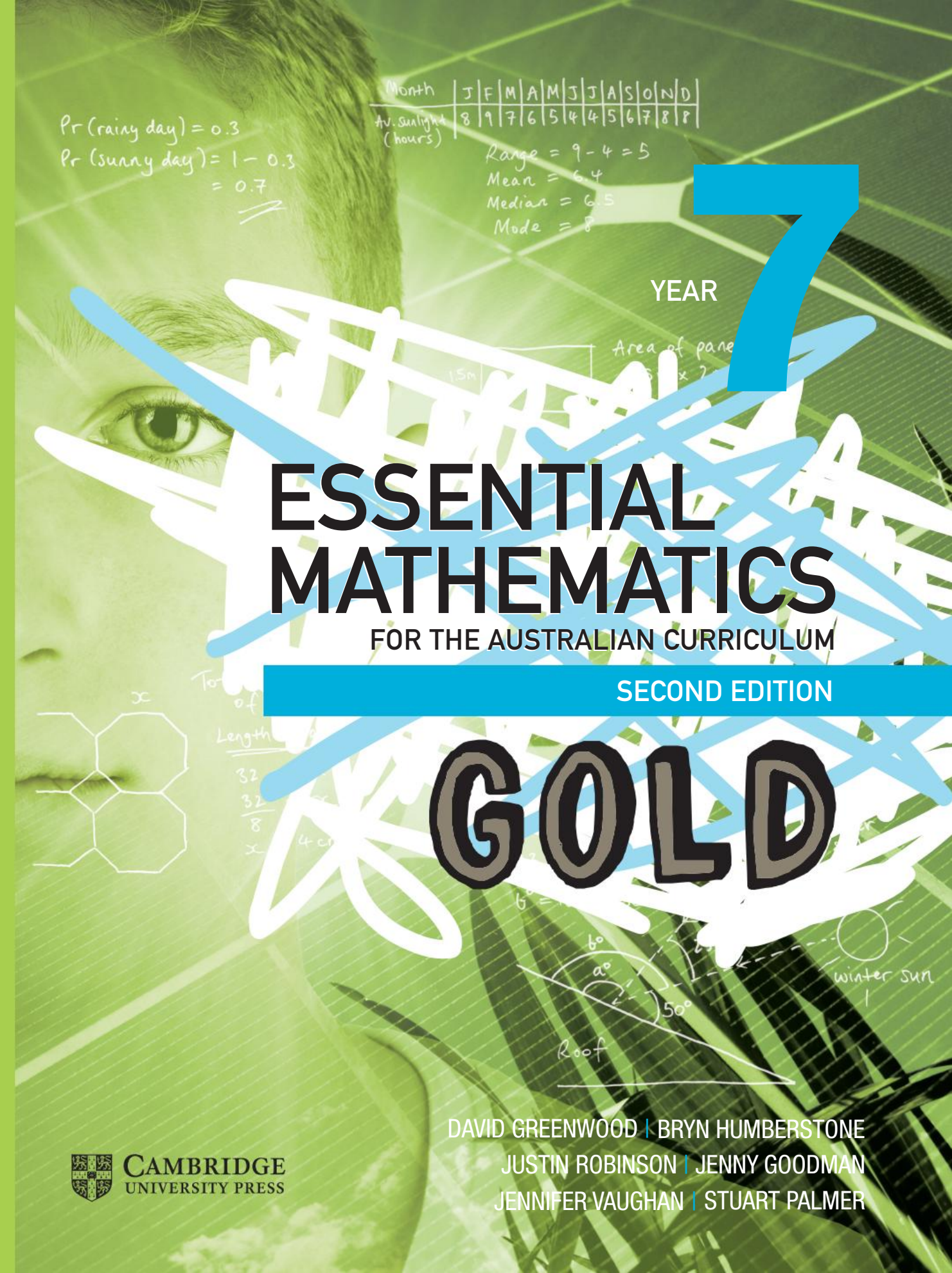


YEAR 7

ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM

SECOND EDITION

GOLD

DAVID GREENWOOD | BRYN HUMBERSTONE
JUSTIN ROBINSON | JENNY GOODMAN
JENNIFER VAUGHAN | STUART PALMER

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About the Authors



David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has more than 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.



Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, is currently teaching both junior and senior mathematics in Victoria and is the Head of Mathematics (Senior School) at Caulfield Grammar School Wheelers Hill. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.



Justin Robinson is the inaugural Director of the Institute of Positive Education, based at Geelong Grammar School, where he leads a team of staff dedicated to promoting the theory and practice of Positive Education. Prior to this, he spent 20 years teaching mathematics, covering all levels of secondary education and with significant experience teaching VCE, IB and A-Levels. Justin is passionate about challenging students within a safe learning environment and instils in his students a growth mindset. He has a keen interest in promoting student engagement through a wide variety of effective teaching methods and materials.



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Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible hence, facilitating student confidence, achievement and enjoyment of maths.



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Introduction

This second edition of *Essential Mathematics Gold for the Australian Curriculum* sees the Gold series revised, updated, redeveloped and seamlessly blended with HOTmaths to create a comprehensive learning package that is specially designed for students who require additional support in mathematics.

An Interactive Textbook (complimentary) for the student and Online Teaching Suite (available separately) for the teacher are new for the second edition. The print textbooks have been carefully revised and updated. New topics have been added to reflect changes to the Australian Curriculum and incorporate selected parts of new state syllabuses, particularly the Victorian Curriculum and the New South Wales Syllabus.

The following four themes have guided the second edition:

Differentiation in the classroom

- All non-core sections are labelled as 'Consolidating' (indicating a revision section) or with a gold star (indicating a topic that could be considered challenging) to help teachers decide on the most suitable way of approaching the course for their class or for individual students.
- Building and Progressing working programs are suggested for all exercises (see 'Guide to the working programs' on the following page for information).
- Two auto-marked quizzes for each section in the Interactive Textbook are aligned with the Building and Progressing learning pathways.
- Two chapter tests per chapter in the Online Teaching Suite have also been aligned to the Building and Progressing learning pathways.
- The Test Generator inside the Online Teaching Suite provides a high level of flexibility for the teacher to create tests from questions containing four difficulty levels.

Mathematical literacy

- In conjunction with margin definitions in the print book, 'enhanced' pop-up definitions in the Interactive Textbook feature visual and audio aids to help students learn definitions in a multi-dimensional way.
- In the Interactive Textbook, simple drag-and-drop and fill-the-gap activities in each chapter allow students to get comfortable using mathematical language and to consolidate definitions.
- In the Online Teaching Suite, maths literacy worksheets featuring a wide range of activities can be downloaded to be used as classwork or homework.

Practical application of mathematics

- A new feature in each chapter called 'Maths@Work' provides case studies to show how the maths learnt in the chapter is relevant to the workplace and to everyday life.

Using technology

- 'Maths@Work' activities feature a section dedicated to giving students practise at using technology such as spreadsheets and calculators in ways they will encounter in the workforce and in everyday life.
- Throughout the textbook, questions best suited to be completed on a calculator are indicated with a calculator icon. 
- In the Interactive Textbook, a fully-functional scientific calculator pops up when students click or tap on the calculator icon.
- CAS calculator activities in the Year 10 Interactive Textbook provide calculator practise for students going to study maths in Year 11.

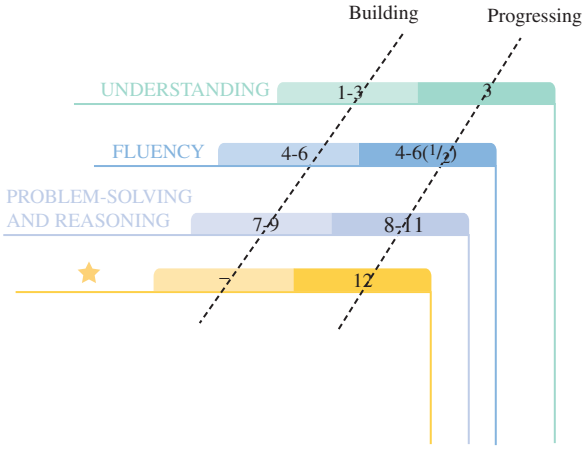
While not part of the ACARA curriculum, Algorithmics is an exciting, powerful way of applying technology to mathematics (and vice versa) and one that is relevant and accessible to students of all ages and ability levels. We have therefore included in each book a new appendix chapter on Algorithmics that takes an introductory, activity-based approach to the subject.

Guide to the working programs

New to the second edition, the working programs that were previously only available to teachers in separate supporting documents have been updated, refined and subtly embedded in the exercises. The suggested working programs provide two pathways through the book to allow differentiation for Building and Progressing students.

As with the first edition, each exercise is structured in subsections that match the Australian Curriculum proficiency strands (with Problem-solving and Reasoning combined into one section to reduce exercise length), as well as 'Gold Star' (★). The questions* suggested for each pathway are listed in two columns at the top of each subsection.

- The left column (lightest shade) shows the questions in the Building working program.
- The right column (darkest shade) shows the questions in the Progressing working program.



Gradients within exercises and proficiency strands

The working programs make use of two gradients that have been carefully integrated into the exercises. A gradient runs through the overall structure of each exercise – where there's an increasing level of sophistication required as a student progresses through the proficiency strands and then to the 'Gold star' question(s) – but also within each proficiency strand; the first few questions in Fluency are easier than the last few, for example, and the first few Problem-solving and Reasoning questions are easier than the last few.

The right mix of questions

Questions in the working programs have been selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway are given extra practise at Understanding and basic Fluency and only the easiest Problem-solving and Reasoning questions. The Progressing pathway, while not meant to be challenging, spends a little less time on basic Understanding questions and a little more on Fluency, Problem-solving and Reasoning. The Progressing pathway also includes the 'Gold star' question(s).

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works best for them.

If required, the chapter pre-tests can be used as a diagnostic tool. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before doing the Building questions, and may require further assistance.
- A student who gets between 40% and 75% should do the Building questions.
- A student who gets 75% and higher should do the Progressing questions.

For schools that have classes grouped according to ability, teachers may wish to set either the Building or Progressing pathways as the default pathway for an entire class and then make individual alternations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, … or b, d, f, …)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : complete none of the questions in this section.

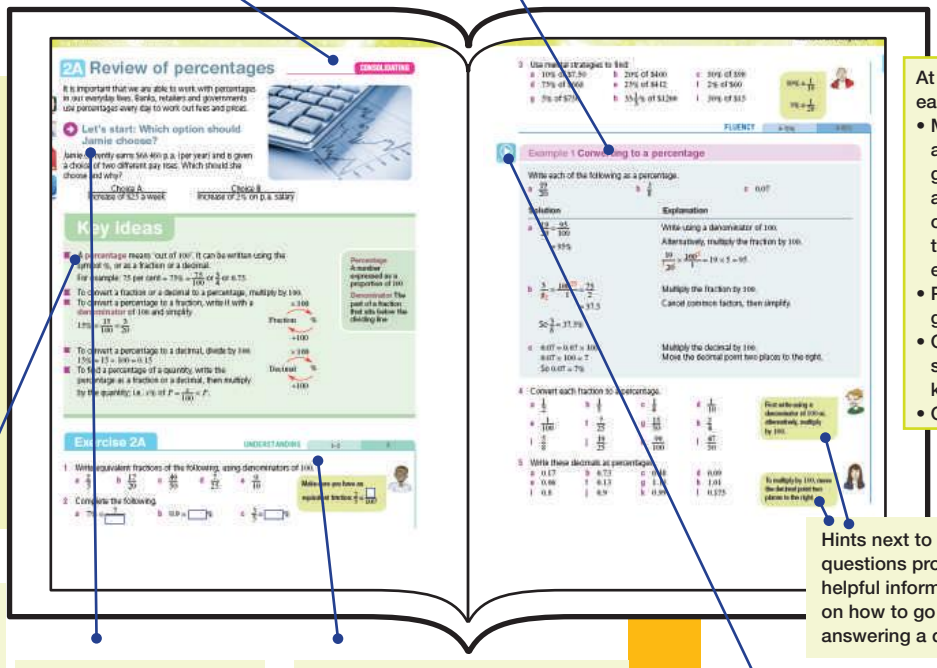
An overview of the *Essential Mathematics Gold for the Australian Curriculum* complete learning suite

For more detail, see the guide in the online Interactive Textbook

Non-core topics in each chapter are labelled as 'Consolidating' or 'Gold Star' to assist with course planning.

Worked examples with Solution and Explanation columns are placed inside the exercises next to the most relevant questions.

PRINT TEXTBOOK



At the end of each chapter:
- Maths@Work and Life Maths give practical applications of maths on-the-job and in everyday life.
- Puzzles and games
- Chapter summary of key knowledge
- Chapter review

Key Ideas summarises the knowledge and skills required for the lesson

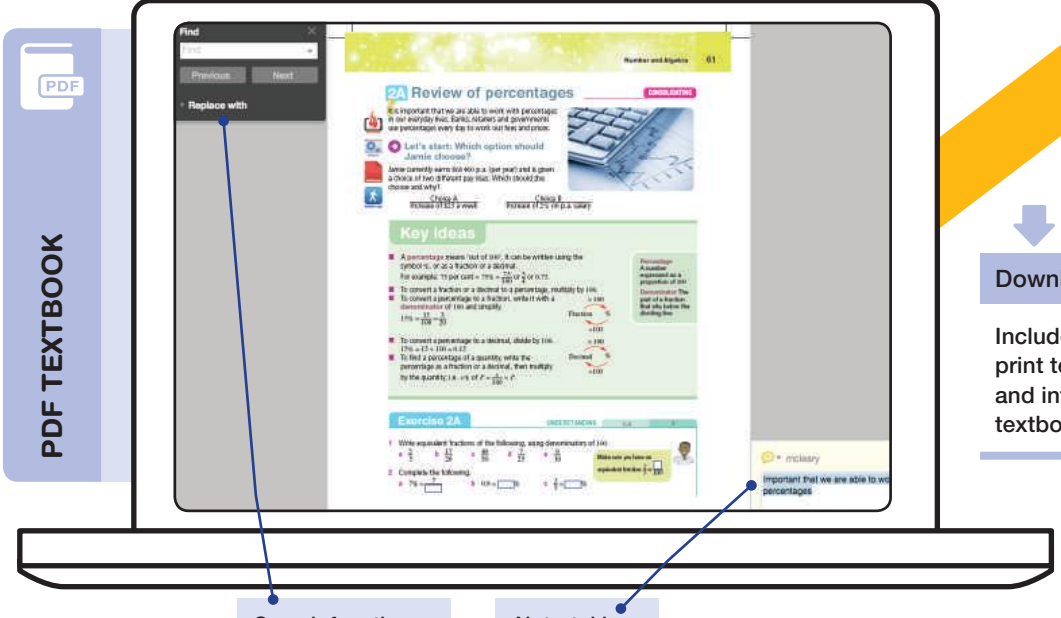
Let's start activities provide an engaging way to start the lesson.

Building and Progressing working programs in every exercise provide two learning pathways depending on student need.

Hints next to exercise questions provide helpful information on how to go about answering a question.

All worked examples in the print book feature a video demonstration in the Interactive Textbook

PDF TEXTBOOK



Downloadable

Included with print textbook and interactive textbook

Search functions

Note-taking

Easy navigation within sections without scrolling

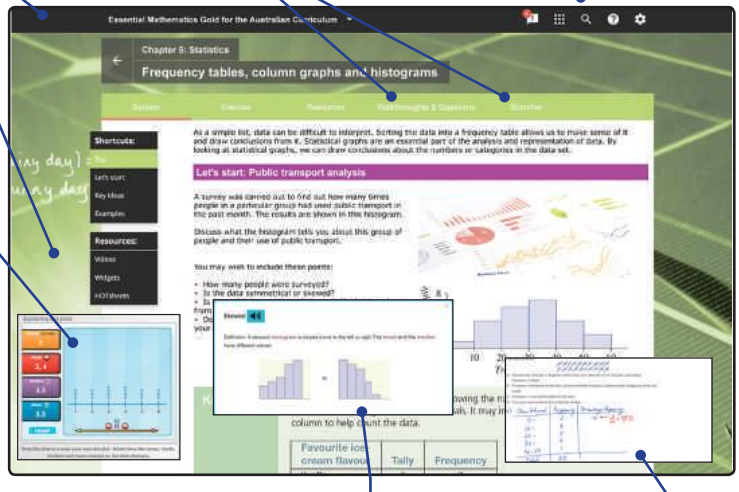
Walkthroughs, Scorcher and levelled question sets in every section

Interactive navigation and searching

Plus:
- Interactive literacy activities
- Pop-up scientific calculator
- Access to *HOTmaths* games library
- View tasks and tests sent by teacher

Access to interactive resources at any time without leaving the page

Hundreds of interactive widgets



INTERACTIVE TEXTBOOK POWERED BY *HOTmaths*

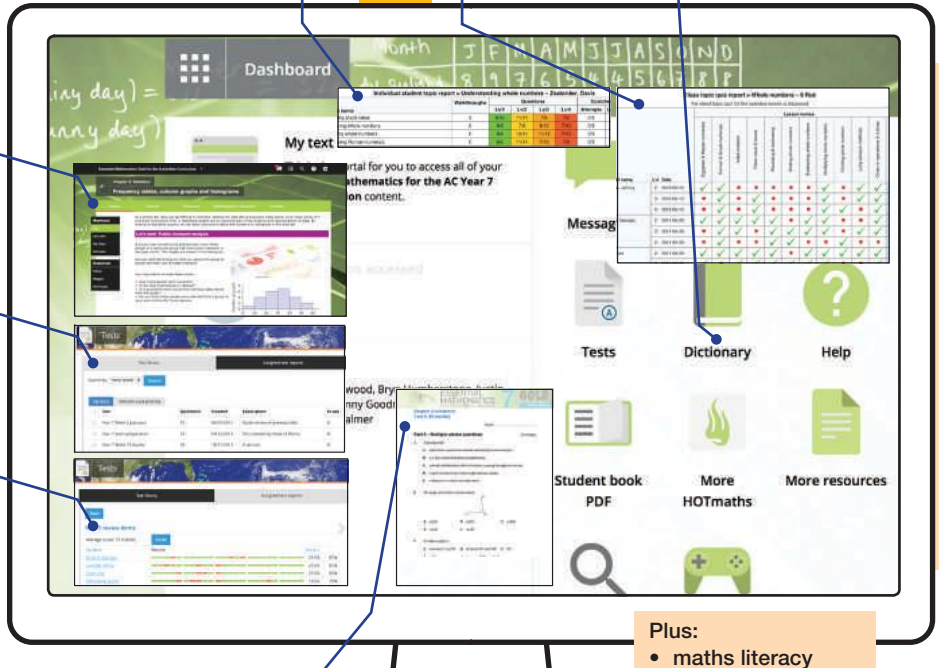
Enhanced pop-up definitions with visual and audio aids to assist with maths literacy

Video demonstration for every worked example

Student reporting

Class reporting

Access to all *HOTmaths* Australian Curriculum courses



ONLINE TEACHING SUITE POWERED BY *HOTmaths*

Teacher's copy of interactive textbook

Test generator and ready-made tests

Student results

Chapter tests, worksheets, skillsheets and investigations

Plus:
- maths literacy worksheets
- teaching program
- curriculum grids and more

Chapter 1

Whole numbers

What you will learn

1A Place value (Consolidating)
1B Addition and subtraction
1C Addition and subtraction of larger numbers
1D Multiplication
1E Multiplying larger numbers
1F Division
1G Estimating and rounding (Consolidating)
1H Order of operations

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Apply the associative, commutative and distributive laws to aid mental and written computation **(ACMNA151)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The Mayan number system

The Mayan people lived in northern Central America from around 2000 BCE until the seventeenth century. Their civilization became highly advanced in language, religion, writing, maths, science and art. Every day people used maths in the markets and scholars made complex calendar and astronomy calculations.

The Mayan's simplest number system used a shell for zero, a pebble for one unit and a stick for five units. For example:

zero	3	5	13
			

The Mayans used addition and subtraction like we do. For example, writing 11 + 7 = 18 in the Mayan number system would look like this:



Writing a subtraction like 10 – 2 = 8 would look like this:



Pre-test

1 Write down the larger number from each pair of numbers.

a 9, 11
b 137, 129
c 99, 104
d 10 102, 9870

2 For each of the following, match the symbol (**A**, **B**, **C** or **D**) to the given word (**a**, **b**, **c**, and **d**).

a add
b subtract
c multiply
d divide

A −
B ÷
C +
D ×

3 Write each of the following as numbers.

a fifty-seven
b one hundred and sixteen
c two thousand and forty-four
d eleven thousand and two

4 Which number is:

a 2 more than 11?
b 5 less than 42?
c 1 less than 100?
d 3 more than 7997?
e double 13?
f half of 56?

5 Complete these patterns, showing the next four numbers.

a 7, 14, 21, 28, 35, ___, ___, ___, ___.
b 9, 18, 27, 36, 45, ___, ___, ___, ___.
c 11, 22, 33, 44, 55, ___, ___, ___, ___.

6 How many equal groups can 48 be divided into if the size of each group is:

a 24?
b 48?
c 16?
d 4?

7 Give the result for each of these additions and subtractions.

a $3 + 11$
b $14 + 9$
c $99 + 20$
d $138 + 12$
e $199 + 11$
f $1010 + 100$
g $396 + 104$
h $837 + 71$
i $20 - 11$
j $41 - 9$
k $96 - 17$
l $101 - 22$
m $136 - 24$
n $421 - 23$
o $783 - 84$
p $1200 - 299$

8 Give the result for each of these multiplications and divisions.

a 5×6
b 9×7
c 12×12
d 8×11
e 7×8
f 10×13
g 100×11
h 2000×4
i $10 \div 2$
j $30 \div 15$
k $66 \div 6$
l $132 \div 12$
m $110 \div 11$
n $63 \div 7$
o $27 \div 9$
p $108 \div 12$

9 Arrange these numbers from smallest to largest.

a 37, 73, 58, 59, 62, 159
b 301, 103, 31, 310, 130
c 29 143, 24 913, 13 429, 24 319, 24 931

10 What is the remainder when these numbers are divided by 3?

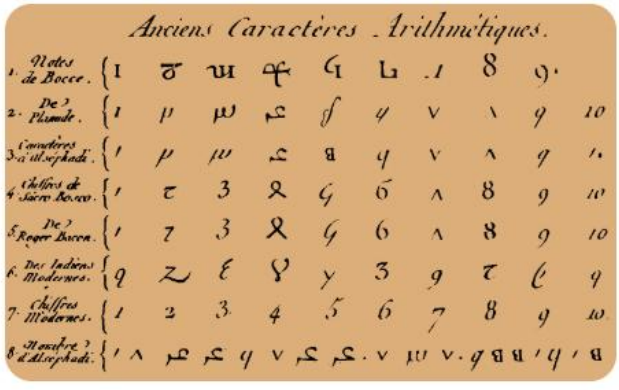
a 12
b 10
c 37
d 62

1A Place value

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthrough

The decimal system is the number system used most often today. It is also called the Hindu-Arabic system. It uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The decimal system came from ancient India about 3000 BCE and spread across Europe through Arabic texts over the next 4000 years.



The famous 'Histoire de la Mathematique' is a French document showing the history of the Hindu-Arabic number system.

Let's start: Write the largest number

Write the largest possible number using these digits.

- 7, 1, 3, 6
- 1, 0, 5, 2, 6
- 9, 1, 2, 8, 4
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Explain why your number is the largest possible.

Key ideas

- The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called digits.
- The value of each digit depends on its place in the number. The **place value** of the digit 2 in the number 3254, for example, is 200.

$3254 = 3000 + 200 + 50 + 4$

Place value The value of a digit in a number, which is determined by its position

	thousands	hundreds	tens	ones
digit	3	2	5	4
value	3000	200	50	4

- Symbols used to compare numbers include the following.

= (equal to)	$1 + 3 = 4$	or	$10 - 7 = 3$
≠ (not equal to)	$1 + 3 \neq 5$	or	$11 + 38 \neq 50$
> (greater than)	$5 > 4$	or	$100 > 37$
≥ (greater than or equal to)	$5 \geq 4$	or	$4 \geq 4$
< (less than)	$4 < 5$	or	$13 < 26$
≤ (less than or equal to)	$4 \leq 5$	or	$4 \leq 4$
≈ (approximately equal to)	$4.02 \approx 4$	or	$8997 \approx 9000$

Exercise 1A

UNDERSTANDING 1–3 3

1 For the number 5207, write down which digit is in the:

a tens place **b** thousands place
c hundreds place **d** ones place



The ten digits used in our number system are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

1A

2 Write down these numbers using digits.
- **a** forty-six
- **b** two hundred and sixty-three
- **c** seven thousand, four hundred and twenty-one
- **d** thirty-six thousand and fifteen

3 Which symbol (**A**, **B**, **C**, **D**, **E** or **F**) matches the given words (**a**, **b**, **c**, **d**, **e** and **f**)?

a not equal to	**A** $=$
b less than	**B** $\neq$
c greater than or equal to	**C** $>$
d equal to	**D** $\geq$
e greater than	**E** $<$
f less than or equal to	**F** $\leq$

FLUENCY 4–5(½), 6 | 4–5(½), 6, 7(½)

Example 1 Finding place value

Write down the place value of the digit 4 in these numbers.

a 437 **b** 1043

Solution	Explanation
a $4 \times 100 = 400$	The 4 has value 4×100 The 3 has value 3×10 The 7 has value 7×1
b $4 \times 10 = 40$	The 1 has value 1×1000 The 0 has value 0×100 The 4 has value 4×10 The 3 has value 3×1

4 Write down the place value of the digit 7 in these numbers.

a 37 **b** 71 **c** 379
d 704 **e** 1712 **f** 7001
g 45 720 **h** 170 966


Write your answer as 7, 70, 700, 7000 or 70 000.

5 Write down the place value of the digit 3 in these numbers.

a 43 **b** 37 **c** 238 **d** 1320
e 2931 **f** 3846 **g** 99 213 **h** 230 040

6 Write down the place value of the digit 2 in these numbers.

a 126 **b** 2143 **c** 91 214 **d** 1 268 804

7 State whether each of these statements is true or false.

a $5 > 4$ **b** $6 = 10$ **c** $9 \neq 99$
d $1 < 12$ **e** $22 \leq 11$ **f** $126 \leq 126$
g $19 \geq 20$ **h** $138 > 137$ **i** $3 \leq 3$
j $7 \neq 7$ **k** $0 \geq 1$ **l** $2013 < 2031$


$<$ less than
$\leq$ less than or equal to
$>$ greater than
$\geq$ greater than or equal to
$=$ equal to
$\neq$ not equal to

PROBLEM-SOLVING AND REASONING 8, 9 8–10

Example 2 Arranging numbers

Arrange these numbers from smallest to largest.
29, 36, 18, 132, 1001, 99, 592, 123, 952

Solution	Explanation
18, 29, 36, 99, 123, 132, 592, 952, 1001	The smallest two-digit number has the smallest number in the tens column. Then choose the next smallest two-digit, and so on.

8 Arrange these numbers from smallest to largest.

a 55, 45, 54, 44
b 729, 29, 92, 927, 279
c 23, 951, 136, 4
d 435, 453, 534, 345, 543, 354
e 12 345, 54 321, 34 512, 31 254
f 1010, 1001, 10 001, 1100, 10 100

9 In the following questions, digits cannot be used more than once, and all the given digits must be used. Do not use a decimal point.

a Write the largest possible number using the digits 2, 7 and 8.
b Write the smallest possible number using the digits 9, 1, 3, 6 and 4.

10 How many numbers can be made using the given digits? Digits are not allowed to be used more than once and all the given digits must be used.

a 2, 8 and 9
b 1, 6 and 7
c 2, 5, 6 and 7

Large numbers

— 11

11 The names of large numbers depend on the number of digits grouped into threes. For example, 1 000 is 1 thousand, 1 000 000 is 1 million and 1 000 000 000 is 1 billion.

a Write these numbers using digits.

i 7 thousand
ii 46 thousand
iii 712 thousand
iv 5 million
v 44 million
vi 6 billion
vii 437 billion
viii 15 trillion

b Research the number 1 googol and write a sentence explaining what it is.



Inflation in Zimbabwe during the last decade has been so high that bank notes need to be issued in hundreds of trillions of dollars. In 2016 one Australian dollar was worth about 250 000 Zimbabwe dollars.

1B Addition and subtraction

Interactive

Widgets

HOTsheets

Walkthrough

The process of finding the total value of two or more numbers is called addition. The words 'plus', 'add' and 'sum' are also used to describe addition.

The process for finding the difference between two numbers is called subtraction. The words 'minus', 'subtract' and 'take away' are also used to describe subtraction.



The difference in height of two things can be found using subtraction.

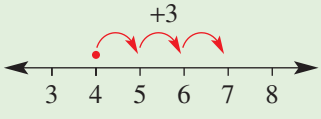
Let's start: Your mental strategy

Many problems that involve addition and subtraction can be solved mentally without the use of a calculator or written working.
Explain your mental strategy for working out the answer to these problems.

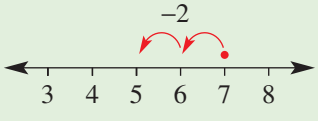
- $132 + 245$
- $99 + 35$
- $73 - 41$

Key ideas

- The symbol + is used to show addition or find a sum.
  For example: $4 + 3 = 7$



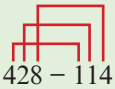
  - Note that the order does *not* matter with addition.
    For example: $5 + 2 = 2 + 5$ and $21 + 12 = 12 + 21$
- The symbol − is used to show subtraction or find a difference.
  For example: $7 - 2 = 5$



  - Note that the order *does* matter with subtraction.
    For example: $5 - 2 \neq 2 - 5$ and $21 - 12 \neq 12 - 21$
- Mental addition and subtraction can be done using different strategies.
  - **Partitioning** (Grouping digits in the same position)



$171 + 23 = 194$



$428 - 114 = 314$

  - **Compensating** (Making a 10, 100 etc. and then adjusting or compensating by adding or subtracting)

$46 + 9 = 46 + 10 - 1$
$= 55$

$138 - 99 = 138 - 100 + 1$
$= 39$

Partitioning A mental strategy where you break up a number into parts, e.g. $123 = 100 + 20 + 3$

Compensating A mental strategy where you round a number and then add or subtract a smaller amount

Exercise 1B

UNDERSTANDING 1–4 | 4

1 **a** List three words that tell you when to use *addition*.
b List three words that tell you when to use *subtraction*.



Choose from these words: *minus, add, sum, subtract, plus, take away*

2 Write the number which is:
a 2 more than 5 **b** 3 more than 7 **c** 58 more than 11
d 5 less than 9 **e** 7 less than 19 **f** 137 less than 157

3 **a** Add to find the sum of these pairs of numbers.
i 2 and 6 **ii** 19 and 8 **iii** 62 and 70
b Subtract (take away) to find the difference between these pairs of numbers.
i 11 and 5 **ii** 29 and 13 **iii** 101 and 93

4 Give the result for each of these problems.
a 7 plus 11 **b** 22 minus 3 **c** the sum of 11 and 21
d 128 add 12 **e** 36 take away 15
f the difference between 13 and 4

FLUENCY 5–8(½) | 5–8(½)

Example 3 Mental addition and subtraction

Use the suggested strategy to mentally work out the answer.

a $132 + 156$ (partitioning) **b** $429 - 203$ (partitioning)
c $25 + 19$ (compensating) **d** $56 - 18$ (compensating)

Solution	Explanation	
a $132 + 156 = 288$	$100 + 30 + 2$ $100 + 50 + 6$ $200 + 80 + 8$	Group the hundreds, tens and ones together.
b $429 - 203 = 226$	$400 - 200 = 200$ $20 - 0 = 20$ $9 - 3 = 6$	
c $25 + 19 = 44$	$25 + 19 = 25 + 20 - 1$ $= 45 - 1$ $= 44$	To add 19, add 20 then take away 1.
d $56 - 18 = 38$	$56 - 18 = 56 - 20 + 2$ $= 36 + 2$ $= 38$	To take away 18, take away 20 then add 2.

1B

5 Mentally find the answers to these sums. Hint: Use the partitioning strategy.

a $11 + 23$ **b** $14 + 32$ **c** $43 + 16$
d $23 + 41$ **e** $71 + 26$ **f** $138 + 441$
g $246 + 502$ **h** $937 + 11$ **i** $1304 + 4293$



Work out the answer by adding the ones, then the tens, and so on.

6 Mentally find the answers to these differences. Hint: Use the partitioning strategy.

a $29 - 18$ **b** $57 - 21$ **c** $94 - 43$
d $249 - 137$ **e** $357 - 124$ **f** $836 - 704$
g $1045 - 1041$ **h** $4396 - 1285$ **i** $10\,101 - 100$

7 Mentally find the answers to these sums. Hint: Use the compensating strategy.

a $15 + 9$ **b** $64 + 11$ **c** $19 + 76$
d $18 + 115$ **e** $31 + 136$ **f** $245 + 52$



Round one of the numbers to the nearest ten, then compensate by adding or subtracting the difference.

8 Mentally find the answers to these differences. Hint: Use the compensating strategy.

a $35 - 11$ **b** $45 - 19$ **c** $156 - 48$
d $244 - 22$ **e** $376 - 59$ **f** $5216 - 199$

PROBLEM-SOLVING AND REASONING 9–12 11–15

9 Mary has $101 in her piggy bank. She takes out $22 to buy a jumper. How much money remains in her piggy bank?

10 Gary worked 7 hours on Monday, 5 hours on Tuesday, 13 hours on Wednesday, 11 hours on Thursday and 2 hours on Friday. What is the total number of hours that Gary worked during the week?



11 In a batting innings, Phil hit 126 runs and Mario hit 19 runs. How many more runs did Phil hit compared to Mario?



12 Mentally find the answers to these mixed problems.

a $11 + 18 - 17$ **b** $37 - 19 + 9$ **c** $101 - 15 + 21$

d $136 + 12 - 15$ **e** $28 - 10 - 9 + 5$ **f** $39 + 71 - 10 - 10$

g $1010 - 11 + 21 - 1$ **h** $5 - 7 + 2$ **i** $10 - 25 + 18$

13 Matt has 36 cards and Andy has 35 more cards than Matt.

a How many cards does Andy have?

b If they combine their cards, how many do they have in total?

14 Are these statements true or false?

a $4 + 3 > 6$ **b** $11 + 19 \geq 30$ **c** $13 - 9 < 8$

d $26 - 15 \leq 10$ **e** $1 + 7 - 4 \geq 4$ **f** $50 - 21 + 6 < 35$

15 Complete these number sentences. (The letters a, b and c represent numbers.)

a $2 + 3 = 5$ so $5 - ___ = 2$ **b** $9 + 6 = 15$ so $15 - ___ = 9$

c $a + b = c$ so $c - ___ = a$ **d** $a + c = b$ so $b - a = ___$

Magic squares — 16

16 A magic square has every row, column and main diagonal adding to the same number, called the magic sum. For example, this magic square has a magic sum of 15.

4	9	2	→ 15
3	5	7	→ 15
8	1	6	→ 15
↓	↓	↓	↘ 15
15	15	15	15

Find the magic sums for these squares, then fill in the missing numbers.

a

6		
7	5	
2		

b

10		
	11	13
		12

c

15	20	
14		
19		

d

1	15		4
	6		9
		11	
13		2	16

1C Addition and subtraction of larger numbers

Interactive

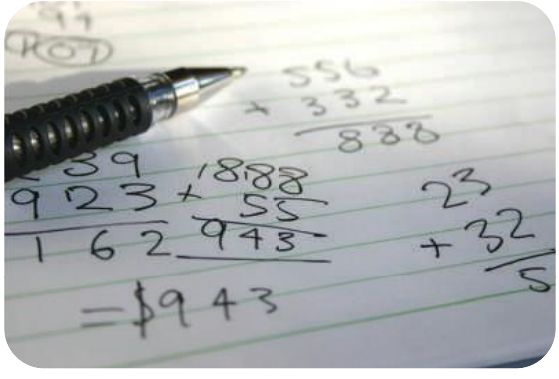
Widgets

HOTsheets

Walkthrough

To add or subtract larger numbers we can use a step-by-step process called an algorithm.

Adding can involve carrying a 'one' to the next column, while subtracting can involve borrowing a 'one' from the next column.



Let's start: The missing digits

Discuss what digits should go in the empty boxes. Give reasons for your answers.

$$\begin{array}{r} \square\,2 \\ +\,9\,\square \\ \hline 1\,2\,6 \end{array} \qquad \begin{array}{r} 4\,\square \\ +\,3\,8 \\ \hline 8\,1 \end{array}$$

$$\begin{array}{r} 1\,\square\,6 \\ -\;\;8\,\square \\ \hline 5\,2 \end{array} \qquad \begin{array}{r} 1\,6\,\square \\ -\;\;\square\,5 \\ \hline 1\,0\,6 \end{array}$$

Key ideas

- To add larger numbers:
  - Arrange the numbers vertically (above each other) so that the digits with similar place value are in the same column.
  - Add digits in the same column, starting on the right.
  - If the digits add to more than 9, carry the 1 to the next column.

Carry the 1

$$\begin{array}{r} {}^{1}2\,3\,4 \\ +\,1\,9\,2 \\ \hline 4\,2\,6 \end{array}$$

$1 + 2 + 1 = 4$ $\quad 4 + 2 = 6$

$3 + 9 = 12$

- To subtract larger numbers:
  - Arrange the numbers vertically so that the digits with similar place value are in the same column.
  - Subtract digits in the same column top-down, starting on the right.
  - If the digits subtract to less than 0, borrow a 1 from the next column to form an extra 10.

Borrow 1

$$\begin{array}{r} {}^{1}2\,{}^{1}5\,9 \\ -\,1\,8\,2 \\ \hline 7\,7 \end{array}$$

$1 - 1 = 0$ $\quad 9 - 2 = 7$

$15 - 8 = 7$

Exercise 1C

UNDERSTANDING 1–3 | 3

1 Mentally find the results to these simple sums.

a $7+6$	**b** $9+4$	**c** $11+9$	**d** $19+3$
e $8+9$	**f** $87+14$	**g** $138+6$	**h** $99+11$
i $998+7$	**j** $19+124$	**k** $102+99$	**l** $52+1053$

2 Mentally find the results to these simple differences.

a $13-6$	**b** $11-9$	**c** $16-11$	**d** $14-8$
e $13-5$	**f** $36-9$	**g** $75-8$	**h** $100-16$
i $37-22$	**j** $104-12$	**k** $46-17$	**l** $1001-22$

3 What is the missing digit in these problems?

a
$$\begin{array}{r} 2\ 7 \\ +\ 3\ 1 \\ \hline 5\square \end{array}$$

b
$$\begin{array}{r} 3\ 6 \\ +\ 1\ 5 \\ \hline 5\square \end{array}$$

c
$$\begin{array}{r} 1\ 2\ 3 \\ +\ \ 9\ 1 \\ \hline 2\square 4 \end{array}$$

d
$$\begin{array}{r} 4\ 6 \\ +\square 5 \\ \hline 1\ 1\ 1 \end{array}$$

e
$$\begin{array}{r} 2\ 4 \\ -\ 1\square \\ \hline 1\ 2 \end{array}$$

f
$$\begin{array}{r} 6\ 7 \\ -\ 4\ 8 \\ \hline \square 9 \end{array}$$

g
$$\begin{array}{r} 1\ 6\ 2 \\ -\ \ \square 1 \\ \hline 8\ 1 \end{array}$$

h
$$\begin{array}{r} 1\ 4\square 2 \\ -\ 6\ 2\ 3 \\ \hline 8\ 0\ 9 \end{array}$$

FLUENCY 4–6(½) | 4–7(½)

Example 4 Adding larger numbers

Give the result for each of these sums.

a
$$\begin{array}{r} 26 \\ +66 \\ \hline \end{array}$$

b
$$\begin{array}{r} 439 \\ +172 \\ \hline \end{array}$$

Solution	Explanation
a $\begin{array}{r} {}^{1}26 \\ +\ 66 \\ \hline 92 \end{array}$	Add the digits vertically. $6+6=12$, so carry the 1 to the tens column.
b $\begin{array}{r} {}^{1}4{}^{1}3\ 9 \\ +\ 1\ 7\ 2 \\ \hline 6\ 1\ 1 \end{array}$	$9+2=11$, so carry a 1 to the tens column. $1+3+7=11$, so carry a 1 to the hundreds column.

4 Give the answer to each of these sums.

a
$$\begin{array}{r} 36 \\ +51 \\ \hline \end{array}$$

b
$$\begin{array}{r} 74 \\ +25 \\ \hline \end{array}$$

c
$$\begin{array}{r} 17 \\ +24 \\ \hline \end{array}$$

d
$$\begin{array}{r} 47 \\ +39 \\ \hline \end{array}$$

e
$$\begin{array}{r} 54 \\ +27 \\ \hline \end{array}$$

f
$$\begin{array}{r} 36 \\ +15 \\ \hline \end{array}$$

g
$$\begin{array}{r} 64 \\ +28 \\ \hline \end{array}$$

h
$$\begin{array}{r} 29 \\ +52 \\ \hline \end{array}$$



For parts **c** to **h**, don't forget to carry the 'one'.

1C

5 Give the answer to each of these sums.

a $\begin{array}{r} 138 \\ +84 \\ \hline \end{array}$ **b** $\begin{array}{r} 257 \\ +65 \\ \hline \end{array}$ **c** $\begin{array}{r} 449 \\ +72 \\ \hline \end{array}$ **d** $\begin{array}{r} 871 \\ +49 \\ \hline \end{array}$

e $\begin{array}{r} 129 \\ +97 \\ \hline \end{array}$ **f** $\begin{array}{r} 458 \\ +287 \\ \hline \end{array}$ **g** $\begin{array}{r} 1041 \\ +882 \\ \hline \end{array}$ **h** $\begin{array}{r} 3092 \\ +1988 \\ \hline \end{array}$



You will need to carry the 'one' twice in these questions.

Example 5 Subtracting larger numbers

Give the result for each of these differences.

a $\begin{array}{r} 74 \\ -15 \\ \hline \end{array}$ **b** $\begin{array}{r} 526 \\ -138 \\ \hline \end{array}$

Solution	Explanation
a $\begin{array}{r} {}^{6}\not{7}{}^{1}4 \\ -\ 1\ 5 \\ \hline 5\ 9 \end{array}$	Borrow 1 from 7 to make $14 - 5 = 9$. Then subtract 1 from 6 (not 7).
b $\begin{array}{r} {}^{4}\not{5}{}^{11}\not{2}{}^{\ 1}6 \\ -\ 1\ 3\ 8 \\ \hline 3\ 8\ 8 \end{array}$	Borrow 1 from 2 to make $16 - 8 = 8$. Borrow 1 from 5 to make $11 - 3 = 8$. $4 - 1 = 3$.

6 Find the answers to these differences.

a $\begin{array}{r} 54 \\ -23 \\ \hline \end{array}$ **b** $\begin{array}{r} 85 \\ -65 \\ \hline \end{array}$ **c** $\begin{array}{r} 46 \\ -27 \\ \hline \end{array}$ **d** $\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$

e $\begin{array}{r} 85 \\ -27 \\ \hline \end{array}$ **f** $\begin{array}{r} 43 \\ -14 \\ \hline \end{array}$ **g** $\begin{array}{r} 82 \\ -56 \\ \hline \end{array}$ **h** $\begin{array}{r} 66 \\ -27 \\ \hline \end{array}$



For parts **c** to **h**, you will need to borrow a 'one' from the tens column.

7 Find the answer to these differences.

a $\begin{array}{r} 235 \\ -\ 86 \\ \hline \end{array}$ **b** $\begin{array}{r} 352 \\ -\ 79 \\ \hline \end{array}$ **c** $\begin{array}{r} 714 \\ -\ 58 \\ \hline \end{array}$ **d** $\begin{array}{r} 932 \\ -\ 44 \\ \hline \end{array}$

e $\begin{array}{r} 125 \\ -\ 89 \\ \hline \end{array}$ **f** $\begin{array}{r} 241 \\ -129 \\ \hline \end{array}$ **g** $\begin{array}{r} 358 \\ -279 \\ \hline \end{array}$ **h** $\begin{array}{r} 491 \\ -419 \\ \hline \end{array}$



You will need to borrow a 'one' twice in each question.

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 Farmer Green owns 287 sheep, Farmer Brown owns 526 sheep and Farmer Grey owns 1041 sheep. How many sheep are there in total?



9 A car's odometer shows 12 138 kilometres at the start of a journey and 12 714 kilometres at the end of the journey. How far was the journey?



10 Give the result for each of these sums.

a
$$\begin{array}{r} 17 \\ 26 \\ +34 \\ \hline \end{array}$$

b
$$\begin{array}{r} 126 \\ 47 \\ +\ 19 \\ \hline \end{array}$$

c
$$\begin{array}{r} 152 \\ 247 \\ +\ 19 \\ \hline \end{array}$$

d
$$\begin{array}{r} 2197 \\ 1204 \\ +\ 807 \\ \hline \end{array}$$

11 Find the missing digits in these sums and differences.

a
$$\begin{array}{r} 3\square \\ +\ 5\ 3 \\ \hline \square\ 1 \end{array}$$

b
$$\begin{array}{r} 1\square 4 \\ +\ \ 7\square \\ \hline \square\ 9\ 1 \end{array}$$

c
$$\begin{array}{r} 6\square \\ -\ 2\ 8 \\ \hline \square\ 4 \end{array}$$

d
$$\begin{array}{r} 2\square 5 \\ -\square\ 8\square \\ \hline 8\ 1 \end{array}$$

12 **a** What are the missing digits in this sum?

$$\begin{array}{r} 2\square 3 \\ +\square\square\square \\ \hline 4\ 2\ 1 \end{array}$$

b Explain why there is more than one possible set of missing digits in the sum above. Give some examples.

More magic squares

—13

13 Complete these magic squares.

a

62	67	60
		65

b

101		114	
	106		109
	110		
113	103	102	116

1D Multiplication

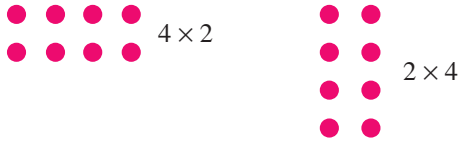
Interactive

Widgets

HOTsheets

Walkthrough

The multiplication of two numbers represents a repeated addition. For example, 4×2 could be thought of as 4 groups of 2 or $2 + 2 + 2 + 2$, or 4×2 could be thought of as 2 groups of 4 or 2×4 or $4 + 4$.



Let's start: Museum tickets

Your teacher purchases 21 tickets at \$9 each for a class excursion to the museum. You need to work out the total cost.

Look at the following strategies. Do any of them give the correct answer?

- 21×9 is the same as 20×10, so the answer is \$200
- 21×9 is the same as $21 \times 10 - 1$, so the answer is $210 - 1 = \$209$
- 21×9 is the same as $20 \times 9 + 1$, so the answer is $180 + 1 = \$181$
- 21×9 is the same as $21 \times 10 - 21$, so the answer is $210 - 21 = \$189$
- 21×9 is the same as $20 \times 9 + 9$, so the answer is $180 + 9 = \$189$



Key ideas

- Finding the **product** of two numbers involves multiplication.
  We say 'the product of 2 and 3 is 6'.
- $a \times b = b \times a$ For example: $2 \times 3 = 3 \times 2$
  - The order does *not* matter when you multiply numbers. For example:
    $2 \times 3 = 3 \times 2$ ← These examples use the **commutative law**.
    $5 \times 11 = 11 \times 5$ ←
    $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ ← This example uses the **associative law**.
- To multiply by a single digit:
  - Multiply the single digit by each digit in the other number, starting from the right.
  - Carry and add any digits with a higher place value to the total in the next column.

$$\begin{array}{r} {}^{1}23 \\ \times\ \ 4 \\ \hline 92 \end{array} \quad \begin{array}{l} 4 \times 3 = 12 \\ 4 \times 2 + 1 = 9 \end{array}$$

- Mental strategies for multiplication include:
  - Knowing your multiplication tables off by heart. For example:
    $9 \times 7 = 63$
    $12 \times 3 = 36$

Product A number that is the result of multiplication

Commutative law When adding and multiplying, the order in which two numbers are combined does not matter

Associative law The result of adding or multiplying three or more numbers does not depend on how they are grouped

- Changing the order. For example:
  $15 \times 3 = 3 \times 15$ (3 groups of 15)
  $= 45$
  $5 \times 13 \times 2 = 5 \times 2 \times 13$
  $= 10 \times 13$
  $= 130$
- Using the **distributive law** by making a 10, 100 etc. and then adjusting by adding or subtracting. For example:

6×20 6×1

$6 \times 21 = 120 + 6$
$= 126$

7×20 7×2

$7 \times 18 = 140 - 14$
$= 126$

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

- Using the doubling and halving strategy by doubling one number and halving the other.

Double the 5 Halve the 4

$5 \times 7 \times 4 = 10 \times 7 \times 2$
$= 70 \times 2$
$= 140$

Exercise 1D

UNDERSTANDING 1, 2, 3(½) 3(½)

1 Write the next three numbers in these patterns.

a 2, 4, 6, 8, ___, ___, ___
b 3, 6, 9, 12, ___, ___, ___
c 7, 14, 21, 28, ___, ___, ___
d 4, 8, 12, 16, ___, ___, ___
e 11, 22, 33, ___, ___, ___
f 17, 34, 51, ___, ___, ___

2 Write the missing number.

a $4 \times 5 = 5 \times$ ___
b $2 \times 7 = 7 \times$ ___
c $15 \times 11 =$ ___ $\times 15$
d $3 \times 2 \times 6 = 6 \times$ ___ $\times 3$
e $12 \times 2 \times 4 = 2 \times 12 \times$ ___
f $7 \times 3 \times 9 = 9 \times 3 \times$ ___

3 Use your knowledge of the multiplication tables to write the answer. Check your answers with your calculator.

a 11×2	**b** 3×9	**c** 8×4	**d** 7×8	**e** 7×4
f 12×5	**g** 4×11	**h** 11×7	**i** 12×9	**j** 9×8
k 3×7	**l** 6×9	**m** 6×5	**n** 10×11	**o** 12×12
p 8×5	**q** 7×7	**r** 9×7	**s** 11×12	**t** 12×6
u 5×11	**v** 2×11	**w** 4×6	**x** 12×8	**y** 6×6

1D

FLUENCY 4–6(½) 4–7(½)

Example 6 Using mental strategies for multiplication

Use a mental strategy to find:

a 3×13 **b** 4×19 **c** 5×24

Solution	Explanation
a $3 \times 13 = 30 + 9$ $= 39$	$3 \times 13 = 3 \times 10 + 3 \times 3$ The distributive law is being used.
b $4 \times 19 = 80 - 4$ $= 76$	$4 \times 19 = 4 \times 20 - 4 \times 1$ The distributive law is being used.
c $5 \times 24 = 10 \times 12$ $= 120$	The doubling and halving strategy is being used.

4 Find the results to these products mentally. Check your answers with your calculator.

a 5×21 **b** 4×31 **c** 3×31
d 6×22 **e** 5×23 **f** 7×31
g 9×22 **h** 6×42 **i** 8×42


For part **a**, work out 5×20 and then add 5×1.

5 Find the answers to these products mentally.

a 3×19 **b** 2×19 **c** 2×29
d 4×29 **e** 5×18 **f** 7×18
g 3×39 **h** 4×49 **i** 6×39


For part **a**, work out 3×20 and then subtract 3×1.

6 Find the answers to these products mentally.

a 5×14 **b** 5×18 **c** 22×5
d 36×5 **e** 4×24 **f** 3×18
g 6×16 **h** 24×3 **i** 18×4


Double one number and halve the other. So $5 \times 14 = 10 \times 7 = 70$

Example 7 Multiplication showing working

Give the result for each of these products.

a 31×4 **b** 197×7

Solution	Explanation
a $\begin{array}{r} 31 \\ \times\ 4 \\ \hline 124 \end{array}$	$4 \times 1 = 4$ $4 \times 3 = 12$
b $\begin{array}{r} {}^{6}1^{4}97 \\ \times\ \ \ 7 \\ \hline 1379 \end{array}$	$7 \times 7 = 49$ (carry the 4) $7 \times 9 + 4 = 67$ (carry the 6) $7 \times 1 + 6 = 13$

7 Give the result of each of these products, showing working.

a 33×2 **b** 43×3 **c** 72×6 **d** 55×3 **e** 37×4

f 51×9 **g** 48×7 **h** 59×8 **i** 129×2 **j** 407×7

k 526×5 **l** 3509×9

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 What is the missing digit in these products?

a $21 \times 3 = 6\square$ **b** $36 \times 5 = 18\square$ **c** $76 \times 2 = 1\square2$ **d** $402 \times 3 = 1\square06$

9 Eight tickets costing $33 each are purchased for a concert. What is the total cost of the tickets?



10 A circular race track is 240 metres long and Rory runs seven laps. How far does Rory run in total?

11 Reggie and Angelo combine their packs of cards. Reggie has five sets of 13 cards and Angelo has three sets of 17 cards. How many cards are there in total?

12 Are these statements true or false?

a $4 \times 3 = 3 \times 4$ **b** $2 \times 5 \times 6 = 6 \times 5 \times 2$ **c** $11 \times 5 = 10 \times 5$

d $3 \times 32 = 3 \times 30 + 3 \times 2$ **e** $5 \times 18 = 10 \times 9$ **f** $21 \times 4 = 2 \times 42$

g $19 \times 7 = 20 \times 7 - 19$ **h** $39 \times 4 = 40 \times 4 - 1 \times 4$ **i** $64 \times 4 = 128 \times 8$

Missing digits — 13

13 Find the missing digits in these products.

a $39 \times 7 = 2\square3$ **b** $25 \times \square = 125$ **c** $79 \times \square = \square37$ **d** $132 \times \square = 10\square6$ **e** $2\square \times 7 = \square89$

f $\square\square \times 9 = 351$ **g** $23\square \times 5 = 1\square60$ **h** $\square\square4 \times \square = \square198$ **i** $\square\square\square \times 3 = 1638$

14 How many different ways can the two spaces be filled in this problem? Explain your answer.

$$2\square3 \times 4 = 8\square2$$

1E Multiplying larger numbers

Interactive
Widgets
HOTsheets
Walkthrough

There are many situations that require the multiplication of large numbers. For example, finding the total amount made from selling 40 000 tickets at $23 each. Another example is finding the area of a rectangular park with length and width dimensions of 65 metres by 122 metres. Doing such calculations by hand requires a number of steps.



The area of a sports stadium can be found using multiplication.

Let's start: Spot the errors

There are three types of errors in the working shown for this problem. Find the errors and describe them.

$$\begin{array}{r} 82 \\ \times\ 16 \\ \hline 482 \\ 82 \\ \hline 464 \end{array}$$

Key ideas

- When multiplying by 10, 100, 1000, 10 000 etc. each digit moves to the left by the number of zeros.
  $2 \times 100 = 200 \quad 41 \times 10 = 410 \quad 279 \times 1000 = 279\,000$
- A strategy for multiplying by multiples of 10, 100 etc. is to first multiply by the number without the zeros then add the zeros to the answer later.
  For example: $21 \times 3000 = 21 \times 3 \times 1000 = 63 \times 1000 = 63\,000$
- To multiply large numbers, divide the problem into smaller products and then add the totals.

$$\begin{array}{r} 37 \\ \times\ 12 \\ \hline 74 \\ 370 \\ \hline 444 \end{array} \begin{array}{l} \\ \\ \leftarrow 37 \times 2 \\ \leftarrow 37 \times 10 \\ \leftarrow 370 + 74 \end{array} \qquad \begin{array}{r} 143 \\ \times\ 14 \\ \hline 572 \\ 1430 \\ \hline 2002 \end{array} \begin{array}{l} \\ \\ \leftarrow 143 \times 4 \\ \leftarrow 143 \times 10 \\ \leftarrow 1430 + 572 \end{array}$$

Exercise 1E

UNDERSTANDING 1–3 | 3

1 Write the missing number: 10, 100 or 1000.

a $35 \times ___ = 350$ **b** $21 \times ___ = 2100$ **c** $49 \times ___ = 49\,000$ **d** $213 \times ___ = 2130$

2 Copy and complete these simple products.

a $\begin{array}{r} 27 \\ \times\ 3 \\ \hline \end{array}$ **b** $\begin{array}{r} 39 \\ \times\ 2 \\ \hline \end{array}$ **c** $\begin{array}{r} 92 \\ \times\ 5 \\ \hline \end{array}$ **d** $\begin{array}{r} 121 \\ \times\ 6 \\ \hline \end{array}$

3 Write the answers to the following.

a 2×60 **b** 3×40 **c** 5×50 **d** 9×80

e 30×6 **f** 90×2 **g** 70×5 **h** 60×9

FLUENCY 4–5(½) | 4–6(½)

Example 8 Multiplying large numbers

Give the result for each of these products.

a 37×100 **b** 21×50 **c** 87×13

Solution	Explanation
a $37 \times 100 = 3700$	Move the 3 and the 7 two places to the left and add two zeros.
b $21 \times 50 = 21 \times 5 \times 10$ $= 105 \times 10$ $= 1050$	First multiply by 5, then multiply by 10. $\begin{array}{r} 21 \\ \times\ 5 \\ \hline 105 \end{array}$
c $\begin{array}{r} 87 \\ \times\ 13 \\ \hline 261 \\ 870 \\ \hline 1131 \end{array}$	First multiply 87×3. Then multiply 87×10. Add the results to give the answer.

4 Give the result of each of these products.

a 4×100 **b** 29×10 **c** 183×10 **d** 46×100

e 37×1000 **f** 192×10 **g** 3010×100 **h** 248×1000

i 50×1000 **j** 630×100 **k** 1441×10 **l** $2910 \times 10\,000$

5 Find these products.

a 12×20 **b** 18×30 **c** 26×20

d 21×30 **e** 17×20 **f** 36×40

g 92×70 **h** 45×500 **i** 138×300

j 92×5000 **k** 317×200 **l** 1043×9000



First multiply by the single non-zero digit, then write the zeros:
$12 \times 20 = 12 \times 2 \times 10$

1E

6 Find these products.

a 21×12 **b** 26×11 **c** 31×14 **d** 43×15 **e** 37×11 **f** 72×19

g 88×14 **h** 57×22 **i** 92×23 **j** 84×27 **k** 46×31 **l** 72×65

PROBLEM-SOLVING AND REASONING 7–9 8–12

7 Mandy buys 28 tickets at $15 each. What is the total cost of the tickets?

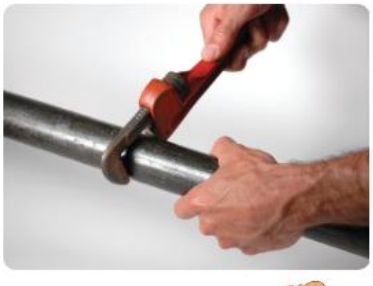
8 A pool area includes 68 square metres of paving at $32 per square metre. What is the total cost of paving?

9 These products involve a number with three digits. Find each answer, showing your working.

a 123×11 **b** 138×22 **c** 126×15 **d** 428×22

e 396×46 **f** 416×98 **g** 380×49 **h** 1026×33

10 Waldo buys 215 metres of pipe at $28 per metre. What is the total cost of piping?



11 How many seconds are there in one day?



There are 60 seconds in a minute.

12 If both numbers in a multiplication problem have at least three digits, then more steps need to be shown. Find these products.

a 294×136 **b** 1013×916 **c** 3947×1204 **d** 47126×3107

Missing digit puzzle — 13

13 Find the missing digits in these products.

a
$$\begin{array}{r} 2\square \\ \times\ 1\ 7 \\ \hline 1\square 1 \\ 2\square 0 \\ \hline \square\square 1 \end{array}$$

b
$$\begin{array}{r} 1\square 3 \\ \times\ \ 1\square \\ \hline \square 2\ 9 \\ 1\square 3\square \\ \hline \square\square 5\square \end{array}$$

c
$$\begin{array}{r} \square\square \\ \times\ \ 3\ 7 \\ \hline 3\ 4\ 3 \\ \square 4\square\square \\ \hline \square\square\square\square \end{array}$$

d
$$\begin{array}{r} \square 2\square \\ \times\ \ \ 2\square \\ \hline 1\ 2\ 6 \\ \square 5\ 2\square \\ \hline \square 6\square\square \end{array}$$

1F Division

Interactive

Widgets

HOTsheets

Walkthrough

Division is used to find the number of equal groups from a given total. Dividing 20 apples among five people or dividing $10 000 between three bank accounts are examples of when division can be used.



Division is often used when we calculate amounts shared between a number of people.

Let's start: Arranging counters

A total of 24 counters sit on a table. Using whole numbers, in how many ways can the counters be divided into equal-sized groups with no counters left over?

- Is it also possible to divide the counters into equal-sized groups but with two counters left over?
- If five counters are to remain, how many equal-sized groups can be formed and why?



Key ideas

- The number of equal-sized groups made by dividing is called the **quotient**.
- The total being divided is called the **dividend**. The size of the equal groups is called the **divisor**.
- Any amount remaining after division into equal-sized groups is called the **remainder**.

$7 \div 3 = 2$ and 1 remainder

total being divided (dividend) | size of equal groups (divisor) | quotient

Quotient A number that is the result of division

Dividend The number being divided

Divisor The number you are dividing by

Remainder The leftover amount after one number has been divided by another

- The order *does* matter when you divide numbers. You cannot swap numbers in a division to make the calculation easier. For example:

$6 \div 3 \neq 3 \div 6$ $\quad (10 \div 5) \div 2 \neq 10 \div (5 \div 2)$

- Use short division to work with larger numbers.
  - Start by dividing the divisor into the first (left) digit, then carry any remainder. This example shows that $413 \div 3 = 137$ and 2 remainder.

$4 \div 3 = 1$ and 1 rem. | $11 \div 3 = 3$ and 2 rem. | $23 \div 3 = 7$ and 2 rem.

$$3\overline{)4^{1}1^{2}3} \quad 1\ 3\ 7$$

So $413 \div 3 = 137$ and 2 remainder

$= 137\frac{2}{3}$

Exercise 1F

UNDERSTANDING 1–3 | 3

1 Find the remainder when 24 is divided by:

a 1 **b** 2 **c** 3 **d** 4
e 5 **f** 6 **g** 7 **h** 8



2 Write the number that is missing in these statements.

a $8 \div 2 = 4$ is the same as $4 \times ? = 8$.
b $36 \div 12 = 3$ is the same as $? \times 12 = 36$.
c $42 \div ? = 6$ is the same as $6 \times 7 = 42$.
d $72 \div 6 = ?$ is the same as $12 \times 6 = 72$.

3 What is the remainder when:

a 2 is divided into 7? **b** 5 is divided into 37?
c 42 is divided by 8? **d** 50 is divided by 9?

FLUENCY 4–9(½) | 4–9(½)

Example 9 Using mental strategies for division

Use a mental strategy to find the answer.

a $56 \div 8$ **b** $93 \div 3$ **c** $57 \div 3$ **d** $128 \div 8$

Solution	Explanation
a $56 \div 8 = 7$	$8 \times ? = 56$ (Use your knowledge from multiplication tables.)
b $93 \div 3 = 31$	$90 \div 3 = 30$ so $93 \div 3 = 31$
c $57 \div 3 = 19$	$60 \div 3 = 20$ so $57 \div 3 = 19$
d $128 \div 8 = 16$	$128 \div 8 = 64 \div 4 = 32 \div 2 = 16$ (Halve both numbers repeatedly.)

4 Use your knowledge of multiplication tables to find the quotient. Check with a calculator.

a $28 \div 7$ **b** $36 \div 12$ **c** $48 \div 8$ **d** $45 \div 9$
e $56 \div 8$ **f** $63 \div 7$ **g** $96 \div 12$ **h** $121 \div 11$
i $110 \div 11$ **j** $36 \div 9$ **k** $77 \div 7$ **l** $48 \div 4$
m $72 \div 9$ **n** $72 \div 6$ **o** $55 \div 5$ **p** $96 \div 8$

5 Find the answer to these using a mental strategy.

a $93 \div 3$ **b** $42 \div 2$ **c** $84 \div 4$ **d** $63 \div 3$
e $210 \div 10$ **f** $220 \div 20$ **g** $96 \div 3$ **h** $64 \div 4$



6 Find the answer to these using a mental strategy.

a $87 \div 3$ **b** $76 \div 4$ **c** $96 \div 4$ **d** $63 \div 7$
e $117 \div 3$ **f** $56 \div 4$ **g** $116 \div 4$ **h** $180 \div 20$



7 Find the answers to these using a mental strategy.

a $88 \div 4$ **b** $124 \div 4$

c $136 \div 8$ **d** $112 \div 16$



Halve both numbers, since they are both even. So, part **a**, $88 \div 4 = 44 \div 2$.

8 Write the answers to these divisions, which involve 0s and 1s.

a $26 \div 1$ **b** $1094 \div 1$ **c** $0 \div 7$ **d** $0 \div 458$

Example 10 Using short division

Use short division to find the quotient and remainder. Then write the result using a fraction.

a $3\overline{)37}$ **b** $7\overline{)195}$

Solution	Explanation
a $3\overline{)37}$ 12 rem. 1	$3 \div 3 = 1$ with no remainder.
	$7 \div 3 = 2$ with 1 remainder.
$37 \div 3 = 12$ and 1 remainder.	
$= 12\frac{1}{3}$	12 rem. $1 = 12\frac{1}{3}$
b $7\overline{)19^{5}5}$ $2\ 7$ rem. 6	7 does not divide into 1.
	$19 \div 7 = 2$ with 5 remainder.
	$55 \div 7 = 7$ with 6 remainder.
$195 \div 7 = 27$ and 6 remainder.	
$= 27\frac{6}{7}$	27 rem. $6 = 27\frac{6}{7}$

9 Use the short division algorithm to find the quotient and remainder. Then write the result using a fraction.

a $3\overline{)71}$ **b** $7\overline{)92}$ **c** $5\overline{)139}$ **d** $6\overline{)247}$

e $5\overline{)217}$ **f** $4\overline{)506}$ **g** $3\overline{)794}$ **h** $9\overline{)814}$

i $4\overline{)2173}$ **j** $3\overline{)61\,001}$ **k** $5\overline{)4093}$ **l** $9\overline{)90\,009}$

PROBLEM-SOLVING AND REASONING 10–12 11–15

10 Write the missing digit in each of these divisions.

a $3\overline{)\square 7}$ $5\ 1$ **b** $7\overline{)\square 2}$ $8\ 4$ **c** $5\overline{)2\square}$ $1\ 2\ 5$ **d** $9\overline{)1\square}$ $1\ 3\ 5$

11 If 117 food packs are divided equally among nine families, how many packs does each family receive?



1F

12 Spring Fresh Company sells mineral water in packs of six bottles. How many packs are there in a truck containing 744 bottles?



Work out $6\overline{)744}$

13 A straight fence has two end posts. It also has other posts that are divided evenly along the fence 4 metres apart. If the fence is to be 264 metres long, how many posts are needed, including the end posts?



14 Friendly Taxis can take up to four passengers each. How many taxis are required to transport 59 people?

15 Pies are purchased wholesale at three for $4. How much will it cost to purchase 153 pies?



Long, short division

— 16, 17

16 Short division can also be used to divide by numbers with more than one digit.
e.g. 215 ÷ 12 = 17 and 11 remainder.

$$12\overline{)21^{9}5} \quad 1\ 7 \text{ rem. } 11$$

Use the short division algorithm to find the quotient and remainder.

a 371 ÷ 11 **b** 926 ÷ 17 **c** 404 ÷ 13
d 1621 ÷ 15 **e** 2109 ÷ 23 **f** 6914 ÷ 56

17 The magic product for this square is 6720. Find the missing numbers.

1	6		56
40		2	3
14			
			10

Progress quiz

1A **1** Write down the place value of the digit 5 in these numbers.

a 356 **b** 5103 **c** 857 412

1A **2** How many numbers can be made using the given digits? Digits are not allowed to be used more than once and all digits must be used.

a 4, 6 and 8

b 1, 2, 3 and 9

1B **3** Mentally find the answers to these problems.

a $146 + 332$

b $754 - 531$

c $85 - 19$

d $21 + 49 - 28$

1B **4** Are these statements true (T) or false (F)?

a $23 - 17 < 8$

b $60 - 18 + 11 > 50$

1C **5** Give the result for each of these problems.

a
$$\begin{array}{r} 37 \\ +\,58 \\ \hline \end{array}$$

b
$$\begin{array}{r} 4584 \\ +\,897 \\ \hline \end{array}$$

c
$$\begin{array}{r} 63 \\ -\,48 \\ \hline \end{array}$$

d
$$\begin{array}{r} 413 \\ -\,296 \\ \hline \end{array}$$

1D **6** Find the answers to these products mentally.

a 6×18

b 32×5

1D **7** Give the result for each of these products.

a
$$\begin{array}{r} 82 \\ \times\ 7 \\ \hline \end{array}$$

b
$$\begin{array}{r} 534 \\ \times\ 3 \\ \hline \end{array}$$

1E **8** Give the result of each of these products.

a 71×1000

b 37×200

c
$$\begin{array}{r} 23 \\ \times 12 \\ \hline \end{array}$$

d
$$\begin{array}{r} 59 \\ \times 23 \\ \hline \end{array}$$

1E **9** Jack purchases 15 basketball tickets at $21 each. What is the total cost of the tickets?

1F **10** Find the answer to these using a mental strategy.

a $63 \div 3$

b $495 \div 5$

c $160 \div 20$

d $240 \div 16$

1F **11** Use the short division algorithm to find the quotient and remainder.

a $5\overline{)227}$ **b** $8\overline{)4237}$

1G Estimating and rounding

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Often, a good estimate is enough to answer a question and it's not necessary to find an exact answer. In such cases we use rounding to help. For example, the approximate total cost of 18 truckloads of soil at \$54 per load could be estimated as $20 \times 50 = 1000$, so the cost is about \$1000.

Let's start: Counting crowds

Here is a photo of a crowd at a sporting event. Describe how you might estimate the number of people in the photo. What is your answer? How different is your answer from those of others in your class?



Key ideas

- **Estimates** or **approximations** can be found by **rounding** numbers to the nearest 10, 100, 1000 etc.
  - If the next digit is 0, 1, 2, 3 or 4, then round down.
  - If the next digit is 5, 6, 7, 8 or 9, then round up.
- Leading digit approximation involves rounding to the first digit. For example:
  For 932 rounds to 900
  For 968 rounds to 1000
- The symbol ≈ means 'approximately equal to'.

Estimate An informed guess

Approximation A value that is close to the real value

Rounding Approximating a number to a specified number of places

Exercise 1G

UNDERSTANDING 1–3 | 3

1 Have these numbers been rounded up or down?

a $59 \approx 60$ **b** $14 \approx 10$ **c** $137 \approx 140$
d $255 \approx 260$ **e** $924 \approx 900$ **f** $1413 \approx 1000$

2 The following numbers are to be rounded to the nearest 10. Should they be rounded up or down?

a 19 **b** 37 **c** 21 **d** 14
e 72 **f** 33 **g** 45 **h** 95
i 132 **j** 176 **k** 288 **l** 304

3 The following numbers are to be rounded to the nearest 100. Decide if they would be rounded up or down.

a 103 **b** 201 **c** 195 **d** 186
e 172 **f** 131 **g** 427 **h** 552
i 956 **j** 349 **k** 198 **l** 359

FLUENCY 4–6(½) | 4–6(½)

Example 11 Rounding

Round these numbers as indicated.

a 86 (to the nearest 10) **b** 4142 (to the nearest 100)

Solution	Explanation
a $86 \approx 90$	The digit after the 8 is greater than or equal to 5, so round up.
b $4142 \approx 4100$	The digit after the 1 is less than or equal to 4, so round down. Keep the 1 and make the following digits zero.

4 Round these numbers as indicated.

a 59 (nearest 10) **b** 32 (nearest 10)
c 124 (nearest 10) **d** 185 (nearest 10)
e 231 (nearest 100) **f** 894 (nearest 100)
g 96 (nearest 10) **h** 584 (nearest 100)
i 1512 (nearest 1000)
j 1492 (nearest 1000) **k** 7810 (nearest 1000)
l 10 200 (nearest 1000)



5 Round these numbers using leading digit approximation.

a 21 **b** 29 **c** 136
d 857 **e** 241 **f** 729
g 98 **h** 962 **i** 5600
j 92 104 **k** 9999 **l** 14



1G

Example 12 Using leading digit approximation

Estimate the answers to these questions by first rounding each number to the leading digit.

a 42×7 **b** 95×326 **c** $302 \div 29$

Solution	Explanation
a $42 \times 7 \approx 40 \times 7$ $= 280$	The leading digit in 42 is the 4 in the 'tens' column.
b $95 \times 326 \approx 100 \times 300$ $= 30\,000$	The nearest 'ten' to 95 is 100, and the leading digit in 326 is in the 'hundreds' column.
c $302 \div 29 \approx 300 \div 30$ $= 10$	302 rounds to 300 and 29 rounds to 30.

6 Use leading digit approximation to estimate the answer.

a 29×4 **b** $124 + 58$
c $232 - 106$ **d** $61 \div 5$
e $103 \div 11$ **f** 32×99
g 59×21 **h** $279 \div 95$ **i** $394 \div 10$
j 97×21 **k** $1390 + 3244$ **l** $999 - 888$



First round each number to the leading digit before making the calculation.

PROBLEM-SOLVING AND REASONING 7–9 9–12

7 You purchase 59 tickets at $21 each. Give an estimate for the total cost of the tickets.



For questions 7 to 10, use leading digit approximation to make your estimate.

8 A digger can dig 29 scoops per hour and work 7 hours per day. Approximately how many scoops can be dug over 10 days?

9 Most of the pens at a stockyard are full of sheep. There are 55 pens and one of the pens has 22 sheep. Give an estimate for the total number of sheep at the stockyard.



10 A whole year group of 159 students is roughly divided into 19 groups. Estimate the number in each group.

11 For the given estimates, decide if the approximate answer is going to give a larger or smaller result compared to the true answer.

a $58 + 97 \approx 60 + 100$ **b** $24 \times 31 \approx 20 \times 30$
c $130 - 79 \approx 130 - 80$ **d** $267 - 110 \approx 270 - 110$

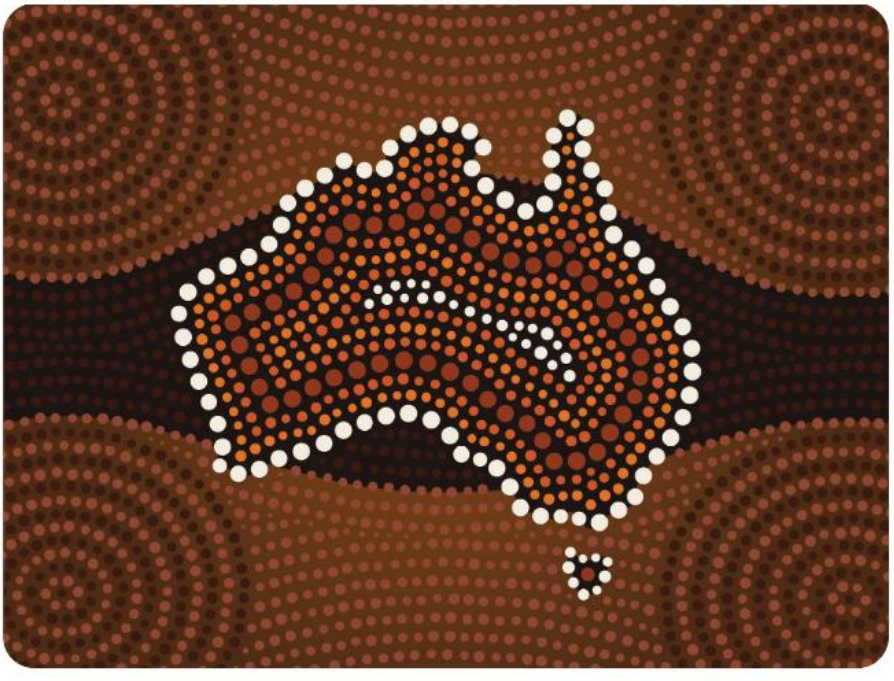
12 It is sensible sometimes to round one number up if the other number is going to be rounded down. Use leading digit approximation to estimate the answers to these problems.

a 11×19 **b** 129×954 **c** 25×36 **d** 1500×2500

Aboriginal dot painting

13

13 Many examples of aboriginal art include dot paintings. Here are two examples. Estimate the number of dots in each one.





1H Order of operations

Interactive

Widgets

HOTsheets

Walkthrough

When combining the operations of addition, subtraction, multiplication and division, a special order needs to be followed. Multiplication and division sit higher in the order than addition and subtraction. This affects how we might make sense of simple mathematical problems put into words.
Consider these two statements.

- 2 groups of 3 chairs plus 5 chairs
- 5 chairs plus 2 groups of 3 chairs



In both cases, there are $2 \times 3 + 5 = 11$ chairs. This means that $2 \times 3 + 5 = 5 + 2 \times 3$.

This also suggests that for $5 + 2 \times 3$, the multiplication should be done first.

Let's start: Make it true!

Can you insert a pair of brackets to make the following true?

- $2 + 3 \times 5 = 25$
- $20 \div 8 - 3 = 4$
- $3 \times 2 + 6 = 24$
- $7 - 1 \div 3 = 2$

Discuss whether you think the following need brackets to make them true.

- $2 + 6 \times 1 = 8$
- $10 \div 2 + 3 = 2$

Key ideas

- When working with more than one operation:
  - Deal with **brackets** first.
  - Do **multiplication** and **division** next, working from left to right.
  - Do **addition** and **subtraction** last, working from left to right.

$7 \div (4 + 3)$ (1st: $4 + 3 = 7$; 2nd: $7 \div 7 = 1$)

So $7 \div (4 + 3) = 7 \div 7$
$= 1$

$4 \times (2 + 3) - 12 \div 6$ (1st: $2 + 3 = 5$; 2nd: $4 \times 5 = 20$; 3rd: $12 \div 6 = 2$; last: $20 - 2 = 18$)

So $4 \times (2 + 3) - 12 \div 6 = 4 \times 5 - 2$
$= 20 - 2$
$= 18$

Exercise 1H

UNDERSTANDING 1–2 | 2

1 Which goes first?

a addition or multiplication

b brackets or division

c subtraction or brackets

d multiplication or subtraction

2 Which operation (addition, subtraction, multiplication or division) is done first in these problems?

a $2+5-3$ **b** $5\div5\times2$ **c** $2\times3\div6$

d $5\times2+3$ **e** $7\div7-1$ **f** $(6+2)\times3$

g $(8\div4)-1$ **h** $4+7\times2$ **i** $8-10\div5$

j $10-2+3$ **k** $6+2\times3-1$ **l** $5\times(2+3\div3)-1$

FLUENCY 3(½) | 3–4(½)

Example 13 Using order of operations

Use order of operations to answer the following.

a $5+10\div2$

b $3\times(2+4)$

c $5\times2-8\div4$

d $6\times(2+10)-24$

e $18-2\times(4+6)\div5$

Solution	Explanation
a $5+10\div2=5+5$ $=10$	Do the division before the addition.
b $3\times(2+4)=3\times6$ $=18$	Deal with brackets before multiplication.
c $5\times2-8\div4=10-2$ $=8$	Do the multiplication and division before the subtraction.
d $6\times(2+10)-24=6\times12-24$ $=72-24$ $=48$	Deal with brackets first. Do the multiplication before subtraction. Do the subtraction last.
e $18-2\times(4+6)\div5=18-2\times10\div5$ $=18-20\div5$ $=18-4$ $=14$	Deal with brackets first. Do the multiplication and division next, working from left to right. Do the subtraction last.

1H

3 Use order of operations to find the answers.

a $1 + 2 \times 3$
b $5 + 7 \times 2$
c $9 - 10 \div 5$
d $4 \times (3 + 2)$
e $21 \div (3 + 4)$
f $18 \div (10 - 1)$
g $(7 + 2) \div 3$
h $(10 - 4) \times 4$
i $(6 - 5) \div 1$
j $2 + 3 \times 7$
k $5 + 8 \times 2$
l $10 - 20 \div 2$
m $22 - 16 \div 4$
n $6 \times 3 + 2 \times 7$
o $1 \times 8 - 2 \times 3$
p $18 \div 9 + 60 \div 3$
q $2 + 3 \times 7 - 1$
r $40 - 25 \div 5 + 3$
s $63 \div 3 \times 7 + 2 \times 3$
t $78 - 14 \times 4 + 6$
u $300 - 100 \times 4 \div 4$



Remember that × and ÷ go before + and −. Work from left to right after you have chosen which operation goes first.

4 Use order of operations to find the answers.

a $2 \times (3 + 2)$
b $18 \div (10 - 4)$
c $(19 - 9) \div 5$
d $2 \times (3 + 2) - 1$
e $10 \div (3 + 2) + 6$
f $13 \times (10 \div 10) - 13$
g $(100 + 5) \div 5 + 1$
h $2 \times (9 - 4) \div 5$
i $50 \div (13 - 3) + 4$
j $16 - 2 \times (7 - 5) + 6$
k $(7 + 2) \div (53 - 50)$
l $14 - (7 \div 7 + 1) \times 2$
m $(20 - 10) \times (5 + 7) + 1$
n $3 \times (72 \div 12 + 1) - 1$
o $48 \div (4 + 4) \div (3 \times 2)$
p $20 - (3 \times 5 + 1) \div 4$



Deal with brackets first, then × and ÷ then + and −.

PROBLEM-SOLVING AND REASONING 5–7 7–10

5 Are these statements true or false?

a $5 \times 2 + 1 = (5 \times 2) + 1$
b $10 \times (3 + 4) = 10 \times 3 + 4$
c $21 - 7 \div 7 = (21 - 7) \div 7$
d $9 - 3 \times 2 = 9 - (3 \times 2)$



Example 14 Using order of operations in worded problems

Find the result if 6 is multiplied by the sum of 2 and 7.

Solution	Explanation
$6 \times (2 + 7) = 6 \times 9$ $= 54$	First, write the problem using symbols and numbers. Use brackets for the sum since this operation is to be completed first.

6 Find the answer to these worded problems by first writing the sentence using numbers and symbols.

a Triple the sum of 3 and 6.
b Double the result of 20 divided by 4.
c 44 divided by 11 plus 4.
d 5 more than the product of 6 and 12.
e The result of 60 divided by 12 is subtracted from the product of 5 and 7.
f 15 less than the difference of 48 and 12.
g The product of 9 and 12 is subtracted from double the product of 10 and 15.



Sum means add.
Difference means subtract.
Product means multiply.

7 A delivery of 15 boxes of books arrives. Each box contains eight books. The bookstore owner removes three books from each box. How many books still remain in total?



8 In a class, eight students have three TV sets at home, four have two TV sets, 13 have one TV set and two students have no TV sets. How many TV sets are there in total?



9 Insert brackets into these equations to make them true.

a $4 + 2 \times 3 = 18$ **b** $9 \div 12 - 9 = 3$ **c** $2 \times 3 + 4 - 5 = 9$

d $3 + 2 \times 7 - 3 = 20$ **e** $10 - 7 \div 21 - 18 = 1$ **f** $4 + 10 \div 21 \div 3 = 2$

10 Decide if the brackets given in each equation are actually necessary. That is, do they make any difference?

a $2 + (3 \times 6) = 20$ **b** $(2 + 3) \times 6 = 30$ **c** $(20 \times 2) \times 3 = 120$

d $10 - (5 + 2) = 3$ **e** $22 - (11 - 7) = 18$ **f** $19 - (10 \div 2) = 14$

Brackets within brackets

— 11, 12

11 These problems involve brackets within brackets. Make sure you work with the inner brackets first. (The first one has already been done.)

a $2 \times [(2 + 3) \times 5 - 1] = 2 \times [6 \times 5 - 1]$

$= 2 \times [30 - 1]$

$= 2 \times 29$

$= 58$

b $[10 \div (2 + 3) + 1] \times 6$ **c** $26 \div [10 - (17 - 9)]$

d $[6 - (5 - 3)] \times 7$ **e** $2 + [103 - (21 + 52)] - (9 + 11) \times 6 \div 12$

12 Insert brackets to make the following true. (You may need to use more than one pair.)

a $20 - 31 - 19 \times 2 = 16$ **b** $50 \div 2 \times 5 - 4 = 1$ **c** $25 - 19 \times 3 + 7 \div 12 + 1 = 6$

Maths@Work: Stock controller at sports events

The 2016 AFL Grand final between the Western Bulldogs and the Sydney Swans saw an official attendance of 99 981, which is extremely close to the capacity crowd of 100 024 people that can fit into the MCG (Melbourne Cricket Ground).

At large sporting events and concerts, there are many food and beverage stands around the grounds. The stock controllers need to predict the crowd size and make sure that enough water and food is available for each event.



1 Complete the table below for these different stadiums and events from around Australia. Use an allowance of 1.5 bottles of water for each person attending the event. After calculating all answers, round to the nearest whole number.

Earnings are the total amount from sales, not the profit.



Venue	Event	Attendance	Water bottles required	Total wholesale cost @ $0.80/bottle	Earnings @ $3 a bottle
Suncorp Stadium, Brisbane	Brisbane Roar vs Perth Glory	35 200			
MCG, Melbourne Cricket Ground	Hawthorn vs Freemantle	100 007			
WACA, WA Cricket Association	T20 cricket India vs Australia	34 527			
SCG, Sydney Cricket Ground	Sydney Swans vs Western Bulldogs	33 386			
Adelaide Oval	Adelaide Crows vs West Coast Eagles	53 445			
NIB Stadium, Perth	Wallabies vs Argentina	16 202			
Darwin Football Stadium	Brumbies vs Reds	4600			
ANZ Stadium, Sydney	Broncos vs Cowboys	82 758			

2 Attendances are often rounded for convenience of reporting. For the events in question **1**, round each of the attendances to the nearest thousand.

3 Average attendance figures help stock controllers predict the needs of events at their venues.

$$\text{Average} = \frac{\text{Total attendance}}{\text{Number of games}}$$

Calculate the average attendance for each of the following events:

- **a** AFL 2014 season with a total attendance of 6 954 585 with 207 games
- **b** Big Bash League 2015–16 season with 1 030 505 people attending 35 games.
- **c** Japanese baseball 2015 season of 848 games with a total attendance of 24 236 920.
- **d** Major league baseball with total attendance 73 760 020 over 2 430 games.



A Big Bash League match between the Brisbane Heat and Hobart Hurricanes at The Gabba

4 List four factors that could cause variation in attendance figures.

Using technology

Use the following example to help you with the question below.

For one MCG cricket event, 6 buckets of hot chips were sold for every 10 people. The following calculation shows how the stock controller predicts the amount of food required for an event.

$$\text{Total number of food items} = \text{seating capacity} \times \frac{\text{number of food items}}{\text{number of people}}$$

$$= 100\,000 \times \frac{6}{10}$$

$$= 60\,000 \text{ buckets of hot chips}$$

5 For the following calculations, you can assume each stadium is filled to capacity.

- **a** Copy the table below into a spreadsheet. Select the shaded cells, right click, select Format Cells/Number/0 d.p./OK

	A	B	C	D	E	F	G
1	Food quantities needed for sports events						
2	Stadium	Seating capacity	Fruit salad	Hot chips	Pies	Mini pizzas	Fresh wraps
3	WACA, Perth	24500					
4	Adelaide Oval	53500					
5	The Gabba, Brisbane	42000					
6	ANZ Stadium, Sydney	83500					
7	Rod Laver Arena, Melbourne	14820					
8	Blundstone Arena, Hobart	20000					
9	Darwin Football Stadium	6000					

- **b** Insert formulas into the shaded cells to calculate the whole number of serves for each food item in this table.
  Use these facts to write the formulas:
  - 1 in 5 people buy fruit salad
  - 3 pies are sold for every 2 fans
  - 3 in 8 people buy fresh wraps
  - half the fans buy hot chips
  - 2 mini pizzas are sold for every 5 people

E3 formula = $B3*3/2$
G3 formula = $B3*3/8$



Puzzles and games

1 Complete these magic squares. Each row, column and main diagonal add up to the same magic total.

a

15		
	16	18
		17

b

		9
	12	14
		13

c

3	17		6
	8		11
		13	
15		4	18

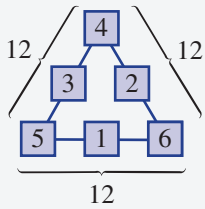
2 Decide where brackets should go to make each statement true.

a $5 + 2 \times 3 = 21$

b $16 - 8 \div 10 - 6 = 2$

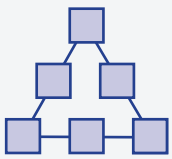
c $4 + 2 \times 7 - 1 \times 3 = 43$

3 Each side on a magic triangle adds up to the same number, as shown in this example with a sum of 12 on each side.

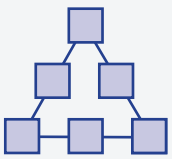


a Place the digits 1 to 6 in a magic triangle with three digits along each side so that each side adds up to the given number.

i 9

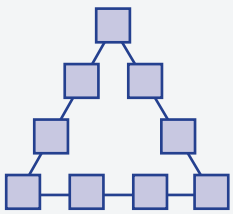


ii 10

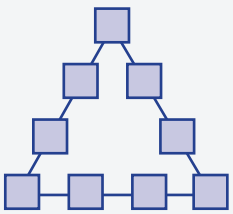


b Place the digits 1 to 9 in a magic triangle with four digits along each side so that each side adds up to the given number.

i 20



ii 23

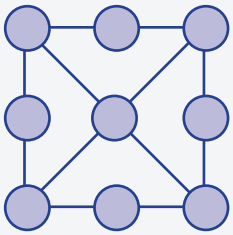


4 Sudoku is a popular logic number puzzle made up of a 9 by 9 square, where each column and row can use the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 only once. Also, each digit is to be used only once in each 3 by 3 square. Solve these puzzles.

	4				2		8	7
2	8		7		9		1	
			6			3		5
	3	7		2				8
	6	5	4	7	8			2
			2	6				
					7	5		
		8	3	9		2	7	

	7			6	9	3		
		4	1		8			7
8					2	9	1	
3		1						
	2	8	5		3			
	5	6		9		2		
	3	9			5			
6				8	4			
5			9		7			

5 The sum along each line is 15. Can you place each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make this true?



6 Find all the missing digits in these products.

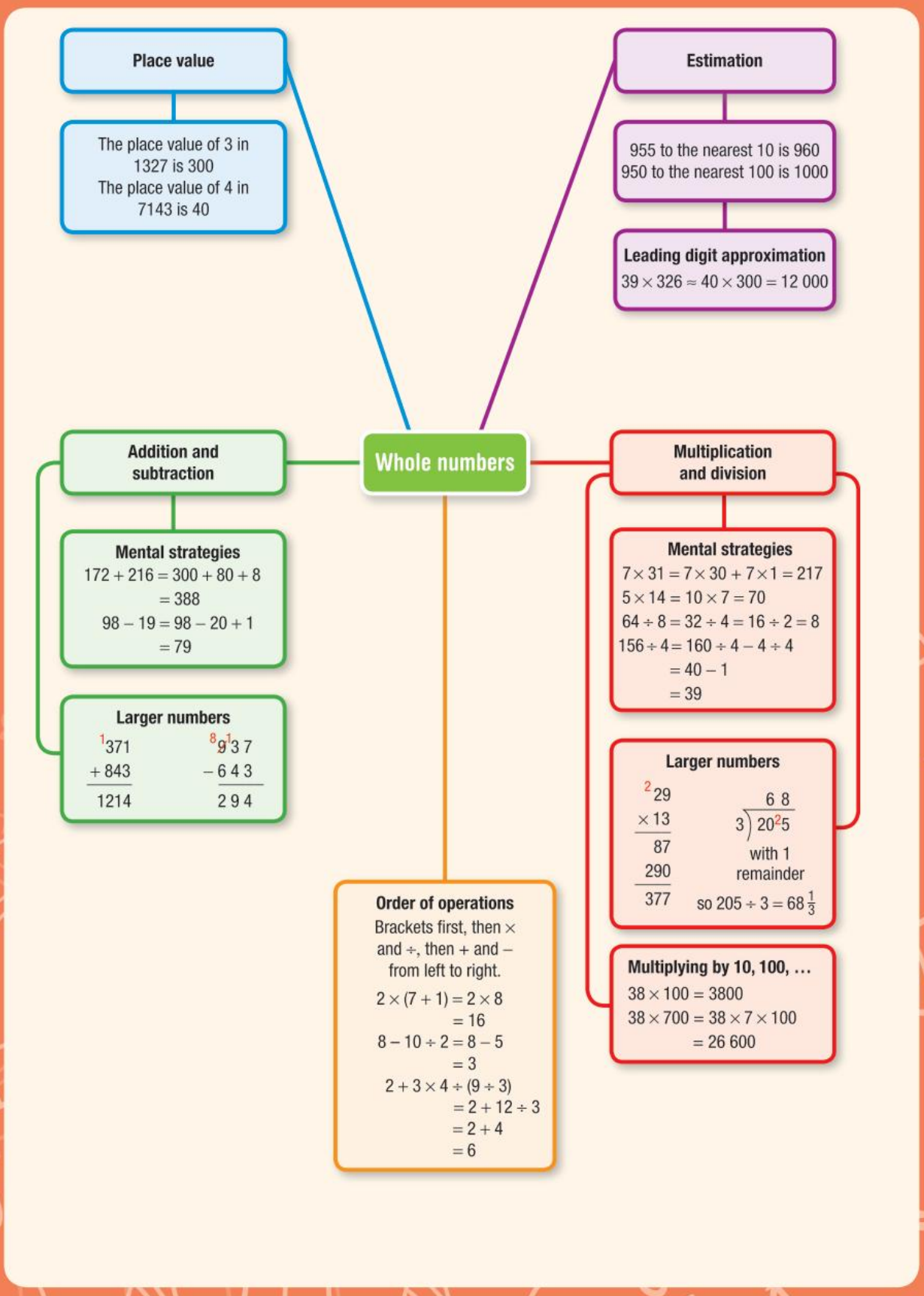
a
$$\begin{array}{r} \square 1\square \\ \times \quad 7 \\ \hline \square\,5\,1\square \end{array}$$

b
$$\begin{array}{r} 2\,9\,\square \\ \times \quad 3 \\ \hline 8\square\square \end{array}$$

7 You decide to start counting backwards by 7, starting at these numbers. If you stop just before passing zero, what is your final number?

a 205 **b** 22 314

Chapter summary



Chapter review

Multiple-choice questions

1A **1** Which of the following is *not* true?
A $2 < 3$ **B** $12 \leq 9$ **C** $15 > 2$ **D** $13 \geq 13$ **E** $7 \neq 8$

1A **2** The place value of 7 in 2713 is:
A 7 **B** 70 **C** 700 **D** 7000 **E** 100

1B/D/F **3** Which of the following is *not* true?
A $2 + 3 = 3 + 2$ **B** $2 \times 3 = 3 \times 2$ **C** $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
D $5 \div 2 \neq 2 \div 5$ **E** $7 - 2 = 2 - 7$

1C **4** The sum of 198 and 103 is:
A 301 **B** 304 **C** 299 **D** 199 **E** 95

1C **5** The difference between 126 and 29 is:
A 102 **B** 97 **C** 103 **D** 98 **E** 99

1D/E **6** The product of 7 and 21 is:
A 147 **B** 141 **C** 21 **D** 140 **E** 207

1F **7** The missing digit in this division $3\overline{)\square 3\ 7}$ $4^{1}1^{2}1$ is:
A 2 **B** 0 **C** 4 **D** 1 **E** 3

1F **8** The remainder when 317 is divided by 9 is:
A 7 **B** 5 **C** 2 **D** 1 **E** 0

1G **9** 458 rounded to the nearest 100 is:
A 400 **B** 500 **C** 460 **D** 450 **E** 1000

1H **10** The answer to $4 \times 3 - 26 \div 13$ is:
A 10 **B** 25 **C** 6 **D** 12 **E** 14

Short-answer questions

1A **1** Arrange these numbers from smallest to largest.
a 317, 713, 731, 371, 173, 137
b 1001, 1010, 199, 999, 1000, 1900, 1090

1A **2** Write down the place value of the digit 5 in these numbers.
a 357 **b** 5249 **c** 356 612

1B **3** Use a mental strategy to find these sums and differences.
a $124 + 335$ **b** $687 - 324$ **c** $59 + 36$ **d** $256 - 39$

1C **4** Find these sums and differences.

a
$$\begin{array}{r} 76 \\ +\,52 \\ \hline \end{array}$$

b
$$\begin{array}{r} 137 \\ +\,218 \\ \hline \end{array}$$

c
$$\begin{array}{r} 329 \\ -\,138 \\ \hline \end{array}$$

d
$$\begin{array}{r} 926 \\ -\,187 \\ \hline \end{array}$$

Chapter review

1D **5** Use a mental strategy to work out the following.

a 5×19 **b** 22×6 **c** 5×44

d $123 \div 3$ **e** $264 \div 8$ **f** $96 \div 4$

g 29×1000 **h** 36×300 **i** $14\,678 \div 1$

1E/F **6** Show your working to find each answer.

a $\begin{array}{r} 39 \\ \times\ 4 \\ \hline \end{array}$ **b** $\begin{array}{r} 21 \\ \times 40 \\ \hline \end{array}$

c $\begin{array}{r} 157 \\ \times\ 9 \\ \hline \end{array}$ **d** $\begin{array}{r} 27 \\ \times 13 \\ \hline \end{array}$

e $3\overline{)135}$ **f** $9\overline{)912}$

g $7\overline{)327}$ **h** $4\overline{)30\,162}$

1E/F **7** Find the missing digits in these problems.

a $\begin{array}{r} 2\square 3 \\ +\,7\ 3\ \square \\ \hline 9\ 6\ 1 \end{array}$

b $\begin{array}{r} \square\ 2\ \square \\ -\ 4\ \square\ 3 \\ \hline 2\ 5\ 6 \end{array}$

c $\begin{array}{r} \square\ 1 \\ \times\ 7 \\ \hline 2\ 8\ \square \end{array}$

d $2\overline{)\square 11\square}$ with no remainder

1G **8** Round these numbers as indicated.

a 72 (nearest 10)

b 3268 (nearest 100)

c 951 (nearest 100)

1G **9** Use *leading digit approximation* to estimate the answers to these problems.

a $289 + 532$

b 22×19

c 452×11

d $99 \div 11$

1H **10** Use *order of operations* to find the answers to these problems.

a $3 \times (2 + 6)$ **b** $6 - 8 \div 4$ **c** $(7 - 4) \div 3$

d $20 \div 10 + 9 \times 10$ **e** $2 \times 8 - 12 \div 6$ **f** $40 \div (5 + 3) - 2$

g $(5 + 2) \times 3 - (8 - 7)$ **h** $0 \times (988\,234 \div 3)$ **i** $1 \times (3 + 2 \times 5)$

Extended-response questions

1 A city tower construction uses 450 tonnes of concrete. The concrete is trucked from a factory that is 2 kilometres from the construction site. Each concrete mixer can carry 5 tonnes of concrete. The concrete costs $350 per truck load for the first 10 loads and $300 per load after that.



a How many loads of concrete are needed?
b Find the total distance travelled by the concrete mixers to deliver all loads. They need to return to the factory after each load.
c Find the total cost of concrete needed for the tower construction.
d If the price of concrete is always $350 regardless of the number of loads, how much more would it cost for the concrete?

2 One night Ricky and her brother Micky decide to have some fun at their father's sweet shop. In the shop they collected 3 tins of 25 jelly beans, 4 packets of 32 choc buds, 5 boxes of 10 smarties and 12 packets of 5 liquorice sticks.
a Find the total number of sweets.
b Find the difference between the number of choc buds and the number of smarties.
c Ricky and Micky decide to divide each type of sweet into groups of 7 and then eat any remainder. Which type of sweet will they eat the most of and how many?



Chapter 2

Geometry

What you will learn

2A Points, lines and angles (Consolidating)
2B Measuring angles (Consolidating)
2C Angles at a point
2D Transversal lines and parallel lines
2E Circles and constructions ⭐
2F Dynamic geometry ⭐

Australian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal **(ACMMG163)**

Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning **(ACMMG164)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Rock geometry

Unusual geometric rock formations are the result of the even cooling of molten rock from a volcano. This happens very slowly. The cooling of molten rock causes the rock to shrink and crack in repeated polygon shapes. Regular hexagons (i.e. 6 equal sides) are a common pattern forming long columns of rock, which are hexagonal prisms.

The volcanic formation known as Sawn Rocks near Narrabri, NSW, was created over 21 million years ago by the Nandewar volcano. Sawn Rocks has huge pentagonal prisms (i.e. 5 sided), up to 40 metres high, looking like very large 'organ pipes'.

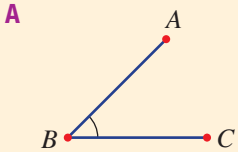
The Giant's Causeway (pictured) in Northern Ireland is another famous example, with most rock columns being pentagonal or hexagonal. Polygons in such rock formations can have as few as 3 sides or as many as 7 sides.

Pre-test

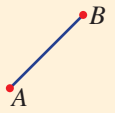
1 Match the object (**A** to **F**) to the given description (**a** to **f**).

a a point
b a circle
c an angle ABC
d a line
e a segment AB (or interval AB)
f an angle DEF

A



B



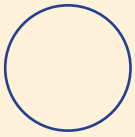
C



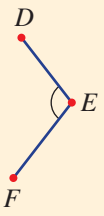
D



E

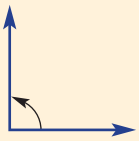


F

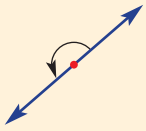


2 State or estimate the size of these angles. Remember there are 360° in a full circle.

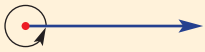
a



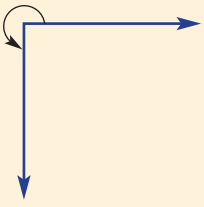
b



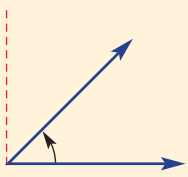
c



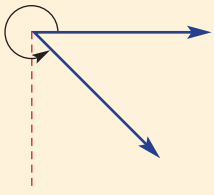
d



e

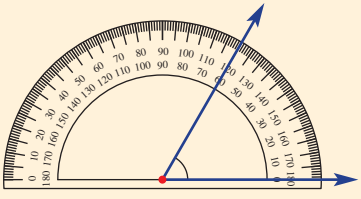


f

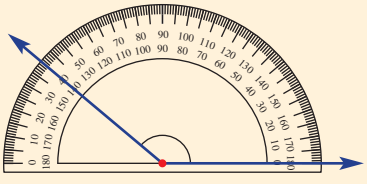


3 What angle measurements are shown on these protractors?

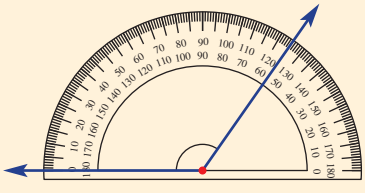
a



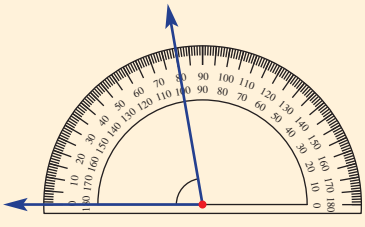
b



c



d



4 Decide if the following pairs of angles add to 180°.

a 75°, 125° **b** 132°, 48° **c** 19°, 151°

5 Find the missing value in these statements.

a ___ + 10° = 90° **b** ___ + 30° = 180° **c** ___ + 210° = 360°

2A Points, lines and angles

CONSOLIDATING

The basic building blocks of geometry are the point, line and plane. They are the objects used to construct angles, triangles and other more complex shapes and objects.

Let's start: Can you draw this?

Before you start learning about the geometrical objects in this topic, see if you can remember what each of the following represent. Draw an example of each of these objects.

- line segment AB
- point P
- angle $\angle FGH$
- line AB
- angle $\angle ABC$
- ray XY



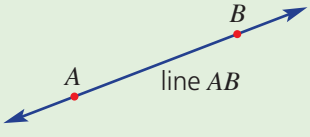
Lines and rays don't take up any area, but they still exist in nature.

Key ideas

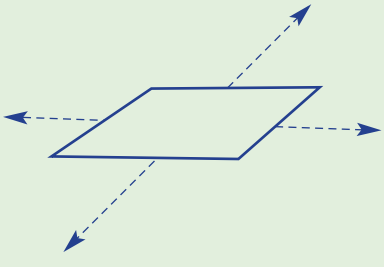
- A **point** is usually labelled with a capital letter.



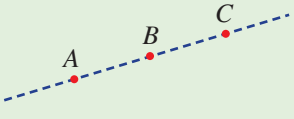
- A **line** can be named using two points.



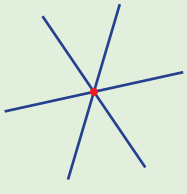
- A **plane** is a flat surface and extends indefinitely.



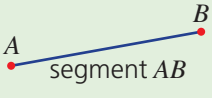
- Points that all lie on a single line are called **collinear**.



- More than two lines that meet at the same intersection point are called **concurrent**.



- A line **segment** (or interval) is part of a line with a fixed length and end points.



Point A position in space, marked with a dot and named with a capital letter

Line A set of points forming a straight path that extends forever in opposite directions

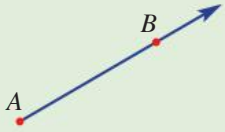
Plane A flat surface that extends forever in all directions

Collinear Lying on the same straight line

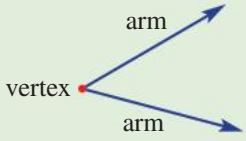
Concurrent Passing through the same point

Segment A section of a straight line (also called an interval)

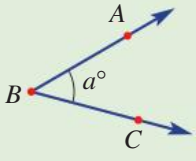
- A **ray** AB is part of a line, with an end point A and passing through point B.



- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called the arms of the angle.



- An angle is named using three points, with the vertex as the middle point. The angle in question is usually marked with an arc. The size of this angle is $a°$.



$\angle ABC$ or $\angle CBA$

Ray Part of a straight line, extending forever in one direction

Vertex A point where two straight lines meet to make an angle

Exercise 2A

UNDERSTANDING 1–4 4

1 Which one of the following objects (**A** to **F**) is:

a a ray? **b** a point? **c** a line segment?
d a line? **e** an angle? **f** a plane?

Look back at the **Key ideas**.

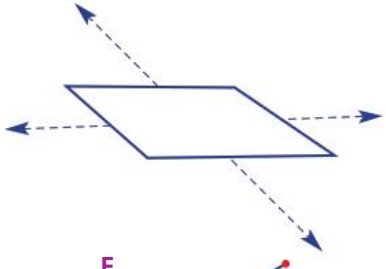


A

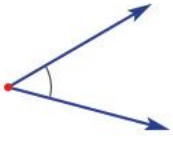
B



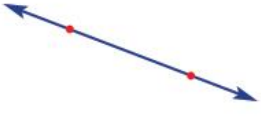
C



D



E



F



2 Draw the following objects.

a a point P **b** a line AN **c** an angle $\angle ABC$
d a ray ST **e** a plane **f** a line segment ST

3 Match the words *line*, *segment* or *ray* to the correct description.

a Starts from a point and extends forever in one direction.
b Extends forever in both directions, passing through two points.
c Starts and ends at two points.

4 Match the words *point, line* or *plane* with the following descriptions.

- **a** the edge of a sheet of paper
- **b** a flat wall
- **c** the surface of a pool of water on a calm day
- **d** where two walls and a floor meet in a room
- **e** where two walls meet in a room
- **f** one side of a cereal packet
- **g** where two sides meet on a box
- **h** where three sides meet on a box



FLUENCY 5–8 5–9

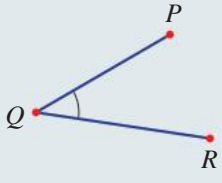
Example 1 Naming objects

Name this line segment and angle, using the given letters.

a



b



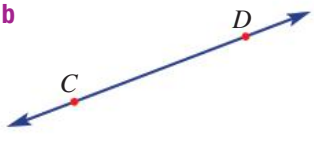
Solution	Explanation
a segment AB	Segment BA, interval AB or interval BA are also acceptable.
b $\angle PQR$ or $\angle RQP$	Point Q is the vertex and sits in between P and R. Either order (PQR or RQP) is correct.

5 Name the following objects, using the given letters.

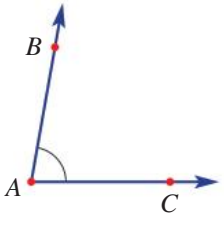
a



b



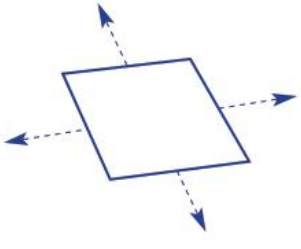
c



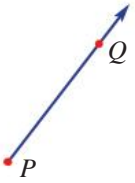


For parts **b**, **c** and **f**, you can reverse the letters and still be correct.

d



e



f



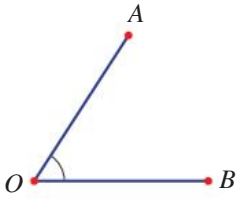
2A

6 Name the angle marked with the arc in these diagrams.

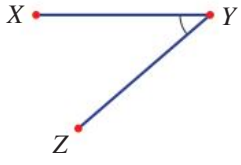


Name the angle which is marked by a small arc: ∠
For example $\angle ABC$.

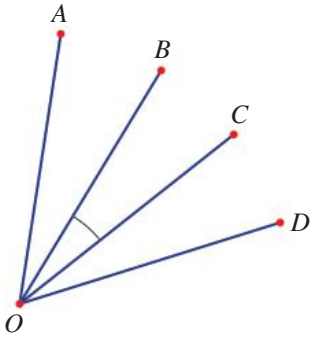
a



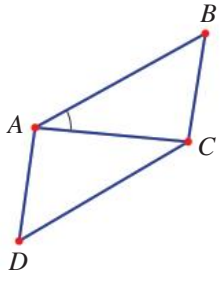
b



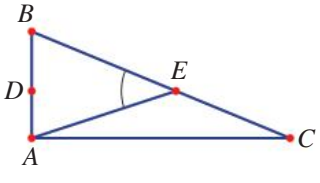
c



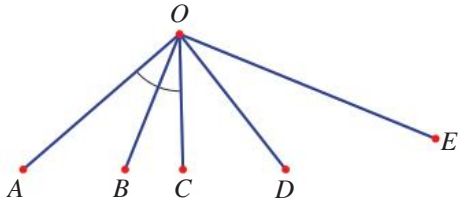
d



e



f



7 Write the missing word.

a Points that sit in a straight line are called _____.

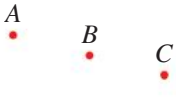
b Lines that meet at the same point are called _____.

8 Do the following sets of points look collinear?

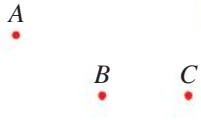


Collinear points lie in a straight line.

a

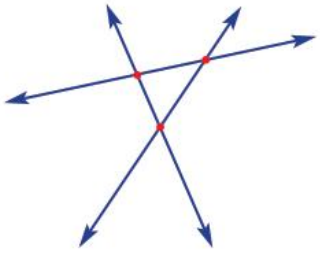


b

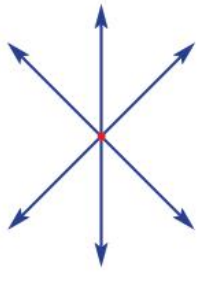


9 Are the following sets of lines concurrent? Explain why or why not.

a



b



PROBLEM-SOLVING AND REASONING 10, 11 10–12

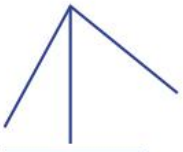
10 Count the number of angles formed inside these diagrams. Count all angles, including those that may be the same size. You should also count those that are divided by another segment.



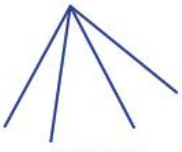
There are three angles in a corner like this:



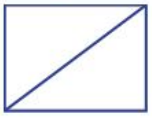
a



b



c



d



11 How many line segments are there on this line? Do not count AB and BA as separate segments since they are the same segment with two different names.



12 A line contains a certain number of labelled points. For example, this line has three points.



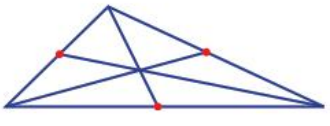
a Complete this table by counting the total number of segments for the given number of labelled points.

Number of points	1	2	3	4	5	6
Number of segments						

b Explain any patterns you see in the table. Is there a quick way of finding the next number in the table?

Are they concurrent? — 13

13 The lines joining each vertex (corner) of a triangle with the midpoint (middle point) of the opposite side are drawn here.



a Draw any triangle and use a ruler to measure and mark the midpoints of each side.

b Join each vertex with the midpoint of the opposite side.

c Are your segments from part **b** concurrent?

d Do you think your answer to part **c** will always be true for any triangle? Try one other triangle of a different size to check.



2B Measuring angles

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Angles can be measured using a simple piece of equipment called a protractor. Some protractors can measure from 0° to 360° (a full turn) and others measure from 0° to 180° (a half turn). The unit for angles is the degree and the symbol used is °.



On this Ferris wheel, what angle is between each spoke and the next?

Let's start: Estimating angles

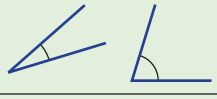
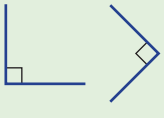
How good are you at estimating the size of angles? Estimate the size of these angles and then check with a protractor.

If your protractor only goes to 180°, discuss how you might measure the second angle.



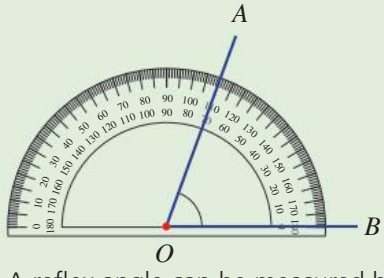
Key ideas

- Angles are classified according to their size.

Angle type	Size	Examples
acute	between 0° and 90°	
right	90°	
obtuse	between 90° and 180°	
straight	180°	
reflex	between 180° and 360°	
revolution	360°	

- A **protractor** can be used to measure angles to about half a degree.
  To use a protractor:
  - Place the centre of the protractor on the vertex of the angle.
  - Align the base line of the protractor along one arm of the angle.
  - Measure the angle using the other arm and the scale on the protractor.

Protractor A semicircular or circular tool for measuring or drawing angles



- A reflex angle can be measured by subtracting a measured angle from 360°.

Exercise 2B

UNDERSTANDING 1–3 3

Example 2 Classifying angles

Classify the following angles as acute, obtuse, straight or reflex.

a 47° **b** 172° **c** 180° **d** 282°

Solution	Explanation
a acute	Angles between 0° and 90° are acute.
b obtuse	Obtuse angles are between 90° and 180°.
c straight	A straight angle is 180°.
d reflex	A reflex angle is between 180° and 360°.

1 Classify the following angles as acute, right, obtuse, straight, reflex or revolution.

a 31° **b** 127° **c** 90°
d 180° **e** 360° **f** 83°
g 291° **h** 320° **i** 93°



acute: between 0° and 90°
right: 90°
obtuse: between 90° and 180°
straight: 180°
reflex: between 180° and 360°
revolution: 360°

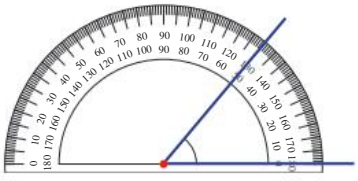
2 Without using a protractor, draw an example of the following types of angles.

a acute **b** right **c** obtuse
d straight **e** reflex **f** revolution

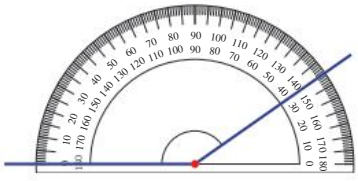
2B

3 What is the size of the angle measured with these protractors?

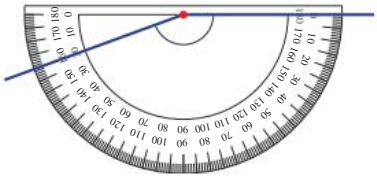
a



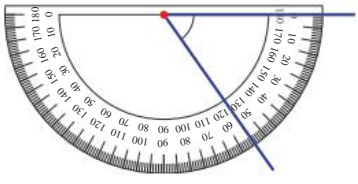
b



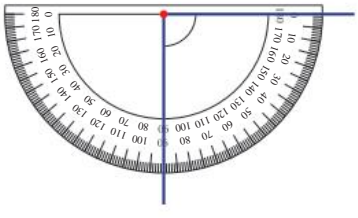
c



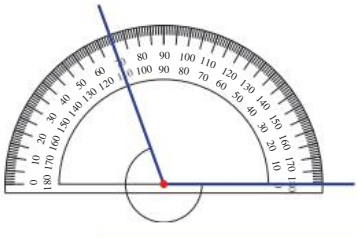
d



e



f





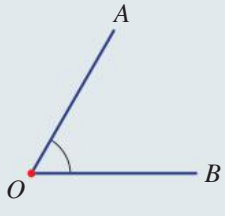
First check the position of the small arc that shows where the angle is.

FLUENCY 4–6(½) 4–6(½)

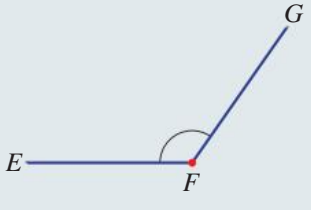
Example 3 Measuring with a protractor

Measure the size of each angle.

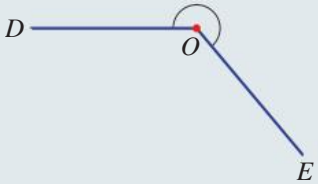
a



b



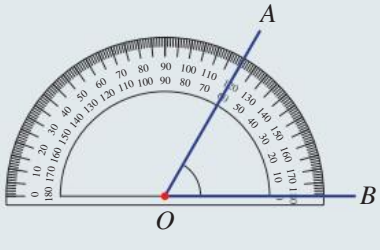
c



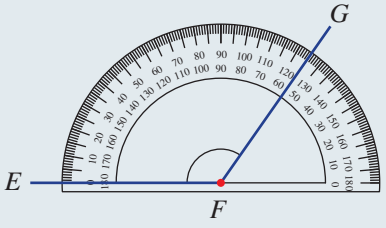
Solution

a $\angle AOB = 60°$

Explanation



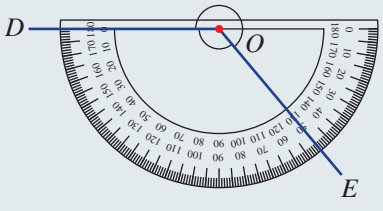
b $\angle EFG = 125°$



c obtuse $\angle DOE = 130°$

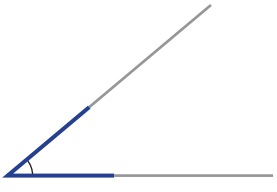
reflex $\angle DOE = 360° - 130°$

$= 230°$

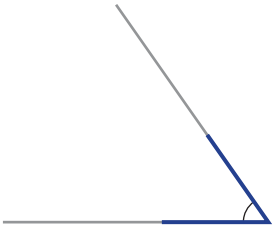


4 Use a protractor to measure the size of each angle.

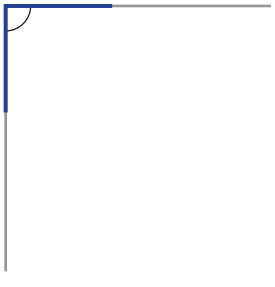
a



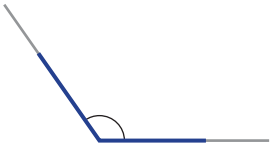
b



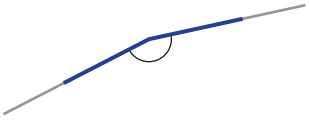
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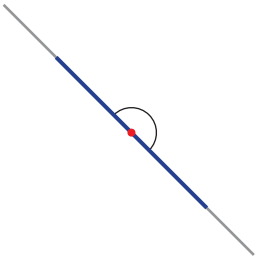
d



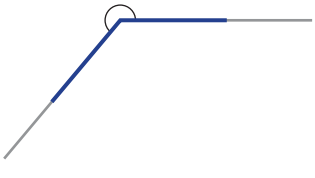
e



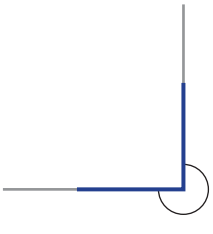
f



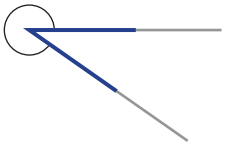
g



h



i



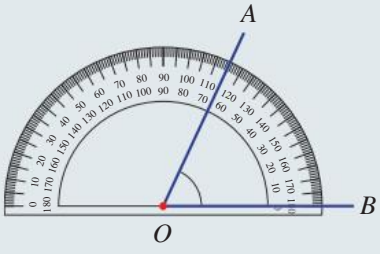
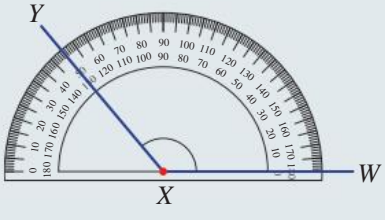
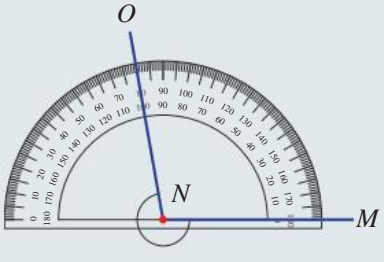
5 Classify each of the angles in question **4** as acute, right, obtuse, straight, reflex or revolution.

2B

Example 4 Drawing angles

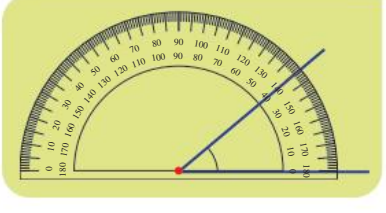
Use a protractor to draw each of the following angles.

a $\angle AOB = 65°$ **b** $\angle WXY = 130°$ **c** $\angle MNO = 260°$

Solution	Explanation
a 	Step 1: Draw a base line OB. Step 2: Align the protractor along the base line with the centre at point O. Step 3: Measure $65°$ and mark a point, A. Step 4: Draw the arm OA.
b 	Step 1: Draw a base line XW. Step 2: Align the protractor along the base line with the centre at point X. Step 3: Measure $130°$ and mark a point, Y. Step 4: Draw the arm XY.
c 	Step 1: Draw an angle of $360° - 260° = 100°$. Step 2: Mark the reflex angle on the opposite side to the obtuse angle of $100°$. Alternatively, draw a $180°$ angle and measure an $80°$ angle to add to the $180°$ angle.

6 Use a protractor to draw each of the following angles.

a $40°$ **b** $75°$
c $90°$ **d** $135°$
e $175°$ **f** $205°$
g $260°$ **h** $270°$
i $295°$ **j** $352°$

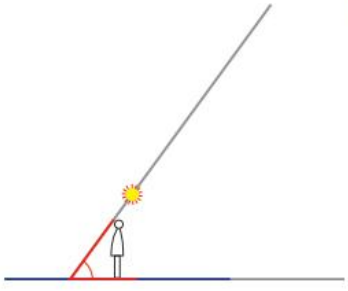




PROBLEM-SOLVING AND REASONING 7, 8 7–10

7 Use a protractor to measure:

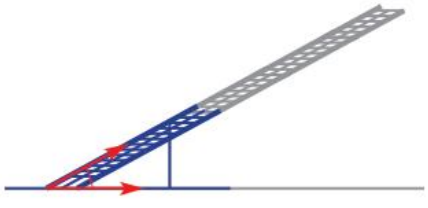
a the angle the sun's rays make with the ground



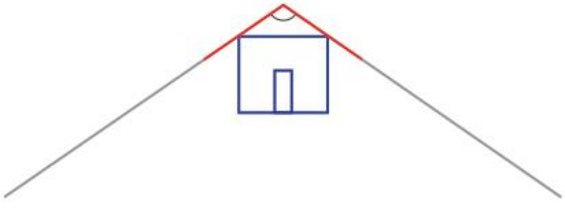


Place the centre of the protractor at the vertex of the marked angle.

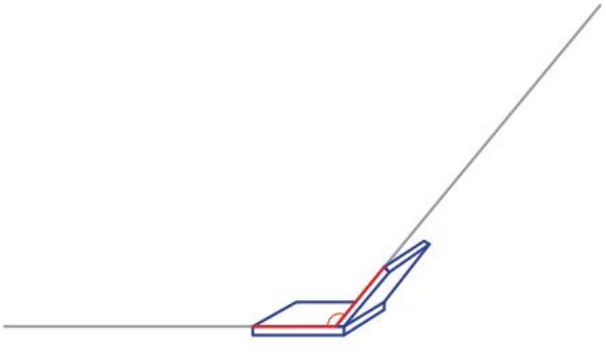
b the angle that this ramp makes with the ground



c the angle of this roof



d the angle between this laptop screen and the keyboard



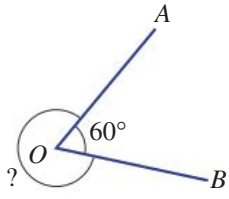
2B

8 How many right angles (i.e. angles of 90°) make up:
a a straight angle? **b** 270°? **c** a revolution?

9 A clock face is numbered 1 to 12. Find the angle the minute hand turns in:
a 30 minutes **b** 1 hour **c** 15 minutes **d** 45 minutes
e 5 minutes **f** 20 minutes **g** 55 minutes **h** 1 minute


There are 360° in a full circle.

10 An acute angle $\angle AOB$ is equal to 60°. Do you need to use a protractor to work out the size of the reflex angle $\angle AOB$? Why not?



Hour hand and minute hand angles

— 11, 12

11 A clock face is numbered 1 to 12. Find the angle between the hour hand and the minute hand at:
a 6 p.m. **b** 3 p.m.
c 4 p.m. **d** 11 a.m.


Draw a clock face for each time. Each 5-minute turn of the minute hand is 360° ÷ 12 = 30°.

12 Find the angle between the hour hand and the minute hand of a clock at these times.
a 10:10 a.m.
b 4:45 a.m.
c 11:10 p.m.
d 2:25 a.m.


Think carefully! The answer to part **a** is not 120°.



2C Angles at a point

Interactive

Widgets

HOTsheets

Walkthrough

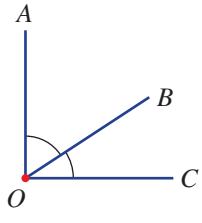
When lines, segments or rays meet at a point, special pairs or groups of angles are formed.

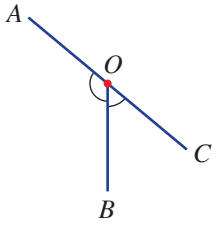
We name these special groups of angles depending on their geometric relationships. Opposite angles formed by intersecting steel beams, for example, are equal and are called vertically opposite angles.

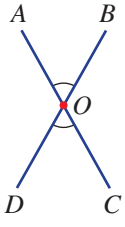


Let's start: Special pairs of angles

By making a drawing or using computer geometry, construct the diagrams below. Measure the two marked angles. What do you notice about the two marked angles?

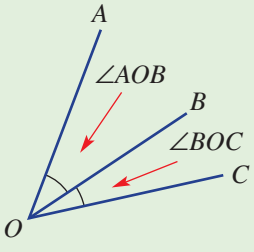




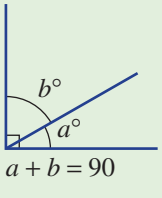


Key ideas

- **Adjacent** angles are side by side and share a vertex and an arm.

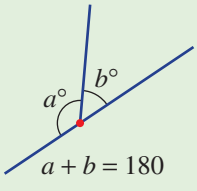


- Angles in a right angle are **complementary**.
  - They sum to 90°
  - $a°$ is the complement of $b°$



$a + b = 90$

- Angles on a straight line are **supplementary**.
  - They add to 180°
  - $a°$ is the supplement of $b°$



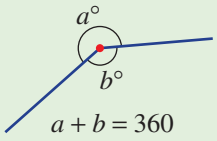
$a + b = 180$

Adjacent Next to each other

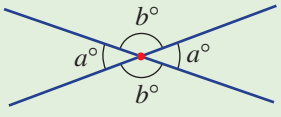
Complementary Having a sum of 90°

Supplementary Having a sum of 180°

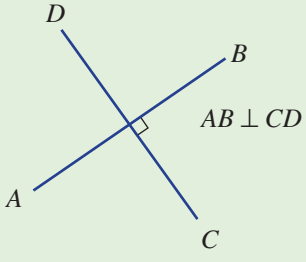
- Angles in a **revolution** sum to 360°.



- **Vertically opposite** angles are formed when two lines intersect. The opposite angles are equal.



- **Perpendicular** lines meet at right angles (90°).



Revolution A full turn or circle (360°)

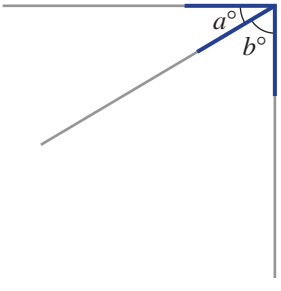
Vertically opposite Opposite each other across a common vertex

Perpendicular At right angles

Exercise 2C

UNDERSTANDING 1–6 6

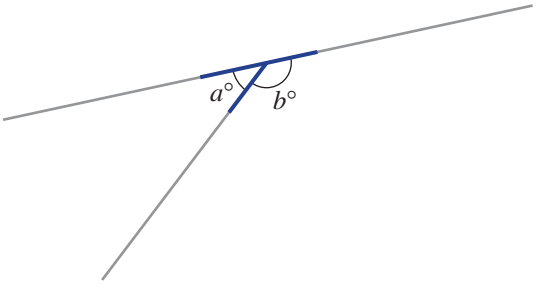
1 **a** Measure the angles to find the values of a and b in this diagram.



b Calculate $a + b$. Is your answer 90? If not, check your measurements.

c Write the missing word: $a°$ and $b°$ are _____ angles.

2 **a** Measure the angles to find the values of a and b in this diagram.



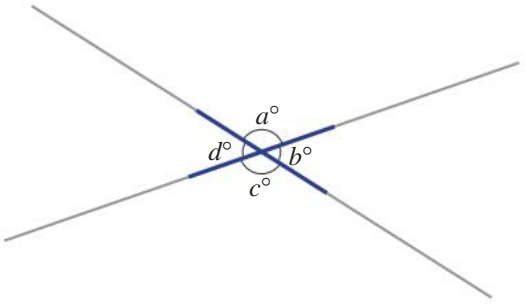
b Calculate $a + b$. Is your answer 180? If not, check your measurements.

c Write the missing word: $a°$ and $b°$ are _____ angles.

3 **a** Measure the angles to find the values of a, b, c and d in this diagram.

b What do you notice about the sum of the four angles?

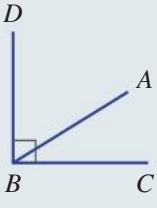
c Write the missing words: $b°$ and $d°$ are _____ angles.

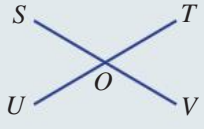


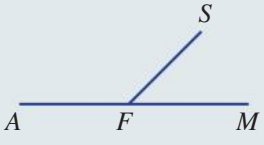
Example 5 Naming angles in relation to other angles

Name the angle which is:

a complementary to $\angle ABC$ **b** vertically opposite to $\angle SOT$ **c** supplementary to $\angle AFS$



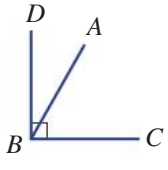




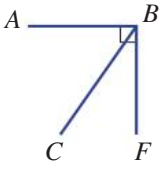
Solution	Explanation
a $\angle ABD$	$\angle ABD + \angle ABC = 90°$ so they are complementary.
b $\angle UOV$	$\angle SOT$ is vertically opposite to $\angle UOV$.
c $\angle SFM$	$\angle AFS + \angle SFM = 180°$ so they are supplementary.

4 Name the angle which is complementary to $\angle ABC$ in each of the diagrams.

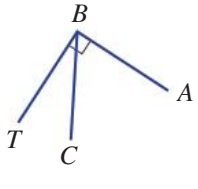
a



b



c



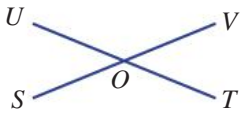


Choose the angle next to $\angle ABC$ that makes up $90°$.

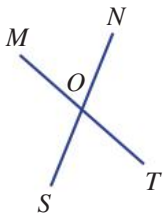
2C

5 Name the angle which is vertically opposite to $\angle SOT$ in each of the diagrams.

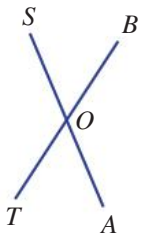
a



b



c





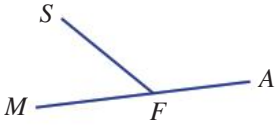
Choose the angle opposite to $\angle SOT$.

6 Name the angle which is supplementary to $\angle AFS$ in each of the diagrams.

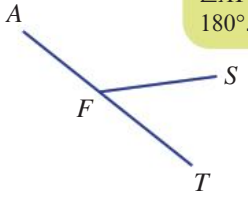


Choose the angle next to $\angle AFS$ that makes up 180°.

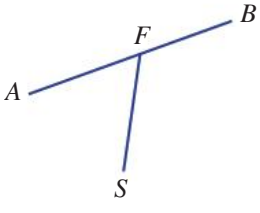
a



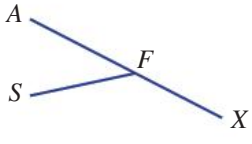
b



c



d

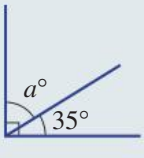


FLUENCY 7(½) 7–8(½)

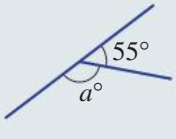
Example 6 Finding angles at a point

Without using a protractor, find the value of a in each diagram.

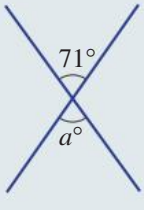
a



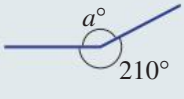
b



c



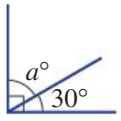
d



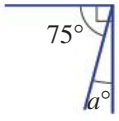
Solution	Explanation
a $a + 35 = 90$ $a = 55$	Angles in a right angle add to 90°. $90 - 35 = 55$
b $a + 55 = 180$ $a = 125$	Angles on a straight line add to 180°. $180 - 55 = 125$
c $a = 71$	Vertically opposite angles are equal.
d $a + 210 = 360$ $a = 150$	The sum of angles in a revolution is 360°. a is the difference between 210 and 360.

7 Without using a protractor, find the value of a in each diagram. (The diagrams shown may not be drawn to scale.)

a



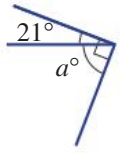
b



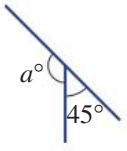


Complementary angles add to 90°. Supplementary angles add to 180°. Vertically opposite angles are equal. Angles in a revolution add to 360°.

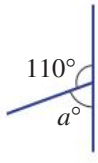
c



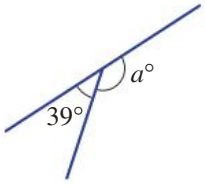
d



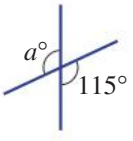
e



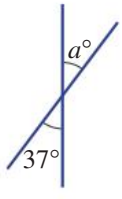
f



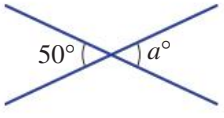
g



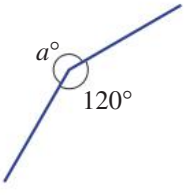
h



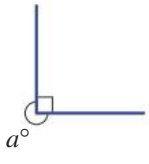
i



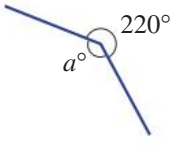
j



k



l

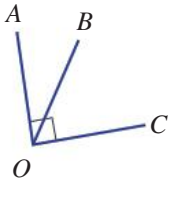


8 For each of the given pairs of angles, write C if they are complementary (add to 90°), S if they are supplementary (add to 180°) or N if they are neither.

a 21°, 79°
b 130°, 60°
c 98°, 82°
d 180°, 90°
e 17°, 73°
f 31°, 59°
g 68°, 22°
h 93°, 87°

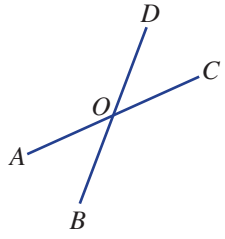
PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 a Name the angle that is complementary to $\angle AOB$ in this diagram.

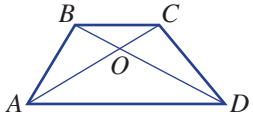


2C

b Name the two angles that are supplementary to $\angle AOB$ in this diagram.

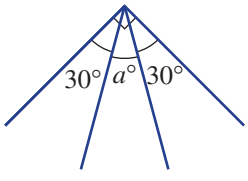


c Name the angle that is vertically opposite to $\angle AOB$ in this diagram.

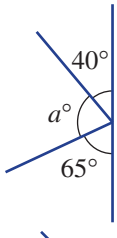


10 Without using a protractor, find the value of a in these diagrams.

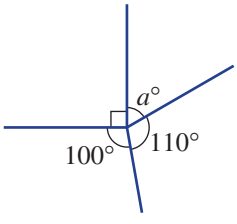
a



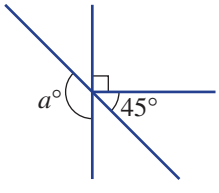
b



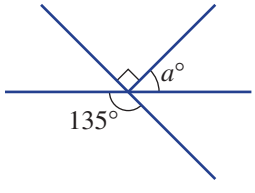
c



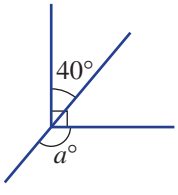
d



e

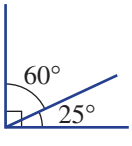


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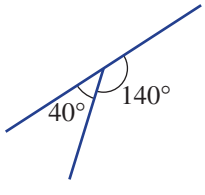


11 Do these diagrams have the correct information? Give reasons.

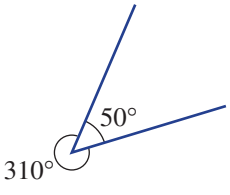
a



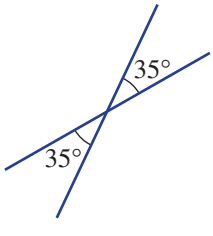
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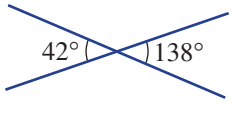
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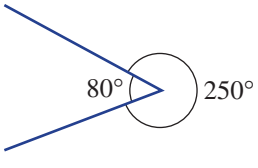
d



e



f



12 A pizza is divided between four people. Bella is to get twice as much as Dom. Dom gets twice as much as Rick. Rick gets twice as much as Marie. If the pizza is cut into triangular pieces, find the angle at the centre of the pizza for Marie's piece.

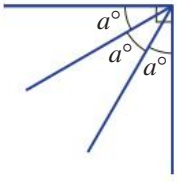


More than one $a°$

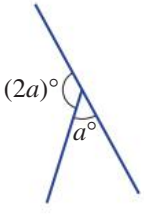
— 13

13 Find the value of a in these diagrams.

a



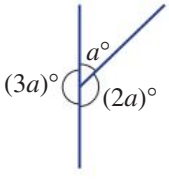
b



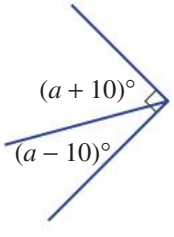


Use trial and error to make a start.

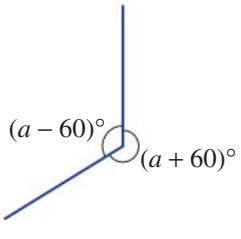
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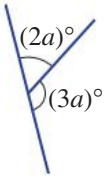
d



e



f



2D Transversal lines and parallel lines

Interactive

Widgets

HOTsheets

Walkthrough

When a line cuts two other lines, it is called a transversal and forms eight angles. If the two other lines are parallel, then special pairs of angles are formed.



Special pairs of angles are formed when lines cross over parallel lines.

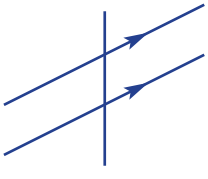
Let's start: What's formed by a transversal?

Draw a pair of parallel lines using either:

- two sides of a ruler; or
- computer geometry (parallel line tool).

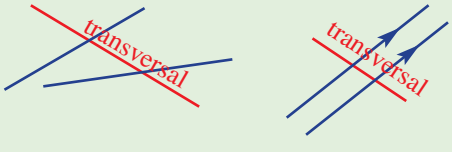
Then cross the two lines with a third line (transversal) at any angle.

Measure each of the eight angles formed and discuss what you find. If computer geometry is used, drag the transversal and see if your observations apply to all the cases that you observe.

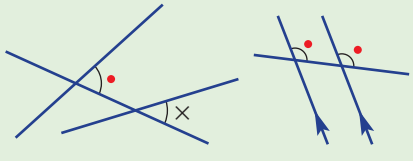


Key ideas

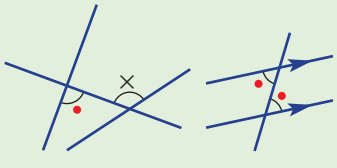
- A **transversal** is a line intersecting two or more other lines that are usually, but not necessarily, parallel.



- A transversal crossing two lines will form special pairs of angles. These are:
  - **Corresponding angles** (in corresponding positions)



  - **Alternate angles** (on opposite sides of the transversal and inside the other two lines)

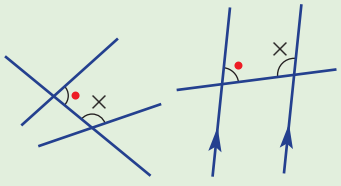


Transversal A line that cuts two or more lines

Corresponding angles Pairs of angles in the same position, formed by two lines cut by a transversal

Alternate angles Two angles that lie between two lines on either side of a transversal

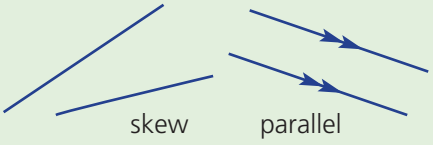
- **Cointerior angles** (on the same side of the transversal and inside the other two lines).



Cointerior angles A pair of angles lying between two lines on the same side of a transversal

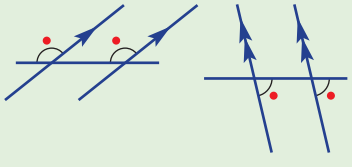
Parallel lines Lines in the same plane that are a fixed distance apart and never intersect

■ **Parallel lines** are marked with the same arrow type.
- Skew lines are not parallel.
- If AB is parallel to CD we write $AB \parallel CD$.

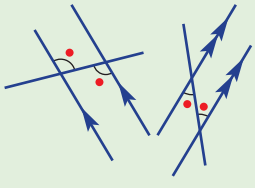


■ If a transversal crosses two parallel lines, then:
- corresponding angles are equal
- alternate angles are equal
- cointerior angles are supplementary (i.e. sum to $180°$).

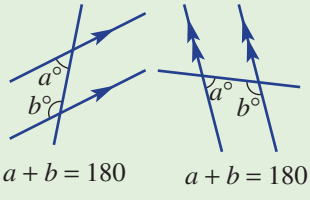
corresponding angles



alternate angles



cointerior angles

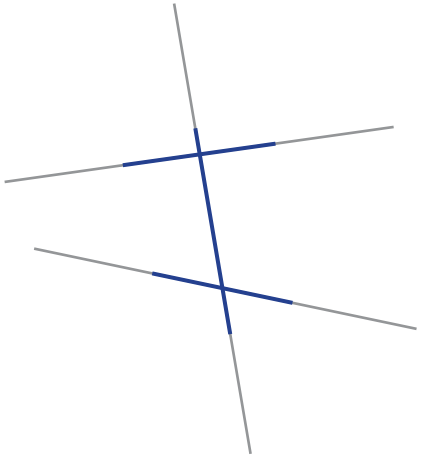


$a + b = 180$ $a + b = 180$

Exercise 2D

UNDERSTANDING 1–3 3

1 Look at each of the eight angles in this diagram.
- **a** How many *different-sized* angles do you see?
- **b** Do you think that the two lines cut by the transversal are parallel?

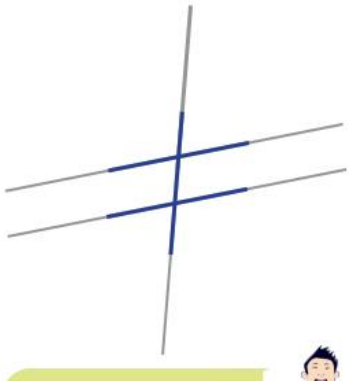


2D

2 Look at each of the eight angles in this diagram.

a How many *different-sized* angles do you see?

b Do you think that the two lines cut by the transversal are parallel?



3 Choose the word *equal* or *supplementary* to complete these sentences. If a transversal cuts two parallel lines, then:

a alternate angles are _____.

b cointerior angles are _____.

c corresponding angles are _____.

d vertically opposite angles are _____.



If angles add to $180°$ they are called supplementary.

FLUENCY 4, 5, 6(½) 4, 5, 6–7(½)

Example 7 Naming pairs of angles

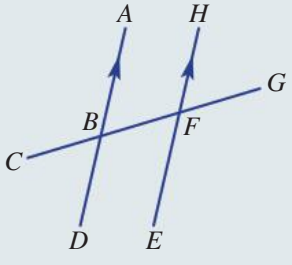
Name the angle that is:

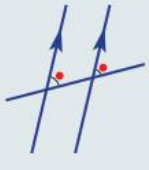
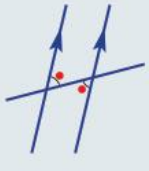
a corresponding to $\angle ABF$

b alternate to $\angle ABF$

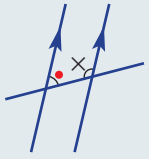
c cointerior to $\angle ABF$

d vertically opposite to $\angle ABF$

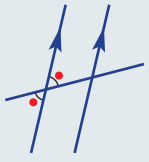


Solution	Explanation
a $\angle HFG$	
b $\angle EFB$	

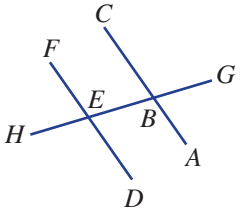
c $\angle HFB$



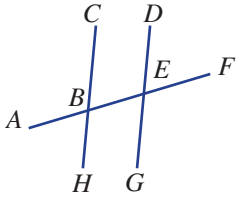
d $\angle CBD$



4 Name the angle that is:
- **a** corresponding to $\angle ABE$
- **b** alternate to $\angle ABE$
- **c** cointerior to $\angle ABE$
- **d** vertically opposite to $\angle ABE$



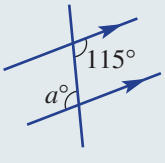
5 Name the angle that is:
- **a** corresponding to $\angle EBH$
- **b** alternate to $\angle EBH$
- **c** cointerior to $\angle EBH$
- **d** vertically opposite to $\angle EBH$



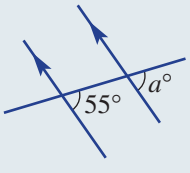
Example 8 Finding angles in parallel lines

Find the angle a in these diagrams and give a reason for each answer.

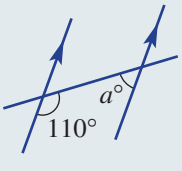
a



b



c

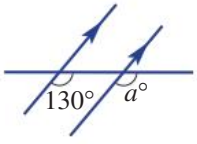


Solution	Explanation
a $a = 115$ alternate angles in parallel lines	Alternate angles in parallel lines are equal.
b $a = 55$ corresponding angles in parallel lines	Corresponding angles in parallel lines are equal.
c $a = 180 - 110$ $= 70$ cointerior angles in parallel lines	Cointerior angles in parallel lines sum to 180°.

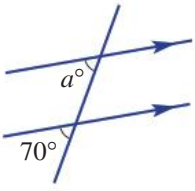
2D

6 Find the value of a in these diagrams, giving a reason.

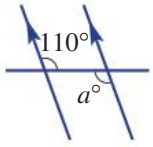
a



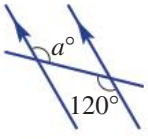
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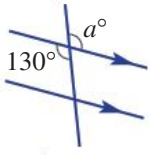
c



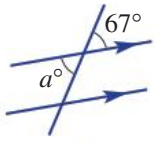
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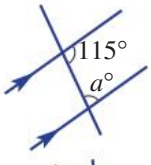
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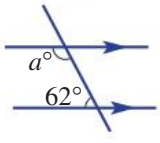
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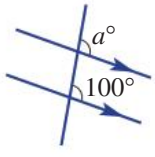
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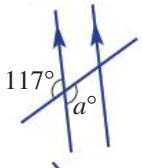
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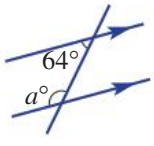
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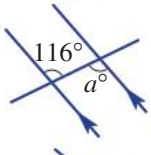
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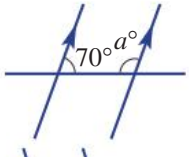
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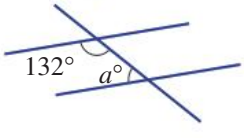
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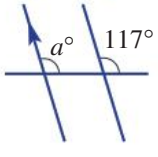
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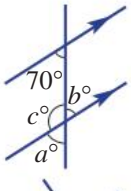


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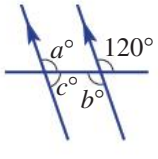


7 Find the value of each unknown pronumeral in the following diagrams.

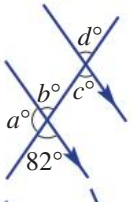
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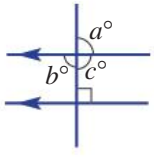
b



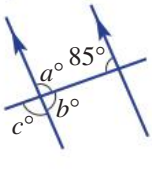
c



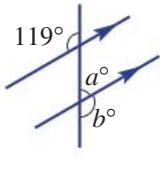
d



e

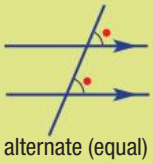


f

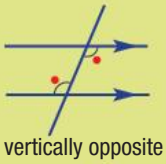




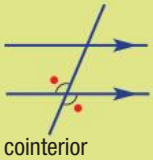
corresponding (equal)



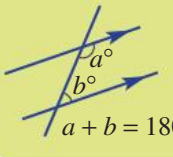
alternate (equal)



vertically opposite (equal)



cointerior (supplementary)



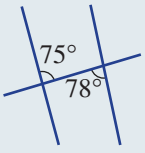
$a + b = 180$

PROBLEM-SOLVING AND REASONING 8, 9 8–11

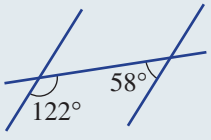
Example 9 Proving lines are parallel

Giving reasons, state whether the two lines cut by the transversal are parallel.

a



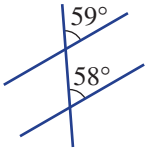
b



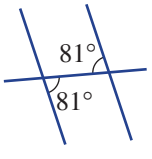
Solution	Explanation
a not parallel Alternate angles are not equal.	Parallel lines have equal alternate angles.
b parallel The cointerior angles sum to 180°.	122° + 58° = 180° Cointerior angles inside parallel lines are supplementary (i.e. sum to 180°).

8 Giving reasons, state whether the two lines cut by the transversal are parallel.

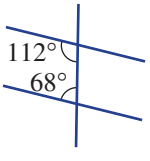
a



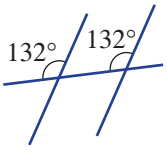
b



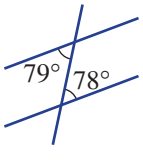
c



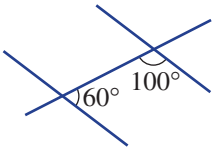
d



e

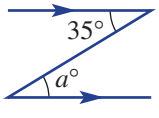


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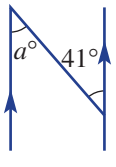


9 Find the value of a in these diagrams.

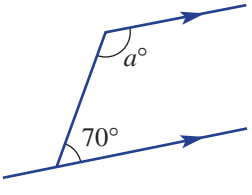
a



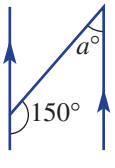
b



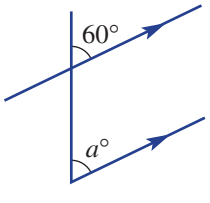
c



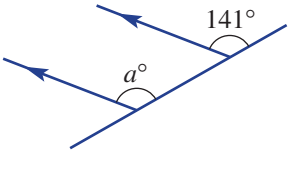
d



e



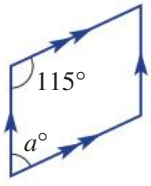
f



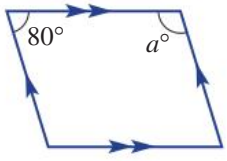
2D

10 Find the value of a in these diagrams.

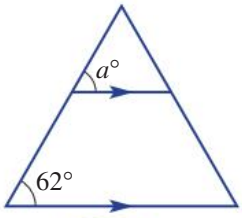
a



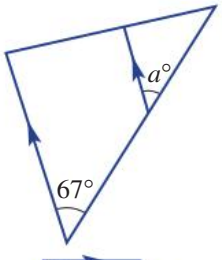
b



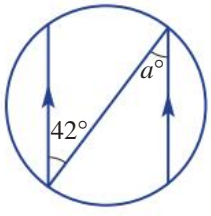
c



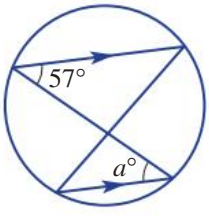
d



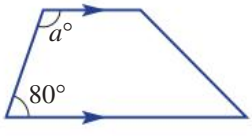
e



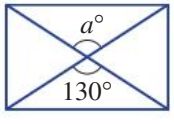
f



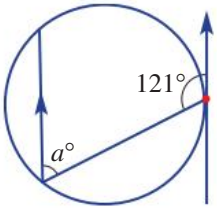
g



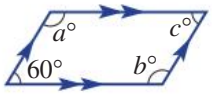
h



i



11 This shape is a parallelogram with two pairs of parallel sides.

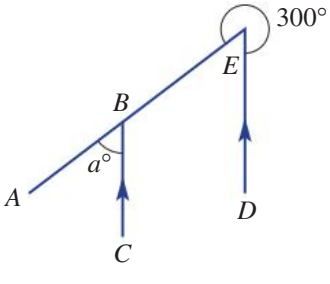


- **a** Use the 60° angle to find the value of a and b.
- **b** Find the value of c.
- **c** What do you notice about the angles inside a parallelogram?

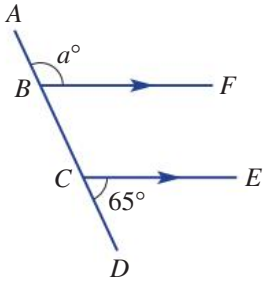
Parallel challenge — 12

12 Find the angle a in these diagrams.

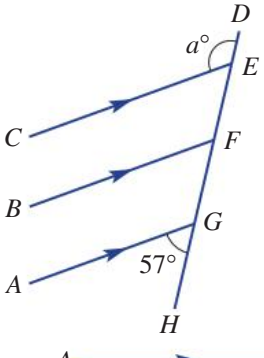
a



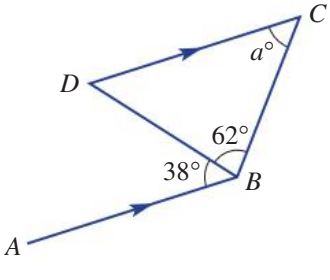
b



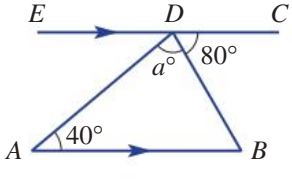
c



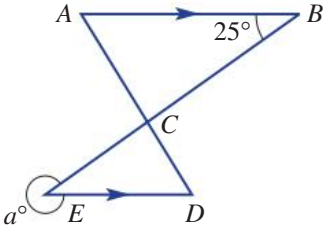
d



e



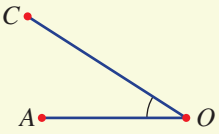
f



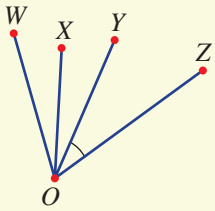
Progress quiz

2A **1** Name the angle marked in these diagrams.

a

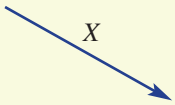


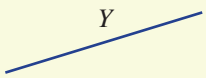
b

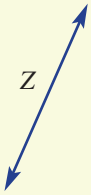


2A **2** Which of the following objects is a:

a line? **b** line segment? **c** ray?







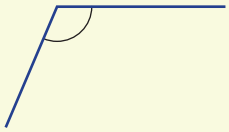
2B **3** Classify the following angles as acute, right, obtuse, straight, reflex or revolution.

a 53° **b** 317° **c** 180° **d** 11°

e 179° **f** 211° **g** 90° **h** 105°

2B **4** The size of the angle shown is:

A 110° **B** 170° **C** 190° **D** 250°

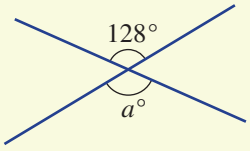


2C **5** For each of the given pairs of angles, write C if they are complementary, S if they are supplementary or N if they are neither.

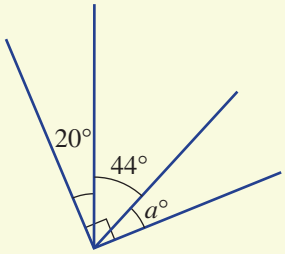
a 32°, 68° **b** 171°, 9° **c** 90°, 90° **d** 40°, 50°

2C **6** Without using a protractor, find the size of each angle marked with the letter a. (The diagrams shown may not be drawn to scale.)

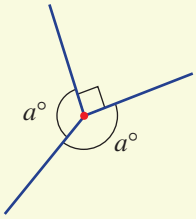
a



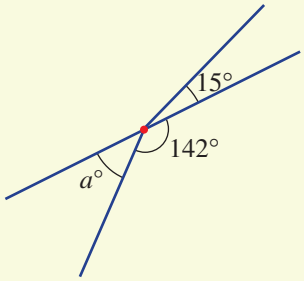
b



c



d



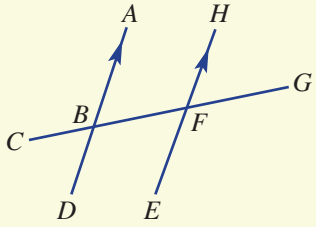
2D **7** Name the angle that is:

a corresponding to $\angle HFG$

b vertically opposite $\angle EFG$

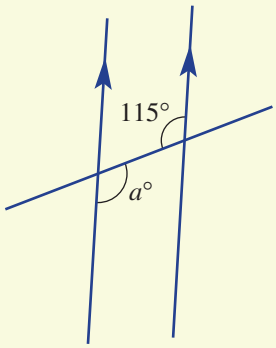
c cointerior to $\angle DBG$

d alternate to $\angle EFC$

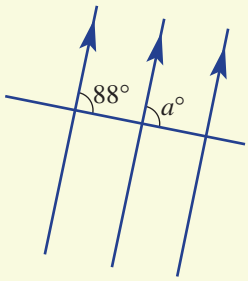


2D **8** Find the value of a in these diagrams, giving a reason (alternate, vertically opposite, cointerior or corresponding).

a

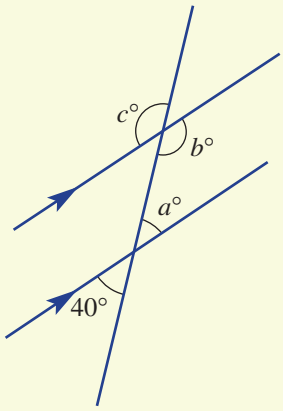


b

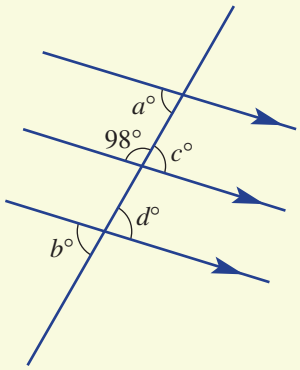


2D **9** Find the value of each unknown pronumeral in the following diagrams.

a



b



2E Circles and constructions

The standard method for constructing accurate geometrical diagrams is with a ruler and a pair of compasses or the equivalent computer software, which will be studied in the next section. Accurate diagrams are important for the design of many objects such as cars and their parts, including the gears in the gearbox.



Gears in an engine are based on circles.

Let's start: Draw it!

Using a pair of compasses and a ruler, see if you can draw:

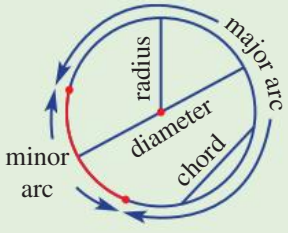
- a circle with radius 2 cm
- a circle showing a chord, diameter and radius
- a minor arc
- a major arc

Now discuss the following.

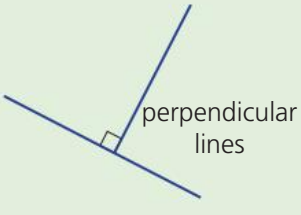
- How is the length of the diameter related to the length of the radius?
- How does a major arc differ from a minor arc?

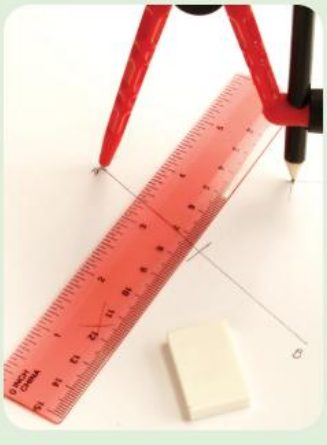
Key ideas

- Common circle features include:
  - centre (point at an equal distance from all points on the circle)
  - **radius** (line interval joining the centre to a point on the circle. Plural: radii)
  - **chord** (line interval joining two points on the circle)
  - **diameter** (longest chord passing through the centre)
  - **arc** (part of a circle. It is possible for a circle to have either a minor or major arc.)



- A pair of compasses (sometimes called a compass) and a ruler can be used to construct geometrical figures precisely.
- The word bisect means to cut in half.
- Perpendicular means at right angles (90°).





Radius A line interval from the centre of a circle to its circumference (boundary), or the length of that interval

Chord A line interval joining two points on the circumference of a circle

Diameter A line interval passing through the centre of a circle with its end points on the circumference, or the length of that interval

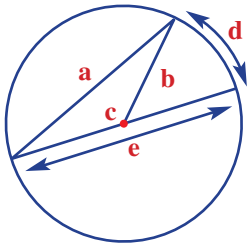
Arc Part of the circumference of a circle between two points

Exercise 2E

UNDERSTANDING 1–3 3

1 Draw the following simple objects.
- **a** A segment AB 5 cm long
- **b** A circle with radius 3 cm
- **c** A circle with radius 2.3 cm
- **d** A circle with diameter 9 cm

2 Name the features **a** to **e** in this diagram. Choose from the words: *radius, diameter, minor arc, centre, chord*.



3 Use a pair of compasses and a ruler to draw a circle with a radius of about 3 cm. Then mark and label these features.
- **a** centre O
- **b** two points, A and B, at any place on the circle
- **c** radius OA
- **d** chord AB
- **e** minor arc AB

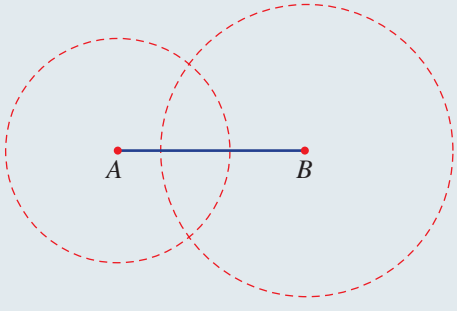
FLUENCY 4 4, 5

Example 10 Constructing a perpendicular line

Use a pair of compasses and a ruler to construct a perpendicular line by following these steps.
- **a** Use a ruler to draw a line segment AB about 5 cm long.
- **b** Construct a circle with centre A and radius about 3 cm.
- **c** Construct a circle with centre B and radius about 4 cm.
- **d** Draw a line connecting the two intersection points of the circles.
- **e** Measure the angle between the two straight lines. What do you notice?

Solution

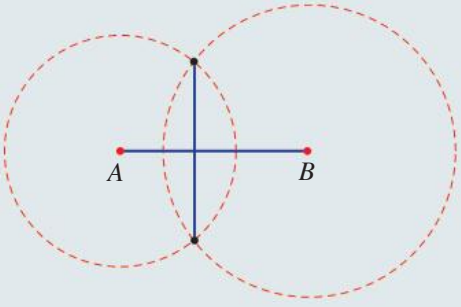
a, b, c



Explanation

First draw segment AB, then use compasses to construct two overlapping circles.

d



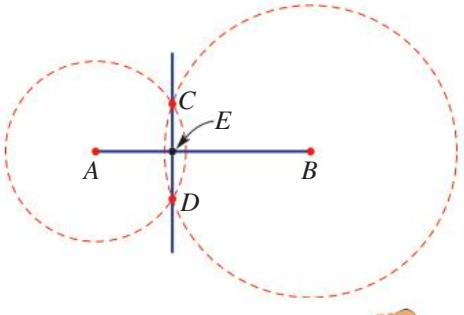
Connect the two circle intersection points with a straight line segment.

e 90° (a right angle)

You have constructed a perpendicular line.

4 Follow steps **a** to **e** to construct two perpendicular lines

- **a** Draw a line segment, AB, of about 5 cm in length.
- **b** Construct overlapping circles of different sizes using the two centres A and B.
- **c** Mark the intersecting points of the circles and label these points C and D.
- **d** Draw the line CD and mark the intersection of line CD and segment AB with the point E.
- **e** Measure $\angle AEC$ with a protractor. What do you notice?

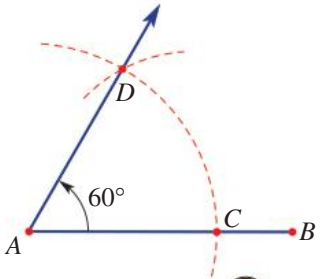




Make your circles large enough to overlap.

5 Follow steps **a** to **d** to construct a 60° angle.

- **a** Draw a line segment, AB, of about 5 cm in length.
- **b** Construct an arc with centre A and intersecting the segment AB at C. Use a radius length less than 5 cm.
- **c** With the same radius construct an arc with centre C and intersecting the first arc at D.
- **d** Draw the ray AD and measure $\angle BAD$. What do you notice?





Use the same radius for each arc, centred at A and then at C.

2E

PROBLEM-SOLVING AND REASONING 6–8 6–10

6 Follow steps **a** to **e** to construct an angle bisector.

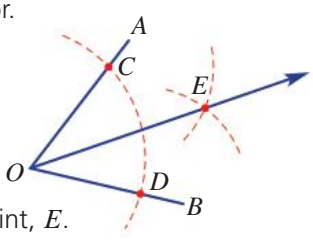
a Draw any angle and label $\angle AOB$.

b Construct an arc with centre O so that it cuts OA and OB at points C and D.

c With the same radius, construct an arc with centre C and another with centre D. Ensure these arcs intersect at a point, E.

d Mark in the ray OE.

e Measure $\angle AOE$ and $\angle BOE$. What do you notice?





We aim to cut the angle $\angle BOC$ in half with ray OE. Use the same radius for each arc.

7 Use a ruler to draw a segment AB of length 6 cm and then complete the following.

a Construct a circle with radius 3 cm with centre A. (Use a ruler to help set the correct length on a pair of compasses.)

b Construct a circle with radius 3 cm with centre B.

c Do your two circles miss, touch or overlap? Is this what you expected?

8 Use a ruler to draw a line segment, AB, of about 5 cm in length.

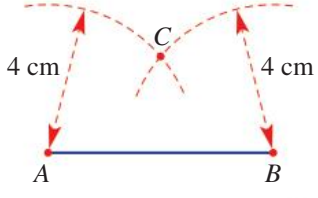
a Using a pair of compasses, construct arcs with radius 4 cm, as shown, using:

i centre A **ii** centre B

b Mark point C as shown and use a ruler to draw the segments:

i AC **ii** BC

c Measure the angles $\angle BAC$ and $\angle ABC$. What do you notice?



9 Using the results from questions **5** and **6**, explain how you could construct the angles below. Try each construction and then check each angle with a protractor.

a 30° **b** 15°



First construct a 60° angle (see question **5**). Then bisect this angle (see question **6**).

10 Show how you could construct these angles. After each construction, measure the angle using a protractor. (You may wish to use the results from questions **4** and **6** for help.)

a 45° **b** 22.5°

The perpendicular bisector

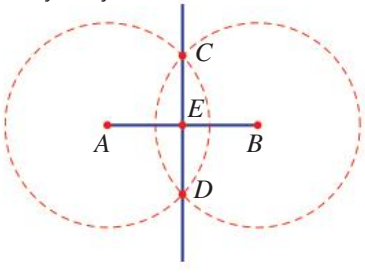
— 11

11 **a** Repeat the full construction from question **4** but this time use circles with the same radius.

b Measure AE and BE. What do you notice?

c Why do you think the line CD is called a perpendicular bisector?





2F Dynamic geometry

Interactive

Widgets

HOTsheets

Walkthrough

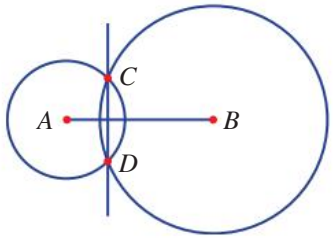
Dynamic computer geometry is an effective tool for constructing geometrical figures. Constructing with dynamic geometry is like constructing with a ruler and a pair of compasses. The computer gives you the added freedom to drag objects and explore variations of the same construction. With dynamic geometry, the focus is on 'construction' as opposed to 'drawing'. Although this can be complicated at first, the results are more precise.

Let's start: The disappearing line

Use computer geometry to construct this figure.

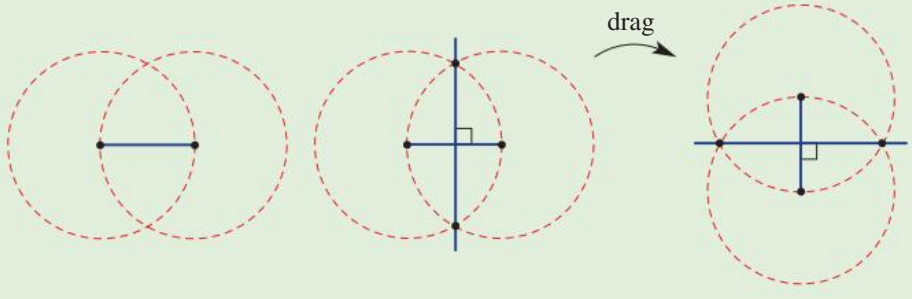
Add the line CD and check that is makes a right angle.
Drag the points A and B or increase the size of the circles.
Can you drag point A or B to make the line CD disappear?

Why would this happen?



Key ideas

- Using dynamic geometry is like using a pair of compasses and a ruler.
- Objects can be dragged to explore different cases.
- The same segment can be used to ensure two circles have exactly the same radius.
- Upon dragging, the geometrical construction should retain the desired properties.



Exercise 2F

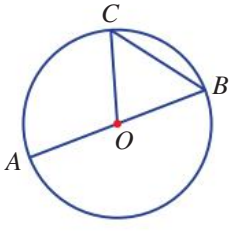
UNDERSTANDING 1, 2 2

1 Use dynamic geometry to construct this diagram.

a Measure the lengths of OC, AB and BC.

b Drag the circle to alter its size.

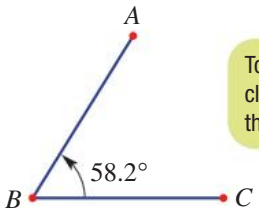
c Drag the point C to alter the length of chord BC.



2F

2 **a** Use computer geometry to construct an angle $\angle ABC$. Any size will do.

b Mark and measure the angle using computer geometry. Drag the point A around B to enlarge the angle. See whether you can form all these types of angles.





To measure the angle, click on A, then B, then C.

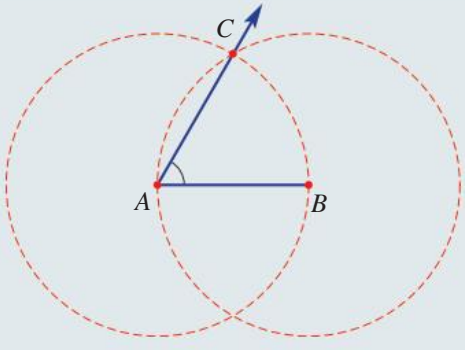
i acute **ii** right **iii** straight
iv reflex **v** revolution

FLUENCY 3 3, 4

Example 11 Constructing a 60° angle

Construct an angle of 60° using computer geometry. Then drag one of the starting points to check the construction.

Solution



Explanation

Step 1: Construct and label a segment AB.

Step 2: Construct two circles with radius AB, one with centre A and one with centre B.

Step 3: Mark the intersection C and draw the ray AC.

Step 4: Measure $\angle BAC$ to check.

3 Construct each of the following using dynamic geometry. If necessary, refer back to Section 2E and Exercise 2E to assist you. Check each construction by dragging one of the starting points.

a perpendicular line **b** 60° angle
c angle bisector



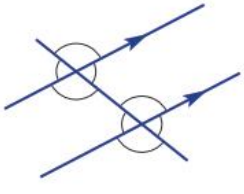
Instead of constructing arcs for parts **b** and **c**, just use full circles.

4 **a** Use the 'parallel line' tool to construct a pair of parallel lines and a transversal.

b Measure the eight angles formed.

c Drag the transversal to change the size of the angles. Check that:

i alternate angles are equal
ii corresponding angles are equal
iii cointerior angles are always supplementary



PROBLEM-SOLVING AND REASONING 5, 6 5–7

5 Look at the 60° angle construction in Example 11.

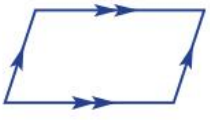
a Why do the two circles have exactly the same radius?

b What other common geometrical object could be easily constructed simply by adding one more segment?

6 Use computer geometry to construct these angles. You may wish to use the 'angle bisector' shortcut tool.

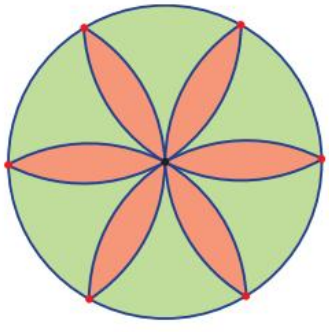
a 30° **b** 15° **c** 45°

7 Construct a parallelogram using the 'parallel line' tool.



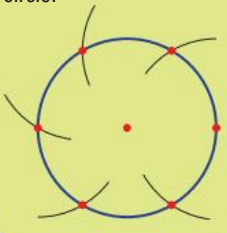
Six-pointed flower — 8

8 Use computer geometry to construct a six-pointed flower. Then drag one of the starting points to increase or decrease its size.





First construct the full circle. Then construct arcs with the same radius to find the position of each point on the circle.





Maths@Work: Glass cutter and design artist for stained-glass windows

Restoration of stained or coloured glass windows and other decorative pieces is a timeless skill requiring artistic ability, a steady hand, good eyesight and fine motor skills. Understanding materials, ratios of colour combinations when mixing and matching paints and stains is vital.

For the restoration of stained glass and the creation of new designs, it is important to understand and apply the geometry of angles and symmetry.

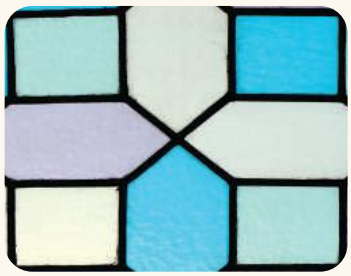


1 Stained-glass windows are made by joining many small pieces of glass. For each stained-glass pattern below, we will copy two of the internal glass pieces.

i Use a protractor to measure the vertex angles of a chosen shape inside the glass designs.

ii Make a neat, enlarged copy of this shape. Carefully copy the angles (use your protractor again) and label the angles, but make the sides larger.

a



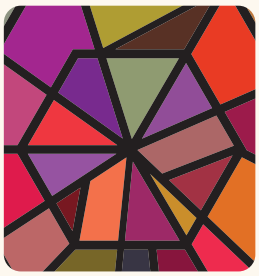
b



c



d



2 Many stained-glass windows are designed using patterns of circles and arcs. Use a pair of compasses and a ruler to design a stained-glass window pattern using circles and arcs. You may wish to use an idea from one of the designs shown below.



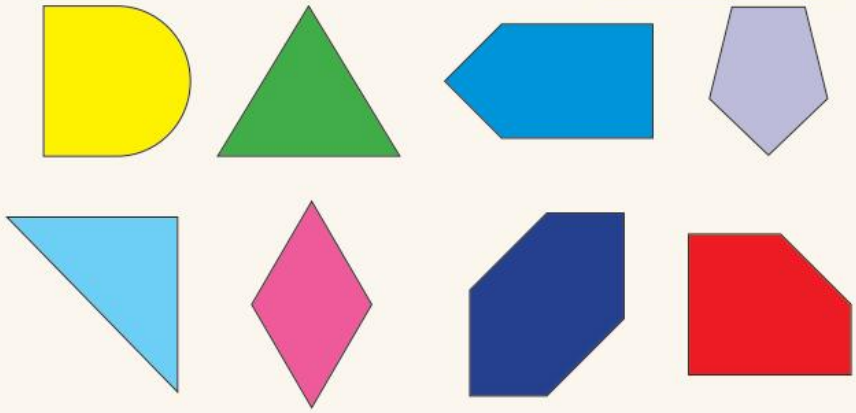


Using technology

3 Use a word processing program or a geometrical drawing application to create your own design for a stained-glass window by combining geometrical shapes.
Some shapes you could use are shown below.



Shapes can be repeated and all shapes need to fit together.

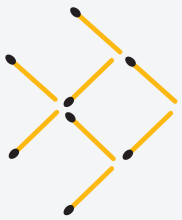




Stained-glass windows have been used for centuries in cathedrals and places of worship to illustrate sacred stories.

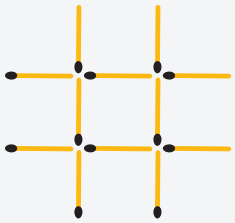
Puzzles and games

1 Move three matchsticks to turn the fish to face the opposite direction.

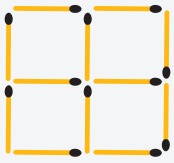


2 Move three matchsticks to get three squares of the same size.

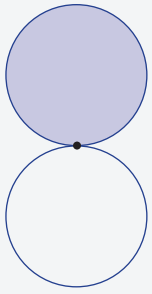
a



b



3 Two circles are the same size. The shaded circle rolls around the other circle. How many degrees will it have turned after returning to its starting position?

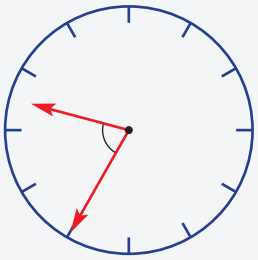


4 Find the angle between the hour hand and minute hand of a clock at:

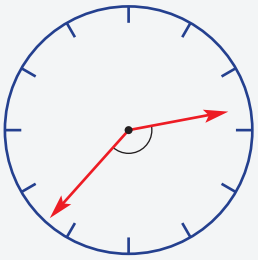
a 1:30 p.m.



b 9:35 a.m.

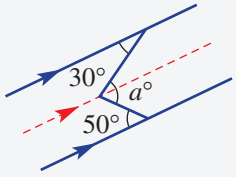


c 2:37 p.m.

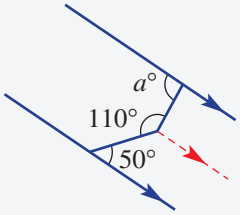


5 Find the value of a.

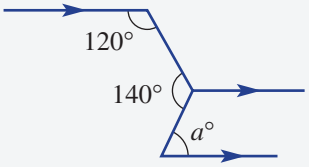
a



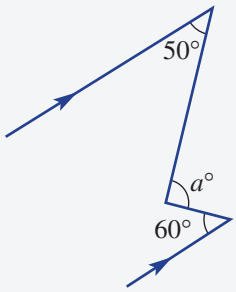
b



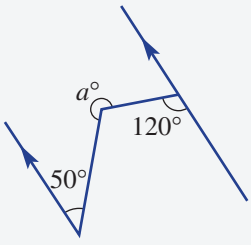
c



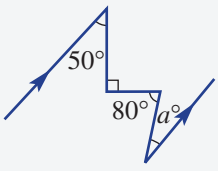
d



e



f



Chapter summary

Geometry

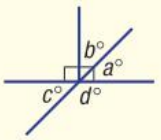
Measuring angles



Angles

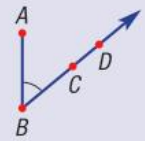
acute between 0° and 90°
right 90°
obtuse between 90° and 180°
straight 180°
reflex between 180° and 360°
revolution 360°

Angles at a point



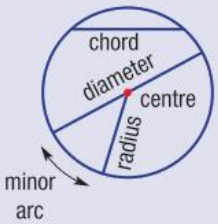
complementary
$a + b = 90$
supplementary
$c + d = 180$
vertically opposite
$a = c$
revolution
$a + b + 90 + c + d = 360$

Geometrical objects

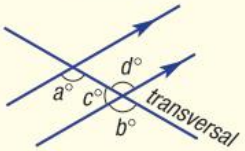


$\angle ABC$
ray BD
segment AB
collinear points B, C, D
vertex B

Circle features



Parallel lines

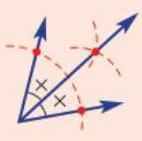


$a = b$ (corresponding)
$a = d$ (alternate)
$a + c = 180$ (cointerior)

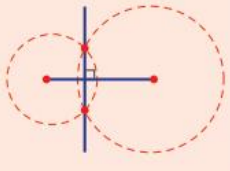
If $a = 120$ then $b = 120$, $d = 120$ and $c = 60$.

Constructions ★

angle bisector



perpendicular line



Chapter review

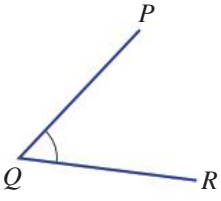
Multiple-choice questions

2A **1** The object shown here would be called:



A ray AB **B** angle AB **C** segment AB
D plane AB **E** line AB

2A **2** The angle shown here can be named:



A $\angle QRP$ **B** $\angle PQR$ **C** $\angle QPR$
D $\angle QRR$ **E** $\angle PQP$

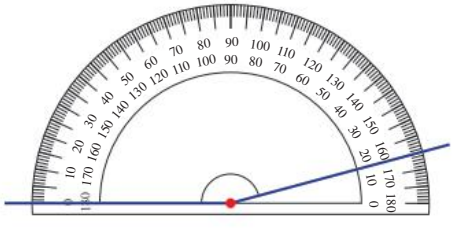
2C **3** Supplementary angles:

A sum to 90° **B** sum to 270° **C** sum to 360°
D sum to 180° **E** sum to 45°

2B **4** An acute angle is:

A 90° **B** 180° **C** between 180° and 360°
D between 90° and 180° **E** between 0° and 90°

2B **5** What is the angle measured on this protractor?



A 15° **B** 30°
C 105° **D** 165°
E 195°

2B **6** The angle in a revolution is:

A 45° **B** 270° **C** 180° **D** 90° **E** 360°

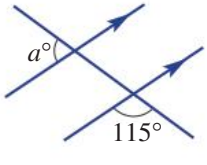
2D **7** If a transversal cuts two parallel lines, then:

A cointerior angles are equal
B alternate angles are supplementary (sum to 180°)
C corresponding angles are equal
D vertically opposite angles are supplementary
E supplementary angles add to 90°

2C **8** The angle between two perpendicular lines is:

A 90° **B** 360° **C** 10 cm **D** 0° **E** 180°

2D **9** The value of a in this diagram is:



A 115 **B** 75 **C** 60 **D** 55 **E** 65

2C **10** The complementary angle to 48° is:

A 132° **B** 42° **C** 312° **D** 222° **E** 48°

Short-answer questions

2A **1** Name each of these objects.

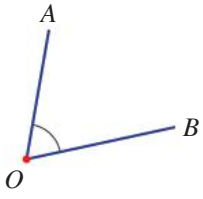


Choose from: *ray* AB, *point* O, *line* AB, *plane*, *angle* AOB, segment AB

a



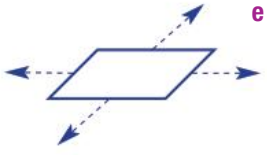
b



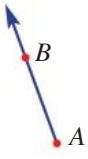
c



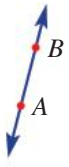
d



e



f

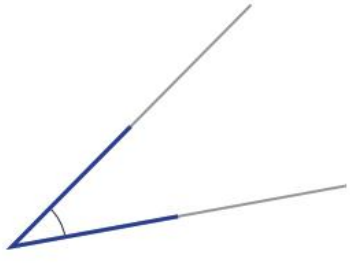


2B **2** For the angles shown, state the type of angle and measure its size using a protractor.



Types of angles include acute, right, obtuse, straight, reflex and revolution.

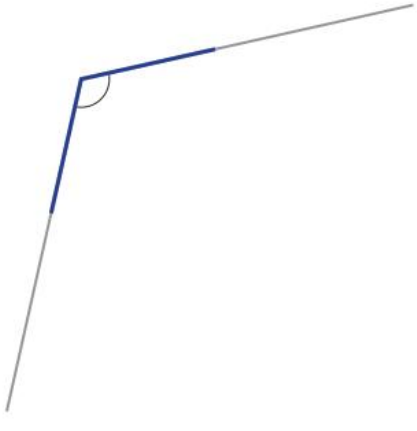
a



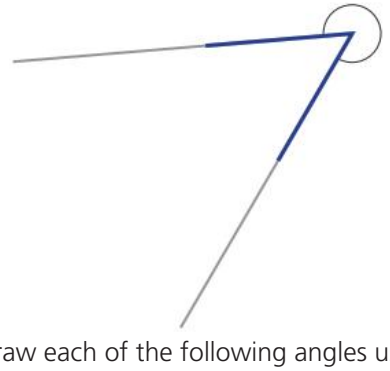
b



c



d



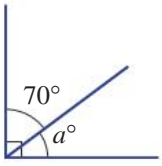
2B **3** Draw each of the following angles using a protractor.

a 60° **b** a right angle **c** 170° **d** 265°

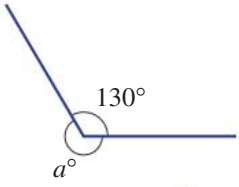
Chapter review

2C **4** Without using a protractor, find the value of a in these diagrams.

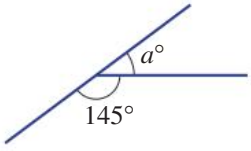
a



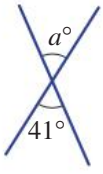
b



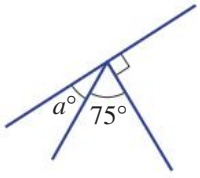
c



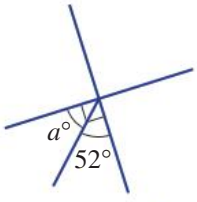
d



e

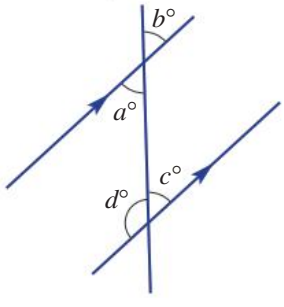


f



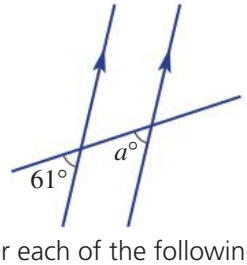
2D **5** Using the letters a, b, c or d given in the diagram, write down a pair of angles that are:

a vertically opposite
b cointerior
c alternate
d corresponding
e supplementary but not cointerior

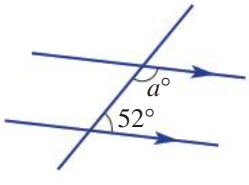


2D **6** Find the value of a in these diagrams, which include parallel lines.

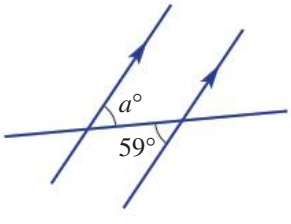
a



b

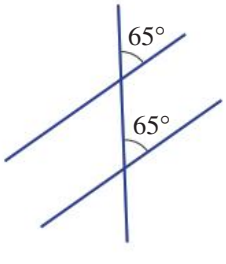


c

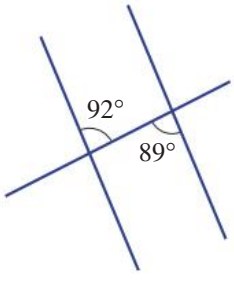


2D **7** For each of the following, state whether the two lines cut by the transversal are parallel. Give reasons for each answer.

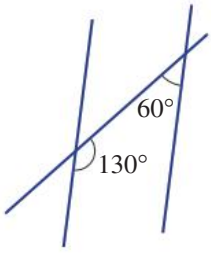
a



b

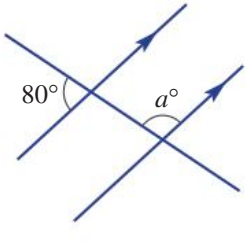


c

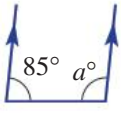


2D **8** Find the value of a in these diagrams.

a



b

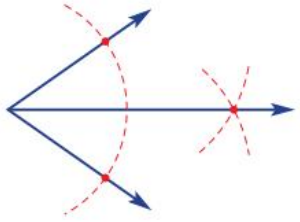


c



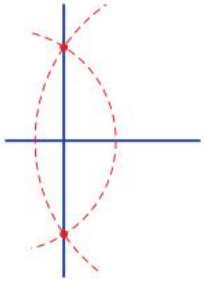
2E/F **9** Use these diagrams to help draw your own construction. You will need a pair of compasses and a ruler or dynamic geometry.

a Angle bisector (start with any angle size)



2E/F

b Perpendicular line (start with a segment of about 6 cm in length)



Extended-response questions

1 A clock face is numbered 1–12.

a Find the angle the minute hand turns in:

i 5 minutes

ii $\frac{1}{2}$ hour

iii 45 minutes

b Find the angle the hour hand turns in:

i 1 hour

ii 7 hours

iii $\frac{1}{2}$ hour

c Find the angle between the hour and minute hands at these times.

i 6 p.m.

ii 3 p.m.

iii 10 p.m.

iv 8:30 a.m.

v 2:30 a.m.

vi 4:45 p.m.



2 A circular birthday cake is cut into pieces of equal size, cutting from the centre outwards. Each cut has an angle of a at the centre. Tanya's family takes four pieces. George's family takes three pieces. Sienna's family takes two pieces. Anita's family takes two pieces. Marcus takes one piece.

a How many pieces were taken all together?

b If there is no cake left after all the pieces are taken, find the value of a.

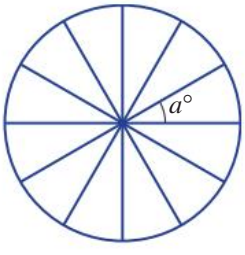
c Find the value of a if:

i half of the cake still remains

ii one-quarter of the cake still remains

iii one-third of the cake still remains

iv one-fifth of the cake still remains



Chapter 3

Number properties and patterns

What you will learn

3A Factors and multiples (Consolidating)
3B Highest common factor and lowest common multiple (Consolidating)
3C Divisibility
3D Prime numbers and composite numbers
3E Index form
3F Prime factors
3G Squares and square roots
3H Number patterns (Consolidating)
3I Patterns with shapes and numbers
3J Tables and rules
3K The number plane and graphs

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Investigate index notation and represent whole numbers as products of powers of prime numbers **(ACMNA149)**

Investigate and use square roots of perfect square numbers **(ACMNA150)**

Linear and non-linear relationships

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point **(ACMNA178)**

Investigate, interpret and analyse graphs from authentic data **(ACMNA180)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Number sequences around us

Number sequences are all around us in everyday life.

Here are some examples of number sequences with a common difference:

- Savings for a holiday increasing by adding the same amount of money each week.
- The amount owing on a debt decreasing by a fixed payment per month.
- The total distance travelled increasing by equal amounts each hour when moving at a constant speed.

Here are some examples of number sequences with a common ratio:

- Australia's annual population numbers are increasing with common ratio 1.014, which is a yearly 1.4% increase. Predicting future population numbers helps governments plan for roads, hospitals, schools etc.
- Threatened animal species have small population numbers that are decreasing with a common ratio of less than 1. For example, Australian spotted tail quolls and tiger quolls are endangered species.

Can you think of other examples of number sequences?

Pre-test

1 Find:

a 10×10 **b** $10 \times 10 \times 10$ **c** $10 \times 10 \times 10 \times 10 \times 10 \times 10$

2 Find:

a $20 \div 5$ **b** $20 \div 2$ **c** $20 \div 1$ **d** $20 \div 20$

e all the numbers that divide into 20 with no remainder

3 Write the quotient and remainder for these divisions.

a $18 \div 4$ **b** $16 \div 5$ **c** $23 \div 10$ **d** $26 \div 7$

4 Find:

a 3 more than 7 **b** the difference between 20 and 16

c the sum of 12 and 7 **d** 6 multiplied by 4

e the product of 2 and 9 **f** 30 divided by 6

5 List all the factors of 24 in ascending order.

6 Replace $\square$ with $<$ or $>$.

a 3 is smaller than 7 so we write $3 \square 7$.

b 12 is bigger than 5 so we write $12 \square 5$.

7 **a** List the next three even numbers: 2, 4, ___, ___, ___

b List the next three odd numbers: 1, 3, ___, ___, ___

8 Find:

a $2+2+2$ **b** $2 \times 2 \times 2$ **c** $3+3$ **d** 3×3

e $1+1+1+1$ **f** $1 \times 1 \times 1 \times 1$ **g** 4×2 **h** $4^2 = ___ \times ___ = ___$

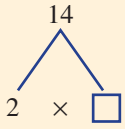
9 Copy and complete the following.

a $8 = 4 \times ___$ **b** $15 = 3 \times ___$ **c** $12 \times ___ = 48$

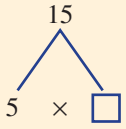
d $4 \times ___ = 16 = 8 \times ___$ **e** $12 \times 3 = ___ = 4 \times ___$

10 State the missing number in these factor trees.

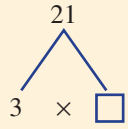
a



b



c



11 **a** Find $33 \div 3$. **b** Is 33 divisible by 3?

c Find $32 \div 3$. **d** Is 32 divisible by 3?

12 Use the order of operations to find the answer.

a $2 \times 10 + 3$ **b** $6 + 3 \times 10$ **c** $20 \div 2 + 12 \div 4$ **d** $3 \times (4 \times 2 - 6)$

13 Choose the correct set of numbers (**A** to **D**) that matches the description (**a** to **d**).

a Factors of 10 **A** 4, 8, 12, 16, …

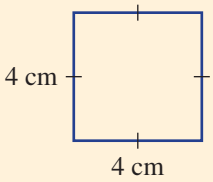
b Multiples of 4 **B** 1, 2, 5, 10.

c Factors of 16 **C** 5, 10, 15, 20, …

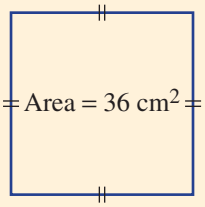
d Multiples of 5 **D** 1, 2, 4, 8, 16.

14 Find:

a the area of this square



b the side length of this square



3A Factors and multiples

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Number patterns can be quite useful when describing groups of objects or relationships between quantities. We start this chapter looking at factors and multiples.

One dozen doughnuts are generally packed into bags or boxes with 3 rows of 4 doughnuts each. Since $3 \times 4 = 12$, we can say that 3 and 4 are factors of 12.



How many factors are there in 12?

Purchasing 'multiple' packs of one dozen doughnuts could result in buying 24, 36, 48 or 60 doughnuts, depending on the number of packs. These numbers are known as multiples of 12.

Let's start: Using factors to plan a party

Shayna wants everyone at her party (including herself) to have the same number of doughnuts.



- What 'party sizes' are possible so everyone has the same number of doughnuts? For example, a party size of 12 means that Shayna invites 11 friends and everyone gets 3 doughnuts each (12 groups of 3 equals 36).
- List the factors of 36 in ascending (increasing) order. (You could arrange 36 counters into various groups of equal size to find all the possible factors.)
- If Shayna wants just one doughnut, then still wants the remaining friends at the party to have an equal number, how many friends should she invite?

Key ideas

- **Factors** of a number divide exactly into that number.
- For example: $20 \div 4 = 5$ exactly, so 4 is a factor of 20.
  - Factors of 20 listed in pairs: $1 \times 20 = 20$, $2 \times 10 = 20$, $4 \times 5 = 20$
  - Factors of 20 in **ascending** order: 1, 2, 4, 5, 10, 20

1 is the smallest factor of any number.

The largest factor of any number is the number itself.

- **Multiples** of a number are made when that number is multiplied by whole numbers.
  - For example, the multiples of 5 in ascending order are 5, 10, 15, 20, ...
  - Another way to find multiples of 5 is to start with 5 and keep adding 5.

+5 +5 +5

5, 10, 15, 20, ...

The smallest multiple of a number is the number itself.

Multiples just keep on getting bigger.

  - A **remainder** is a number left over from division.

Factor A whole number that will divide into another number exactly

Ascending Going up, from smallest to largest

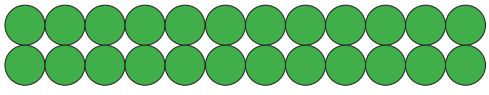
Multiple A multiple of a number is the product of that number and any other whole number

Remainder Leftover amount after one number has been divided by another

Exercise 3A

UNDERSTANDING 1–5 5

1 Think about arranging 24 counters into a rectangle with 2 rows of 12 counters.



a Copy and complete the factor pairs for this rectangle.

$12 \times \square = 24$ or $\square \times 12 = 24$

b Using all 24 counters each time, think about three different sized rectangles and write down a factor pair for each.



Example 1 Finding factor pairs

Write the pairs of factors for 18. Then write the factors in ascending order.

Solution	Explanation
$1 \times 18 = 18, 2 \times 9 = 18, 3 \times 6 = 18$ Factors of 18 are: 1, 2, 3, 6, 9, 18	Think of two numbers that multiply together to equal 18. Ascending means increasing, so start with the smallest factor.

2 Copy and complete:

a $1 \times \square = 12, 2 \times \square = 12, 3 \times \square = 12$
The factors of 12 are 1, ___, ___, ___, ___, 12

b $1 \times \square = 5$
The factors of 5 are ___, ___

c $1 \times \square = 30, 2 \times \square = 30, 3 \times \square = 30, 5 \times \square = 30$
The factors of 30 are ___, ___, ___, ___, ___, ___, ___, ___



List the factors in ascending order (from smallest to largest).

Example 2 Finding multiples

Copy and complete these multiples of 8: 8, ___, ___, 32

Solution	Explanation
8, 16, 24, 32	+8 +8 +8 8, 16, 24, 32

3 Copy and complete:

a The first six multiples of 5 are 5, ___, 15, ___, 25, ___

b The first six multiples of 10 are 10, ___, 30, ___, ___, ___

c The first six multiples of 7 are 7, ___, ___, 28, ___, 42



To make multiples of 5, start with 5 and keep adding 5.

4 Copy and complete:

a The first eight even numbers are 2, ___, 6, ___, 10, ___, 14, ___

b All even numbers are a multiple of ___.

c All odd numbers do not have ___ as a factor.

d The first eight odd numbers are 1, ___, 5, ___, 9, ___, 13, ___

5 Copy and complete:

a $7 \times 6 = 42$ so 7 and 6 are _____ of 42.

b To make multiples of 6, multiply 6 by whole numbers. For example:
$1 \times 6 = 6, \square \times 6 = 12, \square \times 6 = 18, \square \times 6 = 24$

c The first five multiples of 6 are 6, $\square$, 18, $\square$, $\square$

d If you start with 6 and then keep adding 6, you will produce a list of _____ of 6.

3A

FLUENCY 6–8(½) 6–8(½), 9

Example 3 Finding factors

Find the complete set of factors for each of these numbers.

a 15 **b** 40

Solution	Explanation
a Factors of 15 are 1, 3, 5, 15.	$1 \times 15 = 15$, $3 \times 5 = 15$
b Factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40.	$1 \times 40 = 40$, $2 \times 20 = 40$ $4 \times 10 = 40$, $5 \times 8 = 40$ There are no other numbers that divide into 40 with zero remainder.

6 List the complete set of factors for each of the following numbers.

a 10 **b** 24 **c** 17
d 36 **e** 60 **f** 42
g 80 **h** 12 **i** 28



List the factors in ascending order (smallest to largest).

Example 4 Listing multiples

Write down the first six multiples for each of these numbers.

a 11 **b** 35

Solution	Explanation
a 11, 22, 33, 44, 55, 66	The first multiple is always the given number. Add on the given number to find the next multiple. Repeat this process to get more multiples.
b 35, 70, 105, 140, 175, 210	Start at 35 (the given number) and repeatedly add 35 to continue producing multiples.

7 Write down the first six multiples for each of the following numbers.

a 5 **b** 8 **c** 12
d 7 **e** 20 **f** 75
g 15 **h** 100 **i** 37



List the multiples in ascending order.

8 Fill in the gaps to complete the set of factors for each of the following numbers.

a 18 1, 2, ___, 6, 9, ___

b 25 1, ___, 25

c 72 ___, 2, 3, ___, ___, 8, ___, ___, 18, ___, 36, 72

d 120 1, 2, ___, ___, ___, 6, ___, 10, ___, ___, 20, ___, 30, ___, 60, ___

Example 5 Identifying incorrect multiples

Which number is the *wrong* multiple in the sequence 7, 14, 20, 28, 35? Write the correct sequence.

Solution	Explanation
20 is incorrect.	$14 + 7 = 21$ or $3 \times 7 = 21$
7, 14, 21, 28, 35	A multiple of 7 must be a whole number times 7.

9 Find the *wrong* multiple in each of the following. Write the correct sequence.

a 3, 6, 9, 12, 15, 18, 22, 24, 27, 30

b 5, 10, 15, 20, 24, 30

c 11, 21, 33, 44, 55, 66, 77, 88, 99, 110

d 12, 24, 36, 49, 60, 72, 84

PROBLEM-SOLVING AND REASONING 10, 11 10–12

Example 6 Finding factor pairs

Express 96 as a product of two factors, both of which are greater than 6.

Solution	Explanation
$96 = 8 \times 12$	Divide 96 by numbers greater than 6 to find a large factor.

10 Express each of the following numbers as a product of two factors, both of which are greater than 4.

a 30 **b** 36 **c** 42

d 40 **e** 64 **f** 100



11 Zane and Matt are both keen runners. Zane takes 4 minutes to jog around a running track and Matt takes 5 minutes. They start together at the same time and jog 6 laps for training.



a Copy and complete:

i When Zane jogged 6 laps around the track, he crossed the start/finish line after 4, 8, ___, ___, ___ and ___ minutes.

ii Matt crossed the start/finish line after ___, ___, ___, ___, ___ and ___ minutes.

b How many laps had each boy jogged when they crossed the line together?

c For how long had the boys jogged before they crossed the line together?

3A

12 Anson is preparing for his twelfth birthday party. He has invited 5 friends and is making each of them a 'lolly bag' to take home after the party. To be fair, he wants to make sure that each friend has the same number of lollies. Anson has a total of 67 lollies to share among the lolly bags.



a How many lollies does Anson put in each of his friends' lolly bags?

b How many lollies does Anson have left over to eat himself?

Anson then decides that he wants a lolly bag for himself also.

c How many lollies will now go into each of the 6 lolly bags?

d With 6 lolly bags, are there any lollies left over?

Using factors to make squares or rectangles

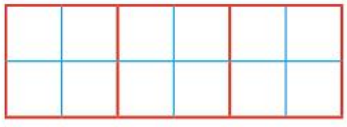
— 13

13 Joanna is a vet. She wants to buy several pens for separating sick animals. There are square or rectangular pens available, all with sides that are 1, 2, 3 or 4 metres long.



a Joanna wants to buy at least two equal-sized pens to cover an area of 12 m^2. Two possibilities are shown below. Find the other four ways Joanna could buy pens.

Option 1: Buy 3 square pens.
Each pen has: L = 2 m,
W = 2 m, A = 4 m^2
Total area = $3 \times 4 = 12$ m^2



Option 2: Buy 2 rectangular pens.
Each pen has: L = 2 m, W = 3 m, A = 6 m^2
Total area = $2 \times 6 = 12$ m^2





Use grid paper and draw the pens. Note that 2 m × 3 m pens are the same as 3 m × 2 m pens, so don't count them twice.

For the following questions, list all possible answers.

b Work out the number of equal-sized *square* pens and their sizes so all these pens join side by side to cover an area of:

i 32 m^2 **ii** 54 m^2

c Work out the number of equal-sized *rectangular* pens and their sizes so all these pens join side by side to cover an area of:

i 32 m^2 **ii** 54 m^2

d A zoo vet wants to buy several larger, equal-sized pens to cover 150 m^2. Find all the possibilities with whole-number sides from 2 to 25 metres.

3B Highest common factor and lowest common multiple

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

In the last section, we worked with factors and multiples. In this section, we look at common factors or multiples that are shared by two numbers.

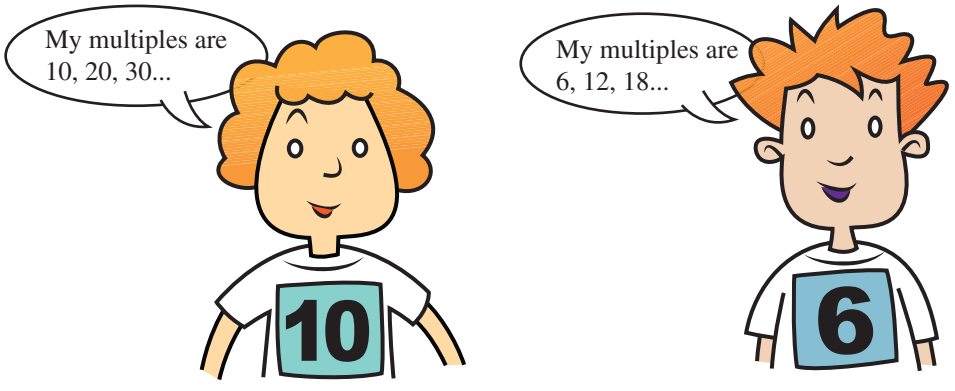
It is often useful to be able to find the highest common factor (HCF) or lowest common multiple (LCM) and these are important in higher levels of mathematics.

Let's start: Finding special numbers



We share some common factors. What are they?

Which is the highest (biggest) common factor we share?



We share some common multiples. What are they?

What is the lowest (smallest) multiple we share?

Key ideas

- **HCF** stands for **highest common factor**. (Common means shared.)
  - For example: Find the HCF of 24 and 40.
    Factors of 24 are (1), (2), 3, (4), 6, (8), 12 and 24.
    Factors of 40 are (1), (2), (4), 5, (8), 10, 20 and 40.
    We can see that the highest common factor is 8 so we write HCF = 8.
- **LCM** stands for **lowest common multiple**.
  - For example: Find the LCM of 20 and 12.
    Multiples of 20 are 20, 40, (60), 80, 100, 120, 140, …
    Multiples of 12 are 12, 24, 36, 48, (60), 72, 84, 96, …
    We can see that the lowest common multiple is 60 so we write LCM = 60.

Highest common factor (HCF) The largest number that is a factor of all the given numbers

Lowest common multiple (LCM) The smallest number that two or more numbers divide into evenly

Exercise 3B

UNDERSTANDING 1–4 | 4

1 Copy and complete the following.

a HCF stands for _____ _____ _____.
b To find the HCF, we first list all the _____ of each number.
c The HCF is the highest _____ (shared) factor.
d LCM stands for _____ _____ _____.
e To find the LCM, we list _____ of each number until we get a _____ (shared) multiple.



Look back at the **Key ideas**.

Example 7 Listing common factors to find the HCF

The factors of 8 are 1, 2, 4, 8. The factors of 20 are 1, 2, 4, 5, 10, 20.

a What are the common factors of 8 and 20?
b What is the HCF of 8 and 20?

Solution	Explanation	
a 1, 2, 4	(1), (2), (4), 8	Circle common
	(1), (2), (4), 5, 10, 20	(shared) factors.
b HCF = 4	Largest circled number is the HCF.	

2 The factors of 12 are 1, 2, 3, 4, 6, 12. The factors of 16 are 1, 2, 4, 8, 16.

a What are the common factors of 12 and 16?
b What is the HCF of 12 and 16?



Common factors: numbers in both lists.

3 Copy and complete to find the HCF of 18 and 30:
Factors of 18 are 1, ___, 3, ___, ___, 18.
Factors of 30 are 1, ___, ___, 5, ___, 10, ___, 30.
HCF = ___



$1 \times 18 = 18$
$___ \times ___ = 18$
$3 \times ___ = 18$

Example 8 Listing common multiples to find the LCM

The first six multiples of 3 are 3, 6, 9, 12, 15, 18.
The first six multiples of 2 are 2, 4, 6, 8, 10, 12.

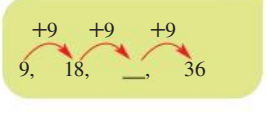
a What are two common multiples of 2 and 3?
b What is the LCM of 2 and 3?

Solution	Explanation
a 6, 12	3, (6), 9, (12), 15, 18 2, 4, (6), 8, 10, (12) Circle common (shared) multiples.
b LCM = 6	The first circled number is the LCM.

4 The first 10 multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80.
The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60.
a What are two common multiples of 8 and 6?
b What is the LCM of 8 and 6?

FLUENCY 5, 6(½) 5, 6–7(½)

5 Copy and complete to find the LCM of 9 and 15:
Multiples of 9 are 9, 18, ___, 36, ___, ___, ___, ___, 81, ___.
Multiples of 15 are ___, 30, ___, 60, 75, ___, ___, 120.
LCM = ___





Example 9 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 36 and 48.

Solution	Explanation
Factors of 36 are: (1), (2), (3), (4), (6), 9, (12), 18 and 36.	Find the factors of 36 ($1 \times 36 = 36$, $2 \times 18 = 36$, etc.) and list them in order.
Factors of 48 are: (1), (2), (3), (4), (6), 8, (12), 16, 24 and 48.	Find the factors of 48 ($1 \times 48 = 48$, $2 \times 24 = 48$, etc.) and list them in order.
HCF = 12	Circle common (shared factors). Pick the HCF (highest shared factor).

3B

6 Find the HCF of the following numbers.

a 4 and 5	**b** 8 and 13	**c** 2 and 12
d 3 and 15	**e** 16 and 20	**f** 15 and 60
g 20, 40 and 50	**h** 12, 15 and 30	**i** 5, 7 and 11



First list the factors of each number.

Example 10 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 6 and 10.

Solution	Explanation
6, 12, 18, 24, (30), 36, …	List multiples of each number.
10, 20, (30), 40, …	Circle the first common (shared) multiple.
LCM = 30	This is the LCM.

7 Find the LCM of the following numbers.

a 4 and 5	**b** 3 and 7	**c** 5 and 6
d 8 and 10	**e** 4 and 6	**f** 5 and 10
g 2, 3 and 5	**h** 3, 4 and 5	**i** 2, 3 and 7



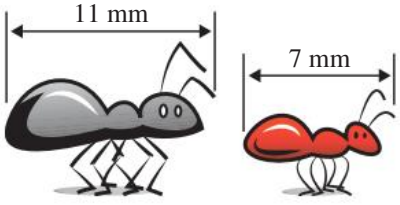
First list some multiples of each number.

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 A trail (line) of red ants runs alongside a trail of black ants. Assume there are no gaps between ants in a trail.



There can only be whole ants in each trail!

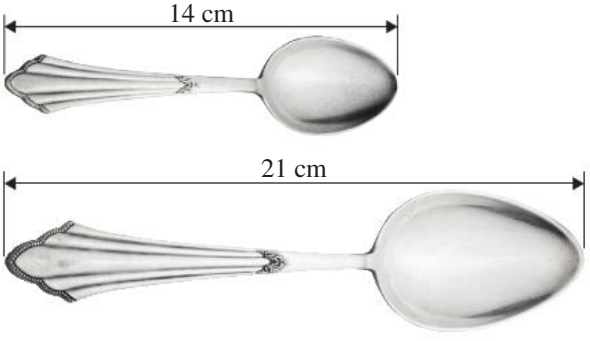


a What is the smallest number of red ants and the smallest number of black ants that would make the trails equal in length?

b How long would that trail be?

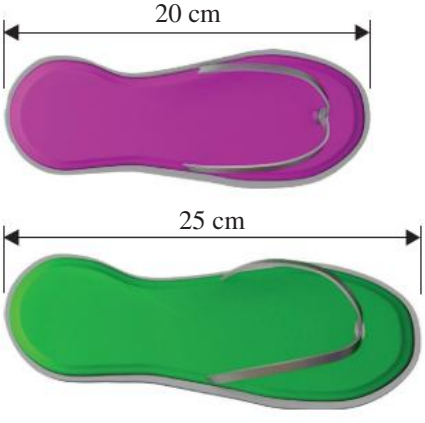


9 A line of large spoons is next to a line of small spoons.



a What is the smallest number of each type of spoon that would make the lines equal in length?
b How long would that line be?
c The answer to part **b** is called the _____ of 14 and 21.

10 A line of pink thongs is next to a line of green thongs.





Remember that LCM means lowest common multiple.

a Suppose that the two lines of thongs are the same length. How long could each line be? Find three possible answers.
b Which of the answers in part **a** is the LCM of 20 and 25?

11 Wendy is a florist who is making up small bunches of roses for sale. She has 36 red roses, 42 pink roses and 30 cream roses. Wendy uses only one colour for each bunch. She wants to use all the roses.
Each bunch must have the same total number of roses.



a What is the largest number of roses Wendy can put in each bunch?
b The answer to part **a** is called the _____ of 36, 42 and 30.
c How many bunches of each colour can she make?

12 Given that the HCF of a pair of different numbers is 8, find the two numbers:
a if both numbers are less than 20
b when one number is in the 20s and the other in the 30s



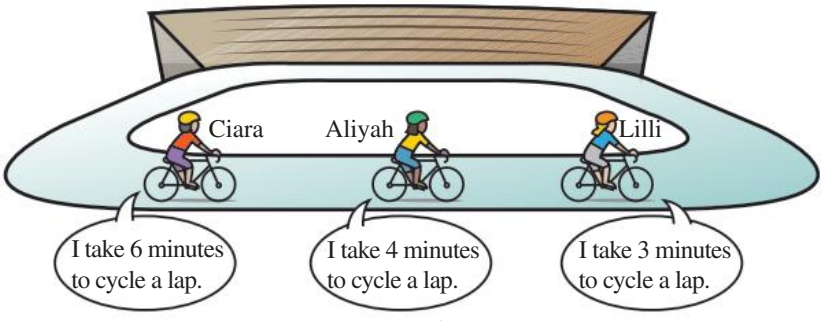
Remember, HCF means highest common factor.

3B

Cycling laps

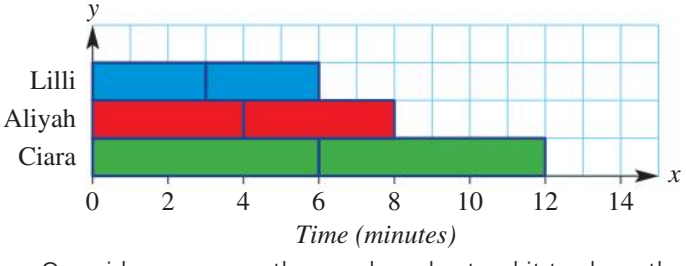
13

13 Three girls are riding their bikes around a circular track. They all start together.



Lilli takes 3 minutes to cycle each lap. (That is, she crosses the starting line every 3 minutes.) Aliyah takes 4 minutes for each lap and Ciara takes 6 minutes.

This bar graph shows the time each girl takes for the first two laps.



a On grid paper, copy the graph and extend it to show the number of laps each girl completes in 24 minutes. (Draw a rectangle for each lap cycled.)

b When do all three girls first cross the starting line together? (Give your answer as the number of minutes after the start.)

c How many full laps has each girl completed after 24 minutes? (Count whole laps only.)

d Suppose that each girl rides 15 laps. How many minutes after Lilli will:

i Aliyah finish? **ii** Ciara finish?



3C Divisibility

It is often useful to know whether a number is exactly divisible by another number. For example, 20 is divisible by 2 because $20 \div 2 = 10$ with no remainder. However, 20 is not divisible by 3 because $20 \div 3 = 6$ with remainder 2.

There are simple divisibility tests for each of the single-digit numbers, with the exception of 7.

Let's start: Exploring remainders

Work with a partner to sort these divisions into two groups:

- Divisions with no remainder
- Divisions with a remainder

$20 \div 4$ $60 \div 12$ $48 \div 9$ $20 \div 7$ $48 \div 10$ $55 \div 11$ $72 \div 7$ $93 \div 3$ $56 \div 5$ $26 \div 8$ $88 \div 4$

Key ideas

- A number is divisible by another number if there is no remainder after the division. For example, 84 is divisible by 4 because $84 \div 21 = 4$ exactly, with no remainder. That is, 4 is a factor of 84.
- **Divisibility tests**
  All numbers are divisible by 1.

2
Numbers which end in 0, 2, 4, 6 or 8 are even and can be divided by 2.
32, 14, 206, 68, 30

3
The sum of the digits must be divisible by 3.
63, 27, 12, 6, 48

4
The number formed from the last two digits must be divisible by 4.
12, 312, 16, 216

5
The last digit must be a 0 or 5.
30, 50, 75, 85, 125

6
The number must be divisible by 2 and 3.
48, 24, 24, 6, 72

7
There is no simple test for divisibility by 7.
49, 56, 63, 70

8
The number formed from the last three digits must be divisible by 8.
64, 40, 120, 320, 4320

9
The sum of the digits must be divisible by 9.
54, 216, 621, 882

10
The last digit must be 0.
90, 200, 650, 2130

Divisibility test A way to work out whether a whole number is divisible by another whole number, without actually doing the division

Exercise 3C

UNDERSTANDING 1–6 6



Look back at the **Key ideas**.

1 Write the missing words or numbers.

a We say that 24 is _____ by 3 because $24 \div 3 = 8$ with no _____.

b Even numbers are all divisible by ___.

c Even numbers end in ___, ___, ___, ___ or ___.

d 432 is _____ by 3 because the sum of the digits is ___ + ___ + ___ = ___ and ___ is divisible by 3.

e 432 is a number divisible by both 2 and 3 so it is also divisible by ___.

2 Who am I? Match each correct description (**A** to **C**) to its clue (**a** to **c**).

a The sum of my digits is divisible by 3.

b I am an even number.

c I am even and the sum of my digits is divisible by 3.

A Divisible by 2

B Divisible by 3

C Divisible by 6

3 Write the missing words or numbers.

a A number that ends in 0 is divisible by both ___ and ___, as well as by 2.

b A number that ends in 5 is divisible by ___.

c The last two digits of 316 form the number ___, and ___ is divisible by 4, so 316 is also divisible by ___.

d The last three digits of 5328 form the number ___, and ___ is divisible by 8, so 5328 is also divisible by ___.

4 Write the missing words or numbers.

a In 2583, the sum of the digits is ___ + ___ + ___ + ___ = ___.

b 2583 is _____ by 3 because the sum of its digits, ___ is divisible by ___.

c 2583 is also _____ by 9 because the sum of its digits, ___ is divisible by ___.

d 2583 is not divisible by 6 because it is an _____ number.

Example 11 Using divisibility rules for 2, 3, 6 and 8

Which of the numbers 148, 63, 462, 6387, 7168 are divisible by:

a 2? **b** 3? **c** 6? **d** 8?

Solution	Explanation
a 148, 462, 7168	Only even numbers are divisible by 2. The numbers 63 and 6387 are odd.
b 63, 462, 6387	Divisible by 3 when the sum of digits is divisible by 3. For 148: $1 + 4 + 8 = 13$ but $13 \div 3 = 4$ with rem. 1 ✘ For 63: $6 + 3 = 9$ and $9 \div 3 = 3$ ✔ In a similar way, test 462 ✔ 6387 ✔ and 7168 ✘
c 462	Divisible by 6 when even and divisible by 3. $4 + 6 + 2 = 12$ and $12 \div 3 = 4$ ✔
d 7168	Divisible by 8 when the number formed from last 3 digits is divisible by 8. $168 \div 8 = 21$ ✔

5 **a** Which of these numbers are divisible by 2?

3, 6, 13, 14, 8, 17, 21, 54, 22, 34, 33, 50, 18, 35, 46

b Which of these numbers are divisible by 3?

12, 14, 18, 20, 22, 30, 27, 23, 54, 50, 36, 42, 13, 24, 43

c Which of these numbers are divisible by 6?

12, 24, 28, 38, 63, 60, 87, 225, 54, 252, 36, 92, 66, 84, 143

d Which of these numbers are divisible by 8?

35, 168, 7168, 40, 5032, 9338, 248, 7831, 6400, 9568



Divisibility tests:
2: last digit even
3: sum of digits is divisible by 3
6: divisible by 2 and 3
8: number formed from last 3 digits is divisible by 8

Example 12 Using divisibility tests for 5, 10, 4 and 9

Which of the numbers 540, 918, 8775, 3924 are divisible by:

a 10? **b** 5? **c** 4? **d** 9?

Solution	Explanation
a 540	Divisible by 10 when number ends in 0.
b 540, 8775	Divisible by 5 when number ends in 0 or 5.
c 540, 3924	Divisible by 4 when last two digits divisible by 4. For 540: $40 \div 4 = 10$ ✔ For 918: $18 \div 4 = 4$ rem. 2 ✘ In a similar way, test 75 ✘ and 24 ✔
d 540, 918, 3924, 8775	Divisible by 9 when sum of digits divisible by 9. For 540: $5 + 4 + 0 = 9$ and $9 \div 9 = 1$ ✔ For 918: $9 + 1 + 8 = 18$ and $18 \div 9 = 2$ ✔ In a similar way, test 8775 ✔ and 3924 ✔

6 **a** Which of these numbers are divisible by 5?

35, 52, 125, 13, 15, 100, 113, 112, 32, 515, 408, 730, 105

b Which of these numbers are divisible by 10?

20, 64, 800, 98, 290, 610, 85, 265, 590, 52, 39, 90, 160

c Which of these numbers are divisible by 4?

16, 32, 220, 10, 12, 28, 213, 432, 72, 316, 424, 1836, 135

d Which of these numbers are divisible by 9?

27, 432, 456, 88, 99, 387, 63, 55, 720, 85, 253, 2799



Divisibility tests:
5: last digit 5 or 0
10: last digit 0
4: number formed from last 2 digits is divisble by 4
9: sum of digits is divisible by 9

3C

FLUENCY 7(½) 7(½), 8

Example 13 Applying divisibility tests

Work out whether the following calculations are possible without leaving a remainder.

a 54 327 ÷ 3 **b** 765 146 ÷ 8

Solution	Explanation
a Digit sum = 21	5 + 4 + 3 + 2 + 7 = 21
Yes, 54 327 is divisible by 3.	21 is divisible by 3.
b $8\overline{)14^{6}6}$ rem. 2	Check whether the number formed by the last three digits is divisible by 8.
No, 765 146 is not divisible by 8.	

7 Work out whether the following calculations are possible without leaving a remainder.

a 23 562 ÷ 3 **b** 39 245 678 ÷ 4 **c** 1 295 676 ÷ 9
d 213 456 ÷ 8 **e** 3 193 457 ÷ 6 **f** 2 000 340 ÷ 10
g 51 345 678 ÷ 5 **h** 215 364 ÷ 6 **i** 9543 ÷ 6
j 25 756 ÷ 2 **k** 56 789 ÷ 9 **l** 324 534 565 ÷ 5
m 2 345 176 ÷ 8 **n** 329 541 ÷ 10 **o** 225 329 ÷ 3
p 356 781 276 ÷ 9 **q** 164 567 ÷ 8 **r** 2 002 002 002 ÷ 4



Apply the divisibility tests rather than doing the entire division.

Example 14 Testing divisibility

Carry out divisibility tests on the given number and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412								

Solution

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412	✔	✔	✔	✘	✔	✘	✘	✘

Explanation

48 569 412 is an even number and therefore is divisible by 2.

48 569 412 has a digit sum of 39 and therefore is divisible by 3, but not by 9.

48 569 412 is divisible by 2 and 3, therefore it is divisible by 6.

The last two digits are 12, which is divisible by 4.

The last three digits are 412, which is not divisible by 8.

The last digit is a 2 and therefore is not divisible by 5 or 10.

8 Copy the table. Carry out the divisibility tests on the given numbers, filling in the table with ticks or crosses.



✔ if divisible.
✘ if not divisible.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567								
28 080								
189 000								
1 308 150								
1 062 347								

PROBLEM-SOLVING AND REASONING

9, 10 | 9–12

9 Give a reason why:

a 8631 is not divisible by 2
b 31 313 is not divisible by 3
c 426 is not divisible by 4
d 5044 is not divisible by 5
e 87 548 is not divisible by 6
f 214 125 is not divisible by 8
g 3 333 333 is not divisible by 9
h 56 405 is not divisible by 10



Think about the rules for divisibility.

10 Give the remainder when:

a 326 is divided by 3
b 21 154 is divided into groups of four
c 72 is divided into six groups
d 45 675 is shared into five groups



The remainder is the amount left over after the division.

11 The game of 'clusters' involves a group getting into smaller-sized groups. Players get into groups as quickly as possible once the cluster size has been called out. If a year level consists of 88 students, which cluster sizes would mean that no students are left out of a group? Give all possible answers.



Think: ☐ groups of ☐ make 88.

12 Blake's age is a two-digit number. It is divisible by 2, 3, 6 and 9. How old is Blake if you know that he is older than 20 but younger than 50?



A very large number

— | 13

13 **a** Is the number 968 362 396 392 139 963 359 divisible by 3?

b Many of the digits in the number above can actually be ignored when calculating the digit sum. Which numbers can be ignored and why?

c To determine if the number above is divisible by 3, only five of the 21 digits actually need to be added together. Find this 'reduced' digit sum.

d Make a list of large numbers. Include some numbers that are divisible by 3 and other numbers that are not.

e Swap lists with a classmate. See how quickly you can find each other's numbers that are divisible by 3.

3D Prime numbers and composite numbers

Interactive

Widgets

HOTsheets

Walkthrough

A prime number has only two factors. For example, 5 is a prime number because its complete list of factors includes only 1 and 5.

Composite numbers have more than two factors. For example, 9 is a composite number because its factors are 1, 3 and 9.

Prime numbers have many important uses in everyday life, such as internet security. There are some interesting prime numbers with remarkable patterns in their digits, such as 12 345 678 901 234 567 891. You can also get palindromic primes, such as 111 191 111 and 123 494 321. Below is a palindromic prime number that reads the same upside down or when viewed in a mirror.



Computers have allowed mathematicians to find larger and larger prime numbers.

$$18880818088881$$

Let's start: Find the prime numbers

Elise is trying to decide which of the following are prime numbers.

$$138 \quad 27 \quad 31 \quad 91 \quad 98 \quad 57 \quad \frac{3}{7} \quad 101$$
$$71 \quad 99 \quad -19 \quad 83 \quad 85 \quad 60 \quad 202$$

- Which numbers could she eliminate straight away? Why?
- How could Elise use divisibility tests to eliminate some of the other numbers?
- Which of the numbers do you think are prime?

Key ideas

- A **prime number** is a positive whole number that has only two factors: 1 and itself.
  - The smallest prime numbers include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, …
- A number that has more than two factors is called a **composite number**.
- The number 1 has only one factor ($1 \times 1 = 1$). It is neither prime nor composite.

Prime number A whole number with only two factors, itself and 1

Composite number A whole number with more than two factors

Exercise 3D

UNDERSTANDING 1–4 | 4

1 Copy and complete the following.
- **a** A _____ number has only two factors: ___ and _____.
- **b** A number that has more than two factors is a _____ number.

2 **a** The factors of 12 are 1, 2, 3, 4, 6 and 12. Is 12 a prime number?
- **b** The factors of 13 are 1 and 13. Is 13 a prime number?

Look at the **Key ideas**.



3 **a** List the first 10 prime numbers.
- **b** List the first 10 composite numbers.

4 **a** What is the first prime number greater than 10?
- **b** What is the first composite number greater than 10?

FLUENCY 5–6(½), 7 | 5-6(½), 7, 8

Example 15 Deciding if a number is prime or composite

Which of these numbers are prime and which are composite: 22, 35, 17, 11, 9, 5?

Solution	Explanation
Prime: 5, 11, 17	5, 11, 17 have only two factors (1 and itself).
Composite: 9, 22, 35	$9 = 3 \times 3$, $22 = 2 \times 11$, $35 = 5 \times 7$

5 Label each of the following as a prime (P) or a composite (C) number.

a 14	**b** 23	**c** 70	**d** 37
e 51	**f** 27	**g** 29	**h** 3
i 8	**j** 49	**k** 99	**l** 59
m 2	**n** 31	**o** 39	**p** 89

A prime number has only two factors: 1 and itself.



Example 16 Finding prime factors

Find the prime numbers that are factors of 30.

Solution	Explanation
Factors of 30 are:	Find all the factor pairs:
1, 2, 3, 5, 6, 10, 15, 30	1×30, 2×15, 3×10, 5×6
Prime factors are 2, 3 and 5.	Determine which factors are prime.
	1 is not a prime number.
	$2 = 2 \times 1$, $3 = 3 \times 1$, $5 = 5 \times 1$

3D

6 Find the prime numbers that are factors of:

a 42 **b** 39 **c** 60
d 25 **e** 28 **f** 36



List all the factors first.

7 List the composite numbers between:

a 30 and 40 **b** 50 and 60 **c** 80 and 90

8 List the prime numbers between:

a 20 and 30 **b** 40 and 50 **c** 70 and 80

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 The following are not prime numbers, yet they are the product (×) of two primes. Find the two primes that multiply to give:

a 15 **b** 21 **c** 35
d 55 **e** 143 **f** 133

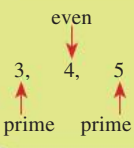
10 Which one of the following numbers has factors that are all prime numbers, except itself and 1?
12, 14, 16, 18, 20



First list all the factors of each number.

11 Twin primes are pairs of primes that are separated from each other by only one even number. For example, 3 and 5 are twin primes. Find three more pairs of twin primes.





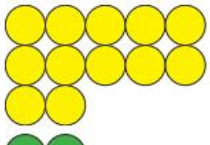
12 Answer these questions for each of the arrays (**a** to **h**) below.

i Is it possible to rearrange the counters into a rectangle with two or more equal rows?
ii If it is possible, state the dimensions of the new rectangle.
iii State the number of counters and whether this is a prime or a composite number.

a



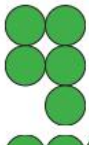
b



c



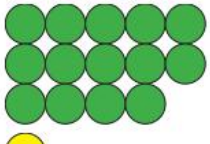
d



e



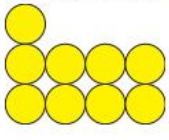
f



g



h





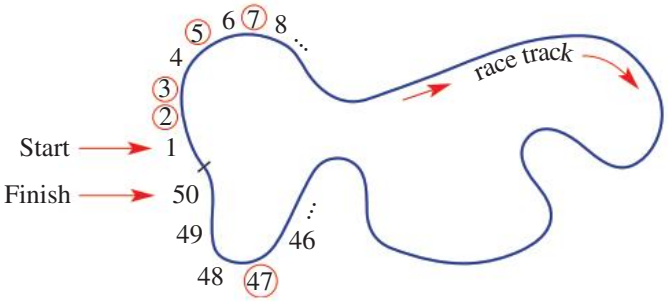
The 'dimensions' are the *number of rows* and the *number of columns*.

Prime car race

— 13

13 Play this game with a classmate. You will need a large sheet of paper, a die and two 'race cars' (counters or erasers).

- On the sheet of paper, draw a curvy line for your race track.
- Write the numbers 1 to 50 along your track and circle all the prime numbers. For example:



- Start with both cars on 1. Take turns to roll the die and drive your car that number of places along the track.
- If your car lands on a prime number, it needs a pit stop (flat tyre, low fuel, etc.) and you miss a turn.
- The car that wins reaches the finish line first. (You must roll the correct number to land exactly on 50.)



3E Index form

Interactive

Widgets

HOTsheets

Walkthrough

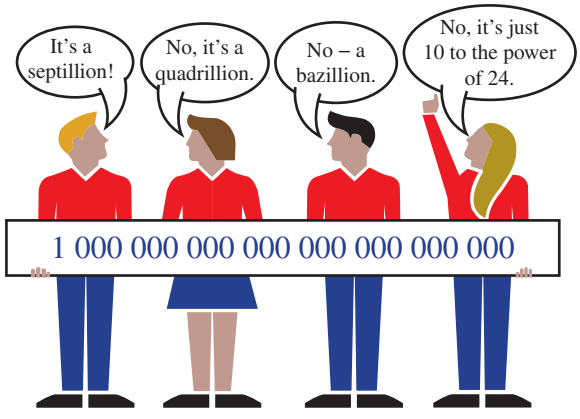
When a number is multiplied by itself, we often write that product in index form.

For example:

$1000 = 10 \times 10 \times 10$ (factor form)

$= 10^3$ (index form)

For 10^3 we say '10 to the power of 3'.



Let's start: An easier way

What is an easier way of writing:

- $10 \times 10 \times 10 \times 10$, other than 10 000?
- $10 \times 10 \times 10 \times 10 \times 10 \times 10$, other than 1 000 000?
- $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, other than 1024?

Computer memory and hard drive storage are measured in bytes. Do you know the size of the hard drive of your computer at school? How many bytes does it have? Copy and complete this table of computer storage units.

kilobyte	kB	1 thousand	
megabyte	MB		
	GB	1 thousand million = 1 billion	10^9
terabyte		1 million _____ = 1 trillion	10^{12}

Key ideas

- We can write the product of repeated factors with a **base** number and an index number. This is called writing an expression in **index form**. For example:

$$\underbrace{32\,768}_{\text{number or value}} = \underbrace{8 \times 8 \times 8 \times 8 \times 8}_{\text{expanded form}} = \underbrace{8^5}_{\text{index form}}$$

(base: 8; index or power: 5)

- The **power** or **index** shows the number of repeated factors. 8^5 reads '8 to the power of 5'.
- Numbers in index form are evaluated first in the order of operations.
  For example: $3 + 2 \times 4^2 = 3 + 2 \times 16$
  $= 3 + 32$
  $= 35$
- Note that $2^3 = 2 \times 2 \times 2 = 8$.
  2^3 does *not* mean $2 \times 3 = 6$.

Base The number that is being raised to a power or index

Index form A method of writing numbers that are multiplied by themselves

Power (index) The number that a base is raised to

Exercise 3E

UNDERSTANDING 1–4 4



Choose from the words: *index, expanded, base, power*

1 Write the missing words.

a The product $2 \times 2 \times 2$ is called the _____ form of 8.

b 2^3 is called the _____ form of 8.

c 2^3 reads: 'two to the _____ of 3'.

d In 2^3, the special name for the 2 is the _____ number.

e In 2^3, the special name for the 3 is the _____ or _____.

2 Copy and complete each product of repeated factors.

a $3^2 = 3 \times \square$

b $2^4 = 2 \times \square \times \square \times \square$

c $5^3 = 5 \times \square \times \square$

d $8^5 = \square \times \square \times \square \times \square \times \square$

Example 17 Relating factor form and index form

Copy and complete this table.

Expanded form	Index form	Base	Index or power
$3 \times 3 \times 3 \times 3$	3^4		
$5 \times 5 \times 5 \times 5 \times 5 \times 5$			
		2	5

Solution

Expanded form	Index form	Base	Index or power
$3 \times 3 \times 3 \times 3$	3^4	3	4
$5 \times 5 \times 5 \times 5 \times 5 \times 5$	5^6	5	6
$2 \times 2 \times 2 \times 2 \times 2$	2^5	2	5

Explanation

The factor 3 (base) is repeated 4 times, so 4 is the index or power.

The factor 5 (base) is repeated 6 (index) times.

The base of 2 is the factor and is repeated 5 times.

3 Copy and complete this table.

Expanded form	Index form	Base	Index or power
	7^3		
$5 \times 5 \times 5$			
		2	6
	6^4		

3E

4 Copy and complete the table.

Index form	Base number	Index or power	Value
2^3	2	3	8
5^2			
10^4			
2^7			
1^{12}			
12^1			
0^5			



index or power

$4^2 = 4 \times 4 = 16$

base value

FLUENCY 5–7(½) 5–8(½)

Example 18 Converting to index form

Simplify these products by writing them in index form.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ **b** $3 \times 3 \times 2 \times 3 \times 2 \times 3$

Solution	Explanation
a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$	The factor 5 is repeated six times.
b $3 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^2 \times 3^4$	2 is repeated two times. 3 is repeated four times.

5 Simplify these products by writing in index form.



The base is the factor.
The power is the number of repeats.

a $3 \times 3 \times 3$
b $2 \times 2 \times 2 \times 2 \times 2$
c $15 \times 15 \times 15 \times 15$
d $10 \times 10 \times 10 \times 10$
e 6×6
f $20 \times 20 \times 20$
g $1 \times 1 \times 1 \times 1 \times 1 \times 1$
h $4 \times 4 \times 4$
i 100×100
j $3 \times 3 \times 5 \times 5$
k $2 \times 2 \times 7 \times 7 \times 7$
l $9 \times 9 \times 12 \times 12$
m $8 \times 8 \times 5 \times 5 \times 5$
n $6 \times 3 \times 6 \times 3 \times 6 \times 3$
o $13 \times 7 \times 13 \times 7 \times 7 \times 7$
p $4 \times 13 \times 4 \times 4 \times 7$
q $10 \times 9 \times 10 \times 9 \times 9$
r $2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 2$

Example 19 Expanding expressions in index form

Write each of the following in factor form and find the value.

a 2^4 **b** $2^3 \times 5^2$

Solution	Explanation
a $2^4 = 2 \times 2 \times 2 \times 2$ $= 16$	The digit 2 is repeated four times. Calculate the value.
b $2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5$ $= 8 \times 25$ $= 200$	The digit 2 is repeated three times, then multiply 5 by 5. Calculate the value.

6 Write in expanded form. (Do not find the value.)

a 2^4	**b** 17^2	**c** 9^3	**d** 3^7
e 14^4	**f** 8^8	**g** 10^5	**h** 54^3
i $3^5 \times 2^3$	**j** $4^3 \times 3^4$	**k** $7^2 \times 5^3$	**l** $4^6 \times 9^3$
m 5×7^4	**n** $2^2 \times 3^3 \times 4^1$	**o** $11^5 \times 9^2$	**p** $20^3 \times 30^2$



7 Copy and complete the following.

a $3 \times 2 =$ ___ but $3^2 =$ ___ $\times$ ___ $=$ ___

b $2 \times 4 =$ ___ but $2^4 =$ ___ $\times$ ___ $\times$ ___ $\times$ ___ $=$ ___

c $5 \times 2 =$ ___ but $5^2 =$ ___ $\times$ ___ $=$ ___

d $6 \times 2 =$ ___ but $6^2 =$ ___ $\times$ ___ $=$ ___

8 Write in expanded factor form and find the value.

a 2^5	**b** 8^2	**c** 10^3	**d** $3^2 \times 2^3$
e 10^4	**f** $2^3 \times 5^3$	**g** $1^6 \times 2^6$	**h** $11^2 \times 1^8$

PROBLEM-SOLVING AND REASONING 9, 10 9–12

Example 20 Evaluating expressions with index form

Evaluate:

a $7^2 - 6^2$ **b** $2 \times 3^3 + 10^2 + 1^7$

Solution	Explanation
a $7^2 - 6^2 = 7 \times 7 - 6 \times 6$	Write powers in factor form.
$= 49 - 36$	Do multiplication before subtraction.
$= 13$	
b $2 \times 3^3 + 10^2 + 1^7$	
$= 2 \times 3 \times 3 \times 3 + 10 \times 10$	Write powers in expanded form.
$+ 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$	
$= 54 + 100 + 1$	Do multiplication first, then addition.
$= 155$	

9 Evaluate:

a $3^2 + 4^2$	**b** $2 \times 5^2 - 7^2$	**c** $8^2 - 2 \times 3^3$
d $(9 - 5)^3$	**e** $2^4 \times 2^3$	**f** $2^7 - 1 \times 2 \times 3 \times 4 \times 5$
g $1^4 + 2^3 + 3^2 + 4^1$	**h** $10^3 - 10^2$	**i** $(1^{27} + 1^{23}) \times 2^2$

3E

10 Find the index number for each of the following.

a $16 = 2^{\square}$ **b** $16 = 4^{\square}$ **c** $64 = 4^{\square}$ **d** $64 = 2^{\square}$

e $27 = 3^{\square}$ **f** $100 = 10^{\square}$ **g** $49 = 7^{\square}$ **h** $625 = 5^{\square}$

11 Write one of the symbols <, = or > in the box to make the following statements true.

a $2^6 \square 2^9$ **b** $8^3 \square 8^2$

c $2^4 \square 4^2$ **d** $3^2 \square 4^2$

e $6^4 \square 5^3$ **f** $12^2 \square 3^4$

g $11^2 \square 2^7$ **h** $1^8 \square 2^3$



< means less than.
> means greater than.
The sign points to the smaller number.

12 Five friends receive the same text message at the same time. Each of the five friends then forwards it to five other friends and each of these people also sends it to five other friends. How many people does the text message reach?

The power of email

— 13

13 A chain email is sent to 10 people. Five minutes later, each of them sends it to 10 other people. That is, after 10 minutes, 110 people (10 + 100) will have received the email.



Assume that everyone uses email and everyone can read!

a How many people will have received the email after:

i 15 minutes? **ii** 30 minutes?

b If the email always goes to a new person, how long would it take until everyone in Australia has received the message? (Australia has approximately 24.5 million people.) Round up to the next 5 minute mark.

c How long would it take until everyone in the world has received the message? (The world population is approximately 7 billion people.)



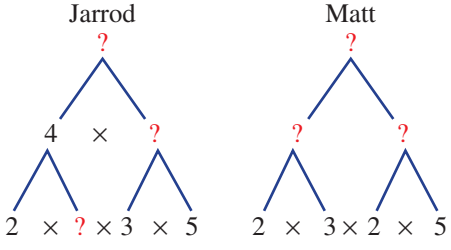
3F Prime factors

Every composite number can be written as the product of prime number factors.

Factor trees help us to work out the prime number factors. When there are repeated factors, we write that product in index form.

Let's start: Factor trees

For homework, Jarrod and Matt each drew a factor tree. Then their little brother rubbed out some of the numbers.



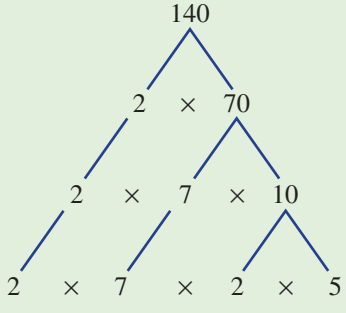
- Can you find the missing numbers? Copy and complete each factor tree.
- What was the boys' homework question?
- How is Jarrod's factor tree different from Matt's?
- What is the same about both factor trees?
- Can you draw a different factor tree that answers the homework question correctly?

Key ideas

- Every composite number can be written as a product of its prime factors.
- We can use a **factor tree** or repeated division to find the prime factors of a composite number.

Factor tree A diagram showing the breakdown of a number into its prime factors

Factor tree



Repeated division with prime factors

2	140
2	70
5	35
7	7
	1

- It is helpful to write the prime factors in ascending (increasing) order, using powers.
  For example: $140 = 2 \times 2 \times 5 \times 7$
  $= 2^2 \times 5 \times 7$

Exercise 3F

UNDERSTANDING 1–4 | 4

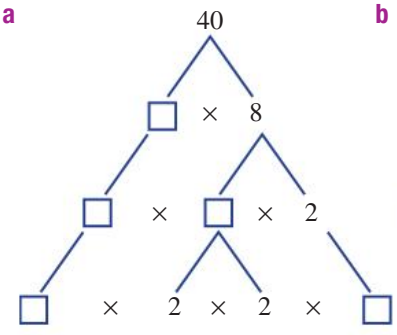
1 Sort the following list of numbers into two groups: composite numbers and prime numbers.
15, 13, 7, 5, 8, 9, 27, 23, 11, 4, 12, 2, 3



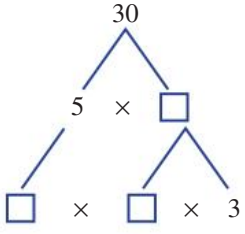
A prime number has two factors: 1 and itself.

2 Copy and complete the following factor trees.

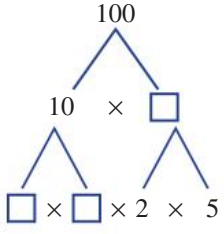
a



b



c



3 Copy and complete these repeated divisions with prime numbers.

a

2	12
2	
	3
	1

b

	30
	15
	5
	1

c

	66
3	33

4 Rewrite these with the prime factors in ascending order. Then write using powers.

a $2 \times 3 \times 3 \times 2 \times 2$ **b** $5 \times 3 \times 3 \times 3 \times 3 \times 5$

c $7 \times 2 \times 3 \times 7 \times 2$ **d** $3 \times 3 \times 2 \times 11 \times 11 \times 2$



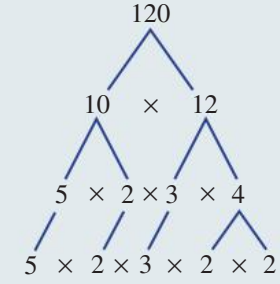
Ascending means smallest to largest.

FLUENCY 5–6(½), 7 | 5–6(½), 7, 8(½)

Example 21 Using a factor tree to find prime factors

Draw a factor tree for the number 120. Then write 120 as the product of prime factors. (Use index form.)

Solution



$120 = 2 \times 2 \times 2 \times 3 \times 5$
$= 2^3 \times 3 \times 5$

Explanation

Start with 120. Find any pair of factors.

$120 = 10 \times 12$

$10 = 5 \times 2$, $12 = 3 \times 4$

All factors are now primes.

Use index form for repeated factors.

The power (3) shows that the factor of 2 is repeated 3 times.

5 For each number, draw a factor tree. Then write the number as a product of prime factors, using powers.

a 72	**b** 24	**c** 38	**d** 44
e 124	**f** 80	**g** 96	**h** 16
i 75	**j** 111	**k** 64	**l** 56



Write prime factors in ascending order. Use powers for repeated factors.

6 For each number, draw a factor tree. Then write the number as a product of prime factors, using powers.

a 600	**b** 800	**c** 5000	**d** 2400
e 1 000 000	**f** 45 000	**g** 820	**h** 690

Example 22 Using repeated division to find prime factors

Use repeated division with prime numbers to find the prime factors of 126. Then write 126 as a product of prime factors with powers.

Solution	Explanation
$2 \mid 126$ $3 \mid 63$ $3 \mid 21$ $7 \mid 7$ 1	$12 \div 2 = 63$ $63 \div 3 = 21$ $21 \div 3 = 7$ $7 \div 7 = 1$
$126 = 2 \times 3 \times 3 \times 7$ $= 2 \times 3^2 \times 7$	Write prime factors in ascending order. $3 \times 3 = 3^2$

7 Use repeated division with prime numbers to find the prime factors of 96. Then write 96 as a product of prime factors with powers.



You could start by dividing 96 by 2.

8 Use repeated division with prime numbers to help you write each of these numbers as a product of prime factors with powers.

a 32	**b** 40	**c** 81	**d** 144
e 120	**f** 500	**g** 1800	**h** 1250

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Match the correct composite number (**a** to **d**) to its set of prime factors (**A** to **D**).

a 120	**A** $2 \times 3 \times 5^2$
b 150	**B** $2^2 \times 3^2 \times 5$
c 144	**C** $2^4 \times 3^2$
d 180	**D** $2 \times 3 \times 2 \times 5 \times 2$



Look for easy ways to multiply

$$\overbrace{2 \times 3}^{10} \times \overbrace{5 \times 5}^{15}$$

10 **a** Express 144 and 96 in prime factor form.

b Determine the HCF of 144 and 96. The prime factor form may help.

3F

11 Draw four different factor trees that each show the prime factors of 24. (For two trees to be different, they must show different combinations of factors, not just the same factors in a different order.)



12 Only one of the following is the correct set of prime factors for 424.

A $2^2 \times 3^2 \times 5$ **B** $2 \times 3^2 \times 5^2$

C 53×8 **D** $2^3 \times 53$

a Explain why **A** and **B** are wrong.

b Why is option **C** wrong?

c Show that option **D** is the correct answer.



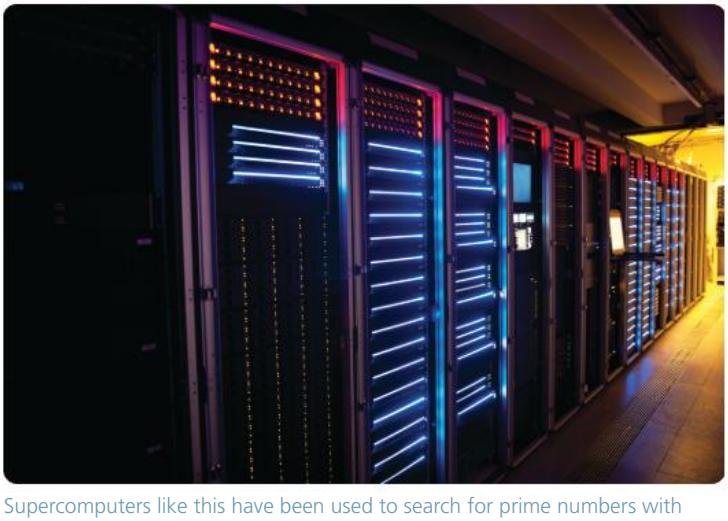
Remember the rules for divisibility.

Four different prime factors

—

13

13 Only 16 composite numbers smaller than 1000 have four prime factors. For example: $546 = 2 \times 3 \times 7 \times 13$.
By considering the prime factor possibilities, see how many of the other 15 composite numbers you can find. Express each of them in prime factor form.



Supercomputers like this have been used to search for prime numbers with millions of digits.

3G Squares and square roots



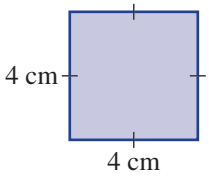






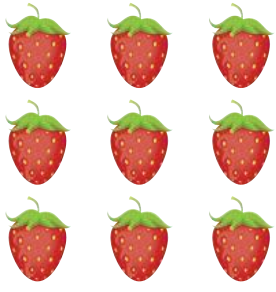
We can picture a square number as the area of a square or the number of dots in a square array.

For example:



Area = 4 cm × 4 cm
= 4 cm^2

16 is a square number.



Number of strawberries = 3 rows of 3
= 3 × 3
= 3^2
= 9

9 is a square number.

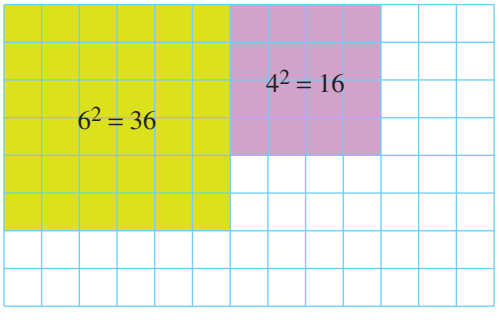
Finding a square root of a number is the opposite of squaring a number. We use the symbol $\sqrt{\ }$ to show the square root of a number.

The pictures above show that $\sqrt{16} = 4$ and $\sqrt{9} = 3$.

Let's start: Shading squares

You will need a sheet of 1 cm grid paper.

- Shade as many different-sized squares as you can fit onto the page.
- Write each area in index form. For example, $6^2 = 36$, $4^2 = 16$, etc.
- Finally, on each square write the side length as the square root of the area. For example, $6 = \sqrt{36}$, $4 = \sqrt{16}$, etc.



Key ideas

- If you multiply a whole number by itself, the result is a **square number**.
  For example: $5^2 = 5 \times 5 = 25$, so 25 is a square number.
  - Square numbers are also known as **perfect squares**.
  - The first 12 square numbers are:

Index form	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
Value	1	4	9	16	25	36	49	64	81	100	121	144

Square number The result of multiplying a number by itself

Perfect square A number with a square root that is a whole number

- The **square root** of a given number multiplied by itself results in the given number.
  - The symbol for square root is $\sqrt{\ }$.
  - Finding a square root of a number is the opposite of squaring a number. For example: $4^2 = 16$, so $\sqrt{16} = 4$.
    We read this as '4 squared equals 16, so, the square root of 16 equals 4'.

Square root (of a given number) The number that, when multiplied by itself, gives the number

Square root form	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$
Value	1	2	3	4	5	6	7	8	9	10	11	12

- Order of operations
  - Squares are powers, so evaluate first. For example: $2 \times 3^2 + 4 = 2 \times 9 + 4$
  - Square roots are like brackets. For example: $\sqrt{16 + 9} = \sqrt{25} = 5$

Exercise 3G

UNDERSTANDING 1–4 4



Look back at the **Key ideas**.

1 Copy and complete this table of square numbers.

Index form	1^2			4^2	5^2		7^2		9^2	
Value		4	9			36		64		100

2 Copy and complete this table of square roots.

Square root form	$\sqrt{1}$	$\sqrt{4}$		$\sqrt{16}$		$\sqrt{36}$			$\sqrt{81}$	
Value			3		5		7	8		10

Example 23 Finding squares and square roots

Write the value of each of these.

a 4^2 **b** 7 squared **c** $(3)^2$ **d** 5 to the power of 2
e square root of 36 **f** side length of square with area 64 cm^2

Solution	Explanation
a $4^2 = 16$	$4^2 = 4 \times 4$
b 7 squared = 49	7 squared is the same as 7^2
c $(3)^2 = 9$	$(3)^2 = 3 \times 3 = 9$
d 5 to the power 2 = 25	5 to the power 2 is the same as 5^2
e square root of 36 = 6	$\sqrt{36} = 6$ because $6 \times 6 = 36$
f side length = 8 cm	side length = $\sqrt{64}$ and $8 \times 8 = 64$

3 Evaluate:

a 6^2 **b** 5 squared

c $(11)^2$ **d** 10 to the power of 2

e 7^2 **f** 12×12

4 Evaluate:

a $\sqrt{25}$ **b** square root of 16 **c** $\sqrt{100}$

d the side length of a square that has an area of 49 cm^2

FLUENCY 5–6(½) 5–7(½)

5 Find the value of:

a 8^2 **b** 7^2

c 1^2 **d** 12^2

e 3^2 **f** 15^2

g 5^2 **h** 0^2

i 11^2 **j** 100^2

k 30^2 **l** 40^2



$8^2 = 8 \times 8 = 64$

Example 24 Evaluating square roots

Find the value of:

a $\sqrt{64}$ **b** $\sqrt{1600}$

Solution	Explanation
a $\sqrt{64} = 8$	$\sqrt{64} = 8$ because $8 \times 8 = 64$
b $\sqrt{1600} = 40$	$\sqrt{1600} = 40$ because $40 \times 40 = 1600$

6 Find the value of:

a $\sqrt{25}$ **b** $\sqrt{9}$

c $\sqrt{1}$ **d** $\sqrt{121}$

e $\sqrt{0}$ **f** $\sqrt{81}$

g $\sqrt{49}$ **h** $\sqrt{16}$

i $\sqrt{4}$ **j** $\sqrt{144}$

k $\sqrt{400}$ **l** $\sqrt{169}$

m $\sqrt{2500}$ **n** $\sqrt{6400}$

o $\sqrt{8100}$ **p** $\sqrt{729}$



For $\sqrt{25}$, think: $\square \times \square = 25$

3G

Example 25 Evaluating expressions involving squares and square roots

Evaluate:

a $\sqrt{64}+\sqrt{36}$ **b** $\sqrt{8^2+6^2}$ **c** $3^2-\sqrt{9}+1^2$

Solution	Explanation
a $\sqrt{64}+\sqrt{36}=8+6$ $=14$	Find square roots first. Then add.
b $\sqrt{8^2+6^2}=\sqrt{64+36}$ $=\sqrt{100}$ $=10$	The square root sign is like a bracket: $\sqrt{(8^2+6^2)}=\sqrt{(8\times 8+6\times 6)}$ Multiply and add to give: $\sqrt{64+36}=\sqrt{100}$
c $3^2-\sqrt{9}+1^2=9-3+1$ $=7$	Evaluate squares and square roots first. $3^2=3\times 3, \sqrt{9}=3, 1^2=1\times 1$

7 Evaluate:

a $\sqrt{9}+\sqrt{16}$ **b** $\sqrt{3^2+4^2}$
c $\sqrt{9}\times\sqrt{16}$ **d** $3^2+5^2-\sqrt{16}$
e 4×4^2 **f** $8^2-0^2+1^2$
g $1^2\times 2^2\times 3^2$ **h** $\sqrt{5^2-3^2}$
i $\sqrt{81}-3^2$ **j** $6^2\div 2^2\times 3^2$
k $\sqrt{9}\times\sqrt{64}\div\sqrt{36}$ **l** $\sqrt{12^2+5^2}$



Remember the order of operations.

PROBLEM-SOLVING AND REASONING 8, 9 8–11

8 This arrangements of dots shows that 9 is a square number.



a Show, using dots, that 6 is *not* a square number.
b Show, using dots, that 16 is a square number.

9 List all the square numbers between 50 and 101.

10 List all the square numbers between 101 and 200. Hint: There are only four.

11 **a** Show that $3^2+4^2=5^2$.
b Does $5^2+6^2=7^2$?
c Does $6^2+8^2=10^2$?
d Find some other true sums of square numbers like the one in part **a**.

Number dot patterns

— 12

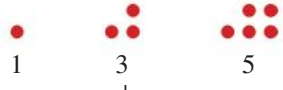
12 For each of the following:

i Copy the dot pattern and draw the next three terms.

ii How many dots are added each time?

iii What patterns did you notice?

a odd numbers



1 3 5

b even numbers



2 4 6

c square numbers



1 4 9

d triangular numbers



1 3 6

iv Can you see a connection between triangular numbers and square numbers?

Progress quiz

3A **1** Find the complete set of factors for each of these numbers.
- **a** 20
- **b** 36

3A **2** Write down the first five multiples for each of these numbers.
- **a** 12
- **b** 21

3B **3** Find the highest common factor (HCF) of the following numbers.
- **a** 24 and 40
- **b** 35 and 70
- **c** 18 and 42

3B **4** Find the lowest common multiple (LCM) of the following numbers.
- **a** 8 and 12
- **b** 11 and 7
- **c** 3, 5 and 10

3C ★ **5** Which of the numbers 75, 14, 141, 52, 88, 1234 are divisible by:
- **a** 2?
- **b** 3?
- **c** 4?
- **d** 5?

3C ★ **6** Work out whether the following calculations are possible without leaving a remainder.
- **a** $32\,689 \div 3$
- **b** $456\,336 \div 8$

3D **7** Which of these numbers are prime (P) and which are composite (C)?
15, 23, 31, 39, 51, 80, 91

3D **8** Find the prime numbers that are factors of:
- **a** 24
- **b** 72

3E **9** Simplify these products by writing them in index form.
- **a** $4 \times 4 \times 4 \times 4 \times 4$
- **b** $6 \times 5 \times 5 \times 6 \times 5 \times 5 \times 6$

3E **10** Write each of the following in expanded form and find the value.
- **a** 3^4
- **b** $2^3 \times 5^2$
- **c** $6^2 - 2^3$

3F **11** Write the following numbers as a product of their prime factors, using powers.
- **a** 40
- **b** 108
- **c** 128

3G **12** Find the value of:
- **a** 13^2
- **b** 30^2
- **c** $\sqrt{49}$
- **d** $\sqrt{2500}$
- **e** $\sqrt{36} \times \sqrt{9}$
- **f** $\sqrt{10^2 - 8^2}$

3H Number patterns

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Number patterns can be found in all sorts of natural and man-made situations. For example, if bacteria cells divide in two every hour, then the number of bacteria follows the pattern 1, 2, 4, 8, 16, 32, 64, 128, and so on. A scientist could use this pattern to predict the number of bacteria after a certain time.

It is helpful to find the rule that describes a number pattern. Entering rules into a spreadsheet makes it easy to perform calculations with large numbers. Scientists, accountants and many other people use number patterns and rules to help them analyse data and make predictions.



Many careers involve using mathematical patterns and rules.

Let's start: Which rule is which?

- Work with a partner to match each of these number patterns to the correct rule. Discuss the thinking you used.

Number pattern		Rule	
A	7, 12, 17, 22, 27, …	**P**	Start with 96 and keep dividing by 2.
B	96, 48, 24, 12, 6, …	**Q**	Start with 7, add 5, add 6, add 7, etc.
C	96, 92, 88, 84, …	**R**	Start with 7 and keep adding 5.
D	7, 12, 18, 25, 33, 42, …	**S**	Start with 96, add 4, take 10, etc.
E	96, 100, 90, 94, 84, 88, 78, …	**T**	Start with 96 and keep subtracting 4.

- Make up some number patterns of your own. Then swap patterns with another group. See if you can work out each other's rules.

Key ideas

- A list of numbers arranged in order according to some rule is called a number pattern or a **sequence**.
- Each separate number in the sequence is called a **term**.
- To find the terms of a sequence, follow the pattern rule. For example: the rule 'start with 3 and add 2 to each term' gives 3, 5, 7, 9, 11, . . .
- To find the pattern rule for a sequence, ask:
  - Is the sequence increasing or decreasing by a fixed amount?

$$25, \overset{+3}{\rightarrow} 28, \overset{+3}{\rightarrow} 31, \overset{+3}{\rightarrow} 34, \ldots \qquad 50, \overset{-5}{\rightarrow} 45, \overset{-5}{\rightarrow} 40, \overset{-5}{\rightarrow} 35, \ldots$$

  - Is each term being multiplied or divided by the same amount?

$$4, \overset{\times 3}{\rightarrow} 12, \overset{\times 3}{\rightarrow} 36, \overset{\times 3}{\rightarrow} 118, \ldots \qquad 64, \overset{\div 2}{\rightarrow} 32, \overset{\div 2}{\rightarrow} 16, \overset{\div 2}{\rightarrow} 8, \ldots$$

Sequence A set of numbers ordered according to a rule

Term One of the numbers in a sequence

Exercise 3H

UNDERSTANDING 1–4 | 4



Look back at the **Key ideas**.

1 Write the missing words/number.

a When a list of numbers follows a pattern that can be described by a rule, it is called a _____.

b Each separate number in a sequence is called a _____.

c The missing number in the sequence ___, 2, 4, 6, 8 is ___.

d The missing number in the sequence ___, 2, 4, 8, 16 is ___.

2 List the next three terms in each of these sequences.

a +4 +4 +4
8, 12, 16, __, __, __

b −5 −5 −5
100, 95, 90, __, __, __

c ÷2 ÷2 ÷2
64, 32, 16, __, __, __

d ×2 ×2 ×2
5, 10, 20, __, __, __

3 List the first five terms of the sequence for each of these pattern rules.

a Start with 8 and keep adding 3.

b Start with 32 and keep subtracting 1.

c Start with 52 and keep subtracting 4.

d Start with 123 and keep adding 7.

4 List the first five terms of the following number patterns.

a Start with 3 and keep multiplying by 2.

b Start with 5 and keep multiplying by 4.

c Start with 240 and keep dividing by 2.

d Start with 625 and keep dividing by 5.

FLUENCY 5–7(½) | 5–8(½)

Example 26 Finding patterns that change by a fixed amount

Find the next three terms for these number patterns.

a 6, 18, 30, 42, ___, ___, ___

b 99, 92, 85, 78, ___, ___, ___

Solution

a 54, 66, 78

Explanation

The same number is being added to each term.
Keep adding 12 to find the next three terms.

+12 +12 +12 +12 +12 +12
6, 18, 30, 42, 54, 66, 78

b 71, 64, 57

Keep subtracting 7 to find the next three terms.

−7 −7 −7 −7 −7 −7
99, 92, 85, 78, 71, 64, 57

5 Find the next three terms for these sequences.

a 3, 8, 13, 18, ___, ___, ___
b 4, 14, 24, 34, ___, ___, ___
c 26, 23, 20, 17, ___, ___, ___
d 106, 108, 110, 112, ___, ___, ___
e 63, 54, 45, 36, ___, ___, ___
f 9, 8, 7, 6, ___, ___, ___
g 101, 202, 303, 404, ___, ___, ___
h 75, 69, 63, 57, ___, ___, ___



Look for a fixed number that is added or subtracted.

Example 27 Finding patterns that involve multiplication or division

Find the next three terms for the following number patterns.

a 2, 6, 18, 54, ___, ___, ___ **b** 256, 128, 64, 32, ___, ___, ___

Solution	Explanation
a 162, 486, 1458	Each term is being multiplied by the same number. Keep multiplying by 3 to find the next three terms. ×3 ×3 ×3 ×3 ×3 ×3 2, 6, 18, 54, 162, 486, 1458
b 16, 8, 4	Keep dividing by 2 to find the next three terms. ÷2 ÷2 ÷2 ÷2 ÷2 ÷2 256, 128, 64, 32, 16, 8, 4

6 Find the next three terms for the following number patterns.

a 2, 4, 8, 16, ___, ___, ___
b 5, 10, 20, 40, ___, ___, ___.
c 96, 48, 24, ___, ___, ___
d 1215, 405, 135, ___, ___, ___
e 11, 22, 44, 88, ___, ___, ___
f 7, 70, 700, 7000, ___, ___, ___
g 256, 128, 64, 32, ___, ___, ___
h 1216, 608, 304, 152, ___, ___, ___



Is each term being multiplied or divided by the same number?

7 Find the missing numbers in each of the following number patterns.

a 62, 56, ___, 44, 38, ___, ___
b 15, ___, 35, ___, ___, 65, 75
c 4, 8, 16, ___, ___, 128, ___
d 3, 6, ___, 12, ___, 18, ___
e 88, 77, 66, ___, ___, ___, 22
f 2997, 999, ___, ___, 37



These patterns can involve + or − or × or ÷.

3H

Example 28 Describing patterns in words

For each of these sequences, write the pattern rule in words.

a 2, 10, 50, 250, …
b 6, 10, 14, 18, …
c 32, 16, 8, 4, …
d 32, 28, 24, 20, …

Solution	Explanation
a Start with 2 and multiply each term by 5.	×5 ×5 ×5 2, 10, 50, 250, ...
b Start with 6 and add 4 to each term.	+4 +4 +4 6, 10, 14, 18, ...
c Start with 32 and divide each term by 2.	÷2 ÷2 ÷2 32, 16, 8, 4, ...
d Start with 32 and subtract 4 from each term.	−4 −4 −4 32, 28, 24, 20, ...

8 Use words to write the pattern rule for each sequence.

a 19, 17, 15, 13, …
b 48, 24, 12, 6, …
c 50, 56, 62, 68, …
d 1, 3, 9, …
e 625, 125, 25, 5, …
f 75, 72, 69, 66, …

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Write the next three terms in each of the following sequences.

a 3, 5, 8, 12, ___, ___, ___
b 1, 2, 4, 7, 11, ___, ___, ___
c 10, 8, 11, 9, 12, ___, ___, ___
d 25, 35, 30, 40, 35, ___, ___, ___



Look at how much each term increases or decreases.

10 A frog has fallen to the bottom of a well that is 6 metres deep.

- On the first day the frog climbs 3 metres up the wall of the well.
- On the second day it slides back 2 metres.
- On the third day it climbs up 3 metres.
- On the fourth day it slides back 2 metres.



Draw a diagram of the well. Use arrows to show the movement of the frog.

The frog continues following this pattern until it reaches the top of the well and hops away.

a Write a sequence of numbers to show the frog's height above the bottom of the well at the end of each day.
b How many days does it take the frog to get out of the well?



11 For each of the following sequences, write the pattern rule.

a 4, 12, 36, 108, 324, …
b 19, 17, 15, 13, 11, …
c 212, 223, 234, 245, 256, …
d 8, 10, 13, 17, 22, …
e 64, 32, 16, 8, 4, …
f 5, 15, 5, 15, 5, …
g 2, 3, 5, 7, 11, …
h 75, 72, 69, 66, 63, …

12 Copy and complete each of the following. Give the special name for each type of numbers.

a 1, 4, 9, 16, 25, 36, ___, ___, ___
b 1, 1, 2, 3, 5, 8, 13, ___, ___, ___
c 1, 8, 27, 64, 125, ___, ___, ___
d 2, 3, 5, 7, 11, 13, 17, ___, ___, ___
e 4, 6, 8, 9, 10, 12, 14, 15, ___, ___, ___
f 121, 131, 141, 151, ___, ___, ___



Choose from: *composite numbers, cube numbers, even numbers, Fibonacci numbers, negative numbers, odd numbers, palindromes, prime numbers, square numbers, triangular numbers*

Human pyramids

— 13

13 When making a human pyramid, each row has one less person than the row below. The pyramid is complete when there is a row of only one person on the top.

Write down a number pattern for a human pyramid with 10 students on the bottom row. How many people are needed to make this pyramid?



Draw a diagram.



3I Patterns with shapes and numbers

Interactive

Widgets

HOTsheets

Walkthrough

Repeated geometric shapes form interesting spatial patterns. Architects often use spatial patterns in the design of buildings. Artists also use repeated geometric shapes in designs to be printed on curtains, tiles and wallpaper.



A pattern rule can be used to find the number of pickets for a given number of fence posts.

Let's start: Stick patterns

Copy these shapes, using matchsticks or toothpicks. Then build the next three shapes in the pattern.



How many sticks would you need to make the shape with:

- 10 triangles?
- 100 triangles?

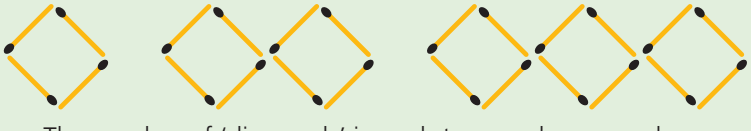
If you know the number of triangles, how could you find the number of sticks? Discuss this with a partner and then write your answer.

Use similar steps to explore the number of squares in the following pattern.



Key ideas

- A spatial pattern is a sequence of geometrical shapes. For example:



  - The number of 'diamonds' in each term makes a number sequence.
  - The number of sticks in each term makes another number sequence.
- A table of values shows the number of shapes and the number of sticks.

Number of shapes	1	2	3	4	5
Number of sticks	4	8	12	16	20

- A pattern rule tells how many sticks are needed for a certain number of shapes.
  For example: number or sticks = 4 × number of shapes

Exercise 3I

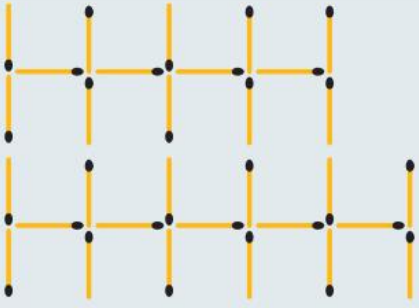
UNDERSTANDING 1–3 3

Example 29 Drawing spatial patterns

Draw the next two shapes in this spatial pattern.



Solution

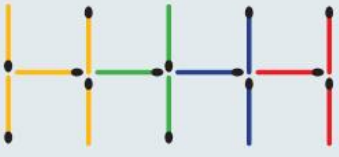


Explanation

Look for the pattern.

5 sticks are used to start with.

Then 3 sticks are added each time.



1 Jack used matchsticks to begin a pattern of rectangles.

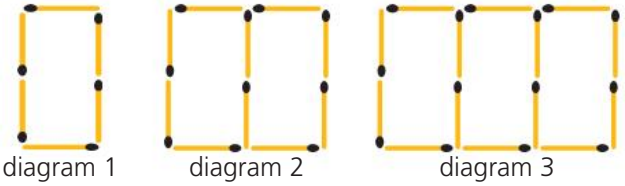


diagram 1 diagram 2 diagram 3

Write the missing words or numbers for each of these.

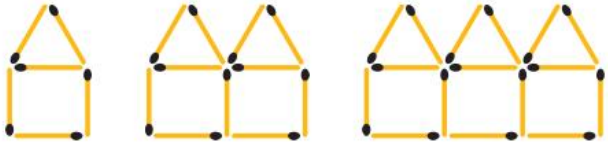
a The shape of the first diagram is called a _____.

b Diagram 1 has ___ rectangle, diagram 2 has ___ rectangles and diagram 3 has ___ rectangles.

c Diagram 1 has ___ sticks, diagram 2 has ___ sticks and diagram 3 has ___ sticks.

d These diagrams follow a sequence that is called a _____ pattern.

2 Jane used matchsticks to make a spatial pattern of houses.



How many extra sticks are needed to change 1 house into 2 houses?



Copy and complete this table.

Number of houses	1	2	
Number of sticks			

3I

3 Draw the next two terms for each of these spatial patterns.

a



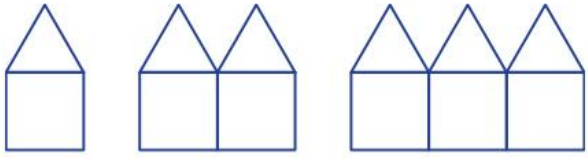
b





What shape is being added each time to make the next diagram?

c



d



FLUENCY 4, 5 4–6

Example 30 Finding a general rule for a spatial pattern

a Draw the next two shapes in this spatial pattern.



b Complete the table.

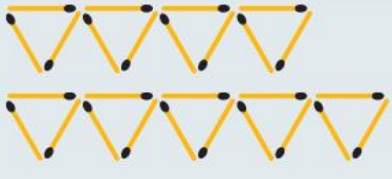
Number of triangles	1	2	3	4	5
Number of sticks	3				

c Complete this pattern rule: number of sticks $\times$ ☐ = number of triangles

d How many sticks would you need for 20 triangles?

Solution

a



b

Number of triangles	1	2	3	4	5
Number of sticks	3	6	9	12	15

c number of sticks = 3 $\times$ number of triangles

d number of sticks = 3 $\times$ 20 triangles = 60 sticks

Explanation

Follow the pattern by adding one triangle each time.

An extra 3 sticks are required to make each new triangle.

3 sticks are required per triangle.

20 triangles $\times$ 3 sticks each

4 **a** Draw the next two shapes for this spatial pattern.





For part **c**, check that your pattern rule works for all values in the table.

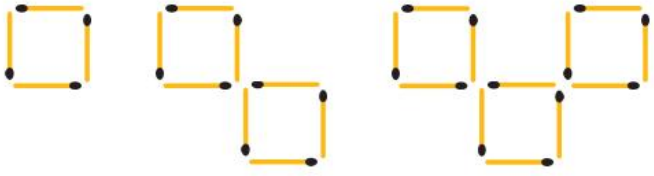
b Copy and complete this table.

Number of crosses	1	2	3	4	5
Number of sticks					

c Copy and complete this pattern rule: number of sticks = ☐ × number of crosses

d How many sticks would you need for 10 crosses?

5 **a** Draw the next two shapes for this spatial pattern.



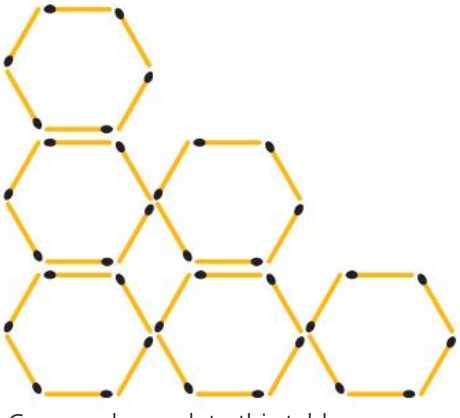
b Copy and complete this table.

Number of squares	1	2	3	4	5
Number of sticks					

c Copy and complete the pattern rule: number of sticks = ☐ × number of squares

d How many sticks would you need for 12 squares?

6 **a** Draw the next two shapes for this spatial pattern.



b Copy and complete this table.

Number of hexagons	1	2	3	4	5
Number of sticks					



How many extra sticks would you need to add another hexagon?

c Copy and complete the pattern rule:
number of sticks = ☐ × number of hexagons

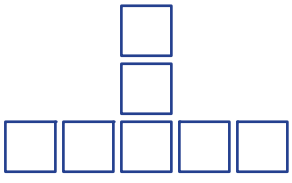
d How many sticks would you need for 20 hexagons?

31

PROBLEM-SOLVING AND REASONING 7–9 9–12

7 List the shapes (**A** to **D**) in the correct order to make a spatial pattern. (Start with the smallest shape.) Then draw the next shape in the sequence.

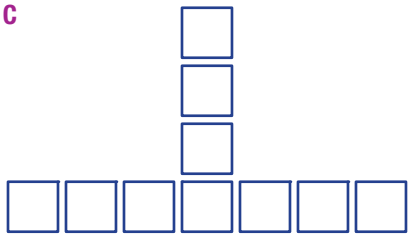
A



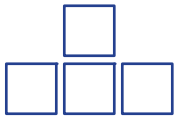
B



C



D



Example 31 Finding more challenging rules

a Draw the next two shapes for this spatial pattern.



b Copy and complete the table.

Number of squares	0	1	2	3	4
Number of sticks	1	1 + □ × 1 = □	1 + □ × 2 = □	1 + □ × 3 = □	1 + □ × 4 = □

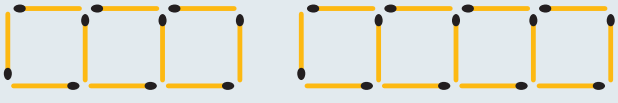
c Copy and complete the rule for the pattern:
number of sticks = 1 + □ × number of squares

d How many sticks are needed to make 30 squares this way?

e How many squares could be made from 25 sticks?

Solution | **Explanation**

a



Add 3 sticks at a time to complete each new square.

b

Number of squares	0	1	2	3	4
Number of sticks	1	1 + 3 × 1 = 4	1 + 3 × 2 = 7	1 + 3 × 3 = 10	1 + 3 × 4 = 13

Count the squares. Complete the calculations, then count sticks in the diagrams to check.

c number of sticks = 1 + 3 × number of squares

The number of sticks is 1 more than 3 times the number of squares.

d 91 sticks

$1 + 3 \times 30 = 91$

e 8 squares

$25 - 1 = 24$,

$24 \div 3 = 8$

8 **a** Draw the next two shapes for this spatial pattern.





How many extra sticks are needed to make 1 stick into a triangle?

b Copy and complete this table.

Number of triangles	0	1	2	3	4
Number of sticks	1	1 + ☐ = ☐	1 + ☐ × 2 = ☐	1 + ☐ × 3 = ☐	1 + ☐ × 4 = ☐

c Copy and complete the rule for this pattern:
number of sticks = 1 + ☐ × number of triangles

d How many sticks are needed to make 12 triangles this way?

e How many triangles could be made from 81 sticks?

9 **a** Draw the next two shapes in this spatial pattern.





Copy the last shape and add more sticks to make the next shape.

b Copy and complete this table.

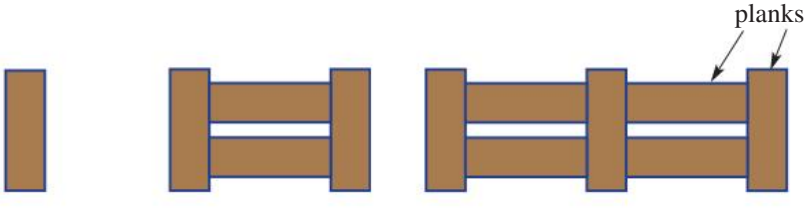
Number of shapes	0	1	2	3	4
Number of sticks	1	1 + ☐ = ☐	1 + ☐ × 2 = ☐	1 + ☐ × 3 = ☐	1 + ☐ × 4 = ☐

c Copy and complete the rule for this pattern:
number of sticks = 1 + ☐ × number of shapes

d How many sticks are needed to make 20 shapes this way?

e How many shapes could be made from 86 sticks?

10 **a** Draw the next two shapes in this spatial pattern.



0 fence sections 1 fence section 2 fence sections

b Copy and complete this table.

Number of fence sections	0	1	2	3	4
Number of planks	1	1 + ☐ = ☐	1 + ☐ × 2 = ☐	1 + ☐ × 3 = ☐	1 + ☐ × 4 = ☐

c Copy and complete the pattern rule:
number of planks = 1 + ☐ × number of fence sections

d How many planks would you need to make 9 fence sections?

e How many fence sections can be made from 43 planks?

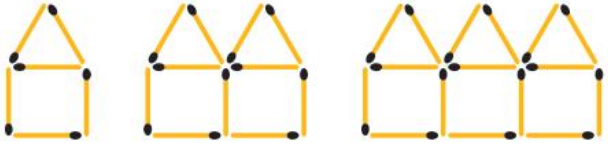
3I

11 Which rule correctly describes this spatial pattern?



A number of sticks = 7 × number of 'hats'
B number of sticks = 7 × number of 'hats' + 1
C number of sticks = 6 × number of 'hats' + 2
D number of sticks = 6 × number of 'hats'

12 Which rule correctly describes this spatial pattern?



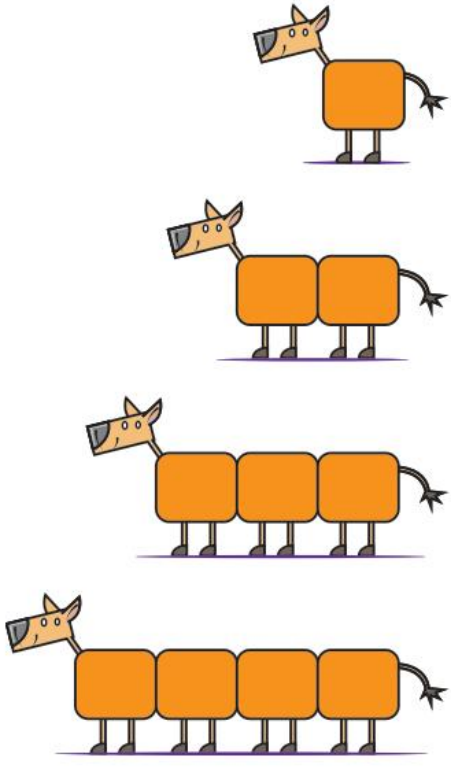
A number of sticks = 5 × number of houses + 1
B number of sticks = 6 × number of houses + 1
C number of sticks = 6 × number of houses
D number of sticks = 5 × number of houses

Design your own spatial pattern

13

13 Design a spatial pattern to fit the following number patterns.

a 4, 7, 10, 13, …
b 4, 8, 12, 16, …
c 3, 5, 7, 9, …
d 3, 6, 9, 12, …
e 5, 8, 11, 14, …
f 6, 11, 16, 21, …



3J Tables and rules









In the last section, we investigated rules for spatial patterns. Rules are also useful for many other everyday situations.

It can be helpful to think of a rule as a 'machine'. You feed in one number (the input), and another number (the output) comes out. For example, Mary was 3 years old when her brother Tim was born. If you know Tim's age (the input), what is a rule for finding Mary's age (the output)?

Showing some values in a table makes it is easy to 'see' the rule.

Tim's age (*input*)	0	1	7	3
Mary's age (*output*)	3	4	10	6

+3

Rule: Mary's age = Tim's age + 3

$output = input + 3$

Let's start: What's the story?

Each of the following stories tells how an input and output are related.

Story	*input*	*output*
1 Connor is 5 years younger than his brother Declan.	Connor's age	Declan's age
2 Liam earns $5 for every car he washes.	number of cars washed	amount ($) earned
3 Jayce and 4 friends share some lollies equally.	total number of lollies	number of lollies each person gets

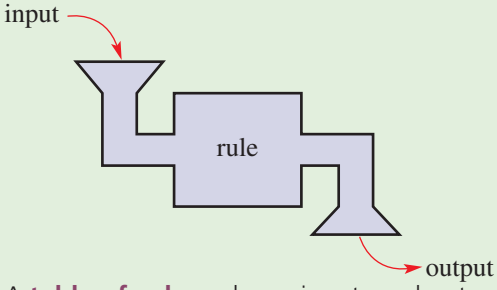
- Which story matches the following table of input and output values? How did you decide?

input	2	1	7	3	6	?
output	10	5	35	15	?	50

- How would you find the missing values?
- Pick one of the other stories and make up your own table of values.

Key ideas

- A rule shows the relationship between two amounts that can vary. It is used to calculate the **output** (answer) from the **input** (starting number).



- A **table of values** shows inputs and outputs. To make a table of values:
  - choose some input values
  - use the rule to calculate the output values.

Output The result from applying a rule to the input

Input A number that is changed according to some rule

Table of values Pairs of numbers related by a rule, often used for plotting points on a number plane

■ To find the rule from a table of values, try different operations (+, −, ×, ÷) until you find a rule that works for *all* the values. For example:

input	*output*
5	15
8	24
12	36

To get the outputs in the table above, has the same number been:
- added to each input? (no)
- subtracted from each input? (no)
- multiplied by each input? (yes)

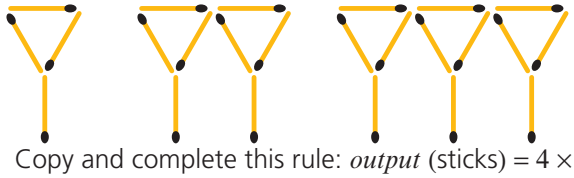
The rule is: $output = 3 \times input$

Exercise 3J

UNDERSTANDING 1–4 4

1 Here is a spatial pattern. Each 'flower' is made with 4 sticks.

1 flower 2 flowers 3 flowers



a Copy and complete this rule: *output* (sticks) = 4 × *input* (_____)

b Copy and complete this table of values.

Number of flowers (*input*)	1	2	3	4	5
Number of sticks (*output*)					

×4

2 Ebony is 3 years older than José.

a Copy and complete this rule: *output* (_____) = *input* (José's age) + 3

b Copy and complete this table of values.

José's age (*input*)	1	3	7	12	15
Ebony's age (*output*)					

+3

3 Jake earns $8 an hour.

a Copy and complete this rule: *output* (amount earned) = 8 × _____ (hours worked)

b Copy and complete this table of values.

Hours worked (*input*)	1	2	3	4	5
Amount earned (*output*)					

×8

Example 32 Substituting an input value to find the output value

Use each rule to find the output when the input = 12.

a $output = input \times 2$
b $output = input - 2$
c $output = input + 2$
d $output = input \div 2$

Solution	Explanation
a $output = 12 \times 2$ $= 24$	Replace *input* by 12, then multiply by 2.
b $output = 12 - 2$ $= 10$	Replace *input* by 12, then subtract 2.
c $output = 12 + 2$ $= 14$	Replace *input* by 12, then add 2.
d $output = 12 \div 2$ $= 6$	Replace *input* by 12, then divide by 2.

4 Use each rule to find the output when the input = 8.

a $output = input \times 2$
b $output = input - 2$
c $output = input + 2$
d $output = input \div 2$

FLUENCY 5, 6 5–7

Example 33 Completing a table of values

Complete each table for the given rule.

a $output = input - 2$

input	3	5	7	12	20
output					

b $output = (3 \times input) + 1$

input	4	2	9	12	0
output					

Solution

a $output = input - 2$

input	3	5	7	12	20
output	1	3	5	10	18

Explanation

Put each *input* value in turn into the rule.

e.g. When *input* is 3:
$output = 3 - 2 = 1$

b $output = (3 \times input) + 1$

input	4	2	9	12	0
output	13	7	28	37	1

Put each *input* value in turn into the rule.

e.g. When *input* is 4:
$output = (3 \times 4) + 1 = 13$

3J

5 Copy and complete each table for the given rule.



Substitute each input number into the rule.

a $output = input + 3$

input	4	5	6	7	10
output					

b $output = input \times 2$

input	5	1	3	21	0
output					

c $output = input - 8$

input	11	18	9	44	100
output					

d $output = input \div 5$

input	5	15	55	0	100
output					

6 Copy and complete each table for the given rule.



Remember to calculate brackets first.

a $output = (10 \times input) - 3$

input	1	2	3	4	5
output					

b $output = (input \div 2) + 4$

input	6	8	10	12	14
output					

c $output = (3 \times input) + 1$

input	5	12	2	9	0
output					

d $output = (2 \times input) - 4$

input	3	10	11	7	50
output					

Example 34 Finding a rule from a table of values

Find the rule for each of these tables of values.

a

input	3	4	5	6	7
output	12	13	14	15	16

b

input	1	2	3	4	5
output	7	14	21	28	35

Solution

a $output = input + 9$

Explanation

Each *output* value is 9 more than the *input* value.

b $output = input \times 7$ or $output = 7 \times input$

By inspection, it can be observed that each *output* value is 7 times bigger than the *input* value.

7 State the rule for each of these tables of values.



The same rule must work for each input/output pair in a table.

a

input	4	5	6	7	8
output	5	6	7	8	9

b

input	1	2	3	4	5
output	4	8	12	16	20

c

input	10	8	3	1	14
output	21	19	14	12	25

d

input	6	18	30	24	66
output	1	3	5	4	11

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 Copy and complete the missing values in the table and state the rule.

input	4	10	13	24			5	11	2
output			39		42	9	15		6

9 Match each rule (**A** to **D**) with the correct table of values (**a** to **d**).

Rule **A**: $output = input - 5$ Rule **B**: $output = input + 1$

Rule **C**: $output = 4 \times input$ Rule **D**: $output = 5 + input$



The rule must be true for each input number in the table.

a

input	20	14	6
output	15	9	1

b

input	8	10	12
output	13	15	17

c

input	4	5	6
output	5	6	7

d

input	4	3	2
output	16	12	8

10 When Zac was born, his grandfather placed \$100 in a special account to save for his education. He adds \$50 to the account every time Zac has a birthday.

a Copy and complete this table for Zac's birthday account:

Zac's age in years (*input*)	0	1	2	3
Amount (\$) in account (*output*)				

b Copy and complete this rule for Zac's birthday account:

$output = \square \times input + \square$

c How much will be in the account when Zac turns 18?

11 Cindy has \$64 saved so far for the school ski trip. She has just started working at the local vet's, helping to clean out pens and feed the animals. Cindy gets paid \$8 an hour and saves all her wages towards the ski trip.

a Copy and complete this table for Cindy's savings:

Hours worked (*input*)	0	2	5	10
Cindy's total savings (*output*)				

b Copy and complete this rule for Cindy's savings:

$output = \square \times input + \square$



For part **c**, first work out how much more Cindy needs to earn.

c Cindy wants to save \$200 to pay for the ski trip. How many hours will she need to work?

12 Complete these two different rules so that they each give an output of 7 when the input is 3.

a $output = \square \times input + \square$

b $output = \square \times input - \square$



3J

Finding harder rules

— 13, 14

13 The following rules all involve two operations. Find the rule for each of these tables of values.

a *output* = ☐ × *input* − ☐

input	4	5	6	7	8
output	5	7	9	11	13

b *output* = ☐ × *input* + ☐

input	1	2	3	4	5
output	5	9	13	17	21

c *output* = ☐ × *input* − ☐

input	10	8	3	1	14
output	49	39	14	4	69

d *output* = ☐ *input* ÷ ☐ + ☐

input	6	18	30	24	66
output	3	5	7	6	13

e *output* = ☐ × *input* + ☐

input	4	5	6	7	8
output	43	53	63	73	83

f *output* = ☐ × *input* − ☐

input	1	2	3	4	5
output	0	4	8	12	16

14 Use a spreadsheet to make each of the tables in part **a**. Each time, enter a formula so that the computer calculates all the output values.
Here is an example of the formulas for **a** above. Try extending your tables by using other input numbers.

	A	B
1	13 (i)	
2	input	output
3	4	=2*A3-3
4	5	=2*A4-3
5	6	=2*A5-3
6	7	=2*A6-3
7	8	=2*A7-3
8		



When entering formulas:
- always start with =
- use * for ×
- instead of typing 'A3' just click on cell A3
- to copy the formula into other cells, fill down by dragging.

3K The number plane and graphs

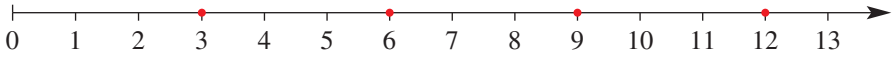
Interactive

Widgets

HOTsheets

Walkthrough

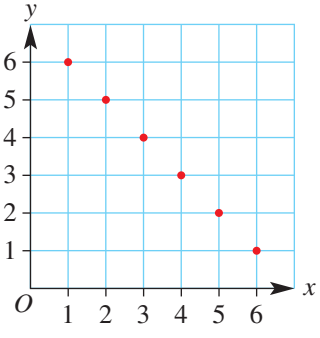
In earlier sections we looked at number sequences and spatial patterns. We used rules and tables of values to describe them. Another way of showing a pattern is by plotting points. For example, we could use a number line to show the simple pattern $3, 6, 9, 12, \ldots$



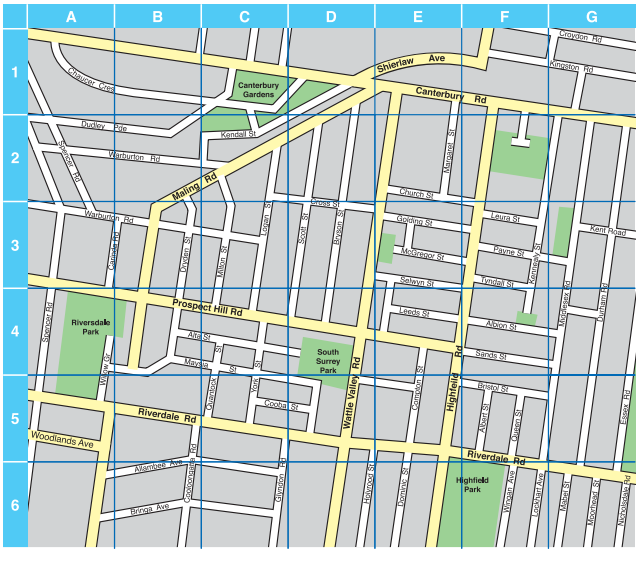
However, when we work with two sets of values (inputs and outputs) we need two dimensions. Instead of a number line, we use a number plane.

Let's start: Locating points

This grid shows a pattern of points on a number plane.

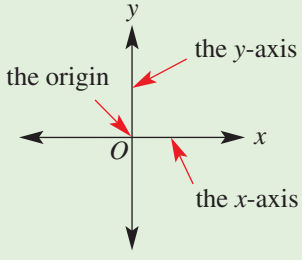


- Describe the behaviour of the y values as the x values increase.
- By continuing the pattern, what would be the y value for $x = 7$ or $x = 0$?
- How would you write a rule linking y (the output) with x (the input)?
- How is the number plane like a street map? How is it different?

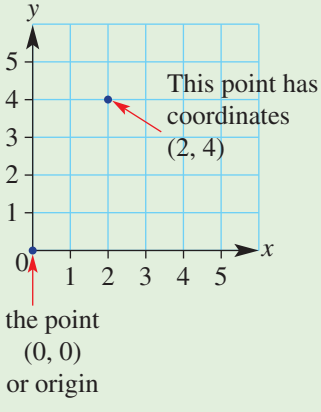


Key ideas

- A **number plane** is a grid for plotting points. Important features of a number plane are:
  - The ***x*-axis** and ***y*-axis**: these are like horizontal and vertical number lines
  - The **origin**: where the x-axis and y-axis meet at $(0, 0)$.



- Points are located by a grid reference system of **coordinates**.
  - The point (x, y) means (x units across from the origin, y units up).
  - For $(2, 4)$ the x-coordinate is 2 and the y-coordinate is 4. To plot this point, start at the origin and go 2 units across, then 4 units up.
- For a rule describing a pattern with *input* and *output*, the x value is the *input* and the y value is the *output*.



Number plane A plane on which every point is related to a pair of numbers called coordinates

***x*-axis** Horizontal axis of the number plane

***y*-axis** Vertical axis of the number plane

Origin The point with coordinates $(0, 0)$ on the number plane

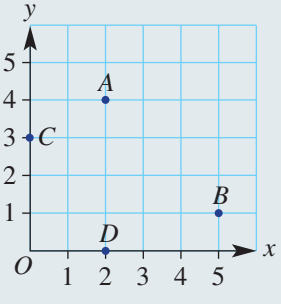
Coordinates Numbers or letters used to give a location or position, often an ordered pair written in the form (x, y)

Exercise 3K

UNDERSTANDING 1–4 4

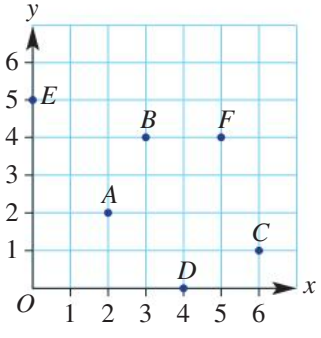
Example 35 Finding coordinates of plotted points

Match each point on the grid with its correct coordinates.
$(0, 3)$ $(2, 4)$ $(5, 1)$ $(2, 0)$



Solution	Explanation
$A(2, 4)$	To locate A, start at 0, go across 2 and up 4.
$B(5, 1)$	To locate B, start at 0, go across 5 and up 1.
$C(0, 3)$	To locate C, start at 0, don't move across at all, then up 3.
$D(2, 0)$	To locate D, start at 0, go across 2 and don't go up at all.

1 Match each point on the grid with its correct coordinates.



$(4, 0)$ $(3, 4)$
$(0, 5)$ $(2, 2)$
$(6, 1)$ $(5, 4)$

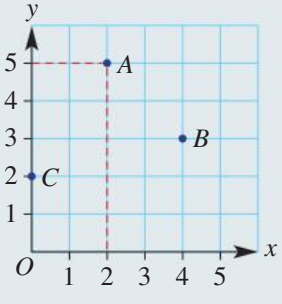


Start at O and then move across and up.

Example 36 Plotting points on a number plane

Draw a number plane and plot these points on it.

$A(2, 5)$ $B(4, 3)$ $C(0, 2)$

Solution	Explanation
	Draw a number plane with both axes labelled from 0 to 5. The numbers go on the grid lines, not in the spaces. Label the horizontal axis x and the vertical axis y. $(2, 5)$ means go across 2 from 0 (along the x-axis) and then go up 5 units. Plot the point and label it A. $B(4, 3)$ means (4 across, 3 up). $C(0, 2)$ means (0 across, 2 up) so C is on the y-axis.

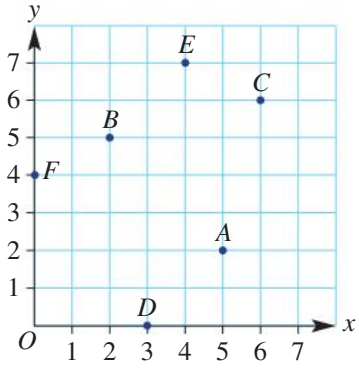
2 On grid paper, draw a number plane, with the numbers 0 to 6 marked on each axis. Plot and label these points:

$A(3, 5)$ $B(4, 2)$ $C(0, 3)$ $D(1, 0)$ $E(6, 6)$

3K

3 Copy and complete the coordinates for each point shown on this number plane.

a $A(5, ?)$
b $B(?, 5)$
c $C(?, ?)$
d $D(3, ?)$
e $E(?, ?)$
f $F(?, 4)$



4 Copy and complete the following sentences.

a The horizontal axis is known as the _____.
b The _____ is the vertical axis.
c The point at which the axes intersect is called the _____.
d The x-coordinate is always written _____.
e The second coordinate is always the _____.
f The letter ___ comes before ___ in the dictionary, and the ___-coordinate comes before the ___-coordinate on the number plane.

Look at the **Key ideas**.



FLUENCY 5–8 5–9

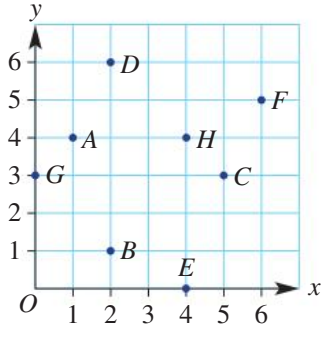
5 Use grid paper to draw a number plane with the x- and y-axis numbered from 0 to 10. Plot each group of points and join them in order, using a ruler.

a $(4, 10)(2, 8)(5, 9)$ — Name this type of angle, less than 90°.
b $(2, 6)(4, 6)(4, 3)$ — Name this type of angle, equal to 90°.
c $(6, 8)(8, 8)(8, 5)(6, 5)(6, 8)$ — Name this shape.
d $(5, 1)(6, 4)(7, 1)(5, 1)$ — Name this type of triangle.
e $(0, 0)(0, 3)(3, 0)(0, 0)$ — Name this type of triangle.
f $(7, 3)(9, 3)(10, 6)$ — Name this type of angle, less than 180°.

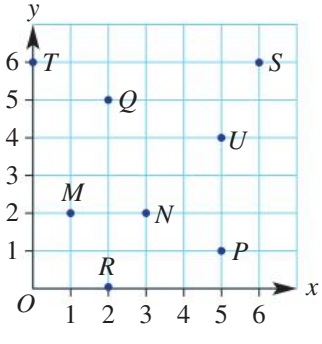
6 Draw a number plane from 0 to 8 on both axes. Plot the following points on the grid and join them in the order they are given.
$(2, 7), (6, 7), (5, 5), (7, 5), (6, 2), (5, 2), (4, 1), (3, 2), (2, 2), (1, 5), (3, 5), (2, 7)$

7 Write down the coordinates of each of these labelled points.

a



b



The coordinates are (x, y) or (across, up).



Example 37 Drawing a graph

For the rule $output = input + 1$:

a Copy and complete the table of values.
b List the coordinates of each point.
c Plot each pair of points on the number plane.

input (x)	*output* (y)
0	1
1	
2	
3	

Solution

a

input (x)	*output* (y)
0	1
1	2
2	3
3	4

Explanation

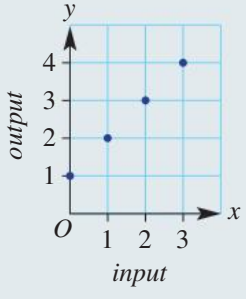
Use the rule to find each *output* value for each *input* value. The rule is:

$output = input + 1$, so add 1 to each *input* value.

b (0, 1), (1, 2), (2, 3) and (3, 4)

The coordinates of each point are (*input*, *output*).

c



Plot each (x, y) pair as a point:

(x units across from 0, y units up).

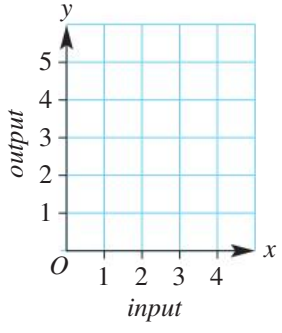
8 For the given rule $output = input + 2$:

a Copy and complete the given table of values.
b List the coordinates of each point.
c Plot each point on a number plane like the one below.



Coordinates are (*input*, *output*) = (x, y).

input (x)	*output* (y)
0	2
1	
2	
3	



3K

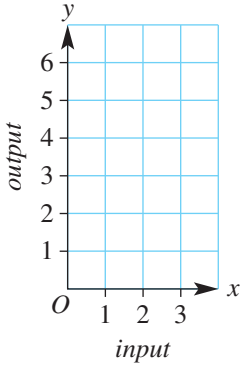
9 For the given rule $output = input \times 2$:

a Copy and complete the given table of values.

input (x)	*output* (y)
0	
1	
2	
3	

b List the coordinates of each point.

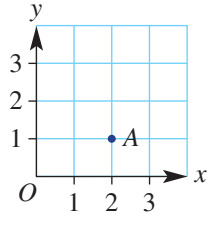
c Plot each point on a number plane like the one below.



PROBLEM-SOLVING AND REASONING 10, 11 10–12

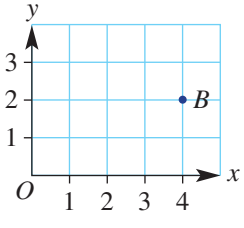
10 Draw a number plane from 0 to 5 on both axes. Place a cross on each point with coordinates that have the same x value and y value.

11 **a** Which of the following is the correct way to describe the position of point A?



A 21
B 2, 1
C (2, 1)
D (x2, y1)
E (2_x, 1_y)

b Which of the following is the correct set of coordinates for point B?



A (2, 4)
B 4, 2
C (4, 2)
D (2, 4)
E $x = 4, y = 2$

12 **a** Plot the following points on a number plane. Join the points in the order given to draw the basic shape of a house.
(1, 5), (0, 5), (5, 10), (10, 5), (1, 5), (1, 0), (9, 0), (9, 5)



Draw a number plane with 0 to 10 on both axes.

b Draw a door and list the coordinates of the four corners of the door.

c Draw a window and list the coordinates of the four corners of the window.

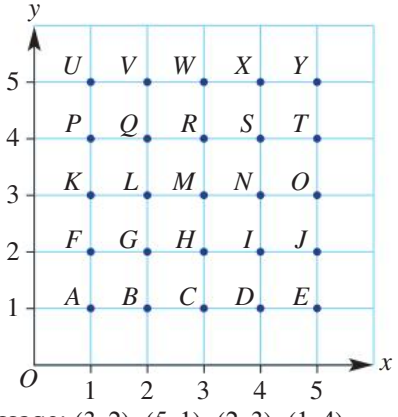
d Draw a chimney and list the coordinates of the four points needed to draw the chimney.



Secret messages

— 13

13 A grid system can be used to make secret messages. Jake decides to arrange the letters of the alphabet on a number plane in the following manner.



a Decode Jake's following message: (3, 2), (5, 1), (2, 3), (1, 4)

b Code the word 'secret'.

To increase the difficulty of the code, Jake does not include brackets or commas and he uses the origin to indicate the end of a word.

c What do the following numbers mean?
13515500154341513400145354001423114354.

d Code the phrase: 'Be here at seven'.

Maths@Work: Computer technician

Computer technicians and computer engineers are people who love technology and its uses. Their jobs range in difficulty, hours and pay. But they all need to have an understanding of patterns and mathematics.

The computer industry is always evolving. New devices and applications require larger and larger storage. Electronic data is represented in digital form using the digits 0 and 1, which are called 'bits' of data. A 'byte' is 8 bits, for example, the capital letter B is 1 byte stored as '01000010', which has 8 bits of data.

- A 'bit' (1 b) is the smallest data unit (either a 1 or 0).
- A 'byte' (1 B) is 8 bits.



Movie rental companies require massive digital storage capacity.

Because there are only two possible bits (i.e. 0 or 1) computer storage data is called binary data and can be written as powers of 2. Note that 'bi' means two of something, for example, a bicycle has 2 wheels.

1 Remembering that a byte has 8 bits, write each of the following in bytes.

a 16 bits (i.e. 16 b) **b** 32 b **c** 64 b
d 256 b **e** 1024 b **f** 1 048 576 b



bits ÷ 8 = bytes

2 Use a factor tree or repeated division to determine the following storage amounts as powers of 2.

a 64 GB
b 512 GB
c 2048 GB (= 2TB)



A computer hard drive disc stores data, for example, 500 GB (gigabytes) or 2 TB (terabytes).

3 Other units are also used in file storage. Complete the table below to find the number of bytes for each of these storage amounts.

Name	Symbol	Powers of 2	Number of bytes
Byte	B	2^0	
Kilobyte	kB	2^{10}	
Megabyte	MB	2^{20}	
Gigabyte	GB	2^{30}	
Terabyte	TB	2^{40}	



Use the power button on your calculator.

4 Use the question 3 answers to help you to write each of the following in kilobytes (kB).

a 1 MB **b** 3 MB **c** 25 MB **d** 100 GB

$1\text{ MB} = \frac{1048576}{1024}$

$= ?\text{ kB}$



5 Before Broadband, download speeds were quite slow. A dial-up internet speed could be 56 k**b**/s (kilo**bits** per second).

Determine the unknown values in the following table to find the download times for the given data files.

Convert 56 kb/s to kB/s

$56 \div 8 = ?$ kiloBytes/s



Convert MB to kiloBytes

$21\text{ MB} = 21 \times 1024$

$= ?$ kiloBytes



	Speed in kilobits/s	Speed in kB/s (kiloBytes/s)	File size in MB (MegaBytes)	File size in kB (kiloBytes)	Download time in seconds	Download time to the nearest hour
a	56		26 MB music video			
b	56		152 MB game			
c	56		740 MB movie			

Using technology

Computer download times depend on the speed of the ISP (Internet Service Provider), the size of the file, the type and quality of the internet connection, the amount of traffic using a website, etc.

6 In this question, you will use a spreadsheet to calculate the download times for various Broadband internet speeds, which are measured in M**b**/s (Mega**bits** per second).

a Set up the following spreadsheet. Format all the shaded cells as Number/1 d.p.

	A	B	C	D	E	F	G	H
1	Download times for various files							
2				Download speeds in Megabits per second				
3				8	20	30	50	100
4	Media type	File size in MB MegaBytes	File size in Mb Megabits	Download times				
5	App	16						
6	Music	24						
7	Short Video	45						
8	Gaming	120						
9	Movie	600						

b In column C, enter formulas to calculate Megabits. Recall that there are 8 bits for each byte, so 8 Megabits for each Megabyte.

Cell C5 formula = B5*8



c In columns D to H, enter formulas to calculate the download times in seconds for each data file for each of the different speeds.

d How much faster, in seconds, is the application download time at 100 Megabits/s compared to 8 Megabits/s?

e How much faster, in minutes and seconds, is the movie download time at 100 Megabits/s compared to 8 Megabits/s?



Formulas for download times

Use $ signs to fix the address of the speed cell for each column.

Cell D5 formula = C5/$D$3

Cell E5 formula = C5/$E$3

To fill formulas down the column, drag the fill handle down.

Puzzles and games

1 To play *Prime Drop* with a partner, you will need a number chart (with circles around the prime numbers and square numbers coloured in) and a die.

Start

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Finish

- Each player must roll a 1 to start.
- Take turns to roll the die and move that number of places on the chart.
- If you land on a prime number, move back to the previous prime number (or to 1).
- If you land on a square number, have another turn.
- To finish, you must land on 50. (If the number you roll is too big, move forward to 50 and then move backwards.)

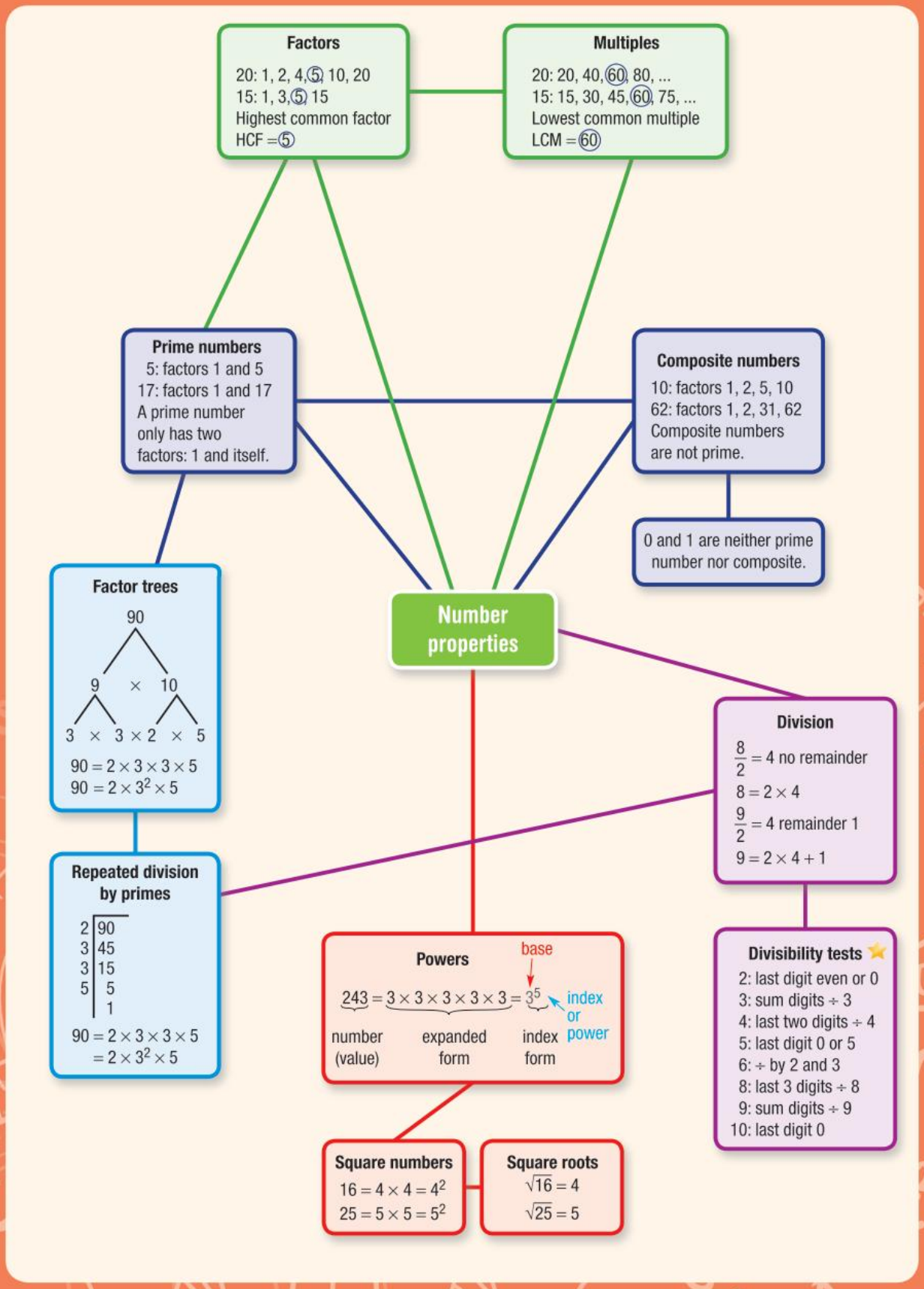
2 *Why did the elephant go, 'Baa, baa'?*

To find out, work out the following powers and square roots. Then decode the answer below.

A 2^2	**E** 8^3	**G** $\sqrt{121}$	**H** 3^2
I $\sqrt{144}$	**K** 4^4	**L** $\sqrt{256}$	**N** 6^2
O 6^3	**P** $\sqrt{196}$	**R** 9^2	**S** 7^2
T 1^3	**U** $\sqrt{225}$	**W** 5^3	

| | | | | | | | | |

Chapter summary



Chapter summary

Sequence

A set of numbers ordered according to a rule

2, 4, 6, 8, ...
↑ ↑ ↑ ↑
terms

+6 +6
4, 10, 16, ...

−3 −3
100, 97, 94, ...

×2 ×2
3, 6, 12, ...

÷2 ÷2
32, 16, 8, ...

Patterns with shapes and numbers

Spatial pattern:  ...

Number pattern: 4 8 12 ...

Pattern rule:
Number of sticks = 4 × number of diamonds

input	1	2	3
output	4	8	12

output = 4 × *input*

Number patterns

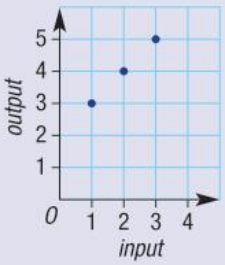
Number plane

Table and graph of patterns

(*input*, *output*) = (*x*, *y*)

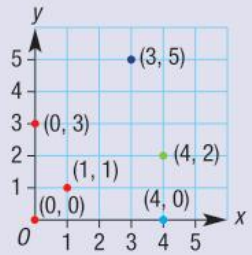
output = *input* + 2

input	*output*
1	3
2	4
3	5



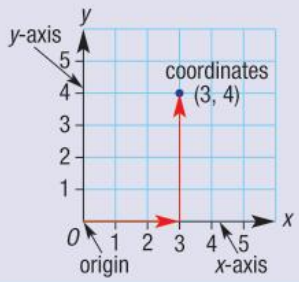
Plotting points with coordinates

(*x*, *y*) = (across from 0, up)



Number plane

x-axis, *y*-axis, origin, coordinates



Chapter review

Multiple-choice questions

3A **1** Which number is the wrong multiple for the following sequence? 3, 6, 9, 12, 15, 18, 22, 24, 27, 30

A 18 **B** 22 **C** 30 **D** 6 **E** 3

3A **2** Which group of numbers contains every factor of 20?

A 2, 4, 5, 10, 20 **B** 2, 4, 5, 15, 20 **C** 1, 2, 4, 5, 10
D 1, 2, 4, 8, 5, 10 **E** 1, 2, 4, 5, 10, 20

3D **3** Which of the following numbers is a prime number?

A 21 **B** 77 **C** 11 **D** 22 **E** 1

3D **4** Which of the following groups of numbers includes one prime and two composite numbers?

A 2, 10, 7 **B** 54, 7, 11 **C** 9, 32, 44 **D** 5, 17, 23 **E** 18, 3, 12

3E **5** $7 \times 7 \times 7 \times 7 \times 7$ can be simplified to:

A 5^7 **B** 7^5 **C** 7×5 **D** 75 **E** 77 777

3G **6** Evaluate $\sqrt{3^2 + 4^2}$.

A 7 **B** 5 **C** 14 **D** 25 **E** 6

3B **7** The HCF and LCM of 12 and 18 are:

A 6 and 18 **B** 3 and 12 **C** 2 and 54 **D** 6 and 36 **E** 3 and 18

3F **8** The prime factor form of 48 is:

A $2^4 \times 3$ **B** $2^2 \times 3^2$ **C** 2×3^3 **D** 3×4^2 **E** $2^3 \times 6$

3E **9** Evaluate $4^3 - 3 \times (2^4 - 3^2)$.

A 427 **B** 18 **C** 43 **D** 320 **E** 68

3A **10** Factors of 189 include:

A 3, 7, 9, 18, 21, 27 **B** 3, 9, 18, 21 **C** 3, 9, 18
D 3, 7, 9, 17, 21 **E** 3, 7, 9, 21, 27, 63

3C **11** Which number is *not* divisible by 3?



A 25 697 403 **B** 31 975 **C** 7 297 008
D 28 650 180 **E** 38 629 634 073

3K **12** Which set of points is in a horizontal line?

A (5, 5), (6, 6), (7, 7) **B** (3, 2), (3, 4), (3, 11)
C (2, 4), (3, 6), (4, 8) **D** (5, 4), (6, 4), (8, 4), (12, 4)
E (1, 5), (5, 1), (1, 1), (5, 5)

Short-answer questions

3A **1** Copy and complete.

a The factor pairs of 12 are $1 \times \square = 12$, $\square \times 6 = 12$, $\square \times 4 = 12$

b The factors of 12 in ascending order are: ___, ___, ___, ___, ___, ___.

3A **2** **a** List all the factors of 24 in ascending order. (Hint: First write the factor pairs.)

b List the first 6 multiples of 5. (Hint: Start with 5 and count in fives.)

Chapter review

3B **3** **a** List the factors of 16.
b List the factors of 20.
c Circle the common (shared) factors of 16 and 20.
d What do the letters HCF stand for?
e Write the HCF of 16 and 20.

3B **4** **a** List the first five multiples of 6.
b List the first five multiples of 8.
c What do the letters LCM stand for?
d State the LCM of 6 and 8.

3C **5** Write the missing words or numbers.
a A number that ends in 0 is divisible by both ___ and ___, as well as 2.
b 264 is _____ by 3 because the sum of the digits is ___ + ___ + ___ = ___ and ___ is divisible by 3.
c 576 is _____ by 9 because the sum of the digits is ___ + ___ + ___ = ___ and ___ is _____ by 9.
d 344 is _____ by 4 because the last two digits are ___ and ___ is _____ by 4.

3C **6** 206, 48, 56, 621, 320, 85, 63, 14, 312
From the list of numbers above, write the numbers that are divisible by:
a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

3D **7** Answer these questions.
a Is the number 1 a prime number?
b Is the number 1 a composite number?
c 5 has only two factors, 1 and 5. Is 5 a prime number?
d List the factors of 10. Is 10 a prime or composite number?
e Sort these numbers into prime or composite numbers: 2, 3, 8, 7, 11, 15, 20.
f Write the factors of 20 in two groups, prime and composite numbers.
g List the prime numbers between 15 and 25.

3E/F **8** Answer these questions.
a For $16 = 4^2$, what number is the base? What number is the index or power?
b Write the product $5 \times 5 \times 7 \times 7 \times 7$ in index form.
c Write $2^3 \times 3^2$ in factor form and find the number value.
d Evaluate: $3^2 + 5^2 - 3 \times 2$ (show steps).
e Find the missing powers: $125 = 5^{\square}$; $32 = 2^{\square}$; $100\,000 = 10^{\square}$.

3F **9** For each number, draw a factor tree. Then write the number with powers of prime factors.
a 50 **b** 16 **c** 2400

3G **10** Copy and complete each of these tables.

Index form	4^2		7^2	
Value		36		81

Square root form	$\sqrt{16}$		$\sqrt{100}$	
Value		5		12

3E/G **11** Find the value of:

a 1^{99} **b** $\sqrt{3^2}+\sqrt{4^2}$ **c** $\sqrt{3^2+4^2}$

d $\sqrt{3^2}\times\sqrt{4^2}$ **e** $4^2-\sqrt{25}+\sqrt{7^2}$ **f** $10^3\div\sqrt{3^2+4^2}$

g 8 to the power of 2

h 11 squared

i the square root of 81

j the side length of a square that has area = 25 cm^2

k the side length of a square that has area = 400 cm^2

3H **12** List the next four terms for these sequences.

a Start with 3 and keep adding 4.

b Start with 64 and keep dividing by 2.

c 3, 6, 12, . . .

d 44, 40, 36, . . .

e 1, 4, 9, . . . (Hint: These are square numbers.)

3I **13** **a** Draw the next two shapes in this spatial pattern.



b Copy and complete this table.

Number of triangles	1	2	3	4	5
Number of sticks	3				

c Write the pattern rule.

d How many sticks would be needed for 12 triangles?

3I **14** Look at this spatial pattern.



a Copy and complete this table.

Number of squares	0	1	2	3	4
Number of sticks	1	1 + ☐ = ☐	1 + ☐ × 2 = ☐	1 + ☐ × 3 = ☐	1 + ☐ × 4 = ☐

b Copy and complete the rule for this pattern:

number of sticks = 1 + ☐ × number of squares

c How many sticks are needed to make 10 squares this way?

d How many squares could be made from 82 sticks?

3J **15** Copy and complete each table for the given rule.

a *output* = *input* + 5

input	3	5	7	12	20
output					

b *output* = 2 × *input* + 7

input	4	2	9	12	0
output					

3J **16** Write the pattern rule for each of these tables.

a *output* = ☐ × *input*

input	3	4	5
output	15	20	25

b *output* = *input* − ☐

input	15	20	28
output	9	14	22

3K **17** **a** State the coordinates of each point plotted on this number plane.

b State the coordinates of the origin.

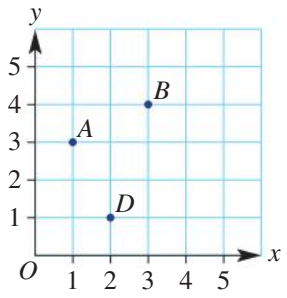
c Which axis is point $(0, 5)$ on?

d Name the vertical axis.

e Which axis is the point $(3, 0)$ on?

f Name the horizontal axis.

g State the coordinates of a point C so that $ABCD$ is a square.



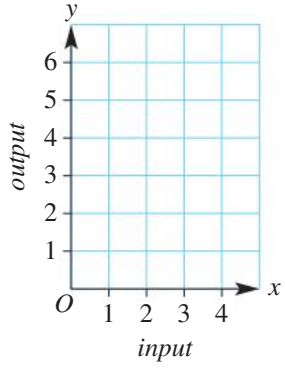
3K **18** For the rule: $output = 2 \times input - 1$

a Copy and complete the table of values below.

input (x)	*output* (y)
1	
2	
3	
4	

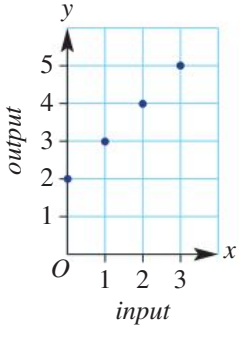
b List the coordinates of each point.

c Plot each point on a number plane like the one below.

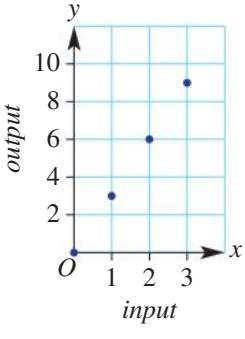


3K **19** Write a rule (e.g. $output = input \times 2$) that would give these graphs.

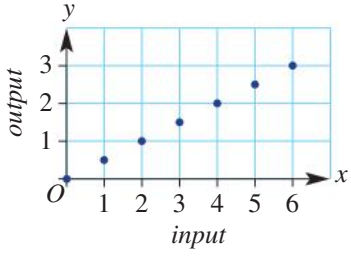
a



b



c



Extended-response questions

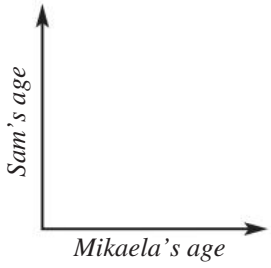
1 Sam is five years older than his sister Mikaela.

a Copy and complete this rule for their ages: Sam's age = Mikaela's age + ☐

b Copy and complete this table showing Sam's age and Mikaela's age in years.

Mikaela's age	0	3	7	13
Sam's age				

c On grid paper, draw horizontal and vertical axes and label each with the numbers from 0 to 20. (Be careful to mark the numbers next to the grid lines, not in the spaces.) Label the axes as follows.



d On your graph, plot the points from the table above. (Hint: Mikaela's age goes *across*, Sam's age goes *up*.)

2 At North Park Primary School, the classrooms have trapezium-shaped tables. Mrs Greene arranges her classroom's tables in straight lines, as shown.



a Copy and complete this table for up to 5 tables joined together.

Number of tables	1	2	3	4	5
Number of students					

b Copy and complete this rule:

number of students = ☐ × number of tables + ☐

Check that the rule works for the numbers in your table.

c The room allows seven tables to be arranged in a straight line. How many students can sit around the tables?

d There are 80 students in Grade 6 at North Park Primary School. Mrs Greene would like to arrange the tables in one straight line for an outside picnic lunch. How many tables will she need?

Chapter 4

Fractions and percentages

What you will learn

4A What are fractions? (Consolidating)
4B Equivalent and simplified fractions
4C Mixed numbers (Consolidating)
4D Ordering fractions
4E Adding fractions
4F Subtracting fractions
4G Multiplying fractions
4H Dividing fractions
4I Fractions and percentages
4J Percentage of a number
4K Expressing a quantity as a proportion
4L Ratios, rates and best buys

Australian curriculum

NUMBER AND ALGEBRA

Real numbers

Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line **(ACMNA152)**

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators **(ACMNA153)**

Multiply and divide fractions and decimals using efficient written strategies and digital technologies **(ACMNA154)**

Express one quantity as a fraction of another, with and without the use of digital technologies **(ACMNA155)**

Connect fractions, decimals and percentages and carry out simple conversions **(ACMNA157)**

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. **(ACMNA158)**

Recognise and solve problems involving simple ratios **(ACMNA173)**

Money and financial mathematics

Investigate and calculate 'best buys', with and without digital technologies **(ACMNA174)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The Australian travel trend

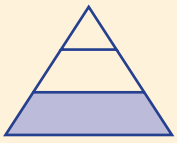
Many people love to travel. During 2015–16, 9.7 million Australians made short-term trips overseas. This was more than double the number in 2005–06. The Australian Bureau of Statistics has looked at other trends associated with short-term travel to and from Australia over the past 10 years and some interesting facts have emerged.

New Zealand is still the leading destination, with 13.3% of all short-term travel, which was 1 289 600 journeys in 2015–16. This is an increase of 54% over 10 years.

The next most popular destination for Australians travelling abroad is Indonesia. Ten years prior it was the USA. Travel to Indonesia has increased from 233 200 trips in 2005–06 to 1 203 500 trips in 2015–16. This is a 416% increase over ten years.

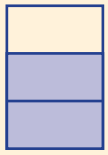
Pre-test

1 In which of the following is one-third of the area shaded?

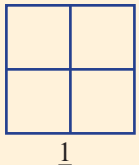
A 

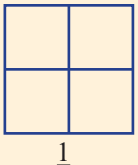
B 

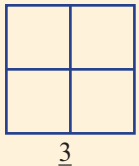
C 

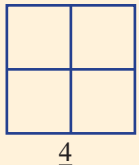
D 

2 How many of the smaller squares would you shade for each given fraction?

a  $\frac{1}{2}$

b  $\frac{1}{4}$

c  $\frac{3}{4}$

d  $\frac{4}{4}$

3 Write the following as fractions.

a one-half **b** one-third **c** two-thirds

d one-tenth **e** three-quarters

4 Which of the following is *not* equivalent to (the same value as) one whole?

A $\frac{2}{2}$ **B** $\frac{6}{6}$ **C** $\frac{1}{4}$ **D** $\frac{12}{12}$

5 Which of the following is *not* equivalent to one-half?

A $\frac{2}{4}$ **B** $\frac{3}{9}$ **C** $\frac{5}{10}$ **D** $\frac{10}{20}$

6 Find:

a $1-\frac{1}{4}$ **b** $1-\frac{1}{2}$ **c** $1-\frac{1}{3}$ **d** $1-\frac{1}{5}$

7 Find:

a $3-\frac{1}{4}$ **b** $2-\frac{1}{2}$ **c** $10-\frac{1}{2}$ **d** $6-\frac{3}{4}$

8 Tom eats half a block of chocolate on Monday and half of the remaining block on Tuesday. How much chocolate is left for Wednesday?

9 Find the next three terms in these number sequences.

a $0, \frac{1}{2}, 1, 1\frac{1}{2}, ___, ___, ___$

b $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, ___, ___, ___$

c $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, ___, ___, ___$

d $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, ___, ___, ___$

10 Write 1, 3 or 4 in each box to make a true statement.

a $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\square\times\frac{1}{2}$

b $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\square\times\frac{3}{4}$

c $\frac{5}{6}\times\square=\frac{5}{6}$

d $\frac{6}{8}\div\square=\frac{6}{8}$

11 Find:

a $\frac{1}{2}$ of \$15 **b** $\frac{1}{4}$ of \$160 **c** $\frac{3}{4}$ of \$1 **d** $\frac{1}{3}$ of \$6

12 State whether each of the following is true (T) or false (F).

a $\frac{1}{2}$ of $16 = 16 \div 2$ **b** $\frac{16}{4}=\frac{1}{4}$ of 16 **c** $\frac{3}{4}$ of $100 = 75$ **d** one-tenth $= \frac{1}{100}$

4A What are fractions?

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

The word fraction comes from the Latin word 'frangere', which means 'to break into pieces'. Fractions are parts of a whole.

We all use fractions every day. Examples could include cooking, shopping, sporting and building construction.



Pieces of a broken object are all fractions of the whole.

Let's start: What strength do you like your cordial?

- Tom uses 40 mL of cordial and 120 mL of water.
- Sally uses 40 mL of cordial with 200 mL of water.
- Who likes their drink the strongest? How can fractions be used to describe the strengths of cordial?

Key ideas

- A **fraction** is made up of a n**u**merator (**u**p) and a **d**enominator (**d**own).

  For example: $\frac{3}{5}$ ← numerator, ← denominator

  - The **denominator** tells you how many parts the whole is divided up into.
  - The **numerator** tells you how many parts we are talking about.
  - The horizontal line separating the numerator and the denominator is called the **vinculum**.



- A **proper fraction** has the numerator less than the denominator.

  For example: $\frac{2}{7}$ is a proper fraction.

- An **improper fraction** has the numerator greater than or equal to the denominator.

  For example: $\frac{5}{3}$ is an improper fraction.

- Whole numbers can be represented as fractions.

  For example: $1 = \frac{4}{4}, 1 = \frac{3}{3}, 2 = \frac{8}{4}$

- Mixed numbers have a whole number and a fraction.

  For example: $1\frac{2}{3}$ is a mixed number.

Fraction Part of a whole

Denominator The bottom part of a fraction

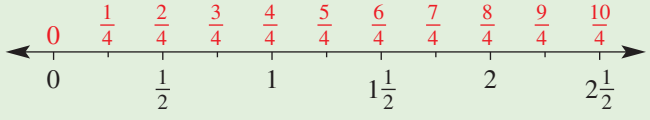
Numerator The top part of a fraction

Vinculum The separating line in a fraction

Proper fraction A fraction where the numerator is less than the denominator

Improper fraction A fraction where the numerator is greater than or equal to the denominator

- We can represent fractions on a number line.
  This number line shows the whole numbers 0, 1 and 2. Each unit has then been divided equally into four segments, therefore creating 'quarters'.

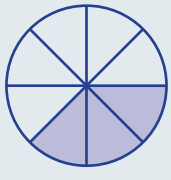


Exercise 4A

UNDERSTANDING 1–4 4

Example 1 Understanding the numerator and the denominator

A pizza has been divided into equal parts.



a Into how many pieces has the whole pizza been divided?
b How many pieces have been selected (shaded)?
c In representing the shaded fraction of the pizza:
  i What must the denominator equal?
  ii What must the numerator equal?
  iii Write the amount of pizza selected (shaded) as a fraction.
  iv Write the fraction in words.
d Is the fraction proper or improper?

Solution	Explanation
a 8	Pizza cut into 8 equal pieces.
b 3	3 of the 8 pieces are shaded.
c i 8	Denominator shows the number of parts the whole has been divided into.
ii 3	Numerator tells how many of the divided parts you have selected.
iii $\frac{3}{8}$	Shaded fraction is the numerator over the denominator; i.e. 3 out of 8 divided pieces.
iv three-eighths	The denominator tells us we have eighths.
d proper	The fraction is less than one.

1 **a** State the denominator of this proper fraction: $\frac{2}{9}$

b State the numerator of this improper fraction: $\frac{7}{5}$

2 Group the following list of fractions into proper fractions or improper fractions. Also state which ones are whole numbers.

a $\frac{7}{6}$ **b** $\frac{2}{7}$ **c** $\frac{50}{7}$

d $\frac{3}{3}$ **e** $\frac{3}{4}$ **f** $\frac{5}{11}$

g $\frac{1}{99}$ **h** $\frac{9}{4}$ **i** $\frac{11}{8}$

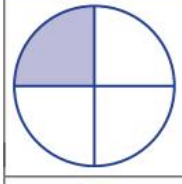
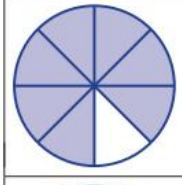
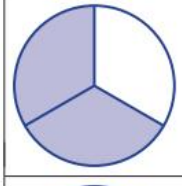
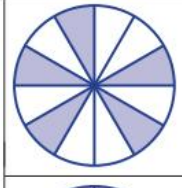
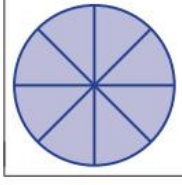
j $\frac{10}{10}$ **k** $\frac{5}{1}$ **l** $\frac{121}{5}$

m $\frac{12}{12}$ **n** $\frac{12}{6}$ **o** $\frac{9}{4}$ **p** $\frac{100}{3}$



Proper fractions:
Numerator is less than denominator.
Improper fractions:
Numerator is greater than or equal to the denominator.

3 Copy and complete the following table.

The whole (divided into equal parts)	Number of equal parts in the whole (denominator)	Number of shaded equal parts (numerator)	Fraction shaded	Name of fraction
	4			one-quarter
				
		2	$\frac{2}{3}$	
				
		8		one whole

4 What fractions are indicated by the dots on these number lines?

a



b





Write any improper fractions as mixed numbers if you wish.

4A

FLUENCY 5–7 5, 6(½), 7, 8

Example 2 Representing fractions on a number line

Show the fractions $\frac{3}{5}$ and $\frac{9}{5}$ on a number line.

Solution



Explanation

Draw a number line starting at 0 and mark on it the whole numbers 0, 1 and 2.

Divide each whole unit into five segments of equal length. Each of these segments has a length of one-fifth.

5 Represent the following fractions on a number line.

a $\frac{3}{7}$ and $\frac{6}{7}$ **b** $\frac{2}{3}$ and $\frac{5}{3}$ **c** $\frac{1}{6}$ and $\frac{5}{6}$



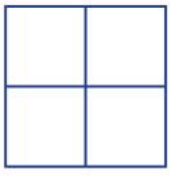
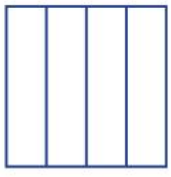
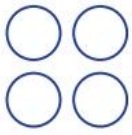
To show sevenths, rule a 7 cm line for one whole and mark each cm for one seventh.

6 Write the following as whole numbers.

a $\frac{5}{5}$ **b** $\frac{6}{6}$ **c** $\frac{8}{8}$ **d** $\frac{16}{8}$

e $\frac{100}{100}$ **f** $\frac{100}{2}$ **g** $\frac{15}{1}$ **h** $\frac{20}{10}$

7 Shade $\frac{3}{4}$ of each of the following diagrams.

a  **b**  **c**  **d**  **e** 

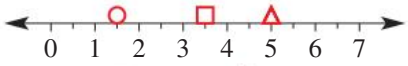
8 Write the next three fractions for each of the following fraction sequences.

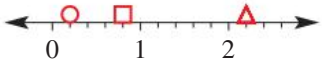
a $\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}, __, __, __$ **b** $\frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, __, __, __$

c $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, __, __, __$ **d** $\frac{11}{7}, \frac{10}{7}, \frac{9}{7}, \frac{8}{7}, __, __, __$

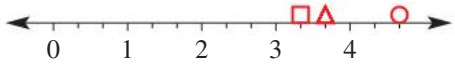
PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 What fraction matches each of the different shapes (○, □ and △) on these number lines?

a 

b 

c 

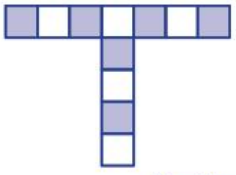
d 



Write any mixed numbers as improper fractions as well.

10 Match each of the following diagrams to one of the fractions in the box below.

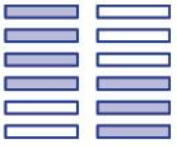
a



b



c



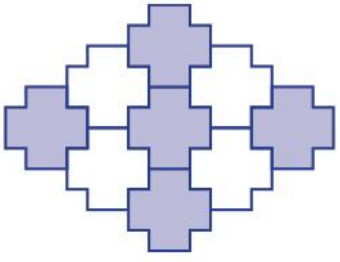
d



e



f



Fractions	$\frac{7}{12}$	$\frac{5}{9}$	$\frac{6}{11}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{6}$

11 For each of the following, write the fraction that is describing part of the total.



a After one day of a 43-kilometre hike, the students had completed 12 kilometres.

b From 15 starters, 13 went on and finished the race.

c Rainfall for 11 months of the year was below average.

d One egg is broken in a carton that contains a dozen eggs.

e Two players in the soccer team (with 11 players) scored a goal.

f The lunch stop was 144 kilometres into the 475-kilometre trip.

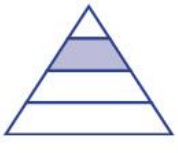
g Seven members in the class of 20 have visited Australia Zoo.

h One of the car tyres (not including the spare) is worn and needs replacing.

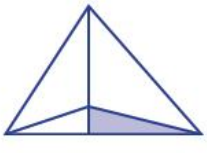
i It rained three days this week.

12 Which diagram has one-quarter shaded?

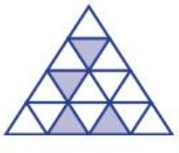
A



B



C



D



Adjusting concentration

13

13 a Callum pours 20 mL of water into this beaker.
What fraction of 200 mL is that?



b Callum adds acid to the same beaker until it holds 200 mL. How much acid did he add? What fraction of 200 mL is that?

c Rosa has a 200 mL beaker that is $\frac{1}{4}$ full of water. How much acid will she need to add to fill the beaker?

4B Equivalent and simplified fractions

Interactive

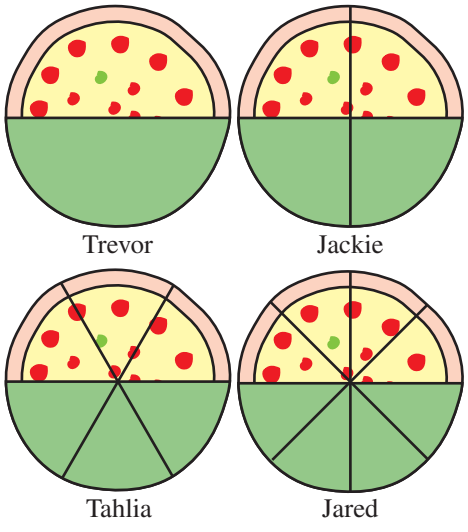
Widgets

HOTsheets

Walkthrough

Fractions may look very different but still have the same value. For example, in an AFL football match, 'half-time' is the same as 'the end of the second quarter'. We can say that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Consider a group of friends eating pizzas during a sleepover. The pizzas are homemade and each person cuts up their pizza as they like. The green shading shows the amount eaten before it is time to start the second movie.



By looking at the pizzas, it is clear to see that Trevor, Jackie, Tahlia and Jared have all eaten the same amount of pizza.

This means that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

Let's start: Odd one out

- Pick the fraction that is the odd one out.

$$\frac{25}{100}, \frac{4}{16}, \frac{2}{8}, \frac{2}{5}, \frac{5}{20}$$

- What could we call the other four fractions?

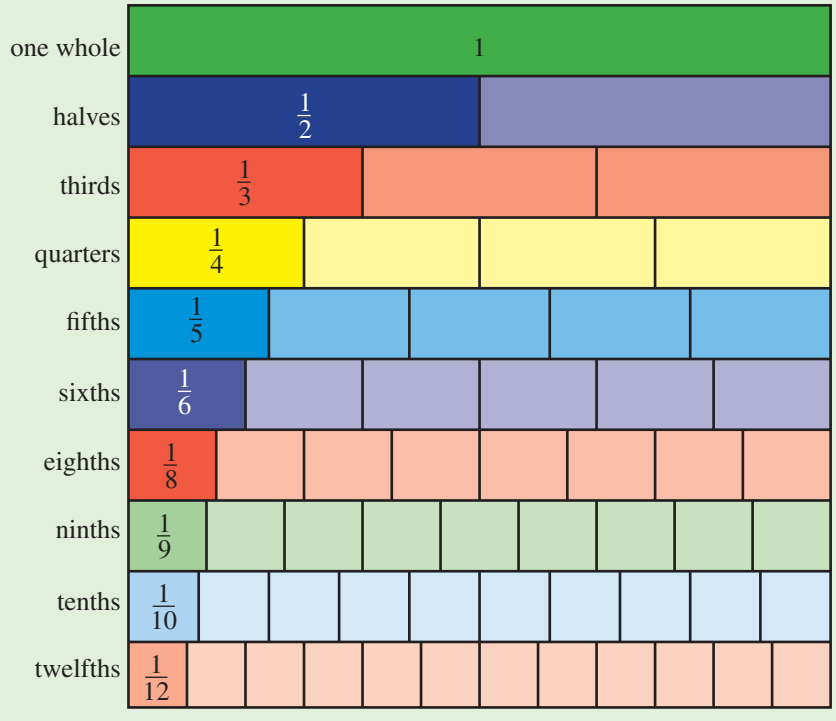
Key ideas

- **Equivalent fractions** are fractions that mark the same place on a number line.
  For example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.
  - Equivalent fractions are produced by multiplying the numerator and denominator by the same number.
  - Equivalent fractions can also be produced by dividing the numerator and denominator by the same number.
- Simplifying fractions involves writing a fraction in its 'simplest form'. To do this, the numerator and the denominator must be divided by their **highest common factor (HCF)**.
  - You should usually write fraction answers in their simplest form.

Equivalent fractions
Fractions that represent the same amount

Highest common factor
The largest number that is a factor of all the given numbers

- A fraction wall can be helpful when comparing fractions.



Exercise 4B

UNDERSTANDING 1–4 4

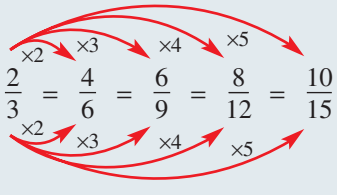
Example 3 Producing equivalent fractions

Write four equivalent fractions for $\frac{2}{3}$ by multiplying the numerator and denominator by 2, 3, 4 and 5.

Solution

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

Explanation



1 Copy and complete the following.

a $\frac{3}{4} \times \frac{2}{2} = \frac{\square}{8}$ **b** $\frac{2}{5} \times \frac{4}{4} = \frac{8}{\square}$ **c** $\frac{1}{3} \times \frac{\square}{\square} = \frac{3}{9}$

d $\frac{1}{2} \times \frac{7}{\square} = \frac{7}{14}$ **e** $\frac{4}{5} \times \frac{\square}{\square} = \frac{8}{10}$ **f** $\frac{3}{4} \times \frac{5}{5} = \frac{15}{\square}$

4B

2 Write four equivalent fractions for $\frac{3}{4}$ by multiplying by $\frac{3}{3}, \frac{5}{5}, \frac{10}{10}$ and $\frac{11}{11}$.



Show the steps each time.
For example:
$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$

3 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\frac{1}{3} = \frac{\square}{6} = \frac{4}{\square} = \frac{\square}{30} = \frac{\square}{60} = \frac{100}{\square}$

b $\frac{2}{8} = \frac{\square}{4} = \frac{\square}{12} = \frac{6}{\square} = \frac{\square}{80} = \frac{10}{\square}$

Example 4 Converting to simplest form

Write these fractions in simplest form.

a $\frac{12}{20}$ **b** $\frac{7}{42}$

Solution	Explanation
a $\frac{12}{20} = \frac{3 \times \cancel{4}^{1}}{5 \times \cancel{4}^{1}} = \frac{3}{5}$	The HCF of 12 and 20 is 4. Both the numerator and the denominator are divided by the HCF of 4. $4 \div 4 = 1$
b $\frac{7}{42} = \frac{\cancel{7} \times 1}{\cancel{7} \times 6} = \frac{1}{6}$	The HCF of 7 and 42 is 7. The 7 is 'cancelled' (i.e. $7 \div 7 = 1$) from the numerator and the denominator.

4 Fill in the gaps to reduce these fractions to their simplest form.



HCF means **H**ighest **C**ommon **F**actor.

a $\frac{10}{30}$ **i** HCF = $\square$ **ii** $\frac{10}{30} = \frac{1 \times \square}{3 \times \square}$. Therefore, simplest form is $\frac{\square}{3}$.

b $\frac{4}{18}$ **i** HCF = $\square$ **ii** $\frac{4}{18} = \frac{2 \times \square}{9 \times \square}$. Therefore, simplest form is $\frac{\square}{9}$.

c $\frac{4}{28}$ **i** HCF = $\square$ **ii** $\frac{4}{28} = \frac{1 \times \square}{7 \times \square}$. Therefore, simplest form is $\frac{1}{\square}$.

d $\frac{9}{15}$ **i** HCF = $\square$ **ii** $\frac{9}{15} = \frac{3 \times \square}{5 \times \square}$. Therefore, simplest form is $\frac{\square}{\square}$.

FLUENCY 5–7(½) 5–8(½)

5 Copy and complete these equivalent fractions.

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{5}{8} = \frac{\square}{80}$ **c** $\frac{6}{11} = \frac{18}{\square}$ **d** $\frac{2}{7} = \frac{16}{\square}$

e $\frac{3}{\square} = \frac{15}{40}$ **f** $\frac{\square}{1} = \frac{14}{7}$ **g** $\frac{\square}{10} = \frac{24}{20}$ **h** $\frac{13}{14} = \frac{\square}{42}$

i $\frac{2}{7} = \frac{10}{\square}$ **j** $\frac{19}{20} = \frac{190}{\square}$ **k** $\frac{11}{21} = \frac{55}{\square}$ **l** $\frac{11}{\square} = \frac{44}{8}$



Always multiply the numerator (up) and denominator (down) by the same number.

$\frac{3}{4} \xrightarrow{\times 3} \frac{\square}{12}$ (also $\times 3$ below)

6 By writing either = or ≠ in the box, state whether each pair of fractions is equivalent or not equivalent. The first one has been done for you.

a $\frac{1}{2} \boxed{\neq} \frac{5}{8}$ **b** $\frac{4}{8} \square \frac{2}{4}$

c $\frac{3}{7} \square \frac{30}{60}$ **d** $\frac{5}{9} \square \frac{15}{18}$

e $\frac{11}{15} \square \frac{33}{45}$ **f** $\frac{1}{2} \square \frac{402}{804}$

g $\frac{12}{36} \square \frac{1}{3}$ **h** $\frac{18}{24} \square \frac{21}{28}$

i $\frac{6}{18} \square \frac{11}{33}$



For 6a think: $\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$ not $\frac{5}{8}$

or: $\frac{1}{2} \overset{\times 4}{=} \frac{4}{8}$ not $\frac{5}{8}$

7 Simplify the following fractions.

a $\frac{5 \times 1}{5 \times 2}$ **b** $\frac{6 \times 1}{6 \times 3}$ **c** $\frac{4 \times 3}{4 \times 4}$

d $\frac{7 \times 2}{7 \times 3}$ **e** $\frac{4 \times 5}{4 \times 6}$ **f** $\frac{10 \times 1}{10 \times 3}$

g $\frac{5 \times 3}{5 \times 5}$ **h** $\frac{8 \times 2}{8 \times 5}$



Remember, $\frac{5}{5} = 5 \div 5 = 1$

8 Write the following fractions in simplest form.

a $\frac{15}{20}$ **b** $\frac{12}{18}$ **c** $\frac{10}{30}$ **d** $\frac{8}{22}$

e $\frac{14}{35}$ **f** $\frac{2}{22}$ **g** $\frac{8}{56}$ **h** $\frac{9}{27}$

i $\frac{35}{45}$ **j** $\frac{36}{96}$ **k** $\frac{120}{144}$ **l** $\frac{700}{140}$



HCF of 15 and 20 is 5.

$\frac{15}{20} \overset{\div 5}{=} \frac{?}{?}$

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 In each group, choose the fraction that is not in its simplest form. What should it be?

a $\frac{1}{3}, \frac{3}{8}, \frac{5}{9}, \frac{7}{14}$

b $\frac{2}{5}, \frac{12}{16}, \frac{15}{19}, \frac{13}{37}$

c $\frac{12}{19}, \frac{4}{42}, \frac{5}{24}, \frac{6}{61}$

d $\frac{7}{63}, \frac{9}{62}, \frac{11}{81}, \frac{13}{72}$

10 Which of the following fractions are equivalent to $\frac{8}{20}$?

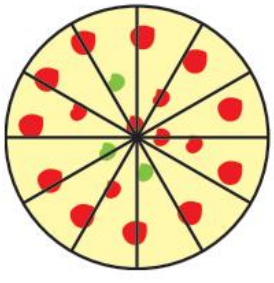
$\frac{4}{10}, \frac{1}{5}, \frac{6}{20}, \frac{8}{10}, \frac{16}{40}, \frac{2}{5}, \frac{4}{12}, \frac{12}{40}, \frac{80}{200}, \frac{1}{4}$

4B

11 A family block of chocolate consists of 6 rows of 6 individual squares. Tania eats 16 individual squares. What fraction of the block, in simplest terms, has Tania eaten?



12 Jason, Joanna and Jack are sharing a large pizza for dinner. The pizza has been cut into 12 equal pieces. Jason would like $\frac{1}{3}$ of the pizza, Joanna would like $\frac{1}{4}$ of the pizza and Jack will eat whatever is remaining.
How much does Jack eat?





Mystery fraction

—

13

13 A fraction when simplified is written as $\frac{3}{5}$.
What could the fraction have been before it was simplified?
Show your fraction on a diagram. Is your fraction the same as the student's sitting next to you?

4C Mixed numbers

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

As we have seen in this chapter, a fraction is a common way of representing part of a whole. For example, a particular car trip may require $\frac{2}{3}$ of a tank of petrol.

On many occasions, you may need whole numbers plus a part of a whole number. For example, a long interstate car trip may require $2\frac{1}{4}$ tanks of fuel.

When you have a combination of a whole number and a fraction, we call this a mixed number.



A long car trip may require a full tank of petrol and another fraction of a tank as well.

Let's start: Pizza frenzy

Tom ate $1\frac{1}{2}$ pizzas. Chandra ate $\frac{3}{2}$ of a pizza. Who ate the most? Discuss, showing each person's pizzas on a separate diagram.

Key ideas

- A number is said to be a **mixed number** when it is a mix of a whole number plus a proper fraction.

$2\frac{3}{5}$ is a mixed number

whole number: 2 — proper fraction: $\frac{3}{5}$

Mixed number A number with a whole number part and a fraction part

- Improper fractions (fractions where the numerator is greater than or equal to the denominator) can be converted to mixed numbers or whole numbers.

$\frac{15}{4} = 3\frac{3}{4}$

improper fraction: $\frac{15}{4}$ — mixed number: $3\frac{3}{4}$ — $15 \div 4 = 3$ rem. 3

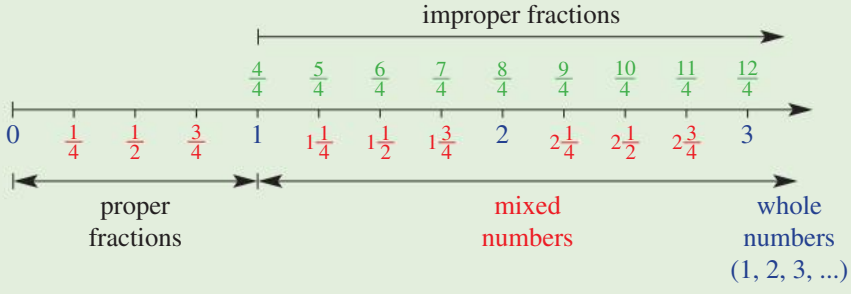
$\frac{16}{4} = 4$

improper fraction: $\frac{16}{4}$ — whole number: 4 — $16 \div 4 = 4$

- Mixed numbers can be converted to improper fractions, and the opposite way.

$\frac{11}{4}$ → $\frac{4}{4}$ $\frac{4}{4}$ $\frac{3}{4}$ ← $\frac{2 \times 4 + 3}{4} = 2\frac{3}{4}$

- A number line helps show the different types of fractions.



Exercise 4C

UNDERSTANDING 1–4 4

Example 5 Placing fractions on the number line

a Show $1\frac{3}{4}$ on a number line.

b Between which two whole numbers does it lie?

Solution	Explanation
a 	Draw a number line starting at zero and going up by quarters. Put a dot at $1\frac{3}{4}$.
b $1\frac{3}{4}$ lies between 1 and 2	The whole number before $1\frac{3}{4}$ is 1 and the whole number after it is 2.

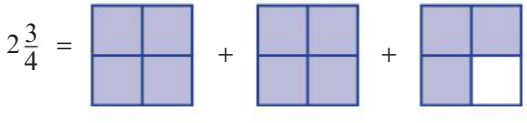
1 Between which two whole numbers do the following mixed numbers lie?

a $2\frac{1}{2}$ **b** $11\frac{1}{7}$ **c** $36\frac{8}{9}$



Draw a number line to help you decide.

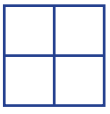
2 The mixed number $2\frac{3}{4}$ can be represented in 'window shapes' as



Represent the following mixed numbers using 'window shapes'.

a $1\frac{1}{4}$ **b** $1\frac{3}{4}$ **c** $3\frac{2}{4}$ **d** $5\frac{2}{4}$

3 A 'window shape' consists of four panes of glass. How many panes of glass are there in the following number of 'window shapes'?





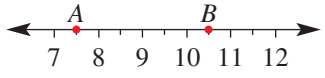
a 2 **b** 3 **c** 7

d 11 **e** $4\frac{1}{4}$ **f** $1\frac{3}{4}$

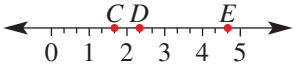
g $2\frac{2}{4}$ **h** $5\frac{4}{4}$

4 What mixed numbers correspond to the letters written on each number line?

a



b



FLUENCY 5–6(½) 5–6(½), 7

Example 6 Converting mixed numbers to improper fractions

Convert $3\frac{1}{5}$ to an improper fraction.

Solution

Method 1

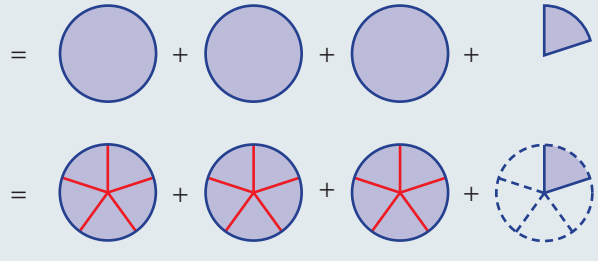
$3\frac{1}{5} = 1 + 1 + 1 + \frac{1}{5}$

$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$

$= \frac{16}{5}$

Explanation

$3\frac{1}{5} = 3$ wholes $+ \frac{1}{5}$ of a whole



Method 2

$3\frac{1}{5} = \frac{15}{5} + \frac{1}{5}$

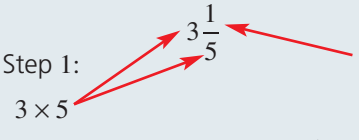
$= \frac{16}{5}$

Short-cut method:
Step 1: Multiply the whole number part by the denominator.
Step 2: Then add the numerator: $3 \times 5 + 1 = 16$

Method 3

$3\frac{1}{5} = \frac{3 \times 5 + 1}{5}$

$= \frac{16}{5}$



This is: $3\frac{1}{5} = \frac{3 \times 5 + 1}{5}$

The denominator remains 5, as we are talking about fifths.

4C

5 Convert these mixed numbers to improper fractions.

a $2\frac{1}{5}$ **b** $1\frac{3}{5}$ **c** $3\frac{1}{3}$ **d** $5\frac{2}{3}$ **e** $4\frac{1}{7}$

f $3\frac{3}{7}$ **g** $2\frac{1}{2}$ **h** $6\frac{1}{2}$ **i** $4\frac{2}{5}$ **j** $11\frac{1}{2}$

k $8\frac{2}{5}$ **l** $10\frac{3}{10}$ **m** $6\frac{1}{9}$ **n** $2\frac{7}{9}$ **o** $5\frac{2}{8}$

p $2\frac{5}{8}$ **q** $1\frac{11}{12}$ **r** $3\frac{5}{11}$ **s** $4\frac{5}{12}$ **t** $9\frac{7}{12}$

Example 7 Converting improper fractions to mixed numbers

Convert $\frac{11}{4}$ to a mixed number.

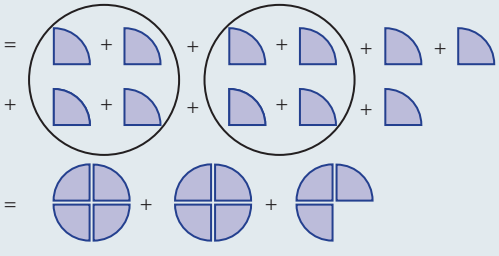
Solution

Method 1

$$\frac{11}{4} = \frac{8+3}{4}$$
$$= \frac{8}{4} + \frac{3}{4}$$
$$= 2 + \frac{3}{4}$$
$$= 2\frac{3}{4}$$

Explanation

$\frac{11}{4} = 11$ quarters



$= 2\frac{3}{4}$

Method 2

$$4\overline{)11} \quad 2 \text{ rem. } 3$$
$$= 2\frac{3}{4}$$

Divide the bottom (denominator) into the top (numerator). The remainder becomes the fraction part.

6 Convert these improper fractions to mixed numbers.

a $\frac{7}{5}$ **b** $\frac{4}{3}$ **c** $\frac{5}{3}$ **d** $\frac{7}{4}$ **e** $\frac{11}{3}$ **f** $\frac{21}{5}$ **g** $\frac{16}{7}$ **h** $\frac{10}{4}$

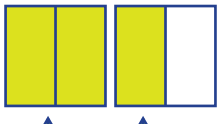
i $\frac{12}{7}$ **j** $\frac{19}{6}$ **k** $\frac{20}{3}$ **l** $\frac{41}{4}$ **m** $\frac{35}{8}$ **n** $\frac{26}{5}$ **o** $\frac{48}{7}$ **p** $\frac{41}{3}$

q $\frac{37}{12}$ **r** $\frac{81}{11}$ **s** $\frac{93}{10}$ **t** $\frac{78}{7}$

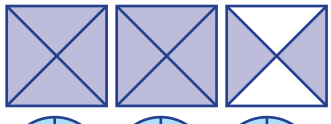
7 For each of these diagrams, write:

i the mixed number **ii** the improper fraction

a



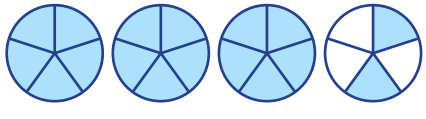
b



c



d



PROBLEM-SOLVING AND REASONING

8, 9 | 8–11

8 $\frac{10}{4}$ simplifies to $\frac{5}{2}$ which equals $2\frac{1}{2}$.
Write the following fractions in two different ways.

a $\frac{8}{6}$ **b** $\frac{12}{8}$ **c** $\frac{6}{4}$ **d** $\frac{20}{16}$



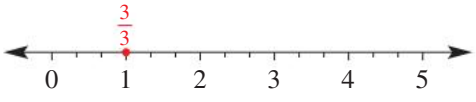
It's easier to simplify first.

9 Copy the number line and show the fractions on it.

a $\frac{2}{3}, 2, \frac{5}{3}, 3\frac{1}{3}, \frac{15}{3}$



This number line is marked in thirds.



b $\frac{1}{5}, \frac{4}{5}, \frac{5}{5}, 1\frac{2}{5}, \frac{10}{5}, 3\frac{4}{5}$



This number line shows fifths.



10 Rewrite each of these patterns, using improper fractions where needed.

a $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$ **b** $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}, 2$

11 Four friends order three large pizzas for their dinner. Each pizza is cut into eight equal slices. Simone has three slices, Izabella has four slices, Mark has five slices and Alex has three slices.



Remember, each pizza has 8 equal slices.

a Draw circles to show the three pizzas ordered.
b How many pizza slices do they eat in total?
c How much pizza do they eat in total? Give your answer as a mixed number.
d How many pizza slices are left uneaten?
e How much pizza is left uneaten? Give your answer as a mixed number.

Writing fractions

— | 12

12 What different fractions can you write using only the digits 1, 2 and 3? Who in your class wrote the most?





You can include improper fractions and mixed numbers. You don't need to use all three digits.

4D Ordering fractions

Interactive

Widgets

HOTsheets

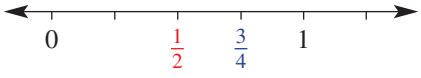
Walkthrough

Just like whole numbers, fractions can be written in order. We can use the ideas from section 4B to help.

Remember that > means *is greater than* and < means *is less than*.

A fraction is greater than another fraction if it lies to the right of that fraction on a number line.

$\frac{3}{4} > \frac{1}{2}$



Let's start: The order of five

- As a warm-up activity, ask five volunteer students to arrange themselves in alphabetical order, then in height order and, finally, in birthday order.
- Each of the five students receives a large fraction card and displays it to the class.
  (For example, $\frac{1}{2}, \frac{1}{10}, \frac{2}{10}, \frac{10}{2}, \frac{2}{3}$.)
- The rest of the class must then attempt to order the students in ascending order, according to their fraction card. It is a group decision and none of the five students should move until the class agrees on a decision.



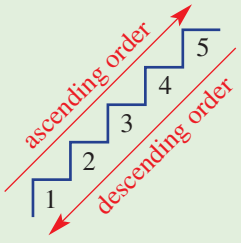
Key ideas

- To order (or arrange) fractions we must know how to compare different fractions. There are three cases to consider.
  1. If the numerators are the same, the smallest fraction is the one with the largest denominator, as the whole has been divided up into the most pieces.
     For example: $\frac{1}{7} < \frac{1}{2}$
  2. If the denominators are the same, the smallest fraction is the one with the smallest numerator.
     For example: $\frac{3}{10} < \frac{7}{10}$
  3. Otherwise, use equivalent fractions to make fractions with the same denominator. (The **lowest common denominator (LCD)** is best.) Then compare numerators as above.
- **Ascending** order is when numbers are ordered going *up*, from smallest to largest.
- **Descending** order is when numbers are ordered going *down*, from largest to smallest.

Lowest common denominator (LCD) The smallest common multiple of the denominators of two or more fractions

Ascending Going up, from smallest to largest

Descending Going down, from largest to smallest



Exercise 4D

UNDERSTANDING 1–4 | 4

1 Circle the largest fraction in each of the following lists.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{4}{3}, \frac{2}{3}, \frac{7}{3}, \frac{5}{3}$

c $\frac{5}{11}, \frac{9}{11}, \frac{3}{11}, \frac{4}{11}$ **d** $\frac{8}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}$



When the denominators are the same, compare numerators: $\frac{6}{7}$ is more than $\frac{5}{7}$, just as 6 is more than 5.

2 State the lowest common multiple of the following sets of numbers.

a 2, 5 **b** 3, 7 **c** 5, 4

d 6, 5 **e** 3, 6 **f** 2, 10

g 4, 6 **h** 8, 6 **i** 2, 3, 5

j 3, 4, 6 **k** 3, 8, 4 **l** 2, 6, 5



2, 4, 6, 8, (10), …
5, (10), 15, 20, 25, …
The LCM of 2 and 5 is 10.

3 State the lowest common denominator of the following sets of fractions.

a $\frac{1}{3}, \frac{3}{5}$ **b** $\frac{2}{4}, \frac{3}{5}$ **c** $\frac{4}{7}, \frac{2}{3}$

d $\frac{2}{10}, \frac{1}{5}$ **e** $\frac{4}{6}, \frac{3}{8}$ **f** $\frac{5}{12}, \frac{2}{5}$

g $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ **h** $\frac{4}{3}, \frac{3}{4}$



Find the LCM of the denominators.

4 Fill in the gaps to produce equivalent fractions.

a $\frac{2}{5} = \frac{\square}{15}$ **b** $\frac{2}{3} = \frac{\square}{12}$ **c** $\frac{1}{4} = \frac{\square}{16}$

d $\frac{3}{7} = \frac{\square}{14}$ **e** $\frac{3}{8} = \frac{\square}{40}$ **f** $\frac{5}{6} = \frac{\square}{18}$

FLUENCY 5–7(½) | 5–7(½), 8

Example 8 Comparing fractions

Write < or > in each box to make a true mathematical statement.

a $\frac{2}{5} \square \frac{4}{5}$ **b** $\frac{1}{3} \square \frac{1}{5}$

Solution	Explanation
a $\frac{2}{5} \boxed{<} \frac{4}{5}$	Denominators are the same, therefore compare numerators. $2 < 4$ so $\frac{2}{5} < \frac{4}{5}$
b $\frac{1}{3} \boxed{>} \frac{1}{5}$	Numerators are the same. The smaller fraction has the larger denominator. So $\frac{1}{3} > \frac{1}{5}$

4D

5 Write < or > to make true statements.

a $\frac{3}{5} \square \frac{1}{5}$ **b** $\frac{7}{9} \square \frac{2}{9}$ **c** $\frac{2}{2} \square \frac{3}{2}$ **d** $\frac{13}{18} \square \frac{17}{18}$

e $\frac{2}{3} \square \frac{1}{3}$ **f** $\frac{7}{10} \square \frac{5}{10}$ **g** $\frac{1}{6} \square \frac{5}{6}$ **h** $\frac{7}{2} \square \frac{1}{2}$

6 Write < or > to make true statements.

a $\frac{1}{4} \square \frac{1}{3}$ **b** $\frac{1}{10} \square \frac{1}{20}$ **c** $\frac{1}{7} \square \frac{1}{5}$

d $\frac{1}{10} \square \frac{1}{2}$ **e** $\frac{1}{4} \square \frac{1}{40}$ **f** $\frac{1}{6} \square \frac{1}{8}$

Example 9 Comparing fractions by using a common denominator

Which is larger, $\frac{2}{3}$ or $\frac{3}{5}$?

Solution	Explanation
$\frac{2}{3} = \frac{10}{15}$ $\frac{3}{5} = \frac{9}{15}$	Find a common denominator. The lowest common multiple (LCM) of 3 and 5 is 15. Produce equivalent fractions with that denominator. $\frac{2}{3} \overset{\times 5}{=} \frac{\square}{15}$ $\quad \frac{3}{5} \overset{\times 5}{=} \frac{\square}{15}$
$\frac{2}{3} > \frac{3}{5}$	Compare numerators: $10 > 9$ so $\frac{10}{15} > \frac{9}{15}$

7 Decide which fraction is the largest in each pair.

a $\frac{1}{2}$ and $\frac{2}{3}$ **b** $\frac{4}{5}$ and $\frac{2}{3}$ **c** $\frac{7}{10}$ and $\frac{4}{5}$

d $\frac{3}{2}$ and $\frac{5}{4}$ **e** $\frac{3}{4}$ and $\frac{4}{9}$ **f** $\frac{1}{2}$ and $\frac{3}{5}$



First write each pair of fractions with the same denominator.

Example 10 Ordering fractions

Write the fractions $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$ in ascending order.

Solution	Explanation
$\frac{45}{60}, \frac{48}{60}, \frac{40}{60}$	LCD of 3, 4 and 5 is 60. Produce equivalent fractions with denominator of 60.
$\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$	Order fractions in ascending order. (Look at the top of each fraction.)
$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$	Rewrite fractions back in original form.

8 Place the following fractions in ascending order.

a $\frac{3}{5}, \frac{8}{5}, 1\frac{2}{5}$ **b** $\frac{5}{9}, \frac{1}{3}, \frac{2}{9}$

c $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ **d** $\frac{5}{6}, \frac{3}{5}, \frac{2}{3}$



Ascending order is from smallest to largest.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Place the following cake fractions in decreasing order of size.

A sponge cake shared equally by four people $= \frac{1}{4}$ cake

B chocolate cake shared equally by eleven people $= \frac{1}{11}$ cake

C carrot and walnut cake shared equally by eight people $= \frac{1}{8}$ cake



10 Sally and Dion ordered two large pizzas. Sally ate $\frac{2}{3}$ of her pizza. Dion ate $\frac{5}{8}$ of his pizza.

a Show each person's pizzas on a separate diagram.

b Who ate the most pizza?

c Should they have ordered only one pizza?

11 Rewrite the fractions in each set with their lowest common denominator. Then write the next two fractions that would continue the pattern.

a $\frac{2}{9}, \frac{1}{3}, \frac{4}{9}, ___, ___$ **b** $\frac{1}{2}, \frac{5}{4}, 2, ___, ___$

12 Write a fraction that lies between the following pairs of fractions.

a $\frac{1}{10}$ and $\frac{7}{10}$ **b** $\frac{1}{2}$ and $\frac{7}{10}$ **c** $\frac{1}{5}$ and $\frac{8}{10}$

d $\frac{3}{5}, \frac{3}{4}$ **e** $\frac{1}{4}, \frac{1}{2}$ **f** $\frac{2}{7}, \frac{1}{6}$

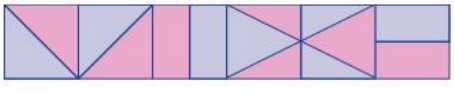


First write each pair with the same denominator.

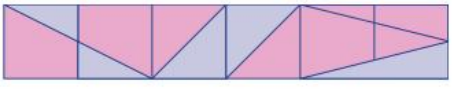
Shady designs — 13

13 **a** For each of the diagrams shown, work out what fraction of the rectangle is coloured purple.

i



ii



b Create your own rectangular design that is three-quarters purple overall.

4E Adding fractions

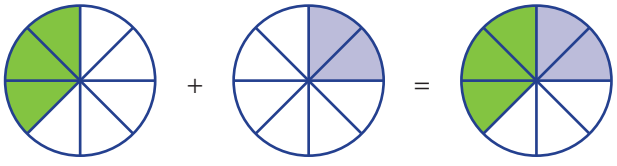
Interactive

Widgets

HOTsheets

Walkthrough

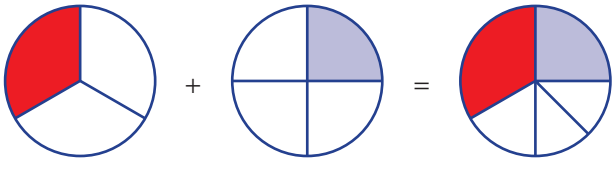
Fractions with the same denominator can be easily added together.



$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

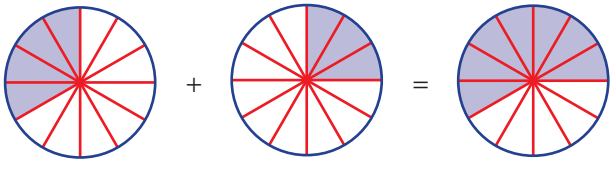
three-eighths + two-eighths = five-eighths

Fractions with different denominators cannot be added together so easily.



$\frac{1}{3} + \frac{1}{4} = ?$

But with a common denominator it is possible.



$\frac{1}{3} + \frac{1}{4} = ?$

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

four-twelfths + three-twelfths = seven-twelfths

Let's start: 'Like' addition

As a class, discuss which of the following pairs of numbers can be simply added together without having to change them in some way.

a 6 goals, 2 goals
b 11 goals, 5 behinds
c 56 runs, 3 wickets
d 6 hours, 5 minutes
e 21 seconds, 15 seconds
f 47 minutes, 13 seconds
g 15 cm, 3 m
h 2.2 km, 4.1 km
i 5 kg, 1680 g
j $\frac{2}{7}, \frac{3}{7}$
k $\frac{1}{4}, \frac{1}{2}$
l $2\frac{5}{12}, 1\frac{1}{3}$

You can see that, when adding, the units need to be the same. With fractions, the 'units' are the denominators.

Key ideas

- Fractions can be simplified using addition if they are 'like' fractions; that is, if they have the same denominator.
- If the denominators are equal, add the numerators and retain the denominator. For example: $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$
- If the denominators are not the same:
  - find the lowest common denominator (LCD)
  - convert fractions to equivalent fractions using the LCD
  - add the numerators and retain the LCD
  - simplify the answer if possible.

  For example: $\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}$

  $=\frac{22}{15}$ or $1\frac{7}{15}$

Exercise 4E

UNDERSTANDING 1–4 4

1 Copy the following sentences into your workbook and fill in the gaps.

a To add two fractions together, they must have the same _____.

b When adding fractions together, if they have the same _____, you simply add the _____.

c When adding two or more fractions where the _____ are different, you must find the _____ _____ _____.

d After carrying out the addition of fractions, you should always _____ your answer.



Choose from:
denominator, common, answer, lowest, simplify, multiple, numerators

2 Use words to complete these additions.

a one-third + one-third = _____ thirds

b one-quarter + two _____ = three-quarters

c three-tenths + _____ tenths = seven-tenths

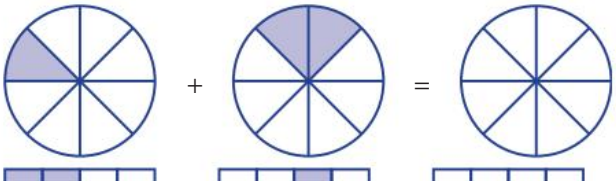
d one-fifth + three-fifths = four _____

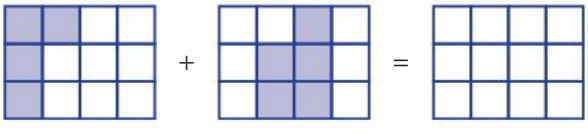
e _____ half + _____ half = two-halves (one whole)

3 Copy and complete the following by shading the shape on the right side.

a 

b 

c 

d 

4E

4 Answer True (T) or false (F) for each of the following.

a $\frac{1}{6}+\frac{3}{6}=\frac{4}{6}$ **b** $\frac{2}{5}+\frac{4}{5}=\frac{6}{10}$ **c** $\frac{1}{11}+\frac{3}{11}=\frac{4}{11}$

d $\frac{3}{5}+\frac{4}{5}=1\frac{2}{5}$ **e** $\frac{2}{7}+\frac{2}{7}=\frac{2}{7}$ **f** $\frac{7}{12}+\frac{4}{12}=\frac{11}{12}$

g $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **h** $\frac{1}{100}+\frac{2}{100}=\frac{3}{200}$

FLUENCY 5–9(½) 5–10(½)

Example 11 Adding 'like' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{3}{5}$ **b** $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}$

Solution	Explanation
a $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$	The denominators are the same ('like'), so simply add the numerators. one-fifth + three-fifths = four-fifths
b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}=\frac{14}{11}$ $=1\frac{3}{11}$	Denominators are the same, so add the numerators. We are adding elevenths. Convert to a mixed number if required.

5 Copy and complete.

a $\frac{1}{8}+\frac{4}{8}=\frac{\square}{8}$ **b** $\frac{2}{7}+\frac{3}{7}=\frac{\square}{7}$ **c** $\frac{1}{5}+\frac{3}{5}=\frac{\square}{5}$

d $\frac{3}{11}+\frac{6}{11}=\frac{\square}{11}$ **e** $\frac{5}{8}+\frac{2}{8}=\frac{\square}{8}$ **f** $\frac{1}{12}+\frac{6}{12}=\frac{\square}{12}$

g $\frac{3}{15}+\frac{4}{15}=\frac{\square}{15}$ **h** $\frac{3}{9}+\frac{2}{9}=\frac{\square}{9}$ **i** $\frac{3}{5}+\frac{1}{5}=\frac{\square}{5}$

j $\frac{2}{7}+\frac{4}{7}=\frac{\square}{7}$ **k** $\frac{1}{10}+\frac{6}{10}=\frac{\square}{10}$ **l** $\frac{77}{100}+\frac{4}{100}=\frac{\square}{100}$

6 Add these fractions and write your answers using mixed numbers.

a $\frac{6}{7}+\frac{3}{7}$ **b** $\frac{7}{10}+\frac{6}{10}$ **c** $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$

d $\frac{12}{19}+\frac{3}{19}+\frac{8}{19}$ **e** $\frac{7}{10}+\frac{4}{10}$ **f** $\frac{4}{5}+\frac{3}{5}$

g $\frac{6}{7}+\frac{4}{7}$ **h** $\frac{8}{11}+\frac{6}{11}$ **i** $\frac{99}{100}+\frac{2}{100}$



When you add these fractions, the denominator stays the same.

Example 12 Adding 'unlike' fractions

Add the following fractions together.

a $\frac{1}{5}+\frac{1}{2}$ **b** $\frac{3}{4}+\frac{5}{6}$

Solution	Explanation
a $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}$	LCD is 10.
$=\frac{7}{10}$	Write equivalent fractions with the LCD.
	$\frac{1}{5}\times\frac{2}{2}=\frac{2}{10}$ $\frac{1}{2}\times\frac{5}{5}=\frac{5}{10}$
	Denominators are the same, so add numerators.
b $\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}$	LCD is 12.
$=\frac{19}{12}$	Write equivalent fractions with the LCD.
	$\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$ $\frac{5}{6}\times\frac{2}{2}=\frac{10}{12}$
$=1\frac{7}{12}$	Denominators are the same, so add numerators.
	Write as a mixed number, if required.

7 Copy and complete the following additions.

a $\frac{3}{10}+\frac{2}{5}$ (LCD = 10)

$=\frac{3}{10}+\frac{\square}{10}$

$=\frac{\square}{10}$

b $\frac{1}{2}+\frac{3}{8}$ (LCD = 8)

$=\frac{\square}{8}+\frac{3}{8}$

$=\frac{\square}{8}$

c $\frac{1}{4}+\frac{2}{3}$ (LCD = 12)

$=\frac{\square}{12}+\frac{8}{12}$

$=\frac{\square}{12}$

d $\frac{2}{8}+\frac{1}{6}$ (LCD = 24)

$=\frac{\square}{24}+\frac{\square}{24}$

$=\frac{\square}{24}$

$=\frac{5}{12}$

8 Add the following fractions.

a $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{3}+\frac{3}{5}$ **c** $\frac{1}{2}+\frac{1}{6}$

d $\frac{1}{4}+\frac{1}{3}$ **e** $\frac{2}{5}+\frac{1}{4}$ **f** $\frac{1}{5}+\frac{3}{4}$

g $\frac{2}{7}+\frac{1}{3}$ **h** $\frac{3}{8}+\frac{1}{5}$ **i** $\frac{3}{5}+\frac{5}{6}$

j $\frac{4}{7}+\frac{3}{4}$ **k** $\frac{8}{11}+\frac{2}{3}$ **l** $\frac{2}{3}+\frac{3}{4}$



Use similar steps to question 7.

4E

Example 13 Adding mixed numbers

Simplify:

a $3\frac{2}{3}+4\frac{2}{3}$ **b** $2\frac{5}{6}+3\frac{3}{4}$

Solution	Explanation
a $3+4+\frac{2}{3}+\frac{2}{3}=7+\frac{4}{3}$	Add the whole number parts together. Add the fraction parts together.
$=8\frac{1}{3}$	Noting that $\frac{4}{3}=1\frac{1}{3}$, simplify the answer.
Alternatively	
$3\frac{2}{3}+4\frac{2}{3}=\frac{11}{3}+\frac{14}{3}$	First convert to an improper fraction.
$=\frac{25}{3}$ or $8\frac{1}{3}$	Add the numerators.
b $2+3+\frac{5}{6}+\frac{3}{4}$	Add the whole number parts together. LCD of 6 and 4 is 12.
$=5+\frac{10}{12}+\frac{9}{12}$	Write equivalent fractions with LCD.
$=5+\frac{19}{12}$	Add the fraction parts together.
$=5+1\frac{7}{12}$	Noting that $\frac{19}{12}=1\frac{7}{12}$, simplify the answer.
$=6\frac{7}{12}$	
Alternatively	
$2\frac{5}{6}+3\frac{3}{4}=\frac{17}{6}+\frac{15}{4}$	First convert to an improper fraction.
$=\frac{34}{12}+\frac{45}{12}$	Convert using a LCD = 12.
$=\frac{79}{12}$ or $6\frac{7}{12}$	Add the numerators.

9 Simplify:

a $1\frac{1}{5}+2\frac{3}{5}$ **b** $3\frac{2}{7}+4\frac{1}{7}$ **c** $11\frac{1}{4}+1\frac{2}{4}$ **d** $1\frac{3}{9}+4\frac{2}{9}$

e $5\frac{2}{3}+4\frac{2}{3}$ **f** $8\frac{3}{6}+12\frac{4}{6}$ **g** $9\frac{7}{11}+9\frac{7}{11}$ **h** $4\frac{3}{5}+7\frac{4}{5}$

10 Simplify:

a $2\frac{2}{3}+1\frac{3}{4}$ **b** $5\frac{2}{5}+1\frac{5}{6}$ **c** $3\frac{1}{2}+8\frac{2}{3}$ **d** $5\frac{4}{7}+7\frac{3}{4}$

e $8\frac{1}{2}+6\frac{3}{5}$ **f** $12\frac{2}{3}+6\frac{4}{9}$ **g** $17\frac{8}{11}+7\frac{3}{4}$ **h** $9\frac{7}{12}+5\frac{5}{8}$

PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 Dad gave you $\frac{2}{5}$ of the money for a movie ticket and Mum gave you $\frac{1}{5}$.



a What fraction of the ticket did your parents pay for?
b What fraction was left for you to pay?
c If the ticket cost $10, how much did you pay?

12 Julie owns $\frac{1}{3}$ of a company and Sean owns $\frac{1}{4}$.



a What fraction of the company do they own together?
b What fraction of the company is left?

13 Mark spends $\frac{1}{3}$ of the day at school and $\frac{3}{8}$ of the day asleep.

a What fraction of the day has been used?
b What fraction of the day is left for fun?
c How many hours of fun does he get?

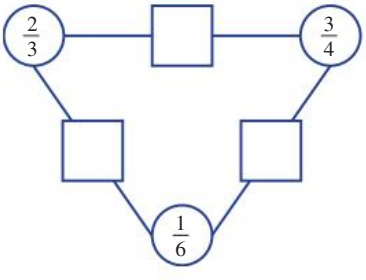


There are 24 hours in a day.

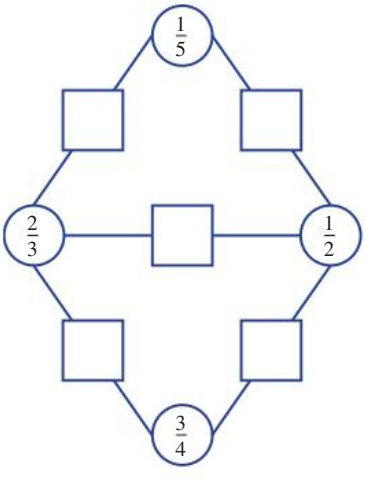
Fraction networks — 14, 15

14 To find the number in each square, add the fractions in the two circles that are joined to it. Fill in the following fraction network.

a

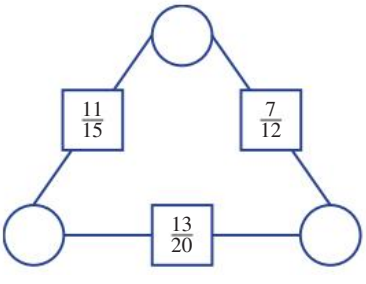


b

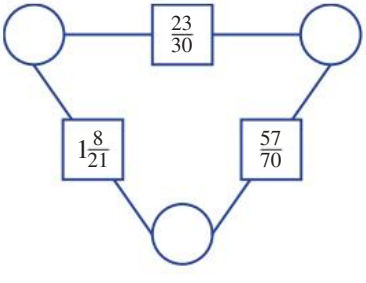


15 Now find the fractions in the circles.

a



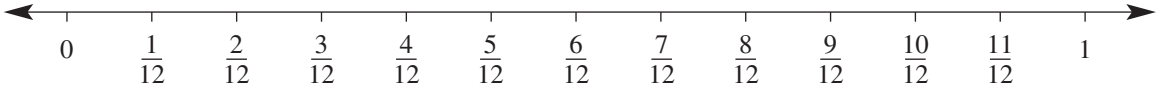
b



4F Subtracting fractions

The rules for subtracting fractions are very similar to adding fractions. Before you can subtract, we aim to write the fractions with the same denominator.

Let's start: Subtraction on a number line



Copy the number line and use it to find:

- $\frac{9}{12} - \frac{2}{12}$
- $\frac{11}{12} - \frac{1}{2}$
- $1 - \frac{9}{12}$

Key ideas

- Fractions can be simplified easily using subtraction if they are 'like' fractions with the same denominator.
- The rules for subtracting fractions are similar to those for adding fractions.
- When subtracting mixed numbers, you may need to borrow a whole.
  For example:

  $7\frac{1}{8} - 2\frac{3}{8}$ $\frac{1}{8}$ is not big enough to have $\frac{3}{8}$ subtracted from it.

  $6\frac{9}{8} - 2\frac{3}{8}$ Therefore, we choose to borrow a whole from the 7. $(6 + 1 = 7)$

  Alternatively, first convert the mixed numbers to improper fractions.

Exercise 4F

UNDERSTANDING 1–4 4

1 Copy the following sentences and fill in the blanks.

a To subtract one fraction from another, you must have a common _____.

b The LCD of 6 and 12 is _____.

c The LCD of 3 and 5 is _____.

2 Copy and complete the following subtractions.

a three-tenths − two-tenths = _____ tenth

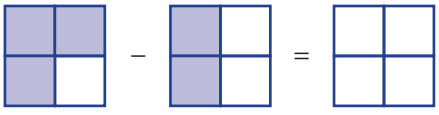
b three-quarters −_____ quarters = one-quarter

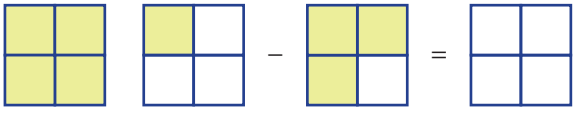
c four _____ − three-fifths = one-fifth

d one whole − two-fifths = _____ fifths

e five-eighths − two _____ = three-eighths

3 Copy and complete the following.

a 

b 

c 

4 Copy and complete these equations.

a $\frac{3}{7} - \frac{2}{7} = \frac{\square}{7}$

b $\frac{8}{13} - \frac{5}{13} = \frac{\square}{13}$

c $\frac{1}{3} - \frac{1}{4} = \frac{\square}{12} - \frac{\square}{12}$

$= \frac{\square}{12}$

d $\frac{4}{5} - \frac{2}{3} = \frac{\square}{15} - \frac{\square}{15}$

$= \frac{\square}{15}$

FLUENCY 5–7(½) 5–8(½)

Example 14 Subtracting 'like' and 'unlike' fractions

Simplify:

a $\frac{7}{9} - \frac{2}{9}$

b $\frac{5}{6} - \frac{1}{4}$

Solution	Explanation
a $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$	Denominators are the same, therefore we are ready to subtract the second numerator from the first.
b $\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12}$ $= \frac{7}{12}$	Find the LCD, which is 12. Write equivalent fractions with the LCD. $\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$ $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$ We have the same denominators now, so subtract second numerator from the first. The denominator stays the same (twelfths).

5 Simplify:

a $\frac{5}{7} - \frac{3}{7}$ **b** $\frac{4}{11} - \frac{1}{11}$ **c** $\frac{12}{18} - \frac{5}{18}$ **d** $\frac{2}{3} - \frac{1}{3}$

e $\frac{3}{5} - \frac{3}{5}$ **f** $\frac{6}{9} - \frac{2}{9}$ **g** $\frac{5}{19} - \frac{2}{19}$ **h** $\frac{17}{23} - \frac{9}{23}$

i $\frac{84}{100} - \frac{53}{100}$ **j** $\frac{41}{50} - \frac{17}{50}$ **k** $\frac{23}{25} - \frac{7}{25}$ **l** $\frac{7}{10} - \frac{3}{10}$

4F

6 Simplify:

a $\frac{2}{3}-\frac{1}{4}$ **b** $\frac{3}{5}-\frac{1}{2}$ **c** $\frac{3}{5}-\frac{3}{6}$ **d** $\frac{4}{7}-\frac{1}{4}$

e $\frac{1}{2}-\frac{1}{3}$ **f** $\frac{3}{4}-\frac{1}{9}$ **g** $\frac{8}{11}-\frac{1}{3}$ **h** $\frac{4}{5}-\frac{2}{3}$

i $\frac{3}{4}-\frac{5}{8}$ **j** $\frac{11}{20}-\frac{2}{5}$ **k** $\frac{5}{12}-\frac{7}{18}$ **l** $\frac{7}{9}-\frac{2}{3}$



First write each fraction using the LCD.

Example 15 Subtracting mixed numbers

Simplify:

a $5\frac{2}{3}-3\frac{1}{4}$ **b** $8\frac{1}{5}-4\frac{3}{4}$

Solution	Explanation
a $5\frac{2}{3}-3\frac{1}{4}=\left(5+\frac{2}{3}\right)-\left(3+\frac{1}{4}\right)$	A mixed number is the addition of a whole number and a proper fraction.
$=(5-3)+\left(\frac{2}{3}-\frac{1}{4}\right)$	Group whole numbers and group proper fractions. Simplify whole numbers.
$=2+\left(\frac{8}{12}-\frac{3}{12}\right)$	Make denominators the same. $\frac{2}{3}\times\frac{4}{4}=\frac{8}{12}, \frac{1}{4}\times\frac{3}{3}=\frac{3}{12}$
$=2\frac{5}{12}$	Borrowing a whole was not required.
Alternatively	
$5\frac{2}{3}-3\frac{1}{4}=\frac{17}{3}-\frac{13}{4}$	First convert to improper fractions.
$=\frac{68}{12}-\frac{39}{12}$	Find and use the LCD.
$=\frac{29}{12}$ or $2\frac{5}{12}$	Subtract the numerators.
b $8\frac{1}{5}-4\frac{3}{4}=8\frac{4}{20}-4\frac{15}{20}$	Convert to twentieths: $\frac{1}{5}\times\frac{4}{4}=\frac{24}{20}, \frac{3}{4}\times\frac{5}{5}=\frac{15}{20}$
$=7\frac{24}{20}-4\frac{15}{20}$	Borrow a whole from the 8 so that the numerators can be subtracted.
$=3\frac{9}{20}$	$8\frac{4}{20}=7+1+\frac{4}{20}=7+\frac{20}{20}+\frac{4}{20}=7\frac{24}{20}$ Subtract whole numbers, subtract fractions.
Alternatively	
$8\frac{1}{5}-4\frac{3}{4}=\frac{41}{5}-\frac{19}{4}$	First convert to improper fractions.
$=\frac{164}{20}-\frac{95}{20}$	Find and use the LCD.
$=\frac{69}{20}$ or $3\frac{9}{20}$	Subtract the numerators.

7 Simplify:

a $3\frac{4}{5} - 2\frac{1}{5}$ **b** $23\frac{5}{7} - 15\frac{2}{7}$ **c** $8\frac{11}{14} - 7\frac{9}{14}$

d $3\frac{5}{9} - \frac{3}{9}$ **e** $6\frac{2}{3} - 4\frac{1}{4}$ **f** $5\frac{3}{7} - 2\frac{1}{4}$

g $9\frac{5}{6} - 5\frac{4}{9}$ **h** $14\frac{3}{4} - 7\frac{7}{10}$



For some of these, you will first need to make denominators the same.

8 Simplify:

a $5\frac{1}{3} - 2\frac{2}{3}$ **b** $8\frac{2}{5} - 3\frac{4}{5}$ **c** $13\frac{1}{2} - 8\frac{5}{6}$

d $12\frac{2}{9} - 7\frac{1}{3}$ **e** $8\frac{5}{12} - 3\frac{3}{4}$ **f** $1\frac{3}{5} - \frac{7}{9}$

g $11\frac{1}{11} - 1\frac{1}{4}$ **h** $6\frac{3}{20} - 3\frac{2}{3}$



You will need to 'borrow a whole' for some of these.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 A family block of chocolate is made up of 60 small squares of chocolate. Marcia eats 10 blocks, Jon eats 9 blocks and Holly eats 5 blocks. What fraction of the block of chocolate is left?

10 Three friends split a restaurant bill. One pays $\frac{1}{2}$ of the bill and one pays $\frac{1}{3}$ of the bill. What fraction of the bill must the third friend pay?



11 A full container of flour weighs $\frac{4}{5}$ kg. The empty container weighs $\frac{1}{20}$ kg. How much does the flour weigh?

12 Copy and complete these two subtractions, which use two different techniques. Decide which technique you prefer.

a $2\frac{1}{4} - 1\frac{2}{3} = \frac{9}{\square} - \frac{\square}{3}$

$= \frac{\square}{12} - \frac{\square}{12}$

$= \frac{\square}{12}$

b $2\frac{1}{4} - 1\frac{2}{3} = \square - 1 + \frac{\square}{4} - \frac{2}{\square}$

$= \square + \frac{\square}{12} - \frac{8}{\square}$

$= \frac{12}{12} + \frac{\square}{12} - \frac{8}{\square}$

$= \frac{\square}{12}$

Finding fractions — 13

13 Two fractions have a sum of $\frac{23}{24}$ and a difference of $\frac{7}{24}$. Can you find the two fractions?



Use a calculator to help.

4G Multiplying fractions

Interactive

Widgets

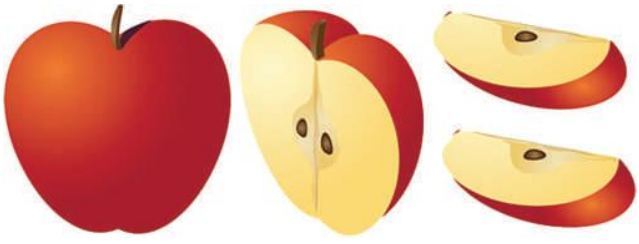
HOTsheets

Walkthrough

When we multiply whole numbers together, we end up with a number larger than (or equal to) the ones we started with.

$2 \times 3 = 6 \quad 5 \times 7 = 35 \quad 15 \times 1 = 15$

But when we multiply fractions together, things can be different. Consider half of half an apple, for example.



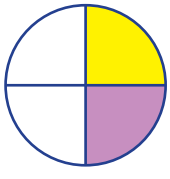




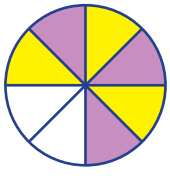
Cutting materials to fit a purpose may involve multiplying a fraction by a fraction.

Let's start: Parts of a circle

- How does this diagram show half of half a circle?



- What is half of three-quarters of a circle?



Use diagrams to investigate other fraction multiplications.

What shortcut (or rule) can you find to help multiply fractions together?

Key ideas

- Fractions do *not* need to have the same denominator to be multiplied together.
- To multiply fractions, multiply the numerators together and multiply the denominators together.
  - In symbols: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$
- If possible, 'simplify' or 'cancel' fractions before multiplying.
  (Remember, you can only cancel numerators with denominators.)
- Mixed numbers must be changed to improper fractions before multiplying.
- Final answers should be written in simplest form.

Exercise 4G

UNDERSTANDING 1–4 4

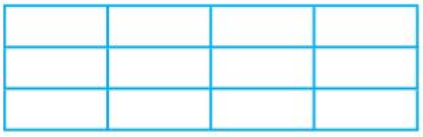


1 Copy these sentences into your workbook and fill in the blanks.

Choose from: *fraction*, *whole*, *number*, *0*, *1*, and *proper*

- **a** A proper fraction has a value that is between _____ and _____.
- **b** An improper fraction is always greater than _____.
- **c** A mixed number consists of two parts, a _____ _____ part and a _____ _____ part.

2 Copy the grid shown here.



- **a** On your diagram, use blue to shade $\frac{1}{3}$ of the grid.
- **b** Now use red to shade $\frac{1}{4}$ of the shaded blue.
- **c** You have now shaded $\frac{1}{4}$ of $\frac{1}{3}$. What fraction is this of the original grid?

Example 16 Finding a simple fraction of a quantity

Find $\frac{2}{3}$ of 15 bananas.

Solution	Explanation
$\frac{2}{3}$ of 15 bananas	Divide 15 bananas into 3 equal groups.
$= \left(\frac{1}{3} \text{ of } 15\right) \times 2$	Therefore, 5 in each group.
$= 5 \times 2$	Take 2 of the groups.
= 10 bananas	
	Answer is 10 bananas.

3 Use drawings to show the answer to these problems.



The denominator tells you how many equal groups to make. The numerator tells you how many of those groups to circle.

- **a** $\frac{1}{3}$ of 12 lollies
- **b** $\frac{1}{5}$ of 10 pencils
- **c** $\frac{2}{3}$ of 18 donuts
- **d** $\frac{3}{4}$ of 16 boxes
- **e** $\frac{3}{8}$ of 32 dots
- **f** $\frac{3}{7}$ of 21 triangles
- **g** $\frac{1}{10}$ of \$20
- **h** $\frac{3}{5}$ of 10 kg
- **i** $\frac{4}{7}$ of 21 triangles
- **j** $\frac{4}{5}$ of 15 stars

4G

4 **a** If $\frac{1}{3}$ of ● = 4, what is $\frac{2}{3}$ of ●?

b If $\frac{1}{10}$ of ▲ = 7, what is $\frac{3}{10}$ of ▲?

c If $\frac{1}{2}$ of ★ = 4, what is $\frac{3}{2}$ of ★?

d If $\frac{1}{5}$ of 10 = 2, what is $\frac{4}{5}$ of 10?

e If $\frac{1}{100}$ of 200 = 2, what is $\frac{21}{100}$ of 200?

f If $\frac{1}{10}$ of P = 3, what is $\frac{10}{10}$ of P?

FLUENCY 5–7(½) 5–8(½)

5 Find:

a $\frac{1}{3}$ of 18 **b** $\frac{1}{5}$ of 45 **c** $\frac{2}{3}$ of 24 **d** $\frac{3}{5}$ of 25

e $\frac{2}{7}$ of 42 **f** $\frac{1}{4}$ of 16 **g** $\frac{4}{5}$ of 100 **h** $\frac{3}{7}$ of 77

i $\frac{1}{3}$ of 6 **j** $\frac{1}{3}$ of 60 **k** $\frac{2}{3}$ of 60 **l** $\frac{10}{9}$ of 18

m $\frac{2}{5}$ of 100 **n** $\frac{3}{4}$ of 16 **o** $\frac{5}{9}$ of 27 **p** $\frac{3}{4}$ of 20



$\frac{1}{3}$ of 18

$= \frac{1}{1\cancel{3}} \times \frac{\cancel{18}^{6}}{1}$

$= \frac{1}{1} \times \frac{6}{1}$

Example 17 Multiplying proper fractions

Evaluate:

a $\frac{2}{3} \times \frac{1}{5}$ **b** $\frac{3}{4} \times \frac{8}{9}$

Solution	Explanation
a $\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5}$ $= \frac{2}{15}$	Multiply the numerators together. Multiply the denominators together. The answer is in simplest form.
b $\frac{3}{4} \times \frac{8}{9} = \frac{1\cancel{3} \times 8^{2}}{1\cancel{4} \times \cancel{9}^{3}}$ $= \frac{2}{3}$	Cancel first. Then multiply numerators together and denominators together.

6 Evaluate:

a $\frac{3}{4} \times \frac{1}{5}$ **b** $\frac{2}{7} \times \frac{1}{3}$ **c** $\frac{2}{3} \times \frac{5}{7}$ **d** $\frac{4}{9} \times \frac{2}{5}$

e $\frac{2}{3} \times \frac{3}{5}$ **f** $\frac{4}{7} \times \frac{1}{4}$ **g** $\frac{3}{4} \times \frac{1}{3}$ **h** $\frac{5}{9} \times \frac{9}{11}$

i $\frac{3}{6} \times \frac{5}{11}$ **j** $\frac{2}{3} \times \frac{4}{8}$ **k** $\frac{8}{11} \times \frac{3}{4}$ **l** $\frac{2}{5} \times \frac{10}{11}$

m $\frac{2}{7}$ of $\frac{3}{5}$ **n** $\frac{3}{4}$ of $\frac{2}{5}$ **o** $\frac{5}{10}$ of $\frac{4}{7}$ **p** $\frac{6}{9}$ of $\frac{3}{12}$



Where possible, cancel first so that you multiply smaller numbers.

Remember that 'of' means multiply.

7 Find:

a $\frac{5}{2} \times \frac{7}{3}$ **b** $\frac{6}{5} \times \frac{11}{7}$ **c** $\frac{6}{4} \times \frac{11}{5}$

d $\frac{9}{6} \times \frac{13}{4}$ **e** $\frac{8}{5} \times \frac{10}{3}$ **f** $\frac{21}{4} \times \frac{8}{6}$

g $\frac{10}{7} \times \frac{21}{5}$ **h** $\frac{14}{9} \times \frac{15}{7}$ **i** $\frac{8}{5} \times \frac{1}{2}$

j $\frac{4}{3} \times \frac{1}{5}$ **k** $\frac{3}{2} \times \frac{1}{4}$ **l** $\frac{3}{4} \times \frac{5}{3}$



Multiply improper fractions just as if they were proper fractions, then simplify your answer.

Example 18 Multiplying mixed numbers

Find:

a $2\frac{1}{3} \times 1\frac{2}{5}$ **b** $6\frac{1}{4} \times 2\frac{2}{5}$

Solution	Explanation
a $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5}$	Convert mixed numbers to improper fractions. Multiply numerators together. Multiply denominators together.
$= \frac{49}{15}$	
$= 3\frac{4}{15}$	Write the answer as a mixed number if required.
b $6\frac{1}{4} \times 2\frac{2}{5} = \frac{{}^{5}\cancel{25}}{{}_{1}\cancel{4}} \times \frac{\cancel{12}^{3}}{\cancel{5}_{1}}$	Convert to improper fractions. Simplify fractions by cancelling. Multiply numerators and denominators together.
$= \frac{15}{1}$	
$= 15$	Write the answer in simplest form.

8 Find:

a $1\frac{3}{5} \times 2\frac{1}{3}$ **b** $1\frac{1}{7} \times 1\frac{2}{9}$ **c** $3\frac{1}{4} \times 2\frac{2}{5}$

d $4\frac{2}{3} \times 5\frac{1}{7}$ **e** $\frac{6}{5} \times \frac{8}{3}$ **f** $\frac{1}{2} \times \frac{3}{8}$

g $\frac{3}{4}$ of $5\frac{1}{3}$ **h** $7\frac{1}{2} \times 4\frac{2}{5}$ **i** $\frac{3}{7} \times \frac{2}{3}$

j $1\frac{1}{2} \times 2\frac{1}{4}$ **k** $\frac{8}{9} \times \frac{6}{20}$ **l** $\frac{15}{4} \times \frac{8}{5}$



First convert mixed numbers to improper fractions.

4G

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 At one secondary college, $\frac{2}{5}$ of the Year 7 students are boys.

a What fraction of the Year 7 students are girls?

b If there are 120 Year 7 students, how many boys and girls are there?

10 Julie was injured during the netball season. She was able to play only $\frac{2}{3}$ of the matches. The season consisted of 21 matches. How many games did Julie miss as a result of injury?



11 **a** Blake spends $\frac{3}{4}$ of an hour on his Maths homework. How many minutes is this?

b Perform this calculation: $60 \div 4 \times 3$. What do you notice?

c Now find the number of minutes in $\frac{2}{3}$ of an hour in a similar way.

12 The diagram shows a plan of Joel's garden. The shaded section is grass. The rest is paved.



a What fraction of the garden is grass?

b If half the grass is removed and replaced with pavers, what fraction of the garden will remain grass?

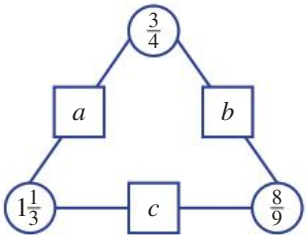
Missing products

— 13

13 **a** To find the number in each square, multiply the numbers in the two nearest circles.

$a = 1\frac{1}{3} \times \frac{3}{4}$





b Challenge yourself. What does $a + b + c$ equal? Check using a calculator.

4H Dividing fractions

Interactive

Widgets

HOTsheets

Walkthrough

Remember that division is the opposite of multiplication. Thinking of division as 'how many' helps us to understand dividing fractions.

For example, to find $\frac{1}{2} \div \frac{1}{4}$, think: How many quarters are in a half?

Consider a strip of paper that is divided into four equal sections. Half the strip is shaded.



There are two quarters in our half. Therefore, $\frac{1}{2} \div \frac{1}{4} = 2$.

When it is half-time in an AFL game, you have played two quarters. This is another way of showing that $\frac{1}{2} \div \frac{1}{4} = 2$.

Let's start: Using division patterns

Use patterns to help you find the missing numbers.

$20 \div 4 = 5$
$20 \div 2 = 10$
$20 \div 1 = 20$
$20 \div \frac{1}{2} = \square$
$20 \div \frac{1}{4} = \square$

Can you see an easy way to find the following?

- $30 \div \frac{1}{2}$
- $12 \div \frac{1}{4}$
- $20 \div \frac{1}{8}$
- $10 \div \frac{1}{3}$

Key ideas

- We use multiplication to help with the division of fractions.
  - Dividing by $\frac{1}{2}$ is like multiplying by 2.

    For example: $20 \div \frac{1}{2} = 40$ and $20 \times 2 = 40$
- The **reciprocal** of x is $\frac{1}{x}$.
  - 2 is the reciprocal of $\frac{1}{2}$ (and vice versa).
  - The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
  - The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$.
- Finding a reciprocal is called inverting, flipping or turning the fraction upside down.

Reciprocal A fraction in which the numerator and denominator have changed places

- To divide by a fraction, you can multiply by its reciprocal.
  For example: $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$ $\quad \frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1}$
- When dividing, or finding reciprocals, mixed numbers must be changed to improper fractions.

Exercise 4H

UNDERSTANDING 1–4 4

1 Copy these sentences, filling in the blanks as you go.

a $10 \div 5$ is the same as $10 _____ \frac{1}{5}$.

b $24 \div \frac{1}{4}$ is the same as $24 \times$ _____.

c To find half of a number you can _____ by 2.

d The _____ of $\frac{3}{5}$ is $\frac{5}{3}$.

e To invert a fraction you _____ it upside down.

f When dividing fractions we change any mixed numbers into _____ fractions.



Choose from: *flip*, *divide*, 4, ×, *reciprocal*, *improper*

2 Which of the following is the correct first step for finding $\frac{3}{5} \div \frac{4}{7}$?

A $\frac{3}{5} \times \frac{7}{4}$ **B** $\frac{5}{3} \times \frac{4}{7}$ **C** $\frac{5}{3} \times \frac{7}{4}$

3 Copy and complete.

a $\frac{5}{11} \div \frac{3}{5} = \frac{5}{11} \times \frac{\square}{\square}$ **b** $\frac{1}{3} \div \frac{1}{5} = \frac{1}{3} \times \frac{\square}{\square}$

c $\frac{7}{10} \div \frac{12}{17} = \frac{7}{10} \times \frac{\square}{\square}$ **d** $\frac{8}{3} \div 3 = \frac{8}{3} \times \frac{\square}{\square}$

e $\frac{7}{10} \div \frac{1}{2} = \frac{7}{10} \times \frac{\square}{\square}$ **f** $1\frac{1}{2} \div \frac{1}{4} = \frac{3}{2} \times \frac{\square}{\square}$

g $\frac{3}{5} \div \frac{1}{10} = \frac{3}{5} \times \frac{\square}{\square}$ **h** $1\frac{1}{2} \div 2\frac{1}{3} = \frac{\square}{\square} \times \frac{3}{7}$



Dividing by a fraction is the same as multiplying by its reciprocal.

4 Make each sentence correct, by inserting the word *more* or *less* in the gap.

a $10 \div 2$ gives an answer that is _____ than 10.

b $10 \div \frac{1}{2}$ gives an answer that is _____ than 10.

c $\frac{3}{4} \div \frac{2}{3}$ gives an answer that is _____ than $\frac{3}{4}$.

d $\frac{3}{4} \times \frac{3}{2}$ gives an answer that is _____ than $\frac{3}{4}$.

e $\frac{5}{7} \div \frac{8}{5}$ gives an answer that is _____ than $\frac{5}{7}$.

f $\frac{5}{7} \times \frac{5}{8}$ gives an answer that is _____ than $\frac{5}{7}$.

FLUENCY 5–8(½) 5–9(½)

Example 19 Finding reciprocals

State the reciprocal of the following.

a $\frac{2}{3}$ **b** 5 **c** $1\frac{3}{7}$

Solution	Explanation
a Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.	The numerator and denominator are swapped. (Flip the fraction upside down.)
b Reciprocal of 5 is $\frac{1}{5}$.	Think of 5 as $\frac{5}{1}$ and then invert (flip) it.
c Reciprocal of $1\frac{3}{7}$ is $\frac{7}{10}$.	Convert $1\frac{3}{7}$ to the improper fraction $\frac{10}{7}$, and then invert.

5 What is the reciprocal of each of the following?

a $\frac{5}{7}$ **b** $\frac{3}{5}$ **c** $\frac{2}{9}$ **d** $\frac{1}{8}$

e $\frac{1}{3}$ **f** $\frac{1}{10}$ **g** $\frac{3}{10}$ **h** $\frac{5}{4}$

i $\frac{12}{1}$ **j** $\frac{101}{1}$ **k** $\frac{1}{9}$ **l** 1

6 First change each of the following to an improper fraction, then find its reciprocal.

a $1\frac{1}{2}$ **b** $1\frac{1}{5}$ **c** $2\frac{1}{2}$ **d** $1\frac{2}{5}$

e $2\frac{3}{4}$ **f** $2\frac{1}{3}$ **g** $4\frac{3}{5}$ **h** $1\frac{5}{6}$

Example 20 Dividing a fraction by a whole number

Find:

a $\frac{5}{8} \div 3$ **b** $2\frac{3}{11} \div 5$

4H

Solution	Explanation
a $\frac{5}{8} \div 3 = \frac{5}{8} \times \frac{1}{3}$	Change the ÷ sign to a × sign and invert (flip) the 3.
$= \frac{5}{24}$	Multiply the numerators and denominators.
b $2\frac{3}{11} \div 5 = \frac{25}{11} \div \frac{5}{1}$	Convert the mixed number to an improper fraction. Write 5 as an improper fraction.
$= \frac{{}^{5}\cancel{25}}{11} \times \frac{1}{\cancel{5}^{1}}$	Change the ÷ sign to a × sign and invert the divisor. Simplify by cancelling.
$= \frac{5}{11}$	Multiply numerators and denominators.

7 Find:

a $\frac{3}{4} \div 2$ **b** $\frac{5}{11} \div 3$ **c** $\frac{8}{5} \div 4$

d $\frac{15}{7} \div 3$ **e** $2\frac{1}{4} \div 3$ **f** $5\frac{1}{3} \div 4$

g $12\frac{4}{5} \div 8$ **h** $1\frac{13}{14} \div 9$



Remember, $2 = \frac{2}{1}$ and flipping $\frac{2}{1}$ gives $\frac{1}{2}$.

Example 21 Dividing a whole number by a fraction

Find:

a $6 \div \frac{1}{3}$ **b** $24 \div \frac{3}{4}$

Solution	Explanation
a $6 \div \frac{1}{3} = \frac{6}{1} \times \frac{3}{1}$	Instead of $\div \frac{1}{3}$, change to $\times \frac{3}{1}$.
$= \frac{18}{1}$	Simplify.
$= 18$	
b $24 \div \frac{3}{4} = \frac{{}^{8}\cancel{24}}{1} \times \frac{4}{\cancel{3}^{1}}$	Instead of $\div \frac{3}{4}$, change to $\times \frac{4}{3}$.
$= 32$	Cancel and simplify.

8 Find:

a $5 \div \frac{1}{4}$ **b** $7 \div \frac{1}{3}$ **c** $10 \div \frac{1}{10}$ **d** $24 \div \frac{1}{5}$

e $12 \div \frac{2}{5}$ **f** $15 \div \frac{3}{8}$ **g** $14 \div \frac{7}{2}$ **h** $10 \div \frac{3}{2}$

Example 22 Dividing a fraction by a fraction

Find:

a $\frac{3}{5} \div \frac{3}{8}$ **b** $2\frac{2}{5} \div 1\frac{3}{5}$

Solution	Explanation
a $\frac{3}{5} \div \frac{3}{8} = \frac{\cancel{3}^{1}}{5} \times \frac{8}{\cancel{3}_{1}}$	Change the ÷ to × and invert the divisor (the fraction *after* the division sign).
$= \frac{8}{5}$	Cancel and simplify.
$= 1\frac{3}{5}$	
b $2\frac{2}{5} \div 1\frac{3}{5} = \frac{12}{5} \div \frac{8}{5}$	Convert mixed numbers to improper fractions.
$= \frac{\cancel{12}^{3}}{\cancel{5}_{1}} \times \frac{5}{\cancel{8}_{2}}$	Change the ÷ sign to a × sign and invert the divisor.
$= \frac{3}{2} = 1\frac{1}{2}$	Cancel, multiply and simplify.

9 Find:

a $\frac{2}{7} \div \frac{2}{5}$ **b** $\frac{1}{5} \div \frac{1}{4}$ **c** $\frac{3}{7} \div \frac{6}{11}$

d $\frac{2}{3} \div \frac{8}{9}$ **e** $2\frac{1}{4} \div 1\frac{1}{3}$ **f** $4\frac{1}{5} \div 3\frac{3}{10}$

g $12\frac{1}{2} \div 3\frac{3}{4}$ **h** $9\frac{3}{7} \div 12\frac{4}{7}$ **i** $1\frac{1}{4} \div \frac{5}{6}$



First convert any mixed numbers to improper fractions.

PROBLEM-SOLVING AND REASONING 10, 11 10–12

10 If $2\frac{1}{4}$ leftover pizzas are to be shared between three friends, what fraction of pizza will each friend receive?

11 A car travels 180 km in $1\frac{1}{2}$ hours. Calculate this car's speed.
Your answer will be in km/h.



speed = distance ÷ time



4H

12 Ceanna colours $\frac{1}{4}$ of her circle pink.



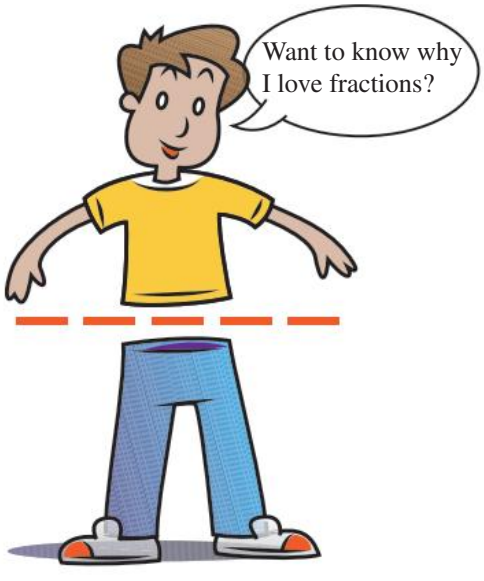
She divides the rest of the circle into three equal sectors and colours them blue, purple and green.
What fraction of the circle is purple?

A puzzling question!

— 13

13 *Why do I love fractions?*
To find out, work out the value of each of these 15 letters.

A $= \frac{2}{3} + 1\frac{1}{3}$	O $= \frac{2}{7} + \frac{2}{7}$	E $= \frac{1}{4} + \frac{1}{5}$
N $= 2 - \frac{3}{4}$	M $= \frac{1}{10} + \frac{1}{2}$	C $= \frac{1}{4} - \frac{1}{5}$
F $= \frac{2}{3}$ of 6	R $= \frac{1}{2} - \frac{3}{10}$	S $= \frac{1}{9}$ of 27
T $= 2\frac{1}{2} + 1\frac{1}{4}$	I $= \frac{5}{6} - \frac{1}{3}$	L $= \frac{7}{12} - \frac{1}{4}$
K $= 1\frac{1}{2} \times 4$	W $= \frac{5}{6} \times \frac{6}{5}$	H $= 2 \div \frac{1}{4}$



Now use those values to decode the answer.

4	$\frac{1}{5}$	2	$\frac{1}{20}$	$3\frac{3}{4}$	$\frac{1}{2}$	$\frac{4}{7}$	$1\frac{1}{4}$	3

$\frac{3}{5}$	2	6	$\frac{9}{20}$

$\frac{3}{5}$	$\frac{9}{20}$

4	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{3}$

1	8	$\frac{4}{7}$	$\frac{1}{3}$	$\frac{9}{20}$

Progress quiz

4A **1** For the following list of fractions, determine if they are proper fractions (P), improper fractions (I) or whole numbers (W).

a $\frac{3}{7}$ **b** $\frac{13}{9}$ **c** $\frac{5}{5}$ **d** $\frac{4}{2}$ **e** $\frac{1}{100}$

4B **2** Write the following fractions in simplest form.

a $\frac{14}{20}$ **b** $\frac{12}{21}$ **c** $\frac{25}{100}$ **d** $\frac{45}{30}$ **e** $\frac{32}{48}$

4B **3** Which of the following fractions is equivalent to $\frac{5}{35}$?

A $\frac{5}{70}$ **B** $\frac{15}{70}$ **C** $\frac{1}{5}$ **D** $\frac{1}{7}$

4C **4** Convert these mixed numbers to improper fractions.

a $3\frac{2}{7}$ **b** $11\frac{4}{9}$

4C **5** Convert these improper fractions to mixed numbers.

a $\frac{33}{10}$ **b** $\frac{25}{3}$

4D **6** Decide which fraction is the largest in each pair.

a $\frac{3}{8}$ and $\frac{5}{8}$ **b** $\frac{1}{3}$ and $\frac{1}{4}$

c $\frac{5}{11}$ and $\frac{3}{5}$ **d** $\frac{3}{4}$ and $\frac{13}{16}$

4D **7** Place the following fractions in ascending order. $\frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{8}{15}$

4E **8** Add the following fractions together.

a $\frac{4}{13}+\frac{5}{13}$ **b** $\frac{3}{7}+\frac{1}{3}$ **c** $3\frac{2}{5}+2\frac{1}{4}$

4F **9** Simplify:

a $\frac{9}{11}-\frac{5}{11}$ **b** $\frac{7}{8}-\frac{2}{3}$ **c** $15\frac{3}{4}-9\frac{5}{12}$

4G **10** Evaluate:

a $\frac{3}{7}\times\frac{3}{4}$ **b** $\frac{10}{3}\times\frac{9}{5}$ **c** $3\frac{1}{3}\times1\frac{2}{5}$

4H **11** State the reciprocal of the following.

a $\frac{2}{9}$ **b** 5 **c** $3\frac{3}{4}$

12 Evaluate:

a $\frac{6}{11}\div3$ **b** $8\div\frac{1}{4}$ **c** $\frac{2}{7}\div\frac{3}{5}$ **d** $5\frac{1}{4}\div2\frac{1}{3}$

4I Fractions and percentages

Interactive

Widgets

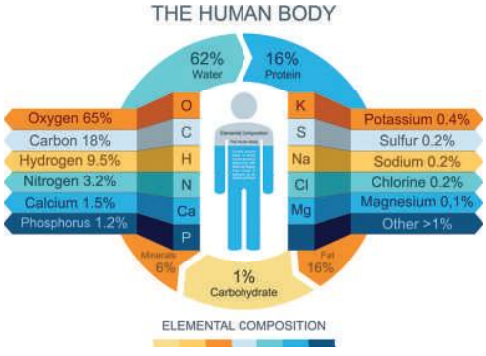
HOTsheets

Walkthrough

Percentages are used in many day-to-day situations including test results, discounts and food ingredients.

A percentage is another way of writing a fraction with a denominator of 100. For example, 87 per cent simply means 87 out of 100, so:

$$87\% = \frac{87}{100}$$



A fraction can be interpreted as a percentage of the total.

Let's start: Student ranking

Five students each completed a different Mathematics test.

- Matthew scored 15 out of a possible 20 marks.
- Mengna scored 36 out of a possible 50 marks.
- Maria scored 17 out of a possible 25 marks.
- Marcus scored 7 out of a possible 10 marks.
- Melissa scored 128 out of a possible 200 marks.

Change these test results to equivalent scores out of 100, and therefore state the percentage test score for each student. Why are percentages useful in this situation?

Key ideas

Per cent (%) Per hundred, out of a hundred

- The % symbol means **per cent**. This comes from the Latin *per centum*, which means out of 100.
  For example: 75 per cent means 75 out of 100. That is, $75\% = \frac{75}{100}$
- We can write percentages as fractions by changing the % sign to a denominator of 100 (meaning out of 100).
  For example: $37\% = \frac{37}{100}$
- We can use equivalent fractions to convert fractions to percentages.
  For example: $\frac{1}{4} = \frac{25}{100} = 25\%$

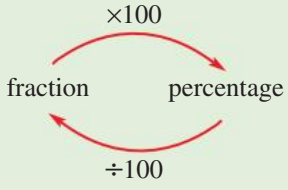
- To convert any fraction to a percentage, multiply by 100.

  For example: $\frac{3}{8} \times 100 = \frac{3}{8} \times \frac{100}{1}$

  $= \frac{75}{2}$

  $= 37\frac{1}{2}$

  $\therefore \quad \frac{3}{8} = 37\frac{1}{2}\%$



- It is useful to know the following common percentages and their equivalent fractions.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$12\frac{1}{2}\%$	$66\frac{2}{3}\%$	75%

Exercise 4I

UNDERSTANDING 1–4 4

1 Change these test results to equivalent scores out of 100, and therefore state the percentage.

a 7 out of 10 = _____ out of 100 = _____ %

b 24 out of 50 = _____ out of 100 = _____ %

c 12 out of 20 = _____ out of 100 = _____ %

d 1 out of 5 = _____ out of 100 = _____ %

e 80 out of 200 = _____ out of 100 = _____ %

f 630 out of 1000 = _____ out of 100 = _____ %

2 Copy and complete these patterns.

a $\frac{1}{4} = 25\%$

$\frac{2}{4} = 50\%$

$\frac{3}{4} = \square$

$\frac{4}{4} = \square$

b $\frac{1}{5} = 20\%$

$\frac{2}{5} = \square$

$\frac{3}{5} = 60\%$

$\frac{4}{5} = \square$

$\frac{5}{5} = 100\%$

c $\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{2}{3} = \square$

$\frac{3}{3} = \square$

3 **a** If 14% of students in Year 7 are absent due to illness, what percentage of Year 7 students are at school?

b If 80% of the Geography project has been completed, what percentage still needs to be finished?

The total is 100%.



4 Zoe scored 100% on her fractions test. The test was out of 25. What was Zoe's mark?

41

FLUENCY 5–7(½) 5–8(½)

Example 23 Converting percentages to fractions

Express these percentages as fractions in their simplest form.

a 17% **b** 36% **c** 140%

Solution	Explanation
a $17\% = \frac{17}{100}$	Change % sign to a denominator of 100.
b $36\% = \frac{36}{100}$	Change % sign to a denominator of 100.
$= \frac{9 \times 4}{25 \times 4}$	Cancel HCF.
$= \frac{9}{25}$	Answer is now in simplest form.
c $140\% = \frac{140}{100}$	Change % sign to a denominator of 100.
$= \frac{7 \times 20}{5 \times 20}$	Cancel HCF.
$= \frac{7}{5} = 1\frac{2}{5}$	Convert answer to a mixed number if required.

5 Express these percentages as fractions in their simplest form.

a 11% **b** 71% **c** 43% **d** 49% **e** 25% **f** 30%
g 15% **h** 88% **i** 7% **j** 19% **k** 21% **l** 50%
m 70% **n** 90% **o** 99% **p** 55%

6 Express these percentages as mixed numbers in their simplest form.

a 120% **b** 180% **c** 237% **d** 401% **e** 175% **f** 110%
g 316% **h** 840% **i** 200% **j** 205% **k** 310% **l** 350%

Example 24 Converting to percentages through equivalent fractions

Convert the following fractions to percentages.

a $\frac{5}{100}$ **b** $\frac{11}{25}$

Solution	Explanation
a $\frac{5}{100} = 5\%$	Denominator is already 100, therefore simply write the number as a percentage.
b $\frac{11}{25} \overset{\times 4}{=} \frac{44}{100} = 44\%$	We need the denominator to be 100. Therefore, multiply numerator and denominator by 4 to get an equivalent fraction.

7 Convert these fractions to percentages, using equivalent fractions.

a $\frac{98}{100}$ **b** $\frac{9}{100}$ **c** $\frac{79}{100}$ **d** $\frac{56}{100}$

e $\frac{8}{100}$ **f** $\frac{15}{100}$ **g** $\frac{97}{100}$ **h** $\frac{50}{100}$

i $\frac{7}{20}$ **j** $\frac{8}{25}$ **k** $\frac{43}{50}$ **l** $\frac{18}{20}$

m $\frac{56}{50}$ **n** $\frac{27}{20}$ **o** $\frac{20}{5}$ **p** $\frac{16}{10}$



First write with a denominator of 100.

Example 25 Converting to percentages by multiplying by 100

Convert the following fractions to percentages.

a $\frac{3}{8}$ **b** $3\frac{3}{5}$

Solution	Explanation
a $\frac{3}{\cancel{8}_{2}} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{2}$	Multiply by 100. Simplify by cancelling HCF.
$= 37\frac{1}{2}$	
$\therefore \quad \frac{3}{8} = 37\frac{1}{2}\%$	
b $3\frac{3}{5} \times \frac{100}{1} = \frac{18}{\cancel{5}_{1}} \times \frac{\cancel{100}^{20}}{1}$	Convert mixed number to improper fraction. Cancel and simplify.
$= 360$	
$\therefore \quad 3\frac{3}{5} = 360\%$	

8 Convert these fractions to percentages by multiplying by 100.

a $\frac{1}{8}$ **b** $\frac{1}{3}$ **c** $\frac{4}{15}$ **d** $\frac{10}{12}$

e $1\frac{3}{20}$ **f** $4\frac{1}{5}$ **g** $2\frac{36}{40}$ **h** $\frac{13}{40}$

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 A bottle of lemonade is only 25% full.

a What fraction of the bottle has been consumed?

b What percentage of the bottle has been consumed?

c What fraction of the bottle is left?

d What percentage of the bottle is left?



41

10 A lemon tart is cut into eight equal pieces. What percentage of the tart does each piece represent?

11 Petrina scores 28 out of 40 on her fractions test. What is her percentage score?

12 The Heathmont Hornets basketball team have won 14 out of 18 games. They still have two games to play. What is the smallest and the largest percentage of games the Hornets could win for the season?



More challenging conversions

— 13

13 a Copy and complete:

i $2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100}$

$= \frac{5}{200}$

$= \frac{\square}{\square}$

ii $3\frac{1}{4}\% = \frac{3\frac{1}{4}}{\square}$

$= \frac{3\frac{1}{4}}{100} \times \frac{\square}{4}$

$= \frac{\square}{400}$

b Write each of the following percentages as fractions using the above technique.

i $8\frac{1}{4}\%$ **ii** $12\frac{1}{2}\%$ **iii** $33\frac{1}{3}\%$

4J Percentage of a number

Interactive
Widgets
HOTsheets
Walkthrough

Throughout life you will come across many examples where you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, personal improvements and salary increases.



Retail sales may involve a percentage of the orginal price being taken away.

Let's start: Percentages in your head

It is a useful skill to be able to calculate percentages mentally. Calculating 10% or 1% is often a good starting point. You can then multiply or divide these values to quickly arrive at other percentage values.

With a partner, and using mental arithmetic only, calculate 10% of each of these amounts.

a	$120	**b**	$35	**c**	$160	**d**	$90	**e**	$300	**f**	$40
g	$80	**h**	$420	**i**	$1400	**j**	$550	**k**	$200	**l**	$60

How did you find 10% of an amount mentally?

Key ideas

- To find the percentage of a number:
  1. Express the required percentage as a fraction.
  2. Change the 'of' to a multiplication sign.
  3. Express the number as a fraction.
  4. Follow the rules for multiplication of fractions.

'of' means $\times$

$$25\% \text{ of } 60 = \frac{25}{100} \times \frac{60}{1}$$
$$= 15$$

- To mentally find a percentage of a number, find the value of 1% or 10% and work from there.
  For example: 10% of 60 = 6 so 25% of 60 $= 2\frac{1}{2} \times 6 = 15$

Exercise 4J

UNDERSTANDING 1–4 | 4

1 Copy and complete the following.

a To find 10% of \$20, you can use $20 \div \square = 2$.

b To find 25% of \$20, you can use $20 \div 4 = \square$.

c To find 50% of \$20, you can use $20 \div \square = \square$.

2 **a** If 10% of ★ = 3, find 20% of ★.

b If 1% of ▲ = 7, find 5% of ▲.

c If 50% of ● = 8, find 100% of ●.

3 Use mental strategies to find:

a 10% of \$500 **b** 1% of \$900 **c** 25% of 84 kilograms

d 50% of 7 days **e** 75% of 84 kilograms **f** 20% of 35 minutes

4 What is 100% of 8 hours?

FLUENCY 5–6(½), 7 | 5–6(½), 7, 8(½)

Example 26 Finding the percentage of a number

Find:

a 30% of 50 **b** 15% of 400

Solution	Explanation
a $30\% \text{ of } 50 = \frac{30}{\cancel{100}_{2}} \times \frac{\cancel{50}^{1}}{1}$	Write % as a fraction. The word 'of' tells us to multiply.
$= \frac{30}{2} = 15$	Cancel and simplify.
Alternatively	
$10\% \text{ of } 50 = 5$	10% is easy to find mentally.
$30\% \text{ of } 50 = 15$	Multiply 10% by 3 to find 30%.
b $15\% \text{ of } 400 = \frac{15}{\cancel{100}_{1}} \times \frac{\cancel{400}^{4}}{1}$	Write % as a fraction. 'Of' means ×. Cancel and simplify.
$= \frac{15 \times 4}{1} = 60$	
Alternatively	
$10\% \text{ of } 400 = 40, \ 5\% \text{ of } 400 = 20$	Find 10% mentally. Halve to find 5%.
$15\% \text{ of } 400 = 60$	Add 10% and 5% to find 15%.

5 Find:

a 50% of 140 **b** 10% of 360 **c** 20% of 50 **d** 30% of 90

e 25% of 40 **f** 25% of 28 **g** 75% of 200 **h** 80% of 250

i 5% of 80 **j** 4% of 1200 **k** 5% of 880 **l** 2% of 9500

m 11% of 200 **n** 21% of 400 **o** 12% of 300 **p** 9% of 700

50% of 140
$= \frac{50}{100} \times \frac{140}{1}$



6 Find:

a 120% of 80 **b** 150% of 400 **c** 110% of 60
d 400% of 25 **e** 125% of 12 **f** 225% of 32
g 146% of 50 **h** 3000% of 20



$120\% = \frac{120}{100}$

7 Match each question with the correct answer.

Questions			Answers	
a 10% of \$200	**b** 5% of \$500		\$8	\$30
c 20% of \$120	**d** 30% of \$310		\$16	\$40
e 10% of \$80	**f** 10% of \$160		\$20	\$44
g 50% of \$60	**h** 1% of \$6000		\$24	\$60
i 20% of \$200	**j** 50% of \$88		\$25	\$93

8 Find:

a 30% of \$140 **b** 10% of 240 millimetres
c 15% of 60 kilograms **d** 2% of 4500 tonnes
e 20% of 40 minutes **f** 80% of 500 centimetres
g 5% of 30 grams **h** 25% of 12 hectares
i 120% of 120 seconds



Remember that the answers need to include units.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Harry scored 70% on his percentages test. If the test is out of 50 marks, how many marks did Harry score?

10 In a student survey, 80% of students said they received too much homework. If 300 students were surveyed, how many students felt they get too much homework?



11 25% of teenagers say their favourite fruit is watermelon. In a survey of 48 teenagers, how many students would write watermelon as their favourite fruit?

12 At Gladesbrook College, 10% of students walk to school, 35% of students catch public transport and the rest are driven to school. If there are 1200 students at the school, find how many students:

a walk to school
b catch public transport
c are driven to school



Percentage challenge — 13

13 **a** Which is larger: 60% of 80 or 80% of 60?

b Tom did the following calculation: $120 \div 4 \div 2 \times 3$. What percentage of 120 did Tom find?

c **i** If 5% of an amount is \$7, what is 100% of the amount?

ii If 25% of an amount is \$3, what is $12\frac{1}{2}\%$ of the amount?

4K Expressing a quantity as a proportion

Interactive

Widgets

HOTsheets

Walkthrough

Sometimes we want to know the proportion of a certain quantity compared to a given total or another quantity. Proportions can be expressed using a fraction, percentage or ratio. The Earth's surface, for example, is about 70% ocean. So, the proportion of land could be written as 30% (as a percentage) or $\frac{3}{10}$ (as a fraction).



The proportion of land to sea in this photo of the Whitsunday Islands, Queensland, could be expressed as a fraction, percentage or ratio.

Let's start: Municipal parkland

A municipal area is set aside in a new suburb and is to be divided into three main parts. The total area is 10 000 m^2 and the three parts are to be divided as follows.

Parkland: 6000 m^2
Shops: 2500 m^2
Playground and skate park: 1500 m^2

- Express each area as a proportion of the total, first as a fraction and then as a percentage.
- If $\frac{3}{5}$ of the parkland is to be a lake, what percentage of the total area is the lake?

Key ideas

Proportion One part considered in relation to the whole

- We can express **proportion** as a fraction or percentage
- To express one quantity as a fraction of another:

$$\text{Fraction} = \frac{\text{amount}}{\text{total}}$$

- To express one quantity as a percentage of another:

$$\text{Percentage} = \frac{\text{amount}}{\text{total}} \times \frac{100}{1}\%$$

- An object or quantity divided into parts can be analysed using fractions and percentages.



$$\text{Purple fraction} = \frac{2}{5}$$

$$\text{Purple percentage} = \frac{2}{5} \times \frac{100}{1}\% = 40\%$$

Exercise 4K

UNDERSTANDING 1–4 4

Example 27 Expressing fractions as percentages

Write $\frac{4}{5}$ as a percentage, by multiplying by 100.

Solution	Explanation
$\frac{4}{5} \times 100 = 80$	$\frac{4}{1\cancel{5}} \times \frac{\cancel{100}^{20}}{1} = \frac{80}{1}$
$\therefore \frac{4}{5} = 80\%$	

1 Write these fractions as percentages by multiplying by 100.

a $\frac{1}{4}$ **b** $\frac{10}{50}$ **c** $\frac{12}{25}$ **d** $\frac{99}{100}$

e $\frac{17}{100}$ **f** $\frac{17}{50}$ **g** $\frac{7}{20}$ **h** $\frac{9}{10}$

2 A cake contains 7 grams of fat. The cake has a mass of 350 grams.

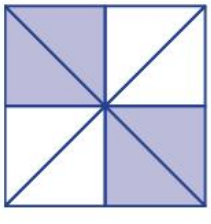
a Write the fraction of the cake which is fat.

b Simplify the fraction in part **a**.

c Use this simplified fraction to find the percentage of fat in the cake.



3 This square shows some coloured triangles and some white triangles.



a How many triangles are coloured?

b How many triangles are white?

c What fraction of the total is coloured?

d What percentage of the total is coloured?

e What fraction of the total is white?

f What percentage of the total is white?

4 A farmer's pen has 2 black sheep and 8 white sheep.

a How many sheep are there in total?

b What fraction of the sheep are black?

c What fraction of the sheep are white?

d What percentage of the sheep are black?

e What percentage of the sheep are white?

4K

FLUENCY 5–6(½), 7 5–6(½), 7, 8

Example 28 Expressing a proportion

Express 24 green ducks out of a total of 30 ducks as a fraction and then as a percentage.

Solution	Explanation
Fraction $= \frac{24}{30}$	24 out of 30 is $\frac{24}{30}$.
$= \frac{4}{5}$	Simplify the fraction.
Percentage $= \frac{24}{30} \times \frac{100}{1}$	Start with the same fraction and multiply by 100.
$= 80\%$	

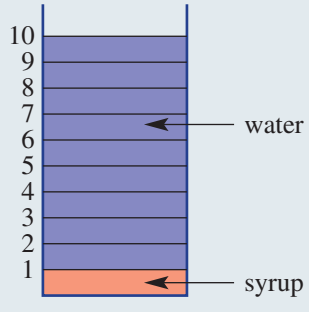
5 Express each of these proportions as a fraction and then as a percentage.

a 30 out of a total of 100
b 3 out of a total of 5
c \$10 out of a total of \$50
d \$60 out of a total of \$80
e 2 kg out of a total of 40 kg
f 14 g out of a total of 28 g
g 3 L out of a total of 12 L
h 30 mL out of a total of 200 mL

Example 29 Finding fractions and percentages from parts

A glass of cordial is 1 part syrup to 9 parts water.

a Express the amount of syrup as a fraction of the total.
b Express the amount of water as a percentage of the total.



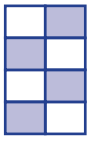
Solution	Explanation
a Fraction $= \frac{1}{10}$	The total is 10 parts, including 1 part syrup.
b Percentage $= \frac{9}{10} \times \frac{100}{1}$	There are 9 parts water in a total of 10 parts.
$= 90\%$	×100 fraction → percentage

6 Write each coloured area as a fraction and then a percentage of the total area.

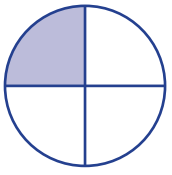
a



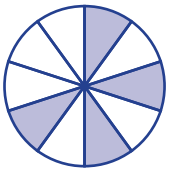
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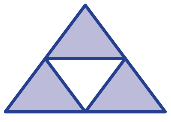
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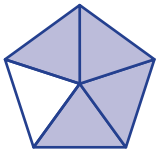
d



e



f



7 A jug of lemonade is made up of 2 parts of lemon juice to 18 parts of water.

a Express the amount of lemon juice as a fraction of the total.

b Express the amount of lemon juice as a percentage of the total.

8 A mix of concrete is made up of 1 part of cement to 4 parts of sand.

a Express the amount of cement as a fraction of the total.

b Express the amount of cement as a percentage of the total.

c Express the amount of sand as a fraction of the total.

d Express the amount of sand as a percentage of the total.



PROBLEM-SOLVING AND REASONING 9, 10 9–13

9 Gillian pays $80 tax out of her income of $1600.

a What fraction of her income is tax?

b What percentage of her income is tax?

c What percentage of her income does she keep?

10 Over summer, a dam's water volume reduces from 20 megalitres to 4 megalitres.

a How many megalitres of water were used?

b What fraction of the water was used?

c What percentage of the water was used?

11 In a new subdivision involving 20 000 m^2, specific areas are set aside for the following purposes.

- Dwellings: 12 000 m^2
- Shops: 1000 m^2
- Roads/Paths: 3000 m^2
- Park: 2500 m^2
- Factories: Remainder



a What area is set aside for factories?

b Express the area of the following as both a fraction and a percentage of the total area.

i shops **ii** dwellings **iii** park

c What fraction of the total area is either dwellings, shops or factories?

d What percentage of the total area is park or roads/paths?

4K

12 Express the following as a fraction and percentage of the total.

a 20 cents out of 500 cents
b 14 days out of 35 days
c 15 centimetres out of 3 metres (300 centimetres)
d 15 seconds out of 5 minutes
e 50 centimetres from a total of 2 metres
f 1500 metres from 2 kilometres



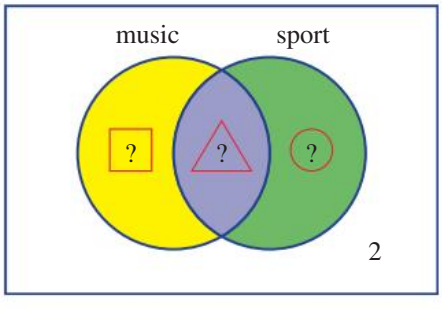
Use the same units:
5 minutes
$= 5 \times 60$ seconds
$= 300$ seconds

13 For a recent class test, Ross scored 45 out of 50 and Maleisha scored 72 out of 100. Use percentages to show that Ross obtained the higher mark.

Sport and music — 14

14 Of 20 students, 10 play sport and 12 play a musical instrument, with some of these students playing both sport and music. Two students do not play any sport or musical instrument.

a How many of the 20 students played music *and* sport?
b Find the missing numbers in this Venn diagram.



c What fraction of the students play both sport and a musical instrument?
d What percentage of the students play a musical instrument but not a sport?



4L Ratios, rates and best buys

Interactive

Widgets

HOTsheets

Walkthrough

Ratios are used to show the relationship between two (or more) related quantities. In general, ratios compare quantities of the same type given in the *same* unit. Therefore a ratio is generally not written with a unit of measurement.

Rates show the relationship between two different types of quantities with *different* units. Therefore, all rates must be expressed with appropriate units, which often include the symbol / to represent the word 'per'. For example, speed is a rate comparing distance and time. Typical units of speed include metres per second (m/s) and kilometres per hour (km/h).



A cricket batter's performance can be measured using a rate called strike-rate.

There are many everyday situations where it is useful to convert rates to the same unit for easy comparison. This can be a good way to determine the best value or best buy.

Let's start: Fair shares

Assuming that everyone gets an equal share, work out how much each person receives in each of the following situations.

- 40 potato gems for 5 children
- 24 party pies for 6 preschoolers at a party
- 480 mini chocolate bars for 160 students

Assuming that the adults need twice as much food as the children, work out how much each child and adult receives in each of the following situations.

- 12 sausages for 2 adults and 2 children
- 80 strawberries for 1 adult and 3 children
- 44 biscuits for 5 adults and 1 child

Key ideas

Ratios

- Ratios show the relationship between two (or more) numbers or quantities measured in the same unit.
  For example: Fleur wrote the ratio 16 : 8 to compare the number of hours she spends awake each day to the number of hours she sleeps.
- The colon : is the mathematical symbol used to represent ratios.
- The ratio a : b is read 'a to b' or 'a is to b'.
- If each value in a ratio is multiplied or divided by the same number, an equivalent ratio is formed.
  For example: 2 : 3 and 4 : 6 and 10 : 15 are equivalent ratios.
- A ratio is in its simplest form when it contains only whole numbers with a highest common factor (HCF) of 1.

Rates

- Rates compare quantities measured in different units.
- Every rate has a unit that compares the two quantities. For example, speed compares distance and time in units such as kilometres per hour (km/h) or metres per second (m/s).
- We usually write rates in their simplest form, giving the amount for one unit of the second quantity.
  For example: Travelling 150 km in 3 hours, would not usually be written as 150 km/3 hours. Instead, it is simplified to 50 km/1 hour, which we write as 50 km/h.

'Best buy'

- The concept of best buy relates to comparing rates to determine the best value for money, the best performance and so on.
- It is easy to compare rates that have the same units.
  For example, the same cheese is sold in packets of 600 g for $9 and 1.75 kg for $29. Converting both rates to price per kilogram gives $15/kg and $16.57/kg. Now we can see that the smaller packet is the better buy.

Exercise 4L

UNDERSTANDING 1–5 | 1, 2, 5

1 Which of the following is an example of a ratio?
A 3.75 seconds **B** 5 m/s **C** $5000 **D** 3 : 7

2 Which of the following is an example of a rate?
A $7.50/kg **B** 2 : 4 : 5 **C** 12:37 p.m. **D** $\frac{3}{4}$

3 Find the HCF for each of the following pairs of numbers.
a 5 and 15 **b** 3 and 7 **c** 4 and 9 **d** 12 and 8

4 Using your answers from Question **3**, state whether the following ratios can be simplified.
a 5 : 15 **b** 3 : 7 **c** 4 : 9 **d** 12 : 8

5 Teddy's resting heart rate is 60 beats per minute (60 bpm). How many times will Teddy's heart beat in:
a 1 minute? **b** 5 minutes? **c** 1 hour? **d** 1 day?

FLUENCY 6–7(½), 8 6–9(½)

Example 30 Writing a ratio in simplest form

Simplify the following ratios.

a 3 : 9 **b** 60 : 24

Solution

a 3 : 9 → (÷ 3) → 1 : 3 (÷ 3)

b 60 : 24 → (÷ 12) → 5 : 2 (÷ 12)

Explanation

The HCF of 3 and 9 is 3.
Therefore divide both numbers by 3 to write the ratio in its simplest form.

The HCF of 60 and 24 is 12.
Therefore divide both numbers by 12 to write the ratio in its simplest form.

6 Simplify the following ratios.

a 3 : 9	**b** 5 : 10	**c** 2 : 18	**d** 6 : 18	**e** 27 : 9
f 16 : 4	**g** 100 : 10	**h** 60 : 12	**i** 10 : 25	**j** 14 : 21
k 32 : 40	**l** 50 : 70	**m** 18 : 14	**n** 36 : 15	**o** 55 : 35
p 60 : 48				

7 There is one mistake in each of the following groups of equivalent ratios. Find the non-equivalent ratio in each group and replace one of its numbers to make a correct ratio.
a 2 : 8, 5 : 20, 4 : 24, 10 : 40, 50 : 200
b 11 : 66, 3 : 18, 7 : 42, 10 : 600, 40 : 240
c 6 : 9, 2 : 3, 20 : 30, 8 : 24, 14 : 21
d 9 : 15, 3 : 5, 15 : 20, 24 : 40, 36 : 60

Example 31 Writing a rate in simplest form

Simplify the following rates.

a Texting 48 words in 3 minutes **b** Flying 900 kilometres in 90 minutes

Solution

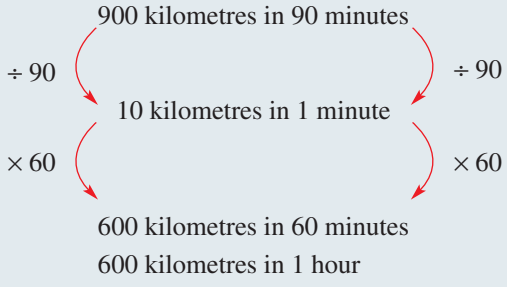
a 16 words/minute

Explanation

48 words in 3 minutes → (÷ 3) → 16 words in 1 minute (÷ 3)

4L

b 10 km/min or 600 km/h



8 Simplify the following rates.

a Riding 24 kilometres in 2 hours
b Clapping 120 times in 30 seconds
c Spending $28 on 4 kilograms of lollies
d Running 140 metres in 20 seconds
e Scoring 24 goals in 8 games
f Travelling 180 kilometres in 90 minutes

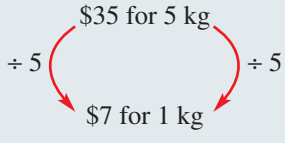
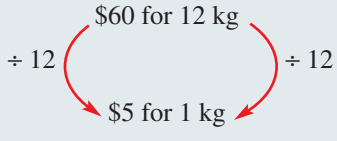
9 Solve the following rate problems.

a How much will 3 kg of salami cost at $8/kg?
b A radio station plays songs at a rate of 9 songs per hour. How many songs would you hear in 8 hours?
c A surfer catches 8 good waves per hour. How many waves would she catch if she surfed for $2\frac{1}{2}$ hours?
d A teenager buys 4 pairs of shoes per year. How many pairs of shoes does he buy during his teenage years?

Example 32 Determining the best buy

By calculating a common rate, determine the best buy for the following brands of dog food.

- A 5 kg bag of King for $35
- A 12 kg bag of Thrive for $60

Solution	Explanation
King dog food costs $7/kg.	
Thrive dog food costs $5/kg.	
The best buy is Thrive dog food.	

PROBLEM-SOLVING AND REASONING 10, 11 10–12

10 By calculating a common rate, determine the best buy for each of the following.
- **a** 2 kg of flour for $3.60 or 5 kg of flour for $8.50
- **b** 12 kg of potatoes for $14.40 or 5 kg of potatoes for $5.50
- **c** 500 g of ham for $9.50 or 2 kg of ham for $39
- **d** 20 L of cooking oil for $120 or 12 L of cooking oil for $84

11 Washing powder can be purchased in many different quantities. By calculating a common rate ($/kg), rank the following options in order of best buy to worst buy.
- 500 g for $2.09
- 2 kg for $7.90
- 5 kg for $19.95
- 10 kg for $38.00

12 Shaun is an apprentice chef. He is required to make a large amount of pastry. The recipe shows the ratio of butter to flour is 1 : 3.
- **a** How much flour will Shaun need if he has 7.5 kg of butter?
- **b** How much butter will Shaun need if he has 7.5 kg of flour?



Reading rates

— 13

13 When Diviesh was 12, he had been learning English for only 3 years and was able to read at a rate of 96 words per minute. Now he is 14, he is able to read at a rate of 160 words per minute.
- **a** Express Diviesh's reading rates at 12 and at 14 as a ratio in simplest form.
- **b** How much quicker would it be for Diviesh to read a 12 288 word novel now that he is 14 than when he was 12?
- **c** At age 14, it took Diviesh a total of 15 hours and 10 minutes to read his longest novel ever. Assuming he read at 160 words per minute, how many words were in the novel?

Maths@Work: Building construction apprentice

The construction industry in Australia is huge where property developers employ many thousands of workers. Apprentices must complete hours of on-the-job training as well as achieve TAFE qualifications. Number and measurement skills are very important, as construction workers apply these skills in everyday projects.

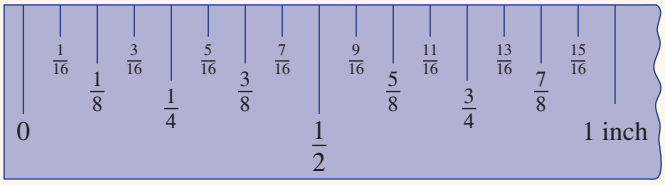


Fractions are studied both at school and at TAFE. Measuring and calculating with fractions is a vital skill in the building industry. Many building components are from the United States, which still uses Imperial measurements. So inches and fractions of an inch often feature in the workplace.

1 Given that 1 inch is 25.4 mm, find the length in millimetres to 1 d.p. for the following:

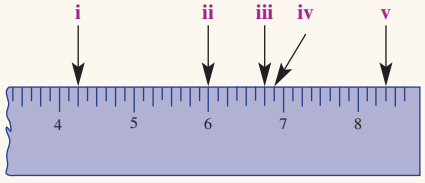
a $\frac{1}{2}$ inch **b** $\frac{1}{4}$ inch **c** $\frac{1}{8}$ inch **d** $\frac{1}{32}$ inch **e** $\frac{3}{4}$ inch

2 Make a neat, enlarged copy of this 1 inch ruler and its scale. Start by ruling an 8 cm line and mark 16 equal spaces on it. Label the ends 0 and 1 inch, label the centre $\frac{1}{2}$ inch, then carefully mark in all the other fractions of an inch as shown on this image.

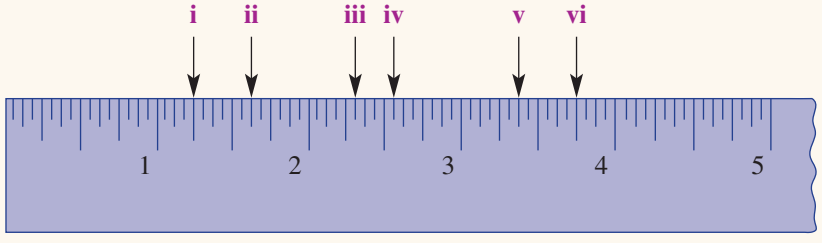


3 Tape measures can show units in both inches and centimetres. What are the fractional measurements in inches indicated on the following tape measures?

a



b



4 TAFE courses for construction apprentices include fraction skills. Write a list in inches for each of the following sets of measurements:

Imperial units
1 foot (1′) = 12 inches (12″)
1 yard = 3 feet = 36 inches

a Increasing in quarters from $10\frac{1}{4}$ inches to 1 foot.

b Decreasing in eighths from 1 foot to $11\frac{1}{4}$ inches.

c $\frac{3}{8}$ of an inch, less than each of 1 inch, 5 inches and 1 foot.

d $4\frac{5}{16}$ inches less than each of 1 foot and 1 yard.



$10\frac{1}{4}''$, $10\frac{1}{2}''$, ... $1'$

5 Travis, a first-year apprentice, is asked to cut off one third of a 12-inch pipe. What length of the pipe in mm is remaining after he cuts it?



Construction workers calculate and measure using fractions.

6 Holly is asked to cut $8\frac{1}{4}$ inches from a pipe $41\frac{1}{4}$ inches long. What proportion of the pipe is cut?

7 Murray has eight lengths of pipe to join together. If each pipe is $4\frac{3}{4}$ inches long, how many mm is the final length of pipe?

Using technology



To type a fraction into a spreadsheet, insert a space between the whole number and the fraction.
e.g. 5 1/2 means $5\frac{1}{2}$

8 Many older buildings have pipes that were installed before Australia changed to the metric system. The replacement of old pipes requires exactly the same length of pipe measured in mm. Set up the following spreadsheet to add fractions and convert inches to mm.
Select all the cells using inches (columns A, B, C) and Format/Fraction/up to two digits.
Enter formulas in the shaded cells.

	A	B	C	D
1	Pipe lengths in inches and mm			
2	Pipe A in inches	Pipe B in inches	Total A + B in inches	Total A + B in mm
3	5 1/2	4 1/16		
4	9 15/16	17 3/16		
5	12 3/16	46 13/32		
6	14 7/8	42 3/4		



Formula for C3 = A3 + B3
Formula for D3 = C3*25.4

Puzzles and games

1 Three cities are known as India's Golden Triangle. To find the names of these cities, complete the puzzle.
Match each of the fractions in the middle row with the equivalent fraction in the bottom row. Place the letter in the code below.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{4}{24}$	$\frac{28}{35}$	$\frac{100}{120}$	$\frac{5}{7}$	$\frac{21}{36}$	$\frac{1}{2}$	$\frac{22}{77}$	$\frac{2}{3}$	$4\frac{2}{5}$	$\frac{81}{90}$	$\frac{25}{3}$	$\frac{43}{9}$
$U = 8\frac{1}{3}$	$A = \frac{5}{6}$	$H = \frac{15}{21}$	$D = \frac{4}{5}$	$G = \frac{1}{6}$	$N = 4\frac{7}{9}$	$I = \frac{7}{12}$	$E = \frac{2}{7}$	$P = \frac{18}{27}$	$J = \frac{9}{10}$	$R = \frac{48}{96}$	$L = \frac{22}{5}$

2	7	9	4	5

3	1	6	3

3	12	2

10	3	5	8	11	6

2 How many words with three or more letters can you and a partner make from the word PERCENTAGES?
You have 15 minutes to come up with as many words as you can.
Score your results as follows:

- 1 point for every three-letter word
- 2 points for every four-letter word
- 3 points for every word with five or more letters

Compare your list with those of other students in the class.

3 *Fraction dice game*
Two different-coloured dice are required. Choose one die for the numerator and one die for the denominator.
For example: $\frac{\text{red die result}}{\text{blue die result}} = \frac{5}{3}$ or $\frac{\text{red die result}}{\text{blue die result}} = \frac{4}{1}$.
Players take turns to throw both dice and record their fraction results.
After an equal number of turns, each player then adds all their results together and the winner is the player with the largest number.

4 In a magic square, the sum of the fractions in each row, column and diagonal is the same.
Find the value of each letter in these magic squares.

a

$\frac{2}{5}$	A	$\frac{4}{5}$
B	C	D
E	$\frac{1}{2}$	1

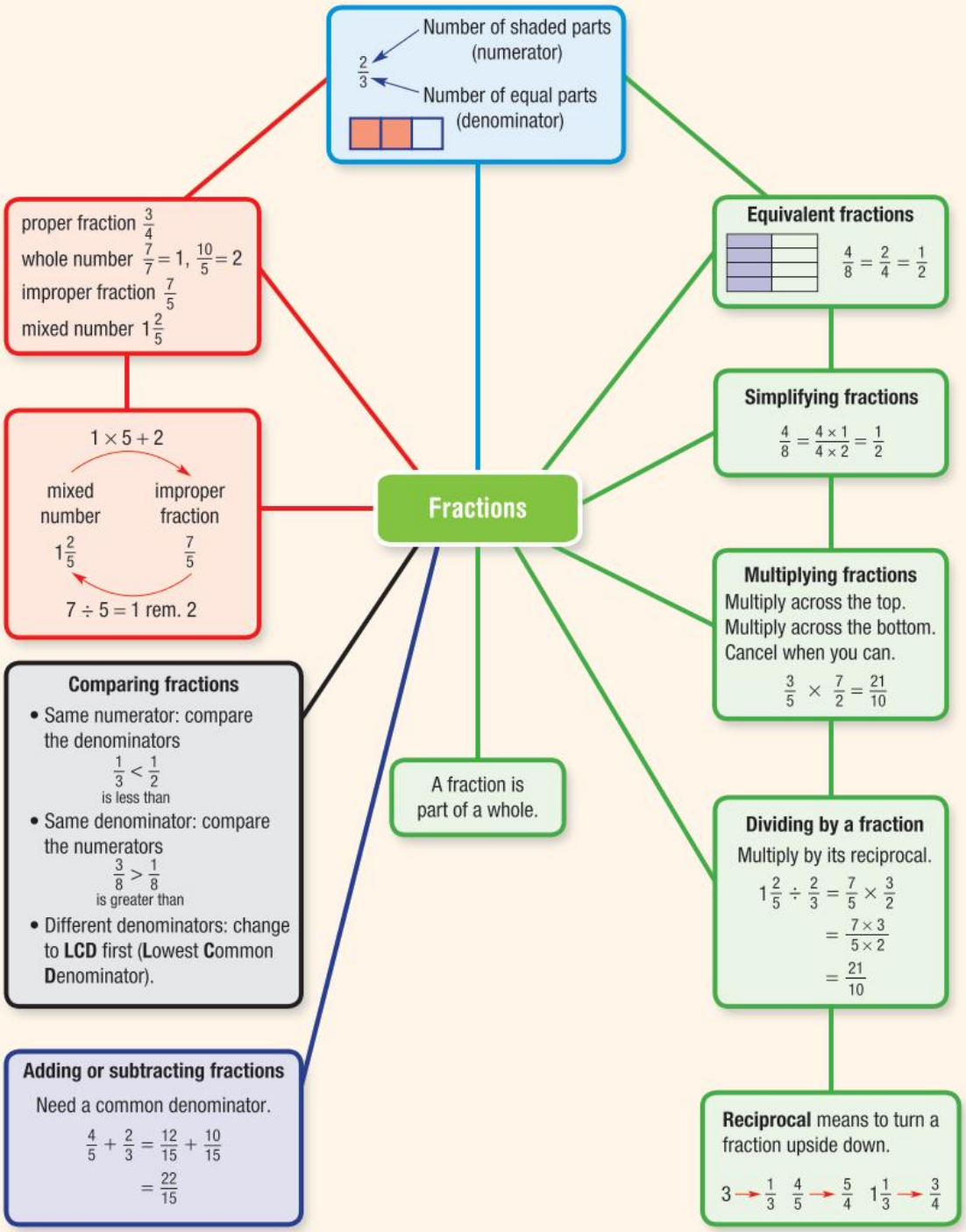
(Magic sum is $2\frac{1}{10}$.)

b

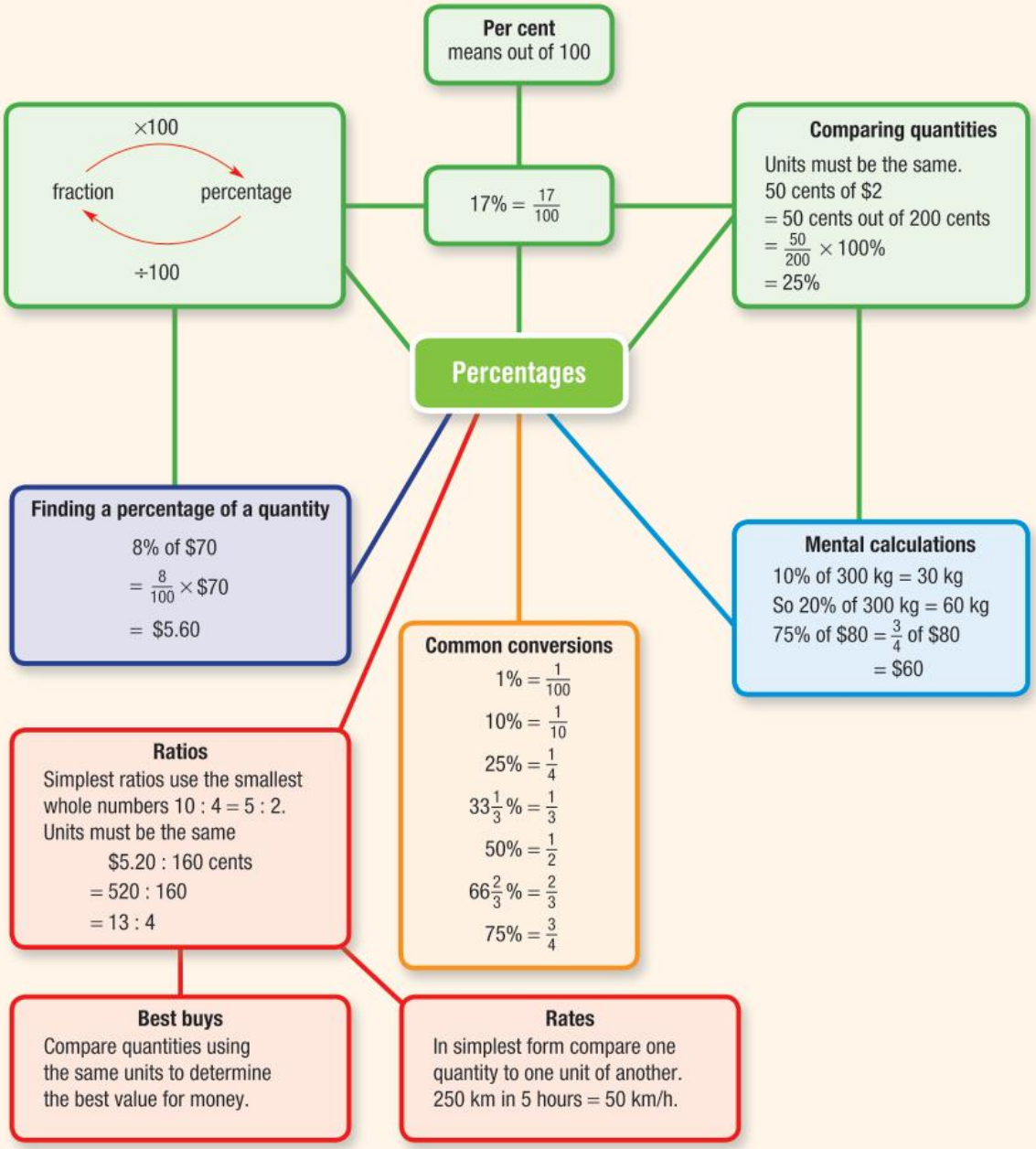
A	B	$2\frac{1}{4}$
C	$1\frac{7}{8}$	D
E	$1\frac{1}{8}$	3

(Magic sum is $5\frac{5}{8}$.)

Chapter summary



Chapter summary



Multiple-choice questions

4A **1** Which fraction is shown on the number line?



A $\frac{2}{3}$ **B** $\frac{4}{5}$ **C** $\frac{12}{5}$ **D** $2\frac{2}{10}$ **E** $\frac{5}{12}$

4B **2** $\frac{3}{4}$ is the same as:

A $\frac{4}{5}$ **B** $\frac{75}{100}$ **C** $\frac{7}{8}$ **D** $1\frac{1}{3}$ **E** $\frac{2}{3}$

4B **3** $\frac{4}{5}$ is smaller than:

A $\frac{7}{10}$ **B** $\frac{3}{4}$ **C** $\frac{1}{2}$ **D** $\frac{9}{10}$ **E** $\frac{79}{100}$

4D **4** Which is the lowest common denominator for this set of fractions? $\frac{1}{3}, \frac{1}{4}, \frac{5}{6}$

A 3 **B** 4 **C** 6 **D** 72 **E** 12

4A **5** Maria has 15 red apples and 5 green apples. What fraction of the apples are green?

A 5 **B** $\frac{1}{3}$ **C** $\frac{2}{3}$ **D** $\frac{1}{4}$ **E** $\frac{3}{4}$

4D **6** Which of the following is true?

A $\frac{1}{2} > \frac{4}{5}$ **B** $\frac{1}{2} < \frac{3}{8}$ **C** $\frac{1}{2} > \frac{2}{5}$ **D** $\frac{1}{2} > \frac{9}{10}$ **E** $\frac{1}{2} < \frac{4}{11}$

4D **7** Which set of fractions is ordered from smallest to largest (ascending order)?

A $\frac{1}{2}, \frac{2}{3}, \frac{5}{12}$ **B** $\frac{1}{2}, \frac{13}{24}, \frac{1}{4}$ **C** $\frac{2}{3}, \frac{1}{2}, \frac{7}{12}$ **D** $\frac{7}{12}, \frac{1}{2}, \frac{1}{4}$ **E** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

4E/F/G **8** Which problem has an incorrect answer?

A $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ **B** $\frac{3}{4} + \frac{5}{12} = \frac{5}{16}$ **C** $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

D $1\frac{3}{4} - \frac{1}{2} = 1\frac{1}{4}$ **E** $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

4I **9** Which fraction is greater than 75%?

A $\frac{1}{4}$ **B** $\frac{3}{4}$ **C** $\frac{15}{24}$ **D** $\frac{4}{5}$ **E** $\frac{6}{8}$

4K **10** What is \$5 as a percentage of \$25?

A 50% **B** 20% **C** 25% **D** 5% **E** 10%

4C **11** $\frac{60}{14}$ can be written as:

A $4\frac{2}{7}$ **B** $2\frac{4}{7}$ **C** $4\frac{2}{14}$ **D** $7\frac{4}{7}$ **E** $5\frac{1}{7}$

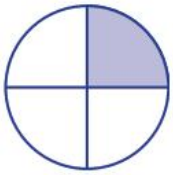
4G **12** $\frac{17}{25}$ of a metre of material is needed for a school project. How many centimetres is this?

A 65 cm **B** 70 cm **C** 68 cm **D** 60 cm **E** 75 cm

Chapter review

Short-answer questions

4A **1** Write the fraction of each circle that is shaded.

a  **b**  **c**  **d** 

e  **f**  **g**  **h** 

4B **2** Given that $24 = 12 \times 2$ and $36 = 12 \times 3$, simplify the fraction $\frac{24}{36}$.

4B **3** Write the following fractions in simplest form.

a $\frac{18}{30}$ **b** $\frac{8}{28}$ **c** $\frac{35}{49}$

4C **4** Convert each of the following to a mixed number in simplest form.

a $\frac{7}{4}$ **b** $\frac{10}{7}$

c $\frac{8}{6}$ **d** $\frac{15}{10}$

4D **5** Place the correct symbol (<, = or >) in between the following pairs of fractions to make true mathematical statements.

a $\frac{2}{7} \square \frac{4}{7}$ **b** $\frac{3}{8} \square \frac{1}{8}$

c $\frac{2}{3} \square \frac{3}{5}$ **d** $3\frac{1}{9} \square \frac{29}{9}$

4D **6** State the largest fraction in each list.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{3}{8}, \frac{2}{8}, \frac{5}{8}, \frac{1}{8}$

4D **7** State the lowest common multiple for each pair of numbers.

a 2, 5 **b** 3, 7 **c** 8, 12

4E/F **8** State the lowest common denominator for each set of fractions.

a $\frac{1}{2}, \frac{3}{5}$ **b** $\frac{2}{3}, \frac{3}{7}$ **c** $\frac{3}{8}, \frac{5}{12}$

4D **9** Rearrange each set of fractions in descending order.

a $\frac{4}{4}, \frac{3}{4}, 1\frac{1}{4}, \frac{1}{4}$ **b** $\frac{1}{8}, \frac{1}{3}, \frac{1}{10}, \frac{1}{5}$ **c** $\frac{2}{3}, \frac{1}{12}, \frac{5}{6}, \frac{4}{3}$

4E/F **10** Determine the simplest answer for each of the following.

a $\frac{3}{8} + \frac{1}{8}$ **b** $\frac{1}{3} + \frac{1}{2}$ **c** $\frac{3}{8} + \frac{5}{6}$

d $2\frac{7}{15} + 3\frac{3}{10}$ **e** $\frac{7}{8} - \frac{3}{8}$ **f** $5\frac{1}{4} - 2\frac{3}{4}$

Chapter review

4G **11** Find:

a $\frac{1}{3} \times 21$ **b** $\frac{4}{5}$ of 100 **c** $\frac{3}{4}$ of 16

d $\frac{8}{10} \times \frac{25}{4}$ **e** $\frac{2}{3}$ of $\frac{1}{4}$ **f** $3\frac{1}{8} \times 2\frac{2}{5}$

4H **12** Determine the reciprocal of each of the following.

a $\frac{3}{4}$ **b** $\frac{7}{12}$ **c** $2\frac{3}{4}$ **d** 8

4H **13** Perform these divisions.

a $\frac{6}{10} \div 3$ **b** $64 \div 3\frac{1}{5}$ **c** $\frac{2}{3} \div \frac{1}{6}$ **d** $1\frac{1}{2} \div \frac{3}{4}$

4I **14** Find the missing percentages and fractions.

Percentage form	10%	A	B	75%	C	150%
Fraction	D	$\frac{1}{4}$	$\frac{1}{2}$	E	$\frac{7}{25}$	F

4J **15** **a** Find 10% of \$200. **b** Find 25% of \$840.
c Find 50% of 96 grams. **d** Find 20% of \$150.
e Find 150% of \$6.

4K **16** Express the following as both a fraction and percentage of the total.
a 6 out of 10 **b** \$4 out of 20
c 50 cents out of \$8 **d** 600 mL out of 2 L

4L **17** Express the following ratios and rates in simplest form.
a 12 : 16 **b** \$2 : 60 cents
c 2.5 m : 10 cm **d** 60 km in 2 hours
e 10 L for 5 m **f** 65 words in 5 minutes

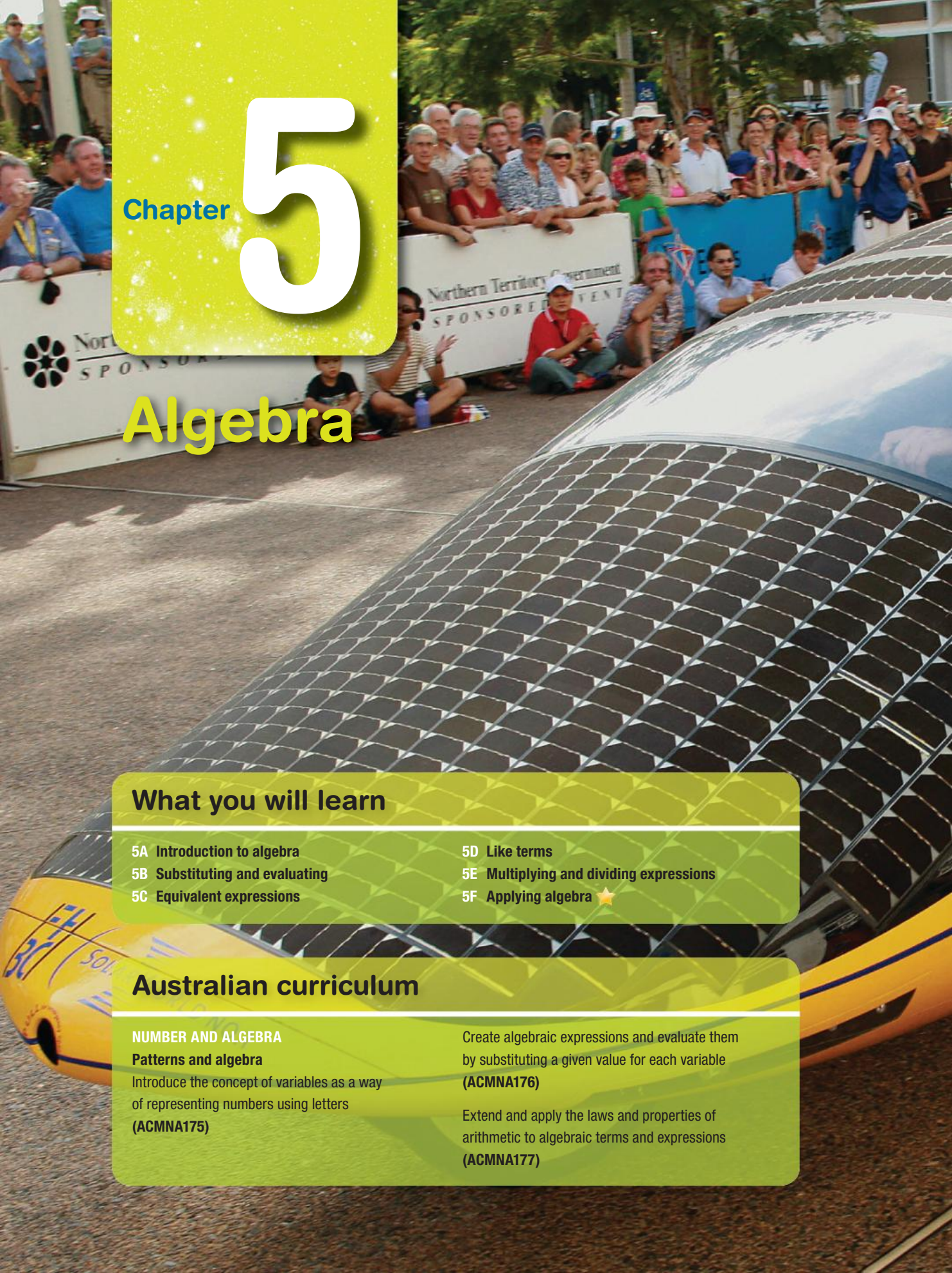
4L **18** Calculate the best buy: 3 kg of potatoes at \$2.40 or 5 kg of potatoes at \$3.50.

Extended-response questions

1 A printer produces 1200 leaflets. One-quarter of the leaflets are on green paper. Half the remaining leaflets are on white paper. There are smudges on 10% of all the leaflets.
a How many leaflets were on green paper?
b What percentage of the leaflets were *not* on green paper?
c How many white leaflets were printed?
d Of the white leaflets, how many had smudges?
e How many leaflets did not have any smudges?

Chapter 5

Algebra



What you will learn

5A Introduction to algebra
5B Substituting and evaluating
5C Equivalent expressions
5D Like terms
5E Multiplying and dividing expressions
5F Applying algebra

Australian curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Introduce the concept of variables as a way of representing numbers using letters **(ACMNA175)**

Create algebraic expressions and evaluate them by substituting a given value for each variable **(ACMNA176)**

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions **(ACMNA177)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Solar energy and algebra

Our sun provides enough energy for all the world's power needs with plenty to spare. Using algebra and advanced technologies, scientists have invented efficient solar panels that convert sunlight into electrical power. Solar power stations are being constructed world-wide, including many in Australia.

Various factors affect the electrical output from a solar panel, such as the hours of sunlight and sunlight intensity. Places north or south of the equator have less intense sunlight due to the curve of the earth. For example, sunlight in Norway, 60° north, is half the intensity of the sunlight on the equator.

Inventions that use solar power include solar lights, signs, vending machines, cars, boats, planes, backpack power, a solar camera strap, a wheel-chair canopy, BBQs and a machine that converts sea water into pure drinking water.

Pre-test

1 Find the value of:

a $3 + 6$ **b** $12 - 5$

c 12×4 **d** $20 \div 5$

2 Write the answer to each of the following problems.

a 4 and 9 are added

b 3 is multiplied by 7

c 12 is divided by 3

d 10 is halved

3 If $\square = 7$, write the value of each of the following.

a $\square + 4$ **b** $\square - 2$

c $12 - \square$ **d** $3 \times \square$

4 Write the value of $\square \times 4$ if:

a $\square = 2$ **b** $\square = 9$

c $\square = 10$ **d** $\square = 2.5$

5 If $\blacktriangle = 10$, write the value of:

a $\blacktriangle + 7$

b $\blacktriangle - 2$

c $\blacktriangle \div 2$

d $\blacktriangle + \blacktriangle$

6 Find the value of each of the following.

a $4 \times 3 + 8$ **b** $4 \times (3 + 8)$

c $4 \times 3 + 2 \times 5$ **d** $4 \times (3 + 2) \times 5$

7 Find the value of each of the following.

a $50 - (3 \times 7 + 9)$ **b** $24 \div 2 - 6$

c $24 \div 6 - 2$ **d** $24 \div (6 - 2)$

8 If $\square = 5$, write the value of each of the following.

a $\square - 4$ **b** $\square \times 2 - 1$

c $\square \div 5 + 2$ **d** $\square \times 7 + 10$

9 Find the value of each of the following.

a 5 more than 12 **b** The sum of 8 and 6

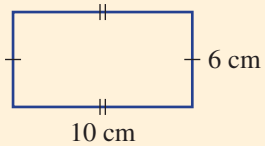
c 4 added to 7 **d** 12 less than 20

e 15 subtracted from 22 **f** double 9

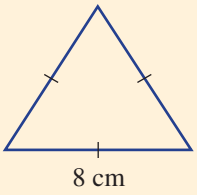
g The product of 7 and 5 **h** 4 is tripled, then 5 is added

10 Find the perimeter (distance around the outside) of each of these shapes.

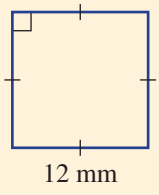
a



b



c



5A Introduction to algebra

Interactive

Widgets

HOTsheets

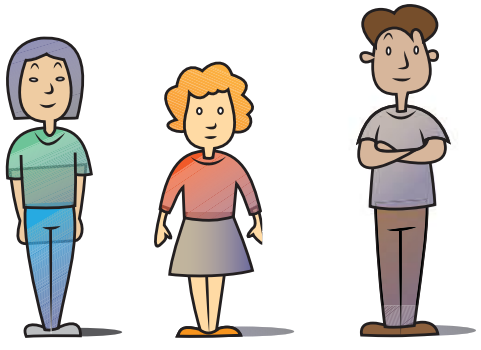
Walkthrough

In algebra, letters can be used to stand for unknown numbers. For example:

- h might stand for someone's height (in cm).
- T might stand for the temperature outside (in °C).
- n could stand for the number of people in Australia.

Letters and numbers can be combined:

- $h + 5$ cm is the height of someone 5 cm taller than h.
- $T - 12^{\circ}$C is the temperature outside if it drops 12°C.
- $5 \times n$ is the number of rabbits in 5 cages if there are n rabbits in each cage.



$h = 180$ $h = 165$ $h = 200$

Let's start: Algebra stories

In the left column are three expressions. An example story is given for the first expression. Try to make a short story for each of the others.

Expression	Story
$3 \times b$	Packets of biscuits each have b biscuits in them. (b is not known.) Ahmed opens three packets of biscuits to serve at a party. The number of biscuits is $3 \times b$.
$3 \times c - 5$	
$2 \times d + 10$	

Key ideas

- In algebra, letters called **pronumerals** or variables stand for numbers.
- $a \times b$ is written ab and $a \div b$ is written $\frac{a}{b}$.

  For example: $5n$ means $5 \times n$ and $\frac{a}{3}$ means $a \div 3$.
- A term consists of numbers and pronumerals combined with multiplication or division. For example:

  5 is a term, x is a term, $9a$ is a term, abc is a term, $\frac{4xyz}{3}$ is a term
- A term that does not contain any pronumerals is called a **constant term**. All numbers by themselves are constant terms.
- An **expression** consists of numbers and pronumerals combined with any mathematical operations. For example: $3x + 2yz$ is an expression and $8 \div (3a - 2b) + 41$ is also an expression. Any single term is also an expression.

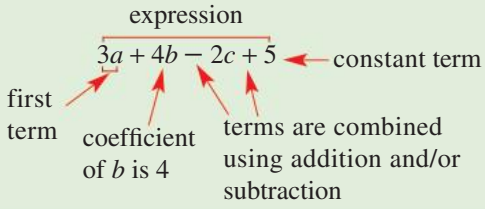
Pronumeral A letter or symbol used to represent a number

Constant term The part of an expression without any pronumerals

Expression A group of mathematical terms that does not contain an equals sign

■ A **coefficient** is the number in front of a pronumeral. For example, the coefficient of y in the expression $8x + 2y + z$ is 2. If there is no number in front, then the coefficient is 1, since $1z$ and z are equal.

Coefficient A numeral placed before a pronumeral



Exercise 5A

UNDERSTANDING 1–4 4

1 Which of the following is the common way to write $7 \times a$?

A seven times a **B** $7a$

C a^7 **D** $\frac{7}{a}$

In algebra, $5 \times b$ is written $5b$.



2 Which of the following is the common way to write $5 \div m$?

A $\frac{m}{5}$ **B** five divided by m **C** $\frac{5}{m}$ **D** $5m$

Example 1 Listing terms

List the individual terms in the expression $3a + b + 13$.

Solution	Explanation
There are three terms: $3a$, b and 13.	Each part of an expression is a term.

3 List the terms in each of the following.

a $2x + 7y$ **b** $3a + 2c + e$ **c** $5q + 3r + 2s$ **d** $7d + 5f + 17$

Example 2 Stating coefficients

State the coefficient of each pronumeral in the expression $3a + 2b + 13c$.

Solution	Explanation
The coefficient of a is 3. The coefficient of b is 2. The coefficient of c is 13.	The coefficient is the number in front of a pronumeral.

4 **a** State the coefficient of x in $3x + 5y$.

b State the coefficient of b in $7a + 13b + 5c$.

c State the coefficient of k in $10 + 4k$.

FLUENCY 5–6(½) 5–7(½)

5 For each of the following expressions, state:
- **i** the number of terms
- **ii** the coefficient of n

a $17n + 24$ **b** $31 + 27a + 15n$
c $15nw + 21n + 15$ **d** $15a - 32b + 2n + 4xy$
e $n + 51$ **f** $5bn - 12 + \frac{d}{5} + 12n$



Terms can be added or subtracted to form an expression.

Example 3 Writing expressions from word descriptions

Write an expression for the following.

a 5 more than k **b** 3 less than m **c** the sum of a and b
d double the value of x **e** the product of c and d

Solution	Explanation
a $k + 5$	5 must be added to k to get 5 more than k.
b $m - 3$	3 is subtracted from m.
c $a + b$	a and b are added to obtain their sum.
d $2 \times x$ or just $2x$	x is multiplied by 2. The multiplication sign is optional.
e $c \times d$ or just cd	c and d are multiplied to obtain their product.

6 Write an expression for the following.
a 3 more than x **b** the sum of k and 5
c 2 is added to b **d** 3 less than g
e 4 is subtracted from H **f** 6 is subtracted from M

7 Write an expression for the following without using $\times$ or $\div$.
a double the value of u
b 4 lots of y
c 3 is multiplied by x
d the product of k and 10
e y is divided by 8
f half of z
g a is tripled, then 4 is added
h p is doubled, then 12 is added



'Product' tells you to use multiplication.
The product of 3 and 10 is $3 \times 10 = 30$.

5A

PROBLEM-SOLVING AND REASONING 8–10 8–12

8 In a room there are k people, and then 5 people leave. How many people are now in the room?
A $k+5$ **B** 5 **C** $k-5$ **D** $5k$

9 Nicholas buys 10 lolly bags from a supermarket.
a If there are 7 lollies in each bag, how many lollies does he buy in total?
b If there are n lollies in each bag, how many lollies does he buy in total?


For part **b**, write an expression involving n.

10 Mikayla is paid $\$x$ per hour at her job. Write an expression for each of the following.
a How much does Mikayla earn if she works 8 hours?
b If Mikayla gets a pay rise of \$3 per hour, what is her new hourly wage?
c If Mikayla works for 8 hours at the increased hourly rate, how much does she earn?



11 Recall that there are 100 centimetres in 1 metre and 1000 metres in 1 kilometre. Write expressions for each of the following.
a How many metres are there in x kilometres?
b How many centimetres are there in x metres?
c How many centimetres are there in x kilometres?

12 If b is an even number greater than 3, decide whether these statements are true or false.
a $b+1$ must be even.
b $b+2$ could be odd.
c $5+b$ could be greater than 10.
d $5b$ must be greater than b.
e $2b$ must be greater than 10.
f $\frac{b}{2}$ is a whole number.

Restaurant algebra

— 13

13 A group of people go out to a restaurant, and the total amount they must pay (in dollars) is A. They split the bill equally. Write expressions to answer the following questions.
a If there are 4 people in the group, how much do they each pay?
b If there are n people in the group, how much do they each pay?
c One of the n people has a voucher that reduces the total bill by \$20.
i How much does each person pay now?
ii If the bill is \$200 and $n = 6$, how much does each person end up paying?

5B Substituting and evaluating



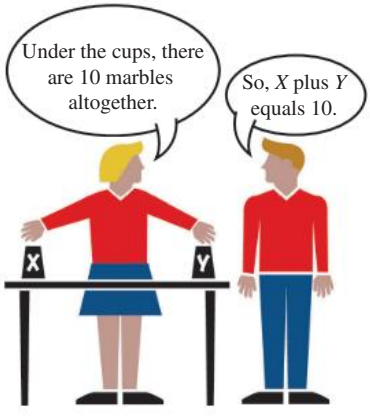






To evaluate an expression, replace the pronumerals (such as x and y) with numbers then simplify to find the answer.

For example, when $a = 11$ we can evaluate $4 + a$ to get 15.



Let's start: Sum to 10

The variables X and Y could stand for any number.

- If $X = 3$ and $Y = 7$, what is $X + Y$?
- If $X + Y = 10$, what other numbers could X and Y be? Try to list as many pairs as possible.
- Can you make $X \times Y$ equal 10 instead?

Key ideas

- To **evaluate** an expression or to **substitute** values, replace each pronumeral in an expression with a number to obtain a final value. For example: if $x = 3$ and $y = 8$, evaluating $x + 2y$ gives $3 + 2 \times 8 = 19$.
- A term like $4a$ means $4 \times a$. When substituting a number we must include the multiplication sign, since the number 42 is very different from the product 4×2.

Evaluate To find the numerical value of an expression

Substitute To replace pronumerals with numbers

Exercise 5B

UNDERSTANDING 1–5 5

1 State the value of:

a $12 + 5$ **b** $40 - 2$ **c** $30 \div 2$ **d** 3×7

2 If $\square = 4$, find the value of:

a $\square + 2$ **b** $7 \times \square$ **c** $\square - 3$ **d** $\square \div 2$

3 If $\blacktriangle = 3$, find the value of:

a $\blacktriangle + 7$ **b** $\blacktriangle \times 5$ **c** $8 - \blacktriangle$ **d** $12 \div \blacktriangle$

4 Use the correct order of operations to evaluate the following.

a $4 + 2 \times 5$ **b** $7 - 3 \times 2$

c $3 \times 6 - 2 \times 4$ **d** $(7 - 3) \times 2$

Remember BODMAS: **B**rackets, **O**f, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction



5 Complete the following

a If a is 4, what is $a + 3$? **b** If b is 5, what is $b + 12$?

c If x is 10, what is $12 \times x$? **d** If r is 7, what is $r - 2$?

$a + 3 = 4 + 3 = ?$



5B

FLUENCY 6–10(½) 6–11(½)

Example 4 Substituting for a single pronumeral

Given that $t = 5$, evaluate:

a $t + 7$ **b** $8t$ **c** $10t + 4 - t$

Solution	Explanation
a $t + 7 = 5 + 7$ $= 12$	Replace t with 5 and then evaluate the expression, which now contains no pronumerals.
b $8t = 8 \times t$ $= 8 \times 5$ $= 40$	Insert a $\times$ between 8 and t, then substitute in 5. If the multiplication sign is not included, we might get a completely wrong answer of 85.
c $10t + 4 - t = 10 \times 5 + 4 - 5$ $= 50 + 4 - 5$ $= 49$	Replace t with 5 before evaluating. Note that the multiplication (10×5) is calculated before the addition and subtraction.

6 Given that $a = 6$, evaluate:
- **a** $a + 2$
- **b** $7 \times a$
- **c** $a - 3$
- **d** $a \div 2$

7 If $b = 5$, find the value of:
- **a** $4b$
- **b** $10b$
- **c** $7b$
- **d** $20b$


$4b$ means $4 \times b$.

8 Calculate the value of $12 + b$ if:
- **a** $b = 5$
- **b** $b = 8$
- **c** $b = 60$
- **d** $b = 0$

9 Substitute $x = 5$ and then evaluate each of the following.

a $x + 3$	**b** $x \times 2$	**c** $14 - x$
d $2x + 4$	**e** $3x + 2$	**f** $13 - 2x$
g $2(x + 2)$	**h** $20 \div x + 3$	**i** $4x - 2$


Substitute means replace x with 5.

10 If $y = 3$, state the value of:

a $5y + 2 - y$	**b** $3 + 4y + 2y$
c $y \times 7 + 5 - y$	**d** $y \times (1 + y)$
e $y + 2y + 3y$	**f** $(10 - y) \times y$

Example 5 Substituting for multiple pronumerals

Substitute $x = 4$ and $y = 7$ to evaluate $5x + y + 8$.

Solution	Explanation
$5x + y + 8 = 5 \times 4 + 7 + 8$ $= 20 + 7 + 8$ $= 35$	Insert the implied multiplication sign between 5 and 4 before completing the calculation.

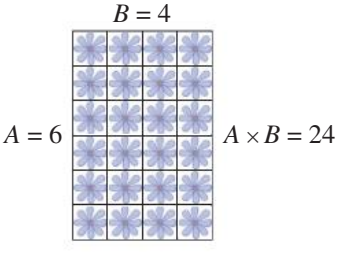
11 Substitute $a = 2$ and $b = 3$ and then evaluate:

a $2a + 4$ **b** $2b - 2$ **c** $a + b$
d $3a + b$ **e** $5a - 2b$ **f** ab
g $a + 2b$ **h** $b - a$ **i** $5a + 2 + b$

PROBLEM-SOLVING AND REASONING 12 12, 13

12 A number is substituted for b in the expression $7 + b$ and gives the result 12. What is the value of b?

13 Assume A and B are two numbers, where $AB = 24$.
What values could A and B equal if they are whole numbers?
Try to list as many pairs as possible.



Working backwards — 14

14 Copy and complete the table.

x	5	9	12			
$x + 6$	11			7		
$4x$	20				24	28

5C Equivalent expressions

Interactive

Widgets

HOTsheets

Walkthrough

Sometimes two expressions will evaluate to give the same result, no matter what numbers the pronumerals stand for.

For example, $B + B$ and $2 \times B$ will always give the same result. For example:

$B = 3 \rightarrow B + B = 6$, $2 \times B = 6$ $\qquad$ $B = 11 \rightarrow B + B = 22$, $2 \times B = 22$

This means that $B + B$ and $2 \times B$ are equivalent.

Let's start: Odd one out

Here are four expressions:

$2 \times B + 6 \qquad 6 + B + B \qquad (B + 3) \times 2 \qquad B + 6$

One of them is not equivalent to the others.

- Copy and complete the table to help you find the odd one out. (The first row has already been done.)

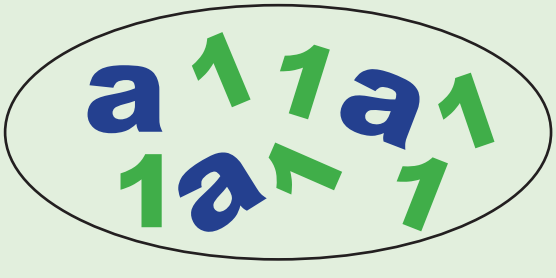
	$2 \times B + 6$	$6 + B + B$	$(B + 3) \times 2$	$B + 6$
$B = 0$	$2 \times 0 + 6 = 6$	$6 + 0 + 0 = 6$	$(0 + 3) \times 2 = 6$	$0 + 6 = 6$
$B = 1$				
$B = 2$				

- Can you draw pictures to show why the 'odd one out' in the table is not equivalent to the other three expressions? (For example, if B is the number of marbles in a bag, you could draw 2 bags and 6 extra marbles to show $2 \times B + 6$.)

Key ideas

- **Equivalent expressions** are *always* equal in value, no matter what numbers are substituted for the pronumerals.
- For example: $3a + 6$ is equivalent to $6 + 3a$ and to $3a + 3 + 3$.

Equivalent expressions
Expressions that are *always* equal in value



= a + a + a + 6
= 3a + 3 + 3
= 2a + 6 + a
= . . .

This collection of terms can be arranged into many different equivalent expressions.

Exercise 5C

UNDERSTANDING 1–5 5



1 **a** If $x = 3$, what does $x + 6$ equal?
b If $x = 3$, what does $4x$ equal?
c Are $x + 6$ and $4x$ equivalent expressions?

2 **a** If $a = 5$, evaluate $a + 4$.
b If $a = 5$, evaluate $2 + a + 2$.
c Are $a + 4$ and $2 + a + 2$ equivalent expressions?

3 Copy and complete:
Two expressions that always have the same value are called _____ expressions.

4 True or false? Explain your answer with a sentence. 'No matter what number you choose for $\square$, the values of $\square + 6$ and $6 + \square$ are equal.'

Example 6 Using tables to decide equivalence

Fill in a table to help you decide if $3a + 6$ and $(a + 2) \times 3$ are equivalent. Use $a = 0$, $a = 1$, $a = 2$, $a = 3$.

Solution

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$a + 6$	6	9	12	15
$(a + 2) \times 3$	6	9	12	15

They are equivalent.

Explanation

$3a + 6$ and $(a + 2) \times 3$ are equal for all values of a, so they appear to be equivalent.

Note

$$(a + 2) \times 3 = 3 \times (a + 2)$$
$$= a + 2 + a + 2 + a + 2$$
$$= 3a + 6$$

5 **a** Copy and complete the following table.

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$2a + 2$				
$(a + 1) \times 2$				

b Fill in the gap: $2a + 2$ and $(a + 1) \times 2$ are _____ expressions.

FLUENCY 6, 7 6–8

6 **a** Copy and complete the following table.

	$B = 0$	$B = 1$	$B = 2$	$B = 3$
$5B + 3$				
$6B + 3$				

b Are $5B + 3$ and $6B + 3$ equivalent expressions?



5C

7 **a** Copy and complete this table.

	$6x + 5$	$4x + 5 + 2x$
$x = 1$		
$x = 2$		
$x = 3$		
$x = 4$		

b Are $6x + 5$ and $4x + 5 + 2x$ equivalent?

8 For each of the following pairs, decide if they are equivalent (E) or not equivalent (N).

a $k + 6$ and $k \times 4$

b $k \times 3$ and $2 \times k + k$

c $k + 2$ and $1 + k + 1$

d $k + 10$ and $k \times 10$

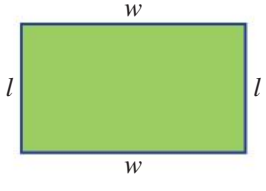


Try making k stand for different numbers ($k = 0$, $k = 1$, $k = 2$, etc.) in a table.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Give an example of an expression that is equivalent to $4y$.

10 The perimeter of this rectangle is given by $w + l + w + l$. Write an equivalent expression for the perimeter.



11 The expressions $a + b$ and $b + a$ are equivalent and only contain two terms. How many expressions are equivalent to $a + b + c$ and contain only three terms and only using a, b and c and addition?

12 Prove that no two of these expressions are equivalent:
$4 + x$, $4x$, $x - 4$, $x \div 4$.



Substitute different values for x.

Matching pairs — 13

13 On the following game board, each box has a partner box. Write all the matches. (For example, A1 and C2 match because $3a + 2a$ is equivalent to $5a$.)

	Column A	Column B	Column C	Column D
Row 1	$3a + 2a$	$6a$	$4a + 2$	$7a$
Row 2	$5 - a - 2a$	10	$5a$	$2 \times (a + 5) - 2a$
Row 3	$(1 + 2a) \times 2$	$2a + 5a$	$2a \times 3$	$5 - 3a$

5D Like terms

Interactive

Widgets

HOTsheets

Walkthrough

Consider boxes of chocolates produced at a factory, with each box containing 12 chocolates. Then 3 boxes added to 5 boxes would contain the same number of chocolates as 8 boxes. So $3 \times 12 + 5 \times 12 = 8 \times 12$.

Similarly, if each box contained n chocolates, then 3 boxes added to 5 boxes would be the same as 8 boxes. So $3n + 5n = 8n$. The terms $3n$ and $5n$ would be considered 'like' terms and can therefore be collected when they are added or subtracted.



Let's start: Simplifying expressions

- Show that $4a + 3a$ is equivalent to $7a$. (For example, you could use a table of values.)
- Is $4a + 3b$ equivalent to $7ab$? Choose values of a and b to substitute and check.
- What is the difference between $4a + 3a$ and $4a + 3b$?
- Write a sentence to explain why $4a + 3b$ can't be simplified.

Key ideas

- **Like terms** have exactly the same pronumerals, although not necessarily in the same order.

Like	Not like
$3x$ and $5x$	$3x$ and $5y$
$-12a$ and $7a$	$11d$ and $4c$
$5ab$ and $6ba$	$-8ab$ and $5a$

- Like terms can be combined. This is called **simplifying**.
  For example:
  $3x + 5x$ simplifies to $8x$
  $12a - 7a$ simplifies to $5a$

Like terms Terms with the same pronumerals

Simplifying Finding the simplest possible expression

Exercise 5D

UNDERSTANDING 1–5 | 5

1 True or false?

a If you add $5a$ to $3a$ you get $8a$.

b If you add $5b$ to $6a$ you get $11b$.

c If you take $6c$ away from $10c$ you have $4c$ left.

d If you add $6c$ to $3a$ you get $9c$.

5D

2 **a** If $a = 7$, what is $2a$?
b If $a = 7$, what is $3a$?
c If $a = 7$, what is $2a + 3a$?
d If $a = 7$, what is $5a$?

$2a = 2 \times a$
$= 2 \times 7$
$= 14$



3 Evaluate the following, using $x = 5$.
a $10x$ **b** $4x$ **c** $10x - 4x$ **d** $6x$

4 For each of the following terms, list all the pronumerals it contains.
a $4xy$ **b** $3abc$ **c** $2k$ **d** pq

5 Copy and complete the following sentences.
a $3x$ and $5x$ are _____ terms.
b $4x$ and $3y$ are not _____ _____.
c $4xy$ and $4yx$ are like _____.
d $12a$ and $5ab$ are not like _____ because they have different pronumerals.

FLUENCY 6–9(½) 6–10(½)

Example 7 Identifying like terms

Which of the following pairs are like terms?

a $3x$ and $2x$ **b** $3a$ and $3b$ **c** $2ab$ and $5ba$
d $2a$ and $4ab$ **e** $7ab$ and $9aba$

Solution	Explanation
a $3x$ and $2x$ are like terms.	The pronumerals are the same.
b $3a$ and $3b$ are not like terms.	The pronumerals are different.
c $2ab$ and $5ba$ are like terms.	The pronumerals are the same, even though they are written in a different order (one a and one b).
d $2a$ and $4ab$ are not like terms.	The pronumerals are not exactly the same (the first term contains only a and the second term has a and b).
e $7ab$ and $9aba$ are not like terms.	The pronumerals are not exactly the same (the first term contains one a and one b, but the second term contains two a terms and one b).

6 Classify the following pairs as like terms (L) or not like terms (N).
a $7a$ and $4b$ **b** $3a$ and $10a$ **c** $18x$ and $32x$
d $4a$ and $4b$ **e** 7 and $10b$ **f** x and $4x$
g $5x$ and 5 **h** $12ab$ and $4ab$ **i** $7cd$ and $12cd$
j $3abc$ and $12abc$ **k** $3ab$ and $2ba$ **l** $4cd$ and $3dce$

Example 8 Adding or subtracting like terms

Simplify:

a $10x + 4x$ **b** $7a - 2a$

Solution	Explanation
a $10x + 4x = 14x$	$10x$ and $4x$ are the terms, so they are combined $(10 + 4 = 14)$.
b $7a - 2a = 5a$	$7a$ and $2a$ are like terms, so they are combined $(7 - 2 = 5)$.

7 Simplify the following by collecting like terms.

a $3x + 2x$ **b** $4a + 2a$ **c** $6q + 10q$
d $b + 2b$ **e** $6cd + 3cd$ **f** $2qr + 4qr$
g $8ab + ab$ **h** $9cf + 2cf$



Remember that b is the same as $1b$ or $1 \times b$.

8 Simplify the following by collecting like terms.

a $7x - 3x$ **b** $8a - 5a$ **c** $12q - 2q$
d $7b - b$ **e** $10cd - 2cd$ **f** $8qr - 6qr$
g $8ab - ab$ **h** $10cf - 7cf$

Example 9 Simplifying by combining like terms

Simplify the following by collecting like terms.

a $7b + 2 + 3b$ **b** $12d - 4d + d$ **c** $5 + 12a + 4b - 2 - 3a$

Solution	Explanation
a $7b + 2 + 3b = 10b + 2$	$7b$ and $3b$ are like terms, so they are combined. They cannot be combined with 2 because it contains no pronumerals.
b $12d - 4d + d = 9d$	All the terms here are like terms. Remember that d means $1d$ when combining them.
c $5 + 12a + 4b - 2 - 3a$ $= 12a - 3a + 4b + 5 - 2$ $= 9a + 4b + 3$	$12a$ and $3a$ are like terms. We subtract $3a$ because it has a minus sign in front of it. We can also combine the 5 and the 2 as they are like terms.

9 Simplify the following by collecting like terms.

a $2a + a + 4b + b$ **b** $5a + 2a + b + 8b$ **c** $3x - 2x + 2y + 4y$
d $4a + 2 + 3a$ **e** $7 + 2b + 5b$ **f** $3k - 2 + 3k$

5D

10 Simplify the following by collecting like terms.

a $7f + 12 - 2f$
b $4a - 4 + 5b + b$
c $3x + 7x + 3y - 4x + y$
d $10a + 3 + 4b - 2a$
e $4 + 10g - 3g$
f $10x + 4x + 31y - y$
g $10 + 7y - 3x + 5x + 2y$
h $11a + 4 - 3a + 9$
i $3b + 4b + c + 5b - c$
j $8 + 3d - 5 + 2d$



For $7f + 12 - 2f$, the sign in front of $2f$ tells us to subtract that term.

PROBLEM-SOLVING AND REASONING

11, 12 | 11–13

11 Ravi and Marissa each work for n hours per week. Ravi earns \$30 per hour and Marissa earns \$35 per hour.

a Write an expression for the amount Ravi earns in one week.
b Write an expression for the amount Marissa earns in one week.
c Write a simplified expression for the total amount Ravi and Marissa earn in one week.



12 The length of the line segment shown could be expressed as $a + a + 3 + a + 1$.



a Write the length as a simplified expression.
b What is the length of the segment if a is equal to 5?

13 **a** Show, using a table of values, that $3x + 2x$ is equivalent to $5x$.
b Prove that $3x + 2y$ is not equivalent to $5x$.
c Prove that $3x + 2y$ is not equivalent to $5xy$.



For part **b**, try choosing different values for x and y.

Algebraic marbles

— | 14

14 Let x represent the number of marbles in a standard-sized bag. Xavier bought 4 bags and Cameron bought 7 bags.

a Write simplified expressions for:
i the number of marbles Xavier has
ii the number of marbles Cameron has
iii the total number of marbles that Xavier and Cameron have
iv the number of *extra* marbles that Cameron has compared to Xavier

b If x is 12, how many marbles do they each have?

Progress quiz

5A **1** **a** State the coefficient of b in the expression $7a + 11b + 4c$.
b State the coefficient of x in the expression $13x + 3y$.

5A **2** Write an expression for the following.
a 7 more than x
b 8 less than t
c Triple the value of f
d Half the value of p, then 4 is added

5A **3** In a room there are p people, and then 7 more people arrive. How many people are now in the room?
A $p - 7$ **B** $7 - p$ **C** $7p$ **D** $p + 7$

5B **4** Given that $y = 4$, evaluate:
a $y + 11$ **b** $5 \times y$ **c** $9 - y$ **d** $24 \div y$

5B **5** Substitute $x = 7$ and then evaluate each of the following.
a $x + 6$ **b** $73 - x$ **c** $3x - 5$ **d** $35 \div x + 18 - x$

5B **6** Substitute $a = 5$ and $b = 3$ and then evaluate:
a $3a + b$ **b** $a + 4b$ **c** ab **d** $7a - 11b - 2$

5C **7** Copy and complete.
Two or more expressions that are always equal are called _____ expressions.

5C **8** For each of the following pairs, decide if they are equivalent (E) or not equivalent (N).
a $r + 5$ and $r \times 5$
b $r \times 3$ and $r + r + r$
c $r + 4$ and $2 + r + 2$
d $6r + 3$ and $4r + 4 + 2r - 1$

5D **9** Classify the following pairs as like terms (L) or not like terms (N).
a $6c$ and $5d$
b $21x$ and $3x$
c 5 and $5t$
d $7xy$ and xy
e $3abc$ and $3ab$
f $6wk$ and $3kw$

5D **10** Simplify the following by collecting like terms if possible.
a $5x + 12x$
b $11kt - 4k$
c $16x + 4y + 7y$
d $5p + 8 - 3p$
e $10a + 4a + 5b + b$
f $17 + 3g - 12 - g$

5E Multiplying and dividing expressions

When we simplify and write expressions involving multiplication and division we usually do not use the $\times$ or $\div$ symbols.

Remember that $4a$ means $4 \times a$ and $6xy = 6 \times x \times y$. Also, we write fractions like $\frac{b}{3}$ to mean $b \div 3$.

Let's start: Putting in the multiplication symbol

- Rewrite the following expression by putting in all the $\times$ signs.
  $7abc + 2de + 3f$
- How can the following expression be written without $\times$ signs? Put the pronumerals in alphabetic order to find a hidden message.
  $s \times i \times m \times c \times h \times p + v \times l \times u + s \times p \times c \times i \times h$

Key ideas

- $a \times b$ is written ab.
- $a \div b$ is written $\frac{a}{b}$.
- For multiplication, numbers should be written first and pronumerals are usually written in alphabetical order.
  For example: $a \times 2 \times b$ is written $2ab$.
- When dividing, we write division using fraction form and any common factors in the numerator and denominator can be cancelled.
  For example: $\frac{{}^{2}\cancel{4}ab^{1}}{{}_{1}\cancel{2}b_{1}c} = \frac{2a}{c}$

Exercise 5E

UNDERSTANDING 1–4 | 4

1 True or false?

a $4 \times n$ can be written as $4n$.

b $n \times 3$ can be written as $3n$.

c $4 \times b$ can be written as $b + 4$.

d $a \times b$ can be written as ab.

e $a \times 5$ can be written as $50a$.

2 Simplify these fractions.

a $\frac{10}{20}$ **b** $\frac{6}{18}$ **c** $\frac{12}{20}$ **d** $\frac{15}{20}$

3 **a** Simplify the fraction $\frac{12}{18}$.

b Simplify the fraction $\frac{2000}{3000}$.

c Simplify $\frac{2a}{3a}$.



$\frac{12}{18} = \frac{2 \times 6}{3 \times 6}$

$\frac{2000}{3000} = \frac{2 \times 1000}{3 \times 1000}$

$\frac{2a}{3a} = \frac{2 \times a}{3 \times a}$

4 Match these expressions (**a** to **e**) with the conventional way to write them (**A** to **E**).

a $2 \times u$	**A** $3u$
b $7 \times u$	**B** $\frac{5}{u}$
c $5 \div u$	**C** $2u$
d $u \times 3$	**D** $\frac{u}{5}$
e $u \div 5$	**E** $7u$

FLUENCY 5–8(½) 5–9(½)

Example 10 Simplifying expressions with multiplication

a Write $4 \times a \times b \times c$ without multiplication signs.
b Simplify $4a \times 2b \times 3c$, giving your final answer without multiplication signs.

Solution	Explanation
a $4 \times a \times b \times c = 4abc$	When pronumerals are written next to each other, they are being multiplied.
b $4a \times 2b \times 3c = 4 \times a \times 2 \times b \times 3 \times c$ $= 4 \times 2 \times 3 \times a \times b \times c$ $= 24abc$	First insert the missing multiplication signs. Rearrange to bring the numbers to the front. $4 \times 2 \times 3 = 24$ and $a \times b \times c = abc$

5 Write each of these expressions without multiplication signs.

a $2 \times x$	**b** $5 \times p$
c $7 \times r$	**d** $11 \times s$
e $10 \times a \times b$	**f** $5 \times c \times d$

6 Write each of these expressions without any multiplication signs.

a $5 \times 2 \times a \times b$	**b** $2 \times 8 \times x \times y$
c $2 \times b \times 5$	**d** $x \times 7 \times z \times 4$
e $2 \times a \times 3 \times b \times 6 \times c$	**f** $8 \times d \times 2 \times e \times 3 \times f$
g $7 \times 3 \times a \times 2 \times b$	**h** $a \times 2 \times b \times 7 \times 3 \times c$
i $7 \times a \times 12 \times b \times c$	**j** $x \times 2 \times 3y$

For part **c**, reorder:
$2 \times b \times 5 = 2 \times 5 \times b$



5E

7 Simplify these expressions.

a $3a \times 12$	**b** $7d \times 9$	**c** $2 \times 4e$
d $3 \times 5a$	**e** $4a \times 3b$	**f** $7e \times 9g$
g $8a \times bc$	**h** $4d \times 7af$	**i** $a \times 3b \times 4c$
j $2a \times 4b \times c$	**k** $4d \times 3e \times 5fg$	**l** $2cb \times 3a \times 4d$



Multiply the numbers and write the pronumerals in alphabetical order.

Example 11 Simplifying expressions with division

a Write $x \div 3$ without a division sign.

b Simplify the expression $\frac{8ab}{12b}$.

Solution	Explanation
a $x \div 3 = \frac{x}{3}$	Divisions are written as fractions in algebra.
b $\frac{8ab}{12b} = \frac{8 \times a \times b}{12 \times b}$	Insert multiplication signs to help spot common factors.
$= \frac{2 \times \cancel{4}^{1} \times a \times \cancel{b}^{1}}{3 \times \cancel{4}^{1} \times \cancel{b}^{1}}$	8 and 12 have a common factor of 4. $4 \div 4 = 1$, $b \div b = 1$
$= \frac{2a}{3}$	Cancel out the common factors of 4 and b.

8 Write each expression without a division sign.

a $x \div 5$	**b** $z \div 2$	**c** $a \div 12$	**d** $b \div 5$
e $2 \div x$	**f** $5 \div d$	**g** $x \div y$	**h** $a \div b$

9 Simplify the following expressions by dividing by any common factors.

a $\frac{2x}{5x}$	**b** $\frac{5a}{9a}$	**c** $\frac{9ab}{4b}$	**d** $\frac{2ab}{5a}$
e $\frac{2x}{4}$	**f** $\frac{9x}{12}$	**g** $\frac{10a}{15a}$	**h** $\frac{30y}{40y}$
i $\frac{4a}{2}$	**j** $\frac{21x}{7x}$	**k** $\frac{4xy}{2x}$	**l** $\frac{9x}{3xy}$

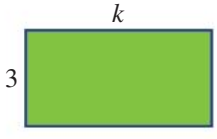


Remember that $\frac{a}{1} = a$ and $\frac{a}{a} = 1$.

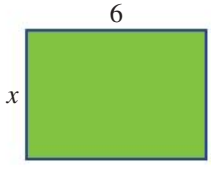
PROBLEM-SOLVING AND REASONING 10, 11 10–12

10 Write a simplified expression for the area of each rectangle.

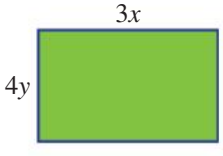
a



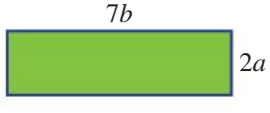
b



c



d





Area of a rectangle = width × length

11 Five friends go to a restaurant. They split the bill evenly, so each pays the same amount.

a If the total cost is $\$100$, how much do they each pay?

b If the total cost is $\$C$, how much do they each pay? Write an expression.



12 The expression $3 \times 2p$ is the same as the expression $\underset{(1)}{2p} + \underset{(2)}{2p} + \underset{(3)}{2p}$.

a What is a simpler expression for $2p + 2p + 2p$?

b $3 \times 2p$ is shorthand for $3 \times 2 \times p$. How does this relate to your answer in part **a**?

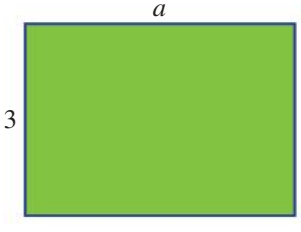


For part **a**, combine like terms.

Doubling rectangles

— 13

13 The area of this rectangular paddock is $3a$.



Both the length and the width of the paddock are now doubled.

a Draw the new paddock, showing its dimensions.

b Write a simplified expression for the area of the new paddock.

c Divide the area of the new paddock by the area of the old paddock. What do you notice?

d What happens to the area of the original paddock if you triple both the length and the width?



5F Applying algebra









An algebraic expression can be used to describe problems relating to many different everyday situations, including costs, speeds and sporting results.

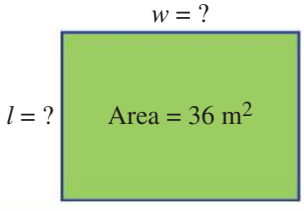


In many sports, results and details can be expressed using algebra.

Let's start: Garden bed area

The garden shown below has an area of 36 m^2, but the width and length are unknown.

- What are some possible values that w and l could equal?
- Try to find the dimensions of the garden that make the fencing around the outside as small as possible.



Key ideas

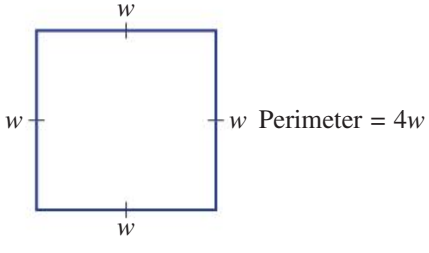
- Many different situations can be modelled with algebraic expressions.
- For example: an algebraic expression for perimeter of a rectangle is $2l + 2w$.
- To apply an expression, the pronumerals should be defined clearly. Then known values should be substituted for the variables.
  For example: If $l = 5$ and $w = 10$,
  Perimeter $= 2 \times 5 + 2 \times 10$
  $= 30$

Exercise 5F

UNDERSTANDING 1–4 4

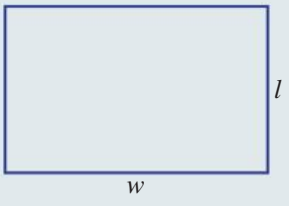
1 **a** If $w = 3$, find the value of $4w$.
b If $c = 2$, find the value of $c + 8$.
c Evaluate $3d$, given that $d = 10$.
d Evaluate $7 + 4f$ if $f = 5$.

2 The perimeter of this square is given by $4w$.
a If $w = 5$, find the value of $4w$.
b Find the perimeter of a square if $w = 7$.
c What is the perimeter of a square if its width is 10?
d If a square has a width of 25, what is its perimeter?



Example 12 Applying an expression

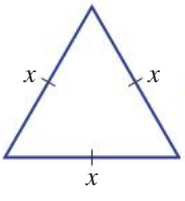
The perimeter of a rectangle is given by the expression $2w + 2l$, where w is the width and l is the length.
Find the perimeter of a rectangle if $w = 5$ and $l = 7$.



Solution	Explanation
Perimeter is given by $2w + 2l = 2 \times 5 + 2 \times 7$ $= 10 + 14$ $= 24$	To apply the rule, we substitute $w = 5$ and $l = 7$ into the expression. Evaluate using the normal rules of arithmetic (BODMAS).

3 The area of a rectangle is given by the expression $w \times l$, where w is its width and l is its length.
- **a** Find the area if $w = 5$ and $l = 7$.
- **b** Find the area if $w = 2$ and $l = 10$.

4 The following diagram shows an equilateral triangle.



- **a** The perimeter is $x + x + x$. Simplify this expression.
- **b** Use your answer to **a** to find the perimeter if $x = 12$.



For part **a**, combine like terms.

FLUENCY 5–9 5–10

5 Exercise books cost $3 each.
- **a** How much does it cost for 12 exercise books?
- **b** How much does it cost for 7 exercise books?
- **c** Write an expression for the cost of n exercise books.



6 Mustafa says that he will work for 2 more hours than Kristina.
- **a** If Kristina works for 4 hours, how long will Mustafa work?
- **b** If Kristina works for 9 hours, how long will Mustafa work?
- **c** If Kristina works for t hours, how long will Mustafa work? Give an expression.



For part **c**, give an expression for 2 more than t.

5F

Example 13 Constructing expressions from situations

Write expressions for each of the following.

a The total cost, in dollars, of 10 bottles, if each bottle costs $\$x$.
b The total cost, in dollars, of hiring a plumber for n hours.The plumber charges a $30 call-out fee plus $60 per hour.

Solution	Explanation
a $10x$	Each of the 10 bottles costs $\$x$, so the total cost is $10 \times x = 10x$.
b $30 + 60n$	For each hour, the plumber charges $60, so must pay $60 \times n = 60n$. The $30 call-out fee is added to the total bill.



7 If pencils cost $\$x$ each, write an expression for the cost of:
a 10 pencils
b 3 packets of pencils, if each packet contains 5 pencils
c k pencils

8 If pens cost $2 each, write an expression for the cost of n pens.

9 A car travels at 60 km/h, so in n hours it has travelled $60n$ kilometres.
a How far does the car travel in 3 hours (i.e. $n = 3$)?
b How far does the car travel in 30 minutes?
c Write an expression for the total distance travelled in n hours for a motorbike with speed 70 km/h.



10 A carpenter charges a $40 call-out fee and then $80 per hour. This means the total cost for x hours of work is $40 + 80x$.
a How much would it cost for a 2-hour job (i.e. $x = 2$)?
b How much would it cost for a job that takes 8 hours?
c The call-out fee is increased to $50. What is the new expression for the total cost of x hours?

PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 A plumber charges a $50 call-out fee and $100 per hour.
a Copy and complete the table below.

Hours	1	2	3	4	5
Total cost ($)					

b Find the total cost if the plumber works for t hours. Give an expression.
c Substitute $t = 30$ into your expression to find how much it will cost for the plumber to work 30 hours.

12 To hire a tennis court, you must pay a $5 booking fee plus $10 per hour.



- **a** What is the cost of booking a court for 2 hours?
- **b** What is the cost of booking a court for x hours? Write an expression.
- **c** A tennis coach hires a court for 7 hours. Substitute $x = 7$ into your expression to find the total cost.

13 In Australian Rules football a goal is worth 6 points and a behind is worth 1 point. This means the total score for a team is $6g + b$, if g goals and b behinds are scored.

- **a** What is the score for a team that has scored 5 goals and 3 behinds?
- **b** What are the values of g and b for a team that has scored 8 goals and 5 behinds?
- **c** If a team has a score of 20, this could be because $g = 2$ and $b = 8$. What are the other possible values of g and b?

Mobile phone mayhem — 14

14 Rochelle and Emma hire two different satellite phones for an extended mountain trek. The costs to make calls on these phones are outlined in this table below.

Rochelle's plan	Emma's plan
20 cents connection fee 60 cents per minute	80 cents connection fee 40 cents per minute

- **a** Write an expression for the cost of making a t minute call using Rochelle's phone.
- **b** Write an expression for the cost of making a t minute call using Emma's phone.
- **c** Whose phone plan would be cheaper for a 7 minute call?
- **d** What length of call would cost exactly the same for both phones?
- **e** Investigate current mobile phone plans and describe how they compare to Rochelle's and Emma's plans.



Maths@Work: Office designs

Planning the design and structure of office spaces requires skill with geometry and number patterns. A business will employ experts to plan their office work and meeting areas. Designing an office or conference space to hold the best arrangement of people and computers is important for productivity and progress.

Algebra skills help in this planning process, using patterns for layouts as well as formulas for costings.



1 Below are some table designs. For each table design answer the following four questions.

- **i** List the sequence for the number of people at 1, 2, 3 and 4 desks. You can draw extra groups of desks if you wish.
- **ii** Write a formula (i.e. a rule) for the number of people that could sit at n desks with each design.
- **iii** Calculate the number of people sitting at 10 desks.
- **iv** If each desk costs \$890, each office chair costs \$345 and there is a delivery fee of \$199 per order, calculate the total cost for 10 desks and accompanying chairs of each design.

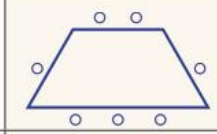
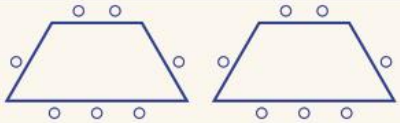
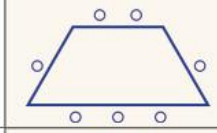
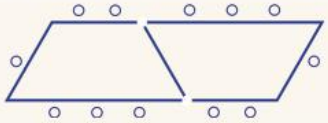
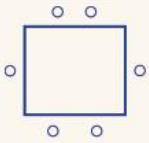
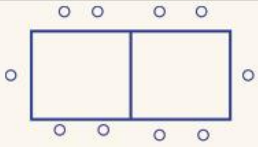
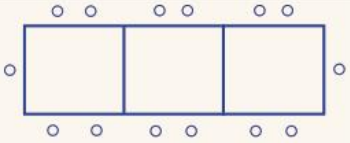


For designs *B* and *C*

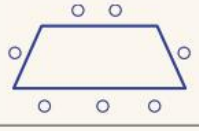
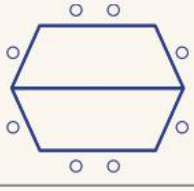
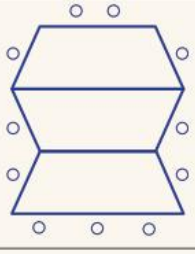
- How many people are always sitting on the ends?
- $\square$ people sit in the middle for 2 desks, 3 desks, n desks?

Formula

Number of people $= 2 + \square \times n$

	One desk	Two desks	Three desks
Design A			
Design B			
Design C			

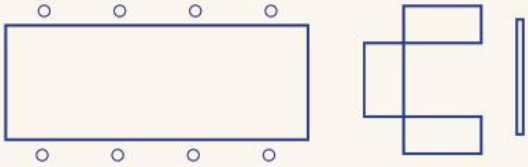
2 Answer the questions below for design D.

	One desk	Two desks	Three desks
Design D			

a Draw 4 desks of design D showing the chairs as small circles.
b List the number sequence for the number of people at 1, 2, 3 and 4 desks.
c Following this number pattern in part **b**, extend the sequence to find the number of people sitting at 10 desks.

Using technology

3 Office designs can include areas for staff to meet and discuss plans in a more relaxed setting.
Design E:
Each row has one table for 8 people, 3 lounge chairs and a large video screen.





When copying a price, only enter the numbers. The computer adds the $ sign and the comma.

a Set up the following spreadsheet for Design E. Select all the cost cells (in columns B to H) right click, select Format/Currency/0 d.p.

	A	B	C	D	E	F	G	H
1	Cost of office furniture							
2	Office	Cost of one large table	Cost of one office chair	Cost of one lounge chair	Cost of one row of furniture	Number of rows	Cost of large video screen	Total cost
3	I	$2,541	$345	$1,500		4	$2,100	
4	II	$4,320	$548	$2,300		9	$2,460	
5	III	$3,875	$470	$1,750		15	$2,040	
6	IV	$2,758	$275	$1,200		24	$1,750	

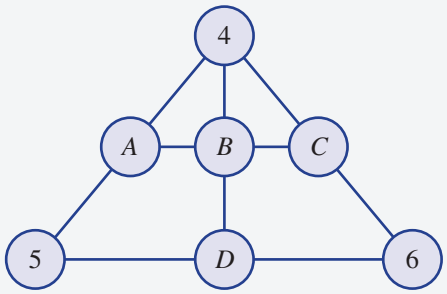
b Enter formulas into the shaded cells to calculate the total furniture cost for each office.



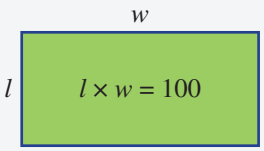
Experts use maths to help design office work spaces.

Puzzles and games

1 Each line of three numbers in the triangle adds to 12.
Find the value of A, B, C and D.



2 A rectangular paddock must have an area of 100 metres^2.
What is the minimum amount of fencing required?



3 Find the values of the pronumerals in the following tables.

a

			Sum
	a	b	c
	d	24	32
Sum	12	e	48

b

			Product
	a	b	18
	2	c	d
Product	12	e	180

4 Copy and complete the following table, in which x and y are always whole numbers.

x	2				
y	7	6		12	
$3x$		6	9		
$x + 2y$			9		7
xy				0	5

5 In a mini-Sudoku, the digits 1 to 4 occupy each square. No row, column or 2×2 block has the same digit twice. Find the value of each of the pronumerals in the following mini-Sudoku.

a	3	2	c
c	d	e	f
2	g	$d + 1$	h
i	1	j	k

6 In a magic square the sum of each row, column and diagonal is the same. Find the value of the unknown variables to make the following into magic squares. Confirm your answer by writing out the magic square as a grid of numbers.

a

5	12	A
B	C	6
D	E	F

(Magic sum is 24.)

b

A	B	C
$A - 1$	$A + 1$	$B - C$
$B - 1$	$C - 1$	$A + C$

(Magic sum is 15.)

Chapter summary

Algebra

Creating expressions
6 more than k: $k + 6$
Product of 4 and x: $4x$
10 less than b: $b - 10$
Half of q: $\frac{q}{2}$
The sum of a and b is tripled: $3(a + b)$

Mathematical convention
$3a$ means $3 \times a$
$\frac{b}{10}$ means $b \div 10$

Algebraic expressions
Combinations of numbers, pronumerals and operations
e.g. $2xy + 3yz$, $\frac{12}{x} - 3$

Pronumerals (or variables)
Letters used to represent numbers
g: number of grapes in a bunch
d: distance travelled by a hockey ball

Terms
Pronumerals and numbers combined with $\times$ or $\div$
e.g. $4x$, $10y$, $\frac{a}{3}$, 12

Equivalent expressions
Always equal when pronumerals are substituted.
e.g. $2a + 3$ and $3 + 2x$ are equivalent.
$4(3x)$ and $12x$ are equivalent.

Substitution
Replacing pronumerals with values
e.g. $5x + 2y$ when $x = 10$ and $y = 3$
becomes $5 \times 10 + 2 \times 3 = 50 + 6 = 56$

To **simplify** an expression, find a simpler expression that is equivalent.

Combining like terms
Gives a way to simplify
e.g. $4a + 2 + 3a = 7a + 2$
$3b + 5c + 2b - c = 5b + 4c$
$12xy + 3x - 5yx = 7xy + 3x$

Like terms have exactly the same pronumerals.
$5a$ and $3a$ ✓
$2ab$ and $12ba$ ✓
$7ab$ and $2a$ ×

Applications
Expressions are used widely
Area is $l \times w$
Perimeter is $2l + 2w$



Cost is $50 + 90x$
call-out fee
hourly rate

Chapter review

Multiple-choice questions

5A **1** In the expression $3x + 2y + 7z$ the coefficient of y is:
A 3 **B** 2 **C** 4 **D** 7 **E** 16

5B **2** If $b = 7$, then $b + 5$ is equal to:
A 5 **B** 7 **C** 57 **D** 12 **E** 75

5B **3** If $t = 5$ and $u = 7$, then $2t + u$ is equal to:
A 17 **B** 32 **C** 24 **D** 257 **E** 70

5C **4** If two expressions are always equal (e.g. $2k$ and $k + k$) then they are called:
A pronumerals **B** equivalent **C** coefficient
D variables **E** constant terms

5D **5** Which of the following pairs does *not* consist of two like terms?
A $3x$ and $5x$ **B** $3y$ and $12y$ **C** $3ab$ and $2ab$ **D** $3d$ and $5c$ **E** $3xy$ and xy

5A **6** How many terms are there in the expression $3a + 4b + 5c + 6d$?
A 1 **B** 2 **C** 3 **D** 4 **E** 6

5D **7** A fully simplified expression equivalent to $2a + 4 + 3b + 5a$ is:
A 4 **B** $5a + 5b + 4$ **C** $10ab + 4$ **D** $7a + 3b + 4$ **E** $11ab$

5E **8** The simplified form of $4x \times 3yz$ is:
A $43xyz$ **B** $12xy$ **C** $12xyz$ **D** $12yz$ **E** $4x3yz$

5E **9** The simplified form of $\frac{21ab}{3ac}$ is:
A $\frac{7b}{c}$ **B** $\frac{7ab}{ac}$ **C** $\frac{21b}{3c}$ **D** 7 **E** $\frac{b}{7c}$

5F **10** A number is doubled and then 5 is added. The result is tripled. The number is represented by k. An expression for this description is:
A $3(2k + 5)$ **B** $6(k + 5)$ **C** $2k + 5$ **D** $2k + 15$ **E** $30k$

Short-answer questions

5A **1** **a** List the four individual terms in the expression $5a + 3b + 7c + 12$.
b What is the constant term in the expression above?

5A **2** Write an expression for each of the following.
a 7 is added to u **b** k is tripled **c** 10 is subtracted from h

5B **3** Find the value of $x + 4$ if:
a x is 2 **b** x is 100 **c** x is 12 **d** x is 17

5B **4** If $k = 8$, evaluate:
a $k + 5$ **b** $3k$ **c** $k - 6$ **d** $2k + 1$

5B **5** If $u = 12$, find the value of:
a $u + 3$ **b** $2u$ **c** $\frac{24}{u}$ **d** $\frac{u}{3}$

5B **6** If $p = 3$ and $q = 5$, find the value of:
a pq **b** $p + q$ **c** $2(q - p)$ **d** $4p + 3q$

5C **7** **a** Copy and complete this table.

b Fill in the gap: $4x$ and $3x + x$ are _____ expressions.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$4x$				
$3x + x$				

5C **8** Classify the following pairs of expressions as equivalent (E) or not equivalent (N).

a $5x$ and $2x + 3x$ **b** $4x + 7 + 2x$ and $13x$

c $3c - c$ and $2c$ **d** $2 + 3b$ and $3 + 2b$

5D **9** Classify the following pairs as like terms (L) or not like terms (N).

a $2x$ and $5x$ **b** $7ab$ and $2a$ **c** $3p$ and p

d $4ab$ and $4aba$ **e** $8t$ and $2t$ **f** $3p$ and 3

5D **10** Simplify the following by collecting like terms.

a $2x + 3 + 5x$ **b** $12p - 3p + 2p$ **c** $12b + 4a + 2b + 3a + 4$

d $12mn + 3m + 2n + 5mn$ **e** $1 + 2c + 4h - 3o + 5c$ **f** $7u + 3v + 2uv - 3u$

5E **11** Simplify the following expressions involving products.

a $3a \times 4b$ **b** $2xy \times 3z$ **c** $12f \times g \times 3h$ **d** $8k \times 2 \times 4lm$

5E **12** Simplify the following expressions involving quotients.

a $\frac{3u}{2u}$ **b** $\frac{12y}{20y}$ **c** $\frac{2ab}{6b}$ **d** $\frac{12xy}{9yz}$

5F **13** If a tin of paint weighs 9 kg, write an expression for the weight of t tins of paint.

5F **14** If there are g girls and b boys in a class, write an expression for the total number of children.

5F **15** Analena owns x fiction books and twice as many non-fiction books. Write an expression for the total number of books that Analena owns.

Extended-response questions

1 To hire a tennis court, Cat must pay $20 per hour. The total cost for n hours is $20n$.

a How much would it cost in total to hire the court for 3 hours?

b If $n = 3$, state the value of $20n$.

c If Cat wants to hire the court at night she must pay $30 per hour to pay for the lights. How much would 3 hours at night cost?

d Write an expression for the total cost of hiring the court for n hours at night.

e On one occasion, Cat hired the court for 2 hours during the day and then another 2 hours during the night. What was the total cost?

2 A taxi driver charges $2 to pick up passengers and then $1.50 per kilometre travelled.

a State the total cost if the trip length is:

i 10 km **ii** 20 km **iii** 100 km

b Write an expression for the total cost of travelling a distance of k kilometres.

c Use your expression to find the total cost of travelling 40 km.

d Another taxi driver charges $6 to pick up passengers and then $1.20 per kilometre. Write an expression for the total cost of travelling k kilometres in this taxi.



Chapter 6

Decimals



What you will learn

- 6A Decimals and place value (Consolidating)
- 6B Rounding decimals
- 6C Addition and subtraction of decimals (Consolidating)
- 6D Multiplying and dividing by powers of 10
- 6E Multiplication of decimals
- 6F Division of decimals
- 6G Decimals and fractions
- 6H Decimals and percentages

Australian curriculum

NUMBER AND ALGEBRA

Real numbers

Money and financial mathematics

Multiply and divide fractions and decimals using efficient written strategies and digital technologies **(ACMNA154)**

Round decimals to a specified number of decimal places **(ACMNA156)**

Connect fractions, decimals and percentages and carry out simple conversions **(ACMNA157)**

Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. **(ACMNA158)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library



Decimals and rally car championships

Decimals and sport go hand in hand. From reporting the statistics to recording the time, decimals are used to handle millions of dollars in sponsorship and the livelihood of rally car drivers and their teams. Championships are decided by tenths of a second, or even less.

The world rally car championships (WRC) is a multimillion dollar sport with 775 000 000 TV viewers over the season, 50 broadcasters in 150 TV markets and 3.8 million people attending the 13 events.

On October 16, 2016, Sebastien Ogier won his fourth race of the season in Spain, winning by 15.6 seconds in his Volkswagen Polo R. The gap between first and third was less than a minute! In fact, 59.4 seconds was all that separated the three podium finishers.

Pre-test

1 Complete the following.

a $\frac{1}{10} = 0.__$ **b** $\frac{3}{10} = 0.__$

c $\frac{17}{10} = 1.__$ **d** $\frac{1}{100} = 0.0__$

e $\frac{1}{1000} = 0._____$ **f** $\frac{47}{10} = __.__$

2 Write the decimal for:

a one-half **b** one-quarter **c** three-quarters

3 Write the following cents as dollars.

a 70 cents **b** 85 cents

c 100 cents **d** 5 cents

e 105 cents **f** 3 cents

4 Find how many cents are in:

a half a dollar **b** one-quarter of \$1

c three-quarters of \$1 **d** half of \$5

5 Find the cost of:

a two labels at 45 cents each **b** 10 pears at \$1.05 each

c $1\frac{1}{2}$ boxes of mangoes at \$15 a box **d** three pens at 27 cents a pen

6 Tom paid \$20 for 200 photos to be printed. What was the cost of each print?

7 \$124 is shared between eight people. If each share is the same amount, how much does each person receive?

8 Complete:

a $\$8.50 \times 10 =$ _____

b $\$6 - \$5.90 =$ _____

c $\$10 - \$7.30 =$ _____

d $\$70 \div 100 =$ _____

e $\$6.90 + \$4.30 =$ _____

f $\$20 - \$19.76 =$ _____

9 **a** Take \$5 away from \$12.

b Take \$2.10 away from \$5.

c Add \$1.70 and \$2.25.

d Add \$12.50 to \$17.25.

e Take \$2.50 away from \$10.

10 Calculate how much change from \$100 Calvin receives when he spends:

a \$12.50 **b** \$7.40 **c** \$79.10

11 Find the total of these amounts: \$7, \$5.50, \$4.90, \$12 and \$9.15.

12 Complete these problems (without decimals).

a
$$\begin{array}{r} 329 \\ +194 \\ \hline \end{array}$$

b
$$\begin{array}{r} 1024 \\ -\ 185 \\ \hline \end{array}$$

c
$$\begin{array}{r} 104 \\ \times\ \ 3 \\ \hline \end{array}$$

d $5\overline{)6185}$

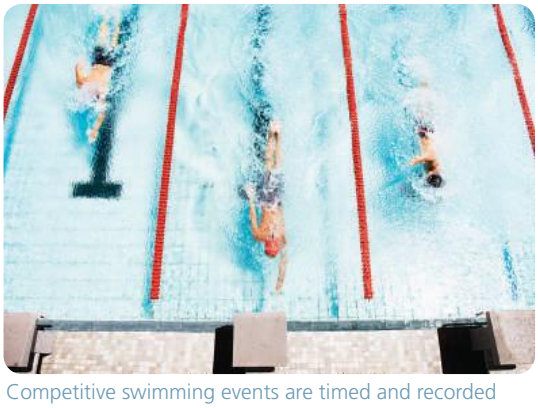
6A Decimals and place value

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthrough

Decimals are used in many aspects of everyday life. For example, Ian Thorpe swam the 400 metres freestyle in 3 minutes 40.08 seconds, which was a new world record at the time.

It's also very important to be able to use decimals, as our money works on a decimal system of dollars and cents.



Competitive swimming events are timed and recorded using decimals.

Let's start: Decimals time trial

Work with a partner. You will need six cards:

0	1	2	3	4	.

How many different numbers can you make in 1 minute?

Key ideas

- A **decimal point** is used to separate the whole number from the decimal or fraction part.
- When dealing with decimal numbers, the place value table can be extended to involve tenths, hundredths, thousandths etc.
  We read the number 428.357 as four hundred and twenty-eight point three, five, seven.

Decimal point
Symbol that separates the whole part of a number from its fractional part

Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousandths
4	2	8	•	3	5	7
4×100	2×10	8×1	•	$3 \times \frac{1}{10}$	$5 \times \frac{1}{100}$	$7 \times \frac{1}{1000}$
400	20	8	•	$\frac{3}{10}$	$\frac{5}{100}$	$\frac{7}{1000}$

whole numbers ← decimal point → fractions

Exercise 6A

UNDERSTANDING 1–4 | 4

1 Match each decimal (**a** to **e**) with the correct wording (**A** to **E**).

a	0.015	**A**	seventeen and three tenths
b	10.835	**B**	five and thirteen hundredths
c	5.13	**C**	zero point zero, one, six
d	0.016	**D**	ten point eight, three, five
e	17.3	**E**	fifteen thousandths



ones	.	tenths	hundredths	thousandths
0	.	0	1	3

$0.013 = \frac{13}{1000}$ = thirteen thousandths

2 For the number 58.237, give the value of the digit:

a 2 **b** 3 **c** 7



Choose from: $\frac{7}{1000}, \frac{2}{10}, \frac{3}{1000}$

3 A stopwatch is stopped at 36.57 seconds.

a What is the digit displayed in the tenths column?
b What is the digit displayed in the ones column?
c What is the digit displayed in the hundredths column?
d Is this number closer to 36 or 37 seconds?

Example 1 Understanding decimal place value

What is the value of the digit 8 in the following numbers?

a 12.85 **b** 6.1287

Solution	Explanation
a The value of 8 is $\frac{8}{10}$.	The 8 is in the first column after the decimal point, which is the tenths column. (Think: First column, one zero in the denominator.)
b The value of 8 is $\frac{8}{1000}$.	The 8 is in the third column after the decimal point, which is the thousandths column. (Think: Third column, three zeros in the denominator.)

4 What is the value of the digit 6 in the following numbers?

a 23.612 **b** 17.46 **c** 80.016 **d** 0.693
e 16.4 **f** 8.568 13 **g** 2.3641 **h** 11.926

FLUENCY 5–6(½) | 5–7(½)

Example 2 Changing to decimals

Express each of the following proper fractions and mixed numbers as decimals.

a $\frac{7}{10}$ **b** $\frac{5}{100}$ **c** $3\frac{17}{100}$

Solution	Explanation
a $\frac{7}{10} = 0.7$	$\frac{7}{10}$ means seven-tenths, so put the 7 in the tenths column.
b $\frac{5}{100} = 0.05$	$\frac{5}{100}$ means five-hundredths, so put the 5 in the hundredths column.
c $3\frac{17}{100} = 3.17$	$3\frac{17}{100}$ means 3 ones and 17 hundredths. 17 hundredths is one-tenth and seven-hundredths.

5 Express each of the following proper fractions as a decimal.

a $\frac{3}{10}$ **b** $\frac{8}{10}$ **c** $\frac{15}{100}$ **d** $\frac{23}{100}$

e $\frac{9}{10}$ **f** $\frac{12}{100}$ **g** $\frac{121}{1000}$ **h** $\frac{174}{1000}$

i $\frac{1}{10}$ **j** $\frac{11}{100}$ **k** $\frac{111}{1000}$ **l** $\frac{3}{100}$

6 Express each of the following mixed numbers as a decimal.

a $6\frac{4}{10}$ **b** $5\frac{7}{10}$ **c** $212\frac{3}{10}$ **d** $1\frac{16}{100}$

e $14\frac{83}{100}$ **f** $7\frac{51}{100}$ **g** $5\frac{7}{100}$ **h** $18\frac{612}{1000}$

7 True (T) or false (F)?

a $0.6 = \frac{6}{10}$ **b** $0.7 = \frac{7}{100}$ **c** $0.70 = \frac{70}{100}$ **d** $0.07 = \frac{7}{100}$

e $\frac{15}{10} = 0.15$ **f** $\frac{15}{10} = 1.5$ **g** $0.6 = 0.60$ **h** $7.0 = 7$

PROBLEM-SOLVING AND REASONING 8–10 8–12



Example 3 Arranging decimal numbers in order

Arrange the following decimal numbers in ascending order (i.e. smallest to largest):
3.72, 7.23, 2.73, 2.37, 7.32, 3.27

Solution	Explanation
2.37, 2.73, 3.27, 3.72, 7.23, 7.32	The ones column has a higher value than the tenths column, and the tenths column has a higher value than the hundredths column. 2.73 is bigger than 2.37 because it has seven-tenths, which is bigger than three-tenths.

6A

8 Choose the larger decimal in each pair.

a 6.1, 0.16 **b** 6.9, 9.6 **c** 0.8, 0.08

d 25.8, 28.5 **e** 0.107, 0.171 **f** 0.032, 0.203

9 Arrange each group of numbers in ascending order (i.e. smallest to largest).

a 3.52, 3.05, 3.25, 3.55

b 30.6, 3.06, 3.6, 30.3

c 17.81, 1.718, 1.871, 11.87

d 26.92, 29.26, 29.62, 22.96, 22.69



To compare decimals, write them underneath each other. Line up the decimal points.

30.60
3.06
3.60
30.30

10 The batting averages for five retired Australian Cricket test captains are: Adam Gilchrist 47.60, Steve Waugh 51.06, Mark Taylor 43.49, Allan Border 50.56 and Kim Hughes 37.41.

a List the five players in descending order of batting averages (i.e. largest to smallest).

b Ricky Ponting's test batting average is 56.72. Where does this rank him in terms of the retired Australian test captains listed above?



11 The depth of a river at 9 a.m. on six consecutive days was:

Day 1: 1.53 m Day 2: 1.58 m
Day 3: 1.49 m Day 4: 1.47 m
Day 5: 1.52 m Day 6: 1.61 m

a On which day was the river level highest?

b On which day was the river level lowest?

c On which day was the water level above 1.6 metres?



12 Which is larger?

a 0.7135 or $7 \times 1 + 1 \times \frac{1}{10} + 3 \times \frac{1}{100} + 5 \times \frac{1}{1000}$

b 1.563 or $1 \times \frac{1}{10} + 5 \times \frac{1}{100}$

Different decimal combinations

— 13

13 For each of the following, write as many different decimal numbers as you can. (Each digit must be used only once, and all digits must be used.) How many different numbers did you get? Circle the largest number and the smallest number.

a Use the digits 0, 1 and a decimal point.

b Use the digits 0, 1, 2 and a decimal point.

c Use the digits 0, 1, 2, 3 and a decimal point.

6B Rounding decimals

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Decimal numbers sometimes contain more decimal places than we need. It is important that we are able to round decimal numbers when working with money, measuring distance or writing answers to some division problems.

For example, the distance around the school oval might be 0.397 km, which rounded to 1 decimal place is 0.4 km or 400 m. The rounded figure, although not as precise, is accurate enough for most applications.

Let's start: Rounding brainstorm

In a group of four, brainstorm times when it may be useful to round or estimate decimal numbers. Aim to achieve more than 5 rounds in each group.



Usain Bolt's 100-metre sprint world record was measured to 2 decimal places.

Key ideas

- **Rounding** involves approximating a decimal number to fewer decimal places.
- To round a decimal, 'cut' the number after the required decimal place. For example, to round to 2 decimal places, imagine a cut after the second decimal place.
- To work out whether to round up or down, consider only the digit *immediately* to the right of the specified place. (We call this the critical digit.) Only the digit to the left of the 'cut' is changed (if necessary).
  - If the critical digit is 5 or more, round up.
  - If the critical digit is less than 5, round down.

Rounding
Approximating a number to a specified number of places

0 1 2 3 4 | 5 6 7 8 9
round down | round up

'cut' | critical digit

Rounding up:	15.63 | ⑤7	becomes	15.64
Rounding down:	21.35 | ④8	becomes	21.35

Rounding *up* makes the decimal number *larger*. E.g. $15.64 > 15.6357$
Rounding *down* makes the decimal number *smaller*. E.g. $21.35 < 21.3548$

Exercise 6B

UNDERSTANDING 1–4 4

1 How many decimal places do these decimals have?

a 0.73 **b** 0.1079 **c** 10.48 **d** 7.900 61

Example 4 Determining the critical digit

The following decimal numbers need to be rounded to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit.

a 23.5398 **b** 1.751 37

Solution	Explanation
a 23.53|(9)8	Draw a line after 2 decimal places. Circle the next digit.
b 1.75|(1)37	The critical digit is always straight after the number of decimal places we want to keep.

2 The following decimals need to be rounded to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit.

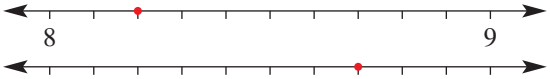
a 12.6453
b 4.819 32
c 157.281
d 4 001 565.384 71
e 0.060 31
f 203.5791
g 66.6666
h 7.995 123

3 State the critical digit in each of the following numbers.

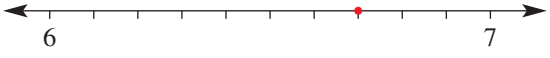
a 25.8(1)74 rounded to 1 decimal place. Critical digit = _____
b 25.81(7)4 rounded to 2 decimal places. Critical digit = _____
c 25.817(4) rounded to 3 decimal places. Critical digit = _____
d 25.(8)174 rounded to the nearest whole number. Critical digit = _____

4 For each of the following, select the closer alternative.

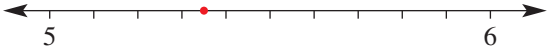
a Is 8.2 closer to 8 or 9?



b Is 6.7 closer to 6 or 7?



c Is 5.35 closer to 5 or 6?



FLUENCY 5–7(½), 8 5–7(½), 8, 9(½)

Example 5 Rounding decimals to 1 decimal place

Round each of the following to 1 decimal place.

a 25.682 **b** 13.5458

Solution	Explanation
a 25.7	25.6(8)2 The critical digit is 8, so round up.
b 13.5	13.5(4)8 The critical digit is 4, so round down.

5 Round each of the following to 1 decimal place.

a 14.82	**b** 7.38	**c** 15.62
d 0.87	**e** 6.85	**f** 9.94
g 55.55	**h** 7.98	**i** 0.68
j 0.72	**k** 0.69	**l** 0.88



6 Write each of the following correct to 2 decimal places.

a 3.7823	**b** 11.8627	**c** 5.9156	**d** 0.932 25
e 123.456	**f** 300.0549	**g** 3.1250	**h** 9.849
i 56.2893	**j** 7.121 999	**k** 29.9913	**l** 0.8971

Example 6 Rounding decimals to different decimal places

a Round 18.347 28 to 3 decimal places. **b** Round 0.439 17 to 2 decimal places.

Solution	Explanation
a 18.347	18.347|(2)8 The critical digit is 2, so round down.
b 0.44	0.43|(9)17 The critical digit is 9, so round up.

7 Round each of the following to the specified number of decimal places, given as the number in the brackets.

a 15.913 (1)	**b** 7.8923 (2)	**c** 235.62 (0)	**d** 0.5111 (0)
e 231.86 (1)	**f** 9.3951 (1)	**g** 9.3951 (2)	**h** 34.712 89 (3)

6B

8 Round each of the following to the specified number of decimal places.

a 23.983 (1) **b** 14.8992 (2)
c 6.954 32 (0) **d** 29.999 731 (3)



To round 7.59(8) to 2 decimal places, think: 59 goes up to 60. Answer = 7.60

9 Round each of the following to the nearest whole number. This is the same as zero decimal places.

a 27.612 **b** 9.458 **c** 12.299 **d** 123.72 **e** 22.26
f 117.555 **g** 2.6132 **h** 10.7532

PROBLEM-SOLVING AND REASONING 10, 11 10–13

10 Round each of the following amounts to the nearest dollar.

a $12.85 **b** $30.50 **c** $7.10 **d** $1566.80
e $120.45 **f** $9.55 **g** $1.39 **h** $36.19

11 Petrol cost 149.9 cents per litre. Yannis put 48 litres in his car. He calculated that this would cost 7195.2 cents.

a What did Yannis type on his calculator to get this answer?

b How much should he pay correct to:

i the nearest cent?
ii the nearest dollar?



12 Lee rounded the decimal 74.74 ☐ 63 to 2 decimal places. She wrote down 74.75. What could the missing digit be?

13 List all the decimal hundredths (such as 0.39) that would round to 0.4.

Rounding with technology — 14

14 Most calculators are able to round numbers correct to a specified number of places. Find out how to do this on your calculator and check your answers to questions **6** and **7**.

6C Addition and subtraction of decimals

CONSOLIDATING

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The techniques used for adding and subtracting decimals are similar to those used for whole numbers. Just as we vertically align digits in place value columns, we also align the decimal point.

Let's start: How fast can you calculate?

- Add $10 + 7 + 13 + 15 + 5$.
  Now add $1 + 0.7 + 1.3 + 1.5 + 0.5$.
- What about using money?
  Add $\$1 + \$0.70 + \$1.30 + \$1.50 + \$0.50$.
- Which calculation was the fastest?
- Now try $10 - 7.85$ and \$10 less \$7.85.
  Counting with money often seems easier!



Key ideas

- When adding or subtracting decimals, the decimal points and each of the matching decimal places must be written under one another.

$$\begin{array}{r} 1.56 \\ +2.70 \\ \hline 4.26 \end{array}$$

Writing an extra zero will help.

Line up the decimal points underneath each other.

Exercise 6C

UNDERSTANDING 1–4 | 4

1 If 7.12, 8.5 and 13.032 are to be added together, which of the following is the best way to write these numbers ready for addition?

A
$$\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$$

B
$$\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$$

C
$$\begin{array}{r} 7.120 \\ 8.500 \\ +13.032 \\ \hline \end{array}$$

D
$$\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \end{array}$$

6C

2 Which of the following is the correct way to present and solve the subtraction problem 77.81 − 6.3?

A
$$\begin{array}{r} 77.81 \\ -\ 6.3 \\ \hline 84.11 \end{array}$$

B
$$\begin{array}{r} 77.81 \\ -\ 6.30 \\ \hline 71.51 \end{array}$$

C
$$\begin{array}{r} 77.81 \\ -6.3 \\ \hline 14.81 \end{array}$$

D
$$\begin{array}{r} 77.81 \\ -\ 6.3 \\ \hline 77.18 \end{array}$$

3 Write down the answer to these sums by putting the decimal point in the correct place.

a
$$\begin{array}{r} 5.16 \\ +\ 3.41 \\ \hline 857 \end{array}$$

b
$$\begin{array}{r} 3.047 \\ 0.522 \\ +\ 1.610 \\ \hline 5179 \end{array}$$

c
$$\begin{array}{r} 15.9 \\ +\ 0.056 \\ \hline 15\,956 \end{array}$$

4 Show how you would write your working for:

a 0.571 + 1.209 + 3.528

b 1.97 − 0.43

c 12.4 − 8.35

FLUENCY 5–7(½) 5–8(½)

Example 7 Adding decimals

Find:

a 8.31 + 5.93 **b** 64.8 + 3.012 + 5.94

Solution	Explanation
a $\begin{array}{r} {}^{1}8.31 \\ +\ 5.93 \\ \hline 14.24 \end{array}$	Write: $\begin{array}{r} 8.31 \\ +\ 5.93 \\ \hline . \end{array}$ Then add.
b $\begin{array}{r} {}^{1}6{}^{1}4.800 \\ 3.012 \\ +\ 5.940 \\ \hline 73.752 \end{array}$	Put decimal points under one another. Fill missing decimal places with zeros. Add, using the procedure for whole numbers.

5 Find each of the following.

a
$$\begin{array}{r} 1.2 \\ +5.6 \\ \hline \end{array}$$

b
$$\begin{array}{r} 12.61 \\ +\ 2.35 \\ \hline \end{array}$$

c
$$\begin{array}{r} 2.83 \\ +\ 1.04 \\ \hline \end{array}$$

d
$$\begin{array}{r} 7.90 \\ +\ 1.09 \\ \hline \end{array}$$

e
$$\begin{array}{r} 13.25 \\ +14.72 \\ \hline \end{array}$$

f
$$\begin{array}{r} 7.23 \\ 16.31 \\ +\ 2.40 \\ \hline \end{array}$$

g
$$\begin{array}{r} 210.0 \\ 22.3 \\ +\ 15.1 \\ \hline \end{array}$$

h
$$\begin{array}{r} 47.81 \\ 6.98 \\ +\ 3.52 \\ \hline \end{array}$$

6 Find each of the following.

a $1.5 + 1.1$	**b** $5.6 + 0.2$	**c** $0.6 + 0.3$
d $0.9 + 0.09$	**e** $6.42 + 2.05$	**f** $9.072 + 6.435$
g $12.45 + 3.61$	**h** $5.37 + 13.81 + 2.15$	**i** $0.012 + 0.001$



Rewrite the sums as shown in Example 7, lining up the decimal points.

Example 8 Subtracting decimals

Find:

a $5.83 - 3.12$ **b** $146.35 - 79.5$

Solution	Explanation
a $\begin{array}{r} 5.83 \\ -3.12 \\ \hline 2.71 \end{array}$	Write: $\begin{array}{r} 5.83 \\ -3.12 \\ \hline . \end{array}$ Then subtract.
b $\begin{array}{r} {}^{13}\not{1}\,{}^{15}\not{4}\,{}^{1}\not{6}.35 \\ -\quad 7\ 9.50 \\ \hline 6\ 6.85 \end{array}$	Put decimal points under one another. Fill missing decimal places with zeros. Subtract, using the procedure for whole numbers.

7 Find:

a $\begin{array}{r} 0.99 \\ -0.20 \\ \hline \end{array}$	**b** $\begin{array}{r} 0.756 \\ -0.240 \\ \hline \end{array}$	**c** $\begin{array}{r} 1.2 \\ -0.8 \\ \hline \end{array}$	**d** $\begin{array}{r} 1.99 \\ -0.26 \\ \hline \end{array}$
e $\begin{array}{r} 17.2 \\ -\ 5.1 \\ \hline \end{array}$	**f** $\begin{array}{r} 128.63 \\ -\ 14.50 \\ \hline \end{array}$	**g** $\begin{array}{r} 23.94 \\ -\ 17.61 \\ \hline \end{array}$	**h** $\begin{array}{r} 158.32 \\ -87.53 \\ \hline \end{array}$

8 Find:

a $5 - 4.5$
b $5.6 - 2.4$
c $23.7 - 2.5$
d $0.98 - 0.5$
e $14.8 - 2.5$
f $234.6 - 103.2$
g $25.9 - 3.67$
h $31.657 - 18.2$
i $412.1 - 368.83$
j $5312.271 - 364.93$



Rewrite the subtractions as shown in Example 8.

6C

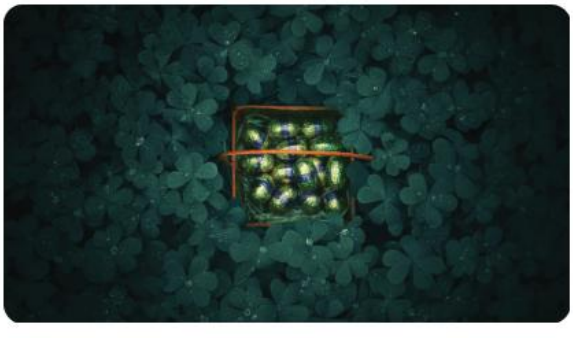
PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Find the total of 0.808, 4.376 and 11.005.



10 Stuart wants to raise $100 for the Rainbow Club charity. He already has three donations of $30.20, $10.50 and $5.00.
- **a** How much has Stuart already raised?
- **b** How much does Stuart still need to raise?

11 Daily rainfalls for four days over Easter were 12.5 mm, 3.3 mm, 0.6 mm and 33 mm. What was the total rainfall over the four-day Easter holiday?



12 Bryce's normal body temperature is 36.9°C. During a fever, Bryce's temperature rose to 40.2°C. What is the difference in temperature?



Decimals magic square

— 13

13 Complete these magic squares. First determine the magic sum.

a

0.6		0.2
0.1		
0.8		

b

1.6	0.5		0.4
0.3		0.6	1.5
	1.1		
1.3		1.2	

6D Multiplying and dividing by powers of 10

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Powers of 10 include 10^1, 10^2, 10^3, 10^4, …, which correspond to the numbers 10, 100, 1000, 10 000, … Note that the number of zeros in the number is the same as the power of 10 for that number.

$10^1 = 10$ (1 zero)
$10^2 = 100$ (2 zeros)
$10^3 = 1000$ (3 zeros)



Powers of 10 are used to describe large numbers.

Let's start: Multiplying by 'tens'

Find:
$24 \times 1 =$ ☐
$24 \times 10 =$ ☐
$24 \times 100 =$ ☐
$24 \times 1000 =$ ☐

Now try:
$2.4 \times 1 =$ ☐
$2.4 \times 10 =$ ☐
$2.4 \times 100 =$ ☐
$2.4 \times 1000 =$ ☐

Think:
$\frac{24}{10} \times 10$
$\frac{24}{10} \times 100$ etc.

Where is the decimal point in the question?

Where is it in the answer?

How does the place value of the 2 and the 4 change when you multiply 24 by a power of 10?

Key ideas

- The decimal places are the number of digits on the RHS of the decimal point. E.g. 7.8 has 1 decimal place and 12.407 has 3 decimal places.
- When multiplying by powers of 10:
  - The decimal point should appear to move to the *right* the same number of places as there are zeros in the multiplier.
    For example: to multiply by 1000, the decimal point should appear to move 3 places to the right.

$5.7839 \times 1000 = 5783.9$
(3 zeros)

- When dividing by powers of 10:
  - The decimal point should appear to move to the *left* the same number of places as there are zeros in the divisor.
    For example: to divide by 100, the decimal point should appear to move 2 places to the left.

$2975.6 \div 100 = 29.756$
(2 zeros)

Exercise 6D

UNDERSTANDING 1–4 | 4

1 How many places does the decimal point appear to move in each of these multiplications and divisions?

a $278.71 \times 10 = 2787.1$
b $15.389 \times 100 = 1538.9$
c $15.985\,13 \times 10\,000 = 159\,851.3$
d $48.9 \div 100 = 0.489$
e $10.076 \div 10 = 1.0076$

2 Fill in the correct number of zeros in the multiplier to make the following product statements correct. The first one has been done for you.

a $56.321 \times 1\,[0\,0] = 5632.1$
b $27.9234 \times 1\,[\quad] = 27\,923.4$
c $0.035\,72 \times 1\,[\quad] = 3.572$
d $3200 \times 1\,[\quad] = 320\,000\,000$

3 Fill in the correct number of zeros in the divisor to make the following division statements correct. The first one has been done for you.

a $2345.1 \div 1\,[0\,0\,0] = 2.3451$
b $7238.4 \div 1\,[\quad] = 72.384$
c $0.003\,67 \div 1\,[\quad] = 0.000\,367$
d $890 \div 1\,[\quad] = 0.0089$

4 How many places, and in what direction, will the decimal point appear to move if the following operations occur?

a $\times 100$ **b** $\div 10$ **c** $\times 1\,000\,000$ **d** $\div 1$
e $\div 1000$ **f** $\times 1000$ **g** $\times 10$ **h** $\div 10\,000\,000$

FLUENCY 5–7(½) | 5–8(½)

Example 9 Multiplying by powers of 10

Evaluate:

a 36.532×100 **b** $4.31 \times 10\,000$

Solution	Explanation
a $36.532 \times 100 = 3653.2$	100 has two zeros, so the decimal point appears to move 2 places to the right. 36.532
b $4.31 \times 10\,000 = 43\,100$	The decimal point moves 4 places to the right and additional zeros are needed. 4.3100

5 Calculate:

a 4.87×10 **b** 35.283×10 **c** 422.27×10
d 14.304×100 **e** $5.699\,23 \times 1000$ **f** $1.259\,63 \times 100$
g 12.7×1000 **h** 154.23×1000 **i** $0.34 \times 10\,000$
j 213.2×10 **k** $867.1 \times 100\,000$ **l** $0.005\,16 \times 100\,000\,000$

Example 10 Dividing by powers of 10

Evaluate:

a $268.15 \div 10$ **b** $7.82 \div 1000$

Solution	Explanation
a $268.15 \div 10 = 26.815$	10 has one zero, so the decimal point appears to move 1 place to the left. 268.15
b $7.82 \div 1000 = 0.007\,82$	The decimal point appears to move 3 places to the left and additional zeros are needed. .00782

6 Calculate:

a $42.7 \div 10$ **b** $353.1 \div 10$
c $24.422 \div 10$ **d** $5689.3 \div 100$
e $12\,135.18 \div 1000$ **f** $93\,261.1 \div 10\,000$
g $2.9 \div 100$ **h** $13.62 \div 10\,000$
i $0.54 \div 1000$ **j** $36.7 \div 100$
k $0.02 \div 10\,000$ **l** $1000.04 \div 100\,000$



Write extra zeros when needed.
$1.6 \div 100 = 0.016$

7 Calculate:

a 22.913×100 **b** $0.031\,67 \times 1000$ **c** $4.9 \div 10$
d $22.2 \div 100$ **e** $6348.9 \times 10\,000$ **f** $1.0032 \div 1000$

Example 11 Working with 'the missing' decimal point

Evaluate:

a $567 \times 10\,000$ **b** $23 \div 1000$

Solution	Explanation
a $567 \times 10\,000 = 5\,670\,000$	As no decimal point is shown in the question, it must be at the very end of the number. Four additional zeros must be put in to move the invisible decimal point 4 places to the right. 5670000.
b $23 \div 1000 = 0.023$	The decimal point appears to move 3 places to the left. 0.023

6D

8 Calculate:

a 156×100	**b** 43×1000	**c** 2251×10	**d** 16×1000
e 2134×100	**f** 2134×1000	**g** 7×1000	**h** $99 \times 100\,000$
i $34 \times 10\,000$	**j** $156 \div 10$	**k** $156 \div 100$	**l** $156 \div 1000$
m $87 \div 10$	**n** $87 \div 100$	**o** $87 \div 1000$	**p** $16 \div 1000$
q $7 \div 1000$	**r** $34 \div 10\,000$		

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 A service station charges \$1.37 per litre of petrol. How much will it cost Tanisha to fill her car with 100 litres of petrol?

10 Wendy is on a mobile phone plan that charges her 3 cents per text message. On average, Wendy sends 10 text messages per day. What will it cost Wendy for 100 days of sending text messages at this rate? Give your answer in cents and then convert it to dollars.



11 Darren wishes to purchase 10 000 shares at \$2.12 per share.

a What is the cost of the shares?

b There is an additional \$200 fee. How much will it cost Darren to buy the shares?

12 Choose 10, 100 or 1000 to complete the following.

a $5.67 \times 10 \div \square = 5.67$	**b** $18.5 \div 100 \times \square = 1.85$	**c** $900 \div \square \times 1 = 9$
d $56 \div \square \div \square = 0.56$	**e** $3.4 \times \square \div 10 = 340$	

Two-step calculations — 13

13 **a** Copy and complete:

i $2.357 \times 200 = 2.357 \times __ \times 100$

$= _____ \times 100$

$= _____$

ii $4.8 \div 20 = 4.8 \div __ \div 10$

$= _____ \div 10$

$= _____$

b Use this method to find:

i 5.05×200	**ii** 9.2×2000
iii 15.09×30	**iv** 7.28×300
v $148 \div 20$	**vi** $484 \div 200$
vii 9.681×4000	**viii** $185.98 \div 200$

6E Multiplication of decimals

Many real-life applications of mathematics involve the multiplication of decimal numbers. Examples include finding the area of a block of land that is 34.5 metres long and 5.2 metres wide or pricing a 4.5 hour job at a rate of $51.75 per hour.

You can use whole-number methods to multiply decimals. However, you must place the decimal point in the correct position in your final answer.



The price of a block of land may depend on its area, which would be calculated by multiplying decimals.

Let's start: Multiplication musings

We can use what we know about multiplying fractions to multiply decimals. For example:

$$0.4 \times 1.6 = \frac{4}{10} \times \frac{16}{10} = \frac{64}{100} = 0.64$$

Now try these.

- 0.7×0.3
- 0.07×0.03
- 0.007×0.03

What do you notice about the decimal places in the question and the decimal places in the final answer?

Key ideas

- When multiplying decimals, we use the following rule.
  The total number of decimal places in the answer is the same as the total number of decimal places in the question.

  For example: $6.25 \times 4.5 = 28.125$ (3 decimal places) $\quad 2.41 \times 6 = 14.46$ (2 decimal places)

- To multiply decimals:
  - Count the total number of decimal places in the question. For example:
    5.34×1.2 ← 3 decimal places in the question
  - Now multiply, ignoring the decimal points.

    $$\begin{array}{r} 534 \\ \times\ 12 \\ \hline 1068 \\ 5340 \\ \hline 6408 \end{array}$$

    Decimal points ignored here.
  - Put the decimal point into your answer.
    $5.34 \times 1.2 = 6.408$ ← decimal places in the answer

Exercise 6E

UNDERSTANDING 1–4 | 4

1 State the number of decimal places for each of the following.

a 5.9	**b** 1.805	**c** 5.12	**d** 0.0072
e 4.9	**f** 0.49	**g** 4.87	**h** 5.29643

2 Work out the total number of decimal places in each of the following product statements.

a 4×6.3	**b** 3.52×76	**c** 42×5.123	**d** 8.71×11.2
e 5.283×6.02	**f** 2.7×10.3	**g** 4.87×3241.21	**h** 0.003×3
i $0.001\,03 \times 0.0045$			

3 Put the decimal point into each of the following answers so that the multiplication is true.

a $6.4 \times 3 = 192$	**b** $6.4 \times 0.3 = 192$	**c** $0.64 \times 0.3 = 192$
d $15.2 \times 0.1 = 152$	**e** $97.3 \times 0.2 = 1946$	**f** $0.18 \times 0.42 = 00756$

4 Copy and complete the rule for multiplying decimal numbers (See the Key ideas in this section).
The total number of decimal places _____ must equal the number of _____ in the answer.

FLUENCY 5–7(½) | 5–8(½)

Example 12 Multiplying decimals

Calculate:

a 0.56×3 **b** 4.13×0.3

Solution	Explanation
a $\begin{array}{r} 56 \\ \times\ 3 \\ \hline 168 \end{array}$ $0.56 \times 3 = 1.68$	Multiply, ignoring decimal point. There are 2 decimal places in the question, so there will be 2 decimal places in the answer.
b $\begin{array}{r} 413 \\ \times\ 3 \\ \hline 1239 \end{array}$ $4.13 \times 0.3 = 1.239$	Ignore both decimal points. Multiply. Total of 3 decimal places in the question, so there must be 3 decimal places in the answer.

5 Calculate:

a 1.2×4	**b** 8.4×2	**c** 75×0.1
d 5.8×5	**e** 9.8×2	**f** 9.8×0.2
g 0.6×4	**h** 0.6×0.4	**i** 0.8×0.7
j 0.9×0.3	**k** 7.4×0.1	**l** 0.9×0.9



We usually write 5.60 as 5.6.

6 Find:

a 5.64×0.2	**b** 18.09×0.3
c 5.08×0.7	**d** 18.5×0.04
e 7.8×0.3	**f** 11.6×0.7



If there are 3 decimal places in the question, there are 3 decimal places in the answer.

7 Calculate:

a 14×7.2	**b** 3×72.82	**c** 1.293×12
d 3.4×6.8	**e** 5.4×2.3	**f** 0.34×16
g 43.21×7.2	**h** 0.023×0.042	**i** 0.001×0.213

8 Calculate and then round your answer to the nearest dollar.

a $5 \times \$6.30$	**b** $3 \times \$7.55$	**c** $4 \times \$18.70$
d $\$5.64 \times 0.5$	**e** $\$10.48 \times 0.2$	**f** $\$7.86 \times 1.5$

PROBLEM-SOLVING AND REASONING

9, 10 | 9–12

9 Calculate:

a 2.52×40	**b** 6.9×70	
c 31.75×800	**d** 1.4×7000	
e 3000×4.8	**f** $7.291 \times 50\,000$	
g 0.0034×200	**h** $0.0053 \times 70\,000$	



$9.8 \times 20\,000$
$= 9.8 \times 2 \times 10\,000$
$= 19.6000$
$= 196\,000$

10 Anita requires 4.21 m of material for each dress she is making. She is planning to make a total of seven dresses. How much material does she need?

11 The net weight of a can of spaghetti is 0.445 kg. Find the net weight of eight cans of spaghetti.

12 **a** If $68 \times 57 = 3876$, what is the answer to 6.8×5.7? Why?
b If $23 \times 32 = 736$, what is the answer to 2.3×32? Why?
c If $250 \times 300 = 75\,000$, what is the answer to 2.5×0.3? Why?
d What is 7×6? What is the answer to 0.7×0.6? Why?
e What about 0.07×0.6?



Remember to count the decimal places. 6.8×5.7 has two places.

Using your calculator

— | 13

13 Yusef buys the following items at the supermarket. He gives the checkout person \$80.

4 chocolate bars @\$1.85 each
3 loaves of bread @\$3.19 each
newspaper @\$1.40
2×2 litres of milk @\$3.70 each
washing powder @\$8.95
toothpaste @\$4.95
2 kg sausages @\$5.99 per kg
tomato sauce @\$3.20
2 packets of chocolate biscuits @\$3.55 each
5×1.25 litres of soft drink @\$0.99 each

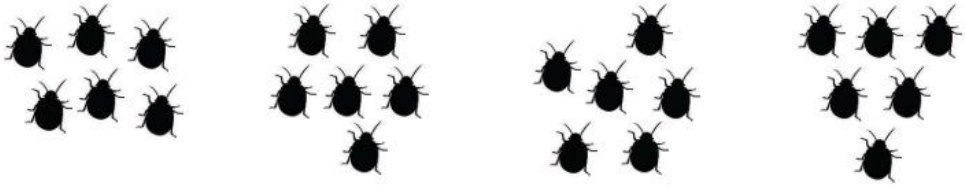


a How much change does Yusef receive?
b How could he be given this change if he receives at least one note?

6F Division of decimals

For multiplication, we treat decimals like whole numbers until the very end of the question. For division, however, we try to change the question so that we divide by a whole number instead of a decimal.

First, let's review the terminology for division. Suppose we divide 24 by 4 to make groups of 6.



We can write this in several different ways.

$24 \div 4 = 6$ or $\frac{24}{4} = 6$ or $4\overline{)24}$ $\overset{6}{}$

- 24 is the dividend (the amount you have, or the number being divided).
- 4 is the divisor (the number doing the dividing).
- 6 is the quotient (the answer).

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

Let's start: Division decisions

Use a calculator to find:

- $100 \div 2$ and $10 \div 0.2$
- $60 \div 3$ and $6 \div 0.3$
- $1.56 \div 0.02$ and $156 \div 2$

What do you notice about each pair? Can you think of an easy way to calculate $21.464 \div 0.02$?

Key ideas

- Division of decimal numbers by whole numbers:
  - Work out as you would any other division question.
  - The decimal point in the quotient (answer) goes directly above the decimal point in the dividend.

  For example: $60.524 \div 4$

$$4\overline{)6^{2}0.5^{1}2\,4}\ \overset{1\,5.1\,3\,1}{}$$

- Division of decimal numbers by other decimals:
  - Change the divisor (number after the ÷ sign) into a whole number.
  - Multiply the dividend and the divisor by the same power of 10 (10, 100, 1000, …).
  - Then carry out the division by a whole number.

  For example: $24.562 \div 0.02$

$$24.562 \div 0.02 = 2456.2 \div 2$$
$$= 1228.1$$

Exercise 6F

UNDERSTANDING 1–4 | 4

1 For $36.52 \div 0.4 = 91.3$, which of the following uses the correct words?
- **A** 36.52 is the divisor, 0.4 is the dividend and 91.3 is the quotient.
- **B** 36.52 is the dividend, 0.4 is the divisor and 91.3 is the quotient.
- **C** 36.52 is the quotient, 0.4 is the dividend and 91.3 is the divisor.
- **D** 36.52 is the divisor, 0.4 is the quotient and 91.3 is the dividend.

2 Copy and complete:
- **a** $0.3 \times \square = 3$
- **b** $0.03 \times \square = 3$
- **c** $0.003 \times \square = 3$
- **d** $1.2 \times \square = 12$
- **e** $0.07 \times \square = 7$
- **f** $0.05 \times \square = 5$

3 Rewrite each question so that you are dividing by a whole number. The first one has been done for you.
- **a** $1.2 \div 0.3 = 12 \div 3$
- **b** $1.8 \div 0.2$
- **c** $15.2 \div 0.1$
- **d** $56.42 \div 0.02$
- **e** $3.8 \div 0.1$
- **f** $3.8 \div 0.01$
- **g** $4.846 \div 0.02$
- **h** $0.07 \div 0.2$
- **i** $7.2 \div 0.9$

4 For each of the following pairs of numbers, move the decimal points the same number of places so that the second number becomes a whole number.
- **a** 3.2456 and 0.3
- **b** 120.432 and 0.12
- **c** 0.003 45 and 0.0001
- **d** 1234.12 and 0.004

FLUENCY 5, 6, 7(½) | 5–8(½)

Example 13 Dividing decimals by whole numbers

Calculate:

a $42.837 \div 3$ **b** $0.0234 \div 4$

Solution	Explanation
a 14.279 $3\overline{)4^{1}2.8^{2}3^{2}7}$ with quotient $1\,4.2\;7\;9$	Simply divide by the whole number. Remember that the decimal point in the answer goes directly above the decimal point in the dividend.
b 0.005 85 $3\overline{)0.02^{2}3^{3}4^{2}0}$ with quotient $0.00\;5\;8\;5$	Remember to write a zero in the answer every time the divisor 'doesn't go'. Make sure the decimal points line up. Write a zero at the end of the dividend so that you can complete the division.

6F

5 Calculate:

a $8.4 \div 2$	**b** $30.5 \div 5$	**c** $64.02 \div 3$
d $2.822 \div 4$	**e** $4.713 \div 3$	**f** $2.156 \div 7$
g $38.786 \div 2$	**h** $1491.6 \div 4$	**i** $0.0144 \div 6$
j $234.21 \div 2$	**k** $3.417 \div 5$	**l** $0.010\,25 \div 4$
m $1.56 \div 4$	**n** $1.85 \div 5$	**o** $1.225 \div 7$
p $53.7 \div 6$	**q** $28.08 \div 3$	**r** $210.2 \div 2$

Example 14 Dividing decimals by decimals

Calculate:

a $62.316 \div 0.03$ **b** $0.031\,52 \div 0.002$

Solution	Explanation
a $62.316 \div 0.03$ $= 6231.6 \div 3$ $= 2077.2$ $3\overline{)62^{2}3^{2}1.6}$ $\quad 20\,7\,7.2$	We want to divide by a whole number, so we need to make the 0.03 into 3. Move each decimal point 2 places to the right. $62.316 \div 0.03$ Carry out the division for $6231.6 \div 3$
b $0.031\,52 \div 0.002$ $= 31.52 \div 2$ $= 15.76$ $2\overline{)3^{1}1.^{1}5^{1}2}$ $\quad 1\,5.\,7\,6$	To change 0.002 into 2, move each decimal point 3 places to the right. $0.03152 \div 0.002$ Carry out the division for $31.52 \div 2$. Remember to line up the decimal points.

6 Calculate:

a $6.14 \div 0.2$	**b** $23.25 \div 0.3$	**c** $2.144 \div 0.08$
d $5.1 \div 0.6$	**e** $0.3996 \div 0.009$	**f** $45.171 \div 0.07$
g $0.0032 \div 0.04$	**h** $0.040\,34 \div 0.8$	**i** $10.78 \div 0.011$
j $4.003 \div 0.005$	**k** $0.948 \div 1.2$	**l** $432.2 \div 0.0002$

7 Find:

a $1500 \div 200$	**b** $1500 \div 20$	
c $1500 \div 2$	**d** $1500 \div 0.2$	
e $1500 \div 0.02$	**f** $1500 \div 0.002$	

$1500 \div 200$
$= 15 \div 2$



8 Calculate the following, rounding your answers to 2 decimal places.

a 35.5 kg ÷ 3	**b** \$213.25 ÷ 7
c 182.6 m ÷ 0.6	**d** 287 g ÷ 1.2
e 482.523 L ÷ 0.5	**f** \$5235.50 ÷ 9

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Calculate:

a $236.14 \div 200$
b $413.35 \div 50$
c $3.712\,44 \div 300$
d $0.846 \div 200$
e $482.435 \div 5000$
f $0.0313 \div 40$



$2.6 \div 20$
$= 2.6 \div 2 \div 10$
$= 1.3 \div 10$
$= 0.13$
Splitting 20 into 2 and 10 makes the question easier.

10 **a** A water bottle can hold 600 mL of water. How many water bottles can be filled from a large drink container that can hold 16 L?
b How many 400 mL bottles are needed for 16 L?



11 Rose paid \$12.72 to fill her ride-on lawnmower with 8 L of fuel.
What was the price per litre of the fuel she purchased?



12 How many 20 cent pieces are needed to make:

a \$0.80?
b \$1.20?
c \$5.40?
d \$7.80?
e \$15.20?
f \$1000?

Calculations without calculators — 13

13 Given that $24.53 \times 1.97 = 48.3241$, write down the value of each of the following. Use a calculator to check your answers.

a $48.3241 \div 1.97$
b $48.3241 \div 2.453$
c $4832.41 \div 1.97$
d $483.241 \div 245.3$
e $0.483\,241 \div 0.197$
f $483\,241 \div 2453$



If $35 \div 5 = 7$ then
$5 \times 7 = 35$ and
$0.5 \times 7 = 3.5$

Progress quiz

6A **1** What is the value of the digit 3 in the following numbers?

a 26.37 **b** 104.6732

6A **2** True (T) or false (F)?

a $0.8 = \frac{8}{100}$

b $\frac{26}{10} = 2.6$

c $0.75 = \frac{75}{100}$

d $\frac{201}{1000} = 0.002\,01$

6A **3** Choose the larger decimal in each pair.

a 5.35, 1.89 **b** 7.9, 9.7

c 0.8, 0.098 **d** 0.237, 0.271

6B **4** Round each of the following to 2 decimal places.

a 13.478 **b** 8.043

c 72.680 99 **d** 45.3951

6B **5** Round each of the following amounts to the nearest dollar.

a \$8.95 **b** \$15.55

c \$104.15 **d** \$1099.50

6C **6** Find each of the following.

a $2.45 + 6.32$ **b** $15.23 + 41.62$

c $181.4 + 35.53 + 12.88$ **d** $5.123 + 25.8 + 32.08$

6C **7** Find each of the following.

a $8.63 - 5.21$ **b** $34.91 - 12.7$

c $41.354 - 19.61$ **d** $408.715 - 375.481\text{p}$

6D **8** Evaluate:

a 73.516×100 **b** $4.92 \times 10\,000$

c $567.89 \div 10$ **d** $5.609 \div 1000$

e 278×100 **f** $278 \div 100\,000$

6E **9** Calculate:

a 0.81×4 **b** 5.23×0.6

c 5.9×30 **d** 4000×7.1

6F **10** Calculate:

a $52.842 \div 3$ **b** $0.0064 \div 4$

c $85.245 \div 0.05$ **d** $0.003\,782 \div 0.02$

6F **11** A water bottle can hold 400 mL of water. How many water bottles can be filled from a large drink container that can hold 40 L?

6G Decimals and fractions

Interactive

Widgets

HOTsheets

Walkthrough

Sometimes we use decimals to show numbers that are not whole numbers, and sometimes we use fractions. It is important to be able to change a fraction to a decimal (for example, $\frac{1}{4} = 0.25$), and change a decimal to a fraction.



Would you use fractions or decimals to measure long jump?

Let's start: How many do you already know?

- You probably know that the decimal 0.5 and the fraction $\frac{1}{2}$ are equivalent. List 10 other decimal–fraction pairs you know.
- Could you use the decimal–fraction equivalences you know to work out some other pairs?

Key ideas

- Converting decimals to fractions
  - Using your knowledge of place value, express the decimal places as a fraction with a denominator that is a power of 10. Remember to simplify the fraction whenever possible.

e.g. $0.25 = \frac{25}{100} = \frac{1}{4}$ (denominator)

- Converting fractions to decimals
  - When the denominator is a power of 10, we can simply change the fraction to a decimal through knowledge of place value.

e.g. $\frac{37}{100} = 0.37$

  - When the denominator is *not* a power of 10, try to find an equivalent fraction with a denominator that has a power of 10 and then convert to a decimal.

e.g. $\frac{2}{5} = \frac{4}{10} = 0.4$

  - A method that will always work for converting fractions to decimals is to divide the numerator by the denominator.

e.g. $\frac{5}{8} = 8\overline{)5.^{5}0^{2}0^{4}0} = 0.625$ (0. 6 2 5)

- Recurring decimals are decimals with a repeated pattern.
  - A dot, a bar or dots above a number indicates a repeated pattern.

For example: $\frac{1}{3} = 0.333\,33\ldots = 0.\dot{3}$ $\quad \frac{13}{11} = 1.181\,818\ldots = 1.\dot{1}\dot{8}$ or $1.\overline{18}$

Exercise 6G

UNDERSTANDING 1–3 3

1 Complete each of these statements, which convert common fractions to decimals.

a $\frac{1}{2} = \frac{\square}{10} = 0.5$

b $\frac{1}{4} = \frac{25}{\square} = 0.25$

c $\frac{3}{4} = \frac{\square}{100} = 0.\square 5$

d $\frac{2}{\square} = \frac{4}{10} = 0.\square$

2 Complete each of these statements, which convert decimals to fractions, in simplest form.

a $0.2 = \frac{\square}{10} = \frac{1}{5}$

b $0.15 = \frac{\square}{100} = \frac{3}{\square}$

c $0.8 = \frac{8}{\square} = \frac{\square}{5}$

d $0.64 = \frac{64}{100} = \frac{\square}{25}$

3 Circle the larger number in each pair.

a $\frac{1}{10}, 0.3$

b $\frac{1}{2}, 0.4$

c $0.8, \frac{8}{100}$

d $1.5, \frac{15}{100}$

e $\frac{3}{4}, 0.9$

f $0.05, \frac{1}{2}$



Convert one of the numbers so that you can compare two fractions or two decimals.

FLUENCY 4–7(½) 4–7(½), 8

Example 15 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.239

b 10.35

Solution	Explanation
a $\frac{239}{1000}$	0.239 has 3 decimal places. So use 3 zeros (1000) in the denominator. 0.239 = 239 thousandths
b $10\frac{35}{100} = 10\frac{7}{20}$	0.35 has 2 decimal places. Use 2 zeros (100) in the denominator. 0.35 = 35 hundredths Simplify by dividing the numerator and denominator by the highest common factor of 5.

4 Convert the following decimals to fractions in their simplest form.

a 0.4	**b** 0.6	**c** 0.8	**d** 0.22	**e** 1.22
f 5.5	**g** 0.15	**h** 0.99	**i** 0.08	**j** 0.01
k 0.001	**l** 0.202	**m** 0.5	**n** 6.4	**o** 10.15
p 18.12	**q** 3.25	**r** 0.05	**s** 9.075	**t** 5.192

Example 16 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $5\frac{3}{5}$ **c** $\frac{7}{12}$

Solution	Explanation
a $\frac{17}{100} = 0.17$	The denominator is a power of 10. So write 17 hundredths as a decimal.
b $5\frac{3}{5} = 5\frac{6}{10} = 5.6$	The denominator (5) is not a power of 10. So change $\frac{3}{5}$ to $\frac{6}{10}$ and write as a decimal.
c $\frac{7}{12} = 0.583\,33\ldots$ or $0.58\dot{3}$	12 is not a factor of 10, 100 or 1000. So divide the numerator (7) by the denominator (12). $12\overline{)7.0^{7}0^{10}0^{4}0^{4}0^{4}0}$ gives $0.5833\ldots$

5 Convert each of these fractions to decimals.

a $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{31}{100}$

d $\frac{79}{100}$ **e** $\frac{121}{100}$ **f** $3\frac{29}{100}$

g $\frac{123}{1000}$ **h** $\frac{3}{100}$ **i** $\frac{7}{100}$

6 Convert the following fractions to decimals by first changing the fraction to an equivalent fraction.

a $\frac{4}{5} = \frac{\square}{10}$ **b** $\frac{1}{2} = \frac{\square}{10}$ **c** $\frac{7}{20} = \frac{\square}{100}$

d $\frac{23}{50} = \frac{\square}{100}$ **e** $5\frac{19}{20} = 5\frac{\square}{100}$ **f** $3\frac{1}{4} = 3\frac{\square}{100}$

g $\frac{5}{2} = \frac{\square}{10}$ **h** $\frac{3}{8} = \frac{\square}{1000}$ **i** $\frac{7}{25} = \frac{\square}{100}$

7 Convert the following fractions to decimals by dividing the numerator by the denominator.

a $\frac{1}{2}$ **b** $\frac{3}{6}$ **c** $\frac{3}{4}$

d $\frac{2}{5}$ **e** $\frac{1}{3}$ **f** $\frac{3}{8}$

g $\frac{5}{12}$ **h** $\frac{3}{7}$ **i** $\frac{2}{9}$



Use a dot to write recurring decimals: $0.333\ldots = 0.\dot{3}$

6G

8 Copy and complete the following tables. Part **c** has already been done for you.
Try to memorise these fractions and their equivalent decimal values.

a halves

Fraction	$\frac{0}{2}$	$\frac{1}{2}$	$\frac{2}{2}$
Decimal			

b thirds

Fraction	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
Decimal				

c quarters

Fraction	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
Decimal	0	0.25	0.5	0.75	1

d fifths

Fraction	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
Decimal						

PROBLEM-SOLVING AND REASONING 9, 10 9–13

9 Arrange $\frac{1}{2}, 0.75, \frac{5}{8}, 0.4, 0.99, \frac{1}{4}$ from smallest to largest.

10 Copy and complete the following table.

Decimal amount	\$0.01	\$0.05	10c	20c		50c		90c	\$0.99
Fraction of \$1	$\frac{1}{100}$				$\frac{1}{4}$		$\frac{3}{4}$		

11 **a** Copy and complete the following fraction ↔ decimal table.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
Decimal									

b What happens to the decimals as the denominator increases?
c Try to explain why this makes sense.



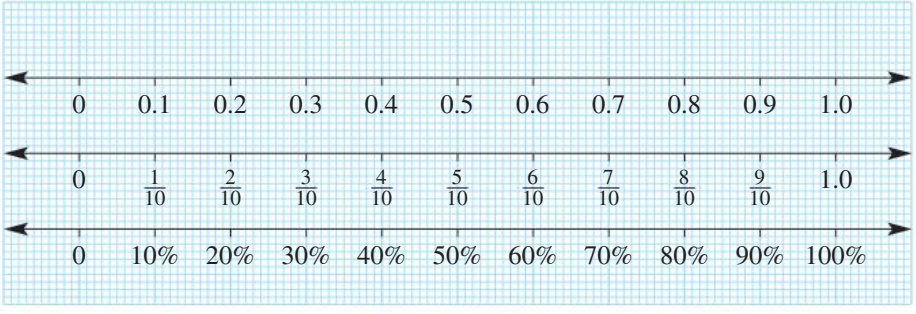
You may need a calculator for questions 11 and 12.

12 **a** Copy and complete the following decimal ↔ fraction table.

Decimal	0.1	0.2	0.25	0.4	0.5	0.6	0.75	0.8	0.9
Fraction									

b How many of the fractions can be expressed in simplest form using a denominator of 5?
c How many of the fractions can be expressed in simplest form using a denominator of 4?

13 Use these number lines to help you answer the following questions.



a List in ascending order: $\frac{7}{10}, 62\%, 0.\dot{6}, \frac{66}{100}, \frac{2}{3}, 65\%, 0.67, \frac{6}{10}$

b List in descending order: $36\%, 0.43, \frac{2}{5}, 45\%, 0.4, \frac{3}{5}, 0.52, \frac{6}{20}, 0.\dot{3}, \frac{9}{25}, \frac{1}{3}$



Put an = sign between any equal numbers in your lists.

Can all fractions be written as decimals?

— 14

14 We know that $\frac{1}{3} = 0.333...$ and that $\frac{11}{7} = 1.\overline{571428}$. These are both infinite recurring decimals, where a pattern continues forever.

a Can you find a fraction that can be written as a decimal which continues indefinitely with no pattern?

b Now write the number $\sqrt{2}$ as a decimal. What do you notice?

c Do you think $\sqrt{2}$ can be written as a fraction?

d Research and write a brief summary of the discovery and existence of irrational numbers.



Pythagoras was famous for discovering and working with irrational numbers (numbers that cannot be expressed as a fraction)

6H Decimals and percentages

Interactive
Widgets
HOTsheets
Walkthrough

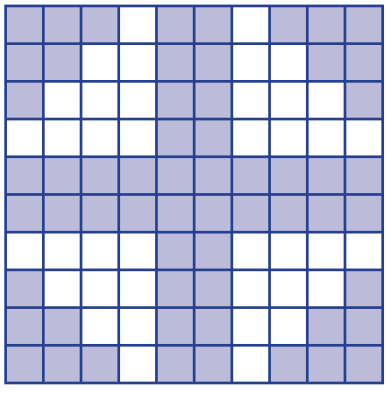
Percentages are used every day. We saw in chapter 4 that per cent means 'out of 100'. This makes it very easy to convert percentages into decimals, and vice versa. We need to do this when working with everyday percentages such as discounts and interest rates.



We see percentages used around us in many different situations.

Let's start: Creative shading

- Draw a square of side length 10 cm and shade exactly 20% or 0.2 of this figure.
- Draw a square of side length 5 cm and shade exactly 60% or 0.6 of this figure.
- Draw another square of side length 10 cm and creatively shade an exact percentage of the figure. Ask your partner to work out the percentage you shaded. You may find it helpful to use grid paper.



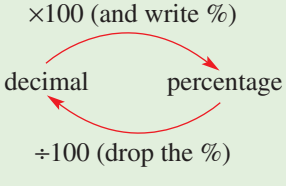
What percentage is shaded?

Key ideas

- The % symbol means per cent. It comes from the Latin words *per centum*, which means 'out of 100'.
  For example: 23% means 23 out of 100

$$23\% = \frac{23}{100}$$
$$= 0.23$$

- To convert a percentage to a decimal, drop the % sign and divide by 100. That is, move the decimal point 2 places to the left.
  For example: $42\% = 42 \div 100 = 0.42$
- To convert a decimal to a percentage, multiply by 100 and write a % sign. That is, move the decimal point 2 places to the right.
  For example: $0.654 = 65.4\%$

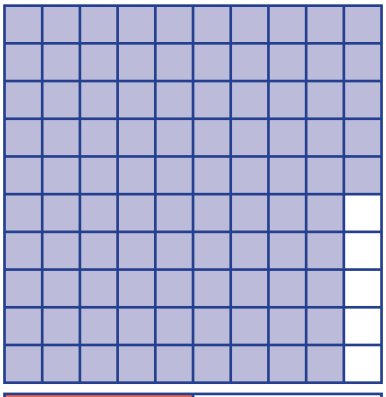


Exercise 6H

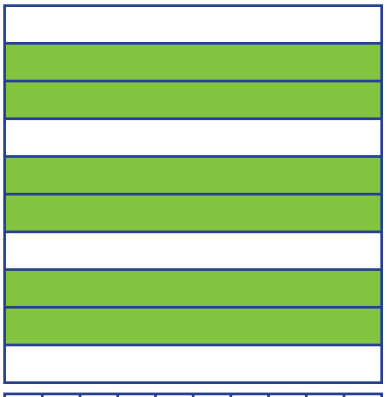
UNDERSTANDING 1–5 5

1 What percentage of each square has been shaded?

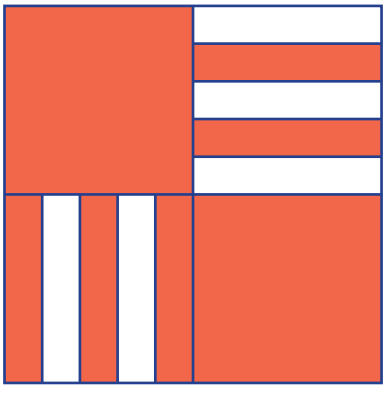
a



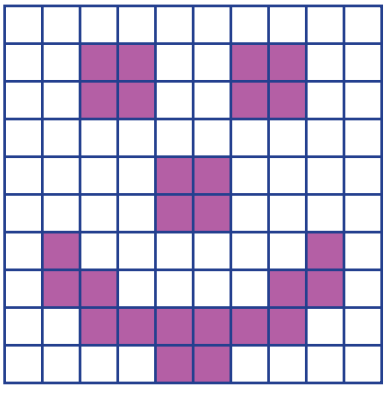
b



c



d



2 72.5% is equivalent to which of the following decimals?

A 72.5 **B** 7.25 **C** 0.725 **D** 725.0

3 0.39 is equivalent to which of the following percentages?

A 39% **B** 3.9% **C** 0.39% **D** 0.0039%

4 Prue answered half the questions correctly for a test marked out of 100.

a What score did Prue get on the test?

b What percentage did Prue get on the test?

c Assuming Prue always gets half the answers correct, find her score if the test was out of:

i 10 **ii** 200 **iii** 40 **iv** 2

d Find Prue's percentage for each of the tests in part **c**.

5 Fill in the empty boxes.

a 58% = 58 out of ☐

b 35% = ☐ out of 100

c 126% = 126☐☐100

d ☐% = 15 out of 100

6H

FLUENCY 6–7(½) 6–8(½)

Example 17 Converting percentages to decimals

Express the following percentages as decimals.

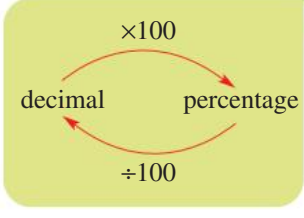
a 30% **b** 240% **c** 12.5% **d** 0.4%

Solution	Explanation
a 30% = 0.3	30 ÷ 100 = 0.3
b 240% = 2.4	240 ÷ 100 = 240. = 2.4
c 12.5% = 0.125	12.5 ÷ 100 Decimal point moves 2 places to the left.
d 0.4% = 0.004	0.4 ÷ 100 Decimal point moves 2 places to the left.

6 Express the following percentages as decimals.

a 32%	**b** 27%	**c** 68%	**d** 54%
e 10%	**f** 12%	**g** 18%	**h** 85%
i 92%	**j** 75%	**k** 11%	**l** 60%
m 6%	**n** 9%	**o** 100%	**p** 1%
q 218%	**r** 142%	**s** 75%	**t** 199%





7 Express the following percentages as decimals.

a 22.5%	**b** 17.5%	**c** 33.33%	**d** 8.25%
e 112.35%	**f** 188.8%	**g** 150%	**h** 520%
i 0.79%	**j** 0.025%	**k** 1.04%	**l** 0.95%

Example 18 Converting decimals to percentages

Express the following decimals as percentages.

a 0.045 **b** 7.2

Solution	Explanation
a 0.045 = 4.5%	0.045 × 100 = 4.5 Decimal point appears to move 2 places to the right.
b 7.2 = 720%	7.2 × 100 = 720.

8 Express the following decimals as percentages.

a 0.8	**b** 0.3	**c** 0.45	**d** 0.71
e 0.416	**f** 0.375	**g** 2.5	**h** 2.314
i 0.025	**j** 0.0014	**k** 12.7	**l** 1.004

PROBLEM-SOLVING AND REASONING

9, 10 | 9–12

9 At a hockey match, 65% of the crowd supports the home team. What percentage of the crowd supports the visiting team?



10 Sarah thinks that the chance of beating her brother, Tim, at tennis is 52%. What is this chance as a decimal?

11 Write the following as:

i percentages
ii decimals

a 50 cents in the dollar
b 25 cents in the dollar
c 90 cents in the dollar
d 10 cents in the dollar
e 100 cents in the dollar

12 **a** Explain why Yuang could not expect to get more than 100% for his Maths test.
b What does it mean to get 50% on a test out of 40?
c If a student received a mark of 0% for a test, how many questions were answered correctly?

AFL ladder

— | 13

13 The Australian Rules football ladder has the following column headings.

Pos	Team	P	W	L	D	F	A	%	Pts
6	Brisbane Lions	22	13	8	1	2017	1890	106.72	54
7	Carlton	22	13	9	0	2270	2055	110.46	52
8	Essendon	22	10	11	1	2080	2127	97.79	42
9	Hawthorn	22	9	13	0	1962	2120	92.55	36
10	Port Adelaide	22	9	13	0	1990	2244	88.68	36

a Using a calculator, can you determine how the percentage column is calculated?
b What do you think the 'F' and the 'A' column stand for?
c In their next match, Essendon scores 123 points for their team and has 76 points scored against them. What will be their new percentage?
d Find out the current For and Against points for your favourite AFL team.

Maths@Work: Retail assistant

Retail assistants use many skills. They need to be confident, polite, pleasant people who like interacting with their customers. Also, they need to have good mathematical skills to quickly and accurately calculate the bill and give correct change.



1 Find the total of each of the following receipts.

a

600 mL lemonade	$2.50
half chicken	$15.00
burger	$9.00
chips	$4.00
ice-cream 2 @	$4.00 each

b

scarf	$24.95
4 cards @ $1.00 each	$4.00
RFID card sleeve	$16.95
pencil case leather	$39.95
orbit key ring	$52.95

c

long black large	$4.50
baby chino	$2.50
kids milkshake	$5.50
small cappuccino	$3.50
banana bread	$4.95

2 In 1990 the Australian government removed all 1 and 2 cent coins. The total of all cash purchases was now rounded to the nearest 5 cents. For example, 71c or 72c rounded down to 70 cents, while 73c and 74c rounded up to 75 cents.
State the rounded price for each of the following cash amounts:
a $5.68 **b** $123.91 **c** $45.66 **d** $2.99



Remember that all cash amounts are rounded to the nearest 5 cents.

3 Customers often give a mixture of notes and coins to minimise the number of coins they receive in change. Complete the following table to find the change due to each customer.

Customer	Sub total	+ or − rounding amount in cents	Total due	Cash tendered	Change due
Adam	$45.67			$50.80	
Blake	$123.03			$130.00	
Christy	$123.03			$125.10	
Dion	$56.92			$57.00	
Eden	$67.56			$70.60	

4 Write the missing values in each of the following receipts. The question parts are shown in brackets beside each missing amount.

a

cheese snack	$1.80
cucumber 0.65 kg @ $8.00/kg	_____ **(i)**
capsicums red 0.45 kg @ $7.00/kg	_____ **(ii)**
Total of the 3 items	_____ **(iii)**
CASH TENDERED	$10.50
CHANGE DUE	_____ **(iv)**

b

honey beef 0.732 kg @ $19.99/kg	_____ **(i)**
lamb mince 1.317 kg @ $17.99/kg	_____ **(ii)**
kebabs 12 pack @ $1.45 EACH kebab	_____ **(iii)**
Total	_____ **(iv)**
Total due to rounding	_____ **(v)**
CASH TENDERED	$55.80
CHANGE DUE	_____ **(vi)**

5 How much is saved by rounding when paying cash for the following?

a sweet chilli red pepper thin sausages 0.236 kg @ $15.99/kg

b thin tasty beef sausages 0.354 kg @ $15.99/kg

6 Complete the missing parts **i–iv** shown on the following receipt.

Description	$
taco shells	2.20
potato chips 53 g	0.95
corn chips 150 g	1.50
frozen fish 375 g	5.60
chicken 400 g	5.00
frozen yoghurt 264 mL	6.79
chocolate 55 g 2 @ $1.00	2.00
cherry tomatoes 200 g	3.00
donuts 12 pk	3.00
cucumber 1 each	1.00
spring onion 1 bunch	2.50
broccoli 0.217 kg @ $5.90/kg	**(i)**
Total for the ____ **(ii)** items	**(iii)**
Change from a $50 note	**(iv)**

Using technology

Retail assistants help with stocktaking. This involves counting the items (called stock) in a store and calculating their total value for the company's financial records. It is usually done once a year in late June as the Australian Tax Office requires profits to be reported at the end of every financial year (June 30).



7 Barney and Noah have completed a stocktake of their hardware store.

a Set up the following spreadsheet for valuing some stock. Select all the value cells (in columns D and E) right click, select Format/Currency/0 d. p.

b Enter formulas in the shaded cells to calculate the total value of this stock.



In cell E10 add the cell values above =SUM(E3:E8)

	A	B	C	D	E
1	Hardware store stocktake				
2	**Assistant**	**Stock**	**Number**	**Value of 1 item**	**Total value**
3	Barney	Bathroom basin tap	25	$82	
4	Barney	Shower mixer tap	34	$119	
5	Barney	Bathroom mirror	8	$77	
6	Noah	Cordless drill	5	$139	
7	Noah	Mitre power saw	6	$380	
8	Noah	Electric sander	12	$48	
9					
10			**Total stock value**		

1 Find the missing numbers in the following sums.

a
$$\begin{array}{r} 3.\square \\ +\,4.6 \\ \hline \square.3 \end{array}$$

b
$$\begin{array}{r} 8.\square 9 \\ +\,\square.75 \\ \hline \square 4.4\square \end{array}$$

c
$$\begin{array}{r} 1.\square 1 \\ +\,\square\square.11 \\ \hline 111.\square \end{array}$$

d
$$\begin{array}{r} \square.3\square 6 \\ 2.\square 43 \\ +\,1.89\square \\ \hline \square 1.395 \end{array}$$

2 Find the missing digits in these questions involving division.

a $3\overline{)2.67}$ $= 0.\square\square$

b $3\overline{)1.\square 2}$ $= 0.64$

c $\square\overline{)10.7\square}$ $= 2.\square 5$

d $\square\overline{)15.\square 29}$ $= 2.14\square$

3 *What room can nobody enter?*
To find out, solve each problem. Then use your answers to unlock the code below.

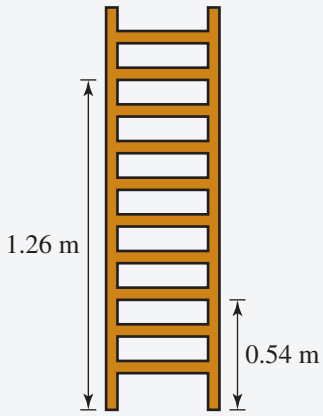
A Find the total of \$7.06, \$24.95, \$1.05 and \$3.50.
O Calculate \$69.97 − \$15.98.
M Find \$100 − 4 × \$16.05.
S Tracy bought five books at \$24.95 each. How much change did she receive from \$150?
H Nensi spent \$216 on nine CDs. How much did each CD cost?
U Find the cost of seven cans of cola if a dozen cans cost \$16.20.
M Find the cost of six slices of cake at \$1.25 per slice.
O Find the total cost of four books at \$4.95 each, three pens at 95 cents each and one eraser at \$1.05.
R Julie spent \$23.15 on items for school and another \$3.95 on chocolate. How much change did she get from \$30?

\$36.56

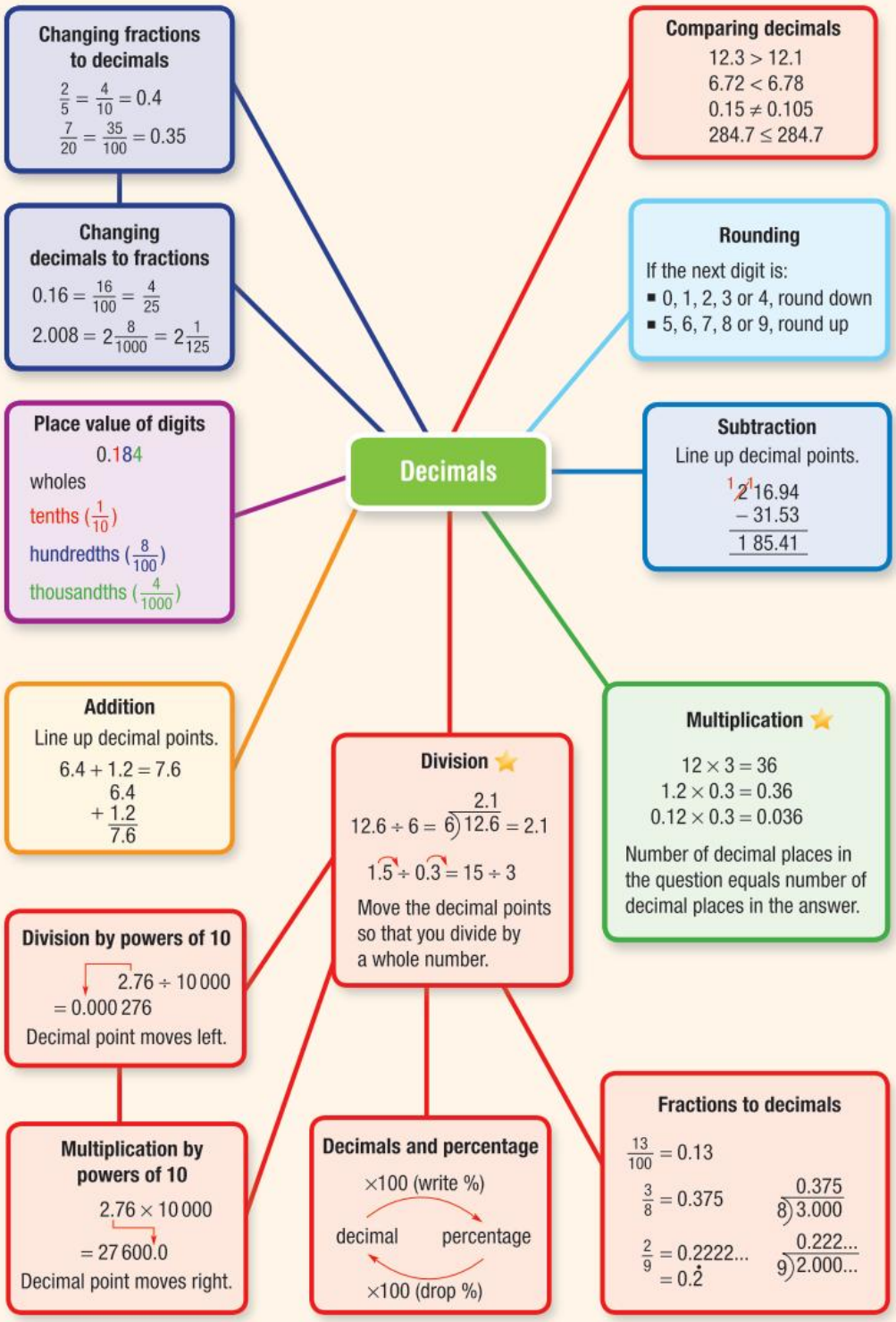
\$7.50	\$9.45	\$25.25	\$24	\$2.90	\$53.99	\$23.70	\$35.80

4 Take your age and double it.
Add 5 to this number.
Multiply the answer by 50, then add your age.
Now add 365 (the days in one year) to this value.
Subtract 615 from your result.
Now divide by 100 and put a dollar sign in front.
What do you notice?

5 The rungs in this ladder are evenly spaced. The height above the ground is shown for two of the rungs. Find the height above the ground of each of the other rungs.



Chapter summary



Multiple-choice questions

6A **1** The next number in the pattern 0.023, 0.025, 0.027, 0.029 is:

A 0.0003 **B** 0.030 **C** 0.0031 **D** 0.031 **E** 0.033

6A **2** 0.05 is equivalent to:

A $\frac{5}{10}$ **B** $\frac{5}{100}$ **C** $\frac{5}{1000}$ **D** $\frac{5}{500}$ **E** 5

6A **3** The smallest number out of 0.012, 10.2, 0.102, 0.0012 and 1.02 is:

A 0.012 **B** 0.102 **C** 0.0012 **D** 1.02 **E** 10.2

6D **4** $0.36 \div 1000$ is equal to:

A 3.6 **B** 360 **C** 0.036 **D** 0.0036 **E** 0.000 36

6E **5** 6.2×0.2 is equal to:

A 1.24 **B** 12.4 **C** 0.124 **D** 124 **E** 0.0124

6E **6** What is the answer to 0.08×0.6?

A 0.48 **B** 4.8 **C** 0.0048 **D** 0.048 **E** 48

6B **7** When rounded to 1 decimal place, 84.553 becomes:

A 80 **B** 84 **C** 84.5 **D** 84.6 **E** 84.55

6G **8** As a decimal, $\frac{3}{9}$ is equal to:

A 0.3 **B** $0.\dot{3}$ **C** 0.39 **D** 0.93 **E** 3.9

6C **9** Sally spends \$7.25 on a sandwich, \$2.55 on a drink and \$2.70 on a doughnut. Her change from \$20 is:

A \$12.75 **B** \$18 **C** \$12.50 **D** \$7.50 **E** \$2

6C **10** Bread rolls at a bakery are 60 cents each or 6 for \$3.50. The lowest price of 8 bread rolls is:

A \$3.60 **B** \$4.80 **C** \$4.70 **D** \$4.10 **E** \$21

Short-answer questions

6A **1** Arrange each group in descending order, from largest to smallest.

a 0.4, 0.04, 0.44 **b** 0.932, 0.98, 0.895

6A **2** Write each fraction as a decimal.

a $\frac{81}{10}$ **b** $\frac{81}{100}$ **c** $\frac{801}{100}$ **d** $\frac{801}{1000}$

6A **3** What is the place value of the digit 3 in the following numbers?

a 12.835 **b** 6.1237 **c** 13.5104

6A **4** Are the following statements true or false?

a $8.34 < 8.28$ **b** $4.668 > 4.67$ **c** $8.2 > 8.18$

d $3.08 \leq \frac{308}{100}$ **e** $\frac{62}{100} \geq 6.20$ **f** $\frac{7}{10} = \frac{70}{100}$

g $0.6 = 0.60$ **h** $1\% = \frac{1}{10}$ **i** $5\% = 0.05$

Chapter review

6B **5** Write \$62.876 to the nearest:

a cent **b** dollar **c** 5 cents

6B **6** Round each of the following to 1 decimal place.

a 12.74 **b** 8.36 **c** 9.41

d 7.45 **e** 0.08 **f** 7.124

6B **7** Round each of these to 2 decimal places.

a 12.814 **b** 423.461 **c** 15.889

d 7.2543 **e** 6.6666 **f** 3.3333 . . .

6C **8** Evaluate:

a $1.2 + 0.4$ **b** $0.36 + 1.2$ **c** $19.4 + 0.194$

d $7.6 + 1.2 + 0.8$ **e** $20 + 1.9$ **f** $6.4 - 3.1$

g $47.06 - 1.12$ **h** $200 - 156.5$ **i** \$15 − \$7.24

j \$1.60 + \$5.40 **k** \$19.46 + \$10.34 − \$5

6E **9** How many decimal places will there be in the answers to these problems?

a A number with 2 decimal places × a number with 3 decimal places

b A number with 1 decimal place × a number with 4 decimal places

c A number with 2 decimal places × a number with 2 decimal places

d A number with 2 decimal places × a number with 0 decimal places

6D-H **10** True (T) or false (F)?

a $1.37 \times 100 = 137$ **b** $9.4 \div 10 = 940$ **c** $8.7 \div 10 = 0.87$

d $18 \div 10 = 1.8$ **e** $5\% = 0.5$ **f** $6\% > 60\%$

g $12 \div 0.4 = 120 \div 4$ **h** $10\% \times 6 = 0.1 \times 6$ **i** $9.81 \times 10 \div 10 = 9.81$

j $\frac{7}{100} = 70\%$ **k** $25\% = 2.5$ **l** $56.1 \div 10 = 5.61$

6E/F **11** Calculate the following:

a 2.4×8 **b** 9×7.11 **c** 2.3×8.4

d $3.8 \div 4$ **e** $12.16 \div 8$ **f** $3 \div 0.5$

g $4 \div 0.25$ **h** $1.2 \div 0.4$ **i** $3.42 \div 0.1$

6G/H **12** Copy and complete this table, stating fractions both with the denominator 100 and in their simplest form.

Decimal	Fraction	Percentage
	$\frac{1}{100}$	
	$\frac{1}{10}$	
	$\frac{100}{100}$	
0.45		
	$\frac{?}{100} = \frac{7}{10}$	
		32%

Decimal	Fraction	Percentage
0.06		
	$\frac{79}{100}$	
1.05		
	$\frac{?}{100} = \frac{7}{20}$	
		65%
	$\frac{?}{1000} = \frac{1}{8}$	

6H **13** Complete these conversions.

a Convert these percentages to decimals.

i 70% **ii** 120% **iii** 37% **iv** 0.21%

b Convert these decimals to percentages.

i 0.4 **ii** 0.02 **iii** 1.65 **iv** 6.2

Extended-response questions

1 Find the answer in these practical situations.

a Jessica is paid $125.70 for 10 hours of work and Jaczinda is paid $79.86 for 6 hours of work. Who receives the higher rate of pay per hour, and by how much?

b Petrol is sold for 124.9 cents per litre. Jacob buys 10 L of petrol for his car. Find the total price he pays, to the nearest 5 cents.

c The cost of a ticket at the movies is $14.50 for an adult and $11.20 for a child. Find the cost for 1 adult and 2 children.

d A pie costs $2.70 at the school canteen. Zara bought 2 pies and a drink. The total was $8.60.

i How much change from $10 did Zara receive?

ii How much was her drink?

e Jen buys one cappuccino every workday, from different shops. Last week the prices were: $2.70, $3.30, $3.50, $3.50, $4.50

i Find the total cost for the week.

ii Find the average cost by dividing the total cost by the number of coffees.

iii How much should Jen budget for the 200 days she works in a year?



Whole numbers

Multiple-choice questions

1 Five thousand, two hundred and six is:
A 526 **B** 50 206 **C** 5260 **D** 5026 **E** 5206

2 The place value of 8 in 4837 is:
A 8 thousand **B** 8 hundred **C** zero **D** 8 tens **E** 8 ones

3 The remainder when 650 is divided by 4 is:
A 0 **B** 4 **C** 1 **D** 2 **E** 3

4 $18 - 3 \times 4$ simplifies to:
A 12 **B** 81 **C** 6 **D** 60 **E** 30

5 $(7 - 2 + 65) \div 10$ is equal to:
A 7 **B** 10 **C** 11 **D** 70 **E** 32

Short-answer questions

1 Write the missing number.
a 0, 2, ____, 6, 8, 10
b 13, 9, 5, ____
c 101, 202, ____, 404

2 Write the number for:
a seven thousand, three hundred and twenty-four
b twelve thousand and ninety-two

3 Calculate:
a $3712 + 1204 + 46$ **b** $1438 - 619$
c 49×3 **d** 380×20
e $525 \div 5$ **f** $411 \div 3$

4 True or false?
a $15 < 6 \times 2$ **b** $9 \times 6 > 45$ **c** $23 = 40 \div 2 + 3$

5 How much more than 9×11 is 11×11?

6 Calculate:
a $4 + 2 \times 3$ **b** $10 \div 5 + 3$ **c** $20 - 15 \div 5$
d $7 \times 6 - 4 \times 3$ **e** $8 \times 8 - 16 \div 2$ **f** $12 \times (6 - 2)$
g $9 \times (2 \times 4) - 10$ **h** $24 \div 6 \times 4$ **i** $56 - (7 - 5) \times 7$

7 Are the following true or false?
a $4 \times 25 \times 0 = 1000$ **b** $0 \div 10 = 0$
c $8 \div 0 = 0$ **d** $8 \times 7 = 7 \times 8$
e $20 \div 4 = 20 \div 2 \div 2$ **f** $8 + 5 + 4 = 8 + 9$

8 Put in brackets to make each of the following true.
a $2 + 3 \times 4 = 20$ **b** $10 - 2 \div 8 = 1$ **c** $4 \times 6 - 2 \div 8 = 2$

9 Round each number to the nearest ten.
a 12 **b** 35 **c** 137

10 Round each number to the nearest hundred.
a 129 **b** 87 **c** 1451

Extended-response questions

1 Tom works as a labourer, earning $25 an hour on weekdays and $60 an hour on weekends.

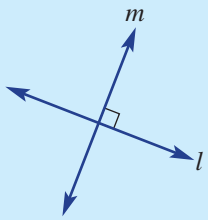


- **a** During a particular week, Tom works 7 a.m. to 2 p.m. Monday to Thursday. How many hours does he work that week?
- **b** How much does Tom earn for this work?
- **c** If Tom works 5 hours on Saturday in the same week, what is his total income for the week?
- **d** How many hours on a Friday must Tom work to earn the same amount as working 5 hours on a Saturday? (Hint: work out how much he earns on Saturday.)

Geometry

Multiple-choice questions

1 Which statement is correct?

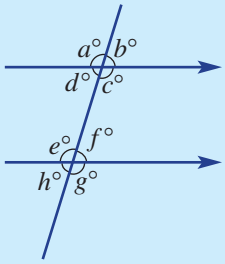


- **A** Line m is perpendicular to line l.
- **B** Line m bisects line l.
- **C** Line m is parallel to line l.
- **D** Line m is shorter than line l.
- **E** Line m is longer than line l.

2 An angle of 181° is classified as:

A acute **B** reflex **C** straight
D obtuse **E** sharp

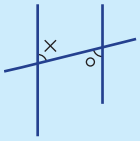
3 Which two pronumerals are equal because of equal alternate angles in parallel lines?



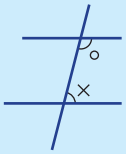
- **A** a and e
- **B** d and f
- **C** a and f
- **D** g and b
- **E** c and f

4 Which of the following shows a pair of cointerior angles?

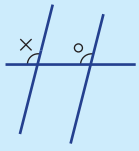
A



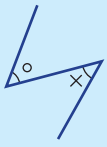
B



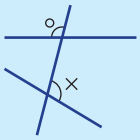
C



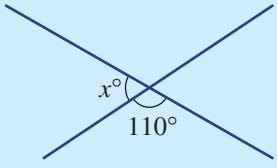
D



E



5 The value of x in this diagram is:



A 140 **B** 110 **C** 220
D 70 **E** 65

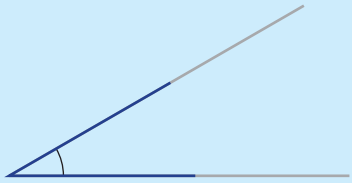
Short-answer questions

1 Give the name of the type of angle which is:

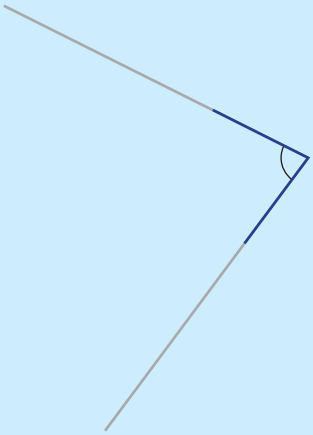
- **a** between 0° and 90°
- **b** exactly 90°
- **c** between 90° and 180°
- **d** exactly 180°
- **e** between 180° and 360°
- **f** exactly 360°

2 Measure these angles using a protractor.

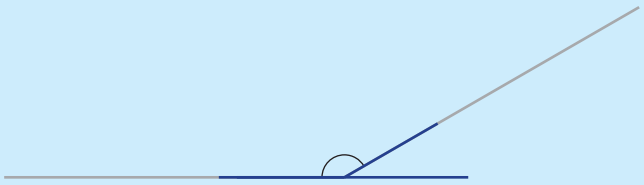
a



b



c

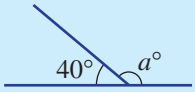


3 What is the complement of 65°?

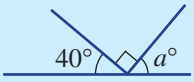
4 What is the supplement of 102°?

5 Find the value of a in each of the following diagrams.

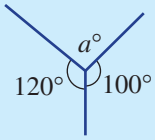
a



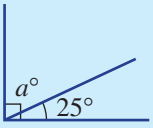
b



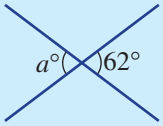
c



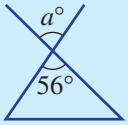
d



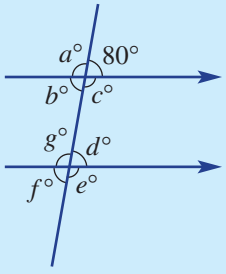
e



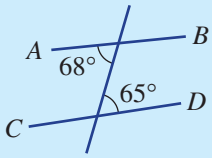
f



6 Find the value of each pronumeral when these two parallel lines are crossed by the transversal, as shown.

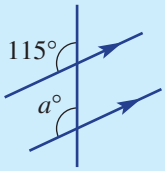


7 Explain why AB is not parallel to CD.

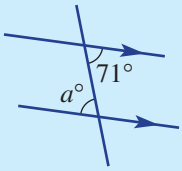


8 Find the value of a in these diagrams.

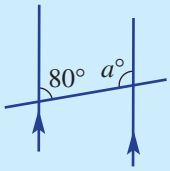
a



b

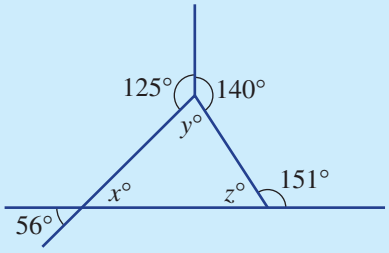


c



Extended-response questions

1 Consider the diagram shown.



a Find the value of:

i x **ii** y **iii** z

b What is the value of $x + y + z$?

Number properties and patterns

Multiple-choice questions

1 The first prime number after 14 is:

A 15 **B** 21 **C** 16 **D** 19 **E** 17

2 The highest common factor (HCF) of 12 and 18 is:

A 6 **B** 12 **C** 4 **D** 2 **E** 9

3 $2 \times 2 \times 2 \times 3$ is the same as:

A 6×3 **B** $2^3 \times 3$ **C** 8^3 **D** 6^3 **E** 4^3

4 Evaluating $3^2 - \sqrt{25} + 3$ gives:

A 8 **B** 5 **C** 4 **D** 17 **E** 7

5 The number 48 in prime factor form is:

A $2^4 \times 5$ **B** $2 \times 3 \times 5$ **C** $2^3 \times 3^2$ **D** $2^4 \times 3$ **E** $2^3 \times 3$

Short-answer questions

1 List the factors of:
a 15 **b** 30 **c** 100

2 List the first five multiples of:
a 3 **b** 7 **c** 11

3 List all factors common to 30 and 36.

4 What is the highest factor common to 36 and 40?

5 Find the value of:
a 11^2 **b** $6^2 \times 2^2$ **c** $33 - 2^3$

6 Find:
a 7 to the power of 2
b 12 squared
c the square root of 81

7 Copy and complete each of these tables.

a

Index form	3^2		6^2	
Value		25		64

b

Square root form	$\sqrt{9}$		$\sqrt{36}$	
Value		5		8

8 Which of the numbers 1080, 536, 135, 930, 316 are divisible by:
a 2? **b** 3? **c** 4?
d 5? **e** 10?

9 A pattern of decorated squares is made with match sticks.



a Copy and complete this table.

Number of squares	0	1	2	3	4
Number of sticks	1	1 + ☐ = ☐	1 + ☐ × 2 = ☐	1 + ☐ × 3 = ☐	1 + ☐ × 4 = ☐

b Copy and complete the rule for this pattern.

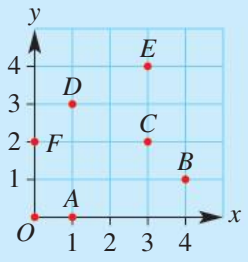
number of sticks = 1 + ☐ × number of squares

c How many sticks are needed to make 9 squares?
d How many decorated squares could be made from 81 sticks?

10 Find the missing values in the table.

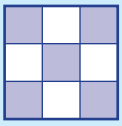
input	4	5	6			100
output	19	23		39	47	403

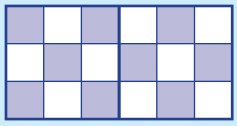
11 Write down the coordinates of each point, including the origin, O.

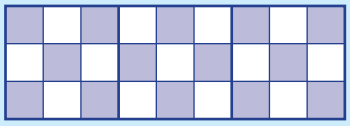


Extended-response questions

1 Purple and white tiles are used to make a pattern for the border of a swimming pool.


tile arrangement 1


tile arrangement 2


tile arrangement 3

a Draw tile arrangement 4 following this pattern.
b How many purple tiles are in arrangement 4 of this pattern?
c Which arrangement would use 41 purple tiles?
d If each purple tile costs \$1 and each white tile costs 50 cents, what is the cost of an arrangement using 41 purple tiles?

Fractions and percentages

Multiple-choice questions

1 Which of the following is equivalent to $\frac{12}{7}$?

A $\frac{24}{7}$ **B** $1\frac{5}{7}$ **C** $1\frac{5}{12}$ **D** $\frac{112}{17}$ **E** $\frac{7}{12}$

2 $\frac{1}{2}+\frac{1}{3}$ is equal to:

A $\frac{2}{5}$ **B** $\frac{2}{6}$ **C** $\frac{5}{6}$ **D** $\frac{1}{5}$ **E** $\frac{7}{6}$

3 $\frac{350}{450}$ in simplest form is:

A $\frac{35}{45}$ **B** $\frac{4}{5}$ **C** $\frac{3}{4}$ **D** $\frac{3.5}{4.5}$ **E** $\frac{7}{9}$

4 What fraction of \$2 is 40 cents?

A $\frac{1}{20}$ **B** $\frac{20}{1}$ **C** $\frac{5}{1}$ **D** $\frac{1}{5}$ **E** $\frac{1}{40}$

5 $2\frac{1}{2}\div\frac{3}{4}$ is the same as:

A $\frac{5}{2}\times\frac{4}{3}$ **B** $\frac{5}{2}\times\frac{3}{4}$ **C** $\frac{2}{5}\div\frac{3}{4}$ **D** $\frac{2}{5}\times\frac{4}{3}$ **E** $\frac{3}{2}\times\frac{3}{4}$

Short-answer questions

1 Arrange $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{3}{10}$ in *ascending* order.

2 Express $5\frac{2}{3}$ as an improper fraction.

3 Find each of the following.

a $\frac{2}{3}+\frac{1}{4}$ **b** $4-1\frac{1}{3}$ **c** $2\frac{1}{2}+3\frac{3}{4}$

d $\frac{2}{5}\times\frac{1}{2}$ **e** $\frac{2}{3}\div\frac{1}{6}$ **f** $1\frac{1}{5}\times\frac{5}{12}$

4 Write 15% as a simple fraction.

5 Find 25% of $480.

6 Find 2% of $400.

7 Are the following true or false?
- **a** To find 25% of an amount, divide the amount by 4.
- **b** 10% of an amount = amount ÷ 10
- **c** 20% of 50 = 2 × 10% of 50
- **d** 1% of $x = 100x$

8 Which is larger, $\frac{2}{3}$ or 60%?

Extended-response questions

1 Caleb's cold and flu prescription states: 'Take two pills three times a day with food.' The bottle contains 54 pills.
- **a** How many pills does Caleb take each day?
- **b** What fraction of the bottle remains after Day 1?
- **c** How many days will it take for the pills to run out?
- **d** If Caleb takes his first dose Friday night before going to bed, on what day will he take his last dose?



Algebra

Multiple-choice questions

1 If $x = 3$ then $5 + x$ equals:
A 2 **B** 8 **C** 3 **D** 53 **E** 15

2 The sum of x and y is written as:
A $x + y$ **B** $2x$ **C** xy **D** $x - y$ **E** $x \div y$

3 If $a = 7$ then $5a + 2$ is equal to:
A 7 **B** 59 **C** 37 **D** 33 **E** 52

4 $4a + 3b + c + 5b + c$ is equivalent to:
A $32ab$ **B** $4a + 8b + 2c$ **C** $8a + 4b$ **D** $64abc$ **E** $4a + 8b$

5 $2a \times 3b \times 4c$ simplifies to:
A $6ab + c$ **B** $7abc$ **C** 24 **D** $24abc$ **E** $24 + abc$

Short-answer questions

1 Consider the expression $5x + 7y + 3x$.
- **a** How many terms are in this expression?
- **b** What is the coefficient of y?
- **c** Simplify this expression by combining the like terms.

2 Write an algebraic expression for each of the following.
- **a** the sum of x and 3
- **b** the product of a and 12
- **c** the sum of double x and triple y
- **d** w divided by 6
- **e** double x taken from y

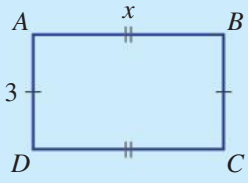
3 For the following pairs, state if they are like (L) or not like (N) terms.
a $2a$ and $3a$ **b** $7k$ and $7m$ **c** $4xy$ and $2xy$ **d** $3ab$ and $5b$

4 If $m = 6$, find the value of each of the following.
a $m + 7$ **b** $2m - 1$ **c** $6m + 3$
d $2(m - 3)$ **e** $\frac{m}{2}$ **f** $5m - 6$

5 Evaluate the expression $6x + 3y$ when $x = 5$ and $y = 2$.

6 Simplify each of the following.
a $6a + 4a$ **b** $7x - 3x$ **c** $9a + 2a + a$
d $m + m - m$ **e** $6 + 2a + 3a$ **f** $x + y + 3x + y$

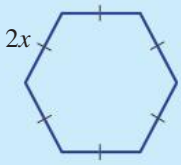
7 **a** Write an expression for the perimeter of rectangle $ABCD$.
b Write an expression for the area of rectangle $ABCD$.



8 Find the missing term.
a $3a \times ____ = 18abc$ **b** $9b - ____ = 4b$ **c** $2p + 2p + 2p = 6____$

9 The cost of one pencil is $\$p$. Write an expression for the cost of:
a 10 pencils **b** 25 pencils

10 Write the simplest expression for the perimeter of this figure.



Extended-response questions

1 A bottle of soft drink costs \$3 and a pie costs \$2.
- **a** Find the cost of:
  - **i** 2 bottles of soft drink and 5 pies
  - **ii** x bottles of soft drink
  - **iii** x bottles of soft drink and 5 pies
- **b** Write an expression for the cost of buying x bottles of soft drink and y pies.
- **c** Substitute $x = 2$ and $y = 5$ into your expression from **b**.

Decimals

Multiple-choice questions

1 $80 + \frac{6}{10} + \frac{7}{100}$ is the same as:

A 8067 **B** 867 **C** 80.67 **D** 80.067 **E** 80.607

2 Select the incorrect statement.

A $0.707 > 0.7$

B $0.770 = \frac{77}{100}$

C $0.07 \times 0.7 = 0.49$

D $0.7 \times \frac{1}{10} = 0.07$

E $0.7 \times 10 = 7$

3 The best estimate for 23.4×0.96 is:

A 234 **B** 230 **C** 0.234 **D** 23 **E** 20

4 $\frac{3}{8}$ is the same as:

A 0.375 **B** 3.8 **C** 0.38 **D** $2.\dot{6}$ **E** 38%

5 $6.8 \div 0.4$ is the same as:

A $68 \div 4$ **B** $680 \div 4$ **C** 1.7 **D** $\frac{4}{68}$ **E** $7 \div 0.05$

Short-answer questions

1 Write each of the following as a decimal.

a two-tenths **b** $\frac{13}{100}$ **c** $\frac{17}{10}$

2 In the decimal 136.094:

a What is the value of the 6?

b What is the value of the 4?

3 Round 18.398 741 correct to:

a the nearest whole

b 1 decimal place

c 2 decimal places

4 Evaluate:

a $15 - 10.93$ **b** $19.7 + 240.6 + 9.03$ **c** $20 - 0.99$

d 0.6×0.4 **e** 0.3×0.3 **f** $\frac{12}{0.2}$

5 Find:

a $1.24 - 0.407$ **b** $1.2 + 1.8 - 0.6$ **c** 0.6×0.07

6 If $369 \times 123 = 45\,387$, write down the value of:

a 3.69×1.23 **b** 0.369×123 **c** 0.369×0.123

7 Find:

a 36.49×1000 **b** $1.8 \div 100$ **c** 19.43×200

8 For each of the following, circle the larger number.

a $\frac{4}{5}$, 0.79 **b** 1.1, 11% **c** $\frac{2}{3}$, 0.6

9 State if each of the following is true or false.

a $0.5 = 50\%$ **b** $0.15 = \frac{5}{20}$ **c** $38\% = 0.19$

d $126\% = 1.26$ **e** $\frac{4}{5} = 0.08$ **f** $1\frac{3}{4} = 1.75$

Extended-response questions

1 The cost of petrol is 116.5 cents per litre.

a Find the cost of 5 L of petrol, correct to the nearest cent.

b Mahir pays cash for 5 L of petrol for his motor bike. What is the amount that he pays, correct to the nearest 5 cents?

c How much change from $10 does Mahir receive?





Chapter 7

Negative numbers

What you will learn

7A Integers (Consolidating)
7B Adding or subtracting a positive integer
7C Adding a negative integer
7D Subtracting a negative integer
7E Substituting integers
7F The number plane

Australian curriculum

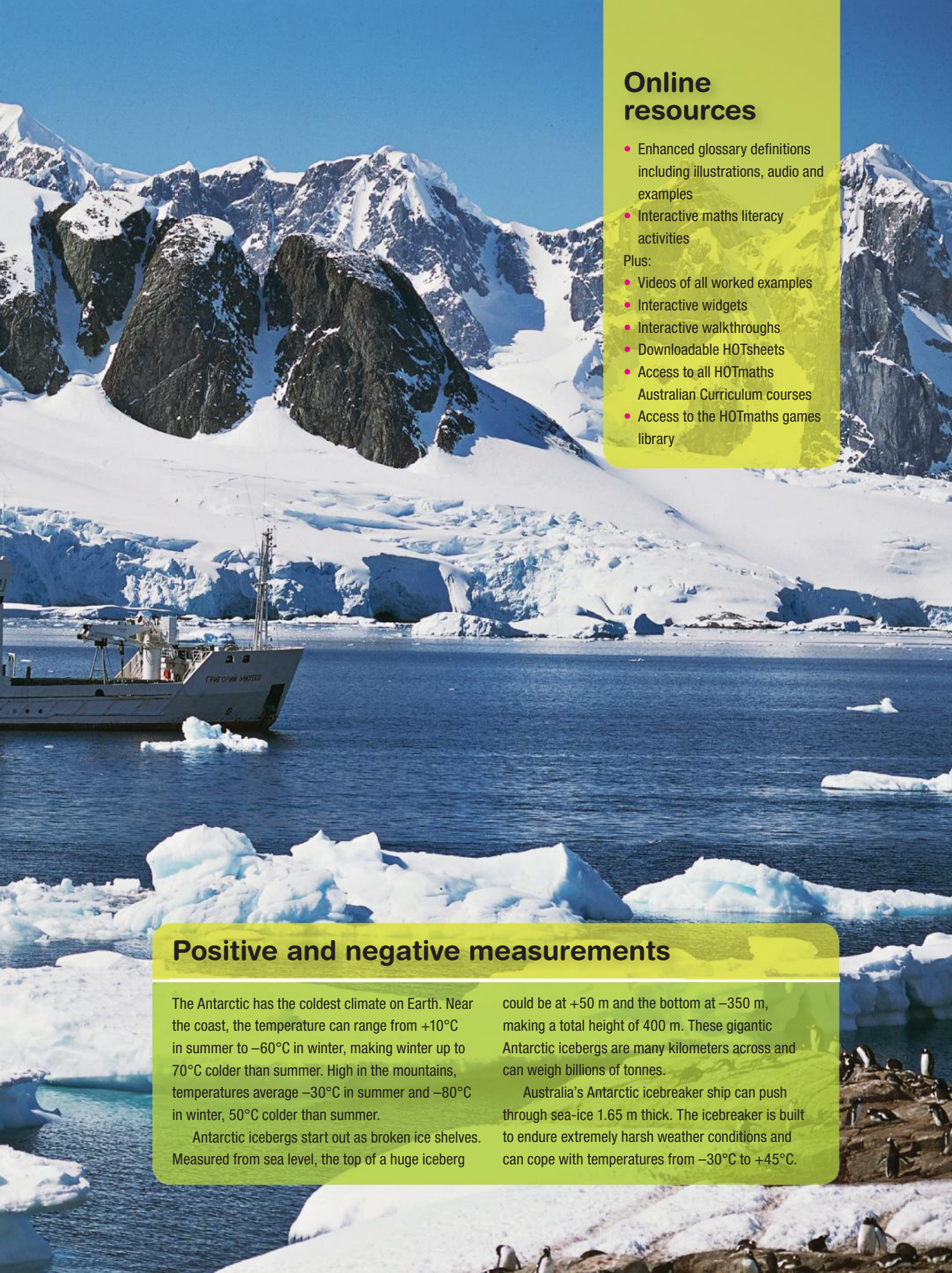
NUMBER AND ALGEBRA

Number and place value

Compare, order, add and subtract integers **(ACMNA280)**

Linear and non-linear relationships

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point **(ACMNA178)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Positive and negative measurements

The Antarctic has the coldest climate on Earth. Near the coast, the temperature can range from +10°C in summer to –60°C in winter, making winter up to 70°C colder than summer. High in the mountains, temperatures average –30°C in summer and –80°C in winter, 50°C colder than summer.

Antarctic icebergs start out as broken ice shelves. Measured from sea level, the top of a huge iceberg could be at +50 m and the bottom at –350 m, making a total height of 400 m. These gigantic Antarctic icebergs are many kilometers across and can weigh billions of tonnes.

Australia's Antarctic icebreaker ship can push through sea-ice 1.65 m thick. The icebreaker is built to endure extremely harsh weather conditions and can cope with temperatures from –30°C to +45°C.

Pre-test

1 Evaluate (find the answer) the following.

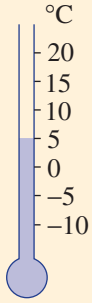
a $7+4$ **b** $0+5$ **c** $7+0$
d $13+9$ **e** $10-10$ **f** $14-8$
g $9-8$ **h** $26-14$ **i** $2+3-4$
j $7-4+5$ **k** $21-16+4$ **l** $36+24-47$

2 Insert the symbols < (less than) or > (greater than) to make each statement true.

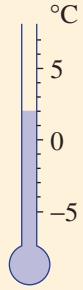
a $5 \square 7$ **b** $0 \square 10$
c $9 \square 11$ **d** $3 \square 0$
e $10 \square 12$ **f** $13 \square 26$
g $2 \square 1$ **h** $101 \square 99$

3 Read the temperature on these thermometers.

a



b



c



4 Evaluate these products.

a 2×15 **b** 11×7 **c** 9×8 **d** 3×13

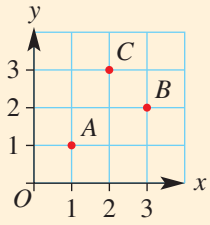
5 Evaluate these quotients.

a $35 \div 7$ **b** $121 \div 11$ **c** $63 \div 7$ **d** $84 \div 12$

6 Substitute $a = 3$ and $b = 4$ into these expressions and then evaluate.

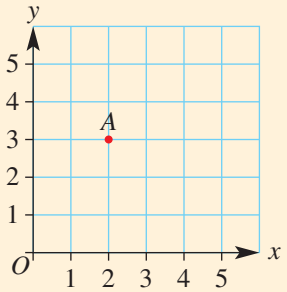
a $a+b$ **b** $2b-a$ **c** $5a-2b$ **d** $ab+4$

7 Write down the coordinates (x, y) of A, B and C for this number plane. Choose from $(1, 1)$, $(2, 3)$ and $(3, 2)$.



8 Plot these points on the given number plane and join A, B, C, D, E, F and A to form a shape. Then find the perimeter of the shape you have formed. (Perimeter is the distance around the outside of the shape.) The first point is done for you.

a $A(2, 3)$ **b** $B(2, 1)$
c $C(5, 1)$ **d** $D(5, 5)$
e $E(3, 5)$ **f** $F(3, 3)$



7A Integers

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Positive numbers are greater than zero. Negative numbers are less than zero. All the positive whole numbers (1, 2, 3, …) and the negative whole numbers (−1, −2, −3, …), together with zero (0), are called integers.

An English mathematician named John Wallis (1616–1703) invented the number line. He also invented the idea that numbers have a direction. This helped define our number system as an infinite set of numbers. This set extends infinitely in both the positive and negative directions.



John Wallis invented the number line.

Let's start: Do negative numbers exist?

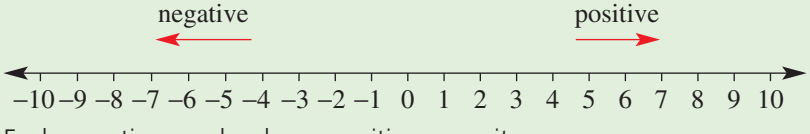
Are negative numbers real?

- Can you think of any situations where negative numbers are used?
- How would you describe what a negative number is?

Key ideas

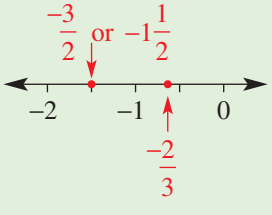
- **Negative numbers** are numbers less than zero.
- **Integers** are whole numbers that can be negative, zero or positive.

  … − 10, −9, −8, −7, −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, …
- A **number line** shows:
  - positive numbers to the right of zero
  - negative numbers to the left of zero.



- Each negative number has a positive opposite.

  3 and −3 are examples of opposite numbers.
- Negative fractions can also be located on the number line.



Negative number A number less than zero

Integers The set of positive and negative whole numbers and zero

Number line A line on which numbers are represented by points

Exercise 7A

UNDERSTANDING 1–3 | 3

1 State the missing number.

a $-3, -2, -1, 0, \square, 2$

b $-1, \square, 1, 2, 3, 4$

c $-6, -5, \square, -3, -2, -1, 0$

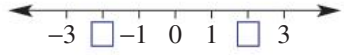
d $-4, -3, -2, \square, 0, 1$

e $-21, -20, -19, \square, -17, -16$

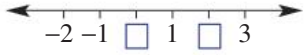
f $-37, \square, -35, -34, -33, -32$

2 What are the missing numbers on these number lines?

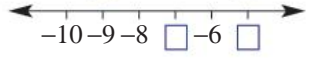
a



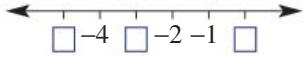
b



c



d



3 Fill in each blank using the word *greater* or *less*.

a 5 is _____ than 0

b −3 is _____ than 0

c 0 is _____ than −6

d 0 is _____ than 1

e −2 is _____ than −3

f −6 is _____ than −2



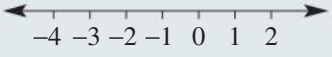
On the number line, numbers to the *right* are greater than numbers to their *left*.

FLUENCY 4, 5, 6–7(½) | 4, 5–8(½)

Example 1 Drawing a number line

Draw a number line, showing all integers from −4 to 2.

Solution



Explanation

Use equally spaced markings and put −4 on the left and 2 on the right.

4 Draw a number line showing all integers:

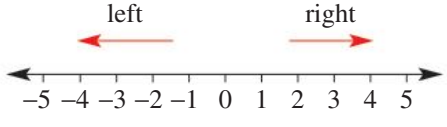
a from −2 to 2

b from −5 to 1

c from −10 to −6

d from −16 to −12

5 Add the word *right* or *left* to make the following statements true. Use this number line to help.



a 2 is to the _____ of 0

b 1 is to the _____ of 3

c −1 is to the _____ of 2

d −4 is to the _____ of −5

e −4 is to the _____ of −1

f 2 is to the _____ of −4

6 −5 is the opposite number of 5, and 5 is the opposite number of −5. Write down the opposite of these numbers.

a 2

b 6

c −3

d −7

e −15

f 21

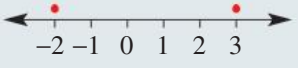
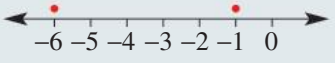
g 132

h −1071

Example 2 Less than or greater than

Insert the symbol < (less than) or > (greater than) into these statements to make them true.

a $-2 \square 3$ **b** $-1 \square -6$

Solution	Explanation
a $-2 < 3$	-2 is to the left of 3 on a number line. 
b $-1 > -6$	-1 is to the right of -6 on a number line. 

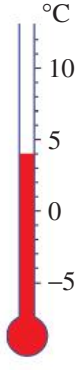
7 Insert the symbol < (less than) or > (greater than) into these statements to make them true.

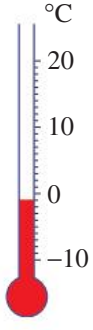
a $7 \square 9$ **b** $3 \square 2$ **c** $0 \square -2$
d $-4 \square 0$ **e** $-1 \square -5$ **f** $-7 \square -6$
g $-11 \square -2$ **h** $-9 \square -13$ **i** $-3 \square 3$
j $3 \square -3$ **k** $-130 \square 1$ **l** $-2 \square -147$
m $-7 \square -6$ **n** $-1 \square -21$ **o** $-116 \square -118$
p $-231 \square -162$

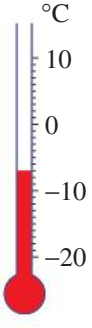


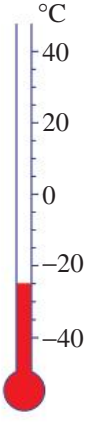
On the number line, numbers to the *left* are less than numbers to their *right*.

8 Give the temperature for these thermometers.

a 

b 

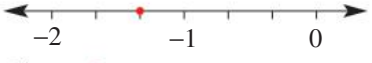
c 

d 

PROBLEM-SOLVING AND REASONING 9–11 10–14

9 Write down the negative fraction illustrated by the dot on these number lines.

a 

b 

c 

d 

7A

10 True or false?

a -3 is the opposite of 6
b 0 is a negative number
c 0 is a positive number
d -5 is less than -2
e -8 is greater than -1
f -2 is equal to 2
g $-11 < -6$
h $-7 > -2$

11 List all the integers that fit the given description.

a from -2 up to 4
b from -7 up to 0
c greater than -3 and less than 2
d greater than -5 and less than 1
e less than 4 and greater than -4
f less than -3 and greater than -10

12 Arrange these numbers in *ascending* order.

a $-3, -6, 0, 2, -10, 4, -1$
b $-304, 126, -142, -2, 1, 71, 0$



Ascending means 'to rise', so arrange numbers from lowest to highest (*left* to right on a number line).

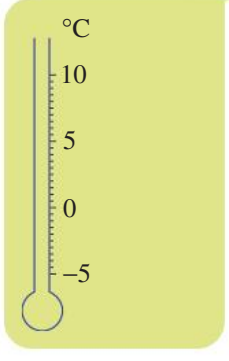
13 Write the next three numbers in these simple patterns.

a 3, 2, 1, ___, ___, ___
b $-8, -6, -4$, ___, ___, ___
c 10, 5, 0, ___, ___, ___
d $-38, -40, -42$, ___, ___, ___
e $-91, -87, -83$, ___, ___, ___
f 199, 99, -1, ___, ___, ___

14 A thermometer shows a temperature of 10 degrees Celsius. What would be the new temperature if it drops by:

a 6 degrees? **b** 10 degrees? **c** 12 degrees? **d** 20 degrees?



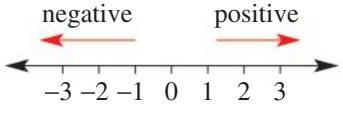




The final position

— 15

15 For these sets of numbers, a positive number means to move right and a negative number means to move left. Start at zero each time and find the final position.



a Move -1, then 4, then -5
b Move 3, -5, -1, 4
c Move -5, -1, 3, 1, -2, -1, 4
d Move -10, 20, -7, -14, 8, -4

7B Adding or subtracting a positive integer

Interactive

When we add a positive number such as 3, we move 3 places to the right on the number line. $2+3$ means start at 2 and move 3 to the right. $-5+3$ means start at -5 and move 3 steps right.

Widgets

To subtract the positive number 4, we move 4 places to the left on the number line. For $7-4$, we start at 7 and move 4 steps left. For $-2-4$, we start at -2 and move 4 steps left.

HOTsheets

So, to add or subtract a positive number, we need to pick a starting point and then move right for addition or left for subtraction.

Walkthrough

Let's start: Walking the number line

Imagine a number line running from one side of the room to the other, large enough to walk along. Decide which half is positive (the right) and which is negative (the left). Each step moves you one whole number along the line.

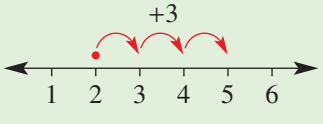
- Stand at zero and move to the right for addition and left for subtraction.
- Another student calls out a command such as 'add 3' or 'subtract 5'. Move the appropriate number of steps on the number line and tell the class your new position.
- If you make a mistake, let another student have a turn.

Key ideas

- If a positive number is added to an integer, you move right on a number line.

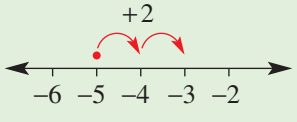
$2 + 3 = 5$

Start here. Move right 3.



$-5 + 2 = -3$

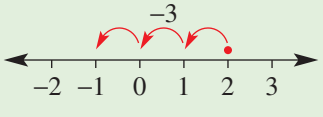
Start here. Move right 2.



- If a positive number is subtracted from an integer, you move left on a number line.

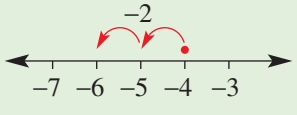
$2 - 3 = -1$

Start here. Move left 3.



$-4 - 2 = -6$

Start here. Move left 2.

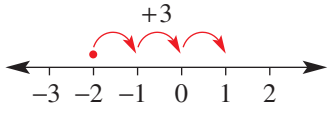


Exercise 7B

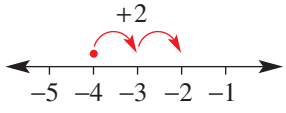
UNDERSTANDING 1–3 3

1 Use the number line to help find the answer to each of the following.

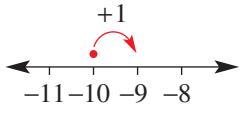
a $-2+3$



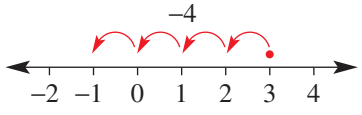
b $-4+2$



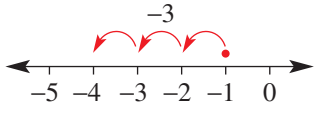
c $-10+1$



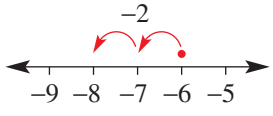
d $3-4$



e $-1-3$



f $-6-2$



2 In which direction (i.e. right or left) on a number line do you move for the following calculations?

a Start at -5, add 2
b Start at -4, add 6
c Start at 2, subtract 4
d Start at -4, subtract 3

3 Match up the problems (**a** to **d**) with the number lines (**A** to **D**).

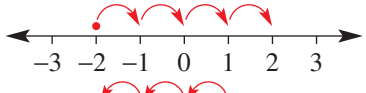
a $5-6=-1$

b $-2+4=2$

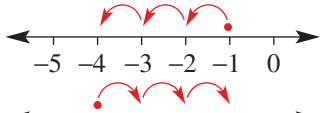
c $-1-3=-4$

d $-6+3=-3$

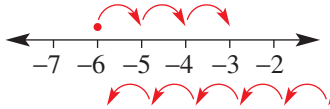
A



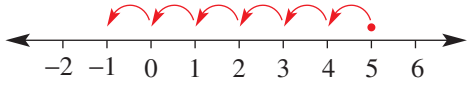
B



C



D

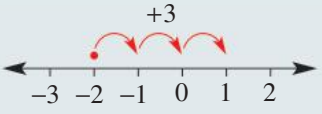
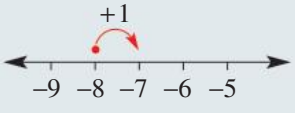
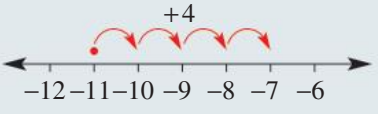


FLUENCY 4–6(½) 4–7(½)

Example 3 Adding a positive integer

Complete the following sums.

a $-2+3$ **b** $-8+1$ **c** $-11+4$

Solution	Explanation
a $-2+3=1$	
b $-8+1=-7$	
c $-11+4=-7$	

4 Complete the following sums.

a $-1+2$ **b** $-1+4$
c $-3+5$ **d** $-10+11$
e $-7+2$ **f** $-10+7$
g $-13+14$ **h** $-9+13$
i $-4+3$ **j** $-5+2$
k $-11+9$ **l** $-20+18$
m $-4+0$ **n** $-8+0$
o $-30+29$ **p** $-39+41$

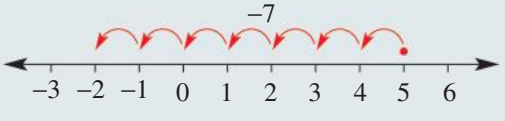
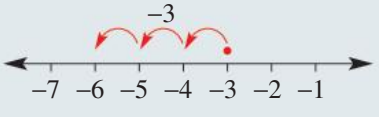
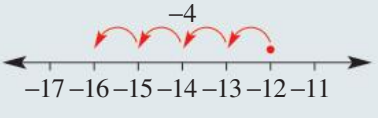


Remember to move *right* on the number line when *adding* a *positive* integer.

Example 4 Subtracting a positive integer

Complete the following differences.

a $5-7$ **b** $-3-3$ **c** $-12-4$

Solution	Explanation
a $5-7=-2$	
b $-3-3=-6$	
c $-12-4=-16$	

7B

5 Complete the following differences.

a	$4-6$	**b**	$7-8$	**c**	$3-11$		
d	$1-20$	**e**	$-1-2$	**f**	$-3-4$		
g	$-5-10$	**h**	$-11-2$	**i**	$-3-1$		
j	$-5-5$	**k**	$-2-13$	**l**	$-7-0$		
m	$-37-4$	**n**	$39-51$	**o**	$62-84$	**p**	$-21-26$



Remember to move left on the number line when subtracting a positive integer.

6 Evaluate:

a	$-130+132$	**b**	$-181+172$	**c**	$-57+63$	**d**	$-99+68$
e	$-100-200$	**f**	$100-200$	**g**	$328-421$	**h**	$-496-138$

Example 5 Combining addition and subtraction

Evaluate the following, working from left to right.

a $2-6+3$ **b** $-6+3-5$

Solution	Explanation
a $2-6+3=-4+3$ $=-1$	First work out $2-6=-4$, then add 3
b $-6+3-5=-3-5$ $=-8$	First work out $-6+3=-3$, then subtract 5

7 Evaluate the following.

a	$3-4+6$	**b**	$2-7-4$				
c	$-1-4+6$	**d**	$-5-7-1$				
e	$-2+7-4$	**f**	$-3+1-6$				
g	$-4-5+2$	**h**	$-16+4-1$				
i	$-3+2-7+9$	**j**	$-6+1-20+3$				
k	$0-9+7-30$	**l**	$-15-20+32-1$				



Remember to work from left to right.

PROBLEM-SOLVING AND REASONING 8–10 10–13

8 Find the missing number.

a	$2+\square=7$	**b**	$-2+\square=7$	**c**	$-2+\square=3$	**d**	$-4+\square=-2$
e	$5-\square=0$	**f**	$3-\square=-4$	**g**	$-9-\square=-12$	**h**	$-20-\square=-30$
i	$-6+\square=-1$	**j**	$-8-\square=-24$	**k**	$\square+1=-3$	**l**	$\square+7=2$
m	$\square-4=-10$	**n**	$\square-7=-20$	**o**	$\square+6=-24$	**p**	$\square-100=-213$

9 Determine how much debt remains in these bank accounts.

a Owes \$300 and pays back \$155
b Owes \$20 and borrows another \$35
c Owes \$21 500 and pays back \$16 250

10 **a** The reading on a thermometer rises 18°C from −15°C. What is the final temperature?

b The reading on a thermometer falls 7°C from 4°C. What is the final temperature?

c The reading on a thermometer falls 32°C from −14°C. What is the final temperature?



11 For an experiment, a chemical solution starts at a temperature of 25°C. It falls to −3°C, rises to 15°C and then falls again to −8°C. What is the total change in temperature? (Hint: it might be helpful to combine all three changes.)

12 An ocean sensor is raised and lowered to different depths in the sea. Note that −100 m means 100 m below sea level.



a If the sensor is initially at −100 m and then raised to −41 m, how far does the sensor rise?

b If the sensor is initially at −37 m and then lowered to −93 m, how far is the sensor lowered?

13 Insert + or − signs into these statements to make them true.

a $3 \square 4 \square 5 = 4$

b $1 \square 7 \square 9 \square 4 = -5$

c $-4 \square 2 \square 1 \square 3 \square 4 = 0$

d $-20 \square 10 \square 7 \square 36 \square 1 \square 18 = -4$

Positive and negative possibilities — 14

14 Decide if it is possible to find an example of the following. If so, give a specific example. It might help to draw a number line.

a A positive number added to a positive number gives a positive number.

b A positive number added to a positive number gives a negative number.

c A positive number added to a negative number gives a positive number.

d A positive number added to a negative number gives a negative number.

e A positive number subtracted from a positive number gives a positive number.

f A positive number subtracted from a positive number gives a negative number.

g A positive number subtracted from a negative number gives a positive number.

h A positive number subtracted from a negative number gives a negative number.

7C Adding a negative integer

Interactive

Widgets

HOTsheets

Walkthrough

So far, we have added or subtracted positive integers to other integers. For example: $-2+4$ or $-3-5$.

We will now look at the case where we add negative integers to other integers. For example: $3+(-4)$ or $-6+(-2)$.

As an example, think of a debt of \$40 as a negative amount (-40). Suppose an account has a balance of \$100. If you add the \$40 debt, you would have a total balance of \$60. So $100+(-40)=60$.

But we know that $100-40=60$. This suggests that to add -40 we should subtract 40.



If I have \$100 of debt and add \$40 of debt, how do I calculate my final balance?

Let's start: What does the pattern tell us?

Look at this addition pattern.

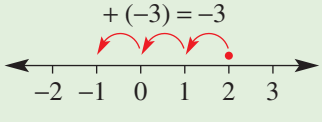
$$\begin{aligned} 2+3&=5 \\ 2+2&=4 \\ 2+1&=3 \\ 2+0&=2 \\ 2+(-1)&=1 \\ 2+(-2)&=0 \\ 2+(-3)&=-1 \end{aligned} \quad \begin{matrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{matrix}$$

- Describe the vertical patterns that you see.
- What do the patterns tell you about adding a negative integer?

Key ideas

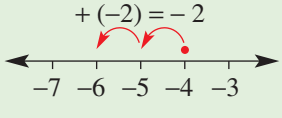
- The opposite of 3 is -3, and the opposite of -2 is 2.
- To add a negative number, subtract its opposite.
  $a+(-b)=a-b$

$2+(-3)=2-3=-1$



To add -3, subtract 3.

$-4+(-2)=-4-2=-6$



To add -2, subtract 2.

Exercise 7C

UNDERSTANDING 1–3 | 3

1 Write the missing number.

a To add −2, subtract ☐.
b To add −6, subtract ☐.
c To add −4, subtract ☐.
d To add −11, subtract ☐.
e To add −37, subtract ☐.
f To add −142, subtract ☐.

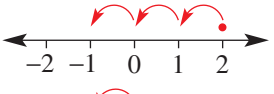
2 Write down the missing numbers in these sentences. The first one has been done for you.

a $2 + 5$ means that $\boxed{5}$ is added to $\boxed{2}$.
b $-3 + 6$ means that ☐ is added to ☐.
c $1 + (-3)$ means that ☐ is added to ☐.
d $-7 + (-11)$ means that ☐ is added to ☐.
e $1 + (-10)$ means that ☐ is added to ☐.
f $-4 + (-2)$ means that ☐ is added to ☐.
g $-3 + (-6)$ means that ☐ is added to ☐.
h $-37 + (-26)$ means that ☐ is added to ☐.

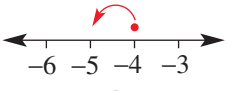
3 Match each addition (**a** to **d**) with the correct diagram (**A** to **D**).

a $4 + (-1)$
b $2 + (-3)$
c $-6 + (-2)$
d $-4 + (-1)$

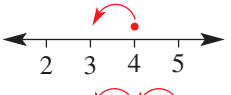
A



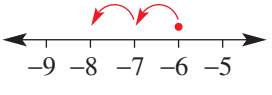
B



C



D



FLUENCY 4–5(½) | 4–6(½)

Example 6 Adding a negative integer

Calculate answers to the following.

a $7 + (-2)$
b $-2 + (-3)$

Solution	Explanation
a $7 + (-2) = 7 - 2$ $= 5$	To add −2, subtract 2.
b $-2 + (-3) = -2 - 3$ $= -5$	To add −3, subtract 3.

7C

4 Calculate:

a $3+(-2)$	**b** $8+(-3)$	**c** $12+(-6)$
d $9+(-7)$	**e** $1+(-4)$	**f** $6+(-11)$
g $20+(-22)$	**h** $0+(-4)$	**i** $-2+(-1)$
j $-7+(-15)$	**k** $-5+(-30)$	**l** $-28+(-52)$
m $-7+(-3)$	**n** $-20+(-9)$	**o** $-31+(-19)$
p $-103+(-9)$	**q** $-99+(-10)$	**r** $-12+(-101)$



To add a negative integer, subtract its opposite.
$-3+(-2)=3-2$

5 Find the answer if:

a -2 is added to 7 **b** -3 is added to 10
c -10 is added to -2 **d** -1 is added to -6
e -13 is added to -2 **f** -31 is added to -11



For part **c**, start at -2 and subtract 10.

Example 7 Combining additions with integers

Calculate the answers to these problems.

a $3+(-2)+5$ **b** $-2+(-3)+(-4)$

Solution	Explanation
a $3+(-2)+5=3-2+5$ $=1+5$ $=6$	To add -2, subtract 2, then add 5 to finish.
b $-2+(-3)+(-4)=-2-3-4$ $=-5-4$ $=-9$	Starting at -2, add -3 by subtracting 3. Then add -4 by subtracting 4.

6 Calculate the answers to these problems.

a $3+(-1)+2$	**b** $7+(-2)+4$
c $2+(-3)+1$	**d** $-1+(-3)+(-4)$
e $-4+(-2)+(-1)$	**f** $-6+(-3)+(-1)$
g $3+(-10)+6$	**h** $-15+2+(-4)$
i $-13+(-17)+(-3)$	

PROBLEM-SOLVING AND REASONING 7–9 8–11

7 Find the missing number.

a $2+\square=-1$	**b** $3+\square=-7$	**c** $-2+\square=-6$
d $\square+(-3)=1$	**e** $\square+(-10)=-11$	**f** $\square+(-4)=0$
g $\square+(-3)=2$	**h** $\square+(-6)=-1$	**i** $-2+\square=-3$
j $-5+\square=-7$	**k** $-10+\square=-12$	**l** $-37+\square=-51$

8 A person has \$120 of debt. If \$70 of debt is added to this, how much debt is now owed by the person?

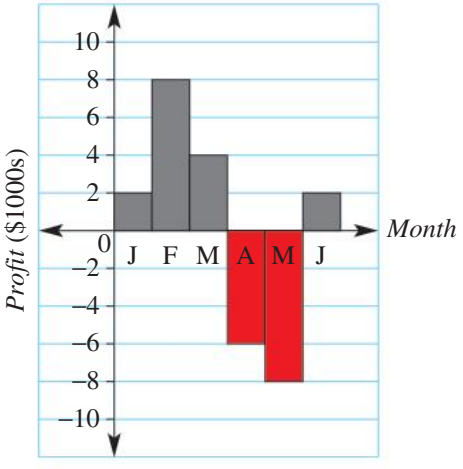
9 Here is a profit graph showing the profit for each month of the first half of the year for a bakery shop.

a What is the profit for:

i February?

ii April?

b What is the overall profit for the 6 months?





10 If a is a positive number and b is a negative number, are the following true?

a $a + b$ is *always* positive.

b $b + a$ is *always* negative.



11 If a is a negative number and b is a negative number, are the following true?

a $a + b$ is *always* positive.

b $b + a$ is *always* negative.

Negative magic squares

— 12

12 Complete these magic squares, using addition. The sum of each row, column and diagonal should be the same.

a

−2		5
	1	
		4

b

		−6
−3		−17
		7

7D Subtracting a negative integer

Interactive

Widgets

HOTsheets

Walkthrough

Imagine that you have \$100 worth of debt and you subtract (pay back) \$30 of that debt. You still owe \$70. This can be written as $-100 - (-30) = -70$.

But to pay back \$30 of your debt, you must *add* \$30 to your account. Also, to move from -100 to -70 on a number line, we *add* 30. So $-100 - (-30) = -100 + 30$.

This suggests that to subtract (-30) we should add 30.

Let's start: Patterns reveal the trick

Look at this subtraction pattern.

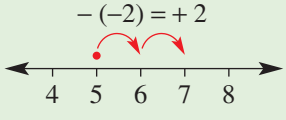
$2 - 3 = -1$ $+1$
$2 - 2 = 0$ $+1$
$2 - 1 = 1$ $+1$
$2 - 0 = 2$ $+1$
$2 - (-1) = 3$ $+1$
$2 - (-2) = 4$ $+1$
$2 - (-3) = 5$

- Describe the vertical patterns that you see.
- What do the patterns tell you about subtracting a negative integer?

Key ideas

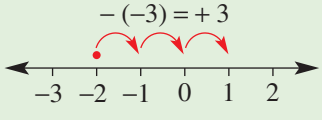
■ To subtract a negative number, add its opposite.
$a - (-b) = a + b$

$5 - (-2) = 5 + 2 = 7$



To subtract -2, add 2.

$-2 - (-3) = -2 + 3 = 1$



To subtract -3, add 3.

Exercise 7D

UNDERSTANDING 1–3 3

1 Write the missing number.

a To subtract -3, add ☐.
b To subtract -6, add ☐.
c To subtract -4, add ☐.
d To subtract -11, add ☐.
e To subtract -15, add ☐.
f To subtract -312, add ☐.

2 Write down the missing numbers in these sentences. Parts **a** and **c** have been started for you.

a $5 - 3$ means that $\boxed{3}$ is subtracted from $\square$.

b $-2 - 6$ means that $\square$ is subtracted from $\square$.

c $7 - (-3)$ means that $\boxed{-3}$ is subtracted from $\square$.

d $-7 - (-11)$ means that $\square$ is subtracted from $\square$.

e $-2 - (-4)$ means that $\square$ is subtracted from $\square$.

f $-6 - (-1)$ means that $\square$ is subtracted from $\square$.

g $-11 - (-7)$ means that $\square$ is subtracted from $\square$.

h $27 - (-12)$ means that $\square$ is subtracted from $\square$.

3 Match each subtraction (**a** to **d**) with the correct diagram (**A** to **D**).

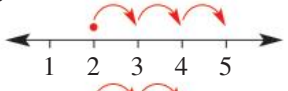
a $5 - (-1)$

b $2 - (-3)$

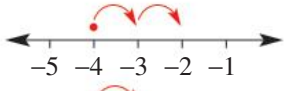
c $-4 - (-2)$

d $-1 - (-1)$

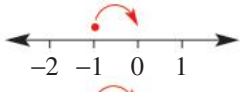
A



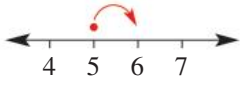
B



C



D



FLUENCY 4–5(½) 4–6(½)

Example 8 Subtracting a negative integer

Calculate:

a $1 - (-3)$ **b** $-6 - (-2)$

Solution	Explanation
a $1 - (-3) = 1 + 3$ $= 4$	To subtract -3, add 3.
b $-6 - (-2) = -6 + 2$ $= -4$	To subtract -2, add 2.

4 Calculate:

a $2 - (-3)$ **b** $5 - (-6)$ **c** $20 - (-30)$

d $29 - (-61)$ **e** $-5 - (-1)$ **f** $-7 - (-4)$

g $-11 - (-6)$ **h** $-41 - (-7)$ **i** $-4 - (-6)$

j $-9 - (-10)$ **k** $-20 - (-20)$ **l** $-96 - (-104)$

m $5 - (-23)$ **n** $28 - (-6)$ **o** $-31 - (-19)$



To subtract a negative integer, add its opposite
$2 - (-3) = 2 + 3$

7D

5 Find the answer if:

a −2 is subtracted from 6
b −1 is subtracted from 10
c −4 is subtracted from −1
d −7 is subtracted from −3
e −2 is subtracted from −7
f −8 is subtracted from −13



For part **a** start with 6 and subtract −2

Example 9 Combining subtractions

Evaluate the following.

a $7 - 9 - (-3)$ **b** $-2 - (-5) - 9$

Solution	Explanation
a $7 - 9 - (-3) = -2 - (-3)$	First work out $7 - 9$.
$= -2 + 3$	To subtract −3, add 3.
$= 1$	−2 plus 3 is 1.
b $-2 - (-5) - 9 = -2 + 5 - 9$	To subtract −5, add 5.
$= 3 - 9$	−2 plus 5 = 3.
$= -6$	3 minus 9 is −6.

6 Evaluate the following.

a $6 - 9 - (-1)$
b $2 - 4 - (-3)$
c $-1 - 3 - (-2)$
d $-3 - (-1) - 4$
e $-4 - (-2) - 5$
f $-10 - (-2) - 6$
g $2 - (-1) - (-3)$
h $-10 - (-4) - (-3)$
i $-16 - (-10) - (-7)$

PROBLEM-SOLVING AND REASONING 7–8(½), 9, 10 10–13

7 Find the missing number.

a $5 - \square = 6$
b $2 - \square = 7$
c $-1 - \square = 3$
d $\square - (-3) = 7$
e $\square - (-10) = 12$
f $\square - (-4) = -20$
g $5 - \square = 11$
h $\square - (-2) = -3$
i $-2 - \square = -4$
j $\square - (-5) = -1$
k $6 - \square = 15$
l $\square - (-2) = -4$

8 Calculate the answer, working from left to right.

a $3 + (-2) + (-1)$
b $2 + (-1) + (-6)$
c $3 - (-1) - (-4)$
d $10 - (-6) + (-4)$
e $-7 - (-1) + (-3)$
f $-20 - (-10) - (-15)$
g $-9 - (-19) + (-16)$
h $-15 - (-20) + (-96)$
i $-13 - (-19) + (-21)$
j $-2 - (-3) - (-5)$
k $-18 - (-16) - (-19)$
l $5 + (-20) - (-26)$

9 A diver is at a height of −19 m from the surface of the sea. During a diving exercise, the diver rises 10 m, falls 18 m and then rises once again by 15 m. What is the diver's final height from sea level?



10 A small business has a bank balance of −$50 000. An amount of $20 000 of extra debt is added to the balance and, later, $35 000 is paid back. What is the final balance?

11 $100 of debt is added to an existing balance of $50 of debt. Later, $120 of debt is removed from the balance. What is the final balance?

12 If a is a positive number and b is a negative number, are the following true?

a $a - b$ is *always* positive.

b $b - a$ is *always* negative.



13 If a is a negative number and b is a negative number, are the following true?

a $a - b$ is *always* positive. **b** $b - a$ is *always* negative.

Make it true

— 14

14 Insert + or − signs to make each statement true.

a $-2 \square 6 \square (-2) = 2$

b $-3 \square (-3) \square 2 = -8$

c $-2 \square (-5) \square 3 = 0$

d $-4 \square (-3) \square (-1) = -6$

e $4 \square (-10) \square (-2) = 12$

f $-3 \square (-16) \square (-2) = -17$

Make up your own and try it on a friend!

Progress quiz

7A **1** Insert the symbol < (less than) or > (greater than) into these statement to make them true.

a $-3 \square 1$

b $-1 \square -6$

c $4 \square -5$

d $-3 \square -2$

7A **2** Write the next three numbers in these patterns.

a 5, 3, 1, ___, ___, ___

b −12, −9, −6, ___, ___, ___

c 4, −1, −6, ___, ___, ___

d −77, −66, −55, ___, ___, ___

7B **3** Evaluate the following.

a $-4+6$ **b** $-7+3$

c $-24+17$ **d** $-15+40$

7B **4** Evaluate the following.

a $8-11$ **b** $-4-5$

c $1-78$ **d** $-32-29$

7B **5** Evaluate the following.

a $4-7+11$ **b** $-3-6-4$

c $6-13-5+2$ **d** $-8+25-17-9$

7B **6** Find the missing number.

a $6-\square=-3$

b $-8-\square=-19$

c $\square+7=-34$

d $\square-81=-16$

7C **7** Calculate answers to the following.

a $8+(-5)$ **b** $12+(-16)$

c $-4+(-3)$ **d** $-31+(-31)$

7C **8** Work from left to right to evaluate the following.

a $4+(-3)+7$ **b** $11+(-21)+5$

c $-13+(-10)+45$ **d** $-83+100+(-27)$

7D **9** Calculate answers to the following.

a $3-(-5)$ **b** $14-(-13)$

c $-27-(-9)$ **d** $-15-(-38)$

7D **10** Work from left to right to evaluate the following.

a $7-12-(-5)$ **b** $-14-21-(-11)$

c $-1+(-5)-(-9)$ **d** $14-(-15)+(-13)$

7D **11** $75 of debt is added to an existing balance of $50 of debt. Later, $100 of debt is removed from the balance. What is the final balance?

7E Substituting integers

Interactive

Widgets

HOTsheets

Walkthrough

Expressions with pronumerals or variables are used to describe many real-world situations. For example, the speed of an accelerating rocket could be shown by $100 + 20t$ metres per second, where t is the time in seconds.

To work out the speed after 5 seconds, we would substitute $t = 5$ into $100 + 20t$ to give $100 + 20 \times 5 = 200$ metres per second.

In this section, we will look at the process of substitution using both positive and negative integers.



Let's start: Order matters

Two students substitute the values $a = -2$, $b = 5$ and $c = -7$ into the expression $ac - bc$. Some of the different answers received are 21, −49, −21 and 49.

- Which answer is correct and what errors were made in the calculation of the other three incorrect answers?

Key ideas

- Substitute into an expression by replacing pronumerals (the letters) with numbers.

  If $a = -2$ and $b = 5$, then:

  $b - a = 5 - (-2)$
  $= 5 + 2$
  $= 7$

  $2b + a = 2 \times 5 + (-2)$
  $= 10 - 2$
  $= 8$

- Use brackets around negative numbers to avoid confusion with other symbols.
- When working with more than one operation and with positive and/or negative numbers:
  - Deal with brackets first.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

$5 \times 2 - (3 - (-2)) \div 5$

2nd: $5 \times 2 = 10$

1st: $3 - (-2) = 5$

3rd: $10 \div 5 = 2$

last: $2 - 1 = 1$

Exercise 7E

UNDERSTANDING 1–3 | 3

1 Find the value of each expression if $a = 3$.

a $a + 3$ **b** $2 - a$
c $2 \times a$ **d** $4 \times a - 10$
e $-2 + a$ **f** $-3 - a$
g $6 \div a$ **h** $-4 - a$

2 Which of the following shows the correct substitution of $a = -2$ into the expression $a - 5$?

A $2 - 5$ **B** $-2 + 5$ **C** $-2 - 5$ **D** $2 + 5$

3 Which of the following shows the correct substitution of $x = -3$ into the expression $2 - x$?

A $-2 - (-3)$ **B** $2 - (-3)$ **C** $-2 + 3$ **D** $-3 + 2$

FLUENCY 4–5(½) | 4–6(½)

Example 10 Substituting integers

Evaluate the following expressions using $a = 3$ and $b = -5$.

a $2 + 4a$ **b** $a - b$ **c** $2a + b$

Solution	Explanation
a $2 + 4a = 2 + 4 \times 3$ $= 2 + 12$ $= 14$	Replace a with 3 and evaluate the multiplication before doing the addition.
b $a - b = 3 - (-5)$ $= 3 + 5$ $= 8$	Replace a with 3 and b with -5. To subtract -5, add 5.
c $2a + b = 2 \times 3 + (-5)$ $= 6 - 5$ $= 1$	Replace a with 3 and b with -5, and evaluate the multiplication before the addition. To add -5, subtract 5.

4 Evaluate the following expressions using $a = 3$ and $b = -2$.

a $2a + 1$ **b** $4a - 3$ **c** $a - 6$
d $3a - 20$ **e** $2 - a$ **f** $4 - 3a$
g $-1 - a$ **h** $-2 - 2a$ **i** $b - 4$
j $b + 8$ **k** $b - a$ **l** $b + 2a$
m $b + 7a$ **n** $a - b$ **o** $2a - b$
p $3a + b$ **q** $9 \div a - b$ **r** $b - 12 \div a$



Substitute $a = 3$ and/or $b = -2$, then work out the answer.

Example 11 Substituting with brackets

Evaluate the following using $x = -2$ and $y = 7$.

a $3 \times (10 - y)$ **b** $(y - x) \div 3$

Solution	Explanation
a $3 \times (10 - y) = 3 \times (10 - 7)$	Substitute $y = 7$.
$= 3 \times 3$	Deal with brackets before other operations.
$= 9$	
b $(y - x) \div 3 = (7 - (-2)) \div 3$	Substitute $x = -2$ and $y = 7$.
$= 9 \div 3$	Evaluate the brackets, using $7 - (-2) = 7 + 2$.
$= 3$	

5 Evaluate the following expressions using $x = 5$ and $y = -3$.

a $2 \times (7 - x)$ **b** $5 \times (8 - x)$
c $(2 + x) \times 4$ **d** $(-2 + x) \div 1$
e $3 \times (1 - y)$ **f** $4 \times (7 - y)$
g $(2 + y) \times 6$ **h** $(4 + y) \div 2$
i $(x + y) \times 4$ **j** $(x + y) \div 1$
k $5 \times (x - y)$ **l** $10 \div (x - y)$



Remember to deal with brackets first, before doing other operations.

6 Evaluate the following if $x = -4$ and $y = 3$.

a $x - y - 6$ **b** $2y + x + 1$
c $3y - x + 7$ **d** $2 - x - y$
e $5y - x$ **f** $2 - x + 3y$
g $-4 + x - 2y$ **h** $-6 + x + 7y$

PROBLEM-SOLVING AND REASONING 7–9 8–11

7 The volume of water running into a tank is given by the expression $10 + 5t$ litres, where t is the time in minutes. Find the volume of water after:

a 2 minutes ($t = 2$)
b 5 minutes ($t = 5$)
c 20 minutes ($t = 20$)

8 Find, by trial and error, the value of a to make each statement true.

a $a + 6$ equals 10 **b** $a - 7$ equals -2
c $2a + 1$ equals 7 **d** $2 - a$ equals 6
e $6 - a$ equals 13 **f** $-1 - a$ equals 0
g $3 + a$ equals -2 **h** $-4 + a$ equals -3
i $-10 - a$ equals -4

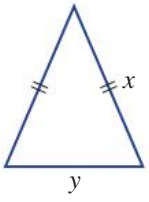


Guess a value for a and substitute to see if you are correct. If not, make a better guess.

7E

9 Albert substitutes $c = -10$ into $10 - c$ and gets 0. Is he correct? If not, what is the correct answer?

10 The formula for the perimeter, P, of this isosceles triangle is $P = 2x + y$.



a Use the formula to find P if:
i $x = 2$ and $y = 1$ **ii** $x = 4$ and $y = -2$

b What problems are there with part **a ii** above?

11 Write two different expressions involving x that give an answer of -10 if $x = -5$.

Negative time

—

12

12 A motorcycle's speed t seconds after passing a particular point on a racing track is given by the expression $20 + t$ metres per second.

a Find the motorcycle's speed after 4 seconds.

b Find the motorcycle's speed at $t = -2$ seconds (i.e. 2 seconds before passing the $t = 0$ point).

c Find the motorcycle's speed at $t = -6$ seconds.



Substitute the value of t into the expression $20 + t$.



7F The number plane

Interactive

Widgets

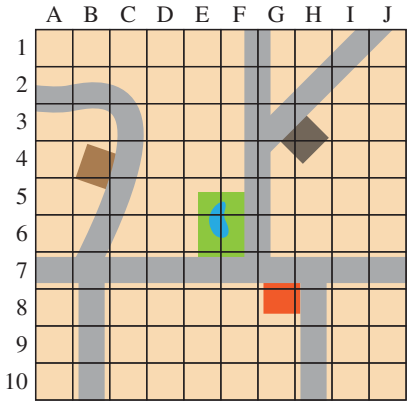
HOTsheets

Walkthrough

Street maps use a coordinate system, where a capital letter and a number, such as D7, are used to locate a square area on the map.

This kind of coordinate system is accurate enough to find a street name or park. In mathematics we need a more precise system so that we can locate points and draw graphs.

The number plane we use today is called the Cartesian plane, after the seventeenth century mathematician René Descartes, and this includes both positive and negative numbers.



Let's start: North, south, east and west

The units for this grid are in metres.

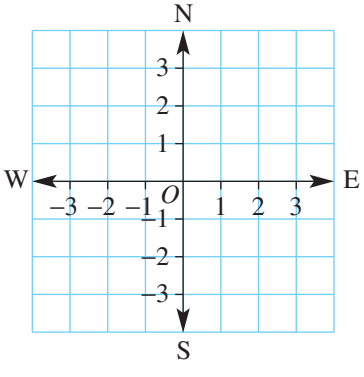
René starts at position O and moves:

- 3 m east
- 4 m west
- 2 m south
- 5 m north.

Pierre starts at position O and moves:

- 1 m west
- 4 m east
- 3 m south
- 5 m north.

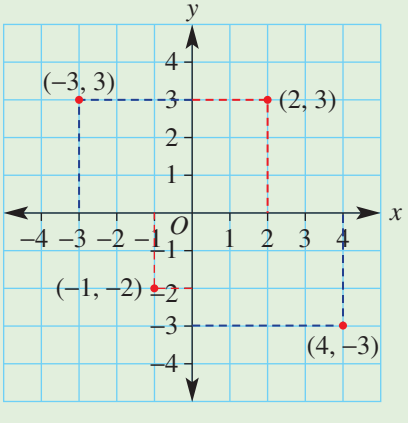
Using the number plane, how would you describe René and Pierre's final positions?



Key ideas

Number plane A plane on which every point is related to a pair of numbers called coordinates

- The **number plane** (also called the Cartesian plane) uses two axes (x-axis and y-axis) at right angles.
- A point plotted on the plane has an x-coordinate and y-coordinate, which are written as (x, y) The x-coordinate is written before the y-coordinate, as in the alphabet.
- The point $(0, 0)$ is called the origin and labelled O.
- To plot points, always start at the origin.
  - For $(2, 3)$ move 2 right and 3 up.
  - For $(4, -3)$ move 4 right and 3 down.
  - For $(-3, 3)$ move 3 left and 3 up.
  - For $(-1, -2)$ move 1 left and 2 down.



Exercise 7F

UNDERSTANDING 1, 2 | 2

1 Put the words *left* or *right* and *down* or *up* into these sentences.

a To move from $(0, 0)$ to $(3, 4)$ you would shift 3 units _____ and 4 units _____.

b To move from $(0, 0)$ to $(2, -1)$ you would shift 2 units _____ and 1 unit _____.

c To move from $(0, 0)$ to $(-3, 2)$ you would shift 3 units _____ and 2 units _____.

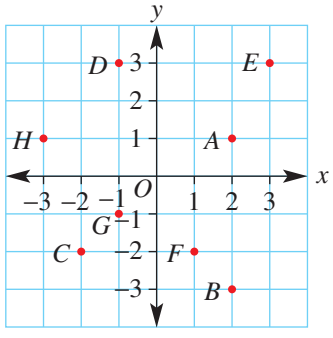
d To move from $(0, 0)$ to $(-1, -4)$ you would shift 1 unit _____ and 4 units _____.

2 Match the points A, B, C, D, E, F, G and H with the given coordinates.

a $(-1, 3)$ **b** $(2, -3)$ **c** $(2, 1)$

d $(-2, -2)$ **e** $(3, 3)$ **f** $(-3, 1)$

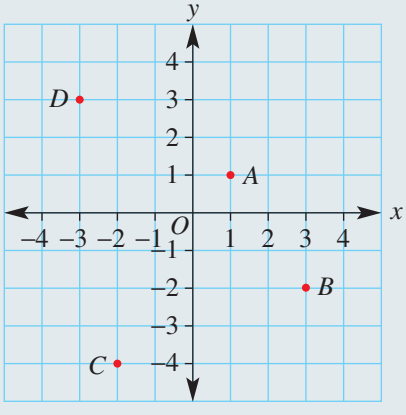
g $(1, -2)$ **h** $(-1, -1)$



FLUENCY 3, 4 | 3–5

Example 12 Finding coordinates

For the number plane shown, write down the coordinates of the points labelled A, B, C and D.



Solution

$A = (1, 1)$

$B = (3, -2)$

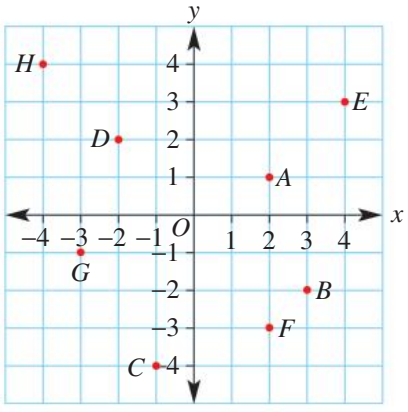
$C = (-2, -4)$

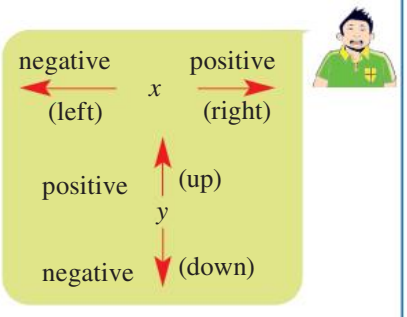
$D = (-3, 3)$

Explanation

For each point, write the x-coordinate first (from the horizontal axis) followed by the y-coordinate (from the vertical axis).

3 For the number plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.

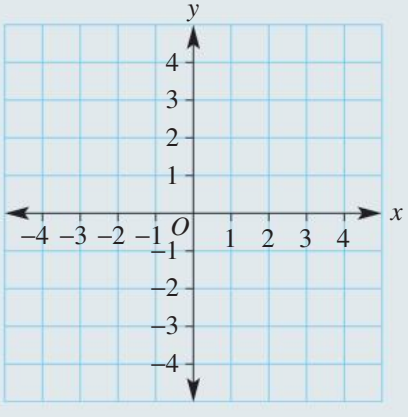




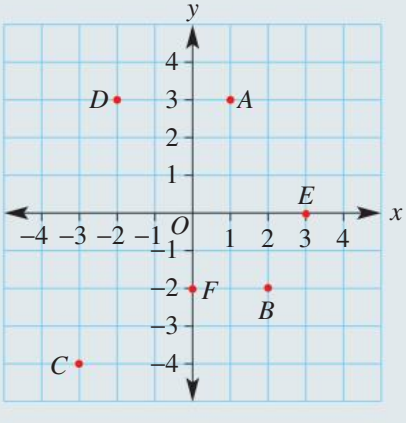
Example 13 Plotting points

Plot these points on the given number plane.

$A(1, 3), B(2, -2), C(-3, -4), D(2, -3), E(3, 0), F(0, -2)$



Solution



Explanation

For each point, start at the origin $(0, 0)$.

First move right (if x is positive) or left (if x is negative).

Then move up (if y is positive) or down (if y is negative).

4 **a** Draw a set of axes like those in the example above. (You can use grid paper to help.)

b Now plot these points.

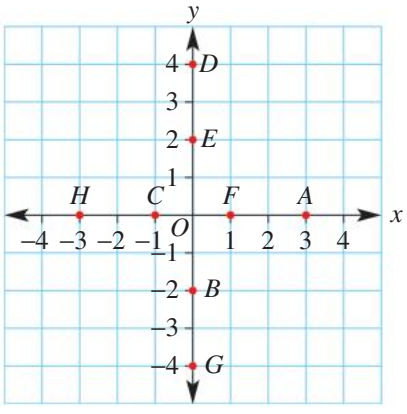
i $A(-3, 2)$ **ii** $B(1, 4)$ **iii** $C(2, -1)$ **iv** $D(-2, -4)$

v $E(2, 2)$ **vi** $F(-1, 4)$ **vii** $G(-3, -1)$ **viii** $H(1, -2)$

ix $I(3, -2)$ **x** $J(-2, 1)$ **xi** $K(-1, -1)$ **xii** $L(1, 2)$

7F

5 For the number plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.

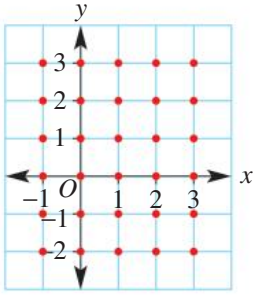




For each point, either the x-coordinate or y-coordinate will be zero.

PROBLEM-SOLVING AND REASONING 6–8 7–10

6 Count the number of points (red dots) on this plane that have:



- **a** both x- and y-coordinates as positive numbers
- **b** an x-coordinate as a positive number
- **c** a y-coordinate as a positive number
- **d** an x-coordinate as a negative number
- **e** a y-coordinate as a negative number
- **f** both x- and y-coordinates as negative numbers
- **g** neither x nor y as positive or negative numbers

7 When plotted on the number plane, what shape does each set of points form?

- **a** $A(-2, 0)$, $B(0, 3)$, $C(2, 0)$
- **b** $A(-3, -1)$, $B(-3, 2)$, $C(1, 2)$, $D(1, -1)$
- **c** $A(-4, -2)$, $B(3, -2)$, $C(1, 2)$, $D(-1, 2)$
- **d** $A(-3, 1)$, $B(-1, 3)$, $C(4, 1)$, $D(-1, -1)$



If you can't 'picture' the points, plot them on a grid.

8 Karen's bushwalk starts at a point $(2, 2)$ on a grid map. Each square on the map represents 1 km. If Karen walks to the points $(2, -7)$, then $(-4, -7)$, then $(-4, 0)$ and then $(2, 0)$, how far has she walked in total?



9 Seven points have the following x- and y-coordinates.

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

a Plot the seven points on a number plane. Use -3 to 3 on the x-axis and -2 to 4 on the y-axis.

b What do you notice about these points on the number plane?

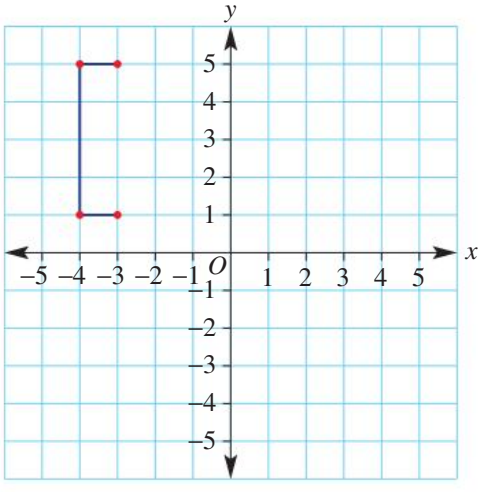
10 The points $A(-2, 0)$, $B(-1, ?)$ and $C(0, 4)$ all lie on a straight line. Find the y-coordinate of point B.

The secret message

—

11

11 Plot these points to decode a secret message. Join points with a line segment if there is a + sign between coordinate pairs. The first one has been done for you.



$(-3, 5) + (-4, 5) + (-4, 1) + (-3, 1)$

$(-2, 2) + (-2, 4) + (-3, 4) + (-3, 2) + (-2, 2)$

$(0, 2) + (0, 4) + (-1, 4) + (-1, 2) + (0, 2)$

$(0, 5) + (1, 5) + (1, 1) + (0, 1)$

$(2, 2) + (2, 4)$

$(3, 2) + (4, 2) + (4, 3) + (3, 3) + (3, 4) + (4, 4)$

$(-1, -2) + (-1, 0)$

$(-2, 0) + (0, 0)$

$(1, 0) + (1, -2)$

$(1, -1) + (2, -1)$

$(2, 0) + (2, -2)$

$(4, 0) + (3, 0) + (3, -2) + (4, -2)$

$(3, -1) + (4, -1)$

$(-4, -3) + (-4, -5) + (-3, -5) + (-3, -3) + (-4, -3)$

$(-2, -5) + (-2, -3) + (-1, -3) + (-1, -4) + (-2, -4) + (-1, -5)$

$(0, -3) + (0, -5)$

$(2, -3) + (1, -3) + (1, -5) + (2, -5) + (2, -4) + (1\frac{1}{2}, -4)$

$(3, -3) + (3, -5)$

$(4, -5) + (4, -3) + (5, -5) + (5, -3)$

Maths@Work: Golfer

Being a golf professional or a golf instructor can be a very exciting career. It is a physically demanding job needing stamina as well as a calculating mind. Distance, slope, angles, velocity of swings, wind speed and direction, as well as club type are all very important factors for success in golf.

Every golf course and each hole is assigned a number called a PAR. This is the number of shots an expert golfer should require to complete that golf course or hole. A player's score is the number of shots above or below the par value. Negative scores are better than positive scores. The person with the lowest score wins!



Golf scoring is based on positive and negative numbers.

1 Different terms are used to describe a player's success at each hole. Write down the positive or negative number that best describes each of the following golfing terms.

a An Eagle which is two under par
b A Birdie which is one under par
c An Albatross which is three under par
d A Bogey which is one over par
e A Double bogey

2 A golfer's handicap is calculated using the average of their best 8 scores from the last 20 games. A handicap of 12 means a person's total shots is adjusted by −12 to give their final score.

- Final score = total shots − handicap
- Very good golfers play 'off scratch' which means they have a zero-handicap.
- Expert golfers have a positive handicap that is added to their total shots.



Write down how the final score will be adjusted for players with:

a a handicap of 7
b a handicap of 20
c a handicap of 5
d playing off scratch
e a positive handicap of 3
f a positive handicap of 2

3 The Lakes Golf club in NSW, where the Australian Open has been held, is rated as a par 72 course. Complete this table for these rounds of golf at Lakes Golf club.

	Total shots	Handicap	Final score	How much over or under the course par is this player's final score?
Chelsea	84	6		
Mitchell	98	16		
Adam	76	6		
Bella	74	+1		

4 The next table has a summary of the scores for two players Rick and Brad at The Lakes.

a Complete the missing values in this score-card.

b Calculate their final scores given Rick's handicap is +3 and Brad's handicap is 7.

c Who won and by how much?

d What was the average number of shots per hole for each of the players? Round your answers to 2 decimal places.

e How many Birdies did each player make on the course?

Hole	PAR	Rick's score	under/over par for Rick	Brad's score	under/over par for Brad
1	4	4		5	
2	4	4		5	
3	4	4		5	
4	4	4		5	
5	4	6		4	
6	4	5		4	
7	3	5		4	
8	5	2		6	
9	3	3		4	
10	4	3		5	
11	5	4		5	
12	4	3		5	
13	4	3		3	
14	5	5		6	
15	3	3		4	
16	4	5		5	
17	5	4		6	
18	3	3		4	
TOTAL					

Using technology

5 The number plane below shows the layout for the first 7 holes of a golf course.



The Green Golf Course	x	y
Tee off 1		
Hole 1		
Tee off 2		
Hole 2		
Tee off 3		
Hole 3		
Tee off 4		
Hole 4		
Tee off 5		
Hole 5		
Tee off 6		
Hole 6		
Tee off 7		
Hole 7		

a Complete the table on the above right for the (x, y) coordinates of each point plotted.

b Using technology such as Excel or GeoGebra, plot and join the points on a number plane graph. (For example, for an Excel spreadsheet: select values/Insert chart/scatter/straight lines and markers. Choose a grid design.)

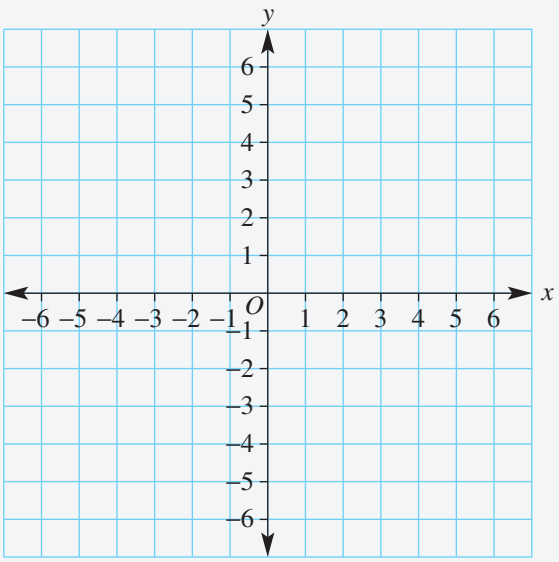
c If you print your graph, you may wish to draw some water hazards, bunkers and sand traps.



A golf ball is placed on a 'tee'. The 'tee off' is the position of the first shot for each hole.

Puzzles and games

1 Plot the following points to find out what I am. Join points with a line segment if there is a + sign between coordinate pairs.
$(-3, 2) + (-3, -4) + (3, -4) + (3, 2)$
$(-1, -4) + (-1, 0) + (1, 0) + (1, -4)$
$(-5, 0) + (0, 5) + (5, 0)$



2 Complete these magic squares. All rows, columns and diagonals have the same sum within a magic square.

a

		−5
	−2	0
	−6	−1

b

−9	5		−6
	−4		−1
		1	
3		−8	6

c

		−1	
−3			
−7	−5	−4	−10
−2		−13	1

3 Insert brackets (if necessary) and symbols (+, −, ×, ÷) into these number sentences to make them true.

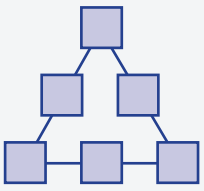
a $-3 \square 4 \square -2 = -6$

b $-2 \square 5 \square -1 \square 11 = 21$

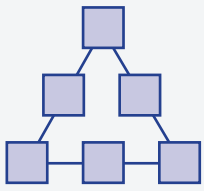
c $1 \square 30 \square -6 \square -2 = -3$

4 Place the integers −3, −2, −1, 0, 1 and 2 into the triangle so that the sum of every side is:

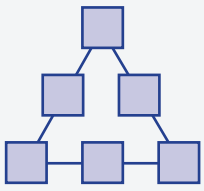
a −3



b 0



c −2



5 Find the next three numbers in these patterns.

a 3, −9, 27, ___, ___, ___

b −32, 16, −8, ___, ___, ___

c 0, −1, −3, −6, ___, ___, ___

d −1, −1, −2, −3, −5, ___, ___, ___

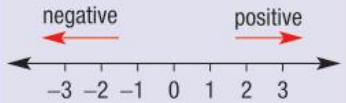
6 **a** The difference between two numbers is 14 and their sum is 8. What are the two numbers?

b The difference between two numbers is 31 and their sum is 11. What are the two numbers?

Chapter summary

Integers
…, –3, –2, –1, 0, 1, 2, 3, …

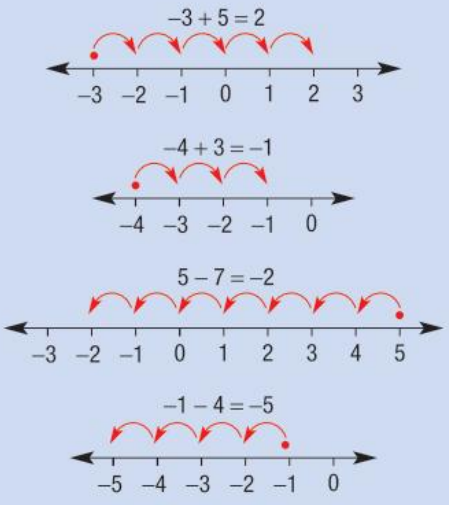
Number line



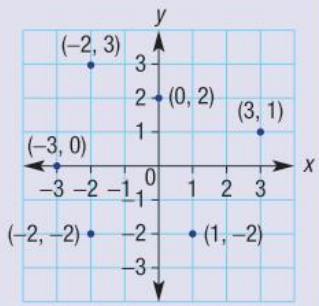
$-2 < 3$ $\quad 1 > -1$

$-5 < -1$ $\quad -2 > -6$

Adding or subtracting a positive integer



Cartesian plane



Substitution

If $a = -2$ and $b = 4$, then:

$b - a = 4 - (-2) = 6$

$2b + a = 2 \times 4 + (-2)$

$= 8 + (-2)$

$= 6$

Order of operations

First brackets, then × or ÷ then + or –, from left to right.

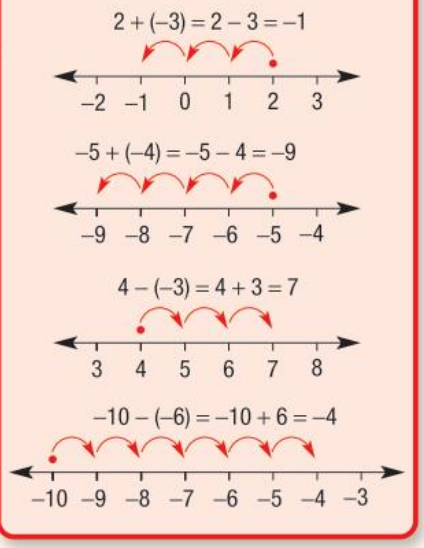
$-2 + 3 \times 2 = -2 + 6$

$= 4$

$(4 - (-2)) \div 6 = 6 \div 6$

$= 1$

Adding or subtracting a negative integer



Multiple-choice questions

7A **1** When the numbers $-4, 0, -1, 7$ and -6 are arranged from lowest to highest, the correct sequence is:

A $0, -1, -4, -6, 7$ **B** $0, -4, -6, -1, 7$ **C** $-6, -4, -1, 0, 7$
D $-1, -4, -6, 0, 7$ **E** $-6, -1, 0, -4, 7$

7B **2** The difference between -3 and 4 is:

A -12 **B** -1 **C** 1
D -7 **E** 7

7D **3** The missing number in $2 - \square = 3$ is:

A 1 **B** -1 **C** 5 **D** -5 **E** 2

7A **4** Which of the following is true?

A $2 < -1$ **B** $-4 > -3$ **C** $0 < -3$ **D** $-4 < -2$ **E** $1 < -4$

7B **5** The temperature inside a mountain hut is $-5°C$. After burning a fire for 2 hours, the temperature rises to $17°C$. What is the rise in temperature?

A $-12°C$ **B** $12°C$ **C** $22°C$ **D** $-85°C$ **E** $-22°C$

7C **6** $-2 + (-3)$ is equal to:

A -5 **B** 1 **C** -2 **D** -1 **E** 5

7D **7** $5 - (-2) + (-7)$ is equal to:

A -4 **B** 10 **C** 7 **D** 0 **E** 14

7E **8** Which operation (i.e. addition, subtraction, multiplication or division) is completed *second* in the calculation of $(-2 + 5) \times 3 + 1$?

A addition **B** subtraction **C** multiplication
D division **E** brackets

7E **9** If $a = -2$ and $b = 5$, then $b - a$ is equal to:

A -3 **B** 3 **C** 7 **D** 10 **E** -7

7F **10** The points $A(-2, 3)$, $B(-3, -1)$, $C(1, -1)$ and $D(0, 3)$ are joined on a number plane. What shape do they make?

A triangle **B** square **C** trapezium
D kite **E** parallelogram

Short-answer questions

7A **1** Write the missing numbers.

a $-4, \square, -2, -1, \square, 1$ **b** $3, 2, \square, 0, -1, \square$
c $-10, -8, \square, -4, \square, 0$ **d** $9, 4, \square, -6, \square, -16$

7A **2** Insert the symbol $<$ (less than) or $>$ (greater than) into each of these statements to make it true.

a $0 \square 7$ **b** $-1 \square 4$ **c** $3 \square -7$ **d** $-11 \square -6$

Chapter review

7B **3** Evaluate:

a $2 - 7$ **b** $-4 + 2$ **c** $0 - 15$
d $-36 + 37$ **e** $-3 + 2$ **f** $-7 + 9$
g $-2 - 5$ **h** $-6 - 19$ **i** $-4 - 7$
j $4 - 11$ **k** $-16 - 31$ **l** $-126 - 5$

7C/D **4** Evaluate:

a $2 + (-1)$ **b** $5 + (-2)$
c $-1 + (-2)$ **d** $-3 + (-4)$
e $5 + (-7)$ **f** $-1 + (-4)$
g $10 - (-2)$ **h** $-21 - (-3)$
i $4 - (-3)$ **j** $-18 - (-1)$
k $-2 - (-5)$ **l** $-15 - (-18)$

7C/D **5** Evaluate:

a $1 - 5 + (-2)$ **b** $-3 + 7 - (-1)$
c $0 + (-1) - 10$ **d** $-2 - (-3) - (-4)$
e $-1 + (-2) - 3$ **f** $-4 - (-1) - 3$
g $-6 - (-4) + 7$ **h** $-17 + (-14) - (-2)$

7C/D **6** Find the missing number for each of the following.

a $-2 + \square = -3$ **b** $-1 + \square = -10$
c $5 - \square = 6$ **d** $-2 - \square = -4$
e $-1 - \square = 20$ **f** $-15 - \square = -13$
g $7 + \square = -80$ **h** $-15 + \square = 15$

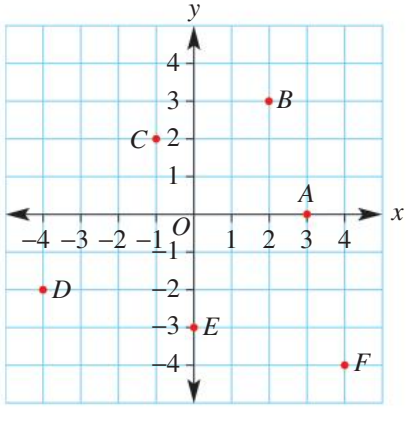
7E **7** Evaluate using order of operations.

a $-2 + 3 \times 2$ **b** $-4 - 7 \div 7$
c $3 \times 4 - 15$ **d** $25 \div 5 - (-2)$
e $(3 - (-2)) \times 6$ **f** $2 \times (4 - (-1))$
g $(7 + (-3)) \div 4$ **h** $20 \times (-1 - (-3))$

7E **8** Evaluate the following expressions if $a = 7$, $b = -3$ and $c = -1$.

a $a + b$ **b** $b + c$
c $c + a$ **d** $b - a$
e $a - b$ **f** $c - b$
g $b - b$ **h** $2a + c$
i $c - 2a$ **j** $b + 3a$
k $c - b - a$ **l** $2a - b + c$

7F **9** For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C, D, E and F.



Extended-response questions

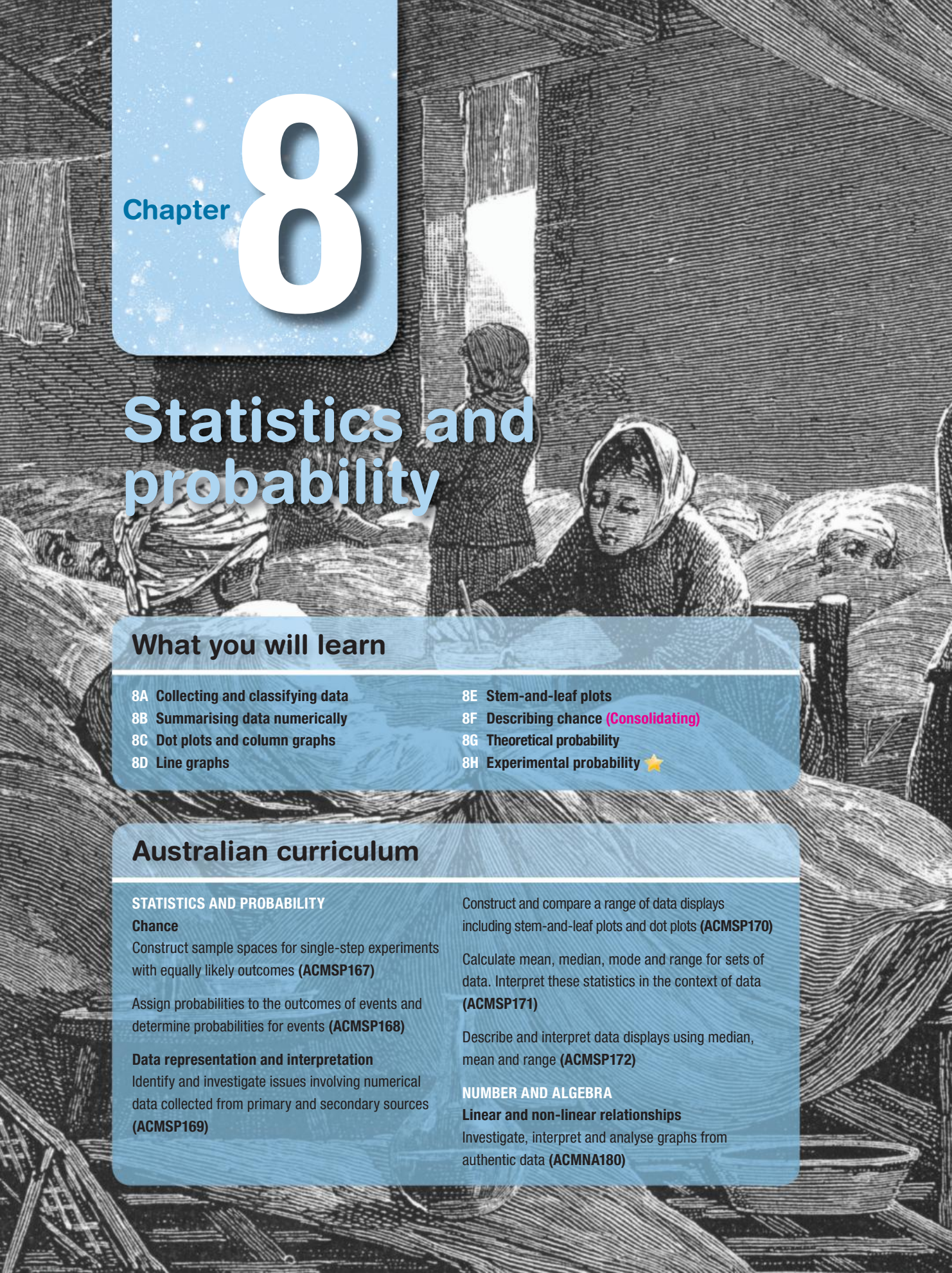
1 A scientist is camped on the ice in Greenland. She records the following details in her notepad about the temperature over five days. Note that 'min' stands for minimum and 'max' stands for maximum.

- Monday: min = −18°C, max = −2°C.
- Decreased 29°C from Monday's max to give Tuesday's min.
- Wednesday's min was −23°C.
- Max was only −8°C on Thursday.
- Friday's min is 19°C colder than Thursday's max.

a What is the overall temperature increase on Monday?
b What is Tuesday's minimum temperature?
c What is the difference between the minimum temperatures for Tuesday and Wednesday?
d What is the overall temperature drop from Thursday's maximum to Friday's minimum?
e By how much will the temperature need to rise on Friday if its maximum is 0°C?



2 When joined, these points form a picture on the number plane.
What is the picture?
$A(0, 5)$, $B(1, 3)$, $C(1, 1)$, $D(2, 0)$, $E(1, 0)$, $F(1, -2)$, $G(3, -5)$, $H(-3, -5)$, $I(-1, -2)$, $J(-1, 0)$, $K(-2, 0)$, $L(-1, 1)$, $M(-1, 3)$, $N(0, 5)$



Chapter 8

Statistics and probability

What you will learn

8A Collecting and classifying data
8B Summarising data numerically
8C Dot plots and column graphs
8D Line graphs
8E Stem-and-leaf plots
8F Describing chance (Consolidating)
8G Theoretical probability
8H Experimental probability ⭐

Australian curriculum

STATISTICS AND PROBABILITY

Chance

Construct sample spaces for single-step experiments with equally likely outcomes **(ACMSP167)**

Assign probabilities to the outcomes of events and determine probabilities for events **(ACMSP168)**

Data representation and interpretation

Identify and investigate issues involving numerical data collected from primary and secondary sources **(ACMSP169)**

Construct and compare a range of data displays including stem-and-leaf plots and dot plots **(ACMSP170)**

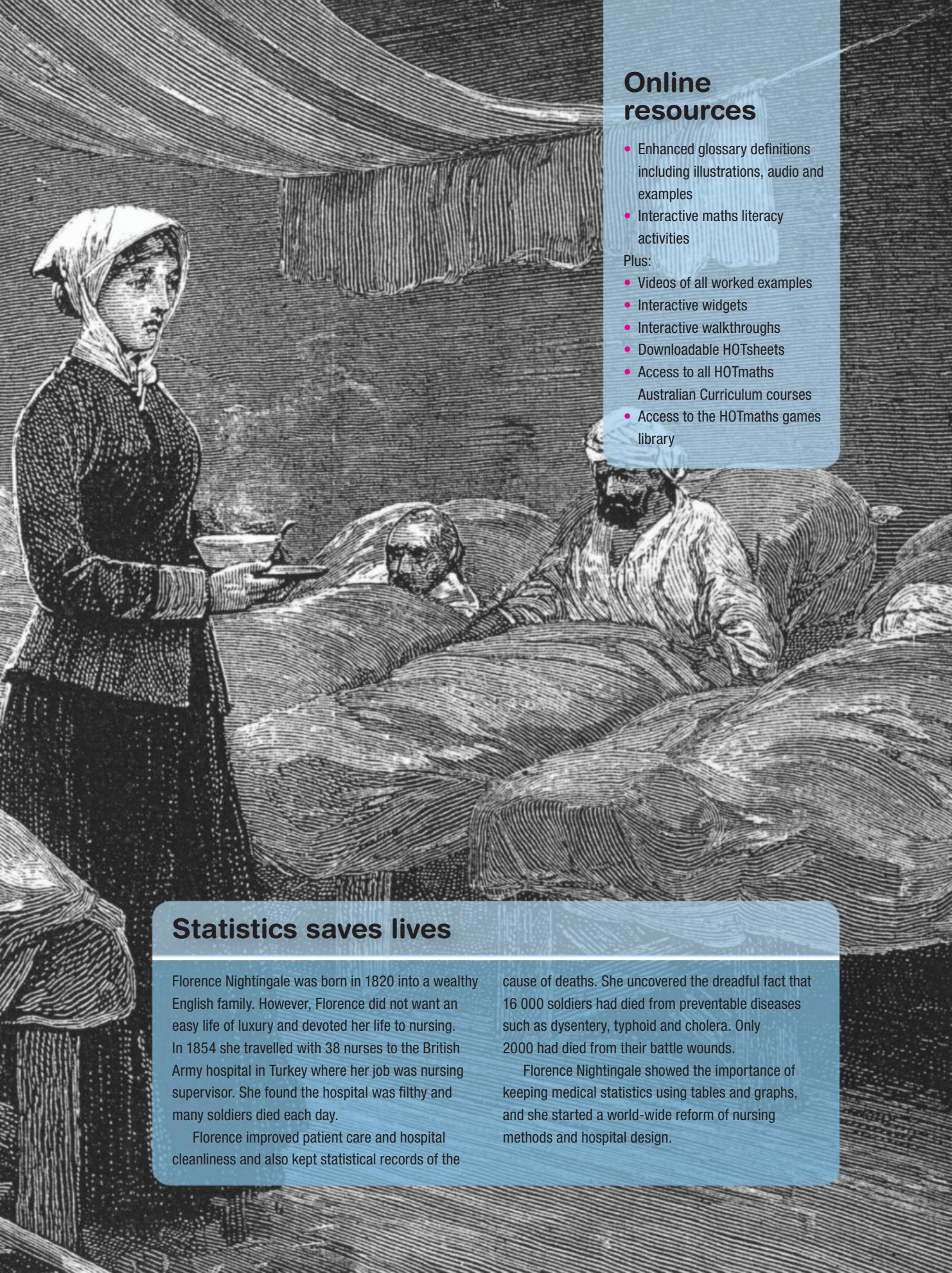
Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data **(ACMSP171)**

Describe and interpret data displays using median, mean and range **(ACMSP172)**

NUMBER AND ALGEBRA

Linear and non-linear relationships

Investigate, interpret and analyse graphs from authentic data **(ACMNA180)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Statistics saves lives

Florence Nightingale was born in 1820 into a wealthy English family. However, Florence did not want an easy life of luxury and devoted her life to nursing. In 1854 she travelled with 38 nurses to the British Army hospital in Turkey where her job was nursing supervisor. She found the hospital was filthy and many soldiers died each day.

Florence improved patient care and hospital cleanliness and also kept statistical records of the cause of deaths. She uncovered the dreadful fact that 16 000 soldiers had died from preventable diseases such as dysentery, typhoid and cholera. Only 2000 had died from their battle wounds.

Florence Nightingale showed the importance of keeping medical statistics using tables and graphs, and she started a world-wide reform of nursing methods and hospital design.

Pre-test

1 Rosemary notes the temperature for a week.

Mon	Tue	Wed	Thu	Fri	Sat	Sun
21°C	24°C	22°C	19°C	20°C	21°C	15°C

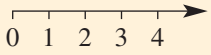
a What was the lowest temperature for the week?
b What was the highest temperature recorded?
c Which temperature(s) occurred on more than one day?

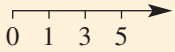
2 Write these fractions in simplest form.

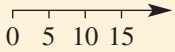
a $\frac{10}{20}$ **b** $\frac{20}{30}$
c $\frac{21}{28}$ **d** $\frac{2}{8}$

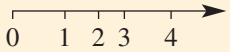
3 Consider the set of numbers 4, 2, 6, 5, 9.
a What is $4 + 2 + 6 + 5 + 9$?
b What is the difference between the smallest and highest number?
c Write the numbers from smallest to largest.

4 For each of the following axes, is the scale correct (C) or incorrect (I)?

a


b


c


d


5 **a** How many faces do dice have?
b How many sides do coins have?

6 Write the following values as decimals.

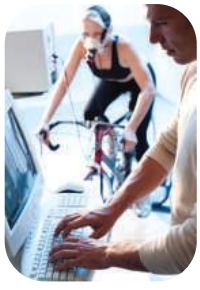
a $2 \div 4$ **b** $20 \div 50$
c $12 \div 60$ **d** $11 \div 55$

7 Order these events from least likely to most likely.
A Rolling a die and it landing on the number 3.
B Flipping a coin and it landing with 'tails' showing.
C The Prime Minister of Australia being struck by lightning tomorrow.
D The internet being used by somebody in the next 20 minutes.

8 Decide if you think these events have a low (L), medium (M) or high (H) chance of occurring.
a Rolling a 6 on a single toss of a die.
b It will rain at least once in the next month.
c Tossing a tail from flipping a coin once.



8A Collecting and classifying data



People collect or use data almost every day. Athletes and sports teams look at performance data, customers compare prices at different stores, investors look at daily interest rates, and students compare marks with other students in their class. Companies often collect and analyse data to help produce and promote their products to customers and to make predictions about the future.

Let's start: Collecting data

Consider, as a class, the following questions and discuss their implications.

- Have you or your family ever been surveyed by a telemarketer at home? What did they want? What time did they call?
- Do you think that telemarketers get accurate data? Why or why not?
- Why do you think companies collect data this way?
- If you wanted information about the most popular colour of car sold in NSW over the course of a year, how could you find out this information?

Key ideas

- In statistics, a **variable** is something that can be measured or observed. It is expected to change over time or between each observation. A variable (or data) can be numerical or categorical.
  - **Numerical data** – is data that can be counted or measured. It can be labelled as discrete or continuous:
    - **discrete numerical** – data that can only be whole number values such as the number of televisions in a house (4 televisions) or the number of cars in a street (10 cars)
    - **continuous numerical** – data that can take any number value, including decimals, such as heights, weights or temperatures. For example, the height of a person could be 172.2 cm, 172.4 cm or 172.215 cm.
  - **Categorical data** – is data that can be placed into categories or types of objects such as colours, genders, brands of car. In a survey, categorical data comes from answers that are given as words (e.g. yellow or female) or ratings (e.g. 1 = dislike, 2 = neutral, 3 = like).
- Data can be collected from primary or secondary sources.
  - Data from a **primary source** are firsthand information collected from the original source by the person or organisation needing the data. For example, a survey an individual student conducts, data collected as a scientist conducts an experiment, or census data collected and then used by an organisation like the Bureau of Statistics.
  - Data from a **secondary source** have been collected, published and possibly summarised by someone else before we use it. Data collected from newspaper articles, textbooks or internet blogs represent secondary source data.

Variable Something that is measurable and observable, which is expected to change over time or between each observation

Numerical data Data that can be counted or measured

Discrete numerical Data that can only be collected in whole number values

Continuous numerical Data that can take any number value

Categorical data Data that can be placed into categories or types of objects

Primary source Information collected from the original source by the person needing the data

Secondary source Information collected, published and summarised by someone else before we use it

- Samples and populations
  - When an entire population (e.g. a maths class, all the cars in a parking lot, a company or a whole country) is surveyed, it is called a **census**.
  - When a subset of the population is surveyed, it is called a **sample**. Samples should be randomly selected and large enough to represent the views of the overall population.
  - When we cannot choose which members of the population to survey, and can record only those visible to us (e.g. people posting their political views on a news website), this is called an **observation**.

Census Collection of data from the whole population

Sample Collection of data from a smaller subset of the whole population

Observation Survey and observation of data from those visible to us

Exercise 8A

UNDERSTANDING 1–3 | 2, 3

1 Look at the following sets of data. Decide if the data is numerical (N) or categorical (C).

a 5, 7, 7, 8, 10, 11, 11, 15
b red, blue, pink, red, yellow, black, black, blue
c high, medium, medium, high, low, medium, low, high
d 0.4, 0.6, −0.4, 0.1, 0.0, −1.3, 7.8, −2.3

2 Choose a word to complete the sentence.

a When an entire _____ is surveyed, it is called a census.
b When a subset of a population is surveyed, it is called a _____.
c Data from a _____ source is firsthand information.
d Data from a _____ source has already been published.

Choose from: *sample*, *secondary*, *primary*, *population*



3 Match each word on the left to its meaning on the right. Check the definitions in the key ideas for help.

a	sample	**i**	only takes on particular numbers within a range
b	categorical	**ii**	a complete set of data
c	discrete numerical	**iii**	a smaller group taken from the population
d	primary source	**iv**	data grouped in categories like ‘male’ and ‘female’
e	continuous numerical	**v**	data collected firsthand
f	population	**vi**	can take on any number in a range

FLUENCY 4–5(½), 6 | 4–5(½), 6, 7

Example 1 Classifying variables

Classify the following variables as categorical, discrete numerical or continuous numerical.

a the gender of a newborn baby
b the length of a newborn baby
c the number of TV sets in people’s homes

Solution	Explanation
a categorical	As the answer is 'male' or 'female' (a word, not a number) the data are categorical.
b continuous numerical	Length is a measurement, so all numbers are theoretically possible.
c discrete numerical	The number of TV sets can only be $0, 1, 2, 3, \ldots$. not 1.385, for example.

4 Classify the following as categorical or numerical.
- **a** the eye colour of each student in your class
- **b** the date of the month each student was born, e.g. the 9th of a month
- **c** the weight of each student when they were born
- **d** the brands of airplanes landing at Sydney's international airport
- **e** the temperature of each classroom
- **f** the number of students in each classroom period one on Tuesday

5 Classify the following variables as categorical, continuous numerical or discrete numerical data.
- **a** the number of cars per household
- **b** the weights of packages sent by Australia Post of the 20th of December
- **c** the highest temperature of the ocean each day
- **d** the favourite brand of chocolate of the teachers at your school
- **e** the colours of the cars in the school car park
- **f** the brands of cars in the school car park
- **g** the number of letters in different words on a page
- **h** the number of advertisements in a time period over each of the free-to-air channels
- **i** the length of time spent doing this exercise
- **j** the arrival times of planes at JFK airport
- **k** the daily pollution levels in the M5
- **l** the number of text messages sent by an individual yesterday
- **m** the times for the 100 m freestyle event at the world championships over the last 10 years
- **n** the number of Blu-ray discs someone owns
- **o** the brands of cereals available at the supermarket
- **p** marks awarded on a maths test
- **q** the star rating on a hotel or motel
- **r** the censorship rating on a movie showing at the cinema



8A

Example 2 Classifying a data source as primary or secondary

Decide if the following data sources are primary or secondary.

a A scientist records the temperature of a chemical every minute for 10 minutes.

b A buyer looking at house price data printed in the newspaper.

Solution	Explanation
a primary data source	The data is collected firsthand from the experiment.
b secondary data source	The data has already been collected and published.

6 Decide if the following data sources are primary or secondary.

a A scientist measures the mass of each mouse in a collection of 20 mice.

b A student records the colours of all the cars that pass in a street over 1 hour.

c Using the data from a website regarding maximum daily temperatures.

d Looking at a graph printed in the newspaper regarding crime rates.



7 Give an example of:

a discrete numerical data

b continuous numerical data

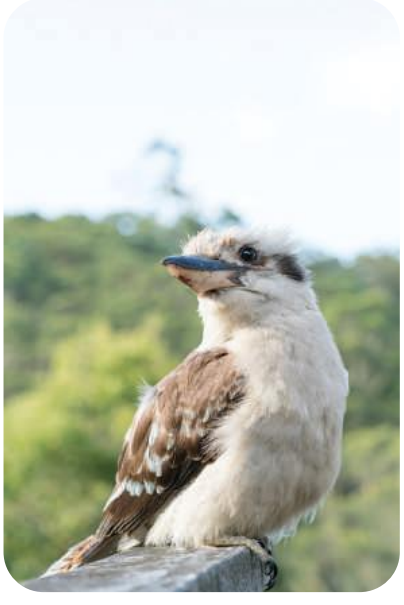
c categorical data

PROBLEM-SOLVING AND REASONING 8–10 8, 10–12



8 Is observation or a sample or a census the most appropriate way to collect data on each of the following?

- **a** the arrival times of trains at central station during a day
- **b** the arrival times of trains at central station over the year
- **c** the heights of students in your class
- **d** the heights of all Year 7 students in the school
- **e** the heights of all Year 7 students in NSW
- **f** the number of plastic water bottles sold in a year
- **g** the religion of Australian families
- **h** the number of people living in each household in your class
- **i** the number of people living in each household in your school
- **j** the number of people living in each household in Australia
- **k** the number of native Australian birds found in a suburb
- **l** the number of cars travelling past a school between 8 a.m. and 9 a.m. on a school day
- **m** the money spent by students during a week at the canteen
- **n** the ratings of TV shows





8A

9 Give a reason why someone might have trouble obtaining reliable and representative data using a primary source to find the following.

- **a** the temperature of the Indian Ocean over the course of a year
- **b** the religions of Australian families
- **c** the average income of someone in India
- **d** drug use by teenagers within a school
- **e** the level of education of different cultural communities within NSW

10 Secondary sources are already published data that are then used by another party in their own research. Why is the use of this type of data not always reliable?

11 When obtaining primary source data you can survey the population or a sample.

- **a** Explain the difference between a 'population' and a 'sample' when collecting data.
- **b** Give an example of a situation where you should survey a population rather than a sample.
- **c** Give an example of a situation where you should survey a sample rather than a population.

12 A Likert-type scale is for categorical data where items are assigned a number; for example, the answer to a question could be 1 = dislike, 2 = neutral, 3 = like.

- **a** Explain why the data collected are categorical even though the answers are given as numbers.
- **b** Give examples of a Likert-type scale for the following categorical data. You might need to reorder some of the options.
  - **i** strongly disagree, somewhat disagree, somewhat agree, strongly agree
  - **ii** excellent, satisfactory, poor, strong
  - **iii** never, always, rarely, usually, sometimes
  - **iv** strongly disagree, neutral, strongly agree, disagree, agree

Australian census

13

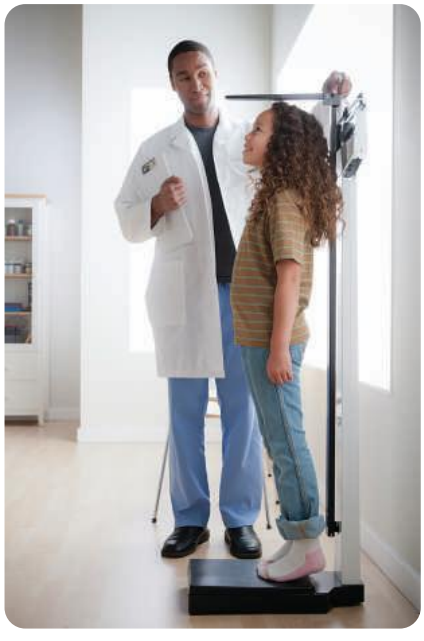
13 Australia's census surveys the entire population every five years.

- **a** Why might Australia not conduct a census every year?
- **b** Over 40% of all Australians were born overseas or had at least one of their parents born overseas. How does this impact the need to be culturally sensitive when designing and undertaking a census?
- **c** The census can be filled out on a paper form or using the internet. Given that the data must be collated in a computer eventually, why does the government still allow paper forms to be used?
- **d** Why might a country like India or China conduct their national census every 10 years?

8B Summarising data numerically

Interactive
Widgets
HOTsheets
Walkthrough

Data sets may contain many numbers and therefore may need to be summarised so we can analyse and interpret the information in a meaningful way. For example, instead of listing the height of every Year 7 student in Australia, you could summarise this by finding and discussing the average height or the difference between the heights of the smallest and tallest people.



Let's start: Average height

- Write down your estimate of the average height (in cm) of students in your class.
- Going round the class, each student should state their height.
- Find the average by adding all the heights and dividing by the number of students in the class.
- What is the difference between the shortest and tallest students in the class?

Key ideas

- The **range** of a set of **data** is given by:
  Range = highest number − lowest number

  1 5 2 7 5 → range = 6
  ↑ ↑
  lowest highest

- The **mean** of a set of data is given by:
  Mean = (sum of all the values) ÷ (total number of values)

  $1 + 5 + 2 + 7 + 5 = 20 \rightarrow \text{mean} = 4$

- The **median** is the middle value if the values are sorted from lowest to highest.
  If there are two middle values, then add them together and divide by 2.

  1 2 (5) 5 7
  ↑
  middle → median = 5

- The **mode** is the most common value. It is the value that occurs most frequently.
  We also say that it is the value with the highest frequency.

  1 2 (5) (5) 7 → mode = 5

Range The difference between the highest and lowest numbers in a set

Data Information (often numerical) gathered by observation, survey or measurement

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Median The middle score when all the numbers in a set are arranged in order

Mode The score that appears most often in a set of numbers

Exercise 8B

UNDERSTANDING 1–6 6

1 Write the numbers 1, 8, 3, 7, 2 from smallest to largest.

2 John finds that the ages of people in a room are 12, 35, 17, 8, 10 and 26 years.
- **a** What is the age of the youngest person?
- **b** How old is the oldest person?

Example 3 Finding the range, mean, median and mode

Consider the ages of seven people who are surveyed in a shop: 15, 31, 12, 47, 21, 65, 12

- **a** Find the range of values.
- **b** Find the mean of this set of data.
- **c** Find the median of this set of data.
- **d** Find the mode of this set of data.

Solution	Explanation
a Range = 65 − 12 = 53	Largest number = 65 Smallest number = 12
b Mean = 203 ÷ 7 = 29	Sum of values = 15 + 31 + 12 + 47 + 21 + 65 + 12 = 203 Number of values = 7
c Values: 12, 12, 15, **21**, 31, 47, 65 Median = 21	Place the numbers in ascending order. The middle value is 21.
d Mode = 12	The most common value is 12.

3 Consider the set of numbers 1, 5, 2, 10, 3.
- **a** Write the numbers from smallest to largest.
- **b** State the largest number.
- **c** State the smallest number.
- **d** What is the range?

4 Consider the numbers 5, 6, 1, 10, 8.
- **a** What is the sum of these numbers?
- **b** How many numbers are there?
- **c** Find the mean of the numbers.



5 Consider the data 2, 5, 6, 9, 10, 12, 20.
- **a** Copy and complete:
  When the numbers are listed in order, the middle number is called the _____.
- **b** State the median of the numbers.

6 The values in a set of data are 2, 8, 3, 5, 2, 7, 2.
- **a** Copy and complete:
  The most common value is called the _____.
- **b** State the mode of the values.

FLUENCY 7–9 7–10

7 State the range of the following sets of numbers.

a 2, 10, 1, 3, 9 **b** 6, 8, 13, 7, 1
c 0, 6, 3, 9, 1 **d** 3, 10, 7, 5, 10



8 State the median for each of the following sets of data.

a 2, 5, 10, 12, 15 **b** 1, 7, 8, 10, 11
c 3, 1, 5, 2, 9 **d** 12, 5, 7, 10, 2
e 12, 18, 31, 15, 19, 10, 12 **f** 17, 63, 4, 13, 97, 82, 56



Example 4 Finding the median with an even number of values

Find the median of 2, 7, 10, 12, 15, 23.

Solution	Explanation
10 and 12 are in the middle. Median = 11	Once the numbers are in ascending order, the two middle values are 10 and 12. The median is $(10 + 12) \div 2 = 11$

9 Find the median of:

a 3, 8, 10, 14, 16, 19 **b** 2, 7, 8, 10, 13, 18
c 1, 5, 2, 9, 13, 17 **d** 5, 2, 3, 11, 7, 15

10 For each of the following sets of data, calculate the mean and the mode.

a 1, 7, 1, 2, 4 **b** 2, 2, 10, 8, 13
c 3, 11, 11, 14, 21 **d** 25, 25, 20, 37, 25, 24
e 1, 22, 10, 20, 33, 10 **f** 55, 24, 55, 19, 15, 36



PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 Brent and Ali make a table to show their test marks for a number of topics in Maths.

Test	1	2	3	4	5	6	7	8	9	10
Brent	58	91	91	75	96	60	94	100	96	89
Ali	90	84	82	50	76	67	68	71	85	57

a Which boy has the higher mean?
b Which boy has the higher median?
c Which boy has the smaller range?
d Which boy do you think is better at tests? Explain why.



8B

12 The number of aces that a tennis player serves per match is recorded over eight matches.

Match	1	2	3	4	5	6	7	8
Number of aces	11	18	11	17	19	22	23	12

a What is the mean number of aces the player serves per match? Round your answer to 1 decimal place.

b What is the median number of aces the player serves per match?

c What is the range of this set of data?



13 The children playing in a room are aged 3, 7, 8 and 10 years.

a What is the mean of these ages?

b An adult enters the room and the mean age is doubled. How old is the adult?



You could guess the adult's age and find the mean. Keep adjusting your guess until you get the correct mean.

Statistical soccer — 14

14 A soccer goalkeeper records the number of saves he makes per game during a season. He presents his records in a table.

Number of saves	0	1	2	3	4	5
Number of games	4	3	0	1	2	2

a In how many games did the keeper make 5 saves?

b How many games did he play this season?

c How many saves in total did he make this season?

d What is the mean number of saves per game this goalkeeper made?

e What is the most common number of saves that he had to make during a game?

f Do these statistics tell you whether the goalkeeper's team was good or bad? Why?



8C Dot plots and column graphs

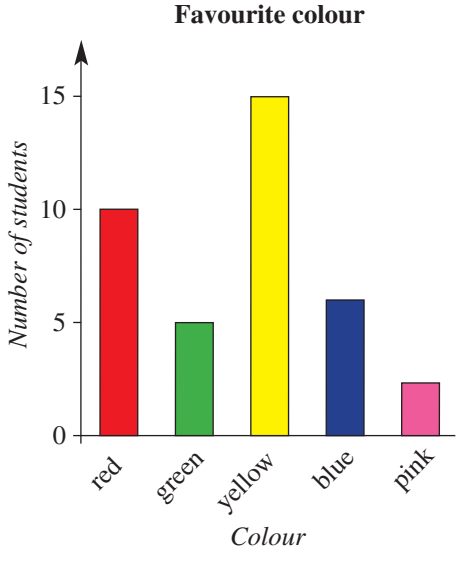
Interactive

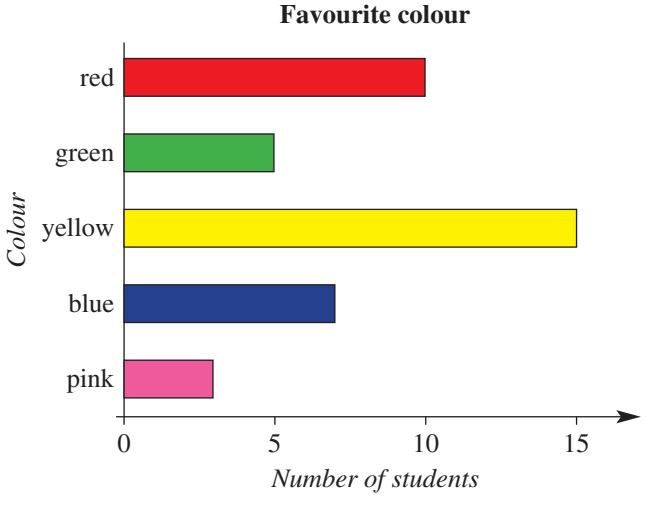
Widgets

HOTsheets

Walkthrough

Graphs are a good way to illustrate data and highlight special information. For example, if students were surveyed on their favourite colours, the results could be shown as a column graph. The 'columns' can be vertical (going up) or horizontal (going across).



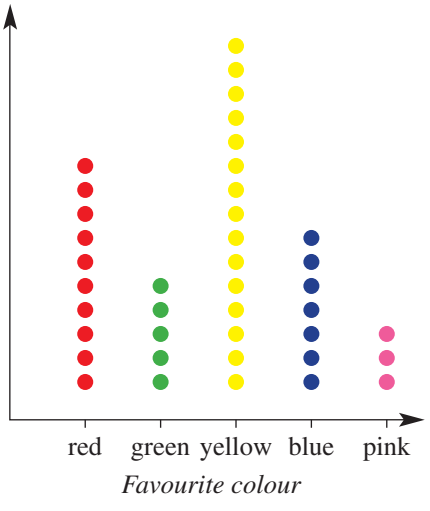


The data could also be shown in a dot plot (below, right).

Let's start: Favourite colours

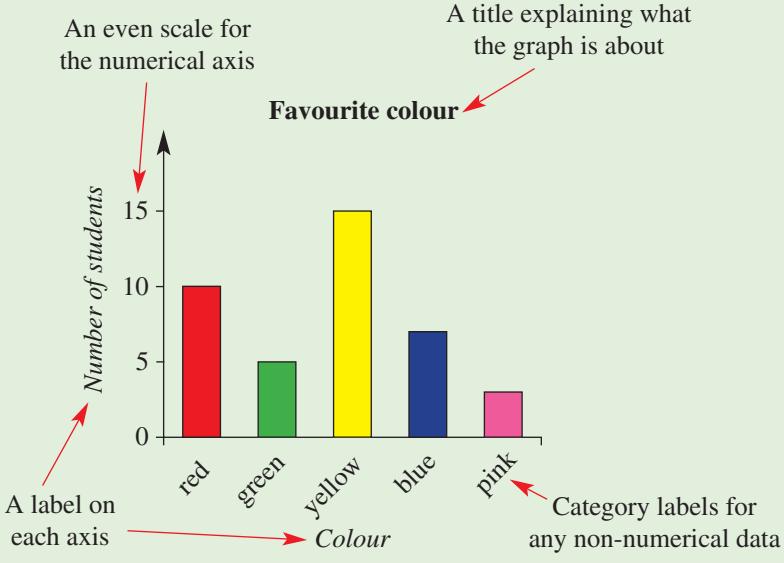
Survey the class to determine each student's favourite colour from the possibilities red, green, yellow, blue and pink.

- Each student should draw a column graph or a dot plot to represent the results.
- Compare your graph with those of other students. Describe any differences you notice.

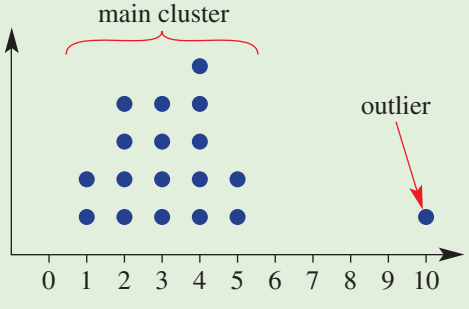


Key ideas

- A **dot plot** can be used to display data, where each dot represents one **datum**.
- A **column graph** is a good way to show data in different categories, and is useful when more than a few items of data are present. Column graphs can be drawn vertically (going up) or horizontally (going across).
- Graphs should have the following features:



- An **outlier** is a value that is noticeably smaller or larger than the main cluster of points.



Dot plot A graph in which each dot represents one score

Datum One score or piece of data

Column graph A graph where the height of each column represents a value

Outlier A value that is much larger or much smaller than the rest of the data

Exercise 8C

UNDERSTANDING 1–3 3

1 Fill in the blanks in the following sentences.

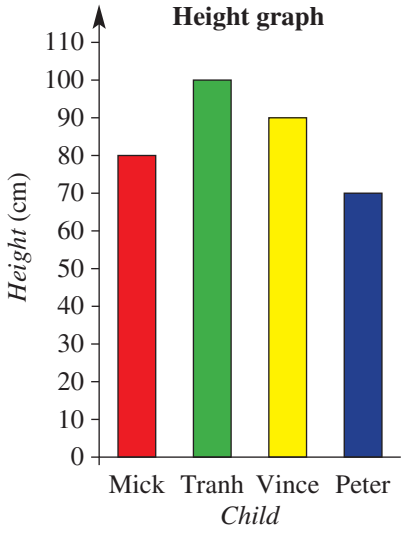
a A _____ _____ is a graph which uses dots to represent data.

b A graph showing data in different categories as rectangles is called a _____ _____ .

c An _____ is a value that is noticeably smaller or larger than the main cluster of points.

2 This column graph shows the height of four boys.
Answer true or false to each of the following statements.

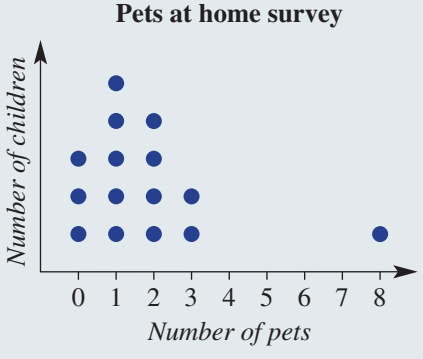
a Mick is 80 cm tall.
b Vince is taller than Tranh.
c Peter is the shortest of the four boys.
d Tranh is 100 cm tall.
e Mick is the tallest of the four boys.



Example 5 Interpreting a dot plot

This dot plot represents the results of a survey that asked some children how many pets they have at home.

a Use the graph to state how many children have 2 pets.
b How many children participated in the survey?
c What is the range of values?
d What is the median number of pets?
e What is the outlier?
f What is the mode?

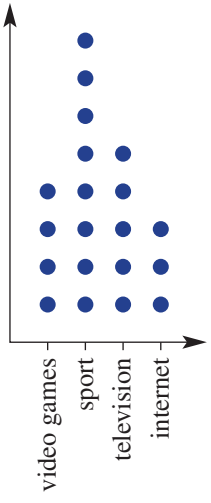


Solution	Explanation
a 4 children	There are 4 dots in the '2 pets' category, so 4 children have 2 pets.
b 15 children	The total number of dots is 15.
c $8 - 0 = 8$	Range = highest − lowest In this case, highest = 8, lowest = 0.
d 1 pet	Write the values in order: 0, 0, 0, 1, 1, 1, 1, (1), 2, 2, 2, 2, 3, 3, 8 Middle value = median = 1

8C

e	8 pets	The main cluster of values is from 0 pets to 3 pets. The dot showing 8 pets is significantly outside this cluster.
f	1 pet	The most common number of pets is 1 pet.

3 The favourite after-school activity of a number of Year 7 students is recorded in this dot plot.



a How many students have chosen television as their favourite activity?

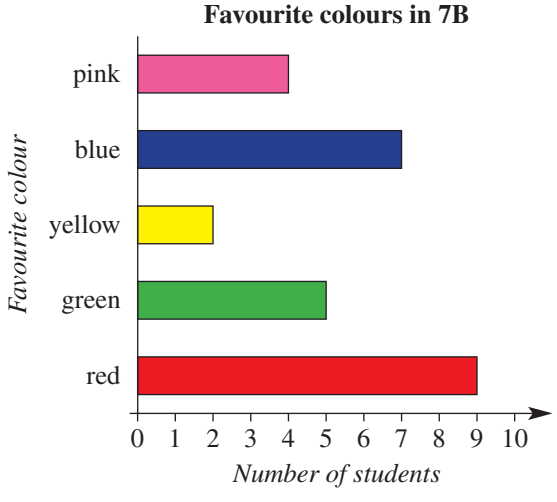
b How many students have chosen surfing the internet as their favourite activity?

c What is the most popular after school activity for this group of students?

d How many students participated in the survey?

FLUENCY 4–7 4–8

4 From a choice of pink, blue, yellow, green or red, each student of Year 7B chose their favourite colour. The results are graphed below.



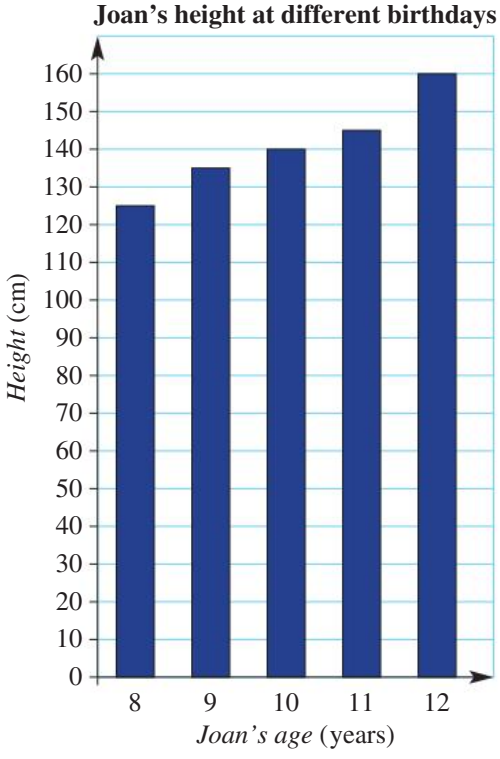
a How many students chose yellow?

b How many students chose blue?

c What is the most popular colour?

d How many students participated in the class survey?

5 Joan has graphed her height at each of her past five birthdays.





a How tall was Joan on her 9th birthday?

b How much did she grow between her 8th birthday and 9th birthday?

c How much did Joan grow between her 8th and 12th birthdays?

d How old was Joan when she had her biggest growth spurt?

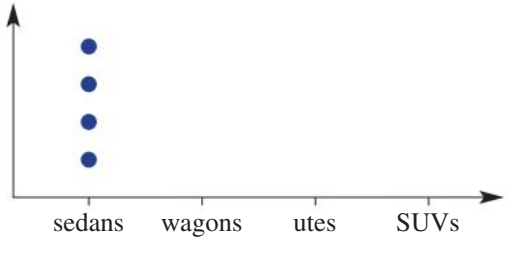
6 The types of cars parked in a small car park were:

Sedan	Wagon	Ute	SUV
4	1	2	3



a How many utes were in the car park?

b Copy and complete the dot plot.



Cars in a car park

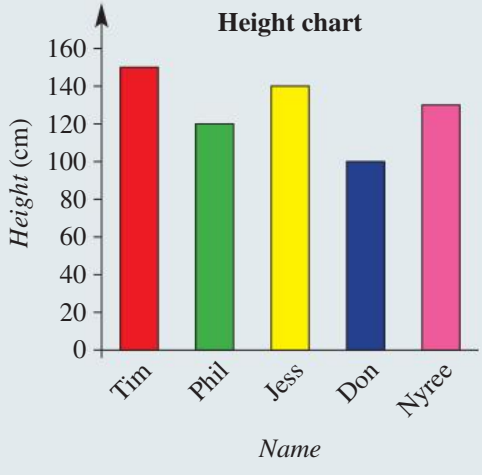
8C

Example 6 Constructing a column graph

Draw a column graph to represent the following people's heights.

Name	Tim	Phil	Jess	Don	Nyree
Height (cm)	150	120	140	100	130

Solution



Explanation

First decide which scale goes on the vertical axis. Maximum height = 150 cm, so axis goes from 0 cm to 160 cm (to allow a bit above the highest value).

Remember to include all the features required, including axes labels and a graph title.

7 Draw a column graph to represent each of these boys' heights at their birthdays.

a Mitchell

Age (years)	Height (cm)
8	120
9	125
10	135
11	140
12	145

b Fatu

Age (years)	Height (cm)
8	125
9	132
10	140
11	147
12	150



The scale on your vertical axis could go 0, 10, 20 … 150.



8 The ages (in years) of children at a party were: 7, 10, 8, 11, 8, 7, 9, 10, 12, 8.

a Represent this as a dot plot.

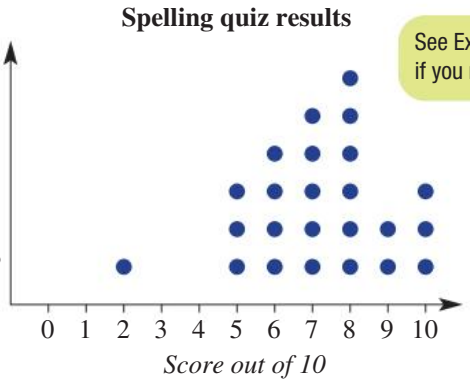
b What is the range of the ages?



Range = largest − smallest.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 The results of a Year 4 spelling quiz are shown as a dot plot.

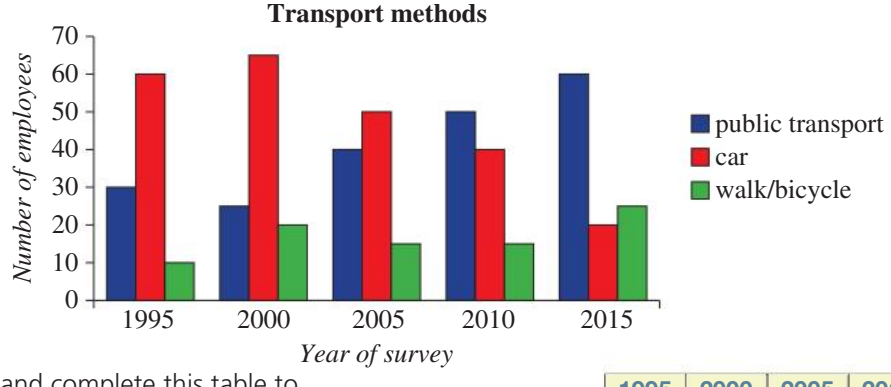




See Example 5 if you need help.

- **a** How many students got a score of 6?
- **b** What is the most common score in the class?
- **c** How many students participated in the quiz?
- **d** What is the range of scores achieved?
- **e** What is the median score?
- **f** Identify the outlier.

10 Every five years, a company in the city conducts a transport survey of the way people get to work in the mornings. The results are graphed below.



- **a** Copy and complete this table to show the data in the graph.
- **b** In which year(s) was public transport the most popular option?

	1995	2000	2005	2010	2015
Use public transport	30				
Drive a car	60				
Walk or cycle	10				

- **c** In which year(s) were more people walking or cycling to work than driving?
- **d** Suggest one reason why the number of people driving to work has decreased.
- **e** What is one other trend that you can see from looking at this graph?





8C

11 **a** Draw a column graph to show the results of the following survey of the number of boys and girls born at a certain hospital. Put the years on the horizontal axis.

	2012	2013	2014	2015	2016	2017
Number of boys born	40	42	58	45	30	42
Number of girls born	50	40	53	41	26	35

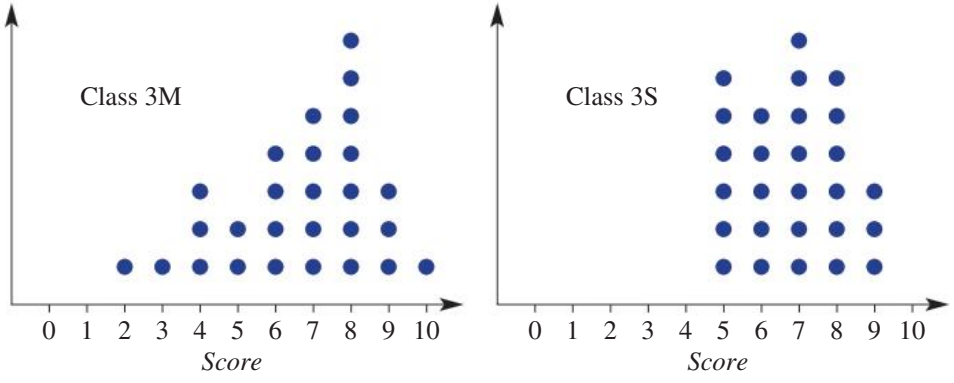
b During which year(s) were more girls born than boys?

c Which year had the smallest number of births?

d Which year had the greatest number of births?

e During the time of the survey, were more boys or girls born?

12 Mr Martin and Mrs Stevensson are the two Year 3 teachers at a school. For the latest arithmetic test, they plotted their students' scores on dot plots.



a What is the median score for class 3M?

b What is the median score for class 3S?

c State the range of scores for each class.

d Based on this test, which class has a greater spread of arithmetic abilities?

e If the two classes competed in an arithmetic competition, where each class is allowed only one representative, which class is more likely to win? Why?

Misleading graphs

13



Check the **Key ideas** to see what features graphs should show.

13 A survey is conducted of students' favourite subjects. Someone has tried to show the results in a column graph.

a What is wrong with the scale on the vertical axis?

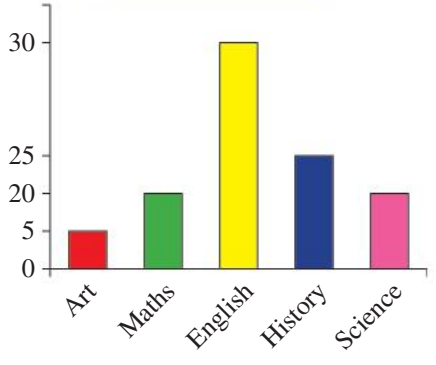
b Give at least two other problems with this graph.

c Redraw the graph with an even scale and appropriate labels.

d The original graph makes Maths look twice as popular as Art, based on the column size. According to the survey, how many times more popular is Maths?

e The original graph makes English look three times more popular than Maths. From the survey, how many times more popular is English?

f Look in a newspaper or magazine for a graph with an uneven scale that makes the graph misleading.



8D Line graphs

Interactive

Widgets

HOTsheets

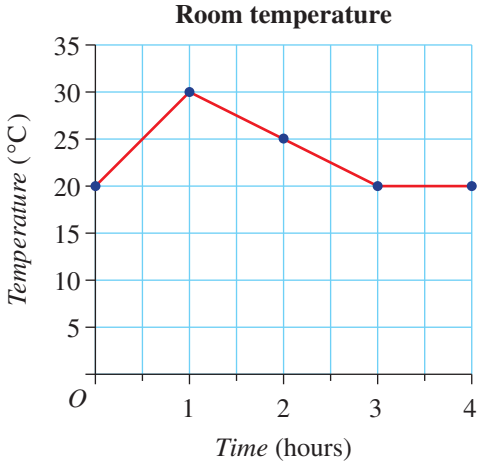
Walkthrough

Line graphs can be used to show how quantities change over time. They clearly show trends in data over time.

Let's start: Room temperature

As an experiment, the temperature in a room is measured hourly over 4 hours. The results are shown in this line graph.

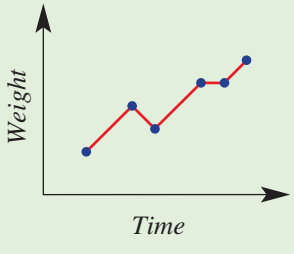
- Describe the temperature changes over the four hours.
- An air conditioner was turned on at some stage. When do you think this happened? Why?
- What was the approximate temperature 90 minutes (1.5 hours) after the experiment started?





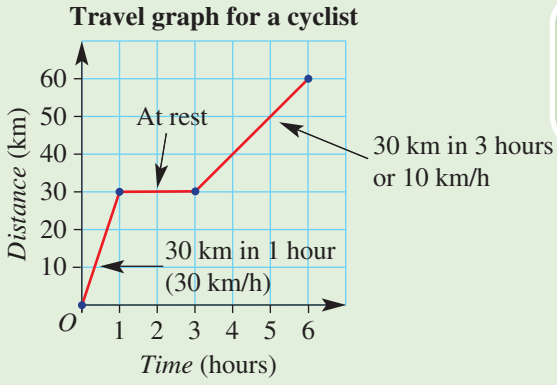
Key ideas

- A **line graph** consists of a series of points joined by straight line segments.
- Time is often shown on the horizontal axis. For example:



Line graph A graph that shows the data as points joined with line segments

- A common type of line graph is a **travel graph**.
  - Time is shown on the horizontal axis.
  - Distance is shown on the vertical axis.
  - The slope of the line shows the rate at which the distance is changing over time. This rate is called speed.



Travel graph A line graph that describes a traveller's position at different times

Exercise 8D

UNDERSTANDING 1–3 3

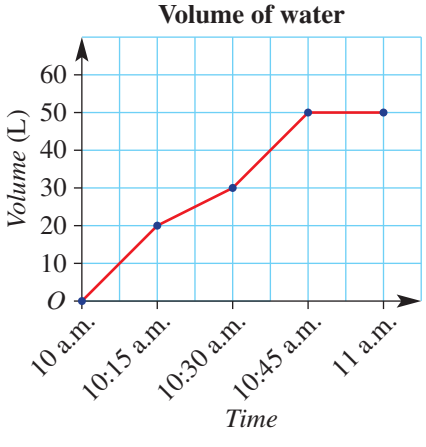
1 The volume of water running into a tank is measured and graphed. State the volume of water at:

a 10:15 a.m.

b 10:30 a.m.

c 10:45 a.m.

d 11 a.m.



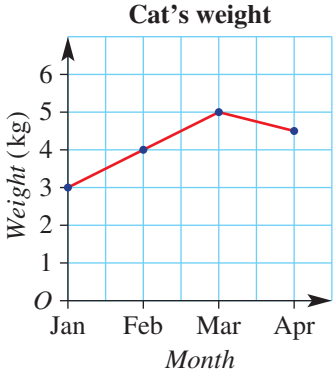
2 This line graph shows the weight of a cat over a 3-month period. The cat is weighed at the start of each month. State the cat's weight at the start of:

a January

b February

c March

d April





Example 7 Drawing a line graph

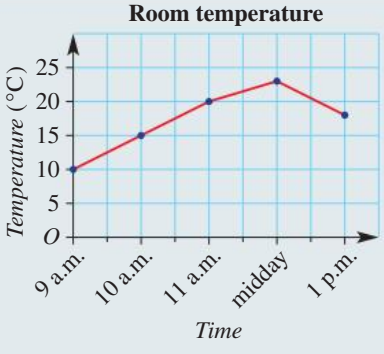
The temperature in a room is noted at hourly intervals.

Time	9 a.m.	10 a.m.	11 a.m.	midday	1 p.m.
Temperature(°C)	10	15	20	23	18

a Draw a line graph of the temperature from 9 a.m. until 1 p.m.
b Use your graph to estimate the room temperature at 12 : 30 p.m.

Solution

a



b About 20°C

Explanation

- The vertical axis is from 0 to 25. The scale is even (i.e. increasing by 5 each time).
- Dots are placed for each measurement and joined with straight line segments.

Look at the graph halfway between midday and 1 p.m. and form an estimate.

3 A dog is weighed over a period of 3 months. Draw a line graph of its weight. January: 5 kg, February: 6 kg, March: 8 kg, April: 7 kg.

Use grid paper to help draw graphs



FLUENCY 4–8 4–9

Example 8 Interpreting a line graph

The graph shows Lillian's height over a 5-year period from birth.

a What was her height when she turned 2?
b Estimate her height at $2\frac{1}{2}$ years.

Solution

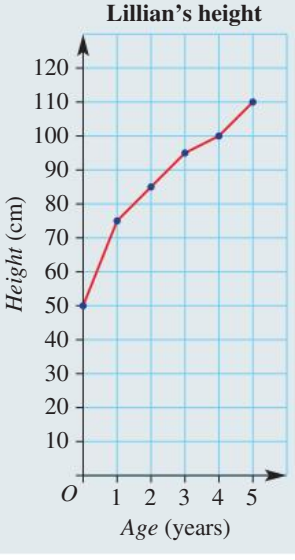
a 85 cm

b About 90 cm

Explanation

Read this directly from the graph.

This is halfway between 2 years (85 cm) and 3 years (95 cm).



8D

4 Look at the graph of Lillian's height in Example 8.

a What was Lillian's height when she was born?

b What was Lillian's height when she turned 3?

c How much did Lillian grow in the year when she was 2 years old?

d Use the graph to estimate her height at the age of $4\frac{1}{2}$ years.



For part **c**, compare Lillian's height at age 2 and age 3.

5 This graph shows the outside temperature over a 24-hour period that starts at midnight.

a What was the temperature at midday?

b When was the hottest time of the day?

c When was the coolest time of the day?

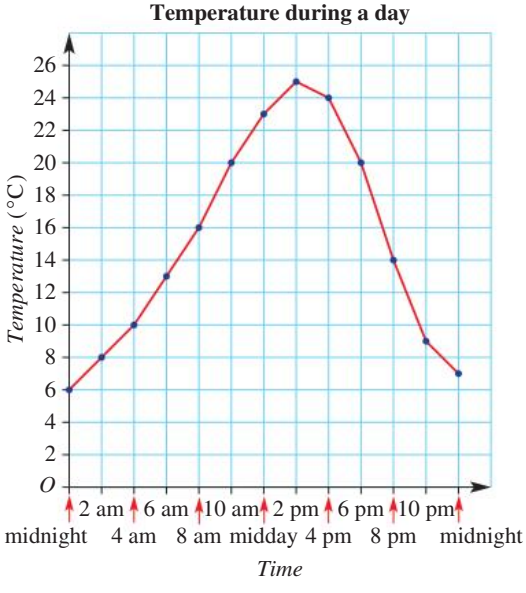
d Use the graph to estimate the temperature at these times of the day.

i 4 a.m.

ii 9 a.m.

iii 1 p.m.

iv 5 p.m.



6 Oliver measures his pet dog's weight over the course of a year. He gets the following results.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Weight (kg)	7	7.5	8.5	9	9.5	9	9.2	7.8	7.8	7.5	8.3	8.5

a Draw a line graph showing this information, making sure the vertical axis has an equal scale from 0 kg to 10 kg.

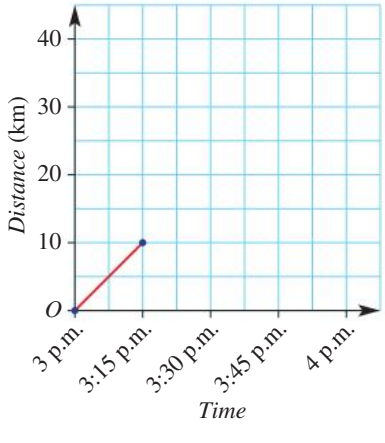
b Describe any trends or patterns that you see.

c Oliver put his dog on a weight loss diet for a period of 3 months. When do you think the dog started the diet? Justify your answer.



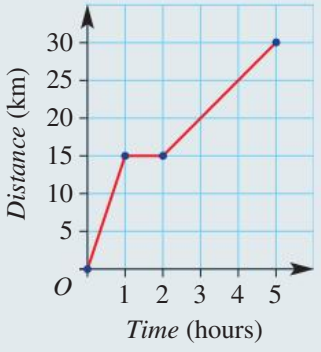
7 This table shows how far Aisha has driven over the course of an hour. Copy and complete the travel graph.

Time	Distance (km) from home
3 p.m.	0
3:15 p.m.	10
3:30 p.m.	15
3:45 p.m.	25
4 p.m.	30



Example 9 Interpreting a travel graph

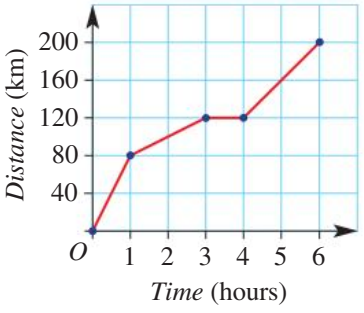
This travel graph shows the distance travelled by a cyclist over 5 hours.



a How far did the cyclist travel in total?
b How far did the cyclist travel in the first hour?
c What is happening in the second hour?
d When is the cyclist travelling the fastest?
e In the fifth hour, how far does the cyclist travel?

Solution	Explanation
a 30 km	The point at the right-hand end of the graph is (5, 30).
b 15 km	At time = 1 hour, the distance covered is 15 km.
c At rest	The distance travelled does not increase in the second hour.
d In the first hour	This is the steepest part of the graph.
e 5 km	In the last 3 hours, the distance travelled is 15 km, so in 1 hour, 5 km is travelled.

8 This travel graph shows the distance travelled by a van over 6 hours.

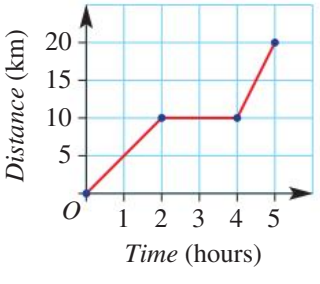


a How far did the van travel in total?
b How far did the van travel in the first hour?
c What is happening in the fourth hour?
d When is the van travelling the fastest?
e In the sixth hour, how far does the van travel?



For part **c**, the fourth hour is from 3 to 4 hours.

9 This travel graph shows the distance travelled by a cyclist over 5 hours.

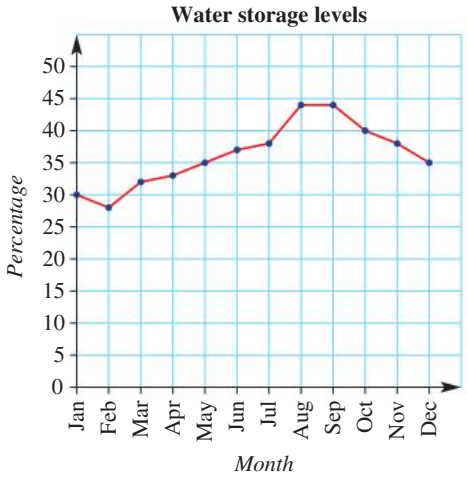


a How far did the cyclist ride in total?
b How far did the cyclist ride in the second hour?
c During which hour did the cyclist ride the fastest?
d For how long did the cyclist rest?

8D

PROBLEM-SOLVING AND REASONING 10 10, 11

10 The graph shows water storage levels for a certain city.



- **a** What was the water level at the start of:
  - **i** January?
  - **ii** May?
  - **iii** December?
- **b** Which month do you think had the highest rainfall? Why?
- **c** What was the maximum water level?
- **d** When did the water storage get to its lowest point?

11 Draw travel graphs to illustrate the following journeys.

- **a** A car travels:
  - 120 km in the first 2 hours
  - 0 km in the third hour
  - 60 km in the fourth hour
  - 120 km in the fifth hour
- **b** A jogger runs:
  - 12 km in the first hour
  - 6 km in the second hour
  - 0 km in the third hour
  - at a rate of 6 km per hour for 2 hours

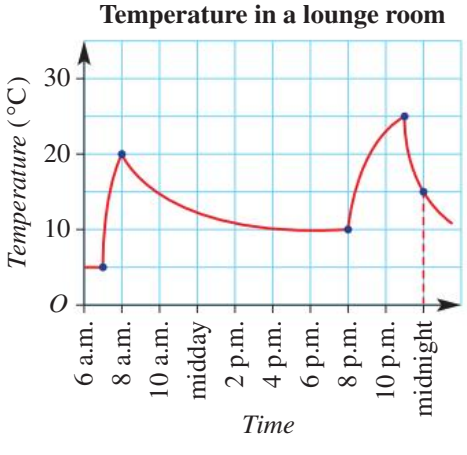


When the distance travelled in an hour is 0 km, draw a horizontal line.

Heating and cooling

— 12

12 The temperature in a lounge room is measured several times on a particular day. The results are shown in a line graph.



- **a** State the room's temperature at:
  i 6 a.m. **ii** 8 a.m. **iii** 10 a.m. **iv** 8 p.m.
- **b** Twice during the day the heating was switched on. At what times do you think this happened? Explain your reasoning.
- **c** When was the heating switched off? Explain your reasoning.
- **d** The house has a single occupant, who works during the day. Describe when you think that person is:
  i waking up **ii** going to work
  iii coming home **iv** going to bed
- **e** These temperatures were recorded during a cold winter month. Draw a graph that shows what the lounge room temperature might look like during a hot summer month. Assume that the room has an air conditioner, which the person is happy to use when at home.

8E Stem-and-leaf plots









A stem-and-leaf plot is a useful way of presenting numerical data. It allows trends to be spotted easily. Each number is split into a stem (the first digit or digits) and a leaf (the last digit).

	Stem	Leaf
53 is	5	3
78 is	7	8
125 is	12	5

Let's start: Test score analysis

In a class, students' results on two recent tests out of 50 are recorded.

Test 1 results
43, 47, 50, 26, 38, 20, 25, 20, 50, 44, 33, 47, 47, 50, 37, 28, 28, 22, 21, 29

Test 2 results

Stem	Leaf
1	8
2	7 8
3	2 2 4 5 5 7 9
4	0 1 2 3 3 6 8 8
5	0 0

- For each test, try to find how many students:
  - achieved a perfect score (i.e. 50)
  - failed the test (i.e. less than 25)
  - achieved a mark in the 40s.
- If there are 100 test results to analyse, would you prefer a list or a stem-and-leaf plot? Why?

Key ideas

Stem-and-leaf plot
A table that lists numbers in order, grouped in rows

- A **stem-and-leaf** plot is a way to display numerical data.
- Each number is split into a stem (the first digit or digits) and a leaf (the last digit).
  For example:

	Stem	Leaf
The number 7 is	0	7
The number 31 is	3	1
The number 152 is	15	2

Key 3|1 means 31

- A key is added to the plot to show the value of the stem and leaf.
  For example: 3|1 means 31 or 2|7 means 2.7.
- Leaves are aligned vertically, getting bigger as you move away from the stem.
- A space is used to separate each leaf value. Commas are not used between leaves. E.g.

Stem	Leaf
3	5 7 7 9

Exercise 8E

UNDERSTANDING 1–4 | 4

1 Copy and complete:
In a stem-and-leaf plot the first digit(s) of a data value is called the _____ and the last digit is called the _____.

2 The number 52 is entered into a stem-and-leaf plot.
a Which digit is the stem?
b Which digit is the leaf?

3 What number is represented by the following combinations if the digit on the right represents ones?
a 3|9 **b** 2|7 **c** 13|4

4 In this stem-and-leaf plot, the smallest number is 35.
What is the largest number?

Stem	Leaf
3	5 7 7 9
4	2 8
5	1 7

FLUENCY 5–9 | 5–10

Example 10 Interpreting a stem-and-leaf plot

Average daily temperatures are shown for some different countries.

a Write out the temperatures as a list.
b How many countries' temperatures are represented?
c What are the maximum and minimum temperatures?
d What is the range of temperatures recorded?
e What is the median temperature recorded?

Stem	Leaf
1	3 6 6
2	0 0 1 2 5 5 6 8 9
3	0 2

2|5 means 25°C

Solution	Explanation
a 13, 16, 16, 20, 20, 21, 22, 25, 25, 26, 28, 29, 30, 32	Each number is converted from a stem and a leaf to a single number. For example, 1|3 is converted to 13.
b 14	The easiest way is to count the number of leaves – each leaf corresponds to one country.

c minimum = 13°C
maximum = 32°C

The first stem and leaf is 1|3.
The last stem and leaf is 3|2.

d range = 19°C

Range = maximum − minimum = 32 − 13 = 19

e median = 23.5°C

The middle value is halfway between the numbers 2|2 and 2|5, so median $= \frac{1}{2}(22 + 25) = 23.5$

5 This stem-and-leaf plot shows the ages of people in a group.

Stem	Leaf
0	8 9
1	0 1 3 5 7 8
2	1 4

1|5 means 15 years old

a Write out the ages as a list.

b How many ages are shown?

c Answer true or false to each of the following.

i The youngest person is aged 10.

ii Someone in the group is 17 years old.

iii Nobody listed is aged 20.

iv The oldest person is aged 4.

6 For each of the stem-and-leaf plots below, state the range and the median. (See Example 10 parts **d** and **e**.)

a

Stem	Leaf
0	9
1	3 5 6 7 7 8 9
2	0 1 9

1|8 means 18

b

Stem	Leaf
1	1 4 8
2	1 2 4 4 6 8
3	0 3 4 7 9
4	2

3|4 means 34

c

Stem	Leaf
3	1 1 2 3 4 4 8 8 9
4	0 1 1 2 3 5 7 8
5	0 0 0

4|7 means 47



7 Copy and complete the stem-and-leaf plot for this set of data.
25, 27, 29, 30, 32, 39, 41, 42, 45, 51

Stem	Leaf
2	
3	
4	
5	

Remember to list the leaves in increasing order across each row.



8E

Example 11 Creating a stem-and-leaf plot

Represent this set of data as a stem-and-leaf plot: 23, 10, 36, 25, 31, 34, 34, 27, 36, 37, 16, 33

Solution

Sorted list:
10, 16, 23, 25, 27, 31, 33, 34, 34, 36, 36, 37

Stem	Leaf
1	0 6
2	3 5 7
3	1 3 4 4 6 6 7

2|5 means 25

Explanation

Sort the list in increasing order so that it can be put directly into a stem-and-leaf plot.

Split each number into a stem and a leaf. Stems are listed in increasing order. Leaves are written vertically, listed in increasing order across each row.

8 Show each of the following sets of data as a stem-and-leaf plot.
- **a** 11, 12, 13, 14, 14, 15, 17, 20, 24, 28, 29, 31, 32, 33, 35
- **b** 20, 22, 39, 45, 47, 49, 49, 51, 52, 52, 53, 55, 56, 58, 58

9 Show each of the following as a stem-and-leaf plot.
- **a** 21, 35, 24, 31, 16, 28, 48, 18, 49, 41, 50, 33, 29, 16, 32
- **b** 32, 27, 38, 60, 29, 78, 87, 60, 37, 81, 38, 11, 73, 12, 14

10 Show each of the following sets of data as a stem-and-leaf plot.
- **a** 80, 84, 85, 86, 90, 96, 101, 104, 105, 110, 113, 114, 114, 115, 119
- **b** 401, 420, 406, 415, 416, 406, 412, 402, 409, 418, 404, 405, 391, 411, 413, 413, 408, 395, 396, 417



Note, 101 is represented as 10|1.

PROBLEM-SOLVING AND REASONING 11 11, 12

11 A company recorded the duration (in seconds) that visitors spent on its website's home page.

Stem	Leaf
0	2 4 6 8 9
1	0 0 1 2 8
2	2 7 9
3	
4	
5	8

2|7 means 27 seconds



An outlier is a value that is not close to the other values.

- **a** How many visitors spent less than 20 seconds on the home page?
- **b** How many visitors spent more than half a minute?
- **c** How many visitors spent between 10 and 30 seconds?
- **d** What is the outlier for this stem-and-leaf plot?

12 A teacher has compiled test scores out of 50 as a stem-and-leaf plot. However, some values are missing, as represented by the letters a, b, c and d.

Stem	Leaf
1	5
2	4 5 a 6 7 9
3	b 0 1 5
4	2 8 c
5	d

3|5 means 35

a How many students took the test?

b How many students passed the test (i.e. achieved a mark of 25 or higher)?

c State the possible values for each of the missing digits a to d.

Back-to-back stem-and-leaf plots — 13

13 Two radio stations poll their audience to determine their ages.



The leaves for station 1 are read in reverse: 8 7|2 means 27 and 28.

Station 1	Stem	Station 2
0	1	2 3 3 4 5 6 8 9
8 7	2	0 0 1 2 4 5 8 8
9 7 5 4 3 3	3	1 1 2
7 6 5 5 4 4 1	4	8
9 3 2 0	5	

3|2 means 32 years old

a Find the age difference between the oldest and youngest listener polled for:
i station 1 **ii** station 2

b One radio station plays modern music that appeals to teenagers. The other plays classical music and broadcasts the news. Which radio station is most likely to be the one that plays classical music and news?

c Advertisers want to know the age of the stations' audiences. This lets them target their advertisements more effectively (e.g. to 38 to 58 year olds). Give a 20-year age range for the audience majority who listen to:
i station 1 **ii** station 2



Progress quiz

8A **1** Consider the ages of five people who are surveyed at a school: 16, 10, 12, 11, 16

a Find the range of values.
b Find the mean of this set of data.
c Find the median of this set of data.
d Find the mode of this set of data.

8A **2** Find the median of:

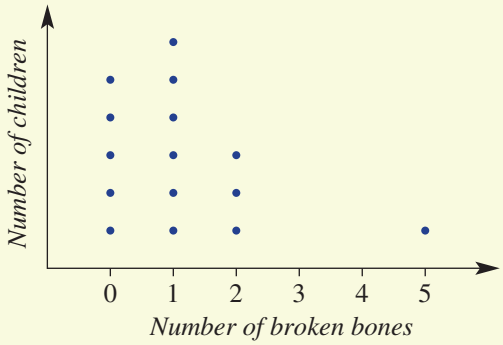
a 5, 12, 15, 17, 24, 26 **b** 40, 32, 6, 18, 24, 41

8A **3** For each of the following sets of data, calculate the mean and the mode.

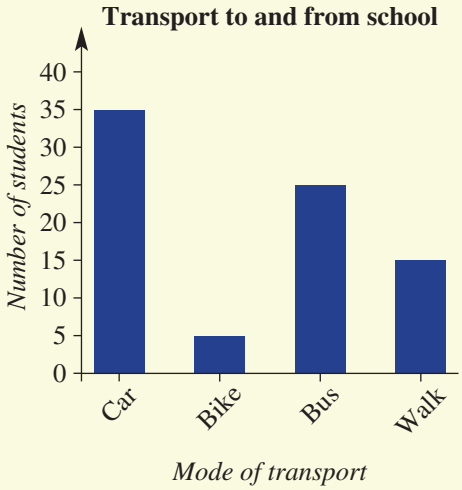
a 4, 12, 21, 36, 12, 11 **b** 20, 30, 30, 12, 20, 20

8B **4** This dot plot represents the results of a survey that asked some children how many broken bones they have had.

a Use the graph to state how many children have had 2 broken bones.
b How many children participated in the survey?
c What is the range of values?
d What is the median number of broken bones?
e What is the outlier?
f What is the mode?



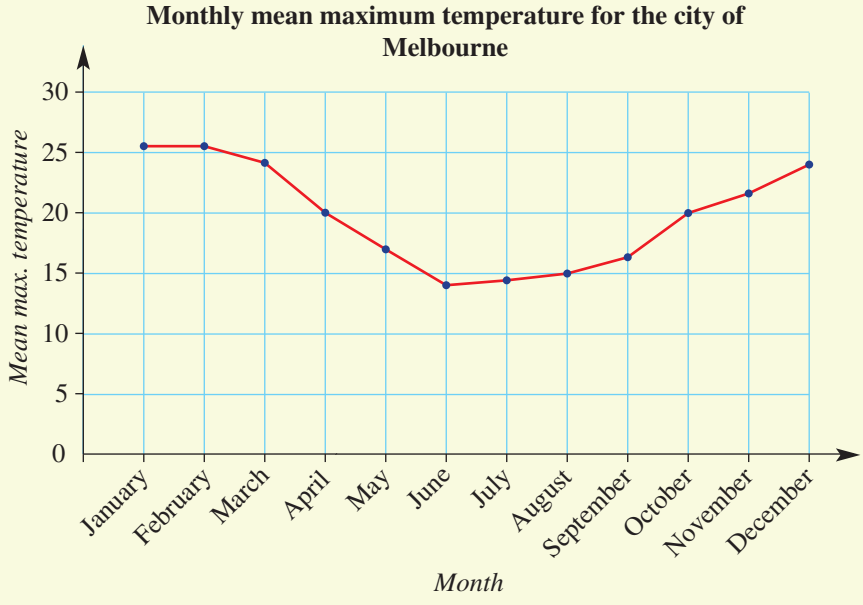
8B **5** A survey was conducted of all the Year 7 students at Brimbrook Secondary College in regards to how they generally commuted to and from school. The answers to the survey are shown in the following column graph.



a How many students were surveyed?
b Write the four modes of transport in order from most common to least common.
c How many more students travelled by bus rather than walked?

8C

6 The line graph shows the monthly mean maximum temperature, rounded to the nearest whole number, for the city of Melbourne.



a Which months have a mean maximum temperature above 25°C?
b What is the range of monthly mean maximum temperatures?
c How much cooler is the mean maximum temperature in May than October?
d What is the average mean maximum temperature for the year? Give your answer rounded to the nearest whole number.

8D

7 The amount of time (in minutes) students spent completing Maths homework on a particular night is represented in the following stem-and-leaf plot.

Stem	Leaf
1	2 5 7
2	1 1 2 4 5 7 8 8
3	0 0 3 6

2|4 means 24 minutes

a Write out the times as a list.
b How many students were surveyed?
c What are the maximum and minimum times spent completing Maths homework?
d What is the range of times spent completing Maths homework?
e What is the median time spent completing Maths homework?

8D

8 Show the following set of data as a stem-and-leaf plot:
57, 63, 60, 71, 78, 60, 54, 59, 74, 69, 68, 65, 66, 60, 73, 86

8F Describing chance

CONSOLIDATING









There may be times when you want to describe how likely it is that a certain event will occur. For example, how likely is your sporting team to win this year's premiership? Or how likely is it that you with receive a total of 12 from rolling two dice? Probability is the study of chance.



Let's start: Likely or unlikely?

Try to rank these events from least likely to most likely. Compare your answers with other students in the class.

- It will rain tomorrow.
- Australia will win the soccer World Cup.
- Tails will land on top when a 20-cent coin is tossed.
- The king of spades is at the top of a shuffled deck of 52 playing cards.
- A diamond card is at the bottom of a shuffled deck of 52 playing cards.
- The sun will rise tomorrow.

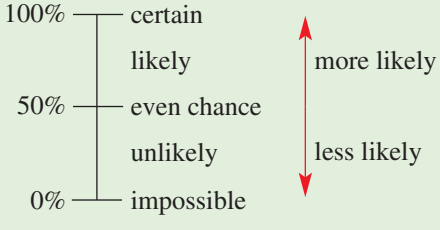


The probability of winning first prize in a lottery is very, very low.

Key ideas

- There are a number of words and phrases we can use to describe **chance**.

Chance The likelihood of an event happening



- If two events have the same chance of occurring, we say they are equally likely.

Exercise 8F

UNDERSTANDING 1, 2 | 2

1 Fill in each blank with the word *likely* or *unlikely*.

a A fair coin is flipped 100 times. It is _____ that these will be 100 tails.

b A page of this book is picked at random. It is _____ that the letter 'e' will be on the page.

c The television is switched on at 5 p.m. and left on for 3 hours. It is _____ that the News will be shown at some stage.

d It is _____ that the next Australian prime minister will be 21 years old.

2 Match each of the events (**a** to **d**) with a description of how likely they are to occur (**A** to **D**).

a A tossed coin landing heads up

b Selecting an ace first try from a fair deck of 52 playing cards

c Obtaining a number other than 6 if a fair 6-sided die is rolled

d Obtaining a number greater than 8 if a fair 6-sided die is rolled

A unlikely

B likely

C impossible

D even chance

FLUENCY 3–6 | 3–7

Example 12 Describing chance

Say whether each of the following statements is true or false.

a It is likely that children will go to school next year.

b It is an even chance for a fair coin to display tails.

c Rolling a 3 on a 6-sided die and getting heads on a coin are equally likely.

d It is certain that two randomly chosen odd numbers will add up to an even number.

Solution	Explanation
a true	Although there is perhaps a small chance that the laws might change, it is (very) likely that children will go to school next year.
b true	There is a 50–50 or even chance of a fair coin displaying tails. It will happen, on average, half of the time.
c false	These events are not equally likely. Flipping heads on a coin is more likely than rolling a 3 on a 6-sided die.
d true	No matter what odd numbers are chosen, they will always add to an even number.

8F

3 Consider a fair 6-sided die with the numbers 1 to 6 on it. Say whether each of the following is true or false.



- **a** Rolling a 3 is unlikely.
- **b** Rolling a 5 is likely.
- **c** Rolling a 4 and rolling a 5 are equally likely events.
- **d** Rolling an even number is likely.
- **e** There is an even chance of rolling an odd number.
- **f** There is an even chance of rolling a multiple of 3.

4 Copy and complete the following, using the special words that describe chance.



Choose from the words: *even chance*, *impossible*, *certain*, *unlikely*

- **a** If an event is guaranteed to occur, we say it is _____.
- **b** An event that is equally likely to occur or not occur has an _____ _____.
- **c** A rare event is considered _____.
- **d** An event that will never occur is called _____.

5 Match each of the events (**a** to **d**) with an equally likely event (**A** to **D**).

- **a** Rolling a 2 on a 6-sided die
- **b** Selecting a heart card from a fair deck of 52 playing cards
- **c** Flipping a coin and tails landing face up
- **d** Rolling a 1 or a 5 on a 6-sided die

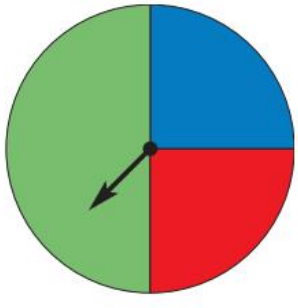
- **A** Flipping a coin and heads landing face up
- **B** Rolling a 5 or a 6 on a 6-sided die
- **C** Selecting a diamond card from a fair deck of 52 playing cards
- **D** Rolling a 4 on a 6-sided die



6 Give an example of:

- **a** an event that is unlikely
- **b** an event that is likely
- **c** an event that has an even chance of occurring
- **d** two events that are equally likely

7 This spinner could land with the arrow pointing to any of the three colours.

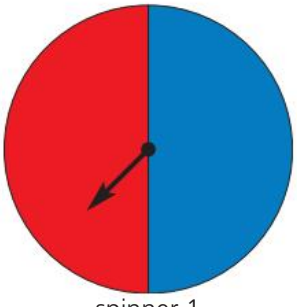


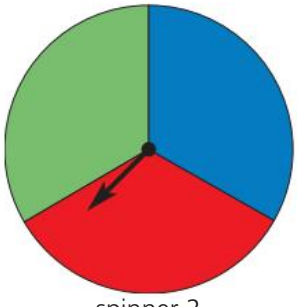
- **a** State whether each of the following is true or false.
  - **i** There is an even chance that the spinner will point to green.
  - **ii** It is likely that the spinner will point to red.
  - **iii** It is certain that the spinner will point to purple.
  - **iv** It is equally likely that the spinner will point to red or blue.

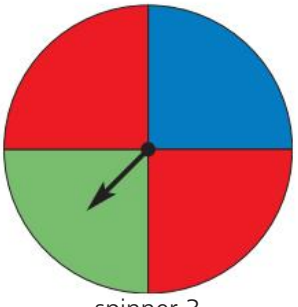
b Use the spinner to give an example of:
- **i** an impossible event
- **ii** a likely event
- **iii** a certain event
- **iv** two events that are equally likely

PROBLEM-SOLVING AND REASONING 8, 9 8–10

8 Three spinners are shown below. Match each spinner with the description.


spinner 1


spinner 2


spinner 3

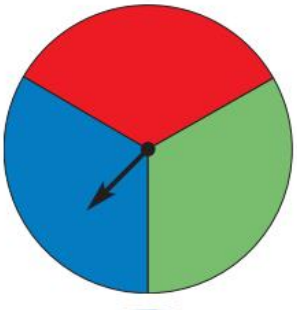
- **a** Has an even chance of red, but blue is unlikely.
- **b** Blue and green are equally likely, but red is unlikely.
- **c** Has an even chance of blue, and green is impossible.

9 Draw spinners to match each of the following descriptions.
- **a** Blue is likely, red is unlikely and green is impossible.
- **b** Red is certain.
- **c** Blue has an even chance, red and green are equally likely.
- **d** Blue, red and green are all equally likely.

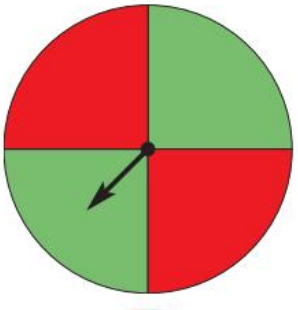


10 For each of the following, describe the chances involved so that someone else could draw the spinner. Use colour names and the language of chance (e.g. *likely*, *impossible*) in your descriptions.

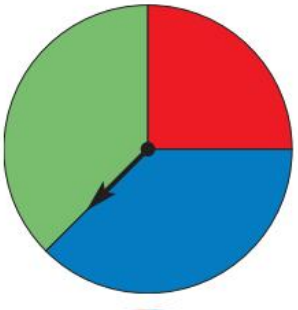
a



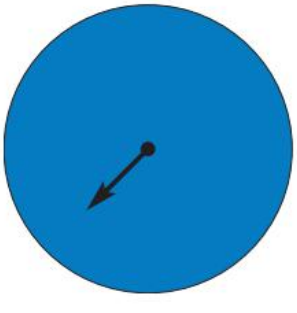
b



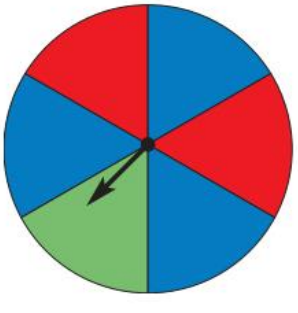
c



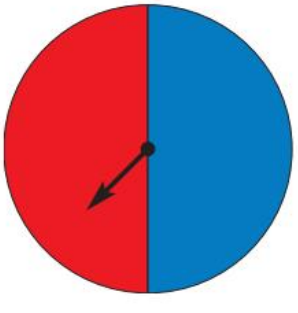
d



e



f

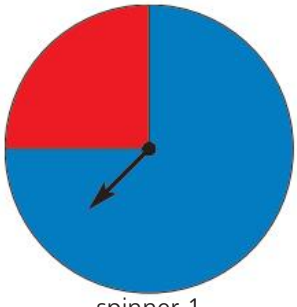


8F

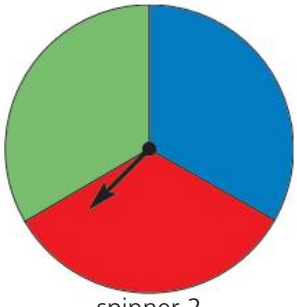
Spinner fractions

— 11

11 The language of chance is a bit vague. For example, for both of the following spinners it is 'unlikely' that you will spin red, but in each case the chance of spinning red is different.



spinner 1



spinner 2

Rather than describing chance in words, we could consider the fraction of the spinner for a certain colour.

a What fraction of spinner 1 is red?

b What fraction of spinner 2 is red?

c What fraction of a spinner would be red if red had an even chance?

d Draw spinners for which red makes up:

- **i** 100% of the spinner
- **ii** 0% of the spinner

e For the sentences below, fill each gap with an appropriate fraction or percentage value.

- **i** You have an even chance of spinning a certain colour if it makes up _____ of the total spinner.
- **ii** It is impossible to spin a certain colour if it makes up ____% of the total spinner.
- **iii** You are unlikely to spin a certain colour if it makes up more than _____ but less than _____ of the total area.
- **iv** You are likely to spin a certain colour if it makes up more than _____ of the total area.

f How can the fractions help determine if two events are equally likely?



8G Theoretical probability

Interactive

Widgets

HOTsheets

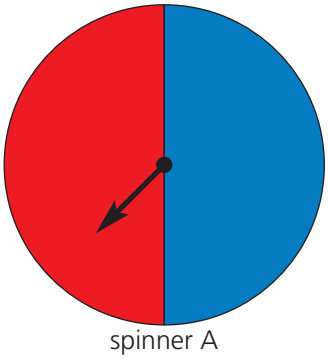
Walkthrough

The theoretical probability of an event is the chance that it will happen. This chance is worked out mathematically.

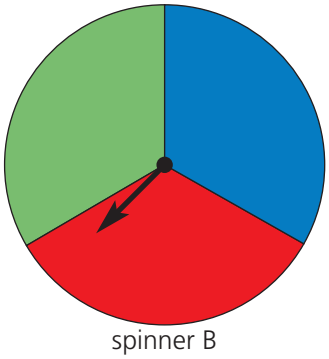
Probabilities are numbers from 0 (impossible) to 1 (certain). They can be written as fractions, decimals or percentages. The larger the probability, the more likely the event will occur.

Let's start: Spinner probabilities

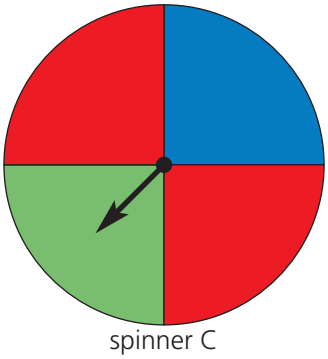
Consider these three spinners.



spinner A



spinner B



spinner C

- Describe the differences between the spinners.
- What is the probability of spinning blue for each of these spinners?
- What is the probability of spinning red for each of these spinners?

Key ideas

- A random experiment is a chance activity that produces varying results.
- A **trial** is a single component of an experiment such as tossing a coin, rolling a die or spinning a spinner.
- An **outcome** is a possible result of an experiment, like rolling a 5 or a coin showing tails.
- An event is either a single outcome (e.g. rolling a 3) or a collection of outcomes (e.g. rolling a 3, 4 or 5).
- The **probability** of an outcome occurring, if all the outcomes are equally likely, is:

$$\frac{\text{number of ways the outcome can occur}}{\text{total number of possible outcomes}}$$

- Probability is often written as a fraction, but it can be written as a decimal or as a percentage. For example: $\frac{1}{2}$ or 0.5 or 50%
- The **sample space** is the set of all possible outcomes of a trial.

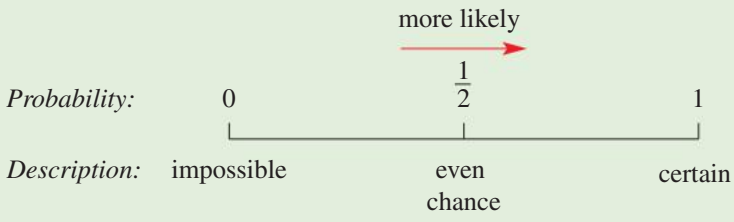
Trial One run of a chance experiment

Outcome One of the possible results of a trial

Probability The likelihood that an event will occur, measured on a scale between 0 and 1

Sample space The list of all the possible outcomes of a trial

- We write Pr(green) to mean 'the probability that a spinner shows green'.



Exercise 8G

UNDERSTANDING 1–4 4

1 Complete the following sentences.

Choose from: *space, zero, more, less, sample, impossible, certain*



- **a** The _____ _____ is the set of possible outcomes.
- **b** An impossible event has a probability of _____.
- **c** If an event has a probability of 1, then it is _____.
- **d** The higher its probability, the _____ likely the event will occur.
- **e** Rolling a 7 on a standard die is _____.
- **f** An event with a lower probability than another is _____ likely to occur.

2 A fair coin is flipped. The probability of flipping tails is $\frac{1}{2}$. Write this probability as:

- **a** a decimal
- **b** a percentage

Example 13 Calculating probability

A fair 6-sided die is rolled.

- **a** List the sample space.
- **b** Find the probability of rolling a 3, giving your answer as a fraction.
- **c** Find the probability of rolling an even number, giving your answer as a decimal.
- **d** Find the probability of rolling a number less than 3, giving your answer as a percentage.

Solution	Explanation
a Sample space = {1, 2, 3, 4, 5, 6}	For the sample space, we list all the possible events using set brackets.
b $\Pr(3) = \frac{1}{6}$	The event can occur in one way (rolling a 3) out of six possible outcomes.
c $\Pr(\text{even}) = \frac{1}{2} = 0.5$	The event can occur in three ways (i.e. 2, 4 or 6). So the probability is $\frac{3}{6} = \frac{1}{2}$. As a decimal this is 0.5.
d $\Pr(\text{less than } 3) = \frac{1}{3} = 33.3\%$	The event can occur in two ways (1 or 2). So the probability is $\frac{2}{6} = \frac{1}{3}$. As a percentage this is 33.3%, rounded to 1 decimal place.

3 Consider a fair 6-sided die.

a List the sample space.

b List the odd numbers on the die.

c State the probability of rolling an odd number.

d State the probability of rolling a 5.



For parts **c** and **d**, begin your answers with Pr(odd) = and Pr(5) =.

4 Match up each event (**a** to **d**) with a set of possible outcomes (**A** to **D**).

a tossing a coin

b rolling a die

c selecting a suit from a fair deck of 52 playing cards

d selecting a letter from the word POWER

A 1, 2, 3, 4, 5, 6

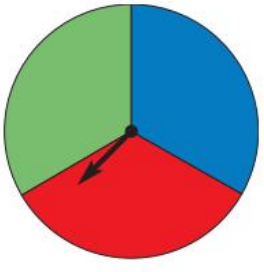
B P, O, W, E, R

C heads, tails

D hearts, diamonds, clubs, spades

FLUENCY 5–7 5–8

5 Consider the spinner shown.



a How many outcomes are there? List them.

b Find Pr(red). That is to find the probability of the spinner pointing to red.

c Find Pr(green).

d Find Pr(not red).

e Find Pr(yellow).



Pr(not red) means the probability that the spinner's arrow does not point to red.

6 A spinner with the numbers 1 to 7 is spun. The numbers are evenly spaced.



a List the sample space.

b Find Pr(6).

c Find Pr(8).

d Find Pr(2 or 4).

e Find Pr(even).

f Find Pr(odd).

g Give an example of an event having the probability of 1.

7 The letters in the word MATHS are written on 5 cards and then one is drawn from a hat.

a List the sample space.

b Find Pr(T), giving your answer as a decimal.

c Find Pr(A), giving your answer as a decimal.

d Find Pr(consonant is chosen), giving your answer as a decimal.

8 The letters in the word PROBABILITY are written on 11 cards and then one is drawn from a hat.

a Find Pr(P).

b Find Pr(I).

c Find Pr(letter chosen is in the word BIT).

d Find Pr(not a B).

e Find Pr(a vowel is chosen).

f Give an example of an event with the probability of $\frac{3}{11}$.



For part **b**, there are 2 cards out of 11 with the letter I.

8G

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 The whole numbers from 1 to 11 are written on 11 cards. If the cards are shuffled and one card is chosen at random, which of the following outcomes is most likely?

A choosing a 5
B choosing an even number
C choosing a two-digit number
D choosing an odd number

10 A bag of marbles contains 3 red marbles, 2 green marbles and 5 blue marbles. They are all equal in size and weight. A marble is chosen at random.

a What is the probability that a red marble is chosen?
b What is the probability that a blue marble is chosen?
c What is the probability that a green marble is *not* chosen?



The probability of choosing red is not $\frac{1}{3}$ because the colours are not equally likely.

11 A box contains different coloured counters, with Pr(purple) $= \frac{1}{10}$, Pr(yellow) $= \frac{2}{3}$ and Pr(orange) $= \frac{1}{7}$.

a Is it possible to obtain a colour other than purple, yellow or orange? If so, state the probability.
b What is the minimum number of counters in the box?
c If the box cannot fit more than 1000 counters, what is the maximum number of counters in the box?

12 Consider this spinner, numbered 2 to 9.



a List the sample space.
b Find the probability that a prime number will be spun, giving your answer as a decimal. (Remember that 2 is a prime number.)
c Giving your answers as decimals, state the probability of getting a prime number if each number in the spinner is:
i increased by 1
ii increased by 2
iii doubled

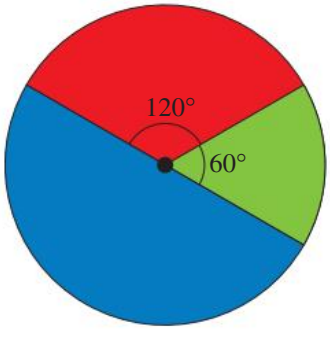
It will help if you draw the new spinners.

d Design a new spinner for which Pr(prime) = 1.

Designing spinners

— 13

13 **a** State the probability of spinning green with this spinner.



b For each of the following, design a spinner using only red, green and blue sectors to obtain the desired probabilities. If it cannot be done, then explain why not.

i Pr(red) $= \frac{1}{2}$, Pr(green) $= \frac{1}{4}$, Pr(blue) $= \frac{1}{4}$
ii Pr(red) $= \frac{1}{2}$, Pr(green) $= \frac{1}{2}$, Pr(blue) $= \frac{1}{2}$
iii Pr(red) $= \frac{1}{4}$, Pr(green) $= \frac{1}{4}$, Pr(blue) $= \frac{1}{4}$
iv Pr(red) $= 0.1$, Pr(green) $= 0.6$, Pr(blue) $= 0.3$

8H Experimental probability









If the probability of an event is unknown, experimental results can help to give you an idea of that probability.

For example, when flipping a coin, the theoretical probability of it landing on heads is $\frac{1}{2}$. If you suspect that the coin is not fair, you could run an experiment and use the results to estimate the experimental probability of heads.

The more trials (flips) you use in your experiment, the more likely you are to get a good estimate of the probability of heads.

Let's start: Tossing coins

For this experiment, each class member needs a fair coin that they can toss.

- Each student should toss the coin 20 times and count how many times heads occurs.
- Tally the total number of heads obtained by the class.
- How close is this total number to the number you would expect that is based on the theoretical probability of $\frac{1}{2}$? Discuss what this means.



Tossing a coin 100 times does not mean it will come up heads 50 times.

Key ideas

- The experimental probability of an event based on a particular **experiment** is:

$$\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$$

Experiment A series of repeated probability trials that lead to outcomes

- The expected number of occurrences = probability × number of trials
- If the number of trials is large, then the experimental probability is likely to be close to the actual theoretical probability.

Exercise 8H

UNDERSTANDING 1–4 4

1 Which of the following experiments would be best to see if a coin was fair?

A Flipping it once **B** Flipping it twice **C** Flipping it 20 times

2 A coin is flipped and the results are:



a How many times did the coin show heads?
b How many times did the coin show tails?
c How many times was the coin flipped?

Example 14 Working with experimental probability

When using a spinner numbered 1 to 4, the following numbers come up.

1, 4, 1, 3, 3, 1, 4, 3, 2, 3.

a What is the experimental probability of getting a 3?
b What is the experimental probability of getting an even number?

Solution	Explanation
a $\frac{2}{5}$ or 0.4 or 40%	$\frac{\text{number of 3s}}{\text{number of trials}} = \frac{4}{10} = \frac{2}{5}$
b $\frac{3}{10}$	$\frac{\text{number of even results}}{\text{number of trials}} = \frac{3}{10}$

3 A 6-sided die is rolled 10 times and the following numbers come up: 2, 4, 6, 4, 5, 1, 6, 4, 4, 3. Find the experimental probability of getting:

a a 3
b a 4
c an odd number



For part **c**, count how many times a 1, 3 or 5 was rolled.

4 When a coin is tossed 100 times, the results are 53 heads and 47 tails.

a What is the experimental probability of getting a head?
b What is the experimental probability of getting a tail?
c The actual probability of getting a tail on a fair coin is $\frac{1}{2}$. Does this experiment prove that the coin is not fair?

FLUENCY 5–8 5–9

5 The table shows the results of spinning a spinner.

Colour	red	green	blue
Number of times	13	5	2

State the experimental probability of getting:

a red **b** green **c** blue

6 A survey is conducted on people's television viewing habits.

Number of hours per week	0–5	5–10	10–20	20–30	30+
Number of people	20	10	15	5	0



a How many people participated in the survey?

b Find the probability that a randomly selected participant watches television for:

i less than 5 hours

ii 20–30 hours

iii between 5 and 20 hours

Example 15 Finding expected numbers

A spinner is found to land on red $\frac{1}{4}$ of the time. If it is spun 200 times, how many times would you expect it to land on red?

Solution	Explanation
50 times	Expected number = probability × number of trials = $\frac{1}{4} \times 200 = 50$

7 A fair coin is tossed.



The theoretical probability of tails is $\frac{1}{2}$.

a How many times would you expect it to show tails in 1000 trials?

b How many times would you expect it to show heads in 3500 trials?

c You start by tossing the coin 10 times to find the probability of the coin showing tails.

i Explain how you could get an experimental probability of 0.7.

ii If you toss the coin 100 times, are you more or less likely to get an experimental probability close to 0.5?

8 A fair 6-sided die is rolled.

a How many times would you expect to get a 3 in 600 trials?

b How many times would you expect to get an even number in 600 trials?

c If you roll the die 600 times, is it possible that you will get an even number 400 times?

d Are you more likely to obtain an experimental probability of 100% from two rolls or to obtain an experimental probability of 100% from 10 rolls?

8H

9 The colour of the cars in a school car park is recorded.

Colour	red	silver	white	blue	purple	black
Number of cars	21	24	25	20	3	7

a Based on this sample, find the probability that a randomly chosen car is:
- **i** white
- **ii** purple
- **iii** silver or black

b How many purple cars would you expect to see in a shopping centre car park with 2000 cars?



PROBLEM-SOLVING AND REASONING 10, 11 10–12

10 The number of children in some families is recorded in the table shown.

Number of children	0	1	2	3	4
Number of families	5	20	32	10	3



For part **f**, think: 3 families each with 4 children makes 12 children.

a How many families have no children?
b How many families have an even number of children?
c How many families participated in the survey?
d Based on this experiment, what is the probability that a randomly selected family has 1 or 2 children?
e Based on this experiment, what is the probability that a randomly selected family has a positive even number of children?
f What is the total number of *children* in this survey?

11 A handful of 10 marbles of different colours is placed into a bag. A marble is selected at random, its colour recorded and then returned to the bag.

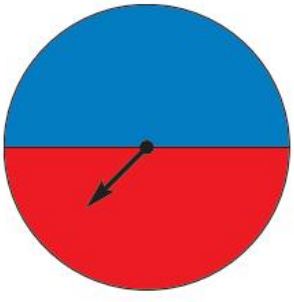
Red marble chosen	Green marble chosen	Blue marble chosen
21	32	47

a Based on the results in the table, how many marbles of each colour do you think there are? Justify your answer in a sentence.
b For each of the following, state whether or not they are possible colours for the 10 marbles in the bag.
- **i** 3 red, 3 green, 4 blue
- **ii** 2 red, 4 green, 4 blue
- **iii** 1 red, 3 green, 6 blue
- **iv** 2 red, 3 green, 4 blue, 1 purple
- **v** 2 red, 0 green, 8 blue

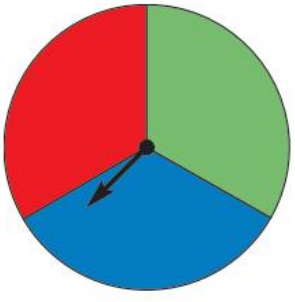
12 Match each of the experiment results (**a** to **d**) with the most likely spinner that was used (**A** to **D**).

	Red	Green	Blue
a	18	52	30
b	27	23	0
c	20	23	27
d	47	0	53

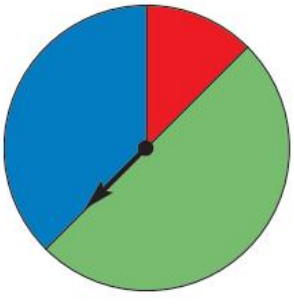
A



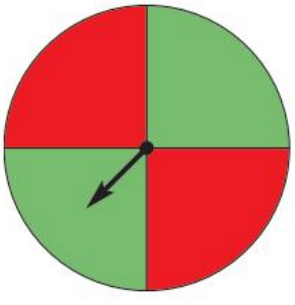
B



C



D



Dice-based basketball

— 13

13 Each time a certain basketball player takes a free throw there is a 4 in 6 chance that the shot will go in. This can be simulated by rolling a 6-sided die and using numbers 1 to 4 to mean 'shot goes in' and numbers 5 and 6 to mean 'shot misses'.



a Use a 6-sided die over 10 trials to find the experimental probability that the shot goes in.

b Use the die over 50 trials to find the experimental probability that the shot goes in.

c Working with a group, use the die over 100 trials to find the experimental probability that the shot goes in.

d Use the die over just one trial to find the experimental probability that the shot goes in. (Your answer should be either 0 or 1.)

e Which of the answers to parts **a** to **d** is closest to the theoretical probability of 66.67%? Why?

f How could you simulate a basketballer who has a 1 in 2 chance of getting a free throw in?

Maths@Work: Teachers

Teachers across all areas of education have many skills. They need to be enthusiastic and good communicators as well as having an in-depth knowledge of their course work. Keeping informed about the educational research and the latest technology helps teachers to offer students a variety of learning experiences.

Teachers across all key learning areas need to be able to generate and interpret statistics. From NAPLAN results to class results, skill with statistics is important in a teacher's career. Teachers can discover the level of their students' and classes' understanding of a concept by analysing test responses.



1 In a recent formal exam of multiple-choice questions, the number of times each answer was selected by Ms Sharma's students for questions 5, 6 and 7 are shown in the tables below:

Question 5		Question 6		Question 7	
A	5	A	2	A	15
B	2	B	17	B	8
C	3	C	3	C	4
D	20	D	7	D	3



a Which answer would you assume to be correct for each question?

b How many students answered each question?

c How many students are in Ms Sharma's class?

Given that the correct answers were:
Question 5 – D, Question 6 – B, Question 7 – A,
complete the following questions.

d What percentage of students in Ms Sharma's class answered each question correctly? Round answers to a whole number.

e Draw a column graph showing the percentage of correct answers for each question.

2 Mr White's class sat the same exam as Ms Sharma's class (in question **1** above).
The responses to question **7** in Mr White's class were:

A	B	A	B	A	A	C	D	D	D	A	A
A	A	A	A	B	A	B	A	C	D	A	D

a Show these results in a table with the number for each response: A, B, C and D.

b What percentage of students in Mr White's class answered question 7 correctly?

c Which class achieved a better result for question 7?

3 A teacher will often pre-test and post-test a topic to determine the understanding gained and to improve future teaching methods.
Results for fractions tests. Girls' marks are in red, boys' in black.

PRE-TEST	12	34	30	25	34	22	9	12	22	45	32	28	48	22	20
Marks out of 50	27	18	8	32	38	26	25	24	34	21	40	27	28	30	41
POST-TEST	23	38	34	35	44	32	18	42	27	46	39	34	47	22	22
Marks out of 50	31	36	19	32	21	45	29	25	44	28	29	46	34	43	38

a Display the pre-test results for the entire class as a stem-and-leaf plot.
b Display two separate stem-and-leaf plots for the girls' results, one for their pre-test results and another for their post-test results.
c Have the girls' results improved from pre-test to post-test? Give at least one observation about their marks to support your conclusion.
d The plot below is called a back-to-back stem-and-leaf plot. It shows the boys' pre-test results on the left and their post-test results on the right.

pre-test results		post-test results
	0	
	1	9
8 7 7 6 5 4 1	2	1 5 8 9 9
8 4 2 0	3	1 2 4 6 8
1 0	4	3 4 5 6

Key
LHS:7|2 = 27
RHS:2|8 = 28



This is a back-to-back stem-and-leaf plot:

Test A (LHS)		Test B (RHS)
2	0	9
8 2	1	3 7

Read test A marks from the centre to the *left*: 2, 12, 18

i List the boys' pre-test results that are less than 20.
ii How many boys got marks in the 20s on each test?
iii List the boys' marks from each test that are 40 or above.
iv Have the boys' results improved from pre-test to post-test? Give at least one observation about their marks to support your conclusion.

Using technology

4 Teachers make good use of the calculation power of spreadsheets. Spreadsheet formulas will convert a result out of any total to a percent. Comparing percentages helps teachers to follow each student's progress.
a Set up the following spreadsheet.
b Select all the cells that will have percentage results, right click and select Format/percentage/0 d. p. The spreadsheet will convert decimal answers to a percentage and automatically insert a % symbol.
c Format average test mark cells to Number/1 d. p.
d Enter formulas into the shaded cells to calculate percentages and averages.



In each percentage cell, calculate a decimal, e.g. $= \frac{B4}{\$B\$3}$



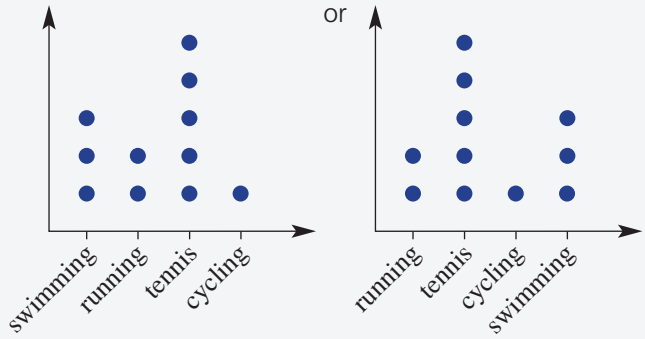
Average formula example
= Average(B4:B10)

	A	B	C	D	E	F	G	H	I	J	K	L
1	Mrs Best's year 7 maths class											
2	Test	Term 1		Term 2		Mental Arithmetic		Term 3		Term 4		Averages
3	Total marks	22	%	19	%	10	%	34	%	26	%	
4	Oscar	20		18		10		32		23		
5	Molly	17		16		9		29		22		
6	Blake	14		12		5		16		16		
7	Angus	19		19		8		33		23		
8	Bhavin	17		14		7		26		18		
9	Scarlett	14		11		6		24		19		
10	Vedika	18		16		8		31		24		
11												
12	Class averages											

Puzzles and games

1 Three brothers are asked about their ages.
The youngest, Andrew, says, 'The range of our ages is 5'.
The middle, Brett, says, 'The median age is 15'.
The oldest, Chris, says, 'But the mean age is 14'.
Assuming the brothers are telling the truth, how old are they?

2 The result of a survey are to be shown in a dot plot with four categories: swimming, running, tennis, cycling. Two ways the graph could be shown are:



How many ways in total are there to draw this dot plot?

3 Six numbers are listed in ascending order and then some are removed.

□ □ 5 □ □ □

The mean and median are both 6, the mode is 2 and the range is 10. Find the missing numbers.

4 In a class of 20 students, a poll was taken of the number of cars owned by each family. The median number of cars owned is 1.5 and the mean number is 1.4 cars. Complete the following table of the results.

Number of cars	0	1	2	3
Number of students	4			



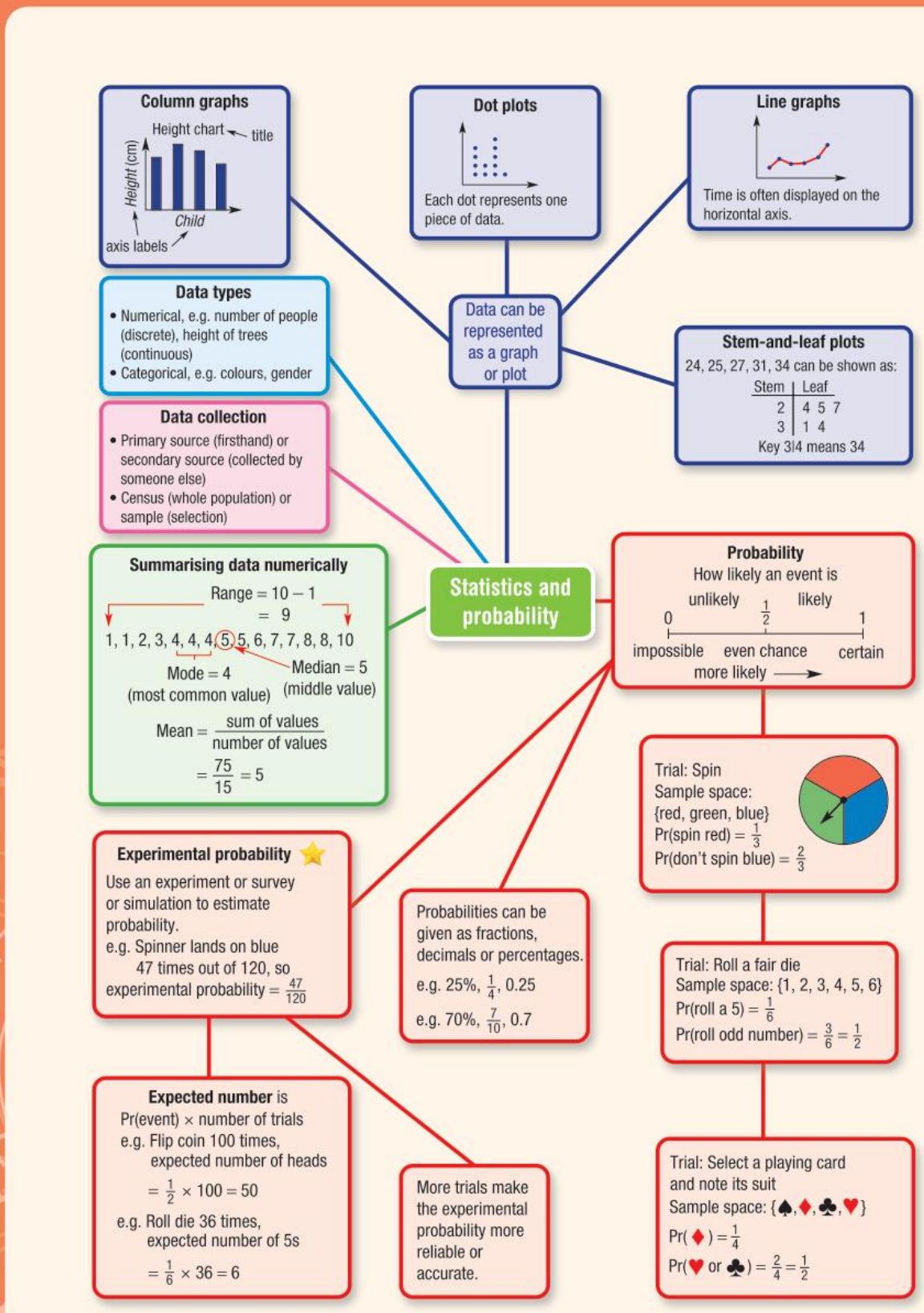
5 When one coin is flipped, Pr(heads) = $\frac{1}{2}$. For two coins, the probability that both are heads is $\frac{1}{4}$. For three coins, Pr(all heads) = $\frac{1}{8}$. How many coins are flipped if the probability of getting all heads is $\frac{1}{64}$?

6 Five people are surveyed. They are asked if they prefer bottled water, tap water or mineral water. Two possible results are shown. How many more possible results are there?

Bottled	Tap	Mineral
3	1	1
2	3	0

7 A bag of 24 marbles contains blue, green and red marbles. If you pick one marble at random (without looking), there is an even chance that it will be blue. Given that there are twice as many green marbles as red marbles, how many of each colour are there?

Chapter summary



Chapter review

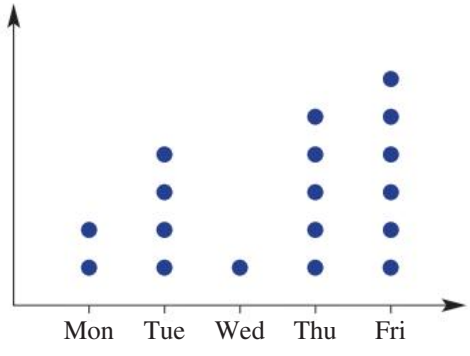
Multiple-choice questions



8A **1** Which one of the following variables is continuous numerical?

A The gender (male or female) of newborn babies
B The number of babies born in a given year
C The number of hairs on a baby's head
D The weight (in kg) of newborn babies
E The length (in number of letters) of a baby's first name

8C **2** Students are asked their favourite weekday. The results are shown in this dot plot.

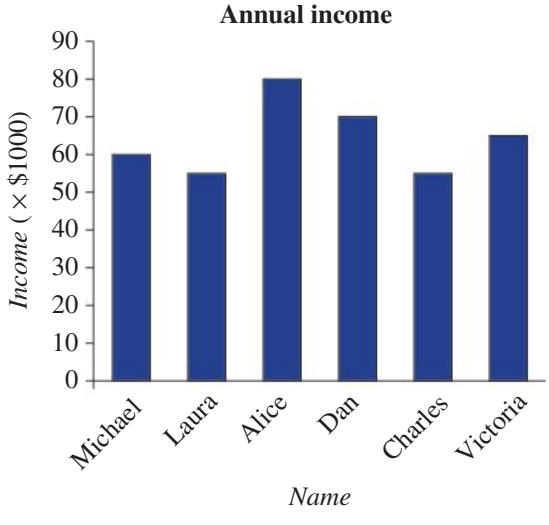


The total number of students asked was:

A 2 **B** 5 **C** 12 **D** 18 **E** 20

8C **3** In the following column graph, the highest income is earned by:

A Michael **B** Alice **C** Dan **D** Laura **E** Victoria



Questions 4 and 5 relate to the following information.
A stem-and-leaf plot shows the ages of various people.

Stem	Leaf
1	7 9
2	3 4 6
3	2 7 9

2|4 means 24

8E **4** The youngest person's age is:
A 1 **B** 17 **C** 7 **D** 2 **E** 39

8E **5** The number of people represented is:
A 8 **B** 11 **C** 39 **D** 3 **E** 26

8B **6** The median of the numbers 2, 4, 7, 9, 11 is:
A 7 **B** 7.5 **C** 9 **D** 8 **E** 11

Questions 7 and 8 relate to the following information.
In a group of 10 students, the number of days each student was absent over a term is recorded.
1, 0, 2, 4, 2, 3, 0, 1, 2, 3

8B **7** The mode is:
A 0 **B** 1 **C** 2 **D** 3 **E** 4

8B **8** The mean number of days a student was absent is:
A 1 **B** 2 **C** 4 **D** 1.8 **E** 18

8G **9** Each of the five letters of the word APPLE is written on a separate card. One card is then chosen at random. Pr(letter P) is:
A 0 **B** 0.2 **C** 0.4 **D** 0.5 **E** 1

8H **10** Michelle scores a goal in netball $\frac{1}{3}$ of the time. The expected number of goals out of 600 shots is:
A 100 **B** 200 **C** 300 **D** 400 **E** 600

Short-answer questions

8C **1** Draw a column graph to represent the following people's ages.

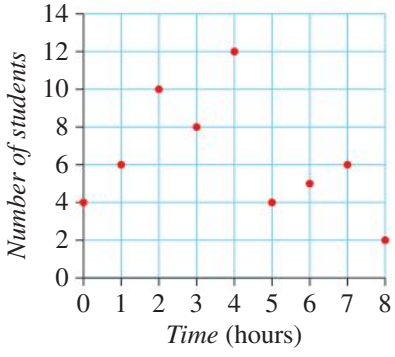
Name	Sven	Dane	Kelly	Hugo	Frankie
Age (years)	20	12	15	22	25

8C **2** A Year 7 group was asked how many hours of television they watch in a week. The results are given in the table.
a How many students participated in the survey?
b How many students watched 11 or 12 hours of television?
c What was the most common amount of television watched?
d Show this information in a column graph.

TV watched (hours)	No. of students
8	5
9	8
10	14
11	8
12	5

Chapter review

8D **3** The number of students in the library is recorded hourly, as displayed in the graph.



- **a** How many students entered the library when it first opened?
- **b** How many students were in the library at 8 hours after opening?
- **c** If the library opens at 9 a.m., at what time are there the most number of students in the library?
- **d** How many students were in the library at 4 p.m.?
- **e** Why do you think these points have not been joined to make a line graph?

8E **4** The weight in grams of various meat patties at a local burger shop are measured. The results are shown in this stem-and-leaf plot.

Stem	Leaf
10	5 8
11	2 6 6 8 9
12	0 2 4 5
13	1

12|4 means 124



- **a** What is the weight of the pattie represented as 11|8?
- **b** What is the weight of the lightest pattie?
- **c** What is the weight of the heaviest pattie?
- **d** Find the range of the weights of the patties.

8B **5** Consider the data 1, 2, 2, 3, 4, 7, 9, 12. State the:

a range **b** mean **c** median **d** mode

8B **6** Consider the data 0, 4, 2, 9, 3, 7, 3, 12. State the:

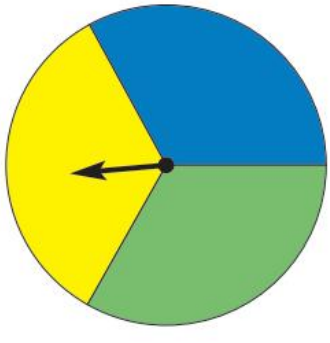
a range **b** mean **c** median **d** mode

8G **7** For each of the following descriptions, choose the probability from the set $0, \frac{1}{8}, \frac{3}{4}, 1, \frac{19}{20}$ that is the best match.

a certain **b** highly unlikely **c** highly likely
d likely **e** impossible

8G **8** List the sample space for each of the following experiments.

- **a** A fair 6-sided die is rolled.
- **b** A fair coin is tossed.
- **c** A letter is chosen from the word DESIGN.
- **d** This spinner is spun.



8G **9** Vin spins a spinner with nine equal sectors, which are numbered 1 to 9.

a How many outcomes are there?

b Find the probability of spinning:

i an odd number **ii** a multiple of 3

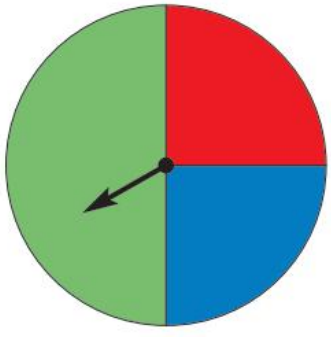
iii a number greater than 10 **iv** an even number

8H ★ **10** A coin is tossed 100 times, with the outcome 42 heads and 58 tails.

a What is the experimental probability of getting heads? Give your answer as a percentage.

b What is the actual probability of getting heads if the coin is fair? Give your answer as a percentage.

8H ★ **11** Consider this spinner.



a State the probability that the spinner lands in the green section.

b State the probability that the spinner lands in the blue section.

c Tanya spins the spinner 100 times. What is the expected number of times it would land in the red section?

d She spins the spinner 500 times. What is the expected number of times it would land in the green section?

Extended-response questions

1 This table shows the number of rainy days in a certain town over one year.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
No. of rainy days	10	11	3	7	2	0	1	5	6	9	7	5

a Show this information in a line graph.

b On how many days of the year did it rain in this town?

c What is the probability that it rained on any day during winter (i.e. during June, July and August)?



2 At a school camp, a survey was conducted to establish each student's favourite dessert.

Ice-cream	Yoghurt	Danish pastry	Jelly	Pudding	Cheesecake
10	5	2	7	4	12

a How many students participated in the survey?

b What is the most popular dessert?

c If a student is picked at random, what is the probability that jelly is their favourite dessert?

d For each of the following graphs and plots, state whether it would be a reasonable way of presenting the survey's results.

i column graph **ii** line graph

iii dot plot **iv** stem-and-leaf plot

e If the campers attend a school with 800 students, how many students from the entire school would you expect to choose pudding as their preferred dessert?

Chapter 9

Shapes and transformations

What you will learn

9A Triangles
9B Triangle angle sum
9C Quadrilaterals
9D Quadrilateral angle sum
9E Symmetry
9F Reflection
9G Rotation
9H Translation
9I Drawing solids

Australian curriculum

MEASUREMENT AND GEOMETRY

Location and transformation

Describe translations, reflections in an axis and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries **(ACMMG181)**

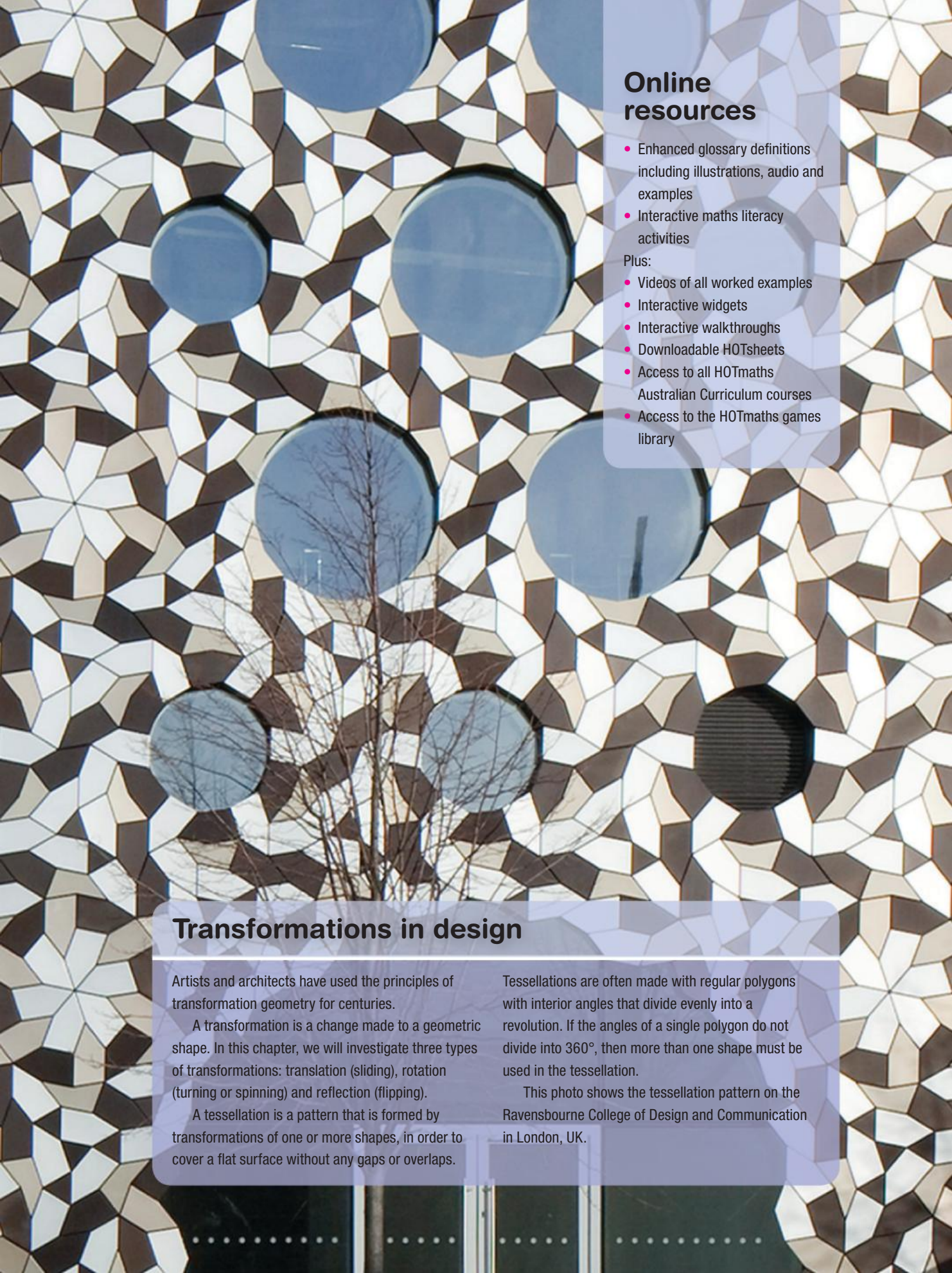
Geometric reasoning

Classify triangles according to their side and angle properties and describe quadrilaterals **(ACMMG165)**

Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral **(ACMMG166)**

Shape

Draw different views of prisms and solids formed from combinations of prisms **(ACMMG161)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Transformations in design

Artists and architects have used the principles of transformation geometry for centuries.

A transformation is a change made to a geometric shape. In this chapter, we will investigate three types of transformations: translation (sliding), rotation (turning or spinning) and reflection (flipping).

A tessellation is a pattern that is formed by transformations of one or more shapes, in order to cover a flat surface without any gaps or overlaps. Tessellations are often made with regular polygons with interior angles that divide evenly into a revolution. If the angles of a single polygon do not divide into 360°, then more than one shape must be used in the tessellation.

This photo shows the tessellation pattern on the Ravensbourne College of Design and Communication in London, UK.

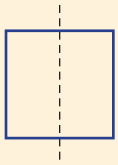
Pre-test

1 **a** How many sides does a triangle have?
b How many angles are there inside a triangle?
c What adds to 110 to give 180?
d What adds to 285 to give 360?
e 180 subtract 99 gives what number?
f 360 subtract 212 gives what number?

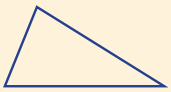
2 **a** How many sides are there on a square?
b What is the size of the interior (inside) angles in a rectangle?
c An equilateral triangle has how many sides of equal length?
d An isosceles triangle has how many sides of equal length?

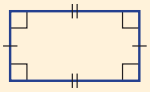
3 Find the missing number.
a $\square + 40 = 90$
b $\square + 125 = 180$
c $\square + 30 + 70 = 180$
d $\square + 124 = 360$
e $\square + 32 + 109 = 180$
f $\square + 95 + 135 = 360$

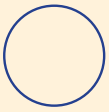
4 This square shows one mirror line. How many other mirror lines could be drawn?



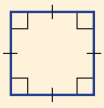
5 Match the names (**A** to **E**) with the given shapes (**a** to **e**).

a 

b 

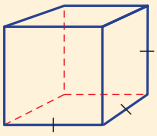
c 

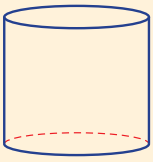
d 

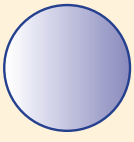
e 

A rectangle
B square
C semicircle
D triangle
E circle

6 Match the names (**A** to **C**) to the solids (**a** to **c**).

a 

b 

c 

A sphere
B cube
C cylinder

7 Complete this table.

Type of angle	acute	right	obtuse	straight	reflex	revolution
Angle			90° – 180°			360°

9A Triangles

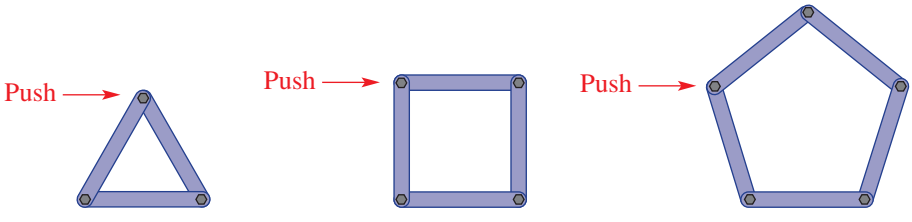
Interactive
Widgets
HOTsheets
Walkthrough

The word triangle, meaning 'three angles', describes a shape with three sides. The triangle is an important building block in mathematical geometry. It's important in the practical world of building and construction because it is a rigid shape.


Interlocking triangles give this bridge strength and stability.

Let's start: Stable shapes

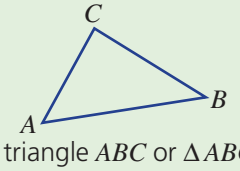
Consider these constructions, which are made from straight pieces of steel held together loosely with bolts. (That is, the bolts have not been tightened.)



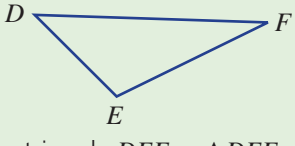
- Which of the constructions do you think could lose their shape if a vertex (corner) is pushed?
- Which of the constructions will not lose their shape when pushed? Why?
- For the constructions that might lose their shape, what could be done to make them rigid?

Key ideas

■ **Triangles** can be named using the vertex labels.



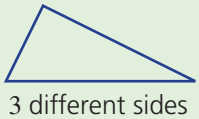
triangle ABC or ΔABC



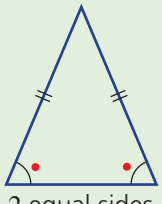
triangle DEF or ΔDEF

■ Triangles are classified by their side lengths.

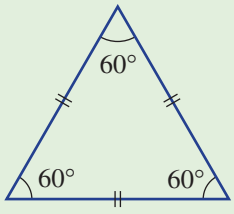
scalene


3 different sides
3 different angles

isosceles


2 equal sides
2 equal angles

equilateral



3 equal sides
3 equal angles (60°)

Triangle A plane (2D) shape with three straight sides and three angles

Scalene triangle A triangle with no equal sides or angles

Isosceles triangle A triangle with two equal sides and two equal angles

Equilateral triangle A triangle with three equal sides and three equal (60°) angles

- Triangles are also classified by the size of their interior angles.

acute



all angles acute

right



one right angle (90°)

obtuse



one obtuse angle

Acute triangle A triangle in which all the angles are acute (less than 90°)

Right triangle A triangle in which one of the angles is a right angle (90°)

Obtuse triangle A triangle with one obtuse angle (between 90° and 180°)

Exercise 9A

UNDERSTANDING 1–3 3

1 Match each word (**a** to **c**) with a diagram (**A** to **C**).

a acute
b right
c obtuse

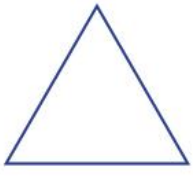
A



B



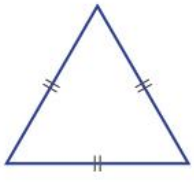
C



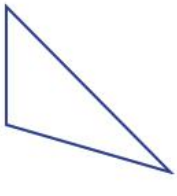
2 Match each word (**a** to **c**) with a diagram (**A** to **C**).

a scalene
b isosceles
c equilateral

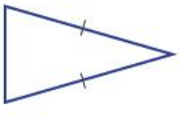
A



B



C



3 Draw an example of each of these triangles.

a scalene
b isosceles
c equilateral
d acute
e right
f obtuse



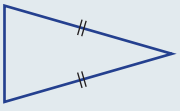
scalene: 3 different sides
isosceles: 2 equal sides
equilateral: 3 equal sides
acute: 3 acute angles
right: 1 right angle
obtuse: 1 obtuse angle

FLUENCY 4–6 4–7

Example 1 Classifying triangles by their sides

Choose the type of triangle (*scalene*, *isosceles* or *equilateral*).

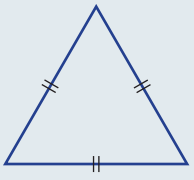
a



b



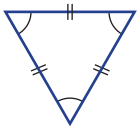
c



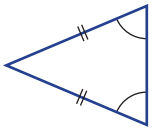
Solution	Explanation
a isosceles	Has 2 sides of equal length.
b scalene	Has 3 different side lengths.
c equilateral	Has 3 sides of equal length.

4 Choose the type of triangle (*scalene, isosceles* or *equilateral*).

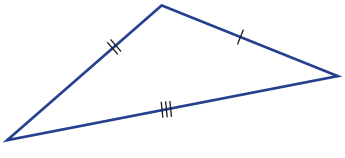
a



b



c



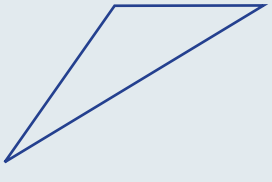
Example 2 Classifying triangles by their angles

Choose the type of triangle (*acute*, *right* or *obtuse*).

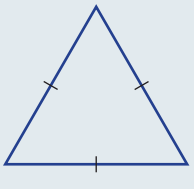
a



b



c

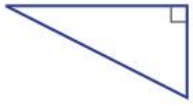


Solution	Explanation
a right	Has 1 right angle (90°).
b obtuse	Has 1 obtuse angle (> 90°).
c acute	All angles are acute (< 90°).

9A

5 Choose the type of triangle (*acute*, *right* or *obtuse*).

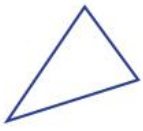
a



b



c



6 Name the type of triangle which fits the description.

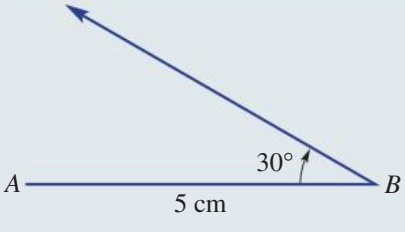
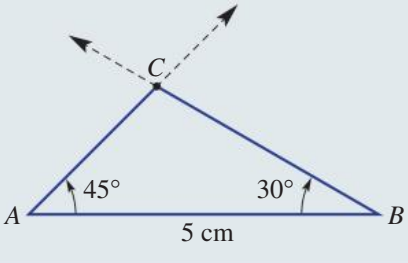
a has one right angle (90°)
b has one obtuse angle (>90°)
c has two sides of equal length
d has three different side lengths
e has three sides of equal length
f has three acute angles (<90°)



acute: between 0° and 90°
right: 90°
obtuse: between 90° and 180°
straight: 180°
reflex: between 180° and 360°
revolution: 360°

Example 3 Drawing triangles

Draw a triangle ABC with $AB = 5$ cm, $\angle ABC = 30°$ and $\angle BAC = 45°$.

Solution	Explanation
	First, measure and draw line segment AB. Then use a protractor to form the angle 30° at point B.
	Next, use a protractor to form the angle 45° at point A. Mark point C and join with A and B.

7 Use a protractor and ruler to draw the following triangles.

a triangle ABC with $AB = 5$ cm, $\angle ABC = 40°$ and $\angle BAC = 30°$
b triangle DEF with $DE = 6$ cm, $\angle DEF = 50°$ and $\angle EDF = 25°$
c triangle GHI with $GH = 4$ cm, $\angle GHI = 70°$ and $\angle HGI = 50°$

PROBLEM-SOLVING AND REASONING 8, 9 8–10

8 Is it possible to draw any of the following? If yes, give an example.

a an acute triangle that is also scalene
b a right triangle that is also isosceles
c an equilateral triangle that is also obtuse
d a scalene triangle that is also right angled



Try sketching with a pencil to see if these are possible.

9 Try to draw an example of a triangle that fits the triangle type in both the row and column. Are there any cells in the table for which it is impossible to draw a triangle?

Triangles	scalene	isosceles	equilateral
acute			
right			
obtuse			



Each triangle must match both the column (scalene, isosceles or equilateral) *and* the row (acute, right or obtuse).

10 Use a protractor and ruler to draw the following.

a triangle ABC with $AB = 5$ cm, $\angle ABC = 35°$ and $BC = 4$ cm

b triangle DEF with $AB = 4$ cm, $\angle CAB = 90°$ and $CA = 3$ cm

c triangle GHI with $HI = 5$ cm, $\angle GHI = 55°$ and $GH = 3.5$ cm

Impossible triangles

— 11

11 Try drawing triangles with the following characteristics. Then explain why they are impossible.

a three sides: 4 cm, 3 cm and 9 cm

b three angles: 70°, 80° and 60°

c two obtuse angles



9B Triangle angle sum

Interactive

Widgets

HOTsheets

Walkthrough

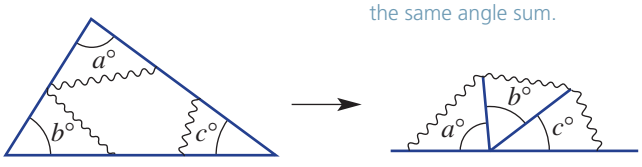
The three interior angles of a triangle have a very important property. No matter the shape of the triangle, the three angles always add to the same total.



The different triangles formed by the bridge's cables all have the same angle sum.

Let's start: A visual perspective on the angle sum

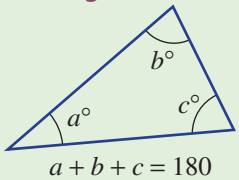
Use a ruler to draw any triangle. Cut out the triangle and tear off the three corners. Then place the three corners together.



What do you notice and what does this tell you about the three angles in the triangle? Compare your results with those of others. Does this work for other triangles?

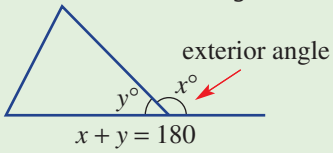
Key ideas

- The **angle sum** of the **interior angles** of a triangle is 180°.



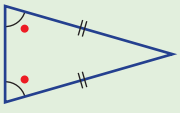
$a + b + c = 180$

- If one side of a triangle is extended, an **exterior angle** is formed.



$x + y = 180$

- An isosceles triangle has one pair of equal angles.



Angle sum The total measure of the interior angles in a 2D shape

Interior angle Any angle inside two adjacent sides of a shape

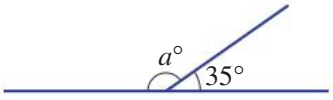
Exterior angle The angle formed between the extended side of a shape and the adjacent side

Exercise 9B

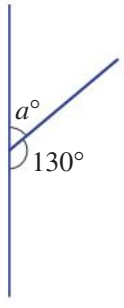
UNDERSTANDING 1–4 4

1 Find the value of a without using a protractor.

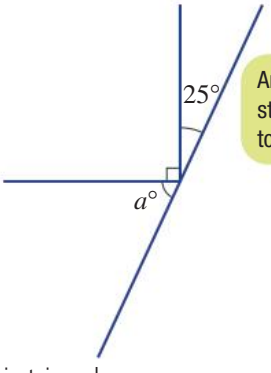
a



b



c

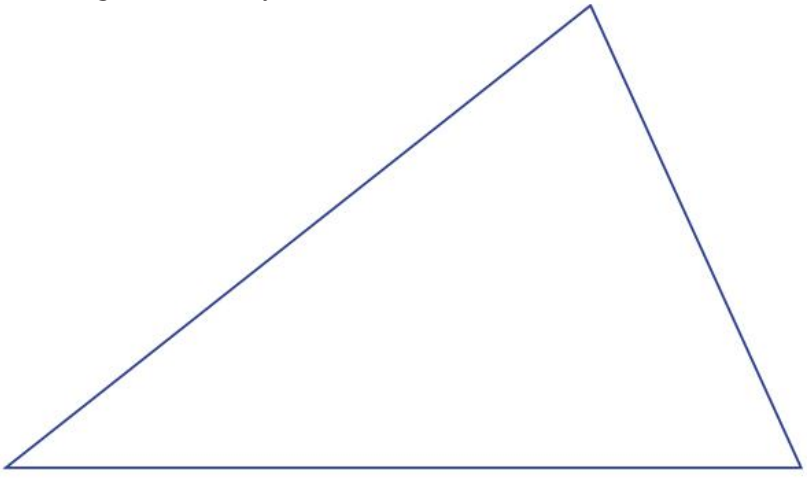




Angles on a straight line add to 180°.

2 **a** Use a protractor to measure the three angles in this triangle.

b Add your three angles. What do you notice?



3 Copy and complete the working for this triangle.



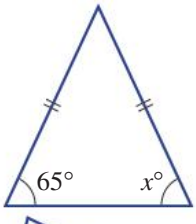
$a + 80 + ___ = 180$

$a + ___ = 180$

$a = ___$

4 State the value of x in these isosceles triangles.

a



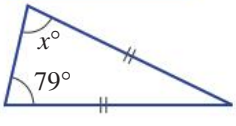
b



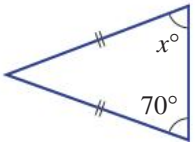


Remember that an isosceles triangle has two equal angles.

c



d



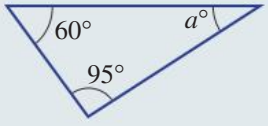
9B

FLUENCY 5, 6 5–7

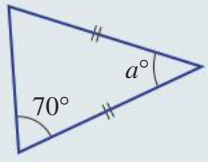
Example 4 Finding an angle in a triangle

Find the value of a in these triangles.

a



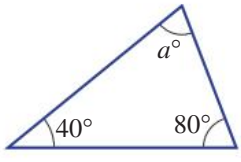
b



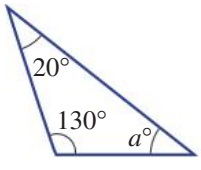
Solution	Explanation
a $60 + 95 = 155$ $155 + a = 180$ So $a = 25$	Add the two known angles. The angle sum is 180°. The difference gives the value of a.
b The unmarked angle is 70° and $70 + 70 = 140$ $140 + a = 180$ So $a = 40$	In an isosceles triangle, the two angles opposite the equal sides are equal. The angle sum is 180. Use addition or subtraction to find a.

5 Find the value of a in each of these triangles.

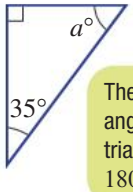
a



b



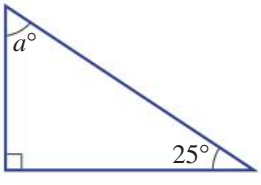
c



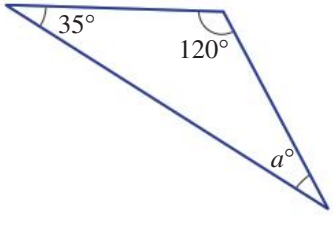


The three angles in a triangle add to 180°.

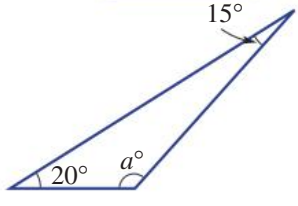
d



e

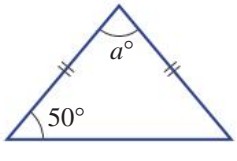


f

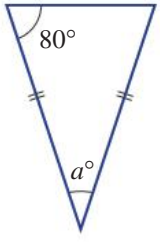


6 Find the value of a in each of these isosceles triangles.

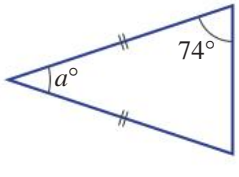
a



b



c

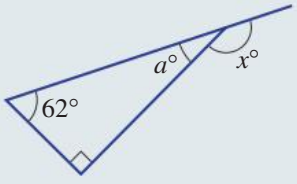




There are two equal angles in an isosceles triangle.

Example 5 Finding an exterior angle

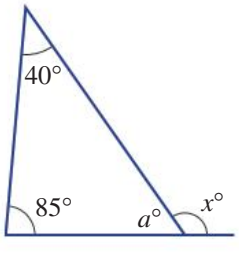
By first finding the value of a, find the value of x.



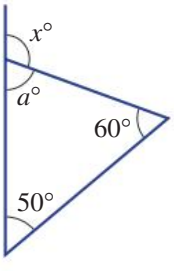
Solution	Explanation
$a + 62 + 90 = 180$ $62 + 90 = 152$ So $a = 28$	First use the angle sum to find a. (There are 90° in a right angle.) Think: $180 - 152$ (or $152 + ? = 180$).
$x + 28 = 180$ $x = 152$	Angles $x°$ and $a°$ are supplementary (i.e. they add to 180°). Add or subtract to find x.

7 By first finding the value of a, find the value of x.

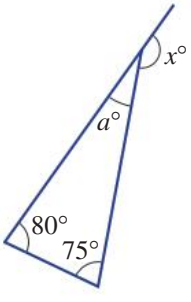
a



b



c



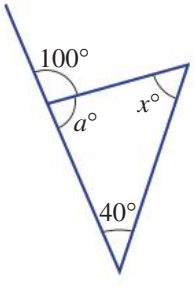


To find a, use the angle sum. To find x, remember that angles on a straight line add to 180°.

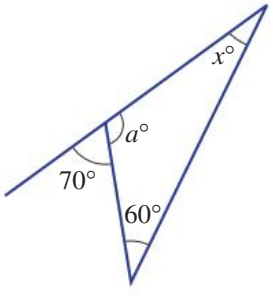
PROBLEM-SOLVING AND REASONING 8–10 9–12

8 The triangles below have exterior angles. Find the value of x.

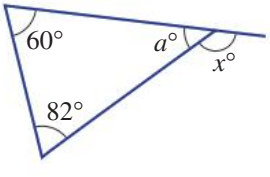
a



b

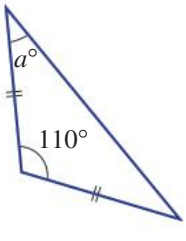


c

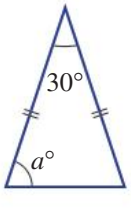


9 Find the value of a in these isosceles triangles.

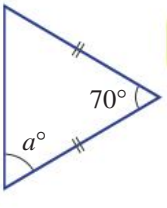
a



b



c



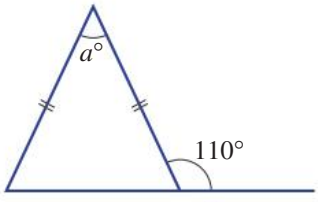


For part a, the two equal angles must total 70°.

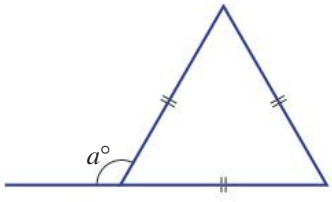
9B

10 Find the value of a in each of these triangles.

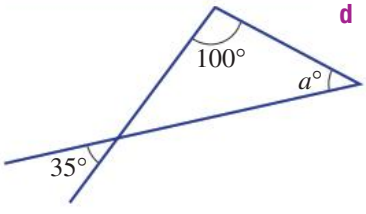
a



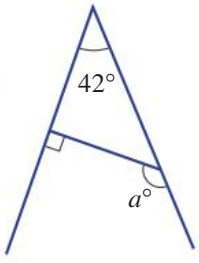
b



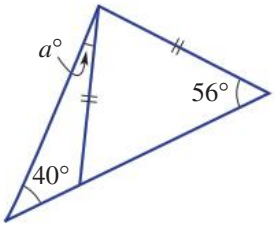
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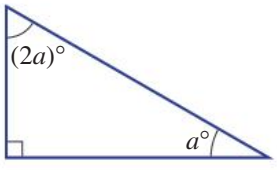
d



e

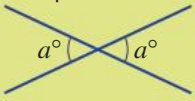


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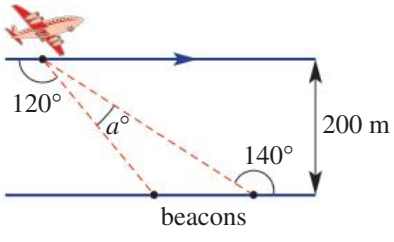




All angles in an equilateral triangle are 60°.
Vertically opposite angles are equal.



11 A plane flies horizontally 200 m above the ground. It detects two beacons on the ground. Some angles are known, and these are shown in the diagram. Find the value of a marked between the line of sight to the two beacons.







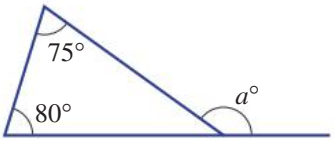
Draw a vertical line to create two right-angled triangles.

12 **a** Work out 75° + 80°.

b Find the value of a in the diagram.

c What do you notice about the answers to parts **a** and **b**?

d Do you think this would be true for other triangles with different angles? Explore.

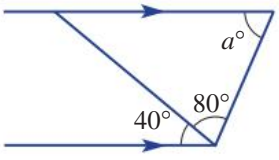


Triangles and parallel lines

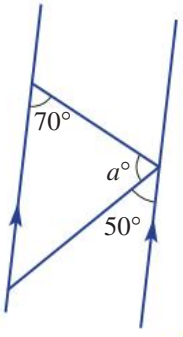
— 13

13 Each of these diagrams has parallel lines. Find the value of a.

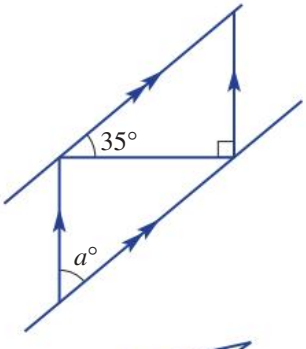
a



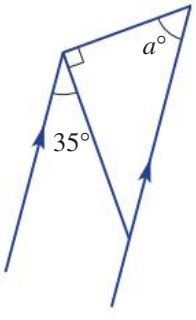
b



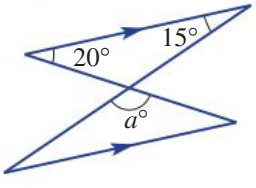
c



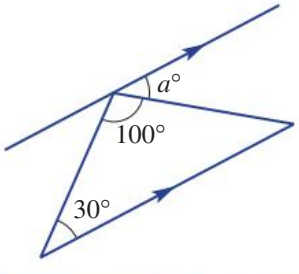
d



e

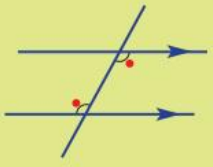


f

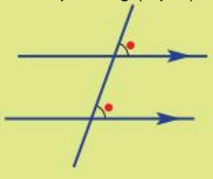




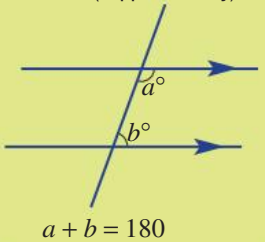
alternate (equal)



corresponding (equal)



cointerior (supplementary)



$a + b = 180$



9C Quadrilaterals

Interactive
Widgets
HOTsheets
Walkthrough

Quadrilaterals are shapes with four straight sides. Special types of quadrilaterals are identified by the number of equal side lengths and the number of pairs of parallel lines.



Streets in major cities often form quadrilaterals, and many buildings on them have a similar shape.

Let's start: Quadrilaterals that you know

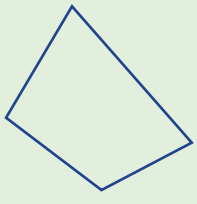
You may already know the names and properties of some of the special quadrilaterals. Which quadrilaterals do you think have:

- 2 pairs of parallel sides?
- all sides of equal length?
- 2 pairs of sides of equal length?

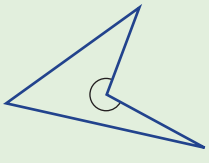
Are there any types of quadrilaterals that you know which you have not yet listed?

Key ideas

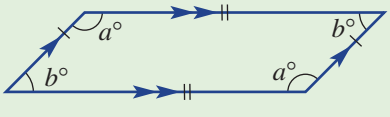
- A convex **quadrilateral** has all four interior angles less than 180°.
  All vertices point outward.



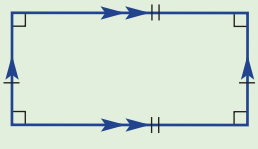
- A non-convex quadrilateral has one interior angle greater than 180°.



- Parallelograms are quadrilaterals with two pairs of parallel sides.
  - **parallelogram**
    2 pairs of parallel sides
    2 pairs of sides of equal length
    opposite angles equal



  - **rectangle** (a parallelogram with all angles 90°)
    2 pairs of parallel sides
    2 pairs of sides of equal length
    all angles 90°

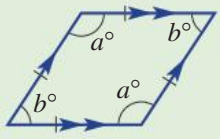


Quadrilateral A four-sided plane (2D) shape with straight sides

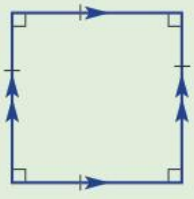
Parallelogram A quadrilateral with two pairs of opposite sides parallel

Rectangle A quadrilateral with both pairs of opposite sides equal and parallel, and with four right angles

- **rhombus** (sometimes called a diamond)
  (a parallelogram with all sides equal)
  2 pairs of parallel sides
  all sides of equal length
  opposite angles equal



- **square** (a rhombus with all angles 90°)
  2 pairs of parallel sides
  all sides of equal length
  all angles 90°



Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal

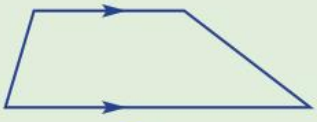
Square A quadrilateral with all sides equal in length and four right angles

Trapezium A quadrilateral with exactly one pair of parallel sides

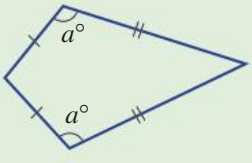
Kite A quadrilateral with exactly two pairs of equal adjacent sides

Other special quadrilaterals include:

- **trapezium**
  1 pair of parallel sides



- **kite**
  2 pairs of sides of equal length
  1 pair of opposite angles that are equal



Exercise 9C

UNDERSTANDING 1, 2 2

1 Draw an example of each of the quadrilaterals **a** to **f**.
Mark any sides of equal length with single or double dashes.
Mark parallel lines with single or double arrows.
Mark equal angles using the letters a and b.

a square
b rectangle
c rhombus
d parallelogram
e trapezium
f kite

Refer back to the **key ideas** if you need help.



2 **a** Draw two examples of a non-convex quadrilateral.
b For each of your drawings, state how many interior angles are greater than 180°.

9C

FLUENCY 3–5 3–6

Example 6 Classifying quadrilaterals as convex or non-convex

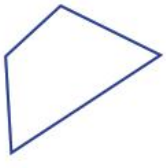
Classify each of these quadrilaterals as convex or non-convex.

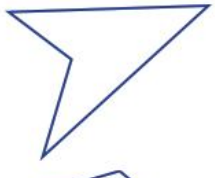
a 

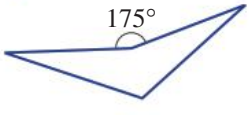
b 

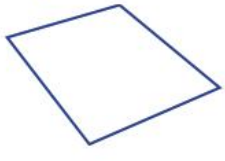
Solution	Explanation
a non-convex	One interior angle is greater than 180°.
b convex	All interior angles are less than 180°.

3 Classify each of these quadrilaterals as either convex or non-convex.

a 

b 

c 

d 

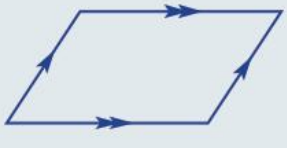


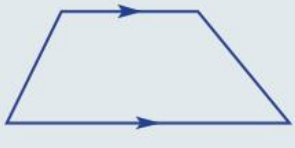
A non-convex quadrilateral has one interior (inside) angle greater than 180°.

4 Which two special quadrilaterals:
- **a** have all angles equal to 90°?
- **b** have all sides equal?
- **c** do not have two pairs of opposite parallel sides?

Example 7 Naming special quadrilaterals

Give the special name for each of these quadrilaterals.

a 

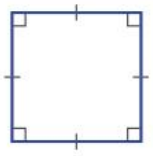
b 

c 

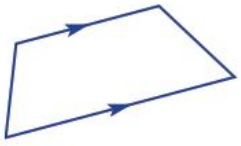
Solution	Explanation
a parallelogram	There are two pairs of parallel sides. There are no right angles.
b trapezium	There is one pair of parallel sides.
c square	There are four equal sides and all angles are 90°.

5 State the special name for each of these quadrilaterals.

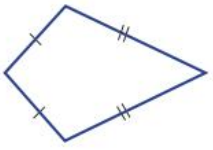
a



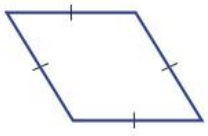
b



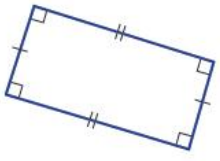
c



d



e

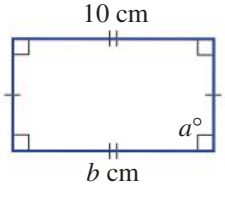


f

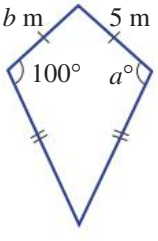


6 Use your knowledge of the properties of quadrilaterals to find the value of the pronumerals in each of these diagrams.

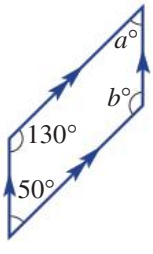
a



b



c



Use the **Key ideas** to check the properties of each shape.



PROBLEM-SOLVING AND REASONING 7, 8 7–10

7 Name all the quadrilaterals that have:

- **a** 2 different pairs of sides of equal length
- **b** 2 different pairs of opposite angles that are equal in size
- **c** 2 different pairs of parallel lines
- **d** only 1 pair of parallel lines
- **e** only 1 pair of opposite angles that are equal in size

8 Consider this 4×4 grid. Using the dots as vertices, how many different shapes of each kind could be drawn? (Do not count shapes that are the same size.)



- **a** squares
- **b** rectangles (not including the squares from part **a**)
- **c** rhombuses (not including the squares)
- **d** parallelograms (not including the shapes from parts **a** to **c**)

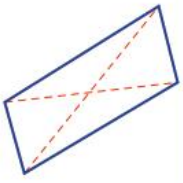


Remember that a square can be drawn like this:



9 The diagonals of a quadrilateral are line segments that join opposite vertices.

- **a** Which quadrilaterals have diagonals of equal length?
- **b** Which quadrilaterals have diagonals intersecting at 90°?





Draw some special quadrilaterals and join opposite vertices to show the diagonals.



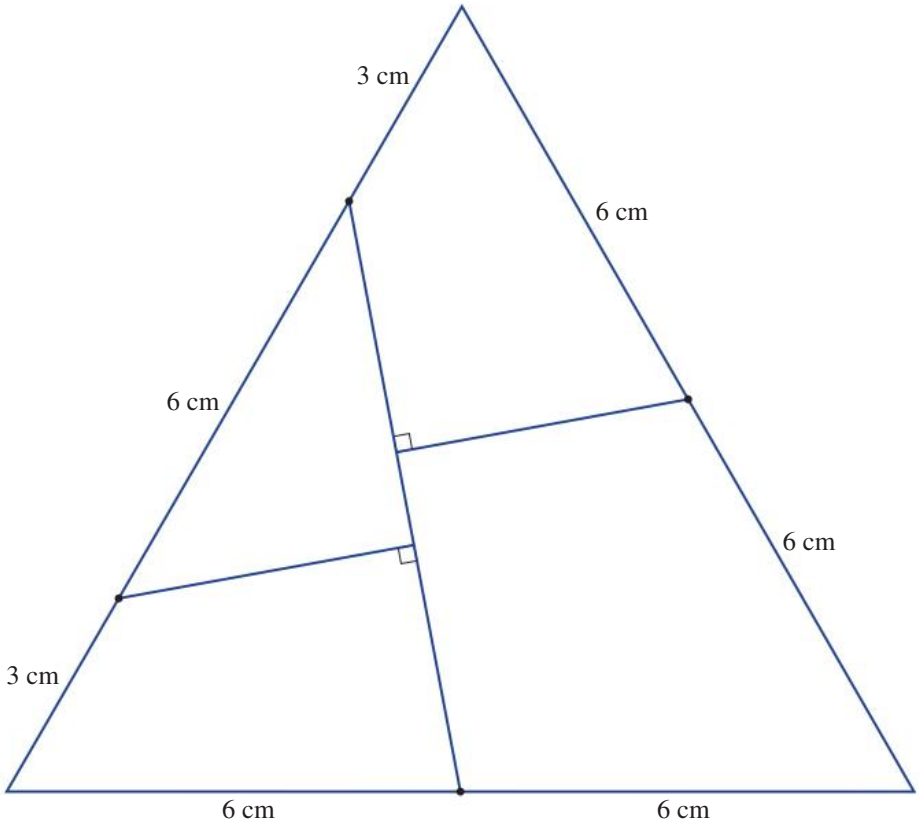
9C

10 **a** Are squares a type of rectangle or are rectangles a type of square? Give an explanation.
b Are rhombuses a type of parallelogram? Explain.
c Are parallelograms a type of trapezium? Explain.
d Is it possible to draw a non-convex trapezium? Give a reason.

A triangle makes a square!

—11

11 Using the given measurements, draw this equilateral triangle accurately on a piece of paper and cut it into the four pieces shown. Can you form a square with all four pieces?
(If you're having trouble drawing it, you could photocopy or print it instead.)



9D Quadrilateral angle sum

Interactive

Widgets

HOTsheets

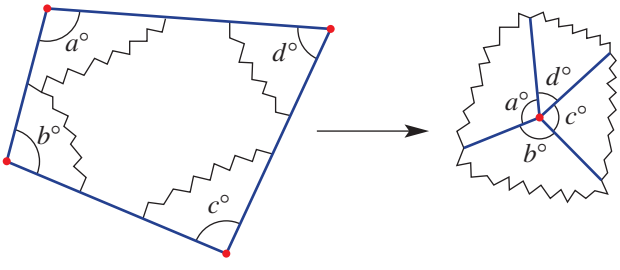
Walkthrough

We have seen that a triangle has an angle sum of 180°. A quadrilateral also has a special angle sum. We use this angle sum to help solve geometry problems that involve four-sided shapes.



Let's start: What do a circle and a quadrilateral have in common?

Use a ruler to draw any quadrilateral. Cut it out and tear off the corners. Arrange them to meet at a point.

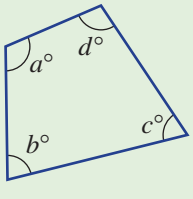


- What does the arrangement tell you about the angles inside a quadrilateral?
- Compare your results with those of others in the class.

Key ideas

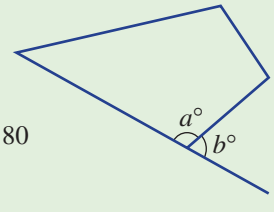
- The interior angle sum of a quadrilateral is 360°.

$a + b + c + d = 360$



- Exterior angles are formed by extending one side.

$a + b = 180$



Exercise 9D

UNDERSTANDING 1–4 | 4

1 Copy and complete:

a The angle sum of a triangle is _____.

b The angle sum of a quadrilateral is _____.

2 **a** Draw any quadrilateral that is convex.

b Measure each interior angle and add them to find the total sum.
Check that your answer is close to 360°.



All the interior angles of a convex shape are less than 180°.



3 **a** Draw any quadrilateral that is non-convex.

b Measure each interior angle and them add to find the total sum.
Check that your answer is close to 360°.

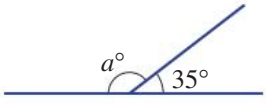


A shape is non-convex when one interior angle is greater than 180°.

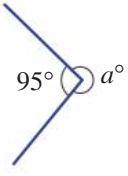


4 Find the value of a in these diagrams.

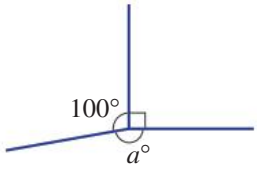
a



b



c

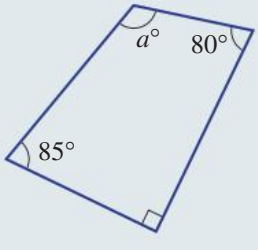


FLUENCY 5 | 5–6

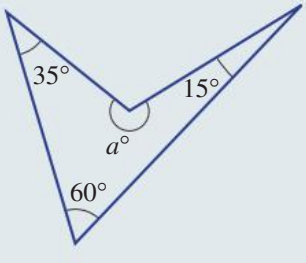
Example 8 Finding unknown angles in quadrilaterals

Find the value of a in each of these quadrilaterals.

a



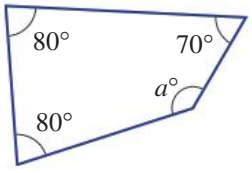
b



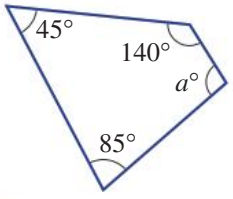
Solution	Explanation
a $85 + 90 + 80 = 255$	Add the known angles.
$a + 255 = 360$	The sum of the interior angles is 360°.
So $a = 105$	Subtract 255 from 360.
b $35 + 60 + 15 = 110$	Add the known angles.
$a + 110 = 360$	Use the angle sum (360°).
So $a = 250$	Subtract to find a.

5 For each of these quadrilaterals, find the size of the unknown angle marked a.

a



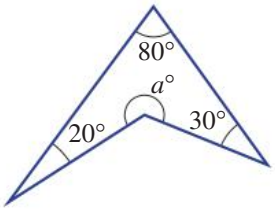
b



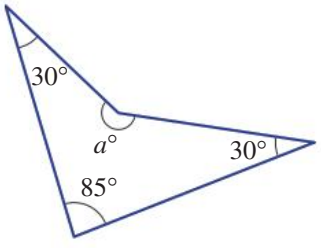
The angle sum in a quadrilateral is 360°.



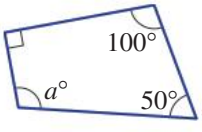
c



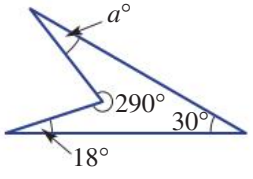
d



e

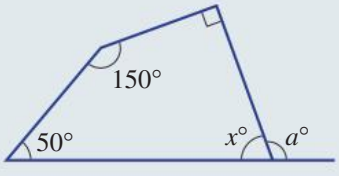


f



Example 9 Using exterior angles

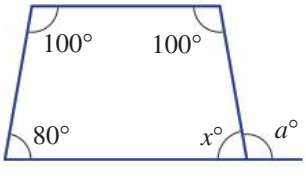
Find the value of a in this quadrilateral by first finding the value of x.



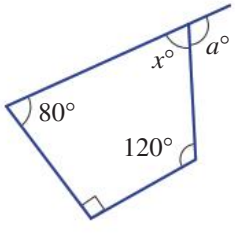
Solution	Explanation
$90 + 150 + 50 = 290$ $x + 290 = 360$ So $x = 70$	First find the value of x using the angle sum (360°).
$a + 70 = 180$ So $a = 110$	Two angles on a straight line add to 180°.

6 Find the value of a in these quadrilaterals by first finding the value of x.

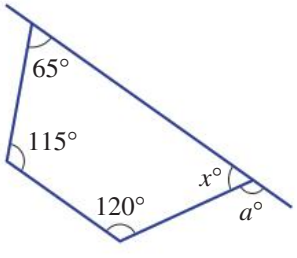
a



b



c

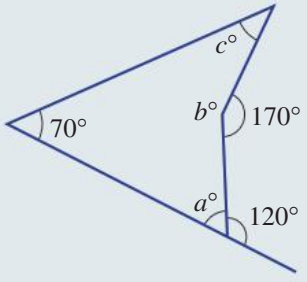


9D

PROBLEM-SOLVING AND REASONING 7, 8 7–10

Example 10 Problem-solving with quadrilaterals

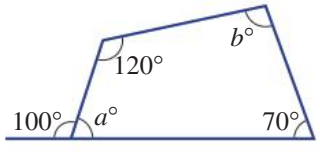
Find the value of a, b and c in this diagram.



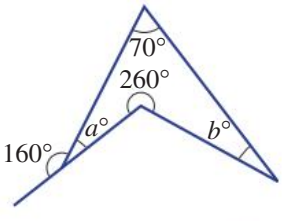
Solution	Explanation
$a = 180 - 120 = 60$	a and 120° are supplementary.
$b = 360 - 170 = 190$	Angles at a point sum to 360°.
$70 + 60 + 190 = 320$	Add the known interior angles.
$c + 320 = 360$	The sum of the interior angles is 360°.
$c = 40$	Subtract to find c.

7 Find the value of a and b in each of the following diagrams.

a



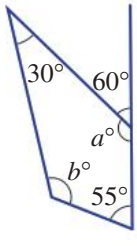
b



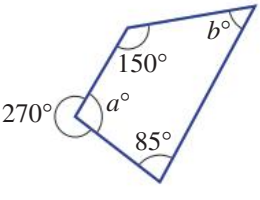


First find the value of a, then use the 360° angle sum to find b.

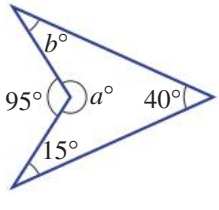
c



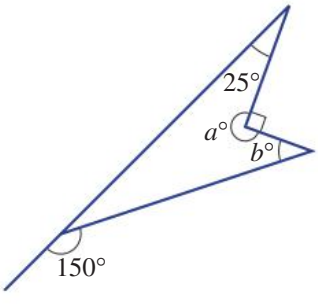
d



e



f



8 Is it possible or impossible for a quadrilateral to have:

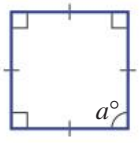
a all interior angles less than $100°$?
b all interior angles less than $90°$?
c more than one interior reflex angle?
d more than one interior obtuse angle?
e more than three interior acute angles?



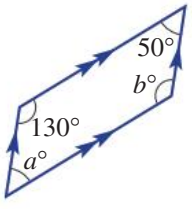
Make drawings to explore these descriptions.

9 Give the value of a (and b) in these special quadrilaterals.

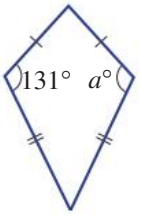
a



b

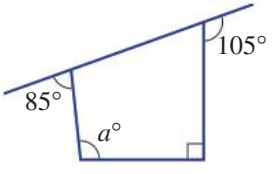


c

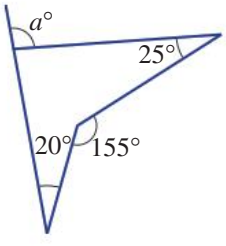


10 For each of these diagrams, find the size of the unknown angle marked a. You may need to find some other angles first.

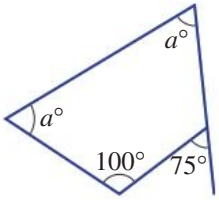
a



b



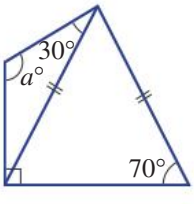
c



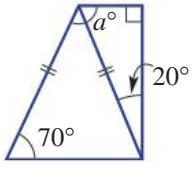
Parallel lines and triangles — 11

11 These diagrams include special triangles or parallel lines. Find the value of a.

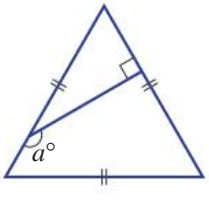
a



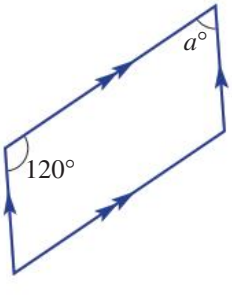
b



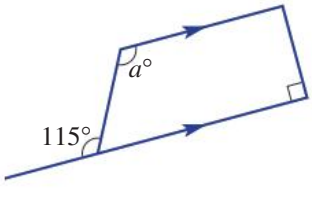
c



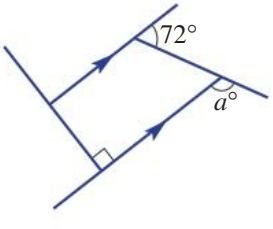
d



e



f



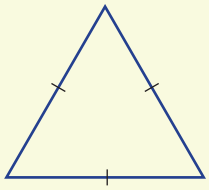
Progress quiz

9A **1** Name the type of triangle that fits the description.

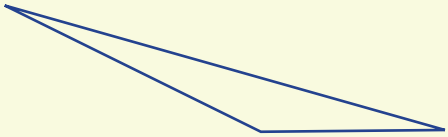
a Has two sides of equal length

b Has three angles each less than 90°

9A **2** **a** Choose the type of triangle (scalene, isosceles, equilateral).

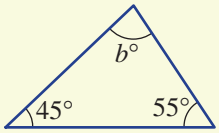


b Choose the type of triangle (acute, right or obtuse).

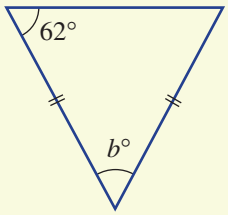


9B **3** Find the value of b in each of these triangles.

a

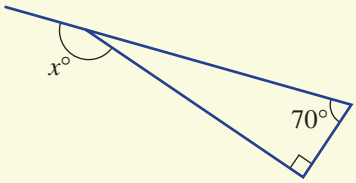


b

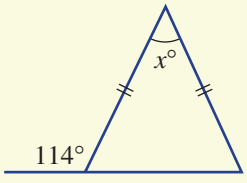


9B **4** Find the value of x in each of these triangles.

a

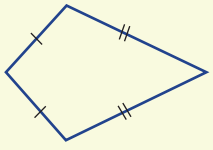


b

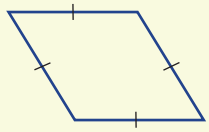


9C **5** State the special name for each of these quadrilaterals.

a



b

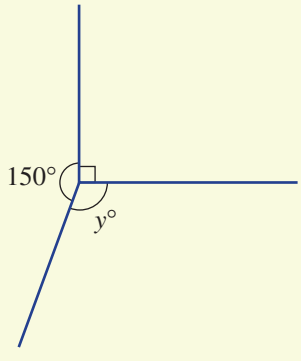


9D **6** Find the value of y in these diagrams.

a

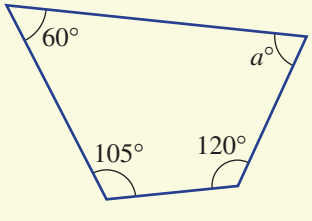


b

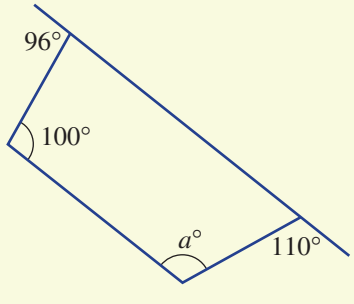


9D **7** Find the value of a in each of these quadrilaterals.

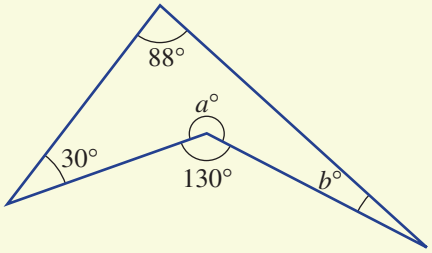
a



b



9D **8** Find the value of a and b in the following diagram.



9E Symmetry

Interactive

Widgets

HOTsheets

Walkthrough

The most familiar form of symmetry, called line symmetry, is connected to the idea of reflection. A flower, for example, might have one or more lines of reflection or mirror lines.

The flower might also have another form of symmetry called rotational symmetry, which will also be studied in this section.

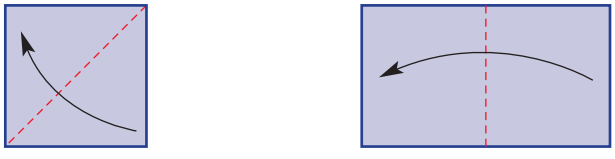


Is this flower perfectly symmetrical?

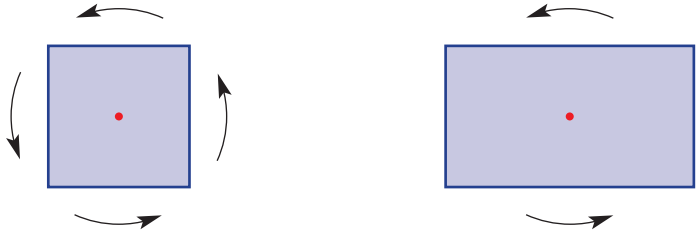
Let's start: Working with symmetry

Cut out a paper square (with side lengths of about 10 cm) and a rectangle (with length of about 15 cm and width of about 10 cm).

- How many ways can you fold each shape in half so that the two halves match exactly? The number of creases formed will be the number of lines of symmetry.

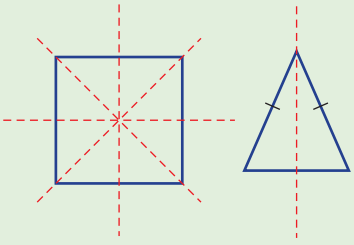


- Now locate the centre of each shape and place a sharp pencil on this point. Rotate the shape 360°. How many times does the shape match its original position exactly? This number describes the rotational symmetry.



Key ideas

- An axis or **line of symmetry** divides a shape into two equal parts. It acts as a mirror line, with each half of the shape being a reflection of the other.

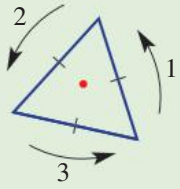


4 lines of symmetry

1 line of symmetry

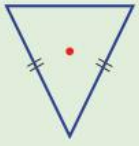
Line of symmetry
The line (axis) along which a figure could be folded to produce identical halves

- The **order of rotational symmetry** is the number of times a shape matches its original position exactly during rotation of 360°.



rotational symmetry of order 3

- We say that there is no **rotational symmetry** if the order is equal to 1.



no rotational symmetry (order 1)

Order of rotational symmetry The number of times a figure matches its original position during rotation of 360°

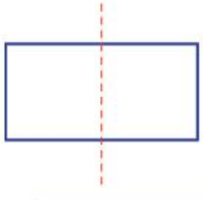
Rotational symmetry When a figure rotated less than 360° matches its original position

Exercise 9E

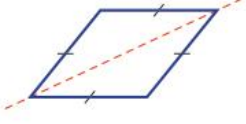
UNDERSTANDING 1–3 3

1 For each shape, decide if the dashed line is a line of symmetry.

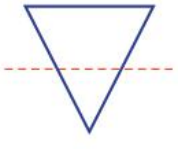
a



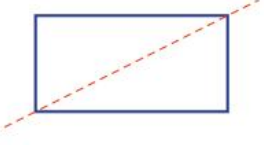
b



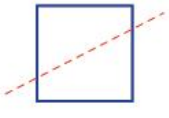
c



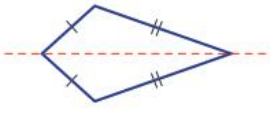
d



e



f



2 In how many ways could you fold each of these shapes in half so that the two halves match exactly?

a square



b rectangle



c equilateral triangle





Try cutting out similar shapes and folding them.

d isosceles triangle



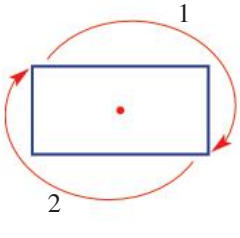
e rhombus



f parallelogram



3 Look again at the shapes in question **2**, and imagine rotating them 360° about their centre. How many times would you get an exact match of the original position?



9E

FLUENCY 4(½), 5 4(½), 5, 6

Example 11 Finding the symmetry of shapes

Find the number of lines of symmetry and the order of rotational symmetry for each of these shapes.

a rectangle



b regular pentagon

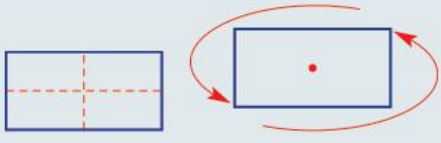


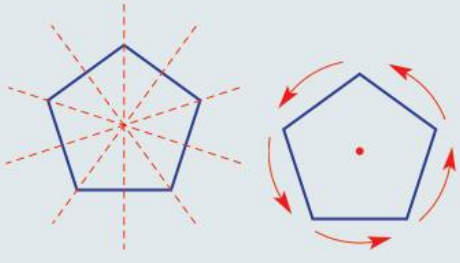
Solution

a 2 lines of symmetry
rotational symmetry of order 2

b 5 lines of symmetry
rotational symmetry of order 5

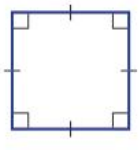
Explanation



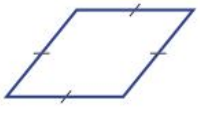


4 State the number of lines of symmetry and the order of rotational symmetry for each shape.

a



b



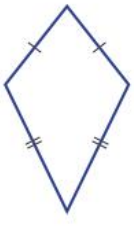
c



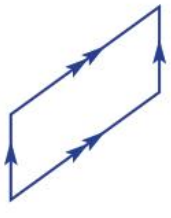
d



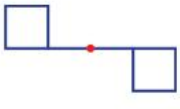
e



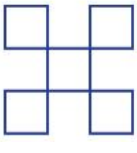
f



g



h



i





The order of rotation is the number of exact 'matches' during a turn of 360°.

5 List the quadrilaterals that have these properties.

a number of lines of symmetry:

i 1 **ii** 2 **iii** 3 **iv** 4

b rotational symmetry of order:

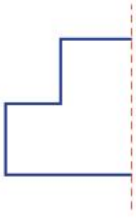
i 1 **ii** 2 **iii** 3 **iv** 4



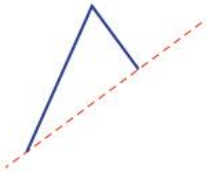
Choose from: *square, rectangle, parallelogram, kite, trapezium, rhombus*

6 Copy each shape and draw the other half for the given axis of symmetry.

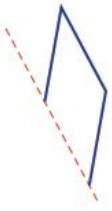
a



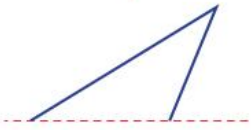
b



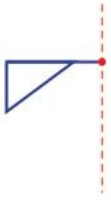
c



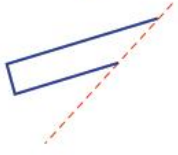
d



e



f



PROBLEM-SOLVING AND REASONING 7, 8 7–10

7 State the number of lines of symmetry and the order of rotational symmetry for each of the following.

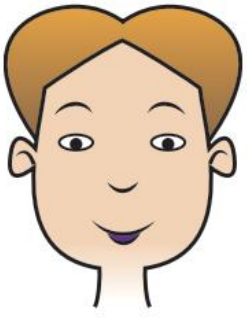
a



b



c



d



8 Name a type of triangle that has:

a 3 lines of symmetry and order of rotational symmetry 3

b 1 line of symmetry and no rotational symmetry

c no line or rotational symmetry

9E

9 Consider these capital letters in the font shown below.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

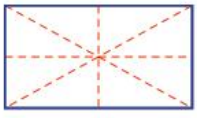
List the letters that have:

a 1 line of symmetry
b 2 lines of symmetry
c rotational symmetry of order 2

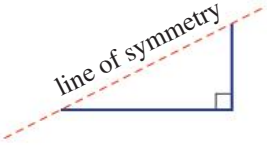


Think carefully about **K** and **Q**.

10 Many people think a rectangle has four lines of symmetry, including the diagonals.



a Complete the other half of this diagram to show that this is not true.



b Using the same method as that used in part **a**, show that the diagonals of a parallelogram are not lines of symmetry.



The answer for part **a** is *not*:

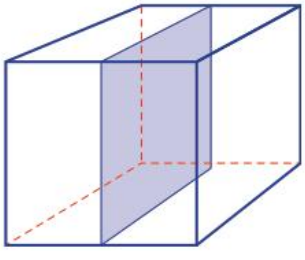


Symmetry in 3D

—

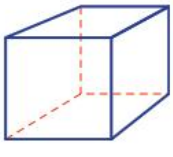
11

11 Some solid objects also have symmetry. Rather than line symmetry, they have plane symmetry. This cube shows one plane of symmetry, but there are more that could be drawn.

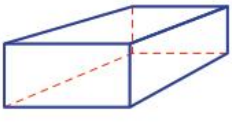


State the number of planes of symmetry for each of these solids.

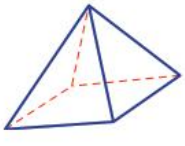
a cube



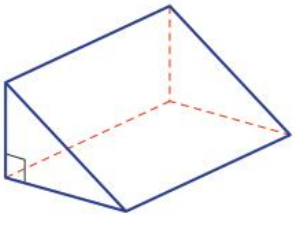
b rectangular prism



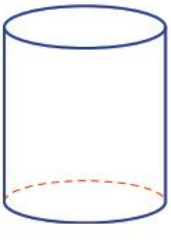
c right square pyramid



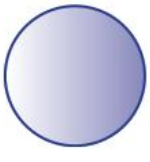
d right triangular prism



e cylinder



f sphere



9F Reflection

Interactive

Widgets

HOTsheets

Walkthrough

From our study of line symmetry, we already know that some shapes can be reflected in their own mirror lines.

Now we will reflect shapes in mirror lines that are not positioned through the centre of the shape. Reflection is one type of transformation that results in an image of equal size and shape to the original.



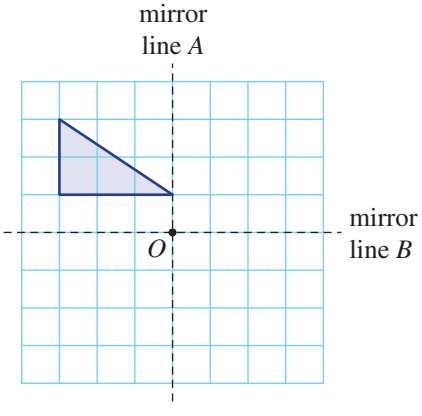
Is this a perfect reflection?

Let's start: Draw the image

Here is a shape on a grid.

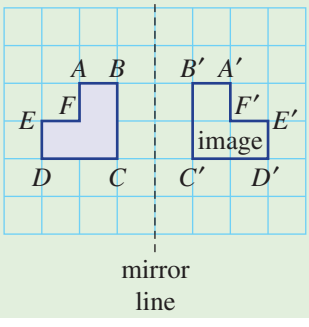
- Draw the image (result) after reflecting the shape in the mirror line A.
- Draw the image (result) after reflecting the shape in the mirror line B.

Discuss what method you used to draw each image and the relationship between the position of the shape and its image after each transformation.



Key ideas

- **Reflection** is a **transformation** that gives an **image** of an object or shape without changing its shape and size.
- The image of point A is denoted A'.
- A reflection involves a mirror line, as shown in the diagram.

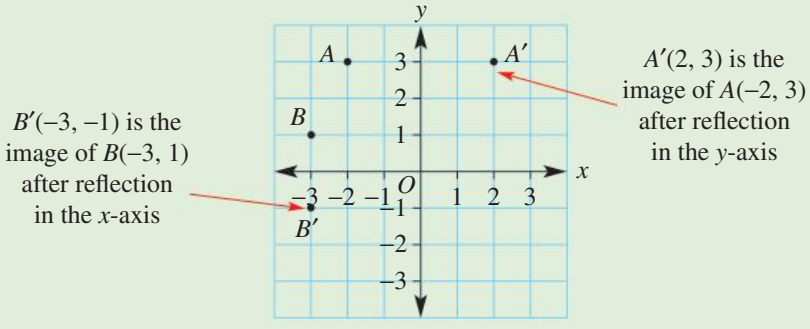


Reflection Flipping a geometrical figure across a line

Transformation Changing a figure's position, size or shape through a mathematical process

Image The result of transforming a figure

- We can use coordinates on the number plane to pinpoint an image after transformation.

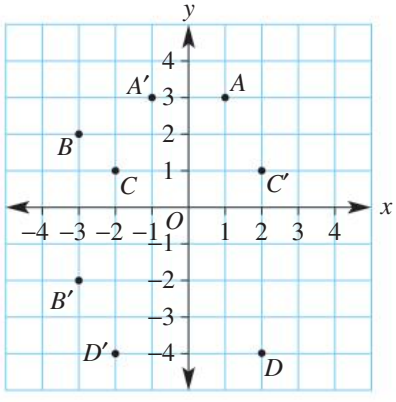


Exercise 9F

UNDERSTANDING 1–2 2

1 Write the coordinates of each of the points shown on this grid.

a A **b** A'
c B **d** B'
e C **f** C'
g D **h** D'

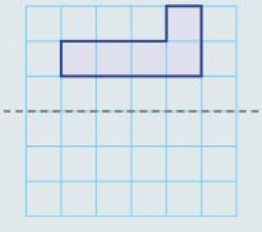




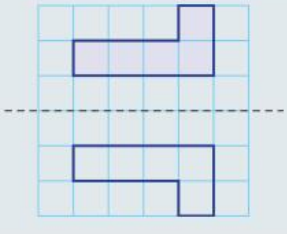
Write the x-coordinate first.

Example 12 Reflecting a shape

Draw the reflected image in the dashed mirror line.



Solution

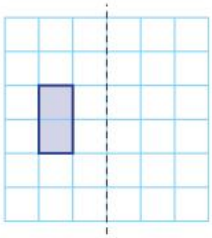


Explanation

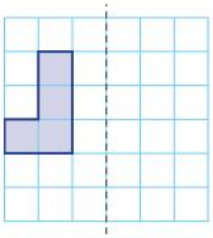
Reflect each vertex (corner) over the mirror line and join to form the image.

2 Copy each grid and reflect the shape in the mirror line.

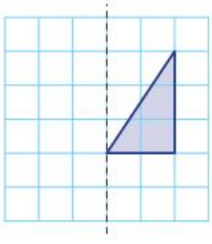
a



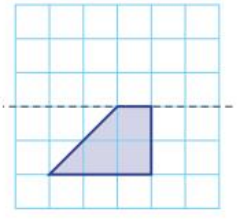
b



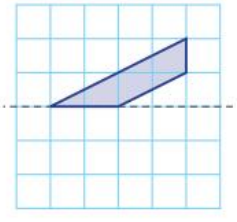
c



d



e



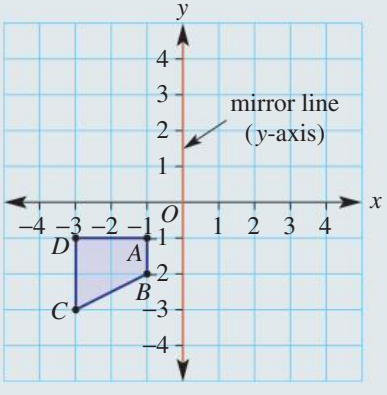


First reflect each vertex (corner point), then join them to form the image.

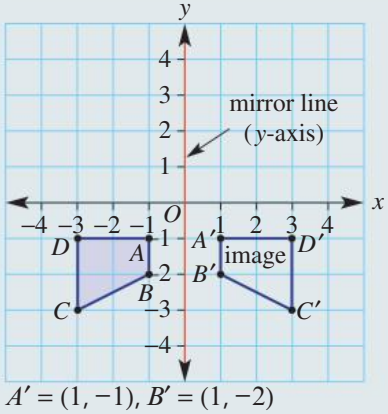
FLUENCY 3–5(½) 4–6(½)

Example 13 Drawing reflections on a number plane

Draw the reflected image of this shape and give the coordinates of A', B', C' and D'. The y-axis is the mirror line.



Solution



$A' = (1, -1)$, $B' = (1, -2)$
$C' = (3, -3)$, $D' = (3, -1)$

Explanation

Reflect each vertex A, B, C and D about the mirror line. The line segment from each point to its image should be at 90° to the mirror line.

9F

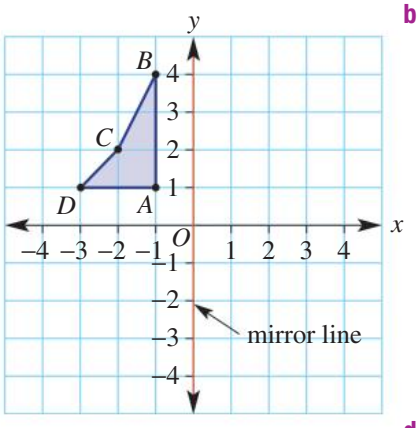
3 Is the size and shape of an object changed after a reflection?

4 Draw the image of each shape and give the coordinates of A', B', C' and D'.

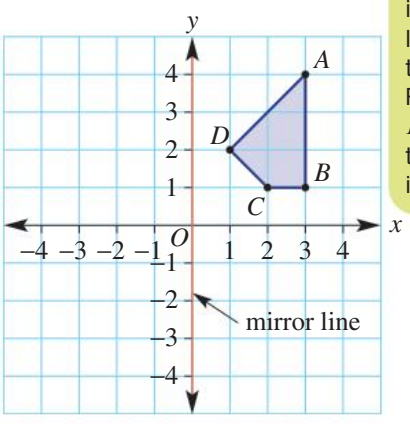


Reflect point A in the mirror line and label the image A'. Repeat to locate B', C' and D' then join the image points.

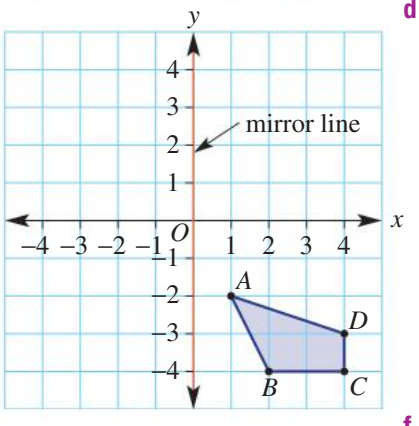
a



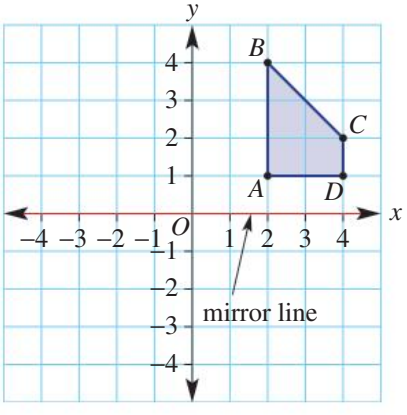
b



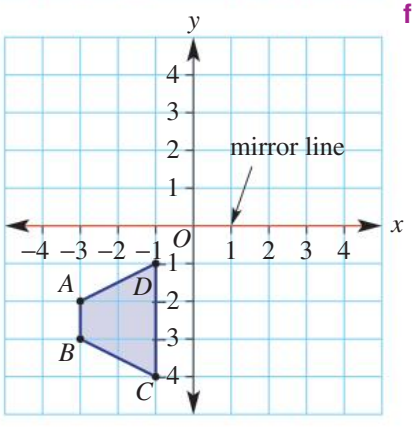
c



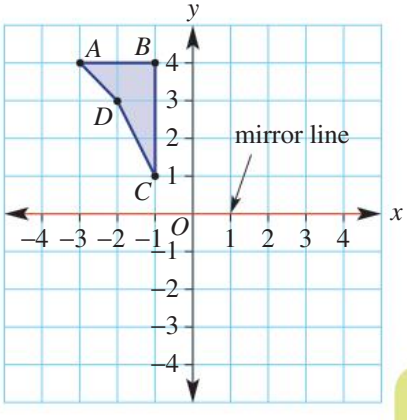
d



e



f



5 Write the coordinates of the following points after reflection in the x-axis.

a $(2, 5)$ **b** $(4, 1)$ **c** $(-3, 2)$ **d** $(-3, 4)$
e $(0, -4)$ **f** $(3, 0)$ **g** $(-2, 0)$ **h** $(-6, -10)$



Reflection in the x-axis is a vertical transformation.

6 Write the coordinates of the following points after reflection in the y-axis.

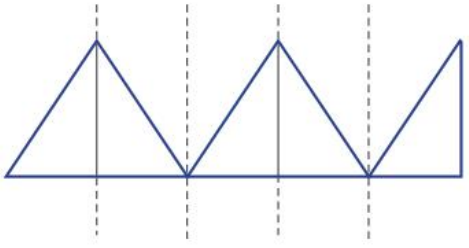
a $(3, 2)$ **b** $(7, 1)$ **c** $(-2, 4)$ **d** $(-4, 6)$
e $(0, 7)$ **f** $(-4, 0)$ **g** $(-4, -6)$ **h** $(0, -3)$



Reflection in the y-axis is a horizontal transformation.

PROBLEM-SOLVING AND REASONING 7, 8 7–10

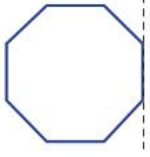
7 By repeatedly reflecting a shape over a moving mirror line, patterns can be formed. For example, the triangle below is reflected 4 times by placing the mirror line vertically and on the right side each time. Create a pattern using these starting shapes by repeatedly placing the vertical mirror line on the right side.



a



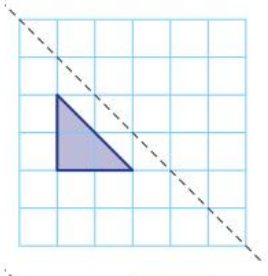
b



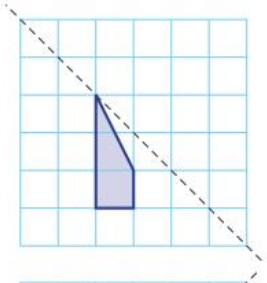
c Create your own using reflection

8 The mirror lines on these grids are at a 45° angle. Draw the reflected image.

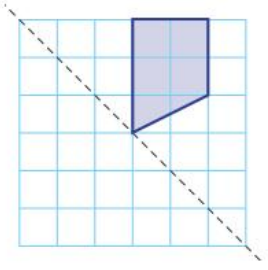
a



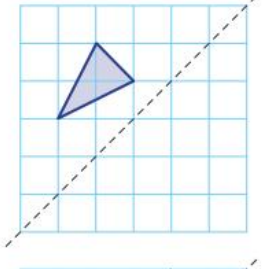
b



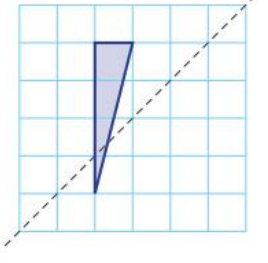
c



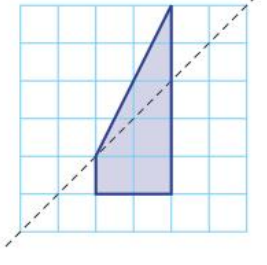
d



e

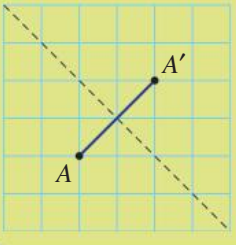


f





The line segment joining a point to its image would be at 90° to the mirror line.



9F

9 On the number plane, the point $A(-2, 5)$ is reflected in the x-axis and this image point is then reflected in the y-axis. What are the coordinates of the final image?

10 What are the coordinates of the image of the point $(2, 3)$ when it is reflected in the line which passes through these pairs of points?

- **a** $(0, -2)$ and $(0, 4)$
- **b** $(1, -3)$ and $(1, 2)$
- **c** $(-2, -4)$ and $(-2, 1)$
- **d** $(-2, 0)$ and $(2, 0)$
- **e** $(-3, 1)$ and $(4, 1)$
- **f** $(-1, -5)$ and $(4, -5)$

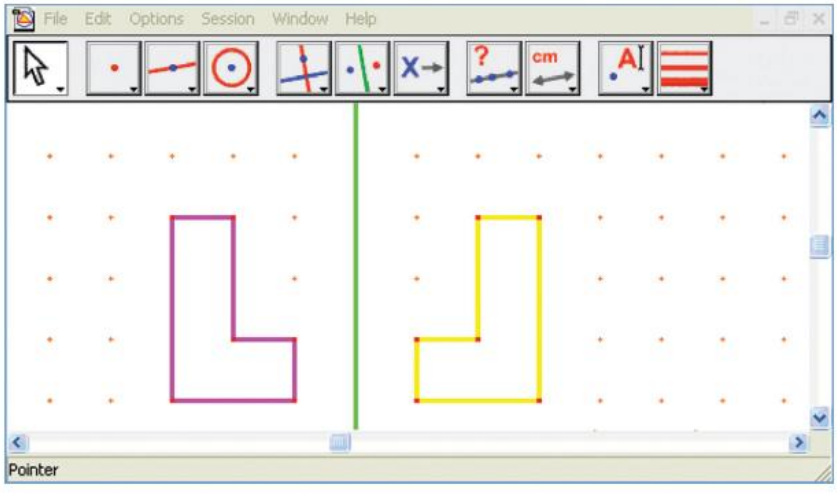
Dynamic geometry exploration

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11

11 Try reflecting shapes using dynamic geometry software.

- **a** On a grid, create any shape using the polygon tool.
- **b** Construct a mirror line.
- **c** Use the reflection tool to create the reflected image about your mirror line.
- **d** Drag the vertices of your original shape and observe the changes in the image. Also try dragging the mirror line.



9G Rotation

Interactive

Widgets

HOTsheets

Walkthrough

We know that rotational symmetry involves turning a shape around its centre. Rotation can also involve moving a shape around a centre of rotation that is outside the shape. This type of transformation still results in an image that is the same size and shape as the original.



What does 'clockwise' mean?

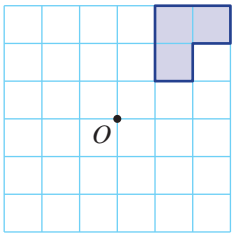
Let's start: Rotation on a grid

Look at the shape on the grid. Draw the image after rotating the shape about point O by:

- 180°
- 90° clockwise
- 90° anticlockwise

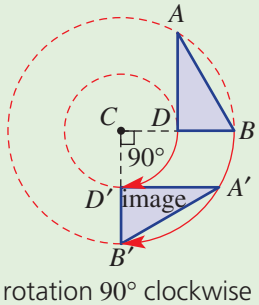
(Anticlockwise means going around a circle in the other direction from clockwise.)

Discuss what method you used to draw each image. What is the relationship between the position of the shape and its image after each rotation?



Key ideas

- **Rotation** is a transformation which involves a turn of every point on a shape about a given point.
- A rotation involves a centre point of rotation (C) and an angle of rotation, as shown.
  - A pair of compasses can be used to draw each circle, to help find the position of image points.



rotation 90° clockwise about C

Rotation A turn around a centre point

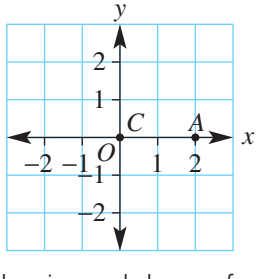
Exercise 9G

UNDERSTANDING 1, 2 2

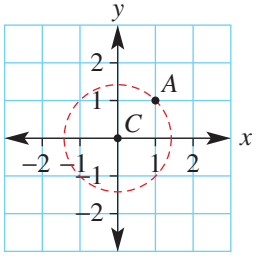
1 Give the coordinates of the point A and its image A' after rotation about point $C(0, 0)$ by:

i 180° clockwise **ii** 180° anticlockwise **iii** 90° clockwise **iv** 90° anticlockwise

a



b



2 Is the size and shape of an object changed after a rotation?

9G

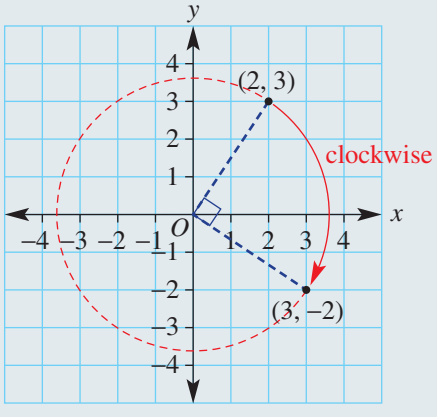
FLUENCY 3, 4 3–5

Example 14 Rotating a point

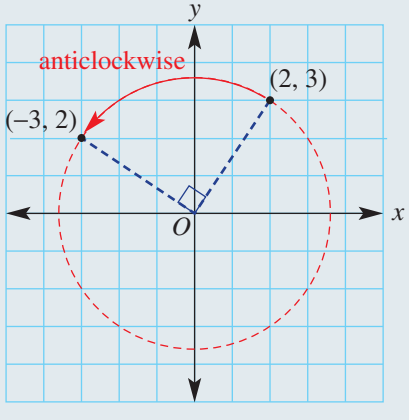
Give the coordinates of the image of the point (2, 3) after each of the following rotations about the origin (0, 0).

a 90° clockwise **b** 90° anticlockwise **c** 180°

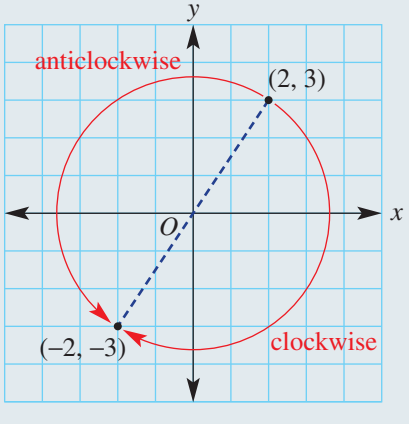
Solution	Explanation
a (3, −2)	Plot the point (2, 3). Use a pair of compasses to draw a circle through the point. Draw a line interval from (2, 3) to the origin. Measure 90° clockwise from the interval. Plot the image point.



b (−3, 2) Use similar steps in the opposite direction.



c (−2, −3) Rotating 180° in either direction gives the same image point.



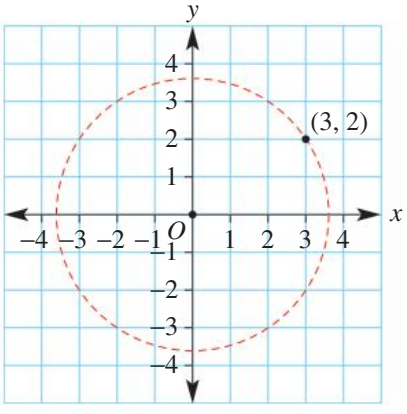
3 Give the coordinates of each of the following points after rotation about the origin $(0, 0)$ by:

i 90° clockwise
ii 90° anticlockwise
iii 180°

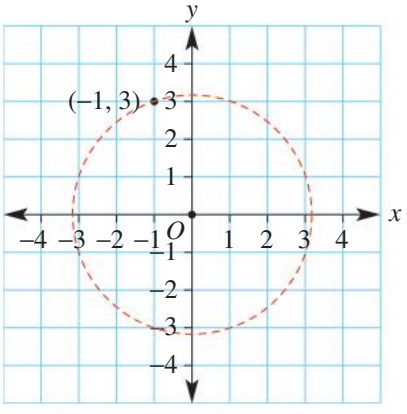


First draw a line interval between the point and $(0, 0)$.

a

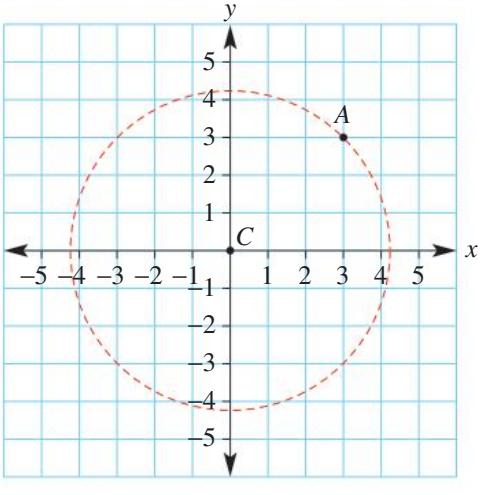


b



4 Give the new coordinates of the image point A' after point A has been rotated around point $C(0, 0)$ by:

a 180° clockwise
b 90° clockwise
c 90° anticlockwise
d 270° clockwise
e 360° anticlockwise
f 180° anticlockwise

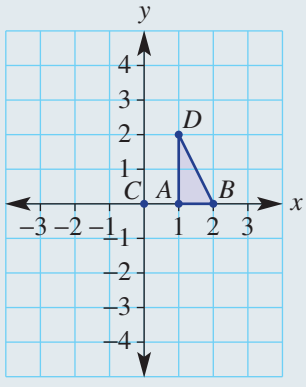


9G

Example 15 Drawing rotations of shapes

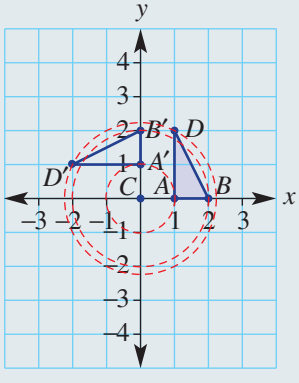
Draw the image of this shape and give the coordinates of A', B' and D' after carrying out the following rotations.

a 90° anticlockwise about C
b 180° about C



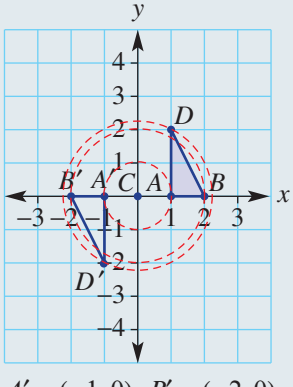
Solution

a



$A' = (0, 1)$, $B' = (0, 2)$, $D' = (-2, 1)$

b



$A' = (-1, 0)$, $B' = (-2, 0)$, $D' = (-1, -2)$

Explanation

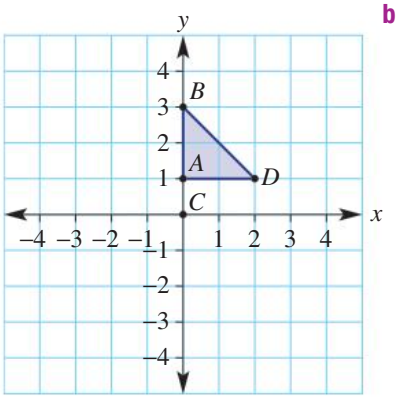
Rotate each point on a circular arc around point C by 90° anticlockwise.

Rotate each point on a circular arc around point C by 180° in either direction.

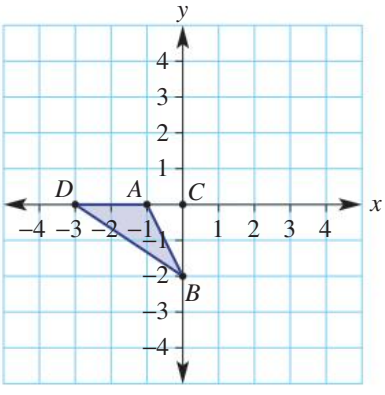
5 Draw the image of each shape and give the coordinates of A', B' and D' after the following rotations.

i 90° anticlockwise about C **ii** 180° about C **iii** 90° clockwise about C

a



b





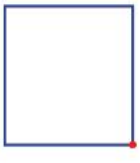
Rotate the points first, then join in form the shape.

PROBLEM-SOLVING AND REASONING 6, 7 6–10

6 By repeatedly rotating a shape about a point, patterns can be formed. This diagram shows a semicircle rotated by 90° three times about the given point. Create a pattern using three 90° rotations about the given point.



a



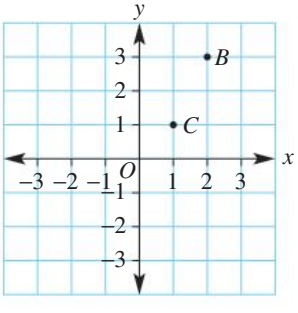
b



c Create your own using rotation

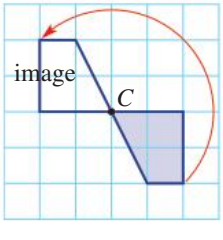
7 The point $B(2, 3)$ is rotated about the point $C(1, 1)$. State the coordinates of the image point B' for the following rotations.

a 180° **b** 90° clockwise **c** 90° anticlockwise

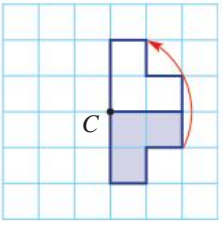


8 How many degrees has each shape been rotated, and in which direction?

a



b

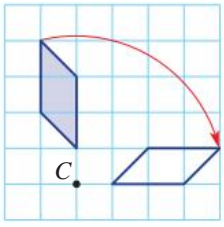




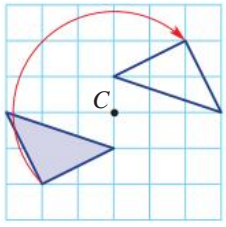
Pick a vertex and identify its image. Draw a line interval joining the vertex to point C, and another joining its image to point C. Then measure the angle between the two intervals.

9G

c



d



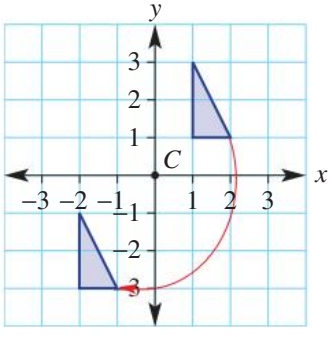
9 Write the missing number in these sentences.

a Rotating a point 90° clockwise is the same as rotating a point _____ anticlockwise.

b Rotating a point 38° anticlockwise is the same as rotating a point _____ clockwise.

c A point is rotated 370° clockwise. This is the same as rotating the point _____ clockwise.

10 Explain what is wrong with this 180° rotation about $C(0, 0)$.



Dynamic geometry exploration — 11

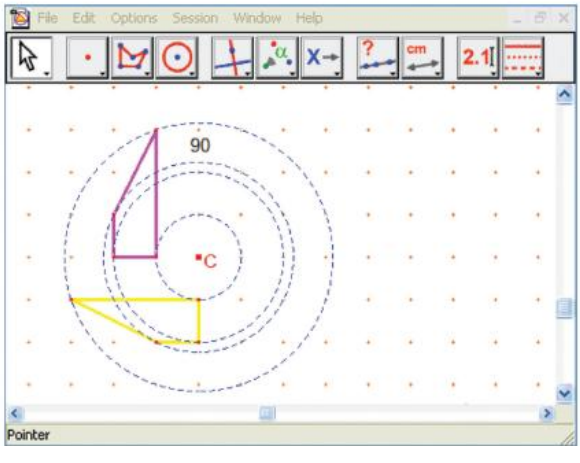
11 Try rotating shapes using dynamic geometry software.

a On a grid, create any shape using the polygon tool.

b Construct a centre of rotation point and a rotating angle (or number). In Cabri, use the Numerical Edit tool to create a dynamic number.

c Use the rotation tool to create the rotated image that has your nominated centre of rotation and angle. You will need to click on the shape, the centre of rotation and your angle.

d Drag the vertices of your original shape and observe the changes in the image. Also try changing the angle of rotation.



9H Translation

Interactive

Widgets

HOTsheets

Walkthrough

Along with rotation and reflection, a third important transformation that does not change the shape or size of the image is called translation. Translation involves a shift in an object left, right, up or down. The orientation of a shape is also unchanged.

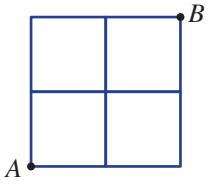


To solve this puzzle you translate numbers left, right, up or down.

Let's start: City grid

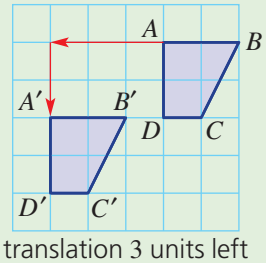
Imagine that a point A on a simple city grid map is your starting point, and point B is your destination.

- Describe one simple way of travelling from A to B.
- Describe two other ways of travelling from A to B.
- How many different ways are there if no section can be travelled more than once?



Key ideas

■ **Translation** is a transformation involving a shift to the left, right, up or down.
- Describing a translation involves saying how many units a shape is shifted left or right and/or up or down.



translation 3 units left and 2 units down

Translation Moving a shape a certain distance in a given direction

Exercise 9H

UNDERSTANDING 1–3 3

1 The point $(7, 4)$ is translated to the point $(0, 1)$.

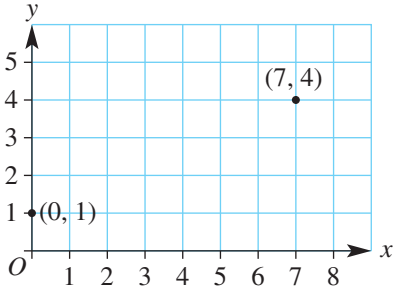
a How far left has the point been translated?

b How far down has the point been translated?

c If the point $(0, 1)$ is translated to $(7, 4)$:

i how far right has the point been translated?

ii how far up has the point been translated?



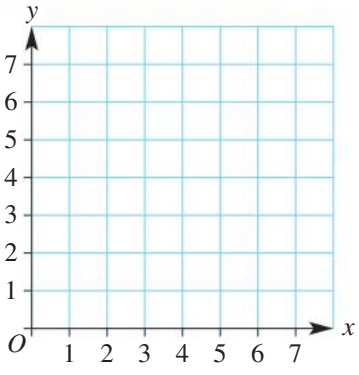
9H

2 A point is translated to its image. Write the missing word (i.e. left, right, up or down) for each of these.

a (1, 1) is translated _____ to the point (1, 3).
b (5, 4) is translated _____ to the point (1, 4).
c (7, 2) is translated _____ to the point (7, 0).
d (3, 0) is translated _____ to the point (3, 1).
e (5, 1) is translated _____ to the point (4, 1).
f (2, 3) is translated _____ to the point (1, 3).
g (0, 2) is translated _____ to the point (5, 2).
h (7, 6) is translated _____ to the point (11, 6).

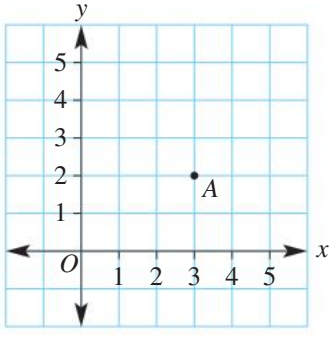


Pencil each pair of points onto a grid to see the translation.



3 Point A has coordinates (3, 2). Write the coordinates of the image point A' when point A is translated in each of the following ways.

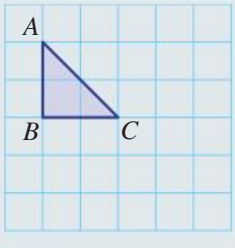
a 1 unit right
b 2 units left
c 3 units up
d 1 unit down
e 1 unit left and 2 units up
f 3 units left and 1 unit down
g 2 units right and 1 unit down
h 0 units left and 2 units down



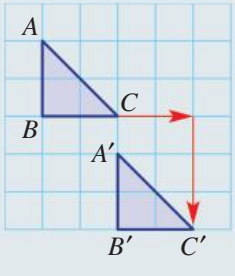
FLUENCY 4 4, 5(½)

Example 16 Translating shapes

Draw the image of the triangle ABC after a translation 2 units to the right and 3 units down.



Solution

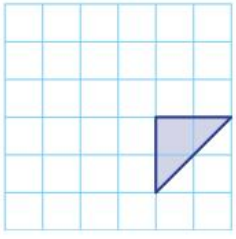


Explanation

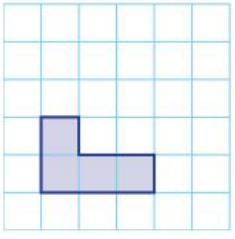
Shift each vertex 2 units to the right and 3 units down. Then join the vertices to form the image.

4 Draw the image of these shapes after each translation.

a 3 units left and 1 unit up



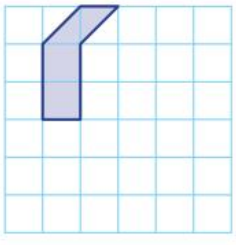
b 1 unit right and 2 units up



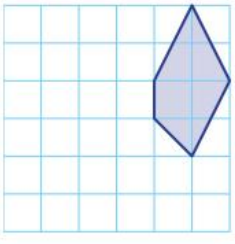


First translate each corner, then join the points to form the image.

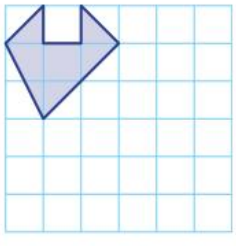
c 3 units right and 2 units down



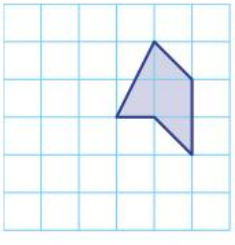
d 4 units left and 2 units down



e 2 units right and 3 units down

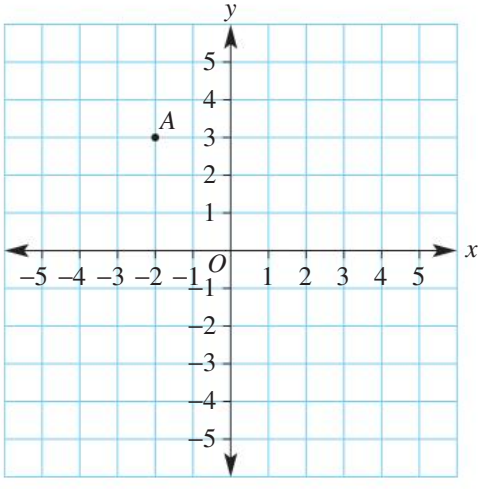


f 3 units left and 1 unit down



5 Point A has coordinates $(-2, 3)$. Write the coordinates of the image point A' when point A is translated in each of the following ways.

a 3 units right
b 2 units left
c 2 units down
d 5 units down
e 2 units up
f 10 units right
g 3 units right and 1 unit up
h 4 units right and 2 units down
i 5 units right and 6 units down
j 1 unit left and 2 units down
k 3 units left and 1 unit up
l 2 units left and 5 units down

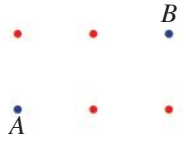


9H

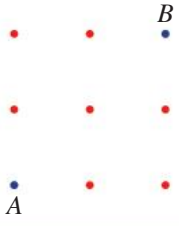
PROBLEM-SOLVING AND REASONING 6, 7 6–9

6 If only horizontal or vertical translations are allowed, how many different paths are there from points A to B on each grid below? No point can be visited more than once.

a



b



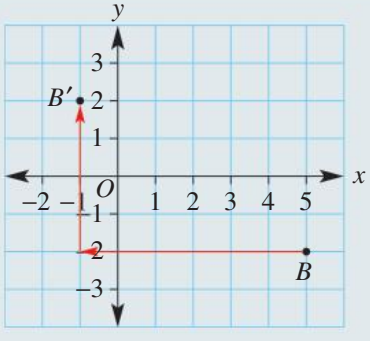
Example 17 Describing translations

A point $B(5, -2)$ is translated to $B'(-1, 2)$ Describe the translation.

Solution

Translation is 6 units left and 4 units up.

Explanation

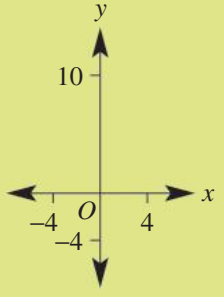


7 Describe the translation from each point to its image.

a $A(1, 3)$ is translated to $A'(1, 6)$.
b $B(4, 7)$ is translated to $B'(4, 0)$.
c $C(-1, 3)$ is translated to $C'(-1, -1)$.
d $D(-2, 8)$ is translated to $D'(-2, 10)$.
e $E(4, 3)$ is translated to $E'(-1, 3)$.
f $F(2, -4)$ is translated to $F'(4, -4)$.
g $G(0, 0)$ is translated to $G'(-1, 4)$.
h $H(-1, -1)$ is translated to $H'(2, 5)$.
i $I(-3, 8)$ is translated to $I'(0, 4)$.
j $J(2, -5)$ is translated to $J'(-1, 6)$.
k $K(-10, 2)$ is translated to $K'(2, -1)$.
l $L(6, 10)$ is translated to $L(-4, -3)$.



Use a diagram to help.



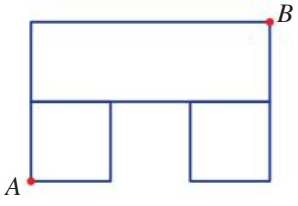
Give answers such as '4 units right' or '2 units left and 3 units up'.

8 The point A is translated to its image, A'. Describe the translation that takes A' to A (i.e. the reverse translation).

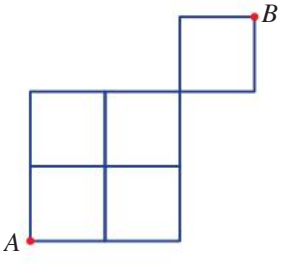
a $A(2, 3)$ and $A'(4, 1)$
b $B(0, 4)$ and $B'(4, 0)$
c $C(0, -3)$ and $C'(-1, 2)$
d $D(4, 6)$ and $D(-2, 8)$

9 If only horizontal and vertical translations are allowed, how many different paths are there from point A to point B? No section can be used more than once in each path.

a



b



Combined transformations — 10

10 Write the coordinates of the image point after each sequence of transformations. (Apply each transformation to the image of the previous transformation.)

a $(2, 3)$
- reflection in the x-axis
- reflection in the y-axis
- translation 2 units left and 2 units up

b $(-1, 6)$
- translation 5 units right and 3 units down
- reflection in the y-axis
- reflection in the x-axis

c $(-4, 2)$
- rotation $180°$ about $(0, 0)$
- reflection in the y-axis
- translation 3 units left and 4 units up

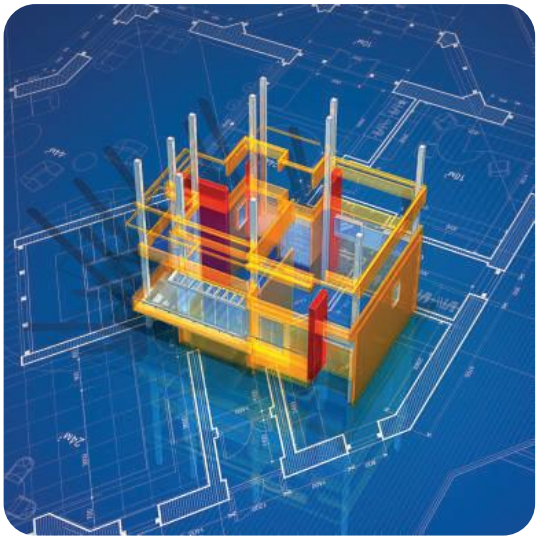


Factory cranes work using horizontal and vertical translations.

9I Drawing solids

Interactive
Widgets
HOTsheets
Walkthrough

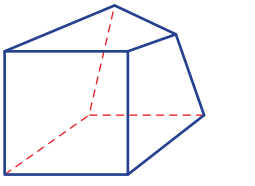
Three-dimensional solids can be represented as a drawing on a two-dimensional surface (such as paper or a computer screen), provided some basic rules are followed.

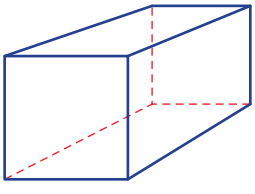


Architects create 3D models of building plans by hand or with computer software.

Let's start: Can you draw a cube?

Try to draw a cube. Here are some bad examples.

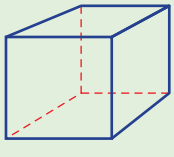


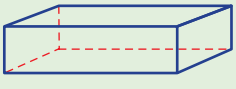


- What is wrong with these drawings?
- What basic rules do you need to follow when drawing a cube?

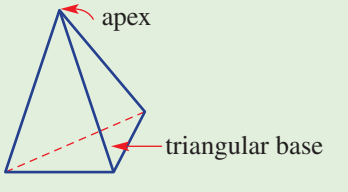
Key ideas

- Draw **cubes** and rectangular **prisms** by keeping:
  - parallel edges pointing in the same direction
  - parallel edges the same length.

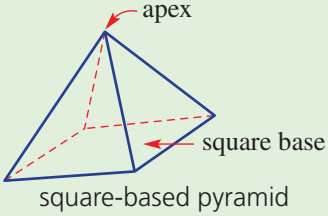




- Draw **pyramids** by joining the apex with the vertices on the base.



triangular pyramid (tetrahedron)



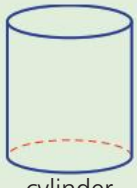
square-based pyramid

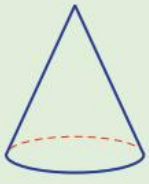
Cube A solid (3D shape) with six square faces that are congruent (the same size and shape)

Prism A solid with two parallel congruent faces, joined by parallelograms (often rectangles)

Pyramid A solid where the base has straight sides and the other faces are triangles meeting at a vertex (the apex)

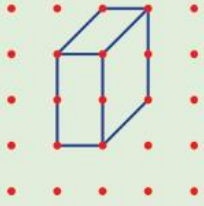
- Draw **cylinders** and **cones** by starting with an oval shape.


cylinder


cone

- Square and isometric dot paper can help you to draw solids accurately. Drawings made on isometric dot paper clearly show the cubes that make up the solid.

square dot paper



isometric dot paper



Cylinder A solid with two circular faces joined by a curved surface

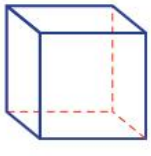
Cone A solid with a circular base and a slanting curved surface that tapers to a point called the apex

Exercise 9I

UNDERSTANDING 1–3 3

1 Name these solids.

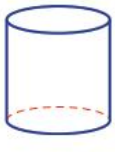
a



b



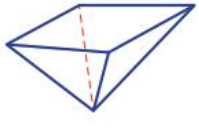
c



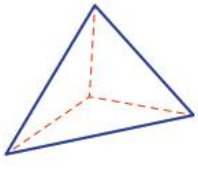
d



e



f

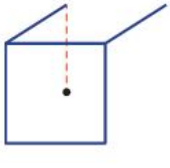




Choose from: *cone, cylinder, triangular pyramid, square pyramid, cube, rectangular prism*

2 Copy these diagrams and add lines to complete the solid. Use dashed lines for invisible edges.

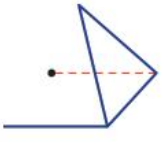
a cube



b cylinder



c square pyramid

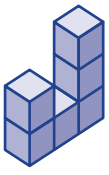


9I

3 Cubes are stacked to form these solids. How many cubes are there in each solid?

a 

b 

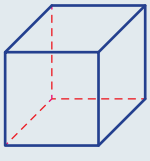
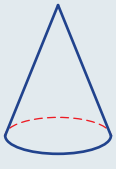
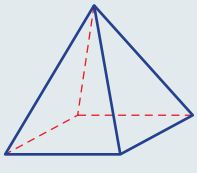
c 

FLUENCY 4, 5 4–6

Example 18 Drawing solids

Draw these solids.

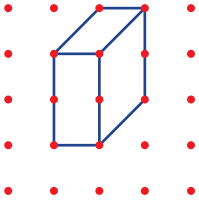
a a cube
b a cone
c a square-based pyramid

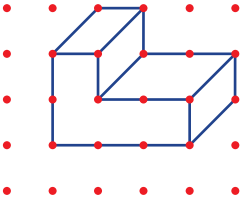
Solution	Explanation
a 	Draw the square front. Draw edges parallel to the top and the right edge. Connect using dashed lines for hidden edges.
b 	Draw an oval shape for the base. Draw the apex point above the centre of the oval. Join the apex to the sides of the base and erase part of the oval to create a dashed (hidden) curve.
c 	Draw a rhombus for the base. Pick a central point above the base for the axis, then connect. Change hidden edges to dashed lines.

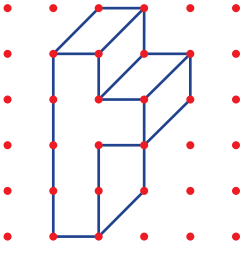
4 On plain paper, draw an example of these common solids.

a cube
b cylinder
c triangular-based pyramid
d cone
e rectangular prism
f square-based pyramid

5 Copy these solids onto square dot paper.

a 

b 

c 

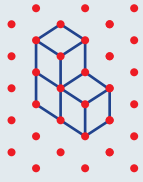
Example 19 Using isometric dot paper

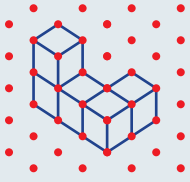
Draw these solids on isometric dot paper.

a 

b 

Solution

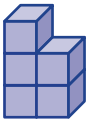
a 

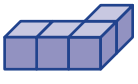
b 

Explanation

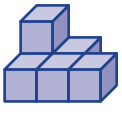
Rotate the solids slightly. Then start at one corner and build your solids piece by piece.

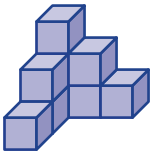
6 Draw these solids on isometric dot paper.

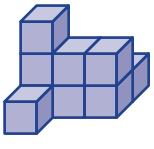
a 

b 

c 

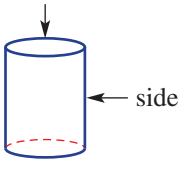
d 

e 

f 

PROBLEM-SOLVING AND REASONING 7, 8 7–10

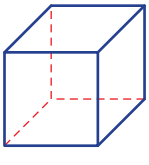
7 Here is a cylinder with its top view (circle) and side view (rectangle).

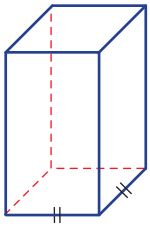


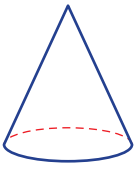




Draw the shapes which are the top view and side view of these solids.

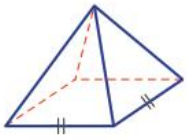
a cube 

b square-based prism 

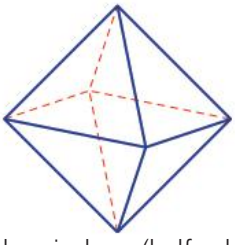
c cone 

9I

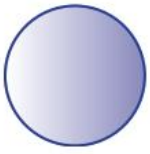
d square-based pyramid



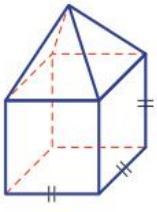
e octahedron



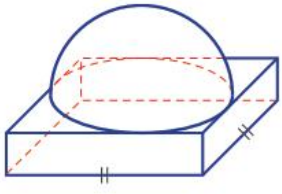
f sphere



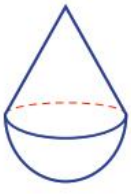
g square pyramid on cube



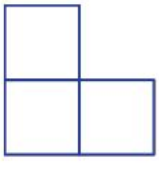
h hemisphere (half sphere) on square prism



i cone on hemisphere

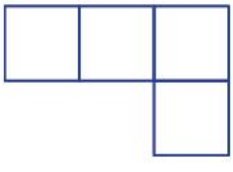


8 Here is the top (or plan) view of a stack of 5 cubes.
How many different stacks of 5 cubes could this represent?





9 Here is the top view of a stack of 7 cubes.
How many different stacks of 7 cubes could this represent?



10 Match each solid (**a** to **d**) with an identical solid chosen from **A**, **B**, **C** and **D**.

a



b



c



d



A



B



C



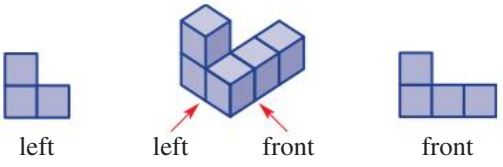
D



3 view points

11

11 These diagrams show the front and left views of a solid.

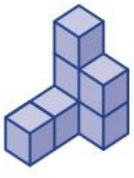


a Draw the front, left and top views of these solids.

i

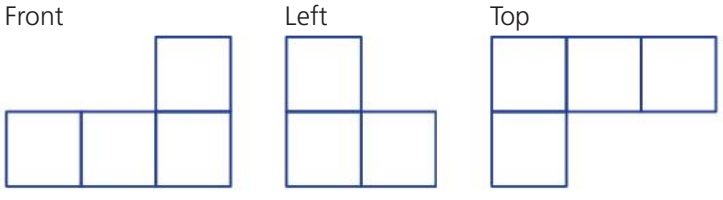


ii

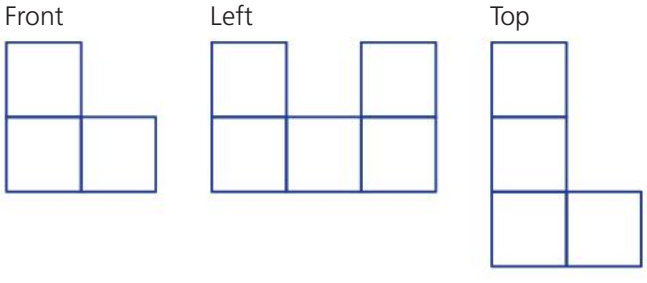


b Draw the solid that has these views.

i



ii



Maths@Work: Quilting

A home-based hobby can become a successful small business. Many communities run markets which are a great way for creative people to sell their handmade items.

Various crafts require an understanding of mathematics. The type of maths needed is dependent on the product created. Designing and making a quilt is a technical process which requires accurate measuring and an understanding of shapes and transformations. Also, an ability to select appealing colour combinations is useful.

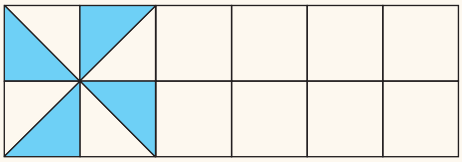


1 The properties of triangles and quadrilaterals are important in quilt design. Many standard quilt patterns use squares and triangles.

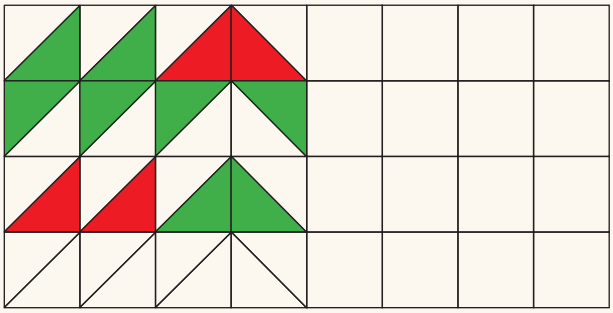
- **a** State the properties of a right-angled isosceles triangle.
- **b** List the names of the quadrilaterals that have at least one line of symmetry.
- **c** Show three ways that a square can have exactly half of its area shaded. A rotation doesn't count as a different method.

2 Below are some quilting designs that have been started. Using grid paper, copy and complete each design.

a

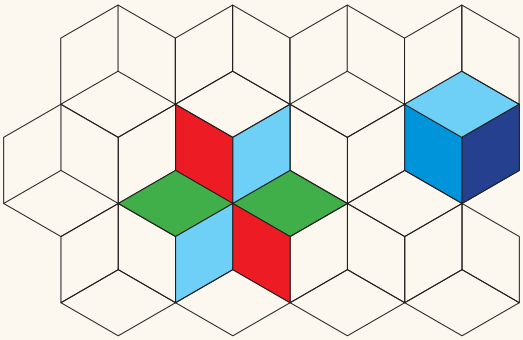


b



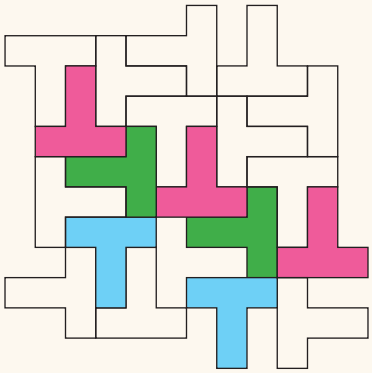
3 For each illustration, explain how the shapes are transformed.

a Rhombus shapes transformed:



i blue to the blue opposite (in the red/blue/green star)
ii blue to green to red (in the red/blue/green star)
iii light-blue to mid-blue to dark-blue (in the right side blue hexagon)

b T shapes transformed:



i pink to the next pink
ii pink to green
iii pink to blue

4 For each quilt design below:

i count how many lines of symmetry there are in the design
ii state the order of rotational symmetry
iii state the types of transformations applied to repeated shapes

a



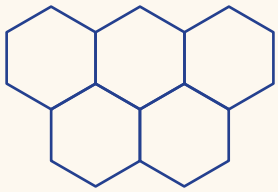
b



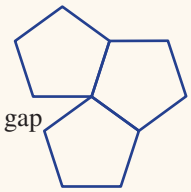
5 Quilts are often designed using a tessellation pattern. A tessellation has polygons joined together on a flat surface without any gaps. Polygons that can tessellate have interior angles that divide evenly into a revolution (360°).

a Find the interior angles in regular hexagons and pentagons and show why one of these shapes can tessellate and why the other shape cannot tessellate.

regular hexagons



regular pentagons



b A popular design has a pinwheel pattern inside squares as shown in this photo of a quilt.

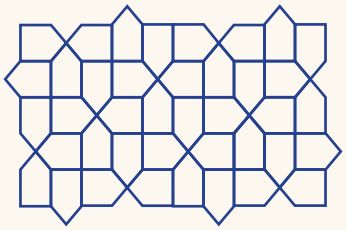
i What type of triangle has been tessellated to make the pinwheel? Draw and label this triangle.

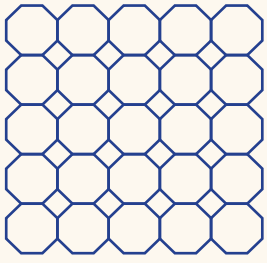


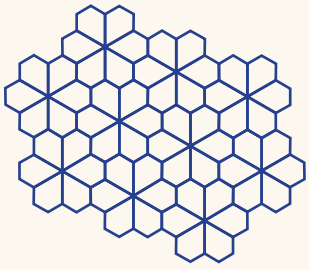
ii Describe the transformations of a triangle that are seen in the pinwheel design.

Using technology

6 Using a geometry application, design and colour your own quilt tessellation pattern. You may wish to use one of the following tessellations.







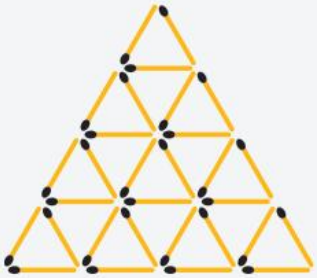
1 Use six matchsticks to create four equilateral triangles. Draw your creation.



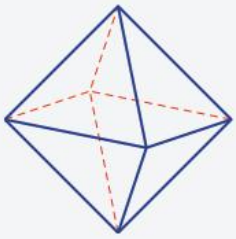


Hint: Use three dimensions.

2 How many equilateral triangles of any size are in this diagram?



3 A regular octahedron has its corners cut off. How many edges are there on the new shape?



4 A polygon's vertices are joined by diagonals. Without counting them, work out how many diagonals can be drawn in each of these polygons.

a quadrilateral



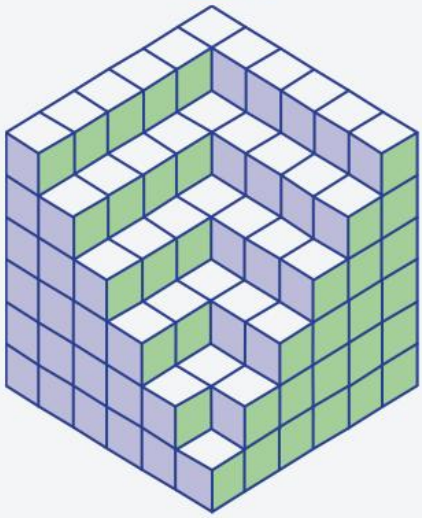
b heptagon



c decagon (10 sides)



5 This solid was made from stacking 1 cm cubes. How many cubes were used?



6 Make your own solid out of 1 cm cubes. Draw the solid on isometric dot paper. Add shading to enhance the 3D effect.

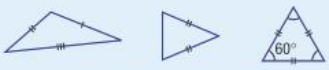
Chapter summary

Shapes and transformations

Triangles

Classifying by sides

scalene isosceles equilateral

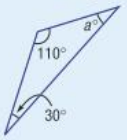


Classifying by angles

acute right obtuse

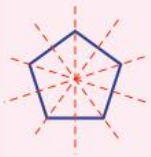


Angle sum



$30 + 110 = 140$
$a + 140 = 180$
So $a = 40$

Symmetry



5 lines of symmetry
rotational symmetry of order 5

regular pentagon

Solids

cylinder square-based pyramid

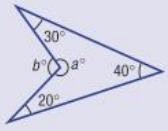


Quadrilaterals

Special quadrilaterals

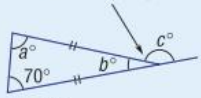
- parallelogram
- rectangle
- rhombus
- square
- trapezium
- kite

Angle sum



$20 + 40 + 30 = 90$
$a + 90 = 360$
So $a = 270$
$b + 270 = 360$
So $b = 90$

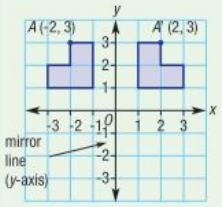
exterior angle



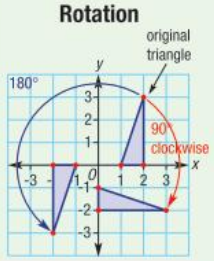
$a = 70$
$70 + 70 + b = 180$
So $b = 40$
$c + 40 = 180$
So $c = 140$

Transformations

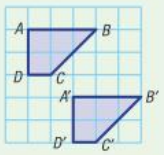
Reflection



Rotation



Translation



2 units right and
3 units down

Chapter review

Multiple-choice questions

9A **1** An equilateral triangle has:

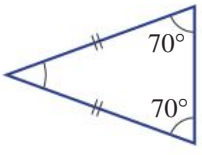
A each interior angle equal to 90°
B a pair of equal sides
C two pairs of equal opposite angles
D one obtuse angle
E each interior angle equal to 60°

9A **2** The three types of triangles all classified by their interior angles are:

A acute, isosceles and scalene
B acute, right and obtuse
C scalene, isosceles and right
D right, obtuse and scalene
E acute, equilateral and right

9B **3** Two angles inside an isosceles triangle are both 70°. The third angle is:

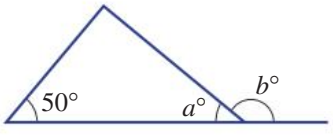
A 70° **B** 30° **C** 40°
D 20° **E** 180°



9B **4** The angle sum of a triangle is always:

A 120° **B** 360° **C** 270° **D** 180° **E** 90°

9B **5** For this triangle, which of the following is true?



A $a + 50 = 180$
B $a + 6 = 360$
C $a + b = 180$
D $b + 50 = 180$
E $a + b + 50 = 180$

9C **6** The quadrilateral that has two pairs of equal sides and one pair of equal angles is called a:

A kite **B** trapezium **C** rhombus **D** triangle **E** square

9D **7** Three angles inside a quadrilateral add to 275°. The fourth angle is:

A 750° **B** 95° **C** 285° **D** 125° **E** 85°

9E **8** How many lines of symmetry does a rhombus have?

A 0 **B** 1 **C** 2 **D** 3 **E** 4

9F **9** The point $T(-3, 4)$ is reflected in the x-axis. The image point T' has coordinates:

A $(3, 4)$ **B** $(-3, 4)$ **C** $(0, 4)$ **D** $(3, -4)$ **E** $(-3, -4)$

9H **10** The translation that takes $A(2, -3)$ to $A'(-1, 1)$ could be described as:

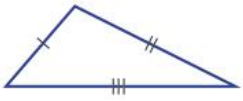
A 3 units left
B 4 units up
C 3 units left and 4 units up
D 1 unit right and 2 units down
E 1 unit left and 2 units down

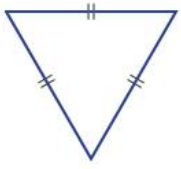
Chapter review

Short-answer questions

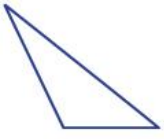
9A **1** Classify these triangles as scalene, isosceles or equilateral.

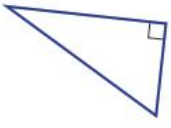
a 

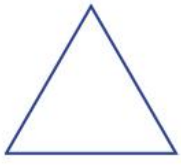
b 

c 

9A **2** Classify these triangles as acute, right or obtuse.

a 

b 

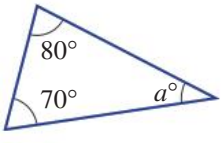
c 

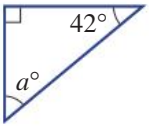
9A **3** Use a protractor and ruler to draw these triangles.

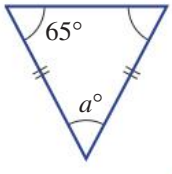
a triangle ABC with $AB = 4$ cm, $\angle CAB = 25°$ and $\angle ABC = 45°$

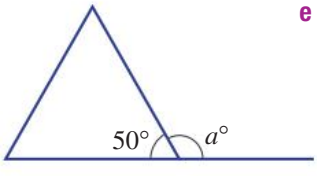
b triangle ABC with $AB = 5$ cm, $\angle BAC = 50°$ and $AC = 5$ cm

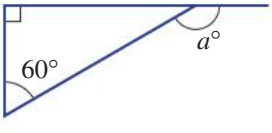
9B **4** Find the unknown angle, a, in each of these shapes.

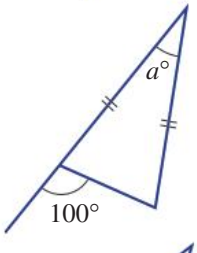
a 

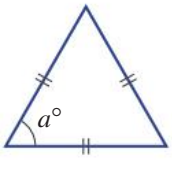
b 

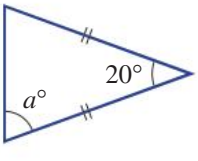
c 

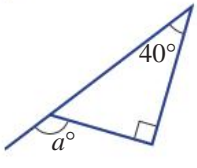
d 

e 

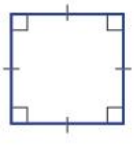
f 

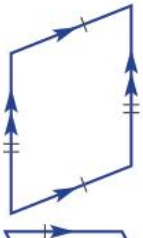
g 

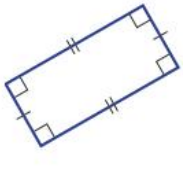
h 

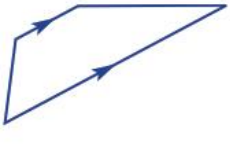
i 

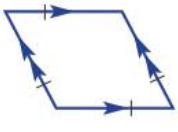
9C **5** Name each of these quadrilaterals.

a 

b 

c 

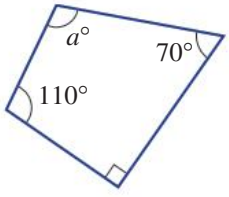
d 

e 

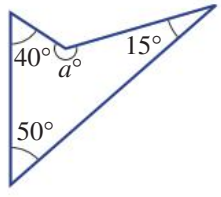
f 

9D **6** Find the size of the unknown angle a in these quadrilaterals.

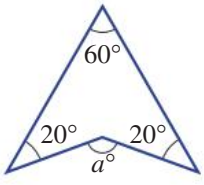
a



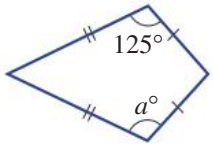
b



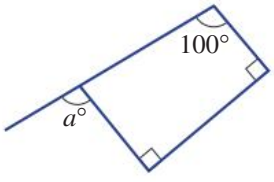
c



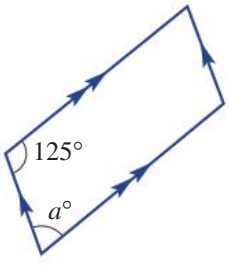
d



e

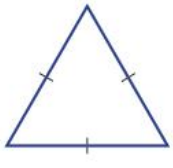


f



9E **7** Give the number of lines of symmetry and the order of rotational symmetry for each of the following.

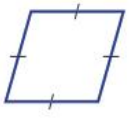
a



b



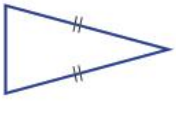
c



d



e



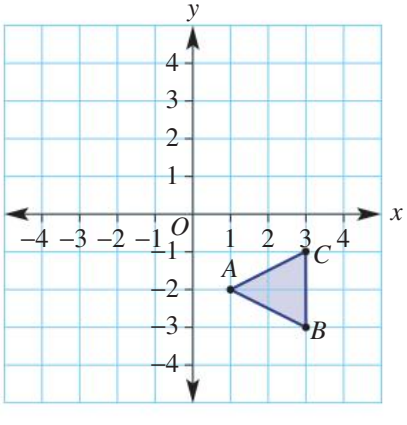
f



9F **8** Write the coordinates of A', B' and C' when the triangle ABC is reflected in the following mirror lines.

a the y-axis

b the x-axis



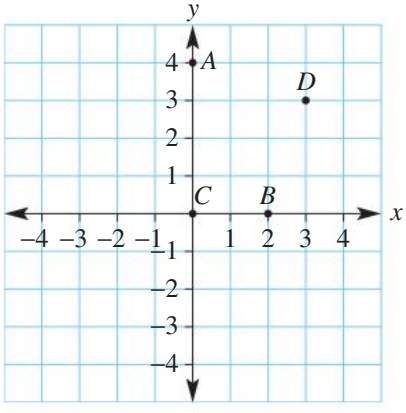
Chapter review

9G **9** Points $A(0, 4)$, $B(2, 0)$ and $D(3, 3)$ are shown here. Write down the coordinates of the image points A', B' and D' after each of the following rotations.

a 180° about $C(0, 0)$

b 90° clockwise about $C(0, 0)$

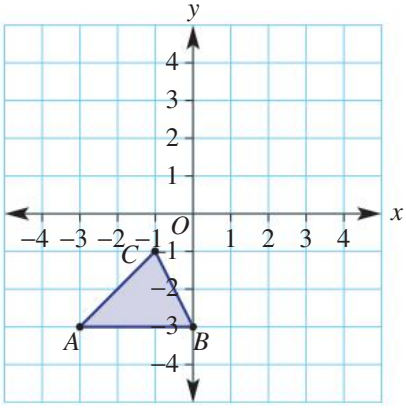
c 90° anticlockwise about $C(0, 0)$



9H **10** Write the coordinates of the vertices A', B' and C' after each of these translations.

a 4 units right and 2 units up

b 1 unit left and 4 units up

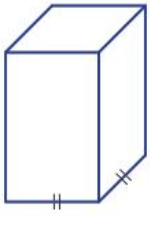


9I **11** Draw a side view and top view for each of these solids.

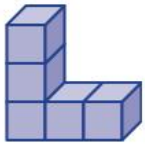
a



b



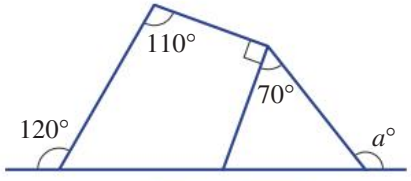
9I **12** Draw this solid on isometric dot paper.



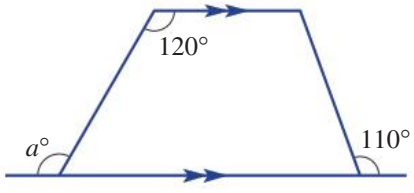
Extended-response questions

1 Here are two designs for a new city sculpture. Can you help the designers find the value of a?

a



b

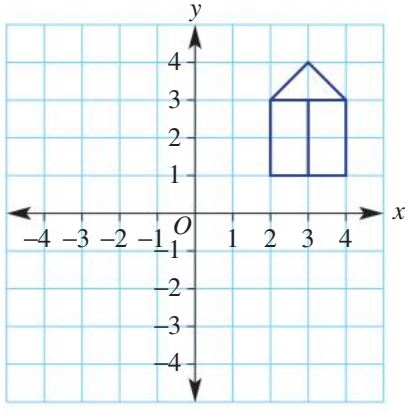


2 Look at this drawing of a simple house. Draw the image of the house after these transformations.

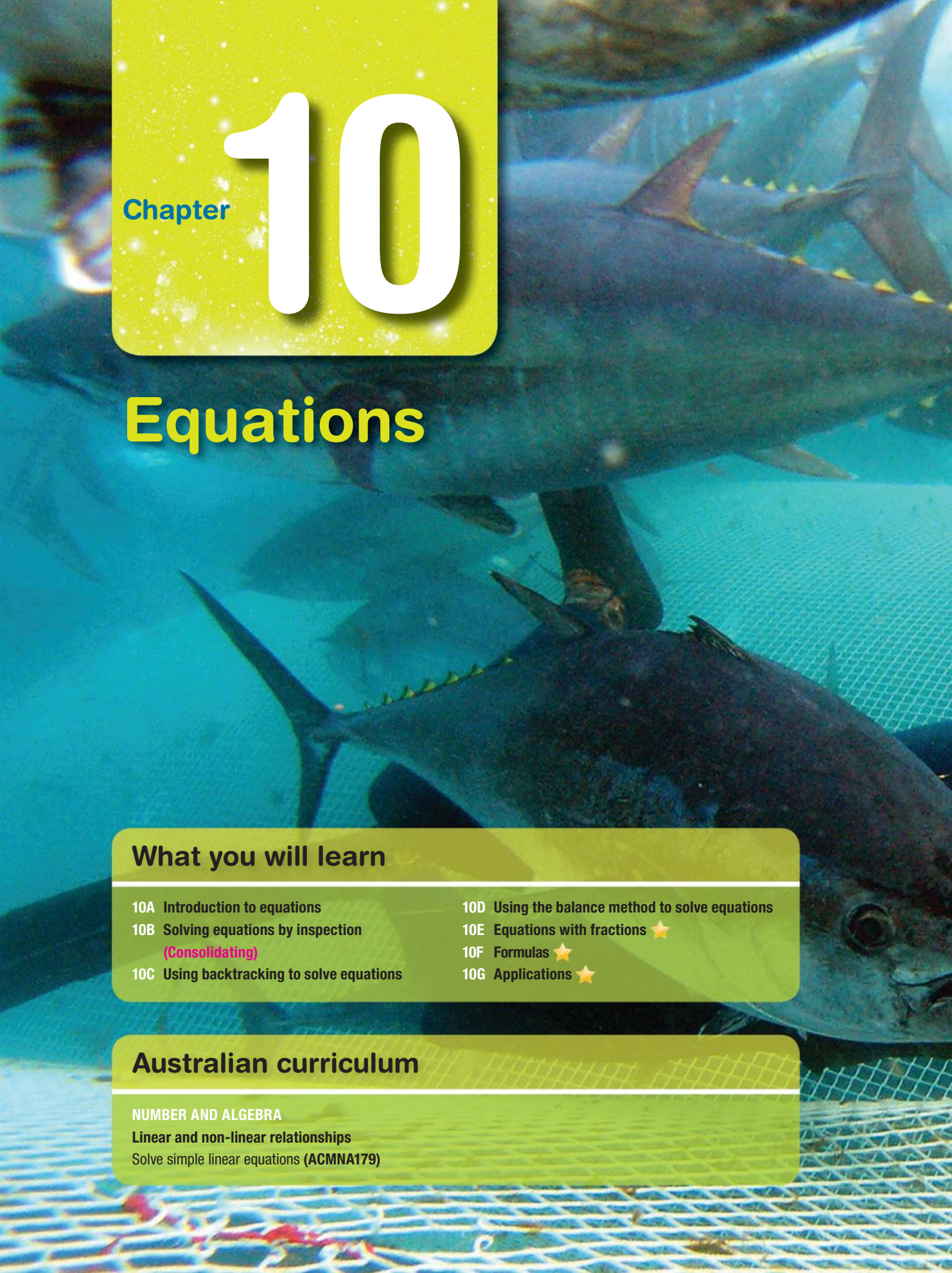
a translation 5 units left and 4 units down

b reflection in the x-axis

c rotation 90° anticlockwise about $C(0, 0)$







Chapter 10

Equations

What you will learn

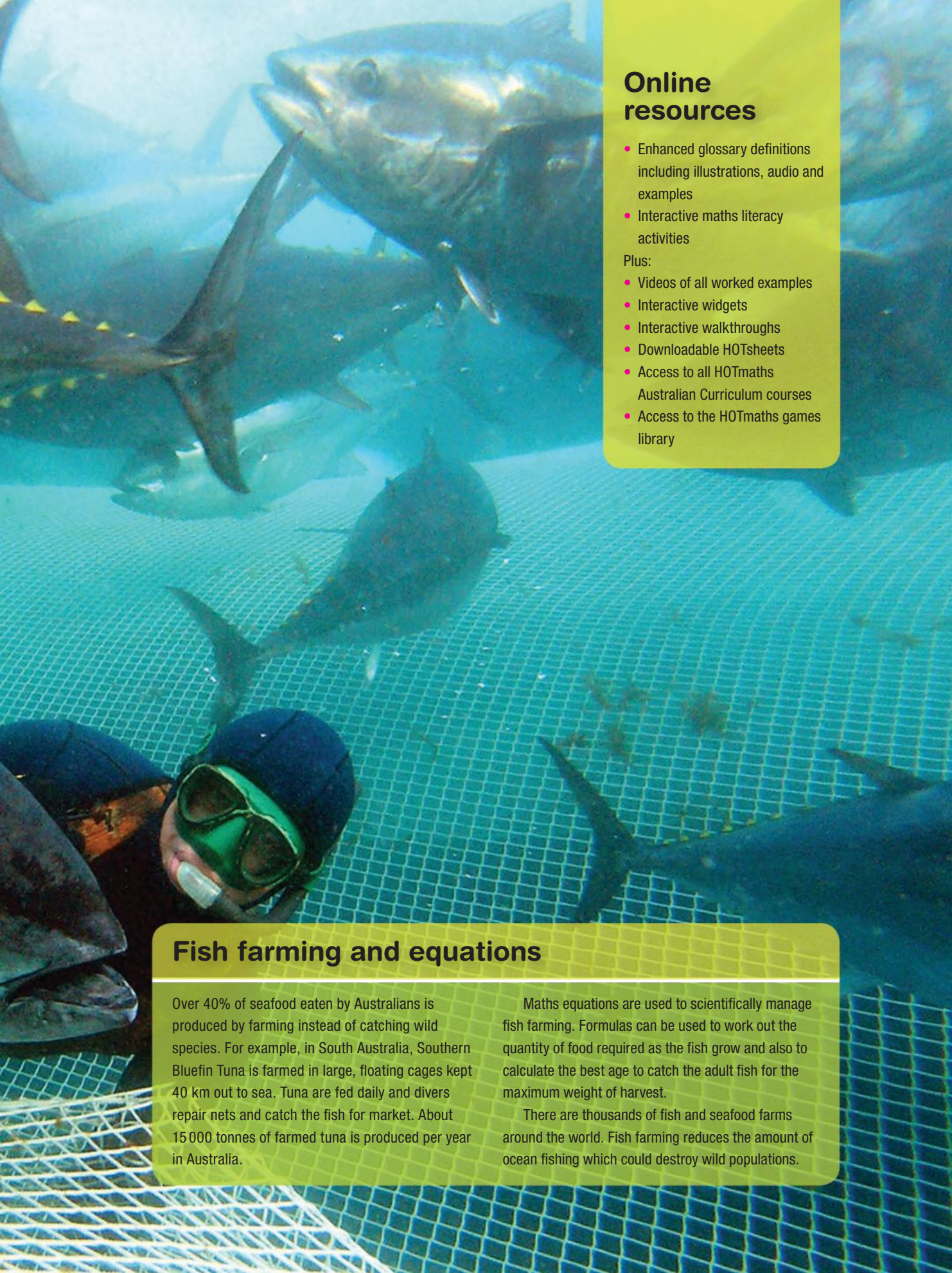
10A Introduction to equations
10B Solving equations by inspection (Consolidating)
10C Using backtracking to solve equations
10D Using the balance method to solve equations
10E Equations with fractions ★
10F Formulas ★
10G Applications ★

Australian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve simple linear equations **(ACMNA179)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Fish farming and equations

Over 40% of seafood eaten by Australians is produced by farming instead of catching wild species. For example, in South Australia, Southern Bluefin Tuna is farmed in large, floating cages kept 40 km out to sea. Tuna are fed daily and divers repair nets and catch the fish for market. About 15 000 tonnes of farmed tuna is produced per year in Australia.

Maths equations are used to scientifically manage fish farming. Formulas can be used to work out the quantity of food required as the fish grow and also to calculate the best age to catch the adult fish for the maximum weight of harvest.

There are thousands of fish and seafood farms around the world. Fish farming reduces the amount of ocean fishing which could destroy wild populations.

Pre-test

1 Fill in each missing number.

a $7 + 5 = \square$ **b** $3 + 12 = \square$

c $2 \times 8 = \square$ **d** $19 - 12 = \square$

2 Fill in the missing numbers.

a $\square + 3 = 10$ **b** $50 - \square = 30$

c $\square + 3 = 19$ **d** $100 \div \square = 20$

3 If $\square = 5$, state whether each of these equations is true or false.

a $\square - 2 = 5$ **b** $\square \times 3 = 15$

c $20 \div 4 = \square$ **d** $7 \times \square = 42$

4 If $a = 3$, find the value of:

a $a + 4$ **b** $8 - a$

c $a \times 5$ **d** $a + 21$

5 If $n = 6$, state the value of:

a $n \div 2$ **b** $n \times 4 + 3$

c $8 - n$ **d** $12 \div n + 4$

6 The expression $n + 3$ can be described as 'the sum of n and 3'. Match the description (**a–c**) with the expression (**A–C**):

a the sum of k and 5 **A** $2p$

b double p **B** $7y$

c 7 lots of y **C** $k + 5$

7 Copy and complete the tables.

a

n	1	2	3	4	5
$5 \times n$	5				

b

n	2	4	6	8	10
$n - 2$					

c

n	1	2	5	8	9
$2n$					

8 Match each of the following operations (**a–d**), with its opposite operation (**A–D**).

a $\times$ **A** $+$

b $+$ **B** $-$

c $\div$ **C** $\times$

d $-$ **D** $\div$

9 Find each of the following.

a The sum of 15 and 12

b The product of 8 and 5

c Triple 6

d Double 8, then add 10

e 12 more than 5

f 20 divided by 5, then add 10

g Add 10 to 20, then divide that answer by 5

10 **a** If Mia is 12 years old and Oliver is 15 years old, what will their ages be in 5 years' time?

b Ethan is paid \$7 per hour for mowing lawns. How much would he earn in 4 hours?

10A Introduction to equations

Interactive

Widgets

HOTsheets

Walkthrough

An equation is a mathematical statement with an equals sign such as $3 + 3 = 6$ or $4 \times 2 = 8$.

When we are using pronumerals, some equations can be true or false, depending on the value of those pronumerals.

For example, $5 + x = 7$ would be true if $x = 2$, but it would be false if $x = 15$.



Let's start: Equations – true or false?

Rearrange the following five symbols to make as many different true and false equations as possible.

$$5, 2, 3, +, =$$

- Which of them are true? Which are false?
- How many more true equations could you form if a subtraction sign is included?

Key ideas

- An **equation** is a mathematical statement that two things are equal.
- Equations have a left-hand side (LHS) and a right-hand side (RHS), with an equals sign in between.

$$\underbrace{2x + 3}_{\text{LHS}} = \underbrace{4y - 2}_{\text{RHS}}$$

equals sign

- Equations can be true (e.g. $2 + 3 = 5$) or false (e.g. $5 + 7 = 21$).

Equation A mathematical statement that two expressions have the same value

Exercise 10A

UNDERSTANDING 1–5 5

1 **a** Is $7 + 4 = 11$ true or false? **b** Is $3 \times 2 = 6$ true or false? **c** Is $6 - 1 = 2$ true or false?

2 Classify each of these equations as true or false.
a $2 + 3 = 5$ **b** $3 + 2 = 6$ **c** $5 - 1 = 6$ **d** $2 + 4 = 6$

3 Put a number in the box to make a true equation.
a $7 + 2 = \square$ **b** $5 \times 3 = \square$ **c** $8 + \square = 10$ **d** $2 \times \square = 20$

4 If k is 5, what is the value of $4 + k$?

10A

Example 1 Classifying equations involving a pronumeral

If k is 5, is $4 + k = 9$ true or false?

Solution	Explanation
True	If k is 5, the equation becomes $4 + 5 = 9$ which is true.

5 **a** If x is 2, is $10 + x = 12$ true or false?
b If y is 10, is $y + 3 = 16$ true or false?

FLUENCY 6(½), 7, 8 6(½), 7–9

Example 2 Classifying equations by comparing sides

State whether the following equations are true or false.

a $10 + 15 = 30 - 5$ **b** $2 + 5 + 6 = 10 - 3$

Solution	Explanation
a True	LHS (left-hand side) is 25. RHS is 25. So LHS = RHS is true.
b False	LHS = 13 and RHS = 7. They are different so the equation is false.

6 For each of the following equations, state whether it is true or false.

a $3 \times 2 = 5 + 1$ **b** $100 - 90 = 2 \times 5$ **c** $30 \times 2 = 32 + 5$
d $12 - 4 = 4$ **e** $2(3 - 1) = 4 + 2$ **f** $5 - (2 + 1) = 7 - 4$
g $2 = 17 - 14 - 1$ **h** $10 + 2 = 12 - 4$ **i** $2 \times 3 = 1 + 2 + 3$

7 If $x = 3$, state whether each of these equations is true or false.

a $5 + x = 7$ **b** $x + 1 = 4$ **c** $13 + x = 10$ **d** $6 = 2x$



8 Consider the equation $4 + 3x = 2x + 9$.

a If $x = 5$, state the value of the left-hand side (LHS).
b If $x = 5$, state the value of the right-hand side (RHS).
c Is the equation $4 + 3x = 2x + 9$ true or false when $x = 5$?

9 If $b = 4$, state whether each of the following equations is true or false.

a $5b + 2 = 22$
b $10 \times (b - 3) = b + b + 2$
c $12 - 3b = 5 - b$
d $b \times (b + 1) = 20$



PROBLEM-SOLVING AND REASONING 10, 11 10–12

Example 3 Writing equations from a description

Write equations for each of the following.

a The sum of x and 5 is 22.
b A deck of cards costs $\$x$. The cost of 7 decks is \$91.
c Priya's age is currently j. In 5 years' time her age will equal 17.

Solution	Explanation
a $x + 5 = 22$	The sum of x and 5 is written $x + 5$.
b $7x = 91$	$7x$ means $7 \times x$ and this number must equal the total cost. We don't include the \$ sign in an equation.
c $j + 5 = 17$	In 5 years' time Priya's age will be 5 more than her current age, so $j + 5$ must be 17.

10 Write equations for each of the following.
a The sum of 3 and x is equal to 10.
b When k is multiplied by 5, the result is 1005.
c The sum of a and b is 22.
d When d is doubled, the result is 78.
e The product of 8 and x is 56.
f When p is tripled, the result is 21.



sum: +
product: ×
doubled: × 2
tripled: × 3

11 Write true equations for each of these problems. You do not need to solve them.
a Chairs cost $\$c$ at a store. The cost of 6 chairs is \$546.
b A plumber charges $\$k$ per hour. The cost of 7 hours' work is \$567.
c Pens cost $\$a$ each and pencils cost $\$b$. Twelve pens and three pencils cost \$28 in total.
d Amy is f years old. In 10 years' time her age will be 27.



12 **a** Find a value of m that would make this equation true: $10 = m + 7$.
b Find a possible value of k that would make this equation true: $k \times (8 - k) = 12$.



For part **b**, there is more than one possible answer.

Equation permutations

— 13

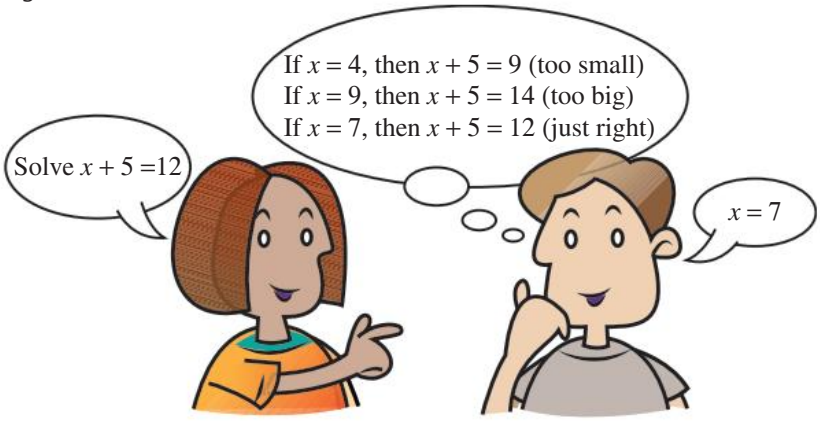
13 For each of the following, rearrange the symbols to make a true equation.
a 6, 2, 3, ×, =
b 1, 4, 5, −, =
c 2, 2, 7, 10, −, ÷, =
d 2, 4, 5, 10, −, ÷, =

10B Solving equations by inspection

CONSOLIDATING

Interactive Widgets HOTsheets Walkthrough

Solving an equation involves finding the value of the variable that makes it true. Simple equations can be solved by trying different numbers until one works.



Let's start: Finding the missing value

- Find the missing values to make the following equations true.
  $16 - \square = 9$ $\quad 27 = 15 + 3 \times \square$ $\quad 2 \times \square + 4 = 17$
- Can you always find a value to put in the place of $\square$ in any equation and make it true?

Key ideas

- **Solving** an equation means finding the values of any pronumerals to make the equation true.
- An **unknown** is a pronumeral (or variable) with a value that needs to be found in order to make an equation true.
- One method of solving equations is by inspection (also called trial and error), which involves inspecting (or trying) different values and seeing which ones make the equation true.

Solving Finding the value of an unknown variable

Unknown A pronumeral with a value that is yet to be found to make the equation true

Exercise 10B

UNDERSTANDING 1–5 5



Use the order of operations:
$2 + \underbrace{5 \times 3}_{15} = 2 + 15$

1 State whether each of the following equations is true or false.

a $10 + 7 = 19$ **b** $5 = 12 - 7$
c $8 \times 2 = 3 + 11$ **d** $2 + 5 \times 3 = 17$

2 If the missing number is 5, classify each of the following equations as true or false.

a $\square + 3 = 8$ **b** $10 \times \square + 2 = 46$
c $10 - \square = 5$ **d** $12 = 6 + \square \times 2$

3 For the equation $\square + 7 = 13$:
- **a** Find the value of the LHS (left-hand side) if $\square = 5$.
- **b** Find the value of the LHS if $\square = 10$.
- **c** Find the value of the LHS if $\square = 6$.
- **d** What value of $\square$ would make the LHS equal to 13?

Example 4 Finding the missing number

For each of these equations, find the value of the missing number that would make it true.

a $\square \times 7 = 35$ **b** $20 - \square = 14$

Solution	Explanation
a 5	Think: What number multiplied by 7 equals 35? $\boxed{5} \times 7 = 35$ is a true equation.
b 6	Think: 20 minus what number equals 14? $20 - \boxed{6} = 14$ is a true equation.

4 Find the value of the missing numbers.
- **a** $4 + \square = 7$
- **b** $2 \times \square = 12$
- **c** $\square \times 4 = 80$
- **d** $\square + 12 = 31$

5 What value should go in the box to make a true equation?
- **a** $12 = 3 + \square$
- **b** $5 = 8 - \square$
- **c** $42 = \square \times 7$
- **d** $8 = \square \div 3$

FLUENCY 6–7(½) 6–7(½), 8

Example 5 Solving equations by inspection

Solve each of the following equations by inspection.

a $c + 12 = 30$ **b** $5 \times b = 20$ **c** $2x + 13 = 21$

Solution	Explanation
a $c + 12 = 30$ $c = 18$	$\boxed{18} + 12 = 30$ is a true equation.
b $5 \times b = 20$ $b = 4$	$5 \times \boxed{4} = 20$ is a true equation.
c $2x + 13 = 21$ $x = 4$	$2x$ means $2 \times x$. Trying a few values: $x = 10$ makes LHS $= 20 + 13 = 33$, which is too large. $x = 3$ makes LHS $= 6 + 13 = 19$, which is too small. $x = 4$ makes LHS $= 21$.

10B

6 Solve the following equations by inspection.

a $8 \times y = 64$ **b** $l \times 3 = 18$ **c** $4 - d = 2$ **d** $l + 2 = 14$ **e** $a - 2 = 4$

f $s + 7 = 19$ **g** $x \div 8 = 1$ **h** $12 = e + 4$ **i** $13 = 5 + s$

7 Solve the following equations by inspection. (See part **c** of Example 5.)

a $2p - 1 = 5$ **b** $3p + 2 = 14$ **c** $4q - 4 = 8$ **d** $4v + 4 = 24$ **e** $2b - 1 = 1$

f $5u + 1 = 21$ **g** $5g + 5 = 20$ **h** $3d - 5 = 13$ **i** $8 = 3m - 4$

8 Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

a $x = 6 - x$ **b** $7 + x = 2 \times x$

c $10 - x = x + 2$ **d** $15 - 2x = x$



An equation is true if its LHS and RHS are equal.

PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Find the value of the number in each of these examples.

a A number is doubled and the result is 22.

b 3 less than a number is 9.

c Half of a number is 8.

d 7 more than a number is 40.

10 Justine is paid \$10 an hour for x hours. During a particular week, she earns \$180.

a Write an equation involving x to describe this situation.

b Solve the equation by inspection to find x.

11 Karim's weight is w kg and his brother is twice as heavy, weighing 70 kg.

a Write an equation involving w to describe this situation.

b Solve the equation by inspection to find w.



Karim's brother weighs $2w$ kg.



12 Yanni's current age is y years old. In 12 years' time he will be three times as old.

a Write an equation involving y to describe this situation.

b Solve the equation by inspection to find y.

Multiple variables — 13

13 When multiple variables are involved, inspection can still be used to find a solution. For each of the following equations find, by inspection, one pair of values for x and y that make them true.

a $x + y = 8$

b $x - y = 2$

c $x \times y = 6$

d $x + y = x \times y$



For $x + y = 8$, you could try:
If $x = 1$ and $y = 1$, $1 + 1 = 8$ (false)
If $x = 1$ and $y = 2$, $1 + 2 = 8$ (false)
If $x = 7$ and $y = 3$, $7 + 3 = 8$ (false)
Keep going until your equation is true.

10C Using backtracking to solve equations

Interactive

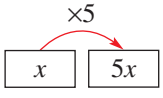
Widgets

HOTsheets

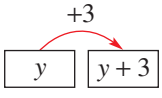
Walkthrough

Flow charts give a systematic way to solve equations. Consider how an expression can be 'built' from a single pronumeral.

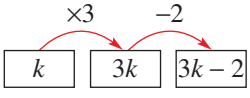
Building $5x$



Building $y + 3$

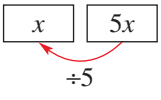


Building $3k - 2$

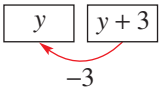


The arrows can be reversed to break down the expression and the label is changed to its opposite.

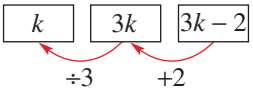
Breaking down $5x$



Breaking down $y + 3$

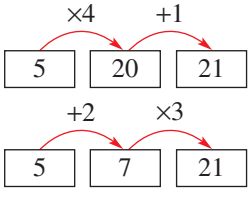


Breaking down $3k - 2$



Let's start: 5 to 21

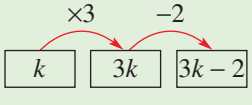
Starting with the number 5, you can get to the number 21 in many different ways. Two ways are shown here.



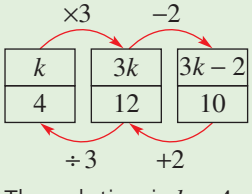
- Try to find other ways to get from 5 to 21. You can use more than two operations.
- What are some ways of getting from 21 to 5?

Key ideas

- Expressions such as $3k - 2$ can be built from a single pronumeral, k.



- Equations can be solved by reversing the arrows in the flow chart and applying the opposite operations (i.e. − and +; ÷ and ×).
  This is called **backtracking**.
  For example, to solve $3k - 2 = 10$, put 10 at the right and follow the arrows to the left.



  The solution is $k = 4$.

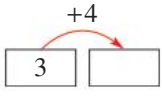
Backtracking A method of solving equations systematically by applying opposite operations

Exercise 10C

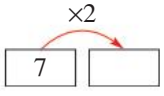
UNDERSTANDING 1–7 7

1 State the missing numbers in these flow charts.

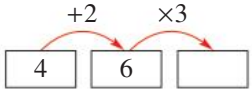
a



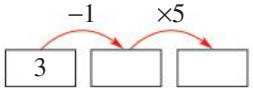
b



c

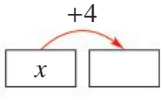


d

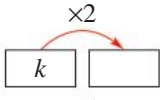


2 Copy and complete these flow charts.

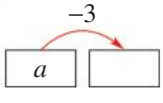
a



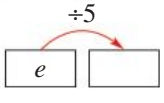
b



c



d



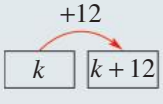
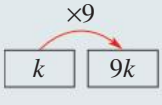


Remember that $k \times 2$ is written $2k$.

Example 6 Building an expression

Use a flow chart starting with k to build the following.

a $k + 12$ **b** $9k$

Solution	Explanation
a 	Starting with k, add 12 to get $k + 12$.
b 	Starting with k, multiply by 9 to get $9k$. Remember that $9k$ means $9 \times k$.

3 Use a flow chart starting with k to build the following.

a $k + 4$ **b** $9k$ **c** $k - 7$ **d** $k + 8$

4 Write down the opposite of these operations.

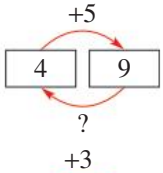
a adding 2 **b** adding 10 **c** subtracting 5
d multiplying by 2 **e** multiplying by 4 **f** dividing by 2



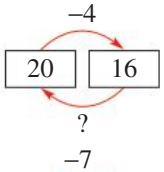
The opposite of adding 2 subtracting 2.

5 For each flow chart, state the opposite operation (on the bottom arrow).

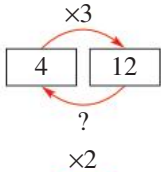
a



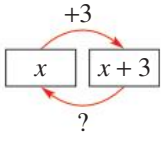
b



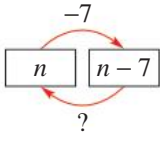
c



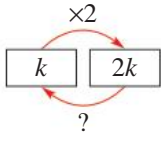
d



e

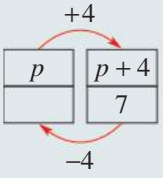


f



Example 7 Using flow charts with two components

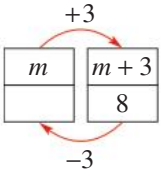
What is the missing number in this flow chart?



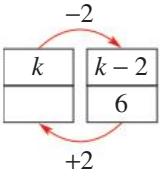
Solution	Explanation
The missing number is 3.	Following the arrow from 7, subtract 4 to get the missing number.

6 Write the missing numbers for these flow charts.

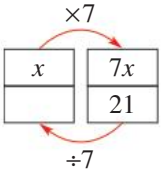
a



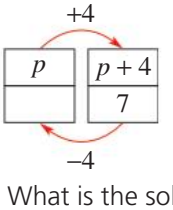
b



c



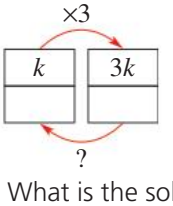
7 **a** Copy and complete this flow chart for the equation $p + 4 = 7$.



b What is the solution to the equation $p + 4 = 7$?

FLUENCY 8–10 8–11

8 **a** Copy and complete this flow chart for the equation $3k = 15$.





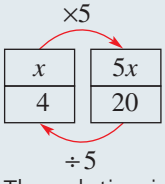
b What is the solution to $3k = 15$?

10C

Example 8 Using backtracking to solve one-step equations

Solve the equation $5x = 20$ by using backtracking.

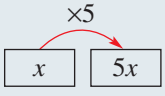
Solution



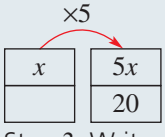
The solution is $x = 4$.

Explanation

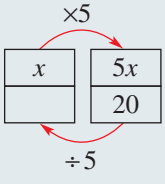
Step 1: Start with x and build $5x$.



Step 2: Put 20 below $5x$.



Step 3: Write ÷ 5 on the bottom arrow.



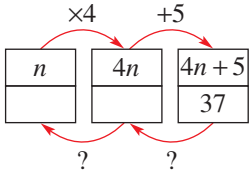
Step 4: Follow the bottom arrow, backtracking to find x.

9 Solve the following equations by using backtracking.

a $5k = 30$ **b** $2x = 20$ **c** $a + 4 = 19$ **d** $m - 3 = 12$ **e** $q \times 7 = 35$ **f** $p + 3 = 12$

10 **a** Copy and complete this flow chart for the equation $4n + 5 = 37$. (Replace each question mark with the correct operation.)

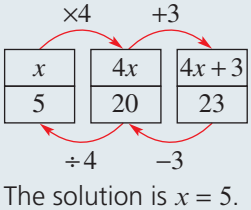
b What is the solution to $4n + 5 = 37$?



Example 9 Using backtracking to solve two-step equations

Solve the equation $4x + 3 = 23$ by using backtracking.

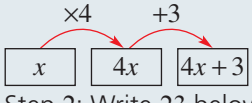
Solution



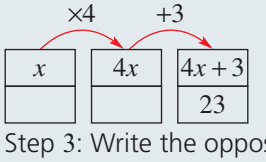
The solution is $x = 5$.

Explanation

Step 1: Build $4x + 3$, starting with x.



Step 2: Write 23 below $4x + 3$.



Step 3: Write the opposite operations on the bottom arrows.

Step 4: Follow the arrows back to x.

11 Solve the following equations by using backtracking.

a $4n + 5 = 21$ **b** $2n + 7 = 29$ **c** $3x - 8 = 22$

d $8a - 4 = 60$ **e** $2y - 4 = 2$ **f** $4m + 12 = 24$

PROBLEM-SOLVING AND REASONING 12, 13 12–14

12 Ezekiel's age is unknown. In 5 years' time he will be 18 years old.



a If x is Ezekiel's current age, which of the following equations can be used to describe this situation?

A $x + 18 = 5$

B $x + 5 = 18$

C $x \times 5 = 18$

D $x - 5 = 18$

b Solve the equation above using a flow chart.

c How old is Ezekiel?



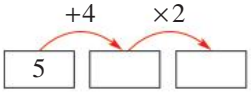
Solving by inspection is when you guess a sensible value for the variable and substitute to check if you were right.

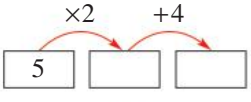
13 **a** Solve the equation $4x + 192 = 340$ using a flow chart.

b In what ways is using a flow chart easier than solving by inspection?

14 Sophie thinks that it does not matter what order the arrows go in a flow chart.

a Prove that she is wrong by filling out these two flow charts.





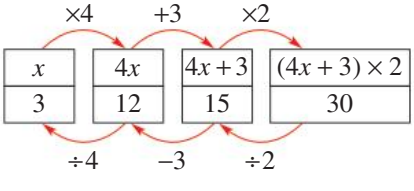
b Give an example of two flow charts where the operations on the arrows are swapped but the result is the same.



Try making a flow chart which involves addition and subtraction only.

Three-step equations — 15

15 Equations with three steps can also be solved using flow charts. For example, $(4x + 3) \times 2 = 30$ can be solved as follows.



Solve the following three-step equations using flow charts.

a $(2x + 5) \times 3 = 33$ **b** $(3m - 1) \times 2 = 28$ **c** $(10p - 3) \times 5 = 35$

10D Using the balance method to solve equations

Interactive

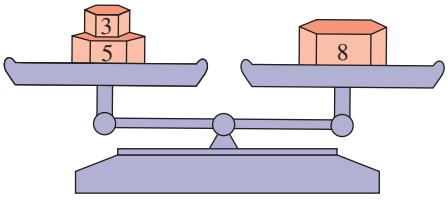
Widgets

HOTsheets

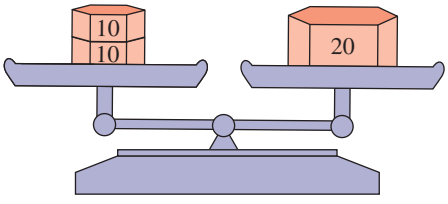
Walkthrough

We can think of an equation as a balanced scale, with two sides that weigh the same.

$5 + 3 = 8$ is like:

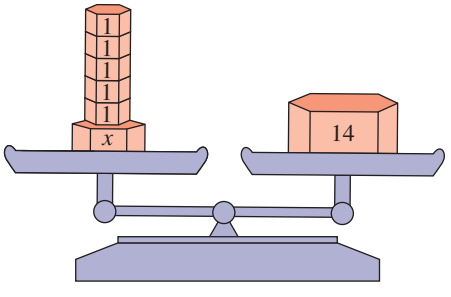


$2 \times 10 = 20$ is like:

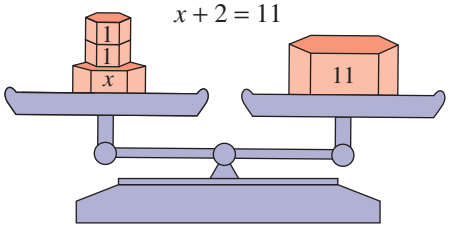


We can do the same thing to both sides of a balanced scale, and it will stay balanced.

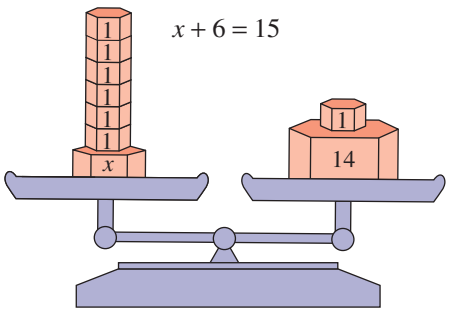
Start with:
$x + 5 = 14$



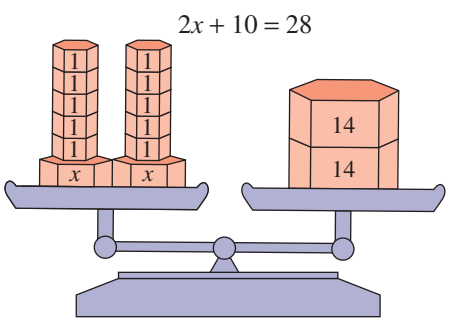
subtract 3 from both sides



add 1 to both sides



double both sides

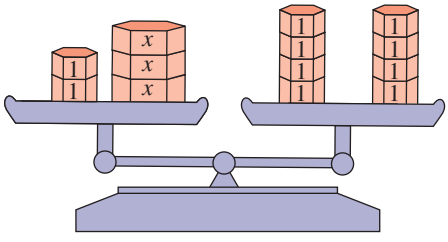


We can write the steps used with arrows on both sides. For example:

$$\begin{matrix} x + 5 = 14 \\ \downarrow \\ x + 2 = 11 \end{matrix} \quad \begin{matrix} -3 \\ \\ -3 \end{matrix}$$

Let's start: Equations as scales

These scales show $2 + 3x = 8$.



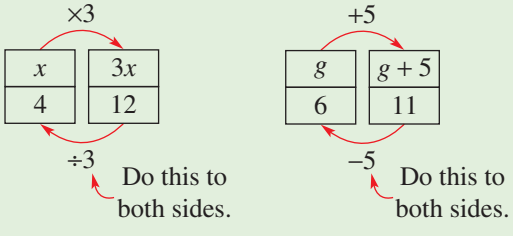
- What would the scales look like if two  were removed from both sides?
- What would the scales look like if the two  were removed just from the left-hand side? (Try to show whether they would be level.)
- Use scales to illustrate $2x = 12$. Then show what would happen if both sides were halved.

Key ideas

- To solve an equation using the balance method, do the same thing to both sides to find a simpler equation. For example:

$$3x = 12 \xrightarrow{\div 3} x = 4 \qquad g + 5 = 11 \xrightarrow{-5} g = 6$$

- Each step to be performed is like backtracking on a flow chart.



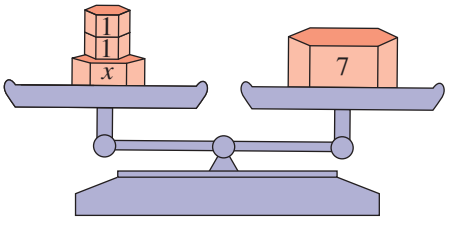
- To check a solution, substitute the value and see if the equation is true. e.g. LHS = $3 \times 4 = 12$ ✔

Exercise 10D

UNDERSTANDING 1–6 | 6

1 Which one of the following equations do the scales represent?

A $x + 7 = 2$
B $x = 7$
C $x + 2 = 7$
D $x - 2 = 7$



10D

2 Copy and complete the following.

a

$3x = 12$

$+5$ ↓ $+5$

$3x + 5 = \square$

b

$4a = 20$

-5 ↓ -5

$4a - 5 = \square$

c

$k = 10$

$\times 2$ ↓ $\times 2$

$2k = \square$

d

$y + 2 = 20$

$+5$ ↓ $+5$

$y + 7 = \square$



For part **a**, $12 + 5 = 17$, so 17 goes in the box.

3 State whether each of the following equations is true or false.

a $x + 4 = 7$, if $x = 3$

b $b - 2 = 7$, if $b = 5$

c $g + 5 = 3g$, if $g = 2$

d $f \times 4 = 20$, if $f = 3$



For part **a**, substitute $x = 3$ in $x + 4 = 7$. Does LHS = RHS?

4 Consider the equation $7x = 42$.

a Copy and complete the following.

$7x = 42$

$\div 7$ ↓ $\div 7$

$x = __$

b What is the solution to the equation $7x = 42$?

5 Copy and complete the following, showing which operation was used.

a

$5 + a = 30$

$?$ ↓ $?$

$a = 25$

b

$10b = 72$

$?$ ↓ $?$

$b = 7.2$

c

$12 = 4c$

$?$ ↓ $?$

$3 = c$

d

$8 = c - 12$

$?$ ↓ $?$

$20 = c$



Remember that the same operation must be used for both sides.

6 For the equation $2x = 18$, write the new equation you get if:

a 2 is added to both sides

b 2 is subtracted from both sides

c both sides are doubled

d both sides are halved



$2x = 18$

$+2$ ↓ $+2$

$2x + 2 = \square$

FLUENCY 7–8(½), 9, 10 | 7–8(½), 9, 10, 11(½)

Example 10 Solving one-step equations

Solve each of the following equations using the balance method.

a $5x = 30$ **b** $17 = y - 21$

Solution	Explanation
a $5x = 30$ $\div 5$ ↓ $\div 5$ $x = 6$ So the solution is $x = 6$.	The opposite of $\times 5$ is $\div 5$. By dividing both sides by 5, we get an equivalent equation. Recall that $5x \div 5$ simplifies to x.

b

$17 = y - 21$

$+21$ $+21$

$38 = y$

The opposite of -21 is $+21$.

So the solution is $y = 38$.

Write the pronumeral on the LHS.

7 Solve the following equations using the balance method.

a $6m = 54$ **b** $g - 9 = 2$ **c** $s \times 9 = 81$ **d** $i - 9 = 1$

e $7 + t = 9$ **f** $8 + q = 11$ **g** $4y = 48$ **h** $7 + s = 19$

8 Solve the following equations using the balance method.

a $24 = j \times 6$ **b** $12 = l + 8$ **c** $1 = v \div 2$ **d** $19 = 7 + y$

e $2 = y - 7$ **f** $13 = 3 + t$ **g** $14 = 2p$ **h** $10 = c - 5$

9 **a** Copy and complete the following to solve the given equations using the balance method.

i

$7a + 3 = 38$

-3 -3

$7a = 35$

$?$ $?$

$__ = __$

ii

$4b - 10 = 14$

$+10$ $+10$

$__ = __$

$?$ $?$

$__ = __$

b Copy and complete the following flow charts.

i

$\times 7$	$+3$	
a	$7a$	$7a + 3$
		38

ii

$\times 4$	-10	
b	$4b$	$4b - 10$
		14

c How are your answers to parts **a** and **b** related?

10 If you were solving these equations, what is the first operation you would apply to both sides?

a $2x + 3 = 9$ **b** $4x - 7 = 33$ **c** $3p + 8 = 38$

The opposite of $+3$ is -3.



Example 11 Solving two-step equations

Solve $4a + 7 = 23$ using the balance method and check the solution.

Solution	Explanation
$4a + 7 = 23$ -7 -7 $4a = 16$ $\div 4$ $\div 4$ $a = 4$	At each step, try to make the equation simpler by applying an operation to both sides.
	Choose the opposite operations based on $4a + 7$: $a \xrightarrow{\times 4} 4a \xrightarrow{+7} 4a + 7$ Opposite operations: -7, then $\div 4$.
Check: $4a + 7 = 4 \times 4 + 7$ $= 16 + 7$ $= 23$ ✔	Check that your solution is correct by substituting $a = 4$ back into the equation.

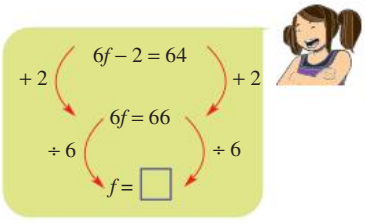
10D

11 For each of the following equations:

i Solve the equation using the balance method.

ii Check your solution.

a $6f - 2 = 64$	**b** $5x - 4 = 41$	**c** $5k - 9 = 31$
d $2n - 8 = 14$	**e** $2g - 7 = 1$	**f** $3q - 3 = 30$
g $3z - 4 = 26$	**h** $9 + 8p = 17$	**i** $10d + 7 = 47$
j $6t - 10 = 38$	**k** $9u + 2 = 47$	**l** $10c - 3 = 7$



PROBLEM-SOLVING AND REASONING 12 12, 13

12 Danny gets paid \$12 per hour, plus a bonus of \$50 for each week. In one week he earned \$410.

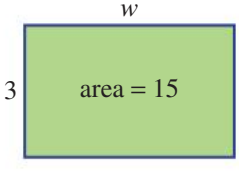
a Write an equation to describe this, using n for the number of hours worked.

b Solve the equation and state the number of hours worked.

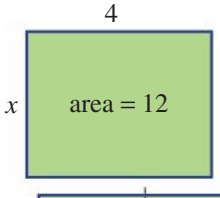


13 Write equations and solve them using the balance method to find the unknown value in each of the following diagrams.

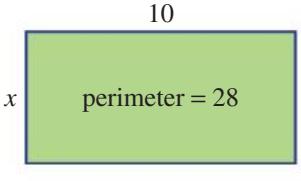
a



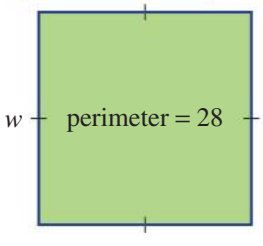
b

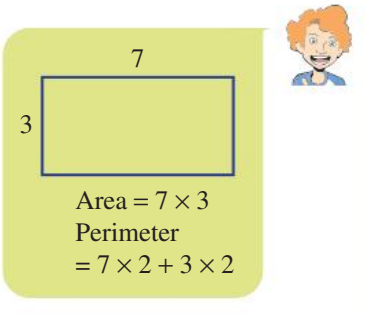


c



d





From solutions to equations

— 14

14 **a** The equations $2x = 12$ and $x - 4 = 2$ each have the solution $x = 6$. Find four other equations with this solution.

b Find an equation with the solution $x = \frac{1}{2}$.

c Is there a limit to the number of equations with the solution $x = 10$? Why, or why not?

Progress quiz

10A **1** State whether the following equations are true or false.

a $12 + 27 = 40 - 1$ **b** $2 + 5 \times 6 = 6 \times 6 + 6$

c $15 - (4 + 9) = 1 \times 1 \times 1 + 1$ **d** $200 \div 40 = 20 \div 4$

10A **2** Write equations for each of the following.

a The sum of 6 and x is equal to 15

b When e is tripled, the result is 39

c The sum of m and n is equal to -9

d When f is taken away from d, the result is 14

10B **3** For each of these equations, find the value of the missing number that would make it true.

a $\square \times 8 = 72$ **b** $36 + \square = 52$

c $12 = \square \div 5$ **d** $35 = 105 \div \square$

10B **4** Solve each of the following equations by inspection.

a $f - 6 = 11$ **b** $4 \times k = 28$ **c** $2y - 4 = 20$ **d** $5t + 11 = 26$

10B **5** Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

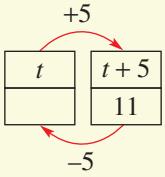
a $x = 8 - x$ **b** $3x = 2x + 7$ **c** $24 - 2x = x$

10C **6** Write down the opposite of these operations.

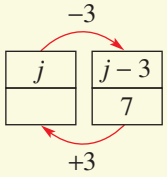
a adding 6 **b** subtracting 5 **c** multiplying by 4 **d** dividing by 10

10C **7** Write the missing numbers for these flow charts.

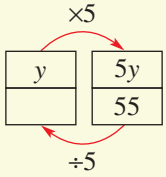
a



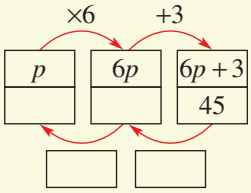
b



c



10C **8** **a** Copy and complete this flow chart for the equation $6p + 3 = 45$ by filling in the blank boxes.



b What is the solution to $6p + 3 = 45$?

10C **9** Solve the following equations by using backtracking.

a $3w - 7 = 8$ **b** $4d + 13 = 53$

10D **10** Solve the following equations using the balance method.

a $8y = 56$ **b** $12 = y - 19$

10D **11** For each of the following equations:

i solve the equation using the balance method

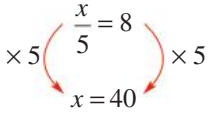
ii check your solution.

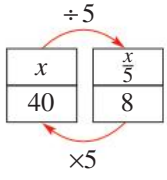
a $2r - 12 = 4$ **b** $8x + 11 = 35$

10E Equations with fractions

Remember that $\frac{x}{5}$ means $x \div 5$.

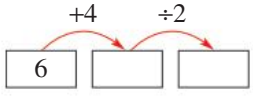
This means we can solve equations such as $\frac{x}{5} = 8$ by using the balance method or backtracking.

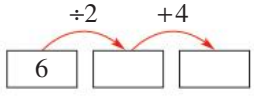




Let's start: Fractional differences

- Starting with the number 6, find the result for each of these flow charts.

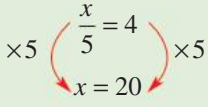


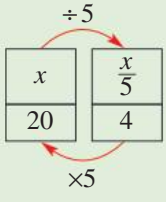


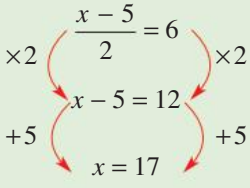
- If you start with 10 instead of 6, what results do you get?
- If you start with x, which of the two flow charts gives $\frac{x}{2} + 4$ and which gives $\frac{x+4}{2}$?

Key ideas

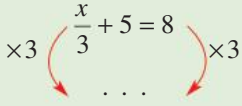
- To solve an equation that has a fraction on one side, multiply both sides by the denominator. Alternatively, use backtracking.



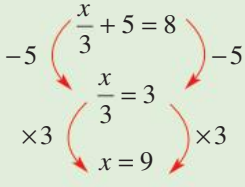




- Do not multiply by the denominator until the fraction is on its own on one side.



✘ Do *not* do this!



✔ Do this.

Exercise 10E

UNDERSTANDING 1–6 | 6

1 Check whether the following equations are true or false.

a $\frac{15}{5} = 3$ **b** $\frac{7+2}{3} = 5$ **c** $\frac{8-6}{2} = 1$ **d** $3 + \frac{10}{2} = 9$



$\frac{15}{5}$ means $15 \div 5$

2 If $x = 24$, classify the following as true or false.

a $\frac{x}{8} = 3$ **b** $\frac{x}{4} = 9$ **c** $\frac{x}{3} = 10$ **d** $\frac{x}{6} = 4$

3 The opposite of adding 5 is subtracting 5. State the opposite of:

a multiplying by 4 **b** subtracting 2 **c** dividing by 7 **d** adding 11

Example 12 Solving one-step equations with fractions

Solve $\frac{a}{7} = 3$.

Solution	Explanation
Balance method: $\times 7$ ($\frac{a}{7} = 3$) $\times 7$ $a = 21$	Multiplying both sides by 7 removes the denominator of 7.
Backtracking: $\div 7$ a | $\frac{a}{7}$ 21 | 3 $\times 7$	To build $\frac{a}{7}$, divide a by 7. The opposite operation is multiplying by 7.

4 Copy and complete the following to solve these equations using the balance method.

a $\times 4$ ($\frac{b}{4} = 11$) $\times 4$, $b = __$

b $\times 5$ ($\frac{d}{5} = 3$) $\times 5$, $d = __$

c $?$ ($\frac{z}{10} = 2$) $?$, $z = __$

5 Copy and complete the following to solve these equations using backtracking.

a

$\div 4$	
x	$\frac{x}{4}$
	7

$\times 4$

b

$\div ?$	
e	$\frac{e}{13}$
	2

$?$

c

$?$	
m	$\frac{m}{2}$
	4

$?$

10E

6 For each of the following equations, choose the appropriate first step (**A** to **D**) needed to solve it using the balance method.

a $\frac{x}{3} = 10$

b $\frac{x}{3} + 2 = 5$

c $\frac{x-3}{2} = 1$

d $\frac{x}{2} - 3 = 5$

A Multiply both sides by 2.

B Multiply both sides by 3.

C Subtract 2 from both sides.

D Add 3 to both sides.

FLUENCY 7–9(½) 7–10(½)

7 Solve the following equations using the balance method or backtracking.

a $\frac{m}{6} = 2$

b $\frac{c}{9} = 2$

c $\frac{s}{8} = 2$

d $\frac{r}{5} = 2$

e $\frac{u}{2} = 6$

f $\frac{y}{5} = 10$

g $\frac{x}{2} = 1$

h $\frac{a}{4} = 3$

Example 13 Solving two-step equations with fractions

Solve each of the following.

a $\frac{d+5}{3} = 12$

b $\frac{m}{5} + 2 = 10$

Solution

a

$\frac{d+5}{3} = 12$

$\times 3$ $\times 3$

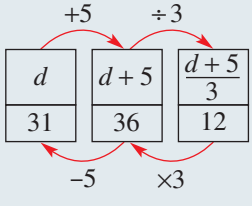
$d + 5 = 36$

-5 -5

$d = 31$

Explanation

Multiply both sides by 3 to remove the denominator of 3.
Then solve $d + 5 = 36$ in the usual way.
Alternatively, use backtracking.



b

$\frac{m}{5} + 2 = 10$

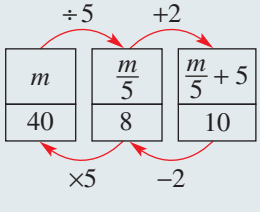
-2 -2

$\frac{m}{5} = 8$

$\times 5$ $\times 5$

$m = 40$

First subtract 2 to get the fraction on its own on the LHS.
Then multiply both sides by the denominator (5).
Alternatively, use backtracking.



8 Solve the following equations.

a $\frac{d+15}{12}=2$ **b** $\frac{y+5}{11}=1$ **c** $\frac{j+8}{11}=1$ **d** $\frac{b-2}{2}=1$

e $\frac{w+2}{6}=3$ **f** $\frac{s+2}{5}=1$ **g** $\frac{v-4}{7}=1$ **h** $\frac{f-2}{7}=1$

9 Solve the following equations.

a $\frac{m}{2}+7=12$ **b** $\frac{q}{3}-2=1$ **c** $\frac{k}{4}-3=7$ **d** $4+\frac{x}{5}=10$

e $\frac{a}{2}+3=5$ **f** $\frac{x}{5}-1=10$ **g** $\frac{y}{2}+5=8$ **h** $\frac{a}{7}-2=1$

10 Solve the following equations.

a $\frac{x+5}{2}=7$ **b** $\frac{x}{2}+5=7$ **c** $\frac{p}{3}-5=2$ **d** $\frac{p-5}{3}=2$ **e** $\frac{q}{2}=10$

f $\frac{r-3}{2}=12$ **g** $\frac{r}{3}+5=12$ **h** $\frac{x}{5}-2=10$


Don't multiply until the fraction is on its own.

PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 In each of the following cases, write an equation and solve it to find the number.

a A number, t, is halved and the result is 9.
b A number, x, is divided by 10 and the result is 8.
c 4 is subtracted from q and this is halved, giving a result of 3.
d 3 is added to x and the result is divided by 4, giving a result of 2.
e A number, y, is divided by 4 and then 3 is added, giving a result of 5.

12 A group of five people go out for dinner and split the bill evenly. They each pay $22.

a If b represents the total cost of the bill, in dollars, write an equation to describe this situation.
b Solve this equation algebraically.
c What is the total cost of the bill?



10E

13 **a** Explain, in one sentence, the difference between $\frac{x+3}{5}$ and $\frac{x}{5}+3$.

b Using the balance method, what is the first operation you would apply to both sides to solve:

i $\frac{x+3}{5}=7$? **ii** $\frac{x}{5}+3=7$?

c Are there any values of x for which $\frac{x+3}{5}$ and $\frac{x}{5}+3$ are equal to each other?



For **c**, try a few values of x and look for a pattern.

Three-step equations with fractions

— 14

14 Sometimes an equation involving fractions takes more than two steps to solve. For example:

$$\frac{2x-3}{5}=3$$

$\times 5$ / $\times 5$

$$2x-3=15$$

$+3$ / $+3$

$$2x=18$$

$\div 2$ / $\div 2$

$$x=9$$



You can also use backtracking to solve these equations.

Solve the following equations, showing all three steps.

a $\frac{3a-4}{2}=1$ **b** $\frac{5k-3}{4}=3$ **c** $\frac{2q+11}{3}=7$

10F Formulas





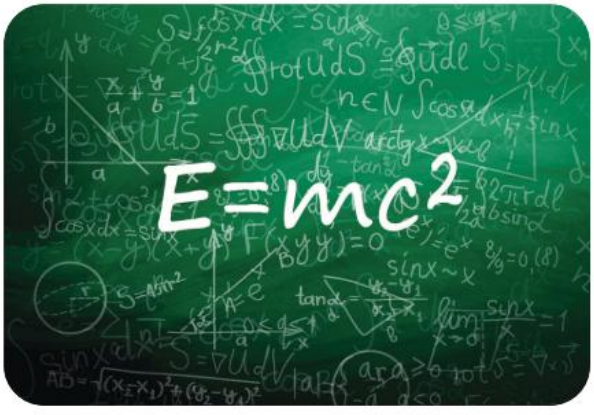




A formula is a rule for finding the value of something. For example, the equation $E = mc^2$ is a formula that relates energy and mass.

Let's start: Different formulas

- List all the formulas you can think of.
- Using the internet or a library, find the longest and/or most complicated formula you can, and try to explain what it does.



Key ideas

- A **formula** or rule is an equation that uses two or more variables. One of these variables is written by itself on the LHS of the equation.
- To use a formula, first substitute all the known values into the equation. Then solve the equation to find the unknown variable.

Formula An equation that shows the relationship between variables

Exercise 10F

UNDERSTANDING 1–4 4

1 List the variables in each of these equations.

a $F = 3g$ **b** $x = 2y - 1$ **c** $A = B + C - 2$ **d** $g = 3.2d + 5$

2 **a** Substitute $g = 5$ into the expression $3g$ and find the value of $3g$.

b If $t = 3$, state the value of $t + 6$.

c What is $5x + 2$ if x is 10?



Remember, $3g$ means $3 \times g$

3 Substitute and find the value of each expression.

a $x = 3$ into the expression $5x$ **b** $x = 7$ into the expression $4(x + 2)$

c $y = 3$ into the expression $20 - 4y$ **d** $y = 10$ into the expression $\frac{y+4}{7}$

4 Find the value of $G + H$ if:

a $G = 2$ and $H = 3$ **b** $G = 7$ and $H = 1$

c $G = 2$ and $H = 12$ **d** $G = 10$ and $H = 100$

10F

FLUENCY 5–9 5–10

Example 14 Applying a formula

Consider the formula $k = 3b + 2$. Find the value of k if:

a $b = 5$ **b** $b = 10$

Solution	Explanation
a $k = 3b + 2$ $= 3 \times 5 + 2$ $= 17$	Copy the formula. Substitute $b = 5$ into the equation, then evaluate.
b $k = 3b + 2$ $= 3 \times 10 + 2$ $= 32$	Copy the formula. Substitute $b = 10$ into the equation, then evaluate.

5 Consider the formula $d = t + 7$. Find the value of d if:

a $t = 2$ **b** $t = 10$ **c** $t = 6$

6 Consider the rule $n = 2m + 1$. Find n if:

a $m = 3$ **b** $m = 4$ **c** $m = 100$



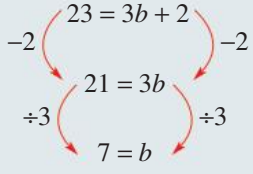
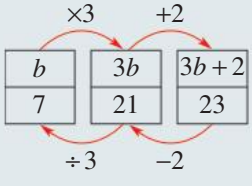
If $m = 3$ then $2m + 1 = 2 \times 3 + 1$

7 Consider the formula $F = ma$. Find F if:

a $m = 3$ and $a = 7$ **b** $m = 9$ and $a = 2$ **c** $m = 1$ and $a = 30$

Example 15 Applying a formula and solving to find the unknown

Consider the formula $k = 3b + 2$. Find the value of b if $k = 23$.

Solution	Explanation
Solve $23 = 3b + 2$.  Therefore, $b = 7$.	Substitute $k = 23$ into the equation. Now solve the equation to find the value of b. Alternatively, use backtracking, starting with b. 

8 You are given the formula $m = 4x$.

a Solve the equation $12 = 4x$. **b** Find the value of x if $m = 12$.

9 You are given the formula $A = 3b + 4$.

a If $b = 2$, what is A?

b If $A = 34$, what is b?

c Find b if $A = 25$.



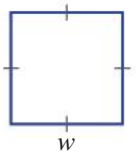
For parts **b** and **c**, set up an equation and then solve it.

10 Consider the formula $y = 5 + 3x$. Find:

a y if $x = 6$ **b** x if $y = 17$ **c** x if $y = 26$

PROBLEM-SOLVING AND REASONING 11 11, 12

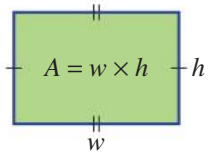
11 The perimeter, P, of a square is given by the formula $P = 4w$ where w is the width.



a Find the perimeter if:
i $w = 7$ **ii** $w = 10$

b If the perimeter is 48, solve the equation $48 = 4w$ to find the width.

12 The formula for the area of a rectangle is $A = w \times h$, where w is the rectangle's width and h is the rectangle's height.



a Find the value of A if $w = 7$ and $h = 5$.

b Set up and solve an equation to find the width of a rectangle with $A = 20$ and $h = 4$.

c A rectangle is drawn for which $A = 25$ and $w = 5$.
i Set up and solve an equation to find h.
ii What type of rectangle is this?

Converting temperatures

— 13

13 To convert between temperatures in degrees Celsius (°C) and degrees Fahrenheit (°F), the rule is $F = 1.8C + 32$.

For parts **c** and **d**, round to the nearest whole degree.



a Find F if $C = 20$.

b Find C if $F = 50$.

c What is the temperature in degrees Fahrenheit if it is 23°C outside?

d In Florida last week, the temperature hit 97°F. What is this in degrees Celsius? Solve an appropriate equation and round your answer to the nearest degree.



e Marieko claims that the temperature in her city varies between 68°F and 95°F. What is the difference in degrees Celsius, between these two temperatures?

10G Applications

Our methods for solving equations can be applied to many everyday situations.



Let's start: Stationery shopping

Sylvia bought 10 pencils and 2 pens for \$25. She knows that the pens cost \$3.50 each. How much did each pencil cost?

- Describe how you got your answer.
- Can you show your steps using equations?

Key ideas

- To solve a problem, follow these steps.

Define variables to stand for unknown numbers.	For example: Let $\$p$ = the cost of a pencil.
Write an equation to describe the problem.	For example: $6p = 12$
Solve the equation.	You could use inspection, backtracking or the balance method.
Make sure that you answer the original question!	Don't forget to include the correct units (e.g. dollars, years, cm).

Exercise 10G

UNDERSTANDING 1–3 3

1 Match each of the descriptions (**a** to **e**) with the correct algebraic expression (**A** to **E**).

a The sum of x and 3 — **A** $2x$

b Twice the value of x — **B** $\frac{x}{2}$

c 2 less than x — **C** $5x$

d Half of x — **D** $x + 3$

e The product of 5 and x — **E** $x - 2$

2 Solve the following equations.

a $5x = 30$ **b** $7a + 2 = 16$ **c** $2k - 3 = 15$

3 Alysha notices that buying 4 pens costs \$12. Which of the following is the cost of 1 pen?

A \$4 **B** \$12 **C** \$1 **D** \$3 **E** \$16

FLUENCY 4–8 4–9

4 The product of k and 7 is 42.

a Write an equation to describe this fact.

b Solve the equation to find the value of k.



5 The sum of x and 19 is 103.

a Write an equation to describe this fact.

b Solve the equation to find the value of x.



Example 16 Solving a problem using equations

The sum of Kate's age now and her age next year is 19. How old is Kate?

Solution	Explanation
Let k = Kate's current age.	Define a variable to stand for the unknown number.
$k + (k + 1) = 19$	Write an equation to describe the situation. Note that $k + 1$ is Kate's age next year.
$2k + 1 = 19$ -1 -1 $2k = 18$ $\div 2$ $\div 2$ $k = 9$	Simplify the LHS and then solve the equation, using the balance method (left) or backtracking. $\times 2$ $+1$ k $2k$ $2k+1$ 9 18 19 $\div 2$ -1
Kate is currently 9 years old.	Answer the original question in a sentence.

6 Millie buys 12 pens for a total cost of \$18.

a Define a variable for the cost of one pen.

b Write an equation to describe the problem.

c Solve the equation.

d What is the cost of one pen?

7 Launz buys four new tyres for his car. He also buys a smaller spare tyre for his trailer, which costs \$160. The total cost is \$1400.

a Define a variable for the cost of a car tyre.

b Write an equation to describe the problem.

c Solve the equation.

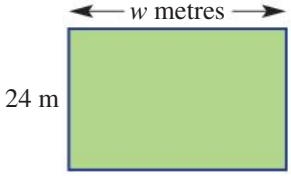
d What is the cost of a car tyre?

10G

8 Jonas is paid $17 per hour and gets paid a bonus of $65 each week. One particular week he earned $643.

- **a** Define a variable for the number of hours Jonas worked.
- **b** Write an equation to describe the problem.
- **c** Solve the equation.
- **d** How many hours did Jonas work in that week?

9 This rectangular paddock has an area of 720 m^2.





- **a** Write an equation to describe the problem, using w for the paddock's width.
- **b** Solve the equation.
- **c** How wide is the paddock?
- **d** What is the paddock's perimeter?



The perimeter is the total distance around the paddock.

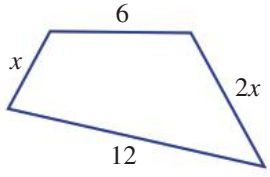
PROBLEM-SOLVING AND REASONING

10, 11 | 10–12

10 A number is doubled, then 3 is added. This gives a final result of 31. Set up and solve an equation to find the original number, showing all the steps clearly.

11 **a** The perimeter of this shape is 30. Find the value of x.

b Is it possible for the perimeter to equal 15? Why, or why not?



12 Marco and Sara's combined age is 30. Given that Sara is 2 years older than Marco, write an equation and find Marco's age.



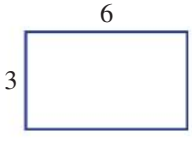
If Marco's age is m, then Sara's age is $m + 2$.

Curious rectangles

— | 13

13 The formulas for the area and perimeter of rectangles are $A = l \times w$ and $P = 2l + 2w$.

- **a** Calculate the area and the perimeter of this rectangle ($l = 3$ and $w = 6$). What do you notice about the numbers you get?
- **b** Find some other solution to the equation $l \times w = 2l + 2w$ by inspection. (There are many solutions.)
- **c** Are there any solutions when the rectangle is a square (that is, $l = w$)? Justify your answer.



Maths@Work: Uber fares

Taxis have competition from Uber and its drivers. Anyone aged 21 and over, with a car less than 10 years old, can apply to be an Uber driver. The Uber company is expanding and is in over 77 countries and 527 cities world-wide.

The cost of your trip is calculated before you even get in the Uber car. The formula usually used is:

Fare = Base fare + $/min × time + $/km × distance



1 UberX is the basic level of Uber transport. It has the following fee rates in Australia.

Base fare	$2.50
Per minute	$0.40
Per kilometre	$1.45
Minimum fare	$8.00
Cancellation fee	$10.00

a How much would the following journeys cost using the fee structure of UberX?
- **i** A 6-minute trip covering 3 km.
- **ii** A 10-minute trip of 12 km.
- **iii** 34-minute trip of 24 km.

b How many kilometres was Sam's Uber trip if it lasted $\frac{3}{4}$ hour and cost her $70?

c How long did it take Tyrone to travel the 34 km from his home to the city centre if it cost $71?

d Write an equation that calculates the UberX fare for a ride of n kilometres taking t minutes.

2 UberSELECT is a more luxurious ride. It has a different fee structure, depending on the city of travel. The prices here are for up to 4 passengers in Brisbane, Queensland.

Base fare	$4.00
Per minute	$0.55
Per kilometre	$1.80
Minimum fare	$10.00
Cancellation fee	$10.00

a How much would a half hour ride covering 40 km cost using UberSELECT?

b What is the maximum distance you can travel in 5 minutes if you only pay the minimum fare?

c Brisbane to the Sunshine Coast is 120.3 km and should take 1 hour 22 minutes. If these figures are used, what should an UberSELECT ride cost you between these two places?

d From Brisbane airport to Australia Zoo is 75 km. If you can only afford $180, how long do you hope the UberSELECT ride will take for a family of 4?

e Write an equation that can be used to calculate the cost of UberSELECT for n kilometres at a time of t minutes.

f A person was charged $10 for a 2 km trip that took 2 minutes. Explain why the formula was not used.

3 UberBLACK operates in Western Australia:

Base fare	$10.00
Per minute	$0.68
Per kilometre	$2.04
Minimum fare	$8.00
Cancellation fee	$10.00



a How much does the 30-minute, 23 km trip from Perth CBD to Fremantle cost?

b How much does the 14-minute, 8 km trip from Fremantle to Cottesloe beach cost?

c How far can a person travel in 30 minutes for $50 using UberBLACK in Western Australia?

d Write an equation that can be used to calculate the cost of n kilometres at a time of t minutes.

4 **a** Research the levels of Uber rides available and their costs in two world cities outside of Australia.

b Research the reasons why Uber fares can sometimes be a lot more expensive for the same route.

c Research the fares charged by the taxi companies operating in your area.

Using technology

5 **a** Set up the following spreadsheet to calculate Uber fares. Select all the cost cells (in columns D, F, H and I) right click, select Format/Currency/2 d.p.

b Enter formulas in the shaded cells to calculate these Uber fares.

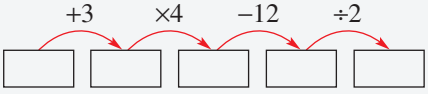
	A	B	C	D	E	F	G	H	I
1	Uber fare calculations								
2	**Uber level**	**Pickup place**	**Destination**	**Base fare**	**Distance in km**	**Cost per km**	**Time in minutes**	**Cost per minute**	**Fare**
3	UberX	Burke and Wills Hotel, Toowoomba, Qld	Byron Bay, NSW	$2.00	268	$1.00	180	$0.35	
4	UberX	Melbourne Airport, Vic	Mornington Peninsula, Vic	$2.00	109	$1.00	90	$0.32	
5	UberXL	Brisbane Airport, Qld	Surfers Paradise, Gold Coast, Qld	$3.10	87	$1.60	70	$0.50	
6	UberXL	Perth Airport, WA	Margaret River, WA	$4.50	274	$1.70	185	$0.50	
7	UberBLACK	Napier Street, Adelaide SA	Adelaide Airport, SA	$8.00	9	$2.05	17	$0.45	
8	UberASSIST	Canberra Airport, ACT	Goulburn, NSW	$2.35	94	$1.35	65	$0.45	

1 Find the unknown number in the following puzzles.

- **a** A number is added to half of itself and the result is 39.
- **b** A number is doubled, then tripled, then quadrupled. The result is 696.
- **c** One-quarter of a number is subtracted from 100 and the result is 8.
- **d** Half of a number is added to 47, and the result is the same as the original number doubled.
- **e** A number is increased by 4, the result is doubled and then 4 is added again to give an answer of 84.

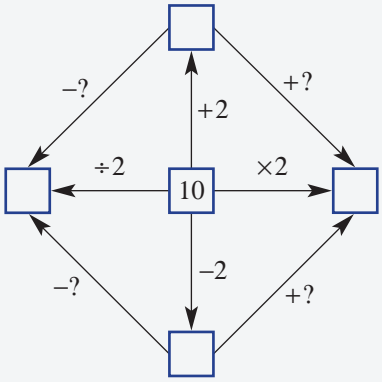
2 Find values of x and y that will make *both* these equations true: $x + y = 20$ and $x \times y = 91$

3 A flow chart is set up like this:



How could you get from the last box back to the start in just one operation?

4 Some numbers in this two-dimensional flow chart are unknown. Find the sum of the four numbers that are shown by question marks.



5 *What did the student expect when she solved the puzzle?*
Find the answer by solving the following equations. If the solution is $x = 1$, the letter is A. If the solution is $x = 2$, the letter is B, and so on.

$3x + 2 = 5$	$16 = 2x + 10$	$5x - 10 = 65$	$\frac{x}{3} + 1 = 7$	$2 = \frac{x+2}{8}$	$2x + 10 = 60$	$\frac{x}{2} + 3 = 8$	$3x + 2 = 47$	$20 - x = 9$	$4x - 3 = 17$

6 In a farmer's paddock there are sheep and ducks. Farmer Jess says to her grandson, 'There are 41 animals in this paddock.' Grandson James says to his grandma, 'There are 134 animal legs in this paddock.'
How many sheep and how many ducks are in the paddock?

7 Michelle looks at how many matchsticks are required to make a pattern of triangles.

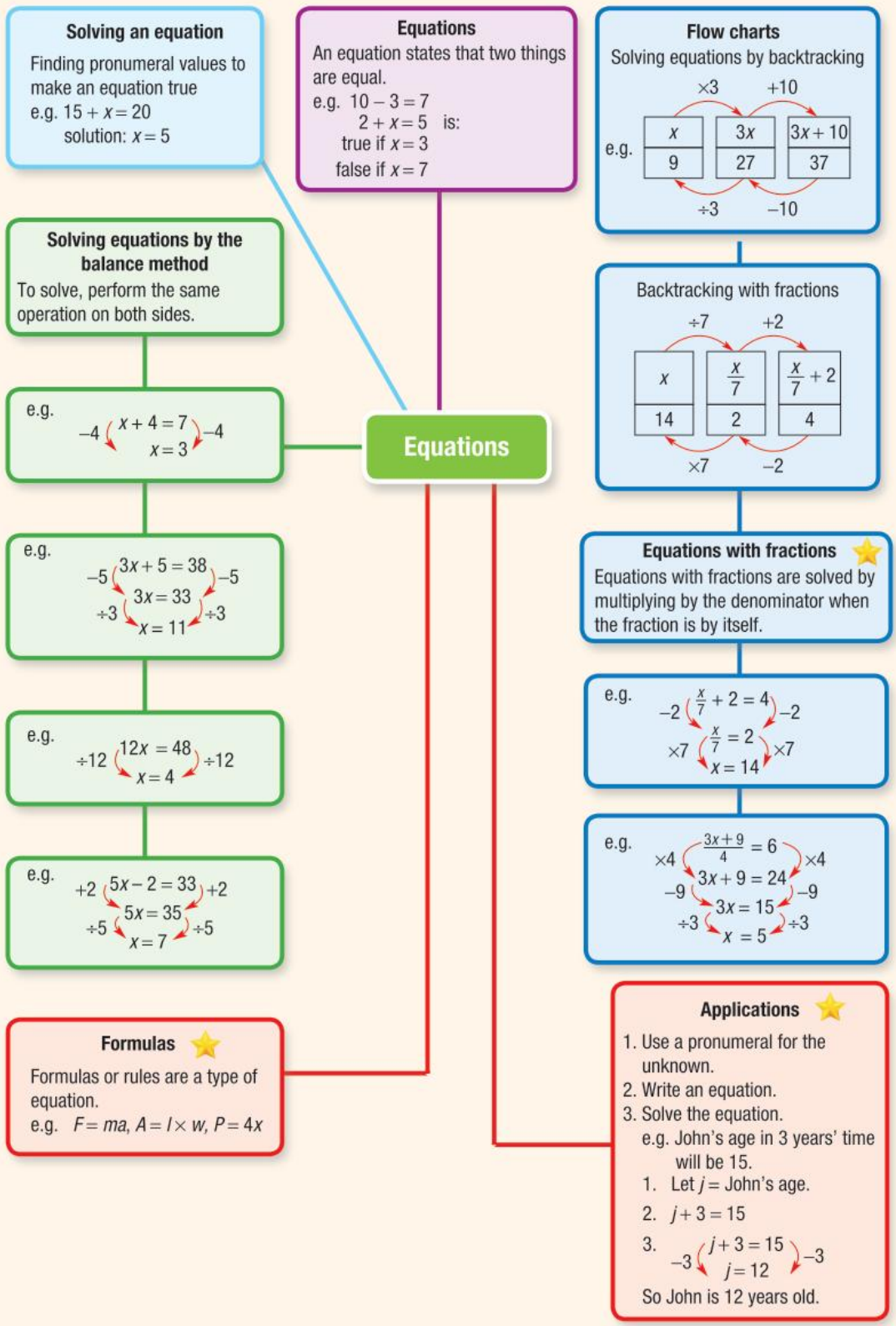


$n = 1$, $M = 3$ $\quad$ $n = 2$, $M = 5$ $\quad$ $n = 3$, $M = 7$

- **a** Find a formula relating M and n.
- **b** How many matchsticks would Michelle need to make 100 triangles?

8 Egan and Joel have ages that are 5 years apart. When their ages are added, the result is 21. How old is each boy?

Chapter summary



Chapter review

Multiple-choice questions

10A **1** Which one of the following equations is *false*?

A $3+4=7$ **B** $10-2=8$ **C** $5+5=2+8$
D $7-4=4$ **E** $5+11=17-1$

10A **2** If $x=3$, which one of the following equations is true?

A $4x=21$ **B** $2x+4=12$ **C** $9-x=6$
D $2=x+1$ **E** $x-3=4$

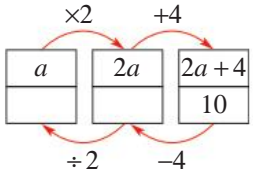
10A **3** When x is tripled and then 11 is added, the result is 53. This can be written as:

A $3x+11=53$ **B** $3(x+11)=53$ **C** $\frac{x}{3}+11=53$
D $\frac{x+11}{3}=53$ **E** $3x-11=53$

10B **4** Which of the following values of x makes the equation $3x=24$ true?

A 2 **B** 4 **C** 6
D 8 **E** 10

10C **5** The equation $2a+4=10$ is solved using backtracking.



The solution is:

A $a=3$ **B** $a=7$ **C** $a=12$
D $a=28$ **E** $a=1$

10D **6** When using the balance method to solve $3a+5=17$, the first step to apply to both sides is:

A add 5 **B** divide by 3 **C** subtract 17
D divide by 5 **E** subtract 5

10C/D **7** The solution to $2t-4=6$ is:

A $t=1$ **B** $t=3$ **C** $t=5$ **D** $t=7$ **E** $t=9$

10E **8** The solution to $\frac{x}{7}=10$ is:

A $x=35$ **B** $x=70$ **C** $x=20$ **D** $x=30$ **E** $x=5$

10C/D **9** The solution to the equation $10=3p-5$ is:

A $p=5$ **B** $p=20$ **C** $p=15$ **D** $p=7$ **E** $p=1$

10F **10** A formula relating F, m and a is $F=ma$. If $F=60$ and $a=6$, then m equals:

A 18 **B** 4 **C** 30 **D** 2 **E** 10

Chapter review

Short-answer questions

10A **1** Classify each of the following equations as true or *false*.

a $4 + 2 = 10 - 2$ **b** $2(3 + 5) = 4(1 + 3)$
c $5w + 1 = 11$, if $w = 2$ **d** $2x + 5 = 12$, if $x = 4$
e $y = 3y - 2$, if $y = 1$ **f** $4 = z + 2$, if $z = 3$

10A **2** Write an equation for each of the following situations. You do not need to solve the equations.

a The sum of 2 and u is 22. **b** The product of k and 5 is 41.
c When z is tripled the result is 36. **d** The sum of a and 12 is 15.

10B **3** Solve the following equations by inspection.

a $x + 1 = 4$ **b** $x + 8 = 14$
c $9 + y = 10$ **d** $y - 7 = 2$
e $5a = 10$ **f** $a \div 5 = 2$

10C **4** Solve each of the following equations by using backtracking.

a $4x = 20$ **b** $7q = 42$
c $a + 4 = 25$ **d** $k - 6 = 12$
e $2k + 3 = 23$ **f** $5w - 7 = 33$
g $3b + 5 = 20$ **h** $10r - 12 = 8$

10D **5** Solve each of the following equations by using the balance method.

a $5x = 15$ **b** $r + 25 = 70$
c $5 = x - 4$ **d** $13 = 2r + 5$
e $10 = 4q + 2$ **f** $8u + 2 = 66$

10E **6** Solve the following equations.

a $\frac{u}{4} = 6$ **b** $\frac{p}{2} = 8$
c $3 = \frac{x + 1}{3}$ **d** $\frac{y}{2} + 10 = 30$
e $4 = \frac{y + 20}{7}$ **f** $\frac{x}{3} + 4 = 24$

10A **7** Consider the equation $4(x + 3) = 2x + 14$.

a Is $x = 2$ a solution? (Check the LHS and RHS.)
b Show that $x = 1$ is a solution.

10F **8** The formula for the area of a rectangle is $A = w \times l$, where w is the width and l is the length.

a Find the area if $w = 10$ and $l = 45$.
b If $A = 24$ and $w = 3$, find the value of l.
c Give an example of a rectangle that has an area of 40.

10F **9** Consider the rule $F = 3a + 2b$. Find:

a F if $a = 10$ and $b = 3$ **b** b if $F = 27$ and $a = 5$ **c** a if $F = 25$ and $b = 8$

10G **10** For each of the following problems, write an equation and solve it to find the unknown value.

a A number is multiplied by 4 and the result is 20. What is the number?
b The sum of 5 and a number is 30. What is the number?
c Juanita's mother is twice as old as Juanita. The sum of their ages is 60. How old is Juanita?
d A square has a perimeter of 20. What is its length?

Extended-response questions

1 Mya is paid $\$12$ per hour at her workplace.

- **a** How much is she paid if she works for 10 hours?
- **b** Which of the following formulas correctly relates her total wage W (in dollars) and the number n of hours worked?
  - **A** $W = 12$
  - **B** $W = 12n$
  - **C** $W = 12 + n$
  - **D** $W = n - 12$
- **c** If $n = 5$, what is the value of her total wage, W?
- **d** If her wage one day is $\$84$, for how long did she work?
- **e** Write an equation (but do not solve it) for the following problem. One week, Mya worked n hours and earned $\$252$.

2 Udhav's satellite phone plan charges a 15-cents connection fee and then 2 cents per second for every call. The total cost is given by $C = 15 + 2t$ if C is the cost in cents and t is the time in seconds.

- **a** How much does a 30-second call cost?
- **b** How much would a 1-minute call cost?
- **c** Solve the equation $15 + 2t = 39$.
- **d** If a call cost 39 cents, how long did it last?
- **e** If a call cost $\$1.15$, how long did it last?
- **f** On a particular day, Udhav makes two calls. The first call lasted 20 seconds and the second lasted twice as long. What was the total cost of these two calls?

For part **b**, $t = 60$





Chapter 11

Measurement

What you will learn

- 11A Using and converting metric lengths (Consolidating)
- 11B Perimeter (Consolidating)
- 11C Areas and rectangles
- 11D Area of a triangle
- 11E Area of a parallelogram
- 11F Volume of rectangular prisms
- 11G Capacity (Consolidating)
- 11H Mass and temperature (Consolidating)

Australian curriculum

MEASUREMENT AND GEOMETRY

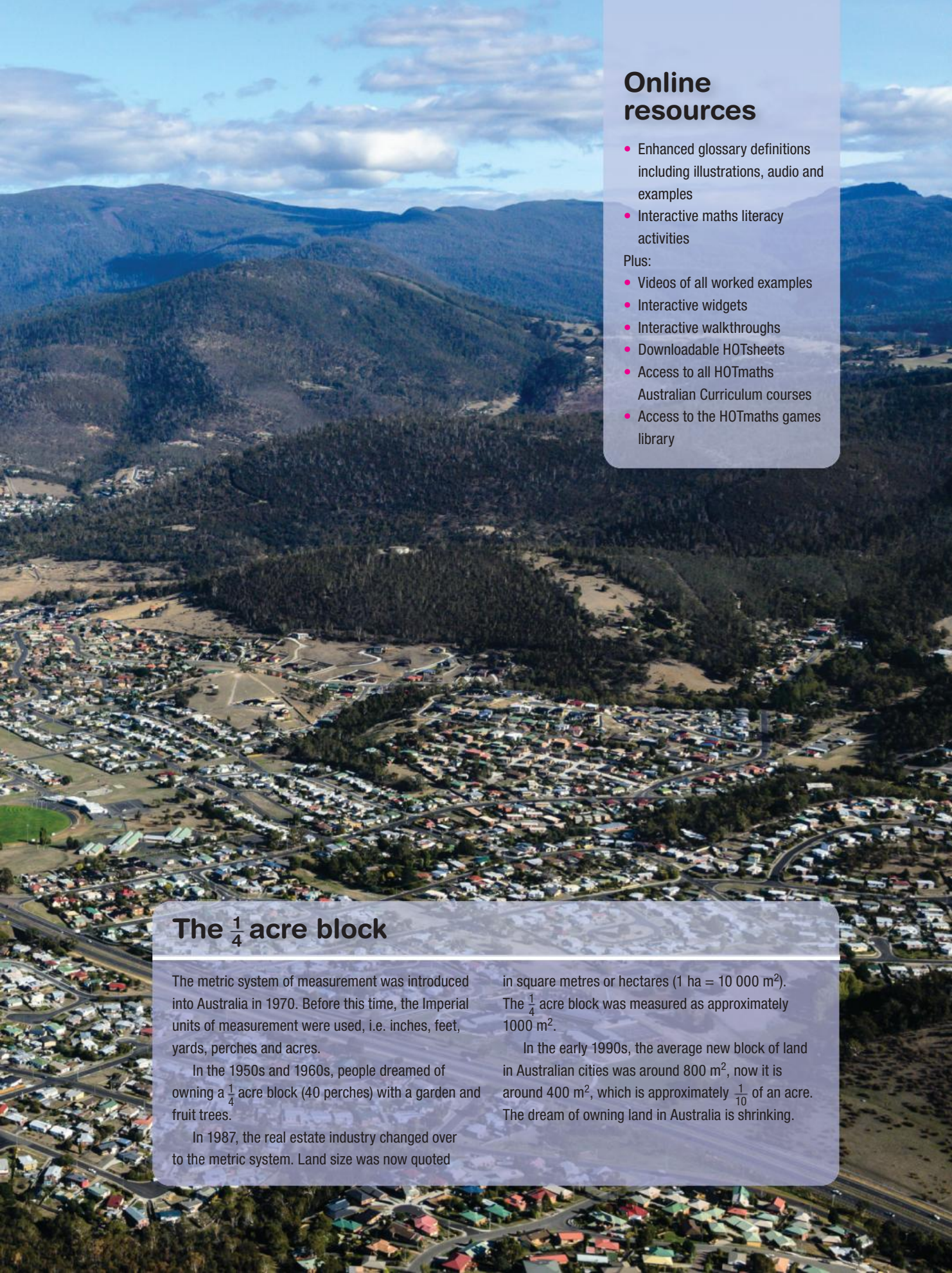
Using units of measurement

Establish the formulas for areas of rectangles, triangles and parallelograms, and use these in problem-solving **(ACMMG159)**

Calculate volumes of rectangular prisms **(ACMMG160)**

Shape

Draw different views of prisms and solids formed from combinations of prisms **(ACMMG161)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The $\frac{1}{4}$ acre block

The metric system of measurement was introduced into Australia in 1970. Before this time, the Imperial units of measurement were used, i.e. inches, feet, yards, perches and acres.

In the 1950s and 1960s, people dreamed of owning a $\frac{1}{4}$ acre block (40 perches) with a garden and fruit trees.

In 1987, the real estate industry changed over to the metric system. Land size was now quoted in square metres or hectares (1 ha = 10 000 m^2). The $\frac{1}{4}$ acre block was measured as approximately 1000 m^2.

In the early 1990s, the average new block of land in Australian cities was around 800 m^2, now it is around 400 m^2, which is approximately $\frac{1}{10}$ of an acre. The dream of owning land in Australia is shrinking.

Pre-test

1 Consider these units of measurement: (A) centimetre, (B) kilogram, (C) litre, (D) metre and (E) minute.

a Which unit measures time?
b Which unit measures volume or capacity?
c Which unit measures mass?
d Which two units measure distance?

2 Measure the length of these lines in cm.

a _____
b _____
c _____

3 Arrange these units from smallest to largest.

a (A) centimetre, (B) kilometre, (C) metre, (D) millimetre
b (A) gram, (B) kilogram, (C) milligram, (D) tonne
c (A) hour, (B) year, (C) second, (D) day, (E) minute

4 Find how many:

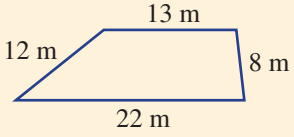
a millimetres are in a centimetre
b centimetres are in a metre
c grams are in a kilogram
d seconds are in a minute
e minutes are in an hour
f millilitres are in a litre

5 Calculate:

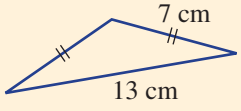
a 2×1000
b $200 \div 100$
c $56\,000 \div 1000$
d 2.5×1000
e 3×60
f $2 \times 60 \times 60$
g $1400 \div 1000$
h $27 \div 1000$

6 Find the total distance around each shape.

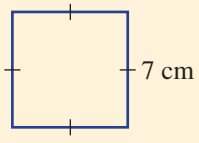
a



b

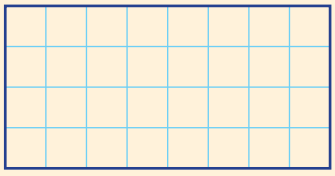


c

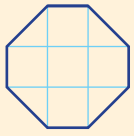


7 How many unit squares make up the area of these shapes?

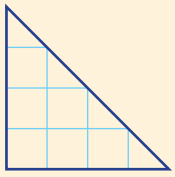
a



b

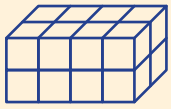


c

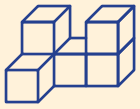


8 How many unit cubes make up the volume of these solids?

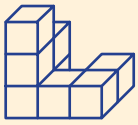
a



b



c



9 Give the most appropriate unit, (A) centimetres, (B) kilometres, (C) kilograms, (D) minutes, for measuring each of the following.

a distance between two towns
b your weight
c length of a school lesson
d width of a large plate

11A Using and converting metric lengths

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

The metric system for measurement was developed in France in the 1790s and is the universally accepted system today. The word *metric* comes from the Greek word *metron* meaning 'measure'. It is a decimal system where length measures are based on the unit called the *metre*.

The definition of the metre has changed over time. Originally it was proposed to be the length of a pendulum that beats at a rate of one per second. It was later defined as 1/10 000 000 of the distance from the North Pole to the equator on a line on the Earth's surface passing through Paris.



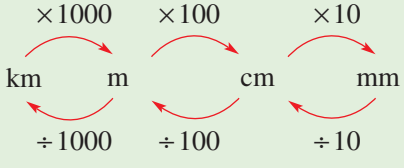
Let's start: How good is your estimate?

In less than 3 seconds, guess the length of your desk, in centimetres.

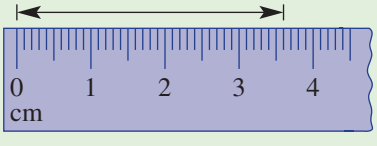
- Now use a ruler to find the actual length in centimetres.
- Convert your answer to millimetres and to metres.
- If you lined up all the class desks end to end, how many desks would be needed to reach 1 kilometre? Explain how you got your answer.

Key ideas

- The **metre** (m) is the basic metric unit of length in the **metric system**.
  - 1 centimetre (cm) = 10 millimetres (mm)
  - 1 metre (m) = 100 centimetres (cm)
  - 1 kilometre (km) = 1000 metres (m)



- Conversion
  - When converting to a smaller unit, multiply by a power of 10 (i.e. 10, 100, 1000). Move any decimal point to the right. For example:
    2.3 m = (2.3 × 100) cm = 230 cm
    28 cm = (28 × 10) mm = 280 mm
  - When converting to a larger unit, divide by a power of 10 (i.e. 10, 100, 1000). Move any decimal point to the left. For example:
    47 m = (47 ÷ 10) cm = 4.7 cm
    4600 m = (4600 ÷ 1000) km = 4.6 km
- When reading scales, be sure about what units are showing on the scale. This scale shows 36 mm or 3.6 cm.



Metre The standard metric unit for length, equal to 100 centimetres

Metric system A measurement system using the base-ten number system

Exercise 11A

UNDERSTANDING 1–3 | 3

1 List the four most common metric units of length.



2 Write the missing number or word in these sentences.
- **a** To convert from metres to centimetres, multiply by _____.
- **b** To convert from metres to kilometres, divide by _____.
- **c** When converting from centimetres to metres, you _____ by 100.
- **d** When converting from kilometres to metres, you _____ by 1000.

3 **a** When multiplying by a power of 10 (e.g. 3.21×100), does the decimal point appear to move left or right?
- **b** When dividing by a power of 10 (e.g. $32.1 \div 100$), does the decimal point appear to move left or right?

FLUENCY 4, 5, 6(½) | 4, 5, 6–7(½)

Example 1 Choosing metric lengths

Which metric unit would be the most appropriate for measuring these lengths?

a width of a large room **b** thickness of glass in a window

Solution	Explanation
a metres (m)	Using mm or cm would give a very large number, and using km would give a number that is very small.
b millimetres (mm)	The thickness of glass is likely to be around 5 mm.

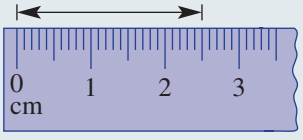
4 Which metric unit would be the most appropriate for measuring the following?
- **a** distance between two towns
- **b** width of a small drill bit
- **c** height of a flag pole
- **d** length of a garden hose
- **e** width of a small desk
- **f** distance across a city



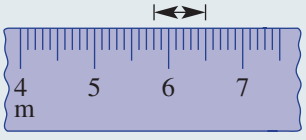
Example 2 Reading length scales

Read the scales on these rulers to measure the marked length.

a



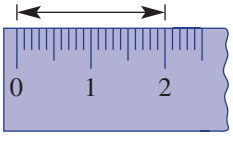
b



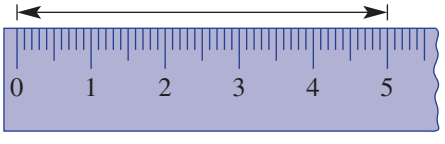
Solution	Explanation
a 25 mm	2.5 cm is also accurate.
b 70 cm	Each division is $\frac{1}{10}$ of a metre, which is 10 cm.

5 These rulers show centimetres with millimetre divisions. Read the scale to measure the marked length.

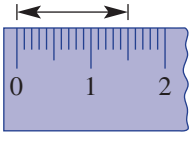
a



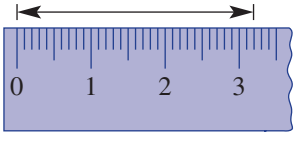
b



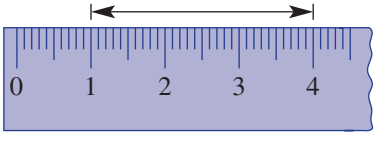
c



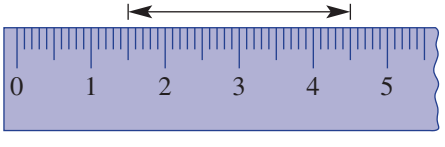
d



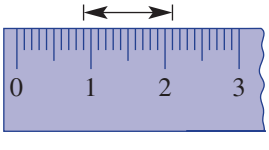
e



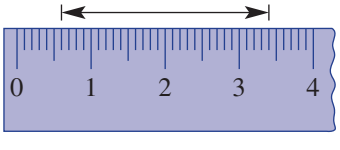
f



g



h



Example 3 Converting to smaller units of length

Convert to the units given in brackets.

a 3 m (cm)

b 2.8 km (m)

Solution	Explanation
a 3 m = (3 × 100) cm = 300 cm	1 m = 100 cm Multiply, as you are converting to a smaller unit.
b 2.8 km = (2.8 × 1000) m = 2800 m	There are 1000 m in 1 km, so multiply by 1000.

11A

6 Convert these measurements to the units shown in brackets.

a 5 cm (mm)
b 2 m (cm)
c 3.5 km (m)
d 26.1 m (cm)
e 2.2 km (m)
f 5.3 cm (mm)
g 6.2 m (cm)
h 20 cm (mm)
i 6.84 m (cm)
j 0.02 km (m)
k 38 m (cm)
l 6.7 m (cm)



Multiply by 10, 100 or 1000.

×1000 ×100 ×10

km m cm mm

Example 4 Converting to larger units of length

Convert to the units given in the brackets.

a 39 mm (cm)
b 580 m (km)

Solution	Explanation
a 39 mm = (39 ÷ 10) cm = 3.9 cm	There are 10 mm in 1 cm so divide by 10.
b 580 m = (580 ÷ 1000) km = 0.58 km	There are 1000 m in 1 km so divide by 1000.

7 Convert these measurements to the units shown in the brackets.

a 40 mm (cm)
b 500 cm (m)
c 4200 m (km)
d 472 mm (cm)
e 360 cm (m)
f 32 mm (cm)
g 50 000 m (km)
h 27 000 m (km)
i 362 mm (cm)
j 0.4 mm (cm)
k 9261 mm (cm)
l 4230 m (km)



Divide by 10, 100 or 1000.

km m cm mm

÷1000 ÷100 ÷10

PROBLEM-SOLVING AND REASONING 8–12 13–16

8 Choose which metric unit (mm, cm, m, km) would be the most suitable for measuring the length indicated in these photos.



Choose the unit for the real-life length, not the length of the arrow in the photo!

a



b



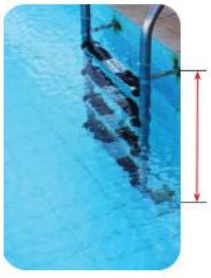
c



d



e



f



9 Add these lengths and give the result in the units shown in brackets.

a 2 cm and 5 mm (cm)
b 8 cm and 2 mm (mm)
c 2 m and 50 cm (m)
d 7 m and 30 cm (cm)
e 6 km and 200 m (m)
f 25 km and 732 m (km)



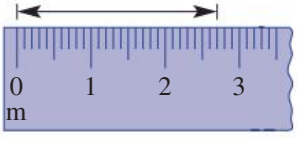
First convert one of the measurements so that both lengths have the same units shown in brackets.

10 Use subtraction to find the difference between the measurements, and give your answer with the units shown in brackets.

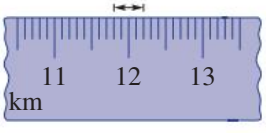
a 9 km, 500 m (km)
b 3.5 m, 40 cm (cm)
c 0.2 m, 10 mm (cm)

11 Read the measurement on each of these scales. Be careful with the units shown!

a



b





Make sure you identify the unit on each scale.

12 Arrange these measurements from smallest to largest.

a 38 cm, 540 mm, 0.5 m
b 0.02 km, 25 m, 160 cm, 2100 mm
c 0.003 km, 20 cm, 3.1 m, 142 mm
d 0.001 km, 0.1 m, 1000 cm, 10 mm

13 Joe widens a 1.2 m doorway by 50 mm. What is the new width of the doorway, in centimetres?

14 Steel chain costs $8.20 per metre. How much does it cost to buy chain of the following lengths?

a 1 km
b 80 cm
c 500 mm

15 Mount Everest is moving with the Indo-Australian plate at a rate of about 10 cm per year. How many years will it take to move 5 km?



16 Convert to the units shown in the brackets.

a 3 m (mm)
b 6 km (cm)
c 2.4 m (mm)
d 0.04 km (cm)
e 47 000 cm (km)
f 913 000 mm (m)
g 216 000 mm (km)
h 0.5 mm (m)
i 0.002 km (m)

11A

Estimating length

— 17, 18

17 Here are the lengths 1 mm and 1 cm.
Use these diagrams as a guide to estimate the length of the following lines.

1 mm 1 cm

a –

b —

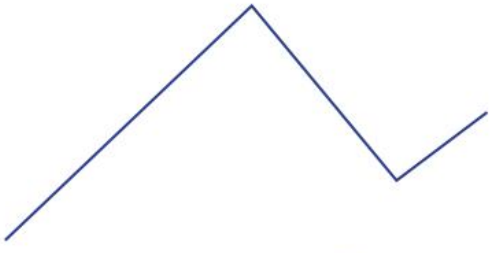
c ———

d ———————————

e ———————————————————

18 Estimate the length of each line or curve in centimetres.

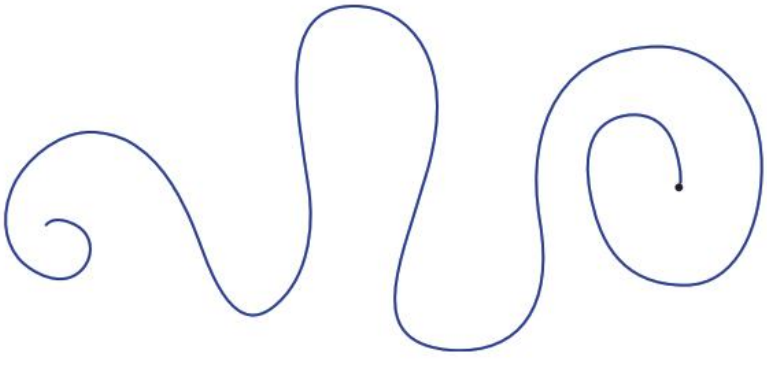
a



b



c



11B Perimeter

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthrough

The distance around the outside of a two-dimensional shape is called the perimeter. The word *perimeter* comes from the Greek words *peri*, meaning 'around', and *metron*, meaning 'measure'.

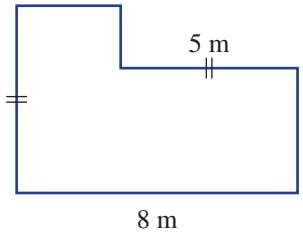
We associate perimeter with the outside of all sorts of regions and objects, like the length of fencing around a block of land or the length of timber required to frame a picture.



This fence marks the perimeter (i.e the distance around the outside) of a paddock.

Let's start: The L-shaped perimeter

This L-shaped area has two given measurements. All angles are 90°.

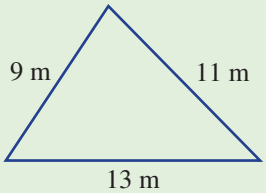


- For each of the other sides, state whether it is possible to find the length. Give reasons for each answer.
- Is it possible to find the perimeter of the entire shape? Explain your answer.

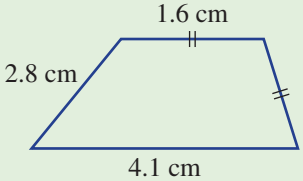
Key ideas

- **Perimeter**, often denoted as P, is the distance around the outside of a two-dimensional shape.
- Sides with the same markings are of equal length.

Perimeter The total distance (length) around the outside of a figure



$P = 9 + 11 + 13$
$= 33$ m



$P = 1.6 + 1.6 + 2.8 + 4.1$
$= 10.1$ cm

Exercise 11B

UNDERSTANDING 1–3 3

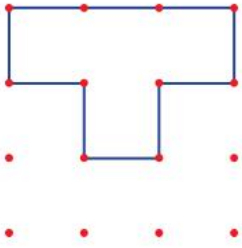
1 Choose a suitable word to complete each sentence.

a The distance around the outside of a shape is called the _____.

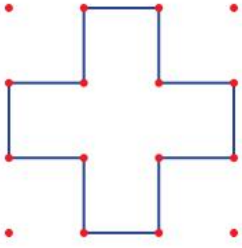
b Sides with the same markings are of _____ length.

2 These shapes are drawn on 1 cm grids. Give the perimeter of each.

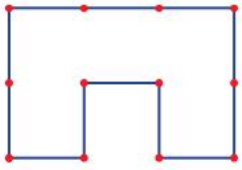
a



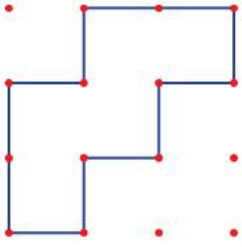
b



c



d

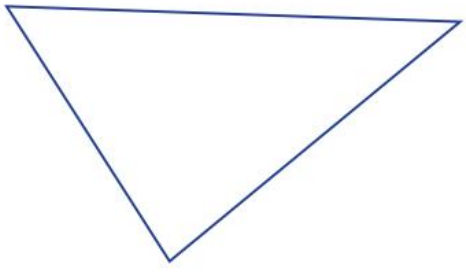




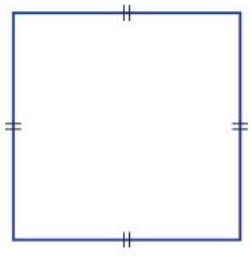
The distance between each pair of red dots is 1 cm.

3 Use a ruler to measure the lengths of the sides of these shapes, and then find the perimeter in centimetres.

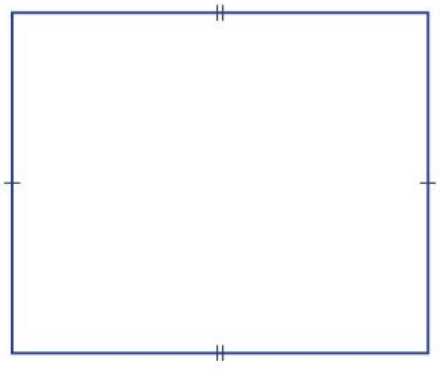
a



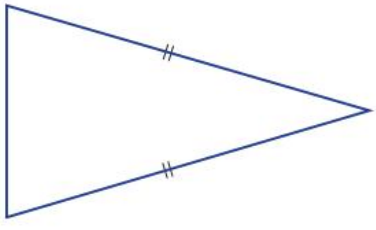
b



c



d

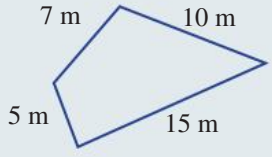


FLUENCY 4 4(½), 5

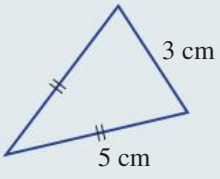
Example 5 Finding the perimeter

Find the perimeter of each of these shapes.

a



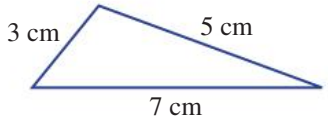
b



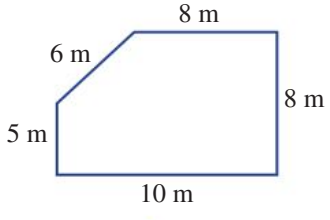
Solution	Explanation
a $P = 5 + 7 + 10 + 15$ $= 37$ m	Add all the side lengths to find the perimeter.
b $P = 2 \times 5 + 3$ $= 13$ cm	There are two equal lengths of 5 cm and one length of 3 cm.

4 Find the perimeter of these shapes. (Diagrams are not drawn to scale.)

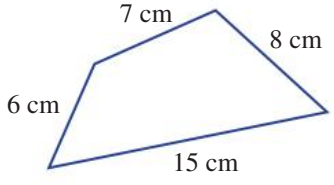
a



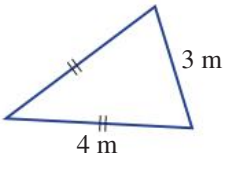
b



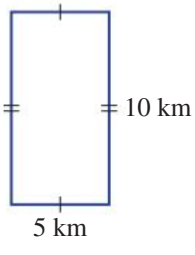
c



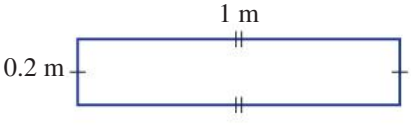
d



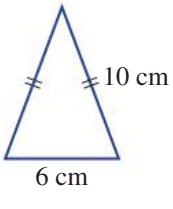
e



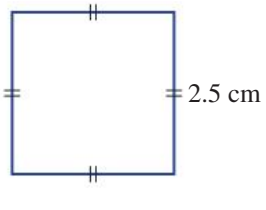
f



g



h



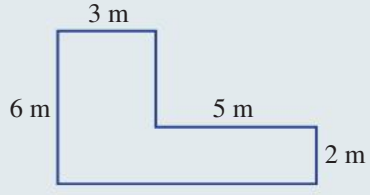


Sides with the same markings are of equal length.
Add all the sides to find the perimeter.

11B

Example 6 Finding perimeter when sides are unknown

Find the perimeter of this L-shape.

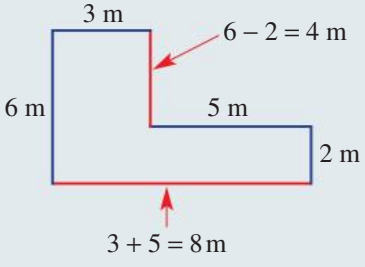


Solution

$P = 6 + 3 + 4 + 5 + 2 + 8$
$= 28$ m

Explanation

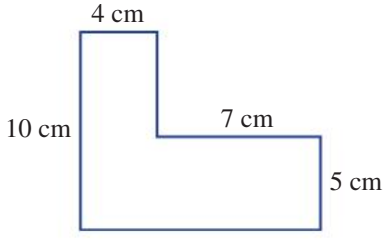
First find the missing side lengths.



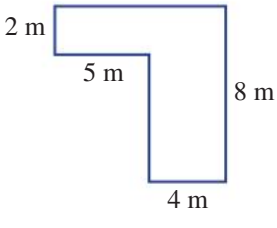
Then add all the side lengths.

5 Find the perimeter of each L-shape. All corner angles are 90°.

a



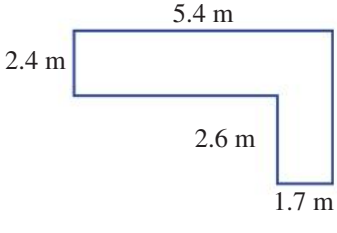
b



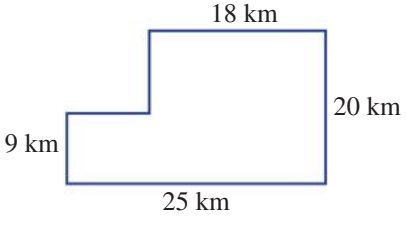
First find the length of the missing sides.



c



d

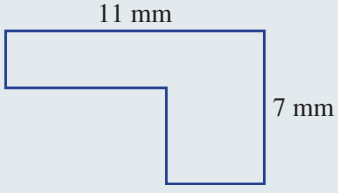


PROBLEM-SOLVING AND REASONING 6–8 7–10

6 **a** A square has a side length of 2.1 cm. Find its perimeter.
b A rectangle has a length of 4.8 m and a width of 2.2 m. Find its perimeter.
c An equilateral triangle has all sides the same length. If each side is 15.5 mm, find its perimeter.

Example 7 Using a rectangle when sides are unknown

Use a rectangle to help find the perimeter of this L-shape.



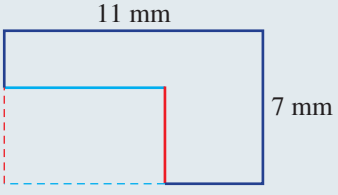
Solution

$P = 2 \times 7 + 2 \times 11$

$= 14 + 22$

$= 36$ mm

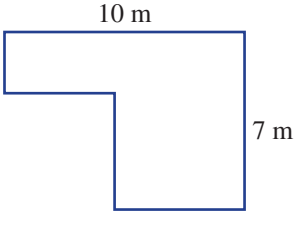
Explanation



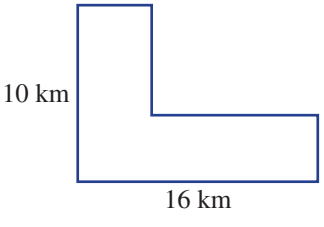
The perimeter of the L-shape is the same as the perimeter of the rectangle.

7 Use a rectangle to help you find the perimeter of each of these shapes.

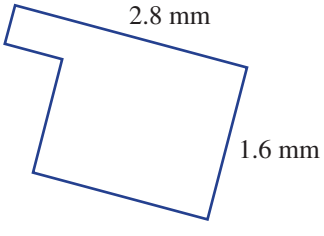
a



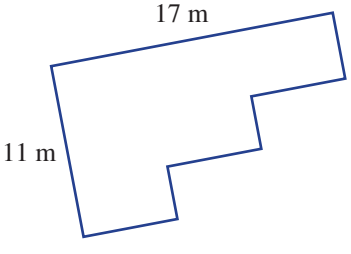
b



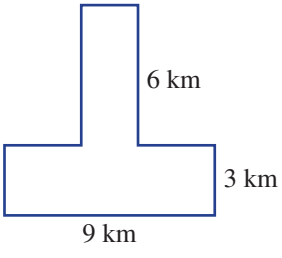
c



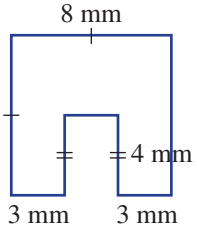
d



e



f



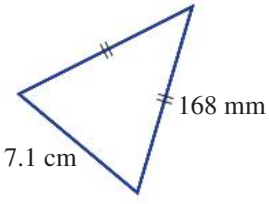
11B

8 A horse paddock is to be fenced on all sides. It is rectangular in shape, with a length of 242 m and a width of 186 m. If fencing costs $25 per metre, find the cost of fencing required.

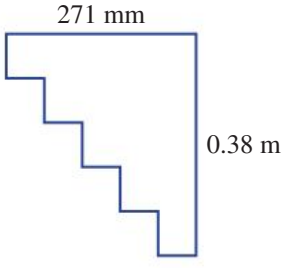


9 Find the perimeter of each of these shapes. Give your answers in centimetres.

a



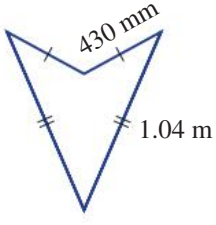
b



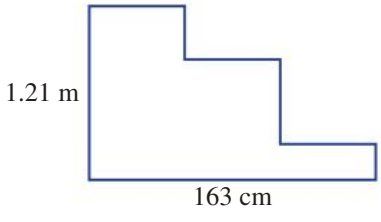


First convert all side lengths to centimetres before adding them.

c

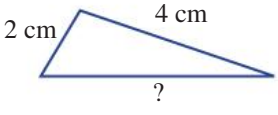


d



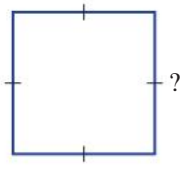
10 The perimeter of each shape is given. Find the missing length of each.

a



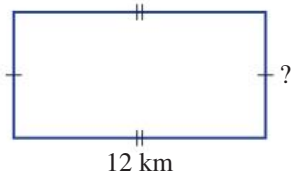
$P = 11$ cm

b



$P = 20$ m

c

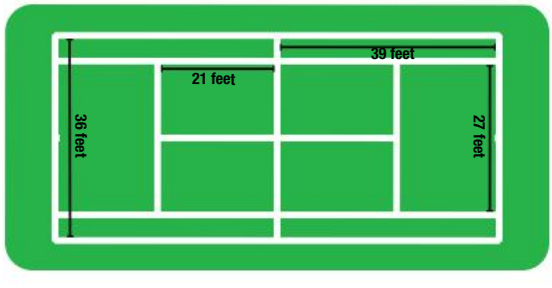


$P = 38$ km

Tennis court perimeters

11

11 A grass tennis court has white chalk lines. All the measurements are shown in the diagram and given in feet.



a Find the total number of feet of chalk required to mark all the lines of the tennis court.

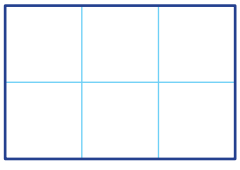
b There are 0.305 metres in 1 foot. Convert your answer to part **a** to metres.

11C Areas and rectangles

Interactive
Widgets
HOTsheets
Walkthrough

The number of square units inside the boundary of a closed shape gives the measurement called area.

The number of square centimetres inside this rectangle is 6, so the area of the rectangle is 6 cm^2.



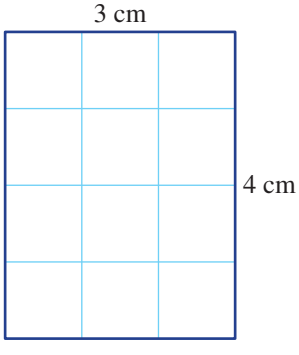


Other shapes may be measured in larger or smaller units. For example, the area of a coin might be 200 mm^2 and the area of a soccer field might be 8000 m^2.

Let's start: The 12 cm^2 rectangle

Consider an area of 12 square centimetres (12 cm^2).

- Draw examples of rectangles that have this area, showing the length and width measurements. One example is shown here. You might find it helpful to draw on grid paper.
- How many different rectangles with whole number dimensions are possible?
- How many different rectangles are possible if there is no restriction on the type of numbers used for length and width?



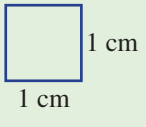
Key ideas

■ The metric units of **area** include:

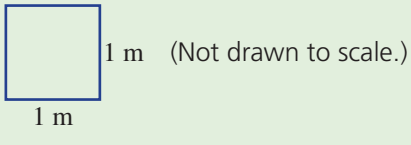
- 1 square millimetre (1 mm^2)



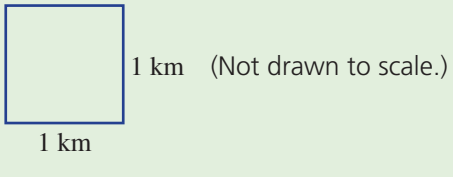
- 1 square centimetre (1 cm^2)



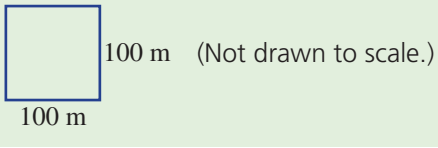
- 1 square metre (1 m^2)



- 1 square kilometre (1 km^2)

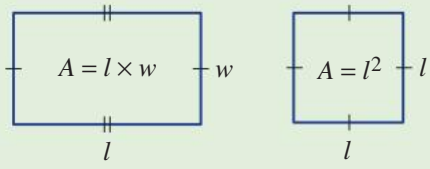


- 1 hectare (1 ha) (10 000 m^2)



Area The amount of surface a shape covers

- The area of a rectangle is given by:
  Area = length × width or $A = l \times w$
- On a grid, you can count squares to find the area of a rectangle or multiply the number of rows by the number of columns.
- The area of a square is given by:
  $A = l \times l = l^2$



Exercise 11C

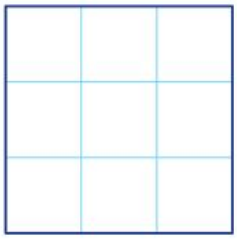
UNDERSTANDING 1–4 4

1 Arrange the following from largest to smallest.
$1\ cm^2$, $1\ m^2$, 1 ha, $1\ km^2$, $1\ mm^2$



1 ha is not as large as $1\ km^2$.

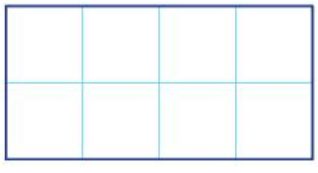
2 For this square drawn on a centimetre grid, find:
- **a** the number of single 1 cm squares
- **b** the length and the width
- **c** length × width





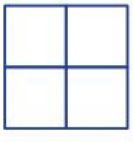
For part **b**, give the side lengths in cm.

3 For this rectangle drawn on a 1 cm grid, find:
- **a** the number of single 1 cm squares
- **b** the length and the width
- **c** length × width

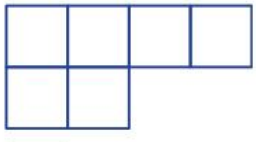


4 Count the number of squares to find each area in square units.

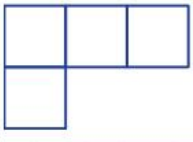
a



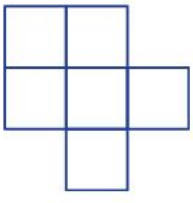
b



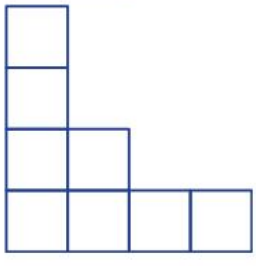
c



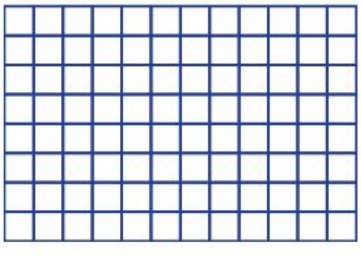
d



e



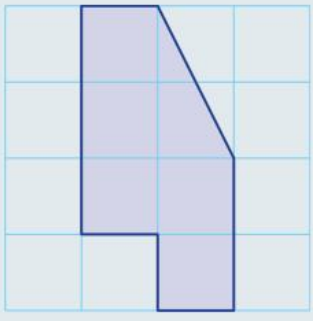
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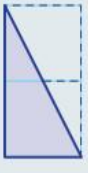
FLUENCY 5–6(½) 5-7(½)

Example 8 Counting areas

Count the number of squares to find the area of the shape drawn on this centimetre grid.



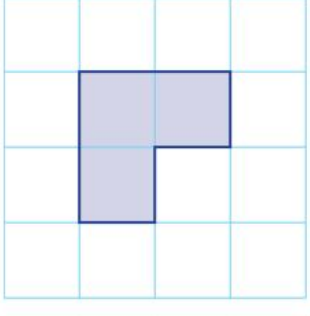
Solution	Explanation
6 cm^2	There are 5 full squares and half of 2 squares in the triangle, giving 5 plus 1 more.



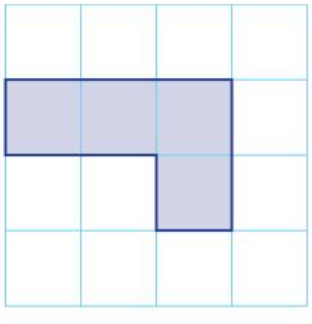
$\frac{1}{2}$ of 2 = 1

5 Count the number of squares to find the area of these shapes on centimetre grids.

a



b

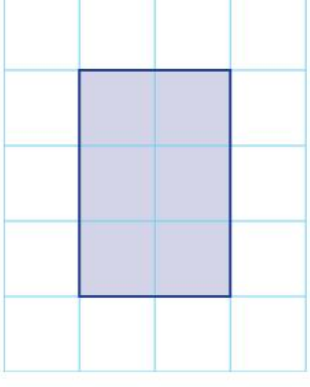




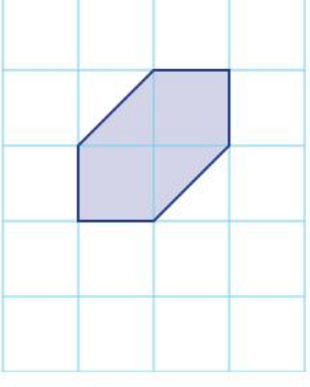
is $\frac{1}{2}$ cm^2



c

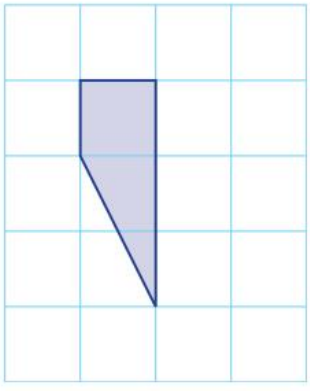


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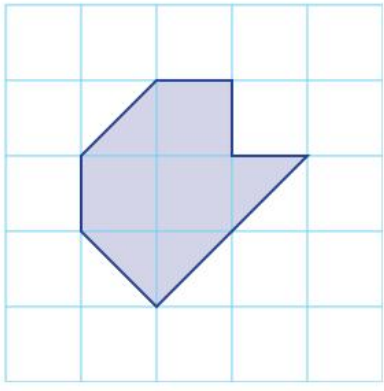


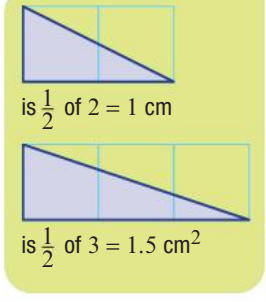
11C

e

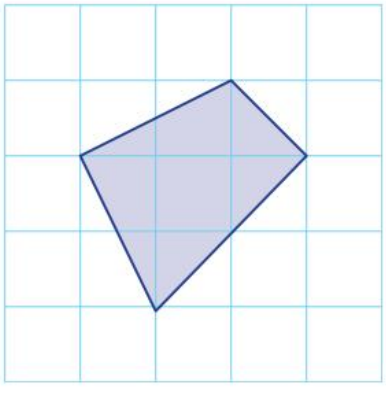


f

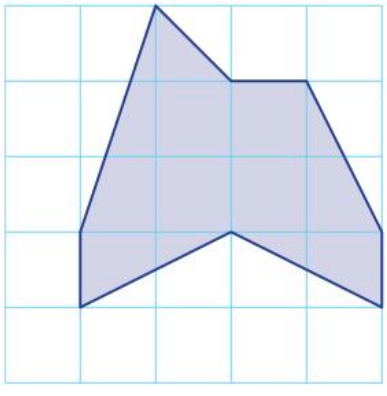




g



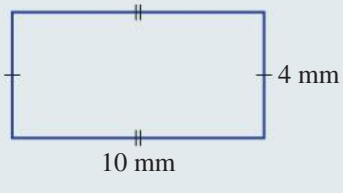
h



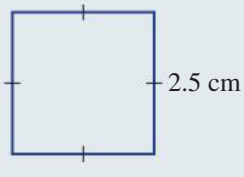
Example 9 Finding areas of rectangles and squares

Find the area of this rectangle and square.

a



b



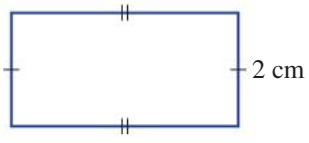
Solution	Explanation
a $A = l \times w$ $= 10 \times 4$ $= 40$ mm^2	Multiply the length and width to find the area of a rectangle.
b $A = l^2$ $= 2.5^2$ $= 6.25$ cm^2	The width is the same as the length, so: $A = l \times l = l^2$ $(2.5)^2 = 2.5 \times 2.5$

6 Find the area of these rectangles and squares. Diagrams are not drawn to scale.

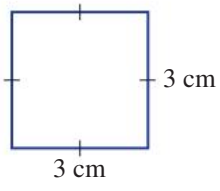


Multiply the length by the width to find the area.

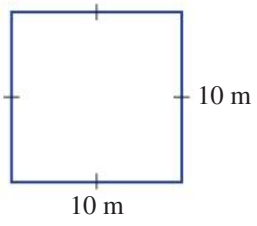
a



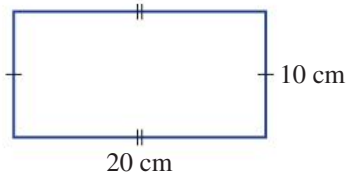
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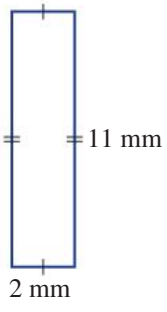
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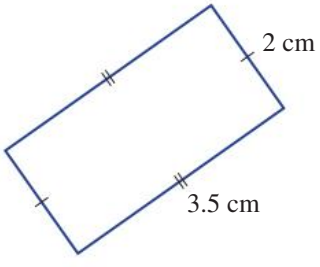
d



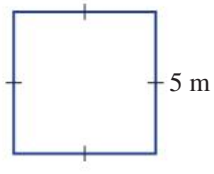
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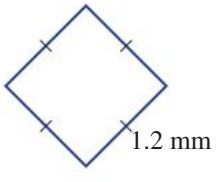
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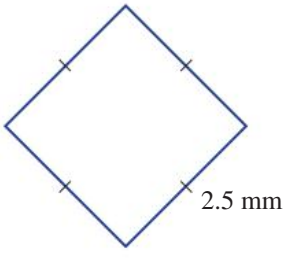
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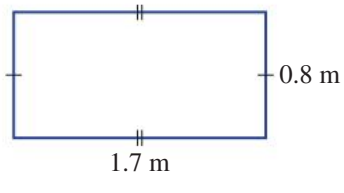
h



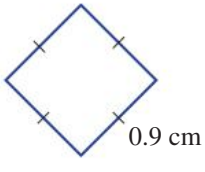
i



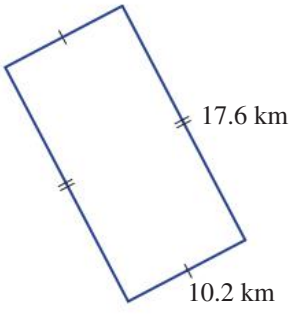
j



k



l



7 Which unit of area (mm^2, cm^2, m^2, ha or km^2) would you choose to measure these areas?

- **a** area of an A4 piece of paper
- **b** area of a wall of a house
- **c** area of a small farm
- **d** area of a large desert
- **e** area of a large football oval
- **f** area of a nail head



Remember, 1 km^2 is much larger than 1 ha.

11C

PROBLEM-SOLVING AND REASONING 8–10 10–13

8 A rectangular soccer field is to be laid with new grass. The field is 100 m long and 50 m wide. Find the area of grass to be laid.

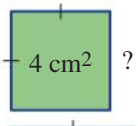
9 Glass is to be cut for a square window of side length 50 cm. Find the area of glass required for the window.



This is a glass cutter

10 Find the side length of each of these squares.

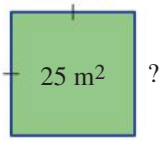
a



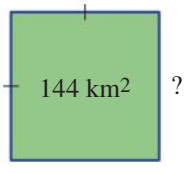


Use trial and error if you are unsure.

b



c





First try to work out the side length of each square.

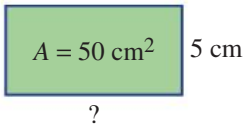
11 **a** A square has a perimeter of 20 cm. Find its area.

b A square has an area of 9 cm^2. Find its perimeter.

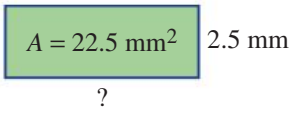
c A square's area and perimeter are the same number. How many units is the side length?

12 **a** Find the missing length for each of these rectangles.

i

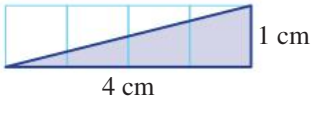


ii



b Explain the method that you used for finding the missing lengths of the rectangles above.

13 Explain why the area shaded here is exactly 2 cm^2.



Renovation work

— 14–16

14 Two hundred square tiles, each measuring 10 cm by 10 cm, are used to tile an open floor area. Find the area of flooring that is tiled.

15 The carpet chosen for a room costs $70 per square metre. The room is rectangular and is 6 m long by 5 m wide. What is the cost of carpeting the room?



16 Tania wants to paint a house wall that is 11 m long and 3 m high. Two coats of paint are needed. The paint suitable to do the job can be purchased only in whole numbers of litres and covers an area of 15 m^2 per litre. How many litres of paint will Tania need to purchase?

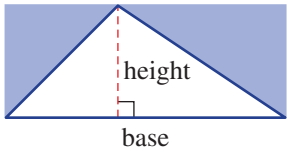
11D Area of a triangle

Interactive
Widgets
HOTsheets
Walkthrough

A triangle can be considered to be half a rectangle. This is why the formula for the area of a triangle looks very much like the formula for the area of a rectangle, but with a factor of $\frac{1}{2}$.

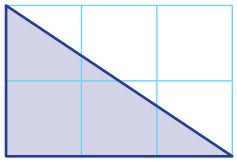
One of the sides of a triangle is called the base (b). The height (h) is the distance between the base and the opposite vertex, perpendicular (i.e. at 90°) to the base.

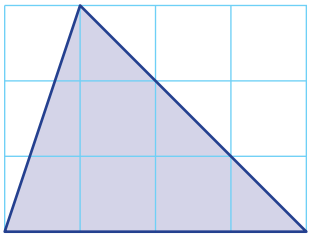




Let's start: Half a rectangle

Look at these triangles. For each one, discuss why the area could be considered as half a rectangle. Give reasons for each case.



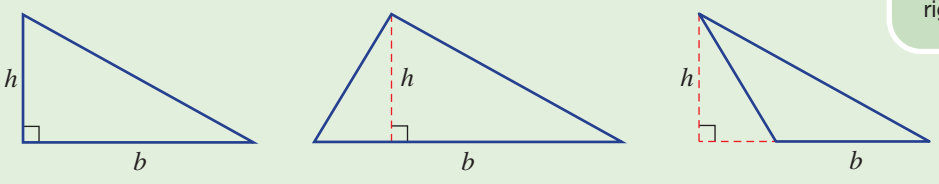


Key ideas

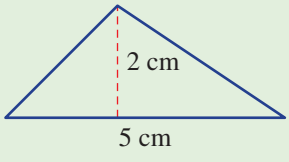
Base One side of a shape, at right angles to the height

Perpendicular At right angles

- One side of a triangle is called the **base**, b.
- The **perpendicular** distance from the base to the opposite vertex is called the height, h



- The area of a triangle is given by the formula:
  Area $= \frac{1}{2} \times$ base $\times$ height or $A = \frac{1}{2}bh$
  Note that $\frac{1}{2}bh$ is the same as $\frac{bh}{2}$.



$A = \frac{1}{2} \times 5 \times 2$

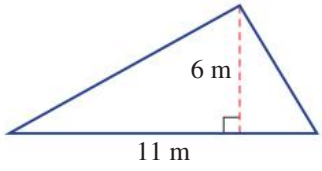
$= 5 \text{ cm}^2$

Exercise 11D

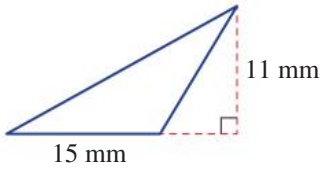
UNDERSTANDING 1–3 3

1 For each of these triangles, what length would be used as the height?

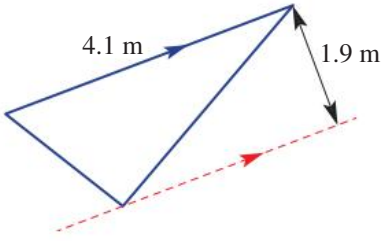
a



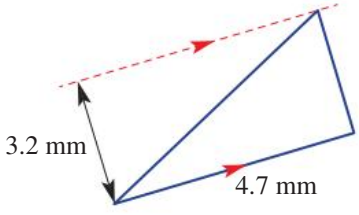
b



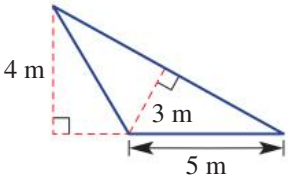
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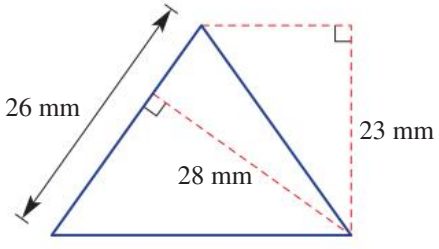
d



e

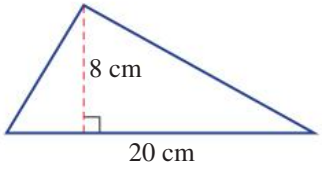


f

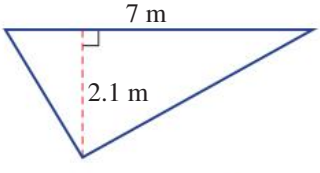


2 For each of these triangles, what length would be used as the base?

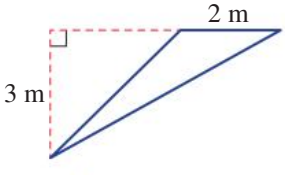
a



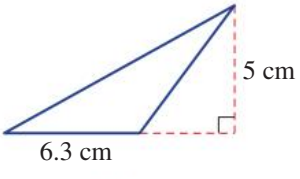
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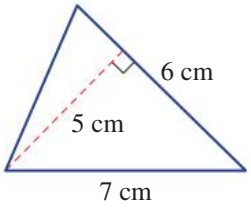
c



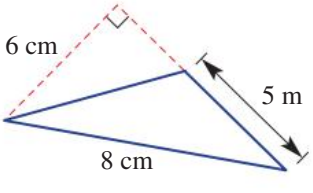
d



e



f



The base is one of the side lengths of the triangle.



3 Using the formula $A = \frac{1}{2}bh$, find the value of A if:

a $b = 5$ and $h = 4$
b $b = 7$ and $h = 16$
c $b = 2.5$ and $h = 10$
d $b = 1.5$ and $h = 7$

Substitute the values for b and h into the formula.

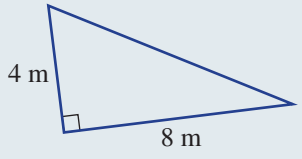


FLUENCY 4(½) 4–5(½)

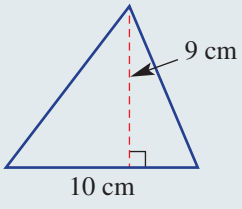
Example 10 Finding areas of triangles

Find the area of each of these triangles.

a



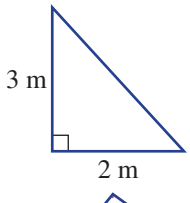
b



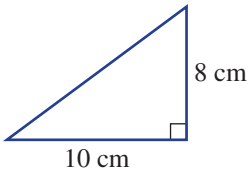
Solution	Explanation
a $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 8 \times 4$ $= 16 \text{ m}^2$	Use 8 m for the base and 4 m for the height.
b $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 9$ $= 45 \text{ cm}^2$	Use the formula and substitute the values for base length and height.

4 Find the area of each of these triangles.

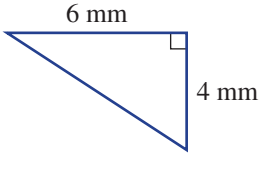
a



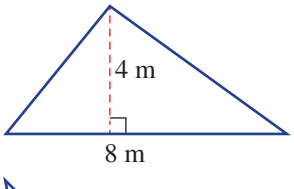
b



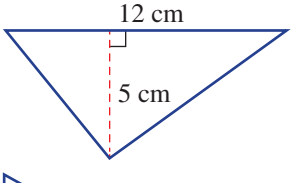
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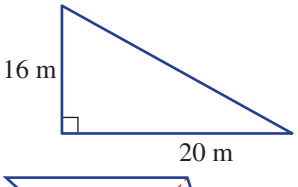
d



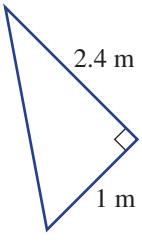
e



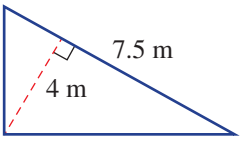
f



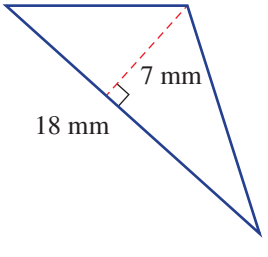
g



h



i

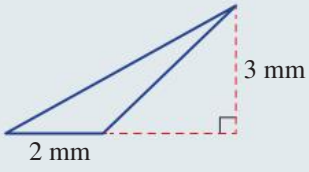


11D

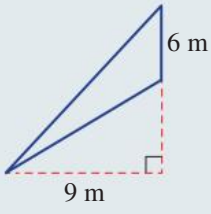
Example 11 Finding area of obtuse triangles

Find the area of each of these obtuse triangles.

a



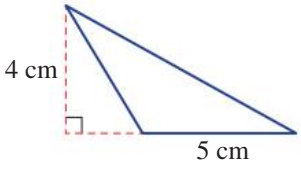
b



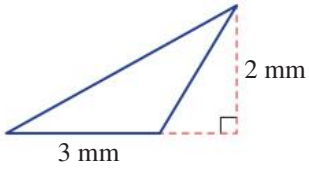
Solution	Explanation
a $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 2 \times 3$ $= 3 \text{ mm}^2$	The base length is 2 mm so use $b = 2$. The height is 3 mm so use $h = 3$.
b $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 9$ $= 27 \text{ m}^2$	The length measure of 9 m is marked at 90° to the side marked 6 m. So 6 m is the length of the base and 9 m is the perpendicular height.

5 Find the area of each triangle.

a



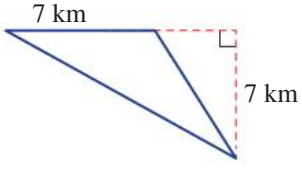
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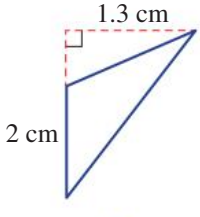


The base is the side with the given length.

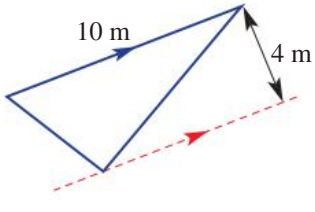
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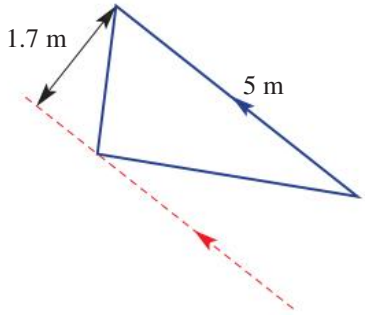
d



e



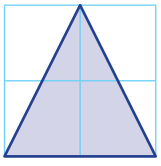
f



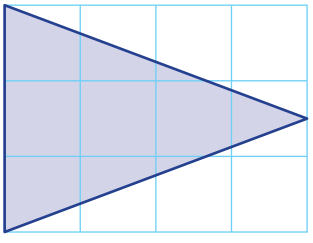
PROBLEM-SOLVING AND REASONING 6–8 8–11

6 Find the area of these triangles, which have been drawn on 1 cm grids. Give your answer in cm^2.

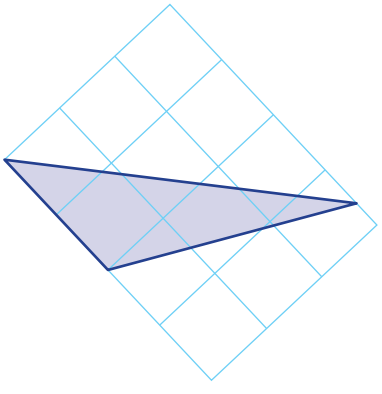
a



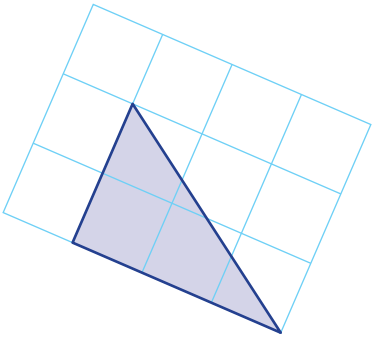
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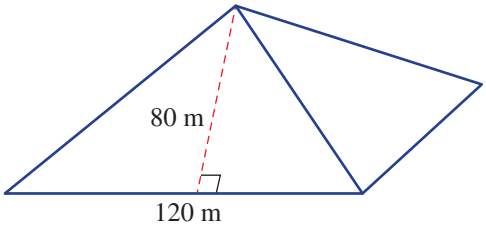
c



d



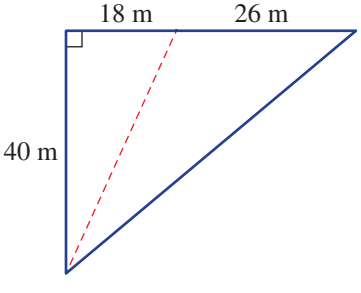
7 A square pyramid has a base length of 120 m and a triangular face of height 80 m. Find the area of one triangular face of the pyramid.





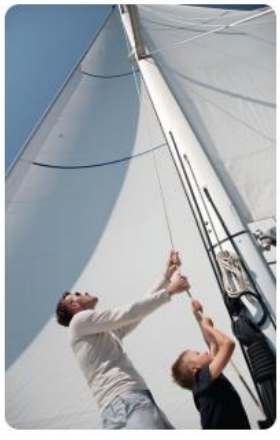
8 A rectangular block of land measuring 40 m long by 24 m wide is cut in half along a diagonal. Find the area of each triangular block of land.

9 A farmer uses fencing to divide a triangular piece of land into two smaller triangles, as shown. What is the difference in the two areas?

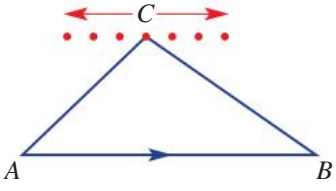


11D

10 A yacht must have two of its sails replaced as they have been damaged by a recent storm. One sail has a base length of 2.5 m and a height of 8 m. The bigger sail has a base length of 4 m and a height of 16 m. If the cost of sail material is $150 per square metre, find the total cost of replacing the yacht's damaged sails.



11 If the vertex C for this triangle moves left or right (to one of the red dots) will the area of the triangle change? Justify your answer.



Try building this in a dynamic geometry computer program.

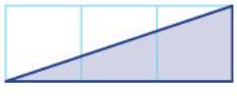


Estimating areas with curves

—

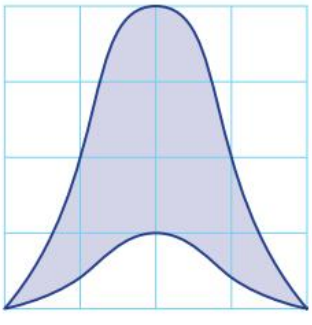
12

12 This diagram shows a shaded region that is $\frac{1}{2}$ of 3 cm^2 = 1.5 cm^2.

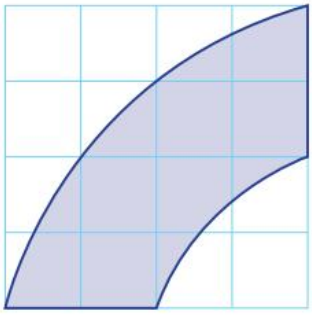


Using triangles like the one shown here, and by counting whole squares also, estimate the areas of these shapes below.

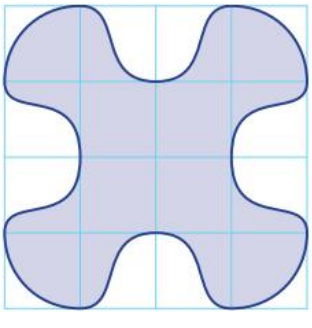
a



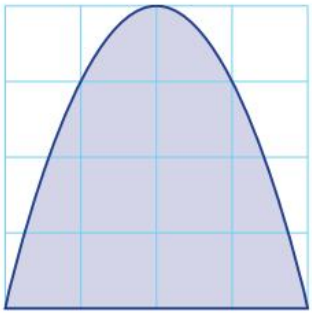
b



c



d



11E Area of a parallelogram

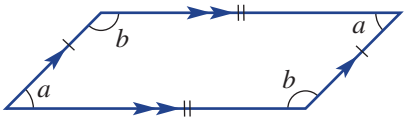
Interactive

Widgets

HOTsheets

Walkthrough

Recall that a parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are of the same length and opposite angles are equal.

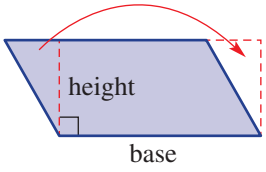


Like a triangle, the area of a parallelogram is found by using the length of one side (called the base) and the height (which is perpendicular to the base.)

Let's start: How is a parallelogram like a rectangle?

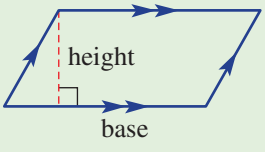
Draw and then cut out a large parallelogram. (Make sure that the opposite sides are parallel.) Label one side as the base and label the height, as shown in the diagram.

- Cut along the dotted line.
- Now shift the triangle to the other end of the parallelogram to make a rectangle.
- Now explain how to find the area of a parallelogram.

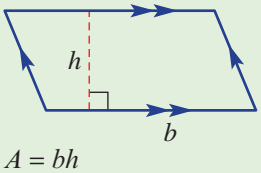


Key ideas

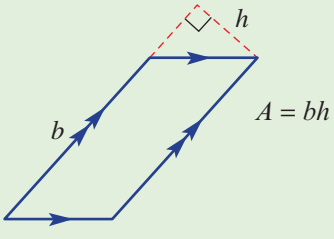
- The height of a parallelogram is perpendicular (at 90°) to the base.



- Area of a parallelogram = base × height or $A = bh$.



$A = bh$



Exercise 11E

UNDERSTANDING 1–3 3

1 Which of the following is the correct formula for the area of a parallelogram?

A $A = \frac{1}{2}bh$ **B** $A = \pi r^2$ **C** $A = bh$ **D** $A = b \div h$

2 Copy and complete the following, using the given values of b and h.

a $b = 5,\ h = 7$

$A = bh$

$= ____ \times ____$

$= 35$

b $b = 20,\ h = 3$

$A = ____$

$= 20 \times ____$

$= ____$

c $b = 8,\ h = 2.5$

$A = ____$

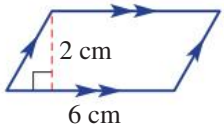
$= ____ \times ____$

$= ____$

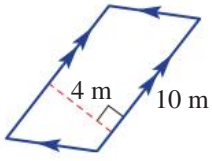
11E

3 For each of these parallelograms, state the base and the height that might be used to find the area.

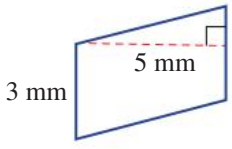
a



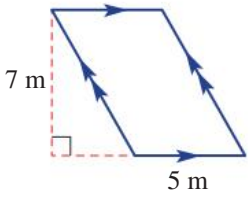
b



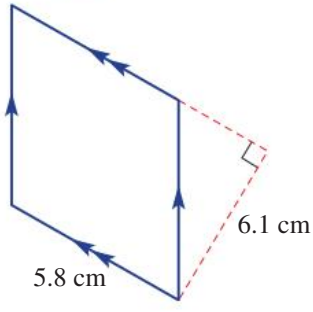
c



d



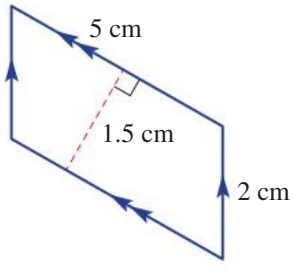
e



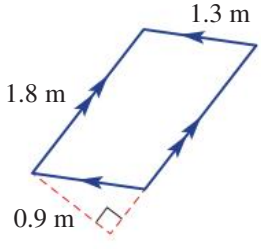


For parts **f** to **h**, choose the base (side) that is at 90° to the height.

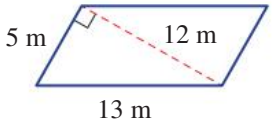
f



g



h

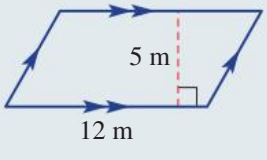


FLUENCY 4(½) 4–5(½)

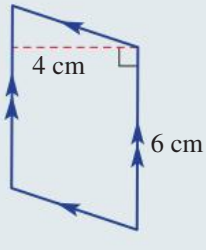
Example 12 Finding the area of a parallelogram

Find the area of these parallelograms.

a



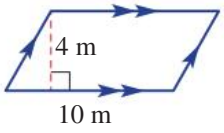
b



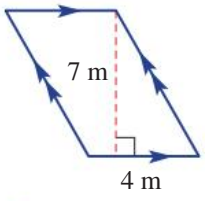
Solution	Explanation
a $A = bh$ $= 12 \times 5$ $= 60 \text{ m}^2$	Choose the given side as the base (12 m) and note the perpendicular height is 5 m.
b $A = bh$ $= 6 \times 4$ $= 24 \text{ cm}^2$	Use the base length 6 cm and the perpendicular height 4 cm.

4 Find the area of these parallelograms.

a



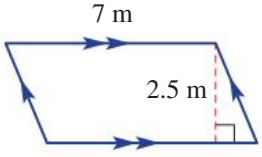
b



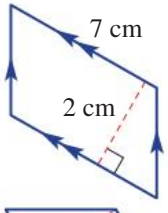


Use the formula $A = bh$, where b is the base and h is the height.

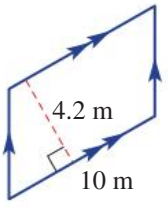
c



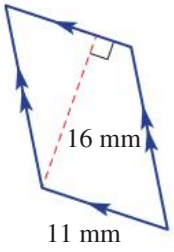
d



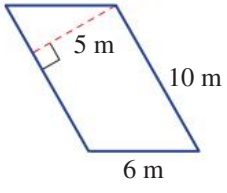
e



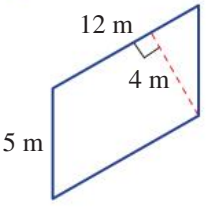
f



g



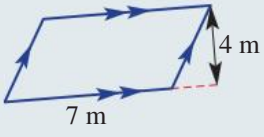
h



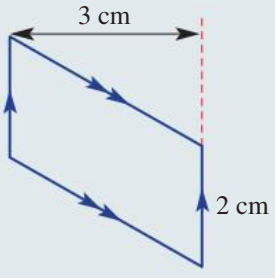
Example 13 Finding area with the height shown outside the shape

Find the area of these parallelograms.

a



b

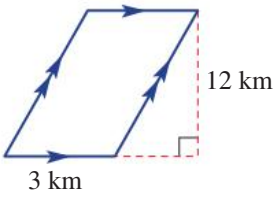


Solution	Explanation
a $A = bh$ $= 7 \times 4$ $= 28 \text{ m}^2$	Choose the base of 7 m and the height 4 m which is at 90° to the base.
b $A = bh$ $= 2 \times 3$ $= 6 \text{ cm}^2$	Use the given side as the base (2 cm), noting that the height is 3 cm.

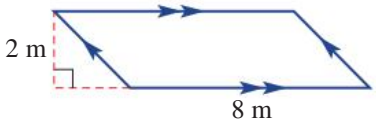
11E

5 Find the area of these parallelograms.

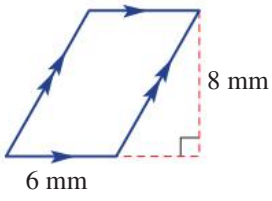
a



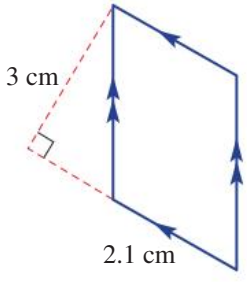
b



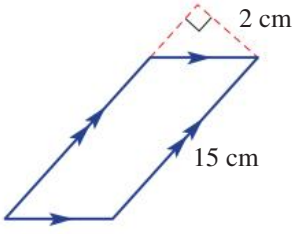
c



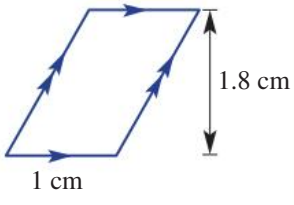
d



e

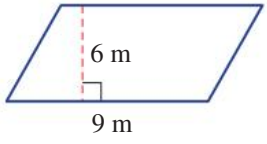


f



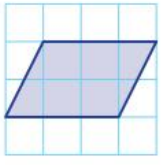
PROBLEM-SOLVING AND REASONING 6–9 9–12

6 The floor of an office space is in the shape of a parallelogram. The longest sides are 9 m and the distance between them is 6 m. Find the area of the office floor.

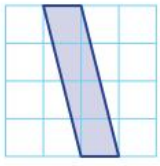


7 These parallelograms are on 1 cm grids (not to scale). Find each area.

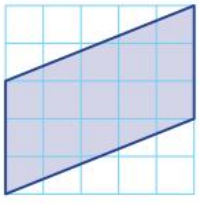
a



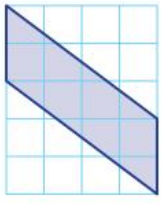
b



c



d



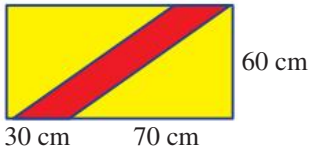


First identify the base and the height.

8 A proposed rectangular flag for a new country is yellow with a red stripe in the shape of a parallelogram, as shown. Find:

a the area of the red stripe

b the yellow area



9 Find the height of a parallelogram when its:

a area = 10 m^2 and base = 5 m

b area = 28 cm^2 and base = 4 cm



Use $A = bh$ and substitute the given information.

10 Find the base of a parallelogram when its:

a area = 40 cm^2 and height = 4 cm

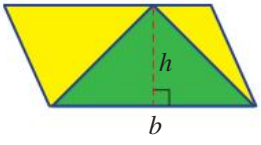
b area = 150 m^2 and height = 30 m

11 A large wall in the shape of a parallelogram is to be painted with a special red paint, which costs $20 per litre. Each litre of paint covers 5 m^2. The wall has a base length of 30 m and a height of 10 m. Find the cost of painting the wall.



First work out the area of the wall. Then find the number of litres of paint needed.

12 A parallelogram includes a green triangular area, as shown. What fraction of the total area is the green area? Give reasons for your answer.





Compare the area formulas for a triangle and a parallelogram.

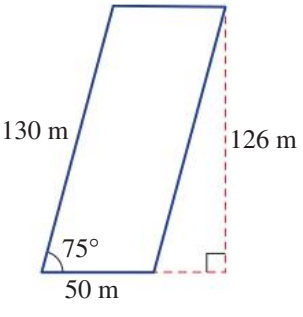
Glass façade — 13

13 The Puerta de Europa (Gate of Europe) towers are twin office buildings in Madrid, Spain. They look like normal rectangular glass-covered skyscrapers but they lean towards each other at an angle of about 15° to the vertical.



For each building:

- The height of the building is 126 metres
- The base is a square with 50 m sides.
- The two vertical walls (i.e. the front and the back) are parallelograms, as shown in the diagram.
- The two slanting walls are rectangles.
- All four walls are covered with glass.



Answer the following for *one* of the towers.

a Find the area of one of the sloping (rectangular) walls.

b Find the area of one of the sides that is vertical (a parallelogram).

c Calculate the total area of all four sides of the tower.

d If glass costs $180 per square metre, find the cost of covering the tower with glass. (Assume that glass covers the entire surface, ignoring the beams.)

Progress quiz

11A **1** Convert to the units given in brackets.

a 5 m (cm)
b 4.2 m (mm)
c 62.1 km (m)
d 14.7 cm (mm)

11A **2** Convert to the units given in brackets.

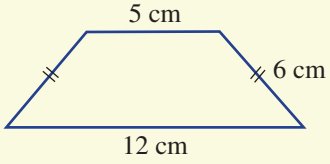
a 72 mm (cm)
b 5.1 cm (m)
c 4132 mm (m)
d 35 000 m (km)

11A **3** Add these lengths and give the result in the units shown in brackets.

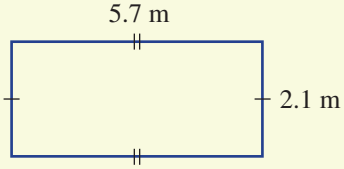
a 3 cm and 8 mm (mm)
b 6.1 m and 72 cm (m)
c 4.5 km + 1200 m + 800 cm (m)

11B **4** Find the perimeter of each of these shapes.

a

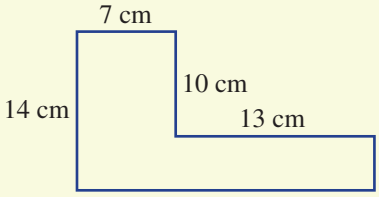


b

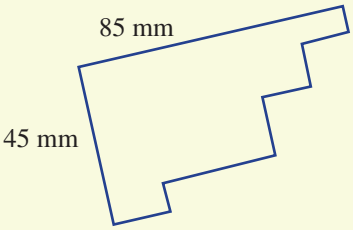


11B **5** Find the perimeter of each of these L-shapes. All corner angles are 90°.

a



b



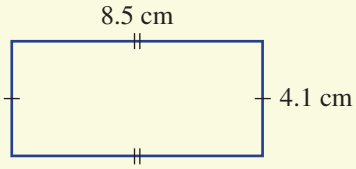
11B **6** A playground is to be fenced on all sides. It is rectangular in shape, with a length of 35 m and a width of 19 m. If fencing costs $120 per metre, find the cost of fencing required.



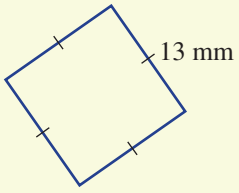
11C

7 Find the area of these rectangles and squares. Diagrams are not drawn to scale.

a



b



11C

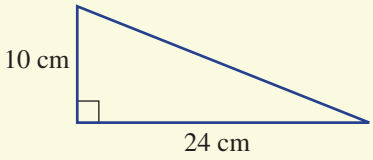
8 **a** A square has a perimeter of 40 cm. Find its area.

b A square has an area of 25 cm^2. Find its perimeter.

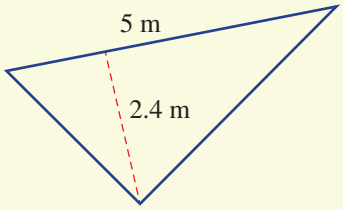
11D

9 Find the area of each triangle given.

a



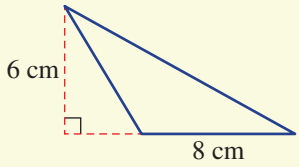
b



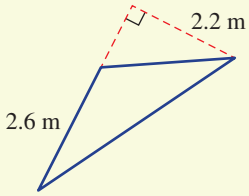
11D

10 Find the area of each of these obtuse triangles.

a



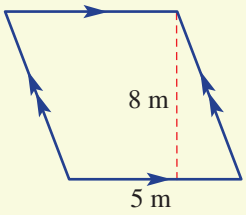
b



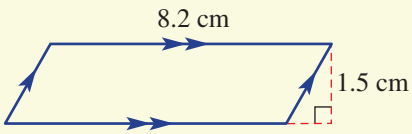
11E

11 Find the area of these parallelograms.

a



b



11F Volume of rectangular prisms

Interactive
Widgets
HOTsheets
Walkthrough

The amount of space inside a three-dimensional (3D) object is called volume. Volume is measured in cubic units such as the cubic centimetre, which is 1 cm long, 1 cm wide and 1 cm high. Just as with length and area, different units can be selected, depending on what is being measured. For example, the volume of water in the sea could be measured in cubic kilometres and the volume of a matchbox could be measured in cubic centimetres.

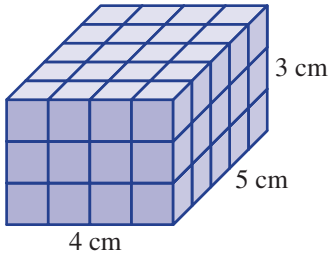


The volume of concrete inside a cement mixer is measured in cubic metres.

Let's start: The quick method

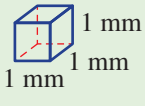
This rectangular prism is made up of small blocks (1 cm^3 cubes). The prism is 4 cm wide, 5 cm long and 3 cm high.

- How many 1 cm^3 cubes are there in one horizontal layer? Explain how you worked this out.
- How many 1 cm^3 cubes are there in total?
- What is the quickest way to find the total number of cubes (i.e. the volume)?



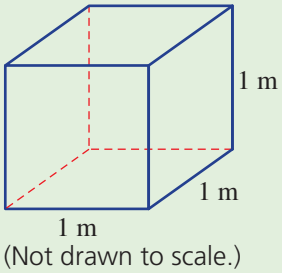
Key ideas

- **Volume** is measured in cubic units.
- The common metric units for volume include:
  - cubic millimetres (mm^3)



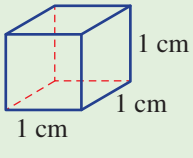
(Not drawn to scale.)

  - cubic metre (m^3)

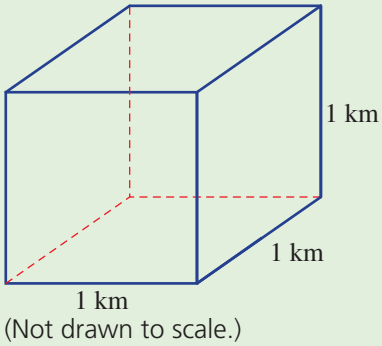


(Not drawn to scale.)

  - cubic centimetre (cm^3)



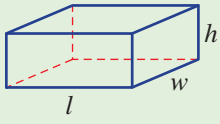
  - cubic kilometre (km^3)



(Not drawn to scale.)

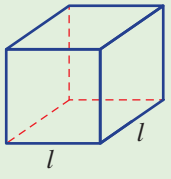
Volume The amount of three-dimensional space in (or occupied by) an object

- The volume of a rectangular prism is given by the formula:
  Volume = length × width × height or $V = lwh$
  - A rectangular prism is also called a cuboid.



$V = lwh$

- The volume of a cube is given by:
  $V = l \times l \times l$
  $= l^3$



$V = l^3$

Exercise 11F

UNDERSTANDING 1, 2 3–4

1 Which of the following is a unit of volume?

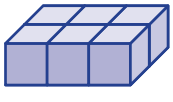
A 3 cm^2
B 5 m^3
C 2 mm
D 7.2 km

2 Which of the following is *not* a unit of volume?

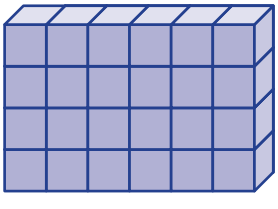
A 7 mm^3
B 6 m^3
C 5 cm^3
D 10 km^2

3 For each of these solids, count the number of cubic units to find the volume.

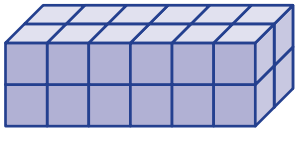
a



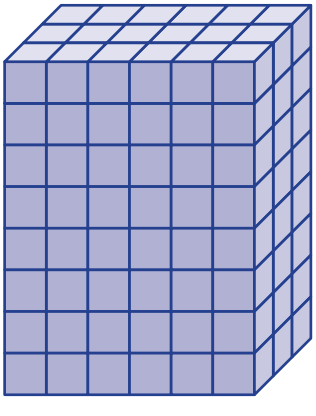
b



c



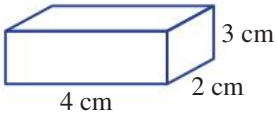
d



11F

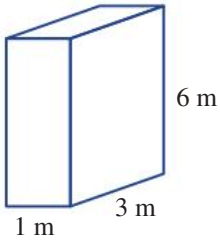
4 Copy and complete the working shown for each of these solids.

a



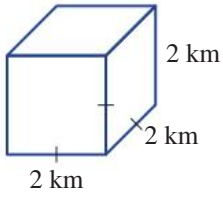
$V = lwh$
$= 4 \times ____ \times ____$
$= ____ \text{ cm}^3$

b



$V = lwh$
$= 1 \times ____ \times ____$
$= 18 ____$

c

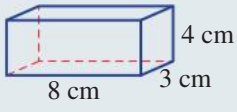


$V = l^3$
$= 2 \times ____ \times ____$
$= ____ \text{ km}^3$

FLUENCY 5, 6 5(½), 6, 7

Example 14 Finding the volume of a rectangular prism

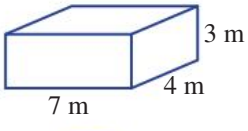
Find the volume of this rectangular prism.



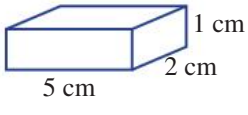
Solution	Explanation
$V = lwh$ $= 8 \times 3 \times 4$ $= 96 \text{ cm}^3$	Use the formula for the volume of a rectangular prism. Substitute the three lengths into the formula. Use the correct units (cm^3).

5 Find the volume of these rectangular prisms.

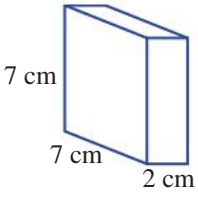
a



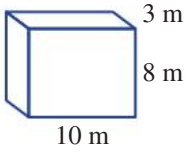
b



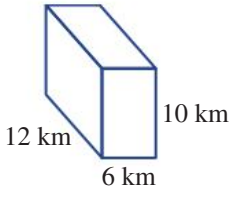
c



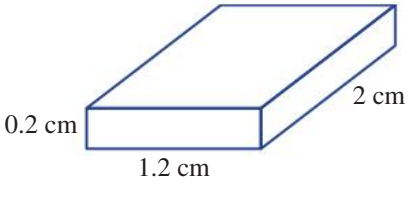
d



e



f

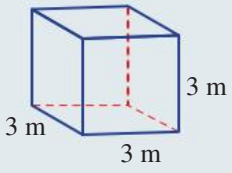




For part **c**, you can use $l = 7$ and $w = 2$ or $l = 2$ and $w = 7$. (If you turn a rectangular prism around, its volume doesn't change.)

Example 15 Finding the volume of a cube

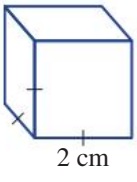
Find the volume of a cube with side length 3 m.



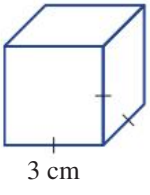
Solution	Explanation
$V = l^3$ $= 3 \times 3 \times 3$ $= 27\ \text{m}^3$	Use the rule $V = l^3$ or $V = l \times w \times h$ and substitute 3 for each side.

6 Find the volume of these rectangular cubes.

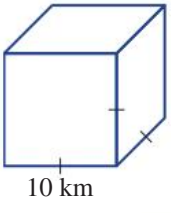
a



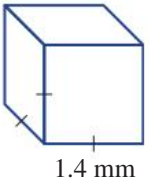
b



c

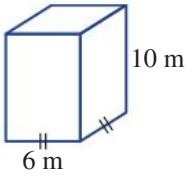


d

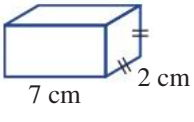


7 Find the volume of these prisms.

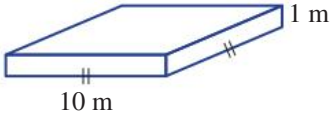
a



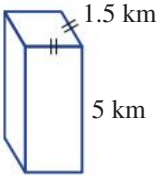
b



c



d





$V = l^3$ is the same as $V = l \times l \times l$.



Edges with the same markings are equal in length.

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 A shipping container is 3 m wide, 4 m high and 8 m long. Find its volume.



9 A fruit box is 40 cm long, 30 cm wide and 20 cm high. Find its volume.

11F

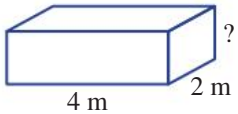
10 There is enough ice on Earth to fill a cube of side length 300 km. Find the approximate volume of ice on Earth.



11 A box measuring 30 cm long, 20 cm high and 30 cm wide is packed with matchboxes, each 5 cm long, 2 cm high and 3 cm wide. How many matchboxes will fit in the box?

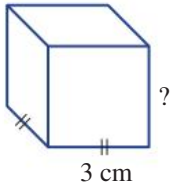
12 **a** Find the height of these rectangular prisms with the given volumes. Use trial and error if you wish.

i

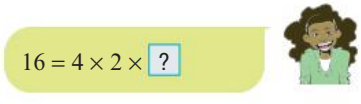


$V = 16 \text{ m}^3$

ii



$V = 45 \text{ cm}^3$



b Can you explain a method that always works for finding the height of a rectangular prism?

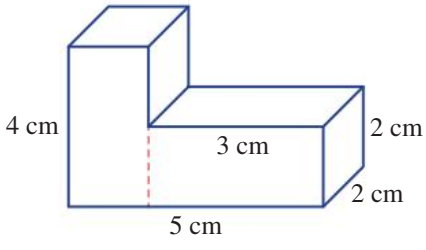
c Use V, l and w to write a rule for h.

Complex solids

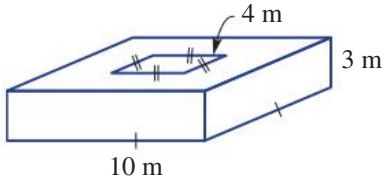
13

13 These solids are made up of more than one rectangular prism. Use addition or subtraction to find the total volume of the solid.

a



b



11G Capacity

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Capacity relates to the volume of fluid or gas that a container can hold. For example, the capacity of a water tank may be 5000 litres or a farmer's water allocation might be 300 megalitres (300 million litres).

The basic unit of capacity is the litre, which fills 1000 cm^3 of space. Other common metric units include the millilitre, kilolitre and megalitre.



Irrigation channels carry many megalitres of water to farms.

Let's start: Who has the most milk?

Megan, Rajesh and Toby each buy milk from their local store.

- Megan buys 1 litre.
- Rajesh buys 1000 cm^3.
- Toby buys 1000 millilitres.

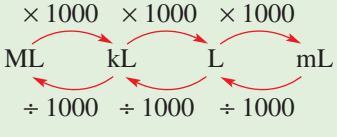
Who bought the most milk? How do these volumes compare?

Key ideas

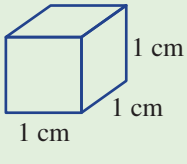
- **Capacity** is the volume of fluid or gas that an object can hold.

Capacity How much liquid or gas a container can hold

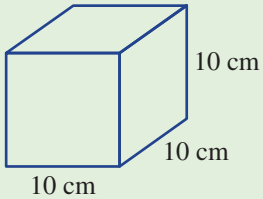
- Common metric units include:
  - 1 litre (L) = 1000 millilitres (mL)
  - 1 kilolitre (kL) = 1000 litres (L)
  - 1 megalitre (ML) = 1000 kilolitres (kL)



- Linking volume and capacity
  - 1 cm^3 = 1 mL

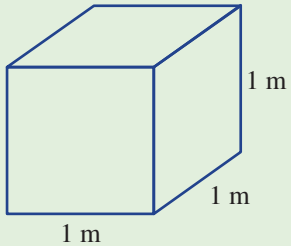


  - 1000 cm^3 = 1000 mL = 1 L



(Not drawn to scale.)

  - 1 m^3 = 1000 L = 1 kL



(Not drawn to scale.)

Exercise 11G

UNDERSTANDING 1–3 | 3

1 Complete these sentences.

a The volume of fluid or gas that an object can hold is called its _____.

b 1 mL contains a volume of _____ cm^3.

c 1 L contains _____ mL.

d 1 L contains _____ cm^3.

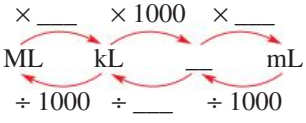
e 1 kL contains _____ L.

f 1 ML contains _____ kL.



Refer back to the **Key ideas** for help.

2 Copy and complete this flow chart.



3 Write the full name for these capacity units.

a L **b** ML **c** mL

d kL **e** cm^3 **f** m^3



Choose from: *cubic centimetre*, *cubic metre*, *kilolitre*, *litre*, *megalitre*, *millilitre*

FLUENCY 4(½), 5, 6 | 4(½), 5, 6, 7(½)

Example 16 Converting units for capacity

Convert to the units shown in brackets.

a 500 mL (L) **b** 3 kL (L)

Solution	Explanation
a 500 mL = (500 ÷ 1000) L = 0.5 L	When converting to a larger unit, divide. There are 1000 mL in 1 L.
b 3 kL = (3 × 1000) L = 3000 L	There are 1000 L in 1 kL, so multiply by 1000.

4 Convert to the units shown in brackets.

a 400 mL (L) **b** 700 mL (L) **c** 2000 L (kL)

d 36 000 L (kL) **e** 4000 kL (ML) **f** 500 kL (ML)

g 2 L (mL) **h** 0.1 L (mL) **i** 6 ML (kL)

j 3 ML (kL) **k** 24 kL (L) **l** 38 kL (L)

m 2000 L (kL) **n** 3500 mL (L) **o** 70 000 mL (L)

p 2500 kL (ML) **q** 0.257 L (mL) **r** 9320 mL (L)

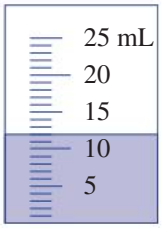
s 3.847 ML (kL) **t** 47 000 L (kL) **u** 5800 kL (ML)



Divide by 1000 to convert: mL to L, L to kL, kL to ML.
Multiply by 1000 to convert: ML to kL, kL to L, L to mL.

5 Read these scales to determine the amount of water in each of the containers.

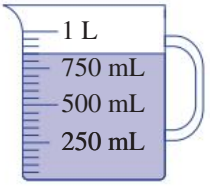
a



b



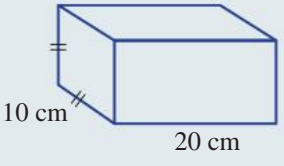
c



Example 17 Finding volumes of rectangular prisms in litres

Find the capacity of this container in:

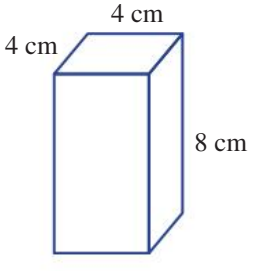
a cm^3
b mL
c litres



Solution	Explanation
a $V = l \times w \times h$ $= 20 \times 10 \times 10$ $= 2000\ cm^3$	$V = lwh$
b $V = 2000$ mL	$1\ cm^3 = 1$ mL, so $2000\ cm^3 = 2000$ mL.
c $V = (2000 \div 1000)$ L $= 2$ L	There are 1000 mL in 1 litre, so divide by 1000.

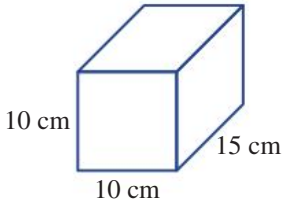
6 Find the capacity of this container in:

a cm^3
b mL
c litres

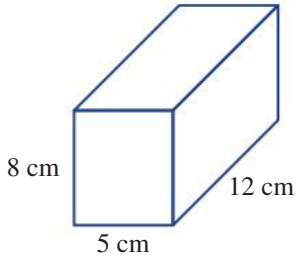


7 Find the capacity of each of these containers, in litres.

a



b

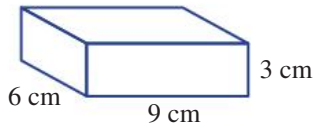




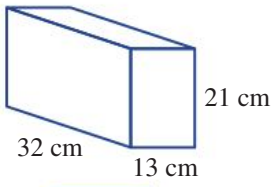
First find the volume in cm^3.
Convert to mL:
$1\ cm^3 = 1$ mL
Then convert to L:
1000 mL = 1 L

11G

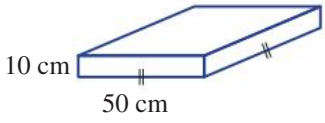
c



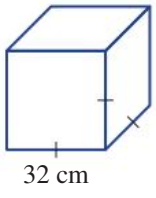
d



e



f



PROBLEM-SOLVING AND REASONING 8–10 10–13

8 Circle or write down which volumes are the same.

a 1 L, 10 kL, 1000 mL, 1 m^3, 1000 cm^3

b 1 m^3, 100 L, 1000 L, 1000 ML, 1 kL

9 Choose the capacity (**A** to **F**) that best matches the container (**a** to **f**).

a teaspoon	**A** 18 L
b cup	**B** 250 mL
c bottle	**C** 10 kL
d kitchen sink	**D** 20 mL
e water tank	**E** 45 ML
f water in a lake	**F** 0.8 L

10 A cup of 200 mL of water is added to a jug already containing 1 L of water. Find the total volume in:

a mL **b** L

11 Convert to the units shown in brackets.

a 6 ML (L) **b** 320 000 L (ML)

c 0.004 kL (mL) **d** 992 700 mL (kL)



You will need to multiply or divide by 1000 twice each time.

12 A farmer purchases 3.3 ML of water for her apple orchard. How many litres is this?



13 A dose of 12 mL of medicine is to be taken twice each day from a 0.36 L bottle. How many days will it take to finish the medicine?





Remember, there are 1000 mL in 1 L.

Water issues

— 14–15

14 A swimming pool in the shape of a rectangular prism has length 50 m, width 25 m and depth 2 m. Find the swimming pool's:

a volume, in m^3

b capacity, in L



15 A dripping tap leaks about 10 mL every minute.

a If there are 50 drips per minute, find the volume of one drip.

b Find the approximate volume of water, in litres, that has leaked from a tap after:

i 100 minutes

ii 1 hour

iii 1 day

iv 1 year

11H Mass and temperature

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Mass relates to how heavy an object is. We use the metric units milligrams, grams, kilograms and tonnes to measure mass. The mass of a large elephant is about 4000 kg or 4 tonnes, while the mass of an ant is about 2 milligrams.

Temperature tells us how hot or cold something is. Anders Celsius (1701–1744), a Swedish scientist, worked to define a scale for temperature. After his death, temperature was officially defined by:

- 0°C (0 degrees Celsius) – the freezing point of water.
- 100°C (100 degrees Celsius) – the boiling point of water.







Let's start: Matching a mass or temperature

Work with a partner or group.

Name an object, place or situation to match each of these temperatures.

- 0°C
- 20°C
- 50°C
- 100°C
- 250°C
- −10°C

Name an object whose mass would be measured using:

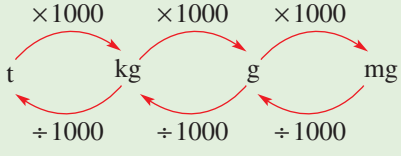
- tonnes
- kilograms
- grams
- milligrams

Key ideas

Kilogram Unit of metric measurement for mass

Celsius Scale used for measuring temperature, where water freezes at 0° and boils at 100°

- The basic unit for mass is the **kilogram** (kg).
  - 1 litre of water has a mass that is very close to 1 kilogram.
- Metric units for mass include:
  - 1 gram (g) = 1000 milligrams (mg)
  - 1 kilogram (kg) = 1000 grams (g)
  - 1 tonne (t) = 1000 kilograms (kg)



- The common unit for temperature is degrees **Celsius** (°C).
  - 0°C is the freezing point of water.
  - 100°C is the boiling point of water.

Exercise 11H

UNDERSTANDING 1–3 | 3

1 Write the missing word or number in these sentences.

a There are _____ grams in 1 kilogram.
b There are 1000 _____ in 1 gram.
c There are 1000 kilograms in 1 _____.
d Water boils at _____ °C.
e Water freezes at _____ °C.

2 Choose the mass (**A** to **F**) that best matches the given object (**a** to **f**).

a human hair	**A** 300 g
b 10-cent coin	**B** 40 kg
c bottle	**C** 100 mg
d large book	**D** 1.5 kg
e large bag of sand	**E** 13 t
f truck	**F** 5 g



mg is milligrams
g is grams
kg is kilograms
t is tonnes

3 Choose the temperature (**A** to **D**) that best matches the description (**a** to **d**).

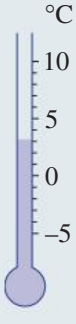
a temperature of coffee	**A** 15°C
b temperature of tap water	**B** 50°C
c temperature of oven	**C** −20°C
d temperature in Antarctica	**D** 250°C



FLUENCY 4, 5, 6(½) | 4, 5, 6–7(½)

Example 18 Reading scales

Read this temperature scale.

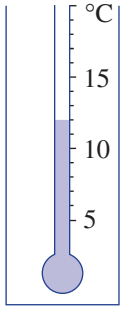


Solution	Explanation
3°C	The markings each represent 1°C and the temperature is 3 markings above zero.

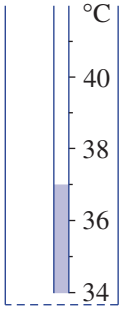
11H

4 Read these temperature scales.

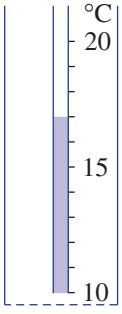
a



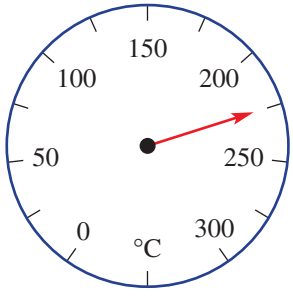
b



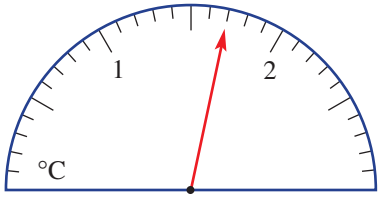
c



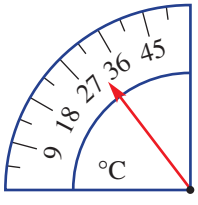
d



e

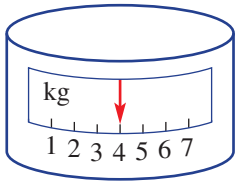


f

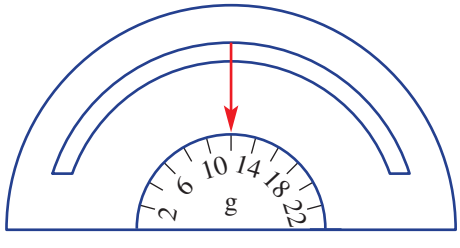


5 Read these mass scales.

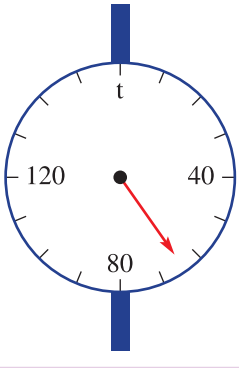
a



b



c



Example 19 Converting units of mass

Convert to the units shown in brackets.

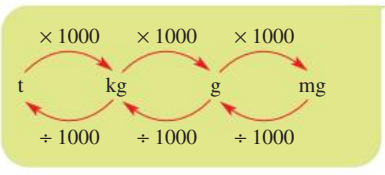
a 2.4 kg (g)

b 170 000 kg (t)

Solution	Explanation
a 2.4 kg = (2.4 × 1000) g = 2400 g	1 kg = 1000 g Multiply because you are changing to a smaller unit.
b 170 000 kg = (170 000 ÷ 1000) t = 170 t	1 t = 1000 kg Divide because you are changing to a larger unit.

6 Convert to the units shown in the brackets.

a 2 kg (g)	**b** 7 kg (g)	**c** 6.2 kg (g)
d 5.8 kg (g)	**e** 6000 g (kg)	**f** 8900 g (kg)
g 900 g (kg)	**h** 450 g (kg)	**i** 5 t (kg)
j 0.6 t (kg)	**k** 2400 kg (t)	**l** 4320 kg (t)
m 3 g (mg)	**n** 4.2 g (mg)	**o** 7500 mg (g)





7 Convert to the units shown in brackets.

a 4620 mg (g)	**b** 21 600 kg (t)	**c** 0.47 t (kg)	**d** 312 g (kg)	**e** 27 mg (g)
f $\frac{3}{4}$ t (kg)	**g** $\frac{1}{8}$ kg (g)	**h** 10.5 g (kg)	**i** 210 000 kg (t)	**j** 0.47 t (kg)
k 592 000 mg (g)	**l** 0.08 kg (g)			

PROBLEM-SOLVING AND REASONING 8–11 11–14

8 The temperature of water in a cup of tea is initially 95°C. After half an hour the temperature is 62°C. How much did the temperature drop?

9 An oven is initially at a room temperature of 25°C. The oven dial is turned to 172°C. What is the expected increase in temperature?



10 A small truck delivers 0.06 t of stone for a garden. Write the mass of stones using these units.

a kg **b** g **c** mg

11 A box contains 20 blocks of cheese, each weighing 150 g. Find the total mass of cheese in the following units.

a g **b** kg



12 Add all the mass measurements and give the result in kg.

a 3 kg, 4000 g, 0.001 t **b** 2.7 kg, 430 g, 930 000 mg, 0.0041 t

13 Arrange these mass measurements from smallest to largest.

a 2.5 kg, 370 g, 0.1 t, 400 mg **b** 0.000 32 t, 0.41 kg, 710 g, 290 000 mg



14 A 10 kg bag of flour is used at a rate of 200 g per day. How many days will the bag of flour last?

Weighing water — 15–16

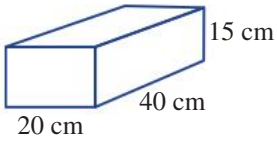
15 Assume that water weighs exactly 1 kg per litre. What is the mass of these volumes of water?

a 1 mL **b** 1 kL **c** 1 ML

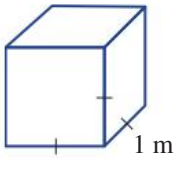


16 Suppose that the containers below are filled with water. Calculate the mass of water in each container, in kg.

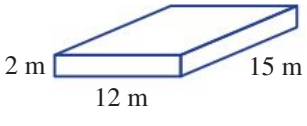
a



b



c



Maths@Work: Landscape gardener

Landscape gardening is a job suited to physically fit people who love the outdoors, plants and design. A friendly, cheerful manner and good communication skills are all important when working with clients.

Landscape gardeners calculate perimeters, areas and volumes to work out costs and order materials. The geometry of angles and symmetry is applied when designing gardens.

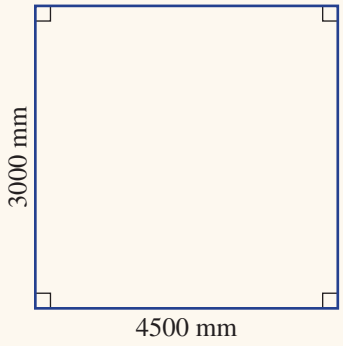


1 For the following garden plots:

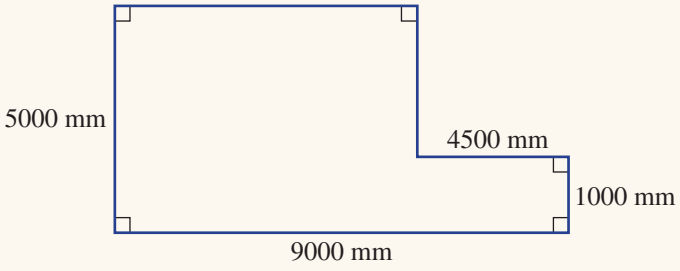
i calculate the perimeter (in metres) and area (in square metres) to 1 decimal place.

ii calculate the cost of a layer of garden mix soil at $68.50 per 10 square metres.

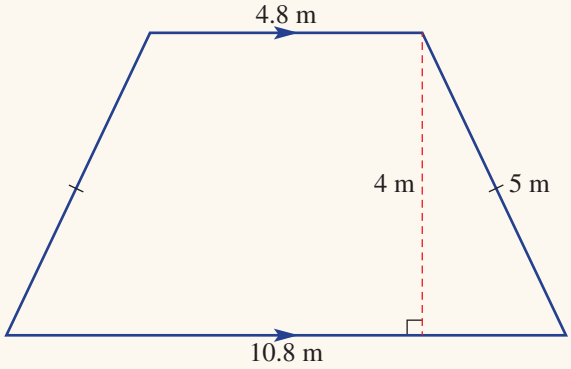
a



b



c



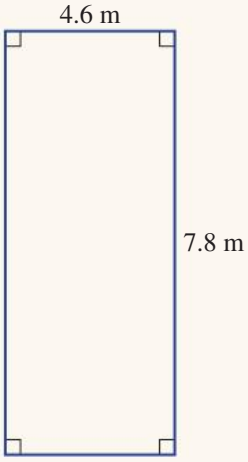
2 Each of the areas below are to be covered with turf (i.e. lawn grass). For each lawn:

- **i** calculate the area in square metres to 2 decimal places.
- **ii** calculate the cost for Aussie Blue Couch turf at $8.75 per square metre.

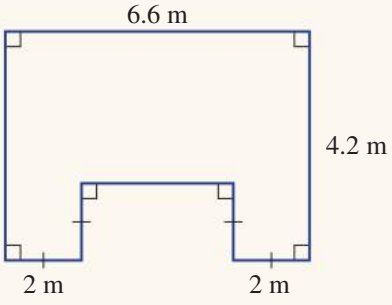


Turf is supplied in rolls that are unrolled to make an instant lawn.

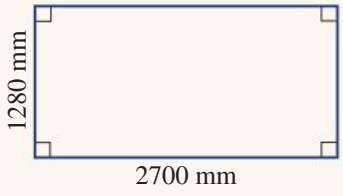
a



b



c



3 Turf is sold in full rolls. Each roll of Wintergreen Couch turf is a rectangle measuring 170 cm by 60 cm when rolled out. Determine the whole number of turf rolls needed to cover a rectangular lawn of length 6 m and width 3 m.



- Work in the same units.
- Sketch a diagram.
- Calculate multiples of turf lengths and turf widths to cover the lawn.

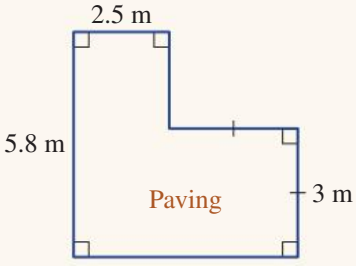
4 Using these prices, calculate the cost of purchasing the materials needed and the total cost for each landscape project (**a** and **b**) shown overpage.

Item	Selling unit	Price
Bedding sand to a depth of 80 mm	Volume	$\$96.40/m^3$
Pavers	Area	$\$66.00/m^2$
Sir Walter Buffalo turf	Area	$\$10.24/m^2$
Top soil over the lawn to a depth of 5 cm	Volume	$\$135.70/m^3$
Mulch to a depth of 100 mm	Volume	$\$106.28/m^3$
Edging – used around the mulch area	Length	$\$19.96/m$



Pavers are laid on a flat layer of bedding sand.

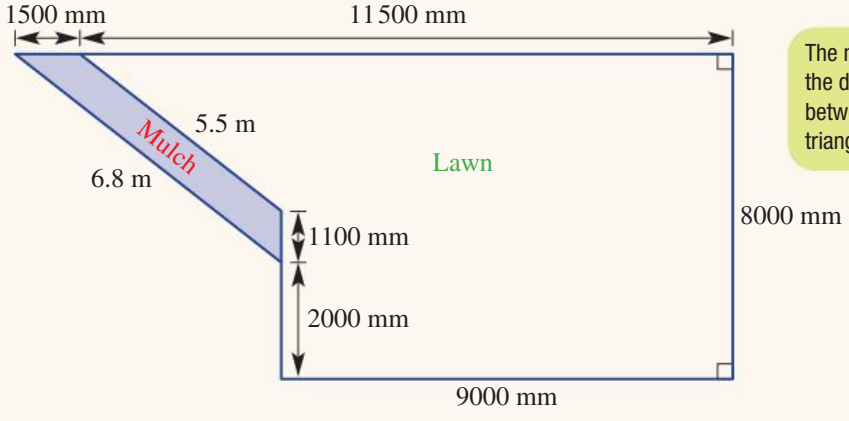
a **Backyard BBQ area**
Bedding sand and pavers required.





Bedding sand is laid under paving.

b **Front yard**
Turf, top soil, mulch and edging around the mulch area required.





The mulch area is the difference in area between two overlapping triangles.

Using technology

Technology is a landscape gardener's 'friend'. Entering data and formulas into spreadsheets helps with planning, ordering and cost calculations and also keeps a record of projects.

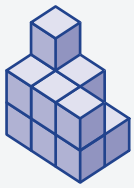
5 Set up this spreadsheet and enter the required formulas. Follow the client's instructions below to calculate relevant quantities. Format cost cells to currency/2 d.p.

	A	B	C	D	E
1	Better Landscaping Company				
2	Client: Jack and Jill Green				
3	Sections	Length in m	Width in m	Area in m^2	Perimeter in m
4	Front lawn	8	5.1		
5	Back garden	5.8	2.2		
6	PLANTS	Number	Price for one	Total cost	
7	Trees	2	$36.00		
8	Shrubs	5	$11.50		
9	Punnets of vegetables	6	$4.55		
10	MATERIALS	Quantity	Cost per unit	Total cost	
11	Legend Couch turf in m^2		$10.75		
12	Garden edge blocks in m		$14.80		
13	Garden soil in m^3		$65.00		
14			Overall cost		

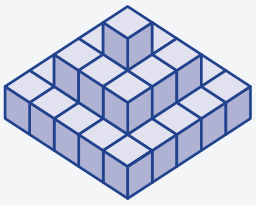
Client's instructions
- Turf laid over the front lawn.
- Edge blocks border the perimeter of the back garden.
- Garden soil to the depth of 120 mm spread over the garden area.

1 How many cubes would you need to make each of these solids?

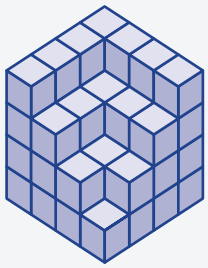
a



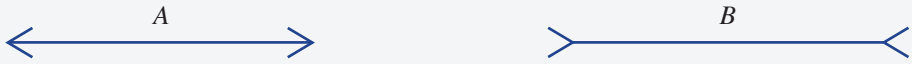
b



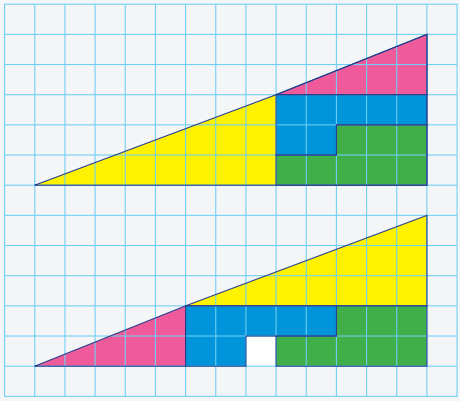
c



2 Without measuring, state which line looks longer: A or B? Then measure to check your answer.

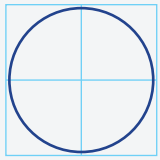


3 Do these two shapes to the right have the same area? Explain the 'hole' in the second shape.

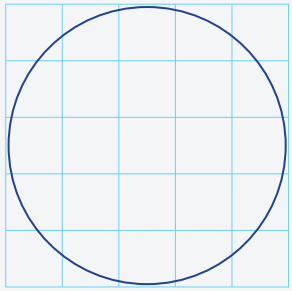


4 Count squares to estimate the area of these circles.

a

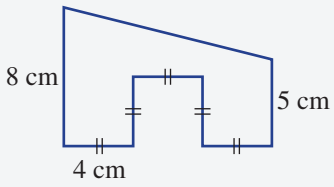


b

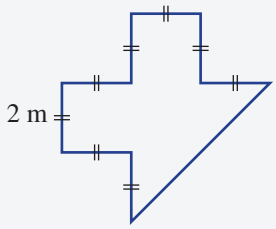


5 Find the areas of these composite shapes.

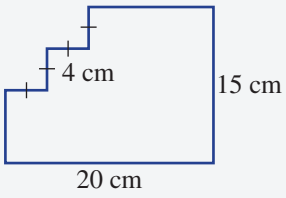
a



b

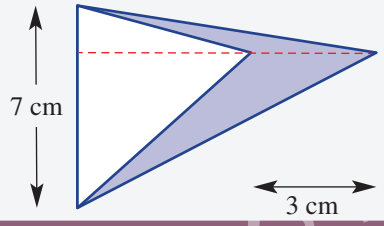


c

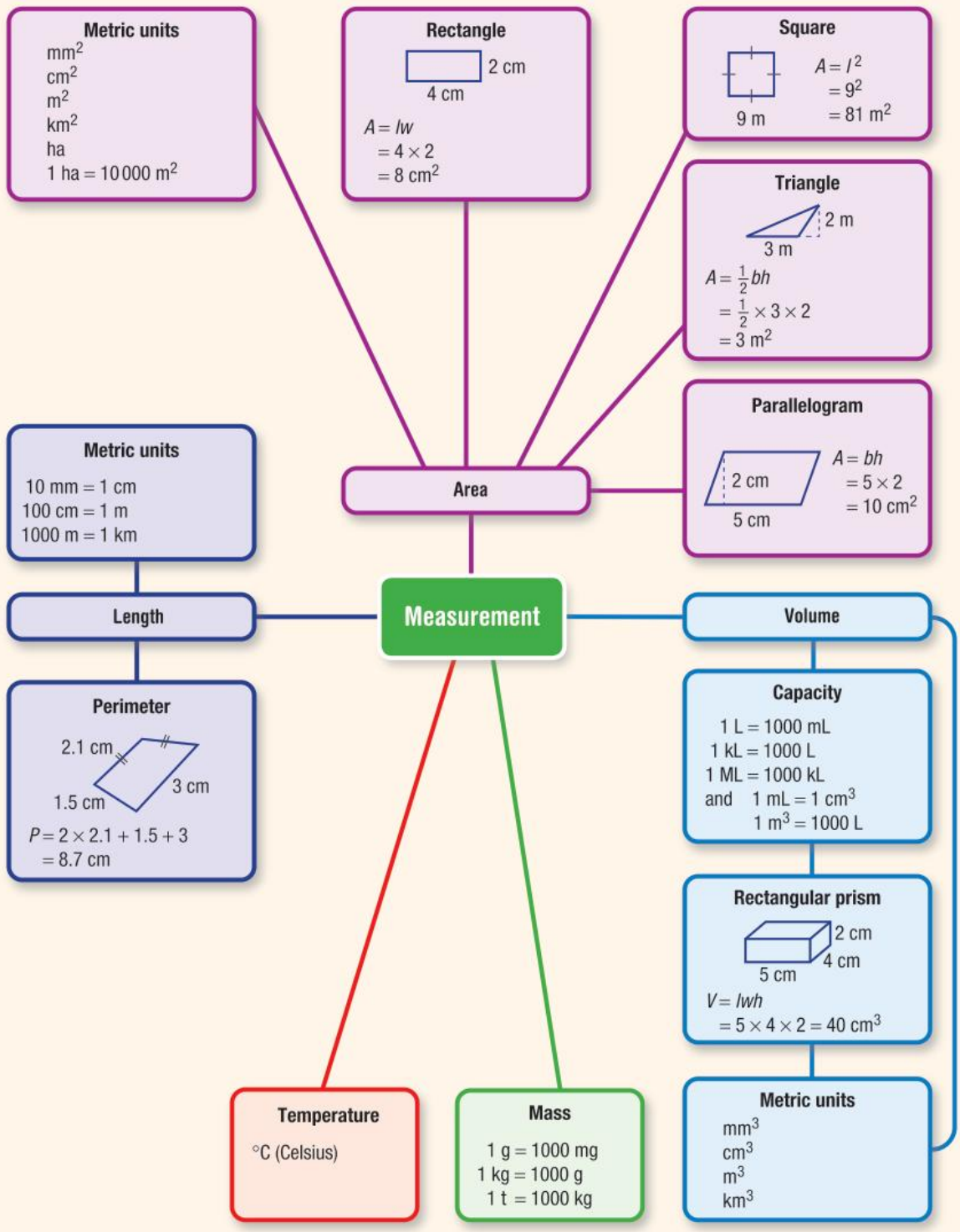


6 You have two sticks of length 3 m and 5 m. Neither stick is marked with a scale. How could you use the sticks to mark a length of 1 m?

7 Find the area of the shaded region.



Chapter summary

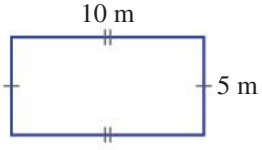


Chapter review

Multiple-choice questions

11B **1** The perimeter of this rectangle is:

A 25 cm
B 40 m
C 15 m
D 30 m
E 30 m^2

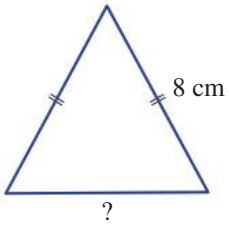


11C **2** A square has side length 3 cm. Its area is:

A 9 cm **B** 12 cm^2 **C** 6 cm
D 6 cm^2 **E** 9 cm^2

11B **3** The triangle has a perimeter of 20 cm. What is the missing base length?

A 6 cm
B 8 cm
C 4 cm
D 16 cm
E 12 cm



11C **4** The area of a rectangle with length 2 m and width 5 m is:

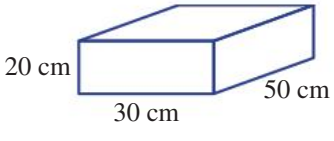
A 10 m^2 **B** 5 m^2 **C** 5 m
D 5 m^3 **E** 10 m

11D **5** A triangle has base length 3.2 cm and height 4 cm. What is its area?

A 25.6 cm^2 **B** 12.8 cm **C** 12.8 cm^2 **D** 6 cm **E** 6.4 cm^2

11F **6** The volume of this rectangular prism in litres is:

A 3 L **B** 30 L **C** 30000 L
D 300 L **E** 3000 L



11F **7** A cube has a side length of 3 cm. Its volume is:

A 27 cm^3 **B** 9 cm^2 **C** 3 cm **D** 9 cm^3 **E** 36 cm^3

11G **8** 2000 cm^3 is the same as:

A 2 m^3 **B** 2 L **C** 2 kL **D** 2 mL **E** 2 t

11H **9** Gravel is being loaded onto a truck at a rate of 20 kg per second. How many minutes will it take to load all of the 9 tonnes of gravel?

A 0.75 min **B** 45 min **C** 7.3 min **D** 450 min **E** 7.5 min

11E **10** The base length of a parallelogram is 10 cm and its height is 4 cm. The parallelogram's area is:

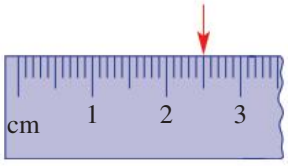
A 20 cm^2 **B** 40 cm^2 **C** 30 cm^2 **D** 4 cm^2 **E** 40 m^2

Chapter review

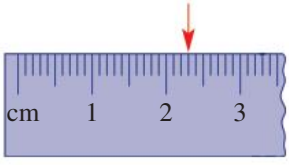
Short-answer questions

11A/11G/11H **1** Read these scales.

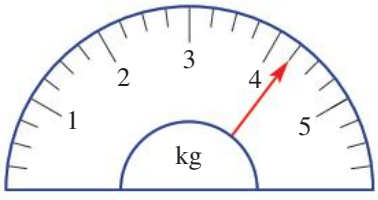
a



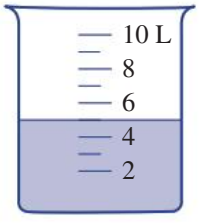
b



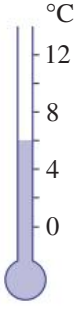
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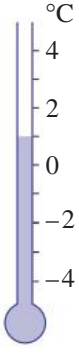
d



e



f

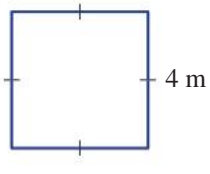


11A/11G/11H **2** Convert to the units shown in brackets.

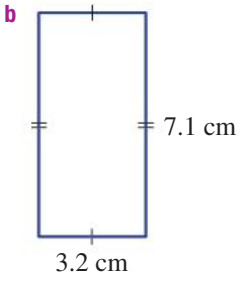
a 5 cm (mm)
b 200 cm (m)
c 3.7 km (m)
d 3600 cm (m)
e 7.1 kg (g)
f 24 000 mg (g)
g 22 000 kg (t)
h 2.5 t (kg)
i 4000 mL (L)
j 40 000 L (kL)
k 0.4 ML (kL)
l 6.5 kL (L)
m 60 mm (cm)
n 3000 kL (ML)
o 0.02 cm (mm)

11B **3** Find the perimeter of these shapes.

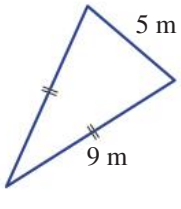
a



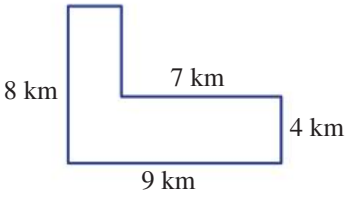
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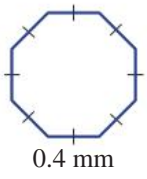
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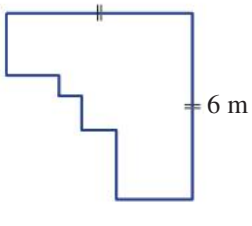
d



e

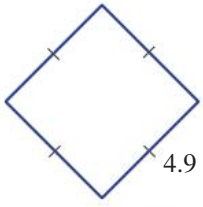


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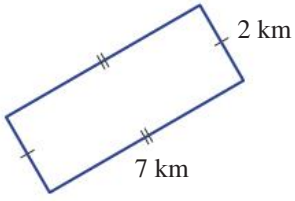


11C/11D/11E **4** Find the area of each of the following shapes.

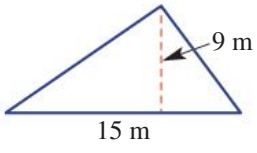
a



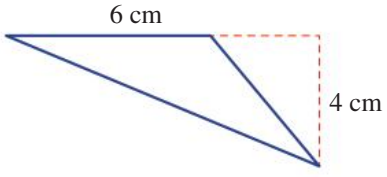
b



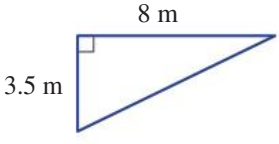
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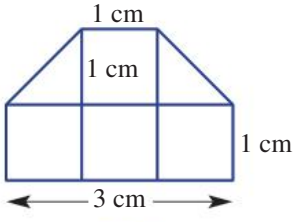
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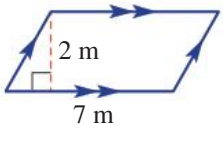
e



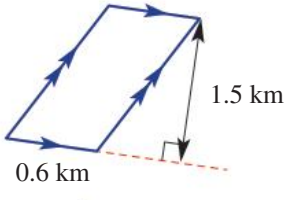
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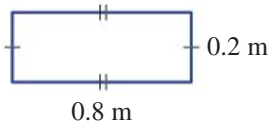
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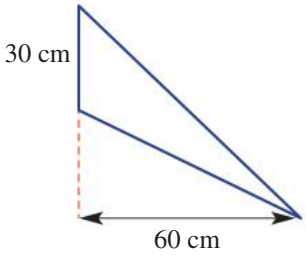
h



i

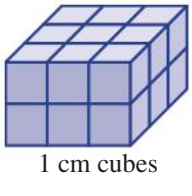


j

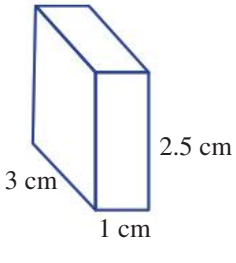


11F **5** Find the volume for each of these solids.

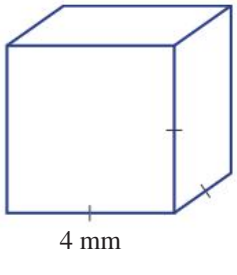
a



b



c



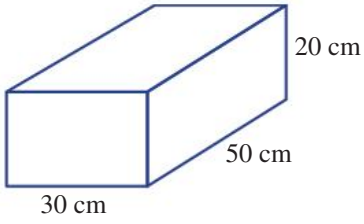
11G **6** A rectangular fish tank is of length 60 cm, width 40 cm and height 30 cm. Give the tank's capacity in:

a cm^3 **b** mL **c** L

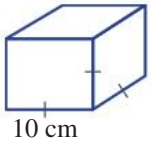
Chapter review

11G **7** Find the volume of these rectangular prisms in litres.

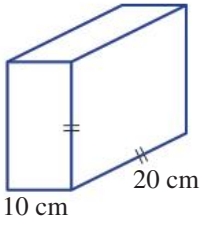
a



b



c



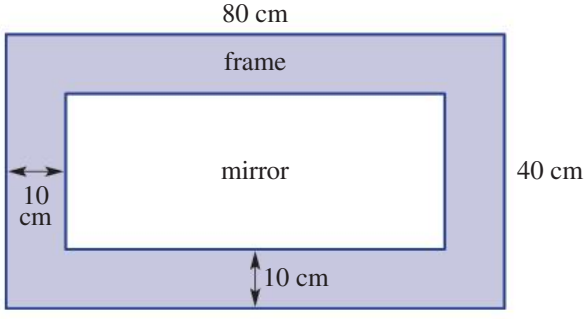
11H **8** Arrange these measurements from smallest to largest.

a 3 t, 4700 kg, 290 000 g, 45 mg

b 50 000 mL, 1 ML, 51 L, 0.5 kL

Extended-response questions

1 A mirror is surrounded by a 10 cm wide frame so that the total dimensions are 80 cm by 40 cm as shown.





a Find the outside perimeter of the frame.

b Find the total area of the framed mirror.

c Find the width and length of the mirror alone.

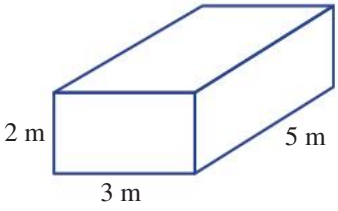
d Find the perimeter of the mirror alone.

e Find the area of the mirror alone.

f Find the area of the frame.

g Find the cost of the frame if it is \$5 per 50 cm^2.

2 A simple rectangular swimming pool is 5 m long, 3 m wide and 2 m deep.



- **a** Find the volume of the pool in m^3.
- **b** Find the perimeter of the rectangular top of the pool.
- **c** By converting the given lengths to cm, find the volume of the pool in cm^3.
- **d** Give the capacity in litres and in kilolitres.
- **e** Given that 1 L weighs 1 kg, how many tonnes of water fill the pool?



Semester review 2

Negative numbers

Multiple-choice questions

1 Which of the following statements is incorrect?
A $-2 > -4$ **B** $0 < 5$ **C** $0 < -10$ **D** $-9 < -8$ **E** $-5 < 3$

2 $5 - 7$ is equal to:
A -2 **B** 2 **C** 12 **D** -12 **E** 0

3 $4 + (-2)$ is equal to:
A -6 **B** 2 **C** 6 **D** -2 **E** -4

4 $9 - (-3)$ is equal to:
A 13 **B** -12 **C** -6 **D** 6 **E** 12

5 The origin is the point at the centre of the number plane with coordinates:
A $(0, 1)$ **B** $(1, 1)$ **C** $(1, 0)$ **D** $(0, 0)$ **E** $(10, 10)$

Short-answer questions

1 For each of the following, insert > (greater than) or < (less than).
a $-3 \square 3$ **b** $-4 \square -5$ **c** $-21 \square -4$

2 Calculate:
a $4 - 5$ **b** $1 - 6$ **c** $0 - 10$ **d** $-4 - 2$ **e** $-7 - 11$ **f** $-37 - 40$

3 Calculate:
a $2 + (-1)$ **b** $11 + (-7)$ **c** $5 + (-7)$
d $-1 + (-4)$ **e** $-10 + (-2)$ **f** $-31 + (-26)$

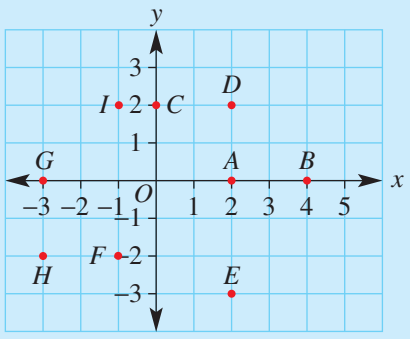
4 Calculate:
a $3 - (-2)$ **b** $6 - (-1)$ **c** $11 - (-13)$
d $-4 - (-3)$ **e** $-13 - (-7)$ **f** $-11 - (-13)$

5 Copy and complete:
a $\square + 9 = -6$ **b** $4 - \square = 7$ **c** $-2 + \square = -10$

6 If $a = 6$ and $b = -4$, find the value of:
a $-a + b$ **b** $a - b$ **c** $2(b - a)$
d $2a - b$ **e** $b + 6a$ **f** $4a - b + b$

Extended-response questions

1 Look at this number plane.



a Give the coordinates of each point (A–I).
b Name any point(s) with a y-coordinate of zero. Where does each point lie?
c Find the distance between points:
i A and B
ii D and E
d What shape is formed by joining the points $IDAG$?
e What is the area of $IDAG$?
f Decode: $(2, 2)$, $(2, -3)$, $(0, 2)$, $(-1, 2)$ $(2, 2)$, $(2, -3)$ using the letters on the number plane.

Statistics and probability

Multiple-choice questions

The following information is relevant for questions 1 and 2.
A survey asked 60 participants their favourite colour. The results are shown below.

blue	pink	green	purple	black
12	20	6	12	10

1 The number of people who listed purple as their favourite colour was:
A 12 **B** 20 **C** 6 **D** 60 **E** 10

2 Based on this survey, the experimental probability that a randomly selected person's favourite colour is black is:
A 1 **B** $\frac{1}{2}$ **C** $\frac{1}{6}$ **D** 10% **E** $\frac{1}{3}$

3 For the set of numbers 1, 5, 2, 10, 9, 7, the median is:
A 5.5 **B** 6 **C** 7 **D** 5 **E** 1

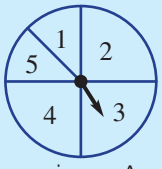
4 For the set of numbers 3, 2, 1, 5, 7, 3, 1, 3, the mean is:
A 3 **B** 5 **C** 6 **D** 22.375 **E** 3.125

5 Which of the following could be used to describe an event which has a probability of 0.9?
A unlikely **B** likely **C** even chance
D certain **E** impossible

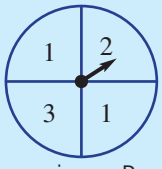
Short-answer questions

1 Consider the set of numbers 1, 2, 5, 5, 8, 9, 10, 5, 3, 8.
a List the numbers in ascending order.
b How many numbers are in the set?
c Calculate the:
i mean **ii** mode **iii** median **iv** range
d If each number in the set is doubled, what is the new mean?

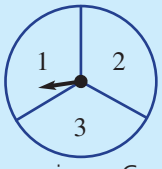
2 Consider the spinners A to D.



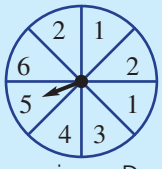
spinner A



spinner B



spinner C



spinner D

a Which spinner has the lowest probability of landing on the number 1 in a single spin?
b Which spinner has a 50% probability of landing on the number 1 in a single spin?
c What is the probability that spinner C will land on the number 2?

3 A standard die is rolled.
a What is the sample space?
b What is the probability of rolling a 2?
c What is the probability of rolling an odd number?

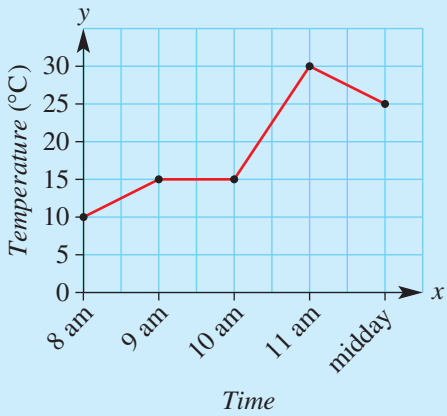
4 This stem-and-leaf plot shows the ages of a group of people in a room.

Stem	Leaf
0	3 5
1	1 7 9
2	0 2 2 3
3	6 9
4	3 7

2 | 3 means 23 years old

a How many people are in the room?
b What are the ages of the youngest and oldest people?
c For the data presented in this plot, find the:
i range **ii** median **iii** mode

5 A line graph is drawn charting the room's temperature at different times of the day.



a What was the temperature at:
i 8 a.m.? **ii** 10 a.m.? **iii** 11 a.m.?
b How much did the temperature increase between 10 a.m. and 11 a.m.?
c What was the approximate temperature at 10:30 a.m.?

Extended-response questions

1 A pack of playing cards contains 13 cards for each suit: hearts, diamonds, clubs and spades. Each suit has an Ace, King, Queen, Jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. One card is drawn at random from the pack.
Find the following probabilities.

a Pr(heart) **b** Pr(club) **c** Pr(diamond or spade)
d Pr(Ace of hearts) **e** Pr(number less than 4) **f** Pr(King)
g Pr(Ace or heart) **h** Pr(Queen or club)

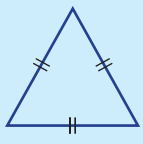
Shapes and transformations

Multiple-choice questions

1 Which of the following shapes has four 90° angles?
A trapezium **B** rhombus **C** pentagon **D** triangle **E** rectangle

2 A quadrilateral with 4 equal sides is best described as a:
A rectangle **B** rhombus **C** trapezium **D** kite **E** plane

3 How many axes of symmetry does an equilateral triangle have?
A 0 **B** 1 **C** 2 **D** 3 **E** 4



4 Which one of the following statements is true?

A All rectangles are squares.
B All isosceles triangles are equilateral triangles.
C A square is a rhombus with all angles equal.
D A trapezium is also a rectangle.
E An isosceles triangle has three equal angles.

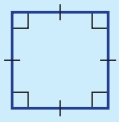
5 The order of rotation of a parallelogram is:

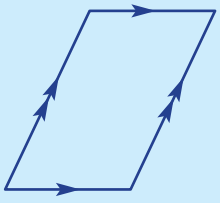
A 2 **B** 4 **C** 3 **D** 0 **E** 1

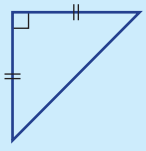


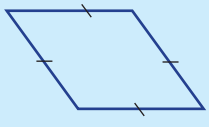
Short-answer questions

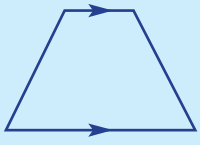
1 Name each of the following shapes.

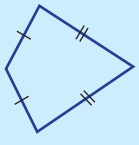
a


b


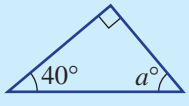
c


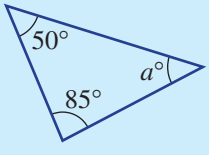
d


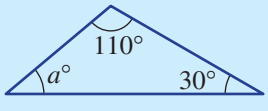
e


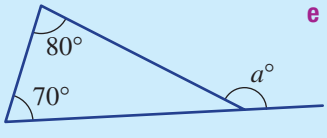
f


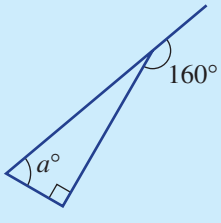
2 Find the value of a.

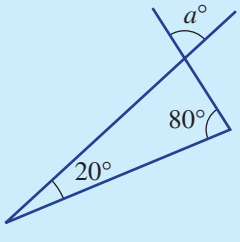
a


b


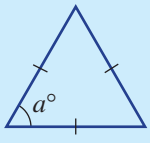
c


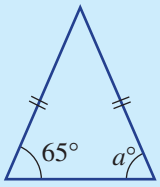
d


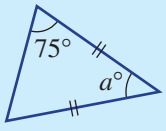
e


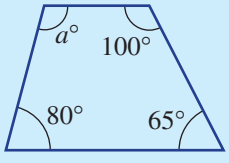
f


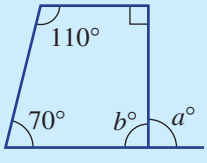
3 For each of the following, find the value of each pronumeral.

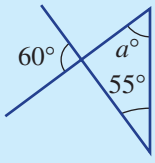
a


b


c


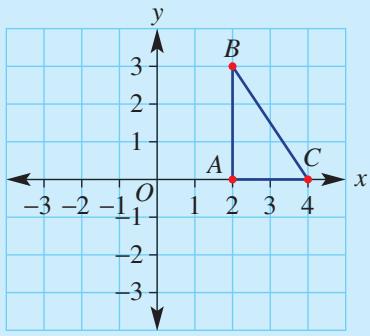
d


e


f


4 State the coordinates of the vertices of the image triangle (A', B' and C') if triangle ABC is transformed in the following ways.

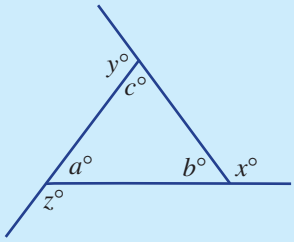
- **a** reflected in the x-axis
- **b** reflected in the y-axis
- **c** rotated about $O(0, 0)$ by 90° clockwise
- **d** rotated about $O(0, 0)$ by 90° anticlockwise
- **e** rotated about $O(0, 0)$ by 180°
- **f** translated 4 units to the left and 1 unit up
- **g** translated 3 units to the left and 2 units down

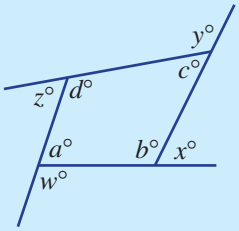


Extended-response questions

1

- **a** Draw any triangle in your workbook and extend each side, as shown at the right.
- **b** Measure the angles a, b and c. What should they add to?
- **c** Find the values of x, y and z (i.e. the exterior angles of the triangle).
- **d** What is the total of angles $x + y + z$?
- **e** Repeat for any quadrilateral, as shown below. What is the value of $w + x + y + z$?





Equations

Multiple-choice questions

1 The solution to the equation $x - 3 = 7$ is:

A $x = 4$ **B** $x = 10$ **C** $x = 9$ **D** $x = 11$ **E** $x = 3$

2 The solution to the equation $2x + 6 = 12$ is:

A $x = 4.5$ **B** $x = 2$ **C** $x = 7$ **D** $x = 6$ **E** $x = 3$

3 $m = 4$ is a solution to:

A $3m + 12 = 0$ **B** $\frac{m}{4} = 16$ **C** $10 - 2m = 2$ **D** $m + 4 = 0$ **E** $3m - 6 = 2$

4 The solution to $2p - 3 = 7$ is:

A $p = 4$ **B** $p = 5$ **C** $p = 2$ **D** $p = 10$ **E** $p = 3$

5 Ying thinks of a number. If he multiplies the number by 5 and then subtracts 4, the result is 35. What equation represents this information?

A $y + 9 = 35$ **B** $5y - 4 = 35$ **C** $5y + 4 = 35$ **D** $5(y + 4) = 35$ **E** $y + 20 = 35$

Short-answer questions

1 Classify the following equations as true or false.

a $2 + 17 = 10 + 8$ **b** $5 = 10 - 5$

c $x + 2 = 10$ if $x = 8$ **d** $a - 3 = 7$ if $a = 4$

e $12 = 20 - k$ if $k = 6$ **f** $y - 3 = 3$ if $y = 6$

2 Solve:

a $x + 9 = 12$ **b** $\frac{x}{9} = 12$ **c** $x - 9 = 12$ **d** $9x = 12$

3 Solve:

a $3x + 3 = 9$ **b** $2y - 7 = 3$ **c** $\frac{x}{2} + 6 = 12$ **d** $\frac{m - 1}{3} = 2$

4 Consider the formula $P = 4w$.

a Find the value of P if:

i $w = 2$ **ii** $w = 5$ **iii** $w = 12$

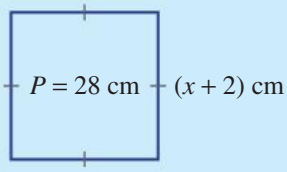
b Solve the equation $24 = 4w$.

c Find the value of w if $P = 24$.

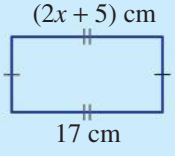
d Find the value of w if $P = 44$.

5 Use your knowledge of geometry and shapes to find the value of x in each of the following.

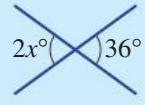
a



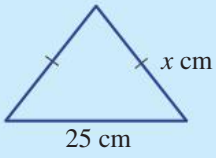
b



c



6 The perimeter of this triangle is 85 cm. Write an equation and then solve it to find the value of x.



Extended-response questions

1 The cost of hiring a hall for an event is \$200 plus \$40 per hour.

a What is the cost of hiring the hall for 3 hours?

b What is the cost of hiring the hall for 5 hours?

c Write a formula for the cost \$$C$ of hiring the hall for n hours.

d If the cost of hiring the hall totals \$460, for how many hours was it hired?

Measurement

Multiple-choice questions

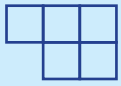
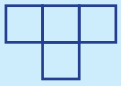
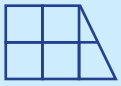
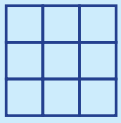
1 17 mm is the same as:

A 0.17 m **B** 0.17 cm **C** 1.7 cm **D** 170 cm **E** 1.7 m

2 0.3 L is the same as:

A 3000 mL **B** 300 mL **C** 30 mL **D** 0.03 mL **E** 0.003 mL

3 Which of the following shapes has the largest perimeter?

A  **B**  **C**  **D**  **E** 

4 The perimeter of this square is 12 cm. Its area is:

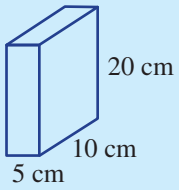
A 4 cm^2 **B** 144 cm^2 **C** 9 cm^2

D 16 cm^2 **E** 8 cm^2

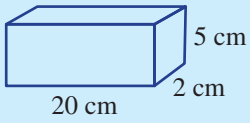


5 Which of the following shapes holds exactly 1 litre? Remember that 1 L = 1000 cm^3.

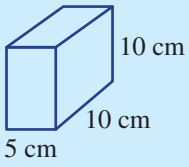
A



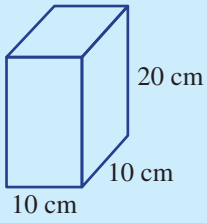
B



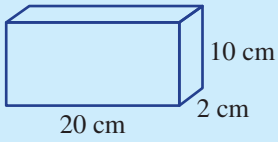
C



D



E



Short-answer questions

1 Complete these conversions.

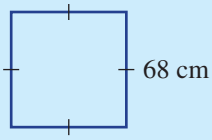
a 5 m = ____ cm **b** 6 km = ____ m

c 180 mm = ____ cm **d** 1.7 cm = ____ m

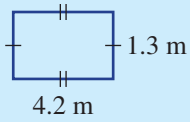
e 180 cm = ____ m **f** $5\frac{1}{2}$ L = ____ mL

2 Find the perimeter of each of the following.

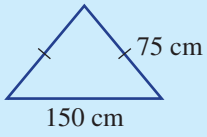
a



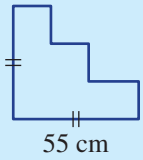
b



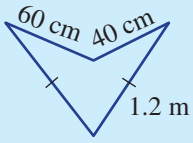
c



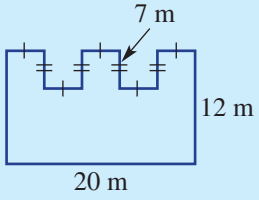
d



e

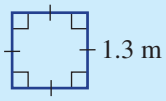


f

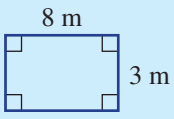


3 Find the area of each of the following.

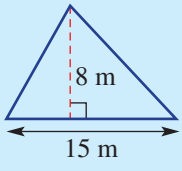
a



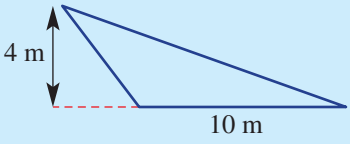
b



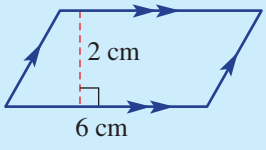
c



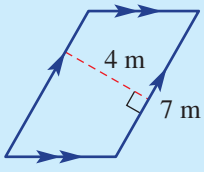
d



e

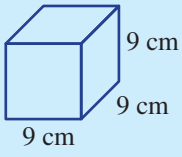


f

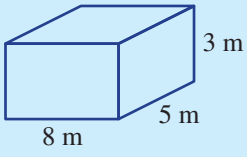


4 Calculate the volume of these solids.

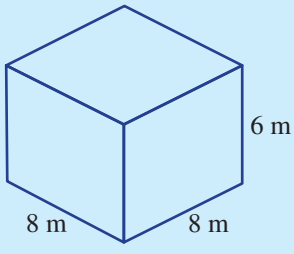
a



b



c



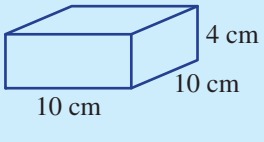
5 Convert:

a 5 L = ____ mL **b** 7 cm^3 = ____ mL **c** 2 ML = ____ kL

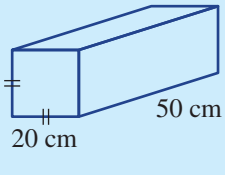
d 3 kL = ____ L **e** 8 m^3 = ____ kL **f** 25 000 mL = ____ L

6 Find the capacity of these rectangular prisms in litres.

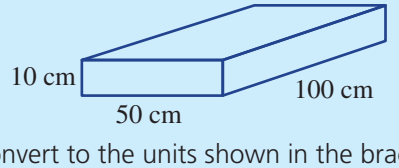
a



b



c



7 Convert to the units shown in the brackets.

a 3 t (kg) **b** 2 kg (g) **c** 6500 g (kg) **d** 500 kg (t)

e 5000 mg (g) **f** 24 g (mg) **g** 0.05 kg (g) **h** 20 mg (g)

8 Copy and complete:

a Water freezes at ____°C. **b** Water boils at ____°C.

Extended-response questions

1 Robert has been given 36 m of fencing with which to build the largest rectangular enclosure that he can, using whole number side lengths.

a Draw three possible enclosures and calculate the area of each one.

b What are the dimensions of the rectangle that gives the largest possible area?

c If Robert chooses the dimensions in part **b** and puts a post on each corner, and then posts every metre along the boundary, how many posts will he need?

d If each post costs $25, what will be the total cost of the posts?

Appendix

Algorithmic thinking



What you will learn

Activity 1: Algorithms for working with data
- 1.1 Finding an average
- 1.2 Generating data: simulation using a spreadsheet

Activity 2: Adding consecutive numbers

Activity 3: Turtle graphics
- 3.1 Basic shapes
- 3.2 Patterns

Australian curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Design and implement mathematical algorithms using a simple general purpose programming language **(VCMNA254)**



Autopilot algorithms

Modern aircraft are fitted with autopilot systems that allow the aircraft to fly safely without direct input from the pilot. Such systems rely on computer algorithms that have been designed to accept a range of inputs and deliver outputs resulting in actions that steer the aircraft safely.

Inputs for such algorithms might include the air speed, direction or tilt angle. The outputs might include engine thrust, flap angle or rudder direction. While looking at the wing on a passenger aircraft during flight you may have noticed the constant movement of a relatively small flap called an aileron. An autopilot algorithm causes this flap to change its angle, which adjusts the aircraft's tilt. A constant feed of information into the algorithm will allow the program to update the output and change the aileron's angle accordingly.

Autopilot systems have become a reliable and essential tool for the aeronautical industry. This would not have been possible without efficient algorithms designed using mathematics and computer code.

Introduction

An **algorithm** is a procedure involving a number of steps. It should have clearly defined inputs and outputs.

Here are some examples that we have explored in earlier chapters.

- The addition algorithm for whole numbers: This algorithm has two inputs; for example, the inputs 37 and 82 would give the output 119. The steps include adding the digits in each place, and checking whether carrying is required. This algorithm is shown in Section 1D.
- The divisibility algorithm for determining if a number is divisible by 10: The input is a single number, such as 80 or 42, and the output is 'yes' or 'no'. This algorithm is shown in Section 3C.
- The averaging algorithm: The input for this algorithm is a list of numbers, such as 5, 2, 8. The algorithm adds the numbers ($5 + 2 + 8 = 15$) and divides by the number of terms in the input list (in this case, 3) to give an output ($15 \div 3 = 5$). This algorithm is shown in Section 8B.

Activity 1: Algorithms for working with data
Statistics and Probability

1.1 Finding an average

Let us look again at the algorithm for finding the average (or mean) of a set of numbers. The algorithm could be described as follows.

Step 1. Add all the numbers in the list.
Step 2. Count how many numbers are in the list.
Step 3. Divide the sum (from step 1) by the count (from step 2). This is the average.

a Using the numbers 10, 1, 4 as the input list, calculate the results of steps 1, 2 and 3 of the averaging algorithm.

b A spreadsheet can be used to implement this algorithm. Open a new spreadsheet and enter the following. Check that the values in cells B2, C2 and D2 match the values you found in **a**.

	A	B	C	D
1	Values	Step 1	Step 2	Step 3
2	10	=SUM(A2:A4)	=COUNT(A2:A4)	=B2/C2
3	1			
4	4			

c Change the values in column A until the average of the three values is 9. Write down the values you chose.

d One advantage of using a spreadsheet is that it is easy to work with a longer list of values. Enter two more values into column A so that there are now five values. You will need to change the formula for step 1 to =SUM(A2:A6) and the formula for step 2 to =COUNT(A2:A6). Step 3 does not change. Check that your spreadsheet is giving the correct answer by calculating the average with a calculator.

e Try to solve each of the following puzzles using your spreadsheet. You can use trial and error to find the missing values.

- **i** The values are 1, 2, ?, 4, 5 and the average is 4. Find the missing number.
- **ii** The values are 1, 10, 100, 1000, ? and the average is 500. Find the missing number.
- **iii** The five values are all whole numbers but the average is 3.2. Find possible values that make this happen.
- **iv** The average of five consecutive numbers is 100. What are the numbers?

1.2 Generating data: simulation using a spreadsheet

A spreadsheet can be used to generate data that simulates an experiment, such as rolling a fair die five times to see how many times the number 1 is rolled.

a Make a new spreadsheet with the following formulas. (If your spreadsheet does not have the function RANDBETWEEN, use =RANDINT(1, 6) or =RAND(1, 6) instead.)

	A
1	Die
2	=RANDBETWEEN(1,6)
3	=RANDBETWEEN(1,6)
4	=RANDBETWEEN(1,6)
5	=RANDBETWEEN(1,6)
6	=RANDBETWEEN(1,6)

Hint: Once you have entered the formula into A2, you can use Fill Down to copy it as many times as you wish.

b To make your spreadsheet 'roll' the dice another five times, choose Calculate Now from the Formulas menu or an equivalent function in different spreadsheets. Alternatively, if you just change the value in any cell, all other values will recalculate.

c Count how many times the number 1 is rolled. You can easily do this by looking at the numbers but it will be more useful to have the spreadsheet count for us. Enter the formula =COUNTIF(A2:A6,1) in B1 and check that the answer is correct.

d Now change your spreadsheet to include 100 dice rolls by dragging the formula in A6 to row 101. Count how many times the number 1 was rolled, remembering to change the formula to =COUNTIF(A2:A101, 1).

e In cell C1, enter the fromula =SUM(A2:A101)/COUNT(A2:A101). This calculates the average of the 100 dice.

- **i** Write down the average of the 100 dice rolls.
- **ii** Why does this average change every time you recalculate?

	A	B	C
1	Die	=COUNTIF(A2:A101,1)	=SUM(A2:A101)/COUNT(A2:A101)
2	=RANDBETWEEN(1,6)		
3	=RANDBETWEEN(1,6)		
4	=RANDBETWEEN(1,6)		
5	=RANDBETWEEN(1,6)		
6	=RANDBETWEEN(1,6)		
7	=RANDBETWEEN(1,6)		

Activity 2 – Adding consecutive numbers

Number and Algebra

One task that an algorithm can perform is adding the consecutive whole numbers 1, 2 and 3. Although it might seem unnecessary to use an algorithm for such a simple task, we can easily change the algorithm to add much longer sequences. The algorithm is written below in **pseudocode**. This is not an actual programming language, but uses some of the same concepts.

Line	Command	Explanation
1	`total ← 0`	`total` is the sum of the numbers so far.
2	`current ← 0`	`current` is the number we are up to as we loop.
3	`loop:`	Performs lines 4–6 forever (until you hit `break`).
4	`  current ← current + 1`	Adds 1 to the current number.
5	`  total ← total + current`	Increases the total by the current number.
6	`  if current = 3: break`	If we reach the number 3, we leave this loop.
7	`display total`	At the end we need to display the total.

A computer can run this program line by line. The execution is shown below and takes 13 steps.

Line	Command	Result
1	`total ← 0`	`total` is set to 0.
2	`current ← 0`	`current` is set to 0.
3	`loop:`	Get ready to repeat lines 4 to 6.
4	`current ← current + 1`	`current` is now 1 (because 0 + 1 = 1).
5	`total ← total + current`	`total` is now 1 (because 0 + 1 = 1).
6	`if current = 3: break`	No effect, because `current` is not 3.
4	`current ← current + 1`	`current` is now 2 (because 1 + 1 = 2).
5	`total ← total + current`	`total` is now 3 (because 1 + 2 = 3).
6	`if current = 3: break`	No effect, because `current` is not 3.
4	`current ← current + 1`	`current` is now 3 (because 2 + 1 = 3).
5	`total ← total + current`	`total` is now 6 (because 3 + 3 = 6).
6	`if current = 3: break`	Exit the loop, because `current` is 3.
7	`display total`	Displays the value of total, which is 6.

a What is the sum of the consecutive numbers from 1 to 3?

b What is the sum of the consecutive numbers from 1 to 5? Show a calculation that could be entered into a calculator or computer to get the result.

c By changing a single line, the program above will add the consecutive numbers from 1 to 5.

i To describe this change, give the line number and write the exact command you think should go there.

ii Create a table like the one at the right to show the steps of this program. At first, the results will be the same as those in the table above, but this loop will exit at a different line.

Line	Result
1	`total` is 0
2	`current` is 0
3	Begin loop
4	`current` is 1
5	`total` is 1
6	No action
4	`current` is 2
5	`total` is 3
6	No action
…	…

d **i** When adding the numbers from 1 to 3, the total execution was 13 steps. How many steps does it take to add the numbers from 1 to 5?

ii Estimate how many steps it would take to add all the whole numbers from 1 to 100.

iii Describe how the program could be changed to add the whole numbers from 1 to 100.

e Adding all the numbers from 1 to 100 is tedious to do by hand. You can perform this algorithm in any programming language, or in a spreadsheet. If you know how to use a programming language then try to complete this part in a programming language, but otherwise you can use the spreadsheet below.

i To set up the spreadsheet, just fill in the top three rows as shown. Then fill down row 3 to row 102.

	A	B
1	Current	Total
2	0	0
3	=A2+1	=A3+B2
4	=A3+1	=A4+B3
5	=A4+1	=A5+B4
6	=A5+1	=A6+B5
7	=A6+1	=A7+B6
8	=A7+1	=A8+B7
9	=A8+1	=A9+B8
10	=A9+1	=A10+B9
11	=A10+1	=A11+B10
12	=A11+1	=A12+B11
13	=A12+1	=A13+B12

ii What is the sum of all whole numbers from 1 to 100?

iii The year 2016 was special because it is the sum of consecutive numbers starting at 1. How many consecutive numbers were added to get 2016?

f Instead of adding consecutive numbers, we could write an algorithm to add numbers in a different sequence, such as 2, 4, 6, 8, 10.

i Using pseudocode, write an algorithm that uses a loop to add the numbers 2, 4, 6, 8 and 10.

ii Describe how to change your algorithm to add all the even numbers from 2 to 1000.

iii Find the sum of all the even numbers up to and including 1000. You could use a spreadsheet like the one above, but change A3 to "=A2 + 2," and then fill down to row 502.

Activity 3 – Turtle graphics

Measurement and Geometry

'Turtle graphics' refers to a style of computer graphics made famous by the Logo programming language. It involves describing the path of a turtle that can draw as it moves along. The turtle faces a given direction (initially upwards) and at each step, it walks forward (or backward) a given distance, or turns by a given angle. A simple set of instructions to use is:

- `FW` x: Move the turtle forward x 'steps'.
- `RT` x: Turn the turtle right x degrees (clockwise).
- `LT` x: Turn the turtle left x degrees (anticlockwise).
- `REP` n [...]: Repeat the instructions in the brackets n times.

3.1 Basic Shapes

a Using a protractor and a ruler, draw an equilateral triangle by following the turtle graphics commands below. (The steps are in centimetres and you should start in the centre of the page, facing the top.)

```
FW 5, RT 120, FW 5, RT 120, FW 5, RT 120
```

The shape could also be formed using the loop `REP 3 [FW 5, RT 120]`.

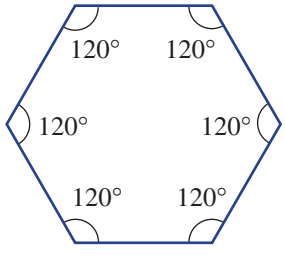
b Explain what would happen if you turned $60°$ rather than $120°$.

c **i** Use your protractor and ruler to show the result of `REP 3 [FW 5, LT 120]`.

ii How is this shape different from the one in **a**? How is it similar?

d Write a set of commands that could be used to draw a square. Try to use the REP instruction.

e A regular hexagon has six sides of the same length with internal angles of $120°$.



i Jack predicts that the following program will draw a regular hexagon. Is he correct? Describe what happens when you follow the steps.

```
REP 6 [FW 5, RT 120]
```

ii To draw a regular hexagon, what is the correct angle that the turtle needs to turn each time? Hint: Extend the line out from one side and measure the angle to turn.

f Sarah wants the turtle to turn right each time. Rewrite the following program so that it draws the same shape but without using `LT`.

```
FW 10, LT 90, FW 10, LT 300, FW 5
```

3.2 Patterns

The following turtle graphics commands are useful for drawing patterns. They allow you to move the turtle without drawing a line.

- `PU`: Lift the pen up off the page
- `PD`: Put the pen back down onto the page

a A simple dashed line can be created by picking up the pen after drawing a section and then putting it down again. For instance, the following commands create a dashed line:

```
FW 2, PU, FW 1, PD, FW 2, PU, FW 1, PD
```

Write a simple program that makes a dashed line with 10 sections that are 2 cm long, with a 1 cm gap in between.

b Another pattern can be formed with parallel lines.

i Draw the result of the following commands using a protractor and ruler.

```
FW 10, PU, RT 180, FW 10, LT 90, FW 5, LT 90, PD, FW 10
```

ii Design a program that draws 10 parallel lines. Hint: Use the REP command.

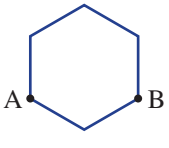
c You can draw a 5 cm wide square using `REP 4 [FW 5, RT 90]`. To draw a second square joined to the first, you could move the pen to the next starting point with the commands: `PU, REP 3 [FW 5, RT 90], PD`.

i The turtle is not facing upwards after these commands. Write a command that would turn it to face upwards again.

ii Write a program that draws 5 squares in a row.



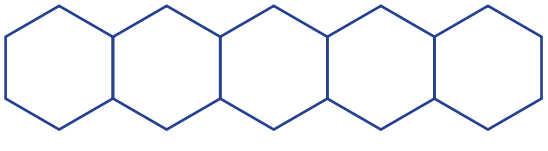
d A similar technique can be used to draw regular hexagons in a row. After one hexagon has been drawn using `REP 6 [FW 5, RT 60]`, the turtle is moved (with the pen up) from location A to location B, and turned to face upwards again, using the commands `PU, REP 4 [FW 5, RT 60], REP 2 [RT 60], PD`



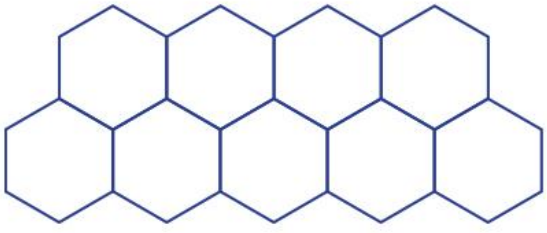
i Follow the commands to check. Why is the command `REP 2 [RT 60]` used?

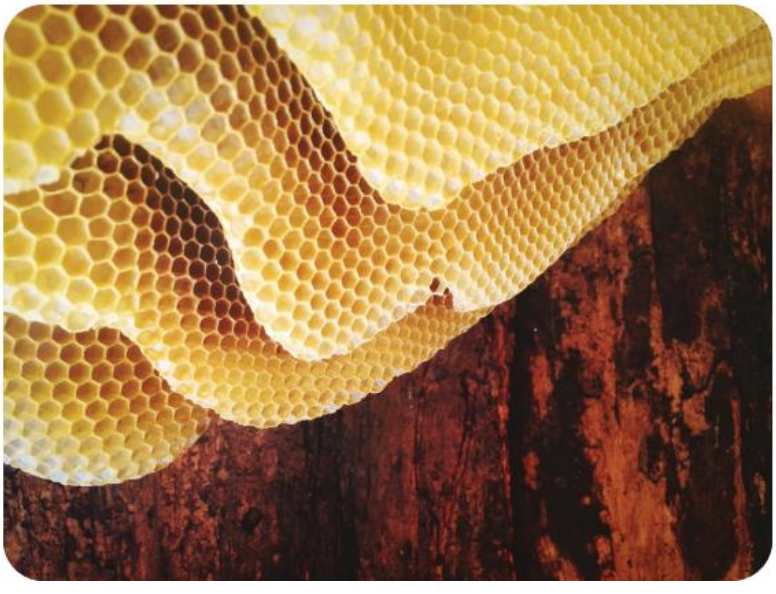
ii How could this be simplified to a single command not involving repetition?

iii Write a program that draws five hexagons in a row to form the pattern below.



e A honeycomb pattern of hexagons can continue indefinitely in each direction. The bottom two rows of a honeycomb pattern are shown below. Design a program that draws these two rows. Hint: Draw the bottom row first. With the pen up, move to the start of the top row.





Glossary

A

Acute triangle A triangle in which all the angles are acute (less than $90°$)

Adjacent Next to each other

Alternate angles Two angles that lie between two lines on either side of a transversal

Angle sum The total measure of the interior angles in a 2D shape

Approximation A value that is close to the real value

Arc Part of the circumference of a circle between two points

Area The amount of surface a shape covers

Ascending Going up, from smallest to largest

Associative law The result of adding or multiplying three or more numbers does not depend on how they are grouped

B

Backtracking A method of solving equations systematically by applying opposite operations

Base (Measurement) One side of a shape, at right angles to the height

Base (Number) The number that is being raised to a power or index

C

Capacity How much liquid or gas a container can hold

Categorical data Data that can be placed into categories or types of objects

Celsius Scale used for measuring temperature, where water freezes at $0°$ and boils at $100°$

Census Collection of data from the whole population

Chance The likelihood of an event happening

Chord A line interval joining two points on the circumference of a circle

Coefficient A numeral placed before a pronumeral

Cointerior angles A pair of angles lying between two lines on the same side of a transversal

Collinear Lying on the same straight line

Column graph A graph where the height of each column represents a value

Commutative law When adding and multiplying, the order in which two numbers are combined does not matter

Compensating A mental strategy where you round a number and then add or subtract a smaller amount

Complementary Having a sum of $90°$

Composite number A whole number with more than two factors

Concurrent Passing through the same point

Cone A solid with a circular base and a slanting curved surface that tapers to a point called the apex

Constant term The part of an expression without any pronumerals

Continuous numerical Data that can take any number value

Coordinates Numbers or letters used to give a location or position, often an ordered pair written in the form (x, y)

Corresponding angles Pairs of angles in the same position, formed by two lines cut by a transversal

Cube A solid (3D shape) with six square faces that are congruent (the same size and shape)

Cylinder A solid with two circular faces joined by a curved surface

D

Data Information (often numerical) gathered by observation, survey or measurement

Datum One score or piece of data

Decimal point Symbol that separates the whole part of a number from its fractional part

Denominator The bottom part of a fraction

Descending Going down, from largest to smallest

Diameter A line interval passing through the centre of a circle with its end points on the circumference, or the length of that interval

Discrete numerical Data that can only be collected in whole number values

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

Dividend The number being divided

Divisibility test A way to work out whether a whole number is divisible by another whole number, without actually doing the division

Divisor The number you are dividing by

Dot plot A graph in which each dot represents one score

E

Equation A mathematical statement that two expressions have the same value

Equilateral triangle A triangle with three equal sides and three equal (60°) angles

Equivalent Equal in value

Equivalent expressions Expressions that are always equal in value

Equivalent fractions Fractions that represent the same amount

Estimate An informed guess

Evaluate To find the numerical value of an expression

Experiment A series of repeated probability trials that lead to outcomes

Expression A group of mathematical terms that does not contain an equals sign

Exterior angle The angle formed between the extended side of a shape and the adjacent side

F

Factor A whole number that will divide into another number exactly

Factor tree A diagram showing the breakdown of a number into its prime factors

Formula An equation that shows the relationship between variables

Fraction Part of a whole

H

HCF Abbreviation of 'highest common factor'

Highest common factor (HCF) The largest number that is a factor of all the given numbers

I

Image The result of transforming a figure

Improper fraction A fraction where the numerator is greater than or equal to the denominator

Index form A method of writing numbers that are multiplied by themselves

Input A number that is changed according to some rule

Integers The set of positive and negative whole numbers and zero

Interior angle Any angle inside two adjacent sides of a shape

Isosceles triangle A triangle with at least two equal sides and two equal angles

K

Kilogram Unit of metric measurement for mass

Kite A quadrilateral with exactly two pairs of equal adjacent sides

L

LCD Abbreviation of 'lowest common denominator'

LCM Abbreviation of 'lowest common multiple'

Like terms Terms with the same pronumerals

Line A set of points forming a straight path that extends forever in opposite directions

Line graph A graph that shows the data as points joined with line segments

Line of symmetry The line (axis) along which a figure could be folded to produce identical halves

Lowest common denominator (LCD) The smallest common multiple of the denominators of two or more fractions

Lowest common multiple (LCM) The smallest number that two or more numbers divide into evenly

M

Mean An average value calculated by dividing the total of a set of numbers by the number of values

Median The middle score when all the numbers in a set are arranged in order

Metre The standard metric unit for length, equal to 100 centimetres

Metric system A measurement system using the base-ten number system

Mixed number A number with a whole number part and a fraction part

Mode The score that appears most often in a set of numbers

Multiple A multiple of a number is the product of that number and any other whole number

N

Negative number A number less than zero

Number line A line on which numbers are represented by points

Number plane A plane on which every point is related to a pair of numbers called coordinates

Numerator The top part of a fraction

Numerical data Data that can be counted or measured

O

Observation Survey and observation of data from those visible to us

Obtuse triangle A triangle with one obtuse angle (between $90°$ and $180°$)

Order of rotational symmetry The number of times a figure matches its original position during rotation of $360°$

Origin The point with coordinates $(0, 0)$ on the number plane

Outcome One of the possible results of a trial

Outlier A value that is much larger or much smaller than the rest of the data

Output The result from applying a rule to the input

P

Parallel lines Lines in the same plane that are a fixed distance apart and never intersect

Parallelogram A quadrilateral with two pairs of opposite sides parallel

Partitioning A mental strategy where you break up a number into parts, e.g. $123 = 100 + 20 + 3$

Per cent (%) Per hundred, out of a hundred

Perfect square A number with a square root that is a whole number

Perimeter The total distance (length) around the outside of a figure

Perpendicular At right angles

Place value The value of a digit in a number, which is determined by its position

Plane A flat surface that extends endlessly in all directions

Point A position in space, marked with a dot and named with a capital letter

Power (index) The number that a base is being raised to

Primary source Information collected from the original source by the person needing the data

Prime number A whole number with only two factors, itself and 1

Prism A solid with two parallel congruent faces, joined by parallelograms (often rectangles)

Probability The likelihood that an event will occur, measured on a scale between 0 and 1

Product A number that is the result of multiplication

Pronumeral A letter or symbol used to represent a number

Proper fraction A fraction where the numerator is less than the denominator

Proportion One part considered in relation to the whole

Protractor A semicircular or circular tool for measuring or drawing angles

Pyramid A solid where the base has straight sides and the other faces are triangles meeting at a vertex (the apex)

Q

Quadrilateral A four-sided plane (2D) shape with straight sides

Quotient A number that is the result of division

R

Radius A line interval from the centre of a circle to its circumference (boundary), or the length of that interval

Range The difference between the highest and lowest numbers in a set

Ray Part of a straight line, extending forever in one direction

Reciprocal A fraction in which the numerator and denominator have changed places

Rectangle A quadrilateral with both pairs of opposite sides equal and parallel, and with four right angles

Reflection Flipping a geometrical figure across a line

Remainder Leftover amount after one number has been divided by another

Revolution A full turn or circle $(360°)$

Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal

Right triangle A triangle in which one of the angles is a right angle $(90°)$

Rotation A turn around a centre point

Rotational symmetry When a figure rotated less than $360°$ matches its original position

Rounding Approximating a number to a specified number of places

S

Sample Collection of data from a smaller subset of the whole population

Sample space The list of all the possible outcomes of a trial

Scalene triangle A triangle with no equal sides or angles

Secondary source Information collected, published and summarised by someone else before we use it

Segment A section of a straight line (also called an interval)

Sequence A set of numbers ordered according to a rule

Simplifying Finding the simplest possible expression

Solving Finding the value of an unknown variable

Square A quadrilateral with all sides equal in length and four right angles

Square number The result of multiplying a number by itself

Square root (of a given number) The number that, when multiplied by itself, gives the number

Stem-and-leaf plot A table that lists numbers in order, grouped in rows

Substitute To replace pronumerals with numbers

Supplementary Having a sum of $180°$

T

Table of values Pairs of numbers related by a rule, often used for plotting points on a number plane

Term One of the numbers in a sequence

Transformation Changing a figure's position, size or shape through a mathematical process

Translation Moving a shape a certain distance in a given direction

Transversal A line that cuts two or more lines

Trapezium A quadrilateral with exactly one pair of parallel sides

Travel graph A line graph that describes a traveller's position at different times

Trial One run of a chance experiment

Triangle A plane (2D) shape with three straight sides and three angles

U

Unknown A pronumeral with a value that is yet to be found to make the equation true

V

Variable Something that is measurable and observable, which is expected to change over time or between each observation

Vertex A point where two straight lines meet to make an angle

Vertically opposite Opposite each other across a common vertex

Vinculum The separating line in a fraction

Volume The amount of three-dimensional space in (or occupied by) an object

X

***x*-axis** Horizontal axis of the number plane

Y

***y*-axis** Vertical axis of the number plane

Answers

Chapter 1

Pre-test

1 **a** 11 **b** 137 **c** 104 **d** 10 102
2 **a** C **b** A **c** D **d** B
3 **a** 57 **b** 116 **c** 2044 **d** 11 002
4 **a** 13 **b** 37 **c** 99 **d** 8000
e 26 **f** 28
5 **a** 42, 49, 56, 63 **b** 54, 63, 72, 81
c 66, 77, 88, 99
6 **a** 2 **b** 1 **c** 3 **d** 12
7 **a** 14 **b** 23 **c** 119 **d** 150
e 210 **f** 1110 **g** 500 **h** 908
i 9 **j** 32 **k** 79 **l** 79
m 112 **n** 398 **o** 699 **p** 901
8 **a** 30 **b** 63 **c** 144 **d** 88
e 56 **f** 130 **g** 1100 **h** 8000
i 5 **j** 2 **k** 11 **l** 11
m 10 **n** 9 **o** 3 **p** 9
9 **a** 37, 58, 59, 62, 73, 159 **b** 31, 103, 130, 301, 310
c 13 429, 24 319, 24 913, 24 931, 29 143
10 **a** 0 **b** 1 **c** 1 **d** 2

Exercise 1A

1 **a** 0 **b** 5 **c** 2 **d** 7
2 **a** 46 **b** 263 **c** 7421 **d** 36 015
3 **a** B **b** E **c** D
d A **e** C **f** F
4 **a** 7 **b** 70 **c** 70 **d** 700
e 700 **f** 7000 **g** 700 **h** 70 000
5 **a** 3 **b** 30 **c** 30 **d** 300
e 30 **f** 3000 **g** 3 **h** 30 000
6 **a** 20 **b** 2000 **c** 200 **d** 200 000
7 **a** true **b** false **c** true **d** true
e false **f** true **g** false **h** true
i true **j** false **k** false **l** true
8 **a** 44, 45, 54, 55 **b** 29, 92, 279, 729, 927
c 4, 23, 136, 951 **d** 345, 354, 435, 453, 534, 543
e 12 345, 31 254, 34 512, 54 321
f 1001, 1010, 1100, 10 001, 10 100
9 **a** 872 **b** 13 469
10 **a** 6 **b** 6 **c** 24
11 **a** **i** 7 000 **ii** 46 000
iii 712 000 **iv** 5 000 000
v 44 000 000 **vi** 6 000 000 000
vii 437 000 000 000 **viii** 15 000 000 000 000
b 1 with 100 zeros

Exercise 1B

1 **a** add, plus, sum
b minus, take away, subtract
2 **a** 7 **b** 10 **c** 69
d 4 **e** 12 **f** 20
3 **a** **i** 8 **ii** 27 **iii** 132
b **i** 6 **ii** 16 **iii** 8
4 **a** 18 **b** 19 **c** 32
d 140 **e** 21 **f** 9
5 **a** 34 **b** 46 **c** 59
d 64 **e** 97 **f** 579
g 748 **h** 948 **i** 5597
6 **a** 11 **b** 36 **c** 51
d 112 **e** 233 **f** 132
g 4 **h** 3111 **i** 10 001
7 **a** 24 **b** 75 **c** 95
d 133 **e** 167 **f** 297
8 **a** 24 **b** 26 **c** 108
d 222 **e** 317 **f** 5017
9 $79
10 38 hours
11 107 runs
12 **a** 12 **b** 27 **c** 107
d 133 **e** 14 **f** 90
g 1019 **h** 0 **i** 3
13 **a** 71 cards **b** 107 cards
14 **a** true **b** true **c** true
d false **e** true **f** false
15 **a** 3 **b** 6 **c** $c - b = a$
d $b - a = c$
16 **a**

6	1	8
7	5	3
2	9	4

b

10	15	8
9	11	13
14	7	12

c

15	20	13
14	16	18
19	12	17

d

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Exercise 1C

1 **a** 13 **b** 13 **c** 20 **d** 22
e 17 **f** 101 **g** 144 **h** 110
i 1005 **j** 143 **k** 201 **l** 1105
2 **a** 7 **b** 2 **c** 5 **d** 6
e 8 **f** 27 **g** 67 **h** 84
i 15 **j** 92 **k** 29 **l** 979
3 **a** 8 **b** 1 **c** 1 **d** 6
e 2 **f** 1 **g** 8 **h** 3

4 **a** 87 **b** 99 **c** 41 **d** 86
e 81 **f** 51 **g** 92 **h** 81
5 **a** 222 **b** 322 **c** 521 **d** 920
e 226 **f** 745 **g** 1923 **h** 5080
6 **a** 31 **b** 20 **c** 19 **d** 58
e 58 **f** 29 **g** 26 **h** 39
7 **a** 149 **b** 273 **c** 656 **d** 888
e 36 **f** 112 **g** 79 **h** 72
8 1854 sheep
9 576 kilometres
10 **a** 77 **b** 192 **c** 418 **d** 4208
11 **a**
$$\begin{array}{r} 3\,\boxed{8} \\ +\,5\;3 \\ \hline \boxed{9}\,1 \end{array}$$
b
$$\begin{array}{r} 1\,\boxed{1}\,4 \\ +\;\;7\,\boxed{7} \\ \hline \boxed{1}\,9\;1 \end{array}$$
c
$$\begin{array}{r} 6\,\boxed{2} \\ -\,2\;8 \\ \hline \boxed{3}\,4 \end{array}$$
d
$$\begin{array}{r} 2\,\boxed{6}\,5 \\ -\,\boxed{1}\,8\,\boxed{4} \\ \hline 8\;1 \end{array}$$
12 **a** Answers may vary.
b Different combinations in the middle column can be used to create the sum.
13 **a**

62	67	60
61	63	65
66	59	64

b

101	115	114	104
112	106	107	109
108	110	111	105
113	103	102	116

Exercise 1D

1 **a** 10, 12, 14 **b** 15, 18, 21
c 35, 42, 49 **d** 20, 24, 28
e 44, 55, 66 **f** 68, 85, 102
2 **a** 4 **b** 2 **c** 11
d 2 **e** 4 **f** 7
3 **a** 22 **b** 27 **c** 32 **d** 56
e 28 **f** 60 **g** 44 **h** 77
i 108 **j** 72 **k** 21 **l** 54
m 30 **n** 110 **o** 144 **p** 40
q 49 **r** 63 **s** 132 **t** 72
u 55 **v** 22 **w** 24 **x** 96
y 36
4 **a** 105 **b** 124 **c** 93 **d** 132
e 115 **f** 217 **g** 198 **h** 252
i 336
5 **a** 57 **b** 38 **c** 58 **d** 116
e 90 **f** 126 **g** 117 **h** 196
i 234
6 **a** 70 **b** 90 **c** 110 **d** 180
e 96 **f** 54 **g** 96 **h** 72
i 72
7 **a** 66 **b** 129 **c** 432 **d** 165
e 148 **f** 459 **g** 336 **h** 472
i 258 **j** 2849 **k** 2630 **l** 31 581
8 **a** 3 **b** 0 **c** 5 **d** 2
9 $264
10 1680 metres
11 116 cards
12 **a** true **b** true **c** false **d** true
e true **f** true **g** false **h** true
i false
13 **a**
$$\begin{array}{r} 3\;9 \\ \times\;\;7 \\ \hline 2\boxed{7}\,3 \end{array}$$
b
$$\begin{array}{r} 2\;5 \\ \times\;\boxed{5} \\ \hline 12\;5 \end{array}$$
c
$$\begin{array}{r} 7\;9 \\ \times\;\boxed{3} \\ \hline \boxed{2}3\;7 \end{array}$$
d
$$\begin{array}{r} 1\;3\;2 \\ \times\;\;\;\boxed{8} \\ \hline 10\,\boxed{5}\,6 \end{array}$$
e
$$\begin{array}{r} 2\,\boxed{7} \\ \times\;\;7 \\ \hline \boxed{1}8\;9 \end{array}$$
f
$$\begin{array}{r} \boxed{3}\,\boxed{9} \\ \times\;\;9 \\ \hline 3\,5\;1 \end{array}$$
g
$$\begin{array}{r} 2\;3\,\boxed{2} \\ \times\;\;\;5 \\ \hline 1\boxed{1}6\;0 \end{array}$$
h
$$\begin{array}{r} \boxed{3}\,\boxed{1}\,4 \\ \times\;\;\;\;7 \\ \hline \boxed{2}1\;9\;8 \end{array}$$
i
$$\begin{array}{r} \boxed{5}\,\boxed{4}\,\boxed{6} \\ \times\;\;\;\;\;3 \\ \hline 1\;6\;3\;8 \end{array}$$
14 Three ways: (0, 1), (1, 5), (2, 9). You cannot carry a number to the hundreds column.

Exercise 1E

1 **a** 10 **b** 100 **c** 1000 **d** 10
2 **a** 81 **b** 78 **c** 460 **d** 726
3 **a** 120 **b** 120 **c** 250 **d** 720
e 180 **f** 180 **g** 350 **h** 540
4 **a** 400 **b** 290 **c** 1830 **d** 4600
e 37 000 **f** 1920 **g** 301 000 **h** 248 000
i 50 000 **j** 63 000 **k** 14 410
l 29 100 000
5 **a** 240 **b** 540 **c** 520 **d** 630
e 340 **f** 1440 **g** 6440 **h** 22 500
i 41 400 **j** 460 000 **k** 63 400
l 9 387 000
6 **a** 252 **b** 286 **c** 434 **d** 645
e 407 **f** 1368 **g** 1232 **h** 1254
i 2116 **j** 2268 **k** 1426 **l** 4680
7 $420
8 $2176
9 **a** 1353 **b** 3036 **c** 1890 **d** 9416
e 18 216 **f** 40 768 **g** 18 620 **h** 33 858
10 $6020
11 86 400 seconds
12 **a** 39 984 **b** 927 908 **c** 4 752 188
d 146 420 482
13 **a**
$$\begin{array}{r} 2\,\boxed{3} \\ \times\;1\;\boxed{7} \\ \hline 1\,\boxed{6}\,1 \\ 2\,\boxed{3}\,0 \\ \hline \boxed{3}\,\boxed{9}\,1 \end{array}$$
b
$$\begin{array}{r} 1\,\boxed{4}\,3 \\ \times\;\;\;1\,\boxed{3} \\ \hline \boxed{4}\,2\;9 \\ 1\,\boxed{4}\,3\,\boxed{0} \\ \hline 1\,\boxed{8}\,5\,\boxed{9} \end{array}$$
c
$$\begin{array}{r} \boxed{4}\,\boxed{9} \\ \times\;\;3\;7 \\ \hline 3\;4\;3 \\ \boxed{1}\,4\,\boxed{7}\,\boxed{0} \\ \hline \boxed{1}\,\boxed{8}\,\boxed{1}\,3 \end{array}$$
d
$$\begin{array}{r} \boxed{1}\,2\,\boxed{6} \\ \times\;\;\;2\,\boxed{1} \\ \hline 1\;2\;6 \\ \boxed{2}\,5\;2\,\boxed{0} \\ \hline \boxed{2}\,6\,\boxed{4}\,\boxed{6} \end{array}$$

Exercise 1F

1 **a** 0 **b** 0 **c** 0 **d** 0
e 4 **f** 0 **g** 3 **h** 0
2 **a** 2 **b** 3 **c** 7 **d** 12
3 **a** 1 **b** 2 **c** 2 **d** 5

4 **a** 4 **b** 3 **c** 6 **d** 5
e 7 **f** 9 **g** 8 **h** 11
i 10 **j** 4 **k** 11 **l** 12
m 8 **n** 12 **o** 11 **p** 12
5 **a** 31 **b** 21 **c** 21 **d** 21
e 21 **f** 11 **g** 32 **h** 16
6 **a** 29 **b** 19 **c** 24 **d** 9
e 39 **f** 14 **g** 29 **h** 9
7 **a** 22 **b** 31 **c** 17 **d** 7
8 **a** 26 **b** 1094 **c** 0 **d** 0
9 **a** 23 rem. 2 = $23\frac{2}{3}$ **b** 13 rem. 1 = $13\frac{1}{7}$
c 27 rem. 4 = $27\frac{4}{5}$ **d** 41 rem. 1 = $41\frac{1}{6}$
e 43 rem. 2 = $43\frac{2}{5}$ **f** 126 rem. 2 = $126\frac{1}{2}$
g 264 rem. 2 = $264\frac{2}{3}$ **h** 90 rem. 4 = $90\frac{4}{9}$
i 543 rem. 1 = $543\frac{1}{4}$ **j** 20 333 rem. 2 = $20\,333\frac{2}{3}$
k 818 rem. 3 = $818\frac{3}{5}$ **l** 10 001 rem. 0 = 10 001
10 **a** 1 **b** 1 **c** 5 **d** 5
11 13 packs
12 124 packs
13 67 posts
14 15 taxis
15 $204
16 **a** 33 rem. 8 **b** 54 rem. 8 **c** 31 rem. 1
d 108 rem. 1 **e** 91 rem. 16 **f** 123 rem. 26
17

1	6	20	56
40	28	2	3
14	5	24	4
12	8	7	10

Progress quiz

1 **a** 50 **b** 5000 **c** 50 000
2 **a** 6 **b** 24
3 **a** 478 **b** 223 **c** 66 **d** 42
4 **a** true **b** true
5 **a** 95 **b** 5481 **c** 15 **d** 117
6 **a** 108 **b** 160
7 **a** 574 **b** 1602
8 **a** 71 000 **b** 7400 **c** 276 **d** 1357
9 $315
10 **a** 21 **b** 99 **c** 8 **d** 15
11 **a** quotient = 45, remainder = 2
b quotient = 529, remainder = 5

Exercise 1G

1 **a** up **b** down **c** up **d** up
e down **f** down
2 **a** up **b** up **c** down **d** down
e down **f** down **g** up **h** up
i down **j** up **k** up **l** down
3 **a** down **b** down **c** up **d** up
e up **f** down **g** down **h** up
i up **j** down **k** up **l** up
4 **a** 60 **b** 30 **c** 120 **d** 190
e 200 **f** 900 **g** 100 **h** 600
i 2000 **j** 1000 **k** 8000 **l** 10 000
5 **a** 20 **b** 30 **c** 100 **d** 900
e 200 **f** 700 **g** 100 **h** 1000
i 6000 **j** 90 000 **k** 10 000 **l** 10
6 **a** 120 **b** 160 **c** 100 **d** 12
e 10 **f** 3000 **g** 1200 **h** 3
i 40 **j** 2000 **k** 4000 **l** 100
7 $1200
8 ≈ 2100 scoops
9 ≈ 1200 sheep
10 ≈ 10 people
11 **a** larger **b** smaller **c** smaller **d** larger
12 **a** 200 **b** 100 000 **c** 800
d 3 000 000
13 Compare your answer with a friend's.

Exercise 1H

1 **a** multiplication **b** brackets
c brackets **d** multiplication
2 **a** addition **b** division
c multiplication **d** multiplication
e division **f** addition
g division **h** multiplication
i division **j** subtraction
k multiplication **l** division
3 **a** 7 **b** 19 **c** 7 **d** 20
e 3 **f** 2 **g** 3 **h** 24
i 1 **j** 23 **k** 21 **l** 0
m 18 **n** 32 **o** 2 **p** 22
q 22 **r** 38 **s** 153 **t** 28
u 200
4 **a** 10 **b** 3 **c** 2 **d** 9
e 8 **f** 0 **g** 22 **h** 2
i 9 **j** 18 **k** 3 **l** 10
m 121 **n** 20 **o** 1 **p** 16
5 **a** true **b** false **c** false **d** true
6 **a** 27 **b** 10 **c** 8 **d** 77
e 30 **f** 21 **g** 192
7 75 books
8 45 TV sets
9 **a** $(4 + 2) \times 3 = 18$ **b** $9 \div (12 - 9) = 3$
c $2 \times (3 + 4) - 5 = 9$ **d** $(3 + 2) \times (7 - 3) = 20$
e $(10 - 7) \div (21 - 18) = 1$ **f** $(4 + 10) \div (21 \div 3) = 2$
10 **a** no **b** yes **c** no **d** yes
e yes **f** no
11 **a** 58 **b** 18 **c** 13 **d** 28
e 22
12 **a** $[20 - (31 - 19)] \times 2 = 16$ **b** $50 \div (2 \times 5) - 4 = 1$
c $(25 - 19) \times (3 + 7) \div 12 + 1 = 6$

Maths@Work: Stock controller at sports events

1 *See table at top of next page.*
2 *See second table at top of next page.*
3

Event	AFL	Big Bash league	Japanese baseball	Major league baseball
Average Attendance	33 597	29 443	28 581	30 354

4 Many correct answers including: the weather; the popularity of the team; a final or earlier in the season; and ticket price.
5 *See third table at top of next page.*

Venue	Event	Attendance	Water bottles required	Total wholesale cost @ $0.80/bottle	Earnings @ $3 a bottle
Suncorp Stadium, Brisbane	Brisbane Roar vs Perth Glory	35 200	52 800	$42 240	$158 400
MCG, Melbourne Cricket Ground	Hawthorn vs Freemantle	100 007	150 011	$120 008	$450 032
WACA, WA Cricket Association	T20 cricket India vs Australia	34 527	51 791	$41 432	$155 372
SCG, Sydney Cricket Ground	Sydney Swans vs Western Bulldogs	33 386	50 079	$40 063	$150 237
Adelaide Oval	Adelaide Crows vs West Coast Eagles	53 445	80 168	$64 134	$240 503
NIB Stadium, Perth	Wallabies vs Argentina	16 202	24 303	$19 442	$72 909
Darwin Football Stadium	Brumbies v Reds	4600	6900	$5520	$20 700
ANZ Stadium, Sydney	Broncos v Cowboys	82 758	124 137	$99 310	$372 411

Event	Brisbane Roar vs Perth Glory	Hawthorn vs Freemantle	T20 cricket India vs Australia	Sydney Swans vs Western Bulldogs	Adelaide Crows vs West Coast Eagles	Wallabies vs Argentina	Brumbies v Reds	Broncos v Cowboys
Attendance	35 000	100 000	35 000	33 000	53 000	16 000	5000	83 000

Stadium	Seating capacity	Fruit salad	Hot chips	Pies	Mini pizzas	Fresh wraps
WACA, Perth	24 500	4900	12 250	36 750	9800	9188
Adelaide Oval	53 500	10 700	26 750	80 250	21 400	20 063
The Gabba, Brisbane	42 000	8400	21 000	63 000	16 800	15 750
ANZ Stadium, Sydney	83 500	16 700	41 750	125 250	33 400	31 313
Rod Laver Arena, Melbourne	14 820	2964	7410	22 230	5928	5558
Blundstone Arena, Hobart	20 000	4000	10 000	30 000	8000	7500
Darwin Football Stadium	6000	1200	3000	9000	2400	2250

Puzzles and games

1 a

15	20	13
14	16	18
19	12	17

b

11	16	9
10	12	14
15	8	13

c

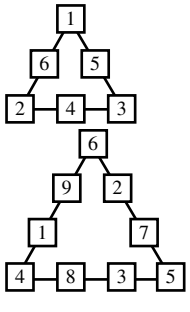
3	17	16	6
14	8	9	11
10	12	13	7
15	5	4	18

2 a $(5 + 2) \times 3 = 21$

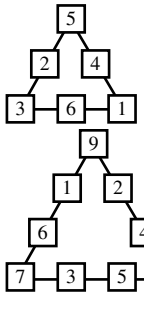
b $(16 - 8) \div (10 - 6) = 2$

c $4 + (2 \times 7 - 1) \times 3 = 43$

3 a i



ii



b i

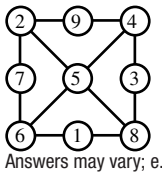
ii

4

5	4	6	1	3	2	9	8	7
2	8	3	7	5	9	6	1	4
7	9	1	6	8	4	3	2	5
8	2	4	5	1	3	7	6	9
1	3	7	9	2	6	4	5	8
9	6	5	4	7	8	1	3	2
3	7	9	2	6	5	8	4	1
6	1	2	8	4	7	5	9	3
4	5	8	3	9	1	2	7	6

1	7	5	4	6	9	3	8	2
2	9	4	1	3	8	6	5	7
8	6	3	7	5	2	9	1	4
3	4	1	2	7	6	8	9	5
9	2	8	5	4	3	7	6	1
7	5	6	8	9	1	2	4	3
4	3	9	6	2	5	1	7	8
6	1	7	3	8	4	5	2	9
5	8	2	9	1	7	4	3	6

5



6 Answers may vary; e.g.

a
$$\begin{array}{r} 217 \\ \times\quad 7 \\ \hline 1519 \end{array}$$

b
$$\begin{array}{r} 295 \\ \times\quad 3 \\ \hline 885 \end{array}$$

7 a 2 **b** 5

Multiple-choice questions

1 B **2** C **3** E **4** A **5** B
6 A **7** D **8** C **9** B **10** A

Short-answer questions

1 a 137, 173, 317, 371, 713, 731
b 199, 999, 1000, 1001, 1010, 1090, 1900

2 a 50 **b** 5000 **c** 50 000

3 a 459 **b** 363 **c** 95 **d** 217

4 a 128 **b** 355 **c** 191 **d** 739

5 a 95 **b** 132 **c** 220 **d** 41
e 33 **f** 24 **g** 29 000 **h** 10 800
i 14 678

6 a 156 **b** 840 **c** 1413 **d** 351
e 45 **f** 101 rem. 3
g 46 rem. 5 **h** 7540 rem. 2

7 a
$$\begin{array}{r} 223 \\ +\,738 \\ \hline 961 \end{array}$$

b
$$\begin{array}{r} 729 \\ -\,473 \\ \hline 256 \end{array}$$

c
$$\begin{array}{r} 41 \\ \times\quad 7 \\ \hline 287 \end{array}$$

d
$$2\overline{)916} \quad 458$$

8 a 70 **b** 3300 **c** 1000

9 a 800 **b** 400 **c** 5000 **d** 10

10 a 24 **b** 4 **c** 1 **d** 92
e 14 **f** 3 **g** 20 **h** 0
i 13

Extended-response questions

1 a 90 loads **b** 360 kilometres
c $27 500 **d** $4000

2 a 313 sweets
b 78
c jelly beans, 5

Chapter 2

Pre-test

1 a C **b** D **c** E
d B **e** A **f** F

2 a 90° **b** 180° **c** 360° **d** 270°
e 45° **f** 315°

3 a 60° **b** 140° **c** 125° **d** 80°

4 a no **b** yes **c** no

5 a 80° **b** 150° **c** 150°

Exercise 2A

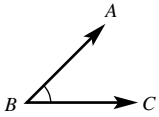
1 a F **b** A **c** B **d** E
e D **f** C

2 a • P

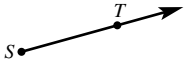
b



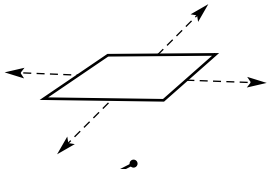
c



d



e



f



3 a ray **b** line **c** segment

4 a line **b** plane **c** plane **d** point
e line **f** plane **g** line **h** point

5 a point T **b** line CD **c** angle BAC
d plane **e** ray PQ **f** segment ST

6 a $\angle AOB$ **b** $\angle XYZ$ **c** $\angle BOC$ **d** $\angle BAC$
e $\angle BEA$ **f** $\angle AOC$

7 a collinear **b** concurrent

8 a yes **b** no

9 a no **b** yes

10 a 3 **b** 6 **c** 8 **d** 14

11 10

12 Missing numbers are 0, 1, 3, 6, 10, 15. For 5 points, add 4 to the previous total; for 6 points, add 5 to the previous total, and so on.

13 All segments should intersect at the same point; i.e. are concurrent.

Exercise 2B

1 a acute **b** obtuse **c** right
d straight **e** revolution **f** acute
g reflex **h** reflex **i** obtuse

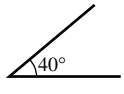
2 Answers may vary.

3 a 50° **b** 145° **c** 160°
d 55° **e** 90° **f** 250°

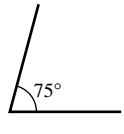
4 a 40° **b** 55° **c** 90° **d** 125°
e 165° **f** 180° **g** 230° **h** 270°
i 325°

5 a acute **b** acute **c** right **d** obtuse
e obtuse **f** straight **g** reflex **h** reflex
i reflex

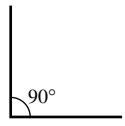
6 a



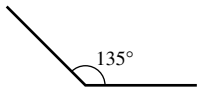
b



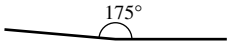
c



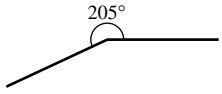
d



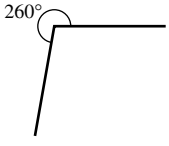
e



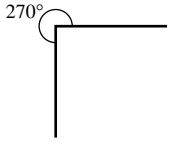
f



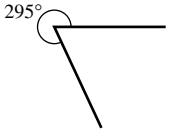
g



h



i



j



7 **a** 55° **b** 29° **c** 110° **d** 130°
8 **a** 2 **b** 3 **c** 4
9 **a** 180° **b** 360° **c** 90° **d** 270°
e 30° **f** 120° **g** 330° **h** 6°
10 Use the revolution to get 360° − 60° = 300°.
11 **a** 180° **b** 90° **c** 120° **d** 30°
12 **a** 115° **b** 127.5° **c** 85° **d** 77.5°

Exercise 2C

1 **a**, **b** angles should add to 90°
c complementary
2 **a**, **b** angles should add to 180°
c supplementary
3 **a**, **b** angles should add to 360°
c vertically opposite angles
4 **a** $\angle ABD$ **b** $\angle CBF$ **c** $\angle CBT$
5 **a** $\angle UOV$ **b** $\angle MON$ **c** $\angle BOA$
6 **a** $\angle MFS$ or $\angle SFM$ **b** $\angle TFS$ or $\angle SFT$
c $\angle BFS$ or $\angle SFB$ **d** $\angle SFX$ or $\angle XFS$
7 **a** 60 **b** 15 **c** 69 **d** 135
e 70 **f** 141 **g** 115 **h** 37
i 50 **j** 240 **k** 270 **l** 140
8 **a** N **b** N **c** S **d** N
e C **f** C **g** C **h** S
9 **a** $\angle BOC$ **b** $\angle AOD$ and $\angle BOC$ **c** $\angle COD$
10 **a** 30 **b** 75 **c** 60 **d** 135
e 45 **f** 130
11 **a** No, should add to 90°. **b** Yes, they add to 180°.
c Yes, they add to 360°. **d** Yes, they are equal.
e No, they should be equal. **f** No, should add to 360°.
12 24°
13 **a** 30 **b** 60 **c** 60 **d** 45
e 180 **f** 36

Exercise 2D

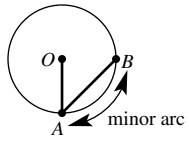
1 **a** 4 **b** no
2 **a** 2 **b** yes
3 **a** equal **b** supplementary
c equal **d** equal
4 **a** $\angle DEH$ **b** $\angle BEF$ **c** $\angle DEB$ **d** $\angle CBG$
5 **a** $\angle FEG$ **b** $\angle DEB$ **c** $\angle GEB$ **d** $\angle ABC$
6 **a** 130, corresponding **b** 70, corresponding
c 110, alternate **d** 120, alternate
e 130, vertically opposite **f** 67, vertically opposite
g 65, cointerior **h** 118, cointerior
i 100, corresponding **j** 117, vertically opposite
k 116, cointerior **l** 116, alternate
m 110, cointerior **n** 48, cointerior
o 117, corresponding
7 **a** $a = 70, b = 70, c = 110$ **b** $a = 120, b = 120, c = 60$
c $a = 98, b = 82, c = 82, d = 82$
d $a = 90, b = 90, c = 90$ **e** $a = 95, b = 85, c = 95$
f $a = 61, b = 119$
8 **a** No, corresponding angles should be equal.
b Yes, alternate angles are equal.
c Yes, cointerior angles are supplementary.
d Yes, corresponding angles are equal.
e No, alternate angles should be equal.
f No, cointerior angles should be supplementary.
9 **a** 35 **b** 41 **c** 110 **d** 30
e 60 **f** 141
10 **a** 65 **b** 100 **c** 62 **d** 67
e 42 **f** 57 **g** 100 **h** 130
i 59
11 **a** $a = 120, b = 120$ **b** 60
c opposite angles are equal
12 **a** 60 **b** 115 **c** 123 **d** 80
e 60 **f** 335

Progress quiz

1 **a** $\angle AOC$ or $\angle COA$ **b** $\angle YOZ$ or $\angle ZOY$
2 **a** Line = Z **b** Line segment = Y **c** Ray = X
3 **a** acute **b** reflex
c straight **d** acute
e obtuse **f** reflex
g right **h** obtuse
4 **A**
5 **a** N **b** S **c** S **d** C
6 **a** 128 **b** 26 **c** 135 **d** 38
7 **a** $\angle ABF$ or $\angle ABG$
b $\angle CFH$ or $\angle BFH$
c $\angle EFB$ or $\angle EFC$
d $\angle ABG$ or $\angle ABF$
8 **a** 115, alternate **b** 88, corresponding
9 **a** $a = 40, b = 140, c = 140$
b $a = 82, b = 82, c = 82, d = 82$

Exercise 2E

1 Check each one using a ruler.
2 **a** chord **b** radius **c** centre
d minor arc **e** diameter
3 **a–e**



4 $\angle AEC$ should be 90°.
5 $\angle BAD$ should be 60°.
6 $\angle AOE$ and $\angle BOE$ should be equal.
7 Your two circles should just touch.
8 $\angle BAC$ and $\angle ABC$ should be equal.

9 a Construct a 60° angle (see question **4**) and then bisect this angle by constructing the angle bisector to form two 30° angles (see question **5**).
b Construct an angle bisector of one of the 30° angles from part **a**.
10 a First, construct a 90° angle by constructing a perpendicular line and then construct the angle bisector of the 90° angle.
b Construct the angle bisector of one of the 45° angles from part **a**.
11 a Circles should overlap and be the same size.
b $AE = BE$
c AB is at 90° to CD and CD cuts AB in half.

Exercise 2F

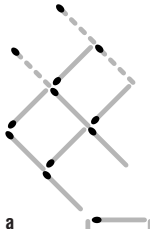
1–8 Answers may vary and can be checked by testing the properties of the constructions.

Maths@Work: Glass cutter and design artist for stained-glass windows

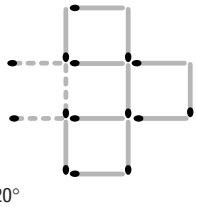
1 Answers will vary.
2 Answers will vary.
3 Answers will vary.

Puzzles and games

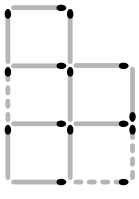
1



2 a



b



3 720°
4 a 135° **b** 77.5° **c** 143.5°
5 a 80 **b** 120 **c** 80
d 110 **e** 250 **f** 40

Multiple-choice questions

1 C **2** B **3** D **4** E **5** D
6 E **7** C **8** A **9** E **10** B

Short-answer questions

1 a segment AB **b** $\angle AOB$
c point O **d** plane
e ray AB **f** line AB
2 a acute, 35° **b** reflex, 210°
c obtuse, 115° **d** reflex, 305°
3 Check by measuring with a protractor. A right angle is 90°.
4 a 20 **b** 230 **c** 35 **d** 41
e 15 **f** 38
5 a $a°$ and $b°$ **b** $a°$ and $d°$
c $a°$ and $c°$ **d** $b°$ and $c°$
e $c°$ and $d°$ or $b°$ and $d°$
6 a 61 **b** 128 **c** 59
7 a Yes, corresponding angles are equal.
b No, alternate angles should be equal.
c No, cointerior angles should be supplementary.
8 a 100 **b** 95 **c** 30
9 a The angle should be divided in half.
b The angle should be 90°.

Extended-response questions

1 a i 30° **ii** 180° **iii** 270°
b i 30° **ii** 210° **iii** 15°
c i 180° **ii** 90° **iii** 60°
iv 75° **v** 105° **vi** 127.5°
2 a 12 pieces
b 30
c i 15 **ii** 22.5 **iii** 20 **iv** 24

Chapter 3

Pre-test

1 a 100 **b** 1 000 **c** 1 000 000
2 a 4 **b** 10 **c** 20 **d** 1
e 1, 2, 4, 5, 10, 20
3 a 4 remainder 2
b 3 remainder 1
c 2 remainder 3
d 3 remainder 5
4 a 10 **b** 4 **c** 19
d 24 **e** 18 **f** 5
5 1, 2, 3, 4, 6, 8, 12, 24
6 a $3 < 7$ **b** $12 > 5$
7 a 6, 8, 10 **b** 5, 7, 9
8 a 6 **b** 8 **c** 6
d 9 **e** 4 **f** 1
g 8 **h** $4 \times 4 = 16$
9 a $8 = 4 \times 2$ **b** $15 = 3 \times 5$
c $12 \times 4 = 48$ **d** $4 \times 4 = 16 = 8 \times 2$
e $12 \times 3 = 36 = 4 \times 9$
10 a 7 **b** 3 **c** 7
11 a 11 **b** yes **c** 10 remainder 2
d no
12 a 23 **b** 36 **c** 13 **d** 6
13 a B **b** A **c** D **d** C
14 a $A = 16$ cm^2
b side = 6 cm

Exercise 3A

1 a $12 \times 2 = 24$ or $2 \times 12 = 24$
b $24 \times 1 = 24$ or $1 \times 24 = 24$
$3 \times 8 = 24$ or $8 \times 3 = 24$
$4 \times 6 = 24$ or $6 \times 4 = 24$
2 a $1 \times 12 = 12$, $2 \times 6 = 12$, $3 \times 4 = 12$,
1, 2, 3, 4, 6, 12
b $1 \times 5 = 5$; 1, 5
c $1 \times 30 = 30$, $2 \times 15 = 30$, $3 \times 10 = 30$, $5 \times 6 = 30$
1, 2, 3, 5, 6, 10, 15, 30
3 a 10, 20, 30
b 20, 40, 50, 60
c 14, 21, 35
4 a 4, 8, 12, 16 **b** 2
c 2 **d** 3, 7, 11, 15
5 a factors
b 2, 3, 4
c 12, 24, 30
d multiples

6 **a** 1, 2, 5, 10
b 1, 2, 3, 4, 6, 8, 12, 24
c 1, 17
d 1, 2, 3, 4, 6, 9, 12, 18, 36
e 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
f 1, 2, 3, 6, 7, 14, 21, 42
g 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
h 1, 2, 3, 4, 6, 12
i 1, 2, 4, 7, 14, 28

7 **a** 5, 10, 15, 20, 25, 30
b 8, 16, 24, 32, 40, 48
c 12, 24, 36, 48, 60, 72
d 7, 14, 21, 28, 35, 42
e 20, 40, 60, 80, 100, 120
f 75, 150, 225, 300, 375, 450
g 15, 30, 45, 60, 75, 90
h 100, 200, 300, 400, 500, 600
i 37, 74, 111, 148, 185, 222

8 **a** 3, 18 **b** 5 **c** 1, 4, 6, 9, 12, 24
d 3, 4, 5, 8, 12, 15, 24, 40, 120

9 **a** 22; 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
b 24; 5, 10, 15, 20, 25, 30
c 21; 11, 22, 33, 44, 55, 66, 77, 88, 99, 110
d 49; 12, 24, 36, 48, 60, 72, 84

10 **a** 5×6 **b** 6×6 **c** 7×6 **d** 5×8
e 8×8 **f** 10×10 or 5×20

11 **a** **i** 4, 8, 12, 16, 20, 24 **ii** 5, 10, 15, 20, 25, 30
b Zane 5 laps, Matt 4 laps
c 20 minutes

12 **a** 13 **b** 2 **c** 11 **d** 1

13 **a** 12 pens ($1\text{ m} \times 1\text{ m}$), 6 pens ($2\text{ m} \times 1\text{ m}$), 4 pens ($3\text{ m} \times 1\text{ m}$) or 3 pens ($1\text{ m} \times 4\text{ m}$)
b **i** 32 pens ($1\text{ m} \times 1\text{ m}$), 8 pens ($2\text{ m} \times 2\text{ m}$) or 2 pens ($4\text{ m} \times 4\text{ m}$)
ii 54 pens ($1\text{ m} \times 1\text{ m}$) or 6 pens ($3\text{ m} \times 3\text{ m}$)
c **i** 16 pens ($1\text{ m} \times 2\text{ m}$), 8 pens ($1\text{ m} \times 4\text{ m}$) or 4 pens ($2\text{ m} \times 4\text{ m}$)
ii 27 pens ($1\text{ m} \times 2\text{ m}$), 18 pens ($1\text{ m} \times 3\text{ m}$) or 9 pens ($2\text{ m} \times 3\text{ m}$)
d 25 pens ($2\text{ m} \times 3\text{ m}$), 15 pens ($2\text{ m} \times 5\text{ m}$), 10 pens ($3\text{ m} \times 5\text{ m}$), 6 pens ($5\text{ m} \times 5\text{ m}$), 5 pens ($5\text{ m} \times 6\text{ m}$ or $3\text{ m} \times 10\text{ m}$ or $2\text{ m} \times 15\text{ m}$), 3 pens ($2\text{ m} \times 25\text{ m}$ or $5\text{ m} \times 10\text{ m}$), or 2 pens ($3\text{ m} \times 25\text{ m}$ or $5\text{ m} \times 15\text{ m}$)

Exercise 3B

1 **a** highest common factor
b factors
c common
d lowest common multiple
e multiples, common

2 **a** 1, 2, 4 **b** 4

3 Factors of 18 are 1, 2, 3, 6, 9 and 18.
Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.
HCF = 6

4 **a** 24, 48 **b** 24

5 Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81 and 90.
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105 and 120.
LCM = 45

6 **a** 1 **b** 1 **c** 2 **d** 3
e 4 **f** 15 **g** 10 **h** 3
i 1

7 **a** 20 **b** 21 **c** 30 **d** 40
e 12 **f** 10 **g** 30 **h** 60
i 42

8 **a** 7 black ants, 11 red ants **b** 77 mm

9 **a** 3 teaspoons, 2 spoons **b** 42 cm **c** LCM

10 **a** 100 cm, 200 cm, 300 cm **b** LCM = 100

11 **a** 6 roses per bunch **b** 6 = HCF
c 6 red, 7 pink, 5 cream bunches

12 **a** 8, 16 **b** 24, 32

13 **b** after 12 minutes
c Lilli 8 laps, Aliyah 6 laps, Ciara 4 laps
d 15 mins more for Aliyah, 45 mins more for Ciara

Exercise 3C

1 **a** divisible, remainder **b** 2
c 0, 2, 4, 6 or 8 **d** divisible, $4 + 3 + 2 = 9$, 9
e 6

2 **a** B **b** A **c** C

3 **a** 5, 10 **b** 5 **c** 16, 16, 4
d 328, 328, 8

4 **a** $2 + 5 + 8 + 3 = 18$ **b** divisible, 18, 3
c divisible, 18, 9 **d** odd

5 **a** 6, 14, 8, 54, 22, 34, 50, 18, 46
b 12, 18, 30, 27, 54, 36, 42, 24
c 12, 24, 60, 54, 252, 36, 66, 84
d 168, 7168, 40, 5032, 248, 6400, 9568

6 **a** 35, 125, 15, 100, 515, 730, 105
b 20, 800, 290, 610, 590, 90, 160
c 16, 32, 220, 12, 28, 432, 72, 316, 424, 1836
d 27, 432, 99, 387, 63, 720, 2799

7 **a** yes **b** no **c** yes **d** yes
e no **f** yes **g** no **h** yes
i no **j** yes **k** no **l** yes
m yes **n** no **o** no **p** yes
q no **r** no

8 *See table at top of next page.*

9 **a** not even
b digits do not sum to a multiple of 3
c 26 is not divisible by 4
d last digit is not 0 or 5
e not divisible by 3 (sum of digits is not divisible by 3)
f 125 is not divisible by 8 and it is not even
g sum of digits is not divisible by 9
h last digit is not 0

10 **a** 2 **b** 2 **c** 0 **d** 0

11 2, 4, 8, 11, 22, 44

12 36

13 **a** yes
b Multiples of 3; adding a multiple of 3 does not change the result of the divisibility test for 3.
c 18

Exercise 3D

1 **a** prime, 1, itself **b** composite

2 **a** no **b** yes

3 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
b 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

4 **a** 11 **b** 12

5 **a** C **b** P **c** C **d** P
e C **f** C **g** P **h** P
i C **j** C **k** C **l** P
m P **n** P **o** C **p** P

6 **a** 2, 3, 7 **b** 3, 13 **c** 2, 3, 5 **d** 5
e 2, 7 **f** 2, 3

7 **a** 32, 33, 34, 35, 36, 38, 39 **b** 51, 52, 54, 55, 56, 57, 58
c 81, 82, 84, 85, 86, 87, 88

8 **a** 23, 29 **b** 41, 43, 47 **c** 71, 73, 79

9 **a** 3, 5 **b** 3, 7 **c** 5, 7
d 5, 11 **e** 11, 13 **f** 7, 19

10 14

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567	✗	✓	✗	✗	✗	✗	✓	✗
28 080	✓	✓	✓	✓	✓	✓	✓	✓
189 000	✓	✓	✓	✓	✓	✓	✓	✓
1 308 150	✓	✓	✗	✓	✓	✗	✓	✓
1 062 347	✗	✗	✗	✗	✗	✗	✗	✗

11 5 and 7, 11 and 13, 17 and 19, as well as other pairs
12 **a** no, 3, prime
b yes, 6×2 or 4×3, 12, composite
c yes, 5×2, 10, composite
d no, 5, prime
e no, 7, prime
f yes, 7×2, 14, composite
g no, 11, prime
h yes, 3×3, 9, composite

Exercise 3E

1 **a** expanded **b** index
c power **d** base
e index, power
2 **a** $3^2 = 3 \times 3$ **b** $2^4 = 2 \times 2 \times 2 \times 2$
c $5^3 = 5 \times 5 \times 5$ **d** $8^5 = 8 \times 8 \times 8 \times 8 \times 8$
3

Expanded form	Index form	Base	Index
$7 \times 7 \times 7$	7^3	7	3
$5 \times 5 \times 5$	5^3	5	3
$2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6	2	6
$6 \times 6 \times 6 \times 6$	6^4	6	4

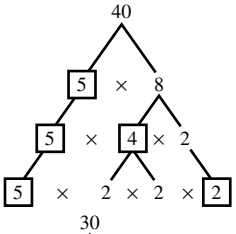
4

Value	Base	Index (power)	Value
2^3	2	3	8
5^2	5	2	25
10^4	10	4	10 000
2^7	2	7	128
1^{12}	1	12	1
12^1	12	1	12
0^5	0	5	0

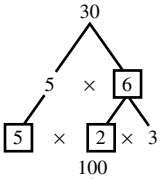
5 **a** 3^3 **b** 2^5 **c** 15^4
d 10^4 **e** 6^2 **f** 20^3
g 1^6 **h** 4^3 **i** 100^2
j $3^2 \times 5^2$ **k** $2^2 \times 7^3$ **l** $9^2 \times 12^2$
m $5^3 \times 8^2$ **n** $3^3 \times 6^3$ **o** $7^4 \times 13^2$
p $4^3 \times 7^1 \times 13^1$ **q** $9^3 \times 10^2$ **r** $2^3 \times 3^2 \times 5^2$
6 **a** $2 \times 2 \times 2 \times 2$
b 17×17
c $9 \times 9 \times 9$
d $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
e $14 \times 14 \times 14 \times 14$
f $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
g $10 \times 10 \times 10 \times 10 \times 10$
h $54 \times 54 \times 54$
i $3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$
j $4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3$
k $7 \times 7 \times 5 \times 5 \times 5$
l $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 9 \times 9 \times 9$
m $5 \times 7 \times 7 \times 7 \times 7$
n $2 \times 2 \times 3 \times 3 \times 3 \times 4$
o $11 \times 11 \times 11 \times 11 \times 11 \times 9 \times 9$
p $20 \times 20 \times 20 \times 30 \times 30$
7 **a** 6, 9 **b** 8, 16 **c** 10, 25 **d** 12, 36
8 **a** 32 **b** 64 **c** 1000 **d** 72
e 10 000 **f** 1000 **g** 64 **h** 121
9 **a** 25 **b** 1 **c** 10 **d** 64
e 128 **f** 8 **g** 22 **h** 900
i 8
10 **a** 4 **b** 2 **c** 3 **d** 6
e 3 **f** 2 **g** 2 **h** 4
11 **a** < **b** > **c** = **d** <
e > **f** > **g** < **h** <
12 125
13 **a** **i** 1110 people **ii** 1 111 110 people
b 40 min
c 50 min

Exercise 3F

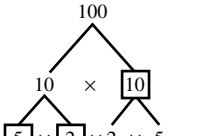
1 composite: 15, 8, 9, 27, 4, 12; prime: 13, 7, 5, 23, 11, 2, 3
2 **a**



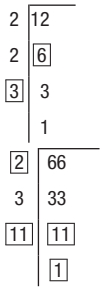
b



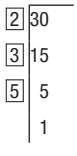
c



3 **a**



b



c

4 **a** $2^3 \times 3^2$ **b** $3^4 \times 5^2$
c $2^2 \times 3 \times 7^2$ **d** $2^2 \times 3^2 \times 11^2$

5 **a** $2^3 \times 3^2$ **b** $2^3 \times 3$ **c** 2×19 **d** $2^2 \times 11$
e $2^2 \times 31$ **f** $2^4 \times 5$ **g** $2^5 \times 3$ **h** 2^4
i 3×5^2 **j** 3×37 **k** 2^6 **l** $2^3 \times 7$

6 **a** $2^3 \times 3 \times 5^2$ **b** $2^5 \times 5^2$
c $2^3 \times 5^4$ **d** $2^5 \times 3 \times 5^2$
e $2^6 \times 5^6$ **f** $2^3 \times 3^2 \times 5^4$
g $2^2 \times 5 \times 41$ **h** $2 \times 3 \times 5 \times 23$

7

2	96
2	48
2	24
2	12
2	6
3	3
	1

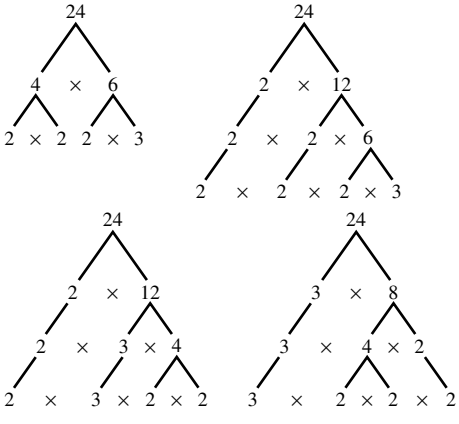
$96 = 2^5 \times 3$

8 **a** $32 = 2^5$ **b** $40 = 2^3 \times 5$
c $81 = 3^4$ **d** $144 = 2^4 \times 3^2$
e $120 = 2^3 \times 3 \times 5$ **f** $500 = 2^2 \times 5^3$
g $1800 = 2^3 \times 3^2 \times 5^2$ **h** $1250 = 2 \times 5^4$

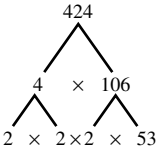
9 **a** D **b** A **c** C **d** B

10 **a** $144 = 2^4 \times 3^2$, $96 = 2^5 \times 3$
b HCF $= 2^4 \times 3 = 48$

11



12 **a** 424 cannot have a factor of 5.
b 8 is not a prime number.
c



13
$2 \times 3 \times 5 \times 7 = 210$ $2 \times 3 \times 7 \times 11 = 462$
$2 \times 3 \times 5 \times 11 = 330$ $2 \times 3 \times 7 \times 13 = 546$
$2 \times 3 \times 5 \times 13 = 390$ $2 \times 3 \times 7 \times 17 = 714$
$2 \times 3 \times 5 \times 17 = 510$ $2 \times 3 \times 7 \times 19 = 798$
$2 \times 3 \times 5 \times 19 = 570$ $2 \times 3 \times 7 \times 23 = 966$
$2 \times 3 \times 5 \times 23 = 690$ $2 \times 5 \times 7 \times 11 = 770$
$2 \times 3 \times 5 \times 29 = 870$ $2 \times 5 \times 7 \times 13 = 910$
$2 \times 3 \times 5 \times 31 = 930$ $2 \times 3 \times 11 \times 13 = 858$

Exercise 3G

1 See key ideas for table.
2 See key ideas for table.

3 **a** 36 **b** 25 **c** 121
d 100 **e** 49 **f** 144

4 **a** 5 **b** 4 **c** 10 **d** 7 cm

5 **a** 64 **b** 49 **c** 1 **d** 144
e 9 **f** 225 **g** 25 **h** 0
i 121 **j** 10 000 **k** 900 **l** 1600

6 **a** 5 **b** 3 **c** 1 **d** 11
e 0 **f** 9 **g** 7 **h** 4
i 2 **j** 12 **k** 20 **l** 13
m 50 **n** 80 **o** 90 **p** 27

7 **a** 7 **b** 5 **c** 12 **d** 30
e 64 **f** 65 **g** 36 **h** 4
i 0 **j** 81 **k** 4 **l** 13

8 **a** A square is not possible.
b Draw 4×4 square.

9 64, 81, 100

10 121, 144, 169, 196

11 **a** $9 + 16 = 25 = 5^2$
b No: $25 + 36 = 61 \neq 49$
c Yes: $36 + 64 = 100 = 10^2$
d Many answers are possible,
e.g. $5^2 + 12^2 = 13^2$, $9^2 + 12^2 = 15^2$

12 **a** **ii** 2 more dots
iii Always one dot by itself
b **ii** 2 more dots
iii Same number of dots in each row
c **ii** 3, 5, 7, 9, 11 dots
iii Dots form a square, alternate odd and even numbers
d **ii** 2, 3, 4, 5, 6 dots
iii One dot at the top, triangle-shaped
iv Each square number is the sum of two triangular numbers
$(4 = 1 + 3, 9 = 6 + 3$ etc.)

Progress quiz

1 **a** 1, 2, 4, 5, 10, 20 **b** 1, 2, 3, 4, 6, 9, 12, 18, 36

2 **a** 12, 24, 36, 48, 60 **b** 21, 42, 63, 84, 105

3 **a** 8 **b** 35 **c** 6

4 **a** 24 **b** 77 **c** 30

5 **a** 14, 52, 88, 1234 **b** 75, 141
c 52, 88 **d** 75

6 **a** not possible **b** possible

7 prime – 23, 31 composite – 15, 39, 51, 80, 91

8 **a** 2, 3 **b** 2, 3

9 **a** 4^5 **b** $5^4 \times 6^3$

10 **a** $3 \times 3 \times 3 \times 3 = 81$
b $2 \times 2 \times 2 \times 5 \times 5 = 200$
c $6 \times 6 - 2 \times 2 \times 2 = 28$

11 **a** $2^3 \times 5$ **b** $2^2 \times 3^3$ **c** 2^7

12 **a** 169 **b** 900 **c** 7
d 50 **e** 18 **f** 6

Exercise 3H

1 **a** sequence **b** term
c 0 **d** 1
2 **a** 20, 24, 28 **b** 85, 80, 75
c 8, 4, 2 **d** 40, 80, 160
3 **a** 8, 11, 14, 17, 20 **b** 32, 31, 30, 29, 28
c 52, 48, 44, 40, 36 **d** 123, 130, 137, 144, 151
4 **a** 3, 6, 12, 24, 48 **b** 5, 20, 80, 320, 1280
c 240, 120, 60, 30, 15 **d** 625, 125, 25, 5, 1
5 **a** 23, 28, 33 **b** 44, 54, 64
c 14, 11, 8 **d** 114, 116, 118
e 27, 18, 9 **f** 5, 4, 3
g 505, 606, 707 **h** 51, 45, 39
6 **a** 32, 64, 128 **b** 80, 160, 320
c 12, 6, 3 **d** 45, 15, 5
e 176, 352, 704 **f** 70 000, 700 000, 7 000 000
g 16, 8, 4 **h** 76, 38, 19
7 **a** 50, 32, 26 **b** 25, 45, 55
c 32, 64, 256 **d** 9, 15, 21
e 55, 44, 33 **f** 333, 111
8 **a** Start with 19 and subtract 2 from each term.
b Start with 48 and divide each term by 2.
c Start with 50 and add 6 to each term.
d Start with 1 and multiply each term by 3.
e Start with 625 and divide every term by 5.
f Start with 75 and subtract 3 from each term.
9 **a** 17, 23, 30 **b** 16, 22, 29
c 10, 13, 11 **d** 45, 40, 50
10 **a** 3, 1, 4, 2, 5, 3, 6 **b** 7 days
11 **a** $\times 3$ **b** -2 **c** $+11$
d start with 8 then increase by 2, then 3, then 4, …
e $\div 2$
f start with 10 then alternately increase or decrease by 10
g list prime numbers in order **h** -3
12 **a** 49, 64, 81; square numbers
b 21, 44, 65; Fibonacci numbers
c 216, 343, 512; cube numbers (i.e. 6^3, 7^3, 8^3)
d 19, 23, 29; prime numbers
e 16, 18, 20; composite numbers
f 161, 171, 181; palindromes
13 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (total = 55)

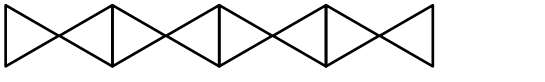
Exercise 3I

1 **a** rectangle **b** 1, 2, 3
c 6, 10, 14 **d** spatial

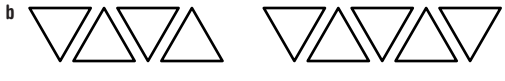
2

Number of houses	1	2	3
Number of matchsticks	6	11	16

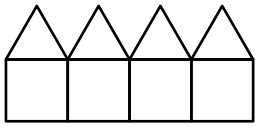
3 **a**

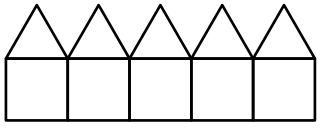


b

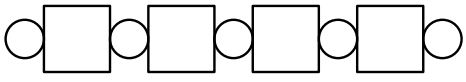


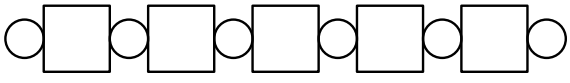
c



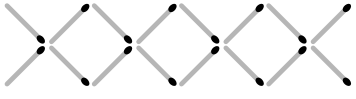


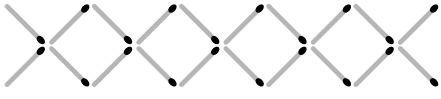
d





4 **a**



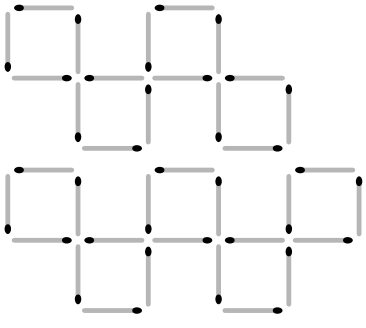


b

Number of crosses	1	2	3	4	5
Number of sticks	4	8	12	16	20

c number of sticks = 4 × number of crosses
d 40 sticks

5 **a**

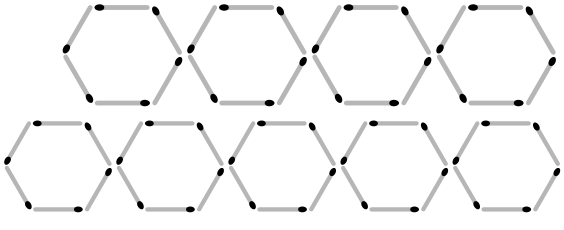


b

Number of squares	1	2	3	4	5
Number of sticks	4	8	12	16	20

c number of sticks = 4 × number of squares
d 48 sticks

6 **a**

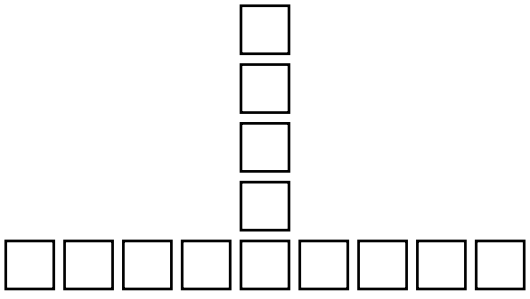


b

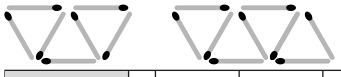
Number of hexagons	1	2	3	4	5
Number of sticks	6	12	18	24	30

c number of sticks = 6 × number of hexagons
d 120 sticks

7 B, D, A, C



8 **a**



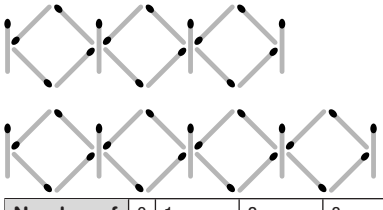
b

Number of triangles	0	1	2	3	4
Number of sticks	1	1 + 2 = 3	1 + 2 × 2 = 5	1 + 2 × 3 = 7	1 + 2 × 4 = 9

c number of sticks = 1 + 2 × number of triangles

d 25 sticks **e** 40 triangles

9 **a**



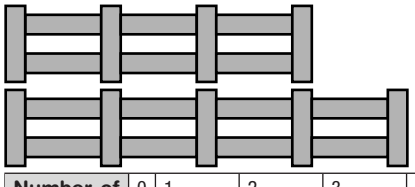
b

Number of shapes	0	1	2	3	4
Number of sticks	1	1 + 5 = 6	1 + 5 × 2 = 11	1 + 5 × 3 = 16	1 + 5 × 4 = 21

c number of sticks = 1 + 5 × number of shapes

d 101 sticks **e** 17 shapes

10 **a**



b

Number of fence sections	0	1	2	3	4
Number of planks	1	1 + 3 = 4	1 + 3 × 2 = 7	1 + 3 × 3 = 10	1 + 3 × 4 = 13

c number of planks = 1 + 3 × number of fence sections

d 28 planks **e** 14 fence sections

11 A

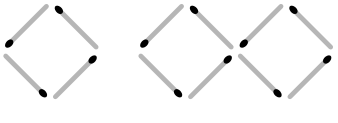
12 A

13 Answers may vary.

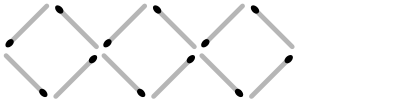
a



b



c



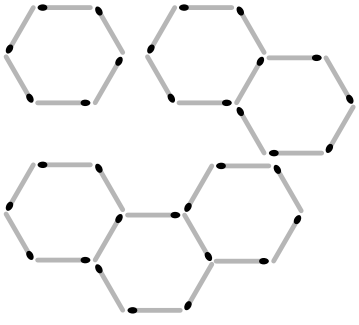
d



e



f



Exercise 3J

1 **a** flowers

b

Number of flowers (*input*)	1	2	3	4	5
Number of sticks (*output*)	4	8	12	16	20

2 **a** Ebony's age

b

José's age (*input*)	1	3	7	12	15
Ebony's age (*output*)	4	6	10	15	18

3 **a** *input*

b

Hours worked (*input*)	1	2	3	4	5
Amount earned (*output*)	$8	$16	$24	$32	$40

4 **a** 16 **b** 6 **c** 10 **d** 4

5 **a**

input	4	5	6	7	10
output	7	8	9	10	13

b

input	5	1	3	21	0
output	10	2	6	42	0

c

input	11	18	9	44	100
output	3	10	1	36	92

d

input	5	15	55	0	100
output	1	3	11	0	20

6 **a**

input	1	2	3	4	5
output	7	17	27	37	47

b

input	6	8	10	12	14
output	7	8	9	10	11

c

input	5	12	2	9	0
output	16	37	7	28	1

d

input	3	10	11	7	50
output	2	16	18	10	96

7 **a** *output* = *input* + 1 **b** *output* = 4 × *input*
c *output* = *input* + 11 **d** *output* = *input* ÷ 6

8 *output* = 3 × *input*

9 **a** A **b** D **c** B **d** C

10 **a**

Zac's age in years (*input*)	0	1	2	3
Amount ($)in account (*output*)	$100	$150	$200	$250

b *output* = 50 × *input* + 100
c $1000

11 **a**

Hours worked (*input*)	0	2	5	10
Cindy's total saving (*output*)	64	80	104	144

b *output* = 8 × *input* + 64 **c** 17 hours

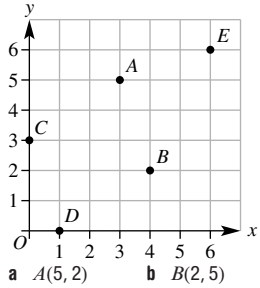
12 **a** *output* = 2 × *input* + 1
b *output* = 3 × *input* − 2
There are other correct answers.

13 **a** *output* = 2 × *input* − 3 **b** *output* = 4 × *input* + 1
c *output* = 5 × *input* − 1 **d** *output* = *input* ÷ 6 + 2
e *output* = 10 × *input* + 3 **f** *output* = 4 × *input* − 4

Exercise 3K

1 $A(2, 2)$, $B(3, 4)$, $C(6, 1)$, $D(4, 0)$, $E(0, 5)$, $F(5, 4)$

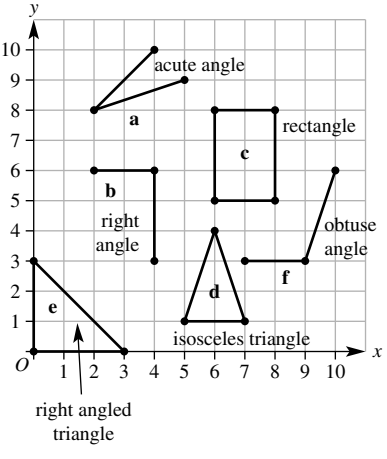
2



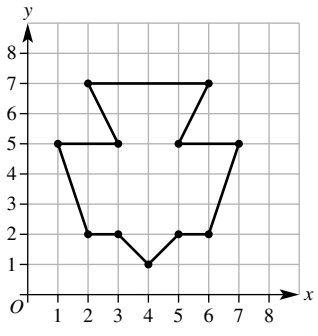
3 **a** $A(5, 2)$ **b** $B(2, 5)$ **c** $C(6, 6)$
d $D(3, 0)$ **e** $E(4, 7)$ **f** $F(0, 4)$

4 **a** x-axis **b** y-axis **c** origin
d first **e** y-coordinate **f** x, y, x, y

5



6

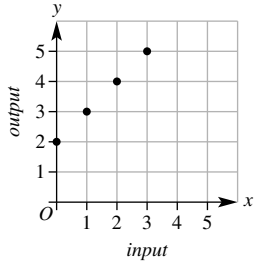


7 **a** $A(1, 4)$, $B(2, 1)$, $C(5, 3)$, $D(2, 6)$, $E(4, 0)$, $F(6, 5)$, $G(0, 3)$, $H(4, 4)$
b $M(1, 2)$, $N(3, 2)$, $P(5, 1)$, $Q(2, 5)$, $R(2, 0)$, $S(6, 6)$, $T(0, 6)$, $U(5, 4)$

8 **a**

input (x)	*output* (y)
0	2
1	3
2	4
3	5

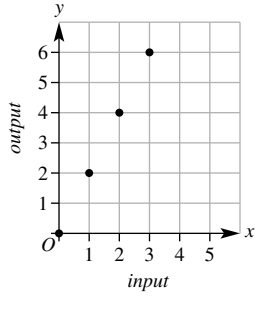
b (0, 2), (1, 3), (2, 4), (3, 5)
c



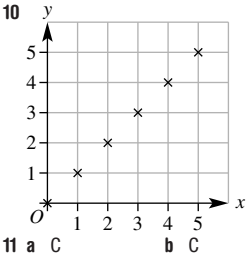
9 **a**

input (x)	*output* (y)
0	0
1	2
2	4
3	6

b (0, 0), (1, 2), (2, 4), (3, 6)
c

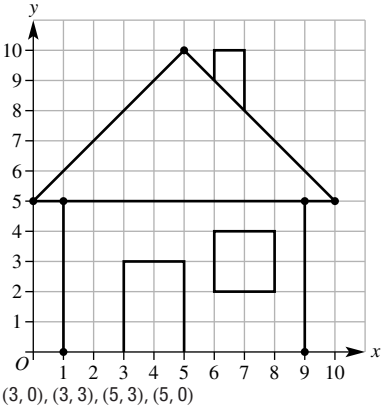


10



11 **a** C **b** C

12 **a** Includes sample door, window and chimney (for **b**–**d**). Other answers are possible for these.



b (3, 0), (3, 3), (5, 3), (5, 0)
c (6, 2), (6, 4), (8, 4), (8, 2)
d (6, 9), (6, 10), (7, 10), (7, 8)

13 **a** HELP **b** (4, 4), (5, 1), (3, 1), (3, 4), (5, 1), (5, 4)
c key under pot plant
d 215100325134510011540044512551 43

Maths@Work: Computer technician

1 **a** 2 **b** 4 **c** 8 **d** 32
e 128 **f** 131 072

2 **a** 2^6 GB **b** 2^9 GB **c** 2^{11} GB (= 2^1 TB)

3

Name	Number of bytes
Byte	1
Kilobyte	1024
Megabyte	1 048 576
Gigabyte	1 073 741 824
Terabyte	1 099 511 627 776

4 **a** 1024 kB **b** 3072 kB **c** 25 600 kB
d 104 857 600 kB

5 *See table at bottom of page.*

6 **a, b and c** *See table at bottom of page.*
d 14.7 seconds
e 9 minutes 12 seconds

Puzzles and games

2 IT WAS SPEAKING ANOTHER LANGUAGE

3 **a** **i** 135 **ii** 624 **iii** 945

Multiple-choice questions

1 B **2** E **3** C **4** E **5** B
6 B **7** D **8** A **9** C **10** E
11 B **12** D

Short-answer questions

1 **a** $1 \times 12 = 12$, $2 \times 6 = 12$, $3 \times 4 = 12$
b 1, 2, 3, 4, 6, 12

2 **a** 1, 2, 3, 4, 6, 8, 12, 24 **b** 5, 10, 15, 20, 25, 30

3 **a** factors of 16: 1, 2, 4, 8, 16
b factors of 20: 1, 2, 4, 5, 10, 20
c 1, 2, 4
d Highest Common Factor
e HCF = 4

4 **a** 6, 12, 18, 24, 30 **b** 8, 16, 24, 32, 40
c Lowest Common Multiple **d** LCM = 24

5 **a** 5, 10 **b** divisible, 2 + 6 + 4 = 12, 12
c divisible, 5 + 7 + 6 = 18, 18, divisible
d divisible, 44, 44, divisible

6 **a** 14, 48, 56, 206, 312, 320 **b** 48, 63, 312, 621
c 48, 56, 320, 312 **d** 85, 320 **e** 48, 312
f 48, 56, 320, 312 **g** 63, 621 **h** 320

	Speed in kB/s (kiloBytes/s)	File size in kB (kiloBytes)	Download time in seconds	Download time to the nearest hour
a	7	26 624	3803	1
b	7	155 648	22 235	6
c	7	757 760	108 251.4	30

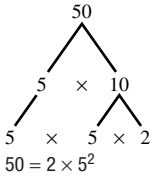
Media type	File size in MB MegaBytes	File size in Mb Megabits	Download times				
App	16	128	16.0	6.4	4.3	2.6	1.3
Music	24	192	24.0	9.6	6.4	3.8	1.9
Short Video	45	360	45.0	18.0	12.0	7.2	3.6
Gaming	120	960	120.0	48.0	32.0	19.2	9.6
Movie	600	4800	600.0	240.0	160.0	96.0	48.0

7 **a** no **b** no **c** yes
d 1, 2, 5, 10, composite number
e prime: 2, 3, 7, 11; composite: 8, 15, 20
f prime: 2, 5; composite: 4, 10, 20
g 17, 19, 23

8 **a** base = 4, index or power = 2 **b** $5^2 \times 7^3$
c $2 \times 2 \times 2 \times 3 \times 3 = 72$ **d** 28
e 3, 5, 5

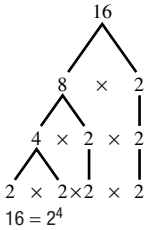
9 Answers may vary.

a



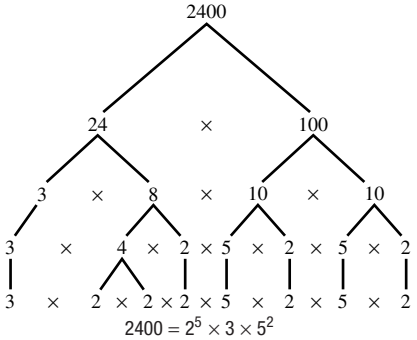
$50 = 2 \times 5^2$

b



$16 = 2^4$

c



$2400 = 2^5 \times 3 \times 5^2$

10

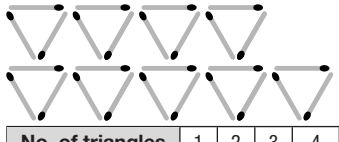
Index form	4^2	6^2	7^2	9^2
Value	16	36	49	81

Square root form	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{100}$	$\sqrt{144}$
Value	4	5	10	12

11 **a** 1 **b** 7 **c** 5 **d** 12
e 18 **f** 200 **g** 64 **h** 121
i 9 **j** 5 cm **k** 20 cm

12 **a** 3, 7, 11, 15, 19 **b** 64, 32, 16, 8, 4
c 24, 48, 96, 192 **d** 32, 28, 24, 20
e 16, 25, 36, 49

13 **a**



b

No. of triangles	1	2	3	4	5
No. of sticks	3	6	9	12	15

c number of sticks = 3 × number of triangles **d** 36 sticks

14 **a**

No. of squares	0	1	2	3	4
No. of sticks	1	1 + 3 = 4	1 + 3 × 2 = 7	1 + 3 × 3 = 10	1 + 3 × 4 = 13

b number of sticks = 1 + 3 × number of squares
c 31 sticks **d** 27 squares

15 **a**

input	3	5	7	12	20
output	8	10	12	17	25

b

input	4	2	9	12	0
output	15	11	25	31	7

16 **a** *output* = 5 × *input* **b** *output* = *input* − 6

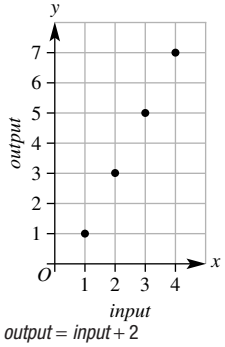
17 **a** $A(1, 3)$, $B(3, 4)$, $D(2, 1)$ **b** (0, 0) **c** y-axis
d y-axis **e** x-axis **f** x-axis **g** $C(4, 2)$

18 **a**

input (x)	*output* (y)
1	1
2	3
3	5
4	7

b (1, 1), (2, 3), (3, 5), (4, 7)

c



19 **a** *output* = *input* + 2 **b** *output* = *input* × 3
c *output* = *input* ÷ 2

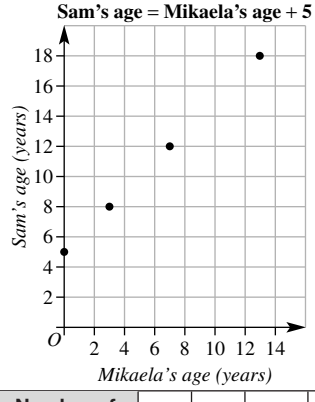
Extended-response questions

1 **a** Sam's age = Mikaela's age + 5

b

Mikaela's age	0	3	7	13
Sam's age	5	8	12	18

c, d



2 **a**

Number of tables	1	2	3	4	5
Number of students	5	8	11	14	17

b number of students = 3 × number of tables + 2
c 23 students
d 26 tables

Chapter 4

Pre-test

1 C

2 **a** 2 **b** 1 **c** 3 **d** 4

3 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{1}{10}$ **e** $\frac{3}{4}$

4 C

5 B

6 **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{4}{5}$

7 **a** $2\frac{3}{4}$ **b** $1\frac{1}{2}$ **c** $9\frac{1}{2}$ **d** $5\frac{1}{4}$

8 one-quarter of a block

9 **a** $2, 2\frac{1}{2}, 3$ **b** $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ **c** $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}$ **d** $\frac{4}{6}, \frac{5}{6}, \frac{6}{6}$

10 **a** $3 \times \frac{1}{2}$ **b** $4 \times \frac{3}{4}$

c $\frac{5}{6} \times 1 = \frac{5}{6}$ **d** $\frac{6}{8} \div 1 = \frac{6}{8}$

11 **a** \$7.50 **b** \$40 **c** 75c **d** \$2

12 **a** T **b** T **c** T **d** F

Exercise 4A

1 **a** 9 **b** 7

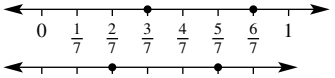
2 proper: b, e, f, g
improper: a, c, d, h, i, j, k, l, m, n, o, p
whole numbers: d, j, k, m, n

3

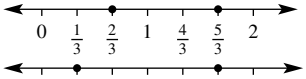
4	1	$\frac{1}{4}$	one-quarter
8	7	$\frac{7}{8}$	seven-eighths
3	2	$\frac{2}{3}$	two-thirds
12	5	$\frac{5}{12}$	five-twelfths
8	8	$\frac{8}{8}$	one whole

4 **a** $\frac{1}{2}, \frac{3}{2} = 1\frac{1}{2}, \frac{5}{2} = 2\frac{1}{2}$ **b** $\frac{1}{3}, \frac{2}{3}, \frac{4}{3} = 1\frac{1}{3}, \frac{7}{3} = 2\frac{1}{3}$

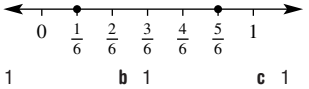
5 **a**



b

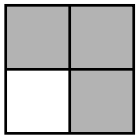


c

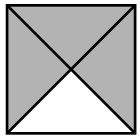


6 **a** 1 **b** 1 **c** 1 **d** 2
e 1 **f** 50 **g** 15 **h** 2

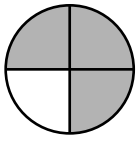
7 **a**



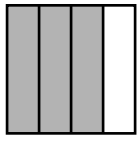
b



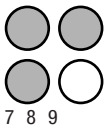
c



d



e



8 **a** $\frac{7}{5}, \frac{8}{5}, \frac{9}{5}$ **b** $\frac{9}{8}, \frac{10}{8}, \frac{11}{8}$

c $\frac{5}{3}, \frac{6}{3}, \frac{7}{3}$ **d** $\frac{7}{7}, \frac{6}{7}, \frac{5}{7}$

9 **a** $\bigcirc = 1\frac{1}{2}, \frac{3}{2}$; $\square = 3\frac{1}{2}, \frac{7}{2}$; $\triangle = 5, \frac{10}{2}$

b $\bigcirc = \frac{1}{5}$; $\square = \frac{4}{5}$; $\triangle = 2\frac{1}{5}, \frac{11}{5}$

c $\triangle = \frac{3}{7}$; $\square = 1\frac{4}{7}, \frac{11}{7}$; $\bigcirc = 2\frac{2}{7}, \frac{16}{7}$

d $\square = 3\frac{1}{3}, \frac{10}{3}$; $\triangle = 3\frac{2}{3}, \frac{11}{3}$; $\bigcirc = 4\frac{2}{3}, \frac{14}{3}$

10 **a** $\frac{6}{11}$ **b** $\frac{4}{8} = \frac{1}{2}$ **c** $\frac{7}{12}$

d $\frac{5}{6}$ **e** $\frac{3}{12} = \frac{1}{4}$ **f** $\frac{5}{9}$

11 **a** $\frac{12}{43}$ **b** $\frac{13}{15}$ **c** $\frac{11}{12}$

d $\frac{1}{12}$ **e** $\frac{2}{11}$ **f** $\frac{144}{475}$

g $\frac{7}{20}$ **h** $\frac{1}{4}$ **i** $\frac{3}{7}$

12 C

13 **a** $\frac{1}{10}$ **b** 180 mL, $\frac{9}{10}$ **c** 150 mL

Exercise 4B

1 **a** 6 **b** 20 **c** $\frac{3}{3}$

d 7 **e** $\frac{2}{2}$ **f** 20

2 $\frac{9}{12}, \frac{15}{20}, \frac{30}{40}, \frac{33}{44}$

3 **a** 2, 12, 10, 20, 300 **b** 1, 3, 24, 20, 40

4 **a** **i** 10 **ii** 10, 10, 1
b **i** 2 **ii** 2, 2, 2
c **i** 4 **ii** 4, 4, 7
d **i** 3 **ii** 3, 3, 3, 5

5 **a** 9 **b** 50 **c** 33 **d** 56
e 8 **f** 2 **g** 12 **h** 39
i 35 **j** 200 **k** 105 **l** 2

6 **a** ≠ **b** = **c** ≠ **d** ≠
e = **f** = **g** = **h** =
i =

7 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$

e $\frac{5}{6}$ **f** $\frac{1}{3}$ **g** $\frac{3}{5}$ **h** $\frac{2}{5}$

8 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$ **d** $\frac{4}{11}$

e $\frac{2}{5}$ **f** $\frac{1}{11}$ **g** $\frac{1}{7}$ **h** $\frac{1}{3}$

i $\frac{7}{9}$ **j** $\frac{3}{8}$ **k** $\frac{5}{6}$ **l** $\frac{5}{1} = 5$

9 **a** $\frac{7}{14} = \frac{1}{2}$ **b** $\frac{12}{16} = \frac{3}{4}$ **c** $\frac{4}{42} = \frac{2}{21}$ **d** $\frac{7}{63} = \frac{1}{9}$

10 $\frac{4}{10}, \frac{16}{40}, \frac{2}{5}, \frac{80}{200}$

11 $\frac{4}{9}$

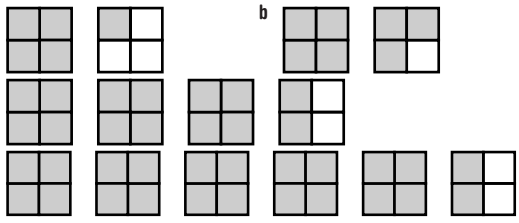
12 Jason 4, Joanna 3, Jack 5

13 Answers may vary; $\frac{6}{10}$

Exercise 4C

1 **a** 2 and 3 **b** 11 and 12 **c** 36 and 37

2 **a**, **b**, **c**, **d**



3 **a** 8 **b** 12 **c** 28 **d** 44
e 17 **f** 7 **g** 10 **h** 24

4 **a** $A\ 7\frac{1}{2}, B\ 10\frac{1}{2}$ **b** $C\ 1\frac{2}{3}, D\ 2\frac{1}{3}, E\ 4\frac{2}{3}$

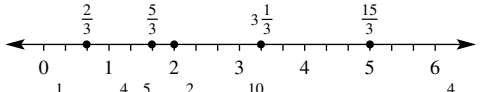
5 **a** $\frac{11}{5}$ **b** $\frac{8}{5}$ **c** $\frac{10}{3}$ **d** $\frac{17}{3}$
e $\frac{29}{7}$ **f** $\frac{24}{7}$ **g** $\frac{5}{2}$ **h** $\frac{13}{2}$
i $\frac{22}{5}$ **j** $\frac{23}{2}$ **k** $\frac{42}{5}$ **l** $\frac{103}{10}$
m $\frac{55}{9}$ **n** $\frac{25}{9}$ **o** $\frac{42}{8}$ **p** $\frac{21}{8}$
q $\frac{23}{12}$ **r** $\frac{38}{11}$ **s** $\frac{53}{12}$ **t** $\frac{115}{12}$

6 **a** $1\frac{2}{5}$ **b** $1\frac{1}{3}$ **c** $1\frac{2}{3}$ **d** $1\frac{3}{4}$
e $3\frac{2}{3}$ **f** $4\frac{1}{5}$ **g** $2\frac{2}{7}$ **h** $2\frac{1}{2}$
i $1\frac{5}{7}$ **j** $3\frac{1}{6}$ **k** $6\frac{2}{3}$ **l** $10\frac{1}{4}$
m $4\frac{3}{8}$ **n** $5\frac{1}{5}$ **o** $6\frac{6}{7}$ **p** $13\frac{2}{3}$
q $3\frac{1}{12}$ **r** $7\frac{4}{11}$ **s** $9\frac{3}{10}$ **t** $11\frac{1}{7}$

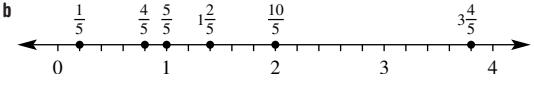
7 **a** **i** $1\frac{1}{2}$ **ii** $\frac{3}{2}$
b **i** $2\frac{1}{2}$ **ii** $\frac{5}{2}$
c **i** $2\frac{1}{3}$ **ii** $\frac{7}{3}$
d **i** $3\frac{2}{5}$ **ii** $\frac{17}{5}$

8 **a** $\frac{4}{3}, 1\frac{1}{3}$ **b** $\frac{3}{2}, 1\frac{1}{2}$ **c** $\frac{3}{2}, 1\frac{1}{2}$ **d** $\frac{5}{4}, 1\frac{1}{4}$

9 **a**

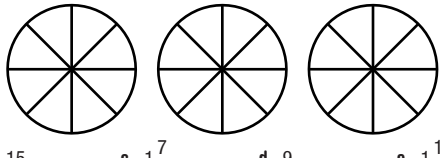


b



10 **a** $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}$
b $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$

11 **a**



b 15 **c** $1\frac{7}{8}$ **d** 9 **e** $1\frac{1}{8}$

12 $\frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{1}, \frac{2}{3}, \frac{3}{2}, \frac{1}{23}, \frac{1}{32}, \frac{2}{13}, \frac{2}{31}, \frac{3}{12}, \frac{3}{21}, \frac{12}{3}, \frac{21}{3}, \frac{13}{2}, \frac{31}{2}, \frac{23}{1}, \frac{32}{1}, 1\frac{2}{3}, 2\frac{1}{3}, 3\frac{1}{2}$

Exercise 4D

1 **a** $\frac{5}{7}$ **b** $\frac{7}{3}$ **c** $\frac{9}{11}$ **d** $\frac{8}{5}$

2 **a** 10 **b** 21 **c** 20 **d** 30
e 6 **f** 10 **g** 12 **h** 24
i 30 **j** 12 **k** 24 **l** 30

3 **a** 15 **b** 20 **c** 21 **d** 10
e 24 **f** 60 **g** 12 **h** 12

4 **a** 6 **b** 8 **c** 4
d 6 **e** 15 **f** 15

5 **a** > **b** > **c** < **d** <
e > **f** > **g** < **h** >

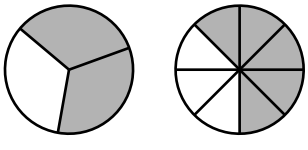
6 **a** < **b** > **c** < **d** <
e > **f** >

7 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{4}{5}$
d $\frac{3}{2}$ **e** $\frac{3}{4}$ **f** $\frac{3}{5}$

8 **a** $\frac{3}{5}, 1\frac{2}{5}, \frac{8}{5}$ **b** $\frac{2}{9}, \frac{1}{3}, \frac{5}{9}$
c $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ **d** $\frac{3}{5}, \frac{2}{3}, \frac{5}{6}$

9 **A** $\frac{1}{4}$, **C** $\frac{1}{8}$, **B** $\frac{1}{11}$

10 **a**



b Sally **c** No: together they ate $1\frac{7}{24}$ pizzas.

11 **a** $\frac{5}{9}, \frac{6}{9}$ **b** $\frac{11}{4}, \frac{14}{4}$

12 Answers will vary. Sample answers:
a $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{2}$
d $\frac{13}{20}$ **e** $\frac{1}{3}$ **f** $\frac{5}{21}$

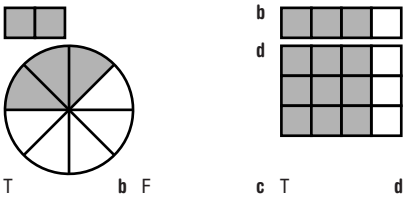
13 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{3}$
b Answers will vary.

Exercise 4E

1 **a** denominator
b denominator, numerators
c denominators, lowest common denominator
d simplify

2 **a** two- **b** -quarters **c** four-
d -fifths **e** one-, one-

3 **a**, **b**, **c**, **d**



4 **a** T **b** F **c** T **d** T
e F **f** T **g** T **h** F

5 **a** $\frac{5}{8}$ **b** $\frac{5}{7}$ **c** $\frac{4}{5}$ **d** $\frac{9}{11}$

e $\frac{7}{8}$ **f** $\frac{7}{12}$ **g** $\frac{7}{15}$ **h** $\frac{5}{9}$

i $\frac{4}{5}$ **j** $\frac{6}{7}$ **k** $\frac{7}{10}$ **l** $\frac{81}{100}$

6 **a** $1\frac{2}{7}$ **b** $1\frac{3}{10}$ **c** $1\frac{4}{5}$ **d** $1\frac{4}{19}$

e $1\frac{1}{10}$ **f** $1\frac{2}{5}$ **g** $1\frac{3}{7}$ **h** $1\frac{3}{11}$

i $1\frac{1}{100}$

7 **a** $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **b** $\frac{4}{8}+\frac{3}{8}=\frac{7}{8}$

c $\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$ **d** $\frac{6}{24}+\frac{4}{24}=\frac{10}{24}$

8 **a** $\frac{3}{4}$ **b** $\frac{14}{15}$ **c** $\frac{2}{3}$ **d** $\frac{7}{12}$

e $\frac{13}{20}$ **f** $\frac{19}{20}$ **g** $\frac{13}{21}$ **h** $\frac{23}{40}$

i $1\frac{13}{30}$ **j** $1\frac{9}{28}$ **k** $1\frac{13}{33}$ **l** $1\frac{5}{12}$

9 **a** $3\frac{4}{5}$ **b** $7\frac{3}{7}$ **c** $12\frac{3}{4}$ **d** $5\frac{5}{9}$

e $10\frac{1}{3}$ **f** $21\frac{1}{6}$ **g** $19\frac{3}{11}$ **h** $12\frac{2}{5}$

10 **a** $4\frac{5}{12}$ **b** $7\frac{7}{30}$ **c** $12\frac{1}{6}$ **d** $13\frac{9}{28}$

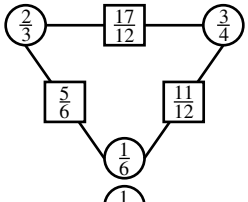
e $15\frac{1}{10}$ **f** $19\frac{1}{9}$ **g** $25\frac{21}{44}$ **h** $15\frac{5}{24}$

11 **a** $\frac{3}{5}$ **b** $\frac{2}{5}$ **c** \$4

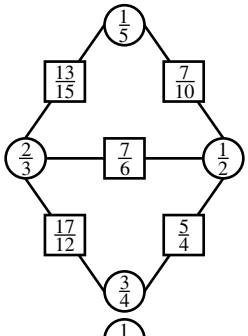
12 **a** $\frac{7}{12}$ **b** $\frac{5}{12}$

13 **a** $\frac{17}{24}$ **b** $\frac{7}{24}$ **c** 7 hours

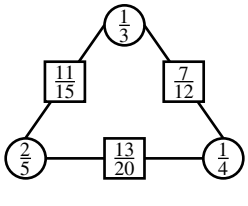
14 **a**



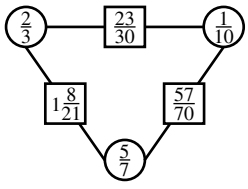
b



15 **a**



b



Exercise 4F

1 **a** denominator **b** 12 **c** 15

2 **a** one- **b** two- **c** -fifths

d three- **e** -eighths

3 **a**  **b**  **c** 

4 **a** 1 **b** 3 **c** 4, 3, 1 **d** 12, 10, 2

5 **a** $\frac{2}{7}$ **b** $\frac{3}{11}$ **c** $\frac{7}{18}$ **d** $\frac{1}{3}$

e 0 **f** $\frac{4}{9}$ **g** $\frac{3}{19}$ **h** $\frac{8}{23}$

i $\frac{31}{100}$ **j** $\frac{12}{25}$ **k** $\frac{16}{25}$ **l** $\frac{2}{5}$

6 **a** $\frac{5}{12}$ **b** $\frac{1}{10}$ **c** $\frac{1}{10}$ **d** $\frac{9}{28}$

e $\frac{1}{6}$ **f** $\frac{23}{36}$ **g** $\frac{13}{33}$ **h** $\frac{2}{15}$

i $\frac{1}{8}$ **j** $\frac{3}{20}$ **k** $\frac{1}{36}$ **l** $\frac{1}{9}$

7 **a** $1\frac{3}{5}$ **b** $8\frac{3}{7}$ **c** $1\frac{1}{7}$ **d** $3\frac{2}{9}$

e $2\frac{5}{12}$ **f** $3\frac{5}{28}$ **g** $4\frac{7}{18}$ **h** $7\frac{1}{20}$

8 **a** $2\frac{2}{3}$ **b** $4\frac{3}{5}$ **c** $4\frac{2}{3}$ **d** $4\frac{8}{9}$

e $4\frac{2}{3}$ **f** $\frac{37}{45}$ **g** $9\frac{37}{44}$ **h** $2\frac{29}{60}$

9 $\frac{3}{5}$

10 $\frac{1}{6}$

11 $\frac{3}{4}$ kg, 750 g

12 **a** $\frac{9}{4}-\frac{5}{3}=\frac{27}{12}-\frac{20}{12}=\frac{7}{12}$

b $2\frac{1}{4}-1\frac{2}{3}=2-1+\frac{1}{4}-\frac{2}{3}=1+\frac{3}{12}-\frac{8}{12}=\frac{12}{12}+\frac{3}{12}-\frac{8}{12}=\frac{7}{12}$

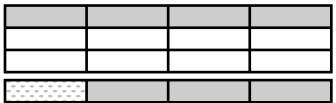
13 $\frac{5}{8}, \frac{1}{3}$

Exercise 4G

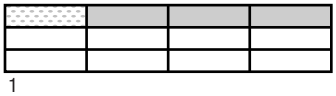
1 **a** 0, 1 **b** 1

c whole number, proper fraction

2 **a**

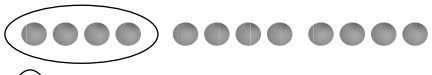


b



c $\frac{1}{12}$

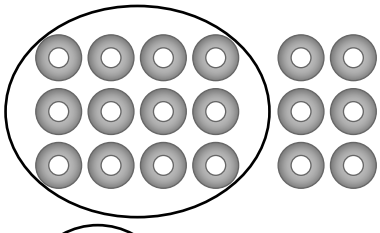
3 a



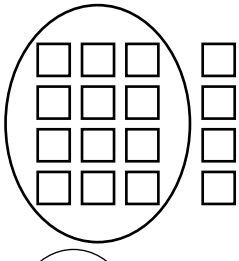
b



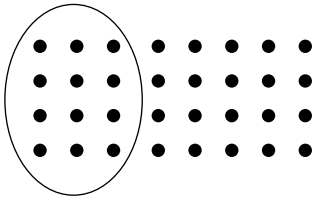
c



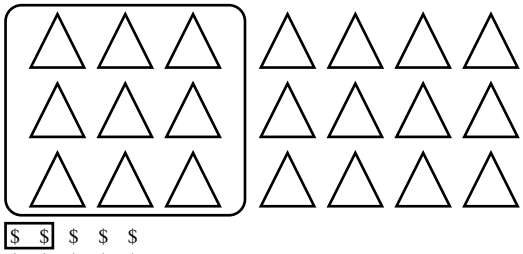
d



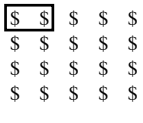
e



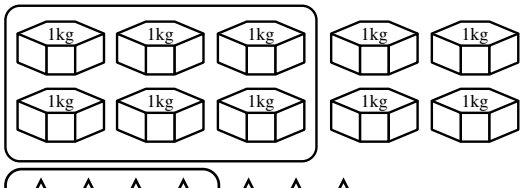
f



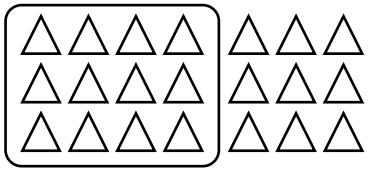
g



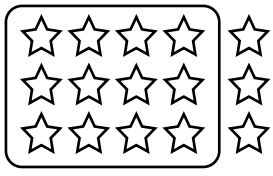
h



i



j



4 **a** 8 **b** 21 **c** 12
d 8 **e** 42 **f** 30

5 **a** 6 **b** 9 **c** 16 **d** 15
e 12 **f** 4 **g** 80 **h** 33
i 2 **j** 20 **k** 40 **l** 20
m 40 **n** 12 **o** 15 **p** 15

6 **a** $\frac{3}{20}$ **b** $\frac{2}{21}$ **c** $\frac{10}{21}$ **d** $\frac{8}{45}$
e $\frac{2}{5}$ **f** $\frac{1}{7}$ **g** $\frac{1}{4}$ **h** $\frac{5}{11}$
i $\frac{5}{22}$ **j** $\frac{1}{3}$ **k** $\frac{6}{11}$ **l** $\frac{4}{11}$
m $\frac{6}{35}$ **n** $\frac{3}{10}$ **o** $\frac{2}{7}$ **p** $\frac{1}{6}$

7 **a** $5\frac{5}{6}$ **b** $1\frac{31}{35}$ **c** $3\frac{3}{10}$ **d** $4\frac{7}{8}$
e $5\frac{1}{3}$ **f** 7 **g** 6 **h** $3\frac{1}{3}$
i $\frac{4}{5}$ **j** $\frac{4}{15}$ **k** $\frac{3}{8}$ **l** $\frac{5}{4} = 1\frac{1}{4}$

8 **a** $3\frac{11}{15}$ **b** $1\frac{25}{63}$ **c** $7\frac{4}{5}$ **d** 24
e $3\frac{1}{5}$ **f** $\frac{3}{16}$ **g** 4 **h** 33
i $\frac{2}{7}$ **j** $3\frac{3}{8}$ **k** $\frac{4}{15}$ **l** 6

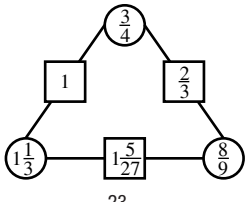
9 **a** $\frac{3}{5}$ **b** 48 boys, 72 girls

10 7 games

11 **a** 45 minutes **b** 45
c $60 \div 3 \times 2 = 40$

12 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$

13 **a**



b $a + b + c = 2\frac{23}{27}$

Exercise 4H

1 **a** × **b** 4 **c** divide
d reciprocal **e** flip **f** improper

2 A

3 **a** $\frac{5}{11} \times \frac{5}{3}$ **b** $\frac{1}{3} \times \frac{5}{1}$ **c** $\frac{7}{10} \times \frac{17}{12}$
d $\frac{8}{3} \times \frac{1}{3}$ **e** $\frac{7}{10} \times \frac{2}{1}$ **f** $\frac{3}{2} \times \frac{4}{1}$
g $\frac{3}{5} \times \frac{10}{1}$ **h** $\frac{3}{2} \times \frac{3}{7}$

4 **a** less **b** more **c** more
d more **e** less **f** less

5 **a** $\frac{7}{5}$ **b** $\frac{5}{3}$ **c** $\frac{9}{2}$ **d** $\frac{8}{1}$
e $\frac{3}{1}$ **f** $\frac{10}{1}$ **g** $\frac{10}{3}$ **h** $\frac{4}{5}$
i $\frac{1}{12}$ **j** $\frac{1}{101}$ **k** 9 **l** 1

6 **a** $\frac{2}{3}$ **b** $\frac{5}{6}$ **c** $\frac{2}{5}$ **d** $\frac{5}{7}$
e $\frac{4}{11}$ **f** $\frac{3}{7}$ **g** $\frac{5}{23}$ **h** $\frac{6}{11}$

7 **a** $\frac{3}{8}$ **b** $\frac{5}{33}$ **c** $\frac{2}{5}$ **d** $\frac{5}{7}$
e $\frac{3}{4}$ **f** $1\frac{1}{3}$ **g** $1\frac{3}{5}$ **h** $\frac{3}{14}$

8 **a** 20 **b** 21 **c** 100 **d** 120
e 30 **f** 40 **g** 4 **h** $6\frac{2}{3}$

9 **a** $\frac{5}{7}$ **b** $\frac{4}{5}$ **c** $\frac{11}{14}$
d $\frac{3}{4}$ **e** $1\frac{11}{16}$ **f** $1\frac{3}{11}$
g $3\frac{1}{3}$ **h** $\frac{3}{4}$ **i** $1\frac{1}{2}$

10 $\frac{3}{4}$ **11** 120 km/h **12** $\frac{1}{4}$

13 FRACTIONS MAKE ME FEEL WHOLE

Progress quiz

1 **a** P **b** I **c** W **d** W **e** P
2 **a** $\frac{7}{10}$ **b** $\frac{4}{7}$ **c** $\frac{1}{4}$ **d** $\frac{3}{2}$ **e** $\frac{2}{3}$
3 **D**
4 **a** $\frac{23}{7}$ **b** $\frac{103}{9}$
5 **a** $3\frac{3}{10}$ **b** $8\frac{1}{3}$
6 **a** $\frac{5}{8}$ **b** $\frac{1}{3}$ **c** $\frac{3}{5}$ **d** $\frac{13}{16}$
7 $\frac{8}{15}, \frac{3}{5}, \frac{2}{3}, \frac{5}{6}$
8 **a** $\frac{9}{13}$ **b** $\frac{16}{21}$ **c** $5\frac{13}{20}$
9 **a** $\frac{4}{11}$ **b** $\frac{5}{24}$ **c** $6\frac{1}{3}$
10 **a** $\frac{9}{28}$ **b** 6 **c** $4\frac{2}{3}$
11 **a** $\frac{9}{2}$ **b** $\frac{1}{5}$ **c** $\frac{4}{15}$
12 **a** $\frac{2}{11}$ **b** 32 **c** $\frac{10}{21}$ **d** $2\frac{1}{4}$

Exercise 4I

1 **a** 70, 70 **b** 48, 48 **c** 60, 60 **d** 20, 20
e 40, 40 **f** 63, 63

2 **a** $\frac{1}{4} = 25\%$, $\frac{2}{4} = 50\%$, $\frac{3}{4} = 75\%$, $\frac{4}{4} = 100\%$
b $\frac{1}{5} = 20\%$, $\frac{2}{5} = 40\%$, $\frac{3}{5} = 60\%$, $\frac{4}{5} = 80\%$, $\frac{5}{5} = 100\%$
c $\frac{1}{3} = 33\frac{1}{3}\%$, $\frac{2}{3} = 66\frac{2}{3}\%$, $\frac{3}{3} = 100\%$

3 **a** 86% **b** 20%
4 She scored full marks: $\frac{25}{25}$

5 **a** $\frac{11}{100}$ **b** $\frac{71}{100}$ **c** $\frac{43}{100}$ **d** $\frac{49}{100}$
e $\frac{1}{4}$ **f** $\frac{3}{10}$ **g** $\frac{3}{20}$ **h** $\frac{22}{25}$
i $\frac{7}{100}$ **j** $\frac{19}{100}$ **k** $\frac{21}{100}$ **l** $\frac{1}{2}$
m $\frac{7}{10}$ **n** $\frac{9}{10}$ **o** $\frac{99}{100}$ **p** $\frac{11}{20}$

6 **a** $1\frac{1}{5}$ **b** $1\frac{4}{5}$ **c** $2\frac{37}{100}$ **d** $4\frac{1}{100}$
e $1\frac{3}{4}$ **f** $1\frac{1}{10}$ **g** $3\frac{4}{25}$ **h** $8\frac{2}{5}$
i 2 **j** $2\frac{1}{20}$ **k** $3\frac{1}{10}$ **l** $3\frac{1}{2}$

7 **a** 98% **b** 9% **c** 79% **d** 56%
e 8% **f** 15% **g** 97% **h** 50%
i 35% **j** 32% **k** 86% **l** 90%
m 112% **n** 135% **o** 400% **p** 160%

8 **a** $12\frac{1}{2}\%$ **b** $33\frac{1}{3}\%$ **c** $26\frac{2}{3}\%$ **d** $83\frac{1}{3}\%$
e 115% **f** 420% **g** 290% **h** $32\frac{1}{2}\%$

9 **a** $\frac{3}{4}$ **b** 75% **c** $\frac{1}{4}$ **d** 25%

10 $12\frac{1}{2}\%$
11 70%
12 70%, 80%
13 **a** **i** $\frac{1}{40}$ **ii** $\frac{13}{400}$
b **i** $\frac{33}{400}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{3}$

Exercise 4J

1 **a** $\boxed{10}$ **b** $\boxed{5}$ **c** $20 \div \boxed{2} = \boxed{10}$
2 **a** 6 **b** 35 **c** 16
3 **a** \$500 ÷ 10 = \$50
b \$900 ÷ 100 = \$9
c 84 kg ÷ 4 = 21 kilograms
d 7 days ÷ 2 = 3 days, 12 hours
e 84 kg ÷ 4 × 3 = 63 kilograms
f 7 minutes
4 8 hours
5 **a** 70 **b** 36 **c** 10 **d** 27
e 10 **f** 7 **g** 150 **h** 200
i 4 **j** 48 **k** 44 **l** 190
m 22 **n** 84 **o** 36 **p** 63
6 **a** 96 **b** 600 **c** 66 **d** 100
e 15 **f** 72 **g** 73 **h** 600
7 **a** 10% of \$200 = \$20 **b** 5% of \$500 = \$25
c 20% of \$120 = \$24 **d** 30% of \$310 = \$93
e 10% of \$80 = \$8 **f** 10% of \$160 = \$16
g 50% of \$60 = \$30 **h** 1% of \$6000 = \$60
i 20% of \$200 = \$40 **j** 50% of \$88 = \$44
8 **a** \$42 **b** 24 millimetres
c 9 kilograms **d** 90 tonnes
e 8 minutes **f** 400 centimetres
g 1.5 grams **h** 3 hectares
i 144 seconds
9 35 marks
10 240 students
11 12 students

12 **a** 120 **b** 420 **c** 660

13 **a** They are the same.

b $37\frac{1}{2}\%$

c **i** \$140 **ii** \$1.50

Exercise 4K

1 **a** 25% **b** 20% **c** 48% **d** 99%
e 17% **f** 34% **g** 35% **h** 90%

2 **a** $\frac{7}{350}$ **b** $\frac{1}{50}$ **c** $\frac{1}{50} \times 100\% = 2\%$

3 **a** 4 **b** 4 **c** $\frac{1}{2}$
d 50% **e** $\frac{1}{2}$ **f** 50%

4 **a** 10 **b** $\frac{1}{5}$ **c** $\frac{4}{5}$
d 20% **e** 80%

5 **a** $\frac{3}{10}$, 30% **b** $\frac{3}{5}$, 60% **c** $\frac{1}{5}$, 20%
d $\frac{3}{4}$, 75% **e** $\frac{1}{20}$, 5% **f** $\frac{1}{2}$, 50%
g $\frac{1}{4}$, 25% **h** $\frac{3}{20}$, 15%

6 **a** $\frac{3}{5}$, 60% **b** $\frac{1}{2}$, 50% **c** $\frac{1}{4}$, 25%
d $\frac{2}{5}$, 40% **e** $\frac{3}{4}$, 75% **f** $\frac{4}{5}$, 80%

7 **a** $\frac{1}{10}$ **b** 10%

8 **a** $\frac{1}{5}$ **b** 20% **c** $\frac{4}{5}$ **d** 80%

9 **a** $\frac{80}{1600} = \frac{1}{20}$ **b** 5% **c** 95%

10 **a** 16 megalitres **b** $\frac{16}{20} = \frac{4}{5}$ **c** 80%

11 **a** 1500 m^2

b **i** $\frac{1}{20}$, 5% **ii** $\frac{3}{5}$, 60% **iii** $\frac{1}{8}$, $12\frac{1}{2}\%$

c $\frac{29}{40}$

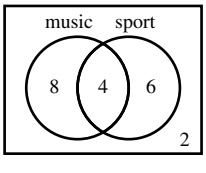
d 27.5%

12 **a** $\frac{1}{25}$, 4% **b** $\frac{2}{5}$, 40% **c** $\frac{1}{20}$, 5%
d $\frac{1}{20}$, 5% **e** $\frac{1}{4}$, 25% **f** $\frac{3}{4}$, 75%

13 $\frac{45}{50} = 90\%$ $\frac{72}{100} = 72\%$ Ross scored more.

14 **a** 4

b



c $\frac{1}{5}$ **d** 40%

Exercise 4L

1 D

2 A

3 **a** 5 **b** 1 **c** 1 **d** 4

4 **a** yes **b** no **c** no **d** yes

5 **a** 60 **b** 300 **c** 3600 **d** 86 400

6 **a** 1 : 3 **b** 1 : 2 **c** 1 : 9 **d** 1 : 3
e 3 : 1 **f** 4 : 1 **g** 10 : 1 **h** 5 : 1
i 2 : 5 **j** 2 : 3 **k** 4 : 5 **l** 5 : 7
m 9 : 7 **n** 12 : 5 **o** 11 : 7 **p** 5 : 4

7 **a** 4 : 24, replace with 4 : 16 or 6 : 24

b 10 : 600, replace with 10 : 60 or 100 : 600

c 8 : 24, replace with 8 : 12 or 16 : 24

d 15 : 20, replace with 15 : 25 or 12 : 20

8 **a** 12 km/h **b** 4 claps/second **c** \$7/kg
d 7 m/s **e** 3 goals/game
f 2 km/min or 120 km/h

9 **a** \$24 **b** 72 songs
c 20 waves **d** 28 pairs of shoes

10 **a** 5 kg of flour for \$8.50

b 5 kg of potatoes for \$5.50

c 500 g of ham for \$9.50

d 20 L of cooking oil for \$120

11 10 kg for \$38.00 (\$3.80/kg), 2 kg for \$7.90 (\$3.95/kg), 5 kg for \$19.95 (\$3.99/kg), 500 g for \$2.09 (\$4.18/kg)

12 **a** 22.5 kg **b** 2.5 kg

13 **a** 3 : 5 **b** 51.2 minutes
c 145 600 words

Maths@Work: Building Construction Apprentice

1 **a** 12.7 mm **b** 6.4 mm
c 3.2 mm **d** 0.8 mm
e 19.1 mm

3 **a** **i** $4\frac{1}{4}$ **ii** 6 **iii** $6\frac{3}{4}$
iv $6\frac{13}{16}$ **v** $8\frac{3}{16}$

b **i** $1\frac{1}{4}$ **ii** $1\frac{5}{8}$ **iii** $2\frac{5}{16}$
iv $2\frac{9}{16}$ **v** $3\frac{3}{8}$ **vi** $3\frac{3}{4}$

4 **a** $10\frac{1}{4}''$, $10\frac{1}{2}''$, $10\frac{3}{4}''$, $11''$, $11\frac{1}{4}''$, $11\frac{1}{2}''$, $11\frac{3}{4}''$, $1'$

b $1'$, $11\frac{7}{8}''$, $11\frac{3}{4}''$, $11\frac{5}{8}''$, $11\frac{1}{2}''$, $11\frac{3}{8}''$, $11\frac{1}{4}''$

c $\frac{5}{8}''$, $4\frac{5}{8}''$, $11\frac{5}{8}''$

d $7\frac{11}{16}''$, $31\frac{11}{16}''$ or $2'\,7\frac{11}{16}''$

5 203.2 mm

6 $\frac{1}{5}$

7 965.2 mm

8

Total A + B in inches	Total A + B in mm
9 9/16	242.9
27 1/8	689.0
58 19/32	1488.3
57 5/8	1463.7

Puzzles and games

1 DELHI AGRA AND JAIPUR

4 **a**

$\frac{2}{5}$	$A = \frac{9}{10}$	$\frac{4}{5}$
$B = \frac{11}{10}$	$C = \frac{7}{10}$	$D = \frac{3}{10}$
$E = \frac{3}{5}$	$\frac{1}{2}$	1

b

$A = \frac{3}{4}$	$B = 2\frac{5}{8}$	$2\frac{1}{4}$
$C = 3\frac{3}{8}$	$1\frac{7}{8}$	$D = \frac{3}{8}$
$E = 1\frac{1}{2}$	$1\frac{1}{8}$	3

Multiple-choice questions

1 C **2** B **3** D **4** E **5** D
6 C **7** E **8** B **9** D **10** B
11 A **12** C

Short-answer questions

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{14}$
e $\frac{1}{10}$ **f** $\frac{1}{8}$ **g** $\frac{1}{5}$ **h** $\frac{1}{3}$

2 $\frac{2}{3}$

3 a $\frac{3}{5}$ **b** $\frac{2}{7}$ **c** $\frac{5}{7}$

4 a $1\frac{3}{4}$ **b** $1\frac{3}{7}$ **c** $1\frac{1}{3}$ **d** $1\frac{1}{2}$

5 a $\frac{2}{7} \boxed{<} \frac{4}{7}$ **b** $\frac{3}{8} \boxed{>} \frac{1}{8}$ **c** $\frac{2}{3} \boxed{>} \frac{3}{5}$ **d** $3\frac{1}{9} \boxed{<} \frac{29}{9}$

6 a $\frac{5}{7}$ **b** $\frac{5}{8}$

7 a 10 **b** 21 **c** 24

8 a 10 **b** 21 **c** 24

9 a $1\frac{1}{4}, \frac{4}{4}, \frac{3}{4}, \frac{1}{4}$ **b** $\frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}$
c $\frac{4}{3}, \frac{5}{6}, \frac{2}{3}, \frac{1}{12}$

10 a $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $1\frac{5}{24}$ **d** $5\frac{23}{30}$
e $\frac{1}{2}$ **f** $2\frac{1}{2}$

11 a 7 **b** 80 **c** 12 **d** 5
e $\frac{1}{6}$ **f** $7\frac{1}{2}$

12 a $\frac{4}{3}$ **b** $\frac{12}{7}$ **c** $\frac{4}{11}$ **d** $\frac{1}{8}$

13 a $\frac{1}{5}$ **b** 20 **c** 4 **d** 2

14 A = 25%, B = 50%, C = 28%, D = $\frac{1}{10}$, E = $\frac{3}{4}$, F = $1\frac{1}{2}$

15 a \$20 **b** \$210 **c** 48 g
d \$30 **e** \$9

16 a $\frac{3}{5}$, 60% **b** $\frac{1}{5}$, 20%
c $\frac{1}{16}$, 6.25% **d** $\frac{3}{10}$, 30%

17 a 3 : 4 **b** 10 : 3 **c** 25 : 1
d 30 km/h **e** 2 L/m **f** 13 words/min

18 5 kg bag is better at 70c per kg compared to 80c per kg.

Extended-response questions

1 a 300 **b** 75% **c** 450
d 45 **e** 1080

Chapter 5

Pre-test

1 a 9 **b** 7 **c** 48 **d** 4
2 a 13 **b** 21 **c** 4 **d** 5
3 a 11 **b** 5 **c** 5 **d** 21
4 a 8 **b** 36 **c** 40 **d** 10
5 a 17 **b** 8 **c** 5 **d** 20
6 a 20 **b** 44 **c** 22 **d** 100
7 a 20 **b** 6 **c** 2 **d** 6
8 a 1 **b** 9 **c** 3 **d** 45
9 a 17 **b** 14 **c** 11 **d** 8
e 7 **f** 18 **g** 35 **h** 17
10 a 32 cm **b** 24 cm **c** 48 mm

Exercise 5A

1 B

2 C

3 a $2x, 7y$ **b** $3a, 2c, e$
c $5q, 3r, 2s$ **d** $7d, 5f, 17$

4 a 3 **b** 13 **c** 4

5 a i 2 **ii** 17
b i 3 **ii** 15
c i 3 **ii** 21
d i 4 **ii** 2
e i 2 **ii** 1
f i 4 **ii** 12

6 a $x + 3$ **b** $k + 5$ **c** $b + 2$
d $g - 3$ **e** $H - 4$ **f** $M - 6$

7 a $2u$ **b** $4y$ **c** $3x$ **d** $10k$
e $\frac{y}{8}$ **f** $\frac{z}{2}$ **g** $3a + 4$ **h** $2p + 12$

8 C

9 a 70 **b** $10n$

10 a $8x$ **b** $x + 3$ **c** $8(x + 3)$

11 a $1000x$ **b** $100x$ **c** $100\,000x$

12 a false **b** false **c** true
d true **e** false **f** true

13 a $\frac{\$A}{4}$

b $\frac{\$A}{n}$

c i $\frac{\$A - 20}{n}$ **ii** \$30

Exercise 5B

1 a 17 **b** 38 **c** 15 **d** 21
2 a 6 **b** 28 **c** 1 **d** 2
3 a 10 **b** 15 **c** 5 **d** 4
4 a 14 **b** 1 **c** 10 **d** 8
5 a 7 **b** 17 **c** 120 **d** 5
6 a 8 **b** 42 **c** 3 **d** 3
7 a 20 **b** 50 **c** 35 **d** 100
8 a 17 **b** 20 **c** 72 **d** 12
9 a 8 **b** 10 **c** 9 **d** 14
e 17 **f** 3 **g** 14 **h** 7
i 18
10 a 14 **b** 21 **c** 23 **d** 12
e 18 **f** 21
11 a 8 **b** 4 **c** 5 **d** 9
e 4 **f** 6 **g** 8 **h** 1
i 15

12 5
13 1 and 24, 2 and 12, 3 and 8, 4 and 6
14

x	5	9	12	1	6	7
$x + 6$	11	15	18	7	12	13
$4x$	20	36	48	4	24	28

Exercise 5C

1 **a** 9 **b** 12 **c** no
2 **a** 9 **b** 9 **c** yes
3 equivalent
4 True. When adding numbers, order does not matter.
5 **a**

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$2a + 2$	2	4	6	8
$(a + 1) \times 2$	2	4	6	8

b equivalent
6 **a**

	$B = 0$	$B = 1$	$B = 2$	$B = 3$
$5B + 3$	3	8	13	18
$6B + 3$	3	9	15	21

b no
7 **a**

	$6x + 5$	$4x + 5 + 2x$
$x = 1$	11	11
$x = 2$	17	17
$x = 3$	23	23
$x = 4$	29	29

b They are equivalent because they are always equal.
8 **a** N **b** E **c** E **d** N
9 $y + y + y + y$; other answers are possible
10 $2(w + l)$; other answers are possible
11 6
12 If $x = 8$, all four expressions have different values.
13 A1 and C2, A2 and D3, A3 and C1, B1 and C3, B2 and D2, B3 and D1

Exercise 5D

1 **a** true **b** false **c** true **d** false
2 **a** 14 **b** 21 **c** 35 **d** 35
3 **a** 50 **b** 20 **c** 30 **d** 30
4 **a** x and y **b** a, b and c
c k **d** p and q
5 **a** like **b** like terms
c terms **d** terms
6 **a** N **b** L **c** L **d** N
e N **f** L **g** N **h** L
i L **j** L **k** L **l** N
7 **a** $5x$ **b** $6a$ **c** $16q$ **d** $3b$
e $9cd$ **f** $6qr$ **g** $9ab$ **h** $11cf$
8 **a** $4x$ **b** $3a$ **c** $10q$ **d** $6b$
e $8cd$ **f** $2qr$ **g** $7ab$ **h** $3cf$
9 **a** $3a + 5b$ **b** $7a + 9b$ **c** $x + 6y$ **d** $7a + 2$
e $7 + 7b$ **f** $6k - 2$
10 **a** $5f + 12$ **b** $4a + 6b - 4$
c $6x + 4y$ **d** $8a + 4b + 3$
e $7g + 4$ **f** $14x + 30y$
g $2x + 9y + 10$ **h** $8a + 13$
i $12b$ **j** $5d + 3$
11 **a** $30n$ **b** $35n$ **c** $65n$
12 **a** $3a + 4$ **b** 19
13 **a**

	$3x + 2x$	$5x$
$x = 1$	5	5
$x = 2$	10	10
$x = 3$	15	15

b For example, if $x = 2$ and $y = 4$, then $3x + 2y = 14$ but $5x = 10$.
c For example, if $x = 5$ and $y = 10$, then $3x + 2y = 35$ but $5xy = 250$.
14 **a** **i** $4x$ **ii** $7x$ **iii** $11x$ **iv** $3x$
b Xavier has 48. Cameron has 84.

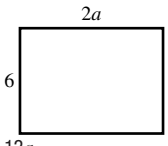
Progress quiz

1 **a** 11 **b** 13
2 **a** $x + 7$ **b** $t - 8$ **c** $3f$ **d** $\frac{p}{2} + 4$
3 D
4 **a** 15 **b** 20 **c** 5 **d** 6
5 **a** 13 **b** 66 **c** 16 **d** 16
6 **a** 18 **b** 17 **c** 15 **d** 0
7 equivalent
8 **a** N **b** E **c** E **d** E
9 **a** N **b** L **c** N **d** L
e N **f** L
10 **a** $17x$ **b** $11kt - 4k$ **c** $16x + 11y$
d $2p + 8$ **e** $14a + 6b$ **f** $5 + 2g$

Exercise 5E

1 **a** true **b** true **c** false **d** true
e false
2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{3}{5}$ **d** $\frac{3}{4}$
3 **a** $\frac{2}{3}$ **b** $\frac{2}{3}$ **c** $\frac{2}{3}$
4 **a** C **b** E **c** B **d** A
e D
5 **a** $2x$ **b** $5p$ **c** $7r$ **d** $11s$
e $10ab$ **f** $5cd$
6 **a** $10ab$ **b** $16xy$ **c** $10b$ **d** $28xz$
e $36abc$ **f** $48def$ **g** $42ab$ **h** $42abc$
i $84abc$ **j** $6xy$
7 **a** $36a$ **b** $63d$ **c** $8e$ **d** $15a$
e $12ab$ **f** $63eg$ **g** $8abc$ **h** $28adf$
i $12abc$ **j** $8abc$ **k** $60defg$ **l** $24abcd$
8 **a** $\frac{x}{5}$ **b** $\frac{z}{2}$ **c** $\frac{a}{12}$ **d** $\frac{b}{5}$
e $\frac{2}{x}$ **f** $\frac{5}{d}$ **g** $\frac{x}{y}$ **h** $\frac{a}{b}$
9 **a** $\frac{2}{5}$ **b** $\frac{5}{9}$ **c** $\frac{9a}{4}$ **d** $\frac{2b}{5}$
e $\frac{x}{2}$ **f** $\frac{3x}{4}$ **g** $\frac{2}{3}$ **h** $\frac{3}{4}$
i $2a$ **j** 3 **k** $2y$ **l** $\frac{3}{y}$
10 **a** $3k$ **b** $6x$ **c** $12xy$ **d** $14ab$
11 **a** \$20 **b** $\frac{\$C}{5}$
12 **a** $6p$
b $3 \times 2p$ also simplifies to $6p$, so they are equivalent.

13 a



b $12a$

c $\frac{12a}{3a}$ simplifies to 4. It has four times the area.

d area is multiplied by 9

Exercise 5F

1 a 12 **b** 10 **c** 30 **d** 27
2 a 20 **b** 28 **c** 40 **d** 100
3 a 35 **b** 20
4 a $3x$ **b** 36
5 a \$36 **b** \$21 **c** $3n$
6 a 6 **b** 11 **c** $t + 2$
7 a $10x$ **b** $15x$ **c** kx
8 $2n$
9 a 180 km **b** 30 km **c** $70n$
10 a \$200 **b** \$680 **c** $50 + 80x$
11 a

Hours	1	2	3	4	5
Total cost (\$)	150	250	350	450	550

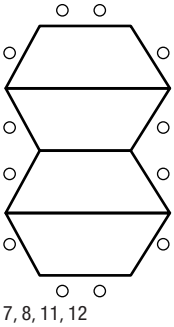
b $100t + 50$ **c** \$3050
12 a \$25 **b** $10x + 5$ **c** \$75
13 a 33 **b** $g = 8$ and $b = 5$
c $g = 3$ and $b = 2$, $g = 1$ and $b = 14$, $g = 0$ and $b = 20$
14 a $0.2 + 0.6t$ **b** $0.8 + 0.4t$
c Emma's **d** 3 min
e Answers will vary.

Maths@Work: Ergonomic office design – Office fitouts

1

	Design A	Design B	Design C
i	7, 14, 21, …	7, 12, 17, 22	6, 10, 14, 18
ii	Number of people $= 7 \times n$	Number of people $= 2 + 5n$	Number of people $= 2 + 4n$
iii	70 people	52 people	42 people
iv	\$33 249	\$27 039	\$23 589

2 a



b 7, 8, 11, 12
c 7, 8, 11, 12, 15, 16, 19, 20, 23, 24
24 people use 10 desks.

3 b

Office	Cost of one row of furniture	Total cost
I	\$9801	\$47 604
II	\$15 604	\$162 576
III	\$12 885	\$223 875
IV	\$8558	\$247 392

Puzzles and games

1 $A = 3, B = 7, C = 2, D = 1$
2 40 metres
3 a $a = 4, b = 12, c = 16, d = 8, e = 36$
b $a = 6, b = 3, c = 5, d = 10, e = 15$
4

x	2	2	3	0	5
y	7	6	3	12	1
$3x$	6	6	9	0	15
$x + 2y$	16	14	9	24	7
xy	14	12	9	0	5

5 $a = 1, c = 4, d = 2, e = 1, f = 3, g = 4, h = 1, i = 3, j = 4, k = 2$
6 a $A = 7, B = 10, C = 8, D = 9, E = 4, F = 11$
b $A = 4, B = 9, C = 2$

Multiple-choice questions

1 B **2** D **3** A **4** B **5** D
6 D **7** D **8** C **9** A **10** A

Short-answer questions

1 a $5a, 3b, 7c, 12$ **b** 12
2 a $u + 7$ **b** $3k$ **c** $h - 10$
3 a 6 **b** 104 **c** 16 **d** 21
4 a 13 **b** 24 **c** 2 **d** 17
5 a 15 **b** 24 **c** 2 **d** 4
6 a 15 **b** 8 **c** 4 **d** 27
7 a

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$4x$	0	4	8	12
$3x + x$	0	4	8	12

b equivalent
8 a E **b** N **c** E **d** N
9 a L **b** N **c** L **d** N
e L **f** N
10 a $7x + 3$ **b** $11p$ **c** $7a + 14b + 4$
d $3m + 17mn + 2n$ **e** $1 + 7c + 4h - 3o$
f $4u + 3v + 2uv$
11 a $12ab$ **b** $6xyz$ **c** $36fgh$ **d** $64klm$
12 a $\frac{3}{2}$ **b** $\frac{3}{5}$ **c** $\frac{a}{3}$ **d** $\frac{4x}{3z}$
13 $9t$ **14** $g + b$ **15** $3x$

Extended-response questions

1 a \$60 **b** \$60 **c** \$90 **d** $30n$
e \$100
2 a i \$17 **ii** \$32 **iii** \$152
b $2 + 1.5k$
c \$62
d $6 + 1.2k$

Chapter 6

Pre-test

1 a 0.1 **b** 0.3 **c** 1.7 **d** 0.01
e 0.001 **f** 4.7
2 a 0.5 **b** 0.25 **c** 0.75

3 **a** \$0.70 **b** \$0.85 **c** \$1.00 **d** \$0.05
e \$1.05 **f** \$0.03
4 **a** 50 **b** 25 **c** 75 **d** 250
5 **a** \$0.90 **b** \$10.50 **c** \$22.50 **d** \$0.81
6 10c
7 \$15.50
8 **a** \$85 **b** \$0.10 **c** \$2.70 **d** \$0.70
e \$11.20 **f** \$0.24
9 **a** \$7 **b** \$2.90 **c** \$3.95 **d** \$29.75
e \$7.50
10 **a** \$87.50 **b** \$92.60 **c** \$20.90
11 **a** \$38.55
12 **a** 523 **b** 839 **c** 312 **d** 1237

Exercise 6A

1 **a** E **b** D **c** B **d** C **e** A
2 **a** $\frac{2}{10}$ **b** $\frac{3}{100}$ **c** $\frac{7}{1000}$
3 **a** 5 **b** 6 **c** 7 **d** 37
4 **a** $\frac{6}{10}$ **b** $\frac{6}{100}$ **c** $\frac{6}{1000}$ **d** $\frac{6}{10}$
e 6 **f** $\frac{6}{100}$ **g** $\frac{6}{100}$ **h** $\frac{6}{1000}$
5 **a** 0.3 **b** 0.8 **c** 0.15 **d** 0.23
e 0.9 **f** 0.12 **g** 0.121 **h** 0.174
i 0.1 **j** 0.11 **k** 0.111 **l** 0.03
6 **a** 6.4 **b** 5.7 **c** 212.3 **d** 1.16
e 14.83 **f** 7.51 **g** 5.07 **h** 18.612
7 **a** T **b** F **c** T **d** T
e F **f** T **g** T **h** T
8 **a** 6.1 **b** 9.6 **c** 0.8 **d** 28.5
e 0.171 **f** 0.203
9 **a** 3.05, 3.25, 3.52, 3.55 **b** 3.06, 3.6, 30.3, 30.6
c 1.718, 1.871, 11.87, 17.81
d 22.69, 22.96, 26.92, 29.26, 29.62
10 **a** Waugh, Border, Gilchrist, Taylor, Hughes **b** first
11 **a** Day 6 **b** Day 4 **c** Day 6
12 **a** $7 \times 1 + 1 \times \frac{1}{10} + \frac{3}{100} + 5 \times \frac{1}{1000}$ **b** 1.563
13 **a** 0.1, 1.0 (2 ways)
b 0.12, 0.21, 1.02, 1.20, 2.01, 2.10, 10.2, 12.0, 20.1, 21.0 (10 ways)
c 0.123, 0.132, 0.213, 0.231, 0.312, 0.321, 1.023, 1.032, 1.203, 1.230, 1.302, 1.320, 2.013, 2.031, 2.103, 2.130, 2.301, 2.310, 3.012, 3.021, 3.102, 3.120, 3.201, 3.210, 10.23, 10.32, 12.03, 12.30, 13.02, 13.20, 20.13, 20.31, 21.03, 21.30, 23.01, 23.10, 30.12, 30.21, 31.02, 31.20, 32.01, 32.10, 102.3, 103.2, 120.3, 123.0, 130.2, 132.0, 201.3, 203.1, 210.3, 213.0, 230.1, 231.0, 301.2, 302.1, 310.2, 312.0, 320.1, 321.0 (60 ways)

Exercise 6B

1 **a** 2 **b** 4 **c** 2 **d** 5
2 **a** 5 **b** 9 **c** 1 **d** 4
e 0 **f** 9 **g** 6 **h** 5
3 **a** 1 **b** 7 **c** 4 **d** 8
4 **a** 8 **b** 7 **c** 5
5 **a** 14.8 **b** 7.4 **c** 15.6 **d** 0.9
e 6.9 **f** 9.9 **g** 55.6 **h** 8.0
i 0.7 **j** 0.7 **k** 0.7 **l** 0.9
6 **a** 3.78 **b** 11.86 **c** 5.92 **d** 0.93
e 123.46 **f** 300.05 **g** 3.13 **h** 9.85
i 56.29 **j** 7.12 **k** 29.99 **l** 0.90
7 **a** 15.9 **b** 7.89 **c** 236 **d** 1
e 231.9 **f** 9.4 **g** 9.40 **h** 34.713
8 **a** 24.0 **b** 14.90 **c** 7 **d** 30.000
9 **a** 28 **b** 9 **c** 12 **d** 124
e 22 **f** 118 **g** 3 **h** 11
10 **a** \$13 **b** \$31 **c** \$7 **d** \$1567
e \$120 **f** \$10 **g** \$1 **h** \$36
11 **a** 149.9×48
b **i** 7195 cents **ii** \$72
12 5, 6, 7, 8 or 9.
13 0.35, 0.36, 0.37, 0.38, 0.39, 0.40, 0.41, 0.42, 0.43, 0.44
14 Depends on your calculator.

Exercise 6C

1 C
2 B
3 **a** 8.57 **b** 5.179 **c** 15.956
4 **a** 0.571 **b** 1.97 **c** 12.40
1.209 −0.43 − 8.35
3.528
5 **a** 6.8 **b** 14.96 **c** 3.87 **d** 8.99
e 27.97 **f** 25.94 **g** 247.4 **h** 58.31
6 **a** 2.6 **b** 5.8 **c** 0.9 **d** 0.99
e 8.47 **f** 15.507 **g** 16.06 **h** 21.33
i 0.013
7 **a** 0.79 **b** 0.516 **c** 0.4 **d** 1.73
e 12.1 **f** 114.13 **g** 6.33 **h** 70.79
8 **a** 0.5 **b** 3.2 **c** 21.2 **d** 0.48
e 12.3 **f** 131.4 **g** 22.23 **h** 13.457
i 43.27 **j** 4947.341
9 16.189
10 **a** \$45.70 **b** \$54.30
11 49.4 mm
12 3.3°C
13 **a**

0.6	0.7	0.2
0.1	0.5	0.9
0.8	0.3	0.4

Magic sum = 1.5

b

1.6	0.5	0.9	0.4
0.3	1.0	0.6	1.5
0.2	1.1	0.7	1.4
1.3	0.8	1.2	0.1

Magic sum = 3.4

Exercise 6D

1 **a** 1 **b** 2 **c** 4
d 2 **e** 1
2 **a** 00 **b** 000 **c** 00 **d** 00000
3 **a** 000 **b** 00 **c** 0 **d** 00000
4 **a** right 2 places **b** left 1 place
c right 6 places **d** no change
e left 3 places **f** right 3 places
g right 1 place **h** left 7 places
5 **a** 48.7 **b** 352.83 **c** 4222.7 **d** 1430.4
e 5699.23 **f** 125.963 **g** 12 700 **h** 154 230
i 3400 **j** 2132 **k** 86 710 000 **l** 516 000
6 **a** 4.27 **b** 35.31 **c** 2.4422
d 56.893 **e** 12.135 18 **f** 9.326 11
g 0.029 **h** 0.001 362 **i** 0.000 54
j 0.367 **k** 0.000 002 **l** 0.010 000 4
7 **a** 2291.3 **b** 31.67 **c** 0.49
d 0.222 **e** 63 489 000 **f** 0.001 003 2
8 **a** 15 600 **b** 43 000 **c** 22 510
d 16 000 **e** 213 400 **f** 2 134 000
g 7000 **h** 9 900 000 **i** 340 000

j 15.6 **k** 1.56 **l** 0.156
m 8.7 **n** 0.87 **o** 0.087
p 0.016 **q** 0.007 **r** 0.0034

9 $137

10 3000 cents, $30

11 **a** $21 200 **b** $21 400

12 **a** 10 **b** 10 **c** 100
d 10, 10 **e** 1000

13 **a** **i** 2, 4.714, 471.4 **ii** 2, 2.4, 0.24
b **i** 1010 ($0.05 \times 100 \times 2$)
ii 18 400 ($9.2 \times 1000 \times 2$)
iii 452.7 ($15.09 \times 10 \times 3$)
iv 2184 ($7.28 \times 100 \times 3$)
v 7.4 ($148 \div 10 \div 2$)
vi 2.42 ($484 \div 100 \div 2$)
vii 38 724 ($9.681 \times 1000 \times 4$)
viii 0.9299 ($185.98 \div 100 \div 2$)

Exercise 6E

1 **a** 1 **b** 3 **c** 2 **d** 4
e 1 **f** 2 **g** 2 **h** 5

2 **a** 1 **b** 2 **c** 3 **d** 3
e 5 **f** 2 **g** 4 **h** 3
i 9

3 **a** 19.2 **b** 1.92 **c** 0.192 **d** 1.52
e 19.46 **f** 0.0756

4 in the question; decimal places

5 **a** 4.8 **b** 16.8 **c** 7.5 **d** 29
e 19.6 **f** 1.96 **g** 2.4 **h** 0.24
i 0.56 **j** 0.27 **k** 0.74 **l** 0.81

6 **a** 1.128 **b** 5.427 **c** 3.556
d 0.74 **e** 2.34 **f** 8.12

7 **a** 100.8 **b** 218.46 **c** 15.516 **d** 23.12
e 12.42 **f** 5.44 **g** 311.112
h 0.000 966 **i** 0.000 213

8 **a** $31.50, $32 **b** $22.65, $23
c $74.80, $75 **d** $2.82, $3
e $2.10, $2 **f** $11.79, $12

9 **a** 100.8 **b** 483 **c** 25 400
d 9800 **e** 14 400 **f** 364 550
g 0.68 **h** 371

10 29.47 m

11 3.56 kg

12 **a** 38.76 **b** 73.6 **c** 0.75
d 42, 0.42 **e** 0.042

13 **a** $13.10
b $10 note, $2, $1 coins and a 10c piece (Other answers are possible.)

Exercise 6F

1 B

2 **a** 10 **b** 100 **c** 1000
d 10 **e** 100 **f** 100

3 **a** $12 \div 3$ **b** $18 \div 2$ **c** $152 \div 1$
d $5642 \div 2$ **e** $38 \div 1$ **f** $380 \div 1$
g $484.6 \div 2$ **h** $0.7 \div 2$ **i** $72 \div 9$

4 **a** 32.456, 3 **b** 12 043.2, 12
c 34.5, 1 **d** 1 234 120, 4

5 **a** 4.2 **b** 6.1 **c** 21.34
d 0.7055 **e** 1.571 **f** 0.308
g 19.393 **h** 372.9 **i** 0.0024
j 117.105 **k** 0.6834 **l** 0.002 562 5
m 0.39 **n** 0.37 **o** 0.175
p 8.95 **q** 9.36 **r** 105.1

6 **a** 30.7 **b** 77.5 **c** 26.8
d 8.5 **e** 44.4 **f** 645.3
g 0.08 **h** 0.050 425 **i** 980
j 800.6 **k** 0.79 **l** 2 161 000

7 **a** 7.5 **b** 75 **c** 750
d 7500 **e** 75 000 **f** 750 000

8 **a** 11.83 kg **b** $30.46
c 304.33 m **d** 239.17 g
e 965.05 L **f** $581.72

9 **a** 1.1807 **b** 8.267
c 0.012 374 8 **d** 0.004 23
e 0.096 487 **f** 0.000 782 5

10 **a** 26.67, 26 can be filled **b** 40

11 $1.59 per L

12 **a** 4 **b** 6 **c** 27
d 39 **e** 76 **f** 5000

13 **a** 24.53 **b** 19.7 **c** 2453
d 1.97 **e** 2.453 **f** 197

Progress quiz

1 **a** $\frac{3}{10}$ **b** $\frac{3}{1000}$

2 **a** F **b** T **c** T **d** F

3 **a** 5.35 **b** 9.7 **c** 0.8 **d** 0.271

4 **a** 13.48 **b** 8.04 **c** 72.68 **d** 45.40

5 **a** $9 **b** $16 **c** $104 **d** $1100

6 **a** 8.77 **b** 56.85 **c** 229.81 **d** 63.003

7 **a** 3.42 **b** 22.21 **c** 21.744 **d** 33.234

8 **a** 7351.6 **b** 49 200 **c** 56.789
d 0.005 609 **e** 27 800 **f** 0.002 78

9 **a** 3.24 **b** 3.138 **c** 177 **d** 28 400

10 **a** 17.614 **b** 0.0016 **c** 1704.9 **d** 0.1891

11 100

Exercise 6G

1 **a** 5 **b** 100 **c** 75, 7 **d** 5, 4

2 **a** 2 **b** 15, 20 **c** 10, 4 **d** 16

3 **a** 0.3 **b** $\frac{1}{2}$ **c** 0.8 **d** 1.5
e 0.9 **f** $\frac{1}{2}$

4 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{4}{5}$ **d** $\frac{11}{50}$
e $1\frac{11}{50}$ **f** $5\frac{1}{2}$ **g** $\frac{3}{20}$ **h** $\frac{99}{100}$
i $\frac{2}{25}$ **j** $\frac{1}{100}$ **k** $\frac{1}{1000}$ **l** $\frac{101}{500}$
m $\frac{1}{2}$ **n** $6\frac{2}{5}$ **o** $10\frac{3}{20}$ **p** $18\frac{3}{25}$
q $3\frac{1}{4}$ **r** $\frac{1}{20}$ **s** $9\frac{3}{40}$ **t** $5\frac{24}{125}$

5 **a** 0.7 **b** 0.9 **c** 0.31 **d** 0.79
e 1.21 **f** 3.29 **g** 0.123 **h** 0.03
i 0.07

6 **a** $\frac{8}{10} = 0.8$ **b** $\frac{5}{10} = 0.5$
c $\frac{35}{100} = 0.35$ **d** $\frac{46}{100} = 0.46$
e $5\frac{95}{100} = 5.95$ **f** $3\frac{25}{100} = 3.25$
g $\frac{25}{10} = 2.5$ **h** $\frac{375}{1000} = 0.375$
i $\frac{28}{100} = 0.28$

7 **a** 0.5 **b** 0.5 **c** 0.75 **d** 0.4
e $0.\dot{3}$ **f** 0.375 **g** $0.41\dot{6}$
h $0.\overline{428571}$ **i** $0.\dot{2}$
8 **a** 0, 0.5, 1 **b** 0, $0.\dot{3}$, $0.\dot{6}$, 1
c 0, 0.25, 0.5, 0.75, 1 **d** 0, 0.2, 0.4, 0.6, 0.8, 1.0
9 $\frac{1}{4}$, 0.4, $\frac{1}{2}$, $\frac{5}{8}$, 0.75, 0.99
10

$0.01	$0.05	10c	20c	25c	50c	75c	90c	$0.99
$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{9}{10}$	$\frac{99}{100}$

11 **a** 0.5, $0.\dot{3}$, 0.25, 0.2, $0.1\dot{6}$, $0.\overline{142857}$, 0.125, $0.\dot{1}$, 0.1
b They get smaller.
12 **a** $\frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{4}{5}, \frac{9}{10}$ **b** 4
c 2
13 **a** $\frac{6}{10}$, 62%, 65%, $\frac{66}{100}$, $0.\dot{6} = \frac{2}{3}$, 0.67, $\frac{7}{10}$
b $\frac{3}{5}$, 0.52, 45%, 0.43, $\frac{2}{5} = 0.4$ $\frac{9}{25} = 36\%$, $\frac{6}{20}$, $0.\dot{3} = \frac{1}{3}$
14 **a** No
b 1.4142135623…, No pattern
c No
d Irrational numbers such as $\sqrt{2}$ and π cannot be expressed as a fraction or as a decimal with a pattern.

Exercise 6H

1 **a** 95% **b** 60% **c** 75% **d** 26%
2 C
3 A
4 **a** 50
b 50%
c **i** 5 **ii** 100 **iii** 20 **iv** 1
d **i** 50% **ii** 50% **iii** 50% **iv** 50%
5 **a** 100 **b** 35 **c** out of **d** 15
6 **a** 0.32 **b** 0.27 **c** 0.68 **d** 0.54
e 0.1 **f** 0.12 **g** 0.18 **h** 0.85
i 0.92 **j** 0.75 **k** 0.11 **l** 0.6
m 0.06 **n** 0.09 **o** 1 **p** 0.01
q 2.18 **r** 1.42 **s** 0.75 **t** 1.99
7 **a** 0.225 **b** 0.175 **c** 0.3333 **d** 0.0825
e 1.1235 **f** 1.888 **g** 1.50 **h** 5.20
i 0.0079 **j** 0.000 25 **k** 0.0104 **l** 0.0095
8 **a** 80% **b** 30% **c** 45% **d** 71%
e 41.6% **f** 37.5% **g** 250% **h** 231.4%
i 2.5% **j** 0.14% **k** 1270% **l** 100.4%
9 35%
10 0.52
11 **a** **i** 50% **ii** 0.5
b **i** 25% **ii** 0.25
c **i** 90% **ii** 0.9
d **i** 10% **ii** 0.1
e **i** 100% **ii** 1
12 **a** 100% is all questions correct.
b 20 out of 40, half the answers correct.
c No questions correct.
13 **a** F ÷ A × 100
b F: points scored for the team; A: points scored against the team
c 100%

Maths@Work: Retail assistant

1 **a** $38.50 **b** $141.80 **c** $20.95
2 **a** $5.70 **b** $123.90 **c** $45.65 **d** $3.00
3

Customer	Rounding amount in cents	Total due	Change due
Adam	−2	$45.65	$5.15
Blake	+2	$123.05	$6.95
Christy	+2	$123.05	$2.05
Dion	−2	$56.90	$0.10
Eden	−1	$67.55	$3.05

4 **a** **i** $5.20 **ii** $3.15 **iii** $10.15 **iv** $0.35
b **i** $14.63 **ii** $23.69 **iii** $17.40
iv $55.72 **v** $55.70 **vi** $0.10
5 **a** 2 cents **b** 1 cent
6 **i** $1.28 **ii** 12 **iii** $34.82 **iv** $15.20
7

Total value
$2050
$4046
$616
$695
$2280
$576
$10 263

Puzzles and games

1 **a** 7, 8 **b** 6, 5, 1, 4 **c** 0, 1, 0, 2
d 7, 5, 1, 6, 1
2 **a** 8, 9 **b** 9 **c** 5, 5, 1 **d** 7, 0, 7
3 A MUSHROOM
4 Both the dollars and the cents match your age.
5 0.3 m, 0.42 m, 0.66 m, 0.78 m, 0.90 m, 1.02 m, 1.14 m, 1.38 m

Multiple-choice questions

1 D **2** B **3** C **4** E **5** A
6 D **7** D **8** B **9** D **10** C

Short-answer questions

1 **a** 0.44, 0.4, 0.04 **b** 0.98, 0.932, 0.895
2 **a** 8.1 **b** 0.81 **c** 8.01 **d** 0.801
3 **a** 3 hundredths = $\frac{3}{100}$ **b** 3 thousandths = $\frac{3}{1000}$
c 3 ones = 3
4 **a** false **b** false **c** true **d** true
e false **f** true **g** true **h** false
i true
5 **a** $62.88 **b** $63 **c** $62.90
6 **a** 12.7 **b** 8.4 **c** 9.4 **d** 7.5
e 0.1 **f** 7.1
7 **a** 12.81 **b** 423.46 **c** 15.89 **d** 7.25
e 6.67 **f** 3.33
8 **a** 1.6 **b** 1.56 **c** 19.594 **d** 9.6
e 21.9 **f** 3.3 **g** 45.94 **h** 43.5
i $7.76 **j** $7 **k** $24.80
9 **a** 5 **b** 5 **c** 4 **d** 2
10 **a** T **b** F **c** T **d** T
e F **f** F **g** T **h** T
i T **j** F **k** F **l** T
11 **a** 19.2 **b** 63.99 **c** 19.32 **d** 0.95
e 1.52 **f** 6 **g** 16 **h** 3
i 34.2

12

Decimal	Fraction	Percentage
0.01	$\frac{1}{100}$	1%
0.1	$\frac{1}{10}$	10%
1	$\frac{100}{100}$	100%
0.45	$\frac{45}{100} = \frac{9}{20}$	45%
0.7	$\frac{70}{100} = \frac{7}{10}$	70%
0.32	$\frac{32}{100} = \frac{8}{25}$	32%
0.06	$\frac{6}{100} = \frac{3}{50}$	6%
0.79	$\frac{79}{100}$	79%
1.05	$\frac{105}{100} = \frac{21}{20}$	105%
0.35	$\frac{35}{100} = \frac{7}{20}$	35%
0.65	$\frac{65}{100} = \frac{13}{20}$	65%
0.125	$\frac{125}{1000} = \frac{1}{8}$	12.5%

13 a i 0.7 **ii** 1.2 **iii** 0.37 **iv** 0.0021
b i 40% **ii** 2% **iii** 165% **iv** 620%

Extended-response questions

1 a Jessica $12.57; Jaczinda $13.31; hence, Jaczinda earns higher pay rate by 74c per hour.
b $12.49, $12.50 to the nearest 5 cents
c $36.90
d i $1.40 **ii** $3.20
e i $17.50 **ii** $3.50 **iii** $700

Semester review 1

Whole numbers

Multiple-choice questions

1 E **2** B **3** D **4** C **5** A

Short-answer questions

1 a 4 **b** 1 **c** 303
2 a 7324 **b** 12 092
3 a 4962 **b** 819 **c** 147 **d** 7600
e 105 **f** 137
4 a false **b** true **c** true
5 22
6 a 10 **b** 5 **c** 17 **d** 30
e 56 **f** 48 **g** 62 **h** 16
i 42
7 a false **b** true **c** false **d** true
e true **f** true
8 a $(2 + 3) \times 4 = 20$ **b** $(10 - 2) \div 8 = 1$
c $4 \times (6 - 2) \div 8 = 2$

9 a 10 **b** 40 **c** 140
10 a 100 **b** 100 **c** 1500

Extended-response questions

1 a 28 h **b** $700 **c** $1000 **d** 12 h

Geometry

Multiple-choice questions

1 A **2** B **3** B **4** B **5** D

Short-answer questions

1 a acute **b** right **c** obtuse
d straight **e** reflex **f** revolution
2 a 30° **b** 80° **c** 150°
3 25°
4 78°
5 a $a = 140$ **b** $a = 50$ **c** $a = 140$
d $a = 65$ **e** $a = 62$ **f** $a = 56$
6 $a = 100$, $b = 80$, $c = 100$, $d = 80$, $e = 100$, $f = 80$, $g = 100$
7 Because the alternate angles are not equal.
8 a 115 **b** 71 **c** 100

Extended-response questions

1 a i $x = 56$ **ii** $y = 95$ **iii** $z = 29$
b $x + y + z = 180$

Number properties and patterns

Multiple-choice questions

1 E **2** A **3** B **4** E **5** D

Short-answer questions

1 a 1, 3, 5, 15 **b** 1, 2, 3, 5, 6, 10, 15, 30
c 1, 2, 4, 5, 10, 20, 25, 50, 100
2 a 3, 6, 9, 12, 15 **b** 7, 14, 21, 28, 35
c 11, 22, 33, 44, 55
3 1, 2, 3 and 6
4 4
5 a 121 **b** 144 **c** 25
6 a 49 **b** 144 **c** 9
7 a

Index form	3^2	5^2	6^2	8^2
Value	9	25	36	64

b

Square root form	$\sqrt{9}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{64}$
Value	3	5	6	8

8 a 1080, 536, 930, 316 **b** 1080, 135, 930
c 1080, 536, 316 **d** 1080, 135, 930
e 1080, 930
9 a

Number of squares	0	1	2	3	4
Number of sticks	1	1 + 5 = 6	1 + 5 × 2 = 11	1 + 5 × 3 = 16	1 + 5 × 4 = 21

b number of sticks = 1 + 5 × number of squares
c number of sticks = 1 + 5 × 9 = 46 matchsticks
d 16 squares

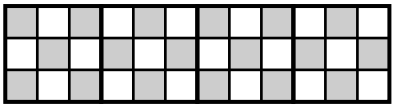
10

input	4	5	6	9	11	100
output	19	23	27	39	47	403

11 $A(1, 0)$, $B(4, 1)$, $C(3, 2)$, $D(1, 3)$, $E(3, 4)$, $F(0, 2)$, $O(0, 0)$

Extended-response questions

1 a



b 18 **c** 9th **d** \$61

Fractions and percentages

Multiple-choice questions

1 B **2** C **3** E **4** D **5** A

Short-answer questions

1 $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$

2 $\frac{17}{3}$

3 a $\frac{11}{12}$ **b** $2\frac{2}{3}$ **c** $6\frac{1}{4}$

d $\frac{1}{5}$ **e** 4 **f** $\frac{1}{2}$

4 $\frac{3}{20}$

5 \$120

6 \$8

7 a true **b** true **c** true **d** false

8 $\frac{2}{3}$

Extended-response questions

1 a 6 **b** $\frac{8}{9}$ **c** 9

d second dose on Sunday week

Algebra

Multiple-choice questions

1 B **2** A **3** C **4** B **5** D

Short-answer questions

1 a 3 **b** 7 **c** $8x + 7y$

2 a $x + 3$ **b** $12a$ **c** $2x + 3y$ **d** $\frac{w}{6}$

e $y - 2x$

3 a L **b** N **c** L **d** N

4 a 13 **b** 11 **c** 39 **d** 6

e 3 **f** 24

5 36

6 a $10a$ **b** $4x$ **c** $12a$ **d** m

e $6 + 5a$ **f** $4x + 2y$

7 a $6 + 2x$ **b** $3x$

8 a $6bc$ **b** $5b$ **c** p

9 a $10p$ **b** $25p$

10 $12x$

Extended-response questions

1 a i \$16 **ii** $3x$ **iii** $3x + 10$

b $3x + 2y$

c 16

Decimals

Multiple-choice questions

1 C **2** C **3** D **4** A **5** A

Short-answer questions

1 a 0.2 **b** 0.13 **c** 1.7

2 a 6 ones **b** $\frac{4}{1000}$

3 a 18 **b** 18.4 **c** 18.40

4 a 4.07 **b** 269.33 **c** 19.01 **d** 0.24

e 0.09 **f** 60

5 a 0.833 **b** 2.4 **c** 0.042

6 a 4.5387 **b** 45.387 **c** 0.045 387

7 a 36 490 **b** 0.018 **c** 3886

8 a $\frac{4}{5}$ **b** 1.1 **c** $\frac{2}{3}$

9 a true **b** false **c** false

d true **e** false **f** true

Extended-response questions

1 a \$5.83 **b** \$5.85 **c** \$4.15

Chapter 7

Pre-test

1 a 11 **b** 5 **c** 7 **d** 22

e 0 **f** 6 **g** 1 **h** 12

i 1 **j** 8 **k** 9 **l** 13

2 a < **b** < **c** < **d** >

e < **f** < **g** > **h** >

3 a 5°C **b** 2°C **c** 7°C

4 a 30 **b** 77 **c** 72 **d** 39

5 a 5 **b** 11 **c** 9 **d** 7

6 a 7 **b** 5 **c** 7 **d** 16

7 $A(1, 1)$, $B(3, 2)$, $C(2, 3)$

8 Perimeter = 14 units

Exercise 7A

1 a 1 **b** 0 **c** −4

d −1 **e** −18 **f** −36

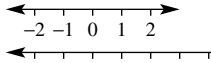
2 a −2, 2 **b** 0, 2 **c** −7, −5

d −5, −3, 0

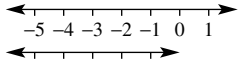
3 a greater **b** less **c** greater

d less **e** greater **f** less

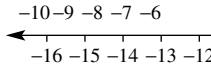
4 a



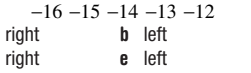
b



c



d



5 a right **b** left **c** left

d right **e** left **f** right

6 **a** -2 **b** -6 **c** 3 **d** 7
e 15 **f** -21 **g** -132 **h** 1071
7 **a** < **b** > **c** > **d** <
e > **f** < **g** < **h** >
i < **j** > **k** < **l** >
m < **n** > **o** > **p** <
8 **a** 4°C **b** -1°C **c** -7°C **d** -25°C
9 **a** $-\frac{1}{2}$ **b** $-1\frac{1}{3}$ or $-\frac{4}{3}$
c $-2\frac{2}{5}$ or $-\frac{12}{5}$ **d** $-13\frac{3}{4}$ or $-\frac{55}{4}$
10 **a** false **b** false **c** false **d** true
e false **f** false **g** true **h** false
11 **a** $-2, -1, 0, 1, 2, 3, 4$
b $-7, -6, -5, -4, -3, -2, -1, 0$
c $-2, -1, 0, 1$ **d** $-4, -3, -2, -1, 0$
e $-3, -2, -1, 0, 1, 2, 3$ **f** $-9, -8, -7, -6, -5, -4$
12 **a** $-10, -6, -3, -1, 0, 2, 4$
b $-304, -142, -2, 0, 1, 71, 126$
13 **a** $0, -1, -2$ **b** $-2, 0, 2$
c $-5, -10, -15$ **d** $-44, -46, -48$
e $-79, -75, -71$ **f** $-101, -201, -301$
14 **a** 4°C **b** 0°C **c** -2°C **d** -10°C
15 **a** -2 **b** 1 **c** -1 **d** -7

Exercise 7B

1 **a** 1 **b** -2 **c** -9 **d** -1
e -4 **f** -8
2 **a** right **b** right **c** left **d** left
3 **a** D **b** A **c** B **d** C
4 **a** 1 **b** 3 **c** 2 **d** 1
e -5 **f** -3 **g** 1 **h** 4
i -1 **j** -3 **k** -2 **l** -2
m -4 **n** -8 **o** -1 **p** 2
5 **a** -2 **b** -1 **c** -8 **d** -19
e -3 **f** -7 **g** -15 **h** -13
i -4 **j** -10 **k** -15 **l** -7
m -41 **n** -12 **o** -22 **p** -47
6 **a** 2 **b** -9 **c** 6 **d** -31
e -300 **f** -100 **g** -93 **h** -634
7 **a** 5 **b** -9 **c** 1 **d** -13
e 1 **f** -8 **g** -7 **h** -13
i 1 **j** -22 **k** -32 **l** -4
8 **a** 5 **b** 9 **c** 5 **d** 2
e 5 **f** 7 **g** 3 **h** 10
i 5 **j** 16 **k** -4 **l** -5
m -6 **n** -13 **o** -30 **p** -113
9 **a** \$145 **b** \$55 **c** \$5250
10 **a** 3°C **b** -3°C **c** -46°C
11 69°C
12 **a** 59 m **b** 56 m
13 Other combinations may be possible.
a $-, +$ **b** $+, -, -$ **c** $+, +, -, +$
d $-, +, +, +, -$
14 Answers may vary.
a $2 + 3 = 5$ **b** not possible
c $-2 + 3 = 1$ **d** $-4 + 2 = -2$
e $7 - 5 = 2$ **f** $2 - 4 = -2$
g not possible **h** $-3 - 4 = -7$

Exercise 7C

1 **a** 2 **b** 6 **c** 4
d 11 **e** 37 **f** 142
2 **b** $6, -3$ **c** $-3, 1$ **d** $-11, -7$
e $-10, 1$ **f** $-2, -4$ **g** $-6, -3$
h $-26, -37$
3 **a** C **b** A **c** D **d** B
4 **a** 1 **b** 5 **c** 6 **d** 2
e -3 **f** -5 **g** -2 **h** -4
i -3 **j** -22 **k** -35 **l** -80
m -10 **n** -29 **o** -50 **p** -112
q -109 **r** -113
5 **a** 5 **b** 7 **c** -12 **d** -7
e -15 **f** -42
6 **a** 4 **b** 9 **c** 0 **d** -8
e -7 **f** -10 **g** -1 **h** -17
i -33
7 **a** -3 **b** -10 **c** -4 **d** 4
e -1 **f** 4 **g** 5 **h** 5
i -1 **j** -2 **k** -2 **l** -14
8 \$190 of debt
9 **a** **i** \$8000 **ii** $-$\$6000
b \$2000
10 **a** false **b** false
11 **a** false **b** true
12 **a**

-2	0	5
8	1	-6
-3	2	4

b

-13	-11	-6
-3	-10	-17
-14	-9	-7

Exercise 7D

1 **a** 3 **b** 6 **c** 4
d 11 **e** 15 **f** 312
2 **a** $3, 5$ **b** $6, -2$
c $-3, 7$ **d** $-11, -7$
e $-4, -2$ **f** $-1, -6$
g $-7, -11$ **h** $-12, 27$
3 **a** D **b** A **c** B **d** C
4 **a** 5 **b** 11 **c** 50 **d** 90
e -4 **f** -3 **g** -5 **h** -34
i 2 **j** 1 **k** 0 **l** 8
m 28 **n** 34 **o** -12
5 **a** 8 **b** 11 **c** 3 **d** 4
e -5 **f** -5
6 **a** -2 **b** 1 **c** -2 **d** -6
e -7 **f** -14 **g** 6 **h** -3
i 1
7 **a** -1 **b** -5 **c** -4 **d** 4
e 2 **f** -24 **g** -6 **h** -5
i 2 **j** -6 **k** -9 **l** -6
8 **a** 0 **b** -5 **c** 8 **d** 12
e -9 **f** 5 **g** -6 **h** -91
i -15 **j** 6 **k** 17 **l** 11
9 -12 m
10 $-$\$35 000
11 $-$\$30
12 **a** true **b** true
13 **a** false **b** false
14 **a** $+, +$ **b** $+, -$ **c** $-, -$
d $+, -$ **e** $-, +$ **f** $+, -$

Progress quiz

1 **a** < **b** > **c** > **d** <
2 **a** $-1, -3, -5$ **b** $-3, 0, 3$
c $-11, -16, -21$ **d** $-44, -33, -22$
3 **a** 2 **b** -4 **c** -7 **d** 25
4 **a** -3 **b** -9 **c** -77 **d** -61
5 **a** 8 **b** -13 **c** -10 **d** -9
6 **a** 9 **b** 11 **c** -41 **d** 65
7 **a** 3 **b** -4 **c** -7 **d** -62
8 **a** 8 **b** -5 **c** 22 **d** -10

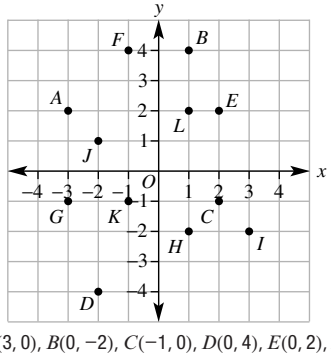
9 **a** 8 **b** 27 **c** −18 **d** 23
10 **a** 0 **b** −24 **c** 3 **d** 16
11 $25 of debt (or −$25)

Exercise 7E

1 **a** 6 **b** −1 **c** 6 **d** 2
e 1 **f** −6 **g** 2 **h** −7
2 C
3 B
4 **a** 7 **b** 9 **c** −3 **d** −11
e −1 **f** −5 **g** −4 **h** −8
i −6 **j** 6 **k** −5 **l** 4
m 19 **n** 5 **o** 8 **p** 7
q 5 **r** −6
5 **a** 4 **b** 15 **c** 28 **d** 3
e 12 **f** 40 **g** −6 **h** $\frac{1}{2}$
i 8 **j** 2 **k** 40 **l** $1\frac{1}{4}$
6 **a** −13 **b** 3 **c** 20 **d** 3
e 19 **f** 15 **g** −14 **h** 11
7 **a** 20 L **b** 35 L **c** 110 L
8 **a** 4 **b** 5 **c** 3 **d** −4
e −7 **f** −1 **g** −5 **h** 1
i −6
9 no, 20
10 **a** **i** 5 **ii** 6
b A negative length is not possible.
11 Answer may vary.
12 **a** 24 metres per second
b 18 metres per second
c 14 metres per second

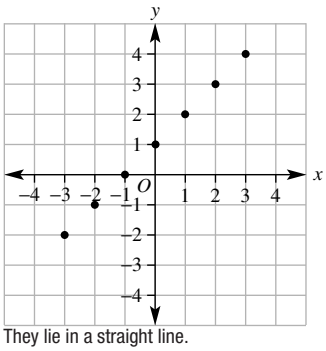
Exercise 7F

1 **a** right, up **b** right, down **c** left, up
d left, down
2 **a** D **b** B **c** A **d** C
e E **f** H **g** F **h** G
3 $A(2, 1)$, $B(3, -2)$, $C(-1, -4)$, $D(-2, 2)$, $E(4, 3)$, $F(2, -3)$, $G(-3, -1)$, $H(-4, 4)$
4a,b



5 $A(3, 0)$, $B(0, -2)$, $C(-1, 0)$, $D(0, 4)$, $E(0, 2)$, $F(1, 0)$, $G(0, -4)$, $H(-3, 0)$
6 **a** 9 **b** 18 **c** 15 **d** 6
e 10 **f** 2 **g** 1
7 **a** triangle **b** rectangle
c trapezium **d** kite
8 28 km

9 **a**



b They lie in a straight line.
10 $y = 2$
11 $[oo]$ is the origin

Maths@Work: Golfer

1 **a** −2 **b** −1 **c** −3 **d** +1
e +2
2 **a** −7 **b** −20 **c** −5 **d** 0
e +3 **f** +2
3

	Final score	under/over par
Chelsea	78	+6
Mitchell	82	+10
Adam	70	−2
Bella	75	+3

4 **a**

hole	PAR	Rick's score	under/over par for Rick	Brad's score	under/over par for Brad
1	4	4	0	5	1
2	4	4	0	5	1
3	4	4	0	5	1
4	4	4	0	5	1
5	4	6	2	4	0
6	4	5	1	4	0
7	3	5	2	4	1
8	5	2	−3	6	1
9	3	3	0	4	1
10	4	3	−1	5	1
11	5	4	−1	5	0
12	4	3	−1	5	1
13	4	3	−1	3	−1
14	5	5	0	6	1
15	3	3	0	4	1
16	4	5	1	5	1
17	5	4	−1	6	1
18	3	3	0	4	1
TOTALS	**72**	**70**	**−2**	**85**	**13**

b Rick 73 and Brad 78
c Rick won by 5 points
d Rick's average 3.89, Brad's average 4.72
e Rick 5 birdies, Brad 1 birdie

5 a

	x	*y*
Tee off 1	−30	−40
Hole 1	−45	40
Tee off 2	−40	45
Hole 2	0	40
Tee off 3	5	35
Hole 3	50	20
Tee off 4	45	15
Hole 4	−30	25
Tee off 5	−30	20
Hole 5	0	0
Tee off 6	10	0
Hole 6	45	−40
Tee off 7	45	−45
Hole 7	−20	−25

Puzzles and games

1 a house

2 a

−3	2	−5
−4	−2	0
1	−6	−1

b

−9	5	4	−6
2	−4	−3	−1
−2	0	1	−5
3	−7	−8	6

c

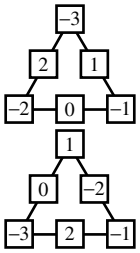
−14	0	−1	−11
−3	−9	−8	−6
−7	−5	−4	−10
−2	−12	−13	1

3 a $-3 \times (4 + -2) = -6$

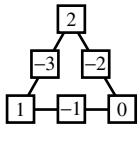
b $-2 \times 5 \times -1 + 11 = 21$ or $-2 \times 5 \div -1 + 11 = 21$

c $(1 - 30 \div -6) \div -2 = -3$ or $1 \times 30 \div -6 - -2 = -3$

4 a



b



c

5 a −81, 243, −729 **b** 4, −2, 1

c −10, −15, −21 **d** −8, −13, −21

6 a 11 and −3 **b** 21 and −10

Multiple-choice questions

1 C **2** E **3** B **4** D **5** C

6 A **7** D **8** C **9** C **10** C

Short-answer questions

1 a −3, 0 **b** 1, −2 **c** −6, −2 **d** −1, −11

2 a < **b** < **c** > **d** <

3 a −5 **b** −2 **c** −15 **d** 1

e −1 **f** 2 **g** −7 **h** −25

i −11 **j** −7 **k** −47 **l** −131

4 a 1 **b** 3 **c** −3 **d** −7

e −2 **f** −5 **g** 12 **h** −18

i 7 **j** −17 **k** 3 **l** 3

5 a −6 **b** 5 **c** −11 **d** 5

e −6 **f** −6 **g** 5 **h** −29

6 a −1 **b** −9 **c** −1 **d** 2

e −21 **f** −2 **g** −87 **h** 30

7 a 4 **b** −5 **c** −3 **d** 7

e 30 **f** 10 **g** 1 **h** 40

8 a 4 **b** −4 **c** 6 **d** −10

e 10 **f** 2 **g** 0 **h** 13

i −15 **j** 18 **k** −5 **l** 16

9 $A(3, 0)$, $B(2, 3)$, $C(-1, 2)$, $D(-4, -2)$, $E(0, -3)$, $F(4, -4)$

Extended-response questions

1 a 16°C **b** −31°C **c** 8°C **d** 19°C

e 27°C

2 rocket

Chapter 8

Pre-test

1 a 15°C **b** 24°C **c** 21°C

2 a $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$

3 a 26 **b** 7 **c** 2, 4, 5, 6, 9

4 a C **b** I **c** C **d** I

5 a 6 **b** 2

6 a 0.5 **b** 0.4 **c** 0.2 **d** 0.2

7 C, A, B, D

8 a L **b** H **c** M

Exercise 8A

1 a N **b** C **c** C **d** N

2 a population **b** sample

c primary **d** secondary

3 a iii **b** iv **c** i **d** v

e vi **f** ii

4 a categorical **b** numerical

c numerical **d** categorical

e numerical **f** numerical

5 a discrete numerical **b** continuous numerical

c continuous numerical **d** categorical

e categorical **f** categorical

g discrete numerical **h** discrete numerical

i continuous numerical **j** continuous numerical

k continuous numerical **l** discrete numerical

m continuous numerical **n** discrete numerical

o categorical **p** discrete numerical

q discrete numerical **r** categorical

6 a primary

b primary

c secondary

d secondary

7 Answers will vary.

8 a observation

b sample of days using observation or secondary source records within each day

c census of the class

d sample
e sample
f sample using secondary source data
g census (this question appears on the population census)
h census of the class
i sample
j results from the population census
k observation
l observation
m sample
n sample

9 a Proximity to the Indian Ocean makes first hand collection of the data difficult.
b Too many people to ask and a sensitive topic means that using the census results as your source would be better.
c Extremely large population makes primary data difficult to collect.
d Sensitive topic might make students less keen to give honest and reliable answers.
e Cultural issues and the different cultural groups that exist in the community makes collection difficult.

10 The data is often collected by a market research company. It is not always possible to know how the data is collected, the areas it is collected from and whether there was a bias introduced in the surveys.

11 a Population is the entire group of people but a sample is a selection from within it.
b If the population is small enough (e.g. a class) or there is enough time/money to survey the entire population (e.g. national census).
c When it is too expensive or difficult to survey the whole population, e.g. television viewing habits of all of NSW.

12 a The answers stand for different categories and are not treated as numbers. They could have been A–E rather than 1–5.
b i 1 = strongly disagree, 2 = somewhat disagree, 3 = somewhat agree, 4 = strongly agree.
ii 1 = poor, 2 = satisfactory, 3 = strong, 4 = excellent.
iii 1 = never, 2 = rarely, 3 = sometimes, 4 = usually, 5 = always.
iv 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, 5 = strongly agree.

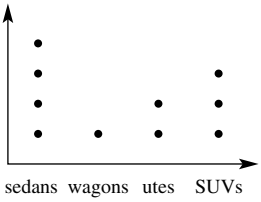
13 a Too expensive and people might refuse to respond if it came too often.
b English as a second language can impact the collection of data (simple, unambiguous English is required). Some people from particular cultures may not be keen to share information about themselves.
c Some people cannot access digital technologies and they would be excluded from the results.
d Larger populations and a greater proportion of people in poverty can make census data harder to obtain.

Exercise 8B

1 1, 2, 3, 7, 8
2 a 8 **b** 35
3 a 1, 2, 3, 5, 10 **b** 10 **c** 1 **d** 9
4 a 30 **b** 5 **c** 6
5 a median **b** 9
6 a mode **b** 2
7 a 9 **b** 12 **c** 9 **d** 7
8 a 10 **b** 8 **c** 3 **d** 7
e 15 **f** 56
9 a 12 **b** 9 **c** 7 **d** 6
10 a mean = 3, mode = 1 **b** mean = 7, mode = 2
c mean = 12, mode = 11 **d** mean = 26, mode = 25
e mean = 16, mode = 10 **f** mean = 34, mode = 55
11 a Brent **b** Brent **c** Ali **d** Brent
12 a 16.6 **b** 17.5 **c** 12
13 a 7 **b** 42
14 a 2 **b** 12 **c** 24 **d** 2
e 0 saves **f** Answers will vary.

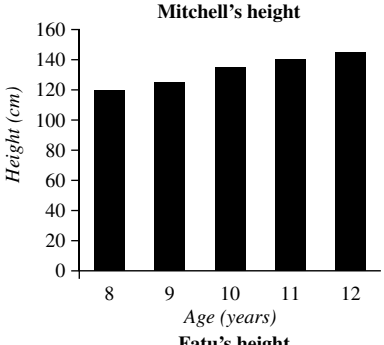
Exercise 8C

1 a dot plot **b** column graph **c** outlier
2 a true **b** false **c** true **d** true
e false
3 a 5 **b** 3 **c** sport **d** 20
4 a 2 **b** 7 **c** red **d** 27
5 a 135 cm **b** 10 cm **c** 35 cm
d 11 years old
6 a 2
b

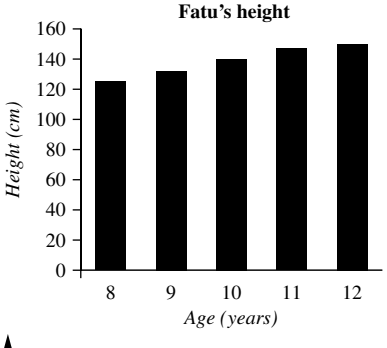


Cars in a car park

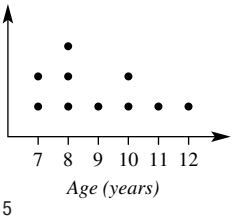
7 a



b



8 a



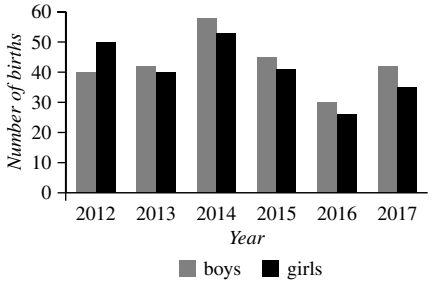
b 5
9 a 4 **b** 8 **c** 24
d 8 **e** 7 **f** 2

10 a

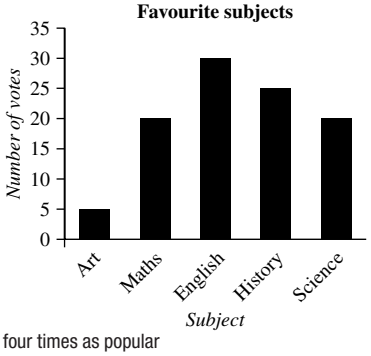
	1995	2000	2005	2010	2015
Use public transport	30	25	40	50	60
Drive a car	60	65	50	40	20
Walk or cycle	10	20	15	15	25

b 2010 and 2015
c 2015
d Environmental concerns; others answers possible.
e Public transport usage is increasing; other answers possible.

11 a



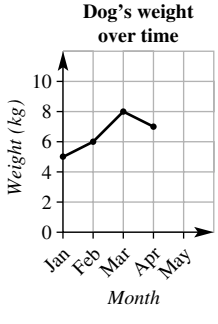
b 2012 **c** 2016 **d** 2014 **e** boys
12 a 7 **b** 7 **c** 3M:8, 3S:4
d 3M
e 3M because the student who got 10 is in that class.
13 a It is unequal.
b The axes have no labels and it does not have a title.
c



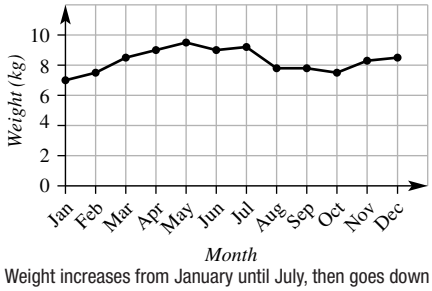
d four times as popular
e one and a half times as popular

Exercise 8D

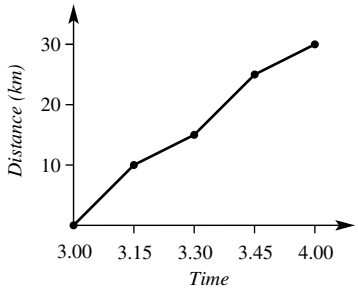
1 a 20 L **b** 30 L **c** 50 L **d** 50 L
2 a 3 kg **b** 4 kg **c** 5 kg **d** 4.5 kg
3



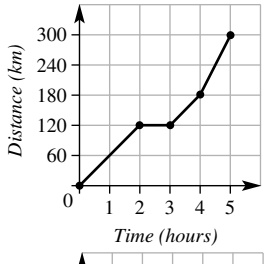
4 a 50 cm **b** 95 cm **c** 10 cm **d** 105 cm
5 a 23°C
b 2 p.m.
c 12 a.m.
d **i** 10°C **ii** 18°C **iii** 24°C **iv** 22°C
6 a



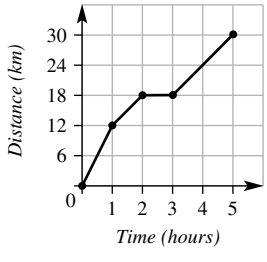
b Weight increases from January until July, then goes down suddenly.
c July, as the weight goes down for the next three months.
7



8 a 200 km **b** 80 km **c** at rest
d in the first hour **e** 40 km
9 a 20 km **b** 5 km **c** fifth hour
d 2 hours
10 a **i** 30% **ii** 35% **iii** 35%
b July, because there was the greatest rise in water level. However, at this time of year the levels of consumption and evaporation would be quite low.
c 44%
d start of February
11 a



b



12 a **i** 5°C **ii** 20°C **iii** 15°C **iv** 10°C
b at 7 a.m. and 8 p.m.
c at 8 a.m. and 11 p.m.
d **i** around 7 a.m. (heater goes on)
ii around 8 a.m. (turns heater off)
iii around 8 p.m. (heater put back on)
iv around 11 a.m. (heater turned off)
e Answers will vary.

Exercise 8E

1 **a** stem, leaf
2 **a** 5 **b** 2
3 **a** 39 **b** 27 **c** 134
4 57
5 **a** 8, 9, 10, 11, 13, 15, 17, 18, 21, 24
b 10
c **i** false **ii** true **iii** true **iv** false
6 **a** range = 20, median = 17
b range = 31, median = 26
c range = 19, median = 40.5

7

Stem	Leaf
2	5 7 9
3	0 2 9
4	1 2 5
5	1

3|2 means 32

8 **a**

Stem	Leaf
1	1 2 3 4 4 5 7
2	0 4 8 9
3	1 2 3 5

2|4 means 24

b

Stem	Leaf
2	0 2
3	9
4	5 7 9 9
5	1 2 2 3 5 6 8 8

4|7 means 47

9 **a**

Stem	Leaf
1	6 6 8
2	1 4 8 9
3	1 2 3 5
4	1 8 9
5	0

3|3 means 33

b

Stem	Leaf
1	1 2 4
2	7 9
3	2 7 8 8
6	0 0
7	3 8
8	1 7

2|9 means 29

10 **a**

Stem	Leaf
8	0 4 5 6
9	0 6
10	1 4 5
11	0 3 4 4 5 9

10|4 means 104

b

Stem	Leaf
39	1 5 6
40	1 2 4 5 6 6 8 9
41	1 2 3 3 5 6 7 8
42	0

41|3 means 413

11 **a** 10 **b** 1 **c** 8 **d** 58
12 **a** 15 **b** 13
c a is 5 or 6, b is 0, c is 8 or 9, d is 0.
13 **a** **i** 49 years **ii** 36 years
b radio station 1
c **i** 33 to 53 years **ii** 12 to 32 years

Progress quiz

1 **a** 6 **b** 13 **c** 12 **d** 16
2 **a** 16 **b** 28
3 **a** mean = 16 mode = 12 **b** mean = 22 mode = 20
4 **a** 3 **b** 15 **c** 5 **d** 1
e 5 **f** 1
5 **a** 80 **b** car, bus, walk, bike **c** 10
6 **a** January, February **b** 12°C **c** 3°C
d 20°C
7 **a** 12, 15, 17, 21, 21, 22, 24, 25, 27, 28, 28, 30, 30, 33, 36
b 15
c maximum = 36 minutes, minimum = 12 minutes
d 24 minutes **e** 25 minutes

8

Stem	Leaf
5	4 7 9
6	0 0 0 3 5 6 8 9
7	1 3 4 8
8	6

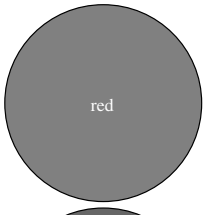
Exercise 8F

1 **a** unlikely **b** likely **c** likely **d** unlikely
2 **a** D **b** A **c** B **d** C
3 **a** true **b** false **c** true
d false **e** true **f** false
4 **a** certain **b** even chance
c unlikely **d** impossible
5 **a** D **b** C **c** A **d** B
6 Answers will vary.
7 **a** **i** true **ii** false **iii** false **iv** true
b (other answers possible)
i spinner landing on yellow
ii spinner not landing on red
iii spinner landing on green, blue or red
iv spinner landing on blue or on red
8 **a** spinner 3 **b** spinner 2 **c** spinner 1
9 Answers will vary.
10 **a** Blue, red and green equally likely.
b Red and green both have an even chance.
c Green and blue equally likely, red and blue are not equally likely.

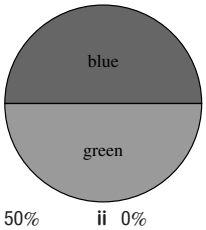
d Blue is certain.
e Blue, red and green all possible, but no two colours are equally likely.
f Red and blue both have an even chance.

11 a $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$

d i



ii Other answers possible.



e i 50% **ii** 0% **iii** 0%, 50% **iv** 50%

f If the two fractions are equal, the two events are equally likely.

Exercise 8G

1 a sample space **b** zero
c certain **d** more
e impossible **f** less

2 a 0.5 **b** 50%

3 a {1, 2, 3, 4, 5, 6} **b** {1, 3, 5}
c $\Pr(\text{odd}) = \frac{1}{2}$ **d** $\Pr(5) = \frac{1}{6}$

4 a C **b** A **c** D **d** B

5 a 3: red, green, blue **b** $\frac{1}{3}$ **c** $\frac{1}{3}$
d $\frac{2}{3}$ **e** 0

6 a {1, 2, 3, 4, 5, 6, 7} **b** $\frac{1}{7}$ **c** 0
d $\frac{2}{7}$ **e** $\frac{3}{7}$ **f** $\frac{4}{7}$
g Number chosen is less than 8; other solutions possible.

7 a {M, A, T, H, S} **b** 0.2 **c** 0.2 **d** 0.8

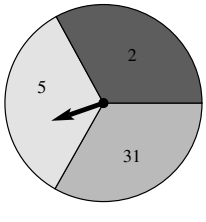
8 a $\frac{1}{11}$ **b** $\frac{2}{11}$ **c** $\frac{5}{11}$ **d** $\frac{9}{11}$ **e** $\frac{4}{11}$
f Choosing a letter in the word TRY; other solutions possible.

9 D

10 a 30% **b** 50% **c** 80%

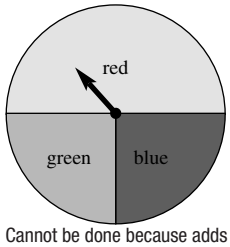
11 a yes, $\frac{19}{210}$ **b** 210 **c** 840

12 a {2, 3, 4, 5, 6, 7, 8, 9}
b 0.5
c i 0.375 **ii** 0.375 **iii** 0
d Possible spinner shown:

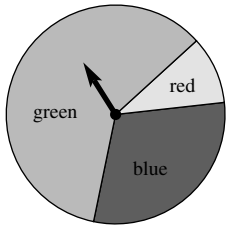


13 a $\frac{1}{6}$

b i



ii Cannot be done because adds to more than 1.
iii Cannot be done because adds to less than 1.
iv



Exercise 8H

1 C

2 a 4 **b** 6 **c** 10

3 a $\frac{1}{10}$ **b** $\frac{2}{5}$ **c** $\frac{3}{10}$

4 a $\frac{53}{100}$ **b** $\frac{47}{100}$ **c** No

5 a $\frac{13}{20}$ **b** $\frac{1}{4}$ **c** $\frac{1}{10}$

6 a 50
b i $\frac{2}{5}$ **ii** $\frac{1}{10}$ **iii** $\frac{1}{2}$

7 a 500
b 1750
c i 7 tails **ii** more

8 a 100 **b** 300
c yes (but this is very unlikely)
d from 2 rolls

9 a i $\frac{1}{4}$ **ii** $\frac{3}{100}$ **iii** $\frac{31}{100}$
b 60

10 a 5 **b** 40 **c** 70
d $\frac{26}{35}$ **e** $\frac{1}{2}$ **f** 126

11 a 2 red, 3 green and 5 blue
b i yes **ii** yes **iii** yes
iv yes **v** no

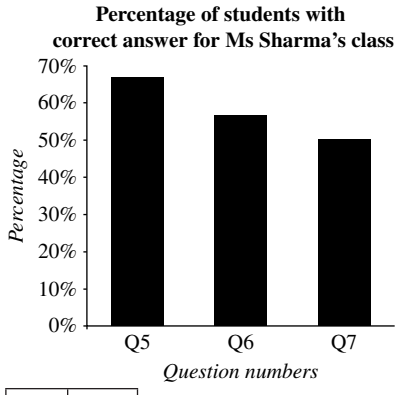
12 a C **b** D **c** B **d** A

13 Answers will vary.

Maths@Work: Teachers

1 a Q5 D, Q6 B, Q7 A
b Q5 30, Q6 29, Q7 30
c 30
d Q5 67%, Q6 57%, Q7 50%

e



2 a

A	13
B	4
C	2
D	5

b 54% **c** Mr White's class

3 a Whole class pre-test results

Stem	Leaf
0	8 9
1	2 2 8
2	0 1 2 2 2 4 5 5 6 7 7 8 8
3	0 0 2 2 4 4 4 8
4	0 1 5 8

b Girl's pre-test results

Stem	Leaf
0	9
1	2 2
2	0 2 2 2 5 8
3	0 2 4 4
4	5 8

Girl's post-test results

Stem	Leaf
1	8
2	2 2 3 7
3	2 4 4 5 8 9
4	2 4 6 7

c The girls have improved their results. Many correct answers. E.g.

Girls: pre-test	Girls: post-test
3 under 20	1 under 20
4 in the 30s	6 in the 30s
2 in the 40s	4 in the 40s

d i 8, 18 **ii** 7, 5
iii pre-test: 40, 41; post-test: 43, 44, 45, 46
iv The boys have improved their results. Many correct answers. E.g.

Boys: pre-test	Boys: post-test
2 under 20	1 under 20
4 in the 30s	5 in the 30s
2 in the 40s	4 in the 40s

4 *See table at bottom of page.*

Puzzles and games

1 Andrew is 11, Brett is 15, Chris is 16
2 24 ways
3 2, 2, 5, 7, 8, 12
4

No. of cars	0	1	2	3
No. of students	4	6	8	2

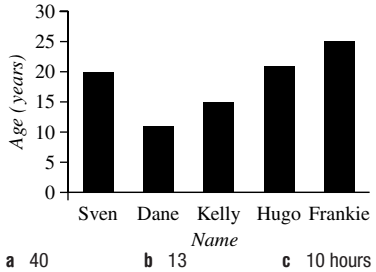
5 6 coins
6 19 more ways (21 in total)
7 12 blue, 8 green, 4 red

Multiple-choice questions

1 D **2** D **3** B **4** B **5** A
6 A **7** C **8** D **9** C **10** B

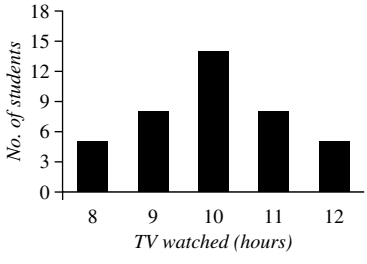
Short-answer questions

1



2 a 40 **b** 13 **c** 10 hours

d



Mrs Best's year 7 maths class											
Test	Term 1		Term 2		Mental Arithmetic		Term 3		Term 4		Averages
Total marks	22	%	19	%	10	%	34	%	26	%	
Oscar	20	**91%**	18	**95%**	10	**100%**	32	**94%**	23	**88%**	**94%**
Molly	17	**77%**	16	**84%**	9	**90%**	29	**85%**	22	**85%**	**84%**
Blake	14	**64%**	12	**63%**	5	**50%**	16	**47%**	16	**62%**	**57%**
Angus	19	**86%**	19	**100%**	8	**80%**	33	**97%**	23	**88%**	**90%**
Bhavin	17	**77%**	14	**74%**	7	**70%**	26	**76%**	18	**69%**	**73%**
Scarlett	14	**64%**	11	**58%**	6	**60%**	24	**71%**	19	**73%**	**65%**
Vedika	18	**82%**	16	**84%**	8	**80%**	31	**91%**	24	**92%**	**86%**
Class averages	17.0	**77%**	15.1	**80%**	7.6	**76%**	27.3	**80%**	20.7	**80%**	**79%**

3 **a** 4 students **b** 2 students
c 1 p.m. **d** 6 students
e The number of students must be a whole number. Joining the dots would include fractions for the number of students.

4 **a** 118 g **b** 105 g **c** 131 g **d** 26 g

5 **a** 11 **b** 5 **c** 3.5 **d** 2

6 **a** 12 **b** 5 **c** 3.5 **d** 3

7 **a** 1 **b** $\frac{1}{8}$ **c** $\frac{19}{20}$ **d** $\frac{3}{4}$
e 0

8 **a** {1, 2, 3, 4, 5, 6} **b** {heads, tails}
c {D, E, S, I, G, N} **d** {blue, yellow, green}

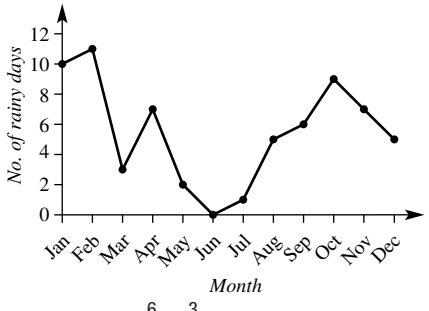
9 **a** 9
b **i** $\frac{5}{9}$ **ii** $\frac{1}{3}$ **iii** 0 **iv** $\frac{4}{9}$

10 **a** 42% **b** 50%

11 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** 25 **d** 250

Extended-response questions

1 **a**



b 66 **c** $\frac{6}{92} = \frac{3}{46}$

2 **a** 40
b cheesecake
c $\frac{7}{40}$
d **i** yes **ii** no **iii** yes **iv** no
e 80

Chapter 9

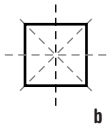
Pre-test

1 **a** 3 **b** 3 **c** 70
d 75 **e** 81 **f** 148

2 **a** 4 **b** 90° **c** 3 **d** 2

3 **a** 50 **b** 55 **c** 80
d 236 **e** 39 **f** 130

4 3



5 **a** D **b** A **c** E **d** C
e B

6 **a** B **b** C **c** A

7

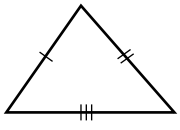
Type of angle	Angle
acute	0°–90°
right	90°
obtuse	90°–180°
straight	180°
reflex	180°–360°
revolution	360°

Exercise 9A

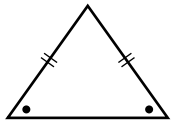
1 **a** C **b** A **c** B

2 **a** B **b** C **c** A

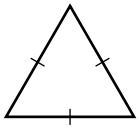
3 **a**



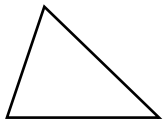
b



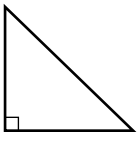
c



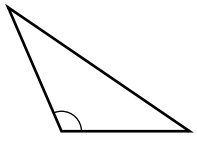
d



e



f



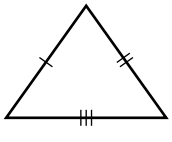
4 **a** equilateral **b** isosceles **c** scalene

5 **a** right **b** obtuse **c** acute

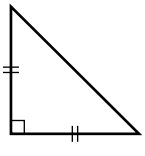
6 **a** right **b** obtuse **c** isosceles
d scalene **e** equilateral **f** acute

7 Check measurements with a ruler and protractor.

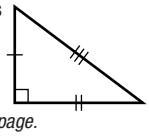
8 **a** yes



b yes



c no **d** yes



9 *See figure at top of next page.*

10 Check measurements with a ruler and protractor.

11 **a** The two shorter sides cannot meet.
b One of the three angles is always wrong.
c Two sides will never meet.

Exercise 9B

1 **a** 145 **b** 50 **c** 65

2 **b** The three angles should add to 180°.

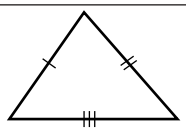
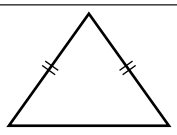
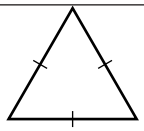
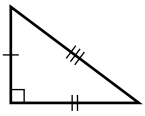
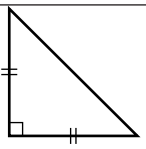
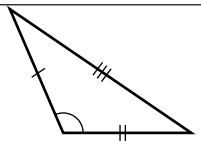
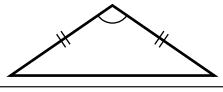
3 $a + 80 + 85 = 180$
$a + 165 = 180$
$a = 15$

4 **a** 65 **b** 25 **c** 79 **d** 70

5 **a** 60 **b** 30 **c** 55
d 65 **e** 25 **f** 145

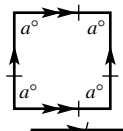
6 **a** 80 **b** 20 **c** 32

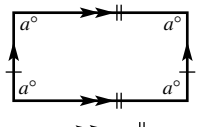
7 **a** $a = 55$, $x = 125$ **b** $a = 70$, $x = 110$
c $a = 25$, $x = 155$

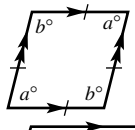
Triangles	scalene	isosceles	equilateral
acute			
right			×
obtuse			×

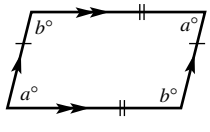
8 **a** 60 **b** 10 **c** 142
9 **a** 35 **b** 75 **c** 55
10 **a** 40 **b** 120 **c** 45 **d** 132
e 16 **f** 30
11 20
12 **a** 155° **b** 155 **c** They are the same.
d yes, always true
13 **a** 60 **b** 60 **c** 55
d 55 **e** 145 **f** 50

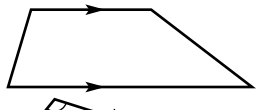
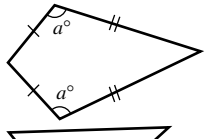
Exercise 9C

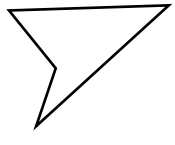
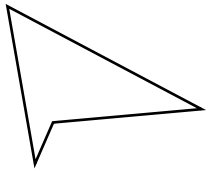
1 **a** 

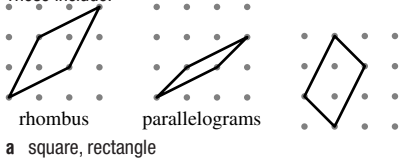
b 

c 

d 

e 
f 

2 **a** 

b 1
3 **a** convex **b** non-convex
c non-convex **d** convex
4 **a** square, rectangle **b** square, rhombus
c kite, trapezium
5 **a** square **b** trapezium **c** kite
d rhombus **e** rectangle **f** parallelogram

6 **a** $a = 90, b = 10$ **b** $a = 100, b = 5$
c $a = 50, b = 130$
7 **a** rectangles, kite, parallelogram
b rhombus, parallelogram
c square, rectangle, rhombus, parallelogram
d trapezium
e kite
8 **a** 5 **b** 4 **c** 1 **d** 11
These include:

rhombus parallelograms
9 **a** square, rectangle
b square, rhombus, kite
10 **a** A square is a type of rectangle because a rectangle can be restricted to form a square.
b Yes, a parallelogram can be restricted to be a rhombus.
c Yes, a parallelogram is a trapezium where the second pair of sides are parallel.
d No, there needs to be a pair of parallel sides.
11 Forming a square is possible.

Exercise 9D

1 **a** 180° **b** 360°
2 Answer may vary, but the sum of angles should be very close to 360°.
3 Answer may vary, but the sum of angles should be very close to 360°.
4 **a** 145 **b** 265 **c** 170
5 **a** 130 **b** 90 **c** 230
d 215 **e** 120 **f** 22
6 **a** $x = 80, a = 100$ **b** $x = 70, a = 110$
c $x = 60, a = 120$
7 **a** $a = 80, b = 90$ **b** $a = 20, b = 10$
c $a = 120, b = 155$ **d** $a = 90, b = 35$
e $a = 265, b = 40$ **f** $a = 270, b = 35$
8 **a** possible **b** impossible **c** impossible
d possible **e** impossible
9 **a** $a = 90$ **b** $a = 50, b = 130$ **c** $a = 131$
10 **a** 100 **b** 70 **c** 77.5
11 **a** 130 **b** 110 **c** 150
d 60 **e** 115 **f** 108

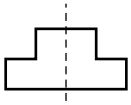
Progress quiz

1 **a** isosceles **b** acute
2 **a** equilateral **b** obtuse
3 **a** 80 **b** 56
4 **a** 160 **b** 48
5 **a** kite **b** rhombus
6 **a** 168 **b** 120
7 **a** 75 **b** 106
8 $a = 230$, $b = 12$

Exercise 9E

1 **a** yes **b** yes **c** no
d no **e** no **f** yes
2 **a** 4 ways **b** 2 ways **c** 3 ways
d 1 way **e** 2 ways **f** 0 ways
3 **a** 4 **b** 2 **c** 3
d 1 **e** 2 **f** 2
4 **a** 4 and 4 **b** 2 and 2 **c** 2 and 2
d 1 and 1 **e** 1 and 1 **f** 0 and 2
g 0 and 2 **h** 4 and 4 **i** 1 and 0
5 **a** **i** kite **ii** rectangle, rhombus
iii none **iv** square
b **i** trapezium, kite
ii rectangle, rhombus, parallelogram
iii none **iv** square

6 **a**



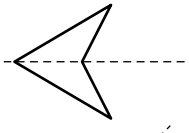
b



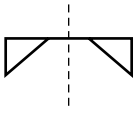
c



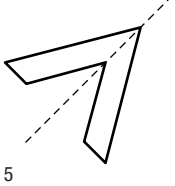
d



e

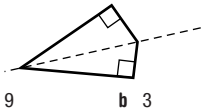


f

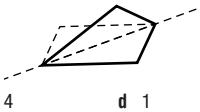


7 **a** line and rotational symmetry of order 5
b line and rotational symmetry of order 1
c line and rotational symmetry of order 1
d line and rotational symmetry of order 4
8 **a** equilateral **b** isosceles
c scalene
9 **a** A, B, C, D, E, M, T, U, V, W, Y **b** H, I, O, X
c H, I, O, S, X, Z

10 **a**



b

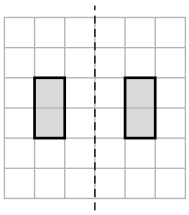


11 **a** 9 **b** 3 **c** 4 **d** 1
e infinite **f** infinite

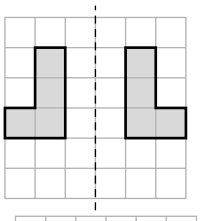
Exercise 9F

1 **a** (1, 3) **b** (−1, 3) **c** (−3, 2) **d** (−3, −2)
e (−2, 1) **f** (2, 1) **g** (2, −4) **h** (−2, −4)

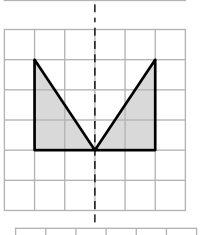
2 **a**



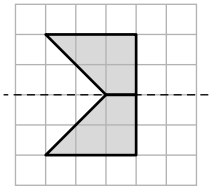
b



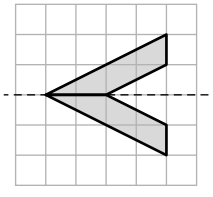
c



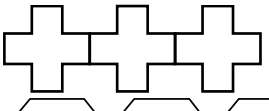
d



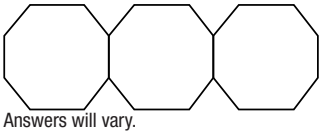
e



3 no
4 **a** $A'(1, 1)$, $B'(1, 4)$, $C'(2, 2)$, $D'(3, 1)$
b $A'(-3, 4)$, $B'(-3, 1)$, $C'(-2, 1)$, $D'(-1, 2)$
c $A'(-1, -2)$, $B'(-2, -4)$, $C'(-4, -4)$, $D'(-4, -3)$
d $A'(2, -1)$, $B'(2, -4)$, $C'(4, -2)$, $D'(4, -1)$
e $A'(-3, 2)$, $B'(-3, 3)$, $C'(-1, 4)$, $D'(-1, 1)$
f $A'(-3, -4)$, $B'(-1, -4)$, $C'(-1, -1)$, $D'(-2, -3)$
5 **a** (2, −5) **b** (4, −1) **c** (−3, −2) **d** (−3, −4)
e (0, 4) **f** (3, 0) **g** (−2, 0) **h** (−6, 10)
6 **a** (−3, 2) **b** (−7, 1) **c** (2, 4) **d** (4, 6)
e (0, 7) **f** (4, 0) **g** (4, −6) **h** (0, −3)
7 **a**

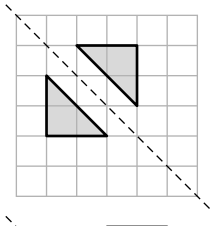


b

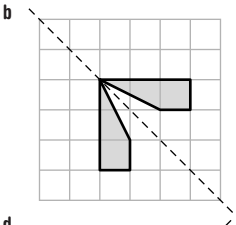


c Answers will vary.

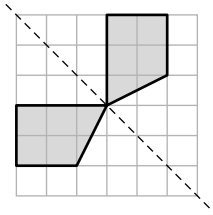
8 **a**



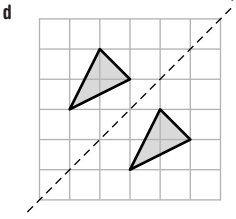
b



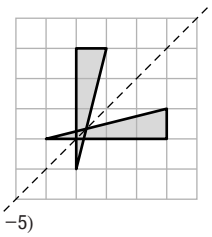
c



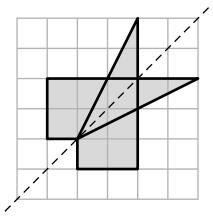
d



e



f



9 (2, −5)

10 **a** (−2, 3) **b** (0, 3) **c** (−6, 3)
d (2, −3) **e** (2, −1) **f** (2, −13)

11 Check with your teacher.

Exercise 9G

1 **a** **i** (−2, 0) **ii** (−2, 0) **iii** (0, −2) **iv** (0, 2)
b **i** (−1, −1) **ii** (−1, −1) **iii** (1, −1) **iv** (−1, 1)

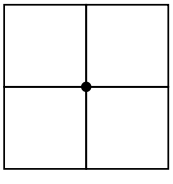
2 no

3 **a** **i** (2, −3) **ii** (−2, 3) **iii** (−3, −2)
b **i** (3, 1) **ii** (−3, −1) **iii** (1, −3)

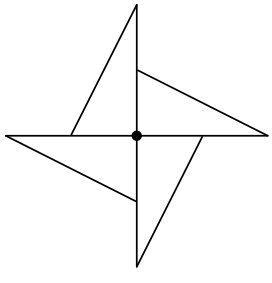
4 **a** (−3, −3) **b** (3, −3) **c** (−3, 3)
d (−3, 3) **e** (3, 3) **f** (−3, −3)

5 **a** **i** $A'(-1, 0)$, $B'(-3, 0)$, $D'(-1, 2)$
ii $A'(0, -1)$, $B'(0, -3)$, $D'(-2, -1)$
iii $A'(1, 0)$, $B'(3, 0)$, $D'(1, -2)$
b **i** $A'(0, -1)$, $B'(2, 0)$, $D'(0, -3)$
ii $A'(1, 0)$, $B'(0, 2)$, $D'(3, 0)$
iii $A'(0, 1)$, $B'(-2, 0)$, $D'(0, 3)$

6 **a**



b



c Answers will vary.

7 **a** (0, −1) **b** (3, 0) **c** (−1, 2)

8 **a** 180° anticlockwise **b** 90° anticlockwise
c 90° clockwise **d** 180° clockwise

9 **a** 270° **b** 322° **c** 10°

10 The triangle has been shifted, not rotated.

11 Check with your teacher.

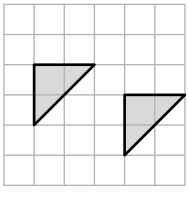
Exercise 9H

1 **a** 7 units
b 3 units
c **i** 7 units **ii** 3 units

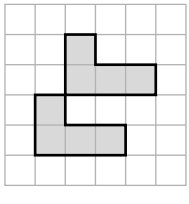
2 **a** up **b** left **c** down **d** up
e left **f** left **g** right **h** right

3 **a** (4, 2) **b** (1, 2) **c** (3, 5) **d** (3, 1)
e (2, 4) **f** (0, 1) **g** (5, 1) **h** (3, 0)

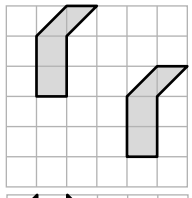
4 **a**



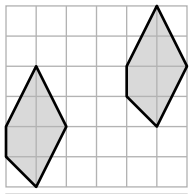
b



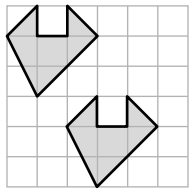
c



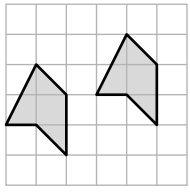
d



e



f



5 **a** (1, 3) **b** (−4, 3) **c** (−2, 1) **d** (−2, −2)
e (−2, 5) **f** (8, 3) **g** (1, 4) **h** (2, 1)
i (3, −3) **j** (−3, 1) **k** (−5, 4) **l** (−4, −2)

6 **a** 4 **b** 12

7 **a** 3 units up **b** 7 units down
c 4 units down **d** 2 units up
e 5 units left **f** 2 units right
g 1 unit left and 4 units up **h** 3 units right and 6 units up
i 3 units right and 4 units down
j 3 units left and 11 units up
k 12 units right and 3 units down
l 10 units left and 13 units down

8 **a** 2 units left and 2 units up
b 4 units left and 4 units up
c 1 unit right and 5 units down
d 6 units right and 2 units down

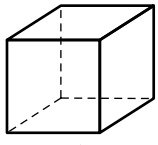
9 **a** 6 **b** 28

10 **a** (−4, −1) **b** (−4, −3) **c** (−7, 2)

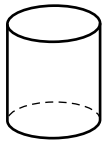
Exercise 9I

1 **a** cube **b** rectangular prism
c cylinder **d** cone
e square-based pyramid **f** triangular-based pyramid

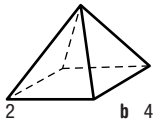
2 **a**



b

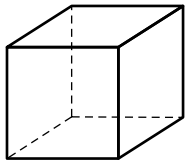


c

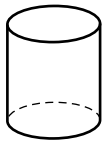


3 **a** 2 **b** 4 **c** 6

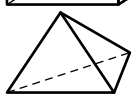
4 **a**



b



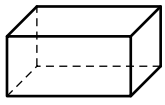
c



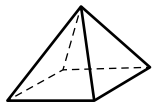
d



e



f

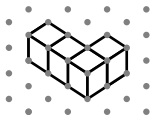


5 See given diagrams.

6 a



b



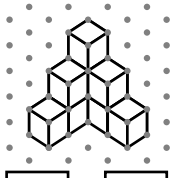
c



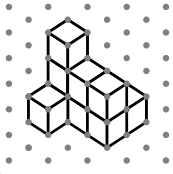
d



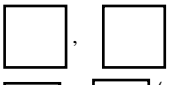
e



f

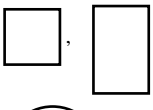


7 a



(two squares)

b



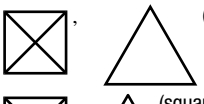
(square and rectangle with the same top side length)

c



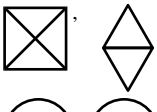
(circle, isosceles triangle)

d



(square, isosceles triangle)

e



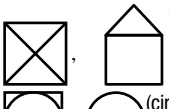
(square, diamond)

f



(2 circles)

g



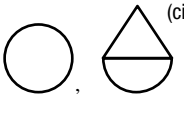
(square, triangle on square)

h



(circle in square, semicircle on rectangle)

i



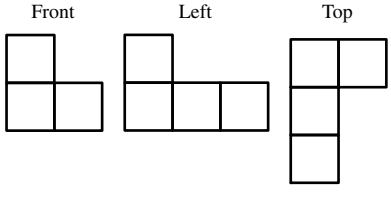
(circle, triangle on semicircle)

8 6

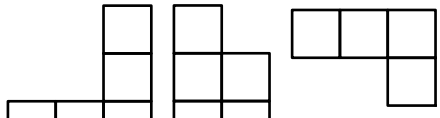
9 20

10 a C **b** A **c** B **d** D

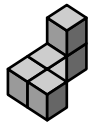
11 a i



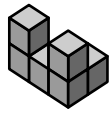
ii



b i



ii

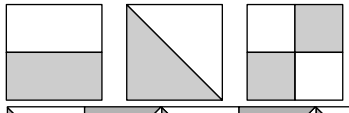


Maths@Work: Quilting

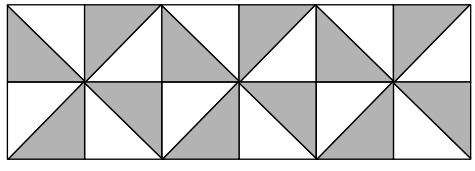
1 a Two equal sides; angles are 90°, 45°, 45°; has one axis of symmetry

b square; rectangle; rhombus; parallelogram; kite

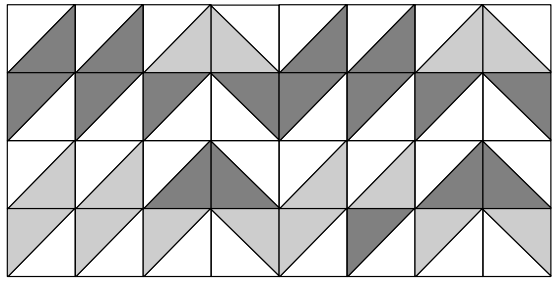
c



2 a



b



3 a i reflection over a line of symmetry or 180° rotation around the centre of the star

ii rotation around the centre of the star or reflection over connecting sides

iii rotation around the centre or reflection over a connecting side

b i translation **ii** rotation

iii reflection then translation (or the reverse order)

4 a i 4 lines of symmetry

ii rotational symmetry of order 4

iii reflection over lines of symmetry, translation and rotation around the centre

b i 8 lines of symmetry

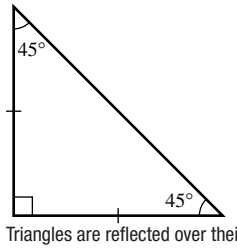
ii rotational symmetry of order 8

iii reflected over lines of symmetry, reflection over connecting sides and rotation around the centre

5 a Hexagon interior angles = 120°, $\frac{360}{120} = 3$ a whole number

Pentagon interior angles = 108°, $\frac{360}{108} = 3.3$, not a whole number.

b i A right-angled isosceles triangle

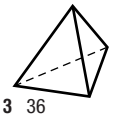


ii Triangles are reflected over their connecting sides and rotated about the centre.

6 Answers will vary

Puzzles and games

1 tetrahedron



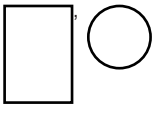
2 27 **3** 36
4 **a** 2 **b** 14 **c** 35
5 161
6 Answer will vary.

Multiple-choice questions

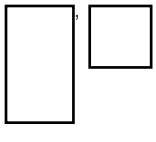
1 E **2** B **3** C **4** D **5** C
6 A **7** E **8** C **9** E **10** C

Short-answer questions

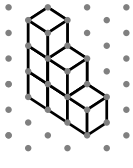
1 **a** isosceles **b** scalene **c** equilateral
2 **a** obtuse **b** right **c** acute
3 Check lengths and angles with a ruler and pair of compasses.
4 **a** 30 **b** 48 **c** 50 **d** 130 **e** 150
f 20 **g** 60 **h** 80 **i** 130
5 **a** square **b** parallelogram
c rectangle **d** trapezium
e rhombus **f** kite
6 **a** 90 **b** 255 **c** 100 **d** 125 **e** 100 **f** 55
7 **a** 3, 3 **b** 0, 1 **c** 2, 2 **d** 2, 2
e 1, 1 **f** 0, 2
8 **a** $A'(-1, -2)$, $B'(-3, -3)$, $C'(-3, -1)$
b $A'(1, 2)$, $B'(3, 3)$, $C'(3, 1)$
9 **a** $A'(0, -4)$, $B'(-2, 0)$, $D'(-3, -3)$
b $A'(4, 0)$, $B'(0, -2)$, $D'(3, -3)$
c $A'(-4, 0)$, $B'(0, 2)$, $D'(-3, 3)$
10 **a** $A'(1, -1)$, $B'(4, -1)$, $C'(3, 1)$
b $A'(-4, 1)$, $B'(-1, 1)$, $C'(-2, 3)$
11 **a**



b

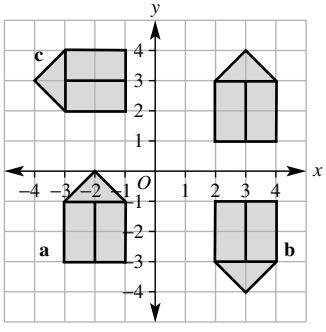


12



Extended-response questions

1 **a** 150 **b** 120
2



Chapter 10

Pre-test

1 **a** 12 **b** 15 **c** 16 **d** 7
2 **a** 7 **b** 20 **c** 16 **d** 5
3 **a** false **b** true **c** true **d** false
4 **a** 7 **b** 5 **c** 15 **d** 24
5 **a** 3 **b** 27 **c** 2 **d** 6
6 **a** C **b** A **c** B
7 **a**

n	1	2	3	4	5
$5 \times n$	5	10	15	20	25

b

n	2	4	6	8	10
$n - 2$	0	2	4	6	8

c

n	1	2	5	8	9
$2n$	2	4	10	16	18

8 **a** D **b** B **c** C **d** A
9 **a** 27 **b** 40 **c** 18 **d** 26
e 17 **f** 14 **g** 6
10 **a** Mia is 17 and Oliver is 20. **b** $28

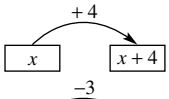
Exercise 10A

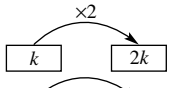
1 **a** true **b** true **c** false
2 **a** true **b** false **c** false **d** true
3 **a** 9 **b** 15 **c** 2 **d** 10
4 9
5 **a** true **b** false
6 **a** true **b** true **c** false
d false **e** false **f** false
g true **h** false **i** true
7 **a** false **b** true **c** false **d** true
8 **a** 19 **b** 19 **c** true
9 **a** true **b** true **c** false **d** true
10 **a** $3 + x = 10$ **b** $5k = 1005$
c $a + b = 22$ **d** $2d = 78$
e $8x = 56$ **f** $3p = 21$
11 **a** $6c = 546$ **b** $7k = 567$
c $12a + 3b = 28$ **d** $f + 10 = 27$
12 **a** $m = 3$ **b** $k = 2$, or $k = 6$
13 **a** $6 = 2 \times 3$; other solutions are possible
b $5 - 4 = 1$; other solutions are possible
c $10 \div 2 = 7 - 2$; other solutions are possible
d $4 - 2 = 10 \div 5$; other solutions are possible

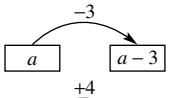
Exercise 10B

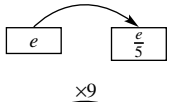
1 **a** false **b** true **c** false **d** true
2 **a** true **b** false **c** true **d** false
3 **a** 12 **b** 17 **c** 13 **d** 6
4 **a** 3 **b** 6 **c** 20 **d** 19
5 **a** 9 **b** 3 **c** 6 **d** 24
6 **a** $y = 8$ **b** $l = 6$ **c** $d = 2$ **d** $l = 12$
e $a = 6$ **f** $s = 12$ **g** $x = 8$ **h** $e = 8$
i $s = 8$
7 **a** $p = 3$ **b** $p = 4$ **c** $q = 3$ **d** $v = 5$
e $b = 1$ **f** $u = 4$ **g** $g = 3$ **h** $d = 6$
i $m = 4$
8 **a** $x = 3$ **b** $x = 7$ **c** $x = 4$ **d** $x = 5$
9 **a** 11 **b** 12 **c** 16 **d** 33
10 **a** $10x = 180$ **b** $x = 18$
11 **a** $2w = 70$ **b** $w = 35$
12 **a** $y + 12 = 3y$ **b** $y = 6$
13 **a** $x = 2$ and $y = 6$; other solutions are possible
b $x = 12$ and $y = 10$; other solutions are possible
c $x = 12$ and $y = 0.5$; other solutions are possible
d $x = 2$ and $y = 2$; other solutions are possible

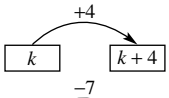
Exercise 10C

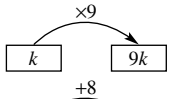
1 **a** 7 **b** 14 **c** 18 **d** 2, 10
2 **a** 

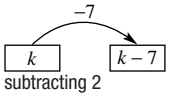
b 

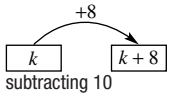
c 

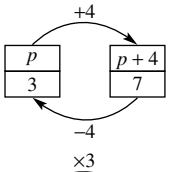
d 

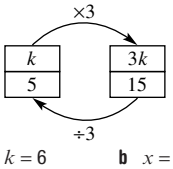
3 **a** 

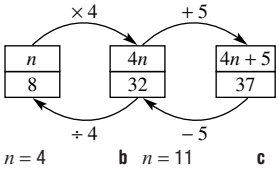
b 

c 

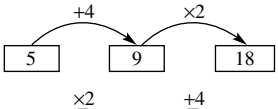
d 

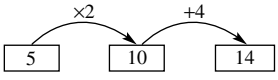
4 **a** subtracting 2 **b** subtracting 10
c adding 5 **d** dividing by 2
e dividing by 4 **f** multiplying by 2
5 **a** −5 **b** +4 **c** ÷ 3
d −3 **e** +7 **f** ÷ 2
6 **a** 5 **b** 8 **c** 3
7 **a** 

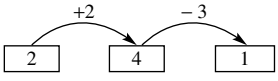
b $p = 3$
8 **a** 

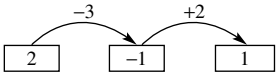
b $k = 5$
9 **a** $k = 6$ **b** $x = 10$ **c** $a = 15$
d $m = 15$ **e** $q = 5$ **f** $p = 9$
10 **a** 

b $n = 8$
11 **a** $n = 4$ **b** $n = 11$ **c** $x = 10$
d $a = 8$ **e** $y = 3$ **f** $m = 3$

12 **a** B **b** $x = 13$ **c** 13 years old
13 **a** $x = 37$ **b** more systematic
14 **a** 

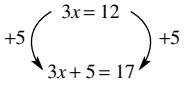


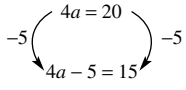
b 

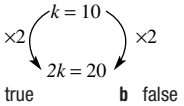


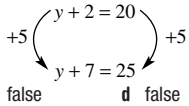
Other answers are possible.
15 **a** $x = 3$ **b** $m = 5$ **c** $p = 1$

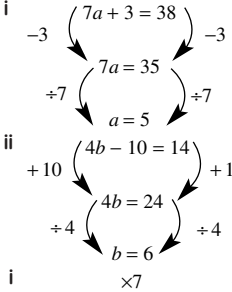
Exercise 10D

1 C
2 **a** 

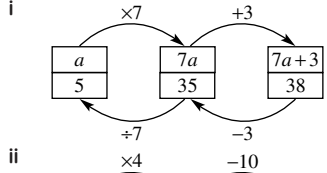
b 

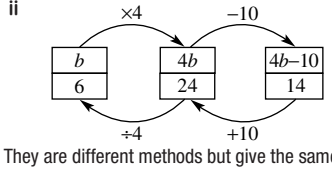
c 

d 

3 **a** true **b** false **c** false **d** false
4 **a** 6 **b** $x = 6$
5 **a** −5 **b** ÷10 **c** ÷4 **d** +12
6 **a** $2x + 2 = 20$ **b** $2x - 2 = 16$
c $4x = 36$ **d** $x = 9$
7 **a** $m = 9$ **b** $g = 11$ **c** $s = 9$ **d** $i = 10$
e $t = 2$ **f** $q = 3$ **g** $y = 12$ **h** $s = 12$
8 **a** $j = 4$ **b** $l = 4$ **c** $v = 2$ **d** $y = 12$
e $y = 9$ **f** $t = 10$ **g** $p = 7$ **h** $c = 15$
9 **a** **i** 

ii

b **i** 

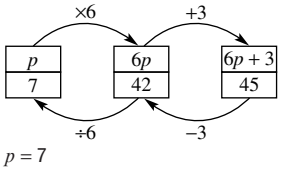
ii 

c They are different methods but give the same answer.
10 **a** subtract 3 **b** add 7
c subtract 8
11 **a** $f = 11$ **b** $x = 9$ **c** $k = 8$ **d** $n = 11$
e $g = 4$ **f** $q = 11$ **g** $z = 10$ **h** $p = 1$
i $d = 4$ **j** $t = 8$ **k** $u = 5$ **l** $c = 1$

12 a $12n + 50 = 410$
b $n = 30$, so he worked 30 hours
13 a $3w = 15 \rightarrow w = 5$ **b** $4x = 12 \rightarrow x = 3$
c $2x + 20 = 28 \rightarrow x = 4$ **d** $4w = 28 \rightarrow w = 7$
14 a Examples include: $x + 2 = 8$, $4x = 24$,
$7 - x = 1$, $x + 15 = 21$
b $6x = 3$; other answers are possible
c No. Consider $x + 1 = 11$, $x + 2 = 12$,
$x + 3 = 13$, etc.

Progress quiz

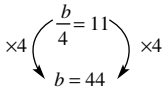
1 a true **b** false **c** true **d** true
2 a $6 + x = 15$ **b** $3e = 39$
c $m + n = -9$ **d** $d - f = 14$
3 a 9 **b** 16 **c** 60 **d** 3
4 a $f = 17$ **b** $k = 7$ **c** $y = 12$ **d** $t = 3$
5 a $x = 4$ **b** $x = 7$ **c** $x = 8$
6 a subtracting 6 **b** adding 5
c dividing by 4 **d** multiplying by 10
7 a 6 **b** 10 **c** 11
8 a



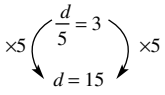
b $p = 7$
9 a $w = 5$ **b** $d = 10$
10 a $y = 7$ **b** $y = 31$
11 a $r = 8$ **b** $x = 3$

Exercise 10E

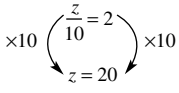
1 a true **b** false **c** true **d** false
2 a true **b** false **c** false **d** true
3 a dividing by 4 **b** adding 2
c multiplying by 7 **d** subtracting 11
4 a



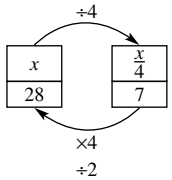
b



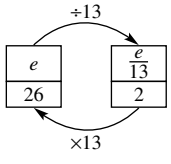
c



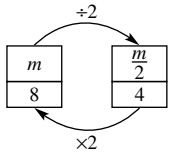
5 a



b



c



6 a B **b** C **c** A **d** D
7 a $m = 12$ **b** $c = 18$ **c** $s = 16$ **d** $r = 10$
e $u = 12$ **f** $y = 50$ **g** $x = 2$ **h** $a = 12$
8 a $d = 9$ **b** $y = 6$ **c** $j = 3$ **d** $b = 4$
e $w = 16$ **f** $s = 3$ **g** $v = 11$ **h** $f = 9$

9 a $m = 10$ **b** $q = 9$ **c** $k = 40$ **d** $x = 30$
e $a = 4$ **f** $x = 55$ **g** $y = 6$ **h** $a = 21$
10 a $x = 9$ **b** $x = 4$ **c** $p = 21$ **d** $p = 11$
e $q = 20$ **f** $r = 27$ **g** $r = 21$ **h** $x = 60$
11 a $\frac{t}{2} = 9 \rightarrow t = 18$ **b** $\frac{x}{10} = 8 \rightarrow x = 80$
c $\frac{q-4}{2} = 3 \rightarrow q = 10$ **d** $\frac{x+3}{4} = 2 \rightarrow x = 5$
e $\frac{y}{4} + 3 = 5 \rightarrow y = 8$
12 a $\frac{b}{5} = 22$ **b** $b = 110$ **c** \$110
13 a The different order in which 3 is added and the result is divided by 5.
b **i** multiply by 5 **ii** subtract 3
c no
14 a $a = 2$ **b** $k = 3$ **c** $q = 5$

Exercise 10F

1 a F, g **b** x, y **c** A, B, C **d** g, d
2 a 15 **b** 9 **c** 52
3 a 15 **b** 36 **c** 8 **d** 2
4 a 5 **b** 8 **c** 14 **d** 110
5 a 9 **b** 17 **c** 13
6 a 7 **b** 9 **c** 201
7 a 21 **b** 18 **c** 30
8 a $x = 3$ **b** 3
9 a $A = 10$ **b** $b = 10$ **c** $b = 7$
10 a $y = 23$ **b** $x = 4$ **c** $x = 7$
11 a **i** $P = 28$ **ii** $P = 40$ **b** $w = 12$
12 a $A = 35$
b $20 = w \times 4 \rightarrow w = 5$
c **i** $25 = 5h \rightarrow h = 5$ **ii** square
13 a $F = 68$ **b** $C = 10$ **c** 73°F
d 36°C **e** 15°C

Exercise 10G

1 a D **b** A **c** E **d** B
e C
2 a $x = 6$ **b** $a = 2$ **c** $k = 9$
3 D
4 a $7k = 42$ **b** $k = 6$
5 a $x + 19 = 103$ **b** $x = 84$
6 a Let p = cost of one pen **b** $12p = 18$
c $p = 1.5$ **d** \$1.50
7 a Let t = cost of a car tyre **b** $4t + 160 = 1400$
c $t = 310$ **d** \$310
8 a Let h = number of hours worked
b $17h + 65 = 643$ **c** $h = 34$ **d** 34 hours
9 a $24w = 720$ **b** $w = 30$ **c** 30 m **d** 108 m
10 $2x + 3 = 31 \rightarrow x = 14$
11 a $x = 4$ **b** No. It must be at least 18.
12 14 years old
13 a Both equal 18.
b Examples: $l = 4$ and $w = 4$, $l = 12$ and $w = 2.4$
c Yes, if the width equals 4.

Maths@Work: Uber fares

1 a **i** \$9.25 **ii** \$23.90 **iii** \$50.90
b 34.14 km
c 48 minutes
d UberX fare = $2.5 + 0.4t + 1.45n$
2 a \$92.50 **b** 1.81 km **c** \$265.64
d 74.55 minutes

e UberSELECT fare $= 4 + 0.55t + 1.8n$
f Formula fare = \$8.70 but the UberSELECT minimum fare is \$10.

3 **a** \$77.32 **b** \$35.84 **c** 9.61 km
d UberBLACK fare $= 10 + 0.68t + 2.04n$

4 Answers may vary.

5 *See table at bottom of page.*

Puzzles and games

1 **a** 26 **b** 29 **c** 368 **d** $31\frac{1}{3}$ **e** 36

2 7 and 13

3 divide by 2

4 30

5 A CORNY JOKE

6 26 sheep, 15 ducks

7 **a** $M = 2n + 1$ **b** 201

8 8 and 13

Multiple-choice questions

1 D **2** C **3** A **4** D **5** A
6 E **7** C **8** B **9** A **10** E

Short-answer questions

1 **a** false **b** true **c** true
d false **e** true **f** false

2 **a** $2 + u = 22$ **b** $5k = 41$
c $3z = 36$ **d** $a + 12 = 15$

3 **a** $x = 3$ **b** $x = 6$ **c** $y = 1$
d $y = 9$ **e** $a = 2$ **f** $a = 10$

4 **a** $x = 5$ **b** $q = 6$ **c** $a = 21$ **d** $k = 18$
e $k = 10$ **f** $w = 8$ **g** $b = 5$ **h** $r = 2$

5 **a** $x = 3$ **b** $r = 45$ **c** $x = 9$
d $r = 4$ **e** $q = 2$ **f** $u = 8$

6 **a** $u = 24$ **b** $p = 16$ **c** $x = 8$
d $y = 40$ **e** $y = 8$ **f** $x = 60$

7 **a** no **b** LHS = 16, RHS = 16 $\therefore$ solution

8 **a** $A = 450$
b $l = 8$
c $w = 5$, $l = 8$; other solutions are possible

9 **a** 36 **b** 6 **c** 3

10 **a** 5 **b** 25 **c** 20 **d** 5

Extended-response questions

1 **a** \$120 **b** B
c \$60 **d** 7 hours
e $252 = 12n$

2 **a** 75 cents **b** \$1.35
c $t = 12$ **d** 12 seconds
e 50 seconds **f** \$1.50

Chapter 11

Pre-test

1 **a** E **b** C **c** B **d** A and D

2 **a** 7.5 cm **b** 13 cm **c** 2 cm

3 **a** D, A, C, B **b** C, A, B, D **c** C, E, A, D, B

4 **a** 10 **b** 100 **c** 1000
d 60 **e** 60 **f** 1000

5 **a** 2000 **b** 2 **c** 56 **d** 2500
e 180 **f** 7200 **g** 1.4 **h** 0.027

6 **a** 55 m **b** 27 cm **c** 28 cm

7 **a** 32 **b** 7 **c** 8

8 **a** 16 **b** 6 **c** 8

9 **a** B **b** C **c** D **d** A

Exercise 11A

1 millimetre, centimetre, metre, kilometre

2 **a** 100 **b** 1000 **c** divide **d** multiply

3 **a** right **b** left

4 **a** kilometres **b** millimetres **c** metres
d metres **e** centimetres **f** kilometres

5 **a** 2 cm **b** 5 cm **c** 1.5 cm **d** 3.2 cm
e 3 cm **f** 3 cm **g** 1.2 cm **h** 2.8 cm

6 **a** 50 mm **b** 200 cm **c** 3500 m **d** 2610 cm
e 2200 m **f** 53 mm **g** 620 cm **h** 200 mm
i 684 cm **j** 20 m **k** 3800 cm **l** 670 cm

7 **a** 4 cm **b** 5 m **c** 4.2 km **d** 47.2 cm
e 3.6 m **f** 3.2 cm **g** 50 km **h** 27 km
i 36.2 cm **j** 0.04 cm **k** 926.1 cm **l** 4.23 km

8 **a** metres **b** millimetres
c kilometres **d** kilometres
e metres **f** centimetres

9 **a** 2.5 cm **b** 82 mm **c** 2.5 m **d** 730 cm
e 6200 m **f** 25.732 km

10 **a** 8.5 km **b** 310 cm **c** 19 cm

11 **a** 2.7 m **b** 0.4 km

12 **a** 38 cm, 0.5 m, 540 mm
b 160 cm, 2100 mm, 0.02 km, 25 m
c 142 mm, 20 cm, 0.003 km, 3.1 m
d 10 mm, 0.1 m, 0.001 km, 1000 cm

13 125 cm

14 **a** \$8200 **b** \$6.56 **c** \$4.10

15 50 000 years

16 **a** 3000 mm **b** 600 000 cm **c** 2400 mm
d 4000 cm **e** 0.47 km **f** 913 m
g 0.216 km **h** 0.0005 m **i** 2 m

17 **a** 2 mm **b** 5 mm **c** 2 cm **d** 5 cm
e 8 cm

18 **a** 9 cm **b** 10 cm **c** 25 cm

Uber level	Pickup place	Destination	Fare
UberX	Burke and Wills Hotel, Toowoomba, Qld	Byron Bay, NSW	\$333.00
UberX	Melbourne Airport, Vic	Mornington Peninsula, Vic	\$139.80
UberXL	Brisbane Airport, Qld	Surfers Paradise, Gold Coast, Qld	\$177.30
UberXL	Perth Airport, WA	Margaret River, WA	\$562.80
UberBLACK	Napier Street, Adelaide SA	Adelaide Airport, SA	\$34.10
UberASSIST	Canberra Airport, ACT	Goulburn, NSW	\$158.50

Exercise 11B

1 **a** perimeter **b** equal
2 **a** 10 cm **b** 12 cm **c** 12 cm **d** 12 cm
3 **a** 15 cm **b** 12 cm **c** 20 cm **d** 12.8 cm
4 **a** 15 cm **b** 37 m **c** 36 cm **d** 11 m
e 30 km **f** 2.4 m **g** 26 cm **h** 10 cm
5 **a** 42 cm **b** 34 m **c** 20.8 m **d** 90 km
6 **a** 8.4 cm **b** 14 m **c** 46.5 mm
7 **a** 34 m **b** 52 km **c** 8.8 mm **d** 56 m
e 36 km **f** 40 mm
8 \$21 400
9 **a** 40.7 cm **b** 130.2 cm **c** 294 cm **d** 568 cm
10 **a** 5 cm **b** 5 m **c** 7 km
11 **a** 516 ft **b** 157.38 m

Exercise 11C

1 1 mm^2, 1 cm^2, 1 m^2, 1 ha, 1 km^2
2 **a** 9 **b** 3 cm and 3 cm **c** 9 cm^2
3 **a** 8 **b** 4 cm and 2 cm **c** 8 cm^2
4 **a** 4 square units **b** 6 square units
c 4 square units **d** 6 square units
e 8 square units **f** 96 square units
5 **a** 3 cm^2 **b** 4 cm^2 **c** 6 cm^2 **d** 3 cm^2
e 2 cm^2 **f** 5 cm^2 **g** 4.5 cm^2 **h** 9 cm^2
6 **a** 8 cm^2 **b** 9 cm^2 **c** 100 m^2 **d** 200 cm^2
e 22 mm^2 **f** 7 cm^2 **g** 25 m^2 **h** 1.44 mm^2
i 6.25 mm^2 **j** 1.36 m^2
k 0.81 cm^2 **l** 179.52 km^2
7 **a** cm^2 **b** m^2 **c** ha **d** km^2
e ha **f** mm^2
8 5000 m^2
9 2500 cm^2
10 **a** 2 cm **b** 5 m **c** 12 km
11 **a** 25 cm^2 **b** 12 cm **c** 4 units
12 **a** **i** 10 cm **ii** 9 mm
b Divide the area by the given length.
13 half of a rectangle with area 4 cm^2
14 20 000 cm^2 = 2 m^2
15 \$2100
16 5 L

Exercise 11D

1 **a** 6 m **b** 11mm **c** 1.9 m **d** 3.2 mm
e 4 m **f** 28 mm
2 **a** 20 cm **b** 7 m **c** 2 m **d** 6.3 cm
e 6 cm **f** 5 m
3 **a** 10 **b** 56 **c** 12.5 **d** 5.25
4 **a** 3 m^2 **b** 40 cm^2 **c** 12 mm^2 **d** 16 m^2
e 30 cm^2 **f** 160 m^2 **g** 1.2 m^2 **h** 15 m^2
i 63 mm^2
5 **a** 10 cm^2 **b** 3 mm^2 **c** 24.5 km^2
d 1.3 cm^2 **e** 20 m^2 **f** 4.25 m^2
6 **a** 2 cm^2 **b** 6 cm^2 **c** 3 cm^2 **d** 3 cm^2
7 4800 m^2
8 480 m^2
9 160 m^2
10 \$6300
11 No, the base and height are always the same.
12 **a** 7 cm^2 **b** 8 cm^2 **c** 11 cm^2 **d** 11 cm^2

Exercise 11E

1 C
2 **a** $A = bh$
$= 5 \times 7$
$= 35$
b $A = bh$
$= 20 \times 3$
$= 60$
c $A = bh$
$= 8 \times 2.5$
$= 20$
3 **a** $b = 6$ cm, $h = 2$ cm **b** $b = 10$ m, $h = 4$ m
c $b = 3$ mm, $h = 5$ mm **d** $b = 5$ m, $h = 7$ m
e $b = 5.8$ cm, $h = 6.1$ cm **f** $b = 5$ cm, $h = 1.5$ cm
g $b = 1.8$ m, $h = 0.9$ m **h** $b = 5$ m, $h = 12$ m
4 **a** 40 m^2 **b** 28 m^2 **c** 17.5 m^2 **d** 14 cm^2
e 42 m^2 **f** 176 mm^2 **g** 50 m^2 **h** 48 m^2
5 **a** 36 km^2 **b** 16 m^2 **c** 48 mm^2 **d** 6.3 cm^2
e 30 cm^2 **f** 1.8 cm^2
6 54 m^2
7 **a** 6 cm^2 **b** 4 cm^2 **c** 15 cm^2 **d** 8 cm^2
8 **a** 1800 cm^2 **b** 4200 cm^2
9 **a** 2 m **b** 7 cm
10 **a** 10 cm **b** 5 m
11 \$1200
12 half; Area (parallelogram) = bh and Area (triangle) = $\frac{1}{2}bh$
13 **a** 6500 m^2 **b** 6300 m^2
c 25 600 m^2 **d** \$4 608 000

Progress quiz

1 **a** 500 **b** 4200 **c** 62 100 **d** 147
2 **a** 7.2 **b** 0.051 **c** 4.132 **d** 35
3 **a** 38 **b** 6.82 **c** 5708
4 **a** 29 cm **b** 15.6 m
5 **a** 68 cm **b** 260 mm
6 \$12 960
7 **a** 34.85 cm^2 **b** 169 mm^2
8 **a** 100 cm^2 **b** 20 cm
9 **a** 120 cm^2 **b** 6 m^2
10 **a** 24 cm^2 **b** 2.86 m^2
11 **a** 40 m^2 **b** 12.3 cm^2

Exercise 11F

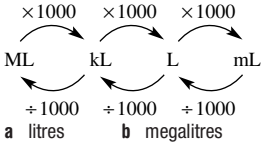
1 B
2 D
3 **a** 6 **b** 24 **c** 24 **d** 144
4 **a** $V = lwh$
$= 4 \times 2 \times 3$
$= 24$ cm^3
b $V = lwh$
$= 1 \times 3 \times 6$
$= 18$ cm^3
c $V = l^3$
$= 2 \times 2 \times 2$
$= 8$ km^3
5 **a** 84 m^3 **b** 10 cm^3 **c** 98 cm^3 **d** 240 m^3
e 720 km^3 **f** 0.48 cm^3
6 **a** 8 cm^3. **b** 27cm^3 **c** 1000 km^3
d 2.744 mm^3
7 **a** 360 m^3 **b** 28 cm^3 **c** 100 m^3
d 11.25 km^3
8 96 m^3
9 24 000 cm^3
10 27 000 000 km^3
11 600

12 a **i** 2 m **ii** 5 cm
b Use: Volume ÷ area of base
c $h = V \div (l \times w)$
13 a 28 cm^3 **b** 252 m^3

Exercise 11G

1 a capacity **b** 1 **c** 1000
d 1000 **e** 1000 **f** 1000

2



3 a litres **b** megalitres **c** millilitres
d kilolitres **e** cubic centimetres
f cubic metres

4 a 0.4 L **b** 0.7 L **c** 2 kL **d** 36 kL
e 4 ML **f** 0.5 ML **g** 2000 mL **h** 100 mL
i 6000 kL **j** 3000 kL **k** 24 000 L **l** 38 000 L
m 2 kL **n** 3.5 L **o** 70 L **p** 2.5 ML
q 257 mL **r** 9.32 L **s** 3847 kL **t** 47 kL
u 5.8 ML

5 a 12 mL **b** 2 cm^3 **c** 850 mL
6 a 128 cm^3 **b** 128 mL **c** 0.128 L
7 a 1.5 L **b** 0.48 L **c** 0.162 L **d** 8.736 L
e 25 L **f** 32.768 L
8 a 1 L, 1000 mL, 1000 cm^3 **b** 1 m^3, 1000 L, 1 kL
9 a D **b** B **c** F
d A **e** C **f** E
10 a 1200 mL **b** 1.2 L
11 a 6 000 000 L **b** 0.32 ML **c** 4000 mL **d** 0.9927 kL
12 3 300 000 L
13 15 days
14 a 2500 m^3 **b** 2 500 000 L
15 a 0.2 mL
b **i** 1 L **ii** 0.6 L **iii** 14.4 L **iv** 5256 L

Exercise 11H

1 a 1000 **b** milligrams **c** tonne
d 100°C **e** 0°C
2 a C **b** F **c** A **d** D
e B **f** E
3 a B **b** A **c** D **d** C
4 a 12°C **b** 37°C **c** 17°C **d** 225°C
e 1.7°C **f** 31.5°C
5 a 4 kg **b** 12 g **c** 65 t
6 a 2000 g **b** 7000 g **c** 6200 g **d** 5800 g
e 6 kg **f** 8.9 kg **g** 0.9 kg **h** 0.45 kg
i 5000 kg **j** 600 kg **k** 2.4 t **l** 4.32 t
m 3000 mg **n** 4200 mg **o** 7.5 g
7 a 4.620 g **b** 21.6 t **c** 470 kg **d** 0.312 kg
e 0.027 g **f** 750 kg **g** 125 g **h** 0.0105 kg
i 210 t **j** 470 kg **k** 592 g **l** 80 g
8 33°C
9 147°C
10 a 60 kg **b** 60 000 g **c** 60 000 000 mg
11 a 3000 g **b** 3 kg
12 a 8 kg **b** 8.16 kg
13 a 400 mg, 370 g, 2.5 kg, 0.1 t
b 290 000 mg, 0.000 32 t, 0.41 kg, 710 g
14 50 days
15 a 1 g **b** 1 t **c** 1000 t
16 a 12 kg **b** 1000 kg **c** 360 000 kg

Maths@Work: Landscape gardener

1 a **i** 15 m, 13.5 m^2 **ii** $92.48
b **i** 28 m, 27 m^2 **ii** $184.95
c **i** 25.6 m, 31.2 m^2 **ii** $213.72
2 a **i** 35.88 m^2 **ii** $313.95
b **i** 22.52 m^2 **ii** $197.05
c **i** 3.46 m^2 **ii** $30.24
3 18 rolls of turf
4 a bedding sand $181; pavers $1551; total $1732
b turf $800; top soil $530; mulch $62; edging $297; total $1689
5 *See table at bottom of page.*

Puzzles and games

1 a 12 **b** 35 **c** 50
2 B
3 No. Although both shapes look like triangles, this is an optical illusion. The 'hypotenuse' (longest side) on each shape is not quite straight.
4 a 3 cm^2 **b** 20 cm^2
5 a 62 cm^2 **b** 16 m^2 **c** 252 cm^2
6 Mark a length of 5 m, then use the 3 m stick to reduce this to 2 m. Place the 3 m stick on the 2 m length to show a remainder of 1 m.
7 10.5 cm^2

Better Landscaping Company				
Client: Jack and Jill Green				
Sections	**Length in m**	**Width in m**	**Area in m^2**	**Perimeter in m**
Front lawn	8	5.1	40.8	26.2
Back garden	5.8	2.2	12.76	16
PLANTS	**Number**	**Price for one**	**Total cost**	
Trees	2	$36.00	$72.00	
Shrubs	5	$11.50	$57.50	
Punnets of vegetables	6	$4.55	$27.30	
MATERIALS	**Quantity**	**Cost per unit**	**Total cost**	
Legend couch turf in m^2		$10.75	$438.60	
Garden edge blocks in m		$14.80	$236.80	
Garden soil in m^3		$65.00	$99.53	
		Overall cost	**$ 931.73**	

Multiple-choice questions

1 D **2** E **3** C **4** A **5** E
6 B **7** A **8** B **9** E **10** B

Short-answer questions

1 **a** 2.5 cm **b** 2.3 cm **c** 4.25 kg
d 5 L **e** 6°C **f** 1°C
2 **a** 50 mm **b** 2 m **c** 3700 m **d** 36 m
e 7100 g **f** 24 g **g** 22 t **h** 2500 kg
i 4 L **j** 40 kL **k** 400 kL **l** 6500 L
m 6 cm **n** 3 ML **o** 0.2 mm
3 **a** 16 m **b** 20.6 cm **c** 23 m **d** 34 km
e 3.2 mm **f** 24 m
4 **a** 24.01 cm^2 **b** 14 km^2 **c** 67.5 m^2 **d** 12 cm^2
e 14 m^2 **f** 5 cm^2 **g** 14 m^2 **h** 0.9 km^2
i 0.16 m^2 **j** 900 cm^2
5 **a** 18 cm^3 **b** 7.5 cm^3 **c** 64 mm^3
6 **a** 72 000 cm^3 **b** 72 000 mL **c** 72 L
7 **a** 30 L **b** 1 L **c** 4 L
8 **a** 45 mg, 290 000 g, 3 t, 4700 kg
b 50 000 mL, 51 L, 0.5 kL, 1 ML

Extended-response questions

1 **a** 240 cm **b** 3200 cm^2
c 20 cm, 60 cm **d** 160 cm
e 1200 cm^2 **f** 2000 cm^2
g $200
2 **a** 30 m^3 **b** 16 m **c** 30 000 000 cm^3
d 30 000 L, 30 kL **e** 30 t

Semester review 2

Negative numbers

Multiple-choice questions

1 C **2** A **3** B **4** E **5** D

Short-answer questions

1 **a** $<$ **b** $>$ **c** $<$
2 **a** -1 **b** -5 **c** -10
d -6 **e** -18 **f** -77
3 **a** 1 **b** 4 **c** -2
d -5 **e** -12 **f** -57
4 **a** 5 **b** 7 **c** 24
d -1 **e** -6 **f** 2
5 **a** -15 **b** -3 **c** -8
6 **a** -10 **b** 10 **c** -20
d 16 **e** 32 **f** 24

Extended-response questions

1 **a** $A(2, 0)$, $B(4, 0)$, $C(0, 2)$, $D(2, 2)$, $E(2, -3)$, $F(-1, -2)$, $G(-3, 0)$, $H(-3, -2)$, $I(-1, 2)$
b A, B, O and G; all lie on the x-axis.
c **i** 2 units **ii** 5 units
d trapezium
e 8 square units
f DECIDE

Statistics and probability

Multiple-choice questions

1 A **2** C **3** B **4** E **5** B

Short-answer questions

1 **a** 1, 2, 3, 5, 5, 5, 8, 8, 9, 10
b 10
c **i** 5.6 **ii** 5 **iii** 5 **iv** 9
d 11.2
2 **a** A **b** B **c** $\frac{1}{3}$
3 **a** {1, 2, 3, 4, 5, 6} **b** $\frac{1}{6}$ **c** $\frac{1}{2}$
4 **a** 13
b 3, 47
c **i** 44 **ii** 22 **iii** 22
5 **a** **i** 10°C **ii** 15°C **iii** 30°C
b 15°C
c 22.5°C

Extended-response questions

1 **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{52}$
e $\frac{2}{13}$ **f** $\frac{1}{13}$ **g** $\frac{4}{13}$ **h** $\frac{4}{13}$

Shapes and transformations

Multiple-choice questions

1 E **2** B **3** D **4** C **5** A

Short-answer questions

1 **a** square
b parallelogram
c isosceles right triangle
d rhombus
e trapezium
f kite
2 **a** 50 **b** 45 **c** 40
d 150 **e** 70 **f** 80
3 **a** $a = 60$ **b** $a = 65$ **c** $a = 30$ **d** $a = 115$
e $a = 90$, $b = 90$ **f** $a = 65$
4 **a** $A'(2, 0)$, $B'(2, -3)$, $C'(4, 0)$
b $A'(-2, 0)$, $B'(-2, 3)$, $C'(-4, 0)$
c $A'(0, -2)$, $B'(3, -2)$, $C'(0, -4)$
d $A'(0, 2)$, $B'(-3, 2)$, $C'(0, 4)$
e $A'(-2, 0)$, $B'(-2, -3)$, $C'(-4, 0)$
f $A'(-2, 1)$, $B'(-2, 4)$, $C'(0, 1)$
g $A'(-1, -2)$, $B'(-1, 1)$, $C'(1, -2)$

Extended-response questions

1 **a** Student's own construction **b** $a + b + c = 180$
c Student's own measurements
d $x + y + z = 360$ **e** 360

Equations

Multiple-choice questions

1 B **2** E **3** C **4** B **5** B

Short-answer questions

1 **a** false **b** true **c** true
d false **e** false **f** true

2 **a** $x = 3$ **b** $x = 108$ **c** $x = 21$ **d** $x = \frac{4}{3}$

3 **a** $x = 2$ **b** $y = 5$ **c** $x = 12$ **d** $m = 7$

4 **a** **i** 8 **ii** 20 **iii** 48
b $w = 6$
c $w = 6$
d $w = 11$

5 **a** $x = 5$ **b** $x = 6$ **c** $x = 18$

6 $2x + 25 = 85 \rightarrow x = 30$

Extended-response questions

1 **a** \$320 **b** \$400 **c** $C = 200 + 40n$
d $6\frac{1}{2}$ hours

Measurement

Multiple-choice questions

1 C **2** B **3** E **4** C **5** A

Short-answer questions

1 **a** 500 **b** 6000 **c** 18
d 0.017 **e** 1.8 **f** 5500

2 **a** 272 cm **b** 11 m **c** 300 cm
d 220 cm **e** 3.4 m **f** 92 m

3 **a** 1.69 m^2 **b** 24 m^2 **c** 60 m^2
d 20 m^2 **e** 12 cm^2 **f** 28 m^2

4 **a** 729 cm^3 **b** 120 m^3 **c** 384 m^3

5 **a** 5000 **b** 7 **c** 2000
d 3000 **e** 8 **f** 25

6 **a** 0.4 L **b** 20 L **c** 50 L

7 **a** 3000 kg **b** 2000 g
c 6.5 kg **d** 0.5 t
e 5 g **f** 24 000 mg
g 50 g **h** 0.02 g

8 **a** 0 **b** 100

Extended-response questions

1 **a** Many answers are possible; e.g. 8 m × 10 m, 4 m × 14 m, 15 m × 3 m
b 9 m × 9 m (area = 81 m^2)
c 36 posts
d \$900