n* = *3* to *n* = *2* produces an L-alpha X-ray while a transition from *n* = *4* to *n* = *2* produces an L-beta X-ray and so on.

Since the electron energy-levels of an atom are characteristic of that atom, then it follows that the characteristic X-rays are unique to the target metal. That is, changing the target metal creates characteristic peaks at different frequencies.

The Line absorption spectra of atoms

Key ideas

1. When light with a continuous spectrum is incident on a gas of an element, discrete frequencies of light are absorbed, resulting in a line absorption spectrum.
2. The frequencies of the absorption lines are a subset of those in the line emission spectrum of the same element.

Figure 3.3.17 shows that when light with a continuous spectrum is incident on the gas of an element and viewed through a spectrometer, a line absorption spectrum results. The spectrum has the appearance of a continuous emission spectrum (a continuous spread of frequencies), crossed with a number of fine dark lines.

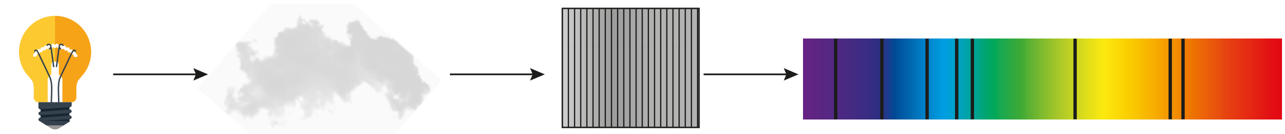


Figure 3.3.17

Helpful online resources

Use the following simulation to demonstrate line absorption spectra of various elements and relate them to emission spectra and energy level diagrams.

https://phet.colorado.edu/sims/cheerpj/hydrogen-atom/latest/hydrogen-atom.html?simulation=hydrogen-atom

Science as a human endeavour

An idea for a possible investigation is to explore how line absorption spectra can be used to make discoveries and reliable predictions about the composition and motion of stars.

Line absorption spectrum of an atom

The line absorption spectrum for hydrogen is shown in Figure 3.3.18.

Figure 3.3.18

When photons with a continuous range of frequencies are incident on the atoms of a gas, electrons in the atoms can **absorb** the **energy of the incident photons** that **exactly match the energy difference between a lower electron energy-level and one of the higher electron energy-levels**. The electrons are elevated to the higher electron energy-level while the incident photons are removed from the incoming light. Since the energy difference between the electron energy-levels is discrete, then these discrete frequencies appear as dark lines on a continuous spectrum and produce a line absorption spectrum.

Figure 3.3.19 shows some transitions that produce line absorption for hydrogen in the ground state.

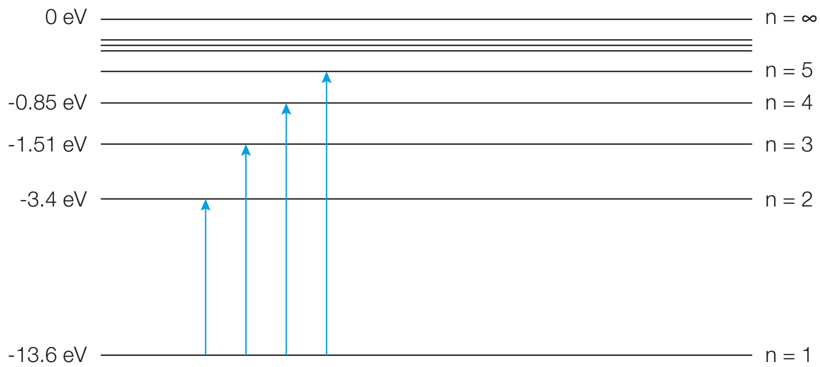


Figure 3.3.19

Since a line absorption spectrum involves the electrons being elevated to higher electron energy-levels, this means that the frequencies of the **line absorption spectrum are a subset of those in the line emission spectrum**. There is only one pathway upwards where as line emission involved electrons making transitions from higher to lower-energy levels. Since there is more than one possible transition down, more emission lines result than absorption.

For hydrogen, the line absorption spectrum consists of thin dark lines crossing a continuous spectrum. The dark lines of the line absorption spectrum exactly match the visible lines in the Balmer series of the line emission spectrum. This is shown in Figure 3.3.20.

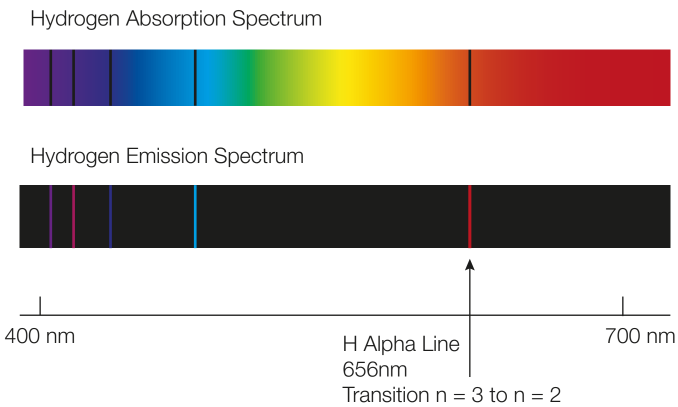


Figure 3.3.20

There are no absorption lines in the visible region for hydrogen at room temperature

When hydrogen gas is at room temperature, the electron is found in the ground state of the atom. Electron transitions from the ground state to any of the higher electron energy-levels fall in the ultra-violet region of the electromagnetic spectrum. This is because the electron in the ground state must absorb a photon of energy of 10.2 eV or more to be elevated. This means that there are no absorption lines are in the visible region of the electromagnetic spectrum when hydrogen gas is at room temperature.

Extra understanding

The transitions corresponding to the line absorption spectrum of hydrogen

In order to produce absorption lines in the visible region of the electromagnetic spectrum, hydrogen needs to be heated so that the electron is in the first excited state ($n = 2$) of the atom. When photons with a continuous range of frequencies are incident on the atoms of hydrogen gas in the first excited state, electrons in the hydrogen atoms can **absorb** the **energy of the incident photons** that **exactly match the energy** of incident photons in the Balmer series. These photons are removed from the incoming light and appear as dark lines in the same position on the absorption spectrum as the spectral lines in the line emission spectrum.

Figure 3.3.21 shows the electron for hydrogen in the first excited state. The four main transitions that produce absorption spectral lines are shown on the diagram.

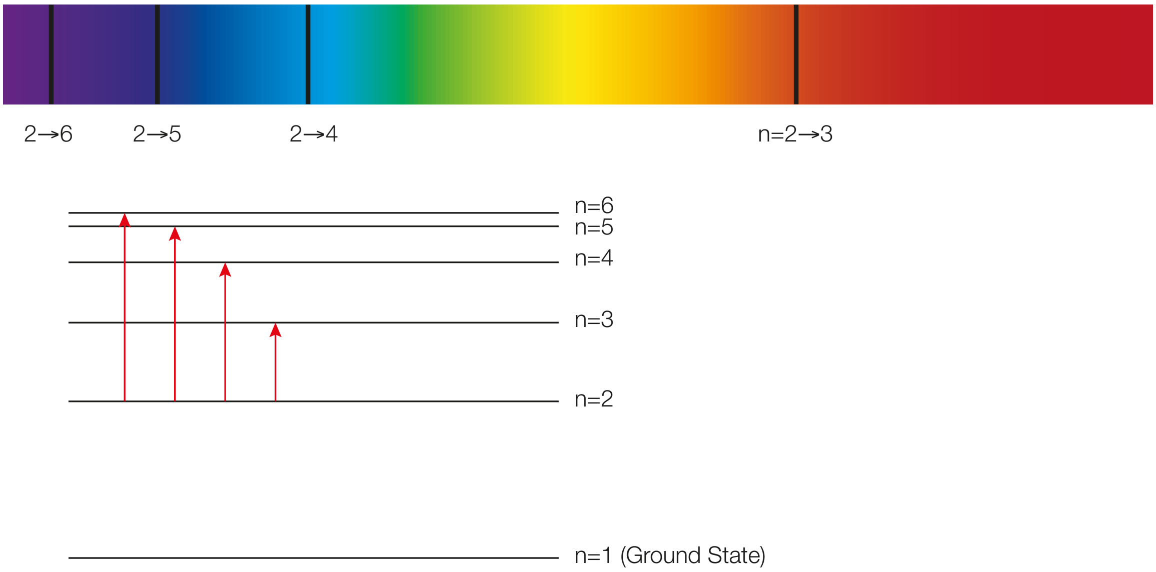


Figure 3.3.21

The presence of absorption lines (Fraunhofer lines) in the Sun's spectrum

Figure 3.3.22 below shows the absorption spectrum for the Sun.

Figure 3.3.22

The line absorption lines produced in the Sun's spectrum are called Fraunhofer lines after the German physicist Joseph von Fraunhofer. While studying the Sun's spectrum he observed 574 dark lines. Modern techniques have allowed astronomers to find thousands more.

The visible surface of the Sun is called the photosphere. The photosphere is about 500 kilometres in thickness. Although the Sun's atmosphere extends for thousands of kilometres beyond this, it is not visible from Earth.

The lower region of the photosphere is at a temperature of about 5500°C and produces white light with a continuous range of frequencies. As the light from the Sun passes through the photosphere the temperature drops by several thousand degrees. Electrons within the atoms of the cooler gases in the Sun's atmosphere absorb photons with energy equal to the energy gap between lower electron energy-levels and higher electron energy-levels. The electrons are elevated to higher electron energy-levels while the incident photons are removed from the incident light. This produces the dark lines in the Sun's spectrum as the intensity of the absorbed photons is too low to be observed.

The solar spectrum has allowed scientists to determine that the two major gases present in the Sun are hydrogen (about 92%) and helium (about 8%). However a small amount of carbon, nitrogen and oxygen are present as well as a very small amount of sodium, neon, iron, silicon, magnesium and sulfur.

Fluorescence

When a photon (usually high-energy) is absorbed by an atom, the atom is excited because electrons are elevated to higher electron energy-levels. Excited states are generally short-lived and the atom reverts to its previous electron energy-level. As the electrons descend, they can make smaller transitions. This results in a number of smaller-energy and hence frequency photons being emitted.

'Fluorescence' is the process where an atom absorbs a photon to reach an excited state, but then reverts to its previous electron energy-level emitting two or more photons with lower frequency and longer wavelength.

Figure 3.3.23 shows the process of fluorescence in hydrogen.

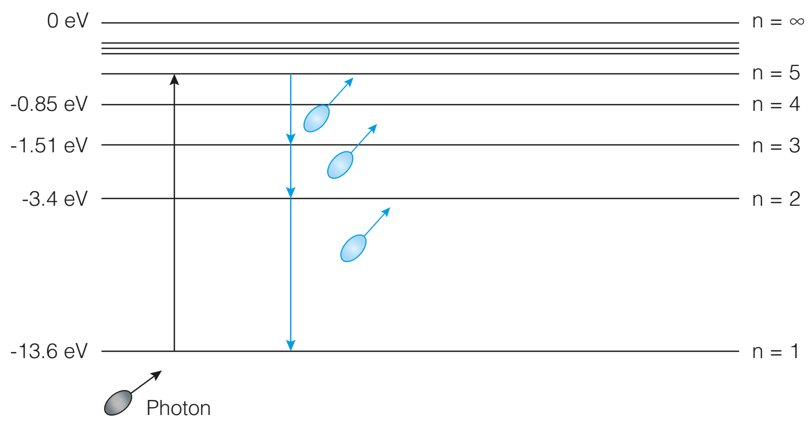


Figure 3.3.23

Science as a human endeavour

A possible idea is to investigate innovative applications of fluorescence, such as:

- biosensors
- currency security features
- forensic science
- mineralogy
- optical brighteners
- gene identification
- defect and leak detection.

Consider the advantages and disadvantages of their use in different contexts.

Spontaneous and stimulated emission

Key ideas

1. When an electron in an atom absorbs a photon and reaches a higher electron energy-level the atom is said to be in an excited state. Excited states are generally short-lived and the electron returns spontaneously to its previous electron energy-level often by emitting a series of lower-energy photons. This is known as 'spontaneous emission'.
2. When a photon is incident on an electron that has been raised to a higher electron energy-level, and the energy of the photon corresponds to a transition to a lower electron energy-level, then the photon can stimulate an electron to transition to the lower electron energy-level. This results in two identical photons; the original photon and a second photon that results from the transition. This is known as 'stimulated emission'.

A comparison of spontaneous and stimulated emission

Figure 3.3.24 illustrates the process of **spontaneous emission**.

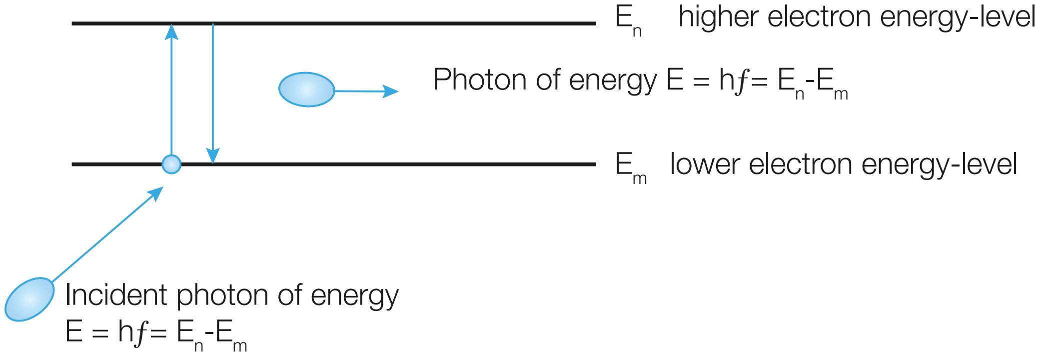


Figure 3.3.24

When a photon is absorbed by an electron in an atom, the electron is elevated to the higher electron energy-level but immediately returns to its previous electron energy-level releasing a series of lower energy photons. This process is called **spontaneous emission**. The emitted photons have random direction and phase.

Figure 3.3.25 illustrates the process of **stimulated emission**.

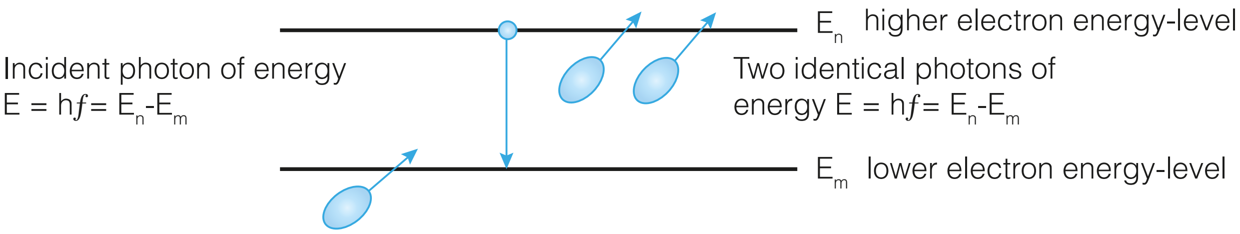


Figure 3.3.25

When a photon of energy corresponding to a transition from a higher electron energy-level to a lower electron energy-level is incident on an electron in the higher electron energy-level it can cause the electron to make a transition to the lower electron energy-level. This process is called **stimulated emission**.

Stimulated emission results in two identical photons, the incident photon and the emitted photon. The emitted photon is identical in energy, direction and phase to the incident photon.

The conditions required for stimulated emission to predominate over absorption when light is incident on a set of atoms.

A **metastable state** is an excited state in an atom that has a longer lifetime than other excited states. That is the electron remains in the electron energy-level longer than it normally would before making a transition to a lower electron energy-level state.

A **population inversion** is produced in a set of atoms whenever there are more atoms in a higher-energy state than in a lower-energy state. For a population inversion to occur, the higher-energy state must be a metastable state.

If stimulated emission is to predominate over absorption and hence spontaneous emission when light is incident on a set of atoms, a population inversion is required. This in turn requires a metastable state in the atom.

Helpful online resources

Illustrate the process of stimulated emission on an energy-level diagram or by using a simulation.

https://phet.colorado.edu/en/simulations/lasers

Lasers

Key ideas

1. Lasers use the process of stimulated emission to produce laser light.
2. The energy carried by a laser beam is concentrated in a small area and can travel efficiently over large distances, giving laser radiation a far greater potential to cause injury than light from other sources.

A laser is a device that produces an intense beam of coherent light with a single frequency. The name laser is an acronym for light amplification by stimulated emission of radiation. As the name implies, lasers use the process of stimulated emission to produce electromagnetic radiation. The first laser was produced in 1960.

All lasers work in a similar fashion. The basic components are a pump, a gain medium which produces the laser light and a cavity as shown in Figure 3.3.26 below.

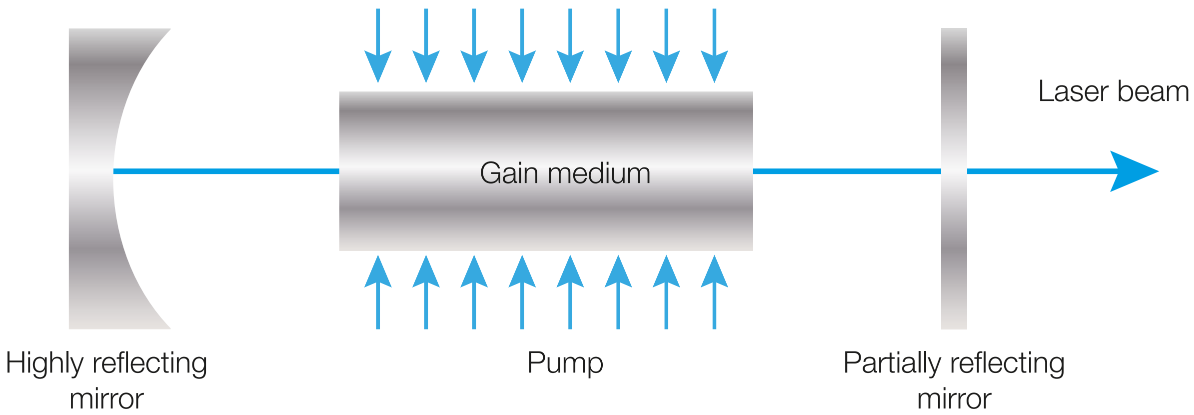


Figure 3.3.26

The production of coherent light in a laser.

The pump is an energy input. This can be in the form of a flash tube, electrical discharge, or another laser. The gain medium can be a solid, liquid, gas or even plasma. The gain medium absorbs energy, which excites the atoms of the gain medium to a higher-energy state. This is usually a metastable state. When there are more atoms in a higher electron energy-level than a lower electron energy-level, a population inversion is achieved. This is the condition for stimulated emission to predominate over absorption. Some photons are released by spontaneous emission and in turn cause electrons in higher electron energy-levels to return to the lower electron energy-levels via stimulated emission. This produces two identical photons, the incident photon and the emitted photon. These photons go on and cause stimulated emission in other atoms. A pair of mirrors at either end of the cavity will amplify the light by continually reflecting photons of light back and forth through the gain medium. One of the mirrors is partially reflective and allows a portion of the laser light to escape. This process produces monochromatic, coherent, and uni-directional light. Figure 3.3.27 illustrates this process.

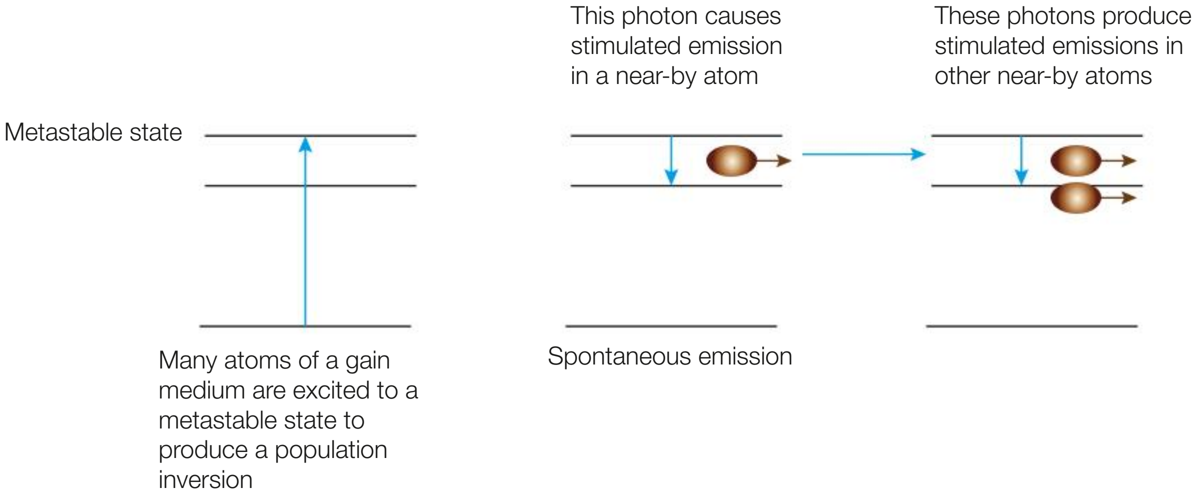


Figure 3.3.27

Figure 3.3.28 shows the structure of a ruby laser. A ruby crystal is placed inside a xenon flash lamp that acts as the pump. Figure 3.3.29 shows the structure of a neon-helium laser. An electrical discharge acts as the pump. Neon is the gain medium but helium atoms facilitate the process. Electrons cross the tube and collide with the helium atoms. This promotes them to the first excited state. The helium atoms then collide with the neon atoms, transferring this energy to the neon atoms which are in turn elevated to their metastable state ($n = 3$).

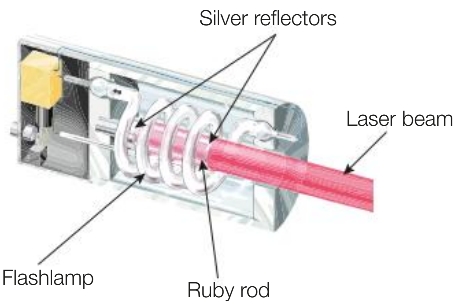


Figure 3.3.28

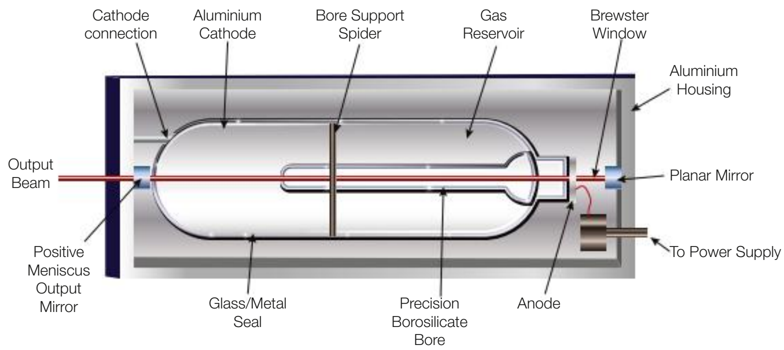


Figure 3.3.29

The useful properties of laser light

- High intensity beam
  The light emitted from a laser may be very intense and carry a large amount of energy.
- Monochromatic
  Laser light consists of one frequency or colour.
- Directional
  Laser light is directional. It is emitted as a narrow beam in one direction only. The beam does not spread and lose energy.
- Coherent
  Laser light is emitted with the same frequency, and maintains a constant phase relationship.

The safe handling of lasers.

Lasers are classified depending on the wavelength of light they produce and their maximum power output.

A laser should never be aimed at anyone's eye. The eye's lens will focus the beam on the retina, increasing its concentration. **Protective eye glasses** absorb laser light and reduce the intensity of the beam. Lasers can also burn skin. It is recommended that **protective gloves** are used especially when working with industrial lasers.

Working in areas with highly reflective surfaces should be avoided and care should be taken when moving lasers. It is not recommended that lasers be used in a dark environment because the pupil is larger.

Warning signs (in addition to those printed on the actual laser) should be used to warn people that a laser may be operating.

Most lasers operate using a large potential difference (pump). **Electrical safety** (e.g. avoiding contact with water, broken leads) also applies.

A high intensity laser beam can produce extremely high temperatures and a significant amount of thermal energy. The work area should be clean and flammable or explosive materials should be avoided. It is recommended that a **fire extinguisher** be available should a fire start.

Some industrial lasers produce fumes and vapours depending on their application. A well **maintained exhaust system** may be required.

Science as a human endeavour

Some possible ideas for investigation are:

1. Research the multiple lines of evidence and international communication that contributed to the development of the laser.
2. Explore the ways in which lasers can be used to solve problems.

   Examples include:

   - LADS for aircraft-based hydrographic surveying
   - laser cutting and welding
   - laser surgery
   - optical data storage
   - communication using fibre optics.
3. Research the international collaboration and communication of scientists from several countries, including Australia, in the joint project LIGO (Laser Interferometer Gravitational-Wave Observatory) to detect gravitational waves.

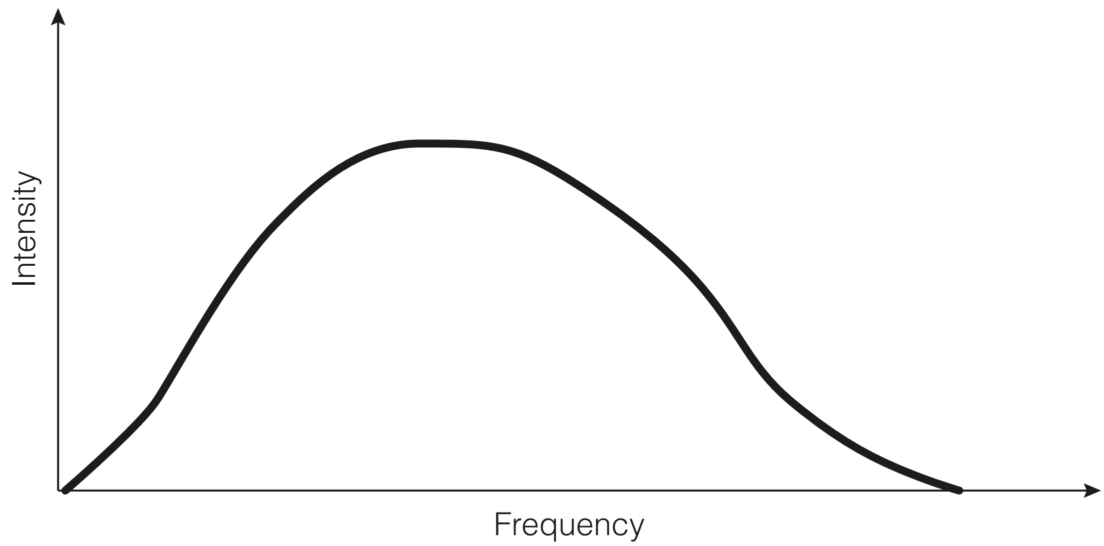
Exercises

1. A continuous spectrum is shown below. A continuous spectrum contains light emitted with a range of frequencies.

(a) Explain why the violet part of the continuous spectrum is produced by the emission of higher-energy photons while the red part of the continuous spectrum is produced by lower-energy photons.

(b) A heated filament globe produces a continuous spectrum. Describe the change in the spectrum of a filament globe as the temperature of the filament increases.

(c) A typical frequency distribution of the intensity of light produced by a heated filament globe is shown below.



On the diagram, draw a second frequency distribution that represents the change in the spectrum of a filament globe when the temperature of the filament is increased.

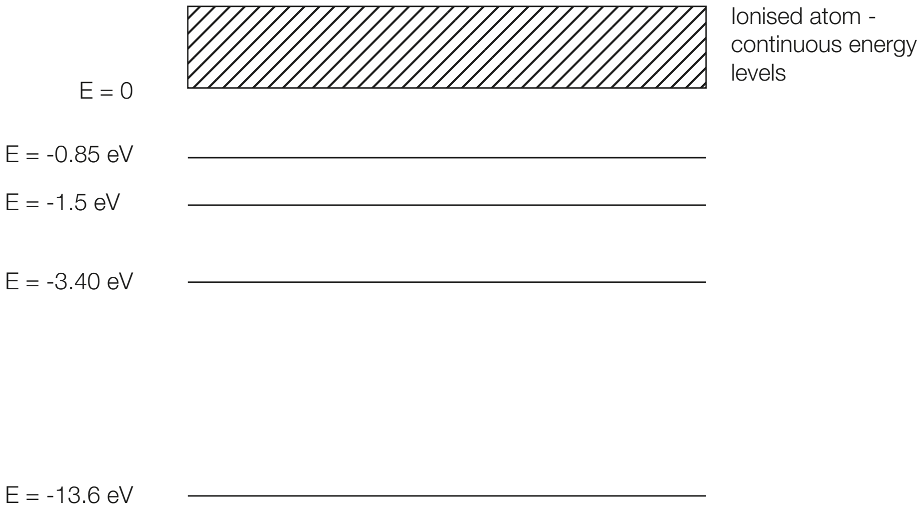
2. (a) The light produced by a heated filament globe is viewed using a spectrometer.

Describe the spectrum produced by the filament globe.

(b) The photograph below shows the same room. Photograph A shows the room being illuminated with filament globes that reach a temperature of 3000 K while photograph B shows the room being illuminated with filament globes that reach a temperature of 6000 K.

Explain how the spectrum of the filament globe changes with increased temperature and hence explain why the light in room B appears whiter.

3. Some of the electron energy-levels of hydrogen are shown below.



(a) State the ionisation energy for hydrogen.

(b) Hydrogen atoms in the ground state are bombarded with photons and raised to $n = 4$.

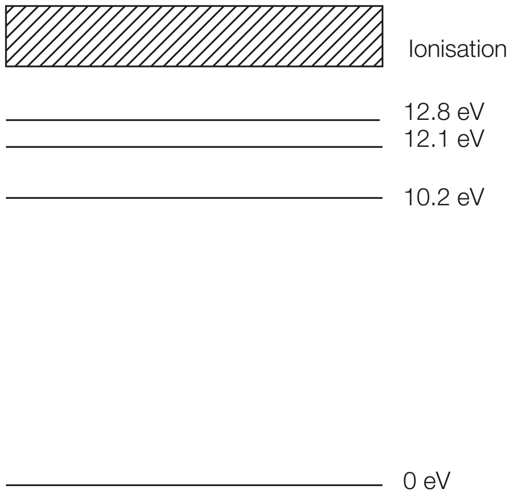
(i) State the energy of the photons needed to raise the hydrogen atoms to $n = 4$.

(ii) As the electrons descend to the ground state fluorescence occurs.

On the diagram, draw one set of energy transitions that show the process of fluorescence.

(iii) Calculate the smallest frequency photon that could be produced as the hydrogen atoms undergo fluorescence from $n = 4$.

4. The diagram below shows some of the electron energy-levels for hydrogen.



(a) Photons of energy 12.0 eV are used to bombard atoms of hydrogen in the ground state.

Explain why the hydrogen atoms cannot be raised to an excited state by photons of energy 12.0 eV.

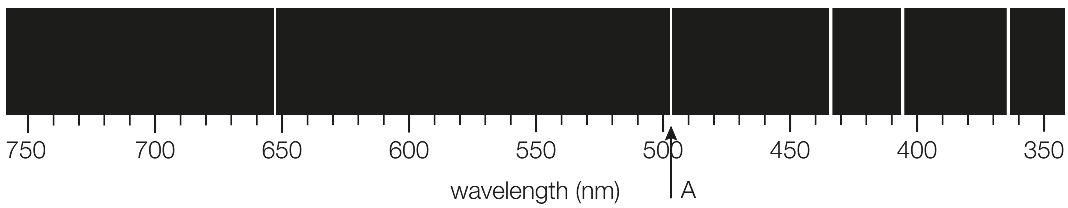
(b) Electrons of energy 12.0 eV are used to bombard atoms of hydrogen in the ground state.

In this situation, it is found that the hydrogen atoms are excited and photons of energy 10.2 eV are released.

(i) Explain why the hydrogen atoms are excited by the incident electrons and why photons of energy 10.2 eV are released.

(ii) Calculate the frequency of the photons emitted with an energy of 10.2 eV.

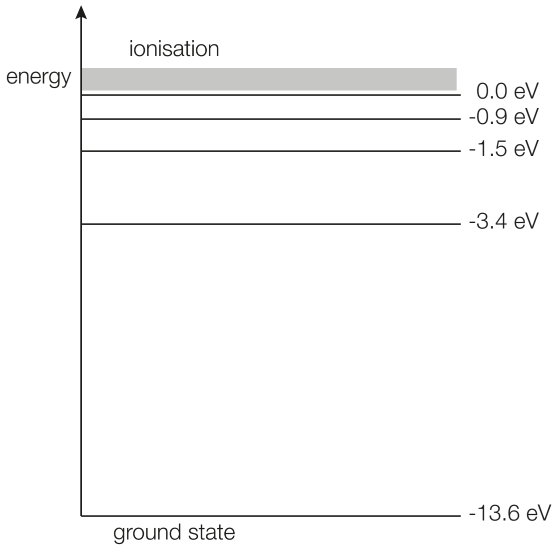
5. The diagram below shows part of the visible line emission spectrum for hydrogen.



(a) Estimate the wavelength of the spectral line marked A.

(b) Calculate the energy of the photons, in eV, that produce the spectral line marked A.

(c) The diagram below shows some of the electron energy-levels of hydrogen.



On the diagram, draw the transition that corresponds to the spectral line marked A.

6. (a) Briefly describe how the line emission spectrum of a pure element can be observed in a laboratory.

....

....

....

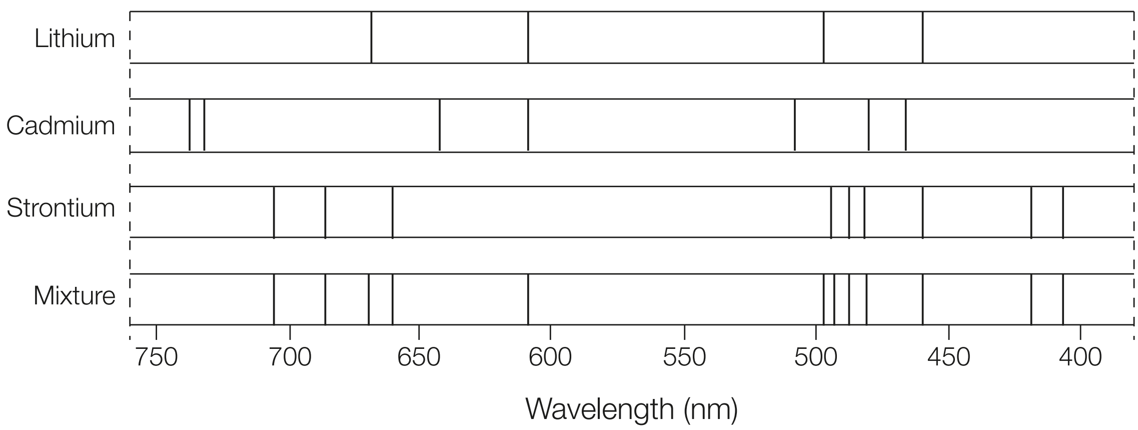
....

(b) Describe the general appearance of the line emission spectra of pure elements.

....

....

(c) The visible line emission spectra of several elements are shown below as well as that of a mixture.



(i) State the two elements that are present in the mixture.

....

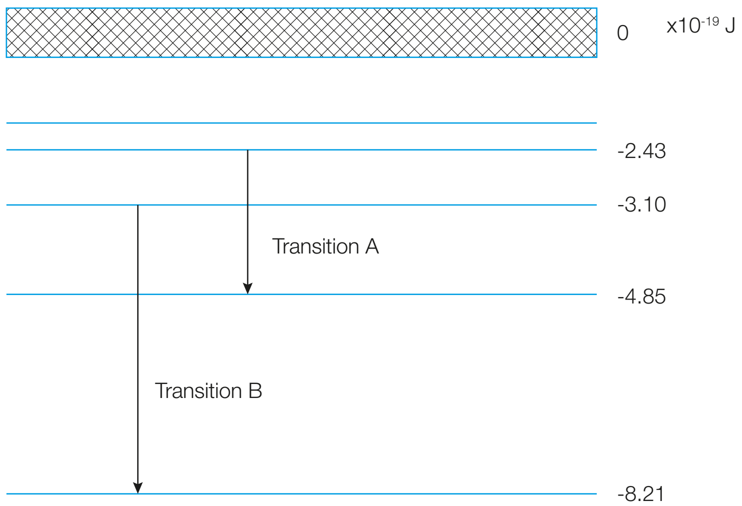
(ii) Explain your answer to (c)(i).

....

....

....

7. The diagram below shows some of the electron energy-levels in atomic sodium.



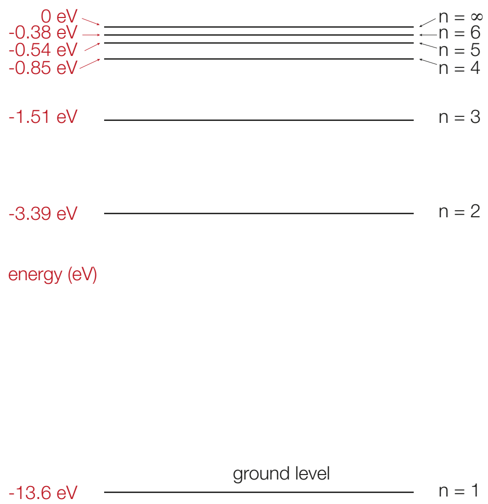
Two transitions, A and B, are shown on the energy level diagram above.

(a) State with reason, which of the two transitions, A or B, produces photons with a larger wavelength.

(b) (i) Define the term ‘ionisation energy’.

(ii) State the ionisation energy for sodium.

8. The diagram below shows some of the electron energy-levels of hydrogen.



One of the visible spectral lines of hydrogen has a wavelength of 4.36×10^{-7} m.

(a) Calculate the energy, in eV, of a photon that has a wavelength of 4.36×10^{-7} m.

(b) On the diagram above, draw the transition within hydrogen atoms that would cause the emission of photons with a wavelength of 4.36×10^{-7} m.

Label this transition E.

(c) On the diagram above, draw the transition within hydrogen atoms that would cause the absorption of photons with a wavelength of 4.36×10^{-7} m.

Label this transition A.

(d) Some of the spectral lines produced by hydrogen do not lie in the visible region. On the diagram, draw

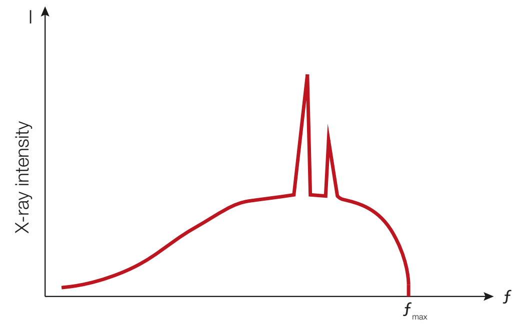
(i) the transition that would cause the emission of the lowest- energy photons in the ultraviolet region of the electromagnetic spectrum.

Label this transition LU.

(ii) the transition that would cause the emission of the highest- energy photons in the infra-red region of the electromagnetic spectrum.

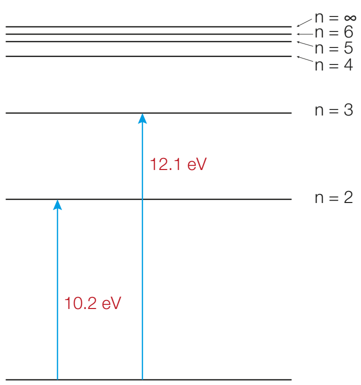
Label this transition HI.

9. A typical X-ray intensity graph for the X-rays produced in an X-ray tube is shown below. One of the main features are the peaks that represent characteristic X-rays.



Explain the production of characteristic X-rays in an X-ray tube.

10. The electron energy-level diagram below shows some of the electron energy-levels of the hydrogen atom. Two transitions within the hydrogen atom are shown.



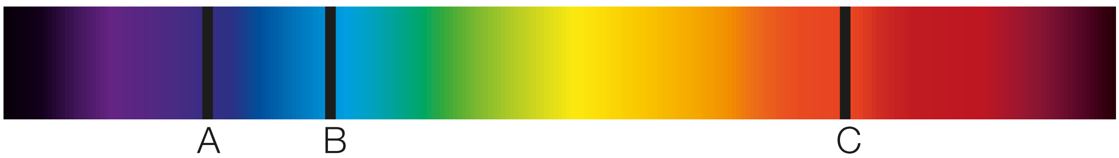
(a) Calculate the wavelength of the photons that would cause the transition from $n = 1$ to $n = 2$.

(b) State the region of the electromagnetic spectrum from which the photons that cause the transition from $n = 1$ to $n = 2$ belong.

(c) With reason, explain the region of the electromagnetic spectrum from which the photons that cause the transition from $n = 1$ to $n = 3$ must belong.

(d) Use your answers to parts (b) and (c), to explain why there are no absorption lines in the visible region of the electromagnetic spectrum for hydrogen at room temperature.

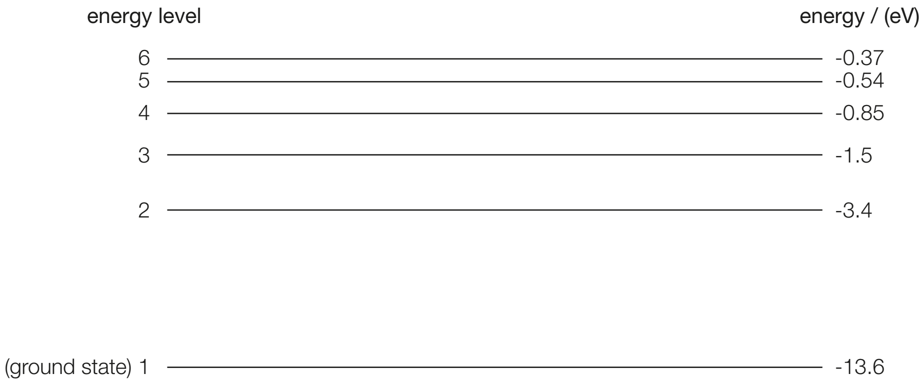
11. The line absorption spectrum of hydrogen consists of many thin lines crossing a continuous spectrum.



Three visible absorption lines A, B and C are labelled on the diagram.

Spectral line	Transition	Wavelength (nm)
A	$n = 2$ to $n = 5$	435
B	....	488
C	....	654

Some of the electron energy-levels of hydrogen are shown below.

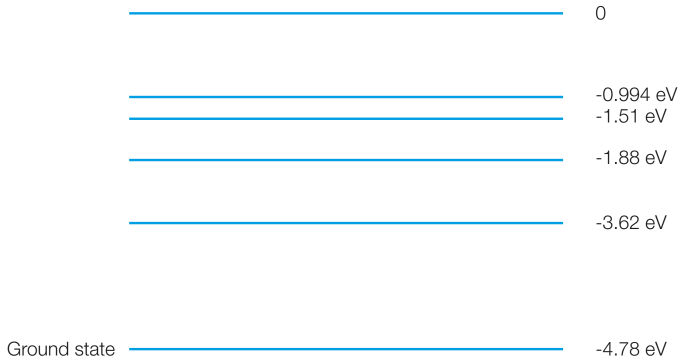


(a) Confirm that the spectral line labelled A results from a transition from $n = 2$ to $n = 5$.

(b) Complete the table above by filling in the middle column with the transition within hydrogen that produces the spectral lines labelled B and C.

12. The element helium was first identified from the absorption spectrum of the Sun.

The diagram below represents some of the electron energy-levels of helium.



(a) Account for the presence of absorption lines in the Sun's spectrum.

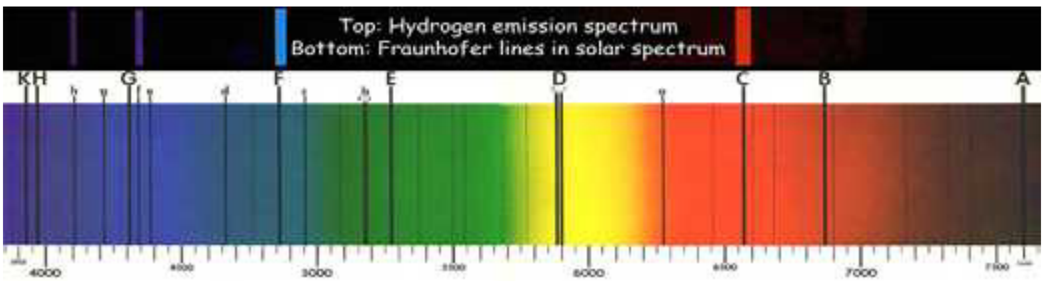
(b) One of the wavelengths in the absorption spectrum of helium occurs at 588 nm.

(i) Calculate the energy, in eV, of the photons absorbed to produce a spectral line with a wavelength of 588 nm.

(ii) Identify the transition involved in the absorption of a photon with a wavelength of 588 nm.

(iii) Mark the transition involved in the absorption of a photon with a wavelength of 588 nm on the diagram above.

13. The diagram below illustrates the line emission spectrum for hydrogen and directly below it, the Fraunhofer absorption lines for the Sun.



Explain why the absorption spectrum for the Sun demonstrates the presence of hydrogen in the solar atmosphere.

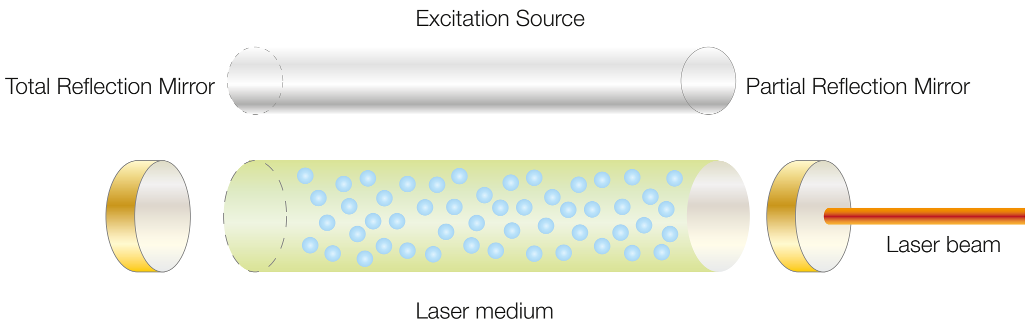
14. The proteins in the exoskeleton of the scorpion shown in the diagram below will cause it to glow a blue colour when it is illuminated with ultra-violet light. This makes the scorpion visible and easy to find even when it is hidden amongst dark rocks.

Explain how the process of fluorescence enables the scorpion to glow a blue colour when illuminated with ultra-violet light.

15. (a) Compare the process of spontaneous emission with that of stimulated emission.

(b) Describe the conditions required for stimulated emission to predominate over absorption when light is incident on a set of atoms.

16. The basic components of a helium-neon laser are shown below.



Helium atoms are excited by a pump and in turn elevate neon atoms to their metastable state, $n = 3$. Coherent laser light is produced as electrons are stimulated to return to $n = 2$.

(a) Explain the term 'metastable state'.

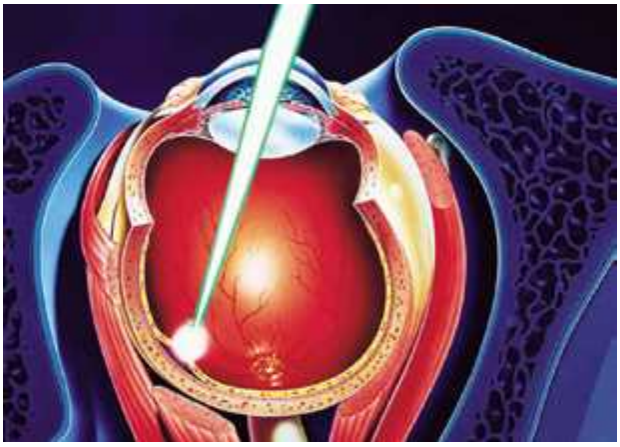
(b) Explain why a metastable state is needed for a population inversion.

(c) Explain how stimulated emission can produce monochromatic coherent light in the neon-helium laser.

(d) Some of the electron energy-levels for neon are shown in the diagram below.

Calculate the frequency of the laser light produced.

17. The diagram below shows how a laser can be used to treat a detached retina in a patient's eye. The scar tissue formed by the laser helps hold the retina in place. Without such surgery, the patient will lose their vision.



State and explain one property of a laser that makes it useful in such surgery.

18. The photograph below shows a robotic arm using a high intensity laser to weld the roof seam of a vehicle.

Such an intense laser beam has the capacity to cause injury.

(a) Describe how the manufacturer of the vehicle has increased the safety of its workers by using a robotic arm to perform the welding.

(b) State one safety precaution that workers should take if they are working within the vicinity of the robotic arm.

19. Science as a human endeavour activity – Structure of the atom

The following text forms part of an article 'Real-Time Tissue Diagnostics with Miniature Photonics Spectrometers'.

Date: 2 May 2018

Source: https://www.novuslight.com/real-time-tissue-diagnostics-with-miniature-photonics-spectrometers_N8051.html#atop

Today physicians are still severely hampered by the lack of precision of the needle tip location during a biopsy. Looking at lung cancers, 25% of the diagnoses suffer a false negative outcome through traditional biopsy methods. In the future, this can be avoided: for the first time, the European project InSPECT developed miniature spectrometers with integrated light sources enabling guided sensing.

By integrating an optical fiber inside a biopsy needle, cancerous and non-cancerous tissue can be illuminated and differentiated by spectral analysis. Backscattered light is collected and led to a spectrometer that identifies spectral fingerprints like water, fat and hemoglobin. The different concentrations, collected by a second optical fiber, give real-time feedback to the physician during the medical intervention.

This method for tissue detection allows a fast and accurate diagnoses that can significantly accelerate the start of the cancer treatment, vital to increase the survival rate and recovery of each patient.

The European-funded H2020 project InSPECT is coordinated by Philips in The Netherlands, bringing together eight partners representing expertise in optics, photonics, device manufacturing, sensors and medical technology: B-PHOT Vrije Universiteit Brussel (Brussels, Belgium), Xenics (Leuven, Belgium), Anteryon (Eindhoven, The Netherlands), Lionix International (Enschede, The Netherlands), Aifotec (Meiningen, Germany), Avantes (Apeldoorn, The Netherlands), and the Fraunhofer Institute for Reliability and Microintegration (Berlin, Germany). The project is funded within the Research and Innovation Action of Horizon2020, and in collaboration with Photonics21.

(a) Describe one key concept of science as a human endeavour that is evident in the above text about miniature spectrometers.

(b) Describe one other example of how miniature spectrometers demonstrate science as a human endeavour.

3.4 Standard Model

Science understanding

1. The Standard Model suggests that there are three fundamental types of particles: gauge bosons, leptons, and quarks.
2. The Standard Model identifies four fundamental forces: electromagnetic, weak nuclear, strong nuclear, and gravitational.
3. Gauge bosons are particles which mediate the four fundamental forces.
4. Photons are the gauge bosons for electromagnetic forces; W or Z particles are the gauge bosons for weak nuclear forces; and gluons are the gauge bosons for strong nuclear forces.
5. The gauge boson for gravitational forces, the graviton, is still to be discovered.
   - Describe the electromagnetic, weak nuclear, and strong nuclear forces in terms of exchange gauge bosons.
   - Solve problems involving the fundamental forces and gauge bosons.
6. Leptons are particles that are not affected by the strong nuclear force. There are six types of leptons – electron, electron- neutrino, muon, muon-neutrino, tau, and tau-neutrino.

   The electron, muon, and tau are negatively charged. Neutrinos do not have charge.
7. Quarks are fractionally charged particles that are affected by all of the fundamental forces.
8. Quarks combine to form composite particles and are never directly observed or found in isolation.
9. There are six types of quark, with different properties, such as mass and charge. Each quark has a charge of either $+\frac{2}{3}e$ or $-\frac{1}{3}e$.

Quark	Symbol	Charge (*e*)
Up	u	$\frac{2}{3}$
Down	d	$-\frac{1}{3}$
Strange	s	$-\frac{1}{3}$
Charm	c	$\frac{2}{3}$
Top	t	$\frac{2}{3}$
Bottom	b	$-\frac{1}{3}$

10. Every particle has an antimatter equivalent. A key difference between a particle and its antimatter equivalent is that their charges are equal magnitude but opposite sign.
    - Identify which types of fundamental particles are affected by each type of fundamental force.
    - Identify the charges of each type of fundamental particle.
    - Describe the properties of a specified antimatter particle.
    - Determine the charge of a specified antimatter particle.
11. All composite matter particles, such as atoms, are thought to be combinations of quarks, antiquarks, and leptons.

    Baryons are composite particles that consist of a combination of three quarks.

    Mesons are composite particles that consist of a combination of one quark and one antiquark.
    - Describe how protons, neutrons ,and other baryons consists of different combinations of quarks.
    - Determine the charge of a baryon, given its quark composition.
    - Describe how pions and other mesons consist of different combinations of quarks and antiquarks.
    - Determine the charge of a meson, given its quark-antiquark composition.
12. Each particle is assigned a lepton number and a baryon number.
13. Lepton numbers can be one of three types:
    - electronic lepton number, L_e
    - muonic lepton number, L_μ
    - tauonic lepton number, L_τ
14. The lepton number, regardless of type, for a lepton is 1. Antileptons have a lepton number –1. All other particles have a lepton number of 0.
15. The baryon number of a quark is $\frac{1}{3}$. Baryons have a baryon number of 1. Antiquarks have a baryon number of $-\frac{1}{3}$. Antibaryons have a baryon number of –1. All other fundamental particles have a baryon number of 0.

16. The laws of the conservation of baryon number, charge, and lepton number determine the types of reactions that can occur between particles.
    - Use the conservation laws to determine the baryon number, lepton number, and charge of particles in reactions.
    - Given a reaction between particles, demonstrate that baryon number, lepton number, and charge are conserved.
17. When a particle and its antiparticle collide, they annihilate, releasing energy according to the mass–energy equivalence formula: $E = \Delta mc^2$.
    - Use the mass–energy equivalence relation to determine the energy released when a particle and antiparticle annihilate.

This subtopic uses the concept of the nucleus developed in Stage 1, Subtopics 6.1: The Nucleus, and 6.2: Radioactive decay.

The Standard Model

All matter is made up of atoms. Up to this point of this course, the atom has been described as having a tiny dense nucleus consisting of protons and neutrons and electrons that orbit the nucleus.

Sub-atomic particles are particles that are smaller than the atom. Particle accelerators are able to accelerate protons and electrons to high kinetic energies. Through collisions, new particles have been discovered. The Standard Model was developed in response to the discovery of these new particles.

The Standard Model describes the particles that form the fundamental building blocks of matter and the forces that govern their behaviour.

Fundamental or elementary particles have no internal structure, that is, they are not made up of smaller constituents.

3

The Standard Model suggests that there are three fundamental types of particles:

1. Bosons (Exchange particles)
2. Leptons (six types plus their antiparticles)
3. Quarks (six types plus their antiparticles)

Helpful online resources

Learn more about The Standard Model.

https://particleadventure.org/standard-model.html

Extra understanding

CERN or European Council for Nuclear Research was founded in 1952 and its name was derived from "Conseil Européen pour la Recherche Nucléaire", French for European Council for Nuclear Research. It is located near Geneva in Switzerland. CERN was established in order for scientists to better understand the structure of the atom.

It is the home of the Large Hadron Collider (LHC). The LHC is the largest and most powerful particle accelerator in the world. It took decades to design and build with thousands of scientists, engineers and technicians contributing from 111 countries. The LHC consists of a 27 kilometre circular ring of superconducting magnets built about 100 m below the ground. Two beams of high-energy particles (travelling close to the speed of light), travel in opposite directions and are forced to collide. Such collisions have enabled scientists to discover many sub-atomic particles.

Figure 3.4.1 shows an aerial photograph of the LHC. ATLAS, CMS, Alice, and LHCb represent the intersection points at which the two beams are made to collide in most experiments.

Figure 3.4.1

Figure 3.4.2 shows part of the beam pipe through which particles are accelerated.

Figure 3.4.2

Helpful online resources

A virtual tour and more information about the LHC and the type of experiments being performed can be found at the home of CERN:

https://home.cern/about

Fundamental forces

The Standard Model identifies four fundamental forces.

1. Electromagnetic
2. Weak nuclear
3. Strong nuclear
4. Gravitational

These four forces are responsible for all interactions but have different characteristics. For instance, the electromagnetic force acts between two protons causing them to repel one another. This force has an infinite range but its magnitude quickly decreases with distance ($F\alpha \frac{1}{r^2}$). The strong nuclear force is responsible for interaction between particles in the nucleus. The nuclear force is very strong but operates over a very short range. Neutrinos emitted during beta decay do not have a charge. They are not affected by the electromagnetic force nor the strong nuclear force. When neutrinos interact, they interact via the weak nuclear force. The gravitational force acts between objects with mass. Although its range is infinite ($F\alpha \frac{1}{r^2}$) it decreases quickly with distance. It is a weak force and therefore generally ignored in particle physics.

Gauge bosons

The Standard Model explains how each of the fundamental forces arise through the exchange of particles called **exchange particles** or **gauge bosons**. This was first proposed in 1935 by a Japanese physicist called Hideki Yakawa. We say that the exchange particles or gauge bosons mediate, transmit or carry the force. Each of the four fundamental forces has a different gauge boson that is responsible for the force.

Exchange particles (or gauge bosons) are elementary particles that mediate the fundamental forces.

The table below summarises the four fundamental forces and their corresponding gauge boson.

Force	Gauge boson
Electromagnetic	photon
Weak nuclear	W, Z
Strong nuclear	gluon
Gravitational	graviton

The gauge boson for gravitational forces, the graviton, is still to be discovered.

Describing the electromagnetic, weak nuclear, and strong nuclear forces in terms of gauge bosons.

To image how a force can be exerted between two particles by the exchange of another particle, an analogy using two ice skaters exchanging pillows can be used.

Figure 3.4.3

Figure 3.4.4

Imaging the two skaters facing each other and about to exchange pillows. Two situations are possible.

1. The skaters can grab the pillows from one another and at the same time pull each other closer. This is analogous to an attractive force as seen in Figure 3.4.3.
2. The two skaters can throw the pillows towards one another and at the same time move backwards. This is analogous to a repulsive force as seen in Figure 3.4.4.

Electromagnetic forces

Electromagnetic forces of attraction and repulsion between charges can be explained through the exchange of **photons**. That is, the photon is the exchange particle or gauge boson for the electromagnetic force. Recall that a photon has an energy of $E = hf = \frac{hc}{\lambda}$.

Consider two electrons repelling one another. A photon emitted by one electron will cause it to recoil as it transfers momentum and energy to the other electron. The second electron also emits a photon and recoils. More energetic photons are exchanged when the electrons are closer to one another. This explains why the electromagnetic force is stronger when the electrons are closer. Conversely, less energetic photons are exchanged when the electrons are further apart and the electromagnetic force is weaker. The exchange occurs almost instantaneously. The photons are not visible and are called virtual photons.

Weak nuclear forces

When neutrinos interact, the mediators of the weak nuclear force that acts are called **exchange bosons W and Z** (W^+, W^- and Z^0). Although the existence of these particles was predicted in the late 1960 they were not discovered until 1983. The W^+ boson has a charge of +1e, the W^- boson has a charge of -1e and the Z^0 is electrically neutral.

Strong nuclear forces

The strong nuclear force explains why the nucleons in the nucleus of an atom are held together despite the presence of a strong repulsive force between positive protons. The existence of a strong nuclear force was first proposed by Yakawa.

The strong nuclear force is due to the exchange of particles called **pions** (short for pi mesons). Nucleons continually exchange mesons with neighbouring nucleons. Yukawa's idea was that the mesons mediated the strong nuclear force. Pions were discovered in 1947 and are present in cosmic rays.

The force between quarks (discussed later in the chapter) is also the strong nuclear force. The exchange particle however, is not the pion but rather the gluon.

In general, the force between nucleons is due to the exchange of pions but it is the **gluons** that are exchanging between the pions and nucleons that mediate the force.

Gravitational forces

The carrier of the gravitational force is called the **graviton**. It has not yet been discovered. This force is an inverse square force with an infinite range.

Antiparticles

For every matter particle there exists an antimatter equivalent referred to as an antiparticle. Antiparticles have the same properties as the particle (mass, spin and lifespan) but their charge and quantum numbers have opposite signs. In this course, it is sufficient to know that that a key difference between a particle and its antimatter equivalent is that their charges have equal magnitude but opposite sign.

The antiparticle of a charged particle is given the same symbol with the opposite sign. The antiparticle of an uncharged particle is given the same symbol with a bar drawn above its symbol.

Some antiparticles that are familiar to you are produced in beta decay. In beta minus decay, and electron (e^-) is emitted along with an antineutrino ($\bar{v}$). In beta-plus decay the positron (e^+) is emitted along with a neutrino (v). The positron is the antiparticle of the electron. It has the same symbol as the electron but is assigned an opposite charge. This is because it has the same properties, including mass, as the electron but its charge, although the same in magnitude, is positive unlike the electron which has a negative charge. The antineutrino is the antiparticle of the neutrino. Since neutrinos do not have a charge, the symbol for the antineutrino is the same as that of the neutrino but it has a bar over the symbol. In the next section, we will see that electrons, positrons and neutrinos are classified as leptons.

Leptons

Leptons are fundamental or **elementary** particles that can be found on their own, that is, they are not trapped inside other particles.

The electron (e^-), a familiar particle, is an example of a lepton. As discussed earlier it is ejected from the nucleus along with an anti electron-neutrino ($\bar{v}_e$) during beta-minus decay.

Leptons consist of the electron (e), tau (τ) and muon (μ), and the neutrinos associated with each particle.

The tau (τ^-) is similar to, but much heavier than the electron (about 3500 times heavier). It has a corresponding tau antineutrino. Its antiparticle is the anti-tau with its corresponding tau antineutrino. Muons (μ^-) are also very similar to the electron but are about 200 times larger.

The electron, tau and muon all have a charge of –1e. The antiparticles (or antileptons) have a charge of +1e. The neutrinos associated with each particle do not have a charge.

The table below shows the six types of leptons, their symbol and their charge.

Lepton name	Symbol	Charge (e)
electron	e^-	–1
electron neutrino	v_e	0
tau	τ^-	–1
tau neutrino	v_τ	0
muon	μ^-	–1
muon neutrino	v_μ	0

The table below shows the six types of antileptons, their symbols and their charge.

Antiparticle name	Symbol	Charge (e)
positron	e^+	+1
electron antineutrino	$\bar{v}_e$	0
anti-tau	τ^+	+1
tau antineutrino	$\bar{v}_\tau$	0
anti-muon	μ^+	+1
muon antineutrino	$\bar{v}_\mu$	0

Leptons interact by exchanging W and Z bosons. Recall that the W and Z boson mediates the weak nuclear force. Leptons that carry a charge, such as the electron, can also interact by exchanging photons. Recall that photons mediate the electromagnetic force. Neutrinos do not have a charge and therefore cannot experience the electromagnetic force.

Leptons are elementary particles that are not affected by the strong nuclear force.

Quarks

All matter is made up of atoms. Atoms are made up of protons, neutrons and electrons. We have already seen that electrons are classified as leptons. Protons and neutrons are made up of smaller particles called quarks. All composite matter particles, such as atoms, are thought to be a combination of quarks, antiquarks and leptons.

Quarks are fractionally charged particles that are affected by all of the fundamental forces.

Quarks have a size less that 10^{-18} m.

Quarks are never found in isolation. They are always found in groups.

Quarks combine to form composite particles called **hadrons**. Proton and neutron are examples of hadrons.

There are six types of quarks and their corresponding six antimatter opposites called antiquarks. Each quark has different properties, such as mass and charge. Quarks have a charge of either $+\frac{2}{3}$e or $-\frac{1}{3}$e. Charge is expressed in terms of 'e' which is the fundamental charge, with a magnitude equal to the charge of a proton.

Table 3.4.1 shows the six types of quarks, their symbol and charge while table 3.4.2 shows the six types of antiquarks, their symbols and charge.

Summary of quarks		
Quark	***Symbol***	***Charge (e)***
Up	u	$+\frac{2}{3}$
Down	d	$-\frac{1}{3}$
Strange	s	$-\frac{1}{3}$
Charm	c	$+\frac{2}{3}$
Top	t	$+\frac{2}{3}$
Bottom	b	$-\frac{1}{3}$

Table 3.4.1

Summary of antiquarks *(the charge is opposite to corresponding quark)*		
Anti-quark	***Symbol***	***Charge (e)***
Anti-up	$\bar{u}$	$-\frac{2}{3}$
Anti-down	$\bar{d}$	$+\frac{1}{3}$
Anti-strange	$\bar{s}$	$+\frac{1}{3}$
Anti-charm	$\bar{c}$	$-\frac{2}{3}$
Anti-top	$\bar{t}$	$-\frac{2}{3}$
Anti-bottom	$\bar{b}$	$+\frac{1}{3}$

Table 3.4.2

Distinguishing between the three types of fundamental particles

Gauge bosons mediate the forces that govern the interaction between all particles. Quarks interact via the strong nuclear force. This distinguishes them from leptons which are elementary particles that are not affected by the strong nuclear force but are acted upon by the weak nuclear force. Quarks have a fractional charge while leptons have a charge of 0 or –1e. Quarks are never found in isolation whereas leptons can exist in isolation.

Hadrons, baryons and mesons

Hadrons are not elementary particles because they are made up of quarks. All hadrons interact by exchanging gluons. Recall that gluons are the particles that mediate the strong nuclear force. Hadrons with charge can interact by exchanging photons (photons mediate the electromagnetic force) but the effect of the gluon is much stronger.

There are **two types** of hadrons; **mesons and baryons.**

Mesons

Mesons are composite particles made up of one quark and one antiquark.

An example of a meson is the pion plus (π^+). It is made up of an up quark and an anti-down quark. Its charge can be determined by adding the charge of the individual quarks ($\frac{2}{3}$e + $\frac{1}{3}$e = + 1e).

Table 3.4.3 shows some of the many mesons, their constituent particles and corresponding charge.

Mesons: made up of $q\bar{q}$			
Name	**Symbol**	**Quark content**	**Charge (e)**
pion-plus	π^+	$u\bar{d}$	+1
pion-minus	π^-	$\bar{u}d$	–1
kaon-minus	k^-	$s\bar{u}$	–1
rho-plus	ρ^+	$u\bar{d}$	+1
ecta-c	η_c	$c\bar{c}$	0
B-zero	B^0	$d\bar{b}$	0

Table 3.4.3

Note: The rho-plus meson has the same quark content as the pion-plus meson but its spin differs.

Baryons

Baryons are composite particles that consist of a combination of three quarks.
Antibaryons are composite particles that consist of a combination of three antiquarks.

Protons (p) and neutrons (n) are baryons. Other examples also include the lambda (Λ), sigma (Σ) and omega (Ω) baryons.

Protons, neutrons and other baryons can be formed from different combinations of quarks

The quarks that make up a proton must have a total charge equivalent to that of a proton (+1e). The proton is formed by two up and a down quark (uud). This combination of quarks has a charge of $\frac{2}{3}$e + $\frac{2}{3}$e – $\frac{1}{3}$e = +1e. The neutron does not have a charge. It is formed from two down and an up quark (ddu). This combination of quarks has a charge of – $\frac{1}{3}$e – $\frac{1}{3}$e + $\frac{2}{3}$e = 0.

Figure 3.4.5 illustrates the combination of quarks that form the proton and the neutron.

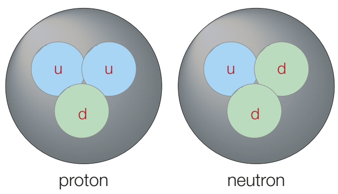


Figure 3.4.5

Table 4 shows some of the many baryons and antibaryons, their constituent particles and corresponding charge.

Name	Symbol	Quark content	Charge (e)
proton	p	*uud*	+1
neutron	n	*ddu*	0
anti-proton	$\bar{p}$	$\bar{u}\bar{u}\bar{d}$	–1
anti-neutron	$\bar{n}$	$\bar{u}\bar{d}\bar{d}$	0
lamda-plus	Λ^+	*udc*	+1
Sigma-minus	Σ^-	*dds*	–1

Table 4

Worked example

1. The D-zero meson is made up of a charm and anti-up quark ($c\bar{u}$). Determine the charge of the D-zero meson.

   Charge = charge of charm quark + charge anti up quark = $\frac{2}{3}e + -\frac{2}{3}e = 0$.

2. The omega-minus particle (Ω^-) is made up of three strange quarks (sss).

   (a) State whether the omega-minus particle is a meson or baryon.

   Baryon

   (b) Show that the charge of an omega-minus particle is –1e.

   Charge = 3 × charge strange quark = $3 \times -\frac{1}{3}e = -1e$

Helpful online resources

Discuss the research using the Large Hadron Collider which has found that some particles are formed from combinations of four and five quarks:

http://www.symmetrymagazine.org/article/july-2015/lhc-physicists-discover-five-quark-particle

3

Baryon number and lepton number

Every particle is assigned a lepton number and a baryon number.

Baryon number

- All particles that are baryons have a baryon number of +1.
- All antibaryons have a baryon number of -1.
- All other particles have a baryon number of 0.
- All quarks have a baryon number of $+\frac{1}{3}$ and antiquarks a baryon number of $-\frac{1}{3}$.

Worked example

1. The antiproton is a composite particle made up of two anti-up ($\bar{u}$) and one anti-down ($\bar{d}$) quark.

   (a) Explain why the antiproton is classified as an antibaryon and not a meson.

   The antiproton is made up of three antiquarks and is therefore an antibaryon. A meson is made up of one quark and one antiquark.

   (b) Show that the charge of an antiproton is –1e.

   The sum of the charge of the individual quarks gives: $-\frac{2}{3}e - \frac{2}{3}e + \frac{1}{3}e = -1e$

   (c) State, with reason, the baryon number of the antiproton.

   All particles that are baryons have a baryon number of +1. All antibaryons have a baryon number of –1. The antiproton is an antibaryon and therefore has a baryon number of –1.

   (d) Use your knowledge of the baryon numbers of quarks and antiquarks to confirm your answer to part (c).

   The antiproton is made up of three antiquarks. The baryon number of an antiquark is $-\frac{1}{3}$.

   The baryon number of the antiproton is given by: $-\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1$

Lepton number

As discussed earlier in this chapter, leptons consist of the electron (e), tau (τ) and muon (μ), and the neutrinos associated with each particle. There are three lepton numbers associated with these particles.

- electronic lepton number, L_e
- muonic lepton number, L_μ
- tauonic lepton number, L_τ

Table 3.4.5 shows the six types of leptons, their symbol, charge and lepton numbers.

Lepton name	**Symbol**	**Charge**	L_e	L_μ	L_τ	**Lepton number**
electron	e^-	-1	+1	0	0	+1
electron neutrino	ν_e	0	+1	0	0	+1
tau	τ^-	-1	0	0	+1	+1
tau neutrino	ν_τ	0	0	0	+1	+1
muon	μ^-	-1	0	+1	0	+1
muon neutrino	ν_μ	0	0	+1	0	+1

Table 3.4.5

Table 3.4.6 shows the six types of antileptons, their symbol, charge and lepton numbers.

Antiparticle name	**Symbol**	**Charge**	L_e	L_μ	L_τ	**Lepton number**
positron	e^+	+1	-1	0	0	–1
electron antineutrino	$\bar{\nu}_e$	0	-1	0	0	–1
anti-tau	τ^+	+1	0	0	-1	–1
tau antineutrino	$\bar{\nu}_\tau$	0	0	0	-1	–1
anti-muon	μ^+	+1	0	-1	0	–1
muon antineutrino	$\bar{\nu}_\mu$	0	0	-1	0	–1

Table 3.4.6

In general:

- All particles that are leptons, regardless of type, have a lepton number of +1.
- All antileptons, regardless of type, have a lepton number of -1.
- All other particles have a lepton number of 0.

Conservation laws

Key idea

The laws of the conservation of baryon number, charge, and lepton number determine the types of reactions that can occur between particles.

Throughout the Stage 1 and 2 course, several conservation laws such as the conservation of charge, momentum and energy have been discussed. These conservation laws still apply when considering particle interactions but it was observed that some interactions and decays could not occur even if the classical conservation laws were obeyed. Particle physicists have discovered two other laws that must be obeyed in addition to the classical conservation laws in order for a reaction or decay to occur.

These two laws are the law of conservation of baryon number and the law of conservation of lepton number.

Conservation of charge

The total charge before an interaction or decay process must be the same as the total charge after an interaction or decay process.

Conservation of baryon number

The total baryon number before an interaction or decay process must be the same as the total baryon number after an interaction or decay process.

Conservation of lepton number

The total lepton number before an interaction or decay process must be the same as the total lepton number after an interaction or decay process.

This course requires you to be able to

- Use the conservation laws to determine the baryon number, lepton number, and charge of particles in reactions and
- Given a reaction between particles, demonstrate that baryon number, lepton number, and charge are conserved.

These requirements are best addressed using worked examples.

3

Worked examples

1. Consider the following reaction for the decay of a neutron into a proton. This process occurs during beta minus decay.

$$n \rightarrow p + e^- + \bar{\nu}_e$$

The table below shows some of the properties of the subatomic particles involved in this reaction.

Particle name	Symbol	Charge (e)	Baryon number	Electronic lepton number L_e
neutron	n	0	+1	0
proton	p	+1	+1	0
electron	e^-	–1	0	+1
electron antineutrino	$\bar{\nu}_e$			

(a) Determine the charge of the electron antineutrino.

The left hand side (LHS) of the equation has a total charge of zero. Using the law of conservation of charge, the right hand side of the equation must also have a total charge of zero. Since the proton has a charge of +1e and the electron has a charge of -1e, this means that the electron antineutrino has a charge of zero.

(b) Determine the baryon number of the electron antineutrino.

The left hand side (LHS) of the equation has a total baryon number of +1. Using the law of conservation of baryon number, the right hand side of the equation must also have a total baryon number of +1. Since the proton has a baryon number +1 and the electron has a baryon number of zero, this means that the electron antineutrino also has a baryon number of zero.

(c) Determine the electronic lepton number (L_e) of the electron antineutrino.

The left hand side (LHS) of the equation has a total lepton number of zero. Using the law of conservation of lepton number, the right hand side of the equation must also have a total lepton number of zero. Since the proton has a lepton number zero and the electron has a lepton number of +1, this means that the electron antineutrino has an electronic lepton number of -1.

(d) Complete the third row of the above table to include the charge, baryon number and electronic lepton number of the electron antineutrino.

Electron antineutrino	$\bar{\nu}_e$	0	0	-1

2. Consider the reaction $\pi^- + p \rightarrow \Sigma^- + K^+$

The table below shows some of the properties of the subatomic particles involved in this reaction.

Name	Symbol	Quark content	Charge (e)	Baryon number
pion-minus	π^-	$\bar{u}d$	–1	0
proton	p	*uud*	+1	+1
sigma-minus	Σ^-	*dds*	–1	+1
kaon-plus	K^+	$u\bar{s}$	+1	0

(a) Identify the two baryons and the two mesons listed in the table.

Baryons: the proton and the sigma-minus particle

Mesons: the pion-minus and kaon-plus particle

(b) Show that the charge of the sigma-minus particle is -1e.

Charge = charge of two down quark + charge of an strange quark = $2 \times -\frac{1}{3}e + -\frac{1}{3}e = -1e$

(c) Show that charge is conserved in this reaction.

LHS = -1e + 1e = 0 RHS = -1e + 1e = 0

The total charge on the left hand side of the equation and the right hand side of the equation are the same. Charge is conserved.

(d) Show that baryon number is conserved in this reaction.

LHS = 0 + 1 = +1 RHS = +1 + 0 = +1

The total baryon number on the left hand side of the equation and the right hand side of the equation are the same. Baryon number is conserved.

(e) State one other quantity that is conserved in this reaction.

Momentum

3. Consider the reaction represented by the equation: $n + v_\tau \rightarrow p + \mu$

The table below shows some of the properties of the subatomic particles involved in this reaction.

Particle name	Symbol	Charge (e)	Baryon number	Tauonic lepton number L_τ	Muonic lepton number L_μ
neutron	n	0	+1	0	0
tau neutrino	v_τ	0	0	+1	0
proton	p	+1	+1	0	0
muon	μ	-1	0	0	+1

Show that the above reaction cannot take place.

	LHS	RHS
Charge	0 + 0 = 0	+1e –1e = 0
Baryon number	+1 + 0 = +1	+1 + 0 = +1
Tauonic lepton number	+1	0
Muonic lepton number	0	+1

Tauonic and muonic lepton numbers are not conserved. The reaction cannot take place.

Extra understanding

Forces involved in different reactions

Basic **collisions** between **charged particles** are caused by the **electromagnetic** force. An example would be the reaction p + p → p + p which represents two protons colliding and repelling away from one another.

Reactions between **hadrons** are caused by the **strong nuclear force**. An example would be the reaction p + p → p + p + π^0. In this case two protons (p) collide and an additional pi-zero pion (π^0) is emitted.

Lepton reactions which produce neutrinos are caused by the **weak nuclear force** because neutrinos do not have a charge and cannot be affected by the electromagnetic force. An example would be the reaction $v_e + \mu^- \rightarrow e^- + v_\mu$. This reaction represents an electron neutrino (v_e) reacting with a pi-minus meson (μ^-) to produce and electron (e^-) and a muon neutrino (v_μ).

Science as a human endeavour

The Higgs boson

Peter Higgs first proposed the existence of an elementary particle now called the Higgs boson (and often referred to as the God particle) in 1964. While Higgs himself did not discover the particle during his career, research by other scientists continued, and it was eventually observed in 2012 at both CERN and Fermilab (Fermi National Accelerator Laboratory) near Chicago in the United States.

It was important to find this particle as it plays an important part in explaining the fundamental forces. The Higgs boson explains why different particles can have different mass. For example the photon does not have mass but the W and Z bosons do. That was not the initial prediction of the Standard Model. Higgs proposed that particles gain mass when they interact with a field called the Higgs field. The equations of the Standard Model were modified to allow the W and Z bosons to have mass.

Helpful online resources

You can learn more abut the Higg's boson at

1. https://home.cern/topics/higgs-boson.
2. http://www.fnal.gov/pub/science/higgs/

Pair annihilation

Pair annihilation refers to the process of a particle and its antiparticle colliding, such that both particles annihilate and release energy. That is, mass is turned into energy.

The process releases two photons that travel in opposite directions so that momentum is conserved.

The energy released is in accordance with the mass–energy equivalence formula: $E = \Delta mc^2$.

Figure 3.4.7 represents an electron and its antiparticle the positron colliding and annihilating to produce two photons.

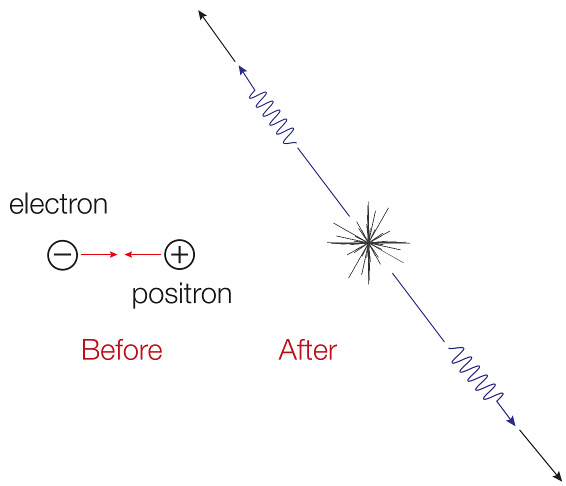


Figure 3.4.7

The reaction can be represented as follows: $e^- + e^+ \rightarrow 2\gamma$

Worked example

Calculate the energy released when an electron and a positron collide and annihilate.

Given the reaction $e^- + e^+ \rightarrow 2\gamma$

$E = \Delta mc^2 = (0 - 2 \times 9.11 \times 10^{-31})\ (3.00 \times 10^8)^2 = 1.64\ 10^{-13}\ J$

Science as a human endeavour

The following are some possible ideas for investigation:

1. Explore the change in understanding of the Standard Model in the light of new information using, for example, high-energy particle accelerators.
2. Explore the benefits and limitations of using positron–electron annihilation in PET scanners, including for the production of gamma rays.
3. Research the economic and social impacts of using the cyclotron at SAHMRI to produce radioisotopes for PET scanning.
4. Explore how beta minus decay involves the conversion of a neutron to a proton accompanied by the production of an electron and an antineutrino.
5. Explore how beta plus decay involves the conversion of a proton to a neutron, accompanied by the production of an electron and an antineutrino.
6. Explore how beta decay can be explained in terms of the conversion of quarks.

Exercises

1. (a) State the three fundamental particles that form the basis of the Standard Model of particle physics.

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(b) State the four fundamental forces that form the basis of the Standard Model of particle physics.

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2. (a) Explain what is meant by a gauge boson.

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(b) Complete the table below which lists the four fundamental forces and the corresponding gauge bosons associated with each force.

Force	Gauge Boson
Electromagnetic	photon
Strong nuclear	....
....	W, Z
....	graviton

(c) Explain how the electromagnetic force of repulsion between two like charges can be explained in terms of the exchange of photons.

3. Explain whether the electron is a fundamental or composite particle.

4. (a) The electron (e^-) is an example of a lepton. Define the term lepton.

(b) All particles have a corresponding antimatter particle equivalent. State the antiparticle of the electron.

(c) The electron (e^-) has a charge of –1e. State the charge of its corresponding antiparticle.

(d) Name two other leptons.

5. (a) Define the term quark.

(b) State the symbol and charge of an anti-bottom quark.

Symbol:.................... Charge:....................

6. Explain the difference between gauge bosons, quarks and leptons.

7. Explain the difference between a baryon and a meson.

8. Look carefully at each of the quark combinations below. Circle the quark combinations that could not form a particle.

$\bar{u}s$ $\bar{u}ss$ uu $\bar{d}\bar{d}\bar{s}$ udc

9. The K-plus kaon (K^+) is made up of an up (u) quark and an anti-strange ($\bar{s}$) quark.
   (a) Classify the K-plus kaon as a baryon or meson.

   ....

   (b) Show that the charge of a K-plus kaon is +1e.

   ....

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   ....

   (c) State the baryon number of the K-plus kaon.

   ....

10. Early in 2017, physicists at CERN reported the discovery of a new Xi baryon called the Xi-c++ baryon. The Xi-c++ particle consists of two charm quarks and an up quark.
    (a) Explain why the Xi-c++ is classified as a baryon.

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    ....

    (b) Show that the charge of the Xi-c++ particle is +2e.

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11. (a) State the quarks that combine to make a proton.

    ....

    (b) Show that the total charge of the combination of quarks in part (a) gives a charge of +1e.

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12. Describe how a neutron can be formed from a combination of quarks.

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13. The following reaction shows the decay of a neutron into a proton, electron and an antineutrino.

$$n \rightarrow p + e^- + \bar{v}_e$$

    (a) Describe how the combination of quarks changes as the neutron decays into a proton.

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    (b) The electron antineutrino ($\bar{v}_e$) has zero charge, a baryon number of zero and an electronic lepton number of –1. Which one of these properties differs for an electron neutrino. Clearly state the difference.

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    ....

    (c) State the name of the fundamental force that would cause this reaction.

    ....

    (d) Name the gauge boson that would be exchanged in this reaction.

    ....

14. Consider the reaction $p + p \rightarrow p + \pi^+$

The proton (p) is classified as a baryon and is made up of two up quarks and one down quark (*uud*). The charge of a proton is +1e. The pion-plus (π^+) is made up of an up quark and an anti-down quark ($u\bar{d}$).

(a) Classify the pion-plus particle as a baryon or meson.

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(b) Calculate the charge on the pion-plus particle.

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(c) Show that charge is conserved in this reaction.

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(d) State, with reason, the baryon number of a proton and the pion-plus particle.

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(e) Show that the reaction cannot occur because baryon number is not conserved.

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15. (a) Consider the following reaction:

$$\bar{v}_e + p \rightarrow n + e^+$$

The table below shows some of the properties of the subatomic particles involved in this reaction.

Particle name	**Symbol**	**Charge (e)**	**Baryon number**	**Electronic lepton number L_e**
electron antineutrino	$\bar{v}_e$	0	0	-1
proton	p	+1	+1	0
neutron	n	0	+1	0
positron	e^+	+1	0	-1

In terms of the law of conservation of charge, baryon number and lepton number, show that this reaction can take place.

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(b) Now consider the reaction: $v_e + p \rightarrow n + e^+$

(i) Charge is conserved during this reaction. Deduce the charge of the electron neutrino (v_e).

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(ii) This reaction cannot take place. Given the baryon number of the electron neutrino is zero and its electronic lepton number is +1, determine which of the conservation laws is violated.

16. Nucleons are made up of quarks.
    (a) Name the fundamental force that acts between nucleons.
    (b) Name the particle that mediates the force that acts between nucleons.
    (c) Name the fundamental force that acts during the decay of nucleons.
    (d) Name the particle that mediates the force that acts during the decay of nucleons.

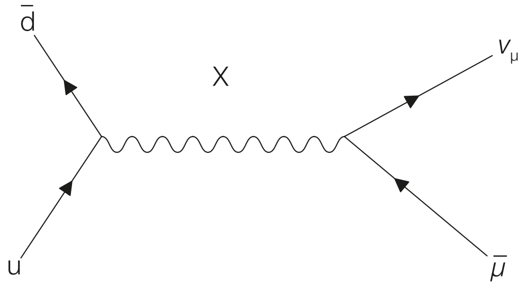
17. The pion-minus particle (π^-) is a meson. It is a combination of an anti-up quark and a down quark.
    (a) State, with reason, the baryon number you'd expect for the pion-minus particle.
    (b) State the baryon number of quarks and antiquarks.
    (c) Use your answer to part (b) to confirm your expected value for the baryon number of the pion-minus particle.

18. The lamda-plus particle is a baryon. Its quark content is udc.

    Show that its baryon number is +1.

19. (a) State the gauge boson that causes the
    (i) electromagnetic interaction between electrons.
    (ii) strong interactions between the protons and neutrons in a nucleus.

(b) The diagram below shows a Feynman diagram. It represents the decay of a meson into an anti-muon and a muon neutrino.



(i) Determine the charge on the meson.

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(ii) The muon neutrino has a charge of zero. Determine the charge on the anti-muon.

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(iii) Identify the gauge boson labelled X.

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20. CERN has been investigating the annihilation of an electron and a positron for many years.

(a) Explain what is meant by the term pair annihilation.

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(b) The annihilation of an electron and a positron produces two photons that travel in opposite directions. State the reason for this.

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(c) Write an equation for the annihilation of an electron and a positron pair.

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(d) Calculate the energy released during the annihilation of an electron and a positron pair.

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(e) Determine the frequency of the photons released during the annihilation of an electron and a positron pair.

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21. Science as a human endeavour activity – Standard model

The text below is an article, 'Nobel-winning discovery of neutrino oscillations, proving that neutrinos have mass'.

Date: December, 2015

Source: US Department of Energy https://phys.org/news/2015-12-nobel-winning-discovery-neutrino-oscillations-neutrinos.html

The 2015 Nobel Prize in Physics was shared by Arthur B. McDonald, the leader of the Sudbury Neutrino Observatory (SNO), and Takaaki Kajita, a leader of the Super-Kamiokande collaboration, "for the discovery of neutrino oscillations, which shows that neutrinos have mass."

The discovery of neutrino oscillations and mass has profoundly affected our understanding of these elusive particles, their role in the theoretical underpinning of physics, and the evolution of the universe. The success of nuclear-physics calculations of solar energy generation has been dramatically confirmed. The discovery, moreover, opens new doors to an understanding of such basic questions as why the universe contains more matter than antimatter, and what properties a new and successful standard model must have.

They have long been thought to be massless, a prediction of the standard model of particles and fields. Beginning in the 1960s, Raymond Davis, Jr. began to measure the flux of neutrinos from the sun. His experiment and subsequent ones at Kamiokande in Japan, Baksan in Russia, and Gran Sasso in Italy all found that the flux was much smaller than expected. In 1985, Herbert Chen observed that if neutrinos oscillated, they would still arrive at earth but in «flavors» undetectable in the Davis experiment, which was designed for electron neutrinos. There are 3 flavors, electron, mu, and tau, but the sun can only make electron neutrinos. He proposed a detector based on heavy water that could detect all flavors equally. The result was the SNO detector, and in 2001 SNO showed that two-thirds of the electron neutrinos had converted to non-electron flavors. Super-Kamiokande had found in 1998 a similar effect in which mu neutrinos produced in the atmosphere converted to a non-electron flavor. These conversions can only occur via a quantum-mechanical effect that requires neutrino mass to be non-zero.

The confirmation of the solar-neutrino flux predictions resolved a problem that had endured for more than 30 years and shows that nuclear processes in the sun's core are understood very accurately. The discovery of neutrino mass forces a revision in our basic model of particles and fields. New theories are being developed, but a decisive choice cannot be made without more information. Experimental work to determine the actual mass (which is not given by oscillations), to answer the question of whether neutrinos and antineutrinos are the same particle, and to see if neutrinos respect a natural symmetry, the reversal of time, need to be carried out. The discovery also means that neutrinos are a part of the dark matter in the universe, but only a small part. Nevertheless, their abundance and small mass mean that they affect the form and evolution of the largest skeins and clusters of galaxies in the universe.

(a) Explain how the article demonstrates that communication and collaboration is required in scientific research.

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(b) Describe one other example of how this article demonstrates science as a human endeavour.

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Physics
Trial paper

Question Booklet 1

- Questions 1 to 10 (60 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 363 if you need more space
- Allow approximately 65 minutes

Examination information

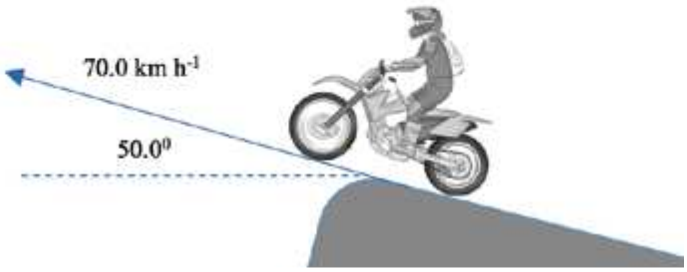
Materials
- Question Booklet 1
- Question Booklet 2
- Formula sheet

Instructions
- Use black or blue pen
- You may use a sharp pencil for diagrams and other representations
- Approved calculators may be used

Total time: 130 minutes

Total marks: 120

1 The diagram below represents a dirt bike rider leaving a dirt ramp with a speed of 70.0 kmh^{-1} at an angle of 50.0^0 above the horizontal. The dirt bike is in the air for 4.20 s before landing on the ground. Ignore the effects of air resistance in this question.



[This diagram is not drawn to scale]

(a) Show that the magnitude of the horizontal component of the dirt bike's velocity as it leaves the ramp is 12.5 ms^{-1}.

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________________________________(2 marks)

(b) Calculate the horizontal distance travelled by the dirt bike.

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________________________________(2 marks)

(c) Show that the dirt bike reaches a height of 11.3 m relative to the top of the ramp.

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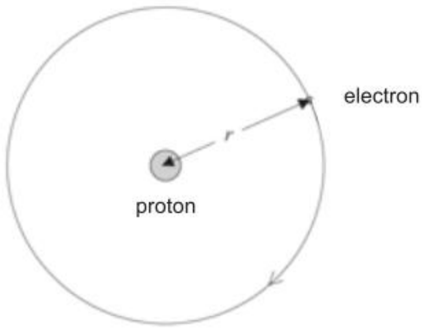
________________________________(2 marks)

2 The diagram below represents a hydrogen atom in which an electron orbits a proton with constant speed. When in the ground state, the radius of orbit of the electron is $r = 5.29 \times 10^{-11}$ m.



[This diagram is not drawn to scale]

(a) Calculate the magnitude and direction of the force that causes the centripetal acceleration of the electron in the ground state.

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____(3 marks)

(b) Calculate the magnitude of the average velocity of the electron in the ground state.

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____(3 marks)

(c) Draw a vector arrow to represent the average velocity of the electron at the position it is shown on the diagram.

(1 mark)

3 When a raindrop falls from a cloud it travels through a large distance towards the ground. When air resistance is not taken into account, calculations show that the speed of a raindrop could be around 400 ms^{-1} by the time it reaches the ground. In reality, the speed of a raindrop when it reaches the ground is much lower than this value.

Explain why the speed of a raindrop is much lower than 400 ms^{-1} as it reaches the ground.

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____(3 marks)

4 (a) Show that the value of the acceleration due to gravity, g, at the surface of a planet is given by the formula

$$g = \frac{GM}{r^2}$$

where M is the mass of the planet and r is the radius of the planet.

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____(3 marks)

(b) The acceleration due to gravity at the surface of Mars is 3.71 ms^{-2}. Calculate the mass of Mars given it has a mean radius of 3.39 x 10^6 m.

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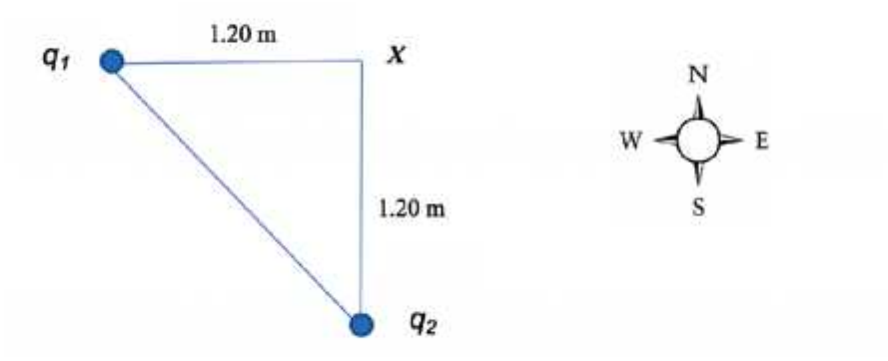
____(3 marks)

5 The diagram below shows two identical point charges q_1 and q_2 of magnitude +3.00 μC in a vacuum. The direction of north is also shown on the diagram. The point ***X***, is located 1.20 m east of q_1 and 1.20 m north of q_2.



[This diagram is not drawn to scale]

(a) Show that the magnitude of the electric field at the point ***X***, due to q_1 is $1.87 \times 10^4\ \mathrm{NC^{-1}}$.

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____(2 marks)

(b) Calculate the magnitude and direction of the electric field at the point ***X***, due to q_1 and q_2.

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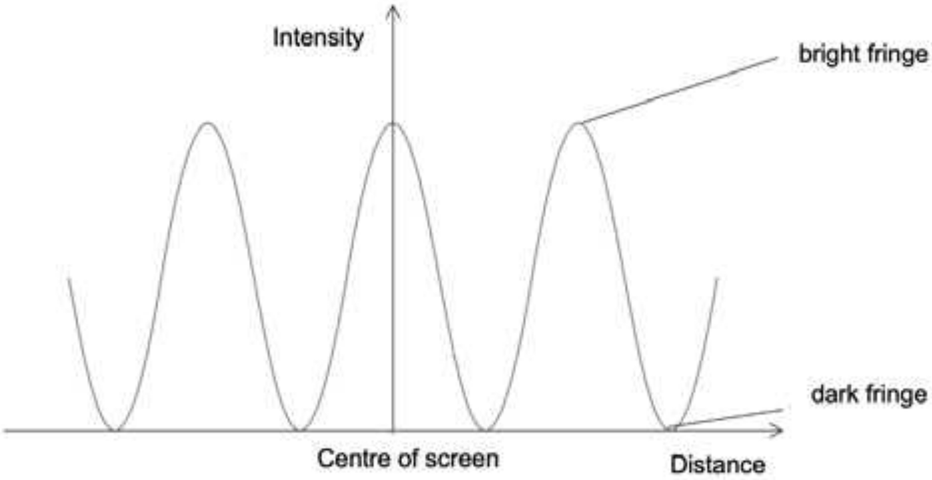
______________________________ *Space for vector diagram* (4 marks)

6 In a Young's double slit experiment, the distance between the slits is 2.40×10^{-4} m and the slit-to-screen distance is 0.280 m. When the two slits are illuminated with coherent monochromatic light, an interference pattern consisting of a series of bright and dark fringes results. The diagram shown below represents the variation in light intensity with distance from the centre of the screen.



[This diagram is not drawn to scale]

The distance between the bright fringe marked on the diagram and the dark fringe marked on the diagram is 3.27×10^{-4} m.

(a) Explain how the bright fringes are produced.

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____(2 marks)

(b) (i) Show that the wavelength of the light used to illuminate the double slits is 5.61×10^{-7} m.

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____(2 marks)

(ii) Light with a smaller wavelength is used to illuminate the same double slits.

State, with reason, the effect this would have on the distance between the bright fringe marked on the diagram and the dark fringe marked on the diagram.

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____(2 marks)

7 The diagram shown below shows some of the energy levels for hydrogen. The diagram is not drawn to scale.

(a) Draw an arrow on the diagram above to represent the transition that results in the lowest frequency emission line in the visible part of the electromagnetic spectrum for hydrogen.

(2 marks)

(b) Calculate the frequency of the photons that are emitted to produce the lowest frequency emission line in the visible part of the electromagnetic spectrum.

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____(3 marks)

8 Two metals, platinum and sodium, are illuminated with monochromatic light with a frequency of 6.67×10^{14} Hz. The work function of each metal is shown in the table below.

Metal	Work function (J)
platinum	1.01×10^{-18}
sodium	3.60×10^{-19}

Electrons are emitted from the surface of sodium but not the surface of platinum.

(a) Deduce that monochromatic light with a frequency of 6.67×10^{14} Hz does not eject electrons from the surface of platinum.

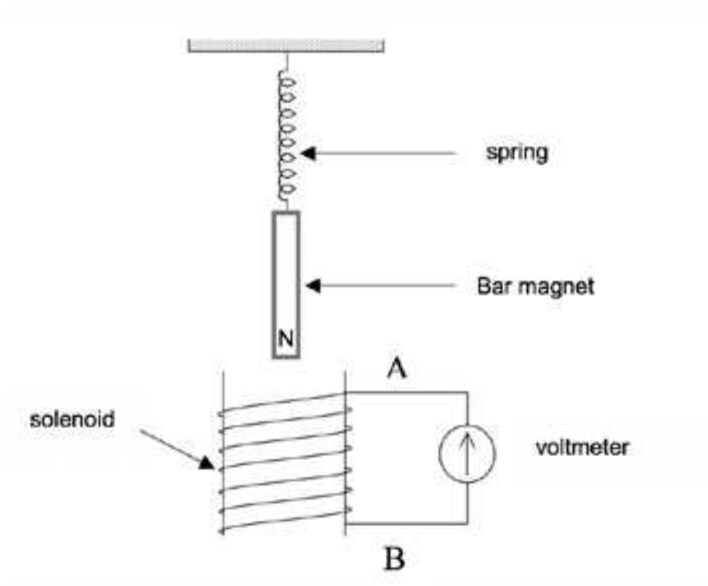
____(2 marks)

(b) Calculate the maximum kinetic energy of the electrons emitted from the surface of sodium when it is illuminated with monochromatic light with a frequency of 6.67×10^{14} Hz.

____(2 marks)

E

9 The diagram below shows a bar magnet suspended above the open end of a solenoid. The bar magnet is attached to a spring and the solenoid is connected to a sensitive voltmeter.



The magnet is pulled down and released so that it oscillates vertically in and out of the solenoid.

(a) Explain why an alternating *emf* results in the solenoid as the magnet oscillates.

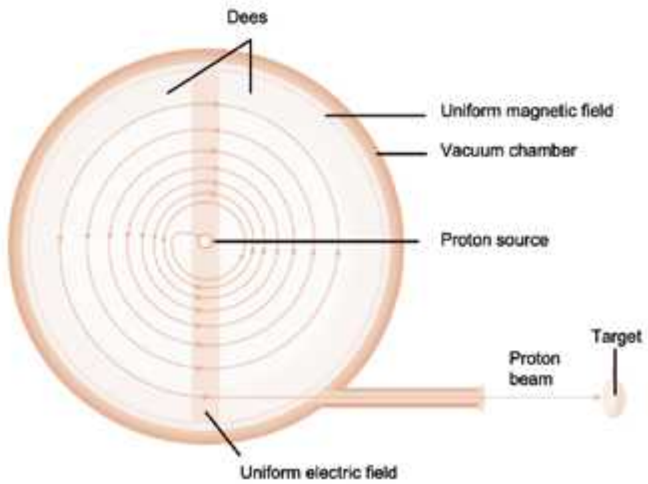
____(3 marks)

(b) Determine the direction (A to B or B to A) of the current in the solenoid as the magnet moves down towards the solenoid.

____(3 marks)

10 The purpose of a cyclotron is to accelerate charged particles to high kinetic energies so that they can collide will a target and produce radioisotopes that are used in medicine and industry.

The diagram below shows main components of a cyclotron and the spiral path traced by protons in the cyclotron.



Source: adapted from https://byjus.com/physics/cyclotron/

[This diagram is not drawn to scale]

The magnitude of the magnetic field acting in the cyclotron shown above is 0.95 T and protons are extracted at a radius of 1.5 m.

(a) State the direction of the magnetic field acting inside the dees of the cyclotron.

____(1 mark)

(b) (i) Explain how the electric field can transfer energy to the protons as they move from one dee to another.

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____(2 marks)

(ii) Explain why the protons move with uniform circular motion inside the dees of the cyclotron causing them to cross the uniform electric field many times.

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____(3 marks)

E

(c) Calculate the kinetic energy of the protons as they emerge from the cyclotron.

(3 marks)

(d) The period of the circular motion of the protons in the cyclotron is 6.9×10^{-8} s.

Calculate the frequency of the alternating potential difference applied across the dees.

(2 marks)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each question carefully (e.g. 6(b)(ii) continued).

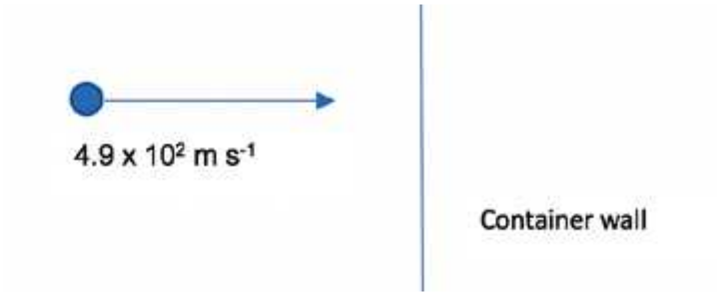
Physics
Trial paper

Question Booklet 2

- Questions 11 to 19 (60 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 376 if you need more space
- Allow approximately 65 minutes

11 A gas molecule will exert a force on a container wall when it collides with the wall of the container.

The diagram below represents an oxygen molecule with a mass of 5.3×10^{-26} kg colliding at normal incidence with the wall of a container at a speed of 4.9×10^{2} ms^{-1}.



The collision is elastic with the oxygen molecule bouncing off the container wall without a loss in speed. The oxygen molecule is in contact with the container wall for a time of 2.0×10^{-4} s.

(a) Calculate the change in momentum of the oxygen molecule.

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____(3 marks)

(b) Determine the force exerted on the wall of the container.

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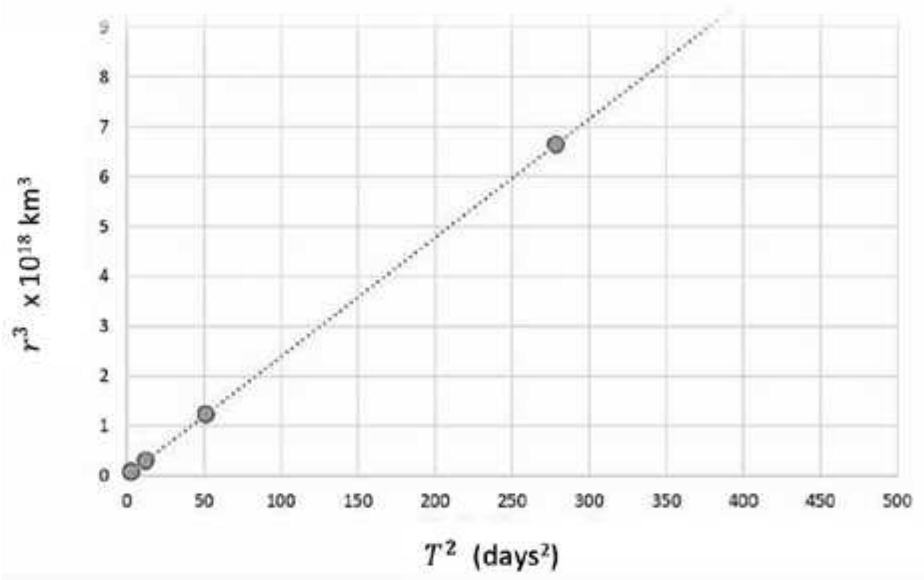
____(3 marks)

E

12 Jupiter has many moons. The graph shown below is a plot of the average radius of orbit cubed against the period squared for four of the larger moons of Jupiter.



(a) Explain why the graph confirms Kepler's Third Law of Planetary Motion.

____(2 marks)

(b) Show that the gradient of the line is 3.22×10^{15} m^3 s^{-2} to three significant figures.

____(3 marks)

(c) Use the value for the gradient to determine the mass of Jupiter.

____(2 marks)

13 Future space travel allows a spacecraft to travel with a speed of 0.95c. The spacecraft travels to our closest star system Alpha Centauri. The astronauts on board the spacecraft measure the time taken to travel to Alpha Centauri as 1.4 years.

(a) Calculate the Lorentz factor for the spacecraft travelling at 0.95c.

(2 marks)

(b) The time taken for the spacecraft to reach Alpha Centauri as measured by an observer on Earth is greater than 1.4 years.

(i) State the name given to this phenomenon.

(1 mark)

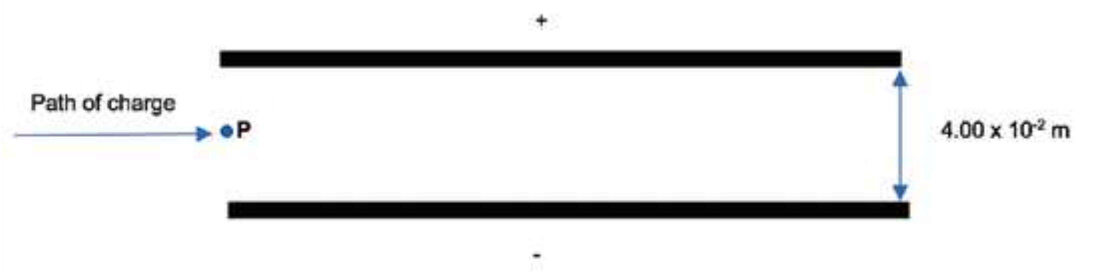
(ii) Calculate the time taken for the spacecraft to travel to Alpha Centauri as measured by an observer on Earth.

(2 marks)

(iii) Calculate the distance to Alpha Centauri as measured by an observer on Earth in light years.

(2 marks)

14 A charge of magnitude -2.70 x 10^{-12} C and mass 1.30 x 10^{-15} kg enters the uniform electric field shown in the diagram below. The electric field outside of the plates may be assumed to be zero. Ignore the gravitational force that may act on the charge.



[This diagram is not drawn to scale]

The potential difference between the plates is 2.00 x 10^{3} V and the plates are separated by a distance of 4.00 x 10^{-2} m.

(a) On the diagram, draw a vector arrow at the point **P** to show the direction of the force acting on the -2.70 x 10^{-12} C charge due to the electric field.

(1 mark)

(b) Determine the magnitude of the acceleration of the -2.70 x 10^{-12} C charge while it is in the electric field.

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____(3 marks)

15 An electric current of magnitude 0.45 A flows through a straight conductor of length 2.5×10^{-2} m which lies perpendicular to the plane of this page. The conductor is shown in the diagram below, as is the direction of the electric current flowing through the conductor.

(a) On the diagram above, sketch magnetic field lines to represent the magnetic field produced by the electric current flowing through the straight conductor.

(3 marks)

(b) Calculate the magnitude and direction of the magnetic field at the point P which is located 0.15 m to the left of the conductor as shown in the diagram below.

____

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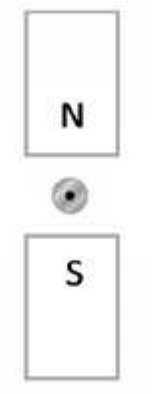
____(2 marks)

(c) The conductor is placed in the uniform magnetic field produced between the opposite poles of two magnets. The magnitude of the magnetic field strength produced by the magnets is 0.18 T. The arrangement is shown in the diagram below.



Calculate the magnitude and direction of the magnetic force acting on the straight current carrying conductor.

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______________________________________(3 marks)

16 X-ray photons are produced in an X-ray tube that operates at a potential difference of 8.00×10^4 V.

(a) Calculate the maximum frequency of the X-rays produced using a potential difference of 8.00×10^4 V.

(2 marks)

(b) The adapted article that follows was published by Scilight, March 22, 2023.

Medical X-ray imaging is perhaps best known for the diagnosis of broken bones. For broader applications, phase-contrast X-ray imaging can produce high-contrast three-dimensional images for the pathological diagnosis of surgically removed tissue and the study of micro-regional anatomy. These uses require high-resolution images for the microscopic examination of multiple tissue sections.

Conventional phase-contrast methods require the object and image detector to be separated to an optimal distance. In principle, increasing that distance lowers the spatial resolution, which poses a challenge for capturing minute details such as the cell nuclei shape needed for pathological diagnosis. Sunaguchi et al. developed the Superimposed Wavefront Imaging of Diffraction-enhanced X-rays (SWIDeX) method for a shorter detector-to-sample distance.

Though this increases the spatial resolution significantly the analyzer must be 20 micrometers thin to achieve the desired one-micrometer goal, requiring high-precision polishing and installation methods that are still in development.

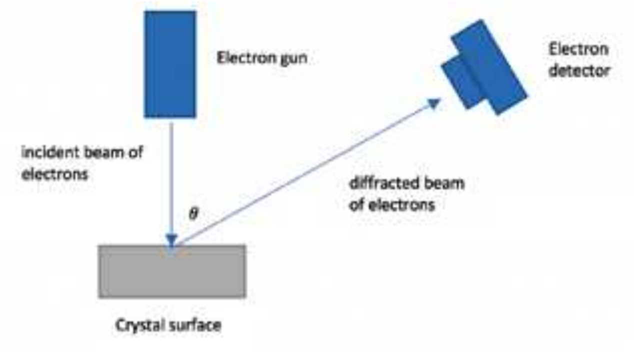
Source: adapted from 'High-resolution phase-contrast X-ray imaging for biological soft tissue'. *Scilight* 2023, 121106 (2023), viewed 20 October 2023, https://pubs.aip.org/aip/sci/article/2023/12/121106/2879492/High-resolution-phase-contrast-X-ray-imaging-for

Explain how the information in the adapted article illustrates *one* key concept of science as a human endeavour.

(3 marks)

E

17 In 1924, Davisson and Germer conducted an experiment, in which the diffraction of low energy electrons by the surface layers of a crystal lattice was observed. The electrons were accelerated through a potential difference of 54.0 V before being scattered from the surface of the crystal. An electron detector was rotated around the crystal and measured the intensity of the scattered electrons at various angles.



[This diagram is not drawn to scale]

(a) Calculate the de Broglie wavelength of the electrons.

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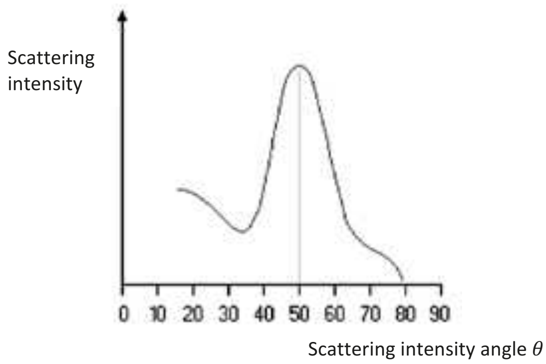
____(4 marks)

(b) The results of the experiment are represented by the graph shown below.



Explain how the results of the Davisson-Germer experiment demonstrated the wave behaviour of the low-energy electrons.

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____(2 marks)

18 Fundamental particles are particles with no internal structure. That is, they are not made up of smaller particles. There are three types of fundamental particles.

An electron is a fundamental particle whereas a proton is not a fundamental particle.

(a) State the name of the group of fundamental particles to which an electron belongs.

____(1 mark)

(b) A proton is classified as a baryon because it is made up of three quarks. A pion-plus particle (π^+) is classified as a meson because it is made up of a quark and an antiquark. The pion-plus particle is made up of an up quark and an anti-down quark ($u\bar{d}$) and has a charge +1e.

(i) State the quark combination that forms a proton.

____(1 mark)

(ii) Use the quark structure of the pion-plus particle to confirm it has a charge of +1e.

____

____(1 mark)

(iii) Consider the reaction $p + p \rightarrow p + \pi^+$.

By considering both charge and baryon number, comment on whether this reaction is possible.

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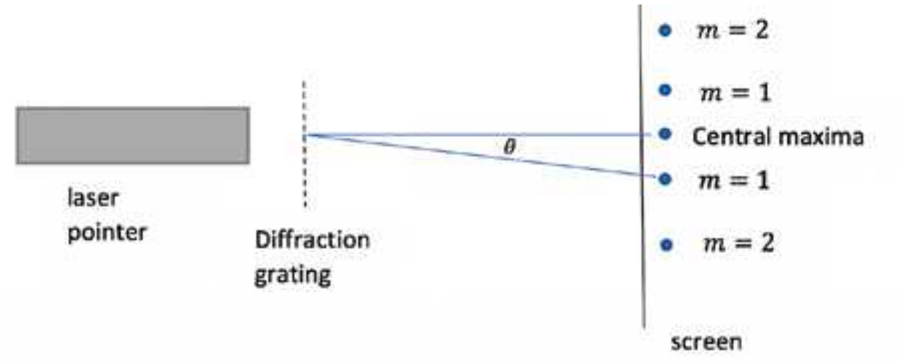
____(3 marks)

19 A student conducts an experiment by passing laser light from a laser pointer through a diffraction grating with 1.00×10^5 lines per metre.

The diagram shown below illustrates the experimental arrangement. Not all of the visible maxima are shown on the diagram.



[This diagram is not drawn to scale]

The student determines the angle between the central maxima and each of the maxima on either side of the central maxima.

The table below shows the data collected by the student in this experiment.

Order m	Angle above the central maxima	Angle below the central maxima	Average angle (^{0})	$sin\theta$
1	3.5	3.7	3.6	0.063
2	7.2	7.2	7.2	0.125
3	11.0	10.7		

(a) (i) The values for the average angle and $sin\theta$ are missing for the third order maxima. Complete the table by filling in these values. (3 marks)

(ii) Describe the benefit of calculating an average angle for each order.

____

____(1 mark)

(b) Calculate the wavelength of the laser pointer using the data for the second order.

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____(3 marks)

(c) The student decides to investigate the relationship between one variable and the angle at which a particular maxima occurs. State one variable that could affect the angle at which a particular maxima occurs and describe a suitable method which would enable the student to collect sufficient and relevant data to determine the relationship between the variable being investigated, and the angle at which a particular maxima occurs.

(4 marks)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 15(b (i) continued).

Solutions

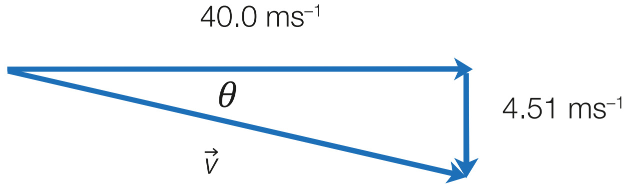
1.1 Projectile motion

1. (a) $v=\frac{s}{t}\therefore t=\frac{s}{v}=\frac{18.4}{40.0}=4.60\times10^{-1}s$

(b) $s=v_o t+\frac{1}{2}at^2=0+\frac{1}{2}\times 9.80\times(4.60\times10^{-1})^2=1.04m$

Height above ground = 1.40 −1.04 = 3.60 × 10^{-1}m

(c) $v_H=40.0ms^{-1}\rightarrow$



$v_v=v_0+at$

$=0+9.80\times4.60\times10^{-1}$

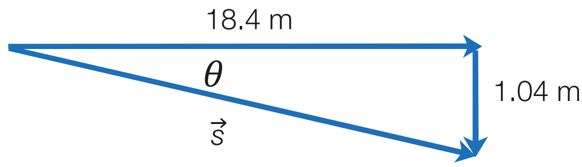
$=4.51\ ms^{-1}\ \downarrow$

$v=\sqrt{40.0^2+4.51^2}=40.3\ ms^{-1}$

$\tan\theta=\frac{4.51}{40.0}\Rightarrow\theta=6.43°$

$\vec{v}=40.3ms^{-1}\ 6.43°$ *below the horizontal*

(d) $s=\sqrt{18.4^2+1.04^2}=18.4m$



$\tan\theta=\frac{1.04}{18.4}\Rightarrow\theta=3.24°$

$\vec{s}=18.4m\ \ 3.43°$ *below the horizontal*

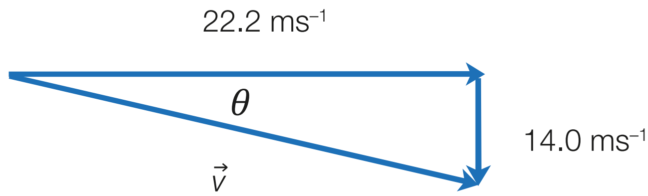
(e) If the pitcher throws the ball harder, the ball reaches the batter in a shorter amount of time. This means that the ball falls a smaller vertical distance ($s=\frac{1}{2}at^2$). The ball reaches the batter at a greater height above the ground.

2. (a) $s=\frac{1}{2}at^2\ \ \therefore\ t=\sqrt{\frac{2s}{a}}=\sqrt{\frac{2\times10.0}{9.80}}=1.43s$

(b) 80.0 kmh⁻¹ = 22.2 ms⁻¹

$s=vt=22.2\times1.43=31.7\ m$

(c) $v_H=22.2ms^{-1}\rightarrow$



$v_v=v_0+at$

$=0+9.80\times1.43$

$=14.0\ ms^{-1}\ \downarrow$

$=\sqrt{22.2^2+14.0^2}=26.2ms^{-}$

$\tan\theta=\frac{14.0}{22.2}\Rightarrow\theta=32.2°$

$\vec{v}=26.2ms^{-1}\ 32.2°$ *below the horizontal*

(d) $v=\frac{s}{t}=\frac{52.0}{1.43}=36.4ms^{-1}$

(e) A greater launch height (taller hill) will produce a greater range because the motorcycle is in the air for a greater length of time as it falls the extra vertical height. Since range is given by the product of the time of flight and the horizontal component of velocity then the range is greater because time is greater while the horizontal component of velocity remains the same.

3. (a) $s = v_o t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.80 \times (11.2)^2 = 615 m$

(b) $s = vt = 10.0 \times 11.2 = 112$ m

(c) $v_H = 10.0 ms^{-1} \rightarrow$

$v_v = v_0 + at$

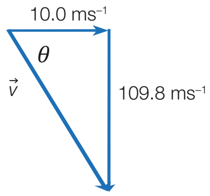
$= 0 + 9.80 \times 11.2$

$= 109.8 = 110 \; ms^{-1} \; \downarrow$

(d) $v = \sqrt{10^2 + 109.8^2} = 110 ms^{-1}$

$\tan\theta = \frac{109.8}{10} \Rightarrow \theta = 84.8°$

$\vec{v} = 110 ms^{-1}$ 84.8° *below the horizontal*



(e) (i) The balloonist will see the marble drop vertically towards the ground.

(ii) A person standing on the ground will see the marble trace a parabolic path.

4. (a) $s = \frac{1}{2}at^2 \; \therefore \; a = \frac{2s}{t^2} = \frac{2 \times 1.80}{0.600^2} = 10.0 \, ms^{-2}$

(b) Ball 2 moves horizontally 0.50 m every 0.100 s. This means that the horizontal motion of the ball is constant.

(c) The vertical positions of Ball 2 exactly match those of Ball 1.

Since Ball 1 accelerated downwards at 10.0 ms^{-2} it can be concluded that Ball 2 is also accelerating downwards at 10.0 ms^{-2}.

(d) $v = \frac{s}{t} = \frac{3.00}{0.600} = 5.00 \; ms^{-1} \rightarrow$

(e) $v_H = 5.00 ms^{-1} \rightarrow$

$v_v = v_0 + at$

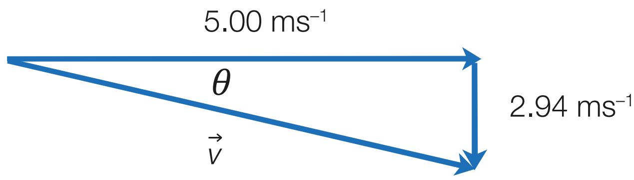
$= 0 + 9.80 \times 0.300$

$= 2.94 \; ms^{-1} \; \downarrow$

$v = \sqrt{5.00^2 + 2.94^2} = 5.80 ms^{-1}$

$\tan\theta = \frac{2.94}{5.00} \Rightarrow \theta = 30.5°$

$\vec{v} = 5.80 ms^{-1}$ 30.5° *below the horizontal*



5. (a) $v_H = v\cos\theta = 40.0\cos30.0 = 34.6 \; ms^{-1}$ $\quad v_V = v\sin\theta = 40.0\sin30.0 = 20.0 \; ms^{-1}$

(b) $v^2 = v_o^2 + 2as$

$0 = 20.0^2 - 2 \times 9.80s$

$s = H = \frac{20.0^2}{2 \times 9.80} = 20.4 \; m$

(c) $v = v_o + at \; \therefore \; t = \frac{v - v_o}{a} = \frac{0 - 20.0}{-9.80} = 2.04 \; s$

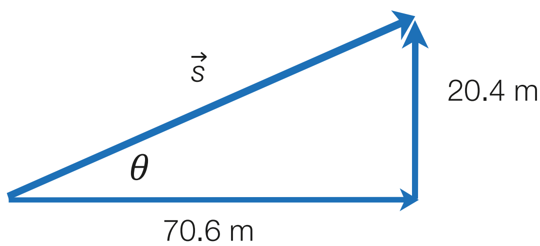
(d) horizontal displacement $s = vt = 34.6 \times 2.04 = 70.6 \; m$

vertical displacement = 20.4 m

$s = \sqrt{70.6^2 + 20.4^2} = 73.5 \; m$

$\tan\theta = \frac{20.4}{70.6} \Rightarrow \theta = 16.1°$

$\vec{s} = 73.5 \; m$ 16.1° *above the horizontal*



(e) $s = vt = 34.6 \times 4.08 = 141 \; m$

Another launch angle that produces this range is 60.0°.

(f) $v_H = 34.6ms^{-1} \rightarrow$

$v_v = v_0 + at$

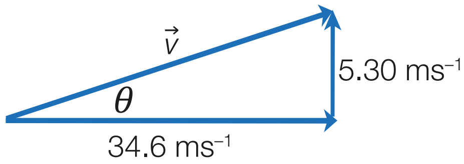
$= 20.0 - 9.80 \times 1.50$

$= -5.30\ ms^{-1}$ *or* $5.30\ ms^{-1} \uparrow$

$v = \sqrt{34.6^2 + 5.30^2} = 35.0ms^{-1}$

$\tan\theta = \frac{5.30}{34.6} \Rightarrow \theta = 8.71°$

$\vec{v} = 35.0ms^{-1}$ 8.71° *above the horizontal*



6. (a) $v_H = v\cos\theta = 25\cos45 = 18\ ms^{-1}$ $v_V = v\sin\theta = 25\sin45 = 18\ ms^{-1}$

(b) $v^2 = v_o^2 + 2as$

$0 = 18^2 - 2 \times 9.80s$

$s = \frac{18^2}{2 \times 9.80} = 17\,m$

(c) $v = v_o + at$ $\therefore$ $t = \frac{v - v_o}{a} = \frac{0 - 18}{-9.80} = 1.84\ s$

$t_{total} = 2 \times 1.84 = 3.7s$

(d) $s = vt = 18 \times 3.7 = 67\ m$

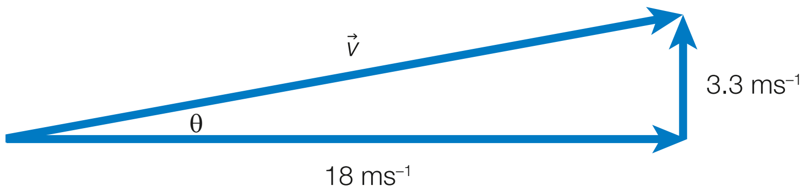
(e) $v_H = 18ms^{-1} \rightarrow$

$v_v = v_0 + at$

$= 18 - 9.8 \times 1.5$

$= 3.3\ ms^{-1}$ *or* $3.3ms^{-1} \uparrow$

scale: 1 cm = 2 ms⁻¹



By measurement $\vec{v} = 18.3 = 18\ ms^{-1}$ 10° *above the horizontal*

(f) (i) smaller

(ii) larger

(iii) larger

7. (a) (i) no change

(ii) decreases

(iii) increases

(b) (i) $v^2 = v_o^2 + 2as$

$0 = v_o^2 - 2 \times 9.80 \times 0.82$

$v_o^2 = 16$ $\therefore$ $v_o = 4.0\ ms^{-1}$

(ii) $v = v_o + at$ $\therefore$ $t = \frac{v - v_o}{a} = \frac{0 - 4.0}{-9.80} = 0.408\ s$

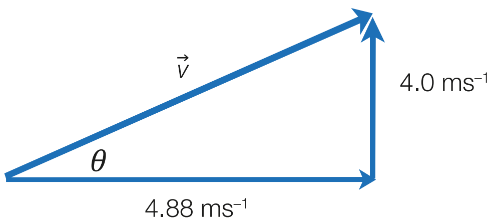
$t_{total} = 2 \times 0.408 = 0.82s$

(iii) $v_H = \frac{s_H}{t} = \frac{4.0}{0.82} = 4.88\ ms^{-1}$

$v = \sqrt{4.88^2 + 4.0^2} = 6.3\ ms^{-1}$

$\tan\theta = \frac{4.0}{4.88} \Rightarrow \theta = 39°$

$\vec{v} = 6.3\ ms^{-1}$ 39° *above the horizontal*



S

(c) Air resistance is the unbalanced force that acts to oppose the motion of an object as it moves through the air due to collisions between the object and air particles.

Air resistance reduces both the horizontal and vertical components of velocity as a projectile moves through the air.

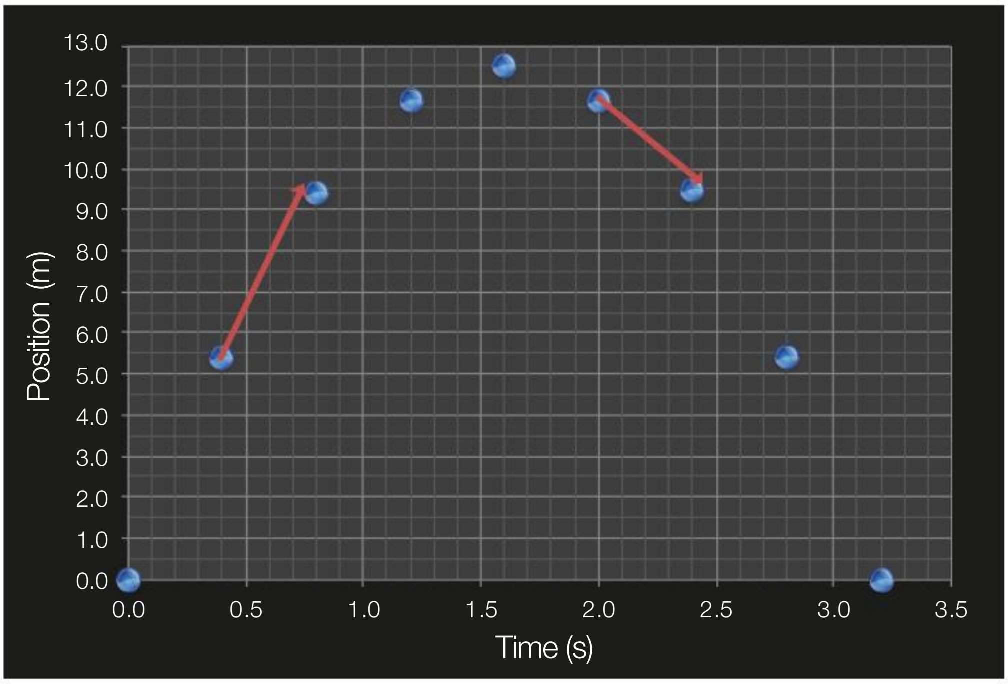
The range of the hockey ball is determined by the product of the horizontal component of velocity and the time of flight. While the time of flight is not reduced significantly, the horizontal component of the hockey ball's velocity is reduced. It follows that the range is also reduced.

The vertical height reached by the hockey ball depends on the initial vertical velocity v_v and the deceleration experienced. When air resistance acts, the vertical height reached is smaller. This is because the hockey ball decelerates more quickly (greater than if gravity was acting alone i.e. greater than 9.80 ms^{-2}) and comes to rest at a smaller height.

8. (a) The object moves the same horizontal distance every 0.5 s. This means that the horizontal motion of the ball is constant.

(b) $s = \frac{1}{2}at^2 \therefore a = \frac{2s}{t^2} = \frac{2\times12.5}{1.6^2} = 9.8ms^{-2}$

(c)



9. (a) $v_H = v\cos\theta = 23.0\cos40.0 = 17.6\ ms^{-1}$ $v_V = v\sin\theta = 23.0\sin40.0 = 14.8\ ms^{-1}$

(b) $v = 17.6ms^{-1} \rightarrow$

The velocity at any instant is a vector sum of the horizontal and vertical components of velocity. The javelin's velocity consists only of the horizontal component which remains constant.

(c) $v^2 = v_o^2 + 2as$

$0 = 14.8^2 - 2\times9.80s$

$s = \frac{14.8^2}{2\times9.80} = 11.2\ m$

Total height above the ground = 2.00 + 11.2 = 13.2 m

(d) $t = \frac{s}{v} = \frac{55.4}{17.6} = 3.15s$

(e) The javelin thrown by the taller athlete is in the air for a greater length of time as it falls the extra vertical height. The range of the javelin is given by the product of its horizontal component of velocity and its time of flight. Since the horizontal component of velocity is the same for both athletes, the range is greater for the taller athlete because the time of flight is greater.

10. (a) $v_V = v\sin\theta = 9.0\sin45 = 6.4\ ms^{-1}$

$v^2 = v_o^2 + 2as$

$0 = 6.4^2 - 2\times9.80\ s$

$s = \dfrac{6.4^2}{2\times9.80} = 2.1\ m$

(b) time to maximum height $v = v_o + at \quad \therefore \quad t = \dfrac{v - v_o}{a} = \dfrac{0 - 6.4}{-9.80} = 0.653\ s$

Time to fall from maximum height $s = \dfrac{1}{2}at^2 \ \therefore\ t = \sqrt{\dfrac{2s}{a}} = \sqrt{\dfrac{2\times(2.1+1.5)}{9.80}} = 0.857s$

total time = 0.653 + 0.857 = 1.51 = 1.5 s

(c) $s_H = v_H t = v\cos\theta t = 9.0\cos45 \times 1.5 = 9.5\ m$

(d) The snowboarder could make the jump with the same speed but at a smaller launch angle (less that 45°). Another way to increase the snowboarder's range is to increase the height of the snow hill.

(e) 9.80 ms^{-2} downwards

(f)

11. (a) $v_V = v\sin\theta \quad \therefore \quad v = \dfrac{v_V}{\sin40.0} = \dfrac{7.00}{\sin40.0} = 10.9\ ms^{-1}$

(b) $v_H = v\cos\theta = 10.9\cos40.0 = 8.35\ ms^{-1} \rightarrow$

$t = \dfrac{s}{v} = \dfrac{10.0}{8.35} = 1.20s$

(c) $v = v_o + at \quad \therefore \quad t = \dfrac{v - v_o}{a} = \dfrac{0 - 7.00}{-9.80} = 0.714\ s$

(d) $v_H = 8.35ms^{-1} \rightarrow$

$v_v = v_0 + at$

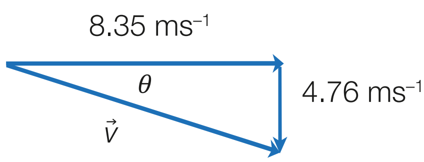
$= 7.00 - 9.80 \times 1.20$

$= -4.76\ ms^{-1} \quad or \quad 4.76ms^{-1} \downarrow$

$v = \sqrt{8.35^2 + 4.76^2} = 9.61ms^{-1}$

$\tan\theta = \dfrac{4.76}{8.35} \Rightarrow \theta = 29.7°$

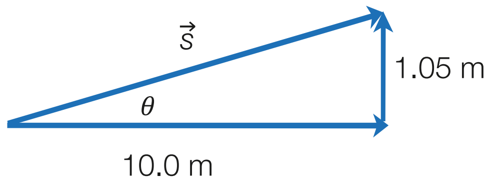
$\vec{v} = 9.61ms^{-1}\ \ 29.7°\ \ below\ the\ horizontal$



(e) $s = \sqrt{10.0^2 + 1.05^2} = 10.1m$

$\tan\theta = \frac{1.05}{10.0} \Rightarrow \theta = 5.99°$

$\vec{s} = 10.1m\ \ 5.99°\ \textit{above the horizontal}$



12. (a) The shorter athlete.

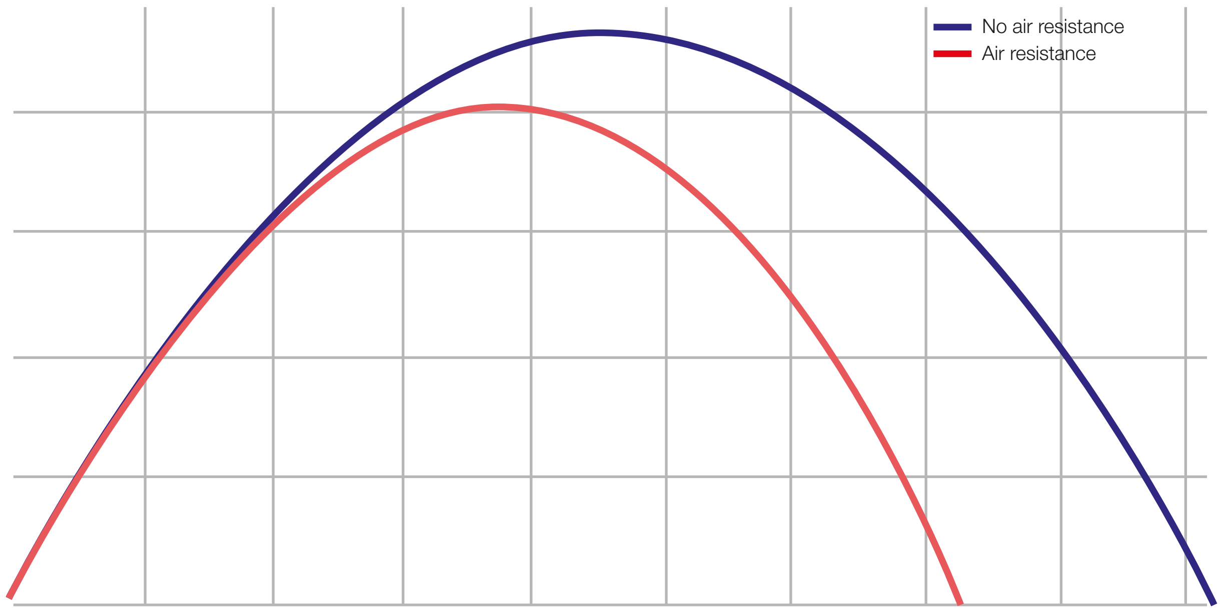
(b) (i) 9.80 ms^{-2}

(ii) 7.66 ms^{-1}

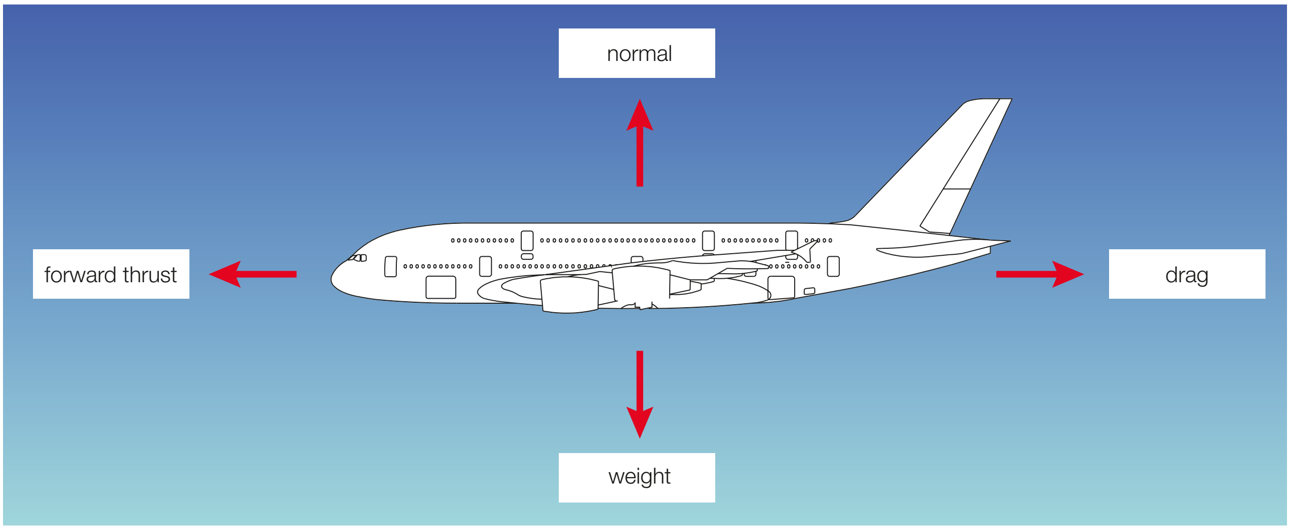
(iii) 0 ms^{-1}

(iv) 7.66 ms^{-1}

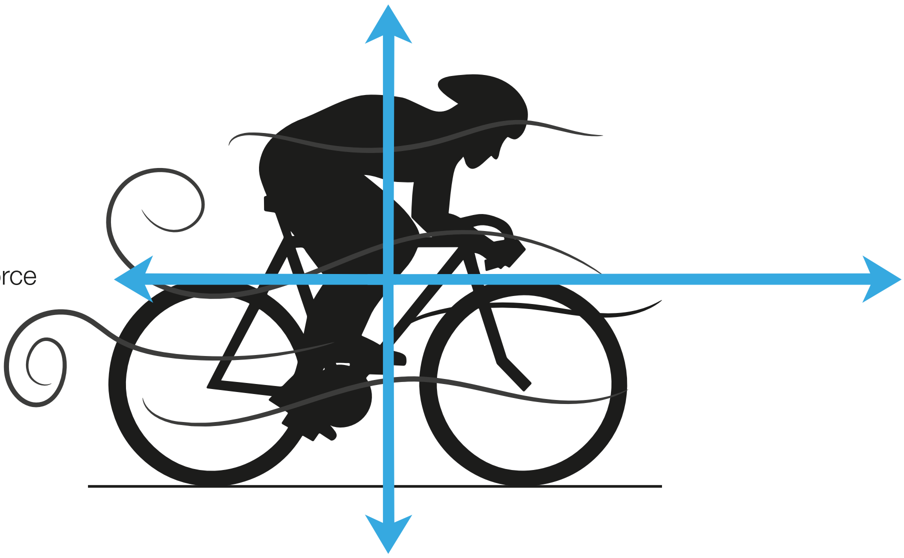
13.



14.



15. (a)

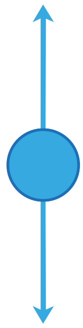


(b) 42.0 N →

(c) $a = \frac{F}{m} = \frac{42.0}{75.0} = 0.560\ ms^{-2}$ →

(d) As the student cycles faster, the magnitude of the total drag force (air resistance and friction) increases. When the magnitude of the drag force is equal in magnitude to the forward thrust of 58 N, terminal velocity is reached because there is no net force acting.

16. (a)



(b) The ball has reached terminal velocity. This means that the magnitude of the drag force acting upwards is equal to the weight of the ball acting down.

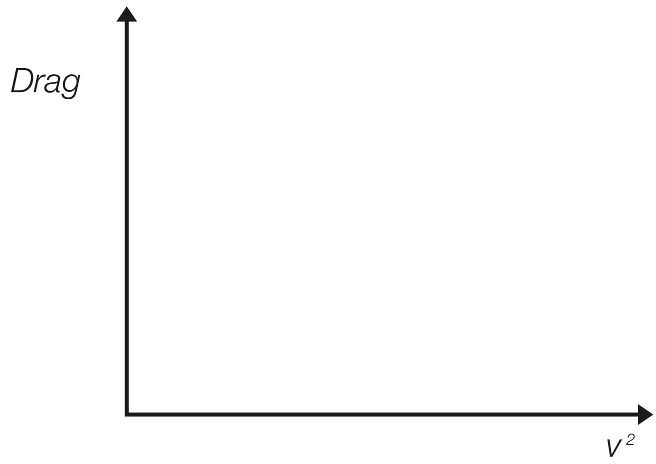
$Drag = mg = 0.2 \times 9.80 = 1.96N$

17. (a) Object 2

(b) Object 1 is not acted upon by air resistance and accelerates down to the ground with a constant acceleration of 10 ms^{-2}. Object 2 experiences a decreasing acceleration because air resistance opposes its motion and increases with speed. At a time of 1.9 seconds, the upward force due to air resistance is equal in magnitude to the downward force due to the object's weight. At this point the forces cancel and there is no net force acting on the object. The object continues to descend with a constant terminal speed of approximately 7.0 ms^{-1}.

18. (a) The drag force is directly proportional to the square of the speed of the toy car.

(b)



(c) A straight line of best fit through the origin should result.

1.2 Forces and momentum

1. (a) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 5.00 \leftarrow - 5.00 \rightarrow = 5.00 \leftarrow + 5.00 \leftarrow = 10.0\ ms^{-1} \leftarrow$

   i.e. 10.0 ms^{-1} 90° away from the wall

   (b) $\Delta\vec{p} = m\Delta\vec{v} = 0.0100 \times 10.0 = 0.100\ kgms^{-1} \leftarrow$ (90° *away from the wall*)

   (c) $\vec{F}_{particle} = \dfrac{\Delta\vec{p}_{particle}}{\Delta t} = \dfrac{0.100}{0.0500} = 2.00N \leftarrow$

   (d) Using Newton's Third Law, $\vec{F}_{wall} = -\vec{F}_{particle} = 2.00N \rightarrow$

   i.e. 2.00 N 90° towards the wall

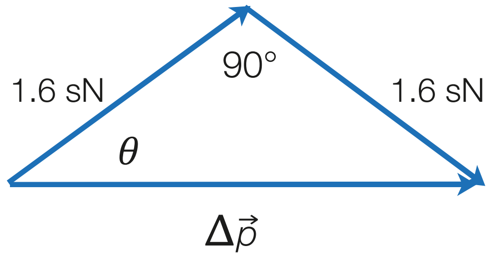
2. (a) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = 0.2 \times 8.0 \nearrow - 0.2 \times 8.0 \nwarrow$

   $\Delta\vec{p} = 1.6 \nearrow + 1.6 \searrow$

   $\Delta\vec{p} = \sqrt{(1.6)^2 + (1.6)^2} = 2.3\ sN$

   $\tan\theta = \dfrac{1.6}{1.6} \quad \therefore \quad \theta = 45.0°$

   $\Delta\vec{p} = 2.3\ sN$ 90° *away from the wall*



   (b) $\vec{F}_{particle} = \dfrac{\Delta\vec{p}_{particle}}{\Delta t} = \dfrac{2.3}{1.00 \times 10^{-2}} = 230N \rightarrow$

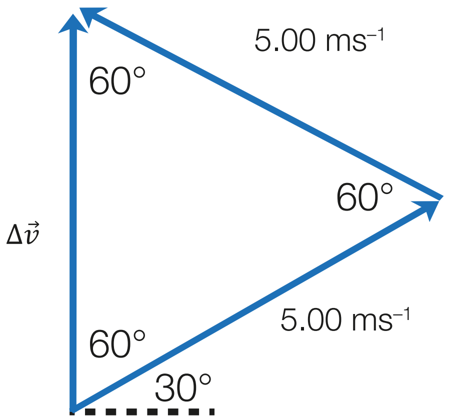
   Using Newton's Third Law, $\vec{F}_{wall} = -\vec{F}_{particle} = 230N \leftarrow$

   i.e. 230 N 90° towards the wall

3. (a) 0 ms^{-1}

   (b) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 5.00 \nearrow - 5.00 \searrow$

   $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 5.00 \nearrow + 5.00 \nwarrow$



   Equilateral triangle

   $\therefore \quad \Delta\vec{v} = 5.00\ ms^{-1}$ 90° away from the ground

   (c) $\vec{F}_{particle} = \dfrac{\Delta\vec{p}_{particle}}{\Delta t} = \dfrac{m\Delta\vec{v}_{particle}}{\Delta t} = \dfrac{0.0650 \times 5.00}{3.00 \times 10^{-2}} = 10.8\ N$ 90° *away from the ground*

   (d) 10.8 N 90° towards the ground

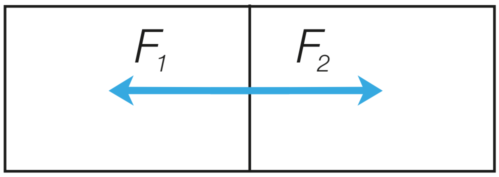
4. (a) The gradient represents the change in momentum of the object per unit time or the time rate of change in momentum. By definition, this is the force acting on the object.

   (b) $Force = gradient = \dfrac{120.0}{6.0} = 20\ N$

5. (a) The law of conservation of momentum states that in an isolated system where no external forces act, the total momentum of the system before an interaction is equal to the total momentum of the system after an interaction.

   (b) An isolated system is one in which no external forces act.

6. Consider two objects colliding head-on.



According to Newton's Third Law of Motion: $\vec{F}_1 = -\vec{F}_2$

According to Newton's Second Law of Motion: $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$

It follows that:

$$\frac{\Delta\vec{p}_1}{\Delta t} = \frac{-\Delta\vec{p}_2}{\Delta t}$$

$$m_1v_{1f} - m_1v_{1i} = -(m_2v_{2f} - m_2v_{2i})$$

$$m_1v_{1f} - m_1v_{1i} = -m_2v_{2f} + m_2v_{2i}$$

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$$

i.e. The total initial momentum is equal to the total final momentum.

7. (a) Using the law of conservation of momentum $\vec{p}_i = \vec{p}_f$

$$m_b\vec{v}_{bi} + m_w\vec{v}_{wi} = m_b\vec{v}_{bf} + m_w\vec{v}_{wf}$$

$$0.05 \times 0.50 \rightarrow + 0.075 \times 0.40 \leftarrow = 0.05 \times v + 0.075 \times 0.30 \rightarrow$$

$$0.005 \leftarrow = 0.05v + 0.0225 \rightarrow$$

$$0.005 \leftarrow - 0.0225 \rightarrow = 0.05v$$

$$0.005 \leftarrow + 0.0225 \leftarrow = 0.05v$$

$$0.0275 \leftarrow = 0.05v$$

$$v = \frac{0.0275 \leftarrow}{0.05} = 0.55 ms^{-1} \leftarrow$$

i.e. $0.55\ ms^{-1}$ to the left

   (b) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = 0.05 \times 0.55 \leftarrow - 0.05 \times 0.50 \rightarrow = 0.0275 \leftarrow - 0.025 \rightarrow = 0.0525\ sN \leftarrow = 0.053\ \text{sN} \leftarrow$

   (c) $\vec{F}_{black} = \dfrac{\Delta\vec{p}_{black}}{\Delta t} = \dfrac{0.0525 \leftarrow}{0.040} = 1.3\ N \leftarrow$

Using Newton's Third Law, $\vec{F}_{white} = -\vec{F}_{black} = 1.3\ N \rightarrow$

8. (a) Total initial momentum: $mv \rightarrow$

   (b) Total final momentum: $mv \rightarrow$

The assumption is that the system is isolated so that the law of conservation of momentum can be applied.

   (c) $mv \rightarrow = mv_f + 3m\frac{1}{5}v \rightarrow$

$$mv \rightarrow - 3m\frac{1}{5}v \rightarrow = mv_f$$

$$v_f = \frac{2}{5}v \rightarrow$$

9. (a) $72.0 \text{ kmh}^{-1} = 20.0 \text{ ms}^{-1}$

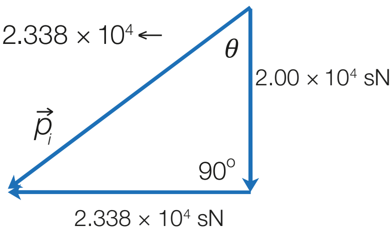
$60.0 \text{ kmh}^{-1} = 16.7 \text{ ms}^{-1}$

$p_i = m_1v_{1i} + m_2v_{2i} = 1.00 \times 10^3 \times 20.0\downarrow + 1.40 \times 10^3 \times 16.7\leftarrow = 2.00 \times 10^4\downarrow + 2.338 \times 10^4\leftarrow$

$p_i = \sqrt{(2.00\times10^4)^2 + (2.338\times10^4)^2} = 3.08\times10^4 \ sN$

$\tan\theta = \dfrac{2.338\times10^4}{2.00\times10^4} \quad \therefore \quad \theta = 49.5°$

$\vec{p}_i = 3.08\times10^4 \ sN \quad S49.5°W$



(b) Using the law of conservation of momentum

$\vec{p}_f = \vec{p}_i = 3.08\times10^4 \ sN \qquad S49.5°W$

(c) Total mass = 2.40×10^3 kg

$p_f = m_{total}v_f = 3.08 \times 10^4$

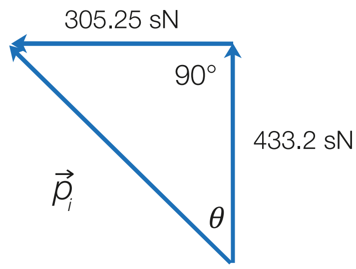
$\therefore v_f = \dfrac{3.08 \times 10^4}{2.40 \times 10^3} = 12.8 \ ms^{-1} \ S49.5°W$

10. (a) $p_i = m_1v_{1i} + m_2v_{2i} = 60.0 \times 7.22\uparrow + 55.0 \times 5.55\leftarrow = 433.2\uparrow + 305.25\leftarrow$

$p_i = \sqrt{(433.2)^2 + (305.25)^2} = 529.9 = 530 \ sN$

$\tan\theta = \dfrac{305.25}{433.2} \quad \therefore \quad \theta = 35.2°$

$\vec{p}_i = 530 \ sN \quad N35.2° W$



(b) Total mass = 115 kg

$p_f = p_i = m_{total}v_f = 530$

$\therefore \ v_f = \dfrac{530}{115} = 4.61 \ ms^{-1} \quad N35.2° W$

11. (a) $p_i = m_3v_{3i} + m_uv_{ui} = 3.00 \times 4.00\uparrow = 12.0 \ sN \ \uparrow$

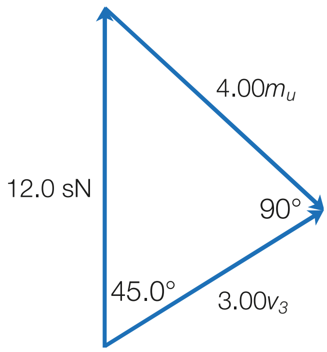
(b) $p_f = p_i = 12.0 \ sN \ \uparrow$

$\cos45 = \dfrac{3.00v_3}{12.0} \ \therefore \ v_3 = 2.83 \ ms^{-1}$

(c) $\sin45.0 = \dfrac{4.00m_u}{12.0} \therefore \ m_u = 2.12 \ kg$

(d) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m_u\vec{v}_f - m_u\vec{v}_i = 2.12 \times 4.00 - 0 = 8.48 \ sN$

(e) $\vec{F}_u = \dfrac{\Delta\vec{p}_u}{\Delta t} = \dfrac{8.48}{0.0500} = 170 \ N$



12. (a) $p_i = m_1v_{1i} + m_2v_{2i} = 6.0\times15\rightarrow = 90 \ sN \ \rightarrow$

$p_f = p_i = 90 \ sN \ \rightarrow$

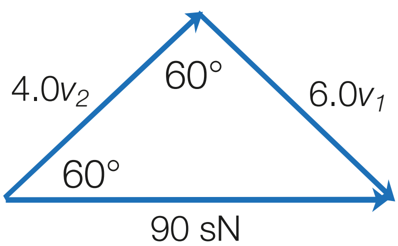
The triangle is equilateral

$6.0v_1 = 90$

$v_1 = 15 \ ms^{-1}$

(b) $4.0v_2 = 90$

$v_2 = 23 \ ms^{-1}$



13. (a) $p_f = p_i = 0\ sN$

(b) $\vec{p}_i = \vec{p}_f$

$0 = m_A v_A + m_B v_B + m_C v_C$

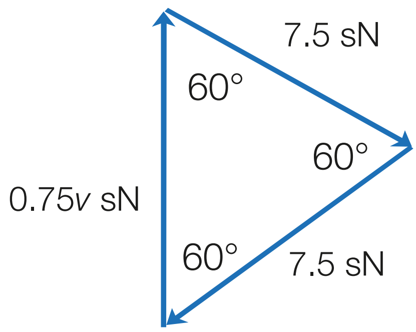
$0 = 0.75v \uparrow + 0.3 \times 25 \searrow + 0.5 \times 15 \swarrow$

$0 = 0.75v \uparrow + 7.5 \searrow + 7.5 \swarrow$

The triangle is equilateral

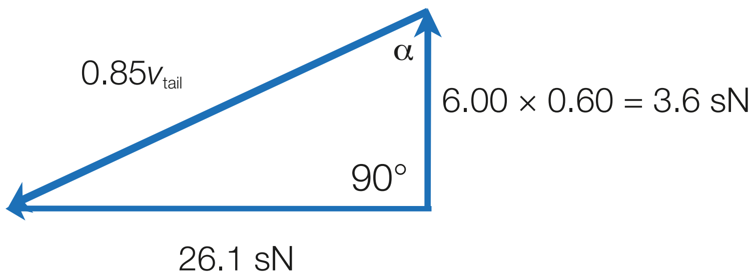
$0.75v = 7.5$

$v = 10\ ms^{-1}$



14. (a) $p_i = mv = 1.45 \times 18.0 \leftarrow = 26.1\ sN \leftarrow$

(b)

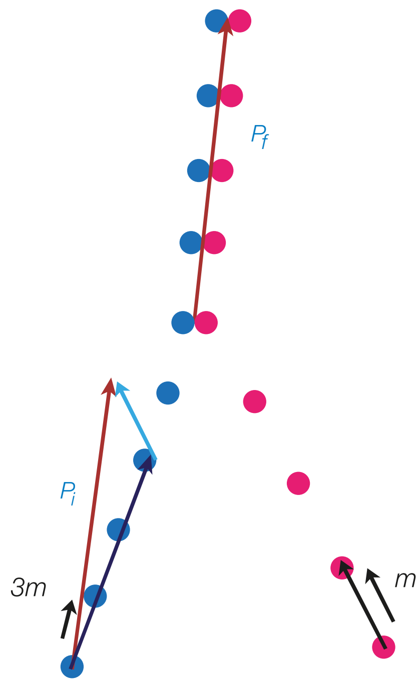


$\tan\alpha = \frac{26.1}{3.6} \quad \therefore \quad \alpha = 82.1°$

$\alpha = 180 - 82.1 = 97.9 = 98°$

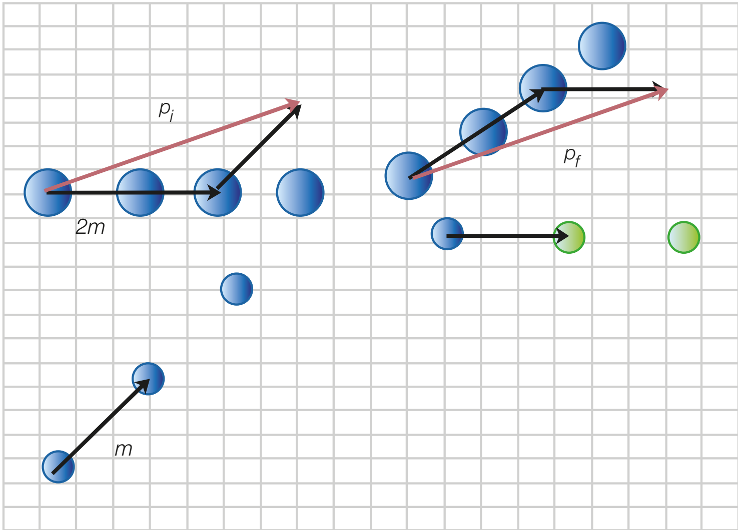
(c) $\sin 82.1 = \frac{26.1}{0.85 v_{tail}} \quad \therefore \quad v_{tail} = 31.0\ ms^{-1}$

15.

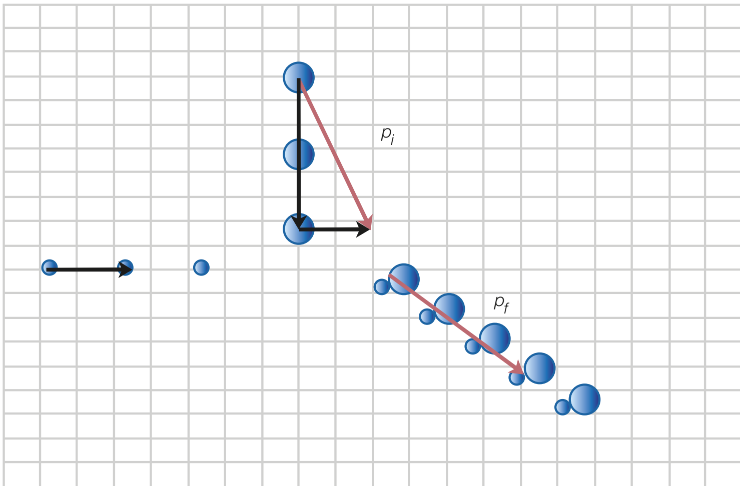


The vector arrow that represents the total initial momentum p_i is equal in length and parallel to the vector arrow that represents the total final momentum p_f. Momentum is conserved during this collision.

16.



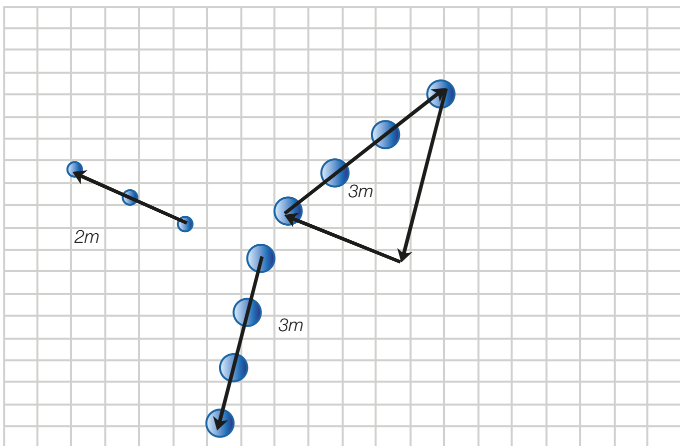
17. (a)



The vector arrow that represents the total initial momentum p_i is not equal in length nor parallel to the vector arrow that represents the total final momentum p_f. Momentum is not conserved during this collision.

(b) The system is not isolated.

18.



The total initial momentum *is zero.* The vector triangle that represents the total final momentum indicates that it is zero. Momentum is conserved during this explosion.

S

19. (a) The spacecraft and its onboard propulsion gas can be considered an isolated system. When the spacecraft ejects charged particles with a large velocity from its rear it will gain momentum and accelerate in the opposite direction to the ejected charged particles. This is because the charged particles gain momentum towards the rear of the spacecraft. Using the law of conservation of momentum, the total momentum of the system must remain constant. As a consequence, the spacecraft gains an equal magnitude of momentum but in the opposite direction to the charged particles. A gain in momentum leads to a gain in speed in the opposite direction to the ionised particles .

(b) Using the law of conservation of momentum

$0 = m_{spacecraft\text{-}ejectedparticles}\vec{v}_{spacecraft\text{-}ejectedparticlef} + m_{particles}\vec{v}_{particlesf}$

$0 = (5.00\times10^4 - m)\,5.00\rightarrow + m\times 50.0\leftarrow$

$0 = 25.0\times10^4\rightarrow -5.00m\rightarrow +50.0m\leftarrow$

$0 = 25.0\times10^4\rightarrow +5.00m\leftarrow +50.0m\leftarrow$

$m = \dfrac{25.0\times10^4}{55.0} = 4550\ kg$

20. (a) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 2p = 2\times2.0\times10^{-27} = 4.0\times10^{-27}\ kgms^{-1}$ (90° *away from the sail*)

(b) $\Delta\vec{p}_{total} = n\Delta\vec{p}_{photon} = 5.0\times10^{20}\times4.0\times10^{-27} = 2.0\times10^{-6}\ kgms^{-1}$ (90° *away from the sail*)

(c) $\vec{F}_{sail} = \dfrac{\Delta\vec{p}_{total}}{\Delta t} = \dfrac{2.0\times10^{-6}}{1} = 2.0\times10^{-6}N$

(d) $a_{sail} = \dfrac{F_{total}}{m} = \dfrac{2.0\times10^{-6}}{850} = 2.4\times10^{-9}\ ms^{-2}$ *in the opposite direction to the ionised particles*

(e) The magnitude of the acceleration would be half that calculated in part (d) i.e. $1.2\times10^{-9}\ ms^{-2}$.

21. (a) When photons are reflected from the solar sail they will experience a change in momentum because their direction of motion has changed. In accordance with the law of conservation of momentum, the total momentum of the system remains unchanged. The solar sail will therefore experiences the same change in momentum (Δp_{sail}) as the incident photons but in the opposite direction. The force on the solar sail is therefore given by $F_{sail} = \dfrac{\Delta p_{sail}}{\Delta t}$ where Δt is the time over which the reflections occurs. In accordance with Newton's Second Law, the solar sail will experience an acceleration given by $a_{sail} = \dfrac{F_{sail}}{m_{sail}}$.

(b) An absorbent solar sail will absorb the incident photons of light. The change in momentum experienced by these photons will be half that experienced by photons that are reflected. The force on the solar sail with be half as large so that acceleration will be half as large.

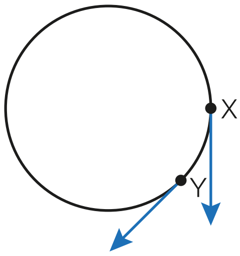
1.3 Circular motion and gravitation

Circular motion

1. (a) Uniform circular motion refers to motion in a circular path at constant speed.

   (b) A centripetal acceleration is an acceleration that acts on an object undergoing uniform circular motion. A centripetal acceleration acts at right angles to the velocity of the object undergoing uniform circular motion and is directed towards the centre of the circular path.

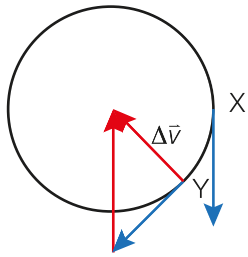
2. (a)



   (b) As the object moves from point X to Y, the speed is constant but the direction changes. This constitutes a change in velocity $\Delta\vec{v}$.

   Since $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, it can be seen from the adjacent vector triangle that $\Delta\vec{v}$ points towards the centre of the circle.

   Since $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ then it follows that the acceleration also points towards the centre of the circular path.



3. (a) The normal force.

   (b) (i) The period is the time taken for the marble to make one complete revolution of the bowl.

   (ii) $T = \frac{8.50}{10} = 0.850\ s$

   (c) $v = \frac{2\pi r}{T} = \frac{2\pi \times 0.190}{0.850} = 1.40\ ms^{-1}$

   (d) $F = \frac{mv^2}{r} = \frac{0.0200 \times 1.40^2}{0.190} = 0.206\ N$ towards the centre of the circular path

4. (a) $v = \frac{2\pi r}{T} \quad \therefore \quad T = \frac{2\pi r}{v} = \frac{2\pi \times 0.54}{(5.0)} = 0.68\ s$

   (b) $a = \frac{v^2}{r} = \frac{5.0^2}{0.54} = 46\ ms^{-2}$ towards the centre of the circular path

   (c) $F = ma = 0.026 \times 46 = 1.2\ N$

   (d) Increasing the speed of the rubber stopper being whirled increases the tension force required to keep the rubber stopper in a circular path ($F \alpha v^2$).

   If the maximum tension force that the string can provide is exceeded, the string will snap.

5. (a) $a = \frac{v^2}{r} = \frac{15^2}{8.0} = 28\ ms^{-2}$ towards the centre of the circular path

   (b) $F = ma = 74 \times 28 = 2072 = 2.1 \times 10^3\ N$

   (c) $v = \frac{2\pi r}{T}\ \therefore\ T = \frac{2\pi r}{v} = \frac{2\pi \times 8.0}{15} = 3.4\ s$

   The time for 10 revolutions = 34 s

6. (a) $a = \frac{v^2}{r} \quad \therefore \quad a \alpha v^2$ since r is constant

   Increasing the speed to $5v$ would increase the centripetal acceleration by a factor of 25.

   $a = 6.0 \times 25 = 150\ ms^{-2}$

   (b) $a = \frac{v^2}{r} \quad \therefore \quad a \alpha \frac{1}{r}$ since v is constant

   If the radius of the circular path is quartered, the centripetal acceleration becomes 4 times longer.

   $a = 6.0 \times 4 = 24\ ms^{-2}$

7. $F = ma = \frac{mv^2}{r} = \frac{1250 \times 17.0^2}{85.0} = 4250\ N$

8. (a) $F = W = mg = 3.75 \times 9.80 = 36.8\ N$

(b) $F = ma = \frac{mv^2}{r} \therefore v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{36.8 \times 0.400}{0.0500}} = 17.2\ ms^{-1}$

(c) $v = \frac{2\pi r}{T} \therefore T = \frac{2\pi r}{v} = \frac{2\pi \times 0.400}{17.2} = 0.146\ s$

9. (a) $F = ma = \frac{mv^2}{r} \therefore F \alpha m\ \ r, v$ constant

If the mass is tripled, so is the force.

New force $3F$

(b) $F = ma = \frac{mv^2}{r} \therefore F \alpha v^2\ \ m, r$ constant

If the speed is reduced by a factor of four, the force is reduced by a factor of 16.

New force $\frac{F}{16}$

(c) $F = ma = \frac{mv^2}{r} \therefore F \alpha \frac{1}{r}\ \ m, v$ constant

If the radius is halved, the force is doubled.

New force $2F$

10. (a) The speed of the satellite is given by $v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

therefore $r^3 = \frac{GMT^2}{4\pi^2}$

$$\therefore r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (3.0 \times 60 \times 60)^2}{4\pi^2}} = 1.06 \times 10^7 m$$

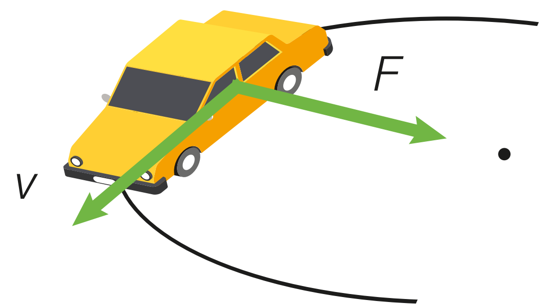
Height $= r - r_E = 1.06 \times 10^7 - 6.37 \times 10^6 = 4.2 \times 10^6$ m

(b) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{1.06 \times 10^7}} = 6.1 \times 10^3\ ms^{-1}$

$F = \frac{mv^2}{r} = \frac{1500 \times (6.1 \times 10^3)^2}{1.06 \times 10^7} = 5.3 \times 10^3\ N$

(c) Gravitational force

11. (a)



(b) 60.0 $kmh^{-1} = 16.7\ ms^{-1}$

$F = \frac{mv^2}{r} = \frac{1600 \times (16.7)^2}{250} = 1.8 \times 10^3\ N$ towards the centre of the circular path

(c) $F = ma = \frac{mv^2}{r} \therefore v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{4000 \times 250}{1600}} = 25\ ms^{-1}$

(d) When the road is banked, the car's normal force is no longer vertical. The normal force has two components. While the vertical component is equal in magnitude but opposite in direction to the weight of the car, the horizontal component points towards the centre of the circular path and provides some or all of the centripetal acceleration needed for uniform circular motion.

12. (a) $r = 3.70 \times 10^7 + 6.37 \times 10^6 = 4.337 \times 10^7$ m

$$v = \frac{2\pi r}{T} = \frac{2\pi \times (4.337 \times 10^7)}{(24.0 \times 60 \times 60)} = 3.15 \times 10^3 \ ms^{-1}$$

(b) $a = \frac{v^2}{r} = \frac{(3.15 \times 10^3)^2}{4.337 \times 10^7} = 0.229 \ ms^{-2}$ towards the centre of the Earth

(c) $F = ma = 1.00 \times 10^3 \times 0.229 = 229\ N$ towards the centre of the Earth

(d) Gravitational force

13. (a) A Friction

B Normal force

C Weight

(b) $T = \frac{60}{24} = 2.5\ s$

(c) A and C friction = weight = $mg = 67 \times 9.80 = 660\ N$

B $v = \frac{2\pi r}{T} = \frac{2\pi \times 6.6}{2.5} = 16.6 \ ms^{-1}$

Normal force $F = \frac{mv^2}{r} = \frac{67 \times (16.6)^2}{6.6} = 2.8 \times 10^3\ N$

14. (a)

$$\frac{a_1}{a_2} = \frac{\frac{v^2}{r_1}}{\frac{v^2}{r_2}} = \frac{r_2}{r_1} = \frac{1}{3}$$

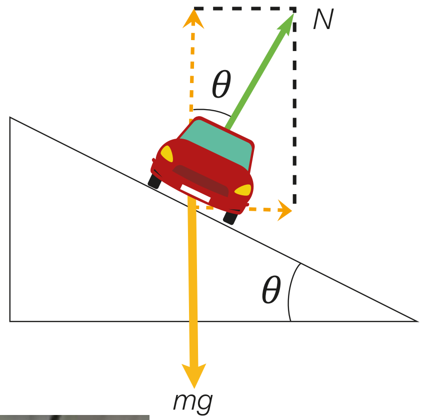
(b)

$$\frac{T_1}{T_2} = \frac{\frac{2\pi r_1}{v}}{\frac{2\pi r_2}{v}} = \frac{r_1}{r_2} = 3$$

15. (a) The vertical component (N_v) of the normal has a magnitude of mg. If the horizontal component (N_H) points towards the centre of the circular path and provides all of the centripetal acceleration, then

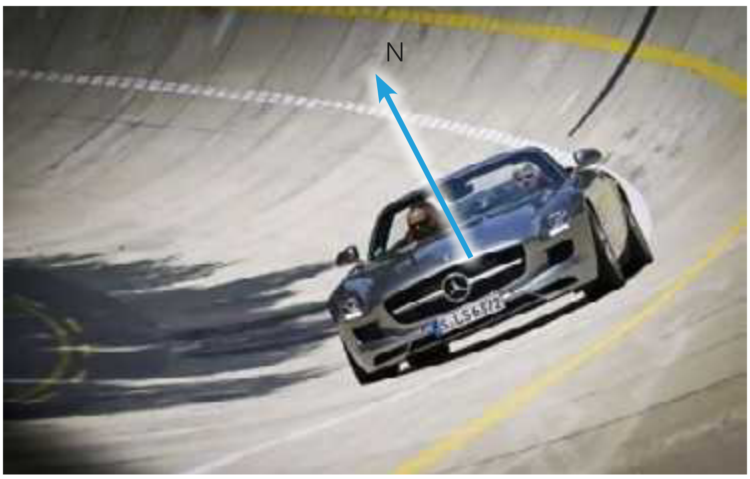
$N_H = ma = \frac{mv^2}{r}$.

It follows that $\tan\theta = \frac{N_H}{N_v} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$ and $\theta = \tan^{-1}(\frac{v^2}{rg})$



(b) $Tan\theta = \frac{v^2}{rg} = \frac{25^2}{120 \times 9.8}$ $\therefore$ $\theta = 28°$

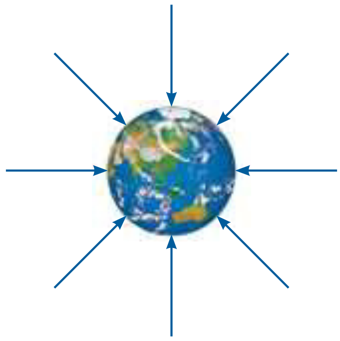
16. (a)



(b) The vertical component of the normal has a magnitude equal to the weight of the car, but acts in the opposite direction (up). It does not affect the motion of the car. The horizontal component points towards the centre of the circular path and provides all of the centripetal acceleration causing the car to travel with uniform circular motion around the banked curve.

Gravitation

1. (a)



(b) The gravitational field strength g at a point is the net force per unit mass at a particular point in the field.

(c) $g = \frac{F}{m} = \frac{37.4}{3.40} = 11.0 \ Nkg^{-1}$

2. $g = \frac{F}{m} \therefore F = mg = 3.7 \times 10.0 = 37\,N$

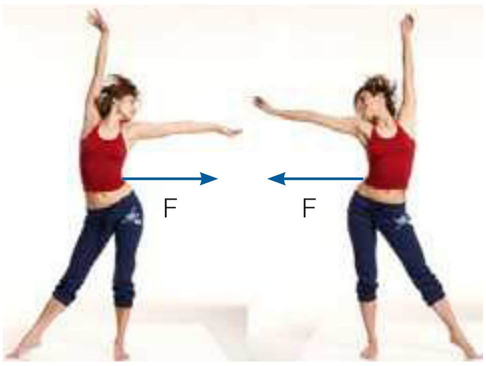
3. (a) The gravitational force of *attraction* between any two masses is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres.

(b) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 20.0 \times 55.0}{1.80^2} = 2.26 \times 10^{-8}\ N\ attraction$

(c) Both masses experience a force of the same magnitude but the force on each mass acts in opposite directions. This means that gravitational forces are consistent with Newton's Third Law which states that if object A exerts a force on object B, then object B exerts an equal and opposite force on object A.

4. (a) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 62.0 \times 55.0}{1.00^2} = 2.27 \times 10^{-7}\ N\ attraction$

(b)



(c) $a = \frac{F}{m} = \frac{2.27 \times 10^{-7}}{62.0} = 3.66 \times 10^{-9} \ ms^{-2}$ towards the 55.0 kg dancer

5. (a) $F = \frac{Gm_1m_2}{r^2} \ \therefore \ m^2 = \frac{Fr^2}{G} \ \therefore m = \sqrt{\frac{5.0 \times 0.050^2}{6.67 \times 10^{-11}}} = 1.4 \times 10^4 kg$

(b) (i) $F \alpha m_1 \quad r, m_2$ constant

If one mass is halved so is the force.

(ii) $F \alpha m_1 m_2 \quad r$ constant

The force becomes $\frac{1}{2} x 6 = 3$ times larger.

(iii) $F \alpha \frac{1}{r^2} \quad m_1, m_2$ constant

If the distance is 4 times smaller, the force becomes 16 times larger.

6. (a) 4F

(b) 4F

7. (a) $F = \frac{Gm_1m_2}{r^2} \ \therefore \ r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.0 \times 10^2}{3.7 \times 10^{-9}}} = 67\ m$

(b) $F \alpha \frac{1}{r^2} \quad m_1, m_2$ constant

If the force is 4 times larger, the distance must have halved.

8. (a) Using Newton's Second Law and the Universal Law of Gravitation, for an object of mass m placed at the surface of the Earth:

$$F = ma = mg = \frac{Gm_1m_2}{r^2} \quad \therefore \quad mg = \frac{GmM_E}{r_E^2} \quad \therefore \quad g = \frac{GM_E}{r_E^2}$$

(b) $g = \frac{GM_E}{r_E^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2} = 9.81 \ ms^{-2}$

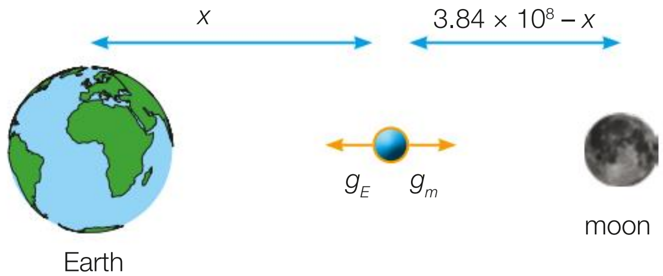
(c) $r = 3.6 \times 10^4 \times 10^3 + 6.37 \times 10^6 = 4.24 \times 10^7$ m

$$g = \frac{GM_E}{r_{orbit}^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(4.24 \times 10^7)^2} = 0.22 \ ms^{-2}$$

9. (a) The gravitational field strength g at a point is the net force per unit mass at a particular point in the field ($g = \frac{F}{m}$). According to Newton's Second Law $a = \frac{F}{m}$. This means that the gravitational field strength is equivalent to the gravitational acceleration.

(b) $g = \frac{GM}{r^2} \ \therefore r = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67 \times 10^{-11} \times 4.90 \times 10^{24}}{8.90}} = 6.06 \times 10^6 \ m$

10.



Consider a mass placed at some point x m from the centre of the Earth. The gravitational fields due to the Earth and moon act in opposite directions. For $g = 0$ then

$$g_E = g_m$$

$$\frac{GM_E}{x^2} = \frac{GM_m}{(3.84 \times 10^8 - x)^2}$$

$$\frac{\sqrt{M_E}}{x} = \frac{\sqrt{M_m}}{(3.84 \times 10^8 - x)}$$

$$\frac{\sqrt{5.97 \times 10^{24}}}{x} = \frac{\sqrt{7.35 \times 10^{22}}}{(3.84 \times 10^8 - x)}$$

rearranging gives $x = 3.46 \times 10^8 m$

11. (a) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1.30 \times 10^3 \times 5.97 \times 10^{24}}{(5.64 \times 10^7)^2} = 163 \ N$

(b) The gravitational force between the space vehicle and the Earth provides the centripetal acceleration required for uniform circular motion.

$$F = \frac{GmM}{r^2} = \frac{mv^2}{r} \quad \therefore \quad \frac{GM}{r} = v^2 \quad v = \sqrt{\frac{GM}{r}}$$ (Where M is the mass of the Earth)

(c) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{5.64 \times 10^7}} = 2.66 \times 10^3 \ ms^{-1}$

(d) The speed of the satellite is given by $v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$

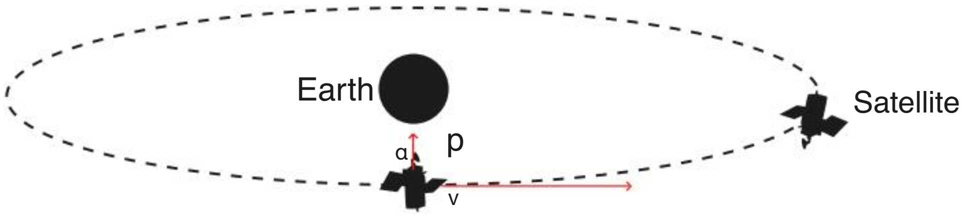
Squaring both sides of the equation gives: $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$

It follows that $GMT^2 = 4\pi^2 r^3 \ \therefore \ T^2 = \frac{4\pi^2}{GM} r^3 \ \therefore \ T = \sqrt{\frac{4\pi^2 r^3}{GM}}$

(e) $T \alpha \sqrt{r^3}$

If the radius of orbit is doubled, the period changes by a factor of $\sqrt{2^3} = 2.83$

12. (a)



This diagram is not to scale

(b) $v=\sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.37\times10^{6}+8000\times10^{3})}} = 5.26\times10^{3}\ ms^{-1} = 5000\ ms^{-1}$

(c) $v\alpha\frac{1}{\sqrt{r}}$ therefore $\frac{v_A}{v_B}=\frac{\sqrt{r_B}}{\sqrt{r_A}}=\frac{\sqrt{\frac{1}{2}}}{\sqrt{1}}=\sqrt{\frac{1}{2}}=0.707$

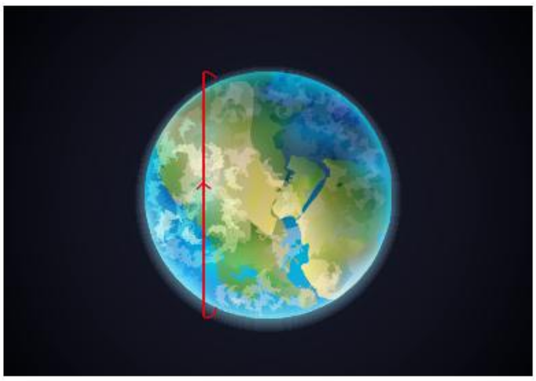
(d) $T=\frac{2\pi r}{v}=\frac{2\pi\times(6.37\times10^{6}+8000\times10^{3})}{5260}=1.71\times10^{4}\ s = 20\ 000\ s$

13. (a) $v=\sqrt{\frac{GM}{r}}\ \therefore\ M=\frac{v^2 r}{G}=\frac{(1.6\times10^{3})^2\times8.44\times10^{6}}{6.67\times10^{-11}} = 3.2\times10^{23}\ kg$

(b) $F=\frac{mv^2}{r}=\frac{850\times(1.6\times10^{3})^2}{8.44\times10^{6}}=260\ \ N$ towards the centre of Mercury

14. (a) The gravitational force between the satellite and the Earth acts towards the centre of the Earth. It must also act towards the centre of the orbit in order to provide the centripetal acceleration for uniform circular motion. The centre of the orbit must coincide with the centre of the Earth for the orbit to be stable.

(b)



(c) Low-altitude polar-orbit satellites are useful for meteorology because they pass over the North and South poles of the Earth many times a day and scan a full picture of the Earth's surface, one strip at a time as the Earth rotates beneath the satellite. Bad weather conditions can be detected and precautions taken.

15. (a) A geostationary satellite is one which remains fixed over one point of the Earth's surface. The Earth rotates from West to East. The only way that a satellite can remain fixed above one position on the Earth's surface is to move in the same direction as the Earth.

(b) A satellite in a geostationary orbit must have an orbit in the Earth's equatorial plane if it is to move in an Easterly direction and remain in a stable orbit. This is because the centre of the orbit must coincide with the centre of the Earth.

(c) Period = 24 h

$T^2=\frac{4\pi^2}{GM}r^3\ \therefore\ r=\sqrt[3]{\frac{GMT^2}{4\pi^2}}=\sqrt[3]{\frac{6.67\times10^{-11}\times5.97\times10^{24}\times(24\times60\times60)^2}{4\pi^2}}=4.22\times10^{7}m$

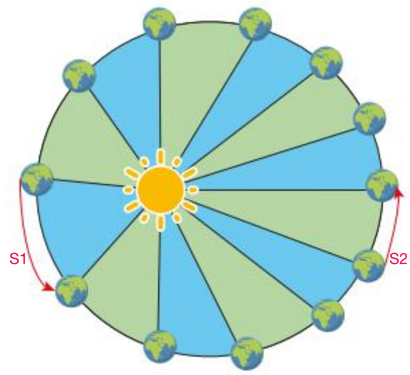
(d) Altitude = $r-r_E = 4.22\times10^{7}-6.37\times10^{6}=3.58\times10^{7}$ m

16. (a) (i) 1st Law: All planets move in elliptical orbits with the Sun at one focus.

(ii) 2nd Law: The radius vector drawn from the Sun to a planet sweeps equal areas in equal time intervals.

(iii) 3rd Law: The period of revolution of a satellite squared is proportional to the radius of orbit cubed.

(b)



The first law describe the motion of planets around the Sun as elliptical.

The second law describes that a planet is faster when it is closer to the Sun and slower when it is further away from the Sun. This is because the radius vector drawn from the planet to the Sun will sweep out an equal area in an equal time. The diagram shows that the distance travelled by the planet when it is close to the Sun (S1) is greater than the distance travelled by the planet when it is further from the Sun (S2). Since the time is the same, the planet must be travelling faster when it is closer to the Sun.

17. (a) $gradient = \frac{rise}{run} = \frac{6.60\times10^{27}}{2.05\times10^{12}} = 3.22\times10^{15}\ m^3s^{-2}$

(b) $T^2 = \frac{4\pi^2}{GM}r^3 \quad \therefore \quad r^3 = \frac{GM}{4\pi^2}T^2$

The gradient = $\frac{GM}{4\pi^2}$

M = $\frac{gradient\times4\pi^2}{G} = \frac{3.22\times10^{15}4\pi^2}{6.67\times10^{-11}} = 1.91\times10^{27}\ kg$

(c) Accuracy is a measure of how close the experimental value is to an accepted value.

1.91×10^{27} kg is very accurate as it is very close to 1.90×10^{27} kg.

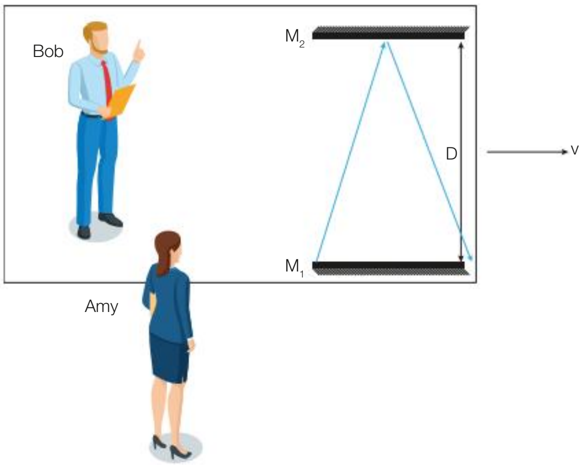
(d) The plotted points lie on or very close to the line of best fit, the data is precise. i.e. there is very little/no scatter in the plotted points.

18. Science as a human endeavour activity – Gravitation 2:

The development of theories often requires a wide range of evidence from more than one source. An impact crater on Proteus was discovered by Voyager 2 images in 1989. Initially, it was believed to be just a crater as suggested by the quote "In 1989, we thought the crater was the end of the story," said Showalter. By using recent evidence from the Hubble Space telescope, it is now known that a small piece of Proteus was left behind, forming Hippocamp. Planetary scientists have therefore used NASA's Hubble Space Telescope to develop an explanation for the mysterious moon (Hippocamp) around Neptune. This study using NASA's Hubble Space Telescope involves a team of astronomers. The team consists of M. Showalter (SETI Institute), I. de Pater (University of California), J. Lissauer (NASA Ames Research Center), and R. French (SETI Institute). This shows that clear communication and collaboration between scientists is required in scientific research. By sharing their ideas, expertise and their analysis of the data an explanation for Hippocamp was finally made possible.

1.4 Relativity

1. (a) 1.2 ms^{-1} to the left
   (b) 13.8ms^{-1} to the right
2. (a) 0 ms^{-1}
   (b) 5 ms^{-1} to the right
   (c) 10 ms^{-1} to the left
3. An inertial frame of reference is one in which Newton's laws of motion apply. An inertial frame is either stationary or moving with a constant velocity.
4. The Theory of Special Relativity is based on two postulates.
   - The speed of light in a vacuum is an absolute constant. That is, it is the same for all inertial frames of reference.
   - The laws of physics are the same in all inertial reference frames.
5. (a)



   (b) Amy sees the pulse of light travel further than Bob. Since the speed of light is constant for all inertial observers, then Amy sees the pulse of light take longer to reach M_2.

   (c) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.500c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.500^2}} = 1.15$

   The light pulse takes 1.15t to reach to M_2 according to Amy.

6. $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.85c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.85^2}} = 1.9$

   $t = \gamma t_0 = 1.9 \times 30 = 57\ s$

7. (a) The observer on the space station is stationary relative to the space probe. The signal time is dilated to this observer. The light signal is observed for a time interval greater than 10.0 s.

   (b) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.600c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.600^2}} = 1.25$

   $t = \gamma t_0 = 1.25 \times 10.0 = 12.5\ s$

8. (a) $\dfrac{2.7 \times 10^8}{3.00 \times 10^8} = 0.90 \quad \therefore \quad 2.7 \times 10^8\ ms^{-1} = 0.90c$

   $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.90c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.90^2}} = 2.3$

   (b) $t = \gamma t_0 = 2.3 \times 3.0 = 6.9\ s$

9. The twin that took the journey is moving relative to the twin that remains on Earth. The time that the travelling twin measures having passed on their journey as observed by the twin that stays on Earth is dilated. The twin on Earth thinks that on the return of the twin that took the journey a greater period of time has passed.

10. (a) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.400c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.400^2}} = 1.09$

$l = \dfrac{l_0}{\gamma} = \dfrac{20.0}{1.09} = 18.3\ m$

(b) The situation is symmetrical. Peter with observe Jane's spacecraft to be 18.3 m long.

11. (a) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.600c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.600^2}} = 1.25$

(b) One of the postulates of the Theory of Special Relativity states that the speed of light is an absolute constant. The speed recorded is 3.00×10^8 ms^{-1}.

(c) (i) L

(ii) $\dfrac{L}{1.25}$

12. (a) $\dfrac{1.6 \times 10^8}{3.00 \times 10^8} = 0.53 \quad \therefore \quad 1.6 \times 10^8\ \text{ms}^{-1} = 0.53c$

(b) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.53c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.53^2}} = 1.2$

$l = \dfrac{l_0}{\gamma} = \dfrac{2.6}{1.2} = 2.2\ m$

(c) $l = \dfrac{l_0}{\gamma} \quad \therefore \quad \gamma = \dfrac{l_0}{l} = \dfrac{2.6}{2.0} = 1.3$

$\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \quad \therefore \quad \gamma^2 = \dfrac{1}{1-\dfrac{v^2}{c^2}} \qquad 1-\dfrac{v^2}{c^2} = \dfrac{1}{\gamma^2} \quad \therefore \quad \dfrac{v^2}{c^2} = 1-\dfrac{1}{\gamma^2} = 1-\dfrac{1}{1.3^2} = 0.41$

Since $\dfrac{v^2}{c^2} = 0.41$ it follows that $v = 0.64c$ or $1.9 \times 10^8\ ms^{-1}$

13. The cylindrical drum is length contracted in the direction that the rocket moves. The horizontal motion of the rocket will only affect the width of the cylindrical drum but not its height. The cylindrical drum is thinner but it has the same height.

14. $l = \dfrac{l_0}{\gamma} = \dfrac{18}{3.2} = 5.6\ cm$

In the alpha particle's frame of reference, the laboratory observer is moving relative to the alpha particle with a speed of 0.95c. The distance that the laboratory observer moves is length contracted to 5.6 cm.

15. $l = \dfrac{l_0}{\gamma} \quad \therefore \quad \gamma = \dfrac{l_0}{l} = 2$

$\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \quad \therefore \quad \gamma^2 = \dfrac{1}{1-\dfrac{v^2}{c^2}} \qquad 1-\dfrac{v^2}{c^2} = \dfrac{1}{\gamma^2} \quad \therefore \quad \dfrac{v^2}{c^2} = 1-\dfrac{1}{\gamma^2} = 1-\dfrac{1}{2^2} = 0.75$

Since $\dfrac{v^2}{c^2} = 0.75$ it follows that $v = 0.866c$ or $2.60 \times 10^8\ ms^{-1}$

16. (a) $s = vt = 0.980 \times 3.00 \times 10^8 \times 2.20 \times 10^{-6} = 647m$

This is much less than 10.0 km – the muons are therefore unlikely to reach the Earth's surface.

(b) $t = \dfrac{s}{v} = \dfrac{10 \times 10^3}{0.980 \times 3.00 \times 10^8} = 3.40 \times 10^{-5} s$

(c) $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(0.980c)^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.980^2}} = 5.03$

$t = \gamma t_0 = 5.03 \times 2.20 \times 10^{-6} = 1.11 \times 10^{-5}\ s$

The muons are moving relative to the Earth. Their average lifespan is dilated as observed from Earth. The Lorentz factor is 5.03 so their lifespan is dilated by a factor of 5.03 to 1.11×10^{-5} s.

(d) The average lifespan of the muon is dilated. This explains why the muons can travel further.

At a speed of 0.980c it would take about three half lives ($\frac{3.40 \times 10^{-5}}{1.11 \times 10^{-5}} = 3.06$) to reach the surface of the Earth.

There would be a significant number of muons that have not decayed in this time and therefore reach the surface of the Earth.

17. (a) $\frac{2.50 \times 10^8}{3.00 \times 10^8} = 0.833 \quad \therefore \quad 2.5 \times 10^8 \text{ ms}^{-1} = 0.833c$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.833c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.833c^2}} = 1.81$$

(b)

Quantity being measured	Affect of relative motion on measurement
mass	increases
width	decreases
Time taken to bounce	increases

18. (a) $m = \gamma m_0 = 1.15 \times 1.67 \times 10^{-27} = 1.92 \times 10^{-27}\ kg$

(b) $p = mv = 1.92 \times 10^{-27} \times 0.5 \times 3.00 \times 10^8 = 2.88 \times 10^{-19}\ sN$

19. $\frac{2.8 \times 10^8}{3.00 \times 10^8} = 0.93 \quad \therefore \quad 2.8 \times 10^8 \text{ ms}^{-1} = 0.93c$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.93c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.93^2}} = 2.7$$

$p = mv = \gamma m_0 v = 2.7 \times 0.2 \times 2.8 \times 10^8 = 1.5 \times 10^8\ sN$

20. (a) Time dilation refers to the slowing down of clocks that are in motion relative to an observer.

(b) Length contraction refers to the decrease in length of an object that is in motion relative to an observer.

(c) The Lorentz factor is a ratio that indicates the factor by which length, time and mass will change in a moving frame of reference when compared to the stationary value.

(d) Rest mass m_0, is defined as the mass of an object when it is at rest relative to the observer.

21. (a) According to the Theory of Special Relativity, the mass of an object increases with speed according to the relationship $m = \gamma m_0$. At relativistic speeds the increase in mass is significant. This is because the Lorentz factor increases with speed.

(b) At the speed of light, the Lorentz factor would be infinite. This would make the mass infinite. This requires an infinite amount of energy and is therefore not possible.

22. **Science as a human endeavour activity - Relativity:**

The development of Starshot probes will enable scientists to collect data from nearby exoplanets. Due to their small size, and the use of a powerful laser beam the probes can travel at speeds of 0.2c. They are likely to replace traditional/existing spacecraft because they can travel vast distances, for example to Proxima b, in far less time (20 years as opposed to several tens of thousands of years). This may reveal new information and enable scientist to further develop their understanding of nearby exoplanets.

The application is that the probes can be used to explore nearby exoplanets with the potential to support life. This may have beneficial or unexpected outcomes that are not yet known. There are however limitations to this project. Their small size will make it hard to pack the necessary electronics on the probe, the signal received on Earth will be weak and the increase in mass due to relativistic speeds will require a more powerful laser beam. These challenges will need to be overcome to make the project a success.

2.1 Electric fields

1. $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times 1.00 \times 2.00}{0.400^2} = 1.12\times10^{11}\ N\ \textit{repulsion}$

2. (a) The electric force of attraction or repulsion between two charged objects is directly proportional to the product of the two charges and inversely proportional to the square of the distance between their centres.

   (b) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times 3.4 \times 10^{-6} \times 4.5 \times 10^{-6}}{0.080^2} = 21\ N\ \textit{attraction}$

   (c) $F \alpha \frac{1}{r^2}$ q_1, q_2 constant

   The distance between the two charges has halved, so the force will increase by a factor of 2^2 or four.

3. (a) 4F

   (b) 6F

   (c) $\frac{F}{2}$

4. (a) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r^2} = \frac{8.99 \times 10^9 \times 500 \times 10^{-3} \times 150 \times 10^{-3}}{0.300^2} = 7.5\times10^9\ N\ \textit{attraction}$

   (b) $F = 7.5 \times 10^9$ *N towards A*

   (c) $F = 7.5 \times 10^9$ *N towards B*

   (d) $F \alpha \frac{1}{r^2}$

   The distance between the two charges is reduced by a factor of 6.The force will increase by a factor of 36 to $36 \times 7.5 \times 10^9 = 2.7 \times 10^{11}$ *N towards A*

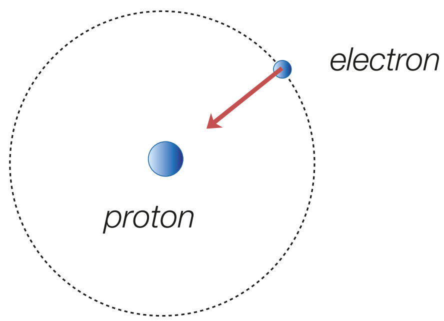
   (e) The charged spheres are conductors and will share the charge equally when they are brought into contact.

   The charge on each sphere is $\frac{500-150}{2} = +175mC$

   $F = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r^2} = \frac{8.99 \times 10^9 \times 175 \times 10^{-3} \times 175 \times 10^{-3}}{0.300^2} = 3.1\times10^9\ N$ *away from B*

5. (a) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times 1.60 \times 10^{-19} \times 1.60 \times 10^{-19}}{(5.92\times10^{-11})^2} = 6.57\times10^{-8}\ N\ \textit{attraction}$

   (b)



   (c) The electron experiences a constant electric force which always acts perpendicularly to its velocity. The electric force points towards the centre of motion and provides the centripetal acceleration for uniform circular motion.

   (d) $F = \frac{mv^2}{r}$ $\therefore$ $v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{6.57\times10^{-8}\times5.92\times10^{-11}}{9.11\times10^{-31}}} = 2.07\times10^6\ ms^{-1}$

6. (a) When more than two point charges are present, the force on any one of them is a vector sum of the electric forces acting due to each of the other point charges present.

(b) (i) Force due to q_1 $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_1^2} = \frac{8.99\times10^9\times2.50\times10^{-3}\times1.00\times10^{-3}}{0.100^2} = 2.25\times10^6\ N\ \rightarrow$

Force due to q_2 $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_2^2} = \frac{8.99\times10^9\times1.00\times10^{-3}\times1.50\times10^{-3}}{0.100^2} = 1.35\times10^6\ N\ \rightarrow$

Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 2.25\times10^6 \rightarrow +1.35\times10^6 \rightarrow = 3.60\times10^6\ N\ \rightarrow$

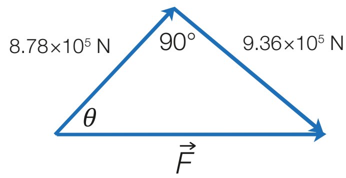
(ii) Force due to q_1 $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_1^2} = \frac{8.99\times10^9\times2.50\times10^{-3}\times1.00\times10^{-3}}{0.160^2} = 8.78\times10^5\ N\ \nearrow$

Force due to q_2 $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_2^2} = \frac{8.99\times10^9\times1.00\times10^{-3}\times1.50\times10^{-3}}{0.120^2} = 9.36\times10^5\ N\ \searrow$

$F = \sqrt{(8.78\times10^5)^2+(9.36\times10^5)^2} = 1.28\times10^6\ N$

$\theta = \tan^{-1}\left(\frac{9.36\times10^5}{8.78\times10^5}\right) = 46.8°$

$\vec{F} = 1.28\times10^6 N\ \ 46.8°$ *clockwise from the line joining q_1 and q_3*



7. (a) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{8.99\times10^9\times q\times2q}{r^2} = \frac{17.98\times10^9\times q^2}{r^2}$

$q = \sqrt{\frac{Fr^2}{17.98\times10^9}} = \sqrt{\frac{25.0\times0.0600^2}{17.98\times10^9}} = 2.24\times10^{-6} C$

(b) (i) $F \alpha\ q_1\ \ q_2$, r constant

The $-2q$ charge has been doubled to become $-4q$. The force doubles.

$F \alpha \frac{1}{r^2}\ \ q_1, q_2$ constant

The distance has halved so the force becomes 4 times larger.

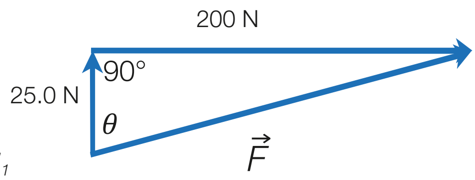
The combined effect is that the force becomes 2 × 4 = 8 times larger i.e. 25.0 × 8 = 200 N.

(ii) Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 25.0 \uparrow +200 \rightarrow$

$F = \sqrt{(25.0)^2 + (200)^2} = 202\ N$

$\theta = \tan^{-1}\left(\frac{200}{25.0}\right) = 82.9°$

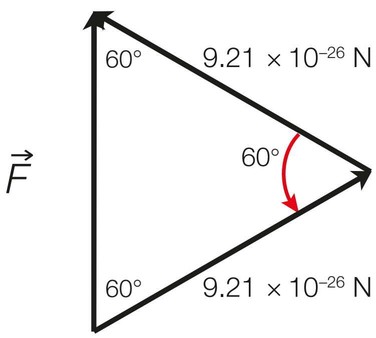
$\vec{F} = 202 N\ \ 82.9°$ *clockwise from the line joining q_2 and q_1*



8. (a) Force due to *electron 1* $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_1^2} = \frac{8.99\times10^9\times1.60\times10^{-19}\times1.60\times10^{-19}}{0.0500^2} = 9.21\times10^{-26}\ N\ \nearrow$

(b) Force due to *electron 2* $F_2 = 9.21\times10^{-26}\ N\ \nwarrow$

(c)



The vector triangle is equilateral:

$F = 9.21\times10^{-26}$ N $\uparrow$

9. (a) $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} = \frac{8.99\times10^9\times Q_1Q_2}{r^2}$

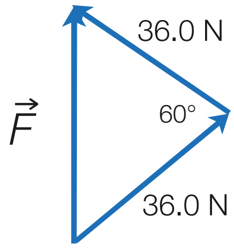
$r = \sqrt{\frac{8.99\times10^9\times Q_1Q_2}{F}} = \sqrt{\frac{8.99\times10^9\times1.58\times10^{-6}\times1.58\times10^{-6}}{36.0}} = 2.50\times10^{-2}m$

(b) Total force $\vec{F} = \vec{F}_1 + \vec{F}_2$

i.e. 36.0 ↗ + 36.0 ↖

The vector triangle is equilateral

F = 36.0 N North



10. (a) The electric field at a point is defined as the electric force per unit charge experienced by a small positive test charge when placed at that point in the field.

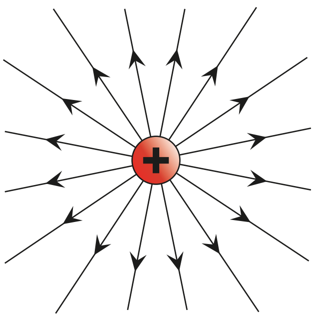
(b) $E = \frac{F}{q} = \frac{2.8}{5.00\times10^{-6}} = 5.6\times10^5\ NC^{-1}\ South$

11. (a) $F = Eq = 1800\times3.2\times10^{-19} = 5.8\times10^{-16}\ N$ *down the plane of the page*

(b) $a = \frac{F}{m} = \frac{5.8\times10^{-16}}{6.645\times10^{-27}} = 8.7\times10^{10}\ ms^{-2}$ *down the plane of the page*

(c) $F\alpha q$ An electron has half the charge of an alpha particle so it would experience half the force. The direction of the electric field is given by the direction of the force experienced by a positive test charge. Since the electron has a negative charge it would move anti-parallel to the electric field.

12. (a)



(b) $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{8.99\times10^9\times4.5\times10^{-6}}{0.100^2} = 4.05\times10^6\ NC^{-1}$ *radially outwards*

(c) Since $E\alpha\frac{1}{r^2}$ it follows that if the distance from the charge creating the electric field is increased by a factor of four, then the electric field becomes 16 times smaller i.e. $2.53\times10^5\ NC^{-1}$

13. (a) $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{8.99\times10^9\times q}{r^2}\ \therefore q = \frac{Er^2}{8.99\times10^9} = \frac{70.0\times0.400^2}{8.99\times10^9} = 1.24\times10^{-9}\ C$

(b) $E\alpha\frac{1}{r^2}$

As the shark approaches the surfboarder the electric field becomes stronger and the discomfort to the shark increases to the point that it is unbearable.

14. (a) $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{8.99\times10^9\times6.00\times10^{-4}}{0.200^2} = 1.35\times10^8\ NC^{-1}$ *radially inwards*

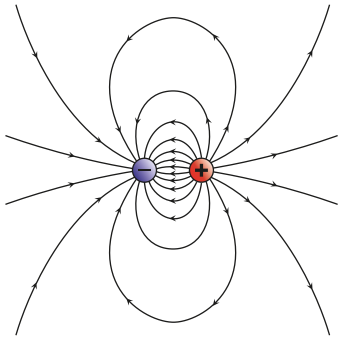
(b) $E\alpha\frac{1}{r^2}$

The electric field at a distance three times greater would be 9 times smaller i.e. $1.50\times10^7\ NC^{-1}$

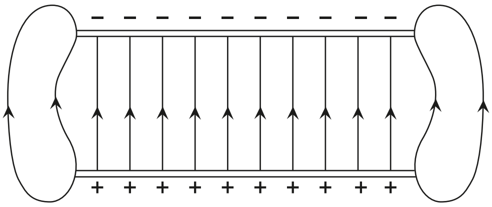
(c) *number of electrons* $= \frac{q_{total}}{q_{electron}} = \frac{6.00\times10^{-4}}{1.60\times10^{-19}} = 3.75\times10^{15}$

S

15. (a)



(b)



16. (a) E field due to q_1 $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{8.99\times10^9\times375\times10^{-9}}{0.5^2} = 1.35\times10^4\ NC^{-1} \leftarrow$

E field due to q_2 $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} = \frac{8.99\times10^9\times262\times10^{-9}}{0.5^2} = 9.42\times10^3\ NC^{-1} \rightarrow$

$\vec{E}_p = 1.35\times10^4 \leftarrow + 9.42\times10^3 \rightarrow = 4.08\times10^3\ NC^{-1} \leftarrow$

(b) $\vec{E}_1 = \vec{E}_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}$$

$$\frac{8.99\times10^9\times375\times10^{-9}}{r^2} = \frac{8.99\times10^9\times262\times10^{-9}}{(1-r)^2}$$

$$\frac{375}{r^2} = \frac{262}{(1-r)^2}$$

$$375(1-r)^2 = 262r^2$$

$$\sqrt{375}(1-r) = \sqrt{262}r$$

$$r = 0.545m$$

17. The electric field at A due to Q_1 acts to the right. The electric field at A due to Q_2 also acts to the right. Since the total electric field at the point A is a vector sum of the electric fields at A due to Q_1 and Q_2, the two fields can never cancel to zero as they act in the same direction.

18. (a) $E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{8.99\times10^9\times10^{-6}}{0.100^2} = 8.99\times10^6\ NC^{-1}$

(b) The magnitude of the electric field is four times greater. Since $E\alpha\frac{1}{r^2}$, then the distance from the charge is halved i.e. 5.0 cm

(c) E field due to q_1 $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{8.99\times10^9\times10.0\times10^{-6}}{0.3^2} = 1.00\times10^6\ NC^{-1} \leftarrow$

E field due to q_2 $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} = \frac{8.99\times10^9\times5.00\times10^{-6}}{0.0500^2} = 1.80\times10^7\ NC^{-1} \rightarrow$

$\vec{E}_x = 1.00\times10^6 \leftarrow + 1.80\times10^7 \rightarrow = 1.70\times10^7\ NC^{-1} \rightarrow$

19. (a) $E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{r^2} = \dfrac{8.99 \times 10^9 \times 1.2 \times 10^{-6}}{(0.15)^2} = 4.8 \times 10^5\ NC^{-1}$

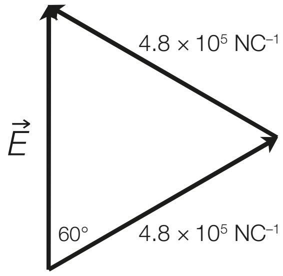
(b) $E_2 = 4.8 \times 10^5\ NC^{-1}$

(c) $\vec{E}_1 = 4.8 \times 10^5\ NC^{-1}$ ↗

$E_2 = 4.8 \times 10^5\ NC^{-1}$ ↖

Equilateral Triangle

$\vec{E}_{Total} = 4.8 \times 10^5\ NC^{-1}$ ↑



20. $\vec{E}_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_A}{r^2} = \dfrac{8.99 \times 10^9 \times 1.5 \times 10^{-7}}{(3.8 \times 10^{-4})^2} = 9.34 \times 10^9\ NC^{-1}$ ↑

$\vec{E}_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_B}{r^2} = \dfrac{8.99 \times 10^9 \times 2.5 \times 10^{-7}}{(5.2 \times 10^{-4})^2} = 8.31 \times 10^9\ NC^{-1}$ ←

$\vec{E}_X = \vec{E}_A + \vec{E}_B$

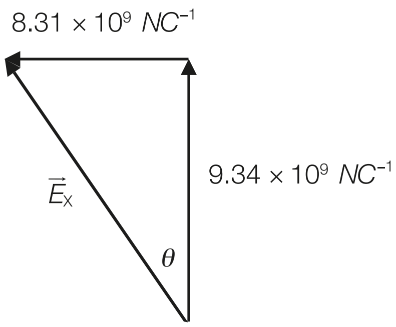
$\vec{E}_X = 9.34 \times 10^9\ NC^{-1}$ ↑ $+\ 8.31 \times 10^9\ NC^{-1}$ ←

$\vec{E}_X = \sqrt{(9.34 \times 10^9)^2 + (8.31 \times 10^9)^2}$

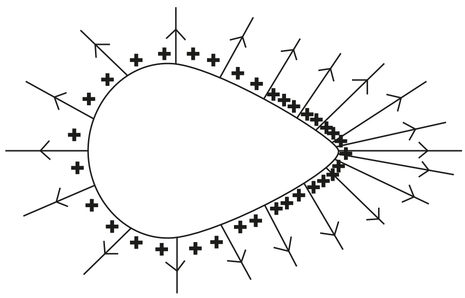
$\vec{E}_X = 1.3 \times 10^{10}\ NC^{-1}$

$\tan\theta = \dfrac{opp}{adj} = \dfrac{8.31 \times 10^9}{9.34 \times 10^9}\ \therefore\ \theta = 42^\circ$

$\vec{E}_P = 1.3 \times 10^{10}\ NC^{-1}\ \ N42^\circ W$



21. (a)



(b) The large electric field near the sharp point will cause a separation of charges in nearby air molecules. This induces a charge on the air molecules and they will be attracted to the charged sharp point. When the air molecules come into contact with the conductor, they will gain the same charge as the conductor becoming ionised.

22. The large electric field created by the thin charged wire will cause a separation of charges in nearby air molecules. This induces a charge on the air molecules and they will be attracted to the wire. When the air molecules come into contact with the wire, they will gain the same positive charge as the wire becoming ionised. The positive ions are then repelled from the wire onto the drum. As the drum rotates, it will become evenly charged.

2.2 Motion of charged particles in electric fields

1. $W = q\Delta V = 1.60 \times 10^{-19} \times 3.8 \times 10^{3} = 6.1 \times 10^{-16}\ J$

2. (a) $W = q\Delta V = 3.20 \times 10^{-19} \times 50.0 \times 10^{3} = 1.60 \times 10^{-14}\ J = 1.00 \times 10^{5}\ eV$

   (b) $W = \Delta E_k = \frac{1}{2}mv^2 \therefore v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-14}}{6.645 \times 10^{-27}}} = 2.19 \times 10^{6}\ ms^{-1}$

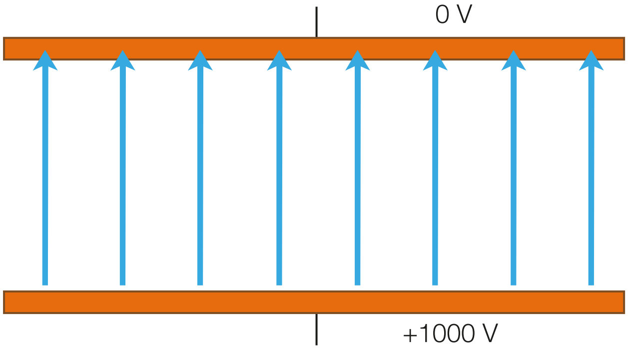
3. (a) $\Delta E_k = W = q\Delta V = \frac{1}{2}mv^2 \quad \therefore \quad \Delta V = \frac{mv^2}{2q}$

   (b) $\Delta V = \frac{mv^2}{2q} = \frac{9.11 \times 10^{-31} \times (2.68 \times 10^{7})^2}{2 \times 1.60 \times 10^{-19}} = 2040\ V$

   (c) From part (a), it can be seen that $\Delta V \alpha v^2 \quad \therefore \quad v \alpha \sqrt{\Delta V}$

   If the potential difference doubles, the speed increases by a factor of $\sqrt{2} = 1.4$

4. (a)



   (b) $E = \frac{\Delta V}{d} = \frac{1000}{2.0 \times 10^{-2}} = 5.0 \times 10^{4}\ Vm^{-1}$ *towards the top plate* ↑

   (c) $W = q\Delta V = 1.60 \times 10^{-19} \times 1000 = 1.6 \times 10^{-16}\ J$

   (d) Using Newton's Second Law, the acceleration of a charge in an electric field is given by: $\vec{a} = \frac{\vec{F}}{m}$

   Since $\vec{E} = \frac{\vec{F}}{q}$ *then* $\vec{F} = q\vec{E}$

   It follows that $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

   (e) $a = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 5.0 \times 10^{4}}{9.11 \times 10^{-31}} = 8.8 \times 10^{15}\ ms^{-2}$ ↓

5. (a) The electric field is uniform. This means that a charge in this electric field will experience the same force both in magnitude and direction at all points within the electric field.

   (b) $E = \frac{\Delta V}{d} \therefore d = \frac{\Delta V}{E} = \frac{1000}{4.50 \times 10^{3}} = 0.222 m$

   (c) $\Delta E_k = W = q\Delta V = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 1000}{1.67 \times 10^{-27}}} = 4.38 \times 10^{5}\ ms^{-1}$

   (d) $a = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 4.50 \times 10^{3}}{1.67 \times 10^{-27}} = 4.31 \times 10^{11}\ ms^{-2}$

   $s = v_0 t + \frac{1}{2}at^2 \quad \therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0.222}{4.31 \times 10^{11}}} = 1.01 \times 10^{-6}\ s$

6. (a) $E=\frac{\Delta V}{d}=\frac{300}{5.0\times10^{-3}}=6.0\times10^{4}\,Vm^{-1}$ *towards the lower plate* ↓

(b) (i) $a=\frac{qE}{m}=\frac{3.4\times10^{-6}\times6.0\times10^{4}}{2.6\times10^{-15}}=7.8\times10^{13}ms^{-2}$ ↑

(ii) $W=q\Delta V=3.4\times10^{-6}\times\frac{2}{5}(300)\ =4.1\times10^{-4}J$

(iii) $s=v_0t+\frac{1}{2}at^2\ \therefore\ t=\sqrt{\frac{2s}{a}}=\sqrt{\frac{2\times2.0\times10^{-3}}{7.8\times10^{13}}}=7.2\times10^{-9}s$

7. (a) $E=\frac{\Delta V}{d}=\frac{2.00\times10^{3}}{0.100}=2.00\times10^{4}Vm^{-1}$

(b) (i) $F=qE=1.60\times10^{-19}\times2.00\times10^{4}=3.20\times10^{-15}\,N$

(ii) The electric field is uniform. This means that a charge in this electric field will experience the same force both in magnitude and direction at all points within the electric field. The force on the proton does not depend on its position.

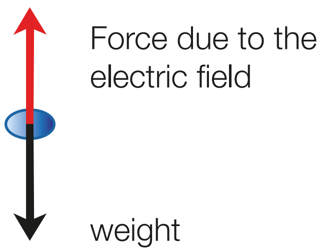
8. (a) $\Delta Ek=W\ =q\Delta V=1.60\times10^{-19}\times1280=2.05\times10^{-16}\,J$

(b) $W=q\Delta V=\frac{1}{2}mv^2\ \therefore\ v=\sqrt{\frac{2q\Delta V}{m}}=\sqrt{\frac{2\times1.60\times10^{-19}\times1280}{2.20\times10^{-25}}}=4.31\times10^{4}ms^{-1}$

(c) $v=v_o+at\quad\therefore\quad a=\frac{v}{t}=\frac{4.31\times10^{4}}{3.00\times10^{-5}}=1.44\times10^{9}\ ms^{-2}$

(d) $s=v_0t+\frac{1}{2}at^2=\frac{1}{2}\times1.44\times10^{9}\times(3.00\times10^{-5})^2=0.648m$

9. (a)



(b) $qE=mg\ \therefore\ q=\frac{mg}{E}=\frac{4.90\times10^{-16}\times9.80}{1.50\times10^{4}}=3.20\times10^{-19}C$

number of electrons $=\frac{3.20\times10^{-19}}{1.60\times10^{-19}}=2$

(c) $\Delta V=Ed=1.50\times10^{4}\times8.00\times10^{-3}=120\,V$

(d) $E=\frac{\Delta V}{d}\quad\therefore\quad E\alpha\frac{1}{d}$

If the distance between the plates is increased by a factor of three, then the magnitude of the electric field deceases by a factor of three to 5000 Vm^{-1}.

10. $\Delta Ek=W=q\Delta V=\frac{1}{2}mv^2\ \therefore\ v=\sqrt{\frac{2q\Delta V}{m}}=\sqrt{\frac{2\times1.60\times10^{-19}\times120\times10^{3}}{9.11\times10^{-31}}}=2.1\times10^{8}ms^{-1}$

11. (a) The electric field exerts a force on the protons due to their charge.

This means that the electric field will do work on the protons. According to the law of conservation of energy, the work done by the electric field is transferred to the protons becoming their kinetic energy.

(b) $W=Nq\Delta V=16000\times1.60\times10^{-19}\times5.5\times10^{3}=1.4\times10^{-11}\,J=88\,Mev$

12. (a) The electron enters in a direction that is perpendicular to the uniform electric field. The electric field exerts a constant force ($F = qE$) towards the upper plate. That is, in the vertical direction. The electric field does not exert a force on the charge in the horizontal direction, so it follows that the component of velocity perpendicular to the electric field remains constant. The result is that the charge follows a parabolic path towards the upper plate.

(b) $a = \frac{qE}{m} = \frac{1.60 \times 10^{-19} \times 3.60 \times 10^{3}}{9.11 \times 10^{-31}} = 6.32 \times 10^{14} ms^{-2}$ *towards the top plate* ↑

(c) $v = \frac{s}{t} \therefore t = \frac{s}{v} = \frac{0.100}{2.00 \times 10^{7}} = 5.00 \times 10^{-9} s$

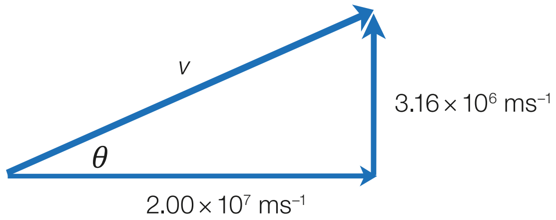
(d) $s = \frac{1}{2}at^2 = \frac{1}{2} \times 6.32 \times 10^{14} \times (5.00 \times 10^{-9})^2 = 7.90 \times 10^{-3} m$

(e) $v_H = 2.00 \times 10^{7}\ ms^{-1}$ →

$v_v = v_0 + at$

$= 0 + 6.32 \times 10^{14} \times 5.00 \times 10^{-9}$

$= 3.16 \times 10^{6}\ ms^{-1}$ ↑



$v = \sqrt{(2.00 \times 10^{7})^2 + (3.16 \times 10^{6})^2} = 2.02 \times 10^{7}\ ms^{-1}$

$\tan\theta = \frac{3.16 \times 10^{6}}{2.00 \times 10^{7}} \Rightarrow \theta = 8.98°$

$\vec{v} = 2.02 \times 10^{7} ms^{-1}$ 8.98° *above the horizontal*

(f) $W = q\Delta V = qEd = 1.60 \times 10^{-19} \times 3.60 \times 10^{3} \times 7.90 \times 10^{-3} = 4.55 \times 10^{-18} J$

13. (a) 2.80×10^{3} eV $= 4.48 \times 10^{-16}$ J

$E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 4.48 \times 10^{-16}}{1.67 \times 10^{-27}}} = 7.32 \times 10^{5} ms^{-1}$

(b) $E = \frac{\Delta V}{d} = \frac{1000}{0.200} = 5.00 \times 10^{3} Vm^{-1}$ *towards the top plate*

(c) length of plates $= s_H = v_H t = 7.32 \times 10^{5} \times 66.0 \times 10^{-9} = 4.83 \times 10^{-2}\ m$

(d) $s = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2} \times \frac{1.60 \times 10^{-19} \times 5.00 \times 10^{3}}{1.67 \times 10^{-27}} \times (66.0 \times 10^{-9})^2 = 1.04 \times 10^{-3} m$

(e) $W = q\Delta V = qEd = 1.60 \times 10^{-19} \times 5.00 \times 10^{3} \times 1.04 \times 10^{-3} = 8.32 \times 10^{-19} J$

14. (a) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.8 \times 10^{-25} \times (4.1 \times 10^{6})^2 = 1.5 \times 10^{-12} J = 9.4 \times 10^{6} eV$

(b) $v_H = v\cos\theta = 4.1 \times 10^{6} \cos 65 = 1.7 \times 10^{6}\ ms^{-1}$

(c) $v_v = v\sin\theta = 4.1 \times 10^{6} \sin 65 = 3.7 \times 10^{6}\ ms^{-1}$

(d) $v^2 = v_o^2 + 2as = \therefore\ a = \frac{v^2 - v_o^2}{2s} = \frac{-(3.7 \times 10^{6})^2}{2 \times 4.0 \times 10^{-3}} = -1.7 \times 10^{15} ms^{-2} = 1.7 \times 10^{15} ms^{-2}$ ↓

i.e. $a = 1.7 \times 10^{15}$ ms^{-2} vertically towards the lower plate

15. (a) $v_{parallel} = v\sin\theta = 2.00 \times 10^{6} \sin 40.0 = 1.29 \times 10^{6}\ ms^{-1}$

(b) $v_{perpendicular} = v\cos\theta = 2.00 \times 10^{6} \cos 40.0 = 1.53 \times 10^{6}\ ms^{-1}$

(c) $v^2 = v_o^2 + 2as = \quad \therefore \quad s = \frac{v^2 - v_o^2}{2a} = \frac{-(1.29 \times 10^{6})^2}{2 \times 1.54 \times 10^{14}} = 5.40 \times 10^{-3} m$

(d) $v = v_o + at \quad \therefore \quad t = \frac{v - v_o}{a} = \frac{0 - 1.29 \times 10^{6}}{-1.54 \times 10^{14}} = 8.37 \times 10^{-9}\ s$

$t_{total} = 2 \times 8.37 \times 10^{-9} = 1.68 \times 10^{-8}\ s$

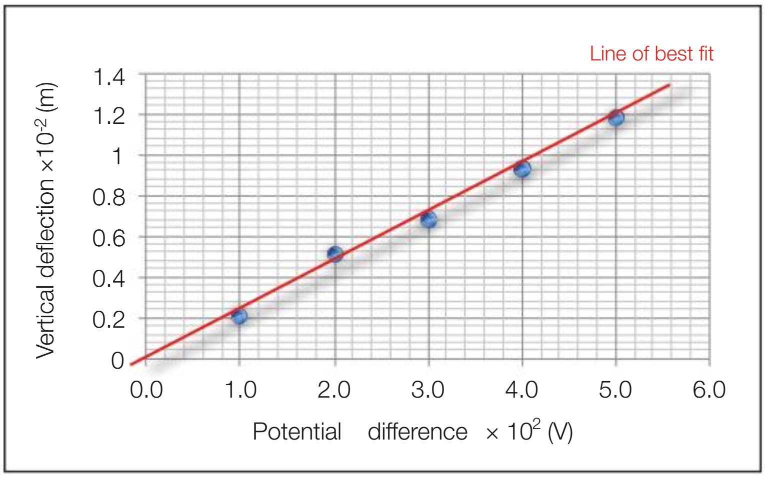
(e) *Length* $= v_{perpendicular} t = 1.53 \times 10^{6} \times 1.68 \times 10^{-8} = 2.57 \times 10^{-2}\ m$

16. (a) The independent variable is the potential difference applied between the plates. It is the variable that is purposely changed by the experimenter.

(b) Repeating and averaging the values of vertical deflection recorded for each potential difference helps reduce the effect of random error in the experiment.

(c) $s = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2$ but $t = \frac{L}{v}$ and $E = \frac{\Delta V}{d}$ so $s = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2}\frac{q\Delta VL^2}{mdv^2} = \frac{e\Delta VL^2}{2mdv^2}$

(d) (i)



(ii) gradient = $\frac{1.2\times10^{-2}}{5.0\times10^{2}} = 2.4\times10^{-5} mV^{-1}$

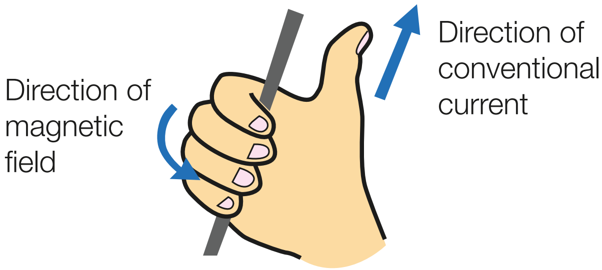
(iii) Comparing the equation $s = \frac{e\Delta VL^2}{2mdv^2}$ to $y = mx$ then it follows that $gradient = \frac{eL^2}{2mdv^2}$

$$v = \sqrt{\frac{eL^2}{2md\times gradient}} = \sqrt{\frac{1.60\times10^{-19}\times0.12^2}{2\times9.11\times10^{-31}\times0.060\times2.4\times10^{-5}}} = 3.0\times10^{7} ms^{-1}$$

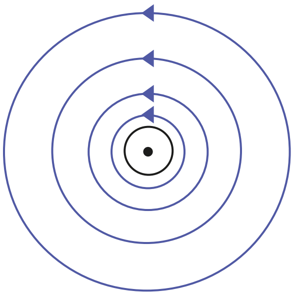
S

2.3 Magnetic fields

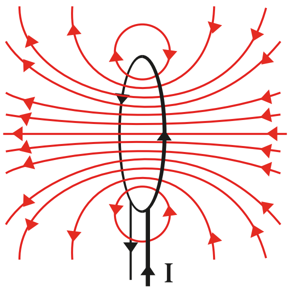
1.



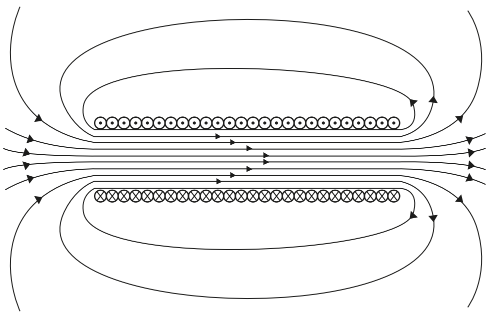
2. (a)



(b)



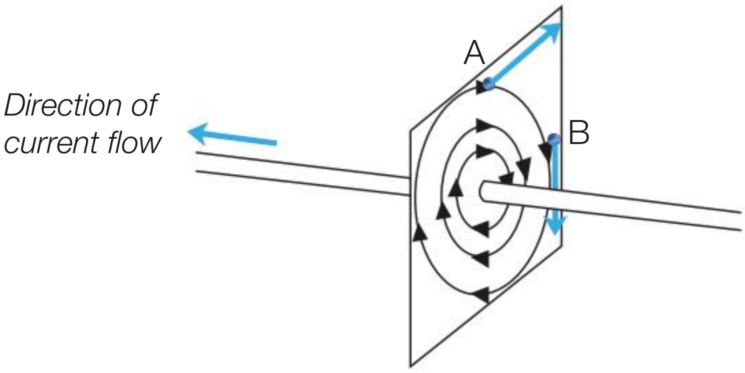
(c)



3. (a) The direction of the magnetic field is given by the direction that the north pole of a small compass needle points. The direction of the field at a given point is at a tangent to the field lines.

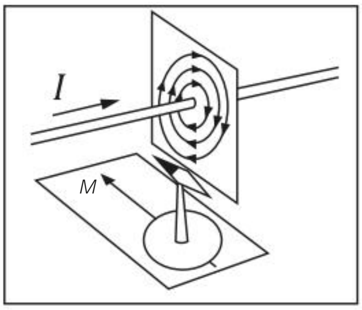
(b) The magnitude of the magnetic field is represented by the number of field lines crossing a unit area perpendicular to the field in the vicinity of the point. The greater the number of field lines crossing a unit area, the stronger the magnetic field.

4. (a)



(b) Arrows indicated on the diagram.

5. (a)



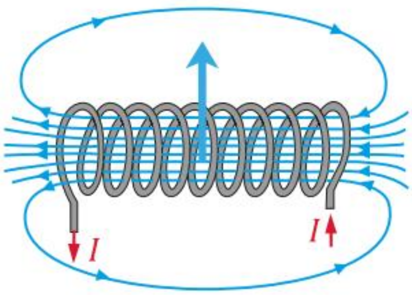
(b) Current I as indicated on the diagram.

6. (a) Out of the page.

(b) The spacing between the magnetic field lines is halved.

(c) $B\alpha\frac{1}{r}$ therefore $\frac{B_P}{B_Q}=\frac{r_Q}{r_P}=\frac{7}{10}=0.7$

7. (a)



South Pole

(b) Up the plane of the page.

8. (a) $B=\frac{\mu_0}{2\pi}\frac{I}{r}=\frac{2.0\times10^{-7}\times45.0}{1.50}=6.00\times10^{-6}\ T$

(b) $B\alpha\frac{1}{r}$

The magnitude of the magnetic field is halved at twice the distance i.e. 3.00×10^{-6} T

(c) Since $B\alpha I$, the magnitude of the magnetic field strength at P will be three times smaller if the current is reduced by a factor of three. i.e. 2.00×10^{-6} T

9. (a) $\frac{B}{10}$

(b) $4B$

10. North

11. The switch in circuit 1 is closed. A current flows through the electromagnet and induces magnetism in the electromagnet that attracts the iron armature. The second switch closes. A current flows through the second circuit.

    When switch 1 is open, the current stops flowing, the electromagnet is no longer magnetised, the armature is released and circuit 2 is open. The current in circuit 2 stops flowing.

12. $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times I}{r} = 2.5 \times 10^{-4}\ T$

    $\therefore I = \frac{Br}{2.00 \times 10^{-7}} = \frac{2.5 \times 10^{-4} \times 0.060}{2.00 \times 10^{-7}} = 75\ A$

13. (a) $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 8.6}{0.030} = 5.7 \times 10^{-5}\ T$ *out of the page*

    (b) The magnitude of the magnetic field due to conductor 1:

    $5.7 \times 10^{-5}\ T$ *out of the page*

    The magnitude of the magnetic field due to conductor 2:

    $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 8.6}{0.050} = 3.4 \times 10^{-5}\ T$ *out of the page*

    5.7×10^{-5} T out of the page + 3.4×10^{-5} T out of the page = 9.1×10^{-5} T out of the page.

14. The magnitude of the magnetic field due to conductor 1:

    $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 20.0}{0.600} = 6.67 \times 10^{-6}\ T$ *into the page*

    The magnitude of the magnetic field due to conductor 2:

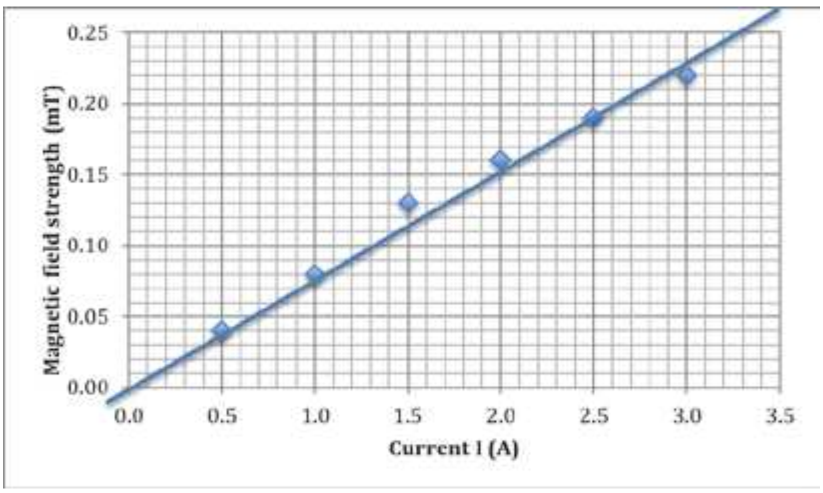
    $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 15.0}{0.300} = 1.00 \times 10^{-5}\ T$ *into the page*

    6.67×10^{-6} T into the page + 1.00×10^{-5} T into the page = 7.67×10^{-5} T into the page.

15. (a)

Average
0.04
0.08
0.13
0.16
0.19
0.22

    (b) (i) Line of best fit is illustrated on the graph.



(ii) The magnetic field strength is directly proportional to the current.

(iii) $gradient = \frac{0.20 \times 10^{-3}}{2.6} = 7.7 \times 10^{-5} TA^{-1}$

(iv) Since $B = \mu_0 nI$ then the gradient = $\mu_0 n$

$\mu_0 n = 7.7 \times 10^{-5} \quad \therefore \quad \mu_0 = \frac{7.7 \times 10^{-5}}{70} = 1.1 \times 10^{-6}$

(c) (i) The experimental value of μ_0 is very close to the accepted value which makes it accurate.

(ii) There is very little scatter between the plotted points and the line of best fit. The data is precise.

16. **Science as a human endeavour activity – Magnetic fields:**

Hybrid magnets show development because they take a material, such as cobalt, that is already magnetic and cover it with molecules of a form of carbon. The strength of the magnet is increased by a factor of five. A smaller magnet could do the same job as a larger non hybrid magnet and could therefore replace their use. This means smaller amounts of rare earth metals are required, reducing the need for mining and processing, and hence the impact on the environment.

An understanding of the environmental damage caused by the mining and processing of rare earth metals needed to make permanent magnets has been an influence for scientists to develop a solution. As explained above, hybrid magnets will reduce the impact on the environment (as less mining and processing is required).

A team of scientists are working on hybrid magnets at the University of Leeds. In addition, the research was funded by the Engineering and Physical Sciences Research Council, Horizon 2020 European Research Infrastructure project OpenDreamKit, Taibah University and the Science and Technology Facilities Council. This demonstrates that clear communication and collaboration i.e. scientists sharing findings and working together, is needed to make discoveries and progress in science.

The applications of hybrid magnets will be the same applications as traditional permanent magnets. Some examples include, renewable energy, electronics and electric-powered vehicles. With their additional strength, new applications may be discovered. The biggest limitation of the hybrid magnets is that the results (increased strength) can only be verified at low temperatures (–195ºC). Further research and modifications are required, but it is hopeful that the same effect will be observed at room temperature.

S

2.4 Motion of charged particles in magnetic fields

1. (a) $F = BIl$

   (b) (i) $F = 2BIl$

   (ii) $F = \frac{BIl}{5}$

   (iii) $F = \frac{2BIl}{5}$

2. (a) Out of the page

   (b) Up the plane of the page (↑)

   (c) To the right of the page (→)

3. $F = IlB\sin\theta = 1.95 \times 0.500 \times 4.00\sin35 = 2.2\ N$

4. (a) 90°

   (b) $F = IlB\sin\theta = 2.0 \times 10^2 \times 1.0 \times 1.8 \times 10^{-5}\ \sin90 = 3.6 \times 10^{-3}\ N$

   (c) $F = IlB\sin\theta$ therefore $\theta = \sin^{-1}\left(\frac{F}{IlB}\right)$

   $\theta = \sin^{-1}\left(\frac{9.0 \times 10^{-4}}{2.0 \times 10^2 \times 1.0 \times 1.8 \times 10^{-5}}\right) = 14°$

5. (a) Using the right hand rule, the force acts into the page, the magnetic field acts to the left so the current must flow down the plane of the page (↓).

   (b) $F = IlB\sin\theta$ therefore $l = \frac{F}{IB\sin\theta} = \frac{5.00 \times 10^{-2}}{2.00 \times 0.360 \times \sin90} = 0.0694\ m$

6. (a) A

   (b) B

7. $F \alpha Bl$ if I and θ remain constant.

   Halving the length and tripling the magnitude of the magnetic field will mean that the force becomes $\frac{3}{2} = 1.5$ times greater.

8. (a) When a current flows through the voice coil, it experienced a force due to the magnetic field ($F = IlB$). The voice coil moves in the direction of the force. When the current reverses, the direction of the force on the voice coil reverses. The alternating current therefore causes the voice coil to vibrate.

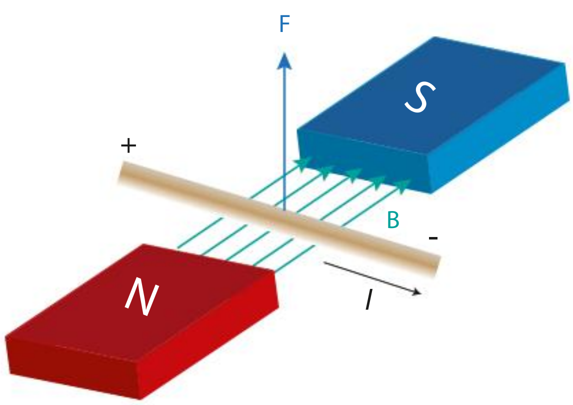
   (b) (i) $l = 10 \times 2\pi r = 10 \times 2\pi \times 0.030 = 1.9\ m$

   (ii) $F = IlB\sin\theta = 0.95 \times 1.9 \times 0.80\ \sin90 = 1.4\ N$

9. (a) The weight of the conductor acting down balances the magnetic force acting upwards.

   $F = mg = IlB\sin\theta$ therefore $I = \frac{mg}{lB\sin\theta} = \frac{7.50 \times 10^{-2} \times 9.80}{1 \times 2.50 \times 10^{-2}} = 29.4\ A$

   (b)



10. (a) Using the right hand rule, the current flows down (↓), the magnetic field acts to the right (→) and the magnetic force on side AB acts out of the plane of the page.

(b) Into the plane of the page

(c) Zero

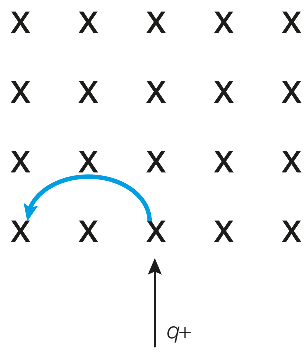
(d) The armature rotates clockwise

(e) (i) The magnetic force on the armature increases because the length of the conductor increases ($F \propto l$). The armature rotates faster.

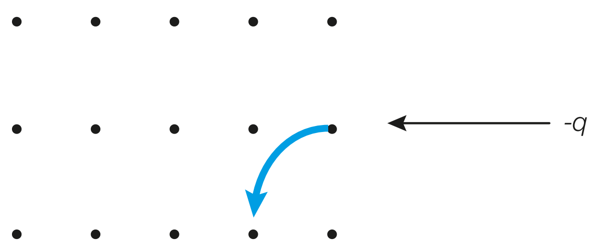
(ii) The magnetic force on the armature increases because the magnitude of the magnetic field increases ($F \propto B$). The armature rotates faster.

(iii) The magnetic force on the armature increases because the magnitude of the current flowing through the conductor increases ($F \propto I$). The armature rotates faster.

11. (a)



(b)



12. (a) $F = qvB\sin\theta = 1.60 \times 10^{-19} \times 2.50 \times 10^{6} \times 1.80 \sin 90 = 7.20 \times 10^{-13}\ N$ into the plane of the page.

(b) The proton is moving at right angles to the uniform magnetic field and experiences a force of constant magnitude at right angles to its velocity. This causes a change in direction without a change in speed. The magnetic force provides the centripetal acceleration required for uniform circular motion.

(c) $r = \dfrac{mv}{qB} = \dfrac{1.67 \times 10^{-27} \times 2.50 \times 10^{6}}{1.60 \times 10^{-19} \times 1.80} = 1.45 \times 10^{-2} m$

13. (a) $F = qvB\sin\theta$ – since q, v, B and θ are all constant, the force on the electron and proton is the same.

(b) $r = \dfrac{mv}{qB} \therefore r \alpha\ m$

The mass of the electron is smaller so it will travel in a circular path of smaller radius.

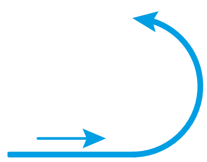
14. (a) The magnetic force provides the centripetal acceleration for uniform circular motion.

$F_{centripetal} = F_{magnetic}$

$\dfrac{mv^2}{r} = qvB$

$r = \dfrac{mv}{qB}$

(b) (i) A circular path upwards.



(ii) $r = \dfrac{mv}{qB} \therefore v = \dfrac{rqB}{m} = \dfrac{5.0 \times 10^{-3} \times 2.70 \times 10^{-9} \times 2.00 \times 10^{-4}}{1.72 \times 10^{-18}} = 1.57 \times 10^{3} ms^{-1}$

$E_k = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 1.72 \times 10^{-18} \times (1.57 \times 10^{3})^2 = 2.12 \times 10^{-12} J = 1.32 \times 10^{7} eV$

15. (a) $r = \dfrac{mv}{qB} \therefore r \alpha\ v$

As the electrons collide with hydrogen atoms they lose kinetic energy and hence speed. The radius of their circular path decreases and they spirals inwards.

(b) Into the plane of the page

16. (a) positive

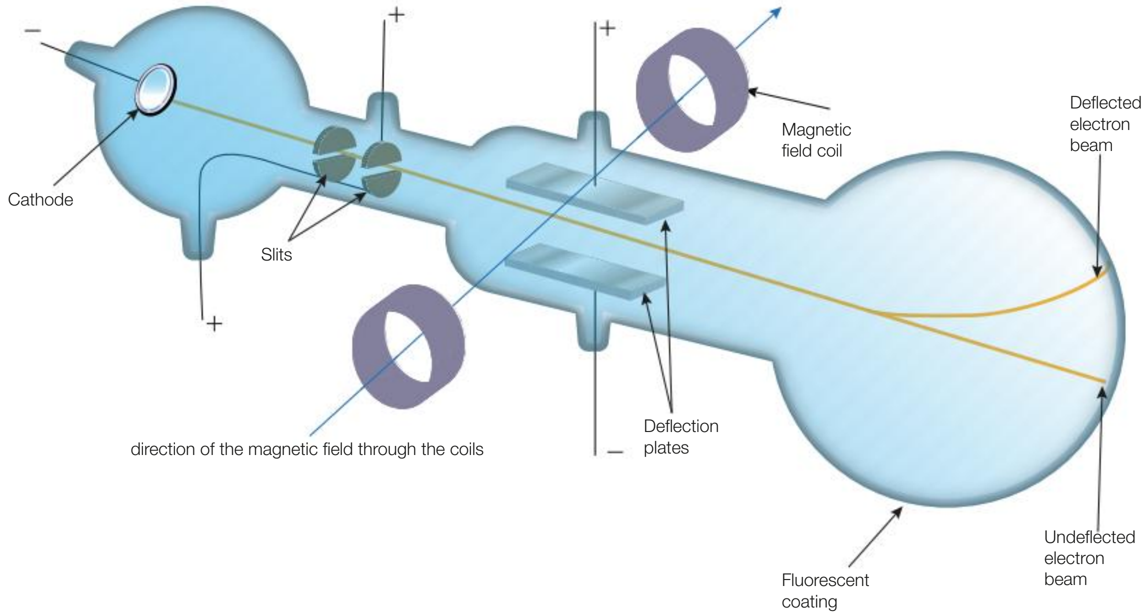
(b) (i) $F \propto v$ The magnetic force on the charged particle increases.

(ii) $r = \frac{mv}{qB} \therefore r \propto v$

The radius of the circular path of the charged particle increases.

17. $r \propto \frac{1}{B}$ therefore $\frac{B_2}{B_1} = \frac{r_1}{r_2} = \frac{r}{3r} = \frac{1}{3}$

18. (a)



(b) The force due to the electric field is greater.

(c) The magnitude of the force due to the electric field is equal to the magnitude of the force due to the magnetic field and acts in the opposite direction.

: $F = qE = qvB$ therefore $v = \frac{E}{B}$

(d) $E = vB = 140 \times 2.5 = 350\ Vm^{-1}$

19. (a) Positive

(b) $r = \frac{mv}{qB} \therefore r \propto m$

The radius of the circular path is directly proportional to the mass of the ions. Ions of mass m_2 travel in a circular path of greater radius and therefore have a greater mass.

(c) $r = \frac{mv}{qB} \therefore \frac{q}{m} = \frac{v}{rB} = \frac{2.00 \times 10^6}{0.250 \times 0.860} = 9.30 \times 10^6 Ckg^{-1}$

20. (a) $T = \frac{2\pi m}{qB} = \frac{2\pi \times 3.34 \times 10^{-27}}{1.60 \times 10^{-19} \times 1.50} = 8.74 \times 10^{-8} s$

(b) $f = \frac{1}{T} = \frac{1}{8.74 \times 10^{-8}} = 1.14 \times 10^7$ Hz

(c) $E_k = \frac{q^2B^2r^2}{2m} = \frac{(1.60 \times 10^{-19})^2 \times 1.50 \times (25.0 \times 10^{-2})^2}{2 \times 3.34 \times 10^{-27}} = 5.39 \times 10^{-13} J = 3.37 MeV$

21. (a) $E_k = \frac{q^2B^2r^2}{2m}\ B = \sqrt{\frac{2mE_k}{q^2r^2}} = \sqrt{\frac{2 \times 1.67 \times 10^{-27} \times (81 \times 10^3 \times 1.60 \times 10^{-19})}{(1.60 \times 10^{-19})^2 \times 0.045^2}} = 0.91\ T$

(b) $T = \frac{2\pi m}{qB} = \frac{2\pi \times 1.67 \times 10^{-27}}{1.60 \times 10^{-19} \times 0.91} = 7.2 \times 10^{-8} s$

(c) $f = \frac{1}{T} = \frac{1}{7.2 \times 10^{-8}} = 1.4 \times 10^7$ Hz

22. (a) The uniform electric field accelerates the protons by doing work on the protons that is transformed into kinetic energy and hence increases the speed of the protons.

(b) The uniform magnetic field acts at right angles to the velocity of the protons. The magnetic force therefore causes the protons to move in a circular path so that they cross the electric field many times.

(c) Out of the page

(d) $E_k = \frac{1}{2}mv^2$ using $r = \frac{mv}{qB}$ it follows that $v = \frac{rqB}{m}$

$$E_k = \frac{1}{2}mv^2 = K = \frac{1}{2}m(\frac{rqB}{m})^2 = \frac{1}{2}m\frac{r^2q^2B^2}{m^2} = \frac{r^2q^2B^2}{2m}$$

(e) $E_k = \frac{q^2B^2r^2}{2m}$ $\therefore$ $r = \sqrt{\frac{2mE_k}{q^2B^2}} = \sqrt{\frac{2mE_k}{q^2B^2}} = \sqrt{\frac{2 \times 1.67 \times 10^{-27} \times (30 \times 10^6 \times 1.60 \times 10^{-19})}{(1.60 \times 10^{-19})^2 \times 1.70^2}} = 4.66 \times 10^{-1} m$

23. (a) $T = \frac{2\pi r}{v}$ but $r = \frac{mv}{qB}$

$$T = \frac{2\pi \frac{mv}{qB}}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$

(b) Ions closer to the centre of the cyclotron travel in a circular path with a smaller radius and therefore travel a smaller distance at a slower speed. Ions closer to the outside of the cyclotron travel in a circular path with a larger radius and therefore travel further at a greater speed. Since $t = \frac{s}{v}$ all the ions reach the electric field at the same time. The period does not depend on the speed of the ions.

24. Science as a human endeavour activity – Motion of charged particles in magnetic fields:

An understanding of the environmental damage caused by increasing urbanisation has been an influence for scientists to develop a solution. Road transport is responsible for three quarters of total carbon dioxide emissions resulting from transport. It also consumes 75% of the total transport energy demand. On-wheel railways use a lot of petroleum products. Road transportation is therefore linked to environmental issues associated with the mining of petroleum products and the issues associated with global warming due to increased levels of carbon dioxide in our atmosphere.

Maglev trains are a fully electrified system and offer a clean and efficient way to transport large number of people. In addition, the system is sustainable as it is fully congruous with the renewable energy resources without the need for any technological modifications. Advances in scientific understanding in the fields of technology and engineering will help to further improve the performance of the magnetic levitation railways.

2.5 Electromagnetic induction

1. (a) Magnetic flux Φ through an area A, is defined as the product of the magnetic field strength B and the area perpendicular to the magnetic field $A_\perp$.

   (b) $\Phi = BA_\perp = 1.2 \times 4.0 \times 10^{-2} = 4.8 \times 10^{-2}\ Wb$

2. (a) $\Phi = BA_\perp = 2.5\pi r^2 = 2.5 \times \pi \times 0.100^2 = 7.85 \times 10^{-2}\ Wb$

   (b) Doubling the magnetic field strength would double the magnetic flux through the circular loop ($\Phi \alpha B$).

3. (a) $A = L^2 = 0.120 \times 0.120 = 1.44 \times 10^{-2}\ \text{m}^2$

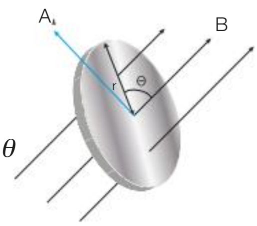
   (b) $\Phi = BA_\perp = 0.980 \times 1.44 \times 10^{-2} \cos 35.0 = 1.16 \times 10^{-2}\ Wb$

   (c) Maximum

   $\Phi = BA_\perp = 0.980 \times 1.44 \times 10^{-2} = 1.41 \times 10^{-2}\ Wb$

   $\Phi = BA_\perp = BA\cos\theta$

   $\cos\theta = \dfrac{\Phi}{BA} \quad \therefore\ \theta = \cos\left(\dfrac{½\,(1.41 \times 10^{-2})}{0.980 \times 1.44 \times 10^{-2}}\right) = 60.0°$



4. $\Phi = BA_\perp$

   The area of the circular coil = πr^2

   The component of area perpendicular to the magnetic field $A\cos(90 - \theta) = A\sin\theta$

   $\Phi = BA_\perp = \pi r^2 B\sin\theta$

5. (a) Faraday's Law states that the induced *emf* in a circuit is equal to the rate of change of the magnetic flux.

   (b) $\varepsilon = \dfrac{\Delta\phi}{\Delta t} = A\dfrac{(B_2 - B_1)}{\Delta t} = 0.15 \times \dfrac{(2.8 - 2.2)}{3.0 \times 10^{-3}} = 3.0 \times 10^{1}\ V$

6. (a) $\varepsilon\ \alpha\ \dfrac{1}{\Delta t}$

   If the time over which the magnetic flux through the loop increases by a factor of three, then the *emf* will decrease by a factor of three.

   The *emf* becomes $\dfrac{\varepsilon}{3}$.

   (b) $\varepsilon = \dfrac{\Delta\Phi}{\Delta t} = \dfrac{B\Delta A}{\Delta t}$

   $\varepsilon\ \alpha \Delta A$

   If the area of the loop halves so does the *emf*. The *emf* becomes $\dfrac{\varepsilon}{2}$.

7. $\varepsilon = \dfrac{\Delta\Phi}{\Delta t} = \dfrac{\Delta(BA)}{\Delta t}$

   Faraday's Law states that the induced *emf* in a circuit is equal to the rate of change of the magnetic flux. There is no change in magnetic flux through the loop because the area of the loop and the magnetic field do not change. This means that there is no *emf* induced in the loop as it is moved from position A to position B.

8. $\varepsilon = N\dfrac{\Delta\Phi}{\Delta t} = 25 \times \dfrac{(0 - 60)}{5.20} = -288\ V$

9. (a) $\Delta\Phi = NB\Delta A_\perp = NB(A_2 - A_1)\cos\theta = 80 \times 5.0 \times 10^{-6} \times (0.12 - 0.089)\cos 23 = 1.1 \times 10^{-5}\ Wb$

   (b) $\varepsilon = \dfrac{\Delta\Phi}{\Delta t} = \dfrac{1.1 \times 10^{-5}}{0.82} = 1.4 \times 10^{-5}\ V$

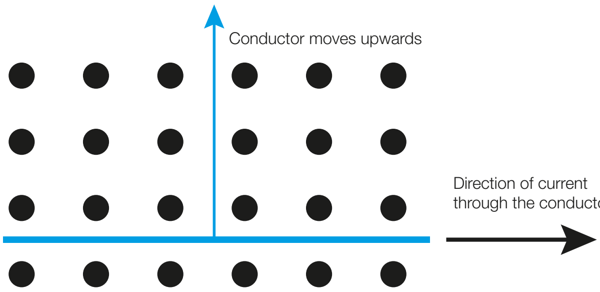
   (c) If the patient breathes twice as fast, the time taken for the change in magnetic flux halves.

   Since $\varepsilon\ \alpha \dfrac{1}{\Delta t}$, this means that the induced *emf* in the coil doubles.

10. (a) Lenz's law states that an induced *emf* creates a current in a direction that opposes the change in magnetic flux producing the *emf*.

(b) (i) Electrons in the conductor are forced to move with the same speed as the conductor and experience a force given by $F = qvB\sin\theta = qvB$ since $\theta = 90°$. Using the right hand rule, electrons flow to the left end of the conductor leaving the right end of the conductor with excess positive charge. This creates an induced potential difference or *emf* between the two ends of the conductor.

(ii)

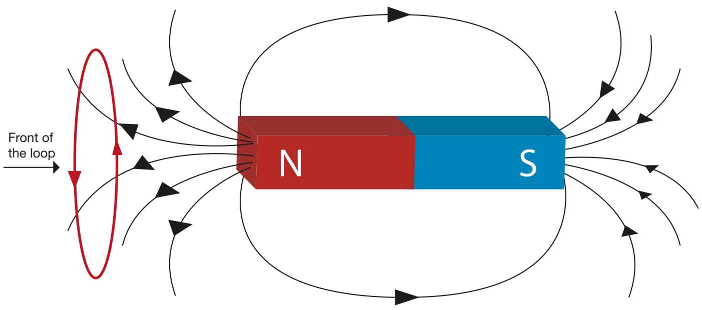


(iii) According to Lenz's Law, the induced current will flow through the conductor to in a direction that opposes the change in magnetic flux producing the *emf*. This means that a force acts on the conductor that is directed down the plane of the page.

Using the right hand rule, the force acts down the plane of the page, the magnetic field acts out of the plane of the page, therefore the current flows to the right of the page.

11. (a) As the magnet moves towards the circular loop, there is a change in magnetic flux (magnetic flux increases). This is because more magnetic field lines pass through the loop as the magnet gets closer to the loop. A change in magnetic flux will induce an *emf*.

(b)



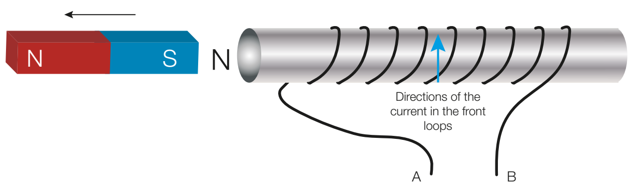
(c) As the magnet moves towards the conducting loop, there is a change in magnetic flux (magnetic flux increases). According to Lenz's Law, the induced current will flow to oppose the change in magnetic flux. The magnetic field of the magnet acts to the left, so the current in the conducting loop will flow to produce a magnetic field through the loop that acts to the right. Using the right hand rule, the current flows anticlockwise (or down the front of the loop) as shown on the diagram.

(d) (i) If the magnet is stationary, there is no change in magnetic flux and therefore no induced *emf* and current.

(ii) If the magnet is moved faster, the time over which the change in magnetic flux occurs decreases. A larger *emf* ($\varepsilon\ \alpha \frac{1}{\Delta t}$) is induced and a larger current flows. The current flows in the same direction.

(iii) As the magnet is withdrawn, there is a change in magnetic flux (magnetic flux decreases). The current flows in the circular loop in the opposite direction to oppose the decrease in magnetic flux.

12. (a) As the south pole of the magnet is moved away from the solenoid, the current flows through the solenoid in a direction that creates a magnetic field that opposes the change in magnetic flux. The magnetic flux through the solenoid is decreasing so the current flows to produce a north pole at the end of the solenoid closest to the magnet. This produces a force that opposes the movement of the magnet.



Using the right hand rule, the current must flow from B to A.

(b) The direction of the current must flow in a direction that opposes the change in magnetic flux creating the *emf* and hence current. In this case, if the current did not flow in such a manner, a south pole (instead of a north pole) would be induced at the end of the solenoid closest to the magnet. The magnetic force of repulsion between the two like poles would cause the magnet to accelerate away from the solenoid. This would induce a greater current in the solenoid and the magnet would further accelerate. This would continue and the speed and hence kinetic energy of the magnet would increase. This means that energy is created and violates the law of conservation of energy. The induce current must flow in a direction to oppose the change in magnetic flux.

13. (a) $\Delta\Phi = B\Delta A = B(A_2 - A_1) = 4 \times (0 - \pi r^2) = 4.0 \times (0 - \pi\, 0.060^2) = -4.5 \times 10^{-2}\ Wb$

(b) $\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{-4.5\times10^{-2}}{1.1\times10^{-2}} = -4.1\ V$

(c) As the coil is moved to the right, there is a change in magnetic flux through the coil (magnetic flux decreases). The current flows in a direction that creates a magnetic field that opposes the change in magnetic flux. That is, the current flows to produce a magnetic field into the page. Using the right hand rule, the current flows clockwise through the coil.

(d) The current stops flowing once the coil is completely removed from the magnetic field and there is no longer a change in magnetic flux.

14. (a) As the magnet is inserted into the coil of wire, there is a change in magnetic flux through the coil (magnetic flux increases). The current flows in a direction that creates a magnetic field that opposes the change in magnetic flux. That is, the current flows to produce a magnetic field out of the page. Using the right hand rule, the current flows anticlockwise through the coil.

(b) $\varepsilon = \frac{\Delta\Phi}{\Delta t}\ \therefore\ \Delta\Phi = \varepsilon \times \Delta t = 3.00 \times 0.450 = 1.35\ Wb$

(c) $\Delta\Phi = \Delta BA = \Delta B\pi r^2\ \therefore\ \Delta B = \frac{\Delta\Phi}{\pi r^2} = \frac{1.35}{\pi(0.100)^2} = 43.0\ T$

15. (a) The sound waves cause the diaphragm to oscillate. This causes the coil to oscillate in the uniform magnetic field. As the coil moves into the uniform magnetic field, there is a change in magnetic flux through the coil. A current is induced in the coil and flows to oppose the change in magnetic flux. As the coil stops and reverses direction there is another change in magnetic flux. The current flows in the opposite direction to oppose the change in magnetic flux. In this way, as the coil oscillates, an oscillating current (electrical signal) is induced in the coil.

(b) $\varepsilon = \frac{\Delta\Phi}{\Delta t}\ \therefore\ \varepsilon\ \alpha \frac{1}{\Delta t}$

If the frequency of the sound waves is larger, the diaphragm oscillates more quickly. This means that the change in magnetic flux occurs over a shorter time and the *emf* increases. It follows that sound waves with a lower frequency induce a smaller *emf*.

16. (a) The gradient of the graph represents the change in magnetic flux per unit time or the rate of change in magnetic flux. Faraday's Law states that the induced *emf* in a circuit is equal to the rate of change of the magnetic flux. It follows that the gradient represents the *emf* induced in the conducting coil.

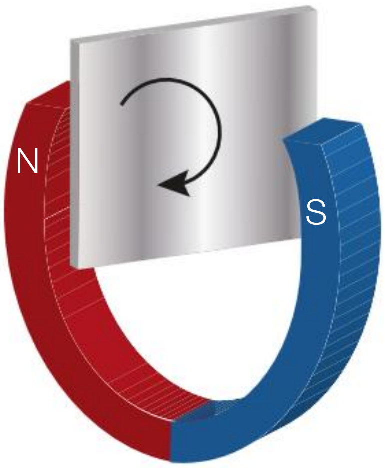
(b) (i) $t = 1$ s

(ii) $t = 2$ s to $t = 6$ s

(iii) $t = 7$ s

(c) $\varepsilon = gradient = \frac{0.8}{1} = 0.8\ V$

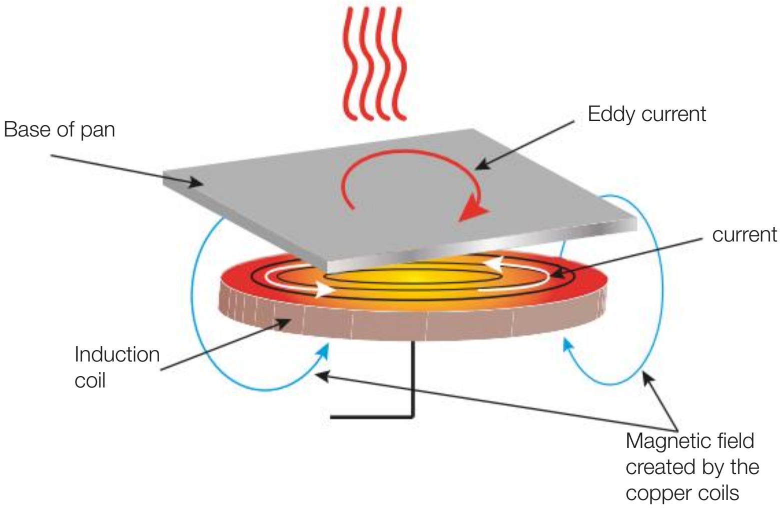
17. (a)



(b) As the metal plate is moved into the magnetic field there is a change in magnetic flux through the metal plate (magnetic flux increases). Eddy currents flow in a direction that produces a magnetic field that opposes the change in magnetic flux. The magnetic field produced by the eddy currents is directed from the south pole of the magnet to the north pole of the magnet so that it acts to reduce the magnetic flux. Using the right hand rule, the eddy currents flow in a clockwise direction.

(c) The eddy currents flow in a direction that produces a force on the plate that opposes its motion. This makes it difficult to push into the magnetic field.

18. (a)

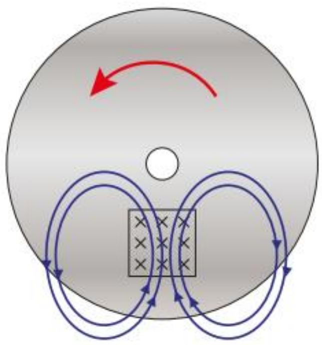


(b) As shown on the diagram for part (a).

(c) (i) Induction coils can only set up eddy currents in a conductor. Heating occurs in the pan only. Eddy currents cannot form in the chocolate as it is not a conductor. The chocolate does not melt.

(ii) If the induction coil is left on, without a pan present the cook top does not get hot. If it is touched it will not burn you like gas and is therefore safer. An induction cooker is energy efficient. Energy is only used to heat the pan and is not lost to the surrounding air like it is when using gas.

19. (a)



(b) The eddy currents flow in a direction that produces a force that opposes the motion of the rotating disc. This causes the rotating disc to stop.

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20. (a) Transformers allow generated voltages to be either increased or decrease before they are used.

(b) (i) step-up transformer

(ii) $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore \frac{V_{input}}{V_{output}} = \frac{50}{250} = \frac{1}{5}$

21. $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore V_{output} = \frac{V_{input} \times N_{output}}{N_{input}} = \frac{1150 \times 650}{1850} = 404\,V$

22. (a) A step-up transformer has more turns in the output coil and fewer in the input coil while a step-down transformer has more turns in the input coil and fewer in the output coil.

(b) A step-up transformer has a greater potential difference in the output coil and a lower potential difference in the input coil while a step-down transformer has a greater potential difference the input coil and a lower potential difference in the output coil.

(c) A step-up transformer produces a lower output current in the output coil while a step-down transformer produces a greater output current in the output coil.

23. (a) Step-down transformer

(b) $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore N_{output} = \frac{V_{output} \times N_{input}}{V_{input}} = \frac{12 \times 2000}{240} = 100$

24. (a) Step-up transformer

(b) $\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}} \therefore N_{input} = \frac{V_{input} \times N_{output}}{V_{output}} = \frac{1 \times 860}{20} = 43$

25. The soldering iron is an example of a step-down transformer. This means that there are more turns in the input coil (N_{input}) that the output coil (N_{output}). A large step ratio would indicate the number of turns differs by a large amount so the ratio $\frac{N_{input}}{N_{output}}$ is large.

This in turn would produce a large voltage difference between the input and output coils ($\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}}$). That is a large potential difference is decreased significantly.

3.1 Wave behaviour of light

1. Electrons in a transmitting antenna are forced to oscillate along the length of the antenna by an alternating potential difference. This creates an oscillating electric field. The oscillating electric field produces an oscillating magnetic field at right angles to the electric field. Both fields are perpendicular to one another and the direction in which the wave travels. Since an oscillating electric field can create an oscillating magnetic field and vice versa, these two fields continually reproduce one another and a plane-polarised electromagnetic radio wave is radiated away from the antenna in all directions. The frequency of the radio wave is the same as the frequency of oscillation of the electrons in the transmitting antenna.

2. (a) Plane-polarisation involves the restriction of vibrations to a single plane. The plane of polarisation of an electromagnetic wave is defined by the plane of the oscillating electric field.

   (b) Television waves are produced when electrons are forced to oscillate in a transmitting antenna. Since the electrons can only oscillate in one plane parallel to the antenna, the emitted waves are said to be plane-polarised. The electromagnetic wave is plane-polarised in the direction of the vibrating charge.

   (c) Horizontal

3. (a) Horizontal

   (b) Vertical

   (c) Country antennae are orientated at right angles to the city antennae. If the receiving antenna is orientated at right angles to the antenna transmitting the radio or television wave, the electric field of the transmitted signal cannot oscillate the electrons in the receiving antenna so that they produce a large oscillating potential across the rods of the antenna. A very weak signal is received if any at all. This reduces the chance of interference between city and country signals.

4. If the magnetic field of the transmitted electromagnetic wave is in a horizontal plane, the electric field oscillates in a vertical plane. The transmitted wave is vertically plane polarised. To receive a strong signal, the receiving antenna must be orientated in the same plane as the plane of polarisation of the transmitted wave. The receiving antenna must be vertical.

5. (a) Electrons in a transmitting antenna are forced to oscillate along the length of the antenna by an alternating potential difference. If the frequency of the alternating potential is 107.1M Hz, the electrons will oscillate at this frequency and produce an electromagnetic wave with a frequency of 107.1M Hz.

   (b) As the electromagnetic wave transmitted from an antenna approaches a receiving antenna, the oscillating electric field exerts a force ($F = Eq$) on the stationary electrons in the receiving antenna. The electrons will oscillate at the same frequency as the electric field. This produces an alternating potential difference between the two rods of the dipole. The frequency of the alternating potential difference is the same as the frequency of oscillation of the electrons. In this way, the radio wave that was originally transmitted is now received. Note: The receiving antenna must be parallel to the antenna transmitting the signal.

   (c) $v = f\lambda \therefore \lambda = \frac{v}{f} = \left(\frac{3.00 \times 10^8}{107.1 \times 10^6}\right) = 2.80$ m

6. (a) (i) Monochromatic light is light composed of a single frequency.

   (ii) Coherent wave sources are wave sources that maintain a constant phase relationship with each other.

   (b) The light produced by an incandescent source is composed of a range of frequencies. In addition, the random oscillations of the charges means that the emitted light does not maintain a constant phase relationship. Light from an incandescent light source is neither monochromatic nor coherent.

7. (a) When waves at a point are in phase, the principle of superposition states that the amplitude of the resultant wave is the vector sum of the individual amplitudes i.e. twice the amplitude of the original waves. This is referred to as constructive interference.

   (b) (i) Lines 1 and 2

   (ii) 2.5 λ

8. (a) Path difference = 1400 – 1260 = 140 m

   (b) Path difference = $\frac{140}{2.8} = 50\ \lambda$

   The path difference is a whole number of wavelength. The waves will meet in phase and undergo constructive interference.

   (c) $2A$

   (d) $4A$

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9. (a) The two-slit interference pattern consists of alternating bright and dark fringes or bands of equal width.

The colour of the bright fringes corresponds to the colour of the monochromatic light used.

(b) (i) $d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{5 \times 5.00 \times 10^{-7}}{2.50 \times 10^{-4}}\right) = 0.573°$

(ii) $\Delta y = \frac{\lambda L}{d} = \frac{5.00 \times 10^{-7} \times 0.350}{2.50 \times 10^{-4}} = 7.00 \times 10^{-4}$ m

(c) If the path difference at X is $\frac{\lambda}{2}$, then the waves from the double slits are half a wavelength out of phase at X and undergo destructive interference. The amplitude of the waves cancels and the result is light of minimum intensity. i.e. a dark fringe

10. (a) The path difference is λ.

The waves from the double slits are in phase at P and undergo constructive interference. The amplitude doubles and the result is light of maximum intensity (i.e. a bright fringe).

(b) $\Delta y = 1.29 \times 10^{-3}$ m

$\Delta y = \frac{\lambda L}{d} = \therefore \lambda = \frac{d\Delta y}{L} = \frac{1.80 \times 10^{-4} \times 1.29 \times 10^{-3}}{0.400} = 5.81 \times 10^{-7}$ m

(c) (i) $\Delta y \alpha L$

Increasing the distance between the double slits and the screen will increase the separation of the maxima observed on the screen. The distance between the centre of the screen and P increases.

(ii) $\Delta y \alpha \lambda$

If a light source with a smaller wavelength illuminates the double slits, the distance between adjacent maxima will decrease. The distance between the centre of the screen and P decreases.

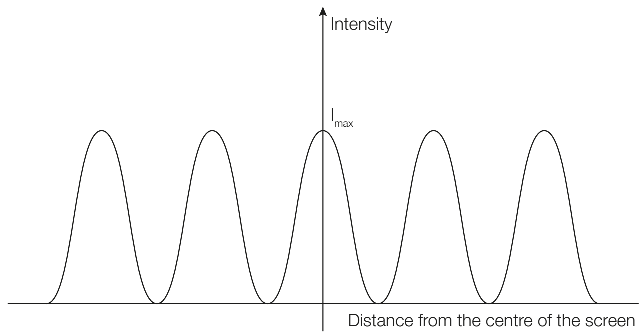
11. (a) Light diffracts at each of the double slits. This produces circular waves that overlap and allow the interference pattern to form on a screen.

(b) $d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3 \times 590 \times 10^{-9}}{6.0 \times 10^{-4}}\right) = 0.17°$

(c) $\Delta y = \frac{\lambda L}{d} = \frac{590 \times 10^{-9} \times 0.45}{6.0 \times 10^{-4}} = 4.4 \times 10^{-4}$ m

(d) $6\Delta y = 6 \times 4.4 \times 10^{-4} = 2.6 \times 10^{-3}$ m

(e)



12. (a) $\Delta y = \frac{\lambda L}{d} = \therefore \lambda = \frac{d\Delta y}{L} = \frac{0.27 \times 10^{-3} \times 4.2 \times 10^{-3}}{2.5} = 4.5 \times 10^{-7}$ m

(b) 2λ

(c) $d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2 \times 4.5 \times 10^{-7}}{0.27 \times 10^{-3}}\right) = 0.19°$

13. (a) $5\Delta y = 7.00 \times 10^{-2} \quad \therefore \Delta y = 1.40 \times 10^{-2}$ m

(b) $\Delta y = \frac{\lambda L}{d} = \therefore d = \frac{\lambda L}{\Delta y} = \frac{633 \times 10^{-9} \times 2}{1.40 \times 10^{-2}} = 9.04 \times 10^{-5}$ m

(c) (i) $\Delta y \alpha L$

Reducing the distance between the double slits and the screen will decrease the distance between adjacent maxima.

(ii) Blue laser has a smaller wavelength than red laser.

Since $\Delta y \alpha \lambda$, the distance between adjacent maxima will decrease.

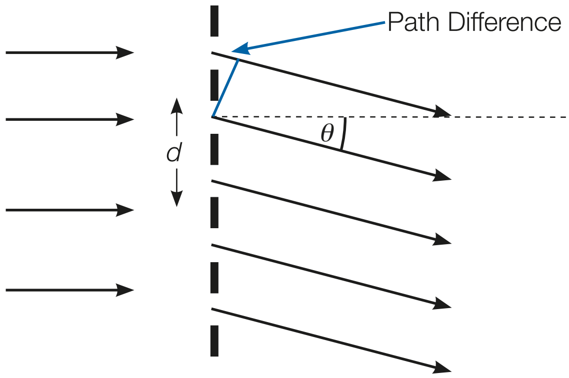
14. (a) $d = \frac{1}{6.00 \times 10^5} = 1.67 \times 10^{-6} m$

(b) $d\sin\theta = m\lambda \therefore \theta = \sin^{-1}(\frac{m\lambda}{d}) = \sin^{-1}(\frac{1 \times 5.40 \times 10^{-7}}{1.67 \times 10^{-6}}) = 18.9°$

(c) $d\sin\theta = m\lambda \therefore m = \frac{d\sin 90}{\lambda} = \frac{1.67 \times 10^{-6}}{5.40 \times 10^{-7}} = 3.1$

A maximum of three orders is possible.

15. (a)



(b) The parallel beams of light that are transmitted through the diffraction grating are focused by the convex lens onto the screen.

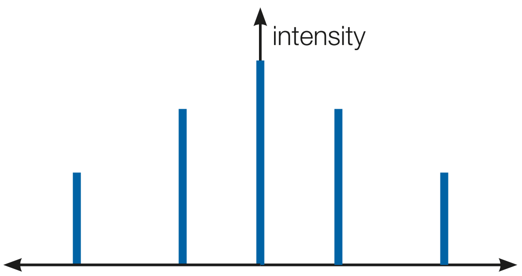
The path difference between rays of light from consecutive slits is given by $d\sin\theta$. For the *mth* order maximum, the path difference is $m\lambda$. It follows that $d\sin\theta = m\lambda$ where $m = 0, 1, 2\ldots$

(c) $d = \frac{1 \times 10^{-3}}{800} = 1.25 \times 10^{-6} m$

$d\sin\theta = m\lambda \therefore m = \frac{d\sin 90}{\lambda} = \frac{1.25 \times 10^{-6}}{5.00 \times 10^{-7}} = 2.5$

A maximum of two orders on either side of the central maximum are possible.

(d)



(e) Fewer lines per millimetre means that the slit separation *d* increases.

Since the maximum number of orders is given by $m = \frac{d\sin 90}{\lambda} = \frac{d}{\lambda}$, then a larger value of *d* will produce a larger number of orders.

16. $d = \frac{1 \times 10^{-3}}{360} = 2.78 \times 10^{-6} m$

$d\sin\theta = m\lambda \therefore \theta = \sin^{-1}(\frac{m\lambda}{d}) = \sin^{-1}(\frac{3 \times 6.0 \times 10^{-7}}{2.78 \times 10^{-6}}) = 40.4 \approx 40°$

17. (a) $d\sin\theta = m\lambda \therefore d = \frac{m\lambda}{\sin\theta} = \frac{4 \times 4.50 \times 10^{-7}}{\sin 76.0} = 1.86 \times 10^{-6}\ m$

(b) *d* is the distance between the lines. If N represents the number of lines per metre, then $d = \frac{1}{N}$.

$N = \frac{1}{d} = \frac{1}{1.86 \times 10^{-6}} = 5.38 \times 10^5$

18. (a) White light contains a range of wavelengths from violet through to red. A white line is seen in the centre of the screen because the path difference between light from each slit is zero for all wavelengths. The light is in phase at the centre of the screen and all wavelengths undergo constructive interference to recombine and produce a white line.

(b) Since $\sin\theta \alpha \lambda$, violet is seen in the first-order followed by all the other colours through to red. This is because the wavelength of violet light is smaller than the wavelength of red light and therefore produces a maximum at a smaller angle than red light. All the other wavelengths produce maxima between the violet and the red in ascending order of wavelength.

19. (a) As the light passes through the transmission diffraction grating, it diffracts at each slit producing circular wavelets. The circular wavelets overlap and interfere producing maxima at positions on a screen where the waves from adjacent slits meet in phase and undergo constructive superposition. This corresponds to positions where the path difference between the light from adjacent slits is $m\lambda$.

(b) The position of the maxima can be analysed using $d\sin\theta = m\lambda$.

Since $\sin\theta \alpha \lambda$, violet light, which has a smaller wavelength than green light will produce maxima at smaller angles.

20. $d = \dfrac{1 \times 10^{-2}}{2000} = 5 \times 10^{-6} m$

The second-order red appears at an angle of

$d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) = \sin^{-1}\left(\dfrac{2 \times 7.00 \times 10^{-7}}{5 \times 10^{-6}}\right) = 16.3°$

The third-order violet appears at an angle of

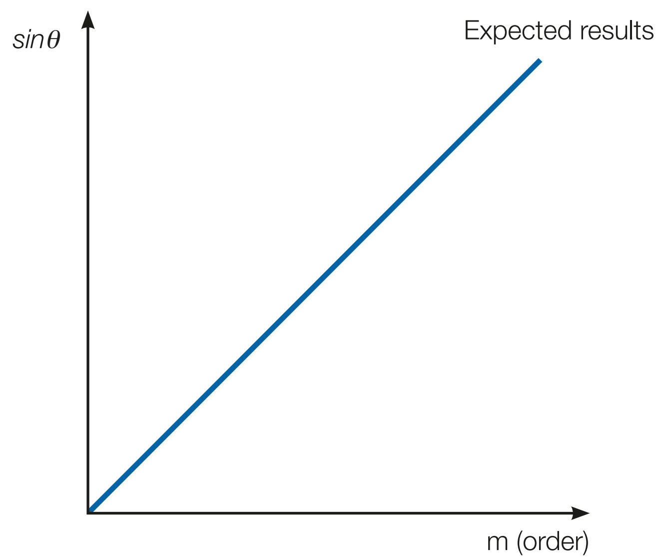
$d\sin\theta = m\lambda \therefore \theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) = \sin^{-1}\left(\dfrac{3 \times 4.00 \times 10^{-7}}{5 \times 10^{-6}}\right) = 13.9°$

The third-order violet appears at a smaller angle than the second order violet. The third-order spectrum therefore overlaps the second.

21. (a) Since $d\sin\theta = m\lambda$, then $\sin\theta \alpha m$

Sine of the angle at which the maxima occur is directly proportional to the order.

(b)



(c) On the graph in part (b)

(d) If $d\sin\theta = m\lambda$ is compared to the equation for a straight line $y = mx$, then the gradient of the graph gives $\dfrac{\lambda}{d}$.

The distance between grating lines $d = \dfrac{\lambda}{gradient} = \dfrac{6.33 \times 10^{-7}}{gradient}$.

The number of lines per metre $= \dfrac{1}{d}$

22. Science as a human endeavour activity – Light magnetometers:

(a) The key concept of development is evident in the quote from Babak Amirsolaimani. The new technology, of using light to measure brain activity has improved the efficiency of data collection and analysis. Data can be safely collected in real-time. A diagnosis and hence treatment can begin faster. This is likely to reduce recovery time and possibly deaths in patients. This light sensor therefore has the potential to replace the process currently used (MRI).

(b) The light sensors are economic, portable and provide a method for safely measuring (no shielding required) brain activity in real-time. This means they have an application on sports fields and conflict zones. The benefit is that patients can be assessed quickly resulting in faster treatment and potentially better recovery outcome. Other applications include predicting volcanic eruptions and earthquakes, better understanding of brain diseases and identifying oil and mineral deposits.

3.2 Wave–Particle Duality

1. (a) $E = hf = 6.63 \times 10^{-34} \times 3.2 \times 10^{9} = 2.1 \times 10^{-24}\ J$

   (b) $v = f\lambda \quad \therefore \quad \lambda = \frac{v}{f} = \frac{3.00 \times 10^{8}}{3.2 \times 10^{9}} = 9.38 \times 10^{-2}\ m$

   $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{9.38 \times 10^{-2}} = 7.1 \times 10^{-33}\ sN$

2. (a) $E = hf \quad \therefore \quad f = \frac{E}{h} = \frac{3.62 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.46 \times 10^{14}\ Hz$

   (b) $v = f\lambda \quad \therefore \quad \lambda = \frac{v}{f} = \frac{3.00 \times 10^{8}}{5.46 \times 10^{14}} = 5.49 \times 10^{-7}\ m$

   $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.49 \times 10^{-7}} = 1.21 \times 10^{-27}\ sN$

3. $E = hf = \frac{hv}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{640 \times 10^{-9}} = 3.1 \times 10^{-19}\ J$

4. $E = hf = \frac{hv}{\lambda} \quad \therefore \quad \lambda = \frac{hv}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{20.0 \times 1.60 \times 10^{-9}} = 6.22 \times 10^{-8}\ m$

   $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.22 \times 10^{-8}} = 1.07 \times 10^{-26}\ sN$

5. Power = energy per second = 0.22 × 60.0 = 13.2 W

   Energy of each photon $E = hf = \frac{hv}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{7.00 \times 10^{-7}} = 2.84 \times 10^{-19}\ J$

   Number of photons $= \frac{13.2}{2.84 \times 10^{-19}} = 4.65 \times 10^{19}$

6. (a) Einstein used the concept of photons and the law of conservation of energy to explain the ejection of electrons. Einstein's explanation was based on the assumption that only ***one electron*** in the metal surface can absorb the energy of ***one*** photon.

   When an electron in the metal absorbs the energy of a photon, the transfer of energy is immediate. The electrons are therefore emitted instantaneously as long as the photons have an energy equivalent to or greater than the work function or the minimum energy needed to release an electron from the surface. Using the law of conservation of energy, part of the energy of the incident photons is used to release an electron, the rest is transformed into the kinetic energy of the electron.

   (b) $W = hf_0 = 6.63 \times 10^{-34} \times 1.1 \times 10^{14} = 7.3 \times 10^{-20}\ J$

   (c) $E_{k_{max}} = hf - W = 6.63 \times 10^{-34} \times 7.5 \times 10^{15} - 7.3 \times 10^{-20} = 4.9 \times 10^{-18}\ J$

7. (a) $W = 2.1 \times 1.60 \times 10^{-19} = 3.36 \times 10^{-19}\ J$

   $W = hf_0 \quad \therefore \quad f_0 = \frac{W}{h} = \frac{3.36 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.1 \times 10^{14}\ Hz$

   (b) $v = f\lambda \quad \therefore \quad f = \frac{v}{\lambda} = \frac{3.00 \times 10^{8}}{520 \times 10^{-9}} = 5.77 \times 10^{14}\ Hz$

   $E_{k_{max}} = hf - W = 6.63 \times 10^{-34} \times 5.77 \times 10^{14} - 3.36 \times 10^{-19} = 4.7 \times 10^{-20}\ J$

   (c) The intensity or brightness of light is directly proportional to the number of photons in the light. Increasing the intensity of the light used to illuminate the metal surface, increases the number of photons incident on the metal without changing their energy. This means that more electrons can absorb the energy of one photon and be emitted. This explains why more electrons are emitted with more intense light.

8. (a) The photoelectric effect is the emission of electrons from the surface of a material when it is illuminated with light of sufficiently high frequency.

(b) (i) $E = hf = 6.63 \times 10^{-34} \times 1.00 \times 10^{15} = 6.63 \times 10^{-19}\ J$

(ii) $W = 3.50 \times 1.60 \times 10^{-19} = 5.60 \times 10^{-19}\ J$

$$W = hf_0 \quad \therefore \quad f_0 = \frac{W}{h} = \frac{5.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 8.45 \times 10^{14}\ Hz$$

(iii) $E_{k_{max}} = hf - W = 6.63 \times 10^{-34} \times 1.00 \times 10^{15} - 5.60 \times 10^{-19} = 1.03 \times 10^{-19}\ J$

(iv) $E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.03 \times 10^{-19}}{9.11 \times 10^{-31}}} = 4.76 \times 10^5\ ms^{-1}$

(v) $E_{k_{max}} = eV_s \quad \therefore \quad V_s = \frac{E_{k_{max}}}{e} = \frac{1.03 \times 10^{-19}}{1.60 \times 10^{-19}} = 0.644\ V$

(c) Doubling the intensity will double the number of photons incident on the metal but will not change their energy. $E_{k_{max}}$ does not change, therefore the stopping voltage does not change.

(d) $v = f\lambda \quad \therefore \quad f = \frac{v}{\lambda} = \frac{3.00 \times 10^8}{600 \times 10^{-9}} = 5 \times 10^{14}\ Hz$

The threshold frequency of the metal plate is $8.45 \times 10^{14}\ Hz$. Light with a frequency below this value does not emit electrons from the metal plate. It follows that light with a wavelength of 600 nm will not emit electrons from the metal plate.

9. (a) $E_{k_{max}} = eV_s = 1.60 \times 10^{-19} \times 3.13 = 5.008 \times 10^{-19}\ J = 5.01 \times 10^{-19}\ J$

(b) $E_{k_{max}} = hf - W \quad \therefore W = hf - E_{k_{max}} = 6.63 \times 10^{-34} \times 6.25 \times 10^{15} - 5.01 \times 10^{-19} = 3.64 \times 10^{-18}\ J$

10. Using the law of conservation of energy, part of the energy of the incident photons is used to release an electron, the rest is transformed into the kinetic energy of the electron. Since electrons deeper in the metal require more energy to be released, the amount of kinetic energy available varies. Electrons close to the surface are emitted with more kinetic energy than those deeper in the metal surface. This means that electrons are emitted with a range of kinetic energies up to a maximum value. Those electrons bound with an energy greater than that of the incident photons are not emitted from the metal.

11. $W = hf_0 = 6.63 \times 10^{-34} \times 1.02 \times 10^{14} = 6.76 \times 10^{-20}\ J$

$E_{k_{max}} = hf - W = 6.63 \times 10^{-34} \times 6.4 \times 10^{14} - 6.76 \times 10^{-20} = 3.57 \times 10^{-19}\ J$

$E_{k_{max}} = eV_s \quad \therefore \quad V_s = \frac{E_{k_{max}}}{e} = \frac{3.57 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.23\ V$

12. (a) $W = hf_0 = 6.63 \times 10^{-34} \times 1.23 \times 10^{14} = 8.15 \times 10^{-20}\ J$

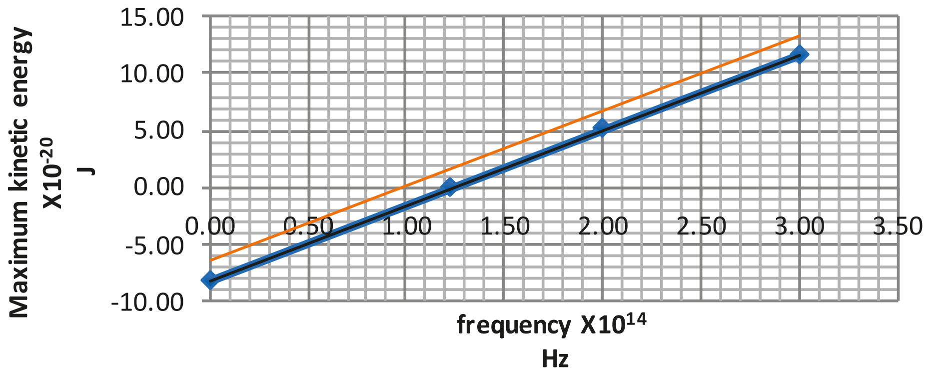
(b) Zinc has the lowest threshold frequency and hence work function. Since the energy of the incident photons ($E = hf$) is the same for all three metals, then it follows that $E_{k_{max}} = hf - W$ is maximised for the metal with the lowest work function. Zinc will emit electrons with the greatest maximum kinetic energy.

(c) (i) The work function of gold

(ii) The threshold frequency

(iii) Planck's constant

(iv)

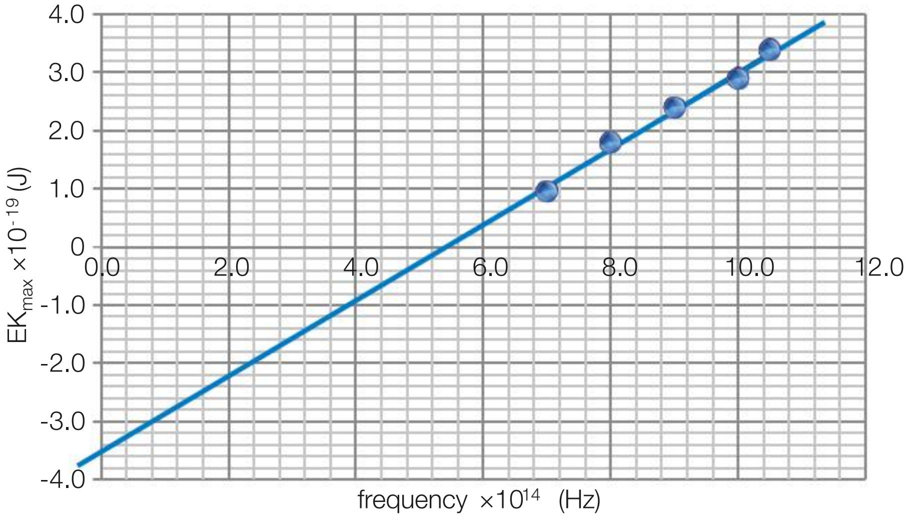


S

13. (a)

$E_{K_{max}}$ (J)
9.6×10^{-20}
1.8×10^{-19}
2.4×10^{-19}
2.9×10^{-19}
3.4×10^{-19}

(b)



(c) (i) $h = gradient = \dfrac{3.8 \times 10^{-19}}{11.2 \times 10^{14} - 5.4 \times 10^{14}} = 6.6 \times 10^{-34}\ Js$

(ii) $5.4 \times 10^{14}\ Hz$

(iii) $3.6 \times 10^{-19}\ J$

14. (a) (i) The minimum energy needed to eject electrons from the surface of a material.

(ii) Using the law of conservation of energy:

$E_{photon} = E_{release} + E_{k_{max}}$

$hf = W + E_{k_{max}}$

$E_{k_{max}} = hf - W$

(b) A negative potential produces an electric field that repels the electrons and stops them from reaching the anode. The potential difference is increased until no electrons reach the anode and the current registered by the sensitive ammeter drops to zero. This potential difference is the **stopping voltage (V_s).** When the stopping voltage is achieved, the work done by the electric field matches the maximum kinetic energy of the emitted electrons.

$$W = q\Delta V = eV_s = E_{k_{max}}$$

The colour of the filter is changed and the corresponding stopping voltage is recorded for a different frequency of light. This is repeated for other coloured filters. The maximum kinetic energy of the emitted electrons for each frequency of light can be calculated using $E_{k_{max}} = eV_s$.

A graph of $E_{k_{max}}$ against frequency is plotted to establish the relationship between the maximum kinetic energy of the emitted electrons and the frequency of light.

15. (a) A Filament

B High voltage supply/potential difference

C Cooling fins

(b)

Feature	Purpose
A	Releases electrons
B	Accelerates electrons
C	Draws heat energy away from the X-ray tube anode

16. A High-intensity peaks
 B Continuous range of frequencies (bremsstrshlung)
 C Maximum frequency

17. (a) $f_{max} = \frac{e\Delta V}{h} = \frac{1.60 \times 10^{-19} \times 5.00 \times 10^{4}}{6.63 \times 10^{-34}} = 1.21 \times 10^{19}\ Hz$

 (b) $f_{max}\ \alpha \Delta V$

 The accelerating potential is doubled. This doubles the maximum frequency of the X-ray photons.

18. (a) As electrons with fixed kinetic energy collide with the target metal, their path is deviated by the electrostatic force between the electron and the nucleus of the target atoms. The electrons slow down and their kinetic energy decreases. Using the law of conservation of energy, the difference in the kinetic energy of the electron before the collision and after the collision with the nucleus is transformed into an X-ray photon.

 The difference in the kinetic energy of the electron before and after the collision varies depending on how close the electron collides with the nucleus. It follows that the energy transformed into an X-ray photon also varies. This results in a continuous range of frequencies for the X-rays.

 When an electron collides head on with the nucleus of a target atom, all of its initial kinetic energy is transferred to an X-ray photon. The X-ray photon has a maximum energy and hence frequency (f_{max}).

 (b) The accelerating potential (ΔV) does work (W) on the electrons and they gain kinetic energy. In a head on collision with the nucleus of the target atoms all of this kinetic energy is transformed into the energy of an X-ray photon.

 $W = \Delta E_k = q\Delta V = e\Delta V = hf_{max}$

 Rearranging yields $f_{max} = \frac{e\Delta V}{h}$

19. The lung cancer tumour is more dense than the surrounding tissue. This means that more X-rays are attenuated and fewer pass through the tumour to expose the X-ray film when compared with surrounding tissue. The tumour appears whiter than the surrounding tissue.

20. (a) $f_{max} = \frac{e\Delta V}{h} \quad \therefore \quad \Delta V = \frac{hf_{max}}{e} = \frac{6.63 \times 10^{-34} \times 2.88 \times 10^{19}}{1.60 \times 10^{-19}} = 1.19 \times 10^{5}\ V$

 (b) The X-rays are being produced with an accelerating potential of almost 120 KV. This is high. The X-rays can be classified as hard X-rays. Their penetration power is high.

 (c) Compact bone e.g. leg or chest X-ray

 (d) Increasing the filament current increases the number of electrons released from the filament. More electrons collide with the target metal to produce more X-ray photons. The kinetic energy of the incident electrons does not change as the potential difference is unchanged. The maximum frequency of the X-rays does not change.

21. (a) $E_k = q\Delta V = 1.60 \times 10^{-19} \times 100 \times 10^{3} = 1.6 \times 10^{-14}\ J$

 $E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-14}}{9.11 \times 10^{-31}}} = 1.87 \times 10^{8}\ ms^{-1}$

 (b) $f_{max} = \frac{e\Delta V}{h} = \frac{1.60 \times 10^{-14}}{6.63 \times 10^{-34}} = 2.41 \times 10^{19}\ Hz$

 $v = f\lambda \quad \therefore \quad \lambda_{min} = \frac{v}{f_{max}} = \frac{3.00 \times 10^{8}}{2.41 \times 10^{19}} = 1.24 \times 10^{-11}\ m$

22. (a) A low intensity beam is achieved with a small filament current. A low filament current releases a smaller number of electrons from the filament. There are fewer collisions between the incident electrons and the target metal. This produces fewer X-ray photons.

 (b) Movement causes blurring of the X-ray photograph.

23. $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.12 \times 10^{-14}} = 5.92 \times 10^{-20}\ m$

24. $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 2.9 \times 10^{7}} = 2.5 \times 10^{-11}\ m$

25. $\Delta E_k = q\Delta V = 1.60 \times 10^{-19} \times 6.50 \times 10^4 = 1.04 \times 10^{-14}\ J$

$E_k = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.04 \times 10^{-14}}{9.11 \times 10^{-31}}} = 1.51 \times 10^8\ \text{ms}^{-1}$

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.51 \times 10^8} = 4.82 \times 10^{-12}\ m$

26. (a) The interference pattern has the same appearance as the two-slit interference pattern for light. The significance is that particles are behaving like light i.e. wavelike.

(b) $d\sin\theta = m\lambda \quad \therefore \quad \lambda = \frac{d\sin\theta}{m} = \frac{1.6 \times 10^{-10} \sin 22}{1} = 6.0 \times 10^{-11}\ m$

27. (a) Low energy electrons were fired towards a nickel crystal. An electron detector was used to measure the intensity of the scattered electrons. Electrons were diffracted by the surface layers of the crystal at preferred angles in a similar way to light. The equation $d\sin\theta = m\lambda$ was able to predict the angles at which the electrons were diffracted.

The crystal spacing d was known and the angle θ of the diffracted electrons was measured for a given order m. Davisson and Germer compared the wavelength found experimentally using the wave relationship $\lambda = \frac{d\sin\theta}{m}$ to the theoretical wavelength calculated using the de Broglie relationship $\lambda = \frac{h}{p}$. The results agreed, confirming the de Broglie relationship.

(b) $d\sin\theta = m\lambda \quad d = \frac{m\lambda}{\sin\theta} = \frac{1 \times 1.6 \times 10^{-10}}{\sin 50} = 2.1 \times 10^{-10}\ m$

(c) $\lambda = \frac{h}{p} = \frac{h}{mv} \quad \therefore \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.6 \times 10^{-10}} = 4.55 \times 10^6\ ms^{-1}$

$\Delta E_k = \frac{1}{2}mv^2 = q\Delta V \quad \therefore \quad \Delta V = \frac{mv^2}{2q} = \frac{9.11 \times 10^{-31} \times (4.55 \times 10^6)^2}{2 \times 1.60 \times 10^{-19}} = 59\ V$

28. $W = \Delta E_k = q\Delta V = \frac{1}{2}mv^2$

therefore $qm\Delta V = \frac{1}{2}m^2 v^2 = \frac{1}{2}p^2$

therefore $p = \sqrt{2qm\Delta V}$

Now $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2qm\Delta V}}$

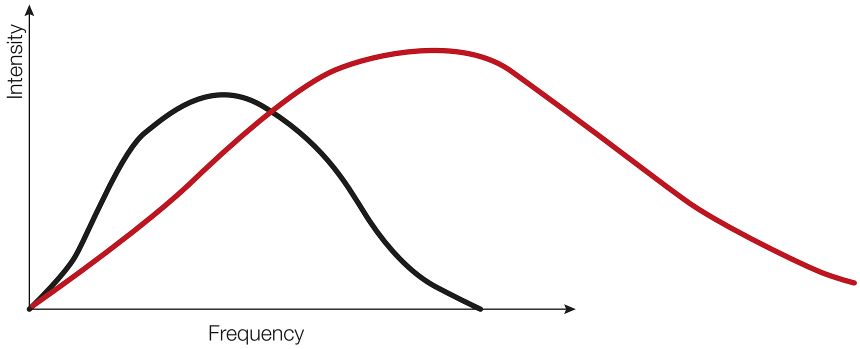
3.3 Structure of the atom

1. (a) The energy of a photon is given by $E = hf = \frac{hc}{\lambda}$

   Violet light has a shorter wavelength than red light. The photons emitted in the violet part of the spectrum therefore have a larger energy than the photons emitted in the red part of the spectrum.

   (b) If the temperature of the filament increases, a larger range of frequencies is produced. The intensity of each frequency increases as does the peak frequency. The spectrum shifts and there is more violet in the spectrum.

   (c)



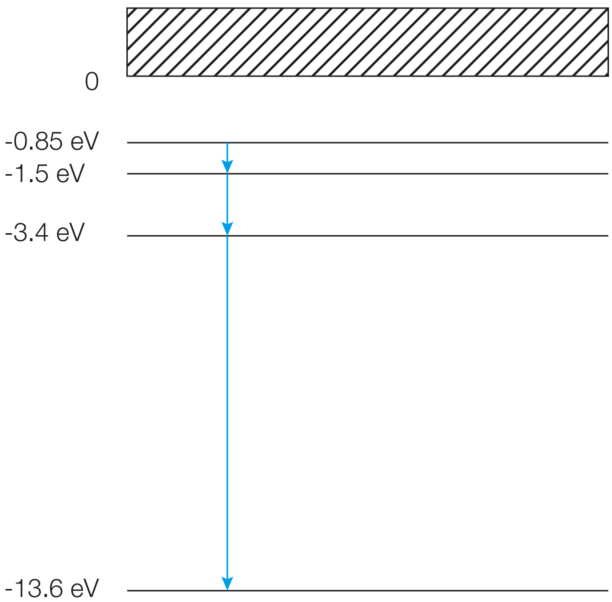
2. (a) A continuous spectrum containing a continuous range of frequencies.

   (b) Hotter filament globes (photograph B), produce light with a larger range of frequencies. The intensity of each frequency increases as does the peak frequency. The continuous spectrum shifts and there is more violet in the spectrum. This produces light with a whiter appearance.

3. (a) 13.6 eV

   (b) (i) 12.75 eV

   (ii)



   (iii) $E_4 - E_3 = -0.85 - (-1.5) = 0.65eV$

   $E = hf \quad \therefore \quad f = \frac{E}{h} = \frac{0.65 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{14}\ Hz$

4. (a) In order for the hydrogen atoms to be raised to an excited state, the energy of the incident photons must exactly match the energy difference between the ground state and one of the electron energy-levels. There is no energy difference of 12.0 eV which means that the incident photons are not absorbed and the hydrogen atoms are not excited.

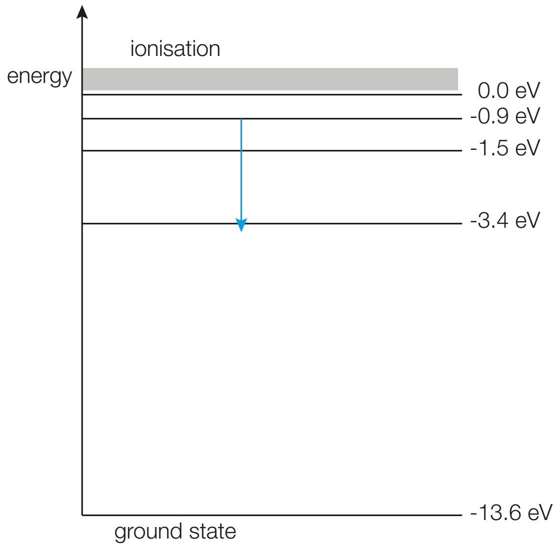
(b) (i) Electrons are particles and can impart some of their kinetic energy to the electrons in the hydrogen atoms. When the incident electrons collide with electrons in the ground state of hydrogen, these electrons can absorb 10.2 eV while the incident electrons scatter with a kinetic energy of 1.80 eV. The ground state electrons are elevated to $n = 2$ but quickly return to the ground state emitting photons of energy 10.2 eV.

(ii) $E = hf \quad \therefore \quad f = \frac{E}{h} = \frac{10.2 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 2.46 \times 10^{15}\ Hz$

5. (a) 498 nm

(b) $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{498 \times 10^{-9}} = 3.99 \times 10^{-19}\ J = 2.50eV$

(c)



6. (a) A line emission spectrum is produced when the atoms of a pure gas are heated to high temperatures or subjected to a large potential difference. The emitted light is viewed through a spectrometer or a diffraction grating.

(b) The spectrum consists of thin discrete coloured lines on a dark background.

(c) (i) Lithium and strontium.

(ii) The position and therefore wavelength and frequency of the spectral lines of lithium and strontium exactly match the position of the spectral lines in the mixture. Since the line emission spectrum of an element is unique to the element it can be concluded that lithium and strontium are present in the mixture.

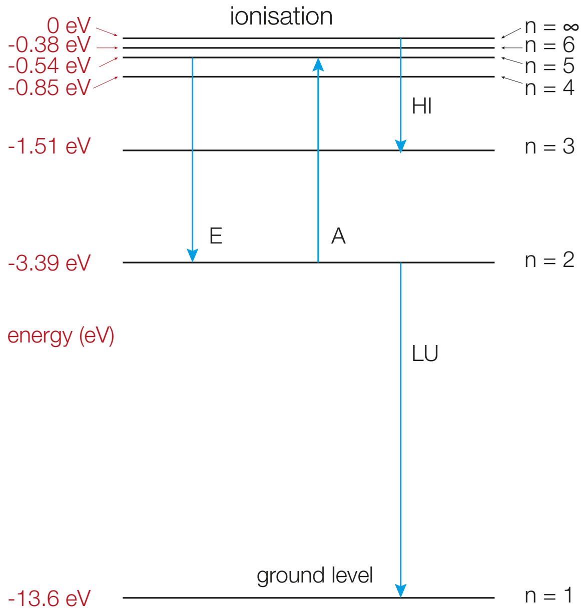
7. (a) Transition A involves a smaller energy than transition B. The photon released in transition A therefore has a smaller energy but a larger wavelength ($E = hf = \frac{hc}{\lambda}$).

(b) (i) The ionisation energy of an atom is the minimum energy required to remove a single electron from the atom in its ground state.

(ii) 8.21×10^{-19} J

8. (a) $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{4.36 \times 10^{-7}} = 4.56 \times 10^{-19}\, J = 2.85 eV$

(b) The transition *n* = *5* to *n* = *2* as shown on the diagram below (E).



(c) The transition *n* = *2* to *n* = *5* as shown on the diagram below (A).

(d) (i) The lowest energy photon emitted in the UV region of the electromagnetic spectrum corresponds to the smallest electron transition that ends on the ground state i.e. the transition *n* = *2* to *n* = *1* as shown on the diagram above (LU).

(ii) The highest energy photon emitted in the IR region of the electromagnetic spectrum corresponds to the largest electron transition that ends on *n* = *3* i.e. the transition *n* = ∞ to *n* = *3* as shown on the diagram above (HI).

9. Electrons are accelerated towards a target metal in an X-ray tube. When these incident electrons collide with electrons in the lower electron energy-levels (inner shell electrons) of the atoms of the target metal, they can transfer energy to these inner shell electrons and knock them out of the target atoms. Electrons from higher energy levels 'drop down' or make descending transitions to fill their place. This causes X-ray photons of energy equal to the energy difference between the two energy levels to be emitted. Since the energy difference is discrete, this results in X-ray photons of discrete energy and hence frequency. The intensity (number) of these discrete X-ray photons is greater than those of all other frequencies and produces the peaks in the X-ray spectrum graph.

10. (a) $E = hf = \frac{hc}{\lambda} \therefore \quad \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{10.2 \times 1.60 \times 10^{-19}} = 1.22 \times 10^{-7}\, m$

(b) Ultra-violet region

(c) The transition from *n* = *1* to *n* = *3* involves a larger energy than the transition from *n* = *1* to *n* = *2*. The photons that cause this transition therefore have a higher frequency and therefore a smaller wavelength than those that cause the transition from *n* = *1* to *n* = *2*. Since the photons that cause the transition from *n* = *1* to *n* = *2* are in the ultra-violet region, photons with a smaller wavelength will also be in the ultra violet region of the electromagnetic spectrum.

(d) When hydrogen gas is at room temperature, the electron is found in the ground state of the atom. Electron transitions from the ground state to any of the higher-energy states fall in the ultra-violet region of the electromagnetic spectrum. This is because the electron in the ground state must absorb a photon of energy 10.2 eV or more to be elevated. This explains why there are no absorption lines are in the visible region of the electromagnetic spectrum when hydrogen is at room temperature.

11. (a) The transition $n = 2$ to $n = 5$ results from the absorption of a photon of energy 2.86 eV. The wavelength of a photon of energy 2.86 eV is given by:

$$E = hf = \frac{hc}{\lambda} \therefore \quad \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{2.86 \times 1.60 \times 10^{-19}} = 4.35 \times 10^{-7}\ m = 435\ nm$$

i.e. the spectral line labelled A results from a transition from $n = 2$ to $n = 5$.

(b)

Spectral line	Transition	Wavelength (nm)
A	$n = 2$ to $n = 5$	435
B	$n = 2$ to $n = 4$	488
C	$n = 2$ to $n = 3$	654

Sample calculation: $\lambda = 488\ nm$ results from the absorption of a photon of energy

$$E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{488 \times 10^{-9}} = 4.07 \times 10^{-19}\ J = 2.55\ eV$$

This corresponds to the transition $n = 2$ to $n = 4$.

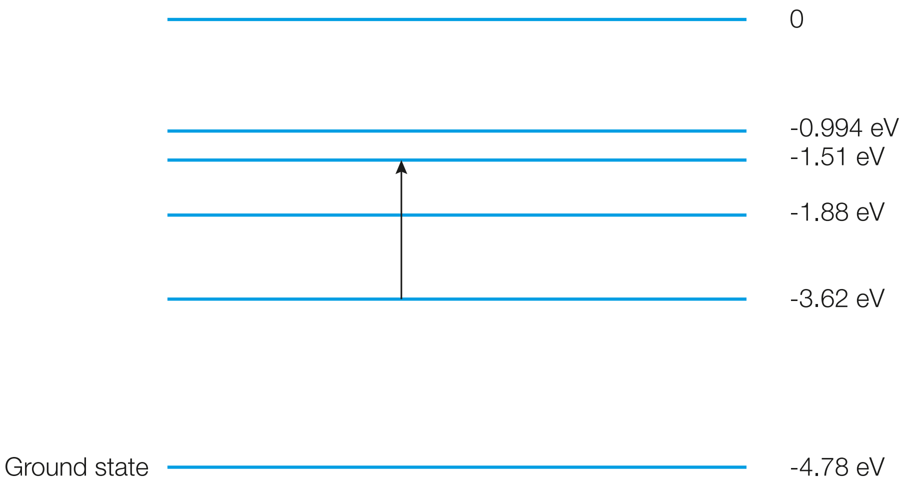
Or absorption lines in the visible region (Balmer) result from transitions beginning in $n = 2$. The absorption line B has a longer wavelength than A. It must result from a shorter transition i.e. $n = 2$ to $n = 4$. Similarly the absorption line C must result from a shorter transition still i.e. $n = 2$ to $n = 3$.

12. (a) Electrons within the atoms of the cooler gases in the Sun's atmosphere absorb photons with energy equal to the energy gap between lower electron energy-levels and higher electron energy-levels. The electrons are elevated to higher electron energy-levels while the incident photons are removed from the incident light. This produces the dark lines in the Sun's continuous spectrum as the intensity of the absorbed photons is too low to be observed.

(b) (i) $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{588 \times 10^{-9}} = 3.38 \times 10^{-19}\ J = 2.11\ eV$

(ii) n = 2 to n = 4

(iii)



13. The position of the spectral lines h, f, F and C in the solar spectrum exactly match the position and therefore frequency of the emission lines for hydrogen. The emission spectrum of a pure element is unique to that element. It follows that the absorption spectrum for the Sun demonstrates the presence of hydrogen in the solar atmosphere.

14. When high-energy ultra-violet photons are absorbed by the atoms in the exoskeleton of the scorpion, electrons in these atoms are elevated to a higher electron energy-levels. These excited states are generally short-lived and the atom returns spontaneously to its ground state. As the electrons descend, they make many smaller transitions. This results in a number of smaller-energy and hence frequency photons being emitted. A significant number of the emitted photons are in the visible region of the electromagnetic spectrum and collectively cause the scorpion to glow a blue colour.

15. (a) When an electron in an atom absorbs a photon and reaches a higher electron energy-level the atom is said to be in an excited state. Excited states are generally short-lived and the electron returns spontaneously to its previous electron energy-level often by emitting a series of lower-energy photons. This is known as 'spontaneous emission'. The emitted photons have random directions and phase.

When a photon is incident on an electron that has been raised to a higher electron energy-level, and the energy of the photon corresponds to a transition to a lower electron energy-level, then the photon can stimulate an electron to transition to the lower electron energy-level. This results in two identical photons; the original photon and a second photon that results from the transition. This is known as 'stimulated emission'. The emitted photon is identical in energy, direction and phase to the incident photon.

(b) If stimulated emission is to predominate over absorption and hence spontaneous emission when light is incident on a set of atoms, a population inversion is required. This in turn requires a metastable state in the atom.

16. (a) A metastable state is an excited state in an atom that has a longer lifetime than other excited states. That is the electron remains in the higher electron energy-level longer than it normally would before making a transition to a lower-energy state.

(b) A population inversion is produced in a set of atoms whenever there are more atoms in a higher-energy state than in a lower-energy state. For a population inversion to occur, the higher-energy state must be a metastable state.

(c) When neon atoms have been excited to their metastable state, and a population inversion occurs, stimulated emission will predominate over absorption. Some photons corresponding to the transition $n = 3$ to $n = 2$ are released by spontaneous emission and in turn cause electrons in the higher-energy state of neon ($n = 3$) to return to the lower-energy state ($n = 2$) via stimulated emission. Each stimulated emission produces two identical photons, the incident photon and the emitted photon. The emitted photon is identical in energy, direction and phase to the incident photon. These photons go on to cause stimulated emission in other neon atoms. This process continues and when it occurs in many atoms a beam of monochromatic coherent light is produced.

(d) Energy of laser photons = 20.66–18.7 = 1.96 eV

$$E = hf \quad \therefore \quad f = \frac{E}{h} = \frac{1.96 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.73 \times 10^{14}\ Hz$$

17. Laser light is directional. It is emitted as a narrow beam in one direction only. The beam does not spread which makes it useful in this sort of eye surgery because the laser can be used to reattach the retina with precision.

18. (a) Workers are not handling the laser. This improves their safety as they avoid contact with the high intensity laser beam as well as the potential hazards associated with reflections, the heat produced and any fly away debris.

(b) Workers should wear protective eye glasses.

19. Science as a human endeavour activity – Structure of the atom:

(a) The European project InSPECT is coordinated by Philips in The Netherlands but is a collaboration between eight partners from various parts of the world all with expertise in optics, photonics, device manufacturing, sensors and medical technology. These partners are: B-PHOT Vrije Universiteit Brussel, Xenics (Belgium), Anteryon (The Netherlands), Lionix International (The Netherlands), Aifotec (Germany), Avantes (The Netherlands), and the Fraunhofer Institute for Reliability and Microintegration (Germany). In addition, the project is funded within the Research and Innovation Action of Horizon2020, and in collaboration with Photonics21. This demonstrates that clear communication and collaboration is involved. By sharing scientific processes and results, the scientists and organisations involved were able to make considerable advancements, and develop miniature spectrometers that will benefit medicine and society.

(b) The lack of precision of the needle tip location during a biopsy often results in false negative diagnosis. The development of integrating an optical fiber inside a biopsy needle and using spectral analysis, has enabled scientists to differentiate cancerous and non-cancerous tissue. Different concentrations in backscattered light collected by a second optical fiber identifies water, fat and haemoglobin. This diagnosis is fast and accurate, enabling real-time feedback to doctors. That is, this technology is important as it enables the treatment of cancer to start much earlier. This has the benefit of increasing survival and or recovery rate in patients.

3.4 Standard Model

1. (a) Gauge bosons, leptons and quarks.

   (b) Electromagnetic, weak nuclear, strong nuclear and gravitational.

2. (a) Exchange particles (or gauge bosons) are elementary particles that mediate the fundamental forces.

   (b)

Force	Gauge Boson
Electromagnetic	photon
Strong nuclear	gluon
Weak nuclear	W, Z
Gravitational	graviton

   (c) For two like charges repelling one another, a photon emitted by one charge will cause it to recoil as it transfers momentum and energy to the other charge. The second charge also emits a photon and recoils. More energetic photons are exchanged when the charges are closer to one another. This explains why the electromagnetic force is stronger when the charges are closer. Conversely, less energetic photons are exchanged when the charges are further apart and the electromagnetic force is weaker.

3. The electron is a fundamental particle. It does not have an internal structure, that is, it is not made up of smaller constituents where as a composite particle is.

4. (a) A lepton is a fundamental particle that is not affected by the strong nuclear force.

   (b) The positron

   (c) +1e

   (d) Tau and the muon

5. (a) Quarks are fractionally charged fundamental particles that are affected by all of the fundamental forces.

   (b) Symbol: $\overline{b}$ Charge: $+\frac{1}{3}$ e

6. Gauge bosons mediate the forces that govern the interaction between all particles. Quarks interact via the strong nuclear force. This distinguishes them from leptons which are elementary particles that are not affected by the strong nuclear force but are acted upon by the weak nuclear force. Quarks have a fractional charge while leptons have a charge of 0 or -1e. Quarks are never found in isolation where as leptons can exist in isolation.

7. A baryon is a composite particle that consists of a combination of three quarks while a meson is a composite particle that consists of one quark and one antiquark.

8. $\overline{u}ss$ and uu

9. (a) Meson

   (b) Charge = charge of an up quark + charge of an anti-strange quark

   $= \frac{2}{3}e + \frac{1}{3}e$

   $= +1e$

   (c) Zero – it is not a baryon.

10. (a) Any particle that consists of a combination of three quarks is called a baryon. The Xi-c++ particle is therefore a baryon.

   (b) Charge = 2 × charge of a charm quark + charge of an up quark

   $= 2 \times \frac{2}{3}e + \frac{2}{3}e$

   $= \frac{6}{3}e$

   $= +2e$

11. (a) The proton is formed by two up and one down quark (*uud*).

(b) Charge = 2 × charge of an up quark + charge of a down quark

$= 2 \times \frac{2}{3}e - \frac{1}{3}e$

$= +1e$

12. The neutron does not have a charge. It is formed from two down and one up quark (*ddu*). This combination of quarks has a charge of $-\frac{1}{3}e - \frac{1}{3}e + \frac{2}{3}e = 0$

13. (a) A neutron is formed from two down and one up quark (*ddu*). The proton is formed by two up and one down quark (*uud*). A down quark turns into an up quark.

(b) The electronic lepton number differs. It would be +1 for an electron neutrino.

(c) The weak nuclear force

(d) W or Z exchange boson

14. (a) The pion-plus particle is made up of one quark and one antiquark, it is a meson.

(b) Charge = charge of an up quark + charge of an anti-down quark

$= \frac{2}{3}e + \frac{1}{3}e$

$= +1e$

(c) LHS = +1e + 1e = +2e RHS = +1e + 1e = +2e

The total charge on the left hand side of the equation and the right hand side of the equation are the same. Charge is conserved.

(d) The proton is a baryon and therefore has a baryon number of +1. The pion-plus particle is a meson and therefore has a baryon number of zero.

(e) LHS = +1 + +1 = +2 RHS = +1 + 0 = +1

The total baryon number on the left hand side of the equation and the right hand side of the equation are not the same. Baryon number is not conserved. The reaction cannot occur.

15. (a)

	LHS	**RHS**
Charge	0 + 1e = +1e	0 + 1e= +1e
Baryon number	0 + 1= +1	+1 + 0 = +1
Electronic lepton number	–1	–1

All conservation laws are obeyed. The reaction can take place.

(b) (i) Charge on the RHS of the reaction = +1e (as in part (a)) According to the law of conservation of charge, the charge on the LHS of the reaction must be +1e. Since the proton has a charge of +1e, the electron neutrino must have a charge of zero.

(ii)

	LHS	**RHS**
Baryon number	0 + 1 = +1	+1 + 0 = +1
Electronic lepton number	+1	–1

The reaction cannot take place because the conservation of lepton number is violated.

16. (a) Strong nuclear force

(b) The gluon

(c) Weak nuclear force

(d) W, Z bosons

17. (a) Zero.

Baryons have a baryon number of +1, antibaryons have a baryon number of -1. All other particles have a baryon number of zero. A meson is not a baryon and would therefore have a baryon number of zero.

(b) All quarks have a baryon number of $+\frac{1}{3}$ and antiquarks a baryon number of $-\frac{1}{3}$.

(c) Baryon number $= -\frac{1}{3} + \frac{1}{3} = 0$

This confirms the answer to part (a).

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18. Baryon number = baryon number of 3 quarks = $3 \times \frac{1}{3} = +1$

19. (a) (i) photon

(ii) gluon

(b) (i) Charge = charge of an up quark + charge of an anti-down quark

$= +\frac{2}{3}e + \frac{1}{3}e$

$= +1e$

(ii) The total charge on the LHS of the reaction is +1e. Using the law of conservation of charge, the charge on the RHS of the equation must also be +1e. Since the muon neutrino does not have a charge, the charge on the anti-muon must be +1e.

(iii) W^+

20. (a) Pair annihilation refers to the process of a particle and its antiparticle colliding, such that both particles annihilate and release energy. That is, mass is turned into energy.

(b) So that momentum is conserved.

(c) $e^- + e^+ \rightarrow 2\gamma$

(d) $E = \Delta mc^2 = (0 - 2 \times 9.11 \times 10^{-31})(3.00 \times 10^8)^2 = 1.64\ 10^{-13}\ J$

(e) Each photon has an energy of $\frac{1.64 \times 10^{-13}}{2} = 8.20 \times 10^{-14}\ J$

$E = hf \quad \therefore \quad f = \frac{E}{h} = \frac{8.20 \times 10^{-14}}{6.63 \times 10^{-34}} = 1.23 \times 10^{20}\ Hz$

21. Science as a human endeavour activity – Standard model

(a) In 2015 the Nobel Prize in Physics was shared between two people, Arthur B. McDonald (the leader of SNO) and Takaaki Kajita (a leader of the Super-Kamiokande collaboration), "for the discovery of neutrino oscillations, which shows that neutrinos have mass." This indicates that clear communication and collaboration between the two and their organisations must have occurred to see them equally recognised. Experiments were conducted across the world (Japan, Italy, Russia), over a 30-year period of time. Scientist shared their data and verified similar results. A SNO detector was proposed. The data collected could only be explained if the neutrino's mass was non-zero. This discovery is unlikely to have been made without collaboration. It will help scientists better understand neutrinos and their role in the evolution of the universe.

(b) The standard model predicts that neutrinos do not have mass. The discovery that they do have mass has sparked a review of the model. The development of new theories is underway, but a more information/data is needed. For example, experimental work to determine the actual mass, symmetry and to determine whether neutrinos and antineutrinos are the same particle is required. When a new theory is developed, it will replace the current standard model. Not only will this information help scientists better understand the evolution of the universe but it may help to answer a long standing questions such as why the universe contains more matter than antimatter.

Trial exam solutions

Question booklet 1

1. (a) $v = \frac{70.0}{3.60} = 19.4\ m\ s^{-1}$

$v_H = v\cos\theta = 19.4 \times \cos 50.0 = 12.5\ m\ s^{-1}$

(b) $s = vt = 12.5 \times 4.20 = 52.5\ m$

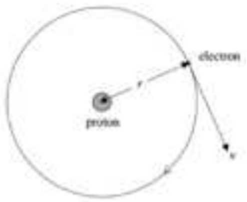
(c) $v_v = v\sin\theta = 19.4 \times \sin 50.0 = 14.9\ m\ s^{-1}$

$v^2 = v_o^{\,2} + 2as. \quad \therefore \quad s = \frac{v^2 - v_o^{\,2}}{2a} = \frac{0^2 - 14.9^2}{2 \times (-9.80)} = 11.3\ m$

2. (a) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{8.99 \times 10^9 \times (1.60 \times 10^{-19})^2}{(5.29 \times 10^{-11})^2} = 8.22 \times 10^{-8}$ N towards the proton

(b) $F = ma = \frac{mv^2}{r} \therefore \quad v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22 \times 10^{-8} \times 5.29 \times 10^{-11}}{9.11 \times 10^{-31}}} = 2.18 \times 10^6\ m\ s^{-1}$

(c)



3. As the raindrop falls from the cloud, it accelerates towards the ground under the action of gravity. As the raindrop falls, it is also acted upon by an opposing drag force or air resistance. As the speed of the raindrop increases, so too does the drag force. When the upward drag force balances the downward weight of the raindrop, the raindrop reaches a constant terminal speed. This value is much lower than 400 ms^{-1}.

4. (a) For an object with a mass m at the surface of a planet of mass M, the force experienced ($F = ma$) is equal to the gravitational force between the object and the planet.

$$F = ma = mg = \frac{GmM}{r^2} \quad \therefore \quad g = \frac{GM}{r^2}$$

(b) $g = \frac{GM}{r^2} \quad \therefore \quad M = \frac{gr^2}{G} = \frac{3.71 \times (3.39 \times 10^6)^2}{(6.67 \times 10^{-11})} = 6.39 \times 10^{23}$ kg

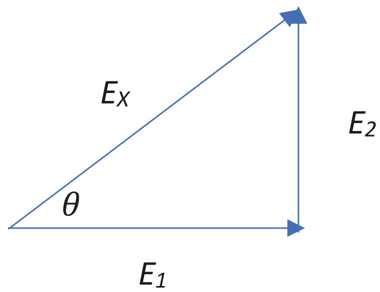
5. (a) $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{8.99 \times 10^9 \times 3.00 \times 10^{-6}}{1.20^2} = 1.87 \times 10^4\ NC^{-1}$

(b) $E_1 = 1.87 \times 10^4\ NC^{-1}$ →

$E_2 = 1.87 \times 10^4\ NC^{-1}$ ↑

$\tan\theta = \frac{1.87 \times 10^4}{1.87 \times 10^4} \therefore \theta = 45.0°$

$E_X = \sqrt{(1.87 \times 10^4)^2 + (1.87 \times 10^4)^2} = 2.64 \times 10^4\ NC^{-1}$ NE



6. (a) Coherent light from each of the two slits reaches a point on the screen in phase. The light undergoes constructive interference and results in a bright fringe.

(b) (i) $\Delta y = \frac{\lambda L}{d}$ where $\frac{\Delta y}{2} = 3.27 \times 10^{-4}$

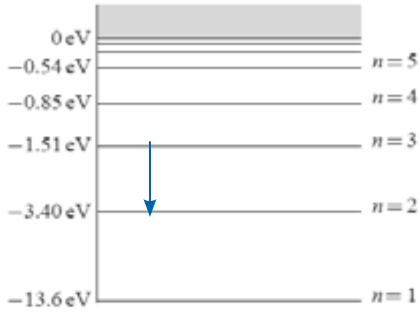
$\therefore\ \Delta y = 3.27 \times 10^{-4} \times 2 = 6.54 \times 10^{-4}\ m$

$\lambda = \frac{\Delta y d}{L} = \frac{6.54 \times 10^{-4} \times 2.40 \times 10^{-4}}{0.280} = 5.61 \times 10^{-7}\ m$

(ii) $\Delta y \propto \lambda$

A smaller wavelength would decrease the distance between the bright fringe marked on the diagram and the dark fringe marked on the diagram.

7. (a)



(b) $E = 1.89\ eV = 3.024 \times 10^{-19}\ J$

$E = hf \therefore \quad f = \frac{E}{h} = \frac{3.024 \times 10^{-19}}{6.63 \times 10^{-34}}\ 4.56 \times 10^{14}\ Hz$

8. (a) $E = hf = 6.63 \times 10^{-34} \times 6.67 \times 10^{14} = 4.42 \times 10^{-19}$ J

The energy of the incident photons is smaller than the work function of platinum. Electrons are not emitted.

(b) $E_{k_{max}} = hf - W = 6.63 \times 10^{-34} \times 6.67 \times 10^{14} - 3.60 \times 10^{-19} = 8.22 \times 10^{-20}$ J

9. (a) As the magnet moves closer to the solenoid, the magnitude of the magnetic field cutting the solenoid increases. This produces a change in magnetic flux. A change in magnetic flux induced an *emf*. As the magnet changes direction, the magnetic flux decreases producing a negative *emf*. As the magnet oscillates up and down, the process repeats and an alternating *emf* results.

(b) Lenz's Law states that the induced *emf* creates a current in a direction that opposes the change in magnetic flux producing the *emf*. As the north pole of the magnet approaches the solenoid, a north pole is induced at the open end of the solenoid closest to the magnet which repels the magnet. Using the right-hand rule, the current flows from B to A.

10. (a) Into the page

(b) (i) The electric field does work on the protons which is transformed into the kinetic energy of the protons in accordance with the law of conservation of energy.

(ii) The protons enter each dee at right angles to the uniform magnetic field. The uniform magnetic field exerts a constant magnetic force that always acts at right angles to the velocity of the protons. The speed of the protons does not change but their direction and hence velocity does change. The magnetic field therefore provides the centripetal acceleration for uniform circular motion.

(c) $E_k = \frac{q^2B^2r^2}{2m} = \frac{(1.60 \times 10^{-19})^2 \times 0.95^2 \times 1.5^2}{2 \times 1.67 \times 10^{-27}} = 1.6 \times 10^{-11}$ J

(d) $f = \frac{1}{T} = \frac{1}{6.9 \times 10^{-8}} = 1.4 \times 10^7$ Hz

Question booklet 2

11. (a) $\Delta p = mv_f - mv_i = 5.3 \times 10^{-26} \times 4.9 \times 10^2 \leftarrow \quad -5.3 \times 10^{-26} \times 4.9 \times 10^2 \rightarrow$

$\Delta p = 2 \times 5.3 \times 10^{-26} \times 4.9 \times 10^2 \leftarrow = 5.2 \times 10^{-23}$ sN$\leftarrow$

(b) Force on oxygen molecule $F = \frac{\Delta p}{\Delta t} = \frac{5.2 \times 10^{-23}}{2.0 \times 10^{-4}} = 2.6 \times 10^{-19}$ N $\leftarrow$

Using Newton's third law $F_{container\ wall} = -F_{gas\ molecule}$

Force on container wall $F = 2.6 \times 10^{-19}$ N $\rightarrow$

12. (a) The straight line of best fit passes through the origin. It therefore indicates that the radius of orbit cubed is directly proportional to the period squared. It therefore confirms Kepler's Third Law which states $T^2 \propto r^3$.

(b) gradient $= \frac{6 \times 10^{18}\ \text{km}^3}{250\ \text{days}^2} = \frac{6 \times 10^{18} \times (10^3)^3}{250 \times (24 \times 60 \times 60)^2} = 3.22 \times 10^{15}\ m^3\ s^{-2}$

(c) Kepler's Third Law of planetary motion can be expressed as: $T^2 = \frac{4\pi^2}{GM} r^3$

Rearranging and comparing $r^3 = \frac{GMT^2}{4\pi^2}$ to $y = mx$ the gradient of the line $= \frac{GM}{4\pi^2}$

Therefore $M = \frac{gradient \times 4\pi^2}{G} = \frac{3.22 \times 10^{15} \times 4\pi^2}{6.67 \times 10^{-11}} = 1.91 \times 10^{27}\ kg$

13. (a) $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.95^2}} = 3.2$

(b) (i) time dilation

(ii) $t = \gamma t_0 = 3.2 \times 1.4 = 4.48 = 4.5\ years$

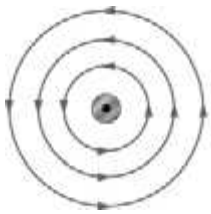
(iii) $s = vt = 0.95c \times 4.5 = 4.3$ light *years*

14. (a) A vertical arrow pointing upwards at P (↑).

(b) $E = \frac{\Delta V}{d} = \frac{2.00 \times 10^3}{4.00 \times 10^{-2}} = 5.00 \times 10^4\ Vm^{-1}$

$a = \frac{F}{m} = \frac{Eq}{m} = \frac{5.00 \times 10^4 \times 2.70 \times 10^{-12}}{1.30 \times 10^{-15}} = 1.04 \times 10^8\ m\ s^{-2}$

15. (a)



(b) $B = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{2.00 \times 10^{-7} \times 0.45}{0.15} = 6.0 \times 10^{-7}\ T \downarrow$

(c) $F = IlB\sin\theta = 0.45 \times 2.5 \times 10^{-2} \times 0.18\ \sin 90 = 2.0 \times 10^{-3}\ N$ *right* ($\rightarrow$)

16. (a) $f_{max} = \frac{e\Delta V}{h} = \frac{1.60 \times 10^{-19} \times 8.00 \times 10^4}{6.63 \times 10^{-34}} = 1.93 \times 10^{19}\ Hz$

(b) *The following is a sample solution.*

One key concept of science as a human endeavour illustrated in the text is development. The resolution achievable in conventional phase-contrast X-ray imaging is limited due to the distance required between the object and the image detector. This distance is reduced by the SWIDeX technology and thus improves resolution. The limitation is that the analyser needs to be thinner than what has currently been achieved. Scientists are still working on methods required to achieve this. Improved resolution results in better medical diagnosis which will lead to better health outcomes and potentially save lives.

S

17. (a) $\Delta E_k = q\Delta V = \frac{1}{2}mv^2 \quad \therefore \quad v = \sqrt{\frac{2q\Delta v}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 54.0}{9.11 \times 10^{-31}}} = 4.36 \times 10^6\ ms^{-1}$

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.36 \times 10^6} = 1.67 \times 10^{-10}\ m$

(b) A large number of the electrons were diffracted at an angle of 50° rather than being scattered randomly. Diffraction is a wave phenomenon. The experiment therefore demonstrated the wave behaviour of low-energy electrons.

18. (a) leptons

(b) (i) Two up quarks and one down quark

(ii) Charge = charge (u) + charge ($\bar{d}$) = $+\frac{2}{3}e + \left(+\frac{1}{3}e\right) = +1e$

(iii) Charge: left hand side of the reaction +1e + (+1e) = +2e
right hand side of the reaction +1e + (+1e) = +2e
charge is conserved

Baryon number: left hand side of the equation +1 + (+1) = +2
right hand side of the equation +1 + (0) = +1
baryon number is not conserved

While charge is conserved, baryon number is not conserved. The reaction is not possible.

19. (a) (i) 10.9 0.189

(ii) Finding an average angle reduces the effect of random error.

(b) $d = \frac{1}{1.00 \times 10^5} = 1.00 \times 10^{-5}\ m$

$d\sin\theta = m\lambda \quad \therefore \quad \lambda = \frac{d\sin\theta}{m} = \frac{1.00 \times 10^{-5} \times \sin 7.2}{2} = 6.27 \times 10^{-7}\ m$

(c) The student could investigate the effect that the line spacing of the diffraction grating has on the angle at which a particular maxima occurs.

The experimental set up would be the same as the first experiment.

The distance between the laser pointer and the diffraction grating is measured using a ruler as is the distance between the diffraction grating and the screen.

The distance between the central maxima and a particular maxima on the screen (say the second) is measured using a ruler.

The angle at which the maxima occurs on the screen is calculated using

$\tan\theta = \frac{\text{distance between the central maxima and the second order maxima}}{\text{distance between diffraction grating and the screen}}$.

This measurement is repeated for the second order maxima on the other side of the central maxima and an average angle is determined.

The diffraction grating is replaced by a different grating having a different number of lines.

The distance between the same laser pointer and the diffraction grating and the distance between the diffraction grating and the screen are checked to ensure that they do not change.

The new angle at which the second order maxima occurs on either side of the central maxima is determined.

The process is repeated with several other diffraction gratings with different spacing (at least five in total).

A graph of angle against the number of lines on the diffraction grating could be plotted to determine the relationship between the two variables.

Appendices

Formula sheet

Vectors are indicated by arrows. If only the magnitude of a vector quantity is used, the arrow is not used.

Symbols of common quantities

acceleration	$\vec{a}$	force	$\vec{F}$	magnetic flux	Φ	time	t
charge	q	frequency	f	mass	m	velocity	$\vec{v}$
displacement	$\vec{s}$	kinetic energy	E_K	momentum	$\vec{p}$	wavelength	λ
electric current	I	length	l	period	T		
electromotive force	ε	magnetic field	$\vec{B}$	potential difference	ΔV		

Magnitude of physical constants

acceleration due to gravity at the Earth's surface	$g = 9.80\ \text{m s}^{-2}$	Planck's constant	$h = 6.63 \times 10^{-34}\ \text{J s}$
constant of universal gravitation	$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$	charge of the electron	$e = 1.60 \times 10^{-19}\ \text{C}$
speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{m s}^{-1}$	mass of the electron	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Coulomb's Law constant	$\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$	mass of the proton	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
		mass of Earth	$5.97 \times 10^{24}\ \text{kg}$
constant for the magnetic field around a conductor	$\frac{\mu_0}{2\pi} = 2.00 \times 10^{-7}\ \text{T m A}^{-1}$	mean radius of Earth	$6.37 \times 10^{6}\ \text{m}$

Topic 1: Motion and relativity

$\vec{v} = \vec{v}_0 + \vec{a}t$	$\vec{v}$ = velocity at time t $\vec{v}_0$ = initial velocity	$v = \frac{2\pi r}{T}$	
$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$		$\vec{g} = \frac{\vec{F}}{m}$	$\vec{g}$ = gravitational field strength
$v^2 = v_0^2 + 2as$		$F = G\frac{m_1 m_2}{r^2}$	r = distance between masses m_1 and m_2
$v_H = v\cos\theta$ $v_V = v\sin\theta$	θ = angle to horizontal	$v = \sqrt{\frac{GM}{r}}$	M = mass of object orbited by satellite r = radius of orbit
$E_K = \frac{1}{2}mv^2$		$T^2 = \frac{4\pi^2}{GM}r^3$	
$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$		$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	γ = Lorentz factor
$\vec{F} = m\vec{a}$		$t = \gamma t_0$	t_0 = time interval in the moving frame of reference
$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$		$l = \frac{l_0}{\gamma}$	l_0 = length in the moving object's frame of reference
$\vec{p} = m\vec{v}$		$p = \gamma m_0 v$	m_0 = mass in the frame of reference where the object is stationary
$a = \frac{v^2}{r}$	r = radius of circle		

Topic 2: Electricity and magnetism

$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$	r = distance between charges q_1 and q_2	$F = qvB\sin\theta$	θ = angle between field $\vec{B}$ and velocity $\vec{v}$
$\vec{E} = \frac{\vec{F}}{q}$	$\vec{E}$ = electric field	$r = \frac{mv}{qB}$	r = radius of circle
$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$	r = distance from charge	$T = \frac{2\pi m}{qB}$	
$W = q\Delta V$	W = work done	$E_K = \frac{q^2B^2r^2}{2m}$	r = radius at which ions emerge from cyclotron
$E = \frac{\Delta V}{d}$	d = distance between parallel plates	$f = \frac{1}{T}$	
		$\Phi = BA_\perp$	$A_\perp$ = area perpendicular to the magnetic field
$\vec{a} = \frac{q\vec{E}}{m}$		$\varepsilon = \frac{\Delta\Phi}{\Delta t}$	
$B = \frac{\mu_0}{2\pi}\frac{I}{r}$	r = distance from conductor	$\varepsilon = \frac{N\Delta\Phi}{\Delta t}$	N = number of conducting loops
$F = IlB\sin\theta$	θ = angle between magnetic field and direction of current	$\frac{V_{input}}{V_{output}} = \frac{N_{input}}{N_{output}}$	V = potential difference in transformer coils

Topic 3: Light and atoms

$v = f\lambda$		$W = hf_0$	W = work function of the metal f_0 = threshold frequency
$d\sin\theta = m\lambda$	d = distance between slits θ = angular position of mth maximum m = integer $(0, 1, 2, \ldots)$	$E_{K\max} = eV_s$	$E_{K\max}$ = maximum kinetic energy of electrons V_s = stopping voltage
$\Delta y = \frac{\lambda L}{d}$	Δy = distance between adjacent minima or maxima L = slit-to-screen distance	$E_{K\max} = hf - W$	
$E = hf$	E = energy of photon	$f_{\max} = \frac{e\Delta V}{h}$	ΔV = potential difference across the X-ray tube
$p = \frac{h}{\lambda}$		$E = \Delta mc^2$	E = energy

Table of prefixes

Prefix	Symbol	Value
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Quarks

Quark	Symbol	Charge (e)
Up	u	$\frac{2}{3}$
Down	d	$-\frac{1}{3}$
Strange	s	$-\frac{1}{3}$
Charm	c	$\frac{2}{3}$
Top	t	$\frac{2}{3}$
Bottom	b	$-\frac{1}{3}$