PEARSON mathematics

# S.B.






Space
Time
Explore
I am Maths
Communicating
And measuring the infinite



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# PEARSON mathematics




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# USING PEARSON mathematics

Bringing the heart and soul of the Australian Mathematics Curriculum to life



Mathspace

Our Global village

Facts about our global village

There are over 6.9 billion people on earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

**Race**
- 0.61 Asian
- $\frac{16}{100}$ European
- 0.08 North American
- 0.05 South American & Caribbean
- 0.13 African
- $\frac{1}{100}$ Oceania (this includes Australia)

**Conditions**
- 0.43 live without basic sanitation
- $\frac{1}{5}$ don't have access to clean, safe drinking water
- 0.13 are hungry and/or malnourished

**Wealth**
- 0.06 of the population own 59% of the entire wealth
- $\frac{1}{4}$ of the population have 75% of the income
- 0.53 of the population live on $2.50 or less per day

**Education and technology**
- 0.14 can't read
- 0.07 have a secondary education
- 0.12 have a computer
- 0.03 have the Internet

**How rich are you?**

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage.*

1.2 − 2.7 + 3.5 − 1.25 =

**How many people?**

Currently there are approximately 6 900 000 000 people living on earth.

3 Using the facts you now know about our world, *estimate* how many people:
- (a) Don't have access to clean water
- (b) Struggle to live on $2.50 or less per day
- (c) Have a computer
- (d) Don't have a secondary school education

**I want to help**

There are many charities out there that are trying to create more equality in the world—Oxfam is one of these. Charities often rely on the generosity of everyday people. Here is a pie chart of the age groups of Oxfam volunteers.

**Volunteers by age group**




Oxfam Annual Report 07-08

4 Using the pie chart above, find out the following:
- (a) What percentage of volunteers are between 18 and 30?
- (b) Why do you think most volunteers are in this age range?
- (c) One of the challenges charities face is the need to pay for fundraising and administration costs. If Oxfam spends 24.4% of their income on fundraising and 8.9% on administration costs, what percentage of their income is available to go directly to community aid projects?

**In the future**

5 Imagine it's the year 2055. How do you hope the 'Global Village' percentages have changed? Write down your predictions and ideas.

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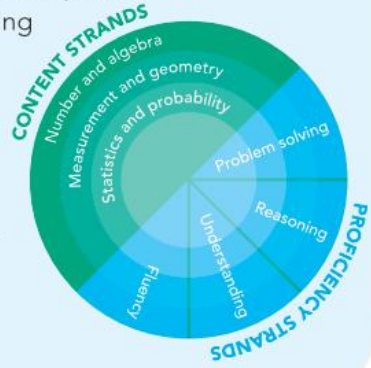
*The Australian Curriculum: Mathematics aims to ensure that students:*

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in* Number and Algebra, Measurement and Geometry, *and* Statistics and Probability
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

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**Pearson Mathematics** shares these aims.

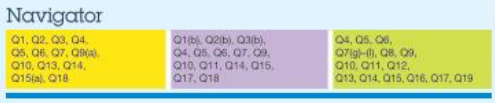
**Pearson Mathematics** is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills, in addition to addressing the Australian Curriculum's General capabilities and Cross-curriculum priorities.



# Making teachers' lives easier

## Differentiation

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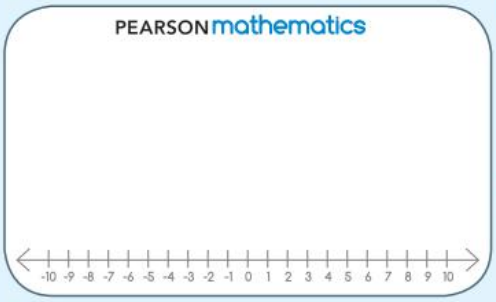



Half-time—A mid-chapter review

Chapter review—An end-of-chapter review

Mixed review—Cumulative revision for every second chapter

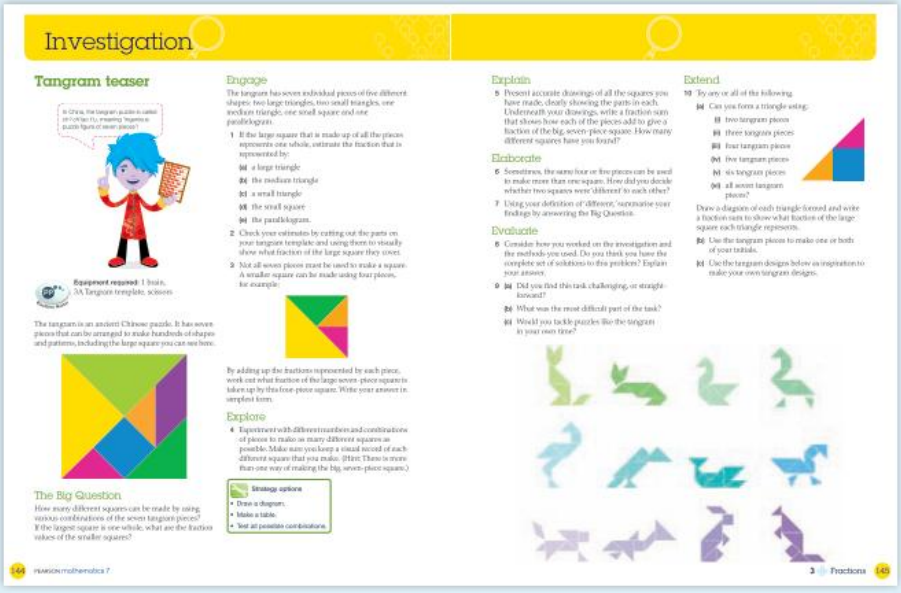
## Active participation




Mini-whiteboard—A modern teaching and learning tool used internationally for full class involvement, allowing every student to participate in giving answers.

Every copy of ***Pearson Mathematics 7*** and ***Pearson Mathematics 8*** has a mini-whiteboard in the back cover pocket.

# Encouraging inquiry and problem solving


Investigation

Tangram teaser

Engage

Explain

Extend

Explore

Elaborate

Evaluate

The Big Question

The Australian Mathematics Curriculum focuses on students developing problem-solving skills, in particular the ability to employ mathematical strategies to solve problems efficiently and to be able to respond to unfamiliar situations. **Pearson Mathematics** is designed to develop these skills.

## Problem solving toolbox




Twelve key problem-solving strategies are included inside the front cover of the Student Book and these **strategy options** are referenced throughout investigations and problem solving.

## Outside the Square: Problem solving

These are located throughout every chapter and give students ample opportunity to develop their problem-solving skills and logical reasoning.

Outside the Square
Problem solving

## Investigations

Investigation




These are scaffolded to be accessible to all learners and structured following a 5e+ format.

## Technology Explorations

Technology Exploration

**Pearson Mathematics** understands the importance the Australian Curriculum places on students developing ICT literacy. Technology Explorations help develop students' deeper understanding of the mathematical concepts covered.

# Engaging students

**Pearson Mathematics** has been designed to capture students' interest, with the incorporation of Mathspaces, Maths 4 Reals, Outside the Squares and Home Pages.

## Maths 4 Reals



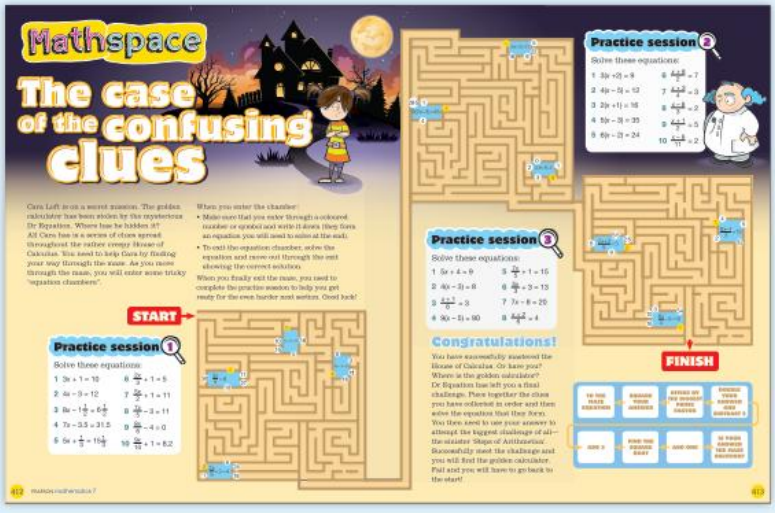

Scenarios that help students make connections with mathematics in the real world.

## Outside the squares



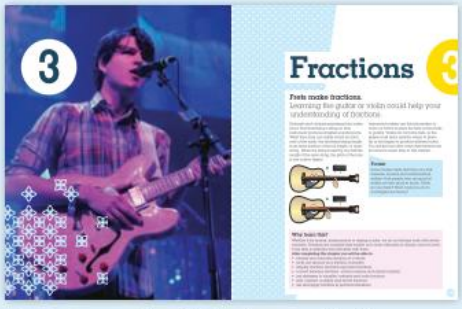

Small activities for early finishers involving problem solving, puzzles and topic-related games.

## Mathspaces




Skill consolidation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games, solve the riddle and find the clue style tasks.

## Home Pages




Anticipate the 'Why learn this?' question and provide a motivating entry into a new topic, including discussion-provoking Forum questions.

Have I seen a similar problem?
Seek an exception
Guess and check
Draw a picture
Make a model
ak problem into manageable parts


1

# 1 Whole numbers

**It's a Santa stampede!** Another Guinness World Record, this time for 'the largest gathering of Santa Clauses'. (12 695 of them!)

For nearly 60 years, *Guinness World Records* has been recording an increasing number of bizarre feats, including 'the most cockroaches eaten' (36), and 'the most consecutive skateboard frontside ollies off a half pipe ramp' (34). The book itself holds the record for being the best-selling copyrighted series of all time. We seem to have an obsession with wanting to know the most, the biggest, the fastest and the tallest.

Keeping records requires a number system with which to count, order, measure and calculate. Of course, we have more serious numbers to work with in our personal lives: how much money we spend, save and earn; how far and how often we travel; and how much of the Earth's resources we use. We find totals, work out differences, divide up quantities, and make estimations of size and number on a daily basis. To operate in this world, you need to be able to work with numbers!

### Forum

Is it important to keep 'world records'? If so, in which areas of achievement should records be kept?

What other kinds of records do we keep?

### Why learn this?

You are heading to the checkout with 2 $18 T-shirts and a $65 pair of jeans. Will $100 be enough to pay for them? If you swim 12 laps of a 50 metre pool 4 times a week, how many metres a week is that? Working confidently with numbers means more than just being able to add, subtract, multiply and divide. It means being able to choose which skill to use, to estimate and round numbers, and to have a range of mental skills to use so you can work things out efficiently and accurately.

**After completing this chapter you will be able to:**

- choose and use a range of mental strategies for calculations
- understand how the properties of numbers can be used to calculate efficiently
- estimate answers to problems using rounding strategies
- interpret and work with numbers in index form
- apply the order of operations
- solve problems involving whole numbers.

# Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R1.1

**1** What is the place value of the red digit?

**(a)** 45 783

**A** eight **B** tens **C** eighty three **D** hundreds

**(b)** 1 264 184

**A** two **B** hundreds **C** ten thousands **D** hundred thousands

Worksheet R1.2

**2** **(a)** Rearrange the following numbers in ascending order (from smallest to largest).

567, 4500, 0, 74, 11 100, 6008, 12, 602

**(b)** Rearrange the following numbers in descending order (from largest to smallest).

1200, 204, 987, 2196, 240, 95, 2400, 1010

Worksheet R1.3

**3** Find:

**(a)** 50 000 + 6000 + 800 + 90 + 5

**(b)** 7 000 000 + 20 000 + 5000 + 70 + 3

Worksheet R1.4

**4** **(a)** Round 1245 off to the nearest:

**(i)** 10 **(ii)** 100 **(iii)** 1000

**(b)** Round 8983 off to the nearest:

**(i)** 10 **(ii)** 100 **(iii)** 1000

Worksheet R1.5

**5** Find:

**(a)** $3 \times 2 \times 3$ **(b)** $5 \times 3 \times 3 \times 2$ **(c)** $2 \times 2 \times 2$ **(d)** $10 \times 10 \times 10 \times 10$

Worksheet R1.6

**6** Set out these calculations in your preferred way and work out the answers.

**(a)** 456 + 56 **(b)** 16 + 2047 **(c)** 90 + 1267 + 341

Worksheet R1.7

**7** Set out these calculations in your preferred way and work out the answers.

**(a)** 298 – 123 **(b)** 854 – 227 **(c)** 1406 – 249

Worksheet R1.8

**8** Set out these calculations in your preferred way and work out the answers.

**(a)** $45 \times 7$ **(b)** $134 \times 5$ **(c)** $34 \times 95$

Worksheet R1.9

**9** Set out these calculations in your preferred way and work out the answers.

**(a)** $844 \div 4$ **(b)** $3708 \div 9$ **(c)** $897 \div 7$

## Key Words

| | | | |
|---|---|---|---|
| associative law | divisor | mathematical conventions | quotient |
| base | estimate | order of operations | remainder |
| commutative law | expanded form | perfect cubes | rounding |
| cube root | index | perfect squares | square root |
| distributive law | index form | power | |
| dividend | indices | product | |

# Mental strategies

# 1.1

This section covers some strategies that are useful for doing calculations in your head. Try writing them out first, then move on to doing them mentally. You might find some methods easier to use than others, or have different strategies of your own. That's okay—people are different in the way they like to approach things.

## Strategy 1 – Make easy numbers

This strategy uses two properties of numbers:

**1** The order in which you add or multiply any two numbers does not change the result.

For example: $2 \times 3 = 6$ and $3 \times 2 = 6$
$4 + 5 = 9$ and $5 + 4 = 9$

This is known as the **commutative law**.




**2** The order in which three or more numbers are added or the order in which they are multiplied is not important. In the following examples, brackets are used to show which pair of numbers is added or multiplied first.

Multiplication:

$(2 \times 3) \times 5$ and $2 \times (3 \times 5)$
$= 6 \times 5$ $= 2 \times 15$
$= 30$ $= 30$

Addition:

$(6 + 7) + 8$ and $6 + (7 + 8)$
$= 13 + 8$ $= 6 + 15$
$= 21$ $= 21$

This is known as the **associative law**.

It is important to realise, however, that the commutative and associative laws do not apply to subtraction and division.

For example: $(9 - 5) - 4$ but $9 - (5 - 4)$
$= 4 - 4$ $= 9 - 1$
$= 0$ $= 8$

$(40 \div 10) \div 2$ but $40 \div (10 \div 2)$
$= 4 \div 2$ $= 40 \div 5$
$= 2$ $= 8$

The order in which you add a group of numbers and the order in which you multiply a group of numbers makes no difference to the result. This is not true for subtraction or division.

Multiples of 10 are easy to add and multiply, so we can use the commutative and associative laws to 'shuffle' the calculation to create multiples of 10.

For example:

$7 + 9 + 3$ is the same as: $3 + 7 + 9$
$= 10 + 9$

$5 \times 37 \times 2$ is the same as: $5 \times 2 \times 37$
$= 10 \times 37$

We can split one number into two parts, then use the associative law to add one part to the other number to create a multiple of 10.

For example: $135 + 46$
$= 135 + 5 + 41$ (splitting 46 into 5 and 41)
$= 140 + 41$ (adding 5 to 135)

## Worked Example 1 WE1

Calculate the following using the 'make easy numbers' strategy.

**(a)** $7 + 32 + 13$ **(b)** $2 \times 13 \times 5$ **(c)** $293 + 568$

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Rearrange the addition to form 'easy' numbers, such as multiples of 10. | (a) $7 + 32 + 13$<br>$= 7 + 13 + 32$ |
| | **2** | Perform these calculations first. | $= 20 + 32$ |
| | **3** | Complete the question. | $= 52$ |
| **(b)** | **1** | Rearrange the multiplication to form 'easy' numbers, such as multiples of 10. | (b) $2 \times 13 \times 5$<br>$= 2 \times 5 \times 13$ |
| | **2** | Perform these calculations first. | $= 10 \times 13$ |
| | **3** | Complete the question. | $= 130$ |
| **(c)** | **1** | Split one number into two parts, then add one part to the other number to create an 'easy' number. (Here, we have split 7 away from 568 and added it to 293.) | (c) $293 + 568$<br>$= 293 + 7 + 561$<br>$= 300 + 561$ |
| | **2** | Perform the calculation to complete the question. | $= 861$ |

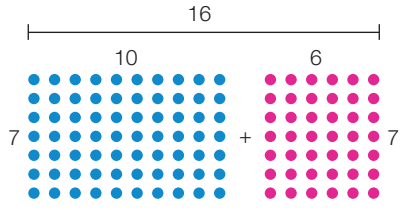
## Strategy 2 – Use the distributive law

This strategy uses a property of numbers called the **distributive law**. The distributive law allows us to multiply a large number by splitting it up into 10s and 1s (or 100s, 10s and 1s), multiplying each part separately, then adding or subtracting each of the products. (When two numbers are multiplied together, the result is called the **product**.)

For example, we can break $7 \times 16$ down into 7 lots of 10 plus 7 lots of 6.

We can represent the multiplication in an array diagram:

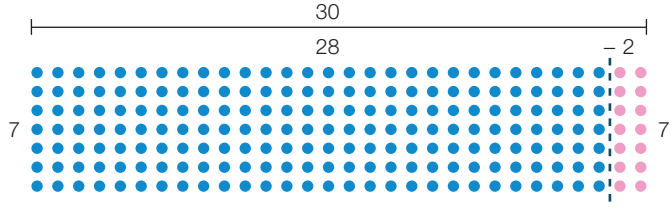
$7 \times 16$
$= 7 \times (10 + 6)$ (because $16 = 10 + 6$)
$= 7 \times 10 + 7 \times 6$
$= 70 + 42$
$= 112$




In a similar way, we can write 19 as $20 - 1$, or 28 as $30 - 2$. For example, we can break $7 \times 28$ down into 7 lots of 30 minus 7 lots of 2.

We can represent this multiplication as an array diagram (the number subtracted is on the right of the dotted line).

$7 \times 28$
$= 7 \times (30 - 2)$ (because $28 = 30 - 2$)
$= 7 \times 30 - 7 \times 2$
$= 210 - 14$
$= 196$



To multiply by a large number, split it up into 10s and 1s (or 100s, 10s and 1s). Multiply by these separately, then add or subtract each of the products.

## Worked Example 2 WE2

Evaluate the following using the distributive law.

**(a)** $7 \times 22$ **(b)** $15 \times 9$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Split the number you are multiplying by into 10s and 1s, writing it in brackets. | $(a) \quad 7 \times 22$<br>$= 7 \times (20 + 2)$ |
| **2** | Multiply the 10s and the 1s by the number in front of the brackets. | $= 7 \times 20 + 7 \times 2$<br>$= 140 + 14$ |
| **3** | Add the two products together. | $= 154$ |
| **(b)** **1** | Round one of the numbers off to the nearest multiple of 10, then write it as a subtraction in brackets. (Here, we write 9 as (10 – 1).) | $(b) \quad 15 \times 9$<br>$= 15 \times (10 - 1)$ |
| **2** | Multiply the numbers inside the brackets by the number in front. | $= 15 \times 10 - 15 \times 1$<br>$= 150 - 15$ |
| **3** | Complete the subtraction. | $= 135$ |

# 1.1 Mental strategies

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Column 1, Q5, Q6, Q7, Q9, Q11, Q15 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Column 2, Q4, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q15 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15 |

Answers page 622

### Fluency

**1** Calculate the following using the 'make easy numbers' strategy. WE1

**(a)** $8 + 23 + 42$ **(b)** $15 + 57 + 35$ **(c)** $64 + 79 + 56$

**(d)** $5 \times 6 \times 2$ **(e)** $4 \times 6 \times 5$ **(f)** $2 \times 42 \times 5$

**(g)** $5 \times 7 \times 6$ **(h)** $5 \times 3 \times 8$ **(i)** $5 \times 14 \times 4$

**(j)** $47 + 73$ **(k)** $124 + 56$ **(l)** $211 + 169$

**(m)** $37 + 128 + 63$ **(n)** $77 + 78 + 23$ **(o)** $89 + 116 + 11$

**2** Evaluate the following using the distributive law. WE2

**(a)** $17 \times 9$ **(b)** $19 \times 8$ **(c)** $49 \times 6$

**(d)** $6 \times 31$ **(e)** $7 \times 52$ **(f)** $5 \times 43$

**(g)** $99 \times 9$ **(h)** $77 \times 3$ **(i)** $57 \times 8$

**(j)** $14 \times 11$ **(k)** $15 \times 13$ **(l)** $16 \times 12$

**(m)** $101 \times 8$ **(n)** $113 \times 5$ **(o)** $124 \times 11$

**3** Use any appropriate mental strategy to work out the following.

**(a)** $23 + 41 + 57$ **(b)** $347 + 156$ **(c)** $335 - 170$

**(d)** $8 \times 9 \times 5$ **(e)** $14 \times 7$ **(f)** $21 \times 9$

**(g)** $103 \times 6$ **(h)** $22 \times 11$ **(i)** $3 \times 194$

**(j)** $147 + 213$ **(k)** $19 \times 14$ **(l)** $4 \times 7 \times 15$

## Understanding




**4** Choose the correct answer to the following question.

$23 \times 7$ could be calculated by:

**A** multiplying 3 and 7, then adding 20

**B** multiplying 20 and 7, then adding 3

**C** multiplying 20 and 7, multiplying 3 and 7, then adding the products together

**D** multiplying 20, 3 and 7 all together.

**5** Bilal has completed the first 3 stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

**(a)** the total distance that Bilal has ridden so far

**(b)** how far Bilal still has to ride, if the total rally distance is 480 km.

**6** The Year 7s at Mountain View Secondary College are undertaking a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings 8 classes of 25 students will need.



**7** Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

**(a)** how much Jason has saved after 17 weeks

**(b)** how much he still has to save if the cricket gear he wants costs $189.

**8** Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

**(a)** Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

**(b)** List two reasons why the actual number of cars might be less than your answer to **(a)**.




**9** Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?

**10** Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

**(a)** 8 notepads at 98 cents each

**(b)** 3 gluesticks at 77 cents each

**(c)** 12 pens at 59 cents each

**(d)** 5 boxes of paperclips at 82 cents each

## Reasoning

**11** Below are some mistakes made by students on a test, and their explanation of the method they used. Write what each student has done incorrectly and what the answer should be.

**(a)** $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So, then I added 21.'

**(b)** $35 \times 3 = 140$. Sam: 'I doubled 35, then doubled my answer to get 140.'

**(c)** $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'

**12** Four students were asked to write down a mental maths strategy for calculating $9 \times 29$. Here are their suggestions:

Alex: $10 \times 29 - 29$ Indrah: $10 \times 29 - 9$
Khalid: $9 \times 30 - 9$ Lucy: $9 \times 30 - 30$

**(a)** Who has written a correct strategy?

**(b)** Why do two different strategies give the same answer?

**(c)** Explain what is wrong with the other strategies.



## Open-ended

**13** Tranh is in the hardware shop buying some supplies. He has 5 picture hooks, which are 28 cents each, 12 curtain rings, which are 15 cents each, and a small hammer costing $6. As he walks to the checkout, Tranh wonders if he has enough money, as he only has $10 in his pocket. Describe the mental calculations Tranh could do to be sure he has enough money to pay for his items. Is $10 enough?

**14** Describe how you could use your calculator to work out $11 \times 23$ if the '1' button was broken.

**15** Brendan has conducted a survey of the number of pets owned by each member of his class. His results are: 2, 2, 2, 1, 1, 2, 3, 3, 2, 2, 1, 4, 1, 3, 5, 2, 4, 1, 2, 1, 3, 2, 1.

Brendan has been trying to use his calculator to add up his list of numbers, but he keeps losing his place in the list. Suggest a method that Brendan could use to organise his numbers and use his calculator more efficiently. Use your method to add up Brendan's list.

## Outside the Square Game

### Greed

**Equipment required:** 2 brains, 2 dice

**How to win:**
The winner is the first player to reach or pass 500.

**How to play:**
On your turn, roll the dice and make a two-digit number (for example, if you roll a 4 and a 3, you can make 34 or 43). Write this number down.

Roll again, make another two-digit number and add it to the first one.

Keep rolling, and adding up the total as you go—BUT, if you roll a 'double' (two of the same number), all your score for that turn is lost, and your turn ends. (This rule does not apply for a player's very first roll.)

You can choose to stop at any time and 'bank' your score. A banked score is carried over into your next turn and cannot be lost. Circle your banked score after each turn to help you keep track of things.

Maths 4 Real




# Numbers around the world



**Throughout history, different civilisations and cultures have developed various systems and symbols for numbers. Some of these systems travelled to other parts of the world as people migrated and traded, while others were (and still are) only used by small groups with a certain language or culture.**

## The Hindu-Arabic system

0 1 2 3 4 5 6 7 8 9

You are very familiar with the above 10 symbols that we use to represent numbers. Have you ever thought about where these symbols come from? They have their origins in the 'Brahmi numerals', which were used in India from about 300 BCE. They were adopted and modified by the Arabic people, who brought them to Europe during the Middle Ages. The Arabs also developed a symbol for zero, calling it *sifr*. Because of its history, we refer to our number system as the 'Hindu-Arabic system'. The Brahmi numerals are shown in the second row of the table below. They have evolved over time to become the more familiar symbols in the top row.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| — | = | ≡ | 𑁬 | 𑁭 | 𑁮 | 𑁯 | 𑁰 | 𑁱 |

## The Roman system

The ancient Romans used letters as their symbols for numbers. Roman numerals are still used today on clocks, watches and monuments, or to show the date when something (such as a movie) was made.

| I | II | III | IV | V | VI | VII |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| VIII | IX | X | L | C | D | M |
| 8 | 9 | 10 | 50 | 100 | 500 | 1000 |

In Roman numerals, the letter **I**, **X** and **C** before a higher value letter means we take it away from the higher value. The rule is that you can only subtract these from the next two higher Roman numerals. For example, 4 is **IV** and 9 is **IX** but 49 is not **IL** (it is **XLIX**; made up of **XL** for 40 and **IX** for 9). So, **I** can be subtracted from **V** and **X**, **X** can be subtracted from **L** and **C**, and **C** can be subtracted from **D** and **M**.

39 is **XXXIX**

(10 + 10 + 10 + 9)

2644 is **MMDCXLIV**

(1000 + 1000 + 500 + 100 + (50 – 10) + (5 – 1))

1 Write out these numbers using Roman numerals:
   - **(a)** 8
   - **(b)** 84
   - **(c)** 439
   - **(d)** 1975

2 Perform the following calculations. Write your answers using Roman numerals.
   - **(a)** XXIX + CLII
   - **(b)** MDCCLX – MCDXCIX

3 What is the largest Roman numeral you can make that uses one of each of the different symbols: **C**, **L**, **M**, **X**, **D**, **V**, **I**? Write your answer in Roman and Hindu-Arabic numerals.

## The Chinese character system

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 〇 | 一 | 二 | 三 | 四 | 五 | 六 |

| 7 | 8 | 9 | 10 | 100 | 1000 | 10000 |
|---|---|---|---|---|---|---|
| 七 | 八 | 九 | 十 | 百 | 千 | 万 |

The Chinese character system uses two symbols to show each of the place values represented. The first symbol indicates how many, and the second symbol indicates the place value. However, when there is only one of a particular place value only one symbol is used. If a place value is zero then no symbols are needed. Chinese numbers are written vertically.

For example:

| 270 | 19 | 8005 |
|---|---|---|
| 二百七十 (two (lots of) one hundred and seven (lots of) ten) | 十九 (ten and nine) | 八千五 (eight (lots of) one thousand and five) |

**4** Write out these numbers using the Chinese character system:

**(a)** 37 **(c)** 1053

**(b)** 823 **(d)** 6400

**5** Perform the following calculations. Write your answers using Chinese characters.

**(a)** 五百六十 + 八百九 **(b)** 千 − 三百二十七

**6** Did you find it difficult to perform calculations with the Roman and Chinese systems? What are the advantages of using the Hindu-Arabic system for calculations?

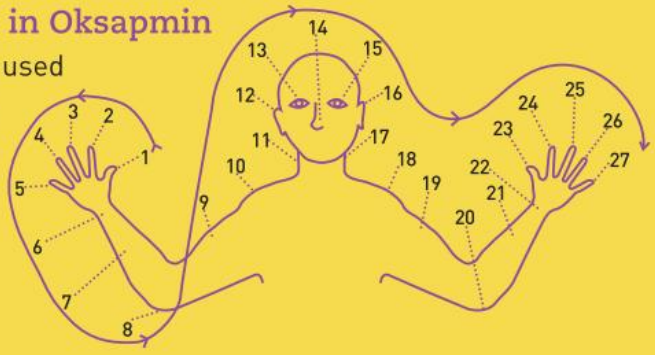
## Non-base 10 systems

The Hindu-Arabic system is a 'base 10' or 'decimal' system. The place value of each digit in a number is 10 times bigger than the next position to the right, for example:
367 = 3 'hundreds' + 6 'tens' + 7 'ones', or 300 + 60 + 7.

Many indigenous cultures of Australia, Torres Strait and Papua New Guinea have counting systems that use body parts to indicate numbers. These systems are not base 10 systems. For example, the people of Sandaun province, Papua New Guinea (who speak the Oksapmin language), use the parts of the upper body to count in lots of 27.

### Counting in Oksapmin

Body parts used to count in Oksapmin




| | | |
|---|---|---|
| 1 *tipun* 'thumb' | *lum* 'nose' | 14 |
| 2 *ləwatipun* 'index finger' | *kin tən* 'other-side eye' | 15 |
| 3 *bumlip* 'middle finger' | *nat tən* 'other-side ear' | 16 |
| 4 *xətlip* 'ring finger' | *gwel tən* 'other-side side of neck' | 17 |
| 5 *xətxət* 'little finger' | *kat tən* 'other-side shoulder' | 18 |
| 6 *xadəp* 'wrist' | *tuwət tən* 'other-side upper arm' | 19 |
| 7 *bes* 'forearm' | *amun tən* 'other-side elbow' | 20 |
| 8 *amun* 'elbow' | *bes tən* 'other-side forearm' | 21 |
| 9 *tuwət* 'upper arm' | *xadəp tən* 'other-side wrist' | 22 |
| 10 *kat* 'shoulder' | *tipun tən* 'other-side thumb' | 23 |
| 11 *gwel* 'side of neck' | *ləwatipun tən* 'other-side index finger' | 24 |
| 12 *nat* 'ear' | *bumlip tən* 'other-side middle finger' | 25 |
| 13 *kin* 'eye' | *xətlip tən* 'other-side ring finger' | 26 |
| | *xətxət tən* 'other-side little finger' | 27 |

**7** **(a)** In Oksapmin what word would you use to say '15'?

**(b)** If you wanted to say 'I have eight chickens' in Oksapmin, what body part would you point to?

**8** The Hindu-Arabic number system is used extensively across the world. List some advantages and disadvantages of having one predominant number system.

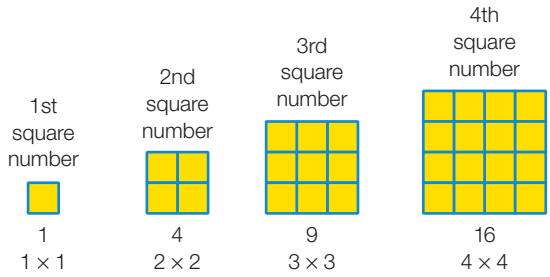
## Research

- Investigate the number systems of other ancient civilisations (such as the Mayans or Babylonians) or other indigenous cultures.
- Find out how to write and say numbers in another language, such as Japanese or Greek.

# 1.2 Indices

## Square numbers

Square numbers (or **perfect squares**) are numbers that can be represented by squares of different side lengths, as shown below. 1, 4, 9, 16 and 25 are square numbers.




The number of small squares in each larger square is equal to the number of smaller squares along one side, multiplied by itself. We can think of it as multiplying the length and width of the larger square.

We have a shorthand way of writing square numbers. $4 \times 4$, for example, is written as $4^2$. The small '2' means that two 4s are multiplied together. We say $4^2$ as 'four squared'.

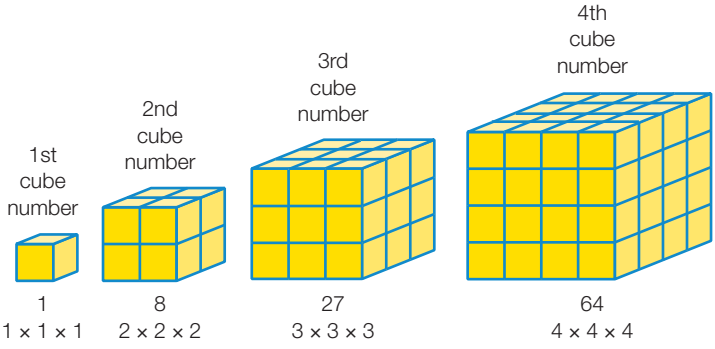
We can write the sequence of perfect squares as:

$1^2, \quad 2^2, \quad 3^2, \quad 4^2, \quad 5^2, \quad 6^2, \quad 7^2 \ldots$

$= 1, \quad 4, \quad 9, \quad 16, \quad 25, \quad 36, \quad 49 \ldots$

## Cube numbers

Cube numbers (or **perfect cubes**) are numbers that can be represented by blocks arranged in a cube pattern, as shown below. 1, 8, 27 and 64 are cube numbers.




The number of smaller cubes in each larger cube is equal to the number of smaller cubes along one side, multiplied by itself, and itself again. We can think of it as multiplying the length, width and height of the cube.

(Notice that when we cube a number, the number is written out three times.)

Using our shorthand, we can write, for example, $2 \times 2 \times 2$ as $2^3$. The small '3' indicates that three 2s are multiplied together. We say $2^3$ as 'two cubed'.

We can write the sequence of perfect cubes as:

$1^3, \quad 2^3, \quad 3^3, \quad 4^3, \quad 5^3 \ldots$

$= 1, \quad 8, \quad 27, \quad 64, \quad 125 \ldots$

To 'square' a number, multiply it by itself:

$$6^2 = 6 \times 6$$
$$= 36$$

To 'cube' a number, multiply it by itself and itself again, so that the number is written three times in all:

$$4^3 = 4 \times 4 \times 4$$
$$= 64$$

Do not confuse $4^3$ with $4 \times 3$.

## Square roots and cube roots

Finding the **square root** or **cube root** of a number is the reverse of squaring or cubing a number. For example, the square root of 16 is the number that gives 16 when multiplied by itself. In this case, that number is 4, because $4^2 = 16$. We write $\sqrt{16} = 4$, where the symbol $\sqrt{\ }$ means 'the square root of'. The cube root of 8 is 2, because $2^3 = 8$. We write $\sqrt[3]{8} = 2$, where $\sqrt[3]{\ }$ means 'the cube root of'.

Only the roots of perfect squares and perfect cubes are whole numbers. Many roots are decimal numbers and a calculator can be used to find them.

## Index notation

We can extend our shorthand for square numbers and cube numbers to write other large numbers produced by repeated multiplication, by continuing a pattern.

| | | |
|---|---|---|
| If: | $3^2 = 3 \times 3$ | $(= 9)$ |
| and | $3^3 = 3 \times 3 \times 3$ | $(= 27)$ |
| then, | $3^4 = 3 \times 3 \times 3 \times 3$ | $(= 81)$ |
| | $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ | $(= 243)$ |

This shorthand notation is called **index form**. The number being multiplied repeatedly is called the **base**. The number written up high to the right of the base is called the **power**, or **index**. (The plural of index is **indices**.) The index tells you how many times the base will appear when written in **expanded form** (that is, written as a series of multiplications).

Index form:

$5^4$ ← index (power)
← base

Expanded form: $5 \times 5 \times 5 \times 5$

Say: '5 to the power of four', or '5 to the fourth' or 'base 5, index 4'.

The value of $5^4$ is 625.

Any number to the power of 1 is itself. $5^1 = 5$.

## Worked Example 3 WE3

Write each of the following numbers in expanded form, then find its value.

**(a)** $4^3$ **(b)** $8^5$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Identify the base and the index. Multiply the base by itself according to what the index is. | (a) $4^3 = 4 \times 4 \times 4$ |
| **2** | Perform the multiplication. | $= 64$ |
| **(b)** **1** | Identify the base and the index. Multiply the base by itself according to what the index is. | (b) $8^5 = 8 \times 8 \times 8 \times 8 \times 8$ |
| **2** | Perform the multiplication. | $= 32\,768$ |

## Indices on the calculator

Many calculators have a key to help you work efficiently with indices. It usually looks like [^] or [$x^y$]. For example, to calculate $12^4$, you would press: [1] [2] [^] [4] [=] or [1] [2] [$x^y$] [4] [=].

Some calculators also have an [$x^2$] key and/or an [$x^3$] key for calculating square and cube numbers. For example, to find $17^2$, press: [1] [7] [$x^2$] [=]. To find $2^3$, press: [2] [$x^3$] [=].

The [^] or [$x^y$] key can also be used to calculate squares and cubes.

## Square roots and cube roots on the calculator

Scientific calculators have a square root key, which usually looks like this: [$\sqrt{}$].

To find the square root of, for example, 256, press: [$\sqrt{}$] [2] [5] [6] [=]. (If you try this, you should find that the answer is 16.) If that doesn't work, try entering the number, then pressing the [$\sqrt{}$] key.

To find a cube root on the calculator, use the [$\sqrt[x]{}$] key. To find the cube root of, for example, 512, press: [3] [$\sqrt[x]{}$] [5] [1] [2] [=]. (If you try this, you should find that the answer is 8.)

## Powers of 10

The number system that we use for counting is based on powers of ten:

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$ etc.

Here, we use the word 'power' to refer to the actual value of the number in index form. So, the first four powers of 10 are: 10, 100, 1000, 10 000.

Notice that when we write the value of the number in index form, the number of zeroes is equal to the index. For example, $10^3$ has an index of 3, and its value, 1000, has 3 zeroes.

# 1.2 Indices

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q10, Q11, Q12, Q13, Q15, Q17, Q19, Q22, Q24 | Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7 Columns 2 & 3, Q8, Q9 Columns 1 & 2, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q22, Q23, Q24 | Q1 Column 3, Q2 Column 2, Q3 Column 3, Q4, Q5, Q6, Q7 Columns 3 & 4, Q9, Q11, Q12 Columns 2 & 3, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24 |

Answers page 622

**Equipment required:** Calculator for Questions **1(j)–(l), 3, 10, 12, 17, 19(a), 21(a)**

## Fluency

**1** Write each of the following numbers in expanded form, then find its value.

WE3

**(a)** $2^3$ **(b)** $2^4$ **(c)** $2^6$

**(d)** $1^8$ **(e)** $0^7$ **(f)** $10^3$

**(g)** $6^4$ **(h)** $5^5$ **(i)** $8^6$

**(j)** $11^3$ **(k)** $12^4$ **(l)** $14^5$

**2** Write each of the following in index form.

**(a)** $8 \times 8 \times 8$ **(b)** $4 \times 4 \times 4 \times 4 \times 4 \times 4$

**(c)** $12 \times 12 \times 12 \times 12 \times 12$ **(d)** $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$

**(e)** seventeen cubed **(f)** nineteen squared

**(g)** eight to the power of 4 **(h)** thirteen to the power of seven

**(i)** base 11, index 7 **(j)** base 9, index 6

**3** Find the value of each of the following.

**(a)** $4^7$ **(b)** $6^6$ **(c)** $9^4$

**(d)** $8^3$ **(e)** $7^5$ **(f)** $15^3$

**(g)** seventeen squared **(h)** fifty cubed **(i)** the cube of 4

**(j)** $5 \times 10^2$ **(k)** $4 \times 10^3$ **(l)** $10^3 \times 2$

**(m)** 2 to the power of 10 **(n)** 3 to the power of 9 **(o)** base 1, index 8

**4** Write in words how we could say each of the following.

**(a)** $5^2$ **(b)** $31^2$ **(c)** $3^3$ **(d)** $27^3$

**(e)** $4^5$ **(f)** $9^6$ **(g)** $1^4$ **(h)** $7^7$

**5** If 1 is the first square number and 4 is the second, write the 5th, 6th and 7th square numbers.

**6** If 1 is the first cube number and 8 is the second, write the 4th, 5th and 6th cube numbers.

**7** Evaluate:

**(a)** $\sqrt{25}$ **(b)** $\sqrt{49}$ **(c)** $\sqrt{100}$ **(d)** $\sqrt{121}$

**(e)** $\sqrt{1}$ **(f)** $\sqrt{0}$ **(g)** $\sqrt{4900}$ **(h)** $\sqrt{400}$

**(i)** $\sqrt[3]{8}$ **(j)** $\sqrt[3]{64}$ **(k)** $\sqrt[3]{1000}$ **(l)** $\sqrt[3]{1}$

**(m)** $\sqrt[3]{0}$ **(n)** $\sqrt[3]{125}$ **(o)** $\sqrt[3]{8000}$ **(p)** $\sqrt[3]{27\,000}$

$\sqrt[3]{5}$ means the cube root of 5. It does not mean $3 \times \sqrt{5}$.



**8** **(a)** In $2^3 = 8$ the base number is:

**A** 2 **B** 3 **C** 6 **D** 8

**(b)** The square of 5 is:

**A** 2.2 **B** 10 **C** 25 **D** 125

**(c)** The cube root of 729 is:

**A** 9 **B** 27 **C** 243 **D** 2187

## Understanding

**9** Evaluate the following without using a calculator.

**(a)** $1^4 + 2^2$ **(b)** $2^3 - 1^6$ **(c)** $3^4 - 2^4$

**(d)** $2^3 \times 2^2$ **(e)** $2^2 \times 2^4$ **(f)** $3^3 \times 3^2$

**(g)** $2^5 + 5^2 - 6^2$ **(h)** $10^2 + 3^2 - 4^3$ **(i)** $5^2 - 1^6 + 3^3$

**10** **(a)** Arrange these numbers in ascending order:

$4^5, 5^4, 1^{200}, 10^3, 4^6, 5^5$

**(b)** Arrange these numbers in descending order:

$100^2, 10^5, 1^{1000}, 0^{100}, 3^2, 2^3$




**11** **(a)** Write down two numbers between 5 and 40 that are both even and square.

**(b)** Write down two numbers between 30 and 90 that are both odd and square.

**12** Evaluate these with your calculator, using the most efficient method possible:

**(a)** $16^3 - 4096$ **(b)** $21^4 - 4481$ **(c)** $15^4 - 5625$

**(d)** $36^3 \times 53$ **(e)** $14^4 \times 14$ **(f)** $19^5 \times 21$

**(g)** $2^{12} + 2^{18}$ **(h)** $3^{10} + 12^4$ **(i)** $4^9 + 3^{11}$

**13** Only the square roots of perfect squares are whole numbers. Square roots of other numbers lie in between the perfect square roots (they are decimal numbers). For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{20}$ would lie in between 4 and 5. Find which two consecutive whole numbers the following lie between. (Consecutive numbers come one after the other; e.g. 8 and 9.)

**(a)** $\sqrt{10}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{20}$ **(d)** $\sqrt{62}$

**(e)** $\sqrt{99}$ **(f)** $\sqrt{2}$ **(g)** $\sqrt{70}$ **(h)** $\sqrt{108}$

**14** Arrange the following in ascending order.

**(a)** $2^4, \sqrt[3]{64}, 5, \sqrt{49}$ **(b)** $7^2, \sqrt[3]{125}, 9, \sqrt{60}$

**(c)** $\sqrt[3]{8}, 10, \sqrt{90}, 4^2$ **(d)** $\sqrt{80}, 8, \sqrt{105}, 3^3$

**15** **(a)** Complete the following table of the powers of 10.

| Index form | Expanded form | Value |
|---|---|---|
| $10^1$ | 10 | 10 |
| $10^2$ | $10 \times 10$ | |
| $10^3$ | | 1000 |
| $10^4$ | | |
| | | |

**(b)** For each row of the table, compare the index numbers in the first column with the number of zeroes in the third column. Describe the pattern.

**16** For each of the following, find the missing power of 10.

**(a)** $20\,000 = 2 \times 10^{\square}$ **(b)** $400 = 4 \times 10^{\square}$ **(c)** $5000 = 5 \times 10^{\square}$

**(d)** $300\,000 = 3 \times 10^{\square}$ **(e)** $80 = 8 \times 10^{\square}$ **(f)** $7\,000\,000 = 7 \times 10^{\square}$

**(g)** $150\,000 = 15 \times 10^{\square}$ **(h)** $91\,500 = 915 \times 10^{\square}$ **(i)** $2\,340\,000 = 234 \times 10^{\square}$

## Reasoning

**17** **(a)** Use your calculator to work out $5^2 \times 5^3$ and $5 \times 5 \times 5 \times 5 \times 5$.

**(b)** Now, try it with $2^3 \times 2^6$ and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

**(c)** What can you conclude?

**18** How many whole numbers have the value of their square root in between:

**(a)** 2 and 3

**(b)** 5 and 6

**(c)** 8 and 9?

**19** **(a)** Use your calculator to answer TRUE or FALSE to each of the following statements.

**(i)** $4^6$ is bigger than $6^4$

**(ii)** $2^{10}$ is bigger than $10^2$

**(iii)** $3^9$ is bigger than $9^3$

**(iv)** $19^2$ is bigger than $2^{19}$

**(b)** Look at your answers for part **(a)** and, *without* using your calculator, answer TRUE or FALSE to each of the following statements.

**(i)** $9^8$ is bigger than $8^9$

**(ii)** $2^{100}$ is bigger than $100^2$

**20** **(a)** The number $10^{100}$ was called a googol by the mathematician Edward Kasner.

**(i)** How many 10s are multiplied together to give a googol?

**(ii)** How many zeroes would follow the 1 in a googol?

**(b)** If you raise the number ten to the power of a googol, you get a number called a googolplex.

**(i)** How many 10s are multiplied together to give a googolplex?

**(ii)** How many zeroes would follow the 1 in a googolplex? How much time do you think you save by writing a googolplex in index form?

**21** **(a)** Complete the following.

$11^2 =$ ____ $111^2 =$ ____ $1111^2 =$ ____

**(b)** Look at the pattern in part **(a)** and, *without* using a calculator, copy and complete the following.

$11\,111^2 =$ ____ $111\,111^2 =$ ____ $1\,111\,111^2 =$ ____

## Open-ended

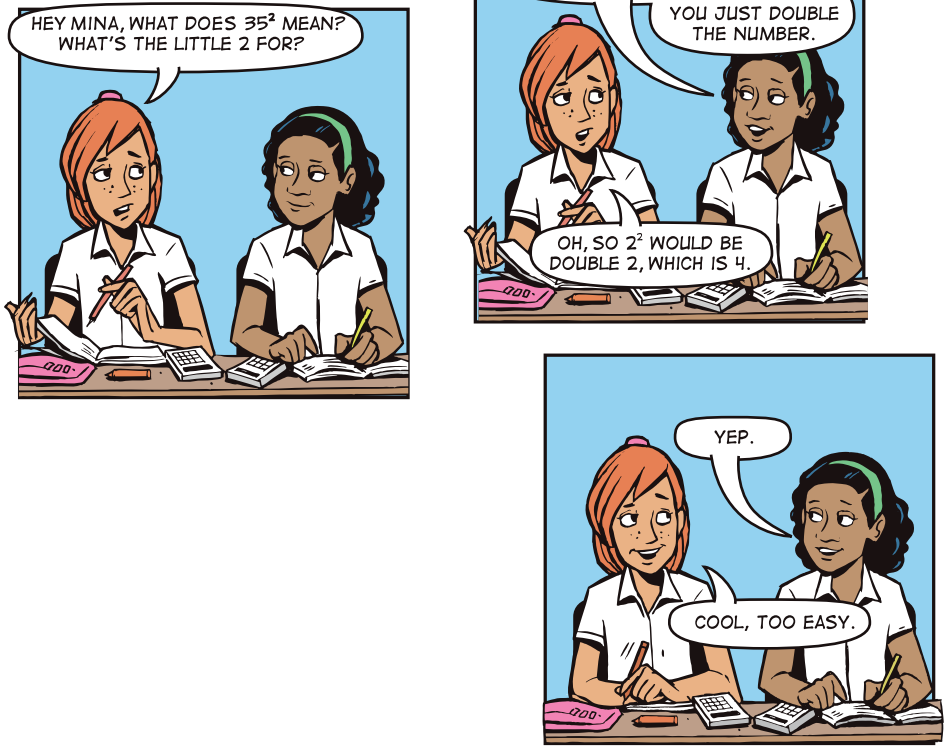



**22** Use the digits 1, 2 and 3 to create at least three numbers that are greater than 500. (Hint: Use index numbers.)

**23** **(a)** A number has two digits when it is squared and three digits when it is cubed. What might be the number?

**(b)** A number has three digits when it is squared and four digits when it is cubed. What might be the number?

**24**




**(a)** What is wrong with Mina's explanation? Write down a better explanation that Mina could give to Jo.

**(b)** What question did Jo get right? Why was this the case?

# Outside the Square

## Problem solving

### Teacher's age

Cheryl, who is a teenager, was curious about her maths teacher's age.

"Well", said the teacher, "Right now the second digit of my age is a square number and the product of my digits is the cube of a number. In ten years' time my age will be a square number and 25 years from now my age will be a number that has been cubed."

"I know your age," Cheryl said with confidence. "It's also interesting that the difference between our ages is one more than an odd number which has been squared."

How old is the teacher?

How old is Cheryl?



**Strategy options**

- Guess and check.
- Make a table.
- Work backwards
- Test all possible combinations.

# Investigation

## How many squares on a chessboard?

**Equipment required:** 1 brain, graph or grid paper

### The Big Question

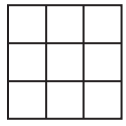
A chessboard is an $8 \times 8$ grid of squares. What is the total number of squares on a chessboard?

### Engage

**1** Shape puzzles, such as the one below, have been appearing in puzzle books for many years.

Write down how many squares there are in this $3 \times 3$ grid.

People who are tricked by this puzzle forget there are squares of more than one size. In addition, several squares overlap each other. By drawing out the grid and shading, show that there are 14 squares of three different sizes in the $3 \times 3$ grid.



### Explore

**2** To solve a puzzle such as this one, it helps to have a system. Use your results from Question **1** to fill in the following table. The $1 \times 1$ and $2 \times 2$ grids have been done for you.

| | Number of squares of each type | | | | |
|---|---|---|---|---|---|
| Grid size | Side of length 1 | Side of length 2 | Side of length 3 | Side of length 4 | Total squares in whole grid |
| $1 \times 1$  | 1 | | | | 1 |
| $2 \times 2$  | 4 | 1 | | | 5 |
| $3 \times 3$  | | | | | |

**3** Now, extend the system of counting the number of squares of each size to work out how many squares there are in a $4 \times 4$, $5 \times 5$ … through to an $8 \times 8$ grid (a chessboard). Extend your table to record the number of each size of square, and the total number of squares in each grid.



**Strategy options**

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

### Explain

**4** Describe any patterns you can see in your table of results. (Consider the numbers of each size square in a grid and how they change as the grid gets larger.)

### Elaborate

**5** State your answer to the Big Question. Present your results so that they clearly show how you arrived at your answer.

**6** Use your pattern to predict the total number of squares on: **(a)** a $10 \times 10$ grid **(b)** any size grid.

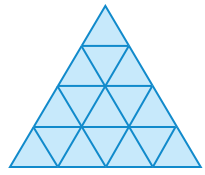
### Evaluate

**7** **(a)** Consider how you worked on the investigation and the methods you used. Describe the strategy or method you used to make sure that you counted all the squares. How confident are you that your answer is correct?

**(b)** Did any of your methods or strategies change as you went? Did you develop any shortcuts that made your working easier? What were they?

### Extend

**8** Using a similar method, work out how many triangles there are in this pattern.



# More strategies for multiplication and division

1.3

The result of a multiplication calculation is called the product. The numbers being multiplied together are sometimes called multipliers.

The result of a division calculation is called the **quotient**. The number being divided is the **dividend** and the number we are dividing by is the **divisor**. Sometimes after a division we are left with a **remainder**.

| | | | | | |
|---|---|---|---|---|---|
| 18<br>multiplier | × | 27<br>multiplier | = | 486<br>product | |
| 32<br>dividend | ÷ | 10<br>divisor | = | 3<br>quotient | rem 2<br>remainder |

You should be very familiar with multiplying any two numbers from 0 to 10 together, such as $4 \times 3$, $8 \times 6$, or $9 \times 5$. If you know a lot of multiplication facts, you also know a lot of division facts; for example, $63 \div 7 = 9$ because $9 \times 7 = 63$. Following on from strategies 1 and 2 covered in section 1.1, here are two more strategies that are useful for multiplying and dividing numbers, including numbers larger than the ones on the multiplication table. You may have other methods that you like to use.

## Strategy 3 – Work in stages

Multiplying a number by 4 is the same as multiplying by 2 twice ($4 = 2 \times 2$); that is, doubling and then doubling again. To multiply by 6, we could multiply by 3, and double the answer ($6 = 3 \times 2$). Similarly, we could divide by 4 by dividing by 2 twice. Dividing by 6 is the same as dividing by 2, and then dividing by 3 (you won't want to use this strategy if the number you are dividing is odd).

Multiply or divide in stages by breaking the multiplier or divisor down and doing a series of simpler multiplications or divisions.

### Worked Example 4 WE4

Calculate the following using the 'work in stages' strategy.

**(a)** $34 \times 8$ **(b)** $900 \div 15$

| Thinking | Working |
|---|---|
| **(a) 1** Break the multiplier down into a series of simpler multiplications. | (a) $34 \times 8$<br>$= 34 \times 4 \times 2$<br>$= 34 \times 2 \times 2 \times 2$ |
| **2** Perform one of these multiplications. | $= 68 \times 2 \times 2$ |

| | | | |
|---|---|---|---|
| | 3 | Perform the next multiplication (× 2 again). | = 136 × 2 |
| | 4 | Keep multiplying until the multiplication is complete (× 2 again). | = 272 |
| (b) | 1 | Break the divisor down into a series of simpler divisions. | (b) 900 ÷ 15<br>= 900 ÷ 3 ÷ 5 |
| | 2 | Perform one of these divisions (÷ 3). | = 300 ÷ 5 |
| | 3 | Perform the next division (÷ 5). Keep dividing until the division is complete. | = 60 |

## Strategy 4 – Use an array

Multiplying two two-digit or three-digit numbers is a little more challenging than multiplying two one-digit numbers. We can extend our 'use the distributive law' strategy from section 1.1 to both numbers and multiply them in an array, or grid.




## Worked Example 5

WE5

Calculate 23 × 47 using the 'use an array' strategy.

| Thinking | Working |
|---|---|
| **1** Split the numbers to be multiplied into 10s and 1s. Draw up a grid. Place one broken-up number on top of the columns and one to the left of the rows. | 23 = 20 + 3 47 = 40 + 7 |
| **2** Multiply each of the 10s and 1s with each of the other 10s and 1s, to get a product in each of the four grid spaces. | |
| **3** Add the two products in each of the rows. (Alternatively, you could add the columns.) | |
| **4** Add these two sums to get your final answer. | 920<br>+ 161<br>1081 |

| | 20 | 3 | |
|---|---|---|---|
| 40 | | | |
| 7 | | | |

| | 20 | 3 |
|---|---|---|
| 40 | 800 | 120 |
| 7 | 140 | 21 |

| | 20 | 3 | |
|---|---|---|---|
| 40 | 800 | 120 | 920 |
| 7 | 140 | 21 | 161 |

## Multiplying by multiples of 10

Multiplying a whole number by a power of 10 such as 10, 100 or 1000 makes the value of each digit in the number 10, 100 or 1000 times bigger. We show this by adding zeroes onto the end of the number. The number of zeroes added is the same as the number of zeroes in the power of 10 (and the same as the power number).

For example: $3 \times 10 = 30$ $(3 \times 10^1)$
$7 \times 1000 = 7000$ $(7 \times 10^3)$
$15 \times 100 = 1500$ $(15 \times 10^2)$

To multiply a whole number by a multiple of 10 such as 40, 800 or 12 000, or to multiply two multiples of 10 together, we can use the associative law to rearrange the calculation and multiply in stages.

For example: $7 \times 40 = 7 \times 4 \times 10$
$= 28 \times 10$
$= 280$

$30 \times 600$
$= 3 \times 10 \times 6 \times 100$
$= 3 \times 6 \times 10 \times 100$
$= 18 \times 1000$
$= 18\,000$

To multiply whole numbers by numbers that are multiples of 10:

1. write each multiple of 10 as the product of a number and a power of 10
2. rewrite the multiplication, grouping the powers of 10 together
3. multiply the other numbers, using the total number of zeroes in the powers of 10 to write the correct number of zeroes in the answer.

## Dividing by multiples of 10

Consider the following divisions.

$50 \div 10 = 5$
$500 \div 100 = 5$
$500 \div 10 = 50$
$50\,000 \div 100 = 500$
$5\,000\,000 \div 10\,000 = 500$

The number of zeroes in the answer is the difference between the number of zeroes in the numbers being divided.

A quick way to show this type of division is to draw a line through them. This is often called 'cancelling zeroes out'. For example: $50\,0\cancel{00} \div 1\cancel{00}$
$= 500 \div 1$
$= 500$

It is important to remember what 'cancelling out zeroes' means—you are dividing both numbers by a power of 10. For example, crossing out three zeroes means that you are dividing by 1000.

To divide by whole numbers that are multiples of 10:

1. divide both numbers by 10 until one number is no longer a multiple of 10. Show this by cancelling zeroes
2. perform the simplified division.

## Worked Example 6 WE6

Calculate the following.

**(a)** $7 \times 900$ **(b)** $6000 \times 30$ **(c)** $4500 \div 90$

| Thinking | | | Working | |
|---|---|---|---|---|
| **(a)** | **1** | Rewrite the multiple of 10 as the product of a power of 10 and another number. | *(a)* | $7 \times 900$<br>$= 7 \times 9 \times 100$ |
| | **2** | Multiply the first two numbers. | | $= 63 \times 100$ |
| | **3** | Multiply by the power of ten. | | $= 6300$ |
| **(b)** | **1** | Rewrite each multiple of 10 as the product of a number and a power of 10. | *(b)* | $6000 \times 30$<br>$= 6 \times 1000 \times 3 \times 10$ |
| | **2** | Rearrange the calculation, placing the powers of 10 together and the other numbers together. | | $= 6 \times 3 \times 1000 \times 10$ |
| | **3** | Perform the multiplications. | | $= 18 \times 10\,000$<br>$= 180\,000$ |
| **(c)** | **1** | Divide both numbers by 10 until one number is no longer a multiple of 10. | *(c)* | $4500 \div 90$ |
| | **2** | Write down the simpler division that results. | | $= 450 \div 9$ |
| | **3** | Perform the division. | | $= 50$ |

# 1.3 More strategies for multiplication and division

## Navigator

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| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4 Column 1, Q5, Q7, Q8, Q12, Q14, Q15 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Column 2, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q15 |

## Fluency

WE4

**1** Calculate the following using the 'work in stages' strategy.

**(a)** $27 \times 4$ **(b)** $41 \times 6$ **(c)** $62 \times 8$

**(d)** $600 \div 4$ **(e)** $153 \div 9$ **(f)** $864 \div 16$

**(g)** $35 \times 9$ **(h)** $390 \div 6$ **(i)** $70 \times 15$

**(j)** $15 \times 12$ **(k)** $450 \div 25$ **(l)** $360 \div 24$

**2** Calculate the following using the 'use an array' strategy. WE5

| | | |
|---|---|---|
| **(a)** $12 \times 41$ | **(b)** $17 \times 26$ | **(c)** $19 \times 68$ |
| **(d)** $23 \times 61$ | **(e)** $31 \times 49$ | **(f)** $39 \times 56$ |
| **(g)** $42 \times 47$ | **(h)** $49 \times 53$ | **(i)** $55 \times 55$ |
| **(j)** $61 \times 82$ | **(k)** $72 \times 77$ | **(l)** $96 \times 51$ |

**3** Calculate the following. WE6

| | | |
|---|---|---|
| **(a)** $700 \times 3$ | **(b)** $400 \times 5$ | **(c)** $2000 \times 13$ |
| **(d)** $360 \div 20$ | **(e)** $5400 \div 200$ | **(f)** $8000 \div 500$ |
| **(g)** $1200 \times 400$ | **(h)** $400 \times 32\,000$ | **(i)** $7000 \times 20\,000$ |
| **(j)** $12\,000 \div 20$ | **(k)** $350\,000 \div 700$ | **(l)** $8\,400\,000 \div 4000$ |

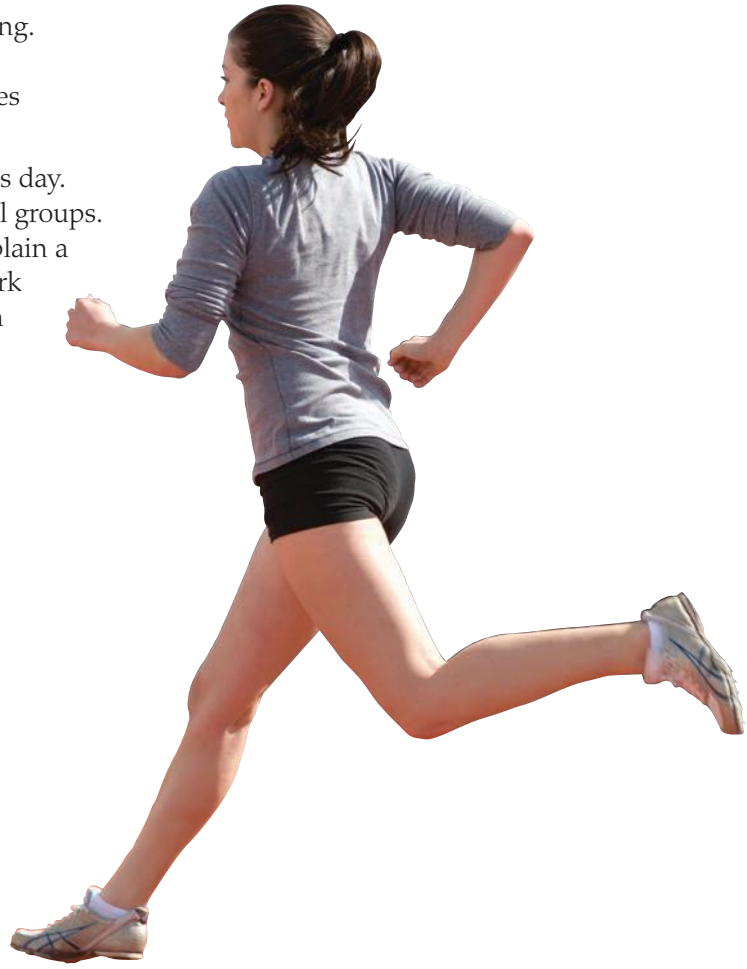
**4** Calculate the following using any suitable strategy.

| | | |
|---|---|---|
| **(a)** $180 \div 20$ | **(b)** $450 \div 90$ | **(c)** $800 \div 25$ |
| **(d)** $40 \times 19$ | **(e)** $53 \times 20$ | **(f)** $102 \times 18$ |
| **(g)** $2700 \div 18$ | **(h)** $1075 \div 25$ | **(i)** $2250 \div 15$ |
| **(j)** $120 \times 12$ | **(k)** $59 \times 72$ | **(l)** $98 \times 25$ |

## Understanding

**5** Charlotte is a distance runner in training. She runs 20 laps of a 400 metre race track 6 times a week. How many metres does Charlotte run in a week?

**6** Year 7 students are practising for sports day. They need to be divided into five equal groups. There are 130 students in the year. Explain a quick way the teacher could use to work out how many students will be in each group, and find the answer.



7 Marie's monthly mobile phone bill is $52. If she continued to pay this amount every month, use a mental or written strategy to calculate how much Marie will have paid in phone bills by the end of her 2 year contract.

8 A group of eight friends have won $2000 and want to divide it equally between them. How much will each receive?

9 Rehan is paying back a bank loan that he used to buy a house. His repayments are set at $1800 per month for 30 years. How much money will Rehan have paid back at the end of this period of time?

10 Dharma has printed 3700 brochures advertising his business. He wants to place them in the letterboxes of houses in the local area. Dharma estimates that there are about 40 houses in each street. How many streets will he be able to cover with 3700 brochures?



## Reasoning

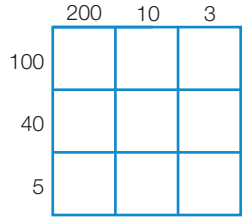
11 A travelling salesperson has lost his calculator. He needs to work out the individual cost of an item that is packaged into groups of 12 items, and costs $660 for the pack.

**(a)** He decides to approximate by dividing by 10. Find the answer he obtains.

**(b)** Calculate the actual cost of each individual item.

**(c)** He sells five items at his approximated cost. How much did he lose or gain?

12 The 'use an array' method of multiplication can be extended from two-digit to three-digit numbers by adding an extra row and an extra column to the grid. For example, 213 × 145 can be calculated using the following array.




Use an extended grid to calculate the following.

**(a)** 162 × 246 **(b)** 356 × 412 **(c)** 107 × 560

13 Which of the following is an incorrect method for calculating 240 ÷ 4?

**A** Halve 240, then halve the answer.

**B** Calculate 2 ÷ 4, 4 ÷ 4 and 0 ÷ 4, then add the results together.

**C** Calculate 200 ÷ 4 and 40 ÷ 4, then add the quotients.

**D** Calculate 24 ÷ 4, then multiply the answer by 10.

## Open-ended

14 ◆ × ● = 1620

**(a)** What could ◆ and ● be? Find at least seven different combinations of whole numbers.

**(b)** Find at least three combinations where both ◆ and ● are two-digit numbers.

**15** Here is part of Kim's maths homework. She had been learning about splitting up numbers to be multiplied, so she decided to use that strategy to complete the following.

Multiplication

(1) $9 \times 24$
$= 9 \times 2 + 9 \times 4$
$= 18 + 36$
$= 54$

(2) $12 \times 43$
$= 12 \times 4 + 12 \times 3$
$= 48 + 36$
$= 84$

**(a)** Kim is a bit worried that her answers don't look big enough. What are the correct answers?

**(b)** Explain to Kim the mistake she has made in both questions.

**(c)** Give Kim some advice so that she can avoid similar mistakes in the future.

# Outside the Square Puzzle

## Wizard maths

Henry and his friends Hilary and Rob are out shopping in Oblique Lane. Oblique Lane is a street of wizard shops in which goods are purchased using wizard money. Wizard money, however, is different to non-wizard money.

Wizard money consists of aureas, denaris and cuprums.

1 aurea is worth 17 denaris.
1 denari is worth 29 cuprums.

**1** Henry, Hilary and Rob each have 10 aureas to spend. Here are their shopping lists. Add up the total cost of each list, then work out the change they will get in aureas, denaris and cuprums. Use the above information to help you with your calculations.



**Henry**

| *Item* | *Price* |
|---|---|
| 1 cloak: | 2 aureas, 4 denaris |
| 1 large bag owl pellets: | 3 aureas, 15 cuprums |
| 2 bottles elderflower juice: | 14 denaris, 27 cuprums |

**Hilary**

| *Item* | *Price* |
|---|---|
| 10 scrolls of parchment: | 1 aurea, 14 denaris, |
| 1 box of quills: | 11 denaris, 23 cuprums |
| *Reading Ancient Runes* book: | 5 aureas, 13 denaris, 10 cuprums |

**Rob**

| *Item* | *Price* |
|---|---|
| *Wizard Monthly* magazine: | 9 denaris, 21 cuprums |
| New wand: | 6 aureas, 16 denaris |
| 1 box of tricky toffee: | 1 aurea, 9 denaris, 17 cuprums |

**2** Invent a wizard shopping list of your own and give it to a friend to add up. (However, sure you can work out the answer too make!)

# Half-time 1

Ex 1.1, 1.3

**1** Calculate the following by using an appropriate mental strategy.

**(a)** $4 \times 17 \times 5$ **(b)** $15 \times 23 \times 2$ **(c)** $33 + 45 + 107$

**(d)** $40 \times 19$ **(e)** $37 + 49 + 153$ **(f)** $127 + 65 + 69$

Ex 1.2

**2** **(a)** Write the value of the following.

**(i)** $6^3$ **(ii)** $5^5$ **(iii)** $10^1$ **(iv)** $7^2$

**(v)** $\sqrt{49}$ **(vi)** $\sqrt[3]{8}$ **(vii)** $\sqrt{100}$ **(viii)** $\sqrt[3]{64}$

**(b)** Which of your answers to **(a)** is:

**(i)** a perfect square **(ii)** a perfect cube?

Ex 1.1, 1.3

**3** Calculate the following. Use mental strategies if possible.

**(a)** $19 \times 9$ **(b)** $41 \times 8$ **(c)** $5 \times 38$ **(d)** $22 \times 17$

**(e)** $147 \times 3$ **(f)** $12 \times 53$ **(g)** $67 \times 71$ **(h)** $92 \times 84$

Ex 1.3

**4** Perform the following divisions. Use mental strategies if possible.

**(a)** $136 \div 4$ **(b)** $174 \div 6$ **(c)** $580 \div 5$ **(d)** $940 \div 20$

**(e)** $128 \div 8$ **(f)** $351 \div 9$ **(g)** $270 \div 15$ **(h)** $1344 \div 24$

Ex 1.2

**5** Write the following in index form.

**(a)** $3 \times 3 \times 3 \times 3 \times 3 \times 3$ **(b)** 6 to the power of 5 **(c)** base 4, index 7

**(d)** fifteen squared **(e)** twenty-one cubed **(f)** $41 \times 41 \times 41$

Ex 1.3

**6** Calculate the following.

**(a)** $70 \times 60$ **(b)** $150 \times 20$ **(c)** $400 \times 300$

**(d)** $250 \times 800$ **(e)** $3600 \div 40$ **(f)** $9000 \div 300$

**(g)** $7500 \div 1500$ **(h)** $124\,000 \div 200$ **(i)** $60\,000 \div 400$

Ex 1.1, 1.3

**7** 13 000 tickets were sold to a music concert at a cost of $52 each. Use a combination of strategies to calculate the amount made by the promoters of the concert from ticket sales.



# Estimating and rounding

1.4

Often, we do not need to know the exact answer to a problem. An **estimate**, or approximate answer, is accurate enough.

We serve about 40 customers every hour. They pay approximately $10 per meal. That's approximately $4800 into the till in a 12 hour day.



It's approximately 750 km from Melbourne to Adelaide. I estimate it will take me 10 hours to drive there, and cost about $450 in fuel.



## Estimating by rounding to the first digit

**Rounding** is an important skill for making good estimations.

Consider the following.

46 is *rounded up* to 50 (because it lies in between 40 and 50, and is closer to 50).

44 is *rounded down* to 40 (because it lies in between 40 and 50, and is closer to 40).

When rounding, we look at the digit to the right of the one being rounded. If it is 0, 1, 2, 3 or 4 we round down. If it is 5, 6, 7, 8 or 9 we round up.

This means that:

173 is rounded *up* to 200

137 is rounded *down* to 100

4500 is rounded *up* to 5000

4499 is rounded *down* to 4000

25 374 is rounded *up* to 30 000

24 985 is rounded *down* to 20 000

This type of rounding is called 'rounding to the first digit'. This means that the rounded number has only zeroes after the first digit.

Rounding to the first digit helps us estimate the answers to multiplication and division questions.

To round a number to the first digit, look at the second digit of the number. If it is 0,1, 2, 3 or 4, keep the first digit the same and replace the other digits with zeroes. If it is 5, 6, 7, 8 or 9, increase the first digit by 1, and replace the other digits with zeroes.

When rounding, we use the symbol ≈ which means 'approximately equal to'.

## Worked Example 7 WE7

Round the following numbers to the first digit.

**(a)** 361 **(b)** 2050 **(c)** 8

| Thinking | Working |
|---|---|
| **(a)** Look at the second digit. Because it is 5 or greater, increase the first digit by one and replace the following digits with zeroes. | (a) $361 \approx 400$ |
| **(b)** Look at the second digit. Because it is less than 5, keep the first digit the same and replace the following digits with zeroes. | (b) $2050 \approx 2000$ |
| **(c)** Because there is only one digit, this number is already rounded to the first digit. | (c) 8 |

## Estimating with multiplication and division

Multiplication and division can be estimated using rounding to the first digit. This will not give an exact answer to a problem, but an approximate one.

## Worked Example 8 WE8

Calculate an approximate answer to the following by first rounding each number to the first digit.

**(a)** $368 \times 52$ **(b)** $77\,483 \div 421$

| Thinking | Working |
|---|---|
| **(a) 1** Round both numbers to the first digit. | (a) $368 \times 52$<br>$\approx 400 \times 50$ |
| **2** Multiply the two rounded numbers together and write the approximate answer. | $\approx 4 \times 100 \times 5 \times 10$<br>$\approx 20 \times 1000$<br>$\approx 20\,000$ |
| **(b) 1** Round both numbers to the first digit. | (b) $77\,483 \div 421$<br>$\approx 80\,000 \div 400$ |
| **2** Divide the two rounded numbers by an appropriate multiple of 10 by 'cancelling zeroes'. (Here, we have divided both numbers by 100.) | $\approx 80\,000 \div 400$<br>$\approx 800 \div 4$ |
| **3** Do the simple division and write the approximate answer. | $\approx 200$ |

## Estimating using other ways of rounding

Instead of rounding off to the first digit, we could round off to a closer multiple of 5 or 10 that is convenient to work with. This would give a more accurate estimate of the actual answer.

## Worked Example 9 WE9

Calculate an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

**(a)** $4587 \div 49$ **(b)** $197 \times 23$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Round off the numbers that you are dividing to convenient numbers. (Here, one number is rounded to the nearest 100, the other to the nearest 10.) | *(a)* $4587 \div 49$<br>$\approx 4600 \div 50$ |
| **2** | Perform the division with the rounded numbers, cancelling zeroes first. | $\approx 460\cancel{0} \div 5\cancel{0}$<br>$\approx 460 \div 5$ |
| **3** | Change the non-zero digits of the larger number to the nearest multiple of the smaller number. | $\approx 450 \div 5$ |
| **4** | Do the simple division and write the approximate answer. | $\approx 90$ |
| **(b)** **1** | Round off the numbers that you are multiplying to convenient numbers. (Here, one number is rounded to the nearest 10, the other to the nearest 5.) | *(b)* $197 \times 23$<br>$\approx 200 \times 25$ |
| **2** | Do the simplified multiplication. | $\approx 2 \times 100 \times 25$<br>$\approx 2 \times 25 \times 100$<br>$\approx 50 \times 100$ |
| **3** | Write the approximate answer. | $\approx 5000$ |

## Overestimating and underestimating

You can use an estimate of the answer to a problem to check whether your calculated answer is 'reasonable'. It is useful to be able to judge whether your estimate is greater than (an 'overestimate') or less than (an 'underestimate') the final answer. To do this, we need to consider the way in which the numbers were rounded.

Consider these examples of multiplication by rounding to the first digit.

**(a)** $157 \times 66$
$\approx 200 \times 70$
$\approx 14\,000$

Actual answer: 10 362

**(b)** $218 \times 46$
$\approx 200 \times 50$
$\approx 10\,000$

Actual answer: 10 028

In **(a)**, the actual answer is a lot less than the estimate! This is because both numbers were rounded up by a large amount.

In **(b)**, the actual answer is 10 028—very close. This is because one number was rounded up, and the other down, by comparable amounts.

When dividing, we need to be aware of the effect of rounding the original numbers up or down.

Consider these examples of division by rounding to the first digit.

**(a)** $2320 \div 9$
$\approx 2000 \div 10$
$= 200$

Actual answer: 257.8

**(b)** $6432 \div 33$
$\approx 6000 \div 30$
$= 200$

Actual answer: 194.9

In **(a)**, the big difference between the approximate answer and the actual answer is because the rounded calculation resulted in a smaller number being divided by a larger one.

In **(b)**, the approximate answer is very close to the actual answer. This is because both numbers were rounded down—a smaller number divided by a smaller number.

To achieve a reasonable approximate answer:

- when multiplying, round one number up and the other number down
- when dividing, both numbers should be rounded up, or both rounded down.

# 1.4 Estimating and rounding

## Navigator

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## Fluency

WE7

**1** Round the following numbers to the first digit.

| | | | |
|---|---|---|---|
| **(a)** 68 | **(b)** 74 | **(c)** 10 | **(d)** 6 |
| **(e)** 619 | **(f)** 650 | **(g)** 483 | **(h)** 970 |
| **(i)** 4846 | **(j)** 3723 | **(k)** 9510 | **(l)** 9643 |
| **(m)** 75 000 | **(n)** 716 599 | **(o)** 800 050 | **(p)** 1 801 021 |

WE8

**2** Calculate an approximate answer to the following by first rounding each number to the first digit.

| | | |
|---|---|---|
| **(a)** $681 \times 41$ | **(b)** $547 \times 84$ | **(c)** $62 \times 819$ |
| **(d)** $2940 \div 51$ | **(e)** $3199 \div 62$ | **(f)** $1955 \div 78$ |
| **(g)** $141 \times 837$ | **(h)** $104 \times 8946$ | **(i)** $7340 \times 250$ |
| **(j)** $9 \times 6511$ | **(k)** $250 \times 950$ | **(l)** $75\,342 \div 80$ |
| **(m)** $11\,397 \div 213$ | **(n)** $27\,496 \div 487$ | **(o)** $2547 \times 30\,995$ |

WE9

**3** Calculate an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

| | | |
|---|---|---|
| **(a)** $41 \times 26$ | **(b)** $122 \times 54$ | **(c)** $191 \times 14$ |
| **(d)** $387 \div 43$ | **(e)** $379 \div 22$ | **(f)** $248 \div 52$ |
| **(g)** $1587 \times 23$ | **(h)** $33\,997 \times 21$ | **(i)** $4036 \times 253$ |
| **(j)** $3527 \div 52$ | **(k)** $10\,803 \div 95$ | **(l)** $44\,895 \div 14$ |
| **(m)** $23\,938 \div 356$ | **(n)** $15\,337 \div 2498$ | **(o)** $5089 \times 2492$ |

**4** In each case, which of these estimates do you think is the closest? Don't try to count or measure.




**(a)** About how many people are there in the photo?

**A** 700

**B** 4000

**C** 30 000

**D** 100 000

**(b)** Approximately how far is it:

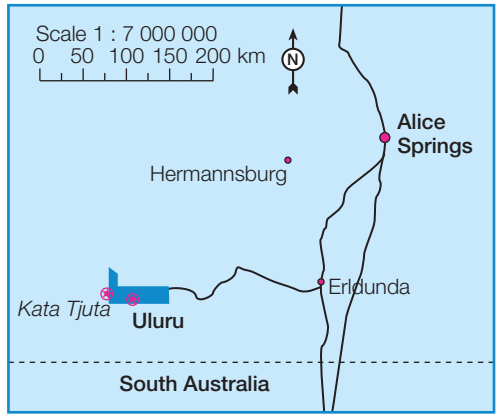
**(i)** from Alice Springs to the South Australian border, by road?

**A** 75 km

**B** 150 km

**C** 300 km

**D** 600 km




**(ii)** directly from Uluru to Alice Springs (i.e. in a straight line)?

**A** 40 km

**B** 350 km

**C** 789 km

**D** 1000 km

## Understanding

**5** Estimate the following by rounding the smaller number to the first digit and the larger number to that same place value (e.g. if the smaller number is rounded to the nearest 10, round the larger number to the nearest 10 also).

**(a)** 1247 + 323 **(b)** 10 290 + 178 **(c)** 14 532 + 2788

**(d)** 823 – 79 **(e)** 1176 – 241 **(f)** 453 – 276

**6** For each of the following, find an estimate for the answer by first rounding each number to the first digit. State whether the actual answer will be higher or lower than the estimate.

**(a)** 364 × 57 **(b)** 2398 × 426 **(c)** 153 × 16

**(d)** 1996 ÷ 24 **(e)** 943 ÷ 86 **(f)** 597 ÷ 136

**7** In each case, choose the best estimate from the alternatives given. Don't do the actual calculation.

**(a)** 321 × 73

**A** 210 **B** 2100 **C** 2163 **D** 21 000

**(b)** 405 × 950

**A** 360 000 **B** 400 000 **C** 450 000 **D** 3 600 000

**(c)** 7865 ÷ 24

**A** 400 **B** 900 **C** 1600 **D** 210 000

**(d)** 999 160 ÷ 527

**A** 220 **B** 330 **C** 660 **D** 2000

**8** Last weekend, 255 345 people attended the AFL matches. They paid, on average, $22 to attend their game.



**(a)** Round the attendance to the first digit.

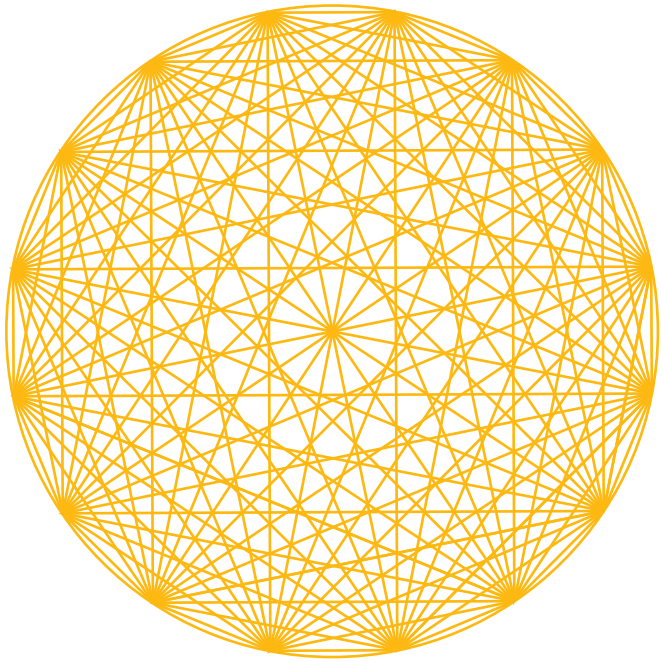
**(b)** Use your answer to **(a)**, and a rounded estimate for the ticket price, to find an estimate for the total entry cost.

**9** **(a)** Tahnee earns $38 295 per year. Approximately how much is this per week? Calculate by rounding to convenient numbers first. (There are 52 weeks in a year.)

**(b)** Catriona earns $175 per week from her part-time job. Approximately how much is this per year? Calculate by rounding to convenient numbers first.

## Reasoning

**10** **(a)** Approximately how many straight lines are there in this picture?



**A** 100

**B** 500

**C** 5000

**D** 10 000

**(b)** Explain a way of finding the exact number of lines without counting them all.

**11** 79 × 5003 and 76 × 5488 both give the same approximate answer of 400 000 when each number is rounded to the first digit. Which of the actual answers to the two calculations will be closer to 400 000? Explain how you arrived at your answer.

**12** **(a)** Use rounding to the first digit to find an approximate answer for 1344 ÷ 21.

**(b)** Find an approximate answer for 1344 ÷ 21 by first rounding to convenient numbers, such as multiples of 10.

**(c)** Which method do you think has given a closer estimate to the actual answer? Give a reason for your prediction.

**(d)** Work out the actual answer and see if you were correct.

**13** Ms Kelly, the assistant principal at Summerhill Secondary, is organising buses to transport students to the pool for the swimming carnival. Each bus can seat 52 people, and she has 428 staff and students going (although she doesn't know how many might be sick or away on the day). How many buses should Ms Kelly order? Why is estimating by rounding not helpful in this situation?

## Open-ended

**14** Natalie is out shopping. She is heading towards the checkout when she suddenly wonders if she has enough money. She mentally rounds her purchases off in her head and adds them up.

Here is Natalie's rounded off, mental list:

T-shirt ≈ $20
Earrings ≈ $10
Necklace ≈ $10
Lip gloss ≈ $10
Total ≈ $50



'Terrific!' thinks Natalie. 'I've got exactly enough!'

The actual prices of the items are: T-shirt $17; earrings $14; necklace $13; lip gloss $9.

**(a)** There is nothing wrong with Natalie's rounding. Why is she in trouble when she gets to the checkout?

**(b)** Describe to Natalie a 'safer' method for rounding money.

**15** Write down three pairs of numbers that when rounded to the first digit, then multiplied together, give:

**(a)** 3200 **(b)** 200 **(c)** 100 000 **(d)** 42 000

## Outside the Square Game

### Estimation frustration

**Equipment required:** 2 brains, 2 dice, 1 calculator

**How to win:**
The winner is the first person to 40 points.

**How to play:**

1. Roll the dice for your partner. Make a two-digit number from the numbers on the dice (e.g. rolling a 4 and a 3 means you can make 34 or 43—your choice). Write this number down.
2. Roll again, make another two-digit number, and write this one down also.
3. Show these two numbers to your partner. They now have just 10 seconds to write down an estimate of what the product of these two numbers will be.
4. Use a calculator to work out the answer, and compare it to your partner's estimate.
5. Score according to the table below.

| How close were they? | Points given |
|---|---|
| Spot on | 20 points |
| Less than 100 off | 10 points |
| Between 100 and 200 off | 5 points |
| More than 200 off | 0 points |

6. Now, get your partner to roll the dice for you.



# 1.5 Order of operations

Jamie, Josh and Ali all worked out the following problem: $1 + 2 \times 3 + 4$.

- Jamie's answer was 13.
- Josh's answer was 11.
- Ali's answer was 15.

Why were there different answers?

We can't have different people getting different answers for the same calculation. Mathematicians have agreed on some rules about the order in which to do the four operations of $\times$, $+$, $-$ and $\div$. Rules that mathematicians agree to work by are called **mathematical conventions**.



The **'order of operations'** rules:

1. Always do calculations in brackets first.
2. Next, do any calculations with indices (powers).
3. Do the multiplication and division next, in order from left to right, as you come to them.
4. Finally, do the addition and subtraction, in order from left to right, as you come to them.

According to the order of operations rules, who had the correct answer to the problem above: Jamie, Josh or Ali?

## Worked Example 10 WE10

Calculate the following, showing all steps of working.

**(a)** $24 + 6 \div 2 - 1 \times 4$ **(b)** $14 - 9 + 6^2 \div (7 + 2) \times 3$

| Thinking | Working |
|---|---|
| **(a) 1** There are no brackets and no indices, so do multiplications and divisions in the order in which they appear. | *(a)* $24 + 6 \div 2 - 1 \times 4$<br>$= 24 + 3 - 1 \times 4$<br>$= 24 + 3 - 4$ |
| **2** Now, do additions and subtractions in the order in which they appear. | $= 27 - 4$<br>$= 23$ |

**(b)** **1** Do the calculation in brackets first.

**2** Evaluate any numbers in index form.

**3** Do multiplications and divisions in the order in which they appear.

**4** Do additions and subtractions in the order in which they appear.

*(b)* $14 - 9 + 6^2 \div (7 + 2) \times 3$
$= 14 - 9 + 6^2 \div 9 \times 3$
$= 14 - 9 + 36 \div 9 \times 3$
$= 14 - 9 + 4 \times 3$
$= 14 - 9 + 12$
$= 5 + 12$
$= 17$

# 1.5 Order of operations

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2, Q3, Q4, Q5 Column 1, Q7, Q9, Q11, Q12, Q13 | Q1 Column 2, Q2, Q4, Q5 Column 2, Q6, Q7, Q8 Column 1, Q9, Q10, Q11, Q12, Q13 | Q1 Column 2, Q4, Q5 Column 2, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13 |

Answers page 624

## Fluency

**1** Calculate the following, showing all steps of working.

WE10

**(a)** $1 + 8 \times 3$
**(b)** $8 - 5 \div 5$
**(c)** $6 \times 2 - 1$
**(d)** $8 \div 4 - 2$
**(e)** $25 - 2 \times 11$
**(f)** $6 \div 3 + 3 \times 5$
**(g)** $8 - 24 \div 12 + 3$
**(h)** $8 \times 3 \div 4 \times 2$
**(i)** $9 \times (10 - 7) \div 3$
**(j)** $24 \div (7 + 5) \times 5^2$
**(k)** $88 \div 8 - 6 \times (5 - 4)$
**(l)** $12 \times 5 + 4 \times (10 - 4)$
**(m)** $38 - 7 \times 2^2 + 13 - 4 \div 2$
**(n)** $9 - 2 + 5 + 3 \times 4 \div 6$
**(o)** $23 - 5 + (17 - 2) \times 3 + 5^2$
**(p)** $28 \div 7 \times 3^2 + (5 - 1) \div 2 + 3$

**2** State TRUE or FALSE for the following.

**(a)** For $2 + 6 \times 4$ we would calculate $2 + 6$ first.
**(b)** For $6 + 12 \div 3$ we would calculate $12 \div 3$ first.
**(c)** For $8 + 40 \div (3 + 5) \times 10^2$ we would calculate $8 + 40$ first.
**(d)** For $24 + 6 \div 2 - 1 \times 4$ we would calculate $6 \div 2$ first.
**(e)** $4 + 12 \div 2$ simplifies to 8.
**(f)** $20 \div 5 - 1$ simplifies to 3.

**3** **(a)** $8 + 6 - 4 \div 2$ is equal to:

**A** 5 **B** 9 **C** 12 **D** 16

**(b)** $12 - 8 \div 4 + 2^2$ is equal to:

**A** 5 **B** 6 **C** 9 **D** 14

## Understanding

**4** **(a)** Which calculation would you do first and which would you do second in each of these questions?

**(i)** $14 - 2 \times 6 + 2 \times 2$
**(ii)** $(15 \div 5) + 6 \times 3$

**(b)** Calculate the value of the expressions in **(a)**.

**5** Put brackets into these statements, where necessary, to make them true.

**(a)** $6 + 6 \times 3 = 36$

**(b)** $10 - 4 \times 5 = 30$

**(c)** $9 - 8 \times 6 + 4 = 10$

**(d)** $12 + 6 \div 7 - 4 = 14$

**(e)** $6 \div 3 + 3 \times 5 = 5$

**(f)** $3 \times 6 \div 8 - 4 + 5 = 2$

**(g)** $3 \times 10 - 7 \div 9 + 12 = 13$

**(h)** $18 \div 3 \times 5 - 3 + 2 = 14$

**(i)** $7 + 3 \div 4 + 1 = 2$

**(j)** $5 - 3 \times 8 - 6 \div 2 = 2$

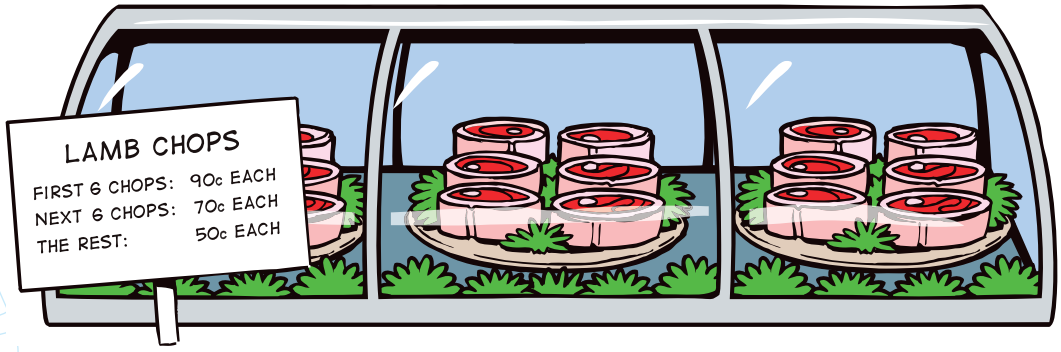
**6** During 'peak' time, Shania's mobile phone company charges 73c per minute, plus a 29c 'flagfall' (a single amount charged as soon as a call is connected). During 'off-peak' time, there is no flagfall, and a rate of 50c per minute is charged per call. Shania has $20 credit on her phone. She makes one 5-minute call during 'peak' time, and one 8-minute call during 'off-peak' time.




**(a)** Calculate the total cost of the two calls. Show your working clearly.

**(b)** Using the symbols +, –, ×, and at least one pair of brackets, write down a calculation that Shania could do to work out how much credit she had left on her phone after the two calls were made.

**7** A butcher had an oversupply of lamb chops. He decided to tempt people to buy more of them by putting up the following sign.




Stuart bought 8 chops, Mahalia bought 11 chops, and Sofia bought 17 chops.

The amount Stuart paid can be worked out using the following calculation.

$6 \times 90 + 2 \times 70$. This gives an answer of 680 cents, which can then be written as $6.80.

**(a)** Write down a calculation similar to the one above to work out how much Mahalia paid, then do the calculation.

**(b)** Write down a calculation similar to the one above to work out how much Sofia paid, then do the calculation.

**(c)** How much more than Stuart did Sofia pay?

**8** Replace each * with one of the four symbols (+, –, ×, ÷) to make the statement true.

**(a)** $2 + 21 * 3 = 9$

**(b)** $15 - 6 * 2 = 12$

**(c)** $14 - 8 * 6 = 0$

**(d)** $7 * 5 * 6 = 29$

**(e)** $14 * 3 * 2 = 15$

**(f)** $(24 * 6) * 10 = 3$

**(g)** $8 * 5 * 2 - 6 = 12$

**(h)** $12 * 2 + 1 * 9 = 15$

**9** Replace each * with either <, > or = to make the statement true.

**(a)** $6 \times (4 \div 2) \times 3 * (6 \times 4) \div 2 \times 3$

**(b)** $(1 + 4) \times 20 \div 5 * 1 + (4 \times 20) \div 5$

**(c)** $100 + 10 \div 10 * (100 + 10) \div 10$

**(d)** $36 \div 6 \times (3 - 3) * 36 \div 6 \times 3 - 3$

Remember < means 'is less than' and > means 'is greater than' when read from left to right.



## Reasoning

**10** Remove the unnecessary brackets from these statements.

**(a)** $(4 \times 3) + (6 - 2) = 16$

**(b)** $8 + (4 \div 2) + (6 \times 2) = 22$

**(c)** $4 + (20 \div 5) \times (3 + 7) = 44$

**(d)** $6 \times (3 + 2) - (12 \div 2) = 24$

**11** Sean typed the following into his calculator: $8 + 6 \div 2 + 3 \times 5$. Shazhad typed the same into the 'calculator' application of his mobile phone. Sean's answer was 26. Shazhad's answer was 50.

**(a)** Do the calculation to decide whose answer was correct.

**(b)** How did the other calculator come up with a different answer?

## Open-ended

**12** By placing one pair of brackets in different positions, show all the possible answers for this question: $8 + 4 \times 6 \div 2 - 1$.

**13** Jing has worked out that the answer to $30 \div 2 \times (2 + 3)$ is 3. Here is his working.

$30 \div 2 \times (2 + 3)$
$= 30 \div 2 \times 5$ Step 1
$= 30 \div 10$ Step 2
$= 3$ Step 3

Jing's teacher has marked it as incorrect. What is the correct answer? Explain to Jing which step of his working is incorrect, what he should have done instead, and why.

# Outside the Square

## Problem solving

### Four 4s = 100?

**1** Using the number 4 exactly four times, together with any of the four operations (+, −, ×, ÷) and brackets if you need them, see if you can make the numbers 0 to 9. A way to get 5 has been done for you. There is more than one way in many cases.

0 =
1 =
2 =
3 =
4 =
5 = $(4 \times 4 + 4) \div 4$
6 =
7 =
8 =
9 =

**2** See how many numbers up to 100 you can find with just four 4s. You might like to work with a partner or in a small group.

**3** Using the digits of the year 2013 exactly once, together with brackets and any of the four operations, see how many of the numbers from 1 to 10 you can make.

e.g. $(3 + 1) \times 2 + 0 = 8$

**Strategy options**

- Guess and check.
- Test all possible combinations.

# Mathspace

# The operation theatre



There are many weird and interesting facts about the human body. Calculate the following expressions using the correct order of operations to discover some of these facts.

**1** The number of muscles it takes to produce human speech.

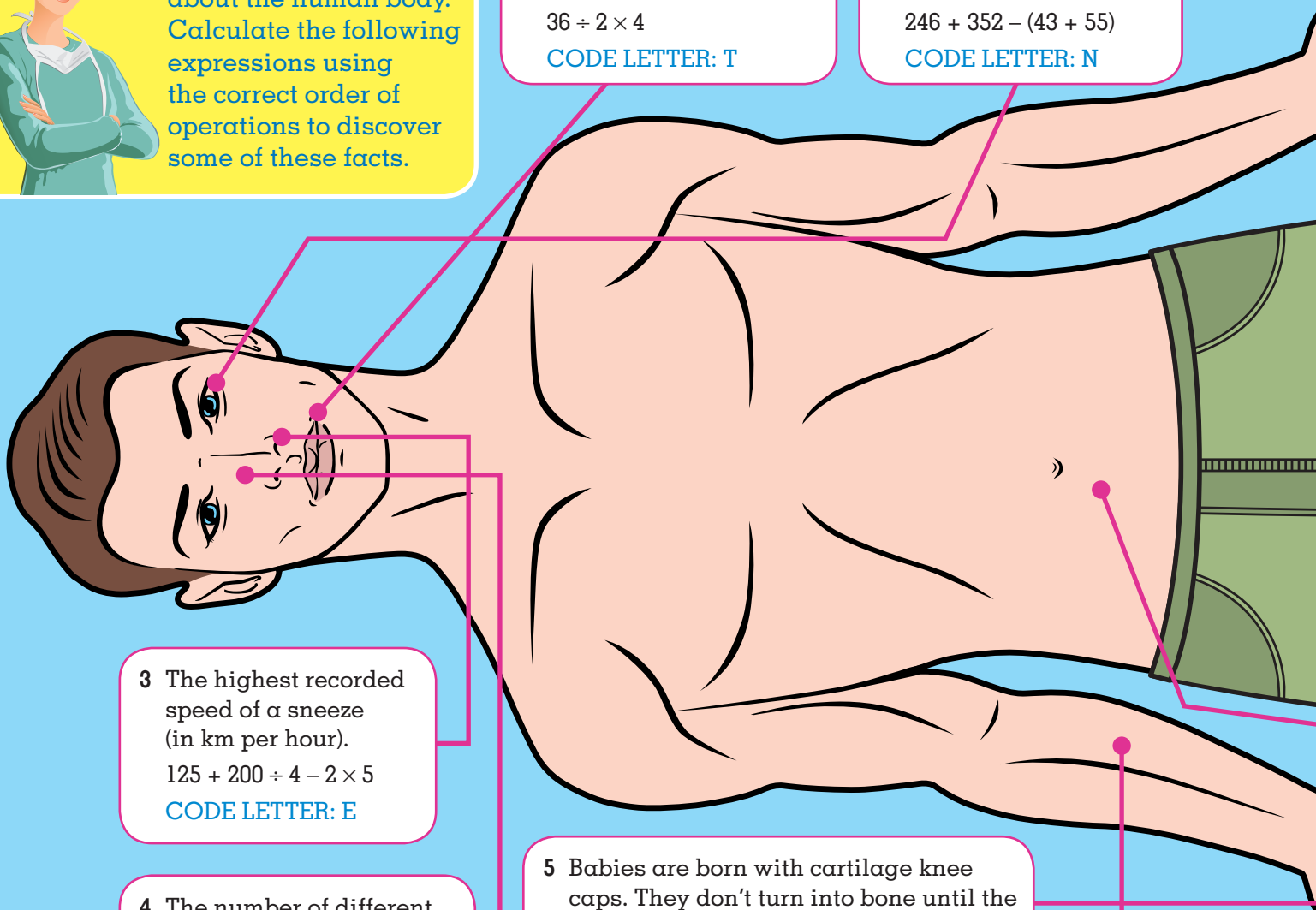
$36 \div 2 \times 4$

CODE LETTER: T

**2** The number of shades of grey a human eye can distinguish

$246 + 352 - (43 + 55)$

CODE LETTER: N



**3** The highest recorded speed of a sneeze (in km per hour).

$125 + 200 \div 4 - 2 \times 5$

CODE LETTER: E

**4** The number of different scents your nose can detect.

$(6000 + 1500 \div 5) - 43 \times 100$

CODE LETTER: W

**5** Babies are born with cartilage knee caps. They don't turn into bone until the child reaches between which two ages?

**(a)** $3 \times 4 \div 12 \times 2$ CODE LETTER: Y

**(b)** $30 - 6 \times 8 \div 12 \times 6$ CODE LETTER: B

**6** **(a)** The number of bones you are born with.

$200 - 50 + (30 - 1) \times 5 + 5$ CODE LETTER: H

**(b)** The number of bones an adult has.

$618 \div (56 - 47) \times 3$ CODE LETTER: G

**7** The average human will shed approximately how many billion flakes of skin a day?

$30 - 7 \times 3 + 10 - 36 \div 4$

CODE LETTER: I

**8** The number of million blood cells destroyed in the human body every second.

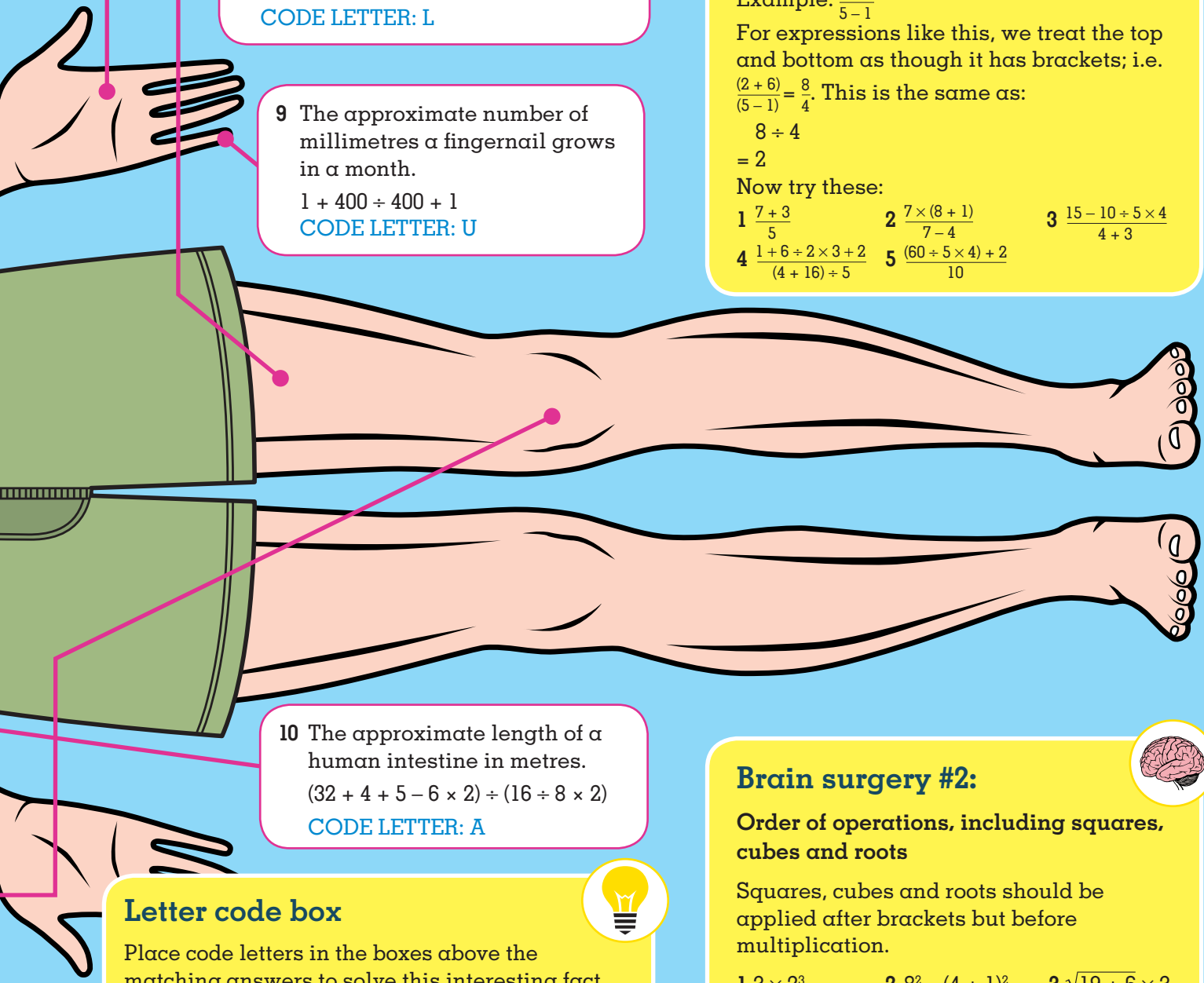
$90 \div (7 + 11) \times 3$

CODE LETTER: L

**9** The approximate number of millimetres a fingernail grows in a month.

$1 + 400 \div 400 + 1$

CODE LETTER: U



**10** The approximate length of a human intestine in metres.

$(32 + 4 + 5 - 6 \times 2) \div (16 \div 8 \times 2)$

CODE LETTER: A

## Letter code box

Place code letters in the boxes above the matching answers to solve this interesting fact. Your brain generates approximately

| | | | | | | |
|---|---|---|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | watts |
| 72 | 2000 | 165 | 500 | 72 | 2 | |

of power while you're awake—enough to illuminate

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| 7 | 15 | 10 | 206 | 300 | 72 | 6 | 3 | 15 | 6 |

## WARNING: Advanced order of operations

Brain surgeons need to be very smart, see if you can calculate these trickier expressions.

## Brain surgery #1:

**Order of operations using a fraction bar**

Example: $\frac{2+6}{5-1}$

For expressions like this, we treat the top and bottom as though it has brackets; i.e. $\frac{(2+6)}{(5-1)} = \frac{8}{4}$. This is the same as:

$8 \div 4$

$= 2$

Now try these:

**1** $\frac{7+3}{5}$ **2** $\frac{7 \times (8+1)}{7-4}$ **3** $\frac{15 - 10 \div 5 \times 4}{4+3}$

**4** $\frac{1 + 6 \div 2 \times 3 + 2}{(4+16) \div 5}$ **5** $\frac{(60 \div 5 \times 4) + 2}{10}$

## Brain surgery #2:

**Order of operations, including squares, cubes and roots**

Squares, cubes and roots should be applied after brackets but before multiplication.

**1** $3 \times 2^3$ **2** $8^2 - (4+1)^2$ **3** $\sqrt{19+6} \times 3$

**4** $\frac{4 + 3^2 \div 3}{\sqrt{16} + 3}$ **5** $\left(\frac{3 \times (8 \div 2 \times 4)}{\sqrt{64}}\right)^2$

# 1.6 Mixed whole number problems

Maths problems in daily life do not come as 'ready made' calculations all set out for you to do. They come in situations that you need to first understand in order to decide which skills to use.

Some words that are associated with the four operations are listed below. You might be able to think of some others. Identifying these in a problem will help you to apply the right operation.

| + Sum | − Difference | × Product | ÷ Quotient |
|---|---|---|---|
| add | subtract | multiply | divide |
| total | less than | how many times more | goes into |
| altogether | minus | lots of | shared between |

This section contains problems that will require you to read the information carefully and decide which skills to use. Many problems require more than one step or calculation. You should use the skills and strategies covered in this chapter to help you.

## 1.6 Mixed whole number problems

### Navigator

Answers page 625

| | | |
|---|---|---|
| Q1, Q2, Q3, Q6, Q7, Q8, Q9, Q11, Q12, Q15 | Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q15, Q16 | Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q15, Q16 |

### Fluency

**1** **(a)** The Australian cricket team made 425 in their first innings and 299 in their second. How many runs did they make in total for the match?

**(b)** A new car cost $28 500. Five years later it was worth $19 275. By how much had the value of the car decreased in that time?

**(c)** A piano teacher charges $27 for each 1-hour lesson. How much would it cost for 13 lessons?

**(d)** Sula has a job that pays her $12 per hour. How many hours did Sula work in a week for which her pay was $372?



2 Harvey 'Scoop' Roberts, a journalist with the *Monthly Farm News*, can type 60 words a minute. How long does it take him to type an article of 1800 words?



3 Wendy is training to be an Olympic swimmer. Every morning she swims 3600 m in a 50 m pool. How many laps is that?

## Understanding

4 Little Lucy is exactly 5 weeks old. How many minutes old is she?

5 The highest mountain in the world, measured from sea level, is the Himalayan peak of Mount Everest. It is 8848 m above sea level. If we measure mountains that start under the ocean, the tallest mountain in the world from base to tip is Mauna Kea on the island of Hawaii. It measures 10 203 m, of which 4205 m is above sea level.



**(a)** How much of Mauna Kea is below sea level?

**(b)** If we don't count the part of Mauna Kea that is under water, how much higher is Mount Everest?

6 Cathy is out shopping for bargains. She bought two pairs of $68 shoes in a store that said 'buy one pair, get the second pair half price'. She bought two $19 T-shirts where the special deal was 'buy one, get $10 off the second'. Cathy also bought a pair of earrings for $25. What was the total cost of her shopping spree?

7 Mick is a baker in a supermarket. He makes hot cross buns and packs them in bags of 6. He stacks 4 shelves with 8 bags on each shelf. At the end of the day, Mike has 3 bags of buns left. How many hot cross buns did he sell?

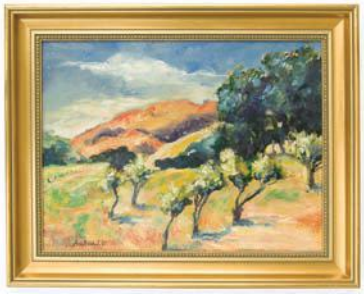
8 The two longest rivers in the world are the Amazon (6448 km) and the Nile (6670 km). The longest river in Australia is the Darling (2739 km).

**(a)** How much longer is the Nile than the Amazon?

**(b)** How much longer is the Nile than the Darling?

**(c)** How much longer is the Amazon than the Darling?

9 The width of a painting including the frame is 85 cm. If the frame is 6 cm wide all the way around, what is the width of the unframed painting?



10 The Pizza Pit-Stop employs five people. The two cooks work 36 hours each per week for $18 an hour. The three waiters work 30 hours each per week for $15 an hour. What does the Pizza Pit-Stop pay its five employees in total per week?

## Reasoning

11 Sam has 28 model planes on a display shelf. Mark has double Sam's number of model planes. Harvey has half as many planes as Sam. What is the difference between the number of model planes that Harvey and Mark have?

**12** Hiram is moving house. He has 13 boxes of possessions, but can only fit 4 at a time in his car.

**(a)** Beginning the day at his old house and ending it at his new house, how many times will Hiram need to travel between the two houses in order to shift all his boxes?

**(b)** If the distance between the two houses is 7 km, how far will he travel?

**13** Jules Verne wrote about travelling around the world in 80 days.

**(a)** Write 80 days as a number of weeks plus a number of days.

**(b)** How many months, weeks and days is it, if we assume there are 30 days in a month?

**(c)** How many months, weeks and days is it, if we assume that there are 4 weeks in a month?

**(d)** Explain why your answers to **(b)** and **(c)** are different.

**14** Weighing Jason's pet dog Scruffy at the vet's was a problem, as Scruffy kept jumping off the scales. The vet held Scruffy and stood on the scales. Then, Jason held Scruffy and stood on the scales. Finally, the vet, Jason and Scruffy all stood on the scales.

The scales read as follows:
The vet and Scruffy = 91 kg
Jason and Scruffy = 56 kg
The vet, Jason and Scruffy = 138 kg

Work out how much Scruffy weighs.



## Open-ended

**15** **(a)** Write two numbers that have a sum greater than 90 but a product less than 1800.

**(b)** Write two numbers that have a difference of less than 10 but have a sum greater than one-quarter of their product.

**16** Simon owns a company that produces fruit juice. The juice comes in three different-sized bottles—300 mL, 600 mL and 1000 mL (1 L), weighing 300 g, 600 g and 1000 g (1 kg) respectively. Simon packs same-size bottles into boxes of 20, and delivers them by truck to shops. Simon's truck can hold a maximum load of 1000 kg.

**(a)** For each of the different-sized bottles, write down the mass in kg of a box of 20 bottles. (Remember, 1000 g = 1 kg.)

**(b)** How many boxes of each type of juice can Simon get on the truck to make a load of 1000 kg?

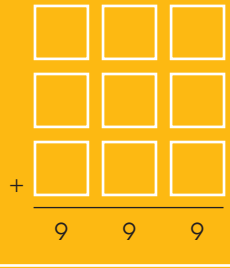
**(c)** Find at least two different combinations of boxes that would make a load of 1000 kg.

# Outside the Square

## Problem solving

### Nine hundred and ninety-nine

Copy the following boxes into your work book.




Using each of the digits from 1 to 9 once in each of the boxes, create three three-digit numbers whose sum is 999.

(It may help to write the digits 1 to 9 on small pieces of paper, so you can move them around the boxes easily.)

How many different combinations can you find?

Aim to find at least four combinations of different three-digit numbers.

**Strategy options**

- Test all possible combinations.
- Break problem into manageable parts.

# Challenge 1

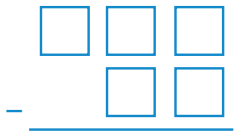
**1** The digits 5, 6, 7, 8 and 9 can be arranged to form five-digit even numbers. The tens digit in the smallest of these numbers is:

**A** 6 **B** 7 **C** 8 **D** 9

**2** How many different numbers can you make by multiplying two one-digit even numbers?

**A** 6 **B** 8 **C** 9 **D** 10

**3** Each of the digits 3, 5, 6, 7 and 8 is placed one to a box in the diagram opposite. If the two-digit number is subtracted from the three-digit number, the smallest difference is:



**A** 261 **B** 269 **C** 271 **D** 278

**4** Kim and Brian are given savings accounts by their grandmother, with the same amount of money in each. Kim is a good saver and puts an extra $5 into her account each week. Brian takes $1 out of his account every week. After 10 weeks, Kim has 3 times as much in her account as Brian has in his account. What was the starting amount in each account?

**5** The product of the ages of two teenagers and their father is 15 181. How old is the father?

**6** Look at the following number arrangement.

1

3 5

7 9 11

13 15 … …

What will be the sum of the seventh row? Look for a pattern in your row totals. What will be the sum of the one-hundredth row?

**7** How many square numbers are there between 8 and 8008?

**8** Find the next three terms in this number pattern 4, 7, 11, 18, 29, ...

**9** The counting numbers are arranged in six columns as shown. In which column will 1001 appear?

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 12 | 11 | 10 | 9 | 8 | 7 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| … | … | … | … | 20 | 19 |

**10** **(a)** Write down the values of $0^2$, $1^2$, $2^2$, $3^2$, … , $9^2$.

**(b)** Consider the last digit of each of the values found in **(a)**. What are the only digits that a square number can end in?

**(c)** What digits can't a perfect square end in?

**(d)** Which of the following numbers can't be perfect squares?

**(i)** 327 **(ii)** 324 **(iii)** 343 **(iv)** 289 **(v)** 732

**(e)** Without using a calculator, write 961 as a perfect square by first deciding what the square root could end in and then thinking about the factors of 900.

# Chapter review 1

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| associative law | divisor | mathematical conventions | quotient |
| base | estimate | order of operations | remainder |
| commutative law | expanded form | perfect cubes | rounding |
| cube root | index | perfect squares | square root |
| distributive law | index form | power | |
| dividend | indices | product | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 For the number $7^3$, 7 is the _____ and 3 is the _____, or _____.

2 1, 9 and 25 are all examples of _____. 1, 8 and 27 are all examples of _____.

3 The _____ of 8 is 2.

4 In the statement $97 \div 3 = 32$ rem 1, 97 is the _____, 3 is the _____, 32 is the _____, and 1 is the _____.

5 One strategy for making multiplication easier is to use the _____; for example, $6 \times 17 = 6 \times (10 + 7)$.

To find the value of a statement such as $(9 + 5) \times 3 - 2$, we would use the _____.

6 The 'long way' of writing a number in index form is in _____.

7 The _____ of 16 is 4, because $4 \times 4 = 16$.

8 $7 \times 5 = 5 \times 7$ is an example of the _____.

9 When two or more numbers are multiplied together, the answer is called the _____.

10 We can obtain an _____ of the answer to a calculation by _____ to the first digit.

## Fluency

1 Use an appropriate mental strategy to help calculate each of the following. Ex 1.1

(a) $89 + 53$ (b) $71 + 67$ (c) $93 + 125 + 7$ (d) $5 \times 12 \times 2$

(e) $21 \times 15$ (f) $19 \times 23$ (g) $4 \times 9 \times 5$ (h) $32 \times 17$

2 Find the value of the following. Ex 1.2

(a) $12^2$ (b) $5^3$ (c) $10^5$

(d) $\sqrt{64}$ (e) $\sqrt{900}$ (f) $\sqrt[3]{27}$

(g) $5^3 + 2^2$ (h) $8^4 - 3^5$ (i) $2^3 \times 3^2$

3 Write each of the following numbers in index form. Ex 1.2

(a) $7 \times 7 \times 7 \times 7 \times 7$ (b) ten cubed

(c) five squared (d) twelve to the power of eight

**4** Calculate the following using any suitable strategy. Ex 1.3

**(a)** $35 \times 40$ **(b)** $850 \times 200$ **(c)** $757 \times 7$ **(d)** $98 \times 31$

**(e)** $4400 \div 200$ **(f)** $27 \times 59$ **(g)** $612 \div 4$ **(h)** $756 \div 7$

**5** Use rounding to the first digit to find approximate answers to the following. Ex 1.4

**(a)** $35 \times 241$ **(b)** $1763 \times 27$ **(c)** $3299 \div 64$ **(d)** $57\,008 \div 192$

**(e)** $159 \times 286$ **(f)** $3427 \times 963$ **(g)** $4398 \div 487$ **(h)** $86\,005 \div 334$

**6** Find approximate answers to the following by first rounding each number to a convenient multiple of 10 or 100. Ex 1.4

**(a)** $554 \div 7$ **(b)** $2136 \div 19$ **(c)** $15\,624 \div 216$ **(d)** $28 \times 604$

**(e)** $157 \times 419$ **(f)** $523 \times 1998$ **(g)** $54\,327 \div 915$ **(h)** $12\,794 \div 183$

**7** Calculate: Ex 1.5

**(a)** $9 \div (2 + 1) - 2$ **(b)** $(3 \times 8) \div 4 + 7$ **(c)** $12 - 6 \times 2 + 11$

**(d)** $7 + 12 \div 4 - 1 \times 2^2$ **(e)** $(13 - 5 \times 2) + (20 \div 10)$ **(f)** $[5 \times (9 + 1)] - 3^2$

**8** Misha pays $112 for two T-shirts and a pair of jeans. If the T-shirts were $23 each, how much did the jeans cost? Ex 1.6

## Understanding

**9** Cartons of milk are packed into boxes that are then stacked into larger crates for transport to supermarkets. If there are 20 cartons in a box, and 9 boxes in a crate, use mental strategies to calculate how many cartons of milk are delivered to a supermarket that ordered 4 crates.

**10** Use rounding to the first digit to estimate the answers to the following, then state whether the actual answer will be higher, lower, or close to your estimate.

**(a)** $3741 \times 22$ **(b)** $265 \times 341$ **(c)** $25\,736 \div 49$ **(d)** $96\,001 \div 17$

**11** Sasha runs a small free-range egg farm. On Tuesday, she collected 283 eggs from her hens.

**(a)** How many cartons will she be able to fill if each carton can hold 12 eggs?

**(b)** How many eggs will she have left over?

**12** In the calculation of $2 \times [30 \div (4 - 1)] + 6$ the first operation to do is:

**A** $+$ **B** $-$ **C** $\times$ **D** $\div$

**13** Put brackets into these statements, where necessary, to make them true.

**(a)** $4 \times 2 + 3 \div 5 - 1 = 3$ **(b)** $5 + 1 \div 6 + 4 + 2 = 7$

**14** Melissa's salary is $57 935 per year.

**(a)** Approximately how much is this per week? Calculate by rounding to the first digit first.

**(b)** Consider how you rounded the numbers in **(a)**. Will Melissa's actual weekly salary be more or less than the approximate answer you calculated?

**15** Which two consecutive numbers do the following lie between?

(Hint: Consider the perfect squares on either side of the number being considered.)

**(a)** $\sqrt{12}$ **(b)** $\sqrt{30}$ **(c)** $\sqrt{88}$ **(d)** $\sqrt{150}$

**16** The local restaurant pays its casual employees $15 per hour. The following table shows the number of hours worked by some of the casual staff. Work out the pay for each of these workers and the total of the wages bill.

| Employee | Number of hours worked |
|---|---|
| Jessica | 12 |
| Amy | 11 |
| Aaron | 9 |
| Nikkita | 6 |
| Josef | 14 |



**17** Replace each * with one of the four operations (+, –, ×, ÷) to make the statement true.

**(a)** $9 * 7 * 3 = 30$ **(b)** $16 * 4 \times 2 * 2 = 12$

**18** As Jared drove into a carpark, he noticed that the cost of parking was $6 for the first hour, then $3 for every hour after that. Jared's car was parked for 6 hours.

**(a)** Write down a calculation that Jared could use to work out how much his parking has cost him.

**(b)** As Jared drives out of the carpark, he gives the attendant $40. Use your answer to part **(a)** to calculate how much change he will receive.

**19** Write these numbers in expanded form and then work out the answer.

**(a)** $5^3 + 2^2$ **(b)** $8^4 - 3^5$ **(c)** $(3^2 - 2^3) \times 6^2$

**20** **(a)** Complete this sequence of the powers of 2:

$2^1 = 2$ $2^4 =$ _____

$2^2 = 4$ $2^5 =$ _____

$2^3 = 8$ $2^6 =$ _____

**(b)** Will $2^{10}$ be larger or smaller than 1000? Write the difference.

**21** Put these numbers in ascending order: $2^4$, $\sqrt{121}$, $10^2$, $3^3$, 4, $\sqrt{81}$

## Reasoning

**22** Michelle has $17 in her purse. Michelle's dad empties out the 'loose change' jar and divides the money equally between his 3 children. Michelle now has $24 in her purse. How much loose change was in the jar?

**23** The train to Suntown has broken down and 224 passengers are stranded. Carly, the train controller, needs to order some buses for them. Each bus can seat 42 people. How many buses should Carly order? Explain why rounding down to the first digit is not useful in this situation.

**24** How many whole numbers have the value of their square root in-between 4 and 5?

**25** Hamish has a collection of 12 alien figurines. His friend Ben has half this number. Hamish's brother Sean has three times the number that Ben has. The difference between the number of aliens in Sean's collection and the number in Hamish's is:

**A** 6 **B** 12 **C** 18 **D** 24

**26** Explain why $3^2$ is not equal to $3 \times 2$.

**27** Will the actual answer to $57 \times 196$ be greater or less than an approximate answer calculated by rounding each number to the first digit? Explain how you decided.

# NAPLAN practice 1

## Numeracy: Non-calculator



**1** Another way of writing $7^3$ is:

**A** $7 + 3$ **B** $7 \times 3$

**C** $7 \times 7 \times 7$ **D** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

**2** What is the best way to estimate the total cost of these items: skateboard, $87; knee pads, $31; helmet $54?

**A** $80 + $30 + $50

**B** $80 + $30 + $60

**C** $90 + $30 + $50

**D** $90 + $40 + $60

**3** The correct answer to $18 + 12 \div (5 + 1)$ is:

**A** 5 **B** 6 **C** 7 **D** 20

**4** The answer to $24 \times 379$ would be closest to:

**A** 1000 **B** 2000 **C** 5000 **D** 10 000

**5** Which one of these has the same value as $13 \times 5$?

**A** $10 + 3 + 5$ **B** $10 \times 3 + 5$ **C** $10 \times 5 + 3$ **D** $10 \times 5 + 15$

## Numeracy: Calculator allowed

**6** 'Mini-Munches' chocolate bars can be bought individually for 85c each or in packs of 6 for $5. You need to buy 34 bars for a party. What is the least amount you can pay?

**7** Tom is stacking a drinks fridge in the supermarket with cans of cola. The fridge has 3 shelves. On each shelf Tom can place 5 rows of 12 cans. How many cans will fit in the fridge?

**8** The area of New Zealand is 268 676 square kilometres. What is this area rounded to the nearest thousand square kilometres?

**A** 268 700 **B** 268 000 **C** 269 000 **D** 270 000

**9** The government of Victoria asked everyone in the State to reduce their water usage to less than 155 L per person per day. Which of the following families has not achieved this target?

| Family name | Number of people in the family | Total water used by the family in 1 day |
|---|---|---|
| Bennett | 5 | 760 L |
| Shwarma | 4 | 632 L |
| Nicolai | 6 | 918 L |
| Zhang | 3 | 459 L |

# 2



# Integers

2

## Texting aliens.

Mathematics is said to be the language that we could use to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same. It is believed that the best way to send a message would be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system. In 1974, the Arecibo telescope in Puerto Rico (pictured here) broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; this is not surprising given the distance it will have to travel. Later in this chapter you can learn about another way prime numbers are used to send information.

### Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10; however, sometimes we count by 2, 7, 60, 360 and 365. What do we count using these numbers?

### Why learn this?

Understanding relationships between numbers allows us to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for our study of many other areas of mathematics. Negative numbers are an important set of numbers that we will also consider in this chapter. Temperatures, elevations, goal differences and money owed are a few examples of the uses of negative numbers.

**After completing this chapter you will be able to:**

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative integers to represent quantities
- compare and order integers
- add and subtract integers.

# Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R2.1

**1** Copy and complete these within 3 minutes.

**(a)** $6 \times 7 =$ $6 \times 6 =$ $6 \times 4 =$ $6 \times 11 =$ $6 \times 8 =$

**(b)** $7 \times 11 =$ $7 \times 7 =$ $7 \times 5 =$ $7 \times 2 =$ $7 \times 3 =$

**(c)** $8 \times 7 =$ $8 \times 6 =$ $8 \times 4 =$ $8 \times 10 =$ $8 \times 8 =$

**(d)** $9 \times 12 =$ $9 \times 3 =$ $9 \times 5 =$ $9 \times 11 =$ $9 \times 8 =$

**(e)** $12 \times 7 =$ $12 \times 6 =$ $12 \times 12 =$ $12 \times 9 =$ $12 \times 11 =$

Worksheet R2.2

**2** **(a)** List all the digits with which an even number can end.

**(b)** List all the digits with which an odd number can end.

Worksheet R2.3

**3** Copy and complete each of the following by writing a < (less than) or > (greater than) sign between the given values.

**(a)** 10 _____ 7 **(b)** 3 _____ 6 **(c)** 2 _____ 0 **(d)** 0 _____ 5

Worksheet R2.4

**4** Calculate:

**(a)** $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

**(d)** $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

Worksheet R2.5

**5** Write the following temperatures in order from coldest to warmest.

**(a)** 15°C, 7°C, 0°C, -4°C, 21°C, -11°C

**(b)** 5°C, -3°C, 10°C, -25°C, 32°C, -14°C

Worksheet R2.6

**6** Write the following in expanded form, then evaluate.

**(a)** $7^2$ **(b)** $3^4$ **(c)** $2^6$ **(d)** $1^9$

Worksheet R2.7

**7** Calculate the following.

**(a)** $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

## Key Words

| | | |
|---|---|---|
| common factor | factor | positive |
| common multiple | Highest Common Factor (HCF) | prime factor |
| composite number | integers | prime number |
| coprime | loss | profit |
| deposit | Lowest Common Multiple (LCM) | withdrawal |
| divisibility | multiple | |
| divisible | negative | |

# Multiples, factors and divisibility

2.1

## Multiples and factors

The numbers 1, 2, 3, 4, 5, ... are called the whole numbers, or the counting numbers. (Any time we use '...' in mathematics, we are saying the pattern is infinite, or goes on forever.)

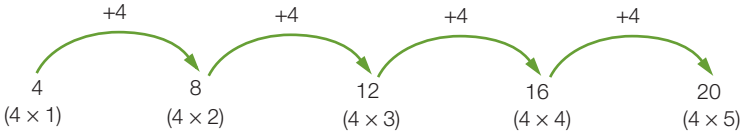
We find the **multiples** of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

| | $1 \times 7$ | $2 \times 7$ | $3 \times 7$ | $4 \times 7$ | $5 \times 7$ | ... |
|---|---|---|---|---|---|---|
| Multiples of 7 | 7 | 14 | 21 | 28 | 35 | ... |

Another way to create a list of multiples of a number is to start at the number and add it repeatedly.

For example, the multiples of 4 are:




The first in the sequence of multiples of a number is always the number itself. We can see from the above table and sequence that the first multiple of 7 is 7 ($1 \times 7$), and the first multiple of 4 is 4 ($1 \times 4$).

A **factor** is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, we can see some factors:

| | 7 | 14 | 21 | 28 | 35 | ... |
|---|---|---|---|---|---|---|
| Some factors | 1, 7 | 2, 7 | 3, 7 | 4, 7 | 5,7 | ... |

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a number has. We can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$
and $28 = 2 \times 14$
and $28 = 1 \times 28$

So, 28 has a total of six factors: 1, 2, 4, 7, 14 and 28.

## Worked Example 1 WE1

Find all the factors of each of the following numbers.

**(a)** 12 **(b)** 110

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc. | (a) $1 \times 12 = 12$<br>$2 \times 6 = 12$<br>$3 \times 4 = 12$ |
| | **2** | List the factors from smallest to largest. | Factors of 12: 1, 2, 3, 4, 6, 12. |
| **(b)** | **1** | Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc. | (b) $1 \times 110 = 110$<br>$2 \times 55 = 110$<br>$5 \times 22 = 110$<br>$10 \times 11 = 110$ |
| | **2** | List the factors from smallest to largest. | Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110. |

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. We include 7 only once in the list of factors for 49. If we reach such a pair, this also tells us we have finished finding the pairs of numbers.

## Divisibility

Another way of considering factors and multiples is to talk about **divisibility**. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all refer to the same idea.

Two factors of 35 are 5 and 7.
35 is divisible by 5 and 7.
Both 5 and 7 go into 35 exactly, without any remainder.
5 multiplied by 7 gives 35.
35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help us determine which numbers are divisible by others. For larger numbers, we can use some tests that enable us to determine whether one number is divisible by another. These tests are summarised in the following table.

| A number is divisible by … | If it passes this divisibility test |
|---|---|
| 2 | The last digit is an even number (0, 2, 4, 6 or 8). |
| 3 | The sum of the digits is divisible by 3. |
| 4 | The number formed by the last two digits is divisible by 4. |
| 5 | The last digit is 0 or 5. |
| 6 | The number is even (divisible by 2) and also divisible by 3. |
| 8 | The number formed by the last 3 digits is divisible by 8. |
| 9 | The sum of the digits is divisible by 9. |
| 10 | The last digit is 0. |

## Worked Example 2

WE2

Determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

**(a)** 3 **(b)** 4 **(c)** 5 **(d)** 6

| Thinking | | Working | |
|---|---|---|---|
| **(a) 1** | Add up the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3. | (a) 75: 7 + 5 = 12<br>98: 9 + 8 = 17<br>110: 1 + 1 + 0 = 2<br>132: 1 + 3 + 2 = 6 | ✓<br>✗<br>✗<br>✓ |
| **2** | State the answer for each number considered. | 75 and 132 are divisible by 3.<br>98 and 110 are not divisible by 3. | |
| **(b) 1** | Look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4. | (b) 75<br>98<br>110<br>132 | ✗<br>✗<br>✗<br>✓ |
| **2** | State the answer for each number considered. | 132 is divisible by 4.<br>75, 98 and 110 are not divisible by 4. | |
| **(c) 1** | Is the last digit 5 or 0? | (c) 75<br>98<br>110<br>132 | ✓<br>✗<br>✓<br>✗ |
| **2** | State the answer for each number considered. | 75 and 110 are divisible by 5.<br>98 and 132 are not divisible by 5. | |
| **(d) 1** | Write down the even numbers (these are divisible by 2). Add up the digits in each of these numbers and see whether the number is divisible by 3. | (d) Using the working from (a):<br>98: 17<br>110: 2<br>132: 6 | <br>✗<br>✗<br>✓ |
| **2** | State the answer for each number considered. | 132 is divisible by 6.<br>75, 98 and 110 are not divisible by 6. | |

Multiples of a whole number are found by multiplying it by another whole number.

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

## Common multiples

A **common multiple** of two numbers is a number that both of them divide into exactly. Changing the multiple table from the start of the section slightly, we get:

| | 1 and 7 | 2 and 7 | 3 and 7 | 4 and 7 | 5 and 7 | ... |
|---|---|---|---|---|---|---|
| A common multiple | 7 | 14 | 21 | 28 | 35 | ... |

This table only gives one common multiple for each pair of numbers. There is an infinite number of others. The **Lowest Common Multiple (LCM)** of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56,… The LCM of 2 and 7 is 14. There is no highest common multiple.

## Worked Example 3 WE3

Find the lowest common multiple (LCM) of the following set of numbers, by first listing the multiples of each: 4 and 6.

| Thinking | Working |
|---|---|
| **1** List the first few multiples of the first number. | 4: 4, 8, 12, 16, 20, 24, ... |
| **2** List the first few multiples of the second number. | 6: 6, 12, 18, 24, 30, 36, ... |
| **3** Circle the first number that appears in both lists. This is the LCM. | LCM of 4 and 6 is 12. |

## Common factors

A **common factor** of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

| | 7 and 14 | 4 and 20 | 9 and 15 | 8 and 40 | 12 and 18 |
|---|---|---|---|---|---|
| Common factors | 1, 7 | 1, 2, 4 | 1, 3 | 1, 2, 4, 8 | 1, 2, 3, 6 |

1 will always be a common factor of any set of numbers. Sometimes it's important for us to find the **Highest Common Factor (HCF)** of two numbers. From the above table, we can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, the smaller number is the HCF. For example, the HCF of 4 and 20 is 4 and the HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

## Worked Example 4 WE4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

| Thinking | Working |
|---|---|
| 1 List all factors of the first number. | 12: (1), (2), (3), 4, (6), 12 |
| List all factors of the second number. | 18: (1), (2), (3), (6), 9, 18 |
| 2 Circle the factors appearing in both lists. These are the common factors. | |
| 3 Select the largest number that appears in both lists. This is the HCF. | HCF of 12 and 18 is 6. |

The lowest common multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The highest common factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is also known as the Greatest Common Divisor (GCD).

# 2.1 Multiples, factors and divisibility

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q4 Columns 1–3, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15, Q18, Q23 | Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2–4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20(a), Q23, Q24 | Q1 Columns 3 & 4, Q2, Q3 Column 3, Q4 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q24, Q25 |

Answers page 627

### Fluency

1 Find all the factors of each of the following numbers. WE1

| | | | |
|---|---|---|---|
| **(a)** 18 | **(b)** 16 | **(c)** 23 | **(d)** 24 |
| **(e)** 20 | **(f)** 35 | **(g)** 36 | **(h)** 42 |
| **(i)** 53 | **(j)** 60 | **(k)** 77 | **(l)** 84 |

2 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. WE2

**(a)** 3 **(b)** 4 **(c)** 5 **(d)** 8 **(e)** 9

WE3

**3** Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

**(a)** 2 and 5 **(b)** 3 and 9 **(c)** 5 and 25
**(d)** 5 and 6 **(e)** 4 and 7 **(f)** 8 and 12
**(g)** 7 and 9 **(h)** 10 and 12 **(i)** 6 and 11
**(j)** 9 and 12 **(k)** 20 and 50 **(l)** 8 and 14
**(m)** 3, 4 and 5 **(n)** 2, 25 and 50 **(o)** 20, 50 and 60

WE4

**4** Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

**(a)** 10 and 15 **(b)** 8 and 24 **(c)** 5 and 12 **(d)** 26 and 36
**(e)** 11 and 33 **(f)** 28 and 70 **(g)** 44 and 22 **(h)** 10 and 30
**(i)** 40 and 70 **(j)** 32 and 60 **(k)** 35 and 70 **(l)** 42 and 48




**5** **(a)** The lowest common multiple of 8 and 1 is:

**A** 8 **B** 16 **C** 24 **D** 80

**(b)** Which of the following is a factor of 34?

**A** 4 **B** 12 **C** 17 **D** 68

**6** **(a)** A number divisible by 2, 3 and 5 is:

**A** 6 **B** 15 **C** 60 **D** 65

**(b)** Which pair of numbers are both divisible by 4?

**A** 38 and 42 **B** 38 and 52 **C** 38 and 60 **D** 52 and 60

## Understanding

**7** **(a)** Which one of the following numbers is not a multiple of 8?

**A** 4 **B** 24 **C** 72 **D** 88

**(b)** Which of the following is not a factor of 42?

**A** 1 **B** 6 **C** 21 **D** 84

**8** State TRUE or FALSE for the following.

**(a)** 346 is a multiple of 3. **(b)** 872 is divisible by 6.
**(c)** 2 is a factor of 348. **(d)** 52 is a multiple of 4.
**(e)** 854 is divisible by 9. **(f)** 3 is a factor of 56 902.

**9** For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

**(a)** 4, 6 and 10 **(b)** 6, 8 and 12
**(c)** 8, 12 and 16 **(d)** 10, 25 and 40

**10** Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

**(a)** 32 is a multiple of 8 because it is _____ by 8.
**(b)** 6 is a _____ of 54, so 54 is a multiple of 6.
**(c)** 72 is divisible by 9, so that makes it a _____ of 9.
**(d)** 4 is a factor of 60, so 60 is _____ by 4.



**11** **(a)** If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

**(b)** If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

**12** Mrs Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

**(a)** How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

**(b)** Mrs Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

**13** Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

**14** The smallest number divisible by 3, 4 and 5 is:

**A** 12 **B** 24 **C** 30 **D** 60

**15** If two events occur at different time intervals, the lowest common multiple (LCM) of the two time intervals is the point when the two events coincide, or occur together. Use this information to answer the following question.

In a city lighting display, one set of lights flashes every 25 seconds and the other set flashes every minute. If they are turned on at the same time, write down the next three times when the two sets of lights flash together.




**16** **(a)** Find the lowest number greater than 50 that is divisible by 7.

**(b)** Find the lowest number greater than 100 that is divisible by 11.

**(c)** Find the first common multiple of 2 and 7 that is greater than 100.

**(d)** Find the first common multiple of 2, 5 and 7 that is greater than 200.

## Reasoning

**17** Peter power-walked around an oval while Mei-ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei-ling jogged 24 laps in the hour.

**(a)** How many times did Mei-ling pass Peter?

**(b)** How many times did Mei-ling pass Peter exactly on the spot where they started?

**(c)** At the beginning of which laps did Mei-ling pass Peter exactly on the spot where they started?

**18 (a)** Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 100 000 | (2) | 3 | (4) | (5) | 6 | (8) | 9 | (10) |
| 202 008 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 12 121 212 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 300 300 300 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 7 500 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 900 090 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 123 456 789 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |

**(b)** Complete the following.

**(i)** If a number is divisible by 4 it will also be divisible by _____.

**(ii)** If a number is divisible by 9 it will also be divisible by _____.

**(c)** Explain your answers to **(b)**.

**19** The test to determine whether a number is divisible by 6 is to test whether it is divisible by 2 *and* 3. Explain why the test works.

**20** A *perfect* number is a number for which the sum of its factors (excluding itself) equals the number. The first perfect number is 6, as $1 + 2 + 3 = 6$.

**(a)** What is the next perfect number? It is less than 40.

**(b)** The next perfect number is between 490 and 510. See if you can find it.

**21** An *abundant* number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than $2 \times 12$. Find the next 2 abundant numbers. (Both are less than 40.)

**22 (a)** How can you always find a common multiple of a pair of numbers?

**(b)** How can you check if this number is the lowest common multiple?

## Open-ended

**23** Darren is designing a box for 60 identical chocolates to be placed in rows.

**(a)** Draw three ways Darren could arrange the chocolates in the box.

**(b)** Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

**24** Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

**25** Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

# Primes and composites

2.2

A number that has exactly two factors, itself and 1, is a **prime number**.

A number that has more than two factors is called a **composite number**.

The number 1 is a special number. It is neither prime nor composite.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **coprime** if their highest common factor is 1.

## The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BCE to 195 BCE. He was the first person to calculate a value for the circumference of the Earth. Another thing he was famous for was his 'sieve'.




See if you can reproduce what he did.

Copy the table and follow the instructions.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Step 1** Cross out the number 1.

**Step 2** Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

**Step 3** Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

**Step 4** Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

**Step 5** Repeat for the next number that isn't crossed out. Keep repeating until there is no 'next number'.

**Step 6** Write the factors of each of the circled numbers. What types of numbers are these?

**Step 7** Write the factors of any five of the crossed out numbers, except for 1.

**Step 8** Which type of number—circled or crossed out—has more factors? Explain why.

# 2.2 Primes and composites

## Navigator

Answers page 628

| | | |
|---|---|---|
| Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14, Q17 | Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18 | Q1, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18 |

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

## Fluency

**1** Write the prime numbers between 1 and 20.

**2** How many single-digit prime numbers are there? List them.

**3** List all the primes between 20 and 60.

**4** **(a)** The first prime number after 50 is:

**A** 51 **B** 53 **C** 55 **D** 57

**(b)** A number coprime with 18 is:

**A** 9 **B** 21 **C** 24 **D** 25

## Understanding




**5** **(a)** What is the next prime number after 60?

**(b)** What is the next composite number after 60?

**(c)** What are the two odd composite numbers less than 20?

**(d)** What is the largest prime number less than 50?

**6** Write TRUE or FALSE for each of the following statements.

**(a)** 21 is prime. **(b)** 38 is composite.

**(c)** 59 is prime. **(d)** 49 is prime.

**(e)** 5 and 7 are coprime. **(f)** 5 and 6 are coprime.

**(g)** All even numbers greater than 2 are composite.

**(h)** All odd numbers are primes.

**7** Name a divisibility test that shows that the following numbers are composites.

**(a)** 410 **(b)** 621 **(c)** 9909

**(d)** 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

**8** Are the following pairs of numbers coprime? Give reasons for your answer.

**(a)** 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27, 63

## Reasoning

**9** Explain why any pair of prime numbers is coprime.

**10** 2 is the only even prime number. Explain why.

**11** Explain why it is easy to tell that 4 567 278 is a composite number.

**12** Explain why 2 and 3 are the only two consecutive prime numbers.

**13** What is the smallest difference between any two consecutive composite numbers?

**14** **(a)** Find the numbers closest to 100 that are coprime with 100.

**(b)** Find the numbers closest to 36 that are coprime with 36.

15 Will a prime number always be coprime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be coprime.

## Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

# Outside the Square Puzzle

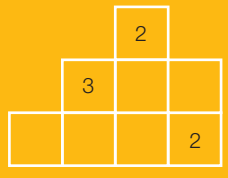
## Gold digger 1

It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. She has marked on two separate maps for you the places next to where the gold lies. If a number 3 is in a box, then it means that there are 3 pieces of gold in adjacent squares, either horizontally, vertically or diagonally. (Adjacent squares share an edge or a corner.) None of the already numbered squares contains a piece of gold, and no square contains more than one piece of gold.

Your task is to find exactly where the gold lies, so your team can get to it first and win the competition. There is only one possible solution for each map.

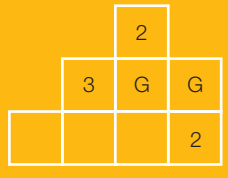
### Basic techniques

Look for an easy opening:

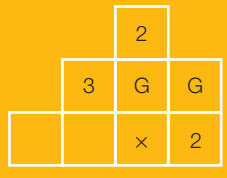



The top square (numbered 2) is only touching 2 other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.

Go back to other squares:



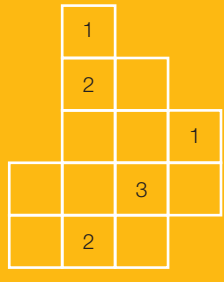

Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it is touching is empty. Mark this square with a cross.



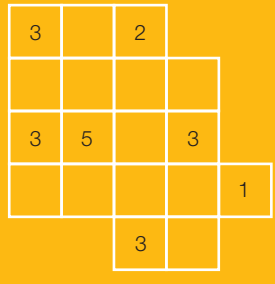

Now, look at the 3, and you can see that it is already next to one piece of gold, and is only touching two other empty squares, so both of these must also contain gold.

Now, copy the following maps and find the gold.

(a)




(b)



Mathspace

# FACTOR-PILLAR

FACTORPILLAR

**Equipment required:** 1 counter per player, 1 die, 2–4 brains, calculator (optional)

## How to play:

- Roll to decide who goes first.
- Players take turns to roll the die and move forward the number of spaces shown.
- The aim is to move from start to finish and get as many points as possible by finding factors of numbers.

When a player lands on a number, they must list all the factors of that number. The number of factors is their score. For example, a player lands on 6: factors are 1, 2, 3 and 6; score = 4.

But beware! If another player notices you have missed one or more factors they gain five points. If they suggest a factor that isn't correct they lose five points.

**LARGE CIRCLE:** If a player correctly finds all the factors of the number in a large circle, they earn double the points for that number.

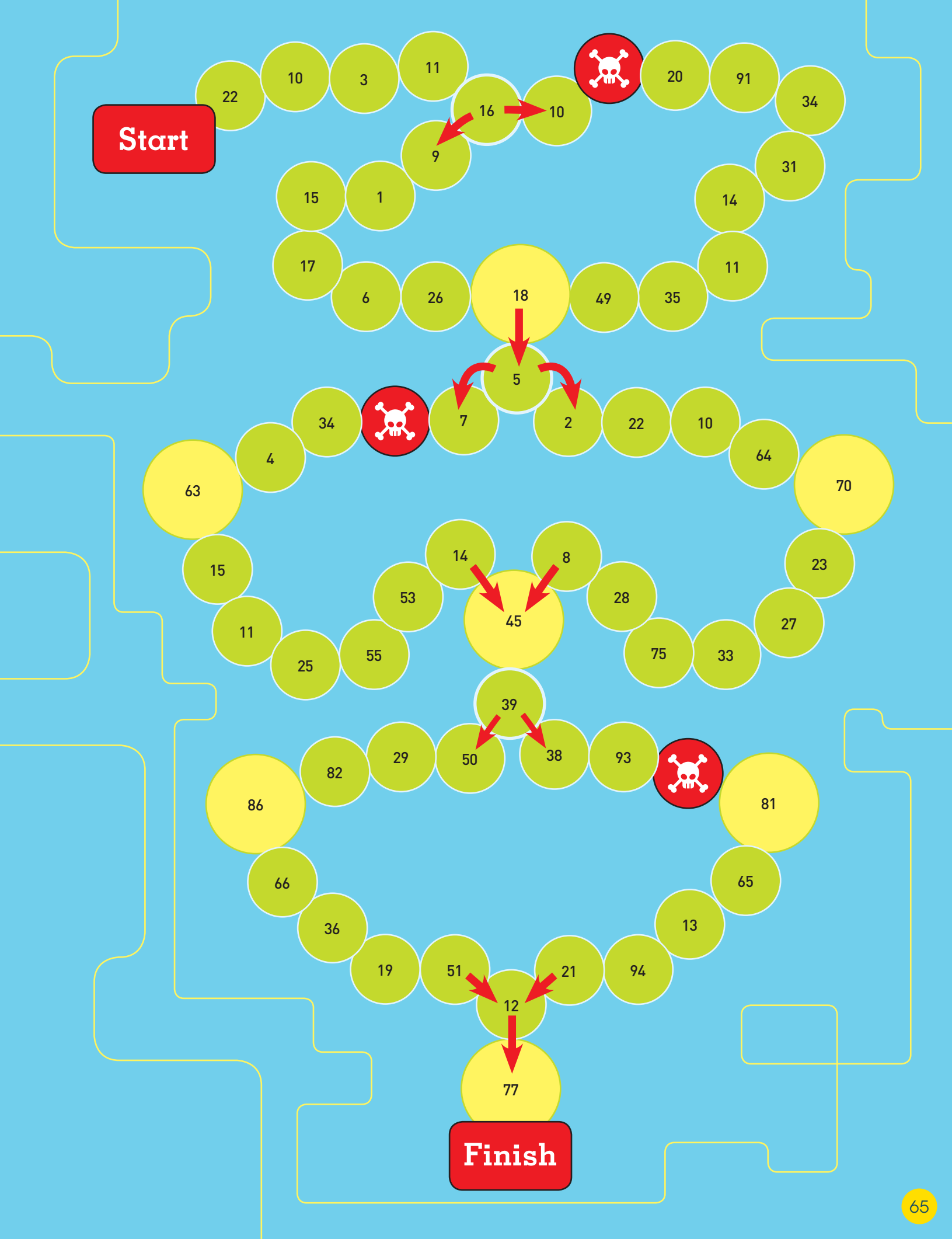
**CROSSBONES:** If a player lands on a circle containing a skull and crossbones, they miss a turn.

**SQUARE NUMBER:** If a player lands on a square number and calls 'Square Power' within two seconds, they have an extra turn.

**PRIME NUMBER:** If a player lands on a prime number and calls 'Prime Power' within two seconds, they can double their points for that number; if they don't call 'Prime Power' they only get two points for the two factors.

Players may not move backwards. When a player reaches a circle with two arrows leading from it they can decide which path to take.

The game ends when one player reaches or crosses the finish line. The player with the highest number of points at that moment is declared the winner (even if they haven't reached the finish line).


Start
22
10
3
11
16
10
9
15
1
17
6
26
18
49
35
11
14
31
34
91
20
5
7
2
22
10
64
70
4
34
63
15
14
8
23
53
28
45
11
27
25
55
75
33
39
82
29
50
38
93
86
81
66
65
36
13
19
51
21
94
12
77
Finish

# 2.3 Prime factors

A **prime factor** of a number is a factor that is also a prime number.

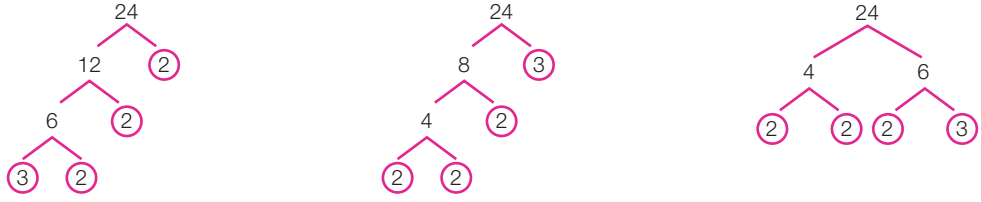
Every whole number can be written as the unique product of its prime factors.

## Using a factor tree to find prime factors

A factor tree is a useful way of finding the prime factors of a number. To construct a factor tree:

1. Draw two lines ('branches') extending out from the number, in an upside-down 'V' shape.
2. Write two factors of the number at the end of the branches.
3. If one of the factors is prime, circle it. If it is composite, draw another two branches and split the factor into two smaller factors.
4. Repeat step **3** until all branches end in a circled prime number.

There is often more than one way for a factor tree to be constructed. Three versions of the factor tree for 24 are shown here. They look different; however, the prime factors that appear on the ends of the branches are the same: 2, 2, 2, 3.



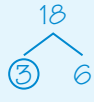
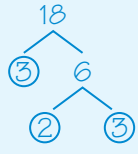

Once all of the branches end in prime factors, we have found all of the prime factors, and we write the number as the product of those factors. We write the factors in ascending order (smallest to largest) and, if any are repeated, we write them in index form.

For example: $24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$

## Worked Example 5 WE5

Draw a factor tree for the number 18, then express the number as a product of its prime factors in index form.

| Thinking | Working |
|---|---|
| **1** Write the number as the product of two factors. | $18 = 3 \times 6$ |
| **2** Show each factor at the end of a branch and circle it if it is prime. |   |
| **3** Split any composite factor into two smaller factors, continuing until the factors at the end of each branch are prime. |   |
| **4** Write the number as a product of the primes in index form and in ascending order (smallest to largest). | $18 = 2 \times 3 \times 3$ $= 2 \times 3^2$ |

## Using repeated division to find prime factors

We can also find the prime factors of a number by doing a series of divisions by small prime numbers, such as 2, 3 or 5. We can use the divisibility rules to identify these smaller prime factors; for example, if the number is even, we can divide by 2. The divisibility rules are listed in section 2.1 on page 55.

We keep dividing by the same, or a different, prime factor until the final answer is 1. The list of all the divisors will be the list of the prime factors.

## Worked Example 6 WE6

Use repeated division to find the prime factors of 84. Express the number as a product of its prime factors in index form.

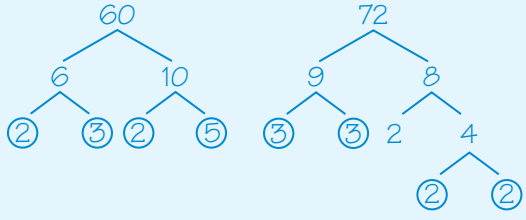
| Thinking | Working |
|---|---|
| **1** Identify a prime factor of the number and divide it into the number. Continue dividing by the same or a different prime factor until the last answer is 1. | $2\overline{)84}$<br>$2\overline{)42}$<br>$3\overline{)21}$<br>$7\overline{)7}$<br>1 |
| **2** The prime factors are listed down the left side of the division calculation. Write the number as a product of these. | $84 = 2 \times 2 \times 3 \times 7$ |
| **3** Write the number as a product of the prime factors in index form and in ascending order. | $84 = 2^2 \times 3 \times 7$ |

## Using prime factors to find the HCF (highest common factor)

If we have two numbers written as a product of their prime factors, the HCF is the product of all the common prime factors.

## Worked Example 7 WE7

Use prime factors to find the HCF of the following pair of numbers: 60 and 72.

| Thinking | Working |
|---|---|
| **1** Use factor trees to find the prime factors. |  |
| **2** Write each number as a product of its prime factors. Do not use index form. | $60 = (2 \times 2 \times 3) \times 5$<br>$72 = 2 \times (2 \times 2 \times 3) \times 3$ |
| **3** Circle the group of factors that is common to both numbers (there must be the same number of factors in each group). | |
| **4** Find the product of the common prime factors. This is the HCF. | $HCF = 2 \times 2 \times 3$<br>$= 12$ |

# 2.3 Prime factors

## Navigator

Answers page 628

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q9, Q10, Q11, Q14 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q9, Q10, Q11, Q12, Q13, Q14 | Q1 Column 4, Q2 Columns 3 & 4, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15 |

## Fluency

WE5

**1** Draw a factor tree for each of the following numbers, then express the number as a product of its prime factors in index form.

| | | | |
|---|---|---|---|
| **(a)** 8 | **(b)** 12 | **(c)** 20 | **(d)** 48 |
| **(e)** 14 | **(f)** 26 | **(g)** 68 | **(h)** 44 |
| **(i)** 64 | **(j)** 72 | **(k)** 108 | **(l)** 144 |
| **(m)** 200 | **(n)** 750 | **(o)** 1000 | **(p)** 1236 |

WE6

**2** Use repeated division to find the prime factors of the following numbers. Express each number as a product of its prime factors in index form.

| | | | |
|---|---|---|---|
| **(a)** 8 | **(b)** 12 | **(c)** 28 | **(d)** 36 |
| **(e)** 39 | **(f)** 77 | **(g)** 51 | **(h)** 38 |
| **(i)** 30 | **(j)** 63 | **(k)** 96 | **(l)** 132 |
| **(m)** 168 | **(n)** 198 | **(o)** 288 | **(p)** 212 |

WE7

**3** Use prime factors to find the HCF of the following pairs of numbers.

| | | |
|---|---|---|
| **(a)** 8 and 20 | **(b)** 12 and 18 | **(c)** 12 and 36 |
| **(d)** 36 and 48 | **(e)** 24 and 56 | **(f)** 28 and 84 |
| **(g)** 64 and 96 | **(h)** 70 and 98 | **(i)** 60 and 105 |
| **(j)** 66 and 110 | **(k)** 80 and 128 | **(l)** 130 and 156 |

**4** **(a)** 120 written as the product of its prime factors is:

**A** $2 \times 4 \times 15$ **B** $3 \times 8 \times 5$ **C** $2 \times 2 \times 5 \times 6$ **D** $2 \times 2 \times 2 \times 3 \times 5$

**(b)** The prime factors of 550 are:

**A** 2, 5, 11 **B** 2, 10, 55 **C** 5, 10, 11 **D** 2, 11, 25

**5** Which numbers are the product of these prime factorisations?

| | | |
|---|---|---|
| **(a)** $2^3 \times 3^2$ | **(b)** $2^2 \times 3 \times 5^2$ | **(c)** $2^4 \times 3^3 \times 5^3$ |
| **(d)** $3^2 \times 5 \times 7^2$ | **(e)** $2^3 \times 5^2 \times 7 \times 11$ | **(f)** $2^6 \times 11^2$ |

**6** $6 = 2 \times 3$ and $8 = 2^3$. Use these facts to write the following numbers as products of prime numbers.

| | | |
|---|---|---|
| **(a)** 48 | **(b)** 96 $(48 \times 2)$ | **(c)** 144 $(48 \times 3)$ |

**7** $33 = 3 \times 11$ and $10 = 2 \times 5$. Use these facts to write the following numbers as products of prime numbers.

| | | |
|---|---|---|
| **(a)** 330 | **(b)** 660 | **(c)** 990 |

## Understanding

**8** The prime factors of 192 are 2 and 3 and the prime factors of 42 are 2, 3 and 7. List the prime factors of $192 \times 42$.

**9** **(a)** The HCF of a pair of numbers can also be used to find the lowest common multiple (LCM). Copy and complete the following method.

Prime factors of 36: $2 \times __ \times __ \times __$

Prime factors of 100: $2 \times __ \times __ \times __$

Highest common factor (HCF) of 36 and 100: $2 \times __ = __$

Factors of 36 not used to find the HCF: $__ \times 3 = __$

Factors of 100 not used to find the HCF: $5 \times __ = __$

HCF × all unused factors of 36 × all unused factors of 100: $__ \times 3 \times __ \times 5 \times __ = 900$

The result is the lowest common multiple of 36 and 100.

**(b)** Use this method to find the LCM for the following pairs of numbers.

| | | |
|---|---|---|
| **(i)** 8 and 18 | **(ii)** 12 and 18 | **(iii)** 8 and 20 |
| **(iv)** 8 and 72 | **(v)** 20 and 26 | **(vi)** 20 and 800 |
| **(vii)** 20 and 1000 | **(viii)** 400 and 800 | **(ix)** 800 and 1000 |

**10** Marcus is making identical balloon arrangements for a party. He has 84 red balloons and 54 blue balloons. He would like to sort his balloons into groups, each group containing the same number of red and blue balloons, with no balloons left over.



**(a)** Use prime factors to find the highest common factor of 84 and 54.

**(b)** If Marcus split his balloons into this number of groups, how many red and blue balloons would be in each group?

## Reasoning

**11** A number less than 550 is the product of four prime factors: 2, 3, 5 and a fourth prime factor between 15 and 20. What is the fourth factor, and what is the number?

**12** **(a)** Find the smallest number that can be written in prime factor form with:

**(i)** exactly two different prime factors

**(ii)** exactly three different prime factors

**(iii)** exactly four different prime factors.

**(b)** Repeat **(a)**, but remove the condition that the factors are different.

**13** 140 tickets were sold to the school concert. Each ticket sold was numbered from 1 to 140 and a lucky door prize was awarded to each person who had a ticket with a number that was a multiple of 15, but not a multiple of 9, 10 or 25. Five friends had the ticket numbers 45, 80, 90, 105 and 135.

**(a)** Do any of them have a winning ticket? If so, what is their winning number?

**(b)** If 250 tickets were sold, how many winning tickets would there be? Find the winning numbers.

## Open-ended

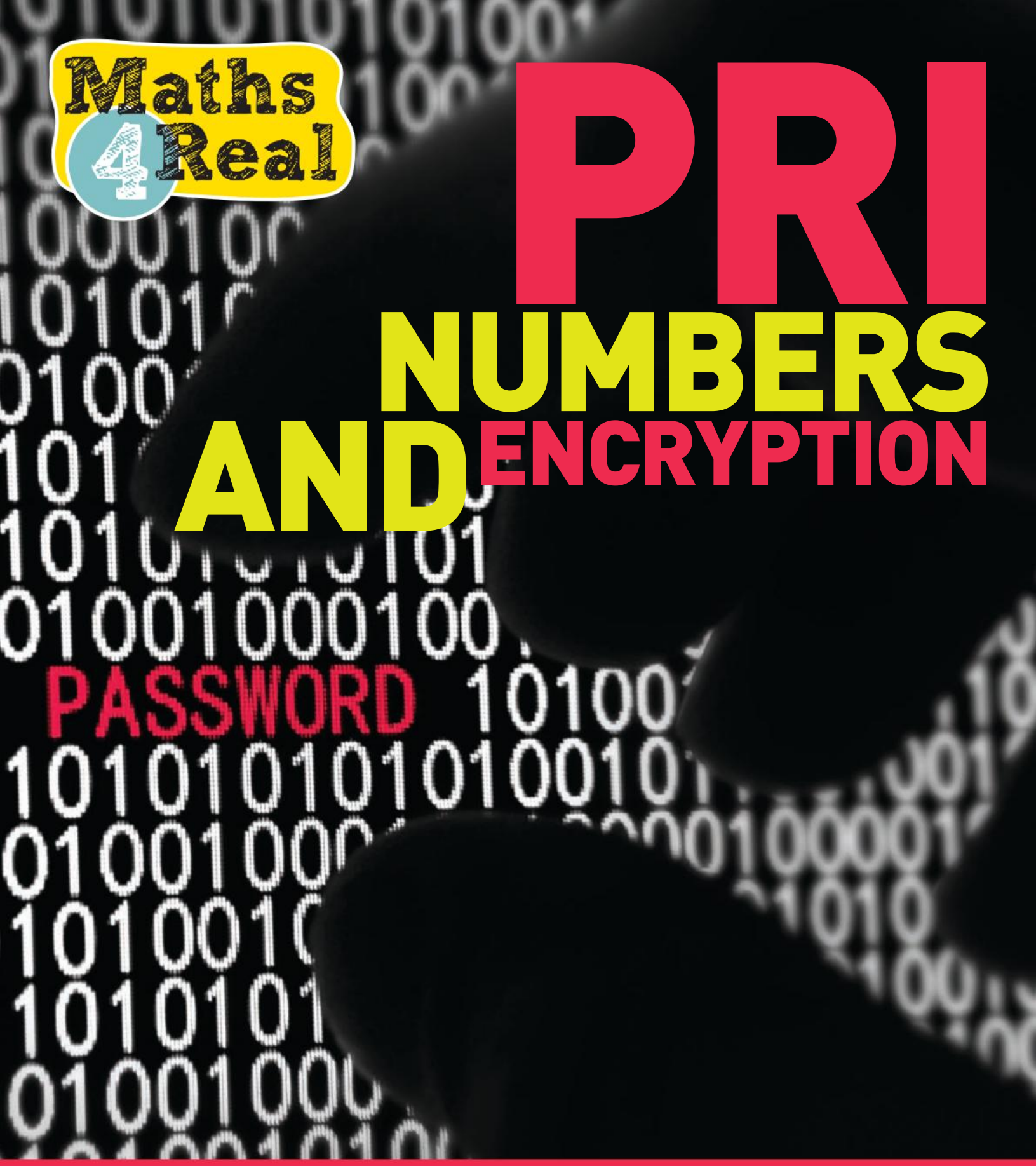
**14** A number between 100 and 200 is the product of four primes and has three prime factors (one factor is repeated). Explain how you might go about finding this number and write down two such numbers.

**15** To find the prime factors of a number, Ray uses the divisibility tests. He then writes down the number from each divisibility test that worked.

**(a)** Using this method, what would Ray say are the prime factors of 20?

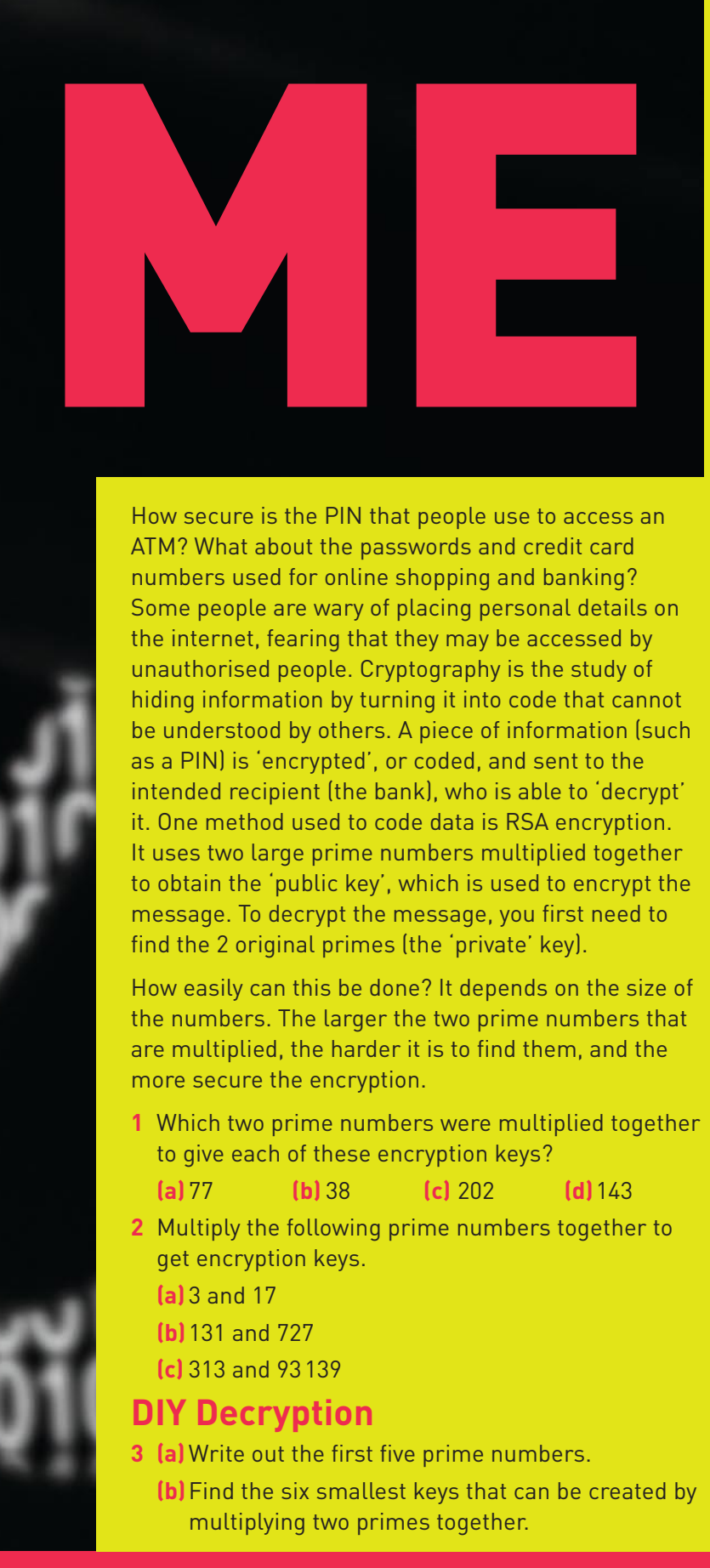
**(b)** What is wrong with Ray's method?

**(c)** Suggest how Ray's method can be improved so that it is accurate.




{A B C D E F G H I J K L M

# ME



How secure is the PIN that people use to access an ATM? What about the passwords and credit card numbers used for online shopping and banking? Some people are wary of placing personal details on the internet, fearing that they may be accessed by unauthorised people. Cryptography is the study of hiding information by turning it into code that cannot be understood by others. A piece of information (such as a PIN) is 'encrypted', or coded, and sent to the intended recipient (the bank), who is able to 'decrypt' it. One method used to code data is RSA encryption. It uses two large prime numbers multiplied together to obtain the 'public key', which is used to encrypt the message. To decrypt the message, you first need to find the 2 original primes (the 'private' key).

How easily can this be done? It depends on the size of the numbers. The larger the two prime numbers that are multiplied, the harder it is to find them, and the more secure the encryption.

1 Which two prime numbers were multiplied together to give each of these encryption keys?

**(a)** 77 **(b)** 38 **(c)** 202 **(d)** 143

2 Multiply the following prime numbers together to get encryption keys.

**(a)** 3 and 17

**(b)** 131 and 727

**(c)** 313 and 93 139

## DIY Decryption

3 **(a)** Write out the first five prime numbers.

**(b)** Find the six smallest keys that can be created by multiplying two primes together.

**(c)** How many factors does each key in part **(b)** have?

**(d)** Will every key created by multiplying two primes together have this number of factors? Why?

4 Here's a simplified version of how decryption using prime numbers works. Suppose the encrypted message is W L R G Y S B E M R K C R E B J C V and your key is 713.

Find the two prime numbers whose product is 713 (23 and 31). Now write the digits of these primes under the code as follows:

| W | L | R | G | Y | S | B | E | M | R | K | C | R | E | B | J | C | V |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 3 |

The digits tell you how many places to shift each letter along the alphabet. The 2 under the W means that you shift two letters forward through the alphabet from W to Y so that the decoded message has Y as its first letter. To get the second letter, you move 3 places forward from L in the alphabet, arriving at the letter O.

**(a)** Continue decrypting the code to get the full message.

**(b)** Describe what you did with the coded letter Y.

5 Decode this message using 943 as the key.

K X P G Q F O N D M N H K B E L N L N S Y K Y D

Note: You will need to decide which order to write the two primes. Only one will work.

6 Encrypt your own message using this method. Use prime numbers under 100 and keep the message under 25 letters. Give your coded message and key to someone to decrypt.

7 Why must prime numbers be used as encryption keys? Why can't we use any number, such as 120, as a key?

### Research

Find out about GIMPS (Great Internet Mersenne Prime Search) and how prime numbers are used in email encryption.

Consider whether governments should ban encryption so that intelligence agencies such as Australian Security Intelligence Organisation (ASIO) can read everyone's emails.

Make a list of the potential problems and dangers that can arise from placing personal information on the internet.

N O P Q R S T U V W X Y Z }

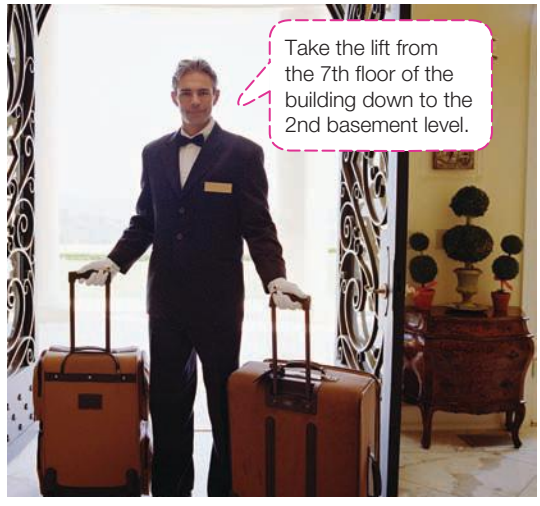
# 2.4 Introduction to integers

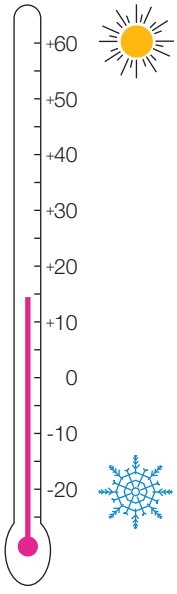









All of the above statements describe more than just the size of a number—they also give a direction. We can use a + (**positive**) sign or a – (**negative**) sign to show this direction. Below is a list of words usually associated with + or –.

| + (positive) | – (negative) |
|---|---|
| up | down |
| increase | decrease |
| gain | lose |
| deposit | withdrawal |
| profit | loss |
| above | below |

The words '**deposit**' and '**withdrawal**' are banking words used to describe the movement of money into and out of a bank account. '**Profit**' and '**loss**' are words used to describe whether money has been made or lost by a business.

## Integers

The **integers** are all of the whole numbers that you are familiar with (1, 2, 3 … ), the negative equivalents of these (-1, -2, -3 … ), and zero, which is neither positive nor negative.

We indicate whether a number is positive or negative by placing a '+' or '-' in front of the number. For example, we write '+4' and say 'positive four'. We write '-9', and say 'negative nine'.

### Worked Example 8 WE8

Write down an integer suggested by each of the following.

**(a)** depositing $660 into your bank account

**(b)** diving 5 m below the surface of the ocean

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Does the statement suggest an increase or a decrease? (In this case, 'deposit' suggests an increase, so use a positive sign.) | *(a)* |
| | **2** | Write the number with the appropriate sign. | *+660* |
| **(b)** | **1** | Does the statement suggest an increase or a decrease? (In this case, 'below' suggests a decrease, so use a negative sign.) | *(b)* |
| | **2** | Write the number with the appropriate sign. | *-5* |

## Comparing integers

The number line is extremely useful when working with integers.

Up until now, we have used only the positive section of the number line, which extended to the right of zero. Now, we will extend the number line to the left of 0 to include negative numbers. Notice that the negative numbers form a mirror image of the positive numbers on the other side of zero.




As we move along the number line, the numbers get larger as we move to the right, and smaller if we move to the left. Look at the positions of +5 and -10 on the number line shown. +5 is further to the right than -10. We say that +5 *is greater than* -10, and write +5 > -10. We could also say that -10 *is less than* +5, and write -10 < +5.

*Note:* Often, the + sign is left off positive numbers; for example, 6 has the same value as +6. From now on, we will leave the + signs off the positive numbers on the number line, because there is no need for them.

If a number has no sign in front of it, we assume it is positive. 7 = +7

'>' is the symbol for 'is greater than'.

'<' is the symbol for 'is less than'.

## Worked Example 9 WE9

Write a > or < symbol between each pair of integers to make a correct statement.

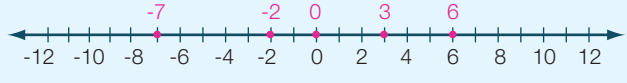
**(a)** -9 _____ -4 **(b)** 0 _____ -10 **(c)** 7 _____ -48

| Thinking | Working |
|---|---|
| **(a)** Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, -9 is to the left of -4.)<br>If it is to the right, insert >.<br>If it is to the left, insert <. | (a) $-9 < -4$ |
| **(b)** Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 0 is to the right of -10.)<br>If it is to the right, insert >.<br>If it is to the left, insert <. | (b) $0 > -10$ |
| **(c)** Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 7 is to the right of -48.)<br>If it is to the right, insert >.<br>If it is to the left, insert <. | (c) $7 > -48$ |

## Worked Example 10 WE10

Arrange the following integers in ascending order (from smallest to largest).

-2, 3, -7, 0, 6

| Thinking | Working |
|---|---|
| **1** Mark the integers on a number line as shown. |  |
| **2** Now, list the marked integers in order, as they appear on the number line, moving from left to right. | -7, -2, 0, 3, 6 |

The further to the right a number is on a horizontal number line, the greater its value.

The further to the left a number is on a horizontal number line, the smaller its value.

Vertical number lines can also represent directed numbers, where the higher a number is on the line, the greater its value. A thermometer is an example of a vertical number line.




# 2.4 Introduction to integers

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q8, Q11, Q12, Q13, Q14, Q16, Q18 | Q1 Column 1, Q2 Columns 2–4, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9 Column 1, Q11, Q12, Q14, Q15, Q16, Q17, Q18 | Q1 Column 2, Q2 Columns 3 & 4, Q3 Column 3, Q4 Column 3, Q5 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18 |

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## Fluency

**1** Write down an integer suggested by each of the following. WE8

**(a)** a profit of $300
**(b)** a loss of $50 000
**(c)** 8 seconds before blast-off
**(d)** 4 hours after take-off
**(e)** 36 m above sea level
**(f)** a deposit of $45
**(g)** a withdrawal of $20
**(h)** a win by 12 points
**(i)** a 2 point loss
**(j)** down 7 floors
**(k)** up 28 floors
**(l)** I owe the bank $40 000.
**(m)** I have $225 in the bank.
**(n)** The water level dropped by 9 m.

**2** Write a > or < symbol between each pair of integers to make a correct statement. WE9

**(a)** +6 _____ -3 **(b)** +1 _____ -3 **(c)** -7 _____ +5 **(d)** -2 _____ +6
**(e)** -12 _____ -4 **(f)** -7 _____ -1 **(g)** -2 _____ -11 **(h)** -9 _____ 0
**(i)** 0 _____ -2 **(j)** -35 _____ +7 **(k)** -4 _____ +67 **(l)** -11 _____ +75

**3** Arrange the following integers in ascending order (from smallest to largest). WE10

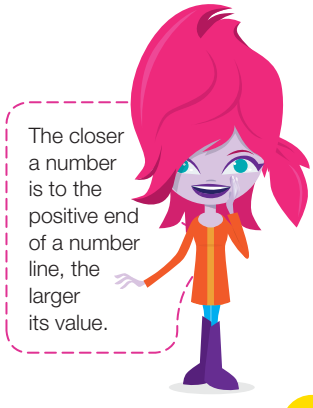
**(a)** -3, -12, 6, -4, -1
**(b)** -11, 10, 9, 2, -10
**(c)** 1, -6, -4, 9, 2
**(d)** -8, -5, 7, 1, -6
**(e)** -1, -4, -7, -8, -12
**(f)** -56, 23, -79, 0, 6

**4** Arrange the following integers in descending order (from largest to smallest).

**(a)** -5, -6, 3, 7, 8
**(b)** -8, 7, 10, 1, -12
**(c)** 1, -8, -4, -9, -6
**(d)** -12, 0, -5, 7, -11
**(e)** -11, -5, -2, -12, -9
**(f)** 0, 79, -54, 4, -9

**5** State the opposite of:

**(a)** west 300 km
**(b)** 3 days late
**(c)** 7°C below zero
**(d)** add 5
**(e)** subtract 22
**(f)** go left 3 m
**(g)** -2
**(h)** +16
**(i)** -350



**6** **(a)** The directed number +2 is suggested by:

- **A** going up two flights of stairs
- **B** going down two flights of stairs
- **C** losing 2 kg in weight
- **D** the temperature going from 19°C to 17°C.

**(b)** The directed number -4 is suggested by:

- **A** going up four flights of stairs
- **B** the rain gauge filling with 4 mm of rain
- **C** gaining 4 kg in weight
- **D** the temperature going from 22°C to 18°C.

**7** Write TRUE or FALSE for each of the following.

**(a)** $58 > -60$ **(b)** $-44 < -50$ **(c)** $-29 > 30$ **(d)** $-31 < -29$

**(e)** $-75 > 70$ **(f)** $-92 > -11$ **(g)** $19 > -100$ **(h)** $82 > -99$

## Understanding

**8** Write down the integers that lie between:

**(a)** -4 and 3 **(b)** -2 and 2 **(c)** -9 and -4

**(d)** -5 and 0 **(e)** -37 and -42 **(f)** -120 and -115

**9** Find the next three numbers in each pattern.

**(a)** -18, -16, -14, _____, _____, _____ **(b)** -20, -15, -10, _____, _____, _____

**(c)** 9, 6, 3, _____, _____, _____ **(d)** -6, -12, -18, _____, _____, _____

**(e)** 30, 20, 10, _____, _____, _____ **(f)** 12, 7, 2, _____, _____, _____

**10** The balances of four traders' bank accounts show the following figures.

| | | | |
|---|---|---|---|
| Ben | $428 | Sue | -$23 |
| Damien | -$260 | Carol | $325 |

**(a)** Who has the largest account?

**(b)** Who owes their bank the most?

**11** The minimum and maximum temperatures recorded at Uluru during one week were:

| Day | Maximum (°C) | Minimum (°C) |
|---|---|---|
| Monday | 19 | -3 |
| Tuesday | 16 | -7 |
| Wednesday | 22 | 0 |
| Thursday | 20 | -1 |
| Friday | 23 | 2 |
| Saturday | 20 | -2 |
| Sunday | 18 | -4 |



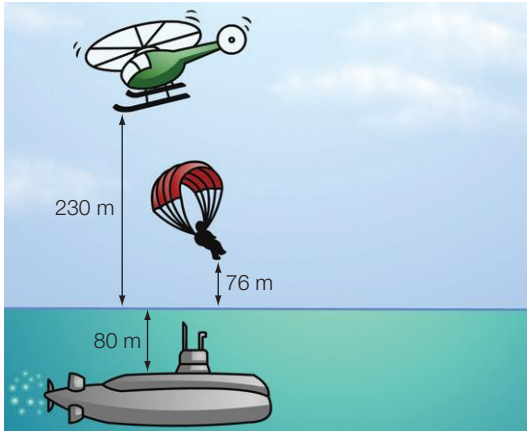
**(a)** On which day was the lowest minimum recorded?

**(b)** When was the highest minimum recorded?

**(c)** Which day had the greatest difference between the minimum and the maximum?

**12** In a combined operation, the airforce and navy have a helicopter and a submarine at the levels shown.

A parachutist has 76 m to fall before hitting the ocean.




**(a)** How far is the parachutist from the submarine?

**(b)** What distance from the helicopter is the parachutist?

**(c)** How far apart are the helicopter and the submarine?

## Reasoning

**13** **(a)** If you cycled 5 km west, then 3 km east, then 2 km west, what single journey could you have taken to get to the finishing point?

**(b)** What is the opposite of this single journey?

**14** **(a)** If you got in a lift on the ground floor, then travelled 7 floors up, then 2 floors up, then 6 floors down and finally 1 floor up, what single journey could you have taken to get to your final floor?

**(b)** What is the opposite of this single journey?

**15** On Monday, Sam withdrew $80 from his bank account at an ATM. On Tuesday, his pay of $350 was deposited into the account, and he also deposited $40 that he had received for his birthday. On Thursday, Sam used the account to pay a $120 bill online, and on Friday he withdrew $50 from an ATM.

**(a)** Did Sam have more or less money at the end of the week than at the start?

**(b)** How much more or less did he have?

## Open-ended

**16** Write any three negative integers greater than -8.

**17** The game of indoor cricket is similar to regular cricket in that batsmen can score 1, 2, 3, 4 or 6 runs off each ball that is bowled to them. It is different in that pairs of batsmen stay in for 4 overs (an 'over' is a group of 6 balls). If they lose their wicket, or 'get out', in these 4 overs, they do not leave the pitch. Instead, 3 runs are subtracted from their score, which is shown as the combined total of the runs scored by both batsmen.

**(a)** In their first over, Angelo and Murray scored as follows: 2, -3 (wicket), 1, 2, 4, -3 (wicket). What was their overall score at the end of the over?



**(b)** At the end of their first over, Saul and Jeremy's score was 4. On each ball of the over, they had either scored runs or lost their wicket. List at least three ways they could have reached 4 in six balls.

**18** Sharif is a computer technician. He works in an office building with 25 floors, helping to fix computer problems for other employees. Write a short story using this sequence of directed numbers, describing a day at work for Sharif.

0, +21, -13, +7, 0, +4, 0, -2, +5, 0, -1, -2, 0

# Outside the Square Puzzle

## Temperature tangle

**Equipment required:** access to the internet or a library may be necessary



Australia is known for its warm summers, but it can get cold here too!

Solve this puzzle to discover the place that holds the record for the coldest recorded temperature in Australia.

1. Write the numbers 1–13 down your page.
2. For the 13 situations listed below, find the matching temperature.
3. For each of the 13 statements in order, write the temperature and the letter next to it in your book.
4. If the temperatures and their corresponding letters are written in the correct order, you will spell out the name of Australia's coldest place.

| | | | |
|---|---|---|---|
| **1** | Boiling point of water | 70°C | P |
| **2** | Oven temperature required to bake a cake | 37°C | E |
| **3** | Temperature inside your fridge at home | 0°C | O |
| **4** | Coldest air temperature recorded on Earth (Vostok Station, Antarctica) | 180°C | H |
| **5** | Temperature of the surface of the Sun | 51°C | A |
| **6** | Freezing point of water | -15°C | T |
| **7** | Hottest air temperature recorded on Earth (Al 'Aziziyah, Libya) | 4°C | A |
| **8** | Temperature inside your freezer at home | 100°C | C |
| **9** | Body temperature of a healthy human | -196°C | S |
| **10** | Drinking temperature of a cup of tea | -40°C | S |
| **11** | Hottest temperature recorded in Australia (Oodnadatta, SA) | 58°C | T |
| **12** | Temperature of a freezer inside an ice-cream factory | 5700°C | L |
| **13** | Temperature of liquid nitrogen | -89°C | R |

# Half-time 2

**1** **(a)** Find the lowest common multiple (LCM) of 9 and 15. Ex 2.1

**(b)** Find the highest common factor (HCF) of 48 and 72.

**2** The common factors of 18 and 30 are: Ex 2.1

**A** 1, 18, 23 **B** 1, 2, 3, 6 **C** 1, 9, 18, 30 **D** 2, 3, 9, 18

**3** Write each of the following as either a positive or a negative integer. Ex 2.4

**(a)** a deposit of $85 into a bank account **(b)** a weight loss of 7 kg

**(c)** a win by 3 goals **(d)** a profit of $28 000

**(e)** 40 m below sea level **(f)** going up 11 floors in a building

**4** List all the prime numbers between 30 and 50. Ex 2.2

**5** Write a < (less than) or > (greater than) symbol between each of the following to make a true statement. Ex 2.4

**(a)** 6 _____ 10 **(b)** -4 _____ 7 **(c)** 3 _____ -12 **(d)** -8 _____ -15

**6** Draw a factor tree for each of the following, then express each number as a product of its prime factors in index form. Ex 2.3

**(a)** 54 **(b)** 60 **(c)** 98 **(d)** 104

**7** **(a)** A number is divisible by 15 if it is divisible by both 3 and 5. Use divisibility tests to decide if 76 905 is divisible by 15. Ex 2.1

**(b)** Show, using two divisibility tests, that 4734 is divisible by 6.

**8** Write the following sets of integers in ascending order (smallest to largest). Ex 2.4

**(a)** -37, 7, -30, 0, -3, 3 **(b)** 54, -20, -1, -5, -40

**9** Use prime factors to find the HCF of the following pair of numbers: 48 and 120. Ex 2.3

**10** The sum of two prime numbers is 30. The numbers cannot be: Ex 2.2

**A** 7 and 23 **B** 11 and 19 **C** 13 and 17 **D** 14 and 16

**11** Three racing cars, Honda, Holden and Ford, take 25 seconds, 30 seconds and 50 seconds, respectively, to complete one lap of a circuit. They start the race at the same time and from the same position. Ex 2.1

**(a)** How many seconds will pass before the cars are at the same position on the track?

**(b)** How many laps had each car completed by this time?

**(c)** How many laps did the Honda complete in the time taken by the Ford to complete 6 laps?



# 2.5 Adding and subtracting positive integers

In the following sections you will learn how to perform calculations with integers. A number line can be very useful for this.

## Adding positive integers

Adding two positive integers is the straightforward addition you have been doing for years.

For example, +1 + +2 = +3, which can simply be written as 1 + 2 = 3.

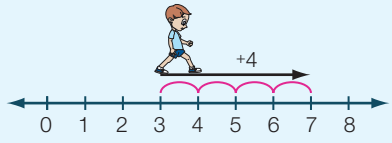
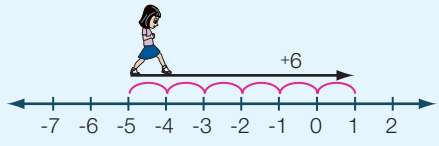
To add a positive number, move that many spaces along the number line to the right (the positive direction) from the starting position.

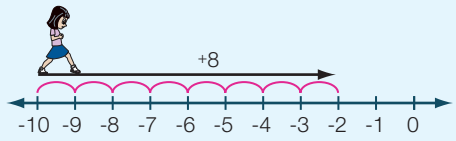
The steps are the same, regardless of whether our starting position is a positive or a negative integer, as we can see in the Worked Example below.

### Worked Example 11 WE11

Use a number line to perform each of the following additions.

**(a)** 3 + 4 **(b)** -5 + 6 **(c)** -10 + 8

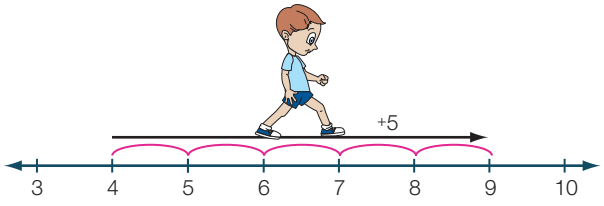
| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Draw or imagine a number line. Start at the first number (3) and walk the number of steps indicated by the second number (4) in the positive direction (to the right). | *(a)*   |
| **2** | Write the addition as a number sentence. | 3 + 4 = 7 |
| **(b)** **1** | Draw or imagine a number line. Start at the first number (-5) and walk the number of steps indicated by the second number (6) in the positive direction (to the right). | *(b)*   |
| **2** | Write the addition as a number sentence. | -5 + 6 = 1 |

| | | | |
|---|---|---|---|
| **(c)** | **1** | Draw or imagine a number line. Start at the first number (-10) and walk the number of steps indicated by the second number (8) in the positive direction (to the right). | *(c)*   |
| | **2** | Write the addition as a number sentence. | *-10 + 8 = -2* |

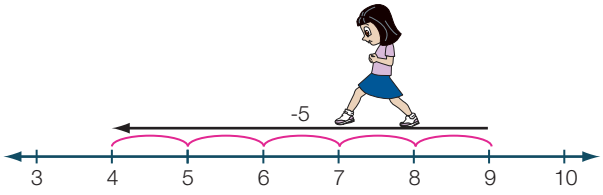
## Subtracting positive integers

Subtraction and addition are opposite operations. We have shown addition as starting on a point on the number line and walking along a number of spaces to the right. Because subtraction is the opposite of addition, we subtract by moving to the left on the number line.

Addition:
Move to the right
4 + 5 = 9




Subtraction:
Move to the left
9 – 5 = 4



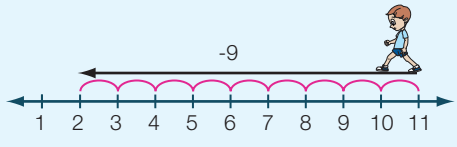
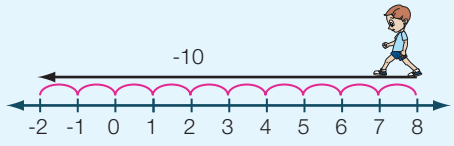

To subtract a positive number, move that many spaces along the number line to the left (the negative direction) from the starting position.

## Worked Example 12

WE12

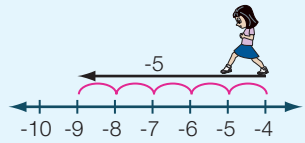
Use a number line to perform each of the following subtractions.

**(a)** 11 – 9 **(b)** 8 – 10 **(c)** -4 – 5

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Draw or imagine a number line. Start at the first number (11) and walk the number of steps indicated by the second number (9) in the negative direction (to the left). | *(a)*   |
| | **2** | Write the subtraction as a number sentence. | *11 – 9 = 2* |
| **(b)** | **1** | Draw or imagine a number line. Start at the first number (8) and walk the number of steps indicated by the second number (10) in the negative direction (to the left). | *(b)*   |
| | **2** | Write the subtraction as a number sentence. | *8 – 10 = -2* |

**(c)** **1** Draw or imagine a number line. Start at the first number (-4) and walk the number of steps indicated by the second number (5) in the negative direction (to the left).

*(c)*



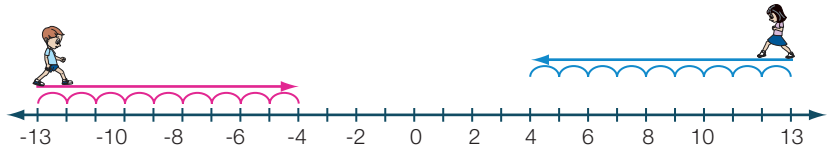

**2** Write the subtraction as a number sentence.

$-4 - 5 = -9$

Part **(b)** of the Worked Example above shows that if we subtract a larger number (such as 10) from a smaller one (such as 8), we will walk past the zero and into the negative side of the number line.

## Using number line symmetry

The number line can be thought of as being a reflection, or mirror image, around 0. This symmetry can be useful when working with integers. For example, the journey to calculate -13 + 9 (moving to the right) is the mirror image of the journey to calculate +13 – 9 (moving to the left), with answers of -4 and +4, respectively.




So, to calculate -85 + 23, we could simply calculate 85 – 23, then place a negative sign in front of the answer.

$85 - 23 = 62$ $\qquad$ $-85 + 23 = -62$

# 2.5 Adding and subtracting positive integers

## Navigator

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| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q5, Q6, Q8, Q12, Q13 | Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13 | Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

## Fluency

WE11

**1** Use a number line to perform each of the following additions.

| | | | |
|---|---|---|---|
| **(a)** 8 + 3 | **(b)** 7 + 1 | **(c)** 3 + 8 | **(d)** 1 + 7 |
| **(e)** -9 + 7 | **(f)** -11 + 6 | **(g)** -10 + 5 | **(h)** -12 + 6 |
| **(i)** -8 + 14 | **(j)** -7 + 19 | **(k)** -5 + 30 | **(l)** -20 + 40 |
| **(m)** -30 + 20 | **(n)** -17 + 13 | **(o)** -75 + 55 | **(p)** -87 + 56 |

WE12

**2** Use a number line to perform each of the following subtractions.

| | | | |
|---|---|---|---|
| **(a)** 5 – 3 | **(b)** 8 – 2 | **(c)** 12 – 7 | **(d)** 11 – 11 |
| **(e)** 8 – 14 | **(f)** 6 – 17 | **(g)** 10 – 18 | **(h)** 11 – 15 |
| **(i)** 30 – 40 | **(j)** -3 – 5 | **(k)** 2 – 22 | **(l)** -5 – 7 |
| **(m)** -9 – 3 | **(n)** -24 – 6 | **(o)** -35 – 34 | **(p)** -66 – 58 |

**3** **(a)** -5 + 3 may be calculated by following which of these instructions?

**A** start at +5 and walk 3 steps to the right, arriving at 8

**B** start at -5 and walk 3 steps to the right, arriving at -2

**C** start at +5 and walk 3 steps to the left, arriving at 2

**D** start at -5 and walk 3 steps to the left, arriving at -8

**(b)** -15 – 10 may be described by which number line journey?

**A** start at -15 and walk 10 steps to the right, arriving at -5

**B** start at +10 and walk 15 steps to the left, arriving at -5

**C** start at +15 and walk 10 steps to the left, arriving at +5

**D** start at -15 and walk 10 steps to the left, arriving at -25

**4** Calculate the following.

**(a)** 3 + 4 + 5 **(b)** 5 + 2 + 4 **(c)** 7 + 1 + 4

**(d)** -8 + 6 + 9 **(e)** -29 + 5 + 6 **(f)** -37 + 7 + 8

**(g)** 6 – 7 – 4 **(h)** 3 – 5 – 8 **(i)** 12 – 16 – 4

**(j)** 14 + 9 – 10 **(k)** 23 – 30 + 6 **(l)** 44 – 51 + 8

## Understanding

**5** The temperature in the desert is -12°C, but rises by 20 degrees. What is the new temperature?

**6** A lift is at the third basement floor (3 floors below ground level). It moves up 14 levels, then stops. On which floor does it stop?

**7** A submarine 30 m below the surface of the ocean rises 16 m, then dives a further 20 m. What depth is it at now?

**8** A diver jumps from a platform 12 m high and dives 17 m to the bottom of a pool. How deep is the pool?

**9** You have $5 in your pocket, but you owe a friend $12 and someone else owes you $20. If everyone pays up, how much will you have at the end?

**10** Write the following number sentences in words.

**(a)** +5 – 7 = -2

**(b)** -8 + 9 = +1

**(c)** -4 – 3 = -7



## Reasoning

**11** **(a)** Show the following pairs of calculations on separate number lines.

**(i)** -1 + 2 and +2 – 1 **(ii)** -7 + 4 and +4 – 7 **(iii)** -13 + 6 and +6 – 13

**(b)** What do you notice? Comment on your observation.

**(c)** Use your observation from **(b)** to write the answers to the following without using a number line.

**(i)** -37 + 86 **(ii)** -72 + 95 **(iii)** -104 + 119

**12** **(a)** Draw a number line from -12 to 12. Draw a number line journey showing 5 – 3 = 2 and the 'mirror image' journey showing -5 + 3 = -2.

**(b)** Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences:

**(i)** 8 – 6 = 2 **(ii)** 5 – 4 = 1 **(iii)** -10 + 6 = -4 **(iv)** -12 + 9 = -3

**(c)** Calculate these additions on the negative side of the number line by considering their positive 'mirror image' journeys.

**(i)** -23 + 17 **(ii)** -48 + 27 **(iii)** -59 + 34 **(iv)** -62 + 43

## Open-ended

**13** The thermometer at a weather station in the desert near Alice Springs read -4°C at 6 a.m. The temperature then rose until it reached a maximum of 25°C at 12 p.m. What might the thermometer readings have been at 7, 8, 9, 10 and 11 a.m.?




**14** Write down three different positive integers that when added to -13 give a negative answer.

## Outside the Square — Problem solving

### The slippery snail

A snail is at the bottom of a low garden wall 40 cm high. If the snail manages to move 4 cm up the wall each day, but slips down 3 cm each night, how long does it take the snail to reach the top of the wall?




**Strategy options**

- Draw a diagram.
- Act it out.
- Look for a pattern.

# Adding and subtracting negative integers

2.6

The '+' and '–' signs can be used to show an operation (addition or subtraction) or a direction (positive or negative). Where necessary, to avoid confusion, brackets are placed around the number and the direction sign in front of it, to separate it from an operation symbol.

For example, +2 + (-3) is said as 'positive 2 add negative 3'.

-4 – (+7) is said as 'negative 4 subtract positive 7'.

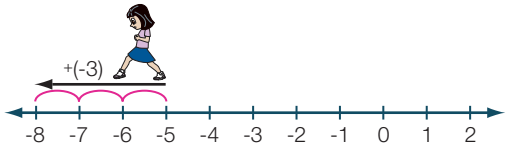
## Adding negative integers

Imagine that you owe a friend $5. We could write this as -5. Now, imagine that you owe a second friend $3, which we can write as -3. How much do you owe altogether? The answer is $8, which we can write as -8.

We 'add up' your total debt by calculating the sum: -5 + (-3) = -8. We can see that adding a negative number gives a 'more negative' result; that is, a number that is further towards the negative end of the number line.

We can show this on the number line in the following way.

1. Start at -5.
2. Walk along the number line 3 steps in the negative direction (to the left).
3. Arrive at the answer of -8.




-5 + (-3) on the number line shows the same journey as the subtraction -5 – 3, or -5 – (+3).

The movement along the number line is the same, and the end result of -8 is the same.

This means that adding -3 and subtracting +3 are exactly the same process.

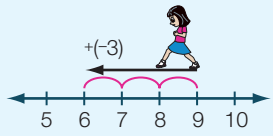
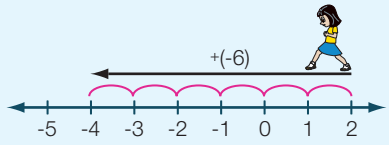
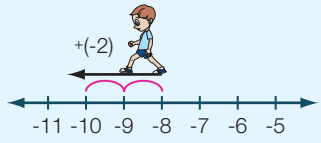
To add a negative number, subtract its opposite.

| | |
|---|---|
| -4 + (-7) | +2 + (-3) |
| = -4 – 7 | = +2 – 3 |
| = -11 | = -1 |

## Worked Example 13 WE13

Use a number line to perform each of the following additions.

**(a)** 9 + (-3) **(b)** 2 + (-6) **(c)** -8 + (-2)

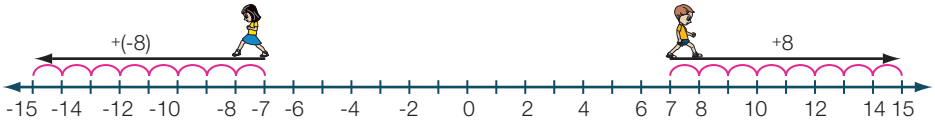
| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Draw or imagine a number line. Start at the first number (9) and walk the number of steps indicated by the second number (-3) in the negative direction (to the left). | *(a)*   |
| **2** | Write the addition as a number sentence. | *9 + (-3) = 6* |
| **(b)** **1** | Draw or imagine a number line. Start at the first number (2) and walk the number of steps indicated by the second number (-6) in the negative direction (to the left). | *(b)*   |
| **2** | Write the addition as a number sentence. | *2 + (-6) = -4* |
| **(c)** **1** | Draw or imagine a number line. Start at the first number (-8) and walk the number of steps indicated by the second number (-2) in the negative direction (to the left). | *(c)*   |
| **2** | Write the addition as a number sentence. | *-8 + (-2) = -10* |

## Using number line symmetry

The symmetry of the number line can be useful when adding negative numbers.

To calculate 7 + 8, we start at 7 and move to the right on the number line 8 steps to 15.

-7 + -8 is the mirror image of this. We start at -7, facing to the left and move to the left on the number line 8 steps to -15.

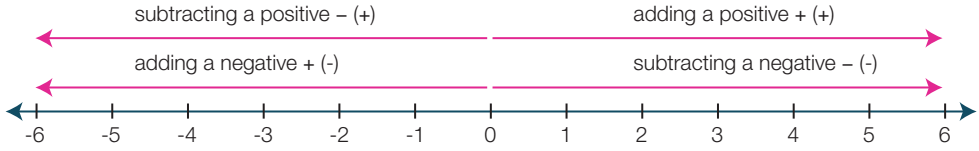



So, to calculate -47 + (-59), we can perform the 'mirror image' calculation of 47 + 59, then place a negative sign in front of the answer.

## Subtracting negative integers

It can be difficult to imagine what it means to subtract a negative number. We have seen, however, that adding a negative number means moving along the number line to the left; that is, in the negative direction. Subtraction is the opposite of addition. So, subtracting a negative number means moving along the number line to the right; that is, in the positive direction.

We can think of subtracting a negative number as making the other number 'less negative', which, in turn, makes it 'more positive'.



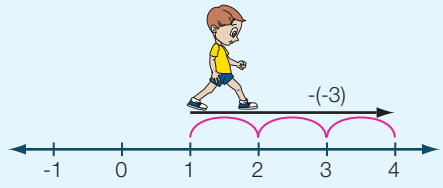
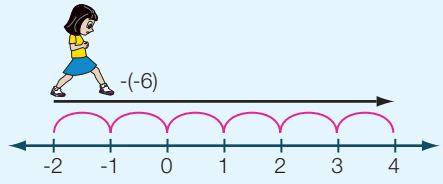
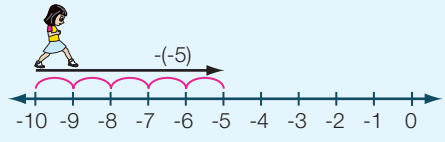

To subtract a negative number, add its opposite.

| | |
|---|---|
| 3 – (-1) | -7 – (-5) |
| = 3 + 1 | = -7 + 5 |
| = 4 | = -2 |

## Worked Example 14 WE14

Use a number line to perform each of the following subtractions.

**(a)** 1 – (-3) **(b)** -2 – (-6) **(c)** -10 – (-5)

| Thinking | Working |
|---|---|
| **(a) 1** Draw or imagine a number line. Start at the first number (1) and walk the number of steps indicated by the second number (-3) in the positive direction (to the right). | *(a)*   |
| **2** Write the subtraction as a number sentence. | 1 – (-3) = 4 |
| **(b) 1** Draw or imagine a number line. Start at the first number (-2) and walk the number of steps indicated by the second number (-6) in the positive direction (to the right). | *(b)*   |
| **2** Write the subtraction as a number sentence. | -2 – (-6) = 4 |
| **(c) 1** Draw or imagine a number line. Start at the first number (-10) and walk the number of steps indicated by the second number (-5) in the positive direction (to the right). | *(c)*   |
| **2** Write the subtraction as a number sentence. | -10 – (-5) = -5 |

Note that we say the above Worked Examples in the following way.

- +1 – (-3) is 'positive one subtract negative three'
- -2 – (-6) is 'negative two subtract negative six'
- -10 – (-5) is 'negative ten subtract negative five'

The number line journeys above show that subtracting a negative number results in a journey in the positive direction of the number line. This is the same journey as adding the positive equivalent of the number.

# 2.6 Adding and subtracting negative integers

## Navigator

Answers page 631

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q5, Q6, Q9, Q10, Q11, Q12, Q15 | Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q15 | Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15 |

## Fluency

WE13

**1** Use a number line to perform each of the following additions.

| | | | |
|---|---|---|---|
| **(a)** 5 + (-4) | **(b)** 3 + (-2) | **(c)** 12 + (-3) | **(d)** 8 + (-2) |
| **(e)** 2 + (-6) | **(f)** 4 + (-9) | **(g)** 10 + (-17) | **(h)** 9 + (-20) |
| **(i)** -2 + (-7) | **(j)** -3 + (-8) | **(k)** -8 + (-12) | **(l)** -10 + (-24) |
| **(m)** -5 + (-5) | **(n)** -30 + (-60) | **(o)** -17 + (-12) | **(p)** -62 + (-71) |

WE14

**2** Use a number line to perform each of the following subtractions.

| | | | |
|---|---|---|---|
| **(a)** 10 – (-1) | **(b)** 4 – (-2) | **(c)** 5 – (-5) | **(d)** 6 – (-6) |
| **(e)** -3 – (-11) | **(f)** -9 – (-12) | **(g)** -4 – (-20) | **(h)** -17 – (-11) |
| **(i)** -3 – (-15) | **(j)** -9 – (-8) | **(k)** -5 – (-13) | **(l)** -19 – (-13) |
| **(m)** -30 – (-20) | **(n)** -55 – (-45) | **(o)** -67 – (-41) | **(p)** -81 – (-53) |

**3** **(a)** The finishing point for the number line journey +6 + (-12) may be found by following which of these instructions?

- **A** start at +6 and walk 12 steps to the left, finishing at -6
- **B** start at +6 and walk 12 steps to the right, finishing at +18
- **C** start at -12 and walk 6 steps to the left, finishing at -18
- **D** start at -6 and walk 12 steps to the right, finishing at +6

**(b)** The answer to +3 – (-20) may be found by following which of these instructions?

- **A** start at +3 and walk 20 steps to the left, arriving at -17
- **B** start at +3 and walk 20 steps to the right, arriving at 23
- **C** start at -20 and walk 3 steps to the right, arriving at -17
- **D** start at -3 and walk 20 steps to the left, arriving at -23

**4** Calculate the following.

| | | |
|---|---|---|
| **(a)** 9 + (-7) + (-2) | **(b)** 6 + (-5) + (-7) | **(c)** 9 + (-12) + (-4) |
| **(d)** -1 + (-4) + (-5) | **(e)** 10 + (-8) + (-7) | **(f)** 24 + (-12) + (-6) |
| **(g)** 2 – (-1) – (-2) | **(h)** 1 – (-6) – (-2) | **(i)** -4 – (-7) + (-6) |
| **(j)** -7 – (-10) + (-5) | **(k)** 12 + (-16) + (-4) | **(l)** 48 + (-32) – (-6) |

## Understanding

**5** The temperature at sunset was 12°C. During the night it fell by 16°C to the minimum temperature. What was the minimum temperature?

**6** A worker steps into a lift on the 7th floor of an office building and travels down 10 levels. Which level does he finish at? Write your answer as an integer.



**7** Your company is $2000 in debt. You then receive notice that you owe the bank $1500. What is your financial state now?

**8** A farmer lowers a bucket from the top of a 1 m high well using a 10 m rope attached to the handle. If she lowers the whole length of rope, how far below ground does the bucket reach?

**9** A bird flying 6 m above sea level suddenly spots a fish and dives 9 m straight down before catching the fish. How far below the surface was the fish?

**10** A miner is sitting on a ledge 8 m below the top of a vertical mine shaft when he drops his torch. If the torch falls 23 m before hitting the bottom of the shaft, how deep is the shaft?

**11** Write the following number sentences in words.

**(a)** $-5 + (-6) = -11$ **(b)** $+7 - (-8) = +15$ **(c)** $-9 - (-14) = +5$

## Reasoning

**12** **(a)** Draw a number line from -12 to +12. Draw number line journeys showing $2 + 3 = 5$ and $-2 + (-3) = -5$.

**(b)** Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences.

**(i)** $1 + 6 = 7$ **(ii)** $4 + 7 = 11$ **(iii)** $-3 + (-8) = -11$ **(iv)** $-5 + (-4) = -9$

**(c)** Work out these calculations on the negative side of the number line by considering their positive 'mirror image' journeys.

**(i)** $-21 + (-15)$ **(ii)** $-28 + (-27)$ **(iii)** $-36 + (-41)$ **(iv)** $-65 + (-29)$

**13** **(a)** Evaluate the following.

**(i)** $12 - (+6)$ **(ii)** $12 - (-6)$ **(iii)** $-3 - (+6)$ **(iv)** $-3 - (-6)$

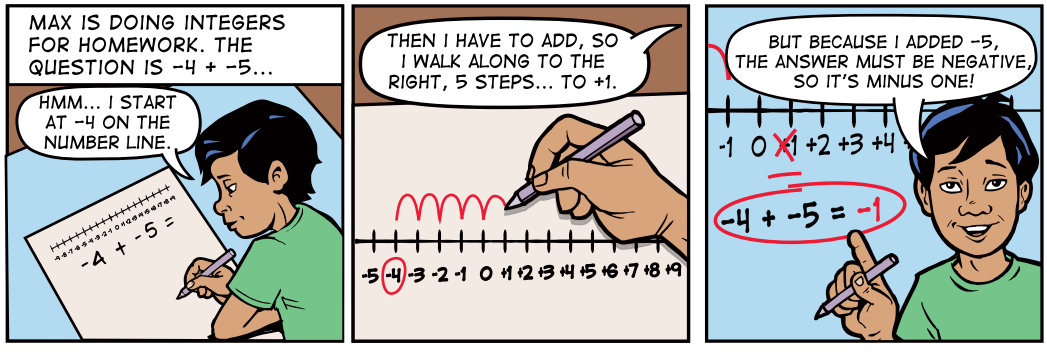
**(b)** Copy and complete the following sentence.

Whether you start with a positive or a negative integer, if you subtract a positive number, then the result is _____ and if you subtract a negative number, then the result is _____.

## Open-ended

**14** Write down three integers that when subtracted from -21 give a positive answer.

**15**




**(a)** What is the correct answer?

**(b)** Explain to Max where he went wrong, and how to do these types of questions correctly in the future.

# Investigation

## Heat beads and ice blocks

**Equipment required:** 1 brain, 10 red and 10 blue counters (or any other pair of colours)

### The Big Question

Positive and negative integers can be represented by different coloured counters. How can we use them to show integer addition and subtraction?

### Engage

We will use the red counters to represent positive integers and the blue counters to represent negative integers.

For example:  = +3

 = -5

Because they are opposites, 1 blue counter will cancel out, or 'annihilate', 1 red counter. In other words, 1 blue + 1 red = 0. We can think of this as 1 'heat bead' (red) cancelling out 1 'ice block' (blue).

 = 0

We can write this as a number sentence: 1 + -1 = 0.

We can use combinations of red and blue counters to represent integers, by cancelling red and blue pairs.

For example:

 -7

+

 -4

= -3

(We have shown here that 4 red and 4 blue pairs cancelled each other.)

**1** Place 3 red counters on the table to represent +3. Now, place 2 blue counters underneath them.

What number is represented now (remembering that 2 blues will cancel 2 reds)?

Complete the number sentence: +3 + -2 = _____ .

**2** Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

**(a)** +3 + -4

**(b)** +2 + -5

**(c)** +4 + -6

**3** Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

**(a)** -4 + +1

**(b)** -7 + +3

**(c)** -9 + +2

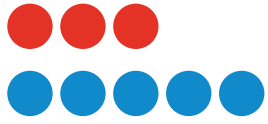
### Explore

**4** So far, we have used the counters to model addition. Modelling subtraction can be a little more challenging.

**Strategy options**

- Draw a diagram.
- Act it out.

**(a)** What number is shown by this collection of counters?



**(b)** If you removed the red counters, what number would be shown now?

Complete the number sentence:
-2 – (+3) = _____

**5** Notice that in order to perform this subtraction, we had to have 3 red counters present, so that we could show the subtraction by removing them. They were balanced or 'annihilated' by 3 of the blue counters, so they had no effect on the starting number.

Use your counters to do the following subtractions. You may need to add pairs of red and blue counters in order to have enough to remove.

**(a)** -4 – +2

**(b)** -1 – +3

**(c)** -2 – +5

6 Use your counters to do the following subtractions. Again, you may need to add pairs of counters at the beginning.

**(a)** +3 – +4

**(b)** +2 – +5

**(c)** +4 – +6

7 Looking at the counters diagram in Question **4**, what number would be shown if you removed all of the blue counters? Write this as a subtraction.

8 Use your counters to do the following subtractions.

**(a)** -4 – -1

**(b)** -7 – -3

**(c)** -9 – -2

9 Use your counters to do the following subtractions.

**(a)** -2 – -4

**(b)** -5 – -1

**(c)** -3 – -7

10 Use your counters to do the following subtractions.

**(a)** +2 – -4

**(b)** +5 – -1

**(c)** +3 – -7

## Explain

11 **(a)** What do you notice about the answers to Question **2** and Question **6**? What conclusion can you draw from this?

**(b)** What do you notice about the answers to Question **3** and Question **8**? What conclusion can you draw from this?

**(c)** What do you notice about the answers to Question **9** and Question **10**? Why is this the case?

12 Why was it necessary to use counters of both colours to represent your starting number in Questions **5** and **6**, and **9** and **10**?

13 Helen wants to use her red and blue counters to show +3 – -5. She has lined up 3 red counters to show +3.



She has no blue counters to remove. Explain how Helen can include 5 blue counters in her representation of +3.

## Elaborate

14 For each of the following, choose an integer between -10 and +10, and use counters of one or both colours to represent it. (For example, you might choose -3, which you could show with 3 blue counters, or 2 red and 5 blue counters.) Then, add or subtract a second number of counters to show the operation described. Write a number sentence for the operation you have modelled, and draw a diagram of the counters.

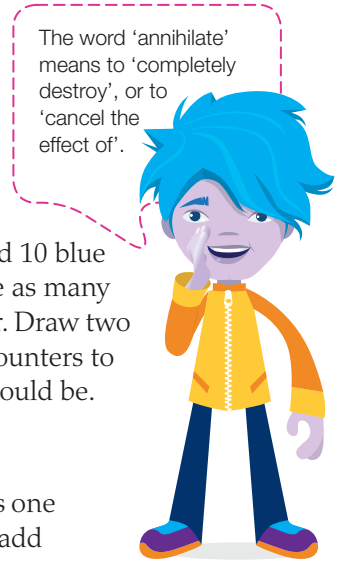
**(a)** adding a positive integer to a negative integer

**(b)** adding a negative integer to a positive integer

**(c)** subtracting a positive integer from a negative integer

**(d)** subtracting a negative integer from a positive integer

15 Explain why this method of using counters to show integer addition and subtraction is called the 'annihilation method'.




16 Sam has made a number using some of his 10 red and 10 blue counters. He has used twice as many of one colour than the other. Draw two different arrangements of counters to show what Sam's number could be.

## Evaluate

17 Using counters in this way is one method of learning how to add and subtract negative numbers. Another method is to use a number line, which is presented elsewhere in this chapter.

Do you find one method easier to work with than the other? Which one? Why do you think this is?

## Extend

18 Can you use your counters to show multiplication?

**(a)** Start by showing 2 × -4 and 3 × -3. It may be useful to remember that multiplication means 'groups of'.

**(b)** Can you show -4 × -2 or -3 × -3? How is this method of using counters limited here?

# 2.7 Simplifying addition and subtraction

You have seen that we can simplify operations with directed numbers by omitting the '+' sign in front of positive numbers. For example, +7 can be written simply as 7. We can use this to write additions and subtractions more simply.

+4 + (+5) = +9 can be written as 4 + 5 = 9

+4 – (+5) = -1 can be written as 4 – 5 = -1

We can simplify other expressions by replacing the two signs between the numbers with a single sign.

We have seen that adding a negative number and subtracting a positive number are the same. Both result in a movement to the left on the number line; that is, subtraction.

7 + (-3) is the same as 7 – (+3). Both can be written as 7 – 3.

+ (-) and – (+) can be replaced by –.

We have also seen that adding a positive number and subtracting a negative number are the same. Both result in a movement to the right on the number line; that is, addition.

6 + (+2) is the same as 6 – (-2). Both are equal to 6 + 2.

+ (+) and – (-) can be replaced by +.




We can summarise these observations with the following rules for adding and subtracting integers:

If the two signs are the same, the result is addition.
+ (+) = +
– (-) = +

If the two signs are opposite, the result is subtraction.
– (+) = –
+ (-) = –

To apply these rules the two signs must be *next* to each other when the calculation is written down. The rules do not apply otherwise.

## A physical method

If you find the above rules a little bit difficult to remember, here is another physical method of adding and subtracting directed numbers:

1. Draw or imagine a number line. Place your pen, or imagine you are standing, on the first number.
2. Look at the operations symbol:

   If it is +, turn and face right (the positive end of the number line).

   If it is –, turn and face left (the negative end of the number line).
3. Look at the sign of the second number:

   If it is positive, walk forward that many paces.

   If it is negative, walk backwards that many paces.

This method can help you become familiar with knowing which combinations of symbols result in addition (movement to the right of the number line), and which result in subtraction (movement to the left of the number line).

## Worked Example 15 WE15

Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

**(a)** 4 + (+9) **(b)** -1 + (-3) **(c)** 10 – (+1) **(d)** -6 – (-5)

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Adding a positive number is straightforward addition. Replace + (+) with +. | (a) 4 + (+9)<br>= 4 + 9 |
| | **2** | Calculate. | = 13 |
| **(b)** | **1** | Adding a negative number is equivalent to subtraction. Replace + (-) with –. | (b) -1 + (-3)<br>= -1 – 3 |
| | **2** | Calculate. | = -4 |
| **(c)** | **1** | Subtracting a positive number is straightforward subtraction. Replace – (+) with –. | (c) 10 – (+1)<br>= 10 – 1 |
| | **2** | Calculate. | = 9 |
| **(d)** | **1** | Subtracting a negative number is equivalent to addition. Replace – (-) with +. | (d) -6 – (-5)<br>= -6 + 5 |
| | **2** | Calculate. | = -1 |

# 2.7 Simplifying addition and subtraction

## Navigator

Answers page 631

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2, Q3 Columns 1–3, Q4, Q5 Columns 1 & 2, Q6, Q7, Q10, Q13 | Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q5 Columns 2 & 3, Q6, Q8, Q9, Q10, Q12, Q13 | Q1 Column 4, Q2, Q3 Column 4, Q4, Q5 Columns 2–4, Q6, Q8, Q9, Q10, Q11, Q12, Q13 |

## Fluency

WE15

**1** Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

| | | | |
|---|---|---|---|
| **(a)** +5 + (+2) | **(b)** -1 + (+4) | **(c)** +5 + (+6) | **(d)** -9 + (+7) |
| **(e)** +5 – (+3) | **(f)** -8 – (+4) | **(g)** +1 – (+12) | **(h)** -3 – (+7) |
| **(i)** +12 + (-3) | **(j)** +10 + (-4) | **(k)** +7 + (-9) | **(l)** +4 + (-8) |
| **(m)** -7 – (-6) | **(n)** -3 – (-9) | **(o)** +2 – (-10) | **(p)** +1 – (-1) |
| **(q)** -7 + (-3) | **(r)** -9 + (-3) | **(s)** -4 – (-9) | **(t)** -2 – (-6) |

**2** State TRUE or FALSE for the following.

| | |
|---|---|
| **(a)** 25 + (+5) = 25 – 5 | **(b)** 30 + (-10) = 30 – 10 |
| **(c)** 15 – (+20) = 15 – 20 | **(d)** 40 – (-50) = 40 – 50 |

**3** Calculate:

| | | | |
|---|---|---|---|
| **(a)** 6 – 18 | **(b)** 9 – 20 | **(c)** -5 + 9 | **(d)** -3 + 6 |
| **(e)** -12 + 7 | **(f)** -10 + 2 | **(g)** -9 – 4 | **(h)** -7 – 5 |
| **(i)** -60 + 30 | **(j)** -52 + 21 | **(k)** -27 + 84 | **(l)** -28 + 93 |

**4** **(a)** -9 – (-6) – (+4) simplifies to:

**A** -9 – 6 – 4 **B** -9 + 6 + 4 **C** -9 + 6 – 4 **D** 9 + 6 + 4

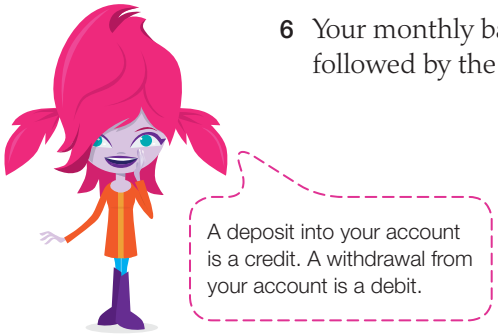
**(b)** 14 – (+4) + (-3) is the same as:

**A** -14 – 4 – 3 **B** 14 – 4 – 3 **C** 14 – 4 + 3 **D** 14 + 4 + 3

**5** Calculate:

| | | | |
|---|---|---|---|
| **(a)** -6 + 7 + 5 | **(b)** -4 + 6 + 5 | **(c)** -12 + 3 + 2 | **(d)** -11 + 3 + 4 |
| **(e)** -8 + 5 – 3 | **(f)** -6 + 2 – 4 | **(g)** -4 + 12 – 4 | **(h)** -3 + 10 – 6 |
| **(i)** -5 – 1 – 6 | **(j)** -2 – 5 – 4 | **(k)** 36 – 40 – 5 | **(l)** 43 – 50 – 2 |

## Understanding




**6** Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions for the month listed below.

| | |
|---|---|
| Credit | $30 |
| Debit | -$80 |
| Credit | $200 |
| Debit | -$60 |
| Debit | -$500 |

What is your account balance at the end of the month?

**7** A maintenance worker in an office block starts work on the ground floor, and then travels as follows.

UP 10 floors
DOWN 7 floors
DOWN 5 floors
UP 4 floors
DOWN 6 floors
UP 12 floors

**(a)** What was the highest floor the worker reached?

**(b)** What was the lowest floor he travelled to?

**(c)** Which floor did he finish on?

**8** Diana makes and sells soft toys. She keeps records of her profits and losses over a 10-week period. These records are shown in the table. Find Diana's overall profit or loss during this time.

| Week | Result | |
|---|---|---|
| 1 | Profit | $120 |
| 2 | Profit | $25 |
| 3 | Loss | -$70 |
| 4 | Profit | $210 |
| 5 | Loss | -$150 |
| 6 | Loss | -$180 |
| 7 | Loss | -$90 |
| 8 | Profit | $40 |
| 9 | Loss | -$160 |
| 10 | Loss | -$95 |



## Reasoning

**9** Complete the following addition and subtraction grids. Begin with the number at the top of each column, and either add or subtract the numbers at the left of each row.

**(a)**

| + | -6 | +20 | | 31 |
|---|---|---|---|---|
| +4 | | | | |
| | | +13 | | |
| | +2 | | | |
| | | | -23 | 22 |

**(b)**

| – | 8 | -11 | | |
|---|---|---|---|---|
| | 5 | | | |
| -7 | | | 26 | |
| +5 | | | | -30 |
| | | -1 | | |

**10** A 'magic square' is a square grid of numbers where the sum of each of the rows, columns and diagonals is the same number.

**(a)** What is the magic sum for this 3 × 3 magic square?

**(b)** Complete the magic square in part **(a)**.

| | 6 | -1 |
|---|---|---|
| | 2 | |
| 5 | | |

**11** You can make new magic squares by adding or subtracting the same number from every number in an existing magic square.

**(a)** Make a new 3 × 3 magic square by subtracting 4 from each number in the magic square in Question **10 (a)**.

**(b)** What is the new magic sum? Explain why the new magic sum is not 4 less than the old magic sum.

## Open-ended

**12** In the sport of golf, each 'hole' has a particular number of 'shots' it should take you to get the ball from the tee to the hole. This is known as the 'par' for the hole. If you take this number of shots to get the ball in the hole, your score for the hole is 0. If your number of shots for the hole is *less than par*, this is shown on the bottom row of the scorecard by a *negative number*. If your number of shots is *greater than par*, this is shown with a *positive number*.



**(a)** Carla played nine holes of golf. She scored under par on four holes, par on three holes and over par for the rest. If Carla took a total of 35 shots to complete her round, fill in what her scorecard may have looked like.

| Hole number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Par for the hole | 4 | 4 | 5 | 3 | 4 | 5 | 4 | 3 | 4 | 36 |
| Shots taken | | | | | | | | | | 35 |
| Score compared to par | | | | | | | | | | |

**(b)** Carla played another round of nine holes. Her overall score was -1 (1 under par). What could her scorecard have looked like, if she did not score par (0) on any hole?

**13** Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

## Outside the Square Puzzle

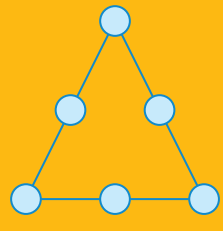
### Pattern sums

Copy and complete each of the following patterns. Arrange the numbers given, one in each circle, so that the sum of each line is equal to the stated value.

**1** Use -4, -3, -2, -1, 0, 1, 2; sum = -3



**2** Use -2, -1, 0, 1, 2, 3; sum = 0



# Challenge 2





**1** What are the next three numbers in each pattern?

**(a)** 17, 13, 9, 5, …

**(b)** -31, -25, -19, -13, …

**2** The product of three brothers' ages is 72 and their sum is 14. The youngest of the brothers are twins. What are the ages of the brothers?

**3** Find three consecutive numbers (numbers that come one after another, such as 5, 6, 7) so that the sum of the first and third numbers is 172.

**4** In a school there are 150 students in Year 7. For a performance in the school hall, chairs are arranged in rows, with the same number of chairs in each row, so that all 150 students are seated with no spare seats. If 10 more chairs were added to each row, everyone could be seated in 4 fewer rows, allowing the people in the back row to be closer to the stage. How many chairs were in each row in the original seating arrangement?

**5** What is the sum of all the digits in the numbers from 1 to 100?

**6** Shannon needs to know the total amount of money her friends have raised for the 'Save the Koala' fund. All they will tell her is this:

Patricia and Tim have collected $130 between them.
If Tim and Jenni pooled their money, they would have $150.
Jenni and Robyn have $100 altogether.
Robyn and Hilary have just $70 in total.
If Hilary and Patricia combined their money, they would have $90.
Tell Shannon what she needs to know.

**7** A number less than 100 gives a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6. Find the number.

**8** 41 is a prime number. If the order of its digits is reversed, it becomes 14 which is NOT a prime number. The number of two-digit prime numbers that DO give a prime number when their digits are reversed is:

**A** 8 **B** 9 **C** 10 **D** 11

**9 (a)** How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 19$?

**A** 1 **B** 2 **C** 3 **D** 4

**(b)** How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 29$?

**A** 3 **B** 4 **C** 5 **D** 6

**(c)** How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 59$?

**A** 9 **B** 10 **C** 11 **D** 12

**(d)** How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 99$?

**A** 18 **B** 19 **C** 20 **D** 21

# Chapter review 2

## D.I.Y. Summary

### Key Words

| | | |
|---|---|---|
| common factor | factor | positive |
| common multiple | Highest Common Factor (HCF) | prime factor |
| composite number | integers | prime number |
| coprime | loss | profit |
| deposit | Lowest Common Multiple (LCM) | withdrawal |
| divisibility | multiple | |
| divisible | negative | |

1 A _____ of 6 is 18. The _____ of 6 and 4 is 12.

2 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the _____s of 18. The _____ of 18 and 27 is 9.

4 A number that is not _____ by any numbers other than 1 and itself is called a _____.

5 A number with more than two factors is called a _____.

6 When you put money into a bank account, you are making a _____.

7 The addition of two negative numbers will always give a _____ answer.

8 If you sell something for less money than you bought it for, you have made a _____.

9 Every whole number greater than 1 can be written as the unique product of its _____s.

10 You make a _____ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a _____.

12 Two numbers are _____ if their highest common factor is 1.

## Fluency

1 Find the LCM of: Ex 2.1

**(a)** 9 and 6 **(b)** 9 and 12 **(c)** 10 and 15

2 List all the factors of: Ex 2.1

**(a)** 36 **(b)** 48 **(c)** 51 **(d)** 100

3 Find the HCF of: Ex 2.1

**(a)** 24 and 56 **(b)** 18 and 72 **(c)** 45 and 80

4 Copy the following table and do the divisibility tests. Circle the number if the original number is divisible by it. Ex 2.1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5301 | 2 | 3 | 4 | 5 | 6 | 9 | 10 |
| 10 000 | 2 | 3 | 4 | 5 | 6 | 9 | 10 |
| 333 333 | 2 | 3 | 4 | 5 | 6 | 9 | 10 |
| 31 700 | 2 | 3 | 4 | 5 | 6 | 9 | 10 |
| 43 521 820 | 2 | 3 | 4 | 5 | 6 | 9 | 10 |

**5** State whether each of the following numbers is a prime number or a composite number, and explain why. Ex 2.2

**(a)** 5 **(b)** 16 **(c)** 77 **(d)** 276 350

**6** By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors. Ex 2.3

**(a)** 24 **(b)** 30 **(c)** 88 **(d)** 200

**7** Use prime factors to find the HCF of: Ex 2.3

**(a)** 27 and 36 **(b)** 72 and 96 **(c)** 108 and 240

**8** Write an integer to represent the following. Ex 2.4

**(a)** 14 degrees below zero **(b)** an altitude of 200 metres

**9** State the opposite of: Ex 2.4

**(a)** north 5 km **(b)** adding 27

**10** Write < or > between the following pairs of numbers to make a true statement. Ex 2.4

**(a)** -52 _____ 25 **(b)** 19 _____ -20

**11** Arrange the following numbers in ascending order. Ex 2.4

**(a)** -7, 12, 0, -9, 7 **(b)** 4, -4000, 40, 400

**12** Calculate: Ex 2.5

**(a)** +16 + 2 **(b)** -3 + 18 **(c)** -15 + 5 **(d)** +9 – 3

**(e)** +1 – 5 **(f)** +16 – 8 **(g)** +7 – 12 **(h)** -14 – 18

**13** Calculate: Ex 2.6

**(a)** +7 + (-10) **(b)** +9 + (-6) **(c)** -11 – (-4) **(d)** -4 – (-4)

**(e)** -12 – (-5) **(f)** +5 + (-3) **(g)** -8 – (-5) **(h)** -5 + (-7)

**14** Rewrite the following with a single sign between the integers, then evaluate. Ex 2.7

**(a)** 9 – (+11) **(b)** -3 + (+10) **(c)** -10 – (-21) **(d)** 8 + (-12)

**(e)** -4 – (-41) **(f)** -14 + (+28) **(g)** -5 – (+8) – (-2) **(h)** 4 + (-9) + (+2)

## Understanding

**15** Use the words 'multiple', 'factor' and 'divisible' to complete the following sentences.

**(a)** 45 is _____ by 9, so that makes it a _____ of 9.

**(b)** 8 is a _____ of 56, so 56 is a _____ of 8.

**(c)** 27 is _____ by 3, so that makes it a _____ of 3.

**16** If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations.

**17** Use the symmetry of the number line to help you calculate the following.

**(a)** -31 + 19 **(b)** -54 – 27 **(c)** -22 + (-38) **(d)** -9 – (61)

**18** Describe the number line journey you could follow to find the value of:

**(a)** +3 + (-8) **(b)** -6 – (+5)

**19** For each pair of numbers, state whether or not they are coprime. If not, explain why.

**(a)** 11 and 27 **(b)** 51 and 63 **(c)** 14 and 35 **(d)** 24 and 55

**20** In the game of indoor cricket, 3 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game.

Ball 1: 2 Runs
Ball 2: Wicket
Ball 3: 1 Run
Ball 4: Wicket
Ball 5: 1 Run
Ball 6: 4 Runs

What was the score at the end of the over?

**21** There are 84 male and 108 female guests at an official dinner. The dinner organiser wants to have an equal number of male and female guests at each table.

**(a)** Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

**(b)** Use your answer from **(a)** to find the number of male and female guests at each table.

**22** Find the first common multiple of 2, 7 and 9 that is greater than 500.

**23** **(a)** What is the first prime number after 70?

**(b)** What is the first composite number after 70?

**24** Michelle made deposits of $210, $25, $45 and $66 into her bank account during one month, and withdrawals of $35, $56, $214 and $102 during the same period.

**(a)** At the end of the period, had her balance increased or decreased?

**(b)** By how much had it increased or decreased?

**25** Joanna and Petra are on two different ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level?

**26** Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers.

## Reasoning

**27** Copy the following and write <, = or > to make true statements.

**(a)** $4 - (-7) ____ 4 + 7$

**(b)** $-3 + 2 ____ 3 - 2$

**(c)** $5 + (-3) ____ 5 - (+3)$

**(d)** $-8 - 9 ____ -8 + 9$

**28** If you know that a number is divisible by 8, what other numbers do you also know it is divisible by?

**29** A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor?

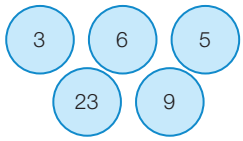
# NAPLAN practice 2

## Numeracy: Non-calculator

**1** Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

**A** -15 **B** -7 **C** 7 **D** 15

**2**




The sum of the composite numbers in the group shown above is:

**A** 8 **B** 15 **C** 18 **D** 31

**3** A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does he get out on?

**4** The number 42 written as a product of prime factors is:

**A** $21 \times 2$ **B** $2 \times 3 \times 7$ **C** $1 \times 42$ **D** $6 \times 7$

**5** On a sunny winter's day in Moscow, the temperature at midday was 3°C. By midnight it had dropped to -9°C. The integer that represents this change is:

**A** -12 **B** -3 **C** 3 **D** 12

## Numeracy: Calculator allowed

**6** A train passes a town every week and every 10 days an aeroplane flies overhead. If the train and the plane were observed to be at the town on a certain day, after how many more days will both appear again at the same time?

**A** 10 **B** 11 **C** 50 **D** 70

**7** Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit: $55
Withdrawal: -$75
Withdrawal: -$33
Deposit: $85
Withdrawal: -$27

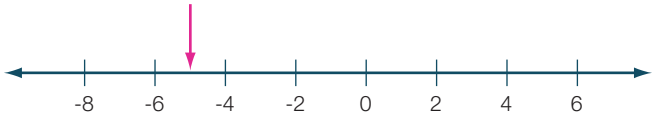
How much does Vin have in his account at the end of the month?

**8** The ages in years of three people are 65, 39 and 52. The highest common factor of the three ages is:

**A** 1 **B** 13 **C** 39 **D** 165

**9** The arrow is pointing to an integer on the number line.

What number is at this position? _____



# Mixed review A

## Fluency

**1** Write these integers in ascending order. Ex 2.4

8, 17, -10, 0, -25, 32, -48

**2** Write the following in index form. Ex 1.2

**(a)** 9 squared **(b)** $7 \times 7 \times 7 \times 7$ **(c)** 4 cubed

**3** List all numbers divisible by both 8 and 6 that are less than 100. Ex 2.1

**4** Write an integer to represent the following. Ex 2.4

**(a)** a bank withdrawal of $570 **(b)** a win by 5 points

**5** Write < or > between the following pairs of numbers to make a true statement. Ex 2.4

**(a)** -27 ____ 14 **(b)** 0 ____ -35

**6** Calculate: Ex 1.3

**(a)** $8000 \div 200$ **(b)** $1200 \div 4$ **(c)** $45\,000 \div 90$

**(d)** $30 \times 120$ **(e)** $400 \times 1500$ **(f)** $2000 \times 5000$

**7** Use a mental strategy to calculate the following. Ex 1.1

**(a)** $4 \times 17 \times 5$ **(b)** $183 + 220$ **(c)** $42 \times 19$

**(d)** $36 \times 11$ **(e)** $169 + 71$ **(f)** $5 \times 24 \times 8$

**8** Evaluate: Ex 1.5

**(a)** $6 \times 4 \div 2 \times 6$ **(b)** $5 + 6 \times 7$ **(c)** $18 + 12 - 7 + 6$

**(d)** $2 + 5 \times 9$ **(e)** $18 \div 6 - 3$ **(f)** $8 \times (15 - 5)$

**9** Calculate the following. Ex 2.5, 2.6

**(a)** $-9 + 7$ **(b)** $5 + (-8)$ **(c)** $-3 - (-7)$ **(d)** $-6 - 11$

**10** Estimate the answers to the following by rounding to the first digit. Ex 1.4

**(a)** $17 \times 93$ **(b)** $46 \times 281$ **(c)** $337 \times 240$

**(d)** $953 \div 11$ **(e)** $8195 \div 237$ **(f)** $12\,495 \div 5400$

**11** Arrange the following numbers in ascending order (from smallest to largest). Ex 2.4

**(a)** 5, 0, -15, 10, -5 **(b)** -300, 3, 0, -30, 3000

**12** Find the lowest common multiple of: Ex 2.1

**(a)** 8 and 12 **(b)** 12 and 16

**13** Find the highest common factor of: Ex 2.1

**(a)** 36 and 27 **(b)** 64 and 72

**14** List the factors of each of these numbers and state whether each number is prime or composite. Ex 2.2

**(a)** 18 **(b)** 23 **(c)** 44 **(d)** 79

**15** Write each number as the product of its prime factors in index form. Ex 2.3

**(a)** 63 **(b)** 48 **(c)** 72 **(d)** 120

**16** Simplify the following by writing a single sign between the values, then calculate the answer.

Ex 2.7

**(a)** -6 + (-7) **(b)** 4 – (-11) **(c)** -2 + (+7)

**(d)** 22 + (-9) **(e)** -5 – (-10) **(f)** 18 + (+3)

## Understanding

**17** A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now?

**18** Which of the following numbers are:

**(a)** prime

**(b)** perfect squares

**(c)** powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

**19** What is the first perfect cube that is divisible by both 3 and 4?

**20** Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower.

**(a)** $256 \times 37$ **(b)** $1379 \times 24$ **(c)** $5498 \div 46$

**21** The Royal Easter Show runs for 7 days. The total attendance at this year's show was 62 982.

**(a)** Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

**(b)** If each person paid an average ticket price of $12, use your answer from **(a)** to calculate how much money the show organisers made from ticket sales. Use some mental or written strategies to calculate your answer.

## Reasoning

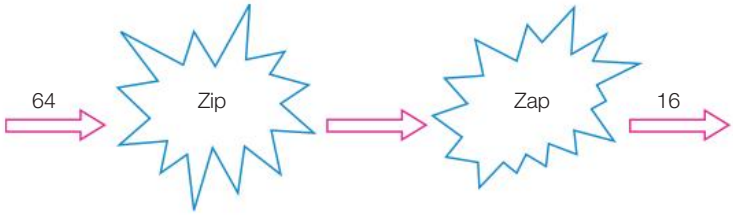
**22** On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account.

**(a)** By the end of the week, did Kiran have more or less money in his bank account?

**(b)** How much more or less?

**23** The number 64 passes through two 'magic clouds' Zip and Zap and emerges as the number 16.

Which of the following can describe what Zip and Zap did to the number passing through?




**A** Zip: square root, Zap: square **B** Zip: cube root, Zap: square

**C** Zip: square root, Zap: nothing **D** Zip: nothing, Zap: cube root


3

# Fractions

3

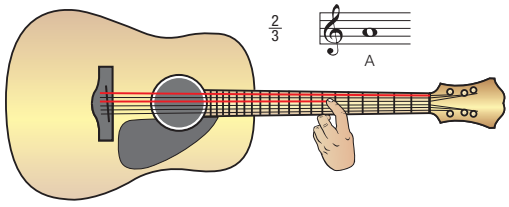
## Frets make fractions.

Learning the guitar or violin could help your understanding of fractions.

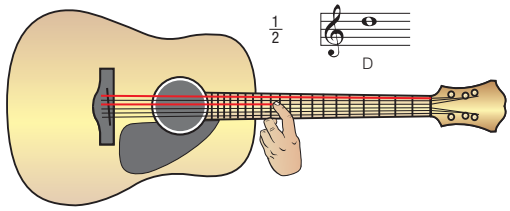
Guitarists and violinists know that shortening a string on their instrument produces a higher sounding note. What they may not realise is that for each note of a scale, the shortened string length is an exact fraction of the full length, or 'open' string. Plucking the third string on a guitar gives the note 'D':




Shortening the string to $\frac{2}{3}$ of its length gives the note 'A'.




Shortening the string to $\frac{1}{2}$ of its length gives the note 'D' an octave higher.




Instrument makers use this information to work out where to place the frets on the necks of guitars. Violins do not have frets, so the player must learn exactly where to place his or her fingers to produce different notes. You will find out other ways in which fractions are involved in music later in this chapter.

### Forum

Some studies claim that there is a link between musical and mathematical ability—that people who are good at maths are also good at music. What do you think? What could you do to investigate this theory?

### Why learn this?

Whether it be money, measurement or mixing a cake, we do not always work with whole numbers. Fractions are numbers that enable us to work with parts of wholes, and we need to be able to estimate and calculate with them.

**After completing this chapter you will be able to:**

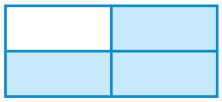
- identify and calculate fractions of a whole
- write one amount as a fraction of another
- simplify fractions and find equivalent fractions
- convert between fractions, whole numbers and mixed numbers
- use strategies to visualise, estimate and order fractions
- add, subtract, multiply and divide fractions
- solve problems involving fractions.

# Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

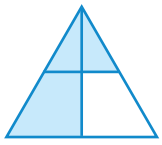
PP Worksheet R3.1

**1** Which of these have three-quarters of the whole shape shaded in?

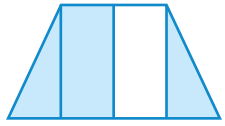
**(a)** 

**(b)** 

**(c)** 

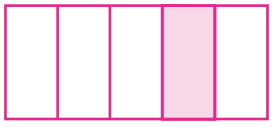
**(d)** 

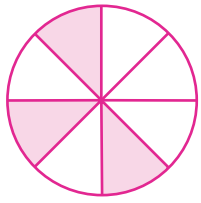
**(e)** 

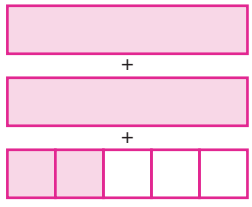
**(f)** 

PP Worksheet R3.2

**2** Write down a fraction to represent each of the following diagrams or descriptions.

**(a)** 

**(b)** 

**(c)** 

**(d)** One person's share of a birthday cake divided equally between 10 people.

**(e)** Joseph eats five out of eight equal-sized slices of a pizza.

**(f)** two-thirds **(g)** twelve-fifths **(h)** seven-sixteenths

PP Worksheet R3.3

**3** **(a)** Write these in order from largest to smallest: $\frac{3}{4}$, 1, 0, 2, $1\frac{1}{4}$

**(b)** Write these in order from smallest to largest: $\frac{3}{8}$, $\frac{1}{8}$, 0, $\frac{7}{8}$, $\frac{11}{8}$, 1, $\frac{9}{8}$

PP Worksheet R3.4

**4** Calculate:

**(a)** $\frac{4}{7} + \frac{1}{7}$ **(b)** $\frac{8}{11} - \frac{2}{11}$ **(c)** $\frac{2}{3} + \frac{2}{3}$

PP Worksheet R3.5

**5** **(a)** Write the first five multiples of 8. **(b)** Write the first five multiples of 12.

**(c)** Write the lowest common multiple (LCM) of 8 and 12.

**(d)** Write all the factors of 24. **(e)** Write all the factors of 36.

**(f)** Write the highest common factor (HCF) of 24 and 36.

## Key Words

| | | | |
|---|---|---|---|
| cancelling | improper fraction | mixed number | simplest form |
| denominator | inverse | numerator | simplify |
| equivalent fractions | Lowest Common Denominator (LCD) | proper fraction | unit fraction |
| fraction | | | |

# Understanding fractions

3.1

## What is a fraction?

A **fraction** is a number that is used to show parts of a whole.

A whole might be an object, a collection of objects or a section of a number line.

If we are dividing a whole into parts to show a fraction, each of the parts must be equal in size.

If we are dividing up a collection of objects into groups to show a fraction, there must be the same number of objects in each group.

To write a fraction, two numbers are required.

$\frac{3}{5}$ ← The **numerator** tells us how many equal parts we have.

← The **denominator** tells us how many equal parts one whole has been divided into.

## Worked Example 1

WE1

How many smileys are there in $\frac{4}{5}$ of this collection?



| Thinking | Working |
|---|---|
| **1** The denominator shows us how many equal parts to divide the collection into. (Dividing 15 into 5 groups gives 3 smileys in each group.) | $15 \div 5 = 3$<br>$\frac{1}{5}$ is 3 smileys<br>  |
| **2** The numerator tells us how many of these 'equal parts' we should count for our total. ($\frac{4}{5}$ means take 4 of the 5 groups.) | $4 \times 3 = 12$<br>  |
| **3** Write the answer. | $\frac{4}{5}$ is 12 smileys |

## Types of fractions

**Proper fractions** have a numerator that is less than the denominator. They have values less than 1. Examples: $\frac{1}{2}$ and $\frac{3}{5}$.

**Improper fractions** have a numerator that is greater than or equal to the denominator. They have values greater than or equal to 1. Examples: $\frac{3}{2}$, $\frac{9}{9}$ and $\frac{17}{6}$.

**Mixed numbers** have whole number parts and fraction parts written separately. Examples: $1\frac{1}{2}$, $2\frac{5}{6}$ and $34\frac{2}{7}$.

## Using a number line to represent fractions

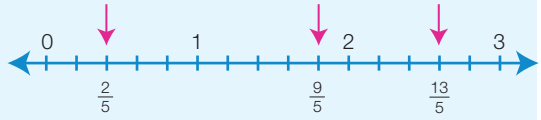
Placing fractions on a number line helps us to understand their size when compared to other numbers.

Drawing up a number line is easier if you can make the distance between whole numbers the same as the denominator. For example, if dividing into thirds, make the distance 3 cm. Then, you can mark off a third for every cm.

### Worked Example 2 WE2

Copy this number line and show the positions of these fractions: $\frac{2}{5}$, $\frac{9}{5}$ and $\frac{13}{5}$.




| Thinking | Working |
|---|---|
| **1** Copy the number line. |  |
| **2** The denominator tells us how to divide up the spaces between the whole numbers (in this case, counting up in fifths). Make sure the distance between each marked division is the same. |  |
| **3** For each fraction, look at the numerator and count that many parts along from zero. Indicate the location of the fraction with an arrow. |  |

## Fractions and division

There is an improper fraction that corresponds to each whole number on the number line.

If we fill in the rest of the fractions on the answer line in the Worked Example above, we can see that: $\frac{5}{5} = 1$, $\frac{10}{5} = 2$, $\frac{15}{5} = 3$ and so on.

We also know that: $5 \div 5 = 1$, $10 \div 5 = 2$, $15 \div 5 = 3$.

Writing $\frac{5}{5}$ is the same as writing $5 \div 5$.

We can write any whole number as an improper fraction by writing the numerator as a multiple of the denominator. For example, $3 = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$, because $6 \div 2 = 3$, $9 \div 3 = 3$ and $12 \div 4 = 3$.

The simplest way of writing a whole number as an improper fraction is to write it with a denominator of 1. For example, $3 = \frac{3}{1}$ $(3 \div 1 = 3)$.

Drawing the line between the numerator and the denominator is equivalent to writing the division sign, ÷.

## Worked Example 3

WE3

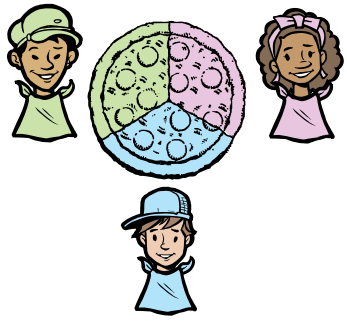
Write the whole number 9 as an improper fraction with a denominator of 4.

| Thinking | Working |
|---|---|
| **1** Which number divided by the denominator gives the required whole number? (Which number divided by 4 gives 9?) | $9 = \frac{\square}{4}$ |
| **2** Multiply the whole number you want by the denominator to find this number. | $9 \times 4 = 36$ |
| **3** Write the answer. | $9 = \frac{36}{4}$ |

We can consider all fractions in terms of division. Consider the following situation: 1 pizza is shared equally between 3 students. How much pizza does each student get?

We can show the answer is $\frac{1}{3}$ by drawing lines to divide the pizza into 3 equal pieces.

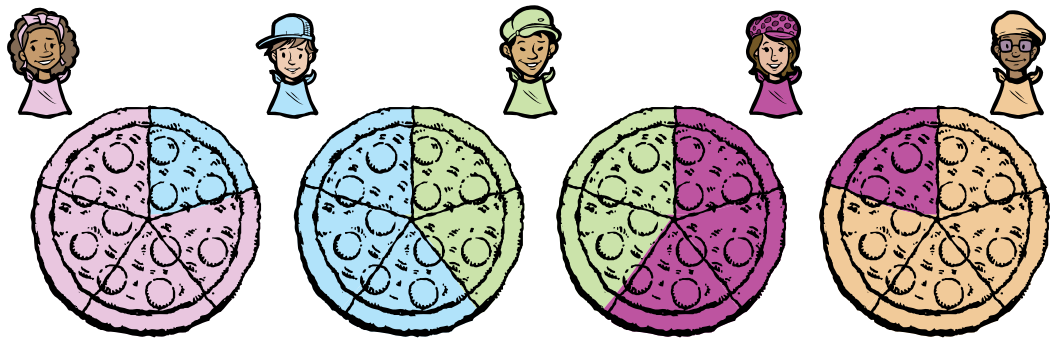
The different colours show which piece goes to which student.



We can write the division of 1 whole pizza among 3 students as $1 \div 3 = \frac{1}{3}$.

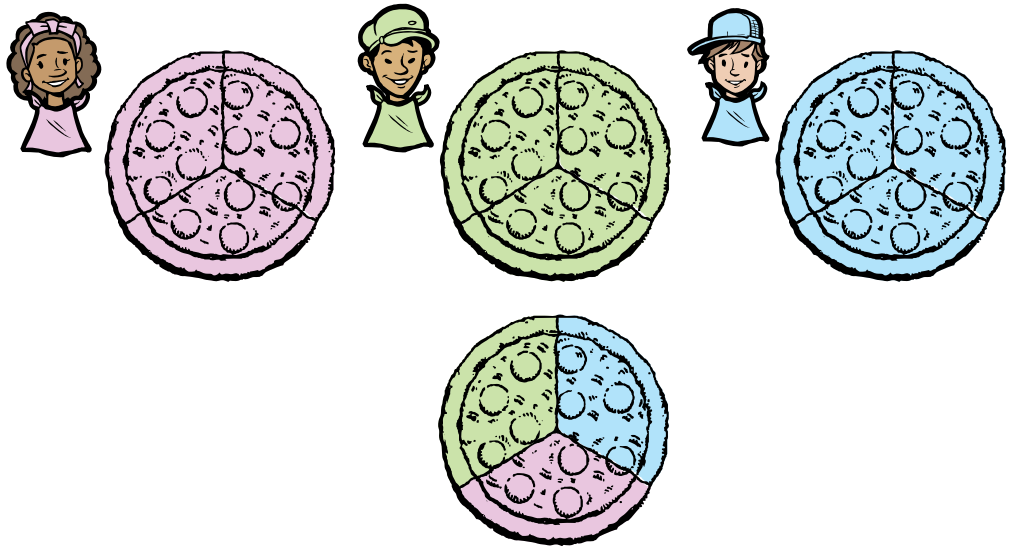
Similarly, if 4 pizzas are shared between 5 students, each student receives $\frac{4}{5}$ of a pizza.

$4 \div 5 = \frac{4}{5}$



If we have a greater number of pizzas than students, each student receives more than a whole pizza. For example, 4 pizzas shared between 3 students gives $4 \div 3 = \frac{4}{3}$ of a pizza each.

From the diagram, we can see that this is equal to 1 whole and $\frac{1}{3}$ of a pizza each, or $1\frac{1}{3}$.



## Worked Example 4

WE4

For the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

5 blocks of chocolate are shared equally between 8 students.

| Thinking | Working |
|---|---|
| **1** Identify the whole and how many it is being shared between. (The whole is 5 and it is being shared between 8.) Write a division that shows this. | $5 \div 8$ |
| **2** Write this division as a fraction. | $= \frac{5}{8}$ |

# 3.1 Understanding fractions

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q11 (a), Q13, Q16, Q17, Q19 (a), Q20 (b), Q22 | Q1, Q2, Q3 (b), Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 (d)–(f), Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19 (b), Q20 (b), Q22 | Q2 (b), Q3 (b), Q4, Q5, Q7 Column 3, Q8 (d)–(f), Q9, Q10, Q11 (b), Q12, Q13, Q14, Q15, Q17, Q18, Q19 (c), Q20, Q21, Q22 |

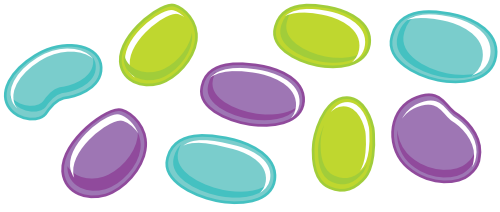
Answers page 634

## Fluency

**1** **(a)** How many smileys are there in $\frac{3}{4}$ of this collection?



**(b)** How many jelly beans are there in $\frac{2}{3}$ of this collection?



WE1




**2** **(a)** Copy this number line and show the positions of these fractions: $\frac{3}{4}$, $\frac{9}{4}$, $\frac{5}{4}$ and $\frac{12}{4}$.

WE2




**(b)** Copy this number line and show the positions of these fractions: $\frac{5}{6}$, $\frac{8}{6}$, $\frac{1}{6}$ and $\frac{14}{6}$.




**3** **(a)** Write the whole number 5 as an improper fraction with a denominator of:

WE3

**(i)** 2 **(ii)** 7 **(iii)** 11 **(iv)** 5 **(v)** 1

**(b)** Write the whole number 13 as an improper fraction with a denominator of:

**(i)** 2 **(ii)** 5 **(iii)** 8 **(iv)** 13 **(v)** 1

**4** **(a)** For each of the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

WE4

**(i)** 1 pizza is shared equally between 2 students.

**(ii)** 2 apples are shared equally between 3 students.

**(iii)** 6 packets of lollies are shared equally between 5 students.

**(iv)** 10 packets of biscuits are shared equally between 7 students.

**(b)** In which of the above situations does a student receive more than one whole?

## Understanding

**5** Here are 16 lollies. How many will you eat if you eat these fractions of the total?

**(a)** $\frac{1}{4}$

**(b)** $\frac{3}{4}$

**(c)** $\frac{1}{8}$

**(d)** $\frac{5}{8}$



**6** **(a)** What fraction of the bananas in this bunch are rotten?

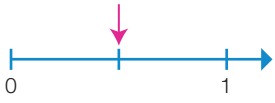


**(b)** What fraction of the flowers in this vase are red?

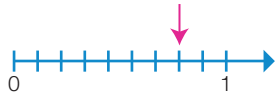


**7** Write the value of the fraction indicated by the arrow on each of the number lines below.

**(a)**



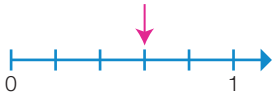

**(b)**




**(c)**




**(d)**



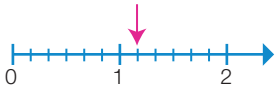

**(e)**



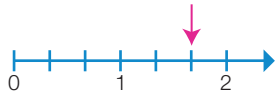

**(f)**




**(g)**




**(h)**




**(i)**




**8** Write a fraction or a mixed number to show each of these:

**(a)** a numerator of 8 and a denominator of 17

**(b)** denominator of 4 and a numerator of 15

**(c)** nine chocolate biscuits in a packet of 20

**(d)** two wholes and two thirds

**(e)** 4 complete pairs of socks and one odd sock

**(f)** 3 whole 24-piece blocks of chocolate, with 7 extra pieces.

**9** Write a fraction to show each of these:

**(a)** the weekend days as a fraction of a whole week

**(b)** 1 hour out of a whole day

**(c)** 1 second out of a whole minute

**(d)** 17 minutes out of a whole hour

**(e)** 157 mL of cola drunk from a 375 mL can

**(f)** 421 L of water in a 500 L rainwater tank.

**10** What fraction of this collection of shapes are:



**(a)** stars

**(b)** stars or hearts

**(c)** not hearts?

**11** **(a)** Draw a diagram to show that if 5 pizzas are shared equally between 6 students, then each student receives $\frac{5}{6}$ of a pizza.

**(b)** Draw a diagram to show that if 5 blocks of chocolate are shared equally between 4 students, then each student receives 1 full block and $\frac{1}{4}$ of a second block.

**12** Tim is 149 cm tall. His dad is 185 cm tall.

**(a)** Write Tim's height as a fraction of his dad's height.

**(b)** Write his dad's height as a fraction of Tim's height.



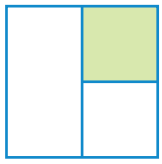
**13** **(a)** After driving 8 km, the Johnson family are $\frac{1}{4}$ of the way to the zoo. How far away from the zoo do the Johnsons live?

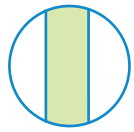
**(b)** Toby and his Mum shared a bag of lollies: $\frac{2}{3}$ for Toby and $\frac{1}{3}$ for Mum. Mum had 6 lollies. How many did Toby have? How many lollies were in the bag?

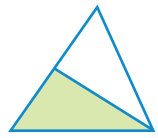
## Reasoning

**14** 4 girls share 5 pizzas evenly between them. 5 boys share 4 of the same size pizzas evenly between them. Write the fraction of pizza that each boy and each girl receives. Who gets more pizza, a boy or a girl?

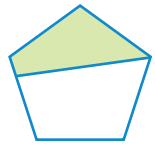
**15** To show fractions, a whole must be divided into equal parts. For each of the shapes below, consider whether the shaded section represents one or more equal parts. State the fraction shown. If you don't think it is possible to state a definite fraction, explain why.

**(a)** 

**(b)** 

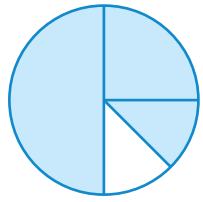
**(c)** 

**(d)** 

**(e)** 

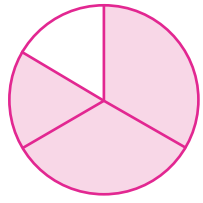
**(f)** 

**16** The fraction of this circle that is shaded is:



**A** $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

**17** The fraction of this circle that is shaded is:



**A** $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

**18** How many whole numbers would appear on the section of the number line between $\frac{1}{2}$ and $\frac{11}{2}$?

## Open-ended

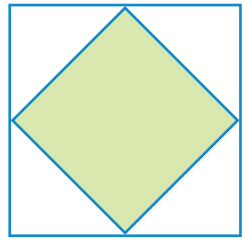
**19** Draw a diagram that shows:

**(a)** three-eighths of an object **(b)** $\frac{5}{6}$ of a collection of objects **(c)** $\frac{9}{3}$

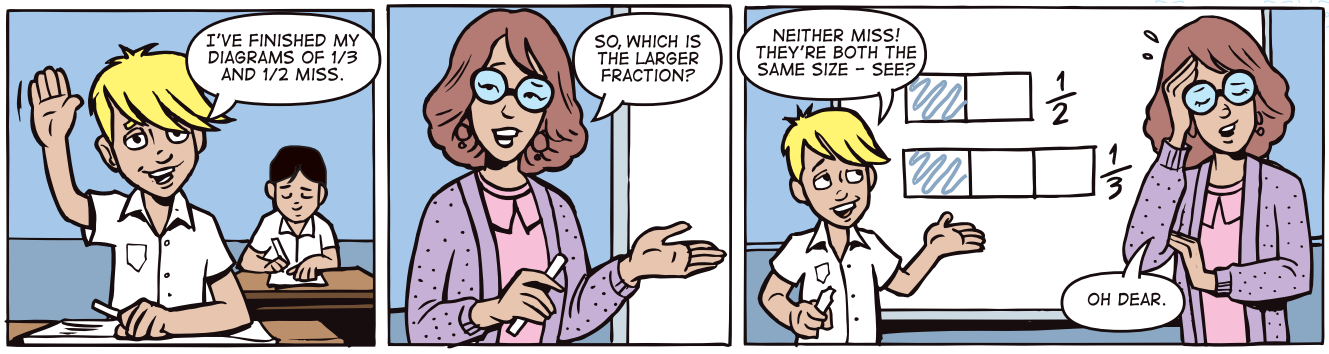
**20 (a)** This shape  represents $\frac{1}{5}$ of a whole. Draw two examples of what one whole could look like.

**(b)** This shape  represents $\frac{1}{4}$ of a whole. Draw two examples of what two wholes could look like.

**21** Draw a diagram to help you explain why the fraction of this tile that is shaded is $\frac{1}{2}$.



22




Oliver's diagram is supposed to show which of the fractions $\frac{1}{2}$ or $\frac{1}{3}$ is the larger fraction.

**(a)** Explain to Oliver what is wrong with his diagram.

**(b)** Draw a more accurate diagram to help you explain where he has gone wrong.

# Outside the Square Puzzle

## Fractionally funny fiction

**Q:** Which English ruler invented fractions?

To find out, write down the letters that correspond to the fractions of each of the following words.

For example, 'the first $\frac{1}{4}$ of 'TERRIFIC' would be the letters 'TE' ('TE-RR-IF-IC')

The letters in each of the 4 boxes form a word.

When you have put together four words, rearrange them to find the answer.

The second $\frac{1}{5}$ of ENTHUSIASM
The middle $\frac{1}{5}$ of WHEEL

The first $\frac{1}{5}$ of KNIFE
The middle $\frac{1}{3}$ of FRINGE
The last $\frac{1}{4}$ of SONG

The last $\frac{1}{6}$ of ENDURE
The third $\frac{1}{6}$ of REFRIGERATOR
The last $\frac{1}{3}$ of HEIGHT
The middle $\frac{1}{7}$ of WASHING

The first $\frac{1}{6}$ of HEAVEN
The middle $\frac{1}{5}$ of CONVENTION
The last $\frac{1}{3}$ of PANTRY

Figured it out?

Make up your own puzzle that uses fractions of words to make the answer to a joke or fact.

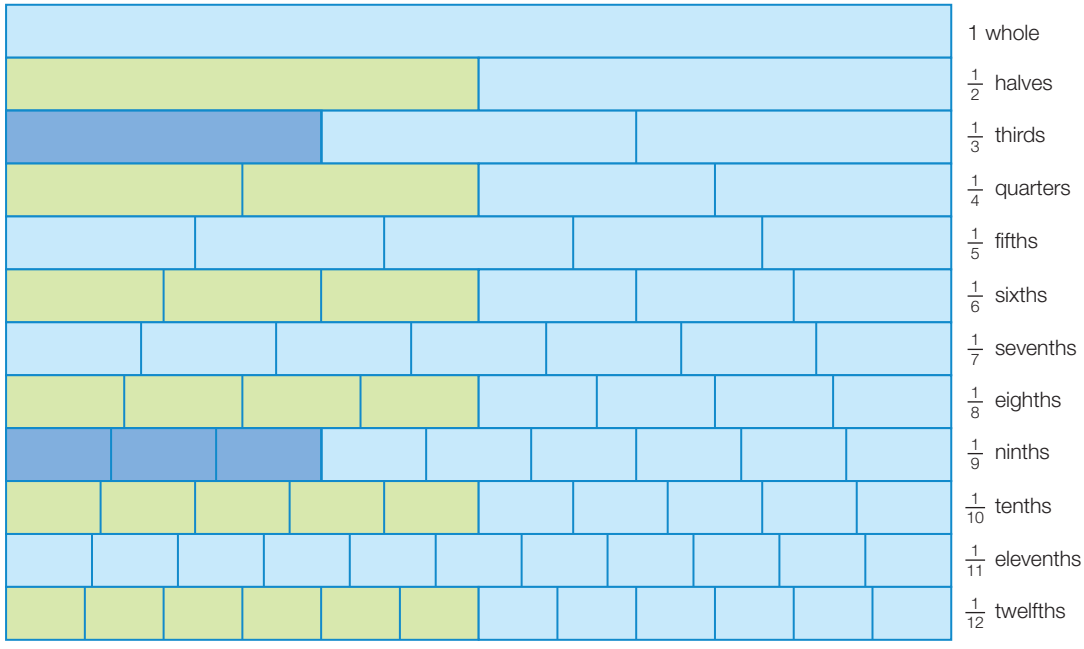


# 3.2 Working with fractions

## The fraction wall

Discovery Task

This fraction wall is made up of 12 layers of identical rectangles. One layer is left whole at the top, then each layer is divided into halves ($\frac{1}{2}$), thirds ($\frac{1}{3}$), quarters ($\frac{1}{4}$), fifths ($\frac{1}{5}$) … all the way down to twelfths ($\frac{1}{12}$). We can see that the size of the individual fractions (the 'bricks' in the wall) get smaller as we divide the whole into a greater number of pieces.




## Equivalent fractions

If we examine the small vertical lines on the fraction wall, we can see that some lie exactly underneath each other (holding a ruler against the lines can help us see this more clearly). These lines in identical positions across the wall indicate **equivalent fractions**—fractions that represent the same amount, but with different numerators and denominators.

For example, if we consider $\frac{1}{2}$, we can see that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (these fractions are all shaded in green on the wall).

If we consider $\frac{1}{3}$, we can see that $\frac{1}{3} = \frac{3}{9}$ ($\frac{1}{3}$ is also equivalent to $\frac{2}{6}$ and $\frac{4}{12}$).

Can you see the pattern that links the numerators and the denominators of the fractions that are equivalent to $\frac{1}{2}$ and $\frac{1}{3}$?

Multiplying the numerator and the denominator of $\frac{1}{2}$ by 2 gives $\frac{2}{4}$: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$.

Similarly, multiplying the numerator and the denominator of $\frac{1}{2}$ by 3, 4, 5 and 6 gives $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and $\frac{6}{12}$.

Dividing the numerator and the denominator by a common factor also produces equivalent fractions: $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$.

(As $\frac{2}{2}$ and $\frac{3}{3}$ are equivalent to 1 whole, multiplying or dividing a fraction by them is the same as multiplying or dividing by 1, which does not change the value of the fraction.)

Equivalent fractions are found either by multiplying both the numerator and the denominator by the same number, or by dividing both the numerator and the denominator by the same number.

## Worked Example 5 WE5

Write pairs of equivalent fractions by copying and completing the following.

**(a)** $\frac{7}{10} = \frac{\square}{40}$ **(b)** $\frac{27}{36} = \frac{3}{\square}$

| Thinking | Working |
|---|---|
| **(a) 1** Compare the two denominators to determine what you need to multiply the first denominator by to get the second. (To get 40, we multiply 10 by 4.) | (a) $\frac{7}{10} \times \frac{\square}{4} = \frac{\square}{40}$ |
| **2** Multiply the numerator by the same number to complete the equivalent fraction. | $\frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$ |
| **(b) 1** Compare the two numerators to determine what you need to divide the first numerator by to get the second. (To get 3, we divide 27 by 9.) | (b) $\frac{27}{36} \div \frac{9}{\square} = \frac{3}{\square}$ |
| **2** We divide the denominator by the same number to complete the equivalent fraction. | $\frac{27}{36} \div \frac{9}{9} = \frac{3}{4}$ |

## Simplifying fractions

$\frac{1}{4}$ and $\frac{5}{20}$ are equivalent fractions: $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$.

Although they both represent the same amount, $\frac{1}{4}$ is the **simplest form** of this pair of fractions.

A fraction in simplest form is often the easiest to understand and visualise.

To write a fraction in its simplest form, or to **simplify** it, we divide the numerator and the denominator by their highest common factor (HCF).

Answers to fraction questions should always be written in simplest form.

For example, the HCF of 5 and 20 is 5, so: $\frac{5}{20} \div \frac{5}{5} = \frac{1}{4}$ gives us the simplest form of $\frac{5}{20}$.

The process of simplifying fractions is often called '**cancelling**'. To cancel a fraction, we show the division in the following way. Draw a line through the numerator. Divide the numerator by the HCF and write the result of the division next to it. Repeat this for the denominator.

For example: $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} = \frac{1}{4}$

Sometimes, if the HCF is not obvious, we can do several divisions by smaller factors, such as 2 or 3. This is shown in Method 2 of the Worked Example below.

## Worked Example 6 WE6

Write the following fraction and mixed number in their simplest form.

(a) $\frac{24}{60}$ (b) $3\frac{12}{28}$

### Method 1: Find the HCF

| Thinking | Working |
|---|---|
| **(a) 1** Find the HCF of the numerator and the denominator. | (a) HCF of 24 and 60 = 12 |
| **2** Divide both the numerator and the denominator by the HCF. (This is shown here using cancelling notation.) | $\overset{2}{\frac{\cancel{24}}{\cancel{60}}}{}_{5}$ |
| **3** Write the answer. | $= \frac{2}{5}$ |
| **(b) 1** Find the HCF of the numerator and the denominator of the fraction part. | (b) HCF of 12 and 28 = 4 |
| **2** Divide both the numerator and the denominator by the HCF. | $3\overset{3}{\frac{\cancel{12}}{\cancel{28}}}{}_{7}$ |
| **3** Write the answer. | $= 3\frac{3}{7}$ |

### Method 2: Do several small divisions

| Thinking | Working |
|---|---|
| **(a) 1** Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so we divide by 2.) | (a) $\overset{12}{\frac{\cancel{24}}{\cancel{60}}}{}_{30}$ |
| **2** Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again, then by 3.) | $= \overset{6}{\frac{\cancel{12}}{\cancel{30}}}{}_{15}$ <br> $= \overset{2}{\frac{\cancel{6}}{\cancel{15}}}{}_{5}$ |
| **3** Write the answer. | $= \frac{2}{5}$ |

| | | |
|---|---|---|
| **(b) 1** | Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so we divide by 2.) | (b) $3\frac{\cancel{12}^{6}}{\cancel{28}_{14}}$ |
| **2** | Continue to divide by this or another small factor until you cannot divide any further. (Here, we divide by 2 again.) | $= 3\frac{\cancel{6}^{3}}{\cancel{14}_{7}}$ |
| **3** | Write the answer. | $= 3\frac{3}{7}$ |

## Mixed numbers and improper fractions

Improper fractions that have a value greater than 1 can be written as mixed numbers. To write an improper fraction as a mixed number, we need to see how many 'wholes' we can make, and what fraction we have left over.

For example, here are 11 slices of pizza. Each slice represents $\frac{1}{4}$, so we have $\frac{11}{4}$.



With 11 quarter slices, we can make 2 whole pizzas with 3 slices left over, or $2\frac{3}{4}$.



This is the same as doing $11 \div 4 = 2$ remainder 3. The 3 becomes the numerator of the fraction part of the mixed number with the same denominator, 4.

To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part. We then write the remainder as the numerator of the fraction part.

## Worked Example 7 WE7

Write the improper fraction $\frac{34}{7}$ as a mixed number.

| Thinking | Working |
|---|---|
| **1** Divide the numerator by the denominator. Include the remainder with your answer. | $34 \div 7 = 4$ rem 6 |
| **2** Write down the whole number part. Write the remainder as the numerator of the fraction part. | $\frac{34}{7} = 4\frac{6}{7}$ |

Writing a mixed number as an improper fraction is the opposite of the above process. Instead of forming wholes from fraction parts, we 'cut up' our whole numbers into fraction parts, and count the total number of parts that we have.

To write a mixed number as an improper fraction, write the whole number part as an improper fraction with the same denominator. Then, add the proper fraction part.

## Worked Example 8 WE8

Write the mixed number $7\frac{3}{5}$ as an improper fraction.

| Thinking | Working |
|---|---|
| **1** Write the whole number part as an improper fraction. (Here, $7 \times 5$ tells us how many fifths there are in 7.) | $7 \times 5 = 35$<br>$7 = \frac{35}{5}$ |
| **2** Add the proper fraction part. | $\frac{35}{5} + \frac{3}{5} = \frac{38}{5}$ |
| **3** Write the mixed number with its equivalent proper fraction. | $7\frac{3}{5} = \frac{38}{5}$ |

You can probably spot a shortcut for these types of questions—multiply the whole number by the denominator, add the numerator, and place the answer over the denominator: $7\frac{3}{5} = \frac{38}{5}$.

# 3.2 Working with fractions

## Navigator

Answers page 635

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1–3, Q4 Columns 1–3, Q5 (a) & (b), Q6, Q7, Q8, Q9, Q10, Q11, Q15, Q16, Q17, Q18 (a)–(c), Q19, Q22 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q21, Q22 | Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q17, Q20, Q21, Q22 |

### Fluency

WE5

**1** Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{3}{4} = \frac{\square}{20}$ (b) $\frac{1}{3} = \frac{7}{\square}$ (c) $\frac{1}{2} = \frac{\square}{52}$ (d) $\frac{2}{5} = \frac{16}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{7}{11} = \frac{\square}{99}$

(i) $\frac{4}{3} = \frac{\square}{27}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{24}{16} = \frac{\square}{4}$ (l) $\frac{72}{36} = \frac{\square}{9}$

(m) $\frac{8}{\square} = \frac{88}{55}$ (n) $\frac{\square}{5} = \frac{42}{35}$ (o) $\frac{54}{\square} = \frac{9}{6}$ (p) $\frac{\square}{36} = \frac{4}{9}$

WE6




**2** Write the following fractions and mixed numbers in their simplest form.

**(a)** $\frac{5}{10}$ **(b)** $\frac{3}{9}$ **(c)** $\frac{3}{12}$ **(d)** $\frac{7}{21}$

**(e)** $\frac{4}{10}$ **(f)** $\frac{8}{22}$ **(g)** $\frac{10}{16}$ **(h)** $\frac{36}{24}$

**(i)** $2\frac{14}{21}$ **(j)** $3\frac{5}{30}$ **(k)** $1\frac{6}{20}$ **(l)** $12\frac{24}{30}$

**(m)** $6\frac{20}{45}$ **(n)** $3\frac{25}{100}$ **(o)** $2\frac{35}{42}$ **(p)** $100\frac{12}{84}$

WE7

**3** Write these improper fractions as mixed numbers.

**(a)** $\frac{7}{5}$ **(b)** $\frac{13}{6}$ **(c)** $\frac{23}{4}$ **(d)** $\frac{15}{7}$

**(e)** $\frac{37}{10}$ **(f)** $\frac{48}{5}$ **(g)** $\frac{44}{7}$ **(h)** $\frac{59}{10}$

**(i)** $\frac{77}{9}$ **(j)** $\frac{91}{12}$ **(k)** $\frac{107}{100}$ **(l)** $\frac{97}{40}$

WE8

**4** Write these mixed numbers as improper fractions.

**(a)** $1\frac{3}{4}$ **(b)** $3\frac{1}{5}$ **(c)** $6\frac{2}{3}$ **(d)** $5\frac{9}{10}$

**(e)** $4\frac{3}{8}$ **(f)** $4\frac{7}{11}$ **(g)** $10\frac{2}{7}$ **(h)** $6\frac{8}{11}$

**(i)** $6\frac{5}{12}$ **(j)** $3\frac{9}{100}$ **(k)** $6\frac{8}{9}$ **(l)** $8\frac{7}{9}$

**5 (a)** Which one of the following is the simplest form of $3\frac{35}{45}$?

**A** $3\frac{5}{9}$ **B** $3\frac{3}{4}$ **C** $3\frac{7}{9}$ **D** $3\frac{70}{90}$

**(b)** Which of the following fractions is not equivalent to $\frac{10}{15}$?

**A** $\frac{2}{3}$ **B** $\frac{30}{45}$ **C** $\frac{40}{60}$ **D** $\frac{20}{25}$

**(c)** As a mixed number, $\frac{33}{4}$ equals:

**A** $3\frac{3}{4}$ **B** $7\frac{4}{5}$ **C** $8\frac{1}{4}$ **D** $8\frac{1}{3}$

**(d)** As an improper fraction, $2\frac{5}{8}$ equals:

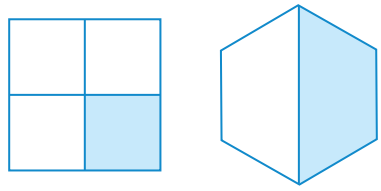
**A** $\frac{7}{8}$ **B** $\frac{21}{8}$ **C** $\frac{25}{8}$ **D** 21

## Understanding

**6** Use the fraction wall on page 116 to identify the fraction or fractions that are equivalent to:

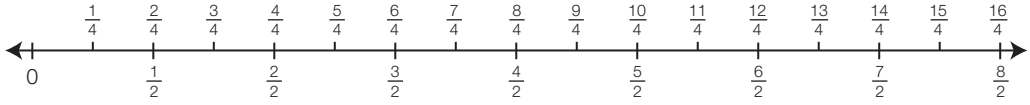
**(a)** $\frac{3}{5}$ **(b)** $\frac{2}{8}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{9}{12}$

**7** **(a)** Copy each of these figures. Write down the fraction that is shaded.



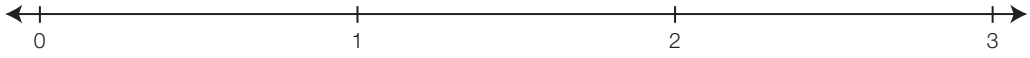
**(b)** In each figure, rule two diagonal lines joining the opposite vertices (corners). Name the equivalent fraction of the shaded part that you have created.

**8** **(a)** The top side of this number line is divided into quarters, and the bottom side into halves.




What do you observe about the positions of the equivalent pairs of fractions $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{2}$ and $\frac{4}{4}$, $\frac{3}{2}$ and $\frac{6}{4}$, and $\frac{4}{2}$ and $\frac{8}{4}$?

**(b)** Copy the number line below. Divide the top side into thirds and the bottom side into ninths. Use your observation from **(a)** to identify pairs of equivalent fractions.






**9** Monica, Michelle and Travis are sharing an apple pie that has been cut into 12 equal slices. Monica and Michelle decide to have $\frac{1}{4}$ of the apple pie each and give Travis the remainder. How many slices of pie did each person receive? Draw a diagram that shows this information.

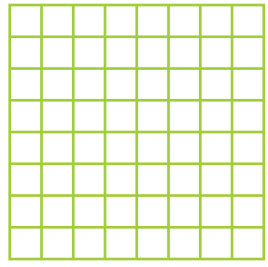
**10** Six pizzas were each sliced into 8 equal pieces. If $4\frac{3}{8}$ of the pizzas were eaten, how many slices is that?

**11** A block of chocolate has 24 individual squares. How many squares of the chocolate will be gone if $\frac{7}{12}$ of the chocolate is eaten?

**12** Talia shaded a fraction of this long rectangle.



How many squares on this grid should Talia shade to show a fraction equivalent to the fraction she shaded in the rectangle above?



**13** Write your answers in simplest form.

**(a)** What fraction of an hour has passed from 3.49 p.m. to 3.51 p.m.?

**(b)** What fraction of an hour has passed from 11.03 a.m. to 11.23 a.m.?

**(c)** What fraction of an hour has passed from 8.13 p.m. to 8.25 p.m.?

**(d)** What fraction of an hour has passed from 2.36 a.m. to 2.45 a.m.?

**14** Imagine that the whole rectangle on the top of the fraction wall on page 116 represents 1 hour. How many minutes would each of the individual fractions on the wall represent? Write any answers that are not whole numbers as mixed numbers.

## Reasoning

**15** How many whole numbers would appear on the section of the number line between $\frac{4}{3}$ and $\frac{51}{9}$? (Hint: Convert these improper fractions to mixed numbers.)

**16** Which of the following fractions could not be the simplified form of an original fraction that had a denominator of 40?

**A** $\frac{3}{8}$ **B** $\frac{2}{5}$ **C** $\frac{3}{7}$ **D** $\frac{9}{10}$

**17** Which rows of fractions in the fraction wall on page 116 have no equivalent fractions on the wall? Explain why.

**18** For each of the following problems, it may be useful to draw and label a number line with the known fractions. Then, locate the position of the unknown fraction and identify it. Equivalent fractions can help you do this.

Which fraction lies exactly in-between the following fractions on the number line?

**(a)** $\frac{2}{5}$ and $\frac{3}{5}$ **(b)** $\frac{3}{7}$ and $\frac{4}{7}$ **(c)** $\frac{1}{4}$ and $\frac{2}{4}$

**(d)** $\frac{6}{10}$ and $\frac{7}{10}$ **(e)** $\frac{1}{9}$ and $\frac{6}{9}$ **(f)** $\frac{2}{11}$ and $\frac{7}{11}$

## Open-ended

**19** Write three fractions that are equivalent to $\frac{4}{5}$.

**20** Write three improper fractions that simplify to $1\frac{2}{3}$.

**21** For each of these pairs of fractions, draw a diagram that shows they are equivalent.

**(a)** $\frac{1}{3}$, $\frac{5}{15}$ **(b)** $\frac{3}{4}$, $\frac{12}{16}$

**22** From the fraction wall on page 116 choose three fractions with three different denominators that have no equivalent fractions in the wall.

## Outside the Square Puzzle

### What fractions are we?

**1** I am greater than $\frac{1}{2}$.

My numerator and denominator are both square numbers.

My numerator is an odd number.

The difference between my numerator and denominator is seven.

What fraction am I?

**2** My numerator is a single-digit odd number.

My denominator is a two-digit number with both digits the same.

If you add 1 to both my numerator and denominator, you get a fraction equivalent to $\frac{1}{2}$.

What fraction am I?

**3** My numerator is a multiple of 5.

My denominator is the LCM of 6 and 9.

I am less than 1.

I am in simplest form.

What fraction am I?

# 3.3 Estimating and comparing fractions

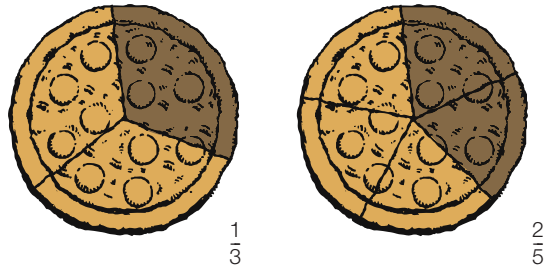
## Comparing fractions by using the Lowest Common Denominator (LCD)

If two fractions have the same denominator, it is clear which fraction is bigger.

$\frac{2}{8}$ of a pizza is obviously more pizza than $\frac{1}{8}$ of a pizza. (It is twice as big.)

Comparing fractions with different denominators is harder. Having different denominators is like having different-sized pieces of pizza.

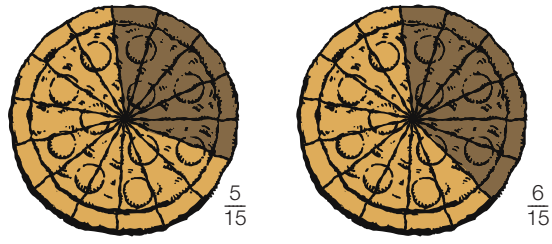
For example, which is larger, $\frac{1}{3}$ or $\frac{2}{5}$?




We need to rewrite the fractions so they have the same denominator. This is like cutting the pizza pieces into smaller, but equal-sized pieces. One way to do this is to find the **Lowest Common Denominator (LCD)**. The LCD is the lowest common multiple of the denominators.

Here, the LCD is 15. We cut both pizzas into 15 pieces, then count how many pieces we have of each:

$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$




$\frac{2}{5}$ is the larger fraction, so we write $\frac{2}{5} > \frac{1}{3}$.

To compare fractions by using the lowest common denominator (LCD):

- find the lowest common multiple of the denominators
- rewrite each fraction as an equivalent fraction with this multiple as the new denominator
- write '>' (greater than), '<' (less than) or '=' (equal to) between the fractions.

If working with mixed numbers, write them as improper fractions first.



## Worked Example 9 WE9

By finding the LCD, determine which fraction in each of the following pairs is larger.

**(a)** $\frac{3}{5}$ or $\frac{4}{7}$ **(b)** $\frac{15}{6}$ or $2\frac{7}{12}$

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Write the list of multiples for each denominator. | (a) $\frac{3}{5}$ or $\frac{4}{7}$<br>Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, ...<br>Multiples of 7: 7, 14, 21, 28, 35, 42, 49, ... |
| | **2** | Find the lowest common multiple of these denominators (LCD). | LCD = 35 |
| | **3** | Multiply each fraction to get an equivalent fraction with the LCD as the denominator. | $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$ |
| | **4** | Use < or > to show which of the two fractions is larger. | $\frac{21}{35} > \frac{20}{35}$ |
| | **5** | Use < or > to show which of the original fractions is larger. | So, $\frac{3}{5} > \frac{4}{7}$ |
| **(b)** | **1** | Write any mixed numbers as improper fractions. | (b) $2\frac{7}{12} = \frac{31}{12}$<br>$\frac{15}{6}$ or $\frac{31}{12}$ |
| | **2** | Find the LCD. | LCD = 12 |
| | **3** | Multiply to get equivalent fractions with the LCD as the denominator. (Only one fraction needs to be multiplied here.) | $\frac{15}{6} \times \frac{2}{2} = \frac{30}{12}$ |
| | **4** | Use < or > to show which of the two fractions is larger. | $\frac{30}{12} < \frac{31}{12}$ |
| | **5** | Use < or > to show which of the original fractions is larger. | So, $\frac{15}{6} < 2\frac{7}{12}$ |

## Comparing fractions by estimating and visualising

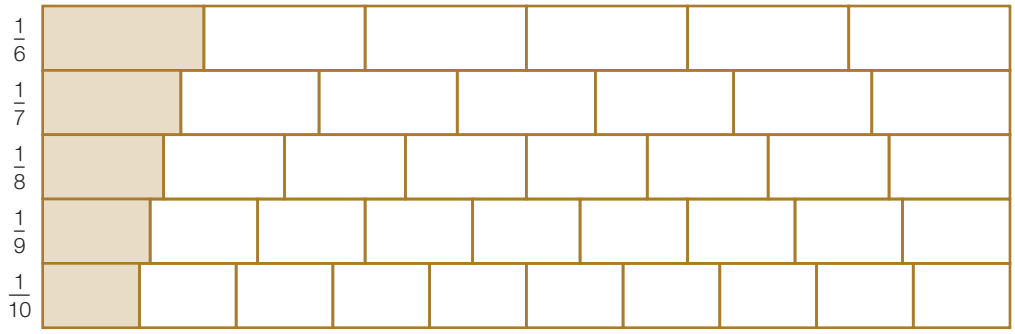
Often, we do not need to know an exact value of a fraction—it is enough to have an estimate. Similarly, when we compare two fractions, we only need to know which fraction is larger, not how much larger it is.

The following strategies can help us visualise what fractions 'look like', so we can estimate and compare them.

## Strategy 1 – Compare the denominators

The fraction wall is very useful for comparing fractions. A section of the fraction wall on page 116 is shown here. It shows that as we divide a whole into more parts, the sizes of the parts (the 'bricks' in the wall) get smaller. (You are possibly already aware of this—the more people you share a pizza with, the smaller the amount that each person gets!)

The shaded parts of each layer of the fraction wall demonstrate that: $\frac{1}{6} > \frac{1}{7} > \frac{1}{8} > \frac{1}{9} > \frac{1}{10}$.




Looking at the fraction wall, we can see that:

$\frac{2}{6}$ is slightly larger than $\frac{2}{7}$, and much larger than $\frac{2}{9}$

$\frac{5}{6}$ is larger than $\frac{5}{8}$ and also larger than $\frac{6}{8}$.

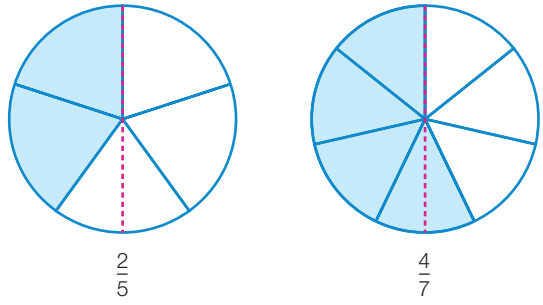
As the denominator numbers get larger, the fraction sizes get smaller.

If two fractions have the same numerator but different denominators, the *larger fraction* is the one with the *smaller number as the denominator*. For example, $\frac{5}{9} > \frac{5}{12}$, $\frac{3}{7} > \frac{3}{8}$.

## Strategy 2 – Compare to a common number

Visualise what a common number such as $\frac{1}{2}$, $\frac{1}{4}$, 1 or 2 'looks like' by imagining a shape, such as a circle or a rectangle, with that fraction shaded in. How would your fraction look in comparison? Would it be bigger or smaller?

For example, we can see that $\frac{2}{5} < \frac{4}{7}$ if we compare them to $\frac{1}{2}$.



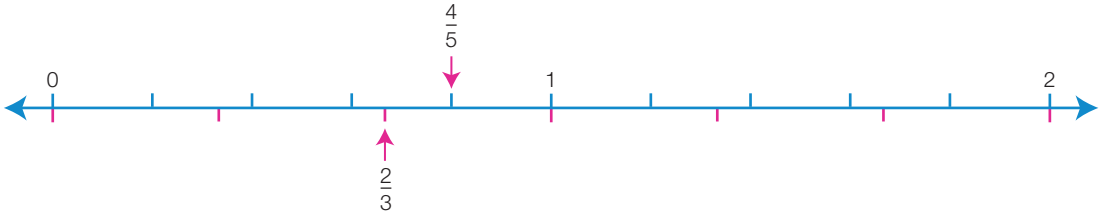

$\frac{2}{5}$ is slightly less than $\frac{1}{2}$ and $\frac{4}{7}$ is slightly greater than $\frac{1}{2}$, so $\frac{2}{5} < \frac{4}{7}$.

### Strategy 3 – Compare by using a number line

One number is larger than another if it lies to the right of it on the number line.

It is not always necessary to measure up the number line exactly—sometimes an accurate sketch will be sufficient.

For example, we can show that $\frac{4}{5} > \frac{2}{3}$ by sketching a number line and dividing one side into fifths, and the other side into thirds. We can then show that $\frac{4}{5}$ lies to the right of $\frac{2}{3}$, which means $\frac{4}{5}$ is larger than $\frac{2}{3}$.




# 3.3 Estimating and comparing fractions

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q17 | Q1 Columns 2 & 3, Q2, Q3 Columns 1–3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q17 | Q1 Column 4, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17 |

Answers page 636

## Fluency

WE9

**1** By finding the LCD, determine which fraction in each of the following pairs is larger.

**(a)** $\frac{3}{5}, \frac{7}{15}$ **(b)** $\frac{13}{12}, \frac{5}{4}$ **(c)** $\frac{7}{8}, \frac{5}{6}$ **(d)** $\frac{5}{9}, \frac{7}{12}$

**(e)** $\frac{2}{3}, \frac{5}{7}$ **(f)** $\frac{1}{6}, \frac{2}{9}$ **(g)** $\frac{3}{8}, \frac{5}{12}$ **(h)** $\frac{7}{4}, \frac{11}{6}$

**(i)** $1\frac{3}{4}, \frac{11}{8}$ **(j)** $\frac{5}{2}, 2\frac{1}{4}$ **(k)** $\frac{16}{7}, 2\frac{1}{5}$ **(l)** $2\frac{2}{7}, 2\frac{4}{9}$




**2** Use the fraction wall on page 116 or 126 to determine which fraction in each of the following pairs is larger.

**(a)** $\frac{3}{7}$ or $\frac{3}{10}$ **(b)** $\frac{7}{9}$ or $\frac{7}{8}$ **(c)** $\frac{4}{5}$ or $\frac{5}{6}$ **(d)** $\frac{1}{3}$ or $\frac{2}{5}$

**(e)** $\frac{9}{12}$ or $\frac{3}{4}$ **(f)** $\frac{5}{11}$ or $\frac{4}{9}$ **(g)** $\frac{6}{7}$ or $\frac{7}{8}$ **(h)** $\frac{3}{5}$ or $\frac{7}{11}$

**3** Copy the following fraction pairs. Use any appropriate strategy to decide which fraction is larger, and place < or > between them. If the fractions are equivalent, write = between them.

**(a)** $\frac{2}{7}$ _____ $\frac{5}{7}$ **(b)** $\frac{4}{5}$ _____ $\frac{4}{7}$ **(c)** $\frac{1}{2}$ _____ $\frac{5}{8}$ **(d)** $\frac{8}{11}$ _____ $\frac{9}{10}$

**(e)** $\frac{2}{4}$ _____ $\frac{3}{6}$ **(f)** $\frac{6}{12}$ _____ $\frac{7}{10}$ **(g)** $\frac{3}{7}$ _____ $\frac{5}{8}$ **(h)** $\frac{5}{9}$ _____ $\frac{2}{5}$

**(i)** $\frac{5}{6}$ _____ $\frac{7}{8}$ **(j)** $\frac{7}{11}$ _____ $\frac{8}{10}$ **(k)** $\frac{6}{10}$ _____ $\frac{3}{5}$ **(l)** $\frac{4}{5}$ _____ $\frac{6}{9}$

**(m)** $\frac{2}{4}$ _____ $\frac{4}{2}$ **(n)** $\frac{24}{6}$ _____ $\frac{8}{2}$ **(o)** $\frac{18}{11}$ _____ $\frac{9}{5}$ **(p)** $\frac{9}{7}$ _____ $\frac{11}{9}$

## Understanding

**4 (a)** Draw a number line from 0 to 2. Indicate the approximate positions of the following fractions on the number line, then use the number line to write the fractions in ascending order (smallest to largest). $\frac{3}{4}, \frac{2}{3}, \frac{6}{4}, 1\frac{1}{3}$

**(b)** Draw a number line from 0 to 3. Indicate the approximate positions of the following fractions on the number line, then use the number line to write the fractions in descending order (largest to smallest). $\frac{5}{2}, \frac{1}{2}, \frac{4}{10}, 1\frac{7}{10}$



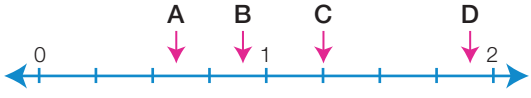
**5** Ali gets 7 out of 8 shots at goal in the basket. Joseph gets 10 out of 12 shots in. By comparing fractions, determine who is the more accurate shooter.

**6** In a box containing 16 chocolates, 5 of them contain nuts. A larger box with 64 of the same brand of chocolates contains 23 nut chocolates. Which box contains the greater fraction of chocolates with nuts?

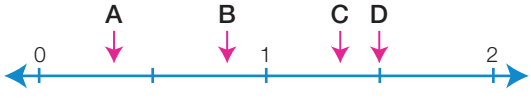
**7** Use the fraction wall on page 116 to determine which fraction or fractions on the wall lie in-between the following (include all equivalent fractions).

**(a)** $\frac{9}{10}$ and $\frac{11}{12}$ **(b)** $\frac{5}{8}$ and $\frac{7}{10}$ **(c)** $\frac{3}{8}$ and $\frac{4}{9}$ **(d)** $\frac{2}{11}$ and $\frac{3}{11}$

**8 (a)** Which arrow is pointing closest to the location of $\frac{9}{10}$ on this number line?




**(b)** Which arrow is pointing closest to the location of $1\frac{1}{3}$?




**9 (a)** Which one of the following is closest to $\frac{5}{8}$?

**A** $\frac{2}{9}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{7}{8}$

**(b)** Which one of the following is closest to 2?

**A** $\frac{11}{12}$ **B** $\frac{3}{2}$ **C** $\frac{7}{4}$ **D** $\frac{7}{3}$

## Reasoning

**10** Write each of these lists in ascending order (from smallest to largest). Explain using a series of brief points, or steps, how you decided the order.

**(a)** $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{10}{11}$

**(b)** $\frac{4}{7}, 1, \frac{7}{4}, \frac{6}{7}, \frac{7}{8}, 0$

**11** Write each of these lists in descending order (from largest to smallest). Explain briefly how you decided the order.

**(a)** $\frac{4}{7}, \frac{2}{9}, \frac{2}{3}, 1, \frac{11}{21}, \frac{45}{63}$

**(b)** $\frac{21}{16}, 1\frac{3}{80}, \frac{5}{4}, 1\frac{7}{20}, \frac{39}{40}, 1\frac{3}{10}$

**12 (a)** Estimate the fraction of the Australian flag that is blue.



**(b)** Estimate the fraction of Australia that is taken up by the state of Queensland.




**(c)** Estimate the fraction of this jug that has cordial in it.

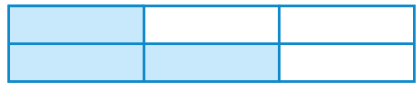


**13** For each of the following pairs of identical shapes, state whether the fraction of the shape that is shaded is larger in **A** or in **B**. If the fraction shaded is the same in **A** and in **B** write 'same'.

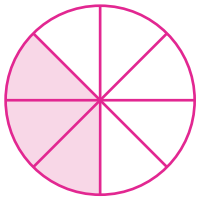
**(a)** **A**



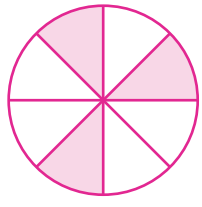
**B**



**(b)** **A**



**B**

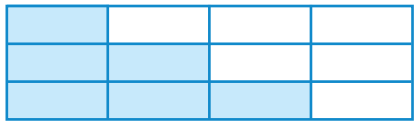
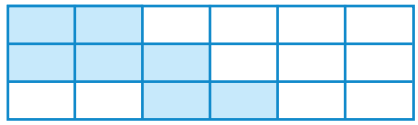


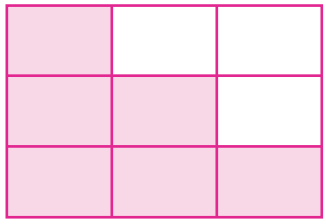
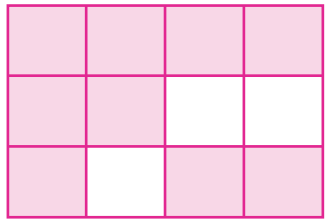
**(c)** **A**

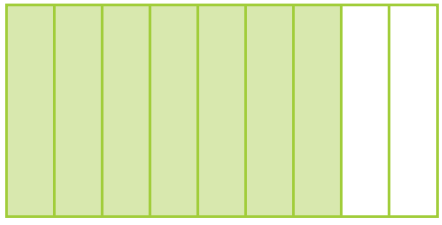
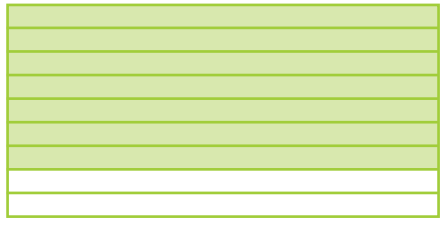


**B**



(d) A  B 

(e) A  B 

(f) A  B 

**14** Angus has 100 mL left of his 500 mL bottle of cola. Gianni has 50 mL left in his 300 mL bottle.

**(a)** Who has drunk more cola?

**(b)** Who has drunk a greater fraction of the cola in their bottle?

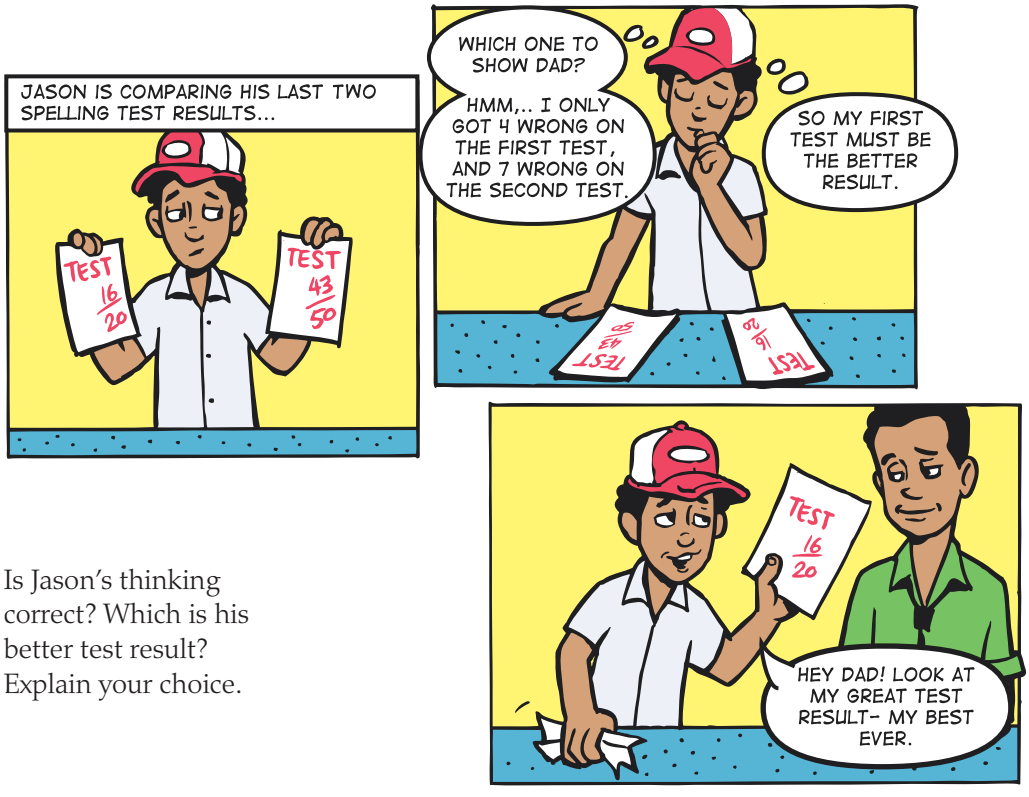
## Open-ended

**15** **(a)** A proper fraction is greater than $\frac{3}{4}$ but less than 1. What might the fraction be? Give two possibilities.

**(b)** An improper fraction is greater than 2 but less than 3. What might the fraction be? Give two possibilities.

**16** Write three fractions that are greater than $\frac{3}{5}$ and less than $\frac{4}{5}$.

**17**




Is Jason's thinking correct? Which is his better test result? Explain your choice.

# Outside the Square Game

## Flatlining

**Equipment required:** 2 brains, 1 die, small counters of two different colours or pieces of paper marked with different colours or the players' initials

**How to win:**
The first player to get three circles in a row on the number line is the winner.

**How to play:**
Decide who will go first.

Either use the number line shown or draw an accurate copy in your book.

On your turn, roll the die and look up the number you rolled in the table. Choose one of the fractions from the list. Locate that fraction's approximate position on the number line and occupy the circle above it with one of your counters or pieces of paper.

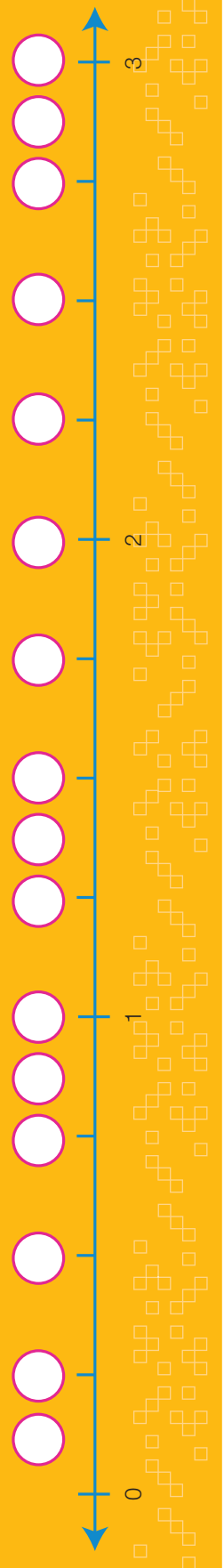
If you can't occupy a circle you miss your turn.

Make sure your opponent locates their fraction correctly.

Once a circle is filled it cannot be taken by another player.

If all of the circles have been occupied and neither player has three circles in a row, then the game is a draw.

| Dice roll | Fractions to choose from | | |
|---|---|---|---|
| 1 | $\frac{1}{4}$ | $1\frac{1}{2}$ | $\frac{7}{4}$ |
| 2 | $\frac{1}{2}$ | $\frac{5}{4}$ | $2\frac{4}{5}$ |
| 3 | $\frac{4}{3}$ | $\frac{3}{4}$ | $\frac{9}{4}$ |
| 4 | 3 | $\frac{4}{5}$ | $\frac{5}{2}$ |
| 5 | 2 | $\frac{11}{4}$ | $\frac{3}{3}$ |
| 6 | 1 | $\frac{1}{5}$ | $1\frac{3}{4}$ |

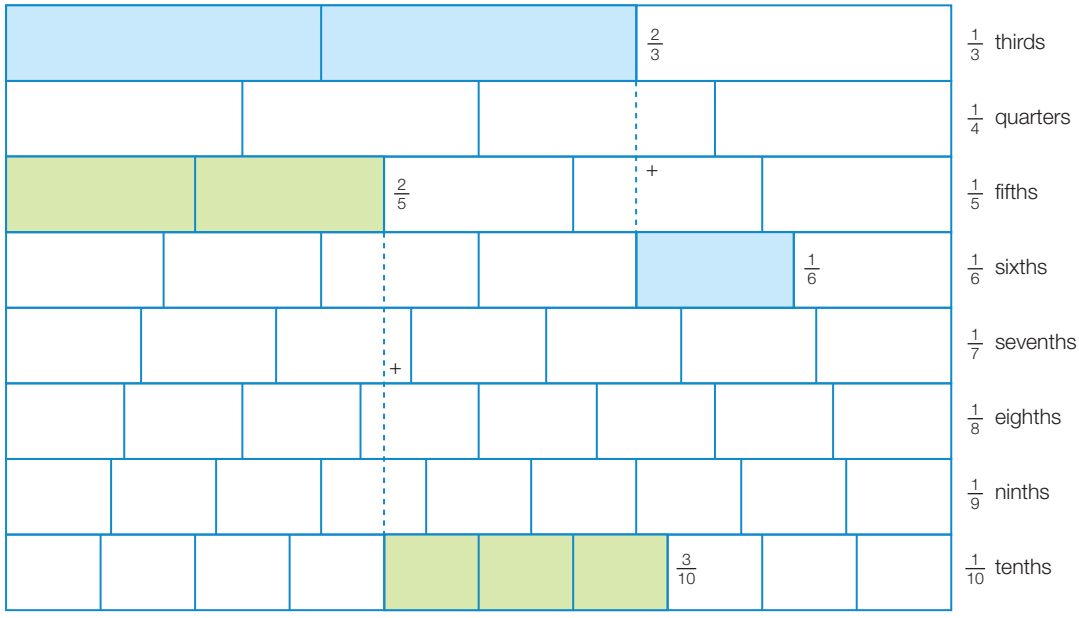


# 3.4 Adding and subtracting fractions

The fraction wall can be useful for learning to add fractions.

For example, we can show $\frac{2}{3} + \frac{1}{6}$ by moving along the wall to $\frac{2}{3}$, then along $\frac{1}{6}$ to arrive at $\frac{5}{6}$. This sum is shaded in blue on the fraction wall below.

We can also show $\frac{2}{5} + \frac{3}{10}$ by moving along the wall to $\frac{2}{5}$, then along $\frac{3}{10}$ to arrive at $\frac{7}{10}$. This sum is shaded in green on the fraction wall below.




These additions are quite straightforward as the second fraction joins exactly onto the end of the first. This is because the denominators of the two fractions are related—we can write one of the fractions as an equivalent fraction with the same denominator as the second fraction.

$$\frac{2}{3} + \frac{1}{6}$$
$$= \frac{4}{6} + \frac{1}{6}$$
$$= \frac{5}{6}$$

$$\frac{2}{5} + \frac{3}{10}$$
$$= \frac{4}{10} + \frac{3}{10}$$
$$= \frac{7}{10}$$

To add or subtract fractions, we must have fractions with the same denominator. If the denominators are the same, the size of the fraction parts is the same, and we can simply count how many of each part we have.

For example: $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$.

Adding or subtracting fractions with different, unrelated denominators requires some additional steps. Before we can add, we need to change both fractions into equivalent fractions with the same denominator. This new denominator will be a multiple of the denominators of the original fractions. The lowest common denominator (LCD) is often the most efficient one to use.

For example: $\frac{1}{3}+\frac{2}{5}$ LCD = 15

$\frac{1}{3}=\frac{5}{15}$ $\frac{2}{5}=\frac{6}{15}$

Each fraction is now written in fifteenths, so we can count how many fifteenths there are:

$\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$

## Estimating answers to fraction problems

To check whether your answer to a fraction addition or subtraction is 'reasonable', estimate the answer before doing the calculation.

For example, the answer to $\frac{3}{5}+\frac{3}{7}$ will be approximately 1, as $\frac{3}{5}$ is slightly bigger than $\frac{1}{2}$ and $\frac{3}{7}$ is slightly smaller. (The actual answer is $\frac{36}{35}$ or $1\frac{1}{35}$, which is very close to 1.)

## Worked Example 10 WE10

Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

**(a)** $\frac{2}{3}+\frac{3}{4}$ **(b)** $\frac{7}{10}-\frac{1}{5}$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Estimate the answer first. (Here, both fractions are greater than $\frac{1}{2}$ but less than 1. The answer should lie between 1 and 2.) | (a) Estimate: between 1 and 2 |
| **2** | Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (In this case, 4 and 3.) | $\frac{2}{3}+\frac{3}{4}$ LCD = 12<br>$=\frac{8}{12}+\frac{9}{12}$ |
| **3** | Add the numerators. | $=\frac{17}{12}$ |
| **4** | Write the answer as a mixed number if appropriate. | $=1\frac{5}{12}$ |
| **5** | Check your answer against your estimate. Is it reasonable? | Reasonable |

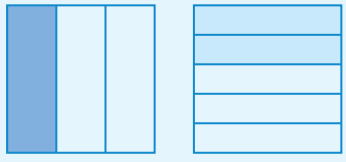
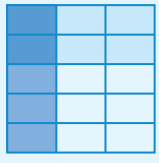
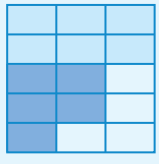
| | | | |
|---|---|---|---|
| (b) | **1** | Estimate the answer first. (Here, we are subtracting $\frac{1}{5}$ from a fraction that is less than 1, so the answer will be about $\frac{1}{2}$.) | (b) Estimate: about $\frac{1}{2}$ |
| | **2** | Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (Here, only one fraction needs to be rewritten.) | $\frac{7}{10} - \frac{1}{5}$ LCD = 10<br>$= \frac{7}{10} - \frac{2}{10}$ |
| | **3** | Subtract the numerators. | $= \frac{5}{10}$ |
| | **4** | Simplify the answer. | $= \frac{1}{2}$ |
| | **5** | Check your answer against your estimate. Is it reasonable? | Exact |

## The grid method

The grid method is a visual way of adding fractions, as demonstrated in the following Worked Example (however, it does not work so well for subtracting fractions).

## Worked Example 11 WE11

Add $\frac{1}{3}$ and $\frac{2}{5}$ by using the grid method.

| Thinking | Working |
|---|---|
| **1** Draw two identical squares or rectangles. Divide them up and shade in each fraction. | $\frac{1}{3} + \frac{2}{5}$  |
| **2** Lay one square or rectangle on top of the other to form a grid. |  |
| **3** Where the grid squares are shaded twice due to overlap, shade an equal number of blank grid squares to 'remove' the overlap. The resultant shaded grid represents the fraction sum. |  |
| **4** Write the answer to the fraction sum. | $\frac{11}{15}$ |

## Adding and subtracting mixed numbers

To add or subtract mixed numbers, we can either:

- add or subtract the whole number parts and fraction parts separately (Method 1), or
- convert the mixed numbers to improper fractions, then add or subtract them in the usual way (Method 2).

## Worked Example 12 WE12

Calculate $4\frac{5}{6} + 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

### Method 1: Add wholes and fractions separately

| Thinking | Working |
|---|---|
| **1** Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are each bigger than $\frac{1}{2}$. The answer lies between 7 and 8. | Estimate: between 7 and 8<br>$4\frac{5}{6} + 2\frac{3}{4}$ |
| **2** Rearrange the addition by separating the whole numbers from the fractions. | $= 4 + 2 + \frac{5}{6} + \frac{3}{4}$ |
| **3** Add the whole numbers. | $= 6 + \frac{5}{6} + \frac{3}{4}$ |
| **4** Rewrite the fractions with the LCD as the denominator. | $= 6 + \frac{10}{12} + \frac{9}{12}$ |
| **5** Add the fractions and simplify if possible. | $= 6 + \frac{19}{12}$ |
| **6** If the fraction sum is an improper fraction, convert it to a mixed number. | $= 6 + 1\frac{7}{12}$ |
| **7** Add the whole numbers. | $= 7\frac{7}{12}$ |
| **8** Check your answer against your estimate. Is it reasonable? | Reasonable |

### Method 2: Use improper fractions

| Thinking | Working |
|---|---|
| **1** Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are bigger than $\frac{1}{2}$. The answer lies between 7 and 8. | Estimate: between 7 and 8<br>$4\frac{5}{6} + 2\frac{3}{4}$ |
| **2** Write the mixed numbers as improper fractions. | $= \frac{29}{6} + \frac{11}{4}$ |
| **3** Rewrite the fractions with the LCD as the denominator. | $= \frac{58}{12} + \frac{33}{12}$ |
| **4** Add the numerators, simplify if possible. | $= \frac{91}{12}$ |
| **5** Convert to a mixed number. | $= 7\frac{7}{12}$ |

# Worked Example 13

WE13

Calculate $6\frac{5}{8} - 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

## Method 1: Subtract wholes and fractions separately

| Thinking | Working |
|---|---|
| **1** Estimate the answer first. Here, we are subtracting $2\frac{3}{4}$ from a number less than $6\frac{3}{4}$, so the answer will be less than 4. | Estimate: less than 4<br>$6\frac{5}{8} - 2\frac{3}{4}$ |
| **2** Rearrange the subtraction by separating the whole numbers from the fractions. | $= 6 - 2 + \frac{5}{8} - \frac{3}{4}$ |
| **3** Subtract the whole numbers. | $= 4 + \frac{5}{8} - \frac{3}{4}$ |
| **4** Rewrite the fractions with the LCD as the denominator. | $= 4 + \frac{5}{8} - \frac{6}{8}$ |
| **5** Subtract the fractions. If you cannot do the subtraction without getting a negative answer, 'borrow' 1 whole. (Here, 1 whole is $\frac{8}{8}$.) | $= 3 + \frac{8}{8} + \frac{5}{8} - \frac{6}{8}$<br>$= 3 + \frac{13}{8} - \frac{6}{8}$ |
| **6** Write the answer and simplify it if possible. | $= 3\frac{7}{8}$ |
| **7** Check your answer against your estimate. Is it reasonable? | Reasonable |

## Method 2: Use improper fractions

| Thinking | Working |
|---|---|
| **1** Make an estimate of the answer first, as for Method 1. | Estimate: less than 4<br>$6\frac{5}{8} - 2\frac{3}{4}$ |
| **2** Write the mixed numbers as improper fractions. | $= \frac{53}{8} - \frac{11}{4}$ |
| **3** Rewrite the fractions with the LCD as the denominator (only one fraction needs to be rewritten here). | $= \frac{53}{8} - \frac{22}{8}$ LCD = 8 |
| **4** Subtract the numerators. | $= \frac{31}{8}$ |
| **5** Simplify, if possible, and write your answer as a mixed number. | $= 3\frac{7}{8}$ |
| **6** Check your answer against your estimate. Is it reasonable? | Reasonable |

## Working with fractions on a calculator

The key that can be used for entering fractions into your calculator will often look something like this $\square\!\!\square$. To enter a fraction, press $\square\!\!\square$, then key in the numerator and the denominator, using the cursor keys to move between them. To enter a mixed number, press SHIFT $\square\!\!\square$.

On some calculators, the key looks like this: $a\,b/c$, or this $a/b$. To enter a fraction, such as $\frac{5}{6}$, type 5 $a\,b/c$ 6. You may need to press = for the fraction to be displayed on the screen. To enter a mixed number, such as $4\frac{3}{8}$, press 4 $a\,b/c$ 3 $a\,b/c$ 8.

The screen will use symbols such as ⌐, ⌙, or _ to separate the whole numbers, numerator and denominator.

Check that you can work with fractions correctly on your calculator by entering some additions and subtractions for which you already know the answers. For example, if you enter $\frac{3}{8} + 1\frac{2}{3}$ correctly, the answer of $2\frac{1}{24}$ will appear on the screen.

If you want to convert a mixed number that you have already entered to an improper fraction, press SHIFT and the $a\,b/c$ key, or SHIFT and the S⇔D key. (You might need to press = to see the result.)

You will also notice that pressing the S⇔D key, or pressing the $a\,b/c$ key twice converts a fraction to its decimal form.

# 3.4 Adding and subtracting fractions

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Column 1, Q6, Q7, Q8, Q10, Q13, Q14, Q16, Q17 | Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17 | Q1 Column 4, Q3 Columns 3 & 4, Q4 Columns 3 & 4, Q5 Column 3, Q6, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17 |

Answers page 637

## Fluency




WE10

**1** Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{5}{14} + \frac{1}{7}$ (b) $\frac{2}{5} - \frac{7}{20}$ (c) $\frac{5}{8} - \frac{7}{40}$ (d) $\frac{4}{55} + \frac{2}{11}$

(e) $\frac{5}{6} + \frac{1}{3}$ (f) $\frac{15}{49} + \frac{5}{7}$ (g) $\frac{3}{5} + \frac{13}{20}$ (h) $\frac{3}{4} + \frac{1}{5}$

(i) $\frac{1}{4} - \frac{1}{6}$ (j) $\frac{5}{9} + \frac{3}{4}$ (k) $\frac{1}{6} + \frac{3}{7}$ (l) $\frac{11}{15} + \frac{7}{20}$

(m) $\frac{11}{12} + \frac{3}{10}$ (n) $\frac{23}{30} - \frac{7}{20}$ (o) $\frac{3}{5} + \frac{9}{14}$ (p) $\frac{41}{45} - \frac{11}{18}$

WE11

**2** Add the following fractions by using the grid method.

(a) $\frac{2}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{1}{7}$

WE12

**3** Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{1}{7}+4\frac{2}{7}$ (b) $8\frac{1}{9}+5\frac{5}{9}$ (c) $2\frac{3}{8}+7\frac{1}{4}$ (d) $6\frac{7}{10}+1\frac{1}{5}$

(e) $1\frac{3}{4}+1\frac{1}{2}$ (f) $3\frac{1}{5}+2\frac{9}{10}$ (g) $4\frac{1}{2}+2\frac{3}{4}$ (h) $1\frac{7}{12}+7\frac{2}{3}$

(i) $6\frac{4}{5}+5\frac{1}{2}$ (j) $3\frac{8}{9}+7\frac{1}{6}$ (k) $1\frac{4}{11}+3\frac{2}{5}$ (l) $1\frac{4}{25}+2\frac{3}{20}$

WE13

**4** Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{5}{7}-2\frac{1}{7}$ (b) $3\frac{2}{9}-2\frac{5}{9}$ (c) $5\frac{7}{9}-1\frac{1}{3}$ (d) $7\frac{11}{12}-6\frac{1}{4}$

(e) $3\frac{1}{12}-1\frac{5}{6}$ (f) $4\frac{7}{10}-2\frac{1}{5}$ (g) $4\frac{1}{10}-2\frac{4}{5}$ (h) $6\frac{1}{18}-4\frac{1}{6}$

(i) $2\frac{1}{9}-1\frac{1}{6}$ (j) $2\frac{1}{4}-1\frac{2}{5}$ (k) $2\frac{1}{8}-1\frac{1}{6}$ (l) $5\frac{1}{8}-2\frac{2}{3}$

Did you know?
5 out of 4 people don't understand jokes about fractions!



**5** Find the following. You might like to use your calculator to check your answers.

(a) $\frac{3}{8}+\frac{1}{6}+\frac{1}{3}$ (b) $\frac{1}{2}+\frac{3}{4}-\frac{1}{3}$ (c) $\frac{7}{10}+\frac{4}{5}-\frac{1}{2}$

(d) $1\frac{1}{2}+\frac{2}{5}+2\frac{3}{10}$ (e) $2\frac{1}{3}+5\frac{3}{4}-1\frac{2}{5}$ (f) $3\frac{1}{4}+2\frac{1}{5}-3\frac{1}{3}$

**6** (a) $\frac{5}{8}-\frac{1}{3}=$

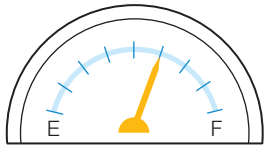
A $\frac{4}{24}$ B $\frac{7}{24}$ C $\frac{4}{5}$ D $\frac{23}{24}$

(b) $1\frac{2}{3}+\frac{3}{20}=$

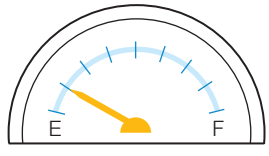
A $1\frac{1}{17}$ B $1\frac{5}{23}$ C $1\frac{5}{20}$ D $1\frac{49}{60}$

## Understanding

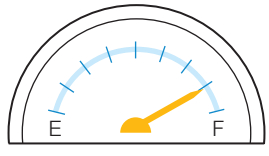
**7** When Eric got in his car and started driving, his fuel gauge looked like this:




Eric stopped to get petrol when his fuel gauge looked like this:




After putting some petrol in his car, Eric's fuel gauge looked like this:




Write a fraction addition or a subtraction to answer the following.

(a) What fraction of the tank of fuel did Eric use between the first two gauge readings? Write your answer in simplest form.

(b) What fraction of the tank of petrol did Eric add when he was at the petrol station? Write your answer in simplest terms.

**8** What fraction of a large pizza was eaten if Steven had $\frac{5}{12}$ of the pizza and Lara had $\frac{1}{3}$ of it? Write your answer in simplest form.

**9** Jaydeep, Corey and Isaac were sharing the driving on a trip along the coast. If Jaydeep drove $\frac{3}{5}$ of the distance and Isaac drove $\frac{3}{20}$, what fraction of the total distance did Corey drive?

**10** Belinda worked part-time at a fast food restaurant. Her hours for the three days she worked one week were: $2\frac{1}{4}$, $3\frac{1}{2}$ and $4\frac{1}{3}$. Calculate the total number of hours she worked for the week.




**11** Five large packets of cereal were bought for a Year 7 camp. On the first day $1\frac{1}{3}$ packets were eaten, on the second day $\frac{3}{4}$ of a packet was eaten and on the third day $2\frac{1}{6}$ packets were eaten. How much cereal was available for the fourth day?

## Reasoning

**12** A fraction is added to $\frac{3}{5}$. The lowest common denominator for the two fractions is 40. Which of the following fractions was added?

**A** $\frac{2}{6}$ **B** $\frac{3}{8}$ **C** $\frac{13}{20}$ **D** $\frac{11}{15}$

**13** A fraction is added to $\frac{3}{10}$. Which of the following would give an improper fraction as the result?

**A** $\frac{7}{15}$ **B** $\frac{7}{12}$ **C** $\frac{3}{5}$ **D** $\frac{18}{25}$

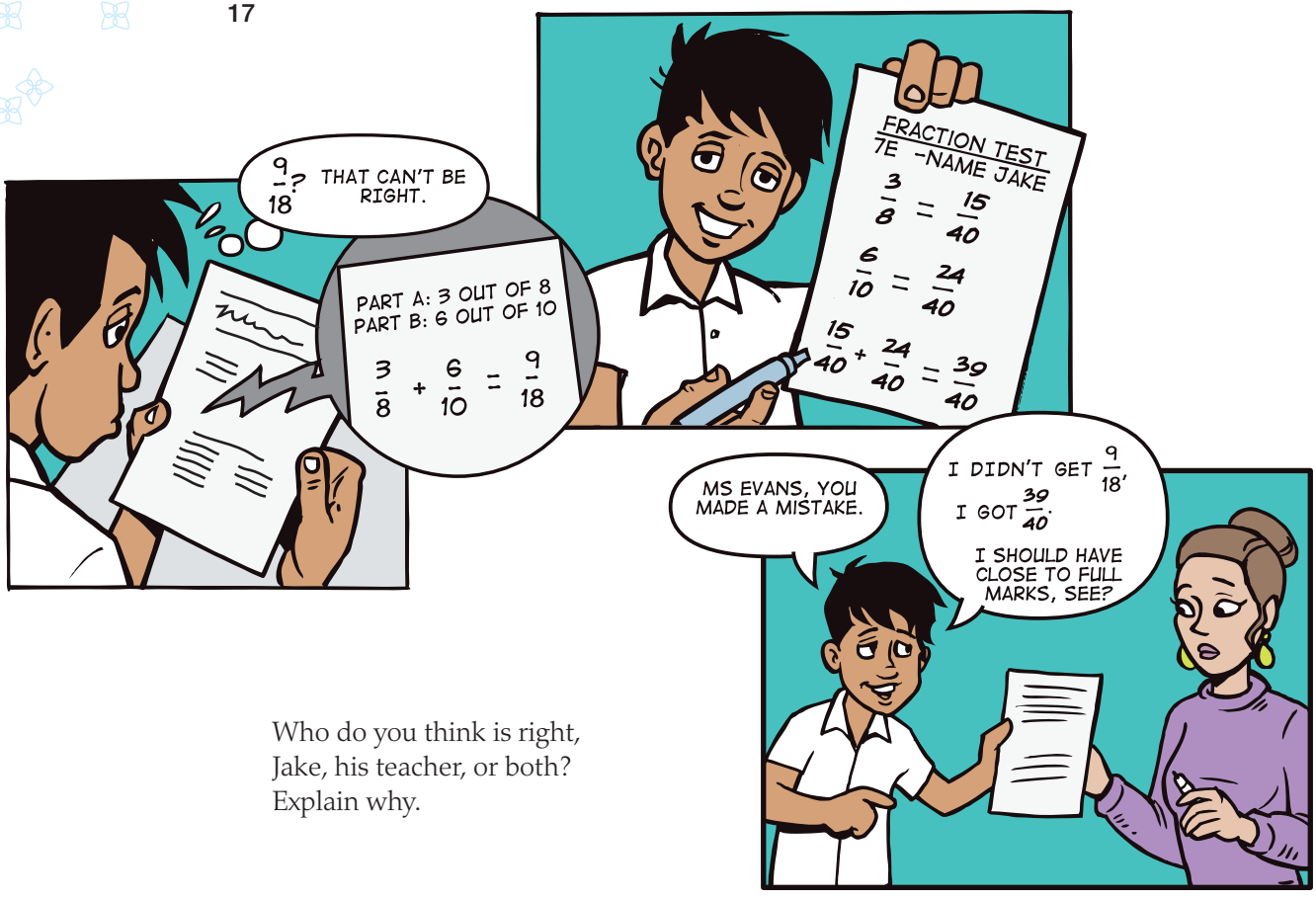
## Open-ended

**14** This question appeared on a Year 7 maths test: 'What whole number is $\frac{7}{8} + \frac{12}{13}$ approximately equal to?' Several students answered 19 or 21, both of which are incorrect. How might the students have arrived at their incorrect answers? Describe how you could use estimation to arrive at the correct answer.

**15** Find two fractions that add to $\frac{3}{4}$ and do not have a denominator of 4.

**16** Use a diagram to help you explain why $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$, not $\frac{7}{20}$. Why do we add the numerators, but not the denominators?

17




Who do you think is right, Jake, his teacher, or both? Explain why.

## Outside the Square Game

### Race to 10

**Equipment required:** 2 brains, 2 dice

**How to win:**
The aim is to be the person to get to a total as close to 10 as possible. Beware, because if you go over 10 you are out!

**How to play:**

On your turn, roll both dice. Choose one number to represent the numerator of a fraction and the other to represent the denominator. It is your choice whether you make a proper or an improper fraction.

For example: Die A rolls 5, Die B rolls 4, so the fraction is either $\frac{5}{4}$ or $\frac{4}{5}$.

Take it in turns to roll and keep a running total by adding your fractions as you go. When your total is as close to 10 as you dare, say 'I'll sit' when it comes to your turn. If you choose to roll and your total goes over 10, you are out.

Play the best of three games.

# Half-time 3

**1** There are 15 cookies in a bag. Ex.3.1



**(a)** If I eat 7 cookies, what fraction of the total cookies is this?

**(b)** If I eat $\frac{2}{5}$ of the cookies in the bag, how many cookies do I eat?

**2** Simplify the following. Ex.3.2

**(a)** $\frac{18}{48}$ **(b)** $\frac{9}{36}$ **(c)** $3\frac{9}{12}$ **(d)** $7\frac{35}{50}$

**3** Convert the following improper fractions to whole numbers or mixed numbers. Ex.3.2

**(a)** $\frac{27}{4}$ **(b)** $\frac{40}{8}$ **(c)** $\frac{50}{10}$ **(d)** $\frac{37}{7}$

**4** Determine which fraction in each of these pairs is the larger number, and write a < or > sign between them. If they are equivalent, write an '=' sign between them. Ex.3.3

**(a)** $\frac{5}{8}$ ____ $\frac{2}{3}$ **(b)** $\frac{4}{9}$ ____ $\frac{15}{27}$ **(c)** $\frac{48}{56}$ ____ $\frac{6}{7}$ **(d)** $\frac{11}{5}$ ____ $\frac{9}{4}$

**5** Draw a number line from 0 to 2. Divide the bottom side of the number line into quarters and the top side into eighths. Ex.3.1, 3.2, 3.3

**(a)** Show the positions of these fractions with a labelled arrow: $\frac{3}{4}, \frac{5}{8}, \frac{5}{4}, \frac{9}{8}, \frac{7}{4}, \frac{14}{8}$.

**(b)** Identify five pairs of equivalent fractions that are shown on the number line.

**(c)** Use the number line to determine which fraction out of the following pairs is larger.

**(i)** $\frac{5}{4}$ or $\frac{9}{8}$ **(ii)** $\frac{11}{8}$ or $\frac{7}{4}$.

**6** Calculate the following, writing your answers in simplest form. Ex.3.4

**(a)** $\frac{3}{8}+\frac{3}{4}$ **(b)** $\frac{7}{15}-\frac{3}{10}$ **(c)** $\frac{7}{9}+\frac{5}{6}$ **(d)** $\frac{11}{18}-\frac{5}{12}$

**7** Place the following in ascending order by first locating them on a number line. Ex.3.3

$\frac{1}{3}, \frac{6}{3}, \frac{9}{4}, \frac{3}{4}, 2\frac{2}{3}$

**8** 7 pizzas were each sliced into 6 equal pieces. Ex.3.2

**(a)** If $4\frac{5}{6}$ of the pizzas were eaten, how many slices was that?

**(b)** If $3\frac{1}{2}$ of the pizzas were eaten, how many slices were left over?

**9** Calculate the following. Ex.3.4

**(a)** $2\frac{7}{12}+2\frac{1}{10}$ **(b)** $1\frac{16}{25}+2\frac{13}{20}$ **(c)** $2\frac{13}{20}-1\frac{3}{100}$ **(d)** $3\frac{9}{10}-2\frac{8}{25}$

**10** Dave is leading a group of 12 people on a bushwalk. He has brought 30 L of water to supply the group. Ex.3.1, 3.2

**(a)** How many litres does each group member receive? Write your answer as both an improper fraction and as a mixed number in simplest form. Ex.3.4

**(b)** One of the group members has drunk $\frac{3}{4}$ of a litre by lunchtime. How many litres do they have left?

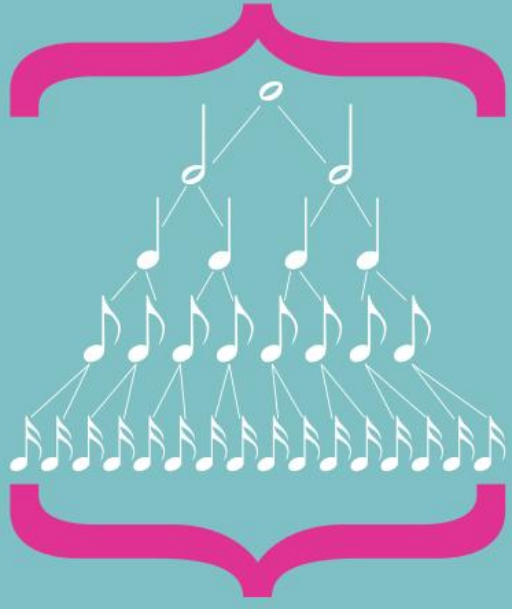
Maths 4 Real

# MUSICAL FRACTIONS

**In western cultures, music is written using symbols called 'notes'. Music notes have different shapes that indicate sounds of different durations, that is, they tell the musician how long the sound should be played for. The note symbols and their names are:**




The note names are based on fractions, because music notes 'add' together in the same way fractions do. This tree diagram shows us how shorter notes add together to give longer notes.



From the diagram, we can see that:

| 2 half notes are equal in length to 1 whole note: | 4 quarter notes are equal in length to 1 whole note: | 2 quarter notes are equal in length to 1 half note: |
|---|---|---|
| $\frac{1}{2}+\frac{1}{2}=1$ | $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$ | $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ |

## Note that:

Eighth notes and sixteenth notes are often joined in pairs, like this:

Or groups of four, like this:

1 Using the tree diagram and examples above, copy and complete the questions below by writing one equivalent note after the equals sign. Write the fraction sum underneath.

(a) (b) (c) (d)

We can also add different types of notes together, e.g.:

$\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$ $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$



**2** Copy and complete the questions below by writing one note in the space. Write the fraction sums underneath.

**(a)** $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \square = \square$ **(b)** $\frac{1}{8} + \frac{1}{8} + \frac{1}{4} + \square = \square$

**(c)** $\square + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$ **(d)** $\frac{1}{16} + \square + \frac{1}{4} = \frac{1}{2}$

**3** Copy and complete the questions below by writing 2 or more different notes in the spaces (there are several ways this can be done).

**(a)** $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \square$ **(b)** $\frac{1}{4} + \frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \square$

**(c)** $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \square = \frac{1}{4} + \frac{1}{2}$ **(d)** $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \square = 1 + 1$

## Time Signatures

Music is written on 5 parallel lines called a 'stave' and in sections called 'bars'. Each bar contains the same number of 'beats', or counts. 'Barlines' are drawn across the lines of the stave. You can see 4 bars and barlines in the piece of music below.

The 'time signature' is placed at the beginning of the first bar of a piece of music, and looks like a fraction. The top number tells us the number of beats in each bar, while the bottom number tells us what type of note counts as one beat.




A common time signature is $\frac{4}{4}$. There are 4 beats in each bar, and each beat is worth 1 quarter note. In $\frac{4}{4}$ the note values in each bar must add up to 4 quarter notes (1 whole note).

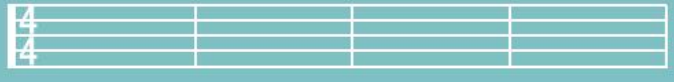
**4 (a)** Copy these bars, then use quarter notes or half notes to fill in the missing beats. Check that the fraction sum for each bar adds up to $\frac{4}{4}$.



**(b)** Copy these bars, then use any combination of half, quarter, eighth or sixteenth notes to fill in the missing beats. Make sure you check the fraction sum of each bar.



**5** Write your own piece of music in the four bars below. Use any number or combination of notes, just make sure that every bar 'adds up' to 4 quarter notes.



## Research

Find out about the music notation of other cultures, such as traditional Chinese or Indian music. Write some simple questions or exercises like the ones in this task to teach others what you have learnt.

# Investigation

## Tangram teaser




PP Blackline Master

**Equipment required:** 1 brain, 3E Tangram template, scissors

The tangram is an ancient Chinese puzzle. It has seven pieces that can be arranged to make hundreds of shapes and patterns, including the large square you can see here.



### The Big Question

How many different squares can be made by using various combinations of the seven tangram pieces? If the largest square is one whole, what are the fraction values of the smaller squares?

### Engage

The tangram has seven individual pieces of five different shapes: two large triangles, two small triangles, one medium triangle, one small square and one parallelogram.

1 If the large square that is made up of all the pieces represents one whole, estimate the fraction that is represented by:

   **(a)** a large triangle

   **(b)** the medium triangle

   **(c)** a small triangle

   **(d)** the small square

   **(e)** the parallelogram.

2 Check your estimates by cutting out the parts on your tangram template and using them to visually show what fraction of the large square they cover.

3 Not all seven pieces must be used to make a square. A smaller square can be made using four pieces, for example:



By adding up the fractions represented by each piece, work out what fraction of the large seven-piece square is taken up by this four-piece square. Write your answer in simplest form.

### Explore

4 Experiment with different numbers and combinations of pieces to make as many different squares as possible. Make sure you keep a visual record of each different square that you make. (Hint: There is more than one way of making the big, seven-piece square.)

**Strategy options**

- Draw a diagram.
- Make a table.
- Test all possible combinations.

## Explain

**5** Present accurate drawings of all the squares you have made, clearly showing the parts in each. Underneath your drawings, write a fraction sum that shows how each of the pieces add to give a fraction of the big, seven-piece square. How many different squares have you found?

## Elaborate

**6** Sometimes, the same four or five pieces can be used to make more than one square. How did you decide whether two squares were 'different' to each other?

**7** Using your definition of 'different,' summarise your findings by answering the Big Question.

## Evaluate

**8** Consider how you worked on the investigation and the methods you used. Do you think you have the complete set of solutions to this problem? Explain your answer.

**9** **(a)** Did you find this task challenging, or straightforward?

**(b)** What was the most difficult part of the task?

**(c)** Would you tackle puzzles like the tangram in your own time?

## Extend

**10** Try any or all of the following.

**(a)** Can you form a triangle using:

**(i)** two tangram pieces

**(ii)** three tangram pieces

**(iii)** four tangram pieces

**(iv)** five tangram pieces

**(v)** six tangram pieces

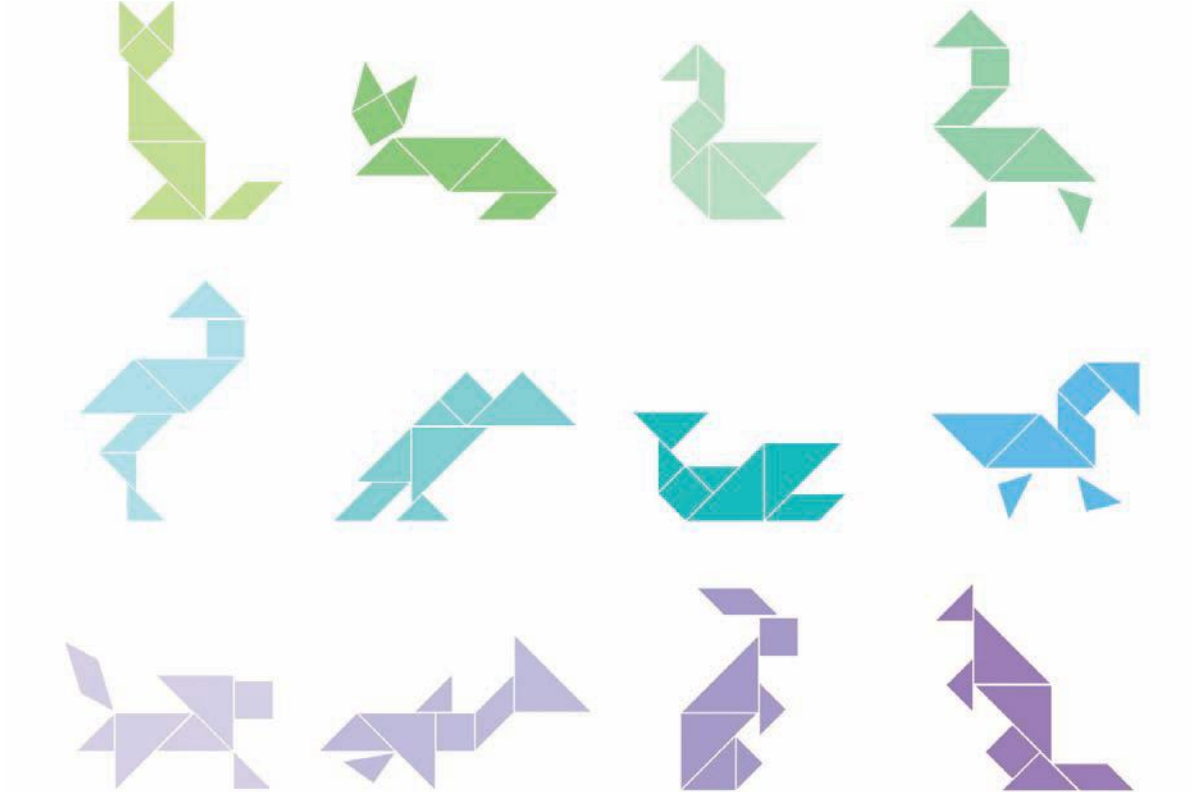
**(vi)** all seven tangram pieces?



Draw a diagram of each triangle formed and write a fraction sum to show what fraction of the large square each triangle represents.

**(b)** Use the tangram pieces to make one or both of your initials.

**(c)** Use the tangram designs below as inspiration to make your own tangram designs.



# 3.5 Multiplying fractions

The multiplication sign, ×, means '*lots of*', or simply '*of*'.
For example, 8 lots of \$5 = 8 × \$5 = \$40.

## Finding a fraction of a whole amount



We know that three-quarters of an hour is 45 minutes: $\frac{3}{4}$ of 60 = 45.

We can show how to calculate this in two ways:

**1** We could find one-quarter of an hour, then multiply by three.

$\frac{1}{4}$ of 60 = 60 ÷ 4 = 15

$\frac{3}{4}$ of 60 = 3 × 15 = 45

This method is called the 'Unit fraction method'.
A **unit fraction** is a fraction with a numerator of 1.
In the above example, the unit fraction is $\frac{1}{4}$.

(This method works well if the denominator of the fraction is a factor of the whole number. This makes the division step straightforward.)

**2** We could replace the word 'of' with the multiplication sign, ×, and multiply:

$$\frac{3}{4} \text{ of } 60 = \frac{3 \times 60}{4 \times 1} \quad \text{(writing 60 as an improper fraction with a denominator of 1)}$$

$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{60}^{15}}{1} \quad \text{(multiplying the numerators together and the denominators together, cancelling common factors first)}$$

$$= \frac{45}{1}$$

$$= 45$$

To find a fraction of a whole number, either:

- find the unit fraction (by dividing by the denominator), then multiply by the numerator

or

- write the whole number as an improper fraction and multiply the numerators together and the denominators together. Cancel any common factors between numerators and denominators before multiplying (this avoids having to work with large numbers).

## Worked Example 14 WE14

Find $\frac{2}{5}$ of 40.

### Method 1: Find the unit fraction, then multiply

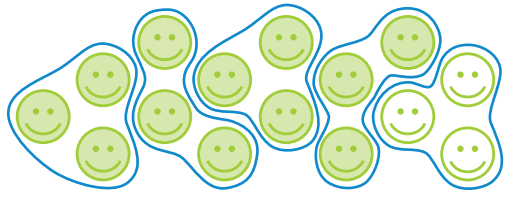
| Thinking | Working |
|---|---|
| **1** Find the unit fraction (in this case, $\frac{1}{5}$) by dividing the whole number by the denominator. | $40 \div 5 = 8$ |
| **2** Multiply the unit fraction amount by the numerator. | $2 \times 8 = 16$ |
| **3** Write the answer. | $\frac{2}{5}$ of $40 = 16$ |

### Method 2: Multiply two fractions

| Thinking | Working |
|---|---|
| **1** Replace 'of' with '×' and write the whole number as an improper fraction with a denominator of 1. | $\frac{2}{5}$ of $40$ |
| **2** Multiply the numerators and denominators together, cancelling common factors first (here, we cancel a common factor of 5). | $= \frac{2}{\cancel{5}_{1}} \times \frac{\cancel{40}^{8}}{1}$<br>$= \frac{16}{1}$ |
| **3** Simplify if possible. | $= 16$ |

## Finding a fraction of a fraction

Our 'smiley collection' from page 107 can help us understand what it means to find a fraction of a fraction.



In Worked Example 1, we found $\frac{4}{5}$ of 15 by dividing the collection into five equal groups and counting how many smileys were in four of those groups.

$\frac{4}{5}$ of 15 = 12



If we wanted to find $\frac{3}{4}$ of $\frac{4}{5}$, we would take the 12 smileys *as our new whole*, ignoring the fifth group. Finding $\frac{3}{4}$ of this new whole means taking three of the four groups, or nine smileys.

$\frac{3}{4}$ of 12 = 9

These nine of the 15 original smileys represent $\frac{3}{4}$ of $\frac{4}{5}$.

$\frac{3}{4}$ of $\frac{4}{5}$ of 15 = 9

They also represent $\frac{3}{5}$ of the original collection: $\frac{3}{5}$ of 15 = 9.

The following Worked Example shows how multiplying $\frac{3}{4}$ and $\frac{4}{5}$ gives $\frac{3}{5}$.

## Worked Example 15 WE15

Find $\frac{3}{4}$ of $\frac{4}{5}$.

| Thinking | Working |
|---|---|
| **1** Change the 'of' to ×. | $\frac{3}{4}$ of $\frac{4}{5}$ |
| **2** Multiply the numerators and denominators together, cancelling common factors before multiplying. (Here, the common factor is 4.) | $= \frac{3}{\cancel{4}_1} \times \frac{\cancel{4}^1}{5}$<br>$= \frac{3}{5}$ |

To find a fraction of a fraction:

- write mixed numbers as improper fractions before multiplying
- replace 'of' with a '×' symbol
- cancel any common factors
- multiply the numerators together and the denominators together
- simplify the answer further if necessary.

## Worked Example 16 WE16

Find $4 \times 2\frac{3}{20}$. Write your answer as a mixed number, if appropriate.

| Thinking | Working |
|---|---|
| **1** Write both numbers as improper fractions. (Whole numbers can be written as fractions with a denominator of 1.) | $4 \times 2\frac{3}{20}$<br>$= \frac{4}{1} \times \frac{43}{20}$ |
| **2** Cancel any common factors between numerators and denominators. (Here, the common factor is 4.) | $= \frac{\cancel{4}^1}{1} \times \frac{43}{\cancel{20}_5}$ |
| **3** Multiply the simplified numerator and the denominator. | $= \frac{1 \times 43}{1 \times 5} = \frac{43}{5}$ |
| **4** Write the answer as a mixed number. | $= 8\frac{3}{5}$ |

# 3.5 Multiplying fractions

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Column 1, Q5, Q6, Q8, Q10, Q11, Q13, Q14, Q16, Q19 | Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q4 Column 2, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q19 | Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 3, Q5, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19 |

Answers page 638

## Fluency

**1** Find the following.

WE14

**(a)** $\frac{1}{3}$ of 18 **(b)** $\frac{1}{4}$ of 12 **(c)** $\frac{1}{6}$ of 18 **(d)** $\frac{1}{9}$ of 36

**(e)** $\frac{3}{4}$ of 28 **(f)** $\frac{4}{5}$ of 30 **(g)** $\frac{6}{7}$ of 56 **(h)** $\frac{3}{8}$ of 72

**(i)** $\frac{3}{2}$ of 20 **(j)** $\frac{6}{5}$ of 100 **(k)** $\frac{11}{10}$ of 50 **(l)** $\frac{5}{4}$ of 60

**2** Find the following.

WE15

**(a)** $\frac{1}{2}$ of $\frac{2}{13}$ **(b)** $\frac{2}{9} \times \frac{1}{2}$ **(c)** $\frac{4}{5} \times \frac{5}{9}$ **(d)** $\frac{3}{7} \times \frac{7}{13}$

**(e)** $\frac{5}{7} \times \frac{1}{10}$ **(f)** $\frac{3}{5}$ of $\frac{10}{11}$ **(g)** $\frac{5}{12}$ of $\frac{6}{7}$ **(h)** $\frac{5}{6}$ of $\frac{1}{11}$

**(i)** $\frac{6}{7} \times \frac{3}{8}$ **(j)** $\frac{9}{10} \times \frac{5}{6}$ **(k)** $\frac{4}{7}$ of $\frac{2}{3}$ **(l)** $\frac{3}{10} \times \frac{9}{25}$

**3** Find the following. Write your answers as mixed numbers, if appropriate.

WE16

**(a)** $3 \times \frac{2}{9}$ **(b)** $\frac{3}{4} \times 8$ **(c)** $\frac{5}{8} \times 2$ **(d)** $3 \times \frac{5}{12}$

**(e)** $5 \times 2\frac{3}{10}$ **(f)** $8 \times 1\frac{3}{4}$ **(g)** $1\frac{4}{5} \times 15$ **(h)** $5\frac{1}{12} \times 8$

**(i)** $5\frac{1}{3} \times \frac{3}{8}$ **(j)** $3\frac{1}{2} \times 3\frac{1}{5}$ **(k)** $4\frac{1}{5} \times 4\frac{2}{7}$ **(l)** $1\frac{1}{8} \times 3\frac{5}{9}$

When cancelling common factors, don't cancel out two things on the same line—cancel something on the bottom with something on the top.



**4** Calculate the following, giving your answers in simplest form.

**(a)** $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ **(b)** $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ **(c)** $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$

**(d)** $1\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ **(e)** $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ **(f)** $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$

**5** **(a)** $\frac{2}{5}$ of $2\frac{1}{2}$ cups of sugar is:

**A** $\frac{1}{5}$ cup **B** $\frac{4}{10}$ cup **C** 1 cup **D** $2\frac{1}{5}$ cups

**(b)** $\frac{1}{5}$ of $\frac{1}{3}$ of 15 is:

**A** 1 **B** 2 **C** 3 **D** 5

## Understanding

**6** A group of 7 workers shared third prize in a local raffle. The prize was worth $2170.

**(a)** What fraction of the prize money does each worker receive?

**(b)** How much money does each worker get?

**7** The proceeds of a $48 000 inheritance were to be shared between four cousins. Each cousin was to receive a specific fraction of the inheritance. Michael was to receive $\frac{5}{24}$, Marcus $\frac{1}{12}$, Lily $\frac{3}{8}$ and Julie $\frac{1}{3}$. How much money did each cousin receive?

**8** A chocolate bar was made up of 45 equal pieces. Ling had to share it equally with her brother and sister, Gao and Chen.



**(a)** How many pieces did each person receive? Copy this chocolate block into your book and divide it up to show each person's share.

**(b)** Ling then decided to divide her share of chocolate equally between herself and four of her friends.

**(i)** How many pieces did each person receive?

**(ii)** What fraction of the original block is this?

**(iii)** On your drawing of the chocolate bar show the fraction that Ling and her friends receive.

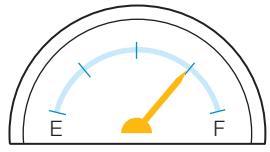
**9** Annika cut one-quarter of an apple pie in halves to share with her brother Arjun. What fraction of the original whole pie do they each now have?

**10** Draw a rectangle that is 5 cm long and 4 cm wide. Divide it lengthways into fifths. Shade in $\frac{1}{5}$. Now, take the shaded section and divide it into quarters. Shade in $\frac{3}{4}$ of this section.

**(a)** What fraction of the original rectangle have you shaded twice?

**(b)** Write the fraction multiplication that is shown by your diagram.

**11** A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.




**(a)** How much petrol has been used?

**(b)** How much petrol is still in the tank?

**12** Melissa surveyed 75 Year 7 students. $\frac{2}{3}$ said they enjoyed studying mathematics. $\frac{4}{5}$ of those who enjoyed mathematics also said they enjoyed studying science.



**(a)** How many students said they enjoyed mathematics?

**(b)** How many students said they enjoyed mathematics and science?

**13** Angelina is making Custard Cream cookies. Below are the ingredients that make 24 cookies.

180 g butter $\frac{1}{2}$ cup icing sugar $1\frac{1}{4}$ cups self-raising flour $\frac{1}{3}$ cup custard powder

Rewrite the ingredients to make 60 cookies. (Hint: What do you need to multiply 24 by to get 60?)

## Reasoning

**14** Without doing the calculation, which is larger: $1\frac{1}{2} \times 12$ or $\frac{1}{3} \times 9$?

**15** Without doing any calculations, say which of the following would give the biggest answer.

**A** $\frac{1}{2} \times \frac{3}{4}$ **B** $\frac{2}{5} \times \frac{2}{3}$ **C** $\frac{3}{4} \times \frac{6}{5}$ **D** $1\frac{1}{2} \times 1\frac{2}{5}$

**16** The missing number in the statement $\frac{5}{12} \times \frac{\square}{10} = \frac{5}{8}$ is:

**A** 4 **B** 5 **C** 10 **D** 15

## Open-ended

**17** Write two fractions that multiply to give $\frac{1}{6}$ as the simplified answer.

**18** $\frac{1}{\square} \times 3\square = 1\square$

Fill each of the boxes with a single digit (0–9) to make the statement correct. There are six possible combinations. Try to find at least three of them.

**19** Asher was asked to find $\frac{3}{5}$ of 60. His answer was 4. Here is his working:

$$\frac{\cancel{3}^{1}}{5} \times \frac{\cancel{60}^{20}}{1} = \frac{20}{5} = 4$$

Asher knows his answer is not correct, because he knows that $\frac{3}{5}$ is bigger than one half, and 4 is nowhere near half of 60! Explain where Asher has gone wrong. Include the correct working and answer to the problem.

## Outside the Square Game

### Fringo!

**Equipment required:** 2 brains, 2 dice

**How to win:**
The winner is the first person to have four fractions in a row (horizontal, vertical or diagonal) crossed off their grid. When they do, they should call out 'Fringo!'

**How to play:**
Fringo is fraction bingo! Draw a $4 \times 4$ grid into your book. Fill in your grid with any 16 of the numbers, fractions and mixed numbers on this list:

$1, 2, 3, 4, 5, 6, \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, 1\frac{1}{3}, 1\frac{2}{3}, \frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{1}{6}, \frac{5}{6}$

Take turns in rolling the dice one at a time. Make the number you roll first into the denominator of a unit fraction and write it down. For example, if you roll a 4, write down $\frac{1}{4}$. Multiply the number you roll second by this unit fraction, and write the answer in simplest form.

For example, Roll 1 = 4, Roll 2 = 2. Write down $\frac{1}{4} \times 2$ and calculate it (in this case, $\frac{2}{4}$, which simplifies to $\frac{1}{2}$).

If you have the answer to the fraction multiplication in your grid, cross it off.

(HINT: You may realise after playing a few rounds that some answers come up more than others. Thinking about where the best places are to put these on the grid will increase your chance of winning. You could also investigate just how many ways the fractions on the list can be made.)

# Mathspace

# FRACTURED RAVINE

**While exploring the mountains with your friends you fall down a ravine! Armed with your fraction skills, can you survive the perils that await you?**

**Equipment:** 1 die, 2–4 brains

**Rules:**

- Each player begins with 100 life points.
- If you lose all your life points you are out (dead).
- Take turns to roll the die and move forward that number of spaces. Follow the directions given in the space that you land on.
- All players must stop at the 'Leap of Faith' regardless of what number they have rolled.
- The goal: to be the first to reach sunlight.

**START**

**You sprain an ankle.**

$1\frac{1}{2} + 3\frac{1}{4} + 4\frac{2}{8} =$

*Deduct the solution from your life points total.*



**You call out 'Cooee' and cause a rock fall.**

$\frac{3}{4}$ of 16 =

*Deduct the answer from your life points total.*



**Your torch runs out of batteries.**

$\frac{1}{3}$ of 60 =

*Deduct the answer from your life points total.*



**Your abseil rope gets caught on a sharp rock.**

$\frac{2}{5}$ of 30 =

*Deduct the answer from your life points total.*



**You run out of drinking water.**

$12\frac{1}{3} + 3\frac{1}{6} - 4\frac{1}{2} =$

*Deduct the solution from your life points total.*



**The oxygen level decreases rapidly.**

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction = $\frac{3}{4}$.

Find the fraction of 60 that this would be,

e.g. $\frac{3}{4}$ of 60 = 45.

*Deduct this from your life points total.*





**You begin to feel claustrophobic.**

Roll the die and multiply the number you rolled by $\frac{2}{5}$. Round the answer to the nearest whole number.

*Deduct this from your life points total.*



**You eat the last of the food supply.**

Roll the die and multiply the number you rolled by $\frac{7}{2}$. Round the answer to the nearest whole number.

*Deduct this from your life points total.*



**A rock falls and breaks all the bones in one of your feet.**

The human body contains 206 bones. If approximately $\frac{1}{8}$ of these are in each foot, how many bones did you break (rounded to the nearest whole number)?

*Deduct this from your life points total.*



## LEAP OF FAITH

**ALL EXPLORERS STOP HERE**

**There is a large crevasse ahead, but it's the only way forward—you need to find the courage to jump!**

Roll the die and substitute the number you roll into this fraction $\frac{3}{\square}$.

If the fraction created is greater than (or equal to) 1 you've made it across.

If the fraction created is less than 1 you must wait until you can roll again.

**You reach a ladder and see sunlight above.**

**All explorers must stop at each rung of the ladder.**

(Players don't need to roll to move.)

**Rung 1:**

Find what fraction of your first name is made up of vowels. e.g. 'Natalie' $= \frac{4}{7}$. Do the same for your last name . Multiply one of these fractions by 50 and round to the nearest whole number.

*Deduct this from your life points total.*

**Rung 2:**

If the fraction of your first name that is made up of vowels is less than or equal to $\frac{1}{2}$, *deduct 10 from your life points total.* If it's greater than $\frac{1}{2}$, *deduct 15.*

**Rung 3:**

**You are on the final rung.**

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction $= \frac{3}{4}$. You need to roll a fraction larger than $\frac{2}{3}$ to make the final step out of the ravine.

*Every time you fail you must deduct 10 points from your life points total.*

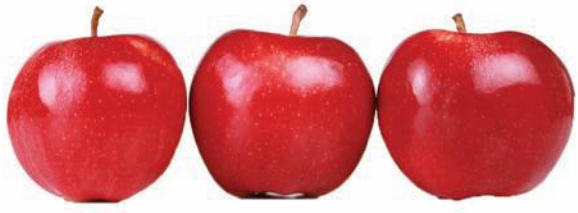




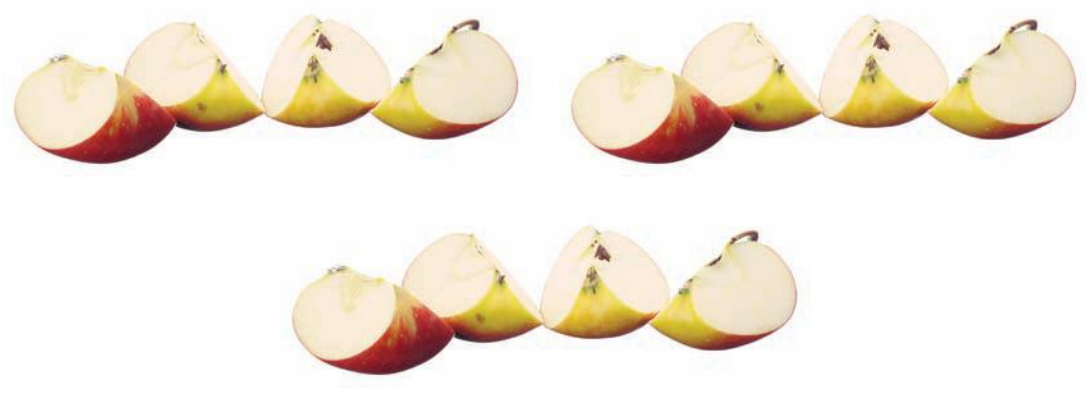
# 3.6 Dividing fractions

## Whole numbers divided by fractions

Here are three whole apples.



If we cut each apple into quarters, how many apple quarters will there be? We can think of this as 'how many quarters in 3' and write it as $3 \div \frac{1}{4} = ?$



We have 4 quarters in each apple, so there are 12 quarters in 3 apples: $3 \div \frac{1}{4} = 12$.

If we cut the apples into fifths, we would have fifteen fifths: $3 \div \frac{1}{5} = 15$.

If we cut the apples into sixths, we would have eighteen sixths: $3 \div \frac{1}{6} = 18$.

We can get our answer by multiplying the whole number by the denominator of the fraction.

i.e. $3 \div \frac{1}{4}$ $\quad$ $3 \div \frac{1}{5}$ $\quad$ $3 \div \frac{1}{6}$

$= 3 \times \frac{4}{1}$ $\quad$ $= 3 \times \frac{5}{1}$ $\quad$ $= 3 \times \frac{6}{1}$

$= 12$ $\quad$ $= 15$ $\quad$ $= 18$

The fractions have been turned upside down, or 'inverted', to show they are being multiplied like whole numbers.

Inverting a fraction (turning it upside down) is also known as 'finding the **inverse**'.

The inverse of $\frac{1}{4}$ is $\frac{4}{1}$ or simply 4.

What about dividing by fractions where the numerator is not 1, such as $3 \div \frac{3}{4}$?

If we put our 12 apple quarters on the previous page into groups of 3 quarters, there will be 4 groups. Because $\frac{3}{4}$ is three times as big as $\frac{1}{4}$, the answer to $3 \div \frac{3}{4}$ is one-third as big as $3 \div \frac{1}{4}$.

We can achieve the same answer by multiplying 3 by $\frac{4}{3}$, the inverse of $\frac{3}{4}$.

$$3 \div \frac{3}{4}$$
$$= \frac{\cancel{3}^{1}}{1} \times \frac{4}{\cancel{3}_{1}}$$
$$= \frac{4}{1}$$
$$= 4$$

To divide a whole number by a fraction, multiply by the inverse of the fraction.

The result of a division calculation is called the 'quotient'.

Another way of dividing a fraction (or a whole number) by a fraction is to convert them both to the same type of fraction; that is, fractions with the same denominator (the LCD). A common denominator means we have fractions of the same type, like having two measurements in the same units. Once we have a common denominator, we can perform the division on just the numerators. This is demonstrated in Method 2 of the Worked Example below.

## Worked Example 17

WE17

Calculate $9 \div \frac{3}{8}$.

### Method 1: Multiply by the inverse

| Thinking | Working |
|---|---|
| **1** Write the whole number as an improper fraction with a denominator of 1. | $\frac{9}{1} \div \frac{3}{8}$ |
| **2** Find the inverse of the second fraction (turn it upside down) and change the ÷ to ×. | $= \frac{9}{1} \times \frac{8}{3}$ |
| **3** Cancel any common factors between numerators and denominators. (Here, the common factor is 3.) | $= \frac{\cancel{9}^{3}}{1} \times \frac{8}{\cancel{3}_{1}}$ |
| **4** Multiply the simplified numerators and denominators. | $= \frac{3 \times 8}{1 \times 1} = \frac{24}{1}$ |
| **5** Write the answer. | $= 24$ |

## Method 2: Use equivalent fractions with the same denominator

| Thinking | Working |
|---|---|
| **1** Write the whole number as an improper fraction with the same denominator as the fraction (in this case, 8). | LCD = 8<br>$9 = \frac{72}{8}$ |
| **2** Rewrite the division using the two fractions. | $9 \div \frac{3}{8}$<br>$= \frac{72}{8} \div \frac{3}{8}$ |
| **3** Now that the denominators are the same, we can ignore them and perform the division with just the numerators. | $= 72 \div 3$<br>$= 24$ |

## Fractions divided by fractions

The 'invert the fraction you are dividing by and multiply' method also applies when dividing one fraction by another.

The 'use equivalent fractions with the LCD' method can also be used.

Convert mixed numbers to improper fractions before dividing.

## Worked Example 18

WE18

Calculate $1\frac{1}{6} \div \frac{2}{3}$.

## Method 1: Multiply by the inverse

| Thinking | Working |
|---|---|
| **1** If there are any mixed numbers, convert them to improper fractions. | $1\frac{1}{6} = \frac{7}{6}$<br>$\frac{7}{6} \div \frac{2}{3}$ |
| **2** Find the inverse of the second fraction (turn it upside down) and change the $\div$ to $\times$. | $= \frac{7}{6} \times \frac{3}{2}$ |
| **3** Cancel common factors and perform the simplified multiplication. | $= \frac{7}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2}$<br>$= \frac{7 \times 1}{2 \times 2}$<br>$= \frac{7}{4}$ |
| **4** Write the answer as a mixed number. | $= 1\frac{3}{4}$ |

## Method 2: Use equivalent fractions with the same denominator

| Thinking | Working |
|---|---|
| **1** Find the LCD. Write the mixed number as an improper fraction with the LCD. Write the fraction as an equivalent fraction with the LCD. | LCD = 6<br>$1\frac{1}{6} = \frac{7}{6}$<br>$\frac{2}{3} = \frac{4}{6}$ |
| **2** Rewrite the division using the fractions with the LCD. | $1\frac{1}{16} \div \frac{2}{3}$<br>$= \frac{7}{6} \div \frac{4}{6}$ |
| **3** Perform the division with just the numerators, and write the answer. | $= 7 \div 4$<br>$= 1\frac{3}{4}$ |

# 3.6 Dividing fractions

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1 & 2, Q4, Q5, Q6, Q8, Q9, Q12, Q15 | Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q15 | Q1 Column 4, Q2 Column 4, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15 |

Answers page 638

## Fluency

**1** Calculate the following.

WE17

**(a)** $6 \div \frac{1}{4}$ **(b)** $2 \div \frac{1}{6}$ **(c)** $3 \div \frac{1}{7}$ **(d)** $4 \div \frac{1}{9}$

**(e)** $4 \div \frac{2}{9}$ **(f)** $4 \div \frac{4}{5}$ **(g)** $5 \div \frac{3}{7}$ **(h)** $7 \div \frac{4}{9}$

**(i)** $6 \div 1\frac{1}{8}$ **(j)** $8 \div 2\frac{2}{3}$ **(k)** $6 \div 3\frac{1}{5}$ **(l)** $3 \div 2\frac{1}{4}$




**2** Calculate the following.

WE18

**(a)** $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{1}{4} \div \frac{1}{6}$ **(c)** $\frac{3}{4} \div \frac{2}{3}$ **(d)** $\frac{7}{10} \div \frac{1}{2}$

**(e)** $\frac{1}{3} \div 2$ **(f)** $\frac{1}{5} \div 4$ **(g)** $\frac{3}{4} \div 3$ **(h)** $\frac{9}{10} \div 6$

**(i)** $\frac{9}{10} \div 1\frac{3}{5}$ **(j)** $\frac{7}{6} \div 2\frac{1}{12}$ **(k)** $\frac{4}{5} \div 1\frac{8}{9}$ **(l)** $\frac{9}{10} \div 2\frac{2}{3}$

**(m)** $2\frac{2}{3} \div 1\frac{1}{3}$ **(n)** $5\frac{1}{2} \div 5\frac{3}{8}$ **(o)** $4\frac{1}{2} \div 1\frac{1}{5}$ **(p)** $4\frac{5}{7} \div 2\frac{3}{4}$

**3** **(a)** $5 \div \frac{15}{8} =$

**A** $\frac{8}{75}$ **B** $\frac{3}{8}$ **C** $2\frac{2}{3}$ **D** $\frac{75}{8}$

**(b)** $\frac{16}{7} \div \frac{20}{49} =$

**A** $\frac{5}{28}$ **B** $\frac{320}{343}$ **C** $\frac{343}{320}$ **D** $5\frac{3}{5}$

**4** Copy and complete the following. Do all working in your head.

**(a)** There are _____ halves in one, so $1 \div \frac{1}{2} =$ _____.

**(b)** There are _____ halves in three, so $3 \div \frac{1}{2} =$ _____.

**(c)** There are _____ quarters in one, so $1 \div \frac{1}{4} =$ _____.

**(d)** There are _____ quarters in two, so $2 \div \frac{1}{4} =$ _____.

**5** The inverse of $\frac{1}{3}$ is:

**A** $\frac{1}{6}$ **B** $\frac{2}{3}$ **C** 1 **D** 3

## Understanding

**6** There is $\frac{5}{8}$ of a metre of ribbon in the textiles class cupboard. Students require $\frac{1}{10}$ of a metre each for their projects. How many students will be able to use the ribbon?



**7** Scientists in a medical laboratory work in shifts of $3\frac{1}{2}$ hours each. How many whole shifts are to be worked during a period of 42 hours?

**8** Hannah is a baker whose specialty is scones. She has a sack containing 80 cups of flour. Her scone recipe uses $2\frac{2}{3}$ of a cup of flour for each batch of scones. How many batches of scones will Hannah get from her sack of flour?

**9** On a 15 km Fun Run course, drinks stations are placed every $2\frac{1}{2}$ km from the start. How many drinks stations will there be on the course? (There are no drinks stations at the start or finish lines.)




**10** The hit movie 'Exterminator 5' runs for $1\frac{3}{4}$ hours. A local cinema complex plans to screen it back to back (alternating the screenings between 2 cinemas). The first screening will start at 1:30 p.m. The last screening must finish by 11 p.m. How many times can the movie be screened within this time period?

## Reasoning

**11** The missing number from the statement $\frac{3}{4} \div \frac{\square}{8} = 2$ is:

**A** 1 **B** 2 **C** 3 **D** 4

**12** Without doing a calculation, state which will give the larger number as a result:
$\frac{4}{5} \div \frac{1}{3}$, or $\frac{4}{5} \div \frac{2}{3}$?

**13** Consider this sequence: $5 \div \frac{3}{4} = 6\frac{2}{3}$, $5 \div \frac{3}{5} = 8\frac{1}{3}$, $5 \div \frac{3}{6} = 10$, $5 \div \frac{3}{7} = 11\frac{2}{3}$.

**(a)** Why does the quotient (the answer to the division) increase as the denominator of the fractions in red is increased?

**(b)** Predict the answer to the next term in the sequence; i.e. $5 \div \frac{3}{8}$ will be ...

## Open-ended

**14** $4 \div \frac{2}{\square} = 1\square$

Use digits from 0 to 9 to fill in the boxes to make a correct statement. Find at least two different combinations.

**15** Here is Gabi's working on a fraction division problem:

$$3 \div \frac{1}{4} = 3 \div 4 = \frac{3}{4}$$

Her friend Natalie has worked it out differently. This is what Natalie wrote:

$$3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$$

'That can't be right', said Gabi—12 is way too big to be the answer!

Which of the girls' working is correct? Explain what the other has done wrong.

# Outside the Square — Problem solving

## Fill in the blanks

**1** Fill in the blanks with positive whole numbers to make mathematically true statements. Do not use the same number twice within a statement. See if you can come up with at least two different combinations.

**(a)** $\frac{\blacklozenge}{4} + \frac{1}{\blacklozenge} = \frac{\blacklozenge}{20}$ **(b)** $\frac{\bigstar}{\bigstar} - \frac{\bigstar}{6} = \frac{\bigstar}{12}$

**2** Use four of the digits 1, 3, 4, 5, 6, 7 to make two fractions whose sum is close to but less than 1.

$\frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} < 1$

Try to come up with at least three combinations. Which combination is the closest to 1?

**Strategy options**

- Guess and check.
- Test all possible combinations

# Technology Exploration Word

**Equipment required:** 1 brain, Microsoft® Word® 2010 (with free Mathematics add-in) or CAS/scientific calculator

## The evil genie and the curse of the infinite fraction

Once upon a time, there were two ladybirds that were very much in love. One day in an antique store, one of them crawled onto a lamp. With a puff of smoke, a genie appeared. The genie roared "Ha, ha, ha, I am an evil genie. You are currently 1 metre apart. For waking me up I curse you with the following rules: You will only be able to move once per day towards your love, and each day you will only be able to travel half of the distance remaining between you and your love. Your love may not step towards you". The genie then disappeared.



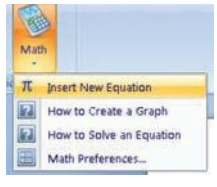
*Were the loving ladybirds ever able to be together again?*

On the first day, one of the ladybirds could travel half a metre forward, then on the next day a quarter of a metre, then an eighth of a metre on the third day.

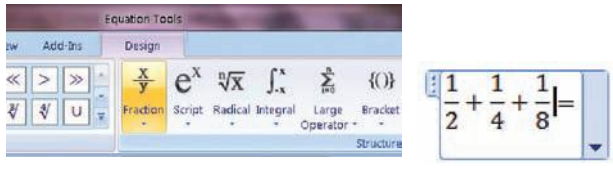
The total distance travelled by the ladybird in the first 3 days is represented by the following addition: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$.

**1** Enter the above expression into Word 2010.

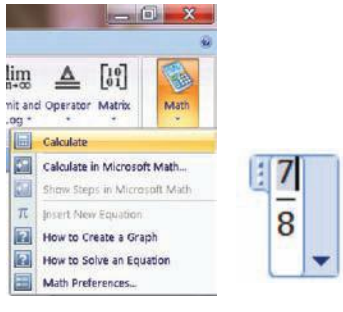
**Step 1:** Select Math from the Add-Ins menu. A drop-down menu will appear; select 'Insert New Equation'.




**Step 2:** Select the Fraction template from the Design ('Equation Tools') menu and enter the necessary fractions.




**Step 3:** To calculate the answer, select Math> Calculate, and the answer should appear.



**2** Explore the distance travelled by the ladybird by copying and pasting the previous fraction sum, and adding the next fraction in the series.

**(a)** What distance does it travel in:

**(i)** the first 5 days

**(ii)** the first 7 days

**(iii)** the first 11 days?

**(b)** At this stage, does it look like the distance between the ladybirds will ever be zero?

**3** Consider the distances travelled over the first 3, 5, 7 and 11 days. What value are your results getting closer to?

**4** To help you visualise the pattern, you can construct a diagram.

Draw a long rectangle to represent the distance between the ladybirds.

Shade in half of the rectangle to represent the distance travelled on the first day, and label it with $\frac{1}{2}$.




Shade in half of the unshaded portion of the rectangle to represent the distance travelled on the second day. Label this $\frac{1}{4}$.




Continue to shade the rectangle according to the pattern.




Continue this process as far as you can.




**5** From your diagram, you can see that you are getting closer and closer to shading in the whole rectangle. If you were able to zoom in and continue shading forever, would you ever be able to shade the entire rectangle? Explain your answer.

**6** Will the distance between the ladybirds ever be zero? Explain why or why not.

**7** Would the situation be different if the ladybirds had started closer together or further apart? Use an example to help you explain.

You considered the problem by calculating distances (in Questions **1**–**3**) and the same problem as a diagram (in Questions **4**, **5**). Which of these methods gave you a better 'sense' of the problem? Explain why.

## Taking it further

**8** When you add together the numbers in a pattern such as this, it is called a series. As you add together more and more terms, you may notice that the series gets closer and closer to a particular value. Series like these are called 'convergent series'. Sometimes, when you add together more and more terms, you may notice that the value just gets larger and larger. Series like these are called 'divergent series'.

**(a)** For the ladybird problem, did it seem to be a convergent or a divergent series?

**(b)** For each of the following series:

write down the next two terms in the series.

Use the Word Mathematics add-in to help you calculate the sum of the series for as many terms as you can.

State whether you think the following series will be convergent or divergent. Explain your answer.

**(i)** $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\ldots$

**(ii)** $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\frac{1}{54}\ldots$

**(iii)** $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\ldots$

**(iv)** $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}\ldots$

**(c)** In English, what does it mean when people say there was 'divergent opinion' about something? How is this similar to what you know about divergent series?

# 3.7 Mixed fraction problems

## Fractions and the order of operations

The order of operations rules that apply to whole numbers also apply to fractions.

The first step in any calculation is to complete any operation within brackets. The next step is to do any multiplication or division as you come to it, working from left to right. Then, do addition or subtraction as you come to it, working from left to right.

### Worked Example 19 WE19

Simplify the following using the correct order of operations.

(a) $3 + \frac{2}{5} \times \frac{1}{2}$ (b) $\frac{6}{7} \times \left(1\frac{2}{3} + \frac{1}{4}\right) - \frac{3}{7}$

| Thinking | Working |
|---|---|
| **(a) 1** As there are no brackets, do the multiplication first. | (a) $3 + \frac{2}{5} \times \frac{1}{2}$<br>$= 3 + \frac{{}^{1}\cancel{2} \times 1}{5 \times {}^{1}\cancel{2}}$<br>$= 3 + \frac{1}{5}$ |
| **2** Do the addition and state your answer in simplest form. | $= 3\frac{1}{5}$ |
| **(b) 1** Perform the operation in the brackets first. | (b) $\frac{6}{7} \times \left(1\frac{2}{3} + \frac{1}{4}\right) - \frac{3}{7}$<br>$= \frac{6}{7} \times \left(\frac{5}{3} + \frac{1}{4}\right) - \frac{3}{7}$<br>$= \frac{6}{7} \times \left(\frac{20}{12} + \frac{3}{12}\right) - \frac{3}{7}$<br>$= \frac{6}{7} \times \frac{23}{12} - \frac{3}{7}$ |
| **2** Next, perform any multiplication or division, cancelling any common factors. | $= \frac{\cancel{6}^{1} \times 23}{7 \times {}^{2}\cancel{12}} - \frac{3}{7}$<br>$= \frac{23}{14} - \frac{3}{7}$ |
| **3** Finally, perform any addition or subtraction. Write the answer in simplest form. | $= \frac{23}{14} - \frac{6}{14}$<br>$= \frac{17}{14} = 1\frac{3}{14}$ |

# 3.7 Mixed fraction problems

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2 (a), Q3 (a) & (b), Q4, Q5, Q6, Q11, Q14 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q13, Q15 | Q1 Columns 2 & 3, Q2, Q3, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15 |

Answers page 639

## Fluency

WE19

**1** Simplify the following using the correct order of operations.

**(a)** $2+\frac{3}{4}\times\frac{1}{3}$ **(b)** $5-\frac{7}{8}\times\frac{1}{7}$ **(c)** $\frac{5}{11}\div(5+4)$

**(d)** $\frac{3}{4}\times\left(\frac{1}{3}+\frac{1}{6}\right)\times\frac{1}{2}$ **(e)** $\frac{7}{8}\times\left(\frac{9}{14}-\frac{2}{7}\right)$ **(f)** $\frac{5}{6}\times\left(\frac{2}{5}+\frac{1}{3}\right)$

**(g)** $1\frac{5}{8}+\left(1\frac{3}{4}-1\frac{1}{8}\right)\times 5$ **(h)** $2\frac{3}{4}+\left(4\frac{1}{2}-3\frac{1}{4}\right)\times\frac{1}{2}$ **(i)** $1\frac{3}{5}+\left(3\frac{3}{10}-1\frac{1}{5}\right)\div 2$

**(j)** $1\frac{1}{4}-\left(3\frac{1}{3}-2\frac{5}{6}\right)\div 2$ **(k)** $3\frac{3}{8}+\left(8\frac{1}{4}-6\frac{3}{8}\right)\div 3$ **(l)** $\frac{7}{9}\div\left(1\frac{5}{6}-1\frac{3}{5}\right)\times\frac{1}{5}$

**2** **(a)** $\frac{3}{7}\times\frac{1}{3}+5=$

**A** $5\frac{1}{7}$ **B** $5\frac{1}{3}$ **C** $5\frac{5}{21}$ **D** $5\frac{7}{9}$

**(b)** $6-\frac{9}{13}\div\frac{3}{4}=$

**A** $5\frac{1}{13}$ **B** $5\frac{25}{52}$ **C** $5\frac{12}{13}$ **D** $7\frac{1}{13}$

## Understanding

**3** Write out the following questions using fractions, the operations symbols (×, ÷, +, –) and brackets where necessary. Then, evaluate using the correct order of operations.

**(a)** Add $\frac{7}{8}$ and $\frac{5}{6}$, and then multiply the total by 2.

**(b)** Find the product of $\frac{4}{5}$ and $\frac{1}{3}$, then add $1\frac{1}{2}$.

**(c)** Find the difference between $2\frac{4}{5}$ and $1\frac{2}{3}$, then multiply by 3.

**(d)** Find out how many $\frac{2}{3}$s in 5, then multiply by 4.

**4** A survey was taken of 120 people as they left Wally's Sandwich Bar. Write your answers to the following in simplest form.

**(a)** If 80 were male, what fraction were male?

**(b)** What fraction were female?

**(c)** If 45 were under 21 years of age, what fraction were under 21?

**(d)** What fraction were 21 or over?

**(e)** If $\frac{3}{4}$ had bought a drink, how many had bought a drink?

**(f)** Of those who bought a drink, $\frac{2}{5}$ had bought a salad roll. How many people had a drink and a salad roll?

**5** Grant is training in his backyard pool for the 1500 m freestyle at the next Olympics. Unfortunately, the pool is only 20 m in length.

**(a)** What fraction of the total 1500 m race would he have swum after completing just 1 lap?

**(b)** How many laps would he have to complete to swim 1500 m?

**(c)** What fraction of the total 1500 m race would he have swum after completing 6 laps?

**(d)** How far would he have swum if he had completed $\frac{1}{10}$ of the race distance?

**(e)** How far would he have swum if he had completed $\frac{3}{4}$ of the race distance?



**6** Valerie is running laps of her school oval to train for Rigby High's Mini Marathon. She ran $4\frac{1}{2}$ laps before having to rest. After her rest, she ran another $3\frac{1}{4}$ laps before stopping again. Then, she struggled through another $\frac{1}{3}$ of a lap. How many laps did she complete altogether?

**7** One weekend, Louisa and Andrew walk from Ferndale to Greenhill and back, a total of $2\frac{3}{4}$ km. The next weekend they walk from Ferndale through Greenhill to Highvale, which is $5\frac{1}{3}$ km.

How far (in km) is it from:

**(a)** Ferndale to Greenhill

**(b)** Greenhill to Highvale

**(c)** Ferndale to Highvale and back again?

**8** Jeremy worked at an amusement park for 4 hours at the normal hourly rate, then another $2\frac{1}{2}$ hours at $1\frac{1}{2}$ times the normal rate. What was the equivalent number of normal hours he was to be paid?



## Reasoning

**9** Theresa, Isabelle and Bryce worked on a mathematical problem and came up with the answers $8\frac{1}{2}$, $8\frac{2}{3}$ and $8\frac{3}{5}$, respectively. The correct answer was $8\frac{7}{12}$. Which of the three students was closest to the correct answer?

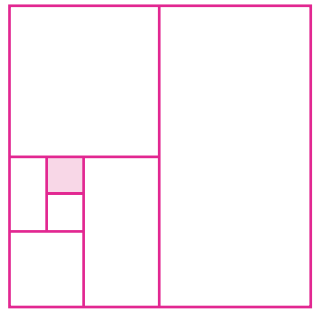
**10** Despina's department store advertises a winter sale in which they claim everything is between $\frac{1}{3}$ and $\frac{1}{2}$ off. For which of the following items is this incorrect?

| Item | Normal price | Sale price |
|---|---|---|
| Bath towels | $22 | $12 |
| Camera | $900 | $600 |
| Cutlery set | $250 | $120 |
| Microwave oven | $540 | $350 |
| Jeans | $60 | $35 |



**11** The Happy Valley Farming Cooperative has divided its square block of land in the way shown in the diagram. Each section of land is exactly half of the section next to it.

The May family has been given the shaded region to farm. If the total block of land is 1, what fraction of the total *don't* the May family farm?



Draw a diagram or flowchart to show this process.



**12** Prashanth has agreed to sell badges for a charity. He receives a large box and places them, with permission, at the school reception. After several days, Prashanth took an estimate of the number of badges he had left. He emptied out the box, divided the pile of badges roughly into halves, and put half back in the box. He then halved the pile he had left, and returned half to the box. He did this one more time, after which he counted the number of badges in the pile, and found he had 13.

**(a)** What fraction of the total number of badges does 13 represent?

**(b)** How many badges did Prashanth have in the box, approximately?

## Open-ended

**13** Write down three numbers in fraction form that are less than $\frac{7}{8}$ but greater than $\frac{1}{2}$.

**14** $A$ and $B$ are two different numbers selected from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where $A$ is always greater than $B$.

**(a)** What values can $\frac{A+B}{A-B}$ have? Give three possibilities.

**(b)** What is the largest value this fraction can have?

**15** 'Ideal fractions' are pairs of fractions whose sum and product are the same. (They give the same number when added as when multiplied.)

**(a)** Show that $\frac{7}{3}$ and $\frac{7}{4}$ are ideal fractions.

**(b)** Show that $\frac{5}{3}$ and $\frac{5}{2}$ are ideal fractions.

**(c)** Look carefully at the numerators and denominators of the fraction pairs in **(a)** and **(b)**. Try to find a pattern.

**(d)** Write down another pair of ideal fractions.

# Outside the Square — Problem solving

## Mega-equivalents

**1** Use four of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 to make a fraction equivalent to $\frac{1}{2}$ that has two digits in the numerator and in the denominator (e.g. $\frac{34}{68}$).

**2** Now, use six digits to make another fraction equivalent to $\frac{1}{2}$ that has three digits in the numerator and denominator.

**3** See if you can form another fraction equivalent to $\frac{1}{2}$ using eight digits (four in numerator and denominator).

**4** Can you make a fraction equivalent to $\frac{1}{2}$ using all 10 digits?

**Strategy options**

- Guess and check.
- Test all possible combinations.

# Challenge 3

**1** If $\frac{1}{3}$ of a number is 16, then $\frac{3}{8}$ of this number is:

**A** 8 **B** 16 **C** 18 **D** 24

**2** Kerry bought a new water tank for her garden. The tank is initially empty, but fills to $\frac{2}{3}$ full of capacity after rain. The tank now holds 480 litres of water. How many litres does the tank hold when full?

**3** If $\frac{\nabla}{16}$ lies between $\frac{1}{4}$ and $\frac{3}{8}$, and $\nabla$ is a whole number, then $\nabla$ equals:

**A** 5 **B** 6 **C** 7 **D** 8

**4** Two proper fractions (in simplest form) are subtracted, the first with a denominator of 10, and the second with a denominator of 6. If the answer is given in simplest form, the largest possible numerator that it could have is:

**A** 8 **B** 9 **C** 11 **D** 22

**5** The number halfway between $\frac{1}{5}$ and $\frac{1}{20}$ is:

**A** $\frac{1}{10}$ **B** $\frac{1}{8}$ **C** $\frac{7}{40}$ **D** $\frac{1}{4}$

**6** Two frogs live together in a backyard pond. One frog croaks every 4 minutes, the other frog croaks every $2\frac{1}{2}$ minutes. If they both croak together at 11 a.m., what is the first time after 1 p.m. that they will croak together?



**7** $2 + \dfrac{6}{1 + \frac{1}{2}}$ equals:

**A** 2 **B** 4 **C** 6 **D** 8

**8** Express this fraction in its simplest form:

$$\cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{3}}}}$$

**9** If 60 is added to one-third of a number, the resulting value is double the number. What is the number?

**10** Five numbers are put in a row from smallest to largest. The difference between each adjacent number (a number and the number next to it) is the same. If the first number is $\frac{1}{8}$ and the last is $\frac{1}{6}$, what are the three numbers in between, in simplest form?

**11** Tom was born into a strange mathematical family. His grandmother sent him a money voucher for his birthday. The card said:

*Dear Tom,*
*I am giving you the following amount to spend as your wish*
$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of $1000.
*Love Grandma*

How much birthday money did Tom receive?

# Chapter review 3

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| cancelling | improper fraction | mixed number | simplest form |
| denominator | inverse | numerator | simplify |
| equivalent fractions | Lowest Common Denominator (LCD) | proper fraction | unit fraction |
| fraction | | | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 In a fraction, the _____ tells us how many parts we have, and the _____ tells us the size of each part.

2 A _____ has a value between 0 and 1.

3 To _____ a fraction, divide the numerator and denominator by a common factor. This process is known as _____.

4 Dividing by a fraction is the same as multiplying by the _____ of the fraction.

5 An _____ has a numerator that is greater than or equal to the denominator.

6 _____ are located at exactly the same position on a number line.

7 A _____ is a fraction with a numerator of 1.

## Fluency

1 Here are 3 dozen (36) eggs. Ex 3.1, 3.2

**(a)** If $\frac{4}{9}$ of the eggs are slightly cracked, how many eggs are damaged?



**(b)** The farmer's 3 best hens laid 15 of the eggs pictured. What fraction of the total is this? Write your answer in simplest form.

2 Write the value of the fractions shown by the arrows on this number line: Ex 3.1




3 Write **(a)** 6 and **(b)** 11 as improper fractions with denominators of: Ex 3.1

**(i)** 3 **(ii)** 10 **(iii)** 1

4 **(a)** 3 pizzas are shared between 5 students. Write the amount of pizza each student gets as a fraction. Ex 3.1

**(b)** 7 packets of lollies are shared between 4 students. Write the number of packets each student receives as a mixed number.

**5** Find equivalent fractions by copying and completing the following. Ex 3.2

**(a)** $\frac{2}{7} = \frac{12}{\square}$ **(b)** $\frac{12}{18} = \frac{\square}{6}$ **(c)** $\frac{15}{30} = \frac{45}{\square}$ **(d)** $\frac{\square}{24} = \frac{5}{4}$

**6** Write the following fractions in their simplest form. Ex 3.2

**(a)** $\frac{12}{15}$ **(b)** $\frac{48}{20}$ **(c)** $3\frac{60}{72}$ **(d)** $1\frac{8}{40}$

**7** **(a)** Write $3\frac{2}{7}$ as an improper fraction. Ex 3.2

**(b)** Write $\frac{50}{9}$ as a mixed number.

**8** Copy and complete the following by inserting <, > or =. Ex 3.3

**(a)** $\frac{5}{13} ____ \frac{1}{2}$ **(b)** $\frac{6}{16} ____ \frac{30}{80}$ **(c)** $\frac{5}{12} ____ \frac{3}{8}$ **(d)** $\frac{8}{9} ____ \frac{9}{10}$

**9** Evaluate the following. Estimate first to check that your answers are reasonable. Ex 3.4

**(a)** $\frac{5}{12} + \frac{7}{8}$ **(b)** $\frac{7}{18} - \frac{2}{9}$ **(c)** $\frac{2}{3} - \frac{2}{7}$ **(d)** $\frac{1}{6} + \frac{7}{10}$

**10** Evaluate the following. Write your answers as mixed numbers in simplest form. Ex 3.4

**(a)** $3\frac{2}{5} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} + 5\frac{11}{12}$ **(c)** $8 - 2\frac{5}{7}$ **(d)** $4\frac{5}{6} - 4\frac{2}{3}$

**11** Calculate the following. Ex 3.5

**(a)** $\frac{3}{7}$ of \$28 **(b)** $\frac{2}{9}$ of $\frac{3}{4}$

**12** Evaluate the following. Ex 3.5

**(a)** $\frac{6}{11} \times \frac{55}{18}$ **(b)** $2\frac{4}{9} \times 5$ **(c)** $\frac{2}{3} \times \frac{9}{16} \times 1\frac{1}{7}$

**13** Evaluate the following. Ex 3.6

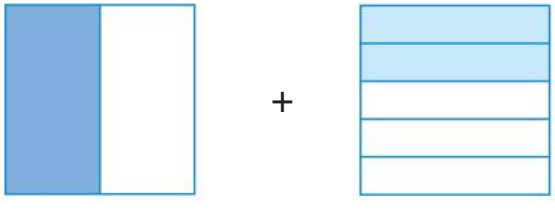
**(a)** $9 \div \frac{3}{4}$ **(b)** $21 \div 3\frac{2}{7}$ **(c)** $\frac{6}{5} \div \frac{32}{15}$ **(d)** $3\frac{1}{4} \div 3\frac{1}{3}$

**14** Simplify by using the correct order of operations. Ex 3.7

**(a)** $4 + \frac{5}{6} \times 2\frac{1}{3}$ **(b)** $7\frac{2}{5} - \left(3\frac{1}{10} - \frac{3}{5}\right)$ **(c)** $\frac{7}{8} \times \left(\frac{3}{4} + 4 \times \frac{3}{4}\right)$

## Understanding

**15** **(a)** What fraction sum is represented by the following two grids?



**(b)** Calculate the fraction sum using the grid method or any other method.

**16** Write a fraction or a mixed number in simplest form to show each of these:

**(a)** eight chocolate biscuits in a packet of 24

**(b)** two complete '6-packs' of soft drink cans, with one can left over

**(c)** one packet of twelve colour pencils and four pencils in a second packet

**(d)** nine hours sleep in one day.

**17** There are 12 boys and 15 girls in class 7A. 6 of the students have blond hair, and 2 have red hair. Write the following as fractions of the whole class, in simplest form.

**(a)** the number of girls

**(b)** the number of blond-haired students

**(c)** the number of students with neither blond nor red hair

**18** Write each set of numbers in order from smallest to largest, by using strategies for comparing fractions.

**(a)** $\frac{3}{4}, \frac{1}{2}, 1, \frac{1}{3}, \frac{3}{5}$

**(b)** $2, \frac{13}{5}, \frac{5}{4}, \frac{30}{50}, 1\frac{2}{5}$

**19** Jamal gets 7 out of 9 shots at goal in the netball ring, whereas Kayla gets 9 out of 12 shots in. Determine who is the more accurate shooter by comparing fractions.

**20** Elise and Reece each have an identical block of chocolate. Elise eats $\frac{2}{5}$ and Reece eats $\frac{2}{3}$.

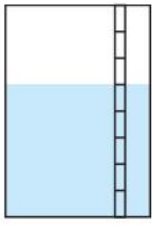
**(a)** How much more has Reece eaten, as a fraction of a block?

**(b)** Together, have Elise and Reece eaten more or less than a whole block of chocolate? Estimate, then calculate your answer.

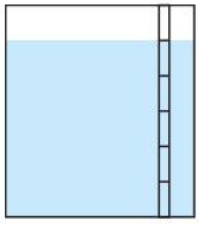
**21** Gary had $1\frac{1}{2}$ bags of cement in his shed. He used $\frac{2}{3}$ of a bag to mix up some concrete. What fraction of a full bag does he have left?

**22** Jarrod owns a petrol station that has three main fuel tanks. There is a dip stick in each tank that shows the level of fuel remaining in each tank. How many litres are left in each tank?

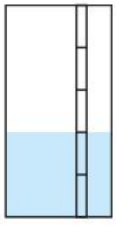
**(a)** Full tank = 48 000 L



**(b)** Full tank = 60 000 L



**(c)** Full tank = 30 000 L



**23** Here are the ingredients for a choc-orange cake:

| | | |
|---|---|---|
| 180 g butter | $\frac{2}{3}$ cup castor sugar | 4 eggs |
| $1\frac{3}{4}$ cups self-raising flour | 2 tablespoons cocoa | $\frac{1}{2}$ cup orange juice |

Rewrite the ingredients to make a cake that is one-quarter of the size of this cake.

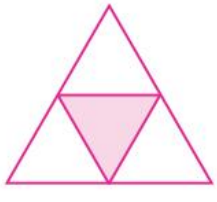
## Reasoning

**24** State the fraction of the area of these shapes that is shaded.

**(a)**



**(b)**



**(c)**



**25** 45 minutes into a Fun Run, Katya had completed 4 km of the 10 km course, whereas Mabok had completed 7 km of the 15 km course.

**(a)** Who had the least distance still to run?

**(b)** Who had completed a bigger fraction of their particular course?

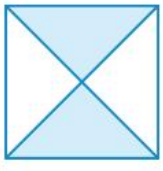
**26** Which of the following will give an answer of 1?

**A** $\frac{1}{4} \div \frac{1}{4}$ **B** $\frac{1}{2} \div \frac{1}{4}$ **C** $1 \div \frac{1}{4}$ **D** $4 \div \frac{1}{4}$

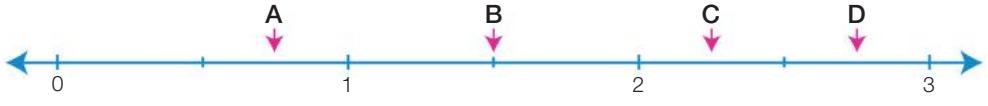
# NAPLAN practice 3

## Numeracy: Non-calculator

**1** Which diagram does not have exactly $\frac{1}{2}$ of the area shaded?

**A**  **B**  **C**  **D** 

**2** Which arrow is pointing closest to the location of $\frac{11}{8}$ on this number line?




**3** Kevin is mixing sand, gravel and cement to make concrete. Sand makes up $\frac{1}{2}$ of the mixture, and gravel is $\frac{1}{3}$. What fraction of the mixture is cement?

**A** $\frac{1}{6}$ **B** $\frac{1}{5}$ **C** $\frac{1}{4}$ **D** $\frac{1}{3}$

**4** Esther is making vanilla ice-cream using these ingredients that make 6 servings:

5 eggs
1 cup sugar
2 cans condensed milk
6 cups milk
$1\frac{1}{2}$ teaspoons vanilla

How many teaspoons of vanilla will Esther need to make 30 servings?

**A** $5\frac{1}{2}$ **B** $7\frac{1}{2}$ **C** $30\frac{1}{2}$ **D** 45

## Numeracy: Calculator allowed

**5** A school has 200 students. 48 of the students are involved in the school production. The fraction of students who are involved in the school production is closest to:

**A** one-fifth **B** one-quarter **C** one-third **D** one-half

**6** Which of the following fractions is greater than 2 but less than 3?

**A** $\frac{3}{4}$ **B** $\frac{12}{9}$ **C** $\frac{17}{7}$ **D** $\frac{15}{5}$

**7** Imran ran $\frac{1}{6}$ of a cross-country course. He ran 3 km. How long is the course?

**A** 6 km **B** 9 km **C** 12 km **D** 18 km

**8** Which fraction lies exactly halfway between $\frac{3}{8}$ and $\frac{4}{8}$ on the number line?

**A** $\frac{3}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{7}{16}$


4
Herald Sun
austral bricks
DAINTY
tab sports
TXT 0409 117 621
4th QTR
31:50
COLL 9.14. 68 STK 10. 8. 68
Herald Sun
Herald Sun

# Decimals, percentage and ratio

4

## 'When is a decimal point not a decimal point?'

A number may contain a point, but this does not make it a decimal point.

Usually, in mathematics, we use numbers to represent quantities, so we can calculate with them. These numbers are called the 'cardinal numbers'. We also use numbers in a less mathematical way—as labels or 'tags' that help to identify things. These numbers are called 'nominal numbers'. Room numbers, telephone numbers, and the footy score in the photo are all examples of nominal numbers. Some nominal numbers include a point that looks like a decimal point, but is actually a 'separator', placed between numbers that are labels for different things. For example, a score of 3.6 in an AFL game means 3 goals and 6 points, not 3 goals and 6 tenths of a fourth goal. A large building such as a hotel might have a room number written as 2.13, meaning the thirteenth room on the second floor, not 2 rooms and 13 hundredths of a third room.

### Forum

We read the time of 7.30 as 'seven thirty' or 'half past seven'. If we wanted to write this time as a proper decimal number, what would we write? How many minutes would 0.30 hours be? Can you think of other examples where numbers may have a point and look like decimal numbers, but are, in fact, nominal numbers?

Amounts of money, such as $4.35, are written with a point. Is it a decimal point?

### Why learn this?

An understanding of decimal numbers helps us to measure the wood we need to build a new shelf, work out who won the 100-m freestyle or calculate a household budget. Percentages have many everyday uses, such as measuring performance, advertising discounts or presenting survey results. Ratios and rates help us compare and calculate quantities of the same and different types, such as determining which product size is the best value for money.

**After completing this chapter you will be able to:**

- compare, order and round decimal numbers
- add, subtract, multiply and divide decimal numbers
- convert between decimals, fractions and percentages
- use estimations to check that answers are reasonable
- use percentages to solve problems
- use ratios and rates to compare and calculate amounts
- understand the relationship between ratios, fractions and percentages
- calculate unit prices and determine 'best buys'.

# Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

**1** Copy and complete each of the following by writing < (less than), > (greater than), or = (equal to) between the given numbers.

Worksheet R4.1

**(a)** 0.1 _____ 0.01 **(b)** 2 _____ 2.2 **(c)** 0.3 _____ 0.1

**(d)** 3.2 _____ 4.1 **(e)** 0.008 _____ 0.09 **(f)** 0.7 _____ 0.07

**2** The number 43 can be written in expanded form as 'four tens and three ones'. Write the following in expanded form.

Worksheet R4.2

**(a)** 72 **(b)** 603 **(c)** 9251 **(d)** 11 080

**3** Write each of the following **(i)** in words and **(ii)** as decimals.

Worksheet R4.3

**(a)** $\frac{7}{10}$ **(b)** $\frac{8}{1000}$ **(c)** $\frac{3}{100}$ **(d)** $1\frac{9}{10}$

**4** Calculate:

Worksheet R4.4

**(a)** $\begin{array}{r} 34 \\ +\ 76 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 925 \\ +\ 610 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 67 \\ 409 \\ +\ 3 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 459 \\ 6013 \\ +\ 27 \\ \hline \end{array}$

**5** Calculate:

Worksheet R4.5

**(a)** $\begin{array}{r} 74 \\ -\ 25 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 823 \\ -\ 376 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 8289 \\ -\ 384 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 2000 \\ -\ 352 \\ \hline \end{array}$

**6** Calculate:

Worksheet R4.6

**(a)** $2 \times 17$ **(b)** $25 \times 96$ **(c)** $3 \times 48$ **(d)** $51 \times 800$

**7** Perform the following divisions.

Worksheet R4.7

**(a)** $362 \div 2$ **(b)** $9459 \div 9$ **(c)** $5600 \div 200$ **(d)** $4500 \div 30$

**8** Calculate:

Worksheet R4.8

**(a)** $70 \times 100$ **(b)** $12 \times 10\,000$ **(c)** $9200 \times 1000$

**(d)** $80 \div 10$ **(e)** $3600 \div 100$ **(f)** $250\,000 \div 1000$

**(g)** $1.2 \times 10$ **(h)** $6.3 \times 100$ **(i)** $0.0427 \times 1000$

**(j)** $58 \div 10$ **(k)** $901 \div 100$ **(l)** $76.2 \div 1000$

**9** Write these percentages as fractions in simplest form.

Worksheet R4.9

**(a)** 50% **(b)** 10% **(c)** 25% **(d)** 20% **(e)** 1% **(f)** 60%

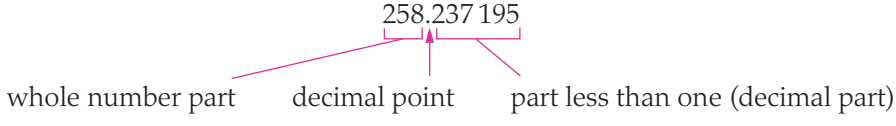
## Key Words

| | | |
|---|---|---|
| decimal places | part : whole ratio | recurring decimal |
| decimal point | per cent | round |
| digit | place value | terminating decimal |
| equivalent ratios | rate | unit price |
| part : part ratio | ratio | unitary method |

# 4.1 Place value and comparing decimals

## Decimal place value

Like fractions, decimals are used to write numbers that have a part that is less than one. The **decimal point** is used to separate the whole number part from the part less than one.




The word decimal comes from the Latin word, *dec* or *decem*, meaning 'tenth part'. December used to be the tenth month on the Roman calendar.



The number 258.237195 is made up of nine individual **digits**. Because six of the digits occur after the decimal point, we say that the number has six **decimal places**.

More examples: 3.25 has three digits and two decimal places
45.079 has five digits and three decimal places.

We can write each digit of the number into a 'place value table' to show this:

| Hundreds 100 | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ (0.01) | Thousandths $\frac{1}{1000}$ (0.001) | Ten-thousandths $\frac{1}{10\,000}$ (0.0001) | Hundred-thousandths $\frac{1}{100\,000}$ (0.000 01) | Millionths $\frac{1}{1\,000\,000}$ (0.000 001) |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 8 | • | 2 | 3 | 7 | 1 | 9 | 5 |

The **place value** of each column (hundreds, tens, ones, tenths, hundredths etc.) is one-tenth of the column to the left of it. As we move across the columns from left to right, we divide by 10 each time. This pattern is continued past the decimal point to get the values of the decimal places. You are probably familiar with the first three decimal places of tenths, hundredths and thousandths; however, we can keep dividing by 10 to get ten-thousandths, hundred-thousandths and millionths.

If there is no digit for a particular place value in a decimal number, we use a zero to show this. For example, five and two tenths and three thousandths is written as 5.203. It is important to include the zero between the 2 and the 3. Leaving it out would mean 5.203 becomes 5.23, which is a different number.

## Expressing decimals in written form

Decimal numbers can be written in several ways:

- Decimal form: 258.237 195
- Expanded fractional form: $258 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10\,000} + \frac{9}{100\,000} + \frac{5}{1\,000\,000}$
- Expanded word form: *two* hundreds, *five* tens, *eight* ones, *two* tenths, *three* hundredths, *seven* thousandths, *one* ten-thousandth, *nine* hundred-thousandths and *five* millionths.

## Expressing decimals in verbal form

- Say each of the digits after the decimal point individually. Do not use whole number language to describe the decimal part of the number. For example, 14.625 is said as 'fourteen point six two five', not 'fourteen point six hundred and twenty five'.
- Fraction equivalents can also be used to describe decimals verbally. For example, 0.63 can be said as 'sixty three hundredths'. This is an equivalent way of saying six tenths and three hundredths.

$$\frac{63}{100} = \frac{6}{10} + \frac{3}{100}$$

## Worked Example 1 WE1

**(a)** Write $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$ as a decimal.

**(b)** Write 6.2807 in expanded fractional form.

### Thinking

**(a)** Imagine the numerators of each fraction placed into their place value columns. You don't actually have to draw them.

(As there are no $\frac{1}{100}$s, we write a zero in the 'hundredths' column.)

| Ones 1 | • | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|
| 4 | • | 3 | 0 | 6 | 5 |

**(b)** **1** Imagine the decimal digits placed into their place value columns.

| Ones 1 | • | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|
| 6 | • | 2 | 8 | 0 | 7 |

**2** Write a series of fractions using the headings of the place value columns. There is no need to include columns with 0 (e.g. $\frac{0}{1000}$).

### Working

*(a)* $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$

$= 4.3065$

*(b)* 6.2807

$= 6 + \frac{2}{10} + \frac{8}{100} + \frac{7}{10\,000}$

## Worked Example 2 WE2

**(a)** Write seven units, three tenths, six hundredths, seven thousandths and four hundred-thousandths as a decimal.

**(b)** Write 28.0045 in expanded word form.

### Thinking

**(a)** Think of, or look at, a place value table. Place the digits described in the corresponding place value columns.

### Working

(a) 7.367 04

| Ones 1 | • | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ | Hundred-thousandths $\frac{1}{100\,000}$ |
|---|---|---|---|---|---|---|
| 7 | • | 3 | 6 | 7 | 0 | 4 |

**(b)** **1** Think of, or look at, a place value table. Place the digits in their place value columns, including zero.

(b)

| Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|
| 2 | 8 | • | 0 | 0 | 4 | 5 |

**2** Write the non-zero digits in words, with the place value column heading after each one.

28.0045
= Two tens, eight ones, four thousandths and five ten-thousandths

## Worked Example 3 WE3

**(a)** Write the value of the 6 in 4.368 as a fraction.

**(b)** Write the value of the 9 in 0.0109 in words.

### Thinking

**(a)** Identify the place value of the digit, then write the digit as the numerator and the place value as the denominator of the fraction.

### Working

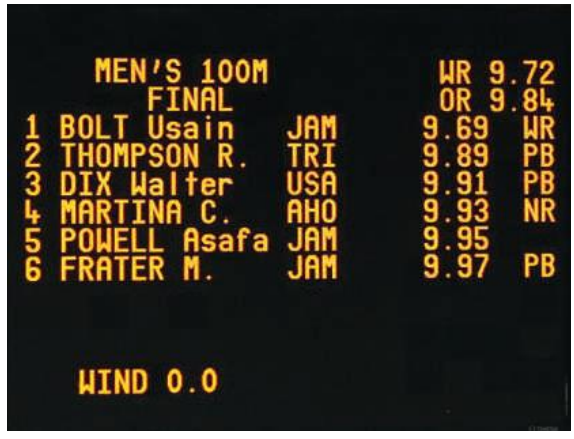
(a) $\frac{6}{100}$

**(b)** Identify the place value of the digit, then write the digit, followed by the place value, in words.

(b) nine ten-thousandths

## Comparing decimals

We often need to compare two decimal numbers to decide which one is greater. We might be comparing prices of goods or working out who won a race. Look at the decimal numbers in this photo of the finishing board of the men's 100 m final at the 2008 Olympics. Usain Bolt's time is placed first because it is the smallest decimal number. How much did Bolt win by?

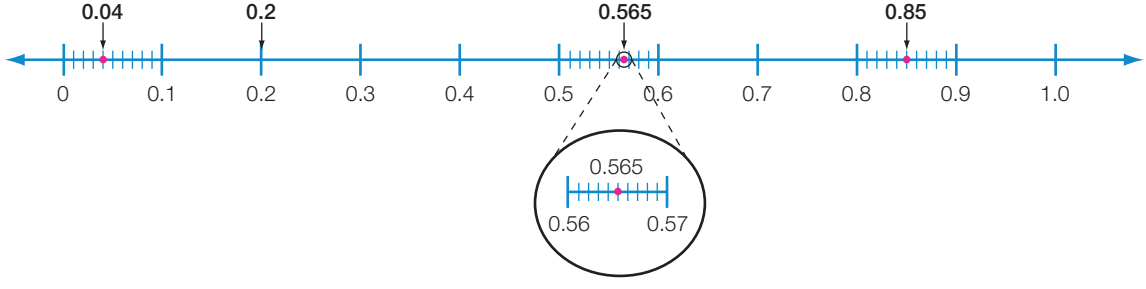



We use the relation symbols of less than (<), greater than (>) and equal to (=) when comparing decimals.

## Comparing decimals using a number line

We can use a number line to compare decimals. Numbers get larger as we move up the number line from left to right. The number line below is marked off in tenths. The space between 0 and 0.1 and between 0.5 and 0.6 shows 1 tenth divided up into 10 parts, each equal to 1 hundredth. $\frac{1}{10} = \frac{10}{100}$

The positions of the decimal numbers 0.04, 0.2, 0.565 and 0.85 are shown with arrows.




The interval between 0.56 and 0.57 has been magnified to show the position of 0.565 more clearly. It shows 1 hundredth divided up into 10 parts, each equal to 1 thousandth. $\frac{1}{100} = \frac{10}{1000}$

From the decimal numbers shown on the number line, we can see that:

The decimal number with the most digits is not necessarily the largest; e.g. 0.565 < 0.85.

A number's size depends on the value and position of the digits; e.g. 0.2 > 0.04.

## Comparing decimals by comparing digits

The higher its place value, the greater the worth of a digit. For example, 0.2 > 0.04, because $\frac{2}{10} > \frac{4}{100}$, so 0.2 lies further to the right on the number line than 0.04. To determine which of two numbers is bigger, we compare the digits in each place value column, beginning with the highest place values.

## Worked Example 4 WE4

Write < or > between each of the following pairs of decimals to make a true statement.

**(a)** 5.173 _____ 5.0731 **(b)** 0.472 39 _____ 0.4731 **(c)** 3.5 _____ 3.51

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Compare the whole number parts of each decimal. (Here, they are the same number, 5.) | *(a)* |
| | **2** | If the whole number parts are the same, compare the tenths digits to see which is greater. (Here, 1 is greater than 0, so 5.173 is the larger number.) | |
| | **3** | Write a relation symbol (< or >) between the two numbers. | *5.173 > 5.0731* |
| **(b)** | **1** | Compare the whole number parts of each decimal. (Here, they are the same, 0.) | *(b)* |
| | **2** | If the whole number parts are the same, compare the tenths digits. (Here, they are the same, 4.) | |
| | **3** | If the tenths digits are the same, compare the hundredths digits. (Here, they are the same, 7.) | |
| | **4** | Repeat until you get different digits. (The thousandths digits are 2 and 3, respectively. So, 0.472 39 is less than 0.4731.) | |
| | **5** | Write a relation symbol beween the two numbers. | *0.472 39 < 0.4731* |
| **(c)** | **1** | Add zeroes to the end of one of the decimals to help you compare. This doesn't change its value (3.5 is the same as 3.50). | *(c)* |
| | **2** | Compare the two decimals as in previous examples. (3.50 has the smaller hundredths digit, so it is the smaller decimal.) | |
| | **3** | Write < or > between the two numbers. | *3.5 < 3.51* |

# 4.1 Place value and comparing decimals

## Navigator

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| | | |
|---|---|---|
| Q1, Q2, Q3, Q4 Column 1, Q5 Column 1, Q7, Q8 (a) & (b), Q9 Column 1, Q10, Q11, Q13, Q14, Q15, Q17, Q19 | Q1, Q2, Q3, Q4 Column 2, Q5 Column 2, Q6, Q8 (c) & (d), Q9 Column 2, Q10, Q11, Q12, Q14, Q15, Q16, Q18, Q19 | Q1 (b), Q2 (b), Q3, Q4 Column 2, Q5 Column 2, Q6, Q7, Q8 (c) & (d), Q9 Column 2, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19 |

## Fluency

WE1

**1 (a)** Write each of the following as a decimal.




**(i)** $45 + \frac{4}{10} + \frac{6}{100} + \frac{2}{1000}$

**(ii)** $12 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000} + \frac{3}{10\,000} + \frac{7}{100\,000} + \frac{2}{1\,000\,000}$

**(iii)** $3 + \frac{7}{10} + \frac{9}{100} + \frac{8}{1000} + \frac{5}{10\,000}$

**(iv)** $1 + \frac{8}{1000} + \frac{3}{10\,000} + \frac{3}{100\,000} + \frac{2}{1\,000\,000}$

**(v)** $\frac{7}{10} + \frac{8}{100} + \frac{6}{1000} + \frac{6}{100\,000}$

**(vi)** $7 + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100\,000} + \frac{7}{1\,000\,000}$

**(b)** Write each of the following in expanded fractional form.

**(i)** 6.63 **(ii)** 0.921 **(iii)** 0.7345

**(iv)** 7.826 **(v)** 23.913 04 **(vi)** 45.004 589

WE2

**2 (a)** Write each of the following as a decimal.

**(i)** six ones and five tenths

**(ii)** nine tenths and seven hundredths

**(iii)** two tenths, seven hundredths and three thousandths

**(iv)** three tens, seven ones, four tenths, two hundredths and one ten-thousandth

**(v)** one ten, four ones, nine tenths, five hundredths, seven thousandths, six ten-thousandths, two hundred-thousandths and three millionths

**(vi)** seven hundreds, four thousandths, five ten-thousandths and nine hundred-thousandths

**(b)** Write each of the following in expanded word form.

**(i)** 5.2 **(ii)** 4.9 **(iii)** 34.17 **(iv)** 0.61

**(v)** 2.794 **(vi)** 7.5092 **(vii)** 35.865 43 **(viii)** 0.820 027

**3** **(a)** Write the value of the 2 in each of the following as a fraction.

WE3

**(i)** 6.012 **(ii)** 0.004 52 **(iii)** 3.287 **(iv)** 2.034

**(b)** Write the value of the 7 in each of the following in words.

**(i)** 5.734 **(ii)** 0.0076 **(iii)** 1.2037 **(iv)** 8.130 037

**4** Write < or > between each of the following pairs of decimals to make a true statement.

WE4

**(a)** 2.4 _____ 0.42 **(b)** 2.32 _____ 1.955

**(c)** 0.65 _____ 0.57 **(d)** 0.3003 _____ 0.333

**(e)** 4.7038 _____ 4.7312 **(f)** 8.251 _____ 8.2501

**(g)** 7.02 _____ 7.002 **(h)** 4.7367 _____ 4.7376

**(i)** 0.927 _____ 0.927 34 **(j)** 6.013 _____ 6.01

**(k)** 3.406 _____ 3.4063 **(l)** 0.9995 _____ 0.9986

**5** Write TRUE or FALSE for each of the following.

**(a)** $7.5 < 5.77$ **(b)** $4.1 > 4.12$

**(c)** $6.08 > 6.8$ **(d)** $67.54 < 67.504$

**(e)** $3.023 < 3.203$ **(f)** $0.547 > 0.547\,08$

**(g)** $2.000\,012 < 2.0001$ **(h)** $4.145\,29 > 4.200\,01$

**6** 53.017 expressed in expanded word form is:

**A** five tens, three ones, one tenth and seven hundredths

**B** five tens, three ones, one hundredth and seven thousandths

**C** five tenths, three hundredths, one ten-thousandth and seven hundred-thousandths

**D** five tens, three ones, one tenth and seven thousandths

**7** Nine hundredths, four thousandths and three ten-thousandths is equal to:

**A** 0.090 43 **B** 0.0943 **C** 0.943 **D** 9.43

**8** For each of the following sets of numbers, copy the number line shown, then mark the positions of the numbers with a labelled arrow. (You may have to estimate the position of some.)




**(a)** 2.05, 2.09, 2.6, 2.12, 2.59 **(b)** 2.0, 2.8, 2.88, 2.9, 2.805

**(c)** 2.7, 2.4, 2.07, 2.04, 2.407 **(d)** 2.2, 2.4, 2.85, 2.35, 2.05

**9** Write each set of decimals in order from smallest to largest.

**(a)** 2.3, 2.03, 2.13 **(b)** 8.7, 8.007, 8.67

**(c)** 6.646, 6.6403, 6.64 **(d)** 0.0095, 0.0905, 0.0509

**(e)** 5.3281, 5.38, 5.003 821 **(f)** 3.616, 3.116, 3.661

**(g)** 0.92, 0.29, 0.092 **(h)** 0.85, 0.815, 0.086

## Understanding

**10** Joel wrote down the five-digit number as he saw it on his electronic stopwatch: 37901. However, he forgot to put in the decimal point. Anna knew that the race was timed to a thousandth of a second and was able to put the decimal point in the correct place. Write the number with the decimal point in the correct place.

**11** The difference in times between two skiers in a downhill race was 0.437 of a second. Write this: **(a)** in expanded fraction form

**(b)** as a single fraction.

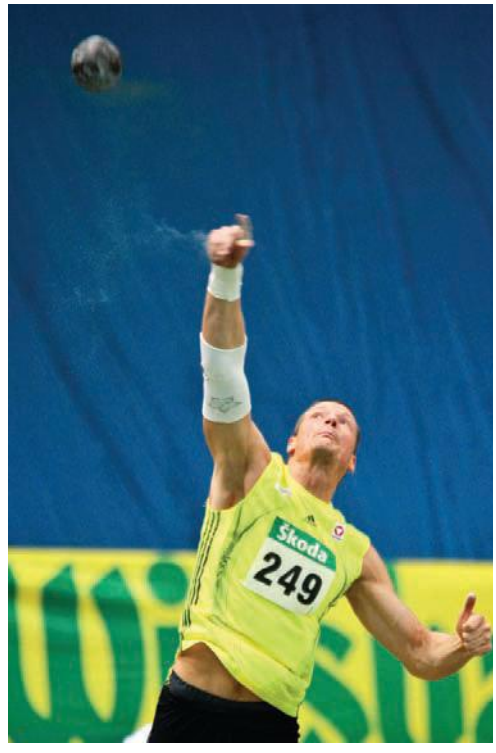
**12** For each of the following sets of decimals, draw a section of the number line, marked in either tenths or hundredths, and indicate the position of each number in the set.

**(a)** 0.4, 0.72, 1.01

**(b)** 1.7, 1.25, 0.95

**(c)** 0.06, 0.045, 0.038

**(d)** 2.9, 3.2, 2.75, 3.06




**13** Jane records her three best practice times for the 100 m sprint. They are as follows. 13.95 seconds, 13.08 seconds and 13.69 seconds

**(a)** Which was Jane's fastest time?

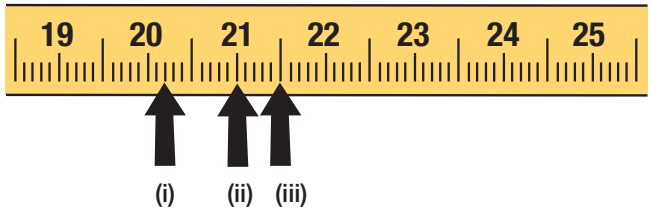
**(b)** Which was Jane's slowest time?

**14** Egor throws a shot-put the following distances during a competition: 25.6 m, 25.56 m and 25.081 m. Which distance was Egor's longest throw?

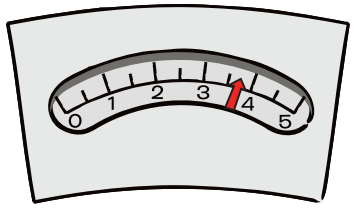
## Reasoning

**15** Write the value of the decimals indicated by the arrows on the following scales.

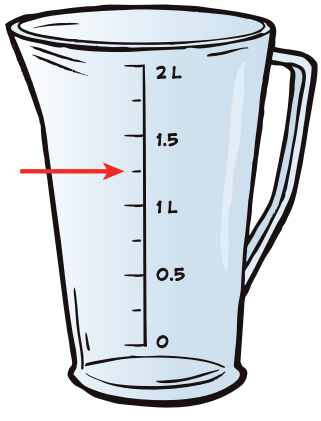
**(a)**



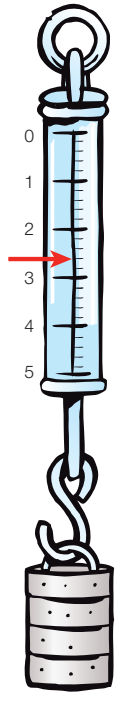

**(b)**




**(c)**




**(d)**




**16** State the mistakes that the following students have made.

**(a)** Minh states that the decimal 34.162 has a 2 in the hundredths column.

**(b)** Al writes four hundreds, nine ones, six tenths and seven hundredths as 409.067.

**(c)** Max writes seven ones, eight hundredths and nine ten-thousandths as 0.7809.

**(d)** Polly writes 0.9056 as $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$.

## Open-ended

**17** **(a)** Write down at least six numbers between 4.5 and 4.7.

**(b)** Put your numbers from **(a)** in ascending order.

**18** **(a)** Use any digit from 0 to 9 in each box to make a correct statement. Digits can be repeated. Find at least three different answers.

$\square.\square\square < 5.\square$

**(b)** Explain any limitations on the other numbers that can be used when certain numbers are used in some boxes.

**19** Donna has cut out the digits 2, 5 and 7, as well as a decimal point, from coloured paper to stick on a poster to show a decimal number. If she uses all three digits:

**(a)** how many different decimal numbers with two decimal places could she make

**(b)** how many decimal numbers greater than 5.4 could she make?

## Outside the Square Game

### Duelling decimals

**Equipment required:** 2 brains, 1 die

**How to win:**
The aim is to be the first player to 5 points.

**How to play:**

**1** One player rolls the die to determine the number of digits in the duel. Each player then puts the appropriate number of boxes on their paper after the decimal point.

e.g. if a 3 is rolled, each would draw:

0 . ☐☐☐ 0 . ☐☐☐

**2** Each player then takes turns to roll the die and write the number from the roll in a box of their choice.

After all of the boxes have been filled, the player who has created the bigger number wins a point for that round.

For example, the players may have filled in their boxes like this:

0 . 2 5 3 0 . 6 3 1

Here, the second player would win a point.

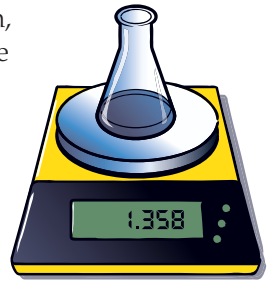
**3** This process is repeated with players taking turns to roll the determining number of digits per round.



# 4.2 Rounding decimals

Decimal numbers sometimes contain more decimal places than we require. It depends on the situation.

A scientist might need to know the mass of a chemical accurate to a thousandth of a gram, or three decimal places. They could place the chemical on an electronic scale and get a reading such as 1.358 g.



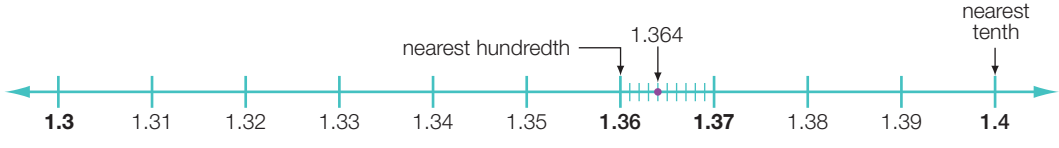




A zookeeper wanting to find the mass of a bear cub could place the cub on a similar electronic scale and get a reading such as 25.427 kg. However, 2 hundredths or 7 thousandths of a kilogram is an insignificant amount compared to the whole mass of the cub. The zookeeper would **round** the bear cub's mass to 25.4 kg, or just 25 kg.

The number line below shows how the number 1.364 is rounded:

- To the nearest tenth (1 decimal place): 1.364 lies between the tenths values of 1.3 and 1.4. The 6 in the hundredths place value column places it closer to 1.4.
  1.364 rounded to the nearest tenth is *rounded up* to 1.4.
- To the nearest hundredth (2 decimal places): 1.364 lies between the hundredths values of 1.36 and 1.37. The 4 in the thousandths column places it closer to 1.36.
  1.364 rounded to the nearest hundredth is *rounded down* to 1.36.




To round decimal numbers:

**Step 1** Determine the number of decimal places that you are required to round to. The digit in this place will either remain as it is, or will increase by one.

**Step 2** Look at the next digit to the right of the one that you are rounding to. If this digit is:

- 0, 1, 2, 3 or 4, then leave the digit considered in step 1 as is and delete all digits after it
- 5, 6, 7, 8 or 9, then increase the digit considered in step 1 by one and delete all digits after it.

If the number in the place value you are rounding to is 9 and you need to increase it, make the 9 a 0, and add one to the digit in front of it. For example, 1.499 rounded to the nearest hundredth is 1.50. We write a 0 in the hundredths column to show this (even though it makes no difference to the value of the number).

## Worked Example 5 WE5

Round the following decimals to the number of decimal places shown in brackets.

**(a)** 3.785 (2) **(b)** 0.958 34 (3) **(c)** 6.214 96 (4)

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 3.78 or 3.79.) | (a) 3.785 |
| | **2** | Look at the digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 5 tells us to round up, so the 8 becomes a 9.) | 3.785 |
| | **3** | Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. | 3.79 |
| **(b)** | **1** | Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 0.958 or 0.959.) | (b) 0.958 34 |
| | **2** | Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 3 tells us that we round down, so we leave the 8 as it is.) | 0.958 34 |
| | **3** | Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. | 0.958 |
| **(c)** | **1** | Decide which digit you will be rounding. (Here, it is the 9, so the answer will either be 6.2149 or 6.2150.) | (c) 6.214 96 |
| | **2** | Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 6 tells us to round up, but rounding the 9 up will give 10 ten-thousandths, which is equal to 1 thousandth. This means that the thousandths digit is increased by 1.) | 6.214 96 |
| | **3** | Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. (Here, we leave the zero in the rounded decimal place.) | 6.2150 |

## Rounding money

We write money using decimal notation, usually with 2 decimal places. Because $1 = 100c, or 10 lots of 10c, the tenths place value column shows how many lots of 10c make up the amount, and the hundredths column shows how many lots of 1c make up the amount.

In the 1990s, Australia stopped using 1 cent and 2 cent pieces, making the 5 cent coin the smallest coin in use. This means that all money amounts paid in cash must be rounded to the nearest 5 cents (5c).

If money is being paid in cash, the following rules apply.

| *If the amount ends in…* | *then it is rounded…* | *the last digit becomes…* |
|---|---|---|
| 1 or 2 cents | down | 0 |
| 3 or 4 cents | up | 5 |
| 6 or 7 cents | down | 5 |
| 8 or 9 cents | up | 0 |

If the amount ends in 0 or 5 cents, then the exact amount is paid.

## Worked Example 6

WE6

Round the following amounts of money to the nearest 5 cents.

**(a)** $8.53 **(b)** $7.02 **(c)** $4.99

| Thinking | Working |
|---|---|
| **(a)** Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 3 up to the nearest 5 cents.) | *(a) $8.55* |
| **(b)** Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 2 down to the nearest multiple if 10 cents, which is 0.) | *(b) $7.00* |
| **(c)** Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 9 up to the nearest multiple of 10 cents, which is 100.) | *(c) $5.00* |

# 4.2 Rounding decimals

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q10, Q12, Q14, Q16 | Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q16 | Q1 Columns 2 & 3, Q2 Columns 3 & 4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16 |

Answers page 642

## Fluency

1 Round the following decimals to the number of decimal places shown in brackets.

WE5

**(a)** 4.88 (1) **(b)** 6.72 (1) **(c)** 7.635 (2)

**(d)** 4.552 (2) **(e)** 0.6416 (3) **(f)** 3.2772 (2)

**(g)** 0.314 42 (4) **(h)** 11.828 55 (2) **(i)** 2.917 96 (2)

**(j)** 18.499 95 (3) **(k)** 90.899 99 (4) **(l)** 18.999 999 (3)

**(m)** 23.4097 (2) **(n)** 57.0804 (3) **(o)** 259.899 99 (4)




2 Round the following amounts of money to the nearest 5 cents.

WE6

**(a)** $4.52 **(b)** $2.76 **(c)** $11.03 **(d)** $23.88

**(e)** $176.35 **(f)** $542.06 **(g)** $789.54 **(h)** $56.75

**(i)** $6774.99 **(j)** $1149.97 **(k)** $8989.99 **(l)** $9999.99

3 0.837 256 rounded to the nearest ten-thousandth is:

**A** 0.837 **B** 0.8372 **C** 0.837 26 **D** 0.8373

## Understanding

4 At the 2009 World Athletics Championships, Usain Bolt set a new 100 m record time of 9.58 seconds. Round this time to the nearest tenth of a second.

5 The total shown on the receipt for Russell's grocery shopping is $53.67. If Russell is paying in cash, how much money will he have to hand over?




6 Sir Donald Bradman's cricket batting average was 99.943 (rounded to the nearest thousandth). What would be his average rounded to the nearest:

**(a)** tenth **(b)** hundredth?

7 The top temperature recorded one day in Mildura was 41.6°C. What figure should the weather presenter read out in his weather report as the day's maximum if he rounds the temperature reading to the nearest whole number (i.e. no decimal places)?

8 The price of a box of chocolates is $11.97. If Beau buys the chocolates and pays with cash, how much will he pay?

9 A physics student calculates the speed of sound to be 352.153 metres per second. In case her equipment is not accurate enough for this result to be reliable, she decides to round it to the nearest tenth. What value does she write in her report, assuming she rounds off correctly?

## Reasoning

**10** Ruth calculates the length of shelf required for a wall unit to be 1.266 66 m. As her tape measure is only accurate enough to measure in centimetres (cm), she needs to round off this figure to the nearest cm. If 1 cm = 0.01 m, what length should she measure?



**11** In a cycling sprint race at a velodrome, the following results were recorded: 12.149 s, 12.143 s, 12.138 s, 12.152 s and 12.157 s.

**(a)** Write these in order from fastest to slowest.

**(b)** It was found that there was a small error in the timing equipment and the results were to be recorded to the nearest hundredth. What were the new times?

**(c)** Does this affect the race result?

**12** **(a)** The total cost of Rani's shopping was $20.34. Round this amount to the nearest 5 cents.

The individual costs of the items Rani bought were:

| | |
|---|---|
| Cereal | $6.93 |
| Yoghurt | $4.78 |
| Bananas | $2.51 |
| Juice | $3.63 |
| Pasta | $2.49 |

**(b)** How much more would Rani have paid if the individual prices of the items were each rounded to the nearest 5 cents?

## Open-ended

**13** Write at least three different decimal numbers that, when rounded, would give the number 3.79.

**14** Write down two prices that would give $100 dollars when rounded to:

**(a)** the nearest 5 cents

**(b)** the nearest dollar.

**15** Taking 2.548 46 and rounding it to four decimal places gives 2.5485, then rounding this to three decimal places gives 2.549.

What answer do you get when rounding 2.548 46 to three decimal places?

Explain why this is different, and what is wrong with doing it the first way.

**16** Why do you think 1 and 2 cent coins were taken out of circulation? List two or three possible reasons why a government might make this decision.

# Decimals and fractions

4.3

## Writing a decimal as a fraction

As we have seen, a decimal can be written in expanded fractional form. This expanded form can be simplified to describe a decimal as a single fraction or as a mixed number.

For example: $3.72 = 3 + \frac{7}{10} + \frac{2}{100}$

$= 3 + \frac{70}{100} + \frac{2}{100}$

$= 3 + \frac{72}{100}$

$= 3\frac{72}{100}$ or $\frac{372}{100}$

$= 3\frac{18}{25}$ or $\frac{93}{25}$

The place value of the last digit (hundredths) gives the denominator of the single, unsimplified fraction (100), and the digits themselves form the numerator (72). Remember that fractions should always be given in simplest form.

### Worked Example 7 WE7

Convert each of the following decimals to fractions, giving your answers in simplest form.

**(a)** 6.28 **(b)** 0.3125

| Thinking | Working |
|---|---|
| **(a) 1** What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator. | (a) $6.28$ $= 6\frac{28}{100}$ or $\frac{628}{100}$ |
| **2** Simplify the fraction if possible. (Here, a common factor of 4 has been cancelled.) | $= 6\frac{7}{25}$ or $\frac{157}{25}$ |
| **(b) 1** What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator. | (b) $0.3125$ $= \frac{3125}{10\,000}$ |
| **2** Simplify the fraction if possible. (Here, we cancel common factors of 5, then 25, then 5 again.) | $= \frac{625}{2000}$ $= \frac{25}{80}$ $= \frac{5}{16}$ |

The decimal point was first used in the 1600s by mathematician Bartholo Maeus Pitiscus. Thanks, Bart!




## Writing a fraction as a decimal

There are two methods we can use to convert a number in fraction form to decimal form:

**Method 1:** Write the fraction as an equivalent fraction (or mixed number) with a denominator of 10, 100, 1000 etc. Then, write the digits of the numerator of this new fraction as the decimal place values.

For example: $\frac{4}{5} = \frac{8}{10} = 0.8$

$\frac{13}{20} = \frac{65}{100} = 0.65$

This method is only convenient for fractions with denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

**Method 2:** The line between the numerator and the denominator is equivalent to the division symbol, ÷. To convert a fraction to a decimal, we simply perform the division. This method can be used for all fractions.

For example: $\frac{7}{8} = 7 \div 8 = 0.875$

$\frac{37}{40} = 37 \div 40 = 0.925$

## Worked Example 8 WE8

Convert each of the following fractions to decimals.

(a) $\frac{7}{5}$ (b) $\frac{3}{8}$ (c) $\frac{4}{3}$

| Thinking | | Working |
|---|---|---|
| (a) 1 | Is the denominator a factor of 10, 100 or 1000? (Yes) | (a) $\frac{7}{5}$ |
| 2 | Write the fraction as an equivalent fraction with a denominator of 10, 100, or 1000 (10 in this case). | $= \frac{14}{10}$ |
| 3 | If the fraction is improper, write it as a mixed number. | $= 1\frac{4}{10}$ |
| 4 | Write any whole numbers on the left of a decimal point. (If there is no whole number part, write a zero.) Write the numerator of the fraction on the right of the decimal point. | $= 1.4$<br>$\frac{7}{5} = 1.4$ |

| | | | |
|---|---|---|---|
| **(b)** | **1** | Is the denominator a factor of 10, 100 or 1000? (No) | (b) $\frac{3}{8}$ |
| | **2** | Write the fraction as a division calculation. | $3 \div 8$ <br> $8\overline{)3}$ |
| | **3** | Perform the division. If you are unable to begin because the divisor is a greater number (here, $8 > 3$), add a decimal point and a zero to the dividend. Place a decimal point in the quotient and a zero in front of it. | $3.0 \div 8$ <br> $8\overline{)3.0}$ |
| | **4** | Continue to divide as though the decimal point was not there, adding extra zeroes where required. | $3.000 \div 8$ <br> $8\overline{)3.0^{6}0^{4}0}$ |
| | **5** | State the answer. | $\frac{3}{8} = 0.375$ |
| **(c)** | **1** | Is the denominator a factor of 10, 100 or 1000? (No) | (c) $\frac{4}{3}$ |
| | **2** | Write the fraction as a division calculation. | $4 \div 3$ <br> $3\overline{)4}$ |
| | **3** | Perform the division. Add a decimal point and zeroes to the dividend to continue the division, placing a corresponding decimal point in the quotient. | $4.000 \div 3$ <br> $3\overline{)4^{1}0^{1}0^{1}0}$ |
| | **4** | If the division gives a repeating pattern, stop. Write the answer with a dot above the repeating digit. | $\frac{4}{3} = 1.\dot{3}$ |

## Recurring decimals

Dividing the numerator by the denominator sometimes results in a **recurring decimal**—one that has a repeating pattern of digits, such as in part **(c)** in the previous Worked Example. To write a recurring decimal, place dots or a line above the digits that form the repeating pattern. If it is a single repeating digit, place a dot or a line above that digit. If more than one digit forms the repeating pattern, place a line above the repeating digits, or a dot above the first and last digit in the repeating pattern.

For example: $\frac{2}{3} = 0.\dot{6}$, $\frac{1}{12} = 0.083\dot{3}$, $\frac{4}{7} = 0.\overline{571\,428}$ or $0.\dot{5}71\,42\dot{8}$

A recurring decimal can also be rounded to give an approximate value; for example, $0.\dot{6}$ rounded to three decimal places is 0.667.

A decimal that is not recurring, but has a finite number of digits (such as 1.4 or 0.375 in the previous Worked Example) is called a **terminating decimal**.

## Fractions and decimals on a calculator

On a scientific calculator, the $a\frac{b}{c}$ or S⇔D key can be used to convert between the fraction and the decimal forms of a number. Pressing the key again will convert it from decimal form back to the fraction.

# 4.3 Decimals and fractions

## Navigator

Answers page 642

| | | |
|---|---|---|
| Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q12 (a) | Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12 | Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12 (b), Q13 |

**Equipment required:** A calculator may be used for Question **2 (i)–(p)**

## Fluency

WE7

**1** Convert each of the following decimals to fractions, giving your answers in simplest form.

| | | | |
|---|---|---|---|
| **(a)** 4.1 | **(b)** 6.5 | **(c)** 8.6 | **(d)** 9.4 |
| **(e)** 4.71 | **(f)** 2.37 | **(g)** 0.35 | **(h)** 2.48 |
| **(i)** 5.009 | **(j)** 0.884 | **(k)** 6.128 | **(l)** 3.172 |
| **(m)** 7.045 | **(n)** 0.088 | **(o)** 6.0015 | **(p)** 1.0075 |




WE8

**2** Convert each of the following fractions to decimals.

| | | | |
|---|---|---|---|
| **(a)** $\frac{7}{10}$ | **(b)** $\frac{9}{100}$ | **(c)** $\frac{17}{100}$ | **(d)** $\frac{123}{1000}$ |
| **(e)** $\frac{1}{5}$ | **(f)** $\frac{3}{25}$ | **(g)** $\frac{13}{50}$ | **(h)** $\frac{7}{4}$ |
| **(i)** $\frac{7}{8}$ | **(j)** $\frac{11}{16}$ | **(k)** $\frac{23}{40}$ | **(l)** $\frac{47}{60}$ |
| **(m)** $\frac{1}{9}$ | **(n)** $\frac{7}{12}$ | **(o)** $\frac{17}{6}$ | **(p)** $\frac{16}{15}$ |

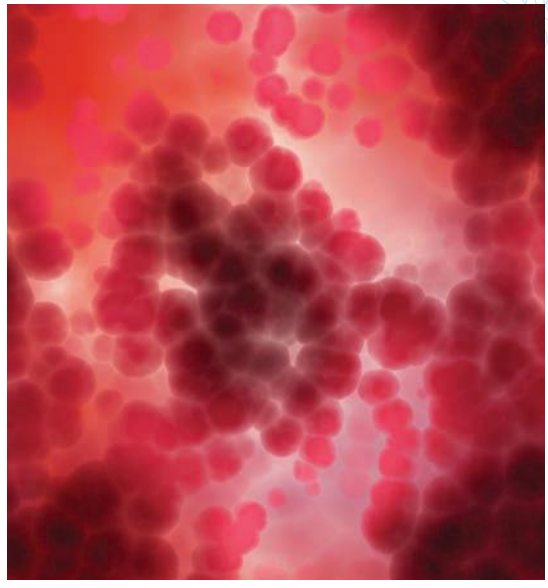
**3** 0.55 expressed as a fraction in simplest form is:

**A** $\frac{55}{1000}$ **B** $\frac{11}{200}$ **C** $\frac{11}{25}$ **D** $\frac{11}{20}$

**4** 2.000 47 is equal to:

**A** $2\frac{47}{100}$ **B** $2\frac{47}{1000}$ **C** $2\frac{47}{10\,000}$ **D** $2\frac{47}{100\,000}$

## Understanding



**5** As part of a science experiment, Galina's reaction time was measured to be 0.48 seconds. What fraction of a second is this?

**6** One inch is approximately equal to 2.54 centimetres. What is this when expressed as a fraction in simplest form?

**7** Hair grows at an average rate of 1.35 cm per month. Write this as a mixed number in simplest form.

**8** A virus particle has a diameter of 0.000024 mm. Write this as a fraction in simplest form.

**9** A red blood cell's diameter is $\frac{1}{200\,000}$ of a metre. Write this measurement in decimal form.

## Reasoning

**10** A bank offers its customers an interest rate of 5.85 per cent. Write this rate in single fraction form. Is this rate higher than the $5\frac{3}{4}$ per cent offered by another bank?

**11** Jeremy has five different containers and is trying to place them in ascending order of size. The volumes of all five containers are measured in litres but some are given using decimals and others using fractions. The tubs have the following volumes: $4\frac{1}{5}$, 4.375, 4.865, $4\frac{9}{10}$, $4\frac{1}{2}$.

Help Jeremy put them in order from the smallest to the largest.

## Open-ended

**12** **(a)** Write three different fractions that have decimal values between 3.4 and 3.5 and have a denominator of 100.

**(b)** Write three different fractions that have decimal values between 3.4 and 3.5 where each fraction has a different denominator that is not a power of 10 (such as 100 or 1000).

**13** Two numbers are rounded to one decimal place to give 5.6. What could those two numbers be? Give your answers in fraction form.

## Outside the Square Puzzle

### What are the neighbours having for dinner?

Four children from the same class in school live on the same street. Find out in which house each child lives, and what they had for dinner yesterday evening.

- Steve didn't have pizza for dinner, which was the meal eaten at house number 15.
- The fish and chips were eaten at house number 1, whereas Joseph lives at house number 11.
- The salad was eaten in a lower house number than the lasagne.
- One of Rowena's classmates lives in house number 5.
- Harriet lives in the highest number house of the four classmates.

# 4.4 Decimal addition and subtraction

Decimal numbers are added and subtracted in the same way as whole numbers. As with whole numbers, we must be sure that we add or subtract digits of the same place value. The best way to ensure this is to line up the decimal points underneath each other.

To add or subtract decimals, line up the decimal points so that digits with the same place value are underneath each other.

If the numbers have different numbers of decimal places, zeroes can be written in the 'empty' place value columns to help them line up correctly.

## Worked Example 9 WE9

Calculate: 12.45 + 6 + 0.3678

| Thinking | Working |
|---|---|
| **1** Place the numbers underneath each other with the decimal points lined up. Fill empty place value columns with zeroes if necessary. | $\begin{array}{rr} & 12.^{1}4500 \\ & 6.0000 \\ + & 0.3678 \\ \hline & 18.8178 \end{array}$ |
| **2** Add as though the values are whole numbers. | |
| **3** Place a decimal point in the answer so it lines up with the other decimal points. | |

## Worked Example 10 WE10

Calculate: 15 – 2.147

| Thinking | Working |
|---|---|
| **1** Write the second number underneath the first with the decimal points lined up. (Write 15 as 15.000) | $\begin{array}{rr} & {\scriptstyle 4\ 9\ 9\ 1} \\ & 15.000 \\ - & 02.147 \\ \hline & 12.853 \end{array}$ |
| **2** Fill empty place value columns with zeroes. | |
| **3** Subtract, lining up the decimal point in the answer with those in the question. | |

# 4.4 Decimal addition and subtraction

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q13, Q14 | Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q15, Q17 | Q1 Column 3, Q2 Column 3, Q4, Q5, Q6, Q8, Q11, Q12, Q13, Q15, Q16, Q17 |

Answers page 642

## Fluency

**1** Calculate:

WE9

**(a)** 8.3 + 7.9 **(b)** 4.85 + 8.09 **(c)** 0.237 + 0.677

**(d)** 25.61 + 0.038 **(e)** 5.098 + 21.32 **(f)** 0.025 + 39.786

**(g)** 9.703 + 5.624 + 7.5 **(h)** 7.35 + 0.609 + 2.3 **(i)** 0.648 + 7.31 + 0.9

**(j)** 6 + 5.017 + 12.9 **(k)** 2.59 + 15 + 0.005 **(l)** 0.0004 + 3.583 + 8

**2** Calculate:

WE10

**(a)** 5.7 – 3.8 **(b)** 7.05 – 2.93 **(c)** 23.982 – 11.735

**(d)** 9.663 – 2.7 **(e)** 4.984 – 1.8 **(f)** 7.238 – 3.4

**(g)** 9.5 – 2.24 **(h)** 16.2 – 8.75 **(i)** 3.7 – 0.931

**(j)** 8 – 7.44 **(k)** 3 – 2.92 **(l)** 93 – 0.698

**3** Which of the following shows the correct way to set out the addition of 0.56, 15.092 and 2.7?

**A**
$$\begin{array}{r} 0.5\,6 \\ 1\,5.092 \\ +\ 2.7 \\ \hline \\ \hline \end{array}$$

**B**
$$\begin{array}{r} 0.\,56 \\ 15.092 \\ +\ 2.7 \\ \hline \\ \hline \end{array}$$

**C**
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\ \ 2.7 \\ \hline \\ \hline \end{array}$$

**D**
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\ \ 2.7 \\ \hline \\ \hline \end{array}$$

## Understanding

**4** Daily rainfall totals for three days over a long weekend were 3.78, 2.5 and 6.42 millimetres. What was the total rainfall over this three-day period?

**5** John loads his shopping trolley with several items priced as shown.

| | |
|---|---|
| 1 frozen lasagna | $5.71 |
| 2 L orange juice | $4.25 |
| 1 tub of yoghurt | $3.77 |
| 1 packet of cereal | $6.14 |

**(a)** Find the total cost of John's purchases.

**(b)** Round the total to the nearest 5 cents.

**6** Cristina's bank account balance was $335.96 just before she withdrew $40.45 to pay a bill. How much did she have left in her account after the withdrawal?



7 In a gymnastics competition, Adelia scores the following from the five judges: 7.5, 8.5, 7.9, 8 and 8.6. Find her total score.

8 A family drives to a holiday resort and records the car's odometer readings as shown.

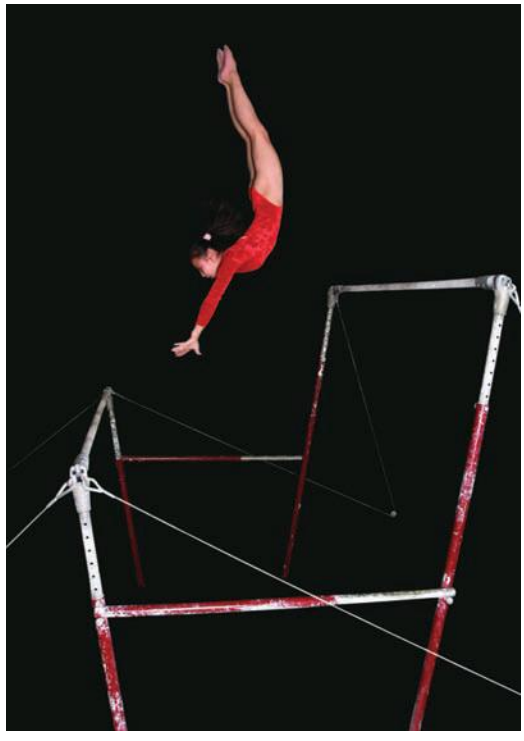
Departure: 234.8 km Arrival: 502.7 km

How far did they travel to get to the resort?

9 During a car tour of Tasmania, Helena travelled distances of 25.64 km, 165.35 km and 5.97 km all in one day. How many kilometres did she travel altogether that day?

10 Georgina pays for $36.35 worth of groceries with a $100 note. How much change should she receive?

11 Nico pours 1.625 litres of milk from a full 2-litre container. How much milk is left in the container?



## Reasoning

12 Amanthi's credit card bill shows the following purchase amounts: $23.56, $40.15, $7.89, $18.48, $45.50, $21.73 and $8.59. The total at the bottom of her statement is $184.38, which she is sure is wrong.

**(a)** What should the total be?

**(b)** How do you think the error was made?

13 Tina sells flowers at a market stall. One day, she takes in $473.50 from the sale of flowers. If the flowers cost Tina $128.35, and she had to pay a fee of $14.50 to rent the stall for 1 day, what profit did she make for the day?



## Open-ended

14 **(a)** Write two numbers that add up to 2.871.

**(b)** Write three numbers, each with three decimal places, that add up to 3.86.

**(c)** Explain how you chose the last digit of the numbers you used in **(b)**.

**15**
$$\begin{array}{r} 4.\square \\ +\ \square.8\square \\ \hline 7.\square 5 \end{array}$$

**(a)** Use one digit from 0 to 9 to fill each box and make a correct sum. Find at least three different combinations.

**(b)** Was your choice of digits restricted? Explain how.

**16 (a)** Write two decimal numbers, each with three decimal places, that have a difference of 8.712.

**(b)** Write two decimal numbers, each with three decimal places, that have a difference of 8.71.

**(c)** Explain how you chose the last digit of the numbers you used in **(b)**.

**17** Two Australian swimmers finish first and second in the women's 100 m freestyle at the Olympics. The difference in their times is 0.04 seconds. The Olympic record was 53.52 seconds. Assuming that they both break this record by less than $\frac{2}{10}$ of a second, what could their times be?

## Outside the Square Game

### 1 Closer than you

**Equipment required:** 2–4 brains, 2 dice

**How to win:**
The aim is to be the person to get a score as close to 10 as possible.

**How to play:**

1. Each player draws up a table like the one on the right.
2. Take turns to roll both dice, and enter the numbers, one each side of the decimal point, in whatever order you like. Add as you go. You can choose to stop at any point in the table.
3. Play the best of three.

| | |
|---|---|
| | ● |
| + | ● |
| = | ● |
| + | ● |
| = | ● |
| + | ● |
| = | ● |
| + | ● |
| | ● |

### 2 Decimal zilch

**Equipment required:**
2–4 brains, 3 dice

**How to win:**
The aim is to be the person to have the closest score to zero (but not less than zero).

**How to play:**

1. Each player draws up a table like the one on the right.
2. Take turns to roll the three dice. Enter the numbers in the boxes in the next empty line in any order you choose. Subtract from the above total. You can choose to stop at any point in the table.
3. Play the best of three.

| 10 | 0 | 0 |
|---|---|---|
| − | | |
| = | | |
| − | | |
| = | | |
| − | | |
| = | | |

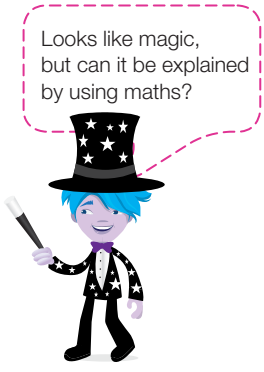
# Investigation

## Ten dollars and eighty-nine cents

**Equipment required:** 2 brains may be used for the 'Explore' section

### The Big Question

Why is the answer to a seemingly random collection of calculations always the same?




### Engage

1 **(a)** Choose an amount of money less than $10.00 (e.g. $7.25) and write it down.

**(b)** Reverse the digits and write down the amount formed, keeping the decimal point after the first digit (e.g. $5.27).

**(c)** Subtract the smaller number from the larger one (e.g. $7.25 – $5.27 = $1.98).

**(d)** Reverse the digits in your answer to part **(c)** and add the resulting number to your answer to part **(c)** (e.g. $1.98 + $8.91 = $10.89).

**(e)** Repeat parts **(a)** to **(d)** with a different starting amount. Do you get the same final answer?

### Explore

2 To investigate further, write down at least 10 different amounts (all less than $10.00) using different combinations of digits. Perform the same steps from Question **1** for each of them. Keep a record of which amounts produced the same final answer and which didn't.

3 Can you see a pattern in the calculations that can explain why the same answer keeps appearing? If you can, try to describe it. If you can't, write down some new amounts and test them. Remember that the three digits you use do not have to all be different—try making some the same and see what happens to the result.

**Strategy options**

- Look for a pattern.
- Seek an exception.
- Break problem into manageable parts.

### Explain

4 For your calculations that gave $10.89 as the final answer, explain the following.

**(a)** Look at the subtractions of the digits in the hundredths column. What do they all have in common? Explain why this is the case, if you can.

**(b)** Look at the middle digit that results after the subtraction step. What do you notice? Again, explain why this is the case, if you can.

**(c)** Did you find amounts for which the answer was not $10.89? What did the digits of these numbers have in common?

### Elaborate

5 **(a)** For amounts that add to $10.89, what is the sum of the first and last digits of the answer to the subtraction? Explain why.

**(b)** Explain why the final addition step produces $10.89. Why does the addition of the tenths digits always result in an 8?

6 Summarise what you have found in this investigation by answering the Big Question.

### Evaluate

7 **(a)** Did you find the results to your first few calculations surprising? Do you have more of an understanding now of how the 'magic' works?

**(b)** How many calculations did it take for you to see patterns emerging? What did you first start to notice?

**(c)** Did you feel frustrated at any time during this investigation? If so, how did you deal with the frustration?

### Extend

8 Try this.

**(a)** Select any three single-digit decimal numbers (e.g. 0.4, 0.6, 0.1).

**(b)** Use these numbers to make six two-digit numbers with one number the units and the other number the tenths (e.g. 4.6, 4.1, 6.1, 6.4, 1.4, 1.6).

**(c)** Add these numbers (e.g. 24.2).

**(d)** Add your original numbers (e.g. 1.1).

**(e)** Divide this into the sum you obtained in part **(c)** (e.g. 24.2 ÷ 1.1).

Did you get 22?

# Decimal multiplication

4.5

## Multiplying a decimal by a whole number

To multiply a decimal by a whole number:

Multiply as though both values are whole numbers. Then, place a decimal point in the answer so that it has the same number of decimal places as the decimal number in the question.

To check that your decimal point is in the correct place, it is a good idea to estimate your answer before calculating it.

### Worked Example 11 WE11

Calculate:

**(a)** $4.172 \times 3$ **(b)** $52.63 \times 14$ **(c)** $0.0003 \times 2$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Use rounding to estimate the answer. | (a) $4.172 \times 3 \approx 4 \times 3$<br>$\approx 12$ |
| **2** | Multiply as though both values are whole numbers. | $\begin{array}{r} 4.172 \\ \times \quad {}^{2}3 \\ \hline 12.516 \end{array}$ |
| **3** | Make sure that there are the same number of decimal places in the answer as in the question (in this case, 3). | $4.172 \times 3 = 12.516$ |
| **4** | Check your answer against your estimate to see whether it is reasonable. | Reasonable |
| **(b)** **1** | Use rounding to estimate the answer. | (b) $52.63 \times 14 \approx 50 \times 14$<br>$\approx 14 \times 5 \times 10$<br>$\approx 700$ |
| **2** | Multiply as though both values are whole numbers. | $\begin{array}{r} 52.63 \\ \times \quad {}^{1}{}^{2}{}^{1}14 \\ \hline 21\,052 \\ 52\,630 \\ \hline 736.82 \end{array}$ |
| **3** | Make sure that there are the same number of decimal places in the answer as in the question (in this case, 2). | $52.63 \times 14 = 736.82$ |

| | | | |
|---|---|---|---|
| | 4 | Check your answer against your estimate to see whether it is reasonable. | Reasonable |
| (c) | 1 | An estimate is not needed here, as we can see that the only non-zero digit will be a 6 ($3 \times 2$). | (c) |
| | 2 | Multiply as though both values are whole numbers. Place a decimal point in the answer so that there are the same number of decimal places as in the decimal number in the question (in this case, 4). | $\begin{array}{r} 0.0003 \\ \times \quad 2 \\ \hline 0.0006 \end{array}$<br>$0.0003 \times 2 = 0.0006$ |

## Multiplying and dividing decimals by powers of 10

When we multiply a number by 10, we are making each digit in the number 10 times its previous value.

In the place value table, each digit is moved up one place value column to the left to show this. Hundredths become tenths, tenths become ones, ones become tens etc.

For example, $43.25 \times 10 = 432.5$.

| Hundreds 100 | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ 0.01 | | Hundreds 100 | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 3 | • | 2 | 5 | $\times 10 =$ | 4 | 3 | 2 | • | 5 |

$\times 10$ $\times 10$ $\times 10$ $\times 10$

When we divide by 10, we are making each digit one-tenth of its previous value. In the place value table, each digit is moved down one place value column to the right to show this. Tenths become hundredths, ones become tenths, tens become ones etc.

For example, $65.1 \div 10 = 6.51$ as shown below.

| Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ (0.01) | | Tens 10 | Ones 1 | • | Tenths $\frac{1}{10}$ (0.1) | Hundredths $\frac{1}{100}$ (0.01) |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 5 | • | 1 | | $\div 10 =$ | | 6 | • | 5 | 1 |

$\div 10$ $\div 10$ $\div 10$

It is convenient to think of this as moving the decimal point one place to the right (for multiplication) or left (for division). This produces the same answer; however, we should remember it is the digits themselves that actually move as their values increase or decrease.

Multiplying or dividing by 100 is like multiplying or dividing by 10 twice (as $100 = 10 \times 10$). Each digit moves two place value columns. Again, we can think of this as moving the decimal point two places to the right or left.

To *multiply* a decimal by a power of 10:

Move the decimal point to the *right* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$0.45 \times 10\ (10^1) = 4.5$

$0.45 \times 100\ (10^2) = 45$

$0.45 \times 1000\ (10^3) = 450$

To *divide* a decimal by a power of 10:

Move the decimal point to the *left* the same number of place values as zeroes in the power of 10. Fill in any empty place values with zeroes.

$78.9 \div 10\ (10^1) = 7.89$

$78.9 \div 100\ (10^2) = 0.789$

$78.9 \div 1000\ (10^3) = 0.0789$

### Decimal 'shorthand'

Each of the powers of 10 has its own 'name', as shown below with the first nine powers:

| | | |
|---|---|---|
| $10^1$ = ten | $10^4$ = ten thousand | $10^7$ = ten million |
| $10^2$ = hundred | $10^5$ = hundred thousand | $10^8$ = hundred million |
| $10^3$ = thousand | $10^6$ = million | $10^9$ = billion |

To write very large numbers, we sometimes write a smaller decimal number and the name of the power. For example, instead of writing 2 300 000, we could write 2.3 million, or $2.3 \times 10^6$. This is equivalent to writing $2.3 \times 1\,000\,000$, or 2 300 000.

## Multiplying decimals by other multiples of 10

Multiples of 10, such as 20, 190, 1700, 8000 etc., can be written as a number multiplied by a power of 10: $20 = 2 \times 10$, $190 = 19 \times 10$, $1700 = 17 \times 100$, $8000 = 8 \times 1000$.

Writing them in this way means we can multiply by the number first, then the power of 10.

## Worked Example 12 WE12

Calculate:

**(a)** $0.6295 \times 7000$ **(b)** $9.81 \times 340\,000$

| Thinking | Working |
|---|---|
| **(a) 1** Rewrite the whole number as a product of a number and a power of 10. | (a) $7000 = 7 \times 1000$ |
| **2** Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied. | $0.6295$<br>$\times\ 7$<br>$4.4065$ |
| **3** Multiply the answer by the power of 10 by moving the decimal point to the right. (In this case, 3 places.) | $4.4065 \times 1000$<br>$= 4406.5$ |

| | | | |
|---|---|---|---|
| **(b)** | **1** | Rewrite the whole number as a product of a number and a power of 10. | (b) $340\,000 = 34 \times 10\,000$ |
| | **2** | Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied. | $\begin{array}{r} 9.81 \\ \times\ {}^{2}{}^{3}\ 34 \\ \hline 3924 \\ {}_{1}2_{1}9430 \\ \hline 333.54 \end{array}$ |
| | **3** | Multiply by the power of 10 by moving the decimal point to the right. (In this case, 4 places.) Write zeroes into the empty place value columns. | $333.54 \times 10\,000$ $= 3\,335\,400$ |

## Multiplying a decimal by another decimal

To develop a method for multiplying one decimal by another, it is useful to look at some number patterns. We have seen that when we multiply by powers of 10, each digit in the number moves up the same number of place values as the power number. We show this by moving the decimal point. For example:

$0.23 \times 10 = 2.3$ $0.23 \times 100 = 23$ $0.23 \times 1000 = 230$

We can reverse the above calculations by dividing by the powers of 10:

$2.3 \div 10 = 0.23$ $23 \div 100 = 0.23$ $230 \div 1000 = 0.23$

We can see that dividing by powers of 10 corresponds to moving the decimal point to the left the same number of places as zeroes in the power of 10. This is used in the Worked Example below.

## Worked Example 13 WE13

Calculate:

**(a)** $0.63 \times 0.4$ **(b)** $0.014 \times 0.002$

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Write both numbers as the division of a whole number by a power of 10. | (a) $0.63 = 63 \div 100$<br>$0.4 = 4 \div 10$ |
| | **2** | Multiply as though both numbers are whole numbers. (We do not need to multiply by the zero in 0.4, as it does not affect the outcome.) Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide by 100 and 10, which is the same as dividing by 1000.) If necessary, place a zero in the units place to clearly show the position of the decimal point. | $\begin{array}{r} 0.6{}^{2}3{}^{1} \\ \times\ 0.4 \\ \hline 0.252 \end{array}$<br>$0.63 \times 0.4 = 0.252$ |

**(b) 1** Write both numbers as the division of a whole number by a power of 10.

**2** Multiply as though both numbers are whole numbers (ignoring the zeroes). Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, we divide twice by 1000, which is the same as dividing by 1 000 000.) Place zeroes in the empty place values and a zero in the units place.

(b) $0.014 = 14 \div 1000$
$0.002 = 2 \div 1000$

$$\begin{array}{r} 0.014 \\ \times \quad 0.002 \\ \hline 0.000\,028 \end{array}$$

$0.014 \times 0.002 = 0.000\,028$

To multiply one decimal by another:

**Step 1** Write both numbers as the division of a whole number and a power of 10.

**Step 2** Multiply as though both numbers are whole numbers.

**Step 3** Divide the product by the powers of 10 identified in **Step 1**. Show this by moving the decimal place to the left.

You may be able to spot a useful shortcut from the Worked Example above: The number of decimal places in the answer to the multiplication is equal to the total number of decimal places in the numbers being multiplied.

# 4.5 Decimal multiplication

## Navigator

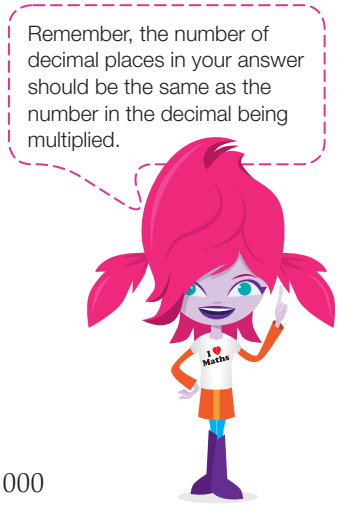
| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q7, Q9, Q11, Q12, Q13, Q14, Q17, Q21, Q23, Q24 | Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q17, Q19, Q21, Q22, Q23, Q24, Q25 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q18, Q19, Q20, Q22, Q23, Q24, Q25 |

Answers page 643

## Fluency

**1** Calculate:

WE11




**(a)** $7.6 \times 4$ **(b)** $5.3 \times 7$ **(c)** $6.5 \times 9$
**(d)** $5 \times 4.62$ **(e)** $0.54 \times 4$ **(f)** $6 \times 12.33$
**(g)** $8.047 \times 8$ **(h)** $0.619 \times 7$ **(i)** $3 \times 18.309$
**(j)** $22.08 \times 13$ **(k)** $48.76 \times 25$ **(l)** $37.09 \times 32$
**(m)** $0.0004 \times 2$ **(n)** $0.0005 \times 7$ **(o)** $1.0006 \times 3$

**2** Calculate:

WE12

**(a)** $2.4 \times 60$ **(b)** $3.27 \times 70$ **(c)** $2.991 \times 40$
**(d)** $0.57 \times 200$ **(e)** $4.34 \times 500$ **(f)** $6.625 \times 400$
**(g)** $4.591 \times 6000$ **(h)** $6.38 \times 4300$ **(i)** $3.14 \times 9300$
**(j)** $0.571 \times 68\,000$ **(k)** $0.9465 \times 71\,000$ **(l)** $7.4832 \times 55\,000$

WE13

**3** Calculate:

**(a)** $0.6 \times 0.8$ **(b)** $0.4 \times 0.6$ **(c)** $0.9 \times 0.4$

**(d)** $9.32 \times 0.7$ **(e)** $4.12 \times 0.3$ **(f)** $5.26 \times 0.8$

**(g)** $7.74 \times 0.31$ **(h)** $9.46 \times 0.22$ **(i)** $3.76 \times 0.18$

**(j)** $0.004 \times 0.6$ **(k)** $0.0308 \times 0.3$ **(l)** $0.072 \times 0.05$

**4** A reasonable estimate for the answer to $1.8 \times 15$ is:

**A** 3 **B** 16.8 **C** 30 **D** 300

**5** How many decimal places will there be in the answer to $12.789 \times 41$?

**A** 1 **B** 2 **C** 3 **D** 4

**6** How many decimal places will there be in the answer to $0.36 \times 1.04$?

**A** 2 **B** 3 **C** 4 **D** 5

## Understanding

**7** Renée buys 52 components from an electronics shop for \$0.35 each. What is the total cost of the components?

**8** A pill bottle contains 37 pills, each of mass 0.287 grams. What is the total mass of the bottle's contents?

**9** Write the following as whole numbers.

**(a)** 3.5 million **(b)** 4.26 billion **(c)** 1.3 trillion (1 trillion = 1000 billion)

**(d)** 57.08 million **(e)** 10.2 trillion **(f)** 20.045 billion

**10** A snail crawls 0.041 metres in an hour. If it kept up this pace, how far could it move in a day?

**11** Amy orders 27.5 metres of timber priced at \$4.29 per metre from her local hardware store. How much will the timber cost her in total? (Round your answer to the nearest cent.)



**12** Gavin sells 300 punnets of berries to a supermarket. If he is paid \$1.72 for each punnet, how much money does he receive?

**13** In 2010, Mexican businessman Carlos Slim was declared the world's richest man, with a fortune worth 53.5 billion US dollars. Write this amount as a whole number.

**14** **(a)** Marita fills her car's fuel tank with petrol at a cost of \$1.35 per litre. If she adds 70 litres, how much must she pay?

**(b)** Ali pumps 9.5 litres of petrol into his car's tank. How much did the fuel cost him if he paid the same price per litre as Marita? Round your answer to the nearest cent.

**15** The instructions on a bag of fertiliser say to spread 0.75 kilograms per square metre of area to be fertilised. Raisa wants to fertilise 2600 square metres of property using this product. How many kilograms of fertiliser should she use?

**16** Bryan has calculated that he needs 3.6 square metres of tiles for his bathroom. What will he have to pay if the tiles he wants cost \$22.39 per square metre? (Round your answer to the nearest cent.)

## Reasoning

**17** Kim works in an ice-cream parlour, and one hot day sells 500 scoops of vanilla. If there is 0.185 litres of ice-cream in each scoop and the day started with 130 litres of vanilla in the freezer, how many litres of vanilla ice-cream are left at the end of the day?



**18** Yoghurt contains 167 kilojoules per 100 grams. How many kilojoules would a 175 gram tub contain?

**19** In April 2010, the Australian Bureau of Statistics calculated that the population of Australia was 22 296 411.

**(a)** Write this number as a decimal number, with the number of millions as the whole number part, and the rest of the number rounded to one decimal place.

**(b)** Why would your answer to **(a)** be the number you would expect to see reported in the news, instead of the 'exact' value?

**20** **(a)** A woodwork teacher needs 50 pieces of pine, each 0.135 metres long, to give to students to make a particular model. What total length (in metres) of timber does the teacher need? (Ignore any lost length due to saw cuts.)

**(b)** If he can buy the pine in 3.5 m lengths, how many pieces does he need, and how much does he have left over?

**21** A multi-bag of mini chocolate bars costs $3.95 and contains 12 bars. Each mini chocolate bar can also be purchased individually for 40 cents.

**(a)** How much would 12 mini chocolate bars cost if they were purchased individually?

**(b)** How much do you save by purchasing the multi-bag?

**(c)** Charlotte wants to buy 30 mini bars for a party. Should she buy them individually, or in multi-bags? Which way is cheaper?



**22** Wayne is following his grandfather's recipe for tomato sauce, which requires 3.5 pounds of tomatoes. (A pound is an old imperial unit of mass.) Because he only has scales in kilograms, Wayne decides to convert the mass to kilograms. A website tells him that 1 pound equals 0.454 kilograms. What answer should he get when he converts the mass of the tomatoes to kilograms? Round your answer to one decimal place.

**23** Francesca read her electricity meter one Saturday, and again at the same time the next Saturday. She recorded the following readings.

First Saturday: 7562.3 kilowatt-hours
Second Saturday: 7638.6 kilowatt-hours

**(a)** How many kilowatt-hours of electricity did Francesca's household use in the week?

**(b)** If electricity costs $0.13 per kilowatt-hour, find the total cost of the electricity supplied in the week.

## Open-ended

**24** Jayapal has been asked to work out the length if every student in Year 7 lay end to end. He has been told that there are 89 students in Year 7 and that their average height is 1.63 m.



This is his working out:

$89 \times 1 = 89$
$89 \times 6 = 534$
$89 \times 3 = 267$

$$\begin{array}{r} 89 \\ 534 \\ +\ 267 \\ \hline = 880 \end{array}$$

Answer: 8.80 m.

**(a)** Where has Jayapal made his error? Explain how he could use estimation to tell that he was incorrect.

**(b)** Explain how he could modify his method to get the correct answer.

**(c)** Show how you would complete this problem.

**25** Two numbers are multiplied together to give 68.4.

**(a)** Find at least two possible pairs of numbers with one number being a multiple of 10.

**(b)** Find at least two possible pairs of numbers with neither being a multiple of 10.

## Outside the Square Game

### Multiplicity

**Equipment required:** 2 brains, 1 die, 1 calculator

**How to win:**
The aim of the game is to get the target number as the whole number part of a decimal.

**How to play:**

1. The first player rolls the die twice to determine the two-digit target number (the first number followed by the second number).
2. The second player then rolls the die twice to determine the starting number.
3. The first player puts the starting number into a calculator and can multiply this by any number of their choice, with the aim of reaching the target number.
4. If they reach a number starting with the two-digit target number, then they're the winner; if not, the other person takes their answer (leave it in the calculator display) and then chooses a number to multiply this by to try to reach the target number.

For example, the first player rolls a 4 and a 3, so the target number is 43.

The second player then rolls two 1s, so the starting number is 11.

The first player chooses to multiply the 11 by 4, which gives an answer of 44—too big.

The second player chooses to multiply the 44 by 0.9, which gives an answer of 39.6—too small.

This continues until a player reaches an answer of 43 point something.

The game can be made more difficult by making the target number a three-digit number, or a two-digit number with one decimal point.

# Half-time 4

**1** Convert the following decimals into fractions (or mixed numbers where appropriate) in simplest form. Ex 4.3

**(a)** 0.25 **(b)** 0.3 **(c)** 3.54 **(d)** 9.345

**2** Put the following decimals in order from smallest to largest. Ex 4.1

9.01, 19.12, 0.03, 0.321, 9.003 24

**3** Calculate: Ex 4.4

**(a)** 12.5 + 7.9 **(b)** 124.54 + 8.379 **(c)** 0.32 + 2.5

**(d)** 15 – 8.98 **(e)** 127.061 – 43.2 **(f)** 6 – 4.9

**4** Round the following numbers to three decimal places. Ex 4.2

**(a)** 4.8765 **(b)** 13.0037 **(c)** 2.3498 **(d)** 23.032 49

**5** Calculate the following. Ex 4.5

**(a)** $7.6 \times 4$ **(b)** $28.3 \times 5$ **(c)** $1.37 \times 9$

**(d)** $45.2 \times 30$ **(e)** $0.26 \times 2000$ **(f)** $6.78 \times 500$

**(g)** $4.3 \times 1.7$ **(h)** $39.82 \times 15.4$ **(i)** $1.2 \times 0.0005$

**6** Franji is saving up to purchase a games console. She has saved the following amounts over the last four weeks: \$25, \$47.50, \$18.70, \$9.35. Ex 4.4



**(a)** How much has she saved in total?

**(b)** If the console is \$149, how much more does Franji have to save?

**7** Write the following in decimal form. Ex 4.1

**(a)** $23 + \frac{4}{10} + \frac{9}{100} + \frac{7}{10\,000}$ **(b)** $14 + \frac{235}{1000}$

**(c)** 2 hundreds, 9 tens, 3 ones, 4 tenths, 1 hundredth and 5 thousandths

**(d)** 9 tens, 0 ones, 6 tenths, 2 hundredths and 7 ten-thousandths

**8** Round the following amounts of money to the nearest 5 cents. Ex 4.2

**(a)** \$1.93 **(b)** \$35.08 **(c)** \$57.44 **(d)** \$189.98

**9** Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. Ex 4.3

**(a)** $\frac{4}{5}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{7}{12}$

**10** The times recorded by the finalists in a track cycling sprint race were: 59.023 s, 59.105 s, 59.23 s, 59.125 s. Write the times in order from fastest to slowest. Ex 4.1

**11** Melinda is out buying some ingredients for a cooking class. She buys: Ex 4.4, 4.5

12 packets of pasta at \$2.49 each
18 tins of tomatoes at \$1.35 each
6 kg of mince at \$8.99 per kg
3 packets of cheese at \$4.78 per packet

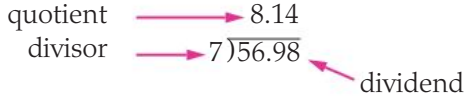
Find the total cost of Melinda's shopping.

# 4.6 Decimal division

When studying division, the following mathematical terms are useful.

- The dividend is the number that is being divided.
- The divisor is the number we are dividing by.
- The quotient is the result of the division (the answer).

These are shown in the example below.




When using long or short division to divide a decimal by a whole number:

Line up the decimal point in the quotient with the decimal point in the dividend.

## Worked Example 14 WE14

Calculate the following. Round your answers to three decimal places if necessary.

**(a)** $5.28 \div 3$ **(b)** $0.41 \div 7$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Write the question with the setting out shown. | (a) $3\overline{)5.28}$ |
| **2** | Perform the division, and place a decimal point in the quotient so it lines up with the one in the dividend. | $3\overline{)5.^{2}2^{1}8}$ 1. 7 6 |
| **(b)** **1** | Write the question with the setting out shown. | (b) $7\overline{)0.41}$ |
| **2** | Perform the division, remembering to place a zero in the quotient if the divisor goes zero times into the dividend. Add zeroes to the divisor to continue the division, until there are four decimal places in the answer. | $7\overline{)0.4^{4}1^{6}0^{4}0}$ 0.0 5 8 5... |
| **3** | Round off the answer to three decimal places. | 0.059 (to 3 d.p.) |

## Dividing decimals by multiples of 10

As we did for multiplying decimals, we can write other multiples of 10 as the product of a number and a power of 10, and do two separate division steps. Remember that we can show division by a power of 10 by moving the decimal point to the left the same number of places as the zeroes in the power of 10.

## Worked Example 15 WE15

Calculate the following. Round your answers to three decimal places where necessary.

**(a)** 9.76 ÷ 400 **(b)** 14.7 ÷ 80

| Thinking | Working |
|---|---|
| **(a) 1** Break the divisor down into a whole number and a power of 10. | (a) 9.76 ÷ 400<br>= 9.76 ÷ 4 ÷ 100 |
| **2** Divide by the whole number first (in this case, by the 4). | $4\overline{)9.^{1}7^{1}6}$ 2.44 |
| **3** Now, divide the quotient of the first division by the power of 10. Show this by moving the decimal point. (Here, it is moved 2 places left.) | 2.44 ÷ 100 = 0.0244 |
| **4** Round your answer to the specified number of decimal places. | = 0.024 (to 3 d.p.) |
| **(b) 1** Break the divisor down into a whole number and a power of 10. | (b) 14.7 ÷ 80<br>= 14.7 ÷ 8 ÷ 10 |
| **2** Divide by the whole number first (in this case, by the 8). Add zeroes to the divisor to continue the division until there are more decimal places in the answer than the rounded answer requires. | $8\overline{)14.^{6}7^{3}0^{6}0^{4}0}$ 1. 8 3 7 5 |
| **3** Now, divide the answer by the power of 10. Show this by moving the decimal point. (Here, it is moved 1 place left.) | 1.8375 ÷ 10 = 0.183 75 |
| **4** Round your answer to the specified number of decimal places. | = 0.184 (to 3 d.p.) |

## Dividing a decimal by another decimal

In order to divide a decimal by another decimal we need to convert it to a problem where we can divide a decimal by a whole number.

Recall that when working with fractions we can multiply or divide the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

For example, $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{10}{20}$. We can apply the same principle to dividends and divisors.

For example, 1 ÷ 2 gives the same answer as 2 ÷ 4 and 10 ÷ 20.

13.52 ÷ 1.2
= 135.2 ÷ 12 (multiplying both numbers by 10)
= 1352 ÷ 120 (multiplying by 10 again)

7.5 ÷ 2.5
= 15 ÷ 5 (multiplying by 2)

To divide one decimal by another, multiply both numbers so that the divisor is no longer a decimal. Then, divide the new dividend by the new, whole number divisor.

Multiplying both numbers by a power of 10 is often the most convenient. Multiply by the smallest power of 10 that gives a whole number divisor.

## Worked Example 16 WE16

Calculate the following. Round your answer to three decimal places if necessary.

**(a)** 3.35 ÷ 0.005 **(b)** 10.52 ÷ 0.9

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 1000.) | (a) $3.35 \div 0.005$<br>$= (3.35 \times 1000) \div (0.005 \times 1000)$ |
| | **2** | Rewrite the question using the new divisor and new dividend. | $= 3350 \div 5$ |
| | **3** | Calculate the quotient. | $5\overline{)33^{3}50}$ 6 70 |
| | **4** | State the answer. | $3.35 \div 0.005 = 670$ |
| **(b)** | **1** | What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 10.) | (b) $10.52 \div 0.9$<br>$= (10.52 \times 10) \div (0.9 \times 10)$ |
| | **2** | Rewrite the question using the new divisor and new dividend. | $= 105.2 \div 9$ |
| | **3** | Calculate the quotient to one more decimal place than the rounded number required. | $9\overline{)10^{1}5.^{6}2^{8}0^{8}0^{8}0}$ 1 1. 6 8 8 8 |
| | **4** | Round off the quotient to the required number of decimal places. | = 11.689 (to 3 d.p.) |

# 4.6 Decimal division

## Navigator

| | | |
|---|---|---|
| Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17, Q19, Q23, Q24 | Q1 Columns 2 & 3, Q2 Column 2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20, Q23, Q24 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24 |

Answers page 643

## Fluency

**1** Calculate the following. Round your answers to three decimal places if necessary.

WE14




**(a)** 8.48 ÷ 4 **(b)** 12.96 ÷ 3 **(c)** 14.22 ÷ 9

**(d)** 3.3 ÷ 6 **(e)** 2.4 ÷ 7 **(f)** 5.1 ÷ 9

**(g)** 39.42 ÷ 6 **(h)** 10.6 ÷ 7 **(i)** 0.47 ÷ 9

**(j)** 0.42 ÷ 6 **(k)** 1.22 ÷ 3 **(l)** 18.46 ÷ 7

**(m)** 154.33 ÷ 5 **(n)** 84.365 ÷ 8 **(o)** 347.507 ÷ 7

**2** Calculate the following. Round your answers to three decimal places if necessary.

WE15

**(a)** 3.6 ÷ 40 **(b)** 8.1 ÷ 90 **(c)** 36.216 ÷ 60

**(d)** 96.58 ÷ 200 **(e)** 2.296 ÷ 400 **(f)** 23.4 ÷ 600

**(g)** 233.4 ÷ 3000 **(h)** 40.6 ÷ 8000 **(i)** 18.9 ÷ 7000

**(j)** 68.4 ÷ 120 **(k)** 3.19 ÷ 110 **(l)** 8.97 ÷ 1300

**3** Calculate the following. Round your answers to three decimal places if necessary.

WE16

**(a)** 5.14 ÷ 0.2 **(b)** 4.12 ÷ 0.4 **(c)** 5.1 ÷ 0.6

**(d)** 15.48 ÷ 0.04 **(e)** 9.018 ÷ 0.09 **(f)** 1.736 ÷ 0.07

**(g)** 2.382 ÷ 0.006 **(h)** 0.3996 ÷ 0.009 **(i)** 0.196 ÷ 0.008

**(j)** 7.7 ÷ 0.011 **(k)** 3.78 ÷ 0.012 **(l)** 1.16 ÷ 0.008

**(m)** 1.74 ÷ 0.0002 **(n)** 8.05 ÷ 0.0007 **(o)** 13.36 ÷ 0.0008

**4** How many decimal places will the answer to 298.12 ÷ 100 have?

**A** 1 **B** 2 **C** 3 **D** 4

**5** Evaluating which of the following will give the answer to 3.2 ÷ 0.005?

**A** 0.0032 ÷ 5 **B** 320.0 ÷ 5 **C** 3200.0 ÷ 5 **D** 32 000.0 ÷ 5

## Understanding

**6** A block of metal of mass 2.312 kg is to be divided equally into 8 pieces. What will the mass of each piece be?

**7** A company makes a profit of 7.86 million dollars. If the profit is to be divided equally among the four company owners, how much would each owner get? (Answer in millions of dollars.)

**8** The total bill for a wedding reception for 100 guests is $9398.55. How much does this work out to per guest? (Round your answer appropriately.)



**9** Anthony needs to mix 0.07 litres of environmentally friendly pest repellent with water each time he fills the tank of his garden sprayer. If he has 0.84 litres of repellent left, how many sprayer tankfuls can he mix?

**10** A jeweller calculated that a popular style of necklace contains 0.009 g of gold. How many necklaces could she make using the 2.332 g of gold in her workshop?

**11** Eleanor's car travels 37.29 km on three litres of fuel. How far would it travel on one litre of fuel?

**12** A 4000 millilitre container of orange juice costs $8.90. Determine the cost per 100 mL. (Round your answer appropriately.)

**13** Simon buys 7 kg of spicy sausages for $32.95. How much is this per kilogram?

**14** Regina has a tub containing 5.8 kilograms of worms which she uses for fishing. If the average worm has a mass of 0.003 kilograms, how many are in her tub?



**15** Hai is making a long-distance call on his mobile. The call costs $0.90 per minute. For how many minutes can he continue his call if he has $5.85 credit left on his phone?

**16** A 1.2 kilometre taxi trip costs Joan $5.70. How much is this per kilometre?

## Reasoning

**17** The captains of two netball teams were discussing which of their teams was taller. The heights of Angie's team were 1.49 m, 1.54 m, 1.65 m, 1.67 m, 1.58 m, 1.73 m and 1.56 m. The heights of Jessica's team were 1.73 m, 1.58 m, 1.59 m, 1.62 m, 1.45 m, 1.51 m and 1.62 m. Add the seven heights and divide the result by seven to find the average (mean) height of each team. Which of the two teams was the taller, on average? Answer correct to two decimal places.

**18** Monica and Celeste were trying to determine the best bargain of three different jars of coffee. The 200 g jar of BestCafe cost $5.95, the 500 g jar of Maxine House was $14.85 and the 300 g tin of Brazilian Roast was $6.75. Which is the best buy in terms of price per 100 grams?

**19** Karen was counting the coins in her cash register drawer and found that she had $1.65 in 5-cent coins, $2.30 in 10-cent coins, $5.20 in 20-cent coins and $7.50 in 50-cent coins. How many of each type of coin did she have?

**20** Jacob received a bill from a rent-a-car firm that gave the total cost as $310. This included the daily cost of $24.80 and $1.20 per kilometre travelled. If Jacob had hired the car for 2 days, how many kilometres did he travel?

## Open-ended

**21** Write three numbers, each with three decimal places, that when divided by 4 give an answer between 0.5 and 1.

**22** For each of the following, write two equivalent divisions by multiplying the divisor by a number that is not a power of 10.

**(a)** $10.05 \div 0.5$

**(b)** $36.04 \div 0.02$

**23** The area of a rectangle is calculated by multiplying the length and the width. If the area of a rectangle is 8.45 $m^2$, what might be the length and width of the sides? Give two possible values for the side dimensions. For each of the side dimensions, work out the related perimeter (distance along the boundary of the rectangle).

**24** Tran divided 5.25 by 5 and wrote down an answer of 1.5.

**(a)** What error did Tran make?

**(b)** Explain to Tran what he needs to do to calculate decimal divisions accurately.

## Outside the Square Puzzle

### Magic square

Copy this 6 × 6 table into your book.

Fill in the blank squares. All the rows and columns must add up to 11.1.

| | | | | | |
|---|---|---|---|---|---|
| 2.1 | | 2.5 | 2.7 | | |
| | 2.5 | 2.8 | | 0.5 | |
| 0.1 | 0.3 | | 1.9 | 3.3 | |
| 0.4 | 0.2 | 1.7 | 1.8 | 3.6 | |
| 2.9 | 3.1 | 0.9 | 1.1 | 1.6 | |
| 3.2 | 3.0 | 1.2 | | 1.3 | 0.8 |

Maths 4 Real



# PERPLEXING PRICES

Imagine you are in your local supermarket, shopping for supplies for a party. You need 7 to 8 litres of soft drink and some lollies, so you move to the soft drink aisle first. A vast array of products and prices confronts you. You see that your favourite brand has three different-sized bottles:




1.25 L for $2.25




1.5 L $2.78




2 L for $3.56

"Hmmm," you think. "The 2 L bottle gives me half a litre more than the 1.5 L, for only 78 cents extra. I could get two 1.5 L bottles, that's 3 L, for $5.56. Maybe I should get a couple of 1.25 L bottles instead. Which do I choose?"

**1** At first glance, which bottle size looks to be the best value for money to you?

You remember that soft drink also comes in 375 mL cans. You move further down the aisle and see the following:




6-pack: $7.00 12-pack: $13.59 18-pack: $19.34 30-pack: $26.34

You begin to wonder: "Should I buy cans instead? How many cans are equivalent to one bottle? Are they better value for money? Help!"

**2** What is it that makes your 'value for money' decision a difficult one? What information would be useful to help you decide?

In December 2009, the Australian government introduced a new, compulsory pricing system in supermarkets, designed to make decisions easier for shoppers. The system is called 'unit pricing'. As well as the regular price, supermarkets are required to display the price for a particular 'unit' of the product, such as 1 L, 1 kg, 100 g or 100 mL. Comparing the prices of different products is much easier when you are comparing the same amount.

We can calculate a unit price for the bottles of soft drink by calculating the price *per litre*. To do this, we divide the price by the capacity of the bottle, in litres:

For example, the unit price for the 1.25 L bottle is: $2.25 ÷ 1.25 L = $1.80 per L.

**3 (a)** Calculate the price per litre of the other two bottle sizes.

**(b)** Which size bottle is the best value for money?

**(c)** Would it always be better to purchase this size? Explain your answer.

**(d)** A bit further down the aisle, you notice that a 1.5 L bottle of a rival brand of cola is on special for $2.39. Calculate the price per litre of this discounted bottle.

**(e)** You also notice that you can buy a 'multipack' of twelve 300 mL bottles for $11.86. Calculate the price per litre of the multipack. (300 mL is 0.3 L)

**(f)** Why is the unit price of the multipack so different to the unit prices for individual bottles?

**4 (a)** Calculate the price per can of each of the packs.

**(b)** Which pack represents the best value for money? Did you find anything surprising in your results?

**(c)** The price of a single can of soft drink is $1.35. Why is there such a difference between the price of a single can and the price per can in the can packs?

**5 (a)** Calculate the price per litre of each of the can packs, remembering that each can contains 375 mL (0.375 L).

**(b)** Suggest a possible explanation for the difference in the price per litre between the cans and the bottles.

**(c)** When might people prefer to buy multipacks of cans instead of bottles?

You move down to the confectionery aisle, and spot a 190 g 'Party Mix' bag of lollies for $2.90, and a 'Jumbo' size 570 g bag for $5.82.




**6** The unit price for products like bags of lollies is the price per 100 g.

**(a)** Calculate the unit price for the 190 g bag and the 570 g bag. (Hint: Work out what you need to divide 190 and 570 by to get 100, and divide the prices by this same amount.)

**(b)** Which is the better buy in this case?

## RESEARCH

- Visit the supermarket, use catalogues or go shopping online to investigate the cost of goods that come in different sizes, such as coffee, Vegemite™, washing powder and pet food. Use a suitable unit, such as 100 g or 1 kg, to calculate the unit prices.
- Comment on your findings. Is there a general rule or conclusion you can make? Discuss, with examples, why different people might buy different-sized products.

# 4.7 Percentages, fractions and decimals

Percentages are used in many everyday situations. Shopping discounts, survey results, business statistics and sporting league ladders all use percentages to communicate information. Percentages make it easier for us to visualise and compare quantities, especially when comparing parts from different-sized wholes.







The word **'per cent'** literally means 'for every hundred' or 'out of one hundred'. The percentage symbol is %. Percentages can be thought of and written as fractions with a denominator of 100. Percentages can also be thought of as division by 100.

For example, 25% means 25 out of 100, and 9% means 9 out of 100. We can write percentages as fractions, some of which can be simplified, and some which can't.

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 9\% = \frac{9}{100}$$

If we perform the division, we get the percentage in decimal form.

$$\frac{25}{100} = 25 \div 100 = 0.25 \qquad \frac{9}{100} = 9 \div 100 = 0.09$$

## Writing percentages as fractions and decimals

*To write a percentage as a fraction …*

- Write the percentage value as the numerator of a fraction with a denominator of 100.
- Simplify the fraction if possible.

*To write a percentage as a decimal …*

- Divide by 100.

### Worked Example 17 WE17

Write 12% as a fraction in simplest form.

| Thinking | Working |
|---|---|
| **1** Write the value of the percentage as the numerator over a denominator of 100. | $12\% = \frac{12}{100}$ |
| **2** Simplify the fraction if possible. (Here, we have cancelled a common factor of 4.) | $= \frac{3}{25}$ |

### Worked Example 18 WE18

Write 45% as a decimal.

| Thinking | Working |
|---|---|
| Divide the value of the percentage by 100. Write your answer without the percentage symbol. | $45\% = 45 \div 100$<br>$= 0.45$ |

## Writing fractions as percentages

To write a fraction as a percentage, we can use one of two methods:

**Method 1:** Write the fraction as an equivalent fraction with a denominator of 100. Use the numerator of this new fraction as the percentage.

**Method 2:** Multiply the fraction by 100%.

Method 1 is convenient, but only works with fractions that have denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2 can be used with any fraction.

Note: As Worked Example 19 shows, not all percentages have whole number values. These percentage numbers are usually written as decimals. If it is necessary to round them, one decimal place usually gives an answer of acceptable accuracy. These percentages can also be written in fractional form; for example, $16.7\% = 16\frac{7}{10}\%$, but decimal form is usually more convenient to work with. Remember that you can use the $a\frac{b}{c}$ or S⇔D key on your calculator to convert answers from decimal to fraction form, and vice versa.

100% is equivalent to 1 whole, $\frac{1}{1}$ and 1.0.

## Worked Example 19 WE19

Write each of the following fractions as percentages.

**(a)** $\frac{4}{5}$ **(b)** $\frac{27}{40}$

| Thinking | Working |
|---|---|
| **(a) 1** Is the denominator a factor of 100? (Yes) Determine what number the denominator must be multiplied by to get 100. | (a) $\frac{4}{5} \times \frac{\quad}{20} = \frac{\quad}{100}$ |
| **2** Multiply the numerator by the same number. | $\frac{4}{5} \times \frac{20}{20} = \frac{80}{100}$ |
| **3** Write the numerator of the new fraction with the percentage symbol. | $= 80\%$ |
| **(b) 1** Is the denominator a factor of 100? (No) | (b) $\frac{27}{40} \times \frac{100}{1}\%$ |
| **2** Multiply the fraction by 100. (Remember that we can write a whole number as an improper fraction with a denominator of 1.) Cancel common factors first to simplify the multiplication. (Here, a common factor of 20 is cancelled.) | $= \frac{27}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$ <br> $= \frac{135}{2}\%$ |
| **3** Divide the numerator of the answer by the denominator. Write your answer as a decimal (rounded if necessary).Write the answer with a percentage symbol. | $= 67.5\ \%$ |

## Writing decimals as percentages

To write a decimal as a percentage, multiply it by 100.

## Worked Example 20 WE20

Write each of the following decimals as percentages.

**(a)** 0.04 **(b)** 0.125

| Thinking | Working |
|---|---|
| **(a)** Multiply the decimal by 100%. | (a) $0.04 = 0.04 \times 100\%$ <br> $= 4\%$ |
| **(b)** Multiply the decimal by 100%. | (b) $0.125 = 0.125 \times 100\%$ <br> $= 12.5\%$ |

# 4.7 Percentages, fractions and decimals

## Navigator

| Q1, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q15, Q16, Q18 | Q1, Q2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18 | Q1, Q2, Q3 Columns 3 & 4, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18 |
|---|---|---|

Answers page 644

## Fluency

**1** Write each of the following percentages as fractions in simplest form. WE17

**(a)** 15% **(b)** 24% **(c)** 18% **(d)** 64%

**(e)** 110% **(f)** 122% **(g)** 140% **(h)** 198%

**2** Write each of the following percentages as decimals. WE18

**(a)** 34% **(b)** 13% **(c)** 78% **(d)** 92%

**(e)** 120% **(f)** 123% **(g)** 254% **(h)** 320%

**3** Write each of the following fractions as percentages. WE19

**(a)** $\frac{2}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{20}$

**(e)** $\frac{11}{10}$ **(f)** $\frac{3}{2}$ **(g)** $\frac{28}{25}$ **(h)** $\frac{6}{5}$

**(i)** $\frac{3}{8}$ **(j)** $\frac{1}{3}$ **(k)** $\frac{5}{6}$ **(l)** $\frac{4}{9}$

**(m)** $\frac{13}{15}$ **(n)** $\frac{19}{30}$ **(o)** $\frac{33}{40}$ **(p)** $\frac{44}{60}$

**4** Write each of the following decimals as percentages. WE20

**(a)** 0.87 **(b)** 0.34 **(c)** 0.65 **(d)** 0.96

**(e)** 1.34 **(f)** 3.58 **(g)** 5.32 **(h)** 1.21

**5** Complete the following table.

| Percentage | Fraction (simplest form) | Decimal |
|---|---|---|
| 40% | | |
| | $\frac{7}{10}$ | |
| | | 0.05 |
| | $\frac{1}{50}$ | |
| 75% | | |
| | $\frac{5}{4}$ | |

## Understanding

**6** $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{5}$ are all common fractions.

**(a)** Change each of these fractions into decimals.

**(b)** Change each of these decimals into percentages.

**7** 95% of all the animal species in the world are insects. Write this as a fraction in simplest form.

**8** At Valley High School, $\frac{5}{8}$ of the students are studying Chinese. Write this fraction as a percentage.

**9** Place the following in ascending order (smallest to largest).

**(a)** 82%, 0.03, $\frac{7}{50}$, $\frac{3}{5}$, 0.8, 57%

**(b)** $\frac{1}{2}$, 73%, 1.05, $\frac{3}{10}$, 43%, 0.05

**10** The Petronas Towers in Kuala Lumpur are approximately 0.6 times the size of the tallest building in the world, the Burj Khalifa in Dubai.



**(a)** Write the height of the Petronas Towers as a fraction of the height of the Burj Khalifa.

**(b)** Now, write the equivalent percentage.

## Reasoning

**11** A rugby stadium contains 43 500 people. 37% of the crowd supports England and the rest supports Australia. What percentage of the crowd supports Australia?

**12** Allison is a journalist. She has surveyed 150 people and found that 6 believe that aliens exist.

**(a)** Write the survey result:

**(i)** as a fraction in simplest form

**(ii)** as a percentage

**(iii)** as a decimal.

**(b)** Which of the three forms of the result would you recommend that Allison uses as the headline for the article she is writing? Give a reason for your answer.

**(c)** Use your answer from **(b)** to write a potential headline.

**13** $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, or $3 \times \frac{1}{4}$. How is this fact reflected in the percentage values for $\frac{1}{4}$ and $\frac{3}{4}$?

**14** 13% of a class have red hair, 62% of the class have black hair.

**(a)** What is the total percentage of the class that have red or black hair?

**(b)** What is this total as a decimal?

**(c)** What percentage of the class have a hair colour that is not red or black?

**(d)** Write your answer to **(c)** as a decimal.

## Open-ended

**15** Write down at least three fractions that have equivalent percentage values between 10 and 40 per cent.

**16** Find at least three percentages that have decimal values between 0.34 and 0.35.

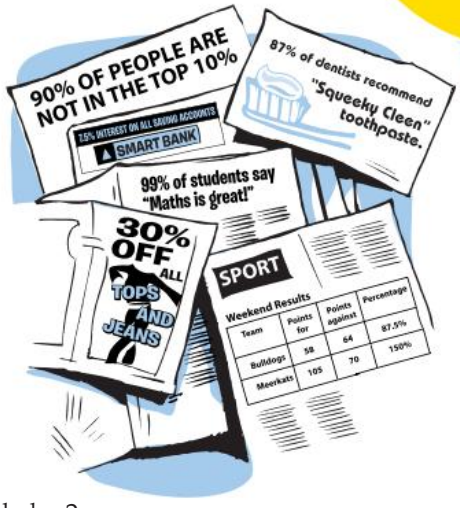
**17** Estimate the percentage of one weekday that you would spend:

**(a)** sleeping **(b)** eating **(c)** studying **(d)** socialising.

# Using percentages

4.8

In this section you will learn two important skills that will enable you to understand and work with two common uses of percentages.




## Finding a percentage of a quantity

Statements involving percentages are common:

'32% of the students in the school have blue eyes.'
'48% of voters voted for a particular candidate.'
'The television is discounted by 20% of the original price.'

Knowing the fraction equivalents of some of the common percentages is very useful.

For example:

50% = $\frac{1}{2}$, so to find 50% of something, we can simply divide by 2.

25% = $\frac{1}{4}$, so we can find 25% easily by dividing by 4 (or by dividing by 2 twice).

10% = $\frac{1}{10}$, so we divide by 10 to find 10%.

20% = $\frac{1}{5}$, so we divide by 5 to find 20% (or divide by 10, then multiply by 2).

We can use these facts to find a percentage of a quantity. This is shown in Method 1 of the following Worked Example.

Another way to find a percentage of a quantity is to use the fact that in mathematics, the word 'of' means 'multiply'. This is shown in Method 2 of the following Worked Example.

## Worked Example 21 WE21

Calculate the following percentage: 15% of 70.

### Method 1: Use basic percentages

| Thinking | Working |
|---|---|
| **1** Break the percentage up into basic percentages, such as 50%, 10%, 5% or 1%. | 15% = 10% + 5% |
| **2** Calculate each of the basic percentages. | 10% of 70<br>= 70 ÷ 10<br>= 7<br><br>5% of 70<br>= (10% of 70) ÷ 2<br>= 7 ÷ 2<br>= 3.5 |
| **3** Use combinations of the basic percentages calculated to obtain the required percentage. | 15% of 70 = 7 + 3.5<br>= 10.5 |

## Method 2: Fraction or decimal multiplication

| Thinking | Working |
|---|---|
| **1** Replace the word 'of' with the multiplication symbol, ×. | 15% of 70<br>$= 15\% \times 70$ |
| **2** Convert the percentage to a fraction or decimal. | $= \frac{15}{100} \times 70$ or $0.15 \times 70$ |
| **3** Perform the multiplication. | $= \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{100}} \times \frac{\cancel{70}}{1}$<br>$= \frac{3 \times 7}{2 \times 1}$<br>$= \frac{21}{2}$<br>$= 10.5$ |

## Writing one amount as a percentage of another

Writing fractions of a whole as percentages makes them easier to visualise, compare and order. The results of surveys, tests and sports teams are often reported using percentages.

To write one amount as a percentage of another:

1. Write the amount we are interested in as the numerator of a fraction, with the other amount as the denominator.
2. Convert this fraction to a percentage. If the percentage is not a whole number, then rounding to one decimal place gives an accurate enough answer in most cases.

## Worked Example 22 WE22

Write the following as percentages. Round your answers to one decimal place, if necessary.

**(a)** Melissa got 21 out of 25 questions on her grammar test correct.

**(b)** 13 of the 24 students in class 7C have brown hair.

| Thinking | Working |
|---|---|
| **(a) 1** Write the result as a fraction. Is the denominator a factor of 100? (Yes) | (a) $\frac{21}{25}$ |
| **2** Determine what number the denominator must be multiplied by to get 100. | $\frac{21}{25} \times \frac{_}{4} = \frac{__}{100}$ |
| **3** Multiply the numerator by the same number. | $\frac{21}{25} \times \frac{4}{4} = \frac{84}{100}$ |
| **4** Write the numerator of the new fraction with a percentage symbol. | 84% |
| **5** Write your answer in a sentence. | Melissa got 84% of the test questions correct. |

**(b) 1** Write the first amount as the numerator, and the second amount as the denominator of a fraction. Is the denominator a factor of 100? (No)

(b) $\frac{13}{24}$

**2** Convert the fraction to a percentage by multiplying by 100. Cancel common factors first to simplify the multiplication. (Here, a common factor of 4 has been cancelled.)

$\frac{13}{\cancel{24}_{6}} \times \frac{\cancel{100}^{25}}{1}\%$

$= \frac{325}{6}\%$

**3** Divide the numerator of the answer by the denominator. Write your answer as a decimal, rounded to one decimal place.

54.2%

**4** Write your answer in a sentence.

54.2% of the students in 7C have brown hair.

# 4.8 Using percentages

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3 Columns 1 & 2, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q18, Q20 | Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q20 | Q1 Columns 2 & 3, Q2 (d)–(i), Q3, Q4, Q5, Q8, Q9, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20 |

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**Equipment required:** A calculator may be used for Questions **2**, **7–12**, **15–17**

## Fluency

**1** Calculate each of the following percentages.

WE21

**(a)** 50% of $36
**(b)** 10% of 450 m
**(c)** 20% of 840 L
**(d)** 80% of 120 m
**(e)** 90% of 500 kg
**(f)** 2% of $8400
**(g)** 13% of 72 kg
**(h)** 27% of $354
**(i)** 21% of 876 m
**(j)** 28% of 231.25 m
**(k)** 34% of 210.4 L
**(l)** 4% of $32.65

**2** Write the following as percentages. Round your answers to one decimal place, if necessary.

WE22

**(a)** 17 out of 20 on a spelling quiz.
**(b)** 5 out of 22 students in the class have blond hair.
**(c)** There are 100 kittens at the animal shelter, 40 of which are female.
**(d)** 4 out of 5 students surveyed own an mp3 player.
**(e)** 50 pairs of shoes were sold, of which 38 pairs were black.
**(f)** 54 out of 70 cars counted on a freeway had only 1 person in them.
**(g)** 15 of the 27 cars in the staff carpark are white or silver.
**(h)** 3 out of every 4 people on the bus said they were travelling to work.
**(i)** 28 out of 34 people surveyed said they were non-smokers.

**3** Calculate the following using the 'basic percentages method' or any other mental strategy.

**(a)** 50% of $25 **(b)** 10% of 75 kg **(c)** 25% of 60 km

**(d)** 20% of 40 L **(e)** 1% of 360 m **(f)** 5% of $120

**(g)** 60% of 350 kg **(h)** 75% of 800 mL **(i)** 35% of $400

**4** Mehmet successfully shot 13 basketball goals out of 18 attempts. The calculation that converts this result to a percentage would be set out as follows.

**A** $\frac{13}{100} \times \frac{18}{1}$ **B** $\frac{13}{18} \times \frac{100}{1}$ **C** $\frac{18}{13} \times \frac{100}{1}$ **D** $\frac{100}{13} \times \frac{18}{1}$

**5** 20% of $420 is:

**A** $8.40 **B** $20 **C** $84 **D** $200

## Understanding

**6** 40% of the 25 students in class 7A have black hair. How many students is this?

**7** 16 out of the 28 biscuits in a packet have cream centres. What percentage of the packet are cream biscuits?

**8** A 2 litre bottle of orange drink contains 35% orange juice. Work out how many litres of orange juice is contained in the bottle. (It may help to work in mL.)

**9** A lion can sleep for up to 20 hours a day. What percentage of a whole day is this?



**10** There are 11 boys and 15 girls in class 7B. Write the number of boys and the number of girls as percentages of the total number of students in the class.

**11** Damian kicked 7 goals from 10 attempts. Franco kicked 11 goals from 16 attempts.

**(a)** Express the boys' results as percentages.

**(b)** Who is the more accurate goalkicker?

**12** Australia scored 381 runs in a cricket match against India. If Shane Watson scored 126 of those runs, what percentage of the team's total did he score?

**13** Shops often uses percentages to advertise savings during sales. The percentage is used to calculate the amount that is removed from the price of the goods. This amount is known as the 'discount'.

**(a)** Calculate the dollar value of the discount on the following.

**(i)** $40 shirt, 20% discount **(ii)** $125 jacket, 25% discount

**(iii)** $65 jeans, 50% discount **(iv)** $239 suitcase, 10% discount

**(v)** $799 camera, 40% discount **(vi)** $1299 sofa, 5% discount

**(b)** Now, find the sale price of the above items by subtracting the discount amount from the original price.

## Reasoning

**14** 8% of the apples in a crate were rotten. What percentage of the apples were not rotten?

**15** Chelsea and Sonia are netballers. In one game, Chelsea scored 17 goals from 25 attempts, and Sonia scored 15 goals from 20 attempts. In the next game, Chelsea scored 19 goals from 27 attempts, and Sonia scored 17 goals from 23 attempts. Use percentages to show which one of the girls improved the accuracy of her goal shooting. Round your figures to one decimal place, if necessary.




**16** The 180 students at St John's College all completed an online survey. The results showed that:

22% of the students had blue eyes

34% spoke two or more languages fluently

7% of them were left handed.

**(a)** Calculate the number of students in each category using the above three percentages.

**(b)** Considering that we are dealing with numbers of students, what is strange about your answer to **(a)**?

**(c)** What might be the actual numbers of students represented by each of the percentages?

**17** The energy in food is measured in kilojoules (kJ). The National Heart Foundation of Australia recommends that no more than 30% of our daily energy intake should come from eating fats.

**(a)** A healthy, active woman should eat up to 9000 kJ of food a day. Up to how many kJ in the form of fats can she eat in 1 day?

**(b)** 1 gram of fat gives the body 37 kJ of energy. How many grams of fat would provide a woman with the maximum daily amount of kilojoules allowed from fat? Round your answer to the nearest whole number.

## Open-ended

**18** Suggest some reasons as to why some stores use percentages to advertise savings, rather than dollar amounts.

**19** Mark saved $5.00 when a store had a sale. Determine at least three different percentage discounts and original prices for the item that Mark purchased.

**20** Fedora and Tanya were asked to show their working to the following question. 'Calculate 15% of 80'.

Here is Tanya's working:

$$\frac{15}{80} \times \frac{100}{1}$$

$$= \frac{150}{8}$$

$$= 18.75$$

Here is Fedora's working:

$$15\% \text{ of } 80$$
$$= 0.15 \times 80$$
$$= 12$$

Which of the girls' working is correct? Give a piece of advice to the other girl to help her avoid similar mistakes in the future.

# Outside the Square — Problem solving

## The great half-price sale

Tricia and Carly were out shopping. They headed towards their favourite clothes shop. On the shop window was a sign that read: 'Take 50% off all marked prices!' Stuck over the top of that sign was another sign, saying: 'Take another 50% off reduced prices!'

'Eh?' said Carly. '50% is one half. If I take half off the price, then take off another half, there won't be any price left! That means everything in the shop is free!'

'I don't think so', said Tricia. 'How can you run a business like that?'

'Well', said Carly, 'If I pick up a pair of $100 jeans and take two lots of 50% off the price, what do they cost now?'

'Errr...', said Tricia.

Can you work it out? How much would the jeans cost?




**Strategy options**

- Draw a diagram.
- Break problem into manageable parts.

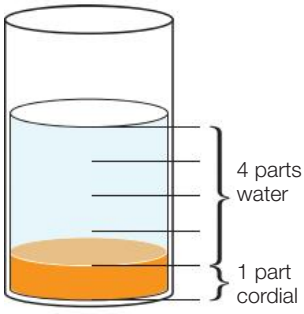
# Ratio

4.9

A **ratio** is a comparison of amounts of the same type of quantity.

When we mix a cordial drink, we add different amounts of cordial and water.

The amount of cordial compared to the amount of water may be written in many ways.




1 part cordial to 4 parts water

or

1 to 4

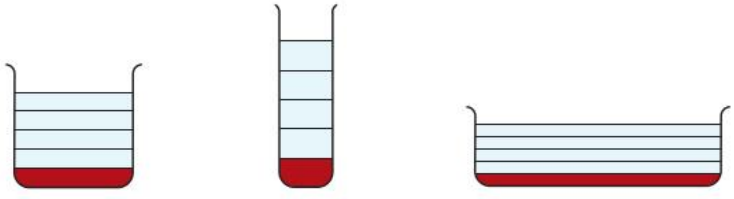
or using a ratio symbol,

1 : 4

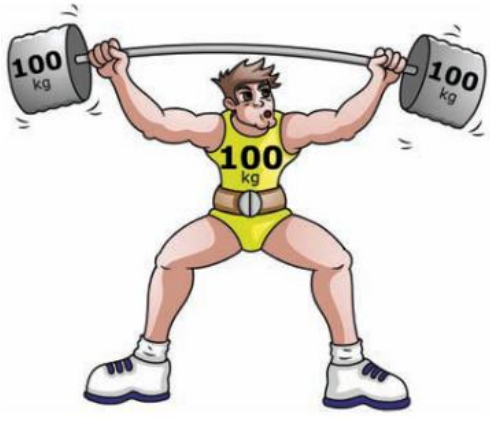
(The ':' can be read as 'to'.)



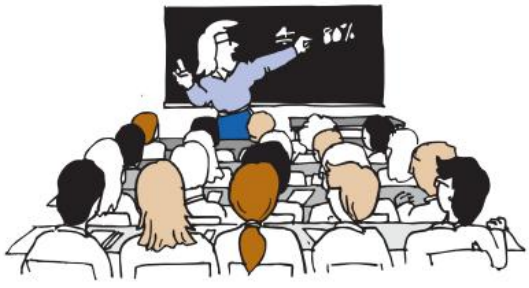

The actual volumes (the number of millilitres) of cordial and water are not given. It is the ratio, or proportion, of cordial compared to water that is important in getting the right taste. Specifying actual quantities would not be very useful, as cordial is made up in glasses and jugs of many different sizes. A sip of cordial from each of the vessels below would taste the same after the mixtures are stirred.



Some more examples of ratios are given below.




The ratio of the mass being lifted to the mass of the lifter is 2 : 1.



The teacher to student ratio in the class is 1 : 20.

A ratio is a comparison of two or more amounts, written using a ':' symbol.

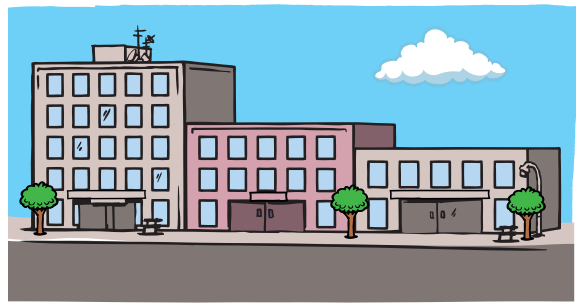
The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

## Worked Example 23 WE23

Write a ratio for each of the following.

**(a)** the number of windows in the middle building compared to the number of windows in the smallest building

**(b)** the number of floors in the buildings, from left to right



| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Write the quantities in the correct order. | (b) Windows in middle building: 12<br>Windows in smallest building: 7 |
| | **2** | Write as a ratio. | 12 : 7 |
| **(b)** | **1** | Write the quantities in the correct order. | (a) 5 floors to 3 floors to 2 floors |
| | **2** | Write as a ratio. | 5 : 3 : 2 |

We can see from the Worked Example above that a ratio can compare more than two amounts.

## Part : part and part : whole ratios

Suppose we had a class of 9 girls and 11 boys. We could write a ratio comparing the number of boys to girls as 11 : 9. This **part : part ratio** compares two parts of a whole. We can add the parts in a ratio to find the total number of parts. Here, 11 + 9 = 20 students in total. We can write another ratio that compares the number of boys to the total number of students in the class as 11 : 20. This **part : whole ratio** compares one part to the whole.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

## Ratios, fractions and percentages

The numbers in part : whole ratios can be written as fractions. For example, the above boys : total students ratio of 11 : 20 could be written as the fraction $\frac{11}{20}$ or as the percentage 55%. The fraction of girls would therefore be $\frac{9}{20}$ of the class, or 45%.

## Worked Example 24 WE24

A large enclosure at the zoo contains 5 giraffes and 12 zebras.

**(a)** Write a ratio comparing the number of zebras to the number of giraffes.

**(b)** Write a ratio comparing the number of giraffes to the total number of animals.

**(c)** Write the number of giraffes as a fraction of the total number of animals.

**(d)** Write the number of zebras as a percentage of the total number of animals.

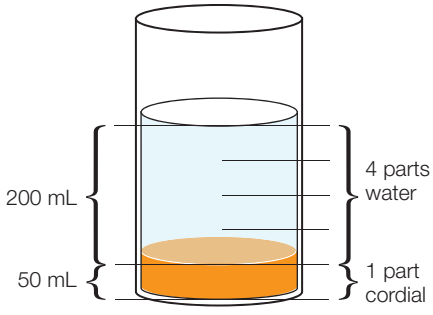
| Thinking | Working |
|---|---|
| **(a)** Write the ratio in words, in the correct order, then write the numbers underneath. | (a) zebras : giraffes<br>= 12 : 5 |
| **(b)** **1** Find the number of parts in the whole (the total number of animals). | (b) 5 + 12 = 17 |
| **2** Write the ratio in words, in the correct order, then write the numbers underneath. | giraffes : total animals<br>= 5 : 17 |
| **(c)** Write one number in the part : whole ratio as a fraction of the other. | (c) $\frac{5}{17}$ |
| **(d)** Write a new part : whole ratio as a fraction, then convert the fraction to a percentage, rounding to a sensible value. | (d) zebras : total animals<br>= 12 : 17<br>$\frac{12}{17} \times \frac{100}{1}$ %<br>= 71% (to nearest whole number) |

## Equivalent ratios

Imagine that you wanted to mix up a large quantity of a cordial drink using the cordial : water ratio of 1 : 4 that we considered at the beginning of this section. Mixing 1 mL of cordial and 4 mL would only give 5 mL of cordial; however, we can make 500 mL of cordial drink that would taste exactly the same if we mixed 100 mL of cordial with 400 mL of water.

1 : 4 and 100 : 400 are **equivalent ratios**. Equivalent ratios are created in the same way as equivalent fractions. We multiply each part in the ratio by the same number. Here, we have multiplied 1 : 4 by 100 to obtain the ratio 100 : 400. Multiplying by 50 would give a ratio of 50 : 200.

$$\begin{array}{ccccc} & 1 & : & 4 & \\ \times 50 & \downarrow & & \downarrow & \times 50 \\ = & 50 & : & 200 & \end{array}$$




## Simplifying ratios

Imagine that you have mixed a cordial drink using the above ratio of 50 : 200, which is equivalent to 1 : 4. Your friend Jack has made his drink by mixing 60 mL of cordial with 300 mL of water. He says his drink is stronger than yours, but it is not easy to tell by comparing the ratios of 50 : 200 and 60 : 300. It would be easier if we could compare the ratios in simplest form.

We can simplify a ratio in the same way we would simplify a fraction—by dividing each part in the ratio by a common factor. If we divide by the HCF, we will obtain the ratio in simplest form.

The HCF of the ratio 60 : 300 is 60.

$$\begin{array}{ccccc} & 60 & : & 300 & \\ \div 60 & \downarrow & & \downarrow & \div 60 \\ = & 1 & : & 5 & \end{array}$$

(we could also have obtained this ratio by doing a series of smaller divisions, such as ÷10, ÷6)

Now, instead of comparing 50 : 200 and 60 : 300, we can compare 1 : 4 with 1 : 5. The ratio 1 : 5 has one extra part of water to the same amount (1 part) of cordial, so Jack's drink is actually weaker than yours. (You may be able to tell this by looking at the colour, or by tasting it.)

Equivalent ratios are created by multiplying or dividing each part in the ratio by the same number.

Ratios can be simplified by dividing each part in the ratio by a common factor. Dividing by the highest common factor (HCF) will give the ratio in simplest form.

# 4.9 Ratio

## Navigator

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| | | |
|---|---|---|
| Q1, Q2, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q9, Q11, Q14, Q15, Q17, Q18, Q19 | Q1 Column 2, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19 | Q1 Column 2, Q2, Q3 Column 4, Q4 Column 4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19 |

## Fluency

WE23

**1** Write a ratio for each of the following.

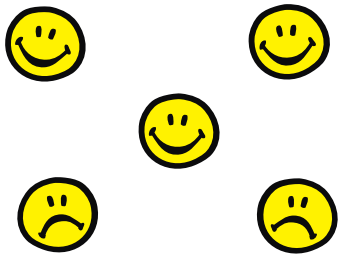
**(a)** circles to squares



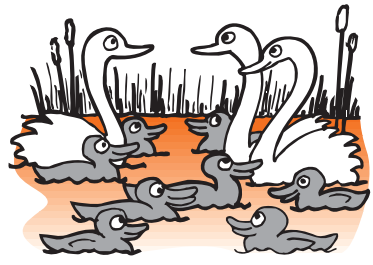
**(b)** blue circles to red circles



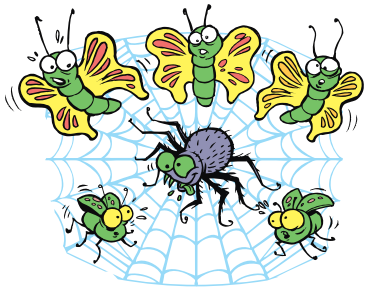
**(c)** happy faces to sad faces



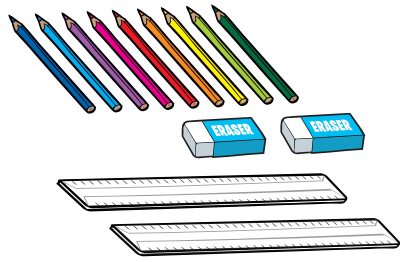
**(d)** ducks to swans



**(e)** spider to flies to butterflies



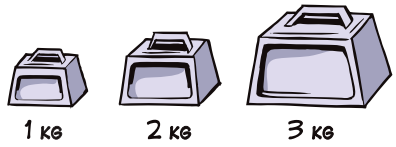
**(f)** rulers to pencils to erasers



**(g)** bees to flowers to leaves



**(h)** large mass to medium mass to small mass




**2** Sumi has 3 canaries and 2 mice as pets.

WE24

**(a)** Write a ratio comparing the number of mice to the number of canaries.

**(b)** Write a ratio comparing the number of canaries to the total number of pets.

**(c)** Write the number of canaries as a fraction of the total number of pets.

**(d)** Write the number of mice as a percentage of the total number of pets.

**3** Create pairs of equivalent ratios by multiplying or dividing each part in the ratio by the number given in brackets beside it.

| | | | |
|---|---|---|---|
| **(a)** 1 : 3 (×2) | **(b)** 1 : 8 (×2) | **(c)** 1 : 11 (×2) | **(d)** 1 : 33 (×2) |
| **(e)** 4 : 5 (×3) | **(f)** 3 : 7 (×4) | **(g)** 6 : 9 (÷3) | **(h)** 8 : 12 (÷4) |
| **(i)** 10 : 8 (÷2) | **(j)** 18 : 9 (÷9) | **(k)** 8 : 5 (×7) | **(l)** 12 : 11 (×5) |
| **(m)** 48 : 18 (÷6) | **(n)** 20 : 6 (×5) | **(o)** 60 : 54 (÷3) | **(p)** 15 : 13 (×5) |

**4** Write the following ratios in simplest form, by dividing by the highest common factor (HCF).

| | | | |
|---|---|---|---|
| **(a)** 2 : 10 | **(b)** 3 : 12 | **(c)** 4 : 18 | **(d)** 6 : 21 |
| **(e)** 21 : 14 | **(f)** 24 : 15 | **(g)** 32 : 20 | **(h)** 48 : 36 |
| **(i)** 54 : 64 | **(j)** 56 : 68 | **(k)** 72 : 81 | **(l)** 38 : 95 |
| **(m)** 75 : 50 | **(n)** 120 : 80 | **(o)** 105 : 60 | **(p)** 240 : 66 |

**5** There are 14 girls and 9 boys in a Year 7 maths class.

**(a)** The ratio of boys to girls is:

**A** 14 : 9 **B** 9 : 14 **C** 14 : 23 **D** 9 : 23

**(b)** The fraction of the whole class that are girls is:

**A** $\frac{9}{23}$ **B** $\frac{9}{14}$ **C** $\frac{14}{23}$ **D** $\frac{14}{9}$

## Understanding

**6** A packet of M&M's contains three colours: 12 red, 7 blue and 16 brown.

**(a)** Write the ratio of red to blue to brown M&M's.

**(b)** How many M&M's are in the packet?

**(c)** What fraction of the M&M's in the packet are red?

**(d)** What percentage of the M&M's are blue?



**7** During a cricket season, Erika's team won 15 matches, drew 2 and lost 7.

**(a)** Write the won : lost : drew ratio for Erika's team for that season.

**(b)** What fraction of their games did Erika's team lose?

**(c)** Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

**8** The students at Hillview High School are voting for a school captain. The results show that for every 4 students who voted for Guy, 5 students voted for Dominic and 3 for Melissa.

**(a)** Write the ratio of the number who voted for Dominic to the number who voted for Guy, to the number who voted for Melissa.

**(b)** What fraction of the total vote did Dominic receive?

**(c)** Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

**9** There must be 1 instructor for every 6 students on a kayaking course.

**(a)** Write the comparison of students to instructors as a ratio.

**(b)** If 24 students want to do the course, use an equivalent ratio to determine how many instructors will be required.

**(c)** If 5 instructors are available, what is the maximum number of students that can do the course?

**10** Tom's recipe for making concrete is to add gravel, sand and cement in the ratio 3 : 2 : 1 before adding water. Tom has 4 buckets of cement. Use an equivalent ratio to show how many buckets of sand and gravel he should add to get the correct concrete mixture.

**11** 'White gold' is commonly made by mixing gold and nickel in the ratio 9 : 1.

**(a)** Write the amount of nickel as a percentage of the total mixture.

**(b)** A jeweller wants to make 20 g of white gold. How much nickel will he require?



**12** A survey found that 75% of Year 7 students were studying music.

**(a)** What percentage were not studying music?

**(b)** Use the two percentages to write a ratio that compares the group studying music to the group not studying music, in simplest form.

## Reasoning

**13**




Which point divides the above line so that the ratio shorter part : longer part is:

**(a)** 1 : 7 **(b)** 5 : 3 **(c)** 1 : 3 **(d)** 1 : 1

**14** There must be at least 1 teacher for every 20 students on a school excursion.

**(a)** If 68 students are going on an excursion, how many teachers are required? Give reasons for your answer.

**(b)** Use your answer to **(a)** to write the actual ratio of teachers : students, in simplest form.

**15** Chloe made a cordial drink by mixing 30 mL of concentrate with 150 mL of water. Her friend Hannah made a cordial drink by mixing 80 mL of concentrate with 200 mL of water. Hannah said her cordial drink was 'stronger' than Chloe's. Chloe could not tell by looking.

**(a)** Write the ratio of concentrate : water for each girl's drink, in simplest form.

**(b)** Use the ratios to state whether Hannah is correct, giving a reason for your answer.

**16** Mia has three cats and five dogs as pets.

**(a)** What fraction of her pets are dogs?

**(b)** What fraction of her pets are cats?

**(c)** If Mia adopts another cat, what fraction of her pets are cats now?

**(d)** After adopting the new cat, Mia moves into a share house with Shona who also has some cats and dogs. If the final ratio of cats to dogs is now 1 : 1, what is the smallest number of cats and dogs that Shona brings to the house?

## Open-ended

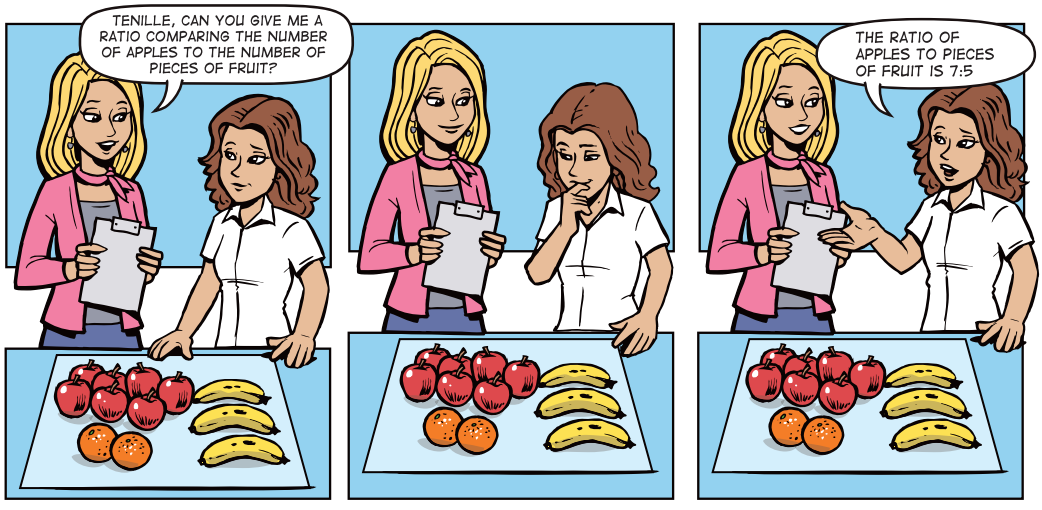
**17** **(a)** What is the ratio of girls to boys in your class?

**(b)** Would the ratio be the same in all the Year 7 classes in your school? Explain your answer by giving examples.

**(c)** Would the ratio be the same for all school classes in Australia? Give some reasons for your answer.

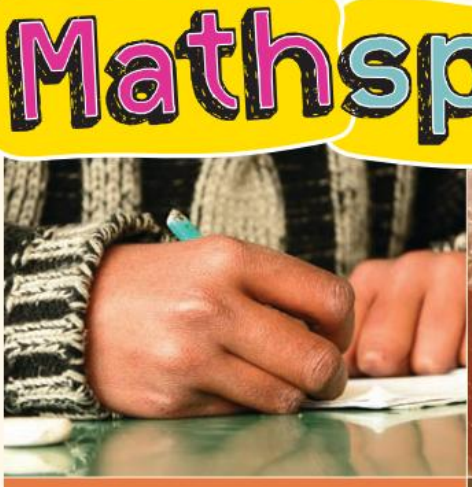
**18** Write three ratios that are equivalent to 4 : 7.

**19**




What mistake has Tenille made? What is the correct answer?

# Mathspace







# Our global village





## Facts about our global village

There are over 6.9 billion people on Earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

### Race

- 0.61 Asian
- $\frac{12}{100}$ European
- 0.08 North American
- 0.05 South American & Caribbean
- 0.13 African
- $\frac{1}{100}$ Oceania (this includes Australia)

### Conditions

- 0.43 live without basic sanitation
- $\frac{1}{3}$ don't have access to clean, safe drinking water
- 0.13 are hungry and/or malnourished

### Wealth

- 0.06 of the population own 59% of the entire wealth
- $\frac{1}{5}$ of the population have 75% of the income
- 0.53 of the population live on $2.50 or less per day

### Education and technology

- 0.14 can't read
- 0.07 have a secondary education
- 0.12 have a computer
- 0.03 have the internet

## How rich are you?

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage*.

$1.2 - 2.7 + 3.5 - 1.25 =$

## How many people?

Currently, there are approximately 6 900 000 000 people living on Earth.

3 Using the facts you now know about our world, calculate how many people:

(a) don't have access to clean water

(b) struggle to live on $2.50 or less per day

(c) have a computer

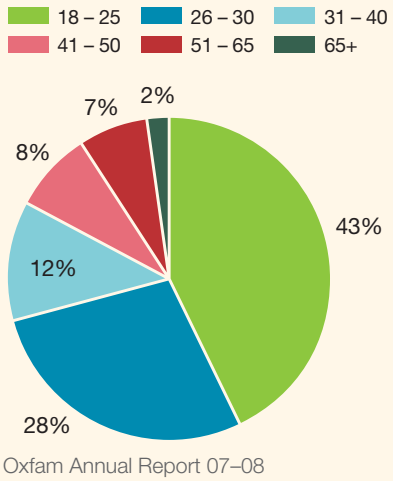
(d) don't have a secondary school education.



## I want to help

There are many charities out there that are trying to create more equality in the world—Oxfam is one of these. Charities often rely on the generosity of ordinary people. Here is a pie chart of the age groups of Oxfam volunteers.

**Volunteers by age group**




Oxfam Annual Report 07–08

4 Using the pie chart above, find out the following.

(a) What percentage of volunteers are between 18 and 30?

(b) Why do you think most volunteers are in this age range?

(c) One of the challenges charities face is the need to pay for fundraising and administration costs. If Oxfam spends 24.4% of its income on fundraising and 8.9% on administration costs, what percentage of its income is available to go directly to community-aid projects?

## In the future

5 Imagine it's the year 2055. How do you hope the 'global village' percentages have changed? Write down your predictions and ideas.

# 4.10 Rates

In the previous section, we used ratios to compare two or more amounts of the same type of quantity (e.g. millilitres of water and millilitres of cordial).

A **rate** is a way of comparing two amounts of different types of quantities. They usually involve the word *per* which means 'for each' and have the symbol / or the letter 'p'. A rate is written as the number of the first quantity for every 1 of the second quantity. Some examples of rates are:

If the price of petrol is $1.29/L, you will pay $1.29 for 1 litre of petrol.



Cricketers scoring at 4 runs per over will score an average of 4 runs in 1 over.



## The unitary method

If we know how much 1 litre of petrol costs, we can calculate the cost of any other volume of petrol by multiplying by the cost of one litre.

If we know that 50 L of petrol costs $75, we can work out the cost of 4 L, 15 L, 37 L or any other volume of petrol by first calculating how much 1 L costs.

Solving problems by first calculating a number (such as cost) per unit is called the **unitary method**.

## Worked Example 25 WE25

**(a)** If 8 kg of oranges costs $24, find the cost of 6 kg of oranges.

**(b)** A machine in a soft drink factory produces 210 bottles every hour. How many bottles would be produced in 40 minutes?

| Thinking | | Working |
|---|---|---|
| **(a)** | **1** Calculate the cost of one 'unit' of the amount (1 kg of oranges). | (a) $24 for 8 kg<br>= $3 for 1 kg (÷ 8) |
| | **2** Multiply the cost/unit by the number of units. | = $18 for 6 kg (× 6) |

| | | |
|---|---|---|
| **(b) 1** | Calculate the number of items produced in one minute. | (b) 210 ÷ 60 = 3.5 bottles/min |
| **2** | Multiply the number produced in one minute by the number of minutes. Round your answer to a sensible value. | 3.5 bottles/min × 40 min<br>= 140 bottles in 40 min |

## Comparing prices




Many supermarket products come in different sizes, which makes it difficult to compare the prices. For example: Which is better value for money, a 500 g box of breakfast cereal for $5.99, or the larger 820 g box for $7.21?

We can use the unitary method to compare the prices. We could divide the prices by the mass of the boxes to find the price per 1 g of the cereal, but, instead, we will compare the price per 100 g, as this is how supermarkets are required to display many of their prices. The price per 100 g (or 100 mL, or 1 kg or 1 L) is called the **unit price**. Unit prices enable shoppers to easily compare sizes and brands to find products that are the best value for money.

To convert the prices to prices per 100 g, we could divide the price by the mass to find the price for 1 g, then multiply by 100. For example: 500 g box: $5.99 ÷ 500 × 100 = $1.20 per 100 g
820 g box: $7.21 ÷ 820 × 100 = $0.88 per 100 g

However, a quicker method is to mentally divide the mass by 100 to find the number of 100 g 'lots'. We can then divide the price by this number; e.g.

500 ÷ 100 = 5
$5.99 ÷ 5 = $1.20 per 100 g

820 ÷ 100 = 8.2
$7.21 ÷ 8.2 = $0.88 per 100 g

## Worked Example 26 WE26

Calculate the unit price (price per 100 g) of the following pair of products, and so determine which one is better value for money: A 500 g packet of pasta for $2.42 or a 750 g packet for $3.44.

| Thinking | Working |
|---|---|
| **1** Divide the mass of the first product by 100. | 500 g ÷ 100 g = 5 |
| **2** Divide the price of the first product by your answer to step **1**. | $2.42 ÷ 5 = $0.48/100 g |
| **3** Repeat steps **1** and **2** for the second product. | 750 g ÷ 100 g = 7.5<br>$3.44 ÷ 7.5 = $0.46/100 g |
| **4** State your answer. | The 750 g packet is slightly better value for money (2c less per 100 g). |

# 4.10 Rates

## Navigator

Answers page 645

| | | |
|---|---|---|
| Q1, Q2, Q3, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q19 | Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q20 | Q1 (d)–(f), Q2 (d)–(f), Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q14, Q15, Q17, Q18, Q19, Q20 |

## Fluency

WE25

**1** **(a)** If 15 kg of potatoes costs $12, find the cost of 7 kg of potatoes.

**(b)** A machine in a food canning factory produces 250 cans every hour. How many cans would be produced in 25 minutes?

**(c)** If 14 pens cost $5.60, find the cost of 3 pens.

**(d)** Farmer Harry buys 8 cows for $6800. How much would 10 cows have cost him?

**(e)** If 21 bags of rice weigh 35 kg, how much do 15 bags of rice weigh?

**(f)** A printing machine prints 70 pages in 8 minutes. How many pages could it print in one hour?

WE26

**2** Calculate the unit price (the price per 100 g) of the following pairs of products, and so determine which one is better value for money.

**(a)** a 600 g jar of jam for $4.09 or a 250 g jar for $2.68

**(b)** a 500 g packet of pasta for $2.81 or a 750 g packet for $3.45

**(c)** a 375 g jar of tomato paste for $1.28 or a 500 g jar for $3.25

**(d)** a 200 g jar of peanut butter for $3.21 or a 375 g jar for $4.93

**(e)** a 500 mL bottle of soy sauce for $2.71 or a 920 mL bottle for $4.29

**(f)** a 450 g bag of cat food for $3.78 or a 1 kg bag for $8.79

**3** Nazeh has saved $2000 in 4 months. If he continues to save at this rate, how much will he have by the end of the year?

**A** $6000 **B** $8000 **C** $12 000 **D** $24 000

**4** An entertainer receives $450 000 for a season of five performances, each of which lasts $1\frac{1}{2}$ hours. The rate of pay per minute is closest to:

**A** $60 000 **B** $5000

**C** $1500 **D** $1000



## Understanding

**5** A caterer knows that 4 loaves of bread can make enough sandwiches for 25 people. How many loaves will he need to make enough sandwiches for 120 people?

**6** **(a)** Which is better value for money, a 280 g jar of jam for $5.15, or a 400 g jar for $6.42?

**(b)** The 280 g jar goes on special for $4.49. Does this change your answer to **(a)**?

**7** A mobile phone carrier charges 75c for 30 seconds. How much will a 4-minute phone call cost?

**8** Rajid's cricket team required 85 runs from 12 overs to win the game. At what rate, in runs per over, must they score to win? (Answer to one decimal place.)

**9** **(a)** Calculate the price per item in each pack of the following products, and so determine which pack is better value for money.

**(i)** a packet of 6 muesli bars for $4.83, or a packet of 10 for $7.51

**(ii)** a 12-pack of cans of soft drink for $14.20, an 18-pack for $20.21, or a 30-pack for $27.51

**(iii)** a box of 50 tea bags for $3.20, or a box of 200 for $7.99

**(b)** Describe another way you could compare the prices of the tea bags in part **(a) (iii)**.




**10** Lucy has bought a 1 year subscription to *Australian Explorer* magazine. She paid $102 for 12 monthly magazines. The regular price at the newsagent for one copy of the magazine is $9.95. How much will Lucy save:

**(a)** per magazine **(b)** over the 1 year period?

**11** Andrea has a leaky tap in her bathroom. She places a container underneath it and collects 1.2 litres in 8 hours. If Andrea does not fix the tap, use a mental or written method to calculate how much water would be lost in:

**(a)** 1 day **(b)** 1 week?

**12** A 200 g serve of yoghurt contains 6.5 g of fat. Use a mental or written method to calculate how much fat there is in:

**(a)** a 100 g serve **(b)** a 400 g serve **(c)** a 500 g serve.

**13** A 5-minute phone call costs $3.00. If the same call rate is charged, use a mental or written method to calculate the cost of:

**(a)** a 10-minute call **(b)** a 1-minute call **(c)** a 13-minute call.

## Reasoning

**14** Charlie's bakery sells hot cross buns in packets of 6. One packet costs $7.20. A customer wants to buy exactly 10 buns. Charlie is happy to take 4 buns from a second packet, but how much should he charge the customer?

**15** Imagine that you are standing in a supermarket, comparing the following pairs of products. Do some mental calculations to determine which product is the better value for money. Rounding prices slightly may help to simplify some calculations. Show how you arrived at your answer.

**(a)** A 1 kg box of washing powder for $5.49, or a 4 kg box for $15.97.

**(b)** Four 250 mL cartons of orange juice for $1.25 each, or a 1 litre carton for $3.99.

**(c)** Buying 500 g of ham from the deli counter where the price is $13 per kg, or buying it in a packet for $5.83.

**(d)** A 400 g jar of honey for $7.99, or a 300 g jar for $6.54.

**(e)** Give some reasons why people might prefer to buy a product even though it is not the best value for money.



**16** A mobile phone carrier charges a rate of 85 cents per minute for pre-paid calls. Calls made on a phone plan are charged at 75 cents per minute, plus a 35 cent connection fee. Which of the two types of charge is cheaper for a 5-minute phone call?

**17** During the month of January, the water level in a Northern Territory dam increased by 78 000 kilolitres (1 kilolitre = 1000 litres).

**(a)** How many kilolitres per day was this? (Round your answer to two decimal places.)

**(b)** If the dam continued to fill at this rate, use your answer to **(a)** to calculate how much water will have flowed into the dam by the end of the year. (Assume there are 365 days in a year.)

**(c)** Would it be sensible to use your answer to **(b)** as a prediction for the amount of water that will flow into the dam over the year? Give some reasons for your answer.

**18** 'Trim and Tasty' cheese comes in three different-sized blocks:

250 g for $4.50
500 g for $7.42
750 g for $9.76

**(a)** How many times bigger than the 250 g block is the 500 g block?

**(b)** If the prices of the 500 g block and the 750 g block were in the same proportion to the price of the 250 g block as their sizes, what would you expect their prices to be?

**(c)** Often (but not always) larger sizes of a product have cheaper unit prices. Suggest a reason why this is the case.

## Open-ended

**19** List four supermarket products whose unit price would be usefully calculated per 100 mL, or per L.

**20** Kumar needs 4.5 kg of rice for the students who are attending his cooking classes. It is available in the following sizes and prices.

500 g for $1.24  1 kg for $2.35  1.5 kg for $3.59  2 kg for $4.52

**(a)** List at least three different combinations of sizes that would give Kumar 4.5 kg of rice.

**(b)** Which combination is the cheapest?

## Outside the Square Puzzle

### Happy numbers

1 Write down your favourite single-digit number.

2 Multiply that number by 9.

3 Multiply this new number by 12 345 679. (Unless your calculator has more than 8 digits in its display you will need to do this by hand to see the full effect.)

4 If you did choose your favourite number, then the answer should make you very happy. Does it work for other single-digit numbers? Can you explain why it works?

**Strategy options**

- Guess and check.
- Work backwards.
- Test all possible combinations.

# Challenge 4

**1** What percentage of whole numbers from 6 to 25 inclusive (i.e. including 6 and 25) are exact multiples of 6?

**A** 5 **B** 20 **C** 25 **D** 30

**2** The numbers 5, 8, 7 and 3 are written on cards. Where should they be placed so that the following product has the greatest possible value?

$\square \times 0.\square\square\square$

**3** Evaluate $0.1^2 - 0.1^3$.

**4** Zinadene has $\frac{3}{4}$ of a dollar and Katrine has $\frac{3}{10}$ of a dollar. Together they have:

**A** \$0.95 **B** \$1.00 **C** \$1.05 **D** \$1.10

**5** $200 \div 2.5 = 80$. Therefore, $20 \div 0.25 =$

**A** 0.80 **B** 8.0 **C** 80 **D** 800

**6** When doing a series of additions using his calculator, Trent noticed that he added 49 095 instead of 49.95. In order to correct his error with a single entry, he should now:

**A** add 49.95 **B** subtract 49 045.05

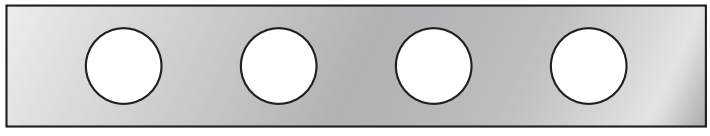
**C** add 49 045.05 **D** subtract 49 095

**7** A decimal with 2 decimal places is multiplied by a decimal with 3 decimal places. Explain with number examples how the solution could have:

**(a)** 5 decimal places **(b)** 4 decimal places **(c)** 3 decimal places.

**8** In Felicity's first six netball games she averaged 7.5 goals per game. If the least number of goals she scored was 4, what is the lowest possible value for the highest number of goals she scored?

**9** Four holes are to be drilled along the centre line of a strip of metal so that their centres are 2.25 cm apart. The centres of the two end holes are to be 3.35 cm from their corresponding ends. What is the length of the strip of metal?



**10** What is the quotient when 0.01 is divided by 0.002?

**11** Mikaela had her salary reduced by 10%. She was later promoted and her salary was increased by 10%. If her original salary was \$30 000, her present salary is:

**A** \$24 300 **B** \$27 000 **C** \$29 700 **D** \$30 000

**12** $\frac{2009}{20.09}$ equals

**A** 100 **B** 10 **C** 0.1 **D** 0.01

**13** A town has 5000 residents, of whom 60% voted in a local council election. The result was that, of those who voted, 37% voted for *P*, 32% for *Q* and 31% for *R*. Under the voting system, *P* was elected. The number of residents who voted for *P* was:

**A** 1850 **B** 1110 **C** 960 **D** 930

# Chapter review 4

## D.I.Y. Summary

### Key Words

| | | |
|---|---|---|
| decimal places | part : whole ratio | recurring decimal |
| decimal point | per cent | round |
| digit | place value | terminating decimal |
| equivalent ratios | rate | unit price |
| part : part ratio | ratio | unitary method |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 4.87 has three _____s and two _____ .

2 When adding or subtracting decimals, it is important to line up the _____s.

3 _____ are created by multiplying each part in the ratio by the same number.

4 _____s compare two or more parts of a whole.

5 A _____ has a finite number of decimal places, a _____ has an infinite number of decimals, in a repeating pattern.

6 _____ literally means 'for every hundred'.

7 Calculating a _____ helps to compare the prices of different sizes and brands of products.

8 Explain the difference between 'round down' and 'round up'.

## Fluency

1 Write in expanded fractional form. Ex 4.1

**(a)** 0.968 **(b)** 5.0702 **(c)** 6.005

2 Write as decimals: Ex 4.1

**(a)** six tenths, five hundredths, two thousandths and four ten-thousandths

**(b)** eight hundredths, nine hundred-thousandths and three millionths

3 Write as decimals: Ex 4.1

**(a)** $4 + \frac{3}{10} + \frac{8}{100} + \frac{6}{10\,000}$ **(b)** $50 + 7 + \frac{3}{100} + \frac{2}{1000} + \frac{9}{100\,000}$

4 Write each of the following in expanded word form. Ex 4.1

**(a)** 1.8531 **(b)** 0.070 06 **(c)** 61.0009

5 Write the value of the 9 in each decimal as **(i)** a fraction **(ii)** in words. Ex 4.1

**(a)** 5.091 **(b)** 0.0659 **(c)** 25.291

6 Write < or > in each of the following pairs of numbers to make a true statement. Ex 4.1

**(a)** 3.0427 _____ 3.0274 **(b)** 0.009 95 _____ 0.01

7 Write in order from smallest to largest. Ex 4.1

**(a)** 0.6055, 0.5506, 0.607 **(b)** 0.071, 0.701, 0.71

**8** Copy this number line and show the positions of the following numbers: 0.25, 1.45, 0.09, 2.05. Ex 4.1




**9** Round each of the following to the number of decimal places shown in the brackets. Ex 4.2

**(a)** 4.398 (1) **(b)** 34.6503 (2) **(c)** 23.098 (1) **(d)** 102.4728 (2)

**10** Round the following to the nearest 5 cents. Ex 4.2

**(a)** $41.21 **(b)** $10.03 **(c)** $79.97 **(d)** $98.98

**11** Write each of the following as a single fraction or mixed number in simplest form. Ex 4.3

**(a)** 3.9 **(b)** 0.62 **(c)** 2.45 **(d)** 0.0018

**12** Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. Ex 4.3

**(a)** $\frac{4}{5}$ **(b)** $\frac{19}{20}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{10}{12}$

**13** Calculate: Ex 4.4

**(a)** 23.6 + 4.79 **(b)** 3.768 + 10.9205 + 0.77 **(c)** 6 + 0.408 + 35.025

**(d)** 10.367 – 2.65 **(e)** 2.082 – 0.394 **(f)** 7 – 2.2198

**14** Find the following products. Ex 4.5

**(a)** $3.376 \times 4$ **(b)** $2.99 \times 35$ **(c)** $6.09 \times 33$

**(d)** $23.6 \times 2000$ **(e)** $0.548 \times 300$ **(f)** $0.069 \times 50$

**(g)** $0.6 \times 0.9$ **(h)** $0.009 \times 0.04$ **(i)** $5.2 \times 3.8$

**15** Calculate the following. Round answers to three decimal places where necessary. Ex 4.6

**(a)** 10.32 ÷ 4 **(b)** 7.028 ÷ 7 **(c)** 3.75 ÷ 8

**(d)** 480.6 ÷ 2000 **(e)** 8.23 ÷ 500 **(f)** 54.63 ÷ 300

**(g)** 0.48 ÷ 0.04 **(h)** 1.2 ÷ 0.5 **(i)** 11.9 ÷ 0.002

**16** Write the following fractions as percentages. Ex 4.7

**(a)** $\frac{7}{10}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{2}{3}$

**17** Convert the following decimals to percentages. Ex 4.7

**(a)** 0.38 **(b)** 0.96 **(c)** 3.55 **(d)** 0.09 **(e)** 3.234 **(f)** 0.5432

**18** Calculate the following percentages. Ex 4.8

**(a)** 40% of $60 **(b)** 15% of 85 L **(c)** 83% of 200 m **(d)** 12.5% of $600

**19** Write the following results as percentages, rounding answers to one decimal place if necessary. Ex 4.8

**(a)** 13 out of 20 **(b)** 35 out of 70

**(c)** 14 out of 40 **(d)** 28 out of 30

**20** There are 56 girls and 64 boys in Year 7. Ex 4.9

**(a)** Write the ratio boys : girls in simplest form.

**(b)** Write the number of girls as a fraction of the total number of students, in simplest form.

**(c)** Write the number of boys as a percentage of the total number of students, to the nearest whole number.

**21** **(a)** A 10 kg box of tomatoes costs $23.50. Use the unitary method to calculate the cost of a 3 kg box.

Ex 4.10

**(b)** If a 250 g block of cheese costs $7.85, find the unit price (the price per 100 g).

## Understanding

**22** Write the populations of the following countries as whole numbers.

**(a)** China: 1.326 billion

**(b)** India: 1.14 billion

**(c)** Indonesia: 228.2 million

**(d)** New Zealand: 4.27 million

**23** An employer wants to buy each of her staff a uniform. If each uniform costs $75.59, how much will the uniforms cost:

**(a)** for 40 staff

**(b)** for 100 staff?

**24** Kayla bought the following items while out shopping: A $30 T-shirt discounted by 10%, and a $60 pair of jeans discounted by 40%.

**(a)** Calculate the dollar value of each of the discounts.

**(b)** Subtract the discount amount to find how much Kayla paid for each item.

**25** **(a)** Which is better value for money: a 250 g jar of honey for $2.98, or a 400 g jar for $4.23?

**(b)** The 250 g jar goes on special for $1.98. Does this change your answer to **(a)**?

**(c)** Give one or two reasons why people might prefer to buy a certain size or product, even though it may not have the cheapest unit price.

**26** A salad dressing is made by mixing oil and vinegar in the ratio 3 : 1.

**(a)** How much oil should be mixed with 25 mL of vinegar?

**(b)** Helen mixes 60 mL of oil with 40 mL of vinegar. Is this an equivalent ratio to the one given above? Demonstrate your answer by writing it in simplest form.

**27** In a springboard diving competition, Claudia needed a score of 52 or more to take first place. For her final dive she chose a dive with a degree of difficulty of 2.7. The scores she received from the judges were 7.0, 8.0, 7.5, 8.5, 7.5, 8.0 and 8.0.

**(a)** Remove the highest and lowest scores and calculate the average of the remaining five scores (add them up and then divide by 5).

**(b)** Calculate the 'three-judge total' by multiplying this average by 3.

**(c)** Calculate her final score by multiplying the 'three-judge total' by the degree of difficulty. Did she do well enough to win?

## Reasoning

**28** Tim buys 6 individual cans of drink for $0.87 per can.

**(a)** If he pays in cash, find the total cost of the cans.

**(b)** A pack of 6 cans costs $4.80. How much could Tim have saved by buying the 6 pack?

**(c)** What is the individual cost of the cans in the 6 pack?

**(d)** Tim needs to buy 40 cans for a party. What is the cheapest way to buy them?

**29** There must be at least one supervisor for every 12 children at a childcare facility. If there are 54 children booked in for a day, how many supervisors will be needed? Give a reason for your answer.

**30** Sally is shopping for shoes. She has found the pair she wants in two different stores. Store A has them priced at $65, with a 'take 25% off' tag on them. Store B has them priced at $75, but the store is having a '30% off everything' sale. Where should Sally buy her shoes? Explain your choice by showing the relevant calculations.

# NAPLAN practice 4

## Numeracy: Non-calculator

**1** Which number is greater than 0.06?

**A** 0.0069 **B** 0.007 **C** 0.05 **D** 0.1

**2** Geraldine buys 400 snack bars for the school camp. If each bar costs $0.64, how much would Geraldine pay for the 400 bars?

**A** $25.60 **B** $256 **C** $464 **D** $25 600

**3** What is the answer to 5.4 ÷ 0.9?

**A** 0.06 **B** 0.6 **C** 6.6 **D** 6

**4** 20% of a class have the flu. The class has 25 students. How many have the flu?

**A** 4 **B** 5 **C** 10 **D** 20

## Numeracy: Calculator allowed

**5** A pack of 10 lollipops costs $4.60.
A pack of 6 costs $3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

**6** The price of grapes at the supermarket is $14 per kg. If Jack paid $3.50 for his grapes, how many grams did he buy?

**A** 0.25 g **B** 25 g **C** 100 g **D** 250 g

**7** Lucy is adding a row of square tiles across the top of a mirror. Each tile is 4.5 cm wide. The mirror is 56 cm wide. Lucy wants to tile right across the top, with no gaps between the tiles. What calculation should she do to work out how many tiles she needs?



**A** 4.5 ÷ 56 **B** 5.6 ÷ 4.5 **C** 56 ÷ 4.5 **D** 4.5 × 56

**8** The material for the curtains in the classroom costs $21.30 per metre. 8.4 m is needed. How much will the material cost?

# Mixed review B

## Fluency

**1** Write the following fractions in simplest form. Ex. 3.2

**(a)** $\frac{12}{40}$ **(b)** $\frac{15}{50}$ **(c)** $\frac{48}{64}$ **(d)** $\frac{20}{65}$

**2** Write a positive or a negative integer to describe the following. Ex. 2.4

**(a)** a withdrawal from a bank account of $74 **(b)** a weight gain of 3 kg

**(c)** 215 m below sea level **(d)** an increase in length of 8 m

**3** For each of the following numbers: Ex. 2.3

**(i)** use a factor tree to find the prime factors

**(ii)** write the number as a product of its prime factors, using index notation.

**(a)** 76 **(b)** 108 **(c)** 250

**4** Calculate approximate answers to the following by rounding to a convenient multiple of 5 or 10. Ex. 1.4

**(a)** $238 \times 21$ **(b)** $107 \times 31$ **(c)** $478 \div 39$ **(d)** $604 \div 153$

**5** For each pair of fractions, determine which one is larger, then write a > or < sign between them. Ex. 3.3

**(a)** $\frac{2}{3}$ _____ $\frac{2}{5}$ **(b)** $\frac{5}{12}$ _____ $\frac{5}{9}$ **(c)** $\frac{11}{15}$ _____ $\frac{3}{4}$

**6** Find: Ex. 4.5, 4.6

**(a)** $0.45 \times 0.6$ **(b)** $45.9 \div 3$ **(c)** $43.5 \div 0.1$

**7** Calculate the following by first writing a single sign between the integers. Ex. 2.7

**(a)** $-1 + (-5)$ **(b)** $3 - (-11)$ **(c)** $-6 - (+4)$

**(d)** $9 + (-17)$ **(e)** $-8 - (-14)$ **(f)** $21 - (+30)$

**8** Write each of the following as a fraction in simplest form. Ex. 4.3

**(a)** 0.32 **(b)** 1.24 **(c)** 3.237

**9** Write the following in index form. Ex. 1.2

**(a)** $7 \times 7 \times 7 \times 7$ **(b)** 4 squared

**(c)** $2 \times 2 \times 3 \times 3 \times 3$ **(d)** 11 cubed

**10** Find the lowest common multiples of the following numbers. Ex. 2.1

**(a)** 5 and 6 **(b)** 3 and 6 **(c)** 2, 3 and 5

**11** Write < or > between the following pairs of numbers. Ex. 2.4

**(a)** $-5$ _____ 3 **(b)** $-4$ _____ $-11$ **(c)** $-7$ _____ 0

**12** Find the HCF of the following numbers. Ex. 2.1

**(a)** 12 and 30 **(b)** 16 and 40 **(c)** 24 and 60

**13** Calculate the following. Write your answers in simplest form. Ex. 3.4

**(a)** $\frac{1}{5} + \frac{3}{4}$ **(b)** $1\frac{2}{3} + 2\frac{3}{8}$ **(c)** $\frac{5}{6} - \frac{3}{8}$ **(d)** $3\frac{1}{4} - 2\frac{1}{2}$

## Understanding

**14** Use an appropriate mental strategy to calculate the following.

**(a)** $29 \times 7$ **(b)** $44 \times 11$ **(c)** $13 \times 15$

**(d)** $260 \div 4$ **(e)** $2 \times 4 \times 7 \times 5$ **(f)** $440 \div 8$

**15** Erin calculates that she needs 9 lots of 0.35 m of ribbon for her textiles project. She wants to round up to the nearest metre so that she makes sure she has enough.

**(a)** Calculate how many metres she should buy.

**(b)** If the ribbon costs \$0.75 per m, how much will she pay?

**16** Write **(i)** a fraction in simplest form and **(ii)** a percentage to represent the following situations.

**(a)** 12 black jelly beans in a bag of 40

**(b)** 120 Year 7 students in a school of 520 students

**17** A 250 g serve of tinned soup contains 7 g of fat. Calculate the amount of fat contained in:

**(a)** a 500 g serve **(b)** a 50 g serve **(c)** a 150 g serve.

**18** 7 pizzas were each sliced into 6 equal pieces.

**(a)** If $4\frac{5}{6}$ pizzas were eaten, how many slices is that?

**(b)** How many slices are left over?

**19** Camilla is on an athlete's diet where $\frac{2}{3}$ of her daily energy must come from carbohydrates, $\frac{1}{4}$ from protein, and the rest from fats. Her daily energy allowance is 4800 kJ.

**(a)** Calculate how many kJ of carbohydrates she must have every day.

**(b)** Calculate how many kJ of protein she must have every day.

**(c)** What fraction of her daily kJ intake is fat?

**20** Explain why $3^2 = 9$, but $3 \times 2 = 6$.

## Reasoning

**21** A lighting display in the city has a set of blue lights that flash every 6 minutes, and a set of red lights that flash every 8 minutes. How many times an hour will the lights flash together?

**22** There is a large block of chocolate sitting in the Williams family pantry. Chloe breaks off and eats $\frac{1}{6}$, and returns the rest. Later, her brother Eamon breaks off and eats $\frac{2}{3}$ of the rest.

**(a)** What fraction of the original block is left?

**(b)** If the original block contained 54 squares of chocolate, who ate more, Chloe or Eamon?

**23** Shania surveyed 130 Year 7 students to find out which charity they wanted to fundraise for. Shania reported back to the Student Council that 65% of the Year 7s wanted to support the Red Cross.

**(a)** What is 65% of 130?

**(b)** Considering that we are working with numbers of students, what is 'strange' about your answer to **(a)**?

**(c)** What might have been the actual number of students who said they wanted to support the Red Cross? Explain how Shania obtained her figure of 65%.

**24** How many whole numbers lie on the number line between $\frac{4}{3}$ and $\frac{38}{4}$?

**25** Jeffery has 46 footy cards in his collection. His little brother Joseph has half as many in his collection, whereas his friend Shahin's collection is twice the size of Jeffery's. How many more cards has Shahin than Joseph?


5

# Algebra

5

## **Algebra rules my kitchen!** Can algebra help you cook the perfect pancake?

A mathematics lecturer at an English university has published a formula she believes will create the perfect pancake. Dr Ruth Fairclough included every detail that could affect the quality of the pancake in her formula. She says that the most important factor, apart from the batter's recipe, is the frying pan's temperature.

The formula is:

$$P = 100 - \frac{10L - 7F + C(k - C) + T(m - T)}{S - E}$$

where
$P$ = the pancake score
$L$ = number of lumps in the batter
$C$ = batter consistency
$F$ = flipping score
$k$ = ideal batter consistency
$T$ = temperature of the pan
$m$ = ideal temperature of the pan
$S$ = length of time the batter stands before cooking
$E$ = length of time the cooked pancake sits before being eaten.

The closer you get to 100, the more delicious the pancake. Yum!

### Forum

Would you use this formula to make pancakes at home?

Would a chef in a restaurant use this formula?

Where would you use a formula around the home or in the garden? Who would use formulas in their work?

### Why learn this?

Algebra is a mathematical language. It uses letters and symbols to communicate general rules found from patterns and to solve problems. Algebra can help engineers to calculate stress forces on bridges, architects to design environmentally friendly buildings, nurses to calculate the correct doses of medicine and accountants to calculate how much tax needs to be paid.

**After completing this chapter you will be able to:**

- write and simplify algebraic expressions using pronumerals
- use a flowchart to describe algebraic rules
- generate tables of values using rules given in words or algebraic formulas
- change rules expressed in words to algebraic formulas using pronumerals
- substitute values into formulas
- use algebra to solve problems
- use tables to plot points on a Cartesian plane
- use a rule to generate linear graphs
- interpret point and line graphs.

# Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.




**1** Find the value of the ☐ in each of the following number sentences. If the ☐ appears more than once in a number sentence, the same number must be used.

**(a)** $3 + 7 = \square + 5$ **(b)** $5 - 1 = 2 \times \square$ **(c)** $3 + \square + 4 = \square \times 8$

**(d)** $5 - 4 + \square = 5$ **(e)** $10 \div \square = 5$ **(f)** $\square \times \square = 16$




**2 (a)** Multiply each of the following by 11 and subtract 3 from your answer.

**(i)** 4 **(ii)** 10 **(iii)** 200

**(b)** Multiply each of the following by itself and then take 6 from your answer.

**(i)** 4 **(ii)** 10 **(iii)** 8




**3** Write the following sentences using only numbers, the four operation signs (+, –, ×, ÷) and the equals sign (=).

**(a)** The sum of eleven and seventeen is twenty-eight.

**(b)** The difference between nine and seven is two.

**(c)** The product of three and four is twelve.

**(d)** The quotient of sixteen and eight is two.




**4** Write the following sentences using only numbers, the four operation signs and brackets. Then, use the order of operations to find the answer.

**(a)** Take the number six, multiply it by two, and then subtract four from it.

**(b)** Add five to the number ten, then divide what you get by five.

**(c)** Multiply the sum of six and five by three.

**(d)** The difference between twenty-three and twenty-one is multiplied by twelve.




**5** Decode the following by writing the letters contained in each grid square.

**(a)** The fastest living creature
E2 E4 A1 E4 B3 A1 D3 C2 E4
A3 A4 A2 C4 D2 C2

**(b)** The largest land carnivore
E3 D2 D4 D3 A4 E3 B4 E4 A4 A1

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 4 | a | b | c | d | e |
| 3 | f | g | h | i | k |
| 2 | l | m | n | o | p |
| 1 | r | s | t | u | y |

## Key Words

| | | | | |
|---|---|---|---|---|
| axes | define | like terms | point graph | table of values |
| Cartesian plane | equation | line graph | pronumeral | terms |
| coefficient | evaluate | linear graph | quadrant | unknown |
| constant | expression | ordered pair | relationship | unlike terms |
| coordinates | formula | origin | substitute | variable |

# Pronumerals and variables

5.1

## The meaning of '*x*' (or '*n*' or ...)

Algebra is a language used by mathematicians to communicate mathematical ideas and information clearly. To become a mathematician, you need to learn the language in the same way that you learn any other language. You will begin this study of algebra by learning words and algebraic conventions (rules that everyone agrees to follow) that will help you to read, speak, write and understand mathematics.

Algebra is used to write general rules to describe patterns that we find with numbers and solve problems using these rules.



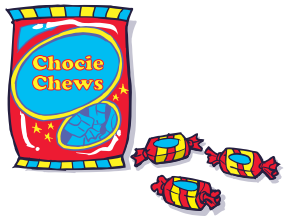

## Writing algebra

- **Pronumerals** are letters or symbols we use for numbers we don't know. Pronumerals represent (stand for) a number the same way that pronouns 'stand for' a noun.
- An **unknown** is the actual number that the pronumeral represents.
- A **variable** describes the unknown number if its value can change.

Using algebra, we can use $x$ as a pronumeral to represent an unknown number of lollies in a packet. If the number of lollies in the packet can vary, $x$ is called a variable.




If we are given 3 extra lollies, we now have $x + 3$ lollies.




If we buy another identical packet of lollies, we have $x + x$, or $2 \times x$ lollies.




If we have 2 identical packets and 3 extra lollies, we have $2 \times x + 3$ lollies in total.




When we have different variables, we need to use different pronumerals. For example, if we represent the number of boys in a class with the pronumeral $b$, we need to use a different pronumeral, such as $g$, to represent the number of girls in the class. The total number of students in the class is the sum of the two unknowns and can be written as $b + g$. Any two letters can be used to represent the number of boys and girls, so the number of students in the class could be written as $x + y$ or $p + r$.

## Algebraic conventions

Mathematicians like to write algebra as simply as possible, so they have decided that it is okay to leave out the '×' sign between a number and a pronumeral and between different pronumerals. That means we can write $2 \times n$ as $2n$ and $x \times y$ as $xy$. It has also been agreed that a '÷' sign can be replaced by a fraction bar, so $n \div 2$ is written as $\frac{n}{2}$. Decisions like this are called algebraic conventions. They ensure that all mathematicians use the same mathematical language. Some conventions that we use in algebra are listed below.

- Leave out the '×' between a number and a pronumeral and between different pronumerals (e.g. $2 \times n$ is written as $2n$ and $x \times y$ as $xy$).
- Replace ÷ with a fraction bar (e.g. $n \div 2$ is written as $\frac{n}{2}$).
- Numbers are always written in front of the pronumeral (e.g. $x \times 2$ is written as $2x$).
- When multiplying more than one pronumeral, write the letters in alphabetical order (e.g. $f \times a \times c \times e$ is written as $acef$).
- Brackets show that addition or subtraction needs to be done before multiplication or division. They change the order of operations.
- Brackets can be left out when using a fraction bar; e.g. $(a + b) \div 2$ can be written as $\frac{a+b}{2}$.
- When a pronumeral is raised to a power, we use the same notation as we do for numbers (e.g. $2 \times 2 \times 2 \times 2 = 2^4$ so we write $c \times c \times c \times c = c^4$ and $p$ squared as $p^2$).
- Numbers are usually written last in an expression (e.g. $6a + 7$).

When working with variables, the same words are used to describe the four operations as when working with numbers:

| | |
|---|---|
| + add, sum, total | – subtract, less than, minus |
| × multiply, product, lots of | ÷ divide, quotient, share equally |

The order of operations, including the use of brackets, is also the same.

## Worked Example 1 WE1

Write the following situations using algebra.

**(a)** Simon has $x$ fish in his aquarium. He buys 10 more. How many fish does he have now?

**(b)** There are $r$ cards in one pack. How many cards are there in three identical packs?

**(c)** Sonia has $n$ dollars in her bank account. She withdraws $100. How much money is left in the account?

**(d)** There are $d$ biscuits in a packet. Half of them are eaten. How many are left?

**(e)** A kitchen cupboard contains $x$ plates and $w$ bowls. What is the total number of plates and bowls in the cupboard?

| Thinking | Working |
|---|---|
| **(a)** 'More' means add onto the unknown amount (add 10 to $x$). | (a) Number of fish now = $x + 10$. |
| **(b)** Multiplication is used to find 'lots of' the unknown amount (multiply $r$ by 3). | (b) Number of cards in 3 packs = $3r$. |

| | | |
|---|---|---|
| **(c)** | 'Withdraw' means subtract from the unknown amount (subtract 100 from $n$). | (c) Amount of money left in Sonia's account $= \$(n - 100)$. |
| **(d)** | 'Half' means divide the unknown amount by 2 (divide $d$ by 2). | (d) Number of biscuits left $= \frac{d}{2}$. |
| **(e)** | 'Total' means add all the unknown amounts together (add $x$ and $w$). | (e) Number of plates and bowls $= x + w$. |

## Worked Example 2 WE2

Write each of the following using algebra.

**(a)** The product of $e$, $f$ and 7.

**(b)** $c$ is divided by 4, then 1 is subtracted.

**(c)** The sum of $x$ and 5 is multiplied by 3.

| Thinking | Working |
|---|---|
| **(a)** 'Product' means multiply. Write the number first, then the pronumerals in alphabetical order with no multiplication signs between them. | (a) $7ef$ |
| **(b)** Division is done first, then subtraction. Use a fraction bar to show division. | (b) $\frac{c}{4} - 1$ |
| **(c)** Addition is done first, then multiplication. Brackets are needed to show this order of operations. | (c) $3(x + 5)$ |

# 5.1 Pronumerals and variables

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14, Q15 | Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15 |

Answers page 648

### Fluency

**1** Write the following situations using algebra. WE1

**(a)** There are $p$ books in Jacob's locker. How many are left after Jacob takes out three for class?

**(b)** There are $k$ people in the line for concert tickets. Nine more people join the end of the line. How many people are lined up now?

**(c)** There are $v$ chocolates in one box. How many chocolates are there in 12 identical boxes?

**(d)** A group of four people won a prize of $a$ dollars, which they shared equally. How much did each person receive?

**(e)** Kyle has $d$ footy cards in his collection, while Khalid has $e$ cards in his. If they combine their collections, how many cards do they have altogether?

WE2

**2** Write each of the following using algebra.

**(a)** The sum of $x$ and 2.

**(b)** The product of 4 and $y$.

**(c)** $t$ *is* subtracted from 5.

**(d)** The quotient when $k$ is divided by 7.

**(e)** The product of $h$, 6 and $g$.

**(f)** $d$ is multiplied by 10, then 8 is added.

**(g)** 8 is added to $d$, and the result is multiplied by 10.

**(h)** The sum of $y$ and 9 is divided by 4.

**(i)** 3 less than 5 lots of $e$.

**(j)** The sum of $r$, $s$ and $t$ is divided by 3, then 11 is added.

**(k)** $y$ is multiplied by itself, then 20 is added.

**(l)** $x$ is squared, then multiplied by 8.

**3** Nadine has 7 bags with $x$ lollies in each bag, plus 9 extra lollies. The total number of lollies Nadine has altogether can be written using algebra as:

**A** $x + 7 + 9$ **B** $7x + 9$ **C** $9x + 7$ **D** $x^7 + 9$

**4** The sum of $z$ and 6 is divided by 2. When written using algebra it is:

**A** $z + 3$ **B** $\frac{z}{2} + 6$ **C** $z + 6 \div 2$ **D** $\frac{z + 6}{2}$

## Understanding

**5** Write each of the following using algebra. Use brackets where necessary.

**(a)** 1 is added to a number, $n$, and the result is multiplied by 5.

**(b)** 2 is added to a number, $d$, and the result is divided by 9.

**(c)** The sum of two different numbers, $a$ and $b$, is multiplied by 12.

**(d)** 3 is subtracted from the product of two different numbers, $m$ and $n$.

**6** There are $x$ people standing in line at the post office. Half of them leave, thinking they will come back later.

**(a)** How many people are in the line now?

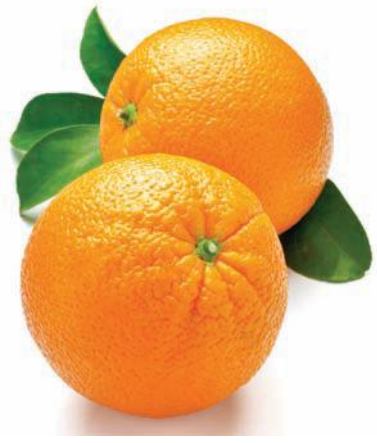
**(b)** Three other people walk in and join the line. How many people are standing in the line now?

**7** **(a)** What is the cost of $d$ metres of timber, if it costs \$6 per metre?

**(b)** What is the cost of $t$ kilograms of oranges, if they cost \$2.50 per kilogram?

**8** What is the cost of hiring the community hall for $x$ hours, if the rate is \$55 per hour, plus a one-off charge of \$300?

**9** What is the cost of buying $b$ movie tickets, at $v$ dollars per ticket?



**10** The cost of 1 desk is $t$ dollars, and the cost of 1 chair is $s$ dollars. 12 new desks and 24 new chairs are bought for a classroom.

**(a)** What is the total cost of the desks?

**(b)** What is the total cost of the chairs?

**(c)** What is the total cost of the furniture?

## Reasoning

**11** A purse contains $n$ 5-cent coins and $r$ 10-cent coins. There is no other money in it. Use algebra to write:

**(a)** the total value in cents of the 5-cent coins in the purse

**(b)** the total value in cents of the 10-cent coins in the purse

**(c)** the total value in cents of the money in the purse.

**12** Emma makes and sells stationery gift packs. Her small pack has $m$ pens, $n$ pencils, $q$ note pads and 1 eraser.

Her large pack has twice as many of each item. Use algebra to write:

**(a)** the total number of items in the small pack

**(b)** the total number of items in the large pack.

**(c)** Emma needs to make up 3 small and 2 large packs. Use algebra to write the number of pens that she will need altogether.

**13 (a)** The length of a rectangle is 5 cm longer than its width. If $w$ represents the width of the rectangle:

**(i)** write the length of the rectangle using $w$ as the variable

**(ii)** draw a rectangle and label each side using $w$ as the variable.

**(b)** Write, in terms of $w$, the total distance around the rectangle (the perimeter).

## Open-ended

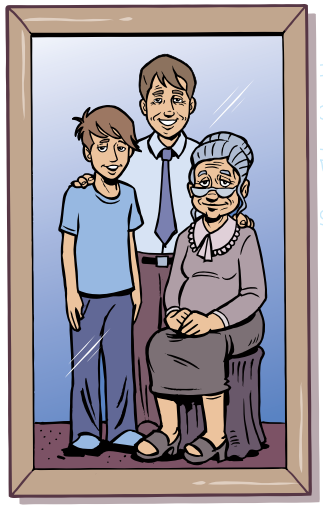
**14 (a)** Draw a diagram that could be represented by $3x + 5$. (Hint: Look at the 'lolly bag' diagrams at the start of this section.)

**(b)** What kind of diagram could you draw to show $\frac{x}{4}$? Draw a diagram, adding any necessary descriptions or labels.

**15** Rory's grandma says she is twice as old as the combined ages of Rory and his dad. Rory is $r$ years old and his dad is $d$ years old.

**(a)** Write Grandma's age using algebra.

**(b)** Write some possible ages for Rory, his dad and his grandma, if Rory is in primary school.



# 5.2 Terms, expressions and equations

## The language of algebra

Here are some more mathematical words that you need to know when using algebra.

### Constants

A number by itself is called a **constant**.

Here are some examples of constants: 2, 9, -37, $\frac{2}{7}$, 4.6

### Terms

**Terms** have one or more pronumerals, or may be just a number. $mn$ means $m \times n$ and is the same as $n \times m$ or $nm$.The pronumerals are often multiplied by a number that is written first.

Here are some examples of terms: $5a$, $7q$, $3pr$, $z$, $abc$, 4, $\frac{3}{11}$, $-2.32x$

### Coefficients

The number written in front of a pronumeral is called the **coefficient**. The sign of the number is included. For example, in $9x$, the coefficient of $x$ is 9. In $9x - 3y$, the coefficient of $y$ is -3.

### Expressions

**Expressions** are made by adding terms. The expression $5a + 7q - 12$ has three terms.

Expressions can also consist of just one term, so $5a$ or 4 could each be called an expression.

In section 5.1, we wrote $x + 3$, $2x$ and $2x + 3$. These are called algebraic expressions. Here are some more examples of expressions: $5a + 7q$, $5a + 7q - 12$, $9xy - z$, $2(p - 7)$.

### Equations

**Equations** contain an equals sign. They are made by writing two expressions equal to each other.

Equations have a left-hand side (LHS) and a right-hand side (RHS) on either side of the equals sign.

Here are some examples of equations: $9xy + z = 17$, $y = 6h$, $2(p - 7) = p + 10$

$3xy - 7z^2 + 5 = 16 + x$

- This is an equation because it has an equals sign.
- $x$, $y$ and $z$ are pronumerals that represent variables.
- 3 is the coefficient of $xy$ and -7 is the coefficient of $z^2$.
- $3xy - 7z^2 + 5$ and $16 + x$ are expressions.
- $3xy$, $-7z^2$, 5, 16 and $x$ are all terms.
- 5 and 16 are constants.

## Representing situations using algebra

When we use algebra to describe practical situations, it is very important to **define** all the variables we are going to use so we know what they represent. For example, we may not know the cost per kilogram of apples and bananas, so they are the variables we need to define. We do know that we purchased 3 kg of apples and 2 kg of bananas and spent $15.20. If we define the cost of apples/kg as $x$ ($) and the cost of bananas/kg as $y$ ($), then we can write:

- $3x$ as the cost of apples ($)
- $2y$ as the cost of bananas ($)
- $3x + 2y = 15.2$ as the equation
- $(3x + 2y)$ and 15.2 as the total cost of the fruit ($).

In this situation, we have examples of:

| | | | |
|---|---|---|---|
| a constant: | 15.2 | expressions: | $3x + 2y$, $3x$, $2y$, 15.2 |
| pronumerals: | $x$, $y$ | an equation: | $3x + 2y = 15.2$ |
| terms: | $3x$, $2y$, 15.2 | | |

## Worked Example 3 WE3

For $3p + 4q - 8pq = 7 + 2r$

**(a)** state whether it is an equation or an expression

**(b)** identify the coefficient of $pq$

**(c)** list all the variables

**(d)** write down any constants

**(e)** list all the terms used.

| Thinking | Working |
|---|---|
| **(a)** Is there an equals sign? If yes, then it is an equation. | (a) It is an equation. |
| **(b)** Look for the number in front of $pq$. Include the sign. | (b) Coefficient of $pq$ is $-8$. |
| **(c)** Look for all the different letters used and list them. (Note that $pq$ is a product of $p$ and $q$.) | (c) $p$, $q$ and $r$ are the variables. |
| **(d)** Look for a number by itself. | (d) The constant is 7. |
| **(e)** Look for all the parts of the equation separated by addition or subtraction. | (e) $3p$, $4q$, $-8pq$, 7 and $2r$ are the terms. |

## Worked Example 4 WE4

For the following situation:

**(a)** identify the variables

**(b)** define each variable using the given pronumerals

**(c)** write an equation.

A farmer has a number of sheep, $x$, and a number of ducks, $y$, in a paddock. There are five times as many sheep as ducks.

| Thinking | Working |
|---|---|
| **(a)** Identify the variables. | (a) The variables are the number of sheep and the number of ducks. |
| **(b)** Define the variables. | (b) Let $x$ = the number of sheep.<br>Let $y$ = the number of ducks. |
| **(c)** Write the equation. (Multiply $y$ by 5 to get $x$.) | (c) $x = 5y$ |

- An algebraic expression may have one term or it may be made up of two or more terms that are added or subtracted.
- An equation uses an equals sign to show that one expression has the same value as another expression.

# 5.2 Terms, expressions and equations

## Navigator

Answers page 648

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4 (a)–(d), Q5 (a) & (b), Q7 | Q1, Q2, Q3, Q4 (b)–(e), Q5 (a)–(c), Q6, Q7 | Q1, Q2, Q3, Q4 (c)–(f), Q5, Q6, Q7, Q8 |

## Fluency

WE3

**1** For each of the following:

**(i)** state whether it is an equation or an expression

**(ii)** identify the coefficient of $x$

**(iii)** list all the variables

**(iv)** write down any constants

**(v)** list all the terms used.

**(a)** $2x + 3y - 7 + 4x^2$

**(b)** $32 - 4x = 3xy + 4z$

**(c)** $2xy - 3x^2 = 9 - 23x$

**(d)** $6 - x + 8y - 4xy$

WE4

**2** For each of the following situations:

**(i)** identify the variables

**(ii)** define each variable using the given pronumerals

**(iii)** write an equation.

**(a)** The number of boys, $x$, in the class is three times the number of girls, $y$.

**(b)** Fred is $g$ years old. This is one less than three times James's age. James is $h$ years old.

**(c)** At the market, an apple costs $\$m$ and a pear costs $\$n$. Two apples cost as much as three pears.

**(d)** A kilogram of potatoes costs $\$a$ and a watermelon costs $\$b$. Three kilograms of potatoes and a watermelon cost \$5.20.

**(e)** The distance, $r$, to Ranko's house is 5 km more than the distance, $s$, to Skye's house.

**(f)** You have $v$ plants and $w$ pavers. To make a path around a row of plants, you need to have twice as many pavers as you have plants plus 6 extra pavers.

**3** Answer TRUE or FALSE for each of these statements.

**(a)** $6y$ is a term

**(b)** $7y - 9$ is an equation

**(c)** $y = 7x$ is a term

**(d)** $ab$ is a term

**(e)** $r = 5t - 9$ is an expression

**(f)** $cd + 4$ is an expression

**(g)** $w = s - 5b$ is an equation

**(h)** $6g$ is an equation

**(i)** $rs = sr$

**(j)** $7 - 6xzy = 7 - 6yxz$

**4** Which expression below matches the instruction given in each case?

**(a)** Choose a number and add any other number to it.

**A** $6 + a$ **B** $u + v$ **C** $c + 9$ **D** $a + b + 1$

**(b)** Choose a number and multiply it by any other number.

**A** $a - b$ **B** $7a$ **C** $4ab$ **D** $ab$

**(c)** Choose a number and multiply it by three, then subtract any other number.

**A** $3a - 1$ **B** $3n - y$ **C** $3m - 3$ **D** $3t$

**(d)** Choose a number and multiply it by two, add any other number, then subtract thirteen.

**A** $2f + 2d - 13$ **B** $2mn - 13$ **C** $2w + g - 13$ **D** $a + b - 13$

**(e)** Choose a number and add it to any other number, then multiply the answer by nine.

**A** $9xy$ **B** $9x + y$ **C** $9(x + y)$ **D** $9 + xy$

**(f)** Choose a number and multiply it by ten, choose any other number and multiply it by four, then add the two answers together.

**A** $w + v + 10 + 4$ **B** $14(v + w)$ **C** $10v + 4w$ **D** $40vw$

## Understanding

**5 (a)** A bottle of mass 120 g contains 25 tablets. The total mass of the bottle and tablets is 195 g.

**(i)** If the variable is the mass of a tablet, use a pronumeral to represent the variable.

**(ii)** Write an equation you would use to find the mass of a tablet.

**(b)** Half Mei Lee's age is the age she was 8 years ago.

**(i)** If the variable is Mei Lee's age, use a pronumeral to represent the variable.

**(ii)** Write an equation you would use to find Mei Lee's age.

**(c)** In 10 years' time, Beth will be half her dad's age now.

**(i)** If one variable is Beth's age and the other other variable is her dad's age, use two pronumerals to represent these variables.

**(ii)** Write an equation you would use to find Beth's age now.

**(d)** In a ball game, a goal is worth 8 points and a penalty goal is worth 1 point. At a match, one team scored 100 points.

**(i)** Define the variables and represent them with pronumerals.

**(ii)** Using these pronumerals, write an equation you would use to show the team's score.

## Reasoning

6 A family on holiday travelled a certain distance on the first day. The next day they were able to travel twice as far as on the first day. On the third day, they travelled 480 km further than on the first day.

**(a)** Define the variable.

**(b)** Write an expression to show the distance travelled on the second day.

**(c)** Write an expression to show the distance travelled on the third day.

**(d)** If the distance travelled on the third day was three times the distance they travelled on the first day, write an equation to show this situation.

## Open-ended

7 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. A team has scored 114 points.

**(a)** Define any variables you need to use.

**(b)** Using these variables, write an equation for the number of points scored.

**(c)** Find at least three different ways that the 114 points could have been scored.



8 In Rugby Union, a try is worth 5 points and a conversion goal is worth 2 points. Penalty goals and drop goals are each worth 3 points. To be able to get a conversion goal you must first score a try.

**(a)** If the Waratahs win a match against the Brumbies by 23 points to 12, write down two different ways these points could have been scored.

**(b)** If $P$ is the total number of points scored, $t$ is the number of tries, $g$ is the number of 2-point goals and $p$ is the number of 3-point goals, write an equation to find the total number of points scored.

## Outside the Square Puzzle

### We all scream for ice-cream

Summer had arrived and four industrious people decided to sell ice-creams at their local beach.

In total, there were four flavours and each person had ice-cream in 2 different flavours.

- Mr Chocolate had vanilla.
- Mr Vanilla did not have mint.
- Mr Mint had strawberry but not chocolate, whereas Mr Strawberry did not have vanilla.
- One person with mint also had chocolate.
- One person with vanilla also had strawberry.
- One of the persons with chocolate had no mint.
- Neither of the persons with vanilla had chocolate.
- No person had two ice-creams of the same flavour, no two persons had the same two flavours and none had the same flavour as their name.

Can you tell who had which flavoured ice-cream?

# Using rules

5.3

## Relationships

One of the main uses of algebra is to describe a **relationship** between two or more variables. A relationship means that the variables are connected in some way so that changing the value of one affects the value of the other.

Consider the following examples. Identify the two variables (values that can change) and decide whether a relationship exists between them. Does changing the value of one cause the other to change?



The number of blocks of chocolate bought, and the total cost of the chocolate.



The number of houses being built, and the number of bricks the builder will require.



The number of students sharing a packet of lollies, and the number of lollies each one receives.



The number of animals in a zoo enclosure, and the amount of space each one has to move around in.

## Rules and flowcharts

Relationships are described by a rule or set of instructions that tells you how to calculate one variable if you know the other.

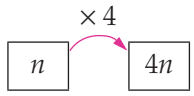
For example, if one block of chocolate costs $4, the rule for calculating the cost of several blocks of chocolate would be:

'Multiply the number of blocks of chocolate you are buying by 4'.

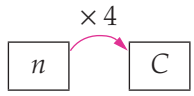
Flowcharts are step-by-step instructions for performing a task. They are used in industry to show the sequence of steps in building a product, such as a car on an assembly production line. Flowcharts are also used to write computer programs.

We can use a flowchart to represent the cost of the chocolate.

There are different ways of writing flowcharts, but, in mathematics, we usually write flowcharts this way:




or




Reading from left to right, this flowchart tells us: 'Take the number of blocks ($n$) and multiply them by 4 to get the total cost ($C$)'.

## Rules using algebra

Rules can be written in words, but it is much quicker to use algebra. Written in algebra, a rule for the cost of the chocolate would look like this:

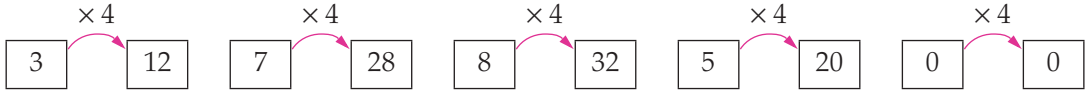
$C = 4n$

where $C$ = total cost of the chocolate
and $n$ = the number of blocks of chocolate bought at $4 a block.

We can use a **table of values** to show the cost of buying different numbers of blocks of chocolate.

| Number of blocks, $n$ | 3 | 7 | 8 | 5 | 0 |
|---|---|---|---|---|---|
| Cost, $C$ | 12 | 28 | 32 | 20 | 0 |

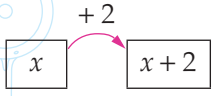
Each of the numbers in the top row of the table has gone through the flowchart (been multiplied by 4) to get the number in the bottom row.



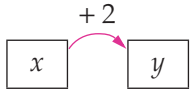

## Creating rules

If we choose any two pronumerals, such as $x$ and $y$, we can create different rules that connect them by constructing different flowcharts.

For example, the rule '$y$ is equal to $x$ plus 2' would have the flowchart:




or



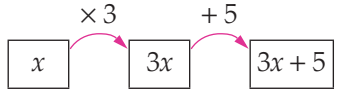

Written in algebra, the rule is: $y = x + 2$.

A 'table of values' for the rule could look like this:

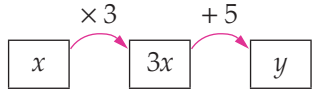
| $x$ | 4 | 10 | 6 | 9 | 2 |
|---|---|---|---|---|---|
| $y$ | 6 | 12 | 8 | 11 | 4 |

The numbers in the bottom row of the table ($y$-values) are the result of adding 2 to each of the numbers in the top row of the table ($x$-values).

Some rules are two-step rules; for example, the rule 'to get $y$, multiply $x$ by 3, then add 5' would have a flowchart that looks like this:



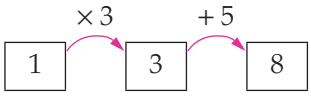

or



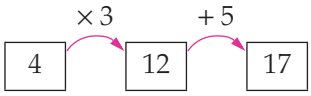

Written in algebra, the rule is: $y = 3x + 5$.

A table of values for this rule could look like this:

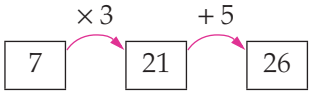
| $x$ | 1 | 4 | 7 | 2 |
|---|---|---|---|---|
| $y$ | 8 | 17 | 26 | 11 |



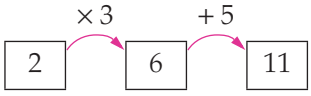
or $3 \times 1 + 5 = 8$



or $3 \times 4 + 5 = 17$



or $3 \times 7 + 5 = 26$



or $3 \times 2 + 5 = 11$

## Worked Example 5

WE5

For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

**(a)** $y$ is equal to $x$ divided by 4.

| $x$ | 20 | 16 | 4 | 12 | 10 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(b)** 2 is added to the product of $x$ and 5 to get $y$.

| $x$ | 2 | 5 | 4 | 3 | 0 |
|---|---|---|---|---|---|
| $y$ | | | | | |

### Thinking

**(a) 1** Decide how many operations are performed on $x$, what operation(s) they are and in which order (one operation, ÷ 4). Draw the flowchart.

**2** Write $y$ on one side of the rule. On the other side, show the operations being performed on $x$ ($\frac{x}{4}$).

**3** For each value of $x$ in the table, apply the rule by passing it through the flowchart (divide it by 4).

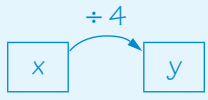
**(b) 1** Decide how many operations are performed on $x$, what operation(s) they are and in which order (× 5, then + 2). Draw the flowchart.

**2** Write $y$ on one side of the rule. On the other side, show the operations being performed on $x$ ($5x + 2$).

**3** For each value of $x$ in the table, apply the rule by passing it through the flowchart (multiply by 5, then add 2).

### Working

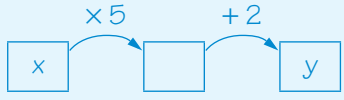
(a)




$y = \frac{x}{4}$

| x | 20 | 16 | 4 | 12 | 10 |
|---|---|---|---|---|---|
| y | 5 | 4 | 1 | 3 | 2.5 |

(b)




$y = 5x + 2$

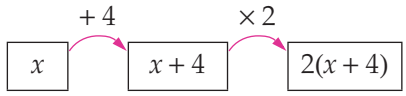
| x | 2 | 5 | 4 | 3 | 0 |
|---|---|---|---|---|---|
| y | 12 | 27 | 22 | 17 | 2 |

## Using brackets

Brackets may be needed to show the correct order of operations.

For example:

If the rule is '$y$ is equal to twice the sum of $x$ and 4', the flowchart would look like this:




The rule written in algebra would be $y = 2(x + 4)$.

Brackets are needed to show that the addition is to be done before the multiplication.

# 5.3 Using rules

## Navigator

Answers page 649

| | | |
|---|---|---|
| Q1 Column 1, Q2, Q3, Q4, Q5 (a) & (b), Q6, Q8, Q10, Q12 | Q1 Column 1, Q2, Q3, Q4, Q5 (c) & (d), Q6, Q7, Q8, Q10, Q13, Q14 | Q1 Column 2, Q2, Q3, Q4, Q5 (e) & (f), Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

## Fluency

WE 5

**1** For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.




**(a)** $y$ is equal to $x$ plus 2.

| $x$ | 13 | 11 | 7 | 28 | 1 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(b)** To find $y$, subtract 5 from $x$.

| $x$ | 6 | 18 | 9 | 85 | 5 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(c)** To find $y$, double $x$.

| $x$ | 2 | 3.4 | 10 | 11 | 101 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(d)** $y$ is equal to $x$ divided by 3.

| $x$ | 18 | 12 | 30 | 9 | 0 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(e)** $y$ is equal to multiplying $x$ by 5, then subtracting 3.

| $x$ | 4 | 2 | 0 | 5 | 20 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(f)** To find $y$, add 5 to $x$, then divide by 10.

| $x$ | 5 | 25 | 45 | 10 | 33 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(g)** To find $y$, divide $x$ by 2, then subtract 1.

| $x$ | 10 | 6 | 24 | 9 | 15 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(h)** $y$ is equal to $x$ multiplied by itself.

| $x$ | 3 | 11 | 7 | 6 | 10 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**2** Choose the correct algebraic rule given in each case.

**(a)** $y$ is equal to the sum of $x$ and twelve.

**A** $y = x + 12$ **B** $y + 12 = x$ **C** $y = 12x$ **D** $y = \frac{x}{12}$

**(b)** To get $y$, subtract 50 from $x$.

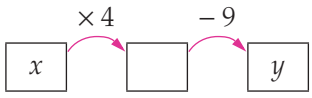
**A** $y = x - 50$ **B** $y = 50 - x$ **C** $y = x \times 50$ **D** $y = x + 50$

**(c)** To get $y$, add thirteen to $x$, then multiply by nine.

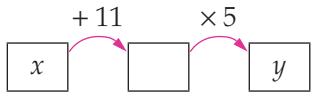
**A** $y = (x + 9) \times 13$ **B** $y = (x \times 13) + 9$ **C** $y = x + 13 \times 9$ **D** $y = 9(x + 13)$

**3** Write the rule shown by each of these flowcharts. Make sure you use brackets where necessary.

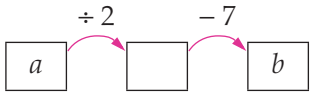
**(a)**



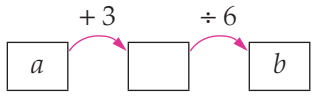

**(b)**



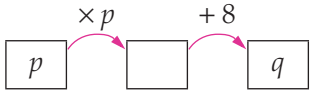

**(c)**



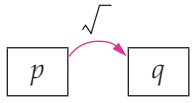

**(d)**




**(e)**




**(f)**




**4** Rewrite each of these rules using algebra.

**(a)** To find $y$, subtract eighteen from $x$.

**(b)** $y$ is equal to sixty multiplied by $x$.

**(c)** Divide $x$ by seven to find $y$.

**(d)** To find $y$, add forty-three to $x$, then multiply by twenty.

**(e)** Multiply $x$ by one hundred, then subtract fifty to find $y$.

**(f)** To find $y$, divide $x$ by sixteen, then add thirteen.

**(g)** $y$ is equal to $x$ multiplied by itself.

**(h)** Subtract twelve from $x$, then divide by nine to find $y$.

**(i)** Multiply $x$ by itself, then take away thirty-seven to find $y$.

## Understanding

**5** For the following rules, draw a flowchart and describe the rule using words.

**(a)** $y = 5x + 11$

**(b)** $y = \frac{x}{7} - 5$

**(c)** $b = 7(a + 6)$

**(d)** $b = \frac{a - 7}{12}$

**(e)** $d = \frac{e + 9}{4} - 13$

(the flowchart for this rule should have three steps)

**(f)** $d = \frac{e^2 - 1}{5}$

(the flowchart for this rule should have three steps)

**6** Joe has some lollies. He gives half to his little brother and then eats four of the rest.

**(a)** Draw a flowchart to show how many lollies he has left. Use $x$ for the number of lollies he starts with and $y$ for the final number.

**(b)** Write the rule to describe the situation.

**(c)** Use the rule to find how many lollies Joe has now if he had 20 to start with.

**7** Anita has some money in her wallet. Her mum gives her $20 for doing some chores and she decides to spend half of the money she now has on a birthday present for her sister. She spends another $4 on a birthday card.



**(a)** Draw a flowchart you would use to find how much money she has spent. Use $x$ for the initial amount in Anita's wallet and $y$ for the amount she has spent.

**(b)** Write a rule to describe the situation.

**(c)** Use the rule to find how much money Anita spent if she had $12 in her wallet initially.

**8** Your Uncle Harry is twice your age, plus 5.

**(a)** Show this information in a flowchart. Use $m$ to represent your age, and $H$ to represent Uncle Harry's age.

**(b)** Write a rule to show this situation.

**(c)** If you are 12 years old, how old is Uncle Harry?

**9** To produce jeans it costs $245 to set up the machine, and then $7 for each pair of jeans. So, for one pair of jeans to be produced, it costs $\$245 + \$7 \times 1 = \$252$.

**(a)** Using this information, fill in the table.

| Number of pairs of jeans, $n$ | 10 | 50 | 150 | 200 |
|---|---|---|---|---|
| Cost to produce the jeans, $C$ | | | | |

**(b)** Write a rule for the cost of producing the jeans. Use $C$ for cost, and $n$ for the number of pairs of jeans.

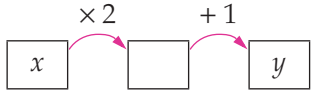
## Reasoning

**10** The following tables of values have only the $y$-values filled in. Work *backwards* along the flowchart to determine the values of $x$ that were used, and complete the tables of values.

To work backwards along the flowchart, do the *opposite* operation to the one shown in the forwards direction.

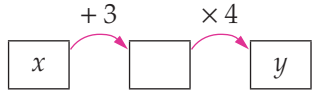


**(a)**




| $x$ | | | | | |
|---|---|---|---|---|---|
| $y$ | 5 | 11 | 15 | 9 | 21 |

**(b)**




| $x$ | | | | | |
|---|---|---|---|---|---|
| $y$ | 52 | 16 | 20 | 28 | 84 |

**11** For each of the following two rules:

**(i)** $y$ is equal to the sum of $x$ and 3, which is then multiplied by 2

**(ii)** to find $y$, multiply $x$ by 2, then add 6

**(a)** draw a flowchart.

**(b)** Write the rule using algebra.

**(c)** Copy and complete the table of values below.

| $x$ | 2 | 5 | 3 | 4 | 7 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(d)** What do you notice about these two rules? Can you explain your observation?

## Open-ended

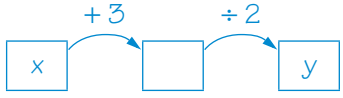
**12** Construct your own table of values and complete it for the rule $y = \frac{x}{2} + 3$. Use at least five different values.

**13** **(a)** Using two pronumerals (such as $x$ and $y$), at least two of the following operations ($+, -, \times, \div$) and some arrows, create at least three different flowcharts that show a rule connecting $x$ to $y$. Use a different combination of operations for each flowchart.

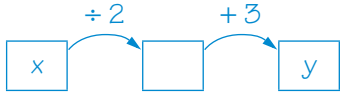
**(b)** Write the rules shown by your flowcharts using algebra.

**14** Kim and Jai are working on the following question: 'Draw a flowchart that represents this rule: to obtain $y$, divide $x$ by 2, then add 3'.

Kim's flowchart looks like this:




Jai's flowchart looks like this:




'Hey', said Jai. 'Yours looks different to mine. I must be wrong.'

'No, mate', said Kim. 'It's just a different way of writing the same thing.'

Is Kim's statement correct? Are they both right? If not, who has the correct flowchart, and why?

# Outside the Square — Problem solving

## Cutting string

**Equipment required:** 1 brain, 2 lengths of string, scissors

**1** Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces again, how many pieces will you now have? Check your answer using some string and scissors, then complete the following table.

| Number of cuts | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $c$ |
|---|---|---|---|---|---|---|---|---|
| Total number of pieces | | | | | | | | |



**2** This time, fold a piece of string in half each time before you cut it. How many pieces at each cut do you have now? *Remember*: Cut only one piece at each time.

| Number of cuts | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $c$ |
|---|---|---|---|---|---|---|---|---|
| Total number of pieces | | | | | | | | |

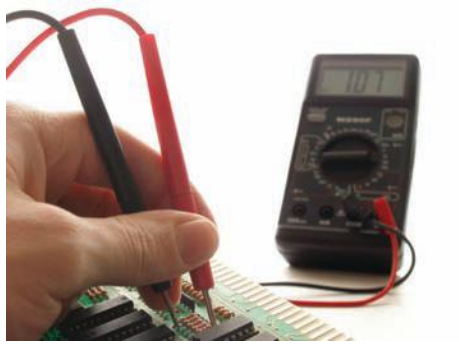
**Strategy options**

- Act it out.
- Look for a pattern.

# 5.4 Formulas and substitution

A **formula** is a mathematical rule that uses two or more variables. A formula is used to calculate the value of one variable when the value of the other variables are known. The plural of formula is formulas or formulae. The rules we have been working with so far can all be called formulas; however, we usually think of a formula as a rule we use in a practical situation. Here are some examples of practical formulas, and what they are used for:

$V = IR$



The voltage ($V$) of an electric circuit is found by multiplying the current ($I$) by the resistance ($R$).

$F = \frac{9C}{5} + 32$




To convert a temperature from Celsius to Fahrenheit, multiply the temperature in Celsius ($C$) by 9, divide by 5, and then add 32.

## Substitution

In mathematics, we can **substitute** a number for a variable. This allows us to find a value for, or **evaluate**, the formula or expression.

Many team sports use substitution. How is it similar to the substitution we're doing here?




### Worked Example 6 WE6

For each of the following formulas, evaluate $y$ by substituting the given value of $x$.

**(a)** $y = 10x - 5, \quad x = 7$

**(b)** $y = \frac{x}{2} + 4, \quad x = 5$

| Thinking | | Working |
|---|---|---|
| **(a)** | **1** Write the formula. | (a) $y = 10x - 5$ |
| | **2** Replace the variable ($x$) in the formula with its given value (7). Insert the $\times$ sign between the coefficient and the number. State the value you will substitute for $x$. | $y = 10 \times 7 - 5, x = 7$ |
| | **3** Calculate the value of the other variable ($y$). | $y = 70 - 5$<br>$= 65$ |
| | **4** State the answer. | $y = 65, x = 7.$ |

| | | | |
|---|---|---|---|
| **(b)** | **1** | Write the formula. | (b) $y = \frac{x}{2} + 4$ |
| | **2** | Replace the variable ($x$) in the formula with its given value (5). | $y = \frac{5}{2} + 4,\ t = 5$ |
| | **3** | Calculate the value of the other variable ($y$). | $= 2\frac{1}{2} + 4$<br>$= 6\frac{1}{2}$ or 6.5 |
| | **4** | State the answer. | $y = 6.5,\ x = 5$ |

When substituting numbers into a formula, write in any 'hidden' multiplication signs.

e.g. $b = 4a + 3$
$= 4 \times 5 + 3,\ a = 5$

## Worked Example 7

WE7

**(a)** The cost of hiring a community hall is calculated using the formula $C = 65t + 200$, where $C$ = cost (in dollars) and $t$ = the hire period (in hours). Find the cost of hiring the hall for 4 hours.

**(b)** The area of a rectangle (in square metres) is given by the formula $A = lw$ where $l$ = length (in metres) and $w$ = width (in metres). Calculate the area of a rectangular room that is 4.5 m long and 3 m wide.

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Write the formula. | (a) $C = 65t + 200$ |
| | **2** | Replace the variable in the formula with its given value (replace $t$ with 4, the number of hours). Insert a multiplication sign between the coefficients and the numbers (65 and 4). | $C = 65 \times 4 + 200,\ t = 4$ |
| | **3** | Evaluate the formula by performing the multiplication and the addition. | $= 260 + 200$<br>$= 460$ |
| | **4** | State the answer. | The cost of hiring the hall for 4 hours is $460. |
| **(b)** | **1** | Write the formula. | (b) $A = lw$ |
| | **2** | Replace the variables in the formula with their given values. (Replace $l$ with 4.5 and $w$ with 3.) Insert a multiplication sign between the numbers (4.5 and 3). | $A = 4.5 \times 3,\ l = 4.5,\ w = 3$ |
| | **3** | Evaluate the formula by performing the multiplication. | $= 13.5$ |
| | **4** | State the answer. | The area is 13.5 square metres. |

# 5.4 Formulas and substitution

## Navigator

Answers page 651

| | | |
|---|---|---|
| Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 (a) & (b), Q6, Q7, Q8, Q10, Q12 | Q1 Column 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q9, Q10, Q11, Q12 | Q1 Column 2, Q2, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 |

## Fluency

WE6

**1** For each of the following formulas, evaluate $y$ by substituting the given value of $x$.

**(a)** $y = x + 3$, $x = 4$

**(b)** $y = x + 5$, $x = \frac{1}{2}$

**(c)** $y = 2x - 7$, $x = 3$

**(d)** $y = 3x - 1$, $x = 8$

**(e)** $y = x - 1$, $x = 3.6$

**(f)** $y = x - 5$, $x = 30$

**(g)** $y = 25 - 3x$, $x = 7$

**(h)** $y = 30 - 2x$, $x = 1$

**(i)** $y = 4x$, $x = \frac{1}{2}$

**(j)** $y = 8x$, $x = 4$

**(k)** $y = 3(x + 5)$, $x = 1$

**(l)** $y = 6(x + 2)$, $x = 18$

**(m)** $y = \frac{x}{5}$, $x = 20$

**(n)** $y = \frac{x}{5}$, $x = 2.5$

**(o)** $y = 11(10 - x)$, $x = 4$

**(p)** $y = 12(10 - x)$, $x = 3$

WE7

**2** **(a)** The cost of hiring power tools from the local hardware store is calculated using the formula $C = 35t + 60$, where $C$ = cost (in dollars), and $t$ = the hire period (in hours). Find the cost of hiring a sander for 6 hours.

**(b)** Body mass index ($B$) is one way of measuring a person's health, and is calculated by the formula $B = \frac{m}{h^2}$, where $m$ is the person's mass in kilograms, and $h$ is their height in metres. Calculate the body mass index of a runner who is 1.7 metres tall and weighs 64 kg. (Round your answer to two decimal places.)

**(c)** The average speed of a moving object (such as a car) can be calculated by the formula $s = \frac{d}{t}$, where $s$ = speed (in kilometres per hour), $d$ = distance (in kilometres) and $t$ = time (in hours). Yoshi drives 210 km from Hobart to Launceston in 3 hours. Calculate her average speed.

**3** Answer TRUE or FALSE for each of the following statements.

**(a)** If we substitute $a = 4$ into $b = 5a$ we get $b = 20$.

**(b)** If we substitute $a = 9$ into $b = a + 11$ we get $b = 20$.

**(c)** If we substitute $u = 4$ into $v = 6u + 1$ we get $v = 65$.

**(d)** If we substitute $q = 10$ into $k = 13q - 8$ we get $k = 122$.

**(e)** If we substitute $x = 6$ into $y = 4(x - 5)$ we get $y = 19$.

**(f)** If we substitute $x = 12$ into $y = 5(14 - x)$ we get $y = 10$.

**4** Use each of the following rules to complete these tables of values.

**(a)** $b = 4a$

| $a$ | 11 | 20 | 5 | 9 | 50 |
|---|---|---|---|---|---|
| $b$ | | | | | |

**(b)** $y = 7x$

| $x$ | 6 | 4 | 10 | 20 | 101 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**(c)** $n = 3m + 2$

| $m$ | 1 | 2 | 10 | 6 | 5 |
|---|---|---|---|---|---|
| $n$ | | | | | |

**(d)** $k = 4j + 7$

| $j$ | 2 | 5 | 11 | 10 | 100 |
|---|---|---|---|---|---|
| $k$ | | | | | |

**(e)** $q = 2p - 10$

| $p$ | 11 | 15 | 10 | 20 | 100 |
|---|---|---|---|---|---|
| $q$ | | | | | |

**(f)** $s = 8r - 2$

| $r$ | 1 | 2 | 3 | 0 | 200 |
|---|---|---|---|---|---|
| $s$ | | | | | |

**(g)** $v = 4(u - 1)$

| $u$ | 5 | 3 | 1 | 201 | 6 |
|---|---|---|---|---|---|
| $v$ | | | | | |

**(h)** $n = 3(m - 2)$

| $m$ | 5 | 10 | 11 | 102 | 52 |
|---|---|---|---|---|---|
| $n$ | | | | | |

**5 (a)** For the rule $m = 3x - 5$, when $x = 3$, $m$ would be equal to:

**A** 1 **B** 2 **C** 3 **D** 4

**(b)** For the rule $y = \frac{x}{3} + 4$, when $x = 6$, $y$ would be equal to:

**A** $\frac{10}{3}$ **B** $4\frac{1}{2}$ **C** 6 **D** 22

**(c)** For the rule $l = 3(n + 2) - 1$, when $n = 1$, $l$ would be equal to:

**A** 5 **B** 6 **C** 8 **D** 9

## Understanding

**6** Sam's new fuel-efficient car uses 7 litres of petrol for every 100 km it travels.

**(a)** Write this as a formula to calculate the amount of petrol ($p$) that is needed to travel a distance ($d$) kilometres.

His father's 4WD uses 11 litres of petrol for every 100 km it travels.

**(b)** Write this as a formula to calculate the amount of petrol ($f$) that is needed to travel a distance of ($d$) kilometres.

**(c)** Use your formulas to calculate the number of litres each car uses to travel a distance of 400 km.

**(d)** Write a formula to calculate the cost ($C$), in dollars, of $y$ litres of petrol if petrol costs $1.30/litre.

**(e)** Calculate how much more Sam's father spent on petrol than Sam to travel 400 km.



7 Jabbok, a mobile phone salesman gets $350 per week and $10 for every phone that he sells.

**(a)** Write this as a formula with $n$ representing the number of phones sold, and $p$ representing the amount of money earned each week.

**(b)** If 15 phones are sold in the first week, use your formula to find how much Jabbok earns this week.

**(c)** If 20 phones are sold the next week, use your formula to find how much money Jabbok earns that week.

## Reasoning

8 I buy three loaves of bread and a $2.60 carton of milk. It costs me $\$D$.



**(a)** If $l$ represents the cost of a loaf of bread, write a formula to find the cost of my shopping ($D$) in terms of $l$.

**(b)** Use the formula to evaluate $D$ if $l = \$3.25$.

9 Jasmine is a caterer. She uses formulas to work out how much food she needs to feed different numbers of people.

**(a)** To calculate the number of sausages she needs at a barbeque, Jasmine uses the following formula: 'Allow 2 sausages for every person and have an extra 10 sausages'. Write Jasmine's formula using algebra using $n$ to represent the number of people and $s$ to represent the number of sausages.

**(b)** Another one of Jasmine's formulas for barbeques is: 'One bowl of salad will feed 8 people'. Write this formula using algebra. Use $n$ to represent the number of people, and $b$ for the number of bowls of salad.

**(c)** Use the formulas you have written to determine the number of sausages and bowls of salad Jasmine will need to cater for 40 people.



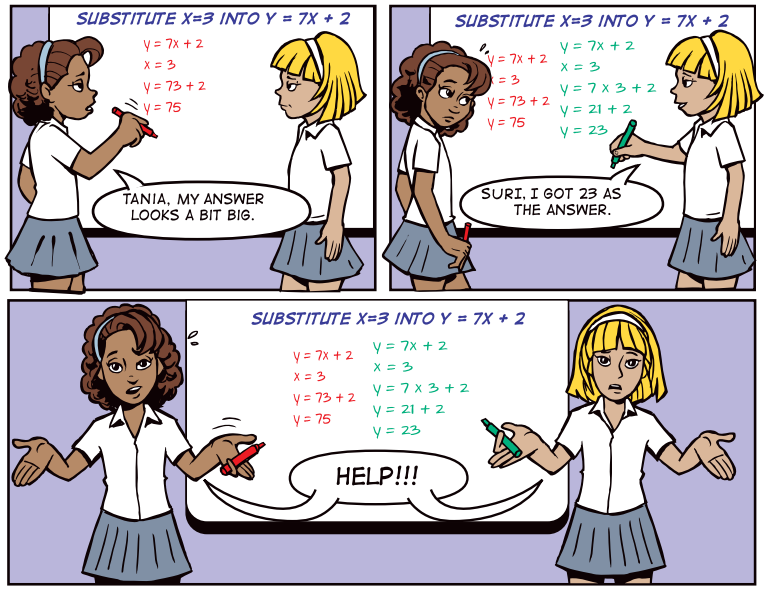
10 Show that the formulas $b = 3(a + 5)$ and $b = 3a + 5$ are different by substituting $a = 4$ into each of them. Explain how the two are different in terms of the order of operations.

## Open-ended

**11** Make up two different formulas connecting $x$ and $y$ that use addition and multiplication. Use them to complete the following table for each one.

| $x$ | 7 | 20 | 13 | 101 |
|---|---|---|---|---|
| $y$ | | | | |

**12**




Which of the two students has substituted correctly? What mistake has the other student made? Give them some advice so they can avoid a similar mistake in the future.

# Outside the Square Game

## Dicey formulas

**Equipment required:** 2 brains, 1 die

**How to win:**
Be the first to 60.

**How to play:**

1. Each person is given a formula.
2. You roll the die in turn. After rolling the die the first time, you must substitute the value shown on the die into your formula and write down the answer.
3. On each later throw, calculate the value from the formula as before, and add this to your previous total. Your opponent checks your answer and if it is wrong you lose a turn.
4. Your game ends when you think that you can get no closer to 60.
5. The other player may keep going until they decide they can get no closer to 60.
6. The winner is the person who gets to exactly 60, or, failing that, the person with the total nearest to, but less than, 60. If you score more than 60 you have lost, so you must decide when to stop.

Three sets of formulas are given:

Game 1:
Person 1, $A = 3n + 2$;
Person 2, $B = 3n - 2$

Game 2:
Person 1, $A = 3n + 4$;
Person 2, $B = 3(n + 2)$

Game 3:
Person 1, $A = 6 - n$;
Person 2, $B = 7 - n$

As a challenge, play this game where you have to get exactly 60. You are able to not include results if you wish. Here, you need to make sure that the last number you need to total 60 is possible to get using your formula when $n = 1$, 2, 3, 4, 5, or 6.

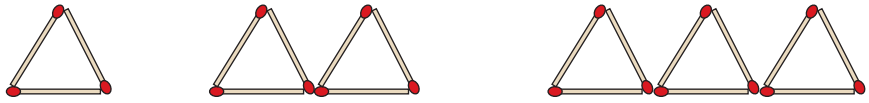
(Use 2 dice and be the first to 150 or more.)

# 5.5 Patterns and rules

A strategy we often use in problem solving is to look for a pattern in our results. Algebra can help us describe a pattern, which can often be written as a general rule, or formula. We can then use the formula to solve problems without having to draw out endless patterns.

## Worked Example 8 WE8

**(a)** Here is a matchstick pattern of triangles. Copy and complete this table of values by continuing the pattern.



| Number of triangles ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of matches ($m$) | | | | | |

**(b)** Find a general rule that connects the number of triangles in the pattern ($t$) to the number of matches used ($m$). Write the rule in words and in algebra.

**(c)** Use your rule to find the number of matches required to make 100 triangles.

### Thinking

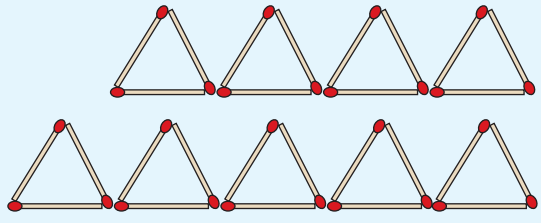
### Working

**(a) 1** Construct the table of values. Fill in any information you know from the pattern provided.

(a)

| Number of triangles ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of matches ($m$) | 3 | 6 | 9 | | |

**2** Continue the pattern by adding matches to create more triangles.



**3** Count the number of matches used for the continued pattern (formed with four and five triangles) and fill in the table.

| Number of triangles ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of matches ($m$) | 3 | 6 | 9 | 12 | 15 |

| | | | |
|---|---|---|---|
| **(b)** | **1** | Identify the number of matches that are being added on each time to make a new shape (3). (This number is also being added on to each number in the second row of the table.) This is the multiplication factor in the rule (× 3). | (b) $m = 3 \times t$ |
| | **2** | Write the rule that links the two variables ($t$ and $m$) in the table, in words and in algebra. | The number of matches is 3 times the number of triangles.<br>$m = 3t$ |
| **(c)** | **1** | Substitute $t = 100$ into the rule, and evaluate. | (c) $t = 100$<br>$m = 3 \times 100$<br>$m = 300$ |
| | **2** | Write the answer in words. | 300 matches are needed to make 100 triangles. |

In the above Worked Example, we see that one 'lot' of 3 matches is added every time to make a new triangle and the next number in the sequence. Therefore, we are multiplying the number of triangles by 3 to find the number of matches.

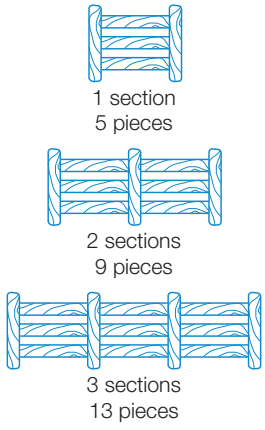
When trying to find a pattern, look for the number that is being added or subtracted every time. This will tell us what to multiply by in our rule.

In some cases, the above procedure may not give us the complete rule. We must then look for the number that is required to complete the first pattern. We then need to add on this number to make our rule work; e.g. $m = 3t + 1$; $m = 3t + 2$; $m = 3t + 3$ etc. Worked Example 9 shows this in detail.

Always try to understand how the rule works by looking at the pattern that has been formed.

## Worked Example 9 WE9

Frederico is building a fence around Farside Farm. The fences are made up of pieces of timber as shown.




**(a)** Copy and complete this table of values by continuing the pattern.

| Number of sections of fence ($F$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of pieces of timber ($P$) | 5 | 9 | | | |

**(b)** Find a general rule that connects the number of sections of fence ($F$) to the number of pieces of timber ($P$) needed. Write the rule in words and in algebra.

**(c)** Use your rule to find the number of pieces of timber required for a fence made up of 50 sections.

| Thinking | Working |
|---|---|
| **(a) 1** Construct the table of values. Fill in any values you know from the pattern provided. Continue the pattern to complete the table. | (a) |

| Number of sections of fence ($F$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of pieces of timber ($P$) | 5 | 9 | 13 | 17 | 21 |

| | | |
|---|---|---|
| **(b) 1** | Identify the number of pieces of the pattern that are needed to complete the new section (4). (This number is also being added on to each number in the second row of the table.) This number is the multiplication factor in the rule (× 4). | (b) 4 pieces are being added to make a new section. Therefore, $4F$ is in our rule. |
| **2** | When we multiply $F$ by 4, we do not have the numbers in the second row, so we now look for a number to add or subtract. How many pieces of the pattern were needed to start the first section? This number is added on to the rule. (1 piece of timber.) | $4F + 1$ |
| **3** | Write the rule that links the variables, both in words and in algebra. | The number of pieces is 4 times the number of sections plus 1 extra piece.<br>$P = 4F + 1$ |
| **(c) 1** | Substitute the value of $F$ into the formula ($F = 50$). | (c) $P = 4F + 1$<br>$= 4 \times 50 + 1$ |
| **2** | Evaluate. | $P = 200 + 1$<br>$= 201$ |
| **3** | Write the answer in words. | To make a fence of 50 sections, 201 pieces of timber are needed. |

# 5.5 Patterns and rules

## Navigator

Answers page 651

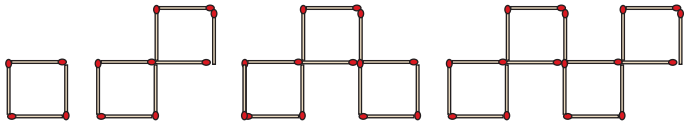
| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q12 | Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 | Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13 |

**Equipment required:** Centimetre grid paper for Question **12**

### Fluency

WE8

**1 (a)** Here is a matchstick pattern of squares. Copy and complete this table of values by continuing the pattern.



| Number of squares ($s$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number of matches ($m$) | | | | | | | | |

**(b)** Find a general rule that connects the number of squares in the pattern ($s$) to the number of matches used ($m$). Write the rule in words and in algebra.

**(c)** Use your rule to find the number of matches required to make 100 squares.

**2** Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds as shown.

WE9




1 garden bed
4 sleepers

2 garden beds
7 sleepers

3 garden beds
10 sleepers

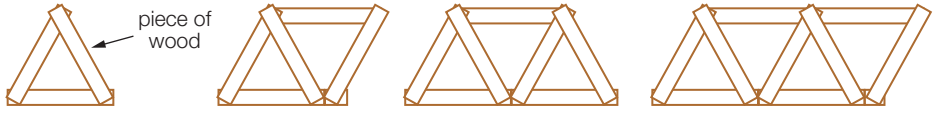
**(a)** Copy and complete this table of values by continuing the pattern.

| Number of garden beds ($B$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of sleepers ($S$) | 4 | 7 | 10 | | |

**(b)** Find a general rule that connects the number of garden beds ($B$) to the number of sleepers ($S$) needed. Write the rule in words and in algebra.

**(c)** Use your rule to find the number of sleepers required to divide the garden into a section made up of 21 beds.

**3** Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the one shown below, running across the walls.




1 triangle
3 pieces of wood

2 triangles
5 pieces of wood

3 triangles
7 pieces of wood

4 triangles
?

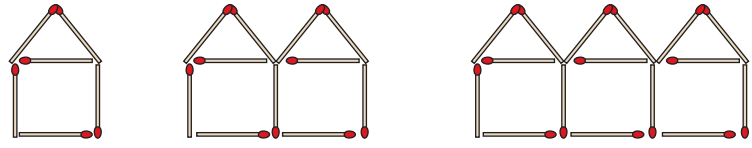
**(a)** Copy and complete this table of values by continuing the pattern.

| Number of triangles ($T$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of pieces of wood ($P$) | 3 | 5 | 7 | | |

**(b)** Find a general rule that connects the number of triangles to the number of pieces of wood required.

**(c)** Use your rule to find how many pieces of wood are required to make a total of 203 triangles.

**4** Here is a matchstick pattern of houses.



1 house
6 matches

2 houses
11 matches

3 houses
? matches

**(a)** Copy and complete the table below.

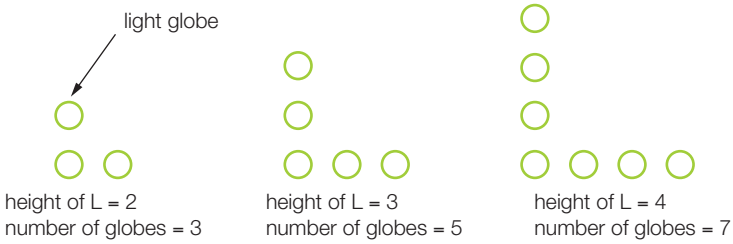
| Number of houses ($h$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number of matches ($m$) | 6 | 11 | | | | | | |

**(b)** Find the general rule that connects the number of matches to the number of houses.

**(c)** Use your rule to find the number of matches required to build 20 houses.

## Understanding

**5** Lightworks International Co., designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.




**(a)** Construct and complete a table of values for this pattern. Use the three patterns given here to begin your table, then continue the pattern for the next two Ls in the pattern.

**(b)** Find the rule that connects the height of the L to the number of globes required. Let $H$ = height of the L and $G$ = number of globes.

**(c)** Use your rule to find how many globes you would need to make an 'L' with a height of 120.

**6** A rule is given as $L = 3n + 1$. Which statement is not true?

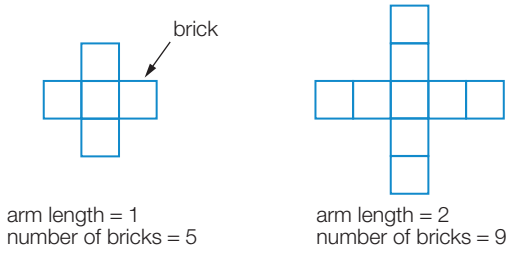
**A** When $n = 3$, $L = 10$.

**B** When $n = 4$, $L = 12$.

**C** When $n = 5$, $L = 16$.

**D** When $n = 6$, $L = 19$.

**7** The White Cross charity organisation has a white cross as its emblem. Their buildings all have a white cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.




**(a)** Draw the next two white crosses in the pattern.

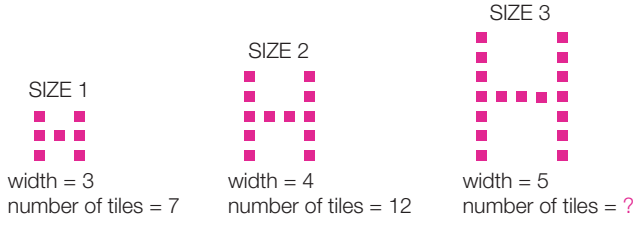
**(b)** Construct and complete a table of values connecting the arm length ($a$) and the number of white bricks ($b$) for these four different-sized crosses.

**(c)** Extend your table for arm lengths of 5 and 6.

**(d)** Find the rule that connects the two pronumerals.

**(e)** Find how many white bricks would be needed to make a cross with an arm length of 52.

**8** Mr Harrison has decided that he wants to use tiles to form large letter Hs, which are built into the brickwork of the walls of his company offices. He must work out how many tiles are needed to make different-sized Hs.



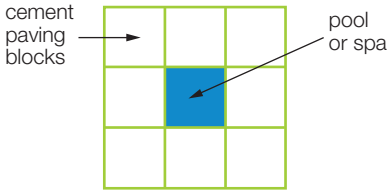

**(a)** Draw the next two Hs in the pattern.

**(b)** Construct and complete a table of values connecting the width of the H ($w$), and the total number of tiles ($t$).

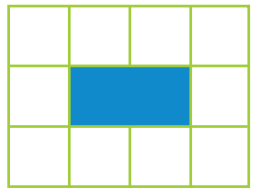
**(c)** Find the rule for these five different sizes that connects the width of the H ($w$) to the total number of tiles ($t$).

**(d)** Find the number of tiles needed to make an H that is 12 tiles wide.

**9** Plunge & Co., the Pool Paving People, specialises in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.




length of spa = 1
number of paving blocks = 8



length of spa = 2
number of paving blocks = 10

**(a)** Draw the next two spas and pavers in the pattern.

**(b)** Construct and complete a table for values of up to 4 spa lengths, connecting the spa length ($l$) with the number of pavers used ($p$) for these four different spa sizes.

**(c)** Find a rule without brackets connecting the two pronumerals.

**(d)** Explain your rule by using diagrams of spas of different lengths. Which paving blocks does the constant represent? Where does the coefficient of the pronumeral representing spa length come from?

**(e)** Find a rule using brackets that also works.

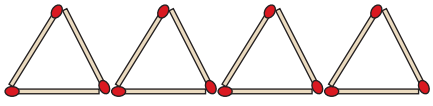
**(f)** Plunge & Co. has recently been asked by a swimming club to build a single-lane lap pool to help train its long-distance swimmers. It has been worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks will be needed to pave around the whole pool.

**(g)** For another job, Plunge & Co. uses 40 paving blocks to surround an outdoor spa. What is the length of this spa?

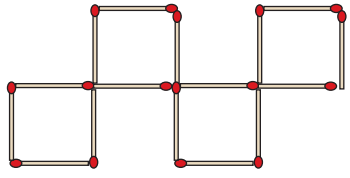
## Reasoning

**10** In each of the following matchstick patterns, the diagrams show the pattern after the fourth set of matchsticks has been added.

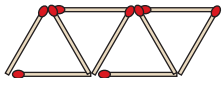
**(i)**



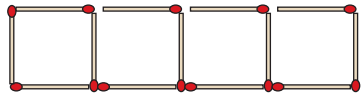
**(ii)**



**(iii)**



**(iv)**

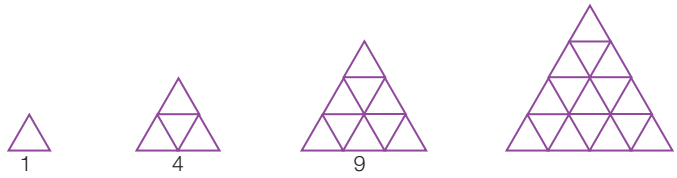


**(a)** Write a general rule for each of the patterns if one more set of matchsticks is added to the pattern each time. Use $s$ = number of shapes and $m$ = number of matchsticks as pronumerals.

**(b)** Patterns **(i)** and **(iii)** both add a new triangle every time. However, the rules are different. Patterns **(ii)** and **(iv)** both add a new square every time. However, the rules are also different. How are patterns **(i)** and **(ii)** different from the patterns in **(iii)** and **(iv)**? How is this difference shown in the rule for each pattern?

Not all patterns grow by a consistent amount each time. For the following question, complete the table and describe how the sequence of numbers formed in the bottom row of the table increases as extra elements of the pattern are added.

**11** **(a)** Count *all* the small triangles in the fourth large triangle.




**(b)** How many small triangles do you think there will be in the fifth large triangle?

**(c)** Copy and complete the following table. Try to find the pattern to help you.

| Large triangle | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of triangles | 1 | 4 | 9 | | | |

## Open-ended

**12** Choose one of the following letters: P, T, A, X, or Y. Create a design for the letter from small centimetre squares, and use it to create a pattern of letters that gradually increases in size. Try to find a rule that describes your pattern.

**13** Shreyas has decided to make a closed matchstick shape with 6 matchsticks. He then adds matchsticks to make a pattern that has a rule $m = 5s + 1$ where $m$ is the number of matchsticks and $s$ is the number of repeated shapes.

**(a)** Construct a shape that Shreyas may have started with.

**(b)** Shreyas changes his shape and adds matchsticks to make a new pattern. He finds that the rule is now $m = 4s + 2$. Draw a second shape that he may have used.

# Outside the Square Puzzle

## Puzzling tables

The table contains multiple patterns, across each row and down each column.

Copy the table and write the correct number in each blank space.

Describe the pattern for each row and for each column.

Patterns like this often exist in tables of information. This table gives the cost to send parcels, up to 20 kg, by air.

Different parts of the table have different patterns. A pattern may only work in part of a row or column. Describe the patterns that you can find.

| | Column 1 | Column 2 | Column 3 | Column 4 |
|---|---|---|---|---|
| Row 1 | 4 | 6 | 8 | 10 |
| Row 2 | 7 | 10.5 | 14 | |
| Row 3 | | 15 | 20 | 25 |
| Row 4 | 13 | | 26 | 32.5 |
| Row 5 | 16 | 24 | | 40 |

| Weight | Zone A NZ | Zone B Asia/Pacific | Zone C USA/Canada/ Middle East | Zone D Rest of World |
|---|---|---|---|---|
| **AIR** | | | | |
| Up to 250 g | $7.20 | $8.40 | $9.55 | $11.35 |
| Over 250 g up to 500 g | $11.00 | $13.40 | $15.70 | $19.30 |
| Over 500 g up to 750 g | $14.80 | $18.40 | $21.85 | $27.25 |
| Over 750 g up to 1000 g | $18.60 | $23.40 | $28.00 | $35.20 |
| Over 1000 g up to 1250 g | $22.40 | $28.40 | $34.15 | $43.15 |
| Over 1250 g up to 1500 g | $26.20 | $33.40 | $40.30 | $51.10 |
| Over 1500 g up to 1750 g | $30.00 | $38.40 | $46.45 | $59.05 |
| Over 1750 g up to 2000 g | $33.80 | $43.40 | $52.60 | $67.00 |
| Extra 500 g or part thereof | $4.00 | $5.20 | $7.45 | $9.95 |

*Postal Charges 2009, Australia Post website.*

# Half-time 5

**1** Which rule describes the given information? Ex.5.3

'Subtract 45 from $x$ to get $y$.'

**A** $y = x + 45$ **B** $y = x - 45$ **C** $y = x \times 45$ **D** $y = x \div 45$

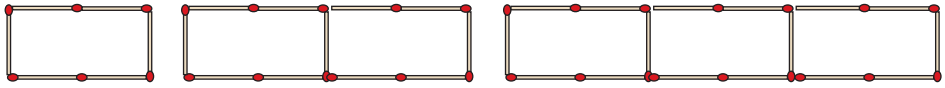
**2** For each of the following rules, evaluate $y$ by substituting the given values of $x$. Ex.5.4

**(a)** $y = x - 4$ when $x = 6$

**(b)** $y = 2(x - 7)$ when $x = -13$

**(c)** $y = 54 - 3x$ when $x = -9$

**3** If $s$ represents the number of shapes formed and $M$ is the number of matchsticks used, which rule is correct for this pattern? Ex.5.5



**A** $M = 5s + 1$ **B** $M = 6s$ **C** $M = 5s - 1$ **D** $M = 4s + 2$

**4** Write an equation to describe each of the following. Ex.5.2

**(a)** Tim has $p$ footy cards. He collects 7 more. He now has 28 cards.

**(b)** 20 is obtained when 4 is subtracted from the product of $x$ and 7.

**(c)** A bird aviary has $w$ parrots and $g$ canaries. When 4 of the canaries are sold, there are 14 birds left in the aviary. How many birds are in the aviary now?

**5** If $y$ is equal to 4 plus the quotient of $x$ divided by 2: Ex.5.3

**(a)** draw a flowchart for this rule

**(b)** write the rule using algebra

**(c)** copy and complete this table of values for the rule.

| $x$ | 2 | 8 | 10 | 5 | 13 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**6** Nicola has 6 identical bags of lollies, plus an extra 11 loose lollies. If $n$ represents the number of lollies in one bag, write an expression for the total number of lollies Nicola has. Ex.5.1

**7** Given the rule $y = 4x - 3$, which statement is not true? Ex.5.4

**A** When $x = 6$, $y = 21$. **B** When $x = 5$, $y = 17$.

**C** When $x = 4$, $y = 13$. **D** When $x = 3$, $y = 0$.

**8** The cost of an international phone call is 35 cents for every minute, plus a one-off 'flagfall' charge of 50 cents. Ex.5.4

**(a)** Write a rule to calculate the total cost of an international phone call. Use the pronumeral $C$ for the cost and $m$ for the length of the call, in minutes.

**(b)** Use your rule to calculate the cost of a 6-minute phone call. Write your answer in dollars.

**(c)** Ravi wants to call his parents in Singapore. He has $8. Will this be enough money for a 20-minute call?

# Investigation

## Richie's Restaurant

**Equipment required:** 1 brain

Richie has just bought 30 new rectangular tables for his restaurant. One table can seat six people. He needs to arrange them according to two conditions.

- The tables must be set up so that he can seat at least one group of 6, 8, 10, 12, 14, 16 and 18 people all at the same time.
- He wants to seat the maximum number of people possible.



### The Big Question

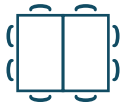
How should Richie arrange his tables to fulfil both of the conditions?

### Engage

To seat more than six people, there are two ways in which Richie can join the tables together:

Lengthways or widthways





1 How many people can be seated around two tables if they are joined together:

**(a)** lengthways **(b)** widthways?

2 For each of the two ways of joining the tables, copy and complete the following table of values. Draw the tables and count the number of people around them if necessary.

| Number of joined tables ($t$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of people seated ($p$) | | | | | | | |

### Explore

3 If Richie had no restrictions on the way he could set up his tables, what is the maximum number of people he could seat?

4 If Richie now sets up his tables to fulfil the given conditions, draw up a seating plan of Richie's restaurant, showing your arrangement of the 30 tables. How many people can be seated using this arrangement?

**Strategy options**

- Draw a diagram.
- Guess and check.
- Make a table.
- Look for a pattern.
- Test all possible combinations.

### Explain

5 Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient (uses fewer tables)? Is this type of arrangement always more efficient?

6 Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

7 For each of the two arrangements, write the rule that connects the number of tables ($t$) to the number of people that can be seated ($p$).

### Elaborate

8 Consider the coefficient and the constant that appears in each of your rules. Explain where they come from.

9 How do the two rules show that one type of table arrangement is more efficient than the other?

10 Given what you now know about the efficiency of different table arrangements, modify your plan from Question **4**, if necessary, to seat a greater number of people, while still fulfilling Richie's condition.

11 Draw your improved seating arrangement to answer the Big Question.

## Evaluate

**12** Consider the way in which you worked on this problem. How did you approach it? Could you have gone about it a different way?

**13** Did you use your tables of values (from Question **2**) or the algebra rules to help you solve the problem?

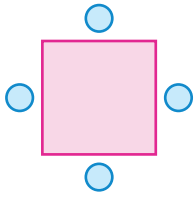
**14** How confident are you that your final solution is the best solution?

**15** Make a list of all the other factors you would need to consider if you were setting up tables in your own restaurant.

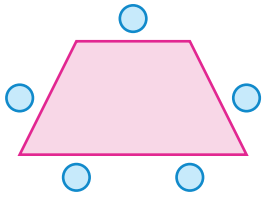
## Extend

**16** For his next restaurant, Richie would like to use some different-shaped tables. He could use:

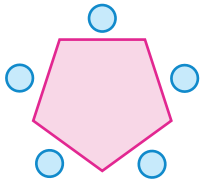
**(a)** square tables that sit 4 people



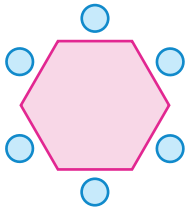
**(b)** trapezoidal tables that seat 5 people



**(c)** pentagonal tables that seat 5 people or



**(d)** hexagonal tables that seat 6 people.



Write a report to explain to Richie why none of these different table shapes would be suitable if he wants to seat the maximum number of people while fulfilling the given conditions and using no more than 30 tables.



# 5.6 Simplifying expressions with addition and subtraction

## Like terms

**Like terms:**

- have the same pronumeral part
- can have different coefficients; $2x$ and $5x$ are like terms
- can have pronumerals in a different order; $ab$ and $ba$ are like terms
- can have pronumerals raised to exactly the same power; $7x^3$ and $4x^3$ are like terms.

## Unlike terms

**Unlike terms:**

- have different pronumeral parts; $2x$ and $2y$ are unlike terms
- can have the the same pronumerals raised to different powers; $x^2$ and $x$ are unlike terms ($2^2 \neq 2$).

Because pronumerals represent numbers, we can use number laws in algebra to change the order of pronumerals.

$2 \times 3 = 3 \times 2$, so $ab = ba$ (commutative law)
$(3 \times 2) \times 5 = 3 \times (2 \times 5)$, so $(ab)c = a(bc)$ (associative law)
$abc = acb = bac = bca = cab = cba$

When terms have different pronumeral parts, we usually write the pronumerals in alphabetical order. This helps to identify like terms.

For example, $6abc$, $3bca$, $4dab$, $8cad$, $11adb$, $7cda$ can be written as $6abc$, $3abc$, $4abd$, $8acd$, $11abd$ and $7acd$. We can now see that $6abc$ and $3bca$ are like terms, $4dab$ and $11adb$ are like terms and $8cad$ and $7cda$ are also like terms.

## Worked Example 10 WE10

Which of the following are like terms?

**(a)** $4x$ and $7x$ **(b)** $5ab$ and $9b$ **(c)** $x$ and $12x$

**(d)** $8$ and $8z$ **(e)** $h^3$ and $6h^3$ **(f)** $xy$ and $x^2y$

| Thinking | Working |
|---|---|
| **(a)** Do the terms have exactly the same pronumeral part? | (a) $4x$ and $7x$ are like terms. |
| **(b)** Do the terms have exactly the same pronumeral part? | (b) $5ab$ and $9b$ are not like terms. |

| | | |
|---|---|---|
| **(c)** | Do the terms have exactly the same pronumeral part? | (c) $x$ and $12x$ are like terms. |
| **(d)** | Do the terms have exactly the same pronumeral part? | (d) $8$ and $8z$ are not like terms. |
| **(e)** | Do the terms have exactly the same pronumeral part? | (e) $h^3$ and $6h^3$ are like terms. |
| **(f)** | Do the terms have exactly the same pronumeral part? | (f) $xy$ and $x^2y$ are not like terms. |

## Adding and subtracting like terms



Maddie bought 6 identical packets of party balloons for her birthday party. Each packet contained $x$ balloons, so she bought $6x$ balloons. At home, Maddie found that she had 4 more identical packets, so she added $4x$ balloons to the $6x$ balloons she has just bought. Now, she had 10 packets, all containing $x$ balloons, so she had a total of $10x$ balloons.

$6x$ and $4x$ are like terms. When we add $6x$ and $4x$ we can simplify them to $10x$ by adding their coefficients.

$6x + 4x = 10x$

Maddie didn't need all the balloons, so she took 2 packets back to the store. She returned $2x$ balloons. She now has 8 packets of balloons, so she has $8x$ balloons for her party.

$10x$ and $2x$ are like terms. When we subtract $2x$ from $10x$ we can simplify them to $8x$ by subtracting their coefficients.

$10x - 2x = 8x$

We can simplify an expression by adding or subtracting like terms. This is called 'collecting like terms'. We add or subtract like terms by adding or subtracting their coefficients.

If Maddy buys 3 identical packets of balloons that contain $x$ balloons and 2 different packets that each contain $y$ balloons, we can show that she now has $3x + 2y$ balloons. $3x + 2y$ cannot be simplified any further, as $3x$ and $2y$ are unlike terms.

Examples:

$6c + 9c = 15c$
$12y + 3y - 4y = 11y$
$3x + 4y + 5x = 8x + 4y$
$7ab + 2ba = 9ab$
$8x - 3x = 5x$
$16abc - 13abc = 3abc$
$6x^2 + 4x^2 + 10x^2 = 20x^2$
$2x + 3x^2 - x = x + 3x^2$

$3ab + 4bc$ cannot be simplified, because $ab$ and $bc$ are not like terms.
$2x + 2x^2$ cannot be simplified because $x$ and $x^2$ are not like terms.

## Checking by substitution

We can replace a pronumeral with any number to check that our simplification is correct.

For example: $6x + 4x = 10x$
$10x - 2x = 8x$

for all values of $x$.

If $x = 20$,
$6 \times 20 + 4 \times 20 = 10 \times 20$
$10 \times 20 - 2 \times 20 = 8 \times 20$

If $x = 7$,
$6 \times 7 + 4 \times 7 = 10 \times 7$
$10 \times 7 - 2 \times 7 = 8 \times 7$

This is an application of the distributive law.

## Worked Example 11

WE11

Simplify each expression by collecting like terms.

**(a)** $4y + 7y$ **(b)** $2mn - 4mn$ **(c)** $8a + 4 - 3a$

| Thinking | Working |
|---|---|
| **(a) 1** Identify like terms by looking for terms that have exactly the same pronumeral part. | (a) $4y + 7y$ |
| **2** Simplify like terms by adding the coefficients of the terms. | $= 11y$ |
| **(b) 1** Identify like terms by looking for terms that have exactly the same pronumeral part. | (b) $2mn - 4mn$ |
| **2** Simplify like terms by subtracting the coefficients of the terms. | $= -2mn$ |
| **(c) 1** Identify the like terms. | (c) $8a + 4 - 3a$ |
| **2** Rearrange the expression to collect the like terms together. When a term is moved, include the sign $(-3a)$. | $= 8a - 3a + 4$ |
| **3** Simplify like terms by subtracting the coefficients of the terms. | $= 5a + 4$ |

## Worked Example 12

WE12

Simplify each expression where possible by collecting like terms.

**(a)** $4x + 3y - 9x + 2y$ **(b)** $6ab - 4a + 3ab - 7$

| Thinking | Working |
|---|---|
| **(a) 1** Rearrange the expression to collect the like terms together. When a term is moved, include the sign $(-9x)$. | (a) $4x + 3y - 9x + 2y$<br>$= 4x - 9x + 3y + 2y$ |
| **2** Simplify the like terms by adding or subtracting the coefficients of the terms. | $= -5x + 5y$ |
| **(b) 1** Rearrange the expression to collect the like terms together. When a term is moved, include the sign $(+3ab - 4a)$. | (b) $6ab - 4a + 3ab - 7$<br>$= 6ab + 3ab - 4a - 7$ |
| **2** Simplify the like terms by adding the coefficients of the terms. | $= 9ab - 4a - 7$ |

- Only like terms can be added together or subtracted.
- The operation sign (+ or –) moves with the term that follows it.

# 5.6 Simplifying expressions with addition and subtraction

## Navigator

| Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q11 | Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q11 | Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 |
|---|---|---|

Answers page 652

## Fluency

**1** Which of the following are like terms? WE10

**(a)** $3k$ and $5k$ **(b)** $4y$ and $5z$ **(c)** $11y$ and $12v$

**(d)** 17 and 8 **(e)** $6ab$ and $7bc$ **(f)** $2xy$ and $14yx$

**(g)** $4xyz$ and $yzx$ **(h)** $mnp$ and $3npm$ **(i)** 3, 5 and $x$

**2** Simplify each expression by collecting like terms. WE11

**(a)** $3x + 4x$ **(b)** $12y + 4y$ **(c)** $11a + 2a$

**(d)** $13y - 9y$ **(e)** $7m - 3m$ **(f)** $-10x - x$

**(g)** $3mn + 2nm$ **(h)** $8xy - 3xy$ **(i)** $-2z + 3z + 4$

**(j)** $x + 4y + 6x$ **(k)** $10w + 3z + 4w$ **(l)** $17a + 2b - 5a$

**3** Simplify each expression where possible by collecting like terms. WE12

**(a)** $3e + 4f + 7e - 6f$ **(b)** $12p + 8q - 4p - 12q$ **(c)** $8t + 15s - 16t - 3s$

**(d)** $6x - 4y + 3x + y^2$ **(e)** $12w - 2z + 3w - 3z^2$ **(f)** $10m + 9n - n - 5m^2$

**(g)** $3a + 17 + 4a - 8$ **(h)** $14x - 4 - 6x + 9$ **(i)** $21y - 36 - y - 17$

**4** **(a)** A like term for $7x$ is:

**A** $12x$ **B** $3 + x$ **C** 7 **D** $xy$

**(b)** A like term for $6wxy$ is:

**A** $6w$ **B** $6x$ **C** $6y$ **D** $12xyw$

## Understanding

**5** Simplify each expression where possible.

**(a)** $5e + 14f + 8e + 6$ **(b)** $18x + 6y - 4y + 2$ **(c)** $4 + 6x + 5y - 3$

**(d)** $2x - 3y + 4$ **(e)** $9m - 2n + 3mn$ **(f)** $6a + 11b - 5c - 7$

**6** Write each of the following rules using algebra, substitute the numbers 2, 3 and 4 into each rule, then answer TRUE or FALSE for each of these statements.

**(a)** These two rules are the same:

Rule 1: 'Take any number and multiply it by four. Take the same number and multiply it by six. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

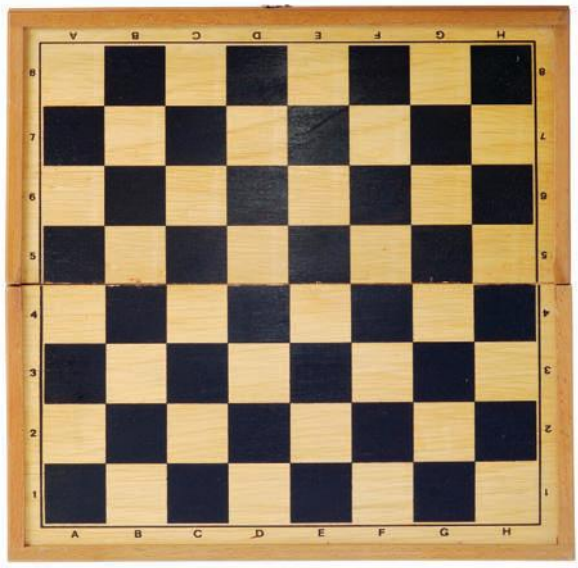
**(b)** These two rules are the same:

Rule 1: 'Take any number and multiply it by fifteen. Take the same number and multiply it by ten. Subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by five.'

## Reasoning

**7** A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.



**(a)** If $x$ is the number of coins on the first square, how many are lying on the:

**(i)** second square

**(ii)** third square

**(iii)** sixth square?

**(b)** How many coins, in total, are there on the first six squares of the board?

**(c)** If each coin's value is $\$v$, what is the total value of the coins on the board?

**(d)** Find a number for $x$ and a number for $v$ that will make the total value of the coins \$378.

**8** On each of her birthdays, Georgia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received $d$ dollars.

**(a)** Write an expression for the amount of money Georgia received on her fourth birthday.

**(b)** How much money, in total, did Georgia receive for her first four birthdays? Write your answer in terms of $d$.

**(c)** How much money did Georgia receive altogether for her first four birthdays if $d = 8$?

**(d)** If Georgia receives \$567 on her fifth birthday, how much did she receive on her first birthday?

**9** Kristian opens a bank account by depositing $\$y$. He then deposits $\$x$ every week and makes no withdrawals or other deposits.

The amount in his account after 5 weeks is:

**A** $x + 5y$ **B** $y + x + 5$ **C** $y + 5x$ **D** $5y + 5x$

## Open-ended

**10** This algebra pattern consists of the terms $x + y$, $2x + 3y$, $3x + 5y$, $4x + 7y$, … where the coefficient of $x$ increases by 1 each time and the coefficient of $y$ increases by 2 each time.

**(a)** Write an expression for the sixth term.

**(b)** Write an expression for the sum of the first five terms.

**(c)** Find two sets of values for x and y so that the sum of the first five terms is a multiple of 10.

**(d)** Find the sum for your chosen values.

**11** A garden bed is made using rectangular timber planks. The planks are arranged to form square and rectangular garden beds as shown in the diagram. Let $N$ be the total number of planks used to make the garden bed, $L$ be the number of planks for the length of the bed and $W$ be the number of planks for the width of the bed.



**(a)** Write down a rule that will find the number of planks needed for square garden beds of any side length.

**(b)** Write down a rule that will find the number of planks needed for a rectangular garden bed of length $L$ and width $W$.

**(c)** Give examples of each type of garden bed using no more than 30 planks.

**(d)** If the length must always be double the width, rewrite the rule for the rectangular garden bed using only $W$ as the pronumeral.

**(e)** Using this new rule, give examples of different types of rectangular garden beds using no more than 30 planks.

## Outside the Square Puzzle

### Latin squares

| | | | |
|---|---|---|---|
| $a$ | $d$ | $b$ | $c$ |
| $c$ | $b$ | $d$ | $a$ |
| $d$ | $a$ | $c$ | $b$ |
| $b$ | $c$ | $a$ | $d$ |

The sum of each row, column and diagonal is represented by $a + b + c + d$.

If you divide the table into quarters, what is the sum of the four squares in each quarter?

Can you find other $2 \times 2$ groups of squares in the table whose sum is $a + b + c + d$?

Draw the two diagonals on the table. What can you say about the remaining rows and columns; that is, the rows and columns that are not on the diagonals?

Draw some $4 \times 4$ tables in your exercise book.

Pick four different values for $a$, $b$, $c$ and $d$ and make up some Latin squares, using the Latin square above to help you. Make sure that you check that your answers are actually Latin squares.

Leave out some of the values and ask your partner to find the missing numbers to complete your Latin square.

Your complete Latin square might look like:

| | | | |
|---|---|---|---|
| 9 | 8 | 14 | 3 |
| 3 | 14 | 8 | 9 |
| 8 | 9 | 3 | 14 |
| 14 | 3 | 9 | 8 |

You might then change it to this one before asking your partner to complete the Latin square.

| | | | |
|---|---|---|---|
| 9 | 8 | | |
| 3 | | 8 | |
| 8 | | 3 | |
| 14 | 3 | | 8 |

# Age matters

**Equipment required:** 1 brain, 1 computer with an Excel spreadsheet program



Poh, Quentin and Rasheed are three friends who share the same birthday. They are now twelve and a half years old (150 months) and they have decided, after learning some algebra, that they may be able to negotiate a better pocket money agreement with their parents than the amount they get at present. They decide that the agreement they reach with their parents will run for 24 months.

Poh thinks that she will do quite well if she can get her parents to agree to pay her using the following formula.

$P = 2m - 285$ where $m$ is her age in months and $P$ is her monthly payment in dollars.

Quentin thinks he has a better plan. His formula is:

$Q = 3m - 440$ where $m$ is his age in months and $Q$ is his monthly payment in dollars.

Rasheed disagrees with the other two. He believes he has the winning formula.

$R = 4m - 595$ where $m$ is his age in months and $R$ is his monthly payment in dollars.

Versions of this Exploration are available for other technologies in Pearson Reader.




1. To see who has the best formula, open an Excel spreadsheet and set up four columns. Use $P$ ($) to represent Poh's pocket money, $Q$ ($) to represent Quentin's pocket money and $R$ ($) to represent Rasheed's pocket money. Enter Age in months ($m$) in A1, $P$ ($) in B1, $Q$ ($) in C1 and $R$ ($) in D1.
2. Enter 150 in A2 and 151 in A3. Highlight A2 and A3 and drag down to A13 using the small black cross on the bottom right-hand side of A3. Enter formulas in the formula bar for cells B2 (=2*A2 – 285), C2 (=3*A2 – 440) and D2 (=4*A2 – 595) as shown.
3. Total the amount of pocket money Poh receives in the first 12 months by entering the formula shown in cell B14 (=SUM(B2:B13)).

This screen shot shows the formulas you need to enter.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Age in months(m) | P($) | Q($) | R($) |
| 2 | 150 | =2*A2-285 | =3*A2-440 | =4*A2-595 |
| 3 | 151 | | | |
| 4 | 152 | | | |
| 5 | 153 | | | |
| 6 | 154 | | | |
| 7 | 155 | | | |
| 8 | 156 | | | |
| 9 | 157 | | | |
| 10 | 158 | | | |
| 11 | 159 | | | |
| 12 | 160 | | | |
| 13 | 161 | | | |
| 14 | | =SUM(B2:B13) | | |

If you have entered each formula correctly your spreadsheet should look like this.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Age in months(m) | P($) | Q($) | R($) |
| 2 | 150 | 15 | 10 | 5 |
| 3 | 151 | | | |
| 4 | 152 | | | |
| 5 | 153 | | | |
| 6 | 154 | | | |
| 7 | 155 | | | |
| 8 | 156 | | | |
| 9 | 157 | | | |
| 10 | 158 | | | |
| 11 | 159 | | | |
| 12 | 160 | | | |
| 13 | 161 | | | |
| 14 | | 15 | | |

**4** To fill the columns B, C and D, highlight B2 to D2, click on the small black cross on the right bottom corner of D2 and drag the formulas down to D13. Drag the formula shown in B14 across C14 and D14 to total the pocket money each received in 12 months.

Use your spreadsheet to answer the following questions.

**5** How much would they each get paid in the first month if their parents agreed to this new arrangement?

**6** Is there any time when they would all get the same amount for the month? If so, when does it happen and how much would they receive that month?

**7** How much would each of them get on their 13th birthday ($m = 156$)?

**8** What is the total amount each of them would receive in the first 12 months of their agreement?

**9** Who has received the most pocket money after 12 months?

**10** Extend your formulas for another 12 months. Who has received the most pocket money over 24 months? Is your answer the same as in Question **9**?

**11** Which student has the best formula? Give reasons for your choice. What do you think each student would say to convince their parents to adopt their formula?

## Taking it further

**12** Poh's mum suggests a different formula. Her formula reads like this. $M = 2y + 2$ where $y$ is the age in years and $M$ is the pocket money in dollars. She says she will round to the nearest dollar.

**(a)** Add this formula to your spreadsheet, use the round function to round to the nearest dollar (=round(2*A2/12+2,0) in cell E2) and determine the total amount she would pay in the first 12 months.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Age in mo | P($) | Q($) | R($) | M($) |
| 2 | 150 | 15 | 10 | 5 | 27 |
| 3 | 151 | | | | |
| 4 | 152 | | | | |
| 5 | 153 | | | | |
| 6 | 154 | | | | |
| 7 | 155 | | | | |
| 8 | 156 | | | | |
| 9 | 157 | | | | |
| 10 | 158 | | | | |
| 11 | 159 | | | | |
| 12 | 160 | | | | |
| 13 | 161 | | | | |
| 14 | | 15 | | | |

**(b)** Compare the amount of pocket money Poh would earn in the first 12 months using this formula with the other three formulas.

**(c)** Poh's older sister advises her to be careful before she decides which formula to use. Which formula would you advise her to use over a 2-year period. Justify your recommendation.

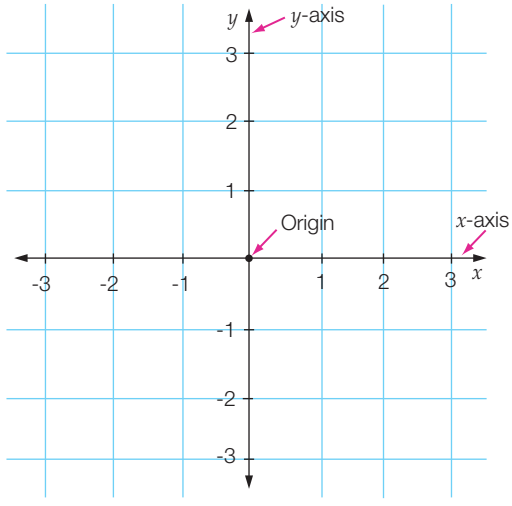
# 5.7 The Cartesian plane

In 1637, the French mathematician René Descartes developed a reference system that allowed any point on a plane to be located accurately. It was called the **Cartesian plane** (a plane is a flat 2-dimensional surface) in his honour, although it is also known as the number plane. This was an exciting idea as now algebra could be used to solve geometrical problems.

A Cartesian plane is constructed by drawing two lines at right angles to each other, one horizontal and the other vertical. The point at which they cross is called the **origin**. The horizontal line is called the $x$-axis and the vertical line is called the $y$-axis. Both **axes** (plural of axis) are number lines that extend infinitely in both directions. The integers on the axes are used to form a grid that allows any point to be located in reference to the origin.

The Cartesian plane is more accurate than alphanumeric grid systems used by street directories and spreadsheets, as the Cartesian plane locates points, whereas the alphanumeric system finds an area within a square or cell.



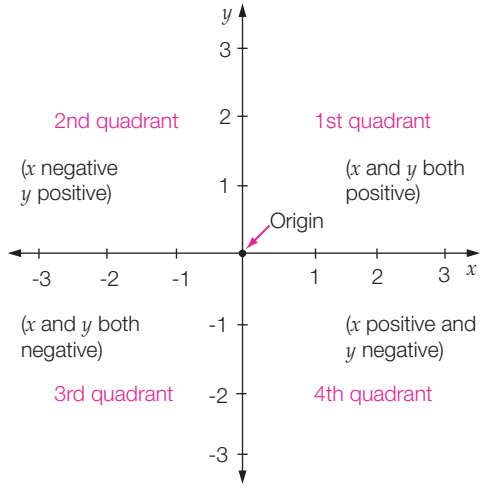

The position of any point on a number plane is described by a pair of numbers called the **coordinates** of the point. Coordinates are always written in brackets as an **ordered pair** $(x, y)$. Locating any point on the plane involves two moves from the origin.

$(x, y)$

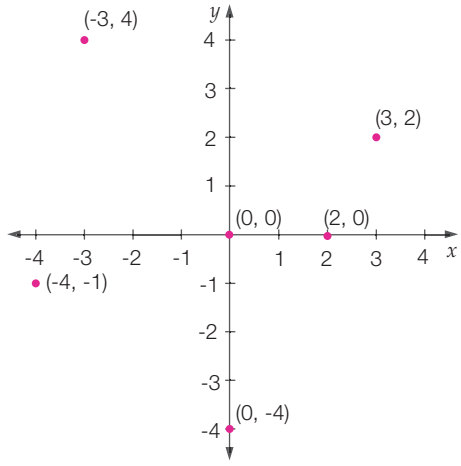
The $x$-coordinate shows the move to the left (–) or to the right (+) of the origin.

The $y$-coordinate shows the move up (+) or down (–) from the origin.

The $x$- and $y$-axes divide any plane into four **quadrants**. We number the quadrants starting with the quadrant in which both the $x$- and the $y$-coordinates have positive values. This is the 1st quadrant. We move in an anticlockwise direction to the 2nd, 3rd and 4th quadrants.



The following points and their Cartesian coordinates are shown on the Cartesian plane at right: (3, 2), (-3, 4), (-4, -1), (0, 0), (2, 0) and (0, -4).



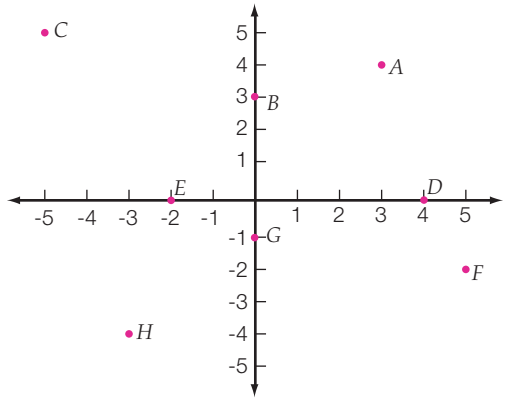

- We always move in the horizontal direction first and then in the vertical direction. This is a mathematical convention.
- The name 'ordered pair' tells you that order is important. The $x$-coordinate is always written first, and the $y$-coordinate is written second $(x, y)$.
- The coordinates of the origin are (0, 0).

## Worked Example 13

WE13

**(a)** Write the coordinates of each of the points $A$ to $H$ shown on the Cartesian plane.

**(b)** State the quadrant in which each point is located.




| Thinking | Working |
|---|---|
| **(a)** For each point, find the number of units it is to the left or right of the origin. This is the $x$-coordinate of the point. Then, find the number of units it is up or down from the origin. This is the $y$-coordinate. The + and - signs indicate direction. (Point $A$ is 3 units to the right of the origin and 4 units up from the origin.) Write the coordinates as an ordered pair. | (a) $A = (3, 4)$<br>$B = (0, 3)$<br>$C = (-5, 5)$<br>$D = (4, 0)$<br>$E = (-2, 0)$<br>$F = (5, -2)$<br>$G = (0, -1)$<br>$H = (-3, -4)$ |
| **(b)** Identify the quadrant that each point is in.<br><br>Points that lie on the $x$-axis or on the $y$-axis are not in a quadrant. | (b) $A$ is in the 1st quadrant, $C$ is in the second quadrant, $H$ is in the 3rd quadrant and $F$ is in the 4th quadrant.<br><br>$B$, $D$, $E$ and $G$ lie on an axis, so they are not in any quadrant. |

# 5.7 The Cartesian plane

## Navigator

Answers page 653

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q12 | Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13 |

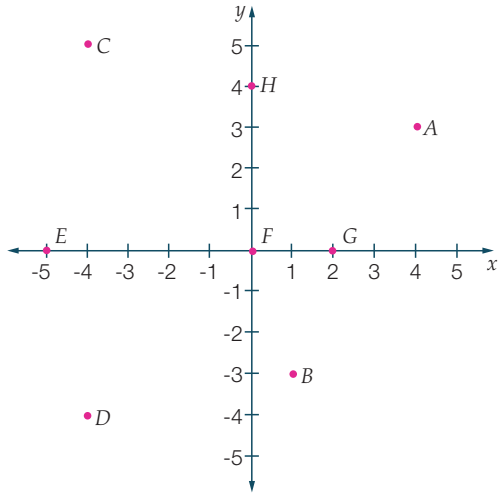
**Equipment required:** Graph paper for Questions **7**, **8** and **13**

## Fluency

WE13

**1** **(a)** Write the coordinates of each of the points $A$ to $H$ shown on the Cartesian plane.

**(b)** State the quadrant in which each point is located.




**2** **(a)** State whether each of the following coordinates is in quadrant 1, quadrant 2, quadrant 3 or quadrant 4.

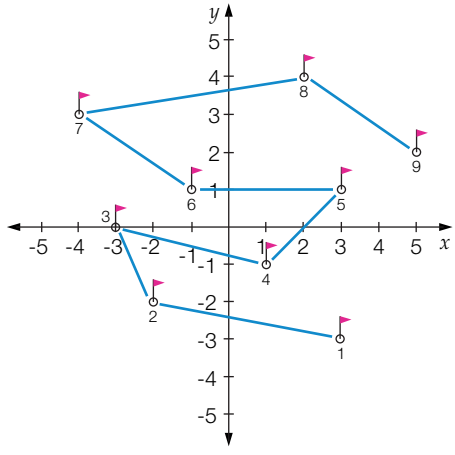
**(i)** (-2, 6) **(ii)** (5, -1) **(iii)** (-7, -4) **(iv)** (12, 10)

**(v)** (-32, 12) **(vi)** (51, -1) **(vii)** (-87, -90) **(viii)** (-2, 21)

**(b)** State whether each coordinate lies on the $x$-axis, the $y$-axis, or on both the $x$-axis and the $y$-axis.

**(i)** (0, 5) **(ii)** (0, -8) **(iii)** (0, 0) **(iv)** (-4, 0) **(v)** (15, 0)

**3** The layout of a nine-hole golf course is shown on the number plane. Write in order the ordered pairs of the nine holes.




**4** **(a)** A point is 1 unit right and 4 units up from the origin of a Cartesian plane. The coordinates of the point are:

**A** (4, 1) **B** (-4, 1)
**C** (1, 4) **D** (-1, 4)

**(b)** A point is 5 units left and 2 units up from the origin of a number plane. The coordinates of the point are:

**A** (-5, 2) **B** (2, -5) **C** (5, -2) **D** (-2, 5)

**(c)** A point is 4 units down and 3 units left of the origin of a Cartesian plane. The coordinates of the point are:

**A** (-4, -3) **B** (-3, -4) **C** (3, -4) **D** (-3, 4)

**5 (a)** The origin has the coordinates:

**A** (1, 0) **B** (0, 1) **C** (1, 1) **D** (0, 0)

**(b)** One coordinate that lies on the $x$-axis is:

**A** (0, 4) **B** (1, 1) **C** (2, 4) **D** (1, 0)

**(c)** One coordinate that lies on the $y$-axis is:

**A** (3, 0) **B** (3, 3) **C** (0, -3) **D** (-3, 0)

## Understanding

**6 (a)** Which of the following points will give a vertical line passing through the $x$-axis when joined in a straight line to (10, -6)?

**A** (-6, -2) **B** (4, -6) **C** (10, 0) **D** (0, 10)

**(b)** Which of the following points will give a horizontal line passing through the $y$-axis when joined in a straight line to (8, 13)?

**A** (4, 2) **B** (8, -3) **C** (-5, 0) **D** (0, 13)

**7** Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -9 to 9 on the $x$-axis and -4 to 4 along the $y$-axis. Plot the following points and join them in the order given to form a picture. You may like to colour your picture when complete.

(-4, -1) (-5, -1) (-5, 0) (-9, 1) (-6, 3) (-1, 2) (-4, -1) (-4, -2) (-1, -4) (0, -3) (-1, 2) (0, 3) (1, 2) (6, 3) (9, 1) (5, 0) (5, -1) (4, -1) (1, 2) (0, -3) (1, -4) (4, -2) (4, -1) STOP

Now join (3, 4) to (0, 2.5) and join (-3, 4) to (0, 2.5).

**8** Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -8 to 9 along the $x$-axis and -11 to 11 along the $y$-axis. Join each of the following sets of points in the order given. When you reach the word STOP, lift your pencil and start again from the next pair of coordinates.

Join (3, 0) (3, 5) (2, 5) (2, 4) (1, 4) (1, 5) (0, 5) (0, 4) (-1, 4) (-1, 5) (-2, 5) (-2, 4) (-3, 4) (-3, 5) (-4, 5) (-4, -9) (3, -9) (3, 0) (4, 0) (4, -1) (5, -1) (5, 0) (6, 0) (6, -1) (7, -1) (7, 0) (8, 0) (8, -9) (3, -9) STOP

Join (5, 0) (5, 7) ($6\frac{1}{2}$, 10) ($6\frac{1}{2}$, 11) (9, 10) ($6\frac{1}{2}$, 10) (8, 7) (8, 0) STOP

Join (-7, -2) (-7, -9) (-4, -9) (-4, 3) (-5, 3) (-5, 2) (-6, 2) (-6, 3) (-7, 3) (-7, 6) ($-7\frac{1}{2}$, 8) (-8, 6) (-8, -1) (-7, -2) STOP

Join (-2, -6) (-1, -5) (0, -5) (1, -6) (-2, -6) (-2, -9) (-3, -11) (2, -11) (1, -9) (1, -6) STOP

Join (3, 5) (3, 7) ($3\frac{1}{2}$, 9) (4, 7) (4, 4) (3, 1) STOP Join (-4, 5) (-4, 9) (-2, 8) (-4, 8) STOP

Join (6, -5) (7, -5) (7, -2) (6, -2) (6, -5) STOP Join (6, 6) (7, 6) (7, 3) (6, 3) (6, 6) STOP

Join (2, -1) (2, 2) (1, 2) (1, -1) (2, -1) STOP Join (-3, -1) (-2, -1) (-2, -4) (-3, -4) (-3, -1) STOP

Join (-6, -3) (-5, -3) (-5, 1) (-6, 1) (-6, -3) STOP Join (3, 7) to (4, 7) STOP

Join (5, 7) to (8, 7) STOP Join (-8, 6) to (-7, 6) STOP

Join (-2, -6) to (-3, -1) STOP Join (1, -6) to (2, -11) STOP

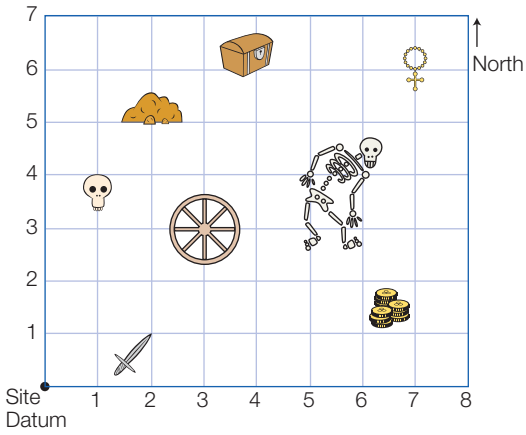
**9 (a)** Which of the following coordinate pairs lies furthest up on the Cartesian plane?

**A** (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

**(b)** Which of the following coordinate pairs is the furthest to the left on the Cartesian plane?

**A** (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

**10** The grid shown allows an archaeologist to reconstruct the layout of an old tomb. (Archaeologists call the point (0, 0) the datum point.) State the point(s) at which each of the following is found.







**(a)** the centre of the base of the treasure chest

**(b)** the middle of the rock face base

**(c)** the tip of the dagger

**(d)** the centre of the chariot wheel

**(e)** the right wrist and the left shoulder of the skeleton

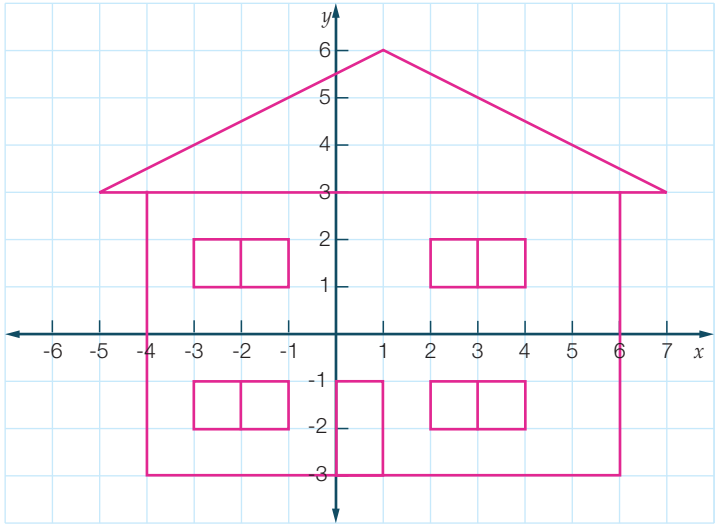
**(f)** the four grid points that enclose all the coins

**(g)** the top of the skull lying by itself

**(h)** the chain at the point where the cross is attached

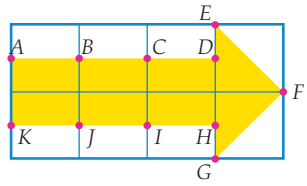
## Reasoning

**11** Write instructions using coordinates so that this drawing could be reproduced by another student.



## Open-ended

**12** Draw this arrow shape on a Cartesian grid so that it lies in at least three quadrants, and give the coordinates of each of the points *A* to *K*.




**13** On a piece of grid or graph paper, rule a set of axes to form a Cartesian plane. Allow for a scale from -5 to +5 on both axes.

**(a)** Using more than one quadrant on your Cartesian plane, use straight lines to draw a simple design of your own.

**(b)** Write instructions using coordinates so that your design could be reproduced by another student.

**(c)** Give your instructions to another student to see if they can draw your design exactly.

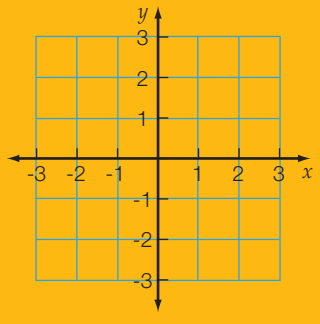
# Outside the Square Game

## Line up

**Equipment required:** 2 brains, grid paper, die

**How to win:**
Five games are played and the player with the greatest total of points is the winner.

**How to play:**




**1** Copy the grid. Take turns to roll the die. The aim is to get three consecutive coordinates in a row, a column or a diagonal.

**2** The number on the die is multiplied by 1 and -1 and then split into a sum or difference of two numbers used as the coordinates. For example, if a 3 is rolled, the coordinates might be (0, 3), (3, 0), (0, -3), (-3, 0), (1, 2), (2, -1), (1, -2), (-1, 2), (-1, -2), (-2, -1).

The player can mark any one of these positions.

**3** Some combinations cannot be used. For example, if a 6 is rolled, the only combinations that fit on the playing board are (3, 3), (-3, -3), (-3, 3), (3, -3) but not, for example, (-5, 1) or (6, 0).

**4** If a player cannot mark a coordinate because it is already marked, they miss a turn.

**5** When a player has three in a row, the number of points earned is the total of the numbers, ignoring the minus sign.

For example, for the coordinates (-3, 2), (-2, 2) and (3, 2), the points awarded are $3 + 2 + 2 + 2 + 3 + 2 = 14$.

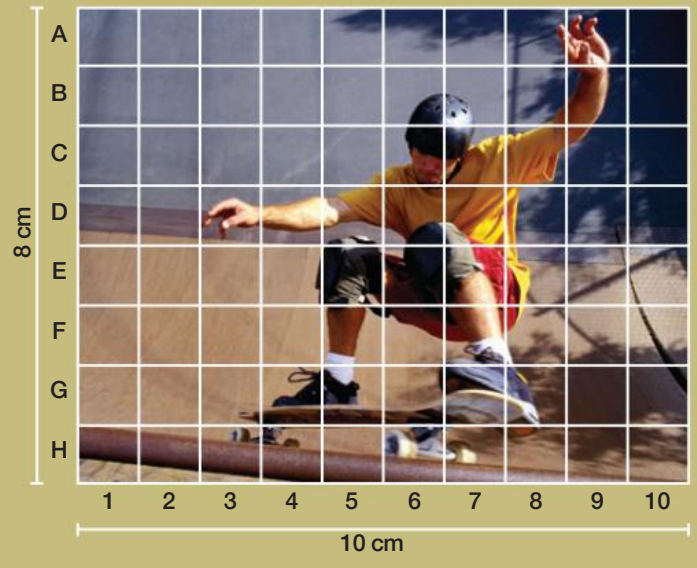
**6** Then, clear the board for the next round.

**7** Five games are played and the player with the greatest total of points is the winner.

Maths 4 Real

# MATHS MEETS ART

Some Art schools teach students to paint using the 'grid method', a method used by Leonardo da Vinci. The method involves overlaying a grid onto the image to be painted and also placing a matching grid pattern onto the canvas. For example, if the image shown is an 8 cm × 10 cm photograph, 1 cm × 1 cm squares can be used to create a grid pattern of eighty squares. If the image is to be copied onto an 80 cm × 100 cm canvas, each square will be 10 cm × 10 cm.




Note that the image and canvas above are not shown to scale.

**1** State the coordinates of each of the following.

**(i)** right foot **(ii)** left ear

**(iii)** left elbow **(iv)** left hand

**2** Suppose you wish to copy the image onto a canvas that is 16 cm × 20 cm.

**(a)** What size square on the canvas is needed to represent one square centimetre on the image?

**(b)** Use centimetre grid paper or graph paper to copy the image.

## The rule of thirds

A basic principle in photography and art is 'the rule of thirds'. The image is divided into thirds horizontally and vertically to obtain nine squares or rectangles.

If points of interest are placed in any of the four marked intersections or along the lines, the image is more balanced and is pleasing to the eye.

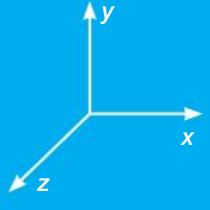
**3** Do you think 'the rule of thirds' was used in the composition shown below?

**4** Use graph or grid paper to create an image of your own using 'the rule of thirds'.



## Research

- The rule of thirds is based on the Fibonacci spiral. Find out how the spiral is created.
- Find out more about the art of Piet Mondrian, and either present a five minute talk with illustrations or prepare a poster.
- Zedism is a painting style that gives a 3-dimensional effect. It uses a third axis (called the *z*-axis) which is at right angles to the Cartesian plane. By using geometry and perspective, the effect is an illusion of form and structure.




"Sky"



Find out how this is achieved and look for other examples of paintings where this method is used.

# 5.8 Patterns and plotting points

When points are plotted on a number plane they often appear to follow a pattern or shape. This shape may be a straight line or it may be a curve. If we can draw a straight line through all the points we form a **linear graph**. Any set of points with a definite shape, such as a straight line, can be described by an algebraic rule.

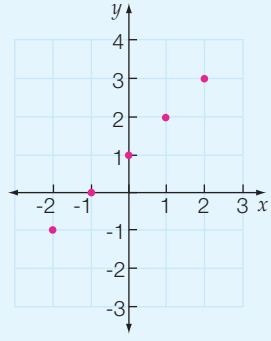
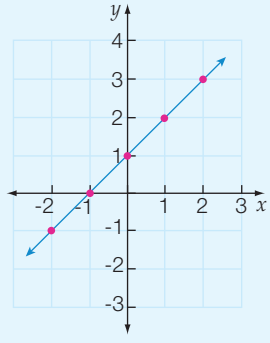
## Worked Example 14 WE14

**(a)** Plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) on a number plane.

**(b)** Rule a straight line passing through all the points.

**(c)** Summarise the set of points in a table of values.

**(d)** Write down the rule linking the $x$- and $y$-values.

| Thinking | Working |
|---|---|
| **(a)** Use a ruler to draw and number the $x$- and $y$-axes on graph or grid paper. Label your axes and plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) moving left or right first, then up or down. | *(a)*   |
| **(b)** Join the points carefully with a ruler. You should have a straight line. | *(b)*   |
| **(c)** Construct a table with $x$-values on the top line and $y$-values on the second line. | *(c)* |
| **(d)** **1** Look for a link between the $x$- and $y$-values. (Each $y$-value is 1 more than the $x$-value.) | *(d)* |
| **2** State the rule. | The rule is $y = x + 1$. |

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | -1 | 0 | 1 | 2 | 3 |

# Worked Example 15

WE15

**(a)** List the coordinates of the points given in the table of values below.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | -3 | -1 | 1 | 3 | 5 |

**(b)** Plot these points on a number plane.

**(c)** Rule a straight line passing through all the points.

**(d)** Write down the rule linking the $x$- and $y$-values.

## Thinking

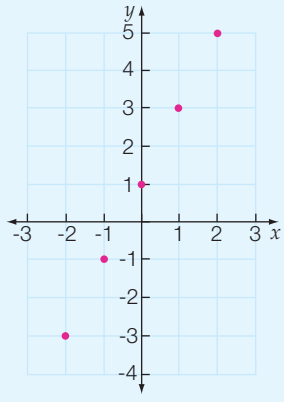
## Working

**(a)** Write the coordinates as an ordered pair with the $x$-coordinate first and the $y$-coordinate next.

*(a)* (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5)

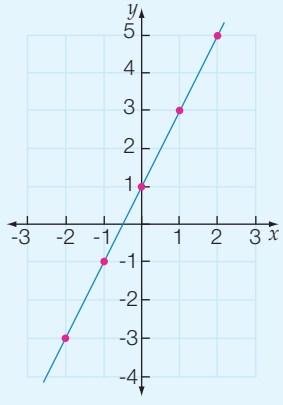
**(b)** Use a ruler to draw and number the $x$- and $y$-axes on graph or grid paper. Label your axes and plot the points (-2, -3), (-1, -1), (0, 1), (1, 3) and (2, 5) moving left or right first, then up or down.

*(b)*




**(c)** Join the points carefully with a ruler. You should have a straight line.

*(c)*




**(d)** **1** Look for a link between the $x$- and $y$-values. (The $y$-values go up by 2, so we are multiplying by 2. Each $y$-value is 1 more than twice the $x$-value.)

*(d)*

**2** State the rule.

*The rule is* $y = 2x + 1$.

# 5.8 Patterns and plotting points

## Navigator

Answers page 653

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 |

**Equipment required:** Graph paper

## Fluency

WE14

**1** **(a)** Plot the points (-2, 0), (-1, 1), (0, 2), (1, 3) and (2, 4) on a number plane.

**(b)** Rule a straight line passing through all the points.

**(c)** Summarise the set of points in a table of values.

**(d)** Write down the rule linking the $x$- and $y$-values.

WE15

**2** **(a)** List the coordinates of the points given in the table of values below.

| $x$ | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | -3 | -1 | 1 | 3 | 5 |

**(b)** Plot these points on a number plane.

**(c)** Rule a straight line passing through all the points.

**(d)** Write down the rule linking the $x$- and $y$-values.

**3** **(a)** Plot the points (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1) on a number plane.

**(b)** Join the points in this order.

**(c)** Summarise the set of points in a table.

**(d)** Write down the rule linking the $x$- and $y$-values.

## Understanding

**4** Which one of the points (-1, -1), (2, 2), (3, 4) and (5, 5) will not lie on the same straight line as the three other points?

**A** (-1, -1) **B** (2, 2) **C** (3, 4) **D** (5, 5)

**5** **(a)** Plot the points (-2, 8), (0, 6) and (5, 1) on a number plane.

**(b)** Draw a straight line passing through all the points.

**(c)** What is the $y$-coordinate of a point on the line if its $x$-coordinate is -1?

**(d)** What is the $x$-coordinate of a point on the line if its $y$-coordinate is 5?

**6** **(a)** Plot the points (1, 1), (1, 5), (5, 5) and (5, 1) on a number plane and join the points in order.

**(b)** Name the geometrical figure drawn in part **(a)**.

## Reasoning

**7** **(a)** Plot the points (0, 0), (2, 2) and (-5, -5) on a number plane.

**(b)** Draw a straight line passing through all the points.

**(c)** Write down the coordinates of three other points through which the line passes.

**(d)** Summarise the set of points in a table.

**(e)** Write down the rule linking the $x$- and $y$-values.

**(f)** By substituting $x = -3$ into the rule, show that the point (-3, -3) lies on the line.

**8** **(a)** Plot the points (-1, -5) and (5, 13) on a number plane and join them with a straight line.

**(b)** Write down the coordinates of three other points through which the line passes.

**(c)** Summarise the set of points in a table.

**(d)** Write down the rule linking the $x$- and $y$-values.

**(e)** Substitute $x = 0$ into the rule to show that (0, -3) does not lie on the line.

**(f)** Plot (0, -3) on the number plane to confirm it does not lie on the line.

## Open-ended

**9** **(a)** Plot the point (3, 3) on a number plane.

**(b)** Draw a straight line passing through this point and one other point.

**(c)** Write down the coordinates of two other points through which the line passes.

**(d)** Write down the rule linking the $x$- and $y$-values for this straight line.

**(e)** Repeat **(b)**, **(c)** and **(d)** using (3, 3) and a different point to find a different rule.

**10** Using -4 to 4 on the $x$- and $y$-axes:

**(a)** choose a point and write down its coordinates

**(b)** choose two other points that will form a line that is not vertical or horizontal

**(c)** find the rule for that line.

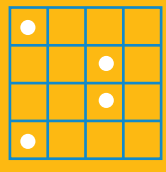
## Outside the Square Puzzle

### Gridlock

**Equipment required:** 1 brain, grid or graph paper

**(a)** In grid 1, mark four squares so that no two marked squares lie in the same row, column or diagonal.

For example, the arrangement shown is not allowed.

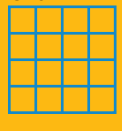


**(b)** In grid 2, mark five squares so that no two marked squares lie in the same row, column or diagonal.

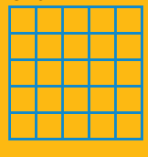
**(c)** Mark six squares in Grid 3 and seven squares in Grid 4 so that no two marked squares lie in the same row, column or diagonal.

**(d)** The challenge: Mark eight squares in Grid 5 so that no two marked squares lie in the same row, column or diagonal.

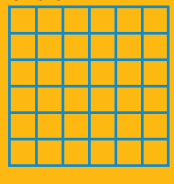
Grid 1



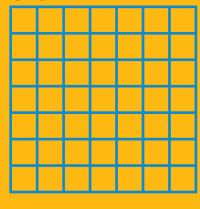
Grid 2



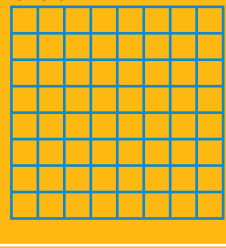
Grid 3



Grid 4



Grid 5



# Mathspace

# SPY vs SPY

## Equipment required:

For task #1: 1 brain, graph paper, ruler

For task #2: 1 brain

For task #3: 2 brains, 1 die, counters or small pieces of paper

While working on Operation Cartesian as a spy for the Confederation of Mathematicians, Freedo has been caught by the Anti-maths guerrilla forces deep in the jungle.

## Task #1: Frame the assassin

Freedo's last communication from headquarters gave him a picture of 'Zero', the most deadly of the Anti-maths guerrillas.

To decipher the picture, Freedo must apply his knowledge of Cartesian graphs. On your graph paper begin by creating a set of axes, from -8 to 8 along the $x$-axis and -10 to 7 along the $y$-axis.

Use a ruler to join each of the following sets of coordinates, like a dot-to-dot drawing. Where there is a •, lift your pen and start a new line without joining to the previous lines drawn.

(0, 7), (2, 7), (4, 6), (6, 4), (6, 1), (7, 2), (8, 0), (7, -2), (6, -3), (5, -5) •

(4, -6), (3, -8), (1, -10), (0, -9) •

(0, -8), (1, -7), (3, -7), (1, -6), (0, -6) •

(3, -7), (1, -5), (0, -5) •

(0, 2), (2, 4), (5, 4), (3, 3), (1, 1), (1, -1), (2, -2), (1, -3), (0, -3) •

(0, -4), (1, -3), (3, -4), (6, -6), (4, -6), (1, -4), (0, -4) •

(6, -1), (7, -1), (7, 1), (6, 0) •

(1, 1), (2, 0), (5, 1), (4, 2), (2, 2), (2, 1), (3, 1), (3, 2) •

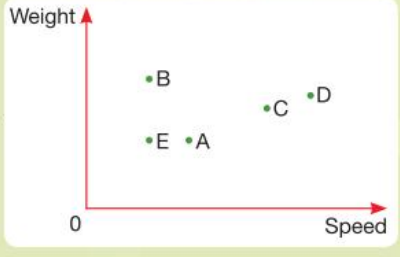
Then complete the image by drawing a reflection of the current image in the $y$-axis.

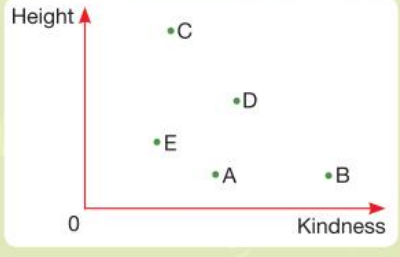
## Task #2: Guard help

Freedo is locked in a cage. He was given the following information from headquarters:

*'The guard who will help you is one of the two shortest guards and one of the two slowest guards.'*

Looking at the following graphs can you figure out which guard will help Freedo?



# Task #3: Quadrant Tic Tac Toe

Play this game to see if you are able to escape from the Anti-maths guerrilla forces.

Choose who will play the role of Zero and who will play the role of Freedo. Will Freedo escape?

Your aim is to be the first to have 3 counters in a row, column or diagonal.

Take turns in rolling. You can place counters on the intersections of the grid lines, according to the following rules.

Rolling a 1 means you must place a counter in quadrant 1.

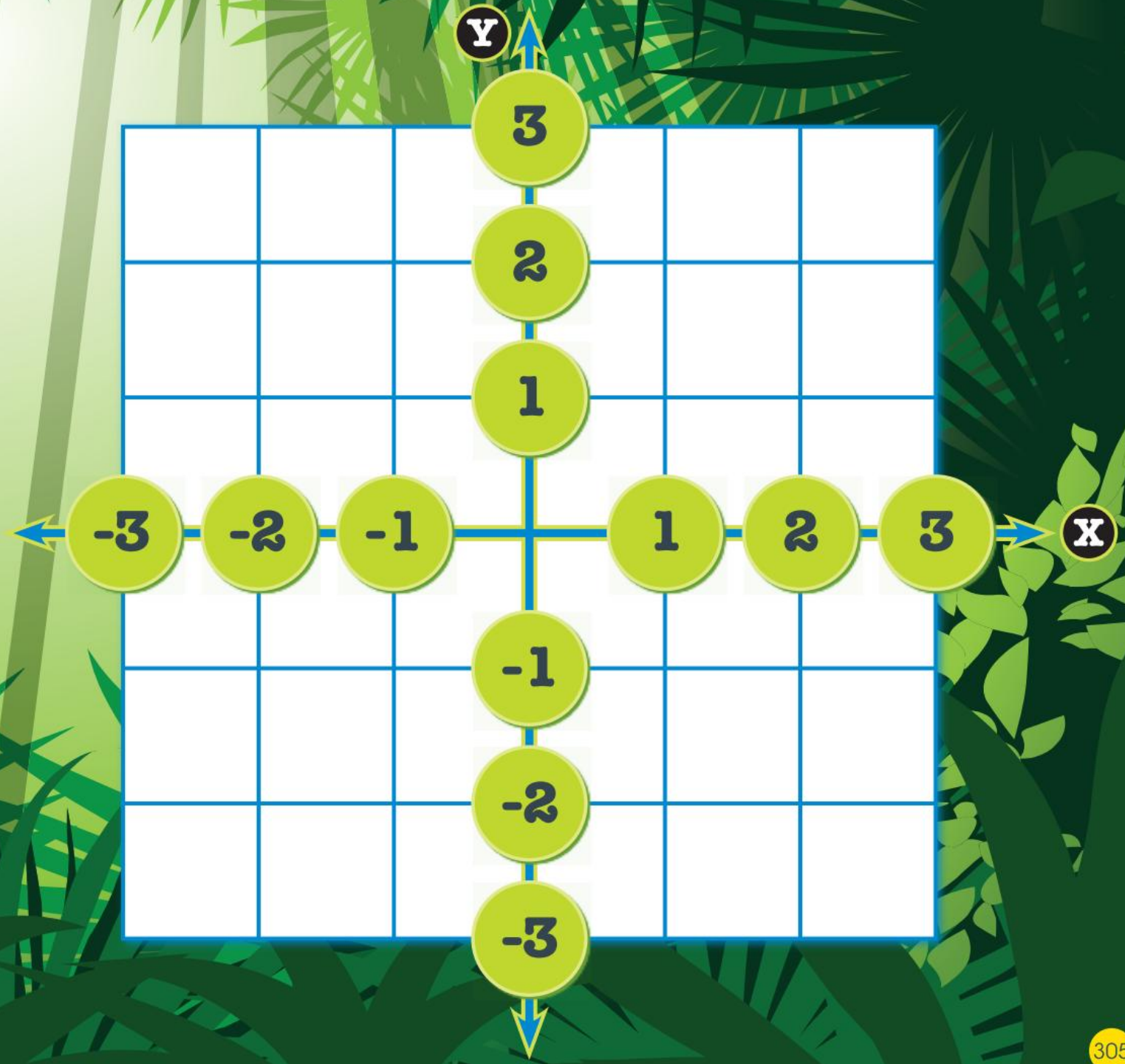
Rolling a 2 means you must place a counter in quadrant 2.

Rolling a 3 means you must place a counter in quadrant 3.

Rolling a 4 means you must place a counter in quadrant 4.

Rolling a 5 means you must place a counter on the $x$-axis.

Rolling a 6 means you must place a counter on the $y$-axis.



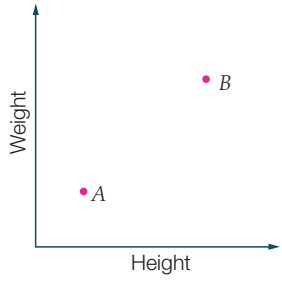
# 5.9 Interpreting graphs

Sometimes, when we plot points showing information relating to two variables, the points should not be joined.

This type of graph is called a **point graph**.

In the graph below, the points labelled *A* and *B* tell us about the height and weight of Oscar and Theodore.

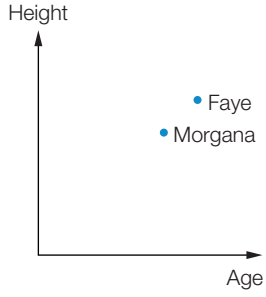
Point *A* matches Theodore (smaller height and lower weight) and point *B* matches Oscar (taller and heavier). The graph shows that Oscar is taller than Theodore and that Theodore weighs less than Oscar.






## Worked Example 16 WE16

This graph shows the age and height of Faye and Morgana.




Answer TRUE or FALSE to each of these statements.

**(a)** Faye is taller than Morgana.

**(b)** Morgana is older than Faye.

**(c)** Faye is younger than Morgana.

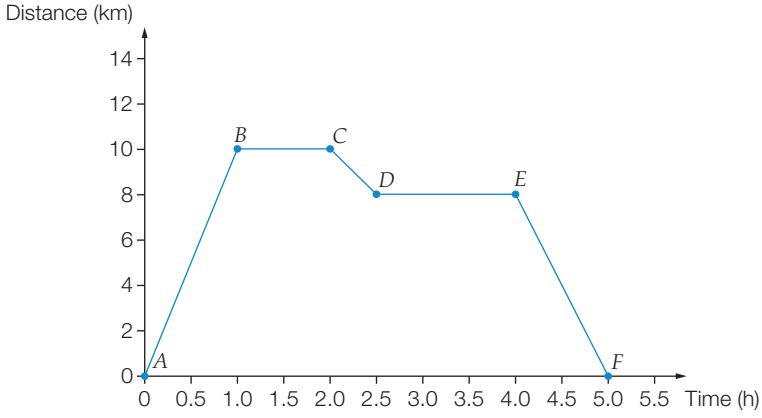
**(d)** Morgana is shorter than Faye.

| Thinking | Working |
|---|---|
| **(a)** Compare heights. Height increases the further you move up the vertical axis. The point for Faye is higher, so she is taller than Morgana. | (a) Faye is taller than Morgana. TRUE |
| **(b)** Compare ages. Age increases the further you move to the right. The point for Faye is further to the right, so she is older than Morgana. | (b) Morgana is older than Faye. FALSE |
| **(c)** Compare ages. Faye is older than Morgana as the point for Faye is further to the right. | (c) Faye is younger than Morgana. FALSE |
| **(d)** Compare heights. The point for Morgana is lower, so she is shorter than Faye. | (d) Morgana is shorter than Faye. TRUE |

If we join points with straight lines, they may not form a linear graph, as the points may not all lie on the same straight line. These graphs are called **line graphs**. An example of line graphs are travel graphs where time is plotted on the $x$-axis and distance is plotted on the $y$-axis. When lines are horizontal, the distance is not changing. When lines are steepest, the speed is greatest.

## Worked Example 17 WE17

May and Kim go jogging. They leave from Kim's house, jog to their friend Jenny's place, stay there for a swim, then jog to May's place. They stay there for a while and watch a DVD before jogging back to Kim's house.




Using the graph given above that shows their journey, answer the following questions.

**(a)** What are the graph coordinates of Kim's house?

**(b)** How far away from Kim's house does Jenny live?

**(c)** How long did it take to reach Jenny's house?

**(d)** How long did they stay at Jenny's house?

**(e)** How far from Kim's place does May live?

**(f)** How long did they stay at May's house?

**(g)** How long were they away from Kim's house?

**(h)** When were they jogging the fastest?

| Thinking | Working |
|---|---|
| **(a)** Identify where the journey begins ($A$). | (a) (0, 0) |
| **(b)** Look for the distance to $B$ on the $y$-axis. The $y$-coordinate of $B$ gives that distance. | (b) 10 km |
| **(c)** Look for the time taken to get from $A$ to $B$ on the $x$-axis. The $x$-coordinate of $B$ gives the time. | (c) 1 h |
| **(d)** The horizontal line $BC$ shows that they stayed in the same place during this time, so the time will be the time difference between $B$ and $C$. | (d) 1 h |
| **(e)** Look for the distance $D$ on the $y$-axis. The $y$-coordinate of $D$ gives the distance. | (e) 8 km |

| | | |
|---|---|---|
| **(f)** | The horizontal line $DE$ shows that they stayed in the same place during this time, so the time will be the time difference between $D$ and $E$. | (f) $1\frac{1}{2}$ h |
| **(g)** | Look for the time to get from $A$ to $F$ on the $x$-axis. The $x$-coordinate of $F$ gives the time. | (g) They were away for 5 hours. |
| **(h)** | Look for the steepest line. This is the line from $A$ to $B$ at the start of the jog. | (h) They jogged fastest from Kim to Jenny's place. |

# 5.9 Interpreting graphs

## Navigator

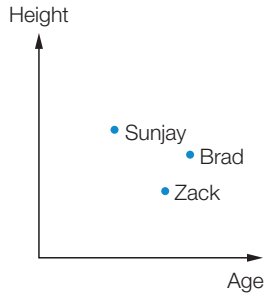
Answers page 655

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q11, Q14 | Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q14 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14 |

## Fluency

WE16

**1** This graph shows the age and height of Sunjay, Zack and Brad.




Answer TRUE or FALSE to each of these statements.

**(a)** Brad is taller than Sunjay.

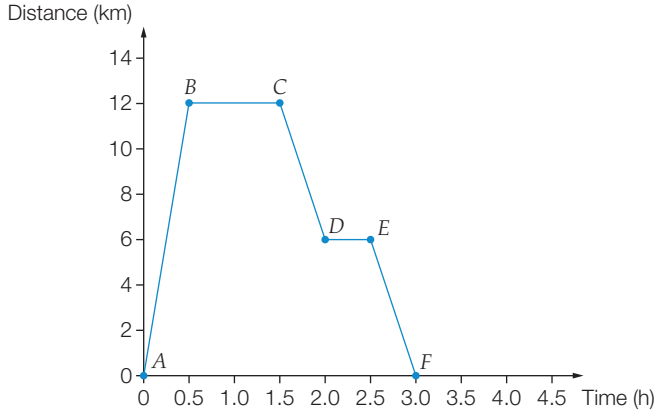
**(b)** Zack is the shortest.

**(c)** Brad is older than Zack.

**(d)** Sunjay is the youngest.

WE17

**2** Ray and Jim go for a bike ride. They leave from Jim's house, ride to their friend Terry's house, stay there for lunch, and then ride to Ray's house. After a short break, they ride back to Jim's house.




Using the graph given above that shows their journey, answer the following questions.

**(a)** What are the graph coordinates of Jim's house?

**(b)** How far away from Jim's house does Terry live?

**(c)** How long did it take to reach Terry's house?

**(d)** How long did they stay at Terry's house?

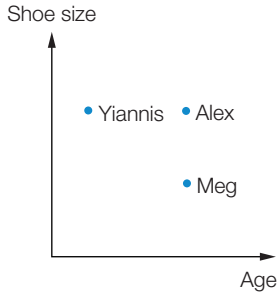
**(e)** How far from Jim's house does Ray live?

**(f)** How long did they stay at Ray's house?

**(g)** How long were they away from Jim's house?

**(h)** When were they riding the fastest?

**3** Use the graph opposite to answer the following questions.



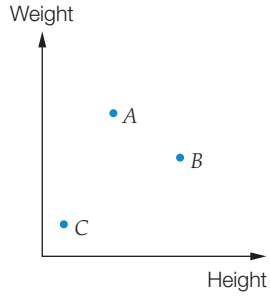

**(a)** Who is the youngest?

**(b)** Who takes the smallest shoe size?

**(c)** What can you say about the shoe sizes of Yiannis and Alex?

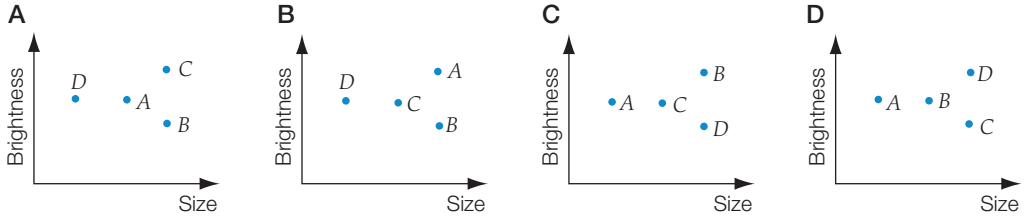
**(d)** What can you say about the ages of Meg and Alex?

**4** Consider this graph showing the weight and height of an elephant, a giraffe and a kangaroo. Match each point with the animal it represents.

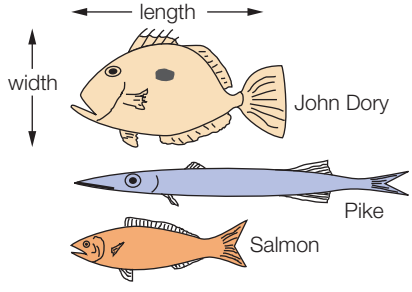


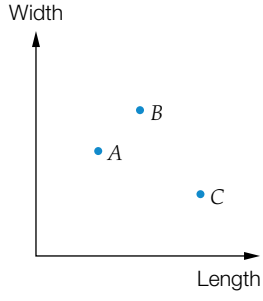

## Understanding

**5** Four light globes, *A*, *B*, *C* and *D* have the properties that globe *A* and globe *B* are equally bright and globe *C* and globe *D* are of equal size. Which graph can represent this situation?



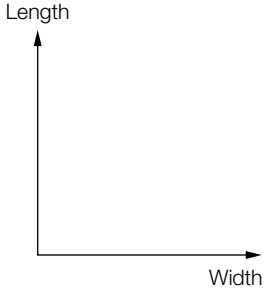

**6** Consider this graph showing the widths and lengths of various fish.



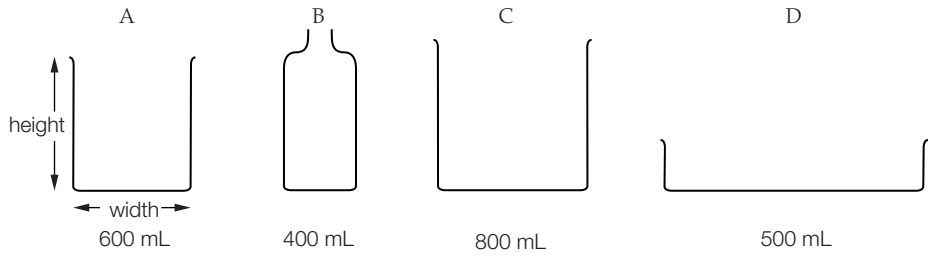




**(a)** Match each point with the fish it represents.

**(b)** Copy and complete this graph to show the positions of the points *A*, *B* and *C* for the three fish.

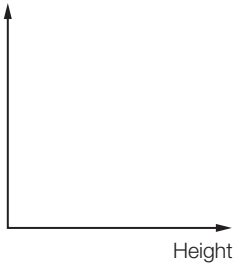


**7**

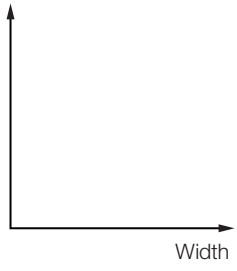



Complete the following point graphs for the containers A, B, C and D.

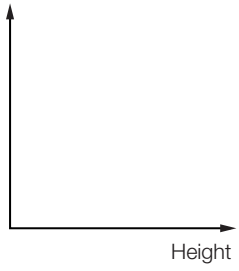
**(a)** Width



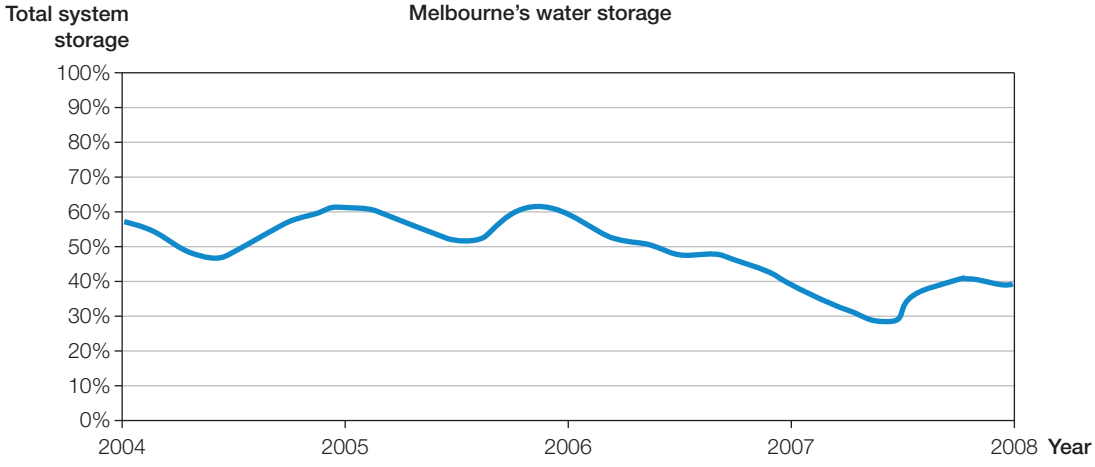

**(b)** Capacity




**(c)** Capacity




**8** Large cities are usually surrounded by dams that collect rain to supply the city with water. The total amount of water stored in the dams is called the water storage and when this amount is calculated as a percentage of the total capacity of all the dams it gives a measure of how full the dams are on average.




Above is a graph of Melbourne's water storage from 2004 to 2008. Use this graph to answer the following questions.

**(a)** What was the storage percentage at the start of 2004?

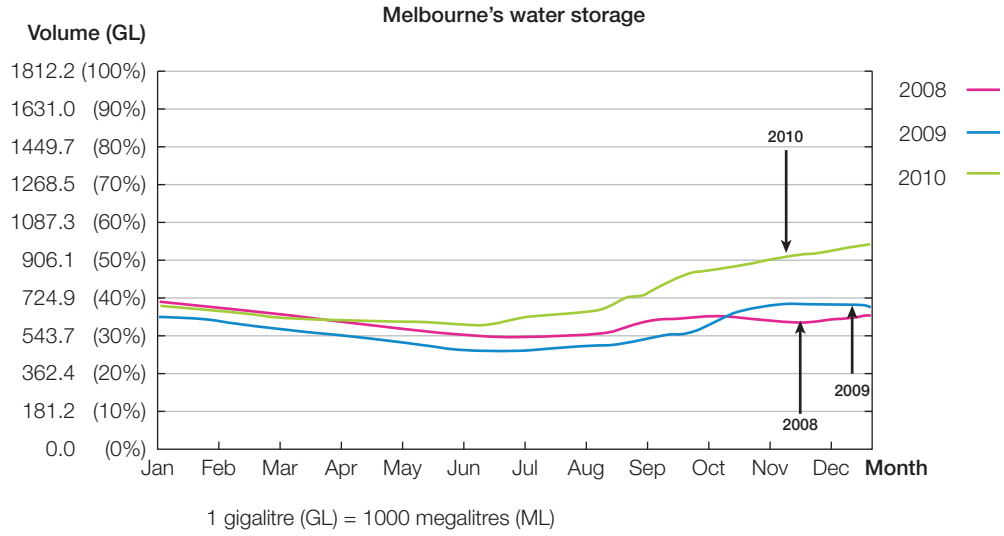
**(b)** Why did the water storage percentage rise in the second half of 2004?

**(c)** Why is there an increase in storage percentage in the second half of each year (except 2006)?

**(d)** What does the graph tell you about Melbourne's rainfall from the start of 2006 to mid 2007?

**(e)** When was the water storage the lowest in the time period 2004–2008? What was the lowest percentage level reached?

9




1 gigalitre (GL) = 1000 megalitres (ML)

Above is a graph of Melbourne's water storage for the years 2008, 2009 and 2010.

Use this graph to answer the following questions.

**(a)** Compare the storage levels at the start of 2008, 2009 and 2010. Were the levels similar? Which year started with the lowest storage level? What was the difference between the highest and lowest storage percentage at the start of each year?

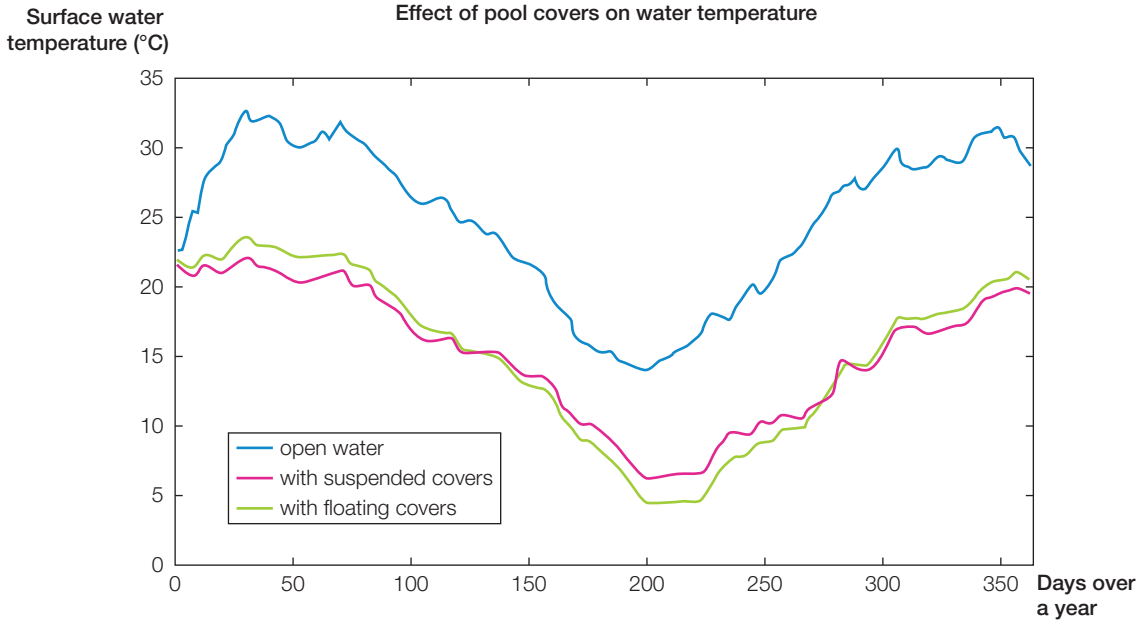
**(b)** Compare the storage levels at the end of 2009 and 2010. What is the difference in storage percentage?

**(c)** During which 3 months over the 3 year period was the storage level the highest?

**(d)** During which 3 months over the 3 year period was the storage level the lowest?

## Reasoning

**10** Evaporation is a major source of water loss from storage facilities. The rate at which water evaporates increases as temperature increases. The lower the temperature of the water, the less water will be lost to evaporation.




Above is a graph showing the effect on water temperature of using covers of different types on a pool. Use this graph to answer the following questions.

**(a)** Do you think covers reduce evaporation? Give reasons for your answer.

**(b)** Is one type of cover more effective than the other at reducing evaporation? Give reasons for your answer.

**(c)** When is a floating cover more effective than a suspended cover?

**(d)** What effect overall does the use of pool covers have on the water temperature?

**11** **(a)** What are the two variables shown in this graph?

**(b)** What happens to the shoe size as the height increases?

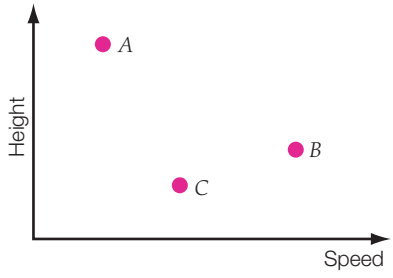


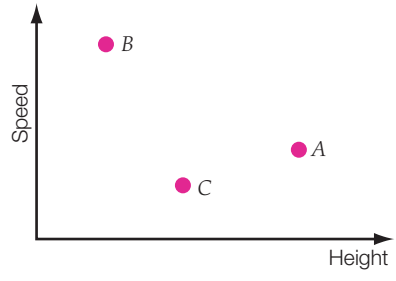




## Open-ended

**12** Agnes transferred the points on the first graph to the graph with swapped axes. She left *C* in the same place but swapped the positions of *A* and *B*.




**(a)** Explain why Agnes' actions are not correct for these graphs. When would it be correct to do the simple swap with *A* and *B*?

**(b)** What would need to be the properties of a point if it is to stay in the same place?

**(c)** Draw what the second graph should look like.

**13** Tom, Adam, Sheena and Toula play basketball. Sheena is shorter than Tom, and Toula scores more points than Adam. One point of view is that the taller the basketball player, the more points they score. Using the variables 'Height' and 'Points scored', draw two possible graphs that show the relationship.

**14** Draw a travel graph of your own and write a few sentences to explain the journey.

## Outside the Square Puzzle

### Solve the grid

Each letter stands for a different number, but it represents the same number each time it is used. The total for the first row and the first two columns is given.

| | | | | |
|---|---|---|---|---|
| D | D | E | D | 39 |
| E | D | D | E | |
| E | E | E | X | |
| X | E | X | X | |
| 40 | 42 | | | |

**1** What number does X represent?

**2** Find the total of each of the other rows and columns.

# Challenge 5

**1** George has 15 coins, all 50c and 20c pieces. If the 20c pieces were 50c pieces and the 50c pieces were 20c pieces, he would have $2.10 more. How many 50c pieces does he actually have?

**2** If $p \bigstar q$ means $3(p + q)$, what is the value of $5 \bigstar (2 \bigstar 8)$?

**3** Steven and Claire live next door to each other. The product of their house numbers is 483. What are their house numbers?

**4** If $x > 4$, put these terms in order from smallest to largest.

$$\frac{x}{4} \qquad \frac{4}{x} \qquad \frac{4}{x+1} \qquad \frac{x+1}{4}$$

**5** Each letter represents one of the digits 1, 2, 3, 4 or 5.

$$\begin{array}{r} \text{K L} \\ \times \quad \text{M} \\ \hline \text{P N} \\ \hline \end{array}$$

The answer to this multiplication is correct.

Which statement is correct?

**A** K = 3 **B** L = 3 **C** M = 3 **D** N = 3

**6** What are possible positive values for $a$ and $b$ if $(a \times b)^2 = a \times b^2$?

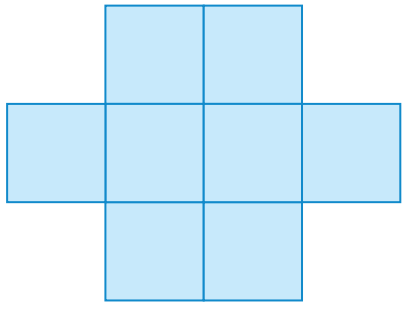
**7** In a 30-question test you earn 9 marks for every correct answer and lose 5 marks for every wrong answer. You must attempt every question.

**(a)** How many marks do you get for 15 correct answers? (Assume you attempt all the questions.)

**(b)** What is the greatest number of marks you can get if you have more answers wrong than correct?

**(c)** Is it possible to score exactly zero on the test? Explain why or why not.

**8** Place one of the numbers 1, 2, 3, 4, 5, 6, 7 and 8 in each square so that no consecutive numbers are beside each other; that is, 2 is not next to a 3 either above, below, to the left or to the right.



**9** If half the number represented by $x$ is 24, what is the value of $2x$?

**10** The sum of three consecutive integers is 90. What is the smallest of the three integers?

**A** 28 **B** 29 **C** 30 **D** 31

**11** Find two numbers which when multiplied together make 1 000 000 if neither of the numbers contains any zeroes.

# Chapter review 5

## D.I.Y. Summary

### Key Words

| | | | | |
|---|---|---|---|---|
| axes | equation | linear graph | relationship | variable |
| Cartesian plane | evaluate | ordered pair | substitute | |
| coefficient | expression | origin | table of values | |
| constant | formula | point graph | terms | |
| coordinates | like terms | pronumeral | unknown | |
| define | line graph | quadrant | unlike terms | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 A _____ is a letter or a symbol that represents a number.

2 Completing a _____ is helpful when finding the _____ that describes a pattern.

3 To find the value of the expression $2x + 3$, when $x = 1$, we need to _____ 1 for $x$, and _____ the expression.

4 The _____ is the point (0, 0). It is the point where the two _____ of the Cartesian plane intersect.

5 A pronumeral represents an _____ or a _____.

6 A number written next to a pronumeral is the _____ of the pronumeral. The pronumeral is multiplied by that number.

7 To simplify the expression $3x + 4x + 5$ you collect _____ .

8 The _____ (3, 5) can be plotted on the _____ by finding the point 3 across and 5 up. Draw a diagram clearly showing this point plotted in its correct location.

## Fluency

1 Write the following situations using algebra. Ex 5.1

(a) There are 12 biscuits in a packet. How many are in $n$ packets?

(b) Tarin has a packet of $p$ lollies. He eats half, then gives 3 to his brother. How many lollies does he have left?

(c) Multiply $p$ by 7, then subtract 9.

(d) Subtract $g$ from 11, then divide by 5.

2 Write an equation for each of the following situations, taking care to define all the variables you have used. Ex 5.2

(a) Joe's age in years is four plus three times Beth's age.

(b) Three bottles of juice and five salad rolls cost $18.60.

**3** For each of the following rules:

Ex 5.3

**(i)** draw a flowchart that shows how to get to $y$ from $x$

**(ii)** write the rule using algebra

**(iii)** copy and complete the table of values for that rule.

**(a)** Add three to $x$ to get $y$.

| $x$ | 57 | 34 | 12 | 4 | 1.1 | 64 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**(b)** To find $y$, multiply $x$ by three, then subtract five.

| $x$ | 4 | 3 | $2\frac{1}{2}$ | 9 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**4** Choose the correct algebraic notation for each rule.

Ex 5.3

**(a)** $y$ is equal to $x$ plus 6.

**A** $y = x - 6$ **B** $y = x + 6$ **C** $y = x \times 6$ **D** $x = y + 6$

**(b)** Subtract five from $x$, then multiply by three to get $y$.

**A** $y = (x - 3) \times 5$ **B** $y = (x + 5) \times 3$ **C** $y = (x - 5) \times 3$ **D** $y = x - 5 \times 3$

**5** For each of the following formulas, evaluate $p$ by substituting the given values of $a$.

Ex 5.4

**(a)** $p = 6 - 20a$ **(i)** $a = 3$ **(ii)** $a = -1$

**(b)** $p = 2(a - 4) + 11$ **(i)** $a = 2$ **(ii)** $a = -5$

**6** If you substitute $x = 3$ into the formula $y = 5x - 2$, the value of $y$ is:

Ex 5.4

**A** $y = 3$ **B** $y = 5$ **C** $y = 13$ **D** $y = 17$

**7** Use each of the following rules to complete the tables of values.

Ex 5.4

**(a)** $y = x - 7$

| $x$ | 7 | 9 | 12 | 20 | 8.1 | 107 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**(b)** $y = 10(x - 3)$

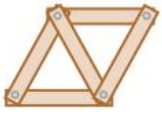
| $x$ | 4 | $5\frac{1}{2}$ | 13 | 7 | 78 | 54 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**8** The frame of a bridge is made up of triangular sections. The triangles are made up of girders held together by bolts.

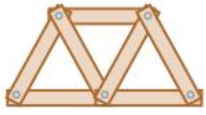
Ex 5.5



1 triangle
3 girders
3 bolts



2 triangles
5 girders
4 bolts



3 triangles
7 girders
5 bolts

**(a)** Complete the table of values by continuing the pattern.

| Number of triangles ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of girders ($g$) | 3 | 5 | | | |
| Number of bolts ($b$) | 3 | 4 | | | |

**(b)** Write down the rule that relates the number of triangles, $t$, to the number of girders, $g$.

**(c)** Write down the rule that relates the number of triangles, $t$, to the number of bolts, $b$.

**(d)** **(i)** How many girders are needed to create each extra triangular section?

**(ii)** How is this reflected in the table of values and the rule?

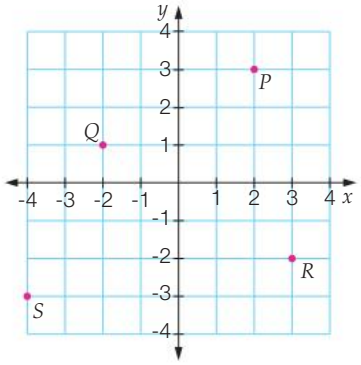
**(e)** **(i)** How many bolts are needed for each new triangular section?

**(ii)** How is this reflected in the table of values and the rule?

**9** Simplify each expression by adding or subtracting like terms. Ex 5.6

**(a)** $12a - 7a$ **(b)** $6a + 12b - 7a + 11b$ **(c)** $x + y + 3x$

**10** Write the coordinates of each of the following points shown on the Cartesian plane. Ex 5.7




**(a)** $P$ **(b)** $Q$

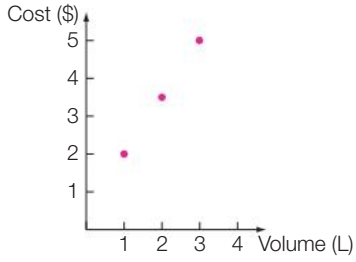
**(c)** $R$ **(d)** $S$

**11** **(a)** Plot the points (-1, -4), (1, 2), (2, 5) and (3, 8) on a number plane and draw a straight line passing through all the points. Ex 5.8

**(b)** Summarise the set of points in a table.

**(c)** Write down a rule linking the $x$- and $y$-values.

**12** Juice is sold in three sizes: 1 L, 2 L and 3 L. Use the graph opposite to answer the following questions. Ex 5.9




**(a)** How much does a 2 L juice cost?

**(b)** Which size is the best value? Why?

**(c)** Why is this information given in a point graph?

## Understanding

**13** Gayle, the kindergarten teacher, has 8 packets of crayons with $n$ crayons in each packet, plus an extra 3 loose crayons.

**(a)** Write an expression for the total number of crayons Gayle has.

**(b)** If all the crayons are to be shared equally between the 15 children in Gayle's class so that none are left over, write an expression to show how many crayons each child receives.

**(c)** Find the smallest possible value for $n$.

**14** Draw a flowchart for each of the following rules.

**(a)** $h = 7g$ **(b)** $d = 9c - 2$ **(c)** $d = \frac{c}{10} + 13$ **(d)** $h = 5(g + 8)$

**15** Tamsin is working out the number of bottles of soft drink she needs for a party. She is going to allow 1 bottle for every 4 people, plus 5 extra bottles.

**(a)** Write out Tamsin's rule using algebraic notation. Let $b$ = number of bottles of drink required and $n$ = number of people at the party.

**(b)** If Tamsin has 16 people at the party, how many bottles of drink will she need?

**16** Simplify if possible:

**(a)** $2a - 7b + 3a + 2$ **(b)** $5a + 12b - 7 - 3b$ **(c)** $x + y + xy$

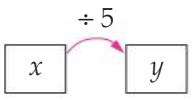
**17** The points (0, 1), (2, 5) and (5, 11) are plotted on a number plane and a straight line drawn through the points. Which one of the following points also lies on the line?

**A** (1, 2) **B** (3, 7) **C** (3, 6) **D** (4, 10)

## Reasoning

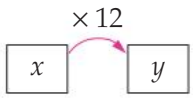
**18** The following tables of values have only the $y$-values filled in. Work backwards along the flowchart to determine the values of $x$ that were used, and complete the tables.

**(a)**




| $x$ | | | | | | |
|---|---|---|---|---|---|---|
| $y$ | 2 | -7 | 20 | 1 | 0 | -6 |

**(b)**




| $x$ | | | | | | |
|---|---|---|---|---|---|---|
| $y$ | 72 | 120 | 36 | -12 | 0 | 60 |

**19** For each of these tables, find the rule that is being used.

**(a)**

| $x$ | 76 | 54 | 8 | 28 | 9 | 103 |
|---|---|---|---|---|---|---|
| $y$ | 69 | 47 | 1 | 21 | 2 | 96 |

**(b)**

| $x$ | 15 | 10 | 2 | 7.3 | 9 | 22 |
|---|---|---|---|---|---|---|
| $y$ | 34 | 24 | 8 | 18.6 | 22 | 48 |

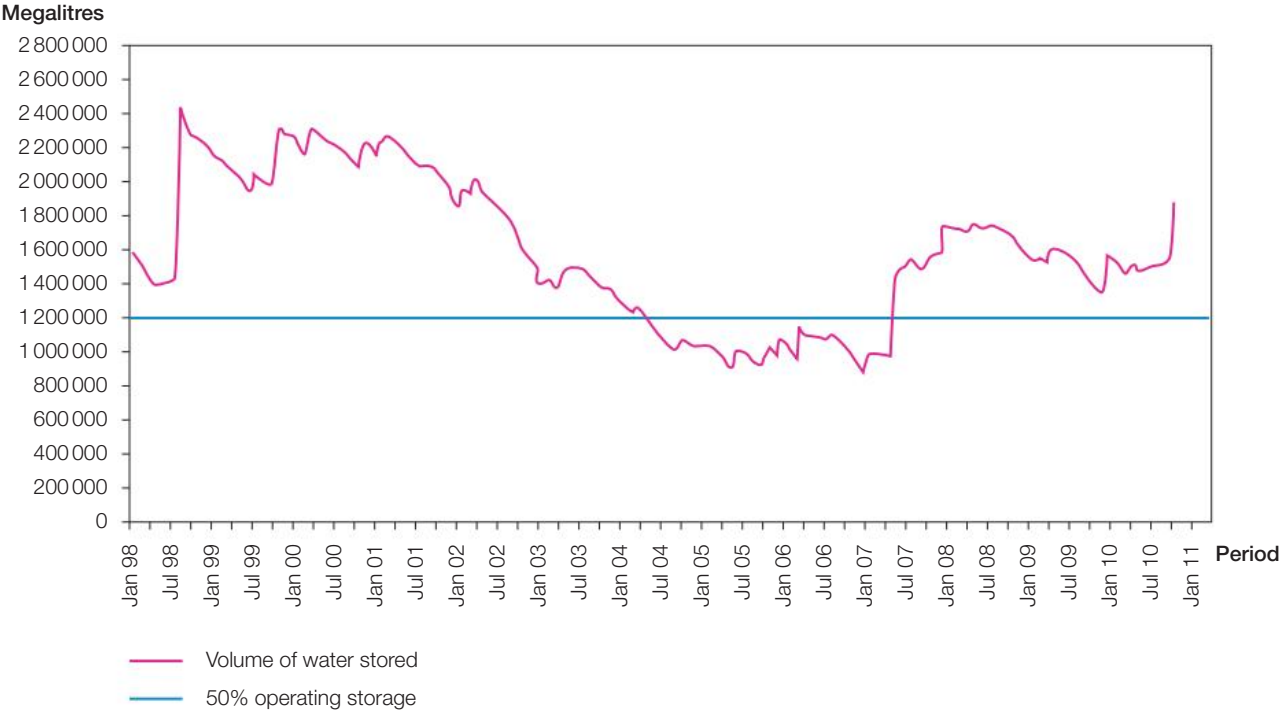
**20** Jack has $21.00 to buy some apples and oranges. An apple costs $1.80 and an orange costs $1.40.

**(a)** Using appropriate pronumerals to represent the number of apples and oranges purchased, write an equation to show the situation when Jack spends all of the $21.00.

**(b)** How many apples and how many oranges could Jack buy using all the money?

**(c)** If Jack buys 4 apples and 5 oranges, how much change will he receive?

**21**




Above is a graph of Sydney's water storage from 1998 to 2010. The volume of water stored is given in megalitres (ML). 1 ML = 1 000 000 litres.

Use this graph to answer the following questions.

**(a)** What was the volume of water (in megalitres) at the start of 2010?

**(b)** During which month and year were the dams the fullest?

**(c)** During which month and year were the dams at their lowest capacity?

**(d)** What does the shape of the graph tell you about Sydney's rainfall from July 1998 to January 1999?

**(e)** What does the shape of the graph tell you about Sydney's rainfall from January 2000 to July 2007?

**(f)** During what period was the storage level below 1 200 000 megalitres (below 50%)?

# NAPLAN practice 5

## Numeracy: Non-calculator

**1** A number is multiplied by eight and then 5 is added. The answer is 61. What is the number?

**2** $x$ and $y$ stand for numbers. $x$ and $y$ are connected by a rule.

| $x$ | 2 | 3 | 5 | 8 |
|---|---|---|---|---|
| $y$ | 8 | 13 | 23 | 38 |

What is the rule?

**A** $y = 4x + 1$ **B** $y = 5x - 2$

**C** $y = 4x$ **D** $y = 10x - 27$

**3** Ting Li followed this rule. She started with 8.

| Starting number 8 | add 4 → | | multiply by 3 → | | subtract 12 → | Final answer |
|---|---|---|---|---|---|---|

**(a)** What is the correct final answer using Ting Li's starting number of 8?

**(b)** Alex followed the same rule using a different starting number.

Alex's starting number is -2. What was his final answer?

## Numeracy: Calculator allowed

**4** Ethan followed a different rule to Ting Li. There were three steps. Ethan started with the number 21 and ended up with a final answer of 30.

| Starting number 21 | Step 1 → | | Step 2 → | | Step 3 → | Final answer |
|---|---|---|---|---|---|---|

**(a)** Write the numbers from 1 to 3 on the lines below to show an order of steps that Ethan followed.

Step _____ add 8

Step _____ divide by 3

Step _____ double the number

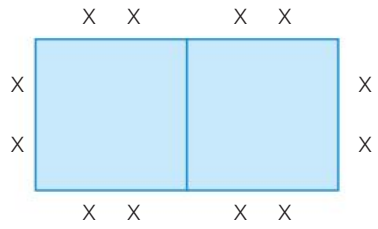
**(b)** Repeat the procedure in part **(a)** to get a final answer of 22.

**5** A pack of 10 lollipops costs $4.60.
A pack of 6 costs $3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

**6** Yusef investigated the seating plan for a restaurant using this table of values.

| Number of tables | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Number of people seated | 12 | 16 | 20 | ? |



The diagram shows the seating arrangement for 2 tables.

What rule did Yusef use to work out how many people can be seated at 5 tables?

**A** number of people seated = number of tables

**B** number of people seated = number of tables + 4

**C** number of people seated = number of tables × 6 − 1

**D** number of people seated = number of tables × 4 + 4

# 6


TIM
27
GENERALI
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TIM

# Measurement 6

**Hat trick at Qatar.** In 2009, 23-year-old Australian Casey Stoner won the Moto GP in Qatar for the third season in a row.

The 22-lap race was postponed because torrential rain made it impossible to see far enough ahead on the twists and turns with the floodlights reflecting off the wet track.

Casey won the race with a comfortable 7.771 second lead over Valentino Rossi. Riding his 800 cc Ducati, he reached speeds of up to 324.7 km/h on the 1.068 km main straight.

In 2007, Casey won his first World Motor Cycle Grand Prix (Moto GP) title, aged just 21. He was the 2008 Young Australian of the Year.

In 2010, Casey won his fourth straight Australian Moto GP at Phillip Island. He led all the way and crossed the line 8.598 seconds clear of Jorge Lorenzo.

### Forum

In swimming and running races, times are measured to 0.01 s. In this fraction of time the swimmer or runner covers about 2 cm. How far would a motorcyclist travel in this time? Why are times for Grand Prix and Moto GP races measured to 0.001 s?

Casey Stoner rides an 800 cc motor bike. What does the 'cc' stand for?

Do you think a knowledge of maths would help you become a good Moto GP racer?

### Why learn this?

Measurement helps us keep records, whether it be the height of a growing child or the volume of water in a dam. Good measurement skills are useful in many practical ways. They help a carpenter to work out how much wood is needed to construct a cupboard, enable a painter to calculate the volume of paint required to paint a room and allow a gardener to determine the area of lawn to be planted or fertilised.

**After completing this chapter you will be able to:**

- choose appropriate units for measuring and convert between them
- estimate common measurements using standard units
- calculate the perimeter of different shapes
- calculate the area of different shapes made up of rectangles and triangles
- calculate the volume of prisms.

# Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R6.1

**1** Calculate the following.

**(a)** $3.4 + 6.8 + 4.2 + 2.1$ **(b)** $45.6 - 5.28$ **(c)** $54 + 11.3 - 26.04$

**(d)** $6 \times 4.2$ **(e)** $9.2 \times 6.1$ **(f)** $18.96 \times 1000$

**(g)** $2780 \div 100$ **(h)** $3 \div 10$ **(i)** $45.9 \div 1000$

Worksheet R6.2

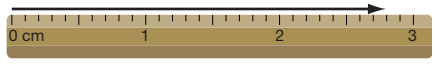
**2** State the length of the arrow shown on each ruler:

**(i)** in millimetres **(ii)** in centimetres.

**(a)**

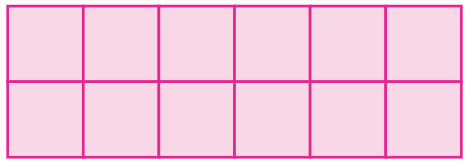



**(b)**




Worksheet R6.3

**3** This rectangle is made up of centimetre squares. Calculate:



**(a)** the perimeter in cm

**(b)** the area in $cm^2$.

Worksheet R6.4

**4** Copy and complete the following conversions.

**(a)** 2 km = _____ m **(b)** 30 mm = _____ cm **(c)** 5 m = _____ cm

**(d)** 1500 g = _____ kg **(e)** 4 L = _____ mL **(f)** 20 kg = _____ g

**(g)** 250 mL = _____ L **(h)** 1.75 m = _____ mm **(i)** 500 g = _____ kg

Worksheet R6.5

**5** How many sugar cubes are in the stack shown here?



## Key Words

| | | | |
|---|---|---|---|
| area | height | metric | perpendicular |
| base | kilometre | millimetre | rectangular prism |
| centimetre | length | parallelogram | volume |
| composite shape | metre | perimeter | width |

# Units of length

# 6.1

The system of measurement used in Australia up until 1970 was the imperial, or British system, which uses units of length such as inches, feet, yards, miles, chains and furlongs. In this system, it is awkward to convert between the different units.

In the 17th century, French scientists created another system of measurement, known as the **metric** system. All lengths in the metric system are derived from dividing or multiplying a standard length, the '**metre**', by powers of 10 (10, 100, 1000, …). This makes it easier to convert between lengths. The metric system was introduced into Australia in 1970. The metric units of **length** used most often are:

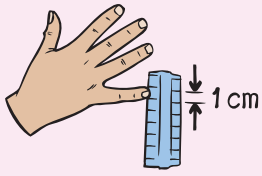
In the 1100s, King Henry I of England decreed that the distance from the tip of his nose to the end of his finger would be called a 'yard'. This unit of length was then used for nearly 900 years!















**kilometre (km)**
1 km = 1000 metres

**metre (m)**
1 m = 100 centimetres

**centimetre (cm)**
1 cm = 10 millimetres

**millimetre (mm)**

When estimating lengths, it helps to be able to picture the size of each unit of length, or compare the object in question to a 'reference' measurement.

## Worked Example 1

WE1

Use the reference measurement to help you estimate:

**(a)** the length

**(b)** the height

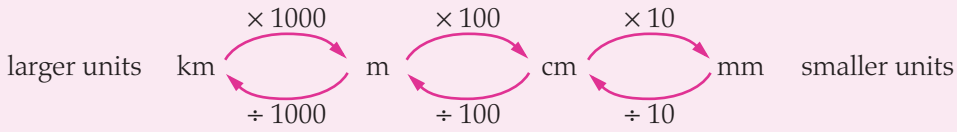
of this motor home.




| Thinking | Working |
|---|---|
| **(a)** Look at the reference measurement. Estimate how many times this length would need to be used to match the length of the motor home. | (a) The motor home is approximately 5 m long. |
| **(b)** Again, estimate how many times the length of the reference measurement would need to be used to match the height of the motor home. | (b) The motor home is about 3 m high. |

## Converting units of length

The ease with which we can convert between units is the great advantage of the metric system. Converting between metric units of length is made easy because we only ever need to multiply or divide by powers of 10 to change to different units.




To convert from a *larger* unit to a *smaller* unit, we *multiply*.
To convert from a *smaller* unit to a *larger* unit we *divide*.

The conversion table above shows that when converting from kilometres to metres, we multiply by 1000. When converting from centimetres to metres, we divide by 100. To convert from kilometres to centimetres we multiply by 1000, then by 100.

## Worked Example 2 WE2

Copy and complete the following conversions.

**(a)** 62.4 km = _____ m **(b)** 87.5 cm = _____ m

| Thinking | Working |
|---|---|
| **(a)** Larger unit to smaller unit, so multiply. There are 1000 m in 1 km, so multiply by 1000. | *(a)* 62.4 km<br>= 62.4 × 1000 m<br>= 62 400 m |
| **(b)** Smaller unit to larger unit, so divide. There are 100 cm in 1 m, so divide by 100. | *(b)* 87.5 cm<br>= 87.5 ÷ 100 m<br>= 0.875 m |

## Worked Example 3 WE3

Copy and complete the following conversions.

**(a)** 8.7 m = _____ mm **(b)** 530 cm = _____ km

| Thinking | Working |
|---|---|
| **(a)** **1** Larger unit to smaller unit, so multiply. First, multiply by 100 to convert m to cm. | *(a)* 8.7 m<br>= 8.7 × 100 cm<br>= 870 cm |
| **2** Then, multiply by 10 to convert cm to mm. | = 870 × 10 mm<br>= 8700 mm |
| **(b)** **1** Smaller unit to larger unit, so divide. First, divide by 100 to convert cm to m. | *(b)* 530 cm<br>= 530 ÷ 100 m<br>= 5.3 m |
| **2** Then, divide by 1000 to convert m to km. | = 5.3 ÷ 1000 km<br>= 0.0053 km |

Notice that in part **(a)** of Worked Example 3, we multiply by 100, then by 10. This is equivalent to multiplying by 1000. In part **(b)** we divide by 100, then by 1000. This is equivalent to dividing by 100 000.

# 6.1 Units of length

## Navigator

| | | |
|---|---|---|
| Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q15, Q16, Q18 | Q1, Q2 Column 2, Q3 Column 2, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18 | Q1, Q2 Column 3, Q3 Column 3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18 |

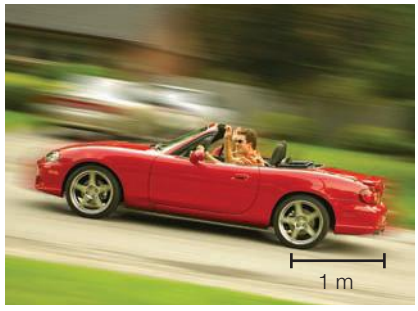
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## Fluency

**1** Use the reference measurement to help you estimate the following.

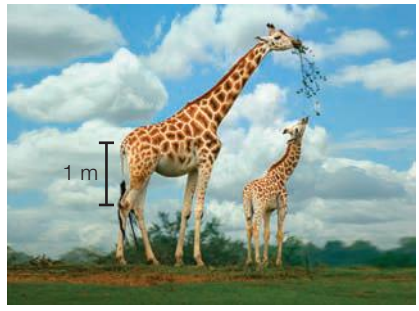
WE1

**(a)**



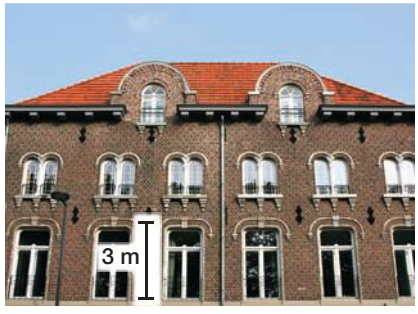

The length of this car.

**(b)**



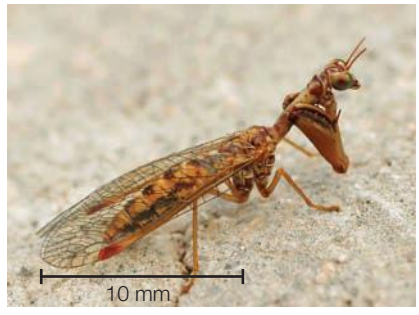

The adult giraffe's height.

**(c)**




The height of the tallest part of this building.

**(d)**




The length of this mantis fly.

**2** Copy and complete the following conversions.

WE2

**(a)** 5 km = _____ m **(b)** 3.6 km = _____ m **(c)** 0.008 km = _____ m

**(d)** 65 m = _____ cm **(e)** 0.55 m = _____ cm **(f)** 1.2 m = _____ cm

**(g)** 2.9 cm = _____ mm **(h)** 6.1 cm = _____ mm **(i)** 0.35 cm = _____ mm

**(j)** 90 cm = _____ m **(k)** 0.3 cm = _____ m **(l)** 3750 cm = _____ m

**(m)** 4200 m = _____ km **(n)** 570 m = _____ km **(o)** 9.2 m = _____ km

**(p)** 80 mm = _____ cm **(q)** 255 mm = _____ cm **(r)** 1.8 mm = _____ cm

**3** Copy and complete the following conversions.

WE3

**(a)** 3.2 m = _____ mm **(b)** 4.95 m = _____ mm **(c)** 9000 cm = _____ km

**(d)** 34 500 cm = _____ km **(e)** 0.003 km = _____ cm **(f)** 560 mm = _____ m

**(g)** 97 mm = _____ m **(h)** 0.342 km = _____ mm **(i)** 7800 mm = _____ km

**(j)** 1.92 km = _____ cm **(k)** 2400 cm = _____ km **(l)** 0.89 m = _____ mm

**4** State which metric unit would be most appropriate for measuring:

**(a)** the length of your foot

**(b)** the distance from Sydney to Melbourne

**(c)** the distance around an athletics track

**(d)** the length of a shoelace

**(e)** the distance run in a marathon

**(f)** the length of your bed.

**5** **(a)** Which of the following gives the best estimate of the distance across a suburban street?

**A** 50 cm **B** 5 m **C** 50 m **D** 5 km

**(b)** Which of the following gives the best estimate of the length of a mouse's tail?

**A** 0.1 cm **B** 1 cm **C** 10 cm **D** 100 cm

**(c)** The diameter of a 20 cent coin is about:

**A** 2.8 mm **B** 2.8 cm **C** 2.8 m **D** 2.8 km

**(d)** The length of a ball-point pen could be:

**A** 15 mm **B** 50 mm **C** 15 cm **D** 15 m

**6** Place the lengths below in order from smallest to largest, by first converting them to a common unit.

**(a)** 0.4 m, 4000 mm, 4 cm, 0.04 km

**(b)** 3.2 km, 360 m, 290 000 cm, 3100 m

**(c)** 71.4 m, 0.64 km, 5600 cm, 820 000 mm

**(d)** 0.9 cm, 90 mm, 0.095 m, 0.0089 km

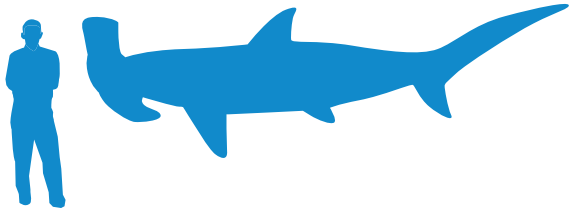
## Understanding

**7** At the Beijing Olympics in 2008, Australia's Steve Hooker won a gold medal and broke the Olympic record in the men's pole vault, clearing the bar at 596 centimetres. How many metres is this?



**8** Situated on Heard Island, Big Ben is Australia's only active volcano. Its summit is 2745 m above sea level. What is its height in kilometres?

**9** The average man is 1.8 metres tall. Use the diagram to estimate the length of the hammerhead shark.



**10** The bird-eating spider of South America has a body length of 89 mm and a leg span of 254 mm. Write these measurements in centimetres.

**11** A giant jellyfish was once measured at 2290 mm across and 36 000 mm long. Convert these dimensions to metres.

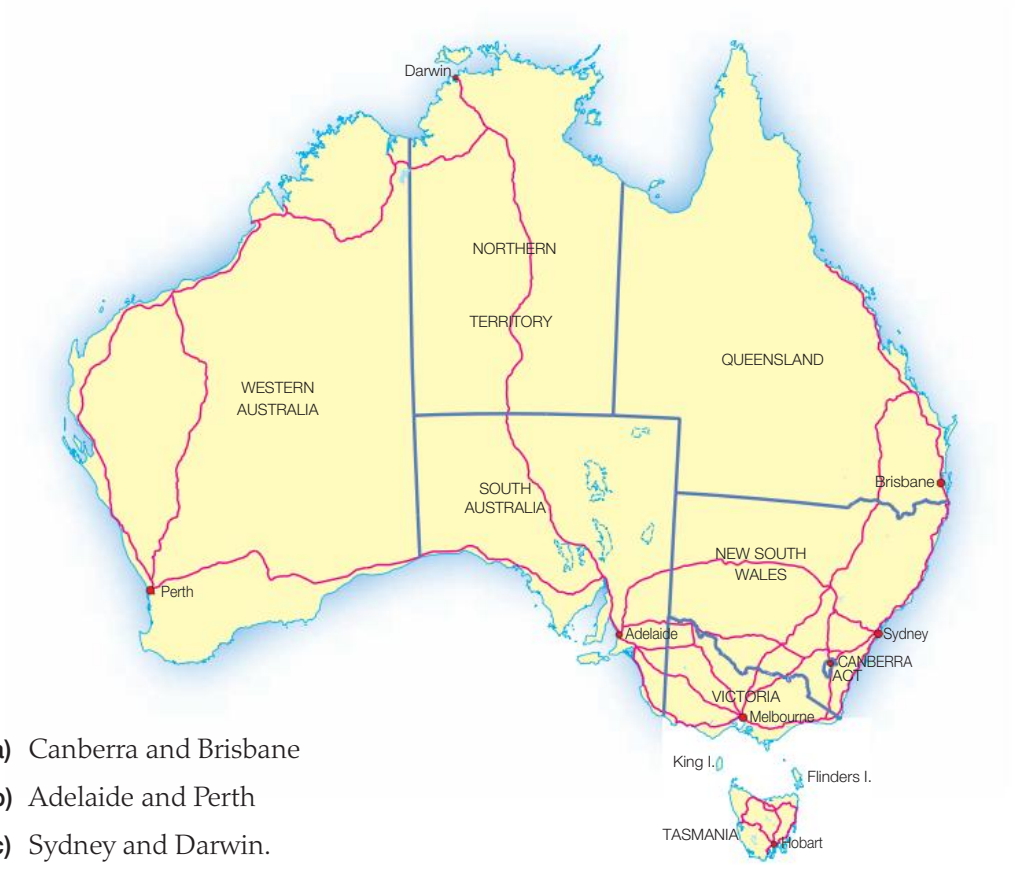
**12** Claire is building a desk and she wants it to be wide enough so that she can fit the length of two A4 sheets across it. If the length of an A4 sheet is 298 mm, how wide does Claire's desk have to be in millimetres? What is this length in centimetres?






## Reasoning

**13** The direct distance 'as the crow flies' between Melbourne and Adelaide is about 800 km. Estimate the direct distance between:




**(a)** Canberra and Brisbane

**(b)** Adelaide and Perth

**(c)** Sydney and Darwin.

**14** One of the fastest growing plants in the world is the bamboo plant. Its stem can grow 300 mm a day. If, on Monday morning, a bamboo plant was one metre tall, and it grew 300 mm every day, how tall will it be on Saturday morning? Write the answer in metres.

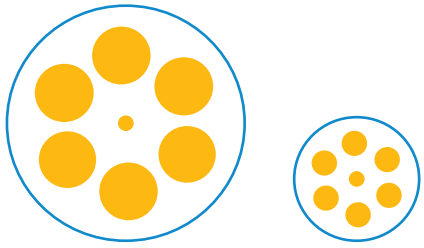


**15** **(a)** Guess the answer to each of the following, then use your ruler to help you decide.

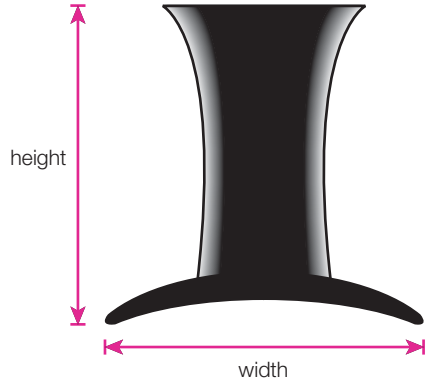
**(i)** Which horizontal line is longer?



**(ii)** Which diagram has the larger central circle?



**(iii)** Which is longer, the height of the hat, or its width?




**(b)** Explain each illusion or effect above and why you think it happens.

## Open-ended

**16** A builder jotted down the measurements needed for a door.

| | |
|---|---|
| height of door | 2032 mm |
| width of door | 821 mm |

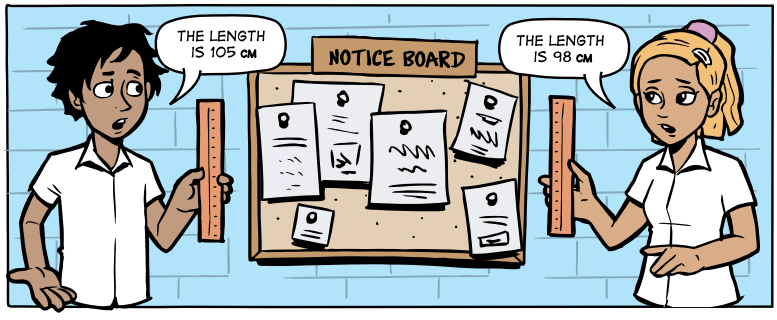
Why did the builder write these measurements using millimetres instead of cm or m?

**17** The following are mistakes made by a student when converting units. For each, explain what mistake has been made and write the correct answer.

(a) 27 m = 0.27 cm ✗ (b) 765 mm = 7.65 cm ✗

(c) 3800 m = 3.08 km ✗ (d) 1356 cm = 1.356 m ✗

**18** Two students have been asked to measure the length of the school notice board.




Suggest possible reasons for the difference in their answers.

# Perimeter

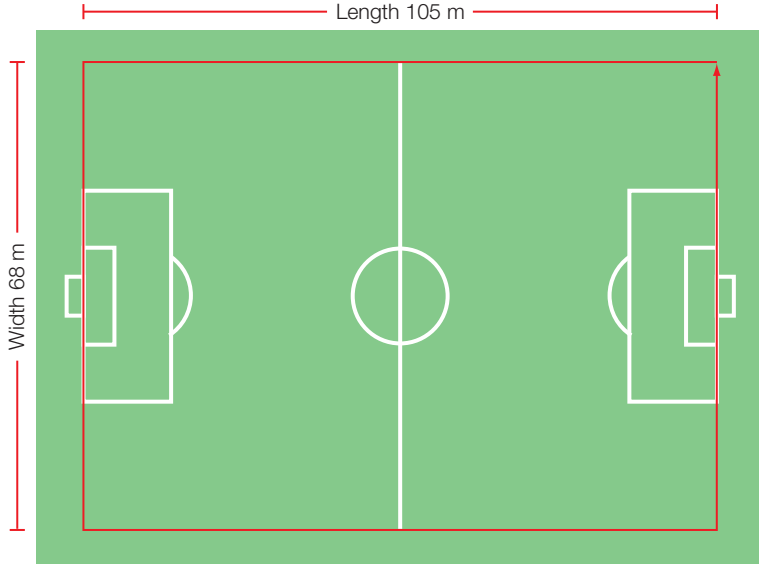
6.2

**Perimeter** is the distance along the boundary of a shape.

The word perimeter comes from two Greek words: *peri*, meaning 'around' and *metron*, meaning 'measure'.

Here are some examples:

- A frame goes along the perimeter of a picture.
- A fence is built along the perimeter of a property.
- A soccer pitch has a boundary line marked along its perimeter (shown here in red).




To find the perimeter of a shape, we simply add up the lengths of the sides (first making sure they are all in the same units).

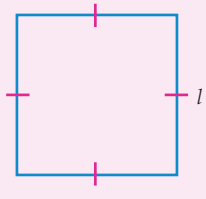
For example, the perimeter of this soccer pitch is 105 + 68 + 105 + 68 = 346 m.

Because the pitch is rectangular in shape, it has 2 pairs of equal sides: 2 lengths and 2 **widths**.

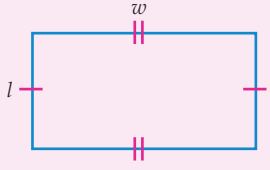
We can use this to write the perimeter of the pitch as $P = 2 \times 105 + 2 \times 68$ m. If we let the pronumerals $l$ and $w$ represent the length and the width, we can write a formula for the perimeter of a rectangle: $P = 2l + 2w$. This formula can also be written as $P = 2(l + w)$: Add the length and the width, then multiply by 2.

A square has 4 equal sides. If we use $l$ to represent the length of each side, we can write the perimeter as: $P = l + l + l + l$. A shorter way to write this is $P = 4l$.

To find the perimeter of a square or a rectangle:




$P = 4l$



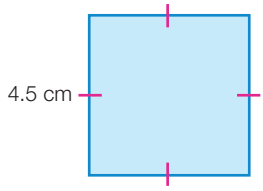

$P = 2l + 2w$
or
$2(l + w)$

Sides that are marked in the same way (with the same number of small dashes) have the same length.

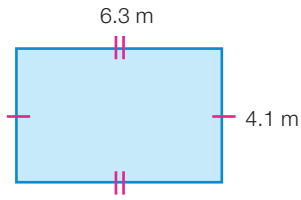
## Worked Example 4 WE4

Find the perimeter of each of these shapes.

(a)



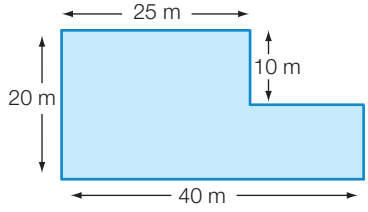

(b)

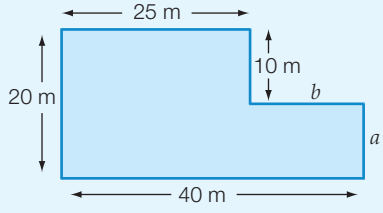



| Thinking | | Working |
|---|---|---|
| (a) 1 | Write down the formula for the perimeter of a square. | (a) $P = 4l$ |
| 2 | Substitute the side length into the formula. | $= 4 \times 4.5$ |
| 3 | Evaluate, writing the answer with the correct units. | $= 18$ cm |
| (b) 1 | Write down the formula for the perimeter of a rectangle. | (b) $P = 2l + 2w$ |
| 2 | Substitute the side lengths into the formula. | $= 2 \times 6.3 + 2 \times 4.1$<br>$= 12.6 + 8.2$ |
| 3 | Evaluate, writing the answer with the correct units. | $= 20.8$ m |

## Worked Example 5 WE5

Calculate the perimeter of the shape below.




| Thinking | | Working |
|---|---|---|
| 1 | First, find the lengths of the unmarked sides. Label them $a$ and $b$, then use the given lengths to find them. |  <br>$a = 20$ m $- 10$ m<br>$= 10$ m<br>$b = 40$ m $- 25$ m<br>$= 15$ m |
| 2 | Add up the lengths of all the sides. | $P = 10 + 25 + 20 + 40 + 10 + 15$<br>$= 120$ m |

# 6.2 Perimeter

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q12 | Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q11, Q12 | Q1 (a)–(c), Q2 (a)–(c), Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12 |

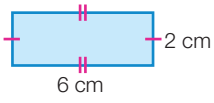
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## Fluency

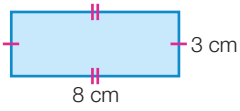
**1** Find the perimeter of each of these shapes.

WE4

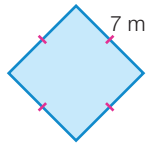
**(a)**



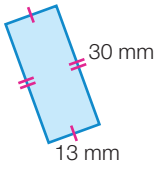

**(b)**



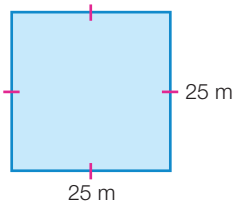

**(c)**



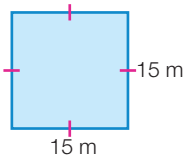

**(d)**




**(e)**



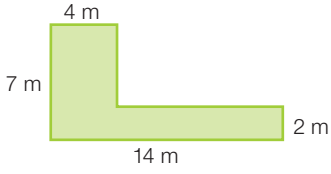

**(f)**



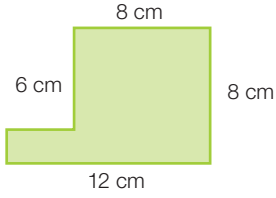

**2** Calculate the perimeter of the shapes below.

WE5

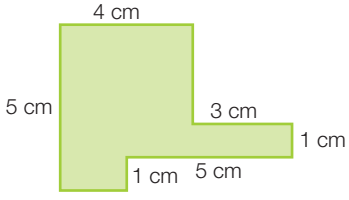
**(a)**



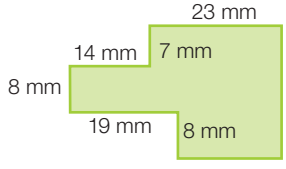

**(b)**



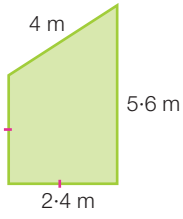

**(c)**



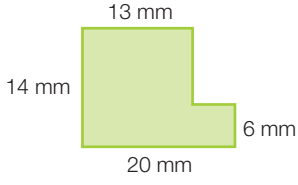

**(d)**




**(e)**




**(f)**




**3 (a)** The perimeter of a rectangle with a length of 25 cm and a width of 10 cm is:

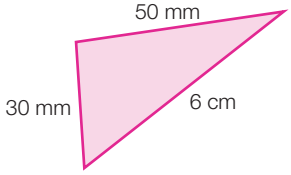
**A** 35 cm **B** 45 cm **C** 70 cm **D** 250 cm

**(b)** The perimeter of a square of side length 460 mm is:

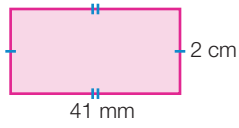
**A** 18.4 m **B** 9.2 m **C** 1.84 m **D** 0.92 m

**4** Calculate the perimeter of each of the following, first converting side lengths to the smaller unit when necessary.

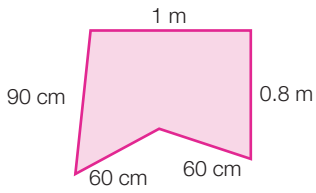
**(a)**



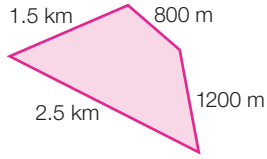

**(b)**



**(c)**




**(d)**

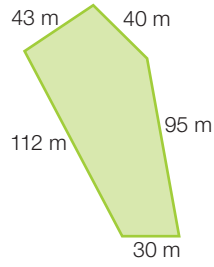



## Understanding

**5** A yacht sails three straight legs of a course which form a triangle. The length of the legs are 2.5 km, 1.7 km and 3.9 km. What distance does the yacht sail to complete the course?

**6** A group of phys. ed. students run around the perimeter of a court in their gymnasium, which is rectangular and has dimensions 38 m by 16 m. What distance will they run if they complete 6 laps?

**7** Juan wishes to run two strands of wire around his property to mark its boundary. A plan of the property is shown on the right. If he can buy the necessary wire for 60 cents per metre, how much will the wire cost him?




**8** Estimate the perimeter of each of the following objects.

**(a)** a 30 cm ruler **(b)** your desk

**(c)** a basketball court **(d)** your school's boundary fence.




## Reasoning

**9** If the perimeter of a rectangle is 240 mm and its width is 20 mm, then its length is:

**A** 100 mm **B** 110 mm **C** 120 mm **D** 220 mm

**10** The local swimming pool is rectangular, with dimensions of 25 m by 12 m. If it has a 1 m wide rectangular path around its perimeter, what is the distance along the outer edge of the path?

## Open-ended

**11** The length and width of a rectangle are both whole numbers in centimetres. Write down some possible dimensions if its perimeter is 84 cm.

**12** "I am holding a picture of a shape with a perimeter of 16 cm" the teacher said, "but it is not a rectangle nor a triangle". Draw two possible examples of the shape the teacher may be holding.

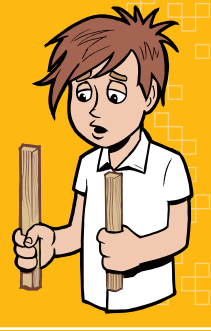
# Outside the Square — Problem solving

### Ram's rulers

Ram is planning a woodwork model and needs to draw a line 6 cm long. He has been using as rulers two straight pieces of wood with no markings, but he knows their lengths are 10 cm and 8 cm. How can he use these pieces of wood to measure a 6 cm line?

**Strategy options**

- Guess and check.
- Work backwards.
- Make a model.



# Area

6.3

Before laying tiles on a bathroom floor, the tiler would first need to find the **area** of the floor so that he or she will know how many tiles to buy.



Area is the amount of surface inside a plane (flat) shape. It is measured in 'square' units. When we calculate the area of a shape we are finding the number of squares that can fit inside the shape.

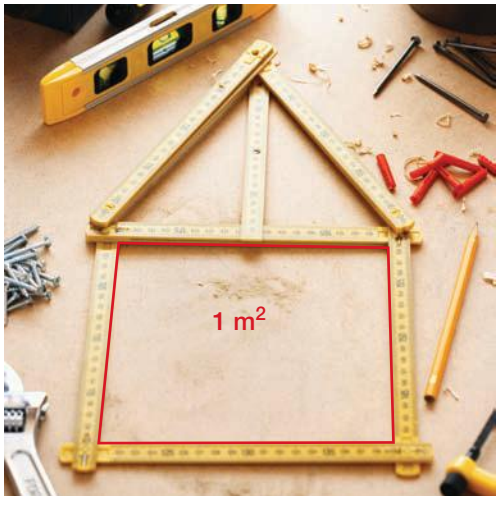
## Units of area



A square millimetre, $mm^2$ (actual size)
(1 mm length × 1 mm width = 1 $mm^2$)



A square centimetre, $cm^2$ (actual size)
(1 cm length × 1 cm width = 1 $cm^2$)




A square metre, $m^2$
(1 m length × 1 m width = 1 $m^2$)

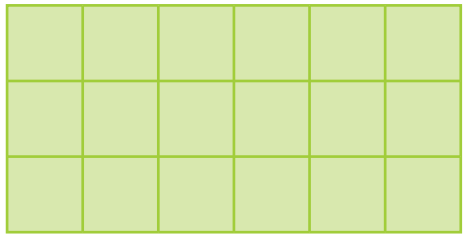



A square kilometre, $km^2$
(1 km length × 1 km width = 1 $km^2$)

We can say the names of square units, such as 1 $cm^2$, in two ways: '1 centimetre squared' or '1 square centimetre'.

## Area of a rectangle

Consider this rectangle of length 6 cm and width 3 cm.



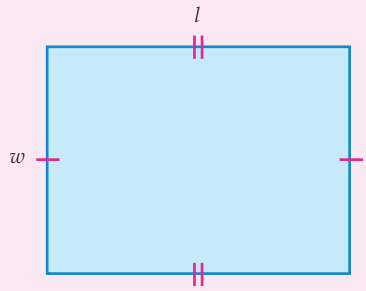
The rectangle contains 3 rows of 6 squares. Its area is equal to 3 cm × 6 cm = 18 $cm^2$.

The area ($A$) of a rectangle is equal to its length ($l$) multiplied by its width ($w$), or $A = l \times w$.

This formula can be written as $A = lw$.

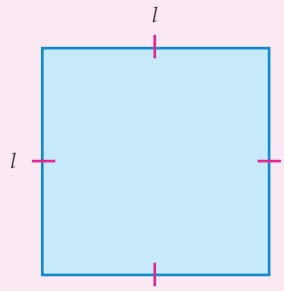
The length and width of a square are equal, so the area of a square can be found using the formula $A = l \times l$, or $A = l^2$.

**Area of a rectangle**




$A = lw$

**Area of a square**



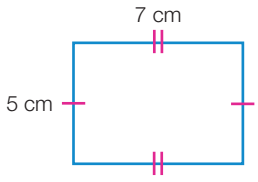

$A = l^2$

## Worked Example 6

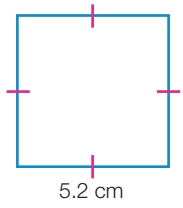
WE6

Calculate the area of the following shapes.

(a)




(b)




| Thinking | | Working | |
|---|---|---|---|
| (a) | 1 Write the formula for the area of a rectangle. | (a) | $A = lw$ |
| | 2 Identify $l$ and $w$, and substitute their values into the formula. | | $A = 7 \times 5$ |
| | 3 Evaluate, writing the answer with the correct units. | | $A = 35\ cm^2$ |
| (b) | 1 Write the formula for the area of a square. | (b) | $A = l^2$ |
| | 2 Identify $l$ and substitute its value into the formula. | | $A = 5.2 \times 5.2$ |
| | 3 Evaluate, writing the answer with the correct units. | | $A = 27.04\ cm^2$ |

# 6.3 Area

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7, Q8, Q11, Q12, Q13, Q16, Q17, Q18, Q22, Q23 | Q1 (a)–(f), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23 | Q1 (g)–(l), Q2, Q3 (b), Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23 |

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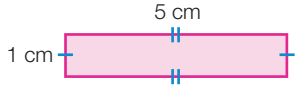
**Equipment required:** A calculator may be used for Questions **11–15**

## Fluency

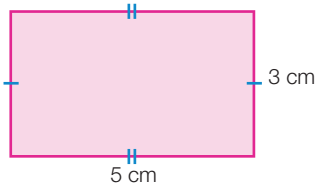
**1** Calculate the area of the following shapes.

WE6

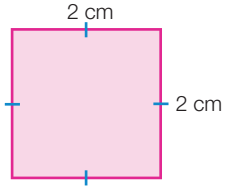
**(a)**



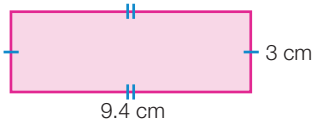

**(b)**



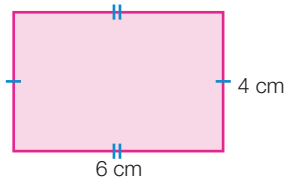

**(c)**



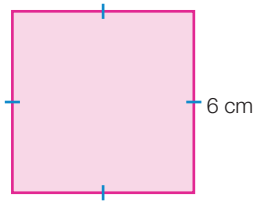

**(d)**



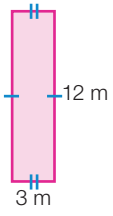

**(e)**



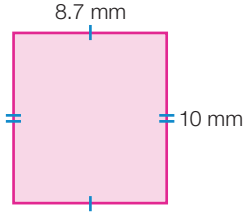

**(f)**



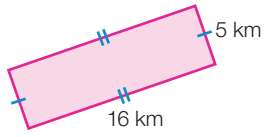

**(g)**



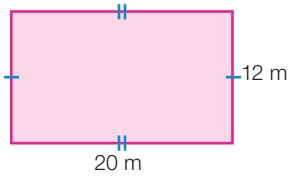

**(h)**



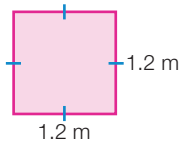

**(i)**



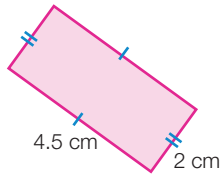

**(j)**




**(k)**

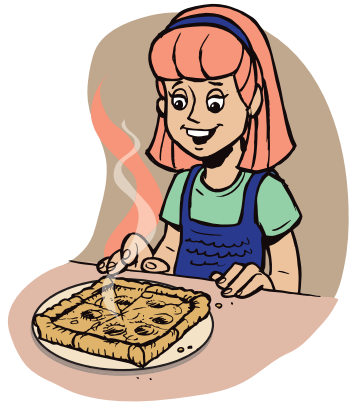



**(l)**




**2** State which of the units of area—$mm^2$, $cm^2$, $m^2$ or $km^2$—would be most suitable for measuring the area of:

**(a)** a sheet of writing paper
**(b)** your little toe-nail
**(c)** a pizza
**(d)** a house block
**(e)** a football oval
**(f)** Tasmania
**(g)** a small watch face
**(h)** the floor of a classroom.



**3** **(a)** A rectangle that has twelve rows of three square centimetres has an area of:

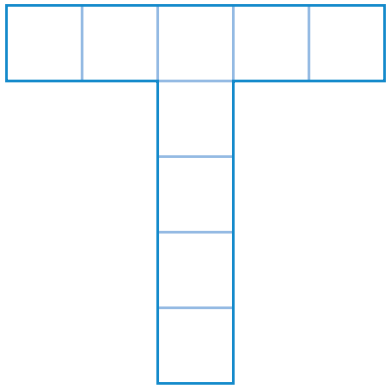
**A** 15 $cm^2$ **B** 30 $cm^2$ **C** 36 $cm^2$ **D** 72 $cm^2$

**(b)** If the area of a rectangle is 24 $cm^2$ and its width is 10 mm, then its length is:

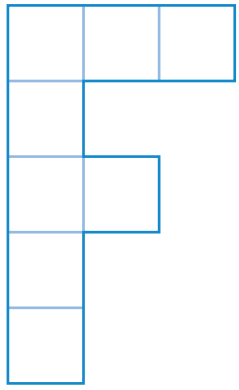
**A** 2.4 mm **B** 2.4 cm **C** 11 cm **D** 24 cm

**4** The following shapes have been drawn on centimetre grid paper. Find **(i)** the perimeter and **(ii)** the area of each one.

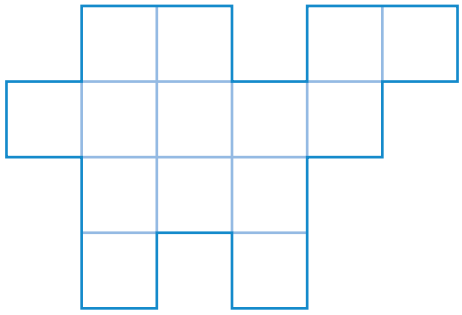
**(a)**



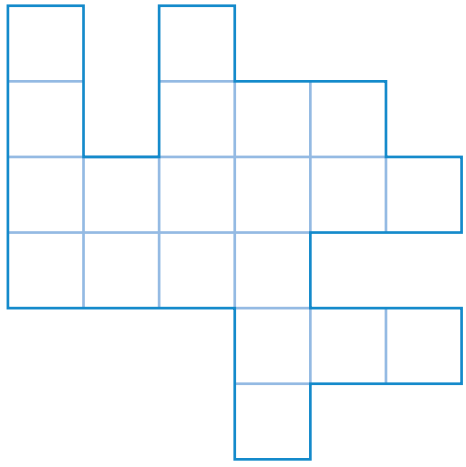
**(b)**



**(c)**

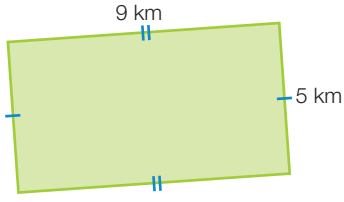


**(d)**

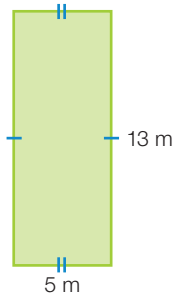


**5** Find **(i)** the perimeter and **(ii)** the area of each shape below.

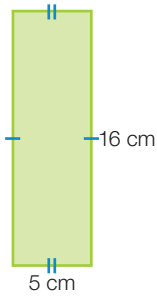
**(a)**



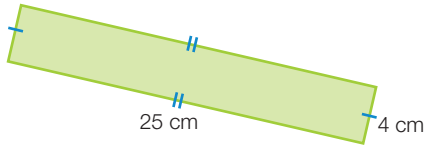

**(b)**




**(c)**




**(d)**







## Understanding

**6** Find the area in $km^2$ of a rectangle of length 3 km and width 800 m.

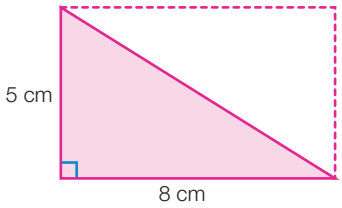
**7** What is the area in $cm^2$ of a rectangle of length 450 mm and width 70 cm?

**8** Find the width of a rectangle with length 7 cm and area 28 $cm^2$.

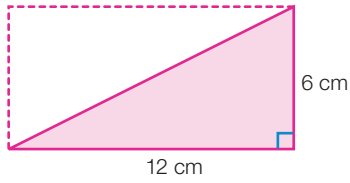
**9** Find the length of a rectangle with width 2.6 m and area 13.52 $m^2$.

**10** Find the area of each of the following triangles by first considering a rectangle.

**(a)**




**(b)**




**11** The Ray family is installing solar power to their house. There will be 6 panels attached to their roof, each with dimensions of 2.4 m × 2.1 m.

**(a)** What is the area of each solar panel?

**(b)** What is the total area of the solar panels?

**(c)** If the solar panels generate a maximum of 6.5 kilowatt hours (kWh) of energy per square metre each day, how much energy could the Ray family's new solar system produce in a day?



**12** Guy wishes to sow a lawn in a rectangular section of his yard, which is 8.5 m long and 6.9 m wide.

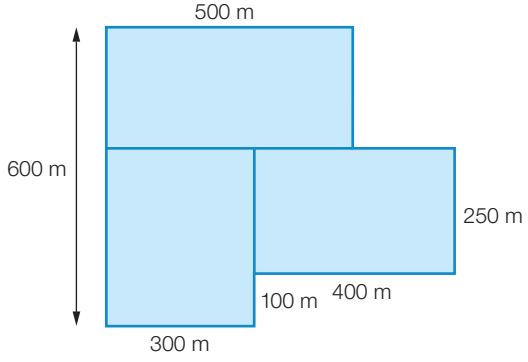
**(a)** How many grams of lawn seed will he need to use if the recommended seed coverage is 60 grams per square metre?

**(b)** What length of wood would Guy need to purchase to create a timber border for the section?

**13** A builder wishes to estimate the number of bricks required to build a wall 4.2 m wide and 2.6 m high. If 1 square metre of brick wall contains 48 bricks, how many bricks are needed?

**14** Jasmin decided to make a 45 cm square pillow for her study chair. One piece of fabric will be required for the front of the pillow, and one for the back. Each piece of fabric will need an extra 1 cm on all edges for the front and back to be sewn together. What area of fabric will Jasmin need to make the pillow?

**15** A farmer wants to create three separate rectangular paddocks on his farm, as shown in the diagram.




**(a)** Find the area of the three paddocks using the plan provided.

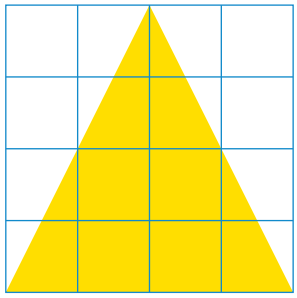
**(b)** To sow his paddocks, the farmer needs to spread 60 g of seed for every square metre. How many kilograms of seed will he need?

**(c)** What is the total length of fencing needed to surround each of the three paddocks?

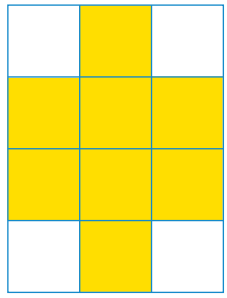
## Reasoning

**16** The following shapes have been drawn on centimetre grid paper. Find the shaded area in each case.

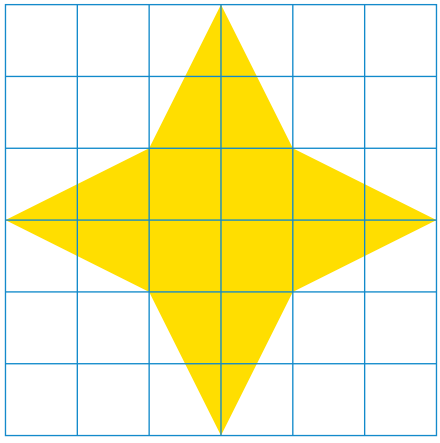
**(a)**



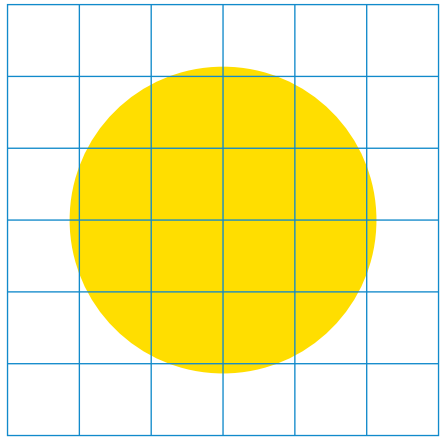
**(b)**



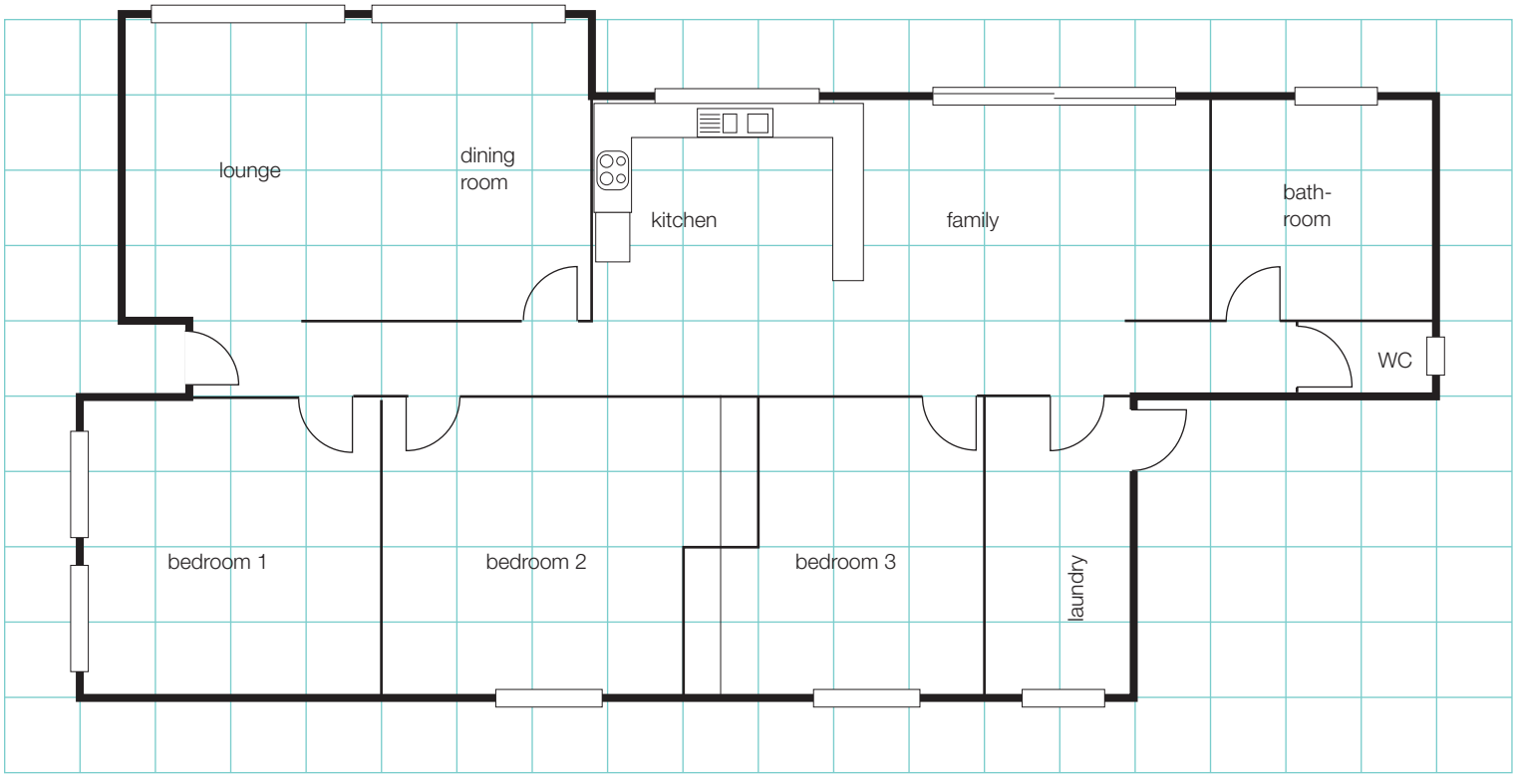
**(c)**



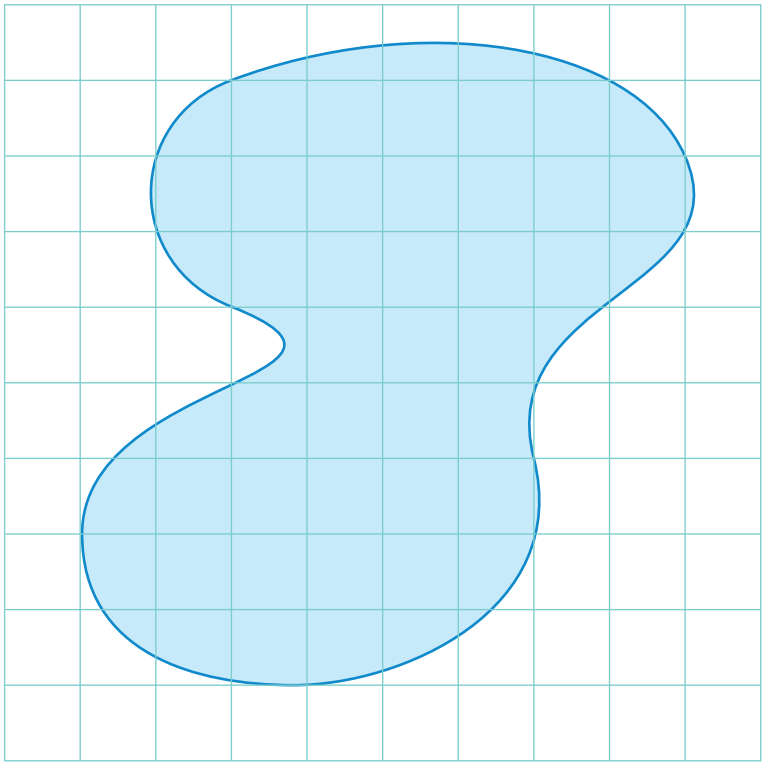
**(d)**



**17 (a)** Use the grid to estimate the area covered by the floor plan of this house. The squares on the plan below represent square metres.



**(b)** Use the grid to estimate the area of the lake. The squares on the grid represent square kilometres.

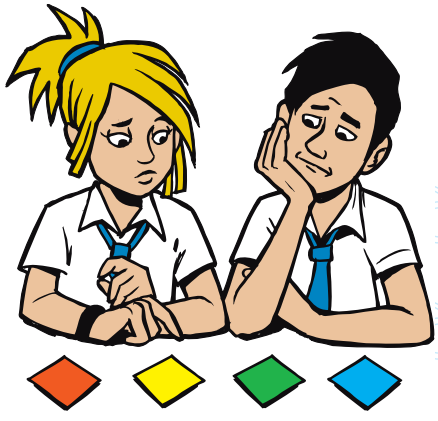


**18** Write down all the possible whole number dimensions of a rectangle with an area of 20 $cm^2$.

**19** Find the length and width of a rectangular room whose area is 60 $m^2$ and perimeter is 34 m.

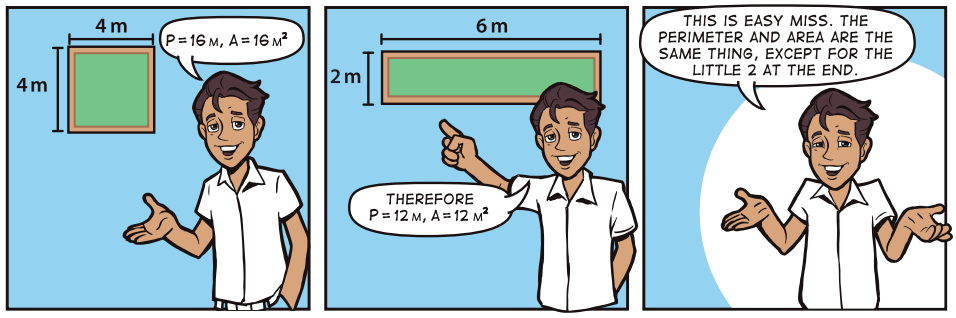
## Open-ended

**20** Ethan and Magda were asked to each make a shape using four 1 cm squares, and then find the perimeter of their shapes. Ethan said the perimeter of his shape was 10 cm, whereas Magda said her shape had a perimeter of only 8 cm. Could they both be right? Draw what their shapes may have looked like.



**21** Peta made a blanket for her newborn baby Mia's cot. The blanket has an area of 2.4 $m^2$. What might the perimeter of the blanket be?

22




**(a)** One of Sam's calculations is not right. What has he done wrong?

**(b)** How would you explain the difference between area and perimeter to Sam?

**(c)** What is meant by the 'little 2' and why must it be written on the answers for area?

**23** List three materials that are sold by the square metre.

# Outside the Square Game

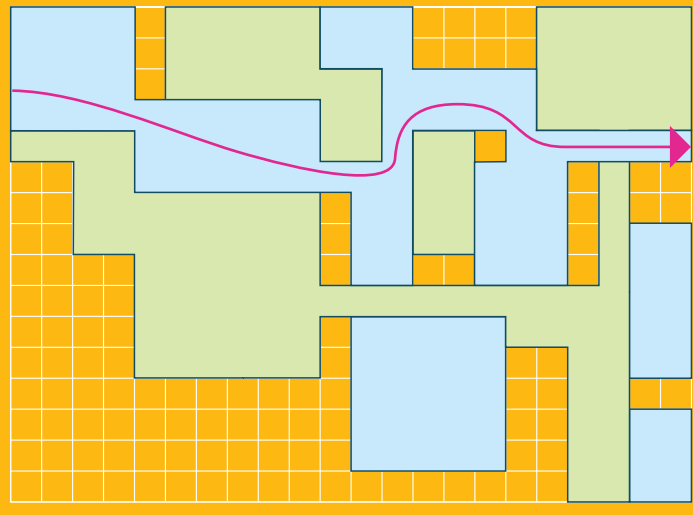
## Blaze a trail

**Equipment required:** 2 brains, A4 graph (or grid) paper, 2 dice, 2 different coloured pens or pencils

There are jungles in the world that have never been crossed. The race is on to be the first to blaze a trail.

**How to win:**
Create an unbroken path from one side of the jungle (graph paper) to the other. Your opponent will be trying to block you, and be the first to achieve this feat for themselves.





**How to play:**

1 Each player has a different coloured pencil or pen. Players take it in turn to roll the dice.

2 The two numbers rolled represent the size of the block to colour in, one for length and one for width. The player may choose which die will be used for width and which will represent length. The block coloured must be a rectangle or a square.

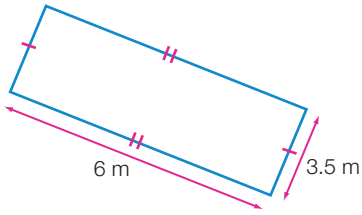
3 A player may place their block anywhere on the graph paper; however, blocks cannot overlap with another block, or go off the graph. If the numbers rolled cannot make a rectangle that will fit on the grid, the player does not place a block for that turn.

# Half-time 6

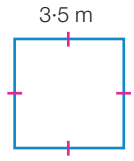
**1** Find **(i)** the perimeter and **(ii)** the area of the following shapes.

Ex 6.2, 6.3

**(a)**




**(b)**




**2** Complete the following conversions.

Ex 6.1

**(a)** 45 cm = ____ m **(b)** 0.015 km = ____ m **(c)** 307 m = ____ km

**(d)** 0.6 km = ____ cm **(e)** 196 000 m = ____ km **(f)** 2640 cm = ____ km

**3** A garden bed is 5.2 m long and 3 m wide.

Ex 6.2, 6.3

**(a)** How much fertiliser will be required to cover the garden bed, if the recommended amount is 0.75 kg per square metre?

**(b)** What length of wooden fencing will be needed to surround the bed?

**4** Which unit of area—$mm^2$, $cm^2$, $m^2$ or $km^2$—would be the most appropriate for measuring the area of:

Ex 6.3



**(a)** a soccer field

**(b)** a postage stamp

**(c)** a table placemat

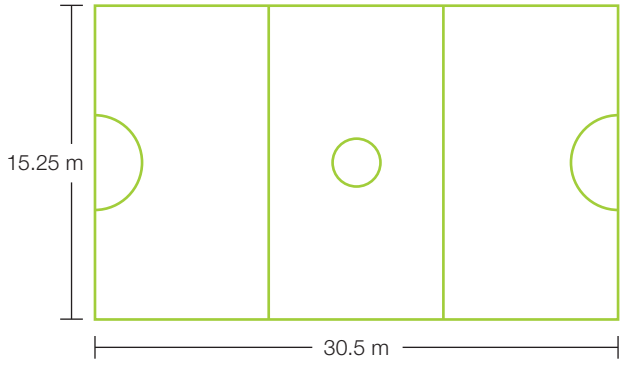
**(d)** a national park?

**5** A rectangle has a perimeter of 26 cm and an area of 30 $cm^2$. What are its dimensions (length and width)?

Ex 6.3

**6** A netball court is 30.5 m long and 15.25 m wide.

Ex 6.2, 6.3




**(a)** At training, players complete five laps around the boundary of the court. How many metres is this?

**(b)** Calculate the area of the court.

**(c)** The player in the position of 'goal attack' is allowed to move around in two-thirds of the court's area. Calculate this area, correct to one decimal place.

# Investigation

## Paddocks

**Equipment required:**
1 brain, graph paper



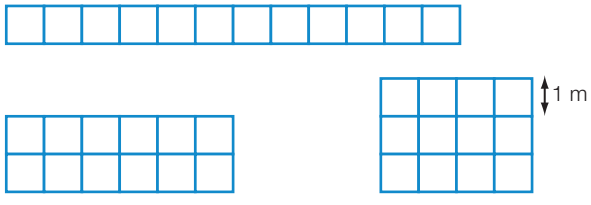
### The Big Question

A farmer is planning to fence off an area of 400 $m^2$ for the calves on his property. What dimensions (length and width) would give the paddock an area that is cheapest to fence?



### Engage

**1** Shapes with the same area can have different perimeters. Each of the rectangles below has been made using 12 squares. Each square has a side length of 1 m.




**(a)** Copy and complete the table below for the three rectangles.

| Dimensions of shape | Area ($m^2$) | Perimeter (m) |
|---|---|---|
| $1 \times 12$ | | |
| $2 \times 6$ | | |
| $3 \times 4$ | | |

**(b)** Which of the rectangles above has the shortest perimeter?

**(c)** Why would the farmer be interested in having the shortest perimeter?

### Explore

**2** List all possible pairs of whole number dimensions that would give a paddock with an area of exactly 400 $m^2$.

**Strategy options**
- Draw a diagram.
- Test all possible combinations.

**3** **(a)** Select three of the paddocks you think are most suitable for containing the calves. Draw the three paddocks on graph paper, showing their dimensions. To show their relative sizes, use a scale of 1 cm on page = 5 m in real life.

**(b)** Find the perimeter of each of the three paddocks.

**4** Use the following information to complete the table on the following page and calculate the total cost of the fencing for each paddock.

**Fencing wire:** Each fence has five strands of wire. Each strand goes all the way around the fence. The wire costs 85c per metre.

**Corner posts:** The paddocks will need a large post at each corner, each costing $40.

**Standard posts:** There must be a post at least every 5 m along a section of fence. If the distance between the corner posts is longer than 5 m, a standard post is placed along the fence. Mark where the standard posts will be required on the diagrams of each of your paddocks, and count the number needed for each paddock. Each standard post costs $15.

This table will help you to calculate the total cost of constructing each paddock. Copy the table and complete the columns for each of your three paddocks.

| Paddock | Dimensions | Perimeter | Metres of fencing wire needed | Cost of fencing wire | No. of corner posts needed | Cost of corner posts | No. of standard posts needed | Cost of standard posts | Total cost |
|---|---|---|---|---|---|---|---|---|---|
| Paddock 1 | | | | | | | | | |
| Paddock 2 | | | | | | | | | |
| Paddock 3 | | | | | | | | | |

## Explain

**5** What happens to the perimeter of a paddock as it gets longer in length and shorter in width? Try to explain why this is. The 12 $m^2$ rectangles you used in Question **1** may help.

## Elaborate

**6** Answer the Big Question by stating which of the three paddocks you think the farmer should choose, giving reasons for your choice.

## Evaluate

**7** **(a)** What information did you discover from the rectangles in Question **1** that could be used to determine the dimensions of a larger paddock with the shortest perimeter?

**(b)** Why did you select the three paddocks that you did?

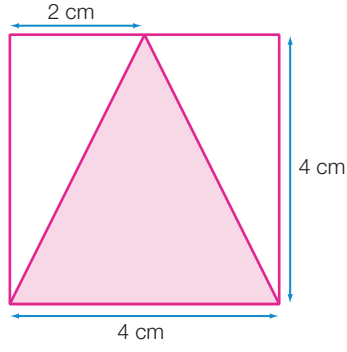
**(c)** Cost is one consideration when designing paddocks. What are other considerations that the farmer would need to take into account?

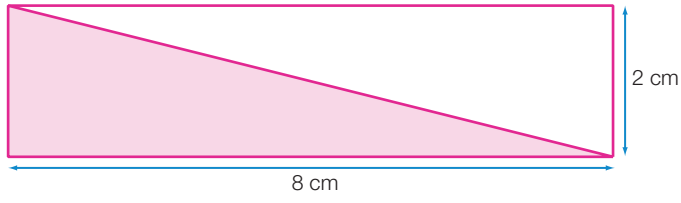
## Extend

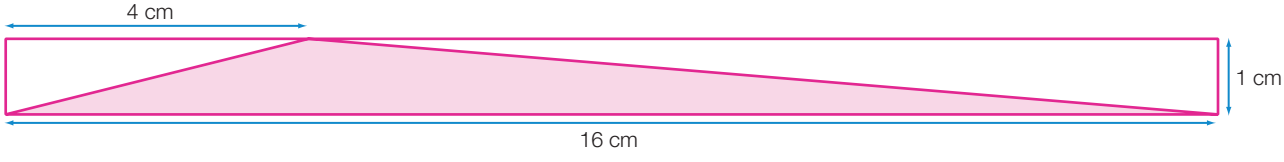
**8** Investigate the perimeter and the area of other shapes by looking at the triangles below. Each triangle has an area of 8 $cm^2$.

**(a)** Copy them onto graph paper using the actual measurements given in the diagrams. Then measure the perimeter of each triangle with a ruler.

**(b)** Which triangle has the shortest perimeter? Is this what you expected? Explain.

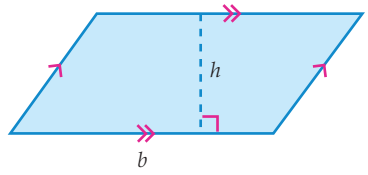


# 6.4 Area of a parallelogram

A **parallelogram** is a four-sided shape that has two pairs of parallel sides. The pairs of parallel sides are marked by > and >> symbols. The parallelogram has a **base**, $b$, and a **height**, $h$, which is **perpendicular** to the base. Perpendicular means 'at right angles to'. This is shown by the symbol for the right angle ⌞.







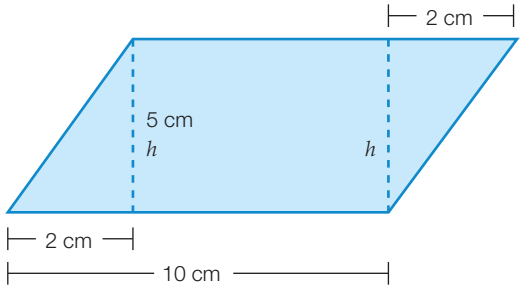


## Developing a formula for area

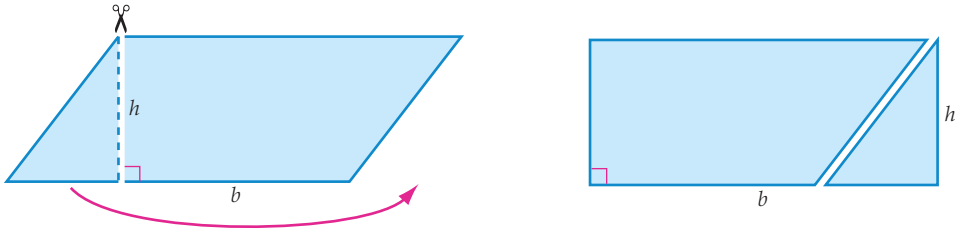
We can find a formula for calculating the area of a parallelogram by cutting and rearranging it into a familiar shape—a rectangle.

Construct a parallelogram by ruling a horizontal line 10 cm long and labelling it $b$. At one end of the line, and 2 cm in from the other end, measure two perpendicular heights of 5 cm. Mark them with dotted lines and label them $h$. Use the marked heights to rule a second 10 cm line parallel to the first. Make sure that this line starts 2 cm in from one end, and extends 2 cm past the other end of the first line.

If you have done this accurately, the two shorter, sloping sides of the parallelogram will also be parallel when you rule them in.



Transform your parallelogram into a rectangle by cutting down one of the dotted lines marking the height and rearranging the two pieces formed into a rectangle, as shown below.



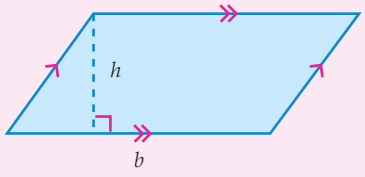

The base and height of the parallelogram become the length and width of a rectangle, and so we can multiply them together to find the area.

**Area of a parallelogram**

Multiply the base ($b$) by its perpendicular height ($h$).

$A = bh$

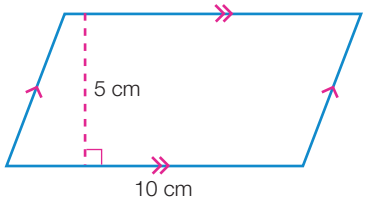
Sides that are marked with the same number of > symbols are parallel.



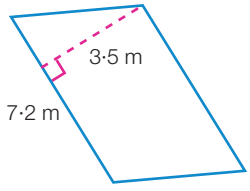

## Worked Example 7 WE7

Find the area of each of the following parallelograms.

(a)




(b)




| Thinking | | Working |
|---|---|---|
| (a) 1 | Write the formula for the area of a parallelogram. | (a) $A = bh$ |
| 2 | Identify $b$ and $h$, and substitute their values into the formula. | $A = 10 \times 5$ |
| 3 | Evaluate, writing the answer with the correct units. | $A = 50\ cm^2$ |
| (b) 1 | Write the formula for the area of a parallelogram. | (b) $A = bh$ |
| 2 | Identify $b$ and $h$, and substitute their values into the formula. | $A = 7.2 \times 3.5$ |
| 3 | Evaluate, writing the answer with the correct units. | $A = 25.2\ m^2$ |

# 6.4 Area of a parallelogram

## Navigator

Answers page 659

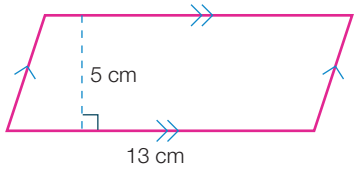
| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q10 | Q1 (c)–(f), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 | Q1 (e)–(f), Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 |

## Fluency

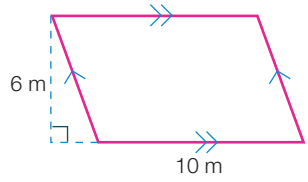
WE7

**1** Find the area of each of the following parallelograms.

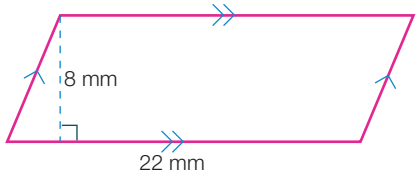
**(a)**



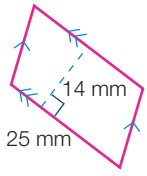

**(b)**



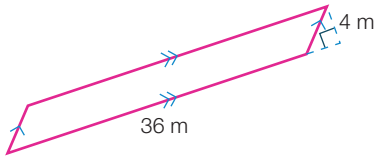

**(c)**



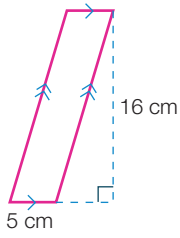

**(d)**




**(e)**

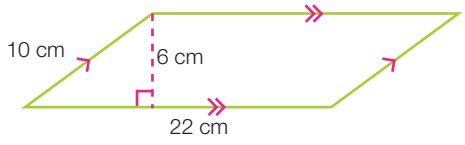



**(f)**




**2** **(a)** Find the area of this parallelogram, in $cm^2$:

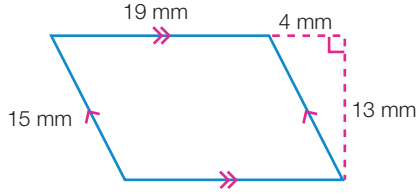
**(b)** Find the perimeter, in cm.




**3** **(a)** The height of this parallelogram is:

**A** 4 mm **B** 13 mm

**C** 15 mm **D** 19 mm




**(b)** A parallelogram with a base length of 14 m, a sloping side length of 5 m and a height of 4 m has an area of:

**A** $20\ m^2$ **B** $28\ m^2$ **C** $56\ m^2$ **D** $70\ m^2$

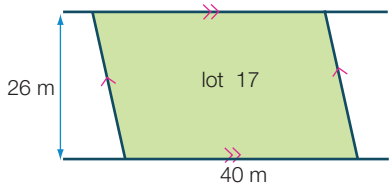
## Understanding

**4** A parallelogram with a base length of 12 cm has an area of 90 $cm^2$. What is the height of the parallelogram?

**5** The picture frame shown has a height of 15 cm and encloses an area of 300 $cm^2$. What is the base length of the picture frame?

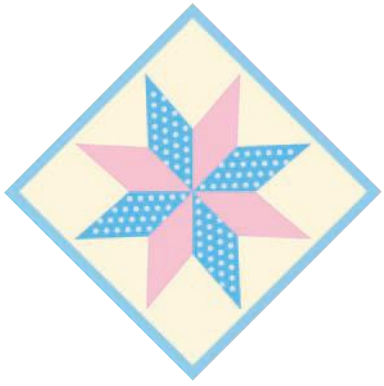


**6** A block of land appears on a council map as shown. Calculate its area in square metres.




**7** **(a)** How many identical parallelograms are shown in the quilt pattern below? (Ignore the different colours.)

**(b)** If a single parallelogram has a base length of 8.5 cm and a height of 6.5 cm, calculate the area of material required for all of the parallelograms on the quilt.

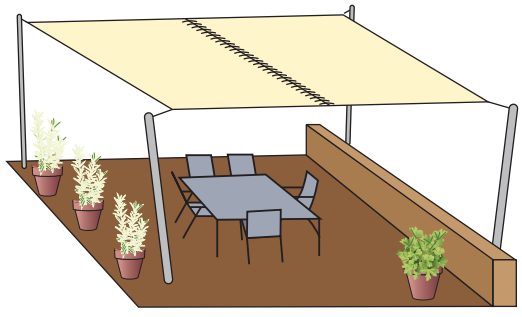


## Reasoning

**8** Two parallelogram-shaped shade sails are sewn together to provide shade for a courtyard, as shown. Each parallelogram has a base length of 4 m and a height of 2.5 m.

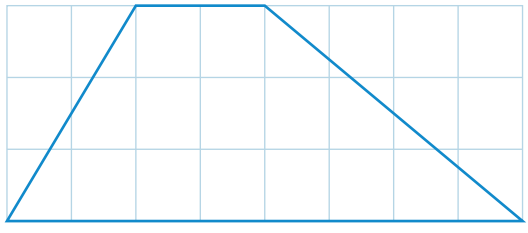
**(a)** At midday when the Sun is directly overhead, what area of shade will the shade sails provide?

**(b)** Show another way that the shade sails could be sewn together. Would this provide the same area of midday shade? Explain.



**9** **(a)** Copy the trapezium and show the position of the cuts and the transformations required to form a rectangle.

**(b)** Calculate the area of the trapezium if one grid square = 1 $cm^2$.



## Open-ended

**10** State the dimensions (base and height) of three parallelograms that have an area of 32 $cm^2$.

**11** Four identical parallelograms are placed together to form a larger parallelogram.

**(a)** Show how this can be done.

**(b)** State two possible dimensions of the smaller parallelogram, so that the larger parallelogram has an area of less than 100 $cm^2$.

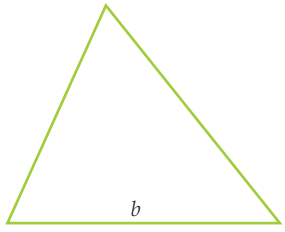
# 6.5 Area of triangles and composite shapes

## Area of a triangle

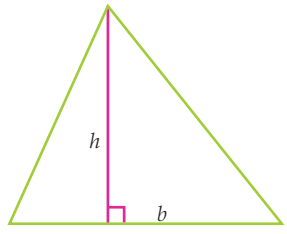
How do you find the area of a triangle? When faced with a new problem, mathematicians often try to use what they already know to solve it.

To find the area of a triangle, we will use a shape we already know how to find the area of: the rectangle. Can triangles be 'rearranged' to form rectangles?

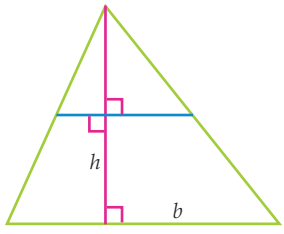


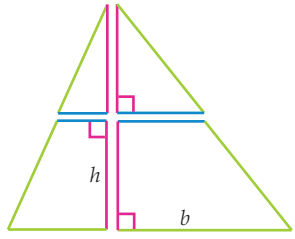

**1** Use a ruler and a pencil to draw a triangle. So it is easy to work with, the side lengths should be at least 10 cm long. Label one side as the base, *b*.




**2** Rule a line from the apex (the point at the top) of the triangle to the base. The line should be perpendicular (at an angle of 90°) to the base. Label this line *h*, the height of the triangle.




**3** Mark a point half-way up this line and use it to rule a line parallel to the base.

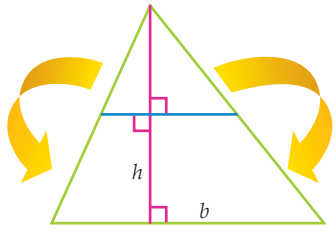


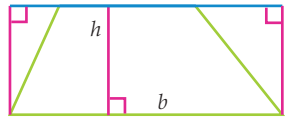

**4** Cut along these two inside lines to create four pieces.

Now:

- Use the four pieces to form a rectangle.
- Reform the triangle.
- Keep moving between the rectangle and the triangle until you are able to do it easily.
- What is the simplest way to move from the triangle to the rectangle? Write down the steps involved, then share them with a friend or the class.

Did you find that the simplest way was to rotate the two top pieces until they touched the bottom two?




We can find the area of the rectangle formed by finding the length and the width and multiplying them.

The length of the rectangle is the length of the base of the triangle, $b$.

The width of the rectangle is half the height of the triangle, $\frac{1}{2}h$, or $\frac{h}{2}$. (Can you see why? Look back at the instructions for step 3.)

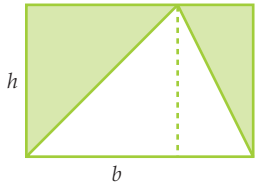
So, instead of writing $A$ = length × width, we can write:

$A = b \times \frac{h}{2}$, or $A = \frac{bh}{2}$ for the area of the rectangle, which is also the area of our original triangle.

This rule will work for all triangles. You can check this by repeating steps 1–4 with differently shaped triangles, then rearranging them into rectangles.

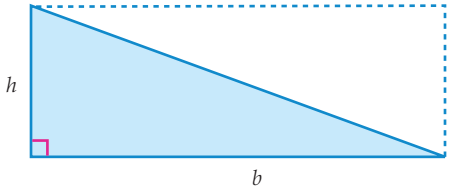
## Another way of looking at it

Another way of showing that the area of a triangle is equal to $\frac{bh}{2}$ is to draw a rectangle around the triangle, touching the points of the triangle, like this:

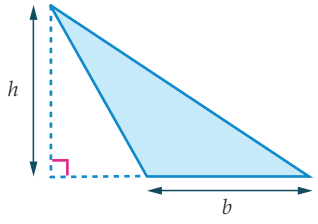



The area of this rectangle is $b \times h$, or $bh$. We can see by using symmetry that the two smaller, shaded triangles together are equal to the original, unshaded triangle. This means that the original triangle occupies half the area of the rectangle, or $\frac{bh}{2}$.

It is easy to see that the area of a triangle is half the area of the enclosing rectangle if you consider a right-angled triangle.



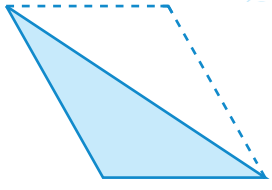

For some triangles, such as this one, the perpendicular height is measured outside of the triangle. We can find the height by extending the base, then drawing a perpendicular line from this extension to the opposite vertex. These triangles also follow the rule $A = \frac{bh}{2}$.




We can draw another identical triangle to form an enclosing parallelogram.

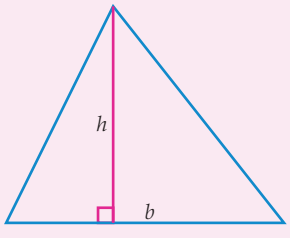
The area of the parallelogram is $bh$.

One triangle forms half of the parallelogram, so its area is $\frac{bh}{2}$.



The area of a triangle is equal to half of the product of the base and the perpendicular height.

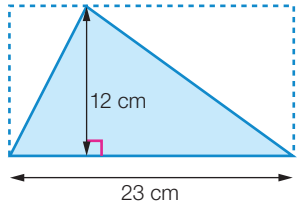
$A = \frac{bh}{2}$



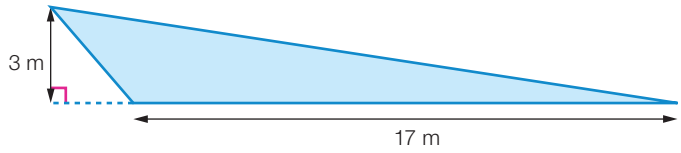
## Worked Example 8 WE8

Calculate the area of the following triangles.

(a)




(b)




| Thinking | | Working |
|---|---|---|
| (a) 1 | Write the formula for the area of a triangle. | (a) $A = \frac{bh}{2}$ |
| 2 | Identify $b$ and $h$, and substitute their values into the formula. | $A = \frac{23 \times 12}{2}$ |
| 3 | Evaluate, writing the answer with the correct units. | $A = 138\ cm^2$ |
| (b) 1 | Write the formula for the area of a triangle. | (b) $A = \frac{bh}{2}$ |
| 2 | Identify $b$ and $h$, and substitute their values into the formula. | $A = \frac{17 \times 3}{2}$ |
| 3 | Evaluate, writing the answer with the correct units. | $A = 25.5\ m^2$ |

## Composite shapes

A **composite shape** is a shape that is made by combining two or more shapes, such as rectangles, squares, parallelograms and triangles. Examples of composite shapes are shown below.

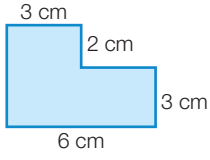


To find the area of a composite shape we divide it up into shapes that we can find the area of, then add or subtract the area of each individual shape.

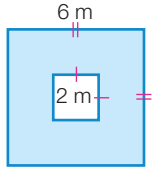
## Worked Example 9 WE9

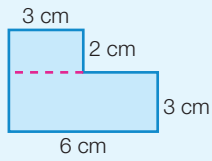
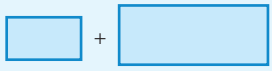
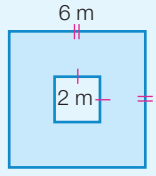
Calculate the shaded area of the following composite shapes.

(a)




(b)



| Thinking | Working |
|---|---|
| **(a) 1** Split the composite shape into recognisable shapes. (This may be done in different ways.) | (a)   |
| **2** Write the area of the composite shape as the sum of the areas of the individual shapes. | Total area =  <br>$= L \times W + l \times w$<br>$= 3 \times 2 + 6 \times 3$ |
| **3** Calculate each of the smaller areas. | $= 6 + 18$ |
| **4** Perform the addition by writing the answer with the appropriate units. | $= 24\ cm^2$ |
| **(b) 1** Write the area of the composite shape as the area of the larger shape minus the area of the smaller shape. | (b)  <br>Total area =  <br>$= L^2 - l^2$ |
| **2** Calculate each of the smaller areas. | $= 6^2 - 2^2$ |
| **3** Perform the subtraction, writing the answer with the appropriate units. | $= 36 - 4$<br>$= 32\ m^2$ |

Note that when we have two of the same type of shape, we use capital letters ($L$, $W$) for the dimensions of the larger shape and lower case letters ($l$, $w$) for the dimensions of the smaller shape. This is demonstrated in Worked Example 9.

# 6.5 Area of triangles and composite shapes

## Navigator

| | | |
|---|---|---|
| Q1, Q2 (a)–(f), Q3, Q4, Q6, Q7, Q9, Q11, Q13 | Q1 Columns 2 & 3, Q2, Q3, Q5, Q6, Q7, Q8 (a) & (b), Q9, Q10, Q11, Q13 | Q1 Column 3, Q2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

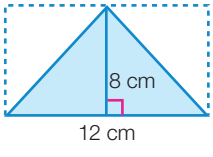
Answers page 660

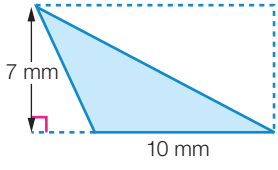
**Equipment required:** A calculator may be used for Questions **2**, **4**, **6**, **8** and **9**

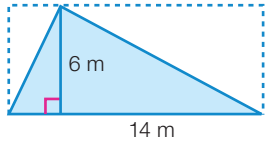
### Fluency

**1** Calculate the area of the following triangles.

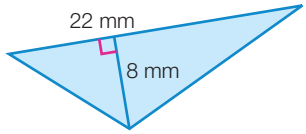
WE8

**(a)**  

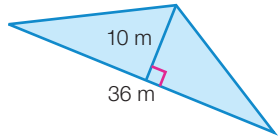
**(b)**  

**(c)**  

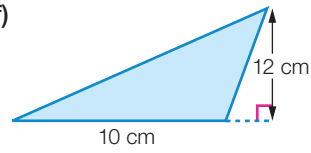
(d)



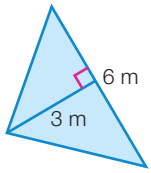

(e)



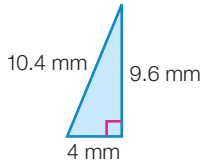

(f)



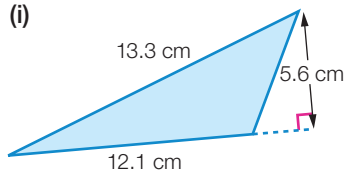

(g)




(h)




(i)



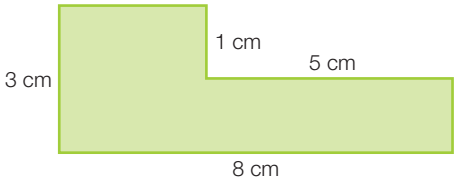

WE9

**2** Calculate the shaded area of the following composite shapes.

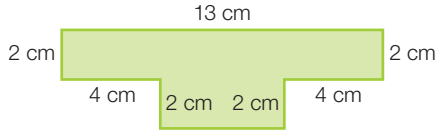
There is sometimes more than one way to split up a composite shape. Look for the easiest way.



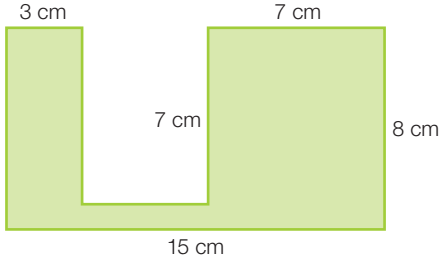
(a)



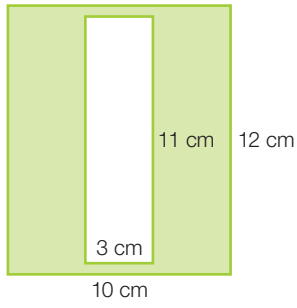

(b)



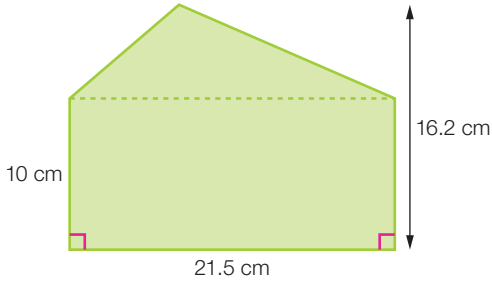

(c)



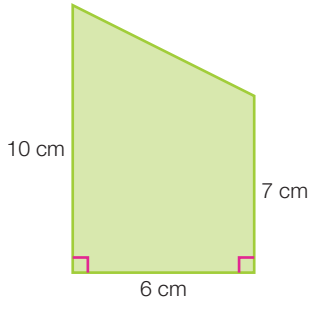

(d)



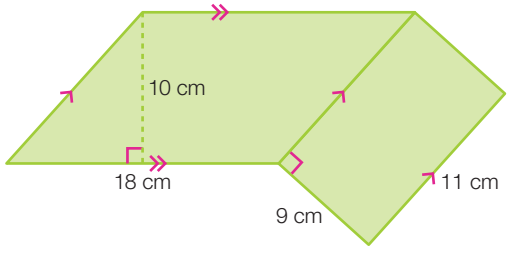

(e)



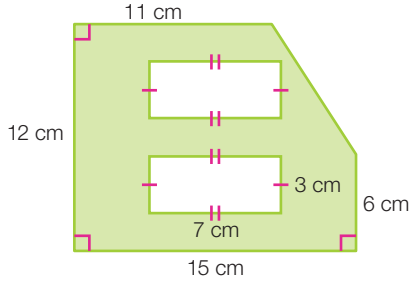

(f)




(g)




(h)



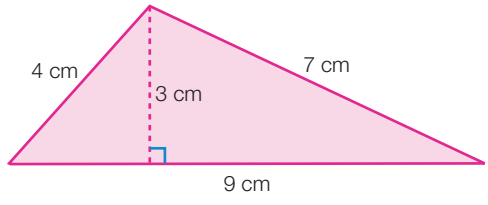

**3** **(a)** In the diagram, the height of the triangle is:

**A** 3 cm

**B** 4 cm

**C** 7 cm

**D** 23 cm



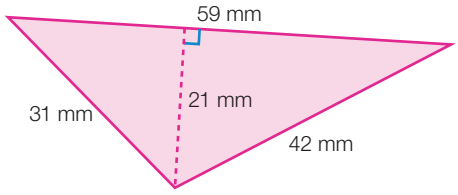
**(b)** The base length of the triangle is:

**A** 21 mm

**B** 31 mm

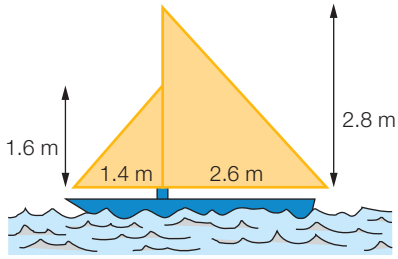
**C** 42 mm

**D** 59 mm

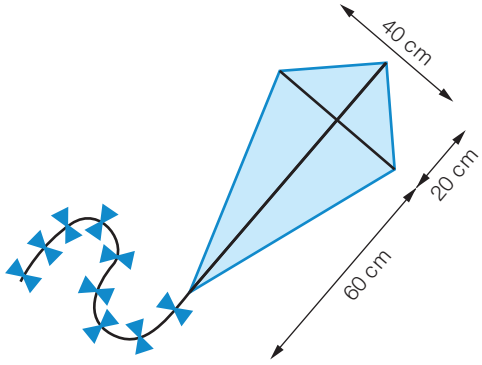



## Understanding

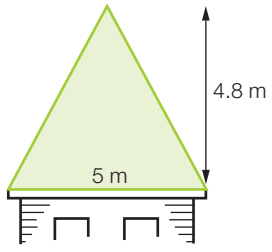
**4** Find the total area of the sails on this yacht.



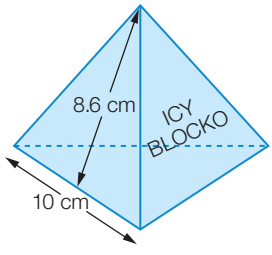

**5** Find the area of paper required to build the kite shown.




**6** The label on a tin of paint claims the contents will cover 4 square metres. How many tins would be needed to paint the triangular roof section pictured?

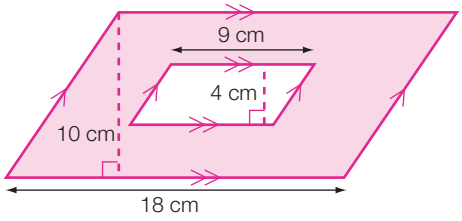



**7** Find the total area of the four identical triangular faces of the container of frozen drink (a Tetrapak) shown.

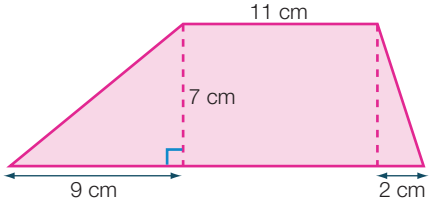



**8** Find the area of each of the following composite shapes.

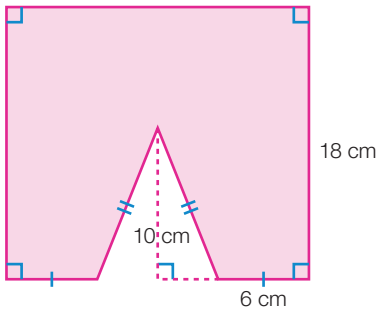
**(a)**



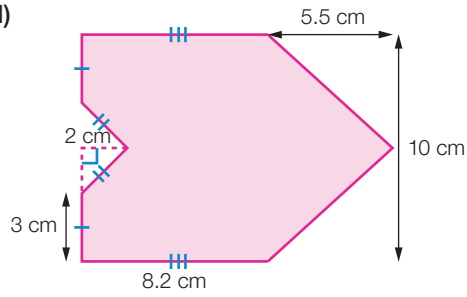

**(b)**



(c)




(d)




**9** Terry is having a doorway put in one of the walls of his house.

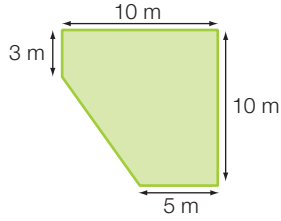
**(a)** If the wall is 5 m long and 2.4 m high, and the doorway measures 0.82 m by 2 m, what area of wall will remain after the doorway is completed?

**(b)** The architraves go along the sides and top of the doorway. What length of architraves is needed?

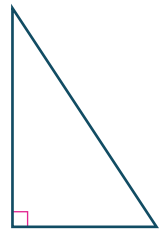
**(c)** Terry installs a door in the doorway, and paints two coats of varnish on both sides of the door. If 1 litre of varnish covers 4 square metres, how many litres of varnish did Terry use?

## Reasoning

**10** Describe a method for finding the area of the composite shape shown without splitting it up into known shapes. Use your method to find the answer.



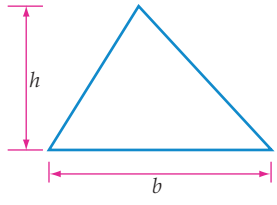

**11** This triangle can be cut into two pieces to form a rectangle.



**(a)** Describe exactly where and how to make the cut.

**(b)** Use a diagram to show how the two pieces fit together to make a rectangle.

**12** This triangle can be cut into four pieces to form a rectangle.



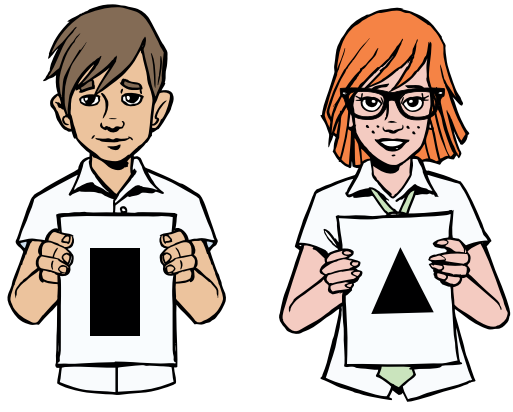

**(a)** Describe exactly where and how to make the cuts.

**(b)** Use a diagram to show how the four pieces fit together to make a rectangle.

## Open-ended

**13** A triangle has an area of 36 $cm^2$. Give two possible combinations for the triangle's base length and height.

**14** The difference in area between a rectangle and a triangle is 12 $cm^2$. Draw a possible example of the pair, showing their dimensions.



# Volume

6.6

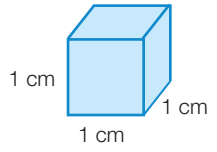
A box is a three-dimensional (3D) object, which means it has length, width and height. The amount of space inside a box (if it is empty) is its **volume**.

Volume is the amount of space occupied by a 3D object. Volume is measured in 'cube' units. When we calculate volume, we are finding the number of cubes inside the object.



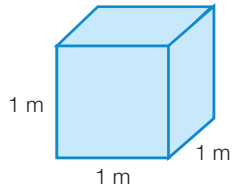
We can measure the volume of an object by counting or calculating the number of cubic centimetres that would fit inside it. A cubic centimetre ($cm^3$) is a cube with a length, width and height of 1 cm. The volume of larger objects may be measured in cubic metres ($m^3$)—a cube with length, width and height of 1 m.

A cubic centimetre



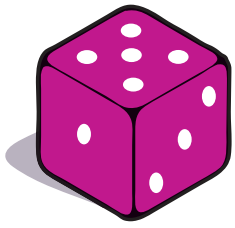
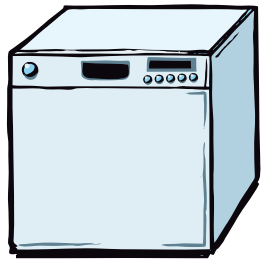

1 cm × 1 cm × 1 cm = 1 $cm^3$

A cubic metre




1 m × 1 m × 1 m = 1 $m^3$

Note that this cube is not drawn to scale.

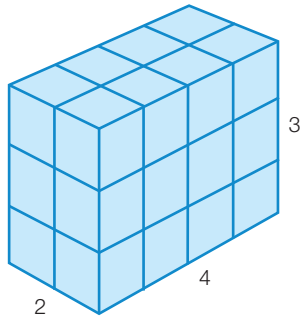
| $mm^3$ | $cm^3$ | $m^3$ |
|---|---|---|
|  |  |  |
| The volume of a red head on a match is about 4 $mm^3$. | The volume of a die is about 1 $cm^3$. | The volume of a standard dishwasher is just over 1 $m^3$. |

The symbol, '$cm^3$', can be said in two ways: 'cubic centimetres' or 'centimetres cubed'.

A **rectangular prism** is made up of layers of cubes. Each layer has the same rectangular shape, meaning that there is the same number of cubes in each layer.

The volume of a prism can be found by finding the number of cubes on the bottom face (or top face, if that is easier to see) and then multiplying this by how many layers of cubes there are.

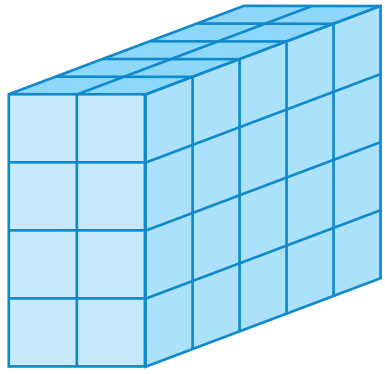
The rectangular prism shown here has 8 cm cubes on its top face, and 3 layers of cubes. Its volume is $8 \times 3 = 24$ $cm^3$.



## Worked Example 10

WE10

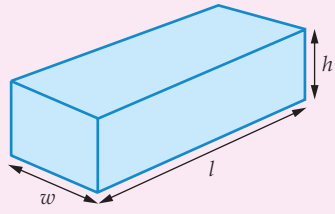
How many cubic centimetres are there in the following rectangular prism? (Each small cube represents 1 $cm^3$.)



| Thinking | Working |
|---|---|
| **1** Find the number of squares on the top face of the prism. | There are 10 squares on the top face. |
| **2** Find the height of the prism. | It is 4 units high. |
| **3** Work out the total number of cubes. | $10 \times 4 = 40$ cubes<br>$= 40\ cm^3$ |

The number of cubes showing on the top or bottom face of a rectangular prism is the same as the area of that rectangular face. Instead of counting the number of cubes in the top or bottom face, we can multiply the length by the width to get the area of the face ($A = l \times w$). We then multiply by the number of layers of cubes. So, to find the volume of a rectangular prism, we multiply the length, width and height values together.

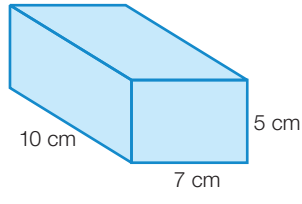
$V = l \times w \times h$




## Worked Example 11

WE11

Find the volume of the following rectangular prism in $cm^3$.




| Thinking | Working |
|---|---|
| **1** State the formula for calculating the volume of rectangular prisms. | $V = lwh$ |
| **2** Substitute the values for $l$, $w$ and $h$ into the formula and evaluate. | $V = 10 \times 7 \times 5$<br>$= 350\ cm^3$ |

# 6.6 Volume

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q6 Column 1, Q7, Q9, Q10, Q12, Q13 | Q1, Q2, Q4, Q5, Q6 Columns 1 & 2, Q7, Q9, Q10, Q11, Q12, Q13 | Q1, Q2, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

Answers page 660

**Equipment required:** A calculator may be used for Questions **7–9**, **11** and **14**

## Fluency

**1** How many cubic centimetres are there in the following rectangular prisms? (Each small cube represents 1 $cm^3$.) WE10

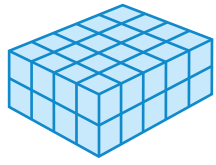
**(a)**



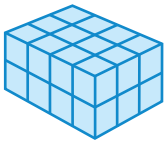
**(b)**



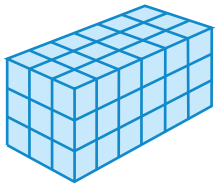
**(c)**



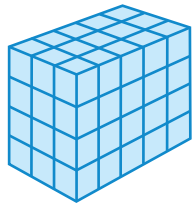
**(d)**



**(e)**

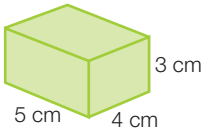


**(f)**

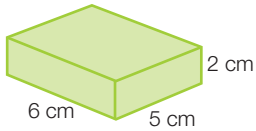


**2** Find the volume of the following rectangular prisms in $cm^3$. WE11

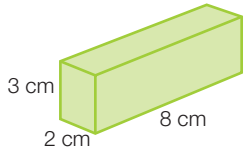
**(a)**



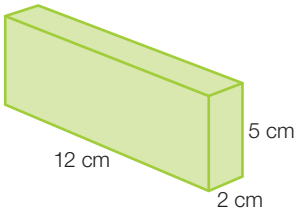

**(b)**



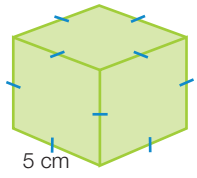

**(c)**



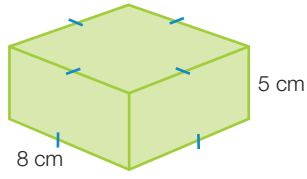

**(d)**




**(e)**




**(f)**




**3** A room is 3 m long, 2 m wide and 2 m high. What is its volume in metres cubed?

**4** A pocket dictionary has a cover 12 cm long and 9 cm wide and is 2 cm thick. Find the volume of this book.

**5** Choose the correct answer for each of the following rectangular prisms.

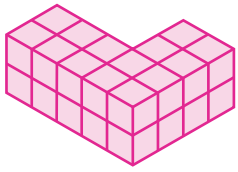
**(a)** Length = 20 cm; width = 10 cm; height = 5 cm. The volume is:

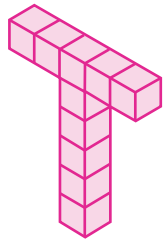
**A** 35 $cm^3$ **B** 100 $cm^3$ **C** 700 $cm^3$ **D** 1000 $cm^3$

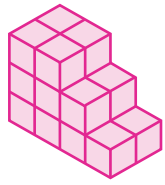
**(b)** Length = 3 m; width = 75 cm; height = 50 cm. The volume is:

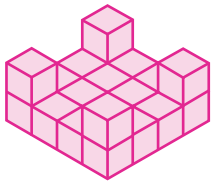
**A** 11.25 $m^3$ **B** 112.5 $cm^3$ **C** 11 250 $cm^3$ **D** 1 125 000 $cm^3$

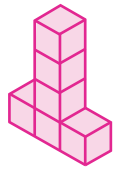
6 Find the volume of each of these compound solids by counting the number of centimetre cubes.

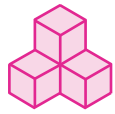
(a) 

(b) 

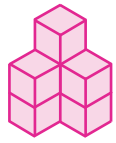
(c) 

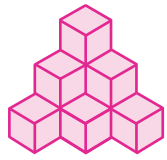
(d) 

(e) 

(f) 

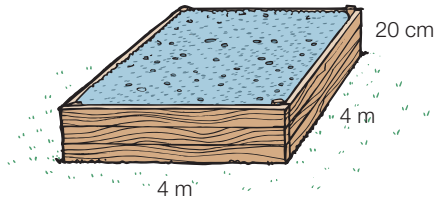
(g) 

(h) 

(i) 

## Understanding

7 A water tank is to be placed on a wooden box filled with gravel to keep it level. How much gravel is needed, in $m^3$, to fill the base shown?

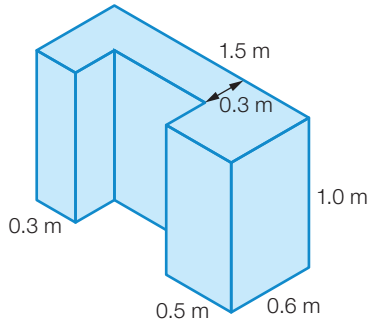



8 If a cake in the shape of a rectangular prism is 20 cm long, 15 cm wide and 6 cm thick when baked, what will its volume be after it is iced all over (including the bottom) with a layer of icing 0.5 cm thick?






9 Lachlan has finished the brickwork for his backyard barbecue, shown below. Find the volume of bricks used in building the barbecue.



## Reasoning

**10** A large toy box is completely full of identical wooden blocks which are 10 cm long, 6 cm wide and 4 cm in height. How many blocks can fit inside the toy box, if the dimensions of the toy box are: length 40 cm, width 30 cm and height 20 cm?

**11** A rectangular prism has a volume of 216 $cm^3$. It is twice as long as it is wide, and its height is 3 cm. Find its length.

## Open-ended

**12** A rectangular prism is made from 30 one-centimetre cubes. Give possible values for the length, width and height of the prism.

**13** At a school fair, a game involves guessing the number of jelly beans in a container. The person with the closest guess will win all the jelly beans.

**(a)** How many jelly beans do you think are in the jar?

**(b)** How did you make an accurate estimate of the number of jelly beans in the jar?

**(c)** If the jelly beans were the same volume but cube-shaped, would more or fewer jelly beans fit in the jar?

**14** A small cube is placed inside a large cube of volume 64 $cm^3$. Give three possible side lengths of the smaller cube and calculate the volume of empty space inside the large cube.



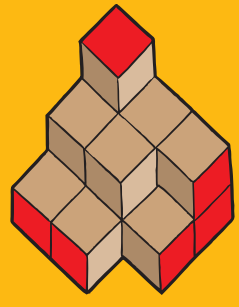
# Outside the Square

## Problem solving

### The red cube

A new water fountain is being constructed as a display for a shopping centre. The water will run smoothly over a large red cube. Construction has started, as shown on the right. How many of the remaining cubes will need one face painted? Two faces painted? Three faces painted?

In the completed water feature, how many cubes will have one, two and three faces painted?



**Strategy options**

- Have I seen a similar problem?
- Make a model.

# Mathspace

# City Mission



Code X, the evil code-breaking master, is on a spree through the city's streets, decoding top secret information that is giving him access to all of the city's finances, gold and valuables. Your mission is to answer the questions he has left on different devices at various locations around the city, starting at the police station. Answering the questions correctly will spell the access code to his computer files and give you the location of the next clue. The path taken will lead you to Code X's hideout. Can you break the code before he escapes?

The volume of air that can fit inside this telephone booth is:

- **K** 12 800 $m^3$ (go to newspaper stand)
- **L** 6400 $m^3$ (go to library)
- **M** 360 $m^3$ (go to police station)
- **N** 1.28 $m^3$ (go to train map)

The length of this single carriage train could be:

- **Y** 0.07 km (go to police station)
- **Z** 200 m (go to train map)
- **A** 2000 cm (go to library)
- **B** 1200 mm (go to bank)

The length of this train line is:

- **R** 4.6 km (go to train carriage)
- **S** 3.8 km (go to L'Hotel)
- **T** 42 km (go to museum)
- **U** 52 km (go to telephone booth)

0.67 km is equal to:

- **N** 6700 cm (go to Pop's Pizza)
- **O** 670 m (go to telephone booth)
- **P** 6.7 m (go to library)
- **Q** 0.0067 m (go to train carriage)

The length of this police sign is:

**A** 40 cm (go to library)
**B** 1600 cm (go to telephone booth)
**C** 400 cm (go to bank)
**D** 800 cm (go to train map)

32 metres is equal to:

**V** 0.32 km (go to post office)
**W** 0.032 cm (go to bank)
**X** 320 cm (go to museum)
**Y** 3200 cm (go to rubbish bin)

The volume of this rubbish bin will give you his hideout:

**E** 0.36 $m^3$ (go to Pig and Whistle Café)
**F** 3.6 $m^3$ (go to park bench)
**G** .72 $m^3$ (go to Ye Olde Bookstore)
**H** 7.2 $m^3$ (go to train platform exit)

The area of this shape is:

**P** 36 $m^2$ (go to L'Hotel)
**Q** 32 $m^2$ (go to telephone booth)
**R** 30 $m^2$ (go to train carriage)
**S** 24 $m^2$ (go to newspaper stand)

The volume of this stack of newspapers is:

**C** 1.3 $m^3$ (go to rubbish bin)
**D** 3 $m^3$ (go to post office)
**E** 0.075 $m^3$ (go to L'Hotel)
**F** 0.75 $m^3$ (go to library)

The volume of this shape is:

**D** 16 $m^3$ (go to museum)
**E** 13 $m^3$ (go to train map)
**F** 15 $m^3$ (go to newspaper stand)
**G** 17 $m^3$ (go to L'Hotel)






Maths 4 Real

# LANDSCAPE GARDENING

Landscape gardening involves designing and creating outside spaces, such as the area around a house or building, and public parks. There are many DIY magazines and TV shows transforming the average backyard into a haven of 'outdoor rooms', lights, waterfalls and vegetable gardens.

Before a garden can be transformed, a detailed design is drawn on grid or graph paper.

**1** **(a)** Why is it helpful to draw the design on a grid?

**(b)** The scale at the top of the grid states that 1 cm on the page represents 1 m in real life. What real life distance would 1 mm represent?

At the top of the following page is a design used to transform an Australian backyard.

Refer to the plan when answering Questions **2**–**8**.

**2** What is the total area of the backyard (not including the house)?

**3** The lawn will be sown by seed. 30 g of grass seed is needed to cover each square metre.

**(a)** What is the area to be covered in grass?

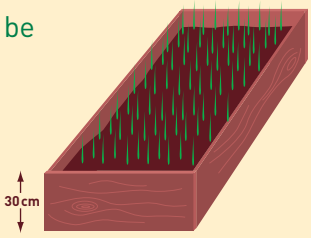
**(b)** How much grass seed will you need to buy? Give your answer in kilograms.

**4** The path that extends from the shed to and around the vegie garden will be paved with bricks. The base dimensions of one brick are 20 cm × 10 cm. Calculate:

**(a)** How many bricks will cover each square metre?

**(b)** What is the total number of bricks required to complete the path?

**5** The vegetable garden will be constructed out of timber sleepers with a height of 30 cm. The box needs to be filled with a mixture of soil and compost.

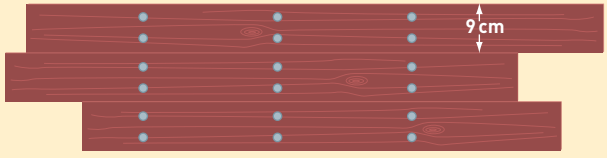



**(a)** What is the total length of the timber sleepers needed to make the vegetable garden box?

**(b)** Find the area of the topsoil of the vegetable garden.

**(c)** What volume of soil mixture will be needed to fill the box to the top?

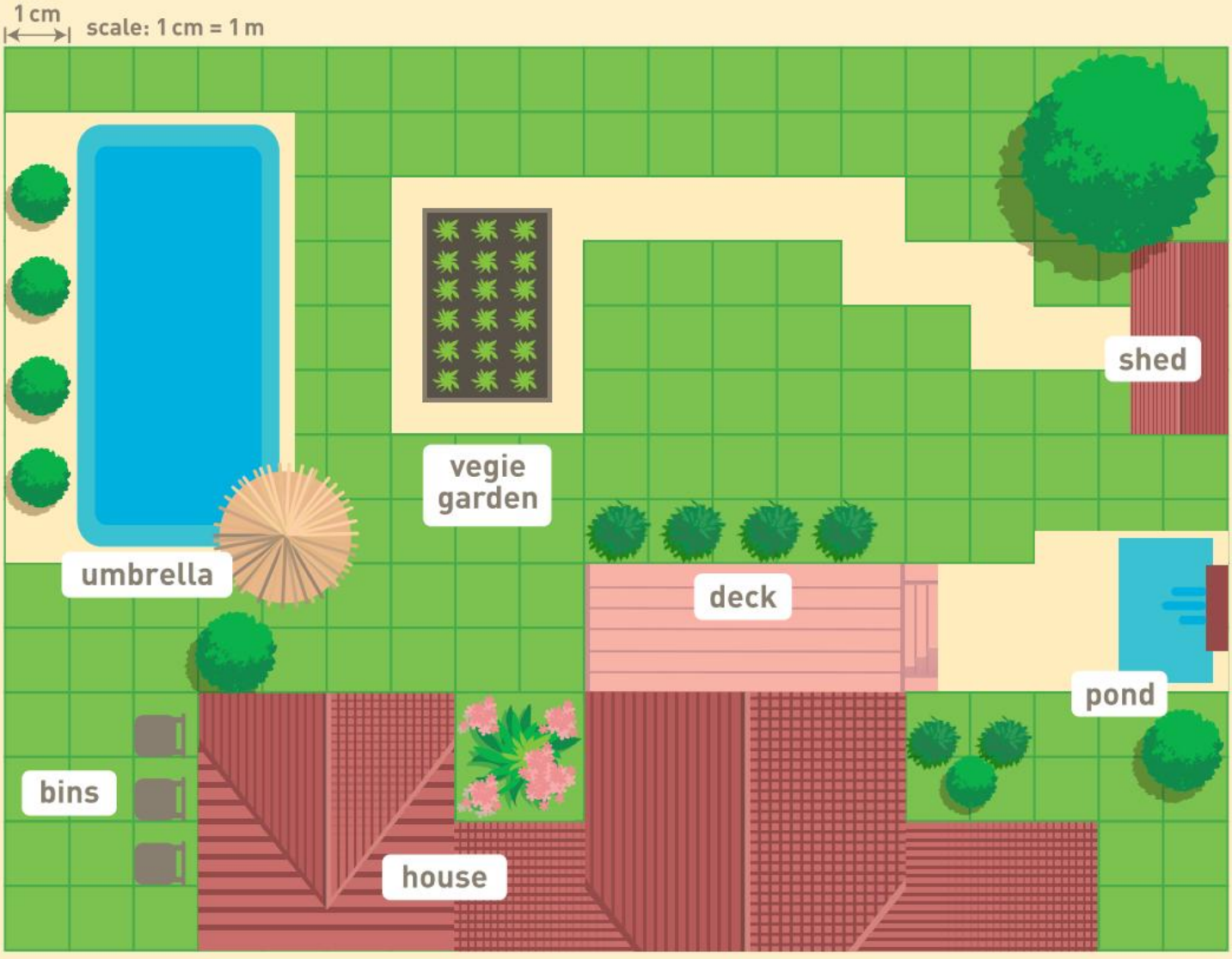
**6** The deck will be made using hardwood boards that are 9 cm wide. Decking boards are sold in different lengths and then cut to size.




**(a)** What area is to be decked? (Include the step in your calculations.)

**(b)** How many boards are needed to cover a width of one metre?

**(c)** What length of decking will need to be bought to make the deck?




An important consideration when designing a backyard is cost. A detailed budget needs to be completed before the project can begin.

**7** Using the figures you have calculated in Questions **2–6**, copy and complete the budget below.

| Item | Number of items required or area to cover ($m^2$) | Cost | Total cost of each item |
|---|---|---|---|
| Grass seed | | $8.95 per 1 kg box | |
| Bricks | | $0.80 each | |
| Timber sleepers | | $12 per metre | |
| Soil mixture | | $42 per $m^3$ | |
| Decking | | $4.50 per metre | |
| | | Total cost | $ |

**8** What other costs in making this backyard have not been included in the above budget?

## Research

Design your own 'ideal' backyard. Make a scale drawing of it using similar symbols as above. Consider aspects such as space, cost, slope of the land, water restrictions, pets and your family's needs.

# Investigation

**Equipment required:** 1 brain, (2 brains may be used for Question **2**), centimetre cubes (optional), graph paper or isometric dot paper

PP Blackline Master

## Blocks of chocolate

The dimensions of a Cadbury Cherry Ripe® chocolate bar are shown below.

The surface area of a chocolate bar is the combined area of the top, bottom and all four sides. It is the area that is covered in chocolate.

The smaller the surface area of a chocolate bar, the less packaging will be required to wrap around it.




### The Big Question

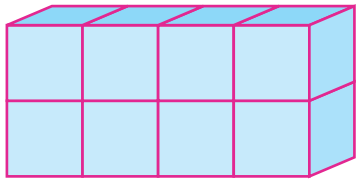
What is the smallest possible surface area for a Cadbury Cherry Ripe® that has a volume of 36 $cm^3$?

### Engage

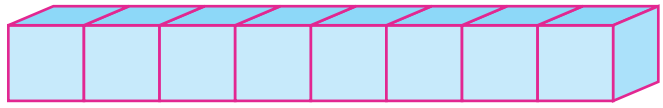
Chocolate comes in blocks that resemble rectangular prisms. The volume of a rectangular prism can be calculated using the formula: Volume = length × width × height. If the volume is known to be 8 $cm^3$ and we want to find the possible dimensions for the rectangular prism, we need to find three numbers that multiply to give 8:

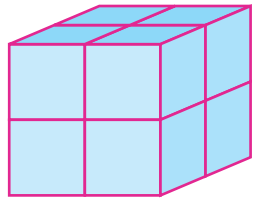
For example: 8 $cm^3$ = 4 × 2 × 1

These dimensions give the following rectangular prism.

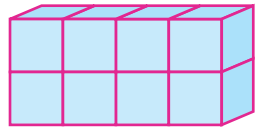
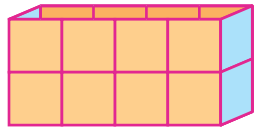
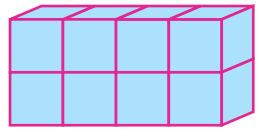


These eight cubes can be arranged to make two other rectangular prisms:





**1** The surface area of a rectangular prism can be found by counting or calculating the number of squares on the surface of each face (including the bottom) and adding them together. The surface area of a rectangular prism with length 4 cm, width 1 cm and height 2 cm is shown in the table below.

| | |
|---|---|
| **Dimensions** |  4 cm × 2 cm × 1 cm |
| **Area of front and back** |  8 + 8 = 16 |
| **Area of top and bottom** |  4 + 4 = 8 |
| **Area of sides** |  2 + 2 = 4 |
| **Total surface area** |  16 + 8 + 4 = **28 $cm^2$** |

(a) Calculate the surface area of the other two rectangular prisms with volume 8 $cm^3$. If you have centimetre cubes, use them to help you.

(b) All three rectangular prisms have a volume of 8 $cm^3$. What are the dimensions of the one with the smallest surface area?

(c) Draw the actual Cherry Ripe® bar as a rectangular prism on graph paper or dot paper.

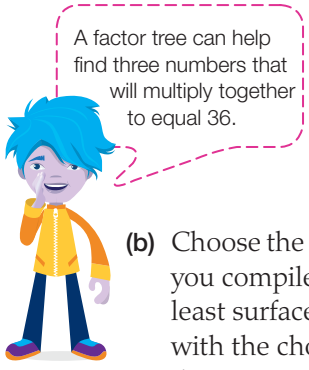
(d) What is the surface area of the Cherry Ripe®?

## Explore

2 The volume of the Cherry Ripe® shown is $12 \times 3 \times 1 = 36$ $cm^3$.

(a) Other whole number dimensions of a rectangular prism will give a volume of 36 $cm^3$. List all the different combinations of whole numbers that will make this statement true.

36 $cm^3$ = ____ × ____ × ____




**Strategy options**

- Make a table.
- Have I seen a similar problem?
- Make a model.

(b) Choose the three sets of dimensions from the list you compiled in **(a)** that you think will have the least surface area. Draw three chocolate bars with the chosen dimensions on graph paper or dot paper.

(c) Copy and complete the table below to find the total surface area of the three prisms:

| Dimensions of chocolate bar | Area of front and back | Area of top and bottom | Area of sides | Total surface area |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |

## Explain

3 (a) Use your results from Question **2** to answer the Big Question. Include dimensions of the bar and its surface area in your answer.

(b) How much less packaging would this new bar require, compared to the current one?

## Elaborate

4 (a) Explain how you can predict which dimensions of a rectangular prism will have the smallest surface area.

(b) Write a statement that explains how you would find the dimensions that give the smallest surface area for a rectangular prism.

## Evaluate

5 (a) Describe, using a step-by-step method, the approach you used to solve this problem. Did you use any of the information you found in Question **1**?

(b) Can this problem be solved another way? What other methods or shortcuts could be used when approaching problems like this one?

(c) Why do you believe current chocolate bars are the shape they are? Do you think that chocolate companies may consider changing the dimensions of chocolate bars? List some advantages and disadvantages of doing this.

## Extend

6 (a) Construct a model of your alternative Cherry Ripe® chocolate bar that has the smallest surface area. Design the packaging and then fold it into a rectangular prism.

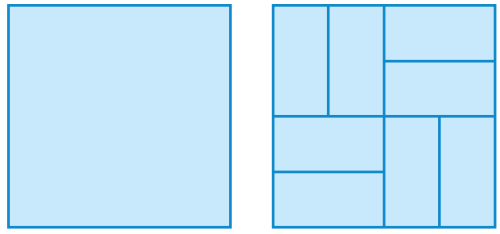
(b) Prepare a report for the Cadbury Schweppes company on the size, dimensions and packaging of your new Cherry Ripe® chocolate bar. Your report should include:

- the dimensions and surface area of your newly designed Cherry Ripe®
- environmental reasons for reducing the amount of packaging used for individual chocolate bars
- financial reasons for reducing the amount of packaging used for individual chocolate bars
- an example of what your new Cherry Ripe® will look like (your design).

# Challenge 6

**1** If a $2 coin is about 2 mm thick, how much is a kilometre-high stack of $2 coins worth?

**2** The perimeter of this square paddock is 800 m. It is divided into eight identical paddocks as shown.



The perimeter, in metres, of each of the eight smaller paddocks is:

**A** 150 **B** 200

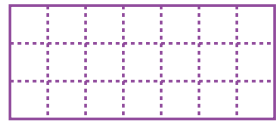
**C** 300 **D** 400

**3** The weight limit for a lift is 1500 kilograms. The average weight of the people in the lift is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the lift?

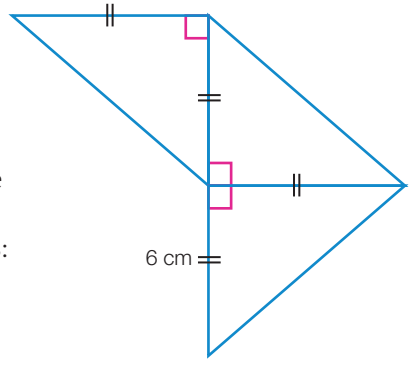
**A** 13 **B** 16 **C** 17 **D** 20

**4** When a square piece of paper is folded in half vertically, the resulting rectangle has a perimeter of 39 cm. Find the area of the original sheet of paper, in square centimetres.

**5** A rectangular area on a wall measuring 7 units by 3 units is covered with 7 rectangular tiles measuring 1 unit by 3 units. In how many ways can this be done?



**6** The area of the entire figure shown is:




**A** 36 $cm^2$ **B** 54 $cm^2$

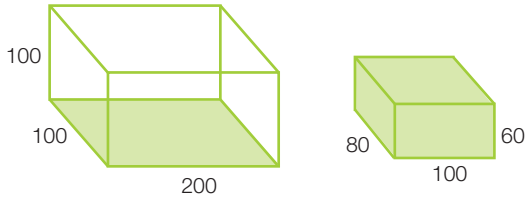
**C** 72 $cm^2$ **D** 108 $cm^2$

**7** A rectangular sign has the dimensions 10 m by 16 m. If a square advertisement to be painted on it must have a border at least 1.5 m wide, the area of the largest square advertisement that can be painted on the sign is:

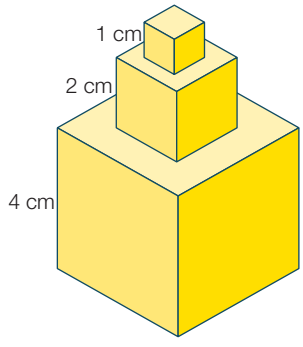
**A** 49 $m^2$ **B** 72.25 $m^2$

**C** 91 $m^2$ **D** 160 $m^2$

**8** A fish tank has a base of 100 cm by 200 cm and a depth of 100 cm. A solid metal rectangular prism with dimensions 80 cm by 100 cm by 60 cm is placed in the tank and water is poured into the tank until it is level with the top of the metal prism. After the prism is carefully removed from the tank, the depth of the water remaining is measured. What are the three possible depths?




**9** Three cubes are sitting on top of each other. The bottom cube has a side length of 4 cm, the cube on top of it has a side length of 2 cm and the very top cube has a side length of 1 cm. What is the visible area of the cube stack?



# Chapter review 6

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| area | height | metric | perpendicular |
| base | kilometre | millimetre | rectangular prism |
| centimetre | length | parallelogram | volume |
| composite shape | metre | perimeter | width |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 _____ is the distance around the outside of a shape.

2 To calculate the _____ of a rectangle, you multiply the _____ by the width.

3 The area of a triangle is half the _____ multiplied by the perpendicular _____.

4 The _____ of a rectangular prism is found by multiplying the length, width and height together.

5 The system of measurement used in Australia is called the _____ system.

6 To find the area of a _____ we break it up into simpler shapes.

7 Multiplying the base length by the perpendicular height will find the area of a _____.

## Fluency

1 **(a)** The height of an average house ceiling is approximately: Ex 6.1

**A** 260 mm **B** 2.6 m **C** 0.26 km **D** 2600 m

**(b)** The length of a classroom desk is approximately:

**A** 18 cm **B** 280 mm **C** 180 cm **D** 180 m

2 Copy and complete the following length conversions. Ex 6.1

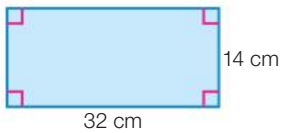
**(a)** 45.9 km = _____ m **(b)** 58 000 cm = _____ km

**(c)** 9.2 m = _____ mm **(d)** 42 m = _____ cm

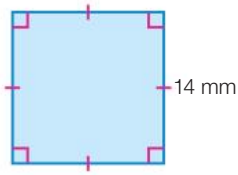
**(e)** 980 m = _____ km **(f)** 0.006 73 km = _____ mm

3 Find the **(i)** perimeter and **(ii)** area of each of the following shapes. Ex 6.2 Ex 6.3 Ex 6.5

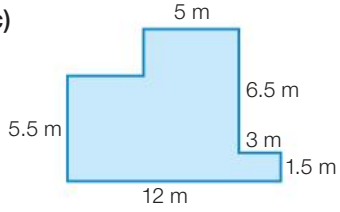
**(a)**



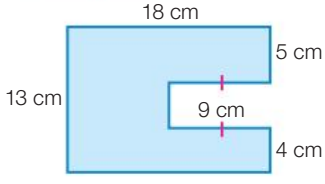

**(b)**




**(c)**




**(d)**



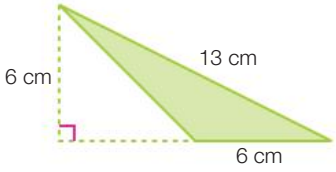
4 Find the perimeter and area of a squash court 9.75 m long and 6.4 m wide.

Ex. 6.2, 6.3

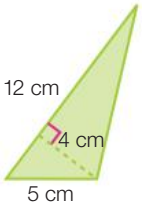
5 Find the area of each of the following shapes.

Ex. 6.3, 6.4

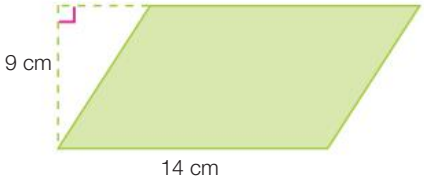
(a)



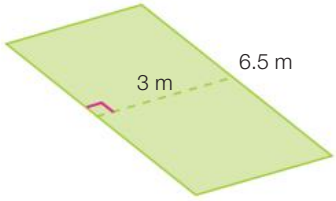

(b)




(c)



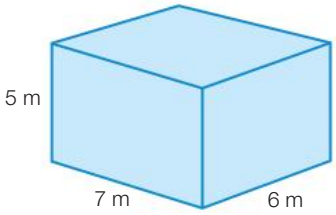

(d)



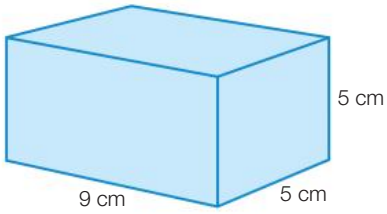

6 Determine the volume of the following solids.

Ex. 6.6

(a)



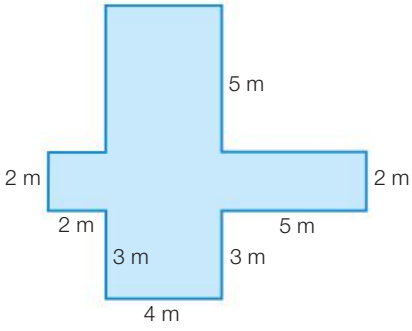

(b)



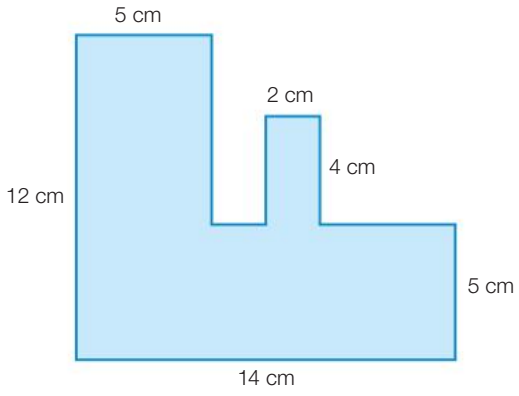

## Understanding

7 Find the area of each composite shape below.

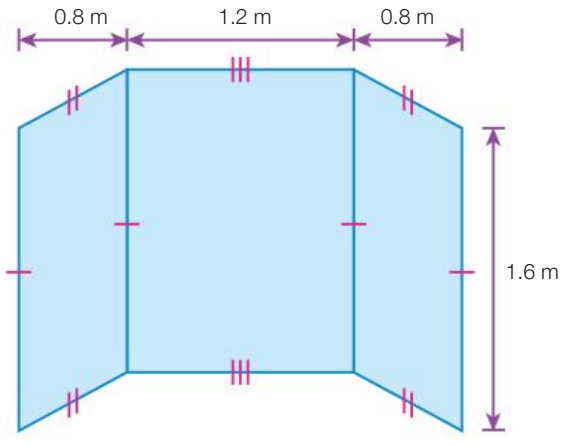
(a)



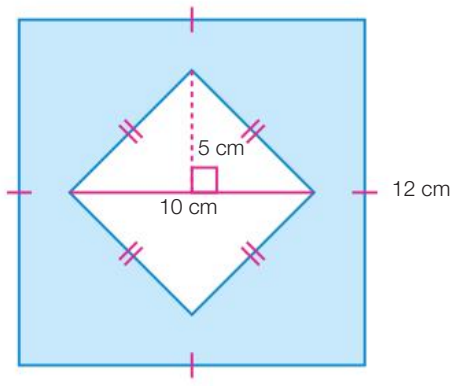

(b)




(c)




(d)



**8** A carpenter calculates that the length of timber she requires to make a book case is 4850 mm.

**(a)** How many metres is this?

**(b)** If timber costs $6 per metre at the hardware store, how much will she pay?



**9** Robbie is tiling the floor of his bathroom, which is a rectangle measuring 3 m by 2.5 m.

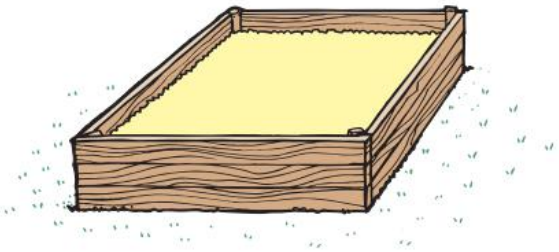
**(a)** Write these measurements in centimetres, then calculate the area of the floor in square centimetres.

**(b)** Robbie is using square tiles that have a side length of 25 cm. Calculate the number of tiles needed to cover the floor.

**(c)** If the tiles cost 80 cents each, calculate how much Robbie will pay for the required number of tiles.



**10** Theresa is constructing a sand pit for her children. She has created a rectangular wooden box that is 2 m long, 1.5 m wide and 0.3 m high.



**(a)** Calculate the perimeter of the box.

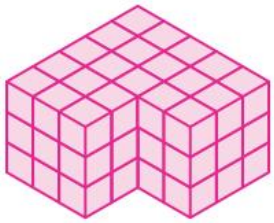
**(b)** Calculate the volume of the box.

**(c)** If one cubic metre of sand costs $15, calculate how much it will cost to fill the box to a depth of 25 cm with sand.

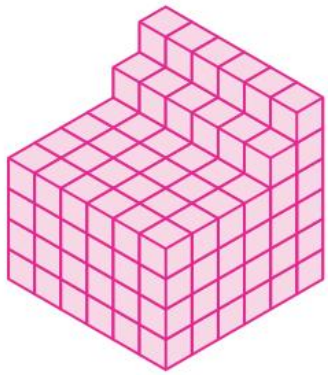
**11** Tricia has obtained a quote of $21 per square metre for her driveway to be resurfaced. If her driveway is rectangular and measures 12 m long and 3.2 m wide, what will the job cost her?

**12** Determine the volume of each of the following solids. (The cubes are centimetre cubes.)

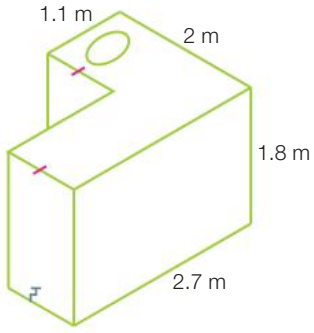
**(a)**



**(b)**



**13** Cameron has had a water tank specially made to fit the side of his house. Calculate the volume of the tank, in cubic metres.

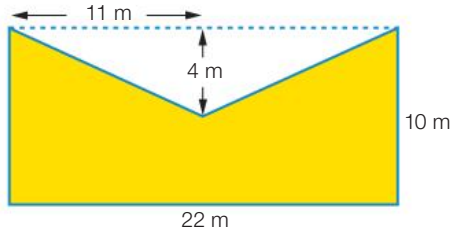



## Reasoning

**14** What is the perimeter of a square with an area of 25 $cm^2$?

**15** What is the area of a rectangle with a perimeter of 32 cm and a length of 10 cm?

**16** At right is a diagram of the available floor space (shaded) in an unoccupied car show-room. Calculate the area available for a display.




**17** Five blocks of dimensions 3 cm × 5 cm × 6 cm are placed in a toy box of length 40 cm, width 20 cm and height 9 cm. Find the volume of space left in the box.

**18** Draw a diagram to help you explain:

**(a)** how to turn a parallelogram into a rectangle

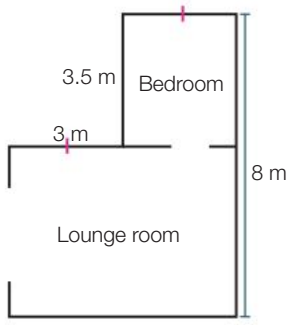
**(b)** how to turn a triangle into a rectangle.

# NAPLAN practice 6

## Numeracy: Non-calculator

**1** Which has the larger area: A rectangle with a length of 20 cm and a width of 15 cm, or a triangle with a base of 20 cm and a height of 15 cm?

**2** Li is having new carpet laid in the lounge room and bedroom of her apartment. What area is to be carpeted?




**3** What is the perimeter of a square that has an area of 100 $cm^2$?

**4** A rectangular picture frame has a perimeter of 80 centimetres. The frame has a length of 25 centimetres.

What is the width of the picture frame? _____ cm

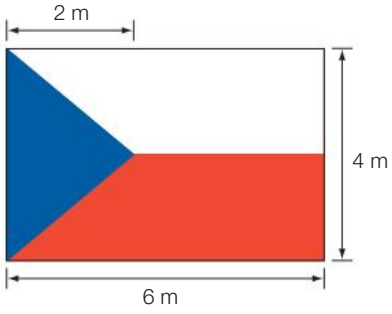
## Numeracy: Calculator allowed

**5** The flag of the Czech Republic is shown here.

The line where the red and white sections meet is halfway up the flag. If the blue triangle has a height of 2 m, the area of the flag that is red is:

**A** 8 $m^2$ **B** 9 $m^2$

**C** 10 $m^2$ **D** 16 $m^2$

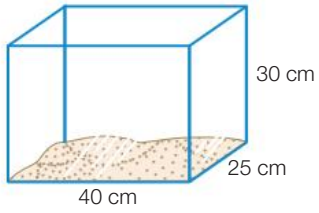



**6** A pack of playing cards is 2.6 cm high. If there are 52 cards in the pack, what is the thickness of each card?

**7** The Hudson family have bought a new fish tank, requiring a 20 mm deep layer of pebbles on the bottom. Which volume of pebbles should they buy?

**A** 1000 $cm^3$ **B** 2000 $cm^3$

**C** 20 000 $cm^3$ **D** 30 000 $cm^3$




**8** Sanjay used identical cubes to build a rectangular prism. There were 18 cubes in its base. She used 54 cubes altogether. Which of these could be the dimensions of Sanjay's prism?

**A** $6 \times 3 \times 2$ **B** $3 \times 2 \times 9$ **C** $9 \times 3 \times 3$ **D** $6 \times 3 \times 9$

# Mixed review C

## Fluency

**1** Write the following in algebra. Ex 5.1

**(a)** Akash has $n$ pencils in his pencil case. He takes 8 out and lends them to a friend. How many pencils are in the case now?

**(b)** $k$ dollars is divided evenly among 6 people. How much money does each person receive?

**(c)** There are 50 matches in one box. How many matches are there in $h$ boxes?

**2** Arrange the following decimals in order from smallest to largest. Ex 4.1

**(a)** 2.3, 2.003, 2.33, 2.323, 2.302 **(b)** 0.089, 0.129, 0.091, 0.0909, 0.0199

**3** Write the following as fractions or mixed numbers in simplest form. Ex 3.1

**(a)** 6 out of 24 students in the class have black hair.

**(b)** 5 full dozen egg cartons and 4 eggs in another carton.

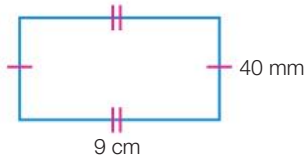
**(c)** 25 minutes out of an hour.

**4** Write each of the following numbers as a product of their prime factors in index form. Ex 2.3

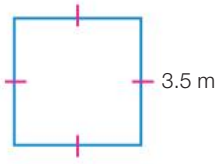
**(a)** 28 **(b)** 200 **(c)** 990

**5** Find **(i)** the perimeter and **(ii)** the area of the following shapes, giving your answer in the larger unit where necessary. Ex 6.2, 6.3, 6.5

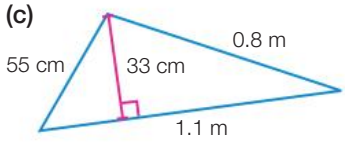
**(a)**




**(b)**




**(c)**




**6** Calculate: Ex 1.5

**(a)** $7 + 32 \div 4 \times 2$ **(b)** $(33 + 12) \div 3 + 19$ **(c)** $6 \times 9 \div 2 + 4$

**7** Calculate the following, writing your answers in simplest form and as a mixed number, if appropriate. Ex 3.5, 3.6

**(a)** $\frac{5}{6} \times \frac{4}{9}$ **(b)** $\frac{9}{10} \div \frac{7}{15}$ **(c)** $1\frac{3}{5} \times 2\frac{7}{8}$ **(d)** $4 \div \frac{2}{3}$

**8** Copy and complete the following conversions. Ex 6.1

**(a)** 6.09 km = _____ m **(b)** 83 cm = _____ m **(c)** 5400 cm = _____ km

**9** Calculate the following. Ex 2.5–2.7

**(a)** -2 + 8 **(b)** 14 – 17 **(c)** -15 + 3 **(d)** -5 – 6

**(e)** 3 + (-9) **(f)** -4 – (-1) **(g)** 11 – (-2) **(h)** -7 – 10

**10** Use a mental or written strategy to calculate the following. Ex 1.1, 1.3

**(a)** $28 \times 43$ **(b)** $990 \div 6$ **(c)** $57 \times 4$

**11** There are 8 red, 12 blue and 4 black jelly beans in a packet. Ex 4.9

**(a)** Write the ratio blue:red:black in simplest form.

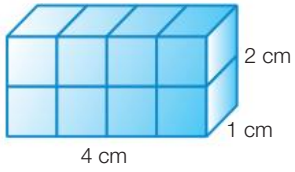
**(b)** Write the number of red jelly beans as a percentage of the total, rounded to one decimal place.

**12** Write the following decimals as fractions in simplest form. Ex 4.3

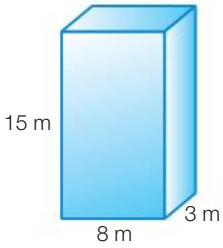
**(a)** 0.23 **(b)** 0.06 **(c)** 0.505

**13** Find the volume of the following rectangular prisms. Ex 6.6

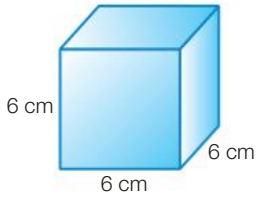
**(a)**




**(b)**




**(c)**




**14** Copy and complete the table using the rule $y = 3(x + 2)$. Ex 5.4

| $x$ | 0 | 2 | 3 | 6 | 10 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**15** Calculate: Ex 4.4

**(a)** 4.67 + 8.8 **(b)** 8.93 + 0.006 35 **(c)** 19.87 + 21.075

**16** Calculate: Ex 4.6

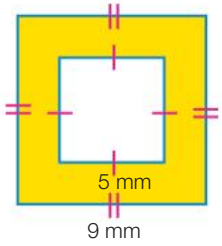
**(a)** 1.86 ÷ 600 **(b)** 0.42 ÷ 1000 **(c)** 10.605 ÷ 0.005

## Understanding

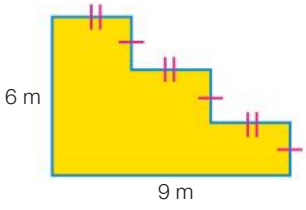
**17** Joseph had $578.97 in his bank account. After his pay was deposited, there was $665.49. How much was Joseph's pay?

**18** Find the area of the shaded regions.

**(a)**




**(b)**




**19** Ben surveyed some Year 7 students to find out their favourite Australian animal. Here are his results:

| | | | |
|---|---|---|---|
| Koala | 8 students | Bilby | 6 students |
| Wombat | 4 students | Kangaroo | 2 students |

Write the preference for each animal **(a)** as a fraction in simplest terms, and **(b)** as a percentage of the total number of students.

## Reasoning

**20** Which is larger: $2^8$ or $8^2$?

**21** The Hawksbury Magpies were playing in the U14 football grand final. All goals were scored by three boys. Johan scored $\frac{1}{3}$ of the goals, Rihbi scored $\frac{1}{2}$ of the goals and Kyle scored the rest. If the Magpies scored 24 goals, how many goals were scored by Kyle?

**22** A rectangle has a length 5 cm greater than its width and a perimeter of 42 cm. What are the dimensions of the rectangle?

**23** At a shop I can buy three cans of soft drink for the same price as a hamburger. How much is one can of drink if I can buy one can and a hamburger for $9?

# 7



# 7 Linear equations

## **Wiis work wonders.** Why do people learn physical tasks at different rates?

Why do some people seem to have 'natural talent' when others struggle to master the basics? University researchers are studying how people learn various skills. They are measuring the motions involved in tasks as simple as playing racquet sports through to more complex activities, such as flying a fighter jet, in the hope that this research will help people to learn these skills faster and more effectively.

One university research project is making use of Nintendo's popular 'Wii' video game technology to create equations that represent skills involved in many human activities from sport to surgery. To do this, a motion-capture device, such as the Wiimote, is used. Data from the Wiimote is used to measure a range of movements that is then used to create mathematical equations. The aim is then to program robots to teach people the most efficient way to learn new tasks. By bringing robotics and virtual reality together, learning will no longer be a matter of trial and error. For example, if you were learning to play tennis, a robotic device, such as a sleeve that prompted you to move your arm the correct way, would make learning easier.

Wiis are already being used in the classroom. How cool is that?

### Forum

Researchers are investigating ways that different movements can be recorded by using a Wiimote. Why might the universities be interested in this? How do you think a Wii could be used in classrooms?

### Why learn this?

Whether you are deciding how much material to buy to make curtains, working to a family budget, or calculating whether a business has made a profit, equations can be used to represent a variety of situations. Solving them can help us to make informed and accurate decisions.

**After completing this chapter you will be able to:**

- write an equation from information given in words
- check solutions to equations by substitution
- solve worded problems
- solve everyday problems using equations
- solve equations using a variety of methods.

# Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R7.1

**1** Answer TRUE or FALSE to each of the following number sentences.

**(a)** $3 + 7 = 5 + 6$ **(b)** $8 \times 3 = 20 + 4$ **(c)** $\frac{24}{8} = 3 \times 5 - 14$

Worksheet R7.2

**2** **(a)** The term $5x$ means:

**A** $5 + x$ **B** $5 - x$ **C** $5 \times x$ **D** $x \div 5$

**(b)** The term $\frac{x}{7}$ means:

**A** $x - 7$ **B** $x + 7$ **C** $x \times 7$ **D** $x \div 7$

**(c)** $4(x + 3)$ means:

**A** $4 + (x + 3)$ **B** $4 \times (x + 3)$ **C** $4 \times x + 3$ **D** $4 - (x + 3)$

Worksheet R7.3

**3** Rewrite each of the following sentences using mathematical symbols and numbers only.

**(a)** Five added to seven is equal to twelve.

**(b)** Subtracting two from the product of four and six is equal to twenty-two.

**(c)** When the sum of eight and six is divided by seven, the answer is equal to two.

Worksheet R7.4

**4** What is the final result after applying each of the following operations to the number in the box?

**(a)** $\boxed{8}$ $\times 2$ then $-4$ then $\div 3$ then $+5$

**(b)** $\boxed{10}$ $\div 2$ then $\div 5$ then $+6$

Worksheet R7.5

**5** **(a)** If $x = 2$, find the value of $4x - 1$.

**(b)** If $x = 7$, find the value of $\frac{x - 3}{2}$.

## Key Words

| | | |
|---|---|---|
| backtracking | false number sentence | solve |
| balance method | guess, check and improve | solving by inspection |
| checking by substitution | inverse operations | true number sentence |
| equivalent equations | solution | |

# Number sentences

# 7.1

Vlado wants change for a $5 note. His friend Perry gives him two $2 coins and a $1 coin in exchange for the note. This can be written as $5 = 2 + 2 + 1$ and is an example of a **true number sentence**.




$2 \times 4 = 5 + 3$ is another example of a true number sentence, as both the left-hand side (LHS) and the right-hand side (RHS) have the same value of 8.

left-hand side (LHS) equals sign right-hand side (RHS)

LHS = RHS

$2 \times 4 = 5 + 4$

LHS ≠ RHS

is an example of a **false number sentence**, as the LHS has a value of 8 and the RHS has a value of 9.

≠ means 'not equal to', < means less than, > means greater than

## Worked Example 1

WE1

The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

**(a)** $5 + 8 = 15$ **(b)** $2 \times 5 - 1 = 7$

| Thinking | | Working |
|---|---|---|
| **(a)** | **1** Find the value of the side that has the coloured number. | (a) $5 + 8 = 13$ |
| | **2** Compare the answer to the other side of the number sentence. | $13 < 15$ |
| | **3** Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number. | $5 + 10 = 15$ |

| | | | |
|---|---|---|---|
| **(b)** | **1** | Find the value of the side that has the coloured number. | (b) $2 \times 5 - 1 = 9$ |
| | **2** | Compare the answer to the other side of the number sentence. | $9 > 7$ |
| | **3** | Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number. | $2 \times 4 - 1 = 7$ |

## Worked Example 2

WE2

Decide which of the following are true number sentences.

**(a)** $8 \times 3 + 4 = 28$

**(b)** $\frac{6 \times 5}{3} = 7 + 2$

| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Evaluate the LHS of the number sentence. | (a) LHS $= 8 \times 3 + 4$<br>$= 28$ |
| | **2** | Compare the LHS and RHS to check whether they are equal. | LHS = RHS |
| | **3** | State your answer. | The sentence is true. |
| **(b)** | **1** | Evaluate the LHS of the number sentence. | (b) LHS $= \frac{6 \times 5}{3}$<br>$= 10$ |
| | **2** | Evaluate the RHS of the number sentence. | RHS $= 7 + 2$<br>$= 9$ |
| | **3** | Compare the LHS and RHS to check whether they are equal. | LHS $\neq$ RHS |
| | **4** | State your answer. | The sentence is false. |

# 7.1 Number sentences

## Navigator

| Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q10(a), Q11 (a–d), Q12, Q14, Q16 | Q1, Q2 Column 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 (a–f), Q12, Q14, Q16 | Q1, Q2 Column 3, Q3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15 |
|---|---|---|

Answers page 662

## Fluency

**1** The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

WE1

**(a)** $3 + 4 = 10$ **(b)** $5 \times 6 = 25$ **(c)** $\frac{24}{4} = 4$

**(d)** $22 - 7 = 8 + 4$ **(e)** $3 \times 7 + 2 = 8 + 9$ **(f)** $1 + \frac{15}{5} = 6$

**2** Decide which of the following are true number sentences.

WE2

**(a)** $\frac{32}{4} + 7 = 16$ **(b)** $4 + 3 \times 2 = 14$ **(c)** $6 + 4 \times 5 = 25$

**(d)** $30 \div 6 = 4 + 1$ **(e)** $23 - 7 = 12 + 4$ **(f)** $8 + 4 = 6 \times 2$

**(g)** $7 - 5 = 9 \div 3$ **(h)** $10 \times 8 = 90 - 10$ **(i)** $\frac{6 + 10 + 12}{7} = \frac{20 + 4}{6}$

**3 (a)** The number missing from the true number sentence $3 \times ____ = 19 + 5$ is:

**A** 3 **B** 6 **C** 7 **D** 8

**(b)** The number missing from the true number sentence $4 \times 2 = ____ + 4$ is:

**A** 0 **B** 2 **C** 4 **D** 8

**(c)** The number missing from the true number sentence $\frac{24}{8} + ____ = 10 - 2$ is:

**A** 4 **B** 5 **C** 6 **D** 8

## Understanding

**4** Rewrite each true number sentence using numbers and mathematical symbols only.

**(a)** Forty-three added to five is equal to forty-eight.

**(b)** Three multiplied by seven is equal to nineteen plus two.

**(c)** Fifty divided by five is equal to the product of five and two.

**(d)** When the sum of the numbers six and eight is subtracted from twenty, the answer is equal to twelve divided by two.

**5** Write each of the following as a number sentence, then state whether it is true or false.

**(a)** Nineteen subtracted from thirteen is equal to six.

**(b)** Four divided by two is equal to eight.

**(c)** Eleven subtracted from eight is equal to six minus three.

**(d)** Twelve divided by four is equal to fifteen divided by five.



6 Jane is 5 years older than Todd. If Todd is 7 years old, then write a true number sentence to show Jane's age.

7 Six pens at 60c each cost the same as buying 4 exercise books at 90 cents each. Write this as a true number sentence.

8 The cost of a new shirt is $25 after a discount of $7 is given. Write a true number sentence to show the original price of the shirt.

9 A bus can carry 48 passengers. The bus is full, then 11 people leave while another 5 get on at the next stop. Write a true number sentence to show how many passengers are left on the bus.

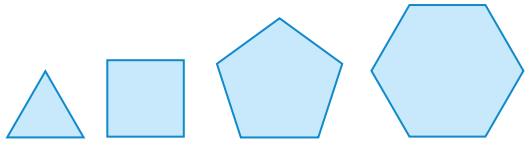
10 Numbers are consecutive if one follows the other. The numbers 8 and 9 are consecutive numbers that add to 17.

**(a)** Write a true number sentence that shows two consecutive numbers that add to 33.

**(b)** Write a true number sentence that shows three consecutive numbers whose sum is 54.

## Reasoning

11 A piece of wire 60 cm long is used to make a triangle shape that has all three sides the same length. Write a true number sentence to answer each of the following.



**(a)** How long is each side of the triangle?

**(b)** The wire is now used to make a square. How long is each side?

**(c)** The wire is bent to make a regular pentagon. How long is each side?

**(d)** The wire is then used to make a regular hexagon. How long is each side?

**(e)** Write a number sentence you could use to determine the length of the side of a shape that has 20 sides of equal length made from the same piece of wire.

**(f)** Write a number sentence you could use to determine the length of the side of a shape that has 50 sides of equal length made from a wire 200 cm long.

**(g)** Use a 100 cm length of wire to make a shape whose sides are of equal length. Find three possible values for the number of sides the shape can have if the length of each side is to be a whole number and shorter than 25 cm.




## Open-ended

12 Use the numbers 1, 2, 4, 8, the equals sign, =, and as many of the symbols +, −, ×, ÷ as you like to write three different true number sentences whose right-hand side is a number from 1 to 10.

13 A length of wire is used to make a shape that has eight sides of equal length. Find five different whole number values for the length of wire if the length of each side is to be greater than the number of sides.

**14** $\square + \square \times \square - \square = 20$

What might the missing numbers be? Give three different answers.

**15** Sam and Ruben have written the following number sentences. Sam wrote $3 + 4 \times 21 = 147$. Ruben wrote $3 + 4 \times 21 = 87$.

Who is correct? Why? Write a sentence so that the person who was incorrect can improve their skills.

**16** Two different students were given the following number sentence.

$38 - 16 = \square - 12$

They presented the following solutions.

$38 - 16 = \square - 12$
$38 - 12 - 4 = \square - 12$
$38 - 4 - 12 = \square - 12$
$34 - 12 = \square - 12$
$\square = 34$

$38 - 16 = \square - 12$
$38 - (12 + 4) = \square - 12$
$38 + 4 - 12 = \square - 12$
$42 - 12 = \square - 12$
$\square = 42$

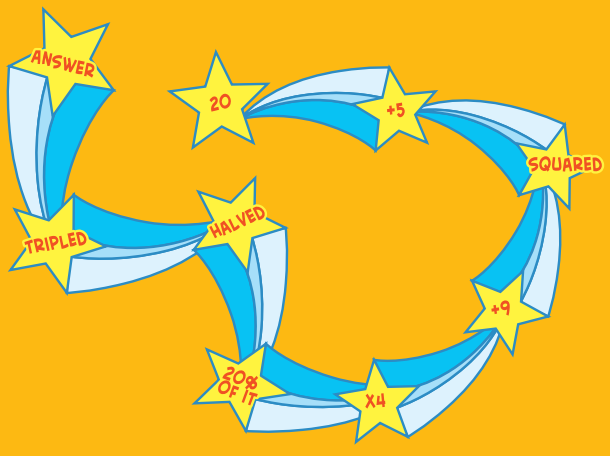
Which of the solutions is correct? Why? What is wrong with the incorrect solution?

## Outside the Square Puzzle

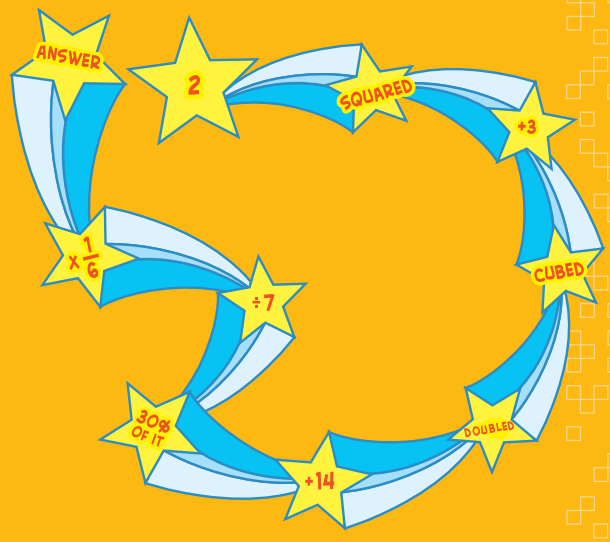
### Star quest

**Equipment required:** 1 brain, 1 watch

**(a)** Start at the first star and follow the shooting stars. Time yourself to see how long it takes to complete all of the operations.




**(b)** What about trying something more challenging? Time yourself completing this star quest.



# 7.2 Introduction to equations

Often, in mathematics, we want to write information in a number sentence but some information is missing. Where a value is unknown, we can use a pronumeral to represent the unknown amount. This number sentence is now called an equation.

$2 \times 4 = 5 + 3$ is a true number sentence. $2 \times 4 = 5 + 4$ is a false number sentence.

$2 \times 4 = 5 + x$ is an equation. Equations always contain an equals sign.

We need to be able to understand and interpret the information given to us in an equation.

The equals sign was invented in 1557 by Welshman Robert Recorde.




## Worked Example 3 WE3

Write each of the following equations in words.

**(a)** $5x - 3 = 1$ **(b)** $6 = \frac{x}{5} + 2$

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | Call the variable 'a number' and write in words the operation that has been performed on it first, using the correct order of operations. | (a) When a number is multiplied by five |
| **2** | Decide what happens next to the LHS and add this on to what you have already written. | and then three is subtracted |
| **3** | State the final result. This is the value shown on the RHS. | the result is equal to one. |
| **4** | Write the complete equation in words. | When a number is multiplied by five, and then three is subtracted, the result is equal to one. |
| **(b)** **1** | Call the variable 'a number' and write in words the operation that has been performed on it first using the correct order of operations. | (b) When a number is divided by five |
| **2** | Decide what happens next to the RHS and add this on to what you have already written. | and then two is added |
| **3** | State the final result. This is the value shown on the LHS. | the result is equal to six. |
| **4** | Write the complete equation in words. | When a number is divided by five, and then two is added, the result is equal to six. |

We **solve** an equation by finding the unknown value that makes an equation a true number sentence. By substituting $x = 3$ into $2 \times 4 = 5 + x$ we get a true number sentence, so the **solution** to $2 \times 4 = 5 + x$ is $x = 3$.

If $x$ is any other value, we would get a false number sentence.

Equations are useful in solving everyday problems by providing a shorthand way of writing information. There are many different methods used to solve equations.

## Solving by inspection

Many equations involve only one operation (+, –, ×, ÷) and are sometimes called one-step equations. For example, $3x = 9$, $\frac{x}{2} = 7$, $x + 3 = 11$, $4 - x = 1$ are one-step equations.

These equations can be solved by simply looking at the equation and guessing the value of $x$. When we do this, we need to check that our solution will make a true number sentence. This method is called '**solving by inspection**'.

Here are some more one-step equations and their solutions:

For $k - 4 = 5$, the solution is $k = 9$. Check: $9 - 4 = 5$ is a true number sentence.

For $\frac{a}{3} = 5$, the solution is $a = 15$. Check: $\frac{15}{3} = 5$ is a true number sentence.

## Worked Example 4 WE4

For the following equation, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

$3x - 4 = 11 \quad (x = 5)$

| Thinking | Working |
|---|---|
| **1** Identify the side of the equation that contains the variable. | LHS $= 3x - 4$ |
| **2** Substitute the given value for the variable and simplify. | $= 3 \times 5 - 4$<br>$= 11$ |
| **3** Check to see whether this answer is the same as the RHS of the equation. | RHS $= 11$<br>Yes, $x = 5$ is the solution. |

## Solving equations using guess, check and improve

We can use '**guess, check and improve**' to solve equations with more than one step.

## Worked Example 5 WE5

Find the solution to the following equation by using the guess, check and improve method.

$$\frac{6x - 1}{5} = 7$$

| Thinking | Working |
|---|---|
| **1** Guess a value for $x$ and substitute it into the LHS of the equation to see if it makes a true number sentence. | Try $x = 5$<br>$\frac{6 \times 5 - 1}{5} = \frac{30 - 1}{5}$<br>$= \frac{29}{5}$<br>$\neq$ RHS |

| | |
|---|---|
| **2** Try a larger value for $x$ and substitute it into the LHS of the equation. | Try $x = 6$<br>$\frac{6 \times 6 - 1}{5}$<br>$= \frac{36 - 1}{5}$<br>$= \frac{35}{5}$<br>$= 7$<br>$= \text{RHS}$ |
| **3** As this value makes a true number sentence, we have a solution. If the LHS still does not equal the RHS, try another value. | $x = 6$ is the solution to the equation. |

# 7.2 Introduction to equations

## Navigator

Answers page 662

| | | |
|---|---|---|
| Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q12 | Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 |

## Fluency

WE3

**1** Write each of the following equations in words.

**(a)** $n + 3 = 10$ **(b)** $4 + p = 7$ **(c)** $a + 5 = 6$

**(d)** $x - 2 = 4$ **(e)** $k - 7 = 2$ **(f)** $a - 8 = 6$

**(g)** $5k = 20$ **(h)** $7p = 14$ **(i)** $7a = 21$

**(j)** $\frac{x}{3} = 4$ **(k)** $\frac{f}{2} = 6$ **(l)** $\frac{y}{6} = 12$

**(m)** $2 + 3x = 7$ **(n)** $2 + 6x = 10$ **(o)** $1 + 3x = 8$

WE4

**2** For the following equations, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

**(a)** $x + 6 = 11$ $(x = 5)$ **(b)** $y + 4 = 10$ $(y = 5)$ **(c)** $m + 2 = 11$ $(m = 9)$

**(d)** $d - 6 = 15$ $(d = 21)$ **(e)** $k - 3 = 8$ $(k = 5)$ **(f)** $a - 6 = 9$ $(a = 15)$

**(g)** $3x = 12$ $(x = 3)$ **(h)** $5a = 20$ $(a = 4)$ **(i)** $7m = 14$ $(m = 2)$

**(j)** $\frac{f}{2} = 16$ $(f = 8)$ **(k)** $\frac{m}{3} = 5$ $(m = 15)$ **(l)** $\frac{x}{7} = 14$ $(x = 2)$

**(m)** $5m + 1 = 9$ $(m = 2)$ **(n)** $2a - 2 = 8$ $(a = 3)$ **(o)** $5s - 4 = 6$ $(s = 2)$

**(p)** $\frac{x}{3} + 2 = 5$ $(x = 9)$ **(q)** $5 + \frac{b}{2} = 9$ $(b = 8)$ **(r)** $\frac{a}{15} + 2 = 5$ $(a = 30)$

WE5

**3** Find the solution to each of the following equations by using the guess, check and improve method.

**(a)** $2x + 1 = 9$ **(b)** $6 + 5x = 41$ **(c)** $7x + 3 = 52$

**(d)** $\frac{x - 8}{3} = 7$ **(e)** $\frac{x - 5}{2} = 6$ **(f)** $\frac{x - 4}{5} = 12$

**(g)** $3(x - 11) = 21$ **(h)** $8(x - 2) = 32$ **(i)** $6(x + 13) = 54$

**(j)** $\frac{x}{2} - 5 = 33$ **(k)** $\frac{x}{6} - 8 = 14$ **(l)** $\frac{x}{3} + 7 = 61$

**4** In words, the equation $2 = 5 - \frac{x}{6}$ can be written as:

**A** two is equal to five added to a number that is divided by six

**B** five subtracted from six times a number is equal to two

**C** five added to two is equal to a number that is divided by six

**D** a number is divided by six and the result is subtracted from five to give an answer of two.

**5 (a)** Which equation describes this sentence?

Four added to a number is equal to ten.

**A** $n + 10 = 4$ **B** $n + 4 = 10n$ **C** $4 + 10 = n$ **D** $n + 4 = 10$

**(b)** Which equation describes this sentence?

A number multiplied by three is equal to six.

**A** $x + 3 = 6$ **B** $6x = 3$ **C** $3x = 6$ **D** $\frac{x}{3} = 6$

**(c)** Which equation describes this sentence?

A number divided by four is equal to three.

**A** $4x = 3$ **B** $x + 4 = 3$ **C** $4 - x = 3$ **D** $\frac{x}{4} = 3$

**6 (a)** The solution to the equation $x + 3 = 18$ is:

**A** $x = 6$ **B** $x = 15$ **C** $x = 16$ **D** $x = 21$

**(b)** The solution to the equation $\frac{x}{2} = 4$ is:

**A** $x = 1$ **B** $x = 2$ **C** $x = 6$ **D** $x = 8$

**(c)** The solution to the equation $4 = x - 11$ is:

**A** -15 **B** 7 **C** 11 **D** 15

## Understanding

**7 (a)** Which equation describes this situation?

The cost, $c$, in dollars, of some grocery items plus a $3 delivery charge totals $55.

**A** $3c = 55$ **B** $c - 3 = 55$ **C** $c + 3 = 55$ **D** $55 + 3 = c$

**(b)** Which equation describes this situation?

Each of 4 people receive the same number of lollies, $n$, when a bag containing 36 lollies is shared between them.

**A** $4n = 36$ **B** $4 + n = 36$ **C** $\frac{n}{4} = 36$ **D** $n - 4 = 36$

**(c)** Which equation describes this situation?

A school minibus seats a total of 12 passengers when full. The number of passengers, $p$, on the bus decreases by 3 at the next stop so that the bus is now half full.

**A** $p - 3 = 12$ **B** $3p = 6$ **C** $p + 3 = 6$ **D** $p - 3 = 6$

**8** Use guess, check and improve to find the solution to the following equations.

**(a)** $\frac{x}{2} = x - 5$ **(b)** $3x = 5x - 8$ **(c)** $\frac{3x - 1}{4} = \frac{2(x + 9)}{5}$

**9** Daniel is three years older than his brother Christopher.

**(a)** Define a variable to represent Daniel's age.

**(b)** Use this variable to write an expression for Christopher's age.

**(c)** Write an expression to show how old Christopher will be in 15 years time.

**(d)** If Christopher will be 32 in 15 years time, write an equation using your answer to part **(c)** as the LHS of the equation and solve it to find Daniel's age now.

## Reasoning

**10** Cameron has some money in his wallet. Libby has $5.

**(a)** Define a variable to represent the amount of money Cameron has in his wallet.

**(b)** If Cameron spends $14, write an expression to show how much money he has left.

**(c)** If Libby and Cameron now have $23 between them, write an equation and solve it to find how much money Cameron had to start with.

## Open-ended

**11** Pete is twice as old as Chee.

**(a)** Choose a value that is a multiple of 3 as the sum of their ages and write the equation that you would use to find Chee's age.

**(b)** Choose another value that is also a multiple of 3 as the sum of their ages and write a new equation you would use to find Chee's age.

**(c)** What age might Chee be if the sum of their ages is less than 30?

**12** Write a short story that could be represented by the following equations.

**(a)** $14 - x = 11$ **(b)** $\frac{x}{5} = 12$ **(c)** $6x = 108$

## Outside the Square Puzzle

### The great number mystery

What number am I?

- I am a prime number.
- If you triple me, I am made up of two digits.
- If I am multiplied by four, and then divided by five, I am one smaller than my original value.
- If I am squared, I am five less than six times my value.



# Solving equations using backtracking

7.3

Some equations are difficult to solve by inspection or guess, check and improve because the solution is not a whole number, or more than one operation is involved.

For example: $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3.75 = 21.8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations but their solutions are not easy to see.

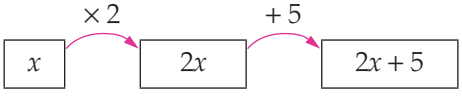
$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x+6}{8} = 2$ are all two-step equations.

We need to use other methods to solve these equations.

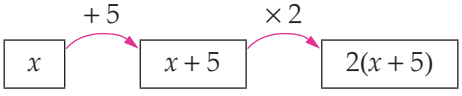
## Using flowcharts

In Chapter 5, flowcharts were used to build expressions. Flowcharts can also be used to help us solve equations. To use a flowchart correctly, we need to apply the order of operations that were used to build the expression.

For the expression $2x + 5$, $x$ is first multiplied by 2 to give $2x$, then 5 is added to give us $2x + 5$, so our flowchart looks like this:




If we wanted to add 5 first before multiplying by 2, the flowchart would look like this:




This expression is $2(x + 5)$, which is quite different from $2x + 5$.




Applying the same operations in a different order produces different expressions.

## Inverse operations

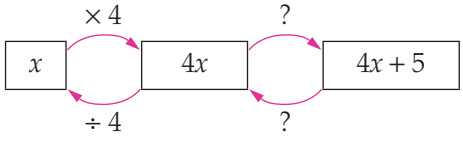
To solve equations using flowcharts, we undo operations used to build the equation by applying **inverse operations** (opposite operations) in the reverse order. We move backwards along the flowchart.

## Worked Example 6

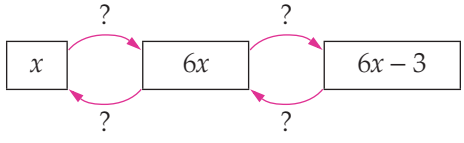
WE6

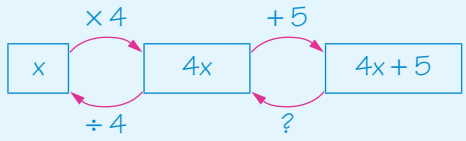
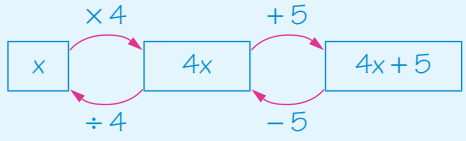
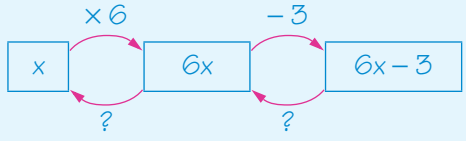
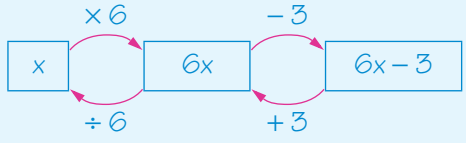
Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

**(a)**



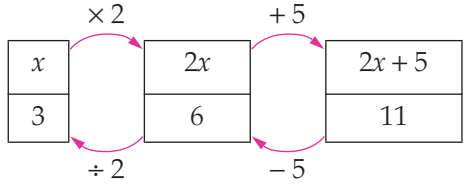

**(b)**



| Thinking | Working |
|---|---|
| **(a) 1** Above the top arrow write the missing operation needed to build the expression. |   |
| **2** Below the bottom arrow write the missing operation that will undo this expression. |   |
| **(b) 1** Above the top arrows write the missing operation needed to build each expression. |   |
| **2** Below the bottom arrows write the missing operation needed to undo this expression. |   |

## Backtracking

To solve $2x + 5 = 11$, we set up the following flowchart.




**Step 1** Draw a flowchart for the expression on the LHS of the equation.

**Step 2** Add boxes underneath the flowchart boxes and arrow signs in the opposite direction.

**Step 3** Write the inverse operations on the arrow signs.

**Step 4** Write the number on the RHS of the equation in the box on the right-hand side of the flowchart underneath the expression.

**Step 5** Complete the flowchart using the inverse operations.

**Step 6** The number under $x$ is the solution.

This method of solving equations is called **backtracking**. Backtracking along this flowchart gives us the solution $x = 3$ to the equation $2x + 5 = 11$.

When backtracking, we undo an operation by using the inverse (opposite) operation.

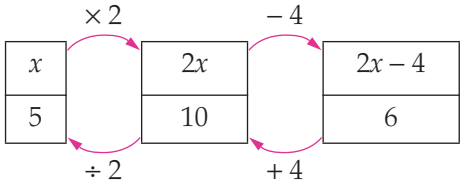
+ and − are inverse operations

× and ÷ are inverse operations

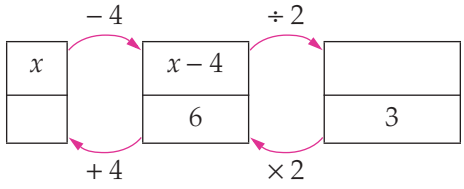
## Worked Example 7 WE7

Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

**(a)**




**(b)**




### Thinking

**(a) (i)** The expression in the last box on the right and the number in the box underneath it form the equation.

**(ii)** The solution is the number under $x$ in the first box on the left of the flowchart.

**(b) (i) 1** Complete the missing parts of the flowchart.

**2** The expression in the last box on the right and the number in the box underneath it form the equation.

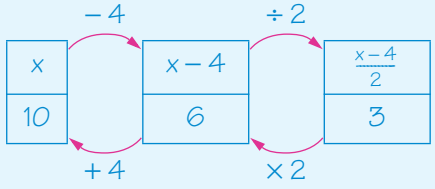
**(ii)** The solution is the number under $x$ in the first box on the left of the flowchart.

### Working

(a) (i) Equation is $2x - 4 = 6$

(ii) Solution is $x = 5$

(b) (i)




$\frac{x-4}{2} = 3$

(ii) $x = 10$

## Worked Example 8 WE8

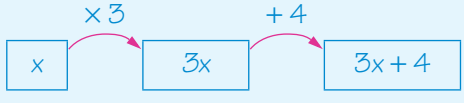
Draw a flowchart and use backtracking to solve the equation: $3x + 4 = 13$.

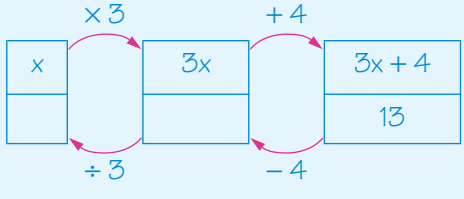
### Thinking

**1** Build the expression on the LHS of the equation with a flowchart. Start with $x$ and identify the operations needed. (× 3, + 4)

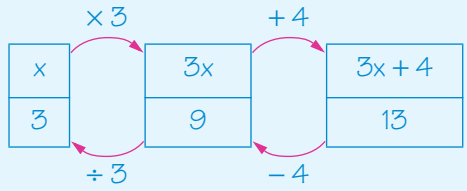
**2** Write the value from the RHS of the equation under the last box on the right of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (÷ 3, − 4)

### Working



**3** Perform the required operations to obtain a value for $x$.




**4** State the solution.

$x = 3$

## Checking by substitution

We can check that the answer obtained is the solution to the equation by substituting it into the equation. We evaluate both sides of the equation. If the LHS = RHS, the answer is the solution. This process is called **checking by substitution**.

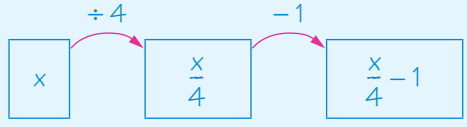
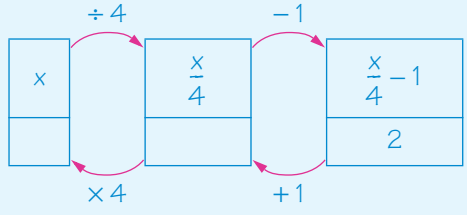
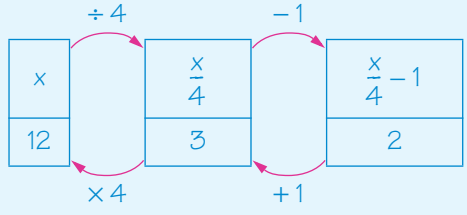
In Worked Example 8, we would check the solution by substituting $x = 3$ into the left-hand side of the equation.

Check: LHS $= 3x + 4$
$= 3 \times 3 + 4$
$= 9 + 4$
$= 13$
$=$ RHS

So, $x = 3$ is the solution.

## Worked Example 9 WE9

Draw a flowchart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

| Thinking | Working |
|---|---|
| **1** Build the expression on the LHS of the equation with a flowchart. Start with $x$ and identify the operations needed. (÷ 4, – 1) |   |
| **2** Write the value from the RHS of the equation under the right-hand box of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (× 4, + 1) |   |
| **3** Perform the required operations to obtain a value for $x$. |   |
| **4** State the solution. | $x = 12$ |

**5** Check the solution by substituting your answer into the left-hand side of the equation. If the left-hand side equals the right-hand side of the equation, you have found the solution.

Check: LHS $= \frac{x}{4} - 1$
$= \frac{12}{4} - 1$
$= 3 - 1$
$= 2$
$= \text{RHS}$

# 7.3 Solving equations using backtracking

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2 Column 1, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Column 1, Q6 Column 1, Q7 Column 1, Q8 Column 1, Q9, Q10 (a) & (b), Q13 | Q1 Column 1, Q2 Column 1, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6 Column 2, Q7 Column 2, Q8 Column 2, Q9, Q10 (a)–(c), Q11, Q12, Q13 | Q1 Column 2, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7 Column 3, Q8 Column 3, Q9, Q10, Q11, Q12, Q13, Q14 |

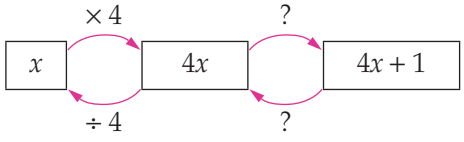
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## Fluency

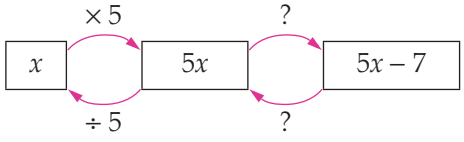
WE6

**1** Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

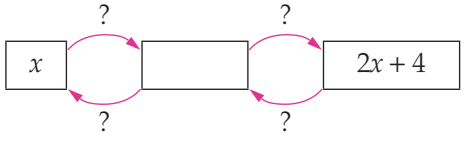
**(a)**



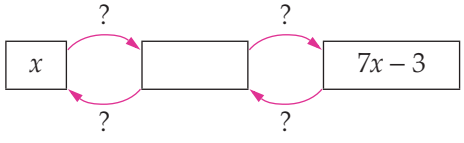

**(b)**



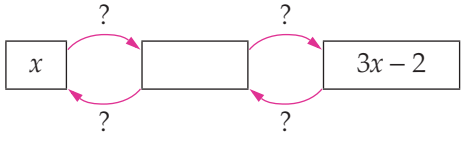

**(c)**



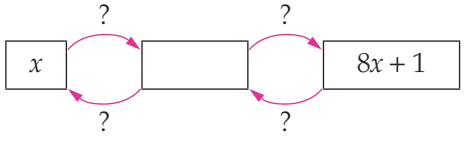

**(d)**




**(e)**



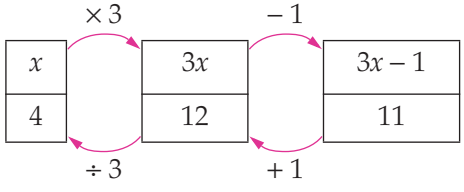

**(f)**



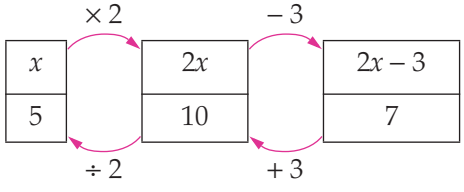

WE7

**2** Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

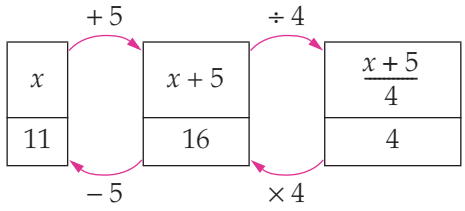
**(a)**



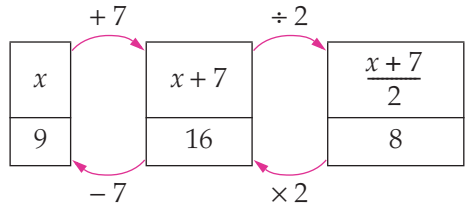

**(b)**



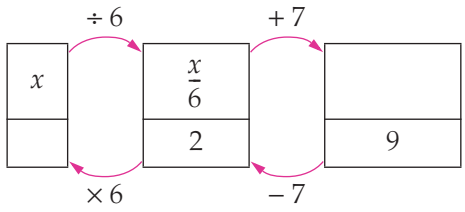
**(c)**



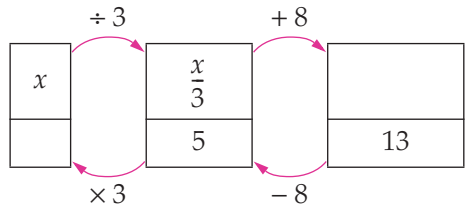

**(d)**



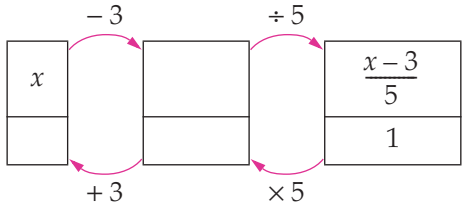

**(e)**



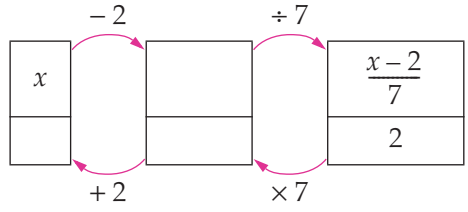

**(f)**



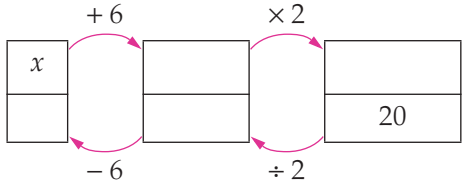

**(g)**



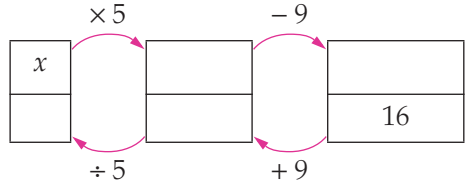

**(h)**



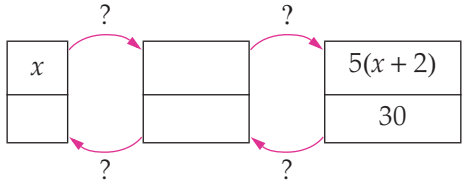

**(i)**



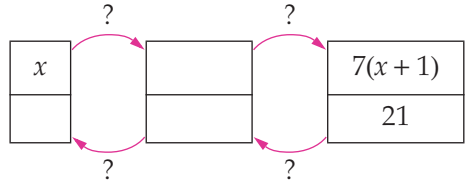

**(j)**




**(k)**




**(l)**




WE8

**3** Draw a flowchart and use backtracking to solve each of the following equations.

**(a)** $5x + 3 = 8$ **(b)** $7x + 3 = 24$ **(c)** $2x + 3 = 11$

**(d)** $2x - 7 = 3$ **(e)** $3x - 2 = 10$ **(f)** $8x - 11 = 13$

WE9

**4** Draw a flowchart and use backtracking to solve each of the following equations. Check your solutions by substitution.

**(a)** $\frac{x}{4} + 5 = 7$ **(b)** $\frac{x}{2} + 6 = 11$ **(c)** $\frac{x}{7} + 3 = 5$

**(d)** $\frac{x}{2} - 2 = 1$ **(e)** $\frac{x}{5} - 1 = 2$ **(f)** $\frac{x}{2} - 4 = 2$

**5** Use backtracking to solve each of the following equations. Check your solutions by substitution.

**(a)** $3(x + 1) = 12$ **(b)** $2(x + 3) = 16$ **(c)** $2(x + 5) = 14$

**(d)** $5(x - 6) = 10$ **(e)** $6(x - 4) = 18$ **(f)** $3(x - 5) = 3$

**6** Use backtracking to solve each of the following equations. Check your solutions by substitution.

**(a)** $\frac{x+8}{3}=3$ **(b)** $\frac{x+5}{2}=4$ **(c)** $\frac{x+10}{7}=2$

**(d)** $\frac{x-2}{5}=1$ **(e)** $\frac{x-3}{4}=2$ **(f)** $\frac{x-5}{2}=4$

**7** Use backtracking to solve each of the following equations. Check your solutions by substitution.

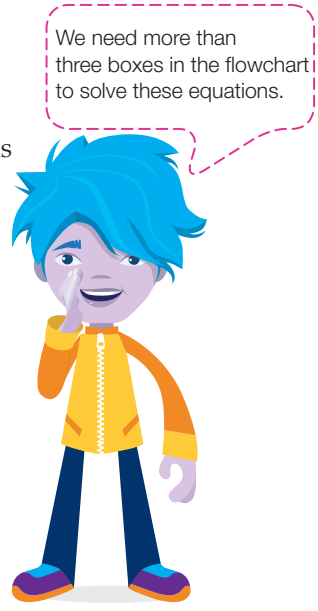
**(a)** $2x+3=13$ **(b)** $3x+2=11$ **(c)** $5x+6=11$

**(d)** $5x-3=17$ **(e)** $4x-1=7$ **(f)** $8x-9=7$

**(g)** $2(x+7)=20$ **(h)** $4(x-3)=8$ **(i)** $6(x-1)=12$

**(j)** $\frac{x}{2}+3=8$ **(k)** $\frac{x}{7}+10=12$ **(l)** $\frac{x}{3}-2=3$

**(m)** $\frac{x+2}{4}=2$ **(n)** $\frac{x-4}{5}=2$ **(o)** $\frac{x-3}{2}=2$




**8** Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

**(a)** $\frac{2x+1}{3}=3$ **(b)** $\frac{3x+7}{2}=11$ **(c)** $\frac{5x-2}{4}=7$

**(d)** $\frac{3x}{5}+6=9$ **(e)** $\frac{2x}{7}+8=12$ **(f)** $\frac{7x}{2}-10=4$

**(g)** $\frac{2(x+4)}{5}=6$ **(h)** $\frac{3(x+2)}{4}=3$ **(i)** $\frac{2(x-3)}{7}=6$

**9** The solution to $\frac{4x}{5}+6=14$ is:

**A** $x=1$ **B** $x=10$ **C** $x=16$ **D** $x=25$

## Understanding

**10** Write an equation using the following information, then solve each equation using backtracking. Use $n$ to represent the unknown number.

**(a)** A number is doubled, then one is added to the result to give an answer of forty-three.

**(b)** When four is added to a number and the result is divided by five, the answer is four.

**(c)** A number is multiplied by seven and the result is divided by three to give an answer of twenty-one.

**(d)** A number is doubled, then six is added. The result is divided by ten to give an answer of two.

**11** Belinda buys 3 packets of pencils. She gives five pencils to her friends. She sells the rest for 10 cents each and makes $6.70. How many pencils were in each packet? Using $n$ as the number of pencils in the packet, write an equation and use backtracking to answer this question.

## Reasoning

**12** Ying repairs televisions. The expression for the amount of money ($) she charges is $30x + 20$, where $x$ is the number of hours she takes to repair a TV. Aldo also repairs televisions. The expression for the amount of money ($) he charges is $20x + 50$ where $x$ is the number of hours he takes to repair a TV.



**(a)** Show the flowchart for the expression $30x + 20$.

**(b)** Show the flowchart for the expression $20x + 50$.

**(c)** Substitute the number $x = 1$ in both flowcharts and write down how much money Ying and Aldo charge for a repair that takes one hour.

**(d)** Ying charges a customer $95. How long did it take her to repair this TV?

**(e)** If Aldo charged $95, how long would he have taken?

**(f)** By trying different values for $x$, find the number of hours for which the amount charged by Ying and Aldo is the same.

## Open-ended

**13** If $x$ is the temperature in degrees Celsius, the expression $\frac{9x}{5} + 32$ gives the value of $x$ in degrees Fahrenheit.




**(a)** Build and complete the flowchart for the expression.

**(b)** Use your flowchart to find two values for the temperature in degrees Celsius that give a whole number for degrees Fahrenheit.

**14** After Ross performs four different operations on a number, he has a result of 10. Write an equation that fits this situation.

## Outside the Square Puzzle

### Gold digger 2

Your aim is to find the gold. The basic rule on how to solve this kind of puzzle can be found on page 63.

Here are the two maps to solve:

**(a)**

| | | | | |
|---|---|---|---|---|
| | 2 | | | |
| | 4 | | 1 | |
| 4 | | | 3 | 1 |
| | | | | 1 |
| | 3 | | | 2 |
| | | | | 1 |

**(b)**

| | | | | | |
|---|---|---|---|---|---|
| 3 | | | | 1 | |
| | | 4 | | 3 | |
| | | | | 3 | |
| 0 | | 3 | | 2 | |
| | | | | | |

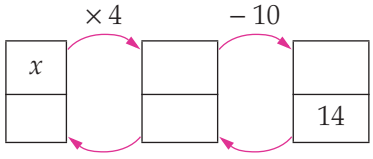
# Half-time 7

**1** Solve the equation $\frac{x}{12} = 7$ by inspection. Ex.7.2

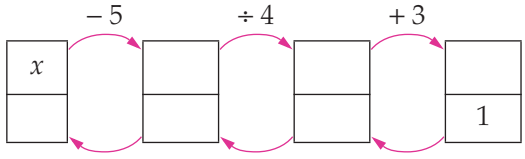
**2** The perimeter of a rectangular room is 18 m. If the length of the room is 5 m, what is the width? Form an equation and solve it with backtracking. Ex.7.3

**3** Copy and complete the following flowcharts. Write the equation and use the flowchart to find the solution for $x$. Ex.7.3

**(a)**




**(b)**




**4** Write a true number sentence that shows two consecutive numbers whose sum is 43. Ex.7.1

**5** **(a)** James is 5 years older than his brother Tom who is 7 years old. If James is $j$ years old: Ex.7.2

**(i)** write an equation to represent this situation with Tom's age on the RHS

**(ii)** solve the equation by inspection to find James' age.

**(b)** Jude is 4 years younger than his sister Amanda who is 16 years old. If Jude is $d$ years old:

**(i)** write an equation to represent this situation with Amanda's age on the RHS

**(ii)** solve the equation by inspection to find Jude's age.

**6** Use backtracking to solve each of the following equations. Ex.7.3

**(a)** $3x + 4 = 11$ **(b)** $\frac{x}{2} + 5 = -3$ **(c)** $3(x - 8) = 15$

**7** Write the equation $5x + 3 = 23$ in words. Ex.7.2

**8** The following number sentences are not true. Rewrite them by changing the coloured number so that you have a true number sentence. Ex.7.1

**(a)** $3 + 6 = 11$ **(b)** $4 \times 3 = 26 - 6$ **(c)** $4 + \frac{28}{4} = 18$

**9** Kate has a piece of cardboard that is $x$ centimetres wide. She joins a 50-centimetre piece of cardboard of the same length to it and then divides the width into 7 strips so she can paint each strip a different colour of the rainbow. Each strip is 10 cm wide. Use backtracking to find the width of the original piece of cardboard. Ex.7.3

**10** State whether the following number sentences are true or false. Ex.7.1

**(a)** $6 - 4 = 4 - 6$ **(b)** $2(3 + 5) = 2 \times 3 + 2 \times 5$ **(c)** $(8 \times 9) \times 2 = 8 \times (9 \times 2)$

**11** Pete is five years older than his brother Sam. The sum of their ages is 23. If Pete is $p$ years old, Ex.7.3

**(a)** write an expression to represent Sam's age

**(b)** write an equation to represent this situation with 23 on the RHS of the equation

**(c)** solve the equation using backtracking to find the ages of Pete and Sam.

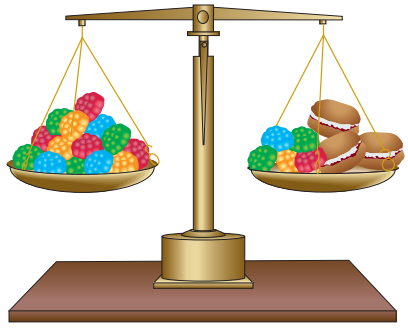
# 7.4 Solving equations using the balance method

Scales can be used to measure the mass of an object if other masses are available for comparison. To balance the scales, the two sides of the scales must hold equal masses.

## Worked Example 10

WE10

This set of scales is balanced. The left-hand side has 14 lollies; and the right-hand side has three biscuits and five lollies.



**(a)** If one lolly is taken from the right-hand side, the scales become unbalanced. Which of the sides is now heavier?

**(b)** How can the scales be balanced without putting the lolly back?

**(c)** Starting again, if all of the lollies are taken from the right-hand side, what should be done to the left-hand side to balance the scales?

**(d)** How many lollies are equivalent to the three biscuits?

**(e)** How many lollies are equivalent to one biscuit?

| Thinking | Working |
|---|---|
| **(a)** Look at the diagram and decide which side is heavier after removing the lolly. | (a) The left-hand side is now heavier. |
| **(b)** Balance the scales by doing the same to the other side. | (b) Take a lolly from the left-hand side. |
| **(c)** Identify how many lollies were on the right-hand side at the start (5). Take the same amount from the left-hand side. | (c) Take five lollies from the left-hand side. |
| **(d)** The beam is horizontal. Look at how many lollies are on the left-hand side (9). | (d) Nine lollies = three biscuits. |
| **(e)** The beam is balanced, so the biscuits on the right-hand side equal the lollies on the left-hand side. | (e) Three biscuits = nine lollies.<br>One biscuit = three lollies. |

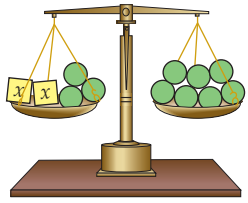
## Equivalent equations

**Equivalent equations** are those that have the same solution. For example, $2x = 4$ is equivalent to $2x + 3 = 7$ as they both have a solution of $x = 2$. This can be checked by substituting $x = 2$ into both equations to show they are both true number sentences.

$2x = 4$ $\qquad$ $2x + 3 = 7$

$2 \times 2 = 4$ $\qquad$ $2 \times 2 + 3 = 7$

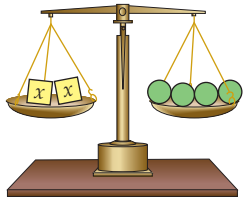
This situation can also be represented with balance scales. The 'equals' sign can be thought of as sitting in the middle of the two balanced sides. These scales represent $2x + 3 = 7$.




$2x + 3 = 7$

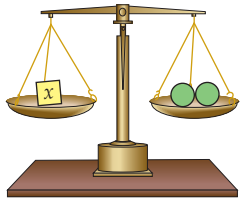
When 3 is taken off both sides, the scales are still balanced and now represent $2x = 4$.

Because the scales are still balanced we say that $2x + 3 = 7$ and $2x = 4$ are equivalent equations.



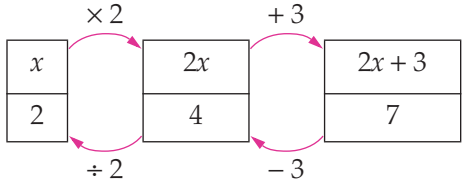

$2x = 4$

As two identical unknown masses equal 4, then one mass equals 2. The scales represent $x = 2$.




$x = 2$

These steps can be shown on a flowchart.




All these equations are equivalent. The simplest equivalent equation is $x = 2$, which is also the solution.

$x = 2$ $\qquad$ $2x = 4$ $\qquad$ $2x + 3 = 7$

We create equivalent equations by performing the same operation on both sides of the equation.

## Worked Example 11 WE11

Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

**(a)** $x - 5 = 7 \quad (+ 6)$ **(b)** $5x + 3 = 8 \quad (- 2)$ **(c)** $\frac{x}{2} = 4 \quad (\times 2)$ **(d)** $8x = 24 \quad (\div 4)$

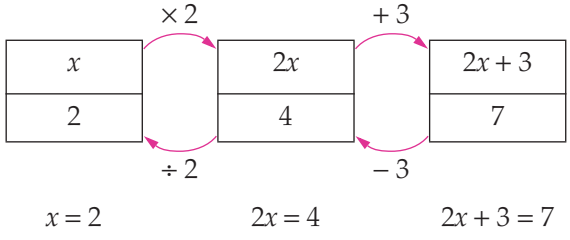
| Thinking | | | Working | |
|---|---|---|---|---|
| **(a)** | **1** | Write the equation. | *(a)* | $x - 5 = 7$ |
| | **2** | Perform the same operation on both sides (add 6). | | $x - 5 + 6 = 7 + 6$ |
| | **3** | Simplify. | | $x + 1 = 13$ |
| **(b)** | **1** | Write the equation. | *(b)* | $5x + 3 = 8$ |
| | **2** | Perform the same operation on both sides (subtract 2). | | $5x + 3 - 2 = 8 - 2$ |
| | **3** | Simplify. | | $5x + 1 = 6$ |
| **(c)** | **1** | Write the equation. | *(c)* | $\frac{x}{2} = 4$ |
| | **2** | Perform the same operation on both sides (multiply by 2). | | $\frac{x}{2} \times 2 = 4 \times 2$ |
| | **3** | Simplify. | | $x = 8$ |
| **(d)** | **1** | Write the equation. | *(d)* | $8x = 24$ |
| | **2** | Perform the same operation on both sides (divide by 4). | | $\frac{8x}{4} = \frac{24}{4}$ |
| | **3** | Simplify. | | $2x = 6$ |



## Solving equations using the balance method

By using a set of scales to represent our equation, we can now solve equations using the **balance method**. Up until now, we have used flowcharts to show us both the order and the operations we need to use to solve equations. With the balance method, we still need to use the same information, but now we show it in a different way.

Consider $2x + 3 = 7$.




$x = 2$ $\quad 2x = 4$ $\quad 2x + 3 = 7$

Notice that the three boxes contain three equivalent equations. We have moved backwards through the flowchart and used inverse (opposite) operations (– 3, ÷ 2) to solve the equation $2x + 3 = 7$. Let's now show the same steps using the balance method. The order of operations in the flowchart tells us how our equation was built. By moving backwards and performing the inverse operation, we will undo our equation to find the solution.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3 \quad \text{(subtracting 3 from both sides)}$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2} \quad \text{(dividing both sides by 2)}$$
$$x = 2$$

Note that the three equivalent equations formed here $2x + 3 = 7$, $2x = 4$, $x = 2$ are the same as the three equations formed from the boxes in the flowchart.

## Worked Example 12 WE12

Solve the equation $3x - 7 = 11$ using the balance method. Check your solution by substitution.

| Thinking | Working |
|---|---|
| **1** Write the equation. Identify the last operation to be performed on the LHS of it. This is the first operation to be undone. (– 7) | $3x - 7 = 11$ |
| **2** Use the inverse operation (+ 7) on both sides of the equals sign and simplify your equation. | $3x - 7 + 7 = 11 + 7$<br>$3x = 18$ |
| **3** Identify the next operation to be undone and apply the inverse operation (÷ 3). If one side of the equation is now the variable by itself, you have found the solution. Otherwise, continue the process until you do have the variable by itself. | $\frac{3x}{3} = \frac{18}{3}$<br>$x = 6$ |
| **4** Check the solution by substitution. | Check: LHS $= 3x - 7$<br>$= 3 \times 6 - 7$<br>$= 18 - 7$<br>$= 11$<br>$=$ RHS |

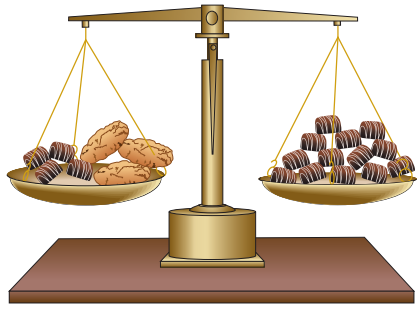
# 7.4 Solving equations using the balance method

## Navigator

| | | |
|---|---|---|
| Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q10 | Q1, Q2 Column 2, Q3 Column 2, Q4, Q6, Q7, Q8, Q9, Q10 | Q1, Q2 Column 2, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 |

Answers page 665

### Fluency



WE10

**1** This set of scales is balanced. The left-hand side has 4 chocolates and 3 biscuits; and the right-hand side has 13 chocolates.

**(a)** If one chocolate is taken from the left-hand side, the scales become unbalanced. Which of the sides is now heavier?

**(b)** How can the scales be balanced without putting the chocolate back?

**(c)** Starting again, if all of the chocolates are taken from the left-hand side, what should be done to the right-hand side to balance the scales?

**(d)** How many chocolates are equivalent to the three biscuits?

**(e)** How many chocolates are equivalent to one biscuit?

WE11

**2** Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

**(a)** $x + 5 = 8$ $(+ 3)$

**(b)** $x + 4 = 12$ $(+ 5)$

**(c)** $x + 1 = 7$ $(+ 6)$

**(d)** $2x + 2 = 10$ $(- 1)$

**(e)** $3x + 5 = 10$ $(- 5)$

**(f)** $6x + 8 = 14$ $(- 5)$

WE12

**3** Solve each of the following equations using the balance method. Check your solutions by subsitution.

**(a)** $3x + 5 = 8$

**(b)** $5x + 1 = 21$

**(c)** $7x + 3 = 17$

**(d)** $2x + 1 = 11$

**(e)** $2x + 5 = 19$

**(f)** $2x + 10 = 8$

**(g)** $3x - 4 = 20$

**(h)** $5x - 1 = 4$

**(i)** $6x - 2 = 16$

**(j)** $4x - 3 = 17$

**(k)** $2x - 3 = 7$

**(l)** $4x - 1 = 31$

**(m)** $\frac{2x}{3} = 8$

**(n)** $\frac{2x}{7} = 2$

**(o)** $\frac{5x}{2} = 20$

**(p)** $\frac{x+5}{3} = 5$

**(q)** $\frac{x+1}{3} = 7$

**(r)** $\frac{x+4}{5} = 3$

**(s)** $2(x - 5) = 14$

**(t)** $3(x + 2) = 24$

**(u)** $7(x - 2) = 35$




## Understanding

**4** This set of scales is balanced with 3 apples and 2 bananas on the left-hand side and 7 apples on the right-hand side.

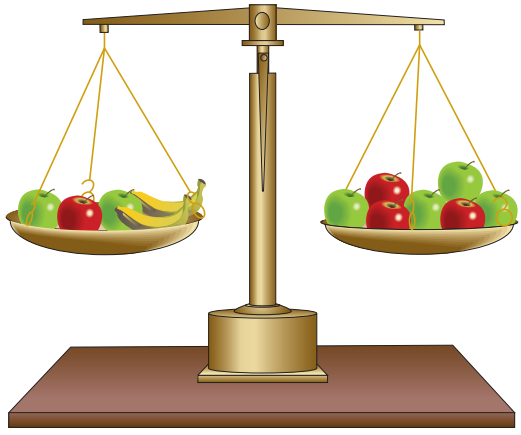
If 4 apples are taken from the right-hand side, the scale can be balanced by:

**A** taking 4 apples from the left-hand side

**B** taking 3 apples and 1 banana from the left-hand side

**C** taking 2 apples and 2 bananas from the left-hand side

**D** taking 2 bananas from the left-hand side.



**5** Trixie's father weighs twice as much as Trixie plus 12 kilograms. If her father weighs 104 kg:

**(a)** write an equation representing the situation using $t$ to represent Trixie's weight

**(b)** solve the equation to find out Trixie's weight.

**6** Naram has shot a number of 3-point goals as well as 18 other points in a game of basketball. If he shot a total of 30 points in the game:

**(a)** write this as an equation with $p$ representing the number of 3-point goals shot by Naram

**(b)** solve your equation to find the number of 3-point goals that he shot.



## Reasoning

**7** Fatima drives to and from work each day for the five-day working week, and drives an additional 85 kilometres on the weekend. She drives a total of 425 kilometres every week in her car. How far is it from her home to her workplace?

**8** George has to take 5 litres of water with him on a boat trip. He has one 1.25 L container and three other containers of equal capacity. Together, the four containers will hold 5 litres. What is the capacity of the other containers?



## Open-ended

**9** Write down three different equations that are equivalent to $5x + 4 = 14$.

**10** Vejay was given the equation $\frac{2x+3}{7} = 3$ to solve and presented the following solution.

$$\frac{2x+3}{7} = 3$$
$$\frac{2x+3}{7} - 3 = 3 - 3$$
$$\frac{2x}{7} = 0$$
$$\frac{2x}{7} \times 7 = 0 \times 7$$
$$2x = 0$$
$$\frac{2x}{2} = \frac{0}{2}$$
$$x = 0$$

**(a)** Explain to Vejay why his working out is incorrect.

**(b)** Show Vejay the process he should have used.

**(c)** How could Vejay avoid making the same mistake next time he's solving equations?

## Outside the Square — Problem solving

### Paper mountain

Imagine a very, very large sheet of paper, 0.1 mm thick.

If you fold it in half, it is 0.2 mm thick.

How thick will it be if you fold it in half again?

Imagine the paper can be folded an unlimited number of times.

How many folds would be needed to create a paper stack that is taller than Mt Everest? (Mt Everest is 8848 m high.)

**Strategy options**

- Act it out.
- Look for a pattern.

Maths 4 Real



# Dance Party dilemma

**Have you ever wondered what is involved in organising a fund-raising event such as a dance party? There would need to be a venue booked, tickets printed, advertising material created and drinks provided, as well as a range of other things.**

Believe it or not, a lot of the costs to be considered can be expressed as an equation. Many costs are variables that can change as the number of people attending the party changes. There are also fixed costs to consider, which remain the same no matter how many people attend. Once you have created an equation that represents all of the costs, it becomes possible to calculate how much you should charge each person to enter the event. This would then determine whether you make a profit or a loss.

**1** The table below represents the total costs associated with running a dance party. It shows that it would cost $50 to organise the party even if no people attend. Why might this be? Explain.

| **Number of people** $n$ | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| **Total cost ($)** $C$ | 50 | 150 | 250 | 350 | 450 | 550 |

**2** **(a)** What happens to the cost every time you increase the number of people attending the dance party by 10?

**(b)** Use your answer to **(a)** to determine the increase in cost for one extra person.

**3** Using the table in Question **1**, what equation could you use to calculate the costs of running the dance party? Use $C$ to represent cost and $n$ to represent the number of people attending the dance party.

$C = __ \times n + __$

**4** Use the equation that you found in Question **3** to calculate the costs of the dance party for the following number of people attending.

**(a)** 25 **(b)** 36 **(c)** 83

**5** Suppose the following table represents the money charged to attend the dance party. As the organiser of the party, this is your income. What equation could you use to represent this? Use $I$ for income and $n$ for the number of people attending.

| **Number of people,** $n$ | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| **Income ($),** $I$ | 0 | 120 | 240 | 360 | 480 | 600 |

**6** Use the formula that you found in Question **5** to calculate the income from the dance party for the following number of people attending.

**(a)** 32

**(b)** 53

**(c)** 95

If your income is bigger than your costs you will make a profit. However, if your costs are greater than your income, you will make a loss.



**7** Use your equations from Questions **3** and **5** to complete the following table showing all the costs and income associated with this dance party. Make sure that you also calculate the profit or loss (the difference between the cost and the incomes). Place a negative sign in front of a number to show a loss.

| **Number of people, *n*** | 0 | 15 | 35 | 45 | 63 | 100 |
|---|---|---|---|---|---|---|
| **Cost ($), *C*** | **50** | **200** | **400** | **500** | | |
| **Income ($), *I*** | **0** | **180** | **420** | | | |
| **Profit/loss ($), *P*** | **-50** | **-20** | **20** | | | |

**8** The profit or loss made at the party is found by subtracting the cost from the income. Using ***P*** to represent the profit or loss, write a formula that shows this.

**9** What is the smallest number of people you would need to attend the dance party in order to make a profit? Show how you worked this out.

## Research

Investigate the costs of organising a dance party for 40 of your friends. You need to find the following.

- cost of a suitable venue
- catering costs
- music costs
- cleaning costs

Present a report that outlines a number of different options for each of the above costs. Which is your preferred option and why? What are the benefits of hiring a venue compared to having a party at home?

# Technology Exploration Excel

**Equipment required:** 1 brain, 1 computer with an Excel spreadsheet

Versions of this Exploration are available for other technologies in Pearson Reader.

## Algebra using a spreadsheet

Did you know that your computer or calculator can do algebra? A spreadsheet program, such as Excel, uses formulas to do a lot of repetitive substitution. Spreadsheets on a CAS calculator work exactly the same way.

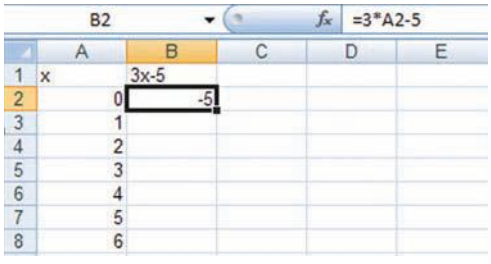
There are many ways to solve algebraic equations. You have been learning some algebraic methods. In this exploration, we are going to use a spreadsheet to find the solution to algebraic equations through a numerical process.

**1** Select an Excel spreadsheet or a spreadsheet from a CAS menu and enter $x$ as a heading in cell A1.

**(a)** To enter $x$-values from 0 to 6 in column A start at cell A2. Enter 0 and 1, and highlight these two cells. With the cursor on the little black cross on the bottom right-hand corner of cell A3, drag down to A8 or enter =A2+1 into cell A3 and drag this formula down to cell A8.

**(b)** Enter the expression 3x–5 as a heading in cell B1.

**(c)** Enter the formula =3×A2–5 into cell B2. This will calculate the value for $3x - 5$ when $x = 0$. The $x$-value in cell A2 is 0.




The formula must be changed in each row to use the $x$-value in column A in that row. The formula will now calculate a new value for $3x - 5$.

**(d)** Copy this formula down to cell B8 by dragging the little black cross on the bottom right-hand corner of cell B2. This will change the formula to refer to the correct $x$-value.

**2** Use your spreadsheet to find the solution to:

**(a)** $3x - 5 = 4$

**(b)** $3x - 5 = 13$

**(c)** $3x - 5 = -2$

Highlight these solutions using any fill colour you want. Select the cell that contains your solution. Then, use the fill colour icon to select a colour from the font group on the Home tab.



In Excel, a formula always starts with an = sign and * represents a multiplication sign.




**3** **(a)** In column C, enter the expression -2x+5 as a heading in C1.

**(b)** Enter the formula required to calculate a value for the expression $-2x + 5$ when $x = 0$ into cell C2.

**(c)** Copy this formula down to C8.

You will need to use a single quotation before the negative sign so that the formula is read as text.




**4** Use your spreadsheet to find the solution to:

**(a)** $-2x + 5 = 5$

**(b)** $-2x + 5 = -3$

**(c)** $-2x + 5 = -5$

Highlight these solutions with a different fill colour.

**5** Use your spreadsheet to find the solution to $3x - 5 = -2x + 5$.

Highlight this solution using another fill colour.

6 We now want to solve $4x - 6 = 30$, so enter 4x–6 as a heading in D1 and =4*A2–6 as a formula in D2.

**(a)** Fill down to D8. You can see that none of the values = 30, so the solution is not in the 0 to 6 values for $x$.

**(b)** We need to extend our values for $x$, so extend your $x$-values to $x = 10$.

**(c)** Now, we can find a solution to $4x - 6 = 30$. Write down the solution and, once again, highlight the solution.

7 Sometimes, our solution might be less than 0, so we will need to insert values smaller than 0 in column A. Insert as many rows as you may want above row 2 and copy the formulas up instead of down.




Left click on row 2, then use 'insert sheet rows' in the 'insert' drop-down menu under the cells group on the Home tab.

Now, find the solution to:

**(a)** $3x - 5 = -11$ **(b)** $-2x + 5 = 11$

Highlight with the same fill colours you chose for these expressions previously.

8 Solutions to equations are not always integers. Suppose we are trying to find the solution to $4x - 6 = 25$. We can see that when $x = 7$, $4x - 6 = 22$ and when $x = 8$, $4x - 6 = 26$, but there is no solution for $4x - 6 = 25$. Because 25 is between 22 and 26, the solution will be between 7 and 8. Insert a row between $x = 7$ and $x = 8$ and enter 7.5. This gives us $4x - 6 = 24$. We now know that the solution is between 7.5 and 8.

**(a)** We can overwrite the 7.5 with 7.7. This gives us $4x - 6 = 24.8$.

**(b)** Overwriting the 7.7 with 7.75, we now get $4x - 6 = 25$.

Highlight this solution.

| | A | B |
|---|---|---|
| 1 | x | 4x-6 |
| 2 | -5 | -26 |
| 3 | -4 | -22 |
| 4 | -3 | -18 |
| 5 | -2 | -14 |
| 6 | -1 | -10 |
| 7 | 0 | -6 |
| 8 | 1 | -2 |
| 9 | 1 | -1 |
| 10 | 2 | 2 |
| 11 | 3 | 6 |
| 12 | 4 | 10 |
| 13 | 5 | 14 |
| 14 | 6 | 18 |
| 15 | 7 | 22 |
| 16 | 7.75 | 25 |
| 17 | 8 | 26 |
| 18 | 9 | 30 |
| 19 | 10 | 34 |

9 Insert new formulas in your spreadsheet to find the solutions to:

**(a)** $5x + 6 = 22$

**(b)** $8x + 1 = 4$

**(c)** $10x + 3.3 = 140$

Highlight each of these solutions.

## Taking it further

10 Try to find the solution to $3x - 4 = 7$. It may look like the solution to this equation is 3.6, but $3 \times 3.6 - 4 = 6.8$, not 7. So, what has gone wrong?




Use the 'increase decimal places' icon  found in the number groups on the Home tab to show four decimal places in column A and the column you have used for the expression $3x - 4$ (do not include the heading row). This gets us closer to a solution, but it is still not an exact solution. $x = 3.6666$ gives 6.9998. Do you think you would ever find an exact solution to this question with a spreadsheet? Explain your answer.

11 Give an example of another equation that will have the same problem.

12 'Spreadsheets are very useful for solving equations, but are not as good as algebraic solutions found by hand.' Do you agree with this statement? Give reasons to support your argument.

# 7.5 Solving problems with equations

Equations can be very useful in solving everyday problems. So far, you have practised solving equations presented to you. Now you will be forming your own equation from information supplied in a worded question. Solving this equation will help you find the unknown value asked for in the question. There may be more than one way to write the equation and solve it.

## Worked Example 13

WE13

Phil has 70 cents. He buys a pear and has 25 cents left. If $x$ represents the cost of a pear (in cents), form an equation and solve it to find the cost of the pear.

| Thinking | Working |
|---|---|
| **1** Define a variable to represent the unknown quantity. | Let $x$ be the cost of a pear. |
| **2** Form an equation using the information given. | $x + 25 = 70$ |
| **3** Identify the first operation to undo and then use the inverse operation on both sides of the equals sign. | $x + 25 - 25 = 70 - 25$<br>$x = 45$ |
| **4** Check the solution. | LHS $= 45 + 25$<br>$= 70$<br>$=$ RHS |
| **5** State the solution in words. | The pear costs 45 cents. |

## Worked Example 14

WE14

Erica has bought two movie tickets and one bucket of popcorn for a total of $21. If the popcorn costs $5, how much does one movie ticket cost? Using $t$ as the cost of a movie ticket, form an equation and solve it to find the value of $t$. (Assume both tickets are the same price.)



| Thinking | Working |
|---|---|
| **1** Define a variable to represent the unknown quantity. | Let $t$ represent the cost in dollars of one movie ticket. |
| **2** Form an equation using the information given. | $2t + 5 = 21$ |
| **3** Identify the first operation to undo and then use the inverse operation on both sides of the equals sign. | $2t + 5 - 5 = 21 - 5$<br>$2t = 16$ |
| **4** Identify the second operation to undo and then use the inverse operation on both sides of the equals sign. | $\frac{2t}{2} = \frac{16}{2}$<br>$t = 8$ |
| **5** Check the solution. | LHS $= 2t + 5$<br>$= 2 \times 8 + 5$<br>$= 16 + 5$<br>$= 21$<br>$=$ RHS |
| **6** State the solution in words. | One movie ticket costs \$8. |

# 7.5 Solving problems with equations

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14 |

Answers page 665

## Fluency

**1** **(a)** Carmen has 95 cents. She buys an ice-cream and has 20 cents left. If $x$ represents the cost of the ice-cream (in cents), form an equation and solve it to find the cost of the ice-cream.

WE13

**(b)** Matthew can run 100 m in 12.8 seconds. In the school sports, Tony ran 1.5 seconds faster. If $t$ represents Tony's time, form an equation and solve it to find Tony's time.

**2** **(a)** Tran bought two sushi rolls and a juice for a total of \$8. If the juice costs \$1.60, how much does one sushi roll cost? Using $d$ as the cost of a sushi roll, form an equation and solve it to find the value of $d$. (Assume both sushi rolls are the same price.)

WE14

**(b)** Asif bought three DVDs and a CD for \$59. If the CD costs \$21.50, how much does one DVD cost? Using $v$ as the cost of a DVD, form an equation and solve it to find the value of $v$. (Assume all DVDs are the same price.)

**3** A bus has the capacity to seat 45 passengers. Eighteen passengers have boarded the bus and taken seats.

**(a)** If $n$ represents the number of seats still available, the equation which can be formed is:

**A** $n - 18 = 45$ **B** $n + 45 = 18$ **C** $18n = 45$ **D** $n + 18 = 45$

**(b)** Solve the equation formed in part **(a)** to find the number of seats still available on the bus.

**4** A roll of fabric is divided evenly between six people. Each person receives 2.5 m of fabric.



**(a)** If $t$ represents the total length of fabric in the roll (in m), the equation that can be formed is:

**A** $t = 6 + 2.5$

**B** $t + 2.5 = 6$

**C** $\frac{t}{6} = 2.5$

**D** $2.5t = 6$

**(b)** Solve the equation formed in part **(a)** to find the total length of the roll of fabric.

## Understanding

**5** The length of a rectangular vegetable garden is 4 m. The perimeter of wire fencing needed to enclose this garden is 13 m.

**(a)** If $w$ represents the width (in m) of the vegetable garden, the equation that can be formed is:

**A** $w + 4 = 13$ **B** $w + 8 = 13$ **C** $2w - 8 = 13$ **D** $2w + 8 = 13$

**(b)** Solve the equation formed in part **(a)** to find the width of the vegetable garden.

**6** Lucas bought a salad sandwich and a container of milk for his lunch.

**(a)** If the salad sandwich cost $\$x$ and the milk cost \$1.50, write an expression for the total cost of his lunch.

**(b)** Form an equation and solve it to find the cost of the salad sandwich if Lucas spent \$5.70 on his lunch.

**7** **(a)** If $x$ represents the length (in cm) of a square floor tile, write an expression for its perimeter.

**(b)** Form an equation and solve it to find the length of the floor tile if its perimeter is 84 cm.



**8** A restaurant has several tables that seat four people, but only three tables that seat two people.

**(a)** If $x$ represents the number of four-person tables available, write an expression for the total number of people that can be seated at the restaurant.

**(b)** A Saturday night booking for 62 people has been made. Six of these people are to be seated at the three two-person tables. Form an equation and solve it to find how many four-person tables will be needed.

**9** At a particular rollerskating rink, it costs \$5 to hire skates and \$3 for each hour spent on the rink. If it costs Natalie \$14, form an equation and solve it to find how many hours she spent rollerskating. Let $h$ represent the number of hours spent rollerskating.



**10** The tallest man in recorded history was Robert Wadlow (1918–1940). At age 22, his height was 54 cm less than twice his height at age 5. If his height at age 22 was 272 cm, form an equation and solve it to find his height at age 5.




**11** A company called Tool Time hires out electrical tools, charging a non-refundable deposit of \$15 for each item plus an amount per day according to the type of tool.

| Tool | Hire charge per day |
|---|---|
| Electric drill | \$8 |
| Electric sander | \$13 |
| Electric circular saw | \$24 |
| Electric jackhammer | \$35 |
| Concrete mixer | \$48 |

Let $d$ = the number of days the tools are hired, then form an equation and solve it to answer each of the following.

**(a)** If Allie hires an electric drill from Tool Time and pays \$63, for how many days did she hire it?

**(b)** If Serena hires a concrete mixer from Tool Time and pays \$111, for how many days did she hire it?

**(c)** Kosta hires an electric sander and an electric saw from Tool Time. For how many days did he hire the tools if he paid \$178?



## Reasoning

**12** The local council has hired a concreting firm to pave some footpaths. Each footpath is to be one metre wide. A wooden frame is needed before the concrete is poured. This frame is made up of one-metre lengths of wood.

The frames for some different lengths of footpath are shown in the diagrams below.



2 m long footpath  3 m long footpath  4 m long footpath

**(a)** How many pieces of wood are needed to construct the frame of a footpath of length:

**(i)** 2 metres **(ii)** 3 metres **(iii)** 4 metres?

**(b)** Write an expression for the number of pieces of wood needed to construct the frame for a footpath of length $x$ metres.

**(c)** Form an equation and solve it to find how many pieces of wood are needed to construct the frame for a footpath of length:

**(i)** 6 metres **(ii)** 10 metres **(iii)** 13 metres.

**(d)** The concreting workers have only 36 pieces of wood. By using your equation and solving it, find the length of the longest footpath that could be poured at the one time.

## Open-ended

**13** Gary the gardener is keen to do some landscaping on a Saturday and has allowed for 7 hours work. It costs $30 per hour for him to hire a jackhammer and $22 per hour to hire a mulcher. He has budgeted to spend no more than $180. Keeping in mind that he can only use one piece of equipment at a time, what are three options for using the equipment and what would each option cost him?

**14** A man is 5 times the age of his son. Given that the sum of their ages is no greater than 70, what are their possible ages? Assume that the answers are whole numbers.

# Outside the Square — Problem solving

## Algebraic puzzles

**1** Four numbers are added together and the result is 255. One of the numbers is the square of one of the others. The largest number is double the square number, and the fourth number is 55. Find the numbers.

**2** If a screw and a washer balance with a bolt, a screw balances with a washer and a nut, and two bolts balance with three nuts, find out how many washers will balance with a screw.

**Strategy options**

- Guess and check.
- Break problem into manageable parts.

# Investigation

## The great escape

**Equipment required:** 1 brain

Farmer Sue has a paddock to which she brings her cows to wait for milking and a large yard next to it for her free-range chickens. Every now and then, some of her chickens escape into the cow paddock.

One day, the gate connecting the chicken yard and the cow paddock was left wide open and all of Farmer Sue's chickens escaped into the cow paddock where all of her cows were waiting to be milked. Farmer Sue raced over and counted 40 heads and 112 legs.

### The Big Question

How many chickens and cows did Farmer Sue own?

### Engage

1 Last week, Farmer Sue found 4 chickens and 8 cows in the paddock and had to race around to catch the chickens and return them to their own yard.

- **(a)** How many heads would she have counted?
- **(b)** How many legs would she have counted?
- **(c)** If there had been 8 chickens and 4 cows:
  - **(i)** how many heads would she have counted?
  - **(ii)** how many legs would she have counted?

### Explore

2 If $n$ represents the number of cows in the paddock and $c$ represents the number of chickens that have escaped into the paddock:

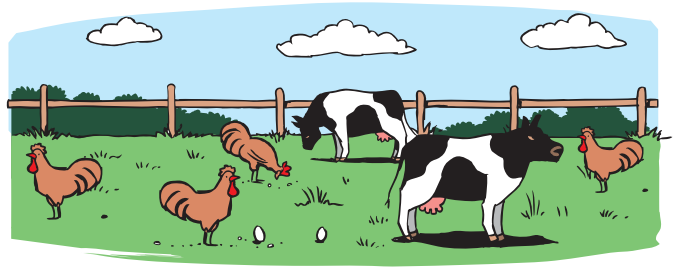
- **(a)** write an equation to show 12 heads altogether
- **(b)** what are all the possible combinations of cows and chickens that would give a total of 12 heads? Present your answers in a list or table.
- **(c)** Write an equation to show 40 legs altogether.
- **(d)** Substitute $n = 8$ and $c = 4$ into your equation to make a true statement.

**Strategy options**

- Guess and check.
- Make a table.
- Make a model.
- Look for a pattern.

3 **(a)** If Farmer Sue counted 40 heads, choose two possible values for the number of cows and chickens she might own.

- **(b)** For each choice, using an appropriate equation, calculate the number of legs she would have counted.
- **(c)** Can you now find the values for $n$ and $c$ that would give 40 heads and 112 legs?



### Explain

4 If the number of heads stays the same but the number of legs changes, explain how the numbers of each animal in the paddock change.

### Elaborate

5 **(a)** State your answer to the Big Question. Explain how it was obtained.

- **(b)** What would be the solution for $n$ and $c$ if the number of heads was still 12 but the number of legs was now only 32? Show how you worked this out.

### Evaluate

6 **(a)** What methods did you use to solve this problem?

- **(b)** Did you use a particular problem-solving strategy or method?
- **(c)** Why did you choose that particular method?
- **(d)** Do you think this was the best method?
- **(e)** Can you think of any other ways to solve this problem?

### Extend

7 Deep in the ocean, a scuba diver discovers a collection of 21 crabs and octopuses together in the same small area. He counts 198 legs altogether. (A crab has 10 legs and an octopus has 8.) How many crabs and octopuses were there in this Octopus' Garden?

# Mathspace

# The case of the confusing clues



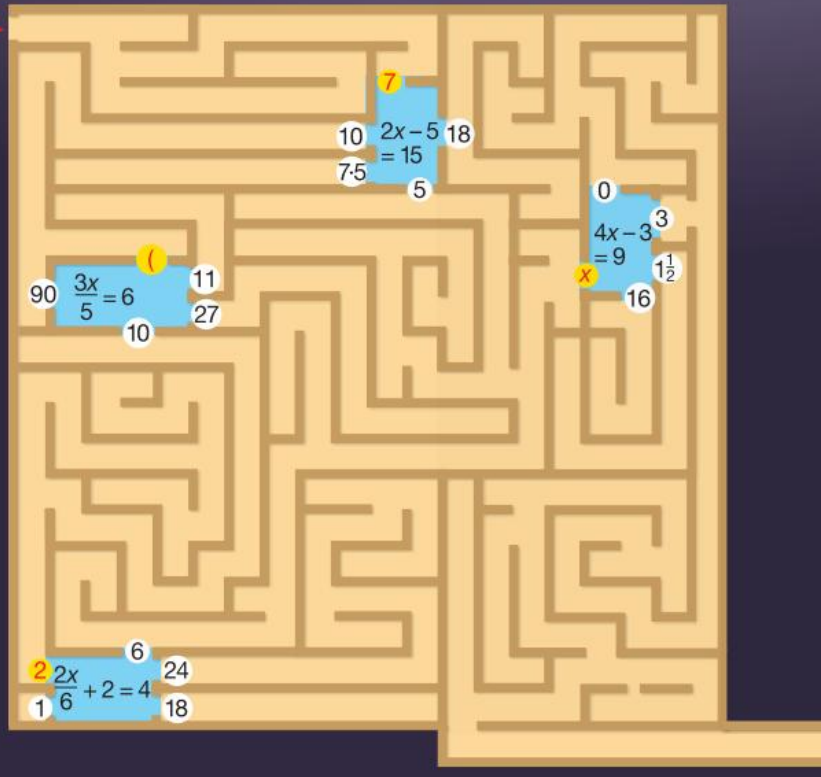

Cara Loft is on a secret mission. The golden calculator has been stolen by the mysterious Dr Equation. Where has he hidden it? All Cara has is a series of clues spread throughout the rather creepy House of Calculus. You need to help Cara by finding your way through the maze. As you move through the maze, you will enter some tricky 'equation chambers'.

When you enter the chamber:

- make sure that you enter through a coloured number or symbol and write it down (they form an equation you will need to solve at the end).
- To exit the equation chamber, solve the equation and move out through the exit showing the correct solution.

When you finally exit the maze, you need to complete the practice session to help you get ready for the even harder next section. Good luck!

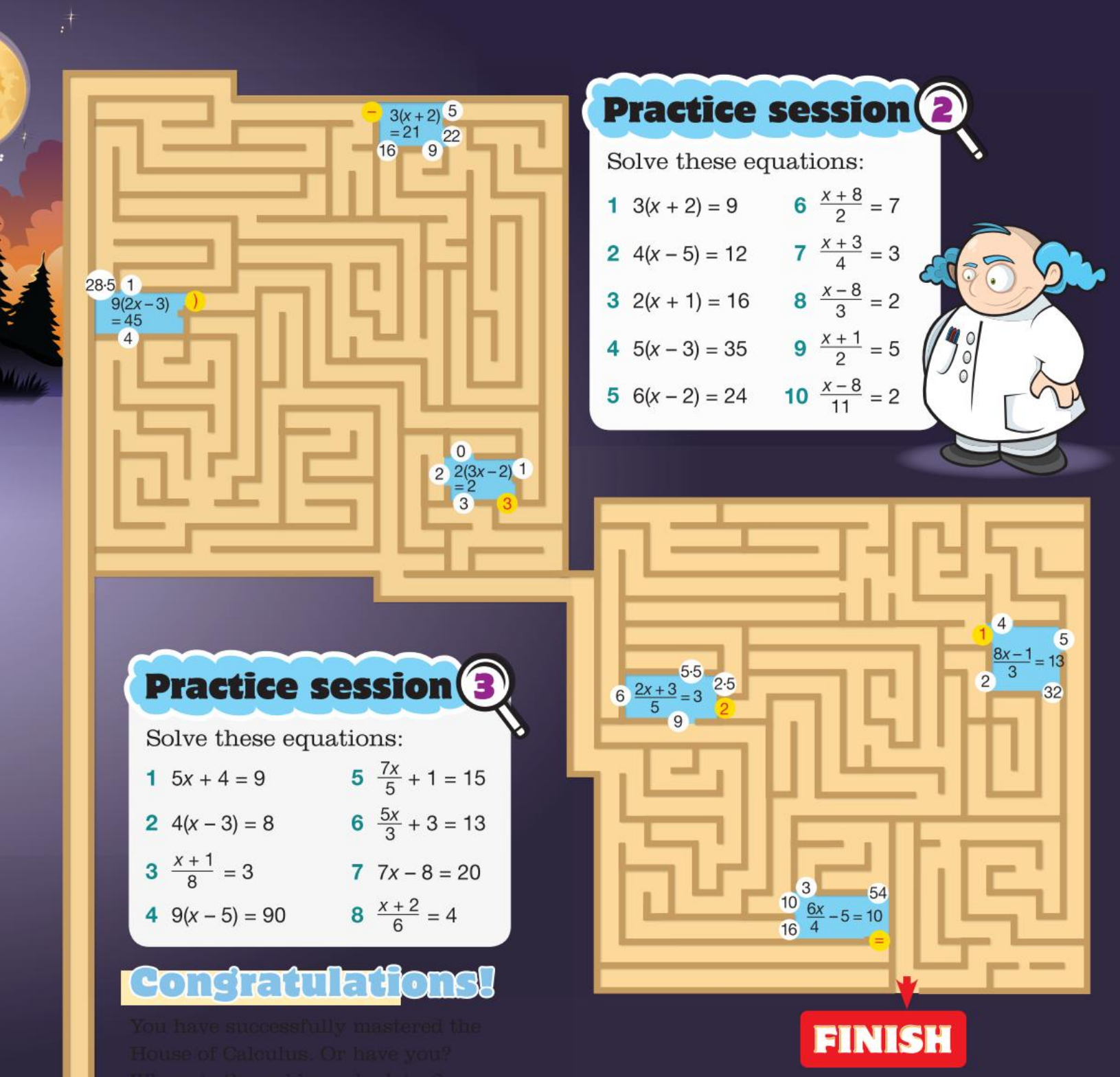
START




## Practice session 1

Solve these equations:

1. $3x + 1 = 10$
2. $4x - 4 = 12$
3. $8x - 1\frac{1}{2} = 6\frac{1}{2}$
4. $7x - 3.5 = 31.5$
5. $5x + \frac{1}{3} = 15\frac{1}{3}$
6. $\frac{2x}{3} + 1 = 5$
7. $\frac{5x}{2} + 1 = 11$
8. $\frac{7x}{5} - 3 = 11$
9. $\frac{8x}{6} - 4 = 0$
10. $\frac{9x}{10} + 1 = 8.2$




## Practice session 2

Solve these equations:

1. $3(x + 2) = 9$
2. $4(x - 5) = 12$
3. $2(x + 1) = 16$
4. $5(x - 3) = 35$
5. $6(x - 2) = 24$
6. $\frac{x + 8}{2} = 7$
7. $\frac{x + 3}{4} = 3$
8. $\frac{x - 8}{3} = 2$
9. $\frac{x + 1}{2} = 5$
10. $\frac{x - 8}{11} = 2$

## Practice session 3

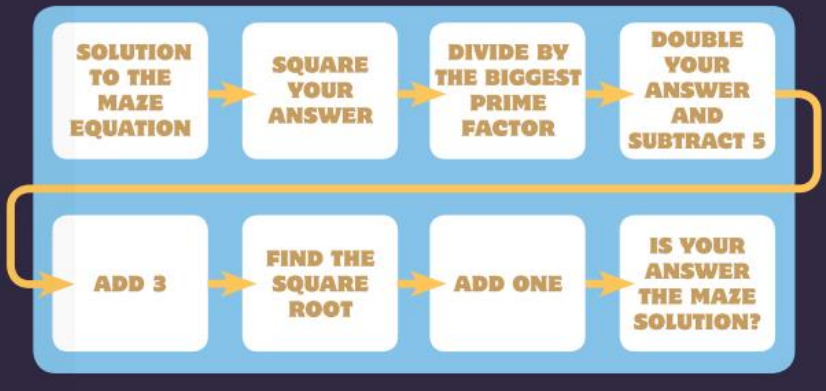
Solve these equations:

1. $5x + 4 = 9$
2. $4(x - 3) = 8$
3. $\frac{x + 1}{8} = 3$
4. $9(x - 5) = 90$
5. $\frac{7x}{5} + 1 = 15$
6. $\frac{5x}{3} + 3 = 13$
7. $7x - 8 = 20$
8. $\frac{x + 2}{6} = 4$

FINISH

## Congratulations!

You have successfully mastered the House of Calculus. Or have you? Where is the golden calculator? Dr Equation has left you a final challenge. Piece together the clues you have collected in order and then solve the equation that they form. You then need to use your answer to attempt the biggest challenge of all—the sinister 'Steps of Arithmetica'. Successfully meet the challenge and you will find the golden calculator. Fail and you will have to go back to the start!

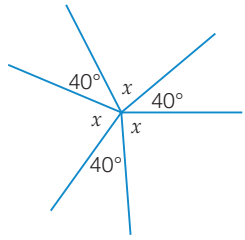


# Challenge 7

**1** When half of a number is increased by 15 the result is 49. The original number is:

**A** 68 **B** 34 **C** 32 **D** 17

**2** Write an equation and solve it to find the value of $x$ in the diagram.




**3** Solve the equation: $\frac{3x-5}{4} = -6$.

**4** If $59 + x = 73 + y$, then:

**A** $x = 14 - y$ **B** $x = 14 + y$

**C** $x = y - 14$ **D** $x = 14y$

**5** Using only the digits 1, 2, 3 and 7 and the two mathematical symbols + and = only once, we can create a true number sentence $2^3 = 1 + 7$.

**(a)** Using the digits 2, 3, 4, 5 and the two mathematical symbols + and = only once and a number as a power, create a true number sentence.

**(b)** Using any different four digits from 0 to 9 and the two mathematical symbols + and = only once and one of the numbers as a power, create five different true number sentences. Do not use $2^3 = 1 + 7$ or your answer to **(a)**. Use 0 and 2 as a power only once.

**(c)** Use once only, any different four digits from 0 to 9 and the two mathematical symbols – and = only once and one of the numbers as a power to create five true number sentences. Use 0 and 1 as a power only once.

**6** The inverse (reciprocal) of $\frac{2}{3}$ is $\frac{3}{2}$. If the inverse of $\frac{4x}{5}$ is $\frac{1}{20}$, then the value of $x$ is:

**A** $\frac{1}{25}$ **B** $\frac{1}{16}$ **C** 16 **D** 25

**7** Solve for $y$:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{y} = 1$$

**8** Vladmir buys an equal number of 55-cent and $1.10 stamps and spends $80.85. How many stamps did he buy altogether? Form an equation and solve it.

**9** A Lotto winner won $\frac{9}{10}$ of the total prize pool. Shortly after, she spent $\frac{3}{4}$ of her winnings, but still had $2700 left. What was the value of the total prize pool? Form an equation and solve it.

**10** If $\frac{1}{3} > \frac{1}{4} > \frac{1}{m}$ and the difference between the first two fractions equals the difference between the last two fractions, find the value of $m$. (*Hint*: Write $\frac{1}{m}$ on one side of the equation before you take the inverse.)

**11** Two integers, $a$ and $b$, do not end in zero. If the product $ab$ is a multiple of 10 and $a > b$, the last digit of $a - b$ cannot be:

**A** 1 **B** 3 **C** 5 **D** 7

# Chapter review 7

## D.I.Y. Summary

### Key Words

| | | |
|---|---|---|
| backtracking | false number sentence | solve |
| balance method | guess, check and improve | solving by inspection |
| checking by substitution | inverse operations | true number sentence |
| equivalent equations | solution | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 An equation can be solved by performing the _____ on both sides of the equation.

2 To _____ an equation is to find the value of the unknown that will make a true number sentence.

3 $4 + 5 = 10$ is a _____.

4 One way of solving an equation is by looking at the equation to see what number might be the answer. This is called _____.

5 Equations that have the same solution are said to be _____.

6 _____ is a method we can use to solve equations. It can be shown by moving backwards along a flowchart.

7 _____ is another method used to solve equations.

## Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. Ex.7.1

**(a)** $5 \times 4 = 30 - 2$ **(b)** $2 \times 7 + 6 = 9 + 5$

2 Find the unknown number in each of the following sentences. Ex.7.1

**(a)** Six subtracted from a number is equal to nine.

**(b)** A number divided by four is equal to seven.

3 Find the solution to each of the following equations by inspection. Ex.7.2

**(a)** $2x = 18$ **(b)** $x + 7 = 10$ **(c)** $\frac{x}{5} = 3$

**(d)** $x - 4 = 11$ **(e)** $x + 3 = 15$ **(f)** $x - 4 = 24$

4 Solve the following equations using guess, check and improve. Ex.7.2

**(a)** $3x + 25 = 16$ **(b)** $\frac{x}{7} + 51 = 63$ **(c)** $\frac{3x + 5}{4} = 8$

5 List the inverse (or opposite) operation to each of the following. Ex.7.3

**(a)** $+ 5$ **(b)** $\div 3$ **(c)** $\times 8$ **(d)** $- 4$

**6** Solve each of the following equations using a flowchart and backtracking.

Ex 7.3

**(a)** $3x - 7 = 2$ **(b)** $\frac{x+6}{5} = 4$ **(c)** $4(x+1) = 20$ **(d)** $\frac{x}{8} - 2 = 3$

**7** Use the balance method to solve the following equations.

Ex 7.4

**(a)** $4x + 7 = 19$ **(b)** $5(x-2) = 20$ **(c)** $\frac{x}{3} + 5 = 10$

**(d)** $\frac{x-8}{6} = 3$ **(e)** $\frac{x+4}{5} = 5$ **(f)** $\frac{x-7}{3} = 10$

**8** The equation $5x - 6 = 4$ has the solution:

Ex 7.4

**A** $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

**9** Danni bought 5 tickets to a show. She was given $27.50 change from $200. Use an equation to find how much each ticket cost. Let $t$ be the cost of a ticket.

Ex 7.5

## Understanding

**10** Which equation describes this situation?

A restaurant seats a maximum of 64 patrons. On Saturday afternoon, it is fully booked for the evening meal service. At 5.30 p.m., a group booking for 16 is cancelled. If $p$ represents the number of people who are still booked, the equation that can be formed is:

**A** $p - 16 = 64$ **B** $p = 64 + 16$ **C** $16p = 64$ **D** $p + 16 = 64$

**11** Ross buys three dim sims and a bottle of juice.

**(a)** If each dim sim costs $x$ cents and the bottle of juice costs $1.40, write an expression for the total cost (in cents) of the food and juice.

**(b)** Use an equation to find the cost of one dim sim if Ross spends a total of $2.75.

**12** The air temperature at 4 p.m. is 28°C. Over the past hour, the temperature has risen by 5°C. What was the temperature at 3 p.m.? (Form an equation to solve by letting $t$ represent the air temperature at 3 p.m.)

## Reasoning

**13** A triangle  made from matchsticks has three sides of equal length. If more matchsticks are added to make another triangle, we obtain this shape  .

**(a)** How many matchsticks of equal length does this shape have?

**(b)** A third triangle is added to the shape. How many matchsticks of equal length does this shape have?

**(c)** If $n$ represents the number of triangles joined together in a row, write, using the pronumeral $n$, the number of matchsticks of equal length the shape has in total.

**(d)** **(i)** Use your answer to **(c)** to write down the equation for $n$ triangles joined together that have 97 matchsticks of equal length.

**(ii)** Solve the equation obtained in part **(d) (i)**.

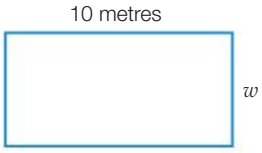
# NAPLAN practice 7

## Numeracy: Non-calculator

**1** Jan is baking biscuits. She puts her biscuits in 3 equal rows. When they are baked, she eats one. She has 44 biscuits left. How many biscuits did she have in each row?

**A** 3 **B** 10 **C** 15 **D** 20

**2** A rectangular garden bed has a perimeter of 30 metres and a length of 10 metres. If the width of the garden can be written as $w$, which equation best represents this situation?




**A** $w + 10 = 30$ **B** $w + 30 = 10$

**C** $2w + 10 = 30$ **D** $2w + 20 = 30$

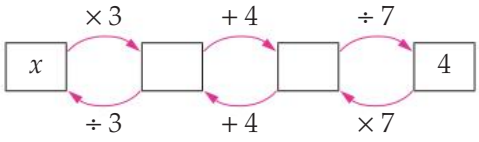
**3** What is the missing number that makes this number sentence true?

$3 \times \square = 12 + 3 \times 6$

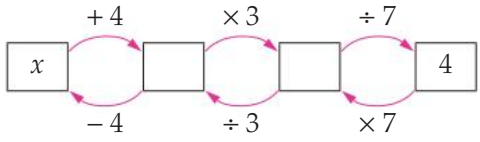
**A** 1 **B** 7 **C** 9 **D** 10

**4** Voula thinks of a number and asks Renza to guess what it is. Voula states that after the number is tripled, four added and the result divided by seven, the answer is four. Which flowchart best represents how Renza might find Voula's mystery number? Let the mystery number = $x$.

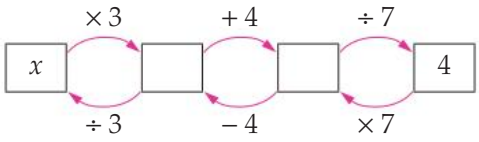
**A**



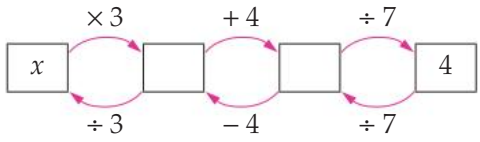

**B**




**C**




**D**




## Numeracy: Calculator allowed

**5** A pack of 4 donuts costs $3.60.
A pack of 6 donuts costs $4.00.
You need to buy 34 donuts.
What is the least amount you can pay?

**A** $23.60 **B** $26.40 **C** $26.50 **D** $29.90

**6** The cooking time for a turkey is 30 minutes plus 20 minutes per kilogram. If $M$ is the mass of the turkey in kilograms, the total cooking time in minutes is:

**A** $20 + 30M$ **B** $20M + 30$ **C** $20 \times 30M$ **D** $30 - 20M$

**7** Julie, Nikki and Romina earned a total of $3400 in the last week. Julie earned twice as much as Nikki whereas Romina earned $200 more than Nikki. How much did Nikki earn in the last week?

**A** $600 **B** $800 **C** $1000 **D** $1600

**8** The solution to $\frac{3(x-4)}{2} = 15$ is:

**A** $x = 6$ **B** $x = 10$ **C** $x = 14$ **D** $x = 58$


8

# 8 Angles and shapes

## Leaning Tower of Pisa stops leaning!

Sometimes, angles can cause all sorts of problems.

The building of the Leaning Tower of Pisa began in the 12th century in Pisa, Italy. The tower started tipping sideways before building had finished and, for centuries, it kept tipping further and further. It would have eventually fallen over if engineers and architects hadn't found a way to stabilise the base. In 1990, it was leaning at an angle of 5.5° from the vertical. Engineers removed soil from underneath the raised end, which straightened it a little. In 2008, even more soil was removed and now the tower leans at 3.97°. The engineers have said that it is now stabilised and will not get any worse for more than 200 years.

### Forum

What problems could be caused if a building starts to lean?

At what angle do you think the Leaning Tower of Pisa would actually fall over?

Discuss how using angles could be important for engineers fixing the Leaning Tower of Pisa.

### Why learn this?

You need to get things straight about angles! They are crucial in sport—whether deciding at what angle to kick a ball, choosing the best club to use for a golf shot or trying to pocket a ball on a pool table. Construction workers need to use angles to make sure that buildings and structures are secure and will not fall down. Surveyors use distances and angles to accurately determine a position on the Earth's surface. They work in construction and establish boundary lines between properties.

**After completing this chapter you will be able to:**

- estimate and measure angles
- construct angles and shapes using a compass or a protractor
- classify and name angles
- calculate the size of complementary and supplementary angles, vertically opposite angles and angles in a revolution
- identify alternate, corresponding and co-interior (allied) angles and calculate their size
- define and classify polygons, especially triangles and quadrilaterals, identifying their properties.

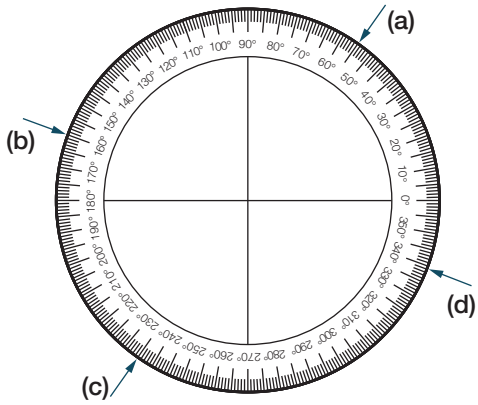
# Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R8.1

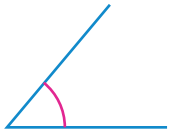
Worksheet R8.2

**1** State the value indicated by each arrow in the opposite diagram.

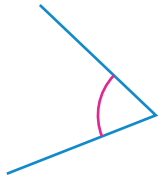



**2** Arrange the following angles in order from smallest to largest.

A



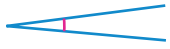
B



C



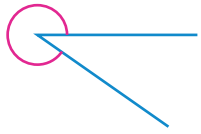
D



E



F



Worksheet R8.3

**3** List the pairs of angles that look to be the same size.

A



B



C



D



E



F

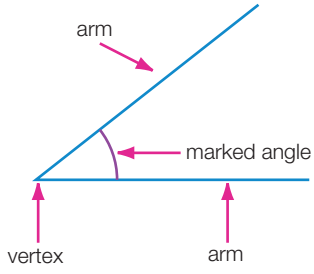


## Key Words

| | | | |
|---|---|---|---|
| acute angle | equilateral triangle | plane | right angle |
| acute-angled triangle | intersect | plane shapes | right-angled triangle |
| alternate angles | irregular | polygon | scalene triangle |
| angle | isosceles triangle | protractor | square |
| arms | kite | quadrilateral | straight angle |
| co-interior (allied) angles | line segment | ray | supplementary angles |
| complementary angles | obtuse angle | rectangle | transversal |
| concave | obtuse-angled triangle | reflex angle | trapezium |
| convex | parallel lines | regular polygon | vertex |
| corresponding angles | perpendicular bisector | revolution | vertically opposite angles |
| degree | perpendicular lines | rhombus | |

# 8.1 Measuring, estimating and drawing angles

An **angle** is the space between two lines that start at the same point. This point is called the **vertex** of the angle and the lines form the **arms** of the angle.

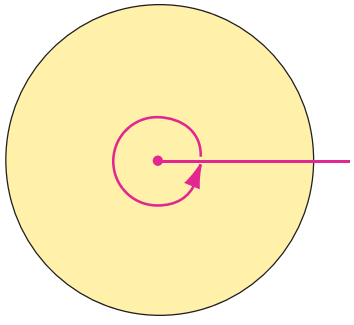
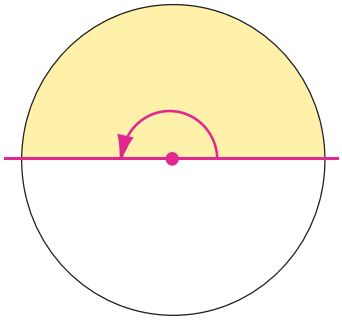
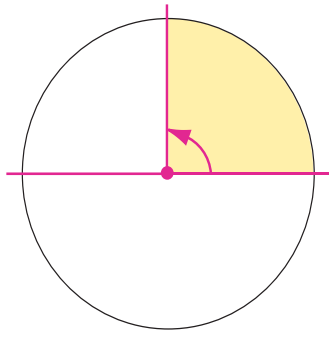


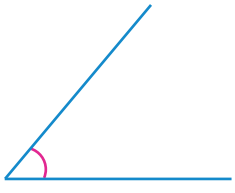

The size of an angle is the amount of turn from one arm to another. One method of measuring the amount of turn is by using **degrees** (°). When an arm is rotated completely around a circle for one full turn, we call this a **revolution** and we divide the revolution into 360 parts.

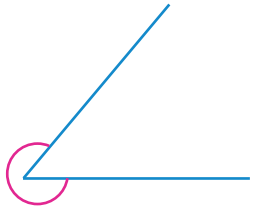
The amount of turn through one part forms a degree.

## Why was a revolution divided into 360 degrees?

It is thought that the ancient Babylonians who counted in lots of 60 and who divided a year into 360 days were responsible for dividing a revolution into 360 parts. A **protractor** has a revolution divided into 360° and is used to draw and measure angles reasonably accurately.

|  |  |  |
|---|---|---|
| Full turn | Half turn | Quarter turn |
| 360° | 180° | 90° |





Because two angles are formed between two arms, we need to mark the angle that is required.

# Estimating and measuring angles

It is often important to be able to estimate angles. It may not be possible or necessary to measure the angles exactly but we need to be able to visualise how big or small an angle is.

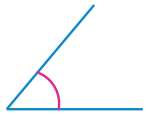
If we need to measure angles, we can use a semicircular protractor or a full-circle protractor.

## Worked Example 1 WE1

Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| | | |

(a)



(b)



### Thinking

**(a)** *For angles less than 180°*

1. As the angle is small and about half of a right angle (90°), estimate the size of the angle.
2. Using a semicircular protractor, place the centre point of the protractor over the vertex of the angle and align the baseline of the protractor with an arm of the angle. Use the scale on the protractor that starts with 0°. This protractor measures the angle shown as 50°.
3. Complete the table.

**Method 1: Using a semicircular protractor**

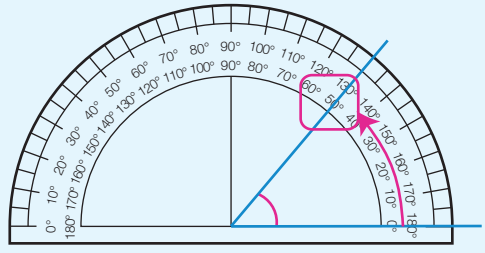
**(b)** *For angles larger than 180°*

1. As the angle is greater than three right angles (270°), but less than four right angles (360°), estimate the size of the angle.
2. Using a semicircular protractor, measure the smaller angle and subtract this angle from 360°.

   The small angle measures 70°, so the marked angle must be 360° – 70° = 290°.

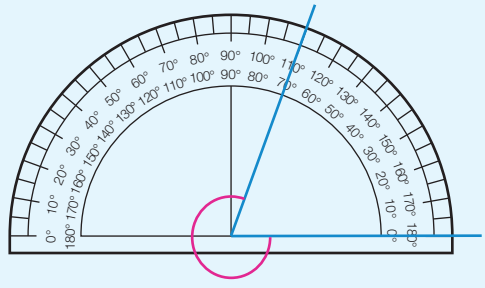
### Working

(a) Estimate: 45°



| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| 45° | 50° | 5° |

(b) Estimate: 300°



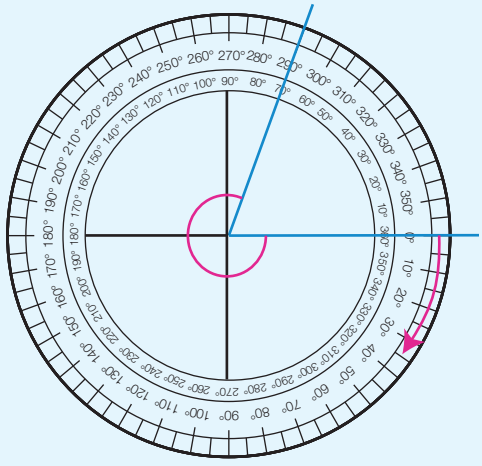
**3** Complete the table.

| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| 300° | 290° | 10° |

**Method 2: Using a full-circle protractor**

**(b) 1** Using a full-circle protractor, line up the centre point with the vertex and the 0° line along one arm, as with the semicircular protractor. Use whichever scale increases from zero. In this case, the outer scale, going clockwise.

(b)



**2** Complete the table.

| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| 300° | 290° | 10° |

## Drawing angles with a protractor

### Worked Example 2 WE2

Use a protractor to draw the following angles.

**(a)** 75° **(b)** 200°

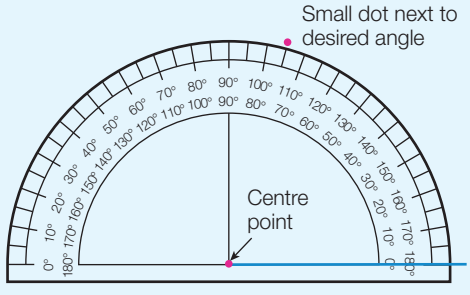
| Thinking | Working |
|---|---|

**(a)** *For angles less than 180°*

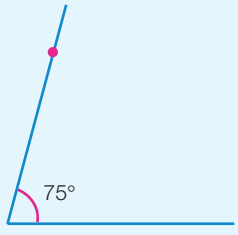
(a)

**1** Draw a straight horizontal line.

**2** Place the centre point of the baseline of the protractor on one end of the straight line. Locate the desired angle and mark a small dot as shown (in this case, 75°). (The base line is not the edge of the protractor. Make sure you use the scale starting at the end of your line segment.)



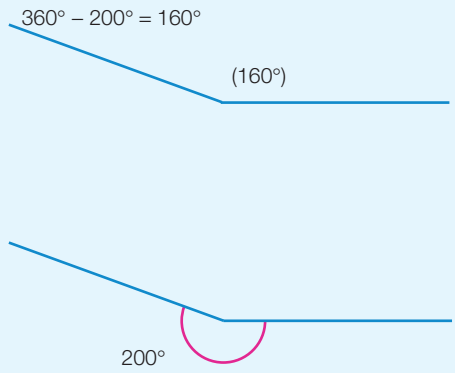

**3** Remove the protractor and join the dot with the end of the line where the centre point was located. Mark the angle.



**(b)** *For angles larger than 180°*

1. Subtract the angle from 360° to get the smaller angle needed to make up a full turn. Now, draw this angle. (For example, to draw 200°, we need to first draw 160°.)
2. Label the correct, larger angle.

(b)




# 8.1 Measuring, estimating and drawing angles

## Navigator

Answers page 667

| | | |
|---|---|---|
| Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11 Column 1, Q12, Q15 | Q1, Q2 Column 2, Q3, Q4 , Q5, Q6, Q7, Q9, Q10, Q11 Column 2, Q12, Q13, Q16 | Q1, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Column 3, Q12, Q14, Q16 |

**Equipment required:** Protractor

## Fluency

WE1

1 Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

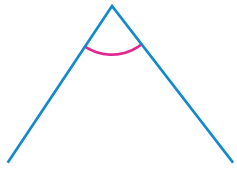



| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| | | |

(a)



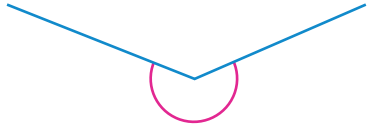
(b)



(c)



(d)



(e)



(f)



WE2

2 Use a protractor to draw the following angles.

(a) 10° (b) 70° (c) 55°

(d) 100° (e) 175° (f) 108°

(g) 290° (h) 265° (i) 318°

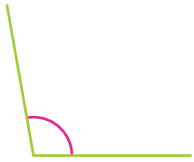
**3** **(a)** The angle shown here is approximately:



**A** 20° **B** 45°

**C** 60° **D** 85°

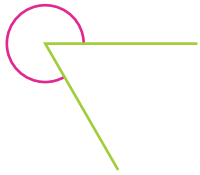
**(b)** The angle shown here is approximately:



**A** 80° **B** 100°

**C** 170° **D** 200°

**(c)** The angle shown here is approximately:



**A** 60° **B** 180°

**C** 240° **D** 300°

**4** **(a)** Which of the angles below is about 170°?

**A**



**B**



**C**



**D**



**(b)** Which of the angles below is about 5°?

**A**



**B**



**C**

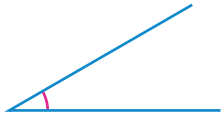


**D**



**(c)** Which of the angles below is about 330°?

**A**



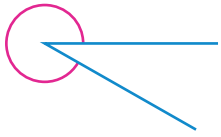
**B**



**C**



**D**



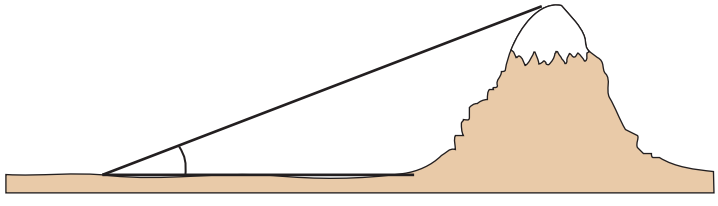
**5** To draw a 250° angle with a semicircular protractor, which of the following would you do first?

**A** Draw a 50° angle. **B** Draw a 90° angle.

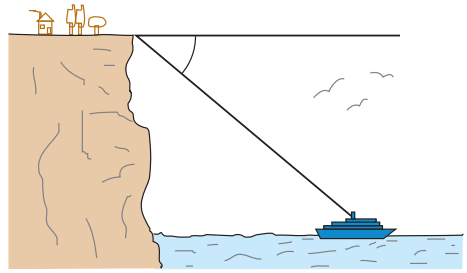
**C** Draw a 110° angle. **D** Draw a 200° angle.

## Understanding

**6** In the diagram, estimate the angle shown from ground level to the mountain top and then measure the angle with your protractor.



**7** In the diagram, estimate the angle shown from the top of the cliff to the ship and then measure the angle with your protractor.

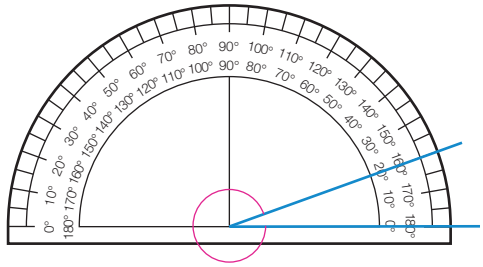


**8** Explain how you would draw an angle of 320° with a semicircular protractor.

**9** Liz uses a semicircular protractor to measure an angle larger than 180°.

Find the size of the actual angle (larger than 180°) if the smaller angle she has measured is:

**(a)** 30° **(b)** 115° **(c)** 160°



**10** Draw the following using your protractor.

**(a)** a wire holding a vertical flagpole attached at 38° to the ground

**(b)** a ramp with a slope of 12°

**(c)** a ladder leaning against a vertical wall at 62° to the ground

**(d)** a round birthday cake sliced into 6 equal pieces

## Reasoning

**11** For each of the following angles:

**(a)** 35° **(b)** 65° **(c)** 83°

**(d)** 120° **(e)** 102° **(f)** 152°

**(g)** 200° **(h)** 265° **(i)** 325°

**(i)** draw an estimate of the angle without the use of a protractor

**(ii)** use your protractor to measure the angles you have drawn to find the actual size of your angles

**(iii)** find the error (the difference between the given angle and your measurement)

**(iv)** copy and complete the following table.

| Estimated angle size | Measured angle size | Difference |
|---|---|---|
| | | |

**(v)** Which angles did you draw the most accurately?

**(vi)** Do you think it is harder to estimate angles greater than 180° than those less than 180°? Why?

## Open-ended

**12** Draw the following angles, estimate the size of the angle you have drawn and then check the size with a protractor.

**(a)** any angle smaller than 90°

**(b)** any angle between 90° and 180°

**(c)** any angle between 180° and 360°

**13** Your friend Nick has just answered the following question.

> Use a protractor to measure the size of the following angle. Give your answer to the nearest degree.
>
> 

Nick says "The answers in the back of the book must be wrong! The book says that the answer is 135°, but my protractor says 45°." Explain how Nick might have found his answer. Give Nick a hint, so that he can measure accurately all the time.

**14** Without using a protractor, draw the following angles and estimate the size of each angle.

**(a)** two angles that add up to 90°

**(b)** three angles that add up to 180°

**(c)** four angles that add up to 360°

**(d)** Check your answers to **(a)**, **(b)** and **(c)** with a protractor.

**15** **(a)** Choose any angle between 40° and 60°, draw it carefully with a protractor and mark the size of your angle on your diagram.

**(b)** Using a protractor, draw an angle 15° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

**(c)** Using a protractor, draw an angle 195° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

**16** **(a)** Choose any angle less than 90°, draw it carefully with a protractor and mark the size of your angle on your diagram.

**(b)** Using a protractor, draw an angle twice the size of the angle you chose in **(a)** and mark the size of your angle on your diagram.

**(c)** Using a protractor, draw an angle twice the size of the angle in **(b)** and mark the size of your angle on your diagram.

## Outside the Square Game

### The Angle Master

**Equipment required:** 2 brains, ruler, protractor

**How to win:**
The winner is the first player to score 15 points, and is proclaimed the 'Angle Master'.

**How to play:**
The aim of the game is to guess the size of different angles more accurately than your opponent.

1 Take it in turns to create an angle by drawing two straight lines without using a protractor. Clearly mark the angle you have created.

2 You and your opponent must then bid to see who can get the closer guess to the size of the angle without guessing over the size of the angle. The bidding ends when one player believes the other has overestimated the angle and calls for the angle to be measured at this guess.

**Scoring**
0–1° under 5 points
2–5° under 4 points
6–10° under 2 points
11+° under 1 point

Correctly claiming your opponent has overestimated the angle: 3 points.

**Sample game play:**
Imagine player 1 draws the following angle.



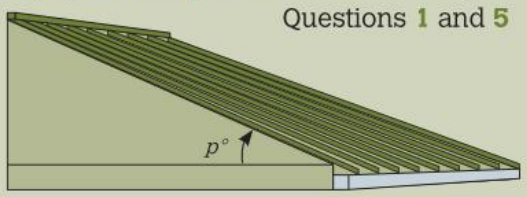
Player 2 starts the bidding at 110°, player 1 ups the bidding to 118°, player 2 ups the bidding to 124°, and player 1 calls for the angle to be measured. The angle is measured at 126°, so player 2 scores 4 points.

Maths 4 Real

# Constructing

The 'pitch' of a roof is the angle that the roof makes with the horizontal. The pitch is determined by a number of factors including visual preference, style of house, the roof 'material', or the weather conditions in the area in which the house is built.

**Equipment required:** 2 rulers and a protractor for Questions 1 and 5




The main reason houses have a pitched roof is to redirect rainwater. A pitch of between 20° and 30° is common.

1 Measure the angle, $p$, of the pitched roof above to determine a common pitch angle for Australian house roofs.

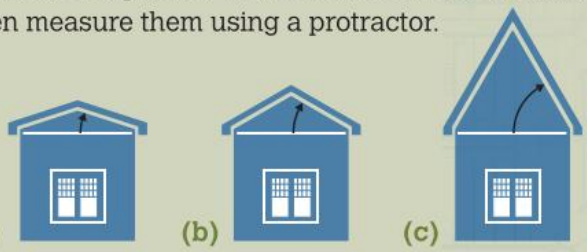
2 Imagine you are building a house in an area that regularly receives a lot of heavy rain. What problems might this cause if the roof is:

(a) too flat? (pitch angle is too small), or

(b) too steep? (pitch angle is too big)

# angles

3 Estimate the pitches of the roofs on these houses, then measure them using a protractor.




4 In parts of North America and Europe, where there is a lot of snow, the pitch of the roof must be at least 30°. Why do you think this is the case?

5 To measure an angle, builders use a *Pitch Angle and Level Finder* for construction.



Construct your own Pitch Angle and Level Finder out of two rulers and a protractor. Use it to measure the pitch of the roof of a building in your area, such as a school building or your house. You can do this by standing on the ground and holding your pitch angle and level finder in line with the roof of the building.

## Research

6 (a) Find out some of the common pitch angles used by builders. Does there seem to be a minimum or maximum value for the pitch angle? What problems might be created if the angle is too steep?

(b) Research the difference in pitch required by different materials, such as a thatched roof, tiled roof or corrugated iron roof.

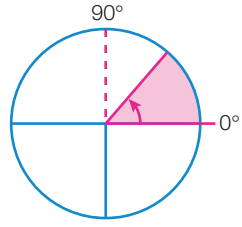


# 8.2 Classifying and naming angles

## Classifying angles

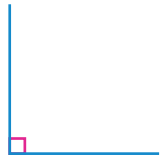
**Acute angle:** greater than 0° but less than 90°

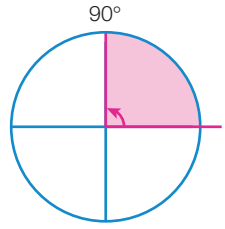





**Right angle:** exactly 90°, a $\frac{1}{4}$ turn.

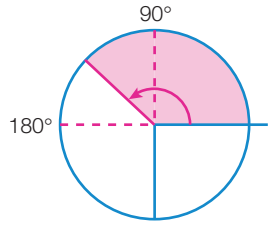
A small square drawn in the corner of an angle means it is a right angle.





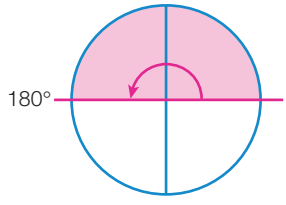

**Obtuse angle:** more than 90° but less than 180°.





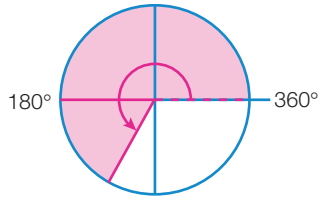

**Straight angle:** exactly 180°, a straight line angle, a half turn.



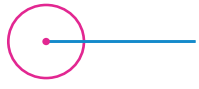


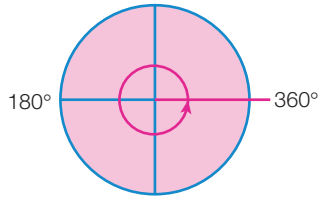

**Reflex angle:** greater than 180° but less than 360°.






Revolution: 360°, a full turn.





## Points, lines and angles

A *point* defines a position in space but it has no size. It has no width or length, so a dot we can see or write is not really a point at all. We represent a point with a dot and label it with upper case letters such as $A$, $B$ and $P$.




Similarly, a *line* we can draw or see is not really a line, because a line has no width and has no beginning or end, extending infinitely in both directions. A drawn line represents a line. A line always means a straight line and does not include curves. We can draw a line through any two points and use arrowheads on each end to show that it goes on forever in both directions.




We name a line that goes through points $A$ and $B$, $\overleftrightarrow{AB}$.

A **ray** is part of a line. Unlike a line, a ray has a beginning. It starts at a point called the vertex and extends infinitely in one direction just like a ray of light from a torch. We draw a ray by starting it at a point and using an arrowhead on the other end.

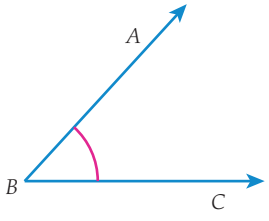


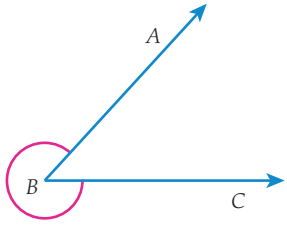

We name a ray that starts at point $A$ and passes through $B$ as $\overrightarrow{AB}$.

A **line segment** is part of a line that has both a beginning and an end. It can also be called an interval. We often refer to line segments as lines, but this is not strictly correct. We name a line segment that starts at $A$ and ends at $B$ as $\overline{AB}$.

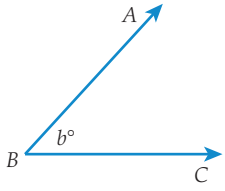



If two rays have a common vertex, two angles are formed. When we draw an angle we have to mark clearly the angle we want.




You will see this notation in geometry software packages; however, we usually use line segments to draw angles. We can name the angles that are formed $\overline{AB}$ and $\overline{BC}$ as $\angle ABC$ or by $\angle CBA$ with the vertex $B$ named as the middle of the three points and use the words acute, obtuse or reflex to indicate which angle we mean.



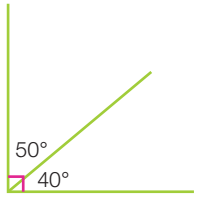

An easier and clearer way to name an angle is to use a letter with units such as $a°$ or $b°$ in the angle space. We usually use the same letter as the letter on the vertex.

## Complementary and supplementary angles

**Complementary angles** add to 90° (a right angle).

40° and 50° are complementary angles because they add to 90°.

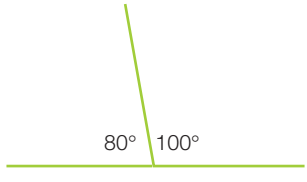
If two angles are complementary, we say one is the complement of the other. For example, 40° is the complement of 50°.




**Supplementary angles** add to 180° (a straight angle).

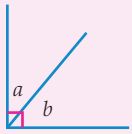
100° and 80° are supplementary angles because they add to 180°.

If two angles are supplementary, we say one is the supplement of the other. For example, 100° is the supplement of 80°.

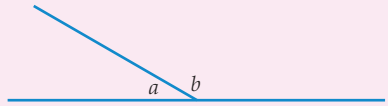





Complementary angles add to 90°:
$a + b = 90$

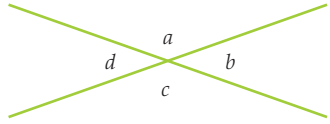



Supplementary angles add to 180°:
$a + b = 180$




## Vertically opposite angles

Whenever two lines intersect, four angles are formed. In the diagram opposite, these angles have been labelled as $a$, $b$, $c$ and $d$. Pairs of angles such as $a°$ and $c°$ are given a special name—they are called **vertically opposite angles**.




There is another pair of vertically opposite angles in the diagram: $b$ and $d$.

If you measured $a$ and $c$ with your protractor you would find they are the same size. The same is true for $b$ and $d$. Check both of these pairs of angles for yourself.

Vertically opposite angles are equal.

# 8.2 Classifying and naming angles

## Navigator

Answers page 668

| | | |
|---|---|---|
| Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13, Q15 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Q12, Q14, Q15 | Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15 |

**Equipment required:** Protractor for Questions **10(a)** and **12**

## Fluency

**1** State the type of angle shown in each case.

**(a)**



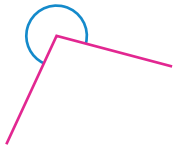
**(b)**



**(c)**



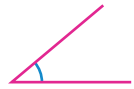
**(d)**



**(e)**



**(f)**



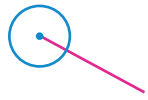
**(g)**



**(h)**



**(i)**

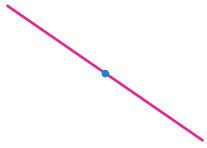


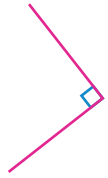
(j) 

(k) 

(l) 

(m) 

(n) 

(o) 

(p) 

(q) 

(r) 

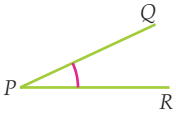
**2** Classify the following angles (acute, obtuse, right etc.).

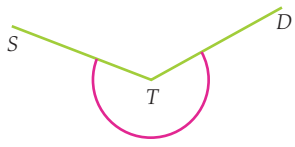
**(a)** 23° **(b)** 117° **(c)** 275°

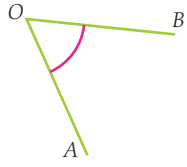
**(d)** 360° **(e)** 180° **(f)** 75°

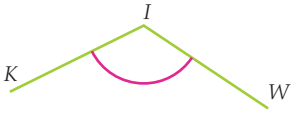
**(g)** 90° **(h)** 165° **(i)** 341°

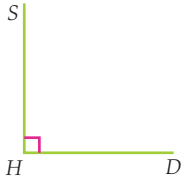
**3** Name the following angles.

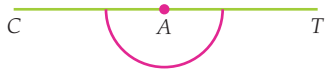
**(a)** 


**(b)** 


**(c)** 


**(d)** 


**(e)** 


**(f)** 

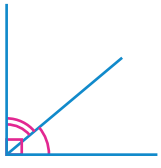

**4 (a)** Which one of the following is a pair of complementary angles?

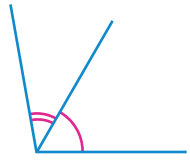
**A** 20° and 40° **B** 330° and 30° **C** 30° and 150° **D** 15° and 75°

**(b)** Which one of the following is a pair of supplementary angles?

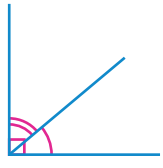
**A** 0° and 90° **B** 45° and 55° **C** 90° and 90° **D** 180° and 20°

**(c)** Which diagram shows supplementary angles?

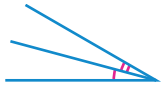
**A** 

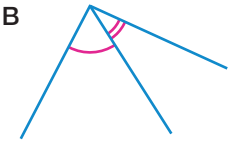
**B** 

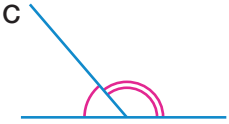
**C** 

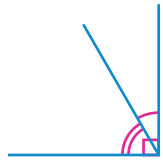
**D** 

**(d)** Which diagram shows complementary angles?

**A** 

**B** 

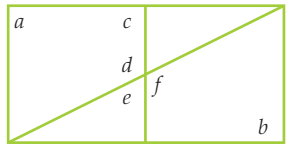
**C** 

**D** 

It might help you to remember that the 'c' of complementary stands for a corner (right angle) and the 's' of supplementary stands for a straight angle.



**5** Which two angles are vertically opposite?

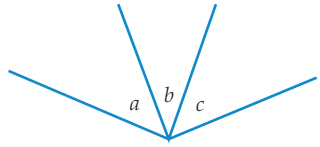



**A** $a$ and $b$ **B** $a$ and $c$

**C** $d$ and $e$ **D** $d$ and $f°$

## Understanding

**6** The angles $a$, $b$ and $c$ in this diagram have been drawn as adjacent angles. They have a common vertex and common arms.




For each of the following:

**(a)** 100°, 210°, 50° **(b)** 32°, 40°, 18° **(c)** 40°, 40°, 100°

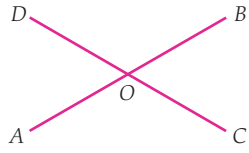
**(d)** 32°, 161°, 85°, 82° **(e)** 44°, 46° **(f)** 39°, 141°

**(i)** draw the angles as adjacent angles

**(ii)** find the angle sum

**(iii)** describe the angle you have formed.

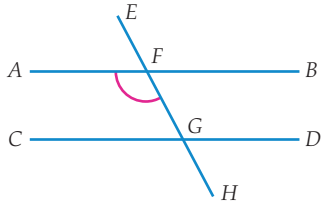
**7** In the diagram opposite:




**(a)** name a pair of supplementary angles

**(b)** name a pair of vertically opposite angles.

**8** In the diagram, what could the marked angle be named?



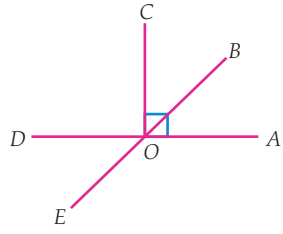

**A** $\angle F$

**B** $\angle AFE$

**C** $\angle AFG$

**D** $\angle BFG$

**9** Using the diagram opposite, find, using letter names:




**(a)** an acute angle

**(b)** an obtuse angle

**(c)** a reflex angle

**(d)** a right angle

**(e)** a straight angle

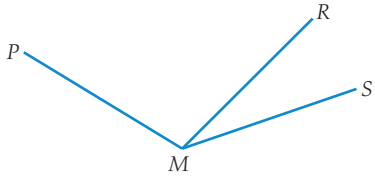
**(f)** one pair of complementary angles

**(g)** one pair of supplementary angles

**(h)** one pair of vertically opposite angles.

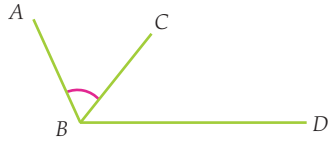
## Reasoning

**10 (a)** Measure the reflex angle $RMS$ in the diagram.




**(b)** Why is it necessary to specify 'reflex' in part **(a)**?

**11 (a)** Why couldn't we refer to the marked angle as $\angle B$?



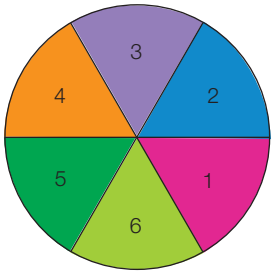

**(b)** Name two acute angles that have $B$ as a vertex.

**(c)** Name an obtuse angle that has $B$ as a vertex.

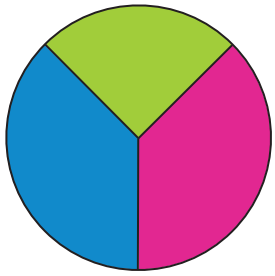
**(d)** Name a reflex angle that has $B$ as the vertex.

**12** Use a protractor to draw accurate copies of the following.

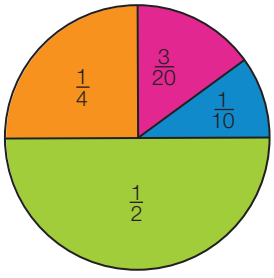
**(a)** A raffle wheel divided into six equal parts.



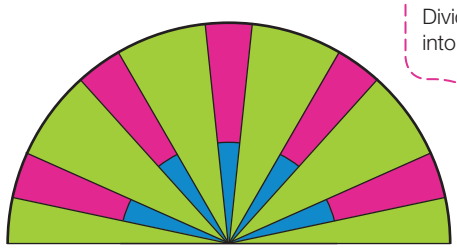

**(b)** This sign.



**(c)** This pie chart.




**(d)** This semicircular stained glass window.






## Open-ended

**13** Draw two examples of:

**(a)** an acute angle **(b)** a reflex angle **(c)** an obtuse angle.

**14** **(a)** Write two examples of pairs of complementary angles.

**(b)** Write two examples of pairs of supplementary angles.

**15** Write two examples of angles that add to make a revolution.

# Outside the Square

## Problem solving

### Count the angles

**Equipment required:** 1 brain, 1 protractor

**1** How many different acute angles can you count in the diagram? Measure each one.



**2** How many different angles can you count above the horizontal line? Measure the size of each one.



**3** How many different angles can you count in the diagram? Measure the size of each one.



**Strategy options**

- Test all possible combinations.
- Break problem into manageable parts.

# 8.3 Calculating angles

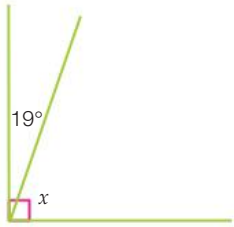
## Complementary and supplementary angles

The sum of complementary angles is 90°, so we can subtract a known angle from 90° to find the unknown complementary angle. In a similar way, we can subtract a known angle from 180° to find the unknown supplementary angle.

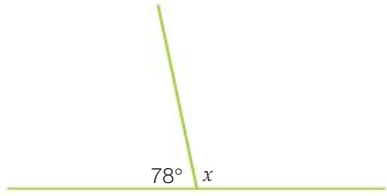
### Worked Example 3 WE3

Find the size of angle $x$ in each diagram.

(a)




(b)




| Thinking | | Working |
|---|---|---|
| (a) 1 | Identify the angles as complementary angles. | (a) $x$ and 19° are complementary angles. |
| 2 | Write an equation with the RHS equal to 90°. | $x + 19 = 90$ |
| 3 | Solve the equation to find the angle. | $x + 19 - 19 = 90 - 19$<br>$x = 71°$ |
| (b) 1 | Identify the angles as supplementary angles. | (b) $x$ and 78° are supplementary angles. |
| 2 | Write an equation with the RHS equal to 180°. | $x + 78 = 180$ |
| 3 | Solve the equation to find the angle. | $x + 78 - 78 = 180 - 78$<br>$x = 102°$ |

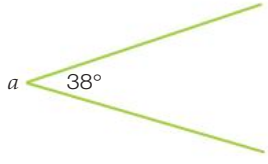
## Angles in a revolution

Angles in a revolution add to 360°, so we can subtract a known angle from 360° to find the unknown angle.

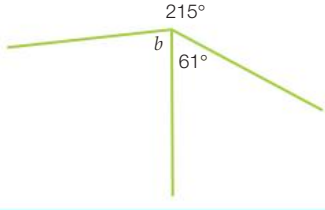
### Worked Example 4 WE4

Find the value of the pronumeral in each diagram.

(a)




(b)



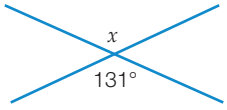
| Thinking | | | Working |
|---|---|---|---|
| **(a)** | **1** | Add all the angles in the revolution, to give 360°. | (a) $a + 38 = 360$ |
| | **2** | Find the value of $a$. | $a + 38 - 38 = 360 - 38$<br>$a = 322°$ |
| **(b)** | **1** | Add all the angles in the revolution, to give 360°. | (b) $215 + 61 + b = 360$<br>$276 + b = 360$ |
| | **2** | Find the value of $b$. | $276 - 276 + b = 360 - 276$<br>$b = 84°$ |

## Vertically opposite angles

Vertically opposite angles are equal, so if we know one angle we know that the vertically opposite angle is the same size.

### Worked Example 5 WE5

Find the value of $x$ in the diagram.




| Thinking | Working |
|---|---|
| These are vertically opposite angles, which means they are equal. | $x = 131°$ |

# 8.3 Calculating angles

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7 Column 1, Q8, Q10 Column 1, Q12 | Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7 Column 2, Q10 Column 2, Q11, Q12, Q13 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7 Column 3, Q9, Q10 Column 3, Q11, Q12, Q13, Q14 |

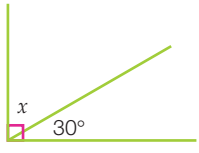
Answers page 668

**Equipment required:** Protractor for Questions **11(a)** and **12**

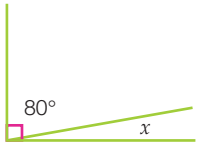
## Fluency

**1** Find the size of angle $x$ in each diagram. WE3

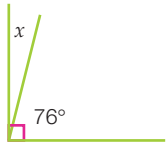
**(a)**



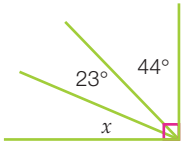

**(b)**



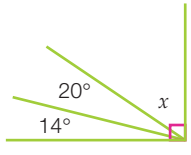

**(c)**



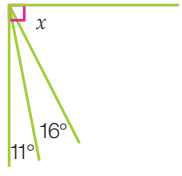

**(d)**

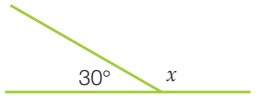


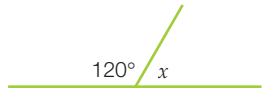

**(e)**



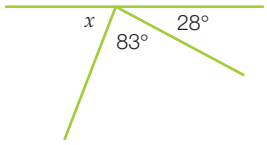

**(f)**

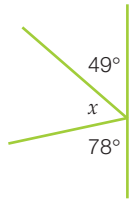


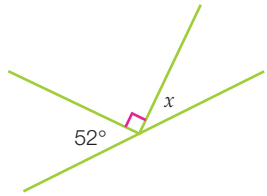
(g) 


(h) 


(i) 

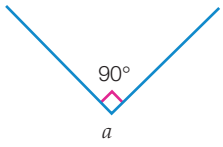

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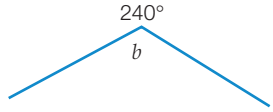

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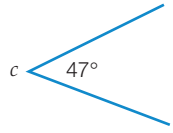

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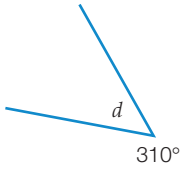

**WE4**

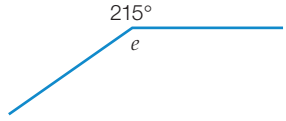
**2** Find the value of the pronumeral in each diagram.

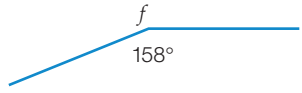
(a) 

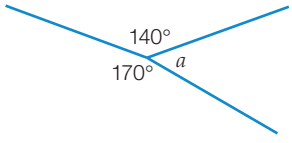

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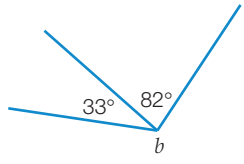

(c) 

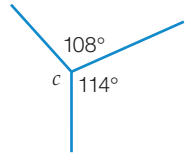

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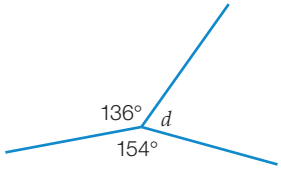

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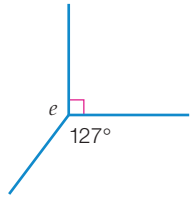

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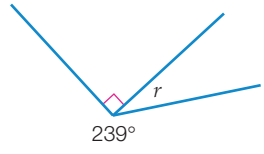

(g) 


(h) 


(i) 

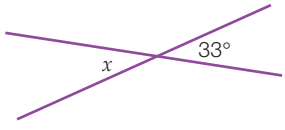

(j) 

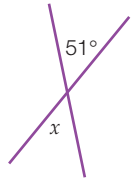

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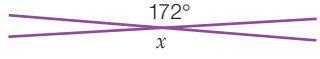

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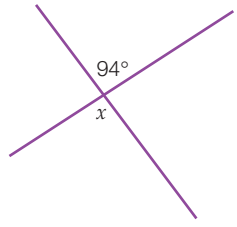

**WE5**

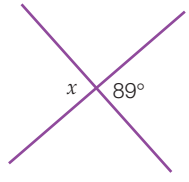
**3** Find the value of $x$ in each diagram.

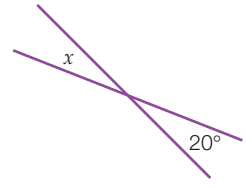
(a) 


(b) 


(c) 


(d) 


(e) 


(f) 





**4** Find the complement of the following angles.

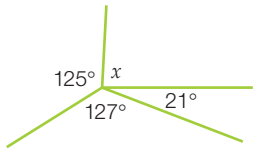
**(a)** 27° **(b)** 45° **(c)** 68° **(d)** 15°

**5** Find the supplement of the following angles.

**(a)** 32° **(b)** 90° **(c)** 124° **(d)** 176°

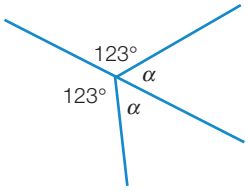
## Understanding

**6** Find the value of $x$ in the diagram.

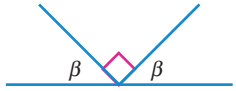



**7** Find the value of the pronumerals in each diagram.

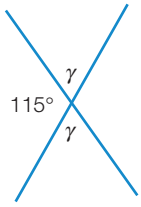
**(a)**




**(b)**



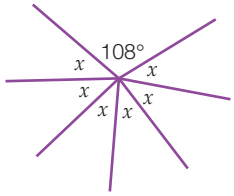

**(c)**




**8** The value of $x$ in the opposite diagram is:

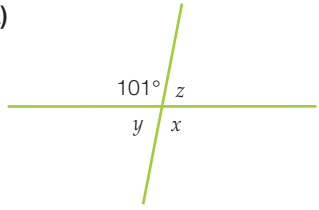
**A** 42° **B** 60°

**C** 108° **D** 360°

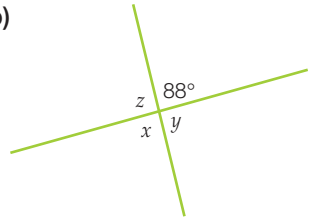



**9** Find the value of the pronumerals in each diagram.

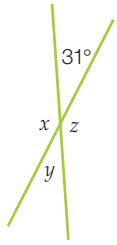
**(a)**




**(b)**



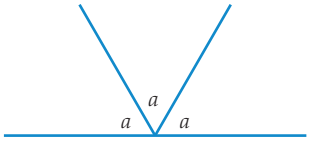

**(c)**



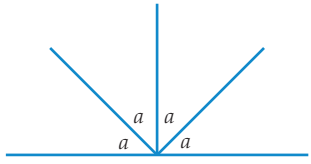

## Reasoning

**10** Determine the size of angle $a$ in each diagram. Give reasons for your answer.

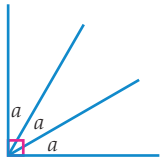
**(a)**



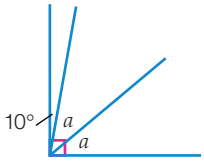

**(b)**



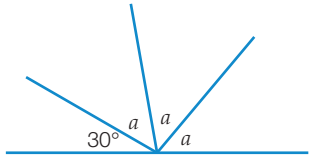

**(c)**



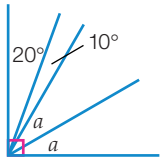

**(d)**



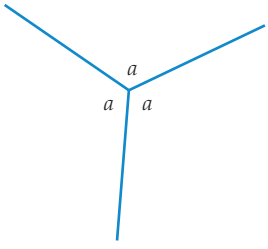

**(e)**



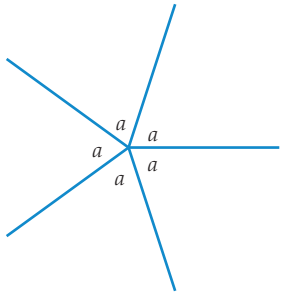

**(f)**



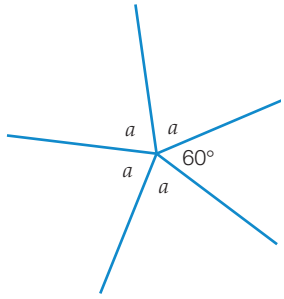

**(g)**



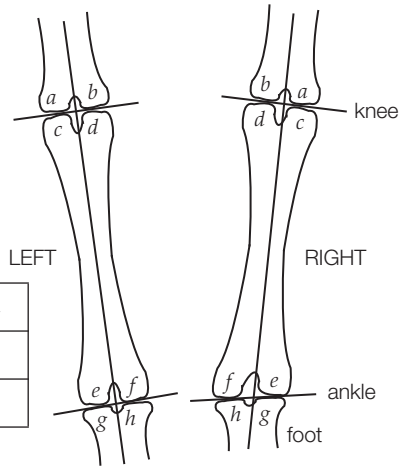

**(h)**




**(i)**



**11** After breaking her leg in an accident, Kira had difficulty walking. She took her X-ray to a specialist, who marked in the following lines. He followed a procedure to decide whether she needed an operation. (You will need a protractor to measure some of the angles.)




**(a)** Measure all the angles and fill in the table below.

| | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ |
|---|---|---|---|---|---|---|---|---|
| **Left leg** | | | | | | | | |
| **Right leg** | | | | | | | | |

**(b)** All angles on Kira's knees must be approximately equal and all the angles on her ankles must also be approximately equal. Which leg do you think Kira broke?

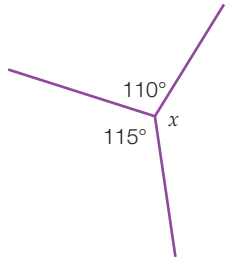
**(c)** The differences between the angles $b$ and $f$, and $c$ and $g$, must be no more than 8°. If they are more, Kira needs an operation. Does she need an operation? Why?

## Open-ended

**12** Sabine was answering the question on the right.

Using her protractor, Sabine measured the angle $x$ and found that it was 140°. This was incorrect. Explain where Sabine made a mistake.

Find the value of the pronumeral.




**13** **(a)** Draw two lines that intersect each other.

**(b)** Measure the two supplementary angles you have created.

**(c)** Use your answers to **(b)** to work out the two angles you did not measure.

**14** Draw two straight lines that intersect, forming two acute angles. What are the sizes of the other two angles?

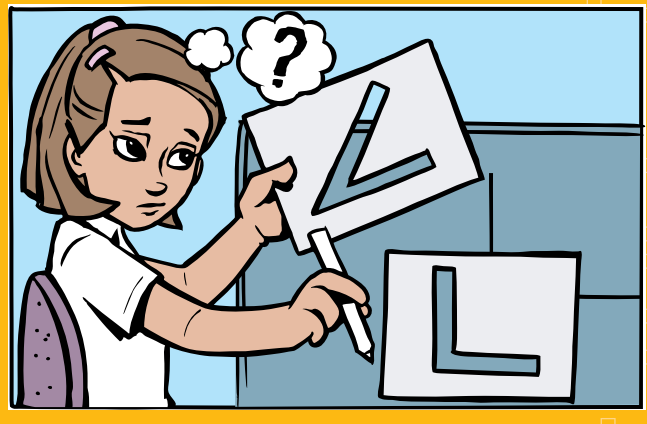
## Outside the Square — Problem solving

### Ella's angles

Ella needs to draw a 60° angle, but she has lost her protractor. She has a stencil that can draw 90° and 40° angles. How can she use these two angle sizes to draw a 60° angle?

**Strategy options**

- Draw a diagram.
- Guess and check.



# Technology Exploration GeoGebra

**Equipment required:** 1 brain, 1 computer with GeoGebra

Versions of this Exploration are available for other technologies in Pearson Reader.

## Investigating angles on parallel lines

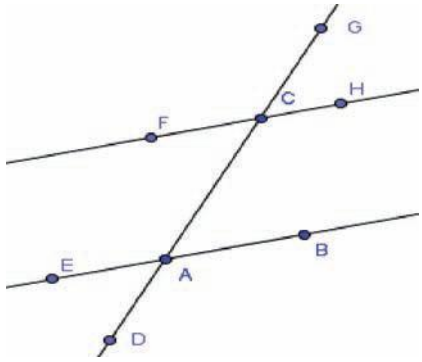
Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are eleven icons called tools. When you hover the mouse over the arrow in the bottom right-hand corner of the tool icon the arrow turns red and the name of the tool appears. By clicking on this arrow a drop-down list of more tools appears. Instructions on how to use the tool will appear in the top right-hand corner of the screen.

1. Click on the View menu. Deselect 'Axes', 'Grid' and 'Algebra View'. (Alternatively, you can right click in the space where the drawings appear, called the graphics view, to do the same for the 'Axes' and the 'Grid'.)
2. Click on the Options menu. Select 'Labelling', then 'New Points only'. Click on Options again, then select 'Rounding', and '0 Decimal Places'.
3. If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided.

### Creating a pair of parallel lines and a transversal

Parallel lines are lines that are equidistant from each other (the distance between the lines is the same all the way along them). A transversal is a line that cuts two or more other lines. (The lines cut by a transversal do not have to be parallel.)

4. Construct a line using the 'Line Through two Points' tool  , then click on any two points on the page.
5. Click on the small arrow on the fourth tool from the left and select the 'Parallel Line' option  . Create a parallel line by clicking on the line and then on anywhere above the line. (You know when you are about to select the line as it becomes darker and the cursor turns to an arrow when you hover over it.)
6. Create a transversal by selecting the 'Line Through two Points' tool  , then clicking on point *A* and then on point *C*.
7. Select the 'New Point' tool  . Use this to place points on the lines as shown below. Keep the points in the same order as we will refer to them in the following steps. They are placed in a clockwise direction from under point *A*. Points will be placed on the line only if the line gets darker when your mouse hovers over it. It does not matter if your transversal or your parallel lines slope in the opposite direction, just ensure that the points are placed in the same positions.



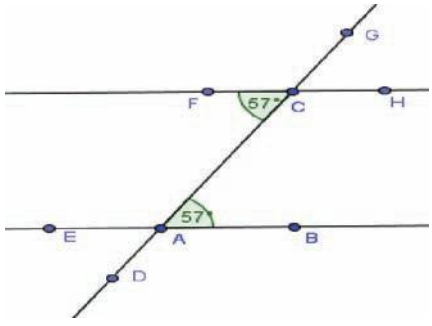

If you make a mistake, you can either:

- right click on the object and select 'Delete', or
- click the 'Edit' menu and select 'Undo', or
- press 'ctrl' + 'z'.

## Alternate angles and parallel lines

**8** Click on the small arrow on the 8th tool from the left.

Select the 'Angle' tool  . Click on point *B*, then *A* and then *C* (∠*BAC*). Notice that the vertex of the angle is the middle letter. Click on ∠*FCA* in the same way. Points must be selected in a clockwise order, so if you get the reflex angle, delete it and try selecting the points in the opposite order. Your diagram should look similar to the one below. You have just marked a pair of alternate angles. Alternate angles sit on opposite sides of a transversal.




**9** Right click inside one of the marked angles and select 'Object Properties'. A pop-up box will appear. On the 'Colour' tab click on a different colour. With the pop-up box still open, click inside the other marked angle and select the same colour (the colour will be stored in the 'Recent' box on the right-hand side of the pop-up box). Close the pop-up box.

**10** Click on the 'Angle' tool  . Mark ∠*CAE* and ∠*ACH* as described in step **8**. You have just marked another pair of alternate angles.

**11** Click on the 'Select' tool  . (Pressing 'Escape' also takes you to the 'Select' tool.) Click on point *B* and drag it about.

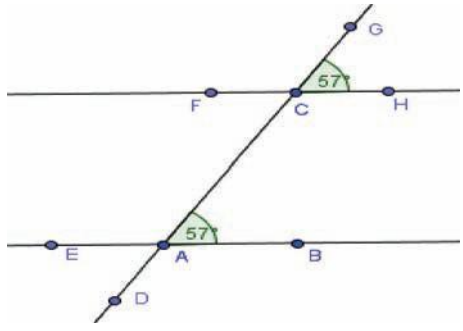
**(a)** You have just marked two pairs of alternate angles between a pair of parallel lines. What do you notice as you move the lines about?

**(b)** Copy and complete the statement: 'Alternate angles between parallel lines are _____'.

**12** Right click inside a marked angle and select delete, repeat for the other marked angles. (If some of the points have disappeared, move point *B* until they reappear.)

## Corresponding angles and parallel lines

**13** Click on the 'Angle' tool  , then mark ∠*HCG* and then ∠*BAC* by clicking on the points. Your diagram should look similar to the one below. You have just marked a pair of corresponding angles. Corresponding angles sit on the same side of the transversal, one underneath the other.




**14** Follow the process in step **9** to change the colour of both angles.

**15** Repeat steps **13** and **14** for ∠*ACH* and ∠*DAB*, for ∠*EAD* and ∠*FCA*, and for ∠*CAE* and ∠*GCF*. Make sure that each pair of angles has a different colour. You have just marked three more pairs of corresponding angles.

**16** Click on the 'Select' tool  (or press escape).

Click on point *B* and drag it about. Take notice of the angle sizes.

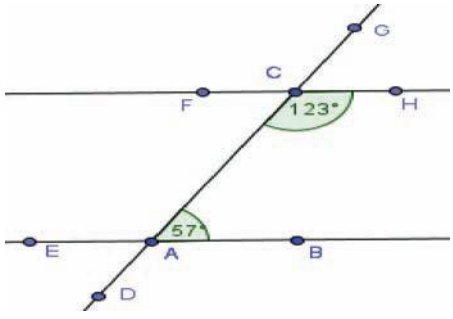
**(a)** What do you notice about each pair of corresponding angles as you move the lines about?

**(b)** Copy and complete the statement: 'Corresponding angles on parallel lines are _____'.

**17** Delete the marked angles as described in step **12**.

## Co-interior angles and parallel lines

**18** Click on the 'Angle' tool, then mark $\angle ACH$ and $\angle BAC$ by selecting the points. Your diagram should look similar to the one below. You have marked a pair of co-interior angles (also known as allied angles). Co-interior angles sit on the same side of the transversal and on the inside of the lines.




**19** Follow the process in step **9** to change the colour of both angles.

**20** Repeat steps **18** and **19** for $\angle CAE$ and $\angle FCA$ to mark another pair of co-interior angles.

**21** Click on the 'Select' tool (or press escape), then click on point $B$ and drag it about. Take note of the angle sizes.

**(a)** What do you notice about each pair of co-interior angles as you move the line about?

**(b)** Copy and complete the statement: 'Co-interior angles between parallel lines sum to _____°'.

**22** Clear the angles as described in step **12**.

## Taking it further

**23** Select the 'New Point' tool and place a point on the interval between points $A$ and $C$. This will be point $I$.

**24** Click on the small arrow on the fourth tool from the left. Select the 'Parallel Line' tool. Create another parallel line by clicking on the line $AB$ and then on the new point $I$.

**25** Select the 'New Point' tool and place a point $J$ on the new line to the left of point $I$ and another point $K$ to the right of point $I$.

**26** Select the 'Angle' tool. Click on $\angle IAE$.

**27** You may use the 'Angle' tool to assist you with the following.

Find two angles that are alternate, two angles that are corresponding, and two angles that are co-interior to $\angle IAE$. Mark each of these angles using different colours to indicate the three different angle types. Use correct terminology to name the angles.

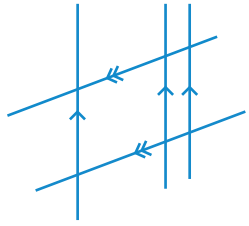
**28** Using the 'Angle' tool, mark and colour all of the angles that are the same size.

**(a)** Can you see any pairs of vertically opposite angles? Name as many pairs as you can.

**(b)** Can you see any pairs of adjacent supplementary angles? Name as many pairs as you can.

# 8.4 Angles and parallel lines

**Parallel lines** are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. The lines on a page of lined paper are parallel.

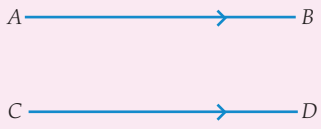


Lines that are parallel are marked with an arrow pointing in the same direction.

If more than one set of parallel lines appears in a diagram we use more than one arrow.

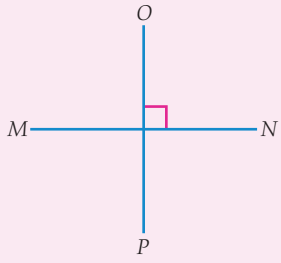
**Perpendicular lines** intersect at right angles.

|| means 'is parallel to'



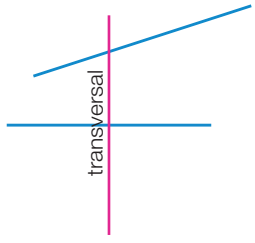

$AB \parallel CD$

$\perp$ means 'is perpendicular to'




$OP \perp MN$

A **transversal** is a line that **intersects** (crosses or transverses) two or more other lines, as shown.

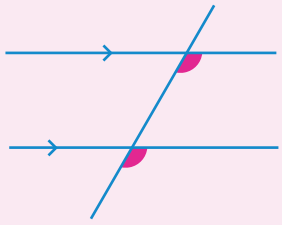



When parallel lines are crossed by a transversal, the pairs of angles on parallel lines described have special properties.






**Corresponding angles**

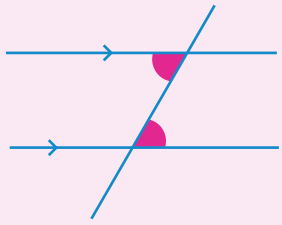


Corresponding angles are *equal*.

These angles both lie above or below parallel lines, and on the same side of the transversal.

'Corresponding' means 'matching', so corresponding angles are in matching positions. There are four pairs of corresponding angles formed when a transversal cuts two lines.

**Alternate angles**

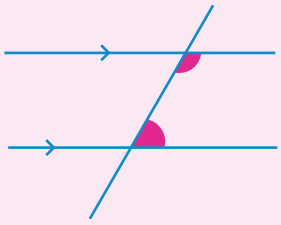


Alternate angles are *equal*.

These angles lie between the parallel lines and on different sides of the transversal between the parallel lines.

'Alternate' means 'swap', so alternate angles swap sides of the transversal. There are two pairs of alternate angles formed when a transversal cuts two lines.

**Co-interior (allied) angles**



Co-interior (allied) angles are *supplementary*. They add to 180°.

These angles lie between the parallel lines and on the same side of the transversal between the parallel lines.

'Co' means 'with' and 'interior' is 'inside', so co-interior angles are on the same side of the transversal and inside the parallel lines. There are two pairs of co-interior angles formed when a transversal cuts two lines.

Co-interior angles are also called allied angles.

The opposite of the above is also true. If corresponding or alternate angles are equal or if co-interior angles are supplementary when two lines are cut by a transversal, the two lines are parallel.

# 8.4 Angles and parallel lines

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6 Columns 1 & 2, Q7, Q8 Column 1, Q9, Q10, Q13 Column 1, Q14 (a) | Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 2, Q9, Q11, Q12, Q13 Column 2, Q14 (a) | Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 3, Q11, Q12, Q13 Column 3, Q14, Q15 |

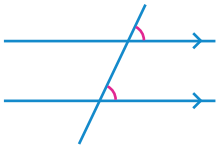
Answers page 669

**Equipment required:** Protractor for Question **6 (a)**

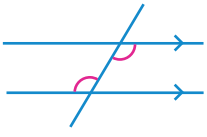
## Fluency

**1** Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

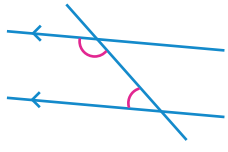
**(a)**

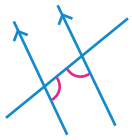


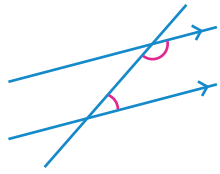
**(b)**

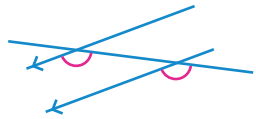


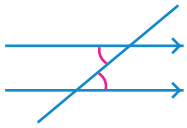
**(c)**

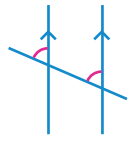


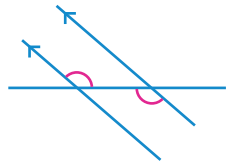
(d) 

(e) 

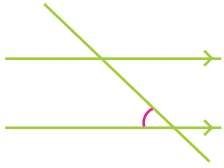
(f) 

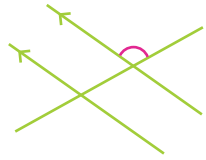
(g) 

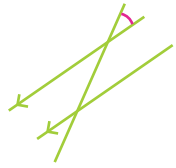
(h) 

(i) 

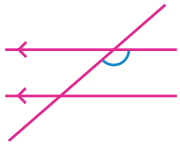
**2** Copy each of the following and mark in an angle corresponding to the one shown.

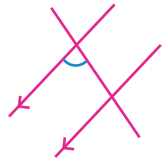
(a) 

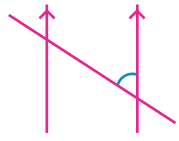
(b) 

(c) 

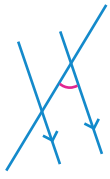
**3** Copy each of the following and mark in an angle alternate to the one shown.

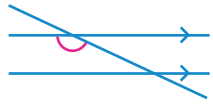
(a) 

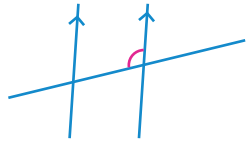
(b) 

(c) 

**4** Copy each of the following and mark in an angle that is co-interior with the one shown.

(a) 

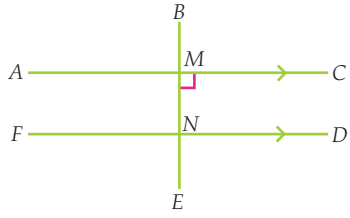
(b) 

(c) 

Remember that ⊥ means perpendicular to and || means parallel to.



**5** Which of the following statements is not true about the diagram shown?




**A** $\angle AMB$ is corresponding to $\angle FNB$

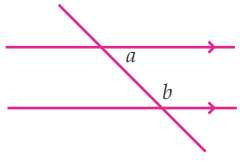
**B** $\angle BNF$ is alternate to $\angle EMC$

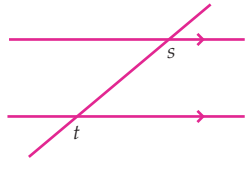
**C** $\angle AMN$ and $\angle CMN$ are co-interior angles

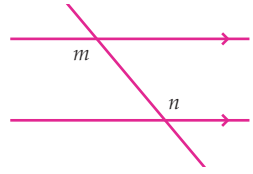
**D** $BE$ is ⊥ to $FD$

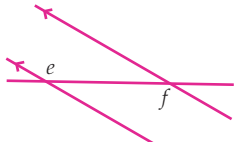
## Understanding

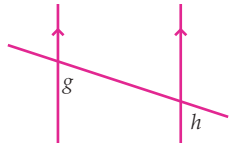
**6 (a)** Use a protractor to measure the labelled angles in each diagram below. State whether each pair of angles is corresponding, alternate or co-interior.

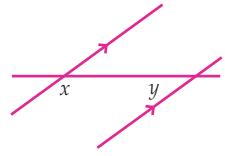
(i) 


(ii) 


(iii) 


(iv) 


(v) 


(vi) 

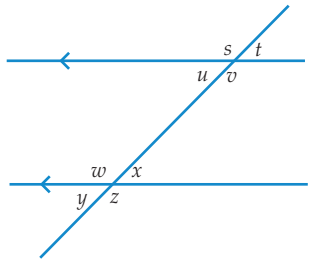

**(b)** State which pairs of angles in part **(a)** are equal. Which pairs of angles add to 180°?

**7** State which angle in the diagram is:

**(a)** corresponding to $u$ **(b)** alternate to $v$

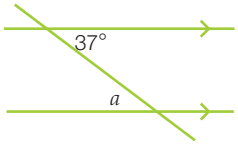
**(c)** co-interior with $w$ **(d)** co-interior with $x$

**(e)** corresponding to $w$ **(f)** alternate to $u$.

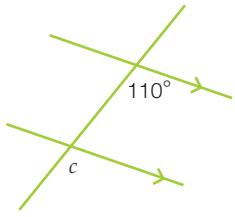



**8** Find the value of the pronumerals in each case, and give a reason for your answer (e.g. corresponding angles).

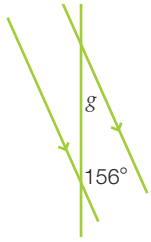
**(a)**



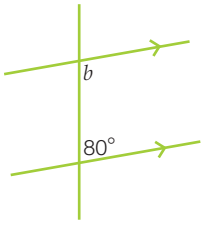

**(b)**



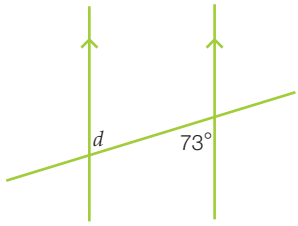

**(c)**



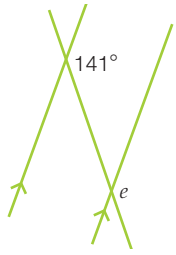

**(d)**



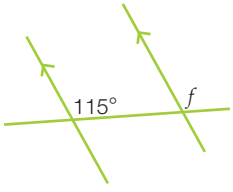

**(e)**



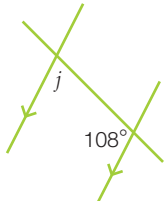

**(f)**



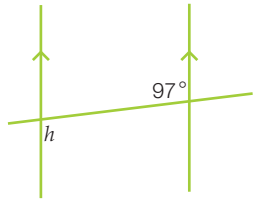

**(g)**



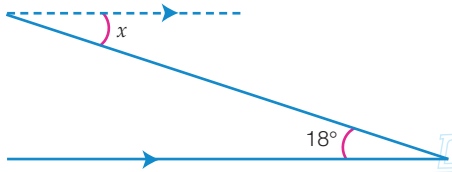

**(h)**



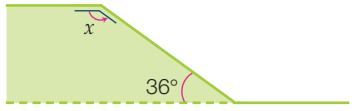

**(i)**




**9** A pilot of a plane flying parallel to the ground can see the runway as it is coming in to land. The angle of approach to the runway is 18°. What angle must the pilot turn the plane down through to start the approach?

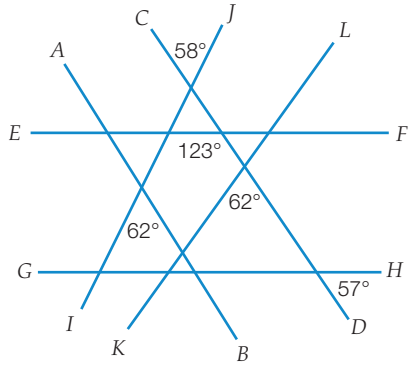



**10** In an underground carpark a ramp is being constructed. The ramp will ascend at an angle of 36°. At what angle must the supports be positioned at the top of the ramp?

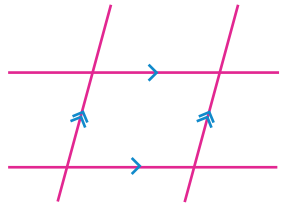



## Reasoning

**11** Choose the pair of lines that are parallel and explain why you choose them.



**12** When a pair of parallel lines is cut by two parallel transversals, how many pairs of angles are formed that are:



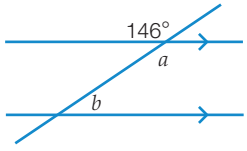
**(a)** corresponding

**(b)** alternate

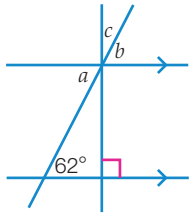
**(c)** co-interior?

**13** Find the value of each pronumeral below, giving a reason for your answer.

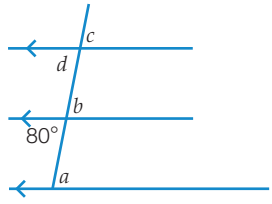
**(a)**



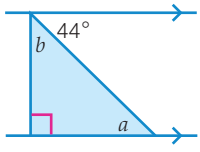

**(b)**



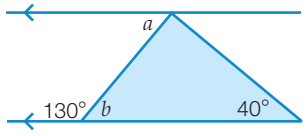

**(c)**



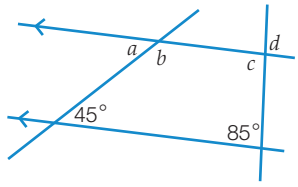

**(d)**



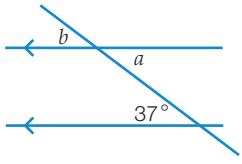

**(e)**



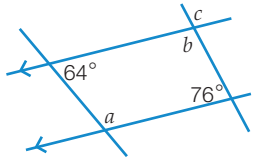

**(f)**



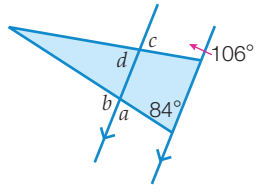

**(g)**




**(h)**

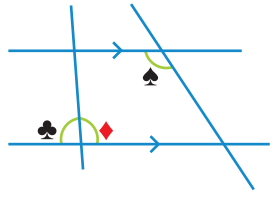



**(i)**




## Open-ended

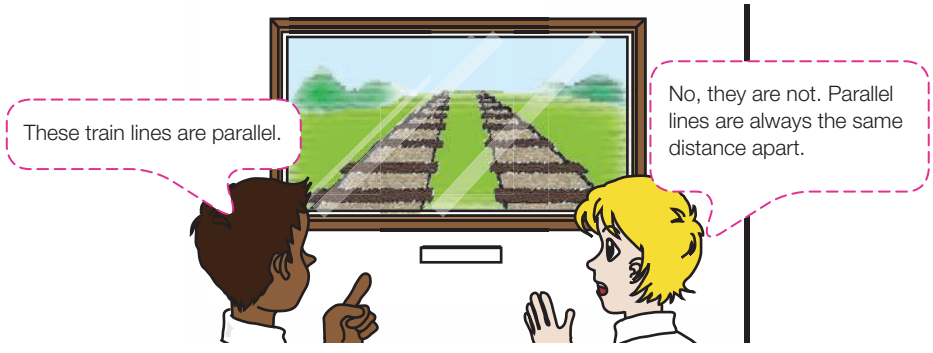
**14** Aileen has drawn a pair of parallel lines with two transversals. She has marked three angles using different symbols.



**(a)** Copy the diagram into your book. Mark another three angles using the same symbols as Aileen so that there is a pair of corresponding angles, alternate angles and co-interior angles.

**(b)** In how many different ways can the three other angles be arranged, with no pair of angles sharing an angle with another pair? Draw a diagram for each combination.

**15**




How would you explain to the students whether the train lines are parallel or not?

# Half-time 8

**Equipment required:** Protractor for Questions **3** and **5**

**1** Draw the following angles by estimating their size. (Do not use a protractor.) Ex 8.1

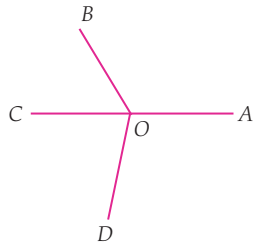
**(a)** 90° **(b)** 180° **(c)** 360°

**(d)** 34° **(e)** 75° **(f)** 120°

**2** **(a)** Name the two acute angles in the diagram opposite. Ex 8.2

**(b)** Name the three obtuse angles in the diagram opposite.

**(c)** Give another name for $\angle AOB$.



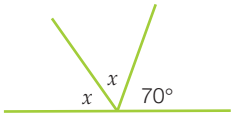

**3** Draw the following angles using a protractor. Ex 8.1

**(a)** 90° **(b)** 180° **(c)** 360°

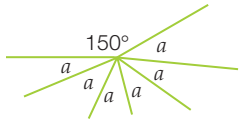
**(d)** 34° **(e)** 75° **(f)** 120°

**4** Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.3

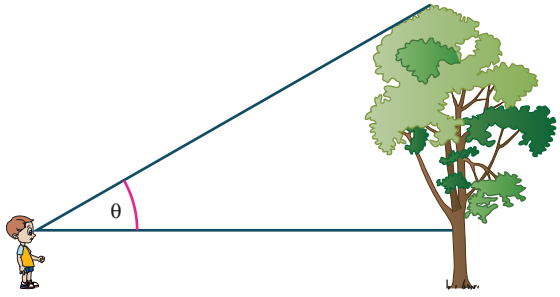
**(a)**




**(b)**

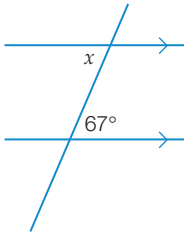



**5** Measure the angle marked in the diagram opposite. Ex 8.1

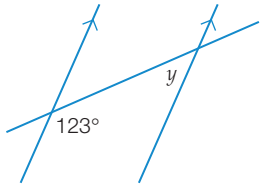



**6** Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.4

**(a)**




**(b)**



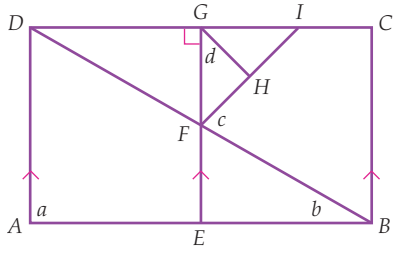

**7** Using letter names such as $\angle ABC$: Ex 8.3, 8.4

**(a)** name an angle corresponding to $a$

**(b)** name an angle alternate to $b$

**(c)** name an angle supplementary to $c$

**(d)** name an angle complementary to $d$.



# Investigation

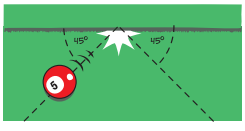
## Billiard ball bounces

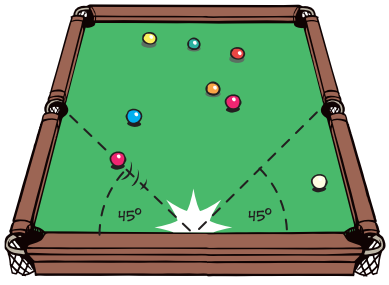
**Equipment required:** 1 or 2 brains, 1-centimetre grid paper, protractor, ruler



The game of billiards is played on a large rectangular table that is covered in felt and surrounded by a cushion along the edges. Players use a long stick, called a cue, to hit balls into pockets around the edges of the table. Snooker and pool are variations of the game that have become extremely popular over time. The size of the table can vary from 12 feet (3.7 m) along the larger side to 9 feet (2.7 m), 8 feet (2.4 m) or 7 feet (2.1 m) in length.

When planning their shots, good billiards players use the fact that a ball bounces off the side of a table at the same angle at which it hits, as shown here:




Here, the ball approaches the cushion at an angle of 45° and rebounds at an angle of 45°.

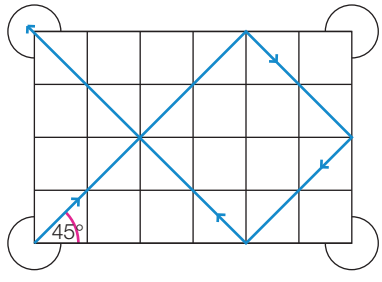
The ball may bounce off the sides several times before it either stops rolling or goes into a pocket. For this investigation, we will assume that the ball keeps rolling until it falls into a pocket.

### The Big Question

Can we predict how many times a billiard ball will bounce on tables of different sizes?

Can we predict which pocket the ball will fall into?

### Engage




Here, the ball is hit from the bottom left corner at an angle of 45° and bounces off the sides of the table three times before falling into the opposite pocket along the short side.

1 **(a)** To see what would happen if we made the table bigger, use your grid paper to draw a similar 'table' to the one above, but make it 6 cm × 8 cm. Starting in the same corner (bottom left), trace the path of a ball hit at an angle of 45°, bouncing it off the sides until the path ends in a pocket. You should find the ball bounces five times and falls into the opposite pocket on the long side.

**(b)** What would happen if the table was square?

## Explore

2 **(a)** Draw up tables of the following dimensions on your grid paper (you might like to share this task with a partner).

$1 \times 2$, $1 \times 4$, $2 \times 3$, $2 \times 4$, $2 \times 8$, $3 \times 4$, $3 \times 6$, $3 \times 8$, $3 \times 12$, $4 \times 5$, $4 \times 6$, $4 \times 7$, $4 \times 8$, $5 \times 6$, $5 \times 7$, $5 \times 8$, $6 \times 7$, $6 \times 8$, $6 \times 9$, $7 \times 9$, $8 \times 10$, $9 \times 12$

**(b)** Starting in the same corner each time, trace the path of the ball hit at an angle of 45°. Count the number of bounces, and also note which pocket the ball falls into.



**Strategy options**

- Make a table
- Look for a pattern.

## Explain

3 **(a)** Collect all of the drawings you have made of the tables and their ball paths. To help you see any patterns or connections between them, group together the tables that have something in common. Some of the groups you could make are:

- tables where the pattern traced by the path of the ball is identical
- tables where the ball passes through every square on the grid.

**(b)** Once you have made these groups, compare the length, width and 'bounce' numbers for each table in the group. Are they connected in some way? Compare the starting and finishing pockets of the balls in each group. Is there a pattern here?

## Elaborate

4 **(a)** Write a couple of sentences that answer the Big Question about predicting the number of bounces for tables of different sizes.

**(b)** Write a couple of sentences that answer the Big Question about predicting which pocket the ball will fall into.

## Evaluate

5 **(a)** Consider how you worked on this task and the methods that you used. How did you organise or keep track of your results? Could you have done this better?

**(b)** Did grouping your results help you to spot patterns and connections between them? Which groups were useful?

## Extend

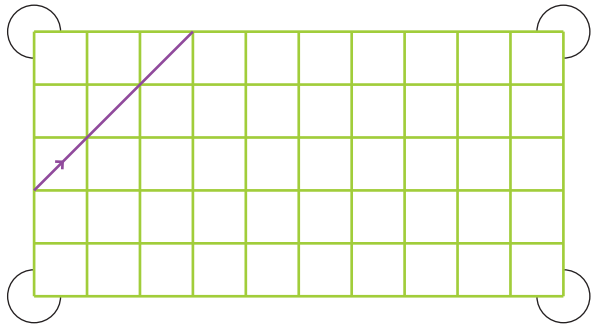
6 **(a)** Predict the number of bounces and which pocket the ball will fall into for the following table sizes.

**(i)** $18 \times 27$ **(ii)** $17 \times 19$

In which of these tables will the ball path pass through every square on the grid?

Draw these tables and check your prediction.

**(b)** Draw a $5 \times 10$ table on grid paper and trace the path of the ball hit from this position.



What do you notice? Try starting at different positions and comment on what you find.

**(c)** A billiard ball bouncing at the same angle at which it hits is an example of 'the law of reflection'. Investigate this law and other examples of where it can be seen.

# 8.5 Polygons

We draw many shapes on a flat surface called a plane. These are called two-dimensional or **plane shapes**. Plane shapes whose sides are all straight lines are called **polygons**.

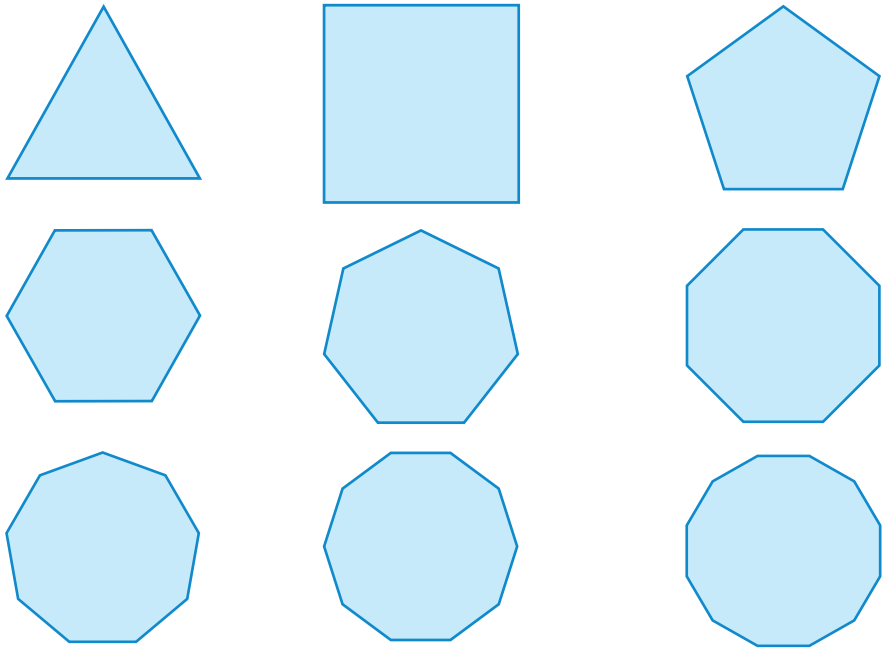
The word polygon is made up of two Greek words—*poly* (meaning many) and *gon* (meaning angle), so a polygon is a many-angled shape. As you can see in the following diagrams of polygons, the number of sides in a polygon is equal to its number of angles.

The table opposite shows the names given to the first 10 polygons. Note that an undecagon is rarely used.

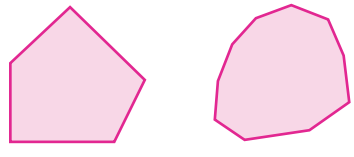
| Number of sides | Polygon name |
|---|---|
| 3 | Triangle |
| 4 | Quadrilateral |
| 5 | Pentagon |
| 6 | Hexagon |
| 7 | Heptagon |
| 8 | Octagon |
| 9 | Nonagon |
| 10 | Decagon |
| 11 | Undecagon |
| 12 | Dodecagon |

A **regular polygon** has all sides of equal length and all angles of equal size. The number of sides gives the name of the polygon.

You are probably familiar with the regular polygons shown below.

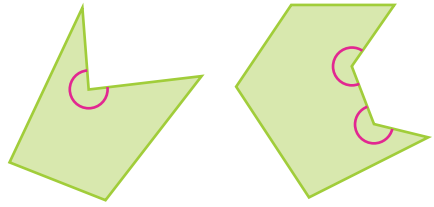


If the sides are not all equal, they are called **irregular** and the polygon is called an irregular polygon.



## Concave and convex polygons

All of the polygons shown so far have been **convex** polygons, as they contain no interior angles greater than 180°. Opposite are examples of **concave** polygons which always contain at least one internal angle greater than 180°. (The sides cave in.)



# 8.5 Polygons

## Navigator

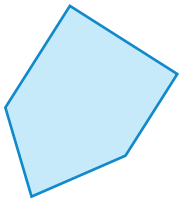
| Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7 | Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7 |
|---|---|---|

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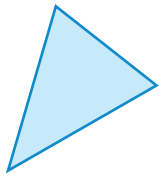
## Fluency

**1** Name each of the polygons below. State whether each one is concave or convex.

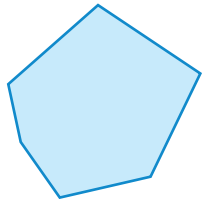
**(a)**



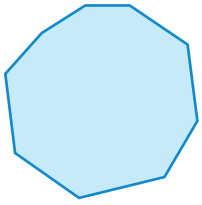
**(b)**



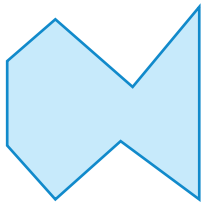
**(c)**



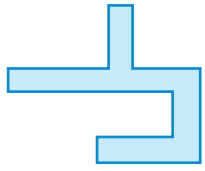
**(d)**



**(e)**



**(f)**



**(g)**



**(h)**

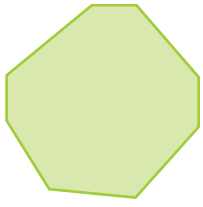


**(i)**



**2** Choose the correct answer.

**(a)** The shape opposite is:



**A** a hexagon

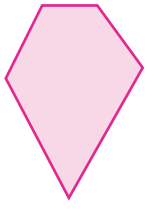
**B** a heptagon

**C** an octagon

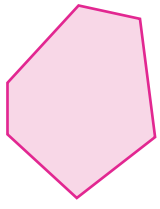
**D** a nonagon

**(b)** Which of the shapes below is a heptagon?

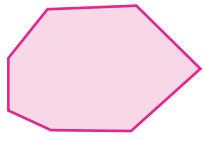
**A**



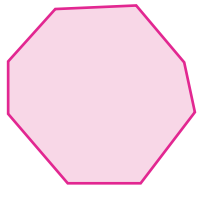
**B**



**C**



**D**



## Understanding

3 Name the type of polygon in each photograph.

(a)




(b)




(c)



(d)



## Reasoning

4 What is the greatest number of right angles that a pentagon can have? Demonstrate your answer.

5 **(a)** If a quadrilateral can have two sets of parallel sides, how many sets of parallel sides can a hexagon have?

**(b)** Based on your results from part **(a)**, how many pairs of parallel sides can an octagon and a decagon have? Explain any pattern you have found.

**(c)** Can this pattern be applied to a pentagon or to a heptagon? Explain why or why not.

## Open-ended

6 Using a ruler, draw any:

**(a)** concave quadrilateral **(b)** concave hexagon **(c)** concave octagon.

7 Name some common objects that contain regular polygons.

## Outside the Square Puzzle

### The hex is gone

**Equipment required:** 1 brain, a pair of scissors

This is a regular hexagon.



On loose paper, draw at least six regular hexagons that are each about a quarter of an A4 page. Cut out the hexagons and use them to assist you with the following questions.

**(a)** How could a regular hexagon be cut into two pieces which, when put together, make a parallelogram?

**(b)** How could a regular hexagon be cut into three pieces which, when put together, make a rhombus?

**(c)** How could a regular hexagon be cut into four pieces which, when put together, make two equilateral triangles?

# Investigation

## Angle sum of polygons

**Equipment required:** 1–2 brains, two pieces of paper, ruler and protractor

### The Big Question

Is there a relationship between the number of sides of a polygon and the angle sum of that polygon?

### Engage

**1** **(a)** On a blank sheet of paper, use a ruler to draw a triangle of reasonable size (about a quarter of a page).

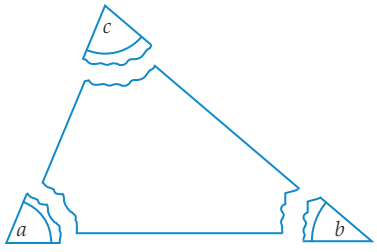
**(b)** Use a protractor to measure each angle.

**(c)** Add these angles together. Did you get 180°?

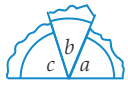
**(d)** Repeat this with a second triangle.

**2** **(a)** Draw a third triangle on a separate piece of paper and cut it out carefully.

**(b)** Tear off the angles as shown.




**(c)** Place all the vertices together with arms touching. (Did they form a straight line?)




**(d)** Paste these angles together on your page.

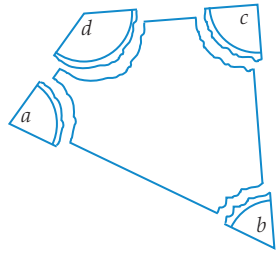
**3** **(a)** On your first sheet of paper, use a ruler to draw a quadrilateral.

**(b)** Measure each angle.

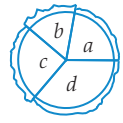
**(c)** Add these together. Did you get 360°?

**(d)** Draw another quadrilateral and cut it out carefully.

**(e)** Tear off the angles as shown.




**(f)** Place all the vertices together with arms touching. (Did they form a revolution?)




**(g)** Paste these angles together on your first page.

**(h)** Draw a second quadrilateral and divide it into two triangles.

**(i)** Calculate the angle sum of the quadrilateral.

### Explore

**4** On a blank sheet of paper, use a ruler to draw a pentagon, a hexagon, a heptagon and an octagon. Make sure that each polygon is a reasonable size (about a quarter of a page).

**5** Use your protractor to measure the angles of each shape.

**6** Find the angle sum for each shape. Did you get a multiple of 180° for each shape?

**7** Use straight lines from vertex to vertex to divide each shape into as many triangles as you can, but do not allow any of the lines to cross. Count the number of triangles in each shape.

**8** Multiply the number of triangles by 180° for each polygon.

**9** Copy the following table and enter your results.

**Strategy options**

- Make a table.
- Make a model.
- Look for a pattern.

| Polygon | Number of angles | Number of triangles | Angle sum (°) | 180° × Number of triangles (°) |
|---|---|---|---|---|
| Triangle | | | | |
| Quadrilateral | | | | |
| Pentagon | | | | |
| Hexagon | | | | |
| Heptagon | | | | |
| Octagon | | | | |

## Explain

**10** Explain how you found a relationship between the number of angles (column 1) and the number of triangles (column 2) for each shape.

**11** Explain how you found a relationship between the number of angles (or sides) of a polygon and the angle sum of that polygon.

## Elaborate

**12** Answer the Big Question by writing the rule you have found, describing carefully any of the variables you have used.

**13** Use your rule to find the angle sum of a nonagon, a decagon and a dodecagon.

**14** Draw each of these and measure each angle.

**15** Find the sum of the angles for each shape and compare your answers with your answers in Question **13**.

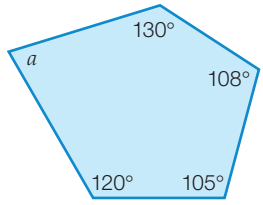
## Evaluate

**16** Did you get exactly the same answer when you measured your angles and added them as you did when you multiplied the number of triangles by 180°. Why could they be different? Which is more accurate?

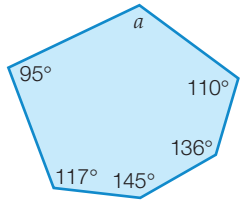
## Extend

**17** Calculate the size of the angle $a$ in each of the following polygons.

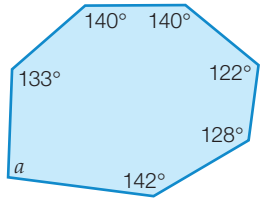
(a)




(b)



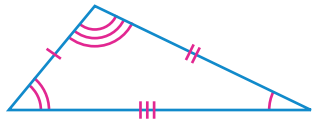

(c)



# Triangles

8.6

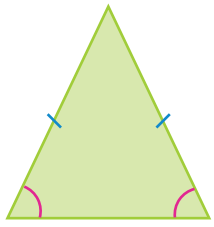
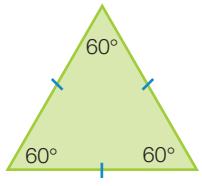
A triangle is the simplest polygon with three straight sides and three angles.



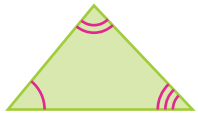
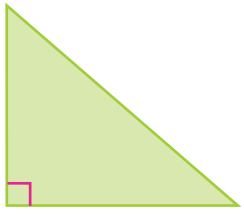
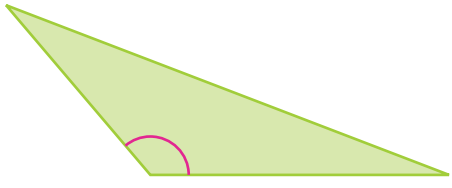
## Classifying triangles

Triangles are classified according to their sides or angles.

When using the side lengths to classify a triangle, there are three main types:

| | |
|---|---|
| **Scalene triangle**—no equal sides and no equal angles. |  |
| **Isosceles triangle**—two equal sides and two equal base angles, which are opposite the two equal sides. |  |
| **Equilateral triangle**—three equal sides and three equal angles of 60° each. |  |

When using angles to classify a triangle, the three main types are:

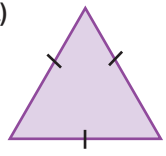
| | |
|---|---|
| **Acute-angled triangle**—all three angles are acute (less than 90°). |  |
| **Right-angled triangle**—one angle is a right angle (equal to 90°). |  |
| **Obtuse-angled triangle**—one angle is obtuse (greater than 90° but less than 180°). |  |

When the angles or lines of a shape are marked in a particular way, it means that all other angles or sides with that same marking are equal in size.

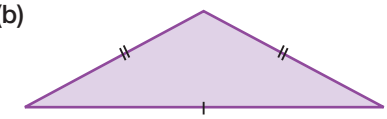
## Worked Example 6 WE6

Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

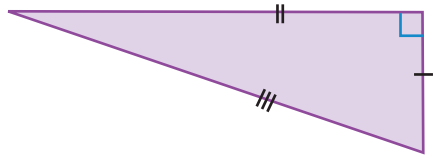
**(a)**



**(b)**



**(c)**



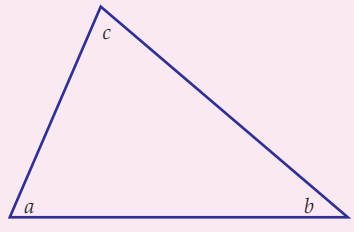
| Thinking | Working |
|---|---|
| **(a) (i)** How many sides are the same length? (three) | (a) (i) equilateral triangle |
| **(ii)** What type of angle is the largest angle? (acute) | (ii) acute-angled triangle |
| **(iii)** Combine the side and angle name. | (iii) an acute-angled equilateral triangle |
| **(b) (i)** How many sides are the same length? (two equal sides) | (b) (i) isosceles triangle |
| **(ii)** What type of angle is the largest angle? (obtuse) | (ii) obtuse-angled triangle |
| **(iii)** Combine the side and angle name. | (iii) an obtuse-angled isosceles triangle |
| **(c) (i)** How many sides are the same length? (none) | (c) (i) scalene triangle |
| **(ii)** What type of angle is the largest angle? (one right angle) | (ii) right-angled triangle |
| **(iii)** Combine the side and angle name. | (iii) a right-angled scalene triangle |

(Note: In **(a)**, it is unnecessary to describe an equilateral triangle with an angle name, as all angles must be 60° and therefore acute angles.)

## Angle sum in a triangle

In any triangle, the three angles have a sum of 180°.

$a + b + c = 180$



## Angles in a triangle using technology

We can explore the sum of the angles in a triangle using the GeoGebra geometry software package, or a CAS calculator. Here we will use GeoGebra.




Open a GeoGebra program. Deselect the axes found under the View menu, as they are not needed here, but select the Algebra View.

To construct a triangle, select the polygon tool  . Now, on the drawing pad, left click for the first point, move the cursor and left click again for the second point, repeat for the third point and return to the first point to complete the triangle. Move in an anticlockwise direction so that the interior angles will be measured.

To measure the angles, select the angle tool  and then 'poly' in the Algebra View (the panel on the LHS of the screen). All the interior angles will now be measured.



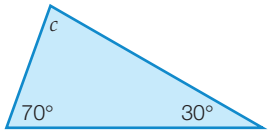

Using the input box at the bottom of the screen, select Sum from the drop-down Command list on the RHS. Use the drop-down variables list (next to Command) to enter the angle variables inside { } brackets; e.g. Sum[{α,β,γ}]. In Algebra View, you will see the sum displayed as $\delta = 180°$.

Use the select tool  to select a vertex of the triangle. Hold down the left click and move the selected point around. You will see the angle change, but the sum remains constant at 180°. Select another vertex and move it around. Repeat this procedure with the third vertex.

## Worked Example 7 WE7

Find the size of the angle labelled $c$ in each of the following triangles.

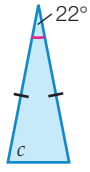
**(a)**




**(b)**




**(c)**

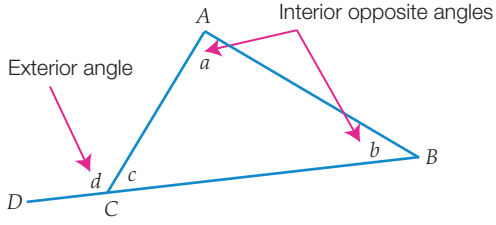



| Thinking | | Working |
|---|---|---|
| **(a)** | **1** The three angles add to 180°. Write an equation that shows this. | (a) $c + 30 + 70 = 180$ |
| | **2** Simplify the equation by adding the given angles. | $c + 100 = 180$ |
| | **3** Solve the equation (subtract 100 from both sides). | $c = 80°$ |
| **(b)** | **1** The three angles add to 180°. Write an equation that shows this. (Remember that the small square in the triangle indicates a 90° angle.) | (b) $c + 90 + 16 = 180$ |
| | **2** Simplify the equation by adding the given angles. | $c + 106 = 180$ |
| | **3** Solve the equation (subtract 106 from both sides). | $c = 74°$ |

| | | |
|---|---|---|
| (c) 1 | The three angles add to 180°. Write an equation that shows this. (Because this is an isosceles triangle, the two angles opposite the equal sides are equal.) | (c) $c + c + 22 = 180$ |
| 2 | Simplify the equation by adding like terms. | $2c + 22 = 180$ |
| 3 | Solve the equation (subtract 22 from both sides, then divide by 2). | $2c = 158$<br>$c = 79°$ |

## Exterior angle of a triangle

An exterior angle of a triangle is formed on the outside of a triangle when one of the sides is extended.




In the diagram, $\angle d$ is the exterior angle.

There can be six different exterior angles formed on any triangle. Can you find them all? What relationship does the exterior angle have to the interior angles of a triangle?

We know that $\angle a + \angle b + \angle c = 180°$ (angles of a triangle add to 180°)

$\angle a + \angle b + \angle c - \angle c = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle a + \angle b = 180° - \angle c$

$\angle c + \angle d = 180°$ (angles on a straight line add to 180°)

$\angle c - \angle c + \angle d = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle d = 180° - \angle c$

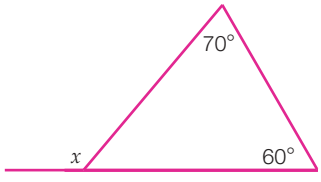
Therefore, $\angle a + \angle b + = \angle d$ (both equal to $180° - \angle c$)

The exterior angle of a triangle is equal to the sum of the two interior opposite angles in the triangle.

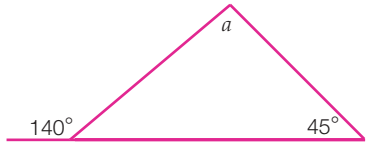
## Worked Example 8 WE8

Calculate the value of the pronumeral in each of the following.

**(a)**




**(b)**



| Thinking | Working |
|---|---|
| **(a)** As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the exterior angle. | (a) $x = 60 + 70$<br>$x = 130°$ |
| **(b)** As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the unknown angle. | (b) $140 = a + 45$<br>$a = 140 - 45$<br>$a = 95°$ |

# 8.6 Triangles

## Navigator

| | | |
|---|---|---|
| Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14, Q15, Q17, Q18, Q19 | Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19 | Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19 |

Answers page 671

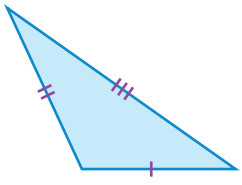
**Equipment required:** Protractor and ruler for Question **4**

## Fluency

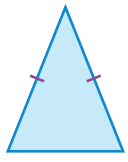
**1** Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

WE6

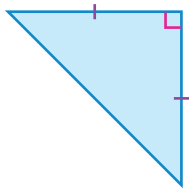
**(a)**



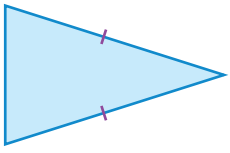
**(b)**



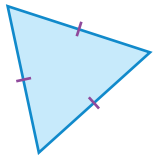
**(c)**



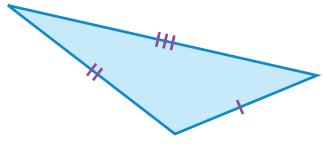
**(d)**



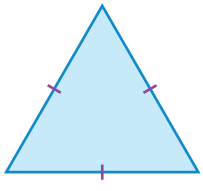
**(e)**



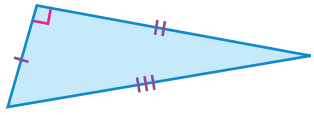
**(f)**



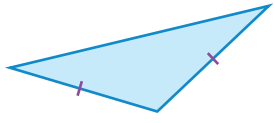
**(g)**



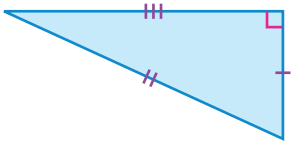
**(h)**



**(i)**



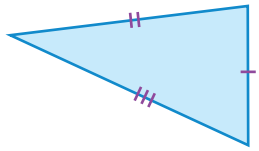
**(j)**



**(k)**

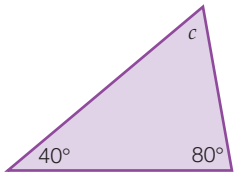


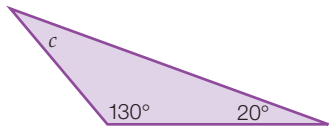
**(l)**

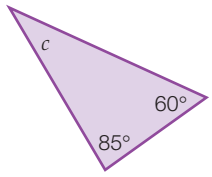


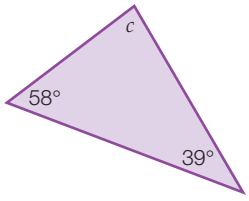
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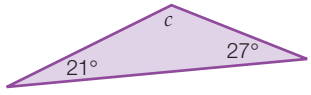
2 Find the size of the angle labelled $c$ in each of the following triangles.

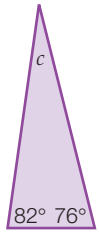
(a)



(b)



(c)


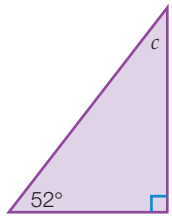

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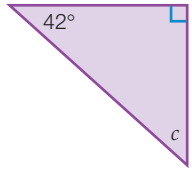

(e)



(f)



(g)

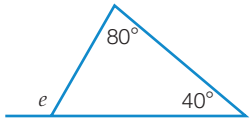


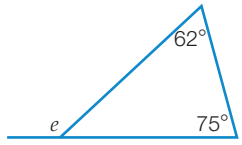
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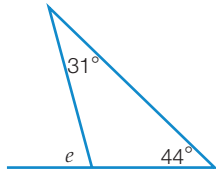

(i)


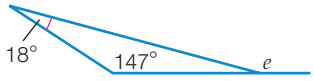

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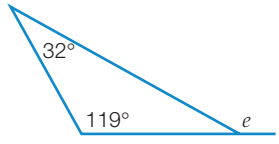
3 Calculate the value of the pronumeral in each of the following.

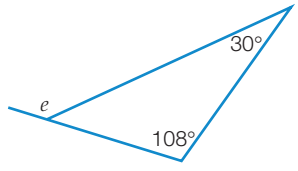
(a) (i)


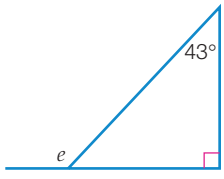

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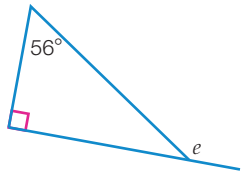

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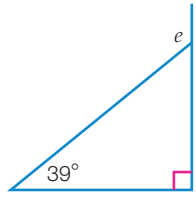

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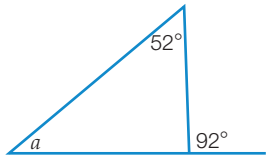

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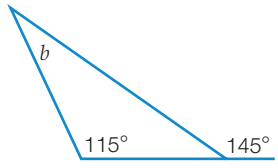

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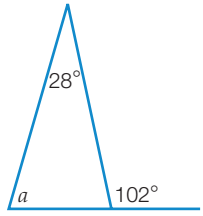

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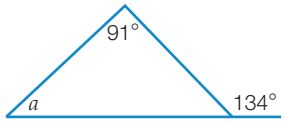

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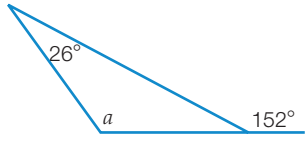

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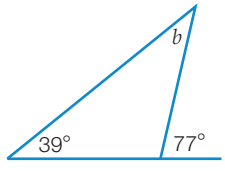

(b) (i)



(ii)



(iii)



(iv)


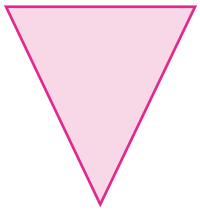

(v)



(vi)


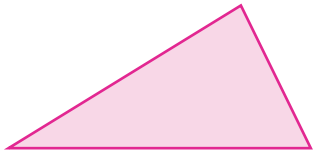
## Understanding

**4** Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one. Give the angle names as well, using a protractor if necessary to check the angle types.

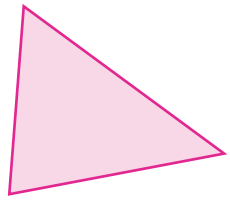
**(a)**



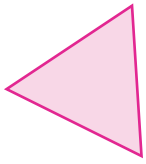
**(b)**



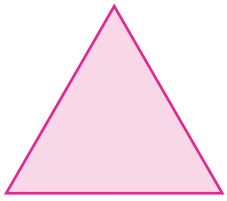
**(c)**



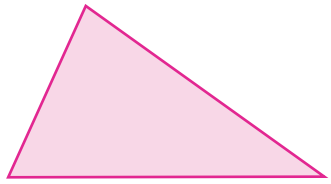
**(d)**



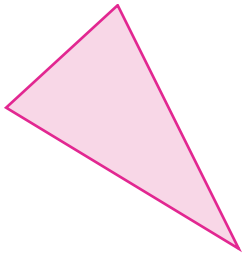
**(e)**



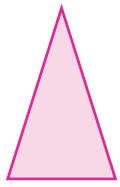
**(f)**



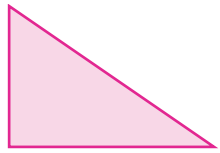
**(g)**



**(h)**



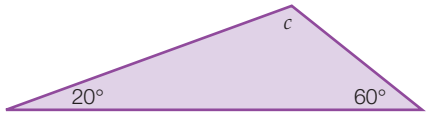
**(i)**



**5 (a)** The size of angle $c$ is:

**A** 20° **B** 60°

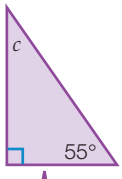
**C** 80° **D** 100°




**(b)** The size of angle $c$ is:

**A** 35° **B** 45°

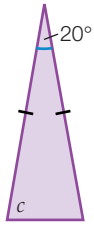
**C** 55° **D** 125°



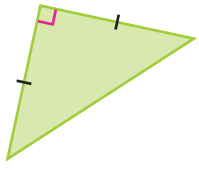

**(c)** The size of angle $c$ is:

**A** 20° **B** 80°

**C** 90° **D** 160°




**6** The triangle shown is best described as:



**A** isosceles, acute-angled

**B** isosceles, right-angled

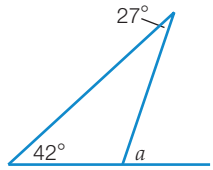
**C** scalene, acute-angled

**D** scalene, right-angled

**7** **(a)** The value of the pronumeral is:

**A** 15° **B** 63°

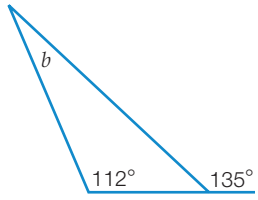
**C** 69° **D** 111°




**(b)** The value of the pronumeral is:

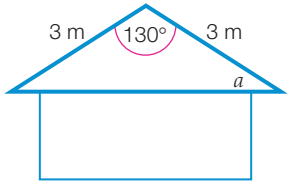
**A** 23° **B** 36°

**C** 42° **D** 68°

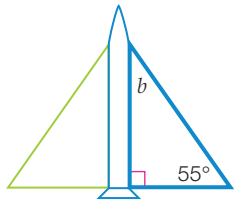



**8** Name the type of triangle shown in bold in each diagram below. (Side or angle names may be used.) Find the value of any unknown angles marked with a pronumeral.

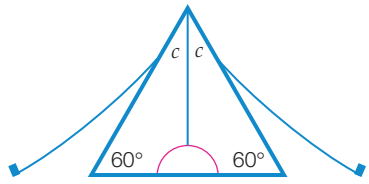
**(a)**



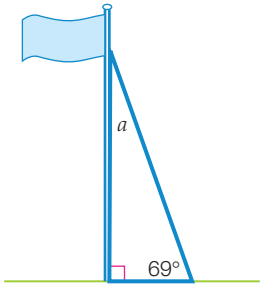

**(b)**



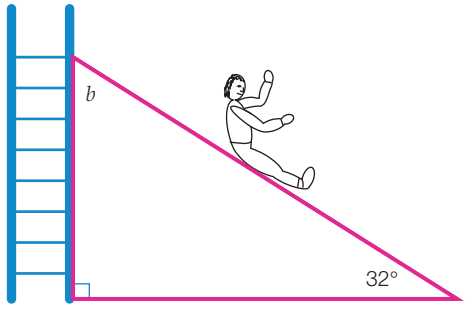

**(c)**



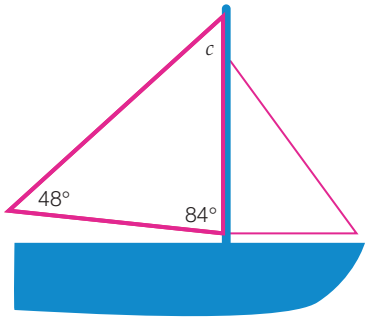

**(d)**



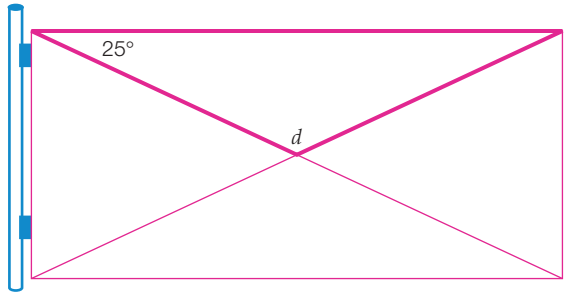

**(e)**




**(f)**




**(g)**




**9** The diagonal of a square cuts the square into two triangles. Give the side and angle name of the triangles formed.

**10** Two equilateral triangles are placed next to each other so that two sides are joined along their whole length to form a quadrilateral.

**(a)** What can you say about the side lengths of the quadrilateral (four-sided shape) formed?

**(b)** What will be the size of each of the angles in the quadrilateral?

## Reasoning

**11** Is it possible to have a triangle with more than one obtuse angle? Explain by drawing a diagram.

**12** What is the minimum number of acute angles a triangle must have?

**13** Give the side and angle names of each of the following triangles, with reasons why.

**(a)** a triangle with one angle measuring 36° and another angle measuring 54°

**(b)** a triangle with one angle measuring 28° and another angle measuring 124°

**(c)** a triangle with two angles measuring 60° each

**(d)** a triangle with one angle measuring 12° and another angle measuring 12°

**14** Explain why the following triangles are not possible.

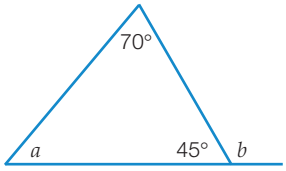
**(a)** a triangle with angles 55°, 65° and 70°

**(b)** an equilateral triangle with three 55° angles

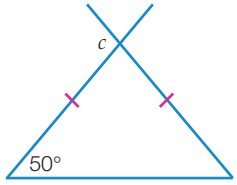
**(c)** isosceles triangle with 45°, 55° and 80° angles

**15** Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

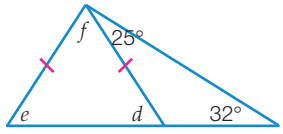
**(a)**



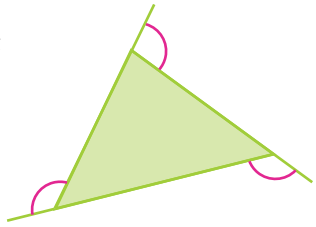

**(b)**




**(c)**




**16** Find the sum of the marked angles in the diagram on the right without measuring the angles.



## Open-ended

**17** Draw three different obtuse-angled, scalene triangles.

**18** Draw an example of:

**(a)** an acute-angled scalene triangle

**(b)** an acute-angled isosceles triangle.

**19** Draw three different triangles with an exterior angle between 80° and 100°.

## Outside the Square Puzzle

### Eggs, spoons and turtles

**1** Jane, Scotty, Jessica, Bruce and Michael all took part in their school egg-and-spoon race. Find the order in which they finished the race.

- Bruce finished the race in between the two girls.
- The winner had six letters in their name.
- Jane finished faster than Michael and slower than Jessica.

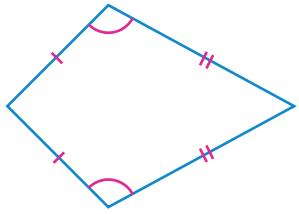
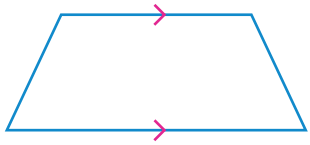
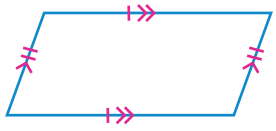
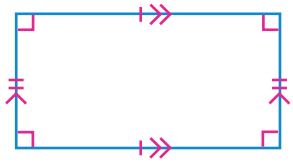
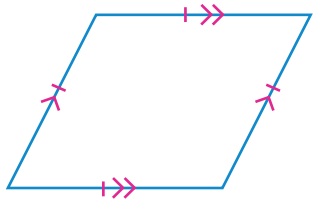
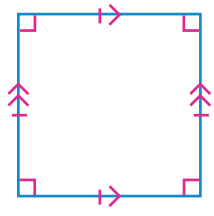
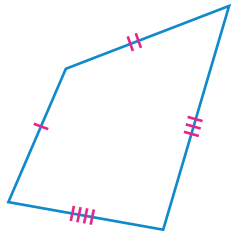
**2** Four students and their turtles took part in the annual School Turtle Championships. Match the students to their turtles, and find the order in which they finished.

- Chloe owns Sandy, who didn't finish in last place.
- Percy finished immediately in front of Vicky's turtle.
- Myrtle finished behind Ernest, who is owned by Vicky.
- Ethan was celebrating as his turtle finished two places higher than Warren's.

# 8.7 Quadrilaterals

A plane shape with four straight sides is called a **quadrilateral**. We name quadrilaterals by looking at the properties of their sides and angles.

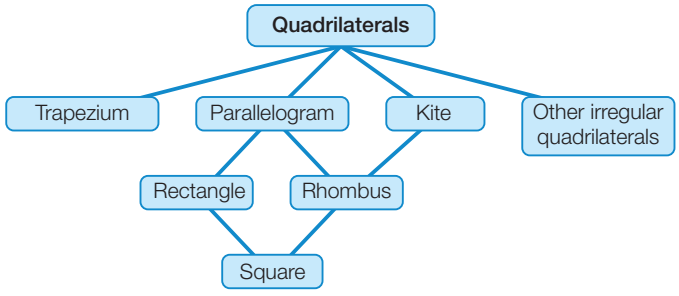
## Quadrilaterals

| Quadrilateral | Description |
|---|---|
| **Kite**  | Two pairs of equal adjacent sides (sides next to each other) and one pair of equal opposite angles. |
| **Trapezium**  | Only one pair of parallel sides. |
| **Parallelogram**  | Two pairs of parallel sides. Opposite sides and angles are equal. |
| **Rectangle**  | Two pairs of opposite sides equal and all internal angles are right angles (equal to 90°). |
| **Rhombus**  | All four sides equal; opposite angles are equal.<br>(This can be thought of as a parallelogram with all sides equal or a 'pushed over' square.) |
| **Square**  | All sides equal and all internal angles are right angles (equal to 90°). |
| **Irregular quadrilateral**  | A four-sided plane shape that has no special qualities. |

Parallel side pairs are indicated by marking the side with the same number of arrows in the same direction, as shown.



A quadrilateral may have more than one correct name. For example, a square may also be called a rectangle, because it has all the angle and side properties of a rectangle (two pairs of parallel sides and 90° angles). It could also be called a rhombus, because its pairs of opposite sides are equal and its opposite angles are equal.




### Angle sum of a quadrilateral

The sum of the angles in a quadrilateral is 360°.

### Angles in a quadrilateral using technology

Use GeoGebra to construct a quadrilateral with the polygon tool as you did to construct a triangle (p. 459). Measure and sum the interior angles. This time, the angle sum will be 360°. Once again, use the select tool to move all four points one by one. The angles will change but the angle sum stays constant at 360°.




**Angle properties of special quadrilaterals**

The diagonally opposite angles of a parallelogram are equal.

The two interior opposite angles of a kite where unequal sides meet are equal.

# 8.7 Quadrilaterals

## Navigator

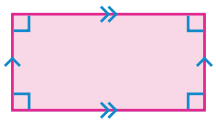
| | | |
|---|---|---|
| Q1, Q2, Q3, Q4 Column 1, Q5, Q6 Column 1, Q7, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q17, Q20 | Q1, Q2, Q3, Q4 Column 2, Q5, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20 | Q1, Q2, Q3, Q4 Column 3, Q5, Q6 Column 3, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21 |

Answers page 672

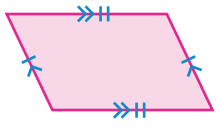
## Fluency

**1** Give the most accurate name for each shape below.

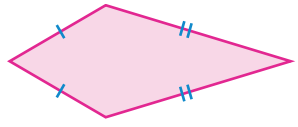
(a)



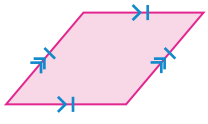
(b)



(c)



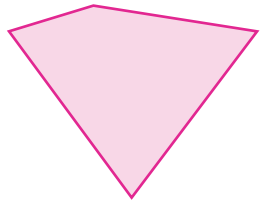
(d)



(e)



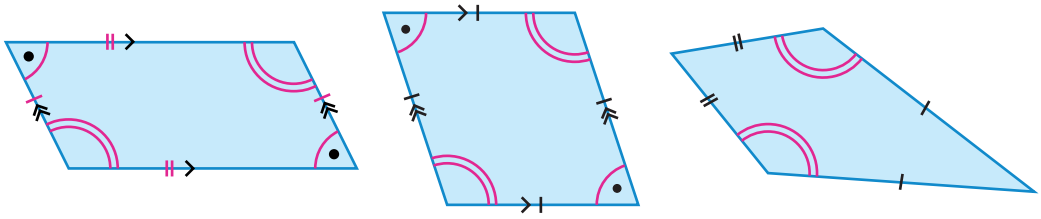
(f)



**2** A quadrilateral has both pairs of opposite sides parallel and equal in length. We can be sure this quadrilateral is a:

**A** parallelogram **B** rhombus **C** rectangle **D** square

**3** State TRUE or FALSE for each of the following, referring to the three quadrilaterals below.



**(a)** The opposite angles in a parallelogram are equal.

**(b)** All four angles in a parallelogram are equal.

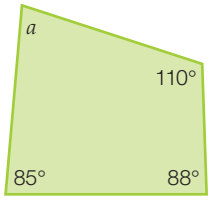
**(c)** The opposite angles in a rhombus are equal.

**(d)** All four angles in a rhombus are equal.

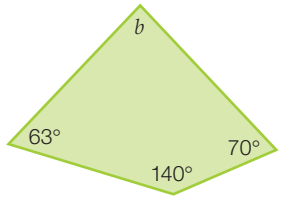
**(e)** One pair of opposite angles in a kite are equal.

**4** Find the size of the unknown angle represented by a letter in each of the following quadrilaterals.

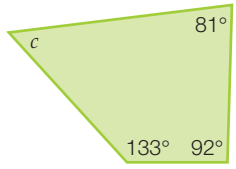
**(a)**



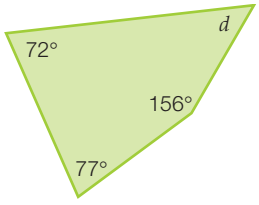

**(b)**



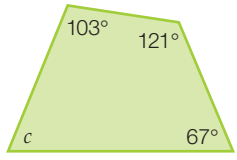

**(c)**



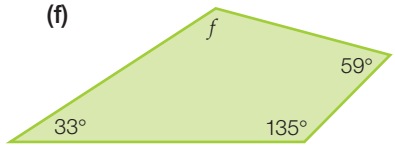

**(d)**



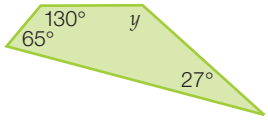

**(e)**



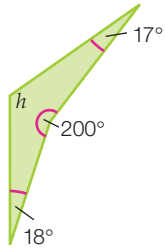

**(f)**



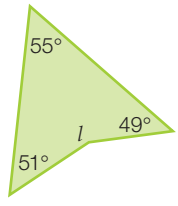

**(g)**




**(h)**



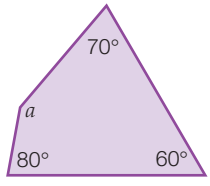

**(i)**




**5 (a)** The size of angle $a$ shown is:

**A** 70° **B** 90°

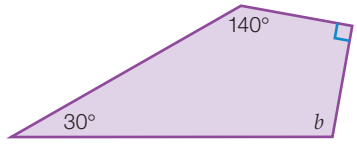
**C** 100° **D** 150°




**(b)** The size of angle $b$ shown is:

**A** 10° **B** 90°

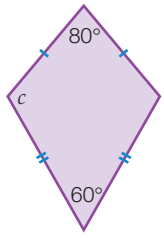
**C** 100° **D** 170°




**(c)** The size of angle $c$ shown is:

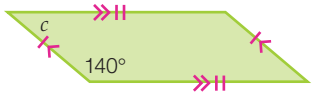
**A** 55° **B** 80°

**C** 110° **D** 140°

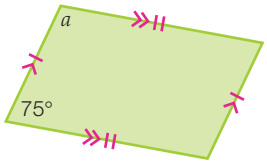


**6** Find the size of the angle labelled with a letter in each quadrilateral.

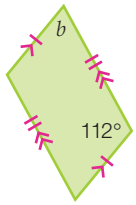
(a)



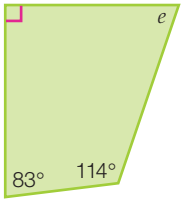

(b)



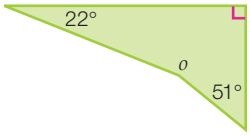

(c)



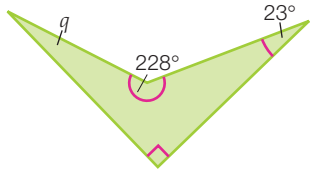

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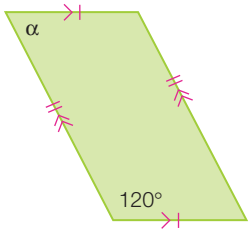

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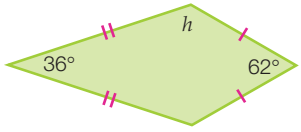

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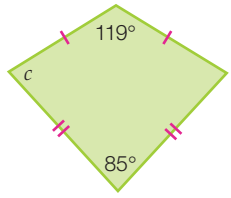

(g)




(h)




(i)



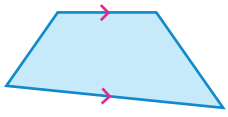

## Understanding

**7** The diagrams below were created by a tired illustrator. What type of shape do you think he meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines are correct.

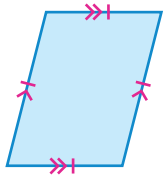
For Question **7**, make sure you do the markings for things such as equal sides, parallel sides and right angles.



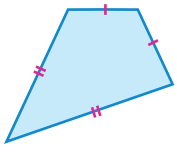
(a)



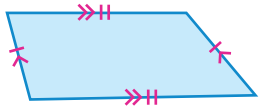
(b)



(c)

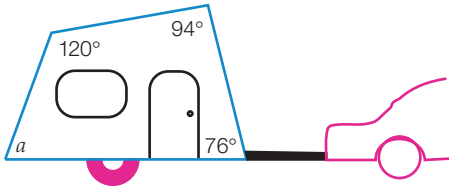


(d)

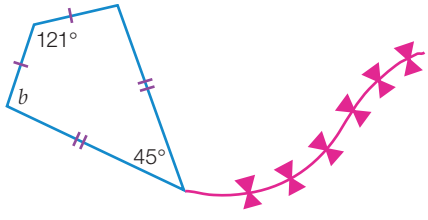


**8** Find the angle represented by the pronumeral in each quadrilateral below. Give reasons to justify your answer.

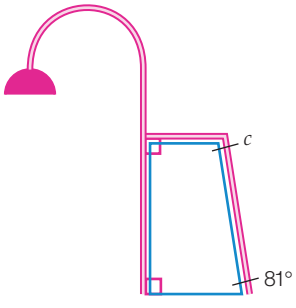
(a)



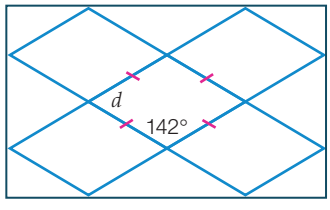

(b)




(c)




(d)



9 An angle in a parallelogram measures 40°. Draw a diagram and mark in the size of each of the other three angles.

10 A quadrilateral has one 45° angle and three unknown angles that are equal to each other. What is the size of each of the three unknown angles?

## Reasoning

11 All rhombuses are parallelograms, but not all parallelograms are rhombuses. Explain.

12 Is a parallelogram a type of trapezium? Explain.

13 All rectangles are parallelograms, but not all parallelograms are rectangles. Is this true? Explain why/why not.

14 What is the maximum number of:

**(a)** obtuse angles a quadrilateral can have? Confirm by drawing a diagram.

**(b)** reflex angles a quadrilateral can have? Confirm by drawing a diagram.

15 A quadrilateral has two angles of 70° and 34°. The other two angles are equal to each other.

**(a)** What is the size of the two unknown angles?

**(b)** What type of quadrilateral could this shape be?

16 Can a quadrilateral have the angles 35°, 85°, 130° and 120°? Explain your answer.

17 What properties of a square make it also classifiable as:

**(a)** a rectangle **(b)** a rhombus **(c)** a parallelogram?

18 Sam is looking at a kite, which has two given angles of 50° and 120°, but it is unclear which angle is which and where they are located on the kite. He thinks that the best way to solve a problem is to draw a diagram. Help Sam by drawing three different kites that could contain the two known angles of 50° and 120°.

## Open-ended

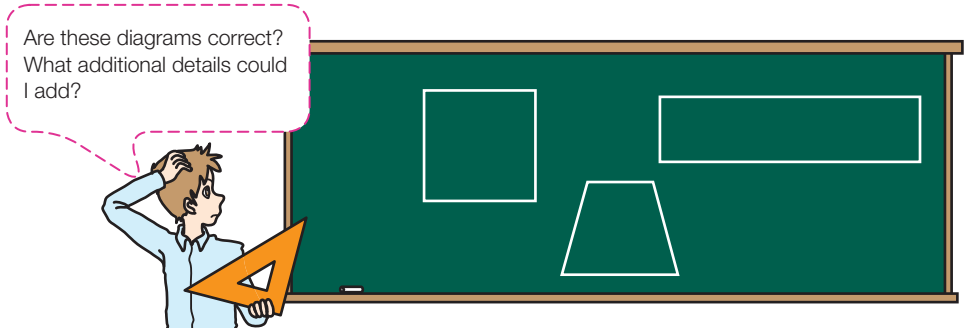
19 A quadrilateral has two angles equal to 80°.

**(a)** Name one type of quadrilateral that this could be and draw it, marking in the 80° angles.

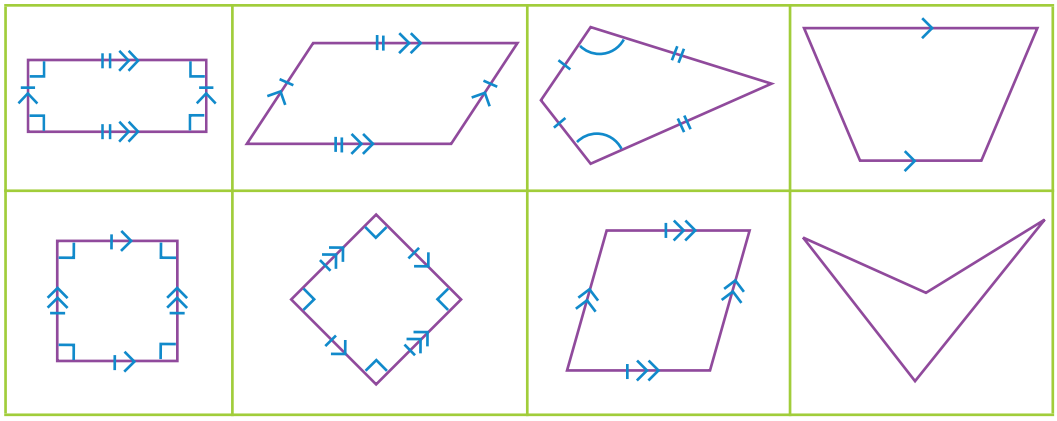
**(b)** How many different types of quadrilaterals can have two angles equal to 80°? List the different types.

**(c)** Draw an example of each, marking in the 80° angles.

20 Jack is asked by his teacher to draw a square, a rectangle and a trapezium. Do Jack's diagrams include enough information to clearly depict a square, a rectangle and a trapezium? Redraw his diagrams, adding any missing details.



**21** Divide these quadrilaterals into two groups. Explain the reason(s) you used to divide them into each group.

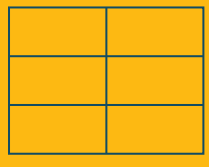


# Outside the Square

## Problem solving

### Quadrilateral quandaries

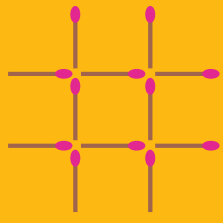
**1** How many rectangles can you count here altogether? (Include rectangles tha are the same size even if they are drawn in different positions.)



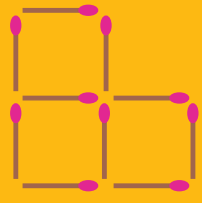
**2** Find how many different size squares can be drawn on the grid shown.



**3** Move three matchsticks in the arrangement shown to create three squares that touch each other.



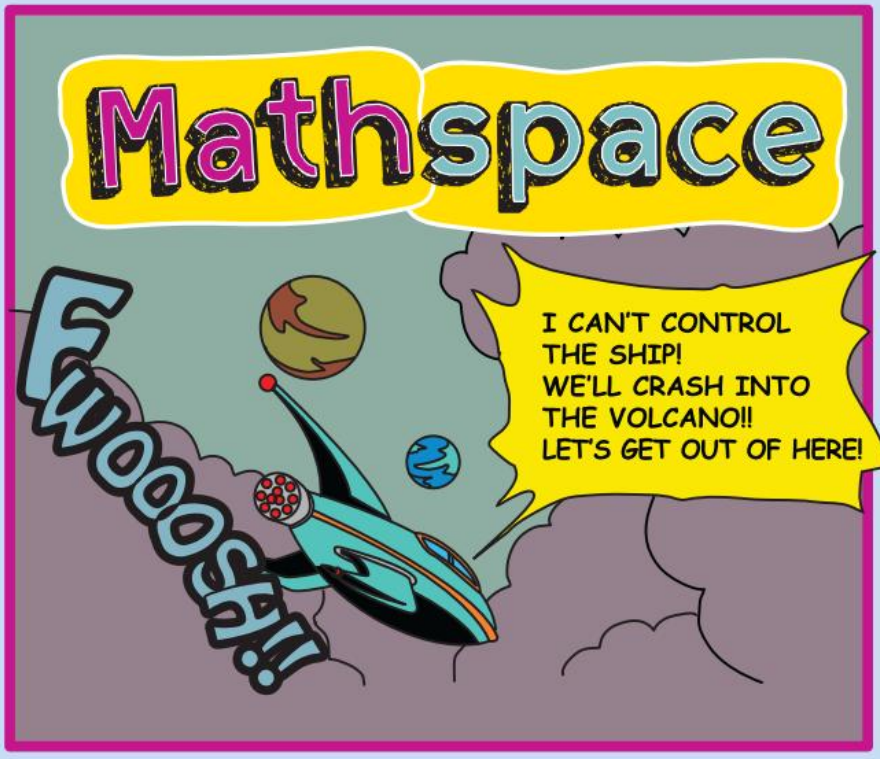
**4** Remove one match and re-position three others to form one square and two parallelograms.

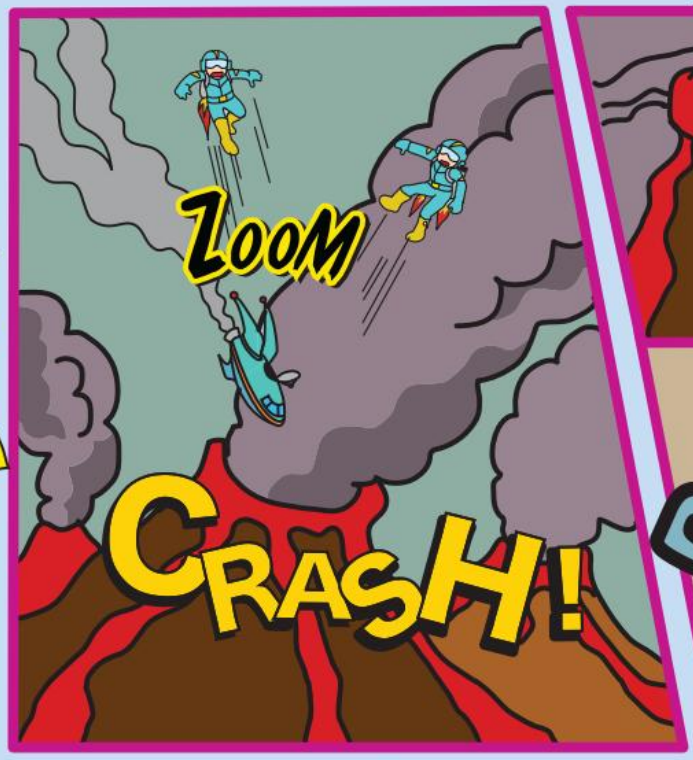





**Strategy options**

- Draw a diagram.
- Look for a pattern.




# PLANET POLYGON

While exploring the universe, you and your co-pilot Violet enter the atmosphere of the strange planet Polygon. Your ship malfunctions, and to avoid certain death you both eject out of the ship. On landing you are knocked unconscious. When you wake, you look around—where is Violet and why is your spacecraft crumpled?

You find a note on your PCD (Personal Communication Device).

## Message #1: 2.15 p.m. Alien code

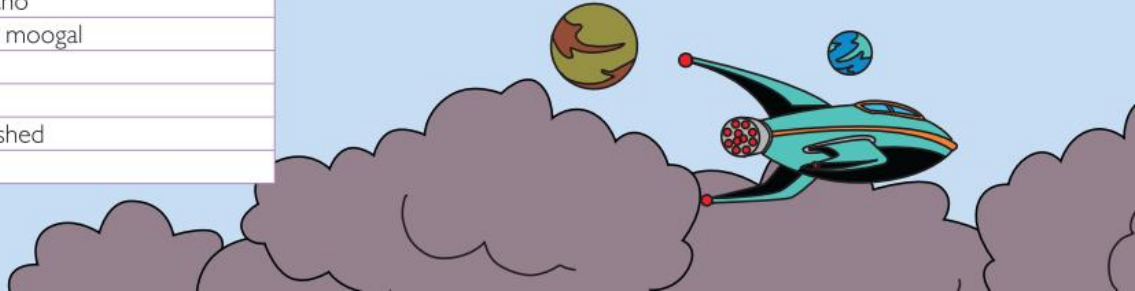
I couldn't find you when I woke. I've met a group of aliens who seem to call themselves Polygons. They don't have a verbal language but communicate instead with shapes. Through our conversation I think I have managed to decode some of their language.

| Shape | Possible meaning |
|---|---|
| square | catapulted |
| kite | flying thing |
| isosceles triangle | volcano |
| rhombus | large moogal |
| rectangle | paw |
| trapezium | into |
| polygon | squashed |
| scalene triangle | hit |

I asked them what happened to our ship, they answered, and I have described the shapes. Here is approximately what they said.

- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only
- quadrilateral with 1 pair of parallel sides only
- 3-sided shape with 2 equal angles
- special rectangle with equal side lengths
- 3-sided shape with 3 different side lengths
- parallelogram that has all equal sides, but is not a square
- 2 pairs of parallel sides, with 90 degree angles (but is not a square)
- hexagonal-looking shape
- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only

**1** Decode what the aliens said happened to the ship.

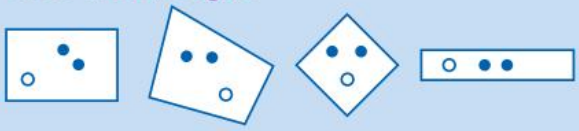




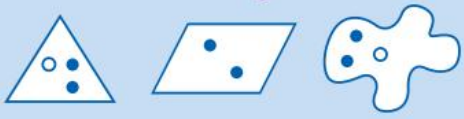

## Message #2: 3.15 p.m.
## Alien intelligence test

The Polygons are willing to give us the necessary materials to fix our spaceship, but we both have to pass their test. Good luck, from Violet.

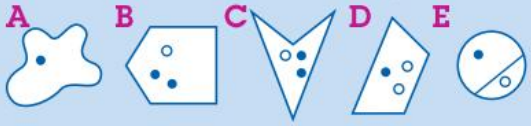
**2 (a)** These are Kwoogals:



These are not Kwoogals:

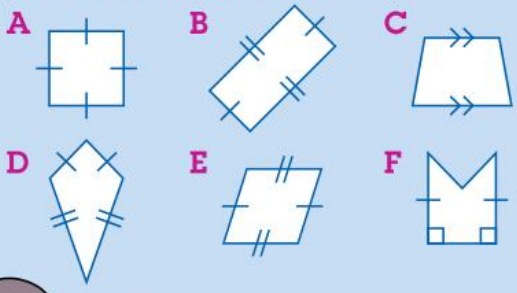


**(i)** Which one of these is a Kwoogal?




**(ii)** Write the definition of a Kwoogal.

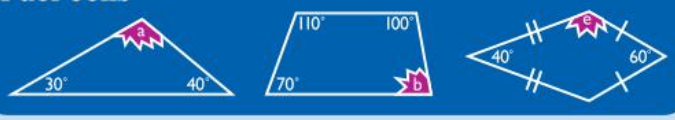
**(b)** Which of these are parallelograms? (More than one answer is okay.)




## Message #3: 4.18 p.m.
## Spaceship blueprints

I've found the blueprints for our spaceship but some of the angles have been burnt in the crash. Can you calculate what the missing angles should be?

**Fuel cells**




**Neutron absorber**




**Black hole combustion unit**




### What is a moogal?

**Use your answers for the spaceship blueprint message to solve the code below.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 110° | | 70° | 100° | 110° | 25° | 25° | 155° |

| | | | | |
|---|---|---|---|---|
| 80° | 130° | 110° | 70° | 335° |

# 8.8 Compass constructions

Using a ruler, a pencil and a compass, we can construct many angles and shapes without measuring any angles with a protractor. Constructions may also be made using a CAS calculator or computer geometry software.

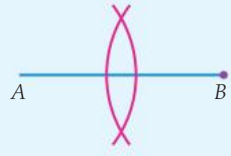
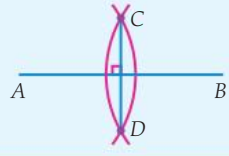
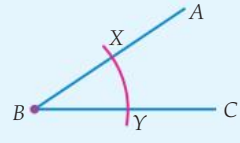
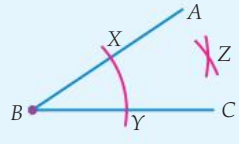
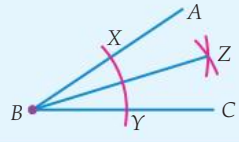
Most constructions are based on two basic techniques: bisecting a line and bisecting an angle. (Bisect means to divide into halves.) A **perpendicular bisector** bisects a line at right angles. We use a sharp pencil for drawing lines and for use in our compass, and we leave all construction lines on our diagrams.

Unless otherwise instructed, lines and compass openings should be between 5 and 8 cm long and angles between 40° and 50°.

## Worked Example 9 WE9

Using only a compass, a ruler and a pencil:

**(a)** bisect a line **(b)** bisect an angle.

| Thinking | Working |
|---|---|
| **(a) 1** Use your ruler to draw a line $\overline{AB}$. Place the point of the compass on point $A$ so that the compass opening is more than half the length of the line and draw an arc above and below the line. | *(a)*   |
| **2** Keeping the compass at the same opening, place your compass point on point $B$ and draw an arc to cut the first arc above and below the line. |   |
| **3** With your ruler join the points $CD$ where the arcs cross. As this line crosses $AB$ at right angles, it is the perpendicular bisector of the line. |   |
| **(b) 1** Draw an acute angle. Place the point of the compass on the vertex of $\angle ABC$ and draw an arc to cut both arms of your angle at $X$ and $Y$. | *(b)*   |
| **2** Keeping the compass at the same opening, place your compass point on point $X$ and draw an arc. Repeat this using point $Y$ so that the arcs cross. |   |
| **3** Using a ruler, draw a line connecting the vertex of $\angle ABC$ with $Z$, the point of intersection of the arcs. This line $BZ$ bisects the angle (cuts the angle in half). |   |

An equilateral triangle not only has three equal sides but also has three equal angles of 60°. A square has four equal sides and all angles are right angles, so we can use our previous constructions to help us construct a square.

## Worked Example 10 WE10

Using only a compass, a ruler and a pencil:

**(a)** construct an equilateral triangle **(b)** construct a square.

### Thinking

**(a) 1** Use your ruler to draw a line $\overline{AB}$. Open your compass to the length of line *AB*. Place the point of the compass on point *A* and draw an arc above the line. Place the point of the compass on point *B* and draw another arc above the line so that the arcs cross at point *C*.

**2** Using a ruler, join points *A* and *B* to point *C*. $\Delta ABC$ is an equilateral triangle. Mark all sides as equal and all angles as 60°.

**(b) 1** Construct a right angle on a horizontal line *AB* by constructing the perpendicular bisector of the line.

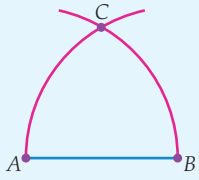
**2** Place your compass point on point *X*, the point of intersection of *AB* and the perpendicular bisector. With the compass opened to length *BX*, cut the perpendicular bisector at point *C*.

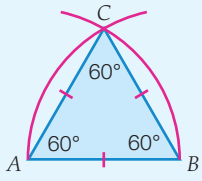
**3** Keeping the compass opening the same, place the compass point on point *B* and make an arc above the line. Place the compass point on point *C* and make an arc to cut the last arc at *D*.

**4** Join *B* to *D* and the point *C* to point *D*. *BXCD* is a square.

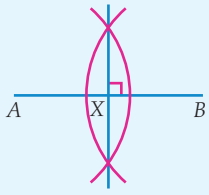
### Working

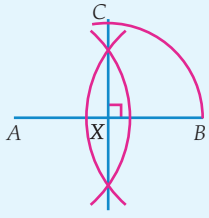
*(a)*

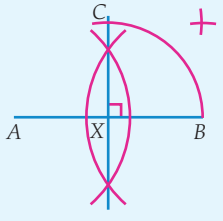


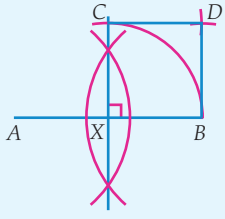




*(b)*



# Worked Example 11

WE11

Using only a compass, a ruler and a pencil:

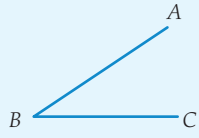
**(a)** construct an angle equal to another angle **(b)** construct a pair of parallel lines.

## Thinking

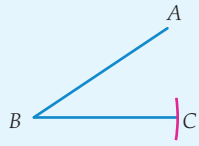
## Working

**(a) 1** Draw an acute $\angle ABC$.

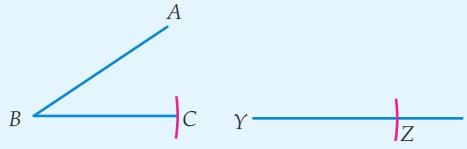
(a)



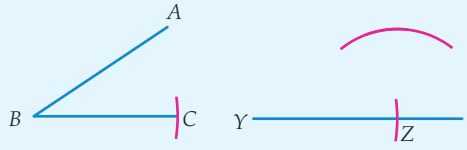

**2** Use a compass and, with the compass point on point $B$, draw an arc through point $C$.



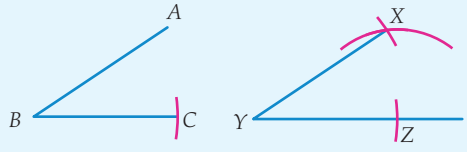

**3** Draw a line segment and label one end point $Y$. Without changing the opening on your compass and, with the compass point on point $Y$, draw an arc to cut the line segment at $Z$.




**4** Place the compass point on point $C$ and alter the compass opening so that it measures the length $\overline{CA}$. Place the compass point on point $Z$ and draw an arc above $\overline{YZ}$.

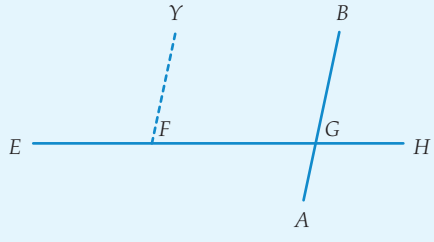



**5** Place the compass point on point $B$ and alter the compass opening so that it measures the length $\overline{BA}$. Place the compass point on point $Y$ and draw an arc to cut the previous arc at $X$. Join $\overline{XY}$.



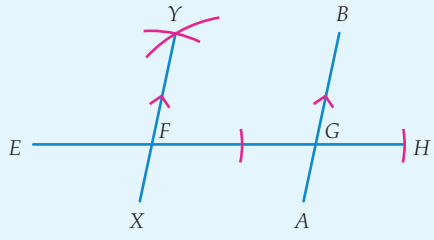

**(b) 1** Draw a line segment $\overline{AB}$. Draw another line $\overline{EH}$ to intersect with $\overline{AB}$. Label the point of intersection $G$. Choose a point $F$ between $E$ and $G$. Using the instructions in part **(a)**, construct acute $\angle YFG$ equal to acute $\angle BGH$.

(b)




**2** Draw a line segment through $\overline{YF}$ and extend to a point $X$.

$\overline{XY}$ will be parallel to $\overline{AB}$, as you have constructed equal corresponding angles.



# 8.8 Compass constructions

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q10, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13 |

Answers page 673

**Equipment required:** Compass, ruler, pencil

### Fluency

WE9

1 Using only a compass, a ruler and a pencil:
   **(a)** bisect a line
   **(b)** bisect an angle.

WE10

2 Using only a compass, a ruler and a pencil:
   **(a)** construct an equilateral triangle
   **(b)** construct a square.

WE11

3 Using only a compass, a ruler and a pencil:
   **(a)** construct an angle equal to another angle
   **(b)** construct a pair of parallel lines

### Understanding

4 Using only a compass, a ruler and a pencil:
   **(a)** construct a 90° angle
   **(b)** bisect this angle to make a 45° angle.

5 Using only a compass, a ruler and a pencil:
   **(a)** construct a 60° angle
   **(b)** bisect the 60° angle constructed in part **(a)** to make a 30° angle
   **(c)** bisect a line 10 cm long.

6 Using only a compass, a ruler and a pencil construct a parallelogram.

### Reasoning

7 Construct a triangle with one angle equal to 30° and another angle equal to 45° and the arm between these two angles equal to 8 cm.

8 **(a)** Construct an angle of 75°.
   **(b)** Construct an angle of 15°.
   **(c)** Construct an angle of 135°.

9 Construct an isosceles right-angled triangle where the two equal sides are 5 cm long.

10 Construct an equilateral triangle on each side of a square.

11 Use a compass to draw a circle. Select a point on the circle. Using the same compass opening, divide the circle perimeter into 6 equal pieces. Join the points of intersection in order to make a hexagon. Explain why this construction works.

12 Use a compass to draw a circle. Construct an octagon by dividing the perimeter of the circle into 8 equal pieces. Explain how your knowledge of constructing a 90° angle and bisecting an angle helped you.

### Open-ended

13 Choose any two polygon shapes and use your construction knowledge to make a design.

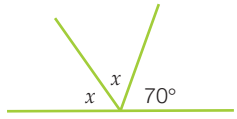
# Challenge 8

**1** The smaller angle between the hands of a clock at half past two is:

**A** 90° **B** 105° **C** 120° **D** 135°

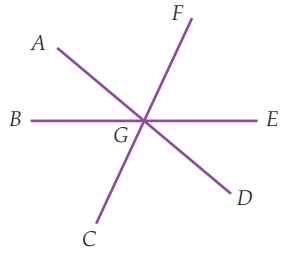
**2** Two angles are complementary and one angle is 46° more than the other. What is the size of the larger angle?

**3** Find the value of the pronumerals in the following.




**4** In the diagram, $\angle BGF = 115°$ and $\angle AGE = 140°$.

What is the size of $\angle CGD$?




**5** Two angles are supplementary and one angle is 54° more than the other. What is the size of the smaller angle?

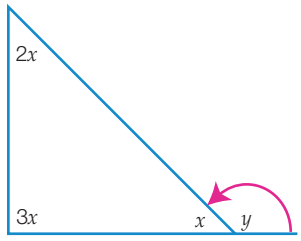
**6** In the diagram, the value of $y$ is:

**A** 30°

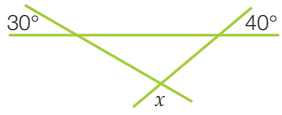
**B** 60°

**C** 90°

**D** 150°




**7** In the diagram, the value of $x$ is:




**A** 110°

**B** 70°

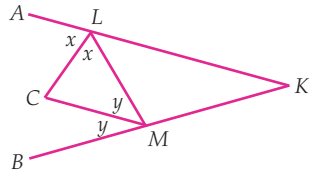
**C** 40°

**D** 30°

**8** **(a)** Shortly after 12 o'clock, the hour and minute hands of a clock form an angle of 55°. What is the time now?

**(b)** Noor observes this and leaves, returning after 40 minutes. What is the acute angle between the hands now?

**9** In the diagram, $ALK$ and $BMK$ are straight lines. The bisector of $\angle ALM$ meets the bisector of $\angle BML$ at $C$. If $\angle LCM$ is 70°, then $\angle LKM$, in degrees, is:




**A** 10°

**B** 20°

**C** 40°

**D** 110°

# Chapter review 8

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| acute angle | equilateral triangle | plane | right angle |
| acute-angled triangle | intersect | plane shapes | right-angled triangle |
| alternate angles | irregular | polygon | scalene triangle |
| angle | isosceles triangle | protractor | square |
| arms | kite | quadrilateral | straight angle |
| co-interior (allied) angles | line segment | ray | supplementary angles |
| complementary angles | obtuse angle | rectangle | transversal |
| concave | obtuse-angled triangle | reflex angle | trapezium |
| convex | parallel lines | regular polygon | vertex |
| corresponding angles | perpendicular bisector | revolution | vertically opposite angles |
| degree | perpendicular lines | rhombus | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 To measure an angle, the centre point of the _____ must match with the _____ of the angle. The lines must match with the _____ of the angle.

2 40° and 50° are _____.

3 _____ add to 180 _____s.

4 Lines drawn at right angles to each other are called _____.

5 The angle names listed in order from largest to smallest are_____, _____, _____, _____, _____ , and _____.

6 The most general name for a four-sided figure is a _____.

7 Lines in the same plane that never meet are called _____.

8 An _____ has two sides equal in length and one unequal side.

9 A closed shape with only straight line sides is called a _____.

10 A square is a _____ with right angles.

11 A _____ has three sides all of different lengths.

12 A line crossing other lines is called a _____.

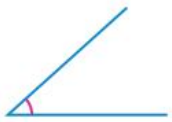
13 A _____ is a quadrilateral with one pair of parallel sides.

14 Angles that are between parallel lines and on the same side of the transversal are _____ and are _____.

**Equipment required:** Protractor for Questions **1**, **3** and **14**, and a compass for Question **13**

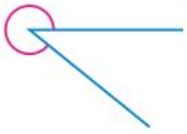
## Fluency

**1** Use a protractor to measure each of the following angles. Ex 8.1

(a) 

(b) 

(c) 

(d) 

**2** Match the following angle sizes with the drawn angles. Do not use a protractor. Ex 8.1

**A** 20° **B** 55° **C** 120° **D** 210° **E** 270° **F** 310°

(a) 

(b) 

(c) 

(d) 

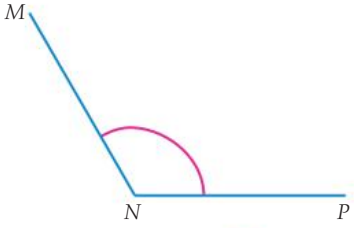
(e) 

(f) 

**3** Use a protractor and a ruler to draw an angle of each size stated. Ex 8.1

**(a)** 56° **(b)** 162° **(c)** 257° **(d)** 304°

**4** Which of the following is not true of the angle shown? Ex 8.2



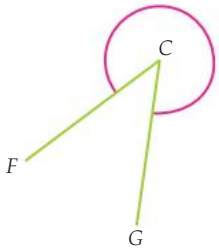

**A** It is named $\angle NPM$.

**B** It is named $\angle MNP$.

**C** It is named $\angle PNM$.

**D** It is an obtuse angle.

**5** Which of the following is true of the angle shown? Ex 8.2




**A** It is obtuse.

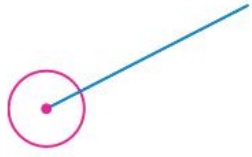
**B** It is reflex.

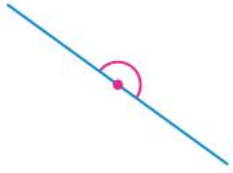
**C** It is less than 180°.

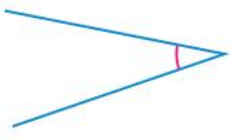
**D** It is called $\angle CFG$.

**6** State the type of each angle below (i.e. acute etc.). Ex 8.2

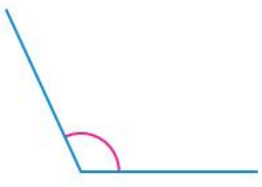
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

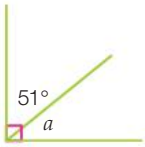
**7** Find the value of $a$ in each of the following diagrams.

Ex 8.3

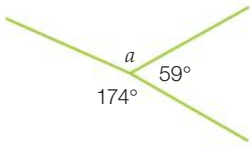
(a)



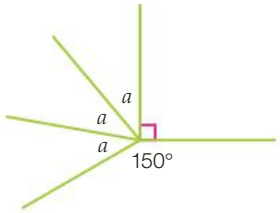

(b)



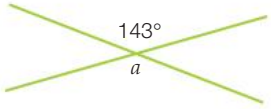

(c)



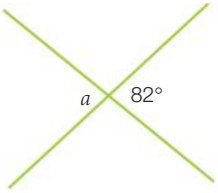

(d)




(e)



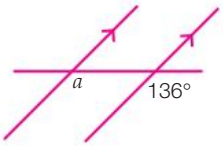

(f)



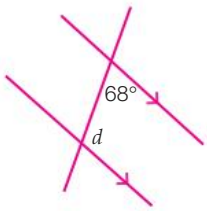

**8** Find the value of the pronumeral in the following diagrams.

Ex 8.4

(a)



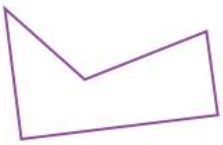

(b)



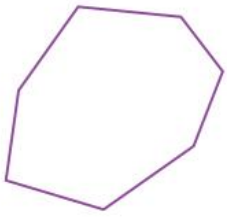

**9** Name the following polygons, also stating whether they are regular or irregular, convex or concave.

Ex 8.5

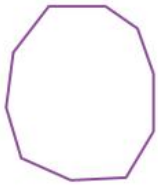
(a)



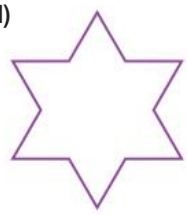
(b)



(c)

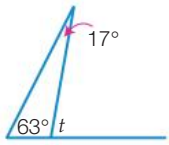


(d)



**10** Find the value of the pronumeral in the diagram.

Ex 8.6




**11** Find the unknown angle in each of the following quadrilaterals. Make sure you support your answer with reason(s).

Ex 8.7

(a)

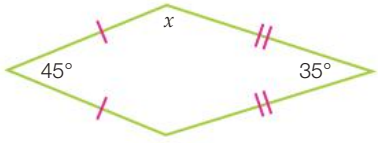



(b)




**12** Find the value of $x$ in the following quadrilateral. Make sure you support your answer with reason(s).

Ex 8.7

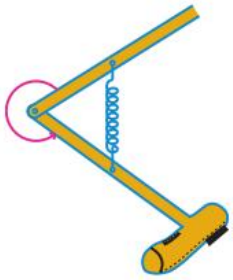



**13** Use a compass to construct a square of side length 4 cm.

Ex 8.8

## Understanding

**14** Eloise has constructed a mechanical leg to kick a football as part of her physical education investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



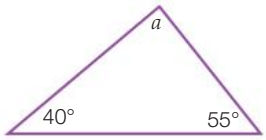
**15** One angle in a right-angled triangle is 45°.

**(a)** What is the value of the other two angles?

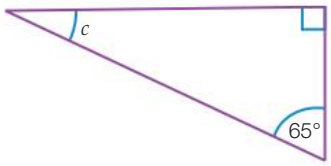
**(b)** The triangle described in **(a)** was named a right-angled triangle. Give another name for this triangle.

**16** Find the missing angle in each of the following triangles. Make sure you support your answer with reason(s).

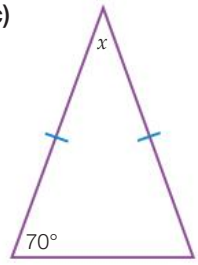
**(a)**




**(b)**

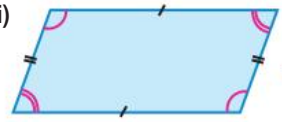



**(c)**

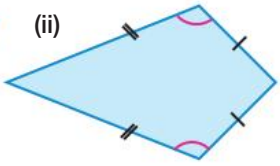



**17** Using the shapes below, answer TRUE or FALSE to the following statements.

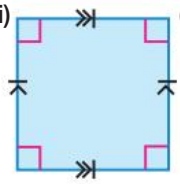
**(i)**



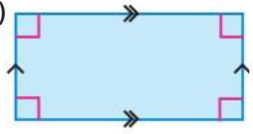
**(ii)**



**(iii)**



**(iv)**



**(a)** All four shapes are quadrilaterals.

**(b)** **(i)** and **(ii)** are kites.

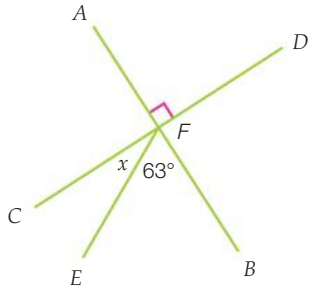
**(c)** **(i)**, **(iii)** and **(iv)** could be classified as parallelograms.

**(d)** **(i)** is a rhombus.

**(e)** **(iii)** is a rectangle.

## Reasoning

**18** Find the value of the angle *x*, giving reasons for each step.




**19** What type of angle would result from adding an acute angle and a right angle?

**20** What type of angle would result from adding a straight angle and an obtuse angle?

**21** What types of angle could result from subtracting a reflex angle from a revolution?

# NAPLAN practice 8

## Numeracy: Non-calculator

**1** Which is the largest angle?

**A**



**B**



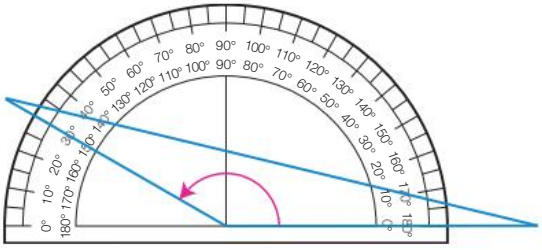
**C**



**D**



**2** What is the size of the angle in the triangle marked by the arrow?



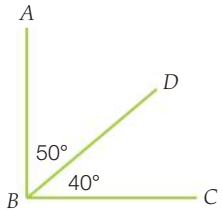
**3** $\angle ABD$ and $\angle DBC$ are best described as:

**A** vertically opposite angles

**B** supplementary angles

**C** angles at a point

**D** complementary angles.



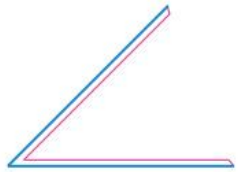

## Numeracy: Calculator allowed

**4** Which diagram shows a book opened to about 30°?

**A**



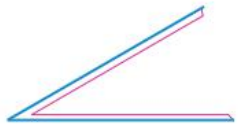
**B**



**C**

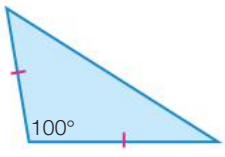


**D**

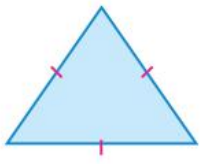


**5** Which one of these is a right-angled isosceles triangle?

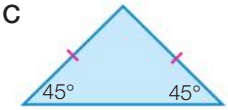
**A**



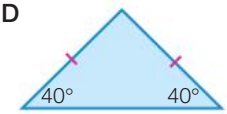

**B**



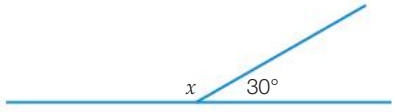
**C**




**D**




**6** What is the value of $x$ in this diagram?




**A** 30° **B** 60° **C** 120° **D** 150°

# Mixed review D

**Equipment required:** Protractor for Question **15(c)**

## Fluency

**1** Simplify:

**(a)** $2a + 3b + 4a - 7b$ **(b)** $6a + 3b - 2c - b + 5c$

Ex. 5.6

**2** Write out factor trees for each of the following numbers, then express each number as the product of its prime factors in index form.

**(a)** 45 **(b)** 24 **(c)** 630

Ex. 2.3

**3** Write the following percentages as fractions in simplest form.

**(a)** 75% **(b)** 50% **(c)** 30% **(d)** 4%

Ex. 4.7

**4** Evaluate without using a calculator:

**(a)** $1.1 \times 0.23$ **(b)** $0.24 + 3.7 - 1.503$

Ex. 4.4, 4.5

**5** Solve the following equations using algebra.

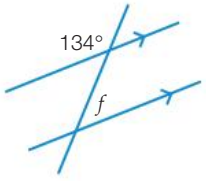
**(a)** $3x - 4 = -7$ **(b)** $\frac{x+5}{3} = -12$

Ex. 7.4

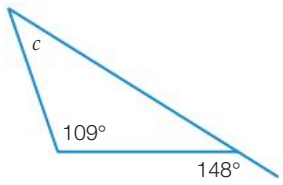
**6** Find the value of the pronumeral in the following diagrams.

Ex. 8.4, 8.6

**(a)**




**(b)**




**7** Evaluate without a calculator:

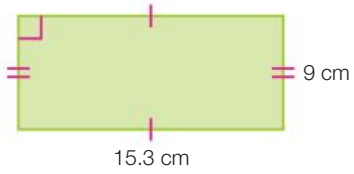
**(a)** $3\frac{1}{5} + 2\frac{1}{2} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} \times 1\frac{1}{3} \div 3\frac{1}{5}$

Ex. 3.4–3.6

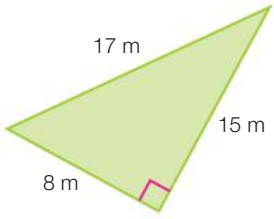
**8** Find the area of the following shapes.

Ex. 6.4, 6.5

**(a)**




**(b)**




## Understanding

**9** The table on the right shows the minimum and maximum temperatures at Frosty Hollow for 1 week in winter.

**(a)** On which day was the lowest maximum recorded?

**(b)** On which day was the highest minimum recorded?

**(c)** Which day had the greatest difference between minimum and maximum?

| Day | Minimum (°C) | Maximum (°C) |
|---|---|---|
| Monday | -4 | 5 |
| Tuesday | -8 | 0 |
| Wednesday | -2 | 1 |
| Thursday | -5 | -1 |
| Friday | -6 | 6 |
| Saturday | -4 | 4 |
| Sunday | -9 | 2 |

**10** Which of the following is the correct answer to $(4 + 3) \times 10 - 4 \div 2$?

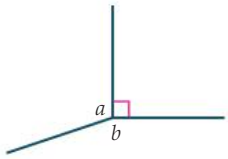
**A** 68 **B** 35 **C** 33 **D** 21

## Reasoning

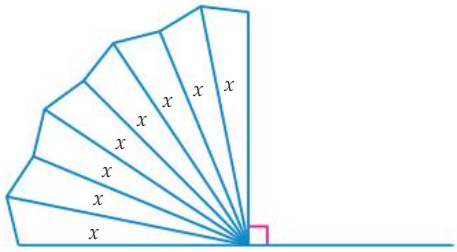
**11** The angles in this diagram can be called:

**A** vertically opposite **B** complementary

**C** supplementary **D** angles at a point.

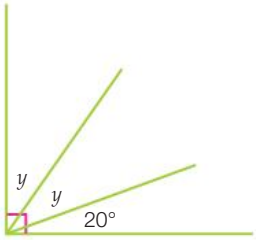



**12** A fan, when opened out, looks like this. What is the size of the angle $x$ between the folds of the fan?

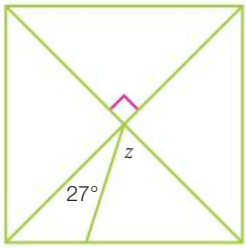



**13** Find the value of the pronumeral.

**(a)**

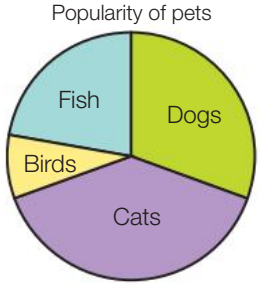



**(b)**




**14** What two angles are formed by the two hands on a clock face at 5.00 p.m.?

**15** John has drawn this pie chart, showing the popularity of pets.



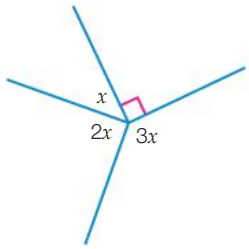

**(a)** Estimate the angle at the centre of each wedge.

**(b)** Without using a protractor, how could you check to see how reasonable the estimates you made in **(a)** are?

**(c)** Using a protractor, measure the size of each wedge.

**16** You enter a lift on the fourth floor. Once inside, you are trapped and the lift takes you on an adventure! First, you go up five floors, then fall six floors, then fall another two floors before finally going up one floor and coming to rest. The doors open, much to your relief. What floor are you on?

**17** Find the size of each angle in the diagram by first finding the value of $x$.

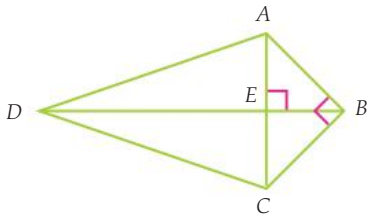



**18** **(a)** Draw and label an acute angle $\angle ABC$.

**(b)** There is another angle in your diagram, apart from the acute angle. Classify it.

**(c)** Name the angle in **(b)**.

**19** Name as many pairs of complementary angles, supplementary angles, vertically opposite angles and angles in a revolution as you can see in the figure.




9

# 9 Statistics and probability

**Caution required when exercising.** We all know that exercise is good for us but we also need to be careful, especially when using gym equipment.

Hospital records indicate that treadmill-related injuries are on the rise. Out of the total number of treadmill-related injuries that occurred in a five-year period, it was found that 38% of the injuries were reported in the fifth year. The most worrying statistic to emerge was that children and teenagers are those most at risk. The age range 0–14 years accounted for 60% of all injuries, with the 0–4 age group accounting for 38% of the injuries alone. Of the 12% of injuries that required hospital admission most were in this youngest age group. The high proportion of young children with these injuries is related to the fact that 70% of the injuries were inflicted in the home.

### Forum

What sport do you think would result in the most injuries? How would you research this issue? Is the sport with the highest number of reported injuries necessarily the most dangerous?

### Why learn this?

In the media, we are often presented with some sort of statistics or probability, such as the average temperature for the month or claims by advertisers about the worth of their products. Understanding probability and statistics helps us to make sense of such figures and to be aware when someone might be trying to mislead us.

**After completing this chapter you will be able to:**

- organise, present and interpret categorical and numerical data
- determine the mean, median, mode and range of a data set using various techniques
- present data appropriately using dot plots, stem-and-leaf plots, bar graphs, histograms, line graphs and sector graphs
- analyse univariate and simple bivariate data using dot plots and stem-and-leaf plots
- construct sample spaces for single-step experiments and use them to assign probabilities.

# Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R9.1

**1** **(a)** List the numbers you get if you count by fives from 0 to 30.

**(b)** List the numbers you get if you count by twos from 16 to 28.

**(c)** List the numbers you get if you count by hundreds from 1300 to 1700.

**(d)** List the numbers you get if you count by twenties from 60 to 200.

Worksheet R9.2

**2** **(a)** List the multiples of 50 from 50 to 250.

**(b)** List the multiples of 25 from 100 to 200.

**(c)** List the multiples of 200 from 1400 to 2000.

Worksheet R9.3

**3** Write the following lists of numbers in order from smallest to largest.

**(a)** 12, 22, 14, 20, 10, 16, 24, 6

**(b)** 15, 0, 50, 20, 55, 100, 25, 5

**(c)** 5, $1\frac{1}{2}$, $6\frac{1}{2}$, 1, $\frac{1}{2}$, 0, $3\frac{1}{2}$, $5\frac{1}{2}$, 2, 3, $2\frac{1}{2}$, 4

Worksheet R9.4

**4** Copy and complete the pattern by adding the same number each time.

**(a)** 12, ____, ____, 21, ____, 27

**(b)** 120, ____, ____, ____, ____, 220

**(c)** 225, ____, ____, 300, ____, 350

Worksheet R9.5

**5** Write the following fractions in simplest form.

**(a)** $\frac{2}{6}$ **(b)** $\frac{6}{8}$

## Key Words

| | | | |
|---|---|---|---|
| average | continuous data | measures of centre | sample space |
| bar graph | data | median | stem-and-leaf plot |
| bimodal | discrete data | mode | tally |
| bivariate data | dot plot | multimodal | theoretical probability |
| categorical data | frequency | outcomes | trend |
| chance | frequency table | outlier | univariate data |
| class interval | histogram | probability | |
| column graph | mean | range | |

# Collecting data

9.1

The study of statistics is the collection, interpretation and presentation of **data**. Data is information such as the colour of the family car or the amount of time Year 7 students spend on the internet every day. In this chapter, you will develop skills so that you can understand how to collect data, present it in useful forms, and interpret it.

## Types of data

We can collect three different types of data.

The first type is **categorical data** because this information is sorted into groups of the same type. An example of categorical data would be the eye colour of a group of students because the data collected would be the number of students with blue eyes, brown eyes, green eyes etc. Another example could be the favourite fruit of a group of students because the data collected would be answers such as apples, bananas, pears, grapes, oranges etc.

The second and third types of data are numerical data, but they are quite different.

The second type of data is **discrete data** and this is data that is counted. Examples of this are the number of runs scored by a cricketer over a season or the number of emails you receive each day.

The third type of data is **continuous data** and this is data that is measured. Examples of continuous data are the weights of newborn babies at a hospital or the volume of water in a dam each day. The accuracy of the measurements depends on the measuring equipment used.

Suppose you asked pet owners the following questions about their pets.

1 What type of pet do you own?

2 How many pets do you own?

3 What does each of your pets weigh?

The first question will collect categorical data because the answers will be cat, dog, rat, tortoise, rabbit etc.

The second question will collect discrete data because the answers will be 1, 2, 3, 4 etc.

This third question will collect continuous data such as 5.3 kg, 34.7 kg, 24.3 kg, 2.6 kg etc.

### Worked Example 1

WE1

For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

**(a)** How many television sets are in your house?

**(b)** What is the height of the tallest person in your family?

**(c)** How far do you have to travel to get to school?

**(d)** What is your favourite dessert?

| Thinking | Working |
|---|---|
| **(a)** Are you counting, measuring or putting things in categories? (Here, we are counting the number of televisions.) | (a) Discrete data |
| **(b)** Are you counting, measuring or putting things in categories?(Here, we are measuring height.) | (b) Continuous data |
| **(c)** Are you counting, measuring or putting things in categories?(Here, we are measuring distance.) | (c) Continuous data |
| **(d)** Are you counting, measuring or putting things in categories? (Here, we are putting things in categories.) | (d) Categorical data |

## Frequency tables

Any data that has been collected can be recorded in a **frequency table**. Here, each time a piece of data occurs, it is recorded in the **tally** column. We count the data in the tally column and write this number in the **frequency** column. Frequency means the number of times an event occurs.

## Worked Example 2 WE2

A Year 7 class of 25 students was surveyed about the number of brothers and sisters they have. The results were: 2, 3, 5, 0, 1, 1, 2, 0, 3, 1, 4, 0, 1, 1, 3, 2, 2, 2, 1, 1, 0, 1, 1, 2, 4

**(a)** Construct a frequency table showing this information.

**(b)** Is this data categorical, discrete or continuous?

**Thinking**

**(a)** **1** Construct a table with three columns that shows the item being surveyed, the tally and the frequency.

**2** Enter the data into the tally column. Start at the left of the list of numbers and work to the right. (Don't count all the 1s, then all the 2s etc.) Notice 𝍸 is used to represent 5.

**3** Enter these tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.

**(b)** Is the data categorical, discrete or continuous?

**Working**

(a) Frequency table

| Number of brothers and sisters | Tally | Frequency |
|---|---|---|
| 0 | 𝍸 \| | 4 |
| 1 | 𝍸 𝍸 \| | 9 |
| 2 | 𝍸 \| | 6 |
| 3 | \|\|\| | 3 |
| 4 | \|\| | 2 |
| 5 | \| | 1 |
| | Total | 25 |

(b) It is discrete data.

## Grouped data

When there is a large amount of numerical data spread out over many values, data is often grouped so that there are between five and 10 different groups. The spread of values is called the **range**.

Range = largest data value – smallest data value.

Data is usually grouped in intervals of 2, 5, 10 etc., so that we get between five and 10 groups when we divide our chosen interval into the range. The spread of the values in each group gives us the **class interval**. The class intervals are the groups in the first column of the frequency table. The class intervals for discrete data are presented in discrete groups; e.g. 0–9, 10–19, 20–29. The class intervals for continuous data are presented in continuous groups; e.g. 0–<10, 10–<20, 20–<30.

## Worked Example 3 WE3

The Exceptional Felines basketball team keeps track of the points they score on the way to the State championship. The point totals are:

73, 84, 68, 45, 52, 44, 45, 52, 66, 42, 43, 40, 53, 47, 82, 76, 42, 57, 65, 81,
80, 40, 56, 72, 74, 83, 41, 66, 76, 75, 68, 81, 82, 79, 58, 81, 78, 80, 78, 76

Group the results and then construct a frequency table showing this information.

### Thinking

**1** Decide on how to group the data by subtracting the smallest score from the highest to find the range. Then divide this number by 2, 5, 10 etc. until you get an interval size that gives between five and 10 groups. (If you get a decimal number, round to the nearest whole number.)

**2** Construct a table with three columns. Write the grouped values in the first column. Be careful with your groupings so that you don't miss values or include them twice. (In this example, we will use 40–44, 45–49, 50–54 and so on.)

**3** Use the tally column to help with the count. Remember to go through the data list systematically from the first piece of data to the last, tallying as you go.

**4** Enter the tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.

### Working

Range = 84 – 40
= 44

44 ÷ 5 = 8.8

Number of groups = $\frac{44}{5}$
= 8.8

So, we will group with an interval size of 5 and have 9 groups.

| Number of points scored | Tally | Frequency |
|---|---|---|
| 40–44 | 𝍸 \|\| | 7 |
| 45–49 | \|\|\| | 3 |
| 50–54 | \|\|\| | 3 |
| 55–59 | \|\|\| | 3 |
| 60–64 | | 0 |
| 65–69 | 𝍸 | 5 |
| 70–74 | \|\|\| | 3 |
| 75–79 | 𝍸 \|\| | 7 |
| 80–84 | 𝍸 \|\|\|\| | 9 |
| | Total | 40 |

# 9.1 Collecting data

## Navigator

Answers page 674

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q12, Q14, Q16 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q16 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15 |

## Fluency

WE1

**1** For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

**(a)** How do you travel to school?

**(b)** What colour hair do you have?

**(c)** How much time did you spend at the beach last summer?

**(d)** What is your height?

**(e)** How many cars passed by the school in a day?

**(f)** What is your favourite book?

**(g)** How many trees are in your front yard?



WE2

**2** A Year 7 class of 25 students was surveyed about the number of pets they own. The results were:

0, 0, 2, 1, 1, 1, 0, 6, 9, 4, 3, 0, 1,
3, 2, 1, 1, 4, 5, 4, 3, 2, 1, 4, 7

**(a)** Construct a frequency table showing this information.

**(b)** Is this data categorical, discrete or continuous?

WE3

**3** The number of calls per day to the emergency help department of an automobile association was recorded for the month of June. The results were:

| | | | | | |
|---|---|---|---|---|---|
| 120 | 100 | 75 | 60 | 66 | 100 |
| 110 | 197 | 55 | 80 | 155 | 179 |
| 154 | 43 | 75 | 98 | 172 | 130 |
| 25 | 55 | 110 | 99 | 72 | 90 |
| 113 | 65 | 87 | 94 | 112 | 18 |

Group the results and then construct a frequency table showing this information.



**4** Students at a school noted the colour of teachers' cars in the car park. They found there were 13 black cars, 21 white cars, 25 silver cars, 7 red cars, 5 blue cars and 11 other colours.

**(a)** How many cars were in the car park?

**(b)** Construct a frequency table showing this information.

**(c)** Is this data categorical, discrete or continuous?

**5** A Year 7 class was surveyed about how many hours of television they watched on a normal week night. The results were:

$3, 2, 1\frac{1}{2}, \frac{1}{2}, 1, 2\frac{1}{2}, 1, 0, 4\frac{1}{2}, 2\frac{1}{2}, 2, 2\frac{1}{2}, 2, 0, 4, 2, 1\frac{1}{2}, 2\frac{1}{2}, 3, \frac{1}{2}, 0, \frac{1}{2}, 1, 2\frac{1}{2}, 2, 2, \frac{1}{2}$

Construct a frequency table showing this information.

**6** Class 7C was surveyed about their favourite takeaway food. The results were:

Sarah—pizza
Whitney—hamburgers
Morgan—Chinese
Kimiko—pizza
Liam—hamburgers
Anne—chicken
David—never eats takeaway
Malcolm—pizza
Gary—hamburgers
Dieter—chicken
Yang—pizza
Brent—fish & chips
Ruby—pizza
Romano—fish & chips
Gareth—chicken
Fleur—never eats takeaway
Jonty—pizza
Carol—Chinese
Tracy—hamburgers
Marcia—pizza
Douglas—pizza
Yusef—fish & chips



Copy and complete the frequency table below to show this information.

| Type of takeaway | Tally | Frequency |
|---|---|---|
| Pizza | | |
| Fish & chips | | |
| ? | | |
| ? | | |
| ? | | |
| None | | |
| | | |

**7** AYear 7 class was surveyed about how much money they had in their pockets at that time. The results were:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1.20 | $0.75 | $1.00 | $2.55 | $1.50 | $4.00 | $0.00 | $0.00 |
| $0.80 | $2.40 | $1.25 | $0.00 | $3.60 | $4.40 | $4.30 | $2.90 |
| $1.80 | $2.35 | $2.20 | $0.50 | $0.40 | $2.50 | $1.50 | $1.75 |

Group the results and then construct a frequency table showing this information.




## Understanding

**8** Look at the following data list.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 8 | 9 | 15 | 22 | 24 | 17 | 16 | 28 | 31 | 40 |
| 15 | 52 | 16 | 51 | 42 | 39 | 27 | 26 | 42 | 55 | 46 |
| 17 | 22 | 18 | 5 | 47 | 46 | 33 | 21 | 19 | 21 | 17 |

Group the data using 0–9, 10–19, 20–29 etc. as the groups.

**(a)** The frequency for the 10–19 group is:

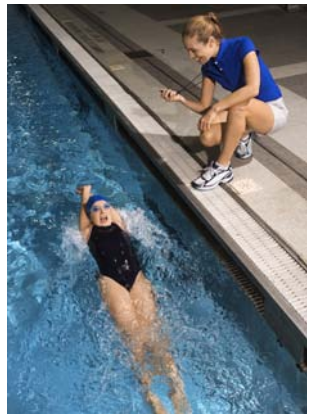
**A** 6 **B** 8 **C** 9 **D** 10

**(b)** The frequency for the last group is:

**A** 2 **B** 3 **C** 4 **D** 5

**9** A class was surveyed about the number of hours spent exercising over a weekend. The results were as follows:

1, 3, 2, 1, 1, 2, 0, 2, 0, 1, 1, 2, 6, 3, 4, 2, 1, 0, 2, 8, 3, 5, 1, 2, 0, 2



**(a)** How many students were surveyed?

**(b)** Construct a frequency table of this data.

**(c)** How many students did not exercise at all?

**(d)** How many students spent more than 2 hours exercising?

**(e)** What number of hours was most commonly spent exercising by the members of the class?

**(f)** What is the greatest number of hours spent exercising by a student?

**(g)** One student hoped to qualify for the next Olympics, one student wanted to be drafted into the AFL and another student's netball team was in the finals. What data values are mostly likely to belong to these three students? Explain your answer.

**10** The cast and crew of a school musical were to be given T-shirts advertising the production. There were five sizes available, XS (extra small), S (small), M (medium), L (large) and XL (extra large). The orders are recorded below:

S, M, S, XL, L, M, L, S, XS, M, L, S, M, L, L, M, XL, S, M, M, XL, L, M, S, XS, L, S, XL, M, L

Five students, all in Year 7, forgot to order a T-shirt.

**(a)** How many students were in the cast and crew of the musical?

**(b)** Construct a frequency table for this data.

**(c)** What type of data is this?

**(d)** What was the most popular T-shirt size ordered?

**(e)** How many T-shirts are smaller than the M size?

**(f)** The music teacher, Ms Campbell, could not decide whether to order one extra T-shirt in each size or five more M size T-shirts for the students who had not submitted an order. What would you advise her to do, explaining your advice.

## Reasoning

**11** Look at the following data set.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 32 | 42 | 53 | 63 | 21 | 33 | 42 | 53 | 65 | 22 | 34 | 43 | 57 | 61 |
| 24 | 34 | 48 | 50 | 61 | 24 | 39 | 40 | 50 | 62 | 29 | 31 | 41 | 52 | 62 |

**(a)** Construct a frequency table using class intervals of 20–29, 30–39 etc.

**(b)** Describe what you find in your frequency table.

**(c)** Construct a frequency table using class intervals of 20–24, 25–29, 30–34 etc.

**(d)** Describe what you find in your frequency table.

**(e)** What does this show?

**12** Sixty 13-year-old students were tested to find their pulse rate (heart beat) when resting. The following beats per minute were measured.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 70 | 68 | 68 | 76 | 79 | 68 | 76 | 55 | 55 | 60 | 60 | 94 | 72 | 65 | 64 |
| 93 | 71 | 62 | 67 | 82 | 76 | 65 | 77 | 82 | 81 | 59 | 74 | 74 | 67 | 68 |
| 78 | 76 | 63 | 82 | 81 | 82 | 74 | 70 | 66 | 63 | 84 | 81 | 69 | 84 | 79 |
| 71 | 70 | 54 | 68 | 64 | 78 | 58 | 84 | 61 | 75 | 72 | 73 | 71 | 91 | 66 |

**(a)** Construct a frequency table using class intervals of 51–60, 61–70 etc.

**(b)** Construct a frequency table using class intervals of 51–55, 56–60, 61–65 etc.

**(c)** Which table shows the spread of results more clearly? Give a brief reason to support your answer.

**13** The following is a list of birth weights (g) of 30 babies.

| | | | | | | |
|---|---|---|---|---|---|---|
| 2900 | 2805 | 2925 | 2010 | 2720 | 3125 | 2670 |
| 2555 | 2963 | 2972 | 3151 | 2515 | 3529 | 3098 |
| 2126 | 2417 | 3000 | 3254 | 2997 | 2986 | 2719 |
| 2842 | 3519 | 3509 | 3218 | 3002 | 2437 | 2222 |
| 2019 | 2113 | | | | | |



Construct a frequency table for this data using a suitable class interval. Briefly explain why you chose this class interval.

## Open-ended

**14** **(a)** A die is rolled 20 times. Construct a possible frequency table for the outcomes 1, 2, 3, 4, 5, 6 with no outcome having a frequency greater than 4.

**(b)** Fifty dogs were weighed. Their weights ranged from 1 kg to 25 kg. Construct a possible frequency table if the data is grouped using class intervals of 1–5, 6–10, 11–15, 16–20 and 21–25 with no data group having a frequency greater than 15.

**15** Find out how many hours of television each of the students in your class watches on a normal week night by surveying your class. Construct a frequency table of your results. Round your answers to the nearest half hour.

**(a)** How many of your class members watch no television on a normal week night?

**(b)** How many of your class members watch less than 1 hour of television on a normal week night?

**(c)** How many of your class members watch 3 or more hours of television on a normal week night?

**(d)** How much television do the majority of the members of your class watch on a normal week night?

**(e)** Do you watch more or less television than the majority of the members of your class?

**16** Find out the favourite school subject of each of the students in your class by surveying your class. Construct a frequency table of your results.

**(a)** What is the favourite subject of the members of your class?

**(b)** What is the second favourite subject of the members of your class?

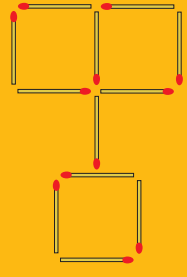
**(c)** Did any student have no favourite subjects?

## Outside the Square Puzzle

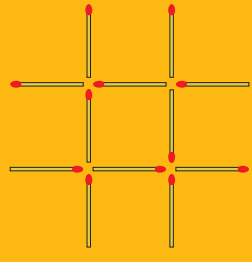
### Matchstick puzzles



Remove six of the fifteen matches to leave ten.



Reposition four matches to make exactly five squares.



Reposition three matches to make exactly three squares.

Mathspace

# Crack the code

You have been captured by your archenemy Dr Code and find yourself in a locked room with the following message written on the door above three symbols—a red square, a yellow circle and a blue triangle. You know the coded message tells you the order in which to press the symbols to open the door, but how do you crack the code?





## Code Box #1

UWJXX YMJ DJQQTB HNWHQJ YMJS YMJ WJI XVZFWJ FSI KNSFQQD YMJ GQZJ YWNFSLQJ



You decide to try a simple letter shift code, knowing that E is the most frequently used letter in the English language. (In a letter shift code, each letter is moved forwards or backwards by the same amount. For example, if the shift is 3, then A would be written as D, B as E, C as F ... X as A, Y as B and Z as C.)

**1 (a)** Which letter in the code appears the most?

**(b)** What are the two possible shifts that would produce this match?

**(c)** Try the simpler of the two shifts. What does the message say?

Now you can escape the locked room!

So far, so good, but you now find yourself facing a much more difficult code to escape the next room. You know this because written above the new code Dr Code has written, 'Aha—not just a letter shift this time!'

You rightly suspect that this time the letter substitution is just random, so you will need to know the frequency with which letters are used in English. Happily, you know this and you write out the following table from memory. It is based on the number of times each letter would appear for every one hundred letters counted.

### Frequency of letters used in English per 100 letters

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 8.17 | 1.49 | 2.78 | 4.25 | 12.70 | 2.23 | 2.02 | 6.09 |
| **I** | **J** | **K** | **L** | **M** | **N** | **O** | **P** |
| 6.97 | 0.15 | 0.77 | 4.03 | 2.41 | 6.75 | 7.51 | 1.93 |
| **Q** | **R** | **S** | **T** | **U** | **V** | **W** | **X** |
| 0.10 | 5.99 | 6.33 | 9.06 | 2.76 | 0.98 | 2.36 | 0.15 |
| **Y** | **Z** | | | | | | |
| 1.97 | 0.07 | | | | | | |

Again, the code requires you to press some buttons that look the same as the last ones.

## Code Box #2

Aha—not just a letter shift this time!

NRCD NCJG EYGDD NRG QOSG NYCFXUOG NRGX NRG VGOOWT ACYAOG NTCAG FXP BCXFOOV NRG YGP DUSFYG BWSY NCJGD



**2 (a)** Looking at the table above, what are the top five most frequently used letters?

**(b)** Draw up a frequency table for Code #2. Which letter is used most frequently? Put this against E in an alphabet chart. You may like to label the second-most frequently used letter too.

**(c)** Using your knowledge from Code #1 about the words Dr Code often uses in his messages, see if you can guess a few of the words. Use these to solve more of the letters in the code.

**(d)** What is the button sequence this time?

If you have broken the code you can escape.

# 9.2 Measures of centre

In everyday life, people talk about 'the average temperature' and 'the average number of runs scored in an over in a cricket match'. The **mean**, commonly known as the **average** of a set of numbers, is a value we use to represent the centre of the data. However, this is not the only measure of the centre that we can use. The **median** and **mode** are two other ways of finding a central value of a whole group of data values. The mean, median and mode are all **measures of centre**, but each is calculated in a different way and gives us different information about the data we are interpreting.

## Mean

The mean is the sum of all the values in the data set divided by the number of values in that data set.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$




## Median

The median is the *middle* value of an ordered data set.

To calculate the median, we arrange the data set in ascending order, then count the number of data values.

If the number of values is odd, select the middle value.

If the number of values is even, select the two middle values. Add them together and divide by 2.

## Mode

The mode is the value that occurs *most often* in a data set.

Sometimes, a set of results can have more than one result that occurs most frequently. As an example, consider the following data: 1, 1, 3, 3, 3, 4, 5, 5, 6, 7, 7, 7, 8, 8, 9.

Here, we can see that both 3 and 7 occur three times. When there are two values that occur most frequently, we say that the results are **bimodal**. If it happens that there are three or more values that each have the maximum frequency, we say that the data is **multimodal**. The mode is not very useful in a multimodal set of data. We sometimes say that such data sets have no mode.

## Worked Example 4 WE4

Find the **(i)** mean, **(ii)** median and **(iii)** mode of each of the following sets of test results.

**(a)** 9, 4, 5, 7, 8, 7, 2 **(b)** 10, 7, 3, 4, 8, 2, 6, 5

| Thinking | | | Working |
|---|---|---|---|
| **(a)** **(i)** | **1** | Write the rule for finding the mean. | (a) (i) Mean = $\frac{\text{sum of data values}}{\text{number of data values}}$ |
| | **2** | Substitute data into the rule. | $= \frac{9+4+5+7+8+7+2}{7}$ |
| | **3** | Simplify and evaluate. | $= \frac{42}{7}$ |
| | **4** | Write down the mean. | Mean = 6 |
| **(ii)** | **5** | Order the values from smallest to largest. Count the number of values. If the number is odd, select the middle value. | (ii) 2, 4, 5, (7), 7, 8, 9<br>↑<br>7 |
| | **6** | Write down the median. | Median = 7 |
| **(iii)** | **7** | Does any value appear more often than any other? This is the mode. (Yes) | (iii) |
| | **8** | Write down the mode. | Mode = 7 |
| **(b)** **(i)** | **1** | Write the rule for finding the mean. | (b) (i) Mean = $\frac{\text{sum of data values}}{\text{number of data values}}$ |
| | **2** | Substitute data into the rule. | $= \frac{10+7+3+4+8+2+6+5}{8}$<br>$= \frac{45}{8}$ |
| | **3** | Simplify and evaluate. | = 5.625 |
| | **4** | Write down the mean. | Mean = 5.625 |
| **(ii)** | **5** | Order the values from smallest to largest. Count the number of values. If the number is even, select the two middle values. | (ii) 2, 3, 4, 5, 6, 7, 8, 10<br>2, 3, 4, (5, 6), 7, 8, 10 |
| | **6** | Find the mean of these two numbers to find the median. | Median = $\frac{5+6}{2}$ |
| | **7** | Write down the median. | Median = 5.5 |
| **(iii)** | **8** | Does any value appear more often than any other? This is the mode. (No) | (iii) |
| | **9** | Write that there is no mode. | There is no mode. |

# 9.2 Measures of centre

## Navigator

Answers page 676

| Q1 Column 1, Q2 (a)–(d), Q3, Q4, Q5, Q6, Q9 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q8, Q9 | Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10 |
|---|---|---|

**Equipment required:** Calculator for Question **2**

## Fluency

WE4

**1** Find the **(i)** mean, **(ii)** median and **(iii)** mode of each of the following sets of test results.

**(a)** 7, 1, 7

**(b)** 5, 8, 5

**(c)** 6, 6, 6, 6, 6

**(d)** 4, 4, 4, 4, 4, 4, 4

**(e)** 7, 6, 7, 5, 7, 9, 7, 8

**(f)** 6, 5, 5, 5, 9, 9, 10

**(g)** 8, 7, 9, 5, 8, 2, 3

**(h)** 2, 8, 2, 6, 8, 6, 1, 2, 1, 4

**(i)** 4, 3, 6, 4, 8, 2, 5, 7, 6, 4

**(j)** 5, 3, 8, 7, 2, 5, 9, 5

**(k)** 1, 6, 2, 7, 9, 8, 9

**(l)** 6, 3, 2, 8, 9, 2, 5

**2** Find the **(i)** mean, **(ii)** median and **(iii)** mode of each of the following sets of data. Round your answers to two decimal places if necessary.

**(a)** 29, 36, 29, 23, 32, 25

**(b)** 67, 73, 72, 82, 50, 45, 76

**(c)** 56, 72, 39, 47, 89, 81, 63, 81

**(d)** 12.8, 6.5, 19.3, 14.2, 4.7, 6.8, 6.5, 11.3

**(e)** 16.3, 15.8, 11.4, 15.2, 12.3, 11.4, 13.5

**(f)** 11.01, 11.001, 10.1, 10.11, 10.01, 1.11, 11.11, 1.01, 11

## Understanding

**3** *Without doing any calculations,* in each case, choose which one of the alternatives could be the mean.

**(a)** 9, 7, 8, 7, 7, 8, 6, 6, 9

**A** 3 **B** 5 **C** 7 **D** 10

**(b)** 6, 6, 6, 7, 6, 7, 4, 7, 2, 7

**A** 2 **B** 4 **C** 6 **D** 8

**(c)** 12, 19, 14, 18, 11, 17

**A** 10 **B** 12 **C** 16 **D** 19

**(d)** 86, 74, 85, 68, 91, 76, 80, 90

**A** 74 **B** 81 **C** 89 **D** 91

**4** The Korea–Japan 2002 World Cup in soccer was played in more stadiums than ever before. This record was not beaten in Germany in 2006 (12 stadiums) or in South Africa in 2010 (10 stadiums). The following table shows the city and capacity of the different stadiums.

| Korea | Capacity |
|---|---|
| Seoul | 63 961 |
| Incheon | 52 179 |
| Suwon | 43 188 |
| Daejeon | 40 407 |
| Daegu | 68 014 |
| Jeonju | 42 391 |
| Gwangju | 42 880 |
| Ulsan | 43 550 |
| Busan | 55 982 |
| Seogwipo | 42 256 |

| Japan | Capacity |
|---|---|
| Sapporo | 42 000 |
| Miyagi | 49 000 |
| Niigata | 42 300 |
| Ibaraki | 42 000 |
| Saitama | 63 000 |
| Shizuoka | 50 600 |
| Kobe | 42 000 |
| Osaka | 50 000 |
| Oita | 43 000 |
| Yokohama | 70 000 |



**(a)** Find the **(i)** mean and **(ii)** median capacity for the stadiums in Korea.

**(b)** Which stadium has closest to the mean capacity in Korea?

**(c)** Find the **(i)** mean and **(ii)** median capacity for the stadiums in Japan.

**(d)** Which stadium has closest to the mean capacity in Japan?

**(e)** Use the mean and median to compare the sizes of the stadiums in the two countries.

**(f)** Find the **(i)** mean and **(ii)** median capacity for all 20 stadiums.

**(g)** Which stadium is closest to the mean capacity for all 20 stadiums?

## Reasoning

**5** Consider the following test results: 7, 5, 6, 8, 3

**(a)** Do you expect the mean to be an actual data value? Explain your answer.

**(b)** Find the mean of these results. Is the mean an actual data value?

**(c)** Do you expect the median to be an actual data value? Explain your answer.

**(d)** Find the median for the test results above.

**6** **(a)** Explain what is meant by the statement 'The average family has 2.3 children'.

**(b)** Explain why the median of a group of whole-number values can only ever either end in .5 or be a whole number.

**(c)** If the data values are all whole numbers, can the mode ever be a decimal? Explain why or why not.

7 A small company has a manager, an assistant manager, two office workers and 10 factory workers. The manager is paid $125 000 per year, the assistant manager $85 000, the office workers $55 000 each, and the factory workers $50 000 each.

**(a)** Find the **(i)** mean, **(ii)** median and **(iii)** mode for the annual income of all the people in the company, rounding your answer to the nearest cent.

**(b)** Which measure of centre gives you the best idea about the salaries paid by this company?

**(c)** Omit the manager's salary and recalculate the **(i)** mean, **(ii)** median and **(iii)** mode.

**(d)** Comment on what you found.

## Open-ended

8 As well as giving a set of results, make sure you include a short explanation of how you found your answers for each of the following.

**(a)** Find five test results for which the mean, median and mode are all equal to 6 if the test results are not all the same.

**(b)** Find five test results for which the mean and the median are 7 and the mode is 9.

**(c)** Find five test results for which the mean is 7 and the median and mode are 6.

9 Three students are finding the median of the following data values.
20, 17, 28, 23, 19, 21, 22, 20

Sam says 'The median is the fourth value, so is 23.'

Veran says 'No, you need to put the data values in order first, so the median is 20.'

Lakme says 'No, you are both wrong.'

Help Lakme explain to Sam and Veran why they are incorrect.

10 Nine friends went tree planting at their local creek. The median number of trees planted was 5, the mean was 6 and the mode was 4. Find a data set for the number of trees each person planted to match the given statistics.

# Outside the Square — Problem solving

## Running the race

1 In a sprint, the first three runners had times of 12.1, 12.1 and 10.9 seconds. Find the mean, median and mode for the three times.

2 In a different race, the first three runners had a mean time of 11.7 seconds, a median time of 10.8 seconds and a modal time of 10.8 seconds. Find the times for the three runners.



3 In a third race, the first three runners had a mean time of 11.8 seconds and a median time of 11.9 seconds. If all three times are different what can you say about the first and third placegetters?

Give at least three pairs of possible times for these two runners, using one decimal place accuracy in your solutions. Explain how you found your answers.

**Strategy options**

- Work backwards.
- Test all possible combinations.

# Graphing univariate data

9.3

Information presented in tables is useful, but we often find it much easier to interpret this information if it is displayed in a graph. There are many types of graphs we can use for displaying data. In this section we will explore graphs of **univariate data**, data that consists of only one variable. A variable in statistics describes data that can change or vary.

## Dot plots

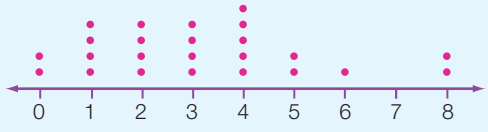
A **dot plot** is a very simple statistical graph where, for each data value, a dot is placed on a scale. When drawing a dot plot it is important to space the dots evenly to give an accurate picture of the data set. A dot plot can be used for categorical or discrete data such as car colour or the number of people who live in each of the houses on a street.

### Worked Example 5

WE5

Draw a dot plot to illustrate the following data that represents the number of pets owned by each student in a class.

2, 5, 8, 6, 2, 4, 3, 4, 2, 1, 8, 3, 4, 0, 3, 1, 0, 3, 5, 1, 4, 2, 1, 4

| Thinking | Working |
|---|---|
| **1** Identify the lowest and the highest values and make these the end points of a scale. |   |
| **2** Complete the plot by marking a dot for each of the data values, being careful to space them evenly. |   |

## Stem-and-leaf plots

When there is a large amount of numerical data it is often convenient to combine several pieces of data into a group. **Stem-and-leaf plots** are useful because they can group data without losing individual data values.

A stem-and-leaf plot consists of two parts, a stem and a leaf. Each piece of data must be divided into two parts so it can be plotted. For example, a score of 23 would be written as:

| STEM | LEAF |
|---|---|
| 2 | 3 |

The smallest place value is used as the leaf with the remaining part of the number as the stem. The leaf can only contain the digits 0 to 9. For example, 34.7 would be written as:

| STEM | LEAF |
|---|---|
| 34 | 7 |

A stem-and-leaf plot should always include a key; e.g. 34 | 7 = 34.7

Note that a decimal point is not used in the plot.

As another example, 536 would be written as:

| STEM | LEAF |
|---|---|
| 53 | 6 |

53 | 6 = 536

Before stem-and-leaf plots can be used to find statistics, we need to arrange the numbers in the leaf section in ascending order to construct an ordered stem-and-leaf plot.

## Worked Example 6 WE6

A class obtained the following percentage results (rounded to the nearest whole number) in a science test.

Express the following data as an ordered stem-and-leaf plot with a class interval of 10.

69, 54, 91, 42, 73, 75, 81, 96, 88, 76, 65, 83, 74, 76, 82, 61, 57, 64, 58, 79, 83, 90, 73, 60, 51

### Thinking

**1** Decide upon the stem you will use for your stem-and-leaf plot. This will be the key for your stem-and-leaf plot.

**2** List the stems down the page in ascending order. In this case, the tens digit is the stem (4, 5, 6, 7, 8, 9). Write the key you are using beside the stem-and-leaf plot.

**3** Work through the data in the order given. List the leaf of each individual result in the appropriate row. The leaf is the units digit.

**4** Order the data in the leaf part of the stem-and-leaf plot, putting the lowest value closest to the stem.

### Working

| STEM | LEAF |
|---|---|
| 4 | 2 |
| 5 | 4 7 8 1 |
| 6 | 9 5 1 4 0 |
| 7 | 3 5 6 4 6 9 3 |
| 8 | 1 8 3 2 3 |
| 9 | 1 6 0 |

Key 5 | 4 = 54

| STEM | LEAF |
|---|---|
| 4 | 2 |
| 5 | 1 4 7 8 |
| 6 | 0 1 4 5 9 |
| 7 | 3 3 4 5 6 6 9 |
| 8 | 1 2 3 3 8 |
| 9 | 0 1 6 |

Key 5 | 4 = 54

It is important that there are enough stems to spread the data. When data values are very close together, each division of the stem is divided into two parts, a lower and an upper section. The lower part contains the digits 0 to 4 in the leaf and the upper part contains the digits 5 to 9 in the leaf. As an example of this, consider the following data set.

21, 32, 41, 22, 25, 29, 33, 42, 33, 44, 21, 28, 37, 40, 39, 36, 24, 32, 44, 30, 34

If we used the tens value as the stem we would have only three stems (2, 3 and 4). To spread out the data we can use $2_L$ for the lower values 20, 21, 22, 23, 24 and $2_U$ for the upper values 25, 26, 27, 28, 29. The stem-and-leaf plot would look like this:

| STEM | LEAF |
|---|---|
| $2_L$ | 1 1 2 4 |
| $2_U$ | 5 8 9 |
| $3_L$ | 0 2 2 3 3 4 |
| $3_U$ | 6 7 9 |
| $4_L$ | 0 1 2 4 4 |

Key 3 | 2 = 32

## Outliers

Sometimes, in a data set, there are one or two values that are quite different from the other data values. These may be called **outliers**. Outliers may simply be errors made in recording or in measuring incorrectly or they may be the result of extremely unusual circumstances. If outlier values can be explained reasonably, they are often excluded from data analysis, as the statistics obtained if they are included can be misleading. Values can often be identified as possible outliers in both dot plots and stem-and-leaf plots.

## Using dot plots and stem-and-leaf plots

Both dot and ordered stem-and-leaf plots can be used to find the median and the range of the data set because the data is organised in both types of plots. The range can also be found, as the maximum (largest) and the minimum (smallest) values are retained in both types of plots.

## Worked Example 7 WE7

Calculate the median and range for each of the following data sets. Comment on whether there are any possible outliers in the data.

**(a)** The students in a class were asked to indicate the number of brothers and sisters they have. These are the results obtained:

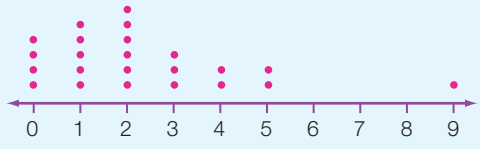
1 1 2 5 2 3 2 0 0 4 2 3 9 1 0 2 1 4 1 5 3 2 0

Draw a dot plot and use it to help you answer the question.

**(b)** A new display at a museum attracts varying numbers of people. A survey was conducted to count the number of people visiting the attraction each day. These are the results obtained for the first 20 days:

235 195 156 188 177 165 189 190 177 166
199 186 165 170 169 173 184 177 192 200

Draw an ordered stem-and-leaf plot and use it to help you answer the question.

| Thinking | | Working |
|---|---|---|
| **(a)** | **1** Construct the dot plot in the normal way. | (a)   |
| | **2** Identify the middle value to find the median. Count from the left. (As there are 23 values here, the middle value is the 12th value.) | Median = 2 |
| | **3** Find the range by subtracting the lowest value from the highest value. | Range: 9 − 0 = 9 |
| | **4** Comment on any possible outliers. | It would seem that 9 is an outlier for this data set. |

**(b) 1** Construct the (ordered) stem-and-leaf plot.

(b)

| STEM | LEAF |
|---|---|
| 15 | 6 |
| 16 | 5 5 6 9 |
| 17 | 0 3 7 7 7 |
| 18 | 4 6 8 9 |
| 19 | 0 2 5 9 |
| 20 | 0 |
| 21 | |
| 22 | |
| 23 | 5 |

Key 15 | 5 = 155

**2** Identify the middle values and use them to find the median. Count from the left.

$$\text{Median} = \frac{177 + 184}{2} = 180.5$$

**3** Find the range by subtracting the lowest value from the highest value.

Range: 235 – 156 = 79

**4** Comment on any possible outliers.

It would seem that 235 is an outlier for this data set.

## Column and bar graphs

**Column graphs** present data in vertical columns, with a space between each column. **Bar graphs** are the same as column graphs except the bars are horizontal. Column or bar graphs should be used for categorical data and for ungrouped discrete data.

### Drawing a column graph

All column graphs should include:

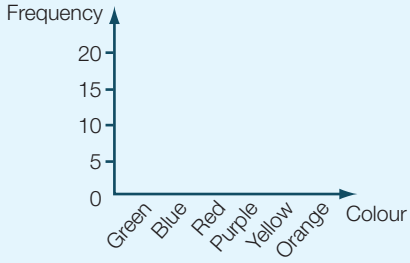
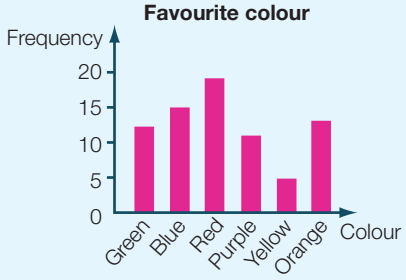
1. a title
2. categories or a numerical scale with equally spaced intervals (gaps) between the numbers on the horizontal axis
3. a numerical scale with equally spaced intervals (gaps) between the numbers on the vertical axis
4. a label for the horizontal axis ($x$-axis) that explains the variable being represented
5. a label for the vertical axis ($y$-axis) that shows the frequency
6. a small gap between each column (usually half a column width) to show that this is categorical or discrete (counted) data.

## Worked Example 8

WE8

The frequency table below shows the responses from a group of children who were asked 'What is your favourite colour?' Only one response per student was recorded. Draw a column graph to represent this information.

| Colour | Green | Blue | Red | Purple | Yellow | Orange |
|---|---|---|---|---|---|---|
| Frequency | 12 | 15 | 19 | 11 | 5 | 13 |

| Thinking | Working |
|---|---|
| **1** Draw axes. Scale and label the $x$- and $y$-axes appropriately. |   |
| **2** Use the frequency values to draw a rectangular column for each category. (Here, each rectangle represents a different colour.) The length of the rectangle indicates the frequency. Ensure that each rectangle has the same width on the $x$-axis. Insert a small space between each category. Add a title. |   |

## Histograms

**Histograms** present data in vertical columns, but they are different from column graphs. The columns have no space between them and each column deals with a range of values rather than a single value. Histograms should be used for continuous data.

### Drawing a histogram

All histograms should include:

1. a title
2. a continuous scale on the $x$-axis for the variable being represented
3. a numerical scale with equally spaced intervals between the numbers on the $y$-axis
4. a label for the horizontal axis that explains the variable that is being represented
5. a label for the vertical axis that shows the frequency
6. no gap between each column
7. a half-column width before the first column even if the first set of data starts at zero.

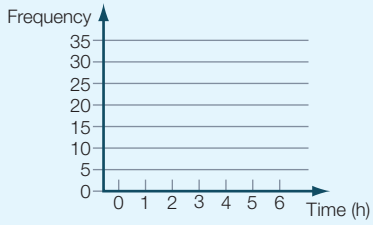
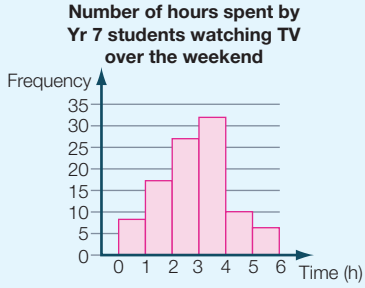
## Worked Example 9

WE 9

100 Year 7 students were surveyed about the number of hours they spent watching TV over the weekend. The data is presented below.

| Hours spent watching TV over the weekend | 0–<1 | 1–<2 | 2–<3 | 3–<4 | 4–<5 | 5 or more hours |
|---|---|---|---|---|---|---|
| No. of students | 8 | 17 | 27 | 32 | 10 | 6 |

Draw a histogram to represent this data.

| Thinking | Working |
|---|---|
| **1** Draw axes; scale and label the $x$- and $y$-axes appropriately. |   |
| **2** Ensure rectangles each have the same width on the $x$-axis but with no gaps between the $x$-axis categories. Add a title. |   |

Section 9.1 described three different types of data. The table below shows the appropriate graph for each data type.

| Question | Data type | Examples of answers | Appropriate graph |
|---|---|---|---|
| What kind of animal do you own? | Categorical | Cat, dog, rat, tortoise, rabbit | Column/bar |
| How many pets do you own? | Discrete | 1, 2, 3, 4 | Column |
| What does each of your pets weigh? | Continuous | 5.3 kg, 34.7 kg, 24.3 kg, 2.6 kg | Histogram |

# 9.3 Graphing univariate data

## Navigator

Answers page 677

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

### Fluency

WE5

**1** Draw a dot plot to illustrate the following data that represents the number of birds counted by a bird watcher in the bush in 15-minute intervals over the course of 6 hours.

12, 18, 12, 16, 19, 13, 19, 20, 17, 15, 20, 21, 18, 15, 20, 18, 21, 21, 17, 12, 17, 19, 12, 21

**2** A class obtained the following percentage results (rounded to the nearest whole number) in a science test. WE6

Express each of the following data as an ordered stem-and-leaf plot with a class interval of 10.

**(a)** 57, 48, 42, 41, 31, 22, 37, 30, 23, 28, 34, 52

**(b)** 25, 44, 15, 29, 37, 26, 33, 35, 14, 8, 2, 27, 30, 42, 34, 21, 18, 3

**3** Calculate the median and range for each of the following data sets. Comment on whether there are any possible outliers in the data. WE7

**(a)** The students in a class were asked to indicate their shoe size. These are the results obtained:

5, 5.5, 7, 4.5, 4.5, 6, 10, 5.5, 4, 4, 5.5, 4.5, 6.5, 7.5, 6, 5.5, 4.5, 4, 6.5

Draw a dot plot and use it to help you answer the question.

**(b)** The number of points scored by each of the teams in the opening round of the local Australian Rules Football competition was recorded. These are the results obtained:

110, 75, 88, 56, 59, 89, 112, 115, 121, 99, 67, 88, 77, 88, 100, 55

Draw an ordered stem-and-leaf plot and use it to help you answer the question.

**4** The frequency table below shows the colour of cars passing the school in a particular timed period. Draw a column graph to represent this information. WE8

| Colour | White | Red | Green | Blue | Silver | Other |
|---|---|---|---|---|---|---|
| Frequency | 42 | 6 | 5 | 12 | 18 | 17 |

**5** The owner of a car dealership wants a graph of the number of cars sold plotted against the value of the cars sold in the last year. The data is presented below. WE9

| Value of car | Under $10000 | $10000– < $20000 | $20000– < $30000 | $30000– < $40000 | $40000– < $50000 | $50000 or more |
|---|---|---|---|---|---|---|
| Number of cars sold | 25 | 73 | 132 | 43 | 18 | 9 |

Draw a histogram to represent this data.

**6** Express the following data as ordered stem-and-leaf plots with class intervals of 5.

**(a)** 23, 28, 34, 27, 38, 42, 31, 21, 22, 27, 20

**(b)** 48, 57, 56, 53, 68, 64, 47, 58, 52, 51, 61, 42, 47

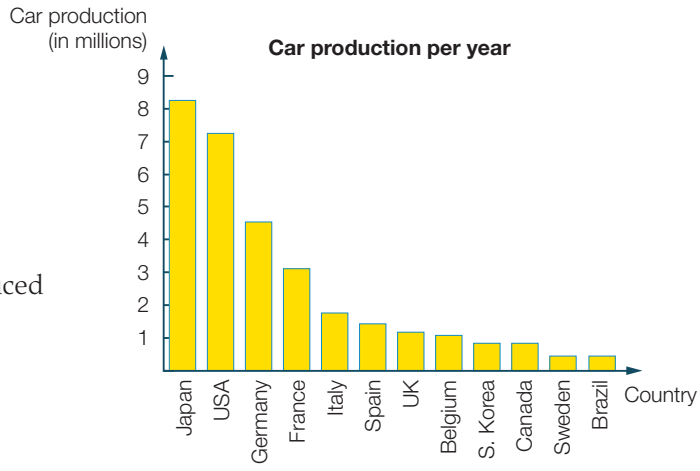
**7** The number of chocolate bars produced at a factory every hour over a 20-hour period is 146, 128, 133, 138, 140, 142, 125, 129, 136, 130, 133, 142, 126, 129, 150, 134, 126, 121, 128, 143.

**(a)** Using class intervals of 5, produce an ordered stem-and-leaf plot of the data.

**(b)** Draw an ordered stem-and-leaf plot of the data if it is expressed in intervals of 10.

## Understanding

**8** This column graph shows the numbers of cars produced in a year by the world's major car-producing countries.




**(a)** Which country produces the most cars?

**A** USA

**B** Germany

**C** Japan

**D** Brazil

**(b)** The number of cars produced by Germany is:

**A** 4.5

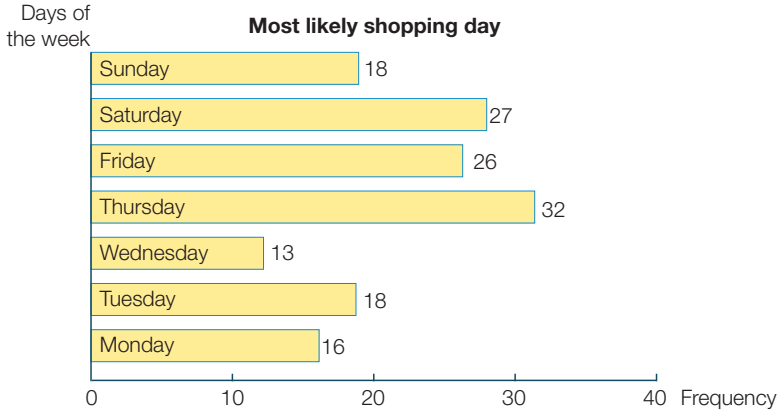
**B** 450 000

**C** 4 500 000

**D** 45 000 000



**9** The bar graph at right shows the results for a survey of 150 people as to which day they were most likely to do their supermarket shopping. The actual numbers are included on the graph to make it easier for you to calculate your answers.

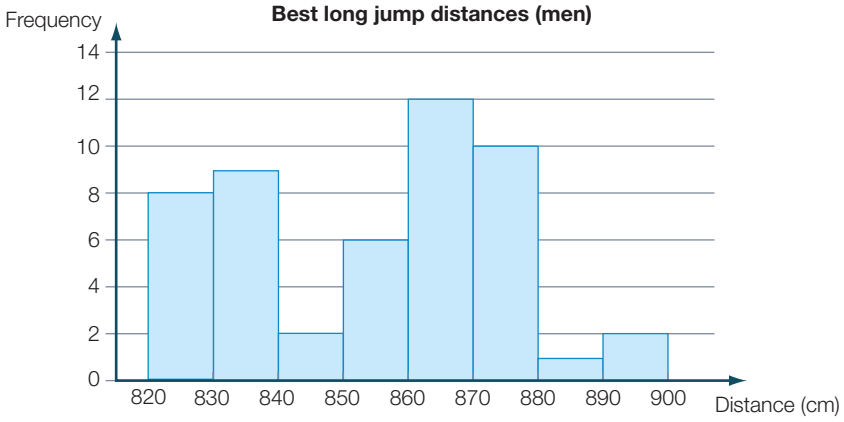



**(a)** Which day was most popular?

**(b)** How many people chose the least popular day?

**(c)** How many people chose the weekend?

**10**




The histogram above shows the best long jump distances for men recorded from 1960 to 2009.

**(a)** What is the class interval?

**(b)** What is the range of the recorded distances?

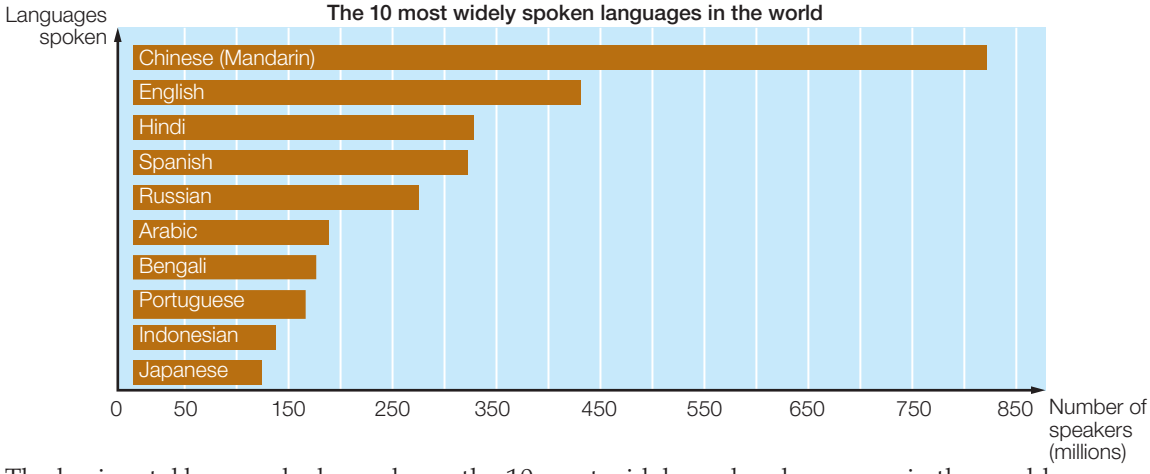
**(c)** Which class interval had six recorded jumps?

**(d)** How many jumps over 870 cm were recorded in this time period?

**(e)** How many jumps have been recorded? (This will be the sum of all the frequencies.)

## Reasoning

**11**




The horizontal bar graph above shows the 10 most widely spoken languages in the world.

**(a)** What are the differences between this and a column graph?

**(b)** Approximately how many people in the world speak Mandarin?

**(c)** Approximately how many people in the world speak English?

**(d)** What is the fourth most-spoken language in the world?

**(e)** What is unusual about the scale at the bottom of the graph?

**(f)** Could this have been drawn as a column graph?

**12** A class of Year 7 students were asked how long it took them to get to school each day.

The times, to the nearest minute, are listed below:
34, 13, 26, 38, 23, 15, 19, 30, 28, 40, 36, 25, 14, 7, 10, 11, 8, 24, 32, 69, 29, 32, 44, 21, 18, 22

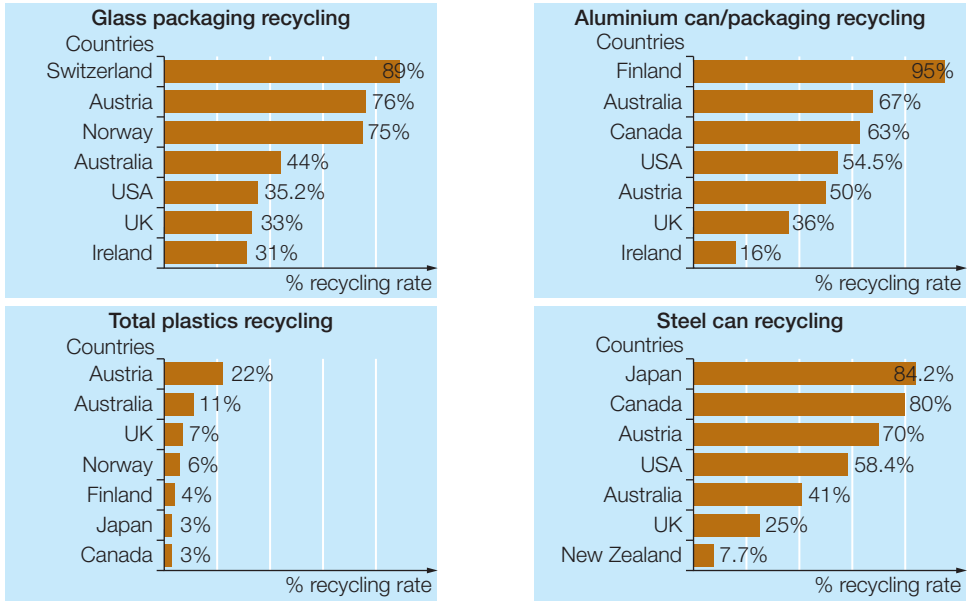
**(a)** Is this discrete or continuous data?

**(b)** Group the data with appropriate class intervals.

**(c)** Draw an appropriate graph to represent the data.

**(d)** Is there a possible outlier? Why or why not?

**13** Compare the four recycling graphs to answer the following questions.




**(a)** Does Australia or Canada have a better recycling rate for aluminium?

**(b)** Does Australia or Canada have a better recycling rate for steel cans?

**(c)** Australia, the UK and Austria appear in all four graphs. Which country seems to be the worst recycler of the three?

**(d)** Which kind of recycling does Australia seem to be best at?

**(e)** Which kind of recycling does Australia seem to be worst at?

**(f)** Compare Australia's recycling rate in all four areas with the best recycling countries on the graphs given.

**(g)** Compare Australia's recycling rate in all four areas with the worst recycling countries on the graphs given.

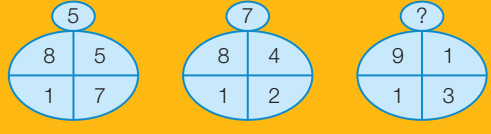
## Open-ended

**14** Collect some **(a)** categorial, **(b)** discrete and **(c)** continuous data from the students in your class and draw an appropriate graph to represent each.

## Outside the Square Puzzle

### Number puzzles A

**1** Find the number that completes the sequence of diagrams:




**2** Find as many solutions as you can for the following, where $A$, $B$ and $C$ are all different and are all positive whole numbers less than ten. Each question can have new values for $A$, $B$ and $C$.

$A \times A = B \times C$ $\quad$ $A \times A = B + C$

$A \times A = B \div C$ $\quad$ $A + A = B \times C$

# Comparing parts of a whole with graphs

9.4

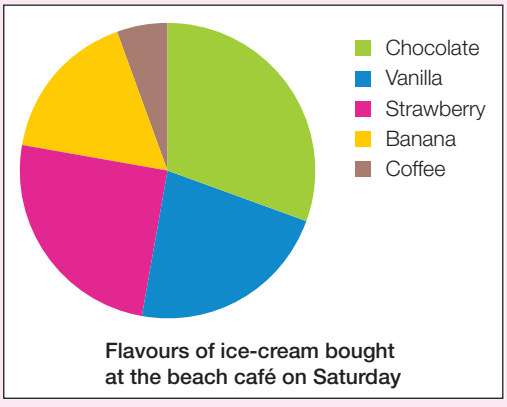
Often, data given to us are the various parts of a whole. For example, a budget shows all the separate items on which a total amount of available money will be spent. The two types of graphs we use to display this type of data are sector graphs and divided bar graphs. Both of these break a whole into parts.

## Sector graphs

A sector graph is often called a pie graph because it resembles a pie divided into pieces from the centre of the circle.

### Drawing sector graphs

1. Add the various parts to find a total.
2. Divide each part by the total to obtain fractional parts of a whole.
3. Mutliply each fractional part by 360° to find the sector angle.
4. Use a protractor to divide a circle into the various parts and draw your graph.
5. Sector graphs should have titles and a key that identifies each of the sectors.




Flavours of ice-cream bought at the beach café on Saturday

## Divided bar graphs

Another way of showing parts of a whole is to divide a rectangle according to the size of the pieces.

### Drawing divided bar or column graphs

1. Draw a rectangle, making the length a multiple or a factor of the total.
2. Multiply each data value by the same factor or multiple to find the length of each section.

The sector graph above can be represented by a divided bar graph as shown below.



Flavours of ice-cream bought at the beach café on Saturday

Chocolate
Vanilla
Strawberry
Banana
Coffee

# Worked Example 10

WE10

For the data in the table opposite:

**(a)** draw a sector graph

**(b)** draw a divided bar graph.

All Year 7 students at a certain school were surveyed about their favourite type of pet and the responses were recorded in the table opposite.

| Type of pet | Number recorded |
|---|---|
| Dog | 78 |
| Cat | 65 |
| Bird | 39 |
| Rabbit | 26 |
| Other | 52 |

## Thinking

## Working

**(a) 1** Add the number of responses to find a total.

(a) 260

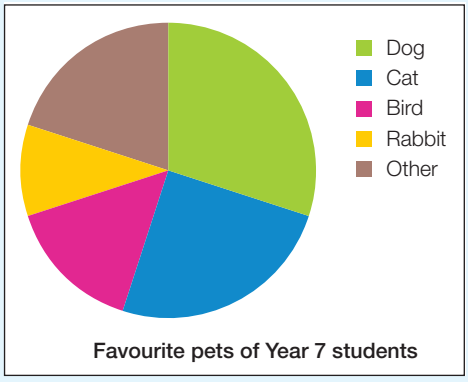
**2** Find the fraction of the whole by dividing the number recorded for each type by the total and record your results in a table.

| Type of pet | Number recorded | Fraction of a whole |
|---|---|---|
| Dog | 78 | $\frac{78}{260}$ |
| Cat | 65 | $\frac{65}{260}$ |
| Bird | 39 | $\frac{39}{260}$ |
| Rabbit | 26 | $\frac{26}{260}$ |
| Other | 52 | $\frac{52}{260}$ |
| Total | 260 | 1 |

**3** Multiply the fraction for each type by 360° and round your answer to the nearest degree. Check that the sum of your angles is 360°. (It may be very slightly more or less after rounding.)

| Type of pet | Number recorded | Fraction of a whole | Sector angle (°) |
|---|---|---|---|
| Dog | 78 | $\frac{78}{260}$ | 108 |
| Cat | 65 | $\frac{65}{260}$ | 90 |
| Bird | 39 | $\frac{39}{260}$ | 54 |
| Rabbit | 26 | $\frac{26}{260}$ | 36 |
| Other | 52 | $\frac{52}{260}$ | 72 |
| Total | 260 | 1 | 360 |

**4** Rule a line from the centre to the edge of the circle. Use this line and a protractor to measure the sector angles and rule each sector. Give your graph an informative title and provide a key for each of the sectors.



Favourite pets of Year 7 students

**(b) 1** Add the number of responses to find a total.

(b) 260

**2** Choose a length that is a multiple or a factor of the total. (As 130 mm is half of 260, draw a line 13 cm long.) Multiply by the same factor or multiple to find the length of each section of the graph. (Multiply each recorded response by $\frac{1}{2}$.)

| Type of pet | Number recorded | Length of line (mm) |
|---|---|---|
| Dog | 78 | 39 |
| Cat | 65 | 32.5 |
| Bird | 39 | 19.5 |
| Rabbit | 26 | 13 |
| Other | 52 | 26 |
| Total | 260 | 130 |

**3** Draw a divided bar graph using a ruler to measure the lengths of each section.

**Favourite pets of Year 7 students**






# 9.4 Comparing parts of a whole with graphs

## Navigator

Answers page 679

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q12 | Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q11, Q12 | Q1, Q2, Q3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13 |

**Equipment required:** A protractor for Questions **8**, **10** and **11**, a computer containing Excel software or similar may be used for Question **13**

## Fluency

WE10

**1** For the data in each of the following tables:

**(i)** draw a sector graph

**(ii)** draw a divided bar graph.

**(a)** Belinda picked some vegetables from her garden and recorded the number of each type of vegetable.



| Vegetable | Number recorded |
|---|---|
| Tomatoes | 20 |
| Carrots | 12 |
| Cucumbers | 8 |
| Lettuce | 10 |

**(b)** A group of tourists was asked to state their nationality and their responses were recorded.

| Nationality | Number recorded |
|---|---|
| Italian | 22 |
| French | 18 |
| Japanese | 12 |
| Indonesian | 8 |

**(c)** The number of pizzas purchased one evening at Abracadabra Pizza Parlour was recorded.



| Pizzas | Number recorded |
|---|---|
| Super Supreme | 6 |
| Capricciosa | 8 |
| Hawaiian | 4 |
| Italiana | 3 |
| Australiana | 2 |

**(d)** The number of each make of car in the staff car park at a small secondary school was recorded.

| Car make | Number recorded |
|---|---|
| Ford | 4 |
| Holden | 5 |
| Mazda | 4 |
| Toyota | 3 |
| Mitsubishi | 2 |
| Other | 5 |



**2** For each of the following data tables:

**(a)** draw a sector graph

**(b)** draw a divided bar graph.

**(i)** Fifty people were asked their favourite music style.

| Music style | Rock & Roll | Hip Hop | Techno | Rap | Punk | Pop |
|---|---|---|---|---|---|---|
| Number | 10 | 9 | 6 | 3 | 5 | 17 |

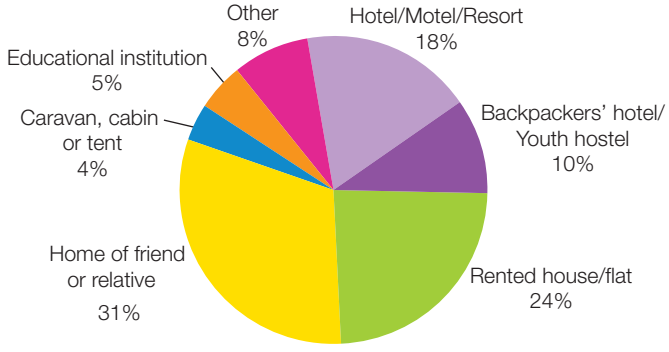
**(ii)** Seventy-five people were asked their favourite vegetable.

| Vegetable | Potato | Carrot | Pumpkin | Cabbage | Capsicum | Cauliflower |
|---|---|---|---|---|---|---|
| Number | 33 | 22 | 9 | 4 | 4 | 3 |

**(iii)** Justin has been working in a full-time job for 6 months. He was asked how he spent his money. He gave the following estimates.

| Item | Rent | Food | Entertainment | Clothes | Other |
|---|---|---|---|---|---|
| Percentage | 47 | 30 | 10 | 10 | 3 |

**3** The sector graph below represents information about the type of accommodation used by overseas visitors to Australia.




**(a)** One of the sectors uses an angle of 64.8°. Which one is it?

**A** Hotel/Motel/Resort **B** Backpackers' hotel

**C** Rented house/flat **D** Other

**(b)** The Educational Institution sector uses an angle closest to:

**A** 5° **B** 10° **C** 13° **D** 18°

4 As part of a project to tackle global warming at a local level, a Victorian local council conducted a survey to see what percentage of greenhouse gas emissions was produced by each of its facilities and services. The following data was recorded: Buildings 28%, Streetlights 47%, Waste 1%, Vehicle fleet 24%.

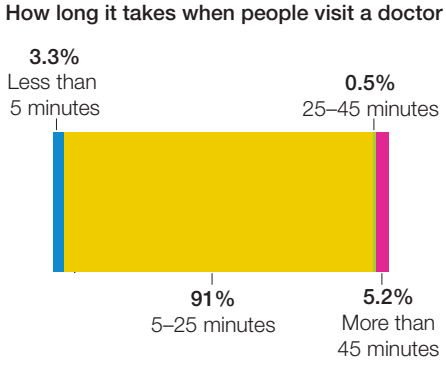
**(a)** Create a sector graph to display the data.

**(b)** Create a divided bar graph to display the data.

**(c)** Which graph do you find easier to understand?

## Understanding

5 The following graph shows how long people take when they visit a doctor.





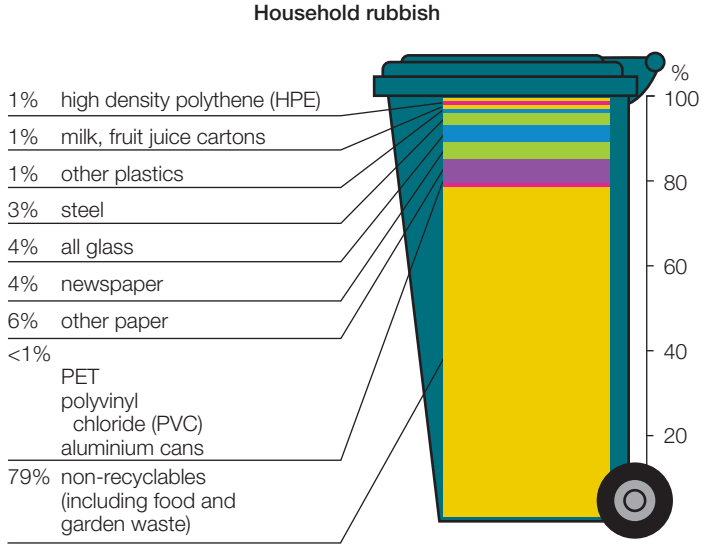

**(a)** What percentage of people spend less than 5 minutes when they visit a doctor?

**(b)** What percentage of people spend more than 45 minutes when they visit a doctor?

**(c)** How long do the majority of people take when they visit the doctor?

**(d)** What percentage of people spend less than 25 minutes when they visit the doctor?

6 Look at the following divided bar graph, which shows how much of different types of rubbish Australians throw out.




**(a)** What percentage of our rubbish is all glass?

**(b)** What percentage of our rubbish is milk and fruit juice cartons?

**(c)** What percentage of our rubbish is newspaper?

**(d)** **(i)** What is the largest category of rubbish?

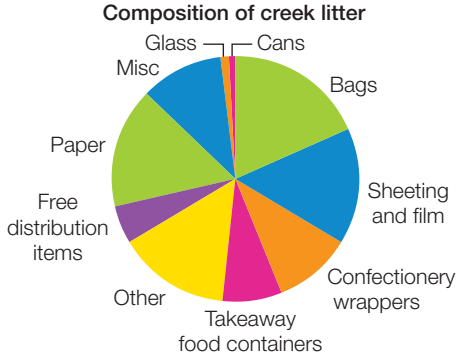
**(ii)** What percentage of rubbish is in this category?

**(e)** Do we throw out more high density polythene or more paper?

**(f)** Do we throw out more steel or more glass?

**(g)** Is the picture of the rubbish bin actually part of the graph?

**7** Look at the following sector graph showing the type of litter in a particular creek.




Total number of litter items collected = 2231

**(a)** What was the most common type of litter?

**(b)** Was there more cans or more sheeting and film?

**(c)** Was there more takeaway food containers or more confectionery wrappers?

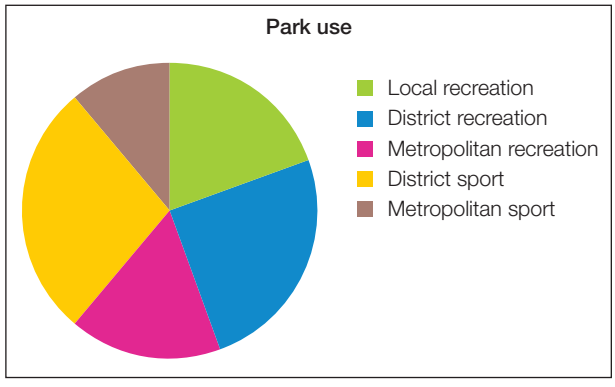
**(d)** When you compare this sector graph with the one in Question **3**, what is missing?

**(e)** Misc stands for miscellaneous. What kinds of rubbish would be classified as miscellaneous?

**(f)** We are told how many litter items were collected in the survey. Is this important? Why or why not?

**(g)** Why are sector graphs like divided bar graphs?

**8**




Above is a sector graph showing park use in a local council area of an Australian city.

**(a)** Use a protractor to find the angle size of each sector.

**(b)** Use your answer from part **(a)** to find the percentage of the park that is used for metropolitan recreation.

**(c)** Use your answer from part **(a)** to find the percentage of the park that is used for district sport, to the nearest whole number.

**(d)** Use your answer from part **(a)** to find the percentage of the park that is used for sport, to the nearest whole number.

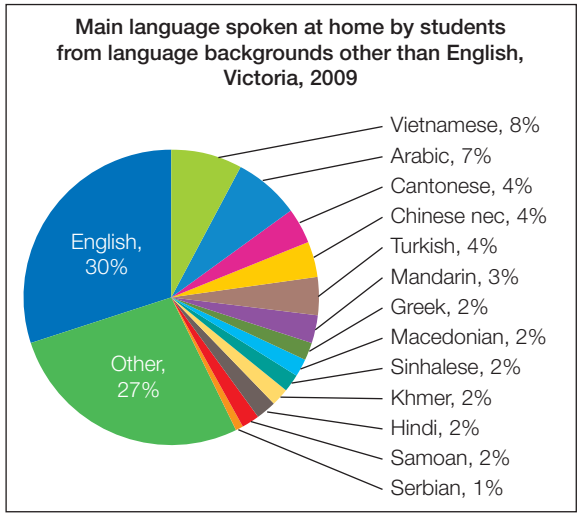
**(e)** Is more parkland used for sport or for recreation? State which is larger and by how much.

**(f)** What type of park use is represented by the smallest sector?

**(g)** Draw a divided bar graph of the data.

## Reasoning

**9** Using the graph below, answer the following questions.




**(a)** What is the main language spoken at home by students from non-English-speaking backgrounds?

**(b)** What is the total percentage of students whose main language spoken at home is some form of Chinese (Cantonese, Mandarin or Chinese nec (not elsewhere classified))?

**(c)** List all the languages that are spoken in 2% of students' homes.

**(d)** Do more students speak Turkish or Arabic at home?

**(e)** Why is the 'Other' section so large?

**(f)** Suggest some of the languages in the 'Other' sector.

**10** The following table shows vehicle sales for Australia in 2008, divided by state or territory.

| State/Territory | Number |
|---|---|
| ACT | 5 462 |
| New South Wales | 106 250 |
| Northern Territory | 3 756 |
| Queensland | 75 228 |

| State/Territory | Number |
|---|---|
| South Australia | 21 492 |
| Tasmania | 5 940 |
| Victoria | 89 826 |
| Western Australia | 39 560 |

**(a)** How many vehicles were sold in total in 2008?

**(b)** Find the sector angles that represent each state or territory (correct to the nearest degree).

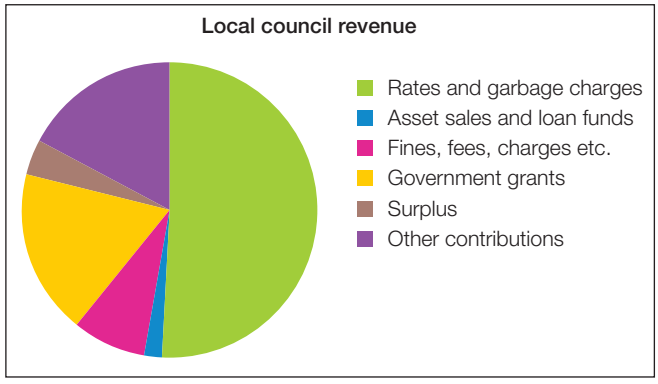
**(c)** Using a protractor, draw a sector graph for this information, clearly labelling each sector.

**(d)** If you were to draw a divided bar chart for this information, how long would you make your graph?

**(e)** Use this length to draw the divided bar chart and clearly show the length of each sector, correct to the nearest mm.

**(f)** Which graph do you think is easier to draw? Give a brief explanation to support your point of view.

**11**




The sector graph above shows the revenue (money) a local council expected to receive in the 2010/2011 financial year. Use this graph to answer the following questions.

**(a)** What is the major source of income for the council?

**(b)** Use a protractor to find the sector angle for the fines, fees, charges etc. sector.

**(c)** Use the angle you found in part **(b)** to find the percentage of revenue (to the nearest %) raised through fines, fees, charges etc.

**(d)** Does the council expect to receive more money from government grants or from other contributions? Explain your answer.

## Open-ended

**12** **(a)** Draw a sector graph to represent one characteristic of the students in your class.

**(b)** Draw a divided bar graph to represent a different characteristic of the students in your class.

**13** Generate sector graphs for Question **10** using Chart Wizard in Excel (or similar) if you have access to a computer. Compare your graphs with the computer-generated ones (you could use 3D graphs also) and comment on any differences.

## Outside the Square — Problem solving

### Missing pieces of the pie

**1** A sector graph has been drawn and labelled, but there is a 60° gap in it. This is to be divided between two pieces, *A* and *B*, so that *A* is twice the size of *B*. How big will the two pieces be? Give your answers correct to one decimal place if necessary.

**2** This time, the 60° piece is to be divided between three pieces, *A*, *B* and *C*, where *A* is twice as big as *B* and *B* is twice as big as *C*. How big will the three pieces be? Give your answers correct to one decimal place if necessary.

**3** This time, the 60° piece is to be divided between four pieces, *A*, *B*, *C*, and *D*, where *A* is twice as big as *B*, *B* is twice as big as *C* and *C* is twice as big as *D*. How big will the four pieces be? Give your answers correct to one decimal place if necessary.

**Strategy options**
- Guess and check.
- Look for a pattern.

# Half-time 9

Ex 9.2

**1** Find the **(i)** mean (correct to one decimal place) **(ii)** median and **(iii)** mode for each of the following sets of test results.

**(a)** 3, 2, 4, 8, 6, 2, 4, 6, 3, 2, 1 **(b)** 4, 6, 3, 4, 3, 6, 5, 9, 8, 7, 6, 3, 2, 4, 5, 6, 4

Ex 9.4

**2**

| Victoria's greenhouse gas emissions (end use allocation of emissions) | |
|---|---|
| Residential | 17% |
| Commercial | 16% |
| Manufacturing | 32% |
| Passenger transport | 14% |
| Primary production net | 12% |
| Freight transport | 5% |
| Waste | 3% |
| Other | 1% |

Using the table given above:

**(a)** draw a sector graph **(b)** draw a divided bar graph.

Ex 9.1

**3** The height of each of the 25 students in a class was recorded.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 142 | 147 | 151 | 142 | 146 | 151 | 157 | 138 | 141 |
| 140 | 129 | 135 | 142 | 151 | 150 | 160 | 141 | 152 |
| 148 | 157 | 130 | 143 | 147 | 149 | 154 | | |

**(a)** Why are class intervals of 125–129, 130–134 etc. appropriate?

**(b)** Using these class intervals, draw a frequency table showing this information.

Ex 9.1

**4** The number of students absent from class was recorded for each of the 21 school days in a month.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 4 | 1 | 0 | 1 | 0 | 2 | 0 |
| 1 | 0 | 0 | 2 | 1 | 3 | 0 | 1 | 2 | 0 | |

Construct a frequency table showing this information.

Ex 9.3

**5** Draw a dot plot to illustrate the following data that represents the number of times the students in a class have travelled interstate.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 4 | 0 | 1 | 2 | 3 | 1 | 2 | 2 | 0 | 0 | 0 |
| 1 | 2 | 4 | 5 | 3 | 2 | 5 | 3 | 0 | 1 | 1 | 2 |

Ex 9.3

**6** The number of marks taken by the top 12 players in the AFL after round 15, 2010, are given below.

86 74 76 91 132 84 67 80 82 62 85 90

**(a)** Draw a stem-and-leaf plot to represent this data.

**(b)** Find the median for this data set.

**(c)** Can you identify any possible outliers? Justify your answer.

# Technology Exploration Excel

**Equipment required:** 1 brain, 1 computer with an Excel spreadsheet

Versions of this Exploration are available for other technologies in Pearson Reader.

## Finding statistics using technology

When we have a very large data set it is best to use some form of technology to calculate statistics such as mean, mode, median and range. Here, we will use Excel to find some of these statistics.

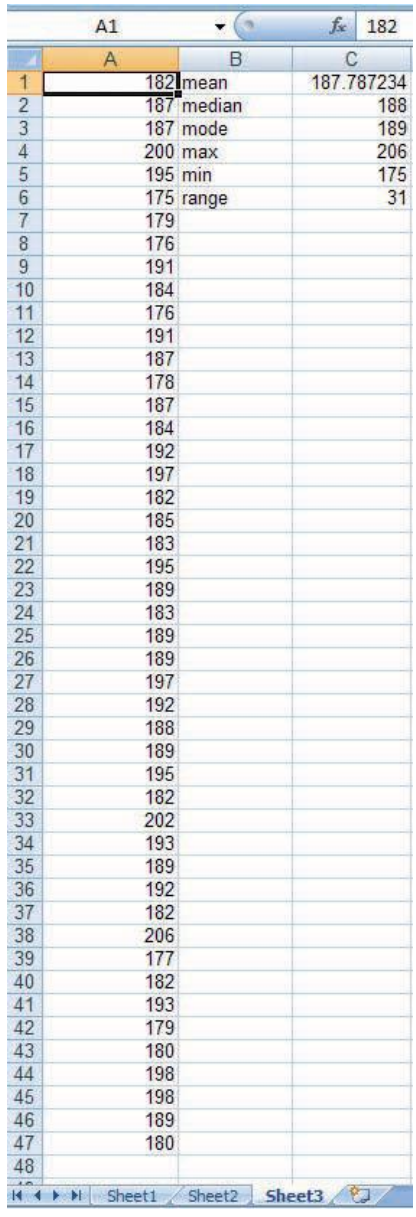
The following table shows the height, in cm, of the 47 players on the list for the Geelong Cats in the AFL for the 2010 playing season. We will use this data to find the mean, median, mode and range using Excel.

Note that Excel uses the word 'Average' to find the mean.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 182 | 187 | 187 | 200 | 195 | 175 | 179 | 176 | 191 | 184 |
| 176 | 191 | 187 | 178 | 187 | 184 | 192 | 197 | 182 | 185 |
| 183 | 195 | 189 | 183 | 189 | 189 | 197 | 192 | 188 | 189 |
| 195 | 182 | 202 | 193 | 189 | 192 | 182 | 206 | 177 | 182 |
| 193 | 179 | 180 | 198 | 198 | 189 | 180 | | | |

1. Open an Excel spreadsheet and enter the data in column A.
2. Enter the names of the statistics to be found in column B, Mean in B1, Median in B2 and Mode in B3.
3. As the range is the difference between the highest (maximum) value and the lowest (minimum) value, Excel is used to find the maximum and the minimum values. Enter Max in B4, Min in B5 and Range in B6.
4. In cell C1, enter the formula = Average (A1:A47) to find the mean.
5. In cell C2, enter the formula to find the median, in cell C3, enter the formula to find the mode, in cell C4, enter the maximum formula and, in cell C5, enter the minimum formula.
6. In cell C6, subtract the minimum value from the maximum value to find the range.

Any of these formulas can be found by selecting Formulas and Insert Function from the toolbar. The fomulas are entered as shown in the screenshot at the bottom right of the page.



| | A | B | C |
|---|---|---|---|
| 1 | 182 | mean | 187.787234 |
| 2 | 187 | median | 188 |
| 3 | 187 | mode | 189 |
| 4 | 200 | max | 206 |
| 5 | 195 | min | 175 |
| 6 | 175 | range | 31 |
| 7 | 179 | | |
| 8 | 176 | | |
| 9 | 191 | | |
| 10 | 184 | | |
| 11 | 176 | | |
| 12 | 191 | | |
| 13 | 187 | | |
| 14 | 178 | | |
| 15 | 187 | | |
| 16 | 184 | | |
| 17 | 192 | | |
| 18 | 197 | | |
| 19 | 182 | | |
| 20 | 185 | | |
| 21 | 183 | | |
| 22 | 195 | | |
| 23 | 189 | | |
| 24 | 183 | | |
| 25 | 189 | | |
| 26 | 189 | | |
| 27 | 197 | | |
| 28 | 192 | | |
| 29 | 188 | | |
| 30 | 189 | | |
| 31 | 195 | | |
| 32 | 182 | | |
| 33 | 202 | | |
| 34 | 193 | | |
| 35 | 189 | | |
| 36 | 192 | | |
| 37 | 182 | | |
| 38 | 206 | | |
| 39 | 177 | | |
| 40 | 182 | | |
| 41 | 193 | | |
| 42 | 179 | | |
| 43 | 180 | | |
| 44 | 198 | | |
| 45 | 198 | | |
| 46 | 189 | | |
| 47 | 180 | | |
| 48 | | | |

Sheet1 Sheet2 Sheet3

| B | C |
|---|---|
| mean | =AVERAGE(A1:A47) |
| median | =MEDIAN(A1:A47) |
| mode | =MODE(A1:A47) |
| max | =MAX(A1:A47) |
| min | =MIN(A1:A47) |
| range | =C4-C5 |

### Taking it further

7. The weights, in kg, of the 47 players on the playing list for the Geelong Cats in the AFL for the 2010 playing season are listed below.

   Use Excel (or another form of technology) to find the mean, median, mode and range of the weights.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 88 | 84 | 88 | 102 | 85 | 77 | 88 | 77 |
| 89 | 72 | 84 | 87 | 82 | 81 | 91 | 85 |
| 88 | 101 | 83 | 100 | 84 | 95 | 87 | 88 |
| 85 | 92 | 99 | 87 | 79 | 92 | 99 | 70 |
| 108 | 102 | 93 | 94 | 86 | 105 | 80 | 73 |
| 92 | 68 | 80 | 94 | 104 | 88 | 80 | |

# 9.5 Graphing bivariate data

**Bivariate data** consists of two variables, an independent variable and a dependent variable. The dependent variable changes as the independent variable changes. An example is the distance travelled by a car in a given time period. The time of the journey is the independent variable and the distance travelled is the dependent variable.

## Line graphs

A line graph is used when we have data that shows changes over time. Line graphs are useful if we are trying to identify a **trend** in the data. That means we are looking for a general pattern being followed that we could use to predict future values in the data set. Line graphs are used only for continuous data, such as mass, height, profits and temperatures. These are all data that can be measured over time.

### Drawing a line graph

All line graphs should include:

1. a title
2. the independent variable always on the horizontal axis
3. a scale across the horizontal axis that uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement
4. a scale on the vertical axis that also uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement.

When drawing line graphs you need to choose a suitable scale. The following data set will be used to illustrate the process.

### Worked Example 11 WE11

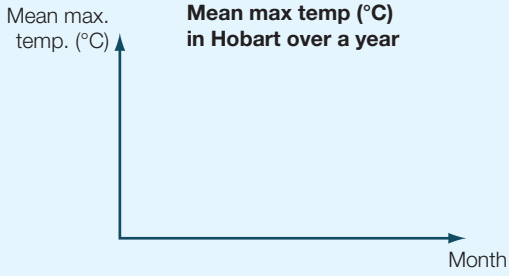
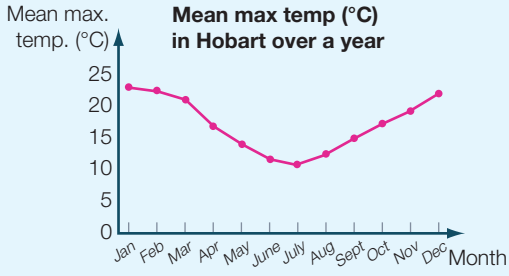
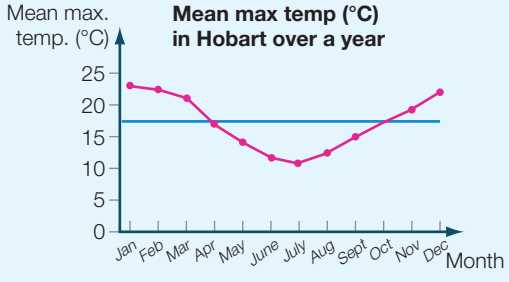
The mean daily maximum temperature (°C) for each month in Hobart (Botanical Gardens) is shown in the table below:

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean max. temp. (°C) | 23.1 | 22.2 | 20.7 | 17.0 | 13.9 | 11.4 | 10.5 | 12.2 | 14.9 | 17.4 | 19.4 | 21.9 |

Bureau of Meteorology

**(a)** Draw a line graph to represent this data.

**(b)** For which months was the mean maximum temperature below 17°C?

| Thinking | Working |
|---|---|
| **(a) 1** Look for the highest value to be graphed. | (a) Highest value = 23.1° |
| **2** Decide how tall you want your graph to be (a vertical height of 10 cm is good) and divide the maximum value by the chosen height to get an idea of the scale required. | Approximate scale $= \frac{23.1}{10} = 2.31$ |
| **3** Use this number to choose an appropriate scale. | Scale 1 cm = 2°C |
| **4** Draw the axes, label them and give the graph a title. |   |
| **5** Scale both axes and plot the pairs of values. (The first one to plot would be January, 23.1. Choose Jan on the horizontal axis and 23.1 on the vertical axis and plot the point.) Join each point to the next with a straight line. |   |
| **(b) 1** Draw a horizontal line across from the vertical axis for the given value (17°). | (b)   |
| **2** Find the points on the line graph under this line and read off the values on the horizontal axis that correspond to these points. | May, June, July, August, September |

# 9.5 Graphing bivariate data

## Navigator

Answers page 680

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q8, Q9, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 |

## Fluency

WE11

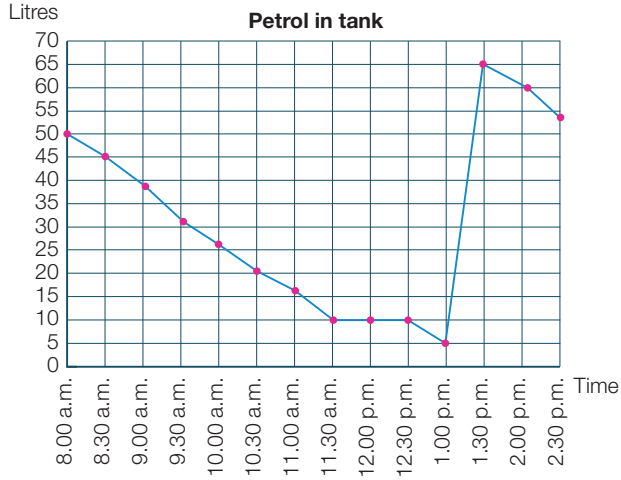
**1** At an athletics competition, the wind speed is measured so that new records can be accepted. The following table shows the wind speed (metres per second) at 10-minute intervals for the hour over which the 100 m sprints were run.

| Time | 1.00 p.m. | 1.10 p.m. | 1.20 p.m. | 1.30 p.m. | 1.40 p.m. | 1.50 p.m. | 2.00 p.m. |
|---|---|---|---|---|---|---|---|
| Wind speed (metres per second) | 0.9 | 0.95 | 1.3 | 1.2 | 1.15 | 1.1 | 0.95 |

**(a)** Draw a line graph to represent this data.

**(b)** When was the wind speed greater than 0.95 metres per second?

**2** The graph shows the number of litres of petrol in a car at various times during the day.



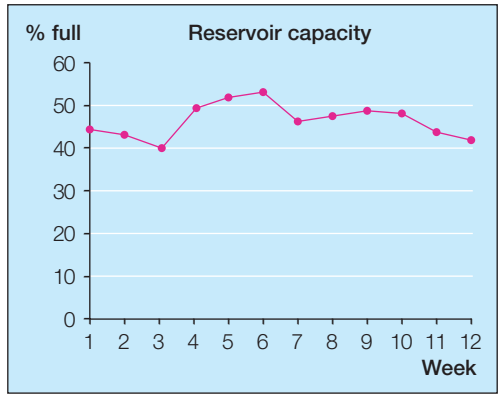

**(a)** How many litres were in the tank at the start of the day?

**(b)** When did the driver visit the petrol station?

**(c)** What was the minimum number of litres of petrol purchased?

**(d)** Between which times is it likely the driver had lunch?

**3** Each week, the amount of water in a reservoir is recorded as a percentage of its maximum capacity. The graph on the right shows the results for a number of weeks.




**(a)** How full was the reservoir when measured in Week 4?

**A** 40% **B** 45%

**C** 50% **D** 55%

**(b)** There was heavy rain during a particular week. The measurement for which week shows the result of this rain?

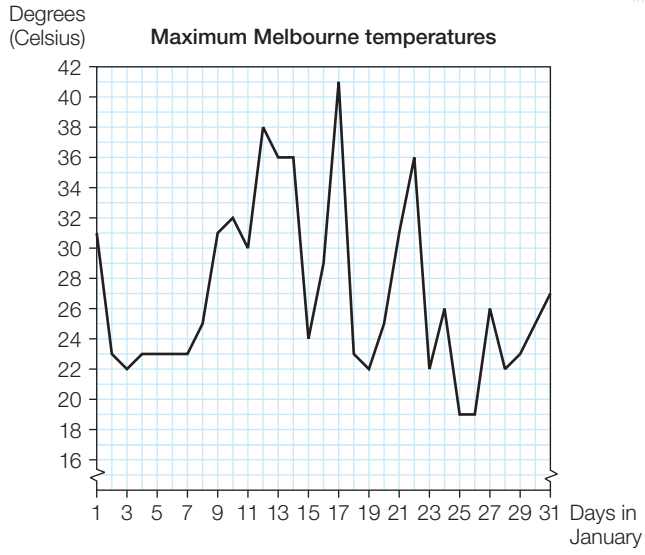
**A** 2 **B** 4 **C** 5 **D** 6

**(c)** A large quantity of water was released from the reservoir to flush out the downstream areas. The measurement for which week most likely shows the result of this action?

**A** 2 **B** 3 **C** 7 **D** 10

## Understanding

**4** The line graph opposite shows the maximum temperatures in Melbourne for January of a particular year.




**(a)** **(i)** What was the highest maximum temperature reached?

**(ii)** On which date was this temperature reached?

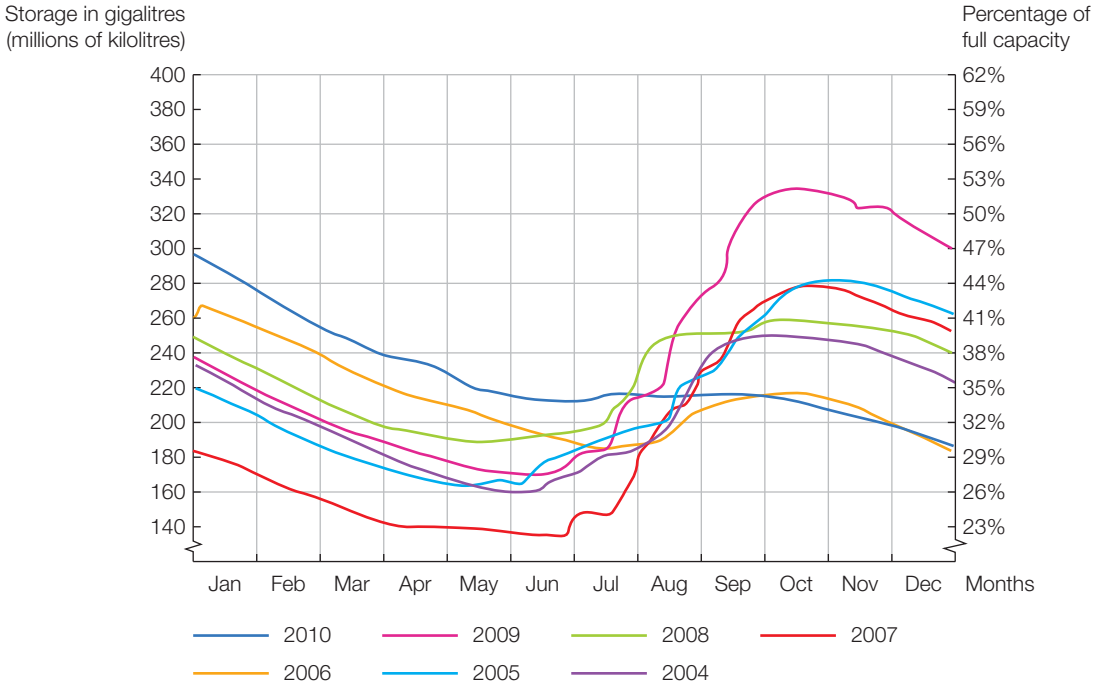
**(b)** **(i)** What was the lowest maximum temperature reached?

**(ii)** On which date(s) was this temperature reached?

**(c)** Up until 15 January, the maximum temperature was always between which two values?

**(d)** After 23 January, the maximum temperature was always between which two values?

**5**




Above is a graph of Perth's water storage from 2004 to 2010. Use this graph to answer the following questions.

**(a)** What was the storage percentage at the start and end of 2008?

**(b)** In which year was there almost no change in storage levels from the beginning of June to the end of September?

**(c)** Which 6-month period over the years 2004–2010 had the lowest water storage? What does this tell you about the rainfall in that period?

**(d)** By what percentage did the water storage levels decrease in December 2009? Check the percentage drop in water storage levels for other years in December. What does this tell you about the rainfall in December in Perth?

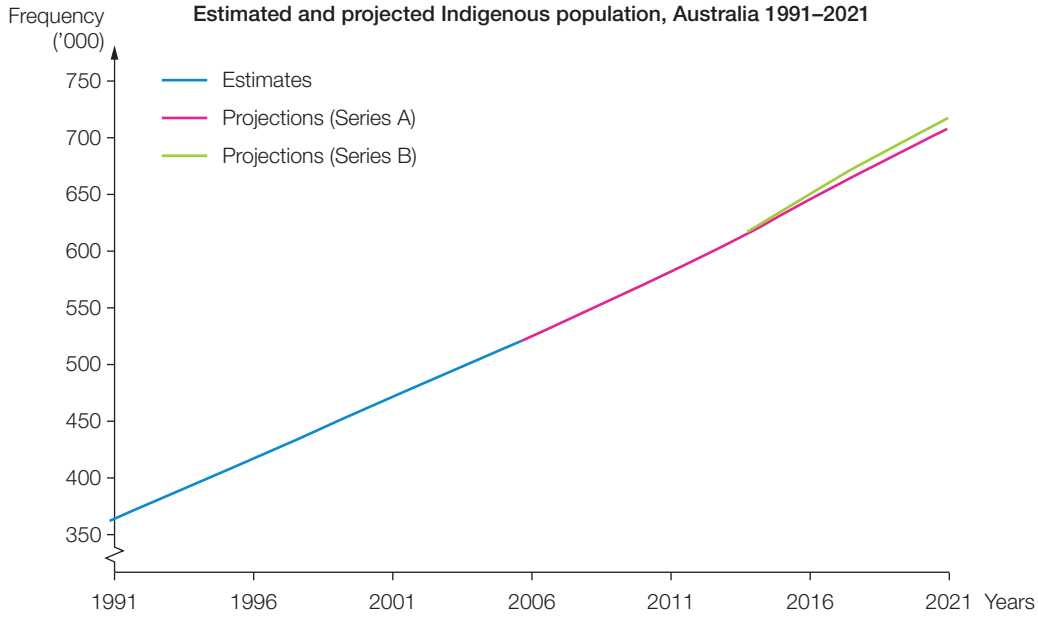
**(e)** Did the water storage levels increase or decrease over the year 2009? Find the increase or decrease percentage.

**(f)** From the data shown here, in which month is Perth likely to be the wettest?

**(g)** From the data shown here, in which month is Perth likely to be the driest?

## Reasoning

**6** This graph shows the Indigenous population of Australia from 1991 to 2006 and the projected (estimated) population for 2006 to 2021. Different projections are made using different estimation methods for future populations.




**(a)** What was the estimated Indigenous population of Australia in 1991?

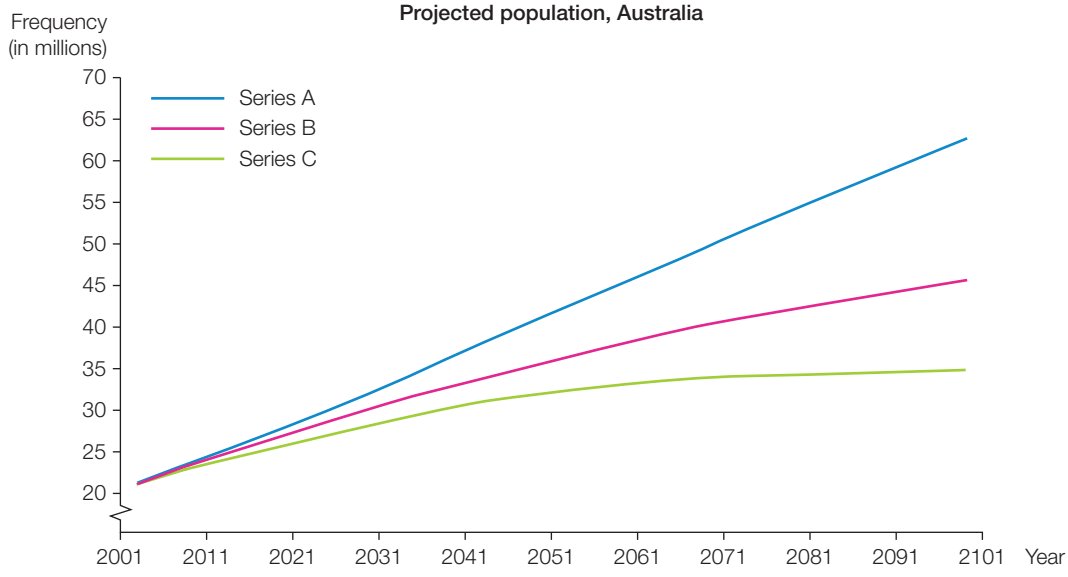
**(b)** What was the estimated Indigenous population of Australia 15 years later?

**(c)** Why do you think the graph from 1991 to 2006 is an estimated graph?

**(d)** What are the two projected values for the Indigenous population in 2021?

**(e)** Give two reasons why you think the Indigenous population has increased so much in the 30 years from 1991 to 2021.

**7** Statisticians use data from the past to predict trends for the future. Below are three graphs that show different ways of predicting (or projecting) the future population of Australia.




**(a)** Use the graph to find the population of Australia in 2007.

**(b)** What is the projected population of Australia in 2041 on the Series A graph?

**(c)** Find the increase in population predicted over the time period shown on the Series A projection.

**(d)** What are the two main factors that would cause the population to increase according to the Series A projection?

**(e)** List at least two problems Australia would face if the Series A projection was correct.

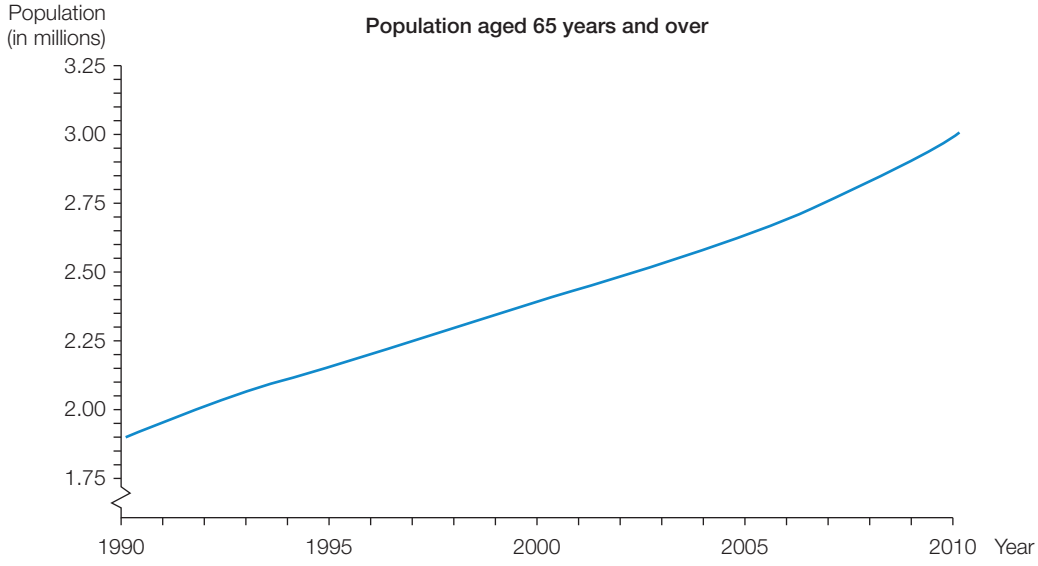
**(f)** What would be the projected population of Australia in 2061 on the Series B projection?

**(g)** If the number of births equals the number of deaths in the Series B projection, why would the population continue to grow?

**(h)** The Series C projection flattens out by 2081. What would be the projected population over the period 2081–2101?

**(i)** What needs to happen if the population of Australia is to stay stable (the same year after year)?

**8**




This line graph shows the number of Australians 65 years and older in the time period 1990–2010.

**(a)** In 2000, approximately how many Australians were 65 years or older?

**(b)** In which year did the number of Australians 65 years and older reach 2.75 million?

**(c)** Between which years does the graph follow an almost perfectly straight line? (Use a ruler to help you.)

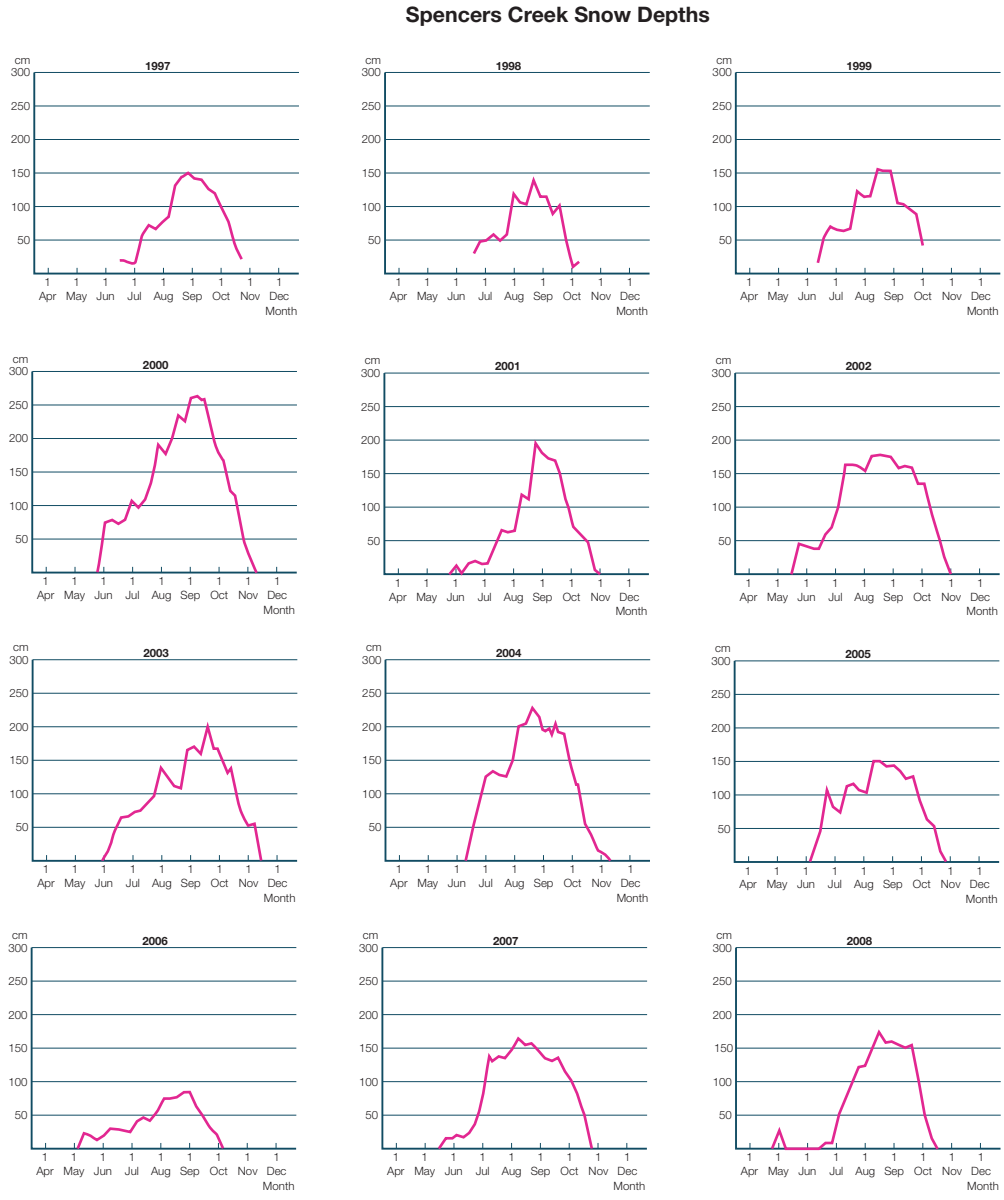
**(d)** Find the population aged 65 years and over in 2002, 2006 and 2010. Use these findings to describe the shape of the graph in the period 2002–2010.

**(e)** Use your answers to parts **(c)** and **(d)** to describe the shape of the graph overall.

**(f)** After World War II ended in 1945, Australia's birth rate increased dramatically. In addition, Australia's immigration policies have changed greatly since then, accepting immigrants from all around the world. Use these facts and your answer to part **(e)** to give two reasons why the shape of the graph has changed.

**(g)** What do you expect the population aged 65 years and over will be in 2015 if the present trend continues?

**9** Look at the following line graphs that show the average snow depths (in centimetres) at Spencers Creek, Mt Kosciuszko, from 1997 to 2008. (The deeper the snow, the better it is for skiing.)




**(a)** Did the snow depth reach 2.5 metres at any time during the 12 years? If so, which year(s) did this happen?

**(b)** What was the greatest snow depth reached in 1997?

**(c)** What was the greatest snow depth reached in 2008?

**(d)** Which year had the most snow in June?

**(e)** Which year had the most snow through the whole of July?

**(f)** Which year had the worst October skiing?

**(g)** Which year had the best October skiing?

**(h)** Which year would have probably had the best November skiing?

**(i)** Which year had the worst skiing overall?

**(j)** Which month usually has the best skiing?

**(k)** Which month is usually the second best skiing month?

**(l)** It has been claimed that New South Wales' overall temperature has been getting higher due to climate change. Is there any evidence in these graphs to back this up?

**(m)** Why do we use line graphs rather than bar graphs to show this information?



## Open-ended

**10** When do you think you feel hungriest during the day? Draw a line graph that shows your level of hunger during the day from the time you wake up until the time you go to sleep. Do you think the shape of the graph would be similar for most people? Explain any differences you can think of.

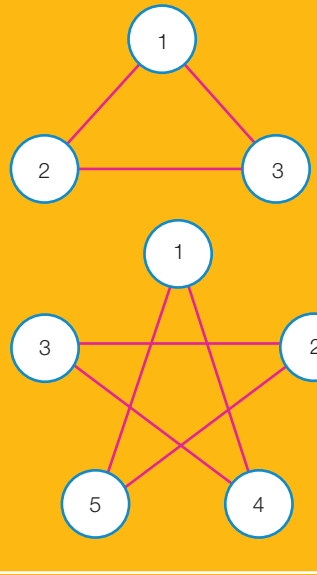
## Outside the Square Puzzle

### Add it up

A game board consists of the numbers 1, 2 and 3 arranged in a triangle as shown. The first player puts a counter on any of the three numbers and takes that number as their score. The other player then moves the counter to one of the other numbers and takes that as their score. The first player moves the counter and adds the value to their total. Play continues in this way until a player gets a total of 12. If you go over 12 you lose.

Explain the strategy you would follow if you were playing this game. You might like to play the game with a friend to help formulate your strategy.

Now, make up a strategy for the second game board shown. What would be a suitable target number for this board?



Maths 4 Real



# The 1-watt initiative

To reduce our carbon footprint (and to save money) we should try to reduce the amount of electricity we use in our households. The unit that measures how much electrical power we use is called the watt and has the symbol W.

When an electronic device such as a television is not on it is often not completely off either. It is usually in what we call standby mode, and can be turned on quickly, often by using a remote control. It has been estimated that standby power is responsible for 5–10 per cent of total electricity use in most homes.

**1** How many devices can you think of in your home that would be in standby mode when not being used? Make a list of them now.

## How big an impact does this have?

As a simple example, you probably have a microwave oven in your house that has a digital clock built into it. If you used the microwave for an average of 10 minutes each day to cook food, then about 21 per cent of its annual energy consumption would be just to display the clock!

The following table shows how much power some electronic devices use per hour.

| Appliance | Off (W) | Passive standby (W) | Active standby (W) | Delay start (W) |
|---|---|---|---|---|
| air conditioner | 0.2 | 1.7 | | 3.0 |
| clothes dryer | 0.2 | | 2.4 | 3.8 |
| dishwasher | 0.6 | | 2.7 | 3.0 |
| front-loading washing machine | 0.9 | | 3.5 | 4.0 |
| top-loading washing machine | 1.2 | | 3.0 | 3.5 |
| microwave oven | | 3.0 | | |
| laptop computer | 1.4 | | | |
| computer monitor | 0.9 | 1.9 | | |
| inkjet printer | 0.8 | 3.8 | | |
| LCD TV | 0.5 | 1.6 | | |
| DVD recorder | | 7.3 | 21.5 | |
| set-top box | | 8.4 | 13.5 | |

'Off' means the device is switched off at the device but not at the wall. Some appliances continue to use electricity even when you think they are off!

'Passive standby' means the device is not performing its main function but is ready to be switched on (in most cases with a remote control), or is performing a secondary function, like the clock on the microwave oven.

'Active standby' means the device is on but not performing its main function. For example, the DVD recorder may be on but is not playing or recording.

'Delay start' means the device has been programmed to start at a later time.

**2** As you go across the table, energy use increases. Suggest a reason for this.

## How much does all of this cost us?

As an example, we will look at an air conditioner in passive standby mode. According to the table this uses 1.7 W per hour, or 1.7 W h (watt hours). We can multiply this by 24 to get a daily use and then by 365 to get a yearly use.

$1.7 \times 24 \times 365 = 14\,892$ W h per year

To avoid using such large numbers we usually deal with kilowatt hours (kW h). 1000 W h = 1 kW h.

Now, let's assume electricity costs 12.2 c/kW h. The air conditioner uses 14.892 kW h, which costs us $14.892 \times 12.2 = 181.6824$ cents per year.

This is an annual cost of approximately $1.82. This may not seem like a lot but it is only one device and one of the more efficient ones. Imagine how much this would be for every air conditioner in Australia.

**3** Calculate the following annual costs.

**(a)** a clothes dryer on active standby

**(b)** an inkjet printer on passive standby

**(c)** a computer monitor switched off

**(d)** the difference in cost between a front-loading washing machine and a top-loading washing machine both switched off

**(e)** the total cost of an LCD TV connected to a DVD recorder and a set-top box all in passive standby mode

**4** In 2006 the Australian government introduced a policy stating that by 2012 all new appliances with a standby mode should use no more than 1 watt of electricity for the standby component. What would be the maximum annual standby cost per machine if the 1-watt initiative was implemented?

## Research

Find out more about James Watt, the person after whom the unit of power is named.

See how many electronic devices you have in your house that are on standby at any given time. Come up with some energy-saving (and therefore money-saving) suggestions for your home.

# 9.6 Comparing data sets

## Comparing column graphs

We can use the shape of column graphs to compare different variables. If the graph shapes are similar, there may be a connection between the variables. If the graph shapes are very different, we would consider that there was no connection.

### Worked Example 12 WE12

Rajit, Steve, Kenny and Dave are keen joggers. The tables below show some data on the four boys. Draw column graphs for each set of data and comment on the likelihood that there is a connection between any of the variables.

| | Distance jogged in a week (km) |
|---|---|
| Rajit | 20 |
| Steve | 28 |
| Kenny | 35 |
| Dave | 24 |

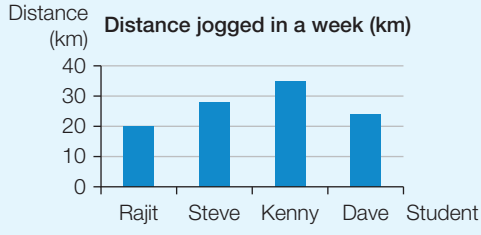
| | Height (cm) |
|---|---|
| Rajit | 152 |
| Steve | 160 |
| Kenny | 157 |
| Dave | 168 |

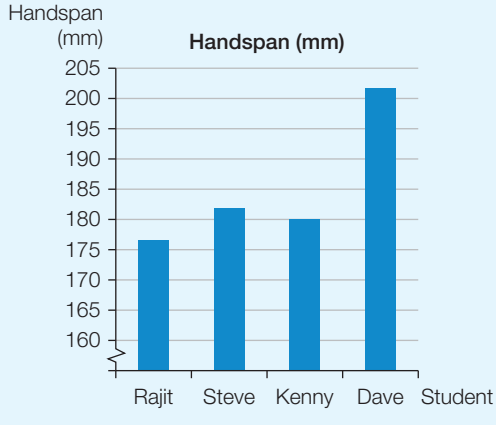
| | Handspan (mm) |
|---|---|
| Rajit | 177 |
| Steve | 182 |
| Kenny | 180 |
| Dave | 202 |

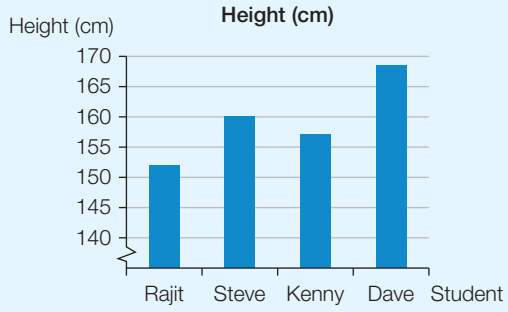
**Thinking**

**Working**

**1** Draw three separate column graphs.






**2** Look at the shape of the graphs and make a comment about similarities and differences.

*The shape of the graph of distance jogged each week is quite different from the shape of the other two graphs. There seems to be no connection between the distance they jog and either their height or their handspan.*

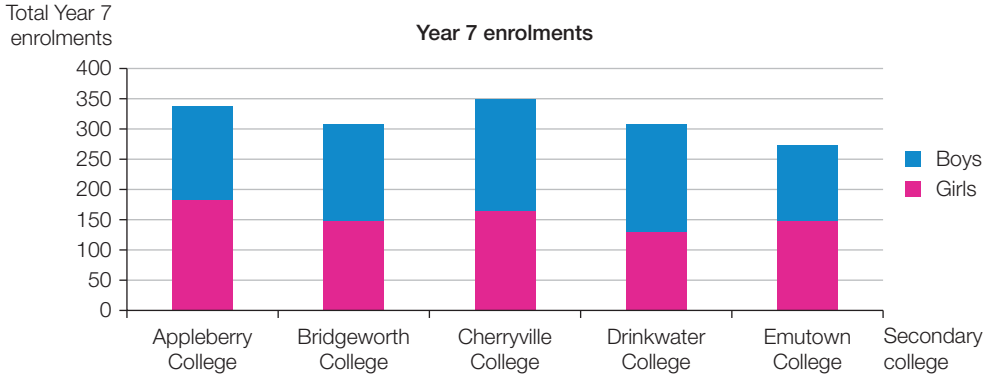
*The shapes of the graphs showing the height of the students and their handspan are very similar, so there could be a connection between these variables.*

## Comparing divided column graphs

We often use several divided column graphs to compare data.

## Worked Example 13

WE13




The graph above shows the Year 7 enrolment in five secondary colleges. Use the graph to answer the following questions.

**(a)** Which school has the highest Year 7 enrolment and what is its total Year 7 enrolment?

**(b)** Which part of the graph displays the number of girls and which school has the highest number of girls enrolled?

**(c)** Which school has the lowest number of boys enrolled?

**(d)** Which school has almost the same number of boys as girls?

**(e)** Which schools have about the same number of students enrolled in Year 7?

| Thinking | Working |
|---|---|
| **(a)** Identify the variable that has the longest column and find the number that corresponds to the length of the column. | *(a) Cherryville College*<br>*350* |

| | |
|---|---|
| **(b)** Identify which part of the column corresponds to the data required and find the column in which this part of the column is longest. | (b) The lower part of each column corresponds to girls.<br>Appleberry College has the highest number of girls enrolled. |
| **(c)** Identify which part of the column corresponds to the data required and find the column in which this part of the column is shortest. | (c) Emutown College has the lowest number of boys enrolled. |
| **(d)** Identify a column where the lengths of each part of the column are about the same. | (d) Bridgeworth College has about the same number of boys and girls enrolled. |
| **(e)** Identify all the columns that have the same length. | (e) Bridgeworth College and Drinkwater College have the same number of students. |

In Section 9.2, we found measures of centre to give us information about data sets. However, they do not give us the complete picture. Consider the following data sets.

Set A: 11, 12, 14, 14, 14, 16, 17 Set B: 1, 2, 14, 14, 14, 26, 27

In both of these sets, the mean = median = mode, but they are clearly different data sets. To help us get a better picture of the data, we need to use the range. In Section 9.1, the range was defined as the difference between the highest and the lowest values in the data set.

For Set A: range = 17 – 11 = 6 For Set B: range = 27 – 1 = 26

As the range is larger in Set B, we can see that the values are spread out much more than in Set A.

## Using tables of information

### Worked Example 14 WE14

The following table gives the monthly mean maximum temperatures, measured in °C, for Adelaide (Kent Town) and Brisbane (Brisbane Aero).

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Adelaide | 29.1 | 29.3 | 26.3 | 22.6 | 19.0 | 16.1 | 15.3 | 16.7 | 19.0 | 21.8 | 25.1 | 27.0 |
| Brisbane | 29.0 | 29.1 | 28.0 | 26.0 | 23.6 | 21.3 | 20.9 | 21.8 | 24.2 | 25.2 | 26.8 | 28.3 |

Compare the monthly mean maximum temperatures for the two cities using measures of centre and the range.

| Thinking | Working |
|---|---|
| **1** Calculate the mean, median and range (correct to one decimal place) in the normal way. | |

| | Mean | Median | Range |
|---|---|---|---|
| Adelaide | 22.3 | 22.2 | 14.0 |
| Brisbane | 25.4 | 25.6 | 8.2 |

| | |
|---|---|
| **2** Compare the means and medians for the two cities and use them to describe any differences or similarities. Also, compare the ranges to explain the pattern of temperatures in each city over the year. | Brisbane is hotter according to both the mean and the median temperatures, although both cities have a very similar highest mean maximum temperature. The range of temperatures is much greater in Adelaide, which means there is a much greater change in the mean maximum temperature from summer to winter than in Brisbane. |

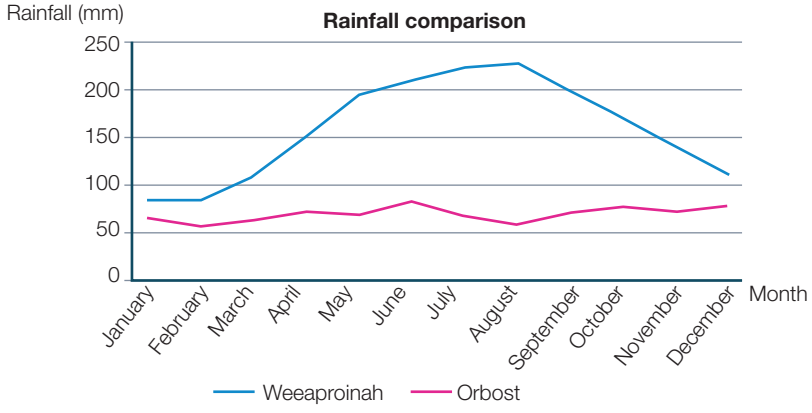
## Using graphical data

If we are presented with a graph we can make some reasonable estimates for the key statistics in which we are interested. The range is the easiest to estimate as you just need to find the difference between the highest and lowest values of each data set. For the median, estimate the value of each data point from the graph, order these values and find the middle value. For the mean you need to draw a horizontal line that would flatten out the graph without changing the mean. Look at the distance between each data value and the corresponding point on the horizontal line. The sum of these distances should be approximately zero.

## Worked Example 15

WE15

Use the following graph to compare the monthly mean rainfall (mm) for the two Victorian towns of Weeaproinah and Orbost. Estimate the mean, median and range to assist your comparison.




### Thinking

**1** To find the range, look at the highest and lowest value for each data point in the set. To find the median, use the graph to find the middle value. (Here, we find the 6th and 7th lowest rainfall for April and October and average them.) To find the mean, draw a horizontal line that would flatten out the line graph. (Here, if a horizontal line was drawn for rainfall = 160 mm and the difference each month was calculated between each value and 160, the sum of these differences would be approximately zero.)

### Working

| | Mean | Median | Range |
|---|---|---|---|
| Weeaproinah | 160 | $\frac{150 + 170}{2}$ = 160 | 230 – 90 = 140 |
| Orbost | 70 | $\frac{75 + 75}{2}$ = 75 | 85 – 55 = 30 |

2 Compare the means and medians for both sets of data and use them to describe any differences or similarities. Compare the ranges to explain what the shapes of the graphs are telling you about the data.

*Weeaproinah is clearly much wetter than Orbost with both the mean and median for Weeaproinah being more than twice the mean and median for Orbost. The range for Weeaproinah is also much larger as it is much wetter in winter than in summer, whereas there is not much difference from month to month in Orbost's rainfall data.*

# 9.6 Comparing data sets

## Navigator

Answers page 681

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13 | Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14 |

## Fluency

WE12

1 Veena, Fay, Kylie and Deanne are keen swimmers. The tables below show some data on the four girls. Draw column graphs for each set of data and comment on the likelihood that there is a connection between any of the variables.

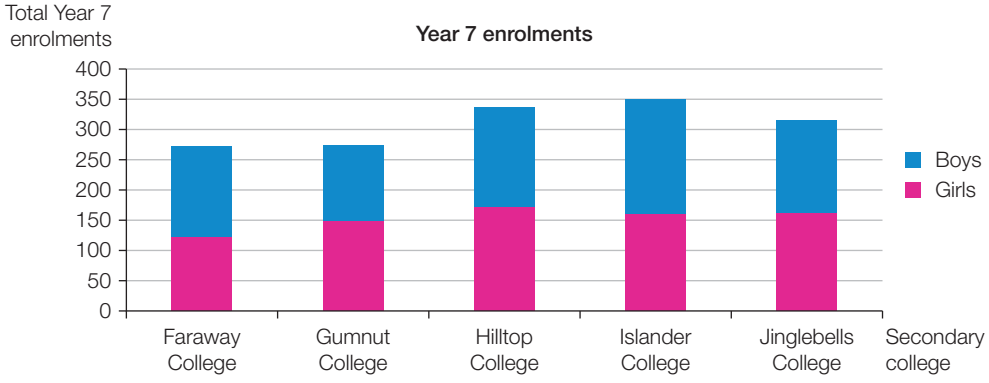
| | Hours spent training each week |
|---|---|
| Veena | 15 |
| Fay | 20 |
| Kylie | 24 |
| Deanne | 18 |

| | Height (cm) |
|---|---|
| Veena | 147 |
| Fay | 153 |
| Kylie | 150 |
| Deanne | 162 |

| | Foot length (mm) |
|---|---|
| Veena | 217 |
| Fay | 236 |
| Kylie | 230 |
| Deanne | 242 |

WE13

2




The graph above shows the Year 7 enrolment in five secondary colleges. Use the graph to answer the following questions.

**(a)** Which school has the highest Year 7 enrolment and what is its total Year 7 enrolment?

**(b)** Which part of the graph displays the number of girls and which school has the highest number of girls enrolled?

**(c)** Which school has the lowest number of boys enrolled?

**(d)** Which school has almost the same number of boys as girls?

**(e)** Which schools have about the same number of students enrolled in Year 7?

**3** The following table gives the monthly mean minimum temperatures, measured in °C, for Adelaide (Kent Town) and Brisbane (Brisbane Aero).

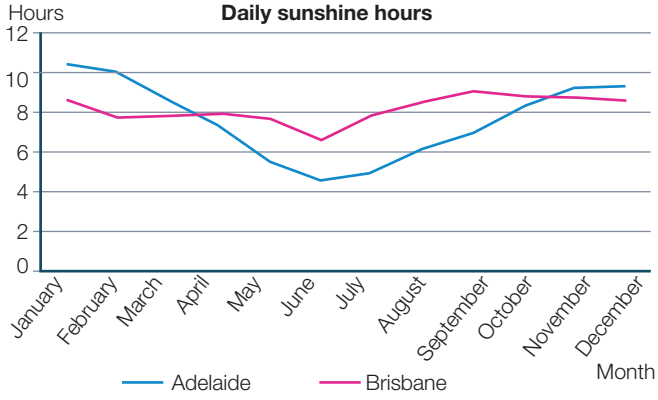
WE14

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Adelaide | 17.0 | 17.1 | 15.2 | 12.4 | 10.2 | 8.1 | 7.5 | 8.2 | 9.7 | 11.4 | 14.0 | 15.5 |
| Brisbane | 21.1 | 21.1 | 19.3 | 16.2 | 12.8 | 10.5 | 8.8 | 9.6 | 12.7 | 15.7 | 18.0 | 20.1 |

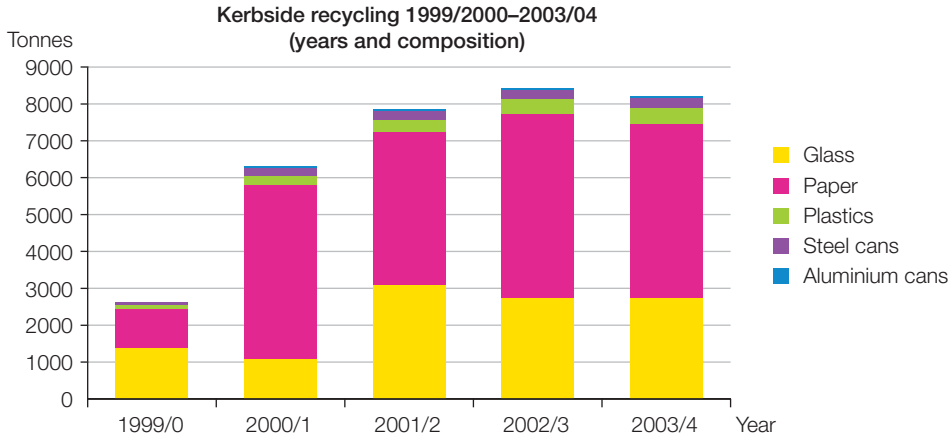
Compare the monthly mean minimum temperatures for the two cities using measures of centre and the range.

**4** Use the following graph to compare the monthly mean sunshine (hours) for Adelaide (Airport) and Brisbane (Airport). Estimate the mean, median and range to assist your comparison.

WE15




**5**




The graph above shows the types of material found in kerbside recycling by Shoalhaven Council over the time period 1999/2000–2003/04. Use the graph to answer the following questions.

**(a)** What material (by weight) was recycled the most in 1999/2000?

**(b)** What material (by weight) was recycled the most over the whole time period shown?

**(c)** Which time period was the best for recycling?

**(d)** Which was the best time period for glass recycling?

**(e)** Comment on the recycling of steel cans in the time period 2001–2004.

**(f)** Comment on the recycling of plastics over the time period shown.

## Understanding

**6** Compare the following data sets using as many statistics as possible.

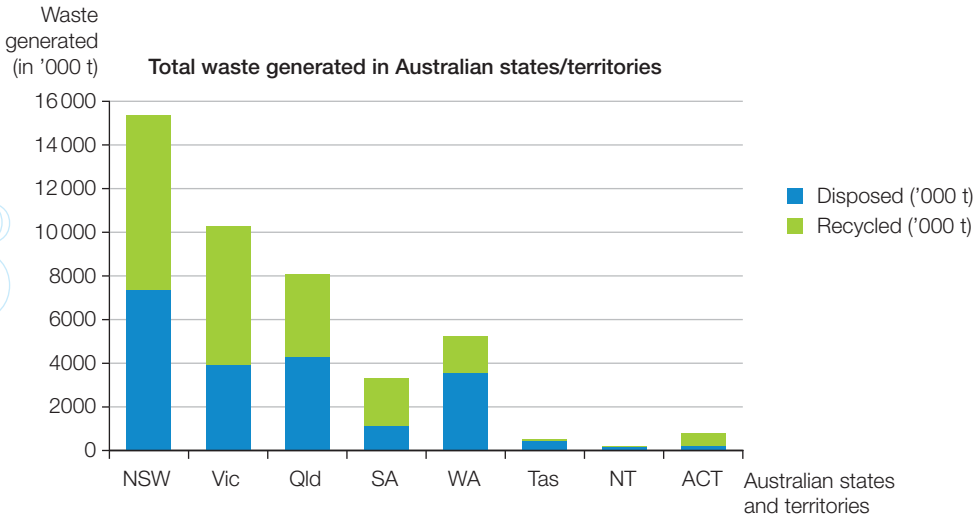
**(a)** Set A: 1, 2, 4, 5, 6, 7, 7, 7, 8, 10 Set B: 2, 2, 2, 2, 4, 5, 8, 9, 9, 9, 9

**(b)** Set A: 2, 4, 1, 5, 6, 2, 3, 7, 8, 2, 4, 1 Set B: 5, 2, 7, 4, 7, 1, 8, 3, 9, 8, 9, 6

7 April went to the market and purchased 5 boxes of bananas at $40 a box, 3 boxes of apples at $55 a box and a box of mangoes at $65 a box. Her friend Izzy bought 3 boxes of bananas at $45 a box, 3 boxes of apples at $50 a box and a box of mangoes for $70. Compare the purchases of the two friends using as many statistics as possible.

## Reasoning

8




Above is a graph of the estimated total waste generated in Australia 2006/07. The divisions show the amount of waste disposed of (sent to landfill) and the amount of waste recycled. Use the graph to answer the following questions.

**(a)** Which state/territory generated the most waste?

**(b)** How much waste did Queensland generate?

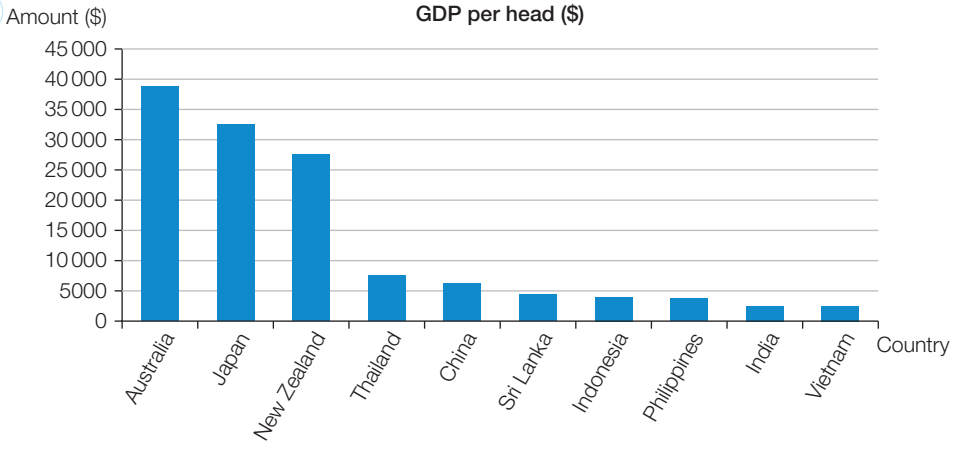
**(c)** Which states/territories recycled more than they disposed of?

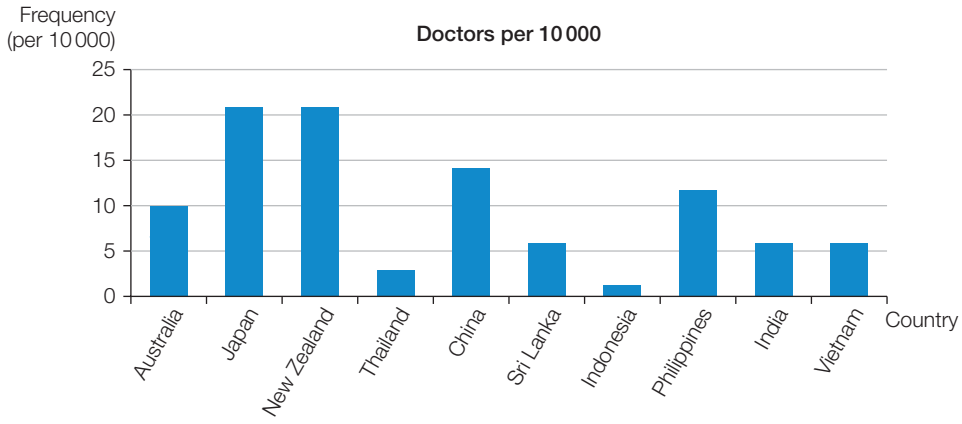
**(d)** Which state/territory recycled the largest proportion of its waste?

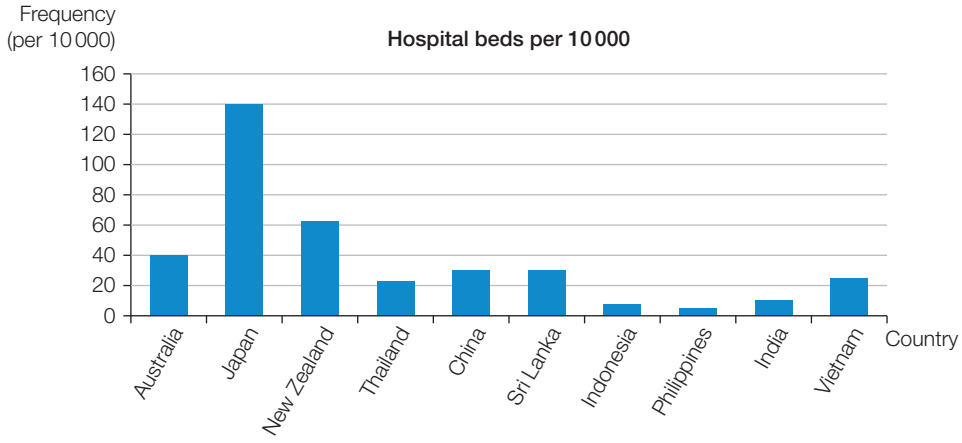
**(e)** Which state/territory recycled the smallest proportion of its waste?

**(f)** Did South Australia or Victoria recycle the larger proportion of its waste?

9






The Gross Domestic Product (GDP) of a country is one measure of a country's wealth. The number of doctors and hospital beds are measures of the health-care facilities of a country. Use the graphs above to answer the following questions.

**(a)** Do any of the graphs have a similar shape?

**(b)** Which country appears to have the best health-care facilities? Give a reason for your answer.

**(c)** Compare the health-care facilities of this country with its GDP per person.

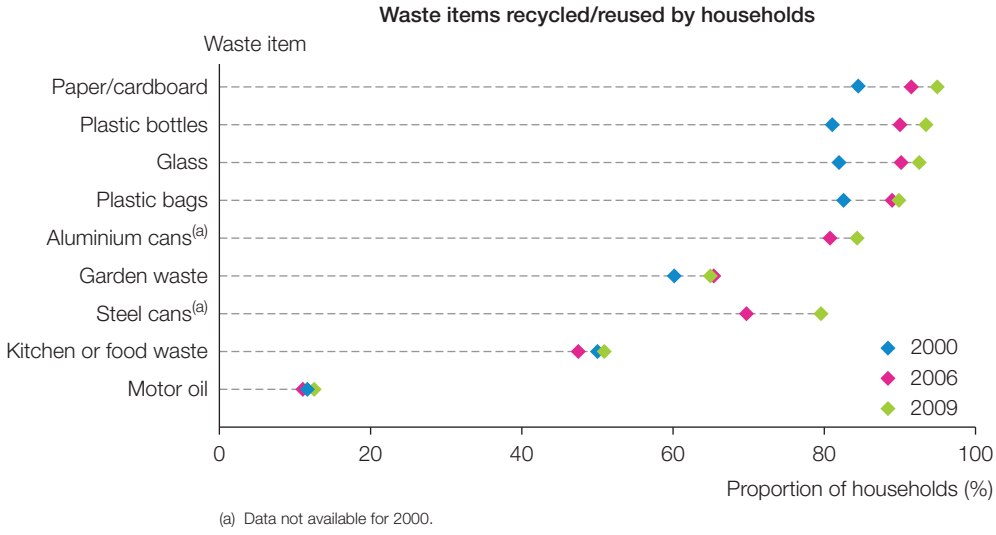
**(d)** Which country appears to have the worst health-care facilities?

**(e)** Compare the health-care facilities of this country to its GDP per person.

**(f)** Comment on Austalia's health-care facilities compared with its GDP per person.

**(g)** Comment on Vietnam's health-care facilities compared with its GDP per person.

**10** This graph shows different types of waste items and the percentage of Australian households that recycle them.




(a) Data not available for 2000.

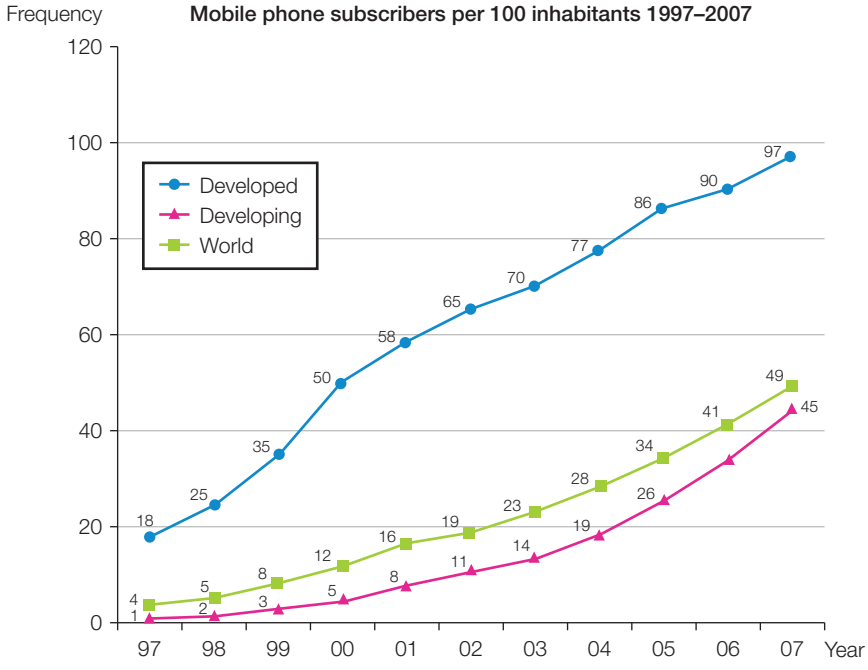
**(a)** **(i)** In 2009, what were the top three types of waste recycled by Australian households?

**(ii)** Suggest a reason for the answer to **(i)**.

**(b)** Which item had a large increase in the level of recycling between 2006 and 2009?

**(c)** Which three items had the most significant increase in the level of recycling between 2000 and 2009?

**11**




This graph shows the number of people out of every hundred who have a mobile phone, worldwide. The 'developed' countries include wealthier countries such as the United States, Australia, Japan and much of Europe. The 'developing' countries include poorer countries, including many in Africa and Asia.

**(a)** Compare the number of people who owned a mobile phone in developed and developing countries in the year 2000.

**(b)** What happened in the developed world over the years 2000–2007 to the number of people who own a mobile phone?

**(c)** What happened in the developing world in the years 2000–2007 to the number of people who own a mobile phone?

**(d)** Why is the world graph closer in shape to the developing world graph than the developed world graph?

**(e)** In which year did the biggest jump in mobile phone ownership in the developed world occur?

**(f)** If the trend of mobile phone ownership in the developing world shown by the graph has continued over the years since 2007, what do you think the number of people per 100 who own a mobile phone would have been in 2010?

**12** The following table shows the number of members for 16 AFL teams for 3 different years. (The V in brackets shows Victorian-based teams.)

| | 2009 | 2005 | 2000 |
|---|---|---|---|
| Adelaide | 46 472 | 43 256 | 42 896 |
| Brisbane Lions | 24 873 | 28 913 | 20 295 |
| Carlton (V) | 42 408 | 33 534 | 27 571 |
| Collingwood (V) | 45 972 | 38 612 | 28 932 |
| Essendon (V) | 40 412 | 32 734 | 34 278 |
| Fremantle | 39 206 | 34 178 | 24 925 |
| Geelong (V) | 37 160 | 30 821 | 25 595 |
| Hawthorn (V) | 52 496 | 29 261 | 26 879 |
| Melbourne (V) | 31 506 | 24 805 | 18 227 |
| North Melbourne (V) | 28 340 | 24 154 | 22 156 |
| Port Adelaide | 30 605 | 36 834 | 34 925 |
| Richmond (V) | 36 981 | 28 029 | 26 869 |
| St Kilda (V) | 31 906 | 32 043 | 17 855 |
| Sydney Swans | 26 269 | 24 955 | 30 177 |
| West Coast Eagles | 43 927 | 42 406 | 38 868 |
| Western Bulldogs (V) | 28 125 | 21 974 | 18 056 |



**(a)** Compare the club membership statistics across the 3 years. What comment could you make about the number of people who are members of AFL clubs?

**(b)** Why is the mode not useful at all in these comparisons?

**(c)** For 2009 only, compare the membership numbers for the Victorian-based teams against the non-Victorian-based teams. On the basis of this comparison, what comment could you make about the popularity of AFL clubs.

**(d)** Comparing 2000 to 2009, which club has been most successful at increasing its membership base?

**(e)** Is there any other information you would like access to before stating which club has been the most successful at turning its fans into members?

## Open-ended

**13** Sahrita measured the heights, in metres, of the 10 girls in her mathematics class. They were:

1.68 1.55 1.67 1.64 1.43 1.77 1.67 1.45 1.59 1.66

**(a)** Work out the mean, median and modal heights.

**(b)** Two new girls joined the class. The modal height did not change, but the mean changed by 0.15 m. Find two possible pairs of values for the heights of the new girls.

**(c)** Using the two pairs of values you used in **(b)** calculate the new median height.

**14** Eric is comparing two sets of data. He has written that the mean of Set A is 1.5 greater than that for Set B, but the median for Set A is 2 less than the median for Set B. The mode and the range are the same for both sets. Construct two data sets that meet these descriptions, given that there are at least 15 results in each data set.

# Outside the Square Puzzle

## Number puzzles B

**1** In each of the following sets of three diagrams one of the numbers is missing. To find it you will need to work out the connection between the numbers in the other two parts of the diagram. As an example:

| 106 | 224 | 333 |
|---|---|---|
| 51 | 169 | ? |

the ? here would be replaced by 278, as 106 – 51 = 55 and 224 – 169 = 55, so we need the number 333 – 55, which is 278.

**(a)**

| 405 | 549 | 693 |
|---|---|---|
| 45 | 61 | ? |

**(b)**

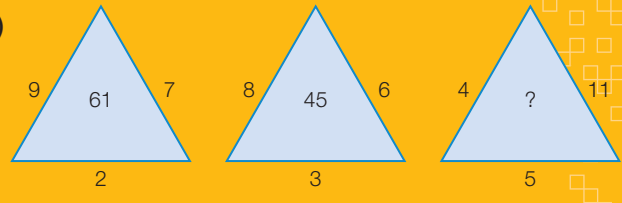
| 275 | 638 | 1001 |
|---|---|---|
| 25 | 58 | ? |

**(c)**

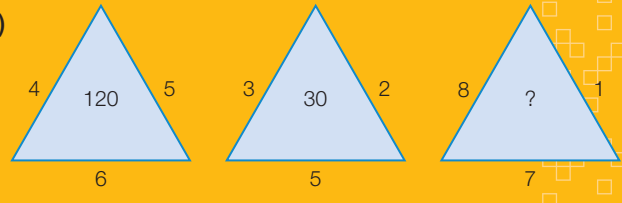
| 301 | 413 | ? |
|---|---|---|
| 43 | 59 | 76 |

**2** On each of the following sets of three triangles there is a number missing. To find it you will need to work out the connection between the outside numbers and the inside number in each of the first two triangles.

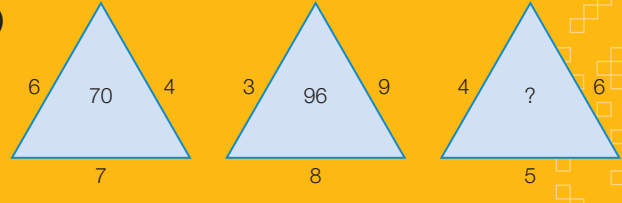
**(a)**




**(b)**




**(c)**



# Probability and sample space

9.7

We use the words **probability** and **chance** to mean the same thing.

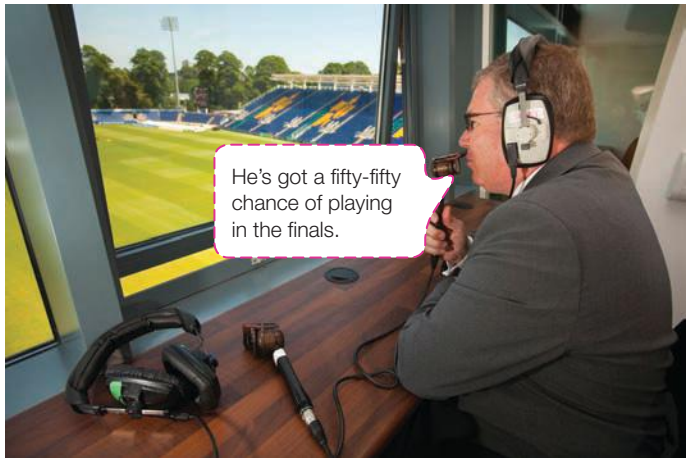
We hear and make statements that involve probability all the time.









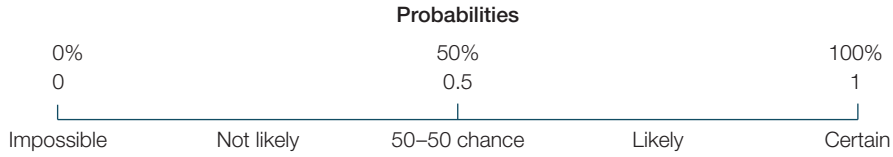




## Theoretical probability

Probabilities can be written as fractions or as decimals from 0 to 1, written in simplest form, or as a percentage from 0% to 100%.

- Events that have no chance of happening (impossible) have a probability of 0 or 0%.
- Events that are certain to happen have a probability of 1 or 100%.
- Events that have a 50–50 chance of happening have a probability of $\frac{1}{2}$ or 0.5 or 50%.

There is a whole range of probabilities between 0 and 1. The easiest way to think of it is as a number line where the closer you get to 0 the less likely an event is to occur and the closer you get to 1 the more likely an event is to occur.




Whenever we determine a probability without actually doing an experiment and collecting data, we are working with **theoretical probability**.

We often use Pr as an abbreviation for probability. The event is then written in brackets. So, if we were spinning a spinner that had different coloured sections, Pr(white) would be written to stand for the probability of the pointer landing in a white section.

To find the probability of an event occurring we do the following calculation.

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

## Sample space

To determine the probability of a particular event occurring, it is useful to list the **sample space**. The sample space is the complete list of all the possible **outcomes** of a probability experiment, such as rolling a die, tossing a coin, or selecting a card from a pack.

When the outcomes of an experiment are equally likely, we can use the sample space to determine the probability of a particular event occurring. A favourable outcome is one where the event for which we are calculating the probability occurs.

## Worked Example 16 WE16

Find the probability of:

**(a)** rolling an odd number with a normal die

**(b)** selecting a vowel (a, e, i, o, u) from a bag containing 26 identical pieces of paper, each containing a different letter of the alphabet.

| Thinking | | Working |
|---|---|---|
| **(a)** **1** | List the sample space and count the number in the sample space. | *(a) Sample space: 1, 2, 3, 4, 5, 6*<br>*There are 6 outcomes in the sample space.* |
| **2** | Identify the favourable outcomes and count the number of favourable outcomes. | *Favourable outcomes are: 1, 3 and 5.*<br>*There are 3 favourable outcomes.* |
| **3** | Find the probability using the rule<br>$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$<br>and simplify the fraction if possible. | $\Pr(\text{odd number}) = \frac{3}{6}$<br>$= \frac{1}{2}$ |

| | | |
|---|---|---|
| **(b)** **1** | List the sample space and count the number in the sample space. | *(b) Sample space: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z*<br>*There are 26 outcomes in the sample space.* |
| **2** | Identify the favourable outcomes and count the number of favourable outcomes. | *Favourable outcomes are: a, e, i, o, u*<br>*There are 5 favourable outcomes.* |
| **3** | Find the probability using the rule<br>$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$<br>and simplify the fraction if possible. | $\Pr(\text{vowel}) = \frac{5}{26}$ |

## Fair games

There are many sorts of games. Some rely mainly on skill to win, such as chess. Some require a mixture of skill and luck, such as poker. Some rely purely on luck, such as lotteries.

When we play a game we expect it to be fair. By this, we mean that each person playing has the same chance of winning. Rules are written to make games fair.

We can use the sample space to help us decide whether a game is fair. For instance, if the game consists of rolling a single die, then the sample space consists of the numbers 1, 2, 3, 4, 5, 6, as these are the events that can occur when the die is rolled. If one player wins if an even number is rolled and the other player wins if a multiple of 3 is rolled, then the game is not fair as the first player has three chances of winning (2, 4 and 6) out of 6, whereas the second player has only two chances (3 and 6) out of 6.

# 9.7 Probability and sample space

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5 (a), Q7, Q9, Q10, Q11 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11 | Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q12 |

Answers page 683

**Equipment required:** A die for Question **6**

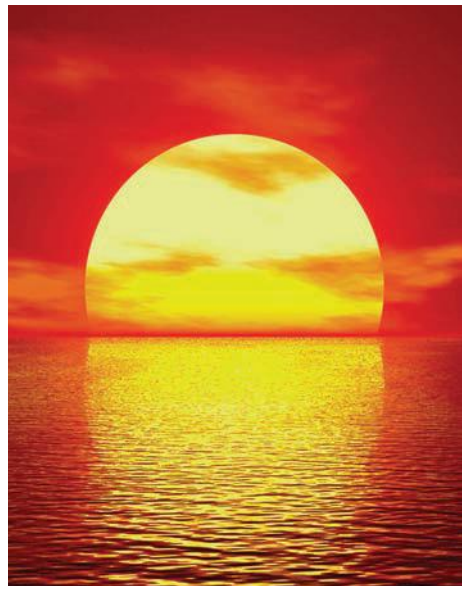
## Fluency

**1** Find the probability of:

WE16

- **(a)** rolling an even number with a normal die
- **(b)** choosing an 'a' from the letters of the word *mathematics*
- **(c)** getting a 4 on a roll of a normal die
- **(d)** drawing a card from a normal pack and getting a diamond
- **(e)** drawing a card from a normal pack and getting a king
- **(f)** drawing a card from a normal pack and getting a black 4
- **(g)** drawing a card from a normal pack and getting a red 7
- **(h)** drawing a card from a normal pack and getting the 9 of spades.

**2** Decide whether you would describe the following events as 'impossible', 'not likely', 'likely' or 'certain'.



**(a)** Tomorrow will have a maximum temperature of -20°C.

**(b)** Christmas Day will be on 25 December.

**(c)** Sri Lanka will win the next cricket World Cup.

**(d)** The Sun will set tonight.

**(e)** You toss a coin and either a head or a tail turns up.

**(f)** You roll a die and a 1 turns up.

**(g)** You roll a die and a 7 turns up.

**(h)** You draw a card from a normal pack of 52 playing cards and it's the ace of clubs.

**(i)** You draw a card from a normal pack of playing cards and it's either red or black.

**3** Choose the value that you think is closest to the correct answer.



**(a)** The probability of winning first prize in Tattslotto is:

**A** 0 **B** 0.5

**C** 0.6 **D** 0.75

**(b)** The probability of Australia winning a gold medal in swimming at the next Olympics is:

**A** 0.3 **B** 0.4

**C** 0.9 **D** 1

**(c)** The probability of rain at some time during the next two months is:

**A** 0 **B** 0.2

**C** 0.5 **D** 1

**(d)** The probability of tossing a coin and getting a tail is:

**A** 0.2 **B** 0.3 **C** 0.5 **D** 0.7

**(e)** The probability of drawing a heart from a pack of cards is:

**A** 0.0001 **B** 0.25 **C** 0.5 **D** 1

**4** Write out the sample space for each of the following games.

**(a)** A ten-sided die, labelled 1 to 10, is rolled once.

**(b)** A letter is drawn at random from the letters of the word *probability*.

**(c)** A coin is tossed twice.

**5** A bag of jelly beans contains 4 blue, 5 red, 3 green, 2 purple and 1 black jelly beans.

**(a)** Rachel puts her hand in the bag and selects one jelly bean without looking. What is the probability of the jelly bean being:

**(i)** red **(ii)** green **(iii)** blue or purple?

**(b)** Rachel pulls out one red jelly bean and eats it. She selects another one from the bag without looking. Recalculate the three probabilities you calculated in **(a)**.

## Understanding

**6** Determine whether the following games are fair by listing the sample space and the successful outcomes for each player. If you are unsure of the answer, or if you want to check if you are right, use a die and play the game with a partner.

**(a)** **(i)** Joe and Henry roll a die. Joe wins if a 1, 2 or 3 turns up, and Henry wins if a 4, 5 or 6 turns up. Is this a fair game?

**(ii)** If Joe and Henry play their game 100 times, how many wins would they probably have each?

**(b)** **(i)** Edward and Irene roll a die. Edward wins if a 3 turns up, and Irina wins if a 3 doesn't turn up. Is this a fair game?

**(ii)** If Edward and Irene play their game a large number of times, and Edward wins 40 times, how many times would Irene probably have won?

**(c)** **(i)** Ravi and Simon roll a die. Branka wins if a multiple of 3 turns up, and Simon wins if a multiple of 3 doesn't turn up. Is this a fair game?

**(ii)** Ravi and Simon score a point each time one of them wins. Suppose the game is the first one to 60 points. Simon wins, but a week later they argue about what the final score was. Simon says he won 60 : 31, while Branka says Simon only won 60 : 55. Who do you think was probably right?

**7** Steve and Nicky play a game based on the spinner shown. They each spin it once. Steve wins if he spins blue, and Nicky wins if she spins red.



**(a)** Is the game fair?

**(b)** Why is there a problem with the 'fairness' of this game?

**8** Maria and Walter are playing a game that is based on the spinner shown. Are these games fair? Think about the sample space and the number of chances of winning.



**(a)** Maria wins if the pointer finishes on blue and Walter wins if the pointer finishes on red. If the pointer finishes on anything else, they spin again until one of them wins.

**(b)** Maria wins if the pointer finishes on blue or yellow and Walter wins if the pointer finishes on red. If the pointer finishes on green, they spin again until one of them wins.

**(c)** Maria wins if the pointer finishes on blue or green and Walter wins if the pointer finishes on red or yellow.

**(d)** Maria wins if the pointer finishes on blue, yellow or green, and Walter wins if the pointer finishes on red.

**(e)** Maria wins if the pointer finishes on yellow and Walter wins if the pointer finishes on red. If the pointer finishes on green or blue, they spin again until one of them wins. However, if green comes up three times in a row, Maria wins, and if blue comes up four times in a row, Walter wins.

## Reasoning

**9** The following is a list of words you may have heard associated with probability.

| | | | |
|---|---|---|---|
| even chance | very likely | virtually impossible | often |
| sure thing | most unlikely | almost always | quite often |
| likely | impossible | sometimes | usually |
| quite frequently | almost never | maybe | more often than not |

Put these words in order from impossible to certain. Be prepared to defend your order in a class discussion.

**10** A drawer contains six identical blue socks and two identical brown socks. How many socks do you have to take from the drawer to be certain of having:

**(a)** a pair of blue socks

**(b)** a pair of brown socks

**(c)** a pair of socks

**(d)** two odd socks?

Explain your answers in each case.

## Open-ended

**11** **(a)** Write down two events of your own that you think are impossible.

**(b)** Write down two events of your own that you think are not likely.

**(c)** Write down two events of your own that you think are likely.

**(d)** Write down two events of your own that you think are certain.

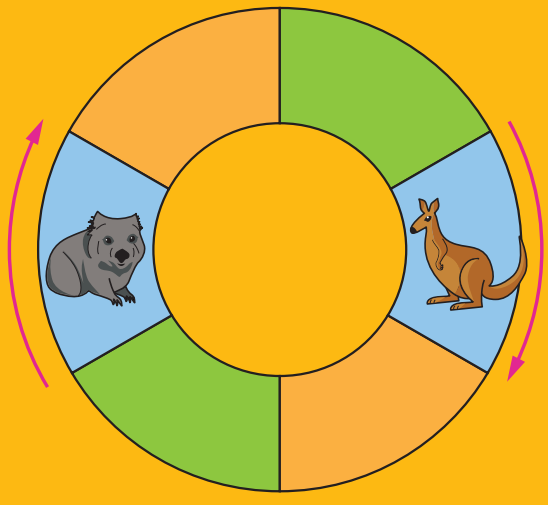
**12** Make up a game between two players involving two dice that has a number of conditions for winning, but that you think is fair.

# Outside the Square Game

## Catch me if you can

**Equipment required:** 1–2 brains, 2 dice, 2 unique counters

**How to win:**
Land on the segment the other player's counter is on.



**How to play:**

Decide which player is the 'kangaroo' and which is the 'wombat' and place your counter on the appropriate start position. Roll the two dice and move the appropriate counter one space, given the following rules.

The wombat moves if the sum of the dice is equal to: 2, 3, 4, 5, 10, 11, 12.

The kangaroo moves if the sum of the dice is equal to: 6, 7, 8 or 9.

Continue rolling the two dice until one of the animals catches the other.

After you have played a few times, decide whether you think the game is fair. Explain your reasons. If you don't think it is fair, see if you can change the rules to make it fair. Play the new game a few times to test it.

# Investigation

## Experimental probability

**Equipment required:** 2 brains, 1 calculator, 1 drawing pin, 1 coin



### The Big Question

Can we use probability to decide whether a game of chance is fair?

### Engage

**1** A drawing pin can land two possible ways. Try dropping the drawing pin a few times and see if it lands point up or point down. Do you think you need to be careful in the way you drop the pin? Should you give it a bit of a 'flip' to help it on its way? Why do you think we might be using the pin instead of a coin or a die?

point up



point down



### Explore

Don and Jacob play a game that involves dropping a drawing pin.

If the drawing pin falls point up, Don scores a point. If the drawing pin falls point down, Jacob scores a point. The first one to get to 50 wins.

**2 (a)** Do you think this is a fair game? Who do you think will win?

**(b)** Predict the final score. Is it harder to make this sort of prediction than the predictions in the previous exercise?

**(c)** Play the game with a partner. Decide who gets a score for point up and who gets a score for point down. Tally your results in a table like this:

| | |
|---|---|
| **Point up** | 𝍸 𝍸 \|\|\| |
| **Point down** | 𝍸 𝍸 𝍸 \|\|\| |

How close were your predictions?

**(d)** Compare your results with other pairs of students by constructing a table like this:

| | Score | |
|---|---|---|
| | **Point up** | **Point down** |
| **Pair 1** | 32 | 50 |
| **Pair 2** | 50 | 44 |
| **Pair 3** | | |

**(e)** Do you notice any pattern in the results of the games?

**(f)** **(i)** Find the point up total by adding all the numbers in the point up column.

**(ii)** Find the point down total by adding up all the numbers in the point down column.

**(iii)** Then, find the total number of throws by adding the point up and point down totals.

**(g)** Find the probability that a drawing pin will fall point up by calculating $\frac{\text{point up total}}{\text{total number of throws}}$:

**(i)** as a fraction

**(ii)** as a decimal

**(iii)** as a percentage.

**(iv)** Could the game still be fair if the percentage is not exactly 50%?

**(h)** Find the probability that a drawing pin will fall point down by calculating $\frac{\text{point down total}}{\text{total number of throws}}$:

**(i)** as a fraction

**(ii)** as a decimal

**(iii)** as a percentage.

(iv) Could the game still be fair if the percentage is not exactly 50%?

(v) Look at the two probabilities you have found in parts **(g) (ii)** and **(h) (ii)**. Add them together. What do you notice?

(vi) If the game is fair, what do you think each of the two probabilities should be equal to? Is the game fair? Is it close to being a fair game?

**Strategy options**

- Make a table.
- Act it out.
- Look for a pattern.

## Explain

**3** How could you change the rules for the drawing pin game to make it fair?

## Elaborate

**4** Omar and Michelle take it in turns to toss a coin four times. If four heads come up, or if two heads and two tails come up, Omar scores a point. If four tails come up, or if three tails and one head come up, Michelle scores a point. If three heads and one tail come up then they both score a point. The first one to 50 points is the winner.

**(a)** Do you think this is a fair game? Who do you think will win?

**(b)** Make a prediction about what you think the final score will be.

**(c)** Play the game with a partner. Decide who is Omar and who is Michelle.

Tally your results in a table. How close were your predictions?

**(d)** Compare your results with other pairs of students by constructing a table.

**(e)** Do you notice any pattern in the results of the games?

**(f)** **(i)** Find Omar's total by adding all the numbers in the Omar column.

**(ii)** Find Michelle's total by adding all the numbers in the Michelle column.

**(iii)** Then, find the total number of throws by adding Omar's and Michelle's totals.

**(g)** **(i)** Find the fraction of points that Omar gets by calculating:

$$\frac{\text{Omar's total}}{\text{total number of throws}}$$

**(ii)** Change this to a decimal. Write the probability that Omar will win as Pr(Omar wins) =

**(iii)** Write the percentage of times Omar will win.

**(h)** **(i)** Find the fraction of points that Michelle gets by calculating:

$$\frac{\text{Michelle's total}}{\text{total number of throws}}$$

**(ii)** Change this to a decimal. Write the probability that Michelle will win as Pr(Michelle wins) =

**(iii)** Write the percentage of times Michelle will win.

**(iv)** Find Pr(Omar wins) + Pr(Michelle wins).

**(v)** By looking at your values for Pr(Omar wins) and Pr(Michelle wins), would you say that the game is fair? Is it close to being fair?

## Evaluate

**5** How could you change the rules for the coin game to make it fair?

## Extend

**6** What other sorts of probability games do you think you could play? Try to come up with some that do not involve coins or dice.

**7** **(a)** Did you affect the result by the way you dropped the pin or tossed the coin?

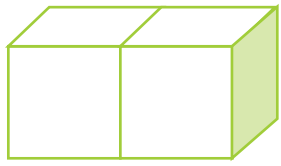
**(b)** Do you think the tasks in this investigation have helped you understand probability? Explain why or why not?

**(c)** Do you think it is important for you to know whether a game is fair? Why?

# Challenge 9

**1** The dots on opposite faces of a die total 7.

Two normal dice are placed next to each other on a table as shown.



Kenny walks around the table.

The greatest number of dots that Kenny can see is:

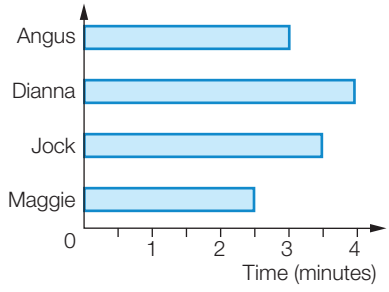
**A** 30 **B** 34 **C** 36 **D** 42

**2** You meet three men at a meeting: Mr Black, Mr Green and Mr White. They are wearing black, green and white bow ties. Mr Black says, "Did you notice that the colour of our ties are different from our names?" The person in the green bow tie says, "Yes, you are correct!" What colour tie is each man wearing?

**3** A die is constructed by labelling the faces of a wooden cube with the numbers 1, 1, 2, 3, 3 and 4. When it is tossed, each face has an equal chance of occurring. If this die is rolled once, the probability of rolling an odd number is:

**A** $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

**4** Four Jack Russell terriers had a race swimming across a pond. The graph shows the time that each dog took to complete the race. Which dog won the race?



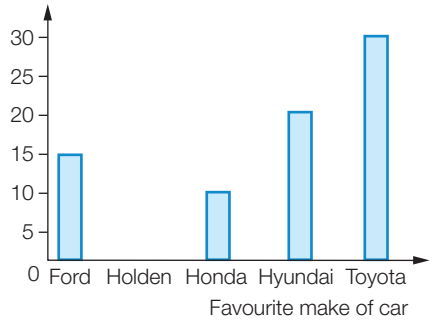

**A** Angus **B** Dianna

**C** Jock **D** Maggie

**5** Vijay has a bag that contains 3 blue marbles, 6 green marbles, 2 red marbles and 6 yellow marbles. Vijay adds a number of white marbles to the bag and tells Samantha that if he now draws a marble from the bag, the probability of it being blue or green is $\frac{3}{7}$. The number of white marbles that Vijay added to the bag is:

**A** 3 **B** 4 **C** 5 **D** 6

**6** In a survey, 1000 people were asked 'What is your favourite make of car?' Their responses were recorded and then graphed as percentages in a column graph. The column representing Holden has been omitted. What percentage of the people selected a Holden as their favourite make of car?




**A** 25 **B** 20

**C** 15 **D** 5

**7** Jade has played 20 games of tennis and has won 95% of the games. If she does not lose any more games, how many more games in a row must she win in order to have then won 96% of the games?

**A** 1 **B** 4 **C** 5 **D** 10

# Chapter review 9

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| average | continuous data | measures of centre | sample space |
| bar graph | data | median | stem-and-leaf plot |
| bimodal | discrete data | mode | tally |
| bivariate data | dot plot | multimodal | theoretical probability |
| categorical data | frequency | outcomes | trend |
| chance | frequency table | outlier | univariate data |
| class interval | histogram | probability | |
| column graph | mean | range | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The graphs in this chapter are all ways of presenting _____.

2 The listing of all the outcomes possible in a probability experiment is called the _____.

3 Of the three measures of central tendency considered, the one that represents the value occurring most frequently is called the _____.

4 The probability of an event occurring is another way of saying what _____ there is of the event happening.

5 In a frequency table for grouped data, each group is called a _____.

6 The _____ is the sum of all the values in the data set divided by the number of values in that data set.

**Equipment required:** Calculator for Question **2**

## Fluency

1 A Year 7 class was surveyed about how many television sets their families had at home. The results were: Ex 9.1

1, 1, 1, 1, 2, 1, 1, 3, 2, 5, 2, 1, 1, 0, 1, 2, 0, 4, 1, 2, 0, 3, 1

Construct a frequency table for this information.

2 Find the mean, median and mode for the following sets of data. Use a calculator when necessary, and give your answers correct to two decimal places. Ex 9.2

**(a)** 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1

**(b)** 40, 20, 60, 30, 50, 10

**(c)** 3.3, 4.1, 3.9, 2.9, 4.4, 4.1, 3.8, 3.3, 4.4, 3.2, 3.9, 4.1, 2.9

**3** Draw a bar graph to show the following data that represents the favourite style of TV program for a sample of 100 people. Ex 9.3

| Style | Number of people |
|---|---|
| Comedy | 35 |
| Drama | 21 |
| Reality | 17 |
| Documentary | 8 |
| Game | 19 |

**4** **(a)** Draw a dot plot to represent the number of customers in a cafe recorded in 15-minute intervals over the course of 8 hours. Ex 9.3

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 4 | 7 | 3 | 6 | 4 | 5 | 9 | 4 | 8 | 8 | 5 | 7 | 9 | 10 | 12 |
| 11 | 8 | 9 | 8 | 6 | 2 | 4 | 4 | 5 | 5 | 2 | 5 | 3 | 0 | 1 | 1 |

**(b)** Calculate the median of the data set.

**(c)** Do there appear to be any outliers?

**5** **(a)** Draw an ordered stem-and-leaf plot to represent the number of runs scored by each of the players at a local cricket club in one round of the season. Ex 9.3

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 46 | 64 | 31 | 0 | 50 | 42 | 11 | 7 | 4 | 0 | 5 |
| 21 | 17 | 115 | 0 | 42 | 61 | 15 | 18 | 5 | 3 | 1 |
| 40 | 0 | 52 | 56 | 77 | 50 | 41 | 20 | 7 | 2 | 2 |
| 88 | 17 | 58 | 63 | 14 | 0 | 18 | 19 | 17 | 6 | 2 |

**(b)** Calculate the median of the data set.

**(c)** Do there appear to be any outliers? If so, which values are they?

**6** The sector graph at right represents information about the money spent by overseas visitors to Australia. Ex 9.4

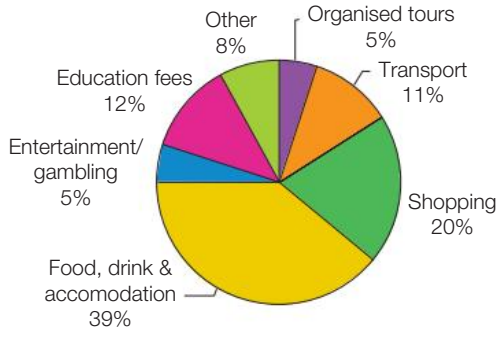
**(a)** One of the sectors uses an angle of 28.8°. Which one is it?

**A** Organised tours

**B** Transport

**C** Entertainment/gambling

**D** Other




**(b)** The Food, drink & accommodation sector uses an angle closest to:

**A** 95° **B** 110°

**C** 130° **D** 140°

**7** The following are the sales figures for Beeper computer games from 2006 to 2011. Ex 9.5

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|
| Units sold | 10 000 | 25 000 | 35 000 | 30 000 | 40 000 | 45 000 |

Draw a line graph for this information.

**8** The following table gives the mean rainfall, measured in mm, for Cairns (Aero) and Brisbane (Aero). Ex 9.6

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Cairns** | 396.8 | 452.5 | 426.1 | 198.2 | 91.4 | 46.1 | 29.3 | 27.0 | 33.4 | 39.7 | 93.4 | 178.5 |
| **Brisbane** | 109.3 | 121.6 | 76.1 | 63.5 | 122.8 | 70.6 | 27.1 | 35.2 | 32.7 | 65.2 | 102.0 | 120.9 |

Compare the mean rainfall for the two cities using as many statistics as you can.

**9** The following table shows the handspan, measured in cm, of twenty 16-year-old students, divided into boys and girls. Ex 9.6

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Boys** | 19.5 | 21.5 | 21 | 22.5 | 19 | 19.5 | 21 | 21.5 | 20.5 | 20 |
| **Girls** | 18 | 18.5 | 17.5 | 17 | 17 | 17.5 | 18 | 18.5 | 17 | 19.5 |

Draw a parallel dot plot and use it to help you describe the data set.

**10** **(a)** When a normal die is rolled, the probability of getting a number less than 5 is equal to: Ex 9.7

**A** $\frac{1}{5}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** 1

**(b)** If the spinner shown on the right is spun once, the probability of the pointer landing on red is:



**A** $\frac{1}{4}$ **B** $\frac{3}{8}$

**C** $\frac{1}{2}$ **D** $\frac{3}{5}$

## Understanding

**11** A number of Year 7 students were surveyed about their shoe size. The results were:

$3, 5, 5\frac{1}{2}, 5, 4\frac{1}{2}, 4, 2\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 6, 2, 5\frac{1}{2}, 1\frac{1}{2}, 1, 3\frac{1}{2}, 4\frac{1}{2}, 3, 4, 5, 5\frac{1}{2}, 2\frac{1}{2}, 2, 3\frac{1}{2}, 4\frac{1}{2}, 3, 1, 3\frac{1}{2}, 1\frac{1}{2}, 4, 3\frac{1}{2}$

**(a)** Construct a frequency table to show this information.

**(b)** Which shoe size occurred most frequently?

**(c)** Which shoe size occurred least frequently?

**12** Look at the following information.

| How much Australians spend on pets each year ($ million) | | | | |
|---|---|---|---|---|
| **Expenditure type** | **Dogs** | **Cats** | **Other** | **Total** |
| Food | 560 | 431 | 90 | 1081 |
| Vet charges and prescriptions | 230 | 148 | 26 | 404 |
| Pet care products/equipment | 153 | 85 | 12 | 250 |
| Pet services | 116 | 35 | – | 151 |
| Other expenses | 109 | 20 | 12 | 141 |
| Total | 1168 | 719 | 140 | 2027 |

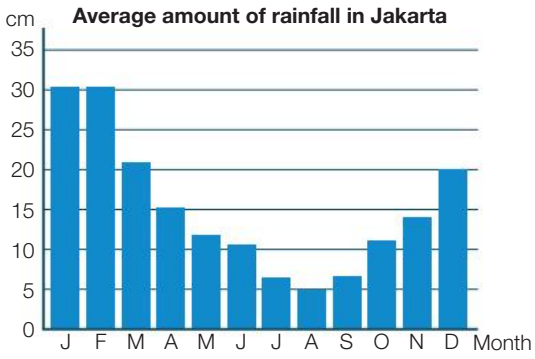
**(a)** Draw a bar chart showing the various expenditure types for dogs.

**(b)** Draw a divided bar chart to show the total expenditure for the three categories of pets.

**(c)** Draw a sector graph to show the various expenditure types for cats.

**(d)** Explain why a line graph could not be used for any of these sets of data.

**13** Look at the following graph, which shows the average amount of rainfall (cm) in the city of Jakarta over 1 year.




**(a)** What type of graph is this?

**(b)** What is Jakarta's second driest month?

**(c)** What is the highest average monthly rainfall?

**(d)** Which 4 months form the rainy season?

**14** Shelley and Douglas play a game in which they toss two coins. Shelley wins if two heads turn up, and Douglas wins if two tails turn up. If both a head and a tail turn up, they throw *one* of the coins again. If this comes up heads, Shelley wins. If it comes up tails, Douglas wins. Do you think this is a fair game?

**15** **(a)** What value would the probability that you will be struck by lightning be close to?

**(b)** What value would the probability that it will rain at some time in the next six months be close to?

**(c)** If something has a probability of $\frac{1}{2}$ and something else has a probability of $\frac{3}{4}$, which one of the two is more likely to occur?

**(d)** If someone tells you something has a probability of 1.2 of happening, how do you know that they have made a mistake?

## Reasoning

**16** Twenty-five Year 7 boys and 25 Year 7 girls had their height measured to the nearest cm. The results are as follows.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Boys: | 140 | 143 | 144 | 142 | 148 | 148 | 152 | 140 | 146 | 144 |
| | 154 | 151 | 145 | 140 | 141 | 147 | 153 | 150 | 149 | 140 |
| | 147 | 152 | 148 | 151 | 149 | | | | | |
| Girls: | 142 | 151 | 157 | 138 | 145 | 148 | 143 | 141 | 146 | 154 |
| | 156 | 154 | 150 | 143 | 150 | 144 | 148 | 144 | 157 | 156 |
| | 146 | 157 | 154 | 145 | 152 | | | | | |

**(a)** Construct an ungrouped frequency table for the boys.

**(b)** Find the **(i)** mode **(ii)** median and **(iii)** mean height for the boys.

**(c)** Construct an ungrouped frequency table for the girls.

**(d)** Find the **(i)** mode **(ii)** median and **(iii)** mean height for the girls.

**(e)** Describe the similarities and differences between the heights of the boys and the girls.

**(f)** Draw a combined ungrouped frequency table for boys and girls.

**(g)** Find the **(i)** mode **(ii)** median and **(iii)** mean height for the students.

**(h)** Describe the heights of the students.

**17** In this game you have 12 counters and three dice. You place as many counters as you like on any number of squares, although you must place all 12. You throw the dice and add the numbers shown. If you have any counters on that number you remove them. You continue until the board is clear.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |

**(a)** On which numbers should you never place any counters? Explain why.

**(b)** On which numbers should you place most counters? Explain why.

**(c)** Once you have decided on a spread of counters, will every game take the same number of turns to clear the board? Explain your answer.

# NAPLAN practice 9

## Numeracy: Non-calculator

**1** Some students were surveyed about the number of hours spent using a computer over the last weekend. The following results were recorded: 4, 7, 3, 2, 4, 6, 8, 5, 9.

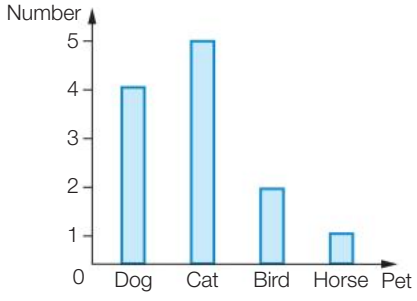
The median result is:

**A** 4 **B** 5 **C** 6 **D** 9

**2** A jar contains 16 red, 12 black, 9 yellow and 8 green lollies.
Without looking, Jen takes one lolly from the jar.
What is the chance that the lolly is yellow?

**A** $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

**3** Ricky surveyed the members of his cricket team to find out what pets they had. The results are shown in the graph on the right.

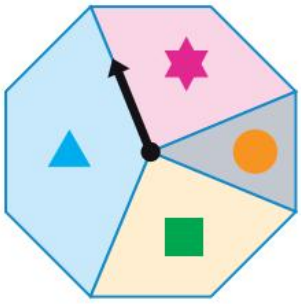



How many pets does the team have altogether?

**A** 4 **B** 5

**C** 12 **D** 15

**4** Kenny spins the arrow 80 times.



Which table is *most* likely to show his results?

**A**

| Shape section | Number of spins |
|---|---|
| ★ | 10 |
| ● | 30 |
| ■ | 20 |
| ▲ | 20 |

**B**

| Shape section | Number of spins |
|---|---|
| ★ | 20 |
| ● | 30 |
| ■ | 10 |
| ▲ | 20 |

**C**

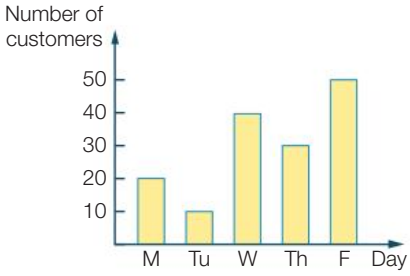
| Shape section | Number of spins |
|---|---|
| ★ | 20 |
| ● | 10 |
| ■ | 20 |
| ▲ | 30 |

**D**

| Shape section | Number of spins |
|---|---|
| ★ | 20 |
| ● | 20 |
| ■ | 10 |
| ▲ | 30 |

## Numeracy: Calculator allowed

**5** A business recorded the number of customers each day for a working week.




**(a)** Which day was the busiest?

**(b)** What was the mean number of customers per day?

**(c)** On which day was the number of customers closest to the mean number of customers per day?

**6** The goal attack for the under 13 Panthers netball team scored the following number of goals in the first six games of the season: 14, 19, 23, 20, 28, 19.

The mean number of goals scored is closest to:

**A** 6 **B** 19.5 **C** 20.5 **D** 126


10

# Transformation and visualisation

10

## Why is The Pentagon a pentagon?

In 1941, a radical new design for a government building impressed the President of the USA.

Located near Washington, DC, the building known as 'The Pentagon' was built during World War II to house the War Department, now the United States Department of Defense. It is one of the largest buildings in the world, despite being only five storeys high (the steel required for extra storeys was needed for warships and weapons). The five-sided shape was chosen because it was the best fit to the shape of the land originally chosen for the location of the building. President Roosevelt changed the location, but kept the shape because, 'nothing like it has ever been done that way before'. The Pentagon is a regular pentagon. Each of the five sides and the five angles formed where the sides meet are equal. Because it is a regular shape, the Pentagon has a high degree of reflectional and rotational symmetry.

### Forum

How do you think the builders of the Pentagon ensured that each of the five sides and angles were equal?

Looking at the Pentagon from above:

How many shapes can you identify within the main pentagon shape?

How many straight lines can you draw that divide the shape into two identical halves?

Many buildings and structures that are considered 'beautiful' are symmetrical. Do you think we are naturally 'drawn' towards symmetry?

### Why learn this?

Everyday we see and work with geometrical figures. We get food and drink from cylindrical cans, pack things into prism-shaped boxes, and play sport on rectangular courts with spherical balls. Each of these objects has been designed so that its shape suits its purpose. Many careers involve design, from architecture (designing buildings), to industrial design (designing products for a mass market) to graphic design (designing the images used in advertising, books, magazines and websites). Knowledge of geometry and appreciation of symmetry is important in these and many other careers.

**After completing this chapter you will be able to:**

- translate, reflect and rotate shapes
- describe the process and compare the results of different transformations
- identify lines of symmetry
- identify the order of reflectional symmetry and the order of rotational symmetry
- recognise symmetry in design and nature
- visualise and draw 3D shapes
- draw elevations and plan views of 3D shapes.

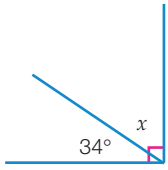
# Recall 10

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

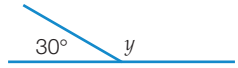
**1** Find the size of the unknown angle in each of the following.

Worksheet R10.1

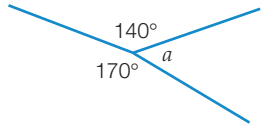
**(a)**




**(b)**



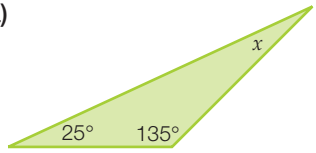

**(c)**



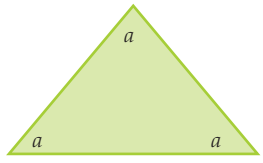

**2** Find the size of the unknown angle in each of the following.

Worksheet R10.2

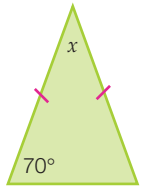
**(a)**




**(b)**



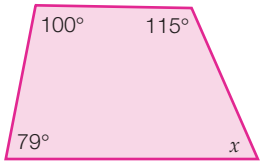

**(c)**



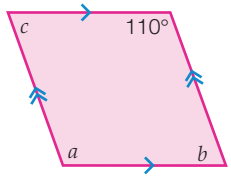

**3** Find the size of the unknown angle in each of the following.

Worksheet R10.3

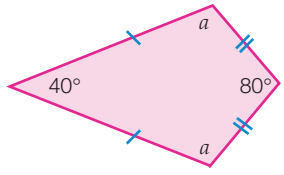
**(a)**




**(b)**




**(c)**




**4** Draw an example of:

Worksheet R10.4

**(a)** a rhombus **(b)** a trapezium **(c)** an equilateral triangle

**(d)** a kite **(e)** an isosceles triangle **(f)** a parallelogram.

Mark all sides that are equal and/or parallel. Use letter names to name your figures.

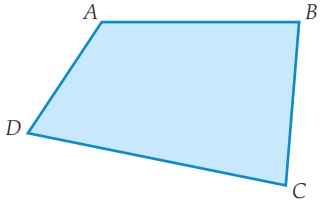
## Key Words

| | | | |
|---|---|---|---|
| asymmetrical | isometric | perpendicular distance | rotation |
| axis of symmetry | line of reflection | plan views | rotational symmetry |
| centre of rotation | order of reflectional symmetry | reflection | transformation |
| elevations | order of rotational symmetry | reflectional symmetry | translation |
| image | original | rotate | |

# Translations

10.1

A **transformation** changes a particular shape (or figure) or set of points according to a defined rule. A transformation may move, turn, flip or change the shape of a figure. The transformations we will deal with here are those that move a shape but retain its original shape and size. The first figure is called the **original** and the transformed shape is called the **image** of the transformation. We give the original shape letter names such as *A*, *B*, *C*, *D* and the image letter names such as *A*′, *B*′, *C*′, *D*′. Transformations may change the position only, or they may involve a change in orientation such as a turn or a reflection.




Original

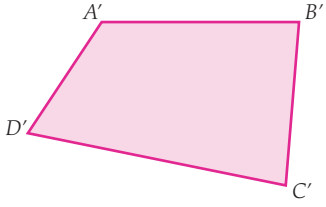



Image 1
Change in position only

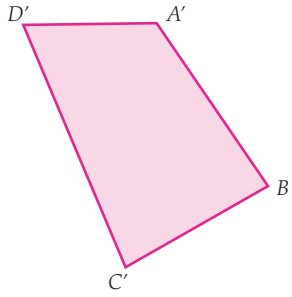



Image 2
Change in position and orientation

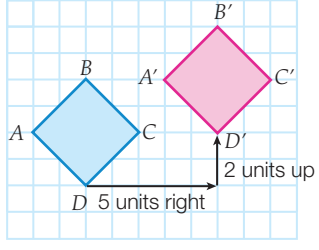
There are three types of transformations in which the shape does not change:

- translation
- reflection
- rotation.

Each type of transformation moves a figure in a particular way. Transformations can be combined to make a complex move and can be reversed to move the image back to its original position and orientation.

## Translations

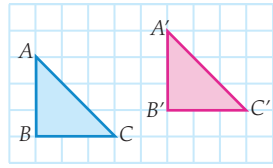
A **translation** occurs when a figure is shifted or slid across the page to change its position. When a figure undergoes translation, the image is moved without turning. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both. The example at right shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as [5, 2].

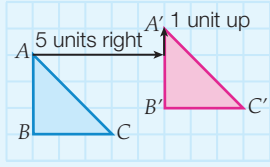



- Vertices of an image are denoted by a dash; e.g. *A*′ or *B*′. We say '*A* dash' or '*A* prime'.
- To distinguish between the original figure and the translated image, you can use a different colour or broken lines.
- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

## Worked Example 1 WE1

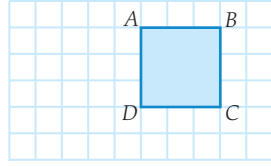
Describe the translation shown in the diagram.

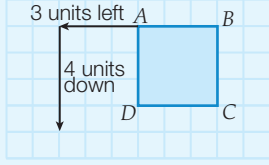
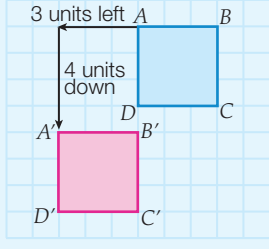



| Thinking | Working |
|---|---|
| **1** Start by selecting one of the vertices and count the units in the horizontal and vertical directions to get to the corresponding translated vertex. |   |
| **2** Write the translation in words or as an ordered pair. | 5 units right and 1 unit up or [5, 1]. |

## Worked Example 2 WE2

Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.




| Thinking | Working |
|---|---|
| **1** Start by selecting one of the vertices ($A$) and move according to the given translation.<br>**2** Label this vertex with a letter using image notation ($A'$). |   |
| **3** Repeat with the remaining vertices and then draw the resulting image in a different colour.<br>**4** Label all vertices of the image using image notation. |   |

If our shape is drawn on the Cartesian plane, we can identify the coordinates of the vertices and then find the coordinates of the transformed shape after a given transformation. In this section, we will identify the coordinates of a shape after a given translation.

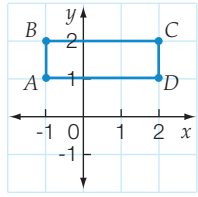
## Worked Example 3 WE3

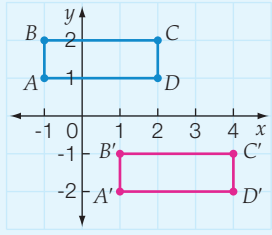
**(a)** Write the coordinates of each vertex on the given shape.

**(b)** Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

**(c)** Write the coordinates of the translated vertices.

**(d)** Explain how the coordinates of the image could be found without drawing the shape.




| Thinking | Working |
|---|---|
| **(a)** Use the $x$- and $y$-axes to identify the coordinates of each of the vertices. | (a) $A = (-1, 1)$, $B = (-1, 2)$, $C = (2, 2)$, $D = (2, 1)$ |
| **(b)** Translate each vertex by the given translation, labelling each vertex with image notation. (Here, we move each point 2 units right and 3 units down. This is a translation of [2, -3].) | (b)  |
| **(c)** Identify the coordinates of each vertex on the resulting image. | (c) $A' = (1, -2)$, $B' = (1, -1)$, $C' = (4, -1)$, $D' = (4, -2)$ |
| **(d)** Look for the connection between the original coordinates, the transformation and the image coordinates. | (d) Adding 2 to the x-coordinate of the original and subtracting 3 from the y-coordinate of the original will give the image coordinates. |



# 10.1 Translations

## Navigator

| | | |
|---|---|---|
| Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q16, Q17 | Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17 | Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q18 |

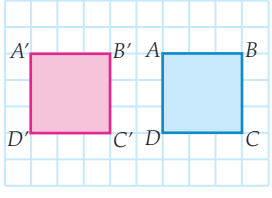
Answers page 686

**Equipment required:** Grid paper for Questions **2**, **3**, **9**, **13(b)**, **15(c)**, **16** and **17**

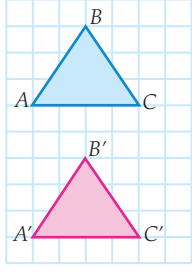
### Fluency

**1** Describe the translation shown in each of the following diagrams.

**(a)**

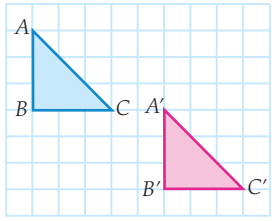



**(b)**

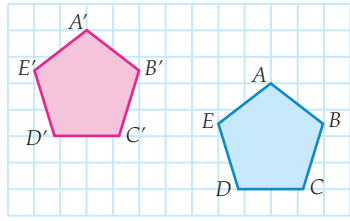





**(c)**



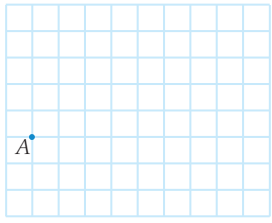

**(d)**



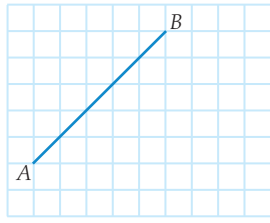

**WE2**

**2** Copy each of the following onto grid paper and draw the resulting image after the translation.

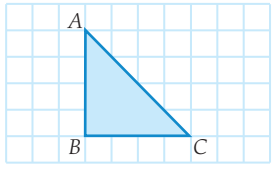
**(a)** 6 units right and 2 units up



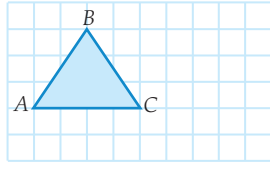

**(b)** 6 units left and 6 units down



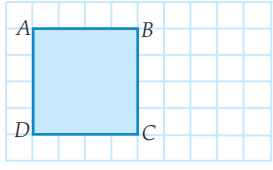

**(c)** 6 units left and 1 unit down



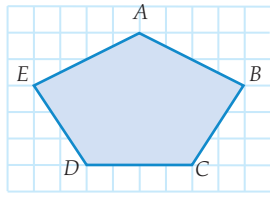

**(d)** 1 unit right and 6 units down




**(e)** [7, -5]




**(f)** [-6, 4]



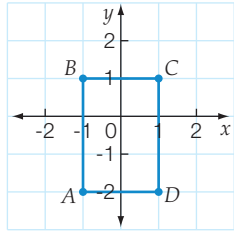

**WE3**

**3** **(a)** Write the coordinates of each vertex on the given shape.

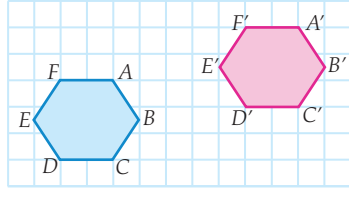
**(b)** Copy the shape onto grid paper and draw the resulting image after the translation of 1 unit to the left and 2 units up.

**(c)** Write the coordinates of the translated vertices.

**(d)** Explain how the coordinates of the image could be found without drawing the shape.




**4** How has the hexagon in the diagram been translated?




**A** 7 units up and 2 units right

**B** 7 units right and 2 units up

**C** 7 units left and 2 units down

**D** 6 units right and 2 units up

**5** What would be the reverse translation of 4 units left and 8 units down?

**A** 4 units up and 8 units right

**B** 4 units left and 8 units up

**C** 4 units left and 8 units down

**D** 4 units right and 8 units up

**6** Write the translation [-2, 6] using direction instructions (i.e. right, left, up, down).

**7** Write the translation of 3 units left and 4 units down using square brackets.

**8** Write the reverse translation of 10 units right and 4 units up using square brackets.

## Understanding

**9** **(a)** Plot the points (-1, 4), (2, 3) and (0, 2) on a Cartesian plane and label them *A*, *B* and *C*.

**(b)** Join the points to form triangle *ABC*, then translate the shape 2 units right and 1 unit down, labelling the image with image notation.

**(c)** Write the coordinates of the translated points.

**10** A figure is translated 4 units left, 3 units down, 6 units left and 7 units up. What would be the final position of the image compared to that of the original figure?

**11** What single translation would be equivalent to 12 units down, 5 units right, 7 units left, 8 units up and 2 units right?

**12** What single translation would be equivalent to a translation of [2, 3] followed by a translation of [-2, 6]?

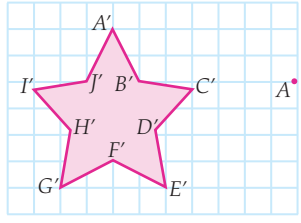
## Reasoning

**13** **(a)** Without plotting points, write the coordinates of the image after the line joining *A*(4, 2) and *B*(6, -1) is translated 2 units left and 2 units up.

**(b)** Plot the line *AB* and its image *A'B'* on a Cartesian plane to check your answer to part **(a)**.

**14** If a particular translation was 2 units right, what would be the single translation that would be equivalent to repeating 2 units right a hundred times?

**15** The following diagram is of a translated image and a single point of the original figure.




**(a)** What translation has occurred to the original figure?

**(b)** What is the reverse translation?

**(c)** Use this to reverse the translation and draw the original figure.

## Open-ended

**16** Plot three points on a Cartesian plane and write their coordinates. Decide upon a particular translation and use this translation to translate your shape. Draw the image after the translation and write down the coordinates of the translated points using image notation.

**17** On grid paper, draw a figure that has 4 vertices. Then, translate that figure in any direction and identify the reverse translation required to get the image back to the original figure.

**18** Provide an explanation either for or against the following statement:

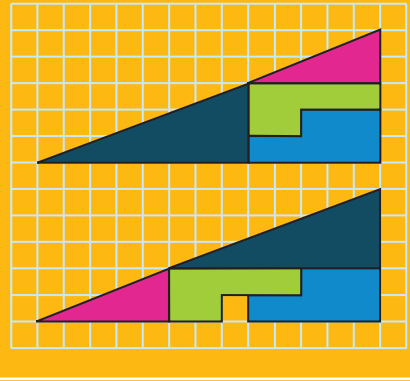
'If a figure undergoes more than one translation, the order in which you translate a figure does not affect the final position of the image.'

## Outside the Square Puzzle

### Jigsaw paradox

This Jigsaw Paradox orginates from the famous Curry's Paradox and was modified by Martin Gardner in 1956. This paradox or puzzle shows two arrangements of four identical shapes to create two right-angled triangles, but one has a 1 × 1 hole in it. Both triangles have 13 units for the base and 5 units for the height.

How can this be true? Can you explain what happened to the missing 1 × 1 square?



# 10.2 Reflections

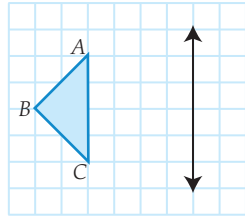


A **reflection** is a transformation that creates an image of the original figure in the same way that a mirror creates an image when you look into it. A reflection is therefore often called a mirror image. It is a reversed or 'flipped over' version of the original about a **line of reflection**. A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed.

Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side. The distance used is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection. To reflect a figure, reflect each vertex on the figure and then join the points in order.

## Worked Example 4 WE4

Copy the following figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



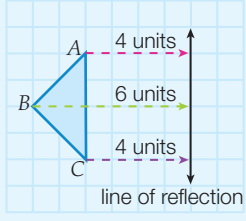

A point on the mirror line or line of reflection doesn't move when you reflect it.



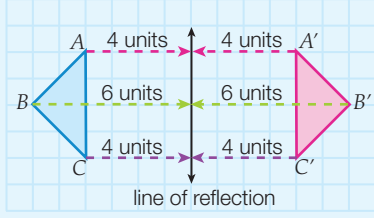
### Thinking

1 Measure the perpendicular distance between each vertex and the line of reflection.

### Working




2 Use the distance between the vertices on the line of reflection to plot the points on the opposite side of the line of reflection and reproduce the original figure.




3 Label the image vertices with image notation.

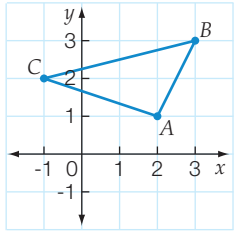
- A reflection always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same; for example, the size of the side lengths or angles will stay the same.

We can find the image of a shape drawn on a Cartesian plane after it is reflected in either the $x$- or $y$-axis.

## Worked Example 5

WE5

**(a) (i)** Write down the coordinates of each vertex on the given shape.

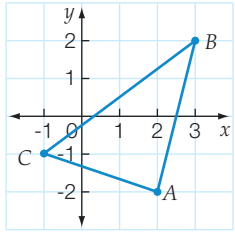



**(ii)** Copy the shape onto grid paper and draw the resulting image after a reflection in the $x$-axis, labelling each vertex with image notation.

**(iii)** Write the coordinates of the reflected vertices.

**(iv)** Explain how the coordinates of the image could be found without drawing the shape.

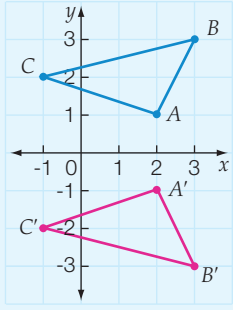
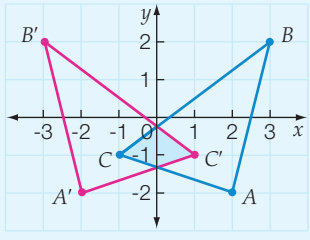
**(b) (i)** Write down the coordinates of each vertex on the given shape.




**(ii)** Copy the shape onto grid paper and draw the resulting image after a reflection in the $y$-axis, labelling each vertex with image notation.

**(iii)** Write the coordinates of the reflected vertices.

**(iv)** Explain how the coordinates of the image could be found without drawing the shape.

| Thinking | Working |
|---|---|
| **(a) (i)** Use the $x$- and $x$-axes to identify the Cartesian coordinates of the points. | (a) (i) $A = (2, 1)$, $B = (3, 3)$, $C = (-1, 2)$ |
| **(ii)** Reflect each point in the $x$-axis and join them in order to form the reflected shape, labelling each vertex with image notation. | (ii)   |
| **(iii)** Identify the coordinates of each vertex on the resulting image. | (iii) $A' = (2, -1)$, $B' = (3, -3)$, $C' = (-1, -2)$ |
| **(iv)** Look for the connection between the coordinates of the original and the image. | (iv) The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original. |
| **(b) (i)** Use the $x$- and $y$-axes to identify the Cartesian coordinates of the points. | (b) (i) $A = (2, -2)$, $B = (3, 2)$, $C = (-1, -1)$ |
| **(ii)** Reflect each point in the $y$-axis and join them in order to form the reflected shape, labelling each vertex with image notation. | (ii)   |

**(iii)** Identify the coordinates of each vertex on the resulting image.

(iii) $A' = (-2, -2)$, $B' = (-3, 2)$, $C' = (1, -1)$

**(iv)** Look for the connection between the coordinates of the original and the image.

(iv) The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.

# 10.2 Reflections

## Navigator

Answers page 687

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13 |

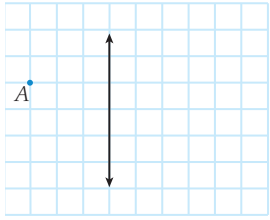
**Equipment required:** Grid paper for Questions **1–3**, **5–8** and **10–13**

## Fluency

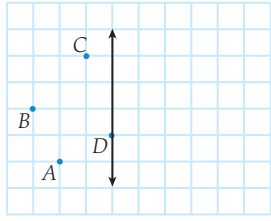
WE4

**1** Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection shown.

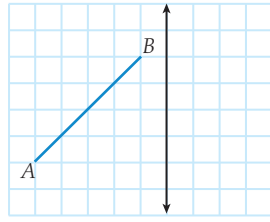
**(a)**



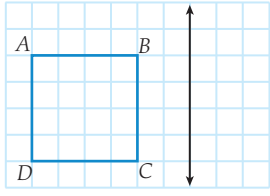

**(b)**



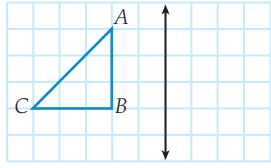

**(c)**



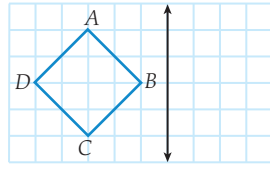

**(d)**




**(e)**

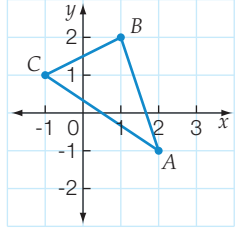



**(f)**




WE5

**2 (a)** **(i)** Write down the coordinates of each vertex on the given shape.

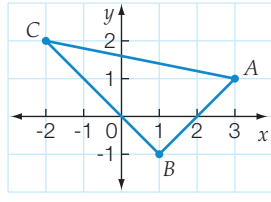



**(ii)** Copy the shape onto grid paper and draw the resulting image after a reflection in the $x$-axis, labelling each vertex with image notation.

**(iii)** Write the coordinates of the reflected vertices.

**(iv)** Explain how the coordinates of the image could be found without drawing the shape.

**(b)** **(i)** Write down the coordinates of each vertex on the given shape.



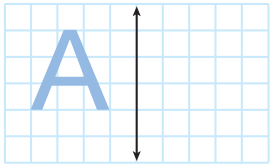

**(ii)** Copy the shape onto grid paper and draw the resulting image after a reflection in the $y$-axis, labelling each vertex with image notation.

**(iii)** Write the coordinates of the reflected vertices.

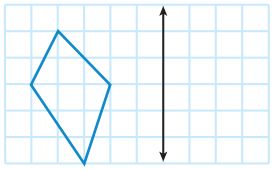
**(iv)** Explain how the coordinates of the image could be found without drawing the shape.

**3** Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection shown.

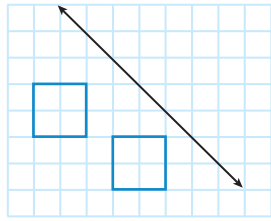
**(a)**



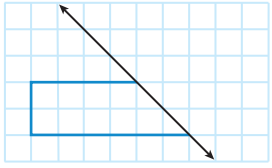

**(b)**



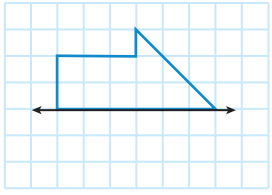
**(c)**



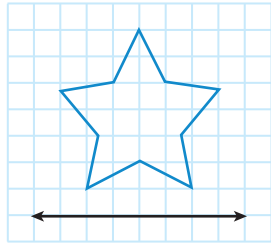
**(d)**



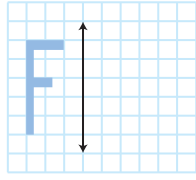
**(e)**



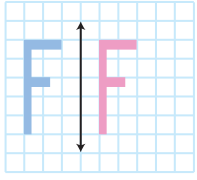
**(f)**



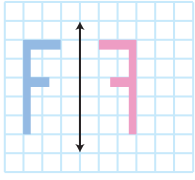
**4** Which of the following options is the correct reflection of the capital F in the vertical line of reflection?



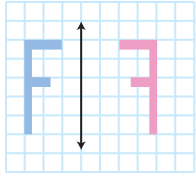

**A**



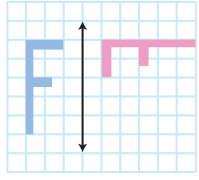

**B**




**C**




**D**




## Understanding

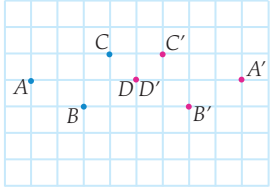
**5** **(a)** Plot the points (-1, 5), (3, 1) and (2, -1) on a Cartesian plane and label them *A*, *B* and *C*.

**(b)** Join the points to form triangle *ABC*, then reflect the shape first in the *x*-axis and then in the *y*-axis.

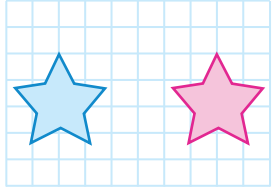
**(c)** Write the coordinates of the points after the second reflection.

**6** Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

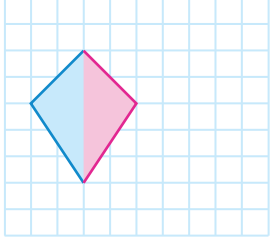
**(a)**



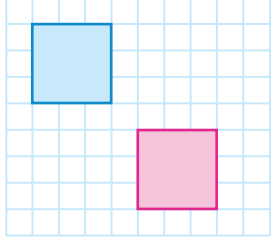

**(b)**



**(c)**

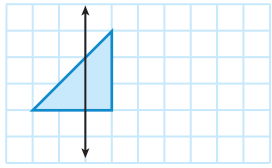


**(d)**

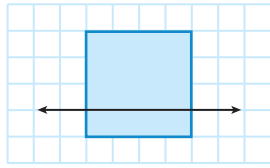


7 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.

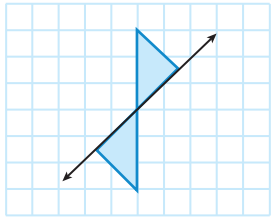
(a)



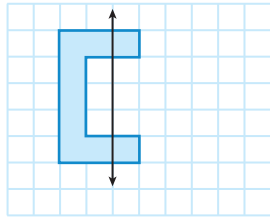
(b)



(c)



(d)



## Reasoning

8 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the $x$-axis.

**(b)** Plot the line $AB$ and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

**(c)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the $y$-axis.

**(d)** Plot the line $AB$ and its image $A'B'$ on a Cartesian plane to check your answer to part **(c)**.

9 Bruno has a digital clock on his glass bedside table. When he looked at the time, he noticed that at that particular instance, the time 8.30 was reflected perfectly on the table. Thinking this to be a little odd, he decided to find out the following.

**(a)** List the number of minutes past the hour when the minute part of the time reflects the correct time (include 00).



**(b)** In a 12-hour mode, list the hours when the hour part of the time reflects the correct time.




**(c)** In a 12-hour period, use your answers to parts **(a)** and **(b)** to find how many times the reflection on the table gives the correct time in a 12-hour mode.

**(d)** In a 24-hour period, how many times does the reflection in the table give the correct time in a 24-hour mode?

**(e)** Is your answer to part **(d)** double your answer to part **(c)**? Explain your answer.

## Open-ended

**10** Plot three points on a Cartesian plane and write their coordinates. Reflect this shape in either the $x$- or $y$-axis. Draw the image after the reflection and write down the coordinates of the reflected points using image notation.

**11** On grid paper, draw a figure with four vertices. Reflect this figure in:

**(a)** a horizontal line of reflection

**(b)** a vertical line of reflection.

**12** Repeat Question **11** with a figure with five, six, or seven vertices.

**13** On grid paper, draw a figure with four vertices.

**(a)** Reflect this figure in a horizontal line of reflection and then in a vertical line of reflection.

**(b)** Reflect the same figure in the vertical line of reflection and then in the horizontal line of reflection.

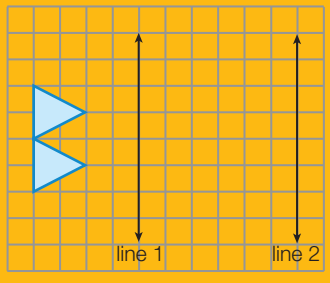
**(c)** Comment on the images found in parts **(a)** and **(b)**.

## Outside the Square — Problem solving

### Mirror, mirror on the wall, does it matter where I stand at all?

**Equipment required:** 1 brain, grid paper

The diagram below consists of two blue triangles and two vertical lines of reflection.




To begin, reflect the two triangles in the line of reflection labelled 'line 1' and then in the line of reflection labelled 'line 2' and note the final location of the image. Next, switch the order in which the two triangles are reflected so that they are reflected in line 2 first, and then in line 1, and note the final location of the image.

Does the order in which these triangles are reflected in either line affect the final location and orientation of the final image? What did you notice about the final image? Can this transformation be described another way?

To justify your decision, experiment with different situations, such as drawing the two triangles originally on the right-hand-side of line 2 or in-between the lines. You can also alter the position of the lines of reflection. For example, line 1 and line 2 could be two horizontal lines or one could be horizontal and the other vertical.

**Strategy options**

- Draw a diagram.
- Test all possible combinations.

# 10.3 Rotations

We also transform a figure when we **rotate** it around a fixed point. This is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter *O*.

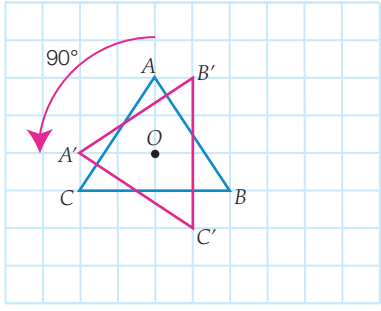
When rotating any figure you need to know:

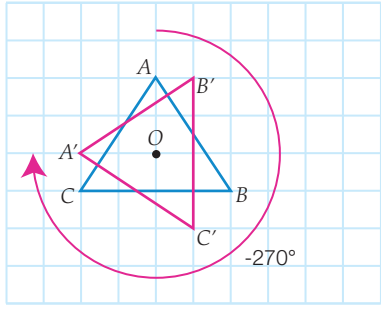
- the location of the centre of rotation
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The centre of rotation can be located inside or outside the figure and is the only point that does not rotate. The size of the angle of rotation is generally in multiples of 30° or 45°. Common angles of rotation are 30°, 45°, 60°, 90°, 180° and 270°. A rotation of 360° will rotate the image of the figure through a full revolution about the centre of rotation and will return the figure to its original position.

- The vertices of a rotated image are labelled in the same order as the original figures.
- Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.
- Two rotations that result in an image in the same position will always add to 360°. For example, 90° clockwise is equivalent to 270° anticlockwise as 90° + 270° = 360°.
- A rotation in an anticlockwise direction is a positive rotation.
- A rotation in a clockwise direction is a negative rotation.

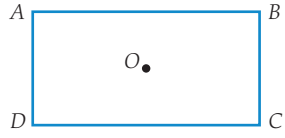


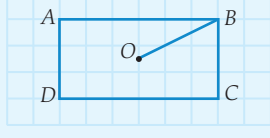



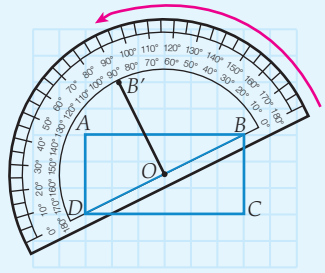

## Worked Example 6 WE6

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about *O*.

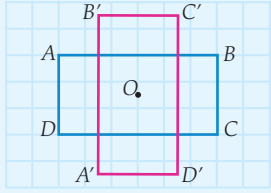



| Thinking | Working |
|---|---|
| **1** Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line. |   |

2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex. Plot the resulting point. The new point must be the same distance from *O* as the original point.




3 Continue the process with the other vertices and connect the resulting points to produce the original figure in its rotated position.

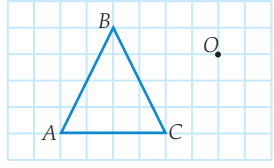



The same steps are used when the centre of rotation is outside the shape. The distance between the vertices and the centre of rotation is the same before and after the rotation, similar to reflection of an object in a line of reflection. Measuring this distance and checking that it is the same before and after the rotation is an easy way to check that the rotation is correct.

## Worked Example 7

WE7

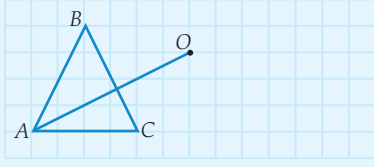
Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (-180°) about *O*.



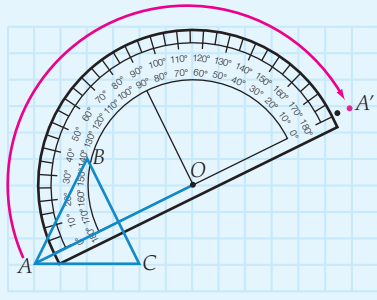

### Thinking

### Working

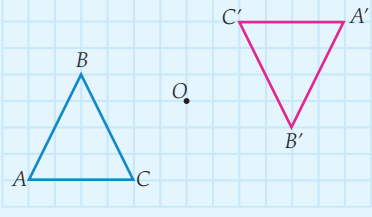
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.




2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex and plot a point. Extend *AO* through this point and mark the point *A′* so that *OA′* is the same length as *OA*.



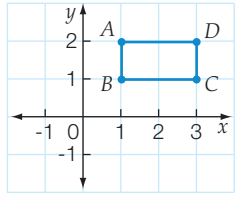

3 Continue the process with the other vertices and connect the resulting points to reproduce the original figure in its rotated position.



We can find the coordinates of an image after a rotation of 90°, 180° or 270° in a clockwise or anticlockwise direction about the origin.

## Worked Example 8 WE8

**(a)** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.




**(b) (i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

**(c) (i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

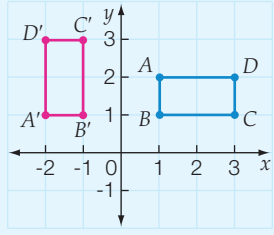
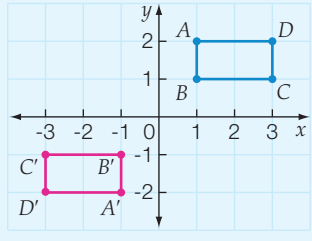
**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

**(d) (i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

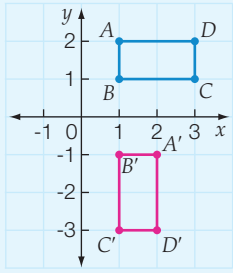
| Thinking | Working |
|---|---|
| **(a)** Use the $x$- and $y$-axes to identify the coordinates of each of the vertices. | (a) $A = (1, 2)$, $B = (1, 1)$, $C = (3, 1)$, $D = (3, 2)$ |
| **(b) (i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 90° in an anticlockwise direction about the origin.) | (b) (i)   |
| **(ii)** Identify the coordinates of each vertex on the resulting image. | (ii) $A' = (-2, 1)$, $B' = (-1, 1)$, $C' = (-1, 3)$, $D' = (-2, 3)$ |
| **(iii)** Look for the connection between the original coordinates, the transformation and the image coordinates. | (iii) The x-coordinates of the image vertices are the negative of the y-coordinates of the original vertices and the y-coordinates of the image vertices are the x-coordinates of the original vertices. |
| **(c) (i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 180° in an anticlockwise direction about the origin.) | (c) (i)   |
| **(ii)** Identify the coordinates of each vertex on the resulting image. | (ii) $A' = (-1, -2)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, -2)$ |

**(iii)** Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

**(d) (i)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 270° or -90°.)

(d) (i)




**(ii)** Identify the coordinates of each vertex on the resulting image.

(ii) $A' = (2, -1)$, $B' = (1, -1)$, $C' = (1, -3)$, $D' = (2, -3)$

**(iii)** Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.

# 10.3 Rotations

## Navigator

| | | | |
|---|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 (a), Q11, Q14, Q15 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q14, Q15 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12 Column 2, Q13, Q14, Q15, Q16 | Answers page 689 |

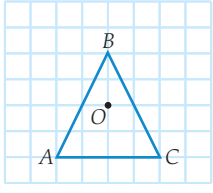
**Equipment required:** Protractor and grid paper for Questions **1**, **2**, **3**, **6**, **9 (b)**, **10**, **14** and **16**

## Fluency

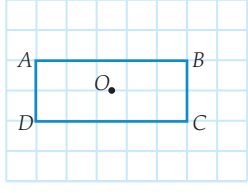
**1** Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

WE6

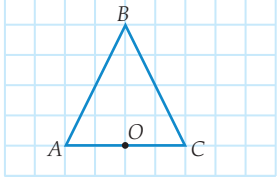
**(a)** Rotate 180° in an anticlockwise direction (180°) about *O*.



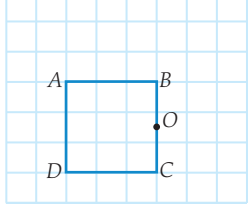

**(b)** Rotate 270° in an anticlockwise direction (270°) about *O*.




**(c)** Rotate 180° in a clockwise direction (-180°) about *O*.



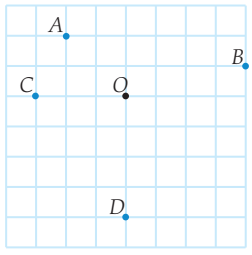

**(d)** Rotate 90° in a clockwise direction (-90°) about *O*.



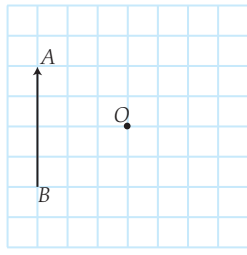
WE7

**2** Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

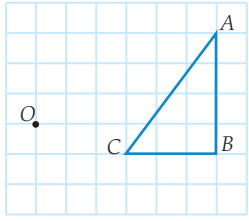
**(a)** Rotate each of the points 90° in a clockwise direction (-90°) about *O*.



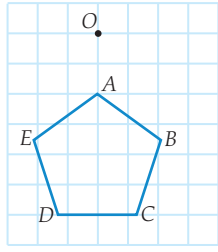

**(b)** Rotate 270° in an anticlockwise direction (270°) about *O*.




**(c)** Rotate 180° in a clockwise direction (-180°) about *O*.

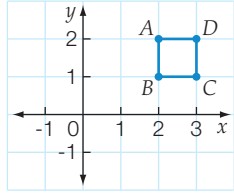



**(d)** Rotate 90° in an anticlockwise direction (90°) about *O*.




WE8

**3** **(a)** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.




**(b)** **(i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

**(c)** **(i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

**(d)** **(i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

**(ii)** Write the coordinates of the rotated vertices.

**(iii)** Explain how the coordinates of the image could be found without drawing the shape.

**4** Which of the following statements about rotation is incorrect?

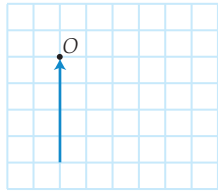
**A** Rotation can be in either a clockwise or an anticlockwise direction.

**B** A rotation of 360° will result in the image located in the same position as the original figure.

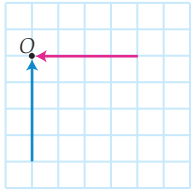
**C** A rotation will always produce an image that is a perfect replica and in the same orientation as the original figure.

**D** A rotation of 90° in a clockwise direction is equivalent to a rotation of 270° in an anticlockwise direction about the same centre of rotation.

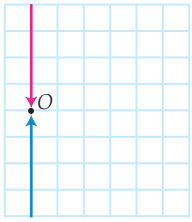
**5** This arrow turns about the centre of rotation $O$. What does the shape look like after a turn of 180° anticlockwise?



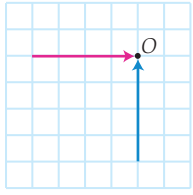

**A**



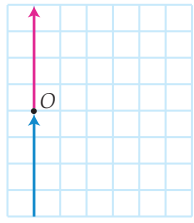

**B**




**C**




**D**




## Understanding

**6** **(a)** Plot the points (1, 4), (2, 4), (2, 2) and (1, 2) on a Cartesian plane and label them $A$, $B$, $C$ and $D$.

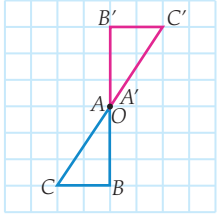
**(b)** Join the points to form a figure $ABCD$, then rotate the figure 90° in a clockwise direction about point $A$.

**(c)** Write the coordinates of the rotated points using image notation.

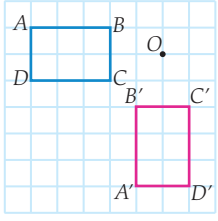
**7** A clockwise rotation of 120° is the same as an anticlockwise rotation of how many degrees?

**8** Find the rotation (the size of the rotation and the direction) that has taken place to produce the following images. Identify more than one rotation that will achieve the same result.

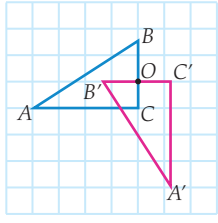
**(a)**



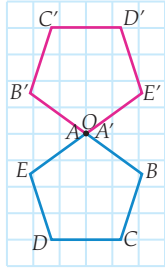

**(b)**




**(c)**




**(d)**



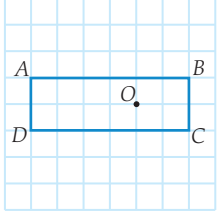

## Reasoning

**9** **(a)** Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(3, -2)$ is rotated 180° clockwise about the origin.

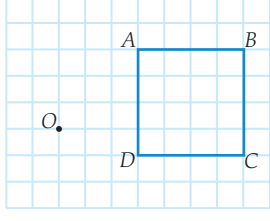
**(b)** Plot the line $AB$ and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

**10** Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

**(a)** Rotate 90° in an anticlockwise direction about $O$.




**(b)** Rotate 90° in a clockwise direction about $O$.



**11** The following are the capital letters of the alphabet.

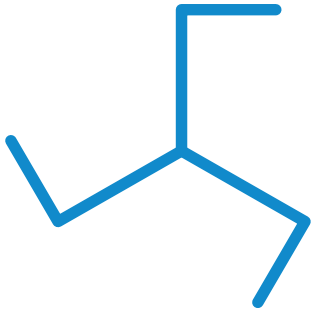
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

**(a)** Which letters of the alphabet are able to be rotated about a centre of rotation in either direction and at an angle of less than 360° to produce an identical image in the same orientation as the original letter?

**(b)** Four of these letters have something else in common with each other. Can you identify these four letters? Describe the similarity you have noticed.

**12** Estimate the smallest angle through which the following figures need to be rotated so the resulting image will match the original figure.

**(a)**



**(b)**



**(c)**



**(d)**



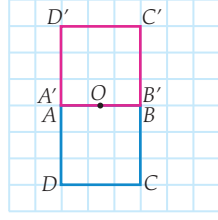
**(e)**



**(f)**



**13** Is the following transformation a rotation of 180° around point *O*, a reflection along the line *AB* or both? Provide an explanation.




## Open-ended

**14** Plot four points on a Cartesian plane and write their coordinates. Decide upon a particular rotation and use this rotation to rotate your shape. Draw the image after the rotation and write down the coordinates of the rotated points using image notation.

**15** Explain whether you agree or disagree with the following statement.

'A rotation of 180° in either direction and about any point of rotation can be equivalent to reflecting the same figure in a line of reflection through that point.'

**16** On grid paper, draw two figures, a regular and an irregular polygon with the same number of sides. Decide upon a centre of rotation for each polygon that is on a vertex of that polygon.

**(a)** Rotate each of the figures by 90° about the centre of rotation in a clockwise direction and draw the image.

**(b)** Repeat this four times.

**(c)** Rotate the figures 45° about the centre of rotation.

**(d)** Repeat this eight times.

**(e)** Comment on the final images in parts **(b)** and **(d)**.

# Outside the Square Puzzle

## Tangram magic

**Equipment required:** 1–2 brains, 2 pieces of grid paper, blank paper, ruler, pens and scissors

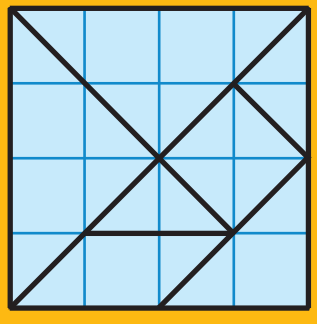
Tangram is an ancient Chinese puzzle. The object of the puzzle is to rearrange the pieces of a square (the puzzle pieces) to form as many different shapes using all seven pieces.

Before you start this puzzle you will need to make the seven pieces required to form the shapes.

**Step 1** On a piece of grid paper, rule a square. The larger the square, the larger the pieces will be to work with. An 8 cm × 8 cm square is recommended.

**Step 2** Use the tangram template below and divide your square into seven pieces as shown.

**Step 3** Cut out your pieces carefully.



The tangram pieces will be:

- one square
- two small congruent (identical) isosceles triangles
- two large congruent isosceles triangles
- one medium isosceles triangle
- one parallelogram.

Once the pieces are cut out, it is time to begin. Form as many of the 12 shapes given below with your tangram pieces. You can translate, reflect and rotate the pieces, but remember to use all seven pieces.

When you have formed a shape, draw it carefully on blank paper, showing how the pieces fit together. Keep a list of the shapes you have made.



Running man



Dancer



House



Christmas tree



Kangaroo



Boat



Shirt



Swan



Double-headed arrow



Letter C



Cat



American Indian

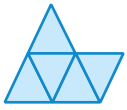
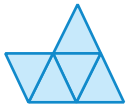
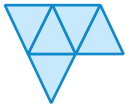
# Investigation

**Equipment required:** 1 or 2 brains, isometric grid paper, isometric dot paper. (A computer drawing program may be used to produce the required shapes.)

## Diamonds are forever!

A frieze is a decorative horizontal strip that consists of a repeating pattern. The repeating pattern is formed by combining shapes with translations, reflections and rotations. A frieze contains a high degree of symmetry.

Throughout this investigation you will be asked for different ways to connect your shapes. Mirror reflections or rotations are not considered to be different.

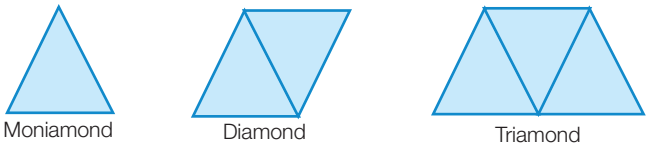
 and  and  are not different ways to connect five equilateral triangles.

### The Big Question

Can you make a frieze with at least five different shapes made from equilateral triangles, diamonds and trapeziums that includes translations, reflections and rotations?

### Engage

If an equilateral triangle or moniamond is reflected along an edge it becomes a diamond (rhombus). If the triangular shape is reflected along another edge it becomes a triamond (trapezium).



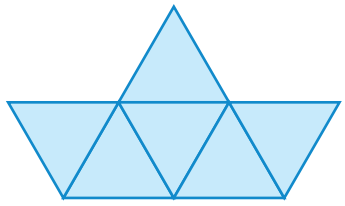

**1** On isometric grid paper, draw the three different ways that four equilateral triangles can be connected to make a new shape. These are called tetriamonds.

**2** On isometric grid paper, draw the four different ways that five equilateral triangles can be connected to make a new shape. These are called pentiamonds.

### Explore

**3** Six equilateral triangles can be connected to make a new shape. These are called hexiamonds. There are 12 different hexiamond shapes that can be made. Here is an example:

On isometric grid paper, copy the hexiamond on the right and find the 11 other hexiamond shapes.



The 12 names given below might help you find the shape. Can you match each name to a shape?

- bar (parallelogram)
- chevron (bat)
- crook (club)
- signpost (pistol)
- crown
- lobster
- sphinx
- hook (shoe)
- snake
- hexagon
- yacht
- butterfly

If a diamond shape (rhombus) is translated by a full side length and then joined to the original shape, it makes a shape we can call a straight bidiamond. If the diamond shape is translated again to make three diamond shapes joined together in a row, the result is a shape we can call a straight tridiamond.

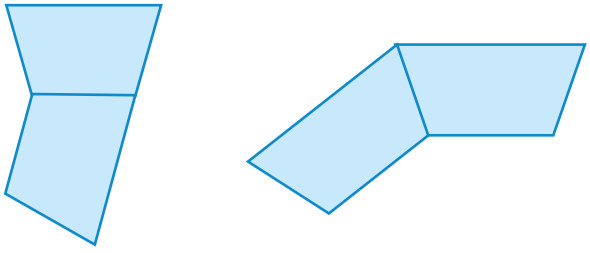



Three diamonds can be connected to make tridiamonds of different shapes, including the straight tridiamond given above.

**4** Nine of the 12 hexiamonds found in Question **3** can be made by reflecting, translating and rotating diamonds to make these tridiamonds. On isometric dot paper, draw the nine tridiamonds showing the diamonds that have been used to form them.

5 One of the hexiamond shapes when reflected along one of its sides will form a six-pointed star. Find the shape, reflect it to form the star and then translate the shape to make a row of three six-pointed stars. Draw your pattern on isometric dot paper.

6 There are nine ways that two triamonds (trapeziums made from three equilateral triangles) can be joined exactly edge to edge to make different shapes. On isometric dot paper, draw the nine triamonds (including the two given below) showing the triamonds that have been used to form them.



7 In how many ways can three triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

8 In how many ways can four triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

**Strategy options**

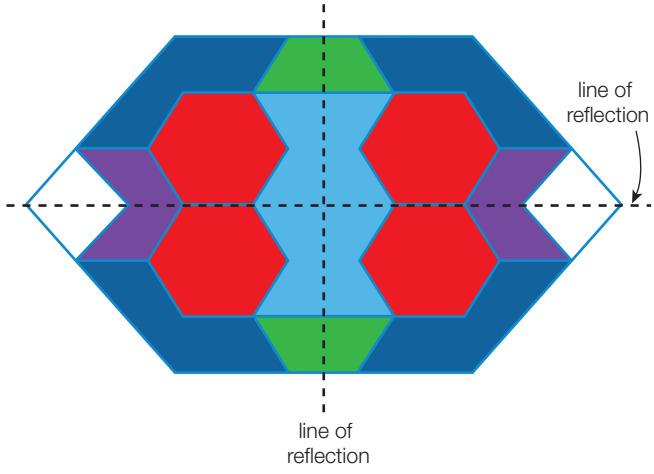
- Draw a diagram.
- Guess and check.
- Look for a pattern.

## Explain

9 For two hexiamonds found in Question **3**, two tridiamonds found in Question **4** and two shapes found in Question **7**, explain how they were formed in terms of translation, reflection and rotation.

## Elaborate

10 Answer the Big Question by making a frieze with at least five different shapes made from equilateral triangles, diamonds and trapeziums that includes translations, reflections and rotations. Your frieze pattern needs to be at least four triangles high and nine triangle lengths long. An example is given below. Indicate any lines of reflection.




## Evaluate

11 Did you find it hard to find all the different shapes? What made it difficult?

12 Did the names help you to find some of the shapes in Question **3**?

## Extend

There are at least 40 ways of using the 12 different shapes found in Question **3** to cover a rhombus of side 6 units. Can you find one?

# 10.4 Combined transformations

We can combine translations, reflections and rotational transformations into one transformation of a figure.

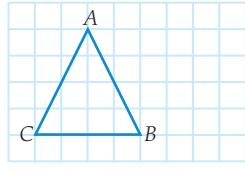
When combining transformations, the transformations are performed in sequence (one after the other) and, after each transformation, the vertices are labelled with an additional dash. For example, the vertex $A'''$ has been transformed by three different transformations.

- Translations, reflections and rotations can be combined into a sequence of transformations.
- A side length can act as the line of reflection during a combined transformation.
- Different colours can be used to distinguish between several transformations.
- The properties of the three different types of transformations still hold true for combined transformations.

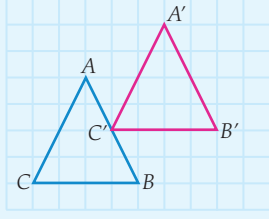
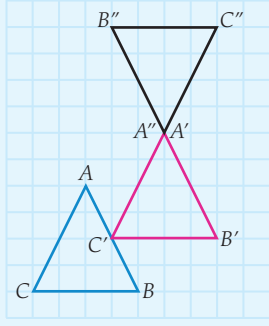
## Worked Example 9

WE9

Copy the following figure onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.



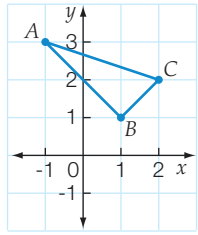

A translation of 3 units right and 2 units up, followed by a rotation of 180° in a clockwise direction about the vertex $A'$.

| Thinking | Working |
|---|---|
| **1** Perform the first identified transformation in the instructions.<br>Make sure you use the correct notation with the transformed vertices. |  |
| **2** Perform each subsequent transformation on the next transformed image.<br>You may want to distinguish each transformation by using a different colour. |  |

If a figure is drawn on a Cartesian plane, we can perform more than one type of transformation on it. For example, we can perform a rotation, then a translation. We could then reflect the figure in the $x$- or $y$-axis. We can then identify the coordinates of the final image.

## Worked Example 10 WE10

For the figure given opposite:




**(a)** write the coordinates of each vertex on the given shape and copy the shape onto grid paper

**(b)** reflect the shape in the $x$-axis

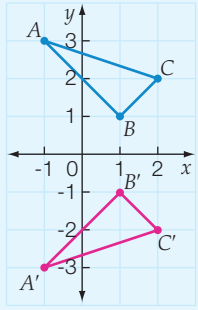
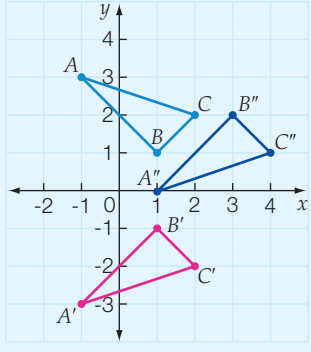
**(c)** write the coordinates of the reflected vertices

**(d)** translate this image 2 units right and 3 units up

**(e)** write the coordinates of the translated vertices

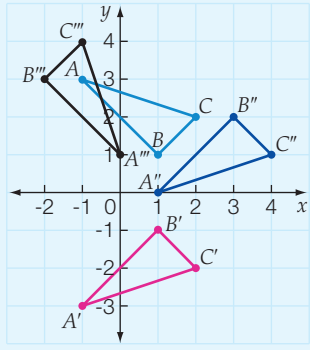
**(f)** rotate this translated image 90° in an anticlockwise direction about the origin

**(g)** write the coordinates of the vertices of the final image.

| Thinking | Working |
|---|---|
| **(a)** Use the $x$- and $y$-axes to identify the coordinates of each of the vertices. | (a) $A = (-1, 3)$, $B = (1, 1)$, $C = (2, 2)$ |
| **(b)** Reflect each vertex in the $x$-axis. | (b)   |
| **(c)** Identify the coordinates of each vertex on the resulting image. | (c) $A' = (-1, -3)$, $B' = (1, -1)$, $C' = (2, -2)$ |
| **(d)** Translate each vertex with the given translation. (Here, we translate each vertex 2 units to the right and 3 units up.) | (d)   |
| **(e)** Identify the coordinates of each vertex on the resulting image. | (e) $A'' = (1, 0)$, $B'' = (3, 2)$, $C'' = (4, 1)$ |

**(f)** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 90°.)

(f)




**(g)** Identify the coordinates of each vertex on the resulting image.

(g) $A''' = (0, 1)$, $B''' = (-2, 3)$, $C''' = (-1, 4)$

# 10.4 Combined transformations

## Navigator

Answers page 691

| | | |
|---|---|---|
| Q1, Q2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8 Column 1, Q9 Column 1, Q10, Q11, Q13, Q15 | Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q15 | Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q14, Q15 |

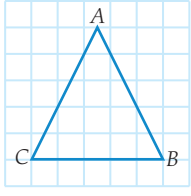
**Equipment required:** Protractor and grid paper for Questions **1–3**, **6**, **10 (b)** and **13–15**

## Fluency

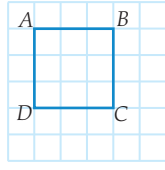
WE9

**1** Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.

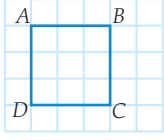
**(a)** A translation of 3 units left and 1 unit down, followed by a reflection in the line $B'C'$.



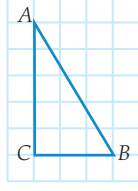

**(b)** A translation of 5 units right and 2 units down, followed by a reflection in the line $D'C'$.




**(c)** A translation of 5 units right, followed by a rotation of 90° about $B'$ in a clockwise direction.

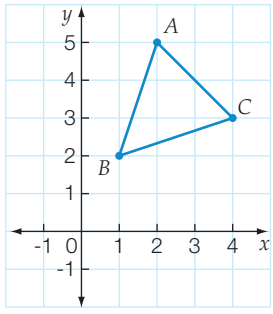



**(d)** A translation of 3 units left and 4 units down, followed by a rotation of 180° about $C'$ in a clockwise direction.



WE10

**2** For the figure given opposite:




**(a)** write the coordinates of each vertex on the given shape and copy the shape onto grid paper

**(b)** reflect the shape in the $x$-axis

**(c)** write the coordinates of the reflected vertices

**(d)** translate this image 5 units left and 3 units up

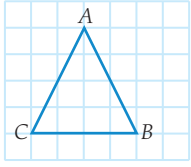
**(e)** write the coordinates of the translated vertices

**(f)** rotate this translated image 180° in a clockwise direction about the origin

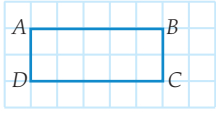
**(g)** write the coordinates of the vertices of the final image.

**3** Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each of the vertices appropriately after each transformation (for **(e)–(h)**, point $O$ is a point on or inside the figure and is reflected with the shape).

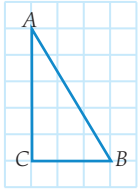
**(a)** Reflect along the line $CB$ and translate 6 units right and 2 units up.



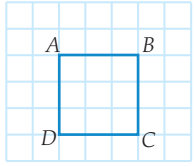

**(b)** Reflect along the line $BC$ and translate 1 unit left and 3 units down.



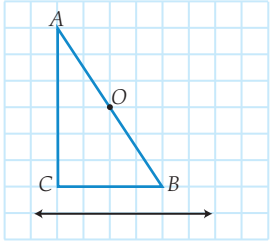

**(c)** Rotate 90° in a clockwise direction about vertex $A$ and translate 8 units down.



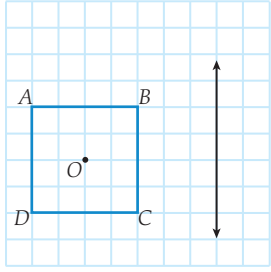

**(d)** Rotate 180° in an anticlockwise direction about vertex $D$ and move 2 units left and 6 units up.



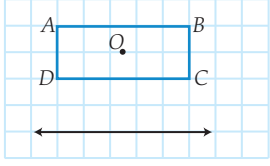

**(e)** Reflect along the line of reflection and then rotate 90° in an anticlockwise direction about the centre of rotation $O'$.



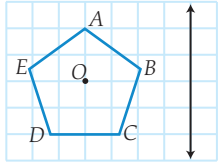

**(f)** Reflect along the line of reflection and then rotate 90° in a clockwise direction about the centre of rotation $O'$.




**(g)** Rotate 90° in an anticlockwise direction about the centre of rotation $O$ and then reflect in the line of reflection.




**(h)** Rotate 180° in a clockwise direction about the centre of rotation $O$ and then reflect in the line of reflection.



4 Which transformation reverses the order in which the vertices are labelled?

**A** rotation and translation
**B** reflection
**C** translation
**D** rotation

5 Which transformation can result in an image in the same position and orientation as the original figure?

**A** rotation and translation
**B** translation
**C** reflection
**D** rotation

## Understanding

6 **(a)** Plot the points (-1, 2), (2, 3) and (3, -1) on a Cartesian plane and label them *A*, *B* and *C*.

**(b)** Join the points to form a triangle *ABC*, then translate the shape 3 units right and 1 unit down.

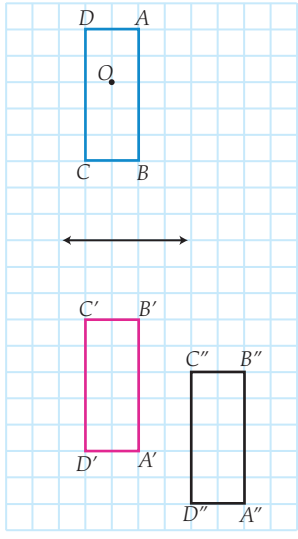
**(c)** Now, rotate the figure 90° in a clockwise direction about the origin.

**(d)** Write the coordinates of the vertices of the final image using image notation.

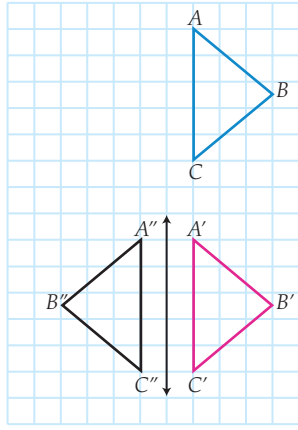
7 For the combined transformations in Question **1**, identify the necessary combined transformation that would move the image back to the position and orientation of the original figure.

8 Describe the transformations that have taken place to move the original figure in blue to the position in pink, then to its final position in black. List the relevant names of each transformation.

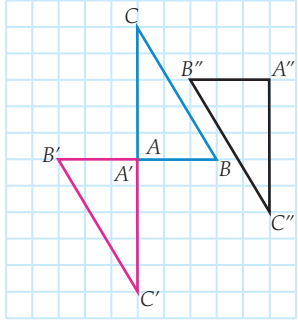
**(a)**



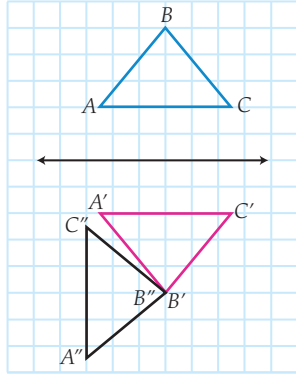

**(b)**



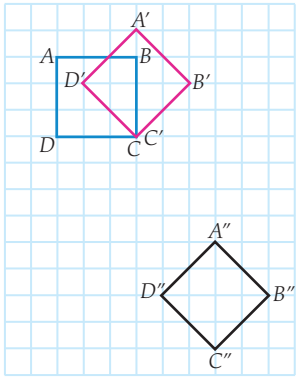

**(c)**



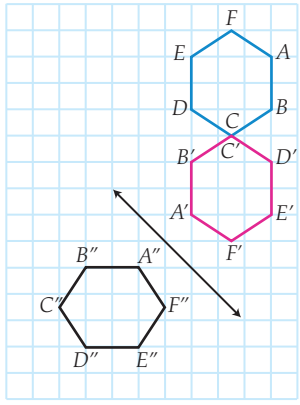

**(d)**



**(e)**




**(f)**




**9** For the combined transformations stated in each part of Question **8**, identify the transformations required to move the final image back to the original position and orientation of the figure.

## Reasoning

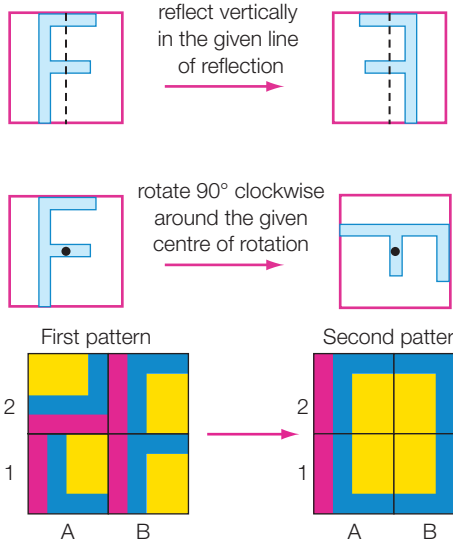
**10** **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 3)$ and $B(4, -1)$ is translated 1 unit right and 5 units down, then reflected in the $y$-axis.

**(b)** Plot the line $AB$, the first image $A'B'$ and the final image $A''B''$ on a Cartesian plane to check your answer to part **(a)**.

**11** Sarah has written a computer program to transform pictures of tiles. There are only two instructions in her program.

1 Reflect vertically in the line of reflection through the centre of each tile.

2 Rotate 90° clockwise around the given centre of rotation.




Sarah wants to transform the first pattern to the second pattern.

Copy and complete the following instructions to transform the tiles B1 and B2. You must only reflect vertically and rotate 90° clockwise through the centre of each tile.

A1 – Tile is in the correct position.

A2 – Reflect vertically, and then rotate 90° clockwise.

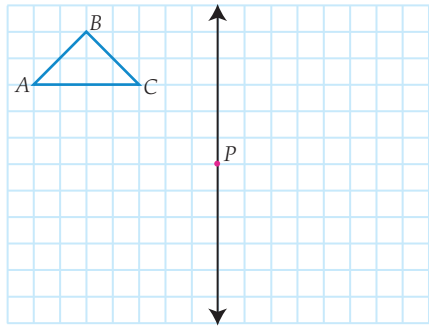
B1 – Rotate 90° clockwise and then …

B2 – …

**12** The following combined transformation is performed on triangle $ABC$.

- A reflection in the given line of reflection.
- A rotation of 180° about point $P$.
- A reflection in the given line of reflection.

Find a single transformation that will move the image $A'''B'''C'''$ back to the original position of the figure. Draw a diagram to show each required transformation.



## Open-ended

**13** Plot three points on a Cartesian plane and write their coordinates. Decide upon two particular transformations and use them to transform your figure. Draw the image after both transformations and write down the coordinates of the transformed vertices using image notation.

**14** Select one of the following capital letters of the alphabet: *B*, *C*, *D*, *E*, *F*, *G*.

**(a)** Perform a translation and then a reflection in a vertical line of reflection. Draw the resulting images on grid paper and label each image appropriately.

**(b)** Find a single transformation that will move the final image back to the original position. Show this transformation on your diagram.

**(c)** Use your answer for part **(b)** to find a single transformation that would produce the same result as the two transformations in part **(a)**.

**15** A shape is rotated through 180° about a point *O* and then its image is reflected in a horizontal line of reflection passing through *O*.

**(a)** Choose any shape and, on grid paper, carry out the transformations described above.

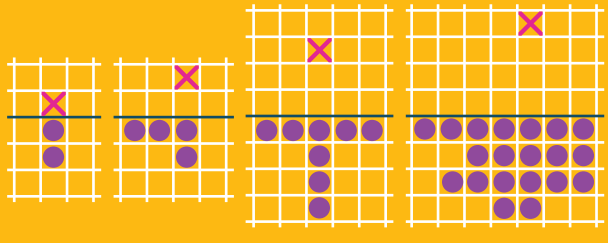
**(b)** Find a single transformation that would have the same results as the two transformations described above.

# Outside the Square — Problem solving

## Conway's army

**Equipment required:** 1–2 brains, grid paper, approximately 40 small counters

Conway's army is a one-person puzzle that was created by mathematician John Horton Conway in 1961. It is a variant of the peg solitaire game and uses a checkerboard of any dimension. The board is divided by a horizontal line; above it are empty cells and below it are a random number of game pieces, which are referred to as the 'soliders'. Examples of different boards and arrangements of soldiers is as follows.



### How to play:

Play proceeds by jumping horizontally or vertically (not diagonally) over other pieces onto an empty space, where jumped-over pieces are then removed.

### How to win:

The goal is to place a soldier as far above the horizontal line as possible. The cross in each of the diagrams at left is the maximum distance that a soldier can be placed from the horizontal line for the given number of original soldiers.

### Questions

**1** Create your own checkerboards with the horizontal line drawn. Begin by investigating how far four counters can be moved beyond the horizontal line. Try at least three different arrangements of the four counters. Is it possible to move a soldier past the second row?

**2** Continue to investigate with 4, 5, 6, 7, 8, 9, 10, 15 and 20 counters to see how far past the horizontal line the counters can be moved.

**3** Use your observations thus far to predict if it is possible to move a soldier to the fifth row above the horizontal line. Explain your answers.

**Strategy options**

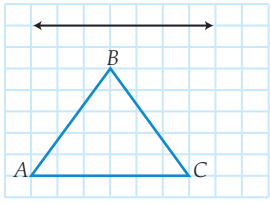
- Act it out.
- Test all possible combinations.

# Half-time 10

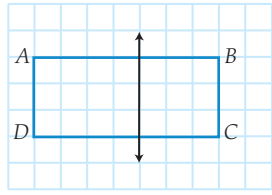
**1** Reflect each of the following figures in the line of reflection shown.

Ex.10.2

**(a)**



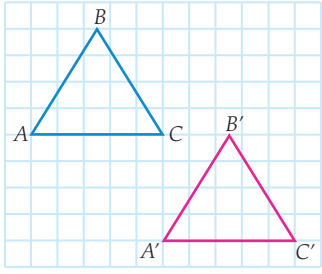

**(b)**



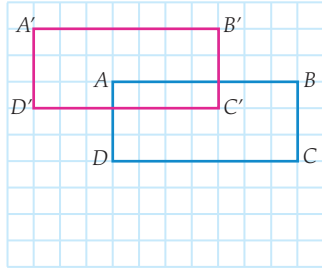

**2** Describe the translation shown in each of the following diagrams.

Ex.10.1

**(a)**




**(b)**



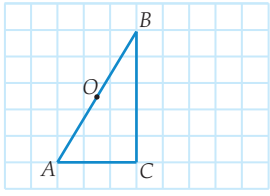

**3** Show the image of each of the following figures after it is rotated about the given centre of rotation.

Ex.10.3

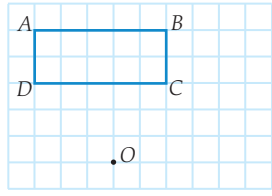
Rotate 180° in an anticlockwise direction about $O$.

Rotate 90° in a clockwise direction about $O$.

**(a)**

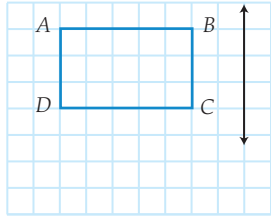



**(b)**




**4** Reflect the following shape in the line of reflection shown, then translate it 2 units right and 1 unit down.

Ex.10.4




**5** **(a)** Plot the points (1, 2), (3, 4) and (5, -2) on a Cartesian plane and join them to form a triangle.

Ex.10.4

**(b)** Draw the image after a translation of 6 units left and 1 unit up.

**(c)** Write the coordinates of the vertices of the translated points.

**(d)** Draw the image after the image drawn in part **(b)** is reflected in the $x$-axis.

**(e)** Write the coordinates of the vertices of this new image.

**(f)** Finally, rotate the image drawn in part **(d)** 180° clockwise about the origin.

**(g)** Write the coordinates of your final image.

# Transformations with GeoGebra

Versions of this Exploration for other technologies are available in Pearson Reader.

**Equipment required:** 1 brain, 1 computer with GeoGebra

When you open the GeoGebra program, you will see seven menu options (File, Edit etc.) at the top of the screen. Below these are eleven icons called tools. By clicking on the small arrow in the bottom right-hand corner, a drop-down list of more tools appears. If you hover over a tool the name and how to use it will appear in the top right-hand corner of the screen.

1 Click on the View menu. Deselect 'Axes' and 'Algebra View' by clicking on them. Select the 'Grid View' (the tick on the left of the word will appear or disappear when you select or deselect an option).

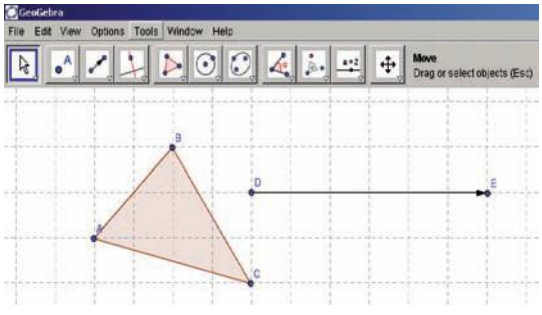
2 Click on the Options menu. Select 'Labelling', then 'New Points only'. Click on Options again, then select 'Point Capturing' and 'On (grid)'.

3 If a larger font is required, click on the Options menu and select 'Font Size'. Choose an appropriate size from the list provided.

## Translating an object

A translation is a slide. We will construct a triangle and translate it along a vector (a line with length and direction).

4 Click on the 'Polygon' tool, . Click on three points on the grid as shown in the diagram below. Notice that a line joins the points as you move around the triangle. To complete the triangle, click on the starting point again. We will be translating (sliding) the triangle ABC 6 units to the right. Can you picture where it will go?




5 Turn off the labelling by clicking on the Options menu, then selecting 'Labelling', then 'No New Objects'. Use the 'Polygon' tool, , to construct a triangle where you think the translated shape will appear.

6 Click on the drop-down arrow of the third tool from the left. Select 'Vector Between two Points'. Click on any grid point, then move the cursor 6 units to the right and click again to complete the vector.

7 Click on the drop-down arrow of the ninth tool from the left. Select the 'Translate Object by Vector' tool . Click anywhere inside the original triangle and then on the vector you have just made.

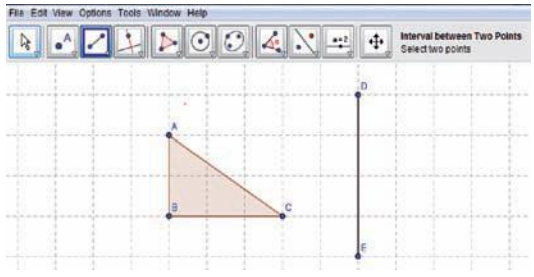
Does your translated image appear on top of the triangle you drew in step **5**? Notice that the points on the translated object are labelled as A′, B′ and C′. What is the relationship between a point on the original triangle and the translated point in the image?

8 Using the 'Select' tool , click on one of the end points of the vector (it does not matter which one). Drag the point about the page and notice how the translated image moves.

## Reflecting an object in a line

A reflection is a flip. We will now construct a right triangle, and reflect it in a line.

9 Click on the File menu. Select 'New'. Unless instructed by your teacher, click 'Don't Save'. Construct a triangle as instructed in step **4**.




10 Construct an interval by clicking on the 'Line through Two Points' tool (third tool from the left). Click on the drop-down arrow to select 'Interval between Two Points'. Construct a straight line segment 2 units to the right of your triangle by clicking on two points, one underneath the other.

We will be reflecting (flipping) the triangle ABC in the line segment DE. Can you picture where it will go?

11 Repeat step **5** to turn off the points labelling, then use the 'Polygon' tool to construct a triangle where you think the reflection will go.

12 To turn the labelling back on, click on the Options menu. Select 'Labelling', then 'New Points only'.

13 Select 'Reflect Object in Line' tool  , the ninth tool from the left. Click anywhere inside the triangle and anywhere along the interval DE to reflect your object. Did your reflected image match up on top of the triangle you drew in step **11**? Notice that the points on the reflected object are labelled as A', B' and C'. What is the relationship between the points on the original triangle and the reflected points in the image?

14 Using the 'Select' tool  (you can get to this tool easily by pressing the 'Escape' key), click and drag one of the end points of the interval DE about the page and notice what happens to the reflected triangle A'B'C'. Keep dragging your point until you find another reflection that has all of the reflected points A'B'C' on the grid. (Note: If your reflected object disappears from the page you can move the page using the 'Move Graphics View' tool  . Press Escape to return to the 'Select' tool.)

## Rotating an object

A rotation is a turn. We will now construct a triangle and rotate it by 180° in a clockwise direction.

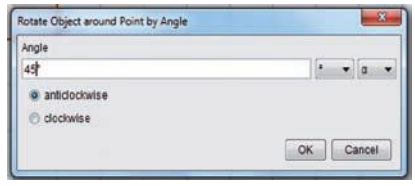
15 Click on the file menu. Select 'New'. Unless instructed by your teacher, click 'Don't Save'. Use the 'Polygon' tool to construct the same triangle you constructed in step **4**.

16 Select the 'New Point' tool  , second from the left. Make a new point that is 1 unit to the right and 1 unit down from point C. This will be your 'centre of rotation'.

We will be rotating (turning) the triangle ABC by 180° about this centre of rotation. Can you picture where it will go?

17 Repeat step **5** to turn off the points labelling, then use the 'Polygon' tool to construct a triangle where you think the rotated triangle will appear.

18 Click on the drop-down arrow of the ninth tool from the left. Select the 'Rotate Object around Point by Angle' tool  . Click anywhere inside the original triangle and then on the point you have just made. A pop-up window will appear. Delete the number 45 and type 180, but be sure to leave the degree symbol there. (If you delete it, select it again from the drop-down menu immediately to the right of the box.) Click clockwise. Click OK.



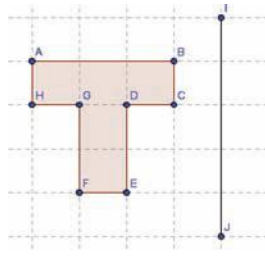

Does your rotated image match up on top of the triangle you drew in step **17**? Notice that the points on the rotated object are labelled as A', B' and C'. What is the relationship between a point on the original triangle and the rotated point in the image?

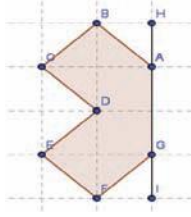
19 Repeat step **18** using angle sizes of 90°, 270° and 360°.

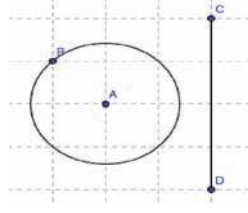
20 You should now have four triangles on the page (not including the unlabelled one you drew). To better understand how the points of the original figure are related to the rotated figures we will now use the 'Circle with Centre Through Point' tool  located as the sixth tool from the left. Click on the centre of rotation and then on one of the original vertices. Repeat this for all three of the original vertices so that you have three circles. What do you notice about the points on each circle?

## Taking it further

21 Perform reflections, translations and rotations with various other shapes such as those shown. Be creative—see if you can create a pattern by performing a series of transformations on a shape, or combining it with other shapes.



# 10.5 Symmetry

## Reflectional symmetry

We have seen how a line of reflection can produce a reversed or mirror image of an object, but many objects in our world, from flowers and insects in nature to company logos, flags and letters of the alphabet, have **reflectional symmetry** or mirror symmetry by themselves. Reflectional symmetry occurs when a line can be drawn through an object to divide it into two identical but reversed parts. Each part is the mirror image of the other. This line is called the **axis of symmetry**. In the photo below, a line can be drawn to divide the building into two identical but reversed parts.



Some objects have multiple axes of symmetry. The **order of reflectional symmetry** is the number of axes of reflectional symmetry that can be drawn through a shape. The Pentagon, as shown on the chapter title page, has five axes of symmetry, so a pentagon has symmetry of order 5. Objects with no line of symmetry are called **asymmetrical**. The letter R has no axes of symmetry and is therefore asymmetrical.

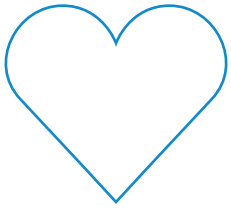


Is a human face symmetrical? Although we think our faces are symmetrical, if we reflect each side of our face separately, we get two different outcomes, neither of which is a perfect reproduction of our own face, one thinner and the other fatter. The face you see in a mirror is the mirror image or reversed view of your actual face.

## Worked Example 11 WE11

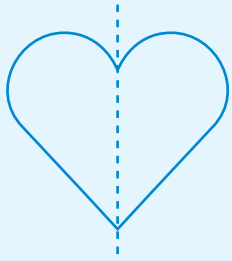
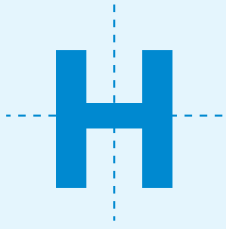
Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

**(a)**



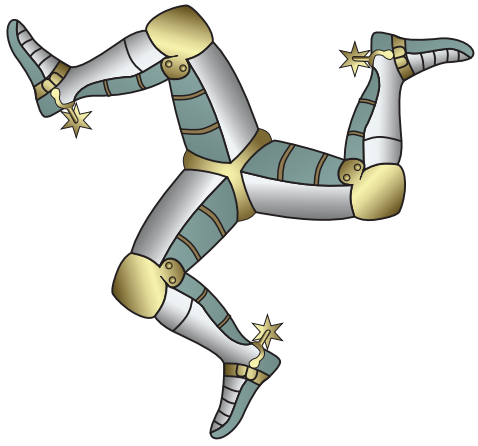
**(b)**



| Thinking | Working |
|---|---|
| **(a) 1** Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves.<br>**2** Count the number of axes of symmetry. This gives us the order of reflectional symmetry. | (a) (i)<br>  <br>(ii) Order of reflectional symmetry = 1. |
| **(a) 1** Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves.<br>**2** Count the number of axes of symmetry. This gives us the order of reflectional symmetry. | (b) (i)<br>  <br>(ii) Order of reflectional symmetry = 2. |

## Rotational symmetry

An object can also have **rotational symmetry**. If an object can be turned about a point less than 360 degrees and still look like the original object, that object is said to have rotational symmetry. The point about which the object is rotated is called the centre of rotation. An example of an object with rotational symmetry is the armoured 'triskelion', which appears on the Isle of Man's flag. The triskelion has three rotational symmetries referred to as 'three-fold symmetry', because every rotation of 120 degrees produces the original image.

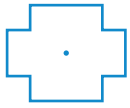


The **order of rotational symmetry** is the number of times in a complete 360° rotation that the original orientation of the shape appears. A pentagon has a rotational order of 5 because every rotation of 72° about the centre of the pentagon produces exactly the same image as the original. This occurs five times in a complete 360° rotation. A pentagon is therefore an interesting shape, as it has an order of 5 in both reflectional and rotational symmetry.

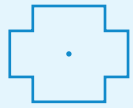
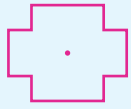
## Worked Example 12

WE12

Determine **(i)** which of the following figures has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

**(a)** 

**(b)** 

| Thinking | Working |
|---|---|
| **(a)** Find the centre of rotation and place your pen/pencil on it. | (a)  |
| Rotate the image around this point to determine whether the original image is reproduced. |  |
| Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation. | (i) Yes, it has rotational symmetry.<br>(ii) Order of rotational symmetry = 2.<br>(iii) $\frac{360}{2} = 180°$ |
| **(b)** Find the centre of rotation and place your pen/pencil on it. | (b)  |
| Rotate the image around this point, to determine whether the original image is reproduced. |  |
| Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation. | (i) Yes, it has rotational symmetry.<br>(ii) Order of rotational symmetry = 5<br>(iii) $\frac{360}{5} = 72°$ |

- An axis of symmetry is a mirror line that can be drawn through a shape to divide it into two identical but reversed halves.
- The order of reflectional symmetry is related to the number of lines of reflectional symmetry.
- An asymmetrical object has no reflectional symmetry.
- If a figure can be turned less than 360° so that it matches the original figure, it has rotational symmetry.
- The order of rotational symmetry is the number of times in a complete 360° rotation that an identical image of the original would be produced.

# 10.5 Symmetry

## Navigator

| | | |
|---|---|---|
| Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q15 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q16 | Q1, Q2, Q3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17 |

Answers page 694

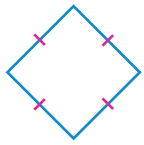
**Equipment required:** Grid paper for Question **12**

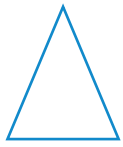
## Fluency

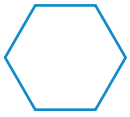
**1** Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

WE11

(a) 

(b) 

(c) 

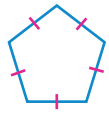
(d) 

(e) 

(f) 

(g) 

(h) 

(i) 

**2** Determine **(i)** which of the figures from Question **1** has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

WE12

**3** How many axes of symmetry do the following objects have?

(a) 

(b) 

(c) 

For Questions **4**, **5** and **6** refer to the letters of the word M A T H S.

**4** Which letter does not have an axis of symmetry?

**A** M **B** A **C** H **D** S

**5** How many of the letters have one axis of symmetry?

**A** 5 **B** 2 **C** 3 **D** 4

**6** Which letter has two axes of symmetry?

**A** H **B** M **C** A **D** S

## Understanding

**7** Copy the lower case letters of the alphabet into your book.

a b c d e f g h i j k l m
n o p q r s t u v w x y z

**(a)** Which lower case letters are asymmetrical?

**(b)** Which lower case letters have one axis of symmetry?

**(c)** Which lower case letters have two or more axes of symmetry?

**8** Copy the upper case letters of the alphabet into your book.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

**(a)** Which upper case letters are asymmetrical?

**(b)** Which upper case letters have one axis of symmetry?

**(c)** Which upper case letters have two or more axes of symmetry?

**9** Do any of the capital letters from Question **8** have rotational symmetry? If so, which ones are they?

## Reasoning

**10** Symmetry is present everywhere in nature. For each of the following images, determine whether it has almost perfect reflectional and/or rotational symmetry and state the order of each.

**(a)** 

**(b)** 

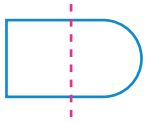
**(c)** 

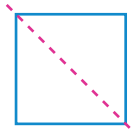
**(d)** 

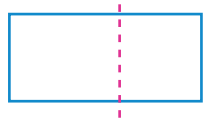
**(e)** 

**(f)** 

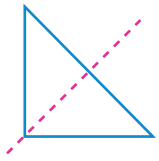
**11** For each of the following shapes, state whether or not the dotted line is an axis of symmetry.

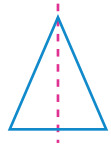
**(a)** 

**(b)** 

**(c)** 

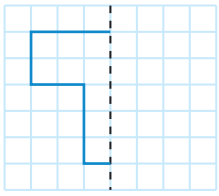
**(d)** 

**(e)** 

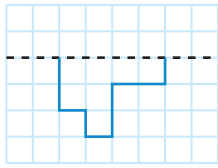
**(f)** 

**12** For each of the following diagrams, use the dotted line as an axis of symmetry to complete the diagram using grid paper.

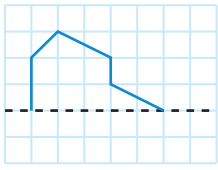
**(a)**



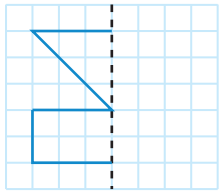
**(b)**



**(c)**



**(d)**



**13** Which two quadrilaterals have exactly two lines of reflectional symmetry?

**14** Which regular polygon has exactly three lines of reflectional symmetry?

## Open-ended

**15** WOW is a symmetrical word. Locate the axis of symmetry and find three more words that are also symmetrical.

**16** The dates of some years have reflectional and/or rotational symmetry. An example of a year that has rotational symmetry is 1961 and a year that has reflectional symmetry is 1881.

Identify an example of a year that has:

**(a)** rotational symmetry

**(b)** reflectional symmetry

**(c)** both reflectional and rotational symmetry.

**17** Describe when the following statement is true and when it is false.

'This parallelogram has reflectional symmetry.'

## Outside the Square — Problem solving

### Symmetry challenge



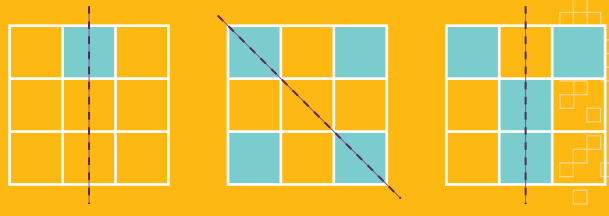
How many symmetrical designs of shaded squares can you find in a blank 3 by 3 square grid?

Hint: Before you shade any squares, can you identify the lines of symmetry present in this 3 by 3 grid shown at left? What symmetries are possible if you shade one square only?

**Strategy options**

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

Here are three examples of possible symmetrical designs. Note the position of the line of symmetry.



Don't include any rotations; e.g.  is considered the same as  .

Maths 4 Real

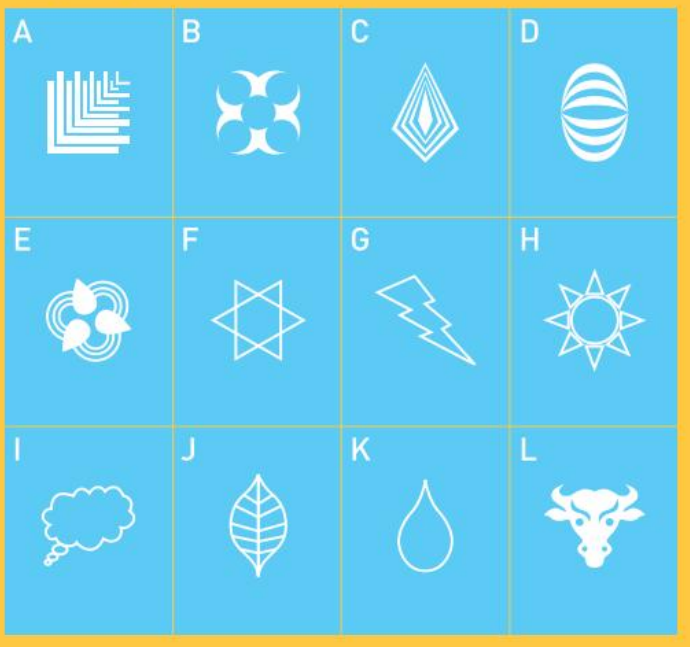


# SNOW LOGO

Symmetry is used in art, architecture, company logos and even in the design of national flags. We seem to be drawn to symmetrical objects, and designers and artists take advantage of our preference for symmetry.

Imagine you are a young marketing executive for a company that manufactures and sells snowboards. You know that many companies succeed or fail on the recognition of their brand by customers. The easiest method for achieving customer recognition is by integrating elements of symmetry into a company logo. Your manager, who is a symmetry fanatic, has given you the task of selecting the most symmetrical (reflectional and rotational symmetry) logo from a collection of sample logos developed by your company's graphic artists.

**Sample logos**




1 To assist you with this task, use the following table as a template and note down in your book the order of reflectional and rotational symmetry for each of the sample logos.

| Logo | Order of reflectional symmetry | Order of rotational symmetry |
|---|---|---|
| A | | |
| B | | |
| C | | |

2 At the end of the selection process, you think that you may be able to develop a better-looking, more symmetrical logo for your company. Your task is to develop your own version of your new company's logo and identify all elements of its symmetry to show the CEO of the company.

3 Apart from the symmetry elements, write down two or three other criteria you would use when deciding which is the best logo for a company.

## Research

- Find as many different examples as you can of well-known company logos or brands that have reflectional and/or rotational symmetry. (Car manufacturers are a good place to start.) Find some examples that are asymmetric (have no symmetry).
- Large companies spend a lot of money on getting the right design for their logo—the symbol by which the company is identified. Why do you think such importance is placed on the logo design?
- Which logo do you think is the most recognised throughout the world? Try to find out.

Mathspace

# Ambigrams and symmetry





**Equipment required:** 1 brain, a small mirror

## Letter ambigrams

An ambigram is a word that you can read in more than one direction or orientation. The word 'ambigram' is derived from *ambi* (meaning 'both', or 'on both sides') and *gram* (meaning 'something written down or drawn').

Ambigrams can have either reflectional symmetry or rotational symmetry.

### Vertical reflectional symmetry

Fantasy

Use your mirror to check

### Horizontal reflectional symmetry

COOKBOOK

### Rotational symmetry

dollop

Turn your book to check the rotational symmetry

**1** For each of the following ambigrams, complete the word (if required) and state whether it has rotational, vertical reflectional, or horizontal reflectional symmetry. (Use your mirror to check.)

**(a)** SWIMS

**(b)** NIDE

**(c)** MUM

**(d)** BID

**(e)** bid

**(f)** su____

**2 (a)** Some pairs of letters, such as 'b' and 'q', have rotational symmetry when used together. (Rotate 'b q' or 'q b' to see how this works.) What other pairs of letters have rotational symmetry when used together?

**(b)** What single letters have rotational symmetry when used by themselves?

**(c)** Create your own ambigram with rotational symmetry or vertical reflectional symmetry.

## Number ambigrams

Some numbers have symmetrical properties. For instance, the number 88 has vertical reflectional and horizontal reflectional symmetry, as well as rotational symmetry.

**3** Find some number ambigrams that have the following symmetries.

**(a)** vertical reflectional symmetry

**(b)** horizontal reflectional symmetry

**(c)** rotational symmetry

**4** Digital clocks often display numbers using a figure-of-eight digital number grid. This means that some digital numbers look the same upside down.

0123456789

**(a)** Which of these digital numbers have rotational symmetry?

**(b)** Which digital square numbers between 1 and 100 are ambigrams? What types of symmetry do they have?

**(c)** What is the largest 4-digit ambigram you can make that:

**(i)** uses only one of the digits 0 to 9

**(ii)** uses two of the digits 0 to 9

**(d)** What is the largest 5-digit ambigram you can make that:

**(i)** uses only one of the digits 0 to 9

**(ii)** uses three of the digits 0 to 9

## Grid symmetry

Draw up a Cartesian grid that goes from 0 to 4 on the horizontal axis, and from 0 to 3 on the vertical axis.

Using the clues below, place the letters R, C, K, H, S, U, Y, M, and B at their correct places on the Cartesian grid.

**Clues**

- The letter at (1, 2) is not symmetrical in any way.
- The letters at (0, 0), (1, 1), (2, 2) and (3, 3) all have horizontal reflectional symmetry.
- The letters at (0, 2) and (3, 3) have rotational symmetry.
- The letters at (3, 0), (3, 1) and (3, 2) all have vertical reflectional symmetry.
- The letters at (1, 1) and (3, 1) contain only curved lines.
- The letters at (2, 2) and (3, 2) contain only straight lines.
- The letters at (3, 0), (3, 1) and (3, 2), in that order, spell a word that means 'tasty'.

What word is spelt by the letters at (1, 2), (3, 1), (0, 0) and (3, 0), in that order?

# 10.6 Drawing and visualising 3D shapes

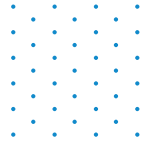
Objects such as buildings, machinery, car parts, people and play stations have three dimensions: length, width and height. It is often necessary to draw three-dimensional objects on a two-dimensional (2D) plane, such as a piece of paper or a computer screen.



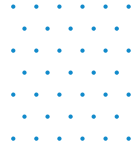
The drawing of the house in this diagram is an isometric drawing used by an architect.

Triangular dot paper, also called **isometric** paper can help to overcome the difficulties of drawing the three dimensions of real objects on a 2D plane. Objects represented on triangular dot paper are also known as isometric drawings.

It is recommended that isometric dot paper be held in this orientation:



This orientation is very difficult to use (not recommended).

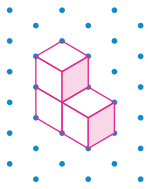


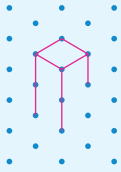
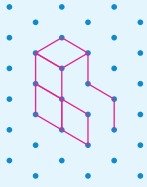
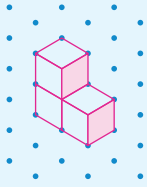
In Greek, 'iso' means 'equal' and 'metric' refers to 'measuring'. In isometric dot paper, the distances between each dot and the dots surrounding it are identical, be it a diagonal or a vertical direction.



## Worked Example 13 WE13

Use isometric dot paper to copy the following shape.



| Thinking | Working |
|---|---|
| **18** Join the points of a parallelogram that will represent the top face.<br>Draw in all the necessary vertical lines from the corners of the face. |  |
| **19** Draw in all the lines from the bottom of the verticals you have just drawn in step **1** that represent horizontal lines.<br>Draw in all additional required vertical lines from the ends of the lines you have just drawn. |  |
| **20** Join the ends of the lines you have just drawn.<br>To create the illusion of 3D, choose one direction (front, side or top) and shade in all faces you would see from that direction. |  |

# 10.6 Drawing and visualising 3D shapes

## Navigator

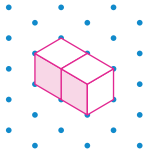
| | | |
|---|---|---|
| Q1, Q2 (a)–(c), Q3, Q4, Q5, Q6 (a)–(c), Q8, Q9 (a), Q10, Q11, Q13 | Q1, Q2 (b),(d),(f), Q4, Q5, Q6 (c)–(e), Q7, Q8, Q9 (b), Q10, Q11, Q14 | Q1, Q2 (d)–(f), Q4, Q6 (d)–(f), Q7, Q8, Q9 (c), Q10, Q11, Q12, Q14 |

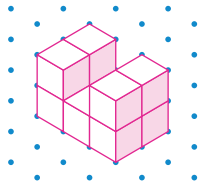
Answers page 696

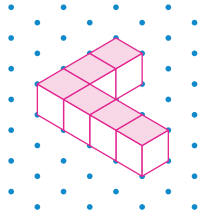
**Equipment required:** Isometric dot paper for Questions **1**, **5**, **8**, **9** and **12–14**

### Fluency

**1** Use isometric dot paper to copy the following shapes.

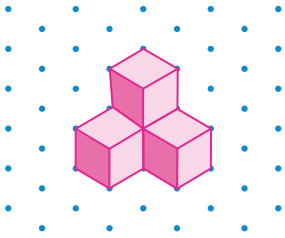
(a) 

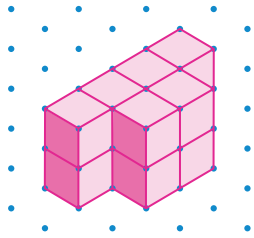
(b) 

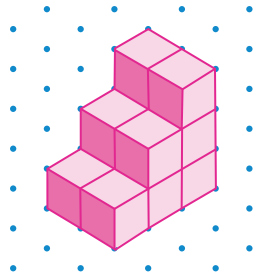
(c) 

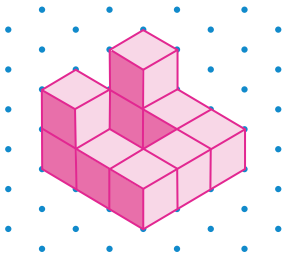


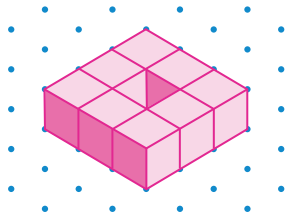
**2** How many cubes would be required to build these solids? (Assume that there are no cubes missing at the back of the solids where you cannot see.)

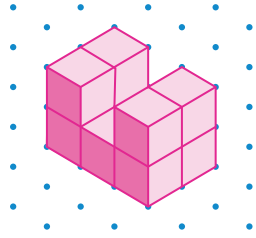
(a) 

(b) 

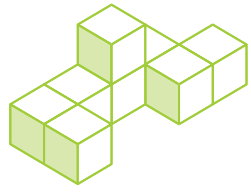
(c) 

(d) 

(e) 

(f) 

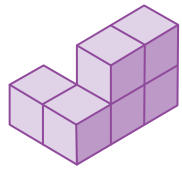
**3** The number of cubes required to build this solid is:

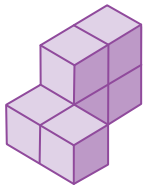


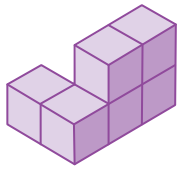
**A** 7 **B** 8

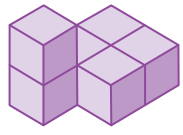
**C** 9 **D** 10

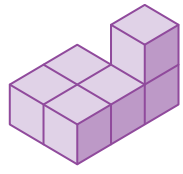
**4** Which of the shapes below is *not* a rotation of the one shown?



**A** 

**B** 

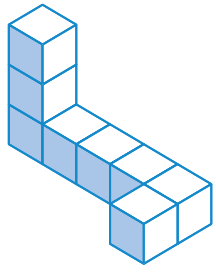
**C** 

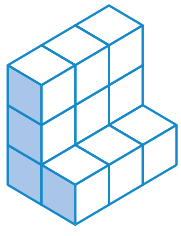
**D** 

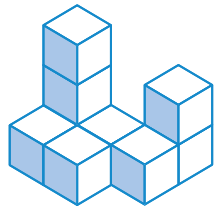
## Understanding

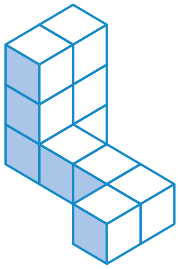
**5** Using isometric dot paper, draw a 3D sketch of a Rubik's cube. (A Rubik's cube is made of 27 cubes stacked three layers high, three layers wide and three layers deep.)

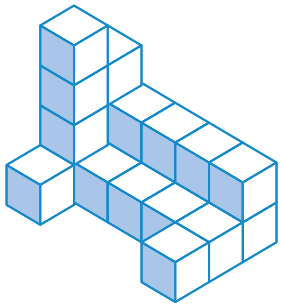
**6** How many cubes are in each solid?

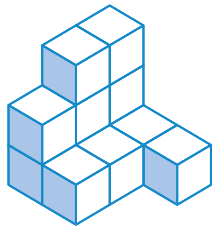
(a) 

(b) 

(c) 

(d) 

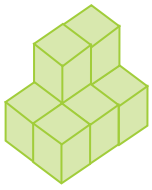
(e) 

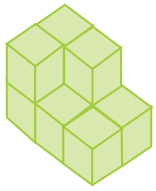
(f) 

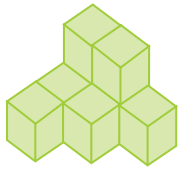
**7** Which of the following shapes could result from joining the two shapes below?

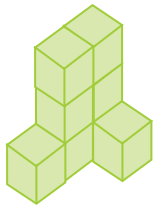




A 

B 

C 

D 

## Reasoning

**8** Using the aerial photograph on page 560 on the opening page of this chapter:

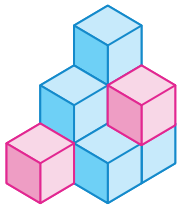
**(a)** draw a top view of the Pentagon

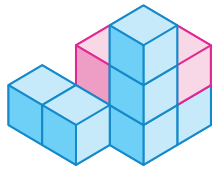
**(b)** draw a side view of the Pentagon.

**(c)** Does it matter which side you choose? Explain.

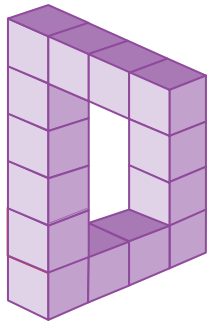
**9** Draw the solid after the pink shaded blocks have been removed.

(a) 

(b) 

(c) 

**10** This shape is known as the impossible rectangle.



**(a)** Why is this shape known as the *impossible* rectangle? (You may like to try to make it with blocks.)

**(b)** Copy the impossible rectangle onto isometric dot paper.

**(c)** Draw a similar rectangle that is possible. Colour your drawing using three colours.

**11** Draw the top view of the house drawn on page 604, the first page of this section.

## Open-ended

**12** A sculptor is designing a sculpture based on an arrangement of 24 cubes. Each cube has a width of 50 cm and the maximum height of the sculpture is to be 2.5 m. The sculptor wants to have a maximum of six cubes at the front and a maximum of four cubes in depth. Each cube must have at least one whole face touching another cube. Use isometric dot paper to design some sculptures that meet all these requirements.

**13** Using isometric dot paper, draw three different solids that can be built from 10 cubes that have whole faces touching.

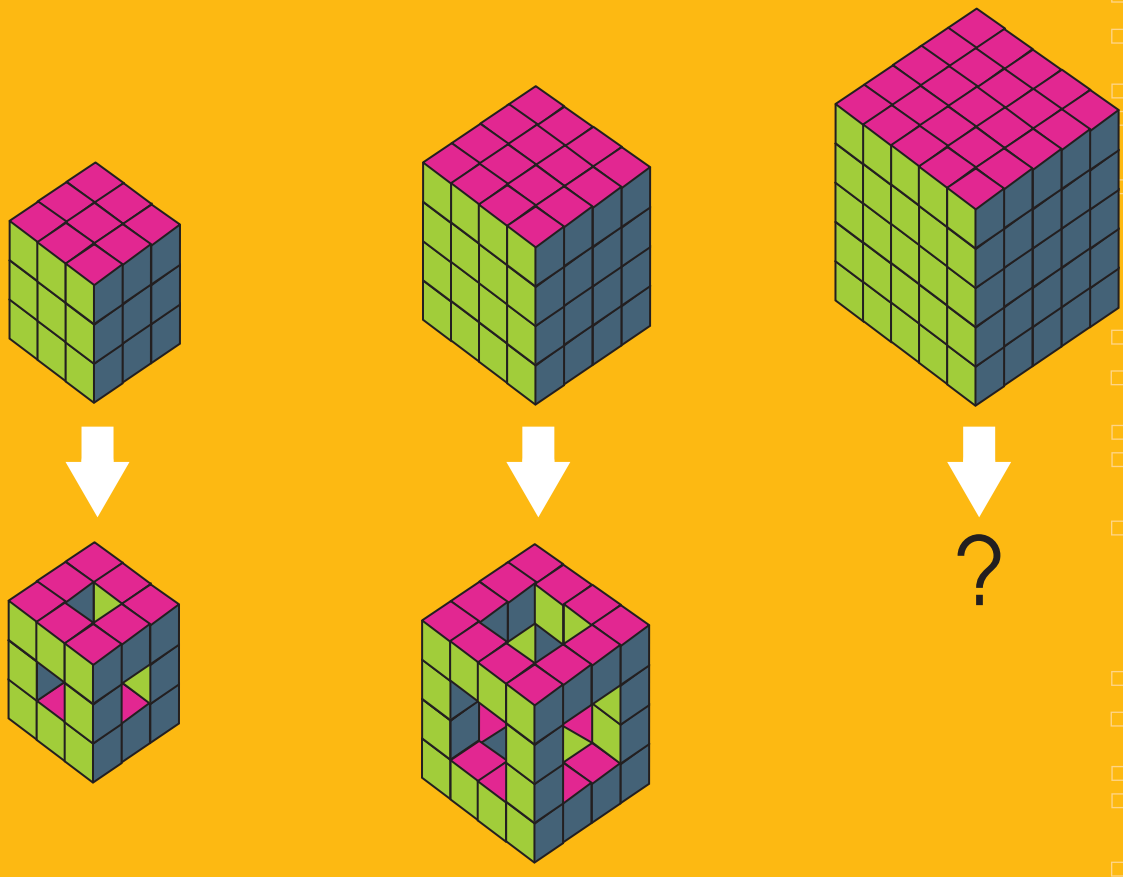
**14** Draw three shapes on isometric dot paper that, when joined together, form a $3 \times 3 \times 3$ cube.

# Outside the Square

## Problem solving

### What comes next?

Look at the pattern below.




A solid $3 \times 3 \times 3$ cube is made using 27 blocks. When one block is removed from the centre of each of the six faces and one block from the centre of the cube, a frame of the same cube is made using 20 cubes.

- Draw the next shape in the pattern.
- How many cubes are needed to make the frame of a $5 \times 5 \times 5$ cube?
- How many cubes are needed to make the frame of a $6 \times 6 \times 6$ cube?

**Strategy options**

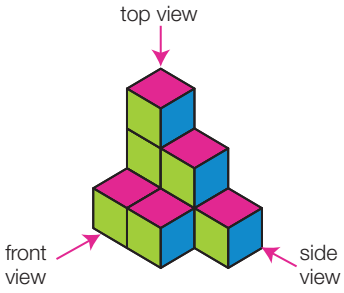
- Make a table.
- Look for a pattern.

# Plan views and elevations

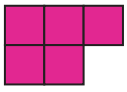
10.7

Three-dimensional objects can be viewed from various positions, or views. Objects looked at from the top, front and side can be drawn in two dimensions, and are known as **plan views**, or orthogonal projections.

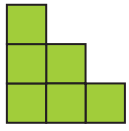
A 3D shape can look different when viewed from different points of view. A top view, or 'bird's eye view' is what you see when looking directly down on the shape, a side view is what you see when looking directly at the side and the front view is what you see when looking at the front.



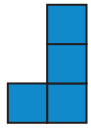

top view



front view



side view



## Worked Example 14

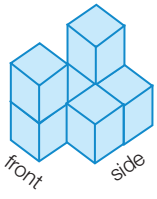
WE14

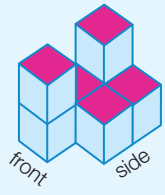
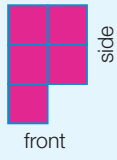
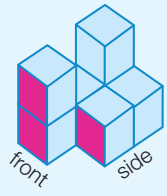
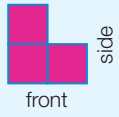
For the following solid, draw the:

**(a)** top view

**(b)** front view

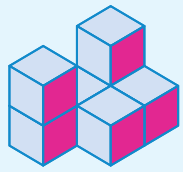
**(c)** side view.



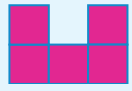

| Thinking | Working |
|---|---|
| **(a) 1** Highlight the faces seen from the top. | *(a)*   |
| **2** Draw squares to make the shape you would see if you were looking down on the solid. |   |
| **(b) 1** Highlight the faces seen from the front. Do not highlight front faces that are behind another block. | *(b)*   |
| **2** Draw squares to make the shape you would see if you were standing in front of the solid. |   |

**(c) 1** Highlight the faces seen from the side.

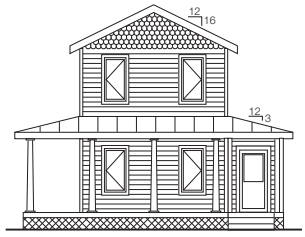
(c)



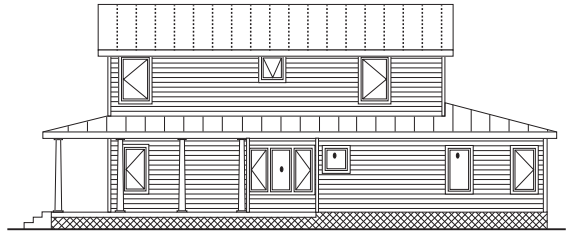
**2** Draw squares to make the shape you would see if you were standing at the side of the solid.



Drawings of houses and buildings viewed from a side are known as **elevations**. Elevations may be referred to as front, rear and side elevations, or by compass direction, such as an east elevation.



FRONT ELEVATION


SIDE ELEVATION

# 10.7 Plans views and elevations

## Navigator

Answers page 696

| | | |
|---|---|---|
| Q1, Q2, Q3, Q5, Q6, Q7, Q9, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10 | Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 |

**Equipment required:** Isometric dot paper for Questions **5**, **7**, **8**, **9**, and **10**

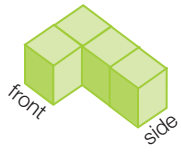
## Fluency

WE14

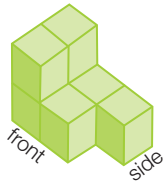
**1** For each of the following solids, draw the:

**(i)** top view **(ii)** front view **(iii)** side view.

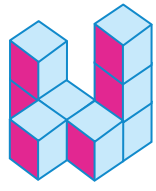
**(a)**



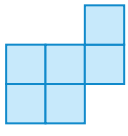

**(b)**



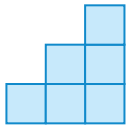

**2** The front view of the solid shown has been shaded. The top view of this solid looks like:



**A**



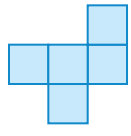
**B**



**C**



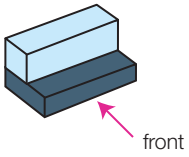
**D**



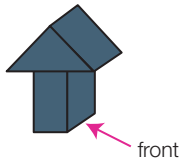
## Understanding

**3** The following structures are made with coloured blocks. Draw the front elevation of each structure by imagining you are standing looking from the position of the arrow, then move 90° to the right and draw the side elevation.

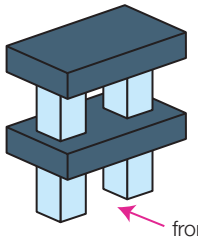
**(a)**



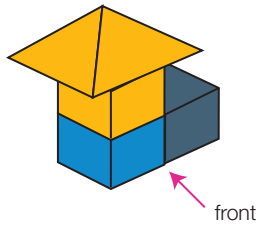

**(b)**



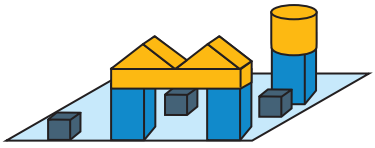

**(c)**




**(d)**

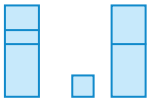



**4**

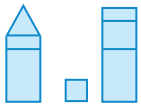


**(a)** Which diagram below shows a possible side elevation of the blocks above?

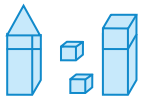
**A**



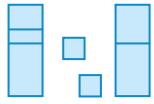
**B**



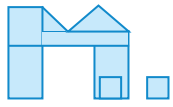
**C**



**D**



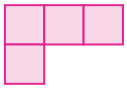
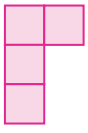
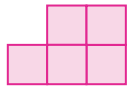
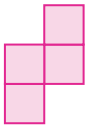
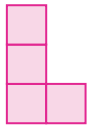
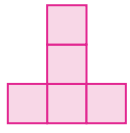
**(b)** Which elevation of the block structure above is shown opposite?



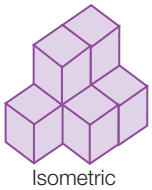
**A** front **B** rear

**C** side **D** top

**5** For each set of the plan views below, draw the 3D solid on isometric dot paper.

| | top view | front view | side view |
|---|---|---|---|
| **(a)** |  |  |  |
| **(b)** |  |  |  |
| **(c)** |  |  |  |

**6** A *mat plan* shows the top view of a shape and the number of cubes in each position.



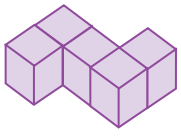
Isometric drawing

| | |
|---|---|
| 2 | 1 |
| 2 | 1 |
| 1 | |

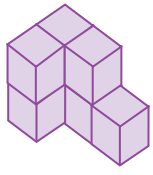
mat plan

Draw the mat plan for each of the shapes below.

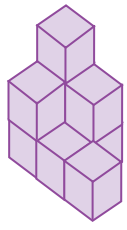
**(a)**



**(b)**



**(c)**



## Reasoning

**7** Draw each of the following mat plans as 3D shapes on isometric dot paper.

**(a)**

| | |
|---|---|
| 1 | 1 |
| 1 | |

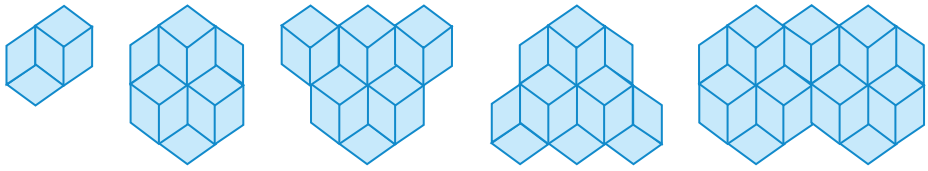
**(b)**

| | |
|---|---|
| 3 | 2 |
| 1 | |

**(c)**

| | | |
|---|---|---|
| 3 | 3 | 3 |
| 2 | 2 | |
| 1 | | |

**8 (a)** Copy the following diagrams onto isometric dot paper.



**(b)** On your drawings, use three colours to shade the different faces of the cubes.

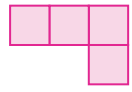
**(c)** How many cubes are in each of the diagrams?

**(d)** There is more than one way to colour the diagrams. If the cubes were coloured differently, would each diagram still look like it has the same number of cubes?

**(e)** Draw the top view of each shape.

## Open-ended

**9** A shape made with small cubes looks like this from above.



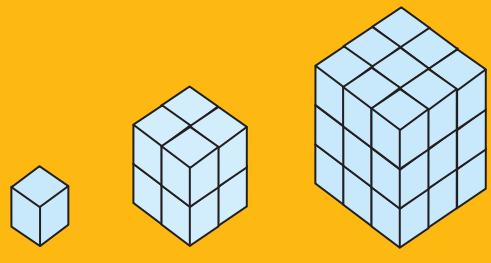

Draw three possible 3D drawings on isometric dot paper, stating the number of cubes in each diagram.

**10** Select a three-dimensional object and draw the front, rear and side elevations.

# Outside the Square — Problem solving

## Bird's eye squares

Ezra glued some 1 cm cube blocks together to form different-sized cubes. He made four cubes of side length 2 cm, four cubes of side length 3 cm and had four 1 cm cubes left over.



Ezra made a solid with his different-sized cubes. The solid's top view was a square.

**(a)** What is the minimum number of cubes Ezra would need to build his solid?

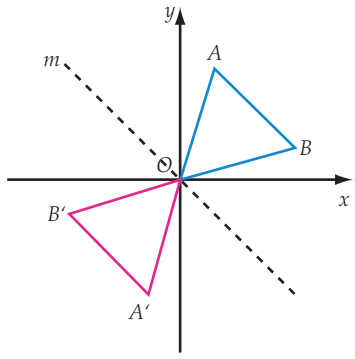
**(b)** How many different solids can be made with the cubes he has made that have a top view of a square and are made with more than one cube? Draw the top view of each solid.

**Strategy options**

- Draw a diagram.
- Make a model.

# Challenge 10

**1** The equilateral triangle $OAB$ has been transformed into the equilateral triangle $OA'B'$.



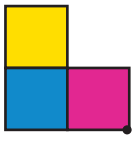

**(a)** Which single transformation changes $OAB$ into $OA'B'$?

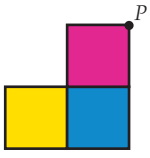
**(b)** Which two different transformations, applied one after the other, would change $OAB$ into $OA'B'$?

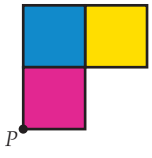
**(c)** $OAB$ is reflected in the line $m$. What is the name of the new position of $A$?

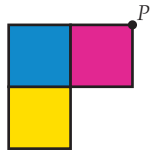
**2** Jemima looks in a mirror and sees the clockface behind her. The hands appear to show 2:45. What will the time appear to be in 15 minutes?

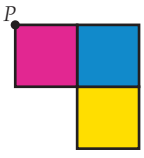
**3** A horizontal line $CD$ is copied three times, and each copy appears on top of the original. Using the terms 'rotation' and 'translation', describe how each copy must be moved in order to create a square. C ——— D

**4** If the shape  P is rotated 90° clockwise about the point $P$, the resulting figure could be:

**A** 

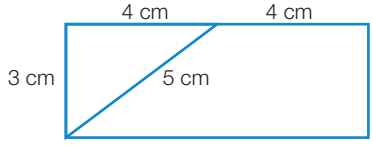

**B** 


**C** 


**D** 

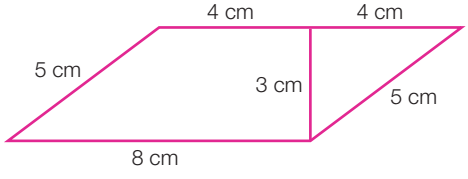

**5** To be decoded, a message has to be read by looking in a mirror. Decode the following message: 'TAƎЯ OT ƆЯAH SI ƎGƎ ƆƎꟻOIꟻƎƆ A'. To make sense of the message, what did you have to do?

**6** A line is drawn on the 8 cm by 3 cm rectangle opposite to make a triangle and a trapezium.

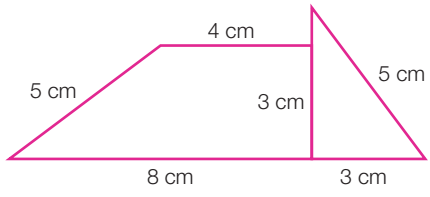



What transformation(s) need to be done to the triangle to create each of the following shapes?

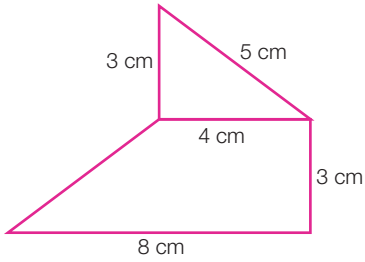
**(a)**



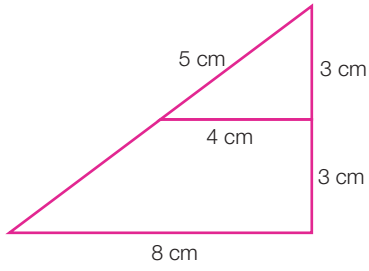

**(b)**




**(c)**




**(d)**



# Chapter review 10

## D.I.Y. Summary

### Key Words

| | | | |
|---|---|---|---|
| asymmetrical | isometric | perpendicular distance | rotation |
| axis of symmetry | line of reflection | plan views | rotational symmetry |
| centre of rotation | order of reflectional symmetry | reflection | transformation |
| elevations | order of rotational symmetry | reflectional symmetry | translation |
| image | original | rotate | |

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 Reflections, rotations and translations are three types of _____s.

2 When an object is turned about a point, a _____ has been performed.

3 When a figure is moved in a horizontal and vertical direction, a _____ has been performed.

4 When a figure is a reversed image of the original, a _____ has been performed.

5 The transformed version of a shape or diagram is called an _____.

6 A _____ acts as a mirror to reflect an object.

7 A point about which a figure is turned is called the _____.

8 The _____ indicates how many times the image is identical to the original in a complete 360° rotation.

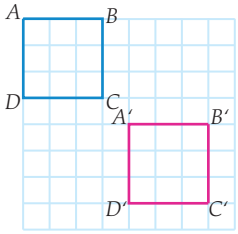
9 A 2D representation of a 3D solid can be drawn on _____ paper.

**Equipment required:** Grid paper for Questions **2**, **4**, **5**, **6**, **7**, **9**, **23** and **28**, isometric dot paper for Questions **14**, **16** and **17**, and a protractor for Questions **7**, **9**, **23** and **28**

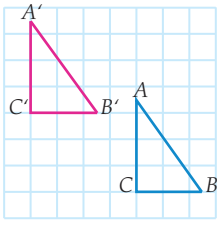
## Fluency

1 Describe the translation shown in each of the following diagrams. Ex 10.1

(a)

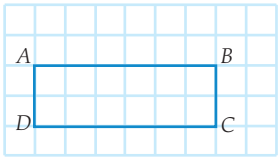



(b)

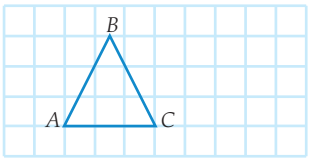



2 Copy each figure below and draw the resulting image after the translation required. Ex 10.1

(a) 3 units right and 1 unit up




(b) 3 units left and 3 units up



**3** What would be the reverse translation of 7 units right and 4 units down?

Ex.10.1

**A** 7 units up and 4 units right **B** 7 units left and 4 units up

**C** 7 units left and 4 units down **D** 4 units left and 7 units up

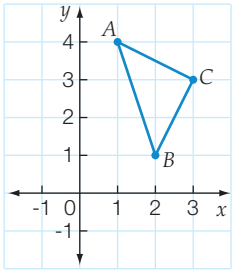
**4** For the figure in the diagram opposite:

Ex.10.1

**(a)** write the coordinates of the vertices

**(b)** copy the figure and translate it 4 units left and 3 units down, marking the image vertices with image notation

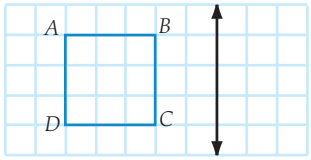
**(c)** write the coordinates of the vertices of the image.



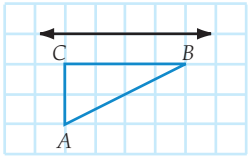

**5** Copy each of the following figures onto grid paper and draw the resulting image when the figure is reflected in the line shown.

Ex.10.2

**(a)**




**(b)**




**6** For the figure in the diagram opposite:

Ex.10.2

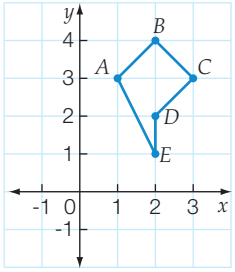
**(a)** write the coordinates of the vertices

**(b)** copy the figure and reflect it in the $x$-axis, marking the image vertices with image notation

**(c)** write the coordinates of the vertices of this image

**(d)** copy and reflect the original figure in the $y$-axis, marking the image vertices with image notation

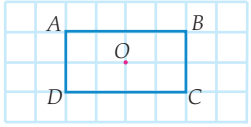
**(e)** write the coordinates of the vertices of this image.



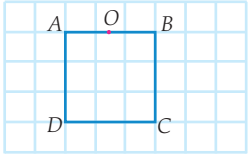

**7** Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

Ex.10.3

**(a)** Rotate 90° in a clockwise direction about $O$.




**(b)** Rotate 180° in an anticlockwise direction about $O$.




**8** A clockwise rotation of 90° is equivalent to an anticlockwise rotation of how many degrees?

Ex.10.3

**A** 180° **B** 360° **C** 90° **D** 270°

**9** For the figure in the diagram opposite:

Ex.10.3

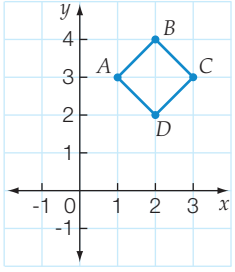
**(a)** write the coordinates of the vertices

**(b)** copy the figure and rotate it 90° in an anticlockwise direction about the origin, marking the image vertices with image notation

**(c)** write the coordinates of the vertices of this image

**(d)** copy and rotate the original figure 180° in an anticlockwise direction about the origin, marking the image vertices with image notation

**(e)** write the coordinates of the vertices of this image



(f) copy and rotate the original figure 270° in an anticlockwise direction about the origin, marking the image vertices with image notation

(g) write the coordinates of the vertices of this image.

**10** What are the reverse transformations for a translation of 5 units right and 8 units down followed by a clockwise rotation of 90°? Ex 10.4

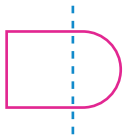
**A** An anticlockwise rotation of 90° and a translation of 5 units right and 8 units down

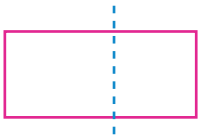
**B** A clockwise rotation of 180° and a translation of 5 units right and 8 units down

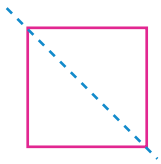
**C** An anticlockwise rotation of 90° and a translation of 5 units left and 8 units up

**D** A clockwise rotation of 90° and a translation of 5 units left and 8 units down

**11** In which of the following shapes is the dotted line an axis of symmetry? Ex 10.5

**A** 

**B** 

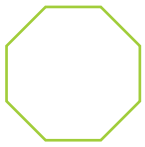
**C** 

**D** 

**12** Copy the following shapes and draw lines of symmetry. Some shapes may have more than one line of symmetry. Ex 10.5

(a) 

(b) 

(c) 

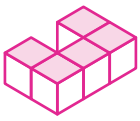
(d) 

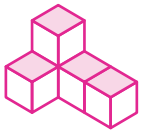
(e) 

(f) 

**13** Identify the shapes from Question **12** that have rotational symmetry. Ex 10.5

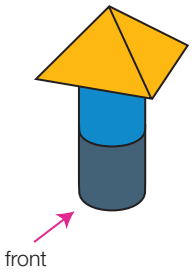
**14** Two solids have been made using wooden cubes, as shown. Ex 10.6

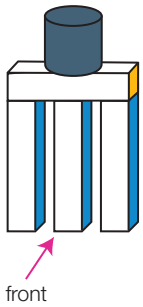
(a) 

(b) 

Use isometric dot paper to copy the shapes.

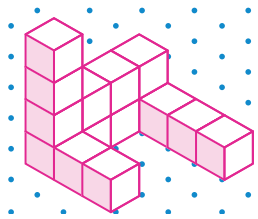
**15** For each of the three-dimensional structures below, draw the front and side elevations. Ex 10.7

(a) 


(b) 

## Understanding

**16** Use isometric dot paper to draw the shape shown.

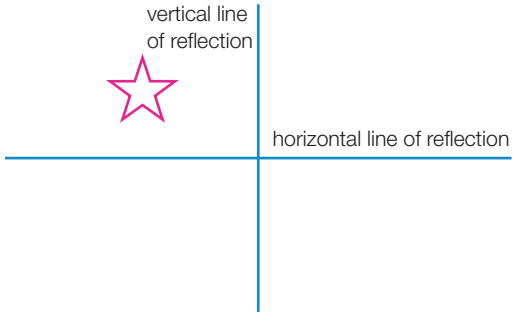


**17** Use the shape above to draw:

**(a)** a front view **(b)** a top view **(c)** a side view.

**18** What single translation would be equivalent to 6 units down, 7 units right, 2 units up, 8 units left, 4 units up and 5 units right?

**19** **(a)** Using the diagram below, reflect the star in the vertical and then the horizontal lines of reflection.



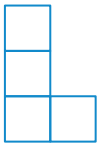

**(b)** Using the same starting position, reflect the star in the horizontal and then the vertical lines of reflection. Does it result in the same outcome as in **(a)**?

**20** What rotation is equivalent to reflecting an object along horizontal, then vertical, lines of reflection?

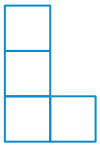
**21** How many letters of the word 'MATHEMATICS' do not have reflectional or rotational symmetry?

**22** The plan views of a three-dimensional shape made with cubes are shown. Draw the solid on isometric dot paper.

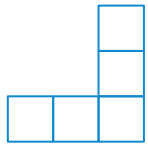
top view



front view



side view



**23** On grid paper:

**(a)** draw the line joining the points $A(-1, 3)$ and $B(2, 5)$

**(b)** translate this line 3 units right and 2 units down

**(c)** reflect this image in the $x$-axis to form a new image

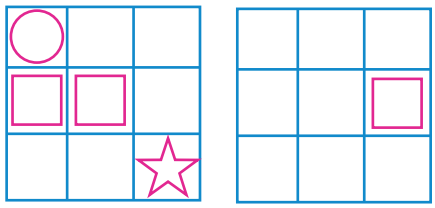
**(d)** rotate this image 90° in a clockwise direction (about the origin) to form a final image

**(e)** write the coordinates of the final image.

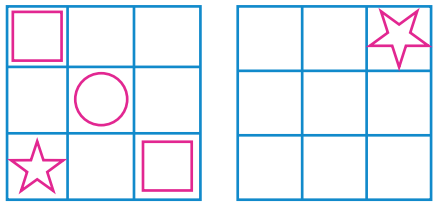
## Reasoning

**24** The following 3 by 3 squares comprise identical symbols, but each one has been reflected horizontally, vertically or along the diagonal and is missing three shapes. Your task is to draw the missing three shapes.

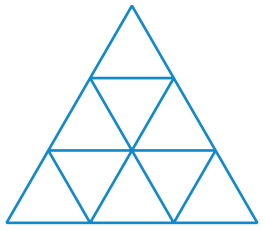
**(a)**



**(b)**



**25** The following equilateral triangle comprises nine smaller equilateral triangles.

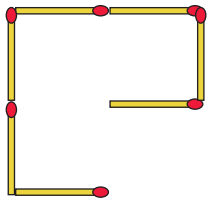


By shading the smaller equilateral triangles, create three different designs that have:

**(a)** one line of symmetry

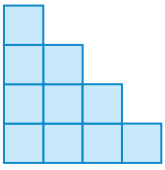
**(b)** three lines of symmetry.

**26** The diagram at right shows a pattern made from sticking seven matchsticks of equal length to a piece of paper. What is the smallest number of matchsticks that need to be added so that the resulting image has at least one line of reflectional symmetry?

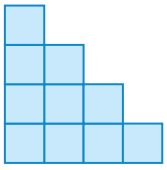


**27** A solid is built using cubed blocks. The front, side and top views of the solid are shown. What is the minimum number of blocks this solid can have?

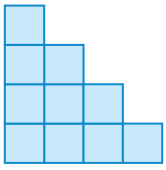
top view



front view



side view



**28** Matt is asked to reflect a particular figure in the $y$-axis. This image is then rotated 180° in a clockwise direction.

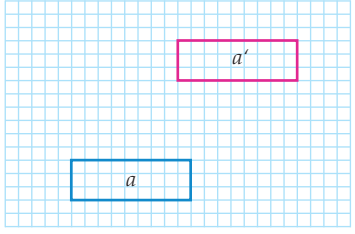
Matt believes he can replace these two combined transformations with one transformation.

Choose a shape of your own and perform the two transformations on it that Matt was required to perform. Is he correct? Can one transformation replace the two transformations? If so, write the required transformation.

# NAPLAN practice 10

## Numeracy: Non-calculator

**1** The translation shown is:



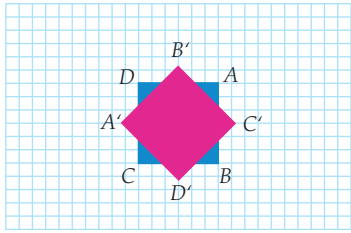

**A** 6 units right, 8 units up

**B** 8 units right, 9 units up

**C** 8 units down, 6 units left

**D** 6 units down, 8 units left

**2** The rotation needed to transform the original shape (blue) to the new rotated image (pink) is:



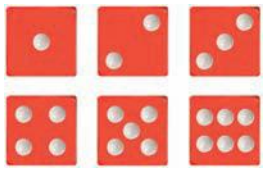

**A** 45° clockwise

**B** 135° anticlockwise

**C** 90° clockwise

**D** 90° anticlockwise

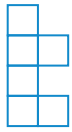
**3** How many of the six faces of a die have fewer than three lines of symmetry?



**A** 1 **B** 2

**C** 3 **D** 4

**4** The top view of a shape is shown at right.

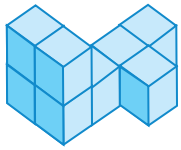


Which solid has this top view?

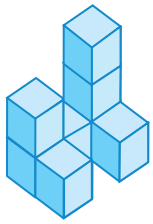
**A**



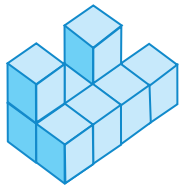
**B**



**C**



**D**

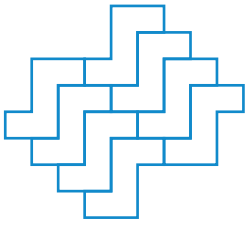


## Numeracy: Calculator allowed

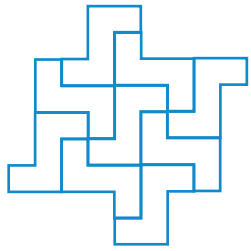
**5** Mia has these S-shaped tiles.
They are blue on one side and red on the other side.
Mia wants to make a pattern with all the tiles blue side up.
Which one of the patterns can Mia not make?



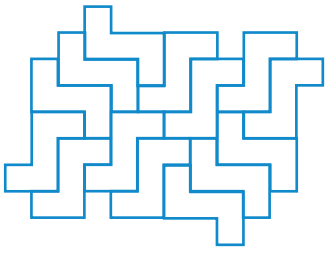
**A**



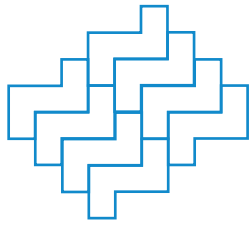
**B**



**C**



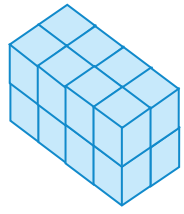
**D**



**6** Which letter in the word SAME does not have an axis of reflectional symmetry?

**A** S **B** A **C** M **D** E

**7** Janita glued small wooden cubes together to form the solid shown below.
She then painted the solid on all surfaces.



How many wooden cubes are painted on just one face?

**A** 0 **B** 2 **C** 4 **D** 8

# Mixed review E

**Equipment required:** Grid paper and protractor for Questions **11**, **17** and **18**

## Fluency

**1** Simplify:

**(a)** $44 - 8 \times 4$ **(b)** $8 \times 6 \div 3 \times 4$ **(c)** $28 \div 4 + 3 \times 6$

Ex 1.5

**2** Simplify:

**(a)** $2^4 - 3^2$ **(b)** $3^3 \times 10^2$ **(c)** $10^5 - 10^3$

Ex 1.2

**3** Calculate:

**(a)** $15.006 + 2.45 + 0.059$ **(b)** $0.2 \times 0.036$ **(c)** $0.597 \div 0.3$

Ex 4.4–4.6

**4** Use the following rules to complete the tables.

Ex 5.3

**(a)** $n = 3m$

| $m$ | 4 | 0 | 11 | 25 | 15 |
|---|---|---|---|---|---|
| $n$ | | | | | |

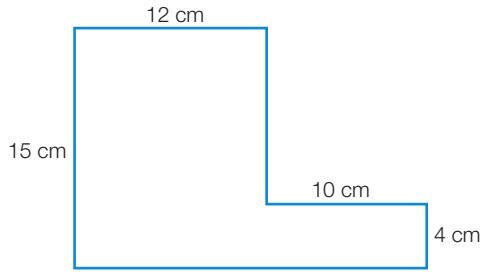
**(b)** $t = s - 12$

| $s$ | 18 | 100 | 12 | 22 | 312 |
|---|---|---|---|---|---|
| $t$ | | | | | |

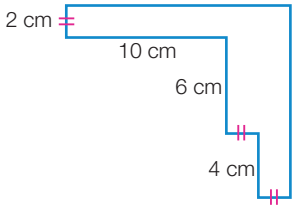
**5** Find the area of each of the following shapes. (All angles are right angles.)

Ex 6.5

**(a)**




**(b)**




**6** Find the following probabilities.

Ex 9.7

**(a)** getting a 6 on one roll of a normal die

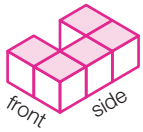
**(b)** getting a red king on a single draw from a normal pack of cards

**(c)** getting two heads when tossing two coins

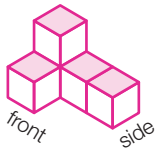
**7** Two shapes have been made using wooden cubes, as shown.

Ex 10.7

**(a)**




**(b)**




For each shape, draw: **(i)** the front elevation **(ii)** the top elevation.

**8** Solve each of the following equations using the balance method. Check your answer by substitution.

Ex 7.4

**(a)** $3(x - 1) = 6$ **(b)** $\frac{x}{4} + 1 = 7$ **(c)** $2(x + 3) = 12$

**9** Find the supplementary angles for each of the following.

Ex 8.3

**(a)** 35° **(b)** 127° **(c)** 90°

**10** The following numbers represent the heights of students in a class, to the nearest centimetre. Ex 9.1

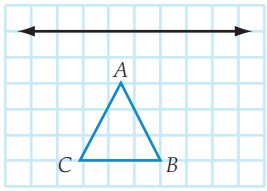
148, 163, 158, 162, 151, 153, 170, 167, 165, 162, 172, 173, 157, 162, 158, 155, 163, 159, 160, 161

Draw the frequency table showing the information. You will need to group the results. Use 145–149, 150–154, 155–159 etc.

**11** Show the image of each of the following figures after each of the following combined transformations. Use a protractor to help you. Ex 10.4

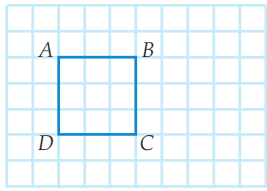
Reflect in the horizontal line of reflection and translate 4 units left and 3 units down.

**(a)**




Reflect in the line $BC$ and rotate 90° in a clockwise direction about point $C'$.

**(b)**




**12** For each of the following, identify the number of axes of symmetry. Ex 10.5

**(a)** isosceles triangle **(b)** square **(c)** rhombus

**13** Find the mean, mode, median and range of the following data, noting any possible outliers. Ex 9.2

23, 34, 45, 26, 35, 42, 6, 34, 22, 36, 21, 29, 32, 34, 41, 39, 25, 27, 31, 30

## Understanding

**14** A figure is translated 7 units left, 4 units down, 8 units right and 3 units up. What would be the final position of the image compared to that of the original figure?

**15** **(a)** Find the prime factors of 24.

**(b)** Find the prime factors of 30.

**(c)** Use the prime factors to find the highest common factor of 24 and 30.

**(d)** Use the prime factors to find the lowest common multiple of 24 and 30.

**16** **(a)** What is the size of each angle in an equilateral triangle?

**(b)** A parallelogram has one angle that measures 126°. What size is each of the three remaining angles?

**17** Which capital letters of the alphabet that have both reflectional and rotational symmetry also have reflectional and rotational symmetry in lower case form?

## Reasoning

**18** A point (1, 1) is translated 2 units right and 1 unit up. It is then reflected in the $y$-axis and then reflected in the $x$-axis. It is then translated 2 units right and 1 unit up. Finally, it is rotated 180° in a clockwise direction about the origin. What are the coordinates of the final image?

**19** **(a)** Find a data set with at least six different values that has a median of 6 and a mean of 8.

**(b)** Describe the process you followed to find these values.

**(c)** Why might it be easier to use an odd number of values in a question of this type?

**20** A game is played in which three different coins are being tossed. It costs 50 cents to play this game. You win $1 (plus your original 50 cents) if more than one of the coins shows heads, otherwise you lose your money.

**(a)** Write down the set of results that are possible.

**(b)** Is the game fair? Explain your reasoning.

# Answers

Worked solutions and answers to ***all*** activities appear in the Teacher Companion.

## Chapter 1

### Recall 1 (p. 4)

1 (a) B (b) D

2 (a) 0, 12, 74, 567, 602, 4500, 6008, 11 100

(b) 2400, 2196, 1200, 1010, 987, 240, 204, 95

3 (a) 56 895 (b) 7 025 073

4 (a) (i) 1250 (ii) 1200 (iii) 1000

(b) (i) 8980 (ii) 9000 (iii) 9000

5 (a) 18 (b) 90 (c) 8 (d) 10 000

6 (a) 512 (b) 2063 (c) 1698

7 (a) 175 (b) 627 (c) 1157

8 (a) 315 (b) 670 (c) 3230

9 (a) 211 (b) 412 (c) 128 rem 1

### Exercise 1.1 (p. 7)

1 (a) 73 (b) 107 (c) 199 (d) 60 (e) 120

(f) 420 (g) 210 (h) 120 (i) 280 (j) 120

(k) 180 (l) 380 (m) 228 (n) 178 (o) 216

2 (a) 153 (b) 152 (c) 294 (d) 186 (e) 364

(f) 215 (g) 891 (h) 231 (i) 456 (j) 154

(k) 195 (l) 192 (m) 808 (n) 565 (o) 1364

3 (a) 121 (b) 503 (c) 165 (d) 360 (e) 98

(f) 189 (g) 618 (h) 242 (i) 582 (j) 360

(k) 266 (l) 420

4 C

5 (a) 245 km (b) 235 km

6 1000

7 (a) $136 (b) $53

8 (a) 720

(b) May be about to begin or end a busy time (e.g. peak hour). A bottleneck, accident or other traffic hold-up may occur.




9 $390

10 (a) $7.84 (b) $2.31 (c) $7.08 (d) $4.10

11 (a) Multiplying 7 by 20 gives one lot less of 7 than needed, not one lot less of 21. Need to multiply 7 by 20, then add 7 to get 147.

(b) Doubling twice is the same as multiplying by 4, not by 3. You can double 35 to get 70, then add another 35 to get 105.

(c) The remaining 9 should have been subtracted to get 191, not added.

12 (a) Alex and Khalid's methods are correct.

(b) They both round off the multiplier (Alex rounds 9 to 10 and Khalid 29 to 30), multiply, then subtract 1 lot of the number they multiplied by (Alex subtracts 1 lot of 29, as he had 10 lots instead of 9, and Khalid subtracts 1 lot of 9, as he had 30 lots instead of 29).

(c) The other two students round off correctly, but then subtract the wrong number.

#### Open-ended – Sample answers

13 5 picture hooks at 28 cents each = $1.40;
12 curtain rings at 15 cents each = $12 \times 10 + 12 \times 5 = \$1.80$.
Total: $\$(1.40 + 1.80 + 6) = \$9.20$.
Tranh should have enough money.

14 Do the calculations $9 \times 23 + 2 \times 23$.

15 Group the numbers and then use multiplication and addition. He has 7 1s (worth $7 \times 1 = 7$), 9 2s (worth $9 \times 2 = 18$) and so on. When the multiplications are finished add together the results. He should get 50.

### Exercise 1.2 (p. 15)

1 (a) $2 \times 2 \times 2 = 8$

(b) $2 \times 2 \times 2 \times 2 = 16$

(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(d) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

(e) $0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$

(f) $10 \times 10 \times 10 = 1000$

(g) $6 \times 6 \times 6 \times 6 = 1296$

(h) $5 \times 5 \times 5 \times 5 \times 5 = 3125$

(i) $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 262\,144$

(j) $11 \times 11 \times 11 = 1331$

(k) $12 \times 12 \times 12 \times 12 = 20\,736$

(l) $14 \times 14 \times 14 \times 14 \times 14 = 537\,824$

2 (a) $8^3$ (b) $4^6$ (c) $12^5$ (d) $16^9$

(e) $17^3$ (f) $19^2$ (g) $8^4$ (h) $13^7$

(i) $11^7$ (j) $9^6$

3 (a) 16 384 (b) 46 656 (c) 6561 (d) 512

(e) 16 807 (f) 3375 (g) 289 (h) 125 000

(i) 64 (j) 500 (k) 4000 (l) 2000

(m) 1024 (n) 19 683 (o) 1

4 (a) five squared (b) thirty-one squared

(c) three cubed (d) twenty-seven cubed

(e) four to the power of five (f) nine to the power of six

(g) one to the power of four

(h) seven to the power of seven

5 25, 36, 49

6 64, 125, 216

7 (a) 5 (b) 7 (c) 10 (d) 11

(e) 1 (f) 0 (g) 70 (h) 20

(i) 2 (j) 4 (k) 10 (l) 1

(m) 0 (n) 5 (o) 20 (p) 30

8 (a) A (b) C (c) A

9 (a) 5 (b) 7 (c) 65 (d) 32

(e) 64 (f) 243 (g) 21 (h) 45

(i) 51

10 (a) $1^{200}$, $5^4$, $10^3$, $4^5$, $5^5$, $4^6$

(b) $10^5$, $100^2$, $3^2$, $2^3$, $1^{1000}$, $0^{100}$

11 (a) 16, 36 (b) 49, 81

12 (a) 0 (b) 190 000 (c) 45 000 (d) 2 472 768

(e) 537 824 (f) 51 998 079 (g) 266 240 (h) 79 785

(i) 439 291

13 (a) 3, 4 (b) 2, 3 (c) 4, 5 (d) 7, 8

(e) 9, 10 (f) 1, 2 (g) 8, 9 (h) 10, 11

14 (a) $\sqrt[3]{64}$, 5, $\sqrt{49}$, $2^4$ (b) $\sqrt[3]{125}$, $\sqrt{60}$, 9, $7^2$

(c) $\sqrt[3]{8}$, $\sqrt{90}$, 10, $4^2$ (d) 8, $\sqrt{80}$, $\sqrt{105}$, $3^3$

15 (a)

| Index form | Expanded form | Value |
|---|---|---|
| $10^1$ | 10 | 10 |
| $10^2$ | $10 \times 10$ | 100 |
| $10^3$ | $10 \times 10 \times 10$ | 1 000 |
| $10^4$ | $10 \times 10 \times 10 \times 10$ | 10 000 |
| $10^5$ | $10 \times 10 \times 10 \times 10 \times 10$ | 100 000 |

(b) The index number in the first column is the same as the number of zeroes after the 1 in the third column.

16 (a) 4 (b) 2 (c) 3 (d) 5

(e) 1 (f) 6 (g) 4 (h) 2

(i) 4

17 (a) $5^2 \times 5^3 = 25 \times 125 = 3125$. $5 \times 5 \times 5 \times 5 \times 5 = 3125$.

(b) $2^3 \times 2^6 = 8 \times 64 = 512$. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$

(c) The sum of the powers is the number of times the base is multiplied with itself to obtain the answer.

18 (a) 4 (b) 10 (c) 16

19 (a) (i) True (ii) True (iii) True (iv) False

(b) (i) False (ii) True

20 (a) (i) one hundred (ii) one hundred

(b) (i) $10^{100}$ (ii) $10^{100}$, more than a lifetime

21 (a) 121; 12 321; 1 234 321

(b) 123 454 321; 12 345 654 321; 1 234 567 654 321

## Open-ended – Sample answers

22 $12^3 = 1728$; $31^2 = 961$; $3^{21} = 10\,460\,353\,203$

23 (a) 5, 6, 7, 8 or 9 (b) 10, 11, ... 21

24 (a) Mina is partly correct. The 'little 2' does mean two 35s, but they are multiplied together, not added. A better explanation for Mina to give Jo would be: 'The 2 tells you how many 35s you need to multiply together.'

(b) $2^2$ is a bit of a special case. $2^2 = 2 \times 2$, which gives the same answer as $2 + 2$ or 'double 2'. However, this is not true for any other number, except 0.

# Exercise 1.3 (p. 24)

1 (a) 108 (b) 246 (c) 496 (d) 150 (e) 17

(f) 54 (g) 315 (h) 65 (i) 1050 (j) 180

(k) 18 (l) 15

2 (a) 492 (b) 442 (c) 1292 (d) 1403 (e) 1519

(f) 2184 (g) 1974 (h) 2597 (i) 3025 (j) 5002

(k) 5544 (l) 4896

3 (a) 2100 (b) 2000 (c) 26 000 (d) 18 (e) 27

(f) 16 (g) 480 000 (h) 12 800 000

(i) 140 000 000 (j) 600 (k) 500 (l) 2100

4 (a) 9 (b) 5 (c) 32 (d) 760

(e) 1060 (f) 1836 (g) 150 (h) 43

(i) 150 (j) 1440 (k) 4248 (l) 2450

5 48 000 m

6 Divide by 10 and double the answer: 26.

7 \$1248

8 \$250

9 \$648 000

10 92 streets, some left over

11 (a) \$66 (b) \$55 (c) He will gain \$55.

12 (a)

| × | 100 | 60 | 2 | + |
|---|---|---|---|---|
| 200 | 20 000 | 12 000 | 400 | = 32 400 |
| 40 | 4000 | 2400 | 80 | = 6480 |
| 6 | 600 | 360 | 12 | = 972 |
| | | | | **39 852** |

(b)

| × | 300 | 50 | 6 | + |
|---|---|---|---|---|
| 400 | 120 000 | 20 000 | 2400 | = 142 400 |
| 10 | 3000 | 500 | 60 | = 3560 |
| 2 | 600 | 100 | 12 | = 712 |
| | | | | **146 672** |

(c)

| × | 100 | 0 | 7 | + |
|---|---|---|---|---|
| 500 | 50 000 | 0 | 3500 | = 53 500 |
| 60 | 6000 | 0 | 420 | = 6420 |
| 0 | 0 | 0 | 0 | = 0 |
| | | | | **59 920** |

13 B

### Open-ended – Sample answers

**14** **(a)** $2 \times 810, 4 \times 405, 5 \times 324, 10 \times 162, 20 \times 81, 30 \times 54, 6 \times 270, 12 \times 135, 18 \times 90$

**(b)** $18 \times 90, 30 \times 54, 20 \times 81$

**15** **(a)** $9 \times 24 = 216$
$12 \times 43 = 516$

**(b)** Kim has forgotten that the first digit in the numbers she is splitting up (24 and 43) is in the 'tens' place value column. She is multiplying them as though they were units.

**(c)** Write out the number being split into tens and units:
$9 \times 24 = 9 \times (20 + 4)$
$12 \times 43 = 12 \times (40 + 3)$

## Half-time 1 (p. 28)

**1** **(a)** 340 **(b)** 690 **(c)** 185
**(d)** 760 **(e)** 239 **(f)** 261

**2** **(a)** **(i)** 216 **(ii)** 3125 **(iii)** 10 **(iv)** 49
**(v)** 7 **(vi)** 2 **(vii)** 10 **(viii)** 4

**(b)** **(i)** 49 and 4 **(ii)** 216

**3** **(a)** 171 **(b)** 328 **(c)** 190 **(d)** 374
**(e)** 441 **(f)** 636 **(g)** 4757 **(h)** 7728

**4** **(a)** 34 **(b)** 29 **(c)** 116 **(d)** 47
**(e)** 16 **(f)** 39 **(g)** 18 **(h)** 56

**5** **(a)** $3^6$ **(b)** $6^5$ **(c)** $4^7$ **(d)** $15^2$
**(e)** $21^3$ **(f)** $41^3$

**6** **(a)** 4200 **(b)** 3000 **(c)** 120 000
**(d)** 200 000 **(e)** 90 **(f)** 30
**(g)** 5 **(h)** 620 **(i)** 150

**7** $676 000

## Exercise 1.4 (p. 32)

**1** **(a)** 70 **(b)** 70 **(c)** 10 **(d)** 6
**(e)** 600 **(f)** 700 **(g)** 500 **(h)** 1000
**(i)** 5000 **(j)** 4000 **(k)** 10 000 **(l)** 10 000
**(m)** 80 000 **(n)** 700 000 **(o)** 800 000 **(p)** 2 000 000

**2** **(a)** 28 000 **(b)** 40 000 **(c)** 48 000 **(d)** 60
**(e)** 50 **(f)** 25 **(g)** 80 000 **(h)** 900 000
**(i)** 2 100 000 **(j)** 63 000 **(k)** 300 000 **(l)** 1000
**(m)** 50 **(n)** 60 **(o)** 90 000 000

**3** **(a)** $40 \times 25 = 1000$ **(b)** $120 \times 50 = 6000$
**(c)** $200 \times 15 = 3000$ **(d)** $400 \div 40 = 10$
**(e)** $400 \div 20 = 20$ **(f)** $250 \div 50 = 5$
**(g)** $1600 \times 20 = 32 000$ **(h)** $34 000 \times 20 = 680 000$
**(i)** $4000 \times 250 = 1 000 000$ **(j)** $3500 \div 50 = 70$
**(k)** $11 000 \div 100 = 110$ **(l)** $45 000 \div 15 = 3000$
**(m)** $24 000 \div 400 = 60$ **(n)** $15 000 \div 2500 = 6$
**(o)** $5000 \times 2500 = 12 500 000$

**4** **(a)** C
**(b)** **(i)** C **(ii)** B

**5** **(a)** $1200 + 300 = 1500$ **(b)** $10 300 + 200 = 10 500$
**(c)** $15 000 + 3000 = 18 000$ **(d)** $820 - 80 = 740$
**(e)** $1200 - 200 = 1000$ **(f)** $500 - 300 = 200$

**6** **(a)** 24 000, actual answer will be lower
**(b)** 800 000, actual answer will be higher
**(c)** 4000, actual answer will be lower
**(d)** 100, actual answer will be lower
**(e)** 10, actual answer will be higher
**(f)** 6, actual answer will be lower

**7** **(a)** D **(b)** B **(c)** A **(d)** D

**8** **(a)** 300 000 **(b)** $6 million

**9** **(a)** $760 **(b)** $9000

**10** **(a)** A **(b)** $16 \times 15 \div 2 = 120$

**11** $79 \times 5003$ will give the closer answer because the numbers are much closer to the rounded ones (80 and 5000). 5488 is rounded a long way down and so gives an estimated answer that is a lot less than the actual one.

**12** **(a)** $1344 \div 21 \approx 1000 \div 20 = 50$

**(b)** $1344 \div 21 \approx 1340 \div 20 = 67$

**(c)** Answer **(b)** will give a closer estimate to the answer because the initial numbers have been rounded far less.

**(d)** $1344 \div 21 = 64$—much closer to answer **(b)**

**13** If she estimated by rounding down, she would run the risk of not having enough seats on the buses for 428 people; e.g. rounding to the first digit gives $400 \div 50 = 8$ buses, but eight 52–seater buses have 416 seats—way too few.

### Open-ended – Sample answers

**14** **(a)** Natalie's rounded down total outweighs her rounded up total. Natalie rounded the T-shirt up by $3 and the lip gloss up by $1 = $4. She rounded the earrings down by $4 and the necklace down by $3 = $7. She is $3 short.

**(b)** A 'safer' method is to round up all purchases to a convenient multiple of 5 or 10.

**15** **(a)** 39 and 82 **(b)** 12 and 19
**(c)** 214 and 487 **(d)** 72 and 573

## Exercise 1.5 (p. 37)

**1** **(a)** 25 **(b)** 7 **(c)** 11 **(d)** 0
**(e)** 3 **(f)** 17 **(g)** 9 **(h)** 12
**(i)** 9 **(j)** 50 **(k)** 5 **(l)** 84
**(m)** 21 **(n)** 14 **(o)** 88 **(p)** 41

**2** **(a)** False **(b)** True **(c)** False **(d)** True
**(e)** False **(f)** True

**3** **(a)** C **(b)** D

**4** **(a)** **(i)** $2 \times 6$, then $2 \times 2$ **(ii)** $15 \div 5$, then $6 \times 3$
**(b)** **(i)** 6 **(ii)** 21

**5** **(a)** $(6 + 6) \times 3 = 36$ **(b)** $(10 - 4) \times 5 = 30$

**(c)** $(9 - 8) \times 6 + 4 = 10$ **(d)** $12 + 6 \div (7 - 4) = 14$

**(e)** $6 \div (3 + 3) \times 5 = 5$ **(f)** $3 \times 6 \div (8 - 4 + 5) = 2$

**(g)** $3 \times (10 - 7) \div 9 + 12 = 13$ **(h)** $18 \div 3 \times (5 - 3) + 2 = 14$

**(i)** $(7 + 3) \div (4 + 1) = 2$ **(j)** $(5 - 3) \times (8 - 6) \div 2 = 2$

**6** **(a)** $73 \times 5 + 29 + 50 \times 8 = 794$, \$7.94

**(b)** $2000 - (5 \times 73 + 29 + 8 \times 50) = 1206$, \$12.06

**7** **(a)** $6 \times 90 + 5 \times 70$
$= 540 + 350$
$= 890$ cents
$= \$8.90$

**(b)** $6 \times 90 + 6 \times 70 + 5 \times 50$
$= 540 + 420 + 250$
$= 1210$ cents
$= \$12.10$

**(c)** \$5.30

**8** **(a)** $2 + 21 \div 3 = 9$ **(b)** $15 - 6 \div 2 = 12$

**(c)** $14 - 8 - 6 = 0$ **(d)** $7 \times 5 - 6 = 29$

**(e)** $14 + 3 - 2 = 15$ **(f)** $(24 + 6) \div 10 = 3$

**(g)** $8 + 5 \times 2 - 6 = 12$ **(h)** $12 \div 2 + 1 \times 9 = 15$

**9** **(a)** $6 \times (4 \div 2) \times 3 = (6 \times 4) \div 2 \times 3$

**(b)** $(1 + 4) \times 20 \div 5 > 1 + (4 \times 20) \div 5$

**(c)** $100 + 10 \div 10 > (100 + 10) \div 10$

**(d)** $36 \div 6 \times (3 - 3) < 36 \div 6 \times 3 - 3$

**10** **(a)** Both pairs of brackets are unnecessary.

**(b)** Both pairs of brackets are unnecessary.

**(c)** First pair is unnecessary.

**(d)** Second pair is unnecessary.

**11** **(a)** 26 is the correct answer.

**(b)** The calculator 'does' order of operations, the phone does every operation as it is entered.

### Open-ended – Sample answers

**12** $(8 + 4) \times 6 \div 2 - 1 = 35$; $8 + (4 \times 6) \div 2 - 1 = 19$;
$8 + 4 \times (6 \div 2) - 1 = 19$; $8 + 4 \times 6 \div (2 - 1) = 32$;
$(8 + 4 \times 6) \div 2 - 1 = 15$; $8 + (4 \times 6 \div 2) - 1 = 19$;
$8 + 4 \times (6 \div 2 - 1) = 16$

**13** Correct answer is 75. Step 2 is incorrect—he needed to do $30 \div 2$, first, then multiply by 5.

## Mathspace (p. 40)

**1** 72 muscles

**2** 500 shades of grey

**3** 165 km/h

**4** 2000 scents

**5** **(a)** 2 **(b)** 6

**6** **(a)** 300 **(b)** 206

**7** 10 billion flakes

**8** 15 million cells

**9** 3 millimetres

**10** 7.25 metres

Brain surgery #1

1 2

2 21

3 1

4 3

5 5

Brain surgery #2

1 24

2 39

3 15

4 1

5 36

## Exercise 1.6 (p. 42)

**1** **(a)** 724 **(b)** \$9225 **(c)** \$351 **(d)** 31 hours

**2** 30 minutes

**3** 72 laps

**4** 50 400 min

**5** **(a)** 5998 m **(b)** 4643 m

**6** Shoes $68 + 34 = \$102$
T-shirts $19 + 9 = \$28$
Earrings = \$25
Total = \$155

**7** $192 - 18 = 174$ buns sold

**8** **(a)** 222 km **(b)** 3931 km **(c)** 3709 km

**9** 73 cm

**10** \$2646

**11** 42

**12** **(a)** 3 trips there and back (12 boxes), plus 1 more trip to new house (with last box), = 7 times

**(b)** 49 km

**13** **(a)** 11 weeks, 3 days

**(b)** 2 months, 2 weeks, 6 days

**(c)** 2 months, 3 weeks, 3 days

**(d)** In **(b)**, 80 was divided by 30 to find the number of months. Dividing by a larger number gives a smaller remainder to split into weeks and days. In **(c)**, 80 was divided by 28 (4 weeks).

**14** Vet + Jason + 2 Scruffys: $91 + 56 = 147$ kg
Scruffy's weight $= 147 - 138 = 9$ kg

### Open-ended – Sample answers

**15** **(a)** The two numbers need to be different by at least 30; e.g. 25 and 66.

**(b)** The two numbers need to be single-digit numbers; e.g. 4 and 7, 5 and 6, 9 and 3.

**16** **(a)** $20 \times 300$ mL bottles: 6 kg
$20 \times 600$ mL bottles: 12 kg
$20 \times 1000$ mL bottles: 20 kg

**(b)** Numbers of each box that give a total mass of 1000 kg:

1 L: 50 boxes
600 mL: 83 boxes
300 mL: 166 boxes

**(c)** $23 \times 1$ L boxes + $30 \times 600$ mL boxes + $30 \times 300$ mL boxes
$26 \times 1$ L boxes + $26 \times 600$ mL boxes + $28 \times 300$ mL boxes

## Challenge 1 (p. 45)

**1** D
Number must end in 6 or 8 to be even, smallest number needs biggest digit at end, so ends in 8. Largest remaining digit needs to be in the tens place.

**2** C
List the possibilities, $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 6 = 12$, $2 \times 8 = 16$, Note: 16 is repeated. $4 \times 4 = 16$, $4 \times 6 = 24$, $4 \times 8 = 32$, $6 \times 6 = 36$, $6 \times 8 = 48$, $8 \times 8 = 64$.

3 B
Three-digit number has to be as small as possible, two-digit number as large as possible: 356 – 87 = 269.

4 $10 \times \$5 = \$50$, Kim has \$50 more, whereas Brian has \$10 less than the amount they each started with.

By trial and error: If they both start with \$50, Kim will now have \$100 and Brian will have \$40. But $\$40 \times 3 = \$120$. Too much, so try a smaller amount. If they both start with \$40, Kim will now have \$90 and Brian will have \$30. $\$30 \times 3 = \$90$. They both started with \$40.

5 Find the factors of 15 181. It helps to realise that the only pairs of numbers that multiply to give a number ending in 1 are: 1 and 1, 3 and 7, 9 and 9. Hence, possible teenage children could be 11, 13, 17 or 19. See which of these numbers are factors of 15 181, then work out the remaining age. The ages are 17, 19 and 47, so the father is 47 years old.

6 Numbers are consecutive odd numbers.

Sum of first row = 1, sum of second row = 3 + 5 = 8, sum of third row = 7 + 9 + 11 = 27

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$. Sum of row is row number cubed.

Sum of 7th row = $7^3 = 343$.
Sum of 100th row = $100^3 = 1\,000\,000$.

7 $\sqrt{8} \approx 2.8$, $\sqrt{8008} \approx 89.5$, so the squares of 3, 4, 5, … 89 are between 8 and 8008. 89 – 2 = 87 numbers.

8 After the first two terms, each term is the sum of the previous two terms: 18 + 29 = 47, 29 + 47 = 76, 47 + 76 = 123. Terms are 47, 76, 123.

9 1001 ÷ 6 = 166.83, so 1001 will appear in the 167th row. The odd-numbered rows run from left to right and the row would start with 997, as row 166 ends with 996 (166 × 6) in column A. Counting across gives 1001 in column E.

10 (a) 0, 1, 4, 9, 16, 25, 36, 49, 64, 81

(b) 0, 1, 4, 5, 6, 9

(c) 2, 3, 7, 8

(d) (i), (iii), (v)

(e) 961: factor ends in 1 or 9. $900 = 30^2$, so try $31^2$.

## Chapter review 1 (p. 46)

1 (a) 142 (b) 138 (c) 225 (d) 120
(e) 315 (f) 437 (g) 180 (h) 544

2 (a) 144 (b) 125 (c) 100 000 (d) 8
(e) 30 (f) 3 (g) 129 (h) 3853
(i) 72

3 (a) $7^5$ (b) $10^3$ (c) $5^2$ (d) $12^8$

4 (a) 1400 (b) 170 000 (c) 5299 (d) 3038
(e) 22 (f) 1593 (g) 153 (h) 108

5 (a) 8000 (b) 60 000 (c) 50 (d) 300
(e) 60 000 (f) 3 000 000 (g) 8 (h) 300

6 (a) 550 ÷ 10 = 55 (b) 2000 ÷ 20 = 100
(c) 15 600 ÷ 200 = 78 (d) 30 × 600 = 18 000
(e) 160 × 400 = 64 000 (f) 500 × 2000 = 1 000 000
(g) 54 000 ÷ 900 = 60 (h) 12 800 ÷ 200 = 64

7 (a) 1 (b) 13 (c) 11 (d) 6 (e) 5 (f) 41

8 \$66

9 720

10 (a) 80 000, close to (b) 90 000, close to
(c) 600, lower (d) 5000, higher

11 (a) 23 (b) 7

12 B

13 (a) 4 × (2 + 3) ÷ 5 – 1 = 3 (b) (5 + 1) ÷ 6 + 4 + 2 = 7

14 (a) \$1200
(b) Melissa's weekly salary will be less than \$1200.

15 (a) 3 and 4 (b) 5 and 6
(c) 9 and 10 (d) 12 and 13

16 Jessica: \$180, Amy: \$165, Aaron: \$135, Nikkita: \$90, Josef: \$210; Total: \$780

17 (a) 9 + 7 × 3 = 30 (b) 16 – 4 × 2 ÷ 2 = 12

18 (a) 6 + 3 × 5 (b) \$19 change

19 (a) 129 (b) 3853 (c) 36

20 (a) $2^4 = 16$, $2^5 = 32$, $2^6 = 64$
(b) $2^{10} = 1024$; therefore, $20^{10}$ will be larger than 1000. The difference is 24.

21 4, $\sqrt{81}$, $\sqrt{121}$, $2^4$, $3^3$, $10^2$

22 \$21

23 224 ÷ 42 = 5 rem 14, so 6 buses will be needed. Rounding to the first digit gives 200 ÷ 40 = 5, meaning the 14 remaining passengers would be stranded—this is because 224 was rounded down.

24 8 numbers: 17, 18, 19, 20, 21, 22, 23, 24

25 A

26 $3^2 = 3 \times 3$, which is not equal to $3 \times 2$

27 Less than, because both of the numbers being rounded are being rounded up, and then multiplied.

### NAPLAN practice 1

1 C 2 C 3 D 4 D
5 D 6 \$28.40 7 180 8 C
9 The Shwarma family

# Chapter 2

## Recall 2 (p. 52)

1 (a) 42, 36, 24, 66, 48 (b) 77, 49, 35, 14, 21
(c) 56, 48, 32, 80, 64 (d) 108, 27, 45, 99, 72
(e) 84, 72, 144, 108, 132

2 (a) 0, 2, 4, 6, 8 (b) 1, 3, 5, 7, 9

3 (a) 10 > 7 (b) 3 < 6 (c) 2 > 0 (d) 0 < 5

4 (a) 23 (b) 34 (c) 65 (d) 5
(e) 47 (f) 76

5 (a) -11°C, -4°C, 0°C, 7°C, 15°C, 21°C
(b) -25°C, -14°C, -3°C, 5°C, 10°C, 32°C

6 (a) 49 (b) 81 (c) 64 (d) 1

7 (a) 225 (b) 8 (c) 100 (d) 32

## Exercise 2.1 (p. 57)

1 (a) 1, 2, 3, 6, 9, 18
(b) 1, 2, 4, 8, 16
(c) 1, 23
(d) 1, 2, 3, 4, 6, 8, 12, 24
(e) 1, 2, 4, 5, 10, 20
(f) 1, 5, 7, 35
(g) 1, 2, 3, 4, 6, 9, 12, 18, 36
(h) 1, 2, 3, 6, 7, 14, 21, 42
(i) 1, 53
(j) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
(k) 1, 7, 11, 77
(l) 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

2 (a) 108
(b) 92, 108, 3100
(c) 245, 3100
(d) None
(e) 108

3 (a) 10 (b) 9 (c) 25 (d) 30 (e) 28
(f) 24 (g) 63 (h) 60 (i) 66 (j) 36
(k) 100 (l) 56 (m) 60 (n) 50 (o) 300

4 (a) 10: 1, 2, 5, 10
15: 1, 3, 5, 15
HCF = 5
(b) 8: 1, 2, 4, 8
24: 1, 2, 3, 4, 6, 8, 12, 24
HCF = 8
(c) 5: 1, 5
12: 1, 2, 3, 4, 6, 12
HCF = 1
(d) 26: 1, 2, 13, 26
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
HCF = 2
(e) 11: 1, 11
33: 1, 3, 11, 33
HCF = 11
(f) 28: 1, 2, 4, 7, 14, 28
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 14
(g) 44: 1, 2, 4, 11, 22, 44
22: 1, 2, 11, 22
HCF = 22
(h) 10: 1, 2, 5, 10
30: 1, 2, 3, 5, 6, 10, 15, 30
HCF = 10
(i) 40: 1, 2, 4, 5, 8, 10, 20, 40
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 10
(j) 32: 1, 2, 4, 8, 16, 32
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
HCF = 4
(k) 35: 1, 5, 7, 35
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 35
(l) 42: 1, 2, 3, 6, 7, 14, 21, 42
48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
HCF = 6

5 (a) A (b) C

6 (a) C (b) D

7 (a) A (b) D

8 (a) False (b) False (c) True (d) True
(e) False (f) False

9 (a) (i) 60 (ii) 2
(b) (i) 24 (ii) 2
(c) (i) 48 (ii) 4
(d) (i) 200 (ii) 5

10 (a) divisible (b) factor (c) multiple (d) divisible

11 (a) 2 bags with 12 lollies in each bag; 3 bags of 8; 4 bags of 6; 6 bags of 4; 8 bags of 3; 12 bags of 2; 24 bags of 1
(b) 2 bags with 18 lollies in each bag; 3 bags of 12; 4 bags of 9; 6 bags of 6; 9 bags of 4; 12 bags of 3; 18 bags of 2; 36 bags of 1

12 (a) 1 row of 96, 2 rows of 48, 3 rows of 32, 4 rows of 24, 6 rows of 16, 8 rows of 12, 12 rows of 8, 16 rows of 6, 24 rows of 4, 32 rows of 3, 48 rows of 2, 96 rows of 1
(b) 8 rows of 12 or 12 rows of 8

13 5 groups of 4 and 2 groups of 3; 2 groups of 4 and 6 groups of 3

14 D

15 5 min, 10 min, 15 min

16 (a) 56
(b) 110
(c) 112
(d) 210

17 (a) 15 times
(b) 7 times
(c) 4, 7, 10, 13, 16, 19, 22




18 (a)

| | |
|---|---|
| 202 008 | 2, 3, 4, 6, 8 |
| 12 121 212 | 2, 3, 4, 6 |
| 300 300 300 | 2, 3, 4, 5, 6, 9, 10 |
| 7 500 | 2, 3, 4, 5, 6, 10 |
| 900 090 | 2, 3, 5, 6, 9, 10 |
| 123 456 789 | 3, 9 |

(b) (i) 2 (ii) 3
(c) 2 is a factor of 4, so if a number is divisible by 4, it is also divisible by 2.
3 is a factor of 9, so if a number is divisible by 9, it is also divisible by 3.

19 6 is the LCM of 2 and 3.

20 (a) 28 (b) 496

21 18 and 20

22 (a) multiply the two numbers together
(b) If the HCF of the numbers is 1, this will be the LCM.

### Open-ended – Sample answers

23 (a) 4 rows of 15; 5 rows of 12; 6 rows of 10
(b) Boxes that are squarer in shape are more practical and require less packaging to make, so $5 \times 12$ or $6 \times 10$ are more likely.

24 Sam's age must be a multiple of 3 and 5; i.e. a multiple of 15: 15, 30, 45, 60, 75.

25 No. Multiplying any common multiple by a whole number greater than 1 gives a higher common multiple.

## Exercise 2.2 (p. 62)

**1** 2, 3, 5, 7, 11, 13, 17, 19

**2** Four (2, 3, 5, 7)

**3** 23, 29, 31, 37, 41, 43, 47, 53, 59

**4** **(a)** B **(b)** D

**5** **(a)** 61 **(b)** 62 **(c)** 9, 15 **(d)** 47

**6** **(a)** False **(b)** True **(c)** True **(d)** False
**(e)** True **(f)** True **(g)** True **(h)** False

**7** **(a)** Divisible by 10, 5 and 2 **(b)** Divisible by 3 and 9
**(c)** Divisible by 3 and 9 **(d)** Divisible by 3 and 9
**(e)** Divisible by 2 and 4 **(f)** Divisible by 5

**8** **(a)** Yes, HCF = 1 **(b)** Yes, HCF = 1
**(c)** No, HCF = 13 **(d)** No, HCF = 9

**9** The only common factors of any two prime numbers is 1.

**10** All other even numbers have 2 as a factor.

**11** It is even, so 2 is a factor.

**12** There are no more even prime numbers.

**13** 1

**14** **(a)** 99 and 101 **(b)** 35 and 37

**15** No, a prime number will only be coprime with any whole number that is not a multiple of the original prime number.

**16** The smaller of the two numbers will be a common factor.

### Open-ended – Sample answers

**17** The first 10 even composite numbers:

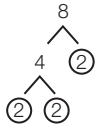
$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 5 + 5$
$12 = 5 + 7$
$14 = 7 + 7$
$16 = 11 + 5$
$18 = 11 + 7$
$20 = 13 + 7$
$22 = 11 + 11$

**18** 3 and 7 $(3 \times 2 + 1 = 7)$
2 and 5 $(2 \times 2 + 1 = 5)$
5 and 11 $(2 \times 5 + 1 = 11)$

## Exercise 2.3 (p. 68)

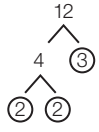
**1** The factor trees can differ slightly from these. The factors at the end should be the same.

**(a)**



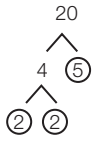

$8 = 2 \times 2 \times 2$
$= 2^3$

**(b)**



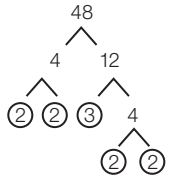

$12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

**(c)**




$20 = 2 \times 2 \times 5$
$= 2^2 \times 5$

**(d)**




$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

**(e)**



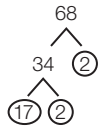

$14 = 2 \times 7$

**(f)**



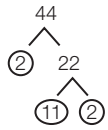

$26 = 2 \times 13$

**(g)**



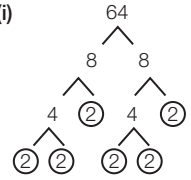

$68 = 2 \times 2 \times 17 = 2^2 \times 17$

**(h)**



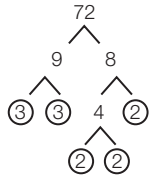

$44 = 2 \times 2 \times 11 = 2^2 \times 11$

**(i)**



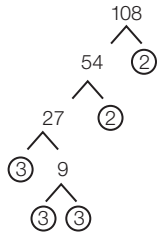

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

**(j)**



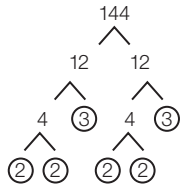

$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \times 3^2$

**(k)**



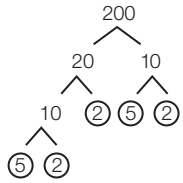

$108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$

**(l)**



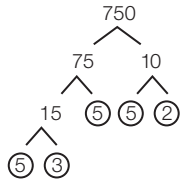

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^4 \times 3^2$

**(m)**



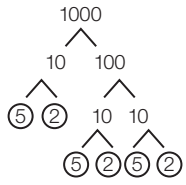

$200 = 2 \times 2 \times 2 \times 5 \times 5$
$= 2^3 \times 5^2$

**(n)**



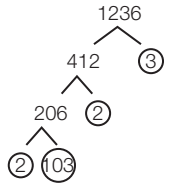

$750 = 2 \times 3 \times 5 \times 5 \times 5$
$= 2 \times 3 \times 5^3$

**(o)**




$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= 2^3 \times 5^3$

**(p)**




$1236 = 2 \times 2 \times 3 \times 103$
$= 2^2 \times 3 \times 103$

**2** **(a)** $8 = 2^3$ **(b)** $12 = 2^2 \times 3$
**(c)** $28 = 2^2 \times 7$ **(d)** $36 = 2^2 \times 3^2$
**(e)** $39 = 3 \times 13$ **(f)** $77 = 7 \times 11$
**(g)** $51 = 3 \times 17$ **(h)** $38 = 2 \times 19$
**(i)** $30 = 2 \times 3 \times 5$ **(j)** $63 = 3^2 \times 7$
**(k)** $96 = 2^5 \times 3$ **(l)** $132 = 2^2 \times 3 \times 11$
**(m)** $168 = 2^3 \times 3 \times 7$ **(n)** $198 = 2 \times 3^2 \times 11$
**(o)** $288 = 2^5 \times 3^2$ **(p)** $212 = 2^2 \times 53$

**3** **(a)** $8 = 2 \times 2 \times 2$
$20 = 2 \times 2 \times 5$
HCF $= 2 \times 2$
$= 4$

**(b)** $12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
HCF $= 2 \times 3$
$= 6$

**(c)** $12 = 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

**(d)** $36 = 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

**(e)** $24 = 2 \times 2 \times 2 \times 3$
$56 = 2 \times 2 \times 2 \times 7$
HCF $= 2 \times 2 \times 2$
$= 8$

**(f)** $28 = 2 \times 2 \times 7$
$84 = 2 \times 2 \times 3 \times 7$
HCF $= 2 \times 2 \times 7$
$= 28$

**(g)** $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

**(h)** $70 = 2 \times 5 \times 7$
$98 = 2 \times 7 \times 7$
HCF $= 2 \times 7$
$= 14$

**(i)** $60 = 2 \times 2 \times 3 \times 5$
$105 = 3 \times 5 \times 7$
HCF $= 3 \times 5$
$= 15$

**(j)** $66 = 2 \times 3 \times 11$
$110 = 2 \times 5 \times 11$
HCF $= 2 \times 11$
$= 22$

**(k)** $80 = 2 \times 2 \times 2 \times 2 \times 5$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
HCF $= 2 \times 2 \times 2 \times 2$
$= 16$

**(l)** $130 = 2 \times 5 \times 13$
$156 = 2 \times 2 \times 3 \times 13$
HCF $= 2 \times 13$
$= 26$




**4** **(a)** D **(b)** A

**5** **(a)** 72 **(b)** 300 **(c)** 54 000
**(d)** 2205 **(e)** 15 400 **(f)** 7744

**6** **(a)** $2^4 \times 3$ **(b)** $2^5 \times 3$ **(c)** $2^4 \times 3^2$

**7** **(a)** $2 \times 3 \times 5 \times 11$ **(b)** $2^2 \times 3 \times 5 \times 11$ **(c)** $2 \times 3^2 \times 5 \times 11$

**8** 2, 3, 7

**9** **(a)** Prime factors of 36: $2 \times 2 \times 3 \times 3$
Prime factors of 100: $2 \times 2 \times 5 \times 5$
Highest common factor (HCF) of 36 and 100: $2 \times 2 = 4$
Factors of 36 not used to find the HCF: $3 \times 3 = 9$
Factors of 100 not used to find the HCF: $5 \times 5 = 25$
HCF × unused factors of 36 × unused factors of 100:
$4 \times 3 \times 3 \times 5 \times 5 = 900$

**(b)** **(i)** 72 **(ii)** 36 **(iii)** 40 **(iv)** 72 **(v)** 260
**(vi)** 800 **(vii)** 1000 **(viii)** 800 **(ix)** 4000

**10** **(a)** 6 **(b)** 14 red balloons and 9 blue balloons

**11** 17; 510

**12** **(a)** **(i)** 6 **(ii)** 30 **(iii)** 210
**(b)** **(i)** 4 **(ii)** 8 **(iii)** 16

**13** **(a)** Yes, 105 is a winning number.
**(b)** There would be four winning tickets: 15, 105, 165 and 195.

## Open-ended – Sample answers

**14** Trial and error; e.g. try $2 \times 2 \times 3 \times 5$. If it is too small, try a bigger prime. $2 \times 2 \times 3 \times 11 = 132$; $2 \times 2 \times 5 \times 7 = 140$

**15** **(a)** 2, 4, 5, 10 (and 20)
**(b)** A factor need not be prime. For example, 6 is a factor of 18, but 6 is a composite number.
**(c)** If a factor is composite, Ray should find its factors.

# Exercise 2.4 (p. 75)

**1** **(a)** +300 **(b)** -50 000 **(c)** -8 **(d)** +4
**(e)** +36 **(f)** +45 **(g)** -20 **(h)** +12
**(i)** -2 **(j)** -7 **(k)** +28 **(l)** -40 000
**(m)** +225 **(n)** -9

**2** **(a)** $+6 > -3$ **(b)** $+1 > -3$ **(c)** $-7 < +5$
**(d)** $-2 < +6$ **(e)** $-12 < -4$ **(f)** $-7 < -1$
**(g)** $-2 > -11$ **(h)** $-9 < 0$ **(i)** $0 > -2$
**(j)** $-35 < +7$ **(k)** $-4 < +67$ **(l)** $-11 < +75$

**3** **(a)** -12, -4, -3, -1, 6 **(b)** -11, -10, 2, 9, 10
**(c)** -6, -4, 1, 2, 9 **(d)** -8, -6, -5, 1, 7
**(e)** -12, -8, -7, -4, -1 **(f)** -79, -56, 0, 6, 23

**4** **(a)** 8, 7, 3, -5, -6 **(b)** 10, 7, 1, -8, -12
**(c)** 1, -4, -6, -8, -9 **(d)** 7, 0, -5, -11, -12
**(e)** -2, -5, -9, -11, -12 **(f)** 79, 4, 0, -9, -54

**5** **(a)** east 300 km **(b)** 3 days early
**(c)** 7°C above zero **(d)** subtract 5
**(e)** add 22 **(f)** right 3 m
**(g)** +2 **(h)** -16
**(i)** +350

**6** **(a)** A **(b)** D

**7** **(a)** True **(b)** False **(c)** False **(d)** True
**(e)** False **(f)** False **(g)** True **(h)** True

**8** **(a)** -3, -2, -1, 0, 1, 2 **(b)** -1, 0, 1
**(c)** -8, -7, -6, -5 **(d)** -4, -3, -2, -1
**(e)** -38, -39, -40, -41 **(f)** -119, -118, -117, -116

**9** **(a)** -12, -10, -8 **(b)** -5, 0, 5
**(c)** 0, -3, -6 **(d)** -24, -30, -36
**(e)** 0, -10, -20 **(f)** -3, -8, -13

**10** **(a)** Ben **(b)** Damien

**11** **(a)** Tuesday **(b)** Friday **(c)** Tuesday

**12** **(a)** 156 m **(b)** 154 m **(c)** 310 m

**13** **(a)** 4 km west **(b)** 4 km east

**14** **(a)** 4 floors up **(b)** 4 floors down

**15** **(a)** Sam has more money at the end of the week.
**(b)** Withdrawals $80 + $120 + $50 = $250
Deposits $350 + $40 = $390
$140 more in the account

## Open-ended – Sample answers

**16** Any three of -7, -6, -5, -4, -3, -2, -1

**17** **(a)** 3

**(b)** -3, -3, 6, 1, 1, 2  1, 1, -3, 1, 2, 2  4, 4, -3, 1, 1, -3

**18** Students' own answers.

# Half-time 2 (p. 79)

**1** **(a)** 45  **(b)** 24

**2** B

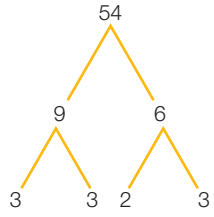
**3** **(a)** +85  **(b)** -7  **(c)** +3  **(d)** +28 000

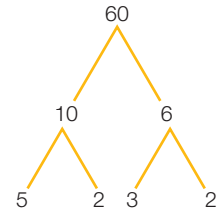
**(e)** -40  **(f)** +11

**4** 31, 37, 41, 43, 47

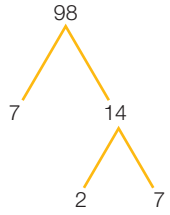
**5** **(a)** $6 < 10$  **(b)** $-4 < 7$  **(c)** $3 > -12$  **(d)** $-8 > -15$

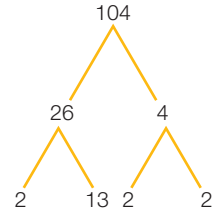
**6** **(a)** $54 = 2 \times 3^3$  **(b)** $60 = 2^2 \times 3 \times 5$




**(c)** $98 = 2 \times 7^2$  **(d)** $104 = 2^3 \times 13$




**7** **(a)** 76 905 is divisible by both 3 and 5, so it is divisible by 15.

**(b)** 4734 is divisible by 2, as it is even. Sum of digits is 18, which is divisible by 3. Therefore, as 4734 is divisible by both 2 and 3, it is divisible by 6.

**8** **(a)** -37, -30, -3, 0, 3, 7

**(b)** -40, -20, -5, -1, 54




**9** 24

**10** D

**11** **(a)** 150 seconds

**(b)** Honda 6 laps,
Holden 5 laps,
Ford 3 laps

**(c)** 12 laps

# Exercise 2.5 (p. 82)

**1** **(a)** 11  **(b)** 8  **(c)** 11  **(d)** 8

**(e)** -2  **(f)** -5  **(g)** -5  **(h)** -6

**(i)** 6  **(j)** 12  **(k)** 25  **(l)** 20

**(m)** -10  **(n)** -4  **(o)** -20  **(p)** -31

**2** **(a)** 2  **(b)** 6  **(c)** 5  **(d)** 0

**(e)** -6  **(f)** -11  **(g)** -8  **(h)** -4

**(i)** -10  **(j)** -8  **(k)** -20  **(l)** -12

**(m)** -12  **(n)** -30  **(o)** -69  **(p)** -124

**3** **(a)** B  **(b)** D

**4** **(a)** 12  **(b)** 11  **(c)** 12  **(d)** 7

**(e)** -18  **(f)** -22  **(g)** -5  **(h)** -10

**(i)** -8  **(j)** 13  **(k)** -1  **(l)** 1

**5** +8°C

**6** the 11th floor

**7** 34 metres below the surface

**8** 5 m

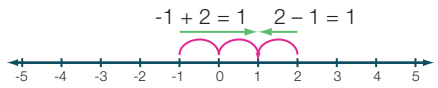
**9** $13

**10** **(a)** positive five subtract positive seven equals negative two

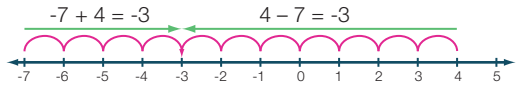
**(b)** negative eight add positive nine equals positive one

**(c)** negative four subtract positive three equals negative seven

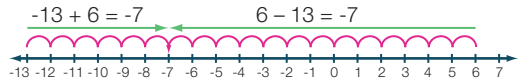
**11** **(a)** **(i)**




**(ii)**




**(iii)**



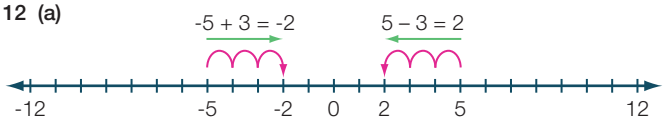

**(b)** The finishing point of the number line journeys for these pairs of additions and subtractions is the same. The pairs are equivalent—each addition can be rewritten as a subtraction. $-1 + 2 = 2 - 1$.

**(c)** **(i)** $-37 + 86 = 86 - 37 = 49$

**(ii)** $-72 + 95 = 95 - 72 = 23$

**(iii)** $-104 + 119 = 119 - 104 = 15$

**12** **(a)**




**(b)** **(i)** $-8 + 6 = -2$  **(ii)** $-5 + 4 = -1$

**(iii)** $10 - 6 = 4$  **(iv)** $12 - 9 = 3$

**(c)** **(i)** -6  **(ii)** -21

**(iii)** -25  **(iv)** -19

## Open-ended – Sample answers

**13** Assuming that the temperature rises fairly gradually after sunrise, a possible answer might be:

| 6 a.m. | 7 a.m. | 8 a.m. | 9 a.m. | 10 a.m. | 11 a.m. | 12 p.m. |
|---|---|---|---|---|---|---|
| -4°C | 0°C | 7°C | 13°C | 17°C | 21°C | 25°C |

**14** Any three positive integers less than 13.

## Exercise 2.6 (p. 88)

**1** (a) 1 (b) 1 (c) 9 (d) 6
(e) -4 (f) -5 (g) -7 (h) -11
(i) -9 (j) -11 (k) -20 (l) -34
(m) -10 (n) -90 (o) -29 (p) -133

**2** (a) 11 (b) 6 (c) 10 (d) 12
(e) 8 (f) 3 (g) 16 (h) -6
(i) 12 (j) -1 (k) 8 (l) -6
(m) -10 (n) -10 (o) -26 (p) -28

**3** (a) A (b) B

**4** (a) 0 (b) -6 (c) -7 (d) -10
(e) -5 (f) 6 (g) 5 (h) 9
(i) -3 (j) -2 (k) -8 (l) 22

**5** -4°C

**6** -3 (3 floors below ground level)

**7** $3500 in debt

**8** 9 m

**9** 3 m

**10** 31 m

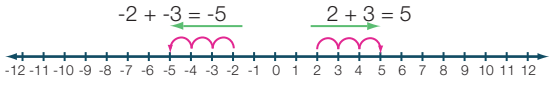
**11** (a) negative five add negative six equals negative eleven

(b) positive seven subtract negative eight equals positive fifteen

(c) negative nine subtract negative fourteen equals positive five




**12** (a)




(b) (i) -1 – 6 = -7 (ii) -4 – 7 = -11
(iii) 3 + 8 = 11 (iv) 5 + 4 = 9

(c) (i) -36 (ii) -55 (iii) -77 (iv) -94

**13** (a) (i) 6 (ii) 18 (iii) -9 (iv) 3

(b) smaller; larger

### Open-ended – Sample answers

**14** Must be -22 or less; e.g. -25, -35, -28.

**15** (a) The correct answer is -9, not -1.

(b) Max has moved in the wrong direction on the number line. To add a negative number, he needs to walk forwards, but in the negative direction.

## Exercise 2.7 (p. 94)

**1** (a) 7 (b) 3 (c) 11 (d) -2
(e) 2 (f) -12 (g) -11 (h) -10
(i) 9 (j) 6 (k) -2 (l) -4
(m) -1 (n) 6 (o) 12 (p) 2
(q) -10 (r) -12 (s) 5 (t) 4

**2** (a) False (b) True (c) True (d) False

**3** (a) -12 (b) -11 (c) 4 (d) 3
(e) -5 (f) -8 (g) -13 (h) -12
(i) -30 (j) -31 (k) 57 (l) 65

**4** (a) C (b) B

**5** (a) 6 (b) 7 (c) -7 (d) -4
(e) -6 (f) -8 (g) 4 (h) 1
(i) –12 (j) -11 (k) -9 (l) -9

**6** -$150

**7** (a) the 10th floor

(b) the 4th floor below the ground floor

(c) the 8th floor

**8** -$350, a loss of $350

**9** (a)

| + | -6 | +20 | -14 | 31 |
|---|---|---|---|---|
| +4 | -2 | +24 | -10 | 35 |
| -7 | -13 | +13 | -21 | 24 |
| +8 | +2 | +28 | -6 | 39 |
| -9 | -15 | +11 | -23 | 22 |

(b)

| – | 8 | -11 | 19 | -25 |
|---|---|---|---|---|
| 3 | 5 | -14 | 16 | -28 |
| -7 | 15 | -4 | 26 | -18 |
| +5 | 3 | -16 | 14 | -30 |
| -10 | 18 | -1 | 29 | -15 |

**10** (a) 6

(b)

| | | |
|---|---|---|
| 1 | 6 | -1 |
| 0 | 2 | 4 |
| 5 | -2 | 3 |

**11** (a)

| | | |
|---|---|---|
| -3 | 2 | -5 |
| -4 | -2 | 0 |
| 1 | -6 | -1 |

(b) -6
Each of the three digits in the row, column or diagonal has been reduced by 4. This means that the magic sum of the digits will be 12 less than the original magic sum.

### Open-ended – Sample answers

**12** (a)

| Hole number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Par for the hole | 4 | 4 | 5 | 3 | 4 | 5 | 4 | 3 | 4 | 36 |
| Shots taken | 4 | 3 | 4 | 5 | 5 | 4 | 3 | 3 | 4 | 35 |
| Score compared to par | 0 | -1 | -1 | +2 | +1 | -1 | -1 | 0 | 0 | -1 |

(b)

| Hole number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Par for the hole | 4 | 4 | 5 | 3 | 4 | 5 | 4 | 3 | 4 | 36 |
| Shots taken | 3 | 6 | 4 | 4 | 5 | 3 | 3 | 4 | 3 | 35 |
| Score compared to par | -1 | +2 | -1 | +1 | +1 | -2 | -1 | +1 | -1 | -1 |

**13** Ignoring the signs, the numbers will add to 31; e.g. -14 and 17, -10 and 21, 18 and -13

## Challenge 2 (p. 97)

**1** **(a)** 1, -3, -7 **(b)** -7, -1, 5

**2** The twins are the youngest and have the same age. Look for three factors of 72 in which the smallest two are the same: $1 \times 1 \times 72$, $2 \times 2 \times 18$, $3 \times 3 \times 8$; $3 + 3 + 8 = 14$. ($6 \times 6 \times 2$ gives sum of 14, but twins are not the youngest.) The brothers are 3, 3 and 8 years old.

**3** $172 \div 2 = 86$ (average of two numbers is the one in the middle) so the middle number is 86. Numbers are 85, 86, 87.

**4** Consider factors of 150. $10 \times 15 = 150$,
$(10 - 4) \times (15 + 10) = 6 \times 25 = 150$, $(10 + 10) \times (15 - 4) = 220$;
$6 \times 25 = 150$, $(6 + 10) \times (25 - 4) = 336$, $(6 - 4) \times (25 + 10) = 70$.
Originally, there were 10 rows of 15 chairs.

**5** $11 \times 0$, $(10 + 10 + 1) \times 1$, $(10 + 10) \times$ each other digits.
Sum $= 0 + 21 + 20 \times (2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)$
$= 21 + 20 \times 44$
$= 901$

**6** The amount that each person raised has been counted twice.
$(\$130 + \$150 + \$100 + \$70 + \$90) \div 2 = \$540 \div 2 = \$270$

**7** Numbers that give a remainder of 2 when divided by 4 are: 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54, 58, 62, ...

Numbers that give a remainder of 3 when divided by 5 are: 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63, ...

Numbers that give a remainder of 4 when divided by 6 are: 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, ...

Looking at the patterns, the first number to appear in all three is 58.

**8** B

Prime: $11 \rightarrow 11$, $13 \rightarrow 31$, $17 \rightarrow 71$, $31 \rightarrow 13$, $37 \rightarrow 73$, $71 \rightarrow 17$, $73 \rightarrow 37$, $79 \rightarrow 97$, $97 \rightarrow 79$

Not prime: $19 \rightarrow 91$

Any primes in the 20s, 40s, 60s and 80s will give even numbers when reversed, so not prime.

Any primes in the 50s will give a number ending in 5 when reversed, so not prime.

**9** **(a)** B $10, 15 \times 2 = 30$

**(b)** C $10, 15 \times 2 = 30, 20, 25 \times 4 = 100$

**(c)** D $10, 15 \times 2, 20, 25 \times 4 = 100, 30, 35 \times 2 = 70, 40, 45 \times 2 = 90, 50 \times 2 = 100, 55 \times 2 = 110$

**(d)** D $10, 15 \times 2, 20, 25 \times 4 = 100, 30, 35 \times 2, 40, 45 \times 2, 50 \times 2 = 100, 55 \times 2 = 110, 60, 65 \times 2 = 130, 70, 75 \times 4 = 300, 80, 85 \times 2 = 170, 90, 95 \times 2 = 190$

## Chapter review 2 (p. 98)

**1** **(a)** 18 **(b)** 36 **(c)** 30

**2** **(a)** 1, 2, 3, 4, 6, 9, 12, 18, 36

**(b)** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

**(c)** 1, 3, 17, 51

**(d)** 1, 2, 4, 5, 10, 20, 25, 50, 100

**3** **(a)** 8 **(b)** 18 **(c)** 5

**4**

| | |
|---|---|
| 5301 | 3, 9 |
| 10 000 | 2, 4, 5, 10 |
| 333 333 | 3, 9 |
| 31 700 | 2, 4, 5, 10 |
| 43 521 820 | 2, 4, 5, 10 |

**5** **(a)** Prime, because it has exactly two factors, 1 and 5.

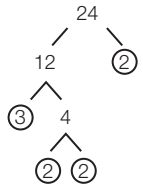
**(b)** Composite, because it has more than two factors; i.e. 1, 2, 4, 8, 16.

**(c)** Composite, because it has more than two factors; i.e. 1, 7, 11, 77.

**(d)** Composite, because it has more than two factors; e.g. 1, 10, 27 635, 276 350.

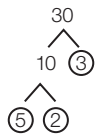
**6** The factor trees can vary, but the factors on the last line should be the same, even though they may be in a different order.

**(a)**



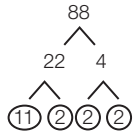

$24 = 2 \times 2 \times 2 \times 3$

**(b)**



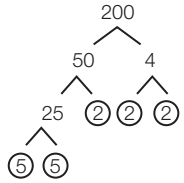

$30 = 2 \times 3 \times 5$

**(c)**




$88 = 2 \times 2 \times 2 \times 11$

**(d)**




$200 = 2 \times 2 \times 2 \times 5 \times 5$

**7** **(a)** $27 = 3 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 3 \times 3$
$= 9$

**(b)** $72 = 2 \times 2 \times 2 \times 3 \times 3$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 3$
$= 24$

**(c)** $108 = 2 \times 2 \times 3 \times 3 \times 3$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
HCF $= 2 \times 2 \times 3$
$= 12$

**8** (a) -14 (b) +200

**9** (a) south 5 km (b) subtracting 27

**10** (a) -52 < 25 (b) 19 > -20

**11** (a) -9, -7, 0, 7, 12 (b) -4000, 4, 40, 400

**12** (a) 18 (b) 15 (c) -10 (d) 6
(e) -4 (f) 8 (g) -5 (h) -32

**13** (a) -3 (b) 3 (c) -7 (d) 0
(e) -7 (f) 2 (g) -3 (h) -12

**14** (a) -2 (b) 7 (c) 11 (d) -4
(e) 37 (f) 14 (g) -11 (h) -3

**15** (a) divisible, multiple (b) factor, multiple
(c) divisible, multiple

**16** 96 (1 packet), 48 (2 packets), 32 (3 packets), 24 (4 packets), 16 (6 packets), 12 (8 packets), 8 (12 packets), 6 (16 packets), 4 (24 packets), 3 (32 packets), 2 (48 packets), 1 (96 packets)

**17** (a) -12 (b) -81 (c) -60 (d) -70

**18** (a) Start at +3 and walk 8 steps to the left, finishing at -5.
(b) Start at -6 and walk 5 steps to the left, finishing at -11.

**19** (a) coprime (b) not coprime, HCF = 3
(c) not coprime, HCF = 7 (d) coprime

**20** 2 runs

**21** (a) HCF = 12, so 12 tables are required
(b) 9 females, 7 males

**22** 504

**23** (a) 71 (b) 72

**24** (a) decreased (b) $61

**25** 150 seconds

**26** -900 + (-250) = -1150 m

**27** (a) = (b) < (c) = (d) <

**28** 2 and 4

**29** 910 and 13

### NAPLAN practice 2

**1** C **2** B **3** 8th floor (-3 + 11 = 8)

**4** B **5** A **6** D

**7** $265 **8** B **9** -5

## Mixed review A (p. 102)

**1** -48, -25, -10, 0, 8, 17, 32

**2** (a) $9^2$ (b) $7^4$ (c) $4^3$

**3** 24, 48, 72, 96

**4** (a) -570 (b) +5

**5** (a) -27 < 14 (b) 0 > -35

**6** (a) 40 (b) 300 (c) 500
(d) 3600 (e) 600 000 (f) 10 000 000

**7** (a) 340 (b) 403 (c) 798
(d) 396 (e) 240 (f) 960

**8** (a) 72 (b) 47 (c) 29
(d) 47 (e) 0 (f) 80

**9** (a) -2 (b) -3 (c) 4 (d) -17

**10** (a) 1800 (b) 15 000 (c) 60 000 (d) 100
(e) 40 (f) 2

**11** (a) -15, -5, 0, 5, 10 (b) -300, -30, 0, 3, 3000

**12** (a) 24 (b) 48

**13** (a) 9 (b) 8

**14** (a) 1, 2, 3, 6, 9, 18 composite
(b) 1, 23 prime
(c) 1, 2, 4, 11, 22, 44 composite
(d) 1, 79 prime

**15** (a) $63 = 3^2 \times 7$ (b) $48 = 2^4 \times 3$
(c) $72 = 2^3 \times 3^2$ (d) $120 = 2^3 \times 3 \times 5$

**16** (a) -6 – 7 = -13 (b) 4 + 11 = 15
(c) -2 + 7 = 5 (d) 22 – 9 = 13
(e) -5 + 10 = 5 (f) 18 + 3 = 21

**17** -180 m

**18** (a) 3, 7, 11, 19, 43, 13, 2 (b) 9, 16, 25
(c) 16, 32, 8, 2

**19** 216

**20** (a) 12 000 (actual answer will be lower)
(b) 20 000 (actual answer will be higher)
(c) 100 (actual answer will be higher)

**21** (a) 9000 (b) $108 000

**22** (a) more (b) $5

**23** B

# Chapter 3

## Recall 3 (p. 106)

**1** (a), (b), (e)

**2** (a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $2\frac{2}{5}$ or $\frac{12}{5}$ (d) $\frac{1}{10}$
(e) $\frac{5}{8}$ (f) $\frac{2}{3}$ (g) $\frac{12}{5}$ (h) $\frac{7}{16}$

**3** (a) $2, 1\frac{1}{4}, 1, \frac{3}{4}, 0$ (b) $0, \frac{1}{8}, \frac{3}{8}, \frac{7}{8}, 1, \frac{9}{8}, \frac{11}{8}$

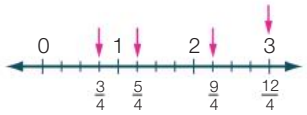
**4** (a) $\frac{5}{7}$ (b) $\frac{6}{11}$ (c) $\frac{4}{3}$ or $1\frac{1}{3}$

**5** (a) 8, 16, 24, 32, 40 (b) 12, 24, 36, 48, 60
(c) 24 (d) 1, 2, 3, 4, 6, 8, 12, 24
(e) 1, 2, 3, 4, 6, 9, 12, 18, 36 (f) 12

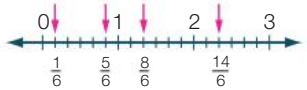
## Exercise 3.1 (p. 111)

**1** (a) 9 (b) 6

**2** (a)




(b)




**3** (a) (i) $\frac{10}{2}$ (ii) $\frac{35}{7}$ (iii) $\frac{55}{11}$ (iv) $\frac{25}{5}$ (v) $\frac{5}{1}$

(b) (i) $\frac{26}{2}$ (ii) $\frac{65}{5}$ (iii) $\frac{104}{8}$ (iv) $\frac{169}{13}$ (v) $\frac{13}{1}$

**4** (a) (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{6}{5}$ or $1\frac{1}{5}$ (iv) $\frac{10}{7}$ or $1\frac{3}{7}$

(b) (iii) and (iv)

**5** (a) 4 (b) 12 (c) 2 (d) 10

**6** (a) $\frac{5}{8}$ (b) $\frac{4}{9}$

**7** (a) $\frac{1}{2}$ (b) $\frac{7}{9}$ (c) $\frac{11}{4}$ (or $2\frac{3}{4}$)

(d) $\frac{3}{5}$ (e) $\frac{3}{14}$ (f) $\frac{25}{8}$ (or $3\frac{1}{8}$)

(g) $\frac{7}{6}$ (or $1\frac{1}{6}$) (h) $\frac{5}{3}$ (or $1\frac{2}{3}$) (i) $\frac{19}{7}$ (or $2\frac{5}{7}$)

**8** (a) $\frac{8}{17}$ (b) $\frac{15}{4}$ (c) $\frac{9}{20}$ (d) $2\frac{2}{3}$

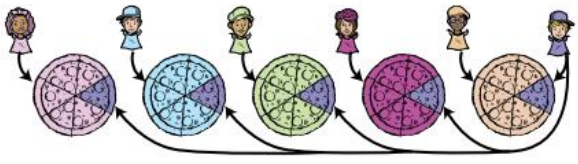
(e) $4\frac{1}{2}$ (f) $3\frac{7}{24}$

**9** (a) $\frac{2}{7}$ (b) $\frac{1}{24}$ (c) $\frac{1}{60}$ (d) $\frac{17}{60}$

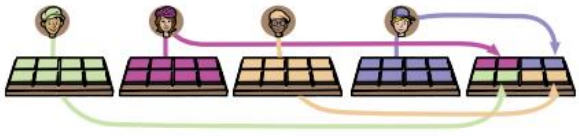
(e) $\frac{157}{375}$ (f) $\frac{421}{500}$

**10** (a) $\frac{7}{14}$ or $\frac{1}{2}$ (b) $\frac{11}{14}$ (c) $\frac{10}{14}$ or $\frac{5}{7}$

**11** (a)



(b)



**12** (a) $\frac{149}{185}$ (b) $\frac{185}{149}$ or $1\frac{36}{149}$

**13** (a) 32 km (b) Toby ate 12 lollies. 18 lollies in bag.

**14** Boys: $\frac{4}{5}$ of a pizza each. Girls: $\frac{5}{4}$ of a pizza each ($1\frac{1}{4}$).
Girls each receive more pizza.

**15** (a) $\frac{1}{4}$

(b) Not possible—not divided into equal parts and unable to create equal parts by further dividing shape up.

(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

(e) Not possible—same reason as (b). (f) $\frac{1}{2}$

**16** D **17** C

**18** 5 whole numbers: 1, 2, 3, 4, 5

### Open-ended – Sample answers

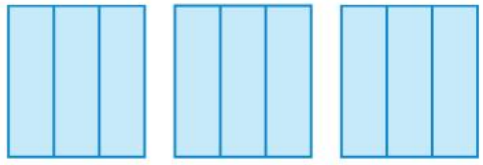
**19** (a) $\frac{3}{8}$ of the shape is shaded



(b) $\frac{5}{6}$ of the collection of stars is shaded.



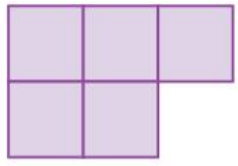
(c) $\frac{9}{3}$



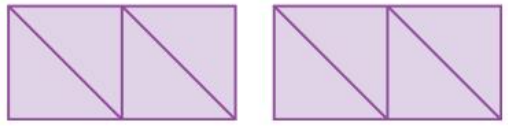
**20** (a)



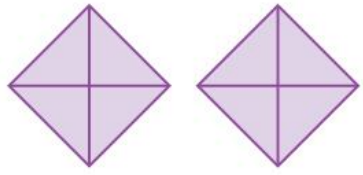
Or

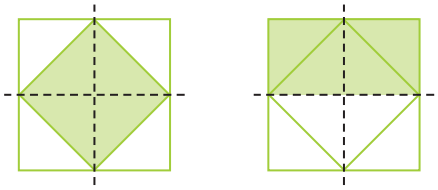


(b)



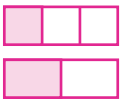
Or



21 

Drawing in a vertical and a horizontal line divides the square into quarters and shows that half of each quarter is shaded. We can rearrange things so that it is more obvious that half the square is shaded.

22 (a) Oliver's shaded areas (representing the fractions $\frac{1}{2}$ and $\frac{1}{3}$) look the same because he is not comparing fractions of the same-sized whole.

(b)  $\frac{1}{3}$

$\frac{1}{2}$

## Exercise 3.2 (p. 120)

1 (a) 15 (b) 21 (c) 26 (d) 40

(e) 2 (f) 3 (g) 10 (h) 63

(i) 36 (j) 3 (k) 6 (l) 18

(m) 5 (n) 6 (o) 36 (p) 16

2 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$

(d) $\frac{1}{3}$ (e) $\frac{2}{5}$ (f) $\frac{4}{11}$

(g) $\frac{5}{8}$ (h) $1\frac{1}{2}$ or $\frac{3}{2}$ (i) $2\frac{2}{3}$ or $\frac{8}{3}$

(j) $3\frac{1}{6}$ or $\frac{19}{6}$ (k) $1\frac{3}{10}$ or $\frac{13}{10}$ (l) $12\frac{4}{5}$ or $\frac{64}{5}$

(m) $6\frac{4}{9}$ or $\frac{58}{9}$ (n) $3\frac{1}{4}$ or $\frac{13}{4}$ (o) $2\frac{5}{6}$ or $\frac{17}{6}$

(p) $100\frac{1}{7}$ or $\frac{701}{7}$

3 (a) $1\frac{2}{5}$ (b) $2\frac{1}{6}$ (c) $5\frac{3}{4}$ (d) $2\frac{1}{7}$

(e) $3\frac{7}{10}$ (f) $9\frac{3}{5}$ (g) $6\frac{2}{7}$ (h) $5\frac{9}{10}$

(i) $8\frac{5}{9}$ (j) $7\frac{7}{12}$ (k) $1\frac{7}{100}$ (l) $2\frac{17}{40}$

4 (a) $\frac{7}{4}$ (b) $\frac{16}{5}$ (c) $\frac{20}{3}$ (d) $\frac{59}{10}$

(e) $\frac{35}{8}$ (f) $\frac{51}{11}$ (g) $\frac{72}{7}$ (h) $\frac{74}{11}$

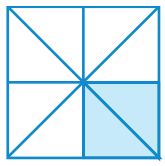
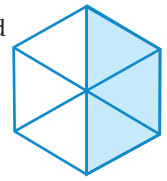
(i) $\frac{77}{12}$ (j) $\frac{309}{100}$ (k) $\frac{62}{9}$ (l) $\frac{79}{9}$

5 (a) C (b) D (c) C (d) B

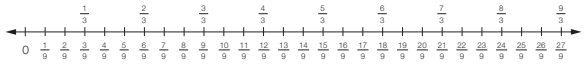
6 (a) $\frac{6}{10}$ (b) $\frac{1}{4}$ and $\frac{3}{12}$

(c) $\frac{10}{12}$ (d) $\frac{3}{4}$ and $\frac{6}{8}$

7 (a) $\frac{1}{4}$, $\frac{1}{2}$

(b)  $\frac{1}{4} = \frac{2}{8}$ and  $\frac{1}{2} = \frac{3}{6}$

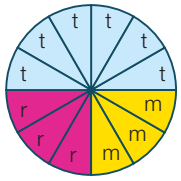
8 (a) They occupy the same position on the number line.

(b) 

$\frac{1}{3}$ and $\frac{3}{9}$, $\frac{2}{3}$ and $\frac{6}{9}$, $\frac{3}{3}$ and $\frac{9}{9}$, $\frac{4}{3}$ and $\frac{12}{9}$, $\frac{5}{3}$ and $\frac{15}{9}$,

$\frac{6}{3}$ and $\frac{18}{9}$, $\frac{7}{3}$ and $\frac{21}{9}$, $\frac{8}{3}$ and $\frac{24}{9}$, $\frac{9}{3}$ and $\frac{27}{9}$

9 Monica 3, Michelle 3, Travis 6




10 35 slices

11 14 squares

12 40 squares, $\frac{5}{8} = \frac{40}{64}$

13 (a) $\frac{1}{30}$ (b) $\frac{1}{3}$ (c) $\frac{1}{5}$ (d) $\frac{3}{20}$

14 $\frac{1}{2}$ = 30 minutes, $\frac{1}{3}$ = 20 minutes, $\frac{1}{4}$ = 15 minutes,

$\frac{1}{5}$ = 12 minutes, $\frac{1}{6}$ = 10 minutes, $\frac{1}{7} = 8\frac{4}{7}$ minutes,

$\frac{1}{8} = 7\frac{1}{2}$ minutes, $\frac{1}{9} = 6\frac{2}{3}$ minutes, $\frac{1}{10}$ = 6 minutes,

$\frac{1}{11} = 5\frac{5}{11}$ minutes, $\frac{1}{12}$ = 5 minutes

15 4 whole numbers: 2, 3, 4, 5

16 C

17 $\frac{1}{7}$ and $\frac{1}{11}$ have no equivalent fractions on the wall. This is because their denominators are prime numbers with no multiples in the other denominators on the fraction wall.

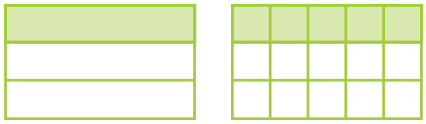
18 (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{13}{20}$ (e) $\frac{7}{18}$ (f) $\frac{9}{22}$

### Open-ended – Sample answers

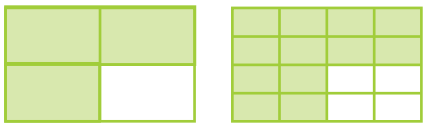
19 $\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$ etc.

20 $\frac{5}{3}$, $\frac{15}{9}$, $\frac{10}{6}$ etc.

21 (a)



(b)



22 $\frac{3}{7}$, $\frac{7}{9}$ and $\frac{5}{11}$

## Exercise 3.3 (p. 127)

1 (a) $\frac{3}{5}$ (b) $\frac{5}{4}$ (c) $\frac{7}{8}$ (d) $\frac{7}{12}$

(e) $\frac{5}{7}$ (f) $\frac{2}{9}$ (g) $\frac{5}{12}$ (h) $\frac{11}{6}$

(i) $1\frac{3}{4}$ (j) $\frac{5}{2}$ (k) $\frac{16}{7}$ (l) $2\frac{4}{9}$

2 (a) $\frac{3}{7}$ (b) $\frac{7}{8}$ (c) $\frac{5}{6}$ (d) $\frac{2}{5}$

(e) equivalent fractions (f) $\frac{5}{11}$ (g) $\frac{7}{8}$ (h) $\frac{7}{11}$

3 (a) $\frac{2}{7} < \frac{5}{7}$ (b) $\frac{4}{5} > \frac{4}{7}$ (c) $\frac{1}{2} < \frac{5}{8}$ (d) $\frac{8}{11} < \frac{9}{10}$

(e) $\frac{2}{4} = \frac{3}{6}$ (f) $\frac{6}{12} < \frac{7}{10}$ (g) $\frac{3}{7} < \frac{5}{8}$ (h) $\frac{5}{9} > \frac{2}{5}$

(i) $\frac{5}{6} < \frac{7}{8}$ (j) $\frac{7}{11} < \frac{8}{10}$ (k) $\frac{6}{10} = \frac{3}{5}$ (l) $\frac{4}{5} > \frac{6}{9}$

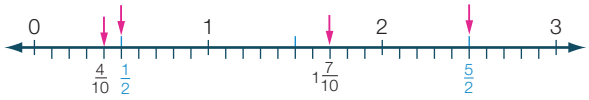
(m) $\frac{2}{4} < \frac{4}{2}$ (n) $\frac{24}{6} = \frac{8}{2}$ (o) $\frac{18}{11} < \frac{9}{5}$ (p) $\frac{9}{7} > \frac{11}{9}$

4 (a)




In ascending order: $\frac{2}{3}$, $\frac{3}{4}$, $1\frac{1}{3}$, $\frac{6}{4}$

(b)




In descending order: $\frac{5}{2}$, $1\frac{7}{10}$, $\frac{1}{2}$, $\frac{4}{10}$

5 $\frac{7}{8} > \frac{10}{12}$

Ali is the more accurate shooter.

6 $\frac{5}{16} < \frac{23}{64}$

The larger box has the greater fraction of nut chocolates.

7 (a) $\frac{10}{11}$ (b) $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, $\frac{7}{11}$ and $\frac{8}{12}$

(c) $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{10}$ and $\frac{5}{12}$ (d) $\frac{1}{5}$, $\frac{2}{10}$, $\frac{2}{9}$, $\frac{1}{4}$, $\frac{2}{8}$ and $\frac{3}{12}$

8 (a) Arrow B (b) Arrow C

9 (a) C (b) C

10 (a) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{10}{11}$

All of the fractions have a numerator that is 1 less than the denominator—they are '1 piece' less than 1 whole. Higher denominator numbers mean that the whole is being cut into more, smaller-sized pieces. This means that the 'missing pieces' that make the fraction less than 1 are getting smaller.

(b) 0, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{7}{8}$, 1, $\frac{7}{4}$

$\frac{7}{4}$ is the only improper fraction and so is greater than 1. All of the other fractions are less than 1, but greater than 0. $\frac{4}{7}$ and $\frac{6}{7}$ have the same denominator, so it is clear that $\frac{6}{7}$ is larger. $\frac{7}{8}$ is greater than $\frac{6}{7}$ for the same reasons described in (a). (Alternatively, they can be converted to fractions with an LCD of 56: $\frac{49}{56} > \frac{48}{56}$.)

11 (a) 1, $\frac{45}{63}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{11}{21}$, $\frac{2}{9}$

All of the fractions are proper fractions, so they are all less than 1. They also all have an LCD of 63. Converting them to equivalent fractions with this LCD gives:

$1 > \frac{45}{63} > \frac{42}{63} > \frac{36}{63} > \frac{33}{63} > \frac{14}{63}$.

(b) $1\frac{7}{20}$, $\frac{21}{16}$, $1\frac{3}{10}$, $\frac{5}{4}$, $1\frac{3}{80}$, $\frac{39}{40}$

Converting all to equivalent fractions with an LCD of 80 gives:

$\frac{108}{80} > \frac{105}{80} > \frac{104}{80} > \frac{100}{80} > \frac{83}{80} > \frac{78}{80}$.

12 (a) approximately $\frac{3}{4}$

(b) Queensland takes up approximately $\frac{1}{5}$ of the area of Australia.

(c) approximately $\frac{2}{3}$

13 (a) same (b) same (c) B

(d) A (e) B (f) same

14 (a) Angus has drunk more cola.

(b) Gianni has drunk a greater fraction ($\frac{5}{6}$ to $\frac{4}{5}$).

### Opened-ended – Sample answers

15 (a) $\frac{4}{5}$, $\frac{5}{6}$ (b) $\frac{5}{2}$, $\frac{11}{4}$

16 $\frac{7}{10}$, $\frac{3}{4}$, $\frac{5}{7}$

17 Jason is comparing fractions of different wholes (different numbers of marks). To do this, he needs to rewrite them out of the same total by finding equivalent fractions with the LCD. $\frac{16}{20} = \frac{80}{100}$, $\frac{43}{50} = \frac{86}{100}$, so $\frac{43}{50}$ is his better result.

## Exercise 3.4 (p. 137)

1 (a) $\frac{1}{2}$ (b) $\frac{1}{20}$ (c) $\frac{9}{20}$

(d) $\frac{14}{55}$ (e) $\frac{7}{6}$ or $1\frac{1}{6}$ (f) $\frac{50}{49}$ or $1\frac{1}{49}$

(g) $\frac{5}{4}$ or $1\frac{1}{4}$ (h) $\frac{19}{20}$ (i) $\frac{1}{12}$

(j) $\frac{47}{36}$ or $1\frac{11}{36}$ (k) $\frac{25}{42}$ (l) $\frac{13}{12}$ or $1\frac{1}{12}$

(m) $\frac{73}{60}$ or $1\frac{13}{60}$ (n) $\frac{5}{12}$ (o) $\frac{87}{70}$ or $1\frac{17}{70}$

(p) $\frac{3}{10}$

2 (a) $1\frac{1}{6}$ (b) $\frac{25}{28}$

3 (a) $7\frac{3}{7}$ (b) $13\frac{2}{3}$ (c) $9\frac{5}{8}$ (d) $7\frac{9}{10}$

(e) $3\frac{1}{4}$ (f) $6\frac{1}{10}$ (g) $7\frac{1}{4}$ (h) $9\frac{1}{4}$

(i) $12\frac{3}{10}$ (j) $11\frac{1}{18}$ (k) $4\frac{42}{55}$ (l) $3\frac{31}{100}$

4 (a) $1\frac{4}{7}$ (b) $\frac{2}{3}$ (c) $4\frac{4}{9}$ (d) $1\frac{2}{3}$

(e) $1\frac{1}{4}$ (f) $2\frac{1}{2}$ (g) $1\frac{3}{10}$ (h) $1\frac{8}{9}$

(i) $\frac{17}{18}$ (j) $\frac{17}{20}$ (k) $\frac{23}{24}$ (l) $2\frac{11}{24}$

5 (a) $\frac{7}{8}$ (b) $\frac{11}{12}$ (c) 1

(d) $\frac{21}{5}$ or $4\frac{1}{5}$ (e) $\frac{401}{60}$ or $6\frac{41}{60}$ (f) $\frac{127}{60}$ or $2\frac{7}{60}$

6 (a) B (b) D

7 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$

8 $\frac{3}{4}$ of the pizza

9 $\frac{1}{4}$

10 $10\frac{1}{12}$ hours

11 $\frac{3}{4}$ of a packet

12 B 13 D

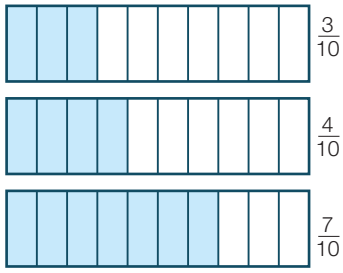



### Open-ended – Sample answers

14 The correct answer is 2, as both fractions are slightly less than 1. Students answering 19 or 21 have just added the numerators or denominators.

15 $\frac{11}{40}$ and $\frac{19}{40}$

16 We add the numerators because we are 'counting up' how many equal-sized parts we have. Adding the denominators would change the part size.




17 Teachers use fraction notation just as a way of keeping track of marks for different sections of the test. The 'whole' is the total number of marks, and the number of correct answers is written as a fraction of that whole.

## Half-time 3 (p. 141)

1 (a) $\frac{7}{15}$ (b) 6

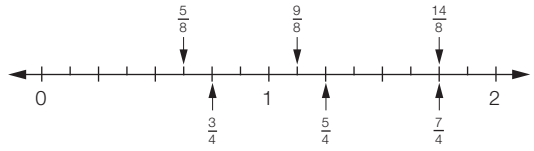
2 (a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $3\frac{3}{4}$ or $\frac{15}{4}$ (d) $7\frac{7}{10}$ or $\frac{77}{10}$

3 (a) $6\frac{3}{4}$ (b) 5 (c) 5 (d) $5\frac{2}{7}$

4 (a) $\frac{5}{8} < \frac{2}{3}$ (b) $\frac{4}{9} < \frac{15}{27}$ (c) $\frac{48}{56} = \frac{6}{7}$ (d) $\frac{11}{5} < \frac{9}{4}$

5 (a)

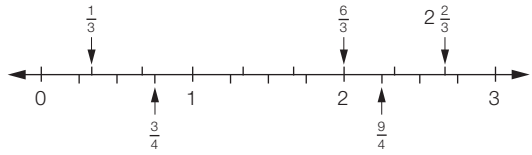



(b) $\frac{1}{4}$ and $\frac{2}{8}$, $\frac{2}{4}$ and $\frac{4}{8}$, $\frac{3}{4}$ and $\frac{6}{8}$, $\frac{5}{4}$ and $\frac{10}{8}$, $\frac{6}{4}$ and $\frac{12}{8}$

(c) (i) $\frac{5}{4}$ (ii) $\frac{7}{4}$

6 (a) $\frac{9}{8}$ or $1\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{29}{18}$ or $1\frac{11}{18}$ (d) $\frac{7}{36}$

7




In ascending order: $\frac{1}{3}, \frac{3}{4}, \frac{6}{3}, \frac{9}{4}, 2\frac{2}{3}$

8 (a) 29 slices (b) 21 slices

9 (a) $\frac{281}{60}$ or $4\frac{41}{60}$ (b) $\frac{429}{100}$ or $4\frac{29}{100}$

(c) $\frac{81}{50}$ or $1\frac{31}{50}$ (d) $\frac{79}{50}$ or $1\frac{29}{50}$

10 (a) $\frac{5}{2}$ of $2\frac{1}{2}$ L (b) $1\frac{3}{4}$ L

## Exercise 3.5 (p. 149)

1 (a) 6 (b) 3 (c) 3 (d) 4

(e) 21 (f) 24 (g) 48 (h) 27

(i) 30 (j) 120 (k) 55 (l) 75

2 (a) $\frac{1}{13}$ (b) $\frac{1}{9}$ (c) $\frac{4}{9}$ (d) $\frac{3}{13}$

(e) $\frac{1}{14}$ (f) $\frac{6}{11}$ (g) $\frac{5}{14}$ (h) $\frac{5}{66}$

(i) $\frac{9}{28}$ (j) $\frac{3}{4}$ (k) $\frac{8}{21}$ (l) $\frac{27}{250}$

3 (a) $\frac{2}{3}$ (b) 6 (c) $1\frac{1}{4}$ (d) $1\frac{1}{4}$

(e) $11\frac{1}{2}$ (f) 14 (g) 27 (h) $40\frac{2}{3}$

(i) 2 (j) $11\frac{1}{5}$ (k) 18 (l) 4

4 (a) $\frac{3}{20}$ (b) $\frac{2}{45}$ (c) $\frac{5}{14}$

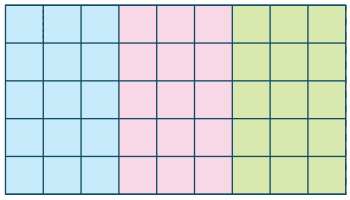
(d) $\frac{10}{13}$ (e) $\frac{7}{6}$ or $1\frac{1}{6}$ (f) $\frac{4}{3}$ or $1\frac{1}{3}$

5 (a) C (b) A

6 (a) $\frac{1}{7}$ (b) \$310

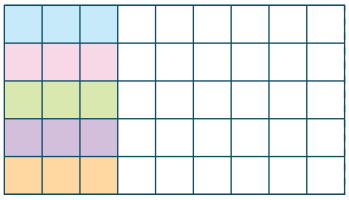
7 Michael \$10 000, Marcus \$4000, Lily \$18 000, Julie \$16 000

8 (a) 15



(b) (i) 3 (ii) $\frac{1}{15}$

(iii)



9 $\frac{1}{8}$ each

10 (a) $\frac{3}{20}$ (b) $\frac{1}{5}\times\frac{3}{4}$ or $\frac{3}{4}\times\frac{1}{5}$

11 (a) 13 L (b) 39 L

12 (a) 50 students (b) 40 students

13 450 g butter, $1\frac{1}{4}$ cups icing sugar, $3\frac{1}{8}$ cups self-raising flour, $\frac{5}{6}$ cup custard powder

14 $1\frac{1}{2}\times 12$

15 D

16 D

### Open-ended – Sample answers

17 $\frac{1}{2}\times\frac{1}{3}$

18 The six possible answers are $\frac{1}{2}\times 30 = 15$, $\frac{1}{2}\times 32 = 16$, $\frac{1}{3}\times 30 = 10$, $\frac{1}{3}\times 33 = 11$, $\frac{1}{3}\times 36 = 12$, $\frac{1}{3}\times 39 = 13$.

19 Asher has cancelled out common factors between the two numerators. Common factors can only be cancelled between numerators and denominators. Correct working:

$$\frac{3}{\cancel{9}_{1}}\times\frac{\cancel{60}^{12}}{1} = \frac{36}{1} = 36$$

## Exercise 3.6 (p. 157)

1 (a) 24 (b) 12 (c) 21

(d) 36 (e) 18 (f) 5

(g) $\frac{35}{3}$ or $11\frac{2}{3}$ (h) $\frac{63}{4}$ or $15\frac{3}{4}$ (i) $\frac{16}{3}$ or $5\frac{1}{3}$

(j) 3 (k) $\frac{15}{8}$ or $1\frac{7}{8}$ (l) $\frac{4}{3}$ or $1\frac{1}{3}$

2 (a) 2 (b) $\frac{3}{2}$ or $1\frac{1}{2}$ (c) $\frac{9}{8}$ or $1\frac{1}{8}$

(d) $\frac{7}{5}$ or $1\frac{2}{5}$ (e) $\frac{1}{6}$ (f) $\frac{1}{20}$

(g) $\frac{1}{4}$ (h) $\frac{3}{20}$ (i) $\frac{9}{16}$

(j) $\frac{14}{25}$ (k) $\frac{36}{85}$ (l) $\frac{27}{80}$

(m) 2 (n) $\frac{44}{43}$ or $1\frac{1}{43}$ (o) $\frac{15}{4}$ or $3\frac{3}{4}$

(p) $1\frac{5}{7}$

3 (a) C (b) D

4 (a) 2, 2 (b) 6, 6 (c) 4, 4 (d) 8, 8

5 D 6 6 students

7 12 whole shifts 8 30 batches of scones

9 5 drinks stations 10 5 complete screenings

11 C 12 $\frac{4}{5}\div\frac{1}{3}$

**13** **(a)** As the denominator number gets larger, the part size gets smaller, so you can fit more of the fraction into the number being divided.

**(b)** $13\frac{1}{3}$

### Open-ended – Sample answers

**14** $4 \div \frac{2}{5} = 10$, $4 \div \frac{2}{6} = 12$, $4 \div \frac{2}{7} = 14$, $4 \div \frac{2}{8} = 16$, $4 \div \frac{2}{9} = 18$

**15** Gabi has not multiplied by the inverse. 12 is a reasonable answer because she is dividing by a number less than 1.

## Exercise 3.7 (p. 163)

**1** **(a)** $\frac{9}{4}$ or $2\frac{1}{4}$ **(b)** $\frac{39}{8}$ or $4\frac{7}{8}$ **(c)** $\frac{5}{99}$

**(d)** $\frac{3}{16}$ **(e)** $\frac{5}{16}$ **(f)** $\frac{11}{18}$

**(g)** $4\frac{3}{4}$ or $\frac{19}{4}$ **(h)** $3\frac{3}{8}$ or $\frac{27}{8}$ **(i)** $2\frac{13}{20}$ or $\frac{53}{20}$

**(j)** 1 **(k)** 4 **(l)** $\frac{2}{3}$

**2** **(a)** A **(b)** A

**3** **(a)** $\frac{41}{12}$ or $3\frac{5}{12}$ **(b)** $\frac{53}{30}$ or $1\frac{23}{30}$ **(c)** $\frac{17}{5}$ or $3\frac{2}{5}$ **(d)** 30

**4** **(a)** $\frac{2}{3}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{5}{8}$

**(e)** 90 **(f)** 36

**5** **(a)** $\frac{1}{75}$ **(b)** 75 **(c)** $\frac{2}{25}$ **(d)** 150 m **(e)** 1125 m

**6** $\frac{97}{12}$ or $8\frac{1}{12}$ laps

**7** **(a)** $\frac{11}{8}$ or $1\frac{3}{8}$ km **(b)** $\frac{95}{24}$ or $3\frac{23}{24}$ km **(c)** $\frac{32}{3}$ or $10\frac{2}{3}$ km

**8** $7\frac{3}{4}$ hours or $\frac{31}{4}$ hours

**9** Bryce

**10** cutlery set

**11** $\frac{63}{64}$

**12** **(a)** $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{8}$ **(b)** $13 \times 8 = 104$

### Open-ended – Sample answers

**13** $\frac{5}{8}, \frac{3}{4}, \frac{2}{3}$

**14** **(a)** $\frac{2+1}{2-1} = \frac{3}{1} = 3$, $\frac{3+2}{3-2} = \frac{5}{1} = 5$, $\frac{4+1}{4-1} = \frac{5}{3}$ or $1\frac{2}{3}$ etc.

**(b)** 17 (when $A = 9$ and $B = 8$)

**15** **(a)** $\frac{7}{3} + \frac{7}{4} = \frac{7}{3} \times \frac{7}{4} = \frac{49}{12}$ or $4\frac{1}{12}$

**(b)** $\frac{5}{3} + \frac{5}{2} = \frac{5}{3} \times \frac{5}{2} = \frac{25}{6}$ or $4\frac{1}{6}$

**(c)** The numerators are the sum of the denominators.

**(d)** Anything that obeys the statement in **(c)**;

e.g. $\frac{9}{4}$ and $\frac{9}{5}$, $\frac{11}{5}$ and $\frac{11}{6}$

## Challenge 3 (p. 167)

**1** C

Find number first, then find required fraction.

**2** 720 L

Fraction given equals volume given. Dividing by the fraction gives total volume.

**3** A

Write fractions with lowest common denominator.

**4** C

List all the possible fractions, remembering that they must already be in simplest form. Subtract the smallest fraction from the second list from the largest fraction from the first list and simplify.

**5** B

Same method as in Q3 but remember to simplify fraction at the end.

**6** 1.20 p.m.

Divide each time into 60 minutes—realise that they both croak on the hour every hour. By listing or otherwise, find times after the hour that the frogs croak. This will give the first time they both croak together.

**7** C

Rewrite the fraction doing the addition in the denominator. Then, do the division, then add.

**8** $\frac{33}{109}$

Simplify each fraction step by step starting with the $3 + \frac{1}{3}$ fraction. Divide, then repeat the process of adding, then dividing until a single fraction remains.

**9** 36

Subtract $\frac{1}{3}$ of the number from 2 times the number. This gives the fraction of the number equal to the amount. Divide this fraction into the amount to find the whole.

**10** $\frac{13}{96}, \frac{7}{48}, \frac{5}{32}$

Find the middle number using the method in Q3. Now, find the middle number between this number and the given numbers by finding equivalent fractions. Don't forget to write the fraction found in simplest form.

**11** $100

Cancelling common factors gives:

$$\frac{1}{\cancel{2}_1}\times\frac{\cancel{2}^1}{\cancel{3}_1}\times\frac{\cancel{3}^1}{\cancel{4}_1}\times\frac{\cancel{4}^1}{\cancel{5}_1}\times\frac{\cancel{5}^1}{\cancel{6}_1}\times\frac{\cancel{6}^1}{\cancel{7}_1}\times\frac{\cancel{7}^1}{\cancel{8}_1}\times\frac{\cancel{8}^1}{\cancel{9}_1}\times\frac{\cancel{9}^1}{10}\times\$1000$$

$$=\frac{1}{10}\times\$1000$$

$$=\$100$$

## Chapter review 3 (p. 168)

**1** (a) 16 (b) $\frac{5}{12}$

**2** $\frac{3}{4}, \frac{5}{4}\left(\text{or } 1\frac{1}{4}\right)$

**3** (a) (i) $\frac{18}{3}$ (ii) $\frac{60}{10}$ (iii) $\frac{6}{1}$

(b) (i) $\frac{33}{3}$ (ii) $\frac{110}{10}$ (iii) $\frac{11}{1}$

**4** (a) $\frac{3}{5}$ (b) $1\frac{3}{4}$

**5** (a) 42 (b) 4 (c) 90 (d) 30

**6** (a) $\frac{4}{5}$ (b) $\frac{12}{5}$ or $2\frac{2}{5}$

(c) $3\frac{5}{6}$ or $\frac{23}{6}$ (d) $1\frac{1}{5}$ or $\frac{6}{5}$

**7** (a) $\frac{23}{7}$ (b) $5\frac{5}{9}$

**8** (a) < (b) = (c) > (d) <

**9** (a) $\frac{31}{24}$ or $1\frac{7}{24}$ (b) $\frac{1}{6}$

(c) $\frac{8}{21}$ (d) $\frac{13}{15}$

**10** (a) $1\frac{13}{20}$ (b) $8\frac{1}{6}$

(c) $\frac{37}{7}$ or $5\frac{2}{7}$ (d) $\frac{1}{6}$

**11** (a) $12 (b) $\frac{1}{6}$

**12** (a) $\frac{5}{3}$ or $1\frac{2}{3}$ (b) $\frac{110}{9}$ or $12\frac{2}{9}$ (c) $\frac{3}{7}$

**13** (a) 12 (b) $6\frac{9}{23}$ (c) $\frac{9}{16}$ (d) $\frac{39}{40}$

**14** (a) $5\frac{17}{18}$ (b) $4\frac{9}{10}$ (c) $3\frac{9}{32}$

**15** (a) $\frac{1}{2}+\frac{2}{5}$ (b) $\frac{9}{10}$

**16** (a) $\frac{1}{3}$ (b) $2\frac{1}{6}$ or $\frac{13}{6}$ (c) $1\frac{1}{3}$ or $\frac{4}{3}$ (d) $\frac{3}{8}$

**17** (a) $\frac{5}{9}$ (b) $\frac{2}{9}$ (c) $\frac{19}{27}$

**18** (a) $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}, 1$ (b) $\frac{30}{50}, \frac{5}{4}, 1\frac{2}{5}, 2, \frac{13}{5}$

**19** Jamal

**20** (a) $\frac{4}{15}$

(b) $\frac{16}{15}$ or $1\frac{1}{15}$ = more than a whole block of chocolate

**21** $\frac{5}{6}$

**22** (a) 30 000 L (b) 50 000 L (c) 12 000 L

**23** 45 g butter, $\frac{1}{6}$ cup castor sugar, 1 egg, $\frac{7}{16}$ cup self-raising flour, $\frac{1}{2}$ tablespoon cocoa, $\frac{1}{8}$ cup orange juice

**24** (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$

**25** (a) Katya (6 km) (b) Mabok

**26** A

### NAPLAN practice 3

**1** D **2** B **3** A **4** B

**5** B **6** C **7** D **8** D

# Chapter 4

## Recall 4 (p. 174)

**1** (a) 0.1 > 0.01 (b) 2 < 2.2 (c) 0.3 > 0.1

(d) 3.2 < 4.1 (e) 0.008 < 0.09 (f) 0.7 > 0.07

**2** (a) seven tens and two ones

(b) six hundreds and three ones

(c) nine thousands, two hundreds, five tens and one one

(d) one ten-thousand, one thousand and eight tens

**3** (a) (i) seven tenths (ii) 0.7

(b) (i) eight thousandths (ii) 0.008

(c) (i) three hundredths (ii) 0.03

(d) (i) one and nine tenths (ii) 1.9

**4** (a) 110 (b) 1535 (c) 479 (d) 6499

**5** (a) 49 (b) 447 (c) 7905 (d) 1648

**6** (a) 34 (b) 2400 (c) 144 (d) 40 800

**7** (a) 181 (b) 1051 (c) 28 (d) 150

**8** (a) 7000 (b) 120 000 (c) 9 200 000 (d) 8

(e) 36 (f) 250 (g) 12 (h) 630

(i) 42.7 (j) 5.8 (k) 9.01 (l) 0.0762

**9** (a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{1}{100}$ (f) $\frac{3}{5}$

## Exercise 4.1 (p. 180)

**1** **(a)** **(i)** 45.462 **(ii)** 12.519 372 **(iii)** 3.7985

**(iv)** 1.008 332 **(v)** 0.786 06 **(vi)** 7.033 047

**(b)** **(i)** $6 + \frac{6}{10} + \frac{3}{100}$

**(ii)** $\frac{9}{10} + \frac{2}{100} + \frac{1}{1000}$

**(iii)** $\frac{7}{10} + \frac{3}{100} + \frac{4}{1000} + \frac{5}{10\,000}$

**(iv)** $7 + \frac{8}{10} + \frac{2}{100} + \frac{6}{1000}$

**(v)** $23 + \frac{9}{10} + \frac{1}{100} + \frac{3}{1000} + \frac{4}{100\,000}$

**(vi)** $45 + \frac{4}{1000} + \frac{5}{10\,000} + \frac{8}{100\,000} + \frac{9}{1\,000\,000}$

**2** **(a)** **(i)** 6.5 **(ii)** 0.97 **(iii)** 0.273

**(iv)** 37.4201 **(v)** 14.957 623 **(vi)** 700.004 59

**(b)** **(i)** five ones and two tenths

**(ii)** four ones and nine tenths

**(iii)** three tens, four ones, one tenth and seven hundredths

**(iv)** six tenths and one hundredth

**(v)** two ones, seven tenths, nine hundredths and four thousandths

**(vi)** seven ones, five tenths, nine thousandths and two ten-thousandths

**(vii)** three tens, five ones, eight tenths, six hundredths, five thousandths, four ten-thousandths and three hundred-thousandths

**(viii)** eight tenths, two hundredths, two hundred-thousandths and seven millionths

**3** **(a)** **(i)** $\frac{2}{1000}$ **(ii)** $\frac{2}{100\,000}$ **(iii)** $\frac{2}{10}$ **(iv)** 2

**(b)** **(i)** seven tenths **(ii)** seven thousandths

**(iii)** seven ten-thousandths **(iv)** seven millionths

**4** **(a)** $2.4 > 0.42$ **(b)** $2.32 > 1.955$

**(c)** $0.65 > 0.57$ **(d)** $0.3003 < 0.333$

**(e)** $4.7038 < 4.7312$ **(f)** $8.251 > 8.2501$

**(g)** $7.02 > 7.002$ **(h)** $4.7367 < 4.7376$

**(i)** $0.927 < 0.927\,34$ **(j)** $6.013 > 6.01$

**(k)** $3.406 < 3.4063$ **(l)** $0.9995 > 0.9986$

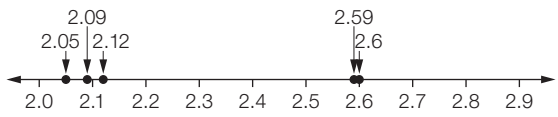
**5** **(a)** False **(b)** False **(c)** False **(d)** False

**(e)** True **(f)** False **(g)** True **(h)** False

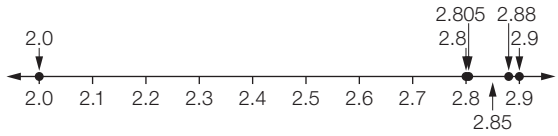
**6** B

**7** B

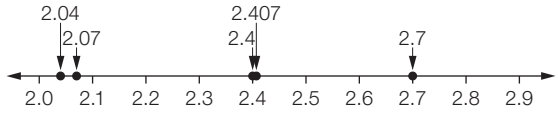
**8** **(a)**



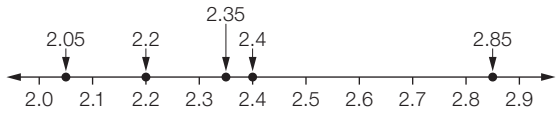

**(b)**




**(c)**




**(d)**




**9** **(a)** 2.03, 2.13, 2.3 **(b)** 8.007, 8.67, 8.7

**(c)** 6.64, 6.6403, 6.646 **(d)** 0.0095, 0.0509, 0.0905

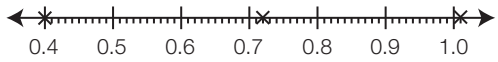
**(e)** 5.003 821, 5.3281, 5.38 **(f)** 3.116, 3.616, 3.661

**(g)** 0.092, 0.29, 0.92 **(h)** 0.086, 0.815, 0.85

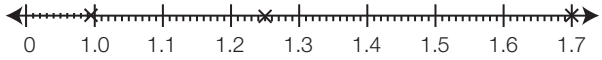
**10** 37.901

**11** **(a)** $\frac{4}{10} + \frac{3}{100} + \frac{7}{1000}$ **(b)** $\frac{437}{1000}$

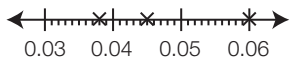
**12** **(a)**



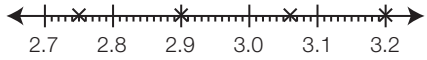

**(b)**




**(c)**




**(d)**




**13** **(a)** 13.08 seconds **(b)** 13.95 seconds

**14** 25.6 m

**15** **(a)** **(i)** 20.2 **(ii)** 21 **(iii)** 21.5

**(b)** 3.75 **(c)** 1.25 L **(d)** 2.60

**16** **(a)** The 2 is in the thousandths place, not in the hundredths place.

**(b)** Should be 409.67. Al has written the 6 in the hundredths place and the 7 in the thousandths place.

**(c)** Max has put the 7 in the tenths place instead of in the ones place. Answer should be 7.0809.

**(d)** $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$ means 0.956. Polly has not accounted for the 0 in the hundredths column. 0.9056 is the same as $\frac{9}{10} + \frac{5}{1000} + \frac{6}{10\,000}$.

### Open-ended – Sample answers

**17** **(a), (b)** 4.51, 4.52, 4.53, 4.6, 4.63, 4.67

**18** **(a)** $2.13 < 5.6$, $3.52 < 5.1$, $4.69 < 5.3$

**(b)** 6, 7, 8, 9 could not be in the ones box.

**19** (a) 2.57, 5.27, 7.25, 2.75, 5.72, 7.52, six different decimals

(b) 25.7, 27.5, 5.72, 52.7, 57.2, 7.25, 7.52, 72.5, 75.2, nine different decimals

## Exercise 4.2 (p. 187)

**1** (a) 4.9 (b) 6.7 (c) 7.64 (d) 4.55

(e) 0.642 (f) 3.28 (g) 0.3144 (h) 11.83

(i) 2.92 (j) 18.500 (k) 90.9000 (l) 19.000

(m) 23.41 (n) 57.080 (o) 259.9000

**2** (a) \$4.50 (b) \$2.75 (c) \$11.05

(d) \$23.90 (e) \$176.35 (f) \$542.05

(g) \$789.55 (h) \$56.75 (i) \$6775.00

(j) \$1149.95 (k) \$8990.00 (l) \$10 000.00

**3** D **4** 9.6 seconds

**5** \$53.65

**6** (a) 99.9 (b) 99.94

**7** 42°C **8** \$11.95

**9** 352.2 metres per second

**10** 1.27 m

**11** (a) 12.138, 12.143, 12.149, 12.152, 12.157

(b) 12.14 s, 12.14 s, 12.15 s, 12.15 s, 12.16 s

(c) Yes—1st and 2nd, and 3rd and 4th, are now tied.

**12** (a) \$20.35 (b) \$20.40—5 cents extra

### Open-ended – Sample answers

**13** 3.792, 3.790 56, 3.787

**14** (a) \$100.02, \$99.98 (b) \$99.60, \$100.48

**15** 2.548, rounding at the 1st step creates errors at the 2nd step.

**16** The coins have no real 'purchasing' power anymore, as prices get higher and higher. They were not worth the cost of producing them.

## Exercise 4.3 (p. 192)

**1** (a) $4\frac{1}{10}$ (b) $6\frac{1}{2}$ (c) $8\frac{3}{5}$ (d) $9\frac{2}{5}$

(e) $4\frac{71}{100}$ (f) $2\frac{37}{100}$ (g) $\frac{7}{20}$ (h) $2\frac{12}{25}$

(i) $5\frac{9}{1000}$ (j) $\frac{221}{250}$ (k) $6\frac{16}{125}$ (l) $3\frac{43}{250}$

(m) $7\frac{9}{200}$ (n) $\frac{11}{125}$ (o) $6\frac{3}{2000}$ (p) $1\frac{3}{400}$

**2** (a) 0.7 (b) 0.09 (c) 0.17 (d) 0.123

(e) 0.2 (f) 0.12 (g) 0.26 (h) 1.75

(i) 0.875 (j) 0.6875 (k) 0.575 (l) $0.78\dot{3}$

(m) $0.\dot{1}$ (n) $0.58\dot{3}$ (o) $2.8\dot{3}$ (p) $1.0\dot{6}$

**3** D **4** D

**5** $\frac{12}{25}$

**6** $\frac{127}{50}$ (or $2\frac{27}{50}$)

**7** $1\frac{7}{20}$

**8** $\frac{3}{125\,000}$

**9** 0.000 005

**10** $5\frac{17}{20}$ per cent, which is higher than $5\frac{3}{4}$ per cent ($5\frac{15}{20}$ per cent)

**11** $4\frac{1}{5}$, 4.375, $4\frac{1}{2}$, 4.865, $4\frac{9}{10}$




### Open-ended – Sample answers

**12** (a) $3\frac{44}{100}$, $3\frac{45}{100}$, $3\frac{46}{100}$ (b) $3\frac{45}{99}$, $3\frac{44}{98}$, $3\frac{46}{97}$

**13** $5\frac{62}{100}$, $5\frac{59}{100}$

## Exercise 4.4 (p. 195)

**1** (a) 16.2 (b) 12.94 (c) 0.914 (d) 25.648

(e) 26.418 (f) 39.811 (g) 22.827 (h) 10.259

(i) 8.858 (j) 23.917 (k) 17.595 (l) 11.5834

**2** (a) 1.9 (b) 4.12 (c) 12.247 (d) 6.963

(e) 3.184 (f) 3.838 (g) 7.26 (h) 7.45

(i) 2.769 (j) 0.56 (k) 0.08 (l) 92.302

**3** D

**4** 12.70 mm

**5** (a) \$19.87 (b) \$19.85

**6** \$295.51

**7** 40.5

**8** 267.9 km

**9** 196.96 km

**10** \$63.65

**11** 0.375 litres

**12** (a) \$165.90 (b) The fourth item (\$18.48) was added twice.

**13** \$330.65

### Open-ended – Sample answers

**14** (a) 0.33 + 2.541 (b) 1.213 + 2.532 + 0.115

(c) The last digits of the numbers have to add up to 10 or 20.

**15** (a) 4.1 + 3.85 = 7.95, 4.3 + 2.85 = 7.15, 4.6 + 2.85 = 7.45

(b) The last digit in the 2nd number must be a 5.

**16** (a) 1.234 and 9.946 (b) 2.324 and 11.034

(c) The last digits must be the same.

**17** 53.47, 53.43

## Exercise 4.5 (p. 203)

**1** (a) 30.4 (b) 37.1 (c) 58.5 (d) 23.10
(e) 2.16 (f) 73.98 (g) 64.376 (h) 4.333
(i) 54.927 (j) 287.04 (k) 1219 (l) 1186.88
(m) 0.0008 (n) 0.0035 (o) 3.0018

**2** (a) 144 (b) 228.9 (c) 119.64 (d) 114
(e) 2170 (f) 2650 (g) 27 546 (h) 27 434
(i) 29 202 (j) 38 828 (k) 67 201.5 (l) 411 576

**3** (a) 0.48 (b) 0.24 (c) 0.36 (d) 6.524
(e) 1.236 (f) 4.208 (g) 2.3994 (h) 2.0812
(i) 0.6768 (j) 0.0024 (k) 0.009 24 (l) 0.0036

**4** C **5** C **6** C

**7** $18.20

**8** 10.619 grams

**9** (a) 3 500 000 (b) 4 260 000 000
(c) 1 300 000 000 000 (d) 57 080 000
(e) 10 200 000 000 000 (f) 20 045 000 000

**10** 0.984 metres

**11** $117.98

**12** $516.00

**13** 53 500 000 000

**14** (a) $94.50 (b) $12.83

**15** 1950 kilograms

**16** $80.60

**17** 37.5 litres

**18** 292.25 kilojoules

**19** (a) 22.3 million

(b) The 'exact' value is not accurate, as births and deaths cause it to change daily. It is not necessary or helpful, for most purposes, to give a more precise number.

**20** (a) 6.75 metres (b) 2, 0.25 m

**21** (a) $4.80 (b) $0.85

(c) Purchasing 3 multi-bags ($11.85) is slightly cheaper than purchasing individual bars ($12).

**22** 1.6 kilograms

**23** (a) 76.3 kilowatt-hours (b) $9.92

### Open-ended – Sample answers

**24** (a) He forgot the place value of the digits.
Estimation: $2 \times 90 = 180$ m.

(b) He could have multiplied 89 by 1, 0.6 and 0.03 and added the answers together.

(c) Students' own answers.

**25** (a) 20 and 3.42, 30 and 2.28

(b) 12.5 and 5.472, 15 and 4.56

## Half-time 4 (p. 207)

**1** (a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) $3\frac{27}{50}$ (d) $9\frac{69}{200}$

**2** 0.03, 0.321, 9.003 24, 9.01, 19.12

**3** (a) 20.4 (b) 132.919 (c) 2.82 (d) 6.02
(e) 83.861 (f) 1.1

**4** (a) 4.877 (b) 13.004 (c) 2.350 (d) 23.032

**5** (a) 30.4 (b) 141.5 (c) 12.33 (d) 1356
(e) 520 (f) 3390 (g) 7.31 (h) 613.228
(i) 0.0006

**6** (a) $100.55 (b) $48.45

**7** (a) 23.4907 (b) 14.235 (c) 293.415 (d) 90.6207

**8** (a) $1.95 (b) $35.10 (c) $57.45 (d) $190.00

**9** (a) 0.8 (b) 0.85 (c) $0.\dot{6}$ (d) $0.58\dot{3}$

**10** 59.023, 59.105, 59.125, 59.23

**11** $122.46

## Exercise 4.6 (p. 211)

**1** (a) 2.12 (b) 4.32 (c) 1.58 (d) 0.55
(e) 0.343 (f) 0.567 (g) 6.57 (h) 1.514
(i) 0.052 (j) 0.07 (k) 0.407 (l) 2.637
(m) 30.866 (n) 10.546 (o) 49.644

**2** (a) 0.09 (b) 0.09 (c) 0.604 (d) 0.483
(e) 0.006 (f) 0.039 (g) 0.078 (h) 0.005
(i) 0.003 (j) 0.57 (k) 0.029 (l) 0.007

**3** (a) 25.7 (b) 10.3 (c) 8.5 (d) 387
(e) 100.2 (f) 24.8 (g) 397 (h) 44.4
(i) 24.5 (j) 700 (k) 315 (l) 145
(m) 8700 (n) 11 500 (o) 16 700

**4** D

**5** C

**6** 0.289 kg

**7** $1.965 million

**8** $93.99 or $94 per guest

**9** 12 tankfuls

**10** 259 necklaces

**11** 12.43 km

**12** $0.22/100 mL

**13** $4.71

**14** 1933 worms

**15** 6.5 minutes

**16** $4.75 per kilometre

**17** Angie's team 1.60 m, Jessica's team 1.59 m. Therefore, Angie's team is taller on average by 0.01 m.

**18** Brazilian Roast at $2.25 per 100 grams

**19** 33 5-cent coins, 23 10-cent coins, 26 20-cent coins, 15 50-cent coins

**20** 217 km

### Open-ended – Sample answers

**21** 2.002, 3.006, 3.666

**22** **(a)** 20.1 ÷ 1 (multiply by 2), 40.2 ÷ 2 (multiply by 4)

**(b)** 18.02 ÷ 0.1 (multiply by 5), 720.8 ÷ 0.4 (multiply by 20)

**23** 10 m and 0.845 m, perimeter 21.69 m; 5 m and 1.69 m, perimeter 13.38 m

**24** **(a)** He didn't consider the place value of the digits to the RHS of the decimal point.

**(b)** To do decimal division more accurately, Tran should perform the division, ignoring the decimal point, and then place a decimal point in the answer, so it lines up with the one in the dividend.

## Exercise 4.7 (p. 219)

**1** **(a)** $\frac{3}{20}$ **(b)** $\frac{6}{25}$ **(c)** $\frac{9}{50}$ **(d)** $\frac{16}{25}$

**(e)** $\frac{11}{10}$ **(f)** $\frac{61}{50}$ **(g)** $\frac{7}{5}$ **(h)** $\frac{99}{50}$

**2** **(a)** 0.34 **(b)** 0.13 **(c)** 0.78 **(d)** 0.92

**(e)** 1.2 **(f)** 1.23 **(g)** 2.54 **(h)** 3.2

**3** **(a)** 40% **(b)** 75% **(c)** 30% **(d)** 35%

**(e)** 110% **(f)** 150% **(g)** 112% **(h)** 120%

**(i)** 37.5% **(j)** $33.\dot{3}\%$ **(k)** $83.\dot{3}\%$ **(l)** $44.\dot{4}\%$

**(m)** $86.\dot{6}\%$ **(n)** $63.\dot{3}\%$ **(o)** 82.5% **(p)** $73.\dot{3}\%$

**4** **(a)** 87% **(b)** 34% **(c)** 65% **(d)** 96%

**(e)** 134% **(f)** 358% **(g)** 532% **(h)** 121%

**5**

| Percentage | Fraction | Decimal |
|---|---|---|
| 40% | $\frac{2}{5}$ | 0.4 |
| 70% | $\frac{7}{10}$ | 0.7 |
| 5% | $\frac{1}{20}$ | 0.05 |
| 2% | $\frac{1}{50}$ | 0.02 |
| 75% | $\frac{3}{4}$ | 0.75 |
| 125% | $\frac{5}{4}$ | 1.25 |

**6** **(a)** 0.25, 0.5, 0.1, 0.2 **(b)** 25%, 50%, 10%, 20%

**7** $\frac{19}{20}$

**8** 62.5%

**9** **(a)** 0.03, $\frac{7}{50}$, 57%, $\frac{3}{5}$, 0.8, 82%

**(b)** 0.05, $\frac{3}{10}$, 43%, $\frac{1}{2}$, 73%, 1.05

**10** **(a)** $\frac{3}{5}$ **(b)** 60%

**11** 63%

**12** **(a)** **(i)** $\frac{1}{25}$ **(ii)** 4% **(iii)** 0.04

**(b)** The percentage, as most members of the public would be able to comprehend and visualise the size of the number.

**(c)** '4% of us believe in aliens'

**13** $\frac{1}{4} = 25\%$ and $\frac{3}{4} = 75\%$, $75 = 3 \times 25$

**14** **(a)** 75% **(b)** 0.75 **(c)** 25% **(d)** 0.25

### Open-ended – Sample answers

**15** $\frac{1}{3}, \frac{1}{4}, \frac{3}{10}$

**16** 34.1%, 34.2%, 34.3%

**17** Students' own answers.

## Exercise 4.8 (p. 223)

**1** **(a)** \$18 **(b)** 45 m **(c)** 168 L **(d)** 96 m

**(e)** 450 kg **(f)** \$168 **(g)** 9.36 kg **(h)** \$95.58

**(i)** 183.96 m **(j)** 64.75 m **(k)** 71.536 L **(l)** \$1.31

**2** **(a)** 85% **(b)** 22.7% **(c)** 40% **(d)** 80%

**(e)** 76% **(f)** 77.1% **(g)** 55.6% **(h)** 75%

**(i)** 82.4%

**3** **(a)** \$12.50 **(b)** 7.5 kg **(c)** 15 km **(d)** 8 L

**(e)** 3.6 m **(f)** \$6 **(g)** 210 kg **(h)** 600 mL

**(i)** \$140

**4** B

**5** C

**6** 10 students

**7** 57.1% are cream biscuits

**8** 0.7 L

**9** $83.\dot{3}\%$

**10** Boys: 42.3%, Girls: 57.7%

**11** **(a)** Damian 70%, Franco 68.75%

**(b)** Damian

**12** 33.1% of the score

**13** **(a)** **(i)** \$8 **(ii)** \$31.25 **(iii)** \$32.50

**(iv)** \$23.90 **(v)** \$319.60 **(vi)** \$64.95

**(b)** **(i)** \$32 **(ii)** \$93.75 **(iii)** \$32.50

**(iv)** \$215.10 **(v)** \$479.40 **(vi)** \$1234.05

**14** 92%

**15** Chelsea—Game 1: 68%, Game 2: 70.4%
Sonia—Game 1: 75%, Game 2: 73.9%

Chelsea increased her percentage, so she improved her accuracy, whereas Sonia's accuracy dropped slightly.

**16** **(a)** 22% of 180 = 39.6, 34% of 180 = 61.2, 7% of 180 = 12.6

**(b)** Each of the answers is a decimal number, which is not possible when working with numbers of people (cannot have 0.6 of a student).

**(c)** Actual number with blue eyes could be either 39 or 40 (both give a percentage that rounds to 22 as the nearest whole number), actual number speaking two or more languages could be 61 or 62 (both give a percentage of 34 when rounded) and the actual number of left-handed students could be 12 or 13 (both give a percentage of 7 when rounded).

**17** **(a)** 2700 kJ **(b)** 73 g of fat

### Open-ended – Sample answers

**18** Savings look more attractive, and are more easily interpreted (e.g. 50% off is easily recognised as half price).

**19** 50%, $10; 25%, $20; 10%, $50

**20** Fedora's working is correct. Tanya has confused the method for finding a percentage with the method for writing one amount as a percentage of another. The question asked here has the percentage sign appearing in the question, which means that the first method (multiplying by the per cent) should be used. If the question contains the words 'out of', then the second method (writing one amount as a percentage of another) should be used.

## Exercise 4.9 (p. 230)

**1** **(a)** 5:3 **(b)** 4:7 **(c)** 3:2 **(d)** 8:3

**(e)** 1:2:3 **(f)** 2:9:2 **(g)** 3:4:7 **(h)** 3:2:1

**2** **(a)** 2:3 **(b)** 3:5 **(c)** $\frac{3}{5}$ **(d)** 40%

**3** **(a)** 2:6 **(b)** 2:16 **(c)** 2:22 **(d)** 2:66

**(e)** 12:15 **(f)** 12:28 **(g)** 2:3 **(h)** 2:3

**(i)** 5:4 **(j)** 2:1 **(k)** 56:35 **(l)** 60:55

**(m)** 8:3 **(n)** 100:30 **(o)** 20:18 **(p)** 75:65

**4** **(a)** 1:5 **(b)** 1:4 **(c)** 2:9 **(d)** 2:7

**(e)** 3:2 **(f)** 8:5 **(g)** 8:5 **(h)** 4:3

**(i)** 27:32 **(j)** 14:17 **(k)** 8:9 **(l)** 2:5

**(m)** 3:2 **(n)** 3:2 **(o)** 7:4 **(p)** 40:11

**5** **(a)** B **(b)** C

**6** **(a)** 12:7:16 **(b)** 35 **(c)** $\frac{12}{35}$ **(d)** 20%

**7** **(a)** 15:7:2 **(b)** $\frac{7}{24}$ **(c)** 29%

**8** **(a)** 5:4:3 **(b)** $\frac{5}{12}$ **(c)** 42%

**9** **(a)** 6:1 **(b)** 6:1 = 24:6, 6 instructors required

**(c)** 30

**10** 3:2:1 = 12:8:4, 8 buckets of sand, 12 buckets of gravel

**11** **(a)** 10% **(b)** 2 g

**12** **(a)** 25% **(b)** 75:25 = 3:1

**13** **(a)** B **(b)** F **(c)** C **(d)** E

**14** **(a)** 4 teachers are required—3 would only be enough for 60 students (1:20 = 3:60).

**(b)** 4:68 = 1:17

**15** **(a)** Chloe: 1:5
Hannah: 2:5

**(b)** Hannah is correct. Looking at the simplified ratios, Hannah has 1 extra part of concentrate for the same number of parts of water.

**16** **(a)** $\frac{5}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{4}{9}$

**(d)** 2 cats and 1 dog

### Open-ended – Sample answers

**17** **(a)** Answer will depend on the class.

**(b)** Unlikely, unless it is a single-sex school.

**(c)** No; there would be a big variation in all coeducational schools; some schools are mixed sex, others are single-sex.

**18** 8:14, 12:21, 16:28

**19** The ratio should be 7:12; the student forgot to count the apples as fruit.

## Mathspace (p. 234)

**1** Asian—61%, European—12%, North American—8%, South American & Caribbean—5%, African—13%, Oceania—1%. Live without basic sanitation—43%, don't have access to clean, safe drinking water—33.$\dot{3}$%, are hungry and/or malnourished—13%, own 59% of the entire wealth—6%, have 75% of the income—20%, live on $2.50 or less per day—53%, can't read—14%, have a secondary education—7%, have a computer —12%, have the internet—3%.

**2** 75%

**3** **(a)** 2 300 000 000

**(b)** 3 657 000 000

**(c)** 828 000 000

**(d)** 6 417 000 000

**4** **(a)** 71%

**(b)** Younger people are less likely to have families to support.

**(c)** 66.7%

**5** Sample answer: I hope that in 2055 everyone has basic sanitation and no one is malnourished. I hope that wealth is more evenly distributed and everyone has access to a good education.

## Exercise 4.10 (p. 238)

**1** **(a)** $5.60 **(b)** 104 cans **(c)** $1.20

**(d)** $8500 **(e)** 25 kg **(f)** 525 pages

**2** **(a)** 600 g: $0.68 per 100 g, 250 g: $1.07 per 100 g;
600 g is the better value

**(b)** 500 g: $0.56 per 100 g, 750 g: $0.46 per 100 g;
750 g is the better value

(c) 375 g: $0.34 per 100 g, 500 g: $0.65 per 100 g; 375 g is the better value

(d) 200 g: $1.61 per 100 g, 375 g: $1.31 per 100 g; 375 g is the better value

(e) 500 mL: $0.54 per 100 mL, 920 mL: $0.47 per 100 mL; 920 mL is the better value

(f) 450 g: $0.84 per 100 g, 1 kg: $0.88 per 100 g; 450 g is the better value

**3** A

**4** D

**5** 19.2, or 20 loaves

**6** (a) the 400 g jar

(b) Yes, the 280 g jar is now slightly better value for money, with both jars approximately $1.60 per 100 g.

**7** $6

**8** 7.1 runs per over

**9** (a) (i) packet of 6: $0.18 per bar, packet of 10: $0.75 per bar; packet of 10 is the better value

(ii) 12-pack: $1.18 per can, 18-pack: $1.12 per can, 30-pack: $0.92 per can; 30-pack is the better value

(iii) box of 50: $0.06 per bag, box of 200: $0.04 per bag, box of 200 is the better value

(b) Multiply the cost of the box of 50 by 4 to get the price of 200, or multiply the cost of the box of 50 by 2 and divide the cost of the box of 200 by 2, to get the prices per 100 bags.

**10** (a) $1.45 (b) $17.40

**11** (a) 3.6 L (b) 25.2 L

**12** (a) 3.25 g (b) 13 g (c) 16.25 g

**13** (a) $6.00 (b) $0.60 (c) $7.80

**14** Individual cost of each bun in the packet = $1.20
$4 \times \$1.20 = \$4.80$
$7.20 (1 packet) + $4.80 (4 loose) = $12.00

**15** (a) 4 kg is better value:
$15.97 ≈ $16
≈ $4/kg

(b) 1 L carton is better:
1 L = 4 × 250 mL
$4 \times \$1.25 = 2 \times \$2.50 = \$5$

(c) packet ham is better value:
500 g of ham at $13/kg
= $6.50

(d) 400 g jar is better value:
$7.99 ≈ $8
≈ $2/100 g, or $6 for 300 g

(e) The product may be Australian made, the packaging may be appealing, or you may prefer a particular brand.

**16** Plan: $4.10 is cheaper, Pre-paid: $4.25 is more expensive

**17** (a) $2516.13 kL/day

(b) 918 387.45 kL

(c) No—cannot assume that the rainfall in January will be the same for the other months of the year. January is part of the wet season, so other months will definitely be drier.

**18** (a) The 500 g block is two times bigger than the 250 g block.

(b) 500 g block: $9.00; 750 g block: $13.50

(c) The packaging of larger items is cheaper; 'bulk' buys are made more attractive so that consumers will purchase more of the product.

### Open-ended – Sample answers

**19** soft drink, milk, fruit juice, yoghurt

**20** (a) $9 \times 500$ g; $4 \times 1$ kg + $1 \times 500$ g; $2 \times 2$ kg + $1 \times 500$ g

(b) $2 \times 2$ kg + $1 \times 500$ g is the cheapest ($10.28 compared to $11.16 and $10.64)

## Challenge 4 (p. 241)

**1** B

**2** $8 \times 0.753$. Try various combinations. Hopefully, the students will only need to investigate $8 \times 0.753$ and $7 \times 0.853$.

**3** $0.01 - 0.001 = 0.009$

**4** C $\$\frac{3}{4} = \$0.75$, $\$\frac{3}{10} = \$0.30$. Add the amounts.

**5** C $200 \div 2.5 = 80$, so $20 \div 2.5 = 8$ and $20 \div 0.25 = 80$

**6** B Amount is $49\,095 - 49.95 = 49\,045.05$ to be subtracted.

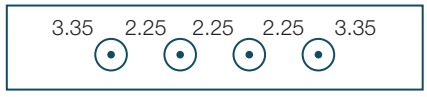
**7** (a) The last digit of both of the decimal numbers must not multiply together to give a number ending in 0, e.g. $3.21 \times 4.547 = 14.595\,87$.

(b) The last digit of both of the decimal numbers must multiply together to give a number ending in 0, e.g. $1.25 \times 3.766 = 4.7075$.

(c) the last two digits of both of the decimal numbers must multiply together to give a number ending in 00, e.g. $7.25 \times 8.324 = 60.349$.

**8** Scores a total of $6 \times 7.5 = 45$ goals. Least number of goals scored = 4, so she scores $45 - 4 = 41$ goals in 5 other games.
$41 = 4 \times 8 + 9$, so she must score at least 9 in one game.
$41 = 7 + 3 \times 8 + 10$, so she could score more than 9. The lowest value for the highest number of goals she scores is 9.

**9** Length of strip of metal = $2 \times 3.35 + 3 \times 2.25 = 13.45$ cm




**10** $0.01 \div 0.002 = 10 \div 2 = 5$

**11** C $0.9 \times \$30\,000 = \$27\,000$; $1.1 \times \$27\,000 = \$29\,700$

**12** A **13** B

## Chapter review 4 (p. 242)

**1** (a) $\frac{9}{10} + \frac{6}{100} + \frac{8}{1000}$ (b) $5 + \frac{7}{100} + \frac{2}{10\,000}$

(c) $6 + \frac{5}{1000}$

**2** (a) 0.6524 (b) 0.080 093

**3** (a) 4.3806 (b) 57.032 09

**4** (a) one one, eight tenths, five hundredths, three thousandths and one ten-thousandth

(b) seven hundredths and six hundred-thousandths

(c) six tens, one one and nine ten-thousandths

**5** (a) (i) $\frac{9}{100}$ (ii) nine hundredths

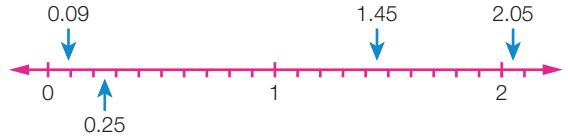
(b) (i) $\frac{9}{10\,000}$ (ii) nine ten-thousandths

(c) (i) $\frac{9}{100}$ (ii) nine hundredths

**6** (a) $3.0427 > 3.0274$ (b) $0.009\,95 < 0.01$

**7** (a) 0.5506, 0.6055, 0.607 (b) 0.071, 0.701, 0.71

**8**




**9** (a) 4.4 (b) 34.65 (c) 23.1 (d) 102.47

**10** (a) \$41.20 (b) \$10.05 (c) \$79.95 (d) \$99.00

**11** (a) $3\frac{9}{10}$ (b) $\frac{31}{50}$ (c) $2\frac{9}{20}$ (d) $\frac{9}{5000}$

**12** (a) 0.8 (b) 0.95 (c) $0.\dot{2}$ (d) $0.8\dot{3}$

**13** (a) 28.39 (b) 15.4585 (c) 41.433

(d) 7.717 (e) 1.688 (f) 4.7802

**14** (a) 13.504 (b) 104.65 (c) 200.97

(d) 47 200 (e) 164.4 (f) 3.45

(g) 0.54 (h) 0.000 36 (i) 19.76

**15** (a) 2.58 (b) 1.004 (c) 0.469

(d) 0.240 (e) 0.016 (f) 0.182

(g) 12 (h) 2.4 (i) 5950

**16** (a) 70% (b) 125% (c) 12.5% (d) $66.\dot{6}\%$

**17** (a) 38% (b) 96% (c) 355% (d) 9%

(e) 323.4% (f) 54.32%

**18** (a) \$24 (b) 12.75 L (c) 166 m (d) \$75

**19** (a) 65% (b) 50% (c) 35% (d) 93.3%

**20** (a) 8:7 (b) $\frac{7}{15}$ (c) 53%

**21** (a) \$7.05 (b) \$3.14

**22** (a) 1 326 000 000 (b) 1 140 000 000

(c) 228 200 000 (d) 4 270 000

**23** (a) \$3023.60 (b) \$7559.00

**24** (a) T-shirt discount \$3 Jeans discount \$24

(b) T-shirt \$27 Jeans \$36

**25** (a) The 400 g jar is better value.

(b) Yes, the 250 g jar is now better value than the 400 g jar.

(c) People might only require a small amount of a product—a larger amount might be wasted, some amounts are in a more convenient size (e.g. travel packs)

**26** (a) 75 mL (b) No; 60:40 = 3:2

**27** (a) 7.8 (b) 23.4 (c) 63.18 Yes

**28** (a) \$5.20 (b) \$0.40 (c) \$0.80

(d) Buy six 6-packs and four individual cans (total cost of \$32.28).

**29** 5 supervisors will be needed; 4 supervisors could only manage 48 children at the required ratio.

**30** Store A discount = \$16.25, sale price = \$48.75

Store B discount = \$22.50, sale price = \$52.50

Sally should buy her shoes at Store A to pay the lowest price.

## NAPLAN practice 4

**1** D **2** B **3** D **4** B

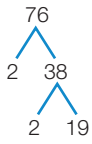
**5** \$11 **6** D **7** C **8** \$178.92

# Mixed review B (p. 246)

**1** (a) $\frac{3}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{4}$ (d) $\frac{4}{13}$

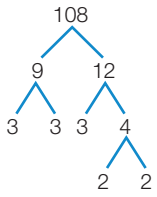
**2** (a) -74 (b) +3 (c) -215 (d) +8

**3** (a) (i)



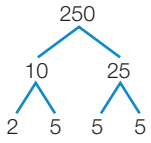

(ii) $2^2 \times 19$

(b) (i)




(ii) $2^2 \times 3^3$

(c) (i)




(ii) $2 \times 5^3$

**4** (a) 4800 (b) 3300 (c) 12 (d) 4

**5** (a) > (b) < (c) <

**6** (a) 0.27 (b) 15.3 (c) 435

**7** (a) -6 (b) 14 (c) -10 (d) -8

(e) 6 (f) -9

**8** (a) $\frac{8}{25}$ (b) $1\frac{6}{25}$ (c) $3\frac{237}{1000}$

**9** (a) $7^4$ (b) $4^2$ (c) $2^2 \times 3^3$ (d) $11^3$

**10** (a) 30 (b) 6 (c) 30

**11** (a) < (b) > (c) <

12 (a) 6 (b) 8 (c) 12

13 (a) $\frac{19}{20}$ (b) $4\frac{1}{24}$ (c) $\frac{11}{24}$ (d) $\frac{3}{4}$

14 (a) 203 (b) 484 (c) 195 (d) 65
(e) 280 (f) 55

15 (a) 4 m (b) $3

16 (a) (i) $\frac{3}{10}$ (ii) 30%

(b) (i) $\frac{3}{13}$ (ii) 23.1%

17 (a) 14 g (b) 1.4 g (c) 4.2 g

18 (a) 29 slices (b) 13 slices

19 (a) 3200 (b) 1200 (c) 400

20 $3^2 = 3 \times 3$, not $3 \times 2$

21 Twice—if the cycles start on the hour, they will flash at 24 and 48 past the hour.

22 (a) $\frac{5}{18}$

(b) Eamon (30 pieces compared to Chloe's 9 pieces).

23 (a) 84.5

(b) It is a decimal number, which is impossible when dealing with numbers of students.

(c) The actual number of students was either 84 or 85. Both numbers give a percentage of 65 when the answer is rounded to the nearest whole number.

24 8

25 69 cards

# Chapter 5

## Recall 5 (p. 250)

1 (a) 5 (b) 2 (c) 1 (d) 4 (e) 2 (f) $\pm 4$

2 (a) (i) 41 (ii) 107 (iii) 2197
(b) (i) 10 (ii) 94 (iii) 58

3 (a) $11 + 17 = 28$ (b) $9 - 7 = 2$
(c) $3 \times 4 = 12$ (d) $16 \div 8 = 2$

4 (a) $6 \times 2 - 4$; 8 (b) $(10 + 5) \div 5$; 3
(c) $(6 + 5) \times 3$; 33 (d) $(23 - 21) \times 12$; 24

5 (a) peregrine falcon (b) kodiak bear

## Exercise 5.1 (p. 253)

1 (a) $p - 3$ (b) $k + 9$ (c) $12v$
(d) $\frac{a}{4}$ (e) $d + e$

2 (a) $x + 2$ (b) $4y$ (c) $5 - t$
(d) $\frac{k}{7}$ (e) $6gh$ (f) $10d + 8$
(g) $10(d + 8)$ (h) $\frac{y + 9}{4}$ (i) $5e - 3$
(j) $\frac{r + s + t}{3} + 11$ (k) $y^2 + 20$ (l) $8x^2$

3 B

4 D

5 (a) $5(n + 1)$ (b) $\frac{2 + d}{9}$ (c) $12(a + b)$ (d) $mn - 3$

6 (a) $\frac{x}{2}$ (b) $\frac{x}{2} + 3$

7 (a) $\$6d$ (b) $\$2.5t$

8 $\$(55x + 300)$

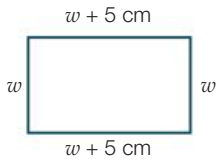
9 $\$bv$

10 (a) $\$12t$ (b) $\$24s$ (c) $\$(12t + 24s)$

11 (a) $5n$ (b) $10r$ (c) $5n + 10r$

12 (a) $m + n + q + 1$ (b) $2m + 2n + 2q + 2$
(c) pens in large packs: $2m + 2m$,
pens in small packs: $m + m + m$
total pens = $3m + 4m$ or $7m$

13 (a) (i) $(w + 5)$ cm (ii)




(b) $w + w + 5 + w + w + 5 = w + w + w + w + 10$ or $4w + 10$

### Open-ended – Sample answers

14 (a)




(b) A packet of lollies shared between 4 people:




15 (a) Grandma's age is $2(r + d)$.

(b) Possible ages are: Rory 10, dad 30, grandma 80; Rory 5, dad 25, grandma 60.

## Exercise 5.2 (p. 258)

1 (a) (i) expression (ii) 2
(iii) $x$ and $y$ (iv) -7
(v) $2x$, $3y$, -7 and $4x^2$

(b) (i) equation (ii) -4
(iii) $x$, $y$ and $z$ (iv) 32
(v) 32, $-4x$, $3xy$ and $4z$

(c) (i) equation (ii) -23
(iii) $x$ and $y$ (iv) 9
(v) $2xy$, $-3x^2$, 9 and $-23x$

(d) (i) expression (ii) -1
(iii) $x$ and $y$ (iv) 6
(v) 6, $-x$, $8y$ and $-4xy$

2 (a) (i) number of girls, number of boys
(ii) Let $x$ = number of boys; let $y$ = number of girls
(iii) $x = 3y$

(b) (i) Fred's age; James' age
(ii) Let $g$ = Fred's age; let $h$ = James' age
(iii) $g = 3h - 1$

(c) (i) cost of an apple, cost of a pear
(ii) Let $m$ = cost of an apple; let $n$ = cost of a pear
(iii) $2m = 3n$

(d) (i) cost of a kilogram of potatoes, cost of a watermelon
(ii) Let $a$ = cost of a kg of potatoes; let $b$ = cost of a watermelon
(iii) $3a + b = \$5.20$

(e) (i) distance to Ranko's house, distance to Skye's house
(ii) Let $r$ = distance to Ranko's house; let $s$ = distance to Skye's house
(iii) $s + 5 = r$

(f) (i) number of plants, number of pavers
(ii) Let $v$ = number of plants, let $w$ = number of pavers
(iii) $2v + 6 = w$

3 (a) True (b) False (c) False (d) True (e) False
(f) True (g) True (h) False (i) True (j) True

4 (a) B (b) D (c) B (d) C (e) C (f) C

5 (a) (i) Let $m$ = mass of 1 tablet
(ii) $25m + 120 = 195$

(b) (i) Let $a$ = Mei Lee's age now
(ii) $\frac{a}{2} = a - 8$

(c) (i) Let $B$ = Beths' age; let $D$ = dad's age
(ii) $B + 10 = \frac{D}{2}$

(d) (i) Let number of goals = $g$;
let number of penalty goals = $p$
(ii) $8g + p = 100$

6 (a) Let $d$ = distance travelled on first day.
(b) $2d$ (c) $d + 480$ (d) $d + 480 = 3d$

## Open-ended – Sample answers

7 (a) Number of goals scored is $g$ and number of behinds is $b$.
(b) $6g + b = 114$
(c) An example is 12 goals and 42 points.

8 (a) 4 tries, 1 penalty goal to 4 penalty goals. 2 converted tries, 3 penalty goals to 2 tries, 1 conversion goal.
(b) $P = 5t + 2g + 3p$

# Exercise 5.3 (p. 264)

1 (a) $x \rightarrow \boxed{+2} \rightarrow y$

$y = x + 2$

| $x$ | 13 | 11 | 7 | 28 | 1 |
|---|---|---|---|---|---|
| $y$ | 15 | 13 | 9 | 30 | 3 |

(b) $x \rightarrow \boxed{-5} \rightarrow y$

$y = x - 5$

| $x$ | 6 | 18 | 9 | 85 | 5 |
|---|---|---|---|---|---|
| $y$ | 1 | 13 | 4 | 80 | 0 |

(c) $x \rightarrow \boxed{\times 2} \rightarrow y$

$y = 2x$

| $x$ | 2 | 3.4 | 10 | 11 | 101 |
|---|---|---|---|---|---|
| $y$ | 4 | 6.8 | 20 | 22 | 202 |

(d) $x \rightarrow \boxed{\div 3} \rightarrow y$

$y = \frac{x}{3}$

| $x$ | 18 | 12 | 30 | 9 | 0 |
|---|---|---|---|---|---|
| $y$ | 6 | 4 | 10 | 3 | 0 |

(e) $x \rightarrow \boxed{\times 5} \rightarrow \boxed{-3} \rightarrow y$

$y = 5x - 3$

| $x$ | 4 | 2 | 0 | 5 | 20 |
|---|---|---|---|---|---|
| $y$ | 17 | 7 | -3 | 22 | 97 |

(f) $x \rightarrow \boxed{+5} \rightarrow \boxed{\div 10} \rightarrow y$

$y = \frac{x + 5}{10}$

| $x$ | 5 | 25 | 45 | 10 | 33 |
|---|---|---|---|---|---|
| $y$ | 1 | 3 | 5 | 1.5 | 3.8 |

(g) $x \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow y$

$y = \frac{x}{2} - 1$

| $x$ | 10 | 6 | 24 | 9 | 15 |
|---|---|---|---|---|---|
| $y$ | 4 | 2 | 11 | 3.5 | 6.5 |

(h) $x \rightarrow \boxed{\times x} \rightarrow y$

$y = x^2$

| $x$ | 3 | 11 | 7 | 6 | 10 |
|---|---|---|---|---|---|
| $y$ | 9 | 121 | 49 | 36 | 100 |

**2** (a) A (b) A (c) D

**3** (a) $y = 4x - 9$ (b) $y = 5(x + 11)$ or $y = (x + 11) \times 5$

(c) $b = \frac{a}{2} - 7$ (d) $b = \frac{a + 3}{6}$

(e) $q = p^2 + 8$ (f) $q = \sqrt{p}$

**4** (a) $y = x - 18$ (b) $y = 60x$

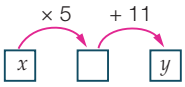
(c) $y = \frac{x}{7}$ (d) $y = 20(x + 43)$ or $y = (x + 43) \times 20$

(e) $y = 100x - 50$ (f) $y = \frac{x}{16} + 13$

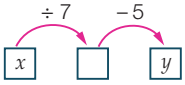
(g) $y = x^2$ (h) $y = \frac{x - 12}{9}$

(i) $y = x^2 - 37$

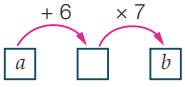
**5** (a)



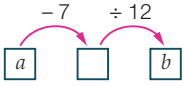

(b)



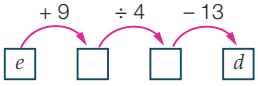

(c)



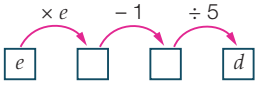

(d)




(e)



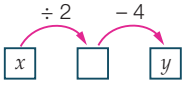

(f)







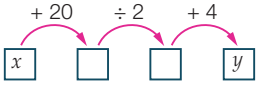
**6** (a)




(b) $y = \frac{x}{2} - 4$

(c) $y = \frac{20}{2} - 4 = 6$ lollies left

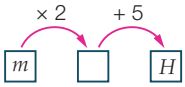
**7** (a)




(b) $y = \frac{x + 20}{2} + 4$ (c) $y = \frac{12 + 20}{2} + 4 = 16 + 4 = 20$

Anita spent $20.

**8** (a)




(b) $H = 2m + 5$ (c) $H = 2 \times 12 + 5 = 24 + 5 = 29$

**9** (a)

| Number of pairs of jeans, $n$ | 10 | 50 | 150 | 200 |
|---|---|---|---|---|
| Cost to produce the jeans, $C$ | $315 | $595 | $1295 | $1645 |

(b) $C = 7n + 245$

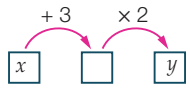
**10** (a)

| $x$ | 2 | 5 | 7 | 4 | 10 |
|---|---|---|---|---|---|
| $y$ | 5 | 11 | 15 | 9 | 21 |

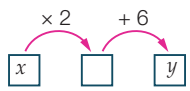
(b)

| $x$ | 10 | 1 | 2 | 4 | 18 |
|---|---|---|---|---|---|
| $y$ | 52 | 16 | 20 | 28 | 84 |

**11** (a) (i)




(ii)




(b) (i) $y = 2(x + 3)$ or $y = (x + 3) \times 2$

(ii) $y = 2x + 6$

(c) (i)

| $x$ | 2 | 5 | 3 | 4 | 7 |
|---|---|---|---|---|---|
| $y$ | 10 | 16 | 12 | 14 | 20 |

(ii)

| $x$ | 2 | 5 | 3 | 4 | 7 |
|---|---|---|---|---|---|
| $y$ | 10 | 16 | 12 | 14 | 20 |

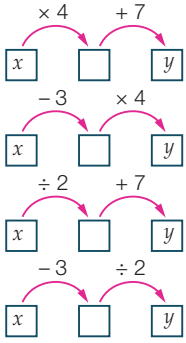
(d) The two tables of values are the same, which means the two rules are equivalent. In the first rule, both $x$ and 3 are doubled in the second step, which gives $2x + 6$.

## Open-ended – Sample answers

**12** One possible table for the rule $y = \frac{x}{2} + 3$ is:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3.5 | 4 | 3.5 | 5 | 5.5 |

**13** (a) Some possible two-step flowcharts are:




(b) The rules that correspond to the above flowcharts are:

$y = 4x + 7$

$y = 4(x - 3)$

$y = \frac{x}{2} + 7$

$y = \frac{x - 3}{2}$

**14** Kim's statement that they are 'different ways of writing the same thing' is incorrect. Both boys have the correct operations of ÷ 2 and + 3 in their flowcharts, but the order in which they are used is important. The rule states that $x$ is first divided by 2, then 3 is added, so Jai's flowchart is correct.

## Exercise 5.4 (p. 270)

**1** **(a)** 7 **(b)** $5\frac{1}{2}$ **(c)** -1 **(d)** 23

**(e)** 2.6 **(f)** 25 **(g)** 4 **(h)** 28

**(i)** 2 **(j)** 32 **(k)** 18 **(l)** 120

**(m)** 4 **(n)** 0.5 **(o)** 66 **(p)** 84

**2** **(a)** \$270 **(b)** 22.15

**(c)** 70 kilometres per hour

**3** **(a)** True **(b)** True **(c)** False

**(d)** True **(e)** False **(f)** True

**4** **(a)**

| $a$ | 11 | 20 | 5 | 9 | 50 |
|---|---|---|---|---|---|
| $b$ | 44 | 80 | 20 | 36 | 200 |

**(b)**

| $x$ | 6 | 4 | 10 | 20 | 101 |
|---|---|---|---|---|---|
| $y$ | 42 | 28 | 70 | 140 | 707 |

**(c)**

| $m$ | 1 | 2 | 10 | 6 | 5 |
|---|---|---|---|---|---|
| $n$ | 5 | 8 | 32 | 20 | 17 |

**(d)**

| $j$ | 2 | 5 | 11 | 10 | 100 |
|---|---|---|---|---|---|
| $k$ | 15 | 27 | 51 | 47 | 407 |

**(e)**

| $p$ | 11 | 15 | 10 | 20 | 100 |
|---|---|---|---|---|---|
| $q$ | 12 | 20 | 10 | 30 | 190 |

**(f)**

| $r$ | 1 | 2 | 3 | 0 | 200 |
|---|---|---|---|---|---|
| $s$ | 6 | 14 | 22 | -2 | 1598 |

**(g)**

| $u$ | 5 | 3 | 1 | 201 | 6 |
|---|---|---|---|---|---|
| $v$ | 16 | 8 | 0 | 800 | 20 |

**(h)**

| $m$ | 5 | 10 | 11 | 102 | 52 |
|---|---|---|---|---|---|
| $n$ | 9 | 24 | 27 | 300 | 150 |

**5** **(a)** D **(b)** C **(c)** C

**6** **(a)** $p = \frac{7}{100}d$ **(b)** $f = \frac{11}{100}d$

**(c)** $p = 28$ litres, $f = 44$ litres

**(d)** $C = 1.30y$ **(e)** \$20.80

**7** **(a)** $\$350 + 10n = p$ **(b)** \$500

**(c)** \$550

**8** **(a)** $D = 3l + 2.6$

**(b)** $D = 3 \times 3.25 + 2.6$
$= 9.75 + 2.60$
$= 12.35$

**9** **(a)** $s = 2n + 10$ **(b)** $b = \frac{n}{8}$

**(c)** Jasmine will need 90 sausages and 5 bowls of salad.

**10** Substituting $a = 4$ into $b = 3(a + 5)$ gives $b = 27$.

Substituting $a = 4$ into $b = 3a + 5$ gives $b = 17$.

In the first formula, the order of operations is add 5, then multiply by 3. In the second formula, the order of operations is multiply by 3, then add 5.

### Open-ended – Sample answers

**11** One possible formula is: $y = 3x + 5$

| $x$ | 7 | 20 | 13 | 101 |
|---|---|---|---|---|
| $y$ | 26 | 65 | 44 | 308 |

Another possible formula is: $y = 2(x + 4)$

| $x$ | 7 | 20 | 13 | 101 |
|---|---|---|---|---|
| $y$ | 22 | 48 | 34 | 210 |

**12** Tania has substituted correctly. Suri has forgotten that $7x$ means $7 \times x$. In the future, Suri could include this in the first step of her working.

## Exercise 5.5 (p. 276)

**1** **(a)**

| Number of squares ($s$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number of matches ($m$) | 4 | 7 | 10 | 13 | 16 | 24 | 28 | 32 |

**(b)** Let $s$ = number of squares, $m$ = number of matches.
$m = 4s$

**(c)** 400 matches are needed to make 100 squares.

**2** **(a)**

| $B$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $S$ | 4 | 7 | 10 | 13 | 16 |

**(b)** $S = 3B + 1$

**(c)** $B = 21$ gives $S = 64$

**3** **(a)**

| $T$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P$ | 3 | 5 | 7 | 9 | 11 |

**(b)** $P = 2T + 1$

**(c)** $T = 203$ gives $P = 407$
Clarence would need 407 pieces of wood.

**4** **(a)**

| Number of houses ($h$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Number of matches ($m$) | 6 | 11 | 16 | 21 | 26 | 31 | 36 | 41 |

**(b)** $m = 5h + 1$

**(c)** If $h = 20$

$m = 5 \times 20 + 1$
$= 101$

101 matches are needed to build 20 houses.

**5** **(a)**

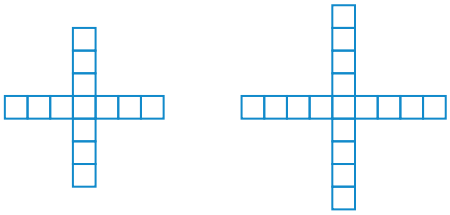
| Height of L ($H$) | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Number of globes ($G$) | 3 | 5 | 7 | 9 | 11 |

**(b)** $G = 2H - 1$

**(c)** $H = 120$ gives $G = 239$. So, 239 globes would be needed.

**6** B

**7 (a)**



**(b)**

| $a$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $b$ | 5 | 9 | 13 | 17 |

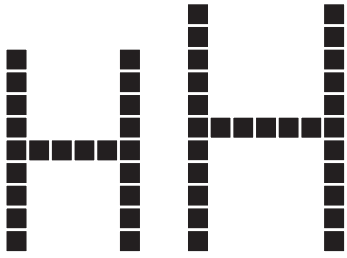
**(c)**

| $a$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $b$ | 5 | 9 | 13 | 17 | 21 | 25 |

**(d)** $b = 4a + 1$

**(e)** $a = 52$ gives $b = 209$
209 bricks would be needed.

**8 (a)**



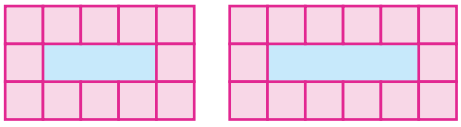
**(b)**

| Width of the H ($w$) | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| Number of tiles ($t$) | 7 | 12 | 17 | 22 | 27 |

**(c)** $t = 5w - 8$

**(d)** If $w = 12$, $t = 52$

**9 (a)**

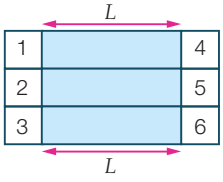


**(b)**

| $l$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p$ | 8 | 10 | 12 | 14 |

**(c)** $p = 2l + 6$

**(d)**




The constant is for the 3 pavers at each end of the pool. The coefficient of the pronumeral '2' is the number of pavers needed to increase the length each time.

**(e)** $p = 2(l + 3)$

**(f)** $l = 345$ gives $p = 696$
696 paving blocks are needed.

**(g)** The spa is 17 paving blocks long.

**10 (a)** **(i)** $m = 3s$ **(ii)** $m = 4s$

**(iii)** $m = 2s + 1$ **(iv)** $m = 3s + 1$

**(b)** In patterns **(i)** and **(ii)**, the number of matchsticks that are required to make each new shape is the same as the number of sides of the shape (3 for triangles and 4 for squares). In patterns **(iii)** and **(iv)** 1 less matchstick is required, because the new triangle or square joins on to an existing shape. This is shown in the formula of $2s + 1$ and $3s + 1$ instead of $3s$ and $4s$.

**11 (a)** 16

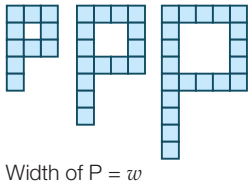
**(b)** 25

**(c)**

| Large triangle | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of triangles | 1 | 4 | 9 | 16 | 25 | 36 |

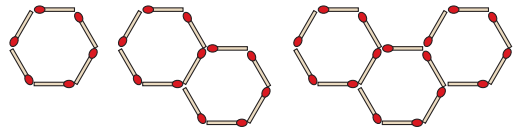
## Open-ended – Sample answers

**12**

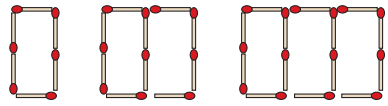


Width of P = $w$
Number of squares = $s$
$s = 5w - 5$

**13 (a)**



**(b)**



## Half-time 5 (p. 283)

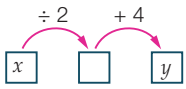
**1** B

**2 (a)** 2 **(b)** -40 **(c)** 81

**3** A

**4 (a)** $p + 7 = 28$ **(b)** $7x - 4 = 20$ **(c)** $w + g - 4 = 14$

**5 (a)**




**(b)** $y = 4 + \frac{x}{2}$

**(c)**

| $x$ | 2 | 8 | 10 | 5 | 13 |
|---|---|---|---|---|---|
| $y$ | 5 | 8 | 9 | 6.5 | 10.5 |

**6** $6n + 11$

**7** D

**8 (a)** $C = 35m + 50$ (cost in cents)

**(b)** 260 cents = $2.60

**(c)** When $n = 20$, $C = 750$ ($7.50), so yes, he has enough money.

## Exercise 5.6 (p. 287)

**1 (a)** Yes **(b)** No **(c)** No **(d)** Yes **(e)** No

**(f)** Yes **(g)** Yes **(h)** Yes **(i)** No

**2** (a) $7x$ (b) $16y$ (c) $13a$ (d) $4y$

(e) $4m$ (f) $-11x$ (g) $5mn$ (h) $5xy$

(i) $z + 4$ (j) $7x + 4y$ (k) $14w + 3z$ (l) $12a + 2b$

**3** (a) $10e - 2f$ (b) $8p - 4q$ (c) $-8t + 12s$

(d) $9x - 4y + y^2$ (e) $15w - 2z - 3z^2$ (f) $10m + 8n - 5m^2$

(g) $7a + 9$ (h) $8x + 5$ (i) $20y - 53$

**4** (a) A (b) D

**5** (a) $13e + 14f + 6$ (b) $18x + 2y + 2$

(c) $1 + 6x + 5y$ (d) $2x - 3y + 4$

(e) $9m - 2n + 3mn$ (f) $6a + 11b - 5c - 7$

**6** (a) Rule 1: $4x + 6x$
Rule 2: $10x$
True

(b) Rule 1: $15y - 10y$
Rule 2: $5y$
True

**7** (a) (i) $2x$ (ii) $4x$ (iii) $32x$

(b) $x + 2x + 4x + 8x + 16x + 32x = 63x$

(c) $\$63xv$

(d) $63 \times 6 = 378$. $x = 3$, $v = 2$

**8** (a) $\$27d$ (b) $d + 3d + 9d + 27d = \$40d$

(c) \$320 (d) $81d = 567$, $d = 7$. She received \$7.

**9** C

## Open-ended – Sample answers

**10** (a) $6x + 11y$ (b) $15x + 25y$

(c) $x$ and $y$ are either both even or both odd; e.g. $x = 1$, $y = 3$: $15x + 25y = 90$; $x = 2$, $y = 4$: $15x + 25y = 130$.

(d) Students' own answers.

**11** (a) $N = 4L$ (b) $N = 2L + 2W$

(c) Square garden bed of length 4 sleepers. Rectangular garden bed length 8 sleepers, width 7 sleepers.

(d) $L = 2W$, $N = 2 \times 2W + 2W = 6W$

(e) $10 \times 5$, $8 \times 4$, $6 \times 3$

# Exercise 5.7 (p. 294)

**1** (a) $A(4, 3)$, $B(1, -3)$, $C(-4, 5)$, $D(-4, -4)$, $E(-5, 0)$, $F(0, 0)$, $G(2, 0)$, $H(0, 4)$

(b) $A$ quadrant 1, $C$ quadrant 2, $D$ quadrant 3, $B$ quadrant 4, $E$–$H$ not in a quadrant

**2** (a) (i) quadrant 2 (ii) quadrant 4 (iii) quadrant 3

(iv) quadrant 1 (v) quadrant 2 (vi) quadrant 4

(vii) quadrant 3 (viii) quadrant 2

(b) (i) $y$-axis (ii) $y$-axis (iii) both (iv) $x$-axis (v) $x$-axis

**3** (3, -3), (-2, -2), (-3, 0), (1, -1), (3, 1), (-1, 1), (-4, 3), (2, 4), (5, 2)

**4** (a) C (b) A (c) B

**5** (a) D (b) D (c) C

**6** (a) C (b) D

**7** The result is a butterfly.

**8** The result is a castle.

**9** (a) C (b) A

**10** (a) (4, 6) (b) (2, 5) (c) (2, 1) (d) (3, 3)

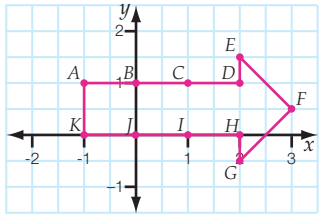
(e) (5, 4) and (6, 4) (f) (6, 1), (6, 2), (7, 2), (7, 1)

(g) (1, 4) (h) (7, 6)

**11** Join (7, 3) (1, 6) (-5, 3) (7, 3) STOP
Join (-4, 3) (-4, -3) (6, -3) (6, 3) STOP
Join (0, -3) (0, -1) (1, -1) (1, -3) STOP
Join (-3, 2) (-1, 2) (-1, 1) (-3, 1) (-3, 2) STOP
Join (-2, 1) (-2, 2) STOP
Join (4, 1) (4, 2) (2, 2) (2, 1) (4, 1) STOP
Join (3, 1) (3, 2) STOP
Join (4, -2) (4, -1) (2, -1) (2, -2) (4, -2) STOP
Join (3, -2) (3, -1) STOP
Join (-1, -2) (-1, -1) (-3, -1) (-3, -2) (-1, -2) STOP
Join (-2, -2) (-2, -1) STOP

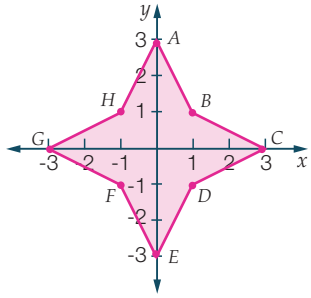
## Open-ended – Sample answers

**12**




$A$ (-1, 1), $B$ (0, 1), $C$ (1, 1), $D$ (2, 1), $E$ (2, 1.5), $F$ (3, 0.5), $G$ (2, -0.5), $H$ (2, 0), $I$ (1, 0), $J$ (0 ,0), $K$ (-1, 0)

**13** (a)

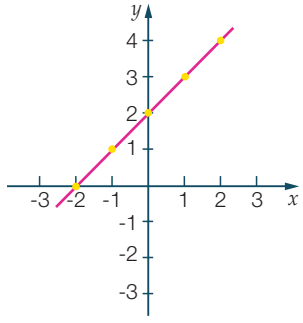



(b) Join the following points with straight lines.
$A(0, 3)$, B(1, 1), $C(3, 0)$, $D(1, -1)$, $E(0, -3)$, $F(-1, -1)$, $G(-3, 0)$, $H(-1, 1)$

(c) Colour the inside of the shape. Student to give to another student.

# Exercise 5.8 (p. 306)

**1** (a,b)



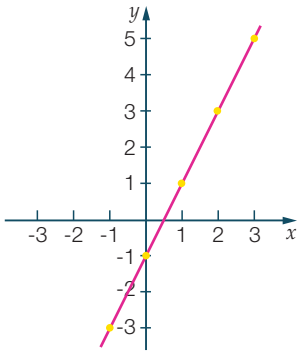

(c)

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 | 4 |

(d) $y = x + 2$

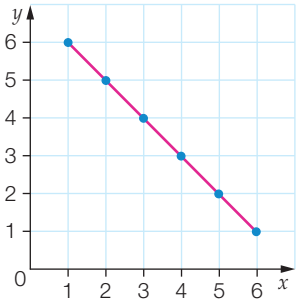
**2** **(a)** (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)

**(b,c)**




**(d)** $y = 2x - 1$

**3** **(a,b)**



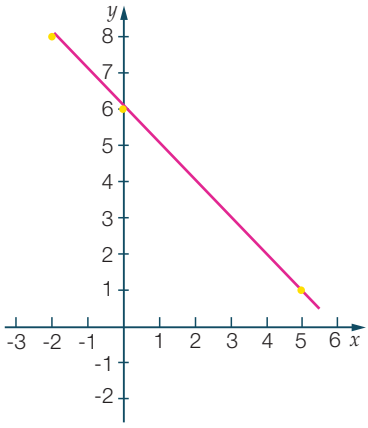

**(c)**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 6 | 5 | 4 | 3 | 2 | 1 |

**(d)** $y = 7 - x$

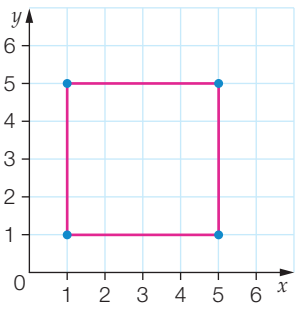
**4** C

**5** **(a,b)**



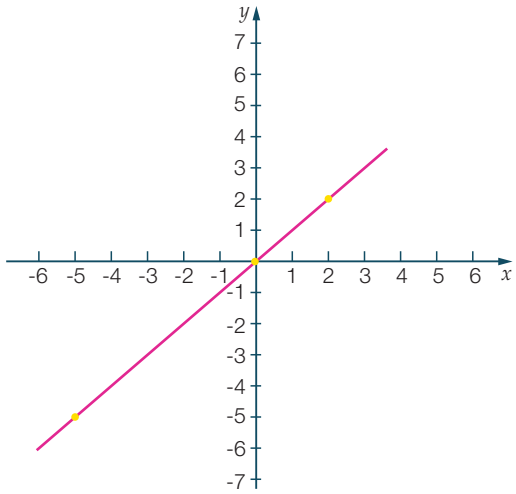

**(c)** $y = 7$ **(d)** $x = 1$

**6** **(a)**




**(b)** square

**7** **(a,b)**




**(c)** Any three of: (1, 1), (3, 3), (-1, -1), (-2, -2), (-3, -3), (-4, -4),

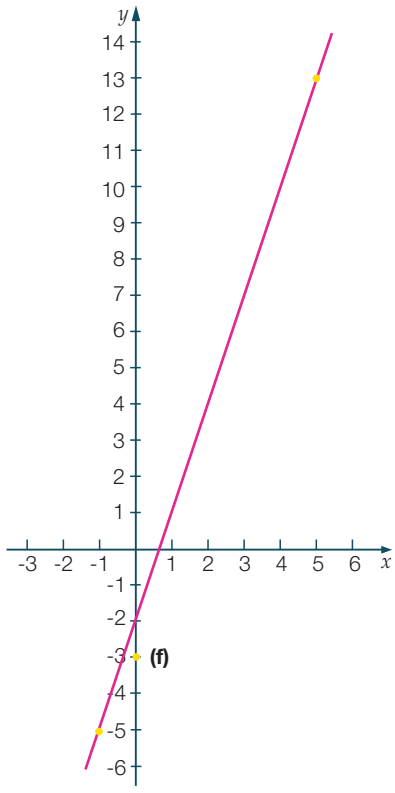
**(d)**

| $x$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |

**(e)** $y = x$

**(f)** When $x = -3$, $y = -3$, therefore (-3, -3) does lie on the line.

**8** **(a)**




**(b)** (2, 4), (3, 7), (4, 10)

**(c)**

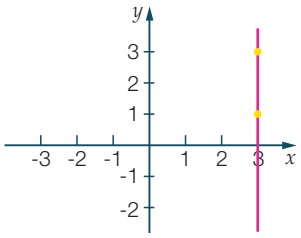
| $x$ | -1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | -5 | 4 | 7 | 10 | 13 |

**(d)** $y = 3x - 2$

**(e)** When $x = 0$, $y = -2 \neq -3$, therefore (0, -3) does not lie on the line.

## Open-ended – Sample answers

**9** **(a,b)**

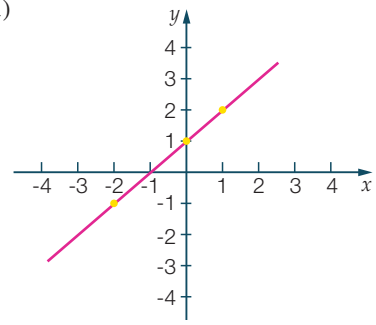



**(c)** (3, -1) and (3, 2)

**(d)** $x = 3$

**(e)** (0, -3), (1, -1) and (2, 1); $y = 2x - 3$

**10** **(a)** (0, 1)

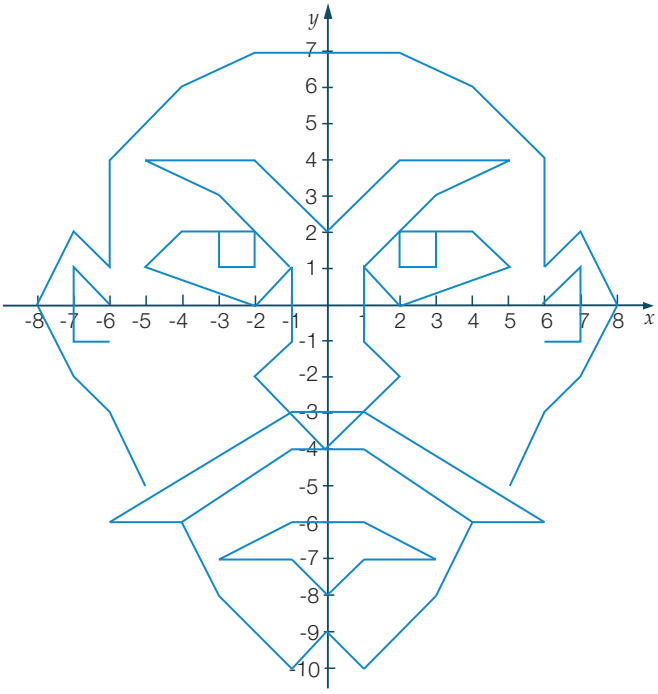



**(b)** (-2, -1) and (1, 2)

**(c)** $y = x + 1$

## Mathspace (p. 304)

Task 1




Task 2

Guard B

## Exercise 5.9 (p. 308)

**1** **(a)** False **(b)** True **(c)** True **(d)** True

**2** **(a)** (0, 0) **(b)** 12 km **(c)** $\frac{1}{2}$ h **(d)** 1 h

**(e)** 6 km **(f)** $\frac{1}{2}$ h

**(g)** They were away for 3 hours.

**(h)** They rode the fastest from Jim to Terry's house.

**3** **(a)** Yiannis

**(b)** Meg

**(c)** Yiannis and Alex take the same shoe size.

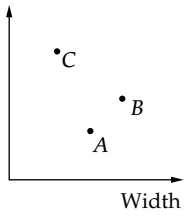
**(d)** Meg and Alex are the same age.

**4** *A*: elephant; *B*: giraffe; *C*: kangaroo

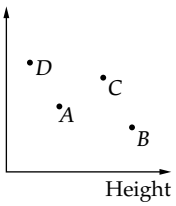
**5** D

**6** **(a)** *A*: salmon; *B*: John Dory; *C*: pike

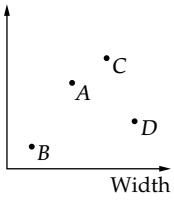
**(b)** Length



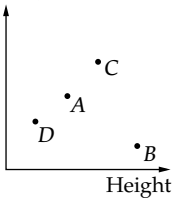

**7** **(a)** Width




**(b)** Capacity




**(c)** Capacity







**8** **(a)** 57%

**(b)** It rained heavily.

**(c)** It usually rains a lot during winter in Melbourne. Melting snow also increases water storage levels.

**(d)** Melbourne's rainfall was well below average—very little rain.

**(e)** June–July 2007, ≈ 28%

**9** **(a)** Yes, although the start of 2009 was a bit lower than the others. Highest ≈ 39%, lowest ≈ 35%, so difference ≈ 4%.

**(b)** 54 – 38 = 16% increase

**(c)** Oct–Dec 2010

**(d)** Jun–Aug 2009

**10** **(a)** Yes, the covers reduce water temperature, so evaporation would also be reduced.

**(b)** The suspended cover seems to be slightly better (≈ 200 days).

**(c)** in winter

**(d)** They reduce water temperature by ≈ 10°C.

**11** **(a)** height and shoe size

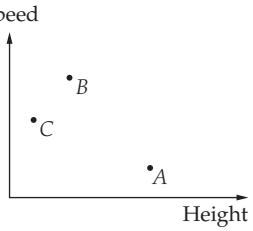
**(b)** The greater the height, the larger the shoe size.

### Open-ended – Sample answers

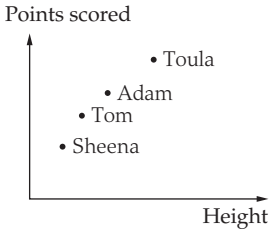
**12** **(a)** The first graph shows that *C* has moderate speed and low height, whereas the second graph shows *C* to have low speed and moderate height. A simple swap of *A* and *B* can only happen when the coordinates of *A* are opposite to the coordinates of *B*.

**(b)** Its height must be the same as its speed.

**(c)**




**13** Students' own answers. For example:




**14** Student's own answers.

## Challenge 5 (p. 313)

**1** Trial and error. As 8 + 7 = 15, start with these numbers as they are nearly the same.

$8 \times 50c = \$4$, $7 \times 20c = \$1.40$. Value = \$5.40.
Reverse numbers: $7 \times 50c = \$3.50$, $8 \times 20c = \$1.60$.

Value = \$5.10. This is 30c less, so, to have more, you need fewer 50c pieces.

$6 \times 50c = \$3$, $9 \times 20c = \$1.80$. Value = \$4.80.
Reverse numbers: $9 \times 50c = \$4.50$, $6 \times 20c = \$1.20$.

Value = \$5.70. This is 90c more, need fewer 50c pieces.

$4 \times 50c = \$2$, $11 \times 20c = \$2.20$. Value = \$4.20.
Reverse numbers: $11 \times 50c = \$5.50$, $4 \times 20c = 80c$.

Value = \$6.30. This is \$2.10 more, as required.

George has four 50c pieces.

**2** $2 ✷ 8 = 3(2 + 8) = 3 \times 10 = 30$

$5 ✷ (2 ✷ 8) = 5 ✷ 30 = 3(5 + 30) = 3 \times 35 = 105$

**3** As house numbers go up by 2 and are even on one side of the street and odd on the other, you have to find two numbers that differ by two and multiply to give 483.
As the product is odd, then each of the numbers is odd.
As $400 = 20 \times 20$, try the next two odd numbers.
$21 \times 23 = 483$. The house numbers are 21 and 23.

**4** If $x > 4$, then $\frac{x}{4} > 1$ and $\frac{x+1}{4} > 1$

As $x + 1 > x$, then $1 < \frac{x}{4} < \frac{x+1}{4}$.

The reverse applies with $x$ in the denominator. If $x > 4$, then $\frac{4}{x} < 1$ and $\frac{4}{x+1} < 1$. As $x + 1 > x$, then the fraction with denominator $x$ is larger than the fraction with denominator $x + 1$, so $\frac{4}{x+1} < \frac{4}{x} < 1$.

Hence, $\frac{4}{x+1} < \frac{4}{x} < 1 < \frac{x}{4} < \frac{x+1}{4}$ and the order is $\frac{4}{x+1}, \frac{4}{x}, \frac{x}{4}, \frac{x+1}{4}$.

**5** B If M = 1, then L = N and K = P, which is not possible as the letters are different numbers.

If L = 1, then M = N, not possible.

N can't be 1 as none of the products of the pairs of numbers ends in 1.

P > 1 as K × M > 1. Hence, K = 1.

L, M, and N can't equal 5. If L = 5 or M = 5, then N = 0 or 5. Hence, P = 5.

Using 2, 3 and 4 the only multiplication is $3 \times 4 = 12$.

Hence, N = 2 and P = 5 = 1 + 4, so M = 4, L = 3.

**6** $a = 1$, and $b$ can be any number.
(Note: zero is neither positive nor negative.)

**7** **(a)** 15 correct means 15 wrong. Marks = $15 \times 9 - 15 \times 5 = 60$

**(b)** 14 correct means 16 wrong, Marks = $14 \times 9 - 16 \times 5 = 46$ is the most.

**(c)** By trial and error getting more wrong than correct.

12 correct, 18 wrong:
Marks = $12 \times 9 - 18 \times 5 = 108 - 90 = 18$

11 correct, 19 wrong:
Marks = $11 \times 9 - 19 \times 5 = 99 - 95 = 4$

10 correct, 20 wrong:
Marks = $10 \times 9 - 20 \times 5 = 90 - 100$ = less than zero (-10)

It is not possible to score zero on the test.

For better students you could discuss the following.

$x$ correct means $(30 - x)$ wrong, and the
marks = $9 \times x - 5 \times (30 - x)$, = $14x - 150$

As 14 is not a factor of 150 you can never score 0.

8 The important thing to realise here is that the numbers 1 and 8 have only one number next to them, so they should be put in the middle two squares.

| | | | |
|---|---|---|---|
| | 5 | 3 | |
| 7 | 1 | 8 | 2 |
| | 4 | 6 | |

9 If half of $x$ is 24, then $x$ is 48, so twice $x$ is 96.

10 B $90 \div 3 = 30$, take integers either side.

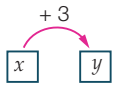
11 This is based on the result $2 \times 5 = 10$, so one of the numbers has only factors of 2 and the other only factors of 5. As there are six zeroes, the numbers are $2^6$ and $5^6$; that is, 64 and 15 625.

## Chapter review 5 (p. 314)

1 (a) $12n$ (b) $\frac{p}{2} - 3$ (c) $7p - 9$ (d) $\frac{11 - g}{5}$

2 (a) Joe's age is represented as $J$; Beth's age is represented as $B$; $J = 3B + 4$

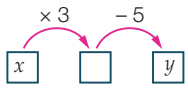
(b) Cost of a bottle of juice = $j$(\$); cost of a salad roll = $r$(\$). $3j + 5r = 18.60$

3 (a) (i) 


(ii) $y = x + 3$

(iii)

| $x$ | 57 | 34 | 12 | 4 | 1.1 | 64 |
|---|---|---|---|---|---|---|
| $y$ | 60 | 37 | 15 | 7 | 4.1 | 67 |

(b) (i) 


(ii) $y = 3x - 5$

(iii)

| $x$ | 4 | 3 | $2\frac{1}{2}$ | 9 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | 7 | 4 | $2\frac{1}{2}$ | 22 | 25 | 31 |

4 (a) B (b) C

5 (a) (i) -54 (ii) 26

(b) (i) 7 (ii) -7

6 C

7 (a)

| $x$ | 7 | 9 | 12 | 20 | 8.1 | 107 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 2 | 5 | 13 | 1.1 | 100 |

(b)

| $x$ | 4 | $5\frac{1}{2}$ | 13 | 7 | 78 | 54 |
|---|---|---|---|---|---|---|
| $y$ | 10 | 25 | 100 | 40 | 750 | 510 |

8 (a)

| No. triangles ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| No. girders ($g$) | 3 | 5 | 7 | 9 | 11 |
| No. bolts ($b$) | 3 | 4 | 5 | 6 | 7 |

(b) $g = 2t + 1$

(c) $b = t + 2$

(d) (i) 2

(ii) In the table of values, as the number of triangles increases by 1, the number of girders increases by 2. In the rule, 2 is the coefficient of $t$.

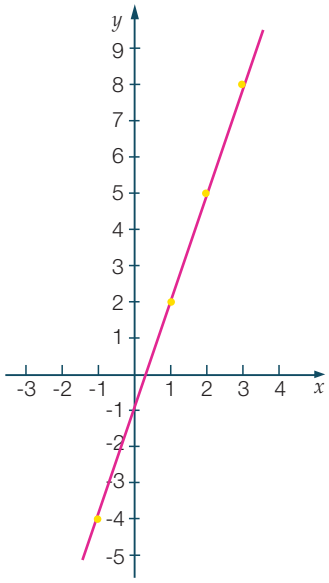
(e) (i) 1

(ii) In the table of values, as the number of triangles increases by 1, so does the number of bolts. In the rule, the coefficient of $t$ is 1.

9 (a) $5a$ (b) $-a + 23b$ (c) $4x + y$

10 (a) (2, 3) (b) (-2, 1) (c) (3, -2) (d) (-4, -3)

11 (a)




(b)

| $x$ | -1 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | -4 | 2 | 5 | 8 |

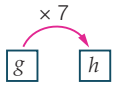
(c) $y = 3x - 1$

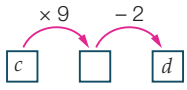
12 (a) \$3.50

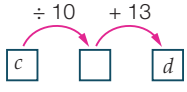
(b) 3 L is the best value because it costs less per litre than the other sizes (\$1.67 compared to \$1.75 and \$2.00).

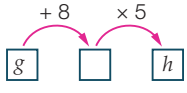
(c) This information is given as a point graph to enable comparison of prices of different sizes.

13 (a) $8n + 3$ (b) $\frac{8n + 3}{15}$ (c) $n = 9$

14 (a) 


(b) 


(c) 


(d) 


15 (a) $b = \frac{n}{4} + 5$ (b) 9

16 (a) $5a - 7b + 2$ (b) $5a + 9b - 7$ (c) $x + y + xy$

**17** B

**18** (a)

| $x$ | 10 | -35 | 100 | 5 | 0 | -30 |
|---|---|---|---|---|---|---|
| $y$ | 2 | -7 | 20 | 1 | 0 | -6 |

(b)

| $x$ | 6 | 10 | 3 | -1 | 0 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 72 | 120 | 36 | -12 | 0 | 60 |

**19** (a) $y = x - 7$ (b) $y = 2x + 4$

**20** (a) $1.4c + 1.8d = 21$

(b) 7 apples and 6 oranges

(c) $6.80

**21** (a) 1 500 000 ML

(b) Aug 98

(c) Jan 07

(d) It rained heavily.

(e) There was very little rain—well below average.

(f) from April 2004 to April 2007

### NAPLAN practice 5

**1** 7

**2** B

**3** (a) 24 (b) -6

**4** (a) 2, 1, 3 (b) 3, 1, 2 or 2, 3, 1

**5** $11.00

**6** D

# Chapter 6

## Recall 6 (p. 322)

**1** (a) 16.5 (b) 40.32 (c) 39.26

(d) 25.2 (e) 56.12 (f) 18 960

(g) 27.8 (h) 0.3 (i) 0.0459

**2** (a) (i) 13 mm (ii) 1.3 cm

(b) (i) 28 mm (ii) 2.8 cm

**3** (a) 16 cm (b) 12 $cm^2$

**4** (a) 2000 m (b) 3 cm (c) 500 cm

(d) 1.5 kg (e) 4000 mL (f) 20 000 g

(g) 0.25 L (h) 1750 mm (i) 0.5 kg

**5** 12 cubes

## Exercise 6.1 (p. 325)

**1** (a) 4 m (b) 5–6 m (c) 8–10 m (d) 20 mm

**2** (a) 5000 m (b) 3600 m (c) 8 m

(d) 6500 cm (e) 55 cm (f) 120 cm

(g) 29 mm (h) 61 mm (i) 3.5 mm

(j) 0.9 m (k) 0.003 m (l) 37.5 m

(m) 4.2 km (n) 0.57 km (o) 0.0092 km

(p) 8 cm (q) 25.5 cm (r) 0.18 cm

**3** (a) 3200 mm (b) 4950 mm (c) 0.09 km

(d) 0.345 km (e) 300 cm (f) 0.56 m

(g) 0.097 m (h) 342 000 mm (i) 0.0078 km

(j) 192 000 cm (k) 0.024 km (l) 890 mm

**4** (a) cm (b) km (c) m

(d) cm (e) km (f) m

**5** (a) B (b) C (c) B (d) C

**6** (a) 4 cm, 0.4 m, 4000 mm, 0.04 km

(b) 360 m, 290 000 cm, 3100 m, 3.2 km

(c) 5600 cm, 71.4 m, 0.64 km, 820 000 mm

(d) 0.9 cm, 90 mm, 0.095 m, 0.0089 km

**7** 5.96 m

**8** 2.745 km

**9** about 4.7 m

**10** length = 8.9 cm, leg span = 25.4 cm

**11** 2.29 m across, 36 m long

**12** 596 mm, 59.6 cm

**13** (a) 1100 km (b) 2500 km (c) 3700 km

**14** 2.5 m

**15** (a) (i) The lines are the same length.

(ii) The central circles are the same size.

(iii) Yes, the hat is as wide as it is tall.

(b) (i) The external arrows make the lines appear longer.

(ii) Being surrounded by bigger dots makes the central dot appear smaller.

(iii) The vertical distance appears bigger by contrast.

### Open-ended – Sample answers

**16** Decimal points are not needed and conversions are not necessary.

**17** (a) used division instead of multiplication; 2700 cm

(b) divided by 100 instead of 10; 76.5 cm

(c) zero added between 3 and 8; 3.8 km

(d) divided by 1000 instead of 100; 13.56 m

**18** Inaccurate measuring with the ruler. The student with the longer length could be including the ends of the ruler before 0 and after 30.

## Exercise 6.2 (p. 331)

**1** (a) 16 cm (b) 22 cm (c) 28 m

(d) 86 mm (e) 100 m (f) 60 m

**2** (a) 42 m (b) 40 cm (c) 24 cm

(d) 120 mm (e) 14.4 m (f) 68 mm

**3** (a) C (b) C

**4** (a) 140 mm (b) 122 mm

(c) 390 cm (d) 6000 m or 6 km

**5** 8.1 km

**6** 648 m

7 $384

8 Students' own answers.

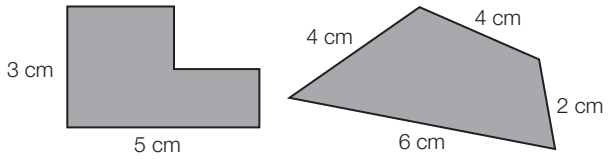
9 A

10 82 m

### Open-ended – Sample answers

11 21 cm and 21 cm; 22 cm and 20 cm; 30 cm and 12 cm

12




## Exercise 6.3 (p. 335)

1 (a) 5 $cm^2$ (b) 15 $cm^2$ (c) 4 $cm^2$
(d) 28.2 $cm^2$ (e) 24 $cm^2$ (f) 36 $cm^2$
(g) 36 $m^2$ (h) 87 $mm^2$ (i) 80 $km^2$
(j) 240 $m^2$ (k) 1.44 $m^2$ (l) 9 $cm^2$

2 (a) $cm^2$ (b) $mm^2$ (c) $cm^2$ (d) $m^2$
(e) $m^2$ (f) $km^2$ (g) $mm^2$ (h) $m^2$

3 (a) C (b) D

4 (a) (i) 20 cm (ii) 9 $cm^2$
(b) (i) 18 cm (ii) 8 $cm^2$
(c) (i) 24 cm (ii) 14 $cm^2$
(d) (i) 32 cm (ii) 20 $cm^2$

5 (a) (i) 28 km (ii) 45 $km^2$
(b) (i) 36 m (ii) 65 $m^2$
(c) (i) 42 cm (ii) 80 $cm^2$
(d) (i) 58 cm (ii) 100 $cm^2$

6 2.4 $km^2$

7 3150 $cm^2$

8 4 cm

9 5.2 m

10 (a) 20 $cm^2$ (b) 36 $cm^2$

11 (a) 5.04 $m^2$ (b) 30.24 $m^2$ (c) 196.56 kW h

12 (a) 3519 grams (b) 30.8 m

13 525 bricks

14 4418 $cm^2$

15 (a) 330 000 $m^2$ (b) 19 800 kg (c) 3350 m

16 (a) 8 $cm^2$ (b) 8 $cm^2$
(c) 12 $cm^2$ (d) 14 $cm^2$ (approx.)

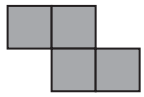
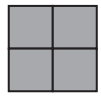
17 Approximate answers are:
(a) 131 $m^2$ (b) 45 $km^2$

18 Possible answers: 5 cm by 4 cm, 10 cm by 2 cm; 20 cm by 1 cm

19 12 m long, 5 m wide

### Open-ended – Sample answers

20 Ethan's shape Magda's shape

21 Possible dimensions: 1 m by 2.4 m, perimeter = 6.8 m; 1.2 m by 2 m, perimeter = 6.4 m.

22 (a) Panel 2: rectangle's perimeter is 16 m.
(b) Perimeter is the length around a shape, whereas area is the space inside a flat shape.
(c) The 'little 2' means 'squared'. Area is measured in squares.

23 wood, carpet, turf (lawn)

## Half-time 6 (p. 341)

1 (a) (i) 19 m (ii) 21 $m^2$
(b) (i) 14 m (ii) 12.25 $m^2$

2 (a) 0.45 m (b) 15 m (c) 0.307 km
(d) 60 000 cm (e) 196 km (f) 0.0264 km

3 (a) 11.7 kg (b) 16.4 m

4 (a) $m^2$ (b) $mm^2$ (c) $cm^2$ (d) $km^2$

5 length 10 cm, width 3 cm

6 (a) 457.5 m (b) 465.125 $m^2$ (c) 310.1 $m^2$

## Exercise 6.4 (p. 346)

1 (a) 65 $cm^2$ (b) 60 $m^2$ (c) 176 $mm^2$
(d) 350 $mm^2$ (e) 144 $m^2$ (f) 80 $cm^2$

2 (a) 132 $cm^2$ (b) 64 cm

3 (a) B (b) C

4 7.5 cm

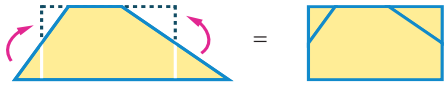
5 20 cm

6 1040 $m^2$

7 (a) 8 (b) 442 $cm^2$

8 (a) 20 $m^2$ (b) same amount of shade

9 (a)



(b) 15 $cm^2$

### Open-ended – Sample answers

10 Possible dimensions: base 8 cm, height 4 cm; base 16 cm, height 2 cm; base 1 cm, height 32 cm

11 (a)



(b) base 4 cm, height 2 cm gives total area of 32 $cm^2$; base 7 cm, height 3 cm gives total area of 84 $cm^2$

## Exercise 6.5 (p. 351)

**1** (a) 48 $cm^2$ (b) 35 $mm^2$ (c) 42 $m^2$
(d) 88 $mm^2$ (e) 180 $m^2$ (f) 60 $cm^2$
(g) 9 $m^2$ (h) 19.2 $mm^2$ (i) 33.88 $cm^2$

**2** (a) 19 $cm^2$ (b) 36 $cm^2$ (c) 85 $cm^2$
(d) 87 $cm^2$ (e) 281.65 $cm^2$ (f) 51 $cm^2$
(g) 279 $cm^2$ (h) 126 $cm^2$

**3** (a) A (b) D

**4** 4.76 $m^2$

**5** 1600 $cm^2$

**6** 3

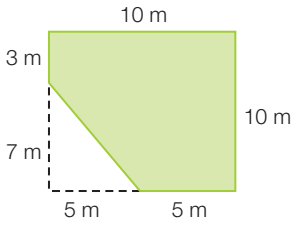
**7** 172 $cm^2$

**8** (a) 144 $cm^2$ (b) 115.5 $cm^2$ (c) 320 $cm^2$ (d) 105.5 $cm^2$

**9** (a) 10.36 $m^2$ (b) 4.82 m (c) 1.64 L

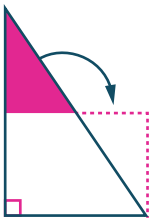
**10** Find the area of the big square and subtract the area of the small triangle.

$A = (10 \times 10) - (\frac{1}{2} \times 5 \times 7)$
$= 100 - 17.5$
$= 82.5\ m^2$



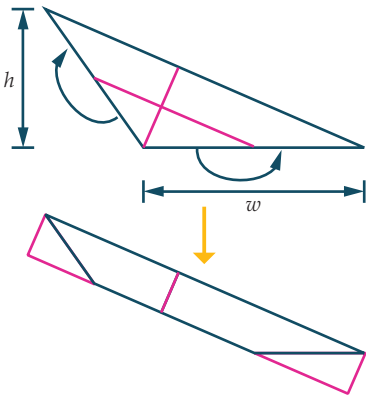

**11** (a) The cut should be made exactly halfway up the height, parallel to the base or exactly halfway along the base, parallel to the height.

(b)



**12** (a) Rule a line from the apex to the bottom side that is perpendicular (at right angles) to the bottom side. Halfway along this line, rule a second line parallel to the bottom line. Cut along these two lines.

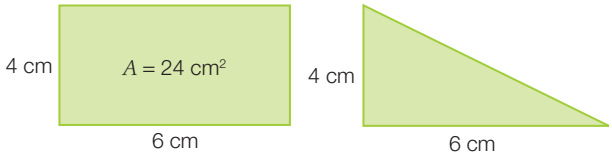
(b)




### Open-ended – Sample answers

**13** 9 cm and 8 cm; 6 cm and 12 cm

**14**




## Exercise 6.6 (p. 357)

**1** (a) 4 $cm^3$ (b) 16 $cm^3$ (c) 40 $cm^3$
(d) 24 $cm^3$ (e) 54 $cm^3$ (f) 60 $cm^3$

**2** (a) 60 $cm^3$ (b) 60 $cm^3$ (c) 48 $cm^3$
(d) 120 $cm^3$ (e) 125 $cm^3$ (f) 320 $cm^3$

**3** 12 $m^3$

**4** 216 $cm^3$

**5** (a) D (b) D

**6** (a) 28 $cm^3$ (b) 10 $cm^3$ (c) 18 $cm^3$
(d) 20 $cm^3$ (e) 6 $cm^3$ (f) 4 $cm^3$
(g) 4 $cm^3$ (h) 7 $cm^3$ (i) 10 $cm^3$

**7** 3.2 $m^3$ **8** 2352 $cm^3$ **9** 0.69 $m^3$

**10** 100 **11** 12 cm

### Open-ended – Sample answers

**12** 3 cm × 2 cm × 5 cm; 6 cm × 5 cm × 1 cm

**13** (a) 900 jelly beans

(b) Calculate the dimensions of the jar (length, width and height) in terms of jelly beans, and then calculate the volume in terms of jelly beans.

(c) More jelly beans would be able to fit into the jar, as they would pack closely together with less space between them.

**14** The side length of the large cube must be 4 cm, to give a volume of 64 $cm^3$. The 3 smaller cubes must have side lengths of 1 cm, 2 cm and 3 cm, giving volumes of 1 $cm^3$, 8 $cm^3$ and 27 $cm^3$ respectively.

| Volume of smaller cube | Volume of empty space in large cube |
|---|---|
| 1 $cm^3$ | 63 $cm^3$ |
| 8 $cm^3$ | 56 $cm^3$ |
| 27 $cm^3$ | 37 $cm^3$ |

## Mathspace (p. 360)

Police station: 400 cm

Telephone booth: 1.28 $m^3$

Train carriage: 2000 cm

Bank: 670 m

Library: 16 $m^3$

Train map: 4.6 km

L'Hotel: 320 cm

Museum: 24 $m^2$

Rubbish bin: 0.36 $m^3$

Newspaper stand: 0.075 $m^3$



## Challenge 6 (p. 366)

**1** 2 mm ↔ \$2

2 m ↔ \$2000

2 km ↔ \$2 000 000

1 km ↔ \$1 000 000

**2** C Sides of square are 200 m each. Each rectangle is 100 m by 50 m. Perimeter = $2 \times (100 + 50)$ m

**3** D

**4** Let square have sides of $2x$ cm. Rectangle has sides $2x$ cm and $x$ cm. Perimeter of rectangle = $6x$ cm.

$6x = 39$, $x = 6.5$ cm. Area of paper = $13 \times 13 = 169$ cm$^2$

**5** The colours are only used to make it easy to see the tiles.

The second pattern can occur 5 ways because the horizontal tiles can be in 5 different places.



The third pattern can occur in 3 ways because the vertical tiles can be in 3 places.



There are 9 different ways in total.

**6** B

**7** A

**8** 60 cm edge vertical,
volume of water = $200 \times 100 \times 60 - 100 \times 80 \times 60 = 720\,000$ cm$^3$

After block removed,
depth of water = $720\,000 \div (200 \times 100) = 36$ cm

80 cm edge vertical,
volume of water = $200 \times 100 \times 80 - 100 \times 80 \times 60 = 1\,120\,000$ cm$^3$

After block removed,
depth of water = $1\,120\,000 \div (200 \times 100) = 56$ cm

100 cm edge vertical,
volume of water = $200 \times 100 \times 100 - 100 \times 80 \times 60 = 1\,520\,000$ cm$^3$

After block removed,
depth of water = $1\,520\,000 \div (200 \times 100) = 76$ cm

**9** Area of each face of bottom cube = $4 \times 4 = 16$ cm$^2$

Area of each face of middle cube = $2 \times 2 = 4$ cm$^2$

Area of each face of top cube = $1 \times 1 = 1$ cm$^2$

Visible area = $5 \times 16 - 4 + 5 \times 4 - 1 + 5 \times 1$
$= 76 + 19 + 5$
$= 100$ cm$^2$

## Chapter review 6 (p. 367)

**1** (a) B (b) C

**2** (a) 45 900 m (b) 0.58 km (c) 9200 mm
(d) 4200 cm (e) 0.98 km (f) 6730 mm

**3** (a) (i) 92 cm (ii) 448 cm$^2$
(b) (i) 56 mm (ii) 196 mm$^2$
(c) (i) 40 m (ii) 66.5 m$^2$
(d) (i) 80 cm (ii) 198 cm$^2$

**4** Perimeter = 32.3 m, Area = 62.4 m$^2$

**5** (a) 18 cm$^2$ (b) 24 cm$^2$
(c) 126 cm$^2$ (d) 19.5 m$^2$

**6** (a) 210 m$^3$ (b) 225 cm$^3$

**7** (a) 54 m$^2$ (b) 113 cm$^2$ (c) 4.48 m$^2$ (d) 94 cm$^2$

**8** (a) 4.85 m (b) \$29.10

**9** (a) 300 cm by 250 cm, area = 75 000 cm$^2$
(b) 120 tiles needed
(c) \$96

**10** (a) 7 m (b) 0.9 m$^3$ (c) \$11.25

**11** \$806.40

**12** (a) 63 cm$^3$ (b) 162 cm$^3$

**13** 6.84 m$^3$

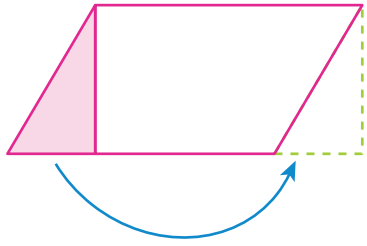
**14** 20 cm

**15** 60 cm$^2$

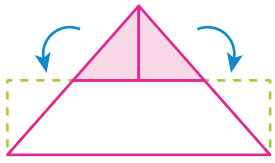
**16** 176 m$^2$

**17** 6750 cm$^3$

**18** (a)



(b)



### NAPLAN practice 6

**1** the rectangle **2** 37.5 m$^2$ **3** 40 cm **4** 15 cm

**5** C **6** 0.05 cm **7** B **8** B

## Mixed review C (p. 372)

**1** (a) $n - 8$ (b) $\frac{k}{6}$ (c) $50h$

**2** (a) 2.003, 2.3, 2.302, 2.323, 2.33
(b) 0.0199, 0.089, 0.0909, 0.091, 0.129

**3** (a) $\frac{1}{4}$ (b) $5\frac{1}{3}$ (c) $\frac{5}{12}$

**4** (a) $2^2 \times 7$ (b) $2^3 \times 5^2$ (c) $2 \times 3^2 \times 5 \times 11$

**5** (a) (i) 26 cm (ii) 36 cm$^2$
(b) (i) 14 m (ii) 12.25 m$^2$
(c) (i) 2.45 m (ii) 0.1815 m$^2$

6 (a) 23 (b) 34 (c) 31

7 (a) $\frac{10}{27}$ (b) $1\frac{13}{14}$ (c) $4\frac{3}{5}$ (d) 6

8 (a) 6090 m (b) 0.83 m (c) 0.054 km

9 (a) 6 (b) -3 (c) -12 (d) -11

(e) -6 (f) -3 (g) 13 (h) -17

10 (a) 1204 (b) 165 (c) 228

11 (a) 3:2:1 (b) 33.3%

12 (a) $\frac{23}{100}$ (b) $\frac{3}{50}$ (c) $\frac{101}{200}$

13 (a) 8 $cm^3$ (b) 360 $m^3$ (c) 216 $cm^3$

14

| $x$ | 0 | 2 | 3 | 6 | 10 |
|---|---|---|---|---|---|
| $y$ | 6 | 12 | 15 | 24 | 36 |

15 (a) 13.47 (b) 8.936 35 (c) 40.945

16 (a) 0.0031 (b) 0.000 42 (c) 2121

17 \$86.52

18 (a) 56 $mm^2$ (b) 36 $m^2$

19 Koala (a) $\frac{2}{5}$ (b) 40%

Bilby (a) $\frac{3}{10}$ (b) 30%

Wombat (a) $\frac{1}{5}$ (b) 20%

Kangaroo (a) $\frac{1}{10}$ (b) 10%

20 $2^8$

21 4 goals

22 8 cm by 13 cm

23 \$2.25

# Chapter 7

## Recall 7 (p. 376)

1 (a) False (b) True (c) False

2 (a) C (b) D (c) B

3 (a) $7 + 5 = 12$ (b) $4 \times 6 - 2 = 22$ (c) $\frac{8+6}{7} = 2$

4 (a) 9 (b) 7

5 (a) 7 (b) 2

## Exercise 7.1 (p. 379)

1 (a) $3 + 7 = 10$ (b) $5 \times 5 = 25$

(c) $\frac{16}{4} = 4$ (d) $22 - 10 = 8 + 4$

(e) $3 \times 5 + 2 = 8 + 9$ (f) $1 + \frac{25}{5} = 6$

2 (a) False (b) False (c) False (d) True

(e) True (f) True (g) False (h) True

(i) True

3 (a) D (b) C (c) B

4 (a) $5 + 43 = 48$ (b) $3 \times 7 = 19 + 2$

(c) $\frac{50}{5} = 5 \times 2$ (d) $20 - (6 + 8) = \frac{12}{2}$

5 (a) $13 - 19 = 6$ False

(b) $\frac{4}{2} = 8$ False

(c) $8 - 11 = 6 - 3$ False

(d) $\frac{12}{4} = \frac{15}{5}$ True

6 $5 + 7 = 12$

7 $6 \times 60 = 4 \times 90$

8 \$25 + \$7 = \$32

9 $48 - 11 + 5 = 42$

10 (a) $16 + 17 = 33$

(b) $17 + 18 + 19 = 54$

11 (a) $\frac{60}{3} = 20$ cm (b) $\frac{60}{4} = 15$ cm

(c) $\frac{60}{5} = 12$ cm (d) $\frac{60}{6} = 10$ cm

(e) Divide 60 by 20 (f) Divide 200 by 50

(g) Three from: 5, 10, 20, 25, 50 and 100

### Open-ended – Sample answers

12 $8 + 1 + 2 - 4 = 7$, $8 \times 1 - 4 + 2 = 6$, $8 - \frac{4}{2} + 1 = 7$

13 72, 80, 88, 96, 104

14 $5 + 6 \times 3 - 3 = 20$; $1 + 3 \times 8 - 5 = 20$; $6 + 4 \times 5 - 6 = 20$

15 Ruben is correct. Sam added four before multiplying; which is incorrect. The order of operations should always be followed.

16 Differences need to be kept the same on both sides. Because 12 is 4 less than 16, the missing number must also be 4 less than 38. Therefore, the first solution is correct.

## Exercise 7.2 (p. 384)

1 (a) Three added to a number is equal to ten.

(b) A number added to four is equal to seven.

(c) Five added to a number is equal to six.

(d) Two subtracted from a number is equal to four.

(e) Seven subtracted from a number is equal to two.

(f) Eight subtracted from a number is equal to six.

(g) A number multiplied by five is equal to twenty.

(h) A number multiplied by seven is equal to fourteen.

(i) A number multiplied by seven is equal to twenty-one.

(j) A number divided by three is equal to four.

(k) A number divided by two is equal to six.

(l) A number divided by six is equal to twelve.

(m) Three times a number added to two is equal to seven.

(n) Six times a number added to two is equal to ten.

(o) Three times a number added to one is equal to eight.

**2** (a) Yes (b) No (c) Yes (d) Yes (e) No

(f) Yes (g) No (h) Yes (i) Yes (j) No

(k) Yes (l) No (m) No (n) No (o) Yes

(p) Yes (q) Yes (r) No

**3** (a) $x = 4$ (b) $x = 7$ (c) $x = 7$ (d) $x = 29$

(e) $x = 17$ (f) $x = 64$ (g) $x = 18$ (h) $x = 6$

(i) $x = -4$ (j) $x = 76$ (k) $x = 120$ (l) $x = 162$

**4** D

**5** (a) D (b) C (c) D

**6** (a) B (b) D (c) D

**7** (a) C (b) A (c) D

**8** (a) $x = 10$ (b) $x = 4$ (c) $x = 11$

**9** (a) $D$ is Daniel's age.

(b) $D - 3$

(c) $D + 12$

(d) $D + 12 = 32$, Daniel is 20 years old.

**10** (a) $C$ is the amount of money Cameron has in his wallet.

(b) $C - 14$

(c) $C - 14 + 5 = 23$, Cameron had $32 to start with.

## Open-ended – Sample answers

**11** (a) $3c = 36$ (b) $3c = 27$ (c) 9 years old

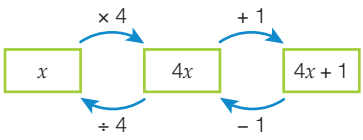
**12** (a) Sean started with 14 marbles. He gave a number to his friend Sasha and was left with 11 marbles.

(b) Tali and 4 of her friends shared a packet of lollies equally between them. They received 12 lollies each.

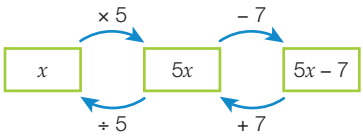
(c) The number of leaves on a tree multiplied by 6 to give 108 leaves in the early days of spring.

# Exercise 7.3 (p. 391)

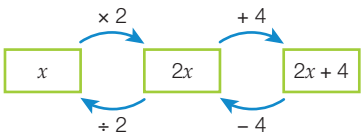
**1** (a)



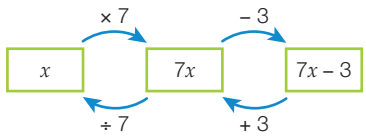

(b)



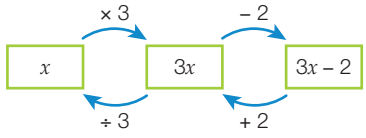

(c)



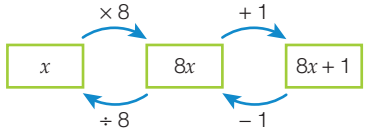

(d)




(e)




(f)




**2** (a) (i) $3x - 1 = 11$ (ii) $x = 4$

(b) (i) $2x - 3 = 7$ (ii) $x = 5$

(c) (i) $\frac{x+5}{4} = 4$ (ii) $x = 11$

(d) (i) $\frac{x+7}{2} = 8$ (ii) $x = 9$

(e) (i) $\frac{x}{6} + 7 = 9$ (ii) $x = 12$

(f) (i) $\frac{x}{3} + 8 = 13$ (ii) $x = 15$

(g) (i) $\frac{x-3}{5} = 1$ (ii) $x = 8$

(h) (i) $\frac{x-2}{7} = 2$ (ii) $x = 16$

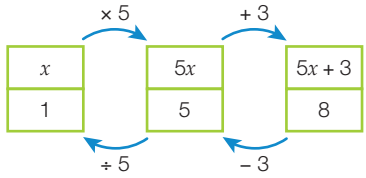
(i) (i) $2(x + 6) = 20$ (ii) $x = 4$

(j) (i) $5x - 9 = 16$ (ii) $x = 5$

(k) (i) $5(x + 2) = 30$ (ii) $x = 4$

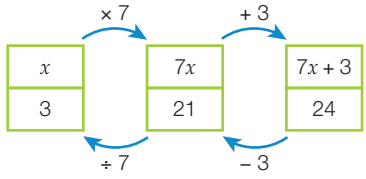
(l) (i) $7(x + 1) = 21$ (ii) $x = 2$

**3** (a)



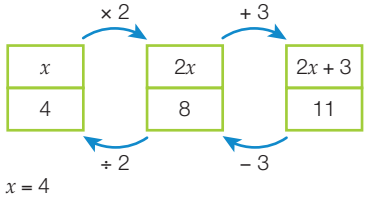

$x = 1$

(b)



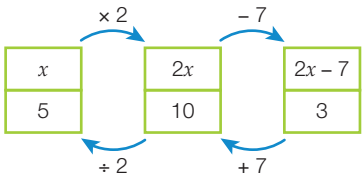

$x = 3$

(c)



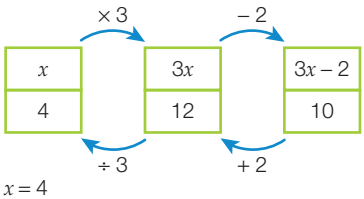

$x = 4$

(d)



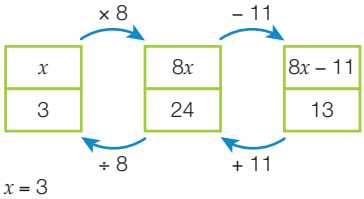

$x = 5$

(e)



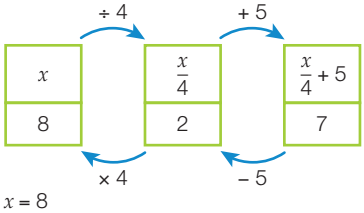

$x = 4$

(f)



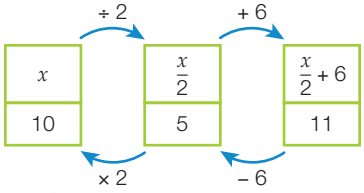

$x = 3$

4 (a)



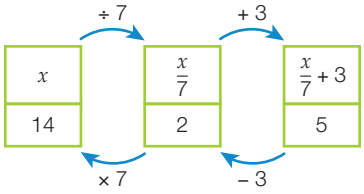

$x = 8$

(b)



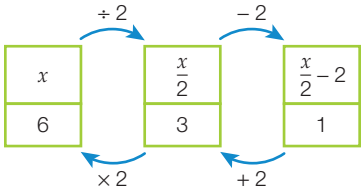

$x = 10$

(c)



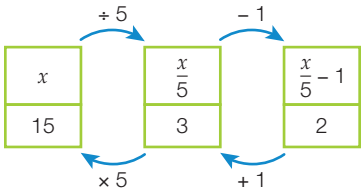

$x = 14$

(d)



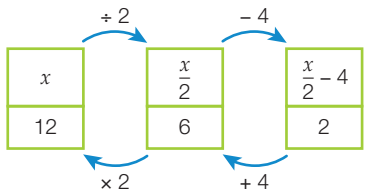

$x = 6$

(e)




$x = 15$

(f)




$x = 12$

5 (a) $x = 3$ (b) $x = 5$ (c) $x = 2$ (d) $x = 8$
(e) $x = 7$ (f) $x = 6$

6 (a) $x = 1$ (b) $x = 3$ (c) $x = 4$ (d) $x = 7$
(e) $x = 11$ (f) $x = 13$

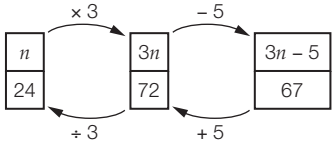
7 (a) $x = 5$ (b) $x = 3$ (c) $x = 1$ (d) $x = 4$
(e) $x = 2$ (f) $x = 2$ (g) $x = 3$ (h) $x = 5$
(i) $x = 3$ (j) $x = 10$ (k) $x = 14$ (l) $x = 15$
(m) $x = 6$ (n) $x = 14$ (o) $x = 7$

8 (a) $x = 4$ (b) $x = 5$ (c) $x = 6$ (d) $x = 5$
(e) $x = 14$ (f) $x = 4$ (g) $x = 11$ (h) $x = 2$
(i) $x = 24$

9 B

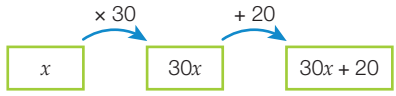
10 (a) $2n + 1 = 43$, $n = 21$ (b) $\frac{n+4}{5} = 4$, $n = 16$
(c) $\frac{7n}{3} = 21$, $n = 9$ (d) $\frac{2n+6}{10} = 2$, $n = 7$

11 $3n - 5 = 67$




There were 24 pencils in each packet.

12 (a)




(b)

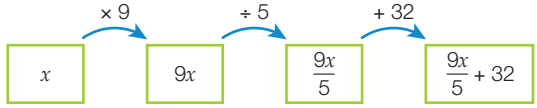



(c) Ying: \$50 Aldo: \$70 (d) $2\frac{1}{2}$ hours

(e) $2\frac{1}{4}$ hours (f) 3 hours

## Open ended – Sample answers

13 (a)




(b) 5, 10, 15, 20, ...

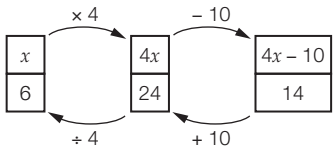
14 $\frac{2x+4}{6} + 6 = 10$

## Half-time 7 (p. 395)

**1** $x = 84$

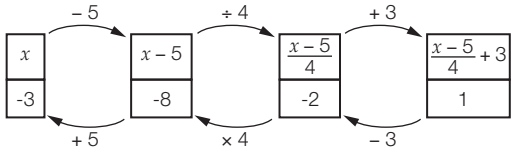
**2** 4 metres

**3 (a)**




$x = 6$

**(b)**




$x = -3$

**4** $21 + 22 = 43$

**5 (a)** (i) $J - 5 = 7$ (ii) 12

**(b)** (i) $d + 4 = 16$ (ii) 12

**6 (a)** $x = \frac{7}{3}$ **(b)** $x = -16$ **(c)** $x = 13$

**7** A number is multiplied by 5 and has 3 added to it. The result is 23.

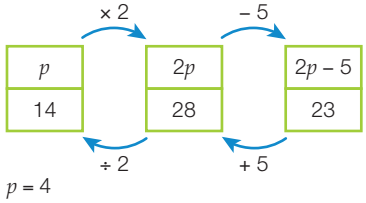
**8 (a)** $3 + 8 = 11$ **(b)** $4 \times 5 = 26 - 6$ **(c)** $4 + \frac{28}{2} = 18$

**9** 20 cm

**10 (a)** False **(b)** True **(c)** True

**11 (a)** $p - 5$ **(b)** $2p - 5 = 23$

**(c)**




$p = 4$

Pete's age = 14
Sam's age = 9

## Exercise 7.4 (p. 399)

**1 (a)** right-hand side (RHS)

**(b)** Take one chocolate from the RHS.

**(c)** Take 4 chocolates from the RHS.

**(d)** 9 chocolates **(e)** 3 chocolates

**2 (a)** $x + 8 = 11$ **(b)** $x + 9 = 17$ **(c)** $x + 7 = 13$

**(d)** $2x + 1 = 9$ **(e)** $3x = 5$ **(f)** $6x + 3 = 9$

**3 (a)** $x = 1$ **(b)** $x = 4$ **(c)** $x = 2$ **(d)** $x = 5$

**(e)** $x = 7$ **(f)** $x = -1$ **(g)** $x = 8$ **(h)** $x = 1$

**(i)** $x = 3$ **(j)** $x = 5$ **(k)** $x = 5$ **(l)** $x = 8$

**(m)** $x = 12$ **(n)** $x = 7$ **(o)** $x = 8$ **(p)** $x = 10$

**(q)** $x = 20$ **(r)** $x = 11$ **(s)** $x = 12$ **(t)** $x = 6$

**(u)** $x = 7$

**4** D

**5 (a)** $2t + 12 = 104$ **(b)** 46 kg

**6 (a)** $3p + 18 = 30$ **(b)** $p = 4$

**7** 34 km

**8** 1.25 L

### Open-ended – Sample answers

**9** $5x = 10$, $5x + 5 = 15$, $5x + 9 = 19$

**10 (a)** Vejay subtracted the 3 too early. He should have done it after multiplying by 7. The order was wrong.

**(b)** $\frac{2x + 3}{7} = 3$

$\frac{2x + 3}{7} \times 7 = 3 \times 7$

$2x + 3 = 21$

$2x + 3 - 3 = 21 - 3$

$2x = 18$

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

**(c)** He could use a flowchart to work out the order of operations used to 'build up' the equation. He should then work backwards along the flowchart, using the inverse operations, to solve the equation.

## Exercise 7.5 (p. 407)

**1 (a)** $x + 20 = 95$, the ice-cream costs 75 cents.

**(b)** $t + 1.5 = 12.8$, Tony ran the 100 m in 11.3 seconds.

**2 (a)** $2d + 1.6 = 8$, a sushi roll costs $3.20.

**(b)** $3v + 21.5 = 59$, a DVD costs $12.50.

**3 (a)** D **(b)** There are 27 seats available.

**4 (a)** C **(b)** The total length of the roll of fabric is 15 m.

**5 (a)** D **(b)** The width of the vegetable garden is 2.5 m.

**6 (a)** $x + 1.5$

**(b)** $x + 1.5 = 5.7$; $x = 4.2$; The salad sandwich costs $4.20.

**7 (a)** $4x$

**(b)** $4x = 84$; $x = 21$; The length of the floor tile is 21 cm.

**8 (a)** $4x + 6$

**(b)** $4x + 6 = 62$; $x = 14$; Fourteen tables that seat four people each are needed.

**9** $3h + 5 = 14$; $h = 3$; Natalie rollerskated on the rink for three hours.

**10** $2x - 54 = 272$; $x = 163$; at age 5, Robert Wadlow's height was 163 cm.

**11 (a)** $8d + 15 = 63$
$d = 6$ days

**(b)** $48d + 15 = 111$
$d = 2$ days

**(c)** $13d + 15 + 24d + 15 = 178$
$d = 4$ days

**12**

| | | Length of footpath | Number of pieces of wood | |
|---|---|---|---|---|
| (a) | (i) | 2 m | 6 | $(2 \times 2 + 2)$ |
| | (ii) | 3 m | 8 | $(2 \times 3 + 2)$ |
| | (iii) | 4 m | 10 | $(2 \times 4 + 2)$ |

(b) A footpath of length $x$ metres would need $(2 \times x + 2)$ or $(2x + 2)$ pieces of wood.

(c) (i) 14 (ii) 22 (iii) 28

(d) If $x$ is the length of the footpath, then the equation to solve is $2x + 2 = 36$. The longest footpath which could be poured at the one time is 17 metres in length.

### Open-ended – Sample answers

**13** 1 hour jackhammer ($30) plus 6 hours mulcher ($132) = $162

**14** $x + 5x \leq 70$
The son is 10 and the father is 50.

## Mathspace (p. 412)

### Practice session 1

**1** $x = 3$ **2** $x = 4$
**3** $x = 1$ **4** $x = 5$
**5** $x = 3$ **6** $x = 6$
**7** $x = 4$ **8** $x = 10$
**9** $x = 3$ **10** $x = 8$

### Practice session 2

**1** $x = 1$ **2** $x = 8$
**3** $x = 7$ **4** $x = 10$
**5** $x = 6$ **6** $x = 6$
**7** $x = 9$ **8** $x = 14$
**9** $x = 9$ **10** $x = 30$

### Practice session 3

**1** $x = 1$ **2** $x = 5$
**3** $x = 23$ **4** $x = 15$
**5** $x = 10$ **6** $x = 6$
**7** $x = 4$ **8** $x = 22$

## Challenge 7 (p. 414)

**1** A $49 - 15 = 34$
$34 \times 2 = 68$

**2** $x + 40 + x + 40 + x + 40 = 360$, $3x + 120 = 360$, $3x = 240$, $x = 80$

**3** $\frac{3x - 5}{4} = -6$, $3x - 5 = -24$, $3x = -19$, $x = -\frac{19}{3}$

**4** B $x = 73 + y - 59$, $x = 14 + y$

**5** (a) $3^2 = 4 + 5$

(b) $4^2 = 7 + 9$, $2^4 = 7 + 9$, $2^3 + 1 = 9$, $3^2 = 1 + 8$, $3^2 = 0 + 9$

(c) $2^3 = 9 - 1$, $3^2 = 9 - 0$, $3^2 - 5 = 4$, $3^2 - 4 = 5$, $4^2 - 9 = 7$

**6** D Reciprocal of $\frac{1}{20}$ is 20. $\frac{4x}{5} = 20$, $x = 25$

**7** $\frac{5+4}{20} + \frac{1}{y} = 1$, $\frac{1}{y} = 1 - \frac{9}{20}$, $\frac{1}{y} = \frac{11}{20}$, $y = \frac{20}{11}$

**8** Let him buy $x$ of each stamp.
$0.55x + 1.1x = 80.85$
$1.65x = 80.85$, $x = 80.85 \div 1.65 = 49$
He bought 98 stamps.

**9** $\frac{1}{4} \times \frac{9}{10}x = 2700$, $x = 300 \times 40 = \$12\,000$

**10** $\frac{1}{3} - \frac{1}{4} = \frac{1}{4} - \frac{1}{m}$, $\frac{1}{m} = \frac{1}{4} + \frac{1}{4} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{2} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{6}$, $m = 6$

**11** C The integers end in 2 and 5, 4 and 5, or 8 and 5. As $a > b$, consider $12 - 5 = 7$, $5 - 2 = 3$, $14 - 5 = 9$, $5 - 4 = 1$, $15 - 8 = 7$, $8 - 5 = 3$. $a - b$ ends in 1, 3, 7 or 9.

## Chapter review 7 (p. 415)

**1** (a) $7 \times 4 = 30 - 2$ (b) $2 \times 4 + 6 = 9 + 5$

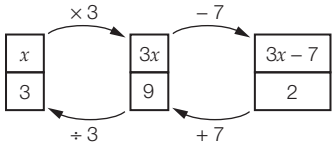
**2** (a) 15 (b) 28

**3** (a) $x = 9$ (b) $x = 3$ (c) $x = 15$ (d) $x = 15$
(e) $x = 12$ (f) $x = 28$

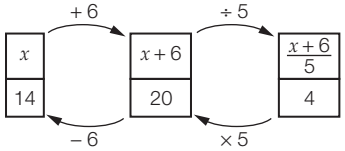
**4** (a) $x = -3$ (b) $x = 84$ (c) $x = 9$

**5** (a) $-5$ (b) $\times 3$ (c) $\div 8$ (d) $+4$

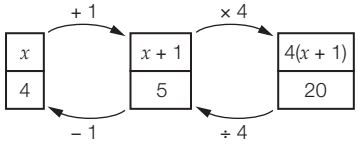
**6** (a) $x = 3$



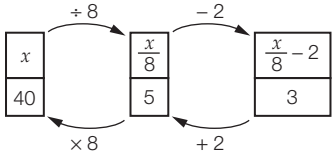

(b) $x = 14$



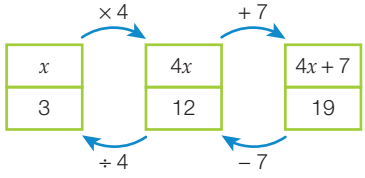

(c) $x = 4$




(d) $x = 40$

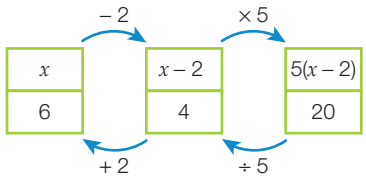



**7** (a)



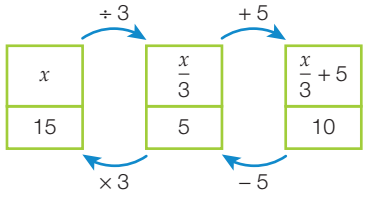

Solution is $x = 3$.

**(b)**



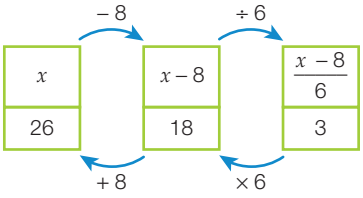

Solution is $x = 6$.

**(c)**



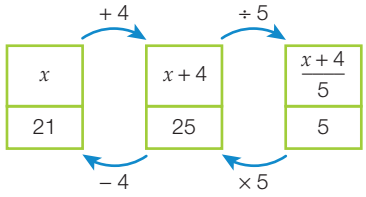

Solution is $x = 15$.

**(d)**



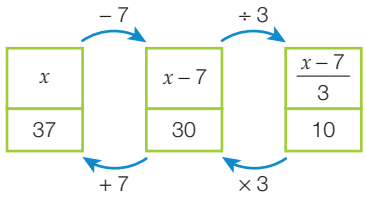

Solution is $x = 26$.

**(e)**




Solution is $x = 21$.

**(f)**




Solution is $x = 37$.

**8** B

**9** $5t + 27.50 = 200$, $t = \$34.50$

**10** D

**11** **(a)** $C = 3x + 140$

**(b)** $275 = 3x + 140$, $x = 45$ cents

Each dim sim costs 45 cents.

**12** $t + 5 = 28$
$t = 23°C$

The temperature at 3 p.m. was 23°C.

**13** **(a)** 5 **(b)** 7

**(c)** $2n + 1$ = number of matchsticks

**(d)** **(i)** $2n + 1 = 97$ **(ii)** $n = 48$

## NAPLAN practice 7

**1** C **2** D **3** D **4** C

**5** A **6** B **7** B **8** C

# Chapter 8

## Recall 8 (p. 420)

**1** **(a)** 55° **(b)** 160° **(c)** 235° **(d)** 339°

**2** D, A, B, E, C, F

**3** A and E, B and F, C and D

## Exercise 8.1 (p. 424)

**1** **(a)** 102° **(b)** 71° **(c)** 129° **(d)** 225°

**(e)** 350° **(f)** 15°

**2** Students' own answers.

**3** **(a)** C **(b)** B **(c)** D

**4** **(a)** B **(b)** B **(c)** D

**5** C **6** 21° **7** 40°

**8** Draw a 40° angle and mark the reflex angle as 320°.

**9** **(a)** 330° **(b)** 245° **(c)** 200°

**10** Students' own answers.

**11** Students' own answers.

### Open-ended – Sample answers

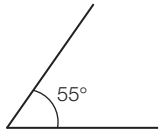
**12** Students' own answers.

**13** Nick has used the incorrect scale on his protractor. He should start from 0 on one arm and follow around the scale to the second arm.

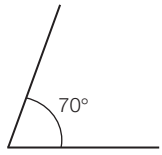
**14** **(a)** 20°, 70° **(b)** 30°, 100°, 50°

**(c)** 110°, 120°, 80°, 50°

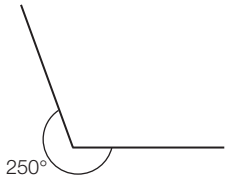
**15** **(a)**



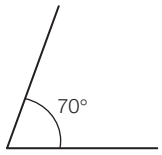

**(b)**



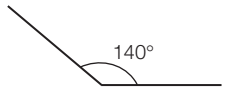

**(c)**



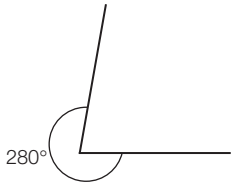

**16** **(a)**




**(b)**




**(c)**



## Exercise 8.2 (p. 432)

**1** **(a)** straight **(b)** reflex **(c)** obtuse
**(d)** reflex **(e)** acute **(f)** acute
**(g)** acute **(h)** obtuse **(i)** revolution
**(j)** revolution **(k)** right **(l)** straight
**(m)** obtuse **(n)** straight **(o)** right
**(p)** right **(q)** revolution **(r)** reflex

**2** **(a)** acute **(b)** obtuse
**(c)** reflex **(d)** revolution
**(e)** straight **(f)** acute
**(g)** right **(h)** obtuse
**(i)** reflex

**3** (*Note:* Other forms of notation may be used.)
**(a)** $\angle QPR$ or $\angle RPQ$ **(b)** $\angle STD$ or reflex angle $\angle DTS$
**(c)** $\angle BOA$ or $\angle AOB$ **(d)** $\angle KIW$ or $\angle WIK$
**(e)** $\angle SHD$ or $\angle DHS$ **(f)** $\angle CAT$ or $\angle TAC$

**4** **(a)** D **(b)** C **(c)** C **(d)** D

**5** D

**6** **(a)** **(i)** Students' own answers.
**(ii)** 360° **(iii)** revolution
**(b)** **(i)** Students' own answers.
**(ii)** 90° **(iii)** right angle
**(c)** **(i)** Students' own answers.
**(ii)** 180° **(iii)** straight angle
**(d)** **(i)** Students' own answers.
**(ii)** 360° **(iii)** revolution
**(e)** **(i)** Students' own answers.
**(ii)** 90° **(iii)** right angle
**(f)** **(i)** Students' own answers.
**(ii)** 180° **(iii)** straight angle

**7** **(a)** $\angle AOD$ and $\angle AOC$ or $\angle AOC$ and $\angle BOC$ or $\angle AOD$ and $\angle BOD$ or $\angle BOC$ and $\angle BOD$
**(b)** $\angle AOD$ and $\angle COB$ or $\angle AOC$ and $\angle DOB$

**8** C

**9** **(a)** $\angle AOB$, $\angle BOC$, $\angle DOE$
**(b)** $\angle AOE$, $\angle BOD$, $\angle COE$
**(c)** $\angle AOB$ (reflex), $\angle AOC$ (reflex), $\angle AOE$ (reflex), $\angle BOC$ (reflex), $\angle BOD$ (reflex), $\angle COD$ (reflex), $\angle COE$ (reflex), $\angle DOE$ (reflex)
**(d)** $\angle AOC$, $\angle COD$
**(e)** $\angle AOD$, $\angle BOE$
**(f)** $\angle AOB$ & $\angle BOC$, $\angle BOC$ & $\angle DOE$
**(g)** $\angle AOB$ & $\angle BOD$, $\angle AOC$ & $\angle COD$, $\angle AOE$ & $\angle DOE$, $\angle BOC$ & $\angle COE$, $\angle BOD$ & $\angle DOE$
**(h)** $\angle AOB$ & $\angle DOE$, $\angle AOE$ & $\angle BOD$

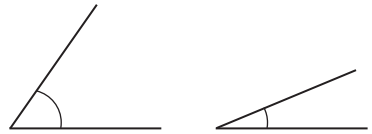
**10** **(a)** 334°
**(b)** $\angle RMS$ could be an acute angle.

**11** **(a)** $\angle B$ could refer to a number of angles.
**(b)** $\angle ABC$, $\angle CBD$
**(c)** $\angle ABD$
**(d)** $\angle ABD$ (reflex) or $\angle ABC$ (reflex) or $\angle CBD$ (reflex)

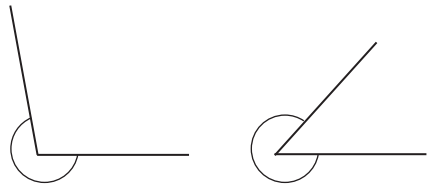
**12** Students to provide accurate copies of the figures in the textbook.

### Open-ended – Sample answers

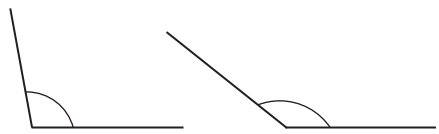
**13** **(a)**



**(b)**



**(c)**



**14** **(a)** 25° and 65°, 12° and 78°
**(b)** 112° and 68°, 92° and 88°

**15** The angles given must add to 360°; 45°, 150° and 165°; 205°, 60° and 95°

## Exercise 8.3 (p. 437)

**1** **(a)** 60° **(b)** 10° **(c)** 14° **(d)** 23°
**(e)** 56° **(f)** 63° **(g)** 150° **(h)** 60°
**(i)** 175° **(j)** 69° **(k)** 53° **(l)** 38°

**2** **(a)** 270° **(b)** 120° **(c)** 313° **(d)** 50°
**(e)** 145° **(f)** 202° **(g)** 50° **(h)** 245°
**(i)** 138° **(j)** 70° **(k)** 143° **(l)** 31°

**3** **(a)** 33° **(b)** 51° **(c)** 172° **(d)** 94° **(e)** 89° **(f)** 20°

**4** **(a)** 63° **(b)** 45° **(c)** 22° **(d)** 75°

**5** **(a)** 148° **(b)** 90° **(c)** 56° **(d)** 4°

**6** 87°

**7** **(a)** 57° **(b)** 45° **(c)** 65°

**8** A

**9** **(a)** $x° = 101°$ (vertically opposite to 101°)
$y° = z° = 79°$ (supplementary to 101°)
**(b)** $x° = 88°$ (vertically opposite to 88°)
$y° = z° = 92°$ (supplementary to 88°)
**(c)** $x° = z° = 149°$ (supplementary to 31°)
$y° = 31°$ (vertically opposite to 31°)

**10** **(a)** 60° (a straight line is 180°)

**(b)** 45° (a straight line is 180°)

**(c)** 30° (a right angle is 90°)

**(d)** 40° (a right angle is 90°)

**(e)** 50° (a straight line is 180°)

**(f)** 30° (a right angle is 90°)

**(g)** 120° (a revolution is 360°)

**(h)** 72° (a revolution is 360°)

**(i)** 75° (a revolution is 360°)

**11** **(a)**

| | Left leg | Right leg |
|---|---|---|
| $a$ | 90° | 91° |
| $b$ | 90° | 89° |
| $c$ | 90° | 89° |
| $d$ | 90° | 91° |
| $e$ | 92° | 80° |
| $f$ | 88° | 100° |
| $g$ | 88° | 100° |
| $h$ | 92° | 80° |

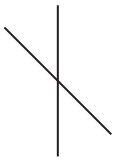



**(b)** right leg **(c)** Yes, the differences are 11°.

## Open-ended – Sample answers

**12** Sabine did not use the information given in the question. She should have used the property that angles at a point add to 360°, but instead used her protractor. The key is that the diagram is not drawn accurately.

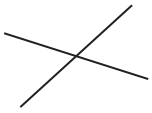
**13** **(a)**



**(b)** 45° and 135°

**(c)** 45° and 135°

**14**



Both angles are 120°

# Exercise 8.4 (p. 445)

**1** **(a)** corresponding **(b)** alternate

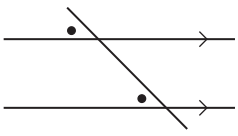
**(c)** co-interior **(d)** co-interior

**(e)** co-interior **(f)** corresponding

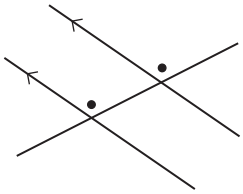
**(g)** alternate **(h)** corresponding

**(i)** alternate

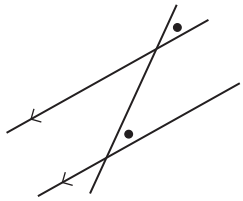
**2** **(a)**



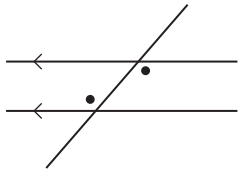
**(b)**



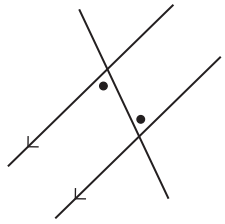
**(c)**



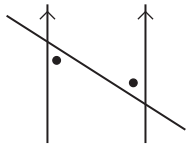
**3** **(a)**



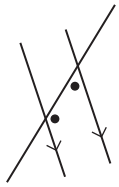
**(b)**



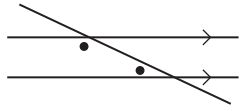
**(c)**



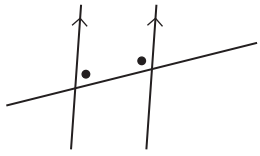
**4** **(a)**



**(b)**



**(c)**



**5** C

**6** **(a)** **(i)** $b° = 135°$, $a° = 45°$ (co-interior)

**(ii)** $s° = 140°$, $t° = 140°$ (corresponding)

**(iii)** $m° = 130°$, $n° = 130°$ (alternate)

**(iv)** $e° = 151°$, $f° = 151°$ (alternate)

**(v)** $g° = 72°$, $h° = 72°$ (corresponding)

**(vi)** $x° = 144°$, $y° = 36°$ (co-interior)

**(b)** **(ii)**, **(iii)**, **(iv)** and **(v)** have pairs that are equal; **(i)** and **(vi)** have pairs that add to 180°.

**7** **(a)** $y°$ **(b)** $w°$ **(c)** $u°$ **(d)** $v°$ **(e)** $s°$ **(f)** $x°$

**8** **(a)** $a° = 37°$ (alternate angles)

**(b)** $c° = 110°$ (corresponding angles)

**(c)** $g° = 24°$ (co-interior angles)

**(d)** $b° = 100°$ (co-interior angles)

**(e)** $d° = 73°$ (alternate angles)

**(f)** $e° = 141°$ (corresponding angles)

**(g)** $f° = 115°$ (corresponding angles)

**(h)** $j° = 72°$ (co-interior angles)

**(i)** $h° = 97°$ (alternate angles)

**9** 18°

**10** 144°

**11** $EF$ and $GH$. Co-interior angles are supplementary.

**12** **(a)** 16 **(b)** 8 **(c)** 8

**13** **(a)** $a° = 146°$ (vertically opposite angle),
$b° = 34°$ (co-interior angle)

**(b)** $a° = 62°$ (alternate angle),
$b° = 62°$ (vertically opposite angle),
$c° = 28°$ (angles in a right angle)

**(c)** $a° = 80°$ (alternate angle),
$b° = 80°$ (corresponding angle),
$c° = 80°$ (corresponding angle),
$d° = 80°$ (vertically opposite angle)

**(d)** $a° = 44°$ (alternate angle),
$b° = 46°$ (angles in a right angle)

**(e)** $a° = 50°$ (co-interior angle),
$b° = 50°$ (angles in a straight line)

**(f)** $a° = 45°$ (alternate angle),
$b° = 135°$ (co-interior angle),
$c° = 95°$ (co-interior angle),
$d° = 95°$ (vertically opposite angle)

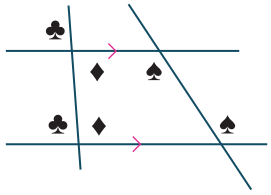
**(g)** $a° = 37°$ (alternate angle),
$b° = 37°$ (vertically opposite angle)

**(h)** $a° = 116°$ (co-interior angle),
$b° = 104°$ (co-interior angle),
$c° = 104°$ (vertically opposite angle)

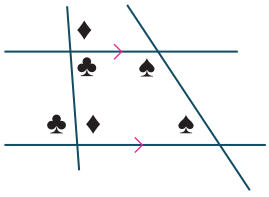
**(i)** $a° = 84°$ (alternate angle),
$b° = 96°$ (angles in a straight line),
$c° = 74°$ (co-interior angle),
$d° = 74°$ (vertically opposite angle)

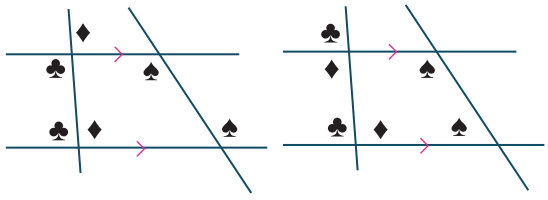
### Open-ended – Sample answers

**14** **(a)**



**(b)** Three:

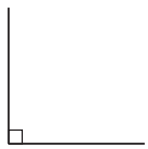




**15** Perspective: in a 2D representation of a 3D situation the parallel lines appear to converge on the horizon to give the impression of depth.

## Half-time 8 (p. 449)

**1** **(a)**



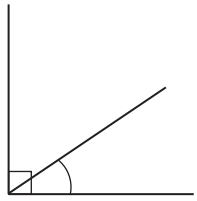
**(b)**



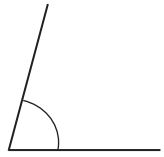
**(c)**



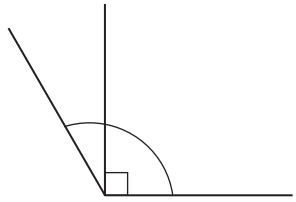
**(d)** About 30° is about one-third of a right angle.



**(e)** 75° is five-sixths of a right angle.



**(f)** 120° is 90° + 30°.



**2** **(a)** $\angle BOC$ (or $\angle COB$), $\angle DOC$ (or $\angle COD$)

**(b)** $\angle BOA$ (or $\angle AOB$), $\angle AOD$ (or $\angle DOA$), $\angle BOD$ (or $\angle DOB$)

**(c)** $\angle BOA$

**3** Students are required to draw angles for this question. The sizes match the diagrams in Question **1**.

**4** **(a)** $x = 55°$ (Angles in a straight line add to 180°.)

**(b)** $x = 35°$ (Angles in a revolution add to 360°.)

**5** 30°

**6** **(a)** $a = 67°$ (Alternate angles are equal.)

**(b)** $y = 57°$ (Co-interior angles add to 180°.)

**7** **(a)** $\angle GEB$ **(b)** $\angle GDF$ **(c)** $\angle DFI$ **(d)** $\angle HGI$

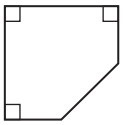
## Exercise 8.5 (p. 453)

**1** **(a)** pentagon (convex) **(b)** triangle (convex)
**(c)** hexagon (convex) **(d)** nonagon (convex)
**(e)** octagon (concave) **(f)** dodecagon (concave)
**(g)** decagon (concave) **(h)** hexagon (concave)
**(i)** hexagon (concave)

**2** **(a)** C **(b)** C

**3** **(a)** octagon **(b)** dodecagon
**(c)** pentagon **(d)** hexagon

4 3



5 (a) 3

(b) 4 and 5. The pattern is: divide the number of sides by 2.

(c) No, a pentagon and a heptagon have an odd number of sides, so they cannot be paired.

### Open-ended – Sample answers

6 (a)  (b)  (c) 

7 50-cent piece (dodecagon), stop sign (octagon), soccer ball (pentagons & hexagons)

## Exercise 8.6 (p. 461)

1 (a) (i) scalene (ii) obtuse
(iii) an obtuse-angled scalene triangle

(b) (i) isosceles (ii) acute
(iii) an acute-angled isosceles triangle

(c) (i) isosceles (ii) right
(iii) a right-angled isosceles triangle

(d) (i) isosceles (ii) acute
(iii) an acute-angled isosceles triangle

(e) (i) equilateral (ii) acute
(iii) an acute-angled equilateral triangle

(f) (i) scalene (ii) obtuse
(iii) an obtuse-angled scalene triangle

(g) (i) equilateral (ii) acute
(iii) an acute-angled equilateral triangle

(h) (i) scalene (ii) right
(iii) a right-angled scalene triangle

(i) (i) isosceles (ii) obtuse
(iii) an obtuse-angled isosceles triangle

(j) (i) scalene (ii) right
(iii) a right-angled scalene triangle

(k) (i) isosceles (ii) obtuse
(iii) an obtuse-angled isosceles triangle

(l) (i) scalene (ii) acute
(iii) an acute-angled scalene triangle

2 (a) 60° (b) 30° (c) 35° (d) 83° (e) 132°
(f) 22° (g) 14° (h) 38° (i) 48°

3 (a) (i) 120° (ii) 137° (iii) 75° (iv) 165° (v) 151°
(vi) 138° (vii) 133° (viii) 146° (ix) 129°

(b) (i) 40° (ii) 30° (iii) 74° (iv) 43° (v) 126°
(vi) 38°

4 (a) isosceles, acute-angled (b) scalene, acute-angled
(c) scalene, acute-angled (d) equilateral, acute-angled
(e) equilateral, acute-angled (f) scalene, acute-angled
(g) isosceles, acute-angled (h) isosceles, acute-angled
(i) scalene, right-angled

5 (a) D (b) A (c) B

6 B

7 (a) C (b) A

8 (a) isosceles; obtuse-angled, $a = 25°$

(b) scalene; right-angled, $b = 35°$

(c) equilateral, $c = 30°$

(d) scalene; right-angled, $a = 21°$

(e) scalene; right-angled, $b = 58°$

(f) isosceles; acute-angled, $c = 48°$

(g) isosceles; obtuse-angled, $d = 130°$

9 isosceles, right-angled

10 (a) all sides equal

(b) Two angles will be 60°, the other two will be 120°

11 No, the three sides won't be able to meet.

12 two

13 (a) right-angled, scalene (third angle 90°)

(b) obtuse-angled, isosceles (third angle 28°)

(c) acute-angled, equilateral (third angle 60°)

(d) obtuse-angled, isosceles (third angle 156°)

14 (a) Angles add up to more than 180° (i.e. 190°).

(b) Angles add up to 165° – must be 180°.

(c) Isosceles triangles must have 2 equal angles

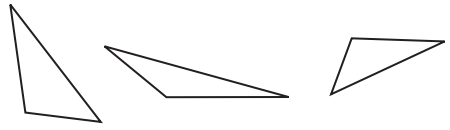
15 (a) $a = 65°$, $b = 135°$

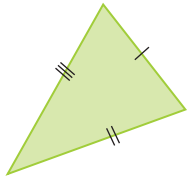
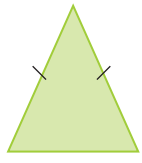
(b) $c = 100°$

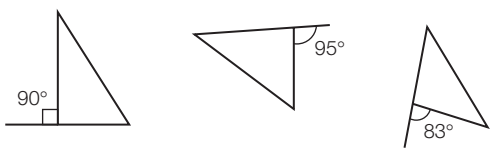
(c) $d = 57°$, $e = 57°$, $f = 66°$

16 360°

### Open-ended – Sample answers

17 

18 (a)  (b) 

19 

## Exercise 8.7 (p. 467)

**1** **(a)** rectangle **(b)** parallelogram **(c)** kite
**(d)** rhombus **(e)** trapezium **(f)** quadrilateral

**2** A

**3** **(a)** True **(b)** False **(c)** True **(d)** False **(e)** True

**4** **(a)** 77° **(b)** 87° **(c)** 54° **(d)** 55° **(e)** 69°
**(f)** 133° **(g)** 138° **(h)** 125° **(i)** 205°

**5** **(a)** D **(b)** C **(c)** C

**6** **(a)** 40° **(b)** 105° **(c)** 68° **(d)** 73° **(e)** 197°
**(f)** 19° **(g)** 60° **(h)** 131° **(i)** 78°

**7** **(a)** trapezium **(b)** rhombus
**(c)** kite **(d)** parallelogram

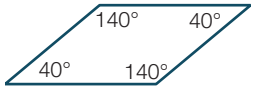
**8** **(a)** $a = 70°$, angles in a quadrilateral add to 360°

**(b)** $b = 97°$, one pair of opposite angles in a kite are equal and angles add to 360°

**(c)** $c = 99°$, co-interior angle

**(d)** $d = 38°$, co-interior angle

**9**



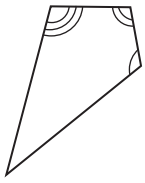

**10** 105°

**11** Parallelograms have opposite sides parallel and equal in length, which is also the case for a rhombus. However, a rhombus has all sides equal, and that is not necessarily true for all parallelograms.

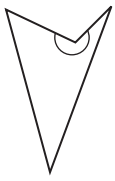
**12** No, a parallelogram must have two pairs of parallel sides whereas a trapezium must only have one pair of parallel sides.

**13** Yes, rectangles have all the properties of a parallelogram and more. Most parallelograms do not have 90° angles.

**14** **(a)** three



**(b)** one



**15** **(a)** 128° **(b)** kite or irregular quadrilateral

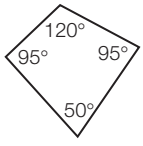
**16** No. Sum of angles in a quadrilateral = 360°, not 370°.

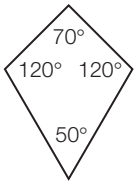
**17** **(a)** 4 right angles

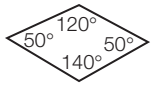
**(b)** 4 equal sides

**(c)** opposite sides are parallel

**18**



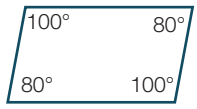




## Open-ended – Sample answers

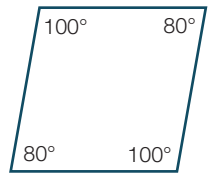
**19** **(a)** Parallelogram, rhombus, kite, trapezium, or irregular quadrilateral.

**(b)** Five—**(i)** parallelogram, **(ii)** rhombus, **(iii)** kite, **(iv)** trapezium, or **(v)** irregular quadrilateral.

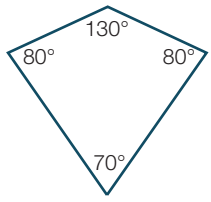
**(c)** **(i)**




**(ii)**



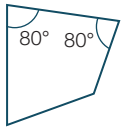

**(iii)**



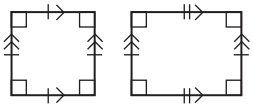

**(iv)**

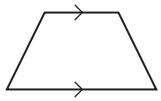



**(v)**




**20** Jack's diagrams lack important angle symbols, congruent and parallel line marking.





**21** Parallelograms: rectangle, square, diamond, rhombus, parallelogram

Non-parallelograms: irregular quadrilateral, trapezium, kite

## Mathspace (p. 472)

**1** Flying thing into volcano catapulted hit large moogal paw squashed flying thing. (The ship crashed into the volcano. It catapulted out and hit a large moogal, whose paw squashed the ship.)

**2** **(a)** **(i)** C

**(ii)** A Kwoogal is a quadrilateral containing two black circles and one clear circle.

**(b)** A, B & E

### Fuel cells

$a = 110°$, $b = 80°$, $e = 130°$

### Neutron absorber

$s = 70°$

### Black hole combustion unit

$g = 25°$, $h = 100°$, $t = 335°$, $y = 155°$

## Exercise 8.8 (p. 477)

1 Follow Worked Example 9 instructions to bisect a line and an angle.

2 (a) Follow Worked Example 10(a) instructions to construct an equilateral triangle.

(b) Follow Worked Example 10(b) instructions to construct a square.

3 Follow Worked Example 11 instructions to construct an angle equal to another angle and a pair of parallel lines.

4 (a) Follow the worked example instructions for bisecting a line to construct a 90° angle.

(b) Bisect the 90° angle to make a 45° angle.

5 (a) Follow the worked example instructions for constructing an equilateral triangle to construct a 60° angle.

(b) Bisect the 60° angle to make a 30° angle.

(c) Begin with a 10 cm line and follow the worked example instructions for bisecting a line.

6 Construct pairs of parallel lines.

7 Begin with an 8 cm line. Construct angles of 60° and 90° at either end. Bisect both angles and join the bisectors to find the 3rd vertex of the triangle.

8 (a) Construct angles of 60° and 90° at the same end of the same line. Bisect the angle between the 60° and 90° angles to form a 75° angle with the initial line.

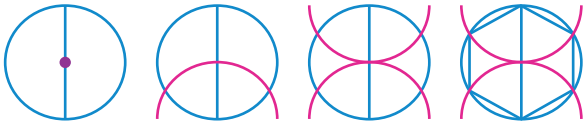
(b) Construct a 60° angle. Bisect this to give a 30° angle. Bisect the 30° angle to give a 15° angle.

(c) Construct a right angle. Bisect the adjacent right angle to give a 45° angle. The right angle and the 45° angle together form a 135° angle.

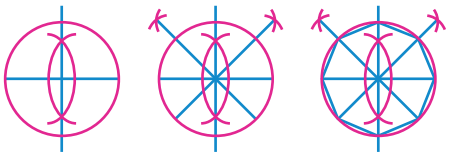
9 Using any base line of length less than 10 cm, open the compass 5 cm to make arcs from either end. Join the intersection of the arcs with the ends of the base line to construct your triangle.

10 Construct a square. Keep the compass at the same side length as the square, and use the sides of the square to construct an equilateral triangle on each side.

11 This construction works because we have constructed 6 equilateral triangles in which all the angles equal 60°.



12 This construction works because we have constructed a 90° angle, then bisected it to make a 45° angle. We have now divided the circle into 8 equal parts.



### Open-ended – Sample answers

13 Use the base and sides of a polygon to start the construction of a new polygon.

## Challenge 8 (p. 478)

1 B At 2.30, the hour hand is halfway between 2 and 3. Total angle is $90° + (30° \div 2) = 105°$.

2 $90° - 46° = 44°$. Small angle $= 44° \div 2 = 22°$, large angle $= 46° + 22° = 68°$.

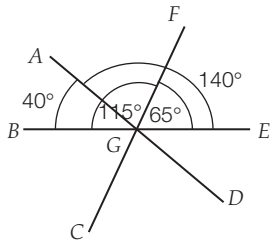
or: $90° \div 2 = 45°$, $46° \div 2 = 23°$. Small angle $= 45° - 23° = 22°$S, large angle $= 45° + 23° = 68°$.

3 $180° - 70° = 110°$. $x + x = 110°$, so $x = 55°$.

4 Draw the diagram and mark the angles on it. Use this information to work out the other angles.

From the diagram, you can get $\angle AGF = 75°$

$\angle CGD = 75°$ (vertically opposite $\angle AGF$)




5 $180° - 54° = 126°$. Small angle $= 126° \div 2 = 63°$, large angle $= 54° + 63° = 117°$.

6 D $6x° = 180°$, $x = 30°$; $y = 180° - 30° = 150°$

7 A $x = 180° - (30° + 40°) = 180° - 70° = 110°$

8 (a) At 12.10, the minute hand makes an angle of 60° with the vertical and the hour hand makes an angle of $\frac{1}{6} \times 30°$ or 5°. The angle between them is $60 - 5° = 55°$.

(b) 40 minutes later, the time is 12.50. The minute hand makes an acute angle of 60° with the vertical and the hour hand makes an angle of $\frac{5}{6} \times 30°$ or 25°. The angle between them is $60° + 25° = 85°$.

9 C $x + y = 110°$, $\angle KLM = 180° - 2x$, $\angle KML = 180 - 2y$
$\angle KLM + \angle KML = 360° - 2(x + y) = 360° - 220° = 140°$,
$\angle LKM = 40°$

## Chapter review 8 (p. 480)

1 (a) 42° (b) 196° (c) 157° (d) 321°

2 (a) E (b) A (c) D (d) B (e) C (f) F

3 Students are required to draw angles for this question.

4 A

5 B

6 (a) reflex (b) revolution (c) straight
(d) acute (e) right (f) obtuse

7 (a) 163° (b) 39° (c) 127°
(d) 40° (e) 143° (f) 82°

8 (a) 136° (b) 112°

9 (a) irregular concave pentagon
(b) irregular convex heptagon
(c) irregular convex decagon
(d) regular concave dodecagon

10 80°

11 (a) 66° (angles in a quadrilateral add to 360°)
(b) 110° (opposite angles in a parallelogram are equal)

12 140°, missing angles are the equal angles in a kite.

13 Students' own diagrams.

14 292°

15 (a) 45° and 90°
(b) isosceles triangle

16 (a) 85° (b) 25° (c) 40°

17 (a) True (b) False (c) True (d) False (e) True

18 $\angle CFB = 90°$, vertically opposite angles are equal.
$x = 27°$, complementary angles add to 90°.

19 An obtuse angle.

20 A reflex angle.

21 An acute angle, a right angle or an obtuse angle.

### NAPLAN practice 8

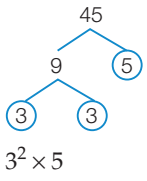
1 B 2 150° 3 D 4 D

5 C 6 D

## Mixed review D (p. 484)

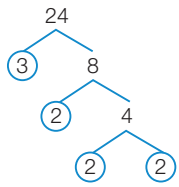
1 (a) $6a - 4b$ (b) $6a + 2b + 3c$

2 (a)



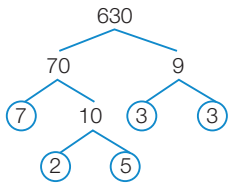

$3^2 \times 5$

(b)




$2^3 \times 3$

(c)




$2 \times 3^2 \times 5 \times 7$

3 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{25}$

4 (a) 0.253 (b) 2.437

5 (a) $x = -1$ (b) $x = -41$

6 (a) 46° (b) 39°

7 (a) $3\frac{19}{20}$ (b) $\frac{15}{16}$

8 (a) 137.7 cm$^2$ (b) 60 m$^2$

9 (a) Thursday (b) Wednesday (c) Friday

10 A 11 D 12 11.25°

13 (a) 35° (b) 63°

14 150°, 210°

15 (a) Students' own answers.
(b) If they add to 360°, then they are likely to be correct.
(c) Dogs 110°, Cats 140°, Birds 30° and Fish 80°.

16 Second floor

17 $x = 45°$, $2x = 90°$, $3x = 135°$

18 (a) Students' own answers.
(b) Reflex angle.
(c) $\angle ABC$ (reflex)

19 Complementary angles: $\angle ABE$ & $\angle CBE$ (also $\angle ABE$ & $\angle BAE$, $\angle EBC$ & $\angle ECB$, $\angle BAE$ & $\angle BCE$, $\angle ECD$ & $\angle EDC$, $\angle EAD$ & $\angle EDA$).
Supplementary angles: $\angle AEB$ & $\angle CEB$, $\angle AED$ & $\angle CED$, $\angle AEB$ & $\angle AED$, $\angle CED$ & $\angle CEB$.
Vertically opposite angles: $\angle CEB$ & $\angle AED$, $\angle AEB$ & $\angle CED$.
Angles at a point: $\angle AEB$, $\angle CEB$, $\angle AED$ & $\angle CED$; $\angle BAE$, $\angle EAD$ and $\angle BAD$ (reflex), there are more around the edge of the kite.

# Chapter 9

## Recall 9 (p. 488)

1 (a) 0, 5, 10, 15, 20, 25, 30
(b) 16, 18, 20, 22, 24, 26, 28
(c) 1300, 1400, 1500, 1600, 1700
(d) 60, 80, 100, 120, 140, 160, 180, 200

2 (a) 50, 100, 150, 200, 250 (b) 100, 125, 150, 175, 200
(c) 1400, 1600, 1800, 2000

3 (a) 6, 10, 12, 14, 16, 20, 22, 24
(b) 0, 5, 15, 20, 25, 50, 55, 100
(c) $0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 5, 5\frac{1}{2}, 6\frac{1}{2}$

4 (a) 15, 18, 24 (b) 140, 160, 180, 200
(c) 250, 275, 325

5 (a) $\frac{1}{3}$ (b) $\frac{3}{4}$

## Exercise 9.1 (p. 492)

1 (a) categorical
(b) categorical
(c) continuous
(d) continuous
(e) discrete
(f) categorical
(g) discrete

**2** (a)

| Number of pets owned | Tally | Frequency |
|---|---|---|
| 0 | \|\|\|\| | 4 |
| 1 | ++++ \|\| | 7 |
| 2 | \|\|\| | 3 |
| 3 | \|\|\| | 3 |
| 4 | \|\|\|\| | 4 |
| 5 | \| | 1 |
| 6 | \| | 1 |
| 7 | \| | 1 |
| 8 | | 0 |
| 9 | \| | 1 |
| | | 25 |

(b) discrete

**3**

| Number of calls | Tally | Frequency |
|---|---|---|
| 0–19 | \| | 1 |
| 20–39 | \| | 1 |
| 40–59 | \|\|\| | 3 |
| 60–79 | ++++ \| | 6 |
| 80–99 | ++++ \| | 6 |
| 100–119 | ++++ \| | 6 |
| 120–139 | \|\| | 2 |
| 140–159 | \|\| | 2 |
| 160–179 | \|\| | 2 |
| 180–199 | \| | 1 |
| | | 30 |

**4** (a) 82

(b)

| Colour | Frequency |
|---|---|
| Black | 13 |
| White | 21 |
| Silver | 25 |
| Red | 7 |
| Blue | 5 |
| Other | 11 |

(c) categorical

**5**

| Hours of television watched | Tally | Frequency |
|---|---|---|
| 0 | \|\|\| | 3 |
| $\frac{1}{2}$ | \|\|\|\| | 4 |
| 1 | \|\|\| | 3 |
| $1\frac{1}{2}$ | \|\| | 2 |
| 2 | ++++ \| | 6 |
| $2\frac{1}{2}$ | ++++ | 5 |
| 3 | \|\| | 2 |
| $3\frac{1}{2}$ | | 0 |
| 4 | \| | 1 |
| $4\frac{1}{2}$ | \| | 1 |
| | | 27 |

**6**

| Type of takeaway | Tally | Frequency |
|---|---|---|
| Pizza | ++++ \|\|\| | 8 |
| Fish & chips | \|\|\| | 3 |
| Hamburgers | \|\|\|\| | 4 |
| Chicken | \|\|\| | 3 |
| Chinese | \|\| | 2 |
| None | \|\| | 2 |
| | | 22 |

**7**

| Amount of money | Tally | Frequency |
|---|---|---|
| \$0.00–\$0.45 | \|\|\|\| | 4 |
| \$0.50–\$0.95 | \|\|\| | 3 |
| \$1.00–\$1.45 | \|\|\| | 3 |
| \$1.50–\$1.95 | \|\|\|\| | 4 |
| \$2.00–\$2.45 | \|\|\| | 3 |
| \$2.50–\$2.95 | \|\|\| | 3 |
| \$3.00–\$3.45 | | 0 |
| \$3.50–\$3.95 | \| | 1 |
| \$4.00–\$4.45 | \|\|\| | 3 |
| | | 24 |

**8** (a) C (b) B

**9** (a) 26

(b)

| Number of hours spent exercising | Tally | Frequency |
|---|---|---|
| 0 | \|\|\|\| | 4 |
| 1 | ++++ \|\| | 7 |
| 2 | ++++ \|\|\| | 8 |
| 3 | \|\|\| | 3 |
| 4 | \| | 1 |
| 5 | \| | 1 |
| 6 | \| | 1 |
| 7 | | 0 |
| 8 | \| | 1 |
| | | 26 |

(c) 4 (d) 7

(e) 2 hours (f) 8 hours

(g) 8, 6 and 5 hours. These students will be training for more hours compared to other students.

**10** (a) 35

(b)

| Size | Tally | Frequency |
|---|---|---|
| XS | \|\| | 2 |
| S | ++++ \|\| | 7 |
| M | ++++ \|\|\|\| | 9 |
| L | ++++ \|\|\| | 8 |
| XL | \|\|\|\| | 4 |
| Did not order | ++++ | 5 |

(c) categorical (d) M (e) 9

(f) Because the students who forgot to order were all in Year 7, she should probably order 5 M size T-shirts, as the L and XL would be too big for most Year 7 students.

**11** (a)

| Class interval | Frequency |
|---|---|
| 20–29 | 6 |
| 30–39 | 6 |
| 40–49 | 6 |
| 50–59 | 6 |
| 60–69 | 6 |

(b) Each of the intervals contains six values.



(c)

| Class interval | Frequency |
|---|---|
| 20–24 | 5 |
| 25–29 | 1 |
| 30–34 | 5 |
| 35–39 | 1 |
| 40–44 | 5 |
| 45–49 | 1 |
| 50–54 | 5 |
| 55–59 | 1 |
| 60–64 | 5 |
| 65–69 | 1 |

(d) The low-half class intervals have 5 values whereas the high-half class intervals have just 1 value. The values tend to appear at the lower end of the 10 groups.

(e) The size of the class interval can have an effect on the way we see the data.

12 (a)

| Pulse rate (beats per minute) | Frequency |
|---|---|
| 51–60 | 7 |
| 61–70 | 21 |
| 71–80 | 19 |
| 81–90 | 10 |
| 91–100 | 3 |

(b)

| Pulse rate (beats per minute) | Frequency |
|---|---|
| 51–55 | 3 |
| 56–60 | 4 |
| 61–65 | 8 |
| 66–70 | 13 |
| 71–75 | 10 |
| 76–80 | 9 |
| 81–85 | 10 |
| 86–90 | 0 |
| 91–95 | 3 |

(c) Probably (b). This table shows that the vast majority of students have a pulse rate between 61 and 85 beats per minute. When looking at table (a), the vast majority of students have a pulse rate between 61 and 90 beats per minute, but the high end of 90 might be a cause for medical concern.

13 The highest weight is 3529 g and the lowest is 2010 g. This is a difference of 1519 g. If we group in 250 g lots, we would have 7 groups. If we group in 200 g lots, we would have 8 groups. If we group in 125 g lots, we would have 14 groups —this is too many. We could choose either of the others. The table below is for lots of 250 g. It starts at 2000 g as this is a multiple of the difference being used.

| Birth weight (g) | Frequency |
|---|---|
| 2000–2249 | 5 |
| 2250–2499 | 2 |
| 2500–2749 | 5 |
| 2750–2999 | 8 |
| 3000–3249 | 6 |
| 3250–3499 | 1 |
| 3500–3749 | 3 |

## Open-ended – Sample answers

14 (a)

| Number rolled | Tally | Frequency |
|---|---|---|
| 1 | \| \| \| | 3 |
| 2 | \| \| | 2 |
| 3 | \| \| \| \| | 4 |
| 4 | \| \| \| \| | 4 |
| 5 | \| \| \| | 3 |
| 6 | \| \| \| \| | 4 |

(b)

| Weight of dogs (kg) | Tally | Frequency |
|---|---|---|
| 1–5 | 𝍸 𝍸 \| \| | 12 |
| 6–10 | 𝍸 \| \| \| | 8 |
| 11–15 | 𝍸 𝍸 \| \| \| \| | 14 |
| 16–20 | 𝍸 𝍸 \| \| \| | 13 |
| 21–25 | \| \| \| | 3 |

15 Students' own answers.

16 Students' own answers.

## Mathspace (p. 496)

1 (a) J

(b) Moving back by 5 or forward by 21.

(c) Press the yellow circle, then the red square and finally the blue triangle.

2 (a) E, T, A, O, I

(b)

| Letter | A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 3 | 2 | 7 | 5 | 1 | 4 | 14 | 0 | 0 | 2 | 0 | 0 | 0 |

| Letter | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 9 | 7 | 2 | 1 | 5 | 3 | 2 | 2 | 2 | 2 | 4 | 6 | 0 |

(c) Look for the words red, yellow and blue.

(d) This time, press the blue triangle, then the yellow circle twice and finally the red square four times.

## Exercise 9.2 (p. 500)

1 (a) (i) 5 (ii) 7 (iii) 7

(b) (i) 6 (ii) 5 (iii) 5

(c) (i) 6 (ii) 6 (iii) 6

(d) (i) 4 (ii) 4 (iii) 4

(e) (i) 7 (ii) 7 (iii) 7

(f) (i) 7 (ii) 6 (iii) 5

(g) (i) 6 (ii) 7 (iii) 8

(h) (i) 4 (ii) 3 (iii) 2

(i) (i) 4.9 (ii) 4.5 (iii) 4

(j) (i) 5.5 (ii) 5 (iii) 5

(k) (i) 6 (ii) 7 (iii) 9

(l) (i) 5 (ii) 5 (iii) 2

2 (a) (i) 29 (ii) 29 (iii) 29

(b) (i) 66.43 (ii) 72 (iii) No mode exists.

(c) (i) 66 (ii) 67.5 (iii) 81

(d) (i) 10.26 (ii) 9.05 (iii) 6.5

(e) (i) 13.7 (ii) 13.5 (iii) 11.4

(f) (i) 8.50 (ii) 10.11 (iii) No mode exists.

3 (a) C; only one within the range of scores

(b) C; 8 too high and 4 too low

(c) C; only one of each score, so mean will be close to median

(d) B; close to median as single results relatively evenly spread

4 (a) (i) 49 480.8 (ii) 43 369

(b) Incheon

(c) (i) 49 390 (ii) 46 000

(d) Miyagi

(e) The mean capacity is slightly higher in Korea (49 480.8 compared to 49 390), but the median capacity is higher in Japan (46 000 compared to 43 369). Half of the stadiums in each country have a capacity less than 45 000 (i.e. smaller than the mean value).

(f) (i) 49 435.4 (ii) 43 369

(g) Miyagi (again) (It is interesting to note there are two stadiums in Japan quite close to the mean—Miyagi and Osaka—but none of the Korean stadiums are very close at all.)

5 (a) No, because the mean is the sum of all the data values divided by the number of values and so it will not necessarily be a data value.

(b) 5.8, no it is not a data value.

(c) Yes, because there is an odd number of values and so the median will be the middle value.

(d) 6

6 (a) This statement means that the average number of children of all families is 2.3. Most families have 2 children, but because many families have more than two children, the average of all these numbers of children is 2.3.

(b) The middle number will always be a whole number. If there are two middle numbers, then the only decimal you can get by dividing them by 2 is one that ends in .5.

(c) No, as the mode is the value that occurs most often in a data set, and the data set only contains whole numbers.

7 (a) (i) $58 571.43 (ii) $50 000 (iii) $50 000

(b) In this case, the mode does a good job as there are so many of this category of worker—well over half. However, the mean does take account of the other workers as well.

(c) (i) $53 461.54 (ii) $50 000 (iii) $50 000

(d) The mean is the only value that changes, and it changes by a large amount (over $5000).

## Open-ended – Sample answers

8 (a) 5, 6, 6, 6, 7. As the median is 6, this is the middle number and we need at least one more 6 to make it the mode. The total of the five results is 30, so we have 18 left, which we can break up any way we like as long as we don't move the median number.

(b) 4, 6, 7, 9, 9. As the median is 7, this has to be the middle number and we need two 9s after that. This adds to 25 and the total required is 35, so we only have 10 left to play with.

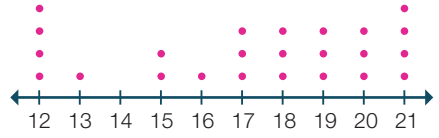
(c) 4, 5, 6, 6, 14. The middle number is 6 and one more is 6. The total is 35, so we still have 23 to play with that can be allocated in many ways.

9 While Veran is correct to say the values must be ordered first when there is an even number of values, the median is found exactly midway between two of the values. The ordered list is: 17, 19, 20, 20, 21, 22, 23, 28, so the median is midway between 20 and 21 and so is 20.5.

10 As the median is 5, we know this must be in the middle; as the mean is 6, we know that the values must add up to 54, and 4 must be the most frequent. Sample answer: 1, 4, 4, 4, 5, 6, 6, 7, 17.

# Exercise 9.3 (p. 508)

1




2 (a)

| STEM | LEAF |
|---|---|
| 2 | 2 3 8 |
| 3 | 0 1 4 7 |
| 4 | 1 2 8 |
| 5 | 2 7 |

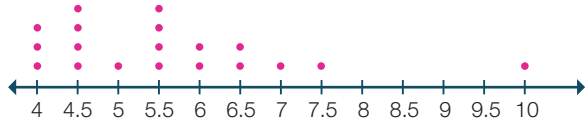
Key 2 | 3 = 23

(b)

| STEM | LEAF |
|---|---|
| 0 | 2 3 8 |
| 1 | 4 5 8 |
| 2 | 1 5 6 7 9 |
| 3 | 0 3 4 5 7 |
| 4 | 2 4 |

Key 1 | 4 = 14

3 (a)




The median is 5.5, the range is 6 and 10 seems to be an outlier.

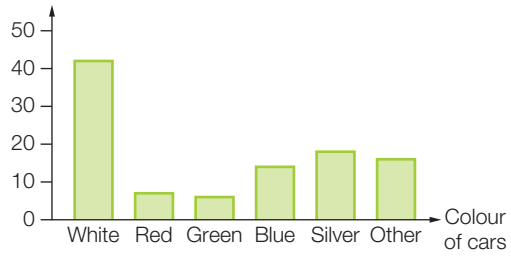
(b)

| STEM | LEAF |
|---|---|
| 5 | 5 6 9 |
| 6 | 7 |
| 7 | 5 7 |
| 8 | 8 8 8 9 |
| 9 | 9 |
| 10 | 0 |
| 11 | 0 2 5 |
| 12 | 1 |

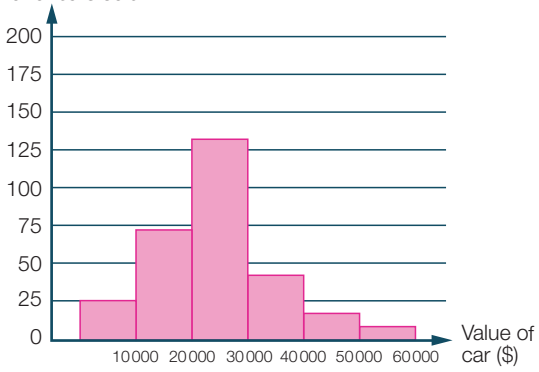
Key 8 | 8 = 88

The median is 88, the range is 66 and there do not appear to be any outliers.

**4** Number




**5** No. of cars sold




**6 (a)**

| STEM | LEAF |
|---|---|
| $2_L$ | 0 1 2 3 |
| $2_U$ | 7 7 8 |
| $3_L$ | 1 4 |
| $3_U$ | 8 |
| $4_L$ | 2 |

Key 2 | 1 = 21

**(b)**

| STEM | LEAF |
|---|---|
| $4_L$ | 2 |
| $4_U$ | 7 7 8 |
| $5_L$ | 1 2 3 |
| $5_U$ | 6 7 8 |
| $6_L$ | 1 4 |
| $6_U$ | 8 |

Key 5 | 2 = 52

**7 (a)**

| STEM | LEAF |
|---|---|
| $12_L$ | 1 |
| $12_U$ | 5 6 6 8 8 9 9 |
| $13_L$ | 0 3 3 4 |
| $13_U$ | 6 8 |
| $14_L$ | 0 2 2 3 |
| $14_U$ | 6 |
| $15_L$ | 0 |

Key 12 | 6 = 126

**(b)**

| STEM | LEAF |
|---|---|
| 12 | 1 5 6 6 8 8 9 9 |
| 13 | 0 3 3 4 6 8 |
| 14 | 0 2 2 3 6 |
| 15 | 0 |

Key 13 | 0 = 130

**8 (a)** C **(b)** C

**9 (a)** Thursday **(b)** 13 **(c)** 45

**10 (a)** 10 cm **(b)** 80 cm **(c)** 850–860 **(d)** 13 **(e)** 50

**11 (a)** The bars are horizontal; the scale is at the bottom.

**(b)** 820 million **(c)** 430 million **(d)** Spanish

**(e)** The first two numbers of the scale are given and then every second number is left off.

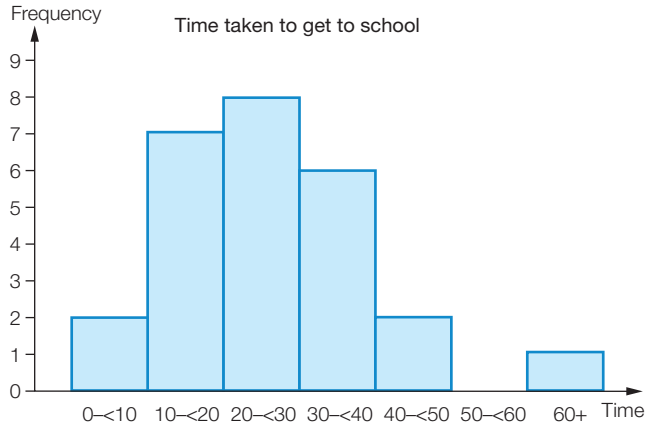
**(f)** Yes

**12 (a)** continuous

**(b)**

| Distance | Tally | Frequency |
|---|---|---|
| 0–<10 | \| \| | 2 |
| 10–<20 | 𝍸 \| \| | 7 |
| 20–<30 | 𝍸 \| \| \| | 8 |
| 30–<40 | 𝍸 \| | 6 |
| 40–<50 | \| \| | 2 |
| 50–<60 | | 0 |
| 60+ | \| | 1 |

**(c)**




**(d)** Most students seem to live locally except for one student who travels 69 minutes. This could appear to be an outlier.

**13 (a)** Australia

**(b)** Canada

**(c)** UK

**(d)** Aluminium cans

**(e)** Plastics




**(f)** Australia's recycling rate for glass is $\approx \frac{1}{2}$ of Switzerland's rate.
Australia's recycling rate for plastics is $\approx \frac{1}{2}$ of Austria's rate.
Australia's recycling rate for steel cans is a little under $\frac{1}{2}$ of Japan's rate.
Australia's recycling rate for aluminium cans/packaging is $\approx \frac{2}{3}$ of Finland's rate.

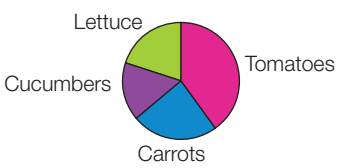
**(g)** Australia's recycling rate for glass is $\approx \frac{4}{3}$ Ireland's rate.
Australia's recycling rate for plastics is ≈ 4 times Canada's rate.
Australia's recycling rate for steel cans is over 5 times NZ's rate.
Australia's recycling rate for aluminium cans/packaging is over 4 times Ireland's rate.

## Open-ended – Sample answers

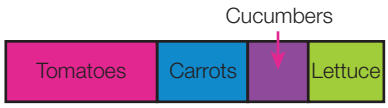
**14** Students' own answers.

## Exercise 9.4 (p. 522)

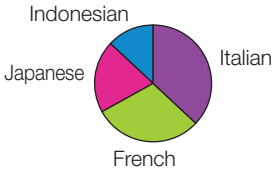
**1 (a) (i)**




**(ii)**



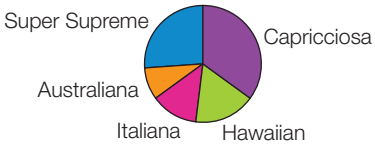

**(b) (i)**



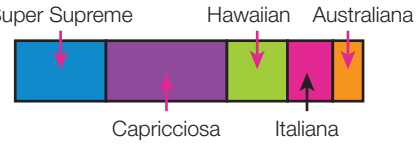

**(ii)**



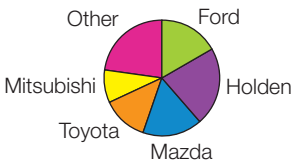

**(c) (i)**



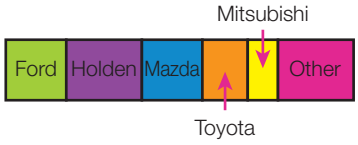

**(ii)**



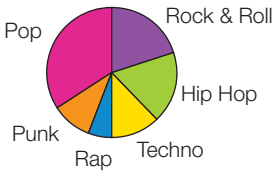

**(d) (i)**



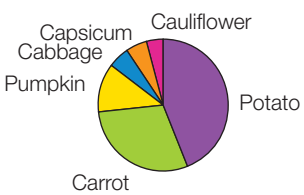

**(ii)**



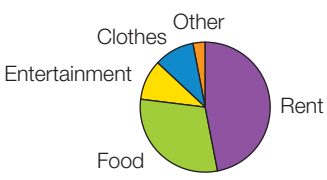

**2 (a) (i)**



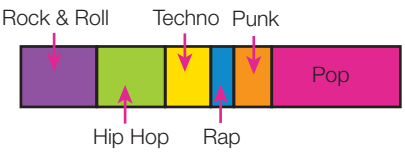

**(ii)**



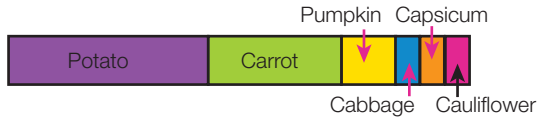

**(iii)**




**(b) (i)**




**(ii)**

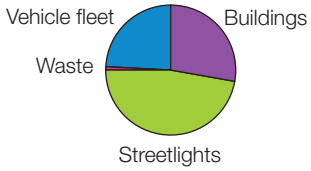



**(iii)**




**3 (a)** A **(b)** D

**4 (a)**




**(b)**




**(c)** Students' own answers.

**5 (a)** 3.3% **(b)** 5.2% **(c)** 5–25 minutes **(d)** 94.3%

**6 (a)** 4% **(b)** 1% **(c)** 4%

**(d)** **(i)** non-recyclables **(ii)** 79%

**(e)** paper **(f)** glass **(g)** No

**7 (a)** bags **(b)** sheeting and film

**(c)** confectionery wrappers **(d)** percentages

**(e)** all items not included in the other categories

**(f)** It is important because the survey would not be of much value if only a small number of items had been collected.

**(g)** They divide something into sections.

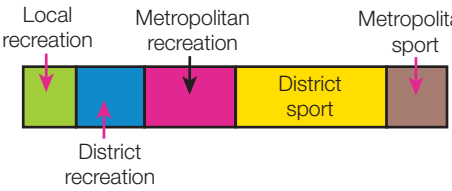
**8 (a)** Local recreation—70°, District recreation—90°, Metropolitan recreation—60°, District sport—100°, Metropolitan sport—40°

**(b)** 17% **(c)** 28% **(d)** 39%

**(e)** Recreation, 22% more

**(f)** Metropolitan sport

**(g)**




**9 (a)** English **(b)** 11%

**(c)** Hindi, Sinhalese, Samoan, Khmer, Macedonian, Greek

**(d)** Arabic

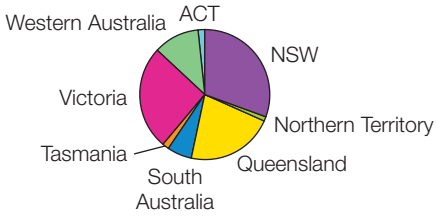
**(e)** There are many other languages spoken at home but each is less than 1% of the population.

**(f)** Students' own answers.

**10** **(a)** 347 514

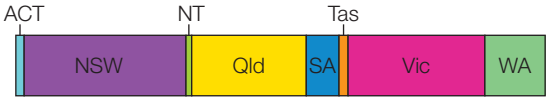
**(b)** ACT—6°, NSW—110°,
NT—4°, Qld—78°,
SA—22°, Tas—6°,
Vic—93°, WA—41°,

**(c)**




**(d)** A length of 70 mm might work reasonably well.

**(e)** Using this length, the pieces, in alphabetical order, would be: 1.1 mm, 21.4 mm, 0.8 mm, 15.2 mm, 4.3 mm, 1.2 mm, 18.1 mm, 8.0 mm. This is a total of 70.1 mm which is about as good as we can do. Your graph should look something like the following, although its length will probably be different.




**(f)** It will probably be easier to draw the sector graph, although neither is very easy. Some rounding is certainly required with the divided bar chart.

**11** **(a)** Rates and garbage charges

**(b)** 29–30°

**(c)** 8% (to nearest %)

**(d)** Government grants; this section is the larger of the two.

## Open ended – Sample answers

**12** Students' own answers.

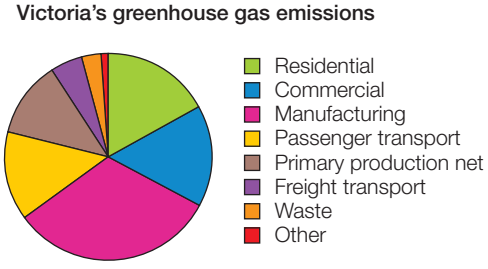
**13** Students' own answers—graphs generated using Excel.

# Half-time 9 (p. 522)

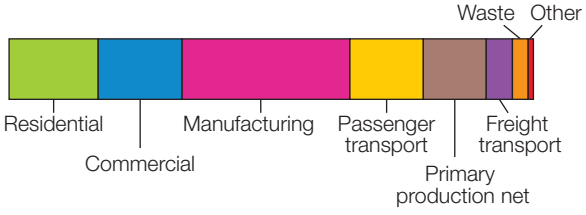
**1** **(a)** **(i)** 3.7 **(ii)** 3 **(iii)** 2

**(b)** **(i)** 5 **(ii)** 5 **(iii)** 4 and 6

**2** **(a)**




**(b)**




**3** **(a)** This gives eight groups, which is within the 5–10 range we prefer.

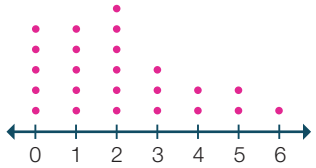
**(b)**

| Height range | Frequency |
|---|---|
| 125–129 | 1 |
| 130–134 | 1 |
| 135–139 | 2 |
| 140–144 | 7 |
| 145–149 | 5 |
| 150–154 | 6 |
| 155–159 | 2 |
| 160–164 | 1 |

**4**

| Number absent | Frequency |
|---|---|
| 0 | 8 |
| 1 | 6 |
| 2 | 4 |
| 3 | 2 |
| 4 | 1 |

**5**




**6** **(a)**

| STEM | LEAF |
|---|---|
| 6 | 2 7 |
| 7 | 4 6 |
| 8 | 0 2 4 5 6 |
| 9 | 0 1 |
| 10 | |
| 11 | |
| 12 | |
| 13 | 2 |

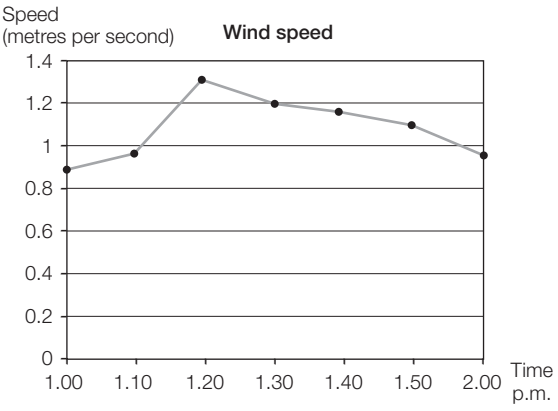
Key 6 | 2 = 62

**(b)** 83

**(c)** 132 appears to be an outlier. The rest of the data in the set is bunched together.

# Exercise 9.5 (p. 526)

**1** **(a)**




**(b)** Between 1.10 p.m. and 2.00 p.m. (not inclusive)

**2** **(a)** 50 **(b)** 1 p.m. **(c)** 60

**(d)** 11.30 a.m. and 12.30 p.m.

**3** **(a)** C **(b)** B **(c)** C

**4** **(a)** **(i)** 41° **(ii)** 17 January

**(b)** **(i)** 19° **(ii)** 25, 26 January

**(c)** 22° and 38° **(d)** 19° and 27°

5 (a) Start 2008 40%, end 2008 38%

(b) 2010 (c) Feb–July 2007

(d) 3% decrease. As the storage levels drop about 3% every year, there is not a lot of rain in December in Perth.

(e) Increase of 9.5%

(f) August

(g) January

6 (a) 360 000

(b) 525 000

(c) Actual numbers were unknown because many Indigenous people live in isolated communities and many move from place to place.

(d) 705 000 and 715 000

(e) More Australians willing to be recognised as of Aboriginal descent. Better health care means less infant mortality and a longer life span.

7 (a) 22 million (b) 37 million (c) 41 million

(d) Number of births > number of deaths, immigration > migration.

(e) Overcrowding of cities, water shortages, environmental damage, not enough food produced.

(f) 37 million

(g) Immigration > migration

(h) 32–33 million

(i) Number of births + number of immigrants = number of deaths + number of migrants.

8 (a) 2.35 million (b) 2007 (c) 1995–2005

(d) 2.45 million, 2.65 million, 3 million

(e) The over 65 population of Australia has been steadily increasing since 1990. The rate of increase from 2006–2010 has been even higher than in previous years.

(f) Babies born after the War are now aged 65 and older. Immigrants who arrived after 1945 are also now aged 65 and older.

(g) ≈ 3.75 million

9 (a) Yes, 2000 (b) 150 cm (c) 175 cm

(d) 2000 (e) 2002 (f) 1999

(g) 2000 or 2003 (h) 2003 (i) 2006

(j) August (k) September

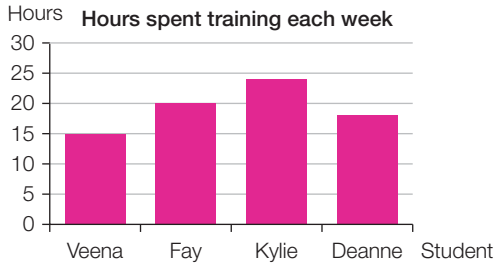
(l) Perhaps—the last 4 years have seen less snow than the previous 6 years, but it could simply be part of a longer cycle.

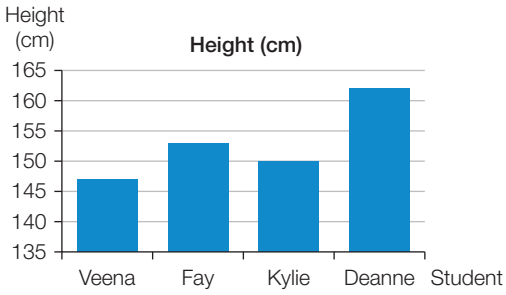
(m) Line graphs are more appropriate to show this information as the depth is changing according to time, and a large number of bars would be needed to show the daily change.

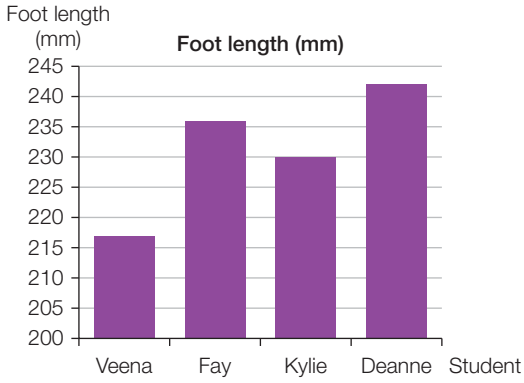
### Open-ended – Sample answers

10 Students' own answers.

## Exercise 9.6 (p. 538)

1 


The shape of the graph of hours spent training each week is quite different from the shape of the other two graphs. There seems to be no connection between the hours spent training and either the height or the foot length.

The shapes of the height graph and the foot length graph are very similar, so there could be a connection between these two variables.

2 (a) Islander College, 350 students

(b) Bottom part; Hilltop College

(c) Gumnut College

(d) Jinglebells College

(e) Faraway and Gumnut Colleges

3

| | Mean | Median | Range |
|---|---|---|---|
| Adelaide | 12.1917 | 11.9 | 9.6 |
| Brisbane | 15.4917 | 15.95 | 12.3 |

Both the lowest mean minimum temperature and the highest mean minimum temperature occur in Brisbane, which accounts for it having a much greater range in temperature. However, both the mean and median indicate that Brisbane is, overall, a warmer place.

4

| | Mean | Median | Range |
|---|---|---|---|
| Adelaide | 7.5 | 7 | 6 |
| Brisbane | 8 | 8 | 3 |

Although Adelaide has the individual highest value, its lack of consistency means that Brisbane has a higher mean and median number of sunshine hours. Adelaide's range is about double that of Brisbane.

5 (a) Glass

(b) Paper

(c) 2002/03

(d) 2001/02

(e) No apparent change in the amount recycled.

(f) A steady increase in the amount recycled, but the amount is still very small.

6 (a)

| | Mean | Median | Mode | Range |
|---|---|---|---|---|
| Set A | 5.7 | 6.5 | 7 | 9 |
| Set B | 5.55 | 5 | 2 & 9 | 7 |

The numbers in Set A are, on average, slightly larger and more spread out. The mode does not really give us any useful information.

(b)

| | Mean | Median | Mode | Range |
|---|---|---|---|---|
| Set A | 3.75 | 3.5 | 2 | 7 |
| Set B | 5.75 | 6.5 | 7, 8 & 9 | 8 |

Although the range of both sets is very close, the numbers in Set A appear, on average, to be quite a bit smaller than those in Set B. Again, the mode gives us no useful information.

7 April paid a mean price of $47.78 for her fruit, whereas Izzy paid $50.71. April bought 9 cases compared to 7 for Izzy with the range of prices paid by both friends being the same. Due to the small numbers, neither the median nor the mode will tell us anything useful.

8 (a) NSW

(b) 8 000 000 t

(c) NSW, Vic, SA, ACT

(d) ACT

(e) NT

(f) SA

9 (a) No

(b) Japan, because they have many more hospital beds per 10 000 people than the other countries.

(c) Because Japan has one of the highest GDPs, the amount it spends on health facilities is to be expected.

(d) Indonesia

(e) Because Indonesia has one of the lowest GDPs, the amount it spends on health facilities is to be expected.

(f) Australia's health-care facilities do not seem to be as good as they should be given its GDP, which is the highest of all the countries shown.

(g) Vietnam's health-care facilities seem to be very good given its GDP, which is the lowest of all the countries shown.

10 (a) (i) Paper/cardboard, plastic bottles, glass.

(ii) Easiest to collect, easiest to recycle.

(b) Steel cans

(c) Paper/cardboard, plastic bottles and glass

11 (a) The number of mobile phones in developed countries is 10 times that of developing countries. Half the population of developed countries has a mobile phone.

(b) The number of mobile phones doubled to nearly 100%.

(c) The number has increased 9 times from 2000 to 2007 to almost 50%.

(d) The population in the developing world is much greater than the population of the developed world.

(e) 1999–2000

(f) 80%–90%

12 (a) The following table summarises the results.

| | Mean | Median | Range |
|---|---|---|---|
| 2009 | 36 666.1 | 37 070.5 | 27 623 |
| 2005 | 31 656.8 | 31 432 | 21 282 |
| 2000 | 27 406.5 | 26 874 | 25 041 |

The total membership numbers have been increasing across the time period shown. This is supported by considerable increases in both the mean and the median values. The range has been a bit variable, which may have been caused by the high membership number for Hawthorn in 2009. However, these figures clearly indicate that the AFL should be quite happy about the trend in membership numbers.

(b) The mode is not useful because it is unlikely that any particular data value is going to be repeated when the range of values is over 20 000 and we only have 16 values.

(c) The following table summarises the results.

| | Mean | Median | Range |
|---|---|---|---|
| Victorian | 37 530.6 | 37 070.5 | 24 371 |
| Non-Victorian | 35 225.3 | 34 905.5 | 21 599 |

On average, the Victorian teams seem to have higher memberships, but this is assisted by the very low numbers for Brisbane Lions and Sydney Swans, which are the two smallest totals overall. Also, Hawthorn's very high total has boosted Victorian numbers. The range for non-Victorian clubs is smaller, indicating more even results for these clubs, although there are still significant differences.

(d) The biggest raw increase is Hawthorn, which has increased by 25 617 members (which is more than the total Brisbane Lions membership in 2009). This also represents the biggest percentage increase of 95.3%, which is followed by St Kilda at 78.7%. The best increase for a non-Victorian club is Fremantle, which increased

by 57.3%. The only clubs to decline over this time period were Port Adelaide and Sydney Swans.

(e) You would also need to know how many people call themselves fans and then decide what percentage of them actually take out memberships.

### Open-ended – Sample answers

**13** (a) mean: 1.611 m median: 1.65 m mode: 1.67 m

(b), (c) Students will produce their own answers but will need to consider the following. The sum of the existing girls is 16.11, so the new total needs to be either 21.132 m or 17.532 m. The new girls cannot be the same height as any of the existing girls unless they are 1.65 m (the median).

**14** Set A: 13, 10, 13, 21, 5, 12, 19, 12, 25, 20, 16, 3, 10, 10, 18, 17
Set B: 4, 2, 10, 19, 1, 23, 18, 17, 10, 3, 16, 18, 10, 14, 18, 17

Set A: median: 13, mode: 10, mean: 14, range: 22
Set B: median: 15, mode: 10, mean: 12.5, range: 22

## Exercise 9.7 (p. 547)

**1** (a) $\frac{1}{2}$ (b) $\frac{2}{11}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

(e) $\frac{1}{13}$ (f) $\frac{1}{26}$ (g) $\frac{1}{26}$ (h) $\frac{1}{52}$

**2** (a) impossible (b) certain

(c) not likely (d) certain

(e) certain (f) not likely

(g) impossible (h) not likely

(i) certain

**3** (a) A (b) C (c) D (d) C (e) B

**4** (a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(b) {a, b, i, l, o, p, r, t, y}

(c) {HH, HT, TH, TT}

**5** (a) (i) $\frac{1}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{5}$

(b) (i) $\frac{2}{7}$ (ii) $\frac{3}{14}$ (iii) $\frac{3}{7}$

**6** (a) (i) Yes (ii) 50

(b) (i) No (ii) 200

(c) (i) No (ii) Simon

**7** (a) No

(b) Nicky's winning colour is two times the area of Steve's.

**8** (a) Yes (b) No (c) Yes (d) No (e) No

**9** Sample answer: impossible, virtually impossible, almost never, most unlikely, sometimes, maybe, even chance, quite frequently, more often than not, quite often, often, likely, usually, very likely, almost always, sure thing.

**10** (a) 4 (Possible: Brown, Brown, Blue, Blue)

(b) 8 (Possible: Blue, Blue, Blue, Blue, Blue, Blue, Brown, Brown)

(c) 3 (Possible: Blue, Brown, Blue)

(d) 7 (all 6 Blues could come out first, then Brown)

### Open-ended – Sample answers

**11** (a) Monday will follow Tuesday. Humans will run 100 m in under 5 seconds.

(b) It will reach 50°C in Australia next year. Australia will win the most gold medals at the next Olympics.

(c) It will rain in Sydney in the next month. Australia will win a swimming gold medal at the next Olympics.

(d) The Sun will rise tomorrow. Someone will be born in Australia today.

**12** Students' own answers.

## Challenge 9 (p. 553)

**1** C If the dice are arranged so that the one-dot and the two-dot faces can't be seen, then the greatest number of dots will be viewable; that is, $42 - 6 = 36$.

Opposite dots on a die total 7
i.e. $1 + 6$, $2 + 5$, $3 + 4$, $4 + 3$, $5 + 2$, $6 + 1$,
there are 6 lots, $6 \times 7 = 42$.

Or: $2 \times (6 + 5 + 4 + 3) = 36$

**2** The reason Mr Black is wearing the white tie and not the green tie is because in the passage, Mr Black is talking to the man wearing the green tie. Therefore, Mr Black must be wearing the white tie. Mr Green cannot be wearing the white tie so he must be wearing the black tie. Therefore, Mr White is wearing the green tie.

**3** D 4 of the six faces will contain odd numbers.

**4** D The shortest time wins the race, so the shortest bar is Maggie.

**5** B $3B + 6G + 2R + 6Y = 17$ marbles. As the probability is $\frac{3}{7}$, the number of marbles in the bag must be a multiple of 7, so at least 21. There are 9 blue or green marbles, $9 = 3 \times 3$, so need 21 marbles in the bag. Vijay adds 4 white marbles to the bag.

**6** A $15 + 10 + 20 + 30 = 75$. $100 - 75 = 25$

**7** C Has won 19 of the 20 games, needs to win 24 out of 25 games to reach 96%, hence next 5 games.

## Chapter review 9 (p. 554)

**1**

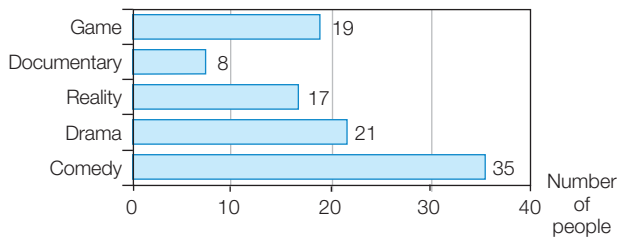
| Number of television sets owned | Tally | Frequency |
|---|---|---|
| 0 | \|\|\| | 3 |
| 1 | 𝍸 𝍸 \| | 11 |
| 2 | 𝍸 | 5 |
| 3 | \|\| | 2 |
| 4 | \| | 1 |
| 5 | \| | 1 |
| | | 23 |

**2** (a) mean = 3.55, median = 3, mode = 2

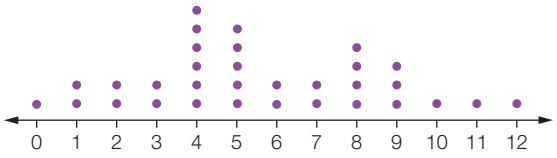
(b) mean = 35, median = 35, there is no mode

(c) mean = 3.72, median = 3.9, mode = 4.1

**3**




**4 (a)**




**(b)** 5

**(c)** No, the data set seems well spread out.

**5 (a)**

| STEM | LEAF |
|---|---|
| 0 | 0 0 0 0 0 1 2 2 2 3 4 5 5 6 7 7 |
| 1 | 1 4 5 7 7 7 8 8 9 |
| 2 | 0 1 |
| 3 | 1 |
| 4 | 0 1 2 2 6 |
| 5 | 0 0 2 6 8 |
| 6 | 1 3 4 |
| 7 | 7 |
| 8 | 8 |
| 9 | |
| 10 | |
| 11 | 5 |

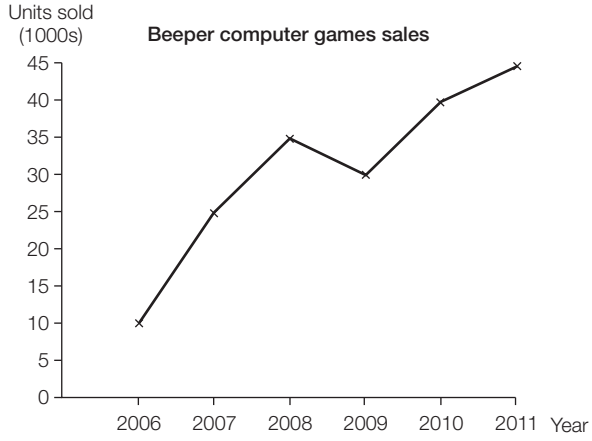
Key 2 | 0 = 20

**(b)** 17.5

**(c)** The score of 115 appears to be an outlier.

**6 (a)** D **(b)** D

**7**

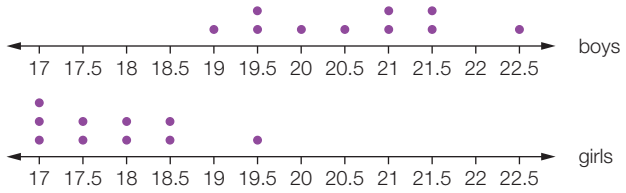



**8**

| | Mean | Median | Range |
|---|---|---|---|
| Cairns | 167.7 | 92.4 | 425.5 |
| Brisbane | 78.9 | 73.35 | 95.7 |

These two cities both receive a lot of rain, although Cairns clearly receives more rain than Brisbane. The wettest month for Cairns is far wetter than the wettest month for Brisbane, as reflected in the mean and range results. The driest months are almost identical. Cairns has a wet summer and a dry winter, whereas Brisbane follows a similar pattern but has a wetter spell in May and June. The median results are not that far apart which is, again, a reflection of the similar patterns, but still shows Cairns to be wetter.

**9**




The boys clearly have larger handspans. The median for boys is 20.75 and for girls it is 17.75. The range for boys is 3.5 compared to 2.5 for girls, so the data for boys are slightly more spread out. The most telling feature, however, is that only one girl has a handspan larger than the boy with the smallest handspan.

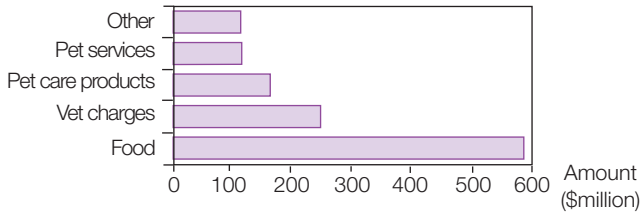
**10 (a)** B **(b)** B

**11 (a)**

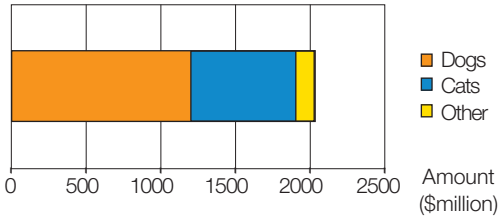
| Shoe size | Frequency |
|---|---|
| 1 | 2 |
| $1\frac{1}{2}$ | 2 |
| 2 | 2 |
| $2\frac{1}{2}$ | 3 |
| 3 | 3 |
| $3\frac{1}{2}$ | 5 |
| 4 | 3 |
| $4\frac{1}{2}$ | 3 |
| 5 | 3 |
| $5\frac{1}{2}$ | 3 |
| 6 | 1 |

**(b)** $3\frac{1}{2}$ **(c)** 6

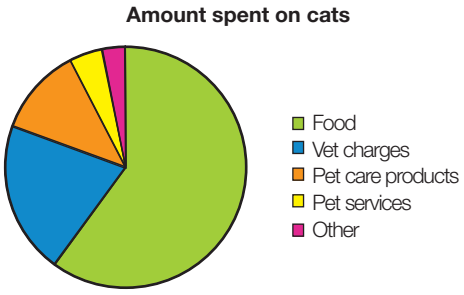
**12 (a)**




**(b)**



(c)




(d) The data sets were not compared over a period of time.

**13** (a) column graph (b) July (c) 30 cm

(d) December, January, February, March

**14** Yes, it is fair.

**15** (a) 0 (b) 1 (c) the second

(d) Probabilities can't be greater than 1.

**16** (a)

| Height (cm) | Frequency |
|---|---|
| 140 | 4 |
| 141 | 1 |
| 142 | 1 |
| 143 | 1 |
| 144 | 2 |
| 145 | 1 |
| 146 | 1 |
| 147 | 2 |
| 148 | 3 |
| 149 | 2 |
| 150 | 1 |
| 151 | 2 |
| 152 | 2 |
| 153 | 1 |
| 154 | 1 |

(b) (i) 140 (ii) 147 (iii) 146.6

(c)

| Height (cm) | Frequency |
|---|---|
| 138 | 1 |
| 141 | 1 |
| 142 | 1 |
| 143 | 2 |
| 144 | 2 |
| 145 | 2 |
| 146 | 2 |
| 148 | 2 |
| 150 | 2 |
| 151 | 1 |
| 152 | 1 |
| 154 | 3 |
| 156 | 2 |
| 157 | 3 |

(d) (i) 154, 157 (ii) 148 (iii) 148.8

(e) On all measures of centre, the girls are taller than the boys. The shortest student, however, is a girl, as is the tallest. The girls' heights are clustered more at the top of the range of values.

(f)

| Height (cm) | Frequency |
|---|---|
| 138 | 1 |
| 140 | 4 |
| 141 | 2 |
| 142 | 2 |
| 143 | 3 |
| 144 | 4 |
| 145 | 3 |
| 146 | 3 |
| 147 | 2 |
| 148 | 5 |
| 149 | 2 |
| 150 | 3 |
| 151 | 3 |
| 152 | 3 |
| 153 | 1 |
| 154 | 4 |
| 156 | 2 |
| 157 | 3 |

(g) (i) 148 (ii) 148 (iii) 147.7

(h) The three measures for the combined group are very close to each other. The average student is about 148 cm tall, the shortest is 138 cm and the tallest is 157 cm.

**17** (a) Don't place any counters on 1, 2, 19 or 20 as these totals will never appear as the sum of the three dice.

(b) The scores 10 and 11 should appear more often than any other total.

(c) The results obtained on any particular game could be quite different from the next game.

## NAPLAN practice 9

**1** B **2** D **3** C **4** C

**5** (a) Friday (b) 30 (c) Thursday

**6** C

# Chapter 10

## Recall 10 (p. 562)

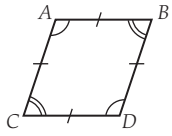
**1** (a) 56° (b) 150° (c) 50°

**2** (a) 20° (b) 60° (c) 40°

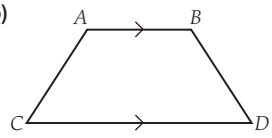
**3** (a) 66° (b) $a° = 110°$, $b° = 70°$, $c° = 70°$

(c) $a° = 120°$

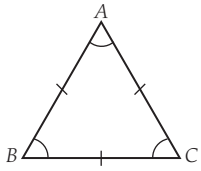
**4** (a)



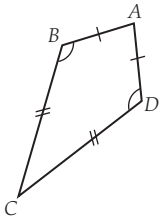

(b)



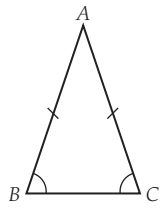

(c)



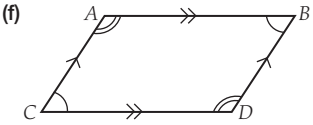

(d)



(e)




(f)




## Exercise 10.1 (p. 565)

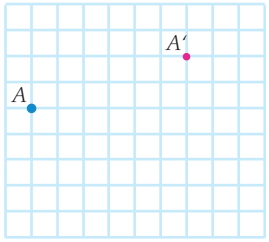
**1** **(a)** 5 units left

**(b)** 5 units down

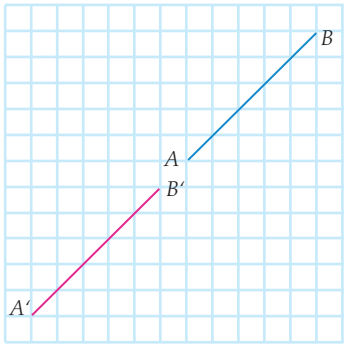
**(c)** 5 units right and 3 units down

**(d)** 7 units left and 2 units up

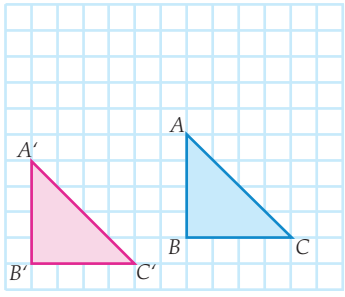
**2** **(a)**



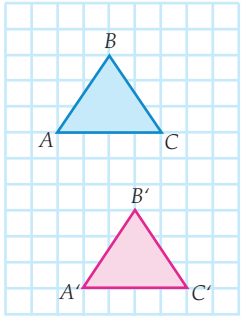

**(b)**



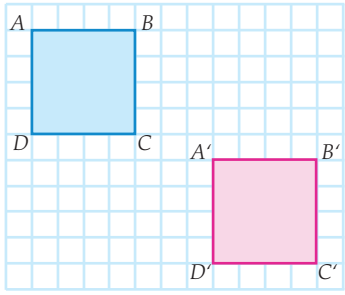

**(c)**



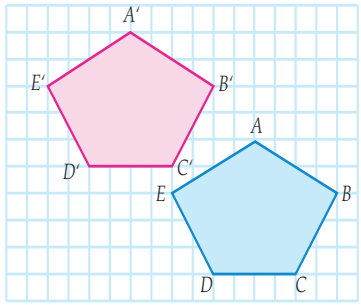

**(d)**




**(e)**

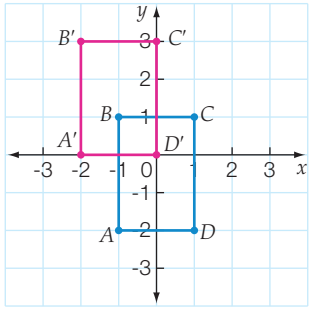



**(f)**




**3** **(a)** $A(-1, -2)$, $B(-1, 1)$, $C(1, 1)$, $D(1, -2)$

**(b)**




**(c)** $A'(-2, 0)$, $B'(-2, 3)$, $C'(0, 3)$, $D'(0, 0)$

**(d)** Subtracting 1 from the $x$-coordinates of the original and adding 2 to the $y$-coordinates of the original will give the image coordinates.

**4** B

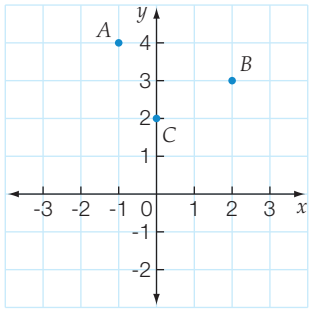
**5** D

**6** 2 units left and 6 units up

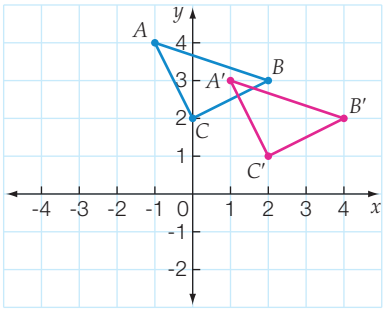
**7** [-3, -4]

**8** [-10, -4]

**9** **(a)**



(b)




(c) $A'(1, 3)$, $B'(4, 2)$, $C'(2, 1)$

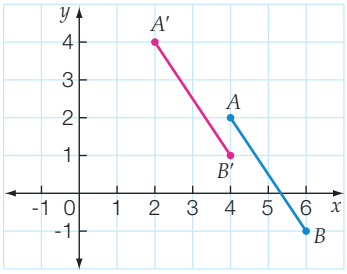
**10** 10 units left and 4 units up

**11** 4 units down

**12** [0, 9]

**13** (a) $A'(2, 4)$, $B'(4, 1)$

(b)

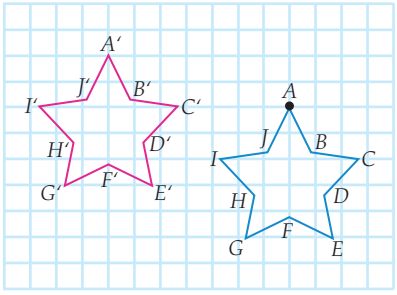



**14** 200 units right

**15** (a) 7 units left and 2 units up

(b) 7 units right and 2 units down

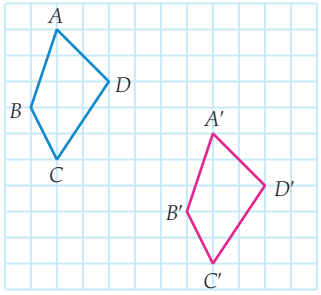
(c)




## Open-ended – Sample answers

**16** Students' own answers.

**17**

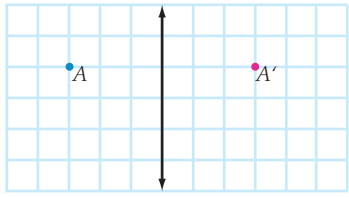



The reverse translation is 6 units left and 4 units up.

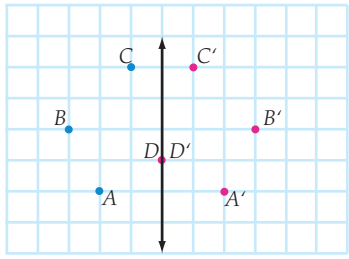
**18** The statement is correct, because the order in which you do translations does not affect the result of the final image. This is because when translating a shape you add or subtract units, and adding or subtracting can be done in any order.

# Exercise 10.2 (p. 570)

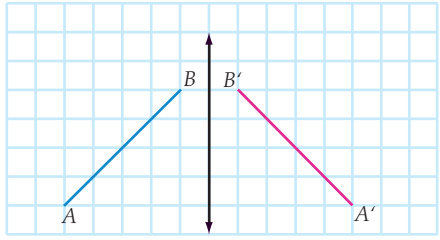
**1** (a)



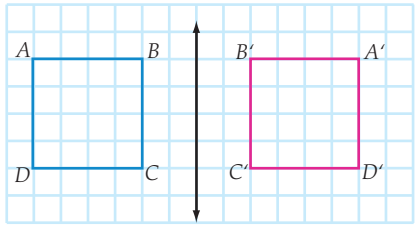

(b)



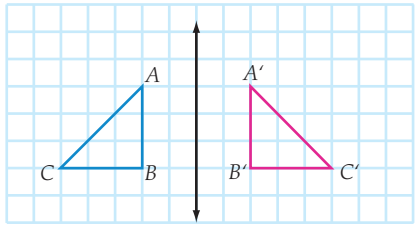

(c)



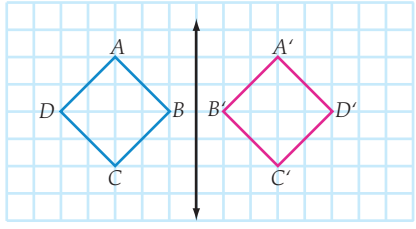

(d)



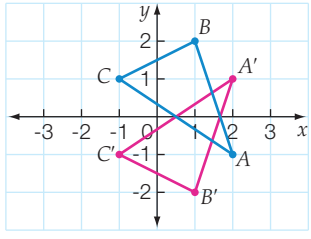

(e)




(f)



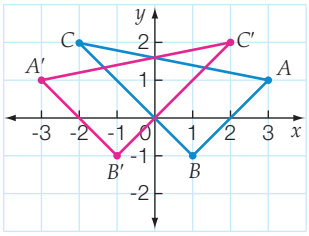

**2** (a) (i) $A(2, -1)$, $B(1, 2)$, $C(-1, 1)$




(iii) $A'(2, 1)$, $B'(1, -2)$, $C'(-1, -1)$

(iv) The $x$-coordinates of the image are the same as the $x$-coordinates of the original but the $y$-coordinates of the image are the negative of the $y$-coordinates of the original.

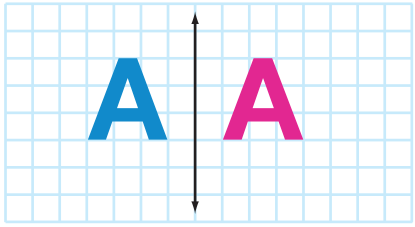
(b) (i) $A(3, 1)$, $B(1, -1)$, $C(-2, 2)$



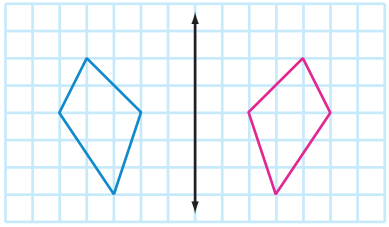

(iii) $A'(-3, 1)$, $B'(-1, -1)$, $C'(2, 2)$

(iv) The $x$-coordinates of the image are the negative of the $x$-coordinates of the original but the $y$-coordinates of the image are the same as the $y$-coordinates of the original.

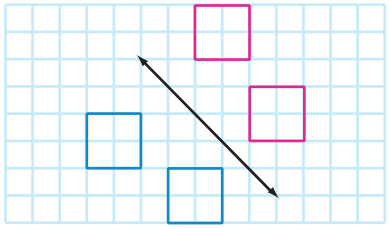
3 (a)



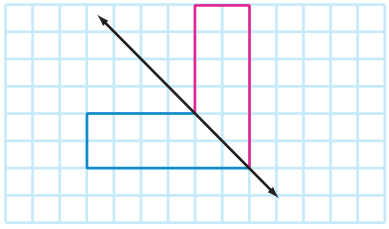

(b)



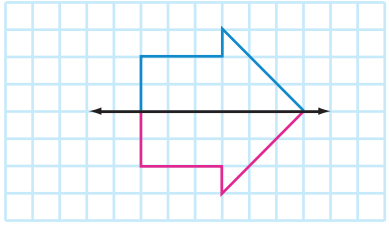
(c)



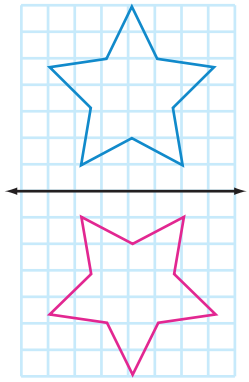
(d)



(e)

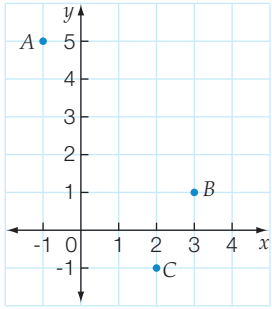


(f)

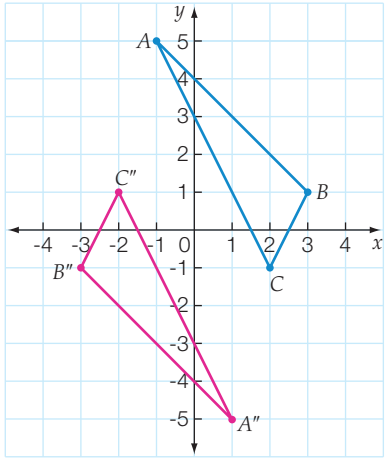


4 B

5 (a)

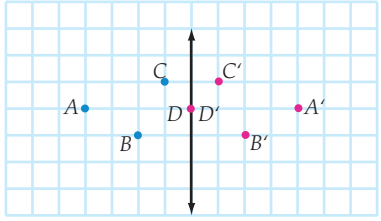



(b)

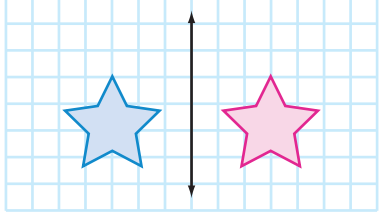



(c) $A''(1, -5)$, $B''(-3, -1)$, $C''(-2, 1)$

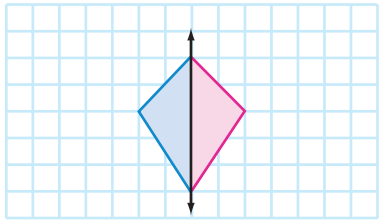
6 (a)



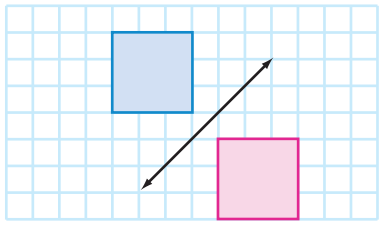

(b)



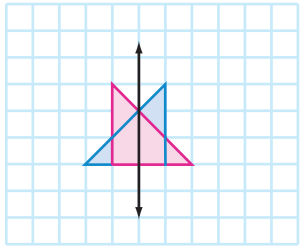
(c)



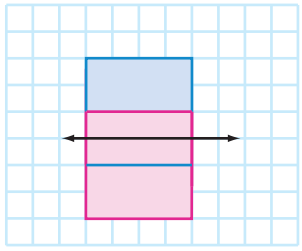
(d)



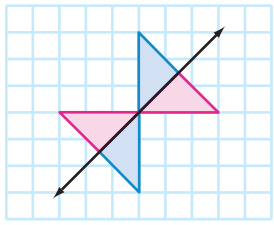
7 (a)



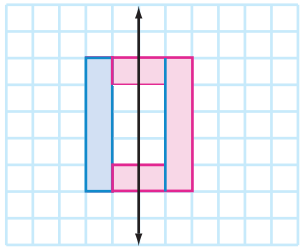
(b)



(c)

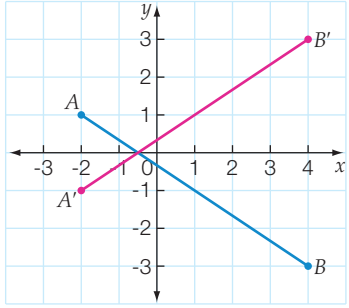


(d)



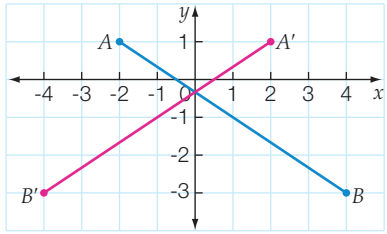
8 (a) $A'(-2, -1)$, $B'(4, 3)$

(b)




(c) $A'(2, 1)$, $B'(-4, -3)$

(d)




9 (a) 0, 1, 3, 8, 10, 11, 13, 18, 30, 31, 33, 38

(b) 0.00, 1.00, 3.00, 8.00, 10.00, 11.00

(c) 72

(d) 96

(e) No, in a 24-hour mode, there are only two more hours (13.00 and 18.00) than the 12-hour mode when the time is correctly reflected.

## Open-ended – Sample answers

10 Students' own answers.

11 Students' own answers.

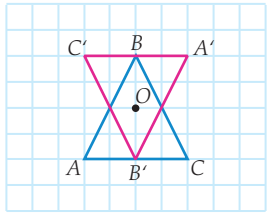
12 Students' own answers.

13 (a), (b) Students' own diagrams.

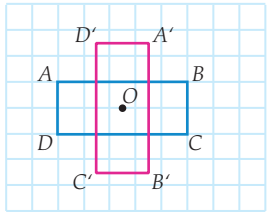
(c) The images found in parts (a) and (b) are the same.

# Exercise 10.3 (p. 577)

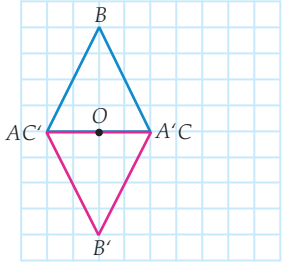
1 (a)



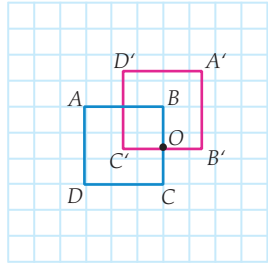

(b)



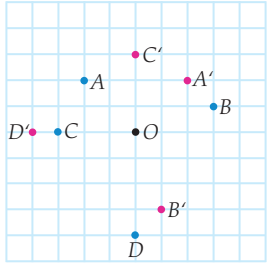

(c)



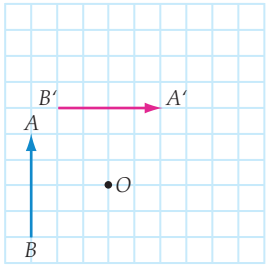

(d)



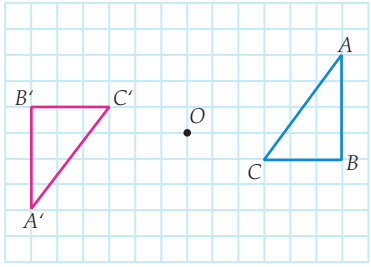

2 (a)



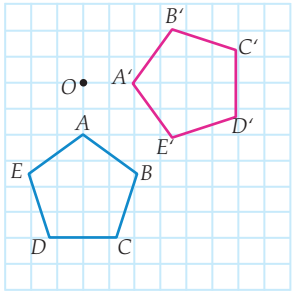
(b)




(c)

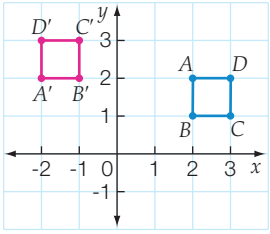



(d)




3 (a) $A(2, 2)$, $B(2, 1)$, $C(3, 1)$, $D(3, 2)$

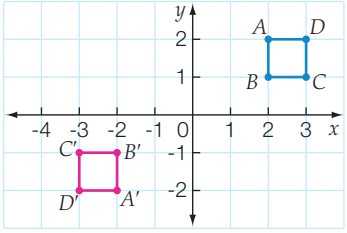
(b) (i)




(ii) $A'(-2, 2)$, $B'(-1, 2)$, $C'(-1, 3)$, $D'(-2, 3)$

(iii) The $x$-coordinates of the image are the negative of the $y$-coordinates of the original vertices and the $y$-coordinates of the image are the $x$-coordinates of the original vertices.

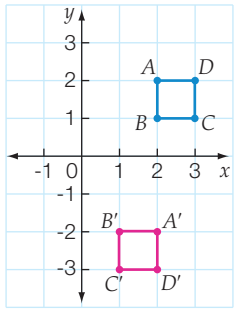
(c) (i)




(ii) $A'(-2, -2)$, $B'(-2, -1)$, $C'(-3, -1)$, $D'(-3, -2)$

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

(d) (i)



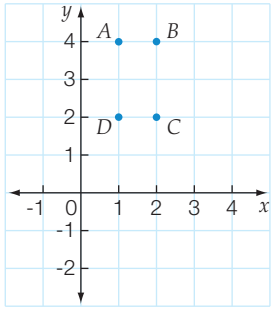

(ii) $A'(2, -2)$, $B'(1, -2)$, $C'(1, -3)$, $D'(2, -3)$

(iii) The $x$-coordinates of the image vertices are the $y$-coordinates of the original vertices and the $y$-coordinates of the image vertices are the negative of the $x$-coordinates of the original vertices.

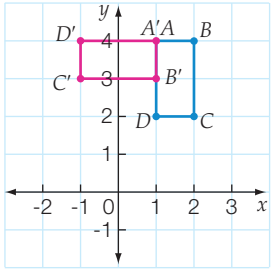
4 C

5 B

6 (a)




(b)




(c) $A'(1, 4)$, $B'(1, 3)$, $C'(-1, 3)$, $D'(-1, 4)$

7 240°

8 (a) Rotated 180° about $O$.

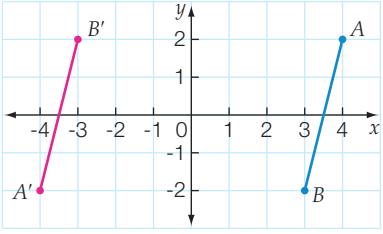
(b) Rotated 90° in an anticlockwise direction or 270° in a clockwise direction about $O$.

(c) Rotated 90° in an anticlockwise direction or 270° in a clockwise direction about $O$.

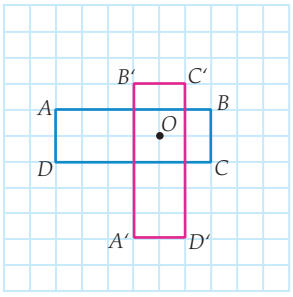
(d) Rotated 180° about $O$.

9 (a) $A'(-4, -2)$, $B'(-3, 2)$

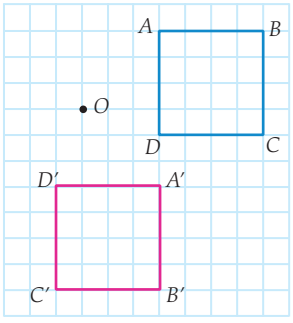
(b)



**10 (a)**




**(b)**




**11 (a)** H, I, N, O, S, X, Z

**(b)** All of these letters are perfectly symmetrical.
H, I, O, X

**12 (a)** 120° **(b)** 180° **(c)** 90°

**(d)** 72° **(e)** 22.5° **(f)** 36°

**13** This is a reflection, because the orientation of the red square's vertices is completed in a manner that indicates a direct reflection. If it was rotated, the order of the vertices would also be rotated.

### Open-ended – Sample answers

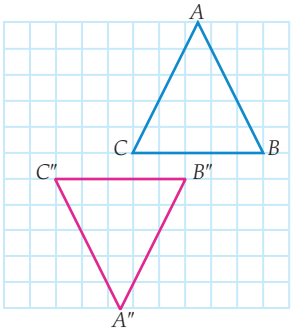
**14** Students' own answers.

**15** Disagree, as the order of the vertices of a figure is shifting in the same direction that the rotation is taking place. In a reflection, the vertices stay in a similar orientation, each an equivalent distance from the line of reflection.

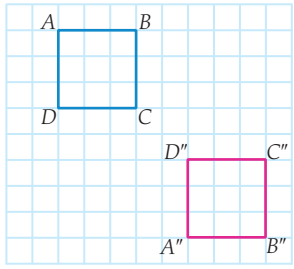
**16 (e)** The images in parts **(b)** and **(d)** are the same as the original polygons.

## Exercise 10.4 (p. 586)

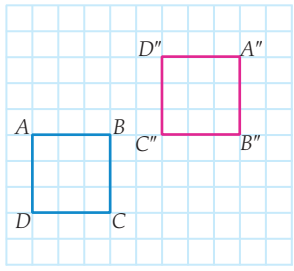
**1 (a)**



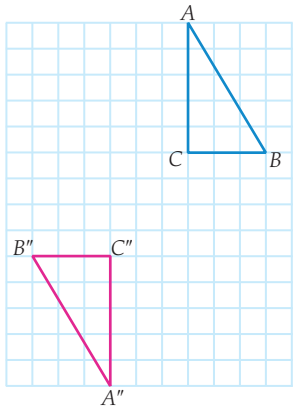

**(b)**



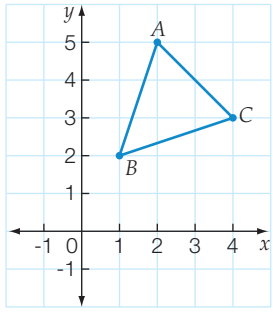

**(c)**



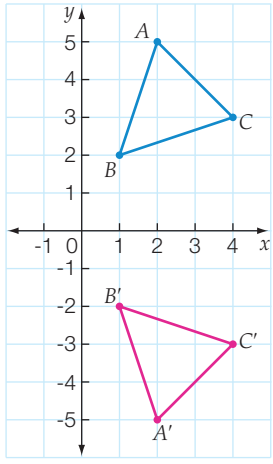

**(d)**




**2 (a)** $A(2, 5)$, $B(1, 2)$, $C(4, 3)$

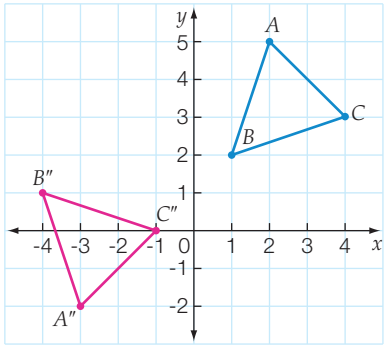



**(b)**



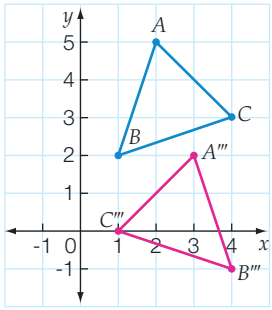

**(c)** $A'(2, -5)$, $B'(1, -2)$, $C'(4, -3)$

(d)



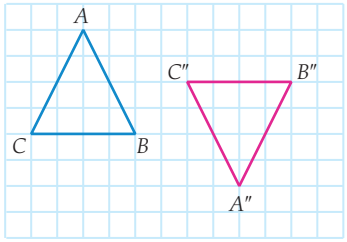

(e) $A''(-3, -2)$, $B''(-4, 1)$, $C''(-1, 0)$

(f)

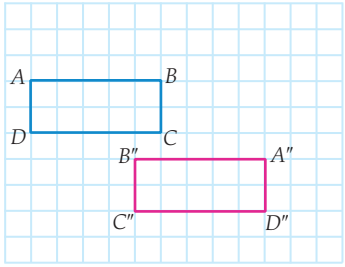



(g) $A'''(3, 2)$, $B'''(4, -1)$, $C'''(1, 0)$

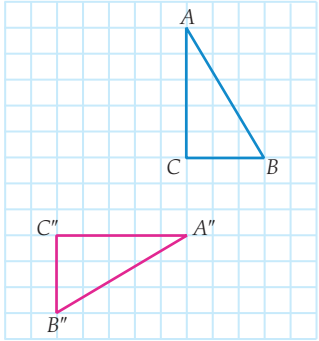
3 (a)



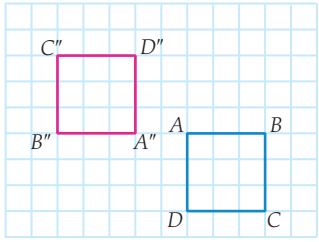

(b)



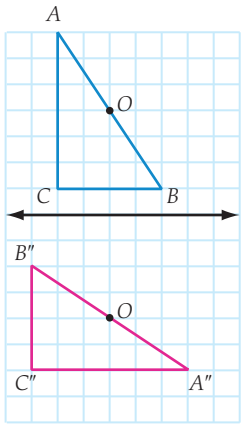

(c)



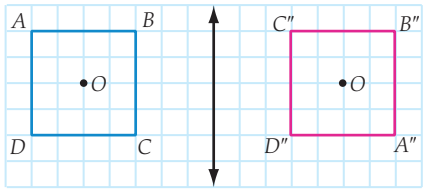

(d)



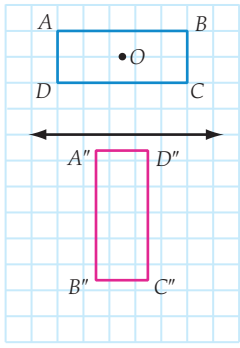

(e)



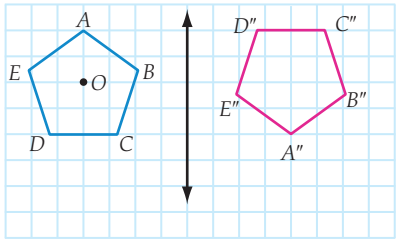

(f)




(g)



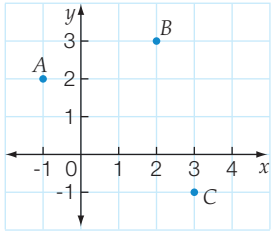

(h)



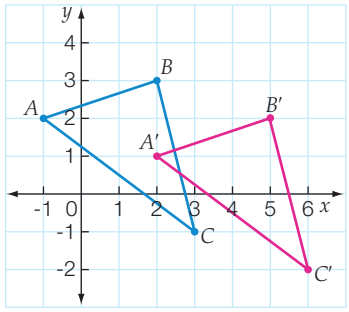

4 B

5 D

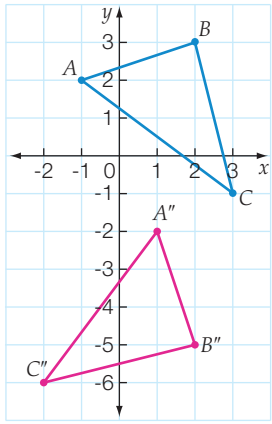
6 (a)



(b)




(c)




(d) $A''(1, -2)$, $B''(2, -5)$, $C''(-2, -6)$

7 (a) Reflect along the line $C''B''$ and translate 3 units right and 1 unit up.

(b) Reflect along the line $D''C''$ and translate 2 units up and 5 units left.

(c) Rotate 90° in an anticlockwise direction about $B''$ and translate 5 units left.

(d) Rotate 180° about $C''$ in either direction and translate 4 units up and 3 units right.

8 (a) Reflected along the axis of reflection and then translated 4 units right and 2 units down.

(b) Translated 8 units down and then reflected along the vertical line of reflection.

(c) Rotated 180° about point $A$ and then translated 5 units right and 3 units up.

(d) Reflected along the axis of reflection and then rotated 90° in an anticlockwise direction about point $B'$.

(e) Rotated 45° in a clockwise direction about point $C$ and then translated 8 units down and 3 units right.

(f) Rotated 180° about point $C$ and reflected along the axis of reflection.

9 (a) Translate 4 units left and 2 units up and reflect along the axis of reflection.

(b) Reflect along the axis of reflection and translate 8 units up.

(c) Translate 3 units down and 5 units left and rotate 180° about point $A'$.

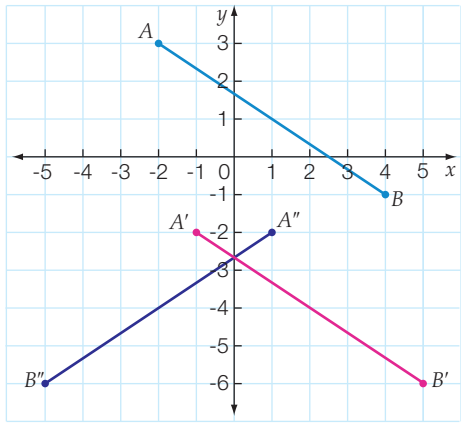
(d) Rotate 90° about point $B''$ in a clockwise direction and reflect in the axis of reflection.

(e) Translate 8 units up and 3 units left and rotate 45° in an anticlockwise direction about point $C'$.

(f) Reflect along the axis of reflection and rotate 180° about point $C'$.

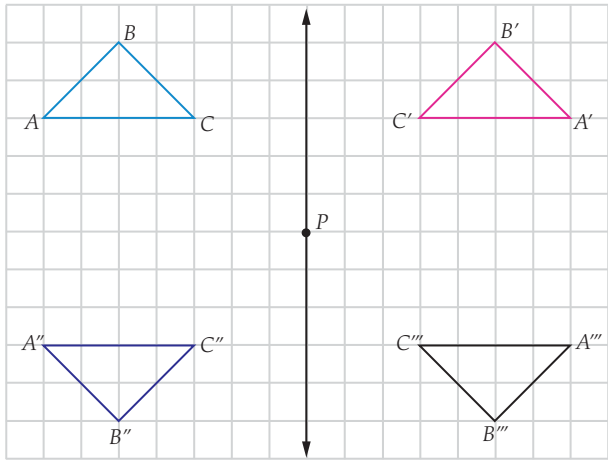
10 (a) $A''(1, -2)$, $B''(-5, -6)$

(b)




11 B1—Rotate 90° clockwise and then rotate 90° clockwise.
B2—Reflect vertically.

12

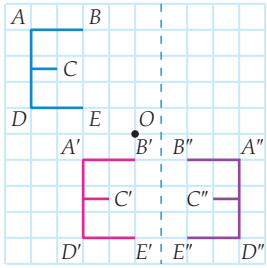



Rotation 180° about point $P$.

## Open-ended – Sample answers

13 Students' own answers.

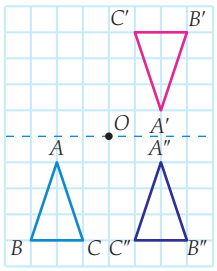
14 (a)




(b) Rotation 180° clockwise about $O$.

(c) Rotation 180° clockwise about $O$.

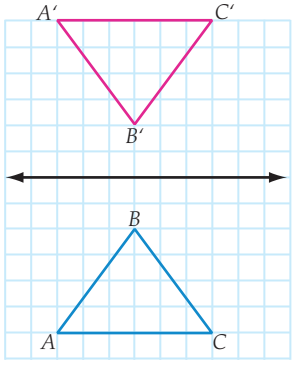
15 (a)



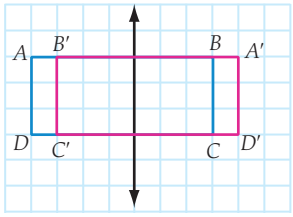

(b) A reflection in a vertical line of reflection passing through $O$.

## Half-time 10 (p. 591)

1 (a)



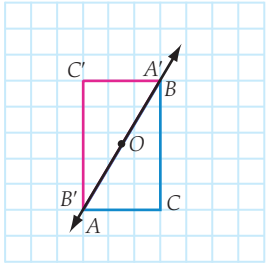

(b)



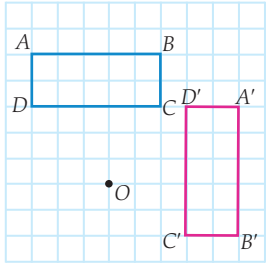

2 (a) Translated 5 units right and 4 units down.

(b) Translated 3 units left and 2 units up.

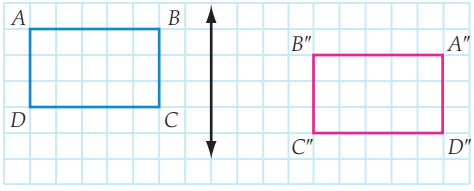
3 (a)



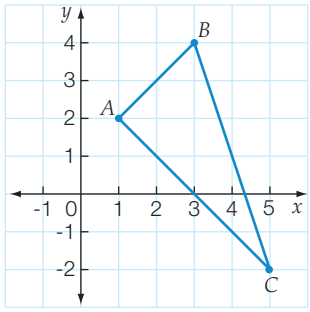

(b)



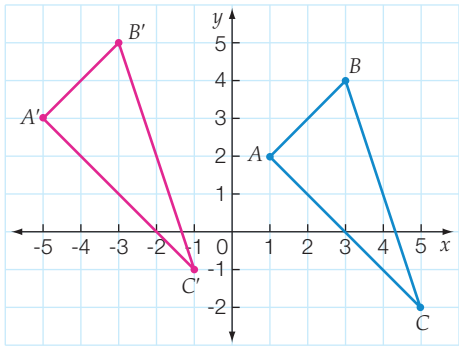

4




5 (a)

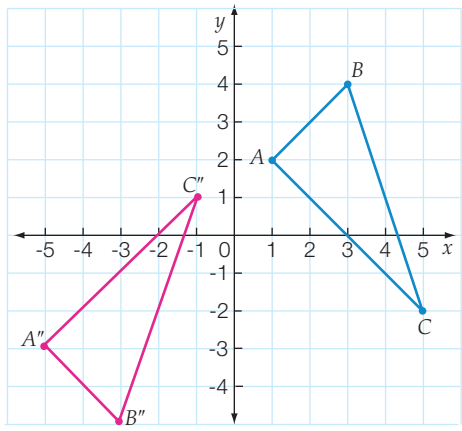



(b)



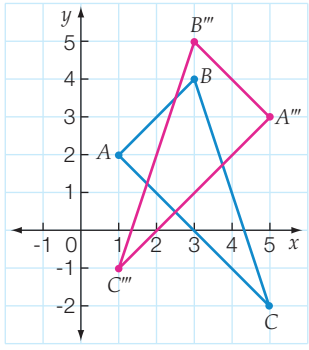

(c) $A'(-5, 3)$, $B'(-3, 5)$, $C'(-1, -1)$

(d)




(e) $A''(-5, -3)$, $B''(-3, -5)$, $C''(-1, 1)$

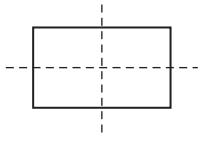
(f)



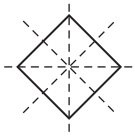

(g) $A'''(5, 3)$, $B'''(3, 5)$, $C'''(1, -1)$

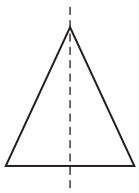
## Exercise 10.5 (p. 597)

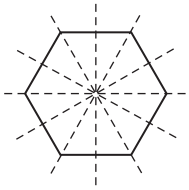
1 (a) (i) Yes



(ii) Order = 2

(b) (i) Yes  (ii) Order = 4

(c) (i) Yes  (ii) Order = 1

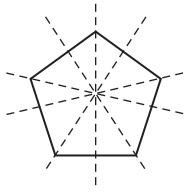
(d) (i) Yes  (ii) Order = 6

(e) (i) Yes  (ii) Order = 1

(f) No 

(g) No 

(h) (i) Yes  (ii) Order = 2

(i) (i) Yes  (ii) Order = 5

2 (a) (i) Yes (ii) 2 (iii) 180°

(b) (i) Yes (ii) 4 (iii) 90°

(c) (i) No Does not have rotational symmetry.

(d) (i) Yes (ii) 6 (iii) 60°

(e) (i) No Does not have rotational symmetry.

(f) (i) No Does not have rotational symmetry.

(g) (i) No Does not have rotational symmetry.

(h) (i) Yes (ii) 2 (iii) 180°

(i) (i) Yes (ii) 5 (iii) 72°

3 (a) 4 (b) 2 (c) 1

4 D

5 C

6 A

7 (a) a, b, d, e, f, g, h, j, k, m, n, p, q, r, s, t, u, y, z

(b) c, i, v, w,

(c) l, o, x

8 (a) F, G, J, K, L, N, P, Q, R, S, Z

(b) A, B, C, D, E, M, T, U, V, W, Y

(c) H, I, O, X

9 Yes —H, I, N, O, S, X, Z

10 (a) Order 1 reflectional symmetry and no rotational symmetry

(b) Order 1 reflectional symmetry and no rotational symmetry

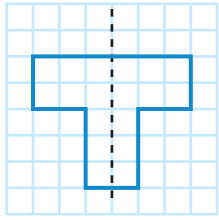
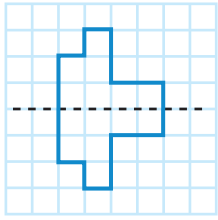
(c) Order 5 reflectional symmetry and order 5 rotational symmetry

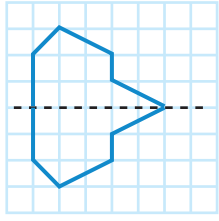
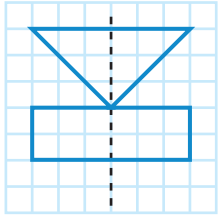
(d) Order 5 reflectional symmetry and order 5 rotational symmetry

(e) Order 1 reflectional symmetry and no rotational symmetry

(f) Order 1 reflectional symmetry and no rotational symmetry

11 (a) No (b) Yes (c) No (d) No (e) Yes (f) Yes

12 (a)  (b) 

(c)  (d) 

13 rectangle and rhombus

14 equilateral triangle

## Open-ended – Sample answers

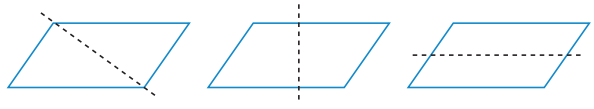
15 O is the axis of symmetry. Other words TOOT, MUM, MOM, HUH.

16 (a) Rotation: 1001, 1111, 1691, 1881, 1961

(b) Reflection: 1001, 1111, 1331, 1881

(c) Rotation and reflection: 1001, 1111, 1881

17 A parallelogram has reflectional symmetry if it is a rhombus, a rectangle or a square. Otherwise, it has no reflectional symmetry.



## Mathspace (p. 602)

1 (a) swims, horizontal reflectional symmetry

(b) HIDE, horizontal reflectional symmetry

(c) vertical reflectional symmetry

(d) horizontal reflectional symmetry

(e) vertical reflectional symmetry

(f) suns, rotational symmetry

2 (a) m and w, d and p, n and u

(b) o, s, x, z, H, I, N

(c) sample rotational: mow
sample vertical reflectional: wow

3 (a) sample: 18181

(b) sample: 3018

(c) sample: 80108

4 (a) 0, 1, 2, 5, 8

(b) 1 (rotational, horizontal and vertical reflectional), 25 (rotational), 81 (horizontal reflectional), 100 (horizontal reflectional)

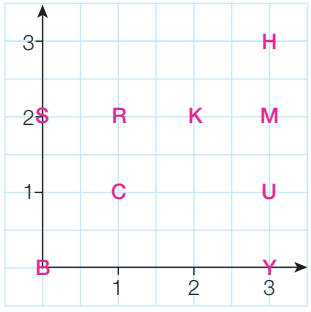
(c) (i) 8888

(ii) 9966

(d) (i) 88888

(ii) 99866

### Grid symmetry



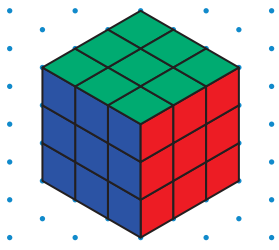

## Exercise 10.6 (p. 605)

1 Check your shapes carefully against the ones in the question.

2 (a) 4 (b) 12 (c) 12 (d) 9 (e) 8 (f) 10

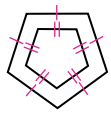
3 B

4 A

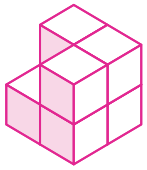
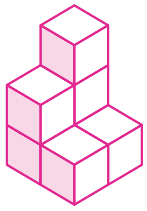
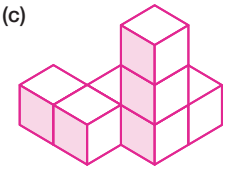
5 

6 (a) 8 (b) 12 (c) 9 (d) 10 (e) 21 (f) 12

7 D

8 (a)  (b) 

(c) No, each side is a rectangle with the same length and height.

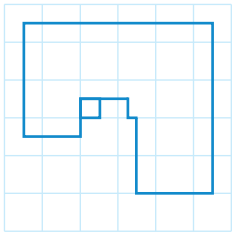
9 (a)  (b)  (c) 

10 (a) It is not a rectangle, as one side has 6 blocks and another has four blocks. It is not possible to make this shape.

(b) Students' own answers.

(c) Students' own answers.

11 Top view



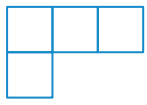
### Open-ended – Sample answers

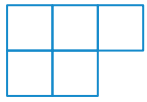
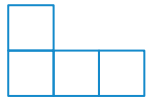
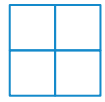
12 Students' own answers.

13 Students' own answers.

14 Students' own answers.

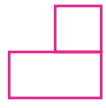
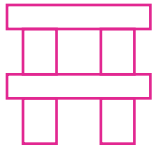
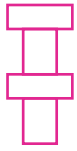
## Exercise 10.7 (p. 610)

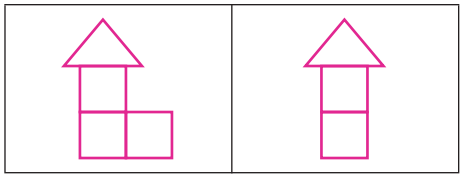
1 (a) (i)  (ii)  (iii) 

(b) (i)  (ii)  (iii) 

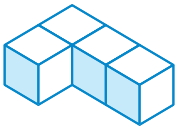
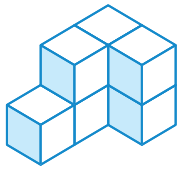
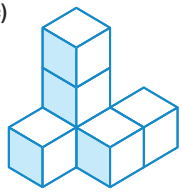
2 D

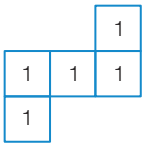
3

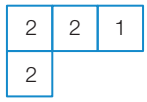
| | Front | Side |
|---|---|---|
| (a) |  |  |
| (b) |  |  |
| (c) |  |  |

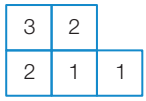
(d) 

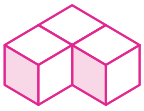
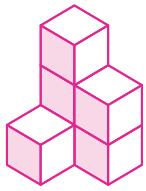
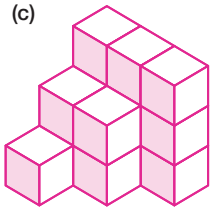
4 (a) A (b) B

5 (a)  (b)  (c) 

6 (a) 

(b) 

(c) 


7 (a)  (b)  (c) 

8 (a, b) Students' own drawings.

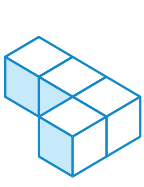
(c) 1; 3; 3 or 5; 3 or 5; 6 or 7

(d) Depending on the way they are shaded, the orientation of the cubes can appear differently. Some cubes may look like concave spaces. The answer to (c) gives the alternative numbers of cubes.

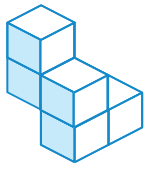
(e) Answers may vary.

### Open-ended – Sample answers

9 Possible answers include:


4 cubes


5 cubes


6 cubes

10 Students' own answers.

## Challenge 10 (p. 613)

1 (a) Rotation through 180° in either direction.

(b) Reflection in the $y$-axis followed by reflection in the $x$-axis, or reflection in the $x$-axis followed by reflection in the $y$-axis.

(c) $B'$

2 2.30

3 One copy is fixed at $C$ and $D$ is rotated counterclockwise through 90°. Another copy is fixed at $D$ and $C$ is rotated clockwise through 90°. The third copy is translated upwards through a distance equal to the length of $CD$.

4 B

5 'A BOILED EGG IS HARD TO BEAT'. The whole message needs to be reflected in a line.

6 (a) Translate 8 cm to the right.

(b) Rotate 90° anticlockwise about the bottom corner, then translate 11 cm to the right.

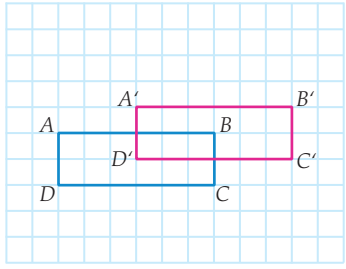
(c) Reflect in the 4 cm side, then translate 4 cm to the right.

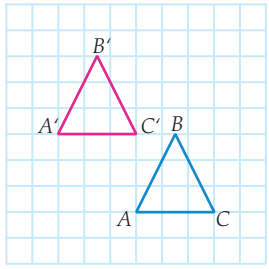
(d) Rotate 180° about the vertex located between the 4 cm and 5 cm sides.

## Chapter review 10 (p. 614)

1 (a) Translated 4 units right and 4 units down.

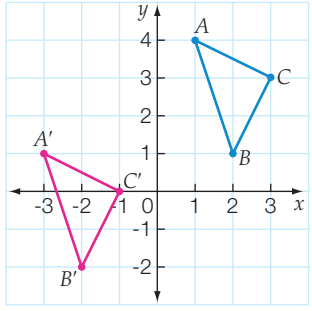
(b) Translated 4 units left and 3 units up.

2 (a) 

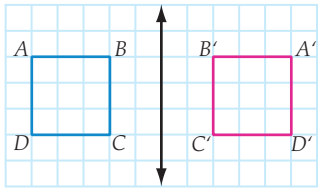

(b) 

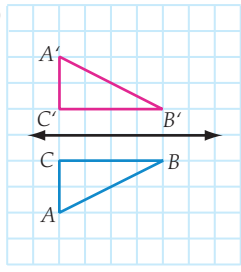

3 B

4 (a) $A(1, 4)$, $B(2, 1)$, $C(3, 3)$

(b) 

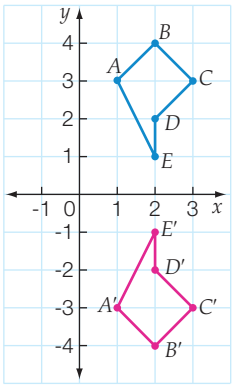

(c) $A'(-3, 1)$, $B'(-2, -2)$, $C'(-1, 0)$

5 (a) 


(b) 

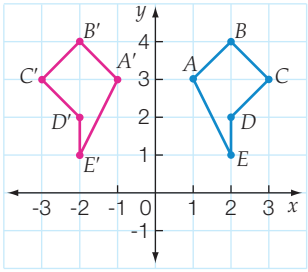
6 (a) $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$, $E(2, 1)$

(b)



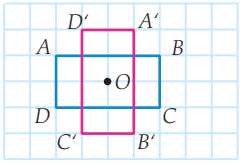

(c) $A'(1, -3)$, $B'(2, -4)$, $C'(3, -3)$, $D'(2, -2)$, $E'(2, -1)$

(d)

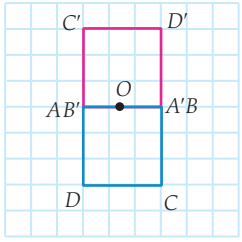



(e) $A'(-1, 3)$, $B'(-2, 4)$, $C'(-3, 3)$, $D'(-2, 2)$, $E'(-2, 1)$

7 (a)



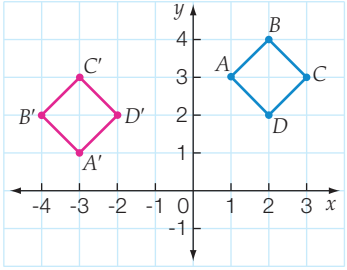

(b)




8 D

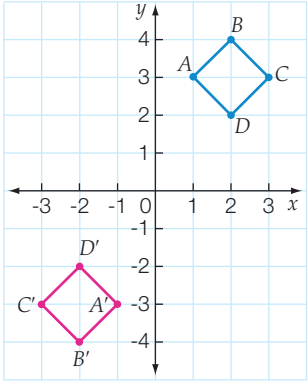
9 (a) $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$

(b)



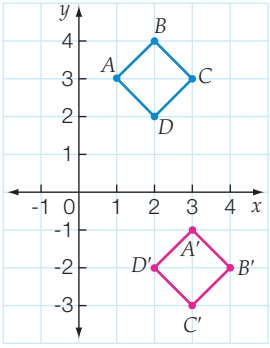

(c) $A'(-3, 1)$, $B'(-4, 2)$, $C'(-3, 3)$, $D'(-2, 2)$

(d)




(e) $A'(-1, -3)$, $B'(-2, -4)$, $C'(-3, -3)$, $D'(-2, -2)$

(f)




(g) $A'(3, -1)$, $B'(4, -2)$, $C'(3, -3)$, $D'(2, -2)$

10 C

11 C

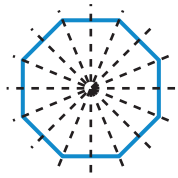
12 (a)



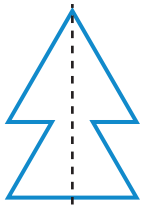
(b)



(c)



(d)



(e)



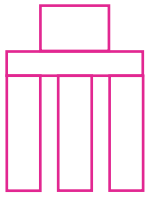
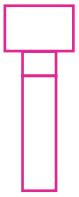
(f)



13 (c) and (e) have rotational symmetry.

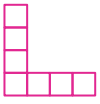
14 Check your drawing carefully against the question.

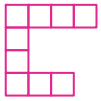
15

| | Front elevation | Side elevation |
|---|---|---|
| (a) |  |  |
| (b) |  |  |

16 Check your drawing carefully against the question.

17

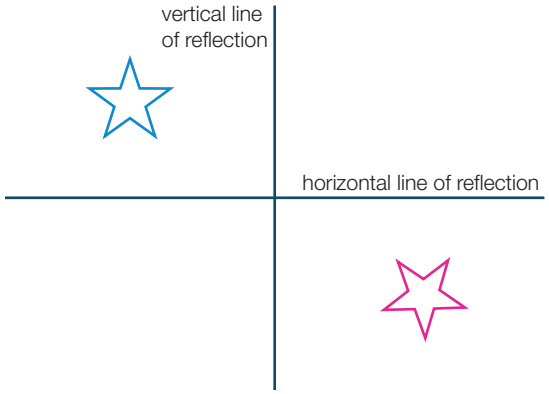

front


top


side

18 4 units right

**19 (a)**

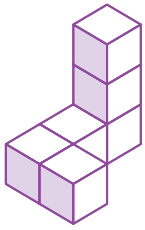



**(b)** Yes

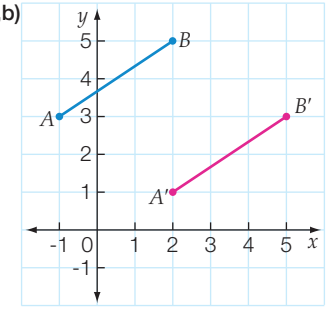
**20** Rotating either 180° in a clockwise or anticlockwise direction about the intersection point of the axes.

**21** None

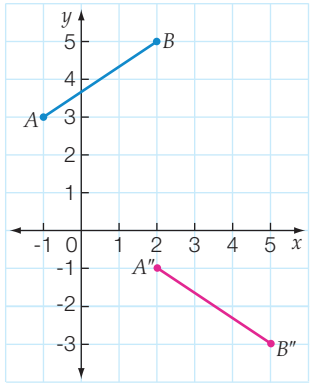
**22**



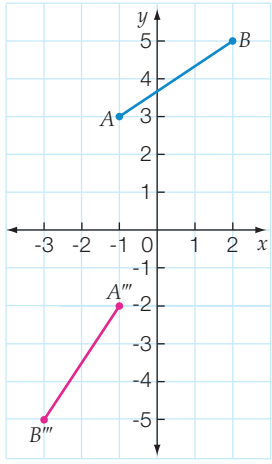
**23 (a,b)**




**(c)**

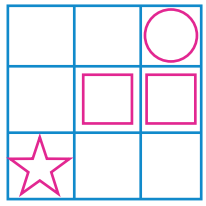



**(d)**

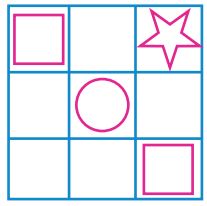



**(e)** $A'''(-1, -2)$, $B'''(-3, -5)$

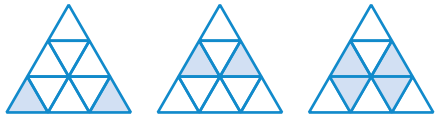
**24 (a)**



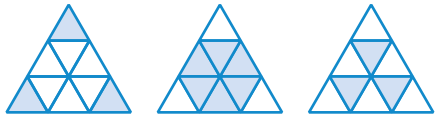
**(b)**



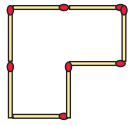
**25 (a)**



**(b)**

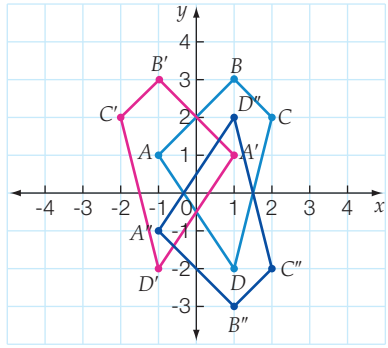


**26** one match



**27** 19

**28**




Yes, Matt is correct. These two transformations could be replaced by a reflection in the $x$-axis.

## NAPLAN practice 10

**1** B **2** B **3** C **4** A

**5** D **6** A **7** A

## Mixed review E (p. 620)

**1** **(a)** 12 **(b)** 64 **(c)** 25

**2** **(a)** 7 **(b)** 2700 **(c)** 99 000

**3** **(a)** 17.515 **(b)** 0.0072 **(c)** 1.99

**4** **(a)**

| $m$ | 4 | 0 | 11 | 25 | 15 |
|---|---|---|---|---|---|
| $n$ | 12 | 0 | 33 | 75 | 45 |

**(b)**

| $s$ | 18 | 100 | 12 | 22 | 312 |
|---|---|---|---|---|---|
| $t$ | 6 | 88 | 0 | 10 | 300 |

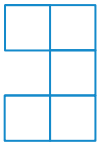
**5** **(a)** 220 $cm^2$ **(b)** 60 $cm^2$

**6** **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{26}$ **(c)** $\frac{1}{4}$

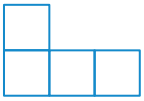
**7** **(a)** **(i)**



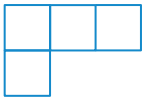
**(ii)**



**(b)** **(i)**



**(ii)**



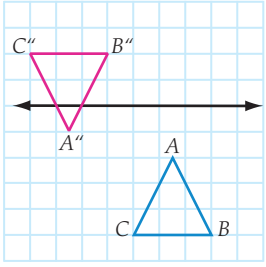
**8** **(a)** $x = 3$ **(b)** $x = 24$ **(c)** $x = 3$

**9** **(a)** 145° **(b)** 53° **(c)** 90°

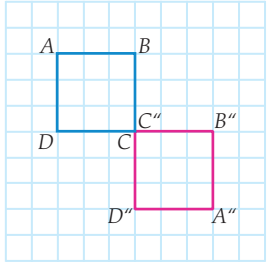
**10**

| Height (cm) | Frequency |
|---|---|
| 145–149 | 1 |
| 150–154 | 2 |
| 155–159 | 5 |
| 160–164 | 7 |
| 165–169 | 2 |
| 170–174 | 3 |
| Total | 20 |

**11** **(a)**




**(b)**




**12** **(a)** 1 **(b)** 4 **(c)** 2

**13** mean = 30.6, median = 31.5, mode = 34, range = 39, outlier = 6

**14** 1 unit right and 1 unit down

**15** **(a)** $24 = 2 \times 2 \times 2 \times 3$ **(b)** $30 = 2 \times 3 \times 5$

**(c)** 6 **(d)** 120

**16** **(a)** 60° **(b)** 126°, 54°, 54°

**17** O, X

**18** (1, 1)

**19** **(a)** Students' own answers.




**(b)** Work out what the numbers need to total using the mean, and then set out some boxes, putting 6 in place to make it the median. The other numbers can then be filled in around the set number(s).

**(c)** It is easier to use an odd number of values because the median is then only one single value.

**20** **(a)** There are eight possible results: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

**(b)** You win with half of the results, and win more than you lose on the other results. So, the game is not fair, but it is not fair to the operator!

# Glossary and index

**acute angle** an angle greater than 0 degrees but less than 90 degrees (p. 430)

**acute-angled triangle** a triangle with all angles less than 90 degrees (p. 457)

**alternate angles** angles found between parallel lines but on opposite sides of the transversal (p. 445)

**angle** the space between two lines that start at the same point (p. 421)

**area** the amount of space contained by a plane shape (p. 333)

**arms** (of an angle) two straight lines that meet at a vertex. The angle is the amount of turn measured between the two arms (p. 421)

**associative law** the order in which numbers are added or multiplied is not important; e.g. $33 \times 5 \times 2$ could be calculated as $165 \times 2$, or $33 \times 10$. Similarly, $29 + 13 + 17$ could be calculated as $42 + 17$, or $29 + 30$. This can be written in algebra as $a + (b + c) = (a + b) + c$ (p. 5)

**asymmetrical** having no symmetry (p. 594)

**average** the word used to collectively describe any of mean, median and mode (p. 498)

**axes** the number lines to which values of a variable are referenced. On the number plane, the $x$-axis runs in the horizontal direction and the $y$-axis runs in the vertical direction (p. 292)

**axis of symmetry** a line that divides a figure into two identical but reversed parts (p. 594)

**backtracking** a method of solving equations that uses a flowchart (p. 388)

**balance method** making equivalent equations by performing the same operation on both sides, keeping them balanced (p. 398)

**bar graph** a graph in which equally spaced bars show the frequency of data in different categories (p. 506)

**base** 1. the number that is raised to an index or power (p. 13); 2. the bottom of a shape (p. 344)

**bimodal** a data set in which exactly two of the data values have the highest frequency (p. 498)

**bivariate data** data relating to two different variables. Time is often used as one variable (p. 524)

**cancelling** the process of dividing the numbers in a fraction by a common factor to produce a simpler fraction (p. 117)

**Cartesian plane** (number plane) a grid made up of two axes that allows any point in the plane to be described by its coordinates (p. 292)

**categorical data** data that is collected according to a description; e.g. eye colour (p. 489)

**centimetre** one-hundredth of a metre (p. 323)

**centre of rotation** the point about which a figure is rotated (p. 574)

**chance** another word for probability (p. 545)

**checking by substitution** the process of checking an answer by substituting the solution into the equation (p. 390)

**class interval** the range of each class of data; used when converting large amounts of raw data into grouped data (p. 491)

**coefficient** a number written before a pronumeral; e.g. in $9x$, the coefficient of $x$ is 9 (p. 256)

**co-interior (allied) angles** angles found between parallel lines and on the same side of the transversal. Co-interior angles are supplementary (p. 445)

**column graph** (or vertical bar graph) a graph where the frequency of collected data is displayed in columns (p. 506)

**common factor** a number that is a factor of all numbers under consideration (p. 56)

**common multiple** a number that is a multiple of all numbers under consideration (p. 56)

**commutative law** the order in which two numbers are added or multiplied can be swapped without changing the result; e.g. $4 + 5 = 9$ and $5 + 4 = 9$, or $2 \times 3 = 6$ and $3 \times 2 = 6$. This can be written in algebra as $a + b = b + a$, and $a \times b$ $(ab) = b \times a$ $(ba)$ (p. 5)

**complementary angles** two angles that add up to 90 degrees (p. 431)

**composite number** a number that has at least two factors other than 1 (p. 61)

**composite shape** a shape made by combining other shapes, such as rectangles and triangles (p. 350)

**concave** having a surface or boundary that curves inwards (p. 452)

**constant** a number by itself; e.g. 2, 7, 9.5 (p. 256)

**continuous data** data that is collected by measuring; e.g. height (p. 489)

**convex** having a surface or boundary that curves outwards (p. 452)

**coordinates** a pair of numbers $(x, y)$ that describes the position of a point on a number plane (p. 292)

**coprime** two or more numbers that have only 1 as common factors (p. 61)

**corresponding angles** angles found either both above or both below parallel lines and on the same side of the transversal (p. 445)

**cube root** a number that is cubed to produce a given number (p. 13)

**data** raw information (p. 489)

**decimal places** the number of digits after the decimal point of a number (p. 175)

**decimal point** a point that separates the whole number part of a number from the decimal part (p. 175)

**define** to identify the variable and represent it with a pronumeral (p. 257)

**degree** the unit that is used to measure the size of angles; a full turn is equal to 360 degrees (p. 421)

**denominator** the number below the line in a common fraction (p. 107)

**deposit** placing an amount of money into a bank account (p. 73)

**digit** any numeral from 0 to 9 (p. 175)

**discrete data** data that is counted; e.g. the number of cans of soup sold by a supermarket over a period of time (p. 489)

**distributive law** a large number can be multiplied by splitting it into two smaller numbers, which can be multiplied individually, and the products added; e.g. $8 \times 17 = 8 \times (10 + 7) = 8 \times 10 + 8 \times 7 = 80 + 56 = 136$. This can be written in algebra as $a(b \pm c) = ab \pm ac$ (p. 6)

**dividend** in a division calculation, the number being divided; e.g. for $169 \div 8 = 21$ rem 1, the dividend is 169 (p. 21)

**divisibility** divisibility tests can be used to find the factors of a particular number (p. 54)

**divisible** able to be divided by a number a whole number of times (there is no remainder) (p. 54)

**divisor** in a division calculation, the number being divided by; e.g. for $169 \div 8 = 21$ rem 1, the divisor is 8 (p. 21)

**dot plot** a graph showing the frequency of each piece of data as a dot (p. 503)

**elevations** 2D drawings of a structure from either the front, rear or side (p. 610)

**equation** two expressions either side of an equals sign (p. 256)

**equilateral triangle** a triangle with all sides equal (p. 457)

**equivalent equations** equations that have the same solution (p. 397)

**equivalent fractions** fractions that represent the same quantity but are written using different denominators; e.g. $\frac{1}{2} = \frac{2}{4}$ (p. 116)

**equivalent ratios** ratios that compare amounts in the same proportion, but have different numbers. One ratio is a multiple of the other (p. 229)

**estimate** a sensible guess that approximates an answer or value (p. 29)

**evaluate** to calculate the value of a formula or expression (p. 268)

**expanded form** a base raised to an index written as a multiplication of the base times itself (p. 13)

**expression** a collection of terms added together or subtracted (p. 256)

**factor** a number that divides exactly into another number, with no remainder (p. 53)

**false number sentence** a sentence written with numbers that is not true (p. 377)

**formula** a rule that is expressed with pronumerals and an equals sign (p. 268)

**fraction** a number that is not a whole number, and is represented using a numerator and a denominator (p. 107)

**frequency** the number of times a piece of data occurs in a data set (p. 490)

**frequency table** a table of data showing a number of observations (p. 490)

**guess, check and improve** solving equations by choosing a number, substituting it into the equation and repeating this process until the LHS equals the RHS (p. 382)

**height** the measurement from the base to the top of a shape or a prism, perpendicular to the base (p. 344)

**Highest Common Factor (HCF)** the largest number that occurs in each of the sets of factors of a given group of numbers (p. 56)

**histogram** a column graph where the width of the bars represents the various classes and the height of the bars represents their relative frequencies (p. 507)

**image** a figure produced by a transformation of the original (p. 563)

**improper fraction** a fraction with a numerator greater than or equal to the denominator (p. 108)

**index (power)** (*plural* **indices**) a number that tells us how many times the base number is multiplied by itself (p. 13)

**index form** a short way of writing a number multiplied by itself several times using a base raised to an index (p. 13)

**integers** all of the positive and negative whole numbers, along with zero (p. 73)

**intersect** the point at which two or more lines meet or cross (p. 444)

**inverse** flipping the positions of the numerator and denominator gives the inverse of a fraction; e.g. the inverse of $\frac{2}{3}$ is $\frac{3}{2}$, the inverse of 4 is $\frac{1}{4}$ (p. 155)

**inverse** (opposite) **operation** a mathematical operation that undoes the effect of another mathematical operation; addition and subtraction, multiplication and division are the most common pairs of opposite operations (p. 387)

**irregular** an irregular shape has varying side lengths and angle sizes (p. 452)

**isometric** a triangular grid that allows 3D solids to be drawn on a 2D plane (p. 604)

**isosceles triangle** a triangle with two equal sides (p. 457)

**kilometre** 1000 metres (p. 323)

**kite** a quadrilateral with two pairs of adjacent sides equal in length (p. 466)

**length** distance from end to end (p. 323)

**like terms** algebraic terms that have exactly the same pronumeral part; e.g. $3ab$ and $5ba$ are like terms, but $2a$ and $3a^2$ are not (p. 284)

**line graph** a straight line representing data, often collected over a period of time (p. 307)

**line of reflection** a line drawn through an object or image that divides it into two identical parts (p. 568)

**line segment** part of a line that has both a beginning and an end (p. 431)

**linear graph** a graph in which a straight line can be drawn through all the points (p. 300)

**loss** money lost by an individual or business selling goods for less than the amount for which they were purchased (p. 73)

**Lowest Common Denominator (LCD)** the lowest common multiple of a set of denominators (p. 124)

**Lowest Common Multiple (LCM)** the smallest number that occurs in each set of multiples of a given group of numbers (p. 56)

**mathematical conventions** rules that mathematicians agree to work by (p. 36)

**mean** the average obtained by adding together all of the scores obtained and dividing by the number of scores (p. 498)

**measure of centre** one of three ways (mean, median, mode) used to explain what is happening around the middle of a data set (p. 498)

**median** the middle value of a set of scores (p. 498)

**metre** the standard unit of length (p. 323)

**metric** the name for the system of measurement used in Australia (p. 323)

**millimetre** one-thousandth of a metre (p. 323)

**mixed number** a number that is made up of a whole number part and a proper fraction (p. 108)

**mode** the most frequently occurring score in a set of scores (p. 498)

**multimodal** a data set in which three or more values have the highest frequency (p. 498)

**multiple** a number that may be divided exactly by a given number (with no remainder) is a multiple of that given number (p. 53)

**negative** used to describe numbers to the left of zero on the number line; also used to describe losses or decreases (p. 72)

**numerator** the number above the line in a common fraction (p. 107)

**obtuse angle** an angle greater than 90 degrees but less than 180 degrees (p. 430)

**obtuse-angled triangle** a triangle with one obtuse angle (p. 457)

**order of operations** a rule that states that when simplifying expressions you must do brackets first, then multiplication and division in the order they appear, then addition and subtraction in the order they appear (p. 36)

**order of reflectional symmetry** the number of axes of symmetry that can be drawn through a shape (p. 594)

**order of rotational symmetry** the number of times in a complete 360 degrees rotation that an identical image of the original would be produced (p. 595)

**ordered pair** the coordinates of any point on the number plane written with the $x$-coordinate first, followed by the $y$-coordinate; i.e. $(x, y)$ (p. 292)

**origin** the point (0, 0) on the Cartesian plane (p. 292)

**original** the first version (p. 563)

**outcomes** the results of a probability experiment (p. 546)

**outlier** a piece of data that is quite different (much higher or much lower) from the rest of the data in a data set (p. 505)

**parallel lines** lines that lie in the same plane and are always the same distance apart (p. 444)

**parallelogram** a quadrilateral with both pairs of opposite sides parallel (p. 344)

**part : part ratio** a ratio that compares two or more parts of a whole (p. 228)

**part : whole ratio** a ratio that compares a part or parts to the whole (p. 228)

**per cent** a number expressed as an amount out of 100 (p. 216)

**perfect cube** found by multiplying a whole number by itself three times; e.g. $4 \times 4 \times 4$ (p. 12)

**perfect square** a whole number that is obtained by multiplying a number by itself; e.g. 25 is the 'square' of 5, as $5 \times 5 = 25$ (p. 12)

**perimeter** the distance along the boundary of a shape (p. 329)

**perpendicular** at right angles to (p. 344)

**perpendicular bisector** a line at right angles to another line that cuts this line in half (p. 474)

**perpendicular distance** is measured by a line that makes a right angle with the line of reflection (p. 568)

**perpendicular lines** lines that are at right angles to each other (p. 444)

**place value** the value of a digit in a number is given by its position (or place) in the number. In the decimal number system, the place value of a digit is 10 times the place value of the digit to the right (p. 175)

**plan views** technical drawings of an object from either the top, front or side (p. 609)

**plane** a flat two-dimensional surface (p. 444)

**plane shapes** enclosed figures drawn on a plane (p. 452)

**point graph** graph showing information relating to two variables where the points are not joined (p. 306)

**polygon** a many-sided (or many-angled) plane shape (p. 452)

**positive** used to describe numbers to the right of zero on the number line; also used to describe gains or increases (p. 72)

**power** *see* index (p. 13)

**prime factor** a factor that is a prime number (p. 66)

**prime number** a number that has only two factors, itself and 1 (p. 61)

**probability** the chance that some event will occur (p. 545)

**product** the result when two or more numbers are multiplied together (p. 6)

**profit** money made by an individual or business selling goods (once costs have been considered) (p. 73)

**pronumeral** a letter or symbol that represents a number (p. 251)

**proper fraction** a fraction in which the numerator is less than the denominator (p. 108)

**protractor** a device for measuring angles (p. 421)

**quadrant** the $x$- and $y$-axes divide the Cartesian plane into four parts, each of which is called a quadrant (p. 292)

**quadrilateral** a plane shape with four straight sides (p. 466)

**quotient** the result of a division calculation (p. 21)

**range** the difference between the highest and lowest value in a data set (p. 491)

**rate** a comparison of two amounts of different quantities, usually written as the number of the first for every one of the second (p. 236)

**ratio** a comparison of two or more amounts of the same quantity, written using the ':' symbol (p. 227)

**ray** part of a line which has a beginning and no end (p. 431)

**rectangle** a plane shape with two pairs of opposite sides equal and all internal angles 90 degrees (p. 466)

**rectangular prism** a prism with a rectangular end (p. 355)

**recurring decimal** a decimal number that has a repeating pattern of digits (p. 191)

**reflection** a transformation of a shape found after drawing the mirror image of it (p. 568)

**reflectional symmetry** if one or more lines of symmetry can be drawn through an object or image, that object or image is said to have reflectional symmetry (p. 594)

**reflex angle** an angle greater than 180 degrees and less than 360 degrees (p. 430)

**regular polygon** a polygon with all sides of equal length and all angles of equal size (p. 452)

**relationship** in algebra, the connection between two or more variables where changing the value of one variable affects the value of the other variable (p. 261)

**remainder** a number 'left over' at the end of a division calculation, usually indicated by 'rem'. (p. 21)

**revolution** a full 360 degree turn (p. 421)

**rhombus** a plane shape with all four sides equal and opposite angles of equal size (p. 466)

**right angle** an angle of exactly 90 degrees (p. 430)

**right-angled triangle** a triangle with one 90 degree angle (p. 457)

**rotate** turn (p. 574)

**rotation** a transformation in which a shape is turned on the flat surface about a point (p. 574)

**rotational symmetry** if an object or shape can be turned about a point less than 360 degrees and still look like the original object or shape, that object is said to have rotational symmetry (p. 595)

**round** to give an approximate value of a number to a specified degree (p. 184)

**rounding** approximating a decimal to a certain number of decimal places, or a number to a certain number of digits or to a certain place value (p. 29)

**sample space** a list of all the events that can occur in a game or experiment (p. 546)

**scalene triangle** all sides of different lengths; used to describe certain triangles (p. 457)

**simplest form** A fraction or ratio in which all common factors have been cancelled (p. 117)

**simplify** to write in simplest form by either cancelling common factors (for fractions and ratios) or by collecting like terms (p. 117)

**solution** the value of a pronumeral that makes the equation true; the answer (p. 382)

**solve** find the solution to (p. 382)

**solving by inspection** a method of solving equations that involves looking at the equation and working out the solution in your head (p. 382)

**square** a quadrilateral with all sides equal and all internal angles 90 degrees (p. 466)

**square root** a number that is squared to become a given number (p. 13)

**stem-and-leaf plot** (stem plot) a display of data in which the last digit in the data becomes a leaf and the rest of the data becomes a piece of the stem (p. 503)

**straight angle** an angle of exactly 180 degrees (p. 430)

**substitute** to replace a pronumeral with a given number (p. 268)

**supplementary angles** two angles that add up to 180 degrees (p. 432)

**table of values** a grid that displays the $x$- and $y$-values of a rule in algebra (p. 262)

**tally** the count of the frequency of different pieces of data (p. 490)

**term** those parts of an algebraic expression that are added or subtracted to make up the expression, consisting of one or more pronumerals multiplied together, possibly multiplied by a number (coefficient of the term) (p. 256)

**terminating decimal** a decimal number that has a finite number of digits (p. 191)

**theoretical probability** the mathematical chance of an event occurring calculated by dividing the number of chances of success by the total number of chances (p. 546)

**transformation** a process (usually translation, rotation or reflection) carried out on a shape that changes its position and/or orientation (p. 563)

**translation** transformation of a shape by moving it in a straight line (p. 563)

**transversal** a line that cuts across each of two or more other lines (p. 444)

**trapezium** a quadrilateral with only one pair of opposite sides parallel (p. 466)

**trend** a general pattern being followed in a graph that can be used to predict other values in a data set (p. 524)

**true number sentence** a sentence that is written with numbers that is true (p. 377)

**unit fraction** a fraction that has 1 as the numerator (p. 146)

**unit price** the price per unit of quantity. The unit is often 100 mL or 1 L, 100 g or 1 kg (p. 237)

**unitary method** a problem-solving method that calculates an amount per unit as a step in the calculation (p. 236)

**univariate data** data that is collected about one variable only (p. 503)

**unknown** in algebra, a number that is not known (p. 251)

**unlike terms** two or more algebraic terms that cannot be simplified as they have different pronumeral parts (p. 284)

**variable** in algebra, an unknown amount that can vary (p. 251)

**vertex** (*plural* **vertices**) the point where two or more lines meet (p. 421)

**vertically opposite angles** the pair of non-adjacent angles formed when two lines intersect (p. 432)

**volume** the amount of space occupied by a three-dimensional object (p. 355)

**width** the distance from one side of a shape to the other, through the centre (p. 329)

**withdrawal** taking an amount of money out of a bank account (p. 73)