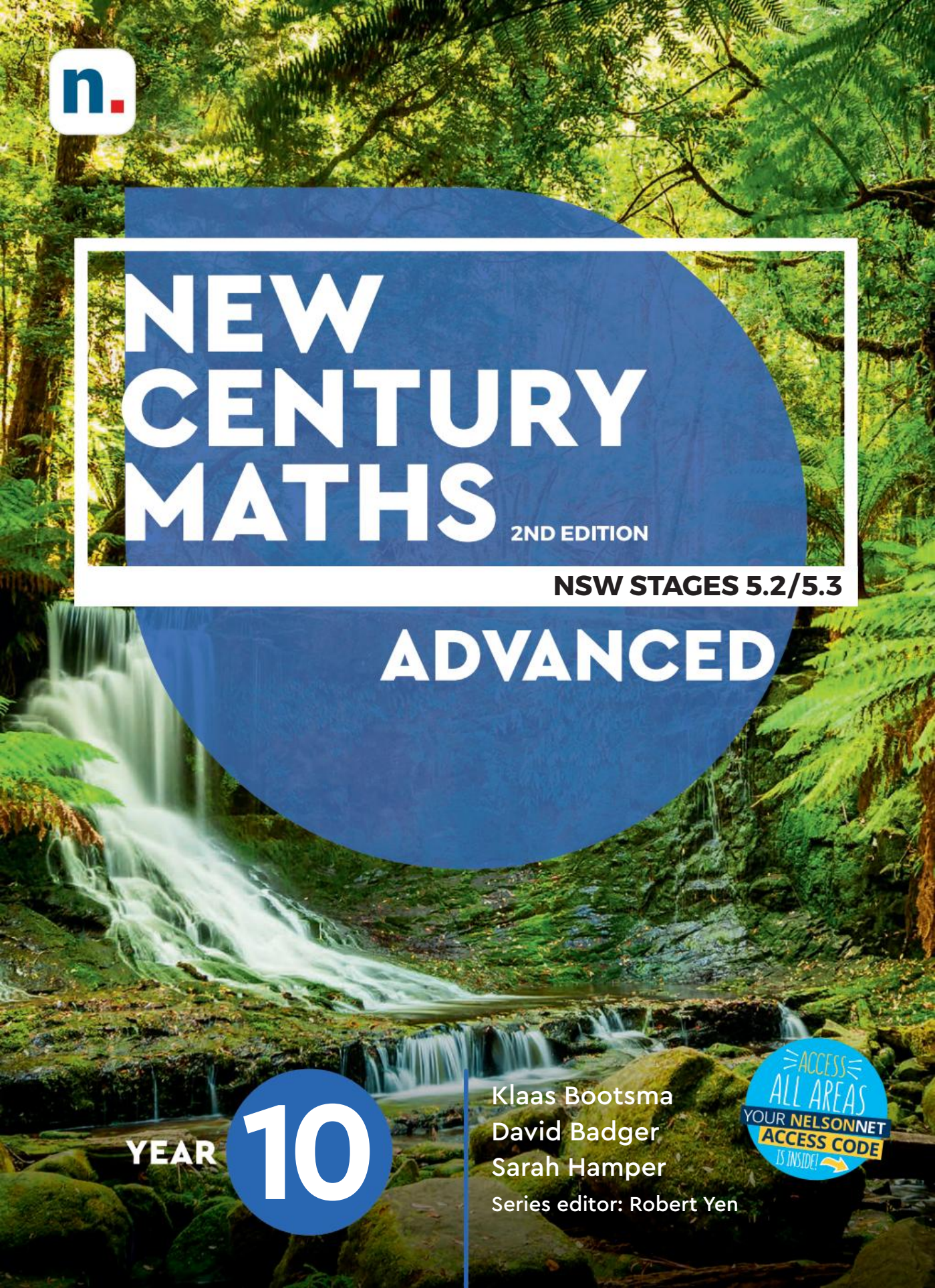


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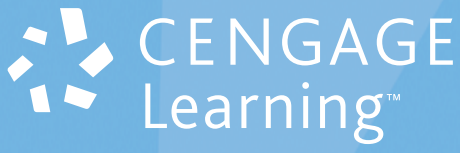
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YEAR 10

Klaas Bootsma
David Badger
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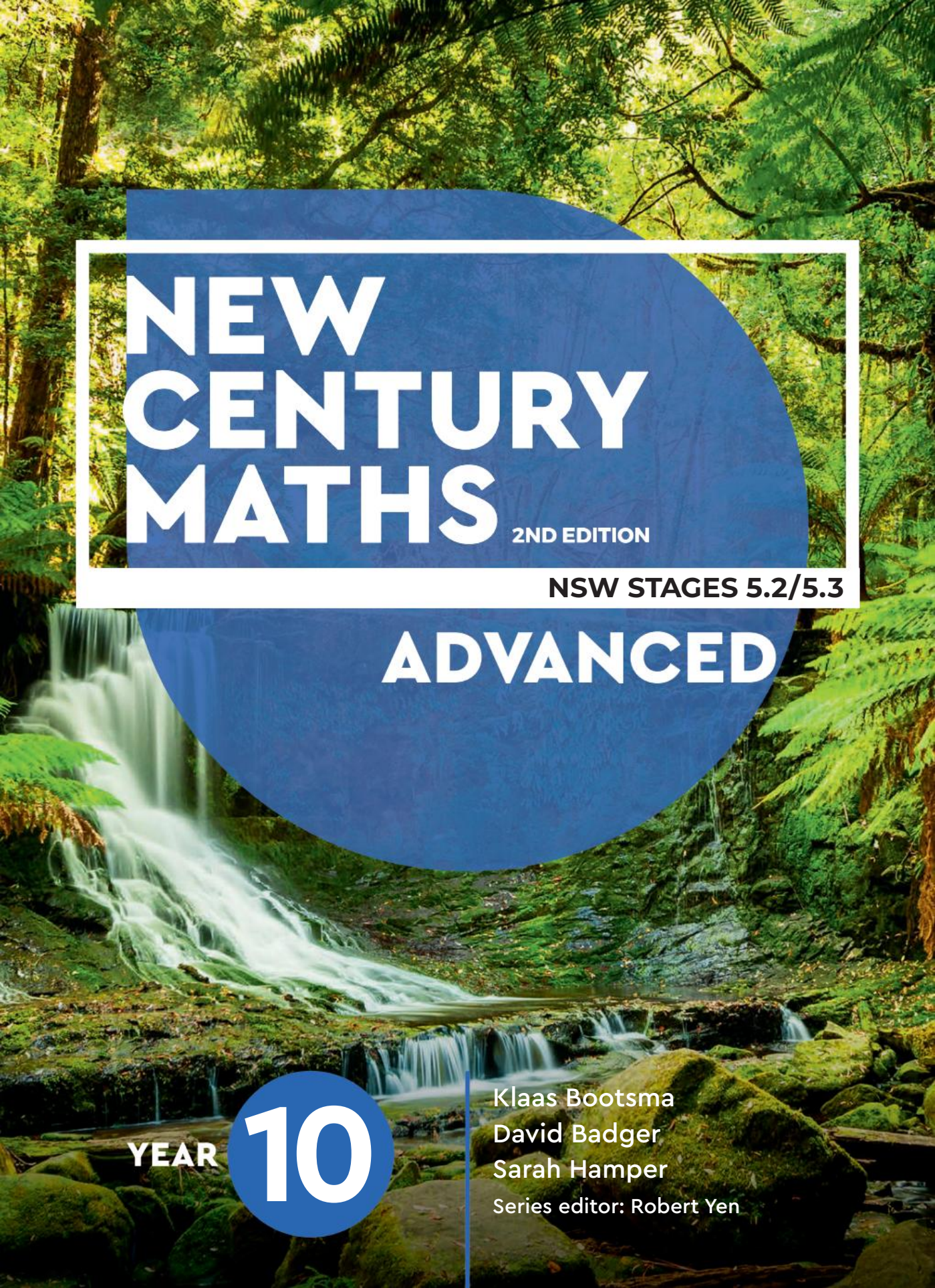
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YEAR 10

Klaas Bootsma
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PREFACE

Teachers, it's time to start teaching maths again. You asked and we listened.

New Century Maths 7-10 has been refreshed for the 2020s classroom, focusing on core skills to discourage 'teaching by syllabus dot points', featuring explicit grading and labelling of exercise questions, more 'flipped classroom' video tutorials, more applications and problem-solving questions, and worked solutions to every question.

In schools for over 25 years, *New Century Maths* is carefully mapped to the NSW syllabus and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

We publish 2 levels of mathematics books for Years 9 and 10 (Stage 5):

- *New Century Maths,* covering NSW Stages 5.1/5.2
- *New Century Maths Advanced,* covering NSW Stages 5.1/5.2/5.3

This book, *New Century Maths 10 Advanced,* has been designed for Year 10 students progressing along Stages 5.2 or 5.3 of the continuum. It also contains **3 optional Stage 5.3 chapters for extension**, especially for students heading towards the Mathematics Extension 1 course in Year 11 (see page vii).

The *NelsonNet* student and teacher websites contain worksheets, video tutorials, topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *New Century Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. So let's get started.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School and has taught at Lurnea and Grantham high schools. He was a senior HSC marker and has worked on the HSC Advice Line. Klaas also co-wrote *New Century Maths 11–12 Mathematics Standard 2.*

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Series editor **Robert Yen** taught at Hurlstone Agricultural High School. He co-wrote *New Century Maths 11–12 Mathematics Standard 2,* co-edited *Reflections,* the MANSW journal, and now works for Cengage as the mathematics publisher.

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Megan Boltze and **Robert Yen** wrote many of the *NelsonNet* worksheets and topic tests.

John Drake, **Katie Jackson**, **Joanne Magner**, **Scott Smith** and **Robert Yen** created the video tutorials.

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	8	Earning money		
Indices (Stages 5.1, 5.2)	5	Indices	5	Products and factors
Linear relationships (Stages 5.1, 5.2, 5.3)	11	Coordinate geometry and graphs	3	Graphing lines
Non-linear relationships (Stages 5.1, 5.2, 5.3)	11	Coordinate geometry and graphs	3	Graphing lines
			8	Graphing curves
			11	Quadratic equations and the parabola
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			10	Simultaneous equations
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	3	Numeracy and calculation	7	Equations and logarithms
	5	Indices	8	Graphing curves
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Patterns and algebra	1	Products and factors	5	Products and factors
	5	Indices	7	Equations and logarithms
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Linear and non-linear relationships	7	Equations	3	Graphing lines
	11	Coordinate geometry and graphs	5	Products and factors
			7	Equations and logarithms
			8	Graphing curves
			10	Simultaneous equations
			11	Quadratic equations and the parabola
			15	Polynomials
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	13	Congruent and similar figures	16	Circle geometry
Pythagoras and trigonometry	2	Surds and Pythagoras' theorem	9	Further trigonometry
	4	Trigonometry		
STATISTICS AND PROBABILITY				
Chance	12	Probability	12	Probability
Data representation and interpretation	9	Investigating data	6	Comparing data

Year 10 content descriptions

Australian Curriculum descriptions (© ACARA 2012)

Content description	*New Century Maths 10 Advanced* chapter	
NUMBER AND ALGEBRA **Money and financial mathematics**		
ACMNA299: Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies	2	Interest and depreciation
PATTERNS AND ALGEBRA		
ACMNA230: Factorise algebraic expressions by taking out a common algebraic factor	5	Products and factors
ACMNA231: Simplify algebraic products and quotients using index laws	5	Products and factors
ACMNA232: Apply the four operations to simple algebraic fractions with numerical denominators	5	Products and factors
ACMNA233: Expand binomial products and factorise monic quadratic expressions using variety of strategies	5	Products and factors
ACMNA234: Substitute values into formulas to determine an unknown	7	Equations and logarithms
LINEAR AND NON-LINEAR RELATIONSHIPS		
ACMNA235: Solve problems involving linear equations, including those derived from formulas	7	Equations and logarithms
ACMNA236: Solve linear inequalities and graph their solutions on a number line	7	Equations and logarithms
ACMNA237: Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology	10	Simultaneous equations
ACMNA238: Solve problems involving parallel and perpendicular lines	3	Graphing lines
ACMNA239: Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate	8	Graphing curves
ACMNA240: Solve linear equations involving simple algebraic fractions	7	Equations and logarithms
ACMNA241: Solve simple quadratic equations using a range of strategies	7	Equations and logarithms
MEASUREMENT AND GEOMETRY **Using units of measurement**		
ACMMG242: Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids	4	Surface area and volume
Geometric reasoning		
ACMMG243: Formulate proofs involving congruent triangles and angle properties	13	Geometry
ACMMG244: Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	13	Geometry

Content description	*New Century Maths 10 Advanced* chapter	
PYTHAGORAS AND TRIGONOMETRY		
ACMMG245: Solve right-angled triangle problems including those involving direction and angles of elevation and depression	9	Further trigonometry
STATISTICS AND PROBABILITY **Chance**		
ACMSP246: Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence.	12	Probability
ACMSP247: Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language	12	Probability
DATA REPRESENTATION AND INTERPRETATION		
ACMSP248: Determine quartiles and interquartile range	6	Comparing data
ACMSP249: Construct and interpret box plots and use them to compare data sets	6	Comparing data
ACMSP250: Compare shapes of box plots to corresponding histograms and dot plots	6	Comparing data
ACMSP251: Use scatter plots to investigate and comment on relationships between two numerical variables	6	Comparing data
ACMSP252: Investigate and describe bivariate numerical data where the independent variable is time	6	Comparing data
ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data	6	Comparing data

Year 10A content descriptions

Australian Curriculum descriptions (© ACARA 2012)

Content description	*New Century Maths 10 Advanced* chapter	
NUMBER AND ALGEBRA **Real numbers**		
ACMNA264: Define rational and irrational numbers and perform operations with surds and fractional indices	1	Surds
	5	Products and factors
ACMNA265: Use the definition of a logarithm to establish and apply the laws of logarithms	7	Equations and logarithms
	14	Functions
PATTERNS AND ALGEBRA		
ACMNA266: Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems	15	Polynomials
LINEAR AND NON-LINEAR RELATIONSHIPS		
ACMNA267: Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations	8	Graphing curves
	11	Quadratic equations and the parabola
ACMNA268: Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation	15	Polynomials
ACMNA269: Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts	5	Products and factors
	11	Quadratic equations and the parabola
ACMNA270: Solve simple exponential equations	7	Equations and logarithms
MEASUREMENT AND GEOMETRY **Using units of measurement**		
ACMMG271: Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids	4	Surface area and volume
GEOMETRIC REASONING		
ACMMG272: Prove and apply angle and chord properties of circles	16	Circle geometry
PYTHAGORAS AND TRIGONOMETRY		
ACMMG273: Establish the sine, cosine and area rules for any triangle and solve related problems	9	Further trigonometry
ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies	9	Further trigonometry
ACMMG275: Solve simple trigonometric equations	9	Further trigonometry
ACMMG276: Apply Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles	9	Further trigonometry
STATISTICS AND PROBABILITY **Data representation and interpretation**		
ACMSP277: Investigate reports of studies in digital media and elsewhere for information on their planning and implementation	6	Comparing data
ACMSP278: Calculate and interpret the mean and standard deviation of data and use these to compare data sets	6	Comparing data
ACMSP279: Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation	6	Comparing data

ABOUT THIS BOOK

Coverage of the syllabus

- *New Century Maths 10* Advanced covers both the NSW syllabus and the Australian curriculum, as shown by the table of contents and syllabus grids on the previous pages.
- This book contains Stages 5.1, 5.2 and 5.3 content. Stage 5.3 content is marked by * and coded in blue, while NSW-only content that is not Australian curriculum is marked with #.
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiencies covered in each section.

Chapter outline

		Working mathematically				
7.01	Equations with algebraic fractions	U	F			
7.02	Quadratic equations $x^2 + bx + c = 0$	U	F		R	C
7.03	Simple cubic equations $ax^3 = c$*#	U	F		R	C
7.04	Equation problems	U	F	PS	R	C
7.05	Equations and formulas	U	F		R	C
7.06	Changing the subject of a formula*#	U	F		R	C
7.07	Graphing inequalities on a number line	U	F			C
7.08	Solving inequalities	U	F		R	C
7.09	Logarithms*	U	F		R	C
7.10	Logarithm laws*	U	F		R	
7.11	Exponential and logarithmic equations*	U	F	PS	R	

*STAGE 5.3

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

U = UNDERSTANDING

Understanding is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.

F = FLUENCY

Fluency is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.

PS = PROBLEM SOLVING

Problem solving is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.

R = REASONING

Reasoning is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

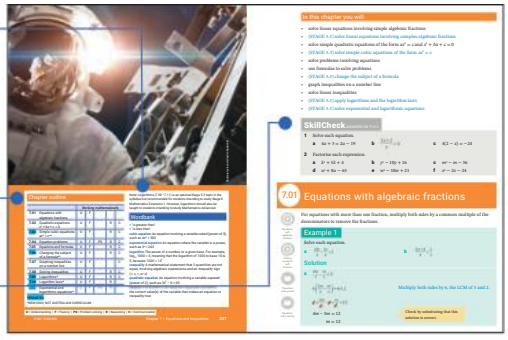
C = COMMUNICATING

Communicating is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

- **Understanding** and **Fluency** can be found in every exercise and activity, while **Problem solving**, **Reasoning** and **Communicating** are found in the **Investigations**, **Technology**, **Mental skills**, **Language of Maths** and **Topic summary** activities, and explicitly labelled in every exercise (see 'In each chapter' next page).

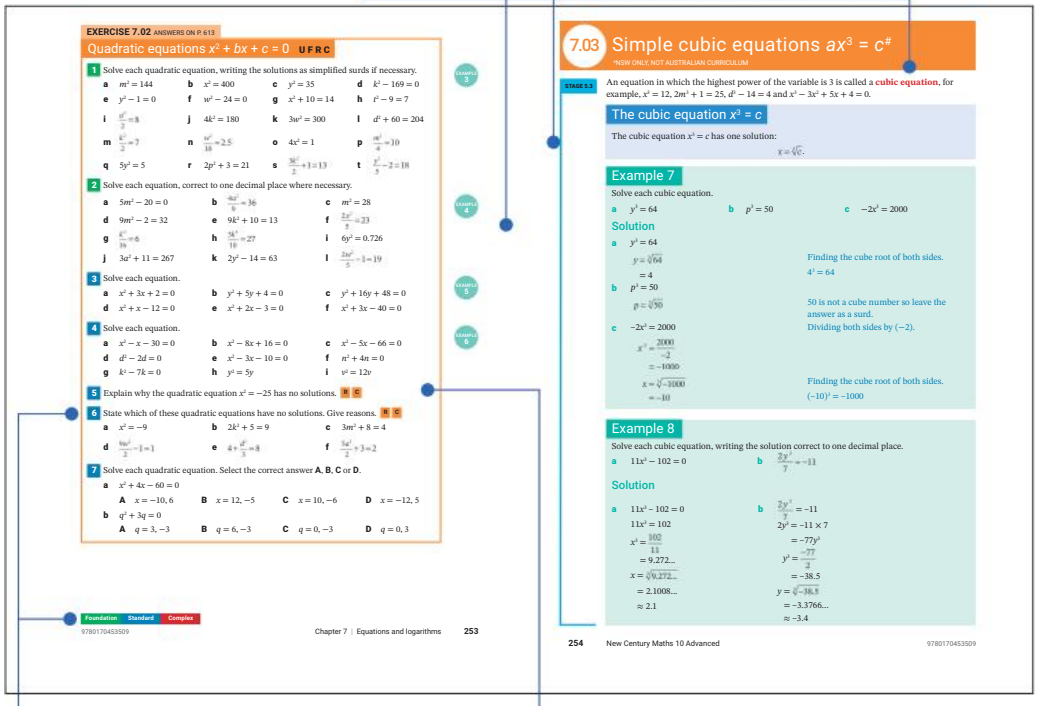
At the beginning of each chapter

- A **Chapter outline** and a **Wordbank** chapter glossary (a full glossary appears at the back of the book)
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter



In each chapter

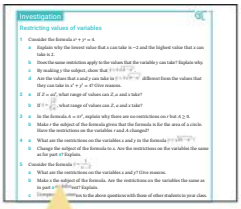
- Important facts and formulas are highlighted in a shaded box.
- Glossary terms are printed in **red**.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.



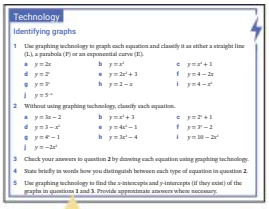
- The exercise questions are colour-coded for level of difficulty:

Foundation **Standard** **Complex**

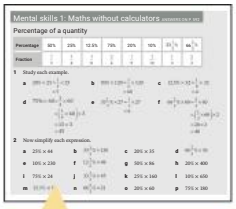
Specific questions are also labelled **PS** for **Problem solving**, **R** for **Reasoning** and **C** for **Communicating**.



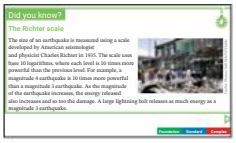
Investigations explore the syllabus in more detail, through group work, discovery and modelling activities.



Technology includes spreadsheets, online calculators, dynamic geometry software and the Internet.



Mental skills reinforce mental calculation strategies ('calculator-free maths')



Did you know? contains interesting facts and applications of the mathematics learned in the chapter.

At the end of each chapter

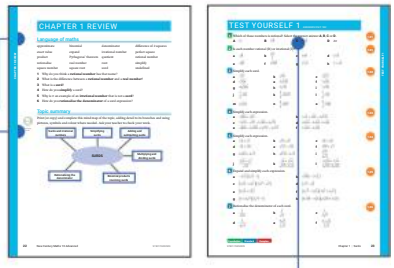
Power plus is an extension/challenge exercise.

Language of maths has a chapter word list and literacy questions.

Topic summary has a mind map activity with downloadable solutions.

Test Yourself contains chapter revision questions linked to the relevant exercise set.

Practice sets after every 3–4 chapters

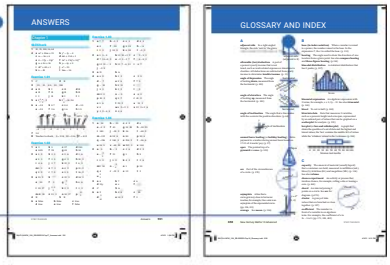


At the end of the book

General practice exercise

Answers (worked solutions are on the teacher website).

Glossary and index.



NelsonNet student website

Visit the *NelsonNet* student website, **www.nelsonnet.com.au** to access 200+ worksheets, 100+ videos and more:

Worksheets and **puzzle sheets**



Worksheet


Puzzle sheet

Skillsheets of examples and exercises


Skillsheet

Video tutorials: worked examples explained by 'flipped classroom' teachers


Video tutorial

Technology worksheets


Technology

Chapter quizzes: interactive and self-marking



Chapter quiz

NelsonNet teacher website

Teachers can access the *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, for:

- a **teaching program**
- **topic tests**
- **worked solutions** to every question
- **quiz generator and questionbank (Cognero)**
- **chapter PDFs**

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

NelsonNetBook

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of NelsonNetBook you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students

New Century Maths 7–10 workbooks

Each *New Century Maths* title has a companion workbook for students to write in, sold separately containing 200 pages of worksheets, puzzle sheets, StartUp topic assignments and weekly homework sheets. Handy for homework, class assessment, practice, revision, relief classes or 'catch-up' lessons.

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MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study something in detail by breaking it into its parts

bisect: cut in half

calculate: *see* **evaluate**

classify, identify: state the type, category or feature of an item or situation

comment: express an observation or opinion about a result

complete: fill in detail to make a statement, diagram or table correct or finished

compare: show how 2 or more things are similar or different

construct: draw an accurate diagram

convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately.

evaluate, calculate: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression, for example, expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

factorise: take out the highest common factor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand**.

give reasons: show the rules or thinking used when solving a problem. See also **justify**.

graph: display on a number line, number plane or statistical graph.

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: *see* **classify**.

interpret: find meaning in an answer or result

justify: give reasons or evidence to support your argument or conclusion. See also **give reasons**.

measure: use an instrument to find the size of something, for example, use a ruler to determine the length of a pen.

prove that: *see* **show that**

rationalise the denominator: simplify a fraction involving a surd by making its denominator rational (that is, not a surd).

recall: remember and state.

reduce (a fraction) to its lowest terms: see **simplify (a fraction)**.

round (a number): find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, \$12.9598 rounded to the nearest cent is \$12.96, 0.166 66 rounded to 3 decimal places is 0.167.

show that, prove that: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

show working: show the steps you used to find the answer

simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

simplify (a fraction): reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$.

simplify (a ratio or rate): reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.

sketch: draw a rough diagram that shows the general shape or idea, less accurate than **construct**

solve: find the value(s) of an unknown variable in an equation or inequality

state: *see* **write**.

substitute: replace a variable by a number and evaluate

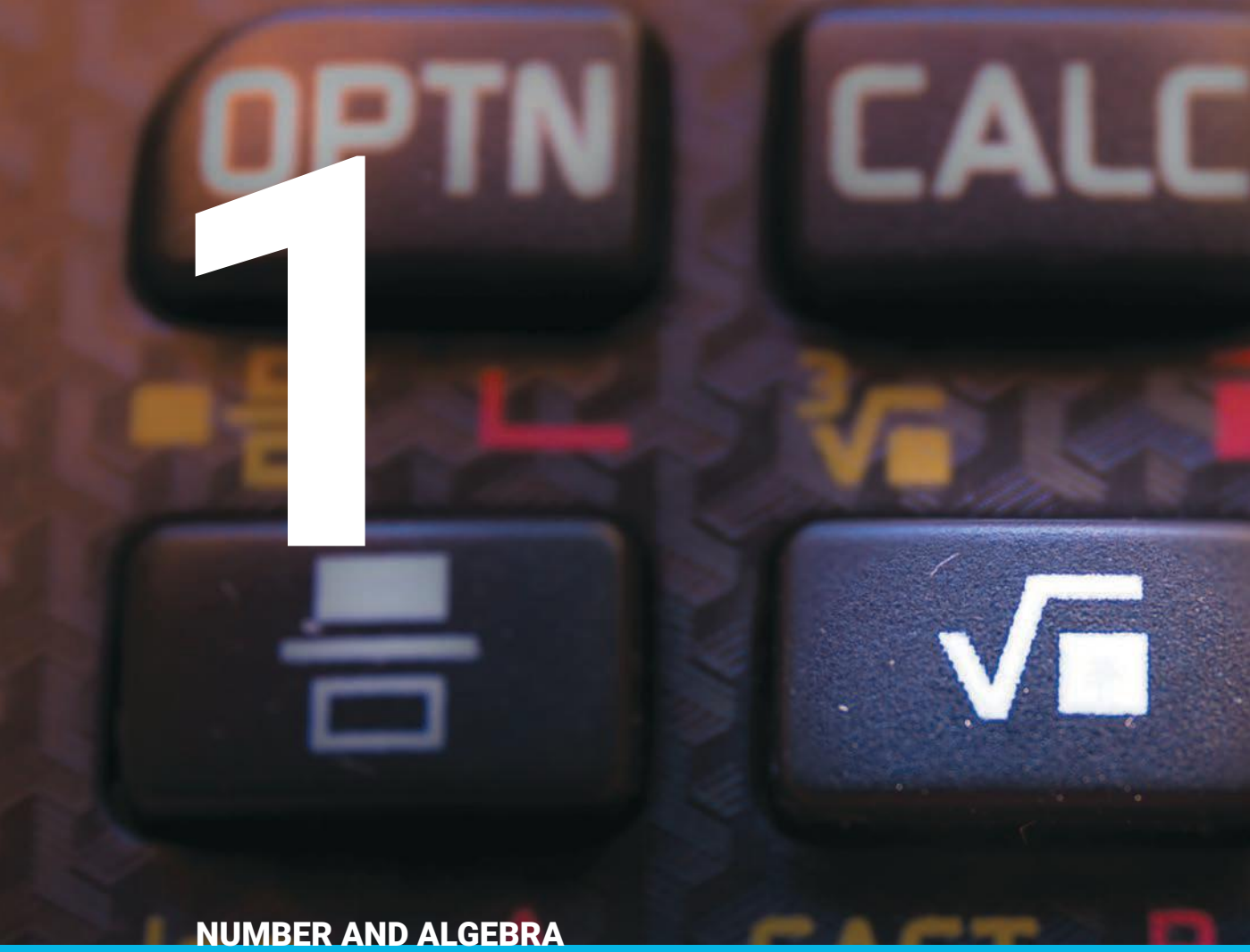
verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write correct to: See **round (a number)**.

write, state: give the answer, formula or result without showing any working or explanation (this usually means that the answer can be found mentally, or in one step)

SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
-3	negative 3
π	pi = 3.14159
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152 ...
$^\circ$	degree
$42^\circ 17' 54''$	42 degrees, 17 minutes, 54 seconds
$\angle A$	angle A
$\triangle ABC$	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\equiv$	is congruent to
$\sim$	is similar to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root
$P(E)$	the probability of event E occurring
$P(\bar{E})$	the probability of event E not occurring
$P(B\|A)$	the probability of B given A
LHS	left-hand side
RHS	right-hand side
%	percentage
p.a.	per annum (per year)
sin	sine ratio
cos	cosine ratio
tan	tangent ratio
$\bar{x}$	the mean (average)
σ	the standard deviation
Σ	the sum of
Q_1	1st quartile or lower quartile
Q_2	median (2nd quartile)
Q_3	3rd quartile or upper quartile
IQR	interquartile range
α	alpha
θ	theta
ϕ	phi
μ	micro-, mu
m	gradient



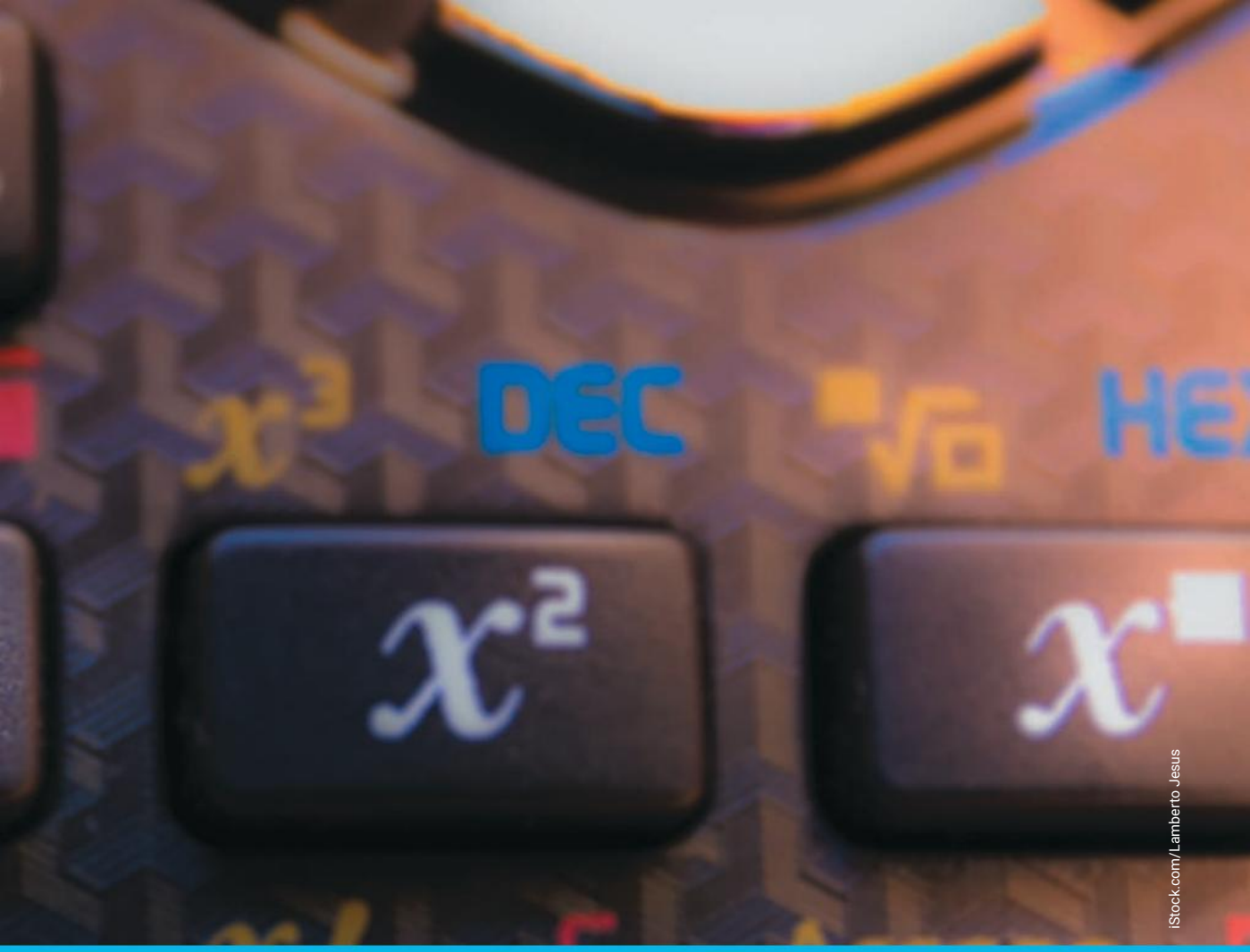
1

NUMBER AND ALGEBRA

SURDS

When applying Pythagoras' theorem, we have found lengths that cannot be expressed as an exact rational number. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit. A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found.

The Pythagoreans explained nature, the universe — in fact everything — in terms of whole numbers. Apparently, they were so upset about the discovery of surds that they tried to keep the discovery a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.



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Chapter outline

		Working mathematically				
1.01	Surds and irrational numbers*	U	F		R	C
1.02	Simplifying surds*	U	F		R	
1.03	Adding and subtracting surds*	U	F			
1.04	Multiplying and dividing surds*	U	F			
1.05	Binomial products involving surds*	U	F			
1.06	Rationalising the denominator*	U	F			

***STAGE 5.3**

Wordbank

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction $\frac{a}{b}$

rational number Any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd)

real number A number that is either rational or irrational and whose value can be graphed on a number line

simplify a surd To write a surd $\sqrt{x}$ in its simplest form so that x has no factors that are perfect squares

surd A square root (or other root) whose exact value cannot be found

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) describe real, rational and irrational numbers and surds
- (STAGE 5.3) simplify, add, subtract, multiply and divide surds
- (STAGE 5.3) expand and simplify binomial products involving surds
- (STAGE 5.3) rationalise the denominator of expressions of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

SkillCheck ANSWERS ON P. 591

1 Select the square numbers from the following list of numbers.

44 81 25 100 75 72 16 50 64 32

2 Expand and simplify each expression.

a $(m+3)(m+7)$ **b** $(y+1)(y-4)$ **c** $(n-2)(n-3)$

d $(2d+3)(1+3d)$ **e** $(1-5p)(4+3p)$ **f** $(3a+2f)(a+5f)$

g $(x+4)^2$ **h** $(y-3)^2$ **i** $(2k+1)^2$

j $(a-5)(a+5)$ **k** $(t+7)(t-7)$ **l** $(3m+4)(3m-4)$

1.01 Surds and irrational numbers

STAGE 5.3

A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

$\sqrt{7}$ is read as 'the square root of 7' or simply 'root 7'.

$\sqrt{7}$ is called the **exact value**, $\sqrt{7} \approx 2.64575$ is the **approximate value**.

Rational numbers such as fractions, decimals and percentages, can be expressed in the form $\frac{a}{b}$, where a and b are integers ($b \neq 0$), but surds are **irrational numbers** because they cannot be expressed in this form.

Rational numbers

can be expressed in the form $\frac{a}{b}$

Integers

$\frac{4}{1}=4, \frac{26}{1}=26, \frac{-3}{1}=-3$

Terminating decimals

$0.5,\ 7\frac{1}{8}=7.125,$

$16\%=0.16,\ 1.32$

Recurring decimals

$\frac{2}{3}=0.666\ldots$

$\frac{5}{6}=0.833\ldots$

$\frac{4}{11}=0.3636\ldots$

Irrational numbers

cannot be expressed in the form $\frac{a}{b}$

Surds

$\sqrt{5}, -\sqrt{2}, \frac{\sqrt{11}}{3}, 8\sqrt{6}$

Transcendental numbers

Have no pattern and are non-recurring

e.g. $\pi = 3.14159...$,

$\cos 38° = 0.78801...$, $e = 2.71828...$

Example 1

Is each number rational or irrational?

a $4\frac{2}{5}$ **b** $\sqrt[3]{-8}$ **c** $-\sqrt{7}$ **d** $0.\dot{6}$ **e** 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$ which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

b $\sqrt[3]{-8} = -2$ which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645751311\ldots$ The digits run endlessly without repeating.

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666\ldots$ which is a recurring decimal

$= \frac{2}{3}$ which is a fraction

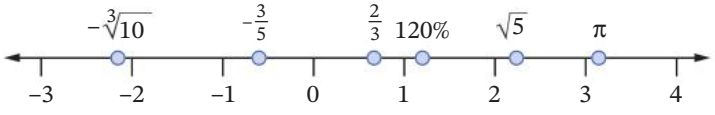
$\therefore 0.\dot{6}$ is a rational number.

e $5\pi = 15.707\,963\,27\ldots$ The digits run endlessly without repeating.

$\therefore 5\pi$ is an irrational number.

Surds on a number line

The rational and irrational numbers together make up the **real numbers**. Any real number can be represented by a point on the number line.



$-\sqrt[3]{10} \approx -2.1544\ldots$ irrational (surd)

$-\frac{3}{5} = -0.6$ rational (fraction)

$\frac{2}{3} \approx 0.6666\ldots$ rational (fraction)

$120\% = 1.2$ rational (percentage)

$\sqrt{5} \approx 2.2360\ldots$ irrational (surd)

$\pi \approx 3.1415\ldots$ irrational (pi)

STAGE 5.3

WS Surds on the number line

Example 2

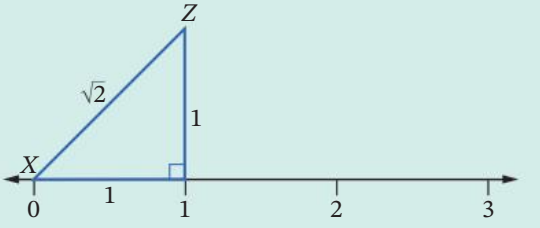
Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

Solution

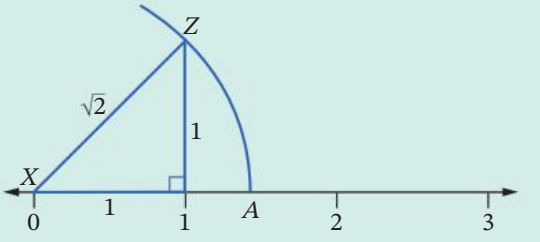
Step 1: Using a scale of 1 unit to 2 cm, draw a number line as shown.



Step 2: Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3: With 0 as the centre, use compasses with radius $XZ\left(\sqrt{2}\right)$ to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142 ...



EXERCISE 1.01 ANSWERS ON P. 591

Surds and irrational numbers U F R C

1 Which one of the following is a surd? Select the correct answer **A**, **B**, **C** or **D**. C

A $\sqrt{9}$ **B** $\sqrt{225}$ **C** $\sqrt{160}$ **D** $\sqrt{81}$

2 Which one of the following is NOT a surd? Select **A**, **B**, **C** or **D**. C

A $\sqrt{77}$ **B** $\sqrt{144}$ **C** $\sqrt{18}$ **D** $\sqrt{200}$

3 Select the surds from the following list of square roots. C

$\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$

$\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

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EXAMPLE 1

1.01

4 Is each number rational (R) or irrational (I)? R C

a $5.\dot{6}$ **b** $\sqrt{8}$ **c** $\sqrt{4}$ **d** $3\frac{1}{7}$

e $\sqrt[3]{27}$ **f** $1.3\dot{5}$ **g** $\sqrt[3]{-64}$ **h** $27\frac{1}{2}\%$

i $\sqrt{5\times10^3}$ **j** $\frac{3}{11}$ **k** $\frac{\sqrt{50}}{3}$ **l** $\sqrt{\sqrt{4}}$

5 Arrange each set of numbers in descending order.

a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ **b** $\sqrt[3]{20}, 2.6\dot{}, 2\frac{7}{9}$

6 Express each real number correct to one decimal place and graph them on a number line.

a $-1\frac{4}{5}$ **b** 74% **c** $\frac{4}{11}$ **d** $-\sqrt{12}$

e $-\sqrt[3]{15}$ **f** $2\frac{5}{9}$ **g** $\frac{\pi}{2}$ **h** 187%

EXAMPLE 2

7 Use the method from Example **2** to estimate the value of $\sqrt{2}$ on a number line.

8 **a** Use the method from Example **2** to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.

b Use a similar method to estimate the following surds on a number line:

i $\sqrt{10}$ **ii** $\sqrt{17}$

Investigation

Proof that $\sqrt{2}$ is irrational

A method of proof sometimes used in mathematics is to assume the **opposite** of what is being proved, and show that it is **impossible**. This is called a **proof by contradiction**, and we will use it to prove that $\sqrt{2}$ is irrational.

First, we assume that $\sqrt{2}$ is rational. So, assume that $\sqrt{2}$ can be written as a simplified fraction $\frac{a}{b}$, where a and b are integers ($b \neq 0$) with no common factor.

$\sqrt{2} = \frac{a}{b}$

$2 = \frac{a^2}{b^2}$ Squaring both sides

$a^2 = 2b^2$

$2b^2$ is an even number because it is divisible by 2, $\therefore a^2$ is even.

$\therefore a$ is even, because an even integer multiplied by itself is always even and an odd integer multiplied by itself is always odd.

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$\therefore a = 2m$, where m is another integer.

$\therefore a^2 = (2m)^2 = 2b^2$

$4m^2 = 2b^2$

$2m^2 = b^2$

$b^2 = 2m^2$

$\therefore b^2$ is even

$\therefore b$ is even.

But a and b can't **both** be even because this contradicts the assumption that a and b have no common factors. Therefore, the assumption that $\sqrt{2}$ is rational is false, so $\sqrt{2}$ must be irrational.

Use proof by contradiction to show that these surds are irrational:

a $\sqrt{3}$ **b** $\sqrt{5}$

1.02 Simplifying surds

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Simplifying surds quiz

Simplifying surds quiz

The square root of x

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x} = 0$.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of x.

For $x \geq 0$, $\left(\sqrt{x}\right)^2 = \sqrt{x} \times \sqrt{x} = x$ and $\sqrt{x^2} = x$.

Example 3

Simplify each expression.

a $\left(\sqrt{7}\right)^2$ **b** $\left(3\sqrt{5}\right)^2$ **c** $\left(-2\sqrt{3}\right)^2$

Solution

a $\left(\sqrt{7}\right)^2 = 7$

b $\left(3\sqrt{5}\right)^2 = 3\sqrt{5} \times 3\sqrt{5}$

$= 3^2 \times \left(\sqrt{5}\right)^2$

$= 9 \times 5$

$= 45$

c $\left(-2\sqrt{3}\right)^2 = (-2)^2 \times \left(\sqrt{3}\right)^2$

$= 4 \times 3$

$= 12$

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into 2 factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, ...

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Simplifying surds 2

Simplifying surds 1

1.02

Example 4

Simplify each surd.

a $\sqrt{8}$ **b** $\sqrt{108}$ **c** $4\sqrt{45}$ **d** $\dfrac{\sqrt{288}}{3}$

Solution

a $\sqrt{8} = \sqrt{4} \times \sqrt{2}$ — 4 is a square number
$= 2 \times \sqrt{2}$
$= 2\sqrt{2}$

b *Method 1*:
$\sqrt{108} = \sqrt{36} \times \sqrt{3}$
$= 6 \times \sqrt{3}$
$= 6\sqrt{3}$

Method 2:
$\sqrt{108} = \sqrt{4} \times \sqrt{27}$
$= 2 \times \sqrt{9} \times \sqrt{3}$
$= 2 \times 3 \times \sqrt{3}$
$= 6\sqrt{3}$

Method 2 involves simplifying surds *twice* ($\sqrt{108}$ and $\sqrt{27}$). Method 1 shows that when simplifying surds, look for the highest square factor possible.

c $4\sqrt{45} = 4 \times \sqrt{9} \times \sqrt{5}$
$= 4 \times 3 \times \sqrt{5}$
$= 12\sqrt{5}$

d $\dfrac{\sqrt{288}}{3} = \dfrac{\sqrt{144} \times \sqrt{2}}{3}$
$= \dfrac{12\sqrt{2}}{3}$
$= \dfrac{12^{4}\sqrt{2}}{\cancel{3}^{1}}$
$= 4\sqrt{2}$

EXERCISE 1.02 ANSWERS ON P. 591

Simplifying surds U F R

EXAMPLE 3

1 Simplify each expression.

a $\left(\sqrt{2}\right)^2$ **b** $\left(\sqrt{5}\right)^2$ **c** $\left(3\sqrt{3}\right)^2$ **d** $\left(5\sqrt{10}\right)^2$

e $\left(\sqrt{0.09}\right)^2$ **f** $\left(-2\sqrt{7}\right)^2$ **g** $\left(-3\sqrt{5}\right)^2$ **h** $\left(-5\sqrt{2}\right)^2$

EXAMPLE 4

2 Simplify each surd.

a $\sqrt{50}$ **b** $\sqrt{27}$ **c** $\sqrt{24}$ **d** $\sqrt{54}$

e $\sqrt{243}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{200}$

i $\sqrt{96}$ **j** $\sqrt{63}$ **k** $\sqrt{288}$ **l** $\sqrt{108}$

m $\sqrt{75}$ **n** $\sqrt{147}$ **o** $\sqrt{32}$ **p** $\sqrt{242}$

q $\sqrt{162}$ **r** $\sqrt{245}$ **s** $\sqrt{125}$ **t** $\sqrt{512}$

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3 Simplify each expression.

a $5\sqrt{50}$ **b** $3\sqrt{8}$ **c** $4\sqrt{27}$ **d** $8\sqrt{98}$

e $\frac{\sqrt{40}}{2}$ **f** $\frac{\sqrt{243}}{9}$ **g** $\frac{\sqrt{28}}{6}$ **h** $3\sqrt{24}$

i $9\sqrt{68}$ **j** $\frac{\sqrt{3125}}{10}$ **k** $\frac{1}{2}\sqrt{72}$ **l** $\frac{3}{4}\sqrt{48}$

m $10\sqrt{160}$ **n** $3\sqrt{75}$ **o** $7\sqrt{68}$ **p** $\frac{\sqrt{52}}{6}$

4 Which surd below is equivalent to $4\sqrt{50}$? Select **A**, **B**, **C** or **D**.

A $8\sqrt{5}$ **B** $20\sqrt{2}$ **C** $8\sqrt{2}$ **D** $20\sqrt{5}$

5 Which surd below is equivalent to $\frac{\sqrt{250}}{10}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{5}}{10}$ **B** $\frac{\sqrt{10}}{2}$ **C** $2\sqrt{10}$ **D** $5\sqrt{10}$

6 Decide whether each statement is true (T) or false (F). R

a $3\sqrt{7} = \sqrt{21}$ **b** $\sqrt{12} = 6$

c $\left(\sqrt{9.4}\right)^2 = 9.4$ **d** $\sqrt{75} = 5\sqrt{3}$

e $\sqrt{3} \approx 1.7$ **f** The exact value of $\sqrt{10}$ is 3.162 277 8

Did you know?

Unreal numbers are imaginary!

$\sqrt{-2}$ is not a real number, because there is no real number which, if squared, equals –2. Numbers such as $\sqrt{-2}$, $\sqrt{-10}$, $\sqrt[4]{-17}$ are called **unreal** or **imaginary numbers** and cannot be graphed on a number line (that is, their values cannot be ordered).

Imaginary numbers were first noticed by the Greek mathematician Hero of Alexandria in the 1st century CE. Imaginary numbers were largely ignored until the 18th century when they were studied by Swiss mathematician Leonhard Euler and the German mathematician Carl Friedrich Gauss.

$\sqrt{-1}$ is defined to be the imaginary number i, so $\sqrt{-1} = i$ and $i^2 = -1$.

$\therefore \sqrt{-36} = \sqrt{36 \times (-1)} = \sqrt{36} \times \sqrt{-1} = 6i$.

Imaginary numbers are useful for solving physics and engineering problems involving heat conduction, elasticity, hydrodynamics and the flow of electric current.

Simplify each imaginary number:

a $\sqrt{-100}$ **b** $\sqrt{-25}$ **c** $\sqrt[4]{-16}$ **d** $\sqrt[6]{-64}$

Adding and subtracting surds

1.03

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'. You may first need to express all the surds in their simplest forms.

STAGE 5.3



Surds code puzzle

1.03

Example 5

Simplify each expression.

a $4\sqrt{2}+5\sqrt{2}$ **b** $7\sqrt{3}-2\sqrt{3}$ **c** $5\sqrt{2}-3\sqrt{3}+\sqrt{2}$

d $\sqrt{50}+\sqrt{32}$ **e** $\sqrt{8}-\sqrt{27}+\sqrt{18}$ **f** $5\sqrt{20}-3\sqrt{125}$

Solution

a $4\sqrt{2}+5\sqrt{2}=9\sqrt{2}$

b $7\sqrt{3}-2\sqrt{3}=5\sqrt{3}$

c $5\sqrt{2}-3\sqrt{3}+\sqrt{2}=6\sqrt{2}-3\sqrt{3}$

d
$$\begin{aligned}\sqrt{50}+\sqrt{32} &= \sqrt{25}\sqrt{2}+\sqrt{16}\sqrt{2}\\ &= 5\sqrt{2}+4\sqrt{2}\\ &= 9\sqrt{2}\end{aligned}$$

e
$$\begin{aligned}\sqrt{8}-\sqrt{27}+\sqrt{18} &= \sqrt{4}\sqrt{2}-\sqrt{9}\sqrt{3}+\sqrt{9}\sqrt{2}\\ &= 2\sqrt{2}-3\sqrt{3}+3\sqrt{2}\\ &= 5\sqrt{2}-3\sqrt{3}\end{aligned}$$

f
$$\begin{aligned}5\sqrt{20}-3\sqrt{125} &= 5\sqrt{4}\sqrt{5}-3\sqrt{25}\sqrt{5}\\ &= 5\times 2\sqrt{5}-3\times 5\sqrt{5}\\ &= 10\sqrt{5}-15\sqrt{5}\\ &= -5\sqrt{5}\end{aligned}$$

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Adding and subtracting surds U F

EXAMPLE 5

1 Simplify each expression.

a $5\sqrt{7}+2\sqrt{7}$ **b** $3\sqrt{2}-8\sqrt{2}$ **c** $7\sqrt{5}-\sqrt{5}$

d $\sqrt{5}+3\sqrt{5}$ **e** $5\sqrt{17}-5\sqrt{17}$ **f** $3\sqrt{10}-2\sqrt{10}$

g $4\sqrt{15}-3\sqrt{15}+7\sqrt{15}$ **h** $5\sqrt{6}-2\sqrt{6}-4\sqrt{6}$ **i** $3\sqrt{3}+4\sqrt{3}-5\sqrt{3}$

j $4\sqrt{5}+7\sqrt{5}-\sqrt{5}$ **k** $8\sqrt{10}-5\sqrt{10}+3\sqrt{10}$ **l** $10\sqrt{3}-3\sqrt{3}-12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5}-8+2\sqrt{5}$ **b** $11\sqrt{10}+3\sqrt{2}+2\sqrt{10}$

c $-4\sqrt{3}+5\sqrt{2}-5\sqrt{3}$ **d** $3\sqrt{15}+3\sqrt{2}+4\sqrt{15}+5\sqrt{2}$

e $\sqrt{7}-3\sqrt{5}-4\sqrt{7}+\sqrt{5}$ **f** $4\sqrt{6}-3\sqrt{3}-2\sqrt{6}-5\sqrt{3}$

g $10\sqrt{11}-5\sqrt{3}+3\sqrt{11}+4\sqrt{3}$ **h** $\sqrt{13}+8\sqrt{7}-7\sqrt{13}+3\sqrt{7}$

i $2\sqrt{5}-3\sqrt{7}-2\sqrt{5}-3\sqrt{7}$ **j** $4\sqrt{10}-3\sqrt{5}-4\sqrt{10}$

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3 For each expression, select the correct simplified answer **A**, **B**, **C** or **D**.

a $\sqrt{3}+\sqrt{12}$

A $5\sqrt{3}$ **B** $\sqrt{15}$ **C** $2\sqrt{6}$ **D** $3\sqrt{3}$

b $4\sqrt{5}-2\sqrt{125}$

A $-6\sqrt{5}$ **B** $\sqrt{5}$ **C** $-\sqrt{45}$ **D** $-46\sqrt{5}$

4 Simplify each expression.

a	$\sqrt{8}+\sqrt{32}$	**b**	$\sqrt{108}-\sqrt{27}$	**c**	$\sqrt{20}-\sqrt{80}$	**d**	$\sqrt{28}-\sqrt{63}$
e	$3\sqrt{6}+\sqrt{24}$	**f**	$2\sqrt{5}+\sqrt{125}$	**g**	$\sqrt{40}-\sqrt{90}$	**h**	$5\sqrt{11}+\sqrt{99}$
i	$3\sqrt{2}+\sqrt{18}$	**j**	$\sqrt{27}+5\sqrt{3}$	**k**	$\sqrt{200}-7\sqrt{2}$	**l**	$\sqrt{50}+\sqrt{32}$
m	$5\sqrt{3}+2\sqrt{27}$	**n**	$3\sqrt{20}-\sqrt{245}$	**o**	$7\sqrt{12}-5\sqrt{48}$	**p**	$4\sqrt{27}+2\sqrt{243}$
q	$3\sqrt{63}-2\sqrt{28}$	**r**	$2\sqrt{98}+3\sqrt{162}$	**s**	$-5\sqrt{6}+2\sqrt{150}$	**t**	$4\sqrt{50}+3\sqrt{18}$
u	$5\sqrt{27}-6\sqrt{75}$	**v**	$3\sqrt{112}-2\sqrt{252}$	**w**	$\sqrt{32}+\sqrt{8}+\sqrt{12}$	**x**	$\sqrt{27}+\sqrt{54}+\sqrt{243}$
y	$\sqrt{98}-3\sqrt{20}-2\sqrt{8}$	**z**	$3\sqrt{96}-2\sqrt{150}+\sqrt{24}$				

1.04 Multiplying and dividing surds

STAGE 5.3

Surds

WS Multiplying and dividing surds

The square root of products and quotients

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Example 6

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{10} \times \sqrt{6}$ **c** $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$ **e** $\sqrt{54} \div \left(-\sqrt{2}\right)$ **f** $\dfrac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b $\sqrt{10} \times \sqrt{6} = \sqrt{60}$
$= \sqrt{4} \times \sqrt{15}$
$= 2\sqrt{15}$

c $3\sqrt{7} \times 5\sqrt{7} = 3 \times 5 \times \sqrt{7} \times \sqrt{7}$
$= 15 \times 7$
$= 105$

d $5\sqrt{27} \times 3\sqrt{6} = 5 \times 3 \times \sqrt{27} \times \sqrt{6}$
$= 15\sqrt{162}$
$= 15 \times \sqrt{81} \times \sqrt{2}$
$= 15 \times 9\sqrt{2}$
$= 135\sqrt{2}$

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e $\sqrt{54} \div (-\sqrt{2}) = -\frac{\sqrt{54}}{\sqrt{2}}$
$= -\sqrt{27}$
$= -\sqrt{9} \times \sqrt{3}$
$= -3\sqrt{3}$

f $\frac{15\sqrt{32}}{5\sqrt{8}} = 3\sqrt{4}$
$= 3 \times 2$
$= 6$

Example 7

Simplify $\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}} = \frac{20\sqrt{24}}{10\sqrt{8}}$
$= 2\sqrt{3}$

EXERCISE 1.04 ANSWERS ON P. 591

Multiplying and dividing surds U F

EXAMPLE 6

1 Simplify each expression.

a $\sqrt{7} \times \sqrt{2}$ **b** $-\sqrt{5} \times \sqrt{7}$ **c** $\sqrt{6} \times \sqrt{8}$

d $\sqrt{12} \times \sqrt{3}$ **e** $\sqrt{10} \times (-\sqrt{5})$ **f** $3\sqrt{3} \times 5\sqrt{3}$

g $5\sqrt{10} \times 3\sqrt{3}$ **h** $-2\sqrt{7} \times 5\sqrt{3}$ **i** $7\sqrt{5} \times 4\sqrt{5}$

j $2\sqrt{3} \times (-5\sqrt{6})$ **k** $4\sqrt{3} \times \sqrt{27}$ **l** $-3\sqrt{5} \times 4\sqrt{10}$

m $-7\sqrt{2} \times 4\sqrt{8}$ **n** $\sqrt{18} \times 8\sqrt{3}$ **o** $10\sqrt{2} \times 2\sqrt{8}$

p $3\sqrt{18} \times 5\sqrt{12}$ **q** $3\sqrt{44} \times (-2\sqrt{99})$ **r** $5\sqrt{8} \times 4\sqrt{40}$

s $8\sqrt{3} \times 3\sqrt{54}$ **t** $-8\sqrt{32} \times \sqrt{27}$ **u** $\sqrt{90} \times \sqrt{72}$

v $-5\sqrt{20} \times 3\sqrt{8}$ **w** $7\sqrt{18} \times 3\sqrt{24}$ **x** $3\sqrt{48} \times 2\sqrt{12}$

2 Simplify each expression.

a $\sqrt{15} \div \sqrt{3}$ **b** $\sqrt{18} \div (-\sqrt{6})$ **c** $\frac{6\sqrt{48}}{2\sqrt{8}}$

d $10\sqrt{54} \div 5\sqrt{27}$ **e** $-3\sqrt{98} \div 6\sqrt{14}$ **f** $\frac{7\sqrt{18}}{\sqrt{2}}$

g $2\sqrt{24} \div 4\sqrt{6}$ **h** $\frac{\sqrt{128}}{\sqrt{2}}$ **i** $15\sqrt{18} \div 3\sqrt{6}$

j $\frac{-20\sqrt{10}}{-4\sqrt{5}}$ **k** $36\sqrt{24} \div 9\sqrt{8}$ **l** $16\sqrt{30} \div 8\sqrt{5}$

m $12\sqrt{14} \div 6$ **n** $\frac{3\sqrt{2}}{-12}$ **o** $\sqrt{80} \div 4\sqrt{5}$

p $5\sqrt{60} \div \sqrt{15}$ **q** $6\sqrt{8} \div 3\sqrt{2}$ **r** $\frac{-42\sqrt{54}}{6\sqrt{3}}$

s $12\sqrt{63} \div 3\sqrt{7}$ **t** $\frac{8\sqrt{50}}{2\sqrt{200}}$ **u** $6\sqrt{3} \div \sqrt{243}$

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3 Simplify:

a $\sqrt{6}\times\sqrt{6}$ **b** $\sqrt{7}\times\sqrt{7}$ **c** $2\sqrt{3}\times\sqrt{3}$

d $5\sqrt{y}\times3\sqrt{y}$ **e** $\sqrt{x}\times\sqrt{x}$ **f** $\sqrt{a^2}\times\sqrt{a}$

4 Simplify $3\sqrt{2}\times\sqrt{6}$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** $6\sqrt{2}$ **C** $6\sqrt{3}$ **D** $12\sqrt{2}$

5 Simplify $20\sqrt{10}\div5\sqrt{2}$. Select **A**, **B**, **C** or **D**.

A $4\sqrt{5}$ **B** $15\sqrt{5}$ **C** 10 **D** 20

EXAMPLE 7

6 Simplify each expression.

a $\dfrac{3\sqrt{5}\times4\sqrt{2}}{3\sqrt{40}}$ **b** $\dfrac{3\sqrt{12}\times8\sqrt{6}}{4\sqrt{27}}$ **c** $\dfrac{5\sqrt{8}\times2\sqrt{90}}{10\sqrt{24}}$

d $\dfrac{4\sqrt{5}}{2\sqrt{15}\times5\sqrt{27}}$ **e** $\dfrac{10\sqrt{686}\times3\sqrt{12}}{5\sqrt{28}\times\sqrt{18}}$ **f** $\dfrac{8\sqrt{80}\times3\sqrt{2}}{4\sqrt{5}\times6\sqrt{8}}$

Mental skills 1: Maths without calculators ANSWERS ON P. 592

Percentage of a quantity

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

1 Study each example.

a $20\%\times25=\frac{1}{5}\times25$
$=5$

b $50\%\times120=\frac{1}{2}\times120$
$=60$

c $12.5\%\times32=\frac{1}{8}\times32$
$=4$

d $75\%\times60=\frac{3}{4}\times60$
$=\left(\frac{1}{4}\times60\right)\times3$
$=15\times3$
$=45$

e $33\frac{1}{3}\%\times27=\frac{1}{3}\times27$
$=9$

f $66\frac{2}{3}\%\times60=\frac{2}{3}\times60$
$=\left(\frac{1}{3}\times60\right)\times2$
$=20\times2$
$=40$

2 Now simplify each expression.

a $25\%\times44$ **b** $33\frac{1}{3}\%\times120$ **c** $20\%\times35$ **d** $66\frac{2}{3}\%\times36$

e $10\%\times230$ **f** $12\frac{1}{2}\%\times48$ **g** $50\%\times86$ **h** $20\%\times400$

i $75\%\times24$ **j** $33\frac{1}{3}\%\times45$ **k** $25\%\times160$ **l** $10\%\times650$

m $12.5\%\times88$ **n** $66\frac{2}{3}\%\times21$ **o** $20\%\times60$ **p** $75\%\times180$

Binomial products involving surds

1.05

STAGE 5.3

Example 8

Expand and simplify each expression.

a $\sqrt{3}\left(\sqrt{5}+\sqrt{7}\right)$ **b** $2\sqrt{11}\left(3\sqrt{11}-5\sqrt{2}\right)$

Solution

a $\sqrt{3}\left(\sqrt{5}+\sqrt{7}\right)=\sqrt{3}\times\sqrt{5}+\sqrt{3}\times\sqrt{7}$

$=\sqrt{15}+\sqrt{21}$

b $2\sqrt{11}\left(3\sqrt{11}-5\sqrt{2}\right)=2\sqrt{11}\times3\sqrt{11}-2\sqrt{11}\times5\sqrt{2}$

$=6\times11-10\times\sqrt{22}$

$=66-10\sqrt{22}$

Example 9

Expand and simplify each binomial product.

a $\left(\sqrt{7}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{3}\right)$ **b** $\left(3-2\sqrt{10}\right)\left(\sqrt{5}-3\sqrt{2}\right)$

Solution

a $\left(\sqrt{7}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{3}\right)=\sqrt{7}\left(3\sqrt{2}-\sqrt{3}\right)+\sqrt{5}\left(3\sqrt{2}-\sqrt{3}\right)$

$=\sqrt{7}\times3\sqrt{2}-\sqrt{7}\times\sqrt{3}+\sqrt{5}\times3\sqrt{2}-\sqrt{5}\times\sqrt{3}$

$=3\sqrt{14}-\sqrt{21}+3\sqrt{10}-\sqrt{15}$

b $\left(3-2\sqrt{10}\right)\left(\sqrt{5}-3\sqrt{2}\right)=3\left(\sqrt{5}-3\sqrt{2}\right)-2\sqrt{10}\left(\sqrt{5}-3\sqrt{2}\right)$

$=3\sqrt{5}-9\sqrt{2}-2\sqrt{50}+6\sqrt{20}$

$=3\sqrt{5}-9\sqrt{2}-2\left(5\sqrt{2}\right)+6\left(2\sqrt{5}\right)$

$=3\sqrt{5}-9\sqrt{2}-10\sqrt{2}+12\sqrt{5}$

$=15\sqrt{5}-19\sqrt{2}$

STAGE 5.3

Special binomial products

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Difference of 2 squares

$(a + b)(a - b) = a^2 - b^2$



Binomial products involving surds

Example 10

Expand and simplify each expression.

a $\left(\sqrt{7}-\sqrt{5}\right)^2$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)$

Solution

a $\left(\sqrt{7}-\sqrt{5}\right)^2=\left(\sqrt{7}\right)^2-2\times\sqrt{7}\times\sqrt{5}+\left(\sqrt{5}\right)^2$ using $(a - b)^2 = a^2 - 2ab + b^2$

$=7-2\sqrt{35}+5$

$=12-2\sqrt{35}$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2=\left(2\sqrt{3}\right)^2+2\times2\sqrt{3}\times3\sqrt{5}+\left(3\sqrt{5}\right)^2$ using $(a + b)^2 = a^2 + 2ab + b^2$

$=(4\times3)+12\sqrt{15}+(9\times5)$

$=12+12\sqrt{15}+45$

$=57+12\sqrt{15}$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$ using $(a + b)(a - b) = a^2 - b^2$

$= 5 - 2$

$= 3$

Note that because of the 'difference of 2 squares', the answer is not a surd but a rational number.

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)=\left(3\sqrt{11}\right)^2-4^2$ using $(a + b)(a - b) = a^2 - b^2$

$= (9 \times 11) - 16$

$= 83$

EXERCISE 1.05 ANSWERS ON P. 592

Binomial products involving surds U F

EXAMPLE 8

1 Expand and simplify each expression.

a $\sqrt{5}(\sqrt{3}+\sqrt{2})$ **b** $\sqrt{6}(\sqrt{2}-1)$ **c** $\sqrt{2}(\sqrt{3}+\sqrt{7})$

d $\sqrt{5}(3\sqrt{2}-\sqrt{5})$ **e** $3\sqrt{2}(\sqrt{2}+2\sqrt{3})$ **f** $-\sqrt{11}(4-\sqrt{5})$

g $2\sqrt{7}(3\sqrt{7}-4)$ **h** $5\sqrt{5}(1+3\sqrt{5})$ **i** $3\sqrt{2}(4\sqrt{2}+\sqrt{3})$

EXAMPLE 9

2 Expand and simplify $(\sqrt{3}+2\sqrt{5})(5\sqrt{2}+\sqrt{3})$. Select the correct answer **A**, **B**, **C** or **D**.

A $20\sqrt{10}$ **B** $2\sqrt{15}+5\sqrt{6}$

C $5\sqrt{6}+3+10\sqrt{10}+2\sqrt{15}$ **D** $5\sqrt{5}+\sqrt{3}+7\sqrt{7}+4\sqrt{2}$

3 Expand and simplify each expression.

a $(\sqrt{5}-3)(2\sqrt{5}+\sqrt{2})$ **b** $(\sqrt{7}-\sqrt{3})(\sqrt{7}+2)$

c $(7\sqrt{3}+2)(4\sqrt{2}+\sqrt{3})$ **d** $(3\sqrt{2}-\sqrt{5})(5\sqrt{2}+2\sqrt{5})$

e $(\sqrt{7}+2\sqrt{11})(3\sqrt{7}+4\sqrt{11})$ **f** $(5\sqrt{3}-2\sqrt{2})(4\sqrt{3}-3\sqrt{2})$

g $(6+2\sqrt{10})(3\sqrt{10}-1)$ **h** $(\sqrt{7}-2\sqrt{5})(3\sqrt{5}+2\sqrt{7})$

EXAMPLE 10

4 Expand $(5+\sqrt{7})^2$. Select **A**, **B**, **C** or **D**.

A 12 **B** 32 **C** $32+10\sqrt{7}$ **D** $32+5\sqrt{7}$

5 Expand and simplify each expression.

a $(\sqrt{5}-\sqrt{3})^2$ **b** $(\sqrt{7}+\sqrt{2})^2$ **c** $(\sqrt{5}-2)^2$

d $(3+\sqrt{10})^2$ **e** $(5\sqrt{2}+3\sqrt{3})^2$ **f** $(5\sqrt{7}-2)^2$

g $(3\sqrt{2}+2\sqrt{5})^2$ **h** $(2\sqrt{5}+\sqrt{3})^2$

6 Expand and simplify each expression.

a $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ **b** $(5+\sqrt{3})(5-\sqrt{3})$

c $(6+2\sqrt{7})(6-2\sqrt{7})$ **d** $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

e $(\sqrt{11}-\sqrt{10})(\sqrt{11}+\sqrt{10})$ **f** $(5\sqrt{7}+3)(5\sqrt{7}-3)$

g $(3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$ **h** $(4\sqrt{2}-5\sqrt{3})(4\sqrt{2}+5\sqrt{3})$

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STAGE 5.3

7 Expand and simplify $(5\sqrt{2}-4\sqrt{3})(5\sqrt{2}+4\sqrt{3})$. Select **A**, **B**, **C** or **D**.

A $25\sqrt{2}-16\sqrt{3}$ **B** $10\sqrt{2}+10\sqrt{6}$ **C** 2 **D** 26

8 Expand and simplify each expression.

a $(3\sqrt{7}-5)^2$ **b** $(5\sqrt{2}-4)(\sqrt{2}+5)$

c $(2\sqrt{7}+3\sqrt{5})(\sqrt{5}+\sqrt{7})$ **d** $(4\sqrt{3}+5)^2$

e $(4\sqrt{2}+\sqrt{3})(4\sqrt{2}-\sqrt{3})$ **f** $(3\sqrt{10}-\sqrt{2})^2$

Investigation

Making the denominator rational

1 What happens when we multiply the numerator and denominator of a fraction by the same number?

2 **a** Simplify $\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$.

b Mentally approximate the value of $\frac{\sqrt{2}}{2}$ given $\sqrt{2}\approx 1.4142$.

c Check using a calculator that $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$. Why is this true?

3 **a** Is it true that $\frac{3}{\sqrt{7}}=\frac{3}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$? Why?

b Simplify $\frac{3}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$. Compare your answer with those of other students.

c Check using a calculator that $\frac{3}{\sqrt{7}}=\frac{3\sqrt{7}}{7}$.

4 **a** Explain why $\frac{3\sqrt{5}}{\sqrt{2}}=\frac{3\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$.

b Show that $\frac{3\sqrt{5}}{\sqrt{2}}=\frac{3\sqrt{10}}{2}$.

Rationalising the denominator

1.06

STAGE 5.3

If $\sqrt{2} \approx 1.4142$, what is the value of $\frac{3}{\sqrt{2}}$? Fractions containing surds in the denominator are difficult to work with. When approximating the value of $\frac{3}{\sqrt{2}}$, it is difficult to mentally divide by 1.4142. We can overcome this by making the denominator **rational** (that is, **not** a surd).

Numbers that have irrational denominators, such as $\frac{1}{\sqrt{5}}, \frac{3}{2\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{2}}, \frac{5\sqrt{7}}{\sqrt{3}}$ can be rewritten with a rational denominator by multiplying both the numerator and denominator by the surd that appears in the denominator. This method is called **rationalising the denominator**.

Example 11

Rationalise the denominator of each surd.

a $\frac{3}{\sqrt{2}}$ **b** $\frac{5}{4\sqrt{3}}$ **c** $\frac{8\sqrt{2}}{3\sqrt{5}}$ **d** $\frac{\sqrt{2}+1}{\sqrt{3}}$

Solution

a $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ because $\frac{\sqrt{2}}{\sqrt{2}} = 1$

$= \frac{3\sqrt{2}}{2}$

Note: Now it is easier to approximate $\frac{3}{\sqrt{2}}$ by approximating $\frac{3\sqrt{2}}{2}$ and mentally multiplying $\frac{3}{2}$ by 1.4142 than by dividing 3 by 1.4142.

b $\frac{5}{4\sqrt{3}} = \frac{5}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{3}}{4 \times 3}$

$= \frac{5\sqrt{3}}{12}$

c $\frac{8\sqrt{2}}{3\sqrt{5}} = \frac{8\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{8\sqrt{10}}{3 \times 5}$

$= \frac{8\sqrt{10}}{15}$

d $\frac{\sqrt{2}+1}{\sqrt{3}} = \frac{\sqrt{2}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}+\sqrt{3}}{3}$

STAGE 5.3

EXERCISE 1.06 ANSWERS ON P. 592

Rationalising the denominator U F

EXAMPLE 11

1 By rationalising the denominator, which surd is equivalent to $\frac{2}{\sqrt{6}}$?
Select the correct answer **A**, **B**, **C** or **D**.

A $2\sqrt{6}$ **B** $\frac{\sqrt{6}}{3}$ **C** $\frac{\sqrt{6}}{6}$ **D** $\frac{2\sqrt{6}}{3}$

2 Rationalise the denominator of each surd.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{7}}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{3}{\sqrt{2}}$

e $\frac{2}{\sqrt{7}}$ **f** $\frac{1}{3\sqrt{2}}$ **g** $\frac{1}{2\sqrt{3}}$ **h** $\frac{1}{4\sqrt{7}}$

i $\frac{7}{3\sqrt{5}}$ **j** $\frac{\sqrt{2}}{3\sqrt{5}}$ **k** $\frac{3\sqrt{2}}{2\sqrt{6}}$ **l** $\frac{5\sqrt{3}}{4\sqrt{5}}$

3 Which surd is equivalent to $\frac{\sqrt{3}}{2\sqrt{5}}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{15}}{10}$ **B** $2\sqrt{15}$ **C** $\frac{\sqrt{15}}{3}$ **D** $\sqrt{5}$

4 Which surd is equivalent to $\frac{\sqrt{27}}{3\sqrt{18}}$? Select **A**, **B**, **C** or **D**.

A $\frac{1}{2}$ **B** $\frac{\sqrt{2}}{2}$ **C** $\frac{\sqrt{5}}{6}$ **D** $\frac{\sqrt{6}}{6}$

5 Rationalise the denominator of each expression.

a $\frac{\sqrt{2}-1}{\sqrt{2}}$ **b** $\frac{1-\sqrt{5}}{\sqrt{5}}$ **c** $\frac{5+\sqrt{3}}{2\sqrt{2}}$ **d** $\frac{\sqrt{2}-\sqrt{3}}{3\sqrt{6}}$

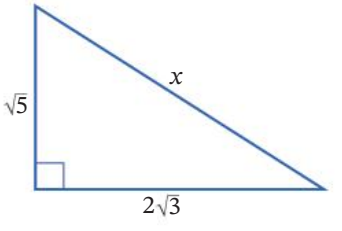
6 Simplify each expression, giving the answer with a rational denominator.

a $\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{2}}$ **b** $\frac{\sqrt{2}}{\sqrt{5}}+\frac{3}{\sqrt{3}}$ **c** $\frac{3}{2\sqrt{3}}-\frac{1}{\sqrt{2}}$

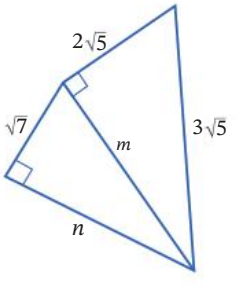
Power plus ANSWERS ON P. 592

1 Find the exact value of each variable.

a



b



2 **a** Is it true that $\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$? Explain.

b Simplify $\frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$. Is the denominator rational?

c Use a calculator to check that the value of your answer to part **b** is equal to the value of $\frac{1}{3+\sqrt{2}}$.

3 The **conjugate** of $3+\sqrt{2}$ is $3-\sqrt{2}$. Find the conjugate of:

a $5+2\sqrt{3}$ **b** $2-\sqrt{3}$

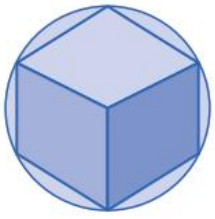
c $\sqrt{5}+1$ **d** $\sqrt{7}-\sqrt{3}$

4 The process shown in question **2** involves rationalising a surd with a **binomial** denominator. By first finding the **conjugate** of the denominator, rationalise the denominator of each expression below.

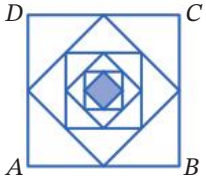
a $\frac{1}{2-\sqrt{3}}$ **b** $\frac{\sqrt{2}}{\sqrt{5}+1}$

c $\frac{1}{\sqrt{7}-\sqrt{3}}$ **d** $\frac{2}{5+2\sqrt{3}}$

5 The largest cube that can fit inside a sphere must have its 8 vertices touching the surface of the sphere. Express the side length, s, of the cube in terms of the diameter, D, of the sphere.



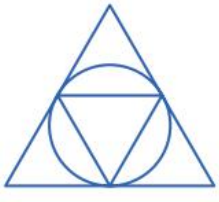
6 Squares are formed inside squares by joining the midpoints of the sides of the squares as shown. If $AB = 4$ cm, find the exact length of the side of the shaded square.



7 6 stormwater pipes, each 2 mm in diameter, are stacked as shown in the diagram. Find the exact height, h, of the stack.



8 An equilateral triangle has an inscribed circle that contains another equilateral triangle inscribed in it. The difference between the areas of the triangles is 40 cm^2. Find the radius of the circle, leaving your answer as a surd.



CHAPTER 1 REVIEW

Language of maths

approximate	binomial	denominator	difference of 2 squares
exact value	expand	irrational number	perfect square
product	Pythagoras' theorem	quotient	rational number
rationalise	real number	root	simplify
square number	square root	surd	undefined

1 Why do you think a **rational number** has that name?

2 What is the difference between a **rational number** and a **real number**?

3 What is a **surd**?

4 How do you **simplify** a surd?

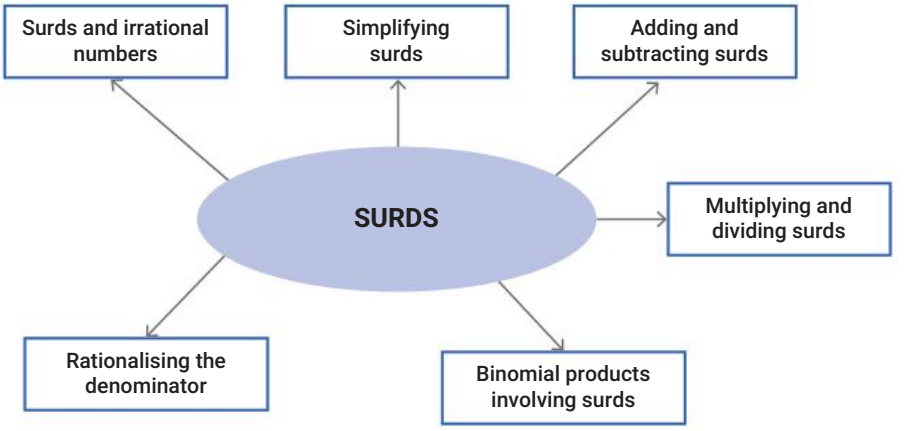
5 Why is π an example of an **irrational number** that is not a **surd**?

6 How do you **rationalise the denominator** of a surd expression?

Topic summary

WS
Mind map: Surds

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 1

ANSWERS ON P. 592

1 Which of these numbers is rational? Select the correct answer **A**, **B**, **C** or **D**. 1.01

A $1\frac{2}{5}$ **B** $\sqrt[3]{9}$ **C** $2\sqrt{5}$ **D** 2π

2 Is each number rational (R) or irrational (I)? 1.01

a $\sqrt{8}$ **b** $\frac{22}{7}$ **c** $0.5\dot{7}$ **d** $3\sqrt{5}$

e $\sqrt{81}$ **f** $\sqrt[3]{125}$ **g** $\sqrt[3]{-8}$ **h** $5+\sqrt{3}$

3 Simplify each surd. 1.02

a $\sqrt{72}$ **b** $\sqrt{98}$ **c** $\sqrt{275}$

d $\sqrt{128}$ **e** $3\sqrt{150}$ **f** $7\sqrt{28}$

g $4\sqrt{288}$ **h** $5\sqrt{45}$ **i** $7\sqrt{48}$

j $\frac{1}{2}\sqrt{24}$ **k** $\frac{2}{3}\sqrt{3125}$ **l** $\frac{1}{2}\sqrt{32}$

m $2\sqrt{44}$ **n** $\frac{2}{5}\sqrt{225}$ **o** $\frac{2}{3}\sqrt{162}$

4 Simplify each expression. 1.03

a $\sqrt{200}+\sqrt{18}$ **b** $3\sqrt{5}+\sqrt{50}-2\sqrt{125}$

c $7\sqrt{32}-\sqrt{27}-2\sqrt{98}+4\sqrt{75}$ **d** $4\sqrt{45}-3\sqrt{63}+5\sqrt{80}$

e $\sqrt{800}-2\sqrt{243}+3\sqrt{72}-2\sqrt{27}$ **f** $7\sqrt{44}-2\sqrt{99}$

5 Simplify each expression. 1.04

a $\sqrt{3}\times\sqrt{7}$ **b** $\sqrt{8}\times\sqrt{5}$ **c** $\sqrt{6}\times\sqrt{8}$

d $\sqrt{5}\times\sqrt{11}$ **e** $\sqrt{72}\div\sqrt{12}$ **f** $\sqrt{98}\div\sqrt{7}$

g $8\sqrt{42}\div 2\sqrt{7}$ **h** $\sqrt{125}\div 5\sqrt{5}$ **i** $\frac{\sqrt{75}}{3\sqrt{3}}$

j $\frac{\sqrt{18}\times\sqrt{3}}{\sqrt{12}}$ **k** $\frac{\sqrt{6}\times\sqrt{24}}{\sqrt{27}\times 2\sqrt{3}}$ **l** $\frac{4\sqrt{90}\times 7\sqrt{8}}{5\sqrt{32}\times 6\sqrt{10}}$

6 Expand and simplify each expression. 1.05

a $-3\sqrt{2}\left(2\sqrt{2}-3\right)$ **b** $\sqrt{10}\left(1-5\sqrt{2}\right)$

c $\left(3\sqrt{5}-2\sqrt{7}\right)\left(3\sqrt{7}+\sqrt{5}\right)$ **d** $\left(\sqrt{7}-4\right)^2$

e $\left(5\sqrt{3}+\sqrt{2}\right)^2$ **f** $\left(3\sqrt{7}-2\sqrt{5}\right)\left(3\sqrt{7}+2\sqrt{5}\right)$

g $\left(3+4\sqrt{7}\right)\left(4\sqrt{7}-3\right)$ **h** $\left(5\sqrt{10}-3\sqrt{2}\right)\left(2\sqrt{10}+5\sqrt{2}\right)$

7 Rationalise the denominator of each surd. 1.06

a $\frac{1}{\sqrt{10}}$ **b** $\frac{3}{\sqrt{2}}$ **c** $\frac{1}{5\sqrt{7}}$

d $\frac{3}{4\sqrt{3}}$ **e** $\frac{5\sqrt{3}}{3\sqrt{2}}$ **f** $\frac{4+\sqrt{2}}{3\sqrt{2}}$

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2

NUMBER AND ALGEBRA

INTEREST AND DEPRECIATION

The value of an investment increases over time as a result of interest being added to it, whether it be simple or compound interest. On the other hand, the value of assets and items such as cars and office equipment decreases over time due to age and wear-and-tear. Compound interest and depreciation use formulas that involve repeated percentage increase and decrease respectively.



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Chapter outline

		Working mathematically				
2.01	Earning an income#	U	F	PS	R	
2.02	Income tax#	U	F	PS		C
2.03	Simple interest	U	F	PS	R	
2.04	Compound interest	U	F			
2.05	The compound interest formula	U	F	PS	R	
2.06	Term payments#	U	F	PS	R	C
2.07	Depreciation#	U	F	PS	R	C

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Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as work-related expenses and donations to charities

compound interest Interest calculated on the principal invested as well as on any accumulated interest

depreciation The decrease in the value of items over time due to ageing

net pay Pay received after deductions from gross pay; 'take-home' pay

per annum (p.a.) Per year

principal The original amount of money invested or borrowed, for the purpose of calculating interest

repayment or **instalment** The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan

simple interest Interest calculated on the original principal invested only

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate weekly, fortnightly, monthly and yearly incomes
- calculate wages, salaries, overtime, commission, piecework and annual leave loading
- use tables to calculate income tax and PAYG tax
- solve problems involving simple interest
- solve problems involving compound interest by repeated percentage increase
- solve problems involving the compound interest formula $A = P(1 + r)^n$
- solve problems involving term payments and depreciation

Mental percentages

Percentage calculations

WS Percentage shortcuts

SkillCheck ANSWERS ON P. 592

1 Increase:

a \$7000 by 5% **b** \$3955 by 2% **c** \$8600 by 1.6%

2 Evaluate, correct to the nearest cent:

a $\$5000 \times (1.045)^4$ **b** $\$28\,000 \times (1.03)^6$

c $\$15\,300 \times (1.065)^3$ **d** $\$32\,400 \times (1.072)^{10}$

2.01 Earning an income#

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Overtime

WS Earning money

Wages, salaries and overtime

A **wage** is calculated from the number of hours worked and is usually paid weekly. Wage earners can make more income by working extra hours (overtime).

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. Salary earners do not earn overtime pay but can receive benefits such as a computer, company car, expense account, shares in the company or paid medical expenses.



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Units of time for wages and salaries

1 year = 12 months

1 fortnight = 2 weeks

1 year = 52 weeks for wage earners

1 year = 52.18 weeks for salary earners

The 2 most common rates of **overtime** pay are:

- **time-and-a-half** = 1.5 × normal hourly rate
- **double time** = 2 × normal hourly rate

Example 1

Noor earns $22.65 per hour at normal rates. Last week, she worked 38 hours at normal rates, 6 hours at time-and-a-half and 3 hours at double time. Calculate Noor's total earnings for the week.

Solution

Normal pay $= \$22.65 \times 38$

$= \$860.70$

Time-and-a-half pay $= 6 \times \$22.65 \times 1.5$

$= \$203.85$

Double time pay $= 3 \times \$22.65 \times 2$

$= \$135.90$

Total earnings $= \$860.70 + \$203.85 + \$135.90$

$= \$1200.45$

Commission, piecework and annual leave loading

Commission is earned by salespeople and agents, and is a percentage of the value of items sold or income made.

Piecework is earned according to the number of items made or tasks completed.

Annual leave loading or **holiday loading** is extra pay given during annual leave (holidays) and is 17.5% of 4 weeks' normal pay.



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Example 2

Georgia is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of $750. How much will Georgia earn if she sells an apartment for $590 000?

A **retainer** is a fixed amount paid regardless of how many items are sold.

Solution

Commission = 2.5% of $590 000

$= \$14\,750$

Total earnings = commission + retainer

$= \$14\,750 + \$750$

$= \$15\,500$

$\therefore$ Georgia earns $15 500.

Example 3

Emad is a jewellery designer. He makes handmade jewellery and is paid at the following rates:

- $278 per necklace
- $72 per pair of earrings
- $105 per bracelet

This month, Emad made 23 necklaces, 7 pairs of earrings and 19 bracelets. How much did he earn?

Solution

Monthly earnings $= 23 \times \$278 + 7 \times \$72 + 19 \times \$105$

$= \$8893$

Example 4

Kham's annual salary is $70 590. For his Christmas holidays, he received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. Calculate Kham's:

a normal weekly pay

b annual leave loading

c total pay for the Christmas holiday.

Solution

a Weekly pay $= \$70\,590 \div 52.18$

$= \$1352.8171...$

$\approx \$1352.82$

b Annual leave loading $= 17.5\% \times \$1352.82 \times 4$ 17.5% of 4 weeks' pay

$= \$946.974$

$\approx \$946.97$

c Total holiday pay $= (4 \times \$1352.82) + \$946.97$ 4 weeks' pay + leave loading

$= \$6358.25$

EXERCISE 2.01 ANSWERS ON P. 592

Earning an income U F PS R

Express all answers correct to the nearest cent where necessary.

1 Find the weekly wage for each person.

a Mary earns $21.85 per hour and works for 40 hours.

b Connor works 8 hours a day, Monday to Friday, and is paid $23.47 per hour.

c Yoshe works on Monday and Tuesday from 8:30 a.m. until 4:00 p.m. and Thursday from midday until 9:00 p.m., and earns $30.60 per hour.

2 Greta earns $19.56 an hour and works for 31 hours each week. Chandler earns $21.44 per hour for his 27 hours of work. Who earns more per week and by how much?

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3 Maggie earns a salary of $180 640 p.a. How much does she earn:

a each week? **b** each fortnight? **c** each month?

4 Rakitu considers 2 jobs, one locally with an annual salary of $57 640 p.a. and the other one in the city with a fortnightly pay of $2320. Calculate the weekly income for each job, determine which one pays more per week, and by how much. R

EXAMPLE 1

2.01

5 Anan works 38 hours at normal rates, 7 hours at time-and-a-half and 4 hours at double time. Calculate Anan's total earnings if he earns $19.60 per hour at normal rates.

6 Jacqui works 8.5 hours per day from Tuesday to Friday. She is paid $21.78 per hour. She also works on Saturday for 4.5 hours at a special rate of $24.59 per hour. How much did Jacqui earn for the week?

7 Idra works the following hours in a week at the clothing chain *Shop til U Drop*.

Day	Hours worked
Monday	9 a.m. – 5 p.m.
Tuesday	9 a.m. – 4 p.m.
Thursday	10 a.m. – 7:30 p.m.
Friday	10 a.m. – 5 p.m.
Saturday	10:30 a.m. – 5 p.m.

She is paid at the following rates.

Day	Rate of pay
Monday to Friday	$19.62 per hour
Saturday Thursday after 4:00 p.m.	$23.15 per hour

What is Idra's total income for the week? PS R

8 Fatimah is paid a commission of 2.5% on the value of goods she sells. She also receives a weekly retainer of $875. How much will Fatimah earn if she sells goods to the value of $41 600 in one week? Select the correct answer **A**, **B**, **C** or **D**.

EXAMPLE 2

A $1915 **B** $1061.88 **C** $2187.50 **D** $1018.13

9 Nathan is a real estate agent whose commission is calculated on the value of the properties he sells:

- 3% paid on first $300 000
- 1.5% paid on next $250 000
- 0.75% paid on any value thereafter

How much commission did Nathan earn for selling a house for $625 000?

10 Briana designed an app, *KeyFinder*, that sells for $2.49. If she makes 70% profit on the sale price of each app sold, how much would she make from selling 800 units of this app?

11 Matt charges $60 for each lawn he mows and $45 for trimming hedges in each yard. In a week, he mows 24 lawns and trims 15 hedges. How much does he earn for the week?

EXAMPLE 3

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12 *Clean 2 Swim* charges $86 to clean backyard pools. If this business earned $4644 in the first week of summer, how many pools were cleaned?

13 Jade makes homemade eco-friendly soaps, shampoos and cleaning products. A customer purchases 3 homemade soaps, 2 bottles of shampoo and 3 of the cleaning sprays. How much does Jade receive for these purchases?



Homemade soaps $5.60



Eco-friendly shampoo $12.70



Natural cleaning spray $7.25

14 Calculate the annual leave loading for each person if it is 17.5% of 4 weeks' pay.

a Peter earns $1220 per week

b Jamilla earns $2000 per fortnight

c Samir earns $5944 per month

d Ellie earns $46 630 p.a.

EXAMPLE 4

15 For his annual holidays, Jake received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. If Jake's annual salary is $50 725, find his:

a normal weekly pay

b annual leave loading

c total pay for the 4-week holiday.

Investigation

Workers' entitlements

The Australian Government sets the minimum standards for pay and conditions for all Australian workers. Different industries can have different needs from employees in terms of:

- normal and overtime hours worked, breaks allowed
- allowances
- dress codes, such as uniforms
- working conditions



1 Visit the Fair Work Ombudsman website **www.fairwork.gov.au** and select **Awards and agreements**.

2 Select 2 industries and identify any similarities and differences in the requirements of those industries.

3 Write a summary of your findings.

4 Give a report in class.

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Income tax#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

2.02

Income tax

Income tax is calculated on a person's **taxable income**, which is the gross income (total earnings) less all allowable deductions, rounded down to the nearest dollar.

Taxable income = gross income − allowable deductions

The more a person earns, the higher the rate of tax to be paid.

Tax rates for Australian residents	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $90 000	$3572 plus 32.5c for each $1 over $37 000
$90 001 – $180 000	$20 797 plus 37c for each $1 over $90 000
$180 001 and over	$54 097 plus 45c for each $1 over $180 000

Source: © Australian Taxation Office for the Commonwealth of Australia

Income tax tables

Income tax and Medicare levy

Income tax

Example 5

Sophia earned $62 348 last financial year and collected bank interest of $440.81. She had allowable deductions of $427.52 in work expenses and $110 in donations to charities.

a Calculate her taxable income.

b Use the tax table to calculate the income tax that Sophie must pay.

Solution

a Taxable income = $62 348 + $440.81 − $427.52 − $110

= $62 251.29

≈ $62 251 Rounded down to the nearest dollar.

b According to the table, a taxable income of $62 251 is in the $37 001 − $90 000 tax bracket.

Income tax = $3572 + 0.325 × ($62 251 − $37 000) '32.5c for each $1' means 32.5% or 0.325

= $11 778.575

≈ $11 778.58

PAYG tax and net pay

Income tax deducted from your pay by your employer every payday is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a tax refund.

Gross pay is the total amount a person earns or receives, but most workers have a variety of deductions taken from their pay before they receive it, and the amount of income left after the deductions is called **net pay**.

Net pay

Net pay = gross pay − tax − other deductions

Example 6

Jayden earns a gross pay of $2290.33 per fortnight. His deductions are for PAYG tax, $44.10 for private health insurance and $55.82 for superannuation.

a Use the PAYG tax table to find Jayden's PAYG tax per fortnight.

b Calculate Jayden's net pay.

c Calculate Jayden's total deductions as a percentage of his gross income (correct to one decimal place).

Fortnightly earnings ($)	PAYG tax withheld ($)
2270–2275	460
2276–2281	462
2282–2287	464
2288–2293	466
2294–2299	468
2300–2305	470

Solution

a In the table, $2290.33 falls in the $2288 – $2293 range.

Fortnightly PAYG tax = $466

b Net pay = $2290.33 – ($466 + $44.10 + $55.82)

= $1724.41

Net pay = gross pay − total deductions

c Total deductions = $565.92

$$\text{Deductions percentage} = \frac{\$565.92}{\$2290.33} \times 100\%$$

$$= 24.7091...\%$$

$$\approx 24.7\%$$

$$\frac{\text{Total deductions}}{\text{Gross pay}} \times 100\%$$

EXERCISE 2.02 ANSWERS ON P. 593

Income tax U F PS C

EXAMPLE 5

1 Shilpa earns $47 628 in a year and has allowable deductions of $1930.46.

a Calculate her taxable income, rounded down to the nearest dollar.

b Use the tax table on page 31 to calculate the income tax that Shilpa must pay.

2 Aiden is an environmental engineer who had a gross income of $118 742 this year and work-related expenses totalling $4022.80, which are tax-deductible. Calculate Aiden's:

a taxable income, rounded down to the nearest dollar

b income tax.

3 Ellie is a graphic designer who earns an annual salary of $90 541 and has collected $1029.45 in bank interest. She has allowable deductions of $379 for tools related to her work and $287 in donations to charity. Calculate: PS

a Ellie's taxable income

b the amount of tax payable.

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4 Riley the builder had a gross income of $56 922 this year. He is entitled to these tax deductions: tools $1538, training courses $445 and outdoor protective clothing $506. How much should Riley pay in tax? Select the correct answer **A**, **B**, **C** or **D**. PS

A $13 046.65 **B** $10 855.58 **C** $9237.73 **D** $6884.27

5 Nicola is a nurse earning $87 996 per year. Her allowable deductions are the cost of non-slip footwear $225, the cost of laundering uniforms $1046, and union fees $297.60. How much should Nicola pay in tax? PS

6 Will owns a photography business and earned $196 000 last year. His allowable deductions were Internet costs for his website $968, photographic equipment $23 672, and travel to photo shoots $15 930. Calculate the amount that Will should pay in tax. PS



Shutterstock.com/Dean Drobot

7 Jackson earns a gross weekly income of $1075.26. His weekly deductions are $309.11 PAYG tax, $44.55 for private health insurance and $25.18 for superannuation. Calculate Jackson's net weekly pay. C

8 Isha earns a gross income of $788.20 per week. Her deductions are $132.44 tax and $32.24 for private health insurance. Calculate Isha's net income. C

EXAMPLE 6

Use the PAYG table from Example 6 on the previous page to answer questions **9** to **12**.

9 Every fortnight, Mr Singh earns $2278 and pays $22.80 in union fees and $94.10 in superannuation.

a Find how much PAYG tax he pays per fortnight.

b Calculate Mr Singh's fortnightly net pay.

c What percentage (correct to one decimal place) of his gross pay do the deductions make up?

10 Holly earns a gross pay of $2295 per fortnight. Her deductions are PAYG tax, $64.35 for superannuation and $30 for life insurance. Find Holly's:

a PAYG tax

b net pay

c total deductions as a percentage of her gross income (correct to one decimal place).

11 Stefan earns $1148 per week.

a If he is paid fortnightly, what is his fortnightly gross pay?

b Find the PAYG tax that is taken out of his gross pay.

c Stefan's deductions are $141.94 for his health fund and $51.33 for superannuation. Calculate Stefan's net fortnightly pay.

12 Agata earns a salary of $60 135 p.a. Each fortnight she has deductions of $256.20 for family health insurance and $35 for superannuation taken from her gross income.

a Calculate Agata's fortnightly gross income.

b How much PAYG tax does she pay per fortnight?

c Calculate Agata's fortnightly net income.

13 Copy and complete this pay slip. PS C

Employee: Ziad Chaker		Hourly pay rate: $19.65	
Hours worked		Deductions	
Normal	39	Tax: $205.72	Other: $168.38
Time-and-a-half	2	Gross weekly income	
Double time	0	Total deductions	
		Net weekly income	

Technology

Online income tax calculators

The Australian Taxation Office (ATO) website **www.ato.gov.au** has online calculators for income tax and PAYG tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

1. Enter the taxable income $63 000 as '63000' (no spaces).
2. Select the current financial year.
3. Select 'Resident for full year' and click 'Next'.
4. The estimated tax payable will be shown on a new screen.
5. Repeat for at least 2 more taxable incomes.
6. Find the PAYG tax calculator and use it to find the PAYG tax payable and net pay for a gross pay of:
   - **a** $1408 weekly
   - **b** $2870 fortnightly
   - **c** $5610 monthly

2.03 Simple interest

WS Simple interest

WS Simple interest table

Simple interest

The simple interest formula

$I = Prn$, where:

I is the **simple interest**

P is the **principal**

r is the interest rate per time period, expressed as a decimal

n is the number of time periods

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Example 7

Find the simple interest on:

a \$4000 at 3.5% p.a. for 6 years

b \$13 500 at 5.5% p.a. for 7 months

c \$75 640 at 0.42% per month for 2 years

Solution

a $P = \$4000$, $r = 3.5\% = 0.035$, $n = 6$ years

$I = Prn$
$= \$4000 \times 0.035 \times 6$
$= \$840$

$r = 0.035$ per year, $n = 6$ years, so the time period is **years**.

b $P = \$13\,500$, $r = 5.5\% = 0.055$, $n = \frac{7}{12}$ years

$I = Prn$
$= \$13\,500 \times 0.055 \times \frac{7}{12}$
$= \$433.125$
$\approx \$433.13$

$r = 0.055$ per year, $n = 7$ *months*, so we must change 7 months to years so that the time period is **years**.

rounded to the nearest cent

c $P = \$75\,640$, $r = 0.42\% = 0.0042$,
$n = 2 \times 12 = 24$ months

$I = Prn$
$= \$75\,640 \times 0.0042 \times 24$
$= \$7624.512$
$\approx \$7624.51$

$r = 0.0042$ per month, $n = 2$ *years*, so we must change 2 years to months so that the time period is **months**.

rounded to the nearest cent

Example 8

Petra invests \$17 400 for 2 years at 3.75% p.a. flat rate interest. To what final value will her investment grow?

Solution

$P = \$17\,400$, $r = 3.75\% = 0.0375$, $n = 2$ years

n and r are in **years**.

$I = Prn$
$= \$17\,400 \times 0.0375 \times 2$
$= \$1305$

Value of investment $= \$17\,400 + \$1305$
$= \$18\,705$

Principal + interest

Example 9

After 4 years, an investment of \$13 000 has earned \$1092 in simple interest.

What is the annual interest rate?

Solution

$I = \$1092$, $P = \$13\,000$, $n = 4$ years

$I = Prn$

$\$1092 = \$13\,000 \times r \times 4$

$\$1092 = \$52\,000r$

$r = \frac{\$1092}{\$52000}$
$= 0.021$
$= 2.1\%$

$\therefore$ Annual interest rate $= 2.1\%$

Example 10

For how many months will \$10 000 need to be invested to earn \$250 in simple interest at 3.25% p.a.?

Solution

$I = \$250$, $P = \$10\,000$, $r = 3.25\% = 0.0325$

$I = Prn$

$\$250 = \$10\,000 \times 0.0325 \times n$

$\$250 = \$325n$

$n = \frac{\$250}{\$325}$

$= 0.7692...$ years — n is in years, so convert to months

$= 0.7692... \times 12$ months

$= 9.230...$ months

≈ 10 months — rounded *up* to the nearest month

EXERCISE 2.03 ANSWERS ON P. 593

Simple interest U F PS R

In this exercise, round all money answers to the nearest cent.

EXAMPLE 7

1 Calculate the simple interest earned on each investment.

a \$35 000 for 4 years at 3.6% p.a.

b \$26 850 at 1.95% p.a. for 2 years

c \$8200 invested for 5 months at 3% p.a.

d \$6590 invested for 16 months at 0.25% per month

e \$5250 invested for 250 days at 1.04% p.a.

f \$18 400 invested for 3 years at 0.18% per month

2 Calculate the flat rate interest charged on each loan.

a \$1250 for 2 years at 3.5% p.a.

b \$18 900 for $5\frac{1}{2}$ years at 5.7% p.a.

c \$1.15 million at 4.5% p.a. for 48 months

d \$12 000 for 10 months at 0.575% per month

e \$9750 for 2.5 years at 0.48% per month

f \$24 720 for 136 days at 7.85% p.a.

3 Harry owed \$783.26 on his credit card. The credit card company charged him one month's simple interest at 21% p.a. How much interest was he charged?
Select the correct answer **A**, **B**, **C** or **D**.

A \$13.71 **B** \$16.45 **C** \$25.38 **D** \$37.30

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4 Find the final value of each investment using simple interest.

EXAMPLE 8

- **a** $10 000 invested for 3 years at 4% p.a.
- **b** $1500 invested for 18 months at 0.19% per month
- **c** $8500 invested for 3.5 years at 0.25% per month
- **d** $9250 invested for 50 months at 3.75% p.a.

5 Liong borrowed $6000 to go on an overseas holiday, at 12% p.a. flat rate interest for 2 years. Calculate:

- **a** the total interest
- **b** the total amount Liong must repay.

6 The interest on a loan of $2500 over 4 years is $450. Calculate the flat rate of interest p.a. R

EXAMPLE 9

7 Katy took out a loan for $22 000 over 5 years. If her total loan repayments amounted to $28 400, calculate: PS R

- **a** the interest charged
- **b** the flat rate of interest p.a., correct to 2 decimal places.

8 After 5 years, the interest on a loan of $8000 amounts to $2340. Calculate the annual simple interest rate.

9 For how many years will $4200 need to be invested to earn $200 interest, if the interest rate is 2.5% p.a.? R

EXAMPLE 10

10 How many weeks will it take for $50 000 to earn $750 in interest if the rate is 2.6% p.a.? R

11 How many days will it take for $20 000 to earn $300 in interest if the rate is 4% p.a.? R

12 An online bank offered the following investment to its customers.

- A rate of 2.35% p.a. simple interest for the first 4 months only
- Then the principal and interest reinvested at 0.45% p.a. simple interest

What will Dariya's investment of $3480 be worth after 7 months?
Select **A**, **B**, **C** or **D**. PS R

A $31.21 **B** $374.10 **C** $3511.21 **D** $3854.10

13 For how many months will $20 000 need to be invested to amount to $22 000, if interest is paid at the rate of 0.33% per month? PS R

14 What is the flat rate of interest (as a percentage p.a., correct to one decimal place) when $1650 earns $85 in interest over 2 years? R

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15 Toula used a credit card to buy a netbook computer for $799 and some extra accessories for $246. She pays off this debt in 30 days. The credit card charges 22% p.a. simple interest. PS R

a Calculate the simple interest charged.

b How much will Toula pay after 30 days?

16 Bhashine earned $185 interest each year for 4 years on an investment account. At the end of the 4 years, she closed her account and withdrew $6000 in total. What was the annual flat rate of interest paid into Bhashine's account? Select **A**, **B**, **C** or **D**. PS R

A 3.1% **B** 3.5% **C** 12.3% **D** 14.1%

2.04 Compound interest

Most investments earn **compound interest**, where the interest earned is **added** to the principal so that next time, the interest is calculated on a larger principal. This means that more interest is earned, because we are also earning interest on the interest we have already earned. The word **compound** means 'combined'.

Example 11

A principal of $23 000 is invested at 4% p.a. interest, compounded yearly for 2 years.

a What is the total value of the investment after 2 years?

b What is the amount of compound interest earned?

Solution

a The interest for each year is calculated separately.

After the 1st year:
$I = \$23\,000 \times 0.04$
$= \$920$

$\text{Investment} = \$23\,000 + \$920$ Principal + interest
$= \$23\,920$

After the 2nd year:
$I = \$23\,920 \times 0.04$
$= \$956.80$

$\text{Investment} = \$23\,920 + \$956.80$ New principal + interest
$= \$24\,876.80$

b Compound interest earned = final investment – principal
$= \$24\,876.80 - \$23\,000$
$= \$1876.80$

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Notice that compound interest involves repeated percentage increase. In the above example, to calculate compound interest on a principal of $23 000 over 2 years at 4% p.a., we are actually increasing $23 000 by 4% twice. Adding 4% to the principal is the same as increasing the principal by 4%, which is the same as multiplying the principal by 104% or 1.04.

Investment after 1st year = $23 000 × 1.04 = $23 920

Investment after 2nd year = $23 920 × 1.04 = $24 876.80

We can even combine these 2 steps into one step by repeated percentage increases:

Investment after 2nd year = $23 000 × 1.04 × 1.04 = $24 876.80

Using repeated percentage increases can simplify our compound interest calculations.

Example 12

A principal of $9000 is invested at 3.7% p.a. compounded yearly over 3 years. What is:

a the value of the investment after 3 years?

b the compound interest earned?

Solution

a Adding 3.7% interest to the principal is the same as multiplying the principal by 1.037.

$\therefore$ Investment after 3 years $= \$9000 \times 1.037 \times 1.037 \times 1.037$

$= \$9000 \times (1.037)^3$

$= \$10\,036.4188...$

$\approx \$10\,036.42$ rounded to the nearest cent.

b Compound interest earned = final investment – original principal

$= \$10\,036.42 - \$9000$

$= \$1036.42$

EXERCISE 2.04 ANSWERS ON P. 593

Compound interest U F

In this exercise, round all money answers to the nearest cent.

EXAMPLE 11

1 A principal of $23 000 is invested at 5% p.a. interest, compounded yearly over 2 years.

a Copy and complete the following working to calculate the value of the investment after 2 years.

After the 1st year:

$I = \$23\,000 \times 0.05$ Investment = $23 000 + $_____

$= \$$_____ $= \$$_____

After the 2nd year:

$I = \$$_____ $\times 0.05$ Investment = $_____ + _____

$= \$$_____ $= \$$_____

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b Copy and complete the following working to calculate the amount of compound interest earned.

Compound interest earned = final investment – principal

= $_____ – $23 000

= $_____

2 Finn invests $15 000 at 2.5% p.a. compounded yearly over 3 years. Show all working (as in question **1**) to find:

a the value of the investment after 3 years

b the total amount of compound interest earned.

3 Selina invests $34 100 at 6.2% p.a. interest compounded yearly over 2 years. Calculate:

a the final value of the investment **b** the compound interest earned.

EXAMPLE 12

4 Use repeated percentage increases to calculate the final value of each investment compounded annually, then calculate the compound interest earned.

a $5000 for 2 years at 4% p.a. **b** $27 800 for 3 years at 2.85% p.a.

c $9600 for 3 years at 5% p.a. **d** $39 500 for 2 years at 3% p.a.

e $18 400 for 4 years at 1.25% p.a.

5 For each investment, calculate the compound interest earned.

a $30 400 at 5% p.a. for 3 years **b** $19 150 at 4.2% p.a. for 2 years

c $8750 at 1.75% p.a. for 2 years **d** $36 000 at 3.5% p.a. for 3 years

e $18 960 at 6.35% p.a. for 5 years

Mental skills 2: Maths without calculators ANSWERS ON P. 593

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

To find 15% of a number, find 10% and 5% of the number separately, then add.

1 Study each example.

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80) = 8 + 4 = 12$

b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170) = \$17 + \$8.50 = \$25.50$

c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600) = 360 + 180 = 540$

d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28) = \$2.80 + \$1.40 = \$4.20$

2 Now find 15% of each amount.

a	120	**b**	$840	**c**	260	**d**	$202
e	$50	**f**	72	**g**	$180	**h**	400
i	$1600	**j**	$22	**k**	6000	**l**	$350

To find $2\frac{1}{2}\%$ of a number, first find 5%, then halve it.

3 Study each example.

a $2\frac{1}{2}\% \times 600$

$10\% \times 600 = 60$

$5\% \times 600 = \frac{1}{2} \times 60 = 30$

$2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2}\% \times \$820$

$10\% \times \$820 = \$82$

$5\% \times \$820 = \frac{1}{2} \times 82 = \$41$

$2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

4 Now find $2\frac{1}{2}\%$ of each amount

a 400 **b** 6640 **c** \$2000 **d** \$880

e 1500 **f** \$232 **g** 5400 **h** \$904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

a $25\% \times 700$

$50\% \times 700 = \frac{1}{2} \times 700 = 350$

$25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$

$50\% \times \$86 = \frac{1}{2} \times \$86 = \$43$

$\therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

6 Now find 25% of each amount.

a 2000 **b** \$80 **c** 18 **d** \$25

e \$324 **f** \$140 **g** 66 **h** 298

i \$780 **j** \$1700 **k** \$126 **l** 1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = \frac{1}{2} \times 400 = 200$

$25\% \times 400 = \frac{1}{2} \times 200 = 100$

$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$

$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$

$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

8 Now find $12\frac{1}{2}\%$ of each amount.

a 1280 **b** \$12 **c** 60 **d** \$260

e \$540 **f** \$250 **g** 304 **h** 1360

2.05 The compound interest formula

Compound interest with annual rests

Compound interest with non-annual rests

Comparing interest rates

Simple and compound interest calculator

The compound interest formula

Simple and compound interest

Interesting facts

Note the following pattern.

- Final amount of \$23 000 at 4% p.a. interest for 2 years = $\$23\,000 \times (1.04)^2$
- Final amount of \$9000 at 3.7% p.a. interest for 3 years = $\$9000 \times (1.037)^3$
- Final amount of \$18 960 at 6.35% p.a. interest for 5 years = $\$18\,960 \times (1.0635)^5$

Compound interest formula

$A = P(1 + r)^n$, where:

A is the total (final) amount of the investment

P is the principal

r is the interest rate per compounding period, expressed as a decimal

n is the number of compounding periods

The compound interest is then calculated using this formula:

Compound interest = total amount – principal

$$I = A - P$$

Example 13

For each investment, calculate:

i the total amount of the investment

ii the compound interest earned if interest is compounded annually

a \$26 750 is invested at 4% p.a. for 3 years

b \$52 000 is invested at 3.8% p.a. for 5 years

Solution

a i $P = \$26\,750$, $r = 4\% = 0.04$, $n = 3$

$A = P(1 + r)^n$

$= \$26\,750(1 + 0.04)^3$

$= \$26\,750(1.04)^3$

$= \$30\,090.112...$

$\approx \$30\,090.11$

The total amount of the investment is \$30.090.11.

a ii Compound interest = \$30 090.11 – \$26 750

= \$3340.11

b i $P = \$52\,000$, $r = 3.8\% = 0.038$, $n = 5$

$A = P(1 + r)^n$

$= \$52\,000(1 + 0.038)^5$

$= \$52\,000(1.038)^5$

$= \$62\,659.9597...$

$\approx \$62\,659.96$

b ii Compound interest = \$62 659.96 – \$52 000

= \$10 659.96

Example 14

Calculate the compound interest when $24 500 is invested at 6.3% p.a. for 5 years:

a compounded annually **b** compounded monthly.

Solution

a $P = \$24\,500$, $r = 0.063$, $n = 5$

$A = \$24\,500\,(1 + 0.063)^5$

$= \$24\,500\,(1.063)^5$

$= \$33\,253.1205...$

$\approx \$33\,253.12$

$I = \$33\,253.12 - \$24\,500$

$= \$8753.12$

b Because interest is compounded monthly, r and n must be expressed in months, not years.

$P = \$24\,500$, $r = \frac{0.063}{12} = 0.00525$ per month, $n = 5 \times 12 = 60$ months

$A = \$24\,500(1 + 0.005\,25)^{60}$

$= \$24\,500(1.005\,25)^{60}$

$= \$33\,543.701\,98$

$\approx \$33\,543.70$

$I = \$33\,543.70 - \$24\,500$

$= \$9043.70$

Note: More interest is earned when it is compounded monthly rather than yearly.

Why do you think this is so?

Compound interest

2.05

EXERCISE 2.05 ANSWERS ON P. 593

The compound interest formula U F PS R

In this exercise, round all money answers to the nearest cent.

EXAMPLE 13

1 An amount of $13 000 is invested at 5% p.a. interest, compounded annually over 2 years. Which expression represents the total value of the investment?
Select the correct answer **A**, **B**, **C** or **D**.

A $13\,000 \times 0.05 \times 2$ **B** $13\,000\,(1 + 0.05)^2$

C $13\,000 \times (0.05)^2$ **D** $13\,000\,(1 - 0.05)^2$

2 For each investment, where interest is compounded yearly, calculate:

i the total amount of the investment, A **ii** the compound interest, I, earned.

a $6500 invested at 7% p.a. for 6 years **b** $10 000 invested at 8.5% p.a. for 4 years

c $12 240 invested at 1.6% p.a. for 2 years **d** $34 600 invested at 4.9% p.a. for 5 years

e $8000 invested at 1.75% p.a. for 3 years

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3 Calculate the amount of interest earned on an investment of $6500 if it is invested at 2.5% p.a. compounded annually for 8 years. Select **A**, **B**, **C** or **D**.

A $131.14 **B** $832.81 **C** $1300 **D** $1419.62

4 Find the amount of interest earned on one million dollars invested at 14.9% p.a. compounded annually for 6 years.

5 Find the amount of interest charged on a loan of $25 000 if it is borrowed over 10 years at 8% p.a. compounded annually. Select **A**, **B**, **C** or **D**.

A $31 250 **B** $28 973.12 **C** $28 589.72 **D** $20 000

6 Yasmin was given $2000 when she turned 3 years old. Her parents invested it at a 2% p.a. compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Yasmin had in the account when she turned 16? Select **A**, **B**, **C** or **D**. R

A $2000\,(1 + 0.02)^{13}$ **B** $2000\,(1 + 0.2)^{13}$

C $2000\,(1 + 0.02)^{16}$ **D** $2000\,(1 + 0.2)^{16}$

EXAMPLE 14

7 For each investment, calculate:

i the total amount **ii** the interest earned.

a $10 000 for 5 years at 2.4% p.a., compounded monthly

b $35 500 for 10 years at 2% per half-year, compounded half-yearly

Half-yearly means 'twice a year' or 'every 6 months'

c $8900 for 2 years at 3% p.a., compounded quarterly

Quarterly means '4 times per year' or 'every 3 months'

d $42 000 for 5 years at 0.225% per month, compounded monthly

e $16 500 for 3 years at 2.6% p.a., compounded half-yearly

f $4900 for 1 year at 0.005% per day, compounded daily

8 Find the total value of an investment of $4300 over 5 years at 4.6% p.a. interest, compounded every 6 months. Select **A**, **B**, **C** or **D**.

A $4817.78 **B** $5384.27 **C** $5397.90 **D** $8506.24

9 **a** Reese invested $6000 for 2 years at a flat rate of 5% p.a. Calculate the amount of interest earned.

b Tegan invested $6000 for 2 years at an interest rate of 5% p.a. compounded annually. Calculate the amount of interest earned.

c Whose investment earned more interest? How much more?

10 A principal of $5000 is invested at 3.5% p.a. for 3 years. Match each compounding period for this investment to its correct expression for the final value of the expression.

a	compounded yearly	**A**	$5000\left(1+\frac{0.035}{2}\right)^{3\times2}$
b	compounded half-yearly	**B**	$5000\left(1+\frac{0.035}{12}\right)^{3\times12}$
c	compounded quarterly	**C**	$5000\left(1+\frac{0.035}{4}\right)^{3\times4}$
d	compounded monthly	**D**	$5000\,(1+0.035)^3$

11 Lisa is setting up a trust account for her new grandson Stefan. In 18 years time, she wants the investment to be worth $30 000, to help with the cost of university fees or the purchase of a car. Suppose the interest rate for the account is 2.94% p.a. compounding yearly. PS R

a How much should Lisa invest now to achieve the $30 000 target?

b If Lisa opened a trust account that earns 2.94% p.a. compounding monthly instead, how much less would she need to invest?

12 Zoe is 5 years old and about to start school. Her parents want to invest $25 000, for her high school education expenses, in an account that earns 3% p.a. over 7 years. R

a Calculate the total interest earned if interest is compounded:

i yearly **ii** half-yearly **iii** quarterly **iv** monthly

b Which compounding period should Zoe's parents choose? Why?

13 A principal of $10 000 is invested for 4 years, earning interest at the rate of 3% p.a., compounded monthly. Which expression represents the total value of the investment? Select **A**, **B**, **C** or **D**.

A $10\,000\left(1+\frac{3}{100}\right)^4$

B $10\,000\left(1+\frac{3}{100}\right)^{48}$

C $10\,000\times\left(1+\frac{3}{1200}\right)^4$

D $10\,000\left(1+\frac{3}{1200}\right)^{48}$

Technology

Comparing simple with compound interest

In this activity, you will compare the interest earned on an investment of $1000 for 10 years at 8% p.a. simple interest and 8% p.a. compound interest, compounded annually.

	A	B	C
1	$ 1,000.00	0.08	0.08
2			
3	Years	Simple Interest	Compound Interest
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		

Spreadsheets

1 Create this spreadsheet. The principal (P) is entered in cell A1 and the annual interest rates (in decimal form) in cells B1 and C1.

2 To calculate the simple interest in column B, in cell B4 enter the formula **=$A$1*$B$1*A4**. Now **Fill Down** from cell B4 to B13.

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3 To calculate the compound interest in column C, in cell C4 enter the formula **=$A$1*(1+$C$1)^ A4-$A$1**. Now **Fill Down** from cell C4 to C13.

4 Highlight cells A3 to C13. **Insert 'Scatter with Smooth lines and markers'**.

5 When the interest rate is the same, which account pays better interest: simple or compound interest? (Type your answer in cell A15)

6 Now compare the interest earned on an investment of $1000 for 10 years at 9% p.a. simple interest and 7% p.a. compound interest, compounded annually. Change the interest rates in cells B1 (0.09) and C1 (0.07) respectively.

Answer the following questions in the spreadsheet cells indicated in brackets.

7 After how many years did the compound interest rate pay more than the simple interest rate? (A16)

8 How much extra interest did the compound interest rate pay at the end of the 10 years? (A17)

9 Change the interest rate in B1 to 10% (0.1) and C1 to 9% (0.09). How does the change in interest rate affect the amount of interest paid? Include calculations to justify your answer. (A18)

10 Change the interest rate in B1 to 12% (0.12) and C1 to 8.5% (0.085). After how many years did the amount of compound interest earned overtake the amount of simple interest earned? (A19)

11 What is the difference in the amount of compound interest earned for the 10-year period compared to the simple interest investment? Is it a significant amount? Justify your answer. (A20)

2.06 Term payments#

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Term paymentss

Many customers buy expensive household items **on terms**, which means 'paying off' the item by regular instalments over time, after paying a deposit. A **term payments** plan is also called **hire-purchase** because the purchaser actually hires the item until it is completely paid off. Special offers can include interest-free periods, but there may be other conditions such as establishment fees and extra charges if the regular **repayments** are not paid on time. Also, if the purchaser fails to keep up with the payments, higher interest may be charged or the item may be **repossessed** (taken back).

Example 15

Sonia purchases a new fridge and dishwasher package valued at $4925. She pays a 10% deposit and repays the balance in monthly repayments over 3 years. Interest on the balance is charged at a flat rate of 12% p.a.

Find:

a the deposit paid

b the balance owing

c the interest charged on the balance

d the total to be repaid

e the amount of each monthly repayment

f the total price paid for the package.

Solution

a Deposit = 10% × $4925
= $492.50

b Balance owing = $4925 – $492.50 or 90% × $4925
= $4432.50

c Interest charged on the balance is flat or simple interest.
P = $4432.50, r = 0.12, n = 3
$I = Prn$
= $4432.50 × 0.12 × 3
= $1595.70

d Total to be repaid = balance + interest
= $4432.50 + $1595.70
= $6028.20

e Number of repayments = 3 × 12
= 36
Monthly repayment = $6028.20 ÷ 36
= $167.45

f Total price paid = deposit + total repayments
= $492.50 + $6028.20
= $6520.70

or cash price + interest
= $4925 + $1595.70

Deferred payment plan

With a **deferred payment plan**, the customer does not make any repayments until a later date, such as after 6 months.

Deferred means 'delayed'.

Example 16

Ilhea and Robert purchase a $1600 home theatre system on a deferred payment plan over 2 years. They make no repayments for the first 3 months, then pay $105 per month.

a Calculate the total repayments.

b How much interest did they pay?

c Ilhea and Robert were also charged the following fees for the plan.

- One-off establishment fee of $25
- Account service fee of $4.95 per month

i How much was paid in fees over the 2-year period?

ii What percentage (correct to one decimal place) of the purchase price was paid in fees?

Solution

a Total cost = $105 × 21
= $2205

2 years − 3 months = 21 months

b Interest = total cost – cash price
= $2205 – $1600
= $605

c **i** Total fees paid $= \$25 + \$4.95 \times 24$
$= \$143.80$

ii Percentage $= \frac{\$143.80}{\$1600} \times 100\%$
$= 8.9875\%$
$\approx 9.0\%$

EXERCISE 2.06 ANSWERS ON P. 593

Term payments U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 15

1 Gaspard purchases a laundry package of washing machine and dryer valued at $1755. He pays a 10% deposit and repays the balance in 18 months. Interest on the balance is charged at a flat rate of 13.85% p.a. Find: PS R C

a the deposit Gaspard paid
b the balance owing
c the interest charged on the balance
d the total to be repaid
e the amount of each monthly instalment
f the total price paid for the package

2 Maree bought a backyard swimming pool for $25 500. She paid a 5% deposit and will repay the balance over 5 years, with flat-rate interest charged 8.6% p.a. Calculate: PS R C

a the deposit
b the balance owing
c the interest charged
d the total to be repaid
e the monthly instalment
f the total price paid.

3 Sarah wants to buy a new 3-piece lounge suite that she saw advertised at the lounge sale pictured, costing $6895. R



a How much deposit will she have to pay?
b What will be the flat-rate interest on the balance?
c Find the size of each monthly repayment.

4 Peter pays a $1200 deposit on his holiday and then $185 per month for a year. The cash price of the holiday is $2700.

a How much did Peter pay for his holiday?
b Calculate the interest paid.
c Calculate the balance owing after the deposit was paid.
d Calculate, as a percentage correct to one decimal place, the flat interest rate charged p.a.

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5 Mrs Mitchell buys a used car for her daughter by paying a 20% deposit and $275 per month for 4 years. If the car has a cash price of $10 400, find: R C

- **a** the deposit Mrs Mitchell paid
- **b** the balance owing after the deposit is paid
- **c** the total amount Mrs Mitchell repaid in instalments
- **d** the interest charged
- **e** the flat interest rate (p.a.), correct to one decimal place.

6 Xuan wanted to buy a new smartphone costing $1044. R C

- **a** How much deposit did she pay?
- **b** What is the total amount repaid in repayments for the smartphone?
- **c** Calculate the total interest paid for the year.
- **d** What was the annual flat interest rate for this purchase, correct to 1 decimal place?



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EXAMPLE 16

7 Diego bought a laptop priced at $800 on a deferred payment plan: nothing to pay for 3 months and then 9 monthly payments of $110. However, a monthly account fee of $6.95 was added to the plan. PS R

- **a** Find the total cost of the laptop to Derek.
- **b** How much in excess of the cash price was paid?
- **c** What was the flat rate of interest p.a. (correct to one decimal place) charged?

'in excess' means 'above' or 'more than'

8 Tahlia bought a new outdoor setting and BBQ for her backyard that retails for $2899. She paid $300 deposit, no payments for 6 months and then fortnightly payments of $63 for 2 years. PS R

- **a** Find the balance owing after Tahlia had paid the deposit.
- **b** Calculate the total cost of the outdoor setting and BBQ.
- **c** How much in excess of the cash price did Tahlia pay under this plan?
- **d** What is the annual flat rate of interest charged, correct to one decimal place?

9 Sofia bought a home cinema system priced at $2100 on interest-free terms for one year with no repayments for the first 4 months. R

- **a** If Sofia makes 8 equal monthly repayments, what is the amount of each payment?
- **b** There is a service charge of $12.95 every month for this deferred payment plan. What percentage (correct to one decimal place) of the purchase price was paid in service charges?

2.07 Depreciation#

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Depreciation is the decrease in value of an item over time. When items lose value because of age or frequency of use, they are said to **depreciate**.

The compound interest formula can be adapted to find the depreciated value of an item. While compound interest involves repeated percentage increases, depreciation involves repeated percentage *decreases*, so its formula has a minus sign.

Depreciation formula

$A = P(1 - r)^n$, where:

A is the final value of the item

P is the original value of the item

r is the rate of depreciation per period, expressed as a decimal

n is the number of periods of depreciation

The amount of depreciation is then calculated using this formula:

$$\text{Depreciation} = \text{original value} - \text{final value}$$

Example 17

An accountant's computer system depreciates by 15% each year.

a If the computer system is currently valued at \$2600, what will its value be in 5 years?

b What is the depreciation over this time?

Solution

a $P = \$2600$, $r = 15\% = 0.15$, $n = 5$

$A = P(1 - r)^n$

$= \$2600\ (1 - 0.15)^5$

$= \$2600\ (0.85)^5$

$= \$1153.6338...$

$\approx \$1153.63$

The value of the computer system in 5 years will be \$1153.63.

b Depreciation $= \$2600 - \$1153.63$ original value − final value

$= \$1446.37$

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Example 18

An industrial oven in a restaurant originally costs $19 800, then depreciates at a rate of 12% p.a.

a Find the value of the oven after 6 years, correct to the nearest dollar.

b Express the depreciated value as a percentage of the cost price, correct to one decimal place.

Solution

a $P = \$19\,800$, $r = 0.12$, $n = 6$

$A = P(1 - r)^n$

$= \$19\,800\,(1 - 0.12)^6$

$= \$19\,800\,(0.88)^6$

$= \$9195.2009...$

$\approx \$9195$

b Percentage of cost price $= \frac{\$9195}{\$19800} \times 100\%$

$= 46.4393...\ \%$

$\approx 46.4\%$

This means that after 6 years, the oven is worth approximately 46% of its original price (or has lost 54% of its original value).



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EXERCISE 2.07 ANSWERS ON P. 593

Depreciation U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 17

1 Find the value of a photocopier after 5 years if its purchase price was $2850 and the annual depreciation rate is 20%.

2 **a** Find the value of a car after 7 years if it is purchased new for $49 990 and it depreciates at 12% p.a.

b Find the amount of depreciation over this time.

3 For each item shown in the table, calculate:

i its value after 4 years of depreciation

ii its value after 4 years as a percentage of its original value, correct to one decimal place.

	Item	Original value	Depreciation rate (p.a.)
a	Stove	$1100	12%
b	Fishing boat	$38 500	18%
c	Library	$8460	12%
d	Computer	$2500	20%
e	Furniture	$27 500	15.5%
f	Bike	$2900	22%
g	Electrical tools	$870	17.5%
h	Air conditioner	$2600	9%



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EXAMPLE 18

4 A smartphone originally valued at $1729 depreciates at 37% p.a. R

a What percentage (to 2 decimal places where necessary) of the original value remains after:

i 1 year? **ii** 3 years? **iii** 6 years?

b Approximately how long would it take the smartphone to halve its original value?

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5 A security system costs a company $12 500 to buy new. It depreciates at a yearly rate of 20%.

a Find the value of the system after:

i 1 year **ii** 2 years **iii** 5 years

b Find the value of the system after 5 years as a percentage of its original value. Answer correct to one decimal place.

6 Parveen pays $25 490 for a new car. The car will depreciate in value by an average of 11% p.a.

a Find correct to the nearest dollar the market value of the car in 3 years.

b Calculate the amount of depreciation in the car after 3 years.

7 Adam has spent $175 000 on equipment to set up his hairdressing salon. The equipment depreciates at 20% per year. R

a Find the value of the equipment after 4 years.

b Find the amount of depreciation in the equipment after 4 years.

c Find, by trial and error, how long it will take for the value to be over $50 000. Answer in years and months.

d Find the value of Adam's equipment after 9 years as a percentage of its original value, correct to one decimal place.

8 Kamal says that, at 10% p.a. depreciation, a car will lose half its value after 7 years.

Is he correct? Show all working to justify your answer. PS R C

9 Office equipment that is worth $12 000 when new, depreciates at 15% p.a. as shown in the table. PS R C

a How much did the office equipment lose in value in the first year?

b After how many years did the office equipment fall below half its original value?

c By how much did the office equipment depreciate between the 5th and 6th years?

d Will the value of the office equipment ever fall below $100?

e Will the value of the office equipment ever be zero?

Year	Depreciated value
0	$12 000
1	$10 200
2	$8670
3	$7369.50
4	$6264.08
5	$5324.46
6	$4525.79
7	$3846.93
8	$3269.89
9	$2779.40
10	$2362.49
11	$2008.12
12	$1706.90
13	$1450.87
14	$1233.24

Power plus ANSWERS ON P. 594

1 How long, in years and days, will it take an investment of $3000 to earn $500 in simple interest at 4% p.a.?

2 What amount should Owen invest to earn $100 in simple interest if the investment will last for 9 months and the interest rate is 3% p.a.?

3 A principal of $10 000 is invested for 5 years at an interest rate of 5% p.a., with interest compounded weekly. Calculate the final value of the investment.

4 Meghan needs $80 000 in 4 years' time. What amount should she invest now at an interest rate of 6% p.a., with interest compounded annually, to reach her target?

5 A painting appreciates in value at a rate of 3% p.a. while a computer depreciates in value at a rate of 10% p.a. If I bought the painting for $1200 and the computer for $1500 new, what would be their combined value in 5 years time?

6 A bacteria colony is growing at a rate of 20% per hour. If there are 10 000 bacteria now, use the compound interest formula to calculate how many there will be after 1 day. (Give your answer correct to the nearest 10 000.)

7 **a** You invest $2000 in a bank account at an interest rate of 4% p.a. with interest compounded annually. How long will it take for your investment to double in value?

b If you invested $4000 instead of $2000 at the same interest rate, how long will it take to double in value?

c Does the size of the principal make any difference to the time taken for it to double?

CHAPTER 2 REVIEW

Money and finance

Language of maths

allowable deductions	annual leave loading	compound interest	deposit
depreciation	double time	flat rate	fortnightly
gross pay	income tax	interest	net pay
overtime	PAYG tax	per annum (p.a.)	principal
quarterly	repayment	salary	simple interest
taxable income	term payments	time-and-a-half	wage

1 When investing, why is compound interest better than simple interest?

2 What do the P and r stand for in the formulas $I = Prn$ and $A = P(1 + r)^n$?

3 What is another name for **flat-rate interest?**

4 What word means a decrease in the value of an item over time?

5 Why is **gross pay** higher than **net pay?**

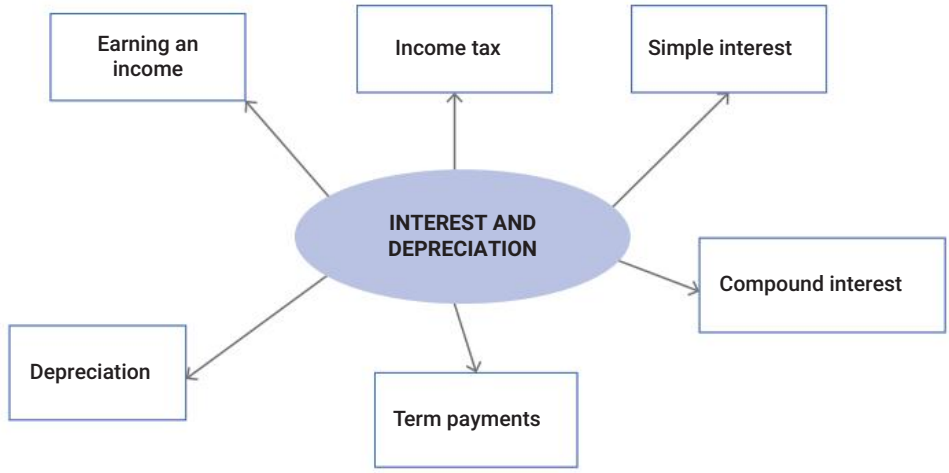
6 Use a dictionary to find at least 2 different meanings of **principal**.

Topic summary

Mind map: Interest and depreciation

- Which parts of this chapter were revision of Year 9 knowledge and skills?
- Which parts of this chapter were new to you?
- Do you know how to use the simple interest and compound interest formulas?
- How is income tax calculated?
- How is the depreciation formula similar to the compound interest formula?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 2 ANSWERS ON P. 594

In this exercise, round all money answers to the nearest cent.

2.01 **1** Hayley is paid a commission of 2.5% on the value of the properties she sells. She also receives a weekly retainer of $1150. How much will Hayley earn if she sells a house for $475 830?

2.01 **2** Caleb earns a salary of $70 400 p.a. How much is he paid each week?

2.01 **3** A supermarket cashier is employed under the following award.

Normal rate: $21.45 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 40 hours **b** 46 hours.

2.01 **4** For his Christmas holidays, Jacan received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Jacan's annual salary is $54 920, find:

a his normal weekly pay

b his leave loading

c his total pay for the 4-week holiday.

2.02 **5** Alia earns a salary of $68 650 p.a. Her allowable deductions are donations to charities of $540 and work-related expenses of $385.

a Calculate Alia's taxable income.

b Use the tax table on page 31 to calculate the income tax Alia should pay.

2.03 **6** Calculate the simple interest earned on each investment.

a $20 000 invested for 3 years at 4% p.a.

b $7850 invested at 2.5% p.a for 15 months

c $4500 invested for 6 months at 0.17% per month

d $25 200 invested for 100 days at 3.45% p.a.

2.04 **7** An amount of $5000 is invested at 2.35% p.a. interest, compounded annually over 3 years.

a What is the total value of the investment after 3 years?

b What is the amount of compound interest earned?

8 Calculate the value of the investment when $34 200 is invested at 3% p.a. for 2 years, with interest compounded annually. 2.05

9 Find the final value if $11 000 is invested for 4 years at 2.4% p.a., with interest compounded monthly. 2.05

10 Find the interest earned when $4895 is invested at 1.95% p.a. for 3 years, with interest compounded annually. 2.05

11 Calculate the interest earned when $46 230 is invested for 9 years at 2.8% p.a., with interest compounded half-yearly. 2.05

12 Yang purchases a furniture package valued at $4875. She pays a 10% deposit and repays the balance in 36 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find: 2.06

- **a** the deposit Yang paid
- **b** the balance owing
- **c** the interest charged
- **d** the total to be repaid
- **e** the amount of each instalment
- **f** the total price Yang paid for the package.

13 Caroline bought a new car for $24 990, which depreciates by 10% p.a. 2.07

- **a** Find correct to the nearest dollar the depreciated value of the car after 5 years.
- **b** What is the depreciation over this time?
- **c** Express the depreciated value as a percentage of the original price (correct to one decimal place).

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3

NUMBER AND ALGEBRA

GRAPHING LINES

Straight lines are an important part of our environment. We play sport on courts using parallel and perpendicular lines, and skyscrapers would not be standing without straight lines. We can also use straight lines to model different types of data and predict future outcomes.



Chapter outline

	Working mathematically				
3.01 Length, midpoint and gradient of an interval	U	F	PS	R	C
3.02 Parallel and perpendicular lines	U	F			
3.03 Graphing linear equations	U	F		R	C
3.04 The gradient–intercept equation $y = mx + c$	U	F		R	
3.05 The general equation $ax + by + c = 0$	U	F		R	C
3.06 The point–gradient equation $y - y_1 = m(x - x_1)$*	U	F		R	C
3.07 Finding the equation of a line	U	F		R	C
3.08 Equations of parallel and perpendicular lines	U	F		R	C
3.09 Coordinate geometry problems*#	U	F	PS	R	C

***STAGE 5.3**

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general form Any linear equation expressed as $ax + by + c = 0$, where a, b and c are integers and a is positive

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

gradient–intercept form Any linear equation expressed as $y = mx + c$, where m is the gradient and c is the y-intercept

linear equation An equation whose graph is a straight line

parallel lines Lines that point in the same direction and have the same gradient

perpendicular lines Lines that cross at right angles (90°) and have gradients whose product is −1

point-gradient form Any linear equation expressed as $y - y_1 = m(x - x_1)$, where m is the gradient and (x_1, y_1) is a point on the line

***x*-intercept** The x-value at which a graph cuts the x-axis

***y*-intercept** The y-value at which a graph cuts the y-axis

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- find the distance between 2 points on a number plane
- find the midpoint and gradient of an interval on a number plane
- (STAGE 5.3) use *formulas* to find the length, midpoint and gradient of an interval on a number plane
- (STAGE 5.3) find the angle of inclination θ of a line using the formula $m = \tan\theta$
- find the properties of the gradients of parallel and perpendicular lines
- graph a linear equation on a number plane
- (STAGE 5.3) graph a line by finding its x- and y-intercepts
- test whether a point lies on a line
- use the gradient–intercept equation of a straight line $y = mx + c$
- use the general form of a linear equation $ax + by + c = 0$ and convert it to the gradient–intercept equation
- (STAGE 5.3) use the point–gradient form of a linear equation $y - y_1 = m(x - x_1)$ to find the equation of a line given the gradient and a point, or 2 points
- find the equation of a line from its graph
- find the equation of a line that is parallel or perpendicular to a given line
- (STAGE 5.3) prove geometrical properties using coordinate geometry methods

Pythagoras' theorem

SkillCheck ANSWERS ON P. 594

1 For this number plane, find:

a the midpoint of interval BC

b the midpoint of interval HE

c the length of interval GC

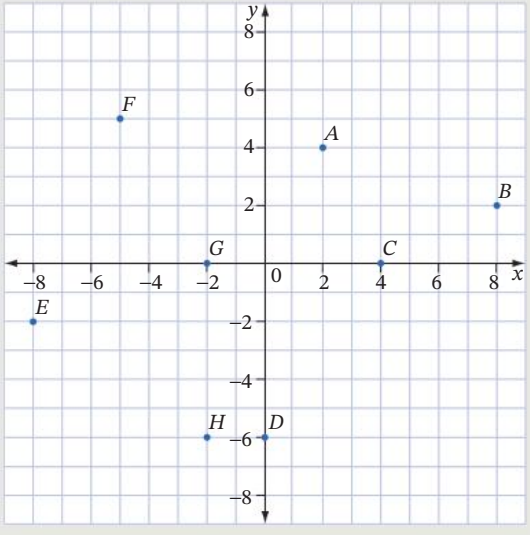
d the length of interval GH

e the lengths of AC and BC, correct to one decimal place

f the type of triangle $\triangle ABC$ is

g the gradient of GE

h the gradient of EH



2 If $(x_1 = 3, y_1 = 4)$, $(x_2 = -5, y_2 = 6)$, then evaluate each expression.

a $x_1 + x_2$ **b** $x_2 - x_1$ **c** $\frac{y_1 + y_2}{2}$

d $(y_2 - y_1)^2$ **e** $\frac{y_2 - y_1}{x_2 - x_1}$ **f** $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

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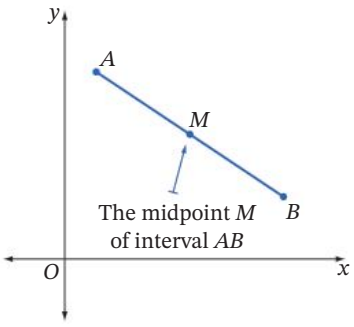
Length, midpoint and gradient of an interval

3.01

The **length** of an interval AB (or the **distance** between A and B) can be calculated using Pythagoras' theorem if we know the coordinates of A and B.

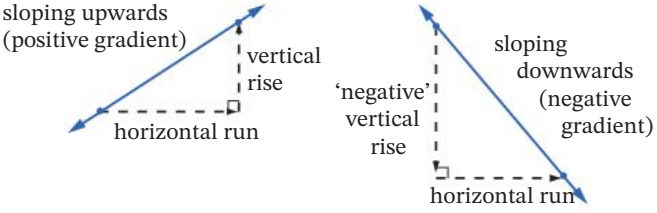
The **midpoint** of an interval AB is the point in the middle of AB or halfway between A and B.

- Its x-coordinate is the average of the x-coordinates of A and B.
- Its y-coordinate is the average of the y-coordinates of A and B.



The **gradient** of an interval measures its steepness. It is given by the formula:

$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$



- A line **sloping upwards** has a **positive rise** and **a positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.

Example 1

For the interval joining $P(-5, 8)$ and $Q(3, 6)$, find:

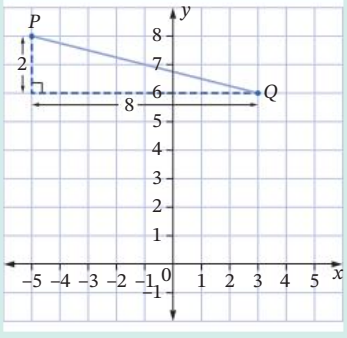
a its length, correct to one decimal place.

b its midpoint

c its gradient

Solution

a Draw a right-angled triangle on the number plane with PQ as the hypotenuse.



The height of the triangle is 2 units.

The base of the triangle is 8 units.

$PQ^2 = 2^2 + 8^2$

$= 68$

$PQ = \sqrt{68}$

$= 8.2462...$

≈ 8.2 units

WS Gradient, midpoint and distance
WS Intervals match-up
Midpoint and distance between 2 points
Midpoint and distance between 2 points
The gradient of a line
WS Gradient between two points

b For $P(-5, 8)$ and $Q(3, 6)$, the average of the x-coordinates is $\frac{-5+3}{2} = -1$.

The average of the y-coordinates is $\frac{8+6}{2} = 7$.

$\therefore$ The midpoint of PQ is $(-1, 7)$.

From the diagram in part **a**, a midpoint at $(-1, 7)$ looks reasonable.

c The rise is -2 units. Line slopes downwards.

The run is 8 units.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-2}{8}$$
$$= -\frac{1}{4}$$

The gradient is negative, the line slopes downward.

STAGE 5.3

Coordinate geometry

The distance, midpoint and gradient formulas

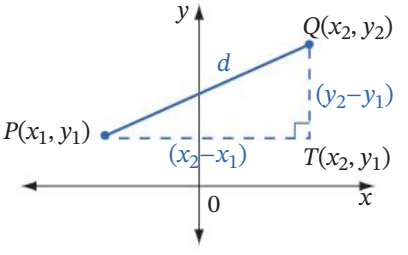
For an interval joining $P(x_1, y_1)$ and $Q(x_2, y_2)$:

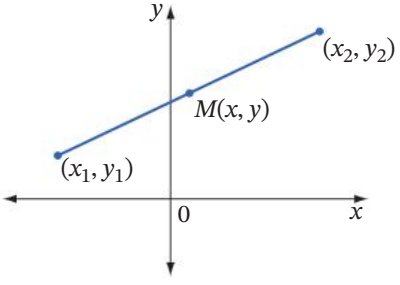
Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

We've used '$\equiv$' ('is identical to') rather than '$=$' because we are referring to the point (x, y), not a number.

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$





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Example 2

For the interval joining $P(-5, 8)$ and $Q(3, 6)$ from Example **1**, use a formula to find:

a its length, correct to one decimal place

b its midpoint

c its gradient.

Solution

For $P(-5, 8)$ and $Q(3, 6)$: $x_1 = -5, y_1 = 8, x_2 = 3, y_2 = 6$.

(x_1, y_1) (x_2, y_2)

a
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-5))^2 + (6-8)^2} \\ &= \sqrt{68} \\ &= 8.2462\ldots \\ &\approx 8.2 \text{ units} \end{aligned}$$

b
$$\begin{aligned} M(x, y) &\equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &\equiv \left(\frac{-5+3}{2}, \frac{8+6}{2}\right) \\ &\equiv (-1, 7) \end{aligned}$$

c
$$\begin{aligned} m &= \frac{\text{difference in } y}{\text{difference in } x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6-8}{3-(-5)} \\ &= \frac{-2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

Example 3

a Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.

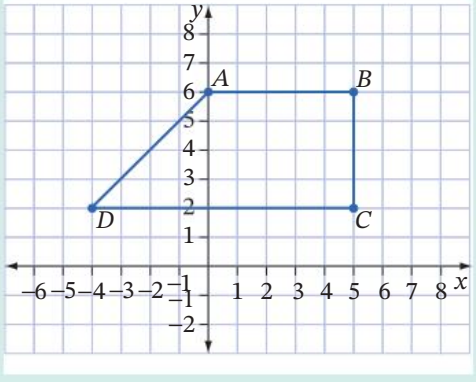
b What type of quadrilateral is $ABCD$?

c Find the exact length of AD.

d Hence find the perimeter of $ABCD$, correct to 2 decimal places.

Solution

a



Join the points in the correct order.

STAGE 5.3

b Since $AB \parallel CD$, the quadrilateral is a trapezium.

c $AD^2 = 4^2 + 4^2$

$= 32$

$AD = \sqrt{32}$ units

In exact surd form.

d By counting grid squares, $AB = 5$, $BC = 4$, $CD = 9$.

Perimeter of $ABCD = 5 + 4 + 9 + \sqrt{32}$

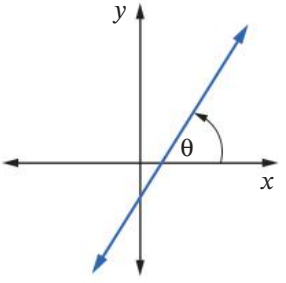
$= 23.656...$

≈ 23.66 units

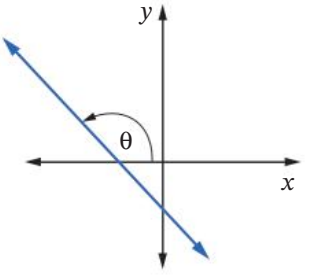
The angle of inclination of a line

The **angle of inclination**, θ, of a line is the angle it makes with the x-axis in the positive direction.

acute angle = positive gradient



obtuse angle = negative gradient

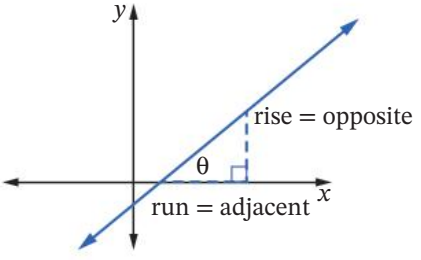


Note from the above diagrams that θ is **acute** when the line has a **positive gradient**, and **obtuse** when the line has a **negative gradient**.

We can use trigonometry to calculate the angle of inclination of a line using its gradient, m.

The diagram below shows that $m = \frac{\text{rise}}{\text{run}}$, but in trigonometry, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{rise}}{\text{run}}$.

$\therefore m = \tan\theta$.



The angle of inclination of a line

The angle of inclination, θ, of a line is related to the gradient, m, of the line by the formula:

$$m = \tan\theta.$$

Example 4

Find, correct to the nearest degree, the angle of inclination of a line with gradient:

a $\frac{1}{3}$ **b** -4

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Solution

a $m = \tan\theta$

$\frac{1}{3} = \tan\theta$

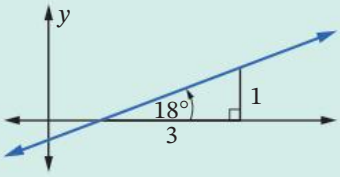
$\tan\theta = \frac{1}{3}$

$\theta = 18.4349...$

$\approx 18°$

On a calculator: SHIFT tan 1 $a^b/_c$ 3 =

The positive gradient means it is an acute angle.



b $m = \tan\theta$

$-4 = \tan\theta$

$\tan\theta = -4$

$\theta = -75.9637...$

$\approx -76°$

On calculator: SHIFT tan (–) 4 =

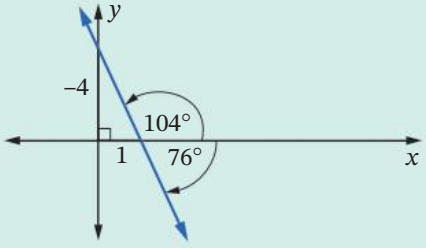
But this negative angle is the angle below the x-axis.

To find the angle of inclination,

$\theta \approx 180° - 76°$

$\approx 104°$

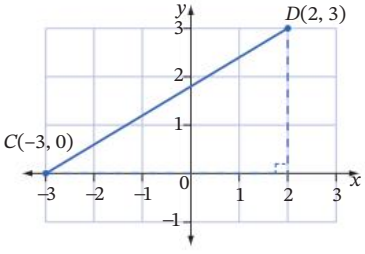
Negative gradient means it is an obtuse angle.



EXERCISE 3.01 ANSWERS ON P.594

Length, midpoint and gradient of an interval U F PS R C

Questions **1**, **2** and **3** refer to this diagram of interval CD.



EXAMPLES 1, 2

1 What is the length of interval CD? Select the correct answer **A**, **B**, **C** or **D**.

A 2 units **B** 5.8 units

C 3.2 units **D** 8 units

2 What is the midpoint of CD? Select **A**, **B**, **C** or **D**.

A $(-1, 3)$ **B** $(-5, 3)$ **C** $(-0.5, 1.5)$ **D** $(-2.5, 1.5)$

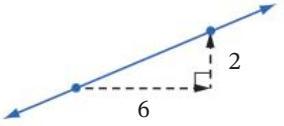
Foundation | Standard | Complex

3 What is the gradient of CD? Select **A**, **B**, **C** or **D**.

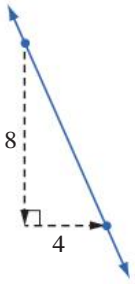
A $\frac{3}{5}$ **B** $-\frac{3}{5}$ **C** $-\frac{5}{3}$ **D** $\frac{5}{3}$

4 Calculate the gradient of each line.

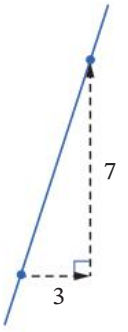
a



b



c



5 For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place

ii the midpoint of the interval

iii the gradient of the interval.

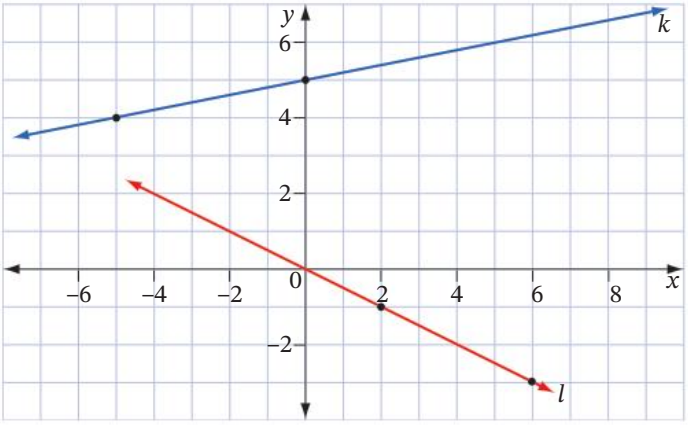
a $A(5, 3)$ and $B(7, 2)$ **b** $J(-1, 0)$ and $K(8, 6)$ **c** $M(0, -3)$ and $N(-5, 2)$

d $R(-3, -6)$ and $S(4, -9)$ **e** $A(-7, 2)$ and $B(-5, -8)$ **f** $U(3, -2)$ and $V(7, 2)$

6 Calculate, in exact (surd) form, the distance between each pair of points.

a $(-8, -1)$ and $(0, 4)$ **b** $(12, -6)$ and $(-1, -1)$ **c** $(7, -2)$ and $(-2, -3)$

7 Find the gradient of the lines labelled k and l.



8 Which expression gives the y-coordinate of the midpoint of the interval joining points $(3, 8)$ and $(-1, 5)$? Select **A**, **B**, **C** or **D**.

A $\frac{-1+5}{2}$ **B** $\frac{8+5}{2}$ **C** $\frac{8-5}{2}$ **D** $\frac{5-8}{2}$

Foundation Standard Complex

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STAGE 5.3

EXAMPLE 3

3.01

9 The vertices of triangle ABC are $A(-1, -1)$, $B(1, 3)$ and $C(3, 1)$. PS R C

a Draw $\triangle ABC$ on a number plane.

b Find the exact length of each side of the triangle.

c Are any sides of the triangle equal in length?

d What type of triangle is ABC?

e Find the perimeter of $\triangle ABC$, correct to one decimal place.

10 The vertices of quadrilateral $KLMP$ are $K(1, 6)$, $L(7, 2)$, $M(3, -4)$ and $P(-3, 0)$. PS R C

a Draw the quadrilateral on a number plane.

b What type of quadrilateral is $KLMP$?

c Find the gradients of sides KL and PM.

d Find the gradients of sides KP and LM.

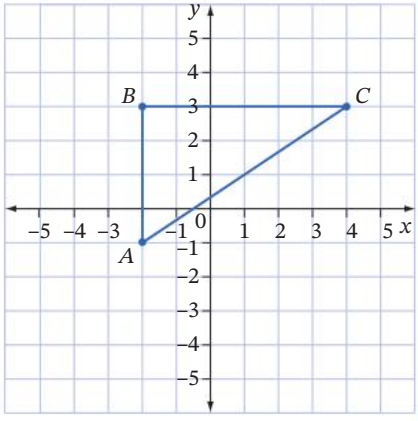
e What do you notice about the gradients of the opposite sides of this quadrilateral? What does that mean about those sides?

f Find the exact length of each side of $KLMP$.

g Find the perimeter of $KLMP$, correct to one decimal place.

h Find the area of $KLMP$.

11 This diagram shows a right-angled triangle with vertices $A(-2, -1)$, $B(-2, 3)$ and $C(4, 3)$. R C



a Copy the diagram and find the coordinates of P and Q, the midpoints of BA and BC respectively. Mark P and Q on your diagram.

b Calculate, correct to one decimal place, the lengths of PQ and AC. What do you notice about your answers?

c Find the gradients of PQ and AC. What do you notice about your answers?

EXAMPLE 4

12 Find, correct to the nearest degree, the angle of inclination of a line with gradient:

a 3 **b** $\frac{1}{2}$ **c** 1 **d** 2.5

e -2 **f** $\frac{3}{4}$ **g** $-\frac{1}{10}$ **h** $-\frac{2}{3}$

13 Find, correct to 2 decimal places, the gradient of the line with angle of inclination:

a 60° **a** 158° **c** 42° **d** 94°

e 8° **f** 135° **g** 177° **h** 0°

Foundation Standard Complex

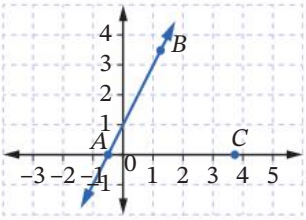
Technology

The angle of inclination

In this activity, we will use dynamic geometry software to calculate the angle of inclination of a line.

1 Use the software to graph the linear equation $y = 2x + 1$.

2 Create points A, B and C as shown and then measure the angle $\angle BAC$.



3 What is the angle of inclination of the line? Answer to the nearest degree.

4 Use the software to measure the angle of inclination of the line with equation:

a $y = -3x - 5$ **b** $y = x + 2$ **c** $y = x - 6$ **d** $y = -2x + 3$

e $y = -5x - 7$ **f** $y = -8x + 1$ **g** $y = 3x - 12$ **h** $y = \frac{2}{3}x + 4$

Investigation

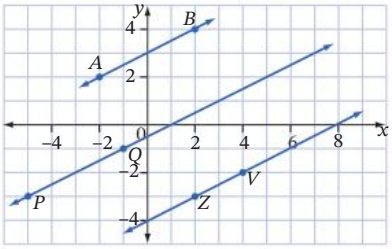
Parallel and perpendicular lines

1 These 3 lines are parallel. Calculate the gradient of:

a AB

b PQ

c ZV

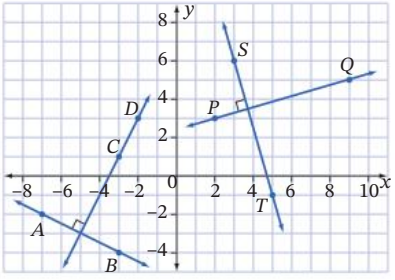


2 What can you conclude about the gradients of parallel lines?

3 This diagram shows 2 pairs of perpendicular lines. $AB \perp CD$ and $PQ \perp ST$. Calculate the gradient of:

a AB **b** CD

c PQ **d** ST



4 Is there a relationship between:

a the gradients of AB and CD? **b** the gradients of PQ and ST?

5 Calculate the product of:

a the gradients of AB and CD **b** the gradients of PQ and ST

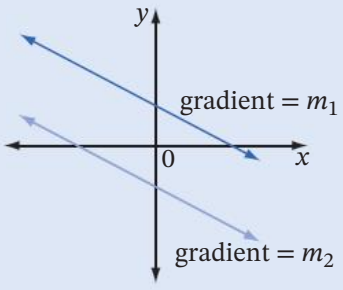
6 What can you conclude about the gradients of perpendicular lines?

Parallel and perpendicular lines 3.02

Parallel lines

Parallel lines have the same gradient.

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.



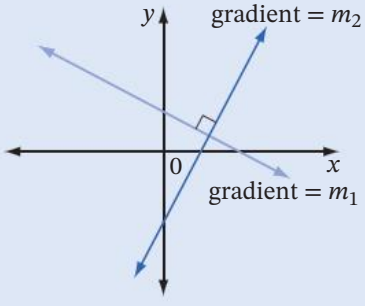
Perpendicular lines

Perpendicular lines have gradients with a product of -1.

If 2 lines with gradients m_1 and m_2 are perpendicular, then

$$m_1 \times m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}.$$

Note that m_2 is the **negative reciprocal** of m_1.



Example 5

State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{2}, m_2 = 2$ **b** $m_1 = 0.4, m_2 = \frac{2}{5}$ **c** $m_1 = 1\frac{3}{5}, m_2 = -\frac{5}{8}$

Solution

a $m_1 \neq m_2$ so the lines are not parallel.

$$m_1 \times m_2 = \frac{1}{2} \times 2$$
$$= 1$$
$$\neq -1$$

so the lines are not perpendicular.

∴ The lines are neither parallel nor perpendicular.

b $m_2 = \frac{2}{5} = 0.4$

$m_1 = m_2$

∴ The lines are parallel.

c $m_1 = 1\frac{3}{5} = \frac{8}{5}$

$$m_1 \times m_2 = \frac{8}{5} \times \left(-\frac{5}{8}\right)$$
$$= -1$$

∴ The lines are perpendicular.

Example 6

Find the gradient of a line that is perpendicular to a line with gradient:

a 2 **b** -3 **c** $\frac{3}{4}$ **d** -0.6

Solution

a $m_1 = 2$

$m_2 = \frac{-1}{m_1}$ for perpendicular lines

$= \frac{-1}{2}$ The negative reciprocal of m_1.

$= -\frac{1}{2}$

The gradient is $-\frac{1}{2}$.

b $m_1 = -3$

$m_2 = \frac{-1}{m_1}$

$= \frac{-1}{-3}$

$= \frac{1}{3}$

The gradient is $\frac{1}{3}$.

c $m_1 = \frac{3}{4}$

$m_2 = \frac{-1}{m_1}$

$= \frac{-1}{\left(\frac{3}{4}\right)}$

$= -\frac{4}{3}$

The gradient is $-\frac{4}{3}$.

d $m_1 = -0.6 = -\frac{3}{5}$

$m_2 = \frac{-1}{\left(-\frac{3}{5}\right)} = \frac{5}{3}$

The gradient is $\frac{5}{3}$.

Example 7

A line passes through the points $A(-2, 5)$ and $B(4, 1)$. What is the gradient of a line:

a parallel to AB? **b** perpendicular to AB?

Solution

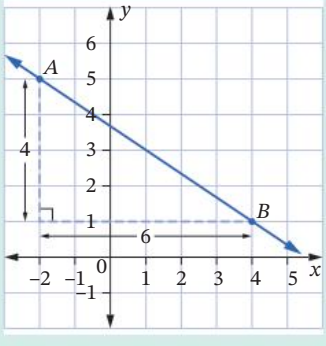
Find the gradient of AB by calculating the rise and run.

Sometimes drawing a diagram helps you to understand the problem.

Rise $= 1 - 5 = -4$

Run $= 4 - (-2) = 6$

Gradient $AB = \frac{-4}{6} = -\frac{2}{3}$



a Any line parallel to AB will have the same gradient as AB. $\therefore m = -\frac{2}{3}$

b The gradient of a line perpendicular to AB will be given by: $m = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$

EXERCISE 3.02 ANSWERS ON P. 594

Parallel and perpendicular lines U F

EXAMPLE 5

1 State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{4}, m_2 = 4$ **b** $m_1 = 3, m_2 = -\frac{1}{3}$ **c** $m_1 = 0.5, m_2 = \frac{1}{2}$

d $m_1 = \frac{2}{7}, m_2 = \frac{7}{2}$ **e** $m_1 = \frac{3}{10}, m_2 = 0.3$ **f** $m_1 = 1\frac{1}{5}, m_2 = -\frac{6}{5}$

3.02

2 Find the gradient of a line that is parallel to a line with gradient:

a 4 **b** -2 **c** $\frac{1}{3}$ **d** -0.2

EXAMPLE 6

3 Find the gradient of a line that is perpendicular to a line with gradient:

a 1 **b** -6 **c** -1.5 **d** $\frac{5}{2}$

4 What is the gradient of a line that is perpendicular to a line with a gradient of 0.8? Select the correct answer **A**, **B**, **C** or **D**.

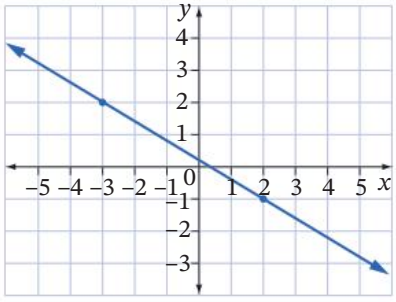
A 0.2 **B** -0.2 **C** 1.25 **D** -1.25

5 What is the gradient of a line that is parallel to a line that goes through $P(0, 3)$ and $Q(5, -2)$? Select **A**, **B**, **C** or **D**.

A 1 **B** -1 **C** $\frac{1}{5}$ **D** $-\frac{1}{5}$

EXAMPLE 7

6 What is the gradient of a line perpendicular to the line shown? Select **A**, **B**, **C** or **D**.



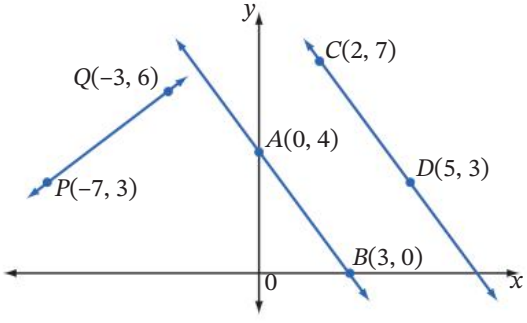
A $\frac{5}{3}$ **B** -5

C $\frac{3}{5}$ **D** $\frac{1}{5}$

7 Calculate the gradient of each line shown and test whether:

a $AB \parallel CD$

b $PQ \perp CD$.



8 A line passes through the points $R(-5, 2)$ and $S(1, 4)$. What is the gradient of a line:

a parallel to RS?

b perpendicular to RS?

Foundation | Standard | Complex

Technology

Parallel and perpendicular lines

Use dynamic geometry software to find out if sets of linear equations represent parallel or perpendicular lines.

1 Parallel lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are parallel using $m_1 = m_2$.

a $5x - 3y = 0$ and $y = \frac{5x}{3}$

b $x + y + 4 = 0$ and $x + y - 6 = 0$

c $x - 2y = 0$ and $y = 0.5x$

d $y = 5x - 9$ and $5x - y - 1 = 0$

2 Perpendicular Lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are perpendicular using $m_1 \times m_2 = -1$.

a $y = 0.6x + 2$ and $y = \frac{5}{3}x$

b $x - 4y + 1 = 0$ and $y = -4x - 3$

c $3x - 2y = 0$ and $y = -\frac{2x}{3}$

d $y = 2x + 4$ and $x - 2y - 1 = 0$

3.03 Graphing linear equations

Graphing linear equations

WS A page of lines

WS A page of number planes

WS Graphing linear equations

Graphing linear equations

WS Graphing linear equations (Advanced)

A relationship between 2 variables, x and y, whose graph is a straight line is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = 3x + 2$, is called a **linear equation**.

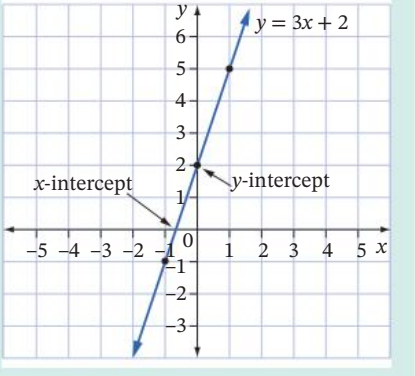
Example 8

Graph $y = 3x + 2$ on a number plane and find its x- and y-intercepts.

Solution

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	2	5



The x-intercept is the x-value where the line cuts the x-axis and can be found by substituting $y = 0$ into the equation $y = 3x + 2$:

$0 = 3x + 2$

$3x = -2$

$x = -\frac{2}{3}$ which agrees with what's shown on the graph

The x-intercept is $-\frac{2}{3}$.

9780170453509

The y-intercept is the y-value where the line cuts the y-axis and can be found by looking at the graph, or looking at the constant term in the equation $y = 3x + 2$, or substituting $x = 0$ into the equation $y = 3x + 2$ (which has already been done for the table of values).

The y-intercept is 2.

Using x- and y-intercepts to graph lines

- To find the x-intercept, substitute $y = 0$ and solve the equation.
- To find the y-intercept, substitute $x = 0$ and solve the equation.

Example 9

Find the x- and y-intercepts of the line $2x - 3y = 6$ to draw its graph.

Solution

For x-intercept, $y = 0$:

$2x - 3 \times 0 = 6$

$2x = 6$

$x = 3$

The x-intercept is 3.

For y-intercept, $x = 0$:

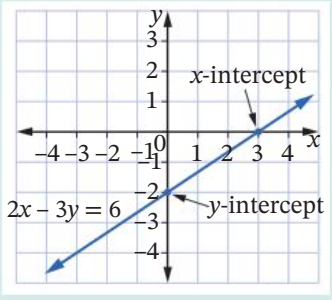
$2 \times 0 - 3y = 6$

$-3y = 6$

$y = -2$

The y-intercept is -2.

Plot both intercepts on the axes, draw a line through the 2 points and label the line with its equation.



Testing if a point lies on a line

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.

Example 10

Test whether each point lies on the line $x - 2y = 5$.

a $(17, 6)$ **b** $(8, -4)$

Solution

- Separate the equation into its left-hand side (LHS) and right-hand side (RHS)
- Substitute the coordinates of the point into both sides
- If LHS = RHS, the point satisfies the equation and so lies on the line
- If LHS $\neq$ RHS, the point does not lie on the line.

STAGE 5.3

3.03

Testing if a point lies on a line

WS

Straight-line equations

a Substitute $x = 17, y = 6$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 17 - 2 \times 6$

$= 5$

RHS $= 5$

LHS = RHS, so (17, 6) lies on the line.

b Substitute $x = 8, y = -4$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 8 - 2 \times (-4)$

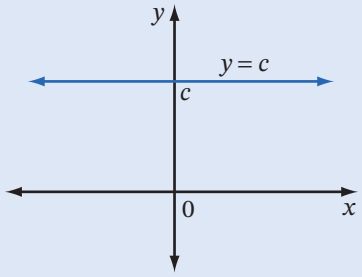
$= 16$

RHS $= 5$

LHS $\neq$ RHS, so (8, −4) does not lie on the line.

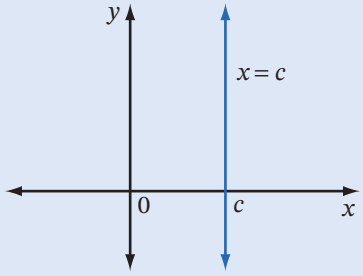
Horizontal and vertical lines

The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).



Note: The x-axis has equation $y = 0$.

The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).



Note: The y-axis has equation $x = 0$.

Example 11

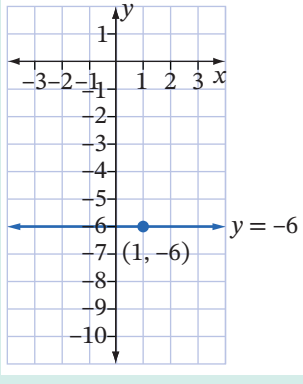
Find the equation of the line that is:

a parallel to the x-axis and passes through the point (1, −6)

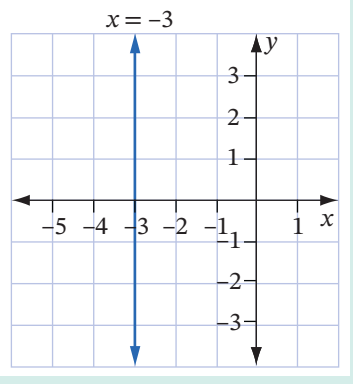
b always 3 units to the left of the y-axis.

Solution

a The equation of the line is $y = -6$.



b The equation of the line is $x = -3$.



EXERCISE 3.03 ANSWERS ON P.594

Graphing linear equations UFRC

EXAMPLE 8

1 Graph each linear equation on a number plane, and find:

i its x-intercept **ii** its y-intercept.

a $y = 3x - 1$ **b** $y = 2x + 5$ **c** $y = -x + 4$

d $y = -2x - 2$ **e** $y = 4x$ **f** $y = \frac{x}{2} + 3$

3.03

STAGE 5.3

EXAMPLE 9

2 Graph each linear equation after finding its x- and y-intercepts.

a $y = 4 - 2x$ **b** $2x = 4y - 8$ **c** $y - x = 6$

d $3x - 2y = 12$ **e** $2x + 2y = 5$ **f** $6 - x = 2y$

g $y = 4 + 2x$ **h** $5x + 3y - 15 = 0$ **i** $3x - y = 6$

j $2x - 5y - 20 = 0$ **k** $4x + 2y - 8 = 0$ **l** $x - 4y - 2 = 0$

EXAMPLE 10

3 Test whether the point $(3, -1)$ lies on each line.

a $y = 2x - 5$ **b** $x - y = 4$ **c** $y + 2x = 5$

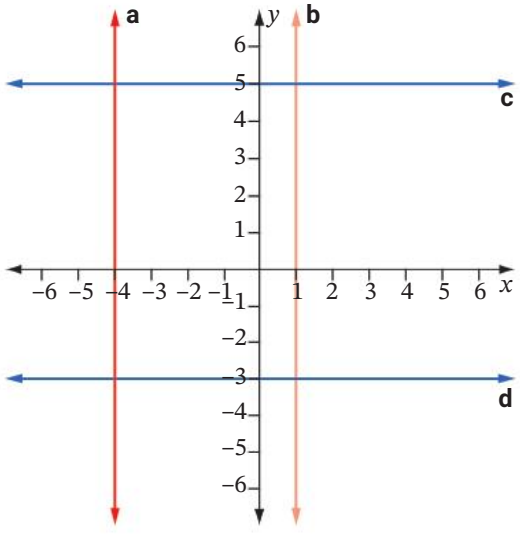
d $y = x - 4$ **e** $x + y = 5$ **f** $3x + y + 8 = 0$

4 Which point lies on the line $y = 6x - 5$? Select the correct answer **A**, **B**, **C** or **D**.

A $(-1, 11)$ **B** $(3, -13)$ **C** $(-2, -17)$ **D** $(-5, 25)$

EXAMPLE 11

5 Find the equation of each line shown.



6 Graph each set of lines on a number plane.

a $x = 2\frac{1}{2}, y = -3, y = 1$ **b** $x = 6, y = -2, x = -\frac{1}{2}$

Foundation Standard Complex

7 Find the equation of the line that is: R C

a horizontal and passes through the y-axis at 2

b vertical with an x-intercept of 4

c parallel to the y-axis and passes through the point $(-1, 4)$

d parallel to the x-axis and passes through the point $(0, -2)$

e 3 units above the x-axis

f 1 unit to the left of the y-axis

g drawn through the points $(-1, 6)$ and $(2, 6)$

h drawn through the points $(-1, 8)$ and $(-1, 2)$.

8 Which point lies on the line $4x + y = 1$? Select **A**, **B**, **C** or **D**.

A $(-1, 5)$ **B** $(-2, 7)$ **C** $(6, 9)$ **D** $\left(-\frac{1}{2}, 1\right)$

9 Which line is horizontal and passes through the point $(8, -2)$? Select **A**, **B**, **C** or **D**.

A $y = 8$ **B** $x = 8$ **C** $y = -2$ **D** $x = -2$

10 a What is another name for the line $y = 0$? C

b What is another name for the line $x = 0$? C

3.04 The gradient–intercept equation $y = mx + c$

Graphing $y = mx + c$

Gradient and y-intercept of a line

The gradient-intercept formula

Equation of a line

Equations in gradient form

The gradient–intercept form of a linear equation

The equation of a straight line is $y = mx + c$, where m is the **gradient** and c is the **y-intercept**.

For this reason, $y = mx + c$ is also called the **gradient–intercept form** of a linear equation.

Example 12

Find the gradient and y-intercept of the line with equation:

a $y = -4x + 9$ **b** $y = 10 - 6x$ **c** $y = \frac{5x+4}{2}$ **d** $3x + 2y - 6 = 0$

Solution

a $y = -4x + 9$ is in the form $y = mx + c$.

$\therefore$ Gradient $m = -4$ and y-intercept $c = 9$.

b $y = 10 - 6x$ can be rewritten as $y = -6x + 10$.

$\therefore$ Gradient $m = -6$ and y-intercept $c = 10$.

Foundation Standard Complex

c $y = \frac{5x+4}{2}$ can be rewritten as $y = \frac{5x}{2} + \frac{4}{2} = \frac{5x}{2} + 2$.

$\therefore$ Gradient $m = \frac{5}{2}$ and y-intercept $c = 2$.

d $3x + 2y - 6 = 0$ can be rearranged in the form $y = mx + c$.

Make y the **subject** of the equation: 'solve' the equation for y.

$$3x + 2y - 6 = 0$$
$$3x + 2y - 6 - 3x = 0 - 3x$$
$$2y - 6 = -3x$$
$$2y - 6 + 6 = -3x + 6$$
$$2y = -3x + 6$$
$$\frac{2y}{2} = \frac{-3x+6}{2}$$
$$y = \frac{-3x}{2} + 3$$

$\therefore$ Gradient $m = -\frac{3}{2}$ and y-intercept $c = 3$.

Example 13

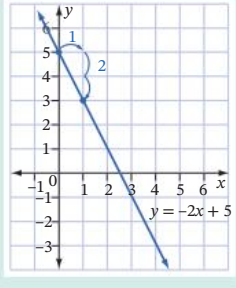
Graph each linear equation by finding the gradient and y-intercept first.

a $y = -2x + 5$ **b** $y = \frac{3}{4}x - 2$

Solution

a $y = -2x + 5$ has a gradient of -2 and a y-intercept of 5.

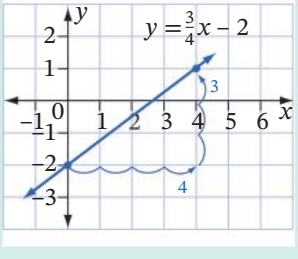
- Plot the y-intercept 5 on the y-axis.
- Make a gradient of $-2 = \frac{-2}{1}$ by moving **across** 1 unit (**run**) and **down** 2 units ('**negative**' **rise**) and marking the point at (1, 3).
- Rule a line through this point and the y-intercept.



Don't forget to label the line with its equation '$y = -2x + 5$'

b $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a y-intercept of -2.

- Plot the y-intercept -2 on the y-axis.
- Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at (4, 1).
- Rule a line through this point and the y-intercept.



Example 14

Test whether each line is parallel to $y = -2x + 3$.

a $y = 2x + 3$ **b** $y = -2x + 1$ **c** $y = -2x$ **d** $y = 5x + 3$

Solution

Parallel lines have the **same gradient**. The line $y = -2x + 3$ has the gradient $m = -2$.

a $y = 2x + 3$ has a different gradient, 2, so it is not parallel to $y = -2x + 3$.

b $y = -2x + 1$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

c $y = -2x$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

d $y = 5x + 3$ has a different gradient, 5, so it is not parallel to $y = -2x + 3$.

$\therefore$ The lines **b** $y = -2x + 1$ and **c** $y = -2x$ are parallel to $y = -2x + 3$.

EXERCISE 3.04 ANSWERS ON P.595

The gradient–intercept equation $y = mx + c$ U F R

EXAMPLE 12

1 Find the gradient and y-intercept of each line.

a $y = 3x - 2$ **b** $y = -2x + 7$ **c** $y = x + 4$

d $y = 9 - x$ **e** $y = \frac{3x}{4} + 6$ **f** $y = x$

g $y = \frac{x}{2} - 11$ **h** $y = \frac{2x + 18}{3}$ **i** $y = \frac{-24 - x}{3}$

j $y = 2(x - 3)$ **k** $11 - 3x = y$ **l** $\frac{2x - 7}{2} = y$

2 Find the equation of a line with:

a a gradient of 2 and a y-intercept of 1 **b** a gradient of $\frac{3}{4}$ and a y-intercept of 2

c a gradient of -7 and a y-intercept of 5 **d** a gradient of $-\frac{2}{5}$ and a y-intercept of 3

e $m = -2, c = -3$ **f** $m = -3, c = \frac{1}{2}$

EXAMPLE 13

3 Graph each linear equation by finding the gradient and y-intercept first.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = 2x$ **d** $y = \frac{x}{2} - 1$

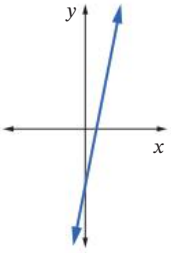
e $y = -2x + 3$ **f** $y = -\frac{3x}{4}$ **g** $y = \frac{-5x + 2}{2}$ **h** $y = \frac{3x - 20}{5}$

4 Write the equation of a line with a gradient of 2 that passes through the origin. R

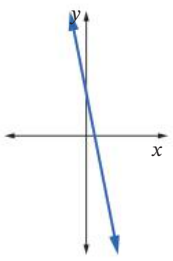
5 Match each equation to its graph below. R

a $y = 4x + 1$ **b** $y = -4x + 1$ **c** $y = -4x - 1$ **d** $y = 4x - 1$

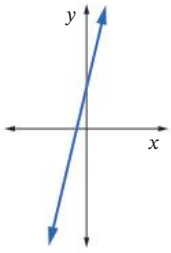
A



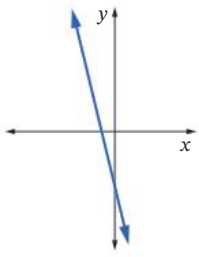
B



C



D



6 For each given line, select the lines that are parallel to it. There may be more than one answer. R

a $y = x + 6$

A $y = 6x$ **B** $y = 6 - x$ **C** $y = x + 1$ **D** $y = 2x$

b $y = 3x + 10$

A $y = 10x + 3$ **B** $y = 3x - 1$ **C** $y = 1 - 3x$ **D** $y = 4 + 3x$

c $y = \frac{x}{2} + 5$

A $y = 2x - 1$ **B** $y = \frac{x+6}{2}$ **C** $y = 1 - \frac{x}{2}$ **D** $y = x + 2$

d $y = 6$

A $y = 2x + 6$ **B** $y = 6x$ **C** $y = -1$ **D** $y = 10$

e $y = 4x$

A $y = 4x - 2$ **B** $y = 4x + 3$ **C** $y = 4$ **D** $y = 1 - 4x$

f $x = 10$

A $y = 10$ **B** $y = 10x$ **C** $x = 2y$ **D** $x = -6$

7 For each set of linear equations, find a pair of equations whose graphs are parallel lines. R

a $y = 4x + 3$ $y = x + 2$ $y = 4x - 6$ $y = 2x$

b $y = 5x + 1$ $3x - y + 7 = 0$ $y = 3x - 2$ $y = -5x + 2$

3.04

EXAMPLE 14

Mental skills 3: Maths without calculators ANSWERS ON P. 596

Time differences

1 Study each example.

a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

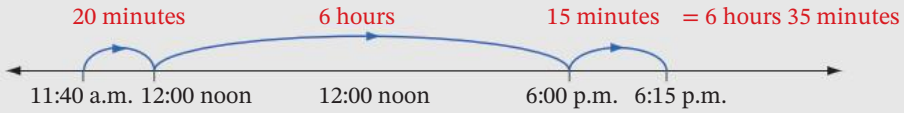
Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

6 hours + 20 min + 15 min = 6 hours 35 min

OR:



Foundation Standard Complex

b What is the time difference between 20:30 and 01:20?

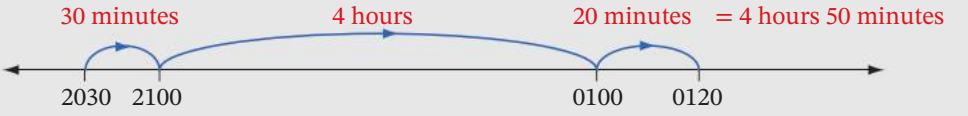
From 20:30 to 00:30 = 4 hours (24 − 20 = 4)

From 00:30 to 01:00 = 30 min

From 01:00 to 01:20 = 20 min

4 hours + 30 minutes + 20 minutes = 4 hours 50 minutes

OR:



2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.

b 6:20 pm. and 12:00 midnight

c 4:45 p.m. and 8:10 p.m.

d 2:35 a.m and 10:50 a.m.

e 1:05 p.m. and 12:30 a.m.

f 9:35 a.m. and 11:15 a.m.

g 04:25 and 09:35

h 14:40 and 20:25

i 7:55 a.m. and 3:50 p.m.

j 2:40 p.m. and 10:20 p.m.

3.05 The general equation $ax + by + c = 0$

WS Parallel and perpendicular lines

A linear equation written in **gradient–intercept form**, such as $y = -\frac{3}{4}x + 2$, can also be written in **general form** ($3x + 4y - 8 = 0$). Note that, for the general form, all of the terms on the left-hand side of the equation are written with no fractions, and only 0 is on the right-hand side. Sometimes the general form is neater and more convenient.

The general form of a linear equation

The **general form of a linear equation** is written as $ax + by + c = 0$, where a, b and c are integers and a is positive.

The general equation $ax + by + c = 0$

Example 15

Write each linear equation in general form.

a $y = 6x + 2$ **b** $y = -\frac{2}{3}x + 2$ **c** $y = 2x - \frac{3}{5}$

Solution

a $y = 6x + 2$

$0 = 6x - y + 2$ Subtracting y from both sides.

$6x - y + 2 = 0$ Swapping sides so that 0 appears on the RHS.

b $y=-\frac{2}{3}x+2$

$3y=3\left(-\frac{2}{3}x+2\right)$ Multiplying both sides by 3 to remove the fraction.

$3y = -2x + 6$

$2x + 3y = 6$ Adding $2x$ to both sides.

$2x + 3y - 6 = 0$ Subtracting 6 from both sides.

c $y=2x-\frac{3}{5}$

$5y=5\left(2x-\frac{3}{5}\right)$ Multiplying both sides by 5.

$5y = 10x - 3$

$0 = 10x - 5y - 3$ Subtracting $5y$ from both sides.

$10x - 5y - 3 = 0$ Swapping sides so that 0 appears on the RHS.

Example 16

Find the gradient and y-intercept of the line whose equation is $5x + 2y - 10 = 0$.

Solution

Rewrite $5x + 2y - 10$ in the form $y = mx + c$.

Make y the subject, solve for y.

$5x + 2y - 10 = 0$

$2y - 10 = -5x$ Subtracting $5x$ from both sides.

$2y = -5x + 10$ Adding 10 to both sides.

$y=\frac{-5x+10}{2}$ Dividing both sides by 2.

Aim to have y on its own on the LHS of the equation.

$y=\frac{-5x}{2}+5$

$\therefore$ Gradient: $m=-\frac{5}{2}$, y-intercept: $c = 5$

EXERCISE 3.05 ANSWERS ON P. 596

The general equation $ax + by + c = 0$ UFRC

1 Write each linear equation in general form. R C

a $y = x + 2$ **b** $y = 3x - 1$ **c** $y = 8 + 5x$

d $x + 2y = 3$ **e** $x - 2y = 6$ **f** $y = 8x + 2$

g $y + 3 = 6x$ **h** $2y = x - 6$ **i** $y=\frac{3}{5}x+2$

2 Find the gradient and y-intercept of the line with equation: R C

a $2x + y = 6$ **b** $8x - 2y = 10$ **c** $3x - 2y + 4 = 0$

d $y + 2x - 1 = 0$ **e** $2x + y + 5 = 0$ **f** $4x + 3y - 12 = 0$

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EXAMPLE 15

EXAMPLE 16

3 Find the gradient, m, and the y-intercept, c, of the line with equation $x - 3y + 5 = 0$. Select the correct answer **A**, **B**, **C** or **D**.

A $m = -1, c = 5$ **B** $m = \frac{1}{3}, c = \frac{5}{3}$ **C** $m = 1, c = -5$ **D** $m = \frac{1}{3}, c = -\frac{5}{3}$

4 Which statement is FALSE about the line whose equation is $3x + y - 6 = 0$? Select **A**, **B**, **C** or **D**. R C

A Its gradient is -3.
B Its y-intercept is -6.
C Its x-intercept is 2.
D It is parallel to the line $y = -3x$.

5 Express each linear equation in general form. R C

a $y = \frac{x}{3} + \frac{1}{4}$ **b** $y = -\frac{5x}{8} + \frac{3}{2}$ **c** $y = \frac{3x}{4} - \frac{2}{5}$

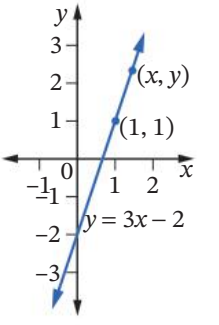
d $y = -\frac{x}{2} - \frac{3}{4}$ **e** $y = \frac{3x}{2} + \frac{2}{5}$ **f** $y = -\frac{9x}{10} - \frac{7}{15}$

STAGE 5.3

Investigation

A line, its gradient and a point

1 The graph shows the line $y = 3x - 2$.



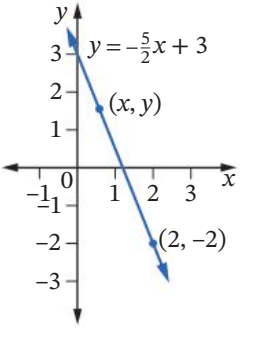
a What is its gradient?

b If (x, y) is any other point on the line, show that $m = \frac{y-1}{x-1}$.

c Explain why $\frac{y-1}{x-1} = 3$

d Hence show that this equation simplifies to $y = 3x - 2$.

2 The graph shows the line $y = -\frac{5}{2}x + 3$.



a What is its gradient?

b If (x, y) is any other point on the line, show that $m = \frac{y+2}{x-2}$.

c Explain why $\frac{y+2}{x-2} = -\frac{5}{2}$

d Hence show that this equation simplifies to $y = -\frac{5}{2}x + 3$.

3 The equation of a line is given by $\frac{y-2}{x-7} = \frac{3}{4}$.

a What is the gradient of the line?

b Can you give the coordinates of a point on this line by looking at its equation? Why?

4 Write the equation of a line that passes through the point $(-3, 5)$ and has a gradient equal to 2. Compare your result with other groups.

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5 A line with gradient m passes through the point (x_1, y_1).

a Show that $m = \dfrac{y - y_1}{x - x_1}$, where (x, y) is any other point on the line.

b Explain why $y - y_1 = m(x - x_1)$.

STAGE 5.3

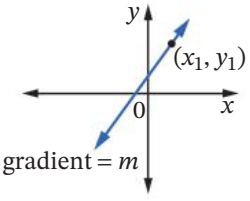
The point–gradient equation $y - y_1 = m(x - x_1)$

3.06

3.05

STAGE 5.3

There is a formula for finding the equation of a line if we know its gradient m and a point on the line (x_1, y_1). Let (x, y) be any other point on the line.



Then $m = \dfrac{y - y_1}{x - x_1}$

or $y - y_1 = m(x - x_1)$.

This is called the **point–gradient form** of a linear equation.

The point–gradient form of a linear equation

The equation of a line with gradient m and which passes through the point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

Example 17

The point –gradient formula

Find the equation of the line with a gradient of $\frac{2}{3}$ that passes through the point $(-2, 1)$.

Solution

$m = \frac{2}{3}, x_1 = -2, y_1 = 1.$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{2}{3}[x - (-2)]$

$3(y - 1) = 2(x + 2)$

$3y - 3 = 2x + 4$

$0 = 2x - 3y + 7$

$2x - 3y + 7 = 0$

In general form

Example 18

The point –gradient formula

Find the equation of the line passing through the points $(1, 3)$ and $(4, -3)$.

Solution

First find the gradient of the line by using the points $(1, 3)$ and $(4, -3)$.

$$m = \frac{-3 - 3}{4 - 1}$$

(x_1, y_1) (x_2, y_2)

$$= \frac{-6}{3}$$

$$= -2$$

STAGE 5.3

Now use $y - y_1 = m(x - x_1)$ with $m = -2$ and $(1, 3)$.

Either point (1, 3) or (4, –3) can be used to find the equation of the line.

$y - 3 = -2(x - 1)$

$= -2x + 2$

$y = -2x + 5$ or $2x + y - 5 = 0$ in general form

OR: using the other point (4, –3) instead:

$y - (-3) = -2(x - 4)$

$y + 3 = -2x + 8$

$y = -2x + 5$ or $2x + y - 5 = 0$ in general form

EXERCISE 3.06 ANSWERS ON P. 596

The point–gradient equation $y - y_1 = m(x - x_1)$ U F R C

In this exercise, express all equations of lines in general form.

EXAMPLE 17

1 Find the equation of each line, given a point on the line and the gradient.

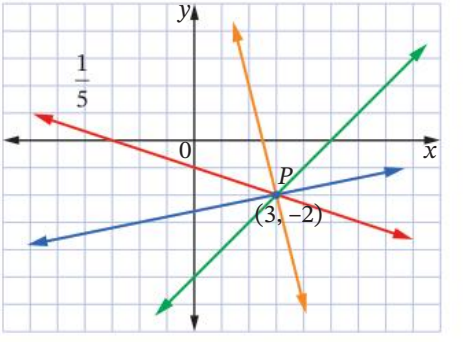
a (2, 5), gradient 2 **b** (–6, 4), gradient –1 **c** (3, –8), gradient 4

d (–1, –2), gradient $\frac{2}{3}$ **e** (2, –8), gradient $-\frac{1}{5}$ **f** (–1, 7), gradient –3

g $\left(\frac{1}{2}, -3\right)$, gradient –4 **h** $\left(-4, -\frac{1}{2}\right)$, gradient $\frac{3}{4}$ **i** (–2, –6), gradient –2

2 4 lines a, b, c and d intersect at $P(3, -2)$.

The gradients of a, b, c and d are 1, $-\frac{1}{3}$, –4 and $\frac{1}{5}$ respectively. R C



a Copy the diagram and correctly label the lines a, b, c and d.

b Find the equation of each line.

EXAMPLE 18

3 Find the equation of the line passing through each pair of points.

a (7, 3) and (10, 6) **b** (8, 10) and (–2, 2) **c** (–1, 3) and (5, 8)

d (2, –2) and (–1, 6) **e** (4, –3) and (6, –6) **f** (–1, –2) and (2, 3)

g (–10, 2) and (1, –4) **h** (–3, 6) and (1, 2) **i** (–4, –9) and (–1, –5)

4 2 lines k and l intersect at (–1, 4). Line k has a gradient of $-\frac{1}{2}$, while line l has a gradient of 3. Find the equations of lines k and l. R C

5 Find the equation of a line with a gradient of –4 and an x-intercept of 5. R C

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6 A line passes through the y-axis at $(0, 6)$ and has a gradient of $\frac{5}{7}$. What is its equation? R C

7 A line with a gradient of $\frac{2}{3}$ passes through the midpoint of $(5, 6)$ and $(1, 10)$.
Find the equation of the line. R C

8 A line with a gradient of $-\frac{3}{5}$ passes through the midpoint of $(-8, -2)$ and $(-2, 20)$.
Find its equation. R C

9 The gradient–intercept form of a line, $y = mx + c$, can also be used to find the equation of a line, given its gradient and a point on the line. R C

a Use $y = mx + c$ to find the equation of the line with gradient 2 that passes through the point $(2, 5)$.

b Compare your equation with your answer to question **1a**.

10 The point–gradient formula can be converted to a formula for finding the equation of a line passing through 2 points (x_1, y_1) and (x_2, y_2). R C

a Prove that the 'two-point formula' is $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$.

b Use the two-point form to find the equation of a line passing through the points $(7, 3)$ and $(10, 6)$.

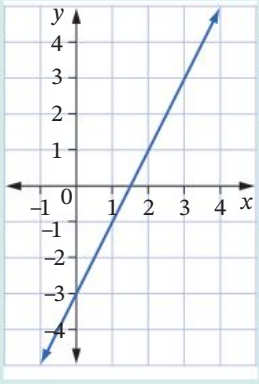
c Compare your equation with your answer to question **3a**.

Finding the equation of a line 3.07

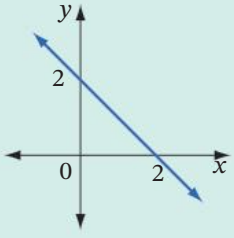
Example 19

Find the equation of each line.

a



b



Finding the equation of a line

WS Finding the equation of a line

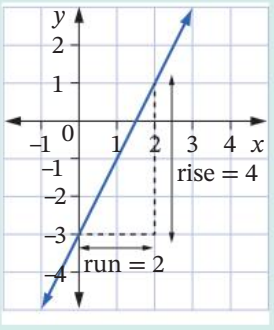
WS A page of lines

Straight-line equations

Foundation Standard Complex

Solution

a Select 2 points on the line to find the gradient, say (0, −3) and (2, 1).



Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ from the graph

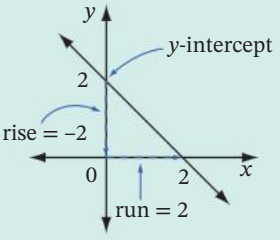
y-intercept: $c = -3$

∴ The equation of the line is $y = 2x - 3$. $y = mx + c$

We can check that this equation is correct for any point on the line, say (3, 3).

When $x = 3$, $y = 2 \times 3 - 3 = 3$.

b Find the gradient of the line passing through (0, 2) and (2, 0).



Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$ from the graph

y-intercept: $c = 2$

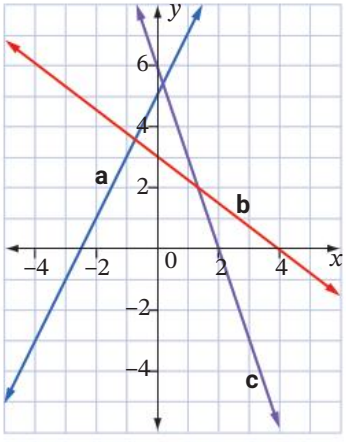
∴ The equation of line is $y = -x + 2$. $y = mx + c$

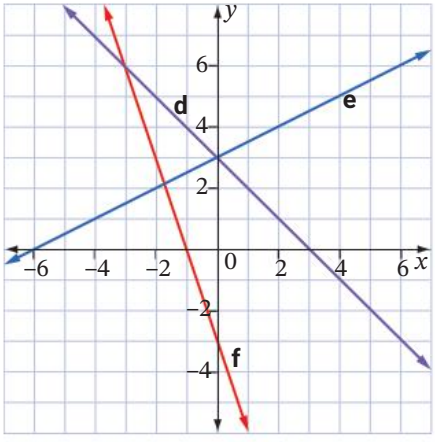
EXERCISE 3.07 ANSWERS ON P. 596

Finding the equation of a line U F R C

EXAMPLE 19

1 Find the equation of each line. R C



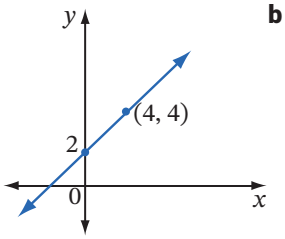


Foundation Standard Complex

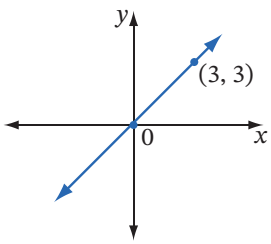
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2 Find the equation of each line. R C

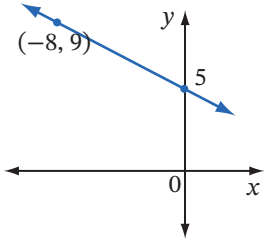
a



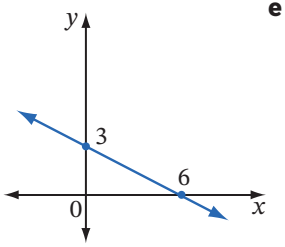
b



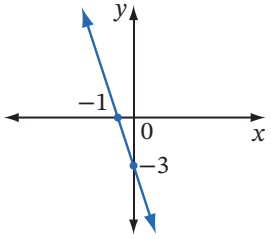
c



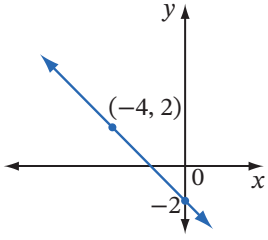
d



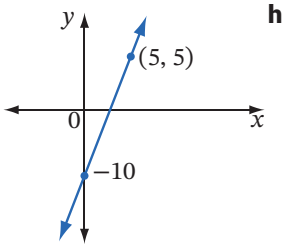
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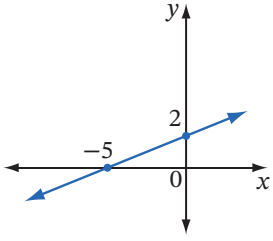
f



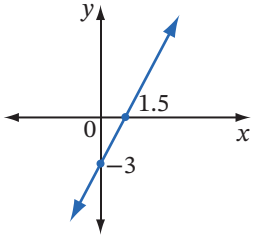
g



h



i



Investigation

Sausage sizzle

A local football club is organising a sausage sizzle on Saturday to raise money to buy new equipment. It costs $50 to hire a gas bottle to run the barbecue and each sandwich costs $1 to make. They hope to sell 100 sandwiches.



Alamy Stock Photo/Boaz Rottem

1 Copy and complete this table to show the cost of making sausage sandwiches. Include the cost of hiring the gas bottle.

No. of sandwiches (x)	0	10	20	30	40	50	60	70	80	90	100
Cost ($\$y$)	50	60									

2 Find the linear equation (formula) for y that represents the cost of making x sausage sandwiches.

Foundation | Standard | Complex

3 Use an appropriate scale to construct a graph that shows the cost of making from $x = 0$ to 100 sandwiches. Label your axes and give your graph an appropriate title.

4 How much will it cost to make 35 sausage sandwiches?

5 How many sandwiches can be made for $132?

6 How much would it cost to make 120 sausage sandwiches?

7 a If the club sold all 100 sausage sandwiches for $5 each, how much money would they take?

b How much profit would the club make?

3.08 Equations of parallel and perpendicular lines

Linear equations match-up

Equations of parallel lines

Gradients of parallel and perpendicular lines

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.

If 2 lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

Example 20

Find the equation of the line parallel to $y = 8 - 3x$ that passes through the point $(-1, 6)$.

Solution

STAGE 5.3

For $y = 8 - 3x$ (or $y = -3x + 8$), the gradient is $m = -3$.

A line parallel to $y = 8 - 3x$, will also have $m = -3$.

Using the point–gradient formula $y - y_1 = m(x - x_1)$ with $m = -3$ and point $(-1, 6)$:

$y - 6 = -3[x - (-1)]$

$= -3(x + 1)$

$= -3x - 3$

$y = -3x + 3$

OR: Using the gradient-intercept equation $y = mx + c = -3x + c$, where c is a constant.

To find the value of c, substitute the point $(-1, 6)$ into the equation:

$y = -3x + c$

$6 = -3 \times (-1) + c$ $\qquad x = -1, y = 6$

$6 = 3 + c$

$c = 3$

$\therefore$ The equation is $y = -3x + 3$.

Example 21

Find the equation of the line perpendicular to $3x - 4y + 6 = 0$ that passes through the point $(5, 4)$.

Solution

To find the gradient of $3x - 4y + 6 = 0$, first convert it to the form $y = mx + c$:

$3x - 4y + 6 = 0$

$3x + 6 = 4y$

$4y = 3x + 6$

$y = \frac{3x+6}{4}$

$y = \frac{3}{4}x + \frac{3}{2}$ $\quad y = mx + c$

$\therefore$ Gradient $= \frac{3}{4}$

$\therefore$ Gradient of perpendicular line $= \frac{-1}{\left(\frac{3}{4}\right)} = -\frac{4}{3}$ The negative reciprocal of $\frac{3}{4}$.

Using the point–gradient formula $y - y_1 = m(x - x_1)$

$y - 4 = -\frac{4}{3}(x - 5)$

$3(y - 4) = -4(x - 5)$

$3y - 12 = -4x + 20$

$4x + 3y - 32 = 0$ In general form.

OR: Using $y = mx + c$, the equation of this line is $y = -\frac{4x}{3} + c$

To find c, substitute $(5, 4)$ into the equation.

$4 = \left(-\frac{4}{3}\right) \times 5 + c$ $\quad x = 5, y = 4$

$c = \frac{32}{3}$

$\therefore$ The equation is $y = -\frac{4x}{3} + \frac{32}{3}$ or $y = \frac{-4x+32}{3}$ or,

in general form: $4x + 3y - 32 = 0$



Shutterstock.com/Rost9

Equations of perpendicular lines

3.08

STAGE 5.3

EXERCISE 3.08 ANSWERS ON P. 597

Equations of parallel and perpendicular lines U F R C

EXAMPLE 20

1 Find the equation of the line that is parallel to: R C

a $y = 2x + 9$ and has a y-intercept of 4

b $y = 3x$ and has an x-intercept of -2

c $y = 5 - \frac{x}{2}$ and passes through $(-1, 6)$.

d $2x - y = 6$ and passes through $(5, -2)$

e $y = -5x - 8$ and passes through the midpoint of $(3, -10)$ and $(-5, -6)$

f $2y = x - 3$ and passes through $(6, -7)$

EXAMPLE 21

2 Find the equation of a line that is perpendicular to: R C

a $y = \frac{x}{2}$ and has a y-intercept of -2

b $y = -5x$ and has an x-intercept of 1

c $y = 3x - 1$ and passes through the x-axis at 4

d $y = \frac{x-6}{3}$ and passes through $(1, -6)$

e $x + y - 6 = 0$ and passes through $(-4, 2)$

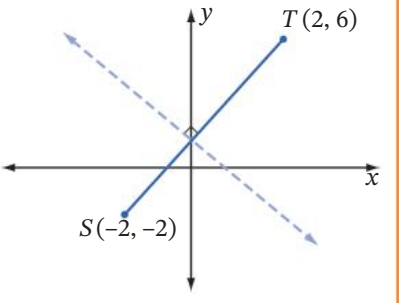
f $3x - y - 9 = 0$ and passes through $(-10, -7)$

3 **a** Find the gradient of interval ST in the diagram.

b Find the midpoint of ST.

c The dotted line is perpendicular to ST and passes through its midpoint. What is its gradient?

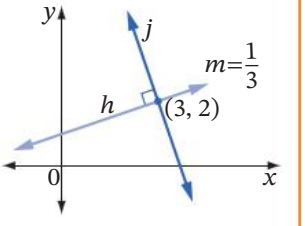
d Find the equation of the dotted line, in the form $y = mx + c$.



4 **a** Find the equation of line h in the diagram.

b Find the gradient of line j (which is perpendicular to line h).

c Find the equation of line j. R C



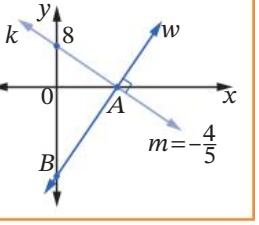
5 **a** Find the equation of line k.

b Find the coordinates of point A.

c Find the gradient of line w.

d Find the equation of line w.

e Find the coordinates of point B. R C



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Coordinate geometry problems#

3.09

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

A variety of problems can be solved by applying coordinate geometry methods, including proving geometrical properties of triangles and quadrilaterals.

STAGE 5.3

WS Geometry problems using coordinates

3.09

Example 22

Lines k and l are shown in the diagram. Find:

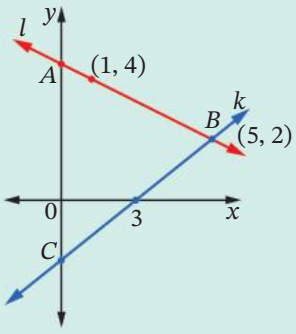
a the equation of line k

b the equation of line l

c the coordinates of point A

d the coordinates of point C

e the area of the triangle ABC



Solution

a Line k passes through (3, 0) and (5, 2).

$$m = \frac{2-0}{5-3}$$
$$= \frac{2}{2}$$
$$= 1$$

Using the point–gradient formula $y - y_1 = m(x - x_1)$:

$$y - 2 = 1(x - 5)$$
$$= x - 5$$
$$y = x - 3$$

(or $x - y - 3 = 0$ in general form)

b Line l passes through (1, 4) and (5, 2).

$$\therefore m = \frac{2-4}{5-1}$$
$$= \frac{-2}{4}$$
$$= -\frac{1}{2}$$

$$y - 4 = -\frac{1}{2}(x - 1)$$
$$2(y - 4) = -1(x - 1)$$
$$2y - 8 = -x + 1$$
$$x + 2y - 9 = 0$$

c A is the y-intercept of line l.

Substitute $x = 0$ into $x + 2y - 9 = 0$.

$$0 + 2y - 9 = 0$$
$$2y = 9$$
$$y = \frac{9}{2}$$
$$= 4.5$$

$\therefore A$ is (0, 4.5)

d C is the y-intercept of line k.

The y-intercept of $y = x - 3$ is -3.

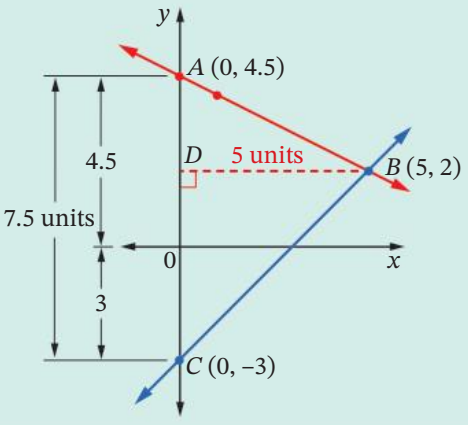
$\therefore C$ is $(0, -3)$

e Area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AC \times BD$ $\quad AC = 4.5 + 3 = 7.5$

$= \frac{1}{2} \times 7.5 \times 5$

$= 18.75 \text{ units}^2$



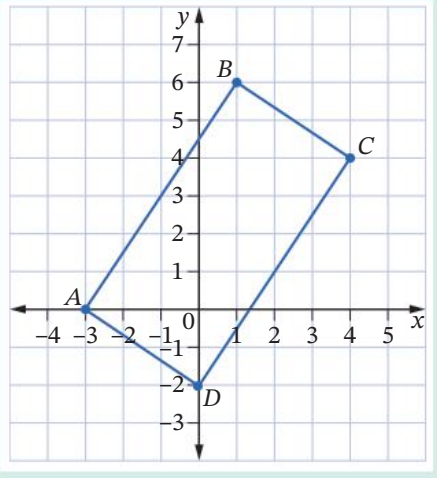
Example 23

$A(-3, 0)$, $B(1, 6)$, $C(4, 4)$ and $D(0, -2)$ are the vertices of a rectangle.

a By finding the lengths of AC and BD, show that the diagonals of the rectangle are equal.

b Find the midpoints of the diagonals AC and BD.

c Show that the diagonals of the rectangle bisect each other.



Solution

a $A(-3, 0)$, $C(4, 4)$

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[4 - (-3)]^2 + (4 - 0)^2}$

$= \sqrt{65}$

$B(1, 6)$, $D(0, -2)$

$BD = \sqrt{(0 - 1)^2 + (-2 - 6)^2}$

$= \sqrt{65}$

$\therefore AC = BD$

$\therefore$ The diagonals are equal.

b Midpoint of $AC \equiv \left(\frac{-3+4}{2}, \frac{0+4}{2}\right) \equiv \left(\frac{1}{2}, 2\right)$

Midpoint of $BD \equiv \left(\frac{1+0}{2}, \frac{6+(-2)}{2}\right) \equiv \left(\frac{1}{2}, 2\right)$

c The midpoints of both diagonals are the same, so the diagonals bisect each other.

EXERCISE 3.09 ANSWERS ON P. 597

EXAMPLE 22

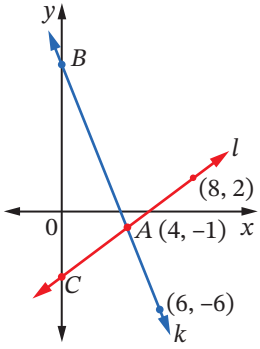
3.09

Coordinate geometry problems U F PS R C

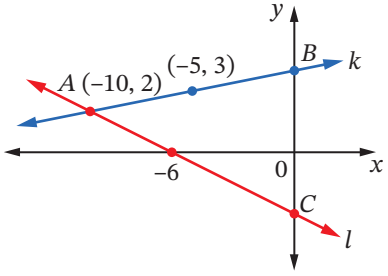
1 For each graph, find: R

- **i** the equation of line k
- **ii** the equation of line l
- **iii** the coordinates of point B
- **iv** the coordinates of point C
- **v** the area of $\triangle ABC$

a

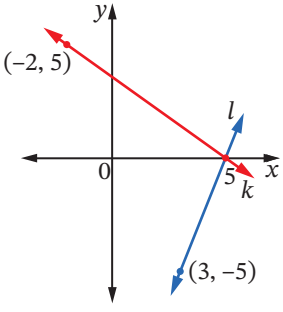


b



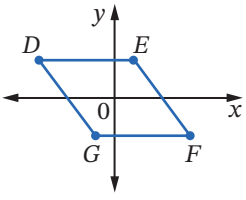
2 For this graph, find: R

- **a** the equation of line l
- **b** the equation of line k
- **c** w if the point $(7, w)$ lies on l
- **d** t if the point $(t, 11)$ lies on



EXAMPLE 23

3 The vertices of a rhombus are $D(-4, 2)$, $E(1, 2)$, $F(4, -2)$ and $G(-1, -2)$. R C



- **a** Show that all sides of the rhombus are equal.

> **'Show that'** means all working out must be provided to fully explain your answer.

- **b** By finding their gradients, show that the opposite sides of the rhombus are parallel.
- **c** Show that the diagonals DF and GE of the rhombus cross at right angles.
- **d** Find the midpoints of the diagonals DF and GE. Do the diagonals bisect each other? Give reasons.
- **e** List the properties of a rhombus that have been demonstrated in this question.

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STAGE 5.3

4 A quadrilateral has vertices $P(-7, 2)$, $Q(2, -7)$, $R(5, -4)$ and $S(-4, 5)$. R C

a Draw a diagram showing the given information.

b Find the lengths of PR and QS in surd form.

c Find the midpoints of PR and QS.

d Is PR perpendicular to QS? Why?

e What type of quadrilateral is $PQRS$? Explain.

5 A quadrilateral has vertices $C(2, 6)$, $D(-5, 2)$, $E(-1, -5)$ and $F(6, -1)$. R C

a Draw a diagram showing the given information.

b Find the length of each diagonal.

c Find the midpoint of each diagonal.

d Show that the diagonals are perpendicular.

e What type of quadrilateral is $CDEF$? Explain.

6 A quadrilateral has vertices $B(1, 7)$, $C(-5, 2)$, $D(2, -2)$ and $E(8, 3)$. R

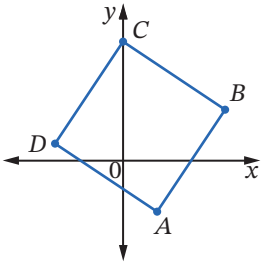
a Find the length of each side.

b Find the gradient of each side.

c Find the midpoint of each diagonal.

d What type of quadrilateral is $BCDE$?

7 A square has vertices $A(2, -3)$, $B(6, 3)$, $C(0, 7)$ and $D(-4, 1)$. R C



a Show that its diagonals are equal.

b Show that its diagonals bisect each other at right angles.

c Hence explain why $ABCD$ is a square.

8 Show that the diagonals of the rhombus with vertices $K(4, 2)$, $L(-1, 4)$, $M(1, -1)$ and $N(6, -3)$ bisect each other at right angles. PS R C

9 Show that $W(-5, -4)$, $X(4, -1)$, $Y(6, 6)$ and $Z(-3, 3)$ are the vertices of a parallelogram by finding the gradients of each side and showing that opposite sides are parallel. R C

STAGE 5.3

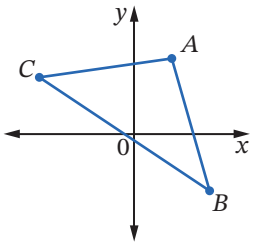
3.09

10 $J(-3, 0)$, $K(3, -2)$, $L(1, 3)$ and $M(-5, 5)$ are the vertices of a quadrilateral. R C

a Find the gradient of each side.

b What type of quadrilateral is $JKLM$? Explain.

11 $A(2, 4)$, $B(4, -3)$ and $C(-5, 3)$ are the vertices of a triangle. R C



a X and Y are the midpoints of AB and AC respectively. Find the coordinates of X and Y.

b Find the gradient of XY and CB. Is it true that $XY \parallel CB$?

c Find the lengths of XY and CB and, hence, show that $CB = 2XY$.

12 $C(-7, -6)$, $N(1, -3)$, $T(4, 5)$ and $W(-4, 2)$ are the vertices of a quadrilateral. PS R C

a Show that its diagonals bisect each other at right angles.

b What type of quadrilateral is $CNTW$? Explain.

13 What type of a quadrilateral is formed by the points $H(-6, 2)$, $I(6, -4)$, $J(4, 2)$ and $K(-2, 5)$? PS R C

14 Show that the points $S(5, -6)$, $T(6, 0)$, $W(-6, 2)$ and $X(-7, -4)$ are the vertices of a rectangle. PS R C



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Foundation Standard Complex

Power plus ANSWERS ON P. 597

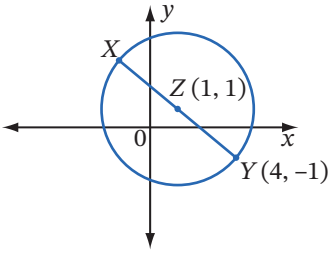
1 A line is drawn through the points $A(0, -2)$ and $B(3, 0)$. The x-coordinate of a point C on AB is 9. Find:

- **a** the gradient of AB
- **b** the equation of AB
- **c** the y-coordinate of C.

2 The point $(-1, 6)$ lies on the line $kx + 3y - 13 = 0$, where k is a constant number. Find k.

3 $Z(-1, 3)$ is the midpoint of the interval joining $A(-4, 7)$ and B. Find the coordinates of B.

4 The circle has XY as a diameter and centre Z.
What are the coordinates of X?



5 Show that the points $(4, 2)$, $(10, -4)$ and $(1, 5)$ are collinear.

6 **a** Find the gradient of any line parallel to $3x + 2y = 4$.

b Find the equation of the line that passes through the point $(0, -1)$ and is parallel to $3x + 2y = 4$.

7 $A(-1, 4)$, $B(4, 6)$, $C(2, 7)$ and D are the vertices of a parallelogram.
Find the coordinates of D.

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CHAPTER 3 REVIEW

Language of maths



Graphing lines

axes	distance	exact answer	general form
gradient	gradient–intercept form	horizontal	interval
length	linear	linear equation	midpoint
parallel	perpendicular	point-gradient form	reciprocal
rise	run	surd	vertical
x-axis	x-intercept	y-axis	y-intercept

1 What is the difference between the y-axis and the y-intercept?

2 When finding the length of an interval on a number plane, what is meant by an **exact** answer?

3 What measurement is the fraction given by the vertical rise of a line divided by the horizontal run?

4 What is the everyday meaning of the word **intercept**? Look it up in a dictionary.

5 What is the property of the gradients of perpendicular lines?

6 What form of the linear equation is $ax + by + c = 0$?

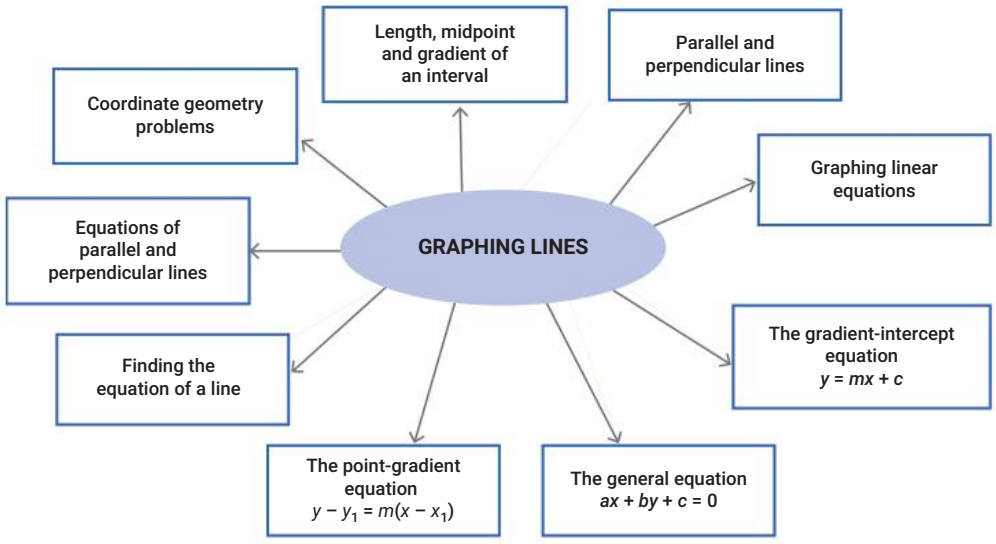
Topic summary



Mind map: Graphing lines (Advanced)

- How can you find the gradient of a line?
- When do you use the formula $y - y_1 = m(x - x_1)$?
- How can you test whether a pair of lines are parallel?
- What parts of this topic did you find difficult?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 3

ANSWERS ON P. 598

1 An interval is formed by joining the points $K(5, 6)$ and $L(-7, 2)$.

a Find, correct to one decimal place, the length of interval KL.

b Find the midpoint of KL.

c Find the gradient of KL.

3.01

2 The vertices of a quadrilateral $HJKL$ are $H(-8, -5)$ $J(-1, -2)$ $K(2, 5)$ $L(-5, 2)$.

a Find the exact length of the sides of the quadrilateral.

b Find the gradient of each side of $HJKL$.

c Find the exact length of the diagonals HK and JL.

d What type of quadrilateral is $HJKL$?

STAGE 5.3

3.01

3 Find, correct to the nearest degree, the angle of inclination of a line with gradient:

a 3 **b** $\frac{5}{4}$ **c** -1 **d** $-\frac{2}{3}$

3.02

4 A line passes through the points $V(8, -1)$ and $W(10, -2)$. What is the gradient of a line:

a parallel to VW? **b** perpendicular to VW?

3.01

5 Graph each linear equation on a number plane.

a $y = -5x - 1$ **b** $x + 2y = 16$ **c** $3x - 4y - 12 = 0$

3.03

6 Which point lies on the line of $3x + y = 2$? Select the correct answer **A**, **B**, **C** or **D**.

A $(1, 0)$ **B** $(2, 4)$ **C** $(-1, 5)$ **D** $(-1, -5)$

3.03

7 What is the equation of the line through $(-2, 3)$ and parallel to the x-axis? Select **A**, **B**, **C** or **D**.

A $x = -2$ **B** $x = 3$ **C** $y = -2$ **D** $y = 3$

3.04

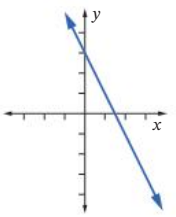
8 Write the gradient, m, and y-intercept, c, for each linear equation.

a $y = 2x - 10$ **b** $y = 4x + 3$ **c** $y = \frac{4 - 3x}{8}$

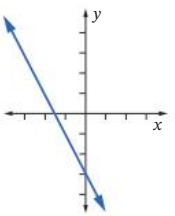
9 Match each equation to its graph below.

a $y = 2x - 3$ **b** $y = -2x - 3$ **c** $y = -2x + 3$ **d** $y = 2x + 3$

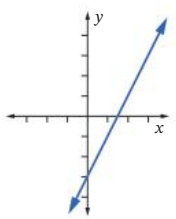
A



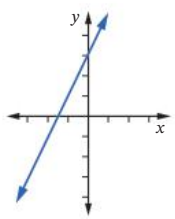
B



C



D



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10 Convert each equation to general form $ax + by + c = 0$.

a $y = 3x + 5$ **b** $y = \frac{2x}{5} - 10$ **c** $x = 3y + 6$

3.05

11 Rewrite each equation in the form $y = mx + c$, then state the gradient, m, and the y-intercept, c.

a $x - y + 2 = 0$ **b** $2x - 8y + 8 = 0$ **c** $3x + y - 9 = 0$

3.05

STAGE 5.3

12 Find in general form the equation of a line that passes through the point:

a (5, 5) and has a gradient of –3 **b** (–1, 8) and has a gradient of $\frac{2}{3}$

3.06

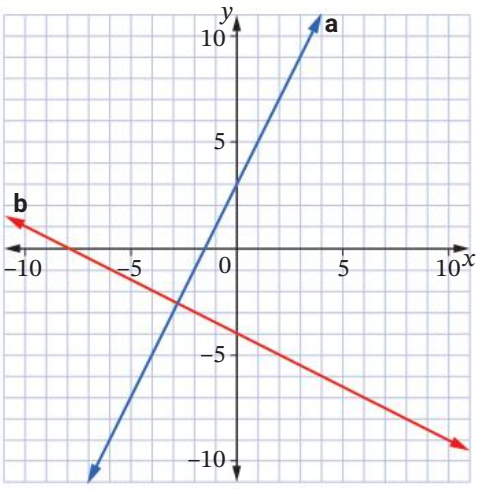
13 Find in general form the equation of a line that passes through the points:

a (10, 2) and (5, –1) **b** (–6, 3) and (–2, –1)

3.06

14 Find the equation of each line and show that they are perpendicular.

3.07



15 Find the equation of a line that is:

a parallel to $y = 3x + 1$ and passes through the x-axis at 2

b perpendicular to $y = \frac{x}{2}$ and passes through the origin.

3.08

16 The line $3x - 8y + 10 = 0$ and another line intersect at right angles at the point (10, 5). Find the equation of the other line.

STAGE 5.3

3.08

17 $L(-1, 2)$, $M(4, 5)$, $N(1, 10)$ and $P(-4, 7)$ are the vertices of a quadrilateral. Show that $LMNP$ is a square.

3.09

TEST YOURSELF 3

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4

MEASUREMENT AND GEOMETRY

SURFACE AREA AND VOLUME

Some theme parks have wave pools, which are big swimming pools that simulate the movement of the water at a beach. A large volume of water is quickly released into one end of the pool, which produces a large wave that moves from one end of the pool to the other. The excess water in the pool is recycled so that it can be used to produce more waves.



Chapter outline

		Working mathematically				
4.01	Surface area of a prism	U	F	PS	R	C
4.02	Surface area of a cylinder	U	F	PS	R	C
4.03	Surface area of a pyramid*	U	F	PS	R	
4.04	Surface areas of cones and spheres*	U	F	PS	R	C
4.05	Surface areas of composite solids	U	F	PS	R	
4.06	Volumes of prisms and cylinders	U	F		R	
4.07	Volumes of pyramids, cones and spheres*	U	F	PS	R	C
4.08	Volumes of composite solids*	U	F	PS	R	
4.09	Areas of similar figures*	U	F		R	C
4.10	Surface areas and volumes of similar solids*#	U	F		R	

***STAGE 5.3**

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

cross-section A 'slice' of a solid, taken across the solid rather than along it

curved surface area The area of the curved surface of a solid, such as a cylinder or sphere. The curved surface of a cylinder is a rectangle when flattened.

hemisphere Half a sphere.

pyramid A solid with a polygon for a base and triangular faces that meet at a point called the apex.

sector A region of a circle cut off by 2 radii, shaped like a piece of pizza

slant height The height of a pyramid or cone from its apex (top) to its base along a side face rather than its perpendicular height.

sphere A ball shape.

surface area The total area of all faces of a solid shape

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate the surface areas, volumes and capacities of right prisms, cylinders and composite solids
- (STAGE 5.3) calculate the surface areas, volumes and capacities of pyramids, cones, spheres and composite solids
- (STAGE 5.3) investigate ratios of areas of similar figures
- (STAGE 5.3) investigate ratios of surface areas and volumes of similar solids

SkillCheck ANSWERS ON P. 598

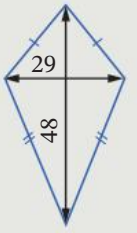
Solid shapes

What is volume?

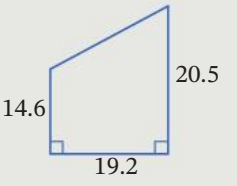
Area

1 Calculate the area of each shape. All measurements are in centimetres.

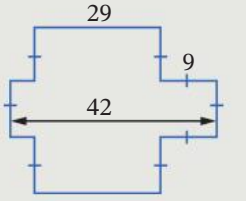
a



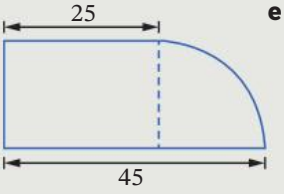
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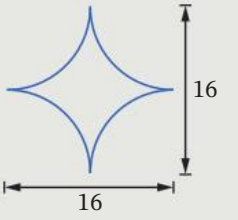
c



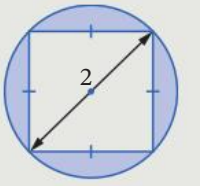
d



e

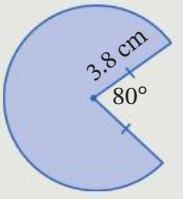


f

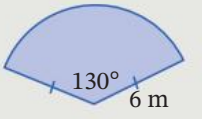


2 Find, correct to one decimal place, the area of each sector.

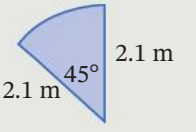
a



b



c



4.01 Surface area of a prism

Surface area

Nets of solids

Surface area

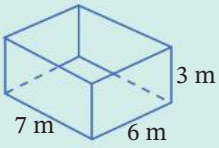
The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

For the surface area of an open solid, we only count the *external* surfaces, not the internal ones (so that we don't count each surface twice).

Example 1

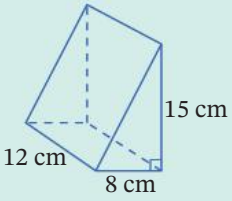
Find the surface area of each prism.

a



Open rectangular prism

b



Closed triangular prism

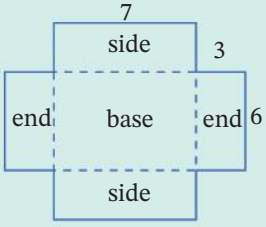
Solution

a This open prism has 5 faces.

Surface area = 2 ends + 2 sides + base

$= 2 \times (3 \times 6) + 2 \times (3 \times 7) + (6 \times 7)$

$= 120 \text{ m}^2$



b This prism has 5 faces.

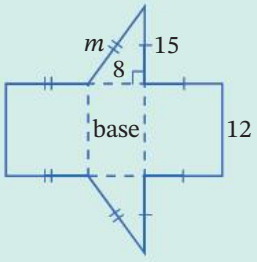
Using Pythagoras' theorem to find m, the hypotenuse of the triangle:

$m^2 = 8^2 + 15^2$

$= 289$

$m = \sqrt{289}$

$= 17$



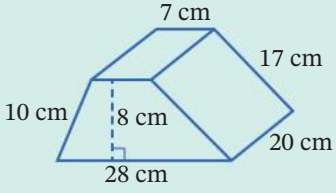
Surface area = 2 triangles + 3 rectangles

$= 2 \times \left(\frac{1}{2} \times 8 \times 15\right) + (17 \times 12) + (8 \times 12) + (15 \times 12)$

$= 600 \text{ cm}^2$

Example 2

Calculate the surface area of this trapezoidal prism.



Solution

This trapezoidal prism has 6 faces:

2 identical trapeziums (front and back) and 4 different rectangles.

Area of each trapezium $= \frac{1}{2} \times (7 + 28) \times 8$

$= 140 \text{ cm}^2$

Surface area $= (2 \times 140) + (20 \times 7) + (20 \times 10) + (20 \times 28) + (20 \times 17)$

$= 1520 \text{ cm}^2$

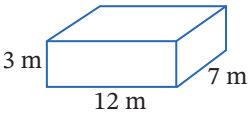
EXERCISE 4.01 ANSWERS ON P. 598

Surface area of a prism U F PS R C

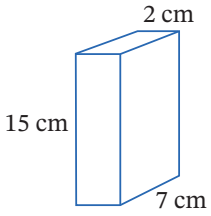
EXAMPLE 1

1 Find the surface area of each prism.

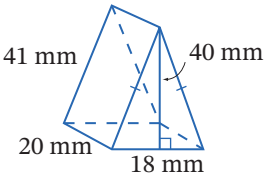
a



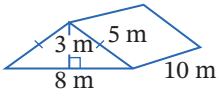
b



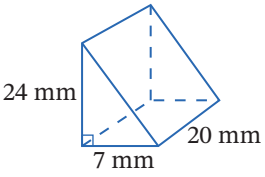
c



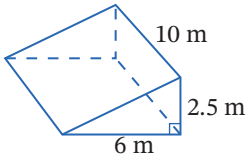
d



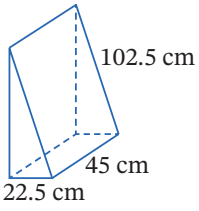
e



f



2 Calculate the surface area of this triangular prism.
Select the correct answer **A**, **B**, **C** or **D**.



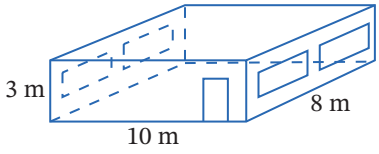
A 12 375 cm^2 **B** 11 250 cm^2

C 10 125 cm^2 **D** 12 431.25 cm^2

3 This classroom is being renovated. Find: PS

a the area of the floor to be carpeted and the total cost, at $55 per square metre.

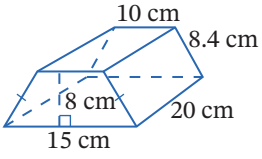
b the ceiling and wall area to be painted, if the room contains 4 windows, each 2.5 m by 1.5 m, and a doorway 2 m by 0.8 m.



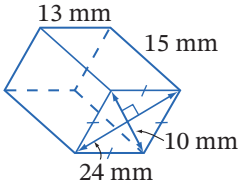
EXAMPLE 2

4 Calculate the surface area of each prism.

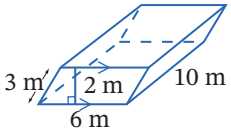
a



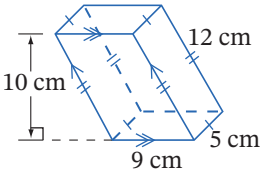
b



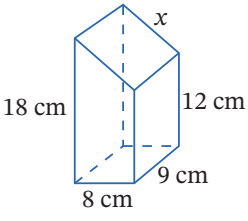
c



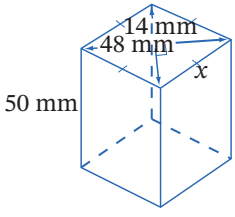
d



e



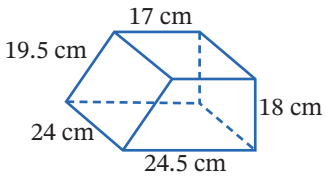
f



Foundation Standard Complex

9780170453509

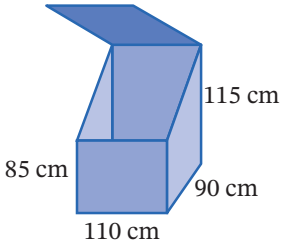
5 Calculate the surface area the trapezoidal prism. Select **A**, **B**, **C** or **D**.



A 10 584 cm^2 **B** 2643 cm^2

C 2082.75 cm^2 **D** 8964 cm^2

6 The wooden toy box is in the shape of a trapezoidal prism.



a Calculate how much timber is required to make the toy box, correct to the nearest cm^2.

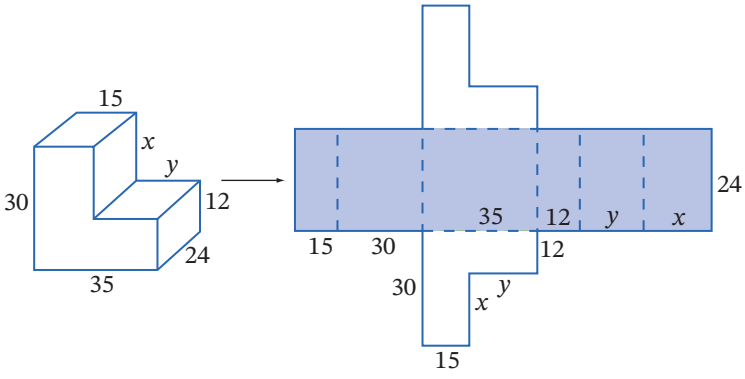
b If the price of the timber is \$25/m^2, calculate the cost of making the box.

4.01

Investigation

A surface area shortcut

1 Consider this L-shaped prism and its net. We will find its surface area.



a Find x and y.

b This prism has 8 faces: 2 'L-shaped' ends and 6 rectangles. Instead of calculating the areas of the 6 rectangles separately, we can combine them into one long rectangle, as shaded in the net above. The length of the rectangle is the same as the perimeter of the L-shape. What is the length of this long rectangle?

c What is the area of this long rectangle?

d Copy and complete:

Length of shaded rectangle = p _____ of the L-shape.

e Find the surface area of the prism by copying and completing the following:

Surface area = 2 'L-shaped' ends + shaded rectangle

$= 2 \times (15 \times 30 + 20 \times 12) +$ _____

$=$ _____

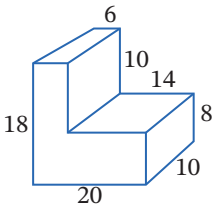
Foundation Standard Complex

2 From question **1**, it can be seen that the surface area of any prism with end faces of area A and perpendicular height (distance between end faces) h can be calculated using the formula:

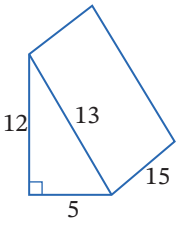
$$SA = 2A + Ph$$

where P = perimeter of the end face.
Use this method to calculate the surface area of each prism. All measurements are in centimetres.

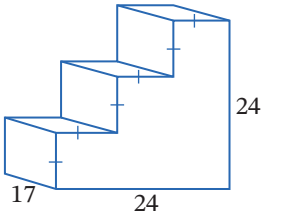
a



b



c



4.02 Surface area of a cylinder

Surface area

Surface area of a cylinder

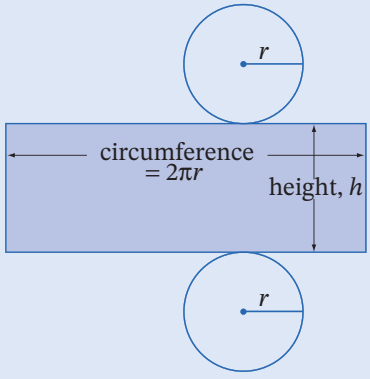
Surface area of a closed cylinder

$$SA = 2\pi r^2 + 2\pi rh$$

where r = radius of circular base

h = perpendicular height

The area of the 2 circular ends = $2\pi r^2$
and the area of the curved surface = $2\pi rh$.

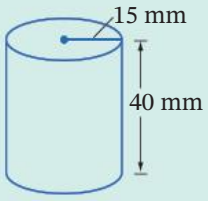




Alamy Stock Photo/Robert Wyatt

Example 3

Find, correct to the nearest mm^2, the surface area of this cylinder.



Solution

Surface area $= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 15^2 + 2 \times \pi \times 15 \times 40$ $\quad r = 15, h = 40$

$= 5183.627...$

≈ 5184 mm^2

Example 4

Calculate in exact form (in terms of π) the surface area of a cylinder open at one end with radius 12 cm and height 15 cm.

Solution

Surface area = circular end + curved surface

$= \pi \times 12^2 + 2 \times \pi \times 12 \times 15$

$= 144\pi + 360\pi$

$= 504\pi$ cm^2

Example 5

Find, correct to 2 **significant figures**, the surface area of:

a a cylindrical tube, open at both ends, with radius 3 cm and height 55 cm

b an open half-cylinder with radius 0.65 m and length 2.4 m.

Solution

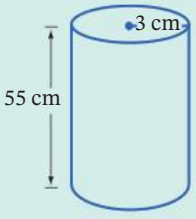
a Surface area = curved surface

$= 2\pi rh$

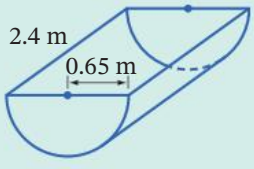
$= 2 \times \pi \times 3 \times 55$

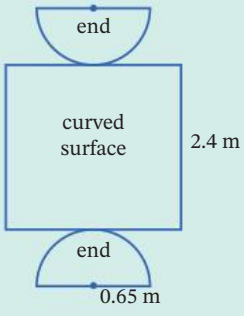
$= 1036.725...$

≈ 1000 cm^2



b





Surface area = 2 semicircle ends + $\frac{1}{2}$ × curved surface

$= 2 \times \left(\frac{1}{2} \times \pi \times 0.65^2 \right) + \frac{1}{2} \times (2 \times \pi \times 0.65 \times 2.4)$

$= 6.2282...$

≈ 6.2 m^2

EXERCISE 4.02 ANSWERS ON P. 598

Surface area of a cylinder U F PS R C

EXAMPLE 3

1 Calculate, correct to one decimal place, the surface area of a cylinder with:

a radius 1.4 m, height 2.2 m
b diameter 45 cm, height 65 cm
c diameter 9 cm, height 24 cm
d radius 1.3 m, height 3.8 m

EXAMPLE 4

2 Find, correct to the nearest whole number, the curved surface area of a cylinder with:

a radius 1.5 m, height 3.75 m
b diameter 27 cm, height 41 cm

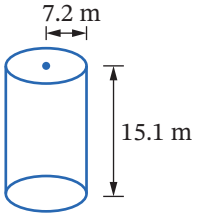
3 Calculate, in exact form, the surface area of: R C

a a closed cylinder, radius 6 cm, height 2.5 cm.
b a cylinder open at one end with radius 1.25 m and height 3.5 m
c a piece of cylindrical tubing, open at both ends, with radius 1.5 cm and length 6.5 m

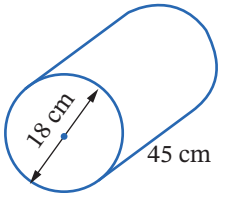
EXAMPLE 5

4 Calculate, correct to the nearest whole number, the surface area of each solid. R

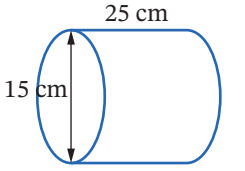
a closed cylinder



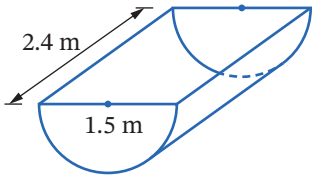
b closed cylinder



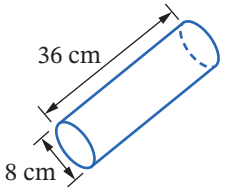
c cylinder with one open end



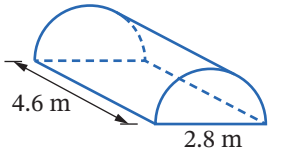
d half cylinder with open top



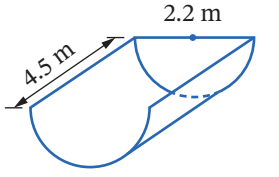
e cylinder open both ends



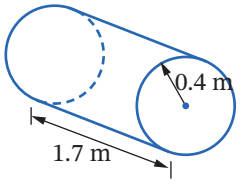
f closed half cylinder



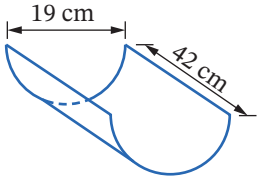
g half cylinder with open top, one end open



h closed cylinder



i half cylinder, open both ends



Foundation Standard Complex

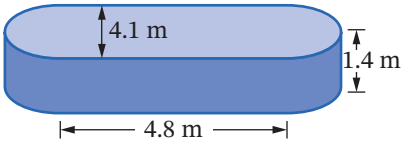
9780170453509

5 A container of potato chips is a cylinder with diameter 7 cm and height 23.2 cm. Calculate its surface area, correct to one decimal place.

6 Find the surface area of a cylinder, open at one end with diameter 12 mm and length 15 cm. Select the closest answer **A**, **B**, **C** or **D**. R

A 678.6 cm^2 **B** 1017 cm^2 **C** 6107.3 mm^2 **D** 5768.0 mm^2

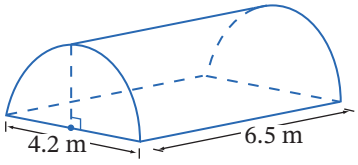
7 The inside of the swimming pool, including the floor, is to be repainted. Find: PS R



a the area to be repainted, correct to one decimal place

b the number of whole litres of paint needed if coverage is 9 m^2 per litre.

8 The diagram shows a tent to be made in the shape of half a cylinder. Find: PS R



a the area of the floor of the tent

b the surface area of the tent, excluding the floor.

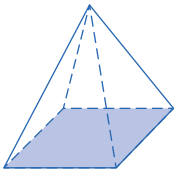
c the total cost of materials for the tent if the material for the flooring costs \$18.50/m^2 and the canvas for the tent costs \$21.75/m^2.

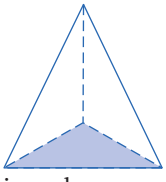
4.02

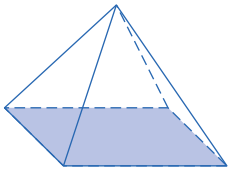
Surface area of a pyramid 4.03

STAGE 5.3

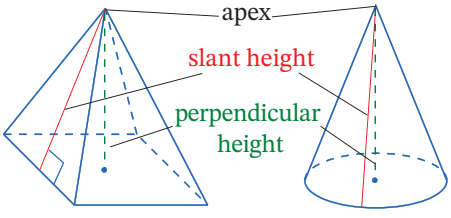
A **pyramid** is a solid shape with a polygon for its base and triangular faces that meet at a point or vertex called its **apex**. Like a prism, a pyramid is named by the shape of its base.


Square pyramid


Triangular pyramid


Rectangular pyramid

A **cone** is a solid shape with a circular base and a curved surface that also has an **apex**. However, a cone is not a pyramid because its base is not a polygon (a circle does not have straight sides).



The **slant height** of a pyramid or cone is the height from the apex to the base, along a side face. It is different from the **perpendicular height** of a pyramid or cone, which is the perpendicular distance from the apex to the base.

The surface area of a pyramid is calculated by adding the areas of its base and its triangular faces.

Foundation Standard Complex

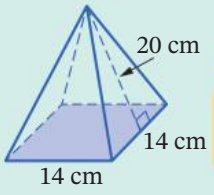
STAGE 5.3

Surface area of a pyramid

Example 6

Find the surface area of each square pyramid.

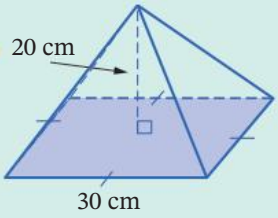
a



This pyramid has a slant height of 20 cm

This pyramid has a perpendicular height of 20 cm

b



Solution

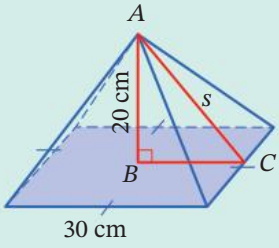
a Surface area = area of square base + area of 4 triangular faces

$$= 14 \times 14 + 4 \times \frac{1}{2} \times 14 \times 20$$
$$= 756 \text{ cm}^2$$

b First find the slant height, s, using Pythagoras' theorem.

In $\triangle ABC$, $s^2 = 20^2 + 15^2$ $\quad BC = \frac{1}{2} \times 30$

$$= 625$$
$$s = \sqrt{625}$$
$$= 25 \text{ cm}$$



Surface area $= 30 \times 30 + 4 \times \frac{1}{2} \times 30 \times 25$

$$= 2400 \text{ cm}^2$$

Example 7

A rectangular pyramid with base 10 cm by 8 cm has a perpendicular height of 15 cm. Find its surface area, correct to one decimal place.

Solution

First find the slant heights AP and AQ.

$AP^2 = AX^2 + XP^2$

$= 15^2 + 4^2$ $\quad XP = \frac{1}{2} \times 8$

$= 241$

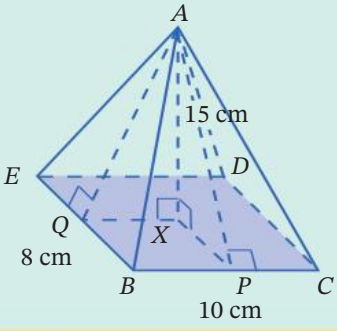
$AP = \sqrt{241}$ cm

$AQ^2 = AX^2 + XQ^2$

$= 15^2 + 5^2$ $\quad XQ = \frac{1}{2} \times 10$

$= 250$

$AQ = \sqrt{250}$ cm



It is better to leave the lengths of AP and AQ in surd form rather than round them to decimals so that the final answer is accurate.

Surface area = area of rectangle base + 2 × Area $\triangle ABC$ + 2 × Area $\triangle ADC$

$$= 10 \times 8 + 2 \times \frac{1}{2} \times 10 \times \sqrt{241} + 2 \times \frac{1}{2} \times 8 \times \sqrt{250}$$
$$= 361.7328...$$
$$\approx 361.7 \text{ cm}^2$$

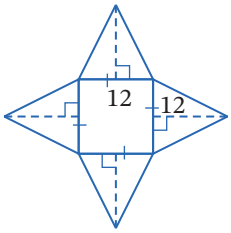
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EXERCISE 4.03 ANSWERS ON P. 598

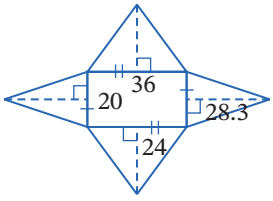
Surface area of a pyramid U F PS R

1 Find the area of each net and hence the surface area of the corresponding pyramid.
All measurements are in centimetres.

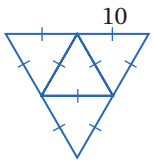
a



b



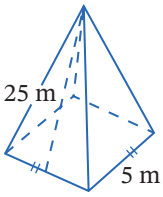
c



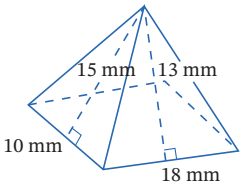
EXAMPLE 6

2 Find the surface area of each pyramid.

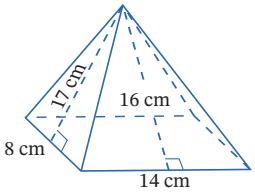
a



b

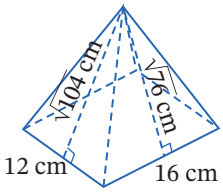


c



3 Find the surface area of this pyramid.
Select the closest answer **A**, **B**, **C** or **D**.

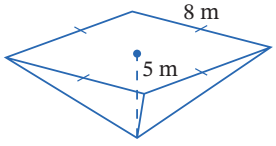
A 288.0 cm^2 **B** 453.9 cm^2
C 459.8 cm^2 **D** 471.0 cm^2



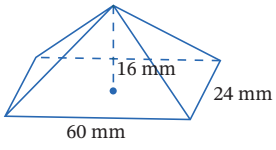
EXAMPLE 7

4 Calculate, correct to one decimal place, the surface area of each pyramid. R

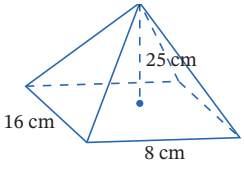
a



b

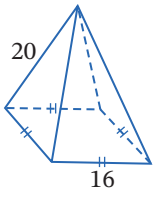


c

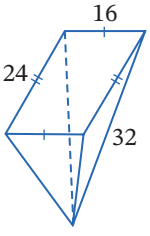


5 Calculate, correct to the nearest square centimetre, the surface area of each pyramid.
All measurements are in centimetres. PS R

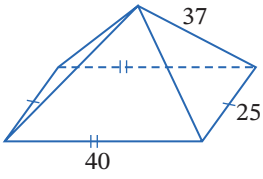
a



b



c



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STAGE 5.3

6 The great pyramid of Khufu (or Cheops) in Egypt has a height of 147 m, and each side of its square base measures 230 m. Find its surface area (excluding the base), correct to the nearest square metre. PS R

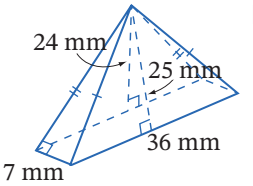
7 The Louvre Art Museum in Paris, France, has a large pyramid at its entrance (built in 1989). It reaches a height of 21.6, has a square base of length 34 m and its sides are covered in rhombus and triangular glass pieces. Calculate the surface area of the glass pyramid (to the nearest m^3). PS R



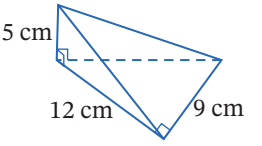
Shutterstock.com/vichie81

8 Calculate, correct to one decimal place, the surface area of each pyramid. PS R

a

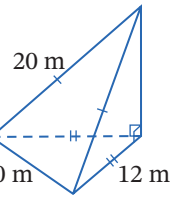


b



All faces are right-angled triangles

c



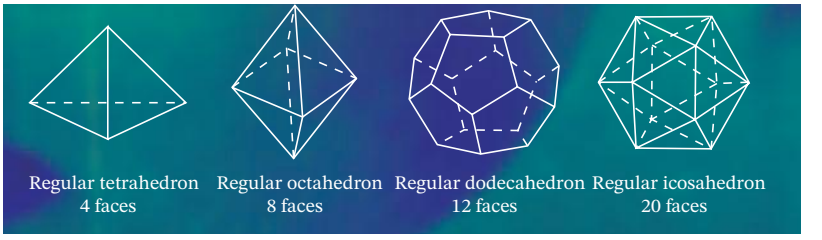
9 A square pyramid has a surface area of 4704 m^2 and a base area of 1764 m^2. Find: R

- **a** the length of its base
- **b** the area of each triangular face
- **c** the slant height of each triangular face
- **d** the perpendicular height of the pyramid

Did you know?

The Platonic solids

The cube is called a **regular polyhedron** or **Platonic solid** because it has 6 identical faces. The more formal name for a cube is **regular hexahedron**, 'hex' meaning 6. There are only 6 possible regular polyhedrons: the other 5 are shown below. Each face on a regular polyhedron is a **regular polygon.** The regular tetrahedron, for example, is a pyramid of equilateral triangles.



The tetrahedron, cube and octahedron occur in nature in the form of certain crystals.

What are the shapes of the faces of each Platonic solid shown?

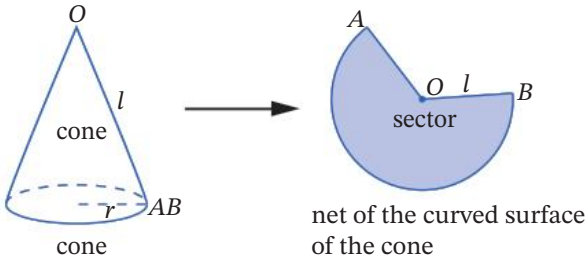
Foundation Standard Complex

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Investigation

The surface area of a cone

The net of a cone is made up of a circle (for the base) and a sector of a circle (for the curved surface). The second diagram below shows the curved surface of a cone.



cone

net of the curved surface of the cone

We can use this fact to find a formula for the curved surface area of a cone. Suppose the cone has base radius r and slant height l. Then the sector must have radius l.

Looking at the sector, the major arc length AB is a fraction of the circumference of the circle and the area of the sector is a fraction of the area of the circle. They should be the same fraction, so:

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference}}$$

1 The major arc length AB is the same as the circumference AB of the base of the cone in the first diagram. Write an algebraic expression for the circumference of the circle in the first diagram.

2 Write an algebraic expression for the circumference of the complete circle in the second diagram.

3 Write an algebraic expression for the *area* of the complete circle in the second diagram.

4 $\dfrac{\text{Area of sector}}{\text{Area of circle}} = \dfrac{\text{Arc length}}{\text{Circumference}}$ becomes $\dfrac{\text{Area of sector}}{\pi l^2} = \dfrac{2\pi r}{2\pi l}$

Copy and complete: $\therefore$ Area of sector $= \dfrac{2\pi r}{2\pi l} \times \pi l^2 =$ _____

5 But the area of the sector is equal to the curved surface area of the cone.

Copy and complete:

Surface area of cone = area of circular base + area of curved surface

= _____ + _____

4.04 Surface areas of cones and spheres

STAGE 5.3

Surface area of a cone

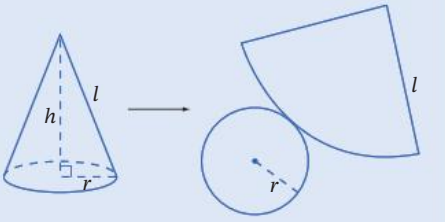
$$SA = \pi r^2 + \pi rl$$

where r = radius of circular base

l = slant height

The area of the circular base = πr^2

and the area of the curved surface = πrl

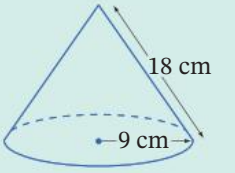


Example 8

For this cone, find correct to one decimal place:

a the curved surface area

b the total surface area



Solution

$r = 9$ cm and $l = 18$ cm

a Curved surface area = πrl

$= \pi \times 9 \times 18$

$= 508.9380\ldots$

≈ 508.9 cm^2

b Total surface area = $\pi r^2 + \pi rl$

$= \pi \times 9^2 + \pi \times 9 \times 18$

$= 763.4070\ldots$

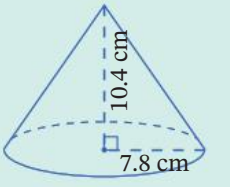
≈ 763.4 cm^2

Example 9

Surface area of a cone and sphere

Surface area and volume of a cone

Find, correct to 2 decimal places, the surface area of this cone.



Solution

First calculate the slant height:

$l^2 = 7.8^2 + 10.4^2$

$= 169$

$l = \sqrt{169}$

$= 13$

Surface area = $\pi r^2 + \pi rl$

$= \pi \times 7.8^2 + \pi \times 7.8 \times 13$

$= 509.6919\ldots$

≈ 509.69 cm^2

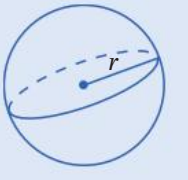
A **sphere** is a ball shape, completely round. A **hemisphere** is half a sphere.

Surface area of a sphere

$SA = 4\pi r^2$

where r = radius of the sphere

Note that the surface area of a sphere is 4 times the area of a circle that shares its centre.

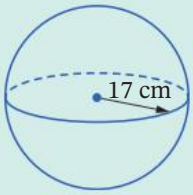


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Surface area of a cone and sphere

Example 10

Find, correct to 2 decimal places, the surface area of this sphere.



Solution

Surface area $= 4\pi r^2$

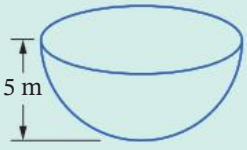
$= 4 \times \pi \times 17^2$

$= 3631.6811...$

$= 3631.68 \text{ cm}^2$

Example 11

Find the surface area of this hemisphere in exact form, in terms of π.



Solution

Surface area = Area of circular base + Curved surface area

$= \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$= \pi r^2 + 2\pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 5^2$

$= 75\pi \text{ m}^2$

Answers written as surds or in terms of π are exact because they are not decimal approximations.

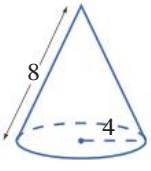
EXERCISE 4.04 ANSWERS ON P. 598

Surface areas of cones and spheres **U F PS R C**

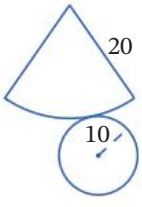
EXAMPLE 8

1 Calculate, correct to the nearest cm², the curved surface area of each cone. All measurements are in centimetres.

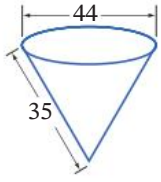
a



b

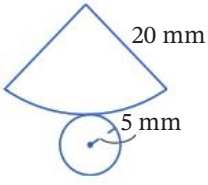


c

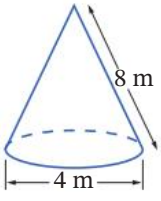


2 Find, correct to one decimal place, the total surface area of each cone.

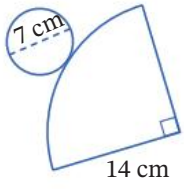
a



b



c



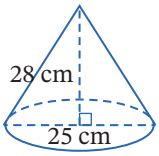
Foundation | Standard | Complex

STAGE 5.3

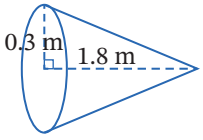
EXAMPLE 9

3 Calculate, correct to one decimal place, the surface area of each cone. R

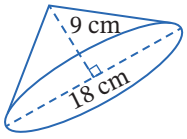
a



b

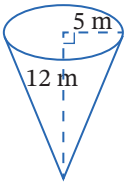


c

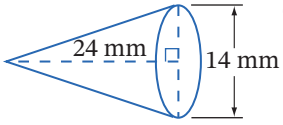


4 Find in exact form (in terms of π) the total surface area of each cone. R C

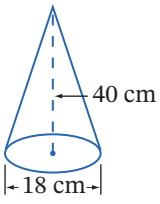
a



b



c



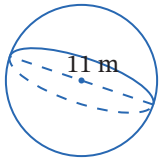
EXAMPLE 10

5 Calculate, correct to one decimal place, the surface area of each sphere.

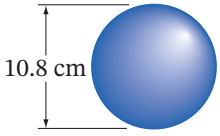
a



b



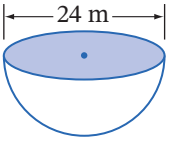
c



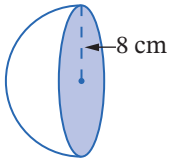
6 Find in exact form the surface area of each closed hemisphere. R C

EXAMPLE 11

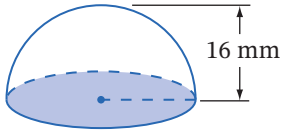
a



b



c



7 Calculate the surface area of a closed hemisphere with radius 1.4 m.
Select the closest answer **A**, **B**, **C** or **D**. R

A 12.32 m^2 **B** 18.47 m^2 **C** 24.63 m^2 **D** 49.26 m^2

8 Find, correct to the nearest square metre, the surface area of: PS R

a a sphere with diameter 10 m

b an open cone with base radius 10 m, slant height 20 m

c an open hemisphere with radius 10 m

d a cone with base diameter 10 m, perpendicular height 20 m

9 The Earth has a radius of approximately 6378 km. PS R C

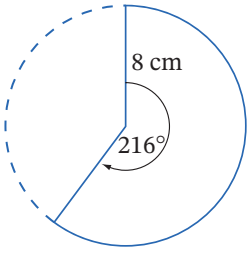
a Calculate the surface area of the Earth in scientific notation, correct to 3 significant figures.

b If 70.9% of the Earth's surface is covered by water, calculate the land area of the Earth's surface, correct to 3 significant figures.

c The land area of Australia is 7 692 000 km^2. Calculate Australia's land area as a percentage of the Earth's land area (correct to 2 decimal places).

10 Find the amount of sheet metal needed to form a conical funnel of base radius 30 cm and vertical height 50 cm, allowing for a 0.5 cm overlap at the join. PS R C

11 A cone is made from a sector of a circle with radius 8 cm and central angle 216°. Find: PS R C



- **a** the length of the arc of the circle which forms the circumference of the cone's base
- **b** the radius of the cone's base
- **c** the slant height of the cone
- **d** the total surface area of the cone, including the base

12 Find the radius of each solid if it has a surface area of 6000 mm^2. Give your answer correct to 3 significant figures. R

a a sphere **b** a closed hemisphere **c** an open hemisphere

13 A cone has a surface area of 2000 cm^2. If the area of its base is 150 cm^2, find: R

a the radius of its base **b** its slant height **c** its perpendicular height

Mental skills 4: Maths without calculators ANSWERS ON P. 599

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$
$= (60 + 20 - 80) + (130 - 30)$
$= 0 + 100$
$= 100$ (Actual answer = 92)

b $78 \times 7 \approx 80 \times 7$
$= 560$ (Actual answer = 546)

c $510 \div 24 \approx 500 \div 20$
$= 50 \div 2$
$= 25$ (Actual answer = 21.25)

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$
b $55 + 34 - 22 - 46 + 136$
c $684 + 903$
d $35 + 81 + 110 + 22 + 7$
e $517 - 96$
f $210 - 38 - 71 + 151 - 49$
g $766 - 353$
h 367×2
i 83×81
j 984×16
k $828 \div 3$
l $507 \div 7$

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3 Study each example involving decimals.

a $20.91 - 11.3 + 2.5 \approx 21 - 11 + 3$

$= 13$ (Exact answer = 12.11)

b $4.78 \times 19.2 \approx 5 \times 20$

$= 100$ (Exact answer = 91.776)

c $\frac{37.6+9.3}{41.2-12.7} \approx \frac{38+9}{40-13}$

$= \frac{47}{27}$

$\approx \frac{50}{30}$

≈ 1.6 (Exact answer = 1.645…)

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$

b $14.807 + 6.6 - 7.22$

c 18.47×9.61

d 4.27×97.6

e $\frac{11.07+18.4}{12.2}$

f $\frac{38.18}{17.2-9.6}$

g $54.75 - 18.6 - 14.4$

h $\frac{18.46 \times 4.9}{39.72-15.2}$

i $62.13 \div 10.7$

j $(4.89)^2$

4.05 Surface area of composite solids

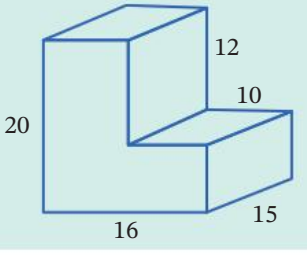
When calculating surface areas of composite solids, remember **not** to include the areas common to both solids.

WS
A page of prisms and cylinders

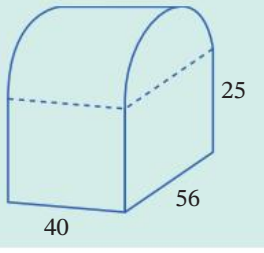
Example 12

Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres.

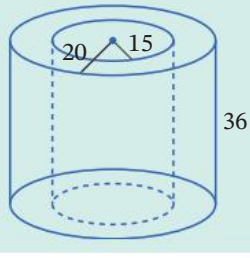
a



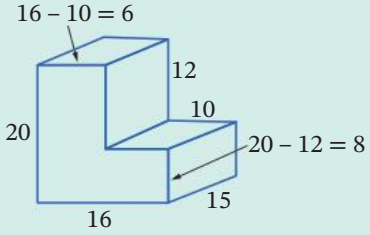
b



c



Solution



a This prism has 8 faces: 2 identical L-shapes (front and back) and 6 different rectangles.

Area of L-shape $= 16 \times 20 - 10 \times 12$

$= 200 \text{ cm}^2$

Surface area $= (2 \times 200) + (6 \times 15) + (12 \times 15) + (10 \times 15) + (8 \times 15) + (16 \times 15) + (20 \times 15)$

$= 1480 \text{ cm}^2$

Note that the 6 rectangles can also be thought of as one long rectangle of width 15 cm.

Length of long rectangle = perimeter of L = 72

Surface area $= (2 \times 200) + (72 \times 15)$

$= 1480 \text{ cm}^2$

b The solid is made up of a half-cylinder (3 faces) and a rectangular prism (5 faces).

Surface area of half-cylinder
= 2 semi-circular ends + curved surface area

$= 2 \times \frac{1}{2} \times \pi \times 28^2 + \frac{1}{2} \times 2 \times \pi \times 28 \times 40$

$= 5981.5924... \text{ cm}^2$

Radius of semi-circle $= \frac{1}{2} \times 56$

$= 28$

Do not round this partial answer, or the final answer will be inaccurate.

Surface area of rectangular prism $= (2 \times 40 \times 25) + (2 \times 56 \times 25) + (40 \times 56)$

$= 7040 \text{ cm}^2$

Total surface area $= 5981.5924... + 7040$

$= 13\,021.5924...$

$\approx 13\,021.6 \text{ cm}^2$

c The hollow cylinder is made up of 2 annulus (ring) faces, an outside curved surface area and an inside curved surface area.

Surface area of annulus faces $= 2 \times (\pi \times 20^2 - \pi \times 15^2)$ $\quad 2 \times$ area between 2 circles

$= 1099.5574... \text{ cm}^2$

Outside curved surface area $= 2 \times \pi \times 20 \times 36$

$= 4523.8334... \text{ cm}^2$

Inside curved surface area $= 2 \times \pi \times 15 \times 36$

$= 3392.9200... \text{ cm}^2$

Total surface area $= 1099.5574... + 4523.8934... + 3392.9200...$

$= 9016.3709...$

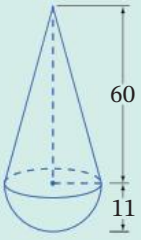
$= 9016.4 \text{ cm}^2$

STAGE 5.3

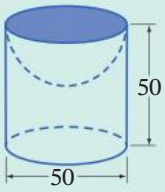
Example 13

Find, correct to the nearest square centimetre, the surface area of each solid.
All measurements are in centimetres.

a



b



Solution

a Surface area = curved surface area of cone + curved surface area of hemisphere

$$= \pi rl + \frac{1}{2} \times 4\pi r^2$$

$r = 11$, $h = 60$ and $l = ?$

$l^2 = 11^2 + 60^2$

$= 3721$

$l = \sqrt{3721}$

$= 61$ cm

Surface area $= \pi \times 11 \times 61 + \frac{1}{2} \times 4 \times \pi \times 11^2$

$= 2868.2740 \ldots$

≈ 2868 cm^2

b Surface area = curved surface of cylinder + circular base + curved surface of hemisphere

$= 2\pi rh + \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$= 2\pi rh + 3\pi r^2$ $\qquad \pi r^2 + 2\pi r^2 = 3\pi r^2$

$= 2 \times \pi \times 25 \times 50 + 3 \times \pi \times 25^2$ $\qquad r = \frac{1}{2} \times 50 = 25$

$= 13\,744.4678 \ldots$

$\approx 13\,744$ cm^2

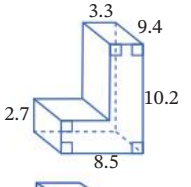
EXERCISE 4.05 ANSWERS ON P. 599

Surface areas of composite solids U F PS R

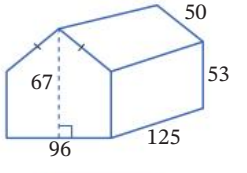
1 Find the surface area of each solid. All measurements are in centimetres. PS R

EXAMPLE 12

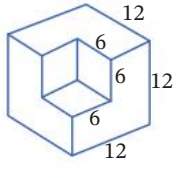
a



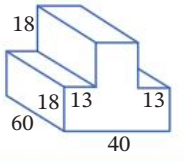
b



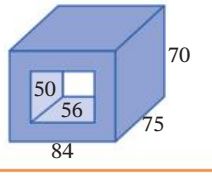
c



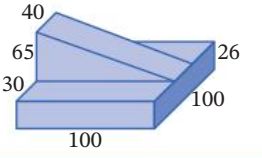
d



e



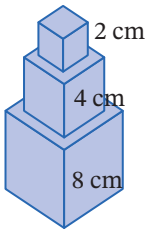
f



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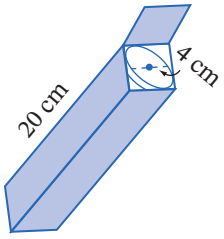
2 3 cubes of length 2 cm, 4 cm and 8 cm are glued on top of each other. Calculate the surface area of the new solid. PS R



3 Circular cracker biscuits of diameter 4 cm are packed in a cardboard box of length 20 cm.

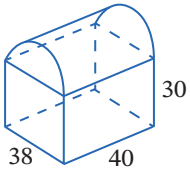
a Calculate the surface area of the box.

b How much cardboard would be saved if the biscuits were packed into a cylindrical box?

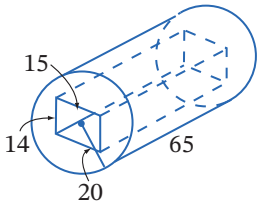


4 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres. PS R

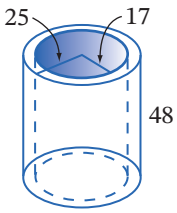
a



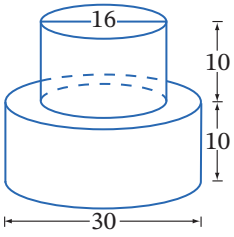
b



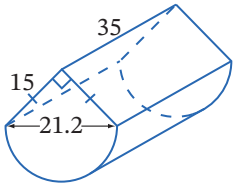
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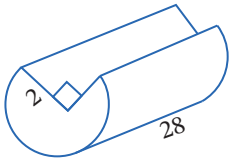
d



e



f

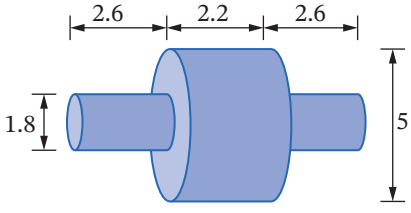


5 Calculate the surface area of the solid, correct to one decimal place. All measurements are in centimetres. R

Select the correct answer **A**, **B**, **C** or **D**.

A 86.0 m^2 **B** 103.2 m^2

C 108.3 m^2 **D** 113.4 m^2



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STAGE 5.3

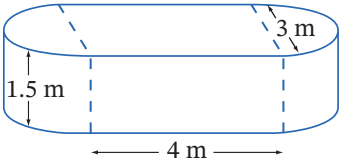
6 A wedding cake with 3 tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total visible surface area, correct to the nearest cm². PS R



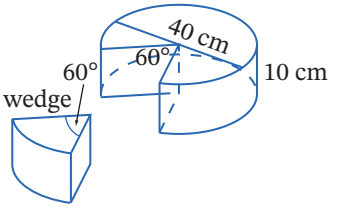
Shutterstock.com/Brian Barney

7 **a** Find, correct to 2 decimal places, the total external area of the wall of this above-ground swimming pool.

b Calculate the area of the water surface, correct to the nearest m². PS R



8 A wedge of cheese is cut from a cylindrical block of height 10 cm and diameter 40 cm. Find the total surface area of the wedge, correct to 2 decimal places. PS R

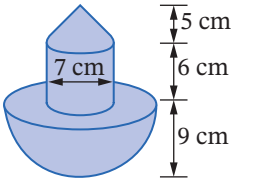


EXAMPLE 13

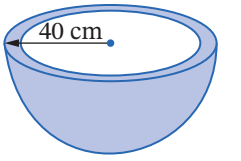
9 Calculate the surface area of the solid. Select **A**, **B**, **C** or **D**. PS R

A 923.98 cm² **B** 962.46 cm²

C 736.62 cm² **D** 669.51 cm²



10 Calculate the total surface of a hemispherical bird bath that is 1 cm thick and has an outer radius of 40 cm. PS R

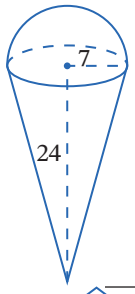


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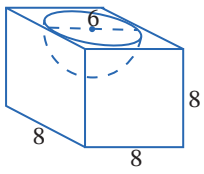
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11 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres. **PS** **R**

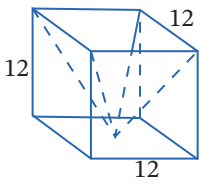
a



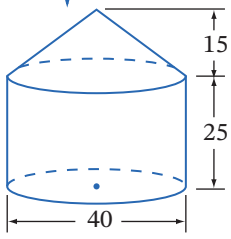
b



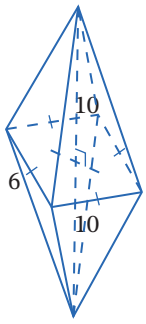
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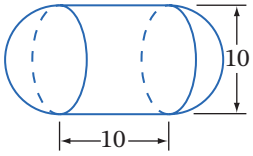
d



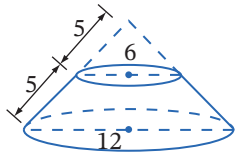
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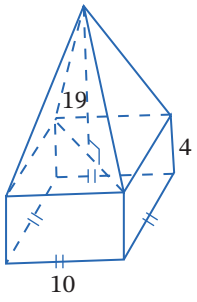
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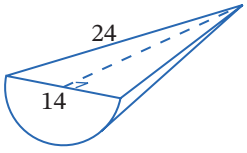
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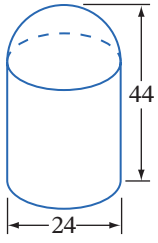
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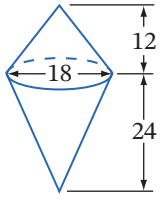
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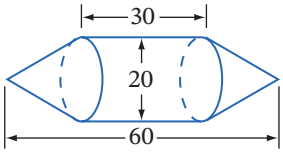
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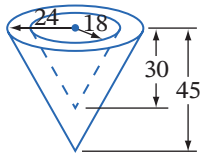
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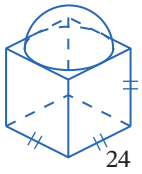
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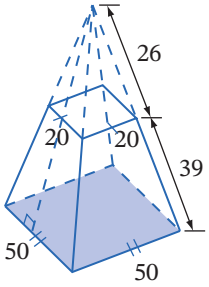
m



n



o



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4.06 Volumes of prisms and cylinders

Volumes of prisms and cylinders

A page of prisms and cylinders

Volumes of solids

Back-to-front problems

Volume and capacity

Biggest volume

Volume of a box

Volume of a box

Formula matching game

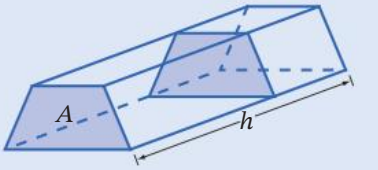
Volumes of shapes

Volume of a prism

$V = Ah$

where A = area of base

h = perpendicular height

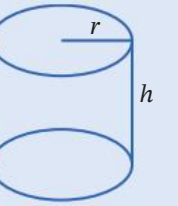


Volume of a cylinder

$V = \pi r^2 h$

where r = radius of circular base

h = perpendicular height



Volume and capacity

1 cm^3 contains 1 mL

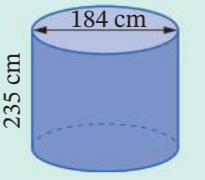
1 m^3 contains 1000 L or 1 kL

Example 14

For this cylinder, calculate:

a its volume correct to the nearest cm^3

b its capacity in kL, correct to 1 decimal place.



Solution

a Radius $= \frac{1}{2} \times 184$

$= 92$ cm

$V = \pi \times 92^2 \times 235$

$= 6\,248\,753.451...$

$\approx 6\,248\,753 \text{ cm}^3$

b Capacity $= 6\,248\,753$ mL

$= 6\,248\,753 \div 1000 \div 1000$ kL

$= 6.248\,753$ kL

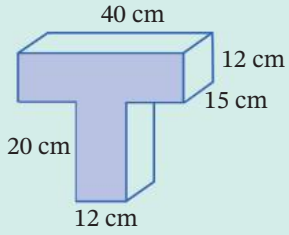
≈ 6.2 kL

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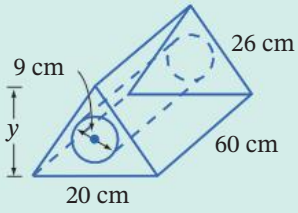
Example 15

Find, correct to the nearest whole number, the volume of each solid.

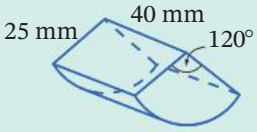
a



b



c



Solution

a $A = 40 \times 12 + 20 \times 12$

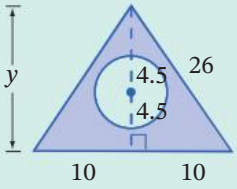
$= 720 \text{ cm}^2$

$V = Ah$

$= 720 \times 15$

$= 10\,800 \text{ cm}^3$

b Cross-section is a triangle minus a circle.



$26^2 = y^2 + 10^2$

$y^2 = 26^2 - 10^2$

$= 576$

$y = 24 \text{ cm}$

Radius of circle $= \frac{1}{2} \times 9 = 4.5$

$A = \frac{1}{2} \times 20 \times 24 - \pi \times 4.5^2$

$= 176.3827... \text{ cm}^2$

$V = Ah$

$= 176.3827 ... \times 60$

$= 10\,582.9649...$

$\approx 10\,583 \text{ cm}^3$

c $A = \frac{120}{360} \times \pi \times 25^2$

$= 654.498... \text{ mm}^2$

$V = Ah$

$= 654.498... \times 40$

$= 26\,179.938...$

$\approx 26\,180 \text{ mm}^2$

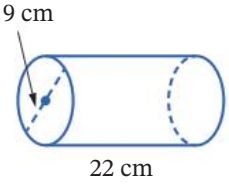
EXERCISE 4.06 ANSWERS ON P. 599

Volumes of prisms and cylinders U F R

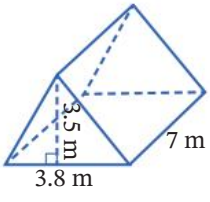
1 Calculate, correct to one decimal place, the volume of each solid.

EXAMPLE 14

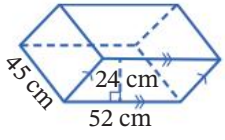
a



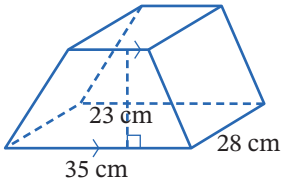
b



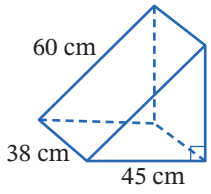
c



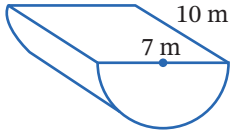
d



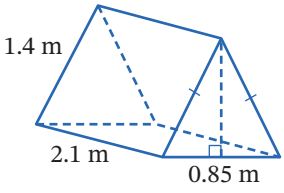
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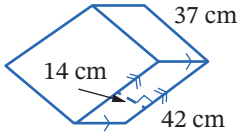
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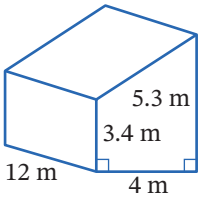
g



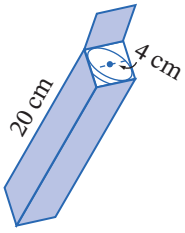
h



i

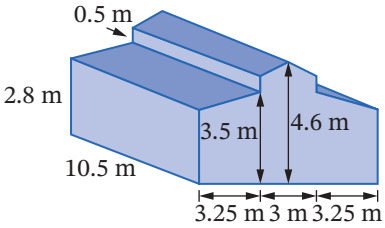


2 Rice crackers of diameter 4 cm are packed in a cardboard box of height 20 cm. Calculate, correct to one decimal place:

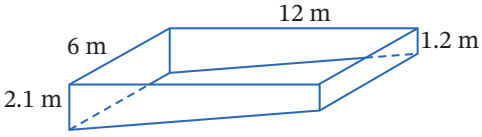


- **a** the volume of the crackers in the box
- **b** the volume of the box
- **c** the percentage of the box that is empty space.

3 Calculate, correct to one decimal place, the volume of the shed. R



4 This swimming pool is 12 m long and 6 m wide. The depth of the pool ranges from 1.2 m to 2.1 m.

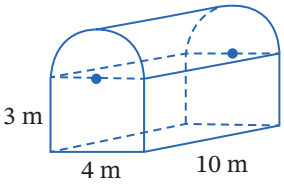


- **a** Calculate the capacity of this pool in litres.
- **b** If the pool is filled so that the water is 8 cm from the top of the pool, calculate the amount of water in the pool (to the nearest litre).

5 An Olympic sized swimming pool is 50 m long, 25 m wide and 2 m deep. What is the capacity of an Olympic pool in litres?

6 **a** Find, correct to 2 decimal places, the volume of this greenhouse.

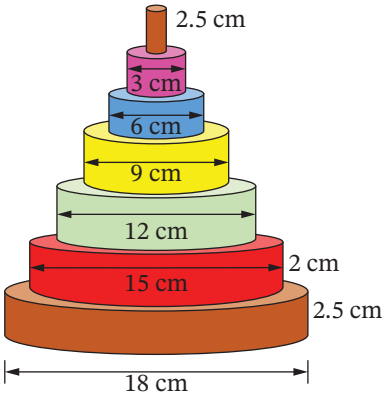
b If this greenhouse costs 0.5c per m^3 per hour to heat, how much is this per day (correct to the nearest cent)?



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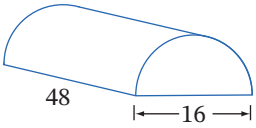
7 A baby's toy involves placing 5 coloured cylinders on a wooden stick (diameter 1 cm) on a base (diameter 18 cm). The cylinders are of varying diameters with a height of 2 cm. Find the volume of the toy, including the base and peg, correct to one decimal place. R



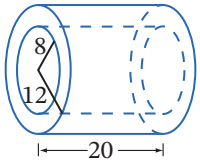
EXAMPLE 15

8 Find, correct to 2 decimal places where appropriate, the volume of each solid. All lengths shown are in centimetres. R

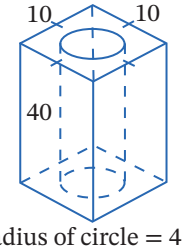
a



b

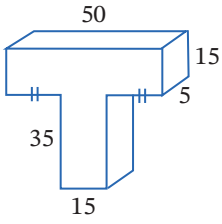


c

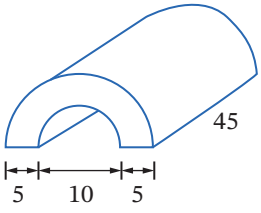


radius of circle = 4 cm

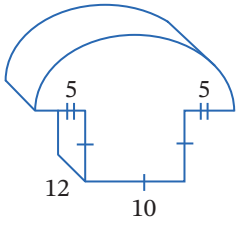
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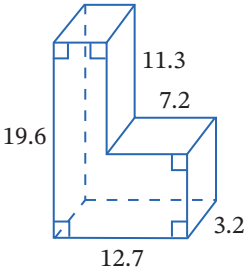
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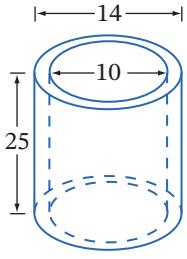
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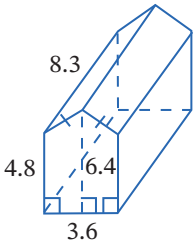
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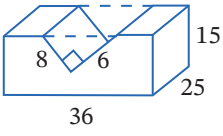
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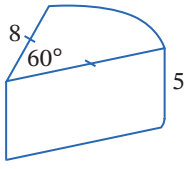
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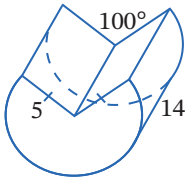
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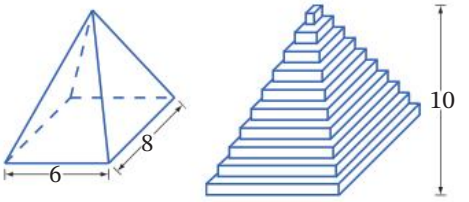


STAGE 5.3

Technology

Approximating the volume of a pyramid

In this activity, we will use a spreadsheet to approximate the volume of a rectangular pyramid by slicing it into tiny layers of rectangular prisms of equal thickness.



Let $L = 6$ be the length of the prism, $W = 8$ be the width and $H = 10$ be the height.

The thickness, T, of each layer is given by the formula $T = \dfrac{H}{\text{number of layers}}$.

Starting at the bottom, the length and width of each layer are decreased by the amounts $\dfrac{L}{\text{number of layers}}$ and $\dfrac{W}{\text{number of layers}}$ with each step.

1 Set up your spreadsheet as shown.

	A	B	C	D	E	F
1						
2			Number of layers =			
3	H	L	W	Thickness of layers	Volume of layer	Sum of volumes
4	10	6	8	=$A$4/$D$2	=B4*C4*D4	=E4
5		=B4-$B$4/$D$2	=$C4-$C$4/$D$2			=E5+F4
6						
7						

2 Let the number of layers be 10. Enter 10 in cell D2.

3 Copy each formula down to row 13.

4 Explain the results in cells E13 and F13.

5 How accurate was your result in F13? Explain.

6 Print out your spreadsheet and paste it into your book.

7 Enter 40 (layers) in cell D2 and copy each formula down to row 43.

8 In 1–2 sentences compare your result in F43 with the previous result in F13 from question 4.

9 Enter each value in cell D2 and copy down the formulas as requested.

a 100 (copy down to row 103)

b 200 (copy down to row 203)

c 400 (copy down to row 403)

10 Use the formula $V = \frac{1}{3}Ah$ to calculate the exact volume of the pyramid.

11 Write a brief report about your results in questions **9** and **10**.

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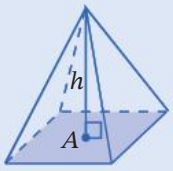
Volumes of pyramids, cones and spheres 4.07

Volume of a pyramid

$$V = \frac{1}{3}Ah$$

where A = area of the base

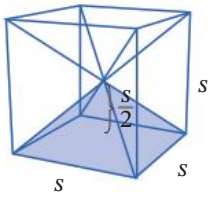
h = perpendicular height.



STAGE 5.3

The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height.

This fact is difficult to prove mathematically, but here's a diagram that might help. This is a square pyramid with dimensions s, s and $\frac{s}{2}$, with an identical pyramid balancing on top of it upside-down. Now look at the big cube that surrounds these 2 square pyramids.

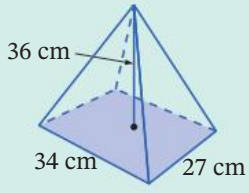


- Can you see how many such pyramids fit into this big cube exactly?
- What is the volume of the big cube?
- So what is the volume of one pyramid?
- Is this the same as $V = \frac{1}{3}Ah$?

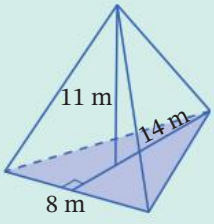
Example 16

Find the volume of each pyramid.

a



b



Solution

a

$A = 27 \times 34$

$= 918$

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 918 \times 36$

$= 11\,016 \text{ cm}^3$

b

$A = \frac{1}{2} \times 8 \times 14$

$= 56$

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 56 \times 11$

$= 205\frac{1}{3} \text{ m}^3$

STAGE 5.3

Example 17

Find the volume of a square pyramid with base length 22 mm and slant height 61 mm.

Solution

First find h, the perpendicular height of the pyramid.

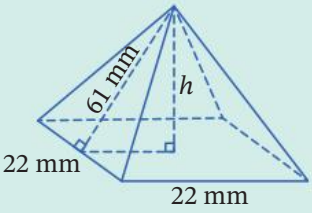
$h^2 = 61^2 - 11^2 = 3600$

$h = 60$ mm

$A = 22^2 = 484$

$V = \frac{1}{3} \times 484 \times 60$

$= 9680$ mm^3



Volume of a cone

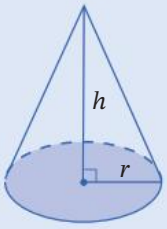
Approximating the volume of a cone

Surface area and volume of a cone

$$V = \frac{1}{3}\pi r^2 h$$

where r = radius of the base

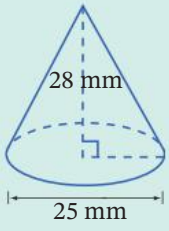
h = perpendicular height.



A cone is like a 'circular pyramid' so $V = \frac{1}{3}Ah = \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3}\pi r^2 h$

Example 18

Find, correct to the nearest cubic centimetre, the volume of this cone.



Solution

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12.5^2 \times 28 \quad r = \frac{1}{2} \times 25 = 12.5$

$= 4581.4892...$

≈ 4581 cm^3

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Example 19

A cone has a slant edge of 50 mm and a base radius of 15 mm.
Find its volume correct to 2 significant figures.

Solution

$h^2 = 50^2 - 15^2$

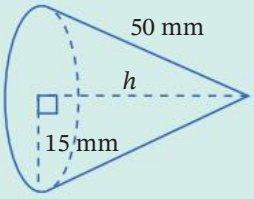
$= 2275$

$h = \sqrt{2275}$

$V = \frac{1}{3} \times \pi \times 15^2 \times \sqrt{2275}$

$= 11\,238.3314...$

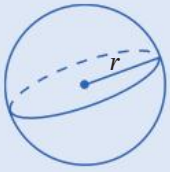
$\approx 11\,000 \text{ mm}^3$



Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

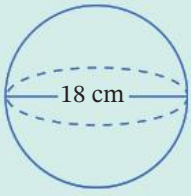
where r = radius of the sphere.



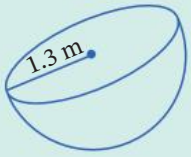
Example 20

Find, correct to 2 significant figures, the volume of each solid.

a



b



Solution

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 9^3$ $\quad r = \frac{1}{2} \times 18 = 9$

$= 3053.6280...$

$\approx 3100 \text{ cm}^3$

b $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 1.3^3$

$= 4.6013...$

$\approx 4.6 \text{ m}^3$

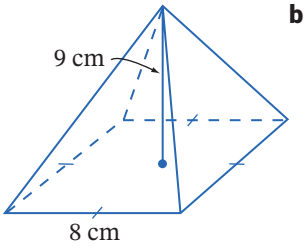
EXERCISE 4.07 ANSWERS ON P. 599

Volumes of pyramids, cones and spheres U F PS R C

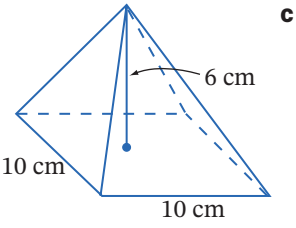
EXAMPLE 16

1 Find, correct to one decimal place, the volume of each pyramid.

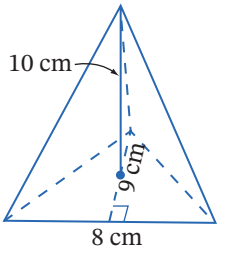
a



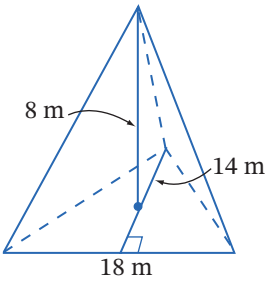
b



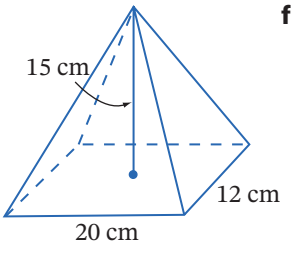
c



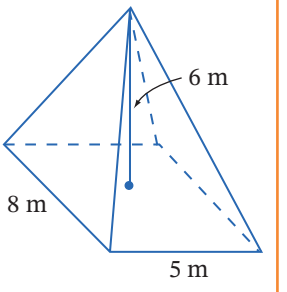
d



e



f

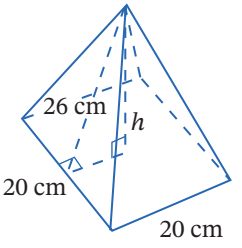


EXAMPLE 17

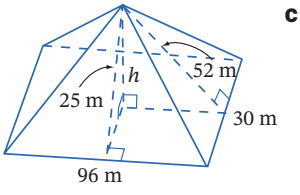
2 For each pyramid, find correct to one decimal place: R

i its perpendicular height **ii** its volume

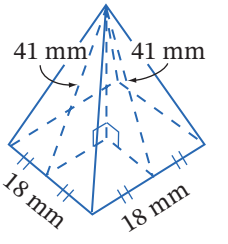
a



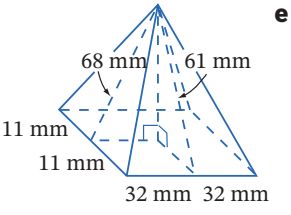
b



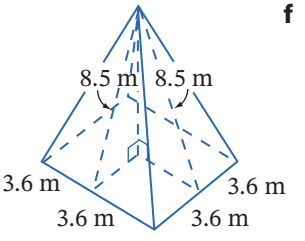
c



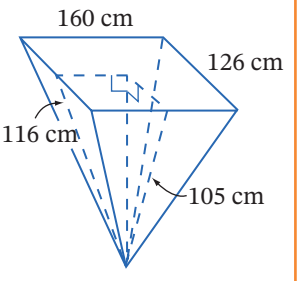
d



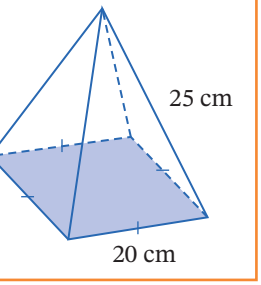
e



f



3 Calculate the volume of this square pyramid. Select the closest answer **A**, **B**, **C** or **D**. R



A 3055.1 cm^3 **B** 2000 cm^3

C 3333.3 cm^3 **D** 2748.7 cm^3

Foundation Standard Complex

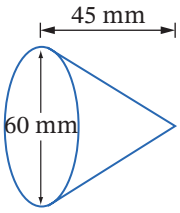
9780170453509

EXAMPLE 18

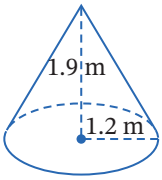
4.07

4 Find, correct to the nearest whole number, the volume of each cone.

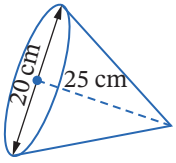
a



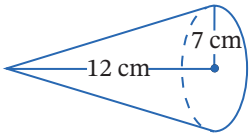
b



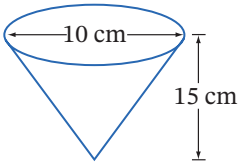
c



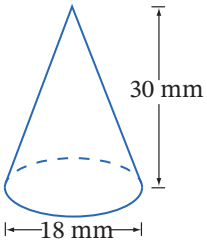
d



e



f

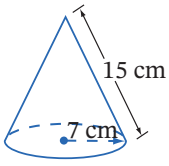


EXAMPLE 19

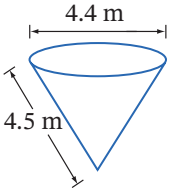
5 For each cone, find correct to one decimal place: R

i its perpendicular height **ii** its volume

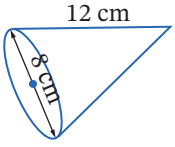
a



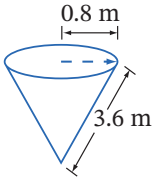
b



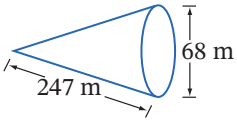
c



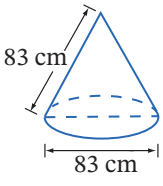
d



e



f



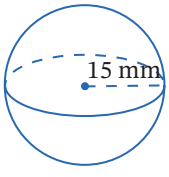
6 Find the volume of a cone with a base diameter of 5.4 cm and a slant height of 9.8 cm. Select **A**, **B**, **C** or **D**. R

A 75 cm^3 **B** 72 cm^3 **C** 288 cm^3 **D** 111.1 cm^3

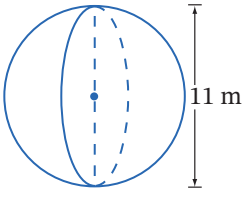
EXAMPLE 20

7 Find the volume of each sphere or hemisphere, correct to one decimal place.

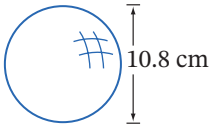
a



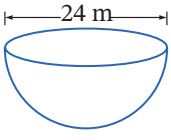
b



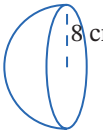
c



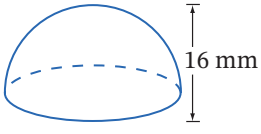
d



e



f

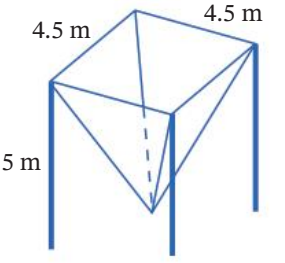


Foundation Standard Complex

STAGE 5.3

8 The Earth has a radius of approximately 6378 km. Calculate its volume in scientific notation, correct to 3 significant figures. C

9 A grain hopper is in the shape of a square pyramid. PS R C



a Find the volume of grain it holds when full.

b If there are 750 kg of wheat per m^3, find the mass of grain in the hopper when it is filled to three-quarters of capacity. Give your answer correct to the nearest tonne.

10 A pyramid has a volume of 360 m^3 and a base area of 48 m^2. Calculate its perpendicular height. R

11 A square pyramid has a volume of 800 cm^3 and a perpendicular height of 12 cm. Calculate, correct to one decimal place, the length of its base. R

12 A cone has a volume of 600 m^3 and a base radius of 10 m. Calculate, correct to 1 decimal place, its perpendicular height. R

13 A cone has a volume of 160 cm^3 and a perpendicular height of 20 cm. Calculate, correct to one decimal place, its radius. R

14 The volume of a spherical light is 2664 cm^3. Calculate its diameter. Select **A**, **B**, **C** or **D**. R

A 17.2 cm **B** 8.6 cm **C** 17.0 cm **D** 8.5 cm

4.08 Volumes of composite solids

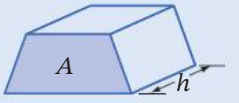
STAGE 5.3

WS Volume and capacity

Volume formulas

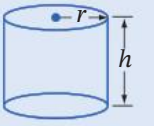
Prism

$V = Ah$



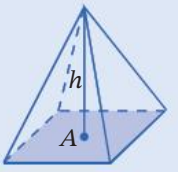
Cylinder

$V = \pi r^2 h$



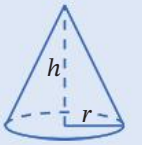
Pyramid

$V = \frac{1}{3}Ah$



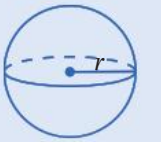
Cone

$V = \frac{1}{3}\pi r^2 h$



Sphere

$V = \frac{4}{3}\pi r^3$



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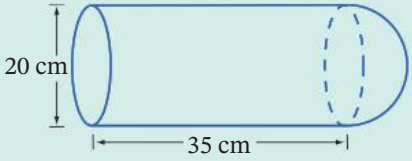
Note that the formulas for **surface area** involve **2 dimensions,** for example, r^2 or rh, while the formulas for **volume** involve **3 dimensions,** for example, r^2h or r^3 or Ah.

Example 21

For this packing capsule, find correct to the nearest whole number:

a its volume

b its capacity in litres



Solution

a Volume $= \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \pi \times 10^2 \times 35 + \frac{2}{3} \times \pi \times 10^3$

$= 13\,089.9693...$

$\approx 13\,090 \text{ cm}^3$

b Capacity $\approx 13\,090$ mL

$= 13.09$ L

≈ 13 L

EXERCISE 4.08 ANSWERS ON P. 599

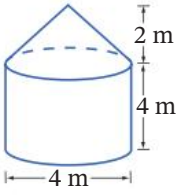
Volumes of composite solids U F PS R

EXAMPLE 21

1 This storage tank shown is completely filled with water.

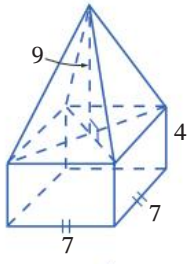
a Calculate to the nearest cubic metre the volume of the tank.

b Find the capacity of the tank, correct to the nearest kilolitre.

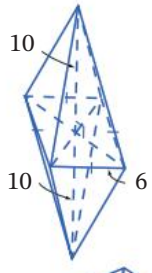


2 Find the volume of each solid. All measurements are centimetres. R

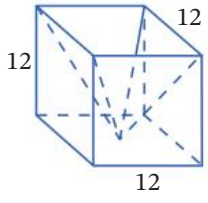
a



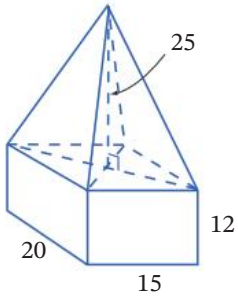
b



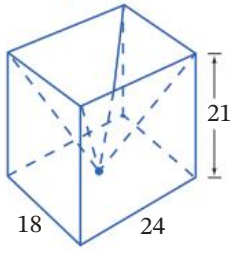
c



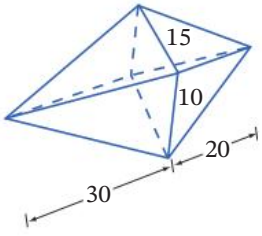
d



e



f



Foundation | Standard | Complex

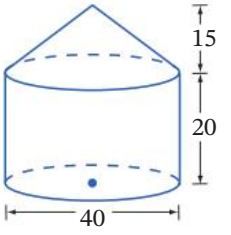
STAGE 5.3

3 For each solid, find:

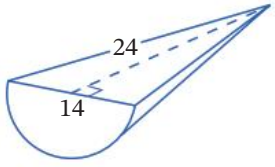
- **i** the volume (to the nearest cm^3)
- **ii** the capacity (in litres, correct to 3 decimal places).

All measurements are in centimetres.

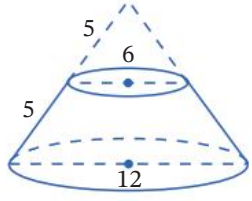
a



b

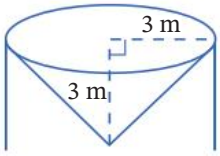


c

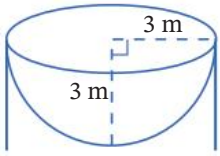


4 A conical tank (A) and a hemispherical tank (B) have measurements as shown. How much more does tank B hold compared to tank A? Answer correct to 2 decimal places.

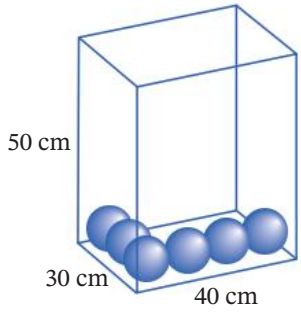
A



B

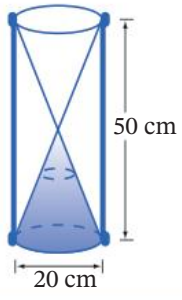


5 Spherical balls of diameter 10 cm are stacked inside a box as shown. PS R



- **a** How many balls will fit in the bottom layer?
- **b** If the balls are stacked in the same manner as the bottom layer until the box is full, how many balls will fit in the box?
- **c** Calculate the volume of the space occupied by the balls when the box is full.
- **d** What percentage of the box is empty space? Give your answer to the nearest whole percentage.

6 The sand in this hourglass takes up $\frac{3}{4}$ of the volume of the bottom cone. PS R



- **a** Calculate, correct to the nearest cubic centimetre, the volume of sand in the hourglass.
- **b** If the sand takes one hour to fall from the top cone to the bottom cone, at what rate is it falling? Give your answer in cm^3/s, correct to 2 significant figures.

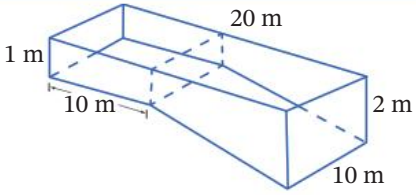
Foundation Standard Complex

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7 **a** Calculate the volume of this swimming pool. PS R

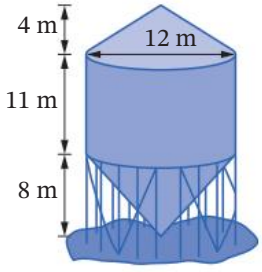
b Calculate the capacity of the pool if it is filled to a depth 20 cm from the top.

c If water costs $1.98/kL, find the cost of filling the pool.



8 **a** Find the capacity of the silo shown, correct to the nearest m^3. PS R

b How many tonnes of barley grain will the silo hold if the density of barley grain is 600 kg/m^3?



STAGE 5.3

4.08

Areas of similar figures 4.09

Areas of similar figures

STAGE 5.3

- If the matching sides of 2 similar figures are in the ratio $1 : k$, then their areas are in the ratio $1 : k^2$.
- If the matching sides are in the ratio $m : n$, then their areas are in the ratio $m^2 : n^2$.

$$A_1 : A_2 = m^2 : n^2 \quad \text{or} \quad \frac{A_1}{A_2} = \frac{m^2}{n^2}$$

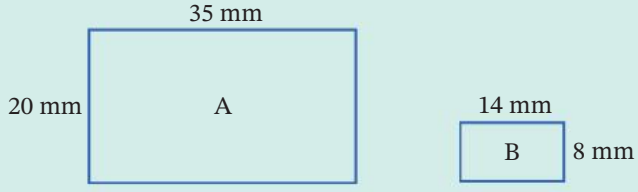
Areas of similar figures

Areas of similar shapes

Areas of similar shapes

Example 22

What is the ratio of the areas of the similar rectangles shown?



Solution

Ratio of matching sides (A to B) $= 35 : 14$
$= 5 : 2$

Ratio of areas $= 5^2 : 2^2$
$= 25 : 4$

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STAGE 5.3

Example 23

2 similar pentagons have areas in the ratio 144 : 169. Find the ratio of the lengths of their matching sides.

Solution

Ratio of areas $= m^2 : n^2 = 144 : 169$

$\therefore$ Ratio of sides $= m : n = \sqrt{144} : \sqrt{169} = 12 : 13$

Example 24

2 similar triangles have matching sides in the ratio 3 : 5. If the area of the larger triangle is 225 cm^2, find the area of the smaller triangle.

Solution

Let the area of the smaller figure be A.

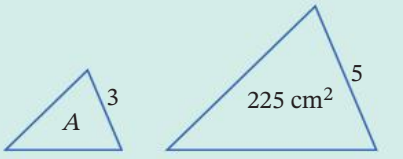
Ratio of matching sides $= 3 : 5$

Ratio of areas $= 3^2 : 5^2 = 9 : 25$

$\therefore \frac{A}{225} = \frac{9}{25}$

$A = \frac{9}{25} \times 225 = 81$ cm^2

The area of the smaller triangle is 81 cm^2.



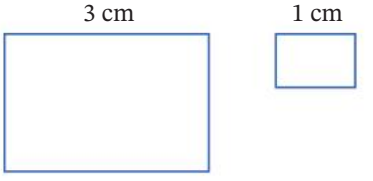
EXERCISE 4.09 ANSWERS ON P. 599

Areas of similar figures U F R C

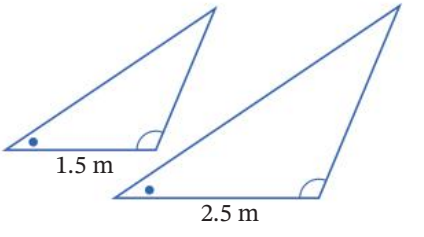
EXAMPLE 22

1 For each pair of similar figures, find the ratio of their areas.

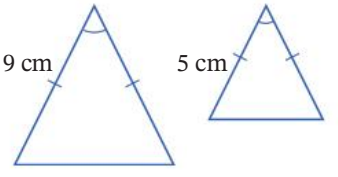
a



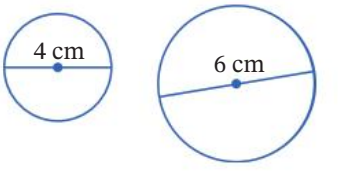
b



c



d



EXAMPLE 23

2 For each ratio of the areas of 2 similar figures, find the ratio of the lengths of their matching sides.

a 9 : 25 **b** 1 : 100 **c** 64 : 25 **d** 16 : 81

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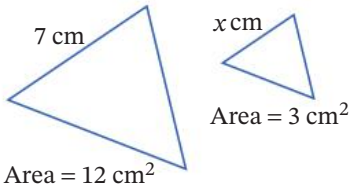
STAGE 5.3

EXAMPLE 24

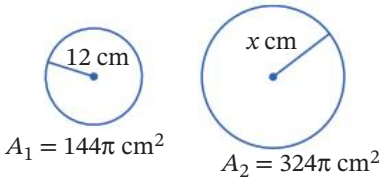
4.09

3 Find x for each pair of similar figures.

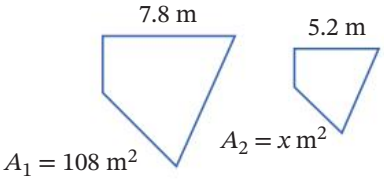
a



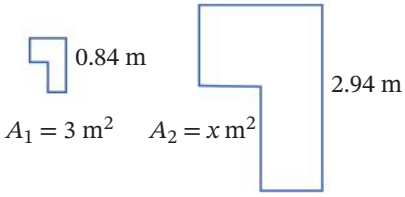
b



c



d



4 2 circles have radii in the ratio 3 : 5. If the larger area is 150 cm², find the area of the smaller circle. R

5 2 squares have sides in the ratio 7 : 4. If the area of the smaller square is 14.4 cm², find the area of the larger square. R

6 2 similar triangles have areas in the ratio 4 : 9. If the length of the base of the smaller triangle is 5 cm, find the length of the base of the larger triangle. R

7 2 similar rectangles have their areas in the ratio 36 : 121. If the width of the smaller rectangle is 84 cm, find the width of the larger rectangle.

8 If the radius of a circle is doubled, how has its area changed? R C

9 If the area of a square is divided by 9, how has its side length changed? R C

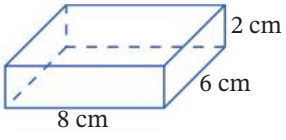
10 If the sides of a triangle are increased by 2.5, how has its area changed? R C

11 If the area of a trapezium is decreased by $\frac{1}{100}$, how have its sides changed? R C

Investigation

Surface areas and volumes of similar solids

1 **a** Calculate the volume of this rectangular prism.



b Calculate the surface area of the rectangular prism.

c If the length, width and height are all doubled, what happens to:

i the volume? **ii** the surface area?

d Copy and complete:

If the length, width and height are all doubled, the volume is increased _____ times and the surface area is increased _____ times.

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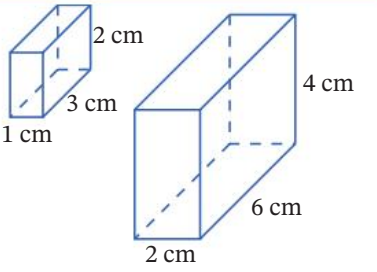
STAGE 5.3

2 **a** Explain why these rectangular prisms are similar solids.

b What is the ratio of their matching sides?

c What is the ratio of their surface areas?

d What is the ratio of their volumes?

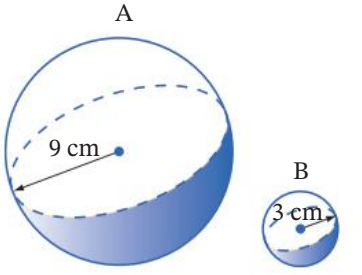


3 For the spheres A and B, find the ratio of:

a their radii

b their surface areas

c their volumes



4 How is the ratio of surface areas of similar solids related to the ratio of matching sides?

5 How is the ratio of volumes of similar solids related to the ratio of their matching sides?

4.10 Surface area and volume of similar solids#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

WS Areas and volumes of similar figures

Surface areas and volumes of similar solids

- If the matching sides of 2 similar solids are in the ratio $1 : k$, then their surface areas are in the ratio $1 : k^2$ and their volumes are in the ratio $1 : k^3$.
- If the matching sides are in the ratio $m : n$, then their surface areas are in the ratio $m^2 : n^2$ and their volumes are in the ratio $m^3 : n^3$.

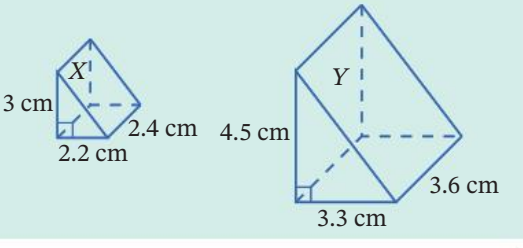
$$\frac{SA_1}{SA_2} = \frac{m^2}{n^2} \text{ and } \frac{V_1}{V_2} = \frac{m^3}{n^3}$$

Example 25

For these 2 similar triangular prisms, find the ratio of their:

a surface areas

b volumes



Solution

a Ratio of sides (X to Y) $= 3 : 4.5$ (or $2.2 : 3.3$ or $2.4 : 3.6$)

$= 2 : 3$

Ratio of surface areas $= 2^2 : 3^2$

$= 4 : 9$

b Ratio of volumes $= 2^3 : 3^3$

$= 8 : 27$

Example 26

2 similar cylinders have their surface areas in the ratio 25 : 36. If the volume of the smaller cylinder is 250 cm^3, find the volume of the larger solid.

Solution

Ratio of surface areas $= 25 : 36$

$\therefore$ Ratio of matching sides $= \sqrt{25} : \sqrt{36}$

$= 5 : 6$

$\therefore$ Ratio of volumes $= 5^3 : 6^3$

$= 125 : 216$

Let the volume of the larger cylinder be V.

$$\frac{V}{250} = \frac{216}{125}$$

$$V = \frac{216}{125} \times 250$$

$$= 432$$

$\therefore$ Volume of the larger cylinder is 432 cm^3.

EXERCISE 4.10 ANSWERS ON P. 599

Surface areas and volumes of similar solids U F R

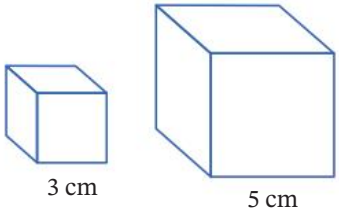
EXAMPLE 25

1 For each pair of similar solids, find the ratio of:

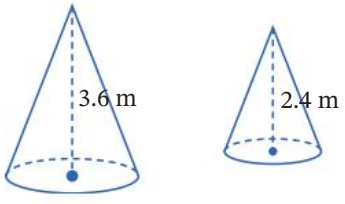
i the smaller surface area to larger surface area

ii the larger volume to the smaller volume

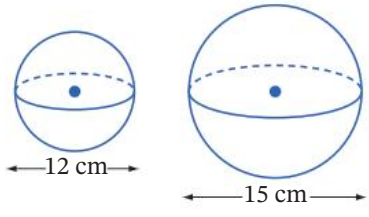
a



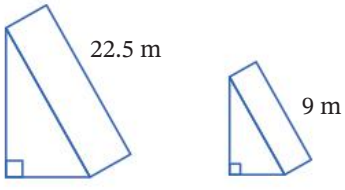
b



c



d



EXAMPLE 25

2 2 similar pyramids have surface areas of 81 cm^2 and 100 cm^2. Find the ratio of their:

a matching side lengths **b** volumes

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STAGE 5.3

3 2 similar prisms have volumes of 125 cm^3 and 343 cm^3. Find the ratio of their:

a matching sides **b** surface areas

4 Blocks of chocolate are sold in the shape of similar triangular prisms. The areas of the triangular faces of two prisms are 6400 mm^2 and 1600 mm^2. If the volume of the smaller prism is 9600 mm^3, find the volume of the larger prism. R

5 2 drink cans have the shapes of similar cylinders. The larger can is 15 cm high and contains 350 mL of drink. If the smaller can is 9 cm high, how much drink does it contain? R

6 A box of washing powder is 12 cm tall and contains 750 g of washing powder. A similar box is 18 cm tall. How much washing powder does it contain? R

7 A large fish tank has a capacity of 624 L. A smaller similar fish tank has half the length, width and depth of the large tank. Find the capacity of the smaller tank. R

8 A cylinder has its height and radius increased 1.5 times. By what factor has its: R

a surface area increased? **b** volume increased?

9 A balloon has a radius of 8 cm. By what factor is the volume decreased if the radius changes to 6 cm? R

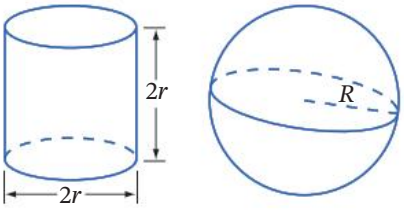
Power plus ANSWERS ON P. 600

1 A square prism and square pyramid have the same base and the same surface area.

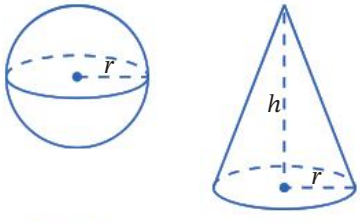
Show that the slant height, l, of the pyramid is $l = \frac{5}{2}s$, where s is the length of the base.

2 A cylinder with diameter and height $2r$ has the same surface area as a sphere of radius R.

Show that $R = \sqrt{\frac{3}{2}}r$.



3 A sphere and a cone have the same radius and volume. Show that the cone's height, h, is 4 times the radius, r.



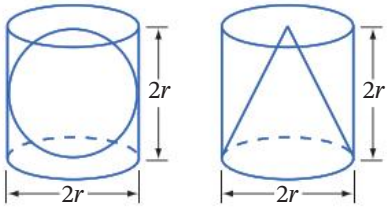
4 A sphere and a cone fit inside identical cylinders with the same base diameter and height.

a Find the ratio

Volume of cone: Volume of sphere : Volume of cylinder.

b Show that

Volume of cone + Volume of sphere = Volume of cylinder.



Foundation Standard Complex

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CHAPTER 4 REVIEW

Language of maths

Qz Area and volume

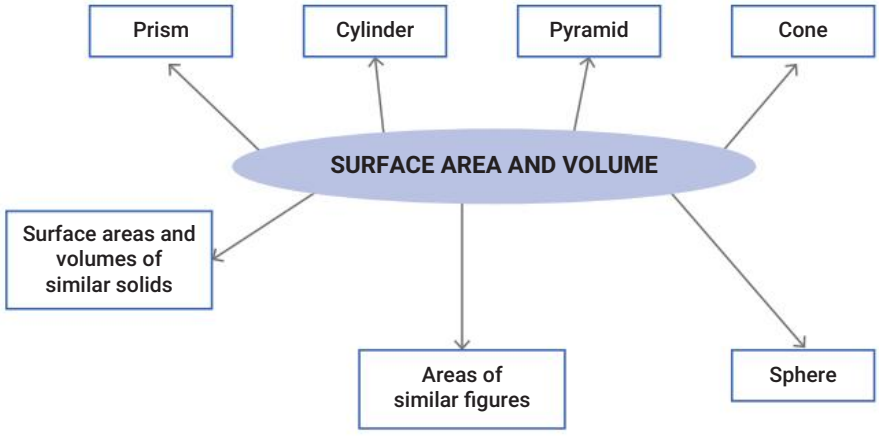
apex	base	capacity	cone
cross-section	cubic	curved surface	cylinder
diameter	hemisphere	kilolitre	litre
perpendicular height	prism	pyramid	radius
ratio	sector	similar figures	similar solids
slant height	sphere	surface area	volume

1. Which word means a 'slice' of a prism or cylinder?
2. Name 3 solids that have a **curved surface area**.
3. What is the formula for the curved surface area of a cone?
4. Explain the difference between the **perpendicular height** and the **slant height** of a pyramid.
5. What is the formula $V = \frac{1}{3}\pi r^2 h$ used for?
6. Describe the relationship between the volumes of similar solids.

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS Mind map: Surface area and volume (Advanced)

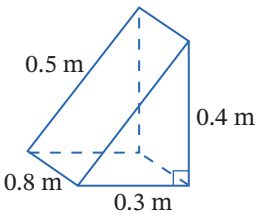


TEST YOURSELF 4 ANSWERS ON P. 600

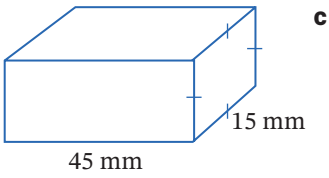
4.01

1 Find the surface area of each prism.

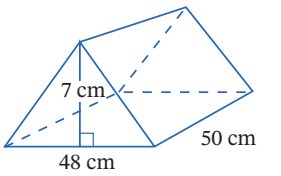
a



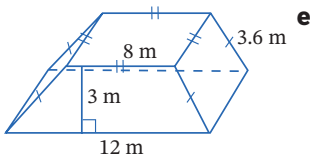
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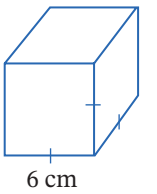
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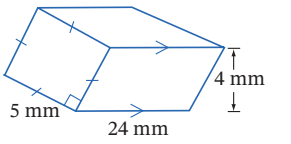
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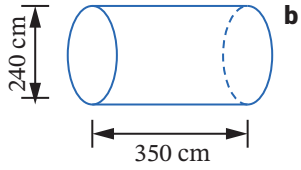
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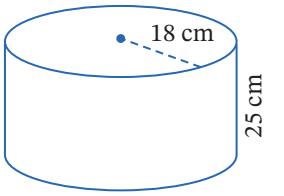
4.02

2 Calculate, correct to one decimal place, the surface area of each solid.

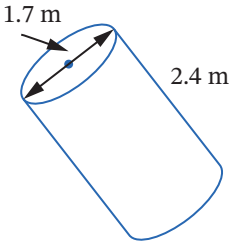
a



b

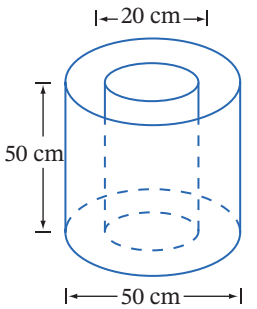


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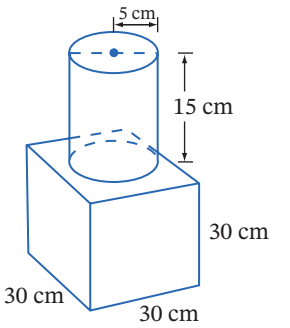


Cylinder, open at one end

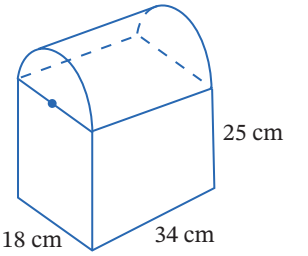
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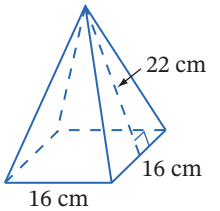
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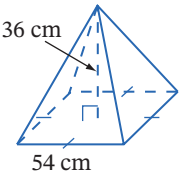
4.03

3 Find the surface area of each pyramid.

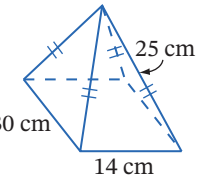
a



b



c

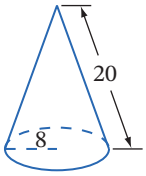


Foundation | Standard | Complex

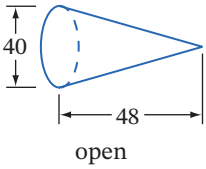
9780170453509

4 Find the surface area of each solid. All measurements are in metres. 4.04

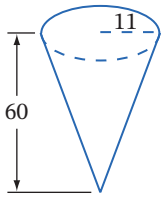
a



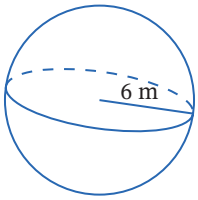
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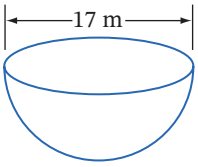
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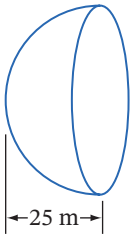
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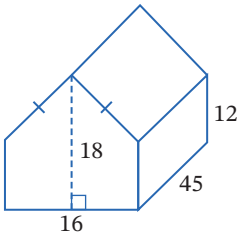


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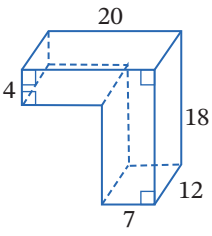


5 Find, correct to the nearest square centimetre, the surface area of each solid. All measurements are in centimetres. 4.05

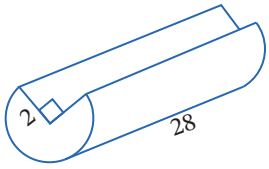
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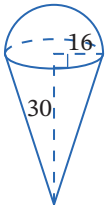
b



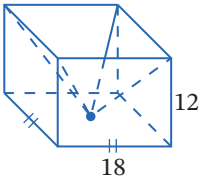
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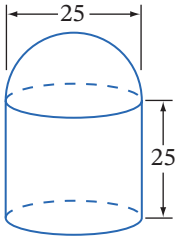
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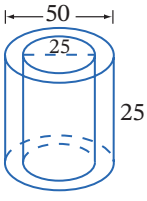


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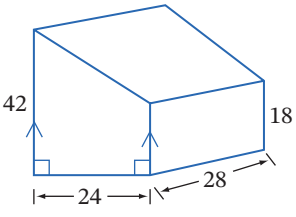


6 Calculate, correct to nearest cubic metre, the volume of each solid. All measurements are in metres. 4.05

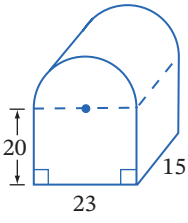
a



b



c



Foundation | Standard | Complex

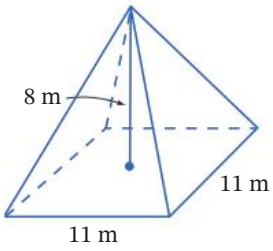
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STAGE 5.3

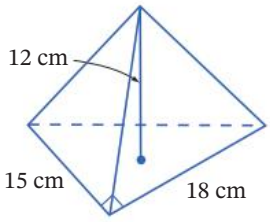
4.05

7 Find, correct to 2 decimal places (where necessary), the volume of each solid.

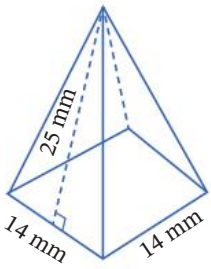
a



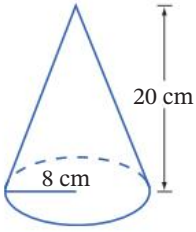
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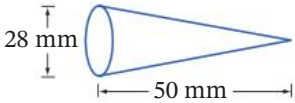
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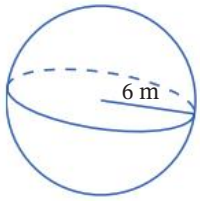
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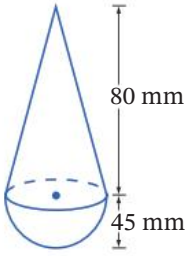
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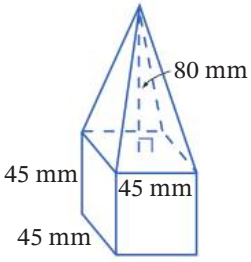
4.08

8 Find, correct to the nearest whole number, the volume of each solid.

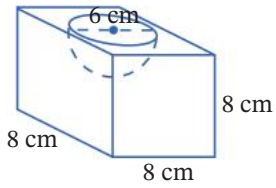
a



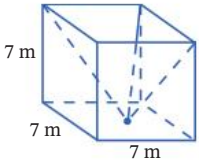
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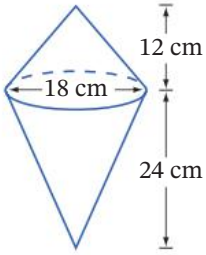
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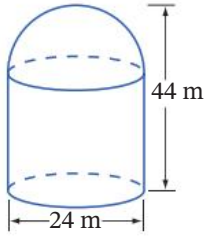
d



e



f



4.09

9 **a** 2 circles have radii in the ratio 4 : 5. If the smaller area is 150 cm^2, find the area of the larger circle.

b The radius of a circle is increased by a factor of $2\frac{1}{2}$. By what factor has the area increased?

4.10

10 **a** The areas of the bases of 2 similar rectangular prisms are in the ratio 25 : 64. If the volume of the larger prism is 1024 cm^2, find the volume of the smaller prism.

b 2 similar pyramids have volumes of 216 cm^3 and 343 cm^3. Find the ratio of their surface areas.

Foundation Standard Complex

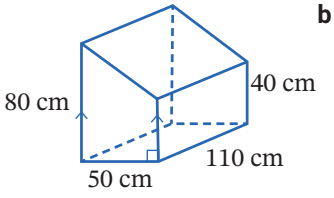
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PRACTICE SET 1

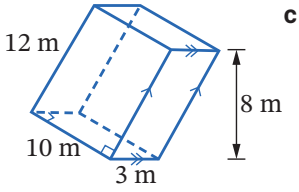
ANSWERS ON P. 600

1 Find the surface area of each prism. 4.01

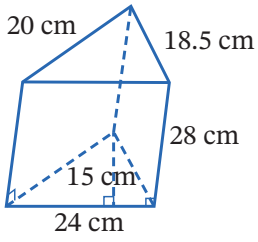
a



b



c



2 Find, correct to the nearest degree, the angle of inclination of a line with gradient $-\frac{3}{4}$. STAGE 5.3 3.01

3 Naved is paid 4 weeks' normal pay plus a 17.5% annual leave loading for his 4-week holiday. If Naved's salary is $78 580, find his: 2.01

a normal weekly pay **b** leave loading

c total pay for the 4-week holiday.

4 A call centre operator is employed under the following award: 2.01

Normal rate: $28.75 per hour	
Normal rate	For 0 to 36 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 20 hours **b** 39 hours

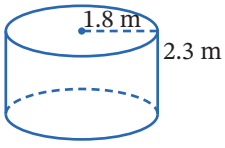
c 45 hours

5 A line passes through the points $H(5, -3)$ and $K(8, -7)$. Calculate the gradient of the line: 3.02

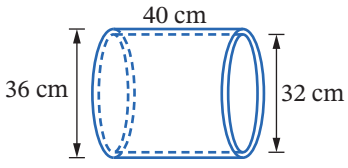
a parallel to HK **b** perpendicular to HK

6 Calculate, correct to one decimal place, the surface area of each solid. 4.02

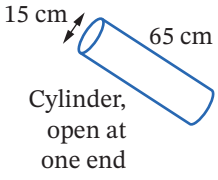
a



b



c



7 Simplify each expression. STAGE 5.3 1.03

a $\sqrt{200}+\sqrt{18}$ **b** $7\sqrt{44}-6\sqrt{99}$

c $3\sqrt{6}+\sqrt{54}-2\sqrt{27}$ **d** $5\sqrt{72}-3\sqrt{45}+3\sqrt{50}$

8 Nikita earns a weekly wage of $1485. She has annual deductions of $1756 for a health fund and $3560 for work expenses. 2.02

a Calculate Nikita's taxable income.

b Use the tax table on page 31 to calculate the income tax Nikita should pay.

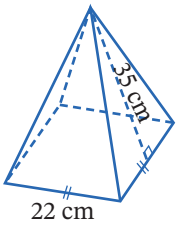
Foundation Standard Complex

3.03 **9** Graph the linear equations $y = 3x - 2$ and $y = -2x + 3$ on a number plane. Where do the lines intersect?

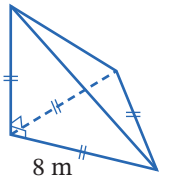
STAGE 5.3

4.03 **10** Find the surface area of each pyramid.

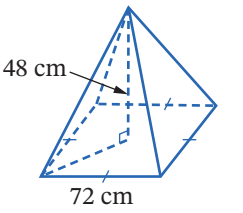
a



b

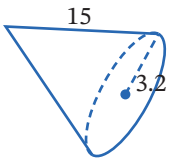


c

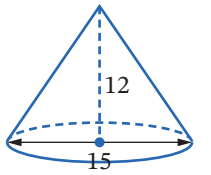


4.04 **11** Find, correct to the nearest square metre, the surface area of each solid. All measurements are metres.

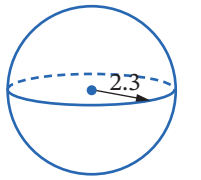
a



b



c



3.06 **12** Find the equation of the line passing through the point $(-8, 3)$ with gradient $-\frac{2}{3}$.

3.03 **13** Which of the following points lie on the line of $2x + y = 3$?

Select the correct answer **A**, **B**, **C** or **D**.

A $(1, 0)$ **B** $(2, -1)$ **C** $(-1, -1)$ **D** $(-1, -5)$

2.05 **14** Calculate the value of an investment if \$46 000 is invested at 4.2% p.a. for 3 years with interest compounded:

a annually **b** quarterly **c** monthly

3.05 **15** Write the linear equation $y = \frac{3}{4}x + 6$ in general form $ax + by + c = 0$.

3.05 **16** Find the gradient and y-intercept of the line with equation $5x - 2y + 3 = 0$.

2.06 **17** Jovan purchases a home theatre system valued at \$7680. He pays a 10% deposit and repays the balance in 48 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find:

a the deposit paid **b** the balance owing

c the interest charged **d** the total amount to be repaid

e the amount of each instalment **f** the total price Jovan paid for the system

2.07 **18** Annieke purchases a new car for \$39 990, which depreciates by 10% p.a.

a Find the depreciated value of the car after 4 years.

b What is the depreciation over this time?

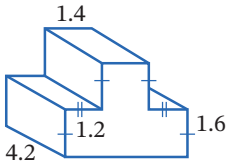
c Express the depreciated value as a percentage of the cost price (to one decimal place).

Foundation Standard Complex

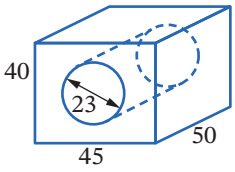
9780170453509

19 Calculate, to nearest cubic metre, the volume of each solid. All measurements are in metres. 4.06

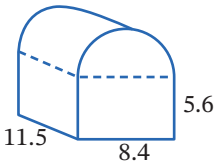
a



b



c



20 The interest earned on \$8500 over 3 years is \$624.75. Calculate the flat rate of interest p.a. 2.03

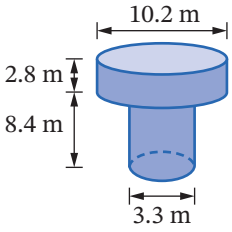
21 Find, in general form, the equation of a line which passes through the points $(3, -2)$ and $(-1, 3)$. STAGE 5.3

22 The line $2x + 5y + 7 = 0$ and another line intersect at right angles at the point $(4, -5)$. 3.06

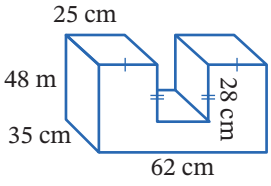
Find the equation of the other line. 3.08

23 Find the surface area of each solid, correct to 1 decimal place. 4.05

a



b



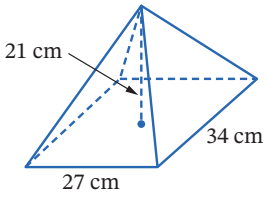
24 Rationalise the denominator of each surd. 1.06

a $\frac{1}{\sqrt{10}}$ **b** $\frac{5\sqrt{2}}{2\sqrt{3}}$ **c** $\frac{4+\sqrt{5}}{3\sqrt{5}}$ **d** $\frac{2\sqrt{6}-\sqrt{3}}{3\sqrt{3}}$

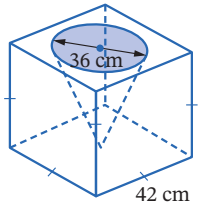
25 Show that the points $P(-2, -1)$, $Q(3, 2)$, $R(0, 7)$ and $S(-5, 4)$ are the vertices of a square. 3.09

26 Find, correct to one decimal place, the volume of each solid. 4.07

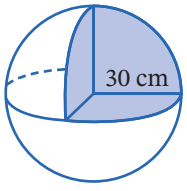
a



b



c



27 Find the equation of the line passing through the pair of points $(7, 3)$ and $(2, -5)$. 3.06

28 2 similar squares have sides in the ratio 3 : 5. If the smaller area is 360 cm^2, find the area of the larger square. 4.09

29 Expand and simplify each expression. 1.05

a $\left(5\sqrt{10}-4\sqrt{2}\right)\left(5\sqrt{10}+4\sqrt{2}\right)$ **b** $\left(\sqrt{6}-7\right)^2$ **c** $\left(2\sqrt{3}+4\sqrt{5}\right)\left(3\sqrt{5}-\sqrt{3}\right)$

30 2 similar containers have volumes 600 cm^3 and 2025 cm^3 respectively. If the height of the larger container is 60 cm, what is the height of the smaller container? 4.10

Foundation Standard Complex



5

NUMBER AND ALGEBRA

PRODUCTS AND FACTORS

In 825 CE, the Persian mathematician al-Khwarizmi used the Arabic word ‘al-jabr’ to describe the process of adding equal quantities to both sides of an equation. When al-Khwarizmi’s book was translated into Latin and introduced to Europe, ‘al-jabr’ became ‘algebra’ and the word was adopted as the name for the branch of mathematics that uses formulas to describe patterns and relationships in our world. The photo above shows a monument of al-Khwarizmi located in Khiva, Uzbekistan.



Chapter outline

	Working mathematically				
5.01 The index laws	U	F		R	C
5.02 Fractional indices*	U	F		R	C
5.03 Adding and subtracting algebraic fractions	U	F			
5.04 Multiplying and dividing algebraic fractions	U	F			
5.05 Expanding and factorising expressions	U	F			
5.06 Expanding binomial products	U	F		R	C
5.07 Factorising special binomial products*	U	F			
5.08 Factorising quadratic expressions $x^2 + bx + c$	U	F		R	
5.09 Factorising quadratic expressions $ax^2 + bx + c$*	U	F		R	
5.10 Mixed factorisations*	U	F		R	
5.11 Factorising algebraic fractions*	U	F		R	

***STAGE 5.3**

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$.

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor; factorising is the opposite of expanding, for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

index laws Rules for simplifying algebraic expressions involving powers of the same base, for example, $a^m \div a^n = a^{m-n}$.

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$

quadratic expression An algebraic expression in which the highest power of the variable is 2, for example, $2x^2 + 5x - 3$ or $x^2 + 2$

quadratic trinomial An algebraic expression that consists of 3 terms, for example, $x^2 + 2x + 6$

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- apply index laws to algebraic expressions with integer indices
- (STAGE 5.3) apply index laws to algebraic expressions with fractional indices
- add, subtract, multiply and divide simple algebraic fractions
- (STAGE 5.3) add and subtract algebraic fractions with binomial numerators
- expand and factorise algebraic expressions involving terms with indices
- expand binomial products and factorise (monic) quadratic expressions of the form $x^2 + bx + c$
- (STAGE 5.3) recognise and expand special binomial products that are perfect squares or the difference of 2 squares
- (STAGE 5.3) factorise an expression with 4 terms by grouping in pairs
- (STAGE 5.3) factorise (non-monic) quadratic expressions of the form $ax^2 + bx + c$, including those that are perfect squares or the difference of 2 squares
- (STAGE 5.3) factorise and simplify algebraic fractions

SkillCheck ANSWERS ON P. 600

1 Simplify each expression.

a $g^4 \times g^5$ **b** $r^8 \div r^2$ **c** $(d^5)^3$ **d** $(-k)^2$

e $h \times h^9$ **f** $m^5 \div m$ **g** a^1 **h** c^0

i $3e^2 \times 2e^5$ **j** $18n^6 \div 6n^2$ **k** $(10w^3)^3$ **l** $25q^0$

m $(vw)^5$ **n** $\left(\frac{m}{n}\right)^3$ **o** y^{-1} **p** p^{-2}

2 Simplify each expression.

a $\frac{a}{5} + \frac{3a}{4}$ **b** $\frac{7p}{2} - \frac{10p}{3}$ **c** $\frac{8}{t} \times \frac{5t}{24}$ **d** $\frac{x}{14} \div \frac{y}{2}$

5.01 The index laws



Index laws review


Index laws

The index laws

$a^m \times a^n = a^{m+n}$ $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{m \times n}$

$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $a^0 = 1$

$a^{-n} = \frac{1}{a^n}$ $a^{-1} = \frac{1}{a}$ $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Example 1

Simplify each expression.

a $\dfrac{24e^6n^{12}}{8en^4}$ **b** $(4d^4q^2r)^3$ **c** $\left(\dfrac{2c^2}{d}\right)^4$

d $31x^0 - (31x)^0$ **e** $9x^{-1}$ **f** $(4q)^{-3}$

Solution

a $\dfrac{24e^6n^{12}}{8en^4} = \dfrac{{}^{3}\cancel{24}e^6n^{12}}{{}_{1}\cancel{8}en^4}$

$= 3e^5n^8$

b $(4d^4q^2r)^3 = 4^3d^{4\times3}q^{2\times3}r^3$

$= 64d^{12}q^6r^3$

c $\left(\dfrac{2c^2}{d}\right)^4 = \dfrac{\left(2c^2\right)^4}{d^4}$

$= \dfrac{2^4c^{2\times4}}{d^4}$

$= \dfrac{16c^8}{d^4}$

d $31x^0 - (31x)^0 = 31 \times 1 - 1$

$= 30$

e $9x^{-1} = 9 \times \dfrac{1}{x}$

$= \dfrac{9}{x}$

f $(4q)^{-3} = \dfrac{1}{(4q)^3}$

$= \dfrac{1}{64q^3}$

Example 2

Simplify:

a $\left(-1\frac{2}{3}\right)^{-5}$ **b** $\left(\dfrac{2}{3r}\right)^{-3}$

Solution

a $\left(-1\frac{2}{3}\right)^{-5} = \left(-\frac{5}{3}\right)^{-5}$

$= \left(-\frac{3}{5}\right)^{5}$

$= -\frac{243}{3125}$

b $\left(\dfrac{2}{3r}\right)^{-3} = \left(\dfrac{3r}{2}\right)^{3}$

$= \dfrac{27r^3}{8}$

EXERCISE 5.01 ANSWERS ON P. 600

The index laws U F R C

1 Simplify each expression. R C

a $8m^5 \times 3m^9$ **b** $30k^{10} \div 25k^2$ **c** $(-y^2)^5$

d $(3q^3)^2$ **e** $32a^7d^4 \div 8ad^3$ **f** $3x^6y^8 \times 7xy^4$

g $5e^4g^3 \times (-e^6g)$ **h** $\dfrac{27w^3y^8}{-9w^2y}$ **i** $(4c^7)^3$

j $(-2p)^5$ **k** $\dfrac{45hk^5m^3}{30hk^3m^2}$ **l** $9u^3vw^2 \times 6uv^2w^8$

2 Simplify each expression. R C

a $(l^3m^5)^6$ **b** $\left(\frac{n}{2}\right)^3$ **c** $\left(\frac{w^4}{m^3}\right)^5$ **d** $\left(\frac{5}{2}\right)^0$

e $(-8ky^5)^2$ **f** $(3p^2q^3r^4)^4$ **g** $5x^0 - (7x)^0$ **h** $(-5d^3y^5)^3$

i $\left(\frac{2b}{3d}\right)^4$ **j** $-9(a^2b^3)^0$ **k** $12h^0$ **l** $\left(-\frac{3k^4}{10}\right)^3$

3 Evaluate each expression. R C

a 7^0 **b** 5×4^0 **c** $(-4)^3$ **d** $(2^3)^2$

e $6^5 \div 6^3$ **f** $(-3^2)^0$ **g** $8^2 \div 8^3$ **h** $5^4 \times 5^{-3}$

i $\left(\frac{3}{5}\right)^0$ **j** $7^5 \div 7^3$ **k** $12^2 \div 12^2$ **l** $3^5 \times 3^{-5}$

m $15^0 \div 15^1$ **n** $(4^{-1})^2$ **o** $(2^3)^0 + (2^0)^3$ **p** $(5^0)^3 \times 5^2$

4 Find the value of $(3x)^0 + 3 \times 2^0$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** 1 **C** 7 **D** 4

5 Express each term as a fraction. R C

a 2^{-3} **b** 10^{-6} **c** 8^{-1} **d** 6^{-2} **e** 3^{-4}

6 Simplify each expression using a positive index. R C

a 5^{-4} **b** 8^{-2} **c** w^{-6} **d** m^{-5}

e $4y^{-3}$ **f** $(ad)^{-2}$ **g** $(2x)^{-5}$ **h** $3mp^{-2}$

i $10g^{-2}h^{-3}$ **j** $(4u)^{-4}$ **k** $2a^5q^{-2}$ **l** $4p^2x^{-3}$

m $10a^{-5}c^3$ **n** $(-3r^6)^{-3}$ **o** $\frac{3}{4}q^{-2}p^{-1}$ **p** $4(5k^3)^{-3}$

7 Simplify $2c^{-3}$. Select **A**, **B**, **C** or **D**.

A $-6c$ **B** $\frac{1}{2c^3}$ **C** $\frac{2}{c^3}$ **D** $\frac{1}{8c^3}$

8 Simplify each expression using positive indices. R C

a $\left(\frac{5}{2d}\right)^{-2}$ **b** $5y^{-2}$ **c** $(5m)^{-3}$ **d** $x^2y^3w^{-2}$

e $\left(\frac{4y^3}{3}\right)^{-2}$ **f** $\left(\frac{a}{7k}\right)^{-1}$ **g** $8(3ab^2)^{-2}$ **h** $\left(\frac{6c}{5k^2}\right)^{-1}$

EXAMPLE 2

9 Simplify each expression. R C

a $\left(\frac{1}{4}\right)^{-2}$ **b** $\left(\frac{5}{3}\right)^{-3}$ **c** $\left(-1\frac{2}{3}\right)^{-4}$ **d** $\left(2\frac{3}{5}\right)^{-2}$

e $\left(\frac{4}{m}\right)^{-3}$ **f** $\left(\frac{3a}{7}\right)^{-2}$ **g** $\left(-\frac{2}{y}\right)^{-5}$ **h** $\left(\frac{8q^2}{5}\right)^{-2}$

i $\left(-\frac{4}{w^3}\right)^{-3}$ **j** $\left(\frac{k^2}{d^5}\right)^{-2}$ **k** $\left(-\frac{10g^4}{3d^3}\right)^{-5}$ **l** $\left(\frac{7h^6}{5}\right)^{-4}$

10 Simplify each expression. R C

a $(10x^{10}y)^3 \times 5x^{-2}y^3$

b $(20k^7m^2 \div 5k^{-4}m)^4$

c $(2q^{-3}r^3)^4 \div 8(qr^2)^2$

d $8w^{-5}m^3 \div (6wm^2)^2$

e $\left(\frac{4a^5x^6}{a^6x^2}\right)^4$

f $\left(\frac{3g^7n^6}{2g^5n^2}\right)^{-2}$

g $(4p^{-3}h^4)^3 \times (-2p^6h^9)$

h $(8d^7n^5)^2 \div (-2d^3n^2)^3$

i $(-2c^3e \times 5c^4e^5)^3$

Fractional indices 5.02

STAGE 5.3

Indices squaresaw (Advanced)

Exponential equations

Fractional indices

$a^{\frac{1}{2}} = \sqrt{a}$ Any number raised to the power of $\frac{1}{2}$ is the square root of that number.

$a^{\frac{1}{3}} = \sqrt[3]{a}$ Any number raised to the power of $\frac{1}{3}$ is the cube root of that number.

$a^{\frac{1}{n}} = \sqrt[n]{a}$ Any number raised to the power of $\frac{1}{n}$ is the nth root of that number.

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$ Any number raised to the power of $\frac{m}{n}$ is the nth root of that number raised to the power of m.

Note: Taking the root first often makes the calculation simpler.

On a calculator, the nth root key is $\sqrt{\square}$ or $\sqrt[x]{\square}$ found by pressing the SHIFT or 2nd F key before pressing $x^{\square}$ or y^x respectively.

Example 3

Evaluate each expression.

a $961^{\frac{1}{2}}$ **b** $1331^{\frac{1}{3}}$ **c** $6561^{\frac{1}{8}}$

Solution

a $961^{\frac{1}{2}} = \sqrt{961}$
$= 31$

b $1331^{\frac{1}{3}} = \sqrt[3]{1331}$
$= 11$

c $6561^{\frac{1}{8}} = \sqrt[8]{6561}$
$= 3$

On a calculator: 8 $\sqrt{\square}$ 6561 =
because $3^8 = 6561$

Foundation Standard Complex

STAGE 5.3



Fractional indices

Example 4

Evaluate each expression.

a $32^{\frac{3}{5}}$ **b** $27^{-\frac{1}{3}}$ **c** $25^{-\frac{5}{2}}$

Solution

a $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3$

$= 2^3$

$= 8$

b $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{27}}$

$= \frac{1}{3}$

c $25^{-\frac{5}{2}} = \frac{1}{25^{\frac{5}{2}}}$

$= \frac{1}{\left(\sqrt{25}\right)^5}$

$= \frac{1}{5^5}$

$= \frac{1}{3125}$

Example 5

Write each expression using a fractional index.

a $\sqrt{w^9}$ **b** $\sqrt[4]{m^5}$ **c** $\frac{1}{\sqrt[3]{y^8}}$

Solution

a $\sqrt{w^9} = \left(w^9\right)^{\frac{1}{2}}$

$= w^{\frac{9}{2}}$

b $\sqrt[4]{m^5} = \left(m^5\right)^{\frac{1}{4}}$

$= m^{\frac{5}{4}}$

c $\frac{1}{\sqrt[3]{y^8}} = \frac{1}{\left(y^8\right)^{\frac{1}{3}}}$

$= \frac{1}{y^{\frac{8}{3}}}$ or $y^{-\frac{8}{3}}$

Example 6

Simplify each expression.

a $\sqrt[4]{\left(81a^2\right)^5}$ **b** $\left(64k^6\right)^{-\frac{5}{3}}$ **c** $4m^{\frac{3}{4}}n^2 \times \sqrt[3]{8m^2n}$

Solution

a $\sqrt[4]{\left(81a^2\right)^5} = \left(81a^2\right)^{\frac{5}{4}}$

$= 81^{\frac{5}{4}} a^{2\times\frac{5}{4}}$

$= 243a^{\frac{5}{2}}$

b $\left(64k^6\right)^{-\frac{5}{3}} = \frac{1}{\left(64k^6\right)^{\frac{5}{3}}}$

$= \frac{1}{64^{\frac{5}{3}} k^{6\times\frac{5}{3}}}$

$= \frac{1}{1024k^{10}}$

c $4m^{\frac{3}{4}}n^2 \times \sqrt[3]{8m^2n} = 4m^{\frac{3}{4}}n^2 \times \left(8m^2n\right)^{\frac{1}{3}}$

$= 4m^{\frac{3}{4}}n^2 \times 8^{\frac{1}{3}} m^{2\times\frac{1}{3}} n^{\frac{1}{3}}$

$= 4m^{\frac{3}{4}}n^2 \times 2m^{\frac{2}{3}} n^{\frac{1}{3}}$

$= 8m^{\frac{3}{4}+\frac{2}{3}} n^{2+\frac{1}{3}}$

$= 8m^{\frac{17}{12}} n^{\frac{7}{3}}$

EXERCISE 5.02 ANSWERS ON P. 601

Fractional indices U F R C

EXAMPLE 3

1 Evaluate each expression. R

a $64^{\frac{1}{2}}$ **b** $-27^{\frac{1}{3}}$ **c** $625^{\frac{1}{2}}$ **d** $1000^{\frac{1}{3}}$

e $16^{\frac{1}{4}}$ **f** $(-0.00032)^{\frac{1}{5}}$ **g** $(0.01)^{\frac{1}{2}}$ **h** $(-512)^{\frac{1}{9}}$

i $(-8)^{\frac{1}{3}}$ **j** $(-729)^{\frac{1}{3}}$ **k** $256^{\frac{1}{8}}$ **l** $3125^{\frac{1}{5}}$

2 Write each expression using a radical (root) sign. R C

a $7^{\frac{1}{3}}$ **b** $15^{\frac{1}{2}}$ **c** $d^{\frac{1}{4}}$ **d** $y^{\frac{1}{2}}$

e $(4x)^{\frac{1}{6}}$ **f** $(10m^3)^{\frac{1}{2}}$ **g** $(2a^7)^{\frac{1}{10}}$ **h** $(12m)^{\frac{1}{3}}$

3 Write $\sqrt[4]{100k}$ using a fractional index. Select the correct answer **A**, **B**, **C** or **D**. R C

A $25k^{\frac{1}{4}}$ **B** $100k^{\frac{1}{4}}$ **C** $(100k)^{\frac{1}{4}}$ **D** $5k^{\frac{1}{4}}$

4 Write each expression using a fractional index. R C

a $\sqrt[3]{80}$ **b** $\sqrt[5]{40}$ **c** $\sqrt{20}$ **d** $\sqrt[4]{c}$

e $\sqrt{b}$ **f** $\sqrt[3]{xy}$ **g** $\sqrt[5]{25w}$ **h** $\sqrt[8]{4y}$

5 Evaluate each expression. R C

a $16^{\frac{3}{4}}$ **b** $25^{\frac{5}{2}}$ **c** $(-27)^{-\frac{5}{3}}$ **d** $8^{\frac{2}{3}}$

e $(-128)^{-\frac{3}{7}}$ **f** $32^{-\frac{1}{5}}$ **g** $1000^{\frac{5}{3}}$ **h** $900^{-\frac{1}{2}}$

i $64^{\frac{5}{6}}$ **j** $9^{-\frac{7}{2}}$ **k** $(-216)^{-\frac{4}{3}}$ **l** $(-243)^{-\frac{2}{5}}$

6 Evaluate the expression $100000^{-\frac{7}{5}}$. Select **A**, **B**, **C** or **D**. R C

A $\frac{1}{1000000}$ **B** $\frac{1}{10000000}$ **C** $\frac{1}{1000}$ **D** $\frac{1}{100000}$

EXAMPLE 4

7 Evaluate each expression correct to 2 decimal places.

a $\sqrt[3]{-250}$ **b** $37^{\frac{1}{2}}$ **c** $14^{-\frac{3}{4}}$ **d** $\sqrt[7]{-138}$

e $4^{-\frac{5}{3}}$ **f** $200^{\frac{3}{10}}$ **g** $(-50)^{-\frac{6}{5}}$ **h** $\sqrt[4]{82}$

EXAMPLE 5

8 Write each expression using a fractional index. R C

a $\sqrt[4]{k^3}$ **b** $\frac{1}{\sqrt{m}}$ **c** $\sqrt[3]{x^7}$ **d** $\sqrt{y^3}$

e $\frac{1}{\sqrt[5]{d^2}}$ **f** $\sqrt{2a^3}$ **g** $\frac{1}{\sqrt[5]{4p}}$ **h** $\frac{1}{\sqrt[4]{vw}}$

EXAMPLE 6

9 Simplify $\sqrt[3]{(8d^2)^5}$. Select **A**, **B**, **C** or **D**. R C

A $32d^{30}$ **B** $4d^{\frac{5}{3}}$ **C** $16d^{\frac{10}{3}}$ **D** $32d^{\frac{10}{3}}$

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STAGE 5.3

10 Simplify each expression. R C

a $(25m^8)^{\frac{5}{2}}$ **b** $\sqrt[3]{(8y^6)^2}$ **c** $\sqrt[4]{(16x^2w^6)^6}$

d $(27c^6)^{-\frac{5}{3}}$ **e** $(8k^9)^{-\frac{4}{3}}$ **f** $\left(\frac{1}{\sqrt[5]{32p^{15}}}\right)^2$

g $(10\,000n^4w^6)^{\frac{1}{2}}$ **h** $\sqrt[3]{(243x^{10}y^5)^4}$ **i** $6p^{\frac{2}{3}}n^{\frac{1}{2}} \times \sqrt{36p^4n^5}$

j $\frac{1}{\left(\sqrt[4]{625y^8d^4}\right)^3}$ **k** $(64m^6)^{-\frac{1}{3}} \times (4m)^2$ **l** $\sqrt[3]{(216a^6b^9)^2}$

5.03 Adding and subtracting algebraic fractions

Adding and subtracting algebraic fractions

Algebraic fractions

Algebraic fractions

Example 7

Simplify each expression.

a $\frac{a}{2}+\frac{a}{3}$ **b** $\frac{2x}{5}-\frac{x}{3}$ **c** $\frac{5}{y}+\frac{7}{2y}$

Solution

a
$$\frac{a}{2}+\frac{a}{3}=\frac{3\times a}{3\times 2}+\frac{2\times a}{2\times 3}$$
$$=\frac{3a}{6}+\frac{2a}{6}$$
$$=\frac{5a}{6}$$

b
$$\frac{2x}{5}-\frac{x}{3}=\frac{3\times 2x}{3\times 5}-\frac{5\times x}{5\times 3}$$
$$=\frac{6x}{15}-\frac{5x}{15}$$
$$=\frac{x}{15}$$

c
$$\frac{5}{y}+\frac{7}{2y}=\frac{2\times 5}{2y}+\frac{7}{2y}$$
$$=\frac{10}{2y}+\frac{7}{2y}$$
$$=\frac{17}{2y}$$

STAGE 5.3

Algebraic fractions

Example 8

Simplify each expression.

a $\frac{6h+1}{8}+\frac{4h-7}{12}$ **b** $\frac{x}{3}-\frac{x+1}{5}$

Solution

a
$$\frac{6h+1}{8}+\frac{4h-7}{12}=\frac{3(6h+1)}{24}+\frac{2(4h-7)}{24}$$
$$=\frac{18h+3}{24}+\frac{8h-14}{24}$$
$$=\frac{18h+3+8h-14}{24}$$
$$=\frac{26h-11}{24}$$

b
$$\frac{x}{3}-\frac{x+1}{5}=\frac{5x}{15}-\frac{3(x+1)}{15}$$
$$=\frac{5x}{15}-\frac{3x+3}{15}$$
$$=\frac{5x-(3x+3)}{15}$$
$$=\frac{5x-3x-3}{15}$$
$$=\frac{2x-3}{15}$$

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EXERCISE 5.03 ANSWERS ON P. 601

Adding and subtracting algebraic fractions U F

EXAMPLE 7

5.03

1 Simplify each expression.

a $\frac{n}{2}-\frac{n}{7}$ **b** $\frac{3c}{2}+\frac{c}{5}$ **c** $\frac{4r}{7}+\frac{5r}{2}$ **d** $\frac{7y}{8}-\frac{5y}{3}$

e $\frac{4t}{3}+\frac{t}{9}$ **f** $\frac{5y}{16}-\frac{3y}{8}$ **g** $\frac{11t}{12}-\frac{5t}{9}$ **h** $\frac{3a}{10}+\frac{4a}{15}$

i $\frac{4}{k}+\frac{3a}{k}$ **j** $\frac{7d}{3y}-\frac{2d}{3y}$ **k** $\frac{12}{5x}-\frac{7}{5x}$ **l** $\frac{11p}{12}+\frac{3p}{12}$

m $\frac{5}{q}+\frac{3}{r}$ **n** $\frac{15}{4h}-\frac{2}{h}$ **o** $\frac{5}{3b}+\frac{3}{4b}$ **p** $\frac{9}{2n}-\frac{4}{3p}$

2 Simplify each expression.

a $\frac{w}{7}+\frac{n}{4}$ **b** $\frac{3x}{8}+\frac{5u}{7}$ **c** $\frac{2a}{15}-\frac{3h}{10}$ **d** $\frac{d}{16}-\frac{r}{24}$

e $\frac{4k}{5}-\frac{g}{9}$ **f** $\frac{5m}{12}+\frac{2r}{5}$ **g** $\frac{3h}{2}+\frac{4}{5}$ **h** $\frac{5u}{4}-\frac{3a}{7}$

i $\frac{a}{5}+\frac{c}{9}$ **j** $\frac{5q}{12}-\frac{11w}{18}$ **k** $\frac{5k}{8}+\frac{3}{11}$ **l** $\frac{4n}{9}+\frac{5n}{6}$

3 Simplify $\frac{5p}{6}-\frac{3p}{8}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{29p}{24}$ **B** $\frac{22p}{48}$ **C** $\frac{11p}{24}$ **D** $\frac{2p}{24}$

4 Simplify $\frac{1}{xw}+\frac{w}{xy}$. Select **A**, **B**, **C** or **D**.

A $\frac{y+w}{xyw}$ **B** $\frac{w^2+y}{xyw}$ **C** $\frac{y+w^2}{xy}$ **D** $\frac{y^2+w}{wx}$

5 Simplify $\frac{4w-3}{5}-\frac{1+2w}{4}$. Select **A**, **B**, **C** or **D**.

A $\frac{6w-17}{20}$ **B** $\frac{6w-7}{20}$ **C** $\frac{26w-17}{20}$ **D** $\frac{26w-7}{10}$

STAGE 5.3

EXAMPLE 8

6 Simplify each expression.

a $\frac{m+3}{5}+\frac{m-2}{4}$ **b** $\frac{3p}{5}+\frac{p+2}{3}$ **c** $\frac{y+3}{4}+\frac{2y-1}{3}$

d $\frac{x-1}{7}+\frac{x-2}{3}$ **e** $\frac{5h+6}{2}-\frac{2h+2}{9}$ **f** $\frac{k+3}{10}-\frac{2k-3}{7}$

g $\frac{a-1}{2}-\frac{a}{3}$ **h** $\frac{3d-1}{7}+\frac{2d-9}{6}$ **i** $\frac{2y+5}{4}-\frac{y-1}{9}$

j $\frac{1-w}{10}-\frac{3w-2}{7}$ **k** $\frac{9-2e}{3}-\frac{e-5}{2}$ **l** $\frac{8-3x}{11}-\frac{2x+5}{4}$

5.04 Multiplying and dividing algebraic fractions

STAGE 5.3

Algebraic fractions puzzle

Algebraic fractions

Upside-down fractions

Example 9

Simplify each expression.

a $\frac{3x}{4} \times \frac{2x}{9}$ **b** $\frac{4}{k} \times \frac{3k}{16}$ **c** $\frac{2}{v} \div \frac{3}{w}$ **d** $\frac{xy}{5} \div \frac{3x}{25}$

Solution

a $\frac{3x}{4} \times \frac{2x}{9} = \frac{3x \times 2x}{4 \times 9}$ or $\frac{3x}{4} \times \frac{2x}{9} = \frac{{}^{1}\cancel{3}x}{{}_{2}\cancel{4}} \times \frac{{}^{1}\cancel{2}x}{{}_{3}\cancel{9}}$

$= \frac{6x^2}{36}$

$= \frac{x^2}{6}$

$= \frac{x^2}{6}$

b $\frac{4}{k} \times \frac{3k}{16} = \frac{{}^{1}\cancel{4}}{\cancel{k}} \times \frac{3\cancel{k}}{{}_{4}\cancel{16}}$

$= \frac{3}{4}$

c $\frac{2}{v} \div \frac{3}{w} = \frac{2}{v} \times \frac{w}{3}$

$= \frac{2w}{3v}$

d $\frac{xy}{5} \div \frac{3x}{25} = \frac{\cancel{x}y}{{}_{1}\cancel{5}} \times \frac{{}^{5}\cancel{25}}{3\cancel{x}}$

$= \frac{5y}{3}$

EXERCISE 5.04 ANSWERS ON P. 601

Multiplying and dividing algebraic fractions U F

EXAMPLE 9

1 Simplify each product.

a $\frac{5}{6} \times \frac{d}{7}$ **b** $\frac{x}{3} \times \frac{w}{8}$ **c** $\frac{5}{k} \times \frac{3}{b}$

d $\frac{5}{3h} \times \frac{4}{m}$ **e** $\frac{15}{w} \times \frac{2}{10y}$ **f** $\frac{4a}{3} \times \frac{5}{12c}$

g $\frac{2x}{3} \times \frac{9}{8y}$ **h** $\frac{6}{5b} \times \frac{10b}{18}$ **i** $\frac{8h}{p} \times \frac{2h}{3g}$

j $\frac{5u}{12c} \times \frac{8c}{15u}$ **k** $\frac{9w}{45pq} \times \frac{5p}{6wh}$ **l** $\frac{20a}{3k} \times \frac{9a}{5k}$

2 Simplify each quotient.

a $\frac{m}{3} \div \frac{g}{7}$ **b** $\frac{xy}{24} \div \frac{5x}{6}$ **c** $\frac{4}{5d} \div \frac{5d}{2}$

d $\frac{8a}{3} \div \frac{5a}{9}$ **e** $\frac{w}{15} \div \frac{w}{5y}$ **f** $\frac{9y}{4h} \div \frac{5y}{12}$

g $\frac{2c^2}{5e} \div \frac{8c}{3}$ **h** $\frac{10g}{7kc} \div \frac{2g}{21k}$ **i** $\frac{4}{3xy} \div \frac{7k}{9y}$

j $\frac{5d^2}{6} \div \frac{d}{10}$ **k** $\frac{4e}{15p^2} \div \frac{6e^2}{5p}$ **l** $\frac{9r}{5} \div \frac{12r^2}{35g}$

3 Simplify the quotient $\frac{5xy}{3k} \div \frac{20}{9ky}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{3y}{4x}$ **B** $\frac{4x}{15y^2}$ **C** $\frac{45k}{60y}$ **D** $\frac{3xy^2}{4}$

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4 Simplify the product $\frac{6ad}{7m} \times \frac{3y}{4d} \times \frac{14gm}{27ay}$. Select the answer **A**, **B**, **C** or **D**.

A $\frac{2ak}{7y}$ **B** $\frac{g}{3}$ **C** $\frac{14gm}{9}$ **D** $\frac{7m}{4}$

5 Simplify each expression.

a $\frac{8}{xy} \div \frac{6}{xw}$ **b** $\frac{3dg}{5k} \div \frac{9kg}{4h} \times \frac{6k}{5h}$ **c** $\frac{12mn}{5} \div \frac{8m}{15}$

d $\frac{10a}{3pf} \div \frac{5}{21p}$ **e** $\frac{8c}{5} \div 4$ **f** $\frac{3n}{v} \div \frac{5}{4v} \div \frac{9nm}{10}$

g $\frac{4ab}{15d} \div \frac{3d}{8b} \times \frac{5bd}{16}$ **h** $24hw \div \frac{4kw}{9y}$ **i** $\frac{8dm}{3} \div \frac{m}{6} \times \frac{1}{12d^2}$

j $\frac{10}{3p} \times \frac{pq}{5h} \div \frac{6q}{5p}$ **k** $\frac{8ac}{w} \div \frac{4w}{9c} \div \frac{2c}{yh}$ **l** $\frac{2u}{7y} \div \frac{3}{4xy} \times \frac{21y}{8x}$

5.04

Mental skills 5: Maths without calculators ANSWERS ON P. 601

Multiplying by 9, 11, 99 and 101

We can use expansion when we multiply by a number near 10 or near 100.

1 Study each example.

a
$$\begin{aligned} 25 \times 11 &= 25 \times (10 + 1) \\ &= 25 \times 10 + 25 \times 1 \\ &= 250 + 25 \\ &= 275 \end{aligned}$$

b
$$\begin{aligned} 14 \times 9 &= 14 \times (10 - 1) \\ &= 14 \times 10 - 14 \times 1 \\ &= 140 - 14 \\ &= 126 \end{aligned}$$

c
$$\begin{aligned} 32 \times 12 &= 32 \times (10 + 2) \\ &= 32 \times 10 + 32 \times 2 \\ &= 320 + 64 \\ &= 384 \end{aligned}$$

d
$$\begin{aligned} 7 \times 99 &= 7 \times (100 - 1) \\ &= 7 \times 100 - 7 \times 1 \\ &= 700 - 7 \\ &= 693 \end{aligned}$$

e
$$\begin{aligned} 27 \times 101 &= 27 \times (100 + 1) \\ &= 27 \times 100 + 27 \times 1 \\ &= 2700 + 27 \\ &= 2727 \end{aligned}$$

f
$$\begin{aligned} 18 \times 8 &= 18 \times (10 - 2) \\ &= 18 \times 10 - 18 \times 2 \\ &= 180 - 36 \\ &= 144 \end{aligned}$$

2 Now evaluate each product.

a 16×11 **b** 33×11 **c** 29×9 **d** 45×9

e 62×11 **f** 7×101 **g** 18×101 **h** 36×99

i 19×8 **j** 45×12 **k** 21×102 **l** 6×98

m 32×9 **m** 7×99 **o** 39×101 **p** 71×12

5.05 Expanding and factorising expressions

Algebra using diagrams
HCF by factor trees
Factorising using diagrams
Expanding and factorising
Algebra 4
The distributive law
Factorising puzzle
Factorising expressions

Example 10

Expand and simplify by collecting like terms.

a $-3r^2(4r + 2) - 5r^3$ **b** $9(m - 3) + m(m - 10)$

Solution

a $-3r^2(4r + 2) - 5r^3 = -3r^2 \times 4r + (-3r^2) \times 2 - 5r^3$ Expanding

$= -12r^3 + (-6r^2) - 5r^3$

$= -17r^3 - 6r^2$ Collecting like terms to simplify.

b $9(m - 3) + m(m - 10) = 9 \times m - 9 \times 3 + m \times m - m \times 10$

$= 9m - 27 + m^2 - 10m$

$= m^2 + 9m - 10m - 27$ It's neater to place m^2 at the front.

$= m^2 - m - 27$ Collecting like terms to simplify.

Example 11

Factorise each expression.

a $18xy^2 - 24xy$ **b** $3m(5 + 2d) + 7(5 + 2d)$ **c** $-5k^2 + 15k$

Solution

a The **HCF** of $18xy^2$ and $24xy$ is $6xy$.

$\therefore 18xy^2 - 24xy = 6xy \times 3y - 6xy \times 4$ Rewrite the expression using the HCF $6xy$.

$= 6xy(3y - 4)$ Write the HCF at the front of the brackets.

b The HCF of $3m(5 + 2d) + 7(5 + 2d)$ is $(5 + 2d)$.

$\therefore 3m(5 + 2d) + 7(5 + 2d) = (5 + 2d) \times 3m + (5 + 2d) \times 7$

$= (5 + 2d)(3m + 7)$

c When factorising expressions that begin with a negative term, we use the 'negative' HCF.

The highest 'negative' common factor of $-5k^2$ and $15k$ is $-5k$.

$\therefore -5k^2 + 15k = (-5k) \times k + (-5k) \times (-3)$ $(-5k) \times (-3) = +15k$

$= (-5k)[k + (-3)]$

$= -5k(k - 3)$

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Example 12

Factorise each expression.

a $8a^3 + 4a^2$ **b** $20h^3k + 25h^4k - 10h^2k$

Solution

a The HCF of $8a^3$ and $4a^2$ is $4a^2$.

$\therefore 8a^3 + 4a^2 = 4a^2 \times 2a + 4a^2 \times 1$ Rewrite the expression using the HCF $4a^2$.

$= 4a^2(2a + 1)$

b The HCF is $5h^2k$.

$\therefore 20h^3k + 25h^4k - 10h^2k = 5h^2k \times 4h + 5h^2k \times 5h^2 - 5h^2k \times 2.$

$= 5h^2k(4h + 5h^2 - 2)$

EXERCISE 5.05 ANSWERS ON P. 601

Expanding and factorising expressions U F

1 Expand each expression.

a $5(d + 11)$ **b** $-3(r + 10)$ **c** $7(x - 9y)$ **d** $-4(a - 5w)$

e $-(2 - p^2)$ **f** $-10e(2e^2 + 3)$ **g** $6y(1 + 7y)$ **h** $4xy(3xy - 1)$

i $8rq(2q - r)$ **j** $3ab(4b - 7a)$ **k** $-6h^2(1 - 3h)$ **l** $-5x(5x^2 + 4y)$

m $-(3 + 8a)$ **n** $-2m^2(3m - 4n)$ **o** $5g(3 + 7g^2)$ **p** $-1(5e - 12)$

2 Expand $-2y(5 + 7y)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3y - 5y^2$ **B** $-10y + 14y^2$ **C** $-10y - 14y^2$ **D** $-10y - 5y^2$

3 Use the substitution $x = 2$ to test whether each equation is correct or incorrect.

a $4(x + 10) = 4x + 40$ **b** $5(x - 1) = 5x - 6$ **c** $x(3 - x) = 3x - x^2$

4 Expand and simplify by collecting like terms.

a $4k(3k - 5) - 9k^2$ **b** $5h - 7h(4 - h)$

c $9w^3 - 3w(5 + 2w^2)$ **d** $24x^3 - 5x^2(2x^2 - 5x)$

e $8 - 3(2 - 7d)$ **f** $4n(3 - 5n) - 6n^2$

g $4y(y - 4) + 5(2y + 1)$ **h** $5(1 - 2a) - a(3 + 4a)$

i $7(3 + 6w) - (5 - 8w)$ **j** $4y^2(5y + 5) + 4(2 - 7y^2)$

k $-v(2v + 7) + 6(v - 1)$ **l** $2(4 - 3a) - a(3 - a)$

m $2c(5c - 1) - 4(7c - 5)$ **n** $3m(m + 5m^2) - m^2(1 - 3m)$

o $4x(2y + 5) - 6y(10 - 3x)$

EXAMPLE 10

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EXAMPLE 11

5 Factorise each expression.

a	$15y - 20$	**b**	$21 + 35w$	**c**	$2p + p^2$
d	$30y - 20y^2$	**e**	$36d^2 + 24d$	**f**	$28k^2 - 21k$
g	$8(c - 5) - c(c - 5)$	**h**	$m(3 + 2m) + 7(3 + 2m)$	**i**	$-q^2 - 36q$
j	$-8x + 12x^2$	**k**	$b(3b + 5) - 2(3b + 5)$	**l**	$-12cd^2 + 8cd$
m	$-hn^2 + h^2 n$	**n**	$-15g^2 - 18g$	**o**	$48q^2 - 54q$

6 Factorise $-16pw + 10xw$. Select **A**, **B**, **C** or **D**.

A $-2w(8p - 10x)$ **B** $-2w(8p - 5x)$ **C** $-8w(2p - x)$ **D** $-4w(5p + 2x)$

EXAMPLE 12

7 Factorise each expression.

a	$8m^2y^2 - 12my$	**b**	$36ab^2c + 27bc$
c	$24m^2n - 108mn^2$	**d**	$20dg^2 - 35ag$
e	$40wy^3 + 24w^2y^2$	**f**	$75g^3h^2 - 125gh$
g	$-4p^3 - 8p^2 + p$	**h**	$6mn^2 + 3mn + 48m^2 n$
i	$32p^3 g + 8pg^2 - 8pg$	**j**	$18a^5 - 12a^2 + 15a^4$
k	$28m^3h^2 - 21mh^2$	**l**	$15kwp - 24wp^2 - 9kw$

Did you know?

CAPTCHA

For security purposes on a website, have you ever been asked to enter letters, words or numbers that have been displayed in a wavy, difficult-to-read format like this?



This process is called **CAPTCHA**, which stands for **Completely Automated Public Turing Test to Tell Computers and Humans Apart**, invented in 1999. It is designed to ensure that the person accessing the website is a human and not another computer that could be hacking into the site, sending spam or viruses. CAPTCHA uses optical character recognition (OCR), which is a technology that can convert images of text into editable text.

How does CAPTCHA prove that you are a human and not a computer?

What is a Turing test, named after English computer scientist and mathematician Alan Turing?

Expanding binomial products

5.06

Binomial means '2 terms'. $(m + 8)$ and $(m - 3)$ are **binomial expressions** because they each have exactly 2 terms. $(m + 8)(m - 3)$ is called a **binomial product** because it is a product of 2 binomial expressions.

Example 13

Expand each binomial product.

a $(m + 8)(m - 3)$ **b** $(4y - 5)(3y - 2)$

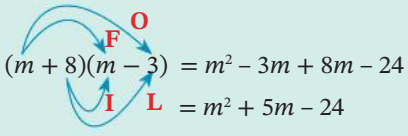
Solution

a $(m + 8)(m - 3) = m(m - 3) + 8(m - 3)$ Each term in $(m + 8)$ is multiplied by $(m - 3)$

$= m^2 - 3m + 8m - 24$ Expanding

$= m^2 + 5m - 24$ Simplifying

One way of remembering which pairs of terms to multiply together in a binomial product is called the **FOIL method** (First-Outer-Inner-Last), as shown.



b $(4y - 5)(3y - 2) = 12y^2 - 8y - 15y + 10$ Using FOIL

$= 12y^2 - 23y + 10$ Simplifying

Perfect squares

The formulas for expanding the perfect square of a binomial are:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example 14

Expand each perfect square.

a $(n - 5)^2$ **b** $(k + 4)^2$ **c** $(3y - 8)^2$

Solution

a $(n - 5)^2 = n^2 - 2 \times n \times 5 + 5^2$

$= n^2 - 10n + 25$

1st term squared – double product + 2nd term squared

b $(k + 4)^2 = k^2 + 2 \times k \times 4 + 4^2$

$= k^2 + 8k + 16$

c $(3y - 8)^2 = (3y)^2 - 2 \times 3y \times 8 + 8^2$

$= 9y^2 - 48y + 64$

STAGE 5.3

5.06

WS Binomial products

WS Area diagrams

Expanding binomial products 1

Expanding binomial products 2

WS Algebra 2

WS Algebra 6

Expanding brackets

Expanding binomials

Expanding binomials

Special binomial products

STAGE 5.3

Difference of 2 squares

Special binomial products

Products and factors squaresaw

Factorominoes

Trinominoes

Enough time

Special products

Difference of 2 squares

$$(a + b)(a - b) = a^2 - b^2$$

The answer is called the **difference of two squares**.

When the sum of 2 terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of 2 squares).

Example 15

Expand each expression.

a $(m + 6)(m - 6)$

b $(10 - e)(10 + e)$

c $(8g - 3)(8g + 3)$

d $(2x + 7w)(2x - 7w)$

Solution

a $(m + 6)(m - 6) = m^2 - 6^2$
$= m^2 - 36$

b $(10 - e)(10 + e) = 10^2 - e^2$
$= 100 - e^2$

c $(8g - 3)(8g + 3) = (8g)^2 - 3^2$
$= 64g^2 - 9$

d $(2x + 7w)(2x - 7w) = (2x)^2 - (7w)^2$
$= 4x^2 - 49w^2$

Example 16

Expand and simplify each expression.

a $x(5y - x)^2$

b $(2a - 3)(2a + 3) - (a + 3)^2$

Solution

a $x(5y - x)^2 = x(25y^2 - 10xy + x^2)$
$= 25xy^2 - 10x^2y + x^3$

b $(2a - 3)(2a + 3) - (a + 3)^2 = 4a^2 - 9 - (a^2 + 6a + 9)$
$= 4a^2 - 9 - a^2 - 6a - 9$
$= 3a^2 - 6a - 18$

EXERCISE 5.06 ANSWERS ON P. 602

Expanding binomial products U F R C

EXAMPLE 13

1 Expand $(5 + 3b)(2b - 7)$. Select the correct answer **A**, **B**, **C** or **D**.

A $6b^2 + 11b - 35$ **B** $6b^2 - 11b - 35$ **C** $6b^2 - 31b - 35$ **D** $11b - 6b^2 - 35$

2 Expand each binomial product.

a $(3m + 1)(m + 7)$ **b** $(4y + 3)(7y + 5)$ **c** $(1 + 4k)(k + 6)$

d $(5h - 2)(2h + 3)$ **e** $(v - 8)(2v - 9)$ **f** $(5a + 2)(4a - 3)$

g $(11 - 3e)(5 - 4e)$ **h** $(8 + g)(5 - 6g)$ **i** $(4y - 9)(7 - 2y)$

j $(2 - 3a)(4a + 3)$ **k** $(12 - 5k)(7 - 4k)$ **l** $(2w + 9)(7w - 10)$

Foundation Standard Complex

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3 Copy and complete the expansion of each perfect square.

EXAMPLE 14

a $(y + 7)^2 = y^2 + _____ + 49$
b $(h - 5)^2 = _____ - 10h + 25$
c $(g - m)^2 = g^2 - 2gm + _____$
d $(u + 11)^2 = u^2 _____ + 121$
e $(3k + 5)^2 = 9k^2 _____ + 25$
f $(8x - 3w)^2 = 64x^2 _____ + _____$

4 Expand each perfect square.

a $(h + 7)^2$
b $(p - 5)^2$
c $(w - 6)^2$
d $(4 - c)^2$
e $(11 + y)^2$
f $(d - w)^2$
g $(10 - x)^2$
h $(d + q)^2$
i $(5u + 1)^2$
j $(3b + 7)^2$
k $(7e - 11)^2$
l $(2 - 5h)^2$
m $(8x - 3y)^2$
n $(6m + 5c)^2$
o $(g + 12w)^2$
p $\left(2+\frac{2}{a}\right)^2$
q $\left(\frac{6}{m}-m\right)^2$
r $\left(\frac{3}{x}+3x\right)^2$

5 Expand $(4a - 5w)^2$. Select **A**, **B**, **C** or **D**.

A $16a^2 + 25w^2$
B $16a^2 - 9aw - 25w^2$
C $16a^2 + 18aw - 25w^2$
D $16a^2 - 40aw + 25w^2$

6 Expand each expression.

EXAMPLE 15

a $(k + 7)(k - 7)$
b $(x + 8)(x - 8)$
c $(g - 1)(g + 1)$
d $(12 + w)(12 - w)$
e $(4 + d)(4 - d)$
f $(7b - 5)(7b + 5)$
g $(3h + 7)(3h - 7)$
h $(10e - 1)(10e + 1)$
i $(11 + 4y)(11 - 4y)$
j $(1 - 5p)(1 + 5p)$
k $(3a - 7c)(3a + 7c)$
l $(9h - 5m)(9h + 5m)$
m $\left(\frac{4}{m}-m\right)\left(\frac{4}{m}+m\right)$
n $\left(3g+\frac{1}{g}\right)\left(3g-\frac{1}{g}\right)$
o $\left(\frac{4a}{3}+2\right)\left(\frac{4a}{3}-2\right)$

7 Expand $(8g - 3y)(8g + 3y)$. Select **A**, **B**, **C** or **D**.

A $16g^2 - 9y^2$
B $64g^2 - 9y^2$
C $16g^2 - 6w^2$
D $64g^2 - 48gy + 25w^2$

8 Expand each expression.

a $(3t - d)^2$
b $(7k + 3)(3 + 7k)$
c $(9 - 4g)(4g + 9)$
d $(5 + 2a)(2a - 5)$
e $(ab - 8)(ab + 8)$
f $(4n + 5p)(5n - 4p)$
g $(10x - 7y)^2$
h $(2 - 11hm)(2 + 11hm)$
i $\left(xy-\frac{1}{y}\right)\left(xy+\frac{1}{y}\right)$

9 Expand and simplify each expression.

EXAMPLE 16

a $8y - (4y - 1)(1 - 2y)$
b $5(3w + 7) - (2w - 7)(w + 5)$
c $(a - b)^2 - (b - 2a)^2$
d $4(p - 5)^2 - 5(p - 3)(p + 8)$
e $(3 - y)^2 - (3 + y)^2$
f $8x^2 - (2x - 5)(2x + 5)$
g $(m + 2)^2 - (m + 2)(m - 2) - (m - 2)^2$
h $2(6 - d)^2 - (d + 5)^2 - (6 - d)(d + 5)$

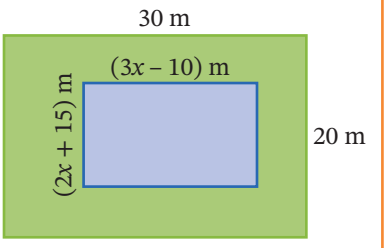
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STAGE 5.3

10 Expand and simplify $(4d - m)^2 - (4d - m)(4d + m)$. Select **A**, **B**, **C** or **D**.

A $8dm$ **B** $32d^2 - 8dm$ **C** $2m^2 - 8dm$ **D** $32d^2 - 2m^2$

11 This diagram shows a house $(3x - 10)$ metres long and $(2x + 15)$ metres wide on a block of land with dimensions 30 m × 20 m. R C

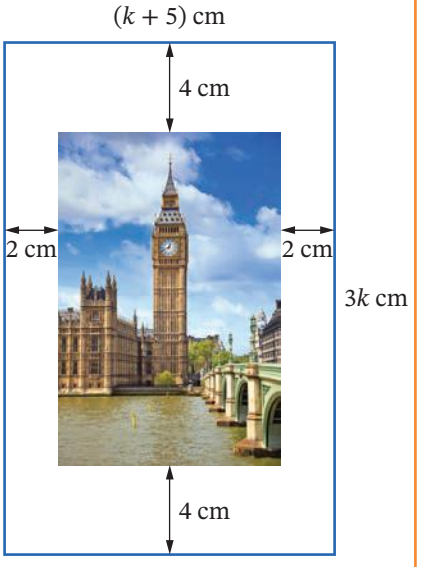


a Write down a binomial expression for the area of the house in square metres.

b Expand and simplify this expression.

c The green area of the block of land not covered by the house is to be turfed. Write a simplified expression for this area in square metres.

12 A photograph frame is $(k + 5)$ cm wide and $3k$ cm long. The gap between the photograph and frame is 4 cm at the top and bottom and 2 cm on each side. R C



a What is the area of the photograph frame?

b Write down expressions for the length and width of the photograph.

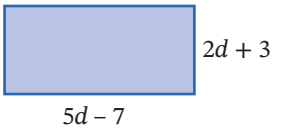
c Write down a binomial expression for the area of the photograph.

d Expand and simplify your expression for the area of the photograph.

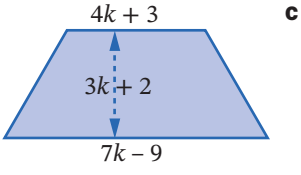
e Find an expression for the area of the frame not taken up by the photograph.

13 Find an expression for the area of each shape in simplest form. R C

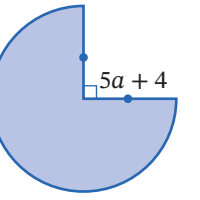
a



b



c



Investigation

Squaring a number ending in 5

Study this mental shortcut for squaring a number ending in 5:

- To evaluate 35^2, calculate $3 \times 4 = 12$, add '25' to the end: $35^2 = 1225$.
- To evaluate 75^2, calculate $7 \times 8 = 56$, add '25' to the end: $75^2 = 5625$.
- To evaluate 105^2, calculate $10 \times 11 = 110$, add '25' to the end: $105^2 = 11\,025$.

Let n stand for the tens digit of the number ending in 5 being squared.

Expand $(10n + 5)^2$ and investigate why the above method works.

Foundation Standard Complex

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Factorising special binomial products 5.07

Factorising by grouping in pairs

STAGE 5.3

Grouping

5.07

An **algebraic expression with 4 terms** can often be factorised in pairs, that is, 2 terms at a time, to make a binomial product.

Example 17

Factorise each expression.

a $6ae + 4cd + 8ad + 3ce$ **b** $3xy - 4px - 6y^2 + 8py$

c $12aw + 20cx - 8cw - 30ax$

Solution

a
$$\begin{aligned} 6ae + 4cd + 8ad + 3ce &= 6ae + 8ad + 4cd + 3ce && \text{Grouping into pairs} \\ &= 2a(3e + 4d) + c(4d + 3e) && \text{Factorising each pair} \\ &= (3e + 4d)(2a + c) && \text{Factorising again} \end{aligned}$$

b
$$\begin{aligned} 3xy - 4px - 6y^2 + 8py &= x(3y - 4p) - 2y(3y - 4p) && \text{Factorising each pair} \\ &= (3y - 4p)(x - 2y) && \text{Factorising again} \end{aligned}$$

c
$$\begin{aligned} 12aw + 20cx - 8cw - 30ax &= 2(6aw + 10cx - 4cw - 15ax) && \text{Factorising all terms first} \\ &= 2(6aw - 15ax - 4cw + 10cx) && \text{Grouping into pairs} \\ &= 2[3a(2w - 5x) - 2c(2w - 5x)] && \text{Factorising each pair} \\ &= 2(2w - 5x)(3a - 2c) && \text{Factorising again} \end{aligned}$$

Factorising the difference of 2 squares

Difference of 2 perfect squares

Difference of 2 squares

The difference of 2 squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 18

Factorise each expression.

a $m^2 - 25$ **b** $36 - 49y^2$ **c** $48k^2 - 3x^2$ **d** $w^4 - 4w^2$

Solution

a
$$\begin{aligned} m^2 - 25 &= m^2 - 5^2 \\ &= (m + 5)(m - 5) \end{aligned}$$

b
$$\begin{aligned} 36 - 49y^2 &= 6^2 - (7y)^2 \\ &= (6 + 7y)(6 - 7y) \end{aligned}$$

c
$$\begin{aligned} 48k^2 - 3x^2 &= 3(16k^2 - x^2) \\ &= 3[(4k)^2 - x^2] \\ &= 3(4k + x)(4k - x) \end{aligned}$$

d
$$\begin{aligned} w^4 - 4w^2 &= w^2(w^2 - 4) \\ &= w^2(w^2 - 2^2) \\ &= w^2(w + 2)(w - 2) \end{aligned}$$

STAGE 5.3

EXERCISE 5.07 ANSWERS ON P.

Factorising special binomial products U F

EXAMPLE 17

1 Factorise each expression.

a $3px + 2qx + 3py + 2qy$

b $2wh + 2wk - 3uh - 3uk$

c $15mk + 20mg + 6nk + 8ng$

d $4yx - 8ay + 7ax - 14a^2$

e $14ak - 35af + 8k - 20f$

f $cd + cy - hd - hy$

g $4am + 4at + 4em + 4et$

h $3yk - 6by + 12k - 24b$

i $12ac - 21mc - 35mw + 20aw$

j $21hk - 28he + 6pk - 8pe$

k $30y^2 - 6yq - 15y + 3q$

l $32gh + 24gk - 4mh - 3mk$

m $6ng - 10ch - 4cg + 15nh$

n $p^2w - p^3 - w + p$

o $m(d - 3) - mx + a(d - 3) - ax$

p $ek + k(x + y) + d(x + y) + ed$

EXAMPLE 18

2 Factorise each expression.

a $d^2 - 16$

b $x^2 - 25$

c $y^2 - 169$

d $p^2 - 81$

e $25 - h^2$

f $121 - a^2$

g $4r^2 - 9d^2$

h $25g^2 - 4e^2$

i $144 - 49m^2$

j $81y^2 - 16k^2$

k $1 - 4d^2$

l $m^2 - 25n^2$

m $25q^2 - 9b^2$

n $64b^2 - u^2$

o $144x^2 - 1$

p $36k^2 - w^2$

q $g^2 - 16p^2$

r $4e^2 - 121d^2$

s $m^2p^2 - 25$

t $a^2 - q^2w^2$

u $x^2 - \frac{1}{16}$

v $\frac{25}{9} - h^2$

w $6\frac{1}{4} - 9c^2$

x $16v^2 - \frac{9}{4}$

3 Factorise each expression.

a $4m^2 - 16p^2$

b $3y^2 - 27$

c $w^3 - 36w$

d $q - 64q^3$

e $12 - 48u^2$

f $rh^2 - r^3$

g $125a^2b^2 - 20v^2$

h $3x^4 - 27p^2$

i $54g^3p - 6gp^3$

j $28c - 63cw^2$

k $144e^5 - 36e^3$

l $9d^2 - 2\frac{1}{4}$

m $100k^2 - 6\frac{1}{4}$

n $98a^2 - 2w^2$

o $c^2 - 1\frac{7}{9}$

p $108m^3n^2 - 27m$

4 Factorise each expression.

a $\frac{1}{25} - x^2$

b $\frac{d^2}{9} - \frac{w^2}{49}$

c $3p^2 - \frac{48}{p^2}$

d $\frac{64b^2}{25} - \frac{4y^2}{81}$

e $u^4 - 16$

f $(m + 3)^2 - 25$

g $(x - y)^2 - y^2$

h $2a^5 - 162a$

i $(p + x)^2 - (p - x)^2$

j $\frac{4c^2}{9} - \frac{e^2}{36}$

k $4g^2 - (g - 2k)^2$

l $5w^5 - 80w$

Foundation Standard Complex

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Factorising quadratic expressions $x^2 + bx + c$

5.08

A **quadratic expression** is an algebraic expression in which the highest power of the variable is 2, such as $y^2 + 12y - 4$, $-m^2 - 8m$, $5d^2 - 7d + 1$ and $3x^2 + 10x$.

A quadratic expression such as $y^2 + 12y - 4$ is called a **trinomial** because it has 3 terms.

The expansion of $(x + 2)(x + 9)$ is $x^2 + 11x + 18$, a quadratic trinomial.

$\therefore$ The factorisation of $x^2 + 11x + 18$ is $(x + 2)(x + 9)$.

Factorominoes

Factorising quadratic trinomials

Trinominoes

In the factorisation of a quadratic trinomial such as $x^2 + 11x + 18$:

- each factor must have an x term to give x^2

$x^2 + 11x + 18 = (x + 2)(x + 9)$

- $2 + 9 = 11$, which is the **coefficient** of x, the number in front of the x

$x^2 + 11x + 18 = (x + 2)(x + 9)$

- $2 \times 9 = 18$, which is the **constant term** with no x

$x^2 + 11x + 18 = (x + 2)(x + 9)$

Example 19

Factorising quadratic expressions1

Factorise each quadratic trinomial.

a $a^2 + 7a + 12$ **b** $x^2 + 9x + 8$

Solution

a Find the 2 numbers that have a sum of 7 and a product of 12.

It is best to test numbers that have a **product of 12** and then check if their sums equal 7.

Pair of numbers	Product	Sum
6, 2	$6 \times 2 = 12$	$6 + 2 = 8$
3, 4	$3 \times 4 = 12$	$3 + 4 = 7$

The correct numbers are 3 and 4.

$\therefore a^2 + 7a + 12 = (a + 3)(a + 4)$

b Find 2 numbers with a sum of 9 and a product of 8.

Test numbers that have a **product of 8** and check if their sums equal 9.

The correct numbers are 8 and 1.

$\therefore x^2 + 9x + 8 = (x + 8)(x + 1)$

Factorising quadratic expressions of the form $x^2 + bx + c$

Factorising quadratic expressions 1

- Find 2 numbers that have a sum of b and a product of c
- Use these 2 numbers to write a binomial product of the form $(x ___)(x ___)$

Example 20

Factorise each quadratic expression.

a $x^2 + 2x - 24$ **b** $c^2 - c - 42$ **c** $k^2 - 11k + 30$

Solution

a $x^2 + 2x - 24$

Find 2 numbers that have a product of -24 and a sum of 2.

Since the product is negative, one of the numbers must be negative.

They are $+6$ and -4.

$\therefore x^2 + 2x - 24 = (x + 6)(x - 4)$

b $c^2 - c - 42$

Product $= -42$, sum $= -1$.

Since the product is negative, one of the numbers must be negative.

They are -7 and $+6$.

$\therefore c^2 - c - 42 = (c - 7)(c + 6)$

c $k^2 - 11k + 30$

Product $= 30$, sum $= -11$.

Since the sum is negative, at least one of the numbers must be negative.

Since the product is positive, *both* of the numbers must be negative.

They are -6 and -5.

$\therefore k^2 - 11y + 30 = (k - 6)(k - 5)$

EXERCISE 5.08 ANSWERS ON P. 603

Factorising quadratic expressions $x^2 + bx + c$ U F R

1 Find 2 numbers whose: R

a product is 24 and their sum is -11 **b** product is -40 and their sum is 6

c product is 15 and their sum is 8 **d** product is -48 and their sum is -8

EXAMPLES 19,20

2 Factorise each quadratic expression. R

a	$x^2 + 12x + 32$	**b**	$w^2 + 9w + 14$	**c**	$y^2 + 12y + 27$
d	$k^2 + 13k + 36$	**e**	$a^2 + 19a + 90$	**f**	$g^2 + 10g + 21$
g	$m^2 - 11m + 28$	**h**	$p^2 - 8p + 15$	**i**	$c^2 - 15c + 54$
j	$u^2 - 5u + 4$	**k**	$h^2 - 14h + 48$	**l**	$e^2 - 8e + 16$
m	$d^2 + 3d - 28$	**n**	$q^2 + 2q - 80$	**o**	$n^2 + 4n - 45$
p	$x^2 - 6x - 7$	**q**	$g^2 - 11g - 80$	**r**	$w^2 - 4w - 96$
s	$r^2 - 12r - 28$	**t**	$b^2 - 3b - 54$	**u**	$m^2 - 4m - 12$
v	$y^2 - 14y + 49$	**w**	$d^2 - d - 30$	**x**	$e^2 + 9e - 90$

Foundation Standard Complex

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Factorising quadratic expressions of the form $ax^2 + bx + c$

5.09

STAGE 5.3

We have factorised quadratic trinomials of the type $x^2 + bx + c$. For example, the factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$. These are called **monic** quadratic expressions because x^2 does not have a **coefficient** (or its coefficient is 1).

We will now factorise quadratic trinomials of the type $ax^2 + bx + c$, such as $8x^2 + 14x - 15$, where x^2 has a coefficient. These are called **non-monic** quadratic expressions.

Example 21

Factorise each quadratic expression.

a $4y^2 + 8y - 140$ **b** $60 + 7d - d^2$

Solution

a $4y^2 + 8y - 140 = 4(y^2 + 2y - 35)$ Taking out the HCF of 4 first.

$= 4(y + 7)(y - 5)$ Product $= -35$, sum $= 2$

b $60 + 7d - d^2 = -d^2 + 7d + 60$ Rearranging the terms to make the d^2 term first.

$= -1(d^2 - 7d - 60)$ Taking out a common factor of -1.

$= -(d - 12)(d + 5)$ Product $= -60$, sum $= -7$

Example 22

Factorise $5k^2 - 12k + 4$.

Solution

There is no HCF, so we need to split up the middle term $-12k$.

Find 2 numbers that have a product of 20 and a sum of -12.

$$5k^2 - 12k + 4 = 5k^2 - 12k + 4$$

sum of -12

product of $5 \times 4 = 20$

Since the sum is negative, at least one of the numbers must be negative.

Since the product is positive, **both** of the numbers must be negative.

The 2 numbers are -10 and -2, so we will split $-12k$ into $-10k$ and $-2k$.

$\therefore 5k^2 - 12k + 4 = 5k^2 - 10k - 2k + 4$

$= 5k(k - 2) - 2(k - 2)$ Factorising by grouping in pairs

$= (k - 2)(5k - 2)$ Factorising again

Factorising quadratic trinomials of the form $ax^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of ac
- Use these 2 numbers to split the middle term bx into 2 terms
- Factorise by grouping in pairs

STAGE 5.3

Factorising quadratic expressions (Advanced)

Advanced algebra

Example 23

Factorise each quadratic expression.

a $3x^2 + 8x + 4$ **b** $3x^2 - 11x + 10$ **c** $4x^2 - 3x - 7$

Solution

a $3x^2 + 8x + 4$

$3 \times 4 = 12$.

Find 2 numbers that have a product of 12 and a sum of 8. They are 6 and 2.

Split $8x$ into $6x$ and $2x$.

$$\begin{aligned} 3x^2 + 8x + 4 &= 3x^2 + 6x + 2x + 4 \\ &= 3x(x + 2) + 2(x + 2) \quad \text{Factorising by grouping in pairs} \\ &= (x + 2)(3x + 2) \end{aligned}$$

b $3x^2 - 11x + 10$

$3 \times 10 = 30$.

Find 2 numbers that have a product of 30 and a sum of –11.

Since the sum is negative and the product is positive, both numbers must be negative.

They are -6 and -5.

Split $-11x$ into $-6x$ and $-5x$.

$$\begin{aligned} 3x^2 - 11x + 10 &= 3x^2 - 6x - 5x + 10 \\ &= 3x(x - 2) - 5(x - 2) \\ &= (x - 2)(3x - 5) \end{aligned}$$

c $4x^2 - 3x - 7$

$4 \times (-7) = -28$.

Find 2 numbers with a product of -28 and a sum of -3.

Since the product is negative, one of the numbers must be negative. They are -7 and 4.

$$\begin{aligned} 4x^2 - 3x - 7 &= 4x^2 - 7x + 4x - 7 \\ &= x(4x - 7) + 1(4x - 7) \\ &= (4x - 7)(x + 1) \end{aligned}$$

Example 24

Factorise each quadratic expression.

a $24k^2 - 54k - 15$ **b** $14 + 29a - 15a^2$

Solution

a
$$\begin{aligned} 24k^2 - 54k - 15 &= 3(8k^2 - 18k - 5) && \text{Taking out the HCF of 3 first} \\ &= 3(8k^2 - 20k + 2k - 5) && \text{Product} = -40, \text{ sum} = -18 \\ &= 3[4k(2k - 5) + 1(2k - 5)] \\ &= 3(2k - 5)(4k + 1) \end{aligned}$$

b
$$\begin{aligned} 14 + 29a - 15a^2 &= -15a^2 + 29a + 14 && \text{Rearranging to make the } a^2 \text{ term first} \\ &= -(15a^2 - 29a - 14) && \text{Taking out a common factor of } -1 \\ &= -(15a^2 + 6a - 35a - 14) && \text{Product} = -210, \text{ sum} = -29 \\ &= -[3a(5a + 2) - 7(5a + 2)] \\ &= -(5a + 2)(3a - 7) \end{aligned}$$

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EXERCISE 5.09 ANSWERS ON P. 603

Factorising quadratic expressions $ax^2 + bx + c$ U F R

1 Factorise each quadratic expression. Look for the highest common factor first. R

EXAMPLE 21

a $2k^2 + 16k + 30$ **b** $3q^2 - 3q - 90$ **c** $5x^2 - 55x + 140$

d $6y^2 + 12y - 18$ **e** $n^3 - 16n^2 + 63n$ **f** $24 + 2a - a^2$

g $4p + 32 - p^2$ **h** $-x^4 - 4x^3 + 21x^2$ **i** $-20 + 24h - 4h^2$

2 Factorise each quadratic expression. R

EXAMPLE 22

a $6d^2 + 19d + 15$ **b** $8m^2 + 10m + 3$ **c** $2y^2 + 7y + 5$

d $2w^2 + 31w + 15$ **e** $6k^2 + 23k + 20$ **f** $3u^2 + 22u + 35$

g $4x^2 + 21x + 27$ **h** $5e^2 + 39e + 28$ **i** $12h^2 + 13h + 3$

3 Factorise each quadratic expression. R

EXAMPLE 23

a $4k^2 - 11k + 6$ **b** $6w^2 - 17w + 5$ **c** $5p^2 - 23p + 12$

d $6g^2 - 35g + 49$ **e** $3x^2 - 32x + 20$ **f** $21m^2 - 26m + 8$

g $20r^2 - 23r + 6$ **h** $10a^2 - 7a - 6$ **i** $20u^2 - 23u - 21$

j $2p^2 - 17p - 9$ **k** $6y^2 - y - 40$ **l** $12b^2 - 17b - 5$

m $12q^2 - 8q - 15$ **n** $4h^2 + 13h - 12$ **o** $10n^2 + 31n - 14$

4 Factorise each expression given it is a perfect square. R

a $16w^2 - 56w + 49$ **b** $121h^2 + 176h + 64$ **c** $9p^2 - 90p + 225$

5 Factorise each quadratic expression by first taking out a common factor. R

EXAMPLE 24

a $6m^2 - 9m - 60$ **b** $8x^3 - 18x^2 + 4x$ **c** $6 - 5e^2 - 29e$

d $-18w^2 - 51w + 42$ **e** $4 - 4k - 48k^2$ **f** $8 + 2n - 15n^2$

g $48d^2 + 156d + 90$ **h** $30h - 24h^2 + 9$ **i** $3 + 5p - 28p^2$

6 Factorise the quadratic expression $8a^2 + 14ax - 15x^2$. Select **A**, **B**, **C** or **D**. R

A $(4a - 3x)(2a + 5x)$ **B** $(2a - 5x)(4a + 3x)$

C $(5a + 4x)(3a - 2x)$ **D** $(4a - 5x)(2a + 3x)$

7 Factorise each quadratic expression. R

EXAMPLE 21

a $28d^2 - 11d + 1$ **b** $6 - 2y - 20y^2$ **c** $30w^2 + 41w - 15$

d $4p^2 + 23p + 15$ **e** $23a - 12 - 10a^2$ **f** $9r^2 + 29r + 20$

g $28k^2 + 19ky + 3y^2$ **h** $72p - 6p^2 - 3p^3$ **i** $4 - 21m - 18m^2$

j $10g - 54g^2 - 36g^3$ **k** $44u^2 - u - 3$ **l** $32h^2 + 28hm - 15m^2$

Foundation Standard Complex

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5.10 Mixed factorisations

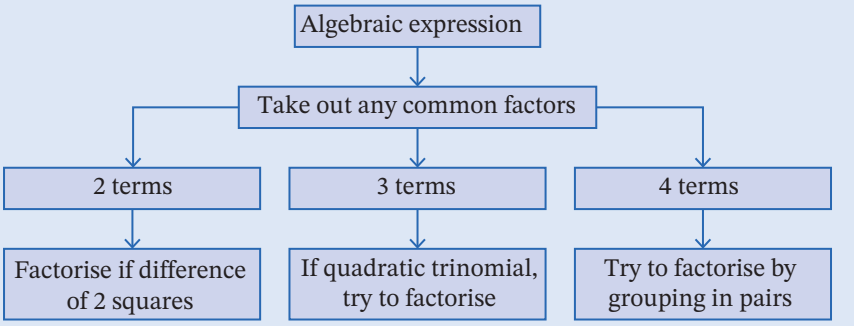
STAGE 5.3

Factor-ominoes

Mixed factorisations

Factorisation strategies

- Look for any common factors and factorise first
- If there are 2 terms, try factorising using the difference of two squares
- If there are 3 terms, try factorising as a quadratic trinomial
- If there are 4 terms, try factorising by grouping in pairs



Example 25

Factorise each quadratic expression.

a $5k^2 - 80$

b $4y^2 + 60d^2$

c $24u^2 - 68u + 20$

d $9m^3 - 18m^2 - 4m + 8$

Solution

a $5k^2 - 80 = 5(k^2 - 16)$ — Taking out the HCF of 5 first.

$= 5(k + 4)(k - 4)$ — Difference of 2 squares

b $4y^2 + 60d^2 = 4(y^2 + 15d^2)$ — 2 terms but not a difference of 2 squares.

c $24u^2 - 68u + 20 = 4(6u^2 - 17u + 5)$ — Taking out the HCF of 4 first.

$= 4(6u^2 - 15u - 2u + 5)$ — Product = 30, sum = −17

$= 4[3u(2u - 5) - 1(2u - 5)]$ — Factorising each pair.

$= 4(2u - 5)(3u - 1)$ — Factorising again.

d $9m^3 - 18m^2 - 4m + 8 = 9m^2(m - 2) - 4(m - 2)$ — Factorising by grouping in pairs

$= (m - 2)(9m^2 - 4)$

$= (m - 2)(3m + 2)(3m - 2)$ — Difference of 2 squares

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EXERCISE 5.10 ANSWERS ON P. 603

STAGE 5.3

Mixed factorisations U F R

EXAMPLE 25

5.10

1 Factorise each expression. R

a $60y^2 - 15$ **b** $15m^2 - 41m + 14$ **c** $k^2 - 14k + 49$

d $11a - 12 - 2a^2$ **e** $6gy + 12cp + 8cy + 9gp$ **f** $15 - 17x - 4x^2$

g $64q^2 - 16q + 1$ **h** $81r^2 - 36w^2$ **i** $3bn^2 - 3np + 2bnd - 2dp$

j $6k^2 + 23k + 20$ **k** $42a^2 + 73a + 28$ **l** $4 - 100h^2$

m $36c^2 + 53c - 63$ **n** $16 - 46d - 35d^2$ **o** $25u^2 + 10u + 4$

p $3xy - 8w + 4x - 6wy$ **q** $1 - b^2 + b^3 - b$ **r** $28c^2 - 9ch - 4h^2$

s $15n^2 - 19n - 56$ **t** $a^3p - ap$ **u** $16r^2 - 146rk + 18k^2$

2 Factorise each expression. R

a $48c^2 - 75$ **b** $8q^2 + 22q - 40$ **c** $15x^2 + x^3 - 2x^4$

d $27p^2 - 24pr - 16r^2$ **e** $9w^2 + 24w - 105$ **f** $n^4 - 4n^2$

g $y - y^2 - y^3 + y^4$ **h** $36u^2 + 105u + 49$ **i** $121k^2 - 198k + 81$

j $4(c - d)^2 - (c + d)^2$ **k** $20y^2 + 18d + 12y + 30dy$ **l** $ah^2 - 6ah - ah^3$

m $18b^2 - 54b + 27b - 81$ **n** $32g - 8g^3y^2$ **o** $8 - 42e - 36e^2$

p $27q^2p - 3p$ **q** $30h - 15h^2 - 15h^3$ **r** $w^4 - w^3 - w^2 + 1$

s $a^5 - 256a^3$ **t** $m^2 - k^2 + 9m - 9k$ **u** $30a^2 + 65a - 25$

3 Factorise the expression $w^6 - w^4 - w^2 + 1$. Select the correct answer **A**, **B**, **C** or **D**. R

A $w^4(w - 1)(w + 1)$ **B** $(w^2 - 1)(w^2 + 1)^2$

C $(w^4 - 1)^2$ **D** $(w^2 - 1)^2(w^2 + 1)$

Factorising algebraic fractions 5.11

STAGE 5.3

Example 26

WS Simplifying algebraic fractions

Simplify each expression.

a $\frac{12x+8y}{4}$ **b** $\frac{30m-18}{25m^2-9}$ **c** $\frac{2h^2+8h}{-3h-12}$ **d** $\frac{5k^2+9k-2}{k^2+7k+10}$

Solution

a $\frac{12x+8y}{4} = \frac{4(3x+2y)}{4}$

$= 3x + 2y$

b $\frac{30m-18}{25m^2-9} = \frac{6(5m-3)}{(5m+3)(5m-3)}$

$= \frac{6}{5m+3}$

c $\frac{2h^2+8h}{-3h-12} = \frac{2h(h+4)}{-3(h+4)}$

$= -\frac{2h}{3}$

d $\frac{5k^2+9k-2}{k^2+7k+10} = \frac{(5k-1)(k+2)}{(k+2)(k+5)}$

$= \frac{5k-1}{k+5}$

Foundation Standard Complex

STAGE 5.3

Example 27

Simplify each expression.

a $\dfrac{5}{x^2+x-2}-\dfrac{2}{x^2-x}$ **b** $\dfrac{6m}{m^2+4m}\times\dfrac{m^2-16}{9}$ **c** $\dfrac{h^2+5h+6}{h^2+4h+3}\div\dfrac{h^2+2h}{h^2-2h-3}$

Solution

a $\dfrac{5}{x^2+x-2}-\dfrac{2}{x^2-x}=\dfrac{5}{(x+2)(x-1)}-\dfrac{2}{x(x-1)}$ Factorising denominators

$=\dfrac{5x}{x(x+2)(x-1)}-\dfrac{2(x+2)}{x(x+2)(x-1)}$ Using common denominators

$=\dfrac{5x-2(x+2)}{x(x+2)(x-1)}$

$=\dfrac{5x-2x-4}{x(x+2)(x-1)}$

$=\dfrac{3x-4}{x(x+2)(x-1)}$

b $\dfrac{6m}{m^2+4m}\times\dfrac{m^2-16}{9}=\dfrac{6m}{m(m+4)}\times\dfrac{(m-4)(m+4)}{9}$

$=\dfrac{{}^{2}\cancel{6m}}{\cancel{m}(\cancel{m+4})}\times\dfrac{(\cancel{m+4})(m-4)}{{}^{3}\cancel{9}}$

$=\dfrac{2(m-4)}{3}$

c $\dfrac{h^2+5h+6}{h^2+4h+3}\div\dfrac{h^2+2h}{h^2-2h-3}=\dfrac{h^2+5h+6}{h^2+4h+3}\times\dfrac{h^2-2h-3}{h^2+2h}$

$=\dfrac{\cancel{(h+2)}(h+3)}{(h+1)\cancel{(h+3)}}\times\dfrac{(h-3)\cancel{(h+1)}}{h\cancel{(h+2)}}$

$=\dfrac{h-3}{h}$

EXERCISE 5.11 ANSWERS ON P. 603

Factorising algebraic fractions U F R

EXAMPLE 26

1 Simplify each expression. R

a $\dfrac{15x+9y}{3}$ **b** $\dfrac{6}{12r-18}$ **c** $\dfrac{4k-8d}{4}$

d $\dfrac{a^2-4}{a+2}$ **e** $\dfrac{3g+3h}{g^2-h^2}$ **f** $\dfrac{4p^2-16}{12p-24}$

g $\dfrac{w^2-49}{4(w+7)^2}$ **h** $\dfrac{7-y}{y-7}$ **i** $\dfrac{c^2+11c+30}{6c+30}$

j $\dfrac{he-hd+pe-pd}{e^2-d^2}$ **k** $\dfrac{k^2+5k-24}{2k^2-18}$ **l** $\dfrac{9y^2-4q^2}{3y^2-4yq-4q^2}$

m $\dfrac{r^2-2r-24}{r^2+12r+32}$ **n** $\dfrac{40+11b-2b^2}{8b^2+34b+35}$ **o** $\dfrac{6m-30-pm+5p}{m^2+3m-40}$

p $\dfrac{25-4a^2}{6a^2-11a-10}$ **q** $\dfrac{5n^2-4n-n^3}{n^2-2n+1}$ **r** $\dfrac{x^2y^2-4}{xy^2-3mxy-2y+6m}$

Foundation Standard Complex

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2 Simplify each expression. R

EXAMPLE 27

a $\frac{7}{k(k-1)} - \frac{4}{k(k+1)}$

b $\frac{8}{(x+4)(x-7)} + \frac{3}{x(x+4)}$

c $\frac{1}{(m+3)(2m+1)} + \frac{4}{(2m+1)(m-2)}$

d $\frac{5}{3h+24} - \frac{1}{h^2-64}$

e $\frac{10}{p^2+4p} + \frac{3}{p^2-4p}$

f $\frac{4}{a^2-25} - \frac{a}{(a+5)^2}$

g $\frac{2}{w^2-9w+20} + \frac{3}{w^2+2w-24}$

h $\frac{d}{d^2-3d-4} - \frac{7}{2d-8}$

i $\frac{9m}{16-25m^2} - \frac{5}{32m-40m^2}$

j $\frac{3u}{u^2+5u-14} - \frac{5}{2u^2-98}$

k $\frac{c+1}{10c^2+7c-12} + \frac{c}{5c^2-39c+28}$

l $\frac{n+1}{4n^3-36n} + \frac{2}{5n^2+15n}$

3 Simplify each expression. R

a $\frac{3r}{5r+25} \times \frac{r+5}{9}$

b $\frac{4y-12}{y} \times \frac{y^2+3y}{8y-24}$

c $\frac{w^2+6w}{2} \div \frac{w^2-36}{10w}$

d $\frac{8a-6}{3a^2+3a} \div \frac{12a-9}{a^3-a}$

e $\frac{5n}{n^2+10n+21} \div \frac{2n^2+4n}{n^2-2n-15}$

f $\frac{x^2+4x-5}{x^2+4x} \times \frac{x^2-16}{3x+15}$

g $\frac{15y-12k}{2y^2+ky-2y-k} \times \frac{8y+4k}{25y^2-16k^2}$

h $\frac{m^2-4m+4}{7m} \div \frac{5m-10}{21m^2+42m}$

i $\frac{1-9m^2}{m^2+9m+20} \times \frac{2m+10}{3m^2+5m-2}$

j $\frac{c^3+8c^2}{6c-18} \times \frac{c^2-6c+9}{c^2-64}$

k $\frac{3e}{e^2+5e-6} \div \frac{5e}{4-4e^2}$

l $\frac{2p-6}{2p^2-7p-4} \div \frac{p^2+3p-18}{p^2+3p-28}$

m $\frac{d^2+10d+21}{5d^2-20} \times \frac{d^2+4d-12}{d^2+6d-7}$

n $\frac{h^2+8h+12}{4h+12} \div \frac{h^2+10h+24}{h^2+4h+3}$

Power plus ANSWERS ON P. 604

1 Expand and simplify each expression.

a $(x+5)^3$ **b** $(y-2)^3$ **c** $(a+b)^3$ **d** $(3d+10)^3$

2 Use the given expansion to evaluate each square number without using a calculator.

a $21^2 = (20+1)^2$ **b** $45^2 = (40+5)^2$

c $29^2 = (30-1)^2$ **d** $59^2 = (60-1)^2$

e $102^2 = (100+2)^2$ **f** $98^2 = 100-2)^2$

3 By expressing 31×29 as $(30+1)(30-1)$, evaluate 31×29 without using a calculator.

4 Use the method of question **3** to evaluate each expression.

a 21×19 **b** 51×49 **c** 89×91 **d** 78×82

5 Expand and simplify each expression.

a $(y+1)^2+(y+2)^2+(y+3)^2$

b $(x-3)(x+3)+(x+3)^2+(x-3)^2$

c $(5n+3)(5n-3)+(3n-5)(3n+5)$

d $2(a-b)(a+b)-(a+b)^2-(a-b)^2$

6 Factorise each expression.

a p^4-16q^4 **b** w^8-256 **c** $(m-2n)^2-(2n+m)^2$

7 **a** Expand $(a-b)(a^2+ab+b^2)$

b Hence, factorise :

i x^3-y^3 **ii** $8m^3-27p^3$

8 **a** Expand $(a+b)(a^2-ab+b^2)$

b Hence, factorise:

i d^3+w^3 **ii** u^3+64h^3

CHAPTER 5 REVIEW

Language of maths

algebraic fraction	base	binomial	binomial product
coefficient	constant term	denominator	difference of 2 squares
expand	factorise	fractional	grouping in pairs
highest common factor (HCF)	index law	indices	perfect square
power	product	quadratic expression	reciprocal
root	sum	term	trinomial

1 Explain the meaning of **quadratic** and **trinomial**. Give an example of a **quadratic trinomial**.

2 Any number raised to the power of 0 is equal to what?

3 What is the difference between **expand** and **factorise**?

4 What power is associated with the **reciprocal** of a term or number?

5 In the quadratic expression $2x^2 - 3x + 6$, what is:

a the constant term? **b** the coefficient of x?

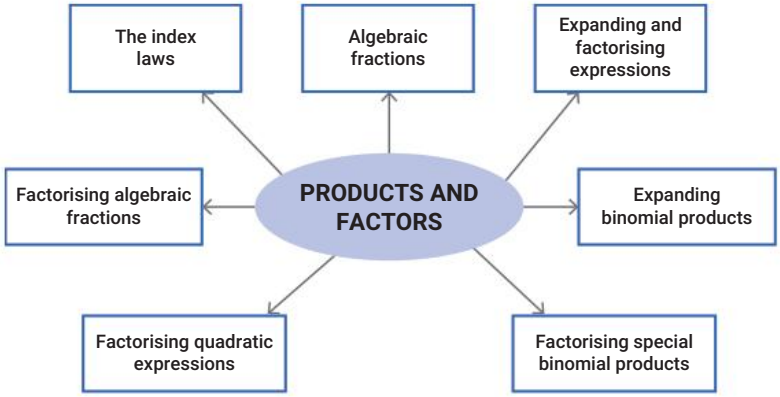
6 Copy and complete:

To factorise quadratic expressions of the form $x^2 + bx + c$, first find 2 numbers that have a _____ of b and a _____ of c.

Topic summary

- What was this topic about? What was the main theme?
- What content was new and what was revision for you?
- Write 3 index laws in both words and symbols
- Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Numbers and powers

Advanced algebra and functions

Algebra

Algebra 6

Mind map: Products and factors

TEST YOURSELF 5

ANSWERS ON P. 604

5.01

1 Simplify each expression.

a $3v^4w^3 \times 2v^5w^8$ **b** $\frac{24r^8h^8}{3rh^2}$ **c** $(5xy^2)^2$

d $\left(\frac{2p}{3}\right)^0$ **e** $(4k)^{-1}$ **f** $\left(\frac{2p}{3}\right)^3$

g $(4m)^{-2}$ **h** $4m^{-2}$ **i** $\left(\frac{5b^8y^6}{b^2y^3}\right)^4$

j $(4t^4u^5)^3 \times 8t^2u$ **k** $45c^6d^8 \div (-3cd^2)^2$ **l** $\left(\frac{45ab^4}{54a^2b^3}\right)^{-1}$

5.01

2 Simplify each expression.

a $\left(\frac{1}{10}\right)^{-5}$ **b** $\left(\frac{3}{5}\right)^{-2}$ **c** $\left(\frac{4g}{7}\right)^{-3}$ **d** $\left(\frac{9}{2d}\right)^{-2}$

STAGE 5.3

5.02

3 Evaluate each expression.

a $400^{\frac{1}{2}}$ **b** $(-27)^{\frac{5}{3}}$ **c** $64^{-\frac{1}{2}}$ **d** $(-32)^{\frac{2}{5}}$

5.02

4 Evaluate each expression.

a $\left(16a^4\right)^{\frac{3}{2}}$ **b** $\sqrt[4]{\left(81m^{12}\right)^5}$ **c** $\frac{1}{\left(\sqrt[3]{8p^6}\right)}$ **d** $\left(125x^3y^6\right)^{-\frac{2}{3}}$

5.03

5 Simplify each expression.

a $\frac{r}{4} - \frac{3r}{5}$ **b** $\frac{5g}{3} + \frac{3g}{2}$ **c** $\frac{11x}{24} - \frac{2x}{3}$ **d** $\frac{3}{4} - \frac{2m}{7}$

STAGE 5.3

5.03

6 Simplify each expression.

a $\frac{3k-4}{5} + \frac{2k}{3}$ **b** $\frac{w+1}{4} - \frac{w-3}{9}$ **c** $\frac{2p+5}{3} - \frac{7p}{12}$ **d** $\frac{5+2y}{11} + \frac{1-y}{8}$

5.04

7 Simplify each expression.

a $\frac{4m}{5} \times \frac{3m}{8}$ **b** $\frac{6}{d} \times \frac{4de}{21}$ **c** $\frac{5}{a} \div \frac{10}{b}$ **d** $\frac{vw^3}{5} \div \frac{3v}{20} \times \frac{4}{3w}$

5.05

8 Expand and simplify each expression.

a $8pq(7p - 5q)$ **b** $-5h^2(3h + 7)$

c $4eg(g - 6e) - 6e^2g$ **d** $12(9 - n) - 5(2n + 3)$

e $x^2(6x + x^2) + 2x(5x^3 + x^2)$ **f** $3(7 - 2y) - 5y(3 + 4y)$

5.05

9 Factorise each expression.

a $16ar^2 + 24ar$ **b** $-24p + 18q$

c $2(5x - 1) - 3x(5x - 1)$ **d** $15xy^2 - 30x^3y^3$

e $6pt^2 + 12p^2t - 48p^3$ **f** $32r^2m^4 + 12r^4m^3$

g $50x^4y^3 - 75x^3y^4$ **h** $-8p^3q^3 + 48p^3q^6$

i $n(n^2 + 6) - (n^2 + 6)$

Foundation Standard Complex

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10 Expand each binomial product.

a $(b + 3)(b + 10)$ **b** $(d + 8)(d - 10)$ **c** $(w - 6)(9 - w)$

d $(5x + 7)(4x - 3)$ **e** $(7y - 3)(7y + 3)$ **f** $(5p - 8)(5p - 8)$

11 Expand each binomial product.

a $(n + 9)(n - 9)$ **b** $(3y + 2d)^2$ **c** $(4n - 11)(4n + 11)$

12 Factorise each expression.

a $3pu + 2qt + 2qu + 3pt$ **b** $4ab + 6bc - 6ad - 9dc$ **c** $b^2 - 100$

d $25 - 16y^2$ **e** $20x^2 - 125$ **f** $3r^3 - 27r$

13 Factorise each expression.

a $y^2 + 10y + 25$ **b** $q^2 - 21q + 20$ **c** $n^2 + 8n - 33$

d $a^2 - 11a + 28$ **e** $m^2 - 5m - 84$ **f** $p^2 + 3p - 54$

14 Factorise each expression.

a $3w^2 + 5w + 2$ **b** $2y^2 - 3y - 9$ **c** $60 - 5b - 5b^2$

d $3p^2 + 10p - 8$ **e** $12x^2 - 46x + 14$ **f** $6n^2 - 13n + 6$

g $40y^2 + 49y - 24$ **h** $48c^2 - 64c - 35$ **i** $10e^2 - 7e + 1$

15 Factorise each expression.

a $5q^2 - 45$ **b** $20x^2 - 52x + 24$ **c** $c^3 - c^2 - c + 1$

d $4 - 47k - 12k^2$ **e** $6m^2 + m - 40$ **f** $a^4 - a^2 + a^3 - a$

16 Simplify each expression.

a $\dfrac{20d - 35}{10}$ **b** $\dfrac{4r + 12y}{7r^2 + 21ry}$

c $\dfrac{u^2 - 4u - 32}{2u^2 - 128}$ **d** $\dfrac{1}{25 - h^2} - \dfrac{3}{h^2 + 13h + 40}$

e $\dfrac{4}{p^2 - 9} + \dfrac{2}{p^2 + 6p + 9}$ **f** $\dfrac{6m^2 - 24}{7m + 14}$

g $\dfrac{3}{y^2 + 4y} - \dfrac{y}{y^2 - 16}$ **h** $\dfrac{3w - 15}{5} \times \dfrac{20w}{3w^2 - 15w}$

i $\dfrac{b^2 + 3b + 2}{b^2 - 9} \div \dfrac{b^2 + b}{3b + 9}$ **j** $\dfrac{6a}{a^2 + 5a + 6} - \dfrac{2}{a^2 - 5a - 14}$

k $\dfrac{3 - 12q^2}{q^2 + 4q + 4} \times \dfrac{q^2 + q - 2}{6q - 12q^2}$ **l** $\dfrac{r^3 - 25r}{r^2 - 9} \div \dfrac{r^2 + 12r + 35}{12r - 36}$

5.06 5.07 5.07 5.08 5.09 5.10 5.11

Foundation Standard Complex

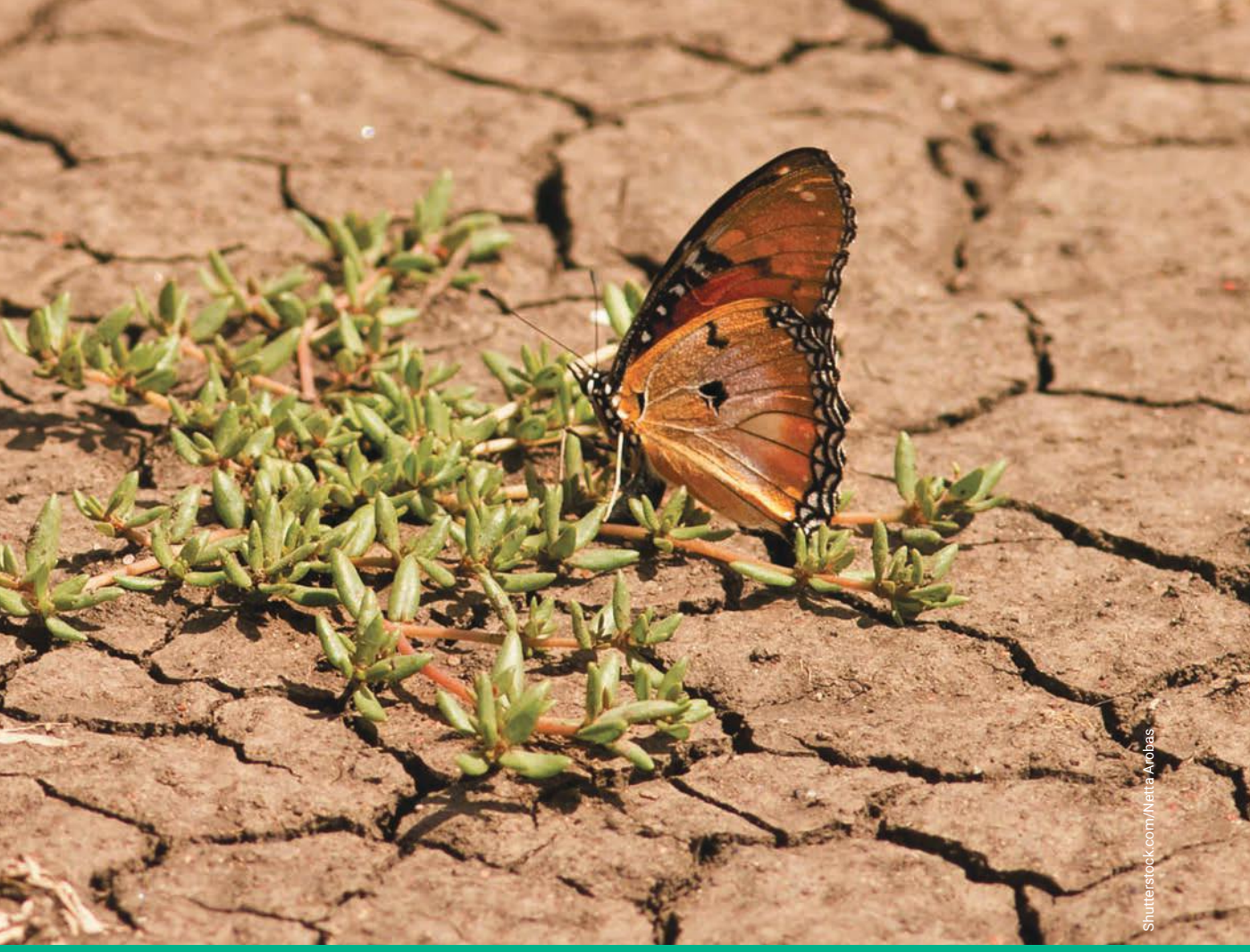


6

STATISTICS AND PROBABILITY

COMPARING DATA

Is climate change affecting the amount of rainfall in different areas? What are the tourist numbers in different parts of Australia, and how much do they spend? How do we ensure accurate medical testing data to monitor the potential for pandemics such as COVID-19? To answer these questions, sets of data need to be collected and then compared by looking at the shape of their displays or by analysing their measures of centre and spread.



Chapter outline

		Working mathematically				
6.01	The shape of a frequency distribution	U	F		R	C
6.02	Quartiles and interquartile range	U	F			
6.03	Standard deviation*	U	F		R	C
6.04	Comparing means and standard deviations*	U	F		R	C
6.05	Box plots	U	F		R	C
6.06	Parallel box plots	U	F		R	C
6.07	Comparing data sets	U	F		R	C
6.08	Scatterplots	U	F		R	C
6.09	Line of best fit*	U	F		R	C
6.10	Bivariate data involving time	U	F		R	C
6.11	Statistics in the media	U	F		R	C
6.12	Investigating statistical studies*	U	F	PS	R	C

*** STAGE 5.3**

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Wordbank

bivariate data Data that measures 2 variables, represented by an ordered pair of values that can be graphed on a scatterplot

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the box contains the middle 50% of values while the lines or 'whiskers' extend to the 2 extremes

five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value

interquartile range (IQR) The difference between the upper quartile and lower quartiles, IQR = $Q_3 - Q_1$, representing the middle 50% of values

quartiles The values Q_1, Q_2, Q_3 that divide a set of data into quarters (4 equal parts)

scatterplot A graph consisting of dots on a number plane that represent bivariate data

standard deviation (symbol σ_n) A measure of spread that depends on every value in the data set and their mean

In this chapter you will:

- describe the shape of a statistical distribution in terms of its symmetry and skewness, using terms such as 'skewed', 'symmetric' and 'bimodal'
- calculate quartiles and the interquartile range of a set of data
- (STAGE 5.3) calculate the mean and standard deviation of a set of data and compare data sets
- calculate the five-number summary of a set of data and use it to construct a box plot
- construct histograms, dot plots, back-to-back stem-and-leaf plots and parallel box plots to compare 2 or more sets of data
- compare shapes of box plots, histograms and dot plots
- graph bivariate data on scatterplots to investigate patterns and relationships between independent and dependent variables, including strength and direction
- (STAGE 5.3) use technology to construct lines of best fit and use them to make predictions
- investigate bivariate data involving time
- evaluate statistics presented in the media and identify biases in sampling, sources, graphs, interpretation, claims and conclusions
- (STAGE 5.3) analyse and evaluate media reports of statistical studies

SkillCheck ANSWERS ON P. 604


Statistical measures

1 For each set of data, find:

i the range **ii** the mean (correct to one decimal place)

iii the median **iv** the mode

a 15 13 18 14 15 18 23 14 20 16 15

b 8°C 3°C −5°C 2°C −4°C 7°C 3°C 0°C

c

Stem	Leaf
1	0 3 6
2	1 4 4 7 8
3	2 3 4 5 5 7 9
4	0 5 7 8
5	2 6 8

d

Score	Frequency
0	2
1	5
2	8
3	4
4	3
5	1

2 A cricketer scored these numbers of runs in 10 innings.

34 21 78 30 26 19 41 36 16 32

a Find:

i the median **ii** the mean **iii** the range.

b Which score is the outlier?

c **i** Calculate the median, mean and range if the outlier is not included in the scores.

ii What effect does the outlier have on the mean, median and range?

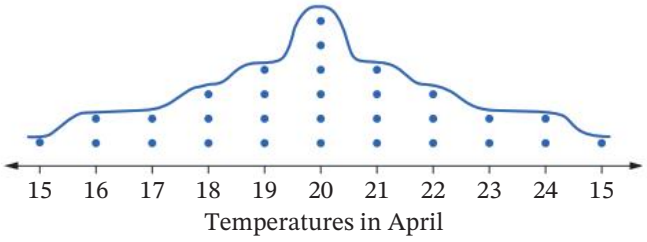
The shape of a frequency distribution 6.01

A **statistical distribution** shows how a **data** set is arranged, especially when graphed. When looking at **histograms, dot plots** and **stem-and-leaf plots**, an overall pattern can be seen from the shape of the display.

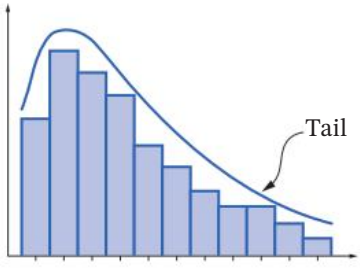
The shape of a statistical distribution shows how the data is spread. This can be seen by drawing a curve around the graph or display.

A distribution is **symmetrical** if the data is evenly spread or balanced about the centre.

Stem	Leaf
3	0 2 4
4	1 8 9 9
5	2 4 5 6 6 7 8 8
6	0 3 4 5 5 6 7 8 9 9
7	2 4 4 4 5 5 5 5
8	2 8 8 8
9	3 5 7



A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other 'end' has a 'tail'.



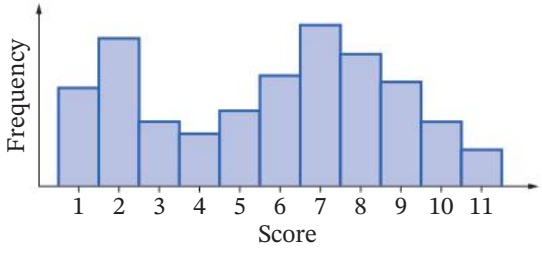
A distribution is **positively skewed** if its tail points to the right (higher values).

Stem	Leaf
0	3 5
1	0 6
2	5 7 8
3	0 3 8 9
4	1 1 2 3 4 8
5	0 0 1 1 2 2 5 5
6	3 5 7 5 6 6 7 7 9
7	0 2 2 4 5

A distribution is **negatively skewed** if its tail points to the left (lower values).

A distribution is **bimodal** if it has 2 peaks. The higher peak is the mode, while the other peak indicates another value that has a high frequency.

For example, this frequency histogram has 2 peaks at 2 and 7 so it is bimodal. The mode, however, is 7.

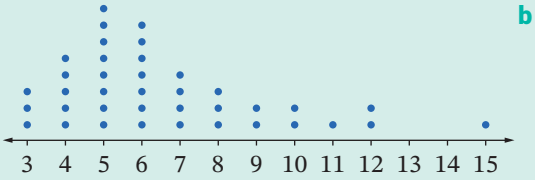


Example 1

For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters

a



b

Stem	Leaf
10	4 5
11	3 4 4 9
12	1 2 2 6 8
13	0 1 5 5 7 9 9 9
14	4 5 6 8 8
15	0 0 1 1
16	0 2

Solution

a **i** The shape is positively skewed (tail points towards the higher values).

ii 15 is an outlier and clustering occurs at 5 and 6.

b **i** The shape is symmetrical (the data is balanced about the stem of 13).

ii There are no outliers, but clustering occurs in the 130s.

EXERCISE 6.01 ANSWERS ON P. 604

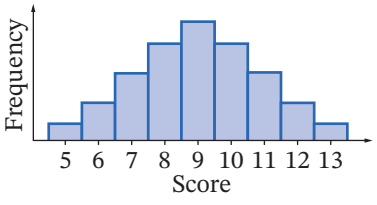
The shape of a distribution U F R C

EXAMPLE 1

1 For each statistical distribution: R C

i describe the shape **ii** identify any outliers and clusters.

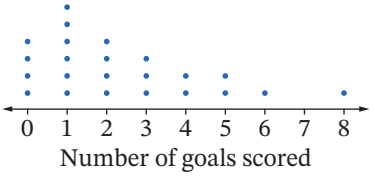
a



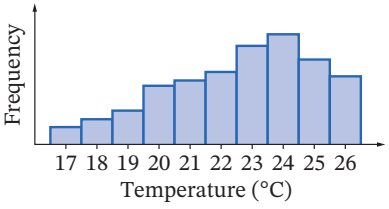
b

Stem	Leaf
2	4 5 6 9
3	1 2 3 3 4 5 7 8
4	0 4 4 6 8 9
5	4 5 5 8
6	0 0 2 3 5 6 7 8 9 9
7	3 5 7 8 8 9 9
8	1 1 3 5 6
9	0 3 5 6

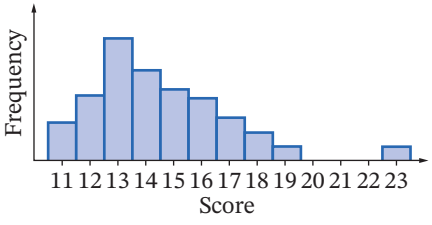
c



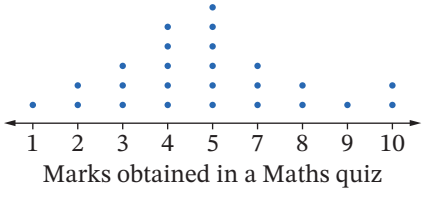
d



e



f



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g

Stem	Leaf
12	0 2 4 9
13	2 4 6 7 8 8 8
14	3 3 4 4 5 5 8 9 9 9
15	0 1 1 5 7 8 9 9
16	1 1 5 6 7
17	2 4 5 8
18	0 3 9
19	5 8
20	6 8

h

Stem	Leaf
5	3 4 4 6 7 8 9
6	0 0 5 9 9
7	2 4 5 6
8	5 7 8
9	3 3 6 7 8
10	2 4 6 8 8 8 8
11	
12	
13	6

2 These are the final round scores for players in a golf tournament. R C

66	70	67	72	75	72	70	74	75	72
74	72	73	71	71	69	70	71	71	74
72	69	75	73	69	75	73	69	69	67
74	72	72	73	71	73	77	68	72	72

a Arrange the data into a frequency table and construct a frequency histogram.

b Are there any outliers?

c Describe the shape of the distribution.

d Give a possible reason for the shape of the distribution.

e Where does clustering occur?

f Find the mode, the mean and the median and show their position in the histogram.

3 The stem-and-leaf plot shows the number of hours that students spend on their computers during the week. R C

Stem	Leaf
0	1 1 1 1 1 2 2 2 2 3 3 3 5 6 6 7 7 7 7 9 9
1	0 1 1 1 2 4 4 5 6 8 8 9
2	0 5 5 5 8 8
3	0 0 0 1 5
4	0 0

a How many students were surveyed?

b Where does the clustering occur?

c Are there any outliers?

d Describe the shape of the distribution.

e Give a possible reason for the shape of the distribution.

f Find the mean (to one decimal place), median and mode.

4 The following values are the heights (in cm) of 30 Year 8 students. R C

162	155	153	162	182	173	165	165	142	167
164	168	150	155	143	153	123	163	170	169
153	162	161	170	160	162	172	151	160	171

a Arrange the data into an ordered stem-and-leaf plot.

b Describe the shape of the distribution.

c Are there any outliers?

d Where does clustering occur?

e Find the mode, median and mean.

5 The daily maximum temperatures (correct to one decimal place) in Brisbane in November are shown in the stem-and-leaf plot. R C

Stem	Leaf
24	1
25	8
26	0 1 2 7 7
27	0 9 9
28	1 1 2 4 4 5 6 7 8 8
29	0 1 3 3 3
30	6 8 9
31	5
32	8

a Describe the shape of the distribution.

b Are there any outliers?

c What is the mode?

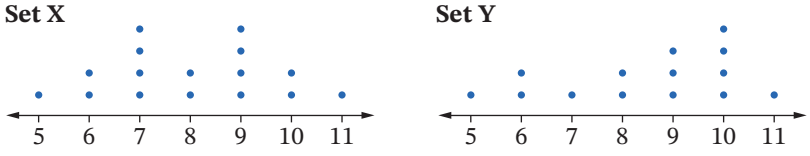
d Find the mean, correct to one decimal place.

e What is the median?

f Find the range.

g Is the range a good indicator of the spread of the temperatures? Give reasons.

6 Which statement is true about the sets of data below? Select the correct answer **A**, **B**, **C** or **D**.



A X is symmetrical and Y is positively skewed.

B X is bimodal and Y is negatively skewed.

C X has 2 peaks and the median of Y is 8.

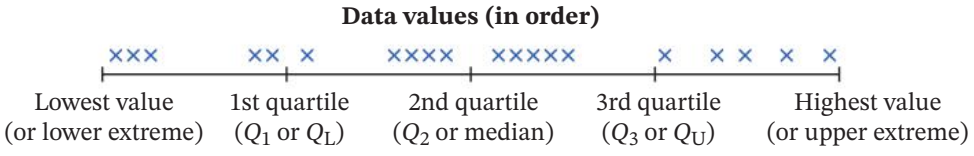
D The median of X and Y is 8.

Quartiles and interquartile range 6.02

Quartiles

The **median**, being the middle value, divides a set of data into 2 equal parts (halves).

Quartiles are the values Q_1, Q_2 and Q_3 that divide the set of data into 4 equal parts (quarters).



The **1st quartile** Q_1, also called the **lower quartile** Q_L, is the value that divides the lower 25% of values. $\frac{1}{4}$ of the values lie below Q_1.

The **2nd quartile** Q_2 is the value that divides the lower 50% of values, so it is also the **median**. $\frac{1}{2}$ of the values lie below Q_2.

The **3rd quartile** Q_3, also called the **upper quartile** Q_U, is the value that divides the lower 75% of values from the upper 25% of values. $\frac{3}{4}$ of the values lie below Q_3, $\frac{1}{4}$ of the values lie above it.

Finding the quartiles of a data set

- Sort the values in order, find the median and call it Q_2
- Find the median of the bottom half of the values and call it Q_1 (or Q_L)
- Find the median of the top half of values and call it Q_3 (or Q_U).

Example 2

Find the quartiles for each set of data.

a	9	3	8	7	6	8	4	6	2	10	9		
b	15	18	7	16	23	9	15	20	16	14	13	11	19
c	65	84	75	82	97	70	68	76	93	48			
	79	54	80	79	82	96	63	85	72	70			

Solution

a Arranging the 11 values in ascending order, we have:

2 3 (4) 6 6 (7) 8 8 (9) 9 10

Lower quartile $Q_1 = 4$ | Median $Q_2 = 7$ | Upper quartile $Q_3 = 9$

Q_2 (median) = 7

Q_1 (lower quartile) = 4 — The middle of the 5 values below 7

Q_3 (upper quartile) = 9 — The middle of the 5 values above 7

b Arranging the 13 values in ascending order, we have:

7 9 11 13 14 15 15 16 16 18 19 20 23

$Q_1 = \frac{11+13}{2} = 12$ $\quad Q_2 = 15$ $\quad Q_3 = \frac{18+19}{2} = 18.5$

c Arranging the 20 values in ascending order, we have:

48 54 63 65 68 70 70 72 75 76 79 79 80 82 82 84 85 93 96 97

$Q_1 = \frac{68+70}{2} = 69$ $\quad Q_2\text{(median)} = \frac{76+79}{2} = 77.5$ $\quad Q_3 = \frac{82+84}{2} = 83$

Interquartile range 1

Interquartile range 2

The interquartile range

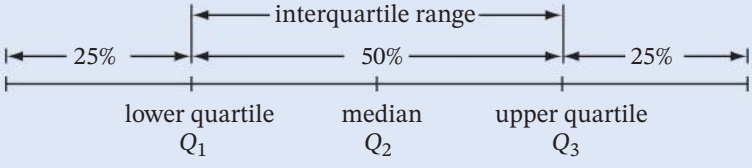
The **range** is a **measure of spread** because it gives an indication of how widely the values are spread in a set of data. Another measure of spread is the **interquartile range**.

The **interquartile range** is the difference between the upper and lower quartiles and so it is the range of the middle 50% of the data.

Interquartile range

Interquartile range (IQR) = upper quartile – lower quartile

$= Q_3 - Q_1$



The interquartile range takes into account the middle 50% of values and ignores very low or very high values (outliers), so sometimes it is better to use than the range as a measure of spread.

Example 3

The number of runs scored by the Sydney Sixers Twenty20 cricket team per match during one season were:

76 143 127 176 142 116 137 104 161 174 149 154 180 137 175

a Find the range.

b Find the interquartile range.

c Which is the better measure of spread of the points scored by the Sixers – the range or interquartile range?

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Solution

First arrange the scores in order:

76 104 116 **127** 137 137 142 **143** 149 154 161 **174** 175 176 180

Lower quartile: $Q_1 = 127$ | Median: $Q_2 = 143$ | Upper quartile: $Q_3 = 174$

a Range $= 180 - 76$
$= 104$

b Interquartile range $= Q_3 - Q_1$
$= 174 - 127$
$= 47$

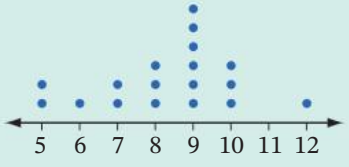
c The interquartile range is the better measure of spread as the outlier of 76 is excluded. The score of 76 has affected the range, making it very big.

6.02

Example 4

Find the interquartile range of each data set.

a

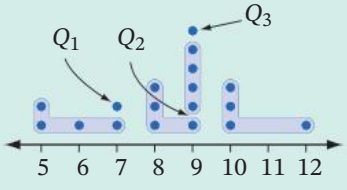


b

Stem	Leaf
1	2 7
2	0 3 4 4 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

Solution

a There are 18 values, so the median is 'between' the 9th and 10th values, which are both 9s (see diagram).



Median, $Q_2 = \frac{9+9}{2} = 9$

Q_1 is the median of the bottom half of 9 values (the 5th value).

$Q_1 = 7$.

Q_3 is the median of the top half of values (the 14th value).

$Q_3 = 9$.

$\therefore$ IQR $= Q_3 - Q_1$
$= 9 - 7$
$= 2$

b There are 21 values, so the median is the 11th value (see diagram).

Stem	Leaf	
1	2 7	Q_1 between 5th–6th values
2	0 3 4 4 5	
3	1 2 2 **4** 6 8 8 9	
4	0 1 3 7	Q_2 11th value
5	1 2	Q_3 between 16th–17th values

$Q_2 = 34$

Lower quartile, $Q_1 = \frac{24+24}{2} = 24$

Upper quartile, $Q_3 = \frac{40+41}{2} = 40.5$

$\therefore$ IQR $= 40.5 - 24$
$= 16.5$

EXERCISE 6.02 ANSWERS ON P. 605

Quartiles and interquartile range U F

EXAMPLE 2

1 Find the quartiles for each set of data.

a 3 7 9 5 5 6 2 8 9 7

b 15 19 18 12 20 34 28 18 28 20 23 25

c 34 45 32 38 29 40 37 33 35
30 34 35 38 37 38 31 30 34

EXAMPLE 3

2 Calculate the range and the interquartile range of each data set in question **1**.

3 Calculate the interquartile range for each set of data below.

a 5 6 6 7 8 9 9 10 14 14 15 16

b 2 0 3 5 2 1 0 6 4 3 8 4 2

4 The monthly rainfall figures in mm for Ulladulla one year were:

31 174 288 89 15 123 26 5 8 275 38 58

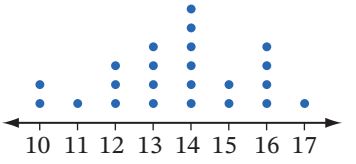
For this data, find:

a the range **b** the interquartile range

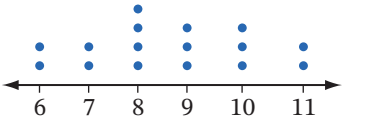
EXAMPLE 4

5 Find the interquartile range for each set of data.

a



b



c

Stem	Leaf
3	2 7
4	0 3 3 5
5	2 4 5 6 7 8 8
6	3 4 7
7	2

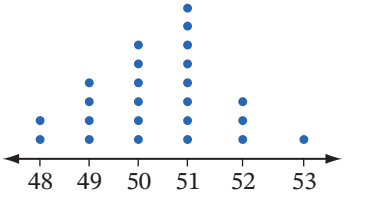
d

Stem	Leaf
1	3 5 8 9
2	0 1 3 3 4 5 6
3	5 8 9 9
4	1 3
5	4

e

Stem	Leaf
10	3 5 5 6 6
11	0 1 2
12	3 4 6 7 8
13	4 7
14	1

f



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6 The pulse rates for a group of students are as follows.

82 81 72 58 79 77 62 66 92 78 80 67 91 75 72 68

a Find the range.

b Find the interquartile range.

c **i** List the values that lie between the lower and upper quartiles.

ii What percentage of values lie between Q_1 and Q_3?

d What percentage of values lie above the lower quartile?

7 The number of points per game scored by a basketball team during one season were:

55 35 49 53 51 55 42 48 63 43 48 48 62

a Find:

i the range **ii** the interquartile range

b Which is the better measure of spread?

c List the values that lie between Q_1 and Q_3. What percentage of the values is this?

Did you know?

How did statistics begin?



Alamy Stock Photo/Adam Ján Figeľ

In prehistoric times, when the number of people and animals was recorded in pictures and symbols on the walls of caves, a simple form of statistics was being used.

Before 3000 BCE, ancient Babylonians used clay tablets to record crop yields and trade data, and around 2650 BCE the Egyptians 'surveyed' the population and wealth of their country before building the pyramids. Forms of statistics were also used in the Bible in the 'Book of Numbers' and the 'First Book of Chronicles'. Numerical records existed in China before 2000 BCE, and the Greeks (to help collect taxes) held a census in 594 BCE. The Roman Empire was the first government to collect information about the population. In 1086 a census was conducted in England. The information obtained in this census was recorded in the Domesday Book.

Use your library or the Internet to find out more about the Domesday Book. Write a one-page report suitable for a classroom presentation.

6.03 Standard deviation

STAGE 5.3

The **standard deviation** is another measure of spread. Like the mean, it is calculated using all the values in a data set.

Standard deviation

- The **standard deviation** of a set of data is an average of how different each value is from the mean.
- The symbol for standard deviation is σ or σ_n. σ is the lower-case Greek letter 'sigma'
- Standard deviation has a complex formula so it is best calculated using the calculator's statistics mode.
- It is a better measure of spread than the range and interquartile range because its value depends on every value in the data set.

Standard deviation on a Casio calculator

Example 5

Calculate, correct to 2 decimal places, the standard deviation of each set of data.

a The daily maximum temperature (in °C) in Campbelltown for 2 weeks in January.

45.0	24.5	24.8	29.1	35.0	26.9	31.8
33.8	32.9	23.6	22.1	29.2	27.1	32.7

b The scores of Year 10 students in a Science quiz.

Score	2	3	4	5	6	7	8	9	10
Frequency	2	1	3	3	2	5	6	4	2

Solution

Follow the instructions for the statistics mode (SD or STAT) of your calculator as shown in the tables below.

a

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data	SHIFT 1 Data to get table 45.0 = 24.5 =, etc. to enter in column AC to leave table	45.0 M+ 24.5 M-, etc.
Calculate the standard deviation (σ_x = 5.75)	SHIFT 1 Var σ_x =	RCL σ_x
Return to normal (COMP) mode.	MODE COMP	MODE 0

$\sigma = 5.75$

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b

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data	SHIFT 1 Data to get table 2 = 3 =, etc. To enter in x column 2 = 1 =, etc. in FREQ column AC to leave table	2 2nd F STO 2 M+ 3 2nd F STO 1 M-
Calculate the standard deviation ($\sigma_x = 2.26$)	SHIFT 1 Var σ_x =	RCL σ_x
Return to normal (COMP) mode.	MODE COMP	MODE 0

$\sigma = 2.26$

EXERCISE 6.03 ANSWERS ON P. 605

Standard deviation UFRC

Note: In this exercise, express all means and standard deviations to 2 decimal places.

EXAMPLE 5

1 Calculate the standard deviation of each set of data.

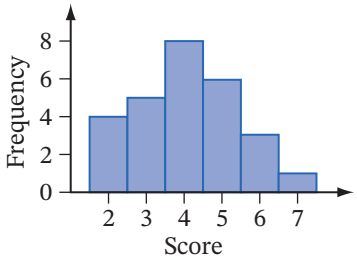
a 5 4 7 8 2 9 10

b 20 23 28 24 19 25 26 24 23

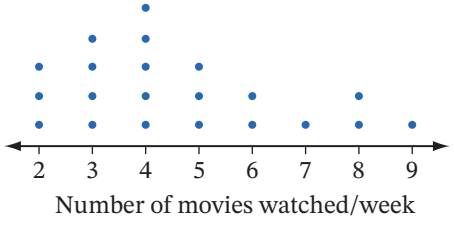
c

x	f
10	2
11	5
12	9
13	8
14	3
15	1

d



e



2 An English class of Year 10 students scored the following marks for their speeches: R C

12 15 14 16 16 12 11 18 7 10

15 14 13 13 18 10 12 12 14 13

a Which mark is the outlier?

b Find the standard deviation of the marks:

i with the outlier **ii** without the outlier

c What effect does removing the outlier have on the standard deviation?

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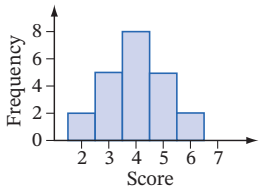
STAGE 5.3

3 For the 3 statistical distributions A, B and C shown, which one has: R

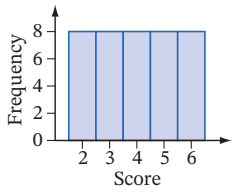
a the highest standard deviation?

b the lowest standard deviation?

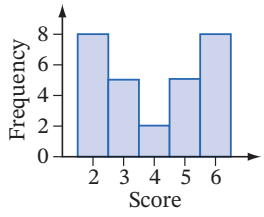
A



B

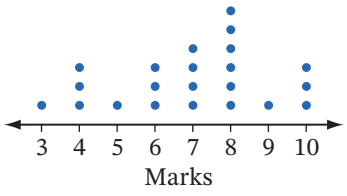


C



4 Find the standard deviation of each data set.

a



b

Stem	Leaf
2	0 2 7
3	5 5 6 8 9
4	1 2 4 5 6 6 7
5	0 3 4 5 9 9
6	1 5 5
7	6

5 The heights of girls in a Year 9 basketball team are: R

151 161 171 175 176 157 175 163 164

a Calculate the mean and the standard deviation of heights of the basketball team.

b Divya joins the basketball team. What is her possible height if the standard deviation:

i increases **ii** decreases?

6 The training times (in seconds) of Harrison sprinting over 100 m were: R

11.2 11.0 10.9 12.3 11.8 11.1 11.4 11.6 11.0

a Find the mean and standard deviation of his training times.

b What training time would Harrison have to do to:

i increase the standard deviation?

ii decrease the standard deviation?

7 Brooke's times (in seconds) for swimming 100 m were: R C

55.7 59.8 58.4 56.7 60.0 55.8 57.4 58.0

An error was made in recording these times and 2 seconds needs to be added to each time.

Which of the following statements is true? Select the correct answer **A**, **B**, **C** or **D**.

A The standard deviation will increase and the mean will stay the same.

B The standard deviation will decrease and the mean will increase.

C The standard deviation will stay the same and the mean will increase.

D The standard deviation and the mean will both stay the same.

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Investigation

The formula for standard deviation

The formula for the standard deviation of a set of data is $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$, where x is each data value, $\bar{x}$ is the mean and n is the number of data values.

The steps for calculating standard deviation are:

- Calculate the mean $\bar{x}$
- For every value in the data set, find the difference between the value and the mean, then square this difference: $(x-\bar{x})^2$
- Calculate the average of these 'squared deviations' by adding them and dividing their sum by the number of values: $\frac{\Sigma(x-\bar{x})^2}{n}$
- Calculate the square root of this average: $\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

We will now use this method to calculate the standard deviation of this set of data:

4 5 6 7 2 8 6 5 2

1 Calculate the mean of this data.

2 Copy and complete the table below by finding for each data value its difference from the mean and the square of this difference.

Score, x	4	5	6	7	2	8	6	5	2
$x-\bar{x}$	−1	0							
$(x-\bar{x})^2$	1	0							

3 Find the mean of the 'squared deviations' calculated in the bottom row of the table.

4 The standard deviation is the square root of this mean. Calculate the standard deviation correct to 2 decimal places.

5 Check your answer by calculating the standard deviation using your calculator's statistics mode and comparing both answers.

6 Use the standard deviation formula to calculate the standard deviation of each set of data.

a 5 4 7 8 2 9 10

b 20 23 28 24 19 25 26 24 23

Check your results by using your calculator.

7 The standard deviation is never negative. Explain why.

8 If the values of a set of data are all the same, what is the standard deviation? Explain.

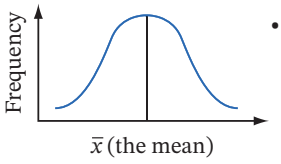
STAGE 5.3

Did you know?

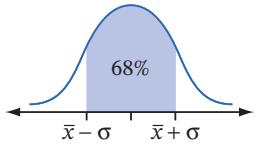
The normal curve

If the heights of all the people in Australia were graphed on a frequency polygon, the graph would be a normal curve, a symmetrical bell-shaped curve that peaks in the middle.

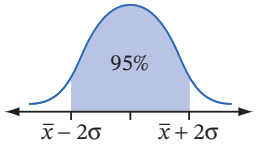
The normal curve has the following features:



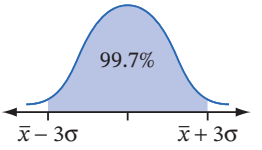
- The mean, median and mode are the same.



- About 68% of scores lie within one standard deviation of the mean.



- About 95% lie within two standard de-viations of the mean.



- About 99.7% lie within three standard deviations of the mean.

Measure and analyse the heights of the students at your school. Do the data follow a normal curve?

6.04 Comparing means and standard deviations

STAGE 5.3

The **mean** and **standard deviation** can be used to compare different sets of data.

Example 6

The heights (in cm) of the girls and boys in a Year 10 PE class at Baramvale High were measured:

Girls:	163	155	171	162	165	158	172	
	166	163	150	160	181	160	156	
Boys:	174	167	164	175	189	145	165	166
	165	168	167	171	169	172	168	

a Calculate, correct to 2 decimal places, the mean and standard deviation for:

i the girls **ii** the boys **iii** the class

b Which group has the greater spread of heights?

c Is there a significant difference between the heights of girls and boys?

Solution

a Using the calculator's statistics mode:

i Girls: $\bar{x} = 163$ cm, $\sigma_n \approx 7.60$

ii Boys: $\bar{x} \approx 168.33$ cm, $\sigma_n \approx 8.64$

iii Class: $\bar{x} \approx 165.76$ cm, $\sigma_n \approx 8.58$

b The data for the boys in the class has the greater spread of heights as its standard deviation is higher.

c The mean height of boys was greater than that of the girls but the girls had the lower spread of heights.

Comparing measures of spread

The **standard deviation** is usually the most appropriate measure of spread as it uses all of the values in the data set.

The **range** is the easiest to calculate but its value only depends upon 2 values: the highest and the lowest.

If there are **outliers** in the data set, then the standard deviation and range will be affected by these extreme scores. In this case, the **interquartile range** is the better measure, because it is the range of the middle 50% of scores and so is not affected by outliers.

Example 7

a The ages of the children using a jumping castle and visiting a petting zoo are shown:

Jumping castle:	3	3	4	5	5	6	8	10	18
Petting zoo:	3	4	5	6	6	7	8	8	10

For each set of data, calculate:

i the range **ii** the interquartile range

iii the standard deviation (to 2 decimal places)

b Which is the best measure of spread for each set of data?

Solution

a For the jumping castle:

i Range $= 18 - 3$
$= 15$

ii IQR $= 9 - 3.5$
$= 5.5$

iii $\sigma_n \approx 4.48$

For the petting zoo:

i Range $= 10 - 3$
$= 7$

ii IQR $= 8 - 4.5$
$= 3.5$

iii $\sigma_n \approx 2.05$

b The jumping castle data has an outlier, 18, which affects the range and standard deviation. The interquartile range is the best measure for this data set.

The petting zoo data do not have an outlier, so the standard deviation is the best measure for this data set.

STAGE 5.3

EXERCISE 6.04 ANSWERS ON P. 605

Comparing means and standard deviations U F R C

Note: In this exercise, express all means and standard deviations correct to 2 decimal places.

EXAMPLE 6

1 The pulse rates (in beats/minute) of a sample of men and women taken at a shopping centre are shown. R C

Men:	68	72	75	73	81	77	69	68	79	83	65	59	60	72	70
Women:	82	61	79	77	75	68	86	81	72	77	78	81	90	83	73

a Find the mean and standard deviation of each group.

b Is there a significant difference between the mean and standard deviation for men and women? Give reasons.

2 The reaction times (in seconds) for the dominant and non-dominant hands of a group of athletes were measured. R C

Dominant hand:	0.41	0.29	0.35	0.42	0.42	0.43	0.39	0.61	0.38
	0.34	0.75	0.34	0.38	0.47	0.34	0.32	0.29	0.30
Non-dominant hand:	0.46	0.34	0.38	0.39	0.39	0.39	0.51	0.50	0.47
	0.40	2.60	0.34	0.39	0.51	0.35	0.37	0.31	0.32

a Find the mean and standard deviation for each data set.

b Is there a significant difference between the results? Explain your answer.

c **i** What are the outliers for the reaction time of the dominant hand?

ii Find the mean and standard deviation without the outliers.

iii What effect does removing the outliers have on the mean and standard deviation?

d Find the mean and standard deviation of the reaction time for the non-dominant hand without the outlier.

e On which group has the removal of outliers had the greater effect on the mean and standard deviation? Justify your answer.

3 The scores of 2 cricket teams were recorded on a back-to-back stem-and-leaf plot. R

Western Tigers		Barrington City
5 2	7	
	8	3
7	9	0 8
8	10	7
	11	4 6
6	12	1 5
9 9 8 5	13	7
7 4	14	6
5	15	6 8

a Find the mean and standard deviation for each team.

b Which team was more consistent with its scores?

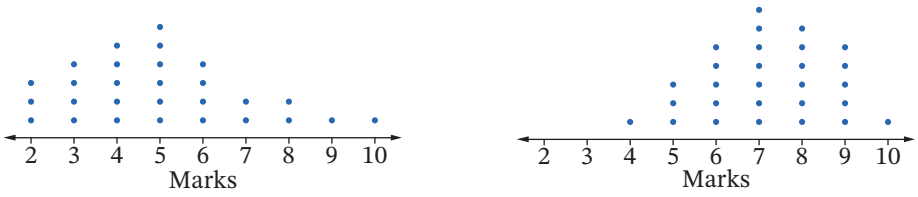
4 Vatha and Ana's times for running 100 m trials are shown. R C

Vatha:	13.0	13.5	14.2	13.7	13.2	14.7	13.5	14.3
Ana:	14.2	13.2	15.1	13.8	14.2	15.2	13.9	13.5

a Find the mean and standard deviation for each runner.

b Which runner is more consistent? Give reasons.

5 The dot plots show the test results of a class before and after using a tutorial website.



Which of the following is true?

A Both the mean and standard deviation increased.

B The mean increased and the standard deviation decreased.

C The mean decreased and the standard deviation increased.

D Both the mean and standard deviation decreased.

6 The marks obtained by students in Maths and Science exams are given below. R C

EXAMPLE 7

Maths: 40 72 76 74 60 64 64 59 74 84 62 84 66 64
71 68 78 63 57 55 73 80 67 86 57 87 62 52

Science: 42 54 61 72 76 54 65 80 39 74 82 54 57 63
64 75 68 76 81 40 37 43 58 68 67 49 54 62

a For each subject, find:

i the range **ii** the interquartile range **iii** the standard deviation

b Find the mean for each subject.

c Determine in which subject the students performed better, giving reasons.

7 The points scored per match by the Roosters and the Dragons during a football season were: R C

Roosters: 10 16 8 50 22 38 34 30 16 12 18 38 12 20 18
36 40 28 42 28 56 22 22 24

Dragons: 10 6 17 25 19 13 10 18 14 32 0 14 14 16 10
0 22 18 20 26 18 18 22 19

a For each team, find:

i the range **ii** the interquartile range

iii the mean **iv** the standard deviation

b By comparing the means and the measures of spread, decide which was the better team.

6.05 Box plots

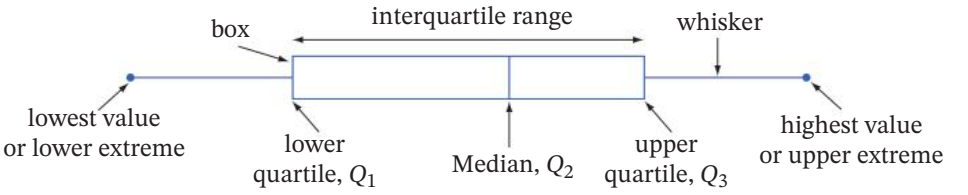
Box-and-whisker plots

Statistics

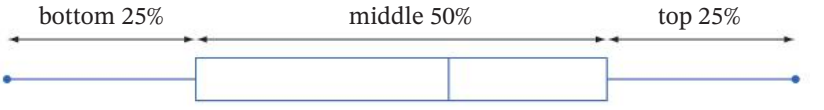
WS Five-number summaries

Mode, median and mean

A **box plot** (or **box-and-whisker plot**) displays the quartiles of a set of data and the lowest and highest values (lower and upper extremes).



The 'box' represents the middle 50% of values and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of values.



Five-number summary

A box plot gives a **five-number summary** of a data set:

- the lower extreme (or lowest value)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the upper extreme (or highest value)

Example 8

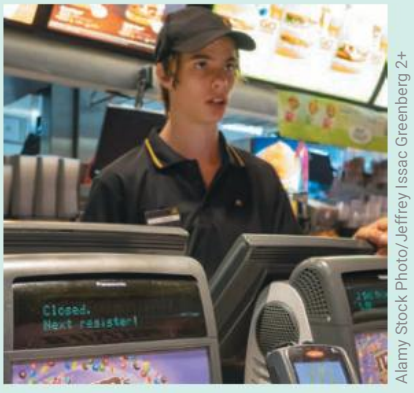
The number of hours per week that Nick worked at the Big Chicken over summer were:

5 5 4 8 10 3 12

7 7 3 8 8 15

a Find a five-number summary for this data.

b Represent this data on a box plot.



Alamy Stock Photo/Jeffrey Issac Greenberg 2+

Solution

a First arrange the values in order.

3 3 4 5 5 7 7 8 8 8 10 12 15

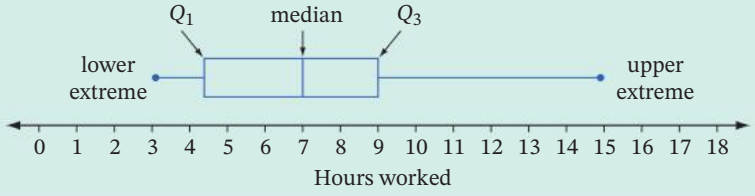
Q_1 Median Q_2 Q_3

Lowest value = 3

$Q_1 = \frac{4+5}{2} = 4.5$ $\quad$ $Q_3 = \frac{8+10}{2} = 9$

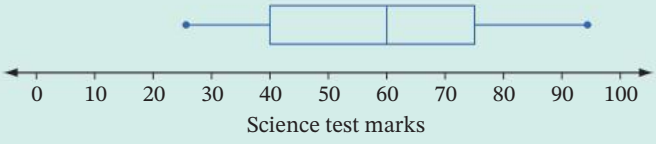
Median = 7 $\quad$ Highest value = 15

b



Example 9

The box plot represents the results of 80 students in a Science test.



a Find the range of the test results.

b Find the median test value.

c What is the interquartile range?

d How many students had a test mark between:

i 25 and 75? $\quad$ **ii** 40 and 60?

e What percentage of students scored more than 75?

Solution

a Range = highest value – lowest value
= 95 – 25
= 70

b Median = 60

c Interquartile range = $Q_3 - Q_1$
= 75 – 40
= 35

d **i** 25 is the lowest value and 75 is Q_3, so 75% × 80 = 60 students had a mark between 25 and 75.

ii 40 is Q_1 and 60 is the median, so 25% × 80 = 20 students had a mark between 40 and 60.

e 75 is Q_3, so 25% × 80 = 20 students scored more than 75.

Five number summary

Five number summary

EXERCISE 6.05 ANSWERS ON P. 606

Box plots UFRC

EXAMPLE 8

1 The number of orders taken per hour at Bernoulli's Pizza on a weekend were:

3	5	1	2	4	6	8	10	7	6
12	15	10	3	5	18	5	8	9	10

a Find the five-number summary for this data.

b Represent this data on a box plot.

2 The daily amount of snow (in cm) that fell at Thredbo during one ski season was:

2	5	5	2	5	7	1	2	2	2	2	2	12
20	12	5	40	50	10	40	13	30	5	35	2	6

a On how many days did it snow?

b Find a five-number summary for this data.

c Represent this data on a box plot.

3 The monthly rainfall figures for Penrith one year were:

98	266	149	94	15	65	19	5	24	34	67	28

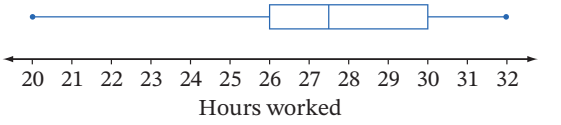
a Find the range.

b Find the five-number summary.

c Represent the data on a box plot.

EXAMPLE 9

4 This box plot represents the number of hours worked in one week by the staff at a supermarket.



a What is the median number of hours worked?

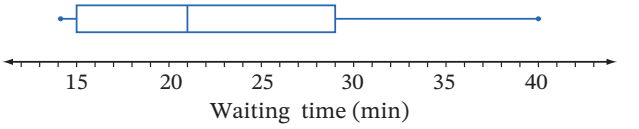
b What is the lower quartile?

c What is the upper quartile?

d Find the interquartile range.

e Estimate the percentage of staff who worked between 26 and 30 hours.

5 The ages of 16 people waiting at a bus stop are displayed in the box plot below.



a What is the range?

b What is the median age?

c Find the interquartile range.

d What percentage of people were aged from:

i 21 to 29?

ii 15 to 40?

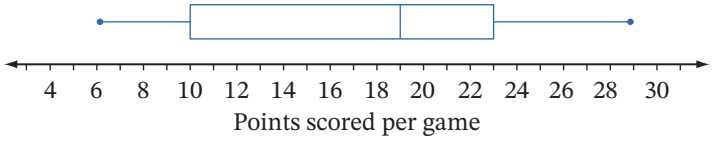


Getty Images/Mark Kolbe

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6 The box plot shows the number of points per game scored by Ben in 28 basketball games during the season.



a What is the five-number summary for the box plot?

b Find the interquartile range.

c In how many games did Ben score:

i more than 19 points? **ii** between 19 and 23 points?

iii less than 10 points? **iv** at least 10 points?

7 For each set of data, find the five-number summary and draw a box plot.

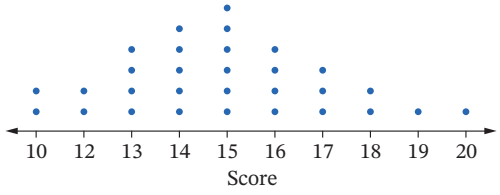
a

Stem	Leaf
2	0 2 3 5
3	3 7
4	4 6 7 8 8 9 9
5	0 1 1 5 6
6	0 3 3 8 8
7	2 5 6
8	5 5 7 8

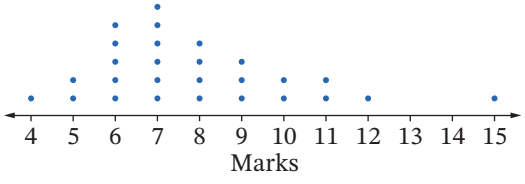
b

Stem	Leaf
3	0 7
4	2 6 6
5	1 2 5 9
6	0 4 7 7 9
7	2 3 5 6 8
8	3 4
9	5

c



8 The results of a general knowledge quiz (out of 15) taken by Year 10 students are displayed in the dot plot. R C



a Find the five-number summary for the dot plot and then draw a box plot.

b Describe the shape of the dot plot and compare it to the shape of the box plot.

c What is the outlier?

d Find the five-number summary for the data in the dot plot without the outlier and draw a box plot.

e Compare the 2 box plots. How are they:

i similar? **ii** different?

6.05

Technology

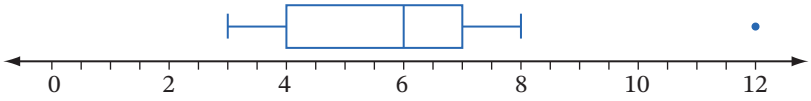
Box plots

Use graphing software or a spreadsheet to draw box plots.

1 Enter this set of data: 3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 12

2 Choose appropriate settings for the scale on your box plot.

3 Your box plot should look similar to the one below.



4 Write down the five-number summary for this data set.

5 Now enter the marks of an English exam completed by 2 classes 10A and 10B and create a box plot for each class:

10A: 21, 81, 33, 58, 67, 76, 64, 74, 56, 60, 54, 74, 49, 83, 66

10B: 77, 63, 63, 35, 51, 42, 54, 55, 71, 43, 41, 41, 40, 76, 72

6 Complete a five-number summary for each class.

7 What is the IQR for each class?

8 Which class had the highest mark?

9 Which class had the lowest mark?

10 Which class performed better? Give reasons for your answer, including explanations using the five-number summaries you found in step 7.

6.06 Parallel box plots

Parallel box-and-whisker plots can be used to compare 2 or more sets of data. They are drawn on the same scale, but above each other.

Double boxplots

Analysing data

Parallel box plots

Example 10

Two sprinters run the following times (in seconds) over 100 metres.

Sam:	10.9	10.5	11.0	9.9	10.7	10.5	10.0	11.2	11.5	10.3
Jesse:	11.0	11.4	10.1	9.8	10.8	11.4	10.7	10.3	11.1	11.6

a Find the five-number summary for each sprinter.

b Draw parallel box plots to display the data for both sprinters.

c Find the interquartile range for each sprinter.

d Find the range for each sprinter.

e Which sprinter is more consistent? Justify your answer.

Solution

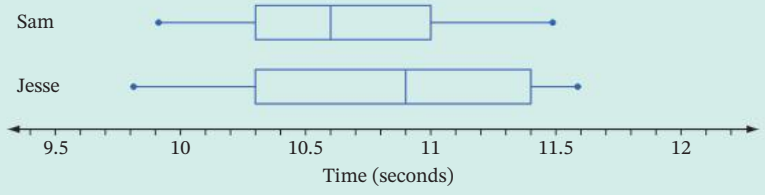
a Sam: 9.9 10.0 10.3 10.5 10.5 10.7 10.9 11.0 11.2 11.5

Lowest value: 9.9 | Q_1: 10.3 | $Q_2 = \frac{10.5+10.7}{2} = 10.6$ | Q_3: 11.0 | Highest value: 11.5

Jesse: 9.8 10.1 10.3 10.7 10.8 11.0 11.1 11.4 11.4 11.6

Lowest value: 9.8 | Q_1: 10.3 | $Q_2 = \frac{10.8+11.0}{2} = 10.9$ | Q_3: 11.4 | Highest value: 11.6

b



c Interquartile range for Sam = 11.0 − 10.3 = 0.7

Interquartile range for Jesse = 11.4 − 10.3 = 1.1

d Range for Sam = 11.5 − 9.9 = 1.6

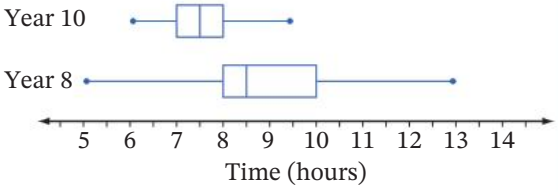
Range for Jesse = 11.6 − 9.8 = 1.8

e Sam is the more consistent sprinter, since both the range and interquartile range of his times are lower than those of Jesse.

EXERCISE 6.06 ANSWERS ON P. 606

Parallel box plots UFRC

1 The parallel box plots show the number of hours of sleep that Year 8 and Year 10 students usually have on a school night.



a For each Year group, find:

i the range **ii** the median **iii** the interquartile range

b What percentage of students usually had at most 8 hours of sleep on a school night in:

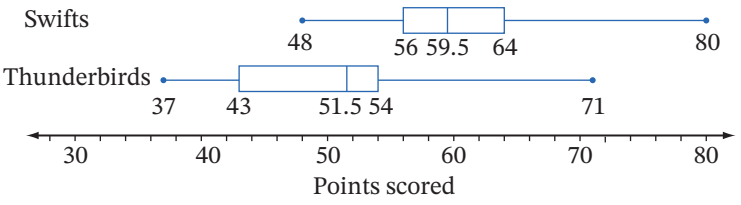
i Year 8? **ii** Year 10?

c 40 students in both Year 8 and Year 10 were surveyed. How many students usually had at least 10 hours of sleep in:

i Year 8? **ii** Year 10?

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2 The number of points scored per match by the Adelaide Thunderbirds and the NSW Swifts netball teams during a season are shown in the parallel box plot. R C



a Find the range of points scored by:

i the NSW Swifts

ii the Adelaide Thunderbirds

b What is the median number of points scored for each team?

c Find the interquartile range for each team.

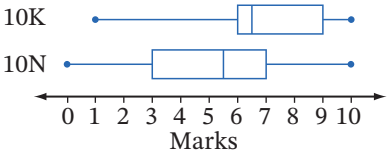
d Which team is more consistent?

e Which team performed better? Give reasons.



Getty Images/Matt King

3 The box plots show the test results of students from 2 different classes. R C



a Find the range of marks for each class.

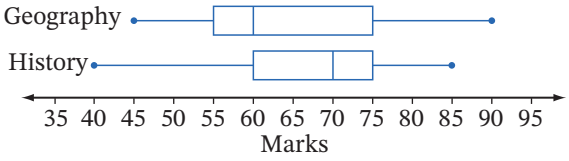
b Find the median mark for each class.

c Find the interquartile range for each class.

d Which class is more consistent?

e Find the percentage of students who scored 6 or more in 10K.

4 In a Year 10 class of 28 students, the marks for History and Geography tests were displayed using a double box plot. R



Which statement below could be true? Select the correct answer **A**, **B**, **C** or **D**.

A In Geography, more students scored between 60 and 75 than between 55 and 60.

B 14 students scored the same or more in History than the median mark in Geography.

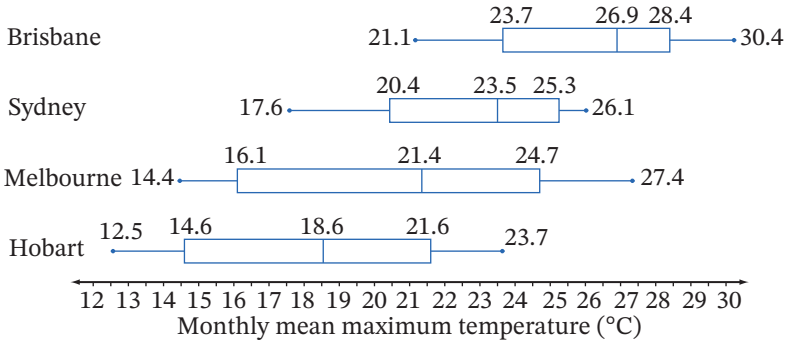
C More students scored 60 or more in History than they did in Geography.

D The interquartile range for Geography is 5 less than the interquartile range for History.

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5 The monthly mean maximum temperatures for 4 state capitals are shown in the box plots. R C



a Find the median, range and interquartile range for each city.

b Which capital city had the most spread in temperature?

c Which capital city had the highest mean monthly temperatures? Justify your answer.

d Which city is warmer – Sydney or Melbourne? Give reasons.

e Which city was more consistent – Sydney or Melbourne? Give reasons.

EXAMPLE 10

6 The number of text messages received by a group of students in one hour are as follows. R C

Male:	2	0	3	0	1	2	5	6	2	1	3	2	3	7	4
Female:	4	5	6	3	7	5	8	7	4	2	4	5	10	4	3

a Find the five-number summary for each sex.

b Draw parallel box-and-whisker plots to display the data.

c Find the interquartile range for each sex.

d Find the range for each sex.

e Compare the number of text messages that males and females receive. Are there any significant differences between the spread of the 2 sets of data?

7 Students in a PE class had their heights measured (in cm). R C

Male:	174	167	164	175	189	145	165	166	165	167	171	169
Female:	163	155	171	162	165	183	172	175	166	163	150	186

a Find the five-number summary for each group and draw a parallel box plot to display the data.

b Find the range and interquartile range for each group.

c How does the spread of heights of male students compare with the spread of heights of female students?

8 Students at a university were asked whether their frequency of exercise was high or low and then had their pulse taken. R C

Low:	90	78	80	84	70	66	92	80	80	77	64	88
High:	96	71	68	56	64	60	50	76	78	49	68	74

a Find a five-number summary for each group and then draw parallel box plots to show the information.

b Find the range and interquartile range for each group.

c Compare the spread between the 2 groups. Are there significant differences between them?

d Which group had the lower pulse rates?

9 The average maximum monthly temperatures (in °C) for Hobart and Darwin are shown. R C

Hobart:	22.7	22.3	20.9	18.2	15.3	13.0	12.5	13.5	15.4	17.4	19.1	20.8
Darwin:	31.8	31.5	32.0	32.7	32.1	30.7	30.6	31.4	32.6	33.3	33.3	32.7

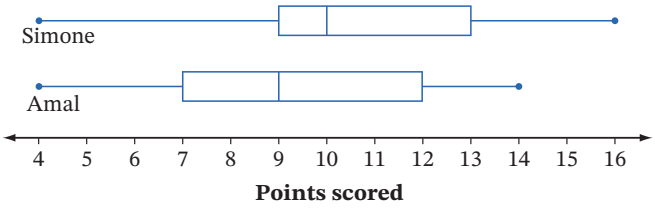
Source: © Bureau of Meteorology

a Find the five-number summary for each city and draw a parallel box plot.

b Find the range and interquartile range for each city.

c Which city had more consistent average maximum monthly temperatures? Give reasons.

10 These box plots show the numbers of points scored by 2 basketball players during the season. R C



a Which player has the most points scored for a single game?

b What is the range of the points scored by each player?

c By just looking at the range, which player would seem to be more consistent? Justify your answer.

d Find the median score of each player.

e Find the interquartile range for each player.

f Which player is more consistent?

g Estimate the percentage of games in which Simone scored 9 or 10 points.

h Estimate the percentage of games in which Amal scored more than 12 points.

Mental skills 6: Maths without calculators ANSWERS ON P. 607

Multiplying and dividing by 5, 15, 25 and 50

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$18 \times 5 = 18 \times \frac{1}{2} \times 10$ (or $9 \times 2 \times 10$)

$= 9 \times 10$

$= 90$

b **To multiply by 50**, halve the number, then multiply by 100.

$26 \times 50 = 26 \times \frac{1}{2} \times 100$ (or $13 \times 2 \times 100$)

$= 13 \times 100$

$= 1300$

c **To multiply by 25**, quarter the number, then multiply by 100.

$44 \times 25 = 44 \times \frac{1}{4} \times 100$ (or $11 \times 4 \times 25$)

$= 11 \times 100$

$= 1100$

d **To multiply by 15**, halve the number, then multiply by 30.

$8 \times 15 = 8 \times \frac{1}{2} \times 30$ (or $4 \times 2 \times 15$)

$= 4 \times 30$

$= 120$

e **To divide by 5**, divide by 10 and double the answer, as there are 2 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$

$= 14 \times 2$

$= 28$

f **To divide by 50**, divide by 100 and double the answer.

$400 \div 50 = 400 \div 100 \times 2$

$= 4 \times 2$

$= 8$

g **To divide by 25**, divide by 100 and multiply the answer by 4.

$600 \div 25 = 600 \div 100 \times 4$

$= 6 \times 4$

$= 24$

h **To divide by 15**, divide by 30 and double the answer.

$240 \div 15 = 240 \div 30 \times 2$

$= 8 \times 2$

$= 16$

2 Now evaluate each expression.

a	32×5	**b**	14×5	**c**	48×5	**d**	18×50
e	52×50	**f**	36×25	**g**	28×5	**h**	12×25
i	12×15	**j**	22×35	**k**	$90 \div 5$	**l**	$170 \div 5$
m	$230 \div 5$	**n**	$1300 \div 50$	**o**	$900 \div 50$	**p**	$300 \div 25$
q	$1000 \div 25$	**r**	$360 \div 45$	**s**	$210 \div 15$	**t**	$360 \div 15$

6.07 Comparing data sets

Comparing data sets

Back-to-back stem-and-leaf plots

WS Comparing city temperatures

WS Comparing word lengths

WS Investigating young drivers

Example 11

The back-to-back stem-and-leaf plot shows the results in Year 10 Maths and Science tests.

Maths		Science
5 2	3	6 8
8 6 3 0	4	4 6
8 7 7 4 1	5	1 5 9
8 8 7 6 6 3 2 0	6	0 2 8 9
6 5 4 2 1 1	7	2 3 4 4 5 8 8
6 4 3	8	0 0 2 4 5 6 7 8 9
6 0	9	0 4 4

a Find the mean mark (correct to one decimal place) for each subject.

b Find the median for each subject.

c Find the range and interquartile range for each subject.

d For each subject:

i describe the shape **ii** identify any outliers and clusters.

e In which subject have the students performed better? Justify your answer.

Solution

a Mean for Maths $= \frac{1919}{30}$

$= 63.9666 \ldots$

≈ 64.0

Mean for Science $= \frac{2151}{30} = 71.7$

b Median for Maths $= \frac{66+66}{2} = 66$

Median for Science $= \frac{74+75}{2} = 74.5$

c Range for Maths $= 96 - 32 = 64$

Interquartile range $= 74 - 54 = 20$

Range for Science $= 94 - 36 = 58$

Interquartile range $= 85 - 60 = 25$

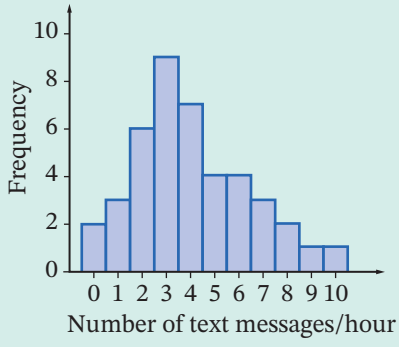
d **i** The results for Maths are symmetrical, while the results for Science are negatively skewed.

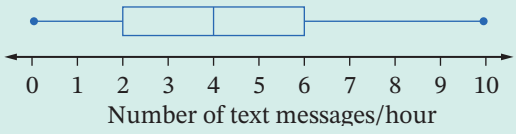
ii No outliers. There is some clustering for the Maths results in the 60s and 70s, and in Science the clustering occurs in the 70s and 80s.

e The students have performed better in Science as the mean and median for it are greater than the mean and median for Maths. This can also be seen by the shape of the data on the stem-and-leaf plot.

Example 12

The number of text messages received per hour by a group of teenagers are displayed in the frequency histogram and the box plot below.





a How many teenagers received more than 6 text messages per hour?

b Find:

i the mode **ii** the median

iii the range **iv** the interquartile range.

c The shape of the distribution is positively skewed. How is this shown by:

i the frequency histogram? **ii** the box plot?

d According to the box plot, what percentage of teenagers received 2 or more text messages?

e What information is better seen on:

i the frequency histogram? **ii** the box plot?

Solution

a Number of teenagers receiving more than 6 text messages

$= 3 + 2 + 1 + 1$ Using the frequency histogram.

$= 7$

b i Mode $= 3$ Using the frequency histogram.

ii Median $= 4$ Using the box plot.

iii Range $= 10 - 0$ Using the frequency histogram or box plot.

$= 10$

iv Interquartile range $= 6 - 2$ Using the box plot.

$= 4$

c i The tail of the frequency histogram leans towards the higher values.

ii The length of the box plot to the right of the median (Q_2) is greater than its length to the left of the median.

d $Q_1 = 2$, so 75% of teenagers received 2 or more text messages per hour.

e i The mode and information regarding the number of text messages received by teenagers can be determined from the frequency histogram.

ii The median, quartiles and interquartile range are easily determined from the box plot.

EXERCISE 6.07 ANSWERS ON P. 607

Comparing data sets U F R C

EXAMPLE 11

1 The back-to-back stem-and-leaf plot shows the amount of cash (in dollars) carried by a sample of Year 11 students at Nelson Senior High. R C

Boys		Girls
5 5 3	0	5 5 6 8 9
8 5 5 2 0	1	0 2 2 5 5 8 8 9
9 6 5 5 5 0 0	2	0 5 6 8 8 8
8 5 5 4 3 2 0 0	3	0 1 4 5 6
5 4 4 2 2 0	4	0 0 5 6
6 6 5 4 3	5	0 3 5
4 2 2	6	5 5 8
5	7	0 4

a Find the mean amount of cash (correct to the nearest cent) carried by each group.

b Find the median amount of cash carried by each group.

c Find the range and interquartile range for each group.

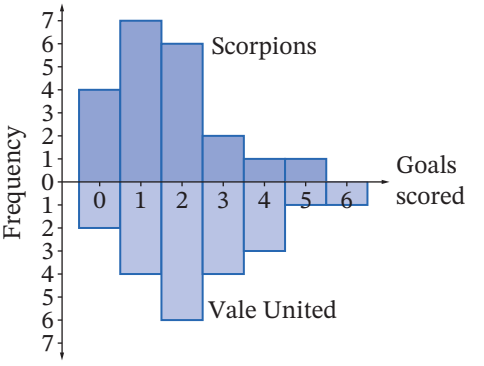
d For each group:

i describe the shape

ii identify any outliers and clusters.

e Who generally carries more cash – boys or girls? Justify your answer.

2 The back-to-back histogram shows the number of goals scored by 2 football teams during a season. R C



a How many games were played by each team?

b How many goals were scored by

i the Scorpions

ii Vale United?

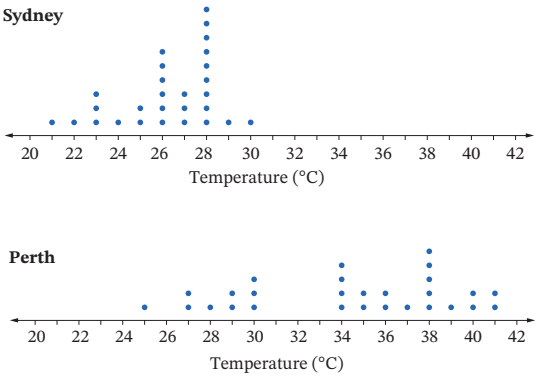
c Find the mean number of goals scored by each team.

d What is the range for each team?

e Describe the shape of each team's results.

f Which team performed better? Give reasons.

3 The daily maximum temperatures for Sydney and Perth in February are shown. R C



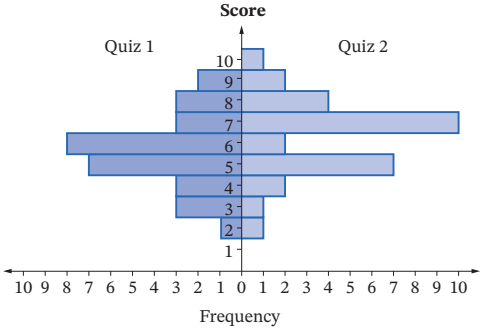
a Find the mean (to 1 decimal place), median and modal temperatures for each city.

b Find the range and interquartile range of temperatures for each city.

c Describe the distribution shape of the temperatures for each city and identify any outliers or clusters.

d Compare the temperatures in Sydney and Perth. Comment on **measures of central tendency** (the mean, median and mode), and measures of spread (range and inter-quartile range).

4 The results for 2 quizzes taken by a Year 10 History class are shown. R C



a How many students are in the Year 10 History class?

b Find the mean and mode for each quiz.

c Find the median for each quiz.

d For each quiz, find the:

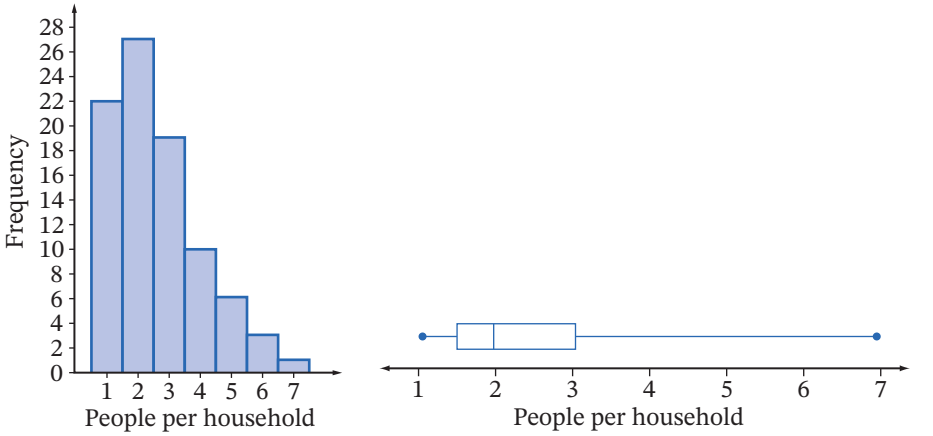
i range **ii** interquartile range.

e Describe the distribution for each quiz, identifying any clusters and outliers.

f Are there significant differences between the results of the 2 quizzes? Justify your answer.

EXAMPLE 12

5 A survey to determine the number of people per household was conducted in several shopping centres. The results are shown in the frequency histogram and box plot. R C



a How many households had 3 or more people?

b Find the:

i mode **ii** median

iii range **iv** interquartile range.

c Describe the shape of the distribution.

d According to the box plot, what percentage of households had 2 or more people?

e Clustering occurs at 1 to 3 people per household. How is this shown on the:

i frequency histogram? **ii** box plot?

f What information is better seen on the:

i frequency histogram? **ii** box plot?

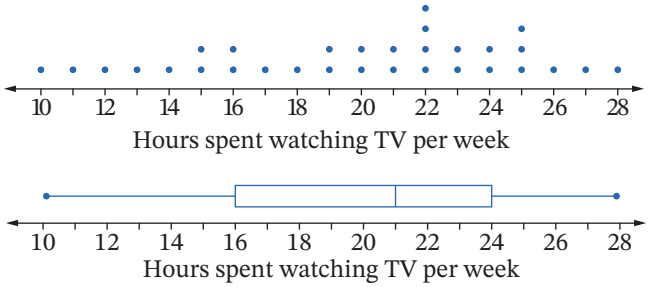


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6 The dot plot and box plot show the number of hours that Year 10 students spent watching TV during one week. R C



a How many students watched TV for:

i fewer than 15 hours per week? **ii** more than 20 hours per week?

b Find the:

i mode **ii** range **iii** interquartile range

c What is the shape of the distribution? How is this shown by the:

i dot plot? **ii** box plot?

d Which display of data, the dot plot or box plot, is better for finding the:

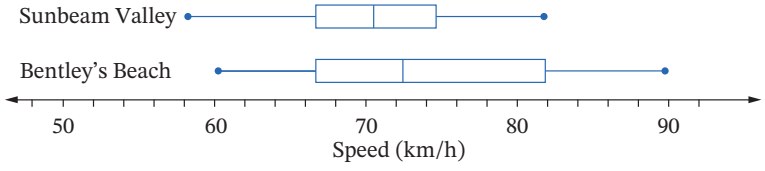
i mode? **ii** median?

iii number of students who watched TV for 25 hours?

iv interquartile range?

7 The speeds of cars were monitored along a main road in 2 different places. The results are shown in the back-to-back stem-and-leaf and parallel box plots. R C

Sunbeam Valley		Bentley's Beach
8	5	
9 8 8 7 4 3 3 3 2 0	6	0 0 1 2 3 5 5 7 8 9
9 9 6 5 5 4 4 3 3 2 2 1 1 0 0 0	7	0 0 2 2 3 3 5 5 5 6 6
2 0 0	8	0 2 3 4 5 5 5 8
	9	0



a Find the range, median and interquartile range for each place.

b What is the shape of the distribution for each place?

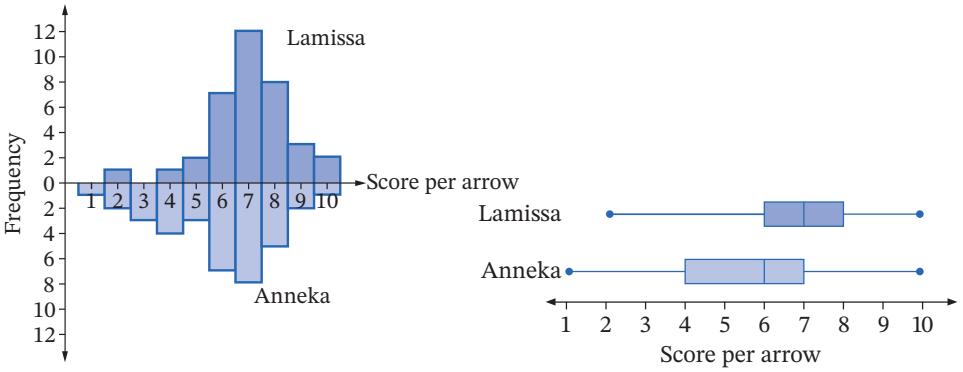
c Are there any clusters or outliers in either place?

d According to the box plot, what percentage of drivers in Bentley's Beach drive faster than all drivers in Sunbeam Valley?

e In which place do drivers generally drive faster? Give a possible reason for your answer.

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8 Lamissa and Anneka shoot arrows at a target 50 m away during an archery contest. They scored 10 for a bullseye down to 1 for the outer ring. Their results are displayed in the back-to-back histogram and the parallel box plots. R C



a How many arrows did Lamissa and Anneka shoot?

b Find the mode and median score per arrow for each contestant.

c Find the range and interquartile range for each contestant.

d Describe the shape of the distribution for each contestant.

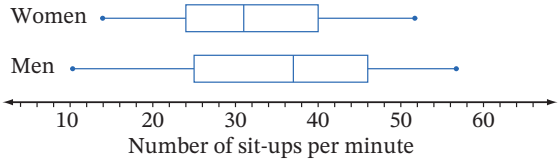
e According to the box plots, on what percentage of arrows shot was a score of 6 or less achieved by:

i Lamissa? **ii** Anneka?

f Who was the better archer during this contest? Justify your answer by referring to the measures of central tendency and spread.

9 The number of sit-ups per minute completed by men and women at the Full On Fitness Centre are displayed in the back-to-back histogram and parallel box plots. R C

Women		Men
8 7 5 4	1	0 6 7 9 9
9 9 9 8 8 7 4 4 3 3 1 0	2	0 2 3 4 4 5 5 7 7 8
7 6 5 5 5 4 3 2 1 0 0	3	0 2 4 5 6 6 7 7 8 8 8 9
7 5 4 3 2 0 0	4	1 3 4 6 6 6 6 7 7 9
2 1 0	5	0 1 3 4 7 7



a Why would a dot plot be an inappropriate way to display the data shown above?

b What is the median number of sit-ups per minute completed by each group?

c Find the range and interquartile range for each group.

d Describe the shape of the distributions for each group.

e Which group has more spread in the number of sit-ups completed per minute? Give reasons for your answer.

Scatterplots

6.08

Bivariate data is data that measures 2 variables, such as a person's height and arm span (distance between outstretched arms). Bivariate data is represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis.

A **scatterplot** is a graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern that may be linear or non-linear. If there is a pattern, then a relationship may exist between the 2 variables.

WS Scatterplots

Scatterplots matching game

Example 13

The heights and arm spans of a group of students are shown in the table.

Height, H cm	162	182	153	145	172	163	150	142	183	145	192	171
Arm span, S cm	158	185	145	143	174	165	151	141	181	158	191	178

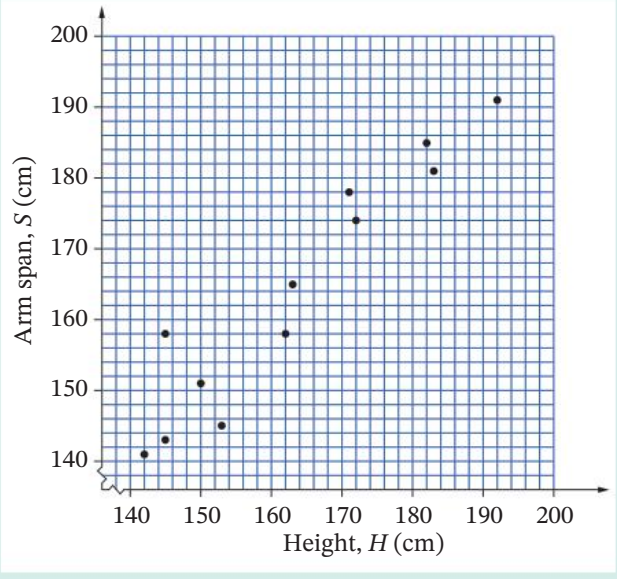
a Plot the data on a scatterplot.

b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and arm spans.

Solution

a



b The points form a linear pattern.

c As the heights of students increase, their arm spans tend to increase.

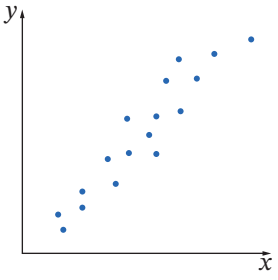
Strength and direction of linear relationships

The linear pattern will indicate the **strength** and **direction** of the relationship between the 2 variables. The strength of a relationship between 2 variables can be described as:

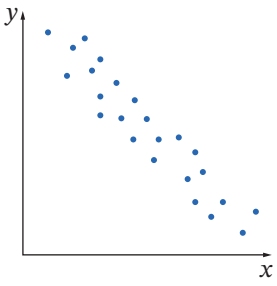
- **strong** if the points are close together
- **weak** if the points are more spread out
- **perfect** if all points lie on a straight line

The direction of a relationship can be described as **positive** or **negative**:

x and y have a **positive relationship** if y increases as x increases.



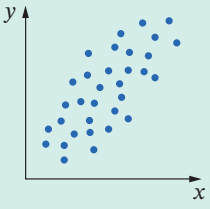
x and y have a **negative relationship** if y decreases as x increases.



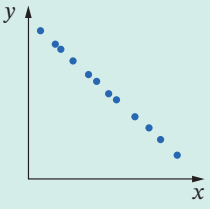
Example 14

Describe the strength and direction of the relationship shown in each scatterplot.

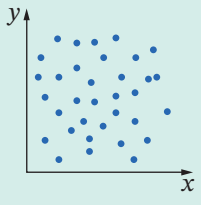
a



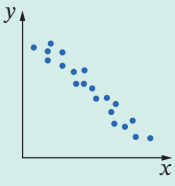
b



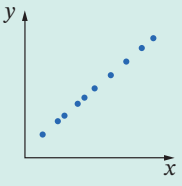
c



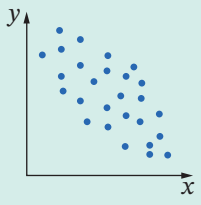
d



e



f



Solution

a	Weak positive relationship	The points can be seen to form a line, but they are spread out.
b	Perfect negative relationship	The points lie on a decreasing straight line.
c	No relationship	The points are very spread out with no pattern.
d	Strong negative relationship	The points can be seen to form a decreasing line and they are close together.
e	Perfect positive relationship	The points lie on an increasing straight line.
f	Weak negative relationship	The points can be seen to form a decreasing line, but they are spread out.

Dependent and independent variables

If a variable y depends on the value of the variable x, y is called the **dependent variable** and x is called the **independent variable**. For example, stride length (the length of a person's walking step or pace) depends on the person's height, so stride length is the dependent variable and height is the independent variable. When graphing, the dependent variable is shown on the vertical (y-) axis while the independent variable is shown on the horizontal (x-) axis.

EXERCISE 6.08 ANSWERS ON P. 608

Scatterplots U F R C

EXAMPLE 13

1 The heights and handspans of a group of students are shown in the table. C

Height, H cm	168	175	175	156	160	173	171	180	185	175	182	180
Handspan, S cm	20.0	21.1	17.6	16.5	17.5	19.0	20.8	22.5	25.0	23.0	20.2	21.1

a Plot the data on a scatterplot.

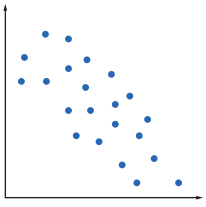
b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and their hand spans.

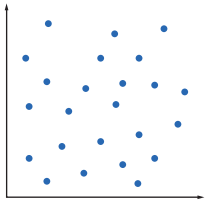
EXAMPLE 14

2 Describe the strength and direction of the relationship shown in each scatterplot. C

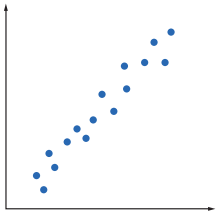
a



b



c



3 Describe the strength and direction between height, H and handspan, S in question **1**. C

4 The height and stride length measurements of some students are shown in the table. R C

Height, H cm	174	160	158	180	169	172	171	171	148	190	166	173
Stride length, L cm	72.2	64.0	66.4	74.7	70	71.5	70.9	71.2	61.4	78.9	68.0	71.9

a Explain why stride length is the dependent variable.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the relationship between the students' heights and stride lengths.

e Describe the strength and direction of the relationship.

f Predict the stride length of a student who is 175 cm tall.

5 The table lists the points scored for and against each NRL team one season. R C

a Graph this data on a scatterplot.

b Is the pattern of the points linear?

c Describe the strength and direction of the relationship between points scored for and points scored against.

Points scored for, *F*	Ponints scored against, *A*
568	369
579	361
559	438
497	403
597	445
545	536
445	441
481	447
405	438
506	551
449	477
448	488
462	626
497	609
409	575
431	674

6 Year 10 students were surveyed on the number of hours in a week they spent doing homework and the number of hours they spent on the computer. R C

Homework, *H*	2	15	12	5	4	2	4	15	14	5	2	5	20	4	2	11
Computer, *C*	25	30	18	35	6	30	20	22	6	40	8	3	20	30	5	8

a Plot the points on a scatterplot.

b Describe the strength and direction of the relationship between hours spent doing homework and hours spent on the computer.

7 A survey was conducted to see whether there was a relationship between height and the age of students in a high school. R C

Age, *A* (years)	14	16	15	13	11	14	17	15	12	11	14	16	13	18
Height, *H* (cm)	162	174	182	162	132	173	187	160	154	145	165	171	151	181

a Graph the points on a scatterplot.

b Which variable could be considered as the dependent variable? Give reasons.

c Describe the strength and direction of the relationship between the age and height of students.



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Technology

Scatterplot patterns

Investigate one of the following pairs of bivariate data for a group of students or people. You will need instruments (measuring tapes and/or trundle wheels) and stopwatches to help you collect your data.

- Height vs arm span
- Reaction time vs hours of sleep
- Stride length vs 50 m sprint time

1. Enter your data into a spreadsheet. Graph it using Scatter with Smooth Lines and Markers.
2. Analyse your graph. What type of linear relationship does it show? Positive or negative? Strong or weak?
3. Write a brief summary describing the relationship between the 2 variables.



Alamy Stock Photo/Kim Karpeles

Line of best fit

6.09

If 2 variables x and y show a strong linear relationship when graphed on a scatterplot, the linear relationship can be approximated by drawing a **line of best fit** through the points and finding its equation $y = mx + c$. This line can be done on paper but it is easier to graph it using technology such as a spreadsheet, dynamic geometry or graphing software.

STAGE 5.3

WS Line of best fit

WS Trendlines

Line of best fit

Line of best fit

Line of best fit

- It represents most or all of the points as closely as possible.
- It goes through as many points as possible.
- It has roughly the same number of points above and below it.
- It is drawn so that the distances of points from the line are as small as possible.

A line of best fit can be used to predict what might happen:

- between the points on the scatterplot, within the range of data (this is called **interpolation**, pronounced 'in-terp-o-lay-shun'), or
- beyond the points on the scatterplot, outside the range of data (this is called **extrapolation**, pronounced 'ex-trap-o-lay-shun').

STAGE 5.3

Lines of best fit

Example 15

The arm span and right foot size of 12 Year 10 students were measured:

Arm span, S (cm)	177	179	162	182	181	171	161	176	175	190	168	165
Right foot size, F (cm)	25	26	24	28	27	25	23	25	24	30	24	24

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

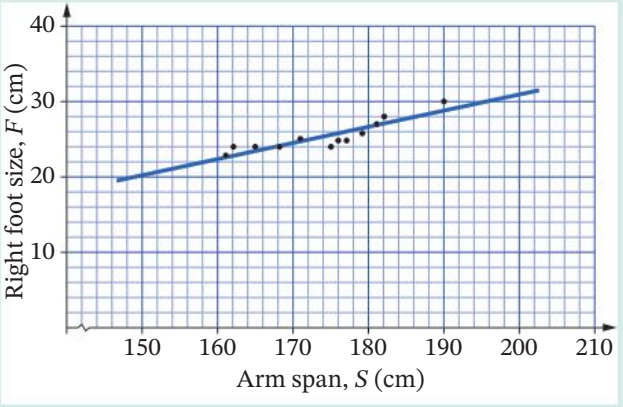
c Use the equation to estimate the foot size of a student with an arm span 173 cm.

d Use the graph to interpolate the foot size of a Year 10 student with an arm span of 185 cm.

e Use the graph to extrapolate the arm span of a Year 10 student who has a foot size of 22 cm.

Solution

a



b Use the point–gradient formula $y - y_1 = m(x - x_1)$ to find the equation of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{27 - 20}{181 - 150}$$ using 2 points on the line (150, 20) and (181, 27)

$$= \frac{7}{31}$$

$$\approx 0.226$$

$y - 20 = 0.226(x - 150)$ using the point (150, 20)

$y = 0.226x - 33.9$

$y = 0.226x - 13.9$

$F = 0.226S - 13.9$ x and y replaced by S and F respectively

c When $S = 173$ cm,

$F = 0.226 \times 173 - 13.9$

$= 25.198$ cm.

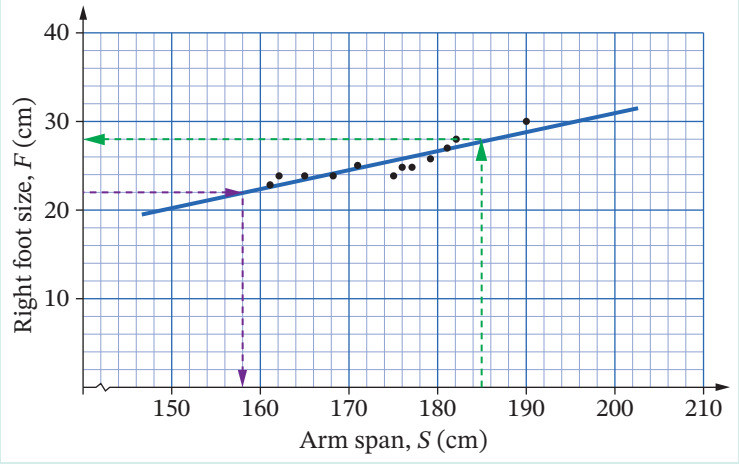
A Year 10 student with an arm span of 173 cm would have a foot size of 25.198 cm

d From the graph, a Year 10 student with an arm span of 185 cm would have a foot size of 28 cm.

This is **interpolating** because we are reading from the graph **between** the given points.

e From the graph, a Year 10 student with a foot size of 22 cm would have an arm span of 158 cm.

This is **extrapolating** because we are reading from the graph **outside** the given points.



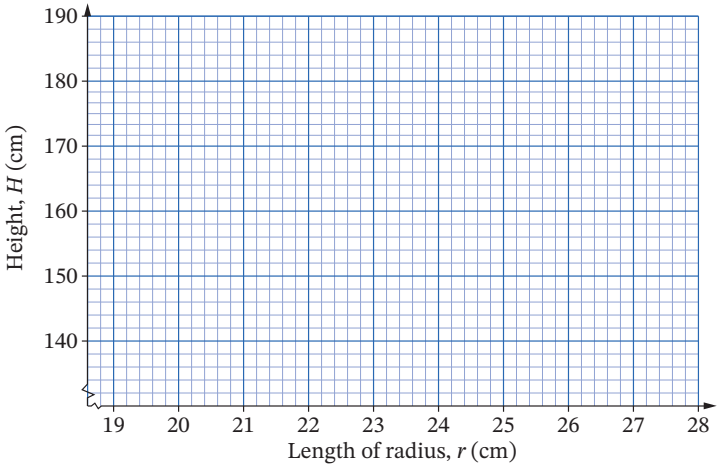
EXERCISE 6.09 ANSWERS ON P. 609

Line of best fit U F R C

EXAMPLE 15

1 Forensic scientists can estimate people's heights from the lengths of their bones such as the tibia, femur, humerus and radius. This table gives the heights of females and the lengths of their radius bones. R

Length of radius, r (cm)	25.2	22	23	22.5	21.8	26.2	20.4	23.5	24.3	21.4
Height, H (cm)	173	158	165	161	158	179	152	167	169	156



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STAGE 5.3

a Use graph paper with axes as shown to plot the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use your equation to find the height of a female whose radius is 25 cm long.

d If the radius is of length 27 cm, use the line of best fit to predict the height of the female.

2 The heights and shoe size of a group of Year 11 students were measured and recorded in the table.

Height, H (cm)	175	174	177	180	179	176	170	175	179	180	178	183	178	173	179	174
Shoe size, S	10.5	10	10	12	11	9.5	7.5	9	11.5	12.5	11	12.5	12	9.5	10.5	9

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the shoe size of a student whose height is 172 cm.

d Use the graph to interpolate the shoe size of a student who is 181 cm tall.

e Use the graph to extrapolate the shoe size of a student with height 185 cm.

3 The air temperature, T (°C) was measured at various heights, h (m) above sea level. R

Height, h (m)	0	500	1000	2000	2500	4000	5900	7500	10000
Temperature, T (°C)	20	14	8	3	−5	−13	−20	−35	−50

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the temperature at a height of 1500 m.

d Use the graph to find the height above sea level for a temperature of −10°C.

4 The results obtained by 18 Year 10 students in Maths and Science exams are shown. R C

Maths	59	52	72	85	75	45	65	64	62	58	78	90	40	70	50	45	82	50
Science	65	54	67	83	75	39	59	64	60	56	80	95	38	65	48	48	85	51

a Graph the points on a scatterplot and construct a line of best fit.

b Simone missed the Science test but obtained a 80% in her Maths exam. Use the line of best fit to predict Simone's Science result.

c If Mario obtained 96% in the Science exam, predict what result he might have achieved in the Maths exam.

5 Angelicki is measuring the amount by which a spring is stretched when different masses are hung from the spring for a Science experiment. Her results are shown. R C

Mass, M (g)	10	20	25	30	35	40	50
Spring Stretch, S (cm)	5.9	11.2	12.3	14.8	17	22.4	25.2

a Graph the points on a scatterplot and construct a line of best fit.

b Use the line of best fit to predict the length the spring stretches for a mass of 45 g.

c What mass would have to be attached to stretch the spring 28 cm?

d Are there limitations to using the line of best fit to predict the length the spring is stretched by different masses?

6 The men's 100 m world record times for running are shown in the table. R C

Year	1964	1968	1983	1988	1991	1994	1996	1999	2005	2006	2007	2008	2009
Time (s)	10.06	9.95	9.93	9.92	9.86	9.85	9.84	9.79	9.77	9.76	9.74	9.69	9.58

a Graph the points and construct a line of best fit.

b Use the line of best fit to predict the time to run the 100 m in 2030.

c What are the limitations of using the line of best fit to predict times to run 100 m?

Did you know?

Lightning Bolt

The current record of 9.58 seconds for running 100 metres is still held by Jamaica's Usain Bolt, who achieved this in 2009 at the World Championships in Berlin, Germany. In 2016, American sprinter Justin Gatlin achieved the time of 9.45 seconds on the Japanese television show *Kasupe!*, but this record was disallowed. In the same year, at the Rio Olympics, Usain Bolt won the gold medal with a winning time of 9.81 seconds, Justin Gatlin won silver with 9.89 seconds, and Canadian Andre de Grasse won bronze with 9.91 seconds.

Find out why Justin Gatlin's record time was disallowed in 2016.



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Technology

Lines of best fit

In this activity, we will use a spreadsheet to create a scatterplot and graph a line of best fit. The heights of men and the lengths of their femur bone are recorded in the table below.

Length of femur, f (cm)	40	42.9	44.2	46.1	46.8	47	48.4	50.3	51.2	57.2
Height, H (cm)	162	165	164	173	174	178	179	182	186	200

1 Enter the data from the table into a spreadsheet. Type **Length of femur** in cell A1 and **Height** in B1.

2 To graph a scatter plot, select all the values in cells B1 to K2, and under the **Insert** menu, select **Scatter** and **Scatter with Straight Lines and Markers**.

3 To draw the line of best fit, select one of the points on the scatterplot and right-click. Select **Add Trendline**, **Linear and Display Equation on chart**, then **Close**.

4 Check your answers to questions **1** to **3** from Exercise 6.09 using a spreadsheet.

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6.10 Bivariate data involving time

Bivariate data involving time, or time series data, is two-variable data that has time as the independent variable. Examples of time series data are population changes over time, weekly share prices, daily rainfall and patients' heart rates.

Example 16

This table shows the average household size between 1961 and 2016, according to the Census.

Year	People	Year	People
1961	3.6	1991	2.8
1966	3.5	1996	2.6
1971	3.3	2001	2.6
1976	3.1	2006	2.6
1981	3.0	2011	2.6
1986	2.9	2016	2.6

Source: CC © Copyright 2016, Commonwealth of Australia aifs.gov.au

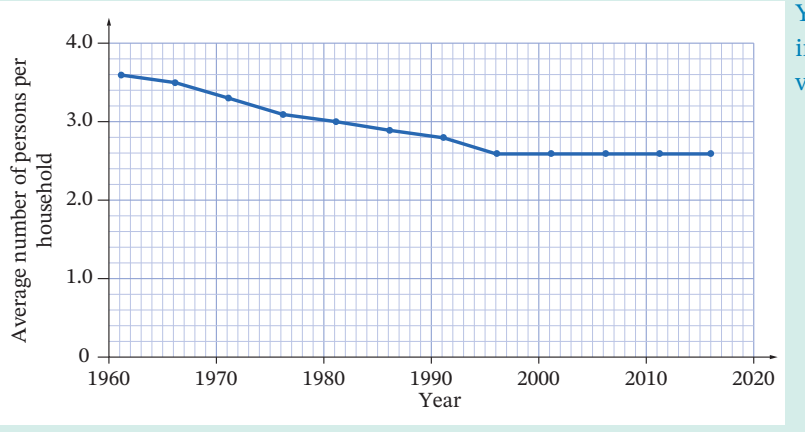
a Graph the data on a scatterplot and join the points.

b Use your graph to describe the change in average household size from 1961 to 2016.

c Based on your time series graph, estimate the household size for 2021.

Solution

a



Year is the independent variable.

b The average household size decreased from 3.6 in 1961 to 2.6 in 1996 and since then there has been no change.

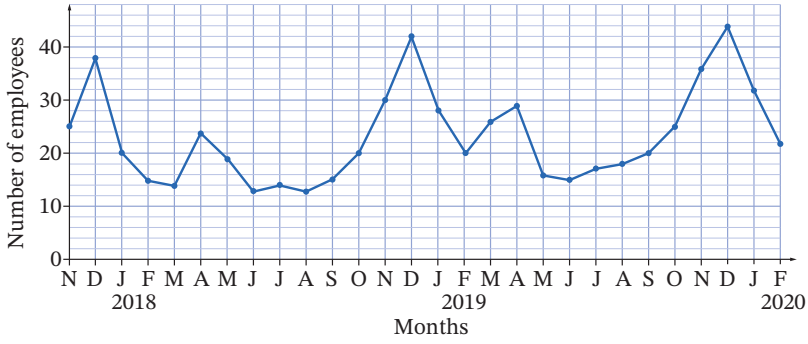
c 2.4 to 2.6 people per household.

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EXERCISE 6.10 ANSWERS ON P. 610

Bivariate data involving time U F R C

1 The number of people employed per month at SupaSave supermarket from November 2017 to February 2020 is displayed on this time series graph. R C



a How many people were employed by the supermarket in:

i November 2017? **ii** December 2018? **iii** June 2019?

b In which month of the year were the most people employed by the supermarket? Suggest a reason why.

c In which month of the year were the least number of people employed? Suggest a reason why.

d Describe how the number of people employed by the supermarket changes from November 2017 to February 2020.

2 The population figures for Australia from 1960 to 2020 are given in the table. R C

EXAMPLE 16

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	2020
Population (millions)	10.28	11.39	12.51	13.89	14.70	15.76	17.07	18.07	19.15	20.39	22.3	23.8	25.4

a Graph the data on a scatterplot and join the points.

b Between which years was the greatest population increase?

c Use your graph to describe the change in Australia's population from 1960 to 2020.

d Based on your time series graph, estimate the population for Australia in:

i 2030 **ii** 2055.

3 This table shows the fatalities on NSW roads from 1950 to 2020. R C

Year	1950	1960	1970	1980	1990	2000	2010	2020
Fatalities	634	978	1309	1303	797	603	405	352

a Draw a time series graph for this data.

b Describe the change in road fatalities from 1950 to 2020.

c Give possible reasons for the reduction in road fatalities from a high of 1309 in 1970 to a low of 352 in 2020.

4 The annual mean maximum temperatures for Sydney from 1995 to 2019 are shown. R C

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003
Temp (°C)	21.8	22.1	22.4	22.7	22.1	22.7	23.1	23.1	22.7

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Temp (°C)	23.4	23.4	23.1	22.7	22.1	22.1	22.6	22.6	22.7

Year	2013	2014	2015	2016	2017	2018	2019		
Temp (°C)	23.7	23.5	23.2	23.9	23.7	23.4	24.0		

Source: Bureau of Meteorology

a Draw a time series graph for temperatures from:

i 1995 to 2005 **ii** 2006 to 2019.

b Has there been much change in Sydney's temperature from:

i 1995 to 2005? **ii** 2006 to 2019?

Justify your answer.

c Are there differences in temperature between the periods 1995–2005 and 2006–2019? Give reasons.

5 The table below shows the annual emissions of carbon (measured in Megatonnes) from 2008 to 2018. R C

Year	2008	2009	2010	2011	2012	2013
Annual emissions (Mt)	554	542.8	577.2	562.5	548.5	533.8

Year	2014	2015	2016	2017	2018
Annual emissions (Mt)	533.7	528.5	529.8	531.6	533

a Draw a time series graph for this data.

b Describe the change in carbon emissions from 2008 to 2018.

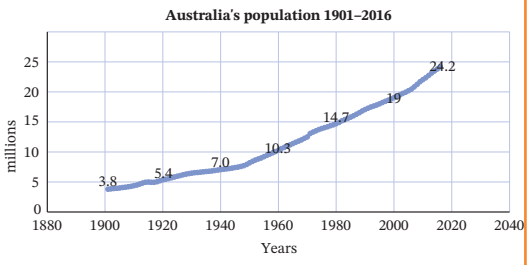
c What happens to the carbon emissions after 2011?

d Give a possible reason for your answer to part **c**.

e What is your estimate of carbon emissions for:

i 2025? **ii** 2035?

6 This graph shows Australia's population from 1901 to 2016. R C



Source: Australian Historical Population Statistics (3105.0.65.001); Australian Demographic Statistics (3101.0).

a What was Australia's population in 1901?

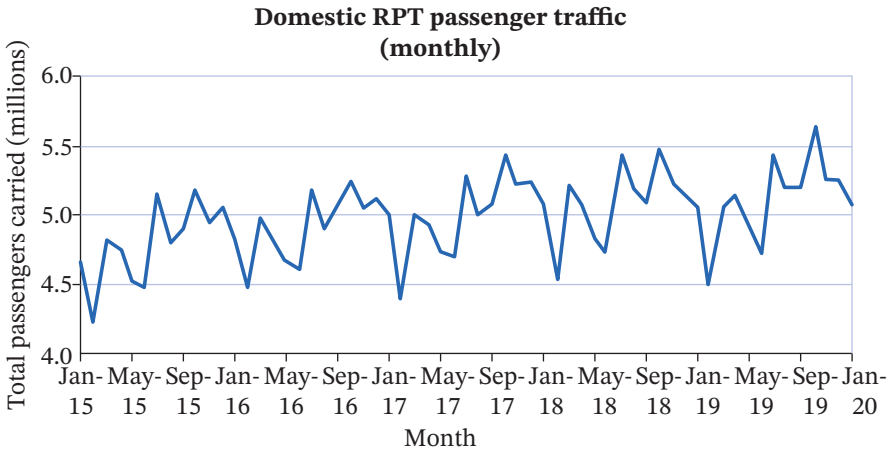
b By how much had Australia's population increased between 1901 and 2016?

c What was the average annual rate of increase in population between 2000 and 2016?

d If this trend continues, what is the expected population in 2030?

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7 This time series graph shows the monthly amount of passenger traffic on Australian domestic commercial airlines. R C



Source: https://www.bitre.gov.au/statistics/aviation/domestic

a Describe the trend in domestic passenger traffic from 2015 to January 2020.

b What was the approximate amount of passenger traffic per month in:

i January 2015? **ii** January 2017?

iii January 2018 ? **iv** January 2020?

c What was the percentage increase in domestic passenger movements from January 2015 to January 2020?

Investigation

Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas – from population, employment, weekly earnings, weight and obesity in adults, to health of children in Australia.

Visit the ABS website www.abs.gov.au to answer the following questions.

1 **a** What is the current population of Australia?

b What is the predicted population for:

i 2020? **ii** 2030? **iii** 2040?

c What is Australia's rate of population increase?

2 Go to 2016 Census Data by Location, and then to Data and analysis.

a What was the population in NSW and its increase from 2011?

b Which state had the:

i largest increase in population?

ii smallest increase in population?

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6.11 Statistics in the media

We live in a world of 24-hour news, whether it is from newspapers, TV or the Internet, that often quote results from surveys. When survey data is used in the media we need to consider:

- where the news comes from and what **samples** the statistics are based on
- who supplied the information
- the number of samples and the sample size used
- the way in which the collected data has been presented



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Example 17

What concerns could be raised about the following claim?

'The Daily Sun newspaper reports that it has an average issue readership of 1.39 million and that its Travel section has a readership of 1.46 million.'

Solution

The newspaper is reporting about its own readership, so it may be biased. It also states that its Travel section has a higher readership than its issue readership.

Example 18

The weights (in kg) of a large group of 18–20-year-olds attending university are:

57	58	62	84	64	74	57	55	56	90
68	63	49	66	63	65	60	60	46	70
85	60	70	41	73	75	67	63	70	85
51	49	75	77	87	54	60	75	58	68
55	65	66	57	85	75	56	60	62	75
74	58	51	62	50	55	71	57	58	100
72	58	103	64	52	55	80	96	45	87
81	80	48	54	65	54	59	50	78	60
74	70	64	59	72	78	104	63	102	95

a How many students were in the group?

b Randomly select 4 groups of 10 and for each sample calculate:

i the mean **ii** the median **iii** the interquartile range.

c Use your results to estimate the mean, median and interquartile range of the **population** from your 4 samples.

d Compare your estimates to the mean, median and interquartile range of the population.

Solution

a There were 90 students in the group.

b Randomly select 4 samples of 10 from the 'population'.

Sample 1:	90	63	75	48	74	85	51	96	60	78
Sample 2:	62	75	103	64	65	54	55	54	60	75
Sample 3:	68	70	57	52	78	74	60	63	58	87
Sample 4:	72	54	52	80	45	87	49	77	54	58

The statistics for each group are:

Sample 1: $\bar{x} = 72$ median = 74.5 interquartile range = 25

Sample 2: $\bar{x} = 66.7$ median = 63 interquartile range = 20

Sample 3: $\bar{x} = 66.7$ median = 65.5 interquartile range = 16

Sample 4: $\bar{x} = 62.8$ median = 56 interquartile range = 25

c Taking averages, population statistics estimates are:

$$\text{Mean} = \bar{\bar{x}} = \frac{72 + 66.7 + 66.7 + 62.8}{4} = 67.1 \quad \text{(correct to 1 decimal place)}$$

$$\text{Median} = \frac{74.5 + 63 + 65.5 + 56}{4} = 64.8 \quad \text{(correct to 1 hbox decimal place)}$$

$$\text{Interquartile range} = \frac{25 + 20 + 16 + 25}{4} = 21.5$$

d The statistics for the population are:
Mean, $\bar{x} = 66.9$ (correct to 1 decimal place)
Median = 64
Interquartile range = 18
The estimates for the mean, median and interquartile range compare very favourably with the population statistics.



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EXERCISE 6.11 ANSWERS ON P. 611

Statistics in the media U F R C

EXAMPLE 17

1 A TV network surveys 300 people in shopping centres between 9 a.m. and 11 a.m. to get feedback on its new game show.

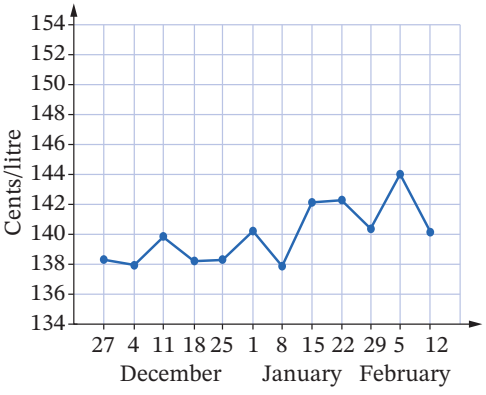
a How is this survey biased?

b Suggest a better method for getting feedback about its game show.

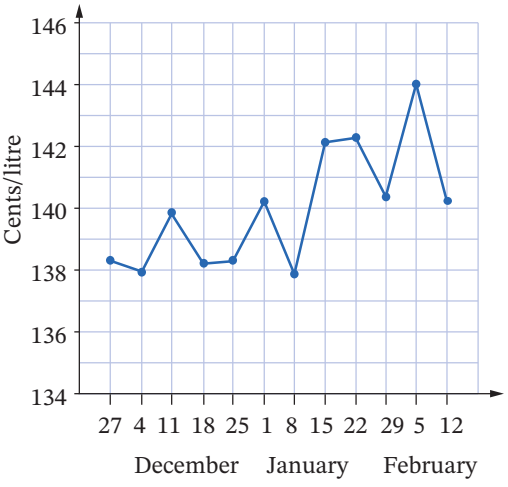
2 A report about hot water systems recommended a heat pump system. The report stated that people in Queensland who had the heat pump hot water system saved 30% on their electricity bills. The company is using this information to advertise their product in NSW and Victoria. How might this information be unsuitable for people in NSW and Victoria? Give reasons.

3 Two reports on petrol prices over a 12-week period were written by 2 different companies. Each company used line graphs to show the price of petrol for the same period. R C

Petrol pricing: Company A



Petrol pricing: Company B



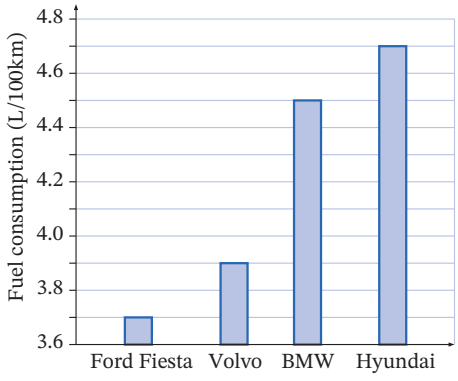
a What message is being suggested about petrol prices in the line graph presented by:

i Company A?

ii Company B?

b How could both graphs be improved to give a true picture of changing petrol prices?

4 A report on the diesel fuel consumption of different cars was published in a motoring magazine, featuring this misleading graph. R C



a What is the graph suggesting about the fuel consumption of the different cars?

b What is the difference in fuel consumption between the:

i Ford Fiesta and the Volvo?

ii Ford Fiesta and the Hyundai?

iii BMW and the Hyundai?

c How should the graph be redrawn so that it is not biased towards the Ford Fiesta and the Volvo?

5 A company sells a new app. After 3 months, they conduct a survey and customers are asked to rate the product as Excellent, Good or Satisfactory. Is the survey biased? Justify your answer. R C

6 A market research company working for a car manufacturer needs to determine the most popular car colours. R C

a Give an example of a biased question for this survey.

b What other information should the market research company use, apart from the survey, to determine the most popular colour car?

7 **a** Randomly select 4 samples of 10 weights from the population shown in Example **14**, and for each sample calculate the: R C

i mean **ii** median **iii** interquartile range.

b Use your results to estimate the mean, median and interquartile range of the population from your 4 samples.

c How do the statistics of your samples compare to the mean, median and interquartile range of the population?

d How do the estimated statistics compare to the population statistics?

8 **a** Repeat the process of question **7** by taking 2 samples of size:

i 5 **ii** 15 **iii** 20

b Do the sample statistics become more accurate and move closer to the population statistics as sample size increases? R C

EXAMPLE 18

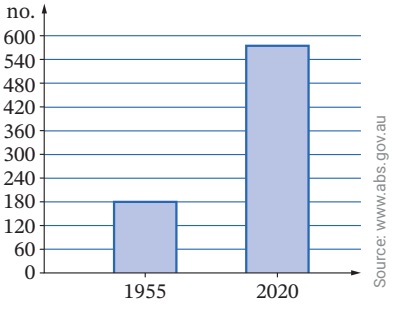
6.12 Investigating statistical studies

STAGE 5.3

EXERCISE 6.12 ANSWERS ON P. 611

Investigating statistical studies U F PS R C

1 This graph compares the number of passenger vehicles per 1000 people in Australia in 1955 and 2020.



a How many passenger vehicles per 1000 people were there in 1955?

b What was the percentage increase in the rate between 1955 and 2020?

2 Visit the Australian Bureau of Statistics (ABS) website www.abs.gov.au and search for Motor vehicle census.

a What was the total number of vehicles registered last year?

b How many passenger vehicles were registered last year?

c What was the average annual growth rate over the last 5 years?

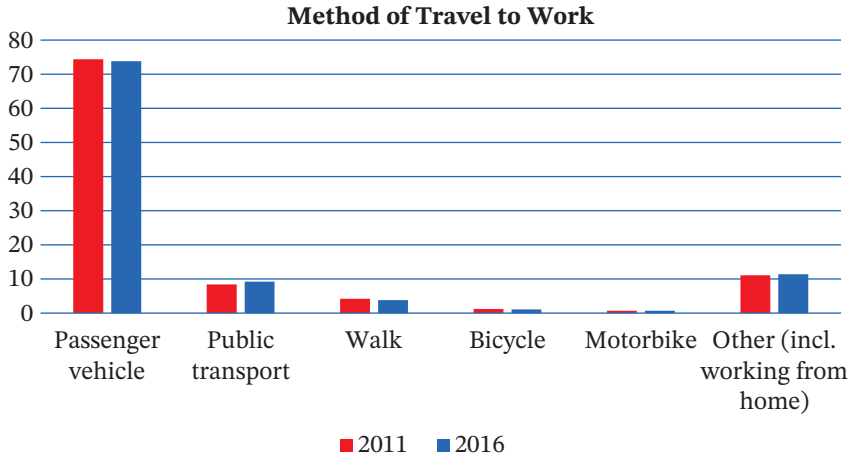


iStock.com/scyther5

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3 This graph compares the types of commuter transport used by Australians in 2011 and 2016. C



Source: www.abs.gov.au

a What percentage of people used a passenger vehicle to get to work in 2011 and what change has occurred from 2011 to 2016?

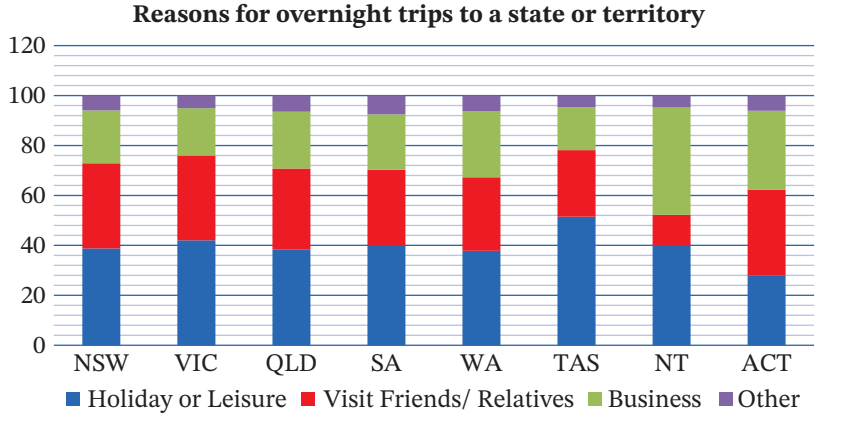
b What percentage of Australians used public transport in 2016? Perform an Internet search to see whether this has changed in recent years.

c Use the Internet to find reasons for Australians choosing not to use public transport.

d List 3 advantages for using public transport.

e Use the Internet to compare Australia's transport use with transport use in other countries (for example, Japan, India, France, UK and USA).

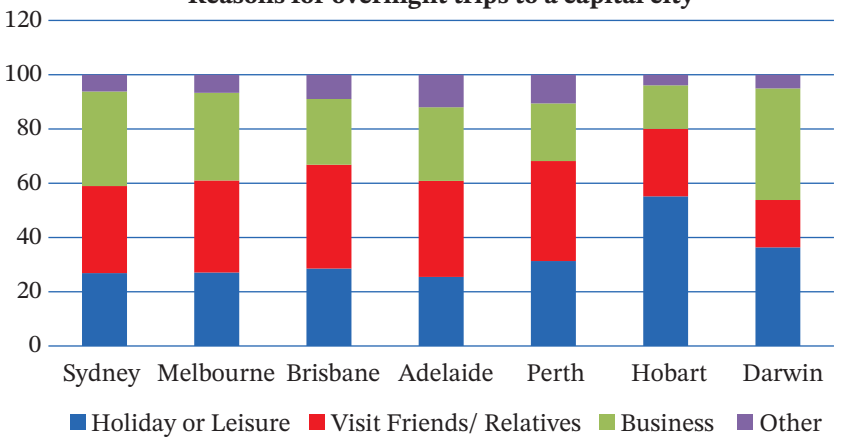
4 These graphs show the reasons Australians went on overnight trips in 2018. R C



Source: www.tra.gov.au

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STAGE 5.3



a Which state/territory had the highest proportion of overnight trips for holiday or leisure?

b What percentage of people visited South Australia for an overnight trip to see friends/ relatives?

c Which capital city showed the highest percentage of business trips and which city had the lowest? Give possible reasons for your answer.

d The percentage of people taking business trips to Sydney and Melbourne is higher than the percentage of people taking business trips to Perth or Hobart. Give a possible reason for this.

e Which state/territory and which capital city had the lowest percentage of people visiting for holiday or leisure? Give a reason for each answer.

5 Summarise your answers to questions **1** to **3** in a brief report about passenger vehicle use in Australia. Using your results, indicate what action governments (Federal, State and local) should do in terms of building roads, accident research and consideration of the environment. R C

6 'Life expectancy at birth is one of the most widely-used and internationally-recognised indicators of population health. High life expectancy at birth indicates low levels of infant mortality, a safe environment in which to live, a good health care system, sufficient food, and the adoption of preventative health measures.' PS R C

Source: Australian Bureau of Statistics, Life Expectancy Trends Australia.

Investigate life expectancy trends in Australia, using ABS and life insurance websites. Investigate data such as:

- life expectancy graphs or tables at birth and for different ages
- comparison of life expectancy between Australia and other countries
- comparison of life expectancy between Australian states and territories
- infant mortality rates
- causes of death (such as cancer)
- life expectancy of Aboriginal and Torres Strait Islander Australians
- implications of higher life expectancy on health costs, disabilities associated with aging population, aged-care facilities, pensions

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7 Is Australia becoming a warmer continent? Investigate this by looking at data from the Australian Bureau of Statistics and the Australian Bureau of Meteorology www.bom.gov.au. PS R C

8 Investigate tobacco and alcohol use by teenagers in Australia. Include tables and graphs in your report. Refer to the National Drug Strategy Household Survey (www.nationaldrugstrategy.gov.au) and search 'alcohol and teenage statistics in Australia' on the Internet. PS R C

Power plus ANSWERS ON P.611

1 The strength and direction of the relationship between 2 variables can be measured by the correlation coefficient (r).

- **a** Between which 2 values does the correlation coefficient lie?
- **b** What is the strength and direction of the relationship if the correlation coefficient is zero?
- **c** Write a possible value for the correlation coefficient for each relationship described.
  - **i** perfect positive
  - **ii** weak negative
  - **iii** strong negative

2 Two variables may have a strong relationship, but this does not mean a change in one variable causes a change in the other. Which of the following pairs of variables have a *causal* relationship?

- **a** height and weight of people
- **b** the time it takes to walk to school and the distance from home to school
- **c** the number of children per household and the number of mobile phones per household
- **d** the age of people and their reaction time
- **e** the price of petrol and the amount of petrol sold
- **f** the interest rate of loans and the number of new home loans

3 The following values are the test results on a History exam for a class of 20 students.

13 14 16 12 14 16 18 13 15 10

9 15 13 14 13 10 8 14 16 14

- **a** Find the mean, median and mode.
- **b** Find the range and interquartile range.
- **c** An error was made in recording the values and 4 marks need to be added to each value. What effect will this have on the statistics calculated in parts **a** and **b**?

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CHAPTER 6 REVIEW

Statistics

Language of maths

bivariate data	box plot	cluster	dependent variable
five-number summary	independent variable	interquartile range	line of best fit
mean	measure of central tendency (or location)	measure of spread	median
mode	negatively skewed	outlier	positively skewed
quartile	range	scatterplot	skewed
standard deviation	strong	symmetrical	weak

1 What is represented by the 'whiskers' on a **box plot**?

2 What are the **measures of central tendency** and the **measures of spread**?

3 What are the 5 things found in a **five-number summary**?

4 Describe a statistical distribution that is **positively-skewed**.

5 What type of graph is used to represent **bivariate data**?

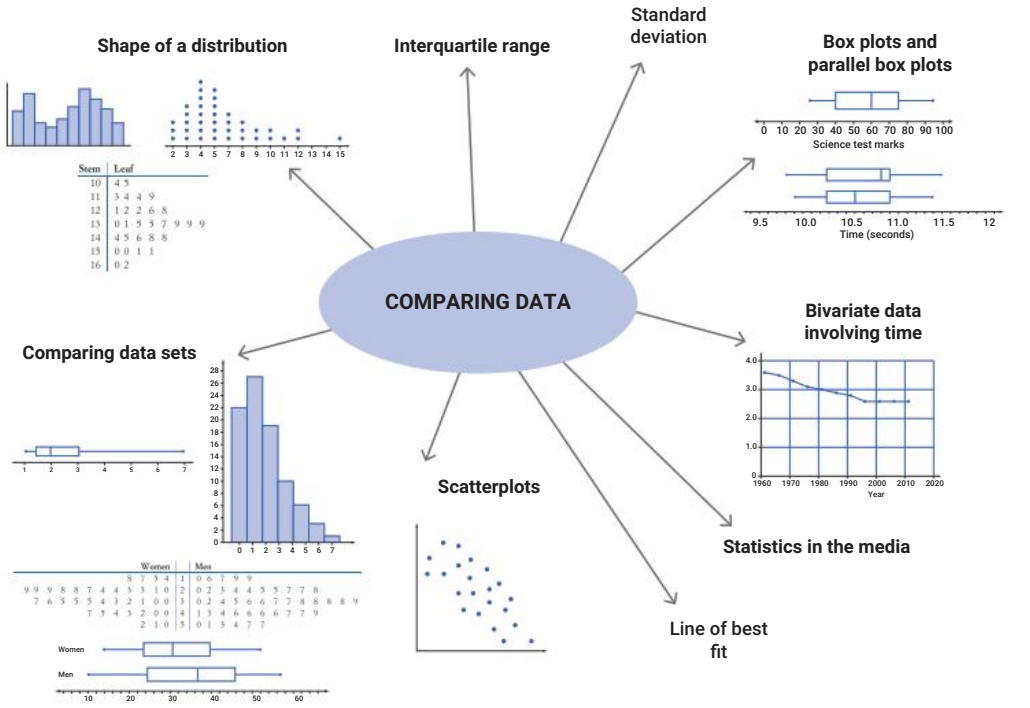
6 Give 2 examples of how statistics can be misleading.

Topic summary

WS

Mind map: Comparing data (Advanced)

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



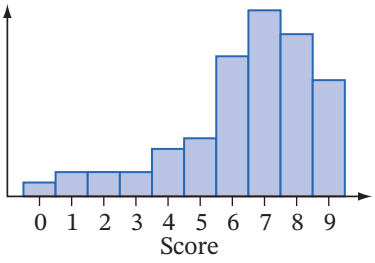
TEST YOURSELF 6 ANSWERS ON P. 611

6.01

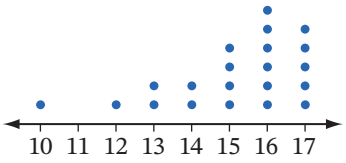
1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



b



c

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

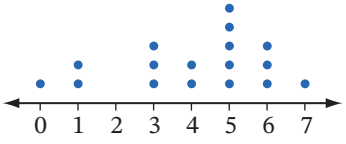
6.02

2 Find the interquartile range of each set of data.

a 5 8 8 10 12 13 14 15 18

b 24 15 23 28 20 20 18 30 21 18

c



d

Stem	Leaf
3	0 1 2
4	3 5 8 8 9 9 9
5	4 5 6 6 8
6	0 1 3 7
7	2

e

Score	Frequency
10	3
11	8
12	15
13	18
14	10
15	5

STAGE 5.3

6.03

3 The reaction times (in seconds) of a sample of truck drivers were measured:

0.34 0.35 0.34 0.37 0.42 0.45 0.43 0.29 0.38 0.40
0.37 0.62

a Find, correct to two decimal places, the mean and standard deviation.

b Find the range and interquartile range.

c Which is the best measure of spread for this set of data? Justify your answer.

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STAGE 5.3

6.04

4 The Health exam results for a class of PE students are shown here.

Girls:	83	78	63	84	65	51	76	69	42	84	60	64	92	73	32
Boys:	65	34	75	68	56	63	79	55	68	52	49	85	64	58	54

a Find the mean and standard deviation of both groups.

b Which group performed better in the exam? Give reasons.

6.05

5 The number of goals scored by the Under-18s Vale soccer team are:

2 0 0 4 2 1 1 2 3

2 3 7 4 3 1 0 4 2

a Find the range and interquartile range of the scores.

b Find the five-number summary for the data.

c Draw a box plot for the data.

6.06

6 The pulse rates of students were taken before and after exercising. The results were:

Before exercise:	78	80	66	70	56	64	68
	65	50	76	80	70	70	59
After exercise:	141	140	89	95	110	126	84
	82	90	88	146	98	96	92

a Find the five-number summary for the pulse rates before and after exercise.

b Construct parallel box plots for the 2 sets of data.

c Find the range and interquartile range of the pulse rates:

i before exercising

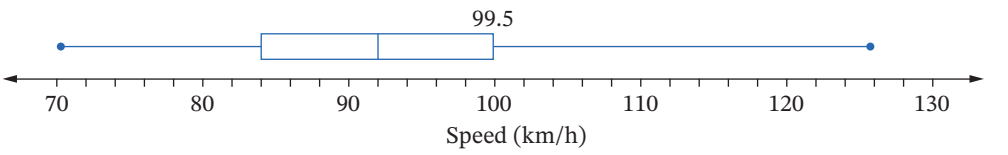
ii after exercising.

d Compare the 2 sets of pulse rates. Are there significant differences between them? Justify your answer.

6.07

7 The speeds of cars (in km/h) were monitored between 1:00 and 1:30 p.m. on a main road. The results are shown in the stem-and-leaf plot and box plot.

Stem	Leaf
7	0 3 7 9
8	0 2 2 3 5 6 8 8 9
9	0 0 1 3 5 5 7 8 9 9 9
10	0 0 4 4 6
11	0 1
12	6



a Which display best indicates:

i skewness?

ii clustering and outliers?

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b Why would a dot plot be unsuitable for displaying the data?

c Find the:

i median

ii interquartile range.

d What percentage of cars were travelling at a speed of at least 92 km/h?

8 11 boxes containing 60 oranges each were placed in cold storage for different periods. After storage, the number of good oranges in each box was counted.

6.08

Weeks in storage (*W*)	2	5	12	8	14	6	5	9	10	3	11
Number of good oranges, (*N*)	58	50	33	40	28	50	52	38	35	55	33

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the relationship between the number of weeks in storage and the number of good oranges.

e Describe the strength and direction of the relationship between the variables *W* and *N*.

9 The heights and weights of 15 people were measured.

STAGE 5.3

6.9

Height, *H* (cm)	152	160	179	180	185	174	165	150	145	142	155	153	175	155
Weight, *W* (kg)	50	65	72	77	81	77	65	57	48	53	61	67	72	56

a Graph the points on a scatter plot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the weight of a student who is 170 cm tall.

d Use the graph to interpolate the weight of a student with height 163 cm.

e Use the graph to extrapolate the height of a student who weighs

i 85 kg **ii** 45 kg

10 The mean maximum temperatures in Perth for January from 2010 to 2020 are shown.

6.10

Year (*t*)	2010	2011	2012	2013	2014	
Temperature (*T*, °C)	33.4	32.5	33.5	31.7	32.3	
Year (*t*)	2015	2016	2017	2018	2019	2020
Temperature (*T*, °C)	33.0	32.0	31.0	30.8	30.5	30.5

Source: Bureau of Meteorology

a What is the independent variable?

b Graph the data on a scatterplot and join the points.

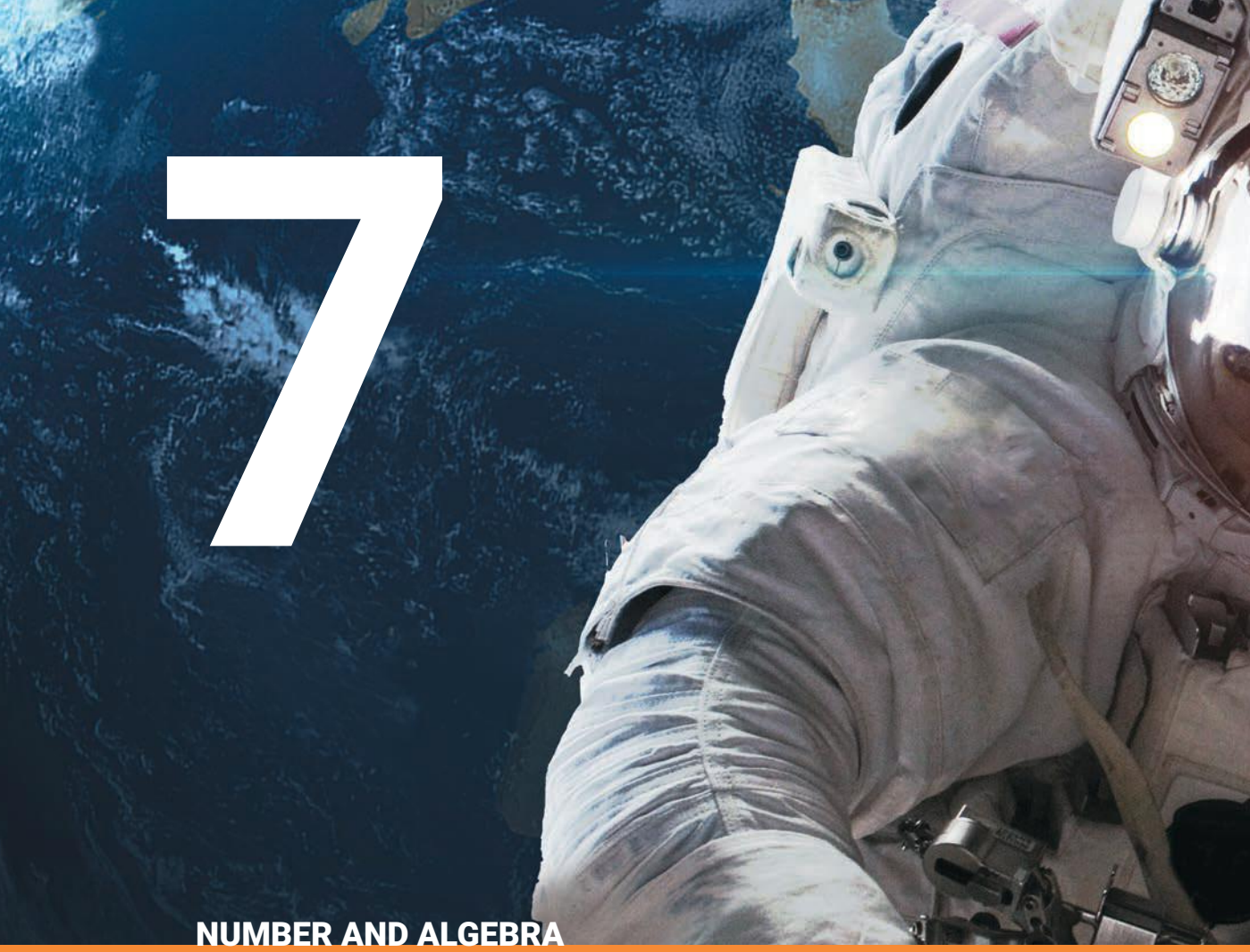
c Has any change occurred to the temperatures in Perth for the month of January over the 11-year period? Give reasons for your answer.

10 An advertisement in a magazine states that a product is 75% fat-free.

6.11

a What impression is the advertisement trying to make about the product?

b What doesn't the advertisement say about the product?



7

NUMBER AND ALGEBRA

EQUATIONS AND LOGARITHMS

In 1962, when astronaut John Glenn was preparing to be the first American to orbit the Earth, he called upon mathematician Katherine Johnson (1918–2020) to check by hand the complex calculations being performed on NASA's new electronic computers.
Johnson worked for NASA for 35 years and her story was told in the 2016 book and film *Hidden Figures*. During her career, she solved equations that guided the paths of spacecrafts and helped the US send the first astronauts to the moon in 1969. Building on the ancient achievements of the Egyptians and Babylonians, Johnson opened the door to space travel and future space tourism.


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Chapter outline

		Working mathematically				
7.01	Equations with algebraic fractions	U	F			
7.02	Quadratic equations $x^2 + bx + c = 0$	U	F		R	C
7.03	Simple cubic equations $ax^3 = c$*#	U	F		R	C
7.04	Equation problems	U	F	PS	R	C
7.05	Equations and formulas	U	F		R	C
7.06	Changing the subject of a formula*#	U	F		R	C
7.07	Graphing inequalities on a number line	U	F			C
7.08	Solving inequalities	U	F		R	C
7.09	Logarithms*	U	F		R	C
7.10	Logarithm laws*	U	F		R	
7.11	Exponential and logarithmic equations*	U	F	PS	R	

***STAGE 5.3**

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Note: Logarithms (7.09–7.11) is an optional Stage 5.3 topic in the syllabus but recommended for students intending to study Stage 6 Mathematics Extension 1. However, logarithms should also be taught to students intending to study Mathematics Advanced.

Wordbank

> 'is greater than'

< 'is less than'

cubic equation An equation involving a variable cubed (power of 3), such as $4x^3 = 500$

exponential equation An equation where the variable is a power, such as $3^x = 243$

logarithm The power of a number, to a given base. For example, $\log_{10} 1000 = 3$, meaning that the logarithm of 1000 to base 10 is 3, because $1000 = 10^3$

inequality A mathematical statement that 2 quantities are not equal, involving algebraic expressions and an inequality sign (>, ≥, <, or ≤)

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$

solution The answer to an equation, inequality or problem; the correct value(s) of the variable that makes an equation or inequality true

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve linear equations involving simple algebraic fractions
- (STAGE 5.3) solve linear equations involving complex algebraic fractions
- solve simple quadratic equations of the form $ax^2 = c$ and $x^2 + bx + c = 0$
- (STAGE 5.3) solve simple cubic equations of the form $ax^3 = c$
- solve problems involving equations
- use formulas to solve problems
- (STAGE 5.3) change the subject of a formula
- graph inequalities on a number line
- solve linear inequalities
- (STAGE 5.3) apply logarithms and the logarithm laws
- (STAGE 5.3) solve exponential and logarithmic equations

Equations with unknowns on both sides

SkillCheck ANSWERS ON P. 612

1 Solve each equation.

a $4a + 5 = 2a - 19$ **b** $\frac{3x+2}{5} = 4$ **c** $4(2 - x) = -24$

2 Factorise each expression.

a $k^2 + 5k + 4$ **b** $y^2 - 10y + 16$ **c** $m^2 - m - 56$

d $u^2 + 8u - 65$ **e** $w^2 - 10w + 21$ **f** $x^2 - 2x - 24$

7.01 Equations with algebraic fractions

Equations with algebraic fractions

Solving equations with fractions

Equations code puzzle

Equations order activity

For equations with more than one fraction, multiply both sides by a common multiple of the denominators to remove the fractions.

Example 1

Solve each equation.

a $\frac{2m}{3} - \frac{m}{2} = 2$ **b** $\frac{2a+4}{5} = \frac{2}{3}$

Solution

a $\frac{2m}{3} - \frac{m}{2} = 2$

$6\left(\frac{2m}{3} - \frac{m}{2}\right) = 6 \times 2$

Multiply both sides by 6, the LCM of 3 and 2.

$\cancel{6}^{2} \times \frac{2m}{\cancel{3}_{1}} - \cancel{6}^{3} \times \frac{m}{\cancel{2}_{1}} = 12$

$4m - 3m = 12$

$m = 12$

Check by substituting that this solution is correct.

b $\frac{2a+4}{5}=\frac{2}{3}$

$\frac{2a+4}{5}\times 15^{3}=\frac{2}{3}\times 15^{5}$

$3(2a+4)=10$

$6a+12=10$

$6a=-2$

$a=\frac{-2}{6}$

$a=-\frac{1}{3}$

Solving linear equations 1

Solving linear equations 2

Example 2

STAGE 5.3

7.01

Equations with algebraic fractions

Solve $\frac{2n+1}{3}-\frac{3n-2}{2}=-5$.

Solution

$\frac{2n+1}{3}-\frac{3n-2}{2}=-5$

$6^{2}\left(\frac{2n+1}{3_1}\right)-6^{3}\left(\frac{3n-2}{2_1}\right)=6\times(-5)$

$2(2n+1)-3(3n-2)=-30$

$4n+2-9n+6=-30$

$-5n+8=-30$

$-5n=-38$

$n=\frac{-38}{-5}$

$=7\frac{3}{5}$

EXERCISE 7.01 ANSWERS ON P. 612

Equations with algebraic fractions U F

1 Solve each equation.

a $\frac{3y}{5}=9$ **b** $\frac{2a}{9}=2$ **c** $\frac{m}{2}+5=6$ **d** $\frac{k}{5}-2=11$

e $\frac{n+5}{3}=-10$ **f** $\frac{y-1}{4}=-2$ **g** $\frac{x+1}{4}+2=10$ **h** $\frac{y-1}{5}-6=3$

i $\frac{m+2}{5}-1=3$ **j** $\frac{x-6}{5}+7=0$ **k** $\frac{2(x+1)}{5}=10$ **l** $\frac{3(m-2)}{4}=6$

m $\frac{8(n+1)}{3}+2=4$ **n** $\frac{5(1-n)}{2}-1=3$ **o** $\frac{4(1+d)}{3}+1=7\frac{1}{3}$

2 Solve each equation.

EXAMPLE 1

a $\frac{2k}{3}=\frac{5}{4}$ **b** $\frac{3w}{10}=\frac{2}{5}$ **c** $\frac{5x}{2}=-\frac{10}{3}$

d $\frac{x-1}{2}=\frac{x+1}{4}$ **e** $\frac{y+2}{5}=\frac{y-1}{2}$ **f** $\frac{a+5}{3}=\frac{a-1}{8}$

g $\frac{p+2}{5}=\frac{p-5}{2}$ **h** $\frac{2y-1}{5}=\frac{y+1}{4}$ **i** $\frac{3y+2}{3}=\frac{2y+1}{4}$

j $\frac{w}{5}+\frac{w}{2}=7$ **k** $\frac{w}{2}-\frac{w}{5}=15$ **l** $\frac{2w}{3}-\frac{w}{4}=4$

m $\frac{3a}{2}+\frac{a}{3}=1$ **n** $\frac{2y}{5}-\frac{y}{3}=4$ **o** $\frac{a}{3}+\frac{3a}{4}=2$

p $\frac{2m}{5}-\frac{m}{10}=1$ **q** $\frac{4h}{3}+\frac{h}{5}=3$ **r** $\frac{5y-2}{7}=\frac{3y+5}{3}$

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3 Solve each equation. Select **A**, **B**, **C** or **D**.

a $\frac{4m}{5}-\frac{m}{3}=2$

A $m=10$ **B** $m=12$ **C** $m=\frac{30}{7}$ **D** $m=\frac{4}{3}$

b $\frac{m+1}{2}=\frac{3+2m}{5}$

A $m=1$ **B** $m=5$ **C** $m=\frac{5}{3}$ **D** $m=\frac{2}{3}$

STAGE 5.3

EXAMPLE 2

4 Solve each equation.

a $\frac{x-1}{4}+\frac{2x}{7}=0$ **b** $\frac{p+2}{3}+\frac{p+1}{4}=10$ **c** $\frac{m+2}{3}+\frac{m+1}{4}=12$

d $\frac{x-3}{5}+\frac{x-2}{2}=6$ **e** $\frac{3x-10}{3}+\frac{x-2}{2}=11$ **f** $\frac{3y+1}{4}-\frac{y+2}{3}=4$

g $\frac{7+2a}{5}-\frac{a-1}{2}=6$ **h** $\frac{6a-1}{4}-\frac{a+2}{3}=8$ **i** $\frac{a-10}{5}-\frac{5-2a}{4}=\frac{1}{2}$

7.02 Quadratic equations $x^2 + bx + c = 0$

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4t^2 - 3t = 8$.

The quadratic equation $x^2 = c$

The quadratic equation $x^2 = c$ (where c is a positive number) has 2 solutions:

$x = \pm\sqrt{c}$ (which means $x = \sqrt{c}$ and $x = -\sqrt{c}$).

Example 3

Simple quadratic equations

Solve each quadratic equation.

a $m^2 = 16$ **b** $3x^2 = 75$ **c** $4m^2 - 12 = 0$

Solution

a $m^2 = 16$

$m = \pm\sqrt{16}$

$= \pm 4$

Finding the square root of both sides.

b $3x^2 = 75$

$x^2 = \frac{75}{3}$

$x^2 = 25$

$x = \pm\sqrt{25}$

$= \pm 5$

c $4m^2 - 12 = 0$

$4m^2 - 12 + 12 = 0 + 12$

$4m^2 = 12$

$m^2 = \frac{12}{4}$

$m^2 = 3$

$m = \pm\sqrt{3}$

Leave the answer as a surd (in exact form).

Foundation Standard Complex

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Example 4

Solve $7x^2 - 88 = 0$, writing the solution correct to one decimal place.

Solution

$7x^2 - 88 = 0$

$7x^2 = 88$

$x^2 = \frac{88}{7}$

$x = \pm\sqrt{\frac{88}{7}}$

$x = \pm 3.54562...$

$\approx \pm 3.5$

The quadratic equation $x^2 + bx + c = 0$

To solve quadratic equations of the form $x^2 + bx + c = 0$, factorise $x^2 + bx + c$ into binomial products.



Quadratic equations by factorising

Example 5

Solve $x^2 + 5x + 6 = 0$.

Solution

To factorise $x^2 + 5x + 6$, find 2 numbers that have a sum of 5 and a product of 6.

The correct numbers are 2 and 3.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

The LHS has been factorised into 2 binomials, $(x + 2)$ and $(x + 3)$, whose product is 0.

If 2 numbers have a product of 0, then one of the numbers *must* be 0.

$\therefore x + 2 = 0$ or $x + 3 = 0$

$\therefore x = -2$ or $x = -3$

$\therefore$ The solution to $x^2 + 5x + 6 = 0$ is

$x = -2$ or $x = -3$.

Check:

When $x = -2$,

LHS $= (-2)^2 + 5 \times (-2) + 6 = 0$

RHS $= 0$

LHS = RHS.

When $x = -3$,

LHS $= (-3)^2 + 5 \times (-3) + 6 = 0$

RHS $= 0$

LHS = RHS.

Example 6

Solve each quadratic equation.

a $x^2 - x - 2 = 0$

b $u^2 + 3u - 28 = 0$

c $3m^2 = 6m$

d $w^2 - 10w + 25 = 0$



Quadratic equations by factorising

Solution

a $x^2 - x - 2 = 0$

Find 2 numbers that have a sum of -1 and a product of -2.

They are -2 and 1.

$(x - 2)(x + 1) = 0$

$\therefore x - 2 = 0$ or $x + 1 = 0$

$\therefore x = 2$ or $x = -1$

$\therefore$ The solution to $x^2 - x - 2 = 0$ is

$x = 2$ or $x = -1$.

Check:

When $x = 2$,

LHS $= 2^2 - 2 - 2 = 0$

RHS $= 0$

LHS = RHS.

When $x = -1$,

LHS $= (-1)^2 - (-1) - 2 = 0$

RHS $= 0$

LHS = RHS.

b $u^2 + 3u - 28 = 0$

$(u + 7)(u - 4) = 0$

$\therefore u + 7 = 0$ or $u - 4 = 0$

$\therefore u = -7$ or $u = 4$

$\therefore$ The solution to $u^2 + 3u - 28 = 0$ is $u = -7$ or $u = 4$.

c $3m^2 = 6m$

$3m^2 - 6m = 0$

This requires a simpler factorisation as there are only 2 terms, both involving m.

You cannot divide both sides by m because if $m = 0$ then you are dividing by 0.

$3m(m - 2) = 0$

$\therefore 3m = 0$ or $m - 2 = 0$

$\therefore m = 0$ or $m = 2$

$\therefore$ The solution to $3m^2 - 6m = 0$ is $m = 0$ or $m = 2$.

d $w^2 - 10w + 25 = 0$

$(w - 5)(w - 5) = 0$

$\therefore w - 5 = 0$ or $w - 5 = 0$

$\therefore w = 5$ or $w = 5$

$\therefore$ The solution to $w^2 - 10w + 25 = 0$ is $w = 5$ (only one solution).

Note: Quadratic equations of the form $ax^2 + bx + c = 0$ will be met in Chapter 11, *Quadratic equations and the parabola*.

EXERCISE 7.02 ANSWERS ON P. 613

Quadratic equations $x^2 + bx + c = 0$ UFRC

1 Solve each quadratic equation, writing the solutions as simplified surds if necessary.

EXAMPLE 3

a $m^2 = 144$ **b** $x^2 = 400$ **c** $y^2 = 35$ **d** $k^2 - 169 = 0$

e $y^2 - 1 = 0$ **f** $w^2 - 24 = 0$ **g** $x^2 + 10 = 14$ **h** $t^2 - 9 = 7$

i $\frac{a^2}{2} = 8$ **j** $4k^2 = 180$ **k** $3w^2 = 300$ **l** $d^2 + 60 = 204$

m $\frac{k^2}{2} = 7$ **n** $\frac{w^2}{10} = 2.5$ **o** $4x^2 = 1$ **p** $\frac{m^2}{4} = 10$

q $5y^2 = 5$ **r** $2p^2 + 3 = 21$ **s** $\frac{3k^2}{2} + 1 = 13$ **t** $\frac{y^2}{5} - 2 = 18$

2 Solve each equation, correct to one decimal place where necessary.

EXAMPLE 4

a $5m^2 - 20 = 0$ **b** $\frac{4a^2}{9} = 36$ **c** $m^2 = 28$

d $9m^2 - 2 = 32$ **e** $9k^2 + 10 = 13$ **f** $\frac{2x^2}{5} = 23$

g $\frac{k^2}{16} = 6$ **h** $\frac{3k^2}{10} = 27$ **i** $6y^2 = 0.726$

j $3a^2 + 11 = 267$ **k** $2y^2 - 14 = 63$ **l** $\frac{2w^2}{5} - 1 = 19$

3 Solve each equation.

EXAMPLE 5

a $x^2 + 3x + 2 = 0$ **b** $y^2 + 5y + 4 = 0$ **c** $y^2 + 16y + 48 = 0$

d $x^2 + x - 12 = 0$ **e** $x^2 + 2x - 3 = 0$ **f** $x^2 + 3x - 40 = 0$

4 Solve each equation.

EXAMPLE 6

a $x^2 - x - 30 = 0$ **b** $x^2 - 8x + 16 = 0$ **c** $x^2 - 5x - 66 = 0$

d $d^2 - 2d = 0$ **e** $x^2 - 3x - 10 = 0$ **f** $n^2 + 4n = 0$

g $k^2 - 7k = 0$ **h** $y^2 = 5y$ **i** $v^2 = 12v$

5 Explain why the quadratic equation $x^2 = -25$ has no solutions. R C

6 State which of these quadratic equations have no solutions. Give reasons. R C

a $x^2 = -9$ **b** $2k^2 + 5 = 9$ **c** $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$ **e** $4 + \frac{d^2}{3} = 8$ **f** $\frac{5a^2}{2} + 3 = 2$

7 Solve each quadratic equation. Select the correct answer **A**, **B**, **C** or **D**.

a $x^2 + 4x - 60 = 0$

A $x = -10, 6$ **B** $x = 12, -5$ **C** $x = 10, -6$ **D** $x = -12, 5$

b $q^2 + 3q = 0$

A $q = 3, -3$ **B** $q = 6, -3$ **C** $q = 0, -3$ **D** $q = 0, 3$

7.02

7.03 Simple cubic equations $ax^3 = c$#

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STAGE 5.3

An equation in which the highest power of the variable is 3 is called a **cubic equation**, for example, $x^3 = 12$, $2m^3 + 1 = 25$, $d^3 - 14 = 4$ and $x^3 - 3x^2 + 5x + 4 = 0$.

The cubic equation $x^3 = c$

The cubic equation $x^3 = c$ has one solution:

$$x = \sqrt[3]{c}.$$

Example 7

Solve each cubic equation.

a $y^3 = 64$ **b** $p^3 = 50$ **c** $-2x^3 = 2000$

Solution

a $y^3 = 64$

$y = \sqrt[3]{64}$

$= 4$

Finding the cube root of both sides.

$4^3 = 64$

b $p^3 = 50$

$p = \sqrt[3]{50}$

50 is not a cube number so leave the answer as a surd.

c $-2x^3 = 2000$

$x^3 = \frac{2000}{-2}$

$= -1000$

Dividing both sides by (-2).

$x = \sqrt[3]{-1000}$

$= -10$

Finding the cube root of both sides.

$(-10)^3 = -1000$

Example 8

Solve each cubic equation, writing the solution correct to one decimal place.

a $11x^3 - 102 = 0$ **b** $\frac{2y^3}{7} = -11$

Solution

a $11x^3 - 102 = 0$

$11x^3 = 102$

$x^3 = \frac{102}{11}$

$= 9.272...$

$x = \sqrt[3]{9.272...}$

$= 2.1008...$

≈ 2.1

b $\frac{2y^3}{7} = -11$

$2y^3 = -11 \times 7$

$= -77$

$y^3 = \frac{-77}{2}$

$= -38.5$

$y = \sqrt[3]{-38.5}$

$= -3.3766...$

≈ -3.4

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EXERCISE 7.03 ANSWERS ON P. 613

STAGE 5.3

Simple cubic equations $ax^3 = c$ U F R C

EXAMPLE 7

1 Solve each cubic equation, writing the solution in exact form where necessary.

a	$x^3 = 1$	**b**	$m^3 = 125$	**c**	$a^3 = 1331$
d	$u^3 = -8$	**e**	$y^3 = -729$	**f**	$n^3 = 20$
g	$h^3 = 11$	**h**	$k^3 = -48$	**i**	$5m^3 = -75$
j	$7m^3 = 448$	**k**	$-4x^3 = 81$	**l**	$12x^3 = -480$

EXAMPLE 8

2 Solve each cubic equation, writing the solution correct to one decimal place.

a	$w^3 - 16 = 0$	**b**	$m^3 + 6 = 22$	**c**	$5m^3 - 1080 = 0$
d	$3t^3 - 10 = 87$	**e**	$\frac{x^3}{3} = 9$	**f**	$\frac{5x^3}{7} = -120$
g	$\frac{3x^3}{4} = -10$	**h**	$\frac{2x^3}{5} = 0.2048$	**i**	$\frac{7a^3}{9} - 10 = 121$
j	$a^3 - 0.064 = 0$	**k**	$-\frac{7x^3}{9} = 10$	**l**	$5t^3 + 46 = -370$

3 **a** Does a cubic equation of the form $ax^3 = c$ always have a solution? R C

b When is the solution to $x^3 = c$ positive?

c When is the solution to $x^3 = c$ negative?

d Can $x^3 = c$ have 2 solutions?

7.03

Equation problems 7.04

Example 9

At a concert, an adult's ticket costs \$5 more than twice the cost of a child's ticket. The total cost for 3 adults and 7 children is \$327. Find the cost of a child's ticket and an adult's ticket.

Solution

Let the cost of a child's ticket be \$$c$.	Using a variable to represent an unknown quantity.
$\therefore$ Cost of an adult's ticket $= \$(2c + 5)$	
$3(2c + 5) + 7c = 327$	Forming an equation.
$6c + 15 + 7c = 327$	Solving the equation.
$13c + 15 = 327$	
$13c = 312$	
$c = 24$	A child's ticket costs \$24.
$\therefore$ Cost of an adult's ticket $= 2 \times \$24 + 5$	$\$(2c + 5)$
$= \$53$	
$\therefore$ A child's ticket costs \$24 and an adult's ticket costs \$53.	*Check*: $3 \times \$53 + 7 \times \$24 = \$327$.

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Example 10

Paris is 7 years older than Amy. 10 years from now, the sum of their ages will be 43.

How old are they now?

Solution

Let x = Amy's age now.

Then Paris' age now = $x + 7$.

Break the information into 'Now' and 'In 10 years time'.

	Now	In 10 years time
Amy	x	$x + 10$
Paris	$x + 7$	$x + 7 + 10 = x + 17$

In 10 years time:

Sum of ages: $(x + 10) + (x + 17) = 43$

$$2x + 27 = 43$$
$$2x = 16$$
$$x = 8$$

Amy is 8 years old now.

Paris is $8 + 7 = 15$ years old now.

Check: In 10 years time, the sum of their ages will be $18 + 25 = 43$.

EXERCISE 7.04 ANSWERS ON P. 613

Equation problems U F PS R C

For each question, write an equation and solve it to answer the problem. PS R C

1 The longer sides of an isosceles triangle are twice as long as the shorter side. The perimeter of the triangle is 90 mm. Find the lengths of the sides of the triangle.

2 At the football match, an adult's ticket costs $6 more than twice the cost of a child's ticket. The total cost for 3 adults and 5 children is $249. Find the cost of a child's ticket and an adult's ticket.

EXAMPLE 9

3 The length of a rectangle is 3 times as long as its width. The perimeter of the rectangle is 152 mm. Find its dimensions.

4 The length of a rectangle is 3 more than twice its width. Find the dimensions of the rectangle if its perimeter is 84 cm.

5 The sum of 3 **consecutive** integers is 186. Find the integers.

EXAMPLE 10

6 Tyson is 9 times the age of his daughter, Charlotte. In 5 years, he will be 4 times the age of Charlotte. How old are they now?

7 When 15 is subtracted from 3 times a certain number, the answer is 63. What is the number?

8 The product of 2 and a number is the same as 12 minus the number. Find the number.

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9 The sum of the present ages of Vatha and Chris is 36. In 4 years time, the sum of their ages will equal twice Vatha's present age. How old are they now?

10 4 consecutive integers have a sum of 858. Find the 4 integers.

11 The sum of 3 consecutive even numbers is 288. Find the numbers.

12 Manori's bag contained 10-cent and 20-cent coins. She had 202 coins, with a total value of $31.90. How many 20-cent coins did Manori have?

13 If 17 more than a number is 5 more than 3 times the number, what is the number?

14 The sum of Scott's age and his mother Kait's age is 45. In 5 years time, 3 times Scott's age less 9 will be the same as Kait's age. Find the present ages of Scott and Kait.

15 One angle in a triangle is double the smallest angle, and the third angle in the triangle is 5 more than 4 times the smallest angle. Find the size of each angle in the triangle.

16 A large container of water is $\frac{7}{8}$ full. After 15 L has been taken out, the container is $\frac{2}{3}$ full. When full, how many litres does the container hold?

17 The total cost of a school camp for Year 10 students was $21 280. Each teacher paid $185 to attend and each student paid $165. There was one teacher for every 15 students. Find the numbers of teachers and students that attended the camp.

Mental skills 7: Maths without calculators ANSWERS ON P. 613

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

b $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.30$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

c By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

2 Now evaluate each product.

a	0.7×5	**b**	12×0.2	**c**	0.4×0.3	**d**	$(0.6)^2$
e	8×0.1	**f**	0.03×0.9	**g**	4×0.05	**h**	1.1×8
i	0.3×0.8	**j**	0.2×0.06	**k**	9×0.2	**l**	0.07×0.4

Foundation | Standard | Complex

3 Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

a $1.5 \times 2.3 = 3.45$

$1 \text{ dp} + 1 \text{ dp} = 2 \text{ dp}$ (*Estimate* $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

$0 \text{ dp} + 2 \text{ dp} \quad = 2 \text{ dp}$

(*Estimate* $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

$2 \text{ dp} + 0 \text{ dp} \quad = 2 \text{ dp}$

(*Estimate* $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a	3.9×17	**b**	39×170	**c**	39×0.17	**d**	0.39×1.7
e	3.9×1.7	**f**	390×1.7	**g**	3.9×0.17	**h**	3.9×170
i	3900×1.7	**j**	39×1.7	**k**	39×0.017	**l**	0.39×0.17

7.05 Equations and formulas

Getting the right formula

A **formula** is an equation that describes a relationship between variables. For example, the formula for the perimeter of a rectangle is $P = 2(l + w)$, where P is the perimeter, l is the rectangle's length and w is the width. Because the formula is for the perimeter, P is called the **subject** of the formula and it is the variable on its own on the left side of the '=' sign.

Equations and formulas

Formulas and equations

Example 11

The cost of a trip, C, in dollars, for a ride share company is $C = 5 + 2.4d$, where d is the distance travelled, in kilometres.

a Find the cost of a trip if the distance travelled is 15 km.

b Find the distance travelled if the cost of the trip was \$78.20.



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Solution

a When $d = 15$:

$C = 5 + 2.4 \times 15$

$= 41$

The cost was \$41.

b When $C = 78.20$:

$78.20 = 5 + 2.4d$

$73.20 = 2.4d$

$d = \frac{73.20}{2.4}$

$= 30.5$

The distance travelled was 30.5 km.

Example 12

The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius.

Find (correct to one decimal place):

a the surface area of a sphere with radius 2.8 cm

b the radius of a sphere with surface area 40 m^2.

Solution

a When $r = 2.8$:

$A = 4 \times \pi \times 2.8^2$

$= 98.520...$

≈ 98.5

The surface area of the sphere is 98.5 cm^2.

b When $A = 40$:

$40 = 4\pi r^2$

$4\pi r^2 = 40$

$r^2 = \frac{40}{4\pi}$

$= 3.183...$

$r = \sqrt{3.183}$ $r > 0$ because the radius is positive.

$= 1.784...$

≈ 1.8

The radius of the sphere is 1.8 m.

EXERCISE 7.05 ANSWERS ON P. 613

Equations and formulas UFRC

1 The formula for the perimeter of a rectangle is $P = 2(l + w)$. R

a Find the perimeter of a rectangle with length 10 cm and width 16 cm.

b Find the width of a rectangle whose perimeter is 58 m and length is 12 m.

2 A formula for converting speed expressed in m/s (metres/second) to a speed expressed in km/h is $k = 3.6M$, where M is the speed in m/s. Convert each speed to km/h. R C

a 10 m/s **b** 24 m/s **c** 50 m/s

3 A car is travelling at a speed of 110 km/h on a freeway. Use the formula from question **2** to calculate how fast this is in m/s.

4 The formula for converting temperatures in °F to °C is $C = \frac{5}{9}(F - 32)$.

Express each temperature in °C, correct to the nearest degree.

a 80°F **b** 32°F **c** 212°F **d** 102°F

5 The average of 2 numbers, m and n, is $A = \frac{m+n}{2}$. If 2 numbers have an average of 28 and one of the numbers is 13, use the formula to find the other number. R

EXAMPLE 12

6 Pythagoras' theorem is $c^2 = a^2 + b^2$. Find, correct to one decimal place where necessary:

a c, if $a = 5$ and $b = 10$ **b** a, if $c = 41$ and $b = 40$ **c** b, if $c = 20$ and $a = 10$

7 The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius. Find, correct to one decimal place: R C

a the circumference of a circle with radius 2.4 m

b the radius of a circle whose circumference is 200 cm.

8 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres. Find, correct to one decimal place: R

a the BMI of Leonie who is 1.85 m tall and has a mass of 72 kg

b the mass of Dane with a BMI of 24, who is 2.1 m tall.

9 The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find, correct to one decimal place: R

a the volume of a sphere with radius 3.2 cm

b the radius of a sphere with volume 500 m^3.

10 The average speed in km/h of a car is given by the formula $S = \frac{D}{T}$, where D is the distance covered in kilometres and T is the time taken in hours. Find, correct to the nearest whole number: R

a the average speed of a car that takes 4.5 hours to travel a distance of 420 km

b the distance travelled, if a car maintains a speed of 87.2 km/h for 5 hours

c the time taken, if a distance of 650 km is covered at a speed of 91 km/h.

11 The cost, C (in dollars), of a hire car is $C = 75 + 2.5d$, where d is the number of kilometres travelled. Calculate: R

a the cost of hiring a car to travel 350 km

b the distance travelled, if the cost is $135.

12 The surface area of a closed cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$. Calculate, correct to one decimal place: R

a the surface area of a cylinder with radius 2.1 m and height 3.5 m

b the height of a cylinder with surface area 1255.38 cm^2 and radius 9 cm.

Changing the subject of a formula#

7.06

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STAGE 5.3



Changing the subject of a formula

7.06

Example 13

Change the subject of the formula:

a $A = \frac{1}{2}bh$ to h **b** $v^2 = u^2 + 2as$ to s **c** $\frac{a+2}{a+10} = k$ to a

Solution

a $A = \frac{1}{2}bh$

$\frac{1}{2}bh = A$ Swapping sides so that h appears on the LHS.

$bh = 2A$ Multiplying both sides by 2.

$h = \frac{2A}{b}$ Dividing both sides by b to make h the subject of the formula.

b $v^2 = u^2 + 2as$

$u^2 + 2as = v^2$ Swapping sides so that s appears on the LHS.

$2as = v^2 - u^2$ Subtracting u^2 from both sides.

$s = \frac{v^2 - u^2}{2a}$ Dividing both sides by $2a$ to make s the subject.

c $\frac{a+2}{a+10} = k$

$a + 2 = k(a + 10)$ Multiplying both sides by $a + 10$.

$= ak + 10k$ Expanding

$a - ak = 10k - 2$ Moving the a-terms to the LHS, the 2 to the RHS.

$a(1 - k) = 10k - 2$ Factorising a from the LHS.

$a = \frac{10k - 2}{1 - k}$ Dividing both sides by $1 - k$ to make a the subject.

EXERCISE 7.06 ANSWERS ON P. 613

Changing the subject of a formula U F R C

1 Make y the subject of each formula. R C

EXAMPLE 13

a $x + 2y = 5$ **b** $m + py = k$ **c** $P - ky = 8$

d $\frac{m}{3} = \frac{y}{5}$ **e** $D = K - My$ **f** $\frac{5+8y}{d} = 4$

g $\frac{ay - k}{2} = c$ **h** $\frac{y+3}{5} = \frac{4m}{3}$ **i** $xy^2 + 5 = w$

j $x = \sqrt{\frac{y}{k}}$ **k** $n = \frac{d}{5 - y}$ **l** $T = \sqrt{\frac{y+k}{c}}$

Foundation Standard Complex

STAGE 5.3

2 Change the subject of each formula to the variable indicated in brackets. R C

a $a^2 + b^2 = c^2$ $[b]$

b $s = ut + \frac{1}{2}at^2$ $[a]$

c $v = u + at$ $[a]$

d $V = \frac{4}{3}\pi r^3$ $[r]$

e $A = \pi(R^2 - r^2)$ $[R]$

f $A = \pi rl + \pi r^2$ $[l]$

g $S = 180(n - 2)$ $[n]$

h $\frac{1}{x} + \frac{1}{r} = \frac{1}{s}$ $[r]$

i $x = \sqrt{b^2 - 4ac}$ $[b]$

j $x + y = 5 - 3x$ $[x]$

k $m = \frac{5A}{2A + n}$ $[A]$

l $S = \frac{a(p-1)}{p}$ $[p]$

m $X(a + b) = Y(a - b)$ $[a]$

n $\frac{5 + x}{3x + a} = 2$ $[x]$

o $y = \frac{u + bx}{u + ab}$ $[b]$

Investigation

Restricting values of variables

1 Consider the formula $x^2 + y^2 = 4$.

a Explain why the lowest value that x can take is -2 and the highest value that x can take is 2.

b Does the same restriction apply to the values that the variable y can take? Explain why.

c By making y the subject, show that $y = \pm\sqrt{4 - x^2}$.

d Are the values that x and y can take in $y = \pm\sqrt{4 - x^2}$ different from the values that they can take in $x^2 + y^2 = 4$? Give reasons.

2 **a** If $Z = ax^2$, what range of values can Z, a and x take?

b If $x = \sqrt{\frac{Z}{a}}$, what range of values can Z, a and x take?

3 **a** In the formula $A = \pi r^2$, explain why there are no restrictions on r but $A \geq 0$.

b Make r the subject of the formula given that the formula is for the area of a circle. Have the restrictions on the variables r and A changed?

4 **a** What are the restrictions on the variables x and y in the formula $y = \sqrt{16 - x^2}$?

b Change the subject of the formula to x. Are the restrictions on the variables the same as for part **a**? Explain.

5 Consider the formula $y = \frac{1}{x - 3}$.

a What are the restrictions on the variables x and y? Give reasons.

b Make x the subject of the formula. Are the restrictions on the variables the same as in part **a** or different? Explain.

c Compare your answers to the above questions with those of other students in your class.

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Graphing inequalities on a number line 7.07

An **inequality** looks like an equation except that the equals sign (=) is replaced by an inequality symbol >, ≥, < or ≤.

$2x - 7 = 15$ is an **equation**. There is only one value of x that makes it true.

$2x - 7 \leq 15$ is an **inequality**. There is a range of values of x that make it true.

Inequality symbols

$>$ 'is greater than'

$<$ 'is less than'

$\geq$ 'is greater than or equal to'

$\leq$ 'is less than or equal to'

For example, '$x \geq 3$' is read 'x is greater than or equal to 3'. It includes 3 and all the numbers above 3, such as 3.01, 4, 10, 20 000. '$x > 3$' is read 'x is greater than 3' and means all the numbers above 3, but **not** 3.

Inequality	In words	Meaning
$x > 3$	x is greater than 3	Values above 3
$x < 3$	x is less than 3	Values below 3
$x \geq 3$	x is greater than or equal to 3	Values above and including 3
$x \leq 3$	x is less than or equal to 3	Values below and including 3

An inequality can be graphed on a **number line**.

Example 14

Graph each inequality on a number line.

a $x \geq 1$ **b** $x < 5$ **c** $x > -3$

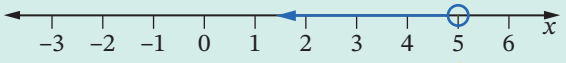
Solution

a $x \geq 1$ means that x can be any number greater than 1 or equal to 1.



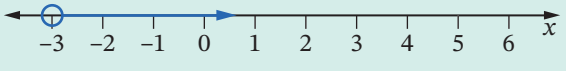
The filled circle at 1 means we include 1.

b $x < 5$ means that x can be any number less than 5, but not including 5.



The open circle on 5 means that 5 is not included.

c $x > -3$ means that x can be any number greater than −3, but not including −3.



Graphing inequalities on the number line

EXERCISE 7.07 ANSWERS ON P. 614

Graphing inequalities on a number line U F C

EXAMPLE 14

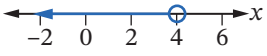
1 Graph each inequality on a separate number line. C

a $x \geq 2$ **b** $x < -3$ **c** $x \leq 1$ **d** $x > 7$

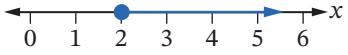
e $x \leq 4$ **f** $x > 0$ **g** $x \geq -2$ **h** $x < 10$

2 Write the inequality shown on each number line. C

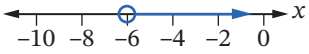
a



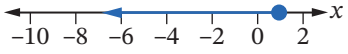
b



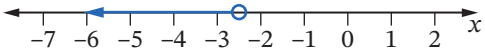
c



d



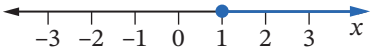
3 Which inequality is graphed below? Select the correct answer **A**, **B**, **C** or **D**. C



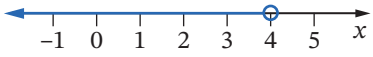
A $x > -2.5$ **B** $x < -2.5$ **C** $x < -3.5$ **D** $x > -3.5$

4 Write the inequality shown on each number line. C

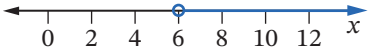
a



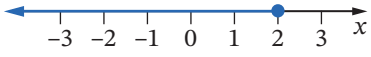
b



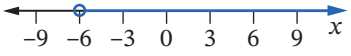
c



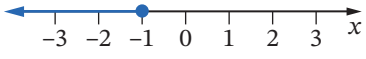
d



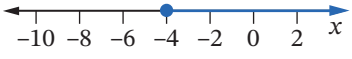
e



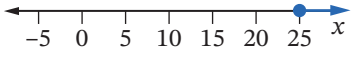
f



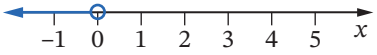
g



h



i



Investigation

The language of inequalities

Work in pairs to complete this activity.

Use inequality symbols to write each statement algebraically.

a The minimum height (H) for rides at an amusement park is 1.3 m.

b The speed limit in a school zone is 40 km/h.

c To be eligible to vote, you must be at least 18 years old (A = age).

d The overseas tour is only for people whose age (A) is from 18 to 35.

e The cost (A) of a tennis racquet will be at least \$95, but no more than \$360.

f A new flute (F) costs at least \$475.

g The price of units (U) in a new block start at \$240 000.

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Investigation

Solving inequalities

We have solved equations by doing the same thing to both sides (keeping the equation 'balanced'). Will this method work with inequalities, such as $x + 4 > 10$ or $6x < 13$?

1 Start with an inequality that is true, such as $7 > 4$.

2 Add 5 (or any number you choose) to both sides of the inequality; for example, $7 > 4$ becomes $12 > 9$. Is the new inequality true or false?

3 Subtract 9 (or any number you choose) from each side of the original inequality; for example, $7 > 4$ becomes $-2 > -5$. Is the new inequality true or false?

4 Multiply both sides of the original inequality by 4 (or any positive number you choose); for example, $7 > 4$ becomes $28 > 16$. Is the new inequality true or false?

5 Divide both sides of the original inequality by 2 (or any positive number you choose); for example, $7 > 4$ becomes $3\frac{1}{2} > 2$. Is the new inequality true or false?

6 Multiply both sides of the original inequality by -3 (or any negative number you choose); for example, $7 > 4$ becomes $-21 > -12$. Is the new inequality true or false?

7 Divide both sides of the original inequality by -4 (or any negative number you choose), for example, $7 > 4$ becomes $-1\frac{3}{4} > -1$. Is the new inequality true or false?

8 Which of the 6 operations used in questions **2** to **7** can be used on inequalities to give a true result?

9 Which of the 6 operations used in questions **2** to **7** cannot be used with inequalities because they give a false result?

10 Copy and complete each set of inequality statements.

a $6 < 8$

$6 \times 3 < 8 \times$ ___ (multiplying both sides by 3)

$\therefore 18$ __ 24

b $10 > -4$

$10 \div 2$ __ $-4 \div$ __ (dividing both sides by 2)

$\therefore$ _____

Does the inequality sign ($<$ or $>$) stay the same when multiplying or dividing by a positive number?

11 **a** Is it true that $5 < 8$?

b Multiply both sides by -2. Is it true that $-10 < -16$?

c What needs to be reversed to change $-10 < -16$ into a true inequality statement?

d Copy and complete to make a true statement: -10 _____ -16.

12 **a** Is it true that $18 > -6$?

b Divide both sides by -3. Is it true that $-6 > 2$?

c What needs to be reversed to change $-6 > 2$ into a true inequality statement?

d Copy and complete to make a true inequality statement: -6 ____ 2.

13 Copy and complete: When multiplying or d_____ both sides of an inequality by a n_____ number, the inequality sign must be r_____.

7.08 Solving inequalities

WS Inequalities review

Example 15

Solve each inequality and graph its solution on a number line.

a $2x - 10 \leq 16$ **b** $2(y - 1) \geq 12$ **c** $\frac{w+3}{2} > -1$

Solution

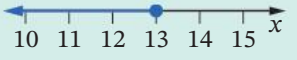
a $2x - 10 \leq 16$

$2x - 10 + 10 \leq 16 + 10$

$2x \leq 26$

$\frac{2x}{2} \leq \frac{26}{2}$

$x \leq 13$



Check: Test a value less than 13, let $x = 11$.

LHS $= 2 \times 11 - 10 = 12$

RHS $= 16$

$12 \leq 16$

LHS $\leq$ RHS

b $2(y - 1) \geq 12$

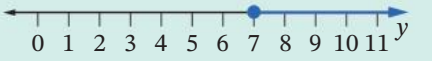
$2y - 2 \geq 12$

$2y - 2 + 2 \geq 12 + 2$

$2y \geq 14$

$\frac{2y}{2} \geq \frac{14}{2}$

$y \geq 7$



Check: Test a value greater than 7, let $y = 8$.

LHS $= 2(8 - 1) = 14$

RHS $= 12$

$14 \geq 12$

LHS $\geq$ RHS

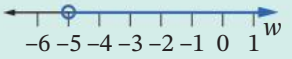
c $\frac{w+3}{2} > -1$

$\frac{w+3}{2} \times 2 > -1 \times 2$

$w + 3 > -2$

$w + 3 - 3 > -2 - 3$

$w > -5$



Check: Test a value greater than −5, let $w = -1$.

LHS $= \frac{-1+3}{2} = 1$

RHS $= -1$

$1 > -1$

LHS > RHS

Solving inequalities

- Inequalities can be solved algebraically in the same way as equations, using inverse operations.
- However, when multiplying or dividing both sides of an inequality by a **negative** number, you must **reverse** the inequality sign.

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Example 16

Solve each inequality.

a $1 - 2x \geq -11$ **b** $4 - r < 7$ **c** $\frac{a+5}{-3} > 4$

Solution

a

$1 - 2x \geq -11$

$1 - 2x - 1 \geq -11 - 1$

$-2x \geq -12$

$\frac{-2x}{-2} \leq \frac{-12}{-2}$ Dividing both sides by a negative number reverses the inequality sign.

$x \leq 6$

Check: Test a value less than 6, let $x = 5$.

LHS $= 1 - 2 \times 5 = -9$

RHS $= -11$

$-9 \geq -11$

LHS $\geq$ RHS

b

$4 - r < 7$

$4 - r - 4 < 7 - 4$

$-r < 3$

$\frac{-r}{-1} > \frac{3}{-1}$

$r > -3$

Check: Test a value greater than −3, let $r = 0$.

LHS $= 4 - 0 = 4$

RHS $= 7$

$4 < 7$

LHS < RHS

c

$\frac{a+5}{-3} > 4$

$\frac{a+5}{-3} \times (-3) < 4 \times (-3)$

$a + 5 < -12$

$a + 5 - 5 < -12 - 5$

$a < -17$

Check: Test a value less than −17, let $a = -20$.

LHS $= \frac{-20+5}{-3} = \frac{-15}{-3} = 5$

RHS $= 4$

$5 > 4$

LHS > RHS

EXERCISE 7.08 ANSWERS ON P. 614

Solving inequalities UFRC

1 Solve each inequality and graph its solution on a number line. R C

EXAMPLE 15

a $x - 1 > 6$ **b** $3y \geq 12$ **c** $m + 4 \leq 2$

d $\frac{x}{5} \geq -20$ **e** $12x < 60$ **f** $5y > -20$

g $4a \geq 2$ **h** $3w \leq -30$ **i** $8a + 5 \geq 45$

j $3a + 1 \leq 10$ **k** $6a + 4 \geq -2$ **l** $3w - 3 < -12$

m $5a + 3 \leq -27$ **n** $5y + 1 \leq 16$ **o** $4a + 5 < 15$

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2 Solve $\frac{3(x-2)}{5} \geq -1$. Select the correct answer **A**, **B**, **C** or **D**.

A $x \geq 0$ **B** $x \geq -1$ **C** $x \geq \frac{1}{3}$ **D** $x \geq -\frac{11}{3}$

3 Solve each inequality.

a $3(x+2) \geq 9$ **b** $5(m-4) \leq 10$ **c** $2(y+5) \leq -6$

d $3+2x < 9+x$ **e** $11-5y \geq 9-6y$ **f** $2(3+5a) \leq 5(4+a)$

g $\frac{x-1}{2} \leq 2$ **h** $\frac{w-2}{5} > -1$ **i** $\frac{2a+1}{3} < 3$

j $\frac{2(m+1)}{3} \leq 3$ **k** $\frac{4(m-2)}{3} \geq -6$ **l** $3+\frac{x}{5} < 10$

EXAMPLE 16

4 Solve each inequality and graph its solution on a number line. R C

a $5-x \leq 2$ **b** $15 > 7-y$ **c** $1-k < 12$

d $7-m \geq 7$ **e** $2-p > 8$ **f** $-t+6 \geq 10$

5 Solve each inequality.

a $-2x < 6$ **b** $\frac{k}{-3} \geq 4$ **c** $-5t > 12$

d $\frac{-x}{3} \leq -4$ **e** $4-3w > 7$ **f** $-4y+3 \leq 11$

g $1-3m < 9-5m$ **h** $8-5a < 3$ **i** $-2d-3 > 8$

j $\frac{5+w}{-3} > 2$ **k** $3(3x+4) \geq 6(1-2x)$ **l** $\frac{-p+2}{-3} < -2$

Investigation

Power tables

1 Copy and complete this table of powers of 2 from 0 to 20.

x	0	1	2	...	20
2^x					

2 Use the table to calculate 32×128. Explain the method you used.

3 Use the table to calculate:

a 16×1024 **b** 128×2048

c 256×64 **d** 4096×32

4 Use the table to calculate $262\,114 \div 8192$. Explain the method you used.

5 Use the table to calculate:

a $16\,384 \div 512$ **b** $128 \div 8$

c $8192 \div 1024$ **d** $1\,048\,576 \div 65\,536$

When powers are used this way in calculations, they may be called **logarithms.**

Logarithms 7.09

The **logarithm** of a number is the power of the number, to a given positive base.

For example, the logarithm of 8 to the base 2 is 3, written $\log_2 8 = 3$, because $2^3 = 8$.

STAGE 5.3

7.09

Logarithms 1

Logarithms 2

Example 17

Evaluate each expression.

a $\log_3 81$ **b** $\log_4 16$ **c** $\log_{10} 10\,000$

Solution

a $\log_3 81$ means $3^? = 81$

'3 to the power of what equals 81?'

Since $3^4 = 81$

then $\log_3 81 = 4$.

b $\log_4 16$ means $4^? = 16$

'4 to the power of what equals 16?'

Since $4^2 = 16$

then $\log_4 16 = 2$.

c $\log_{10} 10\,000$ means $10^? = 10\,000$

Since $10^4 = 10\,000$

then $\log_{10} 10\,000 = 4$.

Logarithms

$$\text{If } y = a^x, \text{ then } \log_a y = x$$

where a is the base, $a > 0$, x is the power, and $y > 0$.

Since $a > 0$, $a^x > 0$ and $y > 0$.

Logarithms are only meaningful for positive numbers, y.

Example 18

Write each expression as a logarithm.

a $243 = 3^5$ **b** $0.01 = 10^{-2}$ **c** $2 = 8^{\frac{1}{3}}$ **d** $p = q^r$

Solution

a $243 = 3^5$

$\therefore \log_3 243 = 5$

b $0.01 = 10^{-2}$

$\therefore \log_{10} 0.01 = -2$

c $2 = 8^{\frac{1}{3}}$

$\therefore \log_8 2 = \frac{1}{3}$

d $p = q^r$

$\therefore \log_q p = r$

STAGE 5.3

Example 19

Rewrite $\log_n m = x$ in index form.

Solution

$\log_n m = x$ — n is the base, x is the power.

$\therefore m = n^x$

EXERCISE 7.09 ANSWERS ON P. 614

Logarithms U F R C

EXAMPLE 17

1 Evaluate each expression. R

a $\log_5 25$	**b** $\log_2 8$	**c** $\log_7 49$	**d** $\log_2 16$
e $\log_3 243$	**f** $\log_{10} 1000$	**g** $\log_5 125$	**h** $\log_6 36$
i $\log_2 64$	**j** $\log_3 6561$	**k** $\log_{10} 1\,000\,000$	**l** $\log_8 512$

EXAMPLE 18

2 Write each expression in logarithmic form. R C

a $5^2 = 25$	**b** $4^3 = 64$	**c** $10\,000 = 10^4$	**d** $25^{\frac{1}{2}} = 5$
e $\frac{1}{16} = 2^{-4}$	**f** $3^{-2} = \frac{1}{9}$	**g** $8^{\frac{2}{3}} = 4$	**h** $0.01 = 10^{-2}$
i $\sqrt{2} = 4^{\frac{1}{4}}$	**j** $16^{\frac{1}{2}} = 4$	**k** $9^{\frac{3}{2}} = 27$	**l** $\frac{1}{\sqrt{6}} = 6^{-\frac{1}{2}}$

EXAMPLE 19

3 Write each expression in index form. R C

a $\log_5 125 = 3$	**b** $\log_{10} 10 = 1$	**c** $\log_{\sqrt{3}} 27 = 6$	**d** $\log_2 8\sqrt{2} = 3.5$
e $\log_2 64 = 6$	**f** $\log_3\left(\frac{1}{81}\right) = -4$	**g** $\log_5\left(\frac{1}{125}\right) = -3$	**h** $\log_8 \sqrt{2} = \frac{1}{6}$
i $\log_{100} 10 = \frac{1}{2}$	**j** $\log_5 5\sqrt{5} = \frac{3}{2}$	**k** $\log_8 2 = \frac{1}{3}$	**l** $\log_{100}\left(\frac{1}{100}\right) = -1$

4 Why can't you find the logarithm of a negative number or zero? R C

Did you know?

The Richter scale

The size of an earthquake is measured using a scale developed by American seismologist and physicist Charles Richter in 1935. The scale uses base 10 logarithms, where each level is 10 times more powerful than the previous level. For example, a magnitude 4 earthquake is 10 times more powerful than a magnitude 3 earthquake. As the magnitude of the earthquake increases, the energy released also increases and so too the damage. A large lightning bolt releases as much energy as a magnitude 3 earthquake.



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Magnitude	0	1	2	3	4	5	6	7	8
Damage	none	micro	minor	minor	light	moderate	strong	major	great

Earthquakes are rare in Australia. The most destructive earthquake occurred in Newcastle in 1989, causing $4 billion in damage.

Research the strength of the Newcastle earthquake on the Richter scale.

Logarithm laws 7.10

STAGE 5.3

The index laws from Chapter 5, *Products and factors*, are related to the logarithm laws.

Logarithm of a product

The logarithm of a product of terms is equal to the sum of the logarithm of each term.

$$\log_a(xy) = \log_a x + \log_a y$$

For example, $\log_2(8 \times 4) = \log_2 8 + \log_2 4$.

This law corresponds to the index law $a^m \times a^n = a^{mn}$.

Proof:

Let $m = \log_a x$ and $n = \log_a y$.

$\therefore x = a^m$ and $y = a^n$

$xy = a^m \times a^n = a^{m+n}$

$xy = a^{m+n}$

$\therefore \log_a(xy) = m + n$

$= \log_a x + \log_a y$

Logarithm of a quotient

The logarithm of a quotient of terms is equal to the difference between the logarithm of each term.

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

For example, $\log_3\left(\frac{243}{27}\right) = \log_3 243 - \log_3 27$

This law corresponds to the index law $a^m \div a^n = a^{m-n}$.

Proof:

Let $m = \log_a x$ and $n = \log_a y$.

$\therefore x = a^m$ and $y = a^n$

$\therefore \frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$

$\therefore \log_a\left(\frac{x}{y}\right) = m - n$

$= \log_a x - \log_a y$

Logarithm of a power

The logarithm of a term raised to a power is equal to the power multiplied by the logarithm of the term.

$$\log_a x^n = n \log_a x$$

STAGE 5.3

For example, $\log_4 8^2 = 2 \log_4 8$

This law corresponds to the index law $(a^m)^n = a^{mn}$.

Proof:

Let $m = \log_a x$

$\therefore x = a^m$

$\therefore x^n = (a^m)^n$

$= a^{mn}$

$\therefore \log_a x^n = mn$

$= (\log_a x) \times n$

$= n \log_a x$

Properties of logarithms

$\log_a a^x = x$

$\log_a a = 1$, because $a^1 = a$

$\log_a 1 = 0$, because $a^0 = 1$

$\log_a \left(\frac{1}{x}\right) = -\log_a x$

Proof:

$\log_a \left(\frac{1}{x}\right) = \log x^{-1}$ Using the law $\log_a x^n = n \log_a x$

$= -\log_a x$

Logarithm laws

Example 20

Evaluate each expression.

a $\log_5 0.04$

b $\log_2 5 - \log_2 10$

c $2 \log_3 6 - \log_3 4$

d $\log_5 10 + \log_5 2 - \log_5 4$

Solution

a $\log_5 0.04 = \log_5 \left(\frac{4}{100}\right)$

$= \log_5 \left(\frac{1}{25}\right)$

$= \log_5 \left(\frac{1}{5^2}\right)$

$= \log_5 5^{-2}$

$= -2$ since $\log_a a^x = x$

b $\log_2 5 - \log_2 10 = \log_2 \left(\frac{5}{10}\right)$

$= \log_2 \left(\frac{1}{2}\right)$

$= \log_2 2^{-1}$

$= -1$ since $\log_a a^x = x$

c $2 \log_3 6 - \log_3 4 = \log_3 6^2 - \log_3 4$

$= \log_3 36 - \log_3 4$

$= \log_3 \left(\frac{36}{4}\right)$

$= \log_3 9$

$= 2$

d $\log_5 10 + \log_5 2 - \log_5 4 = \log_5 (10 \times 2) - \log_5 4$

$= \log_5 20 - \log_5 4$

$= \log_5 \left(\frac{20}{4}\right)$

$= \log_5 5$

$= 1$

Example 21

Simplify each expression using the logarithm laws.

a $6\log_a a + \log_a a^4 - \log_a a^9$ **b** $\log_2 x + \log_2 w - 2\log_2 y$

c $\dfrac{\log_3 a^3}{5\log_3 a}$

Solution

a $6\log_a a + \log_a a^4 - \log_a a^9 = 6 \times 1 + 4 - 9$

$= 1$

b $\log_2 x + \log_2 w - 2\log_2 y = \log_2 (xw) - \log_2 y^2$

$= \log_2\left(\dfrac{xw}{y^2}\right)$

c $\dfrac{\log_3 a^3}{5\log_3 a} = \dfrac{3\log_3 a}{5\log_3 a}$

$= \dfrac{3}{5}$

Example 22

Given $\log_{10} 7 \approx 0.8451$, use logarithm laws to evaluate each expression.

a $\log_{10} 49$ **b** $\log_{10} 700$ **c** $\log_{10} (0.07)$

Solution

a $\log_{10} 49 = \log_{10} 7^2$

$= 2\log_{10} 7$

$\approx 2 \times 0.8451$

$= 1.6902$

b $\log_{10} 700 = \log_{10} (7 \times 100)$

$= \log_{10} 7 + \log_{10} 100$

$\approx 0.8451 + 2$

$= 2.8451$

c $\log_{10} (0.07) = \log_{10}\left(\dfrac{7}{100}\right)$

$= \log_{10} 7 - \log_{10} 100$

$\approx 0.8451 - 2$

$= -1.1549$

EXERCISE 7.10 ANSWERS ON P. 615

Logarithm laws U F R

1 Evaluate each expression. R

a $\log_2 128$ **b** $\log_{10} 1000$ **c** $\log_8 64$

d $\log_5 \dfrac{1}{5}$ **e** $\log_2 \sqrt{2}$ **f** $\log_3 \dfrac{1}{9}$

g $\log_{10} 0.0001$ **h** $\log_2 \dfrac{1}{16}$ **i** $\log_8 2 + \log_8 4$

j $\log_4 32 - \log_4 2$ **k** $\log_{10} 4 + \log_{10} 25$ **l** $\log_5 200 - \log_5 8$

m $\log_2 18 - 2\log_2 3$ **n** $3\log_{10} 2 + \log_{10} 12.5$

EXAMPLE 20

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EXAMPLE 21

2 Simplify each expression. R

a $\log_x 5 + \log_x 6$ **b** $\log_x 10 - \log_x 2$ **c** $3\log_x 2$

d $2\log_x 4 - \log_x 8$ **e** $\log_x 10 + \log_x 4$ **f** $\frac{1}{2}\log_x 100$

g $-\log_x 4$ **h** $\log_x 8 - (\log_x 10 + \log_x 4)$ **i** $\frac{1}{2}(\log_x 8 + \log_x 18)$

EXAMPLE 22

3 If $\log_{10} 4 = 0.6021$, evaluate each expression. R

a $\log_{10} 16$ **b** $\log_{10} 400$ **c** $\log_{10} 4000$ **d** $\log_{10} 2$

e $\log_{10} 0.4$ **f** $\log_{10} 160$ **g** $\log_{10} 2.5$ **h** $\log_{10} \sqrt{40}$

4 Evaluate each expression. R

a $\log_3 4 + \log_3 15 - \log_3 20$ **b** $\log_3 270 - (\log_3 2 + \log_3 5)$

c $\log_4 20 + (\log_4 32 - \log_4 10)$ **d** $2\log_{10} 25 - \log_{10} 6.25$

e $2\log_{10} 2 - (\log_{10} 5 + \log_{10} 8)$ **f** $\log_{100} 50 - \log_{100} 5$

g $2\log_5 10 + (\log_5 50 - \log_5 40)$ **h** $5\log_8 2 + \frac{1}{2}\log_8 4$

i $\frac{1}{2}\log_4 25 - 2\log_4 \sqrt{20}$ **j** $\frac{1}{3}\log_2 125 - 3\log_2 \sqrt[3]{80}$

5 Simplify each expression. R

a $\log_a a^2 + 3\log_a a$ **b** $\log_a (a^3)$ **c** $5\log_a a - \log_a a^4$

d $\frac{\log_a (x^7)}{\log_a x}$ **e** $\log_a y^3 - 3\log_a y$ **f** $\log_a \sqrt{x} - \log_a \frac{1}{x}$

7.11 Exponential and logarithmic equations

STAGE 5.3

Exponential equations

Exponential equations are equations like $3^x = 243$, where the variable or unknown is a **power.**

Logarithms can be used to solve exponential equations rather than using a 'guess-and-check' method. The log key on your calculator calculates logarithms to the base 10, $\log_{10} x$. Logarithms to the base 10 are called **common logarithms.**

Example 23

Solve each exponential equation.

a $3^x = 243$ **b** $4^{m+1} = \frac{1}{8\sqrt{2}}$

Solution

a $3^x = 243$

$\log_{10} 3^x = \log_{10} 243$ Taking $\log_{10}$ of both sides.

$x \log_{10} 3 = \log_{10} 243$

$x = \frac{\log_{10} 243}{\log_{10} 3} = 5$ On a calculator, enter: log 243 ÷ log 3 =

Note: The log key means $\log_{10}$, and for convenience we will write log to mean $\log_{10}$.

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b

$$4^{m+1} = \frac{1}{8\sqrt{2}}$$

$$\log\left(4^{m+1}\right) = \log\left(\frac{1}{8\sqrt{2}}\right)$$

$$(m+1)\log 4 = \log\left(\frac{1}{8\sqrt{2}}\right)$$

$$(m+1)\log 4 = -\log\left(8\sqrt{2}\right)$$

$$m+1 = \frac{-\log\left(8\sqrt{2}\right)}{\log 4}$$

$$= -1.75$$

$$m = -2.75$$

Example 24

Solve $5^x = 17$, writing the solution correct to 3 decimal places.

Solution

$$5^x = 17$$

$$\log 5^x = \log 17$$

$$x \log 5 = \log 17$$

$$x = \frac{\log 17}{\log 5}$$

$$= 1.7603...$$

$$\approx 1.760$$

Logarithmic equations are equations like $\log_5 x = -3$, which can be solved by rewriting the equation in index form.

Example 25

Solve each logarithmic equation.

a $\log_5 x = -3$

b $\log_x 18 = 3$

Solution

a $\log_5 x = -3$

$\therefore x = 5^{-3}$

$= \frac{1}{5^3}$

$= \frac{1}{125}$

b $\log_x 18 = 3$

$\therefore 18 = x^3$

$x = \sqrt[3]{18}$

$= 2.6207...$

≈ 2.62

EXERCISE 7.11 ANSWERS ON P. 615

Exponential and logarithmic equations U F PS R

1 Solve each exponential equation. R

EXAMPLE 23

a $2^k = 512$

b $5^m = 78\,125$

c $3^d = 59\,049$

d $5^x = 25\sqrt{5}$

e $2^y = \frac{1}{16\sqrt{2}}$

f $4^a = 128$

g $3^{k+2} = 27\sqrt{3}$

h $6^{n-2} = \frac{1}{216\sqrt{6}}$

i $9^{1-d} = \frac{1}{27\sqrt{3}}$

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STAGE 5.3

EXAMPLE 24

2 Solve each exponential equation, writing the solution correct to 3 decimal places. R

a $7^x = 16$ **b** $5^x = 36$ **c** $11^x = 420$

d $2^x = 0.52$ **e** $3^x = 1.6$ **f** $4^x = \frac{2}{5}$

g $2^{x-2} = 47$ **h** $3^{x+4} = 72$ **i** $6^{x+3} = 29$

j $8^{5-x} = 4000$ **k** $5^y = 4.8$ **l** $7^{k+5} = 300$

3 Solve each exponential equation by expressing both sides to base 2. R

a $2^{x+2} = 16$ **b** $8^x = 32$ **c** $4^{x-1} = \sqrt{2}$

d $8^{1-x} = 16\sqrt{2}$ **e** $4^{2-x} = \frac{1}{8}$ **f** $8^{x+1} = \frac{1}{8\sqrt{2}}$

g $\left(\frac{1}{4}\right)^{1-x} = \sqrt{2}$ **h** $5\left(\frac{1}{2}\right)^x = 20$

EXAMPLE 25

4 Solve each logarithmic equation, expressing your answer correct to 3 decimal places where necessary. R

a $\log_2 x = 3$ **b** $\log_{10} x = 3$ **c** $\log_5 x = -2$

d $\log_4 x = -3$ **e** $\log_{27} x = \frac{1}{3}$ **f** $\log_4 x = \frac{1}{2}$

g $\log_{10} x = -3$ **h** $\log_8 x = \frac{3}{2}$ **i** $\log_{10} x = -\frac{1}{2}$

j $\log_4 x = -\frac{3}{2}$ **k** $\log_4 x = 3\frac{1}{2}$ **l** $\log_{\sqrt{5}} x = -4$

m $\log_x 4 = 2$ **n** $\log_x 5 = -1$ **o** $\log_x \frac{1}{4} = 2$

p $\log_x 0.01 = 2$ **q** $\log_x 16 = 1$ **r** $\log_x 8 = 3$

s $\log_x 60 = 3$ **t** $\log_x 4.8 = \frac{1}{2}$

5 Use the compound interest formula $A = P(1 + r)^n$ to determine the number of years (to the nearest year) it will take an investment of \$1000 to grow to \$2000, if it earns compound interest at a rate of 6% p.a. PS R

6 Samaria invests \$12 000 at 1% per month compound interest. How many whole months will it take for her investment to grow to \$15 000? PS R

7 A radioactive substance with a mass of 150 grams decays according to the equation $A = 150 \times \left(2^{\frac{-t}{20}}\right)$, where A (grams) is the amount remaining after t days.

Find, correct to the nearest whole number: PS R

a the mass of substance remaining after 10 days

b the time taken for the substance to decay to half its original mass

c the time taken for the substance to decay to a mass of 20 g.

Power plus ANSWERS ON P. 615

1 Solve each equation.

a $\frac{2-x}{3}-\frac{x-5}{6}=\frac{5+x}{4}+\frac{x-2}{3}$

b $\frac{1}{x-1}+\frac{2}{x+1}=0$

c $x^2(x-2)-2(x+1)=x(x^2-3)-2(5+x^2)$

2 Graph each inequality on a number line.

a $1 \leq x \leq 4$

b $-2 \leq x \leq 3$

c $-12 < 4x \leq 4$

3 The number of diagonals, D, in a polygon with n sides is $D=\frac{1}{2}n(n-3)$. Show that there is no polygon that has exactly 100 diagonals.

4 The 2 solutions of $x^2-8x-11=0$ are in the form $x=p\pm q\sqrt{3}$, where p and q are integers. Find p and q.

5 Solve each logarithmic equation.

a $\log a+\log 3=\log 21$

b $\log x-\log 4=\log 5$

c $\log 7+\log m=\log(m+12)$

d $\log(h+7)-\log 3=\log(h-1)$

STAGE 5.3

CHAPTER 7 REVIEW

Language of maths

Qz Equations

check	cubic equation	exact
expand	exponential equation	factorise
formula	fraction	greater than
inequality	LHS	less than
logarithm	logarithmic equation	lowest common multiple (LCM)
number line	quadratic equation	RHS
solution	solve	subject
surd	variable	

1 What type of equation has 2 as the highest power of x? Write an example of this type of equation.

2 Write $\log_7 a = -3$ in index form.

3 What is the difference between an **equation** and an **inequality**?

4 Why is it possible for a **quadratic equation** to have more than one solution?

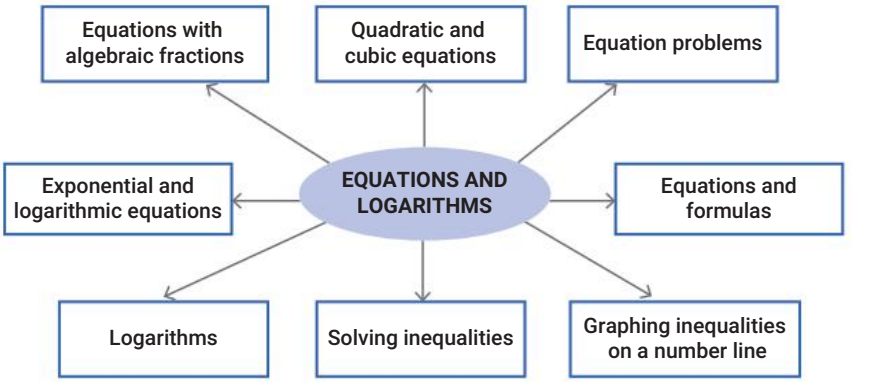
5 When checking the solution to an equation, you need to show that 'LHS = RHS'. What does that mean?

6 What does the inequality symbol '$\leq$' mean?

Topic summary

WS Mind map: Equations and logarithms

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 7

ANSWERS ON P. 615

1 Solve each equation. 7.01

a $\frac{3w+2}{5}=4$ **b** $\frac{y}{5}=\frac{7}{4}$ **c** $\frac{2a+1}{2}=\frac{3a-1}{4}$

d $\frac{3m+5}{6}=\frac{10-m}{3}$ **e** $\frac{2s}{3}-\frac{s}{6}=2$ **f** $\frac{x}{10}+\frac{x}{2}=1$

STAGE 5.3

2 Solve each equation. 7.01

a $\frac{m-1}{4}+\frac{m}{2}=6$ **b** $\frac{y+1}{3}+\frac{y-1}{4}=\frac{1}{2}$ **c** $\frac{2m-1}{4}-\frac{m-4}{3}=\frac{4}{3}$

3 Solve each quadratic equation. 7.02

a $y^2=4$ **b** $p^2-100=0$

c $4x^2=40$ **d** $3m^2-3=0$

e $\frac{2w^2}{5}=20$ **f** $x^2+8x+7=0$

g $h^2-8h-9=0$ **h** $u^2+4u-77=0$

i $k^2+5k=0$ **j** $m^2-2m=0$

k $b^2+20b+100=0$ **l** $w^2=9w$

STAGE 5.3

4 Solve each cubic equation, correct to one decimal place. 7.03

a $u^3-7=0$

b $5m^3-125=0$

c $\frac{x^3}{2}=1.5$

5 Grace is 3 years younger than her sister Jamiela. Twice the sum of their ages is 4 more than their father's age. If their father is 54, find the ages of Grace and Jamiela. 7.04

6 The braking distance (in metres) of a bicycle travelling at a speed of v metres/second is $d=\frac{v(v+1)}{2}$. Calculate the braking distance when the speed of the bicycle is 15 m/s. 7.05

7 The volume of a pyramid is given by the formula $V=\frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid with a base area of 48 mm^2 and a perpendicular height of 10 mm

b the base area of a pyramid with a volume of 500 m^3 and a perpendicular height of 5 m

STAGE 5.3

8 Make a the subject of each formula. 7.06

a $y=ax+b$

b $P=\sqrt{\frac{a}{m}}$

c $M(1+a)=1-a$

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7.07 **9** Graph each inequality on a number line.

a $x \geq 0$ **b** $x < 3$ **c** $x \leq -2$ **d** $x > -5$

7.08 **10** Solve each inequality.

a $y - 6 \geq 10$ **b** $2y \leq -15$ **c** $3a + 10 > -5$

d $10 - 6x < 28$ **e** $\frac{a+2}{-4} > \frac{7}{2}$ **f** $\frac{3-5x}{2} \geq 9$

STAGE 5.3

7.09 **11** Write each expression in index form.

a $\log_6 216 = 3$

b $\log_2 \frac{1}{16} = -4$

c $\log_7 7\sqrt{7} = \frac{3}{2}$

7.10 **12** Evaluate each expression.

a $\log_7 84 - \log_7 12$

b $\log_2 3 + \log_2 \frac{1}{3}$

c $2 \log_3 9 + \log_3 2 - \log_3 6$

7.10 **13** If $\log_{10} 3 \approx 0.4771$, find the value of:

a $\log_{10} 9$ **b** $\log_{10} 300$ **c** $\log_{10} \frac{10}{3}$ **d** $\log_{10} \sqrt{90}$

7.11 **14** Solve each exponential equation, correct to 3 decimal places.

a $5^x = 11$ **b** $2^x = 0.52$ **c** $3^{x+4} = 105$ **d** $16^{2-x} = 5$

PRACTICE SET 2

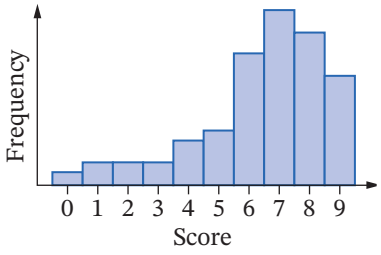
ANSWERS ON P. 616

1 For each statistical distribution:

6.01

i describe its shape **ii** identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

2 Solve each equation.

7.01

a $\frac{3b+1}{4}=\frac{2b-1}{3}$ **b** $\frac{2x}{3}-\frac{x}{6}=5$ **c** $\frac{1}{5}(4-x)+\frac{1}{3}(x-7)=5$

3 Simplify each expression.

5.01

a $(3nm^2)^4$ **b** $\frac{32a^5b^6}{8ab^3}$ **c** $(3m^{-3}n^5 \div 9mn^4)^2$

4 Simplify each expression.

STAGE 5.3

5.03

a $\frac{a}{15}+\frac{a-4}{3}$ **b** $\frac{x+3}{3}-\frac{x-4}{8}$ **c** $\frac{6r-3}{5}-\frac{r}{2}$

5 The heart rate (in beats/seconds) of a random sample of shoppers were measured:

6.02

84 85 84 87 92 95 83 79 88 90 87 112

a Find the range and interquartile range.

b Which is the better measure of spread for this set of data? Justify your answer.

6 Solve each quadratic equation.

7.02

a $\frac{x^2}{3}=36$ **b** $n^2=64n$ **c** $u^2+11u-60=0$

7 Simplify each expression.

5.04

a $\frac{21e}{10}\times\frac{15d}{7e}$ **b** $\frac{3m^2}{8}\div\frac{9m}{16}$ **c** $\frac{12p^2v}{5}\times\frac{1}{24v}\div\frac{3p}{10v}$

8 The maximum daily temperatures (°C) in Perth for the first 14 days in April were:

6.05

27.1 24.1 23.7 24.6 26.5 31.8 33.6 32.6 34.0 35.6 39.5 26.2 26.7 24.9

a Find the range and interquartile range for the temperatures.

b Find the five-number summary for the data.

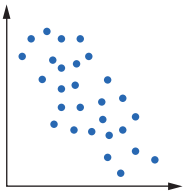
c Draw a boxplot for the data.

d Identify any outliers.

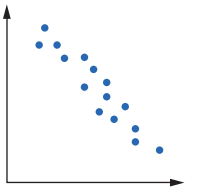
6.08

9 Describe the strength and direction of the relationship between the variables shown in each scatterplot.

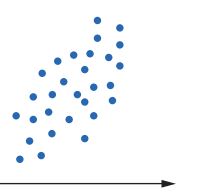
a



b



c



STAGE 5.3

5.02

7.03

10 Evaluate each expression.

a $64^{\frac{1}{3}}$ **b** $1024^{\frac{2}{5}}$ **c** $144^{-\frac{1}{2}}$ **d** $100^{-\frac{3}{2}}$

11 Solve $5y^3 + 1000 = 0$, correct to one decimal place.

5.05

12 Expand and simplify each expression.

a $9gh(6 - 5h) - gh$ **b** $y^2(4y + y^2) + 2y(3y^3 - y^2)$

STAGE 5.3

6.04

13 The test results for 2 groups of students are shown.

Girls: 78 73 58 79 60 46 71 64 37 79 55 59 87 68 27

Boys: 70 39 80 73 61 68 84 60 73 60 54 90 69 63 59

a Find the mean and standard deviation of each group.

b Which group performed better in the exam? Give reasons.

STAGE 5.3

5.06

14 Expand each binomial product.

a $(x + 7)^2$ **b** $(5m - 2n)^2$ **c** $(3n - 10)(3n + 10)$

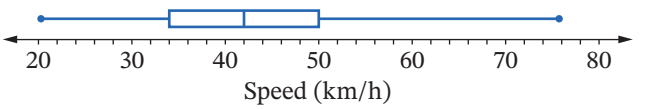
7.04

15 The sum of 3 consecutive numbers is 63. Find the numbers by solving an equation.

6.07

16 The speeds of cars (in km/h) were monitored from 2:30 to 4:00 p.m. in a school zone. The results are shown in the stem-and-leaf plot and box plot.

Stem	Leaf
2	0 3 7 9
3	0 2 2 3 5 6 8 8 9
4	0 0 1 3 5 5 7 8 9 9
5	1 1 4 4 6
6	0 1
7	6



a Which plot gives the better indication of:

i skewness? **ii** clustering and outliers?

b Find the median.

c Find the interquartile range.

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17 Factorise each expression.

5.08

a $y^2 - 18y + 32$ **b** $n^2 + 2n - 48$ **c** $a^2 - a - 72$

18 10 boxes containing 60 apples each were placed in cold storage for different periods. After each period of storage, a box was withdrawn and the number of good apples was counted.

6.08

Weeks in storage (W)	2	3	5	6	7	8	9	10	12	14
Number of good apples (N)	56	53	48	48	50	39	36	30	34	26

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the relationship between the weeks in storage and number of good apples.

d Describe the strength and direction of the relationship between the variables W and N.

19 Factorise each expression.

STAGE 5.3

5.09

7.06

a $5n^2 + 7n + 2$ **b** $2a^2 - 7a - 15$ **c** $8x^2 + 14x - 15$

20 Make n the subject of the formula $t = \sqrt{\frac{n}{3}}$.

21 Graph each inequality on a number line.

7.07

a $x \geq 0$ **b** $x < 4$ **c** $x \leq 1$ **d** $x > -6$

22 Solve each inequality.

7.08

a $n - 5 \geq 0$ **b** $21 + 4k < -7$

c $1 - 2y < 2(5 + 2y)$ **d** $\frac{2(4-x)}{3} \geq 1$

23 Simplify each expression.

STAGE 5.3

5.11

a $\frac{d^2 - d - 12}{d^2 + 2d - 3}$ **b** $\frac{4a-16}{3} \times \frac{12a}{a^2 - 4a}$ **c** $\frac{y^2 + 4y + 3}{y^2 - y} \div \frac{y^2 - 9}{3y - 3}$

24 The heights and shoe sizes of 10 people were measured.

STAGE 5.3

6.09

Weight, W (kg)	60	78	82	88	90	94	75	66	58	65
Height, H (cm)	142	168	184	170	180	178	170	158	150	146

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the graph to extrapolate the weight of a person with a height 130 cm.

d Use the graph to interpolate the height of a person who weighs 80 kg.

25 Simplify and evaluate each expression.

7.10

a $\log_9 108 - \log_9 12$ **b** $\log_6 5 + \log_6 \frac{1}{5}$ **c** $3 \log_5 10 - \log_5 8$

26 Solve each equation.

7.11

a $3\sqrt{3} = 9^x$

b $4^x = 120$, correct to 3 decimal places.

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8

NUMBER AND ALGEBRA

GRAPHING CURVES

When an object is thrown upwards, its path is a curve called a parabola. The shape and length of the path will depend on the initial speed of the object. Furthermore, car headlights, and satellite dishes all use mirrors or reflectors that have the shape of a parabola.



Chapter outline

		Working mathematically				
8.01	Direct proportion	U	F	PS	R	C
8.02	Inverse proportion#	U	F	PS	R	C
8.03	Conversion graphs	U	F		R	C
8.04	Distance–time graphs*#	U	F		R	C
8.05	Graphs of variable change*#	U	F		R	C
8.06	The parabola $y = ax^2 + c$	U	F	PS	R	C
8.07	The parabola $y = a(x - b)^2 + c$*#	U	F		R	
8.08	The cubic curve $y = kx^3 + c$*#	U	F		R	C
8.09	The cubic curve $y = k(x - a)(x - b)(x - c)$*#	U	F		R	C
8.10	The power curves $y = kx^n + c$*#	U	F		R	C
8.11	The hyperbola $y = \frac{k}{x}$*	U	F		R	C
8.12	The exponential curve $y = a^x$	U	F		R	C
8.13	The circle $(x - a)^2 + (y - b)^2 = r^2$	U	F		R	C
8.14	Identifying graphs	U	F		R	C

*STAGE 5.3

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Wordbank

asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve

direct proportion A relationship between 2 variables of the form $y = kx$, where k is a constant; for example, if $y = 8.5x$, then y is directly proportional to x

exponential equation An equation involving a variable as a power, such as $y = 3^x$, whose graph is an exponential curve

hyperbola The graph of $y = \frac{k}{x}$, where k is a constant, which has 2 branches and 2 asymptotes

inverse proportion A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve problems involving direct proportion and inverse proportion
- read and interpret conversion graphs
- (STAGE 5.3) read and interpret distance–time (travel) graphs where the speed is variable
- (STAGE 5.3) read and interpret graphs of variables changing at different rates
- graph parabolas of the form $y = ax^2 + c$
- (STAGE 5.3) graph parabolas of the form $y = a(x - b)^2$
- (STAGE 5.3) graph cubic curves of the form $y = kx^3 + c$ and $y = k(x - a)(x - b)(x - c)$
- (STAGE 5.3) graph higher-power curves of the form $y = kx^n + c$ and $y = k(x - b)^n$
- (STAGE 5.3) graph hyperbolas of the form $y = \frac{k}{x}$, $y = \frac{k}{x} + c$ or $y = \frac{k}{x - b}$
- graph exponential curves of the form $y = a^x$
- graph circles of the form $x^2 + y^2 = r^2$
- (STAGE 5.3) graph circles of the form $(x - a)^2 + (y - b)^2 = r^2$
- match graphs to their equations

SkillCheck ANSWERS ON P. 616

1 If $A = 2x^2 - 3$, find A if:

a $x = 1$ **b** $x = 4$ **c** $x = 0$ **d** $x = -6$

2 If $R = \frac{8}{u}$, find R if:

a $u = 2$ **b** $u = -16$ **c** $u = 5$ **d** $u = -2.5$

3 If $y = 5^x$, find y if:

a $x = 4$ **b** $x = 5$ **c** $x = 0$ **d** $x = -2$

8.01 Direct proportion

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other; when one variable changes, the other one changes by the same factor.

Direct proportion

If y is directly proportional to x, then $y = kx$, where k is a constant (number) called the constant of proportionality or constant of variation.

- A direct linear relationship exists between x and y
- If x increases (or decreases), y increases (or decreases)
- If x is doubled (or halved), y is doubled (or halved)
- Another way of saying 'y is directly proportional to x' is 'y varies directly with x'
- The graph of direct proportion is a straight line going through (0, 0) with gradient k

Example 1

The distance (d) in metres travelled by a car is directly proportional to the number of rotations (r) of its tyres. After 540 rotations, a distance of 950 m is travelled.

a What distance (correct to the nearest metre) will be travelled after 800 rotations?

b How many full rotations will be needed to cover 360 m?

Solution

a d is directly proportional to r

$\therefore d = kr$

To find k, substitute the information given for r and d.

When $r = 540$, $d = 950$:

$950 = k \times 540$

$$k = \frac{950}{540}$$

$= 1.759...$

$\therefore d = 1.759... r$

When $r = 800$,

$d = 1.759... \times 800$

$= 1407.4074...$

≈ 1407 m

After 800 rotations, the distance travelled will be 1407 m.

b When $d = 360$,

$360 = 1.759... \times r$

$$r = \frac{360}{1.759...}$$

$= 204.661$

≈ 205 rotations

Rounding up for full rotations

For a distance of 360 m, there will be 205 rotations.

Solving direct proportion problems

1 Identify the 2 variables (say x and y) and form a proportion equation, $y = kx$

2 Substitute values for x and y to find k, the constant of proportionality

3 Rewrite $y = kx$ using the value of k

4 Substitute a value for x or y into $y = kx$ to solve the problem

EXERCISE 8.01 ANSWERS ON P. 616

Direct proportion U F PS R C

1 The distance, D, travelled by Connor, a marathon runner, varies directly with time, T. PS R C

Time, T (min)	1	2	3
Distance, D (m)	190	380	570

a Find the constant of proportionality and write the equation for D.

b How far, in kilometres, will Connor run in:

i 20 minutes? **ii** 45 minutes?

c How long would it take Connor to run 12.35 kilometres? Answer in hours and minutes.

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Direct proportion

Direct linear variation

8.01

EXAMPLE 1

2 Mehta's earnings for working a shift at the local nursery are directly proportional to the number of hours she works. Yesterday, she earned $222.70 for working an 8.5-hour shift. PS R C


iStock.com/Lordn

a If Mehta's earnings are represented by E, and the number of hours worked is represented by h, find the constant of proportionality and write an equation for E.

b How much will she earn for working a 7-hour shift?

c How many hours did she work today if she earned $144.10 for the shift?

3 The amount of interest, I, earned for one year on an investment account varies directly with the size of the deposit, D. PS R C

a If Caterina earns $16 interest on an investment of $425, find the variation equation for I, including the constant of variation.

b Hence, how much will she earn on an investment of $900?

c If she doubles the size of her investment in **b**, how much will she earn in interest?

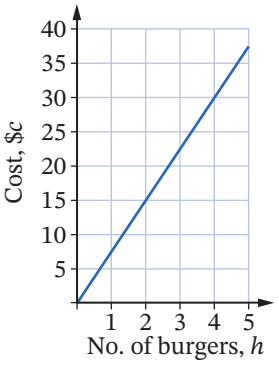
4 S varies directly with t. If when $t = 14$, $S = 106.4$, what is the value of S when $t = 0.3$? Select the correct answer **A**, **B**, **C** or **D**. R C

A 2.28 **B** 27.72 **C** 36.12 **D** 446.88

5 Find the linear formula for b in terms of a for this table of values.

a	4	8	12	16	20
b	10	20	30	40	50

6 The line graph shows that the cost of hamburgers purchased from the local takeaway store depends directly on the number of burgers purchased. PS R C



a Copy this table and use the graph to complete it.

No. of burgers, h	1	2	3
Cost, c ($)			

b Find the variation equation to represent the relationship between the cost ($\$c$) and the number of burgers (h).

c If Kendall buys 6 hamburgers, what is the total cost of the hamburgers?

d The total cost of one order of hamburgers is $82.50. How many hamburgers were ordered?

e Find the gradient of the line. How is it related to the constant of variation?

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7 K varies directly with L. If $L = 9.5$ when $K = 1045$, what is the value of K when $L = 1.65$? Select **A**, **B**, **C** or **D**. R C

A 0.015 **B** 93.7 **C** 181.5 **D** 1708.575

8 A linear relationship exists between the mass of a car (m kg) and its rate of fuel consumption (F L/100 km). PS R C

a Find the variation equation for F if a 1000 kg car uses fuel at a rate of 6 L/100 km.

b Find the fuel consumption of a 2500 kg car.

9 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C? Select **A**, **B**, **C** or **D**. PS R C

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

10 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars. PS R C

a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.

Inverse proportion#

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8.02

Two variables are **inversely proportional** to each other if, when one variable increases, the other one decreases by the same factor.

The table below shows the different speeds of a car (s km/h), and the time it takes to travel 100 km (t min). As the speed increases, the time taken decreases.

Speed (s km/h)	50	60	80	100
Time (t min)	120	100	75	60

Inverse proportion

If y is **inversely proportional** to x, then $y = \frac{k}{x}$, where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- If x increases, y decreases ('inverse' means 'opposite')
- If x decreases, y increases
- If x is doubled, y is halved
- If x is halved, y is doubled
- Another way of saying 'y is inversely proportional to x' is 'y **varies inversely** with x'



Direct and inverse proportion


Inverse variation



Variation problems

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Example 2

The time (t) in minutes taken by a car to travel 100 km is inversely proportional to the speed (s km/h) of the car, as shown in the table on the previous page. At 50 km/h, the time taken is 120 minutes.

a Find the constant of proportionality and the inverse variation equation for t.

b How long did the car take to travel 100 km at:

i 40 km/h? **ii** 110 km/h?

c Find the car's speed if it took 45 minutes to travel 100 km.

Solution

a t is inversely proportional to s.

$\therefore t = \frac{k}{s}$ Forming an inverse proportion equation

To find k, substitute the information given for s and t.

When $s = 50$, $t = 120$: Finding k, the constant of proportionality.

$120 = \frac{k}{50}$

$k = 120 \times 50$

$= 6000$

The constant of proportionality is 6000.

$\therefore t = \frac{6000}{s}$ Rewriting the equation $t = \frac{k}{s}$

b **i** When $s = 40$, $t = \frac{6000}{40} = 150$ min

At 40 km/h, the trip takes 150 min (or 2 h 30 min).

ii When $s = 110$,

$t = \frac{6000}{110}$

$= 54.5454...$

≈ 55 min

At 110 km/h, the trip takes 55 min.

c When $t = 45$,

$45 = \frac{6000}{s}$

$45s = 6000$

$s = \frac{6000}{45}$

$= 133\frac{1}{3}$ km/h

For a travel time of 45 min, the speed must be $133\frac{1}{3}$ km /h.

Solving inverse proportion problems

1 Identify the 2 variables (say x and y) and form a proportion equation, $y = \frac{k}{x}$

2 Substitute values for x and y to find k, the constant of proportionality

3 Rewrite $y = \frac{k}{x}$ using the value of k

4 Substitute a value for x or y into $y = \frac{k}{x}$ to solve the problem

Example 3

The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 600 m above sea level, the temperature is 8°C.

a What is the temperature at 1000 m above sea level?

b Graph the relationship between temperature and height above sea level, for heights between 0 and 5000.


iStock.com/Easyturn

Solution

a T is inversely proportional to h.

$T = \frac{k}{h}$

Substitute $h = 600$ and $T = 8$ to find k.

$8 = \frac{k}{600}$

$k = 8 \times 600$

$= 4800$

$\therefore T = \frac{4800}{h}$

When $h = 1000$,

$T = \frac{4800}{1000} = 4.8°C$

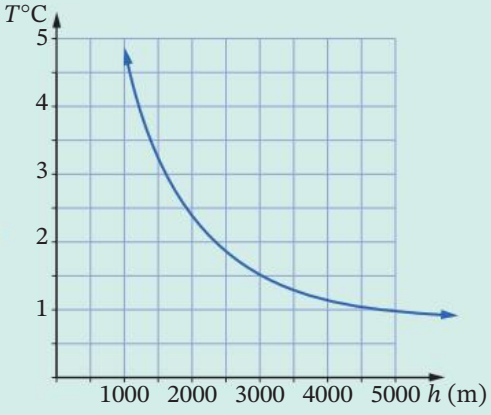
The temperature at a height of 1000 metres above sea level is 4.8°C.

b Draw a table of values for $T = \frac{4800}{h}$

h	1000	2000	3000	4000	5000
T	4.8	2.4	1.6	1.2	0.96

Note that as h increases, T decreases.

This graph is called a **hyperbola**, which we will learn more about later in this chapter at 8.11.



EXERCISE 8.02 ANSWERS ON P. 617

Inverse proportion U F PS R C

1 The time taken, T hours, to travel from Sydney to Melbourne varies inversely with the average speed, s km/h. PS R C

EXAMPLE 2

a If it takes 11.5 hours at an average speed of 80 km/h, find the constant of proportionality and the variation equation for T.

b If the average speed is increased to 90 km/h, how long will the journey take? Answer in hours and minutes.

c Find the average speed needed to complete the trip in 10 hours.

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EXAMPLE 3

2 The temperature, T (in degrees Celsius), of the air varies inversely with the height, h (in metres), above sea level. At 150 m above sea level, the temperature is 30°C. PS R C

a Find the constant of variation and write a variation equation for T.

b What is the temperature at:

i 300 m above sea level? **ii** 2500 m above sea level?

c What is the height above sea level when the temperature is:

i 8°C? **ii** 22.5°C?

d Graph the relationship between temperature and height above sea level. Use T on the vertical axis and h on the horizontal axis with $h = 0, 500, 1000, 1500, ..., 3000$.

3 The maximum number of diners, N, in Yen's restaurant varies inversely with the amount of floor space, S m^2, allocated to each diner. If 1.4 m^2 is allowed per diner, the restaurant can have 80 diners. PS R C

a Write a variation equation for N.

b How many diners could the restaurant contain if only 1.1 m^2 was allocated per person?

c How much space (correct to one decimal place) is allocated to each diner if the restaurant has a maximum capacity of 55 diners?

d During the COVID-19 pandemic of 2020, social distancing measures introduced to minimise the spread of the virus allocated 4 m^2 to each diner. How many diners could Yen's restaurant serve under this restriction?

4 The rate of vibration of a string varies inversely as its length. A string that is 8 cm long vibrates at 9375 Hz (hertz). What length of string will vibrate at 6250 Hz?
Select the correct answer **A**, **B**, **C** or **D**. PS R C

A 5 cm **B** 7 cm **C** 12 cm **D** 73 cm

5 Which equation represents this table of values? Select **A**, **B**, **C** or **D**. R

x	2	5	8	10
y	2.5	1	0.625	0.5

A $y = \frac{10}{x}$ **B** $y = \frac{5}{x}$ **C** $y = \frac{2.5}{x}$ **D** $y = \frac{1}{x}$

6 K is inversely proportional to L. If $L = 2$ when $K = 7$, find K when $L = 15$. R

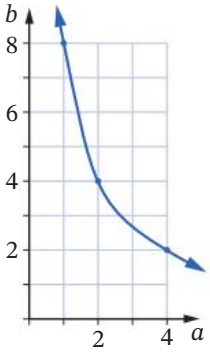
7 Ankit believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 16 people waiting, the train will arrive in 2.5 minutes. PS R C

a When will the train arrive if there are 5 people waiting?

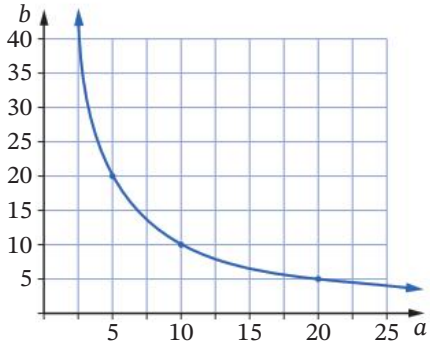
b How many people are waiting at the station 10 minutes before the train arrives?

8 Each graph shows an inverse relationship between a and b. Find each variation equation. R C

a



b



9 The frequency, F beats per second, that a bird beats its wings varies inversely as the length, L cm, of its wings. A bird with wings of length 14 cm beats them at a frequency of 8 beats per second.

a Find the variation equation for F in terms of L. PS R C

b Calculate, to the nearest whole number, the wingbeat frequency for wings of length 18 cm.

c A bird beats its wings with a frequency of 4.5 beats per second. What is the length of its wings, correct to the nearest centimetre?

10 For a certain equation, y varies inversely with x. R

a Given $x = 0.2$ when $y = 10$, find y when $x = 32$.

b Find x when $y = 1.6$.

11 The amount of time it takes Sarah to move house is inversely proportional to the number of friends she has to help her. When she has 4 friends helping, the job takes $3\frac{3}{4}$ hours. PS R C

a How long will it take if she has 6 friends helping?

b How many friends must she have to help her to move house in 3 hours?

8.02

Conversion graphs 8.03

A **conversion graph** is used to convert from one unit to another; for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0), so it has an equation in the form $y = kx$ and is an example of **direct proportion**.

STAGE 5.3

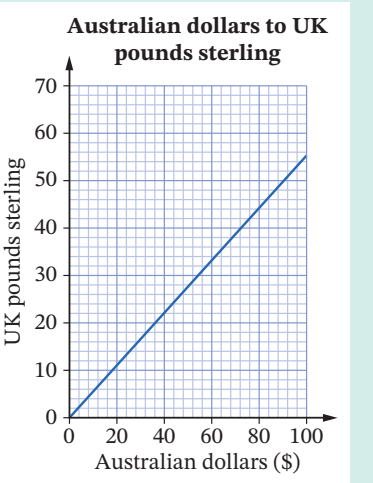
WS Currency conversion graph

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STAGE 5.3

Example 4

Exchange rates change daily, but suppose the exchange rate between the Australian dollar and the UK pound sterling is \$A1 = £0.553, then \$A100 = £55.30 sterling. These values are used to draw this conversion graph.



Use the graph to convert:

a \$A50 to pounds

b £10 to Australian dollars.

Solution

Reading from the graph:

a \$A50 ≈ £28

b £10 ≈ \$A18

EXERCISE 8.03 ANSWERS ON P. 617

Conversion graphs UFRC

EXAMPLE 4

1 Use the graph in Example **4** for this question.

a Convert to pounds:

i \$A40 **ii** \$A88

b Convert to Australian dollars:

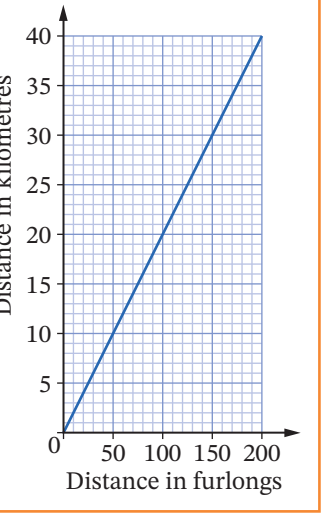
i £18 **ii** £60

c In 2015, \$A1 = £0.50. R C

i How much less was \$A40 worth in UK pounds sterling in 2015 than it is using this more recent conversion graph?

ii How much more was £60 worth in Australian dollars in 2015 for UK tourists visiting Australia, than it is using this more recent conversion graph?

2 The furlong is an imperial measure once used to measure length. This conversion graph shows distances in furlongs converted to kilometres.



a Convert to kilometres:

i 10 furlongs **ii** 100 furlongs

iii 170 furlongs

b Convert to furlongs:

i 10 km **ii** 25 km

iii 36 km

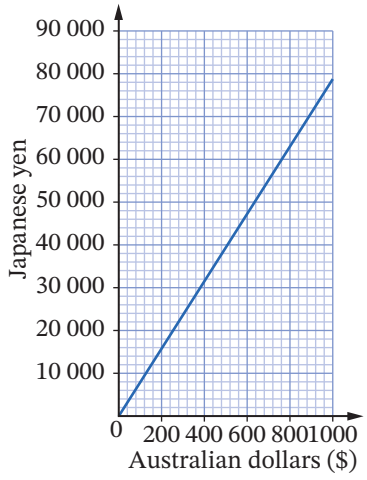
c Use an answer from part a to convert 300 furlongs to kilometres. R

d Use an answer from part b to convert 100 kilometres to furlongs. R

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3 This graph converts between Australian dollars and Japanese yen (¥).



a Convert to Japanese yen:

i $200

ii $800

iii $1000

b Convert to Australian dollars.

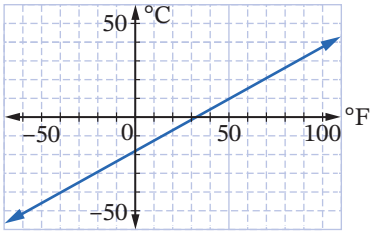
i ¥20 000

ii ¥60 000

iii ¥72 000

4 This graph converts temperatures between degrees Fahrenheit and degrees Celsius. Convert:

Degrees Fahrenheit to degrees Celsius



a 0°F to °C

b 50°F to °C

c 80°F to °C

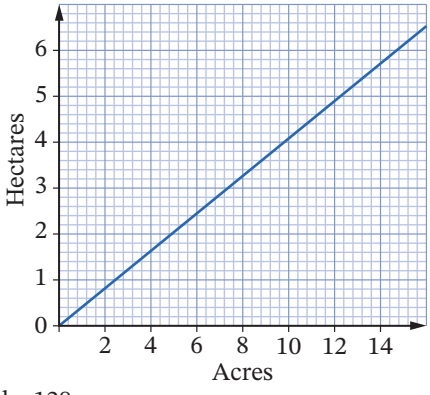
d 0°C to °F

e −10°C to °F

f 30°C to °F

5 This conversion graph converts between acres and hectares. The acre is an imperial measure of land area.

Converting acres to hectares



a Use the graph to convert 12 acres to hectares.

b A garden has an area of 5 acres. What is this area in hectares?

c Use the graph to convert 4.4 hectares to acres.

d Mr Ferguson has a property with an area of 5 hectares. How big is this in acres?

e A rectangular playing field measures 250 m by 128 m.

i What is the area of the field in square metres?

ii What is the area of the field in hectares?

iii What is the area of the field in acres?

STAGE 5.3

6 This graph is used to convert Australian dollars to Philippine pesos (₱).

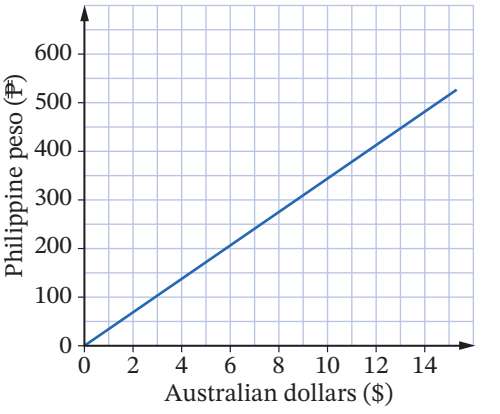
a Change each Australian dollar amount into Philippine pesos.

i \$15 **ii** \$6 **iii** \$9

b Change ₱350 to Australian dollars.

c How many Australian dollars would you receive for ₱200?

d Estimate the number of pesos you should get for \$120. R



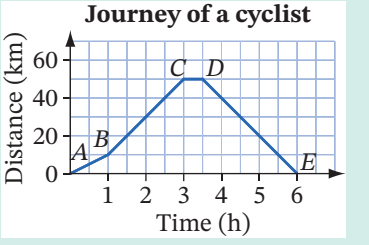
8.04 Distance–time graphs#

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The distance travelled by a moving object can be shown on a **distance–time graph**, also called a **travel graph.**

Example 5

This distance–time graph shows the journey of a cyclist on a training ride.



a What was the total distance covered?

b The cyclist's speed increases at *B*, after the first hour. How is this shown by the graph?

c Calculate the speed of the cyclist from:

i *B* to *C* **ii** *D* to *E*

What do you notice?

d When was the cyclist stationary?

Solution

a The cyclist travelled 50 km, then returned to the starting point, so the total distance covered was 100 km.

b The graph becomes steeper after *B*.

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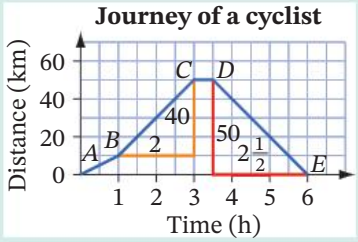
c Speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$ or the gradient of the interval on the graph

i Speed from B to $C = \dfrac{40}{2}$

$= 20$ km/h

ii Speed from D to $E = \dfrac{50}{2\frac{1}{2}}$

$= 20$ km/h



The speeds were the same.

d The cyclist was stationary (not moving) where the travel graph was flat, that is, between 3 and 4 hours (CD on the graph).

Distance–time graphs

- The gradient of the line shows the speed of the object.
- The steeper the graph, the greater the speed.
- If the gradient of the line is positive, the object is moving away from the starting point.
- If the gradient of the line is negative, the object is moving back towards the starting point.

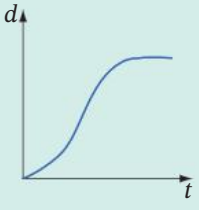
Distance–time graphs with variable speed

Example **5** showed a distance–time graph with straight lines, indicating that the speed of the object did not change much, staying fairly constant. However real-life situations are more complex and involve variable speed or a rate of change that is **not** constant. In these cases, the graph will be curved.

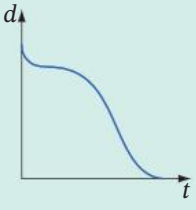
Example 6

Describe the journey represented by each distance–time graph.

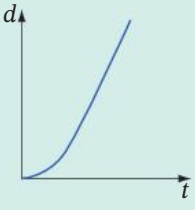
a



b

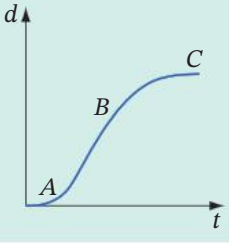


c

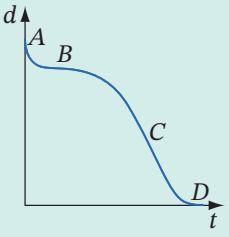


Solution

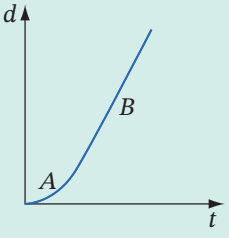
a The person starts the journey slowly (at A, the graph is not very steep), then increases his speed to a maximum (at B, the graph is the steepest), then slows down and stops (at C, graph becomes horizontal).



b The person going home starts the journey at high speed (at A, the graph is very steep) before slowing down and stopping briefly (at B, the graph is flat). The person then speeds up again (at C, the graph becomes very steep) before slowing down again and arriving home (at D).



c The person starts the journey slowly, then speeds up (at A) before reaching a constant speed (at B, the graph is a straight line) and continuing on the journey.



Example 7

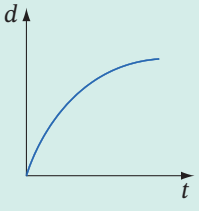
Draw distance–time graphs to represent each situation described.

a Kobi was running quickly but then his speed decreased until he stopped.

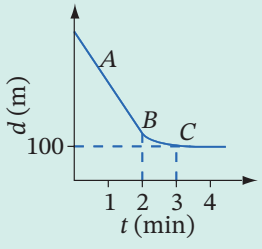
b Susanna was running home at a constant speed for 2 minutes, before slowing down and stopping 100 m from home after a further 1 minute.

Solution

a Kobi's speed is gradually decreasing, so the steepness (or gradient) of the graph must be steep at first, then decreasing.



b The graph should be a decreasing straight line (at A) for 2 minutes, indicating a constant speed. Then for one minute (at BC), the gradient of the graph should gradually decrease, until becoming flat (at C) to show when Susanna stops, 100 m from home.



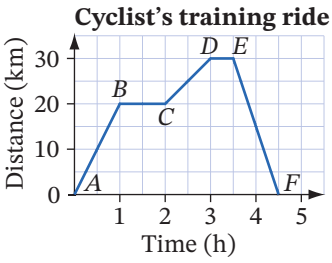
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EXERCISE 8.04 ANSWERS ON P. 617

Distance–time graphs U F R C

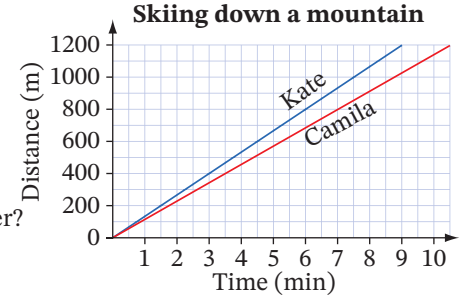
EXAMPLE 5

1 This graph shows a cyclist's journey on a training ride. R C



- **a** Describe the journey of the cyclist, giving the speeds at each stage.
- **b** Do any intervals of the graph indicate that the cyclist is travelling at the same speed? Give reasons.
- **c** The gradient of the interval *EF* is –30, but the speed at this stage is 30 km/h. What does the negative gradient indicate?

2 Kate and Camila are downhill skiers. Here is the distance–time graph for their 1200 m trip down the mountain. R C

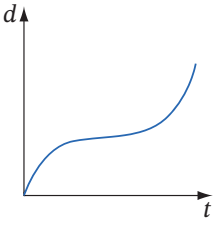


- **a** What was the speed of each skier?
- **b** Who reached the base of the mountain first and how many minutes did it take her?
- **c** How many minutes later did the second skier arrive?
- **d** How far had Camila gone after 6 minutes?
- **e** How far *ahead* was Kate after 6 minutes?
- **f** If Camila and Kate were skiing *down* the mountain, why do the graphs go *up*?

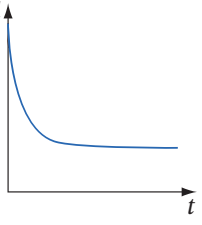
EXAMPLE 6

3 Describe the journey represented by each distance–time graph. R C

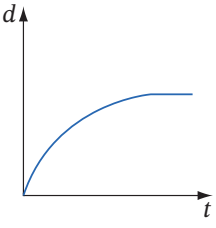
a



b

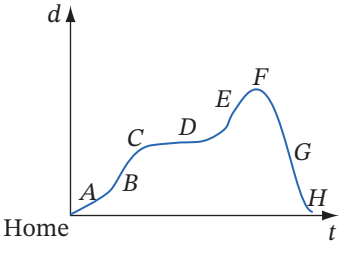


c



4 Match each statement to the correct part of the distance–time graph. R C

- **a** the person slows down because he is almost home
- **b** the person is not moving
- **c** after moving at a constant speed, the person increases his speed
- **d** the person changes direction for the return trip
- **e** after stopping, the person increases his speed
- **f** the person gradually slows down

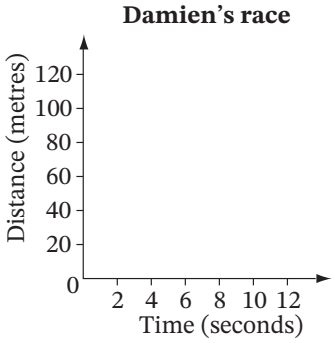


Foundation | Standard | Complex

EXAMPLE 7

5 Copy and complete this distance–time graph, using the following information about Damien completing a 100 m race. R C

- Damien covered 5 m at 2 seconds and 20 m at 4 seconds as his running speed increased
- Damien then ran at a constant speed until reaching the 80 m mark at 9 seconds
- Damien slowed down over the last 20 m, completing the 100 m race in 11 seconds
- Damien finally stopped after running a further 20 m



6 Sketch a distance–time graph for each situation described. R C

a Starting at home and running away from home at a decreasing speed

b Starting at home and running away from home at an increasing speed

c Starting away from home and running home at an increasing speed

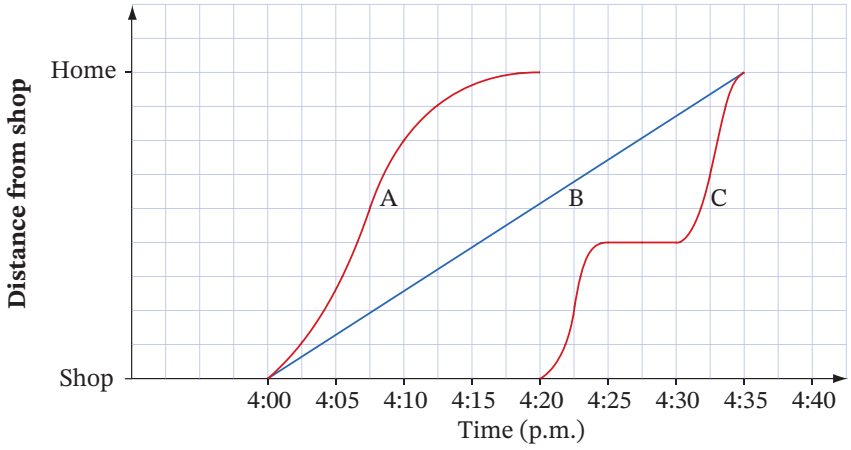
d Starting away from home and running home at a decreasing speed

7 These 3 statements can be matched to a graph below: R C

I Jade rode her bicycle home

II Cameron walked home

III Kiet ran home



a Match each statement to its correct graph.

b Explain in words why you made each match.

c What could have caused the level section at about 4:25 p.m. in graph **C**?

d Describe how the speed changes in graph **A**.

8 Match each description to its correct graph. R C

a The speed increases at an increasing rate and then stops.

b The speed increases slowly, slows down to a stop, then increases to a constant rate.

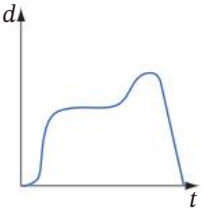
c The speed increases at an increasing rate, slows down and stops.

d The speed starts high, decreases, then stops.

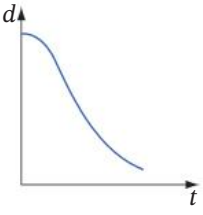
e The speed increases to a maximum, then slows down.

f The speed increases, then slows down and stops, then begins to increase again before stopping for an instant and returning to the start at a constant speed.

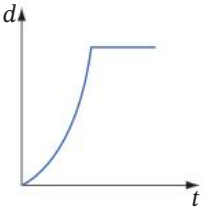
A



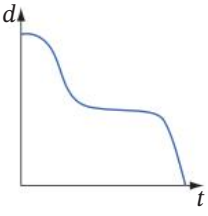
B



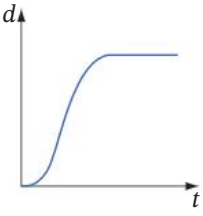
C



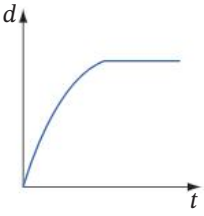
D



E



F



8.04

Graphs of variable change#

8.05

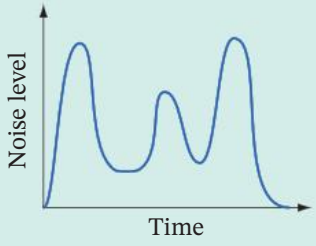
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STAGE 5.3

WS Graphs of change

Example 8

This graph shows the noise level of a classroom during a lesson. Describe what may have happened in the classroom during the lesson.



Solution

- The noise level increased as the students entered the classroom.
- The noise level lowered as the students settled down to work.
- There may have been a classroom discussion (the noise level increased) and then the class settled down again.
- The noise level increased towards the end of the lesson as students became restless.
- The period ended and the noise level decreased after the students left the classroom.

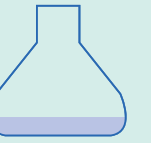
Foundation Standard Complex

STAGE 5.3

Graphs of variable change

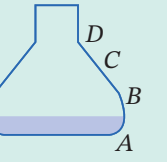
Example 9

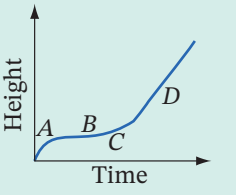
Water is poured into this container at a constant rate. Draw a graph to show the height of water rising in the container over time.



Solution

- The container is wider at B than at A, so the rate of increase in the water level will slow down.
- As the container narrows at C, the water level will increase faster (the graph becomes steeper).
- The container is cylindrical at D, so the water level will rise at a constant rate (the graph is a straight line).



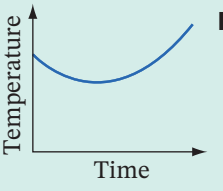


Graphs of variable change

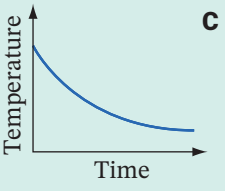
Example 10

A bowl of hot soup sits on the kitchen bench cooling. At first it loses heat quickly but, as time passes, it loses heat more slowly until it is at room temperature. Which of these graphs best illustrates this?

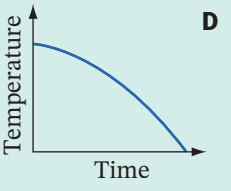
A



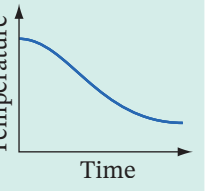
B



C



D



Solution

The correct graph is B. The temperature decreases rapidly initially (steep graph), but then this rate of decrease slows (graph becoming flatter, the temperature decreases at a decreasing rate).

A is incorrect, because it shows the temperature decreasing, then increasing.

C is incorrect because it shows the temperature decreasing slowly initially (flat graph), then decreasing quickly to 0 (steep graph, decreasing at an increasing rate).

D is incorrect because it shows the temperature decreasing slowly initially (flat graph), then quickly, then slowly again.

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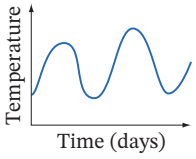
EXERCISE 8.05 ANSWERS ON P. 618

Graphs of variable change U F R C

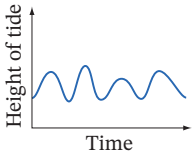
EXAMPLE 8

1 Describe what may be happening as represented in each graph. R C

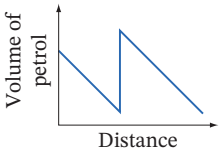
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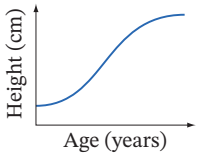
b



c



d



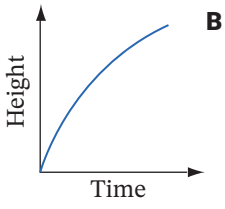
EXAMPLE 9

2 For each container, select the graph that best describes the height of the water as it is poured at a constant rate into the container. R

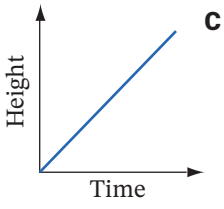
a



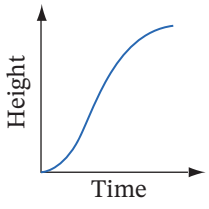
A



B



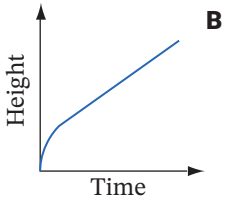
C



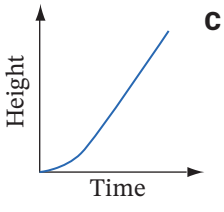
b



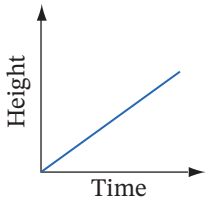
A



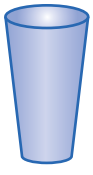
B



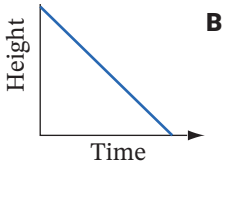
C



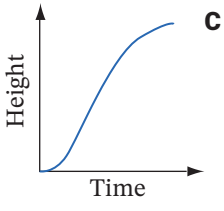
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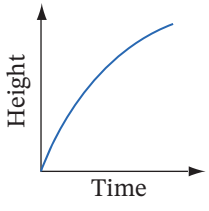
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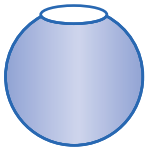
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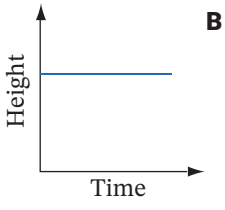
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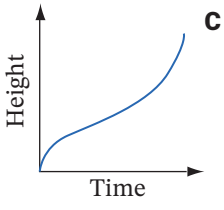
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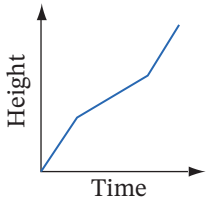
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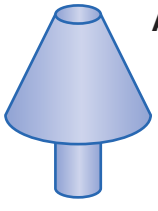
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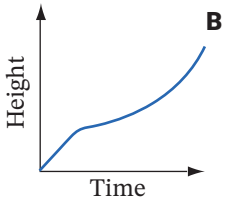
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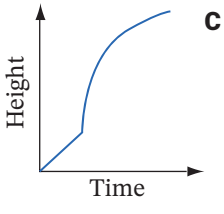
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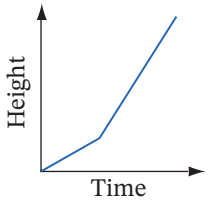
A



B



C



Foundation | Standard | Complex

STAGE 5.3

EXAMPLE 9

3 Draw a graph that models each situation described. Use the variables given in brackets. R C

a The water level in a pool with people swimming in it on a hot summer day. (height, time)

b The height of a girl above the ground as she climbs up the ladder of a slide, sits and then slides down. (height, time)

c The distance from a shop of a person walking at a steady speed up and down the street past that shop. (distance, time)

EXAMPLE 10

4 Match each story to its graph below. (The variables are given in brackets.) R

a People were purchasing from the drink-vending machine until it broke down. (number of cans in machine, time)

b The tank was half full of water all day. (depth of water, time)

c The cost of a mobile phone call is charged at a constant rate. (rate, time)

d Jo was watching a movie but stopped to take a phone call halfway through. (amount of movie seen, time)

e Interest rates rise rapidly at a constant rate, then steadily fall at a constant rate and stay at a constant low. (rate, time)

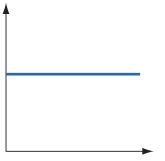
f A taxi fare includes the hire charge plus a constant amount per kilometre. (cost, kilometres travelled)

g The car is consuming petrol at a steady rate until it runs out of fuel. (litres in tank, time)

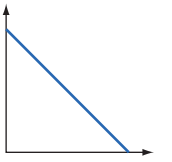
h Dario jogs at a steady rate, then stops and rests. (distance travelled, time)

i Jordan jogs at a steady rate, then walks home. (distance travelled, time)

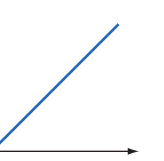
A



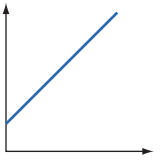
B



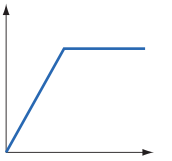
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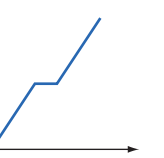
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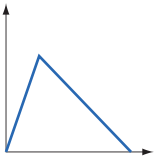
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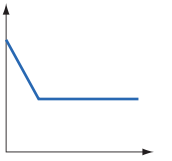
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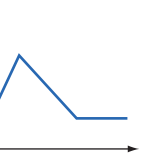
G



H



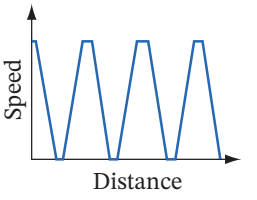
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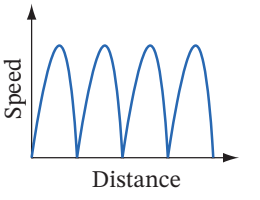
5 For each description, select the graph that best describes the situation. Select the correct answer **A**, **B** or **C**. R

a The speed of a bus that stops 3 times

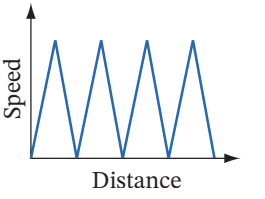
A



B



C

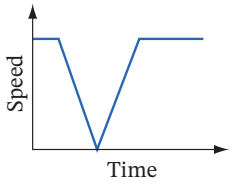


Foundation Standard Complex

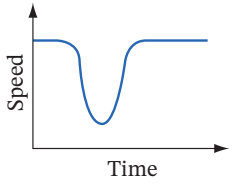
9780170453509

b The speed of a car as it goes around a sharp corner

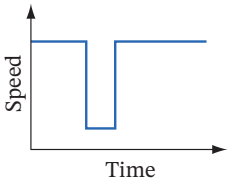
A



B

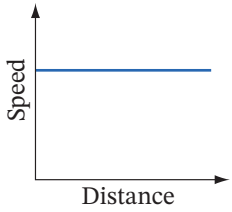


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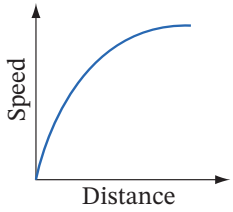


c The speed of a runner going up a steep hill from a standing start

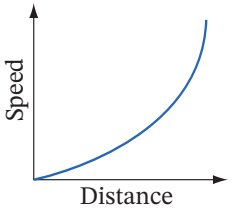
A



B

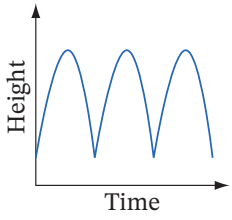


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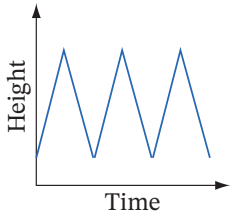


d The height of a person jumping on a trampoline

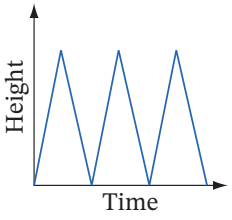
A



B



C



6 The 5 containers below are filled at a constant rate. Match a graph to each container, showing the water height against time. R

a



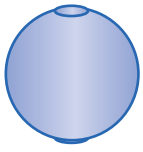
b



c



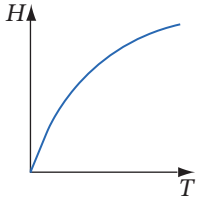
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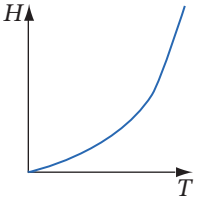
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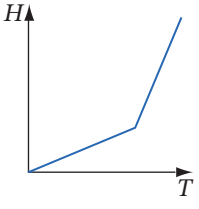
A



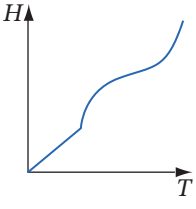
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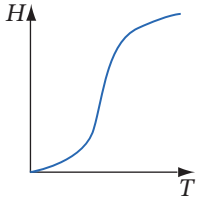
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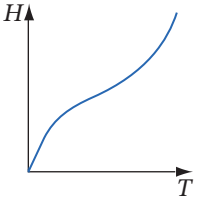
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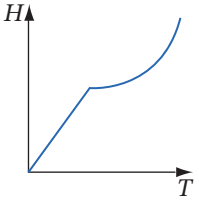
E



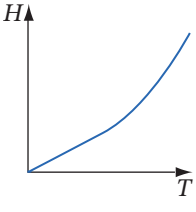
F



G



H



Foundation | Standard | Complex

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STAGE 5.3

7 Match each description to its correct temperature–time graph. R

a The temperature increases at a constant rate.

b The temperature decreases at a constant rate.

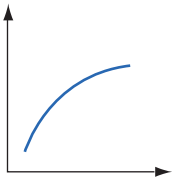
c The temperature increases at an increasing rate.

d The temperature decreases at a decreasing rate.

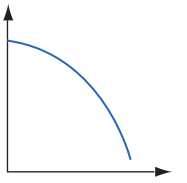
e The temperature increases at a decreasing rate.

f The temperature decreases at an increasing rate.

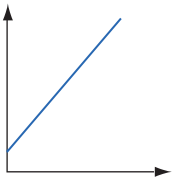
A



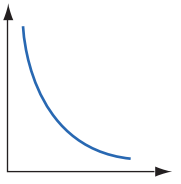
B



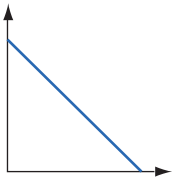
C



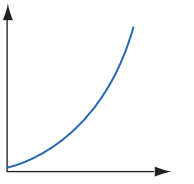
D



E



F



8 Select the speed–time graph that best describes: R

a a car accelerating until it reaches a constant speed

b a car stopped at traffic lights

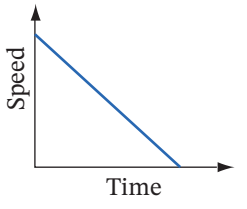
c a bus travelling at a constant speed before stopping for passengers

d a car travelling at a constant speed

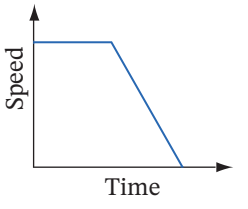
e a train slowing down at a constant rate until it stops

f a rocket launched into space

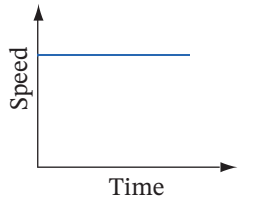
A



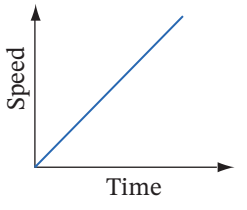
B



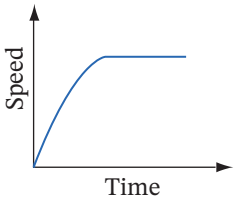
C



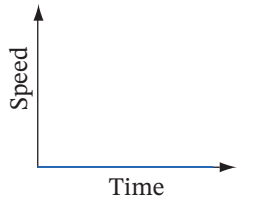
D



E



F



Foundation Standard Complex

Mental skills 8: Maths without calculators ANSWERS ON P. 618

Divisibility tests

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 3 or 4.

- Sum of digits $= 7 + 4 + 8 = 19$, which is not divisible by 3,
  $\therefore$ 748 is not divisible by 3
- 48 is divisible by 4, $\therefore$ 748 is divisible by 4 $(748 \div 4 = 187)$

b Test whether 261 is divisible by 9.

- $2 + 6 + 1 = 9$, which is divisible by 9, $\therefore$ 261 is divisible by 9. $(261 \div 9 = 29)$.

c Test whether 570 is divisible by 6.

- 570 is even and $5 + 7 + 0 = 12$, which is divisible by 3,
  $\therefore$ 570 is divisible by 6 $(570 \div 6 = 95)$

2 Test whether each number is divisible by 2, 3, 5 or 6.

a	250	**b**	189	**c**	78	**d**	465
e	1024	**f**	840	**g**	715	**h**	627

3 Test whether each number is divisible by 4, 9 or 10.

a	144	**b**	280	**c**	522	**d**	4170
e	936	**f**	726	**g**	342	**h**	5580

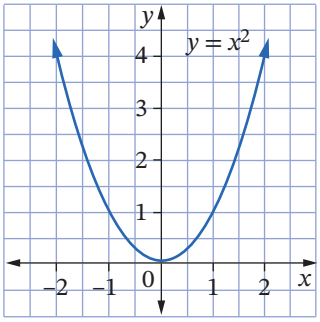
The parabola $y = ax^2 + c$ 8.06

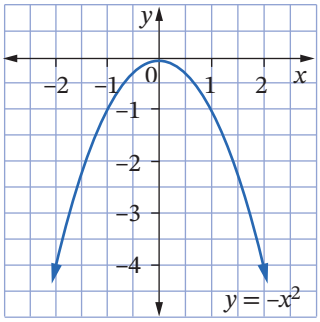
An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$. The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**.


Investigating parabolas 1


Investigating parabolas 1

The graphs of $y = x^2$ and $y = -x^2$ are shown below.





Concave up (looks like a smile ☺)

Minimum value of the parabola is 0.

Concave down (looks like a frown ☹)

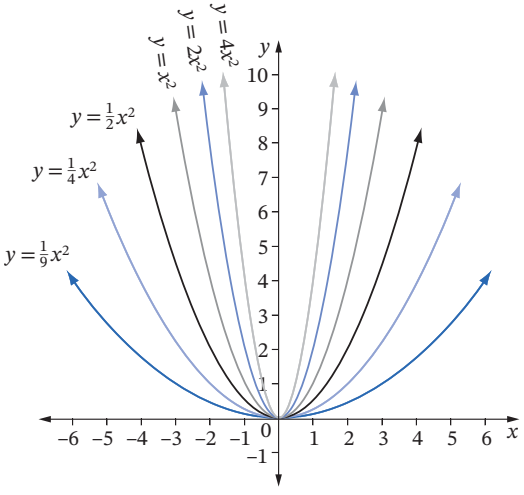
Maximum value of the parabola is 0.

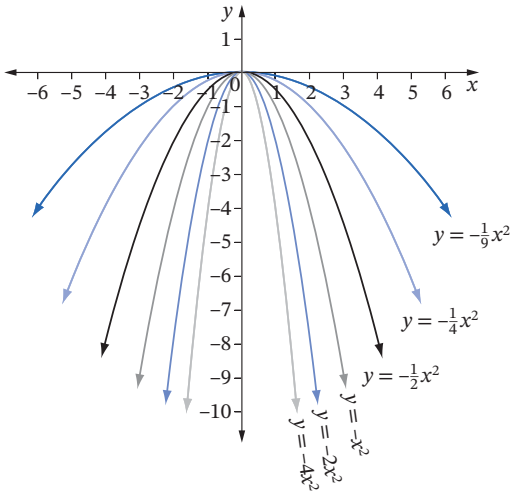
Both parabolas have one axis of symmetry, the y-axis, and a turning point, called the **vertex**, at the origin (0, 0).

The graph of $y = ax^2$

For the graph of $y = ax^2$, where a is a constant (number), the size of a (the **coefficient** of x^2) affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and taller, being dilated (stretched) vertically. As the size of a decreases, the parabola 'widens' and becomes shorter, being compressed vertically. If a is negative, then the parabola is concave down.





The graph of $y = ax^2$ is a parabola with the y-axis as its **axis** (of symmetry) and (0, 0) as its **vertex**.

The graph of $y = ax^2 + c$

For the graph of $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the origin. Also, c is the y-intercept of the parabola.



Example 11

Graph each set of quadratic equations on the same number plane, showing the vertex of each parabola.

a $y = x^2, y = x^2 - 4, y = x^2 + 2$

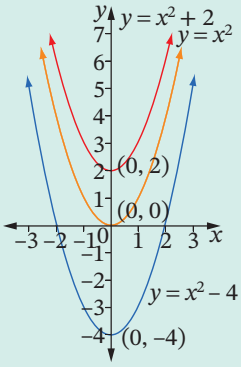
b $y = -x^2, y = -x^2 - 4, y = -x^2 + 5$

Solution

a First draw the graph of $y = x^2$. Its vertex is at (0, 0).

The graph of $y = x^2 - 4$ is like that of $y = x^2$, but moved 4 units down. Its vertex is at (0, −4).

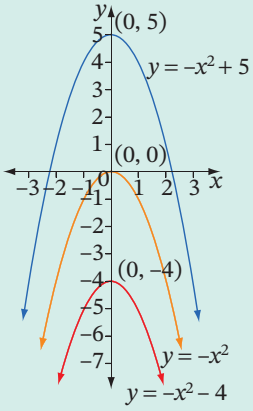
The graph of $y = x^2 + 2$ is like that of $y = x^2$, but moved 2 units up. Its vertex is at (0, 2).



b The graph of $y = -x^2$ is the graph of $y = x^2$ reflected across the x-axis. Its vertex is at (0, 0) as well.

The graph of $y = -x^2 - 4$ is like that of $y = -x^2$, but it is moved 4 units down. Its vertex is at (0, −4).

The graph of $y = -x^2 + 5$ is like that of $y = -x^2$, but it is moved 5 units up. Its vertex is at (0, 5).



Note:
- In part **a**, all parabolas are concave up, because a is positive (1)
- In part **b**, all parabolas are concave down, because a is negative (−1)
- For $y = ax^2 + c$, the y-intercept of the parabola is c

Example 12

For the graph of each quadratic equation, state:

- **i** whether the parabola is wider or narrower than the graph of $y = x^2$
- **ii** whether the parabola has moved up or down when compared to the graph of $y = x^2$
- **iii** the y-intercept.

a $y = 3x^2 - 1$ **b** $y = \frac{1}{3}x^2 + 2$

Solution

a i The coefficient of x^2 is 3, while the coefficient of x^2 in $y = x^2$ is 1.

$\therefore$ The parabola will be narrower than $y = x^2$.

ii The constant term is -1.

$\therefore$ The parabola has moved down.

iii The y-intercept is -1. Vertex at $(0, -1)$

b i The coefficient of x^2 is $\frac{1}{3}$.

$\therefore$ The parabola will be wider than $y = x^2$.

ii The constant term is 2.

$\therefore$ The parabola has moved up.

iii The y-intercept is 2. Vertex at $(0, 2)$

Example 13

A parabola has the equation $y = 3x^2 - 1$.

Find the x-coordinate of the point on the parabola that has a y-coordinate of:

a 11 **b** 191

Solution

a Substitute $y = 11$ into $y = 3x^2 - 1$.

$11 = 3x^2 - 1$

$12 = 3x^2$

$3x^2 = 12$

$x^2 = \frac{12}{3}$

$= 4$

$x = \pm\sqrt{4}$

$= \pm 2$

This means that there are 2 points on the parabola with a y-coordinate of 11.

They are $(2, 11)$ and $(-2, 11)$.

b Substitute $y = 191$ into $y = 3x^2 - 1$

$191 = 3x^2 - 1$

$192 = 3x^2$

$3x^2 = 192$

$x^2 = \frac{192}{3}$

$= 64$

$x = \pm\sqrt{64}$

$= \pm 8$

This makes sense because a parabola is symmetrical: $(2, 11)$ and $(-2, 11)$ are on either side of the y-axis.

EXERCISE 8.06 ANSWERS ON P. 618

The parabola $y = ax^2 + c$ U F PS R C

This exercise can be completed using graphing technology.

EXAMPLE 11

8.06

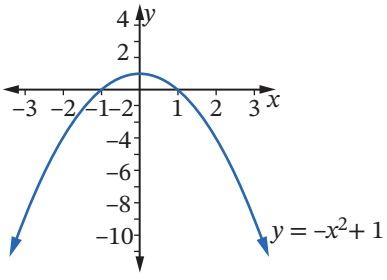
1 **a** Graph each quadratic equation, showing the vertex of each parabola.

$y = x^2$ $y = -x^2$ $y = x^2 + 2$ $y = -2x^2$ $y = x^2 - 1$

b State which graphs you have drawn in part **a**: C

i are concave up **ii** are concave down **iii** have a turning point at (0, 0).

2 Which statement is false about this parabola? Select the correct answer **A**, **B**, **C** or **D**. C



A Its axis of symmetry is the x-axis.

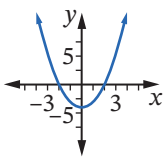
B It is concave down.

C Its vertex is (0, 1).

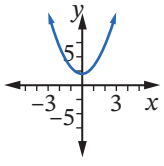
D It has a maximum value.

3 Which diagram shows the graph of $y = x^2 - 2$? Select **A**, **B**, **C** or **D**. R

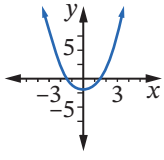
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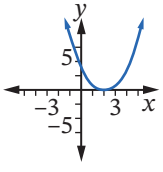
B



C



D



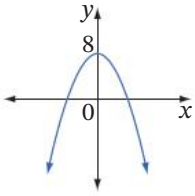


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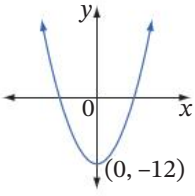
Foundation Standard Complex

4 Match each graph with its correct quadratic equation. R C

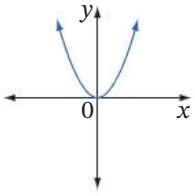
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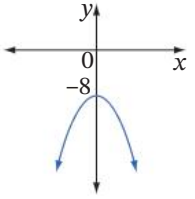
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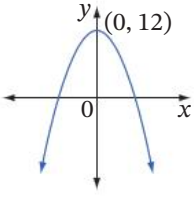
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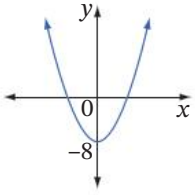
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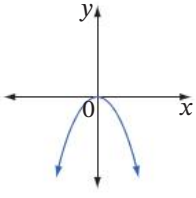
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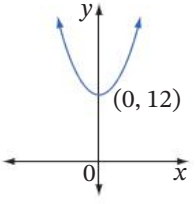
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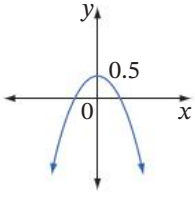
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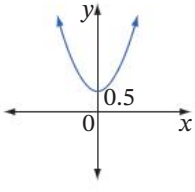
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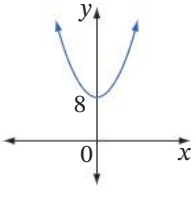
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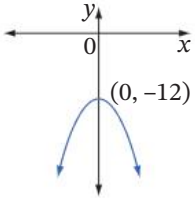
j



k



l



A $y = x^2$ **B** $y = -x^2$ **C** $y = x^2 - 8$ **D** $y = -12 - x^2$

E $y = \frac{1}{2} + x^2$ **F** $y = 8 - x^2$ **G** $y = 8 + x^2$ **H** $y = -x^2 + \frac{1}{2}$

I $y = x^2 - 12$ **J** $y = 12 - x^2$ **K** $y = -x^2 - 8$ **L** $y = x^2 + 12$

5 Find the equation of each parabola, in the form $y = x^2 + c$ or $y = -x^2 + c$. R C

a vertex (0, 0), concave down

b concave up, turning point (0, 0)

c axis of symmetry $x = 0$, maximum $y = -\frac{1}{4}$

d concave down, maximum $y = -9$

e turning point $\left(0, \frac{1}{2}\right)$, concave down

f axis of symmetry y-axis, minimum $y = 9$

Foundation Standard Complex

9780170453509

6 **a** Graph $y = 2x^2 + 1$ after copying and completing this table. R C

x	−2	−1	0	1	2
y					

b State the coordinates of the vertex.

c Is the parabola concave up or concave down?

d What is its minimum value?

7 **a** Graph $y = -3x^2 + 2$ after copying and completing this table. R C

x	−2	−1	0	1	2
y					

b Find the vertex.

c Write the equation of its axis of symmetry.

d Find its maximum value.

8 Which statement is false about the graph of $y = 4x^2 - 1$? Select **A**, **B**, **C** or **D**. R C

A Its axis of symmetry is $y = 0$.

B It is concave up.

C The vertex is $(0, -1)$.

D It has a minimum value of $y = -1$.

9 For the graph of each given quadratic equation, state: R C

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 2x^2 + 3$

b $y = \frac{1}{2}x^2 + 1$

c $y = 6x^2 - 5$

d $y = 0.2x^2 - 12$

10 A parabola has the equation $y = x^2 - 5$. Find the x-coordinates for the points on the parabola that have a y-coordinate of:

a 11

b 116.

11 A stone is dropped from a cliff and its height (h metres) at any time (t seconds) is given by $h = 80 - 4.8t^2$. PS R C

a Draw a graph of the equation for values of t from 0 to 5.

b What is the height of the cliff?

c What is the height of the stone after 3 seconds?

d When will the stone hit the ground?

e How long after it is dropped is the stone 5 metres above the ground? Answer correct to 2 decimal places.

8.06

EXAMPLE 12

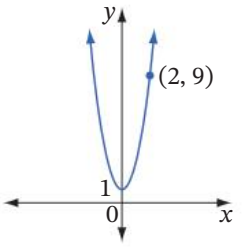
EXAMPLE 13

Foundation | Standard | Complex

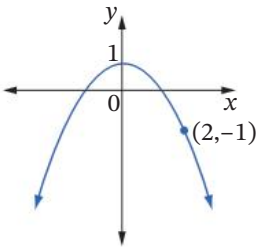
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12 Match each graph with its correct quadratic equation. R C

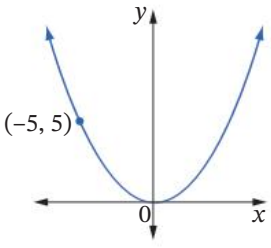
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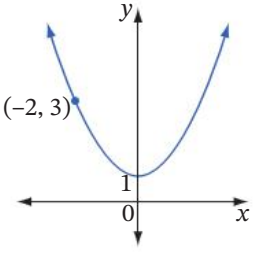
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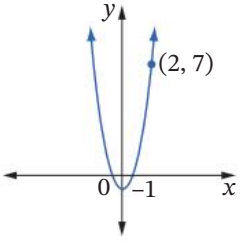
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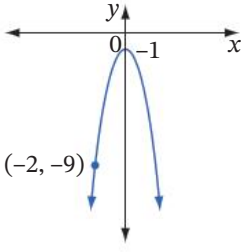
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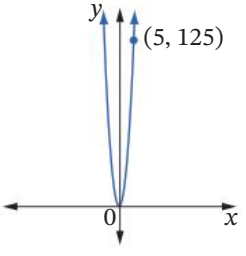
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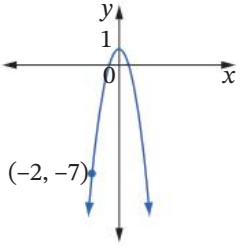
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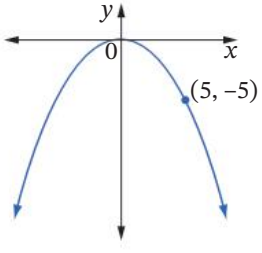
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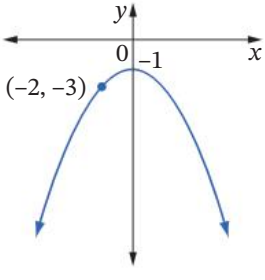
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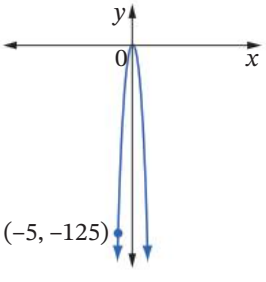
i



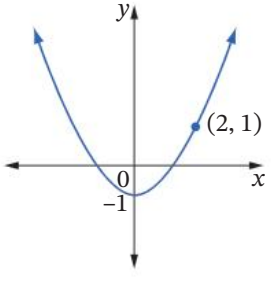
j



k



l



A $y = 5x^2$

B $y = 2x^2 + 1$

C $y = \frac{1}{2}x^2 - 1$

D $y = \frac{1}{5}x^2$

E $y = 2x^2 - 1$

F $y = -5x^2$

G $y = -\frac{1}{2}x^2 + 1$

H $y = -\frac{1}{5}x^2$

I $y = -2x^2 - 1$

J $y = \frac{1}{2}x^2 + 1$

K $y = -2x^2 + 1$

L $y = -\frac{1}{2}x^2 - 1$

Foundation Standard Complex

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Technology

Graphing $y = a(x - b)^2$

In this activity, use graphing technology to compare the shapes of parabolas of the form $y = a(x - b)^2$.

1 **a** Use the software to graph the parabolas $y = (x-1)^2, y = (x-2)^2, y = (x+2)^2, y = (x+3)^2$.

b Compare and contrast the parabolas. What is the same and what is different?

c Are the parabolas concave up or down?

d Do they have a common vertex?

e What are their axes of symmetry?

f Given that these parabolas are of the form $y = (x + b)^2$, what does b represent?

2 **a** Use the software to graph the parabolas $y = 3(x - 2)^2, y = -(x - 4)^2, y = \frac{1}{2}(x+3)^2$, $y = -0.1(x + 1)^2$.

b Compare and contrast the parabolas. What is the same and what is different?

c Which parabola is the widest? Which parabola is the narrowest?

d Which parabolas are concave up and concave down? Why?

e What is the vertex of each parabola?

f Given that these parabolas are of the form $y = a(x - b)^2$, what does b represent?

g What is the effect of the size of a on the shape of the parabola?

The parabola $y = a(x - b)^2 + c$#

8.07

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

The parabola $y = a(x - b)^2 + c$

STAGE 5.3

- The graph of $y = a(x - b)^2$ is a parabola with a vertex at $(b, 0)$.
- The parabola $y = a(x - b)^2$ is the graph of $y = ax^2$ translated b units to the right (or left if b is negative)
- The graph of $y = a(x - b)^2 + c$ is the parabola $y = a(x - b)^2$ translated c units up (or down if c is negative), so its vertex is at (b, c).

WS Matching parabolas with their equations

STAGE 5.3

Investigating parabolas 2

Graphing the parabola $y = a(x - b)^2 + c$

Investigating parabolas 2

Example 14

Graph each parabola, clearly showing the vertex and y-intercept.

a $y = (x - 1)^2$ **b** $y = -(x + 2)^2$ **c** $y = 2(x + 1)^2 - 2$

Solution

a $y = (x - 1)^2$

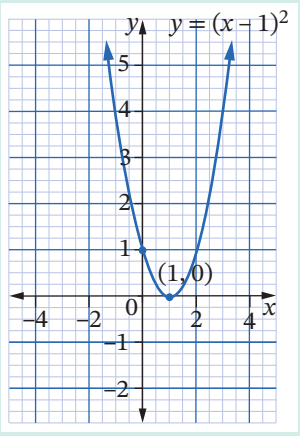
This parabola is the graph of $y = x^2$ translated 1 unit to the right, with a vertex at (1, 0).

Substitute $x = 0$ for the y-intercept:

$y = (0 - 1)^2$

$= (-1)^2$

$= 1$



b $y = -(x + 2)^2$

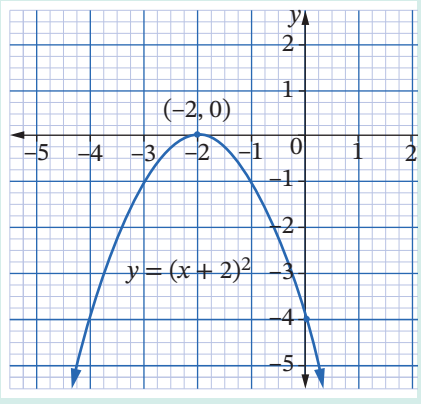
This parabola is the graph of $y = -x^2$ translated 2 units to the left, with a vertex at (–2, 0).

It is concave down because a is negative.

Substituting $x = 0$ for the y-intercept:

$y = -(0 + 2)^2$

$= -4$



c $y = 2(x + 1)^2 - 2$

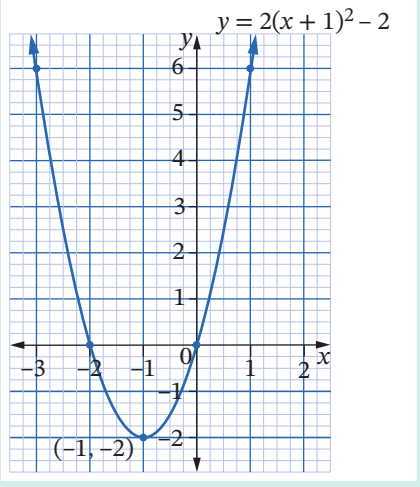
This is $y = 2x^2$ translated 1 unit to the left, with a vertex at (–1, 0), then translated 2 units down, with a vertex at (–1, –2).

Substituting $x = 0$ for the y-intercept:

$y = 2(0 + 1)^2 - 2$

$= 2 - 2$

$= 0$



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EXERCISE 8.07 ANSWERS ON P. 619

The parabola $y = a(x - b)^2 + c$ U F R

EXAMPLE 14

8.07

1 Graph each parabola, clearly showing the vertex and y-intercept. R

a $y = (x - 3)^2$ **b** $y = (x - 2)^2$ **c** $y = (x + 1)^2 + 3$

d $y = -(x - 3)^2$ **e** $y = -(x + 1)^2$ **f** $y = -(x - 5)^2 - 1$

g $y = 3(x + 4)^2$ **h** $y = -2(x - 1)^2$ **i** $y = -\frac{1}{4}(x + 6)^2 + 2$

2 Which statement is true about the parabola $y = (x + 6)^2$?
Select the correct answer **A**, **B**, **C** or **D**. R

A Its axis of symmetry is $x = 6$ **B** It has a y-intercept at 36

C Its vertex is $(6, 0)$ **D** It passes through the point $(-1, 5)$

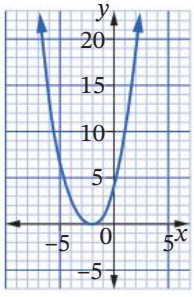
3 Match each equation to its graph below. R

a $y = (x + 4)^2$ **b** $y = -(x - 1)^2$ **c** $y = (x + 2)^2$

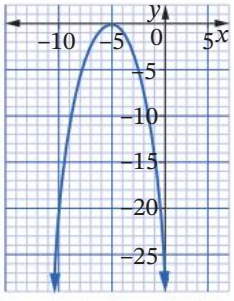
d $y = (x - 3)^2 + 4$ **e** $y = -(x + 5)^2$ **f** $y = -2(x + 1)^2$

g $y = -0.5(x - 4)^2$ **h** $y = 4(x - 1)^2 - 3$

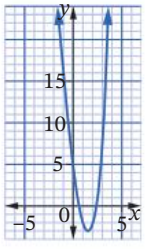
A



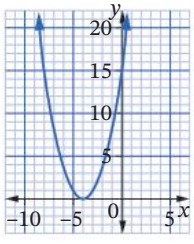
B



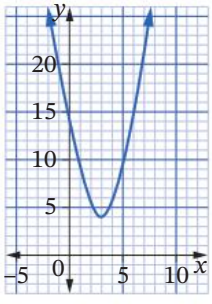
C



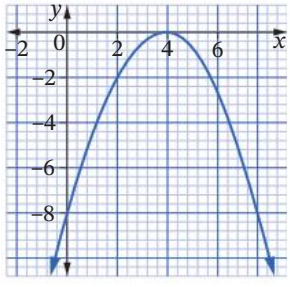
D



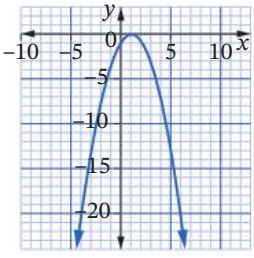
E



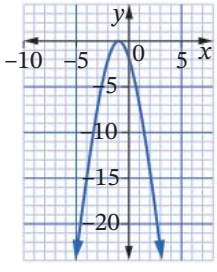
F



G



H



Foundation | Standard | Complex

STAGE 5.3

Technology

Graphing $y = kx^3 + c$

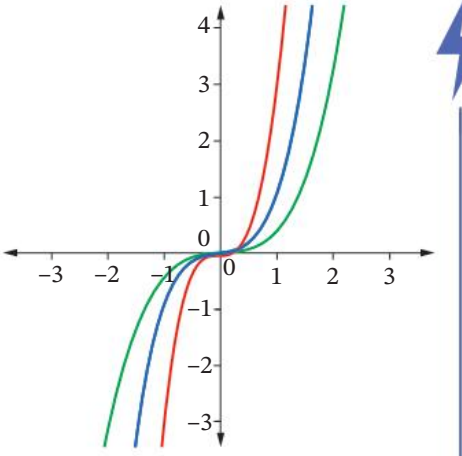
1 Use graphing technology to graph the cubic equations $y = 0.4x^3$, $y = x^3$ and $y = 3x^3$ in different colours.

2 What is the effect of k in $y = kx^3$?

3 Now graph $y = 2x^3$, $y = 2x^3 - 4$ and $y = 2x^3 + 1$ and compare the curves.

4 Graph $y = -\frac{1}{2}x^3$, $y = -\frac{1}{2}x^3 - 2$ and $y = -\frac{1}{2}x^3 + 3$ and compare the curves.

5 What is the effect of c in $y = kx^3 + c$?



8.08 The cubic curve $y = kx^3 + c$#

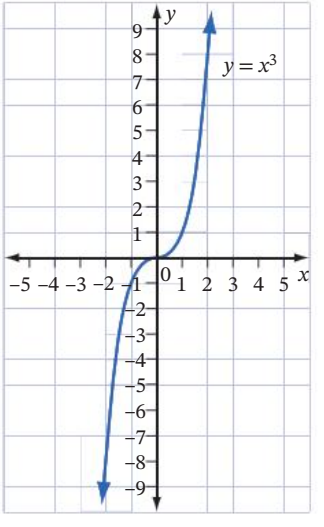
#NSW ONLY, NOT AUSTRALIAN CURRICULUM

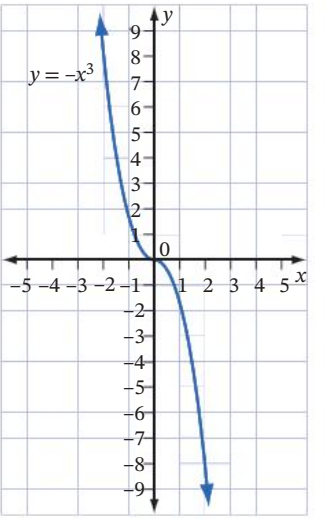
STAGE 5.3

WS Graphing cubics 1

An equation in which the highest power of the variable is 3 is called a **cubic equation**, for example, $y = 3x^3$, $y = -x^3 + 9$ and $y = x^3 + 2x^2 - 5x + 10$. The graph of a cubic equation is called a **cubic curve**.

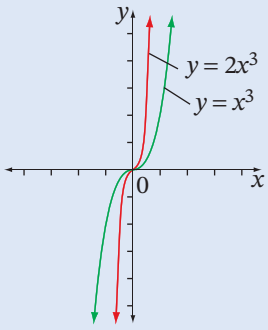
The graphs of $y = x^3$ and $y = -x^3$ are shown below.

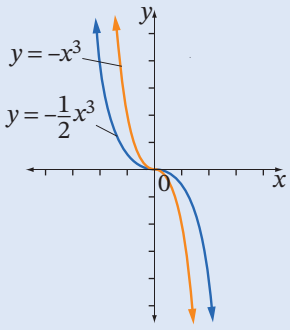




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The cubic curve $y = kx^3$

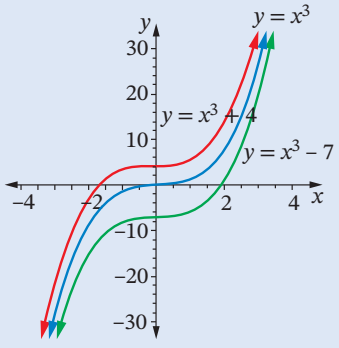




- The graph has no line symmetry.
- The graph has rotational symmetry of 180° about (0, 0): if you spin the curve upside-down, it maps onto itself.
- If k is positive, the graph is increasing, but flat at (0, 0)
- If k is negative, the graph is decreasing, but flat at (0, 0)
- The size of k (the coefficient of x^3) determines whether the cubic curve is 'wider' or 'narrower' (compared to $y = x^3$).

The cubic curve $y = kx^3 + c$

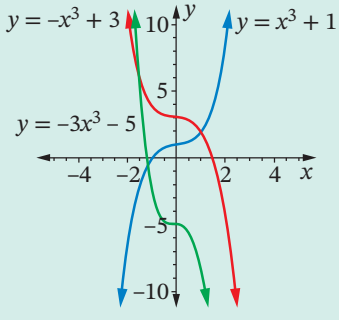
The graph of $y = kx^3 + c$ is the cubic curve $y = kx^3$ translated c units up (or down if c is negative), so its y-intercept is c).



Example 15

Graph the cubic equations $y = x^3 + 1$, $y = -x^3 + 3$ and $y = -3x^3 - 5$, showing the y-intercept of each curve.

Solution



STAGE 5.3

EXERCISE 8.08 ANSWERS ON P. 619

The cubic curve $y = kx^3 + c$ UFRC

EXAMPLE 15

1 Graph each cubic equation.

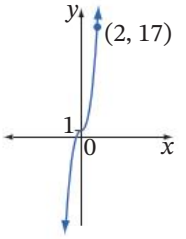
a $y = x^3 - 2$ **b** $y = -x^3$ **c** $y = 2x^3$

d $y = x^3 + 3$ **e** $y = -x^3 - 4$ **f** $y = -x^3 + 2$

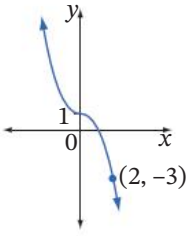
g $y = 2x^3 - 3$ **h** $y = -3x^3 - 2$ **i** $y = \frac{1}{2}x^3 + 4$

2 Match each cubic curve to its equation below. R

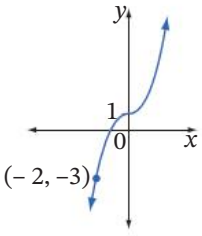
a



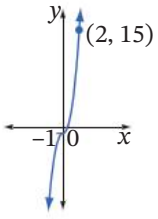
b



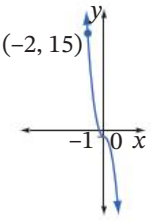
c



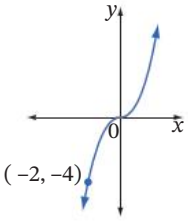
d



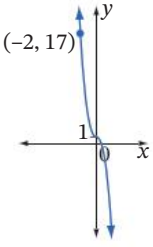
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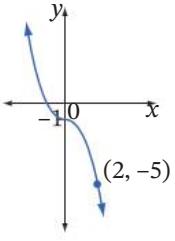
f



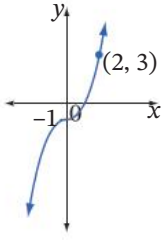
g



h



i



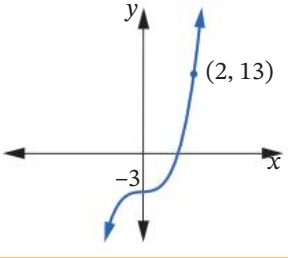
A $y = 2x^3 + 1$ **B** $y = \frac{1}{2}x^3 - 1$ **C** $y = 2x^3 - 1$

D $y = \frac{1}{2}x^3$ **E** $y = -\frac{1}{2}x^3 + 1$ **F** $y = -2x^3 - 1$

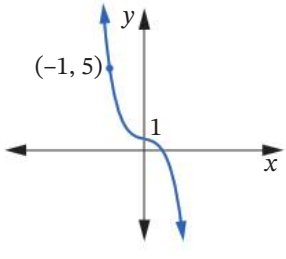
G $y = \frac{1}{2}x^3 + 1$ **H** $y = -2x^3 + 1$ **I** $y = -\frac{1}{2}x^3 - 1$

3 Find the equation of each cubic curve in the form $y = kx^3 + c$. R C

a



b



Foundation Standard Complex

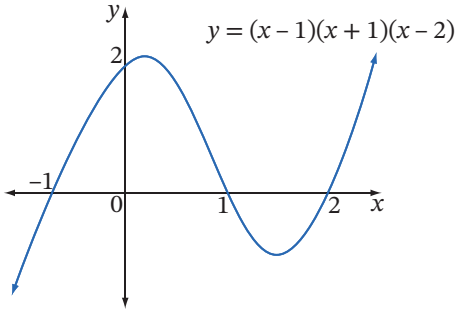
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The cubic curve $y = k(x - a)(x - b)(x - c)^{\#}$ 8.09

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STAGE 5.3

$y = (x - 1)(x + 1)(x - 2)$ is a **cubic equation** because if you expand the RHS, you get $y = x^3 - 2x^2 - x + 2$, and the first term is x^3. Its graph is a more complicated cubic curve:



$y = k(x - a)(x - b)(x - c)$ is called the **intercept form of a cubic equation**. Looking at the graph of $y = (x - 1)(x + 1)(x - 2)$ above, can you see why?

In Chapter 3, *Graphing lines*, we graphed the line $y = mx + c$ by using its x- and y-intercepts and substituting $y = 0$ and $x = 0$ respectively. We can graph the cubic curve $y = k(x - a)(x - b)(x - c)$ in a similar way.

Example 16

Graph the cubic equation $y = x(x + 5)(x - 1)$.

Solution

Substitute $y = 0$ to find the x-intercepts.

$0 = x(x + 5)(x - 1)$

This is true if $x = 0$ or $x + 5 = 0$ or $x - 1 = 0$

$\therefore x = 0, -5$ or 1

If 3 numbers multiplied together equal 0, then one of them must be 0.

The x-intercepts are -5, 0 and 1.

Substitute $x = 0$ to find the y-intercept.

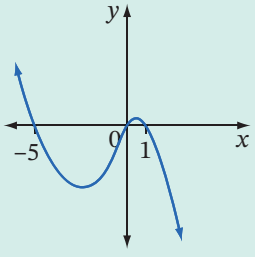
$y = 0(0 + 5)(0 - 1)$

$= 0$

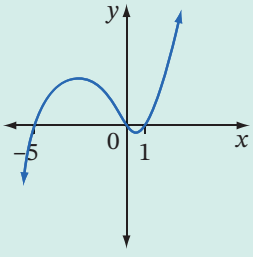
The y-intercept is 0.

Possible graphs are:

A



B



STAGE 5.3

To determine which graph is correct, we look at k, the coefficient of the cubic equation $y = k(x - a)(x - b)(x - c)$.

In the expansion of $y = x(x + 5)(x - 1)$, the coefficient of x^3 is 1, which is positive, so the correct graph is B, an increasing cubic curve.

(If the coefficient of x^3 is negative, the correct graph is A, a decreasing cubic curve)

We can check this by substituting a value of x, say $x = -3$, into the equation to find a point on the curve:

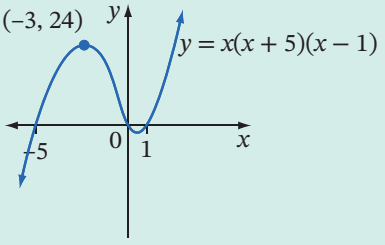
$y = -3(-3 + 5)(-3 - 1)$

$= 24$

This means $(-3, 24)$ lies on the curve.

So B must be the correct graph.

Use graphing technology to check the shape of this graph.



The cubic curve $y = k(x - a)(x - b)(x - c)$

- The cubic curve $y = k(x - a)(x - b)(x - c)$ has x-intercepts of a, b and c.
- If k is positive, the graph is increasing, but decreasing in the middle.
- If k is negative, the graph is decreasing, but increasing in the middle.

EXERCISE 8.09 ANSWERS ON P. 620

The cubic curve $y = k(x - a)(x - b)(x - c)$ U F R C

Graphing technology can be used to check the shapes of the graphs in this exercise.

EXAMPLE 16

1 For each cubic equation, find the x- and y-intercepts and sketch its graph. R

a $y = (x - 2)(x + 2)(x + 3)$ **b** $y = x(x - 2)(x + 1)$

c $y = (x - 1)(x - 3)(x + 1)$ **d** $y = (4 - x)(x + 1)(x - 5)$

e $y = (1 - x)(2 + x)(x + 3)$ **f** $y = 2x(x - 6)(x - 3)$

g $y = -(x + 3)(x - 2)(x - 5)$ **h** $y = -2(x - 1)(x + 2)(x - 3)$

i $y = (x + 2)(x - 1)(x - 2)$

2 What are the x-intercepts of the graph of $y = 2x(x + 1)(x - 3)$?
Select the correct answer **A**, **B**, **C** or **D**.

A $x = 2, 1, -3$ **B** $x = 0, -1, 3$ **C** $x = 0, 1, -3$ **D** $x = 2, -1, 3$

3 Which cubic equation has a graph with x-intercepts -2, -5 and 6? Select **A**, **B**, **C** or **D**.

A $y = (x + 2)(x + 5)(x - 6)$ **B** $y = (x - 2)(x - 5)(x + 6)$

C $y = (x + 2)(x - 5)(x + 6)$ **D** $y = (x - 2)(x + 5)(x + 6)$

4 **a** Prove that the y-intercept of $y = k(x - a)(x - b)(x - c)$ is $-kabc$. R C

b Prove that the x-intercepts of $y = k(x - a)(x - b)(x - c)$ are a, b and c.

5 Sketch the graph of each cubic equation. R

a $y = x(x - 3)(x + 1)$ **b** $y = -(x - 3)(x - 1)(x + 1)$

c $y = -2x(x - 1)(x + 6)$ **d** $y = (2x - 3)(x - 1)(x + 2)$

e $y = (x + 1)(x + 1)(x + 1)$ **f** $y = (x - 3)^2(x + 2)$

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Technology

STAGE 5.3

Graphing $y = kx^n$

1 Use graphing technology to graph on the same axes $y = x^2$, $y = x^4$ and $y = x^6$.

2 How are the graphs similar? How are the graphs different?

3 Graph on the same axes $y = x^3$ and $y = x^5$.

4 How are the graphs similar? How are the graphs different?

5 Describe the differences between the graphs of $y = x^n$ when n is even and when n is odd.

8.09

The power curves $y = kx^n + c^{\#}$

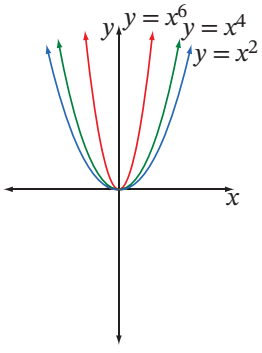
8.10

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

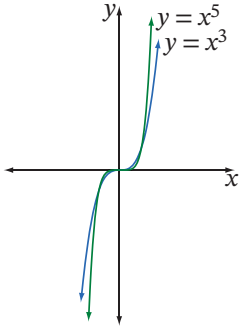
STAGE 5.3

We have already graphed $y = ax^2 + c$ and $y = ax^3 + c$. Now we will graph equations containing higher powers of x, of the form $y = ax^n + c$, where n is a positive integer.

The graphs of $y = x^2$, $y = x^4$ and $y = x^6$ are shown: even powers of x.



The graphs of $y = x^3$ and $y = x^5$ are shown: odd powers of x.



We can call these **power curves** as they are graphs of **power equations**.

The graph of $y = kx^n$ if n is even

- The even power curve is like a steeper parabola, symmetrical about the y-axis, with a vertex at (0, 0)
- The graph is concave up if k is positive and concave down if k is negative
- The higher the power (the value of n), the narrower the graph

The graph of $y = kx^n$ if n is odd

- The odd power curve is like a steeper cubic curve, with rotational symmetry of 180° about (0, 0)
- The graph has a steep gradient, except around (0, 0) where it is flat.
- The higher the power (the value of n), the narrower the graph

Power functions

Power functions

STAGE 5.3

The graph of $y = kx^n + c$

- The graph of $y = kx^n + c$ is the power curve $y = kx^n$ translated c units up (or down if c is negative), a vertical translation, so its y-intercept is c.

The graph of $y = k(x - b)^n$

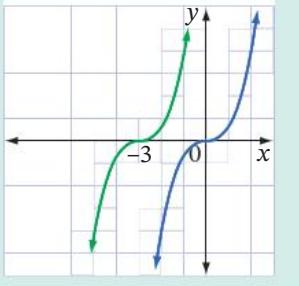
- The graph of $y = k(x - b)^n$ is the power curve $y = kx^n$ translated b units to the right (or left if b is negative), a horizontal translation.

Example 17

Sketch the graphs of $y = \frac{1}{2}x^3$ and $y = \frac{1}{2}(x+3)^3$ on the same axes.

Solution

The graph of $y = \frac{1}{2}(x+3)^3$ is obtained by moving the graph of $y = \frac{1}{2}x^3$ to the left by 3 units.



EXERCISE 8.10 ANSWERS ON P. 621

The power curves $y = kx^n + c$ U F R C

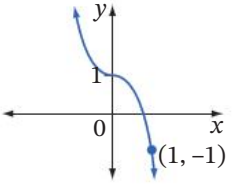
1 Match each equation to its correct graph. R

a $y = x^2 - 1$ **b** $y = (x - 3)^3$ **c** $y = 2x^4 + 2$

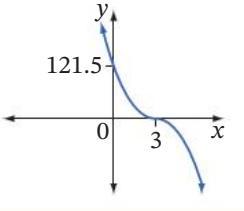
d $y = \frac{1}{3}(x+2)^3$ **e** $y = 2x^5 - 3$ **f** $y = -2x^3 + 1$

g $y = -(x - 4)^4$ **h** $y = 3x^3 + 1$ **i** $y = -\frac{1}{2}(x-3)^5$

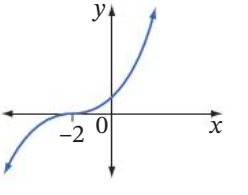
A



B



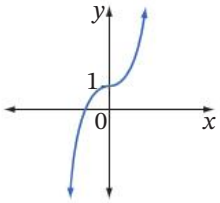
C



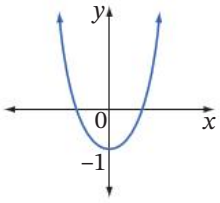
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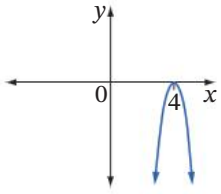
D



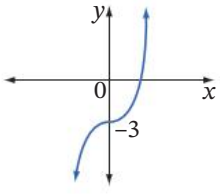
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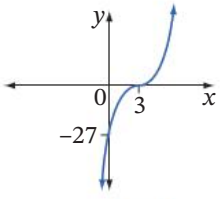
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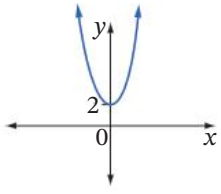
G



H



I



2 Sketch the graphs of each pair of equations. R

a $y = x^2$ and $y = (x - 2)^2$

b $y = 3x^2$ and $y = 3x^2 + 1$

c $y = -x^3$ and $y = -x^3 + 2$

d $y = -2x^4$ and $y = -2(x + 2)^4$

e $y = x^4$ and $y = (x - 1)^4$

f $y = -x^5$ and $y = -x^5 - 2$

EXAMPLE 17

3 For each pair of equations, explain how the second equation can be transformed using the graph of the first equation, for example, move left 4 units. R C

a $y = -x^5$ and $y = -x^5 + 4$

b $y = x^2$ and $y = (x - 5)^2$

c $y = 5x^6$ and $y = 5(x + 3)^6$

d $y = \frac{1}{4}x^3$ and $y = 4 + \frac{1}{4}x^3$

e $y = -x^4$ and $y = -(3 - x)^4$

f $y = -x^3$ and $y = -(x + 2)^3$

Investigation

Graphing $y = \frac{1}{x}$

1 Copy and complete this table for $y = \frac{1}{x}$. Explain why no y value exists for $x = 0$.

x	−5	−4	−3	−2	−1	−0.5	−0.2	−0.1	0	0.1	0.2	0.5	1	2	3	4	5
y									–								

2 Hence graph $y = \frac{1}{x}$ on a number plane.

3 There are 2 parts or 'branches' to your graph. In which quadrants of the number plane are the branches?

4 Use your graph to explain what happens to the y value as x becomes very large.

5 Explain what happens to the y value as x approaches 0.

6 The graph of $y = \frac{1}{x}$ has 2 axes of symmetry. Draw them on your graph.

7 Copy and complete the table from question **1** for $y = -\frac{1}{x}$.

8 Hence graph $y = -\frac{1}{x}$ on a number plane.

9 How does the graph of $y = -\frac{1}{x}$ compare with that of $y = \frac{1}{x}$?

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STAGE 5.3

Technology

Graphing $y = \frac{k}{x}$

1 Use graphing technology to graph each equation.

a $y = \frac{1}{x}$ **b** $y = \frac{2}{x}$ **c** $y = \frac{5}{x}$ **d** $y = \frac{10}{x}$

2 Compare the graphs from question **1**. What happens to the graph of $y = \frac{k}{x}$ as k increases?

3 Graph $y = \frac{2}{x}$ and $y = \frac{-2}{x}$ and compare them.

4 Graph $y = \frac{4}{x}$ and use **Trace** to complete this table of values.

x	1	2	5	10	100	200	1000
y							

5 What happens to the y values when the x values become very large?

6 For $y = \frac{4}{x}$ use the **Trace** function to complete this table of values.

x	0.0001	0.01	0.1	0.5	1	5
y						

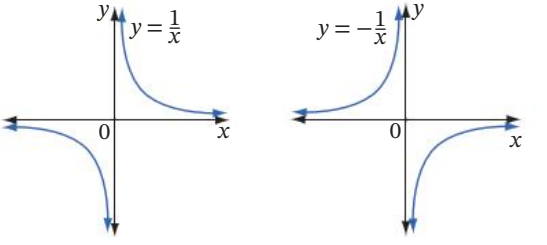
7 What happens to the y values when the x values become very small and close to 0?

8.11 The hyperbola $y = \frac{k}{x}$

STAGE 5.3

The graph of $y = \frac{k}{x}$, where k is a constant, is a curve with 2 branches called a **hyperbola** (pronounced 'hy-perb-o-la').

The graphs of $y = \frac{1}{x}$ and $y = -\frac{1}{x}$ are shown.



The hyperbola $y = \frac{k}{x}$

- The hyperbola has 2 separate branches in different quadrants.
- If k is positive, the graph is in the 1st and 3rd quadrants.
- If k is negative, the graph is in the 2nd and 4th quadrants.
- The graph has 2 axes of symmetry: their equations are the straight lines $y = x$ and $y = -x$.
- The graph has rotational symmetry of 180° about (0, 0).
- The higher the value of k, the further the hyperbola is from the x- and y-axes.
- As x becomes very large, y gets closer to 0.
- As x becomes closer to 0, y gets very large.
- The graph gets very close to the x- and y-axes but never crosses them. The x- and y-axes are called **asymptotes** because the graph approaches them but never touches them.

Example 18

Graph each hyperbola and mark the coordinates of one point on the curve.

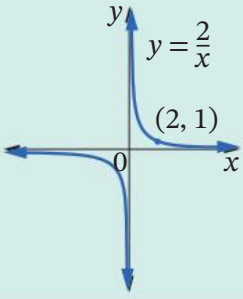
a $y = \frac{2}{x}$ **b** $y = -\frac{3}{x}$

Solution

a Let $x = 2$ Choosing any value of x

$y = \frac{2}{2} = 1$

A point on the curve is (2, 1).

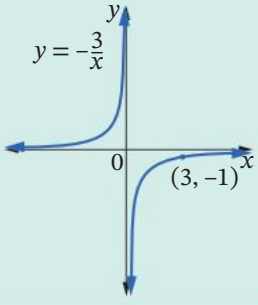


b Let $x = 3$

$y = \frac{-3}{3} = -1$

A point on the curve is (3, –1).

As $k = -3$ is negative, the hyperbola is in the 2nd and 4th quadrants.



The hyperbola $y = \frac{k}{x} + c$

The graph of $y = \frac{k}{x} + c$ is the hyperbola $y = \frac{k}{x}$ translated c units up (or down if c is negative).

The hyperbola $y = \frac{k}{x - b}$

The graph of $y = \frac{k}{x - b}$ is the hyperbola $y = \frac{k}{x}$ translated b units to the right (or left if b is negative).

Example 19

Graph each hyperbola, find any intercepts and mark the coordinates of one point on the curve.

a $y = \frac{2}{x} + 1$ **b** $y = \frac{-3}{x - 2}$

Solution

a Let $x = 2$ Choosing any value of x

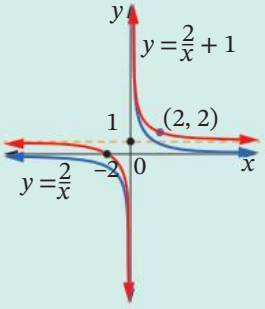
$y = \frac{2}{2} + 1 = 2$

A point on the curve is (2, 2).

STAGE 5.3

The graph of $y=\frac{2}{x}+1$ is the graph of $y=\frac{2}{x}$ translated up 1 unit. This means that the horizontal asymptote is now at $y=1$.
An x-intercept now occurs when $y=0$.

$0=\frac{2}{x}+1$ Multiplying both sides by x.
$0=2+x$
$x=-2$
The x-intercept is -2.



b Let $x=3$

$y=\frac{-3}{3-2}=-3$

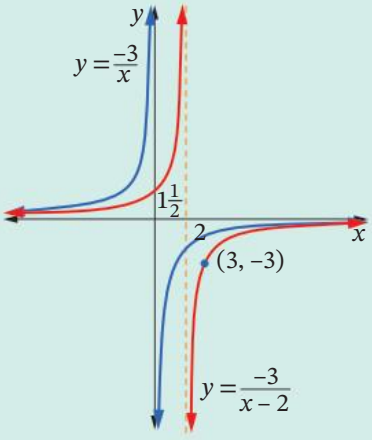
A point on the curve is $(3,-3)$.

The graph of $y=\frac{-3}{x-2}$ is the graph of $y=\frac{-3}{x}$ translated right 2 units. This means that the vertical asymptote is now at $x=2$.

A y-intercept now occurs when $x=0$.

$y=\frac{-3}{0-2}=\frac{-3}{-2}=1\frac{1}{2}$

The y-intercept is $1\frac{1}{2}$.



EXERCISE 8.11 ANSWERS ON P. 621

The hyperbola $y=\frac{k}{x}$ UFRC

EXAMPLE 18

1 **a** Copy and complete this table for $y=\frac{2}{x}$.

x	−3	−2	−1	0	1	2	3
y				–			

b Graph $y=\frac{2}{x}$, showing the coordinates of one point on the hyperbola.

c On your diagram, draw in the axes of symmetry for the hyperbola.

d What are the equations of these axes?

2 Graph each hyperbola and mark the coordinates of one point on the curve.

a $y=\frac{4}{x}$ **b** $y=-\frac{2}{x}$ **c** $y=\frac{3}{x}$

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3 **a** The distance from Sydney to Melbourne is close to 1000 km. Using the equation $s = \frac{1000}{t}$, copy and complete the following table that relates time (t hours) and speed (s km/h) for the trip. Round your answers to the nearest km/h.

t	1	2	3	...	10
s					

b Hence graph the equation $s = \frac{1000}{t}$.

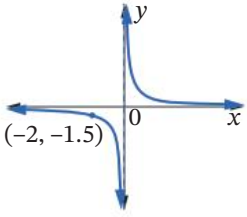
c Why are the values for t only positive numbers? Explain why t cannot be equal to 0. R C

d If the time is doubled, is the speed halved? Use the information from your graph to support your answer. R C

4 This curve is a hyperbola of the form $y = \frac{k}{x}$.

a Find the value of k.

b Hence state the equation of this hyperbola.



EXAMPLE 19

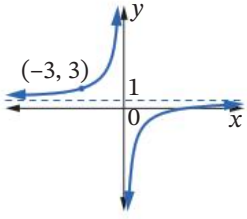
5 Graph each hyperbola and mark the coordinates of one point on the curve.

a $y = \frac{1}{x} + 2$ **b** $y = -\frac{2}{x} - 3$ **c** $y = \frac{2}{x-1}$ **d** $y = \frac{-3}{x+2}$

6 This curve is a hyperbola with equation $y = \frac{k}{x} + c$.

a Find the values of c and k.

b Hence state the equation of this hyperbola.



7 Sarah and David want to buy a rectangular block of land that has an area of 800 m^2. There are several blocks available with this area. R C

a Copy and complete this table that relates the length (L metres) and width (W metres) of the block of land.

L	10	20	30	...	100
W					

b What is the formula for W?

c Explain why the length or width cannot be equal to 0 metres.

d Graph the formula for W.

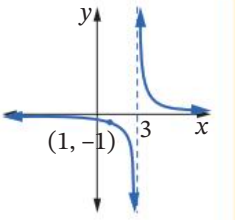
e What happens to the width as the length continues to increase? How is this shown on the graph?

f What happens to the width as the length approaches 0? How is this shown on the graph?

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STAGE 5.3

8 Which equation best represents this hyperbola?
Select the correct answer **A**, **B**, **C** or **D**.



A $y=\frac{2}{x-3}$ **B** $y=\frac{-1}{x+3}$

C $y=\frac{2}{x}+3$ **D** $y=-\frac{1}{x}-3$

9 Find the asymptotes of the hyperbola with equation: R C

a $y=\frac{k}{x}$ **b** $y=\frac{k}{x}+c$ **c** $y=\frac{k}{x-b}$ **d** $y=\frac{k}{x-b}+c$

Investigation

Graphing $y = 2^x$

This activity can be completed using graphing technology.

1 Copy and complete this table of values for $y = 2^x$.

x	−3	−2	−1	0	1	2	3	4
y								

2 Graph the points from the table and join them with a smooth curve. The equation $y = 2^x$ is called an **exponential equation** and its graph is called an **exponential curve** (exponent means 'power').

3 Graph $y = 2^{-x}$ in a similar way.

4 Compare the graphs of $y = 2^x$ and $y = 2^{-x}$. Describe any similarities and differences.

5 The y-intercept of any graph with equation $y = a^x$ (where a is a positive constant) is always 1. Explain why.

6 The graph of $y = 2^x$ is increasing. Is the graph of $y = 2^{-x}$ increasing or decreasing? Give reasons.

7 Describe what happens to the graph of $y = 2^x$ when:

a x approaches a large positive number

b x approaches a large negative number.

Technology

STAGE 5.3

Exponential curves

Use graphing technology to complete this activity.

1 Graph $y = 2^x$ using the software. Adjust the colour or line thickness if you need to.

2 Now graph these exponential curves, using different colours.

$y = 2^{-x}$, $y = -2^x$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$

3 **a** Which graphs are similar?

b Identify any features such as y-intercepts.

c Which graphs are similar as:

i x becomes larger? **ii** x becomes smaller?

4 Repeat steps **1** to **3** for these exponential curves:

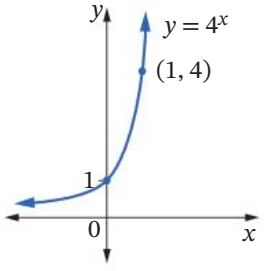
$y = 3^x$, $y = 3^{-x}$, $y = -3^x$, $y = -3^{-x}$, $y = 3^x + 1$, $y = 3^x - 1$

The exponential curve $y = a^x$ 8.12

An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, is called an **exponential equation**, for example, $y = 5^x$, $y = 2^x$ and $y = 3^x$. The graph of a quadratic equation is called an **exponential curve**.

The table of values and graph of $y = 4^x$ is shown.

x	−2	−1	0	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64	256



- The y-intercept of $y = a^x$ is always 1 since $a^0 = 1$.
- As x increases (to the right, in the positive direction), a^x becomes very large. Graphically, this means that the graph of $y = a^x$ increases sharply with a steep positive gradient.
- As x decreases (to the left, in the negative direction), a^x approaches 0. This means that the graph of $y = a^x$ flattens out and approaches the x-axis as x becomes a large negative number. The x-axis is an **asymptote** because the curve approaches it but never touches it.
- The exponential curve is always above the x-axis because the value of a^x is always positive.

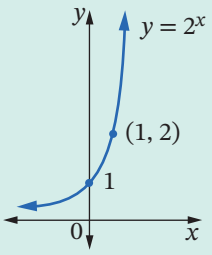
Example 20

Sketch each exponential equation and mark the y-intercept on each curve.

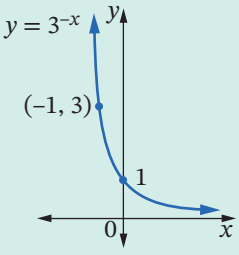
a $y = 2^x$ **b** $y = 3^{-x}$

Solution

a
- The y-intercept of $y = 2^x$ is 1
- When $x = 1$, $y = 2$, so (1, 2) is a point on the curve
- As x increases (to the right in the positive direction), 2^x becomes very large (steep positive gradient)
- As x decreases (to the left in the negative direction), 2^x approaches 0.
- The x-axis is an asymptote.



b
- The y-intercept of $y = 3^{-x}$ is 1
- When $x = -1$, $y = 3$, so (–1, 3) is a point on the curve
- As x decreases (to the left in the negative direction), 3^{-x} becomes very large (steep negative gradient)
- As x increases (to the right in the positive direction), 3^{-x} approaches 0.
- The x-axis is an asymptote.



Note that the graph of $y = 3^{-x}$ (and of $y = a^{-x}$ in general) is decreasing, and is actually a reflection of $y = 3^x$ in the y-axis.

EXERCISE 8.12 ANSWERS ON P. 622

The exponential curve U F R C

Some of this exercise may be completed using graphing technology.

EXAMPLE 20

1 **a** Graph each exponential equation on the same axes. R C

i $y = 2^x$ **ii** $y = 3^x$ **iii** $y = 5^x$

b What is the y-intercept of each curve?

c Describe what happens to the graph of $y = a^x$ as a increases.

2 **a** Graph $y = 4^x$ and $y = 4^{-x}$ on the same axes.

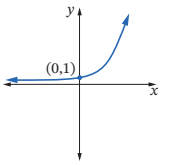
b Copy and complete:

i The reflection of $y = 4^x$ in the y-axis is ...

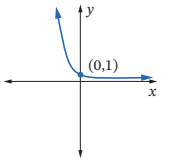
ii The reflection of $y = a^x$ in the y-axis is ...

3 Which graph represents $y = 2^{-x}$? Select the correct answer **A**, **B**, **C** or **D**. R C

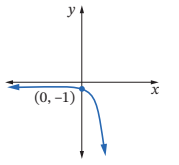
A



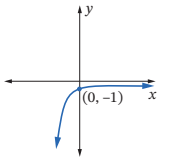
B



C



D



4 **a** Graph $y = 2^x$ and $y = -2^x$ on the same axes.

b How are the 2 graphs related? R C

c Copy and complete: The reflection of $y = a^x$ in the x-axis is ...

Foundation Standard Complex

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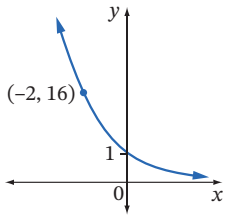
5 Graph $y = 3^x + 1$ and $y = 3^x - 1$ on the same axes and describe how they are related. R C

6 Sketch each exponential curve, showing the y-intercept.

a $y = 2^x$ **b** $y = 3^{-x}$ **c** $y = -4^x$

d $y = -2^{-x}$ **e** $y = 4^x + 1$ **f** $y = 4^x - 1$

7 Find an exponential equation for this graph. R C



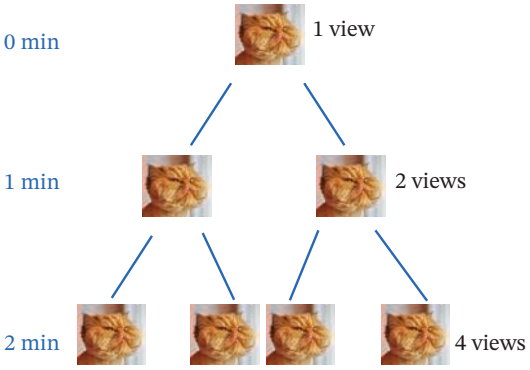
Investigation

Videos going viral

Use a spreadsheet to help you with this investigation.

With the invention of social media and social networks, we can now communicate with each other instantly. However, what often starts off as a simple message between friends or an online video post can quickly multiply at an alarming rate, be seen by the general public worldwide and 'go viral' (spread like a virus).

Suppose a video of an angry cat is shared with a friend who one minute later shares it with 2 other friends. This occurs every minute so that every minute the number of *new* views doubles. This is an example of something growing exponentially, according to the formula $y = 2^x$, where x represents number of minutes.



Shutterstock.com/Zanna Pesnina

1 Use a spreadsheet to calculate the number of new views after:

a 15 min **b** 30 min **c** 1 hour **d** 2 hours

e 4 hours **f** 10 hours **g** 20 hours **h** 30 hours

2 Find how long it will take until the total number of new views reaches:

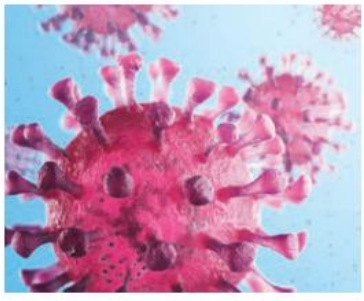
a 64 **b** 256 **c** over 500 **d** over 1000

e over 3000 **f** over 10 000 **g** over 1 million **h** over 4 million

3 Use the spreadsheet's **Graph Option** to graph the viral pattern. Describe the shape of the graph.

Did you know?

Exponential growth



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When an increase can be described using an exponential equation, it is called exponential growth. Examples include the growth of a virus, population (people or animals) and compound interest investments.

Population growth is monitored in different countries through the fertility (birth) and mortality (death) rates as well as migration. The data collected for these figures can often be modelled as an exponential function. By modelling the changes in population, predictions of future changes in population can be simulated and towns and cities can prepare for possible expansion in the numbers of schools, hospitals, housing and other necessary infrastructure.

At what rate is the population of Australia growing? What about the world's population?

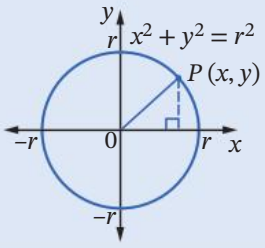
8.13 The circle $(x - a)^2 + (y - b)^2 = r^2$

Circle equations

The equation of a circle, centre (0, 0)

The equation of a circle with centre (0, 0) and radius r units is

$$x^2 + y^2 = r^2$$



Example 21

Find the equation of a circle with centre (0, 0) and diameter 14 units.

Solution

Radius $= \frac{1}{2} \times 14 = 7$ units. The equation of the circle is

$$x^2 + y^2 = 7^2$$

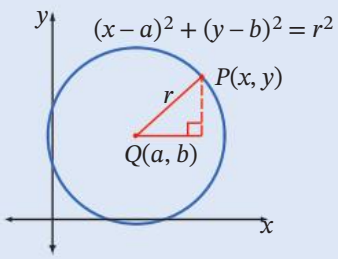
$$x^2 + y^2 = 49$$

If the centre of the circle is shifted from (0, 0) to new coordinates (a, b), then the equation of the circle changes from $x^2 + y^2 = r^2$ to $(x - a)^2 + (y - b)^2 = r^2$.

The equation of a circle, centre (a, b)

The equation of a circle with centre (a, b) and radius r units is

$$(x-a)^2+(y-b)^2=r^2.$$



Proof

Let $P(x, y)$ be any point on the circle and $Q(a, b)$ be its centre so that the distance $PQ = r$ units.
Using the distance formula:

$$d=\sqrt{(x-a)^2+(y-b)^2}=r$$
$$(x-a)^2+(y-b)^2=r^2$$

Example 22

Find the centre and radius of the circle with equation:

a $(x-2)^2+(y-5)^2=9$ **b** $(x+3)^2+(y+1)^2=16$

c $x^2+(y-6)^2=1$ **d** $(x+1)^2+(y-4)^2=40$

Solution

a Centre is $(2, 5)$, radius $=\sqrt{9}=3$ units

b Centre is $(-3, -1)$, radius $=\sqrt{16}=4$ units

c Centre is $(0, 6)$, radius $=\sqrt{1}=1$ unit

d Centre is $(-1, 4)$, radius $=\sqrt{40}=2\sqrt{10}$ units

Completing the square

$(x-2)^2+(y-5)^2=9$ can be expanded to become $x^2-4x+y^2-10y-20=0$

This is called the **general form of the equation of a circle.**

To find the centre and radius of a circle written in the general form, we must 'complete the square' on both x and y to arrange it in the form $(x-a)^2+(y-b)^2=r^2$.

The method of completing the square depends on these formulas for **perfect squares** studied in Chapter 5, *Products and factors*:

$$(x+a)^2=x^2+2ax+a^2$$
$$(x-a)^2=x^2-2ax+a^2$$

Note that the last term, a^2, is the square of 'half the coefficient of x'.

STAGE 5.3

Example 23

Find the numbers that complete the square in each equation.

a $x^2 + 10x + ___ = (x + ___)^2$

b $x^2 - 14x + ___ = (x - ___)^2$

Solution

a The coefficient of x is 10.

$\frac{1}{2}$ of 10 is 5, and $5^2 = 25$.

$\therefore$ The perfect square is $x^2 + 10x + 25 = (x + 5)^2$

b The coefficient of x is -14.

$\frac{1}{2}$ of -14 is -7, and $(-7)^2 = 49$.

$\therefore$ The perfect square is $x^2 - 14x + 49 = (x - 7)^2$

Example 24

Graph the circle with equation $x^2 + 4x + y^2 - 6y + 4 = 0$.

Solution

We need to rewrite the general equation into the form $(x - a)^2 + (y - b)^2 = r^2$.

$$x^2 + 4x + y^2 - 6y + 4 = 0$$

$$x^2 + 4x + y^2 - 6y = -4$$

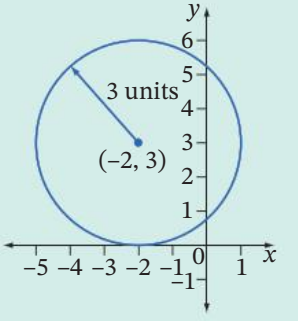
Moving the constant, 4, to the RHS

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -4 + 4 + 9$$

Complete the square on $x^2 + 4x$ and on $y^2 - 6y$ and adding the same values to the RHS.

$$(x + 2)^2 + (y - 3)^2 = 9$$

This is the equation of a circle with centre $(-2, 3)$ and radius $\sqrt{9} = 3$ units.



EXERCISE 8.13 ANSWERS ON P. 623

The circle $(x - a)^2 + (y - b)^2 = r^2$ UFRC

Some of this exercise may also be completed using graphing technology.

EXAMPLE 21

1 Find the centre and radius of the circle with equation:

a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 64$

d $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 81$ **f** $2x^2 + 2y^2 = 50$

2 What is the equation of a circle with centre (0, 0) and radius 3 units? Select the correct answer **A**, **B**, **C** or **D**.

A $x^2 + y^2 = -9$ **B** $x^2 + y^2 = 3$ **C** $x^2 + y^2 = -3$ **D** $x^2 + y^2 = 9$

3 **a** Show that the point (8, 6) lies on the circle $x^2 + y^2 = 100$. R

b Show that the point (5, 9) does not lie on the circle $x^2 + y^2 = 100$.

c Does (5, 9) lie inside or outside this circle?

4 Given the equation of the circle $x^2 + y^2 = 4$, substitute each of the following points into the equation and determine whether the points are *inside*, *on* or *outside* the circle: R

a (0, 0) **b** (2, 0) **c** (3, 1) **d** (1, 1) **e** (−5, 2)

EXAMPLE 22

5 Find the centre and radius of the circle with equation:

a $(x + 2)^2 + (y - 4)^2 = 49$ **b** $(x - 3)^2 + (y - 1)^2 = 1$

c $(x - 9)^2 + (y - 12)^2 = 225$ **d** $x^2 + (y + 3)^2 = 4$

e $(x + 6)^2 + (y + 1)^2 = 10$ **f** $(x + 5)^2 + (y + 8)^2 = 16$

g $x^2 + y^2 = 72$ **h** $(x + 2)^2 + (y - 1)^2 = 50$

i $(x - 2)^2 + y^2 = 64$ **j** $(x - 4)^2 + (y + 3)^2 = \frac{25}{4}$

k $(x - 3)^2 + (y - 4)^2 = 81$ **l** $3x^2 + 3(y + 1)^2 = \frac{1}{3}$

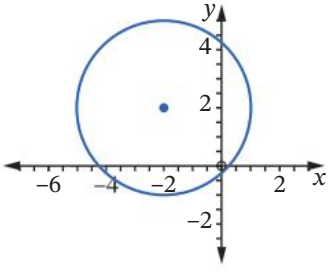
6 Find the equation of this circle. Select **A**, **B**, **C** or **D**.

A $(x - 2)^2 + (y + 2)^2 = 3$

B $(x + 2)^2 + (y - 2)^2 = 9$

C $(x + 2)^2 + (y - 2)^2 = 4$

D $(x - 2)^2 + (y + 2)^2 = 16$



7 Find the equation of the circle whose centre and radius are respectively:

a (1, −2), 3 **b** (10, −11), 2 **c** (−3, 2), 10

d (0, −1), 1 **e** (−6, 2), $\sqrt{5}$ **f** (−1, 5), $2\sqrt{2}$

8 Graph each circle.

a $(x + 1)^2 + (y + 1)^2 = 1$ **b** $x^2 + (y - 4)^2 = 25$

c $(x - 1)^2 + y^2 = 16$ **d** $(x + 5)^2 + (y + 2)^2 = 4$

STAGE 5.3

EXAMPLE 23

9 Find the numbers that complete the square in each equation. R C

a $x^2 + 2x + _____ = (x + _____)^2$ **b** $p^2 - 6p + _____ = (p - _____)^2$

c $m^2 - 8m + _____ = (m - _____)^2$ **d** $k^2 + 4k + _____ = (k + _____)^2$

e $y^2 - 7y + _____ = (y - _____)^2$ **f** $w^2 - 3w + _____ = (w - _____)^2$

g $x^2 + x + _____ = (x + _____)^2$ **h** $h^2 - 5h + _____ = (h - _____)^2$

EXAMPLE 24

10 Find the centre and radius of the circle given with equation: R C

a $x^2 + 6x + y^2 - 2y = 15$ **b** $x^2 - 8x + y^2 - 4y = 29$

c $x^2 + 4x + y^2 - 10y = 7$ **d** $x^2 + 20x + y^2 - 12y + 135 = 0$

e $x^2 + y^2 = 4x - 8y + 5$ **f** $x^2 - 12x + y^2 + 6y + 29 = 0$

g $x^2 + y^2 = 20y - 6x - 28$ **h** $h^2 + y^2 + 5 = 8x - 2y$

8.14 Identifying graphs

Matching graphs

Matching graphs (Advanced)

STAGE 5.3

Graphs

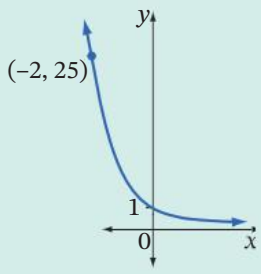
Straight line:	$y = mx + c,\ ax + by + c = 0$
Parabola:	$y = ax^2 + c$
Exponential curve:	$y = a^x$
Circle:	$x^2 + y^2 = r^2$
Parabola:	$y = a(x - b)^2 + c$
Cubic curve:	$y = kx^3 + c,\ y = k(x - a)(x - b)(x - c)$
Power curves:	$y = kx^n + c, y = k(x - b)^n$
Hyperbola:	$y = \frac{k}{x}, y = \frac{k}{x} + c, y = \frac{k}{x - b}$
Circle:	$(x - a)^2 + (y - b)^2 = r^2$

When matching graphs with equations, the x value and y value of a point on the graph may need to be substituted into the equation to verify that the equation represents the graph.

Example 25

Match each graph to its equation.

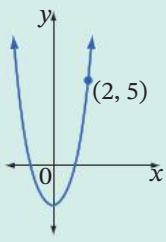
a



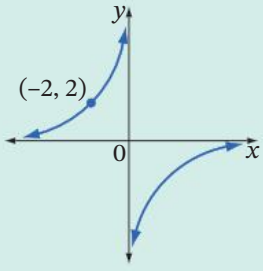
b



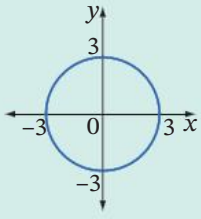
c



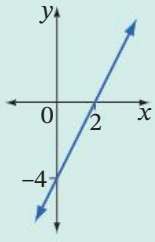
d



e



f



A $y = 2x - 4$ **B** $x^2 + y^2 = 9$ **C** $y = -\frac{4}{x}$

D $y = 2x^2 - 3$ **E** $y = 5^{-x}$ **F** $y = -x^3 + 8$

Solution

a An exponential curve that matches with E, $y = 5^{-x}$

Test point: (–2, 25)

LHS = 25

RHS = $5^{-(-2)} = 5^2 = 25$ = LHS

b A decreasing cubic curve with y-intercept 8 that matches with F, $y = -x^3 + 8$

Test point: (2, 0)

LHS = 0

RHS = $-2^3 + 8 = -8 + 8 = 0$ = LHS

c A concave-up parabola that matches with D, $y = 2x^2 - 3$

Test point: (2, 5)

LHS = 5

RHS = $2 \times 2^2 - 3 = 5$ = LHS

d A 'negative' hyperbola that matches with C, $y = -\frac{4}{x}$

Test point: (–2, 2)

LHS = 2

RHS = $y = -\frac{4}{-2} = 2$ = LHS

e A circle with centre (0, 0) and radius 3 that matches with B, $x^2 + y^2 = 9$

f A straight line with gradient 2 and y-intercept –4 that matches with A, $y = 2x - 4$

STAGE 5.3

STAGE 5.3

EXERCISE 8.14 ANSWERS ON P. 624

Identifying graphs U F R C

1 For each equation state whether its graph is a straight line (L), a parabola (P), an exponential curve (E) or a circle (C). R C

a $y = 9x^2 - 4$ **b** $y = 9x$ **c** $y = 9^x$ **d** $y = 9$

e $x^2 + y^2 = 81$ **f** $y = 3x - 8$ **g** $y = 3x^2 - 8$ **h** $y = 2x + 5$

i $y = -x^2 + 6$ **j** $y = 10^{-x}$ **k** $y = 7x^2 + 2$ **l** $x^2 + y^2 = 36$

EXAMPLE 25

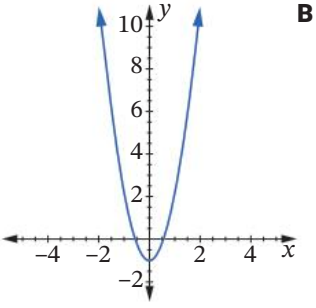
2 Match each equation with its graph. R C

a $x = 4$ **b** $y = -\frac{1}{2}x + 1$ **c** $y = 1 - x^2$ **d** $y = 5$

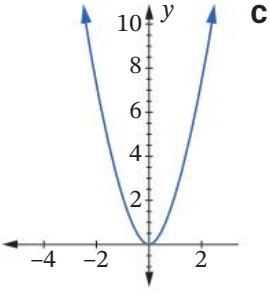
e $y = 3x^2 - 1$ **f** $y = 3^x$ **g** $x^2 + y^2 = 9$ **h** $y = 3^{-x}$

i $y = 2x^2$ **j** $y = 9x^2 - 4$

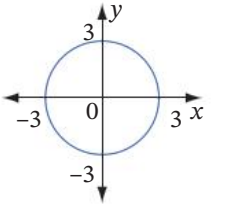
A



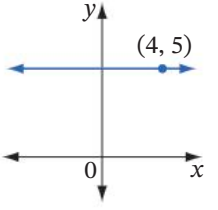
B



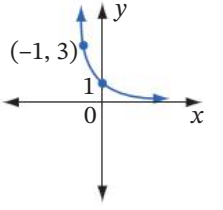
C



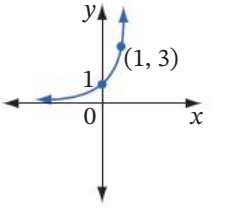
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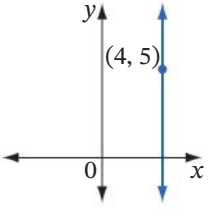
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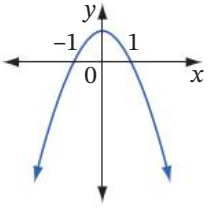
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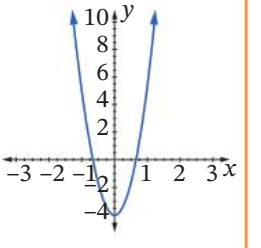
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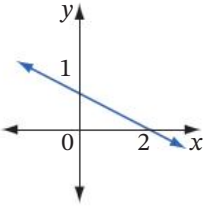
H



I



J



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3 Sketch each equation, showing a point on the curve.

a $y = x^2 - 3$ **b** $y = 5^x$ **c** $y = -x^2 + 4$ **d** $x^2 + y^2 = 49$

e $y = \frac{1}{2}x^2$ **f** $y = -2x + 4$ **g** $x^2 + y^2 = 144$

4 Find the y-intercept of the graph of each equation.

a $y = 3^x$ **b** $y = 2x^2 + 3$ **c** $y = -7x^2 - 6$ **d** $y = 5^{-x}$

5 For each equation, state whether its graph is a parabola (P), a cubic (Q), a hyperbola (H), an exponential (E) or a circle (C). R C

a $y = \frac{9}{x}$ **b** $(x-2)^2 + y^2 = 4$ **c** $y = 2(x-2)^2$

d $y = \frac{-2}{x-4}$ **e** $y = 2x(x+4)(x-1)$ **f** $(x+4)^2 + (y-1)^2 = 15$

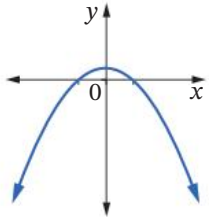
g $y = 4^x + 2$ **h** $y = -\frac{8}{x} - 1$

6 Match each equation with its graph. R C

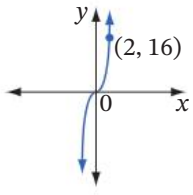
a $y = -\frac{4}{x}$ **b** $y = (x-3)^3$ **c** $y = -x^3 + 1$ **d** $y = 1 - x^2$

e $y = \frac{1}{2}x^3 - 1$ **f** $y = 2x^3$ **g** $y = \frac{4}{x}$ **h** $y = (x+4)^2$

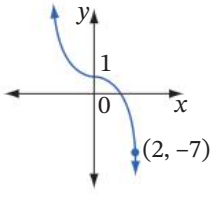
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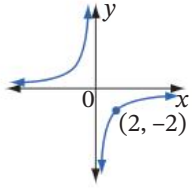
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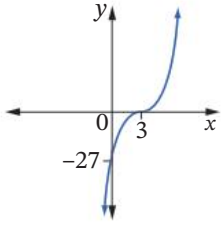
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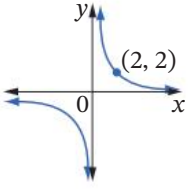
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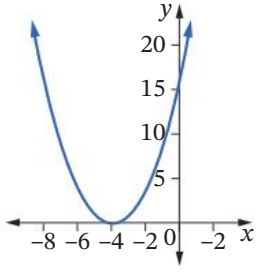
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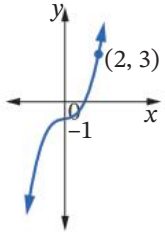
F



G



H



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STAGE 5.3

7 For each equation: R C

i write the name of its graph

ii find its y-intercept

iii find the equation of its asymptote(s)

a $y = 2^x$ **b** $y = 5^x + 1$ **c** $y = 10^x - 3$

d $y = \frac{4}{x}$ **e** $y = \frac{2}{x-3}$ **f** $y = -\frac{1}{x} + 2$

8 Sketch the graph of each equation, showing all main features. R C

a $y = \frac{6}{x}$ **b** $y = 3^x + 2$

c $y = x^3 + 3$ **d** $y = 2(x-5)^2$

e $y = -\frac{1}{x+4}$ **f** $(x+5)^2 + (y-5)^2 = 4$

g $y = -(x+2)(x-3)(x+4)$



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Power plus ANSWERS ON P. 625

1 Graph the equation $y = \frac{1}{x-1} + 2$, showing all intercepts and asymptotes.

2 Sketch the graph of each equation and find the centre and radius of the semicircle.

a $y = \sqrt{16 - x^2}$

b $y = \sqrt{25 - x^2}$

c $y = -\sqrt{9 - x^2}$

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CHAPTER 8 REVIEW

Language of maths

asymptote	axis	centre	circle
coefficient	concave down	concave up	constant
conversion graph	curve	direct proportion	exponential
inverse proportion	parabola	quadratic	radius
table of values	turning point	variable	vertex
x-intercept	y-intercept		

1 What is the coefficient of x^2 in the quadratic equation $y = 3x^2 + 10$?

2 What is the graph of a **quadratic equation** called?

3 True or false: The exponential curve $y = 2^x$ passes through the point (0, 0).

4 In the variation equation $y = \frac{k}{x}$, which is the **constant of proportionality**?

5 Write down the equation of a parabola that is concave down and has a y-intercept of 3.

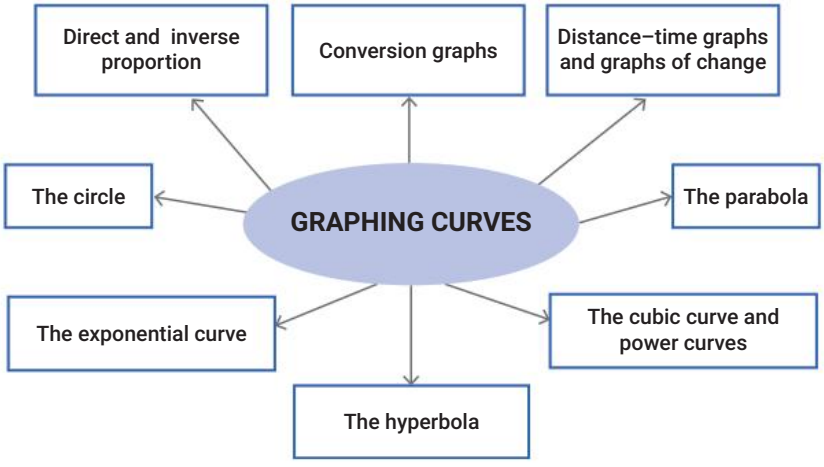
6 What is the asymptote of the exponential curve $y = a^x$?

Topic summary

- Which parts of this chapter were new to you?
- What is the difference between direct and inverse proportion?
- Do you know the equations of a parabola, exponential curve and circle, and how to graph them?
- Explain how the graph of $y = 2x^2 + 3$ is different from the graph of $y = -2x^2 + 3$. How are they similar?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS Mind map: Graphing curves (Advanced)



TEST YOURSELF 8 ANSWERS ON P. 625

8.01

1 H is directly proportional to t. If when $t = 12$, $H = 138$, find H when $t = 27$.

STAGE 5.3

8.02

2 The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 400 m above sea level, the temperature is 15°C. What is the temperature at 600 m above sea level?

8.03

3 The graph in Example **4** on page 294 converts Australian dollars to UK pounds sterling. Use the graph to convert:

a \$A70 to £ **b** £56 to Australian dollars

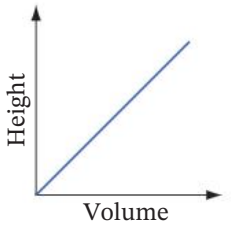
STAGE 5.3

8.04

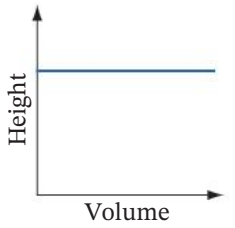
4 Draw a distance–time graph to show a person leaving home at an increasing speed and then slowing down.

5 The graphs below show the height of water in a bathtub over time. Match each graph to the correct description.

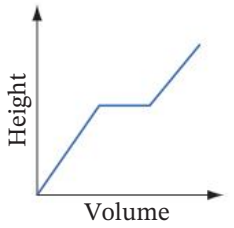
a



b



c



A The bath is almost full of water.

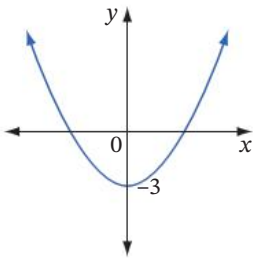
B Hot water is accidentally run into the bath, so the tap is turned off to let it cool. Then it is filled with cold water only.

C The bath is filled at a steady rate.

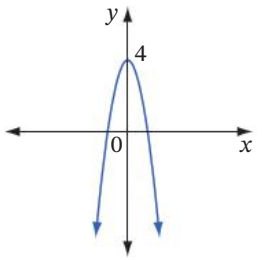
8.06

6 Match each graph to its correct equation.

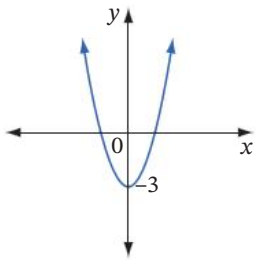
a



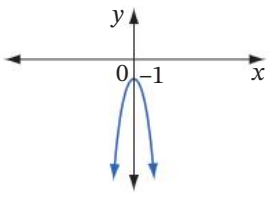
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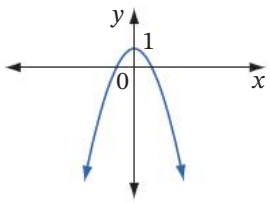
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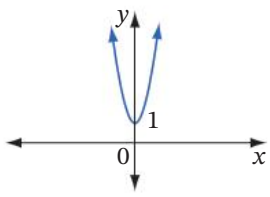
d



e



f



A $y = x^2 - 3$ **B** $y = 3x^2 + 1$ **C** $y = \frac{1}{2}x^2 - 3$

D $y = -x^2 + 1$ **E** $y = -4x^2 - 1$ **F** $y = 4 - 3x^2$

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7 Graph the equation $y = 2(x + 3)^2$, showing all main features.

8 Graph the equation $y = x^3 + 2$, showing all main features.

9 Graph the equation $y = 2x(x + 6)(x - 2)$, showing all main features.

10 Sketch the graphs of the equations $y = -3x^4$ and $y = -3(x - 2)^4$, showing all main features.

11 Sketch the graph of $y = \frac{2}{x+1}$, showing the x- and y-intercepts and a point on the curve.

12 Sketch each curve described below.

a $y = 4^x$ **b** $y = 4^{-x}$ **c** $y = -4^x$ **d** $y = -4^{-x}$

13 Find the centre and radius of each circle described below.

a $x^2 + y^2 = 100$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 49$

d $(x + 5)^2 + (y - 6)^2 = 81$ **e** $(x - 10)^2 + y^2 = 225$ **f** $(x + 7)^2 + (y + 10)^2 = 80$

14 Match each equation with its correct graph.

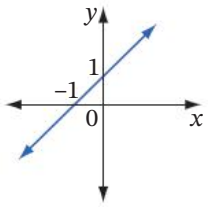
a $y = \frac{1}{4}x^2$ **b** $y = 3^x$ **c** $y = -2x^2 - 1$

d $x = -5$ **e** $y = -3x^2$ **f** $y = 2x^2 - 1$

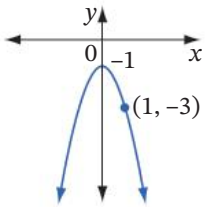
g $y = 3^{-x}$ **h** $x^2 + y^2 = 25$ **i** $y = x^2$

j $y = x + 1$ **k** $y = -5$ **l** $y = -2 - 2x$

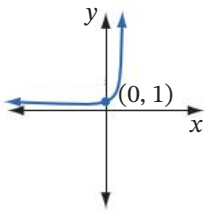
A



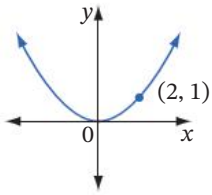
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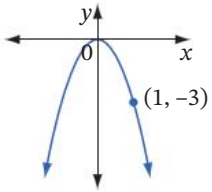
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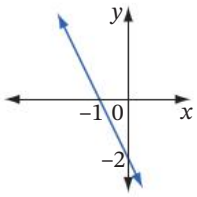
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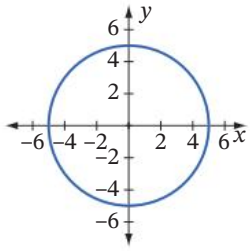
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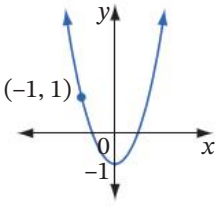
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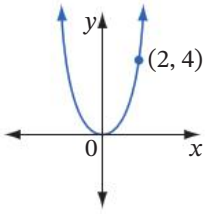
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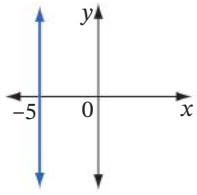
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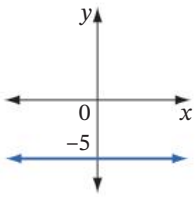
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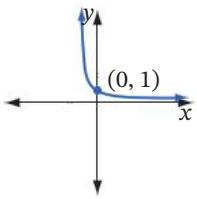
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9

MEASUREMENT AND GEOMETRY

FURTHER TRIGONOMETRY

Trigonometry is a branch of mathematics that uses the relationship between the angles and sides of triangles. Trigonometry is used in navigation when locating the position or bearing of a destination from a known location, and finding the distances between places that cannot be physically measured.

Trigonometry helps us understand physical phenomena that are periodic or cyclic, such as tidal movements, phases of the moon, average monthly temperatures and sound waves. It is also used in construction, engineering, physics and gaming.



Chapter outline

		Working mathematically				
9.01	Right-angled trigonometry	U	F	PS		
9.02	Bearings	U	F	PS	R	C
9.03	Pythagoras' theorem and trigonometry in 3D*	U	F	PS	R	C
9.04	Complementary relations and exact ratios*#	U	F		R	
9.05	The trigonometric functions*	U	F		R	C
9.06	Trigonometric equations*	U	F			
9.07	The sine rule*	U	F	PS	R	
9.08	The sine rule for angles*	U	F	PS	R	C
9.09	The cosine rule*	U	F	PS	R	C
9.10	The cosine rule for angles*	U	F	PS	R	
9.11	The area of a triangle*	U	F	PS	R	
9.12	Problems involving the sine and cosine rules*	U	F	PS	R	C

***STAGE 5.3**

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

bearing The angle used to show the direction of one location from a given point

complementary angles 2 angles that add to 90°

cosine rule A rule that relates the 3 sides and one of the angles of any triangle: $c^2 = a^2 + b^2 - 2ab\cos C$

exact ratio The sine, cosine and tangent of the special angles 30°, 45° and 60°, which can be expressed as exact fractions or surds rather than decimal approximations

included angle The angle between 2 known sides

quadrant A quarter of the number plane with the x- and y-axes as borders

sine rule A rule that relates the sides of any triangle to the sine of their opposite angles: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

unit circle A circle on the number plane with centre (0, 0) and radius 1

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- use trigonometry to solve problems involving right-angled triangles
- solve trigonometry problems involving angles of elevation and depression, and bearings
- (STAGE 5.3) use Pythagoras' theorem and trigonometry to solve three-dimensional problems
- (STAGE 5.3) learn the complementary relations and the exact trigonometric ratios for 30°, 45° and 60°
- (STAGE 5.3) use the unit circle to define and graph trigonometric functions
- (STAGE 5.3) solve trigonometric equations
- (STAGE 5.3) use the sine and cosine rules to find an unknown side or angle in any triangle
- (STAGE 5.3) use the formula $A = \frac{1}{2}ab\sin C$ to find the area of a triangle with side lengths a, b and included angle C

The trigonometric ratios

Trigonometry

Trigonometric calculations

Investigating trigonometry ratios

Trigonometry match-up

Trigonometry equations 1

Trigonometry equations 2

Finding an unknown angle

SkillCheck ANSWERS ON P. 626

1 Round each angle to the nearest degree.

a 64° 27′ **b** 25° 43′ **c** 12° 8′ 50″ **d** 85.85°

2 Evaluate each expression correct to 4 decimal places.

a cos 32° **b** sin 50.9° **c** tan 8° 45′ **d** 200 tan 18°

e 14 sin 87° 40′ **f** $\frac{13}{\cos 18° 27′}$ **g** $\frac{\sin 72° 43′}{28.4}$ **h** 5.9 tan 56.2°

3 Convert each angle to degrees and minutes, correct to the nearest minute.

a 45.80° **b** 33.1750° **c** 5.3460° **d** 84.375°

4 Find the size of angle A, correct to the nearest minute.

a $\cos A = \frac{3}{7}$ **b** $\tan A = 2.7$ **c** $\sin A = 0.4716$

9.01 Right-angled trigonometry

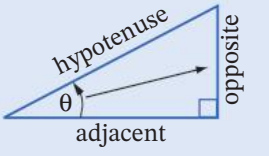
Finding an unknown side

Finding the hypotenuse

Finding an unknown angle

The trigonometric ratios

Ratio	Abbreviation	Meaning
sine	sin	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
cosine	cos	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
tangent	tan	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

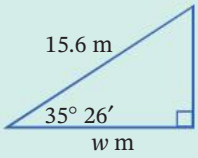


 9780170453509

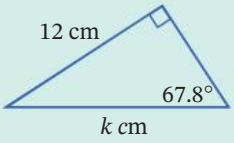
Example 1

Find the value of each variable, correct to one decimal place.

a



b



Solution

a $\cos 35° 26' = \frac{w}{15.6}$

$w = 15.6 \cos 35° 26'$

$= 12.710...$

≈ 12.7 m

b $\sin 67.8° = \frac{12}{k}$

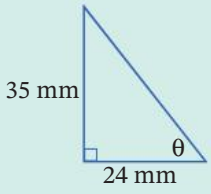
$k = \frac{12}{\sin 67.8°}$

$= 12.960...$

≈ 13.0 cm

Example 2

Find the value of θ, correct to the nearest minute.



Solution

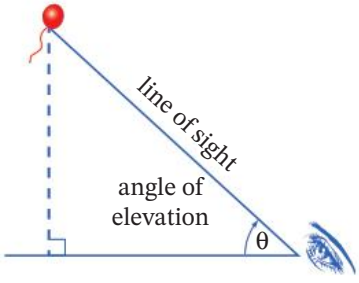
$\tan \theta = \frac{35}{24}$ 35 is opposite and 24 mm is the adjacent side

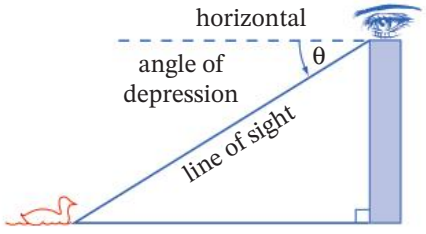
$\theta = 55.561...$ SHIFT tan 35 ÷ 24 =

$= 55° 33' 39.64''$ Press °′″ or DMS

$\approx 55° 34'$ From the diagram, 55° 34′ seems reasonable.

Angles of elevation and depression

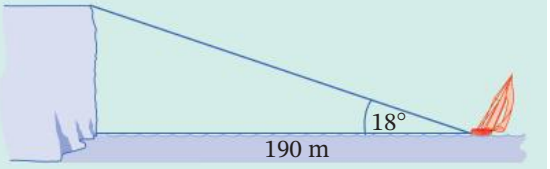




Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

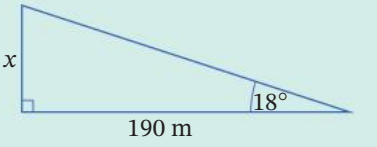
Angles of elevation and depression

Example 3



The angle of elevation from a yacht to the top of a cliff is 18°. If the yacht is 190 m from the base of the cliff, find correct to one decimal place the height of the cliff.

Solution



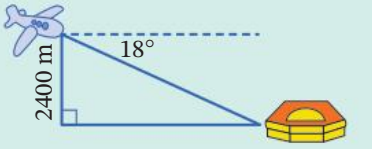
Let the height be x metres.

$$\tan 18° = \frac{x}{190}$$
$$x = 190 \tan 18°$$
$$= 61.73474...$$
$$\approx 61.7 \text{ m}$$

The height of the cliff is 61.7 m.

Angle of depression 2

Example 4

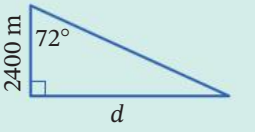


A pilot flying at a height of 2400 m saw a life raft in the sea at an angle of depression of 18°. Calculate the horizontal distance from the plane to the life raft, correct to the nearest metre.

Solution

Angle of depression 1

Let d be the horizontal distance from the plane to the life raft.



The angle in the triangle adjacent to the angle of depression = 90° − 18° = 72°.

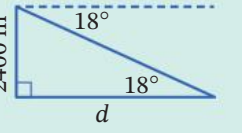
Angles of elevation and depression

$$\tan 72° = \frac{d}{2400}$$
$$d = 2400 \tan 72°$$
$$= 7386.4404...$$
$$\approx 7386 \text{ m}$$

The horizontal distance from the plane to the life raft is 7386 m.

Alternative method

By alternate angles, the angle of elevation of the plane from the life raft is also 18°.



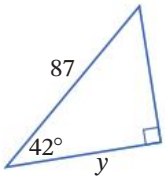
$$\tan 18° = \frac{2400}{d}$$
$$d = \frac{2400}{\tan 18°}$$
$$= 7386.4404...$$
$$\approx 7386 \text{ m}$$

EXERCISE 9.01 ANSWERS ON P. 626

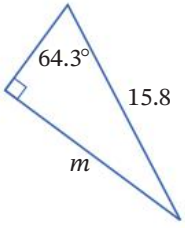
Right-angled trigonometry U F PS

1 Calculate, correct to one decimal place, the value of each variable. All measurements are in centimetres.

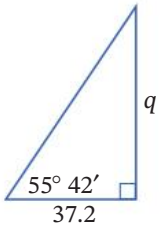
a



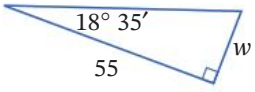
b



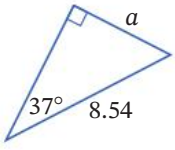
c



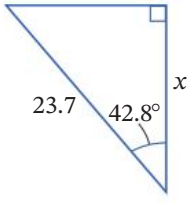
d



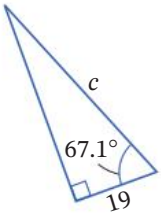
e



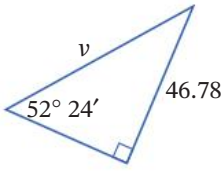
f



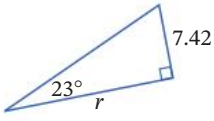
g



h



i



2 In $\triangle PQR$, $\angle P$ is a right angle, $\angle Q = 46° 12'$ and $PR = 8.7$ m. Find the length of QR.

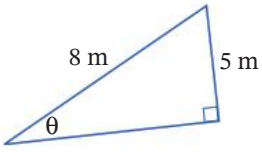
Select the correct answer **A**, **B**, **C** or **D**.

A 6.3 m **B** 6.0 m **C** 12.1 m **D** 8.3 m

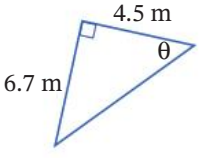
3 Find the value of θ, correct to the nearest minute.

EXAMPLE 2

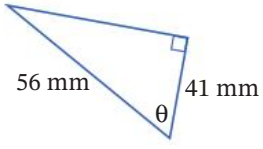
a



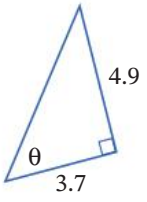
b



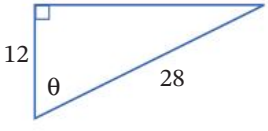
c



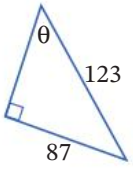
d



e



f



4 A 6-metre ladder rests against a wall. The foot of the ladder is 1.8 m from the base of the wall. Find: PS

a the angle (to the nearest degree) that the ladder makes with the ground

b the distance (correct to one decimal place) that the ladder reaches up the wall.

5 A walking trail falls 11 m for every 125 m travelled along the trail. At what angle is the trail inclined to the horizontal? Select **A**, **B**, **C** or **D**. PS

A 5° **B** 8.5° **C** 7° **D** 5.4°

6 Calculate the take-off angle (to the nearest minute) of a passenger plane if its altitude after 30 seconds is 600 m and it has flown a distance of 2500 m. PS

7 A kite attached to a string is flying at a height of 75 m. If the string makes an angle of 49° with the vertical, what is the length of the string, correct to the nearest metre? PS

8 The entrance to a bank has a ramp for wheelchair access that is 3.6 m long. If the ramp is inclined at an angle of 10° to the ground, what is the height of the entrance (to the nearest cm)? PS

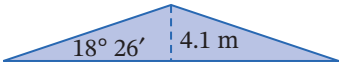
9 A section on a water slide has a vertical rise of 50 m and a horizontal run of 40 m. PS

a At what angle is the slide descending, correct to the nearest minute?

b What is the gradient of the section?

10 The training of a football team involves running up and down a sandhill 25 times. If the hill is 30 m high and inclined at an angle of 35° to the horizontal, how far does the team run during training? Answer correct to the nearest 0.1 km. PS

11 A roof has a pitch of 18° 26′. The height of the roof is 4.1 m. Calculate the overall width of the roof (correct to one decimal place). PS

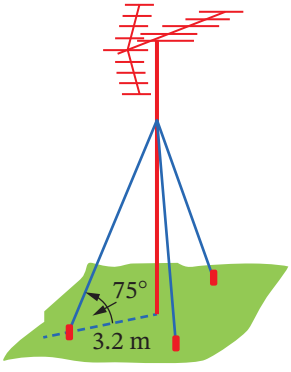


12 A glider is descending at an angle of 25° to the horizontal. The length of its flight path until it lands is 3.5 km. What was the altitude (to the nearest 0.1 km) of the glider? PS

13 A radio tower is supported by cables as shown. The cables make an angle of 75° with the ground and are fixed 3.2 m from the base of the tower. Calculate, correct to one decimal place: PS

a the length of each cable

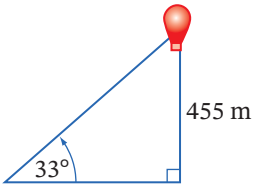
b how far up the tower each cable is fastened.



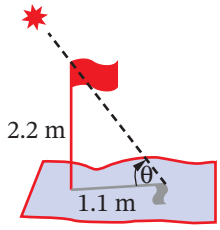
Foundation Standard Complex

9780170453509

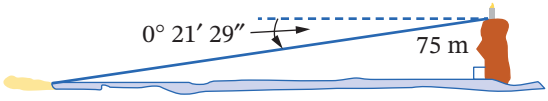
14 The angle of elevation of a hot air balloon at a height of 455 m is 33°. How far (to the nearest metre) has the balloon has drifted from its starting point? PS



15 A 2.2 m flag on a golf course casts a shadow 1.1 m long. Find the angle of elevation of the sun (to the nearest degree). PS



16 The angle of depression of Collingwood Beach from Point Perpendicular is 0° 21′ 29″. If the height of Point Perpendicular above sea level is 75 m, calculate the distance to the beach, correct to the nearest kilometre. PS



17 The angle of depression of a car from the top of a building is 41°. If the car is 110 m from the base of the building, find the height of the building (correct to one decimal place). PS

18 From a lookout, the angle of depression to the centre of a town is 5° 47′. The horizontal distance from the centre of town to the lookout is 1.72 km. If the elevation (height above sea level) of the town is 690 m, calculate the elevation of the lookout. PS

19 Emily is walking towards a 72 m tall building. The angle of elevation to the top of the building is first 22° and 5 minutes later, 68°. How far has Emily walked towards the building? PS

20 From the window of Joshua's apartment, the angles of elevation and depression to the top and bottom of another apartment building are 42° and 32° respectively. If the buildings are 85 m apart, how tall, correct to the nearest metre, is the building that Joshua is looking at? PS

9.02 Bearings

WS A page of bearings

WS NSW map bearings

WS Elevations and bearings

WS 16 points of the compass

WS Bearings match-up

Qz Trigonometry

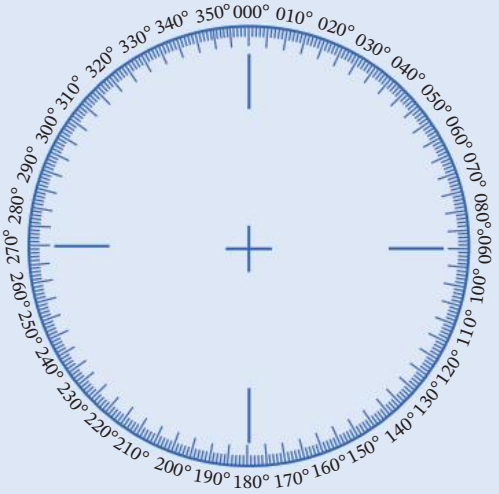
Trigonometry 2

True bearings

Bearings are used in navigation. A **bearing** is an angle measurement used to precisely describe the direction of one location from a given reference point.

Three-figure bearings

Three-figure bearings, also called **true bearings**, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°**. Note that the angles are always written with 3 digits.

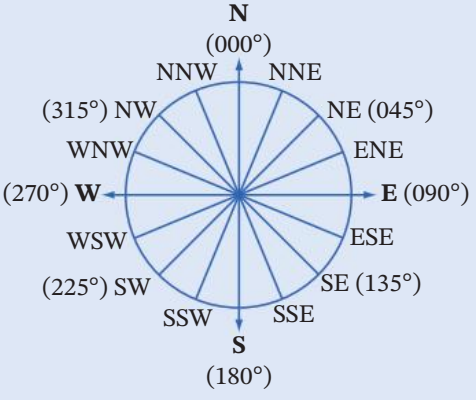


True bearings

Compass bearings

Compass bearings refer to the 16 points of a mariner's compass.

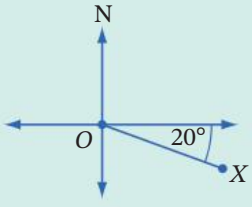
N = north	NE = northeast
E = east	SE = southeast
S = south	SW = southwest
W = west	NW = northwest
NNE = north-northeast	ENE = east-northeast
ESE = east-southeast	SSE = south-southeast
SSW = south-southwest	WSW = west-southwest
WNW = west-northwest	NNW = north-northwest



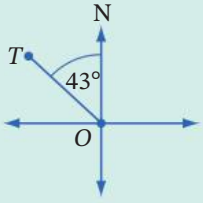
Example 5

Write the three-figure bearing of each point from O.

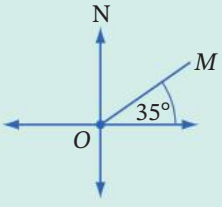
a



b



c



Solution

a Bearing of X from O is $90° + 20° = 110°$.

b Bearing of T from O is $360° - 43° = 317°$.

c Bearing of M from O is $90° - 35° = 055°$. Must be written as a 3-digit angle

Example 6

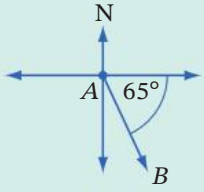
Sketch point B on a compass rose if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is measured from.

155° is between 90° and 180°, so B is in the southeast (SE) quadrant.

$155° - 90° = 65°$, so B is 65° from east (E).



Example 7

A plane leaves a town and remains on a bearing of 122° for 260 km.

a How far south of the town is the plane, correct to one decimal place?

b What is the bearing of the town from the plane?

Solution

a Let d km = distance south

$\angle SOP = 180° - 122°$ (angles on a straight line)

$= 58°$

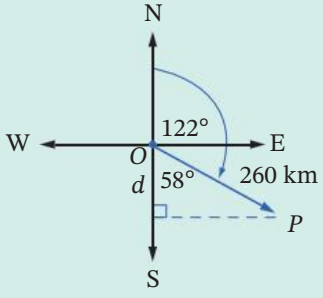
$\cos 58° = \frac{d}{260}$

$d = 260 \cos 58°$

$= 137.7790...$

≈ 137.8 km

The plane is 137.8 km south of the town.



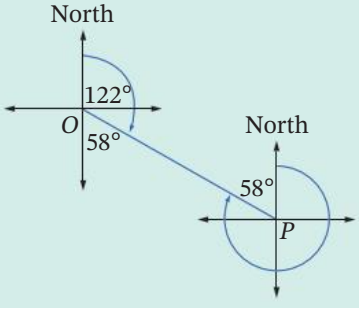
b Draw a compass rose with its centre at P.

$\angle OPN = 58°$ (alternate angles on parallel lines)

Bearing of O from $P = 360° - 58°$

$= 302°$

Bearing of the town from the plane = 302°



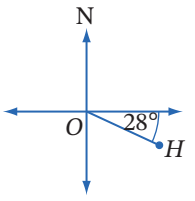
EXERCISE 9.02 ANSWERS ON P. 626

Bearings U F PS R C

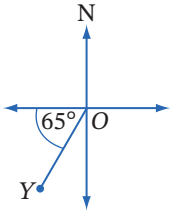
EXAMPLE 5

1 Write the bearing of each point from O. C

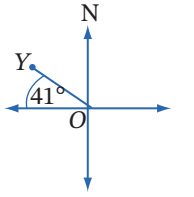
a



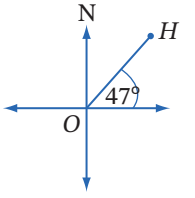
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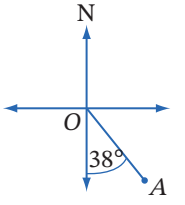
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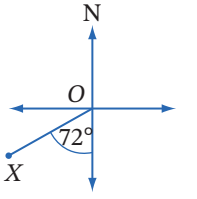
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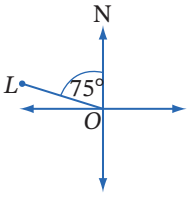
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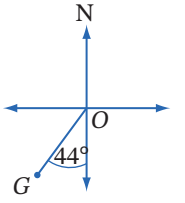
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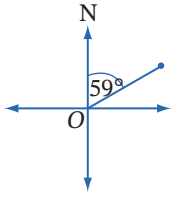
g



h

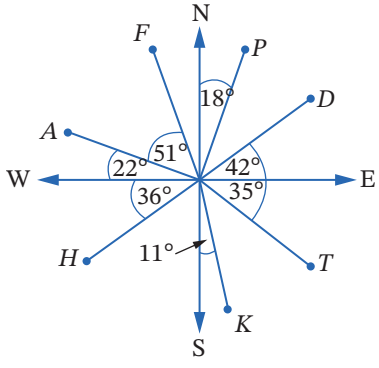


i



2 What is the bearing of each point from O? C

a	N	**b**	E	**c**	S
d	W	**e**	F	**f**	P
g	T	**h**	A	**i**	H
j	K	**k**	D		



3 **a** What is the compass direction halfway between east and southwest?

b What is the three-figure bearing of this compass direction? C

4 What is the three-figure bearing of the following compass bearings? C

a	SW	**b**	E	**c**	NE	**d**	ESE
e	WNW	**f**	W	**g**	SE	**h**	WSW

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5 Sketch P on a compass rose if P has a bearing of: C

a 212° from G **b** 114° from X **c** 082° from A **d** 324° from W

EXAMPLE 6

6 If the bearing of A from B is 140°, what is the bearing of B from A? R

7 The bearing of H from P is 100°. What is the bearing of P from H? R

8 What is the (smallest) angle between:

a NW and SW? **b** NE and SSW **c** E and WSW **d** SW and WNW

9.02

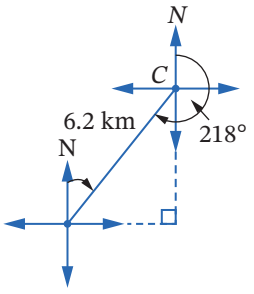
9 The compass bearing of H from M is WNW. Find the compass bearing of M from H. R

10 Annieke leaves a campsite (C) and rests after she has walked 6.2 km on a bearing of 218°. PS C

a How far south (correct to 2 decimal places) of the campsite did she stop to rest ?

b What is the bearing of the campsite from Annieke?

EXAMPLE 7



11 A yacht leaves Sydney and sails 780 km on a bearing of 071° to Lord Howe Island. PS C

a How far east (to the nearest km) of Sydney is Lord Howe Island?

b What is the bearing of Sydney from Lord Howe Island?

12 Sienna cycles due east for 18 km and then due south for 35 km. PS C

a How far is Sienna from her starting point (correct to the nearest km)?

b What is the bearing (correct to one decimal place) of the starting point from where Sienna finishes?

13 A plane flies from Sydney to Canberra on a bearing of 231.01°. If the distance from Sydney to Canberra is 247 km, how far south is Canberra from Sydney? PS C

14 Klaas Creek is on a bearing of 314° from Hampervale and the distance between them is 158 km. Badgerton is 325 km from Hampervale and its bearing from Klaas Creek is 044°. Find the bearing of Badgerton from Hampervale. PS C

15 A plane flies 1200 km on a NE compass bearing and then changes direction to a bearing of NW and flies for a distance of 450 km. PS C

a How far, correct to the nearest km, is the plane from its starting point?

b What is the bearing (to the nearest minute) of the plane from its starting point?

16 A fishing boat leaves a harbour on a compass bearing of ESE and after 15 nautical miles, changes direction to a bearing of SSW and travels a further 10 nautical miles.

Find the bearing of the harbour from the boat's final position, correct to the nearest minute. PS C

9.03 Pythagoras' theorem and trigonometry in 3D

STAGE 5.3

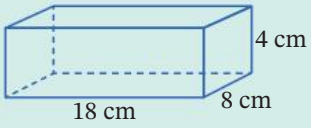
Pythagorean two-step problems

Pythagorean triads

2D and 3D applications of trigonometry

Example 8

A wooden rectangular box has dimensions 18 cm × 8 cm × 4 cm.



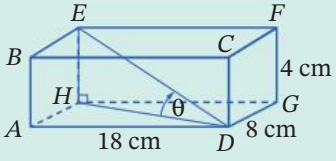
a Find, correct to one decimal place, the length of the longest pencil that can lie flat in the base of the box.

b Find, correct to one decimal place, the length of the longest diagonal of the box.

c Find, correct to the nearest degree, the angle that the longest diagonal makes with the base of the box.

Solution

Label the box as shown. HD is the length of the longest pencil that can lie flat in the base of the box, while ED is the longest diagonal of the box.



a Using the right-angled triangle DAH:

$$\begin{aligned} HD^2 &= DA^2 + AH^2 \\ &= 18^2 + 8^2 \\ &= 388 \\ HD &= \sqrt{388} \\ &= 19.6977... \\ &\approx 19.7 \text{ cm} \end{aligned}$$

The longest pencil that can lie flat in the base of the box is 19.7 cm.

b

$$\begin{aligned} ED^2 &= HD^2 + HE^2 \\ ED^2 &= \left(\sqrt{388}\right)^2 + 4^2 \\ &= 404 \\ ED &= \sqrt{404} \\ &= 20.0997... \\ &\approx 20.1 \text{ cm} \end{aligned}$$

The longest diagonal of the box is 20.1 cm.

c In the right-angled triangle EHD, θ is the angle that the longest diagonal makes with the base of the box.

$$\begin{aligned} \tan\theta &= \frac{EH}{HD} \\ &= \frac{4}{\sqrt{388}} \\ \theta &= 11.4789...^\circ \\ &\approx 11^\circ \end{aligned}$$

The longest diagonal makes an angle of 11° with the base of the box.

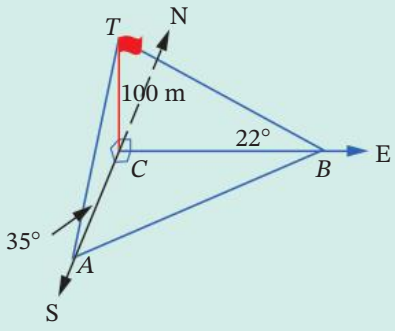
or use sin or cos since ED is also known

4 [▭] [√] 388 SHIFT tan =

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Example 9

A 100 m flagpole is observed from 2 different locations. From point A, due south of the flagpole the angle of elevation of the top of the flagpole is 35°; from point B, due east of the flagpole the angle of elevation is 22°. Find the distance between A and B, correct to the nearest metre.



Solution

There are 3 right-angled triangles in this diagram. To find AB, we must first find AC and CB using trigonometry.

In $\triangle ACT$,

$$\tan 35° = \frac{100}{AC}$$

$$AC = \frac{100}{\tan 35°}$$

$$= 142.8148...$$

Do not round yet.

In $\triangle BCT$,

$$\tan 22° = \frac{100}{CB}$$

$$CB = \frac{100}{\tan 22°}$$

$$\approx 247.5086...$$

In $\triangle ABC$,

$$AB^2 = AC^2 + CB^2$$

$$= (142.8148...)^2 + (247.5086...)^2$$

$$= 81656.6166...$$

$$AB = \sqrt{81656.6166...}$$

$$= 285.7562...$$

$$\approx 286 \text{ m}$$

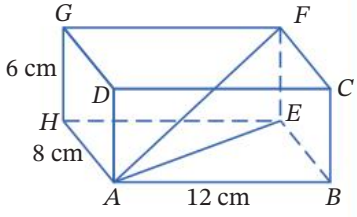
From the diagram, 286 m seems to be a reasonable answer.

EXERCISE 9.03 ANSWERS ON P. 626

Pythagoras' theorem and trigonometry problems in 3D

U F PS R C

1 For this rectangular prism, calculate:

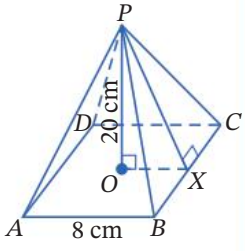


- **a** the length of AE in surd form
- **b** the length of AF, correct to one decimal place
- **c** the size of $\angle FAE$, correct to the nearest degree.

2 Sketch a diagram of a cube of length 20 cm, then find:

- **a** the exact length of the longest diagonal on any face
- **b** the length of the longest diagonal of the cube, correct to 2 decimal places
- **c** the angle that the longest diagonal makes with the base, correct to the nearest minute.

3 The diagram shows a square pyramid with base length 8 cm and perpendicular height 20 cm. PX is the slant height of the pyramid. Calculate: R



- **a** the length of PX, correct to 2 decimal places
- **b** the angle of inclination of PX, correct to the nearest degree.

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EXAMPLE 7

STAGE 5.3

4 A cone has a base diameter of 2.8 m and a slant height of 2.5 m. Find the angle that the cone makes with the vertical at the top of the cone. R

5 A fruit juice container is a square prism with dimensions 8 cm by 3 cm by 3 cm. R

a Find, correct to one decimal place, the length of the longest drinking straw that fits inside the container.

b To the nearest degree, what angle does the longest straw make with the vertical?

6 From a point X, 37 m from the base of a tree, the angle of elevation is 55° while the angle of elevation of the tree, from a point Y due east of the tree, is 25°. Find, correct to the nearest metre: PS R

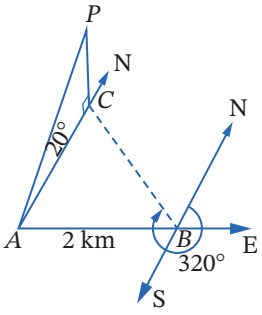
a the height of the tree

b the distance of the tree from point Y

EXAMPLE 9

7 A 15 m flagpole stands on level ground. From point P, due west of the flagpole the angle of elevation of the top of the pole is 38°. From point Q, due north of the flagpole, the flagpole has an angle of elevation of 25°. Find PQ, correct to one decimal place. PS R

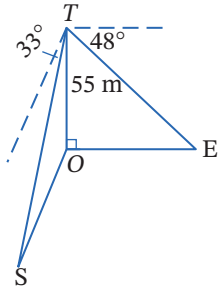
8 From a point, A, at the base of a mountain, the mountain peak, P, is due north and has an angle of elevation of 20°. From a point, B, 2 km due east of A on the same level, the mountain peak has a bearing of 320°. PS R



a What is the size of $\angle CBA$?

b Calculate the height, PC, of the mountain correct to the nearest metre.

9 From the top of her 55-metre office building, Madison observes 2 cars parked at ground level. The angle of depression of the car due east of the building is 48° and the angle of depression of the car parked due south of the building is 33°. PS R



Calculate, correct to the nearest metre, how far:

a each car is from the building

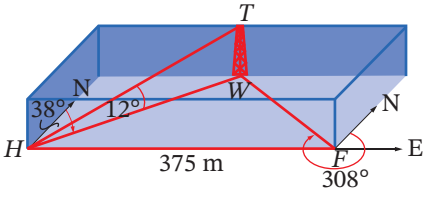
b the cars are apart.

2D and 3D applications of trigonometry

10 Hassan observes a transmission tower at an elevation angle of 12° and bearing 038°. Fatima stands 375 m due east of Hassan and observes the tower at a bearing of 308°. PS R

a Find the sizes of the angles of $\triangle FHW$.

b Find the height of the tower TW, correct to one decimal place.

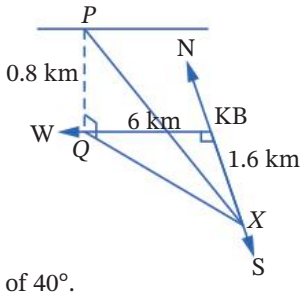


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STAGE 5.3

11 A plane flies 6 km due west of Keira Bay at a constant height of 800 m. Xander sees the plane from his house 1.6 km south of Keira Bay.



Find, correct to the nearest degree: PS R C

a the bearing of the plane from Xander

b the angle of elevation of the plane from Xander.

12 Sophia observes a plane due north at an angle of elevation of 40°. The plane is flying due east at an altitude of 4000 m and at a speed of 200 km/h. After 2 minutes, find: PS R C

a the angle of elevation and the bearing of the plane from Sophia to the nearest degree

b the distance from Sophia to the plane to the nearest metre

13 From a lighthouse 95 m above sea level, a boat is observed on a compass bearing of SSE and at an angle of depression of 2°. Another boat is seen on a bearing of ENE and at angle of depression of 1.5°. PS R C

a Calculate the distance (correct to one decimal place) between the boats.

b What is the bearing of the first boat from the second boat?

9.03

Complementary relations and exact ratios# 9.04

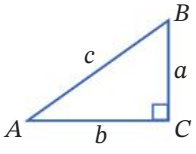
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STAGE 5.3

Trigonometric ratios of complementary angles

In $\triangle ABC$, $\angle C = 90°$ so $\angle A + \angle B = 90°$ due to the angle sum of a triangle.

$\therefore A$ and B are complementary angles.



Now $\sin A = \frac{a}{c}$ and $\cos A = \frac{b}{c}$

But $\sin B = \frac{b}{c}$ and $\cos B = \frac{a}{c}$

$\therefore \sin A = \cos B$ and $\cos A = \sin B$

But $B = 90° - A$ (the complement of A)

$\therefore \sin A = \cos(90° - A)$ and $\cos A = \sin(90° - A)$

The sine and cosine ratios of complementary angles

$$\sin A = \cos(90° - A)$$
$$\cos A = \sin(90° - A)$$

Because of this complementary relationship, the sine and cosine ratios are called **complementary ratios**. That is why **co-sine** is named as the complement of **sine**.

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STAGE 5.3

Example 10

a If $\sin 35° = \cos \alpha$, find α.

b If $P + Q = 90°$ and $\sin Q = \frac{15}{17}$, find:

i $\cos P$ **ii** $\cos Q$

Solution

a $\sin 35° = \cos (90° - 35°)$ — $\sin A = \cos (90 - A)$

$= \cos 55°$

$\alpha = 55°$ — 35° and α are complementary angles

b i Since $P + Q = 90°$

$\cos P = \sin Q$ — P and Q are complementary angles

$= \frac{15}{17}$

ii Since $\sin Q = \frac{15}{17}$, draw a right-angled triangle to find $\cos Q$.

$x^2 = 17^2 - 15^2 = 64$

$x = \sqrt{64} = 8$

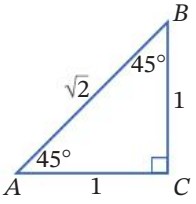
$\therefore \cos Q = \frac{8}{17}$



Exact trigonometric ratios of 30°, 45° and 60°

WS The exact ratios

The value of most trigonometric ratios can only be approximated in decimal form. However, the trigonometric ratios of the special angles 30°, 45° and 60° can be written in exact (surd) form. These are called the **exact ratios**.



For 45°, consider an isosceles right-angled triangle with 2 equal sides 1 unit, 2 equal angles 45° and hypotenuse h units.

By Pythagoras' theorem,

$h^2 = 1^2 + 1^2 = 2$

$h = \sqrt{2}$

$\therefore \sin 45° = \frac{1}{\sqrt{2}}, \cos 45° = \frac{1}{\sqrt{2}}, \tan 45° = \frac{1}{1} = 1$

For 30° and 60°, consider an equilateral triangle ABC with equal sides 2 units long, equal angles of 60° and an axis of symmetry CD. This means $\angle ACD = 30°$, $\angle ADC = 90°$ and side $AD = 1$ (half of 60°, 180° and 2 respectively).

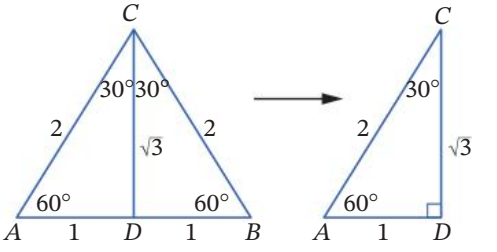
By Pythagoras' theorem,

$CD^2 = 2^2 - 1^2 = 3$

$CD = \sqrt{3}$

$\therefore \sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2}, \tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}, \cos 60° = \frac{1}{2}, \tan 60° = \sqrt{3}$



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The trigonometric ratios

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

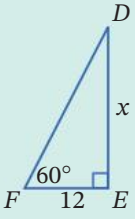
Note that $\sin 30° = \cos 60°$, $\sin 60° = \cos 30°$, and $\sin 45° = \cos 45°$, because 30° and 60° are complementary angles and 45° is the complement of itself.

Learn the values in this table by looking for patterns in the exact values.

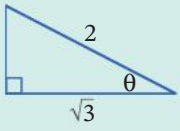
Example 11

Find the exact value of each variable.

a



b



Solution

a $\tan 60° = \frac{x}{12}$

$x = 12 \tan 60°$

$= 12\sqrt{3}$ $\quad \tan 60° = \sqrt{3}$

b $\cos \theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

The exact trigonometric ratios

EXERCISE 9.04 ANSWERS ON P. 626

Complementary relations and exact ratios U F R

EXAMPLE 10

1 Find the value of each variable.

a $\sin 47° = \cos X$ **b** $\cos 74° = \sin Y$ **c** $\sin 2.55° = \cos P$

d $\cos 55.2° = \sin \theta$ **e** $\sin 38° 17' = \cos M$ **f** $\cos 65° 26' = \sin T$

g $\sin (A + 28°) = \cos 37°$ **h** $\cos (M - 35°) = \sin 67°$ **i** $\sin 49° = \cos (T + 12°)$

2 Simplify each expression.

a $\cos 22° - \sin 68°$ **b** $\frac{\sin 57°}{\cos 33°}$

c $\frac{\sin (90° - \theta)}{\cos \theta}$ **d** $\frac{\cos 28° + \sin 62°}{\sin 62°}$

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STAGE 5.3

3 If $\sin\alpha = \frac{5}{13}$ and $\alpha + \beta = 90°$, find $\cos\beta$, $\cos\alpha$ and $\sin\beta$. R

4 Given $\cos E = \frac{40}{41}$, and E and F are complementary angles, find $\sin F$, $\sin E$ and $\cos F$. R

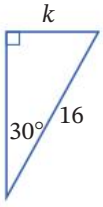
5 If $X + Y = 90°$, find the exact values of $\cos Y$, $\sin Y$ and $\sin X$, given $\cos X = \frac{2}{3}$. R

6 Find the exact values of $\cos\phi$, $\sin\phi$ and $\cos\theta$ if $\sin\theta = \frac{\sqrt{5}}{4}$ and $\theta = 90° - \phi$. R

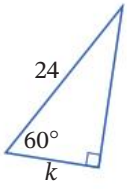
EXAMPLE 11

7 In each triangle, find the exact value of the variable.

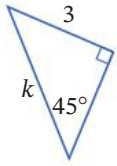
a



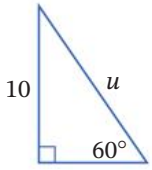
b



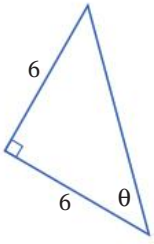
c



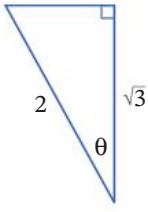
d



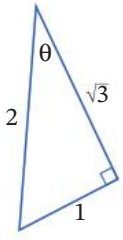
e



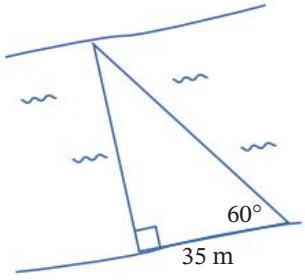
f



g

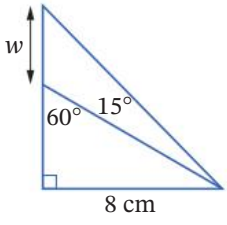


8 Find the width of this river in exact form.

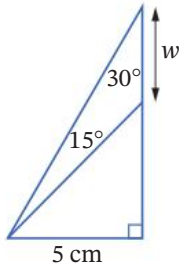


9 Find the exact value of w in each diagram.

a



b



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Mental skills 9: Maths without calculators ANSWERS ON P. 626

Simplifying fractions and ratios

When simplifying a fraction or a ratio, look for a common factor to divide into both the numerator and the denominator, preferably the highest common factor (HCF).

1 Study each example.

a Simplify $\frac{27}{45}$

$\frac{27^{9}}{45^{15}} = \frac{9}{15}$ Dividing numerator and denominator by 3.

$\frac{9^{3}}{15^{5}} = \frac{3}{5}$ Dividing numerator and denominator by 3 again.

Note: This fraction could be simplified in one step if you divided by 9, the highest common factor (HCF) of 27 and 45.

b Simplify $\frac{160}{400}$

$\frac{160^{16}}{400^{40}} = \frac{16}{40}$ Dividing numerator and denominator by 10.

$\frac{16^{2}}{40^{5}} = \frac{2}{5}$ Dividing numerator and denominator by 8.

Note: This fraction could be simplified in one step if you divided by 80, the HCF of 160 and 400.

c Simplify 24 : 36.

$24^{4} : 36^{6} = 4 : 6$ Dividing both terms by 6

$4^{2} : 6^{3} = 2 : 3$ Dividing both terms by 2

Note: This fraction could be simplified in one step if you divided by 12, the HCF of 24 and 36.

d Simplify 135 : 90.

$135^{27} : 90^{18} = 27 : 18$ Dividing both terms by 5

$27^{3} : 18^{2} = 3 : 2$ Dividing both terms by 9

e Calculate $\frac{3}{8} \times \frac{2}{15}$ in simplest form.

$\frac{3}{8^{4}} \times \frac{2^{1}}{15} = \frac{3}{4} \times \frac{1}{15}$ Dividing 2 and 8 by 2

$\frac{3^{1}}{4} \times \frac{1}{15^{5}} = \frac{1}{20}$ Dividing 3 and 15 by 3

f What fraction is 36 minutes of 1 hour?

$\frac{36}{1 \text{ hour}} = \frac{36 \text{ min}}{60 \text{ min}} = \frac{3}{5}$

2 Now simplify each fraction or ratio.

a $\frac{10}{15}$	**b** $\frac{16}{20}$	**c** $\frac{30}{42}$	**d** $\frac{8}{16}$
e $\frac{20}{80}$	**f** $\frac{6}{36}$	**g** $\frac{20}{24}$	**h** $\frac{12}{30}$
i 20 : 36	**j** 25 : 45	**k** 18 : 40	**l** 28 : 35
m 27 : 21	**n** 16 : 12	**o** $\frac{5}{6} \times \frac{18}{25}$	**p** $\frac{12}{50} \times \frac{10}{21}$

3 Express each as a simplified fraction.

a 425 g of 1 kg	**b** 8 months of 1 year	**c** 64 cm of 1 m
d 750 mL of 3 L	**e** 10 hours of 2 days	**f** 80c of $10

9.05 The trigonometric functions

STAGE 5.3



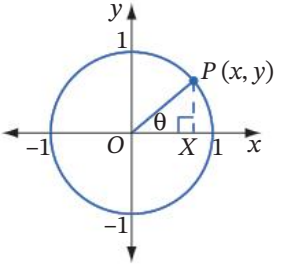
Unit circle investigation


Trigonometric graphs


Trigonometric graphs

Trigonometric ratios of any angle

The sine, cosine and tangent ratios can be extended to include angles that are over 90°, that is, obtuse and reflex angles. The trigonometric ratios for angles of any size can be best explained using a **unit circle**.



A unit circle is a circle of radius 1 drawn on a number plane, with the origin as the centre of the circle. Starting from the positive direction of the x-axis, angles can be measured around this circle in an anti-clockwise direction.

Let $P(x, y)$ be any point on the unit circle as shown and θ the angle that PO makes with the positive x-axis.

Let the vertical interval from P meet the x-axis at X to make the right-angled triangle OXP.

Since P has coordinates (x, y), $OX = x$ and $XP = y$.

In $\triangle XOP$, $\cos\theta = \frac{OX}{OP}$

$= \frac{x}{1}$

$\therefore \cos\theta = x$

$OP = 1$ because it is the radius of the unit circle

The x-coordinate of point P on the unit circle

Also, $\sin\theta = \frac{XP}{OP}$

$= \frac{y}{1}$

$\therefore \sin\theta = y$

The y-coordinate of the point P on a unit circle

and $\tan\theta = \frac{XP}{OX}$

$\therefore \tan\theta = \frac{y}{x}$

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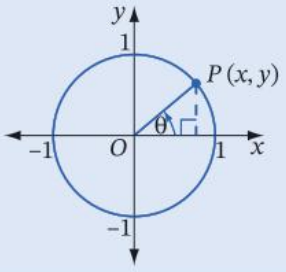
Trigonometric ratios on the unit circle

If $P(x, y)$ is any point on the unit circle, and θ is the angle that PO makes with the positive x-axis, then:

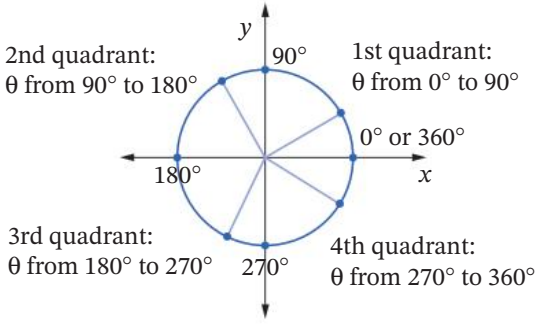
$\sin\theta = y$-coordinate of P

$\cos\theta = x$-coordinate of P

$\tan\theta = \dfrac{y\text{-coordinate of } P}{x\text{-coordinate of } P}$



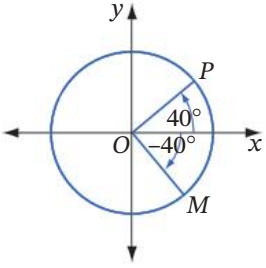
The x- and y-axes divide the number plane into 4 equal **quadrants**. Now we can investigate the trigonometric ratios for all angles from $\theta = 0°$ to $360°$, by looking at $P(x, y)$ on the unit circle in the 1st, 2nd, 3rd and 4th quadrants.



The unit circle can also be used to define the trigonometric ratios for angles below 0° and above 360°.

Negative angles (below 0°) are measured in a clockwise direction on the unit circle. In this diagram, M represents $-40°$ but it could also represent $360° - 40° = 320°$.

Angles above 360° are measured on the unit circle by going around the circle more than once. In the diagram, P represents $40°$, but it could also represent $360° + 40° = 400°$.



The tangent ratio

The tangent ratio can be expressed in terms of the sine and cosine ratios.

Since $\sin\theta = y$ and $\cos\theta = x$,

$$\frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

But $\tan\theta = \dfrac{y}{x}$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$$

The tangent ratio

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

STAGE 5.3

Example 12

Given that $\sin\alpha = \frac{2}{\sqrt{13}}$ and $\cos\alpha = \frac{3}{\sqrt{13}}$, find $\tan\alpha$.

Solution

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$
$$= \frac{2}{\sqrt{13}} \div \frac{3}{\sqrt{13}}$$
$$= \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{3}$$
$$= \frac{2}{3}$$

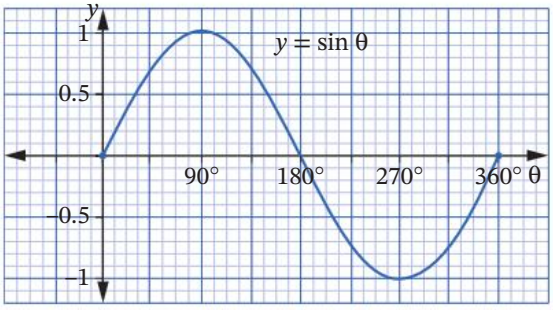
The sine curve

$\sin\theta$ = y-coordinate of P (the height of P above the x-axis), so note its value as θ moves around the unit circle (in an anti-clockwise direction) from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
sin θ	0	from 0 to 1	1	from 1 to 0	0	from 0 to −1	−1	from −1 to 0	0

Note that the value of $\sin\theta$ always lies between 1 and −1.

The graph of $y = \sin\theta$ for θ from 0° to 360° is a 'wave curve' that repeats itself after 360°.



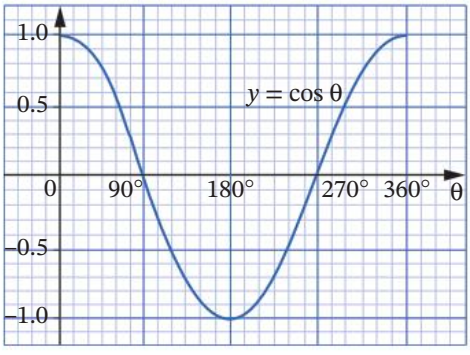
The cosine curve

$\cos\theta$ = x-coordinate of P (the length of P from the y-axis), so note its value for θ from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
cos θ	1	from 1 to 0	0	from 0 to −1	−1	from −1 to 0	0	from 0 to 1	1

Note that the value of $\cos\theta$ always lies between 1 and −1.

The graph of $y = \cos\theta$ for θ from 0° to 360° is another 'wave curve' that repeats itself after 360°.



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The tangent curve

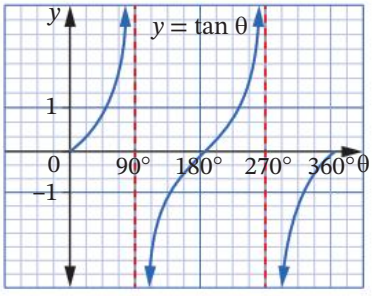
$\tan\theta = \frac{y\text{-coordinate of } P}{x\text{-coordinate of } P} = \frac{\sin\theta}{\cos\theta}$, so note its value for θ from 0° to 360°.

θ	0°	1st quadrant 0° to 90°	90°	2nd quadrant 90° to 180°	180°
tan θ	$\frac{0}{1} = 0$	$\frac{+}{+}$ = positive	$\frac{1}{0}$ = undefined	$\frac{+}{-}$ = negative	$\frac{0}{-1} = 0$

θ	3rd quadrant 180° to 270°	270°	4th quadrant 270° to 360°	360°
tan θ	$\frac{-}{-}$ = positive	$\frac{-1}{0}$ = undefined	$\frac{-}{+}$ = negative	$\frac{0}{1} = 0$

Note that the value of tan θ has no value at 90° and 270°.

The graph of $y = \tan\theta$ for θ from 0° to 360° is a curve that repeats itself after 180°, with asymptotes at θ = 90° and 270°.

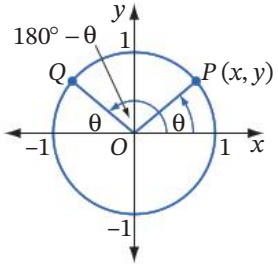


Trigonometric ratios of supplementary angles

For obtuse angles (between 90° and 180°, represented by the 2nd quadrant in the unit circle), we can use this diagram where Q is a reflection of P across the y-axis.

QO makes an angle of $180° - \theta$ with the positive x-axis.

In the 2nd quadrant, Q has a negative x-coordinate and a positive y-coordinate, so if the coordinates of P are (x, y), then the coordinates of Q are $(-x, y)$.



$\therefore \cos(180° - \theta) = -x$
$= -\cos\theta$

$\therefore \sin(180° - \theta) = y$
$= \sin\theta$

$\therefore \tan(180° - \theta) = \frac{y}{-x}$
$= -\frac{y}{x}$
$= -\tan\theta$

Supplementary relations

For obtuse angles (in the second quadrant), sine is positive while cosine and tangent are negative.

$$\sin(180° - A) = \sin A$$

The sine of an obtuse angle is equal to the **sine** of its supplement.

$$\cos(180° - A) = -\cos A$$

The cosine of an obtuse angle is equal to the **negative cosine** of its supplement.

$$\tan(180° - A) = -\tan A$$

The tangent of an obtuse angle is equal to the **negative tangent** of its supplement.

STAGE 5.3

Example 13

If θ is acute, find θ if:

a $\tan 140° = -\tan \theta$ **b** $\sin 100° = \sin \theta$ **c** $\cos 120° = -\cos \theta$

Solution

a $\theta = 180° - 140°$
$= 40°$
$\therefore \tan 140° = -\tan 40°$

b $\theta = 180° - 100°$
$= 80°$
$\therefore \sin 100° = \sin 80°$

c $\theta = 180° - 120°$
$= 60°$
$\therefore \cos 120° = -\cos 60°$

Example 14

Find the exact value of each expression.

a $\sin 120°$ **b** $\tan 135°$ **c** $\cos 135°$

Solution

a $\sin 120° = \sin (180° - 120°)$
$= \sin 60°$
$= \frac{\sqrt{3}}{2}$

b $\tan 135° = -\tan (180° - 135°)$
$= -\tan 45°$
$= -1$

c $\cos 135° = \cos (180° - 135°)$
$= -\cos 45°$
$= -\frac{1}{\sqrt{2}}$

EXERCISE 9.05 ANSWERS ON P. 627

The trigonometric functions U F R C

EXAMPLE 12

1 a If $\sin A = \frac{60}{109}$ and $\cos A = \frac{91}{109}$, find $\tan A$

b If $\sin Y = 0.2$ and $\cos Y = 0.15$, find $\tan Y$

c If $\sin X = \frac{2}{\sqrt{13}}$ and $\cos X = \frac{3}{\sqrt{13}}$, find $\tan X$

d If $\cos P = \frac{40}{41}$ and $\sin P = \frac{9}{41}$, find $\tan P$

e If $\cos Q = \frac{3}{7}$ and $\sin Q = \frac{\sqrt{40}}{7}$, find $\tan Q$

f If $\cos X = \frac{60}{61}$ and $\tan X = \frac{11}{60}$, find $\sin X$

g If $\tan X = \frac{24}{7}$ and $\sin X = \frac{24}{25}$, find $\cos X$

h If $\tan X = \frac{2}{\sqrt{5}}$ and $\cos X = \frac{\sqrt{5}}{3}$, find $\sin X$

2 State whether each acute or obtuse angle is positive (P) or negative (N).

a sin 95° **b** cos 46° **c** tan 153° **d** cos 171°

e sin 142° **f** tan 91° **g** tan 130° **h** cos 87°

3 Evaluate, correct to 2 decimal places, each trigonometric expression.

a cos 153° **b** tan 349° **c** sin 230°

d tan 173° 42′ **e** cos 300.9° **f** sin 324.8°

g sin 176° 54′ **h** cos 245° 23′ **i** tan (−38°)

j sin (−61°) **k** tan 370° **l** cos 434°

4 **a** Copy and complete this table of values for $y = \sin\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	...	360°
y	0	0.5	0.87	...	0

b Graph $y = \sin\theta$, either by using graphing technology or on paper using a scale of 1 cm = 30° on the θ-axis and a scale of 4 cm = 1 unit on the y-axis.

c Comment on the shape of the graph $y = \sin\theta$. What are the maximum and minimum values of the graph and when do they occur?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is sin θ:

i positive **ii** negative?

5 **a** Copy and complete this table of values for $y = \cos\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	...	360°
y	1	0.87	0.5	...	1

b Graph $y = \cos\theta$, either by using graphing technology or on paper.

c Comment on the shape of the graph $y = \cos\theta$. What are the maximum and minimum values of the graph and when do they occur?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is cos θ:

i positive **ii** negative?

g Comment on the similarities and differences between the graphs of $y = \sin x$ and $y = \cos x$.

6 If θ is acute, find θ if:

a $\cos 170° = -\cos\theta$ **b** $\sin 110° = \sin\theta$ **c** $\tan 130° = -\tan\theta$

d $\tan 97° = -\tan\theta$ **e** $\cos 115° = -\cos\theta$ **f** $\sin 168° = \sin\theta$

EXAMPLE 13

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STAGE 5.3

EXAMPLE 14

7 Write each expression in terms of sin A, cos A or tan A, where A is acute.

a	cos 142°	**b**	sin 105°	**c**	cos 155°	**d**	tan 102°
e	cos 172.7°	**f**	sin 115.5°	**g**	cos 139° 35′	**h**	tan 170.8°
i	sin 120° 35′	**j**	tan 160° 10′	**k**	sin 95.5°	**l**	tan 139.5°

8 Find the exact value of each expression.

a	sin 150°	**b**	tan 135°	**c**	sin 135°	**d**	cos 120°
e	tan 150°	**f**	sin 120°	**g**	cos 150°	**h**	tan 120°
i	sin 90°	**j**	cos 135°				

9 **a** Copy and complete this table of values for $y = \tan\theta$, evaluating y correct to 2 decimal places. R C

θ	0°	30°	60°	...	360°
y	0	0.58	1.73	...	0

b Graph $y = \tan\theta$, either by using graphing technology or on paper.

c Comment on the shape of the graph $y = \tan\theta$. When does it start to repeat itself?

d Does the graph have an axis of symmetry? If so, what is it?

e Does the graph have rotational symmetry? If so, what is the centre of symmetry?

f For what range of values of θ is tan θ:

i positive **ii** negative?

9.06 Trigonometric equations

STAGE 5.3

WS Trigonometric equations

Example 15

Solve each trigonometric equation, giving all possible acute and obtuse solutions correct to the nearest degree.

a $\sin\theta = 0.7538$ **b** $\tan\theta = -2.5$

Solution

a $\sin\theta = 0.7538$

$\theta = 48.9206...$

$\approx 49°$

But θ could be obtuse, because sin θ is also positive in the second quadrant.

$\theta \approx 180° - 49°$

$= 131°$

$\therefore \theta \approx 49°$ or 131°.

On a calculator: SHIFT sin 0.7538 =

On some calculators, 2nd F instead of SHIFT

(Check: sin 49° = sin 131° = 0.7547...)

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b $\tan\theta = -2.5$

$\theta = -68.1985...$

$\approx -68°$

But θ is obtuse, because $\tan\theta$ is negative in the second quadrant.

$\theta \approx 180° - 68°$

$= 112°$

On a calculator: SHIFT tan (−) 2.5 =

On a calculator: 180 + Ans =

(Check: $\tan 112° = -2.4750...$)

Example 16

Solve each trigonometric equation correct to the nearest minute, if x is obtuse.

a $\cos x = -0.09$ **b** $\sin x = 0.64$

Solution

SHIFT cos automatically gives the obtuse angle when you enter a negative value.

a $\cos x = -0.09$

$x = 95.1636...$

$= 95° 9' 48.99''$

$\approx 95° 10'$

On a calculator: SHIFT cos (−) 0.09 =

On a calculator: °'" or DMS

b $\sin x = 0.64$

$x = 39.7918...$

But x is obtuse, so:

$x = 180 - 39.7918...$

$= 140.2081...$

$= 140° 12' 29.45''$

$\approx 140° 12'$

On a calculator: SHIFT sin 0.64 =

180 − Ans =

°'" or DMS

EXERCISE 9.06 ANSWERS ON P. 627

Trigonometric equations U F

1 Solve each trigonometric equation, giving all possible acute and obtuse solutions, correct to the nearest degree.

EXAMPLE 15

a $\sin\theta = 0.84$ **b** $\tan\theta = -\frac{3}{4}$ **c** $\cos\theta = -0.342$

d $\cos\theta = -\frac{7}{11}$ **e** $\sin\theta = 0.1164$ **f** $\tan\theta = -1$

g $\tan\theta = -5.8671$ **h** $\sin\theta = \frac{3}{7}$ **i** $\cos\theta = -0.4$

j $\sin\theta = \frac{3.8}{7}$ **k** $\cos\theta = -\frac{21}{80}$ **l** $\tan\theta = -\frac{15}{8}$

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STAGE 5.3

EXAMPLE 16

2 Solve each trigonometric equation, correct to the nearest minute, if x is obtuse.
Note: some equations have no solution.

a $\sin A = \frac{4}{7}$ **b** $\sin A = 0.7438$ **c** $\sin A = 0.3514$

d $\sin A = 0.108$ **e** $\sin A = \frac{5}{11}$ **f** $\sin A = 0.9$

g $\cos x = 0.6$ **h** $\cos x = -0.6$ **i** $\tan x = 0.3$

j $\tan x = -0.3$ **k** $\sin x = 0.8$ **l** $\sin x = \frac{3}{7}$

3 Solve each trigonometric equation correct to the nearest degree, if A is between 0° and 180°.

a $\cos x = \frac{-8}{11}$ **b** $\tan x = -0.95$ **c** $\sin x = \frac{7}{8}$

d $4\cos x = \sqrt{2}$ **e** $3\sin x = 2$ **f** $-4\tan x = 3$

g $\tan x = 1$ **h** $\cos x = \frac{1}{2}$ **i** $\sin x = \frac{1}{\sqrt{2}}$

Investigation

Sides and opposite angles

This activity can also be done using dynamic geometry software.

1 Construct 3 triangles of different sizes.

2 Label the angles of each triangle A, B and C and the sides opposite them a, b and c respectively.

3 Measure the sides of each triangle correct to the nearest millimetre and the angles to the nearest degree. Copy the table below and record your results in it.

	Side a	Angle A	$\frac{a}{\sin A}$	Side b	Angle B	$\frac{b}{\sin B}$	Side c	Angle C	$\frac{c}{\sin C}$
Triangle 1									
Triangle 2									
Triangle 3									

4 Complete the table by calculating $\frac{a}{\sin A}$, $\frac{b}{\sin B}$ and $\frac{c}{\sin C}$ correct to 2 decimal places.

5 Compare the values of $\frac{a}{\sin A}$, $\frac{b}{\sin B}$ and $\frac{c}{\sin C}$ for each triangle. What do you notice?

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The sine rule

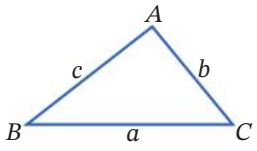
9.07

STAGE 5.3

9.07

WS Discovering the sine rule

The angles of a triangle are labelled with capital letters while the sides are labelled with lowercase letters. By convention, we use a to label the side opposite $\angle A$, b to label the side opposite $\angle B$, and so on.



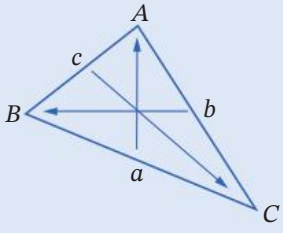
There is a relationship between each angle in a triangle and its opposite side. The longest side is always opposite the largest angle, the next smallest side is opposite the next smallest angle and so on. This relationship is called the **sine rule**.

The sine rule

For any triangle ABC:

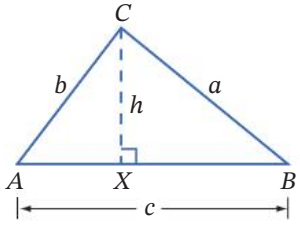
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The ratios of the sides in a triangle to the sine of their opposite angles are equal.



Proof:

In $\triangle ABC$, draw CX for the perpendicular height, h, of the triangle. CX divides $\triangle ABC$ into 2 right-angled triangles.



In $\triangle AXC$, $\sin A = \frac{h}{b}$

$\therefore h = b \sin A$

In $\triangle BXC$, $\sin B = \frac{h}{a}$

$\therefore h = a \sin B$

$\therefore a \sin B = b \sin A$

$$\frac{a \sin B}{\sin B} = \frac{b \sin A}{\sin B}$$ Dividing both sides by $\sin B$

$$a = \frac{b \sin A}{\sin B}$$

$$\frac{a}{\sin A} = \frac{b \sin A}{\sin B \sin A}$$ Dividing both sides by $\sin A$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

By drawing the perpendicular from A to BC, it can be shown that $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine rule allows us to apply trigonometry to any triangle, not just right-angled triangles.

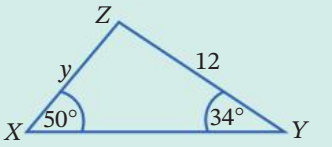
The sine rule is used in problems involving 2 sides of a triangle and the 2 angles opposite them.

STAGE 5.3

The sine rule

Example 17

Find y, correct to one decimal place.



Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Sides and opposite angles

$$\frac{y}{\sin 34°} = \frac{12}{\sin 50°}$$

$$y = \frac{12 \sin 34°}{\sin 50°}$$

$$= 8.7596\ldots$$

$$\approx 8.8 \text{ cm}$$

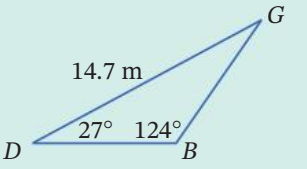
From the diagram, an answer of 8.8 cm looks reasonable.

Example 18

In $\triangle BDG$, $\angle B = 124°$, $\angle D = 27°$ and $DG = 14.7$ m. Find BD, correct to one decimal place.

Solution

Draw a diagram.



$$\frac{BD}{\sin G} = \frac{14.7}{\sin 124°}$$

$\angle G = 180° - 27° - 124°$ (angle sum of a triangle)

$= 29°$

$$\frac{BD}{\sin 29°} = \frac{14.7}{\sin 124°}$$

$$BD = \frac{14.7 \sin 29°}{\sin 124°}$$

$$= 8.5963\ldots$$

$$\approx 8.6 \text{ m}$$

EXERCISE 9.07 ANSWERS ON P. 627

The sine rule U F PS R

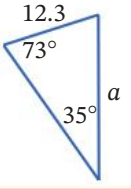
EXAMPLE 17

1 Evaluate each expression, correct to one decimal place.

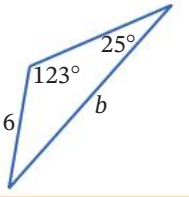
a $\frac{14.7 \sin 64°}{\sin 46°}$ **b** $\frac{34.5 \sin 33.4°}{\sin 115.7°}$ **c** $\frac{69 \sin 107° 33'}{\sin 38° 47'}$

2 Find the value of each variable, correct to 2 decimal places.

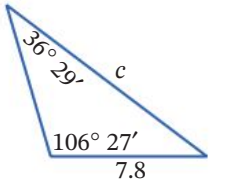
a



b



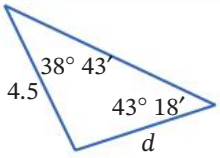
c



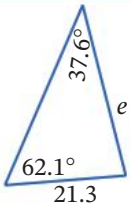
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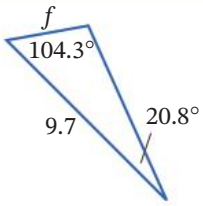
d



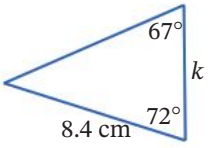
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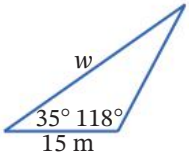
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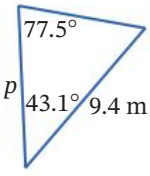
g



h



i

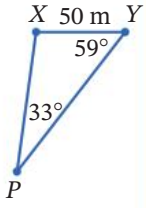


3 In $\triangle PTM$, $\angle P = 67°$, $\angle M = 52°$ and $PM = 12.5$ m. Find the length of PT.
Select the closest answer **A**, **B**, **C** or **D**.

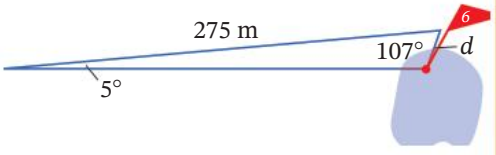
A 10.7 m **B** 14.6 m **C** 11.3 m **D** 13.9 m

EXAMPLE 18

4 X and Y are 2 light towers 50 m apart on one side of a park. P is a light tower on the other side of the park. If $\angle Y = 59°$ and $\angle P = 33°$, find PX correct to the nearest metre. PS R



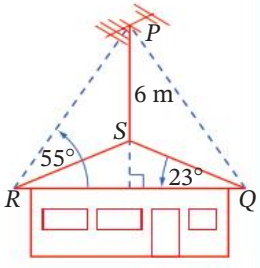
5 A golfer drives a ball 275 m at an angle of 5° off centre. The ball lands at an angle of 107° from the hole. Calculate the distance of the ball from the hole, correct to the nearest metre. PS R



6 A 6 m television antenna is mounted on a roof pitched at an angle of 23°. It is supported by 2 wires, PQ and PR, inclined at 55° to the horizontal. PS R

a Show that $\angle PSR = 113°$.

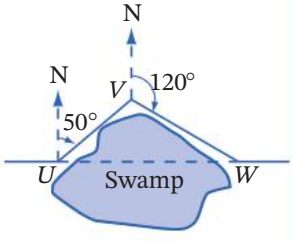
b Calculate the length of the wire PR, correct to the nearest centimetre.



7 To avoid a swamp, Jesinta runs 70 m on a bearing of 050° to V. She then turns and runs to W on a bearing of 120°.
If W is directly east of U: PS R

a find $\angle UVW$

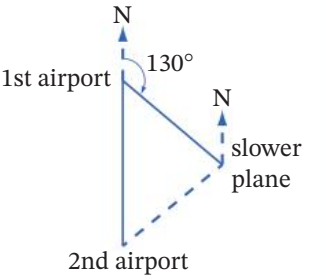
b calculate UW, correct to one decimal place.



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STAGE 5.3

8 2 planes leave the airport at the same time. One flies due south at 400 km/h and lands at a second airport after $1\frac{1}{2}$ hours. The other flies on a bearing of 130° and after $1\frac{1}{2}$ hours is at a bearing of 075° from the second airport. How far (to the nearest km) is the slower plane from the second airport? PS R



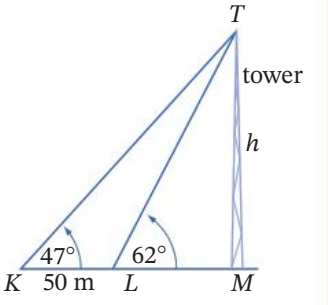
9 The angle of elevation of a tower from a point L is 62°. From a point K, 50 m further from the tower, the angle of elevation is 47°. PS R

a Use the sine rule in $\triangle KTL$ to show that $TL = \frac{50 \sin 47°}{\sin 15°}$.

b Let the height of the tower be h. In the right-angled $\triangle LMT$, show that $TL = \frac{h}{\sin 62°}$.

c Hence show that $h = \frac{50 \sin 47° \sin 62°}{\sin 15°}$

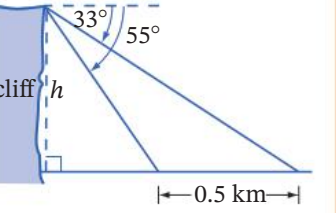
d Hence calculate the height of the tower, correct to one decimal place.



10 From the top of a cliff, the angles of depression of 2 boats at sea that are 0.5 km apart are 55° and 33°. PS R

a Let the height of the cliff be h. Show that $h = \frac{0.5 \sin 33° \sin 55°}{\sin 22°}$.

b Hence calculate the height, to the nearest metre.



11 A yacht leaves Fremantle and sails on a bearing of 312° for 650 km. It then changes direction and sails on a bearing of 235° until it is due west of Fremantle. How far is the yacht from Fremantle (to the nearest km)? PS R



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The sine rule for angles

9.08

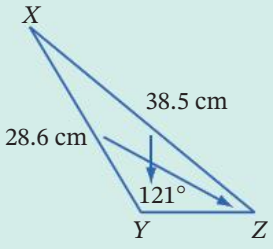
STAGE 5.3

WS Sine rule problems

Obtuse angles using the sine rule

Example 19

Find angle Z, correct to the nearest degree.



Solution

$$\frac{28.6}{\sin Z} = \frac{38.5}{\sin 121°}$$

Sides and opposite angles

$$\frac{\sin Z}{28.6} = \frac{\sin 121°}{38.5}$$

Inverting both sides so that Z is in the numerator.

$$\sin Z = \frac{28.6 \sin 121°}{38.5}$$
$$= 0.636\ldots$$

On a calculator: SHIFT sin Ans =

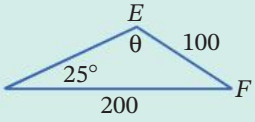
$$Z = 39.55\ldots$$

From the diagram, an answer of 40° looks reasonable.

$$\approx 40°$$

Example 20

Find θ correct to the nearest minute if it is an obtuse angle.



Solution

$$\frac{200}{\sin\theta} = \frac{100}{\sin 25°}$$
$$\frac{\sin\theta}{200} = \frac{\sin 25°}{100}$$
$$\sin\theta = \frac{200 \sin 25}{100}$$
$$= 0.845\ldots$$

$$\theta = 57.697\ldots$$

But θ is obtuse, so:

$$\theta = 180 - 57.697\ldots$$
$$= 122.3027\ldots$$
$$= 122° 18' 9.77''$$
$$\approx 122° 18'$$

STAGE 5.3

The ambiguous case (when there are 2 possible answers)

When we use the sine rule to find an angle, it is possible to find both an acute angle and an obtuse angle as solutions. Likewise, there could be 2 possible triangles: one acute-angled, the other obtuse-angled. However, the obtuse-angled triangle may not be possible. We need to check that the sum of the angles in the triangle is not greater than 180°.

The sine rule

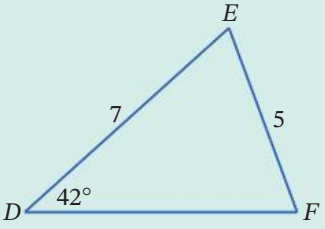
Example 21

a In $\triangle DEF$, $\angle D = 42°$, $d = 5$ cm and $f = 7$ cm. Find $\angle F$ correct to the nearest degree.

b In $\triangle LMN$, $\angle M = 130°$, $LN = 15$ cm and $LM = 7$ cm. Find $\angle N$, correct to the nearest degree.

Solution

a Draw a rough diagram.



$$\frac{7}{\sin F} = \frac{5}{\sin 42°}$$

$$\frac{\sin F}{7} = \frac{\sin 42°}{5}$$

$$\sin F = \frac{7 \sin 42°}{5}$$

$$= 0.93678\ldots$$

$$F = 69.5181\ldots .$$

$$\approx 70°$$

But F could be obtuse.

$$F = 180° - 70°$$

$$= 110°$$

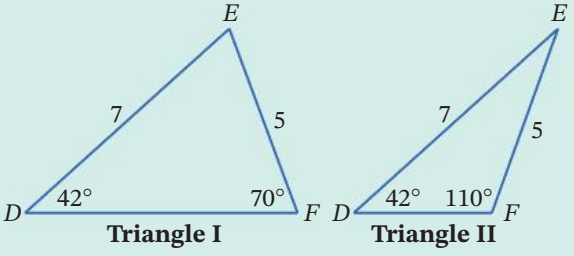
Checking the third angle of the obtuse-angled triangle:

$$\angle E = 180° - 42° - 110°$$

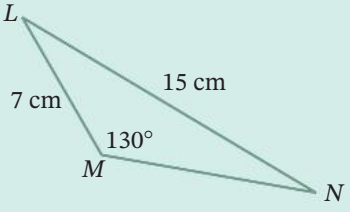
$$= 28°$$

$\therefore$ The obtuse-angled solution is possible.

$\therefore \angle F = 70°$ or $110°$



b Draw a rough diagram.



$$\frac{\sin N}{7} = \frac{\sin 130°}{15}$$

$$\sin N = \frac{7 \sin 130°}{15}$$

$$= 0.3574\ldots$$

$$N = 20.9459\ldots$$

$$\approx 21°$$

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But N could be obtuse.

$N = 180° - 21°$

$= 159°$

Checking the third angle of the obtuse-angled triangle:

$\angle L = 180° - 130° - 159°$

$= -109°$ Impossible

$\therefore$ The obtuse-angled solution is not possible.

$\therefore \angle N = 21°$

EXERCISE 9.08 ANSWERS ON P. 627

The sine rule for angles U F PS R C

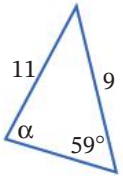
1 Find the acute angle X in each equation, correct to the nearest degree.

a $\sin X = \frac{5.3 \sin 123°}{9.7}$ **b** $\sin X = \frac{39 \sin 85°29'}{64}$ **c** $\sin X = \frac{467 \sin 63.8°}{518}$

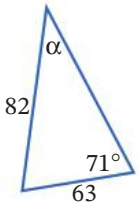
2 Find α in each triangle if α is acute, correct to the nearest 0.1 degree.

EXAMPLE 6

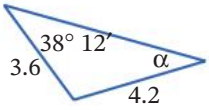
a



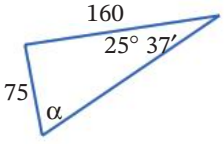
b



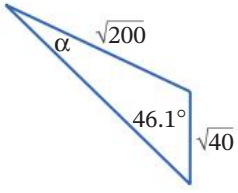
c



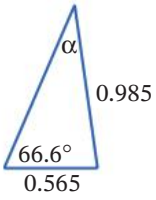
d



e



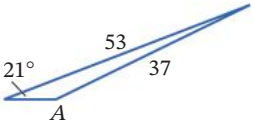
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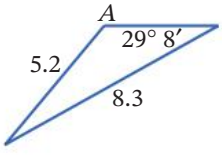
3 Find the size of $\angle A$ to the nearest minute if $\angle A$ is obtuse.

EXAMPLE 20

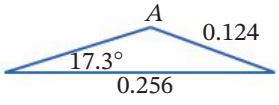
a



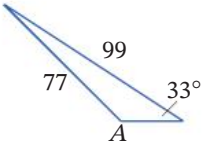
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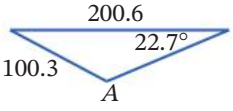
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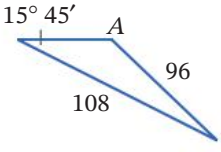
d



e



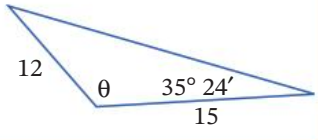
f



4 Find the size of θ. Select the correct answer **A**, **B**, **C** or **D**.

A 46° 24′ **B** 98° 12′

C 133° 36′ **D** 81° 48′



Foundation Standard Complex

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STAGE 5.3

EXAMPLE 21

5 In $\triangle BGP$, $\angle G = 121°$, $BG = 5.2$ m, and $BP = 12.8$ m. Find $\angle B$ (to the nearest degree).

6 Find all possible angles for each triangle, correct to the nearest degree. R C

a In $\triangle PQR$, $\angle P = 35°$, $p = 8$ cm, and $q = 10$ cm. Find $\angle Q$.

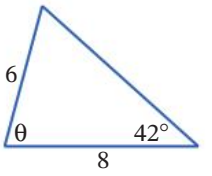
b In $\triangle UVW$, $\angle W = 95°$, $w = 16$ cm, and $v = 10$ cm. Find $\angle V$.

c In $\triangle XYZ$, $\angle Y = 24°$, $y = 3.4$ km, and $z = 5.7$ km. Find $\angle Z$.

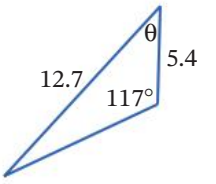
d In $\triangle DEF$, $\angle E = 37°$, $e = 107$ mm, and $d = 121$ mm. Find $\angle D$.

7 Find θ in each triangle correct to the nearest degree, given that θ is acute.

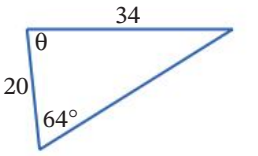
a



b



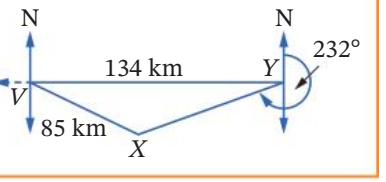
c



8 Max walked 5.6 km due east from his campsite. He then turned and walked on a bearing of 213° until he was 6 km from the campsite. What is the bearing of the campsite from Max? Select **A**, **B**, **C** or **D**. PS R C

A 52° **B** 289° **C** 341° **D** 308°

9 Yorker (Y) is 134 km due east of Vendor (V). Xena (X) is on a bearing of 232° from Yorker and is also 85 km from Vendor. $\angle VXY$ is an obtuse angle. Find the bearing of Xena from Vendor. PS R C



9.09 The cosine rule

STAGE 5.3

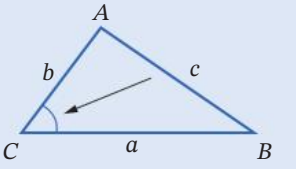
The **cosine rule** is a relationship between the 3 sides of a triangle and one of its angles. It is an extension of Pythagoras' triangle that can be applied to any triangle, not just right-angled ones.

The cosine rule

For any triangle ABC:

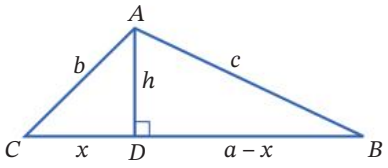
$$c^2 = a^2 + b^2 - 2ab\cos C$$

where c is the unknown side, C is the angle opposite c, and a and b are the other 2 sides.



Proof:

In $\triangle ABC$, draw AD for the perpendicular height, h, of the triangle. AD divides $\triangle ABC$ into 2 right-angled triangles.



Let $CD = x$, $\therefore DB = a - x$

In $\triangle ADC$, $b^2 = h^2 + x^2$

$\therefore h^2 = b^2 - x^2$ [1]

In $\triangle ABD$, $c^2 = h^2 + (a - x)^2$

$\therefore h^2 = c^2 - (a - x)^2$

$\therefore b^2 - x^2 = c^2 - (a - x)^2$

$b^2 - x^2 = c^2 - (a^2 - 2ax + x^2)$

$b^2 - x^2 = c^2 - a^2 + 2ax - x^2$

$b^2 = c^2 - a^2 + 2ax$

$\therefore c^2 = a^2 + b^2 - 2ax$ [2]

In $\triangle ADC$, $\cos C = \frac{x}{b}$

$\therefore x = b \cos C$

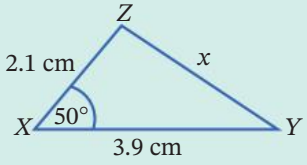
Substituting for x in [2]:

$c^2 = a^2 + b^2 - 2ab \cos C$

The cosine rule can be used in problems involving 3 sides of a triangle and one of the angles.

Example 22

Find x correct to 2 decimal places.



Solution

$c^2 = a^2 + b^2 - 2ab \cos C$

$x^2 = 2.1^2 + 3.9^2 - 2 \times 2.1 \times 3.9 \cos 50°$

$= 9.091138...$

$x = \sqrt{9.091138...}$

$= 3.01515...$

≈ 3.02 cm

50° is the angle opposite x.

From the diagram, an answer of 3.02 cm looks reasonable.

EXERCISE 9.09 ANSWERS ON P. 627

The cosine rule U F PS R C

1 Solve each equation for x, correct to one decimal place.

a $x^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 38°$

b $x^2 = 11.3^2 + 9.7^2 - 2 \times 11.3 \times 9.7 \times \cos 76.9$

c $x^2 = 17^2 + 20.1^2 - 2 \times 17 \times 20.1 \times \cos 149° 45'$

The cosine rule

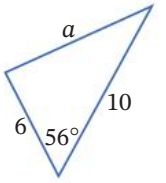
Foundation | Standard | Complex

STAGE 5.3

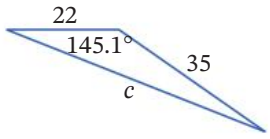
EXAMPLE 22

2 Find, correct to 2 decimal places, the value of each variable.

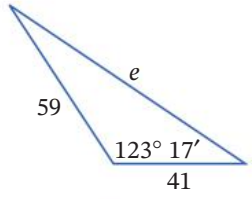
a



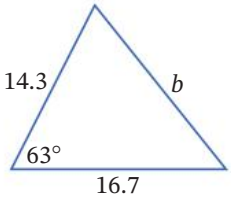
b



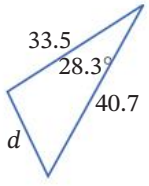
c



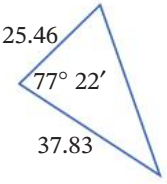
d



e



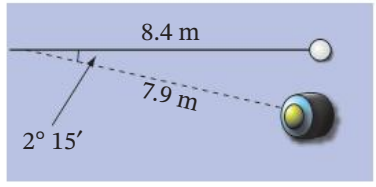
f



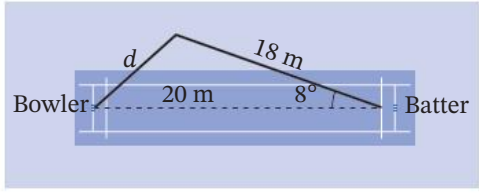
3 In $\triangle DEF$, $\angle E = 38° 46'$, $d = 8.7$ m and $f = 15.6$ m. What is the length of DF? Select the closest answer **A**, **B**, **C** or **D**.

A 10.4 m **B** 19.5 m **C** 17.9 m **D** 12.8 m

4 In a game of lawn bowls, Jayden is aiming to hit the jack (target ball) 8.4 m away. If he bowls 2° 15′ off centre and his bowl travels 7.9 m, how far is his bowl from the jack? Answer correct to one decimal place. PS R

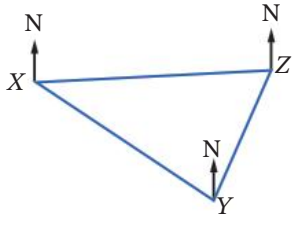


5 In a cricket match, the distance between the bowler and the batter was 20 m. During one bowl, the batter hit the ball at an angle of 8° to the line of the pitch and the bowler ran and caught the ball after it had travelled 18 m. How far did the bowler run to catch the ball? Select **A**, **B**, **C** or **D**. PS R



A 1.1 m **B** 2.0 m **C** 3.3 m **D** 4.0 m

6 A yacht sails from X to Y on a bearing of 130° for 4.2 km. It then turns and travels to Z on a bearing of 025° for 2.9 km. PS R C



a Copy the diagram and mark the given information on it.

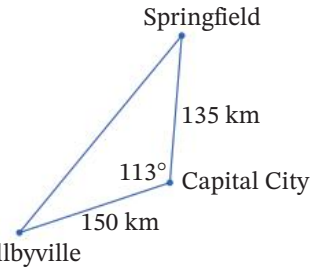
b Explain why $\angle XYZ = 75°$.

c Calculate the distance XZ, correct to one decimal place.

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7 3 towns are joined by straight roads. What distance (correct to the nearest kilometre) is saved by going directly from Springfield to Shellbyville instead of travelling via Capital City? PS R C



8 **a** What is the value of $\cos 90°$?

b What does $c^2 = a^2 + b^2 - 2ab\cos C$ simplify to if $C = 90°$?

c Hence what happens to the cosine rule when it is applied to a right-angled triangle?

9.09

The cosine rule for angles 9.10

STAGE 5.3

If we rewrite the cosine rule so that $\cos A$ is the subject, then we will have a formula for finding an unknown angle when the 3 sides of a triangle are known.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 + 2ab\cos C = a^2 + b^2$$ Adding $2ab\cos C$ to both sides so that $\cos C$ appears on the LHS

$$2ab\cos C = a^2 + b^2 - c^2$$

$$\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

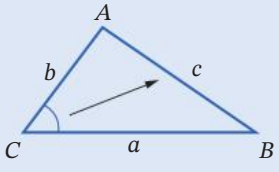
WS Cosine rule problems

The cosine rule for angles

For any triangle ABC:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

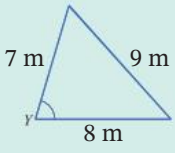
where C is the unknown angle, c is the side opposite C, and a and b are the other 2 sides.



The cosine rule can be used to find an unknown angle if the lengths of the 3 sides are known.

Example 23

Find the size of the marked angle Y, correct to the nearest degree.



Solution

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos Y = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8}$$

$$= \frac{32}{112}$$ 9 m is opposite angle Y

$$Y = 73.398...$$

$$\approx 73°$$ From the diagram, an answer of 73° looks reasonable.

The cosine rule for angles 2

Foundation Standard Complex

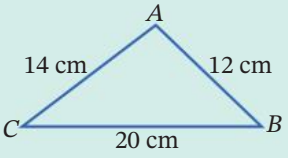
STAGE 5.3

The cosine rule

The cosine rule for angles 1

Example 24

Calculate, correct to the nearest minute, the size of the largest angle in this triangle.



Solution

The largest angle is opposite the longest side, so it is $\angle A$.

$$\cos A = \frac{14^2 + 12^2 - 20^2}{2 \times 14 \times 12}$$

$$= \frac{-60}{336}$$

Cos is negative so the angle will be obtuse.

$$A = 100.28656...$$

$$= 100° 17' 11.6''$$

$$\approx 100° 17'$$

From the diagram, an answer of 100° 17′ looks reasonable.

EXERCISE 9.10 ANSWERS ON P. 627

The cosine rule for angles U F PS R

1 Solve each equation for X, correct to the nearest degree.

a $\cos X = \frac{12^2 + 14^2 - 15^2}{2 \times 12 \times 14}$

b $\cos X = \frac{5.7^2 + 6.8^2 - 3.7^2}{2 \times 5.7 \times 6.8}$

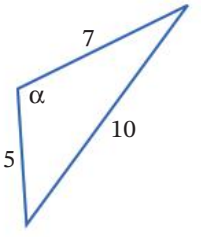
c $\cos X = \frac{5^2 + 6^2 - 9^2}{2 \times 5 \times 6}$

d $\cos X = \frac{9.2^2 + 4.7^2 - 12.8^2}{2 \times 9.2 \times 4.7}$

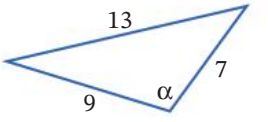
EXAMPLE 23

2 Find the size of α in each triangle, correct to the nearest degree.

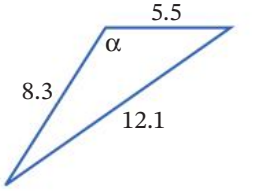
a



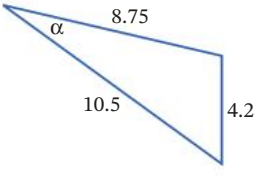
b



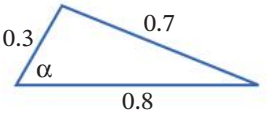
c



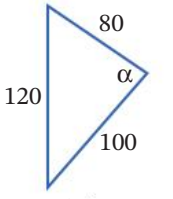
d



e



f



3 In $\triangle ABC$, $a = 8.5$ cm, $b = 12.1$ cm and $c = 17.6$ cm. What is the size of $\angle B$? Select the correct answer **A**, **B**, **C** or **D**.

A 25° 40′ **B** 141° 57′ **C** 116° 17′ **D** 38° 3′

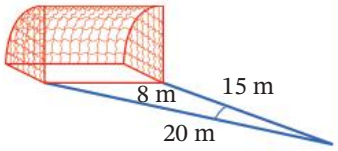
Foundation Standard Complex

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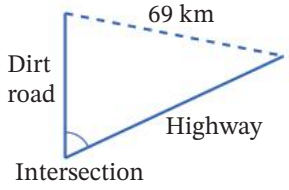
STAGE 5.3

EXAMPLE 24

4 A soccer goal is 8 m wide. A player shoots for goal (along the ground) when 20 m from one post and 15 m from the other post. Within what angle (correct to the nearest 0.1 degree) must the shot be made for the player to have a chance of scoring a goal? PS R



5 2 cars leave an intersection at the same time. Car A drives down the dirt road at 60 km/h and car B drives down the highway at 100 km/h. After 45 minutes they are 69 km apart. Find the angle between the 2 roads, correct to the nearest minute. PS R



6 A triangle has sides of 21 m, 17 m and 10 m. Find the size of the largest angle, correct to the nearest degree. R

7 A plane flies from Alice Springs on a bearing of 245°. After 450 km, the plane changes to a northwesterly direction for 1100 km. When the plane stops it is 1380 km from Alice Springs. What is the bearing (to the nearest degree) of the plane from Alice Springs? PS R

9.10

The area of a triangle 9.11

STAGE 5.3

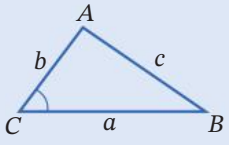
We already know that the formula for the area of a triangle is $A = \frac{1}{2}bh$, but there is also a trigonometric formula if we know the lengths of 2 sides of the triangle and the size of the **included angle** between them.

We've learned about **included angles** with the congruent and similar triangles tests.

The area of a triangle

$$A = \frac{1}{2}ab\sin C$$

where C is the included angle between sides a and b.



Proof:

In $\triangle ABC$, draw AX for the perpendicular height, h, of the triangle.

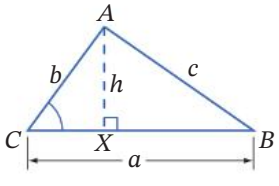
Area of $\triangle ABC = \frac{1}{2}ah$ [1]

But in $\triangle AXC$, $\sin C = \frac{h}{b}$

$\therefore h = b \sin C$

Substituting this into [1]:

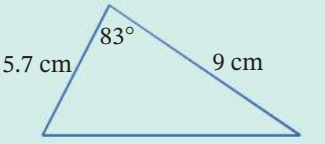
$A = \frac{1}{2}ab\sin C$



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Example 25

Find, correct to one decimal place, the area of this triangle.



Solution

$A = \frac{1}{2}ab \sin C$

$= \frac{1}{2} \times 5.7 \times 9 \times \sin 83°$

$= 25.458...$

$\approx 25.5 \text{ cm}^2$

83° is the included angle between 5.7 cm and 9 cm.

STAGE 5.3

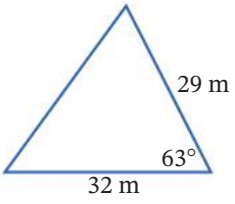
EXERCISE 9.11 ANSWERS ON P. 628

The area of a triangle U F PS R

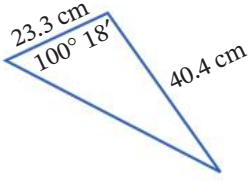
EXAMPLE 25

1 Find, correct to one decimal place, the area of each triangle.

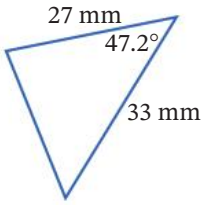
a



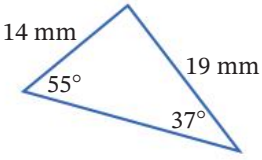
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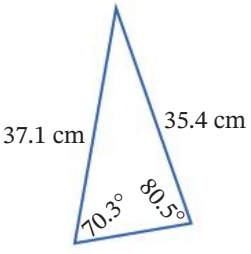
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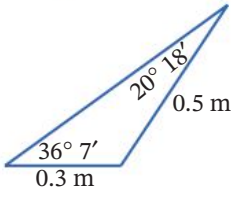
d



e

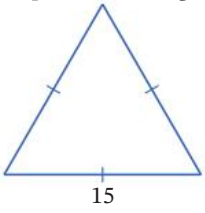


f

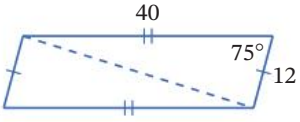


2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres. R

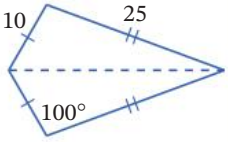
a Equilateral triangle



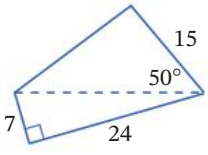
b Parallelogram



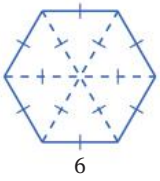
c Kite



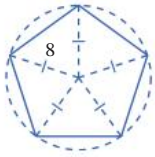
d Quadrilateral



e Regular hexagon



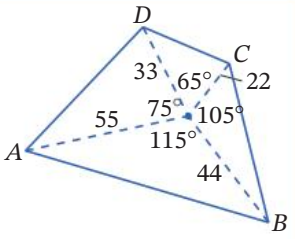
f Regular pentagon inscribed in a circle of radius 8



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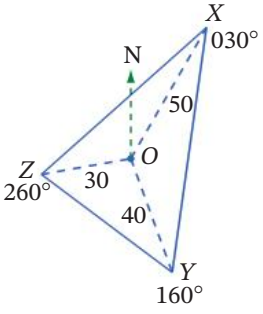
3 The diagram shows the results of a radial survey of a block of land. All distances are in metres. PS R



a Use the cosine rule to find the lengths of AB, BC, CD and AD and, hence, find the perimeter of the block of land (to the nearest metre).

b Use the area formula to find the area of each triangle and, hence, calculate the area of the block of land (to the nearest m^2).

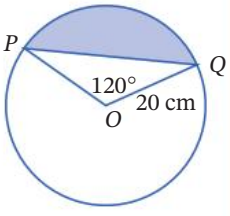
4 The results of a radial survey are shown in the diagram. All measurements are in metres. PS R



a Find the size of $\angle XOY$.

b Calculate, correct to 2 decimal places, the area of $\triangle XOY$.

5 O is the centre of a circle of radius 20 cm. Calculate, correct to one decimal place, the area of: PS R

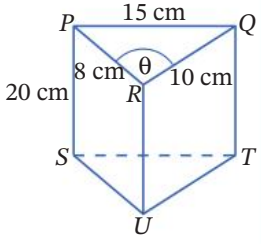


a sector OPQ

b triangle OPQ

c the shaded segment.

6 A triangular prism has base edges of 8 cm, 10 cm and 15 cm, and a height of 20 cm. PS R



a Calculate the size $\angle PRQ$, correct to nearest degree.

b Find the area of $\triangle PQR$, correct to the nearest cm^2.

c Find the volume of the prism.

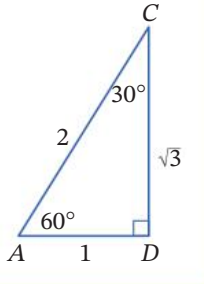
7 The area of an equilateral triangle is 600 cm^2. Calculate the perimeter of the triangle, correct to one decimal place.

Did you know?

Triangle area formulas

There are many formulas for calculating the area, A, of a triangle:

- Heron's formula: given 3 sides a, b, c:
  $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter $\frac{a+b+c}{2}$
- Given 2 sides, a, b and the included angle, C: $A = \frac{1}{2}ab\sin C$
- Given one side, a and 3 angles, A, B, C: $A = \frac{1}{2}a^2\frac{\sin B \sin C}{\sin A}$



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1 **Find the area of this triangle using the 3 formulas, and check that your answers agree.**

2 **Show how $A = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$ can be proved from $A = \frac{1}{2}ab\sin C$ using the sine rule.**

3 **Research other formulas for the area of a triangle.**

9.12 Problems involving the sine and cosine rules

STAGE 5.3

WS Finding an unknown side

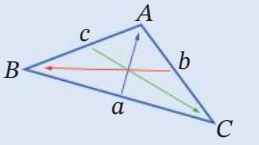
WS The sine and cosine rules

The sine and cosine rules

The sine and cosine rules

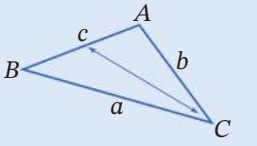
The sine rule is used for triangle problems involving 2 sides and the 2 angles opposite them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

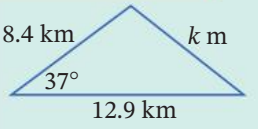


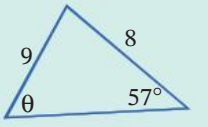
The cosine rule is used for triangle problems involving 3 sides and one angle.

$$c^2 = a^2 + b^2 - 2ab\cos C \text{ and } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Example 26





a Find the value of k, correct to one decimal place.

b Find the value of θ, correct to the nearest minute.

Solution

a The problem involves 3 sides and one angle so use the cosine rule.

$k^2 = 8.4^2 + 12.9^2 - 2 \times 8.4 \times 12.9 \times \cos 37°$

$= 63.889...$

$k = \sqrt{63.889...}$

$= 7.993...$

≈ 8.0 m

b The problem involves 2 sides and the 2 angles opposite them, so use the sine rule.

$$\frac{\sin\theta}{8} = \frac{\sin 57°}{9}$$

$$\sin\theta = \frac{8\sin 57°}{9}$$

$$= 0.7454...$$

$\theta = 48.2007...$

$= 48° 12' 2.77''$

$\approx 48° 12'$

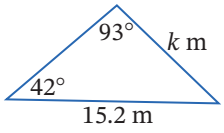
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Problems involving the sine and cosine rules U F PS R C

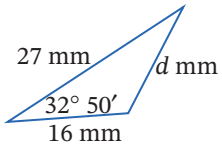
EXAMPLE 26

1 Find, correct to one decimal place, the value of each variable.

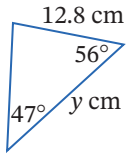
a



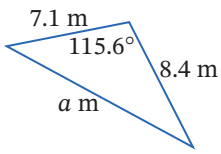
b



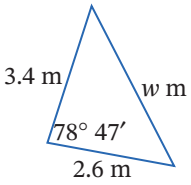
c



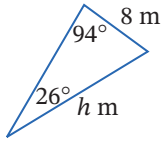
d



e

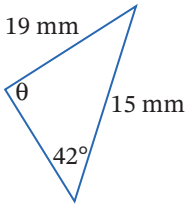


f

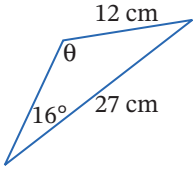


2 Find the value of θ to the nearest degree. Use diagrams to note whether θ is acute or obtuse.

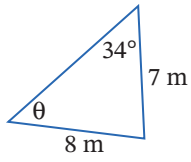
a



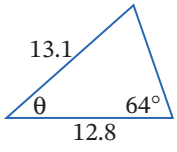
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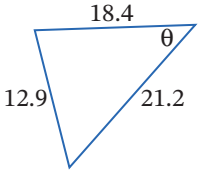
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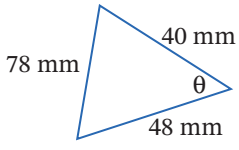
d



e



f

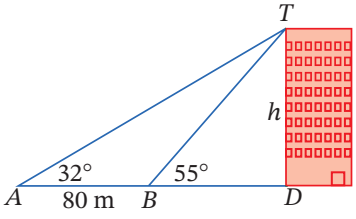


3 The angles of elevation of a building measured from 2 positions 80 m apart are 32° and 55°. PS R C

a Explain why $\angle ATB = 23°$.

b Find, correct to 2 decimal places, the length of BT.

c Hence find the height, h, of the building, correct to the nearest metre.



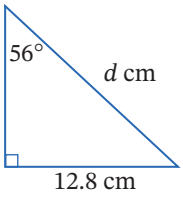
4 **a** What is the value of sin 90°? PS R C

b Find, correct to one decimal place, the value of d using:

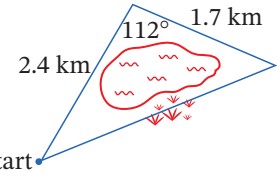
i the sine rule

ii the sine ratio for right-angled triangles.

c What do you notice about your results? Give reasons.



5 Mikayla needs to run around a cross-country course as shown. What is the length of the course, correct to one decimal place? PS R

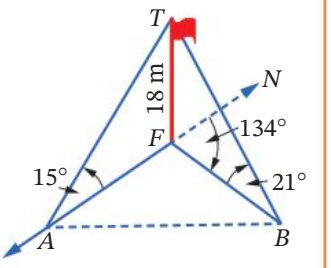


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STAGE 5.3

6 A plane flew on a bearing of 150° for 370 km. It then changed direction and flew another 285 km on a bearing of 235°. How far, correct to the nearest kilometre, is the plane from its starting point? PS R

7 From a point A, south of a flagpole, the angle of elevation to the top of the pole is 15°. From B, the angle of elevation to the top of the flagpole is 21°.

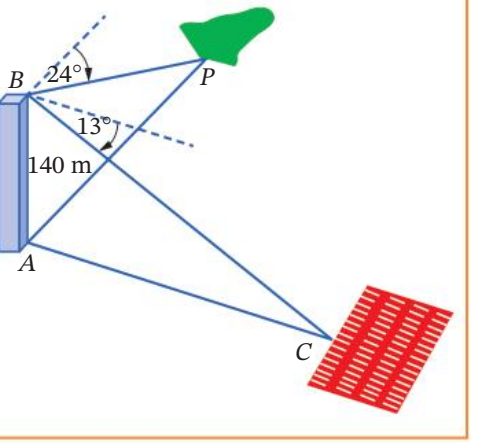


The bearing of B from the flagpole is 134°. Find correct to one decimal place: PS R

a the length of AF and BF

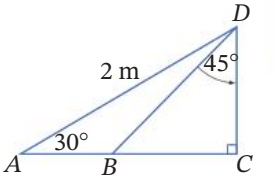
b the distance between A and B

8 From the top of a building, the angles of depression to a park (P) and a car park (C) are 24° and 13° respectively. The height of the building is 140 m, the park is due north of the building and the compass bearing of the car park is ESE from the building. Calculate the distance between the park and the car park, correct to the nearest metre. PS R



Power plus ANSWERS ON P. 628

1 For this diagram, find the exact value of:



a the length of AB

b the area of $\triangle ADB$

2 Find θ if θ is between 0° and 180°, and:

a $\sin\theta = \cos\theta$ **b** $\sin\theta = \sqrt{3}\cos\theta$ **c** $\sqrt{3}\sin\theta = \cos\theta$

d $\cos\theta = -\sqrt{3}\sin\theta$ **e** $\sin\theta = -\sqrt{3}\cos\theta$ **f** $\cos\theta = -\sin\theta$

g $\sin^2\theta = \frac{3}{4}$ **h** $\tan^2\theta = \frac{1}{3}$ **i** $\cos^2\theta = \frac{1}{2}$

3 Show that:

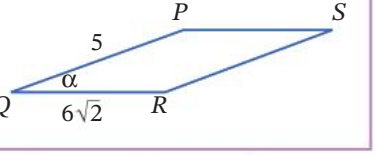
a $\sin(180° - B) + \cos(90° - B) = 2\sin B$

b $\dfrac{\sin(180° - B)}{\cos(180° - B)} = -\tan B$ **c** $\sin(180° - B)\cos(180° - B)\tan(180° - B) = \sin^2 B$

d $\dfrac{(\sin A)^2 + \sin A\cos A}{(\cos A)^2 + \sin A\cos A} = \tan A$

4 Express $\cos 45° + \sin 45°$ as a surd in simplest form.

5 The area of the parallelogram shown is 30 cm². Find 2 possible values for α, correct to the nearest degree.



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CHAPTER 9 REVIEW

Language of maths

angle of depression	angle of elevation	bearing	compass bearing
complementary	cosine rule	degree (°)	denominator
exact ratio	included angle	minute (′)	obtuse
Pythagoras' theorem	right-angled	second (″)	sine rule
slant height	supplementary	surd	tangent (tan)
theta (θ)	three-figure bearing	trigonometric ratio	unit circle

1 Copy and complete: A **bearing** is an _____ used to describe precisely the _____ of one location from a given reference point.

2 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?

3 If 2 angles have a sum of 90°, what special name do we give them?

4 What is an **exact ratio**? Give an example of one.

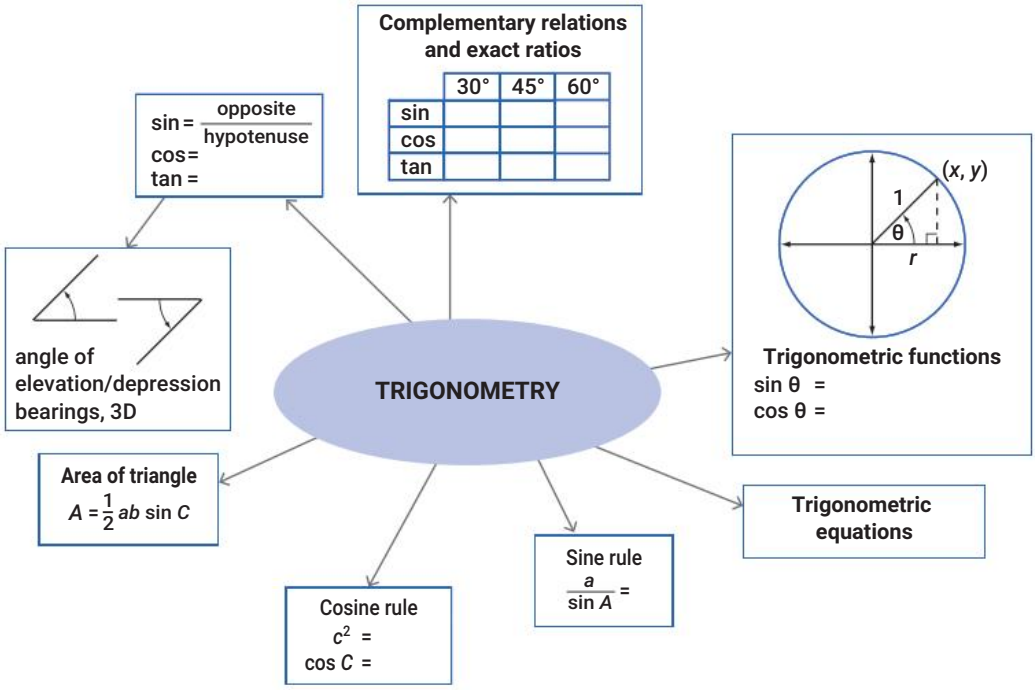
5 Describe the shape of the **sine curve**.

6 What is the name given to the angle that is between 2 known sides?

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS
Mind map: Trigonometry (Advanced)



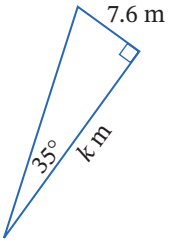
TEST YOURSELF 9

ANSWERS ON P. 628

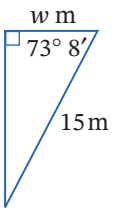
9.01

1 Find, correct to one decimal place, the value of each variable.

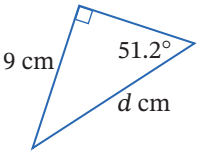
a



b



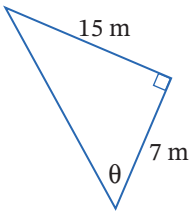
c



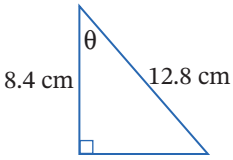
9.01

2 Find the value of θ, correct to the nearest minute.

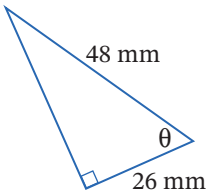
a



b



c



9.01

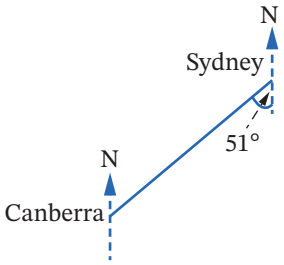
3 From the top of a 300 m cliff Renee observes a boat at a distance of 1.3 km from the base of the cliff. Find, correct to the nearest degree, the angle of depression of the boat.

9.02

4 What is the bearing of:

a Canberra from Sydney?

b Sydney from Canberra?



9.02

5 2 planes leave an airport at the same time. The first travels on a bearing of 206° at 450 km/h. The second travels on a bearing of 296° at 600 km/h.

a How far apart are the planes after 3 hours, to the nearest km?

b Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.

STAGE 5.3

9.03

6 A box in the shape of a square prism has a base of 10 cm by 10 cm and is 30 cm tall. Find to the nearest whole number:

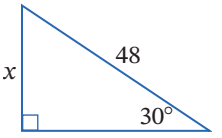
a the length of the longest diagonal of the box

b the angle that the longest diagonal makes with the base.

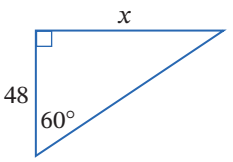
9.04

7 Find the exact value of x in each triangle.

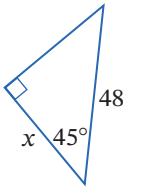
a



b



c



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8 Graph $y = \cos\theta$ for θ from 0° to 360°. 9.05

9 Find the acute angle α if: 9.05

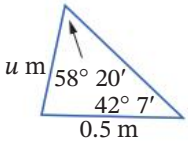
a $\sin 123° = \sin\alpha$ **b** $\tan 93° = -\tan\alpha$ **c** $\cos 110° = -\cos\alpha$

10 Solve each trigonometric equation, giving all possible acute and obtuse solutions correct to the nearest minute. 9.06

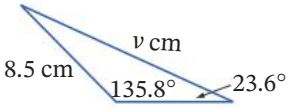
a $\cos x = -\frac{4}{5}$ **b** $\tan x = \frac{15}{7}$ **c** $\sin x = \frac{3}{14}$

11 Find the value of each variable, correct to one decimal place. 9.07

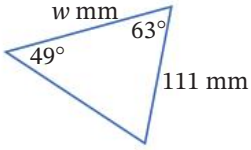
a



b

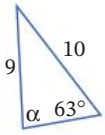


c

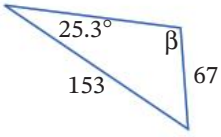


12 Find the value of each variable, correct to the nearest minute. 9.08

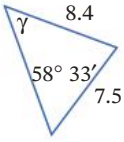
a



b

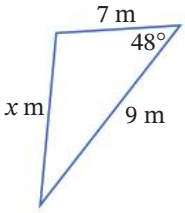


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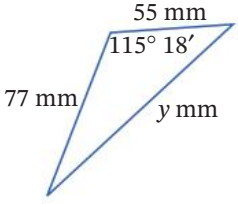


13 Find the value of each variable, correct to one decimal place. 9.09

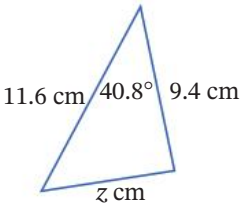
a



b

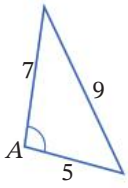


c

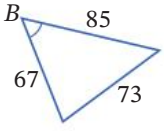


14 Find the size of each marked angle, correct to the nearest degree. 9.10

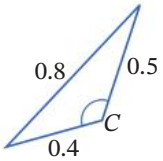
a



b

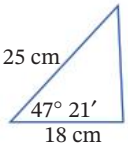


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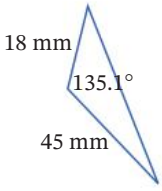


15 Find, correct to the nearest whole number, the area of each triangle. 9.11

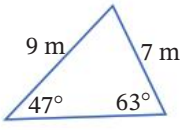
a



b

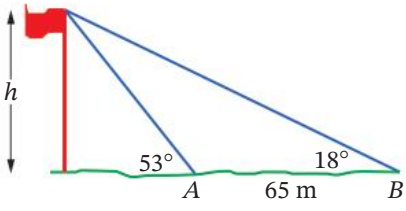


c



16 A parallelogram has sides of 12 cm and 6 cm and one interior angle of 65° 35′. Find the length of the longer diagonal, correct to one decimal place. 9.12

17 The distance AB is 65 m. Calculate the height of the flagpole, correct to one decimal place. 9.12



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10

NUMBER AND ALGEBRA

SIMULTANEOUS EQUATIONS

Many scientific, natural, economic and social phenomena can be modelled by equations. Often these models consist of more than one equation. For example, when manufacturing milk, equations can be written that describe relationships between quantity, cost and income. These equations can then be solved simultaneously to obtain information about pricing and the quantities that need to be produced and sold to make the most profit.



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Chapter outline

		Working mathematically				
10.01	Solving simultaneous equations graphically*	U	F		R	C
10.02	The elimination method*	U	F			
10.03	The substitution method*	U	F			
10.04	Problems involving simultaneous equations*	U	F	PS	R	C

Wordbank

coefficient The numerical part of an algebraic term. For example, in $3x^2 + 7x - 1$ the coefficient of x is 7.

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables

graphical method A method of solving simultaneous equations that involves graphing them on a number plane and identifying the point(s) of intersection

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$.

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- Solve linear simultaneous equations graphically, including using graphing technology
- Solve linear simultaneous equations algebraically using the elimination and substitution methods
- Solve problems using linear simultaneous equations

SkillCheck ANSWERS ON P. 628

1 Given the equation $y = 2x + 5$, find y when:

a $x = 0$ **b** $x = 4$ **c** $x = \frac{1}{2}$ **d** $x = -3$

2 Given the equation $y = 4 - 3x$, find y when:

a $x = 5$ **b** $x = 1$ **c** $x = -1$ **d** $x = -\frac{1}{2}$

3 Test whether the point $(-2, 3)$ lies on the line represented by each equation.

a $y = 1 - x$ **b** $x + y = 3$ **c** $2x - y = 7$

d $\frac{1}{2}x + y = 2$ **e** $y = 3x + 7$ **f** $2y = 3x$

4 **a** Show that the point $(2, 5)$ lies on both the lines $y = 2x + 1$ and $x + y = 7$.

b At what point do these 2 lines intersect?

5 Use the y-intercept and the gradient to graph each equation.

a $y = -2x + 3$ **b** $y = \frac{5}{2}x - 2$ **c** $y = -\frac{4}{3}x + 5$

Investigation



When 2 lines meet

1 Copy and complete the table of values for each equation.

a $x + 2y = 5$

x	−2	−1	0	1	2
y					

b $y = x + 4$

x	−2	−1	0	1	2
y					

2 Which coordinates satisfy both equations?

3 On the same set of axes, draw the graphs of $x + 2y = 5$ and $y = x + 4$.

4 **a** Do the lines you drew in question **3** intersect?

b What are the coordinates of the point of intersection?

5 Repeat questions **1** to **4** for these pairs of equations.

a $x - y = 5$
$2x + y = 1$

b $3x + y = 8$
$x + 2y = 1$

6 Copy and complete.

a The coordinates of the p_____ of intersection between 2 lines satisfy both equations.

b The values of x and y that satisfy both equations are the coordinates of the _____.

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Solving simultaneous equations graphically

10.01

A linear equation such as $3x + 5 = 11$ is in one variable (x) and has only one solution ($x = 2$). However, a linear equation in **2** variables, such as $x + 3y = 5$, has more than one solution (for example, $x = 2, y = 1$, or $x = 5, y = 0$, and so on). The equation actually has an **infinite number** of solutions.

We will now look at solving 2 equations simultaneously to see if there is a solution that satisfies **both** equations. These are called **simultaneous equations**, which can be solved **graphically** or **algebraically**.

Solving simultaneous equations graphically

- Linear simultaneous equations can be graphed as lines on the same number plane.
- If 2 lines are drawn, the lines will intersect (unless they are parallel).
- At the point of intersection, the x-coordinate and y-coordinate represent the solution to the simultaneous equations.

Example 1

On the same set of axes, graph $3x + y = 4$ and $x + y = -2$, then solve the equations simultaneously.

Solution

Step 1

Construct tables of values (or convert each equation to $y = mx + c$ form to find the gradient and y-intercept).

$3x + y = 4$

x	0	1	2
y	4	1	−2

$x + y = -2$

x	0	1	2
y	−2	−3	−4

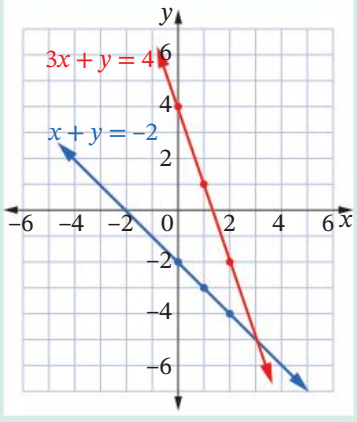
Step 2

Graph the equations.

The lines intersect at $(3, -5)$.

$\therefore$ The solution of the simultaneous equations $3x + y = 4$ and $x + y = -2$ is $x = 3, y = -5$.

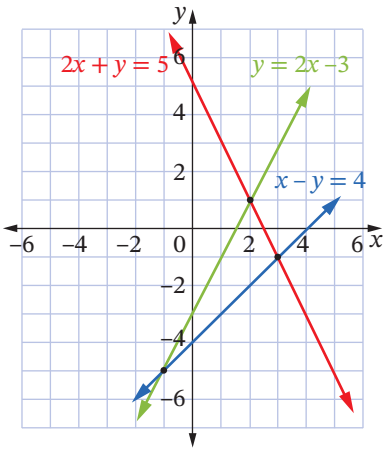
Check that $x = 3, y = -5$ satisfies both equations.



EXERCISE 10.01 ANSWERS ON P. 628

Solving simultaneous equations graphically UFRC

1 Use the graph to write the solution to each pair of simultaneous equations. C



a $x - y = 4$ and $2x + y = 5$

b $2x + y = 5$ and $y = 2x - 3$

c $x - y = 4$ and $y = 2x - 3$

EXAMPLE 1

2 Graph each pair of equations on the same set of axes. Then find the solution to the pair of simultaneous equations.

a $y = 2x$ and $y = 3 - x$

b $y = 2x + 1$ and $y = x - 4$

c $x + y = 3$ and $4x + y = 6$

d $y = -x + 2$ and $y = 3x + 4$

e $y = 2x - 5$ and $y = 5x + 1$

f $2x + y = 6$ and $y = 1 - x$

g $y = 7 - x$ and $y = 3x + 5$

h $x + 2y = 7$ and $2x - y = 4$

i $3x - 2y = 12$ and $x + 2y = 8$

j $y = x + 3$ and $2x - y = 2$

k $5x - y = 5$ and $x + y = 4$

l $5x + 3y = 20$ and $y = x - 4$

3 **a** On the same set of axes, draw the graphs of $y = 1 - 2x$ and $2x + y = 4$.

b Why isn't there a solution to the simultaneous equations $y = 1 - 2x$ and $2x + y = 4$? R C

Technology

Solving simultaneous equations graphically

You can use graphing technology to solve simultaneous equations graphically.
Write each answer as coordinates in the form (x, y) representing the point of intersection.

1 Use the graphing software to graph these linear equations.

$y = -x + 1$

$y = x + 3$

2 Find the coordinates of the point of intersection of the lines.

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3 Repeat steps **1** and **2** to solve each pair of simultaneous equations.

a $y = 2$, $x = 3$ **b** $y = -2x + 4$, $y = x - 5$ **c** $y = 5x + 2$, $y = 3x - 1$ **d** $y = -1$, $x = 0$

e $y = -x - 8$, $y = -3x + 4$ **f** $y = 2x + 6$, $y = x + 9$ **g** $2x - y = 5$, $x + y = 4$ **h** $3x + y = 4$, $x + 2y = 3$

4 a Use the graphing software to graph each pair of simultaneous equations.

i $y = x + 4$, $y = x + 6$ **ii** $y = -2x + 2$, $y = -2x$

b What do you notice about these 2 pairs of equations? Do they intersect?

5 Graph each set of equations and find their point of intersection.

a $y = 3x$, $y = -x + 2$ and $x = 0.5$ **b** $y = -4x + 1$, $y = -5x$ and $y = x + 6$

The elimination method 10.02

Using graphs to solve simultaneous equations can be time-consuming and inaccurate.

Algebraic methods provide a better way of solving things. There are 2 algebraic methods: the **elimination method** and the **substitution method**.

In the elimination method, equations are added or subtracted to eliminate one of the variables.

Example 2

Simultaneous equations

Solve the simultaneous equations $x + 3y = 7$ and $4x - 3y = 13$ using the elimination method.

Solution

Label each equation.

$x + 3y = 7$ [1]

$4x - 3y = 13$ [2]

Since there is the same number of ys in each equation, and since they are opposite in sign ($3y$ and $-3y$), add equations [1] and [2] to eliminate the variable y.

$5x = 20$ [1] + [2]

$\frac{5x}{5} = \frac{20}{5}$

$\therefore x = 4$

Substitute $x = 4$ into equation [1] to find the y-value.

$x + 3y = 7$

$4 + 3y = 7$

$4 + 3y - 4 = 7 - 4$

$3y = 3$

$\therefore y = 1$

$\therefore$ The solution is $x = 4$, $y = 1$.

Check that this solution works for both equations [1] and [2].

Example 3

Solve the simultaneous equations $2k + 3m = 9$ and $2k - 5m = 1$.

Solution

Label each equation.

$2k + 3m = 9$ [1]

$2k - 5m = 1$ [2]

Since there is the same number of ks in each equation, and because they have the same sign ($2k$ and $2k$), subtract equation [2] from [1] to eliminate k.

$8m = 8$ [1] – [2]

$\frac{8m}{8} = \frac{8}{8}$

$\therefore m = 1$

Substitute $m = 1$ into equation [1] to find the value of k.

$2k + 3m = 9$

$2k + 3 \times 1 = 9$

$2k + 3 = 9$

$2k + 3 - 3 = 9 - 3$

$2k = 6$

$\frac{2k}{2} = \frac{6}{2}$

$\therefore k = 3$

$\therefore$ The solution is $m = 1$, $k = 3$.

Check that this solution works for both equations [1] and [2].

Example 4

Solve $3a + 4c = 8$ and $2a - 3c = 11$.

Solution

Label each equation.

$3a + 4c = 8$ [1]

$2a - 3c = 11$ [2]

In this case, neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate c. We need to make the **coefficient** of c the same in both equations ($12c$).

The **coefficient** of c is the number in front of the c in the equation

$9a + 12c = 24$ [3] Multiplying both sides of [1] by 3

$8a - 12c = 44$ [4] Multiplying both sides of [2] by 4

$17a = 68$ [3] + [4]

$\therefore a = 4$

Substitute $a = 4$ in [1] to find c.

$3a + 4c = 8$

$3 \times 4 + 4c = 8$

$12 + 4c = 8$

$4c = -4$

$c = -1$

$\therefore$ The solution is $a = 4$, $c = -1$.

EXERCISE 10.02 ANSWERS ON P. 629

The elimination method U F

1 For each pair of simultaneous equations, eliminate one variable by adding the equations, then solve the equations. EXAMPLE 2

a $4k + d = 5$, $2k - d = 7$

b $2x - w = 6$, $x + w = 9$

c $3g + 5h = 4$, $2g - 5h = 6$

d $7p - 4n = -20$, $3p + 4n = 10$

e $4q + 3r = 8$, $-q - 3r = 7$

f $-5k - 3x = 8$, $5k + 4x = -3$

2 For each pair of simultaneous equations, eliminate one variable by subtracting the equations, then solve the equations. EXAMPLE 3

a $5k + d = 16$, $3k + d = 4$

b $4a + 3c = 7$, $a + 3c = 4$

c $4h + 3y = 24$, $4h - y = 8$

d $3x + 5e = 16$, $3x - 2e = -5$

e $4q - 2w = -1$, $7q - 2w = 8$

f $6p + 5c = 39$, $4p + 5c = 31$

3 Solve each pair of simultaneous equations. EXAMPLE 4

a $3w + q = 6$, $2w - 3q = 15$

b $2x + m = 5$, $3x + 2m = 3$

c $2d + 3h = 25$, $d + 4h = -5$

d $-3g + 2n = 9$, $g + 5n = 14$

e $5m - h = 10$, $m - 3h = 2$

f $2y + 3e = -6$, $5y - 2e = 23$

g $3q - 2w = 11$, $2q - 5w = 22$

h $5a + 3d = 4$, $4a + 2d = 3$

i $-2p + 3k = 19$, $7p + 4k = 6$

j $5a + 2f = -14$, $2a - 3f = 2$

k $5r - 3c = 2$, $-3r + 2c = -14$

l $5y - 4x = 1$, $2y - 3x = 6$

m $3x + 4y = 20$, $2x - 5y = 21$

n $7g + 3h = 39$, $3g + 5h = 26$

o $5w - 3k = 25$, $3w - 7k = 28$

p $3x + 5y = 13$, $4x - 3y = 18$

q $m + 8p = 25$, $4m + 5p = -11$

r $9k - 4d = 12$, $5k - 7d = 8$

10.03 The substitution method

With the **substitution method**, **substitute** the x or y from one equation into the other equation.

Simultaneous equations order activity

Simultaneous equations by substitution

Example 5

Solve the simultaneous equations $y = x + 4$ and $y = 3x - 2$.

Solution

Label each equation.

$y = x + 4$ [1]

$y = 3x - 2$ [2]

Use equation [1] to substitute for y in equation [2] and solve for x.

$$x + 4 = 3x - 2$$
$$x + 4 - 3x = 3x - 2 - 3x$$
$$-2x + 4 = -2$$
$$-2x + 4 - 4 = -2 - 4$$
$$-2x = -6$$
$$\frac{-2x}{-2} = \frac{-6}{-2}$$
$$x = 3$$

Now substitute $x = 3$ into equation [1] to find y.

$y = x + 4$

$y = 3 + 4 = 7$

$\therefore$ The solution is $x = 3$ and $y = 7$.

Simultaneous equations

Example 6

Solve the simultaneous equations $5x + 3y = 9$ and $y = 7 - 3x$.

Solution

Label each equation.

$5x + 3y = 9$ [1]

$y = 7 - 3x$ [2]

Since y is the subject in [2], substitute equation [2] into equation [1] to give an equation using x only.

$$5x + 3(7 - 3x) = 9$$
$$5x + 21 - 9x = 9$$
$$-4x = -12$$
$$\frac{-4x}{-4} = \frac{-12}{-4}$$
$$x = 3$$

Now substitute $x = 3$ into equation [2] to find y.

$y = 7 - 3x$

$y = 7 - 3 \times 3$

$= -2$

$\therefore$ The solution is $x = 3$ and $y = -2$.

EXERCISE 10.03 ANSWERS ON P. 629

The substitution method U F

EXAMPLE 5

EXAMPLE 6

10.03

1 Use the substitution method to solve each pair of simultaneous equations.

a $y = 2x + 1$ and $y = x + 3$

b $y = 5 - 2x$ and $y = 3x + 2$

c $x = 3 + 2y$ and $x = 9 - y$

d $y = -x$ and $y = 3x - 8$

2 Solve each pair of simultaneous equations.

a $y = 2x + 3$ and $3x - y = 6$

b $y = x - 2$ and $3x + y = 18$

c $y = 1 - 4x$ and $4x + 2y = 3$

d $x = 2y - 5$ and $4x - y = -13$

e $x = 3y - 4$ and $5x - 4y = 2$

f $x = 5 - 3y$ and $4y - x = 23$

g $2x - 5y = -1$ and $y = 10 - x$

h $6y - 2x = 9$ and $y = \frac{x+2}{2}$

i $x = \frac{9-y}{3}$ and $3x + 2y = 10$

j $y = 3x + 5$ and $4x - 3y = 1$

k $3y + 5d - 20 = 0$ and $d = 7 - 2y$

l $w = \frac{3a+2}{5}$ and $2w + 7a = 16$

Investigation

Elimination or substitution method?

With 2 algebraic methods of solving simultaneous equations, often it is more efficient to use one method than the other.

1 Consider these pairs of simultaneous equations.

a $x - 2y = 9$ [1]
$3x + 2y = 11$ [2]

b $4a + 3c = 18$ [1]
$4a - 3c = -6$ [2]

c $3a - 2y = -5$ [1]
$2a + 5y = 3$ [2]

i Why might the **elimination method** be the more appropriate method to use with these equations?

ii What feature in the pairs of equations do you look for to decide if the elimination method is the best one to use?

iii Solve the 3 pairs of simultaneous equations using the elimination method.

2 Consider these pairs of simultaneous equations.

a $m = 2p$ [1]
$m + p = 15$ [2]

b $m = 4 - p$ [1]
$4m - 3p = -6$ [2]

c $p = 2m - 5$ [1]
$5m - 3p = 11$ [2]

i Why might the **substitution method** be the more appropriate method to use with these equations?

ii What feature in the pairs of equations do you look for to decide if the substitution method is the best one to use?

iii Solve the 3 pairs of simultaneous equations using the substitution method.

Foundation | Standard | Complex

3 Using whichever method is more efficient, solve each pair of simultaneous equations.

a $7c + 2y = 13$ [1]
$3c + 2y = 1$ [2]

b $m = 5 - k$ [1]
$2m - k = 4$ [2]

c $3x + 8y = 10$ [1]
$x = 3 - 2y$ [2]

d $4h - 3w = 8$ [1]
$4h + 7w = 12$ [2]

e $3d = q$ [1]
$q + 4d = 14$ [2]

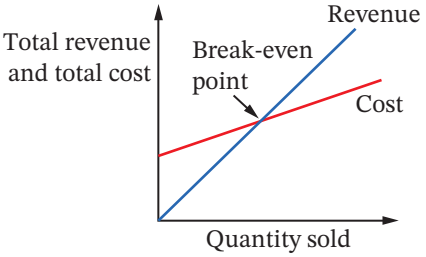
f $3h + 5r = 7$ [1]
$2h - 3r = -8$ [2]

Did you know?

Break-even point

Manufacturers use simultaneous equations to make decisions about how many products they should make and sell. Linear equations can be formed to determine total revenue (the amount made from selling products) and total costs (the cost of making the products).

Total revenue = cost per item × number of items made, while total costs includes rent and production costs.



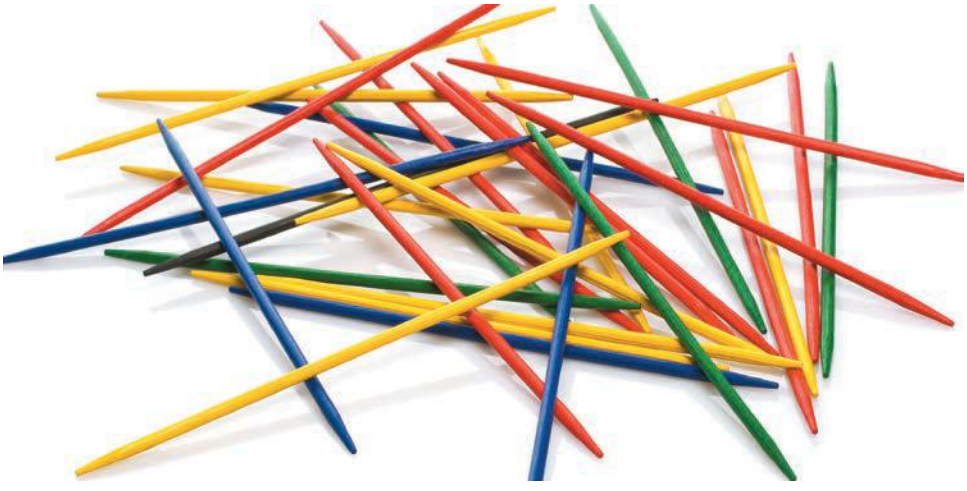
The equations can be graphed as shown. The point where the 2 lines intersect is called the **break-even point** and occurs when total revenue is equal to total cost.

A publisher receives $35 per book sold. There are fixed costs of $110 000 and production costs per book are $8.50.

a **Determine the equations for total revenue and total costs.**

b **Graph the equations to find the break-even point.**

c **How many books must be sold before the publisher makes a profit?**



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Problems involving simultaneous equations

10.04

Sometimes, worded problems can be solved using simultaneous equations. In these situations, follow these steps:

- Read the problem carefully
- Identify the variables to be used
- Use the variables to write simultaneous equations from the information given in the problem
- Solve the equations
- Solve the problem by answering in words

WS Simultaneous equations problems

Simultaneous equations

Example 7

At an art show there were 520 guests. If there were 46 more women than men, how many women attended the show?

Solution

Let the number of women attending be w.

Let the number of men attending be m.

$w + m = 520$ [1] 520 people altogether.

$w = m + 46$ [2] 46 more women than men.

Use equation [2] to substitute for w in equation [1].

$$m + 46 + m = 520$$
$$2m + 46 = 520$$
$$2m + 46 - 46 = 520 - 46$$
$$2m = 474$$
$$m = 237$$

Substitute $m = 237$ into equation [2] to find w.

$w = 237 + 46 = 283$

$\therefore$ There were 283 women who attended the art show.



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Example 8


Simultaneous equations problems

Masi and Amir spent $1582 on shrubs and trees to plant on their large block of land. Altogether they bought 73 plants. The shrubs cost $19 each while the trees cost $32 each. How many of each plant did they buy?

Solution

Let x be the number of shrubs.

Let y be the number of trees.

$\therefore x + y = 73$ [1]

and $19x + 32y = 1582$ [2]

Neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate x.

We will need to make the coefficient of x the same in both equations ($19x$).

$19x + 32y = 1582$ [2]

$19x + 19y = 1387$ [3] Multiplying both sides of equation [1] by 19.

$13y = 195$ [2] – [3]

$y = 15$

Substitute $y = 15$ in [1] to find the value of x.

$x + y = 73$

$x + 15 = 73$

$x = 58$

So Masi and Amir bought 58 shrubs and 15 trees.

EXERCISE 10.04 ANSWERS ON P. 629

Problems involving simultaneous equations U F PS R C

EXAMPLE 7

1 At a school concert there were 640 guests. There were 70 more women than men. How many of the audience were men? R C

2 At a circus, there were twice as many children as there were adults in attendance. Altogether, 1020 attended the circus. How many were children? R C

EXAMPLE 8

3 Tickets to a concert cost $15 for children and $28 for adults. Altogether, 650 people attended the concert and ticket sales totalled $13 052. Let a stand for the number of adults and c stand for the number of children in the audience. R C

a Explain why the equations $a + c = 650$ and $28a + 15c = 13\,052$ correctly match the information.

b Solve the equations simultaneously to find the number of children that attended the concert.

4 Ashleigh bought a total of 130 meat pies and sausage rolls for the canteen of the local football club. Each meat pie cost her $3 and the sausage rolls were $2 each. Altogether, Ashleigh spent $335. How many sausage rolls did she buy? PS R C

5 Tayyab is 3 times as old as Sejuti. The sum of their ages is 48. How old are Tayyab and Sejuti? PS R C

6 The sum of the ages of Hayley and her mother is 70. The difference between their ages is 38 years. How old is Hayley? PS R C

7 A business bought a total of 60 ink cartridges. Some of them were black, costing $35 each. The others were colour, each costing $49. How many of each type did the business buy if the total cost of the ink cartridges was $2422? PS R C

8 The cost of going to the movies for 3 adults and 5 children is $142, while the cost for 5 adults and 8 children is $231.50. Find the cost for an adult and the cost for a child. PS R C

9 Pete's Pizzas sells Supreme pizzas for $15.90 each and vegetarian pizzas for $13.50 each. If 45 pizzas were sold at lunchtime, totalling $684.30, how many of each pizza were sold? PS R C

10 Ronel bought 4 punnets of strawberries and 7 punnets of blueberries for $40.30 and Nikita bought 5 punnets of strawberries and 2 punnets of blueberries for $26.75. What was the cost of each punnet of strawberries and blueberries? PS R C

11 A money box contains only 20-cent coins and 50-cent coins. Altogether, there are 853 coins in the money box and they amount to $281. Let x be the number of 20c coins and y be the number of $2 coins. R C

a Explain why the equations $x + y = 853$ and $20x + 50y = 28\,100$ correctly match the information.

b Solve the equations to determine the number of 20-cent and 50-cent coins in the money box.

12 The initial cost for producing bottles of fresh orange juice is $135 plus $1.20 for each bottle. The bottles of juice are sold for $3 each. C is the cost in dollars, R is the total sales in dollars and n is the number of bottles produced and/or sold. PS R C

a Explain why the equations $C = 135 + 1.2n$ and $R = 3n$ correctly match the information.

b Copy and complete the tables of values below for both equations.

$C = 135 + 1.2n$

n	0	50	100
C			

$R = 3n$

n	0	50	100
R			

c Draw the graphs of both equations on the same axes for values of 0 to 100 for n on the horizontal axis and values of $0 to $300 on the vertical axis.

d For what value of n is total sales equal to total cost (the break-even point)?

Mental skills 10: Maths without calculators ANSWERS ON P. 629

Percentage increase and decrease

1 Study each example.

a Increase 360 by 25%.

$25\% \text{ of } 360 = \frac{1}{4} \times 360$

$= 360 \div 4$

$= 90$

$360 + 90 = 450$

b Increase \$80 by 5%.

$5\% \text{ of } \$80 = \frac{1}{20} \times \$80$

$= \$80 \div 20$

$= \$4$

$\$80 + \$4 = \$84$

or $10\% \text{ of } \$80 = \$8$

$\therefore 5\% \text{ of } \$80 = \$8 \div 2$

$= \$4$

2 Now increase:

a \$340 by 20%

b 66 by 50%

c 150 by $33\frac{1}{3}\%$

d \$400 by 1%

e 640 by 5%

f \$72 by $12\frac{1}{2}\%$

g \$470 by 10%

h 180 by 25%

i 420 by $33\frac{1}{3}\%$

j \$80 by 5%

k \$280 by 25%

l 70 by 20%

3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$.

$33\frac{1}{3}\% \text{ of } \$225 = \frac{1}{3} \times \$225$

$= \$225 \div 3$

$= \$75$

$\$225 - \$75 = \$150.$

b Decrease \$70 by 15%

$10\% \text{ of } \$70 = \frac{1}{10} \times \$70 = \$7$

$\therefore 5\% \text{ of } \$70 = \frac{1}{2} \times \$7 = \$3.50$

$\therefore 15\% \text{ of } \$70 = (10\% \times \$70) + (5\% \times \$70)$

$= \$7 + \$3.50$

$= \$10.50$

$\$70 - \$10.50 = \$59.50$

4 Now decrease:

a 440 by 25%

b \$300 by 20%

c 2400 by $33\frac{1}{3}\%$

d \$500 by 15%

e \$250 by 10%

f \$120 by 50%

g \$72 by $12\frac{1}{2}\%$

h 80 by 5%

i \$85 by 20%

j \$3800 by 1%

k \$440 by 15%

l \$150 by $33\frac{1}{3}\%$

Power plus ANSWERS ON P. 629

1 With simultaneous equations in 2 variables, we have 2 equations to solve.
With simultaneous equations in 3 variables, we have 3 equations to solve.

Step 1: Take 2 of the equations and eliminate one of the variables.

Step 2: Take another 2 of the equations and eliminate the same variable.

Step 3: Solve the 2 new simultaneous equations from Steps **1** and **2**.

Step 4: Use substitution to find the values of the other 2 variables.

Use the above steps to solve the following sets of simultaneous equations.

a $2x + y - 3w = -16$
$x - y + 4w = 25$
$3x - y + 2w = 19$

b $3a - 2c + d = 5$
$5a + 2c + d = 25$
$4a + 3c - d = 10$

c $2m + 3n - p = 9$
$3m - 2n + 5p = 27$
$4m + 3n + 2p = 13$

2 **a** Show that the solutions to the simultaneous equations $ax + by = c$ and $dx + ey = f$ are $x = \frac{ce - bf}{ae - bd}$ and $y = \frac{af - cd}{ae - bd}$.

b The above solutions do not work when $ae = bd$. Explain why.

c Solve the equations $3x - 2y = 11$ and $5x + y = 14$ by either the substitution or elimination method. Check that the results in part **a** give the same answer.

d Set up a spreadsheet to solve simultaneous equations of the form $ax + by = c$ and $dx + ey = f$ using the solutions $x = \frac{ce - bf}{ae - bd}$ and $y = \frac{af - cd}{ae - bd}$.

Use your spreadsheet to solve each pair of simultaneous equations.

i $3x + y = 4$
$2x - y = 6$

ii $3x - 5y = 4$
$2x - 3y = 8$

iii $15x + 6y = 17$
$2x + 3y = 8$

CHAPTER 10 REVIEW

Language of maths

algebraic	axes	coefficient	elimination method
graphical	linear	point of intersection	satisfy
simultaneous equations	solution	substitution method	variable

1 How do you think **simultaneous equations** got their name?

2 What are the 2 algebraic methods for solving simultaneous equations?

3 Which algebraic method involves cancelling one of the variables?

4 What word means the answer to an equation or problem?

5 What does '**linear**' mean?

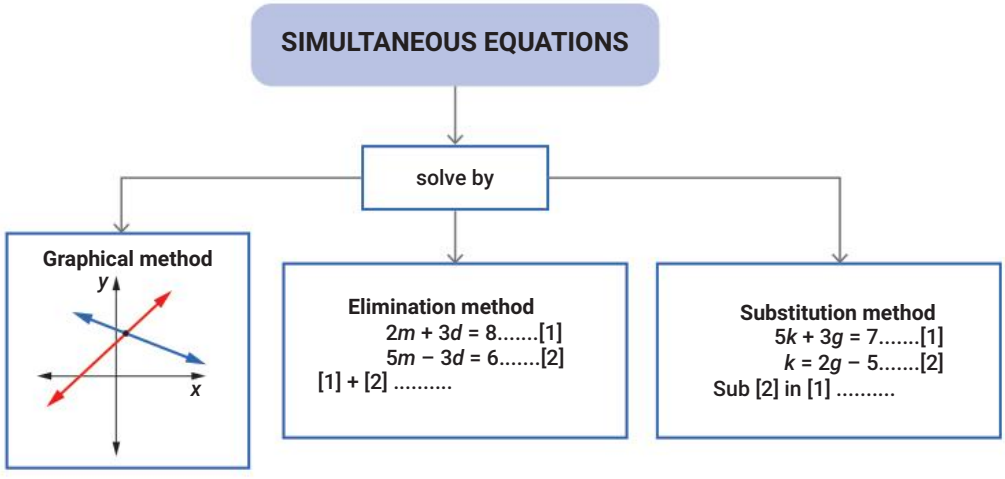
6 Which method of solving simultaneous equations involves finding the point of intersection of lines on a number plane?

Topic summary

WS
Mind map: Simultaneous equations

- In your own words, write down the new things you have learnt about simultaneous equations.
- What parts of this topic did you like?
- What parts of the topic did you find difficult or not understand?
- Copy and complete the following topic overview, and refer to the **Language of maths** word list for keywords you might like to include.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 10

ANSWERS ON P. 629

1 Graph each pair of simultaneous equations on the same set of axes. By finding their point of intersection, write the solution to each pair of equations. 10.01

a $y = x + 2$
$y = 6 + 2x$

b $y = 3 - \frac{x}{2}$
$y = 2x - 7$

c $y = 4 - 3x$
$y = x$

d $y = 2x + 3$
$y = 9 - x$

e $x + y = 7$
$y = 2x + 1$

f $y = 5 - 2x$
$y = -1 - x$

2 Use the elimination method to solve each pair of simultaneous equations. 10.02

a $5m + 2c = 6$
$3m + 2c = -4$

b $2x + 3y = 5$
$5x - 3y = 9$

c $3a + 4d = 7$
$3a + d = 4$

d $4x - y = 9$
$x - y = -9$

e $x - 4y = 3$
$x + 2y = -9$

f $3d - 2w = 11$
$2d - 5w = 44$

3 Use the substitution method to solve each pair of simultaneous equations. 10.03

a $y = 7x - 3$
$y = x + 9$

b $m = 4 - p$
$m = -2 + p$

c $h = 3t - 2$
$h = t + 6$

d $a = 4 - 2c$
$a = 6c$

e $x + 2y = 3$
$y = 2 - x$

f $p = 4 - 2q$
$p = 3q + 24$

4 Solve each problem using simultaneous equations. 10.04

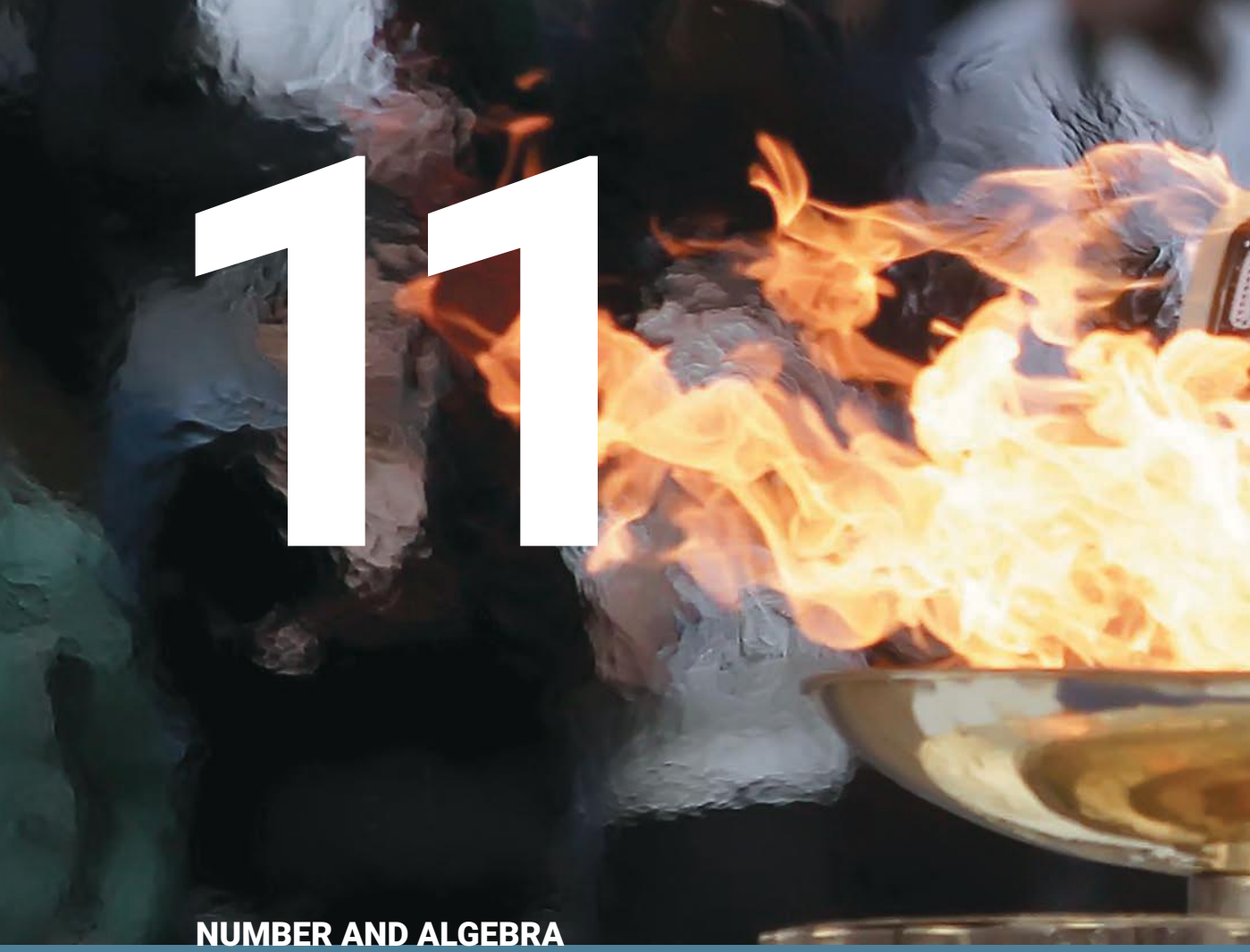
a In an audience of 2500, there were 700 more adults than children. Find the number of adults and the number of children that were in the audience.

b Rosanna bought 30 movie passes as raffle prizes at a school fete for a total cost of $322.50. Movie passes for children cost $8.25 each and the adult price was $14.50.
How many of each did she buy?

c It costs 2 adults and 5 children $191 to go to a football game, while the cost for 3 adults and 2 children is $160. Find the cost of an adult ticket.

d At the cake stall, the student council sell 2 types of cakes: cheesecakes for $4 each and mudcakes for $3 each. In total, they sold 75 cakes for a total of $253. How many of each cake did they sell?

e In Year 10, there are 213 students. There are 27 more boys than girls. Find the number of boys and girls in Year 10.



11

NUMBER AND ALGEBRA

QUADRATIC EQUATIONS AND THE PARABOLA

Every 4 years, a few months before the Olympic Games Opening Ceremony, the Olympic flame is lit in a symbolic ceremony near the Temple of Hera, at the sacred site of the Ancient Olympics in Greece.

The flame is lit by traditional means, using the sun's rays, to demonstrate its purity.

To ensure it is lit by natural means, a parabolic mirror is used, which acts as a giant magnifying glass.

The Olympic flame has been used to light the Olympic cauldron for every Olympic Games Opening ceremony since the Summer Olympic Games held in Berlin, Germany in 1936.



Chapter outline

		Working mathematically				
11.01	Quadratic equations $ax^2 + bx + c = 0$*	U	F	PS	R	C
11.02	Completing the square*	U	F		R	C
11.03	The quadratic formula*	U	F			C
11.04	Mixed problems*	U	F		R	
11.05	Higher-order quadratic equations*#	U	F		R	
11.06	Quadratic equation problems*	U	F	PS	R	C
11.07	The parabola $y = ax^2 + bx + c$*	U	F		R	C
11.08	The axis of symmetry and vertex of a parabola*	U	F		R	C
11.09	Non-linear simultaneous equations*#	U	F		R	C

***STAGE 5.3**

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

axis of symmetry The vertical line that divides a parabola in half and passes through the parabola's vertex

coefficient The number in front of a variable, for example, in $y = 3x^2 + 4x - 6$, the coefficient of x^2 is 3

quadratic equation An equation in which the highest power of the variable is 2, for example, $2x^2 - 12x + 10 = 0$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $3x^2 + 4x - 6$

quadratic formula A formula for solving quadratic equations of the form $ax^2 + bx + c = 0$

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true

vertex A turning point of a curve; a parabola has one vertex

***x*-intercept** The x value(s) at which a graph intersects the x-axis

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) solve quadratic equations of the form $ax^2 + bx + c = 0$ by factorising, completing the square and using the quadratic formula
- (STAGE 5.3) solve problems involving quadratic equations
- (STAGE 5.3) solve higher-order quadratic equations such as $x^4 - 2x^2 - 8 = 0$
- (STAGE 5.3) graph parabolas of the form $y = ax^2 + bx + c$ and identify their features
- (STAGE 5.3) find the axis of symmetry and the vertex of a parabola
- (STAGE 5.3) find the point of intersection of a line with a parabola, circle or hyperbola by solving linear and non-linear simultaneous equations

SkillCheck ANSWERS ON P. 630

1 Solve each quadratic equation.

a $4x^2 = 100$ **b** $3m^2 = 12$ **c** $x^2 - 81 = 0$

d $12k^2 = 3$ **e** $4u^2 - 49 = 0$ **f** $7w^2 - 63 = 0$

2 Factorise each expression.

a $16 - m^2$ **b** $d^2 - 121$ **c** $14y - 2y^2$

d $10p^2 + 25p$ **e** $5x^2 - 320$ **f** $18w^2 - 50$

g $k^2 + 5k + 4$ **h** $y^2 - 10y + 16$ **i** $m^2 - m - 56$

j $u^2 + 8u - 65$ **k** $w^2 - 10w + 21$ **l** $x^2 - 2x - 24$

3 If $y = x^2 + 4x - 7$, find the value of y if:

a $x = 1$ **b** $x = -1$ **c** $x = 2$ **d** $x = -\frac{1}{2}$

4 Factorise each expression.

a $3a^2 + 10a + 3$ **b** $5x^2 - 13x - 6$ **c** $6y^2 + y - 40$

d $15t^2 + 7t - 4$ **e** $5v^2 - 32v - 21$ **f** $8y^2 + 34y + 35$

g $15h^2 - 23h + 4$ **h** $12p^2 + 11p - 15$ **i** $16d^2 + 40d + 25$

11.01 Quadratic equations $ax^2 + bx + c = 0$

STAGE 5.3

Factorising quadratic equations

In Chapter 7, *Equations and logarithms*, we learnt that in a **quadratic equation**, the highest power of the variable is 2; for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4y^2 - 3y = 8$.

The general form of a quadratic equation is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b and c are constants (numbers) and $a \neq 0$.

Solving $ax^2 + bx + c = 0$ by factorising

STAGE 5.3

To solve quadratic equations of the form $ax^2 + bx + c = 0$, we need to factorise the quadratic expression on the LHS, which we learned in Chapter 5, *Products and factors*).

Example 1

Solve each quadratic equation.

a $4w(3w + 2) = 0$ **b** $(5m + 2)(2m - 7) = 0$

Solution

a $4w(3w + 2) = 0$

$\therefore 4w = 0$ or $3w + 2 = 0$

$w = 0$ or $3w = -2$

$w = 0$ or $w = -\frac{2}{3}$

b $(5m + 2)(2m - 7) = 0$

$5m + 2 = 0$ or $2m - 7 = 0$

$5m = -2$ or $2m = 7$

$m = -\frac{2}{5}$ or $m = 3\frac{1}{2}$

Example 2

Solve each quadratic equation. Use substitution to check your solutions.

a $6p^2 - 9p = 0$ **b** $2x^2 - x - 15 = 0$ **c** $6y^2 = -7y + 5$

Solution

a $6p^2 - 9p = 0$

$3p(2p - 3) = 0$ Factorising

$3p = 0$ or $2p - 3 = 0$

$p = 0$ or $2p = 3$

$p = 0$ or $p = 1\frac{1}{2}$

Check: When $p = 0$, LHS $= 6 \times 0^2 - 9 \times 0 = 0 =$ RHS

When $p = 1\frac{1}{2}$, LHS $= 6 \times \left(1\frac{1}{2}\right)^2 - 9 \times 1\frac{1}{2} = 0 =$ RHS

b $2x^2 - x - 15 = 0$

$2x^2 - 6x + 5x - 15 = 0$

$2x(x - 3) + 5(x - 3) = 0$

$(x - 3)(2x + 5) = 0$

$x - 3 = 0$ or $2x + 5 = 0$

$x = 3$ or $2x = -5$

$x = 3$ or $x = -2\frac{1}{2}$

Check: When $x = 3$, LHS $= 2 \times 3^2 - 3 - 15 = 0 =$ RHS

When $x = -2\frac{1}{2}$, LHS $= 2 \times \left(-2\frac{1}{2}\right)^2 - \left(-2\frac{1}{2}\right) - 15 = 0 =$ RHS

11.01



Quadratic equations by factorising

STAGE 5.3

c

$$6y^2 = -7y + 5$$
$$6y^2 + 7y - 5 = 0$$
$$(2y - 1)(3y + 5) = 0$$

$2y - 1 = 0$ or $3y + 5 = 0$

$2y = 1$ or $3y = -5$

$y = \frac{1}{2}$ or $y = -1\frac{2}{3}$

Check: When $y = \frac{1}{2}$:

$\text{LHS} = 6 \times \left(\frac{1}{2}\right)^2 = 1\frac{1}{2}$

$\text{RHS} = -7 \times \left(\frac{1}{2}\right) + 5 = 1\frac{1}{2}$

LHS = RHS

When $y = -1\frac{2}{3}$:

$\text{LHS} = 6 \times \left(-1\frac{2}{3}\right)^2 = 16\frac{2}{3}$

$\text{RHS} = -7 \times \left(-1\frac{2}{3}\right) + 5 = 16\frac{2}{3}$

LHS = RHS

EXERCISE 11.01 ANSWERS ON P. 630

Quadratic equations $ax^2 + bx + c = 0$ U F PS R C

EXAMPLE 1

1 Solve each quadratic equation.

a	$(m + 7)(m + 3) = 0$	**b**	$(d - 3)(d - 7) = 0$	**c**	$(y - 3)(y + 5) = 0$
d	$k(k - 3) = 0$	**e**	$t(t + 7) = 0$	**f**	$2p(p - 3) = 0$
g	$w(3w - 2) = 0$	**h**	$(2n + 1)(n - 3) = 0$	**i**	$(5a - 3)(2a - 1) = 0$
j	$(3x + 1)(2x + 3) = 0$	**k**	$(2c - 5)^2 = 0$	**l**	$(1 - 2f)^2 = 0$
m	$(3c + 1)(4c + 1) = 0$	**n**	$(1 - 2h)(h + 1) = 0$	**o**	$(5 - 7e)(1 - e) = 0$

2 What is the solution to the quadratic equation $(5x - 2)(x + 3) = 0$?
Select the correct answer **A**, **B**, **C** or **D**.

A $x = -\frac{2}{5}$ and $x = 3$

B $x = \frac{2}{5}$ and $x = 3$

C $x = \frac{2}{5}$ and $x = -3$

D $x = \frac{5}{2}$ and $x = -3$

EXAMPLE 2

3 Solve each quadratic equation.

a	$2y^2 + 7y + 6 = 0$	**b**	$2g^2 + 5g + 3 = 0$	**c**	$3d^2 + 5d + 2 = 0$
d	$5t^2 + 16t + 11 = 0$	**e**	$2p^2 - 11p + 12 = 0$	**f**	$10x^2 - 19x + 6 = 0$
g	$8y^2 - 2y - 3 = 0$	**h**	$6a^2 - 5a - 4 = 0$	**i**	$4w^2 - 7w - 15 = 0$
j	$5c^2 + 2c - 7 = 0$	**k**	$8e^2 + 10e - 3 = 0$	**l**	$3q^2 + 4q - 15 = 0$
m	$4g^2 - 20g + 25 = 0$	**n**	$18m^2 - 3m - 10 = 0$	**o**	$16 - 8w - 3w^2 = 0$
p	$36 + 3y - 3y^2 = 0$	**q**	$2f^2 - 24f + 72 = 0$	**r**	$12h^2 + 3h - 9 = 0$

Foundation Standard Complex

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4 Express each quadratic equation in the form $ax^2 + bx + c = 0$ and solve.

a	$2x^2 = x + 15$	**b**	$4t(t + 2) = 5$	**c**	$41u = -8u^2 - 5$
d	$7m^2 = 8m - 1$	**e**	$p(p - 3) = 28$	**f**	$(e - 2)^2 = 9$
g	$t(2t - 13) = -15$	**h**	$7 = 6d^2 + 11d$	**i**	$5h^2 = 125$
j	$8f^2 = 4f$	**k**	$6w^2 + 3 = 19w$	**l**	$4a(3a + 5) - 8 = 0$

5 A certain positive number, plus its square, minus 72, equals 0. Find the number. PS R C

STAGE 5.3

11.01

Completing the square 11.02

STAGE 5.3

Quadratic equations can be solved by using a method called **completing the square**. We try to make the LHS of the equation a perfect square (see Chapter 5, *Products and factors* and 8.13 in Chapter 8, *Graphing curves*).

Complete the square order activity

The method of **completing the square** is based on the following results for perfect squares.

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

We note that the last term, a^2, is the square of 'half the coefficient of x'.

Example 3

Find the numbers that complete the square in each equation.

a $x^2 + 10x + ... = (x + ...)^2$ **b** $x^2 - 14x + ... = (x - ...)^2$

Solution

a The coefficient of x is 10.
Half of 10 is 5, and $5^2 = 25$.
The perfect square is
$x^2 + 10x + 25 = (x + 5)^2$

b The coefficient of x is -14.
Half of -14 is -7, and $(-7)^2 = 49$.
The perfect square is
$x^2 - 14x + 49 = (x - 7)^2$

Example 4

Solve $(k + 3)^2 = 7$.

Solution

$(k + 3)^2 = 7$

$k + 3 = \pm\sqrt{7}$ Taking the square root of both sides.

$k + 3 - 3 = \sqrt{7} - 3$ or $k + 3 - 3 = -\sqrt{7} - 3$

This solution is usually written as:

$k = -3 + \sqrt{7}$ or $k = -3 - \sqrt{7}$

Notice that the 2 solutions are surds (not rational), of the form $a + \sqrt{b}$ and $a - \sqrt{b}$.

Simple quadratic equations

Foundation Standard Complex

STAGE 5.3

Example 5

Solve $x^2 + 6x - 2 = 0$ by completing the square.

Solution

Step 1: Move the constant term to the RHS.

$x^2 + 6x = 2$

Step 2: Halve the coefficient of x, square it and then add the square to both sides.

$x^2 + 6x + 3^2 = 2 + 3^2$

$x^2 + 6x + 9 = 11$

Step 3: Express the LHS as a perfect square.

$(x + 3)^2 = 11$

Step 4: Solve the resulting equation.

$x + 3 = \pm\sqrt{11}$ Taking the square root of both sides.

$x = -3 \pm \sqrt{11}$ This answer in surd form is called the **'exact answer'**.

$x = -3 + \sqrt{11}$ or $x = -3 - \sqrt{11}$

Example 6

Solve $2x^2 - 3x - 4 = 0$, writing the solution correct to 2 decimal places.

Solution

$2x^2 - 3x - 4 = 0$

$2x^2 - 3x = 4$ Moving the constant term to the RHS.

$x^2 - \frac{3}{2}x = 2$ Dividing both sides by 2, the coefficient of x^2.

$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = 2 + \left(-\frac{3}{4}\right)^2$ Completing the square.

$\left(x - \frac{3}{4}\right)^2 = \frac{41}{16}$ Expressing the LHS as a perfect square and leaving the RHS as an improper fraction.

$x - \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$ Taking the square root of both sides.

$x - \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$ Simplifying the RHS.

$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$ This is the exact answer in surd form.

$x = \frac{3 \pm \sqrt{41}}{4}$

$x = \frac{3+\sqrt{41}}{4}$ or $x = \frac{3-\sqrt{41}}{4}$

$x = 2.35078...$ or $x = -0.85078...$

$x \approx 2.35$ or $x \approx -0.85$ This is an approximate answer in decimal form.

EXERCISE 11.02 ANSWERS ON P. 631

Completing the square UFRC

EXAMPLE 3

1 Find the value of $\square$ in $x^2 - 6x + 9 = (x + \square)^2$. Select the correct answer **A**, **B**, **C** or **D**.

A -3 **B** 3 **C** -9 **D** 9

2 Find the numbers that 'complete the square' in each equation. R C

a $x^2 + 2x + ___ = (x + ___)^2$ **b** $p^2 - 6p + ___ = (p - ___)^2$

c $m^2 - 8m + ___ = (m - ___)^2$ **d** $k^2 + 4k + ___ = (k + ___)^2$

e $y^2 + 7y + ___ = (y + ___)^2$ **f** $w^2 - 3w + ___ = (w - ___)$

g $x^2 + x + ___ = (x + ___)^2$ **h** $h^2 - 5h + ___ = (h - ___)^2$

i $a^2 + \frac{7}{2}a + ___ = (a + ___)^2$ **j** $v^2 - \frac{5}{3}v + ___ = (v - ___)^2$

EXAMPLE 4

3 Find the value of $\square$ in $a^2 + \square a + \frac{49}{16} = \left(a + \frac{7}{4}\right)^2$. Select **A**, **B**, **C** or **D**.

A $\frac{7}{16}$ **B** $\frac{7}{8}$ **C** $\frac{7}{4}$ **D** $\frac{7}{2}$

4 Solve each equation, writing the solution in surd form. R C

a $(d + 3)^2 = 7$ **b** $(x - 5)^2 = 5$ **c** $(p + 1)^2 = 10$

d $(y - 1)^2 = 2$ **e** $\left(m - \frac{1}{2}\right)^2 = 5$ **f** $\left(t + \frac{2}{3}\right)^2 = 3$

g $(c+1)^2 = \frac{21}{2}$ **h** $(w-3)^2 = \frac{41}{2}$ **i** $\left(n + \frac{2}{3}\right)^2 = \frac{7}{9}$

j $\left(e - \frac{3}{2}\right)^2 = \frac{71}{4}$ **k** $(d - 2)^2 = 5$ **l** $\left(x - \frac{3}{4}\right)^2 = 2$

EXAMPLE 5

5 Solve each equation by completing the square. Leave your answers in exact form. R C

a $h^2 + 2h - 5 = 0$ **b** $r^2 - 2r - 1 = 0$ **c** $m^2 + 6m + 2 = 0$

d $w^2 - 4w - 60 = 0$ **e** $a^2 - 10a - 5 = 0$ **f** $y^2 + 8y - 3 = 0$

g $p^2 + 12p - 5 = 0$ **h** $x^2 - 4x + 2 = 0$ **i** $u^2 + 9u + 14 = 0$

j $d^2 + d - 7 = 0$ **k** $c^2 - 9c + 2 = 0$ **l** $e^2 + 5e + 2 = 0$

m $y^2 - 3y - 8 = 0$ **n** $b^2 - b - 5 = 0$ **o** $q^2 - 3q + 1 = 0$

p $2g^2 + 7g = 3$ **q** $2x^2 = 7 - 5x$ **r** $f(3f + 4) = 6$

EXAMPLE 6

6 Solve each equation (correct to 2 decimal places if irrational) by completing the square. R C

a $x^2 + 12x + 9 = 0$ **b** $m^2 - 16m - 7 = 0$ **c** $g^2 + 4g - 3 = 0$

d $2h^2 + 3h - 7 = 0$ **e** $5w^2 - 4w - 3 = 0$ **f** $3y^2 + y - 5 = 0$

g $p(3p + 2) = 8$ **h** $4e^2 - 4 = e$ **i** $2n^2 = 5 - 3n$

11.03 The quadratic formula

STAGE 5.3



Investigating quadratic equations


The quadratic formula


The quadratic formula


The quadratic formula


The quadratic formula


Solving quadratic equations

There is a formula for solving a quadratic equation of the form $ax^2 + bx + c = 0$ that involves the coefficients a, b and c.

The quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 7

Solve each quadratic equation using the quadratic formula.

a $x^2 - 3x + 2 = 0$ **b** $6x^2 + x - 2 = 0$ **c** $3x^2 + 11x + 2 = 0$

Solution

a For $x^2 - 3x + 2 = 0$:

$a = 1$, $b = -3$, and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{1}}{2}$$

$$= \frac{3 \pm 1}{2}$$

$x = \frac{3+1}{2}$ or $x = \frac{3-1}{2}$

$x = 2$ or $x = 1$

b For $6x^2 + x - 2 = 0$:

$a = 6$, $b = 1$, and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 6 \times (-2)}}{2 \times 6}$$

$$= \frac{-1 \pm \sqrt{49}}{12}$$

$$= \frac{-1 \pm 7}{12}$$

$x = \frac{6}{12}$ or $x = \frac{-8}{12}$

$x = \frac{1}{2}$ or $x = -\frac{2}{3}$

c For $3x^2 + 11x + 2 = 0$:

$a = 3$, $b = 11$ and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{-11 \pm \sqrt{97}}{6}$$

In exact surd form.

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Example 8

Solve $2x^2 - 6 = 9x$ using the quadratic formula, expressing the answer correct to 2 decimal places.

Solution

$2x^2 - 6 = 9x$

$2x^2 - 9x - 6 = 0$

Writing the quadratic equation in the form $ax^2 + bx + c = 0$

Substitute $a = 2$, $b = -9$, $c = -6$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times (-6)}}{2 \times 2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{9 \pm \sqrt{129}}{4}$$

$x = 5.0894...$ or $x = -0.5894...$

$x \approx 5.09$ or $x \approx -0.59$

EXERCISE 11.03 ANSWERS ON P. 631

The quadratic formula U F C

1 Solve each quadratic equation using the quadratic formula. C

EXAMPLE 7

a	$x^2 + 6x + 2 = 0$	**b**	$m^2 - 5m - 3 = 0$	**c**	$w^2 - 8w + 3 = 0$
d	$k^2 + 3k - 5 = 0$	**e**	$y^2 - 4y - 1 = 0$	**f**	$p^2 + p - 5 = 0$
g	$u^2 - 7u - 3 = 0$	**h**	$2a^2 + 3a - 7 = 0$	**i**	$5q^2 - 6q + 1 = 0$
j	$3c^2 + 2c - 2 = 0$	**k**	$4e^2 - 5e - 2 = 0$	**l**	$3x^2 + 8x + 2 = 0$
m	$2d^2 - 4d - 5 = 0$	**n**	$3a^2 - 10a - 2 = 0$	**o**	$2t^2 + 3t - 5 = 0$
p	$3y^2 + 8y + 4 = 0$	**q**	$6k^2 - 11k + 5 = 0$	**r**	$2n^2 - 5n - 11 = 0$

2 Solve $2a^2 - 5a - 1 = 0$. Select the correct answer **A**, **B**, **C** or **D**.

A $a = \frac{5 \pm \sqrt{33}}{4}$ **B** $a = \frac{-5 \pm \sqrt{33}}{4}$

C $a = \frac{5 \pm \sqrt{17}}{4}$ **D** $a = \frac{-5 \pm \sqrt{17}}{4}$

3 Solve each quadratic equation using the quadratic formula, expressing the solution as a surd. C

a	$5y^2 - 9y = 3$	**b**	$3m^2 = 7 - 2m$	**c**	$4x^2 = 3x + 2$
d	$1 - 4k - k^2 = 0$	**e**	$3m^2 - 1 - 3m = 0$	**f**	$1 - 2g - 5g^2 = 0$
g	$8 = 9h - 2h^2$	**h**	$2w + 2 = 3w^2$	**i**	$4p - 3p^2 = -1$
j	$2 - 4u - 5u^2 = 0$	**k**	$6a^2 = 9 - 4a$	**l**	$10 = 3y + 2y^2$

4 Find the solutions to $7x^2 + 3x - 2 = 0$, correct to 2 decimal places. Select **A**, **B**, **C** or **D**.

EXAMPLE 8

A $x = 0.79, -0.36$ **B** $x = 0.36, -0.79$

C $x = 0.72, -1.58$ **D** $x = 0.82, -0.40$

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STAGE 5.3

5 Solve each quadratic equation, writing the solutions correct to 2 decimal places. C

a	$k^2 - 9k + 1 = 0$	**b**	$c^2 - 2 = 0$	**c**	$m^2 - 5 = 2$
d	$2n^2 + 2 = 7n$	**e**	$2p^2 + 3p - 4 = 0$	**f**	$6w^2 + 5w - 2 = 0$
g	$3x^2 + 2 - 8x = 0$	**h**	$h^2 = 7 + 2h$	**i**	$1 + x - x^2 = 0$
j	$36 = 13a - a^2$	**k**	$5v^2 - 11 = 0$	**l**	$5c^2 + 8 = 15c$
m	$t^2 = 5(t + 5)$	**n**	$(x - 6)^2 = 3$	**o**	$12 = 2d^2 - 3d$

Investigation

Proving the quadratic formula

The 'completing the square' method can be used to prove the quadratic formula. Copy and complete the following proof that solves $ax^2 + bx + c = 0$ by completing the square.

$ax^2 + bx + c = 0$

$ax^2 + bx = _____$	Moving c to the RHS
$x^2 + \frac{b}{a}x = _____$	Dividing both sides by a
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = _____ + \left(\frac{b}{2a}\right)^2$	Completing the square
$\left(x + ___\right)^2 = ___ + \frac{___}{4a^2}$	Expressing the LHS as a perfect square
$\left(x + ___\right)^2 = \frac{___}{4a^2}$	Simplifying the RHS
$\therefore x + \frac{b}{2a} = \pm\sqrt{\frac{___ + b^2}{4a^2}}$	Taking the square root of both sides
$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - ___}}{2a}$	Simplifying the RHS
$x = -\frac{___}{2a} \pm \frac{\sqrt{b^2 - ___}}{2a}$	Moving $\frac{b}{2a}$ to the RHS
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Technology

The quadratic formula

Set up your spreadsheet as shown below.

	A	B	C	D	E
1	a	b	c	x_1	x_2
2					
:					
:					
10					

Foundation Standard Complex

- a, b and c are the coefficients from the quadratic equation $ax^2 + bx + c = 0$.
- x_1, x_2 are the possible solutions.

1 Enter the formula **=(-B2+SQRT(B2^2-4*A2*C2))/2*A2** in cell D2 and enter the formula **=(-B2–SQRT(B2^2-4*A2*C2))/2*A2** in cell E2.

2 Copy the formulas down to row 10.

3 Use your spreadsheet to solve each quadratic equation.

a $x^2 + 9x - 20 = 0$ **b** $2x^2 + 6x + 1 = 0$ **c** $3x^2 + 2x - 5 = 0$

d $x^2 + 2x - 15 = 0$ **e** $8x^2 - 10x + 3 = 0$ **f** $x^2 + 2x - 3 = 0$

g $4x^2 - 20x + 4 = 0$ **h** $x^2 - 6x - 8 = 0$ **i** $10x^2 - 3x - 1 = 0$

4 Use your spreadsheet to solve $x^2 - x + 3 = 0$. Explain your result.

Investigation

How many solutions?

1 A quadratic equation can have 0, 1 or 2 solutions. Looking at the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can you determine when a quadratic equation has:

a no solutions? **b** only one solution?

2 In the quadratic formula, the expression under the square root symbol, $b^2 - 4ac$, has a special name, the **discriminant**. The table below shows 12 quadratic equations and the number of solutions for each one. Copy and complete the table by calculating the value of $b^2 - 4ac$ for each equation.

	Equation	Number of solutions	$b^2 - 4ac$
a	$x^2 - 4x + 4 = 0$	1	
b	$x^2 - 3x + 6 = 0$	0	
c	$x^2 + 4x + 1 = 0$	2	
d	$9x^2 - 60x + 100 = 0$	1	
e	$x^2 + 6x + 10 = 0$	0	
f	$5x^2 - 2x + 3 = 0$	0	
g	$49x^2 - 28x + 4 = 0$	1	
h	$x^2 + 5x + 6 = 0$	2	
i	$x^2 - 4x + 3 = 0$	2	
j	$2x^2 + 4x + 5 = 0$	0	
k	$3x^2 + 7x + 4 = 0$	2	
l	$x^2 - 16x + 64 = 0$	1	

3 What do you notice about the value of the discriminant, $b^2 - 4ac$, when the quadratic equation has:

a no solutions? **b** 1 solution? **c** 2 solutions?

4 Explain how the discriminant, $b^2 - 4ac$, in the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, 'discriminates' whether a quadratic equation has 2, 1 or no solutions.

STAGE 5.3

5 Without solving the equations, determine the number of solutions for each quadratic equation.

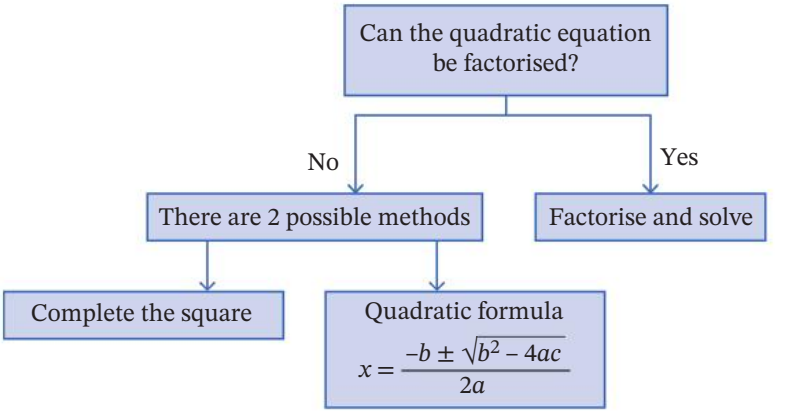
a $3x^2 - 2x + 5 = 0$	**b** $2x^2 - 6x + 1 = 0$	**c** $4x^2 - 12x + 9 = 0$
d $25x^2 + 10x + 1 = 0$	**e** $x^2 - 6x + 5 = 0$	**f** $x^2 + x + 4 = 0$
g $x^2 = 3 - 8x$	**h** $20x = 2 - 5x^2$	**i** $x(x + 6) = 12$

11.04 Mixed problems

STAGE 5.3

We have learned 3 methods of solving quadratic equations.

Factorising is the simplest, but only some quadratic equations can be factorised, while the **quadratic formula** works for all quadratic equations. **Completing the square** also works for all equations, but the algebra can be messy.



EXERCISE 11.04 ANSWERS ON P. 631

Mixed problems U F R

1 Solve each quadratic equation using the most suitable method. You may want to try 2 methods to compare your answers and determine how effective the methods were. R

a $x(2x - 3) = 0$	**b** $m^2 = 36$	**c** $3w^2 - 5w - 1 = 0$
d $u^2 - u - 12 = 0$	**e** $(5k - 3)(2k + 7) = 0$	**f** $16 = 25a^2$
g $x(x - 2) = 63$	**h** $n^2 - 6n = 10$	**i** $(a + 3)^2 = 5$
j $(d + 3)(d - 1) = 0$	**k** $t^2 = 4t$	**l** $(2y - 3)^2 = 11$
m $\frac{30}{x} - x = 7$	**n** $2a = \frac{6 - 5a}{3a}$	**o** $\frac{2x + 1}{2x - 1} - \frac{2x - 1}{2x + 1} = 6$

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2 Solve each quadratic equation using an appropriate method, giving your answers correct to 2 decimal places where necessary. R

STAGE 5.3

a $y^2 = 5y$ **b** $144 = a^2$ **c** $(u-2)^2 = 11$

d $x^2 - 6x = -9$ **e** $18 + 7x - x^2 = 0$ **f** $x^2 + 10x + 5 = 0$

g $a(2a-1) = 7$ **h** $2w^2 = 3w + 7$ **i** $(5m-1)^2 = 18$

j $(3-2k)^2 = 27$ **k** $\frac{36}{x} + 1 = 2x$ **l** $\frac{h}{3h+2} = \frac{h-3}{4h}$

11.04

Higher-order quadratic equations#

11.05

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

Higher-order equations have the highest power of the variable being greater than 2. Equations such as $x^4 - 6x^2 + 8 = 0$, $2y^6 + 15y^3 - 8 = 0$ are examples of higher-order 'quadratic' equations. These equations can be 'simplified' or reduced to a quadratic equation.

Example 9

Solve each quadratic equation.

a $x^4 - 11x^2 + 18 = 0$ **b** $m^4 + 3m^2 - 4 = 0$ **c** $y^6 - 9y^3 - 10 = 0$

Solution

a The equation can be written as:
$(x^2)^2 - 11(x^2) + 18 = 0$.
This is a quadratic equation in x^2.
Let $u = x^2$.
$\therefore x^4 - 11x^2 + 18 = 0$ becomes
$u^2 - 11u + 18 = 0$
$(u-9)(u-2) = 0$
$u - 9 = 0$ or $u - 2 = 0$
$u = 9$ or $u = 2$
But $u = x^2$
$\therefore x^2 = 9$ or $x^2 = 2$
$x = \pm 3$ or $x = \pm\sqrt{2}$
The solutions of the equation are:
$x = \pm\sqrt{2}, \pm 3$

This quadratic equation has 4 solutions: $x = \sqrt{2}, -\sqrt{2}$, 3 or –3.
You could substitute back to check.

b $(m^2)^2 + 3(m^2) - 4 = 0$ is a quadratic equation in m^2.
Let $u = m^2$
$\therefore m^4 + 3m^2 - 4 = 0$ becomes
$u^2 + 3u - 4 = 0$
$(u+4)(u-1) = 0$
$\therefore u + 4 = 0$ or $u - 1 = 0$
$u = -4$ or $u = 1$
But $u = m^2$
$\therefore m^2 = -4$ or $m^2 = 1$
which has no solution or $m = \pm 1$
The solutions of the equation are
$m = \pm 1$.

This quadratic equation has 2 solutions: $m = 1$ or –1.
You could substitute back to check.

Foundation Standard Complex

STAGE 5.3

c $(y^3)^2 - 9(y^3) - 10 = 0$ is a quadratic equation in y^3.

Let $u = y^3$

$\therefore y^6 - 9y^3 - 10 = 0$ becomes

$u^2 - 9u - 10 = 0$

$(u + 1)(u - 10) = 0$

$\therefore u + 1 = 0$ or $u - 10 = 0$

$u = -1$ or $u = 10$

But $u = y^3$

$\therefore y^3 = -1$ or $y^3 = 10$

$y = -1$ or $y = \sqrt[3]{10}$

The solutions of the equation are $y = -1$ and $\sqrt[3]{10}$.

This quadratic equation has 2 solutions.

EXERCISE 11.05 ANSWERS ON P. 632

Higher-order quadratic equations U F R

EXAMPLE 9

1 Use the substitution given to solve each quadratic equation. R

a $y^4 - 12y^2 + 32 = 0$ let $u = y^2$

b $m^4 - 13m^2 - 48 = 0$ let $u = m^2$

c $w^4 - 8w^2 + 12 = 0$ let $u = w^2$

d $m^6 - 9m^3 + 8 = 0$ let $u = m^3$

e $k^6 - 26k^3 - 27 = 0$ let $u = k^3$

f $w^6 - 4w^3 - 32 = 0$ let $u = w^3$

g $(3x + 5)^2 - 8(3x + 5) + 12 = 0$ let $u = 3x + 5$

h $(5y - 1)^2 - (5y - 1) - 30 = 0$ let $u = 5y - 1$

i $(2a + 7)^2 - 10(2a + 7) + 24 = 0$ let $u = 2a + 7$

j $p^6 + 6p^3 + 5 = 0$ let $u = p^3$

k $8g^6 - 65g^3 + 8 = 0$ let $u = g^3$

l $125c^6 + 999c^3 - 8 = 0$ let $u = c^3$

2 Solve each equation, correct to one decimal place where necessary. R

a $2m^4 - 19m^2 + 24 = 0$

b $5x^4 - 14x^2 + 8 = 0$

c $7a^6 - 9a^3 - 10 = 0$

d $2y^6 + y^3 - 10 = 0$

e $3w^6 - 16w^3 - 35 = 0$

f $12e^4 + 5e^2 - 2 = 0$

g $w^4 - 21w^2 - 100 = 0$

h $x^6 + 6x^3 - 16 = 0$

i $k^6 - 64 = 0$

j $m^4 - 4m^2 + 4 = 0$

k $2y^4 + 7y^2 - 15 = 0$

l $v^6 + 5v^3 - 24 = 0$

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Mental skills 11 Maths without calculators ANSWERS ON P. 632

STAGE 5.3

11.05

Dividing a quantity in a given ratio

1 Study this example.

Divide \$5600 between Alice and Peter in the ratio 5 : 3.

Total number of parts = 5 + 3 = 8.

1 part = \$5600 ÷ 8 = \$700

Alice's share = 5 × \$700 = \$3500

Peter's share = 3 × \$700 = \$2100

Check: \$3500 + \$2100 = \$5600 (original amount)

2 Now divide each of these quantities in the given ratio.

a Divide \$150 between Mark and Jenni in the ratio 2 : 1

b Divide \$2100 between Simon and Sunil in the ratio 4 : 3

c Divide \$720 between Lisa and Bree in the ratio 2 : 7

d Divide \$2000 between William and Adriana in the ratio 1 : 3

e Divide \$4500 between Ed and Pete in the ratio 3 : 2

f Divide \$3000 between Sharanya and Asam in the ratio 3 : 7

g Divide \$3600 between Cindy and Carmen in the ratio 5 : 1

h Divide \$1600 between Nancy and John in the ratio 3 : 5

i Divide \$990 between Carol and Louis in the ratio 5 : 4

j Divide \$4000 between Yvette and Andre in the ratio 1 : 4

k Divide \$4900 between Arden and Ivan in the ratio 3 : 4

l Divide \$3200 between Tan and Mai in the ratio 5 : 3



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11.06 Quadratic equation problems

STAGE 5.3

WS Problems involving quadratic equations

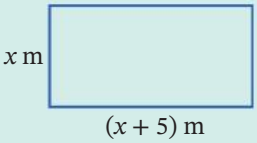
Example 10

A rectangular garden is 5 m longer than it is wide. The area of the rectangle is 84 m^2. Find the dimensions of the garden.

Solution

Let the width of the rectangular garden be x cm.

$\therefore$ The length of the garden is $(x + 5)$ cm.



Area = length × width

$= x(x + 5)$ m^2

$\therefore x(x + 5) = 84$

$x^2 + 5x = 84$

$x^2 + 5x - 84 = 0$

$(x + 12)(x - 7) = 0$

$x + 12 = 0$ or $x - 7 = 0$

$\therefore x = -12$ or $x = 7$

Since x represents a measurement of width, $x \neq -12$.

$\therefore$ The width is 7 m and the length is $7 + 5 = 12$ m

Check: Area = 7 m × 12 m = 84 m^2

Example 11

Quadratic equation problems

A ball is thrown upwards and its height, h metres, after t seconds is given by the formula

$$h = 30t - 5t^2$$

At what times did the ball reach a height of 24 m? Express the answer correct to 2 decimal places.



Alamy Stock Photo/ Science Photo Library

Solution

The equation is $h = 30t - 5t^2$

When $h = 24$,

$24 = 30t - 5t^2$

Rearranging the equation,

$5t^2 - 30t + 24 = 0$

Using the quadratic formula,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \times 5 \times 24}}{2 \times 5}$$

$$= \frac{30 \pm \sqrt{420}}{10}$$

$t = \frac{30 + \sqrt{420}}{10}$ or $t = \frac{30 - \sqrt{420}}{10}$

$t = 5.049...$ or $t = 0.950...$

$t \approx 5.05$ or $t \approx 0.95$

The ball reaches a height of 24 m at 0.95 s and 5.05 s.

Why are there 2 answers to this problem? Why is the ball at a height of 24 m at 2 different times?

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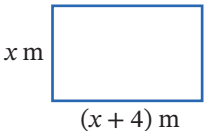
EXERCISE 11.06 ANSWERS ON P. 632

Quadratic equation problems U F PS R C

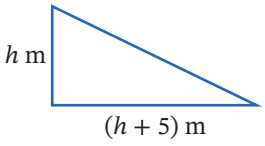
To answer each problem, first form a quadratic equation and then solve it.

EXAMPLE 10

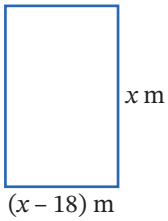
1 A garden is in the shape of a rectangle and its length is 4 m longer than its width. If the area of the garden is 96 m^2, find the dimensions of the garden. PS R C



2 A park is in the shape of a right-angled triangle, with its base 5 m longer than its perpendicular height. If the area of the park is 700 m^2, find the dimensions of the park. PS R C



3 A rectangular block of land has its width 18 m shorter than its length. If the area of the block is 1008 m^2, find the dimensions of the block of land. PS R C



4 A block of land is in the shape of a rectangle with its width 20 m shorter than its length. If the area of the block is 2204 m^2, find the length and width of the block of land. PS R C

5 A rectangular garden bed is twice as long as it is wide. Its area is 84.5 m^2. Find the length of the garden bed. PS R C

6 Find the dimensions of a rectangle whose length is 3 m longer than its width and its area is 180 m^2. PS R C

EXAMPLE 11

7 A golf ball is thrown upwards and its height, h metres, after t seconds is given by the formula $h = 18t - 5t^2$. At what times did the ball reach a height of 8 m? Answer correct to 2 decimal places. PS R C

8 After jumping from a plane, the height (in metres) of a skydiver above the ground is given by $h = 4000 - 5t^2$, where t is the time (in seconds) after jumping. R C



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a How high was the plane at the moment the skydiver jumped?

b What was the skydiver's height after 20 seconds?

c The skydiver opened his parachute at 1000 m. How long did it take the skydiver to reach this height? (Give your answer correct to one decimal place.)

9 A ball is thrown from a balcony and its height (in metres) after t seconds is given by the formula $h = 30 + 12t - 5t^2$. R C

a What is the height of the ball after 2 seconds?

STAGE 5.3

b When does the ball hit the ground? Answer correct to one decimal place.

c At what time (correct to one decimal place) is the ball at a height of:

i 35 m? **ii** 10 m?

10 The sum of a number and its square is 72. What is the number? PS R C

11 The product of 2 consecutive integers is 600. Find the integers. PS R C

12 When a number is subtracted from its square, the result is 1190. What is the number? PS R C

13 The difference between 2 positive integers is 12 and their product is 405. Find the integers. PS R C

14 The production costs, $\$C$, of a factory producing n toy boats each week is given by:

$$C = 0.05n^2 - 12n + 2700$$

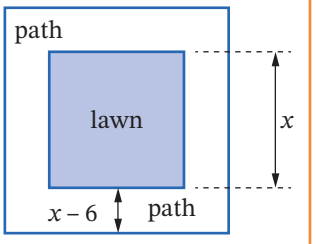
Find: R C

a the cost of producing 500 toy boats

b the profit made on 500 toy boats if each one sells for \$29.50

c the (whole) number of toy boats that can be produced at a production cost of \$8150.

15 In a garden, a square lawn area is surrounded by a path of width 6 metres less than the side of the square lawn. The area of the path is 32 square metres. Find the area of the lawn. PS R C



11.07 The parabola $y = ax^2 + bx + c$

STAGE 5.3

Investigating parabolas 1

Investigating parabolas 1

The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**. We have already graphed simple parabolas of the form $y = ax^2 + c$ and $y = a(x - b)^2 + c$ in Chapter 8, *Graphing curves*. Now we will graph parabolas of the general form $y = ax^2 + bx + c$.

Example 12

For each quadratic equation:

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts.

a $y = x^2 + 6x + 5$ **b** $y = 2x^2 - x - 10$

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Solution

a **i** $y = x^2 + 6x + 5$

x	−6	−5	−4	−3	−2	−1	0	1
y	5	0	−3	−4	−3	0	5	12

ii The x-intercepts are -5 and -1.

iii The y-intercept is 5.

Note that this is the constant term, $c = 5$, in $y = x^2 + 6x + 5$.

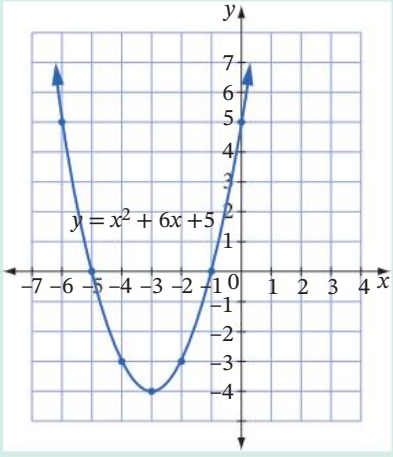
iv $x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x + 5 = 0$ or $x + 1 = 0$

$x = -5$ or $x = -1$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = x^2 + 6x + 5$.



b **i** $y = 2x^2 - x - 10$

x	−3	−2	−1	0	1	2	3
y	11	0	−7	−10	−9	−4	5

ii The x-intercepts are -2 and $2\frac{1}{2}$.

iii The y-intercept is -10.

Note that this is the constant term, $c = -10$, in $y = 2x^2 - x - 10$.

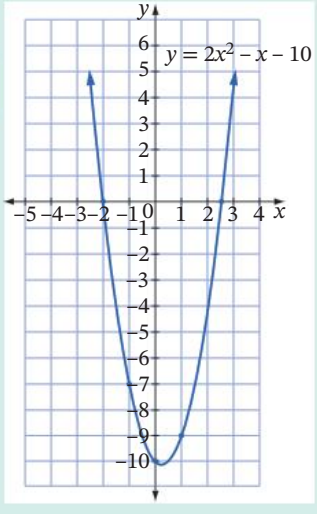
iv $2x^2 - x - 10 = 0$

$(2x - 5)(x + 2) = 0$

$2x - 5 = 0$ or $x + 2 = 0$

$x = 2\frac{1}{2}$ or $x = -2$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = 2x^2 - x - 10$.



The graph of the parabola $y = ax^2 + bx + c$

- If $a > 0$ (positive), the parabola is **concave up**
- If $a < 0$ (negative), the parabola is **concave down**
- The ***y*-intercept** of the parabola is c
- The ***x*-intercepts** of the parabola are the solutions to the quadratic equation $ax^2 + bx + c = 0$

STAGE 5.3

Example 13

Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(2x - 5)$ **b** $y = 2x^2 + x - 6$ **c** $y = -2x^2 - 3x + 9$

Solution

a $y = x(2x - 5)$

$= 2x^2 - 5x$

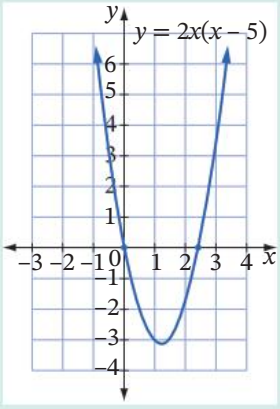
$a = 2 > 0$, so the parabola is concave up

x-intercepts: $x(2x - 5) = 0$

$x = 0$ and $2x - 5 = 0$

$x = 0$ and $x = 2\frac{1}{2}$

y-intercept: $c = 0$



b $y = 2x^2 + x - 6$

$a = 2 > 0$, so the parabola is concave up

x-intercepts:

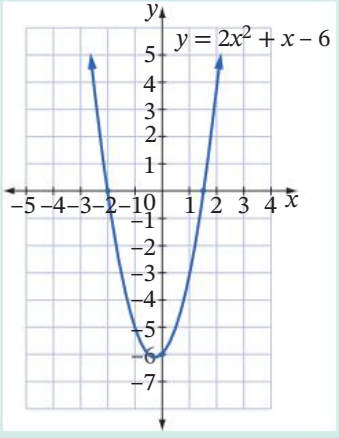
$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ and $x + 2 = 0$

$x = 1\frac{1}{2}$ and $x = -2$

y-intercept: $c = -6$



c $y = -2x^2 - 3x + 9$

$a = -2 < 0$, so the parabola is concave down

x-intercepts:

$-2x^2 - 3x + 9 = 0$

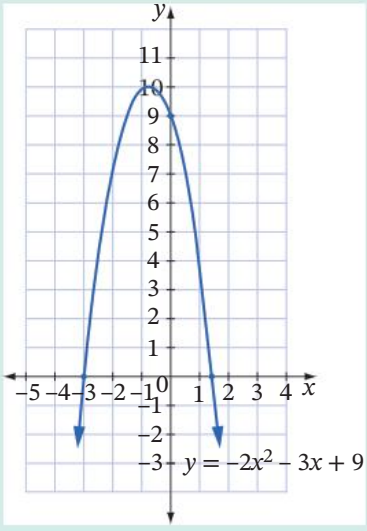
$2x^2 + 3x - 9 = 0$ Dividing by –1

$(2x - 3)(x + 3) = 0$

$2x - 3 = 0$ and $x + 3 = 0$

$x = 1\frac{1}{2}$ and $x = -3$

y-intercept: $c = 9$



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EXERCISE 11.07 ANSWERS ON P. 632

The parabola $y = ax^2 + bx + c$ U F R C

EXAMPLE 11

1 For each quadratic equation: R C

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts

a $y = x^2 + 4x + 3$ **b** $y = 2x^2 + 7x + 3$

c $y = 2x^2 - 3x - 9$ **d** $y = x^2 - 2x$

2 Find the y-intercept of the parabolas with equation:

a $y = 3x^2 - 2x - 5$ **b** $y = 2x^2 + 6x + 3$ **c** $y = 5x^2 - 10x$

3 Which quadratic equation below represents a parabola with x-intercepts 2 and -5? Select the correct answer **A**, **B**, **C** or **D**.

A $y = x^2 + 7x - 10$ **B** $y = x^2 - 7x - 10$

C $y = x^2 + 3x - 10$ **D** $y = x^2 - 3x - 10$

4 Which quadratic equation below represents a parabola with only one x-intercept, 1, and a y-intercept of 2? Select **A**, **B**, **C** or **D**.

A $y = (x - 1)(x - 2)$ **B** $y = 2(x - 1)^2$

C $y = (x + 1)(x + 2)$ **D** $y = (2x - 1)^2$

EXAMPLE 13

5 Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(x - 4)$ **b** $y = 3x(x + 2)$

c $y = (x + 3)(x - 5)$ **d** $y = (x - 2)(2x - 5)$

e $y = -(x + 1)(3x + 2)$ **f** $y = x^2 + 6x + 8$

g $y = -x^2 + 4x + 5$ **h** $y = 3x^2 + 11x - 20$

Did you know?

Parabolas in action

When an object is thrown or a bullet is fired from a gun, its path is a parabola. The length of the path will depend on the initial speed. The photo shows parabolas of lava being expelled from a volcano.

Parabola-shaped mirrors or reflectors are used in car headlights, spotlights and satellite dishes.

Investigate the property of parabolas that makes them useful as reflectors of light and radio signals.



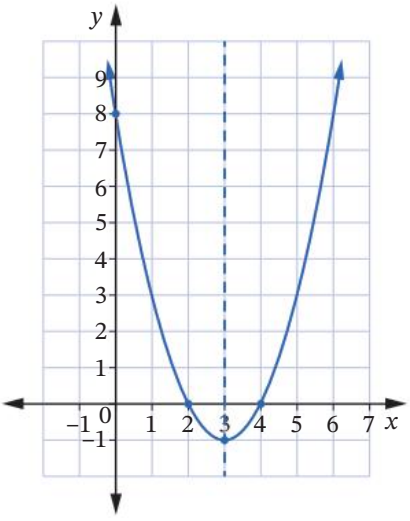
Shutterstock.com/Daniel Lopez Toriello

STAGE 5.3

Investigation

The axis of symmetry and vertex of a parabola

1 The graph of $y = x^2 - 6x + 8$ is shown.



- **a** Write the equation of the axis of symmetry of the parabola.
- **b** What are the x-intercepts of the parabola?
- **c** What is the midpoint of the interval joining the x-intercepts?
- **d** How can the x-intercepts be used to find the equation of the axis of symmetry?
- **e** Copy and complete: The equation of the axis of symmetry can be found by finding the _____ of the x-intercepts.

2 According to the quadratic formula, the solutions to $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This means that the x-intercepts of the parabola $y = ax^2 + bx + c$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, that is, $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.
Hence find the equation of the axis of symmetry of $y = ax^2 + bx + c$.

3 **a** For $y = x^2 - 6x + 8$, what are the values of a (the coefficient of x^2) and b (the coefficient of x)?

- **b** Find the value of $-\frac{b}{2a}$.
- **c** How is the value of $-\frac{b}{2a}$ related to the axis of symmetry of the parabola $y = x^2 - 6x + 8$?

Technology

The vertex of the parabola

In this activity, we will use dynamic geometry software to graph parabolas and identify the vertex.

1 Graph $y = (x + 2)^2 + 6$ and find the coordinates of its vertex.

2 Graph each equation and identify its vertex.

- **a** $y = (x + 3)^2 + 7$
- **b** $y = (x + 4)^2 - 5$
- **c** $y = (x - 1)^2 + 6$
- **d** $y = (x - 5)^2 - 6$
- **e** $y = -(x + 2)^2 - 3$
- **f** $y = -(x - 6)^2 - 8$

3 Do you see a pattern? By comparing the graphs drawn and the equations identify the features that represent the vertex for each equation in question **2** above.

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The axis of symmetry and vertex of a parabola

11.08

STAGE 5.3

11.08

WS Features of a parabola

WS A page of parabolas

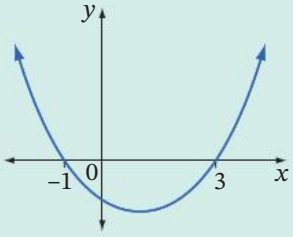
WS Parabolas

The axis of symmetry and vertex of $y = ax^2 + bx + c$.

- The **axis of symmetry** of the parabola is the vertical line: $x = -\frac{b}{2a}$
- The **axis of symmetry** passes through the point halfway between the 2 x-intercepts of the parabola
- The **vertex** of the parabola lies on the axis of symmetry, so its ***x*-coordinate** is $x = -\frac{b}{2a}$ and its ***y*-coordinate** can be found by substituting the x-coordinate into the equation of the parabola

Example 14

Find the equation of the axis of symmetry of this parabola.



Solution

The x-intercepts are -1 and 3.

The equation of the axis of symmetry is:

$$x = \frac{-1+3}{2}$$
$$= 1$$

Finding the average of the x-intercepts.

Example 15

A parabola has the equation $y = 2x^2 - 4x + 3$.

a Find the equation of its axis of symmetry

b Find the coordinates of the vertex of the parabola.

Solution

a For $y = 2x^2 - 4x + 3$, $a = 2$, $b = -4$ and $c = 3$.

The axis of symmetry is:

$$x = -\frac{b}{2a}$$
$$= -\frac{-4}{2 \times 2}$$
$$= 1$$

$x = 1$ is the equation of the axis of symmetry.

b The vertex lies on the axis of symmetry.

Substitute $x = 1$ in $y = 2x^2 - 4x + 3$

$y = 2 \times 1^2 - 4 \times 1 + 3$

$= 1$

The vertex is $(1, 1)$.

STAGE 5.3

Graphing the parabola $y = ax^2 + bx + c$

Example 16

Sketch the parabola $y = 2x^2 + 5x + 1$.

Solution

$a = 2 > 0$, so the parabola is concave up.

y-intercept $= 1$.

$y = 2x^2 + 5x + 1$ cannot be factorised, so the x-intercepts are not rational.

For the vertex:

$x = -\dfrac{b}{2a}$ where $a = 2$, and $b = 5$

$= -\dfrac{5}{2 \times 2}$

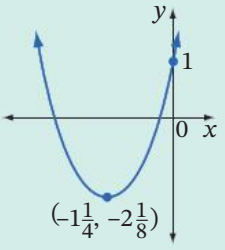
$= -1\dfrac{1}{4}$

Substitute $x = -1\dfrac{1}{4}$ into $y = 2x^2 + 5x + 1$

$y = 2 \times \left(-1\dfrac{1}{4}\right)^2 + 5 \times \left(-1\dfrac{1}{4}\right) + 1$

$= -2\dfrac{1}{8}$

$\therefore$ The vertex is $\left(-1\dfrac{1}{4}, -2\dfrac{1}{8}\right)$.



Graphing the parabola $y = ax^2 + bx + c$

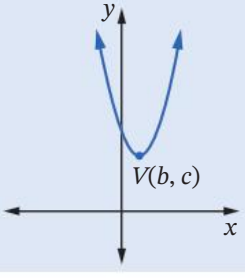
- Use the sign of a (the coefficient of x^2) to determine whether the parabola is **concave up** or **down**
- Find the ***y*-intercept**, c
- Find the ***x*-intercepts** where possible by solving a quadratic equation
- Use $x = -\dfrac{b}{2a}$ to find the **axis of symmetry** and the **vertex**

The vertex equation of a parabola $y = (x - b)^2 + c$

In Chapter 8, *Graphing curves*, we learned about the graph of $y = a(x - b)^2 + c$.

The parabola $y = a(x - b)^2 + c$

- The parabola $y = a(x - b)^2 + c$ is the graph of $y = ax^2$ translated b units to the right (or left if b is negative) and c units up (or down if c is negative)
- The parabola $y = a(x - b)^2 + c$ has its vertex at (b, c).
  $y = a(x - b)^2 + c$ is called the **vertex form** of the quadratic equation.



Just as a circle with equation $(x - a)^2 + (y - b)^2 = r^2$ has its centre at (a, b), a parabola with equation $y = a(x - b)^2 + c$ has its vertex at (b, c).

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Example 17

Find the vertex of the parabola with equation:

a $y = (x - 1)^2 + 3$ **b** $y = -2(x + 3)^2 - 5$

c $y = x^2 + 8x + 5$ by completing the square

Solution

a $y = (x - 1)^2 + 3$

Comparing against $y = (x - b)^2 + c$, this means $b = 1$ and $c = 3$, so the vertex is at $(1, 3)$.

b $y = -2(x + 3)^2 - 5$

$b = -3$ and $c = -5$ so the vertex is at $(-3, -5)$.

c Complete the square to get the equation in the form $y = (x - b)^2 + c$.

$y = x^2 + 8x + 5$

$= (x^2 + 8x + 4^2) - 4^2 + 5$ Completing the square on $x^2 + 8x$

$= (x^2 + 8x + 16) - 16 + 5$

$= (x + 4)^2 - 11$

$b = -4$ and $c = -11$, so the vertex is at $(-4, -11)$

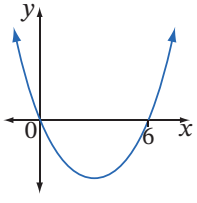
EXERCISE 11.08 ANSWERS ON P. 633

The axis of symmetry and vertex of a parabola U F R C

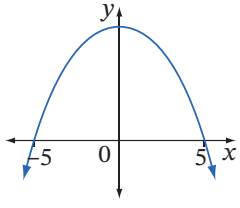
1 Write the equation for the axis of symmetry in each parabola. R C

EXAMPLE 14

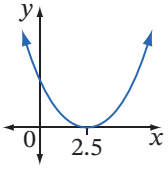
a



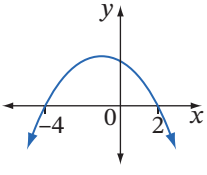
b



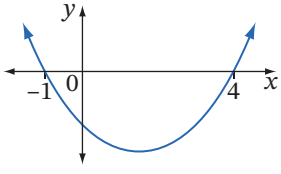
c



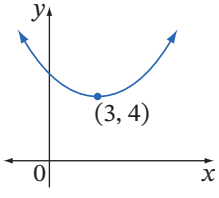
d



e



f



2 For each parabola with the equation given below:

i find the equation of its axis of symmetry

ii find the coordinates of its vertex

EXAMPLE 15

a $y = x^2 - 6x + 8$ **b** $y = -x^2 + 10x - 9$ **c** $y = x^2 - 2x + 10$

d $y = -x^2 + 8x + 9$ **e** $y = -x^2 + x - 25$ **f** $y = 5x^2 + 40x$

g $y = 24x - 3x^2$ **h** $y = 4x^2 + 2x - 1$ **i** $y = 1 - 3x - 9x^2$

Foundation Standard Complex

STAGE 5.3

EXAMPLE 16

3 Graph each quadratic equation, showing:

- **i** the x-intercepts (correct to one decimal place where necessary)
- **ii** the y-intercept
- **iii** the equation of the axis of symmetry
- **iv** the coordinates of the vertex
- **v** the concavity

a $y = x^2 - 6x - 40$ **b** $y = x^2 - 3x$ **c** $y = 2x^2 + 3x + 4$

d $y = -x^2 + 6x + 5$ **e** $y = -4x^2 - 12x + 21$ **f** $y = x^2 - 8x + 3$

g $y = 5x^2 + 7x + 4$ **h** $y = 8x - 2x^2$ **i** $y = -2x^2 + 7x - 6$

EXAMPLE 17

4 Which one of these equations is of a parabola with vertex $(-2, 9)$?
Select the correct answer **A**, **B**, **C** or **D**.

A $y = (x + 2)^2 + 9$ **B** $y = (x - 2)^2 + 9$

C $y = (x + 2)^2 - 9$ **D** $y = (x - 2)^2 - 9$

5 Find the vertex of the parabola with equation:

a $y = (x + 2)^2 + 3$ **b** $y = (x + 5)^2 - 2$ **c** $y = (x - 3)^2 + 1$

d $y = -(x - 1)^2 + 5$ **e** $y = -(x + 7)^2 - 2$ **f** $y = -(x - 10)^2 - 8$

6 By completing the square, find the vertex of the parabola with each equation.
Check your answers using the $x = -\frac{b}{2a}$ method.

a $y = x^2 + 4x + 1$ **b** $y = x^2 + 5x - 4$ **c** $y = -x^2 + 2x + 5$

d $y = x^2 - 6x$ **e** $y = 2 - 2x + x^2$ **f** $y = 2x^2 - 5x + 2$

Technology

Graphing parabolas

1 Use dynamic geometry software to graph $y = x^2 + 5x + 6$.

a What is the axis of symmetry?

b What is the vertex?

2 Graph $y = 2x^2 - 9x - 35$ and find its axis of symmetry and vertex.

3 Enter each equation and identify its axis of symmetry and vertex.

a $y = x^2 - 14x + 49$ **b** $y = 2x^2 - 11x - 21$ **c** $y = 8x^2 + 28x + 20$

d $y = 8 + 2x - x^2$ **e** $y = 20 - x - x^2$ **f** $y = 40 + 2x - 3x^2$

Foundation Standard Complex

Non-linear simultaneous equations#

11.09

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

The points of intersection of a line with a parabola, circle or hyperbola may be found either graphically or algebraically. The algebraic method involves solving simultaneous equations using the substitution method, which you learned about in Chapter 10, *Simultaneous equations*.

Example 18

Find the points of intersection of:

a the line $y = 2x - 3$ and the parabola $y = x^2 - 3x + 1$

b the line $y = x + 2$ and the circle $x^2 + y^2 = 4$

c the line $y = x + 5$ and the hyperbola $y = \frac{6}{x}$

Solution

a $y = 2x - 3$ [1]

$y = x^2 - 3x + 1$ [2]

Use [2] to substitute for y in [1].

$x^2 - 3x + 1 = 2x - 3$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4$ or $x = 1$

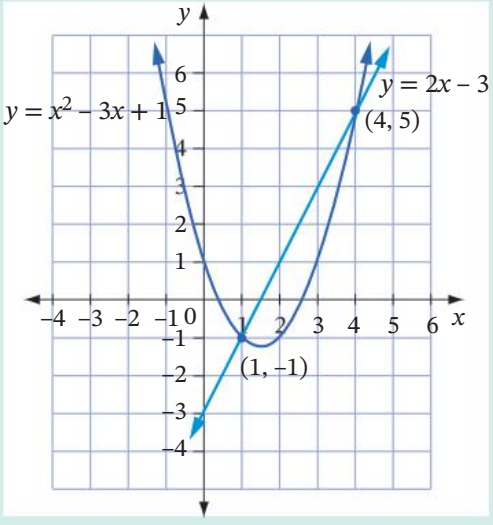
Substitute $x = 4$ and $x = 1$ into [1] to find y.

$y = 2 \times 4 - 3$ and $y = 2 \times 1 - 3$

$= 5$ $= -1$

The solutions are $x = 4, y = 5$ and $x = 1, y = -1$.

The points of intersection are (4, 5) and (1, −1).



This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.

b $y = x + 2$ [1]

$x^2 + y^2 = 4$ [2]

Use [1] to substitute for y in [2].

$x^2 + (x + 2)^2 = 4$

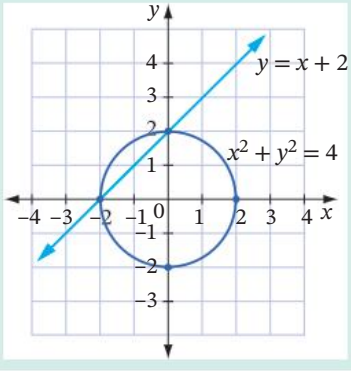
$x^2 + x^2 + 4x + 4 = 4$

$2x^2 + 4x = 0$

$2x(x + 2) = 0$

$2x = 0$ or $x + 2 = 0$

$x = 0$ or $x = -2$



Substitute $x = 0$ and $x = -2$ into [1] to find y.

$y = 0 + 2$ and $y = -2 + 2$

$y = 2$ and $y = 0$

The points of intersection are $(0, 2)$ and $(-2, 0)$.

This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.

c $y = x + 5$ [1]

$y = \frac{6}{x}$ [2]

Use [1] to substitute for y in [2].

$$x + 5 = \frac{6}{x}$$
$$x(x + 5) = x \times \frac{6}{x}$$
$$x^2 + 5x = 6$$
$$x^2 + 5x - 6 = 0$$
$$(x + 6)(x - 1) = 0$$

$x + 6 = 0$ or $x - 1 = 0$

$x = -6$ or $x = 1$

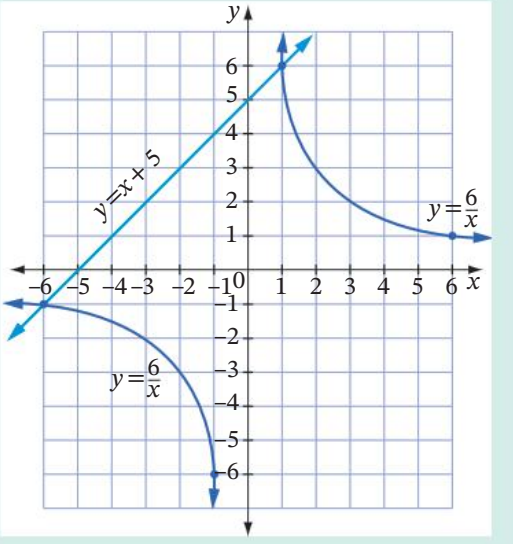
Substitute $x = -6$ and $x = 1$ into [1] to find y.

$y = -6 + 5$ and $y = 1 + 5$

$y = -1$ and $y = 6$

The points of intersection are $(-6, -1)$ and $(1, 6)$.

(Also, see their graphs).



EXERCISE 11.09 ANSWERS ON P. 634

Non-linear simultaneous equations UFRC

EXAMPLE 18

1 Find the points of intersection of the line and the parabola with equations: R

a $y = x^2$ and $y = 5x - 6$

b $y = 2x^2$ and $y = 8x$

c $y = x^2 + 10$ and $y = 2x + 18$

d $y = 5x^2$ and $y = x + 6$

e $y = 3x^2 + 2x + 10$ and $y = 12 - 3x$

f $y = 9x - x^2$ and $y = 6x - 10$

g $y = 6x - 3$ and $y = x^2 - 8x + 46$

h $x + y = 2$ and $y = x^2 - 2x$

2 Find the points of intersection of the line: R

a $y = x - 3$ and the hyperbola $y = -\frac{2}{x}$

b $y = 1 - x$ and the circle $x^2 + y^2 = 1$

c $x + y = 3$ and the circle $x^2 + y^2 = 9$

d $y = x + 5$ and the circle $x^2 + y^2 = 25$

e $y = 5x + 2$ and the hyperbola $y = \frac{7}{x}$

f $2x - y = 1$ and the hyperbola $y = \frac{15}{x}$

g $y = 2x + 5$ and the hyperbola $y = -\frac{3}{x}$

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3 Explain why there are no points of intersection between the equations $y = -3x - 1$ and $y = \frac{2}{x}$. Show working to help justify your answer. R C

Technology

Points of intersection

In this activity, we will use dynamic geometry software to find the point of intersection of a line and a curve.

1 Enter the following equations of a line and a parabola:

$$y = 3x - 4 \text{ and } y = 2x^2 + x - 16$$

2 Click on each point of intersection of the line and the curve to display its coordinates.

3 Check your answers to Exercise 11.09 by graphing the pairs of equations to find the points of intersection.

Power plus ANSWERS ON P. 634

1 **a** Show that $(3x^2 - 7x)^2 + 5(3x^2 - 7x) + 4 = 9x^4 - 42x^3 + 64x^2 - 35x + 4$.

b Hence, solve the equation $9x^4 - 42x^3 + 64x^2 - 35x + 4 = 0$.

2 Solve each equation.

a $(2^x - 32)(3^x - 27) = 0$

b $(2^x - 1)\left(2^x - \frac{1}{2}\right) = 0$

c $\left(2^x - \frac{1}{8}\right)\left(10^x - \frac{1}{100}\right) = 0$

d $2^{2x} - 2 \times 2^x + 1 = 0$

e $3^{2x} + 4 \times 3^x - 5 = 0$

f $4^x - 2^{x+2} - 32 = 0$

g $9^x - 10 \times 3^x + 9 = 0$

3 The solutions to the quadratic equation $(x - \alpha)(x - \beta) = 0$ are α and β.

a Prove that $(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta$.

b Hence find a quadratic equation of the form $ax^2 + bx + c = 0$ with solutions:

i 2 and 7

ii 5 and −8

iii −4 and $\frac{1}{2}$

iv $\sqrt{3}$ and $-\sqrt{3}$

v $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$

4 Does the line $y = 8x + 5$ intersect once, twice or completely miss the parabola $y = 2x^2 + 7x - 6$?

5 A gift box is to be made from a rectangular piece of cardboard that is 3 times as long as it is wide. 5 cm squares are cut from each corner and the ends are then folded up to make the box. If the volume of the box is 4340 cm^3, find the length and width of the piece of cardboard.

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CHAPTER 11 REVIEW

Language of maths

axis	axis of symmetry	check	circle
coefficient	complete the square	concave down	concave up
exact	factorise	hyperbola	left-hand side (LHS)
parabola	quadratic equation	quadratic formula	right-hand side (RHS)
simultaneous equations	solution	square root	substitution
surd	vertex	x-intercept	y-intercept

1 What is a **quadratic equation**?

2 What is the **coefficient** of x in $ax^2 + bx + c = 0$?

3 What are the 3 methods for solving a quadratic equation?

4 How do you write the solution to a quadratic equation in **exact form**?

5 How many solutions can a quadratic equation have?

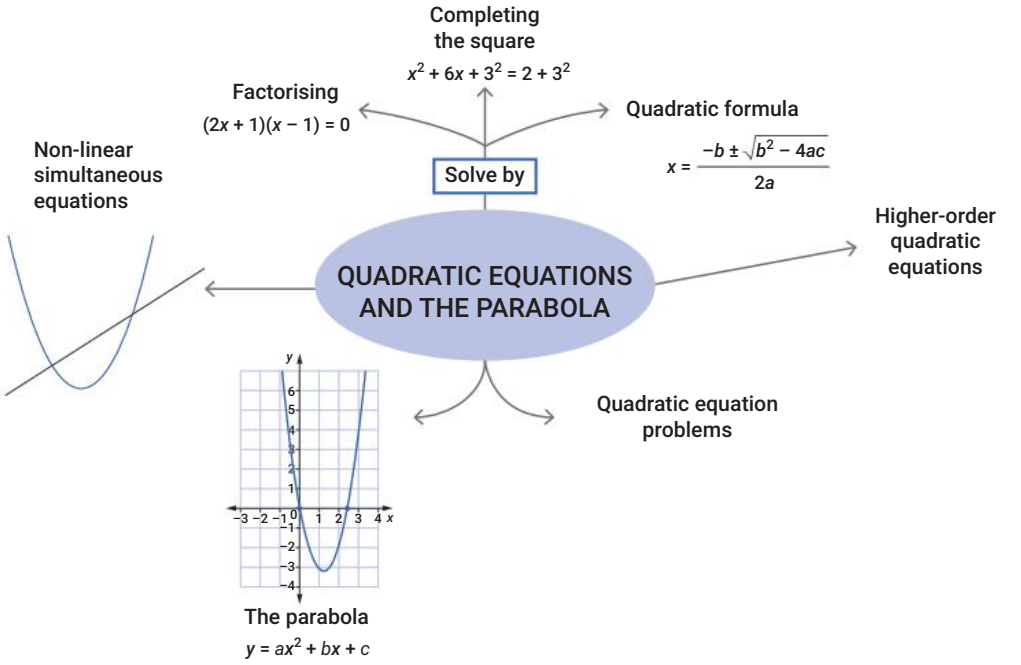
6 What are higher-order quadratic equations?

Topic summary

- Write in your own words what you have learned in this chapter.
- Write which parts of this chapter were new to you.
- What parts of this chapter did you find difficult?

WS
Mind map: Quadratic equations and the parabola

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 11

ANSWERS ON P. 634

1 Solve each quadratic equation. 11.01

a $(m-2)(2m+3)=0$ **b** $3x(x-5)=0$ **c** $p^2-2p-48=0$

d $y^2-8y+12=0$ **e** $3w^2+19w+20=0$ **f** $8k^2=2k+3$

g $n(2n-4)=30$ **h** $(2d+3)^2=36$ **i** $4x^2-15=4x$

2 Solve each quadratic equation by completing the square. 11.02

a $y^2+4y-7=0$ **b** $2p^2-3p-3=0$ **c** $5w^2+2w-1=0$

3 Solve each quadratic equation using the quadratic formula. 11.03

a $m^2+5m-3=0$ **b** $2d^2-7d-11=0$ **c** $3k^2-k=21$

4 Solve each quadratic equation, writing the solutions correct to 2 decimal places. 11.03

a $x^2+3x-1=0$ **b** $4h^2-3h-5=0$ **c** $w^2=5w+2$

5 Use the most suitable method to solve each quadratic equation. 11.04

a $x^2=3x$ **b** $h^2-10h-11=0$ **c** $5a^2=15a+20$

d $5x^2+7x-1=0$ **e** $y^2-6y=15$ **f** $8-5t-10t^2=0$

6 Solve each quadratic equation. 11.05

a $m^4-6m^2+8=0$ **b** $2y^4-5y^2-12=0$ **c** $x^6-7x^3-8=0$

7 Write a quadratic equation for each problem, then solve the problem. 11.06

a The product of 2 consecutive even numbers is 1088. Find the numbers.

b A rectangular garden bed is 8 m longer than it is wide. If the area of the garden bed is 308 m^2, find the dimensions of the garden bed.

8 Graph $y=2x^2+5x-7$, showing its x- and y-intercepts. 11.07

9 Find the vertex of the parabola with equation: 11.08

a $y=(x+5)^2+1$ **b** $y=-(x-7)^2+4$

c $y=x^2+6x$ by completing the square

10 Graph each quadratic equation, showing the x-intercepts (correct to one decimal place where necessary), the y-intercept, the equation of the axis of symmetry and the coordinates of the vertex. 11.08

a $y=x^2-2x-24$ **b** $y=-8x-3x^2$ **c** $y=5x^2+3x-8$

11 Find the point of intersection of the line: 11.10

a $y=x+3$ and parabola $y=2x^2$ **b** $y=3-2x$ and parabola $y=x^2+2x-2$

c $y=4-x$ and circle $x^2+y^2=16$ **d** $y=x-6$ and hyperbola $y=-\frac{8}{x}$

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PRACTICE SET 3

ANSWERS ON P. 634

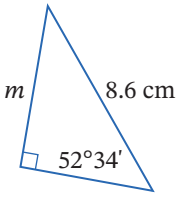
10.01

1 Graph the simultaneous equations $y = 2x - 3$ and $x + y = 6$ on a number plane to solve them.

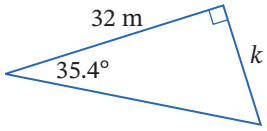
9.01

2 Find the value of each variable, correct to one decimal place.

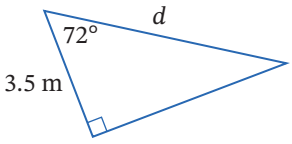
a



b



c



8.02

3 The time taken, t seconds, to complete a race is inversely proportional to the speed, S m/s, of the sprinter.

a If it takes Leilani 9.8 seconds to complete a race at a speed of 10.2 m/s, find the variation equation for t.

b If it took Leilani 10.5 seconds to complete a similar race, what was her speed?

c If Leilani runs at a speed of 10.3 m/s, how long will it take her to complete the race?

11.08

4 For the parabola with equation $y = -3x^2 + 6x - 1$, find:

a the equation of its axis of symmetry

b its vertex

10.02

5 Solve each pair of simultaneous equations.

a $3m + 5d = 14$
$3m - 5d = 10$

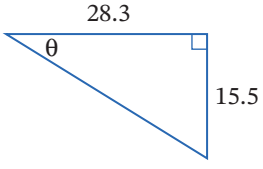
b $-2w + 3g = 8$
$w - 4g = 12$

c $2y + 3f = 15$
$5y - 2f = 9$

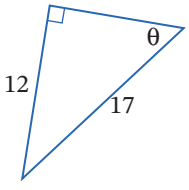
9.01

6 Find θ, correct to the nearest degree.

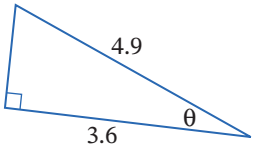
a



b



c



STAGE 5.3

11.03

7 Solve $3x^2 - 4x - 1 = 0$, writing the solution in exact form.

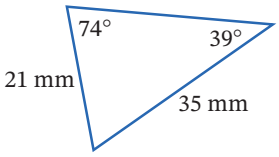
8.08

8 Sketch the graph of $y = 2x^3 - 2$.

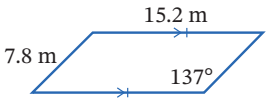
STAGE 5.3

9 Find the area of each shape, correct to 2 significant figures. 9.11

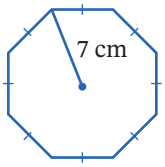
a



b



c

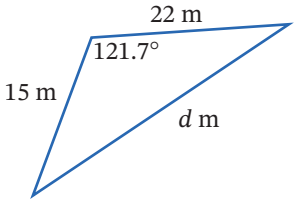


10 Solve each quadratic equation. 11.01

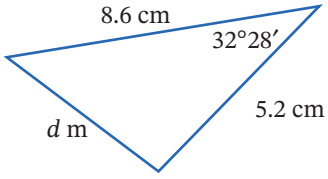
a $4m^2 + 5m + 1 = 0$ **b** $y(10y - 3) = 4$ **c** $2w^2 = 3w + 5$

11 Find the value of d, correct to one decimal place. 9.09

a



b



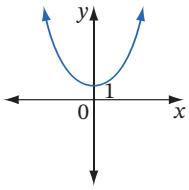
12 Tickets to the school play cost \$20 for adults and \$15 for children. Altogether, 395 people attended and ticket sales totalled \$6700. 10.04

a Write a pair of simultaneous equations to represent this situation.

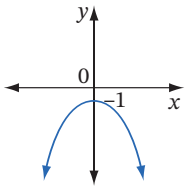
b Solve the simultaneous equations to find the number of children who attended the play.

13 Which of these could be a graph of $y = 1 - 2x^2$? Select the correct answer **A**, **B**, **C** or **D**. 8.06

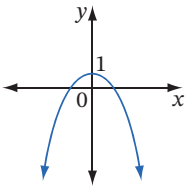
A



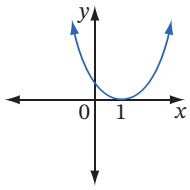
B



C



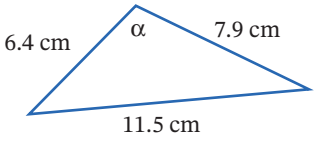
D



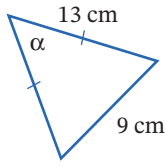
STAGE 5.3

14 Find α, correct to the nearest minute. 9.10

a



b



15 Find the centre and radius of the circle with the equation $x^2 - 10x + y^2 + 6y - 2 = 0$. 8.13

16 A fishing trawler leaves Ulladulla harbour on a bearing of 080° for 30 km and then changes direction to a bearing of 127° and stops when it is due east of the harbour. 9.07

How far from Ulladulla, correct to the nearest km, is the trawler?

10.03

17 Solve each pair of simultaneous equations.

a $y = x + 3$
$y = 5x - 7$

b $2h + 5k = 9$
$h = \frac{3k - 7}{2}$

c $3m - 2w = 12$
$m = 3w - 4$

STAGE 5.3

9.08

18 In $\triangle ABC$, $\angle A = 38°$, $a = 7$ cm and $c = 9$ cm. Find the possible values for the size of $\angle C$ (correct to the nearest degree).

8.12

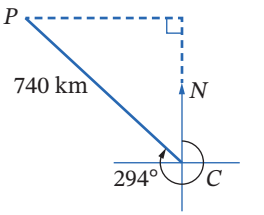
19 Sketch the exponential curve with equation $y = 4^{-x}$, showing the y-intercept.

9.02

20 A plane flies on a bearing of 294° from Canberra for a distance of 740 km.

a How far north has the plane travelled (to the nearest kilometre)?

b What is the bearing of Canberra from the plane's position?



8.01

21 P varies directly with T. When $P = 56$, $T = 32$.

a Write a variation equation to represent the relationship between P and T.

b Find P when $T = 150$.

c Find T when $P = 120$.

STAGE 5.3

9.04

22 Find the exact value of each expression.

a cos 45° **b** sin 60° **c** tan 30° **d** cos 30°

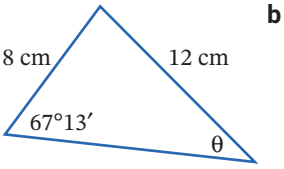
23 Substitute k for x^2 to solve $x^4 - 13x^2 + 36 = 0$.

11.05

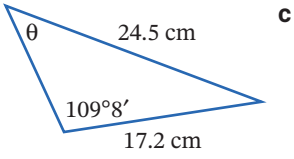
9.08

24 Find θ, correct to the nearest minute.

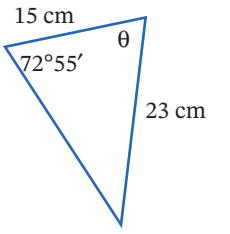
a



b



c



11.06

25 The length of a block of land is 5 metres less than 4 times its width. The area of the block is 1071 m^2.

a Write a quadratic equation to show this information.

b Solve the equation to find the dimensions of the block of land.

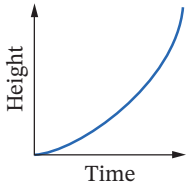
11.07

26 Find the point of intersection of the line $y = x - 3$ and the hyperbola $y = \frac{4}{x}$.

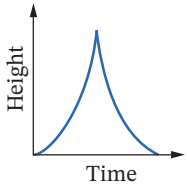
STAGE 5.3

27 Select the graph that best describes the height of a ball thrown upwards. 8.05

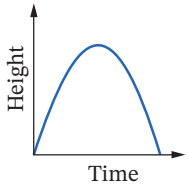
A



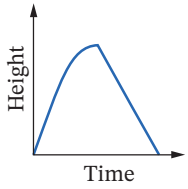
B



C



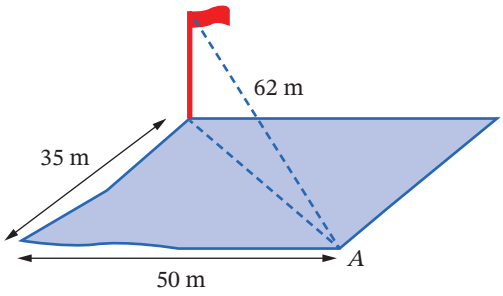
D



28 A flagpole is at the corner of a rectangular courtyard in a school. The distance from A to the top of the flagpole is 62 metres. 9.03

a Calculate, correct to one decimal place, the height of the flagpole.

b Calculate, correct to the nearest minute, the angle of elevation from A to the top of the flagpole.



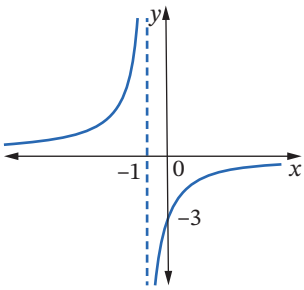
29 Which equation best represents this hyperbola? Select **A**, **B**, **C** or **D**. 8.11

A $y = \frac{3}{1-x}$

B $y = \frac{-1}{x+3}$

C $y = \frac{-3}{1-x}$

D $y = \frac{-3}{x+1}$



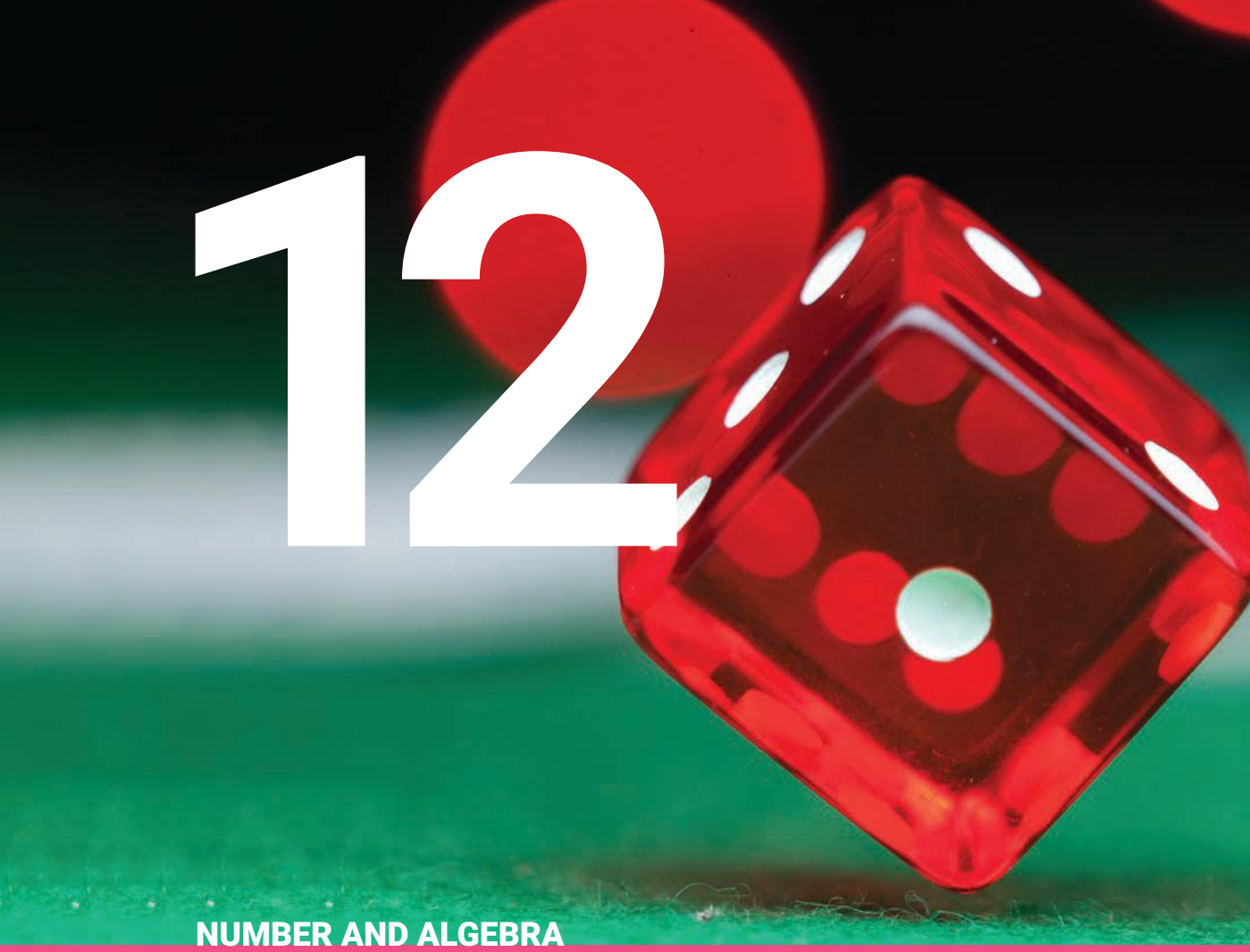
30 Solve $\tan A = -2.368$, giving all possible acute and obtuse solutions, correct to the nearest degree. 9.06

31 Solve each equation by completing the square. 11.02

a $d^2 + 5d + 2 = 0$ **b** $m^2 - 6m - 3 = 0$ **c** $3w^2 + 7w - 2 = 0$

32 Graph the parabola $y = -\frac{1}{2}(x-4)^2$, showing the vertex and y-intercept. 8.07

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12

NUMBER AND ALGEBRA

PROBABILITY

Probability theory, the study of chance, began in the 17th century when 2 great mathematicians, Blaise Pascal and Pierre de Fermat, wrote to each other to discuss mathematical problems about games of chance. Since then, probability has become an essential branch of mathematics that is used widely in fields such as weather forecasting, finance, insurance, politics and medical testing.



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Chapter outline

		Working mathematically				
12.01	Relative frequency	U	F		R	C
12.02	Venn diagrams	U	F		R	C
12.03	Two-way tables	U	F		R	C
12.04	Tree diagrams	U	F	PS	R	C
12.05	Selecting with and without replacement	U	F	PS	R	C
12.06	Dependent and independent events	U	F	PS	R	C
12.07	Conditional probability	U	F	PS	R	C

Wordbank

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

tree diagram A diagram of branches for listing all the possible outcomes of a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with 2 steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into 2 overlapping categories, such as gender and the ability to drive a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- calculate the probabilities and relative frequencies of events
- describe compound events using terminology such as 'and', exclusive 'or', inclusive 'or' and 'at least'
- use Venn diagrams and two-way tables to represent sample spaces and compound events to solve probability problems
- use tree diagrams and tables (arrays) to represent the sample space of two- and three-step chance experiments, with and without replacement, to solve probability problems
- identify dependent and independent events, and use the product rule for independent events
- solve problems involving conditional probability

WS Probability review

SkillCheck ANSWERS ON P. 635

1 For this spinner, the red sector is twice as large as each of the other sectors. Find the probability that when the spinner is spun, the arrow lands on:



a red

b purple or blue

c not green

2 A normal die is rolled. What is the probability of rolling:

a a 7? **b** a number less than 7?

3 Aditi bought 10 tickets in a raffle in which 400 tickets were sold and there is only one prize. What is the probability that Aditi will win the prize? Select **A**, **B**, **C** or **D**.

A $\frac{1}{400}$ **B** 2.5% **C** 0.0025 **D** 0.975

4 The probability of cloudy skies tomorrow is 0.85. What is the probability of clear skies?

12.01 Relative frequency

Experimental probability

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability**.

WS Relative frequencies

Experimental probability or **observed probability**, is based on **relative frequency**, the number of times an **event** occurred as a fraction of the total frequency of outcomes.

Dice probability

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

Long-run proportion

Long-run proportion

12.01

Expected frequency

Expected frequency = theoretical probability × number of trials.

Example 1

James rolled a die 100 times and recorded the results in a table.

Outcome	Frequency
1	23
2	19
3	11
4	12
5	18
6	17

a Find the experimental probability of rolling:

i an even number

ii an even number or a number greater than 4

'Even number or a number greater than 4' is an example of a **compound event**.

iii an even number less than or equal to 4.

b Calculate the probability of rolling a 2 or 3

i as an experimental probability **ii** as a theoretical probability.

c If the die is rolled 100 times, what is the expected frequency of rolling a 2 or a 3?

How does this compare with James' observed frequency?

Solution

a i Rolls of even numbers = 19 + 12 + 17

= 48

Experimental $P(\text{even}) = \frac{48}{100} = \frac{12}{25}$

ii Rolls of even numbers or numbers greater than 4 = 19 + 12 + 17 + 18

= 66

Experimental $P(\text{even or} > 4) = \frac{66}{100} = \frac{33}{50}$

iii Rolls of even numbers less than or equal to 4 = 19 + 12

= 31

Experimental $P(\text{even and} \leq 4) = \frac{31}{100}$

b i Rolls of 2 or 3 = 19 + 11 = 30

Experimental $P(2 \text{ or } 3) = \frac{30}{100} = \frac{3}{10}$

ii Theoretical $P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$

c Expected frequency of 2 or 3 $= \frac{1}{3} \times 100$ Probability × number of trials

$= 33.333...$

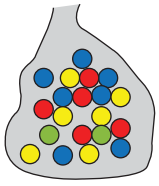
≈ 33

From the table, the observed frequency = 19 + 11 = 30, which is close to 33.

EXERCISE 12.01 ANSWERS ON P. 635

Relative frequency U F R C

1 A bag contains 7 blue, 6 yellow, 5 red and 2 green marbles. Lamisa selects a marble at random, records its colour and then returns it to the bag. Lamisa repeats this process 100 times and the results are shown. R C



Outcome	Frequency
Red	20
Blue	38
Yellow	33
Green	9

a Find the relative frequency of selecting a marble that is:

i red **ii** blue

iii yellow **iv** green

b What is the theoretical probability that the marble is:

i red? **ii** blue?

iii yellow? **iv** green?

c Are the experimental and theoretical probabilities similar?

d If the process is repeated 100 times, what is the expected frequency of a selecting a yellow or green marble? How does this compare with the observed frequency?

EXAMPLE 1

2 A coin is tossed. R C

a What is the expected number of heads if the coin is tossed 100 times?

b Toss a coin 100 times. Copy this table and record your results in it.

Outcome	Frequency
Head	
Tail	

c Calculate, as a decimal:

i the experimental probability of tossing a head

ii the theoretical probability of tossing a tail.

d Are the experimental and theoretical probabilities similar?

3 A die was repeatedly rolled and the results are shown in the table. R C

Outcome	Frequency
1	95
2	119
3	108
4	87
5	78
6	113

a How many times was the die rolled?

b Find the experimental probability (as a decimal) of rolling:

i an odd number

ii a number less than 4

iii a 2 or a 3

iv a number less than 4 or an even number.

c Find the theoretical probability (as a decimal) of rolling:

i an odd number **ii** a number less than 4

iii a 2 or a 3 **iv** a number less than 4 or an even number.

d Compare the experimental probabilities to the theoretical probabilities.

4 Place 5 blue counters, 2 red counters and 3 yellow counters in a bag. Select a counter at random from the bag, note its colour and return the counter to the bag. Repeat this 60 times. Copy this table and record your results in it. R C

Event	Tally	Frequency
Blue		
Red		
Yellow		

a What is the experimental probability of selecting:

i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

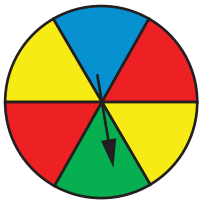
b What is the theoretical probability of selecting:

i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

c Are the experimental probabilities similar to the theoretical probabilities?

5 Tara spun this spinner 50 times and the results are shown. R C



Event	Frequency
Red	15
Blue	6
Yellow	24
Green	5

a What is the experimental probability (as a decimal) of the arrow stopping on

i red? **ii** blue? **iii** yellow? **iv** green?

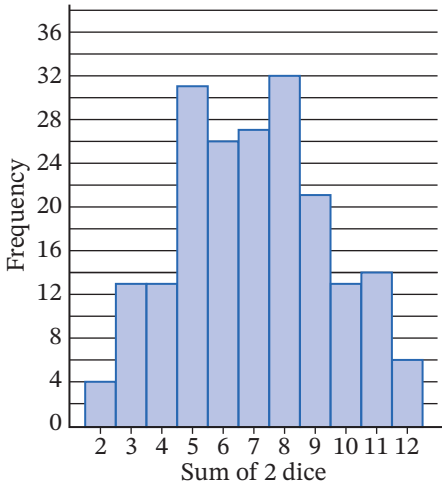
b What is the theoretical probability (as a decimal) of the arrow stopping on

i red? **ii** blue? **iii** yellow? **iv** green?

c Are the experimental and theoretical probabilities similar?

d What is the expected frequency of the arrow stopping on a colour that is not yellow? How does this compare with Tara's observed frequency?

6 2 dice are rolled and the sum of the numbers rolled was recorded in the frequency histogram.



a How many times were the dice rolled?

b Based on these results, what is the experimental probability of rolling a sum:

i of 2? **ii** of 7?

iii of 10? **iv** greater than 7?

v less than 7?

vi of 7 or 8?

vii that is even and greater than 6?

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7 Children at a shopping mall were asked how they travelled to school. R C

Mode of transport	Frequency
Walk	27
Bus	80
Car	62
Train	21
Bicycle	5
Skateboard	1
Other	4

a How many students were surveyed?

b Based on these results, find the probability that a student chosen at random will:

i walk to school

ii be driven to school

iii catch a bus to school

iv catch a train to school

v ride a skateboard to school

c What mode of transport could 'Other' include?

d Survey 100 students at your school and make up a table showing the results. How do the results from your school compare with the results from the survey?

8 A die is rolled 100 times. R C

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a What is the probability of rolling a 6? (Express your answer as a fraction and as a decimal.)

b How many times would you expect a 6 to appear if the die was rolled 100 times?

c Roll a die 100 times and record your results in a table similar to the one shown.

d What is the relative frequency of rolling a 6? (Express your answer as a fraction and as a decimal.)

e How does the theoretical probability of rolling a 6 compare with the experimental probability?

9 'If there are 10 horses in a race, the probability of each horse winning is 1 in 10'. Explain why this statement is not true. Give at least 2 reasons. R C



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Venn diagrams

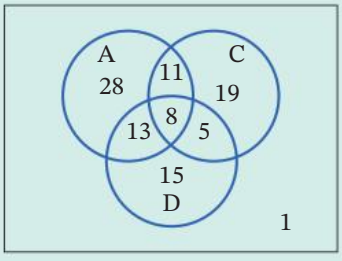
12.02

A **Venn diagram** is a diagram of circles (usually overlapping) that is used to group items into categories.

Venn diagrams 1
Venn diagrams 2
Venn diagrams matching activity
And/or problems
Venn diagrams

Example 2

This Venn diagram shows the results of a survey on what type of movies – action (A), comedy (C) or drama (D) – students prefer to watch.



a How many students were surveyed?

b How many students preferred to watch 2 types of movies only?

c Calculate, as a decimal, the probability of selecting a student who prefers to watch:

i action movies only **ii** action or comedy movies, but not dramas

iii action and drama movies **iv** all types.

d A student is chosen from those who like action and comedy movies. What is the probability that they also like to watch drama movies?

e What is the probability of selecting a student who does not like watching any of the 3 movie types?

Solution

a Number of students $= 28 + 11 + 8 + 13 + 5 + 19 + 15 + 1$
$= 100$

b 29 students preferred 2 types of movies only. $11 + 13 + 5 = 29$

c **i** Students preferring action movies only $= 28$ — The region of A that doesn't overlap C or D.

$P(\text{action only}) = \frac{28}{100} = 0.28$

ii Students preferring action or comedy only $= 28 + 19 + 11 = 58$ — The regions of A and C that don't overlap with D.

$P(\text{action or comedy only}) = \frac{58}{100} = 0.58$

'Action or comedy only' is an example of a **compound event**.

iii Students preferring action and drama $= 13 + 8$
$= 21$ — The regions where A and D intersect.

$P(\text{action and drama}) = \frac{21}{100} = 0.21$

iv Students preferring all types $= 8$ — The region where the 3 circles intersect.

$P(\text{all types}) = \frac{8}{100} = 0.08$

d Students preferring action and comedy $= 11 + 8$
$= 19$

Students preferring action and comedy and drama $= 8$

$P(\text{drama if preferring action and comedy}) = \frac{8}{19}$
≈ 0.42

e There is one student who doesn't prefer action, comedy or drama.

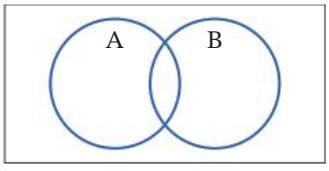
$P(\text{not action, comedy or drama}) = \frac{1}{100}$
$= 0.01$

When we combine 2 or more simple events, we get a **compound event**. In the above example, 'action and drama' and 'all types' are compound events.

'And' vs 'or'

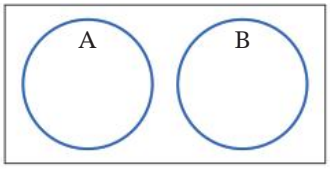
For 2 categories or events A and B, the compound event '**A and B**' means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

If A and B are **overlapping**, the compound event '**A or B**' means to have A or B or both. For example, 'to drive a car' or 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.



Overlapping events:
'A or B' means A or B or both

If A and B are **mutually exclusive**, they are **not overlapping** and the phrase '**A or B**' means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.



Mutually exclusive events:
'A or B' means A or B but not both

Example 3

A survey of 110 students at Hamper Valley College showed that 34 students study Art, 65 students study Chemistry, and 23 students study both Chemistry and Art.



Alamy Stock Photo/Ian Shaw

a Represent this information on a Venn diagram.

b How many students study Art and Chemistry, but not both?

c What is the probability of randomly selecting a student from this group who takes:

i Chemistry? **ii** Art and Chemistry?

iii Art or Chemistry? **iv** neither Art nor Chemistry?

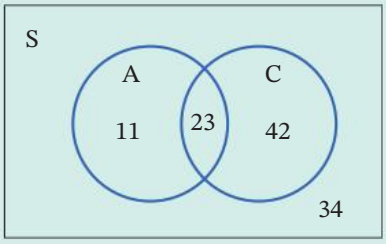
Solution

a S = students surveyed

A = students doing Art

C = students doing Chemsitry

There are 23 students who take both Art and Chemistry.



∴ Students doing Art only = 34 – 23 = 11

∴ Students doing Chemistry only = 65 – 23 = 42

∴ Students who take neither Art nor Chemistry = 110 – 11 – 42 – 23 = 34

b Number of students studying Art and Chemistry only = 11 + 42 = 53

'Art and Chemistry only' is a compound event.

c **i** 65 students take Chemistry.

$P(\text{Chemistry}) = \frac{65}{110} = \frac{13}{22}$

ii $P(\text{Art and Chemistry}) = \frac{23}{110}$

iii Number of students who take Art or Chemistry = 11 + 23 + 42 = 76

$P(\text{Art or Chemistry}) = \frac{76}{110} = \frac{38}{55}$

iv $P(\text{neither Art nor Chemistry}) = \frac{34}{110} = \frac{17}{55}$

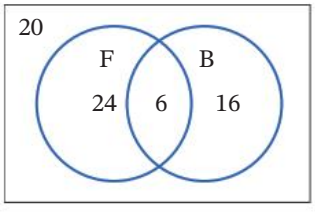
12.02

EXERCISE 12.02 ANSWERS ON P. 636

Venn diagrams U F R C

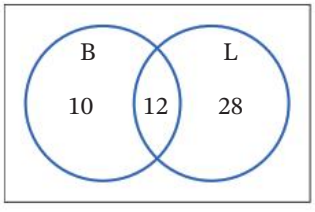
EXAMPLE 2

1 This Venn diagram shows the number of competitors who swam in freestyle (F) and butterfly (B) events at the school swimming carnival. How many students swam in Freestyle or Butterfly, but not both?
Select the correct answer **A**, **B**, **C** or **D**.



A 28 **B** 34 **C** 40 **D** 60

2 50 people were asked whether they had breakfast (B) or lunch (L) today. C



a What is the probability of selecting a person from this group who had:

i breakfast?

ii lunch?

iii breakfast but not lunch?

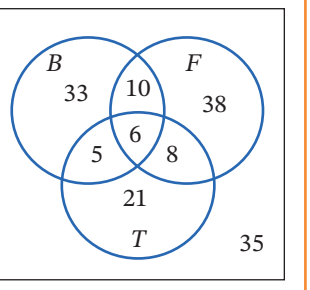
iv breakfast and lunch?

v breakfast or lunch only?

b Of the people who had lunch, find the probability that a person also had breakfast.

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3 The Venn diagram shows the number of Year 10 students who play basketball (B), touch football (F) or tennis (T). C



a How many students are in Year 10?

b Find the probability of selecting a student who plays:

i basketball only **ii** tennis only

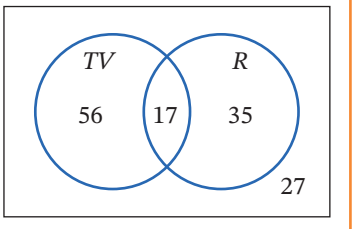
iii touch football and tennis

iv touch football or tennis

v basketball but not touch football **vi** all 3 sports.

c Of the students that play touch football, find the probability of selecting a student who also plays tennis.

4 This Venn diagram shows the results of a survey asking people whether they watch TV or read (R) at home. R



a How many people were surveyed?

b Find the probability of selecting a person who watches TV only.

c What is the probability of selecting a person who doesn't watch TV or read?

d Of the people who read, find the probability that they also watch TV.

EXAMPLE 3

5 Of the 54 Year 10 Music students, 23 students sing (S), 43 students play a musical instrument (P) and 12 students sing and play a musical instrument. R C

a Show this information on a Venn diagram.

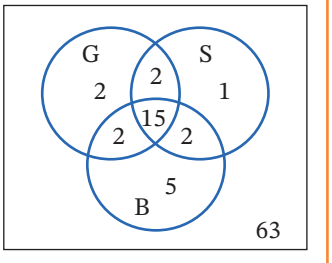
b Find the probability of selecting a Music student who:

i sings or plays an instrument **ii** sings only

iii plays a musical instrument only

iv sings or plays an instrument, but not both.

6 This Venn diagram shows the number of countries that won medals at the Winter Olympic Games in Pyeongchang, South Korea in 2018. R



a How many countries competed at these games?

b What is the probability of randomly selecting a country that:

i won only gold medals?

ii won gold, silver and bronze medals?

iii won gold or silver medals, but not bronze?

iv did not win a gold or silver medal?

c Of the countries that won medals, what is the probability of selecting a country that:

i won gold medals? **ii** won bronze, but no gold or silver?

7 At Riverside College, Year 10 students are asked what language they are studying. 64 students take French (F), 47 students take Japanese (J), 15 students take both French and Japanese, and 27 do not study a language. R C

a How many students are in Year 10?

b Show the information on a Venn diagram.

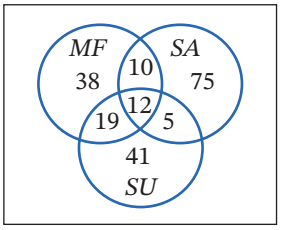
c How many students studied only one language?

d Find the probability of selecting a Year 10 student at random who studies:

i French but not Japanese **ii** Japanese but not French

iii no languages **iv** only one language.

8 People were surveyed on the day they preferred to shop: Monday to Friday (MF), Saturday (SA) or Sunday (SU). The results are shown in the Venn diagram. R C



a How many people were surveyed?

b What is the probability of selecting a person who prefers to shop on:

i Monday to Friday? **ii** Saturday?

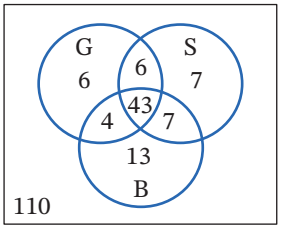
iii Sunday? **iv** on the weekend only?

v on Saturday or Sunday? **vi** any day (has no preference)?

c Find the probability of selecting a person who only prefers Saturday or Sunday, but not both.

d Is it necessary to include the rectangle in this Venn diagram? Give reasons.

9 This Venn diagram shows the number of countries that won gold, silver, bronze or no medals at the 2016 Olympic Games in Rio de Janeiro, Brazil.



a Find the total number of countries that competed.

b What is the probability of selecting a country that:

i won a silver medal only?

ii won one medal only?

iii won at least 2 medals?

iv won at most one medal only?

c Out of the countries that won gold medals, find the probability of selecting a country that won:

i gold and silver, but not bronze **ii** gold, silver and bronze.

12.03 Two-way tables

WS Two-way probability tables

Combined events: Two-way tables

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by a Venn diagram.

Example 4

Year 11 students at Southbank College were surveyed on whether they had part-time jobs.

	Part-time work	No part-time work
Male	43	27
Female	35	31

a How many students are in Year 11 at Southbank College?

b How many students had part-time work?

c How many male students were in Year 11?

d What is the probability of selecting a student at random that:

i works part-time? **ii** is female and works part-time?

iii is male and doesn't work? **iv** doesn't work?

e What is the probability of selecting a student working part-time given that:

i the student is male? **ii** the student is female?

Solution

a Number of Year 11 students $= 43 + 27 + 35 + 31 = 136$

b Students with part-time work $= 43 + 35 = 78$

c Male students in Year 11 $= 43 + 27 = 70$

d **i** $P(\text{student works part-time}) = \frac{78}{136} = \frac{39}{68}$

ii There are 35 female students who work part-time.

$P(\text{female and part-time}) = \frac{35}{136}$

'Female and works part-time' is a compound event.

iii There are 27 males who don't work.

$P(\text{male and not working}) = \frac{27}{136}$

iv Number of students not working $= 27 + 31 = 58$

$P(\text{not working}) = \frac{58}{136} = \frac{29}{68}$

e **i** There are 70 male students and 43 of them work part-time.

$P(\text{working part-time given that student is male}) = \frac{43}{70}$

ii There are 66 female students and 35 of them work part-time.

$P(\text{working part-time given that student is female}) = \frac{35}{66}$

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EXERCISE 12.03 ANSWERS ON P. 636

Two-way tables U F R C

EXAMPLE 4

12.03

1 People attending the Staying Alive Fitness Centre early on a Saturday morning either went swimming or did a workout in the gym. The numbers are shown in the table. R C

	Swimming	Gym
Male	32	53
Female	24	41

a How many people went to the fitness centre?

b Find the probability that a person selected at random:

i was female and went swimming

ii was male and did a workout in the gym

iii went swimming.

c Find the percentage (to the nearest whole number) of females who did a workout in the gym.

2 Year 10 students at Nusentry High School were given a choice of 2 activities on a wet sports afternoon: bowling or indoor soccer. Their selections are shown in the table. R C

	Bowling	Indoor soccer
Boys	25	48
Girls	43	12

a How many students are in Year 10?

b How many students:

i went bowling? **ii** played indoor soccer?

c What is the probability of randomly selecting a student who went bowling?

d What is the probability of randomly selecting a girl who played indoor soccer?

3 The composition of the Legislative Assembly (the lower house) in the NSW State Parliament is shown in the table. R C

	Liberal/Nationals	Labor	Greens/Other
Male	41	20	5
Female	11	14	6

The Liberal/Nationals are in government and the others are in opposition.

a How many members of parliament (MPs) are there in the Legislative Assembly?

b Find the percentage probability of randomly selecting an MP who is:

i female **ii** male and in the Opposition **iii** Greens/Other

c What percentage probability of:

i Government MPs are female? **ii** Opposition MPs are female?

d Compare your answers to part **c** and comment on the differences between the 2 results.

4 People were asked to name their favourite takeaway food. The results are shown in the table. R C

	Pizza	Hamburger	Fish and Chips
Men	22	35	18
Women	43	26	6

a How many people were surveyed?

b Find the probability (as a decimal) that a person selected at random:

i is male

ii is female and likes fish and chips

iii likes pizza

iv is male and likes hamburgers.

c If a male is selected at random, what is the probability that his favourite takeaway food is pizza?

5 Year 7 students were asked about their favourite drink. The results are in the table. R C

	Boys	Girls
Water	21	35
Milk	11	12
Juice	15	17
Soft drink	31	18

a How many students were in Year 7?

b What is the probability of randomly selecting a student that:

i prefers water?

ii is a boy and likes milk?

iii is a girl and likes soft drinks?

c What is the probability that if a girl is randomly selected, she prefers water?

6 A survey looked at whether people ate breakfast and whether they exercised regularly. R C

	Exercise	No exercise
Ate breakfast	72	27
Did not eat breakfast	38	63



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a How many people were surveyed?

b What percentage of people exercised?

c Find the percentage probability of picking a person at random who:

i eats breakfast

ii does not exercise regularly

iii eats breakfast and exercises regularly

iv does not eat breakfast and does not exercise.

d Of the people who exercise regularly, what is the probability of picking someone who eats breakfast?

7 Students at Mt Badger College were asked to indicate their preference for dark or milk chocolate in a survey. R C

		Milk chocolate	
		Like	Dislike
Dark chocolate	Like	545	134
	Dislike	157	42

a How many students attended the college?

b What is the probability (to 3 decimal places) of selecting a student at random who:

i likes dark chocolate?

ii likes both milk chocolate and dark chocolate?

iii likes dark chocolate, but dislikes milk chocolate?

iv dislikes both dark and milk chocolate?

Tree diagrams 12.04

A **two-** or **three-step experiment** is a **chance experiment** that has 2 or 3 parts or stages, such as:

- rolling 2 or 3 dice
- drawing 2 or 3 prizes in a raffle
- observing the weather each day over a weekend or a long weekend
- throwing 2 or 3 coins together

Tables and tree diagrams

Combined events: Tree diagrams

Example 5

A coin is tossed and a die is rolled.

a Use a table to display the sample space.

b Find the probability of rolling:

i a tail and a 3 **ii** a head and an even number.

Solution

a The sample space of a coin is a head (H) and a tail (T).

The sample space for a die is 1, 2, 3, 4, 5 and 6.

The sample space of tossing a coin and rolling a die is shown in the table below.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

Using a table ensures that all outcomes are counted

b i There are 12 outcomes in the sample space.

$\therefore P(\text{a tail and a 3}) = P(\text{T3}) = \frac{1}{12}$

'Tail and 3' is a compound event.

ii There are 3 outcomes that make up the event a head and an even number: H2, H4, H6

$\therefore P(\text{a head and an even number}) = \frac{3}{12} = \frac{1}{4}$

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The sample space for two-step experiments can be displayed using lists, tables or tree diagrams, but the sample space for three-step experiments is best displayed using a tree diagram.
A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each step or stage. An outcomes column at the end of the diagram lists the sample space.

Tree diagrams 3

Tree diagrams 1

Tree diagrams 2

Example 6

2 coins are tossed.

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

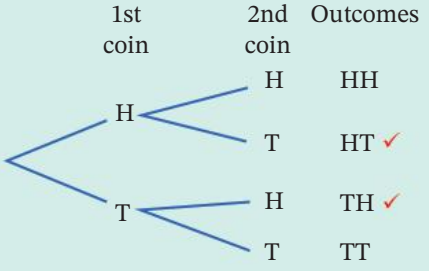
i 2 heads

ii a head and a tail (in any order).

Solution

a There are 2 outcomes for the first coin, followed by 2 outcomes for the second coin. There are $2 \times 2 = 4$ possible outcomes.

Using a tree diagram ensures that all outcomes are counted.



b **i** There is one outcome out of a possible 4 for 2 heads.

$\therefore P(\text{2heads}) = \frac{1}{4}$

ii There are 2 outcomes for a head and a tail (✓ on the tree diagram).

$\therefore P(\text{a head and a tail}) = \frac{2}{4} = \frac{1}{2}$

EXERCISE 12.04 ANSWERS ON P. 636

Tree diagrams U F PS R C

EXAMPLE 5

1 A boy and a girl are to be chosen from this shortlist to represent the school at a conference:

Boys: Ben, Christian, Ewan, William.

Girls: Becky, Cassandra, Emily, Marianne, Roz, Siana. R C

a List all the possible pairs of a boy and a girl.

b Find the probability of selecting:

i Christian and Roz

ii a boy and a girl whose names begin with a B or a C

iii a pair that includes Emily.

2 2 coins are tossed together. R C

a Copy and complete the table.

b What is the probability of tossing:

i 2 tails? **ii** a head and a tail?

iii at least one head?

		1st coin	
		H	T
2nd coin	H		
	T		

3 2 dice are rolled.

a Copy and complete this table to list the sample space. PS R C

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1				
	2						
	3						
	4			3, 4			
	5						
	6						

b How many possible outcomes are there?

c Find the probability of rolling:

i two 6s **ii** at least one 6

iii doubles **iv** 2 even numbers

v at least one 2 **vi** 2 numbers greater than 3

vii one odd and one even number

viii 2 numbers where the first number is greater than the second number.

4 3 coins are tossed. PS R C

a Copy and complete the tree diagram.

b How many outcomes are there in the sample space?

c Use the tree diagram to find:

i P(3 heads) **ii** P(2 heads)

iii P(3 tails)

iv P(head, then tail and then head)

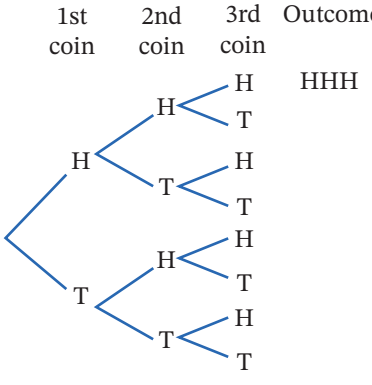
v P(2 heads or 3 heads)

d Find the probability of tossing:

i at least 1 tail **ii** at most 2 tails.

e If 3 coins are tossed 200 times, find the expected frequency of:

i tossing 2 heads **ii** tossing no tails.



5 Use a tree diagram to display all possible outcomes when a coin and die are tossed together. R C

12.04

EXAMPLE 6

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6 A 6-sided die and a 4-sided die (numbered 1, 2, 3 and 4) are rolled together. PS R C

a Construct a table to list the outcomes in the sample space.

b How many outcomes are in the sample space?

c Find the probability of rolling:

i doubles **ii** 2 even numbers

iii one even and one odd number

iv a pair of numbers that are both less than 4

v a pair of numbers that are both greater than 4.

7 2 dice are rolled and the sum of the 2 numbers is calculated. PS R C

a Copy and complete this table to show all possible sums.

		1st die					
		1	2	3	4	5	6
2nd die	1	2					
	2						
	3				7		
	4						
	5						
	6					11	

b Find the of probability of rolling a sum:

i of 5 **ii** of 12 **iii** of 7

iv that is even **v** less than 2 **vi** more than 7

vii at least 7 **viii** between 4 and 8.

8 4 coins are tossed. PS R C

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

i 4 heads **ii** 1 head **iii** 2 tails

iv at least 1 tail **v** 2 heads and then 2 tails **vi** not more than 1 tail

c If 4 coins are tossed 1000 times, find the expected frequency of having:

i 4 heads **ii** 2 heads and 2 tails **iii** at least one tail.

9 The weather on a long weekend will either be fine or rain each day, with each outcome being equally likely. PS R C

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that:

i it rains on all 3 days?

ii it is fine on 2 of the 3 days?

iii it is fine on Saturday and Sunday, but rains on Monday?

iv it rains on at least one day of the long weekend?

Mental skills 12: Maths without calculators ANSWERS ON P. 637

The unitary method with percentages

1 Study each example.

a If 8% of a number is 24, what is the number?

8% of the number = 24

$\therefore$ 1% of the number = 24 ÷ 8 = 3

$\therefore$ 100% of the number = 3 × 100 = 300

The number is 300. *Check*: 8% × 300 = 24

b If 15% of an amount is \$90, what is the whole amount?

15% of the amount = \$90

$\therefore$ 1% of the amount = \$90 ÷ 15 = \$6

$\therefore$ 100% of the amount = \$6 × 100 = \$600

The amount is \$600. *Check*: 15% × \$600 = \$90

2 Find the whole amount if:

a 5% of the amount is \$35

b 11% of the amount is \$88

c 20% of the amount is 80

d 6% of the amount is 42

e 90% of the amount is \$270

f 15% of the amount is \$60

g 40% of the amount is 100

h 120% of the amount of \$360

i 25% of the amount is \$75

j 8% of the amount is 40

Investigation

The birth month paradox

A **paradox** is a statement or proposition that seems impossible but is actually true.

1 Copy this table.

Group	Outcome (Y or N)
1	
2	
3	
4	
5	

2 Randomly select a group of 5 people and ask them what month they were born in. If 2 or more people have the same birth month, record a Y in the table for Group 1, otherwise write N.

3 Repeat this process 4 more times, recording your results in the table.

4 Combine your results with those of 6 other students so that you have the outcomes for 30 groups.

5 What fraction of the groups had repeated birth months?

6 Collect the results of another group of 6 students. What fraction of the groups had a repeated birth month?

7 The birth month paradox is that in any randomly selected group of 5 people, the probability that at least 2 people have the same birth month is greater than 0.5. Have your results shown this to be true?

8 Can you show the following?

- **a** For every 23 people selected at random, the probability that at least 2 people will share the same birthday is 50%.
- **b** If 30 people are selected at random, this probability is 70%.
- **c** If 50 people are selected at random, this probability is 97%

12.05 Selecting with and without replacement

WS Multi-step experiments

In two- and three-step experiments where an item is selected repeatedly, the outcome of the second or third step may be affected by the outcome of the previous step. This depends on whether each selected item is **returned** to the set of items before the next item is selected. If it is, then this is called **selecting 'with replacement'**. If it isn't, then it is called **selecting 'without replacement'**.

Example 7

2 cards are selected at random from a set of cards numbered 1 to 5, to form a 2-digit number.



a Make a list of all possible outcomes if the cards are drawn:

i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that the number formed is:

i even **ii** greater than 30 **iii** divisible by 5.

c If the first card is not replaced, find the probability that the number formed is:

i even **ii** odd **iii** less than 20

Solution

a i The possible outcomes, **with replacement**, are:

		1st digit				
		1	2	3	4	5
2nd digit	1	11	21	31	41	51
	2	12	22	32	42	52
	3	13	23	33	43	53
	4	14	24	34	44	54
	5	15	25	35	45	55

There are $5 \times 5 = 25$ different outcomes possible.

ii The possible outcomes, **without replacement**, are:

2nd digit \ 1st digit	1	2	3	4	5
1	-	21	31	41	51
2	12	-	32	42	52
3	13	23	-	43	53
4	14	24	34	-	54
5	15	25	35	45	-

There are fewer outcomes for 'without replacement' because numbers with repeated digits such as 11 and 44 are not allowed.

There are $5 \times 4 = 20$ different outcomes possible.

b **i** With replacement, there are 10 even numbers.

$P(\text{even number}) = \frac{10}{25} = \frac{2}{5}$

ii There are 15 numbers greater than 30.

$P(\text{number} > 30) = \frac{15}{25} = \frac{3}{5}$

iii There are 5 numbers divisible by 5. 15, 25, 35, 45 and 55

$P(\text{number divisible by 5}) = \frac{5}{25} = \frac{1}{5}$

c **i** Without replacement, there are 8 even numbers.

$P(\text{even}) = \frac{8}{20} = \frac{2}{5}$

ii There are 12 odd numbers.

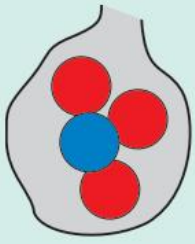
$P(\text{odd}) = \frac{12}{20} = \frac{3}{5}$

iii There are 4 numbers less than 20.

$P(\text{number} < 20) = \frac{4}{20} = \frac{1}{5}$

Example 8

A bag contains 3 red badges and one blue badge.
3 badges are drawn at random without replacement.



a Use a tree diagram to display all possible outcomes.

b Find the probability of drawing:

i 2 red badges

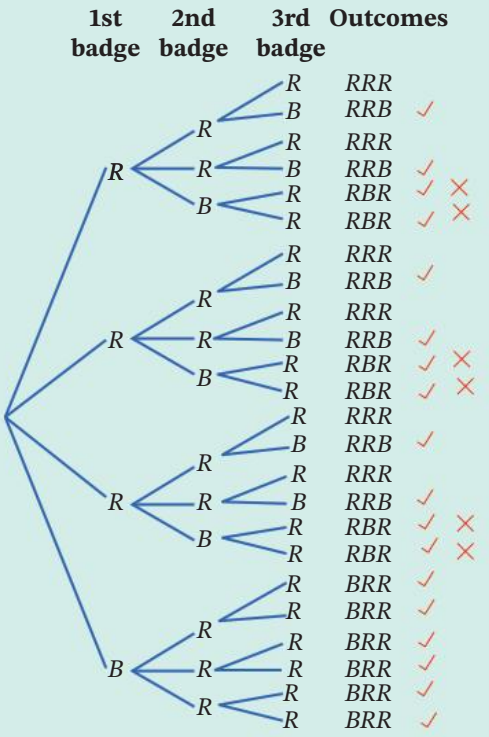
ii a red, blue and red in that order

iii at least one red badge.

Solution

a The tree diagram will have 4 branches for the first step or stage, followed by 3 branches for the second step, followed by 2 branches for the third step.

So there are $4 \times 3 \times 2 = 24$ outcomes in the sample space.



b **i** There are 18 outcomes with 2 red badges (✓ on the tree diagram)

$\therefore P(\text{2 red badges}) = \frac{18}{24} = \frac{3}{4}$

ii Red, blue, red (RBR) occurs 6 times (× on the tree diagram).

$\therefore P(\text{red, blue, red}) = \frac{6}{24} = \frac{1}{4}$

iii All outcomes contain at least one red badge.

$\therefore P(\text{at least one red badge}) = \frac{24}{24} = 1$

EXERCISE 12.05 ANSWERS ON P. 638

Selecting with and without replacement U F PS R C

EXAMPLE 7

1 The positions of captain and vice-captain of a netball team are to be selected from Cate, Amal, Gemma, Josie, Evangeline and Rukshana. R C

a List the possible pairings of captain and vice-captain.

b What is the probability of Amal being captain or vice-captain?

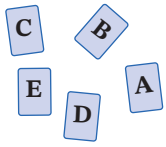
c What is the probability of Evangeline becoming vice-captain?



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2 2 cards are drawn from a set of cards labelled A, B, C, D and E. PS R C



a Make a list of all possible outcomes if the cards are drawn:

i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that:

i both letters are the same **ii** both letters are vowels

iii one letter is a vowel and the other is a consonant.

c If the first card is not replaced, find the probability that:

i both letters are vowels

ii one letter is a vowel and the other is a consonant

iii the first letter is a B or a D **iv** the last letter is not A.

3 When staying at a hotel, David and Sarah can select one item from each course of a breakfast menu. R C

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Fruit (F)	Ham and cheese croissants (H)
Yoghurt (Y)	Pancakes (P)
	Sausages and tomatoes (S)
	Toast and jam (T)

a Copy and complete the table to list all the different 2-course breakfasts available.

		2nd course				
		B	H	P	S	T
1st course	C					
	F					
	Y					

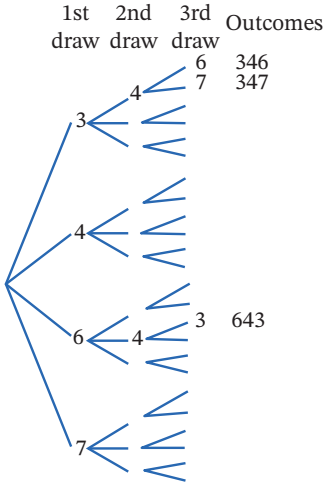
b If one of the combinations of breakfasts is chosen at random, what is the probability that it includes:

i fruit? **ii** cereal but not bacon and eggs?

iii fruit and croissants?

4 The numbers 3, 4, 6 and 7 are written on separate cards and placed in a bag. 3 cards are drawn at random without replacement to form a 3-digit number. PS R C

EXAMPLE 8



a Copy and complete the tree diagram to show all 24 possible outcomes.

b Find the probability of forming a number:

i that is even

ii greater than 400

iii between 400 and 700

iv that is even and greater than 400.

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5 The numbers 4, 5 and 9 are written on separate cards and placed in a bag. 3 cards are drawn at random with replacement to form a 3-digit number. PS R C

a Use a tree diagram to show all 27 possible outcomes.

b Find the probability of forming a number:

i with all digits the same

ii that is odd

iii less than 500

iv with all digits different.

6 A bag contains 2 red marbles, 1 green marble and 1 yellow marble. 3 marbles are drawn from the bag at random without replacement. PS R C

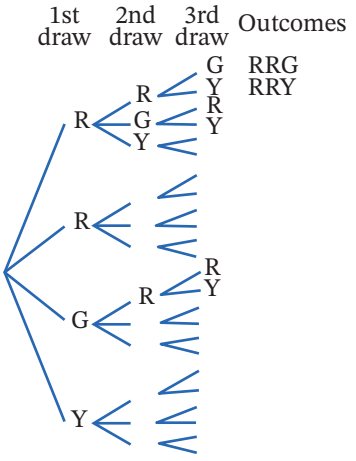
a Copy and complete the tree diagram to list the sample space.

b Find the probability of drawing:

i 2 red marbles

ii a red, green, and red marble in that order

iii at least one red marble.



7 A family has 3 children. PS R C

a Use a tree diagram to list all possible outcomes in the sample space.

b What is the probability that the family consists of:

i 3 boys?

ii 3 girls?

iii 2 girls and a boy?

iv a girl and then 2 boys?

8 2 ice cream flavours are selected from vanilla, strawberry and chocolate. PS R C

a Draw a tree diagram to show the sample space if the flavours are selected:

i with replacement (repetition allowed)

ii without replacement.

b If the flavours are selected with replacement, find the probability of selecting:

i 2 identical flavours

ii 2 different flavours

iii no vanilla

iv at least one strawberry flavour.

c If the flavours are selected without replacement, find the probability of selecting:

i 2 identical flavours

ii 2 different flavours

iii no vanilla

iv at least one strawberry flavour.

Investigation

Dependent or independent?

Work in pairs.

You will need: a coin, 3 blue pens and 2 red pens.

1 **a** **i** Toss a coin and record the outcome (head or tail).

ii What is the probability of obtaining your outcome?

b **i** Toss the coin a second time and record the outcome.

ii What is the probability of obtaining the second outcome?

c Is the outcome of the second toss affected by the outcome of the first toss? Is the probability of the second outcome independent or dependent on the first outcome? Justify your answer.

2 **a** Copy this table.

With replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Put 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Put back the pen you drew and shake the bag. Draw a pen again and record its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i P(blue pen drawn first) **ii** P(blue pen drawn second)

f **i** Are your 2 results for part **e** the same?

ii Would you expect the results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw?

3 **a** Copy this table.

Without replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Again, place 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Do not return the pen, shake the bag and draw a second pen, recording its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i P(blue pen drawn first) **ii** P(blue pen drawn second)

f **i** Are your 2 results for part **e** the same?

ii Would you expect your results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw? Compare your results with those of other students in your class.

12.06 Dependent and independent events

Two events are **independent** if the outcome of one event **does not affect** the outcome of the other event. So, one event occurring does not change the probability of the other event. For example, if a coin and a die are tossed together, the 2 events are independent as the outcome on the coin does not affect the outcome on the die. Wearing blue socks and passing a driving test are also independent events.

Two events are **dependent** if the outcome of one event **does affect** the outcome of the other event. So, one event occurring changes the probability of the other event occurring. For example, when selecting 2 coloured pencils from a pencil case without replacement, the 2 events are dependent because the outcome of the second draw is affected by the outcome of the first draw. Raining on a school sports day and sport being cancelled are also dependent events.

Example 9

A coin and a die are tossed together.

a List the outcomes in the sample space.

b Find:

i $P(\text{head on the coin})$ **ii** $P(\text{even number on the die})$

iii $P(\text{head and even number})$

c Is $P(\text{head and even number}) = P(\text{head}) \times P(\text{even number})$?

d Are the 2 events dependent or independent?

Solution

a The outcomes are H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5 and T6.

b **i** $P(\text{head}) = \frac{1}{2}$ **ii** $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

iii $P(\text{head and even}) = \frac{3}{12} = \frac{1}{4}$ H2, H4 and H6

c Yes, since $P(\text{head and even}) = \frac{1}{4}$ and $P(\text{head}) \times P(\text{even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

d The 2 events are independent since the outcome when tossing a coin does not affect the outcome when rolling a die.

The product rule for independent events

Two events are **independent** if the outcome of one event does not affect the outcome of the other event.

If A and B are 2 independent events,
then $P(A \text{ and } B) = P(A) \times P(B)$.

Example 10

A bag contains 3 brown chocolates and 1 white chocolate. 2 chocolates are drawn from the bag, without replacement.

a Find the probability of:

i selecting a brown chocolate with the first draw

ii selecting a brown chocolate with the second draw if the first chocolate was brown.

b Are selecting a brown chocolate with the first and second draws dependent or independent?

Solution

a **i** $P(\text{brown on the first draw}) = \frac{3}{4}$

ii After drawing a brown chocolate, there are 3 chocolates left, of which 2 are brown.

$\therefore P(\text{brown on the second draw}) = \frac{2}{3}$

b The bag contains 2 brown chocolates and 1 white chocolate for the second draw, so $P(\text{brown})$ decreases from $\frac{3}{4}$ to $\frac{2}{3}$. The second event is dependent on the first event.

EXERCISE 12.06 ANSWERS ON P. 639

Dependent and independent events U F PS R C

1 State whether each pair of events are dependent or independent. R C

a Rolling 4 on a die and rolling an even number on another die

b Rolling 6 on a die and rolling 6 again on the same die

c Training hard at soccer and winning a soccer match

d Drawing a red ball from a bag containing red and blue balls, replacing it and then drawing a blue ball from the bag

e Electing a team captain from a group of players and then electing a vice-captain from the same group

f Tossing 2 coins and obtaining a head on the first coin and a head on the second coin

g Finding $20 on the street and getting a phone call from your parent

2 In Lotto, 6 balls are drawn without replacement. Are the events of drawing each of the balls dependent or independent events? Give reasons. R C

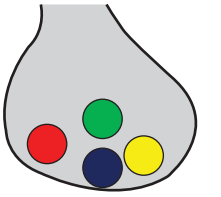
3 A coin is tossed 3 times and the result is heads each time. R C

a Are each of the 3 coin tosses dependent or independent events?

b The coin is tossed a 4th time. What is the probability of obtaining a head on the 4th toss?

EXAMPLE 9

4 A normal die is rolled and a marble is drawn from a bag containing a yellow marble, green marble, blue marble and red marble. R C





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a Find the probability of:

i rolling a number less than 3 with the die

ii drawing a green marble from the bag.

b List the outcomes for rolling the die and drawing a marble from the bag.

c What is the probability of rolling a number less than 3 and drawing a green marble?

d Is $P(\text{rolling a number less than 3}) \times P(\text{drawing a green marble}) = P(\text{a number less than 3 and a green marble})$?

e Are the events of rolling a number less than 3 and drawing a green marble dependent or independent?

5 A red die and a blue die are rolled. R

a Find:

i $P(5 \text{ on the red die})$ **ii** $P(\text{an even number on the blue die})$

b Hence find $P(5 \text{ on the red die AND an even number on the blue die})$.

EXAMPLE 10

6 A bag contains 5 red balls and 4 yellow balls. 2 balls are drawn at random without replacement. R C

a What is the probability of drawing a red ball first?

b What is the probability of drawing a red ball on the second draw if the first ball was red?

c Are the 2 events dependent or independent? Give reasons.

7 A bag contains 5 yellow counters and 3 red counters. 2 draws are made with no replacement. Find the probability of drawing: R C

a **i** a yellow counter on the first draw

ii a yellow counter on the second draw after a yellow counter was drawn

b **i** a red counter on the first draw

ii a yellow counter on the second draw after a red counter was drawn

c **i** a yellow counter on the first draw

ii a red counter on the second draw after a yellow counter was drawn

d **i** a red counter on the first draw

ii a red counter on the second draw after a red counter was drawn.

8 3 children in a family are all girls. What is the probability that the next child in this family will be a girl? R

9 A pair of dice are rolled. Use the product rule to find the probability of rolling: PS R

a a 6 on the first die and an odd number on the second die

b a 1 on both dice

c a prime number on the first die and a factor of 6 on the second die

d an even number on the first die and a number between 2 and 5 on the second die

10 The probability that Natalie wins her tennis match is $\frac{13}{20}$. The chance that she will pass her Science exam is $\frac{4}{5}$. What is the probability that Natalie: PS R

a wins her tennis match and passes her Science exam?

b loses her tennis match and passes her Science exam?

11 The probability that it will rain on Saturday is 0.6. The probability that it will rain on Sunday is 0.25. What is the probability that it will rain: PS R

a on both Saturday and Sunday? **b** on Sunday only?

c on neither Saturday nor Sunday?

12 A main road has 3 sets of traffic lights, each with a probability of 0.7 of being green. Use the product rule to find the probability of: PS R

a green on the first 2 lights, red (or amber) on the 3rd light

b green on all 3 lights

c red (or amber) on all 3 lights **d** only one of the 3 lights being green

Did you know?

Lotteries and Lotto

A lottery is a game of chance in which numbered tickets are drawn from tickets that have been sold. Lotteries were introduced by the State Government to raise money for hospitals. The first lottery was drawn on 20 August, 1931 with a first prize of £5000.

Lotteries have been used to celebrate special events and to help finance special projects. The Opera House Lottery, which commenced selling on 25 November 1957, was used to finance the construction of the Sydney Opera House.



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Other games of chance have been introduced, including Lotto (1979), Instant Scratchies (1982), OzLotto (1994) and Powerball (1996).

Research the probability of winning Lotto, OZ lotto and Powerball.

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12.07 Conditional probability

Conditional probability

In many practical situations, events are **dependent**. For example, the probability of a student arriving at school on time when catching a bus may be dependent on the amount of traffic.

Conditional probability is used to calculate probabilities for dependent events.

Conditional probability: Two-way tables

The **conditional probability** of an event B *given event* A, also written as $P(B|A)$, is the probability that event B occurs, given that event A has already occurred.

Example 11

A bag contains 3 red marbles and 2 yellow marbles. 2 marbles are drawn at random from the bag without replacement. What is the probability that the second marble is yellow, given that the first marble was also yellow?

Solution

If the first marble is yellow, then there are 3 red marbles and 1 yellow marble left in the bag.

$\therefore P(\text{second marble yellow, given the first marble is yellow}) = \frac{1}{4}$

Conditional probability

Example 12

2 dice are rolled and their total is calculated.

a Use a table to show all possible totals.

b Given that the total is 7, what is the probability that one of the dice shows a 3?

c Given that one of the dice shows a 4, what is the probability that the total is 10?

d Given that the total is 6, what is the probability of a double?

e Given that a double is rolled, what is the probability of:

i a total of 12? **ii** a total less than 10?

Solution

a

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

36 possible outcomes in the sample space

b There are 6 outcomes that give a total of 7. (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 2)

If one of the dice shows a 3, the possible outcomes are (3, 4) and (4, 3).

The conditional event 'total of 7' reduces the sample space from 36 to 6 outcomes.

$P(\text{one die is 3, given total is 7}) = \frac{2}{6} = \frac{1}{3}$

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c There are 11 outcomes that have 4 showing on one die. (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)

Of these outcomes, only 2 have a total of 10. (6, 4) and (4, 6)

$P(\text{total is 10, given one die is 4}) = \frac{2}{11}$ The conditional event 'one die is 4' reduces the sample space to 11 outcomes.

d There are 5 outcomes that give a total of 6. (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)

There is only 1 double. (3, 3)

$P(\text{double} \mid \text{total is 6}) = \frac{1}{5}$ '|' means 'given that'; 'total of 6' reduces the sample space to 5 outcomes.

e **i** There are 6 doubles that can be rolled. (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

(6, 6) is the only double with a total of 12. The conditional event 'a double' reduces the sample space to 6 outcomes.

$P(\text{total is 12} \mid \text{double}) = \frac{1}{6}$

ii (1, 1), (2, 2), (3, 3) and (4, 4) are the doubles with a total less than 10.

$P(\text{total} < 10 \mid \text{double}) = \frac{4}{6} = \frac{2}{3}$

12.07

EXERCISE 12.07 ANSWERS ON P. 639

Conditional probability U F PS R C

EXAMPLE 11

1 A bag contains 4 yellow and 3 red marbles. 2 marbles are drawn from the bag without replacement. R

a Given that the first marble was red, what is the probability that the second marble is also red?

b Given that the first marble was yellow, what is the probability that the second marble is red?

2 10 remote-controlled cars are delivered to a toy store, 3 of which are defective. 2 cars are randomly selected and tested. What is the probability that the second car tested is defective, given that the first car tested was defective and was not replaced? R C



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3 In a physics class, there are 5 boys and 7 girls. 2 students are randomly selected to help with an experiment. If the first student was a boy, what is the probability that: R

a both students chosen were boys? **b** a boy and a girl were chosen?

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4 In Aiden's pencil case, there are 3 red pens, 4 blue pens and 5 black pens. Aiden takes out 2 pens at random. Find: R C

a P(2nd pen red | 1st pen red)

b P(2nd pen blue | 1st pen red)

c P(2nd pen black | 1st pen black)

d P(2nd pen black | 1st pen blue)

5 A die is rolled and a number less than 4 is the result. What is the probability that the number is even?

6 A coin is tossed and a die is rolled at the same time. Knowing that an even number has been rolled, what is the probabililty of the result being a head and a 4? R C

EXAMPLE 12

7 2 dice are rolled and the sum of the numbers is calculated. PS R C

a Draw up a table to show all possible sums.

b Given that the sum is 9, find the probability that:

i one of the dice shows a 4

ii one of the dice shows an even number.

c Knowing that one of the dice shows a 6, find the probability that the sum is 11.

d Given that one of the dice shows an even number, find the probability that:

i the sum is even

ii the sum is 7.

e If the dice show a double, what is the probability of a sum of 4?

8 A drawer contains 5 different pairs of coloured socks: black, blue, red, brown and white.

a Haylee randomly takes 2 socks from the drawer. Find: R C

i P(2nd sock blue | 1st sock blue)

ii P(2nd sock not blue | 1st sock blue)

b Haylee selects a red sock and a blue sock. What is the probability that the third sock she selects will form a matching pair?

c What is the maximum number of socks that Haylee will need to take from the drawer before she has at least one matching pair of socks?

9 Lotto is a game of chance in which 6 balls are selected at random from a barrel containing balls numbered 1 to 45. What is the chance of Yuri winning Lotto with the 6th ball, given that he has the first 5 numbers? R

10 3 cards are chosen at random from a normal deck of 52 cards. Given that the 3 cards are hearts, what is the probability that the 4th card is also a heart? R

11 A card is drawn at random from a deck of 52 cards. What is P(Queen | heart card)? R C

12 2 dice are rolled. What is P(2nd dice = 6 | 1st die = 6)? R C

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13 2 dice are rolled and the difference between the numbers is calculated. PS R C

a Copy and complete this table to show all possible outcomes.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2					3	
	3						
	4			1			
	5						
	6						

$6 - 1 = 5$



The difference between these numbers is 5.

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b What is the probability of rolling a difference:

i of 0? **ii** of 5? **iii** greater than 3?

c Knowing that the difference is 4, what is the probability that one of the dice shows:

i a 1? **ii** a 3?

d If one of the dice shows a 4, find the probability that the difference is:

i 0 **ii** 1

e Given that the difference is odd, find the probability that one of the dice shows:

i a 3 **ii** an even number.

Power plus ANSWERS ON P. 640

1 Students at Nelson Secondary College were surveyed about which sport they like to watch and what type of movies they like to see.

	Horror/Drama	Fantasy	Comedy	Action
Football	23	34	30	48
Cricket	25	12	45	34
Tennis	8	12	32	17

a How many students were surveyed?

b If a student is selected at random, what is the probability that the student likes to watch:

i horror/drama movies? **ii** comedy and football?

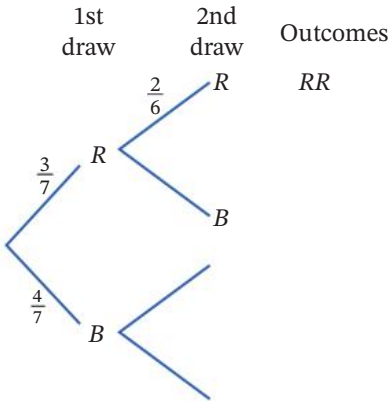
iii tennis, but not fantasy? **iv** action, but not cricket or tennis?

c Given that a student likes to watch football, find the probability that the student also likes to watch action movies.

d Of the students who like comedy, what is the probability that they also like to watch cricket?

2 A bag contains 3 red and 4 blue marbles. 2 marbles are taken out of the bag without replacement.

a A probability tree is a tree diagram that has the probability of each step or stage listed on the branches. Copy and complete the probability tree shown to list all possible outcomes.



b Use the probability tree diagram to find the probability of drawing:

i 2 red marbles

ii 2 blue marbles

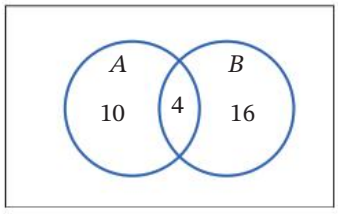
iii a blue and a red marble

iv at least one blue marble.

3 **a** Find:

i $P(A)$ **ii** $P(B)$ **iii** $P(A \text{ and } B)$

iv $P(A \mid B)$ **v** $P(B \mid A)$



b **i** Find the value of $\frac{P(A \text{ and } B)}{P(B)}$

ii Is $P(A|B)=\frac{P(A \text{ and } B)}{P(B)}$?

c Show, by calculation, that $P(B|A)=\frac{P(A \text{ and } B)}{P(A)}$.

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CHAPTER 12 REVIEW

Language of maths

at least	compound event	conditional probability
dependent event	die/dice	event
expected frequency	experimental probability	independent event
mutually exclusive	observed frequency	overlapping
random	relative frequency	sample space
table	theoretical probability	three-step experiment
tree diagram	trial	two-step experiment
two-way table	Venn diagram	with/without replacement

1 What is the meaning of **expected frequency**?

2 What term from the above list is another name for **experimental probability**?

3 On a **Venn diagram**, what does the rectangle represent?

4 Give an example of **dependent events**.

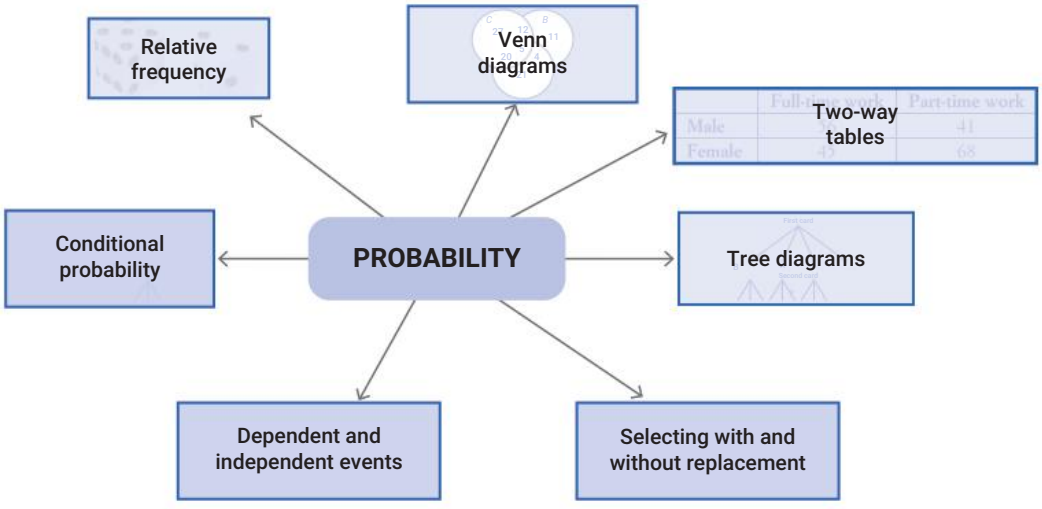
5 When are **tree diagrams** used in probability?

6 For 2 events A and B, what is the difference between '**A or B**' and '**A and B**'?

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

WS
Mind map: Probability



TEST YOURSELF 12 ANSWERS ON P. 640

12.01

1 3 coins are tossed 150 times and the number of heads at each trial is recorded in the table.

Number of heads	Frequency
0	20
1	53
2	64
3	13

a Find correct to 3 decimal places the relative frequency of tossing:

i one head

ii 2 heads

iii 3 heads

iv at least 2 heads.

b Find correct to 3 decimal places the experimental probability of:

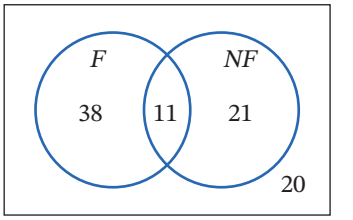
i at least one head

ii 3 tails.

c Are the answers in part **b** the same or different? Explain why.

12.02

2 People at a train station were asked whether they prefer to read fiction (F) or non-fiction (NF) books. The results are shown in the Venn diagram.



a How many people were surveyed?

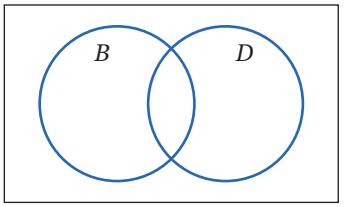
b Find the probability of selecting a person from this group who only reads fiction books.

c What is the probability of selecting a person who doesn't read fiction or non-fiction books?

d Of the people who read fiction books, find the probability that they read non-fiction books.

12.02

3 Of 20 people in an office, 6 have blue eyes (B), 8 have dark hair (D) and 3 have blue eyes and dark hair.



a Copy and complete the Venn diagram to show the given information.

b What is the probability of selecting a person at random from the office who has:

i blue eyes only?

ii dark hair?

iii blue eyes and dark hair?

iv hair that is not dark?

c What is the probability of selecting a person at random who has neither blue eyes nor dark hair?

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4 Students were asked what type of activities they would like to do on a camp. 12.03

	Hiking	Rock climbing	Kayaking
Boys	25	38	47
Girls	45	23	22

a How many students were surveyed?

b Find the probability (as a decimal) that a student selected at random:

i likes rock climbing

ii likes kayaking and is a girl

iii is a girl who likes hiking

iv is a boy who likes rock climbing or kayaking

c If a boy is selected at random, what is the probability that his favourite activity is hiking?

d If a student who likes kayaking is chosen, what is the probability that the student is:

i a boy? **ii** a girl?

5 A pair of dice are rolled and the sum of the 2 numbers is calculated. Find the probability of rolling a sum of 6. Select the correct answer **A**, **B**, **C** or **D**. 12.04

A $\frac{1}{5}$ **B** $\frac{1}{9}$ **C** $\frac{5}{6}$ **D** $\frac{5}{36}$

6 A pair of 4-sided dice (numbered 1, 2, 3 and 4) are rolled.

a Copy and complete this table.

		1st die			
		1	2	3	4
2nd die	1				
	2		2, 2		
	3				
	4			3, 4	

b How many possible outcomes are there?

c Find the probability of rolling:

i one odd and one even number

ii 2 even numbers

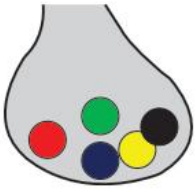
iii at least one 3

iv 2 numbers less than 3

v a double

vi 2 numbers so that the first number is odd.

7 2 marbles are drawn from a bag containing a red, a blue, a green, a yellow and a black marble. 12.05



a Make up a list to show all the possible outcomes if the marbles are taken:

i with replacement

ii without replacement.

b If the first marble is replaced before the second marble is drawn, find the probability of drawing:

i 2 red marbles **ii** 2 marbles of the same colour

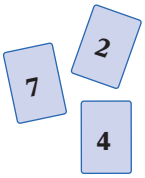
iii a yellow and a black marble **iv** at least one green marble.

c If the first marble is not replaced, find the probability of drawing:

i a green marble and a yellow marble **ii** no red marbles.

12.05

8 The numbers 2, 4, and 7 are written on separate cards and placed in a bag. 3 cards are drawn at random to form a 3-digit number.



a Make up a tree diagram to list the sample space if the cards are drawn:

i with replacement

ii without replacement.

b If the cards are drawn with replacement, find the probability of forming:

i an even number

ii a number less than 400

iii the numbers 222, 444, or 777

iv an odd number greater than 400.

c If the cards are drawn without replacement, find the probability of forming:

i an odd number

ii a number greater than 400

iii a number beginning with 7

iv a number divisible by 4.

12.06

9 State whether the following pairs of events are dependent or independent.

a Tossing a tail from a coin and then a head from the same coin.

b Drawing a ticket in a raffle and winning a first prize and then drawing a second ticket and winning a second prize.

c Electing a president for a cricket club and then electing the vice-president of the cricket club.

d A family's first 3 children are girls and then the 4th child is also a girl.

e Rain in your town or city today and a car accident.

f The Roosters winning a football match and you winning Lotto.

12.07

10 2 hair ties are drawn without replacement from a bag containing 3 purple and 2 red hair ties.

a Copy and complete the tree diagram to show the sample space.

b Given that the first hair tie drawn is purple, what is the probability that the second hair tie will be:

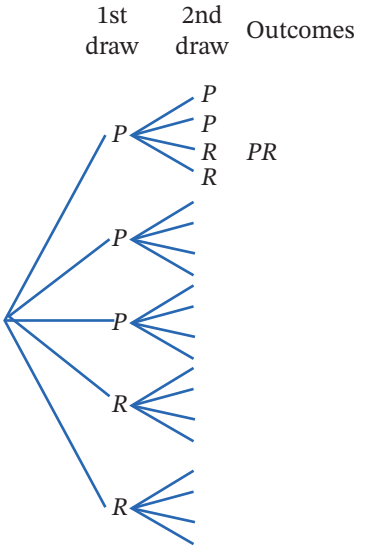
i red? **ii** purple?

c Given that the first hair tie drawn is red, find the probability that the second hair tie is:

i red **ii** purple.

d Given that the drawn hair ties are the same colour, what is the probability that they are:

i both red? **ii** both purple?



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12.07

11 The pair of 4-sided dice from question **6** are rolled and the sum of the numbers is calculated.

a Draw up a table to show all possible sums.

b Given that the sum is 5, find the probability that one of the dice shows:

i a 2 **ii** an odd number.

c Find $P(\text{sum of } 7 \mid \text{one die is } 4)$.

d Given that one of the dice shows an even number, find the probability that the sum:

i is even **ii** is 5.

e Find $P(\text{sum of } 2 \mid \text{dice show a double})$.



13

MEASUREMENT AND GEOMETRY

GEOMETRY

The word ‘geometry’ comes from the Greek word *geometria*, which means ‘earth measuring’. The principles and ideas of geometry are evident in many aspects of our lives. For example, geometry can be seen in the design of buildings, bridges, roads and transport networks, such as this footbridge over the Yarra River in central Melbourne.



Chapter outline

		Working mathematically				
13.01	Angle sum of a polygon#	U	F		R	C
13.02	Congruent triangle proofs	U	F		R	C
13.03	Tests for quadrilaterals*	U	F	PS	R	C
13.04	Proving properties of triangles and quadrilaterals	U	F		R	C
13.05	Formal geometrical proofs*	U	F	PS	R	C
13.06	Finding unknown sides in similar figures	U	F	PS	R	
13.07	Tests for similar triangles	U	F		R	C
13.08	Similar triangle proofs*	U	F		R	C

***STAGE 5.3**

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Wordbank

congruence test One of 4 tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical, exactly the same (symbol: ≡)

enlargement An increase in the size of a shape

included angle The angle between 2 given sides of a shape

convex polygon A polygon whose vertices all point outwards

quadrilateral test A property of a quadrilateral that proves that it is a particular type of quadrilateral, for example, if opposite angles are equal, then it *must* be a parallelogram.

regular polygon A polygon with all angles equal and all sides equal, such as an equilateral triangle or a square

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||)

similarity test One of 4 tests for proving that triangles are similar: 'SSS', 'SAS', 'AA' and 'RHS'

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve geometrical problems involving the angle sum of a polygon and the exterior angle sum of a convex polygon
- write formal proofs for congruent triangles
- (STAGE 5.3) identify and use the definitions and tests for the special quadrilaterals
- investigate properties of special triangles and quadrilaterals using congruent triangles and solve related geometry problems
- formulate proofs involving congruent triangles and angle properties
- (STAGE 5.3) write formal geometrical proofs using logical reasoning
- use scale factors to find unknown sides in similar figures
- identify and use the 4 tests for similar triangles
- (STAGE 5.3) write formal proofs for similar triangles

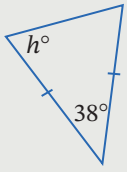
SkillCheck ANSWERS ON P. 641

Geometry

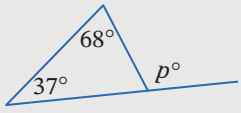
Finding angles

1 Find the value of each variable.

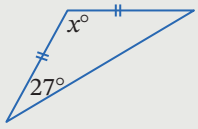
a



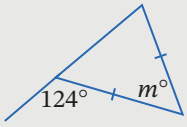
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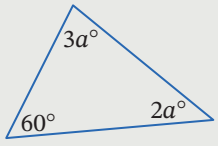
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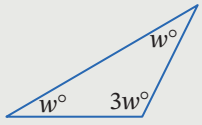
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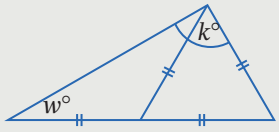
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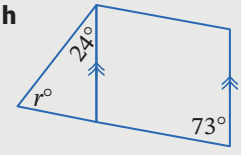
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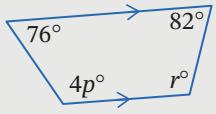
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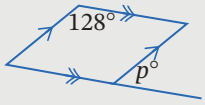
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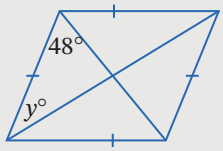
i



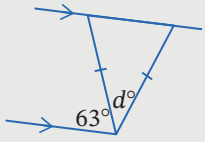
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k

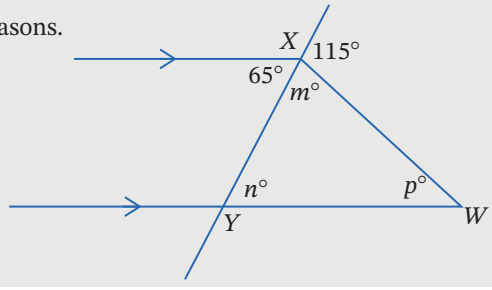


l



2 **a** Find the value of m, n *and* p, giving reasons.

b What type of triangle is $\triangle XYW$?



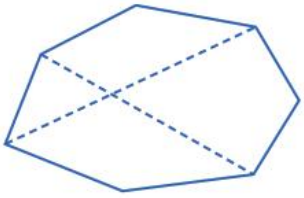
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Angle sum of a polygon#

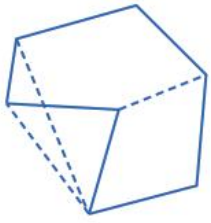
13.01

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Convex polygon



Non-convex polygon



WS Angle sum of a polygon

WS Equal angles

WS Angles in polygons

Angle sum of a polygon

The angle sum of a polygon with n sides is given by the formula

$$A = 180(n - 2)°.$$

This formula applies to both convex and non-convex polygons.

Example 1

Find the angle sum of a 15-sided polygon.

Solution

Angle sum $= 180\,(15 - 2)°$ $\qquad n = 15$

$= (180 \times 13)°$

$= 2340°$

Example 2

Find the number of sides in a polygon that has an angle sum of 1080°.

Solution

$180(n - 2) = 1080$

$180n - 360 = 1080$

$180n = 1440$

$n = \frac{1440}{180}$

$= 8$

∴ The polygon has 8 sides (octagon).

Angle in a regular polygon

The size of each angle in a **regular polygon** with n sides is:

$$\frac{\text{Angle sum}}{\text{Number of sides}} = \frac{180(n-2)°}{n}$$

Example 3

Find the size of one angle in a regular pentagon.

Solution

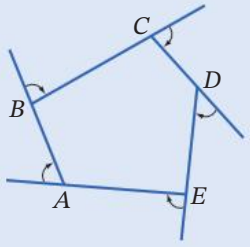
A pentagon has 5 sides ($n = 5$).

$$\text{Size of one angle} = \frac{180(5-2)^\circ}{5}$$
$$= \frac{(180 \times 3)^\circ}{5}$$
$$= 108^\circ$$

Each angle in a regular pentagon is 108°.

Exterior angle sum of a convex polygon

The sum of the exterior angles of a convex polygon is 360°.

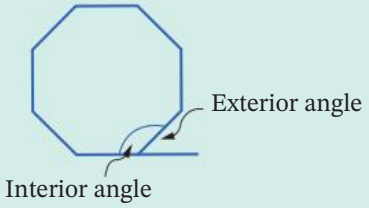


Example 4

For a regular octagon, find the size of:

a each exterior angle

b each (interior) angle.



Solution

a Sum of exterior angles = 360°

One exterior angle = 360° ÷ 8 = 45°

b Each angle = 180° – 45° (angles on a straight line)

= 135°

OR: Each angle $= \frac{180(8-2)^\circ}{8}$

= 135°

Example 5

Find the number of sides of a regular polygon if:

a each exterior angle is 24°

b each (interior) angle is 140°.

Solution

a Number of exterior angles = 360 ÷ 24

= 15

∴ The regular polygon has 15 sides.

b Exterior angle = 180° – 140° (angles on a straight line)
= 40°
Sum of exterior angles = 360°
Number of exterior angles = 360° ÷ 40°
= 9
∴ The regular polygon has 9 sides.

OR: $\frac{180(n-2)}{n} = 140$
$180(n-2) = 140n$
$180n - 360 = 140n$
$40n = 360$
$n = \frac{360}{40}$
$= 9$

13.01

EXERCISE 13.01 ANSWERS ON P. 641

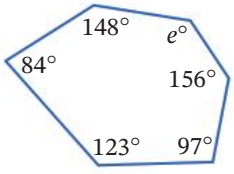
Angle sum of a polygon UFRC

1 Find the angle sum of a polygon with: EXAMPLE 1

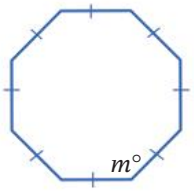
a 12 sides **b** 10 sides **c** 9 sides **d** 20 sides **e** 15 sides

2 Find the value of each variable. R

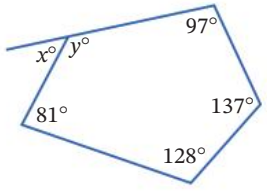
a



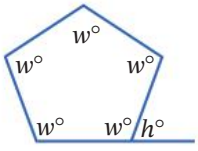
b



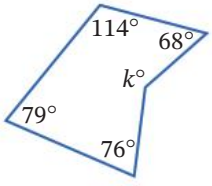
c



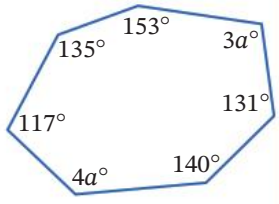
d



e



f



3 Find the number of sides in a polygon that has an angle sum of: R EXAMPLE 2

a 720° **b** 3420° **c** 1980° **d** 5040° **e** 1260°.

4 The angle sum of a regular polygon is 2520°. R

a How many sides does the polygon have?

b Find the size of each angle.

5 Find the size of one angle in a regular: R EXAMPLE 3

a decagon **b** octagon **c** hexagon **d** dodecagon.

6 Find the size of one angle in a regular polygon with 15 sides. Select **A**, **B**, **C** or **D**. R

A 142° **B** 150.5° **C** 156° **D** 165°

7 How many sides does a regular polygon have if each of its angles is: R

a 168°? **b** 172°? **c** 165.6°?

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EXAMPLE 4

8 Find the size of each exterior angle in a regular:

a pentagon **b** dodecagon

c 18-sided polygon **d** hexagon.

9 Find the size of each angle in a regular: R

a nonagon **b** 20-sided polygon

c decagon **d** 30-sided polygon.

EXAMPLE 5

10 Find the number of sides in a regular polygon if each exterior angle is:

a 15° **b** 72° **c** 20° **d** 40° **e** 5°

11 Find the number of sides in a regular polygon with an exterior angle of 12°. Select the correct answer **A**, **B**, **C** or **D**.

A 12 **B** 15 **C** 30 **D** 45

12 Find the number of sides in a regular polygon if each angle is: R

a 135° **b** 144° **c** 156°

d 178° **e** 165° **f** 150°.

Did you know?

The Beijing 'Water Cube'



Alamy Stock Photo/Arcaid Images

The National Aquatic Centre in Beijing, China, was specifically designed by Australian architects for the 2008 Summer Olympic Games. Also called the 'Water cube', it held the Olympic swimming, diving and synchronized swimming events.

This aquatic centre was built using a groundbreaking geometric and sustainable design for its time, inspired by the geometry of the soap bubble, which has the properties of a polygon. The polygonal panels are made from ETFE and are semi-translucent. They allow natural daylight into the centre and save on lighting energy.

Beijing is a city that suffers from water scarcity and water conservation is also a key feature in the geometric design. Pipes in the polygon structure allow water to be harvested from roof catchment areas and reused throughout the building.

1 **Find out why the geometry of the polygon was useful for the design of the aquatic centre, and the special properties of the material ETFE.**

2 **The polygons are what gives the building its name 'Water cube', find out why.**

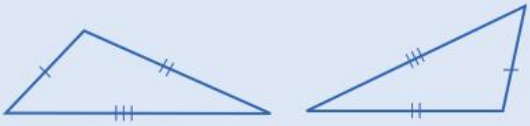
Congruent triangle proofs 13.02

Congruence tests

There are 4 tests for congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

Two triangles are congruent if:

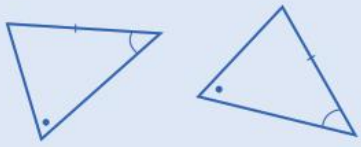
- the 3 sides of one triangle are respectively equal to the 3 sides of the other triangle (**SSS** rule)



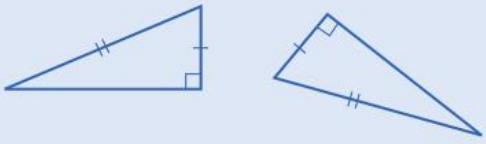
- 2 sides and the **included angle** of one triangle are respectively equal to 2 sides and the **included angle** of the other triangle (**SAS** rule)



- 2 angles and one side of one triangle are respectively equal to 2 angles and the matching side of the other triangle (**AAS** rule)



- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle (**RHS** rule).



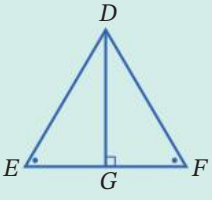
$\triangle ABC \equiv \triangle XYZ$ also means $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

Example 6

In this diagram, $\angle DEF = \angle DFE$ and $DG \perp EF$.

a Prove that $\triangle DEG \equiv \triangle DFG$.

b Hence show that G is the midpoint of EF.



Solution

a In $\triangle DEG$ and $\triangle DFG$:

	Identifying the triangles in matching order of vertices.
$\angle DEF = \angle DFE$ (given)	Stating each part of the congruence test, giving reasons.
$\angle DGE = \angle DGF = 90°$ $(DG \perp EF)$	
DG is common	
$\therefore \triangle DEG \equiv \triangle DFG$ (AAS)	Concluding the congruence proof, stating the test used.

b $\therefore EG = GF$ (matching sides of congruent triangles)

$\therefore G$ is the midpoint of EF.

Example 7

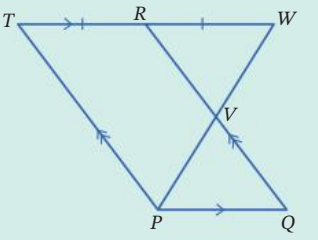
In the diagram, $PQRT$ is a parallelogram.

TR is extended to W so that $TR = RW$.

Prove that:

a $\triangle PQV \equiv \triangle WRV$

b V is the midpoint of PW.



Solution

a In $\triangle PQV$ and $\triangle WRV$:

$\angle Q = \angle WRV$ (alternate angles, $PQ \parallel TW$)

$\angle PVQ = \angle WVR$ (vertically opposite angles)

$PQ = TR$ (opposite sides of a parallelogram equal)

$TR = RW$ (given)

$\therefore PQ = RW$

$\therefore \triangle PQV \equiv \triangle WRV$ (AAS)

b $\therefore PV = WV$ (matching sides of congruent triangles)

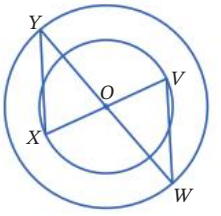
$\therefore V$ is the midpoint of PW.

EXERCISE 13.02 ANSWERS ON P. 641

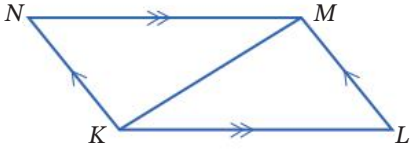
Congruent triangle proofs U F R C

EXAMPLE 6

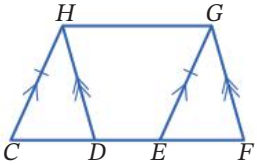
1 O is the centre of both circles. Prove that $\triangle XOY \equiv \triangle VOW$. R C



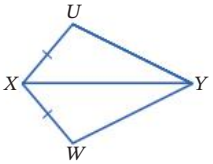
2 $KLMN$ is a quadrilateral with opposite sides parallel. Prove that $\triangle KLM \equiv \triangle MNK$. R C



3 $CH \parallel EG$, $DH \parallel$ FG and $CH = EG$. Prove that $\triangle CDH \equiv \triangle EFG$. R C



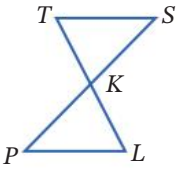
4 YX bisects $\angle UXW$ and $UX = WX$. Prove that $\triangle UXY \equiv \triangle WXY$. R C



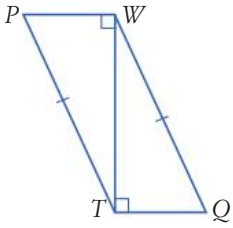
Foundation Standard Complex

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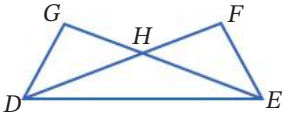
5 $TK = LK$ and $SK = PK$. Prove that $\triangle TSK \equiv \triangle LPK$. R C



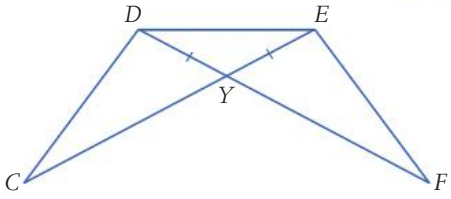
6 $QT \perp WT$, $PW \perp WT$ and $QW = PT$. Prove that $\triangle QTW \equiv \triangle PWT$. R C



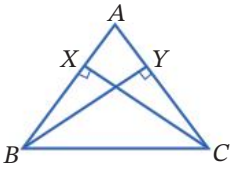
7 If $\angle GDE = \angle FED$ and $GD = FE$, prove that $\triangle GDE \equiv \triangle FED$ and hence $\angle DGE = \angle EFD$. R C



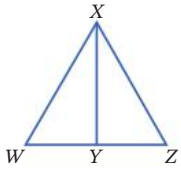
8 If $\angle CDE = \angle FED$ and $DY = EY$ in the diagram, prove that $\triangle CDE \equiv \triangle FED$ R C



9 If $CX \perp AB$, $BY \perp AC$ and $XC = YB$, prove that $\triangle BCX \equiv \triangle CBY$. R C



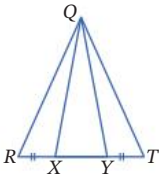
10 $XW = XZ$ in this isosceles triangle and Y is the midpoint of WZ. Prove that $\triangle WYX \equiv \triangle ZYX$. R C



11 $\triangle QRT$ is isosceles with $QR = QT$. If $RX = TY$, prove that:

a $\triangle QRX \equiv \triangle QTY$

b $\triangle QXY$ is isosceles.

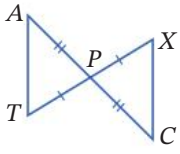


12 $TP = XP$ and $AP = CP$. Prove that:

a $\triangle TAP \equiv \triangle XCP$

b $TA = XC$

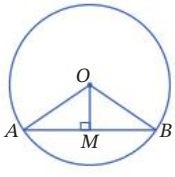
c $TA \parallel XC$



13 O is the centre of the circle and $OM \perp AB$. Prove that: R C

a $\triangle OAM \equiv \triangle OBM$

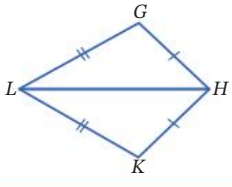
b OM bisects AB.



14 $GHKL$ is a kite with $GL = KL$ and $GH = KH$. Prove that: R C

a $\triangle GLH \equiv \triangle KLH$

b LH bisects $\angle GLK$ and $\angle GHK$.

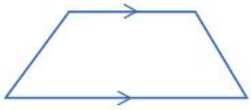
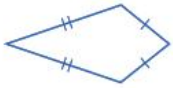
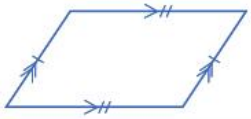
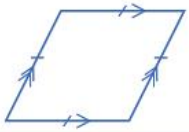
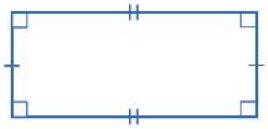
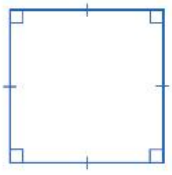


13.03 Tests for quadrilaterals

STAGE 5.3

WS Quadrilaterals: True or false?

The special quadrilaterals

Quadrilateral	Formal definition	Properties
Trapezium 	A quadrilateral with at least one pair of opposite sides parallel.	None
Kite 	A convex quadrilateral with 2 pairs of equal adjacent sides.	• One pair of opposite angles equal • Diagonals intersect at right angles
Parallelogram 	A quadrilateral with both pairs of opposite sides parallel.	• Opposite sides equal • Opposite angles equal • Diagonals bisect each other
Rhombus 	A parallelogram with 2 adjacent sides equal in length.	• All sides equal • Diagonals bisect each other at right angles • Diagonals bisect the angles of the rhombus
Rectangle 	A parallelogram with one angle a right angle.	• All angles are right angles • Diagonals are equal in length
Square 	A rectangle with 2 adjacent sides equal in length.	• All sides equal • All angles are right angles • Diagonals are equal in length • Diagonals bisect each other at right angles • Diagonals bisect the angles of a square

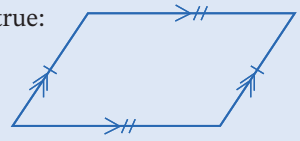
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Note that the definitions of the quadrilaterals give *minimum conditions*. For example, a rhombus is defined as 'a parallelogram with 2 adjacent sides equal in length'. However, opposite sides are equal in a parallelogram so that means the rhombus has *all* sides equal in length.

Some properties of the quadrilaterals can be used as minimum conditions to prove or test that a quadrilateral is a parallelogram, rectangle, square or rhombus. For example, if opposite angles are equal, then it *must* be a parallelogram.

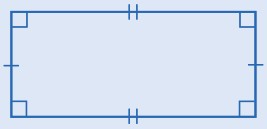
Tests for quadrilaterals

A quadrilateral is a **parallelogram** if one of these conditions is true:



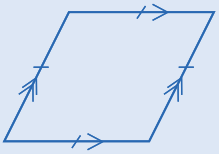
- both pairs of opposite angles are equal, or
- both pairs of opposite sides are equal, or
- both pairs of opposite sides are parallel, or
- one pair of opposite sides are equal and parallel, or
- the diagonals bisect each other.

A quadrilateral is a **rectangle** if one of these conditions is true:



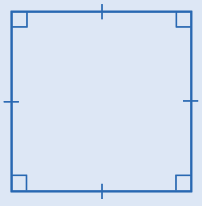
- all angles are 90°, or
- diagonals are equal and bisect each other.

A quadrilateral is a **rhombus** if one of these conditions is true:



- all sides are equal, or
- diagonals bisect each other at right angles.

A quadrilateral is a **square** if one of these conditions is true:



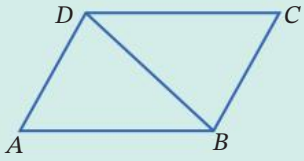
- all sides are equal and one angle is 90°, or
- all angles are 90° and 2 adjacent sides are equal, or
- diagonals are equal and bisect each other at right angles.

STAGE 5.3

Example 8

$ABCD$ is a quadrilateral with BD as a diagonal.

Prove that if the opposite sides of $ABCD$ are equal, then it must be a parallelogram.



Solution

In $\triangle ABD$ and $\triangle CDB$:

$AD = CB$	(opposite sides of $ABCD$ are equal)
$AB = CD$	(opposite sides of $ABCD$ are equal)
BD is common	
$\therefore \triangle ABD \equiv \triangle CDB$	(SSS)
$\therefore \angle ABD = \angle CDB$	(matching angles of congruent triangles)
$\therefore AB \parallel CD$	(alternate angles are equal)
Also, $\angle ADB = \angle CBD$	(matching angles of congruent triangles)
$\therefore AD \parallel CB$	(alternate angles are equal)
$\therefore ABCD$ is a parallelogram	(opposite sides are parallel)

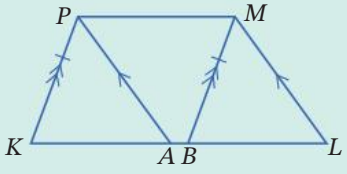
Example 9

In the diagram, $KP \parallel BM$, $AP \parallel LM$ and $KP = BM$.

Prove that:

a $\triangle KAP \equiv \triangle BLM$

b $ALMP$ is a parallelogram



Solution

a In $\triangle KAP$ and $\triangle BLM$:

$\angle PKA = \angle MBL$	(corresponding angles, $KP \parallel BM$)
$\angle KAP = \angle BLM$	(corresponding angles, $AP \parallel LM$)
$KP = BM$	(given)
$\therefore \triangle KAP \equiv \triangle BLM$	(AAS)

b

$\therefore AP = LM$	(matching sides of congruent triangles)
$AP \parallel LM$	(given)
$\therefore ALMP$ is a parallelogram	(one pair of opposite sides are parallel and equal)

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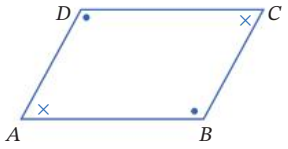
EXERCISE 13.03 ANSWERS ON P. 641

Tests for quadrilaterals U F PS R C

EXAMPLE 8

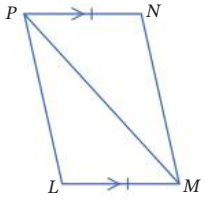
13.03

1 *ABCD* is a quadrilateral in which opposite angles are equal. Prove that *ABCD* must be a parallelogram. PS R C

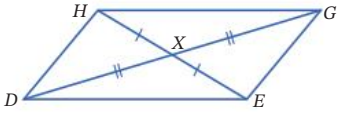


2 *LMNP* is a quadrilateral in which $LM = NP$ and $LM \parallel NP$.

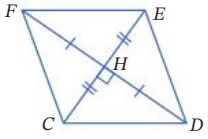
Prove that if a pair of opposite sides in a quadrilateral are equal and parallel, then the quadrilateral must be a parallelogram. PS R C



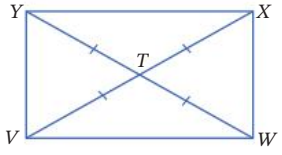
3 *DEGH* is a quadrilateral whose diagonals *DG* and *EH* bisect each other. Prove that it must be a parallelogram. PS R C



4 *CDEF* is a quadrilateral whose diagonals *CE* and *DF* bisect each other at right angles. Prove that *CDEF* must be a rhombus. PS R C

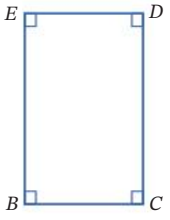


5 *VWXY* is a quadrilateral whose diagonals *VX* and *WY* are equal and bisect each other. Prove that it must be a rectangle. PS R C



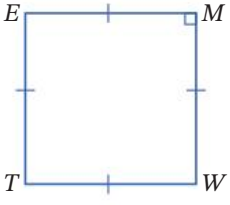
6 *BCDE* is a quadrilateral with all angles equal to 90°.

Prove that its opposite sides are parallel as well, and hence it must be a rectangle. PS R C



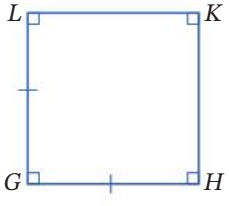
7 *TWME* is a quadrilateral with all sides equal and $\angle M = 90°$.

Prove that the other angles are 90° as well and hence it must be a square. PS R C



8 *GHKL* is a quadrilateral with all angles equal to 90° and $GH = GL$.

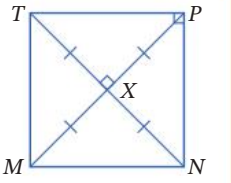
Prove that all sides are equal as well and hence it must be a square. PS R C



Foundation | Standard | Complex

STAGE 5.3

9 $MNPT$ is a quadrilateral whose diagonals MP and NT are equal and bisect each other at right angles. Prove that $MNPT$ must be a square. PS R C

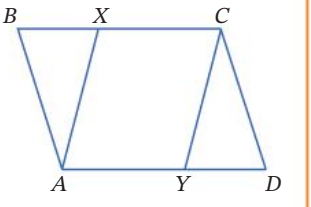


EXAMPLE 9

10 $ABCD$ is a parallelogram and $BX = DY$. Prove that: R C

a $\triangle ABX \equiv \triangle CDY$

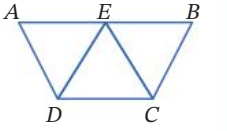
b $AXCY$ is a parallelogram.



11 $AECD$ is a rhombus and $AE = EB$. Prove that: R C

a $\triangle CBE \equiv \triangle DAE$

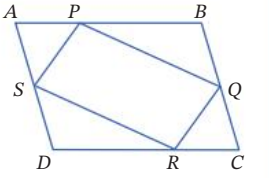
b $BCDE$ is a parallelogram.



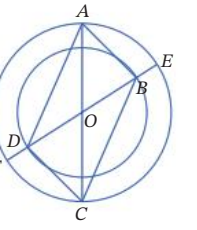
12 $ABCD$ is a parallelogram and $AP = AS = CQ = CR$. Prove that: PS R C

a $RQ = PS$ and $PQ = RS$

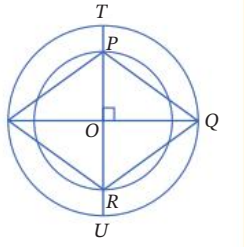
b $PQRS$ is a parallelogram



13 AC and DB are diameters of concentric circles with centre O. Prove that $ABCD$ is a parallelogram. PS R C



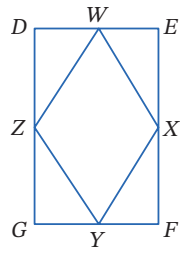
14 PR and SQ are diameters of concentric circles, centre O and $TU \perp SQ$. Prove that $PQRS$ is a rhombus. PS R C



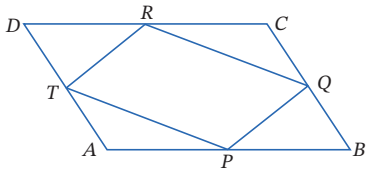
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15 *DEFG* is a rectangle. *W*, *X*, *Y* and *Z* are the midpoints of the sides. Prove that *WXYZ* is a rhombus. PS R C



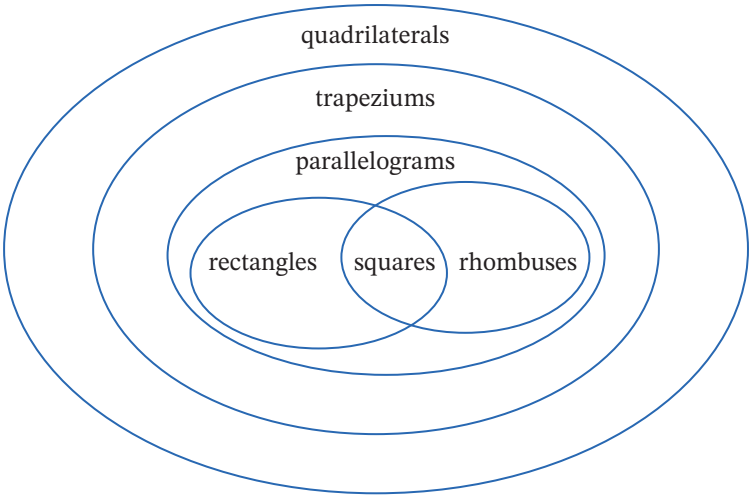
16 *ABCD* is a parallelogram. *P*, *Q*, *R* and *T* are the midpoints of the sides. Prove that *PQRT* is a parallelogram. PS R C



Investigation

Is a square a rhombus?

Using the definitions of the special quadrilaterals, we see that a parallelogram can also be classified as a trapezium since it has at least one pair of opposite sides parallel. This means that trapeziums are **inclusive** of parallelograms. The set of trapeziums includes all parallelograms, or put another way, a parallelogram is a special type of trapezium. Similarly, parallelograms are inclusive of rectangles and rectangles are inclusive of squares. This can be represented by a Venn diagram:



1 Why is a rectangle a special type of parallelogram but a parallelogram is not always a rectangle? How can you use the Venn diagram to explain this?

2 Where would you put **kites** on the Venn diagram?

Foundation Standard Complex

13.04 Proving properties of triangles and quadrilaterals

WS Quadrilaterals: True or false?

WS Congruent and similar triangle proofs

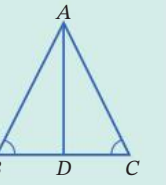
Geometric problems and proofs

Proving properties of a rectangle

Properties of triangles and quadrilaterals can be proved using the congruence tests.

Example 10

$\triangle ABC$ is an isosceles triangle with $AB = AC$. D is the midpoint of BC.



a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?

b Explain why $\angle B = \angle C$.

c What geometrical result about isosceles triangles does this prove?

Solution

a For $\triangle ABD$ and $\triangle ACD$:

$AB = AC$ (given)

AD is common.

$BD = CD$ (D is the midpoint of BC)

$\therefore$ The congruence test is SSS.

b $\angle B = \angle C$ because they are matching angles of congruent triangles.

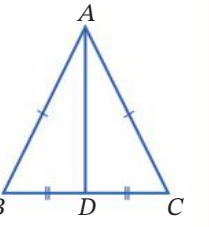
c The angles opposite the equal sides of an isosceles triangle are equal.

EXERCISE 13.04 ANSWERS ON P. 642

Proving properties of triangles and quadrilaterals UFRC

EXAMPLE 10

1 $\triangle ABC$ is an isosceles triangle, with $AB = AC$. D is the midpoint of BC. R C

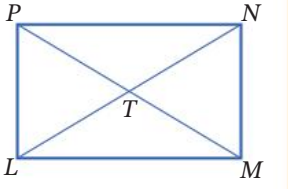


a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?

b Explain why $\angle ADB = \angle ADC$.

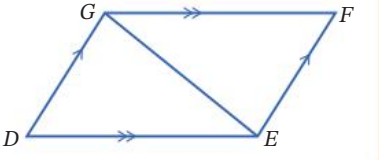
c Hence prove that $AD \perp BC$.

2 **a** If $LMNP$ is a rectangle, prove that $\triangle PNT \equiv \triangle MLT$.



b Prove that the diagonals of a rectangle bisect each other.

3 $DEFG$ is a parallelogram. R C



a Prove that $\triangle DEG \equiv \triangle FGE$.

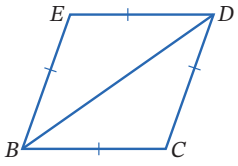
b Draw the other diagonal DF and prove that $\triangle DGF \equiv \triangle FED$.

c Hence prove that the opposite angles of a parallelogram are equal.

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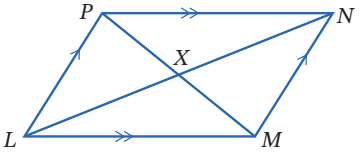
4 *BCDE* is a rhombus, so all of its sides are equal and opposite sides are parallel. R C



a Prove that $\triangle BED \equiv \triangle BCD$.

b Hence prove that the diagonal *BD* bisects $\angle EBC$ and $\angle EDC$.

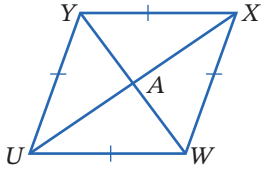
5 *LMNP* is a parallelogram, so its opposite sides are parallel and equal. R C



a Prove that $\triangle LXM \equiv \triangle NXP$.

b Hence prove that the diagonals of a parallelogram bisect each other.

6 *UWXY* is a rhombus. R C

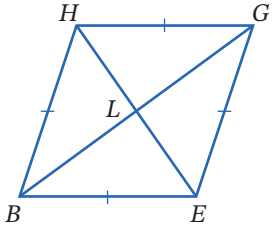


a Prove that $\triangle UAW \equiv \triangle XAW$.

b Prove that $\triangle UAW \equiv \triangle UAY$.

c Hence prove that the diagonals of a rhombus bisect each other at right angles.

7 *BEGH* is a rhombus (a parallelogram with equal sides) whose diagonals *BG* and *EH* intersect at *L*. R C



a Prove that $\triangle BEL = \triangle GHL$.

b Prove that the diagonals of a rhombus bisect each other.

c $\triangle BEH$ is isosceles, so which equation is true? Select **A**, **B**, **C** or **D**.

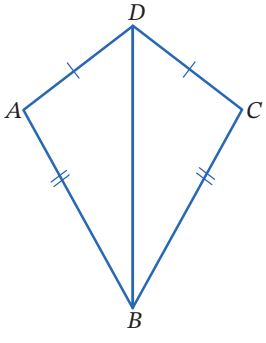
A $\angle BEH = \angle BHE$ **B** $\angle BLE = \angle BEH$

C $\angle HBE = \angle BEH$ **D** $\angle BEH = \angle EBH$

d Hence prove that $\triangle BEL \equiv \triangle BHL$.

e Hence prove that the diagonals of a rhombus cross at right angles.

8 *ABCD* is a kite, so pairs of adjacent sides are equal. R C



a Prove that $\triangle ABD \equiv \triangle CBD$.

b Prove that $\angle A = \angle C$.

c Prove that diagonal *DB* bisects $\angle ADC$ and $\angle ABC$.

d Copy the diagram and draw the other diagonal *AC*, intersecting *DB* at point *X*.

e Prove that $\triangle DAX \equiv \triangle DCX$.

f Prove that diagonal *DB* bisects diagonal *AC*.

g Prove that $DB \perp AC$.

Mental skills 13: Maths without calculators ANSWERS ON P. 642

Time before and time after

1 Study each example.

a What is the time 7 hours and 40 minutes after 11:45 a.m.?

11:45 a.m. + 7 hours = 6:45 p.m.

Count: '11:45, 12:45, 1:45, 2:45, 3:45, 4:45, 5:45, 6:45'

$$\begin{aligned} 6:45 \text{ p.m.} + 40 \text{ minutes} &= 6:45 \text{ p.m.} + 15 \text{ minutes} + 25 \text{ minutes} \\ &= 7:00 \text{ p.m.} + 25 \text{ minutes} \\ &= 7:25 \text{ p.m.} \end{aligned}$$

OR:



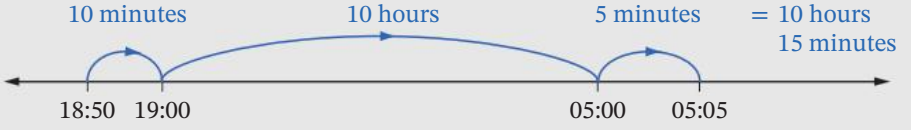
b What is the time 10 hours and 15 minutes after 18:50?

18:50 + 10 hours = 04:50 (next day).

Count: '18:50, 19:50, 20:50, 21:50, 22:50, 23:50, 00:50, 01:50, 02:50, 03:50, 04:50'

$$\begin{aligned} 04:50 + 15 \text{ min} &= 04:50 \text{ hours} + 10 \text{ min} + 5 \text{ min} \\ &= 05:00 + 5 \text{ min} \\ &= 05:05 \text{ (next day)} \end{aligned}$$

OR:



2 Now find the time of day:

a 3 hours 20 minutes after 9:05 a.m.
b 5 hours 40 minutes after 7:30 p.m.
c 4 hours 35 minutes after 6:15 p.m.
d 11 hours 10 minutes after 11:45 a.m.
e 2 hours 45 minutes after 03:25
f 7 hours 5 minutes after 17:05
g 8 hours 30 minutes after 12:40 a.m.
h 4 hours 55 minutes after 10:20 p.m.
i 6 hours 25 minutes after 04:35
j 2 hours 15 minutes after 20:50
k 9 hours 50 minutes after 2:30 p.m.
l 3 hours 10 minutes after 8:25 a.m.

3 Now find the time of day:

a 1 hour 15 minutes before 7:20 p.m.
b 4 hours 40 minutes before 11:20 a.m.
c 3 hours 20 minutes before 3:30 p.m.
d 5 hours 35 minutes before 8:25 a.m.
e 2 hours 10 minutes before 14:55
f 3 hours 45 minutes before 07:40
g 5 hours 25 minutes before 4:15 a.m.
h 9 hours 30 minutes before 9:45 p.m.
i 4 hours 20 minutes before 20:05
j 2 hours 15 minutes before 06:15

Formal geometrical proofs

13.05

STAGE 5.3

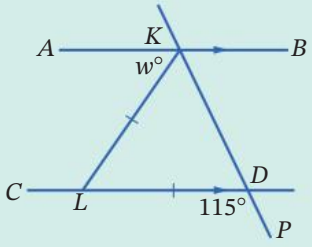
13.05

Geometrical proofs order activity

General geometrical results can be proved by writing a geometrical argument, where reasons are given at each step of the argument. This is called **deductive geometry.**

Example 11

In the diagram, $AB \parallel CD$, $KL = DL$ and $\angle LDP = 115°$.
Find the value of w, giving reasons.

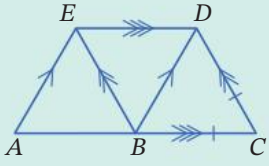


Solution

$\angle KDL = 180° - 115°$ (angles on a straight line)
$= 65°$
$\therefore \angle LKD = 65°$ (equal angles of isosceles $\triangle KLD$)
$\angle AKD = 115°$ (corresponding angles, $AB \parallel CD$)
$\therefore w = 115° - 65°$
$= 50°$

Example 12

In the diagram, $AC \parallel ED$, $AE \parallel BD$, $BE \parallel CD$ and $CB = CD$.
Prove that $\triangle ABE$ is an isosceles triangle.



Solution

$\angle EBD = \angle CDB$ (alternate angles, $BE \parallel CD$)
and $\angle EBD = \angle AEB$ (alternate angles, $AE \parallel BD$)
$\therefore \angle CDB = \angle AEB$
But $\angle CDB = \angle CBD$ (equal angles of isosceles $\triangle CBD$)
and $\angle BAE = \angle CBD$ (corresponding angles, $AE \parallel BD$)
$\therefore \angle CDB = \angle BAE$
$\therefore \angle AEB = \angle BAE$
$\therefore \triangle ABE$ is an isosceles triangle (2 equal angles)

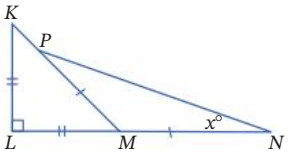
STAGE 5.3

EXERCISE 13.05 ANSWERS ON P. 642

Formal geometrical proofs U F PS R C

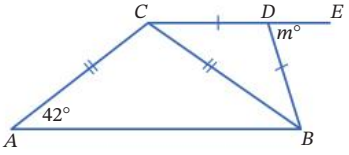
EXAMPLE 11

1 $KL = ML$ and $MN = MP$. Find x, giving reasons. PS R C

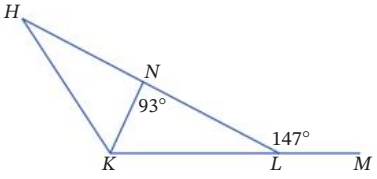


2 $CE \parallel AB$, $CD = BD$ and $AC = BC$.

Find m, giving reasons. PS R C

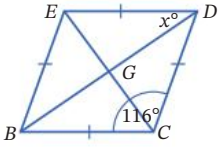


3 NK bisects $\angle HKL$. Find the size of $\angle NHK$, giving reasons. PS R C



4 $BCDE$ is a rhombus with the diagonals intersecting at G.

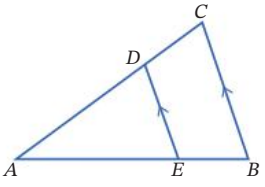
Find the value of x, giving reasons for each step. PS R C



EXAMPLE 12

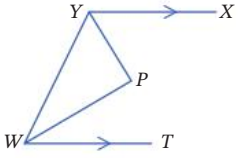
5 $\triangle ABC$ is an isosceles triangle where $AB = AC$ and $BC \parallel ED$.

Prove that $\triangle ADE$ is an isosceles triangle. PS R C



6 PY bisects $\angle XYW$, PW bisects $\angle TWY$ and $YX \parallel WT$.

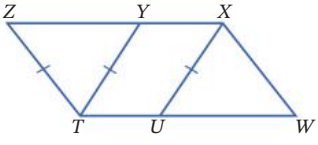
Prove that $\angle YPW = 90°$. PS R C



7 $TWXZ$ is a parallelogram and $TZ = TY = UX$. R C

a Prove that $\triangle TZY \equiv \triangle XWU$.

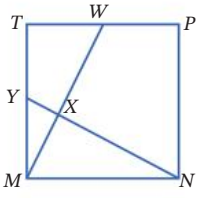
b Hence prove that $TUXY$ is a parallelogram.



8 $MNPT$ is a square. W and Y are the midpoints of sides TP and MT. R C

a Prove that $\triangle MNY \equiv \triangle MTW$.

b Prove that $MW \perp NY$.

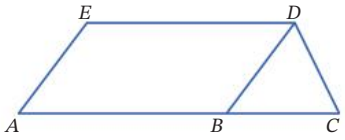


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9 $ABDE$ is a parallelogram and $BC = BD$.

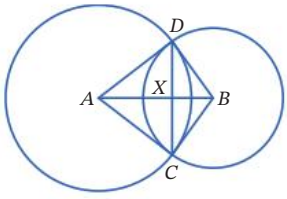
Prove that $\angle AED = 2\angle BCD$. PS R C



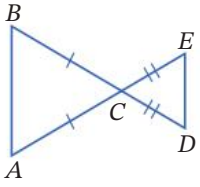
10 A and B are the centres of 2 circles that intersect at C and D. R C

a Prove that $\triangle ADB \equiv \triangle ACB$.

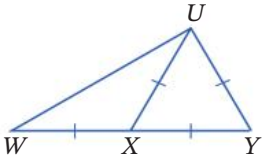
b Hence prove that $\triangle DXB \equiv \triangle CXB$ and $DX = CX$.



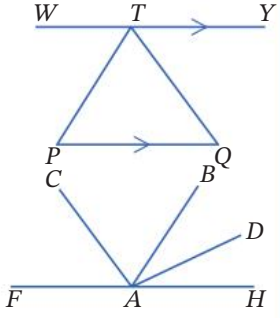
11 If $AC = BC$ and $DC = EC$, prove that $AB \parallel DE$. PS R C



12 $\triangle UXY$ is an equilateral triangle and $WX = XU$.
Prove that $\angle WUY$ is a right angle. PS R C

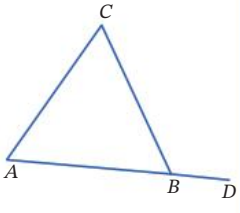


13 If $WY \parallel PQ$, prove that the angle sum of $\triangle PQT$ is 180°.
PS R C

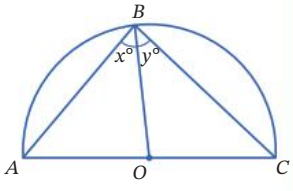


14 CA bisects $\angle FAB$ and DA bisects $\angle HAB$.
Prove that $\angle CAD = 90°$. PS R C

15 Prove that the exterior angle of a triangle is equal to the sum of the interior opposite angles (that is, prove that $\angle CBD = \angle CAB + \angle BCA$). PS R C



16 AC is the diameter of a semicircle with centre O.
B is a point on the semicircle. Let $\angle ABO = x°$ and $\angle CBO = y°$. Prove that $\angle ABC$ is a right angle.
PS R C



Foundation Standard Complex

13.06 Finding unknown sides in similar figures

WS A page of similar figures

WS Enlargements and reductions

WS Enlarging an icon

Cartoon enlargement

Finding sides in similar triangles

WS Finding sides in similar figures

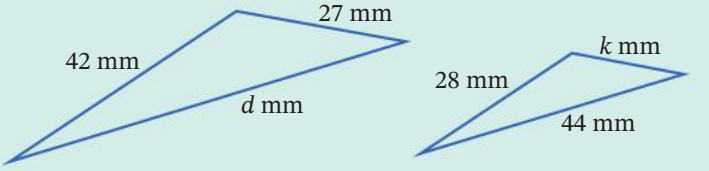
Similar triangles

Properties of similar figures

- Matching angles are equal
- Matching sides are in the same ratio

Example 13

The 2 triangles are similar. Find the values of d and k.



Solution

Since the triangles are similar, the ratios of matching sides are equal.

$\frac{d}{44} = \frac{42}{28}$

$d = \frac{42}{28} \times 44$

$= 66$

$\frac{k}{27} = \frac{28}{42}$

$k = \frac{28}{42} \times 27$

$= 18$

Alternative method:

Scale factor $= \frac{28}{42} = \frac{2}{3}$

$d = 44 \div \frac{2}{3}$

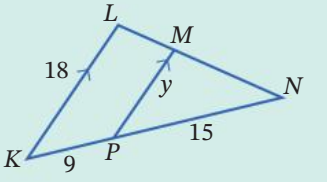
$= 66$

$k = 27 \times \frac{2}{3}$

$= 18$

Example 14

$\triangle KLN \,|||\, \triangle PMN$. Find the value of y.



Solution

$\frac{MP}{LK} = \frac{PN}{KN}$ Ratios of matching sides are equal.

$\frac{y}{18} = \frac{15}{24}$ $KN = 9 + 15 = 24$

$y = \frac{15}{24} \times 18$

$= 11\frac{1}{4}$

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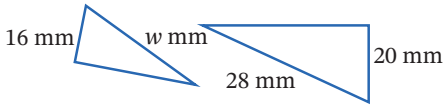
EXERCISE 13.06 ANSWERS ON P. 642

Finding unknown sides in similar figures U F PS R

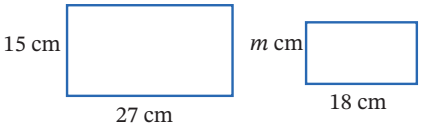
1 Find the value of each variable in each pair of similar figures. R

EXAMPLE 13

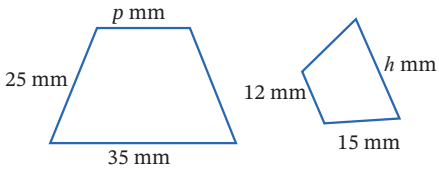
a



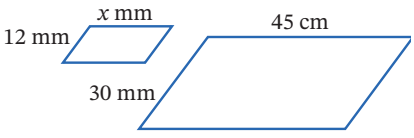
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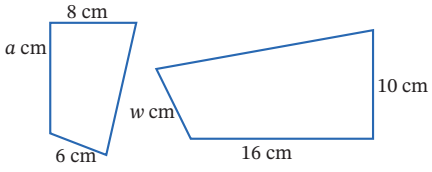
c



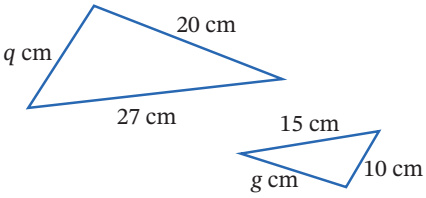
d



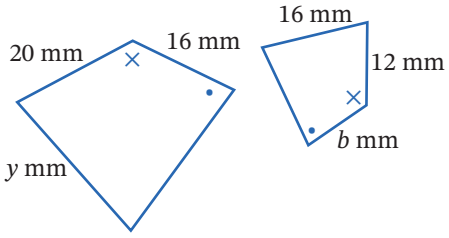
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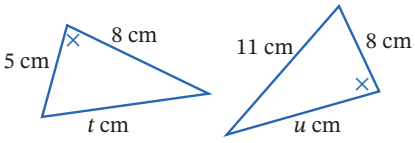
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g

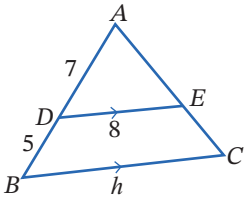


h

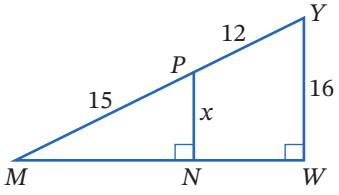


2 $\triangle ABC \,|||\, \triangle ADE$. Find the value of h. R

EXAMPLE 14



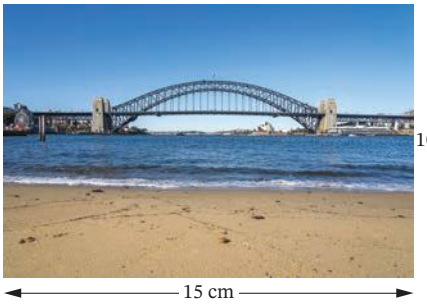
3 $\triangle MNP \,|||\, \triangle MWY$. Find the value of x. R



Foundation | Standard | Complex

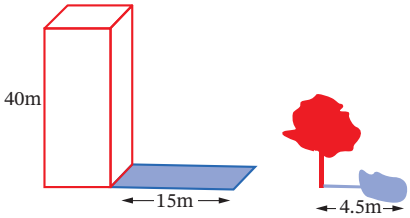
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4 This photograph of the Sydney Harbour Bridge has been enlarged so that its length is 24 cm. If the dimensions of the original photo were 15 cm × 10 cm, what is the width of the enlargement? R



Shutterstock.com/Taki O

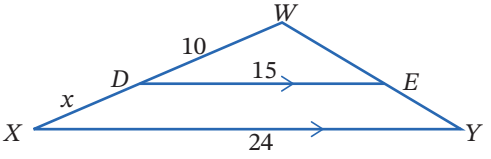
5 A building that is 40 m high casts a shadow 15 m long. At the same time, the shadow of a tree is 4.5 m long. What is the height of the tree? PS R



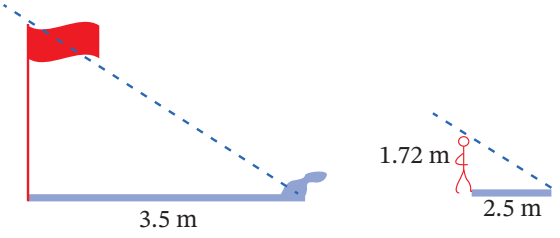
6 $\triangle WXY \,|||\, \triangle WDE$. What is the value of x? Select the correct answer **A**, **B**, **C** or **D**. R

A 4 **B** 6

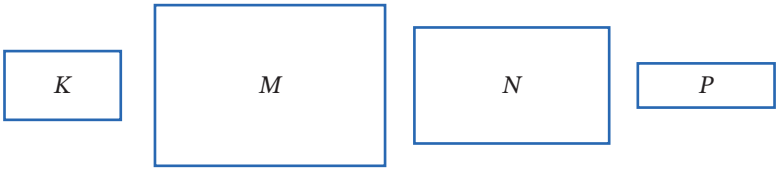
C 8 **D** 10



7 Katrina is 1.72 m tall and casts a shadow that is 2.5 m long. At the same time, a flagpole casts a shadow that is 3.5 m long. How long is the flagpole? PS R



8 Which 2 rectangles are similar? Select **A**, **B**, **C** or **D**.



A M and N **B** K and P **C** M and P **D** K and N

9 A 2 m high fence casts a shadow of 1.4 m. How long is the shadow cast by a pole that is 3.2 m high at the same time? PS R

Tests for similar triangles

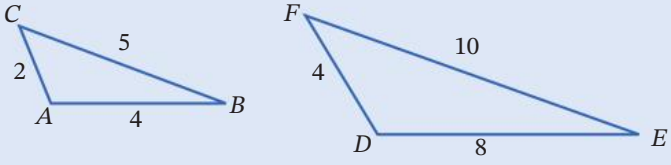
13.07

Similarity tests

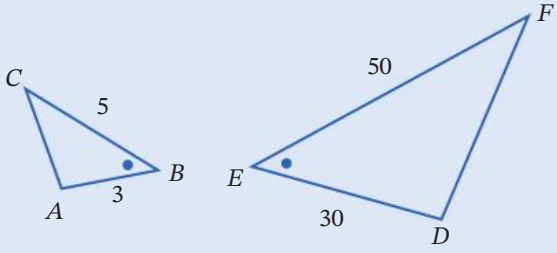
There are 4 tests for similar triangles.

Two triangles are similar if:

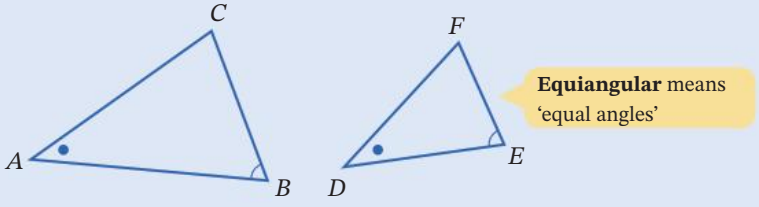
- the 3 sides of one triangle are proportional to the 3 sides of the other triangle (**'SSS'**)



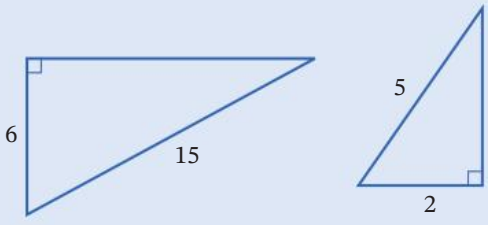
- 2 sides of one triangle are proportional to 2 sides of the other triangle, and the **included angles** are equal (**'SAS'**)



- 2 angles of one triangle are equal to 2 angles of the other triangle (**'AA'** or **'equiangular'**)



- they are right-angled and the hypotenuse and a second side of one triangle are proportional to the hypotenuse and a second side of the other triangle (**'RHS'**).



WS Congruent and similar triangle proofs

WS Congruence and similarity review

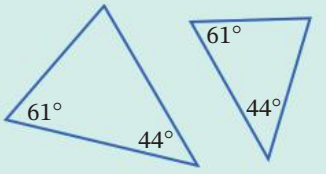
13.07

Tests for similar triangles

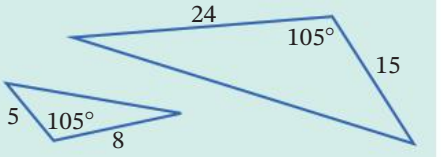
Example 15

Which test can be used to prove that each pair of triangles are similar?

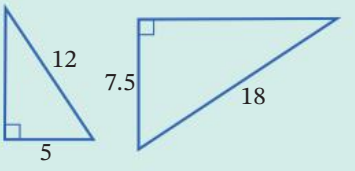
a



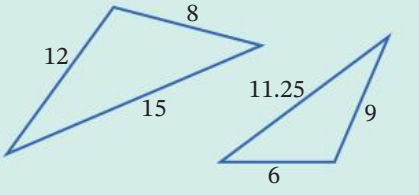
b



c



d



Solution

a 2 pairs of angles are equal, or equiangular ('AA').

b 2 pairs of matching sides are in the same ratio and the included angles in both triangles are equal ('SAS'). $\frac{15}{5}=\frac{24}{8}=3$

c Both have right angles, and the pairs of hypotenuses and second sides are in the same ratio ('RHS'). $\frac{7.5}{5}=\frac{18}{12}=\frac{3}{2}$

d All 3 pairs of matching sides are in the same ratio ('SSS'). $\frac{11.25}{15}=\frac{9}{12}=\frac{6}{8}=\frac{3}{4}$

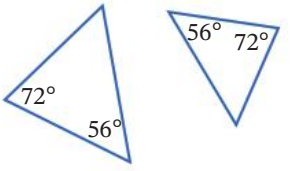
EXERCISE 13.07 ANSWERS ON P. 642

Tests for similar triangles U F R C

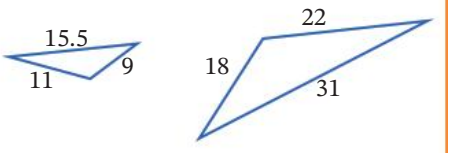
EXAMPLE 15

1 Which test can be used to prove that each pair of triangles are similar? R C

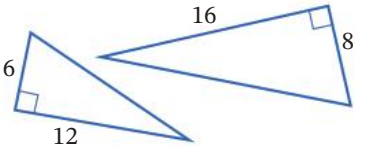
a



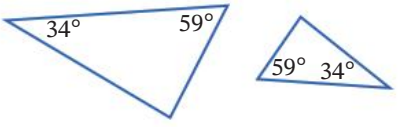
b



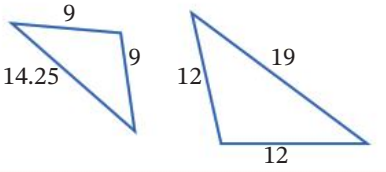
c



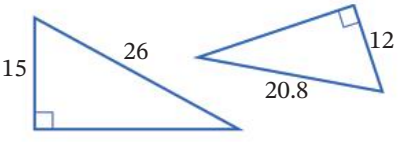
d



e



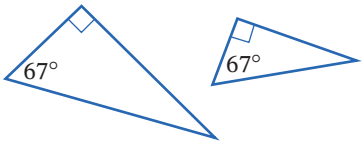
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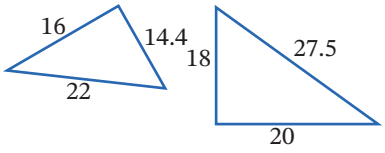
Foundation Standard Complex

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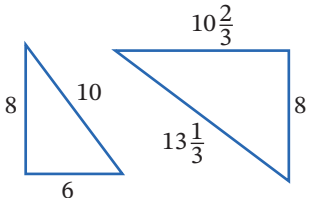
g



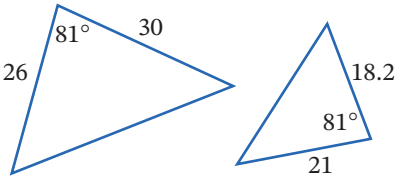
h



i

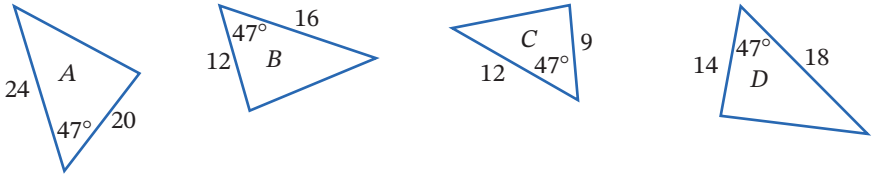


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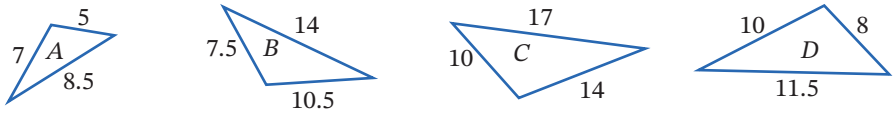


2 For each set of triangles, find the pair of similar triangles. R

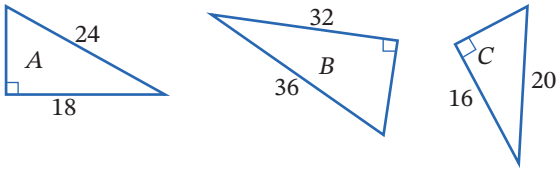
a



b

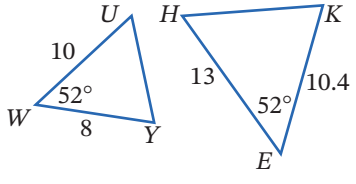


c

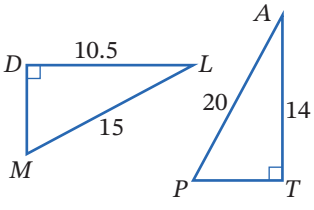


3 Use the correct notation to write a similarity statement relating each pair of similar triangles. R C

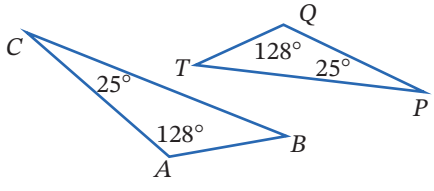
a



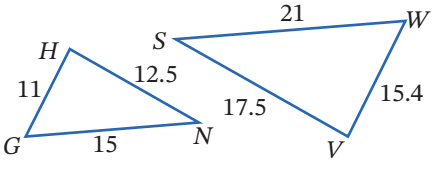
b



c



d



13.08 Similar triangle proofs

STAGE 5.3

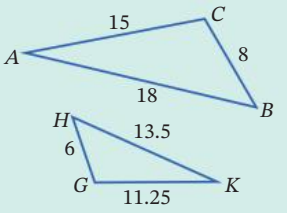
WS Congruent and similar triangle proofs

Similar triangle proofs

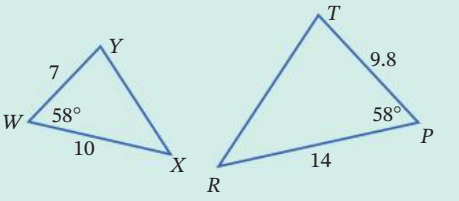
Example 16

Prove that each pair of triangles are similar.

a



b



Solution

a In $\triangle ABC$ and $\triangle KHG$:

$$\frac{AB}{KH} = \frac{18}{13.5} = \frac{4}{3}$$

$$\frac{AC}{KG} = \frac{15}{11.25} = \frac{4}{3}$$

$$\frac{BC}{HG} = \frac{8}{6} = \frac{4}{3}$$

$$\therefore \frac{AB}{KH} = \frac{AC}{KG} = \frac{BC}{HG}$$

$\therefore \triangle ABC \,|||\, \triangle KHG$ (3 pairs of matching sides in proportion, or 'SSS')

b In $\triangle WXY$ and $\triangle PRT$:

$$\frac{WX}{PR} = \frac{10}{14} = \frac{5}{7}$$

$$\frac{WY}{PT} = \frac{7}{9.8} = \frac{5}{7}$$

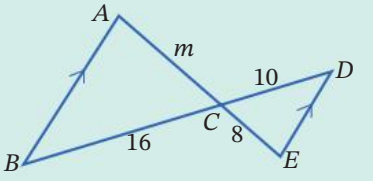
$$\therefore \frac{WX}{PR} = \frac{WY}{PT}$$

$\angle W = \angle P = 58°$

$\therefore \triangle WXY \,|||\, \triangle PRT$ (2 pairs of matching sides in proportion and the included angles equal, or 'SAS')

Example 17

Prove that $\triangle ABC \,|||\, \triangle EDC$ and hence find the value of m.



Solution

In $\triangle ABC$ and $\triangle EDC$:

$\angle A = \angle E$ (alternate angles, $AB \parallel DE$)

$\angle B = \angle D$ (alternate angles, $AB \parallel DE$)

$\therefore \triangle ABC \,|||\, \triangle EDC$ (equiangular, or 'AA')

$\therefore \frac{m}{8} = \frac{16}{10}$ (matching sides in similar triangles)

$m = \frac{16}{10} \times 8$

$= 12.8$

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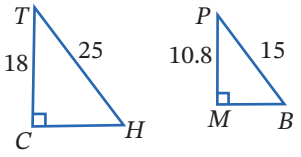
EXERCISE 13.08 ANSWERS ON P. 642

Similar triangle proofs U F R C

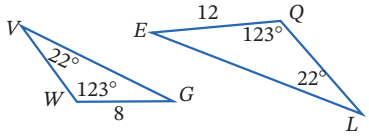
EXAMPLE 16

1 Prove that each pair of triangles are similar. R C

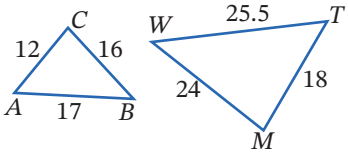
a



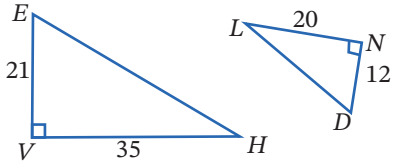
b



c

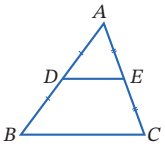


d

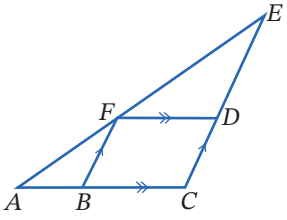


EXAMPLE 17

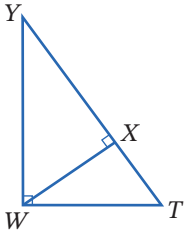
2 D and F are the midpoints of AB and AC.
Prove that $\triangle ADE \,|||\, \triangle ABC$. R C



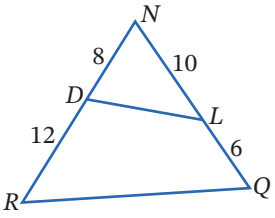
3 $AC \,||\, FD$ and $BF \,||\, CE$.
Prove that $\triangle ABF \,|||\, \triangle FDE$. R C



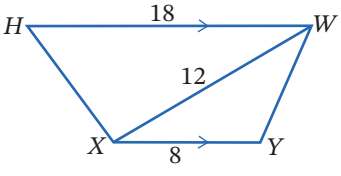
4 Prove that $\triangle WXY \,|||\, \triangle TXW$. R C



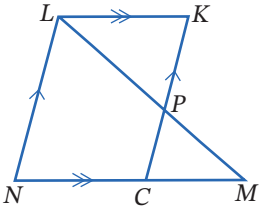
5 Prove that $\triangle NDL \,|||\, \triangle NQR$. R C



6 $HW \,||\, XY$. Prove that $\triangle XWH \,|||\, \triangle XYW$. R C



7 $NCKL$ is a parallelogram.
Prove that $\triangle NML \,|||\, \triangle KLP$. R C

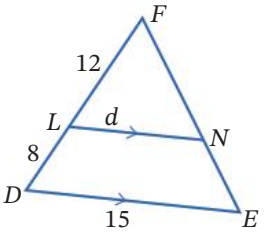


Foundation | Standard | Complex

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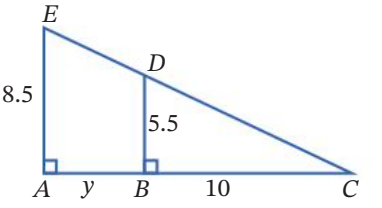
8 **a** Prove that $\triangle FLN \,|||\, \triangle FDE$. R C

b Find the value of d.



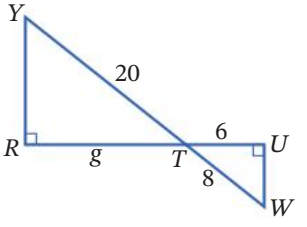
9 **a** Prove that $\triangle ACE \,|||\, \triangle BCD$. R C

b Find the value of y.



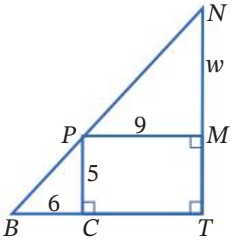
10 **a** Prove that $\triangle YRT \,|||\, \triangle WUT$. R C

b Find the value of g.



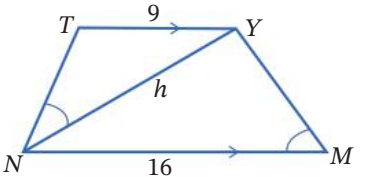
11 **a** Prove that $\triangle NMP \,|||\, \triangle PCB$. R C

b Find the value of w.



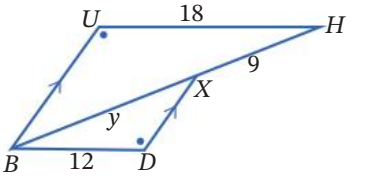
12 **a** Prove that $\triangle TYN \,|||\, \triangle YNM$. R C

b Find the value of h.



13 **a** Prove that $\triangle BHU \,|||\, \triangle XBD$. R C

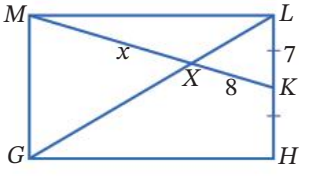
b Find the value of y.



14 $GHLM$ is a rectangle and K is the midpoint of HL. R C

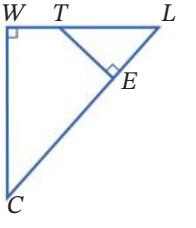
a Prove that $\triangle MXG \,|||\, \triangle KXL$.

b Find the value of x.



15 **a** Prove that $\triangle CLW \,|||\, \triangle LTE$. R C

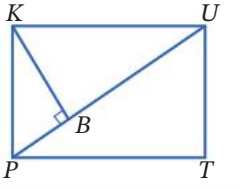
b If $WT = 5$ cm, $CE = 15$ cm and $EL = 6$ cm, find the length of TL.



16 $PTUK$ is a rectangle and $KB \perp PU$. R C

a Prove that $\triangle PTU \,|||\, \triangle KBP$.

b If $BU = 21$ cm and $KP = 10$ cm, find the length of PB.



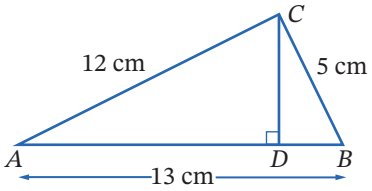
Foundation Standard Complex

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Power plus ANSWERS ON P. 643

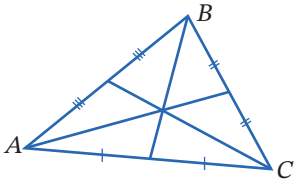
1 **a** Prove that the 3 triangles ABC, ACD and CBD are similar.

b Find the length of CD.



2 $ABCD$ is a parallelogram. HG is any interval joining parallel sides AB and DC and passing through the midpoint of the diagonal BD. Prove that the interval through the midpoint of a diagonal in a parallelogram divides opposite sides equally. (That is, prove that $DG = BH$.)

3 The median is a line joining a vertex of a triangle to the midpoint of the opposite side. Prove that the medians of a triangle are concurrent (that is, they meet at one intersection point).



CHAPTER 13 REVIEW

Language of maths

Geometry

Congruence and similarity

AAS	angle sum	congruence test
congruent (≡)	convex polygon	enlargement
equiangular	exterior angle	hypotenuse
image	included angle	matching
original	polygon	proof
proportional	reduction	regular polygon
RHS	SAS	scale factor
similar (|||)	similarity test	SSS

1 What is a **convex polygon**?

2 Explain the difference between the **interior** and **exterior** angles of a polygon.

3 What is the symbol and meaning of 'is similar to'?

4 What happens to a figure that is changed by a scale factor of $\frac{1}{2}$?

5 What are the 4 tests for similar triangles?

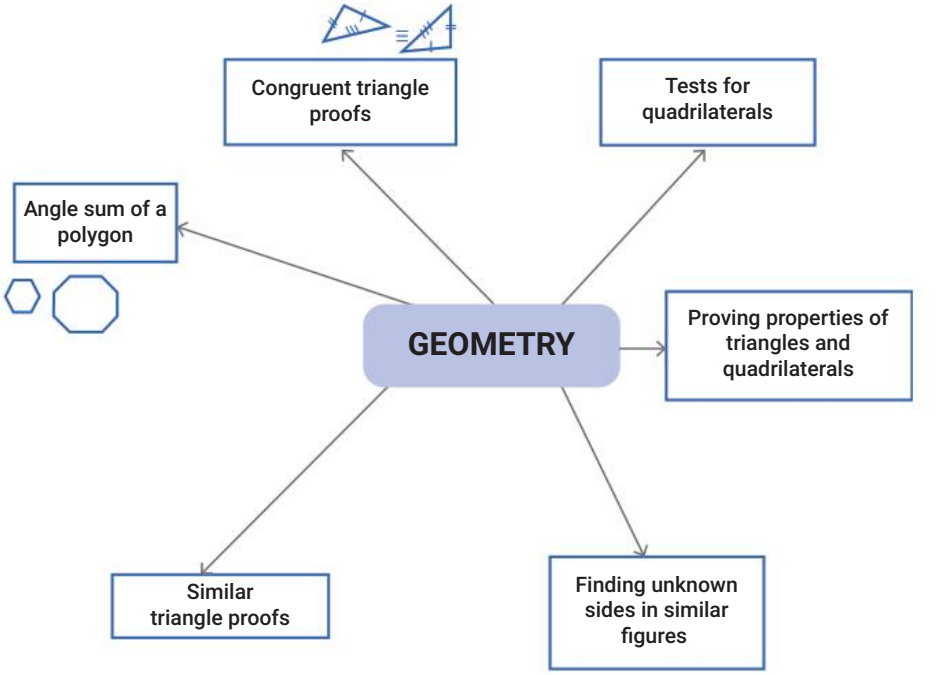
6 What is the meaning of the 'A' in the SAS test for congruent triangles?

7 What does **equiangular** mean in the similarity tests?

Topic summary

Mind map: Geometry (Advanced)

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 13

ANSWERS ON P. 643

1 Find the angle sum of a polygon with: 13.01

a 15 sides **b** 24 sides **c** 12 sides **d** 48 sides

2 Find the size of one angle in a regular 15-sided polygon. 13.01

3 The angle sum of a polygon is 6120°. How many sides does the polygon have? 13.01

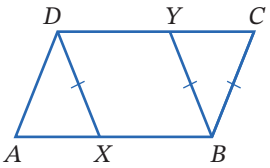
4 Find the number of sides in a regular polygon if each exterior angle is: 13.02

a 10° **b** 24° **c** 45° **d** 15°

5 Each angle of a regular polygon is 162°. How many sides does the polygon have? Select the correct answer **A**, **B**, **C** or **D**. 13.02

A 18 **B** 20 **C** 22 **D** 24

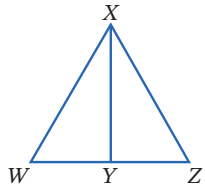
6 $ABCD$ is a parallelogram and $BC = BY = DX$. 13.03



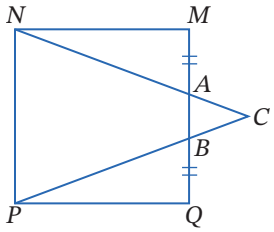
a Prove that $\triangle DAX \equiv \triangle BCY$.

b Hence, prove that $BXDY$ is a parallelogram.

7 In $\triangle WXY$, $\angle W = \angle X$ and $YZ \perp WX$. Prove that $\triangle WZY \equiv \triangle XZY$. 13.04

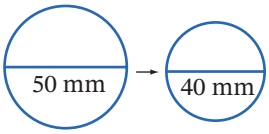


8 $PNMQ$ is a square and $AM = BQ$. Prove that $\triangle NPC$ is isosceles. 13.05

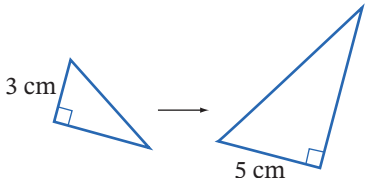


9 Calculate the scale factor between each pair of similar figures. 13.06

a

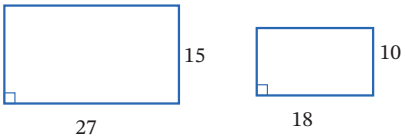


b

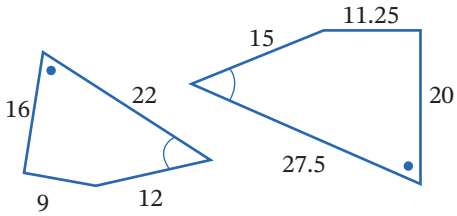


10 Test whether each pair of figures are similar. 13.06

a



b



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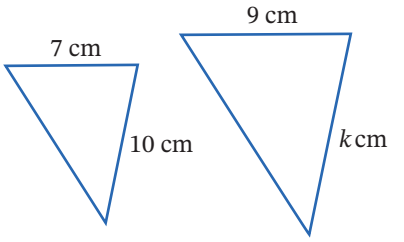
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STAGE 5.3

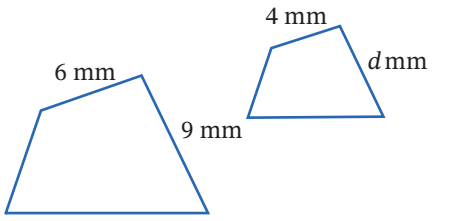
13.06

11 Find the value of the variable in each pair of similar figures.

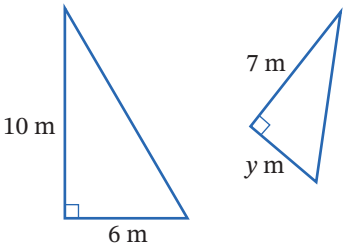
a



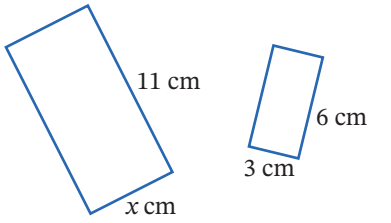
b



c

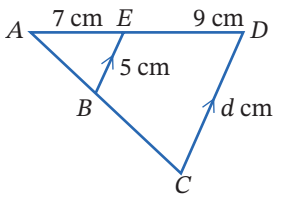


d



13.06

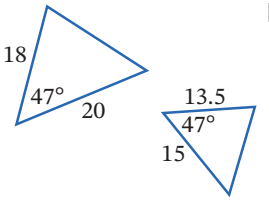
12 If $\triangle ABE \,|||\, \triangle ACD$, find the value of d.



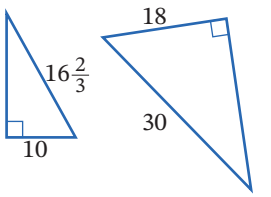
13.07

13 Which test can be used to prove that each pair of triangles are similar?

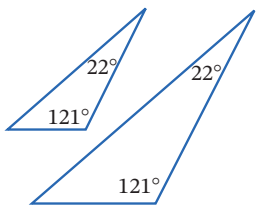
a



b

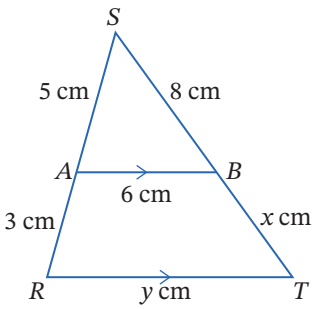


c



13.08

14 Prove that $\triangle ASB \,|||\, \triangle RST$ and hence find the values of x and y.



13.08

15 Use similar triangles to prove that the interval joining the midpoint of 2 sides in a triangle is parallel to the third side and is half its length.

Foundation Standard Complex

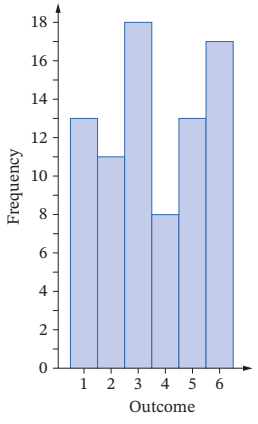
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PRACTICE SET 4

ANSWERS ON P. 644

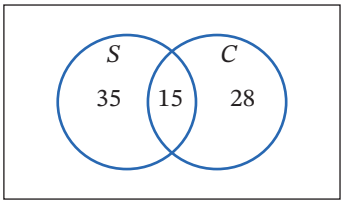
1 Each angle of a regular polygon is 171°. How many sides does the polygon have? 13.01

2 A die was rolled and the outcomes were recorded in this frequency histogram. 12.01



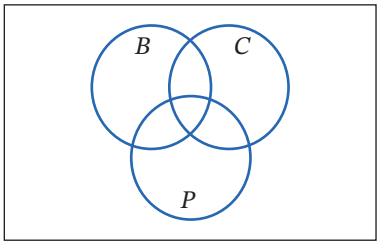
- **a** How many times was the die rolled?
- **b** Find the relative frequency of rolling:
  - **i** a 1
  - **ii** an even number
  - **iii** a number less than 4
  - **iv** at least a 3
- **c** What is the theoretical probability of rolling a 6? How does this compare with the experimental probability of rolling a 6?

3 This Venn diagram shows the results of a survey asking people whether they prefer to watch movies by streaming (S) or at the cinema (C). 12.02



- **a** How many people were surveyed?
- **b** Find the probability of selecting a person who prefers to watch movies:
  - **i** by streaming
  - **ii** only at the cinema
- **c** What is the probability of selecting a person who likes to stream movies only?

4 Of 160 Year 11 students at Westvale High, 54 do Biology (B), 75 take Chemistry (C) and 68 study Physics (P). 55 students do both Chemistry and Physics, 20 take Biology and Chemistry and 10 students study all three. 12.02

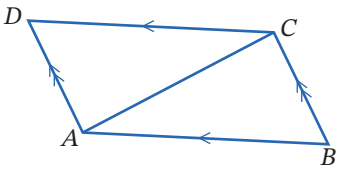


- **a** Copy and complete the Venn diagram to show this information.
- **b** Find the probability of selecting a student who:
  - **i** only takes Physics
  - **ii** does not do Biology, Chemistry or Physics
  - **iii** takes Chemistry and Physics but not Biology
  - **iv** studies Chemistry or Biology
  - **v** only does one Science subject
- **c** From the students who take Biology, what is the probability that the student also does Physics?

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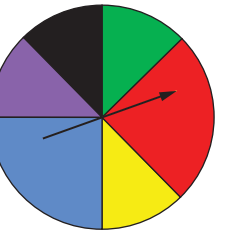
13.02

5 Prove that $\triangle ABC \equiv \triangle CDA$.



12.06

6 Olivia spun this spinner twice, landing on red both times. Is the second outcome dependent or independent of the outcome of the first spin?



12.01

7 A bag contains 4 red, 5 black and 3 green marbles. Sadiya selects a marble at random, records its colour and then returns the marble to the bag. Sadiya does this process 150 times. The results are shown in the table.

Outcome	Frequency
red	58
black	65
green	27

a Find, correct to 2 decimal places, the relative frequency of selecting a marble that is:

i red **ii** black **iii** green.

b Find, correct to 2 decimal places, the theoretical probability of selecting a marble that is

i red **ii** black **iii** green

c For which outcome are the experimental and theoretical probabilities most similar?

d What is the expected frequency of selecting a red or green marble? How does this compare with the observed frequency?

13.01

8 The angle sum of a regular polygon is 6120°.

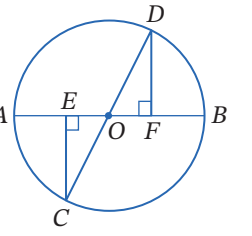
a How many sides does the polygon have?

b Find the size of each angle.

13.02

9 O is the centre of the circle shown. $CE \perp AB$ and $DF \perp AB$.

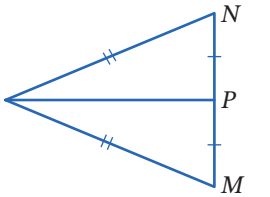
Prove that $\triangle CEO \equiv \triangle DFO$.



13.02

10 In the diagram, $LM = LN$ and P is the midpoint of MN.

Prove that $\triangle LMP \equiv \triangle LNP$ and hence that $\angle LPM = \angle LPN = 90°$.



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11 Shoppers in a mall were asked whether they had a pet dog or cat. The results are shown in the two-way table. 12.03

	Cat	No cat
Dog	28	35
No dog	32	40

a How many shoppers were surveyed?

b Find the probability of randomly selecting a shopper from the survey who does not have a cat or dog.

c What is the probability of randomly selecting a shopper who:

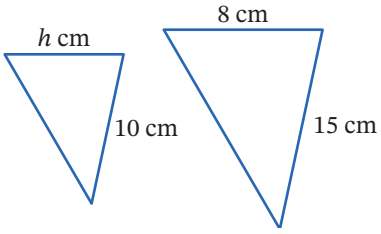
i has only a dog or a cat (not both)?

ii has a cat as a pet?

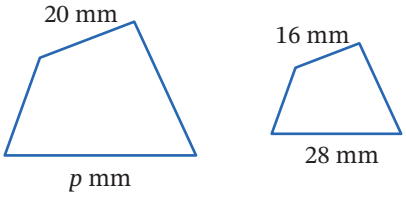
iii does not have a dog?

12 Find the value of the variable in each pair of similar figures. 13.07

a

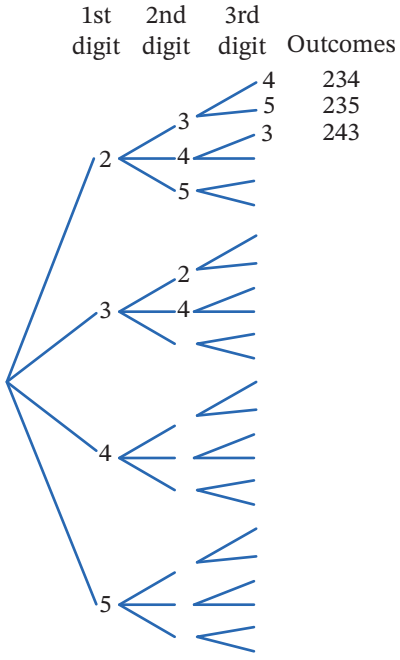


b



13 3 cards are drawn from a set of cards numbered 2, 3, 4 and 5, without replacement, to form a 3-digit number. 12.05

a Copy and complete the tree diagram to list all possible outcomes.



b Find the probability of forming:

i an even number

ii a number ending in 3

iii a number greater than 400

iv a number between 200 and 500

v a number divisible by 5

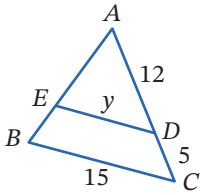
14 Find the angle sum of a polygon that has: 13.01

a 22 sides **b** 7 sides

c 32 sides **d** 19 sides

15 Find the number of sides in a regular polygon if each exterior angle is 10°. 13.01

16 If $\triangle ABC \,|||\, \triangle AED$, find the value of y (correct to one decimal place). 13.07



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12.07

17 2 dice are rolled and the product of the 2 numbers is calculated.

		1st die					
		1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

a Copy and complete the table to show all possible outcomes.

b Given that one of the numbers was 3, what is the probability of obtaining a product that is odd?

c Given that one of the numbers is 4, what is the probability of obtaining a product:

i that is even?

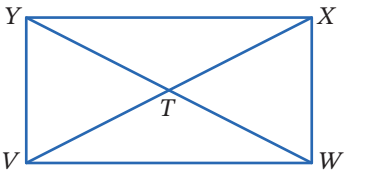
ii that is odd?

d Find $P(\text{an even product} \mid \text{the first number is odd})$

e Find $P(\text{an even product} \mid \text{an even number and an odd number are obtained})$

13.04

18 $VWXY$ is a rectangle and the diagonals, VX and WY intersect at T.



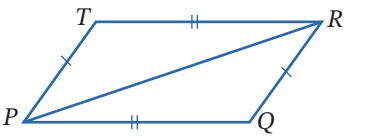
a Prove that $\triangle YXT \equiv \triangle WVT$.

b Hence prove that the diagonals of a rectangle bisect each other.

STAGE 5.3

13.03

19 $PQRT$ is a quadrilateral in which $PQ = RT$ and $PT = RQ$. Prove that:

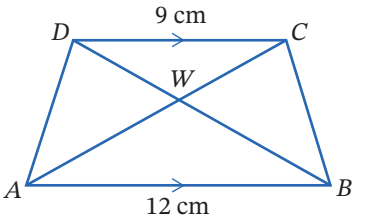


a $\triangle PRT \equiv \triangle RPQ$

b $PQRT$ is a parallelogram.

13.09

20 $ABCD$ is a trapezium.



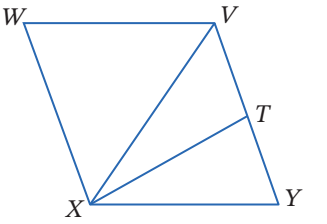
a Prove that $\triangle CDW \,|||\, \triangle ABW$.

b If $AC = 11$ cm, find the length of CW.

STAGE 5.3

13.05

21 $XYVW$ is a rhombus and TX bisects $\angle VXY$. Prove that $\angle XTY = 3 \times \angle TXY$.



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GENERAL PRACTICE ANSWERS ON P. 644

STAGE 5.3 — CHAPTER 1, CHAPTER 1, CHAPTER 1

1 Simplify $3\sqrt{8}-\sqrt{32}+\sqrt{50}$.

2 Rationalise the denominator of $\dfrac{3\sqrt{2}}{2\sqrt{5}}$.

3 Expand and simplify $\left(3\sqrt{2}+4\sqrt{5}\right)^2$

4 Kim is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of \$1250. How much will Kim earn if she sells an apartment for \$578 000?

CHAPTER 2

5 Find the gradient and y-intercept of the line with equation $4x + 5y - 10 = 0$.

CHAPTER 3

6 Factorise each expression.

a $6x^2 + 12x$ **b** $a^2 - 6a - 40$ **c** $25 - 100y^2$ **d** $10p + 6 - 4p^2$

CHAPTER 5

STAGE 5.3

7 Solve the quadratic equation $x^2 + 7x + 2 = 0$, writing the solution in exact form.

CHAPTER 11

8 Find correct to one decimal place the mean and standard deviation of this set of data.

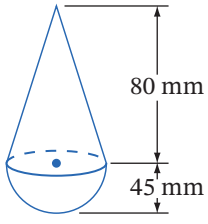
Stem	Leaf
3	2 5 8 9
4	3 4 4 8 9
5	0 3 4
6	5 8
7	0 2

CHAPTER 6

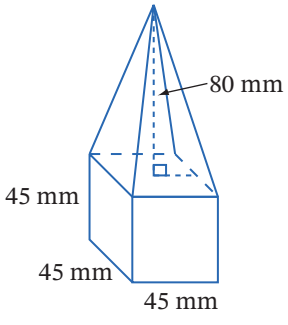
CHAPTER 4

9 Calculate the volume of each solid, correct to the nearest cubic millimetre.

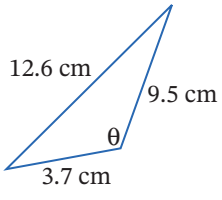
a



b



10 Find θ correct to the nearest minute.



CHAPTER 9

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CHAPTER 12

11 Students in Year 10 at Nelson Secondary College were asked if they had studied Japanese.

	Male	Female
Japanese	35	87
No Japanese	67	21

a How many students are in Year 10?

b What is the probability of selecting a Year 10 student at random who is:

i male and has studied Japanese? **ii** female or has studied Japanese?

c Find the probability, expressed as a percentage to the nearest whole number, of randomly selecting a male in Year 10 who has not studied Japanese.

d Given that a student is a Year 10 female, what is the probability that she has studied Japanese?

CHAPTER 5

12 Simplify each expression, writing the answer with a positive index where necessary.

a $7x^5 \times 8x^7$ **b** $4x^2 \div 16x^{-3}$

c $(3y)^{-2}$ **d** $m^{-6}n^3 \times mn^{-1} \div m^2n$

CHAPTER 7

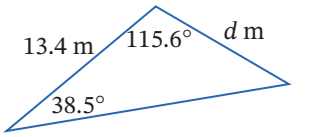
13 Solve each equation.

a $\frac{4y}{5} - 2 = 3$ **b** $\frac{5k}{3} - \frac{k}{2} = 4$ **c** $3x^2 = 27$

STAGE 5.3

CHAPTER 9

14 Find d correct to one decimal place.

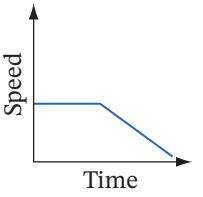


STAGE 5.3

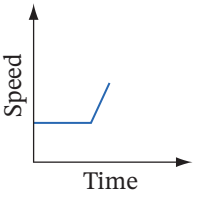
CHAPTER 8

15 A hiker climbs a hill at a steady speed and then runs down the hill. Which graph matches this situation? Select the correct answer **A**, **B**, **C** or **D**.

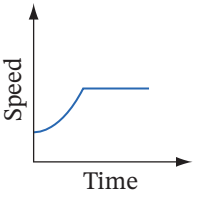
A



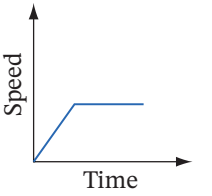
B



C



D



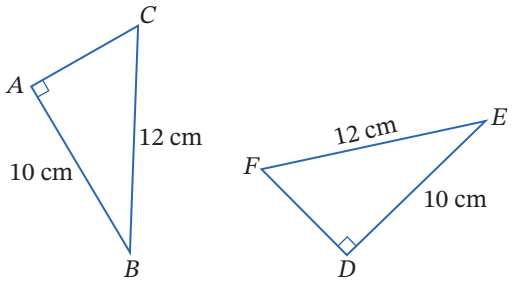
16 Evaluate each logarithm.

a $\log_{10} 0.1$ **b** $\log_3 81$ **c** $\log_6 \sqrt{6}$

17 Simplify and evaluate each expression.

a $\log_3 9 + \log_3 27$ **b** $\log_5 125 - \log_5 25$ **c** $\log_2 128 - 5\log_2 4$

18 Prove that $\triangle ABC \equiv \triangle DEF$.



19 Solve each inequality and graph its solution on a number line.

a $5y + 3 \geq -2$ **b** $\frac{2x+5}{2} < 4$ **c** $5 - 4x > 17$

20 Find, in general form, the equation of a line that is parallel to $2x + 3y - 4 = 0$ and passes through $(2, -3)$.

21 Solve this pair of simultaneous equations.

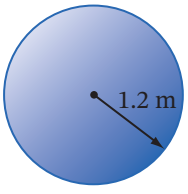
$$3x - y = 4 \text{ and } 2x + y = 6$$

22 Find the equation of a circle with centre $(0, 0)$ and radius 8.

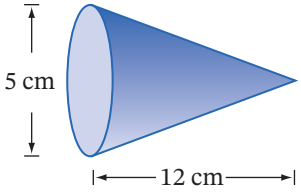
23 Graph the quadratic equation $y = 2x^2 - x - 15$, showing x- and y-intercepts, the axis of symmetry and the vertex.

24 Find, correct to one decimal place, the volume of each solid.

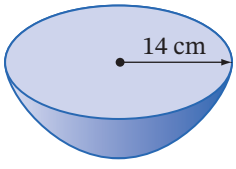
a



b



c



STAGE 5.3 CHAPTER 8 CHAPTER 8 CHAPTER 13 CHAPTER 7 STAGE 5.3 CHAPTER 3 CHAPTER 10 CHAPTER 8 STAGE 5.3 CHAPTER 11 CHAPTER 4

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CHAPTER 2

25 Calculate the final amount if $1260 is invested at 8.2% p.a. for 3 years, compounded quarterly.

CHAPTER 3

26 $A(1, 3)$, $B(-2, 1)$, $C(4, -1)$ and $D(7, 1)$ are the vertices of a quadrilateral.

a Calculate the exact length and gradient of side:

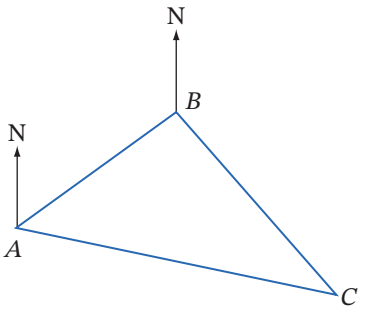
i AB **ii** BC **iii** CD **iv** AD

b What type of quadrilateral is $ABCD$?

STAGE 5.3

CHAPTER 9

27 A ship sails from port A for 100 km on a bearing of 040° to port B and then sails on a bearing of 162° for 324 km to port C.



a Copy the diagram and mark on it all of the given information.

b Explain why $\angle ABC = 58°$.

c What is the distance between port A and port C?
Answer to the nearest km.

d What is the bearing of port C from port A, correct to the nearest minute?

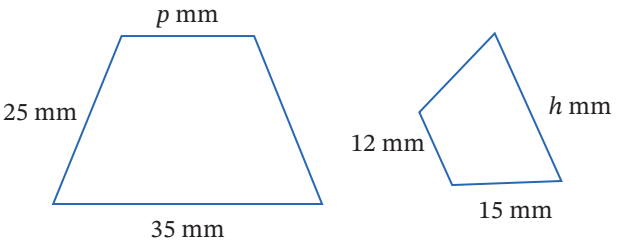
CHAPTER 8

28 Graph each equation on a number plane.

a $y = \frac{3}{x}$ **b** $y = 3^x$

CHAPTER 13

29 Find the value of h and p in this pair of similar figures.



CHAPTER 12

30 The weather on a long weekend will either be fine or rain each day, with each outcome being equally likely.

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that is fine:

i on all 3 days? **ii** on exactly 2 of the days?

iii on at least one of the days?

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31 For an object that is cooling, the drop in temperature is directly proportional to the time. If the temperature drops 5°C in 12 minutes, how long would it take to drop 8°C?

CHAPTER 8

32 Solve this pair of simultaneous equations:

$$y = 5 - 2x \text{ and } y = 4x - 7$$

CHAPTER 10

33 A computer depreciates by 25% each year. If the computer originally cost $4950, what will its value be in 5 years and by how much will it depreciate?

CHAPTER 2

34 Find the five-number summary for this data and construct a box-and-whisker plot.

2 4 8 5 5 10 12 7 7 8 8 13 3

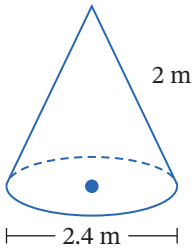
CHAPTER 6

35 Find, correct to one decimal place, the surface area of each solid.

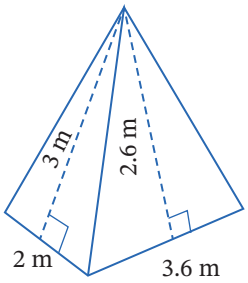
STAGE 5.3

CHAPTER 4

a



b



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14

NUMBER AND ALGEBRA

FUNCTIONS

OPTIONAL STAGE 5.3 TOPIC

RECOMMENDED FOR STAGE 6 MATHEMATICS EXTENSION 1 AND MATHEMATICS ADVANCED

Functions are used in many areas of mathematics and science and can be used to describe the relationship between 2 quantities such as the distance run by an athlete and the time taken. In business, computing, economics and engineering, finding the right model or function is important for solving problems. The German mathematician, Peter Dirichlet, was the first person to use the word *function* in 1837.



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Chapter outline

STAGE 5.3		Working mathematically				
14.01	Functions#	U	F		R	C
14.02	Function notation#	U	F		R	C
14.03	Inverse functions#	U	F		R	C
14.04	Graphing $y = a^x$ and $y = \log_a x$	U	F		R	C
14.05	Graphing translations of functions#	U	F		R	

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

domain The set of permissible x values for the function $y = f(x)$

function A rule or relationship between 2 variables where, for each value of the independent variable (input value), x, there is only one value of the dependent variable (output value), y

function notation A way of writing a function, using the form $y = f(x)$

inverse function The reverse of a function, which 'undoes' the original function, found by interchanging the dependent and independent variables (x and y)

range The set of permissible y values for the function $y = f(x)$

vertical line test A test applied to a graph to determine whether it is the graph of a function

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) understand the definition of a function and use function notation
- (STAGE 5.3) use the vertical line test on a graph to test whether it represents a function
- (STAGE 5.3) find the domain and range of a function
- (STAGE 5.3) understand and graph inverse functions
- (STAGE 5.3) graph exponential and logarithmic functions
- (STAGE 5.3) sketch $y = f(x) + c$ and $y = f(x - a)$ given the graph of $y = f(x)$

SkillCheck ANSWERS ON P. 645

1 Sketch each parabola.

a $y = x^2$ **b** $y = x^2 - 3$ **c** $y = x^2 + 3x$

2 Sketch each cubic curve.

a $y = x^3$ **b** $y = x^3 + 3$ **c** $y = x^3 - 1$

3 Sketch each hyperbola.

a $y = \frac{1}{x}$ **b** $y = \frac{1}{x-1}$ **c** $y = \frac{1}{x} + 2$

4 Sketch each exponential curve.

a $y = 2^x$ **b** $y = 3^{-x}$ **c** $y = -4^x$

5 Make x the subject of each equation.

a $y = 2x - 1$ **b** $y = \frac{3}{x+1}$ **c** $y = x^2 + 4$ **d** $y = \sqrt{x-2}$

14.01 Functions#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

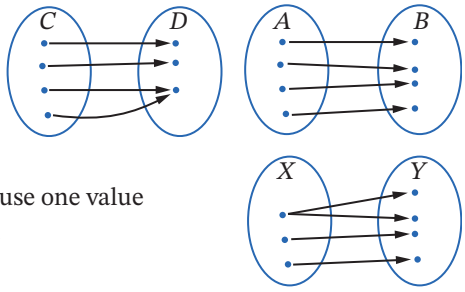
STAGE 5.3

A **function** is a relationship between 2 variables, usually described by an equation such as $y = 3x^2 - 5x + 1$. In this example, the value of y depends on the value of x, so y is called the **dependent variable** and x is called the **independent variable**.

Functions

- A **function** is a relationship between 2 variables where, for each value of the independent variable, there exists **only one** value of the dependent variable.
- If x is the independent variable and y is the dependent variable, then for each x value, there is **only one** y value.

A function can also be visualised as a relationship between 2 sets of values where every value of one set associates with (or maps to) **exactly one** value of the second set. Two examples of functions are shown here:



This next diagram does *not* represent a function because one value of set X associates with (or maps to) **2** values of set Y.

Example 1

Do the points (3, 5), (4, 3), (4, −2) represent a function?

Solution

When $x = 4$, $y = 3$ and $y = -2$.

So, one x value gives 2 y values.

∴ The points do not represent a function.

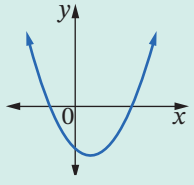
Vertical line test

- Any vertical line drawn on the graph of a function will cut the graph **only once.**
- If any vertical line can cut a graph in more than one place, then the graph is **not** a function.
- This is a visual representation of the definition: 'for each x value, there is **only one** y value'.

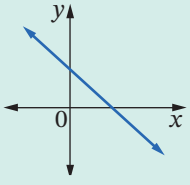
Example 2

Use the vertical line test to determine whether each graph represents a function.

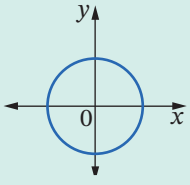
a



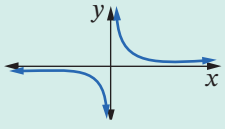
b



c



d



Solution

a Any vertical line will only cut the parabola once.
∴ The graph represents a function.

For every x value, there is only one y value.

b Any vertical line will only cut the line once.
∴ The graph represents a function.

c A vertical line can cut the circle twice.
∴ The graph does not represent a function.

For this x value, there are 2 y values.

d Any vertical line will only cut the hyperbola once.
∴ The graph represents a function.

STAGE 5.3

EXERCISE 14.01 ANSWERS ON P. 646

Functions U F R C

EXAMPLE 1

1 Determine whether each set of points represents a function. R C

a (2, 4), (−2, 7), (5, 2)

b (−1, 7), (−1, 3), (4, 6)

c (−6, 3), (5, 1), (4, −5), (−3, −1)

d (2, 3), (−5, −3), (4, 4), (8, −3)

e (5, 8), (−1, −1), (3, 8), (4, 0)

f (7, 2), (−6, 0), (0, 3), (−3, 2)

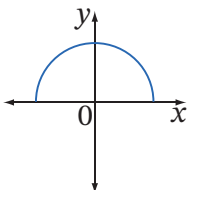
g (3, −1), (3, 5), (3, −6), (3, 0)

h (0, 2), (7, 2), (−4, 2), (9, 2), (5, 2)

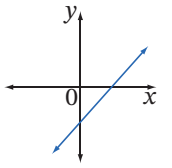
EXAMPLE 2

2 Test whether each graph represents a function. R C

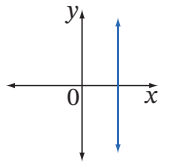
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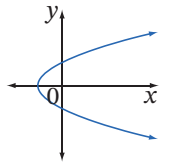
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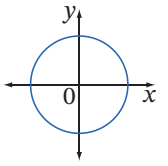
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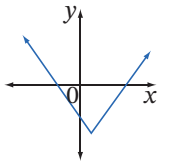
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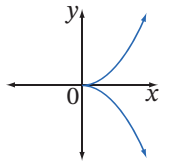
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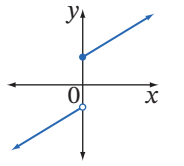
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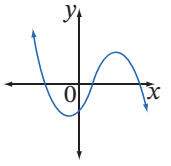
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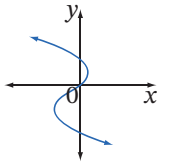
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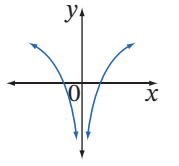
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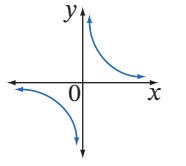
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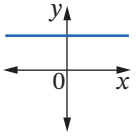
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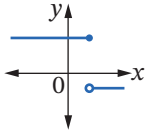
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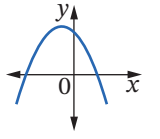
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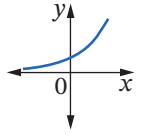
n



o



p



3 **a** Graph each linear equation and find its gradient. R C

i $y = 3x - 2$

ii $x + 2y = 3$

iii $y = 4$

iv $x = -2$

b Which of the lines drawn in part **a** represent a function?

c For what values of the gradient is a linear equation:

i always a function?

ii never a function?

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Function notation#

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14.02

STAGE 5.3

For the function $y = 3x - 1$, y depends on x.

We say that 'y is a function of x'. This is written as $y = f(x)$, which is read as 'y is f of x' and the 'f' stands for function.

So, for $y = 3x - 1$, we may write $y = 3x - 1$ or $f(x) = 3x - 1$.

Note that other variables can be used in functions. For example, if the rule $h = 12t - 5t^2$ represents the height, h metres, of a ball after time t seconds, then h is a function of t and we can write $h(t) = 12t - 5t^2$.

The value of a function

If $y = f(x)$, then $f(c)$ is the value of the function when $x = c$ (or the y value when $x = c$).

Example 3

If $y = f(x) = x^2 - 2x$, find:

a $f(3)$ **b** $f(-4)$ **c** $f(2a)$

Solution

a $f(x) = x^2 - 2x$

$\therefore f(3) = 3^2 - 2 \times 3$

$= 9 - 6$

$= 3$

b $f(-4) = (-4)^2 - 2 \times (-4)$

$= 16 + 8$

$= 24$

c $f(2a) = (2a)^2 - 2 \times 2a$

$= 4a^2 - 4a$

Example 4

For the function $g(h) = 3h + 5$, find:

a $g(-2)$ **b** $g(3) + g(-1)$

c $g(k + 4)$ **d** the value of h for which $g(h) = 20$

Solution

a $g(-2) = 3(-2) + 5$

$= -6 + 5$

$= -1$

b $g(3) = 3(3) + 5$

$= 14$

$g(-1) = 3(-1) + 5$

$= 2$

$\therefore g(3) + g(-1) = 14 + 2$

$= 16$

c $g(k + 4) = 3(k + 4) + 5$

$= 3k + 12 + 5$

$= 3k + 17$

d $g(h) = 3h + 5 = 20$

$3h = 20 - 5$

$= 15$

$h = 5$

STAGE 5.3

The domain and range of a function

For some functions $y = f(x)$, there may be restrictions on the x values and y values. The set of permissible x values is called the **domain**, while the set of permissible y values is called the **range**.

Example 5

Draw the graph of each function and find its domain and range.

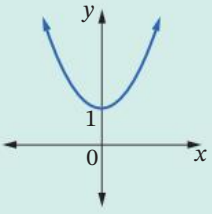
a $f(x) = x^2 + 1$ **b** $y = \frac{3}{x}$ **c** $y = 3^{-x}$

Solution

a The graph of $y = f(x) = x^2 + 1$ is a parabola with a vertex at $(0, 1)$.

Note from the equation and graph that there is no restriction on the x values, so the **domain** of the function is 'all x'.

Note from the equation and graph that the y values are all greater than or equal to 1. So the **range** of the function is $y \geq 1$.



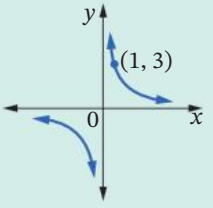
b The graph of $y = \frac{3}{x}$ is the hyperbola as shown.

The permissible x values are all values of x, except 0, which is written as $x \neq 0$.

The **domain** of the function is $x \neq 0$.

The permissible y values are all y values except 0.

The **range** of the function is $y \neq 0$.

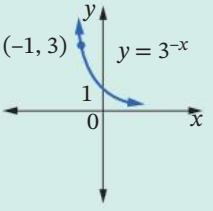


c The graph of $y = 3^{-x}$ is the exponential curve as shown.

The **domain** is 'all x'.

The permissible y values are all positive y values.

The **range** is $y > 0$.



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EXERCISE 14.02 ANSWERS ON P. 646

Function notation U F R C

EXAMPLE 3

WS Algebra 5

14.02

1 For each function below, evaluate:

i $f(3)$ **ii** $f(-1)$ **iii** $f(0)$

a $f(x) = 2x$ **b** $f(x) = 1 - x$ **c** $f(x) = x^2 + 5x$

d $f(x) = \frac{1}{2x+1}$ **e** $f(x) = \sqrt{2x+3}$ **f** $f(x) = \frac{2x+1}{2x-1}$

g $f(x) = 1 - \frac{2}{1-2x}$ **h** $f(x) = x^2 - 4x + 1$ **i** $f(x) = (x+5)^2 - 4$

2 If $f(x) = 3x - x^2$, find: R C

a $f(3)$ **b** $f(-4)$ **c** $f(3k)$

3 If $f(x) = 6x - 2$, find: R C

EXAMPLE 4

a $f(3)$ **b** $f(-2) + f(1)$ **c** $f(d-3)$

d $f(d+1) - f(d-1)$

e the value of x for which $f(x) = 15$

4 If $h(x) = \frac{x+2}{x-1}$, find: R C

a $h(2)$ **b** $h(-1)$ **c** $h(0)$ **d** x, if $h(x) = 3$

5 If $g(t) = \frac{t}{2} + t^2$, find: R C

a $g(2)$ **b** $g(0)$ **c** $g(-4)$

d the values of t for which $g(t) = 3$

6 Given $p(y) = 11 - 2y$, find: R C

a **i** $p(0)$ **ii** $p(-4)$ **iii** $p(0.5)$

b **i** $p(y+1)$ **ii** $p(y+1) - p(y)$

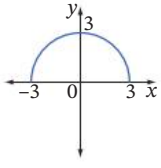
c y given that $p(y) = -21$.

7 For each graph of a function, find: R C

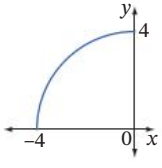
EXAMPLE 5

i the domain **ii** the range

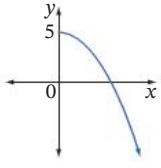
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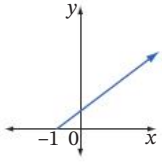
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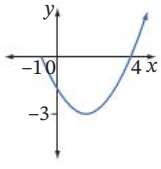
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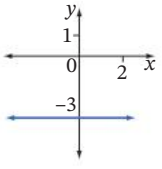
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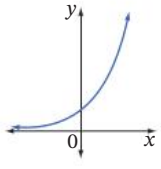
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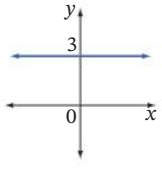
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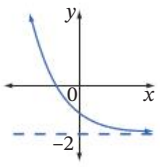


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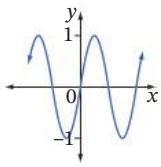


Foundation Standard Complex

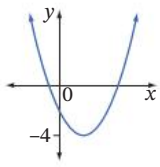
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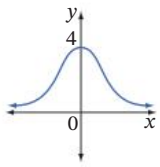
j



k



l



8 Graph each function and write its domain and range. R C

a $f(x) = 3x + 1$ **b** $y = 1 - x^2$ **c** $f(x) = 2^x$

d $y = \frac{2}{x}$ **e** $y = 1 - 2x$ **f** $f(x) = x^2 - 2x - 15$

g $f(x) = x^3$ **h** $f(x) = 4^x + 3$ **i** $y = (x + 3)^2 + 2$

9 $g(m) = m^2 + 2$ and $h(m) = 2m + 5$. R C

a Find:

i $g(3)$ **ii** $h(-5)$ **iii** $g(-1) + h(-1)$

b Show that $h(w^2) - 2g(w) = 1$.

c Solve $g(m) = h(m)$.

10 $k(x) = \sqrt{2x+1}$. R C

a Find $k(4)$. **b** Solve $k(x) = 5$.

c Explain why there is no value for $k(-1)$.

d What is the smallest value for x such that $k(x)$ exists?

11 If $h(t) = 3t^2 - 2t + 3$, find: R C

a $h(-1)$ **b** $h(2)$ **c** $h(4) - h(0)$

d $h(k^2)$ **e** $h(-k)$ **f** $h(k) - h(-k)$

12 $H(x) = 2^x + 2^{-x}$. R C

a Find the values of $H(3)$, $H(-2)$ and $H(a + 1)$.

b Show that $H(x) = H(-x)$.

13 $F(x) = x - 3 + \frac{1}{x}$. R C

a Evaluate:

i $F(3)$ **ii** $F(-3)$ **iii** $F\left(\frac{1}{3}\right)$

b Does $F(a) = F(-a)$ for all possible values of a?

c Does $F(a) = F\left(\frac{1}{a}\right)$ for all possible values of a?

Inverse functions#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

14.03

STAGE 5.3

14.03

Inverse functions code puzzle

The inverse function can be considered as the 'reverse operation' of the original function. The inverse function *undoes* the original function. For example, the inverse function to $y = x + 3$ is $y = x - 3$. The inverse function $y = x - 3$ will 'undo' the original function $y = x + 3$. Substituting $x = 10$ into $y = x + 3$ gives 13, and substituting $x = 13$ into $y = x - 3$ brings the answer back to 10.

Other examples of inverse functions are:

$y = 2x$ and $y = \frac{x}{2}$

$y = x^3$ and $y = \sqrt[3]{x}$

$y = \sin x$ and $y = \sin^{-1} x$ (shift sin)

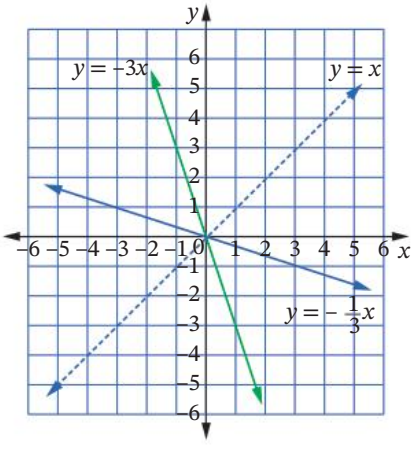
$y = a^x$ and $y = \log_a x$

Inverse function notation and graph

The inverse function for $y = f(x)$ is written as $\boldsymbol{y = f^{-1}(x)}$. Note that f^{-1} in this case means 'inverse' and **not** 'to the power of -1'.

The graph of $f(x) = -3x$ is shown as a green line on the diagram.

The graph of its inverse function $f^{-1}(x) = -\frac{1}{3}x$ is shown as a light blue line.



The points (1, –3) and (–2, 6) lie on the line $y = -3x$.

The points (–3, 1) and (6, –2) lie on the line $y = -\frac{1}{3}x$, the inverse function.

Note that on the inverse function, the x and y values of each point are swapped.

The graph of $f^{-1}(x) = -\frac{1}{3}x$ is a reflection of $f(x) = -3x$ across the dotted line $y = x$.

The graph of $y = f^{-1}(x)$ is always a reflection of the graph $y = f(x)$ in the line $y = x$.

Inverse functions

To find the inverse function of a function $y = f(x)$, rewrite the function with x and y interchanged, then make y the subject.

When the coordinates (x, y) are swapped to (y, x), the point is reflected in the line $y = x$. This explains why the graph of $y = f^{-1}(x)$ is $y = f(x)$ reflected in this line.

STAGE 5.3

Example 6

a If $f(x) = 3x - 4$, find the inverse function $f^{-1}(x)$.

b Graph $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

Solution

a $y = 3x - 4$

To find the inverse function, interchange the x and y variables.

$\therefore x = 3y - 4$

Make y the subject.

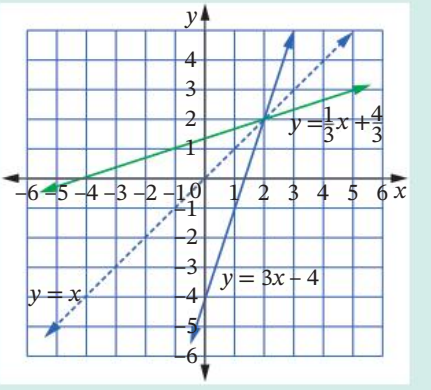
$3y - 4 = x$

$\therefore 3y = x + 4$

$y = \frac{x}{3} + \frac{4}{3}$

$f^{-1}(x) = \frac{x}{3} + \frac{4}{3}$

b



Condition for a function to have an inverse function

Not all functions have an inverse function.

Consider $f(x) = x^2 + 1$.

$y = x^2 + 1$

Swapping x and y to find the inverse function:

$x = y^2 + 1$

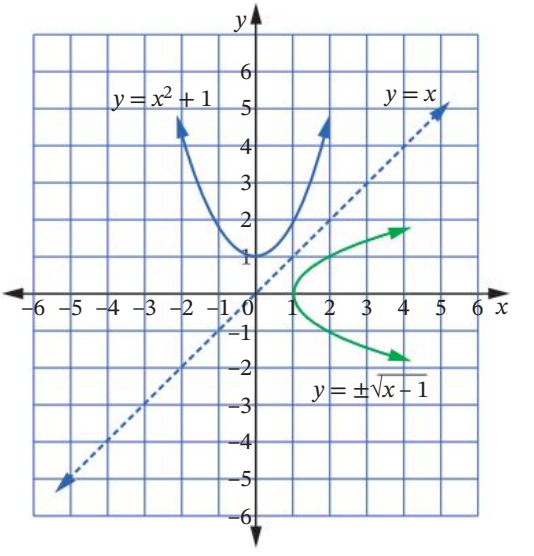
$y^2 + 1 = x$

$y^2 = x - 1$

$y = \pm\sqrt{x-1}$

$y = \pm\sqrt{x-1}$ is not a function because y takes on a positive and negative value (2 values) for every value of x. Furthermore, when graphed on the number plane, it is a 'sideways parabola' (shown in green) that fails the vertical line test.

Note that when a graph is reflected in the line $y = x$, it is flipped sideways, so for an inverse function to exist, it must also pass a **horizontal line test** so that for every value of y there is only one value of x.



Horizontal line test

For a function to have an inverse function:

- for each y **value**, there is **only one** x value
- any **horizontal line** drawn on the graph of the function will cut the graph only once.

Example 7

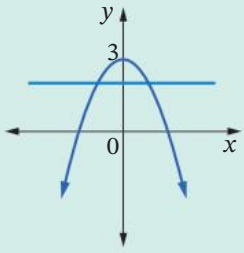
Does the function $f(x) = 3 - x^2$ have an inverse function?

Solution

The graph of $f(x) = 3 - x^2$ is shown.

A horizontal line cuts the graph of the function twice.

$\therefore f(x) = 3 - x^2$ does **not** have an inverse function.



EXERCISE 14.03 ANSWERS ON P. 647

Inverse functions U F R C

1 For each function: R C

EXAMPLE 6

i find the inverse function $f^{-1}(x)$

ii sketch $y = x$, $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $y = 2x - 5$ **b** $f(x) = \frac{x}{3}$ **c** $f(x) = 3 - \frac{x}{2}$

d $f(x) = \frac{4}{x}$ **e** $y = \frac{2x-3}{2}$ **f** $y = 1 - \frac{3x}{2}$

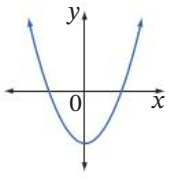
2 **a** If $f(x) = 2 - x$, show that $f^{-1}(x) = 2 - x$. R C

b By sketching $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ on the same set of axes, explain why $f(x) = f^{-1}(x)$.

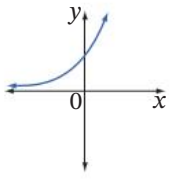
3 Which of the following represent functions whose inverse is a function?

EXAMPLE 6

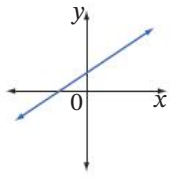
a



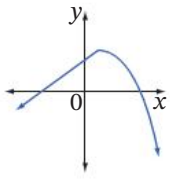
b



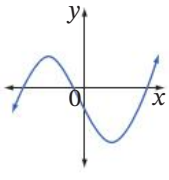
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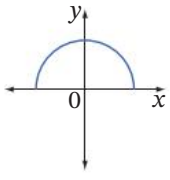
d



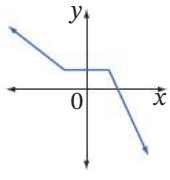
e



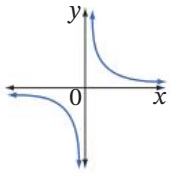
f



g



h



4 **a** Sketch $y = x^2 - 2$. R C

b Does $y = x^2 - 2$ have an inverse function?

c Sketch $y = x^2 - 2$ for $x \geq 0$. Hence find and graph its inverse function.

d Sketch $y = x^2 - 2$ for $x \leq 0$. Hence find and graph its inverse function.

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STAGE 5.3

5 a Sketch $y = x^2 + 3$. R C

b Does $y = x^2 + 3$ have an inverse function?

c What restriction needs to be placed on $y = x^2 + 3$ so that it does have an inverse?

6 a Sketch $y = x^2 - 4x$ and find the coordinates of the vertex. R C

b What restriction needs to be placed on $y = x^2 - 4x$ so that it does have an inverse function?

7 a Sketch the parabola $y = (x - 2)(x + 3)$ and find the coordinates of the vertex. R C

b Does $y = (x - 2)(x + 3)$ have an inverse function?

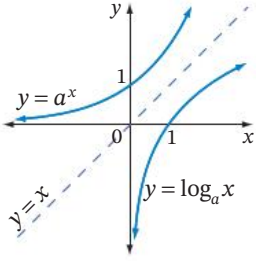
c What restriction needs to be placed on $y = (x - 2)(x + 3)$ so that it does have an inverse function?

14.04 Graphing $y = a^x$ and $y = \log_a x$

STAGE 5.3

The **exponential function** $y = a^x$ and the **logarithmic function** $y = \log_a x$ are inverse functions of each other. Remember from Chapter **7**, *Equations and logarithms*, that the **logarithm** of a number is the power of the number, to a given positive base. For example, the logarithm of 32 to the base 2 is 5, written $\log_2 32 = 5$, because $2^5 = 32$. (This shows that $y = \log_2 x$ is the inverse function of $y = 2^x$).

Since $y = \log_a x$ is the inverse function of $y = a^x$, the graph of $y = \log_a x$ can be drawn by reflecting $y = a^x$ in the line $y = x$.

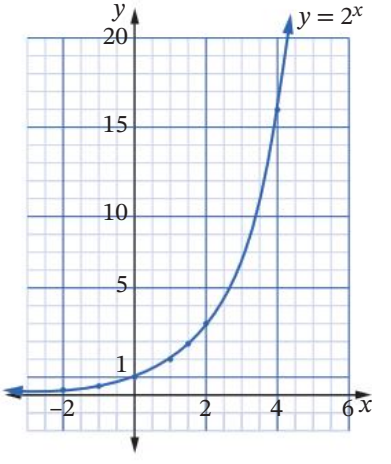


Note that as x increases, the graph of $y = \log_a x$ increases slowly because y is a *power*. For example, since $\log_2 32 = 5$, this means that (32, 5) is a point on the graph of $y = \log_2 x$.

The table of values and graph for $y = 2^x$ and $y = \log_2 x$ are shown below for comparison.

$y = 2^x$

x	−2	−1	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	4
y	$\frac{1}{4}$	$\frac{1}{2}$	1	1.4	2	2.8	4	16

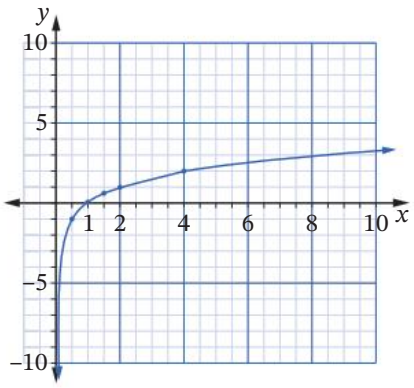


Foundation Standard Complex

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$y = \log_2 x$

x	−2	−1	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	4
y	–	–	–	−1	0	0.6	1	2



EXERCISE 14.04 ANSWERS ON P. 649

Graphing $y = a^x$ and $y = \log_a x$ U F R C

Graphing technology can be used in this exercise.

1 Show that the inverse function of $y = 2^x$ is $y = \log_2 x$ by making x the subject and then interchanging the x and y variables. R C

2 **a** Find the domain and range of $y = 2^x$. R C

b Find the domain and range of $y = \log_2 x$.

c What do you notice about the domain and range of $y = 2^x$ and $y = \log_2 x$?

3 Describe the shape of the graph of $y = 2^x$ and $y = \log_2 x$. What are their x- and y-intercepts and asymptotes? R C

4 On the same set of axes, graph the functions $y = 3^x$ and $y = 4^x$. Draw the reflections of each graph in the line $y = x$ to illustrate $y = \log_3 x$ and $y = \log_4 x$. Describe the differences and similarities in the graphs:

a $y = 2^x$, $y = 3^x$ and $y = 4^x$

b $y = \log_2 x$, $y = \log_3 x$ and $y = \log_4 x$

5 Which is larger, 2^x, 3^x or 4^x, when: R C

a $x < 0$?

b $x > 0$?

6 Which is larger, $\log_2 x$, $\log_3 x$ or $\log_4 x$, when: R C

a $x < 1$?

b $x > 1$?

7 The log key on a scientific calculator is used to find $\log_{10} x$, where $x > 0$. Evaluate each logarithm correct to 4 decimal places.

a $\log_{10} 20$

b $\log_{10} 627$

c $\log_{10} 5426$

d $\log_{10} 8.2$

e $\log_{10} 2.5$

f $\log_{10} 0.76$

Foundation Standard Complex

STAGE 5.3

8 Graph $y = \log_{10} x$ for values of x from 0 to 10.

9 Use the graph of $y = \log_{10} x$ to estimate: R

a $\log_{10} 3$ **b** $\log_{10} 6.5$ **c** $\log_{10} 8.4$ **d** $\log_{10} 0.5$

Compare your answers with those found using your calculator.

14.05 Graphing translations of functions#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

Vertical and horizontal translations of $y = f(x)$

- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ translated vertically, shifted up c units (or down if c is negative).
- The graph of $y = f(x - b)$ is the graph of $y = f(x)$ translated horizontally, shifted right b units (or left if b is negative).

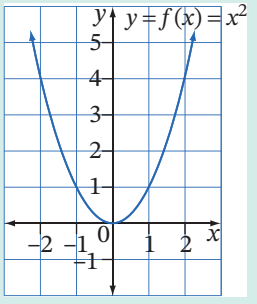
Example 8

If $y = f(x) = x^2$, then graph:

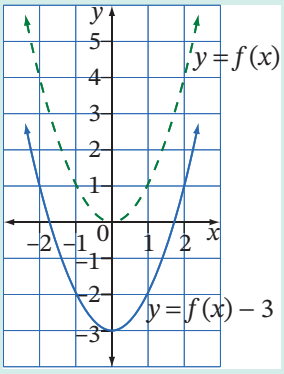
a $y = f(x)$ **b** $y = f(x) - 3$ **c** $y = f(x + 5)$

Solution

a

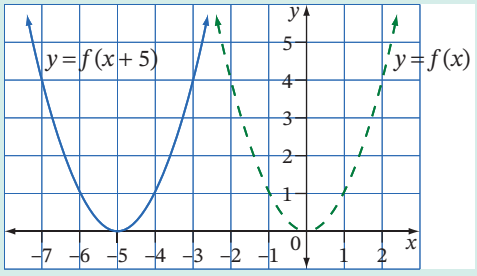


b



For $y = f(x) - 3$, the graph of $y = f(x)$ is shifted down 3 units.

c



For $y = f(x + 5)$, the graph of $y = f(x)$ is shifted 5 units to the left.

EXERCISE 14.05 ANSWERS ON P. 649

Graphing translations of functions U F R

EXAMPLE 7

1 If $y = f(x) = 2x$, then graph: R

a $y = f(x)$ **b** $y = f(x) + 1$ **c** $y = f(x) - 3$

d $y = f(x - 2)$ **e** $y = f(x + 4)$

2 If $y = f(x) = x^2$, then graph: R

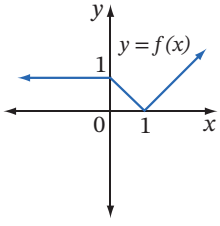
a $y = f(x) - 3$ **b** $y = f(x) + 2$ **c** $y = f(x) - 1$

d $y = f(x - 3)$ **e** $y = f(x + 1)$ **f** $y = f(x + 2) - 1$

3 If $y = f(x) = x^3$, then graph: R

a $y = f(x - 1)$ **b** $y = f(x) + 2$ **c** $y = f(x + 2) + 1$

4 The graph of $y = f(x)$ is shown. Sketch the graph of: R

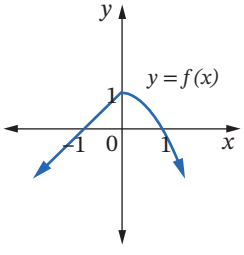


a $y = f(x) - 2$ **b** $y = f(x - 3)$

c $y = f(x + 2)$ **d** $y = f(x) + 3$

e $y = f(x + 1) - 3$

5 The graph of $y = f(x)$ is shown. Sketch the graph of: R



a $y = f(x) + 2$ **b** $y = f(x) - 3$

c $y = f(x - 1)$ **d** $y = f(x - 2) + 1$



15

NUMBER AND ALGEBRA

POLYNOMIALS

OPTIONAL STAGE 5.3 TOPIC

RECOMMENDED FOR STAGE 6 MATHEMATICS EXTENSION 1

We have analysed and graphed linear equations $y = mx + c$, quadratic equations $y = ax^2 + bx + c$ and simple cubic equations $y = kx^3 + c$. In this topic we will look at equations involving higher powers of x, their graphs and the methods for factorising them and sketching their graphs.



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Chapter outline

STAGE 5.3		Working mathematically				
15.01	Polynomials	U	F			C
15.02	Adding and subtracting polynomials	U	F			
15.03	Multiplying polynomials	U	F			
15.04	Dividing polynomials	U	F		R	C
15.05	The remainder theorem	U	F			
15.06	The factor theorem	U	F		R	C
15.07	Graphing polynomials	U	F	PS	R	C
15.08	Transforming graphs of polynomials	U	F		R	C

Wordbank

degree of a polynomial The highest power in a polynomial. For example, the degree of $8x^3 + 4x - 7$ is 3.

polynomial An algebraic expression involving powers of x that are positive integers. For example, $P(x) = 8x^3 + 4x - 7$.

quotient The 'whole' part of the answer when a polynomial is divided by another polynomial

remainder The 'left-over' part of the answer when a polynomial is divided by another polynomial

root of an equation A value of x that is a solution to the equation $P(x) = 0$. A root of $P(x) = 0$ is the same as a *zero* of $P(x)$.

zero of a polynomial A value of x that makes the value of the polynomial $P(x)$ equal to 0

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- (STAGE 5.3) recognise a polynomial and use polynomial notation
- (STAGE 5.3) add and subtract polynomials
- (STAGE 5.3) multiply and divide a polynomial by a linear polynomial
- (STAGE 5.3) understand and apply the remainder and factor theorems
- (STAGE 5.3) factorise polynomials and solve polynomial equations
- (STAGE 5.3) graph quadratic, cubic and quartic polynomials, including those with multiple factors
- (STAGE 5.3) graph transformations of a basic polynomial $y = P(x)$

15.01 Polynomials

STAGE 5.3

A **polynomial** is an algebraic expression involving powers of x that are **positive integers**, for example, $8x^3 + 4x - 7$. It is written using the notation $P(x)$, meaning 'a polynomial using the variable x'.

Polynomials

A **polynomial** has the general form $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$, where the powers of x are positive integers, n and a_n, a_{n-1}, a_{n-2}, ..., a_1, a_0 are called **coefficients**.

Terminology	For $P(x) = 2x^5 + 8x^3 - 7$:	Definition
leading term of a polynomial	$2x^5$	The term with the highest power.
leading coefficient of a polynomial	2	The coefficient of the leading term.
degree of a polynomial	5	The highest power of the polynomial.
monic polynomial	Not monic.	A polynomial with a leading coefficient of 1, such as $P(x) = x^4 - 7x^2 + x + 8$
constant term	-7	The term 'at the end' of the polynomial that is independent of x

Names of polynomials

- **Linear polynomials** have degree 1 and their graph is a straight line, for example, $P(x) = 2x - 7$.
- **Quadratic polynomials** have degree 2 and their graph is a parabola, for example, $P(x) = 3x^2 - 2x + 5$.
- **Cubic polynomials** have degree 3 and their graph is a cubic curve, for example $P(x) = 6x^3 - 7x^2 + 9x - 10$.
- **Quartic polynomials** have degree 4, for example $P(x) = -2x^4 + x^3 - 8x^2 - 4x + 13$.

Example 1

State whether each expression is a polynomial.

a $A(x)=\sqrt{2}x^4-7x^5+\frac{3}{x^2}$

b $B(x)=5x+2$

c $C(x)=2\sqrt{x}+x^3+7$

d $D(x)=(x^2-2)(x+5)$

Solution

a $A(x)=\sqrt{2}x^4-7x^5+\frac{3}{x^2}=\sqrt{2}x^4-7x^5+3x^{-2}$ is **not** a polynomial because powers of x must be positive integers and $3x^{-2}$ has a negative power.

b $B(x)=5x+2$ is a polynomial.

c $C(x)=2\sqrt{x}+x^3+7=2x^{\frac{1}{2}}+x^3+7$ is **not** a polynomial because $2x^{\frac{1}{2}}$ has a fractional power.

d $D(x)=(x^2-2)(x+5)=x^3+5x^2-2x-10$ is a polynomial.

Example 2

For the polynomial $P(x)=7x^6+5x^3+9x-2$, state:

a the degree

b the leading term

c the leading coefficient

d the constant term

e whether the polynomial is monic.

Solution

a The degree is 6.

b The leading term is $7x^6$.

c The leading coefficient is 7.

d The constant term is -2.

e Since the leading coefficient is not 1, the polynomial is not monic.

$P(x)$ notation

$P(c)$ is the value of a polynomial $P(x)$ when $x=c$. This means that the number c is substituted for x in the polynomial. This is the same as with function notation $f(x)$ from Chapter 14.

Example 3

For the polynomial $P(x)=3x^4-6x^2+4$, find:

a $P(3)$

b $P(-2)$

c $P(0)-P(-1)$

Solution

a $P(3)=3(3)^4-6(3)^2+4$
$=193$

b $P(-2)=3(-2)^4-6(-2)^2+4$
$=28$

c $P(0)=3(0)^4-6(0)^4+4=4$
$P(-1)=3(-1)^4-6(-1)^2+4=1$
$\therefore P(0)-P(-1)=4-1=3$

STAGE 5.3

EXERCISE 15.01 ANSWERS ON P. 650

Polynomials U F C

EXAMPLE 1

1 Determine whether each expression is a polynomial. If it is a polynomial, state whether it is monic. C

a $9x^2 - 5x$ **b** $\frac{5}{x^2} + \frac{2}{x^3} + x$ **c** $3x^5 + x^6 - \sqrt{7}$

d $10 - \frac{x}{2}$ **e** $4x + 2\sqrt{x}$ **f** $x^3 + x^2 - 11$

g $\sqrt{5}x^2 - \sqrt{10}x^4 + x^3$ **h** $3\sqrt{x} - 2x^4 + x^5$ **i** $(20 - x)(20x - x^2)$

j $3^x - \frac{x}{3} + x^3$ **k** $(x^2 - 2)^2(x + 3)$ **l** $\frac{9x^3 + x^2 + x}{x}$

EXAMPLE 2

2 For each polynomial, state: C

i the degree **ii** the leading coefficient **iii** the constant term.

a $9x^5 - 2x^4 + 5x^3 + 1$ **b** $7x^4 - 6x^5 + 3$ **c** $11x^2 - 10$

d $-6x$ **e** $8x - \frac{1}{2}x^2 + 7x^5 + 3$ **f** 9

g $4x^3 + \sqrt{2}x^4 - 11 + x^6$ **h** $22 - \frac{5x}{4}$ **i** $\frac{x^3}{3} + \frac{x^2}{2} + x$

EXAMPLE 3

3 If $P(x) = 3x^3 - 2x^2 + x - 1$, evaluate: C

a $P(0)$ **b** $P(2)$ **c** $P(-1)$ **d** $P\left(\frac{1}{2}\right)$ **e** $P\left(\sqrt{2}\right)$

4 If $P(x) = x^2 - x$ and $Q(x) = 1 - x^3$, find: C

a $P(0) + Q(0)$ **b** $P(2) + Q(1)$ **c** $P(4) + Q(-1)$ **d** $P(-2) - Q(-2)$

e $P\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)$ **f** $P\left(2\sqrt{3}\right) + Q\left(\sqrt{3}\right)$ **g** $P[Q(2)]$ **h** $Q[P(2)]$

15.02 Adding and subtracting polynomials

STAGE 5.3

Example 4

If $P(x) = x^3 + 3x^2 - 2x - 5$ and $Q(x) = 3x^2 - 5x + 7$, simplify each expression.

a $P(x) + Q(x)$ **b** $P(x) - Q(x)$

Solution

a $P(x) + Q(x) = x^3 + 3x^2 - 2x - 5 + (3x^2 - 5x + 7)$

$= x^3 + 6x^2 - 7x + 2$

b $P(x) - Q(x) = x^3 + 3x^2 - 2x - 5 - (3x^2 - 5x + 7)$

$= x^3 + 3x - 12$

EXERCISE 15.02 ANSWERS ON P. 650

STAGE 5.3

Adding and subtracting polynomials U F

EXAMPLE 4

1 Simplify each expression.

a $(9x^3 + x^2 + x) + (7x^2 + 5x - 2)$ **b** $(3x + 2) - (x^3 - x)$

c $(8x - 2) - (11x + 4x^2)$ **d** $2(x^4 - x^2 + 2) - 3(x^3 + x^2 - x^4)$

e $x^4 - (x^3 - x^2 + 5x)$ **f** $2(5x^4 + x^3 + 2x) + 2(11 - 2x^2 + x^5)$

g $(7x^6 + x^5) + (x^2 + x^3) - (2 - 3x)$

h $8x^4 - x^3 + 6x^2 + 5x - (x + 6) + (x^3 + 9x^2 - 1)$

i $x^4 - 8x^3 + 2x^2 + 7x - 2 - (6x - 3x^2 + 5x^4)$ **j** $6x^3 - 2x^2 - x - (2x - 3) - (3x + 2)$

2 If $P(x) = 4x - 3$ and $Q(x) = x^2 + 7x + 2$, simplify each expression.

a $P(x) + Q(x)$ **b** $P(x) - Q(x)$ **c** $Q(x) - P(x)$ **d** $3P(x) + 2Q(x)$

3 If $A(x) = x^2 - 6$, $B(x) = 11x - 2x^2$ and $C(x) = 9 - 4x$, simplify each expression.

a $C(x) - B(x)$ **b** $A(x) - C(x)$ **c** $B(x) - A(x)$

d $A(x) + B(x) + C(x)$ **e** $A(x) - B(x) + C(x)$ **f** $A(x) - [B(x) + C(x)]$

4 If $R(x) = x^2 - 6x + 5$, $P(x) = x - 1$ and $Q(x) = R(x) - P(x)$, simplify each expression.

a $Q(x)$ **b** $Q(3\sqrt{2})$ **c** the values of x if $Q(x) = 0$

15.02

Multiplying polynomials 15.03

STAGE 5.3

Example 5

If $P(x) = 3x + 1$ and $Q(x) = x^2 - 5x - 2$, find $P(x) \cdot Q(x)$.

Solution

$$\begin{aligned} P(x) \cdot Q(x) &= (3x + 1)(x^2 - 5x - 2) \\ &= 3x(x^2 - 5x - 2) + 1(x^2 - 5x - 2) \\ &= 3x^3 - 15x^2 - 6x + x^2 - 5x - 2 \\ &= 3x^3 - 14x^2 - 11x - 2 \end{aligned}$$

EXERCISE 15.03 ANSWERS ON P. 650

Multiplying polynomials U F

EXAMPLE 5

1 Expand each product.

a $(3x + 2)(x^3 - x)$ **b** $(11x + 4x^2)(8x - 2)$

c $(9x^3 + x^2 + x)(5x - 2)$ **d** $(6x^3 - 2x^2 - x - 3)(3x + 2)$

e $(2 - 3x)(7x^6 + x^5 + x^3 + x^2)$ **f** $(x + 6)(8x^4 - x^3 + 6x^2 + 5x - 2)$

Foundation | Standard | Complex

STAGE 5.3

2 If $P(x) = 4x - 3$ and $Q(x) = x^2 + 7x + 2$, find $P(x) \cdot Q(x)$.

3 If $A(x) = x^2 - 6$, $B(x) = 11x - 2x^2$ and $C(x) = 9 - 4x$, find:

a $A(x) \cdot B(x)$ **b** $A(x) \cdot C(x)$ **c** $B(x) \cdot C(x)$

15.04 Dividing polynomials

STAGE 5.3

Finding the quotient

The **long division** process for $9947 \div 65$ is shown on the right.

- 65 into 99 is 1, remainder 34
- Bring down the 4 from 9947
- 65 into 344 goes 5, remainder 19
- Bring down the 7 from 9947
- 65 into 197 goes 3, remainder 2

$$\begin{array}{r} 153 \\ 65\overline{)9947} \\ -65\downarrow \\ \underline{344} \\ -\ 325\downarrow \\ \underline{197} \\ \underline{195} \\ 2 \end{array}$$

$\therefore 9947 \div 65 = 153$, remainder 2

$= 153\frac{2}{65}$

$\therefore 9947 = 65 \times 153 + 2$

9947 is the **dividend**, 65 is the **divisor**, 153 is the **quotient**, 2 is the **remainder**.

The long division process can also be used to divide polynomials.

For example, $(x^3 + 5x^2 - 6x - 4) \div (x + 3)$ is

- $(x + 3)$ into $(x^3 + 5x^2)$ is x^2, remainder $2x^2$ $\quad x^3 \div x = x^2$
- Bring down the $(-6x)$ from the dividend
- $(x + 3)$ into $(2x^2 - 6x)$ goes $2x$, remainder $-12x$ $\quad 2x^2 \div x = 2x$
- Bring down the (-4) from the dividend
- $(x + 3)$ into $(-12x - 4)$ goes (-12), remainder 32 $\quad -12x \div x = -12$

$$\begin{array}{r} x^2+2x-12 \\ x+3\overline{)x^3+5x^2-6x-4} \\ x^3+3x^2\downarrow \\ \underline{2x^2-6x} \\ 2x^2+6x\downarrow \\ \underline{-12x-4} \\ -12x-36 \\ \underline{32} \end{array}$$

$\therefore (x^3 + 5x^2 - 6x - 4) \div (x + 3) = x^2 + 2x - 12$, remainder 32

$\therefore (x^3 + 5x^2 - 6x - 4) = (x + 3)(x^2 + 2x - 12) + 32$

$(x^3 + 5x^2 - 6x - 4)$ is the **dividend**, $(x + 3)$ is the **divisor**, $(x^2 + 2x - 12)$ is the **quotient**, 32 is the **remainder**.

Note that we can express the dividend $P(x)$ as the product of its factors plus the remainder:

$P(x) =$ divisor $\times$ quotient + remainder.

Foundation Standard Complex

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Example 6

Divide $P(x) = x^3 - x + 4$ by $A(x) = x - 3$. Then write $P(x)$ in the form $P(x) = A(x) \cdot Q(x) + R(x)$, where $Q(x)$ is the quotient and $R(x)$ is the remainder.

Solution

$$\begin{array}{r} x^2+3x+8 \\ x-3\overline{)\,x^3+0x^2-x+4} \\ \underline{x^3-3x^2} \\ 3x^2-x \\ \underline{3x^2-9x} \\ 8x+4 \\ \underline{8x-24} \\ 28 \end{array}$$

Write $0x^2$ as there is no x^2 term

$\therefore x^3 - x + 4 = (x-3)(x^2+3x+8)+28$

Example 7

Show that $(2x + 5)$ is a factor of $2x^3 + 5x^2 - 2x - 30$.

Solution

If $2x^3 + 5x^2 - 12x - 30 = (2x + 5) \cdot Q(x)$ with no remainder, then $(2x + 5)$ is a factor.

$$\begin{array}{r} x^2 \quad -6 \\ 2x+5\overline{)\,2x^3+5x^2-12x-30} \\ \underline{2x^3+5x^2} \\ 0x^2-12x-30 \\ \underline{-12x-30} \\ 0 \end{array}$$

The remainder is 0, which means that $2x^3 + 5x^2 - 12x - 30 = (2x + 5)(x^2 - 6)$

$\therefore (2x + 5)$ is a factor of $2x^3 + 5x^2 - 12x - 30$.

EXERCISE 15.04 ANSWERS ON P. 650

Dividing polynomials U F R C

EXAMPLE 6

1 Perform the following divisions, then write the first polynomial in the form: dividend = divisor × quotient + remainder. R C

a $(x^2 + 7x + 4) \div (x + 2)$

b $(x^2 - 6x + 2) \div (x - 3)$

c $(4x^2 + 3x + 10) \div (x - 1)$

d $(8x^2 + 9x + 11) \div (2x + 1)$

e $(x^3 + 6x^2 + 5x - 4) \div (x - 3)$

f $(4x^3 + 2x^2 + x) \div (x + 4)$

g $(2x^3 - x^2 + 5x + 3) \div (x + 6)$

h $(11 - x^2 + 3x^3) \div (x + 2)$

i $(x^5 - x^4 + 8x^3 + 2x^2 - x - 1) \div (x + 1)$

j $(x^4 - x^2 - 10) \div (x + 3)$

2 If $P(x) = 3x^2 - 7x + 5$, $W(x) = x^2 + 5x$ and $T(x) = x - 2$, then find the quotient and remainder in each expression. R C

a $P(x) \div T(x)$

b $W(x) \div T(x)$

c $[W(x) \times P(x)] \div T(x)$

d $[P(x) + W(x)] \div T(x)$

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STAGE 5.3

EXAMPLE 7

3 Show that $(2x - 1)$ is a factor of each polynomial. Express $P(x)$ as a product of 2 factors. R C

a $P(x) = 6x^2 + x - 2$ **b** $P(x) = 2x^3 + x^2 + x - 1$

c $P(x) = 8x^2 + 10x - 7$ **d** $P(x) = 6x^3 + x^2 - 1$

e $P(x) = 2x^4 - 7x^3 - 5x^2 + 8x - 2$ **f** $P(x) = 2x^4 - x^3 - 2x^2 + 7x - 3$

g $P(x) = 6x^3 - 3x^2 + 2x - 1$ **h** $P(x) = 11x + x^5 - 6x^2 - 2x^6 - 4$

15.05 The remainder theorem

STAGE 5.3

The remainder theorem

If a polynomial $P(x)$ is divided by the linear expression $(x - a)$, then the remainder is $P(a)$.

Proof:

Since $(x - a)$ is a polynomial of degree 1:

$P(x) = (x - a) \cdot Q(x) + R$, where the remainder R is a constant.

Substituting $x = a$, gives:

$P(a) = (a - a) \cdot Q(a) + R$

$= 0 + R$

$= R$

Example 8

Find the remainder when $P(x) = 6x^4 - 3x^3 + 2x + 5$ is divided by each linear expression.

a $(x - 2)$ **b** $(x + 3)$

Solution

a Dividing by $(x - 2)$ will give the remainder $P(2)$.

$P(2) = 6(2)^4 - 3(2)^3 + 2(2) + 5$

$= 81$

b Dividing by $(x + 3)$ will give the remainder $P(-3)$.

$P(-3) = 6(-3)^4 - 3(-3)^3 + 2(-3) + 5$

$= 566$

EXERCISE 15.05 ANSWERS ON P. 651

The remainder theorem U F

EXAMPLE 8

1 Find the remainder when $P(x) = 2x^3 - 3x^2 + x - 1$ is divided by each linear expression.

a $(x - 2)$ **b** $(x + 4)$ **c** $(x - 1)$ **d** $(x - 5)$

e $(x + 1)$ **f** $(x - 10)$ **g** $(x + 3)$ **h** $(x - 3)$

Foundation Standard Complex

2 Determine the remainder when the first polynomial is divided by the second polynomial.

a $x^2 + 11x - 6, x - 4$
b $x^2 - 2x + 3, x - 1$
c $3x^2 - x + 4, x - 2$
d $x^3 + x^2 - x, x + 2$
e $3x^3 + 2x^2 - 11, x + 1$
f $2x^2 - x - 16, x - 10$
g $5x^3 - x^2 - 6x, x + 1$
h $x^6 - 3x^4 + x^3 - 2, x + 2$
i $2x^4 - 3x^2 + 6x - 2, x + 3$
j $5x^5 - 2x^4 - 3x^2 + 7x - 8, x + 5$

STAGE 5.3

15.05

The factor theorem 15.06

STAGE 5.3

If a polynomial $P(x)$ is divided by $(x - a)$ and the remainder is 0, then $(x - a)$ is a factor of $P(x)$. Furthermore, by the remainder theorem, $P(a) = 0$.

Factorising cubic functions

The factor theorem

If $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factorising polynomials

Example 9

a Show that $(x - 2)$ is a factor of $P(x) = 2x^3 - 7x^2 - 3x + 18$.

b Hence express $P(x)$ as a product of its factors.

Solution

a $P(2) = 2(2^3) - 7(2^2) - 3(2) + 18$

$= 0$

$\therefore (x - 2)$ is a factor of $P(x)$.

b Use long division to find the other factors.

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 9 \\
\hline
x - 2 & 2x^3 - 7x^2 - 3x + 18 \\
 & \underline{2x^3 - 4x^2} \\
 & -3x^2 - 3x \\
 & \underline{-3x^2 + 6x} \\
 & -9x + 18 \\
 & \underline{-9x + 18} \\
 & 0
\end{array}
$$

Foundation | Standard | Complex

STAGE 5.3

$\therefore P(x) = (x-2)(2x^2 - 3x - 9)$

$= (x-2)(x-3)(2x+3)$ Factorising $2x^2 - 3x - 9$

$\therefore 2x^3 - 7x^2 - 3x + 18 = (x-2)(x-3)(2x+3)$

Zeroes of a polynomial

If $(x - a)$ is a factor of $P(x)$, $x = a$ is called a **zero** of the polynomial $P(x)$.

A **zero** of a polynomial $P(x)$ is a value of x that makes $P(x)$ equal to zero.

It is a solution of $P(x) = 0$.

In Example **9** above, $x = 2$ is a **zero** of the polynomial $P(x) = 2x^3 - 7x^2 - 3x + 18$ because $P(2) = 0$.

Solving polynomial equations

Example 10

a Factorise $2x^3 - 3x^2 - 29x - 30$.

b Hence solve the equation $2x^3 - 3x^2 - 29x - 30 = 0$.

Solution

a Let $P(x) = 2x^3 - 3x^2 - 29x - 30 = 2(x-a)(x-b)(x-c)$ where a, b, c are constants.

Use 'guess and check' to find a zero of $P(x)$.

Note that if you expanded $2(x-a)(x-b)(x-c)$, then the last term would be the constant term $2(-a)(-b)(-c) = -2abc$.

That means any zero of $P(x)$ must be a factor of the constant term of $P(x)$, which is (-30), because $-2abc = -30$.

The factors of 30 are 1, –1, 2, –2, 3, –3, 5, –5, 6, –6, 10, –10, 15, –15, 30 and –30.

Try these values to find a zero of $P(x)$ by guessing and checking:

$P(1) = 2 - 3 - 29 - 30$

$= -60$

$\neq 0$ $\therefore (x-1)$ is not a factor

$P(-1) = -2 - 3 + 29 - 30$

$= -6$

$\neq 0$ $\therefore (x+1)$ is not a factor

$P(-2) = -16 - 12 + 58 - 30$

$= 0$ $\therefore (x+2)$ is a factor

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Now, by long division:

$$\begin{array}{r} 2x^2-7x-15 \\ x+2\overline{)\,2x^3-3x^2-29x-30} \\ 2x^3+4x^2 \\ \hline -7x^2-29x \\ -7x^2-14x \\ \hline -15x-30 \\ -15x-30 \\ \hline 0 \end{array}$$

$\therefore P(x) = (x+2)(2x^2-7x-15)$

$= (x+2)(x-5)(2x+3)$

$\therefore 2x^3-3x^2-29x-30 = (x+2)(x-5)(2x+3)$

b $2x^3-3x^2-29x-30=0$

$(x+2)(x-5)(2x+3)=0$

$x+2=0$ or $x-5=0$ or $2x+3=0$

$x=-2$ $x=5$ $2x=-3$

$x=-1\frac{1}{2}$

$\therefore x=-2, x=5$ or $x=-1\frac{1}{2}$

EXERCISE 15.06 ANSWERS ON P. 651

The factor theorem UFRC

EXAMPLE 9

1 For each polynomial $P(x)$, test whether each linear polynomial, **A**, **B** and **C**, is a factor.

	$P(x)$	A	B	C
a	x^2-8x+7	$(x-2)$	$(x-7)$	$(x-1)$
b	x^3+2x^2-x-2	$(x+3)$	$(x+1)$	$(x-1)$
c	$2x^3+x+3$	$(x+1)$	$(x-2)$	$(x-3)$
d	$x^3-5x^2-22x-16$	$(x+1)$	$(x+2)$	$(x-8)$
e	$x^3+2x^2-13x+10$	$(x+5)$	$(x-2)$	$(x-3)$

2 Show that the second polynomial is a factor of the first polynomial.

a $x^2+10x-24, x-2$

b $x^3+5x^2+11x+10, x+2$

c $x^3+3x^2-x-3, x-1$

d $x^4-x^3+2x^2-x-1, x-1$

e $x^3-x^2-12x, x+3$

f $x^5-4x^4+3x^3+x^2+4, x-2$

g $8x^4+2x^2-1, x-\frac{1}{2}$

h $2x^4+7x^3-56x^2-37x+84, x-4$

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STAGE 5.3

EXAMPLE 10

3 Factorise each polynomial. R C

a $x^3 + 6x^2 + 8x$

b $x^3 - x^2 - 2x$

c $x^3 + 2x^2 - x - 2$

d $2x^3 + 3x^2 - 18x + 8$

e $x^3 - 6x^2 + 11x - 6$

f $x^3 + x^2 - 46x + 80$

g $3x^3 - 16x^2 - 13x + 6$

h $6x^3 - 13x^2 + x + 2$

i $2x^4 - 5x^3 + 2x^2$

j $2x^4 - 3x^3 - 18x^2 + 17x + 30$

4 Solve each equation.

a $(2x - 1)(x - 3)(x + 4) = 0$

b $(x + 4)(x^2 + 1) = 0$

c $(2x - 5)(x^2 - x - 6) = 0$

d $(x^2 - 25)(x^2 + 3) = 0$

e $(x - 2)(x + 3)(x^2 - 16) = 0$

f $x^3 + 5x^2 - 14x = 0$

g $x^3 + 2x^2 - 9x - 18 = 0$

h $x^3 - x^2 - 10x - 8 = 0$

i $x^3 - 21x - 20 = 0$

j $2x^3 - 7x^2 + 2x + 3 = 0$

k $12x^3 - 23x^2 + 13x - 2 = 0$

l $x^3 - x - 24 = 0$

15.07 Graphing polynomials

STAGE 5.3

WS Polynomials review

Graphing polynomials

To graph the polynomial $y = P(x)$:

- substitute $y = 0$ to find the x-intercepts
- substitute $x = 0$ to find the y-intercepts
- use the sign of the leading coefficient to sketch the shape of the curve

Example 11

Sketch $y = x^3 - x^2 - 10x - 8$.

Solution

Substitute $y = 0$ to find the x-intercepts.

$0 = x^3 - x^2 - 10x - 8$

Use the factor theorem to factorise the RHS.

Let $P(x) = x^3 - x^2 - 10x - 8$

Test factors of -8.

$P(1) = 1 - 1 - 10 - 8$

$= -18$

$\neq 0$

$P(-1) = (-1)^3 - (-1)^2 - 10(-1) - 8$

$= 0$

$\therefore (x + 1)$ is a factor.

$$
\begin{array}{r|l}
x+1 & x^2 - 2x - 8 \\
\hline
 & x^3 - x^2 - 10x - 8 \\
 & x^3 + x^2 \\
\hline
 & -2x^2 - 10x \\
 & -2x^2 - 2x \\
\hline
 & -8x - 8 \\
 & -8x - 8 \\
\hline
 & 0
\end{array}
$$

$\therefore P(x) = (x + 1)(x^2 - 2x - 8)$

$= (x + 1)(x - 4)(x + 2)$

Foundation Standard Complex

If $P(x) = 0$: $(x + 1)(x - 4)(x + 2) = 0$,

$\therefore$ the x-intercepts are -2, -1 and 4.

Substitute $x = 0$ to find the y-intercept.

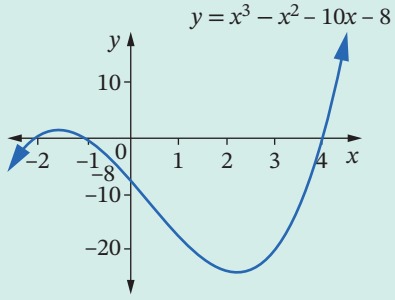
$y = 0^3 - 0^2 - 10(0) - 8$

$= -8$

$\therefore$ The y-intercept is -8.

The leading coefficient is 1, which is positive, so the cubic curve is increasing at the end.

Use graphing technology to check the shape of this graph.



Single, double and triple roots of a polynomial equation

If $x = a$ is a solution of the polynomial equation $P(x) = 0$, then $x = a$ is called a **root** of the equation.

Root is another name for **solution.**

Roots of P(x) = 0	Example
If $(x - a)$ is a factor of a polynomial $P(x)$, then $x = a$ is a **single root** of $P(x) = 0$.	$H(x) = (x - 1)(x + 2)(x - 3)$. $x = 1$, $x = -2$, $x = 3$ are all single roots of $H(x) = 0$.
If $(x - a)^2$ is a factor of a polynomial $P(x)$, then $x = a$ is a **double root** of $P(x) = 0$.	$P(x) = (x - 3)^2(x^2 - x - 1)$. $x = 3$ is a double root of $P(x) = 0$.
If $(x - a)^3$ is a factor of a polynomial $P(x)$, then $x = a$ is a **triple root** of $P(x) = 0$.	$A(x) = (x + 1)^3(x - 2)$ $x = -1$ is a triple root of $A(x) = 0$.

Example 12

Sketch $P(x) = -(x - 3)^3(x + 1)$.

Solution

Solve $P(x) = 0$ to find the x-intercepts.

$\therefore x = 3$ and $x = -1$

$x = 3$ is a triple root and $x = -1$ is a single root, so the graph will cross the x-axis at $x = 3$ (and be flat similar to $y = x^3$ there) and at $x = -1$.

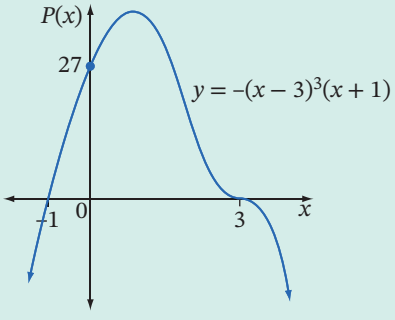
Substitute $x = 0$ to find the y-intercept.

$y = -(0 - 3)^3(0 + 1)$

$= 27$

The polynomial is a quartic (degree 4), the leading term is $-x^3 \times x = -x^4$ so the leading term is -1, which is negative, which means the quartic curve will decrease at the end.

Use graphing technology to check the shape of this graph.



STAGE 5.3

Roots of a polynomial equation

- If $P(x) = 0$ has a **single root** at $x = a$, then the graph of the polynomial **crosses** the x-axis at $x = a$.
- If $P(x) = 0$ has a **double root** at $x = a$, then the graph of the polynomial **touches** the x-axis at $x = a$ with a flat gradient, taking the shape of a parabola's vertex there.
- If $P(x) = 0$ has a **triple root** at $x = a$, then the graph of the polynomial **crosses** the x-axis at $x = a$ with a flat gradient, taking the shape of a cubic curve there.

EXERCISE 15.07 ANSWERS ON P. 651

Graphing polynomials U F PS R C

Graphing technology can be used to check the shapes of the graphs in this exercise.

EXAMPLE 12

1 Sketch the graph of each polynomial. PS R C

a $P(x) = x^3 + 6x^2 + 8x$
b $P(x) = x^3 + 2x^2 - x - 2$
c $P(x) = 2x^3 + 3x^2 - 18x + 8$
d $P(x) = 2x^4 - 5x^3 + 2x^2$
e $P(x) = 2x^3 + 17x^2 + 31x - 20$
f $P(x) = -x^3 + 5x^2 + 8x - 12$
g $P(x) = -x^4 + 3x^3 + 6x^2 - 8x$
h $P(x) = -x^4 + 13x^2 - 36$

EXAMPLE 13

2 Sketch the graph of each polynomial. PS R C

a $y = (x - 2)(x + 4)^2$
b $y = -(x + 2)(x - 3)^2$
c $y = -(x + 1)^2(x - 2)^2$
d $y = x(x + 1)(x - 3)^2$
e $y = -(x - 1)(x - 2)(x + 2)^2$
f $y = -x(x - 4)^2$
g $y = -(x - 2)^2(x + 1)^2$
h $y = (x - 4)^3(x + 1)$
i $y = (x^2 - 4)(x - 4)^2$
j $y = -x^3 - x^2 + 5x - 3$
k $P(x) = x^4 - 5x^2 + 4$
l $y = -x^4 - 3x^3 - 12x^2 - 44x - 48$

Technology

Transforming graphs

1 Use graphing technology to graph $P(x) = x^2 - 3x + 2$.

2 Write the equations of each polynomial.

a $-P(x)$ **b** $P(-x)$ **c** $P(x) - 3$ **d** $2P(x)$

3 Graph the polynomials shown in question **2**.

4 Describe how the graphs of the polynomials $y = -P(x)$, $y = P(-x)$, $y = P(x) - 3$, $y = 2P(x)$ can be drawn from the graph of $y = P(x)$.

Foundation Standard Complex

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Transforming graphs of polynomials 15.08

STAGE 5.3

WS Advanced graphs

15.08

Transformations of polynomials

Compared to the graph of the polynomial $y = P(x)$, the graph of:

- $y = -P(x)$ is a reflection of $y = P(x)$ in the x-axis
- $y = P(-x)$ is a reflection of $y = P(x)$ in the y-axis
- $y = P(x) + c$ is a vertical translation of the graph $y = P(x)$ shifted up or down
- $y = kP(x)$ is $y = P(x)$ either 'stretched' or 'compressed' vertically, with the same x-intercepts

Example 13

The graph of $y = P(x)$ is shown.

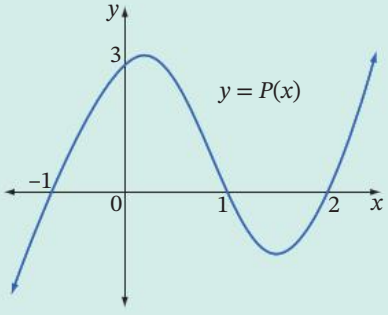
Draw the graphs of each polynomial.

a $y = -P(x)$

b $y = P(-x)$

c $y = P(x) + 2$

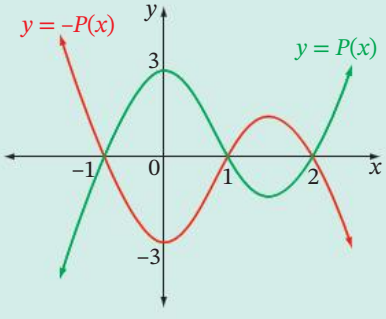
d $y = 3P(x)$



Solution

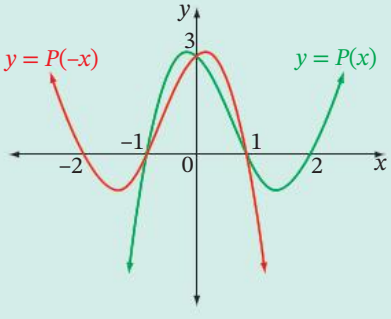
a $y = -P(x)$ is a reflection of $y = P(x)$ in the x-axis.

For each point on $y = P(x)$, the y-value changes sign.



b $y = P(-x)$ is a reflection of $y = P(x)$ in the y-axis.

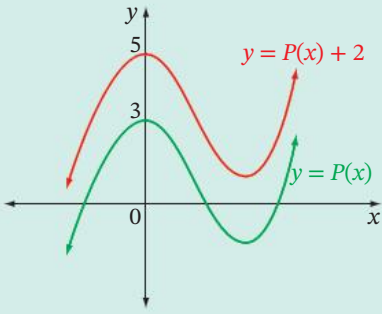
For each point on $y = P(x)$, the x value changes sign.



STAGE 5.3

c $y = P(x) + 2$ is $y = P(x)$ vertically translated 2 units up.

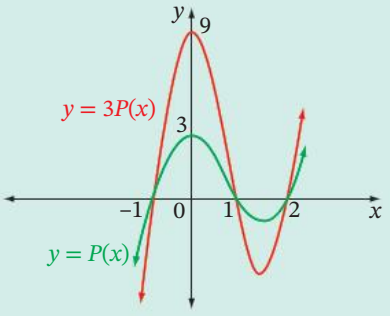
For each point on $y = P(x)$, the y-value increases by 2.



d $y = 3P(x)$ is $y = P(x)$ stretched or dilated vertically by a factor of 3.

For each point on $y = P(x)$, the y-value is multiplied by 3.

A stretch or compression of a graph is called a **dilation**.

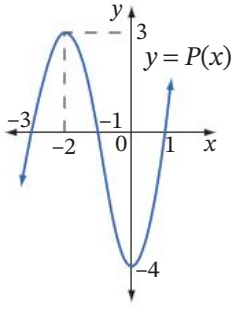


EXERCISE 15.08 ANSWERS ON P. 652

Transforming graphs of polynomials U F R C

EXAMPLE 13

1 The graph of the polynomial $y = P(x)$ is shown.



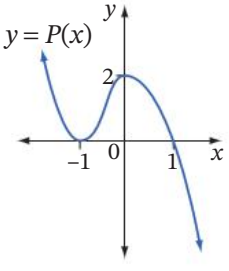
Sketch the graph of each polynomial. R C

a $y = 2P(x)$ **b** $y = P(x) + 2$

c $y = \frac{1}{2}P(x)$ **d** $y = P(x) - 3$

e $y = -P(x)$ **f** $y = P(-x)$

2 Use the graph of $y = P(x)$ shown to sketch each polynomial. R C



a $y = P(x) + 1$ **b** $y = P(x) - 2$

c $y = 2P(x)$ **d** $y = -P(x)$

e $y = P(-x)$ **f** $y = -3P(x)$

Foundation Standard Complex

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STAGE 5.3

3 Graph $P(x) = (x - 1)(x - 3)^2$ and use it to sketch the graph of each polynomial. R C

a $y = -P(x)$ **b** $y = P(-x)$ **c** $y = P(x) - 3$ **d** $y = 2P(x)$

4 Describe the transformation needed to graph each cubic equation using the graph of $y = x^3$. R C

a $y = x^3 - 2$ **b** $y = x^3 + 1$ **c** $y = 2x^3$

d $y = -x^3 + 3$ **e** $y = -3x^3$ **f** $y = 2 - x^3$

g $y = 2x^3 - 5$ **h** $y = 4 - 3x^3$ **i** $y = \frac{1}{2}x^3 + 4$

15.08

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16

MEASUREMENT AND GEOMETRY

CIRCLE GEOMETRY

OPTIONAL STAGE 5.3 TOPIC

RECOMMENDED FOR STAGE 6 MATHEMATICS EXTENSION 1

The ancient Greek philosopher, Thales of Miletus (624–546 BCE), is believed to have been the first mathematician and scientist to attempt to explain things by reasoning and deduction rather than by resorting to mythology and superstition. He lived before Pythagoras and proved the first theorem in mathematics: that any angle drawn on the circumference of a semicircle is a right angle. This rule is often called **Thales' theorem.**



Chapter outline

STAGE 5.3		Working mathematically				
16.01	Parts of a circle	U	F			C
16.02	Chord properties of circles	U	F		R	C
16.03	Angle properties of circles	U	F		R	C
16.04	Tangent and secant properties of circles#	U	F		R	C
16.05	Proofs using circle theorems#	U	F		R	C

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

Wordbank

collinear points Points that lie on a straight line.

converse A rule or statement turned back-to-front; the reverse statement

cyclic quadrilateral A quadrilateral bounded by a circle, whose vertices lie on the circle's circumference

intercept The distance between the points where a line intersects other lines or curves

perpendicular bisector An interval or line that bisects another interval at right angles

secant A line that intersects a curve at 2 points

subtend an angle To sit opposite an angle and 'hold' its arms

tangent A line that touches a curve at one point but does not cross it

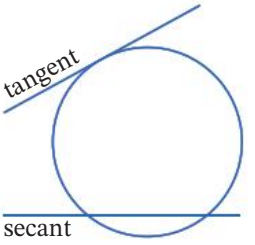
In this chapter you will:

- (STAGE 5.3) identify the parts of a circle
- (STAGE 5.3) prove chord and angle properties of circles and use them to find unknown lengths and angles in circles
- (STAGE 5.3) prove tangent and secant properties of circles and use them to find unknown lengths and angles in circles

16.01 Parts of a circle

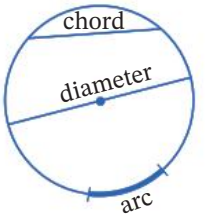
STAGE 5.3

WS Parts of a circle



A **tangent** is a line touching the circle at one point.

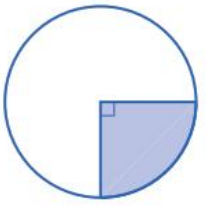
A **secant** is a line that cuts the circle at 2 points.



A **chord** is an interval joining 2 points on the circle.

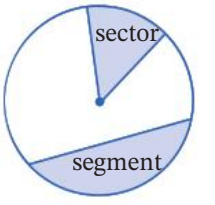
A **diameter** is a chord that passes through the centre.

An **arc** is a part of the circumference of the circle.



Quadrant

A **quadrant** is quarter of a circle.



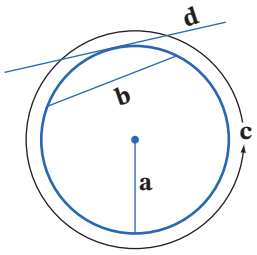
A **sector** is part of a circle bounded by 2 radii and an arc.

A **segment** is part of a circle cut off by a chord.

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EXERCISE 16.01 ANSWERS ON P. 652

Parts of a circle U F C

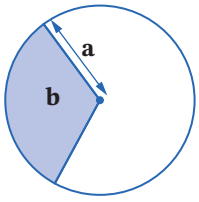


1 Name each part of the circle marked by a letter. C

2 **a** Draw a sector and a segment in a circle.

b What is the difference between a sector and a segment? C

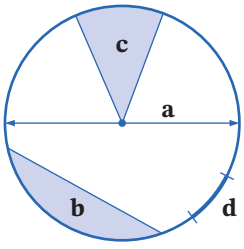
3 State the mathematical rule connecting d (diameter) and r (radius) as an algebraic formula. C



4 Which parts of a circle are labelled **a** and **b**? Select the correct answer **A**, **B**, **C** or **D**. C

A diameter and segment **B** radius and segment

C diameter and sector **D** radius and sector



5 Name each part of the circle marked by a letter. C

6 Write the correct word to match each description. C

a The distance from the centre of a circle to its side

b Quarter of a circle

c A line that touches the outside of a circle once

d An interval from one side of a circle to the other side, through the circle's centre

e An interval from one side of the circle to the other side, not through the centre

f Part of the circumference of a circle

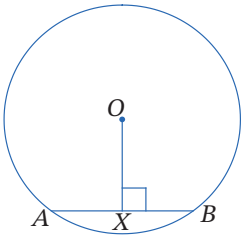
g The area inside a circle formed by 2 radii and an arc

h The complete distance around a circle

i The area inside a circle formed by a chord and an arc

Technology

Perpendicular to a chord



1 Use dynamic geometry software to draw a circle with a centre, O.

2 Mark 2 points A and B on the circumference of the circle.

3 Join the 2 points to make a chord AB.

4 Select the centre of the circle and the chord.

5 Draw a **perpendicular line** from the centre to the chord, and make the point of intersection X.

6 Measure AX and BX. What do you notice?

7 Move points B and C to draw different chords. Measure AX and BX again. What do you notice again?

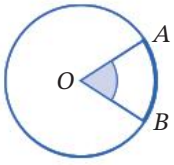
Foundation | Standard | Complex

16.02 Chord properties of circles

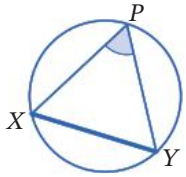
STAGE 5.3

WS
A page of circles

When an arc or chord of a circle **subtends** an angle, it sits opposite or under the angle and 'holds' the angle's arms. The angle stands on the ends of the arc or chord. The angle is **subtended by** the arc or chord.



The arc AB subtends the angle at O.



The chord XY subtends the angle at P.

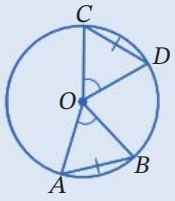
A **theorem** is a rule or statement that can be proved from basic principles. We will now look at circle theorems relating to chords that can be proved by congruent triangles. These theorems can also be demonstrated using dynamic geometry software or pencil-and-paper constructions.

Equal chords subtend equal angles

Theorem 1: Chords of equal length in a circle subtend equal angles at the centre of the circle.

If $AB = CD$,

$\angle AOB = \angle COD$.



Proof:

In $\triangle AOB$ and $\triangle COD$:

$AB = CD$	(given)
$OA = OC$	(equal radii)
$OB = OD$	(equal radii)
$\therefore \triangle AOB \equiv \triangle COD$	(SSS)
$\therefore \angle AOB = \angle COD$	(matching angles of congruent triangles)

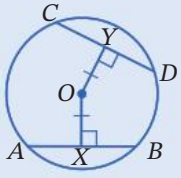
So chords of equal length subtend equal angles at the centre of a circle.

Equal chords equidistant from centre

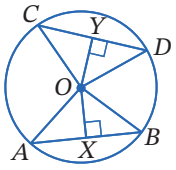
Theorem 2: Chords of equal length in a circle are equidistant from the centre of the circle.

If $AB = CD$,

then $OX = OY$.



Proof:



Draw radii OA, OB, OC and OD.

In $\triangle AOX$ and $\triangle COY$:

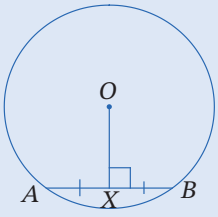
$OA = OC$	(equal radii)
$\angle A = \angle C$	(matching angles of congruent $\triangle AOB$ and $\triangle COD$ proved in Theorem 1)
$\angle OXA = \angle OYC = 90°$	(given)
$\therefore \triangle AOX \equiv \triangle COY$	(AAS)
$\therefore OX = OY$	(matching sides of congruent triangles)

So equal chords are equidistant from the centre of a circle.

Perpendicular from centre to chord

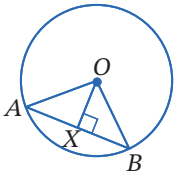
Theorem 3: The perpendicular from the centre of a circle to a chord bisects the chord.

The **converse** of this is also true: The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.



The word 'converse' means to 'turn around' or 'reverse'. So the converse of a theorem is the theorem written 'back-to-front'.

Proof:



Draw radii OA and OB.

In $\triangle AOX$ and $\triangle BOX$:

$OA = OB$ (equal radii)

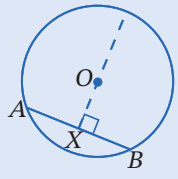
OX is common.

$\angle OXA = \angle OXB = 90°$	($OX \perp AB$)
$\therefore \triangle AOX \equiv \triangle BOX$	(RHS)
$\therefore AX = BX$	(matching sides of congruent triangles)

So the perpendicular from the centre of a circle to a chord bisects the chord.

Perpendicular bisector of a chord

Theorem 4: The perpendicular bisector of a chord in a circle passes through the centre of the circle.



Proof:

The perpendicular from O to AB bisects AB. (proved in Theorem 3)

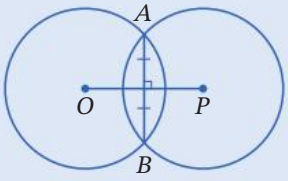
$\therefore$ OX is the perpendicular bisector of AB.

$\therefore$ The perpendicular bisector of AB passes through the centre O.

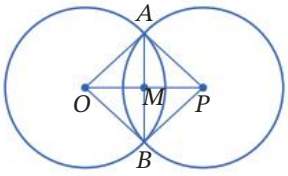
STAGE 5.3

Common chords

Theorem 5: When 2 circles intersect, the line joining their centres bisects their common chord at right angles.



Proof:



Draw radii OA, OB, PA and PB and let OP and AB bisect at M.

In $\triangle OAP$ and $\triangle OBP$:

$OA = OB$ (equal radii)

$PA = PB$ (equal radii)

OP is common.

$\therefore \triangle OAP \equiv \triangle OBP$ (SSS)

$\therefore \angle AOM = \angle BOM$ (matching angles of congruent triangles)

In $\triangle OAM$ and $\triangle OBM$:

$OA = OB$ (equal radii)

OM is common.

$\therefore \angle AOM = \angle BOM$ (proved above)

$\therefore \triangle OAM \equiv \triangle OBM$ (SAS)

$\therefore AM = MB$ (matching sides of congruent triangles)

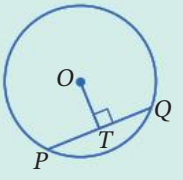
$\therefore AB \perp OP$ (line from the centre of a circle to the midpoint of a chord)

So the perpendicular from the centre of a circle to a chord bisects the chord.

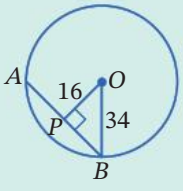
Example 1

In each diagram, O is the centre of a circle.

a If $PQ = 35$ cm, find PT.



b If $OP = 16$ cm, find AB.



Solution

a $PT = \frac{1}{2}(PQ)$ (the perpendicular from the centre bisects the chord)

$= \frac{1}{2} \times 35$

$= 17.5$ cm

b $PB^2 = 34^2 - 16^2$

$= 900$

$PB = \sqrt{900}$

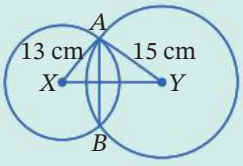
$= 30$ cm

$\therefore AB = 2 \times PB$ (the perpendicular from the centre bisects the chord)

$= 60$ cm

Example 2

The length AB of the common chord of 2 circles is 24 cm. If the radii of the circles are 13 cm and 15 cm, find the distance between their centres.

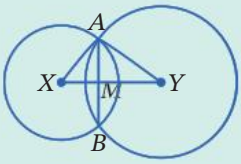


$AB = 24$ cm

Solution

We need to find XY.

Let XY and AB cross at M.



$XY \perp AB$ (common chord of circles)

$AM = \frac{1}{2} \times AB$

$= \frac{1}{2} \times 24$

$= 12$ cm

$XM^2 = 13^2 - 12^2 = 25$ by Pythagoras' theorem

$XM = \sqrt{25} = 5$ cm

$YM^2 = 15^2 - 12^2 = 81$

$YM = \sqrt{81} = 9$ cm

$\therefore XY = 5 + 9 = 14$ cm

The distance between the centres is 14 cm.

EXERCISE 16.02 ANSWERS ON P. 652

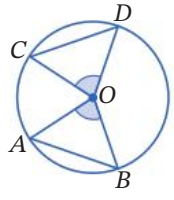
Chord properties of circles U F R C

EXAMPLE 1

1 The converse of Theorem 1 is: 'Chords subtending equal angles at the centre of a circle have equal lengths'. R C

a Prove that $\triangle AOB \equiv \triangle COD$, given that $\angle AOB = \angle COD$.

b Hence, prove that $AB = CD$.



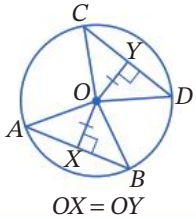
$\angle AOB = \angle COD$

2 The converse of Theorem 2 is: 'Chords that are equidistant from the centre of a circle are equal'. R C

a Prove that $\triangle OXA \equiv \triangle OYC$ and therefore $AX = CY$.

b Prove that $\triangle OXB \equiv \triangle OYD$ and therefore $BX = DY$.

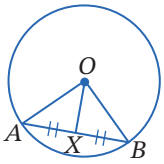
c Hence show that $AB = CD$.



$OX = OY$

Foundation Standard Complex

3 The converse of Theorem 3 is: 'The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.' R C



$AX = BX$

a Prove that $\triangle OXA \equiv \triangle OXB$ and therefore $\angle OXA = \angle OXB$.

b Prove that $OX \perp AB$.

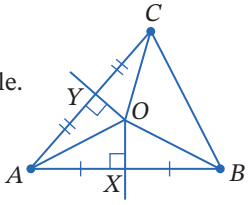
4 **a** Draw a circle and construct any 2 non-parallel chords. R C

b Bisect these chords.

c Through what point must each perpendicular bisector pass? (Use Theorem 4.)

d Explain why the point of intersection between the 2 perpendicular bisectors is the centre of the circle.

5 The diagram shows the perpendicular bisectors of 2 sides AB and CA of a triangle, intersecting at O. It can be proved that O is the centre of a circle passing through all 3 vertices of the triangle.



The full statement on this theorem is: 'Given any 3 non-collinear* points, the point of intersection of the perpendicular bisectors of any 2 sides of the triangle formed by the 3 points is the centre of the circle through all three points.

(*Non-collinear means *not* on the same straight line.)

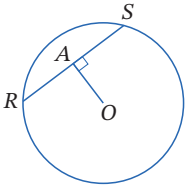
Prove this theorem as follows: R C

a Prove that $\triangle AOY \equiv \triangle COY$ and therefore $OA = OC$

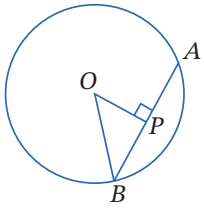
b Prove that $\triangle AOX \equiv \triangle BOX$ and therefore $OA = OB$

c Hence prove that the circle with centre O and radius OA also passes through B and C.

6 This circle has a radius of 7 cm and the chord RS is 8 cm. Find, correct to 2 decimal places, the length of OA. R



7 In the diagram, $AB \perp OP$. If $OB = 32.5$ cm and $OP = 12.5$ cm, find AB. R

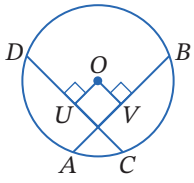


8 In each problem, state which chord theorem you use. R C

a $OU = OV$

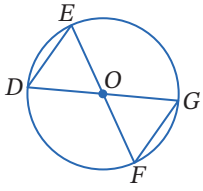
$AB = 9$ m

Find the length of UC.



b $GF = 12$ m

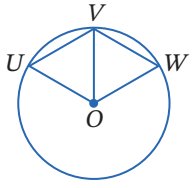
Find the length of DE.



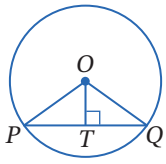
c $UV = VW$

$\angle VOW = 64°$

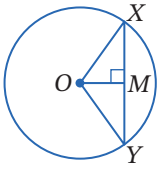
Find the size of $\angle UVO$.



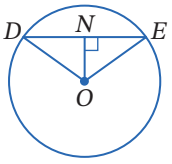
d $OT \perp PQ$
$OQ = 17$ mm
$OT = 8$ mm
Find PQ.



e $OM \perp XY$
$XY = 40$ cm
$OX = 29$ cm
Find OM.



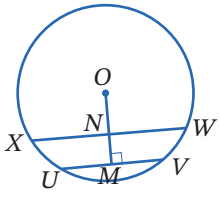
f $DE \perp ON$
$ON = 18$ m
$DE = 36$ m
Find the radius.



9 Find the radius of a circle in which a chord of length 96 cm is 20 cm from the centre. R

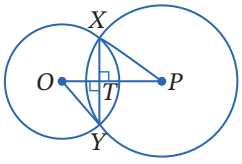
10 On a clear day, you can see 11.5 km in any direction from a particular lookout. If a straight road is 6.9 km from the lookout, what length of the road can be seen from the lookout? R

11 In the diagram, $OM \perp UV$, $XW \parallel UV$, $UV = 24$ cm and $XW = 32$ cm. If the radius of the circle is 20 cm, find the length of MN. R

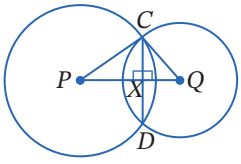


EXAMPLE 2

12 O and P are the centres of circles. If $OY = 40$ cm, $XP = 51$ cm and $XY = 48$ cm, find the distance between the centres. R



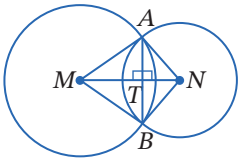
13 P and Q are the centres of circles. If $DC = 32$ cm, $PX = 30$ cm and $XQ = 12$ cm, find the radius of each circle. R



14 M and N are the centres of circles. $MT = 36$ cm, $NT = 20$ cm and $AM = 39$ cm. Find: R

a AB

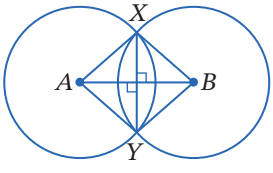
b the area of kite $ANBM$



15 A and B are the centres of circles with the same radius. $XY = 40$ cm and $AB = 96$ cm. Find: R C

a the radius of each circle

b the area of rhombus $AXBY$



16.03 Angle properties of circles

STAGE 5.3

We will now look at proofs of theorems relating to angles in a circle. These theorems can also be demonstrated using dynamic geometry software or pencil-and-paper constructions.

WS A page of circles

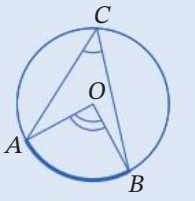
WS Angle problems with algebra

Finding angles in circles

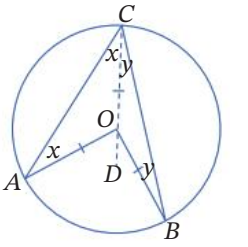
Angles at circumference and centre

Theorem 6: The angle at the centre of a circle is twice the angle at the circumference, standing on the same arc.

$$\angle AOB = 2 \times \angle ACB$$



$\angle AOB$ is the **angle at the centre** and $\angle ACB$ is the **angle at the circumference.** Both angles stand on the same arc AB.



Proof:

Draw radius CO and produce (extend) to point D.

Let $\angle ACD = x$ and $\angle BCD = y$.

$\therefore \angle ACB = x + y$

$OA = OC = OB$ (equal radii)

$\therefore \angle OAC = x$ (equal angles in isosceles $\triangle OAC$)

Similarly, $\angle OBC = y$ (equal angles in isosceles $\triangle OBC$)

$\angle AOD = x + x$ (exterior angle of $\triangle OAC$)

$= 2x$

Similarly, $\angle BOD = y + y$ (exterior angle of $\triangle OBC$)

$= 2y$

$\therefore \angle AOB = 2x + 2y$

$= 2(x + y)$

$= 2 \times \angle ACB$

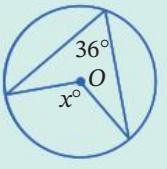
So the angle at the centre of the circle is twice the angle at the circumference, standing on the same arc.

Circle geometry

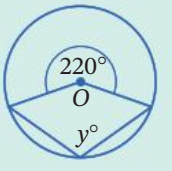
Example 3

Find the value of each variable.

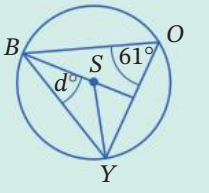
a



b



c



Solution

a $x = 2 \times 36$ (angle at centre is twice angle at circumference)
$= 72$

b $y = \frac{1}{2} \times 220$ (angle at centre is twice angle at circumference)
$= 110$

c $\angle BSY = 2 \times 61°$ (angle at centre is twice angle at circumference)
$= 122°$

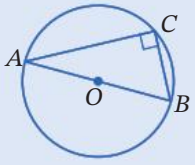
$SB = SY$ (equal radii)

$\therefore \angle SBY = \angle SYB = d$ (equal angles in isosceles $\triangle SYB$)

$2d + 122 = 180$ (angle sum of $\triangle SYB$)
$2d = 58$
$d = 29$

Angle in a semicircle (Thales' theorem)

Theorem 7: The angle in a semicircle is a right angle.



$\angle ACB = 90°$

Proof:

$\angle AOB = 180°$ (a straight angle)

$\angle ACB = \frac{1}{2}$ of $\angle AOB$ (angle at centre is twice angle at circumference)

$= \frac{1}{2} \times 180°$
$= 90°$

The angle in a semicircle is a right angle.

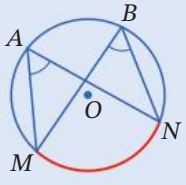
Angles in the same segment

Theorem 8: Angles at the circumference of a circle, standing on the same arc, are equal.

Or put another way, angles in the same segment are equal.

$\angle A = \angle B$

Angles $\angle A$ and $\angle B$ both stand on the same arc MN.

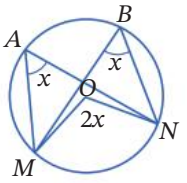


STAGE 5.3

Proof:

Join radii OM and ON.

Let $\angle MON = 2x$



$\therefore \angle A = \frac{1}{2} \times \angle MON$ (angle at the centre is twice the angle at the circumference)

$= \frac{1}{2} \times 2x$

$= x$

Similarly, $\angle B = \frac{1}{2} \times \angle MON$

$= \frac{1}{2} \times 2x$

$= x$

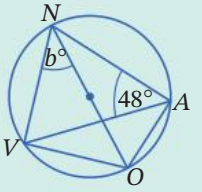
$\therefore \angle A = \angle B$

So angles standing on the same arc are equal.

Circle geometry

Example 4

Find b, giving reasons.



Solution

$\angle NAO = 90°$ (angle in a semicircle)

$\therefore \angle VAO = 90° - 48°$

$= 42°$

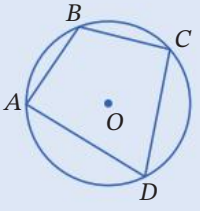
$\therefore b = 42$ (angles standing on the same arc)

A **cyclic quadrilateral** is a quadrilateral whose 4 vertices lie on the circumference of a circle.



Cyclic quadrilaterals

Theorem 9: The opposite angles in a cyclic quadrilateral are supplementary.



$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$

9780170453509

Proof:

Draw radii OA and OC.

Let $\angle B = x°$ and $\angle D = y°$.

$\therefore$ Reflex $\angle AOC = 2x°$ (angle at centre is twice angle at the circumference)

$\therefore$ Obtuse $\angle AOC = 2y°$ (angle at centre is twice angle at the circumference)

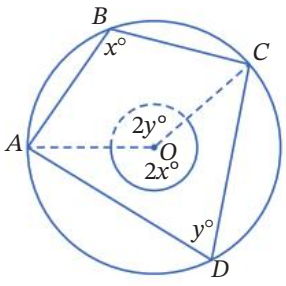
$2x + 2y = 360$ (angles at a point)

$x + y = 180$

$\therefore \angle B + \angle D = 180°$

Hence, $\angle A + \angle C = 180°$ (angle sum of a quadrilateral)

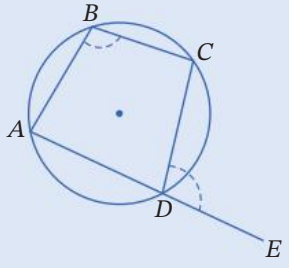
So the opposite angles of a cyclic quadrilateral are supplementary.



Exterior angle of a cyclic quadrilateral

Theorem 10: The exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle.

$$\angle CDE = \angle B$$



Proof:

Let $\angle B = x°$.

$\angle ADC = 180° - x°$ (opposite angles of a cyclic quadrilateral)

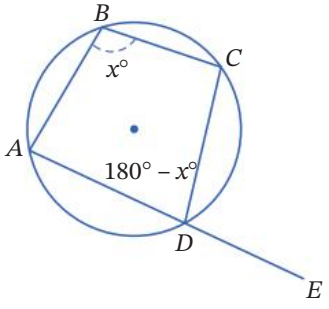
$\angle CDE = 180° - (180° - x°)$ (angles on a straight line)

$= 180 - 180 + x°$

$= x°$

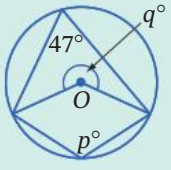
$\therefore \angle ABC = \angle CDE = x°$

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



Example 5

Find p and q, giving reasons.



Solution

$p = 180 - 47$ (opposite angles of a cyclic quadrilateral)

$= 133$

$q = 2 \times 133$ (angle at centre is twice angle at circumference)

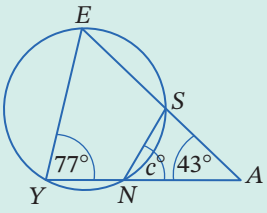
$= 266$

STAGE 5.3

Circle geometry

Example 6

Find c, giving reasons.



Solution

$\angle ASN = 77°$ (exterior angle of cyclic quadrilateral $YESN$)

$c = 180 - 43 - 77$ (angle sum of $\triangle SAN$)

$= 60$

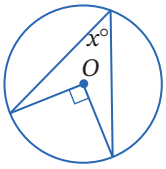
EXERCISE 16.03 ANSWERS ON P. 653

Angle properties of circles U F R C

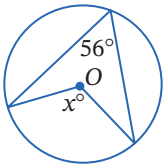
EXAMPLE 3

1 Find x, giving reasons, if O is the centre of the circle. R C

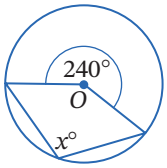
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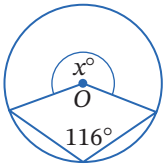
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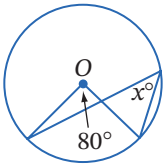
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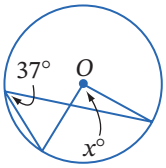
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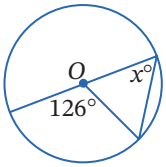
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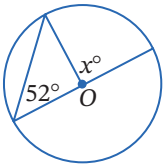
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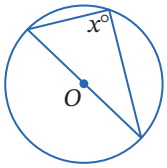
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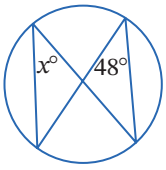
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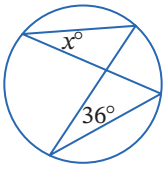
EXAMPLE 4

2 Find x, giving reasons. R C

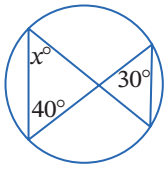
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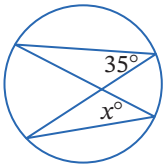
b



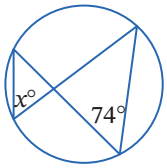
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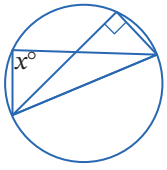
d



e



f



Foundation Standard Complex

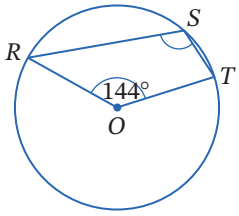
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STAGE 5.3

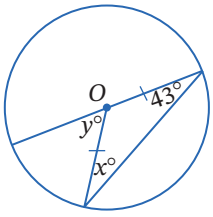
16.03

3 Find the value of each variable, giving reasons. R C

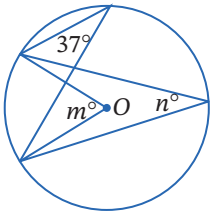
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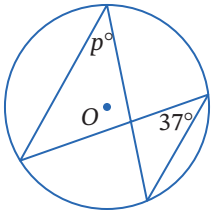
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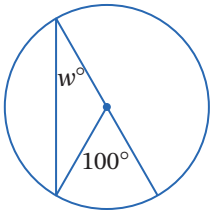
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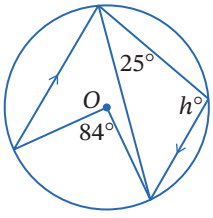
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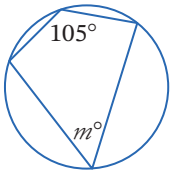
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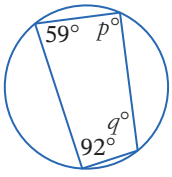
4 Find the value of each variable, giving reasons. R C

EXAMPLE 5

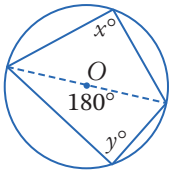
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b



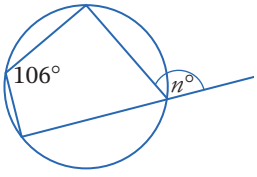
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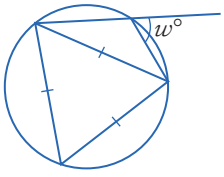
5 Find the value of each variable, giving reasons. R C

EXAMPLE 6

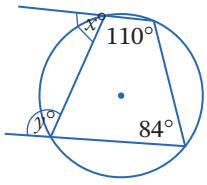
a



b

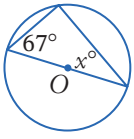


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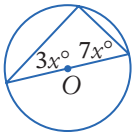


6 Find x, giving reasons. R C

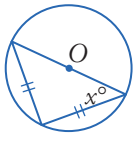
a



b



c

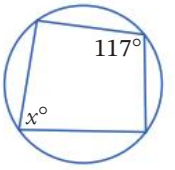


Foundation | Standard | Complex

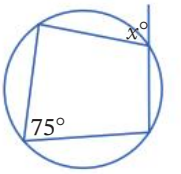
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STAGE 5.3

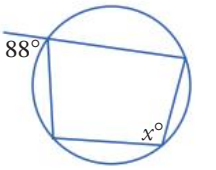
d



e

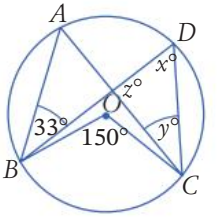


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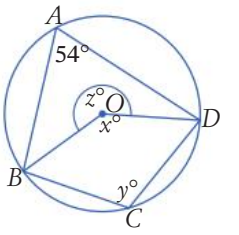


7 Find the value of each variable, giving reasons. R C

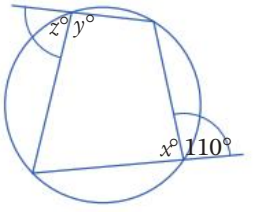
a



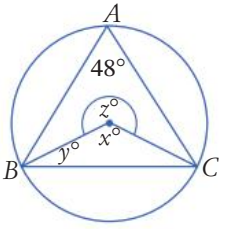
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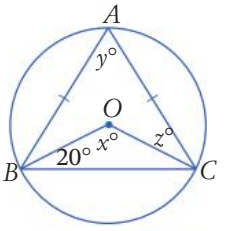
c



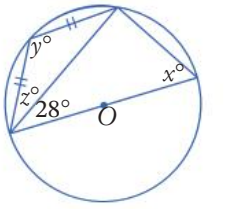
d



e

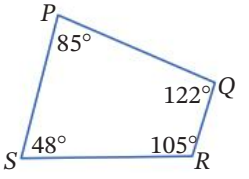


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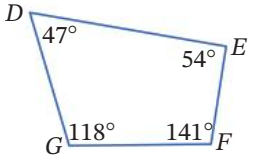


8 Which of these is a cyclic quadrilateral? Why? R C

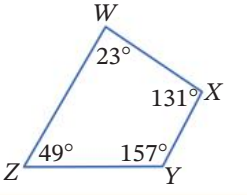
A



B



C



16.04 Tangent and secant properties of circles#

#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

WS Circle geometry cards

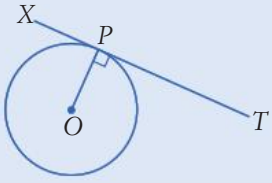
A **tangent** to a circle is a line that touches the circle at only one point. The point of intersection is called the **point of contact**.

At any point on a circle, there is only one tangent to the circle at that point.

Tangent theorems

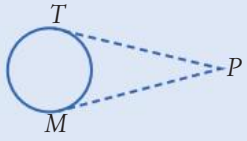
Theorem 11: A tangent to a circle is perpendicular to the radius drawn to the point of contact.

$\angle OPT = 90°$

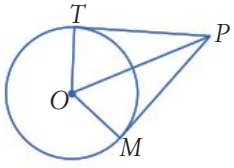


Theorem 12: The 2 tangents drawn to a circle from an external point are equal in length.

$PT = PM.$



Proof of Theorem 12:



Draw radii OT and OM, and join OP.

In $\triangle PTO$ and $\triangle PMO$:

OP is common.

$OT = OM$ (equal radii)

$\angle OTP = \angle OMP = 90°$ (angle between tangent and radius)

$\therefore \triangle PTO \equiv \triangle PMO$ (RHS)

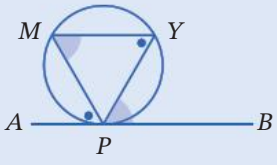
$\therefore PT = PM$ (matching sides of congruent triangles)

So tangents drawn from an external point are equal in length.

Alternate segment theorem

Theorem 13: The angle between a tangent and a chord drawn to the point of contact is equal to the angle in the **alternate segment.**

$\angle YPB = \angle M$



(To be proved in question **1** of Exercise 16.04)

AB is a tangent to the circle. The chord PY divides the circle into 2 segments.

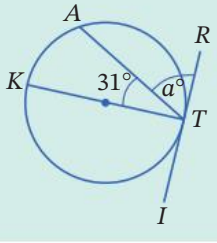
$\angle M$ is in the alternate (opposite) segment to $\angle YPB$.

Similarly, $\angle Y$ is in the alternate segment to $\angle APM$, so $\angle APM = \angle Y$.

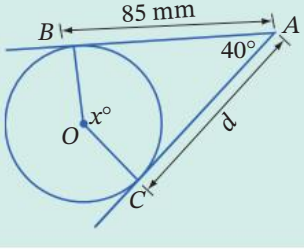
Example 7

Find the value of each variable, giving reasons.

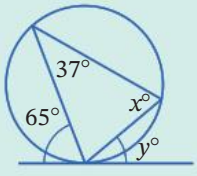
a



b



c



STAGE 5.3

Solution

a $\angle KTR = 90°$ (angle between a tangent and a radius)

$a + 31 = 90$

$a = 59$

b $d = 85$ (tangents from an external point)

$\angle ABO = \angle ACO = 90°$ (angle between a tangent and a radius)

$\therefore x + 90 + 40 + 90 = 360$ (angle sum of a quadrilateral)

$x + 220 = 360$

$x = 140$

c $x = 65$ (angle in alternate segment)

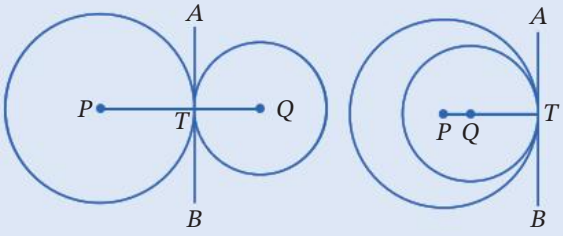
$y = 37$ (angle in alternate segment)

Touching circles

Theorem 14: When 2 circles touch, their centres and the point of contact are **collinear**.

The circles have a common tangent AB.

P, T and Q lie on a **straight** line (to be proved in question **2** of Exercise 16.04).

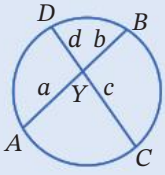


When a line crosses another line or curve at 2 points, the distance between the points is called an **intercept**.

Intercepts of intersecting chords

Theorem 15: The products of the intercepts of 2 intersecting chords are equal.

$$ab = cd$$



a, b, c and d are called **intercepts**.

Proof:

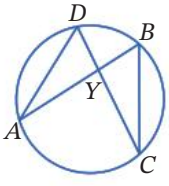
Draw AD and CB.

In $\triangle ADY$ and $\triangle CBY$:

$\angle D = \angle B$ (angles in the same segment)

$\angle A = \angle C$ (angles in the same segment)

$\therefore \triangle ADY \,|||\, \triangle CBY$ (equiangular)



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$\therefore \frac{AY}{CY} = \frac{YD}{YB}$ (matching sides of similar triangles)

$\therefore AY \times YB = CY \times YD$

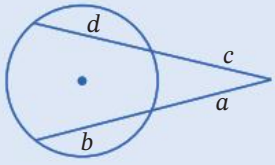
$\therefore ab = cd$

Therefore, the products of intercepts of 2 intersecting chords are equal.

Intersecting secants and the tangent-secant theorem

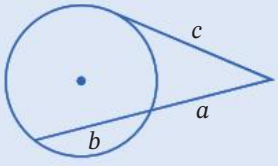
Theorem 16: The product of the intercepts of 2 intersecting secants to a circle from an external point are equal.

$$a(a + b) = c(c + d)$$



Theorem 17: The square of a tangent to a circle from an external point equals the product of the intercepts of any secant from the point.

$$c^2 = a(a + b)$$

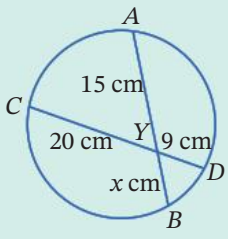


(To be proved in question **3** of Exercise 16.04.)

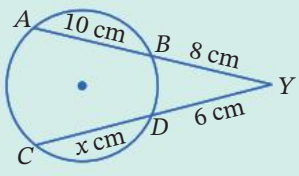
Example 8

Find x, giving reasons.

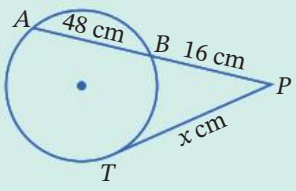
a



b



c



Solution

a $x \times 15 = 9 \times 20$ (intercepts of intersecting chords)

$15x = 180$

$x = 12$

b $6 \times (6 + x) = 8 \times 18$ (intercepts of intersecting secants)

$36 + 6x = 144$

$6x = 108$

$x = 18$

c $x^2 = 16 \times (16 + 48)$ (tangent-secant theorem)

$= 1024$

$x = \sqrt{1024}$

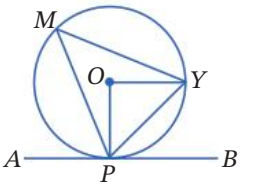
$= 32$

STAGE 5.3

EXERCISE 16.04 ANSWERS ON P. 653

Tangent and secant properties of circles U F R C

1 AB is a tangent to a circle at P, with centre O. R C



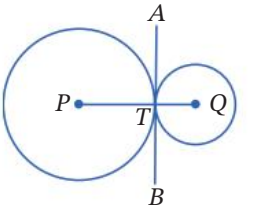
a If $\angle YPB$ is an angle between a tangent and chord, what is the angle in the alternate segment?

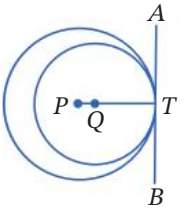
b If $\angle YPB = x°$, why is $\angle OPY = 90° - x°$?

c Show that $\angle POY = 2x°$.

d Hence show that $\angle PMY = \angle YPB$.

2 2 circles with centres P and Q touch at T with a common tangent AB. R C

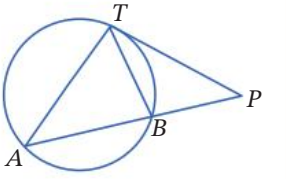




a Explain why $\angle ATP$ and $\angle ATQ$ are right angles.

b Hence, show that P, T and Q are on the same line.

3 PT is a tangent and PA is a secant. R C



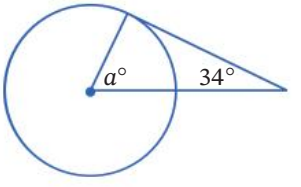
a Prove that $\triangle PTA \,|||\, \triangle PBT$.

b Hence, show that $\frac{PT}{PB} = \frac{PA}{PT}$ and $PT^2 = PB \times PA$.

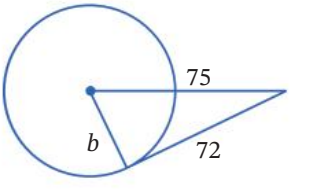
EXAMPLE 7

4 Find the value of each variable, giving reasons. R C

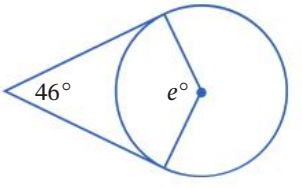
a



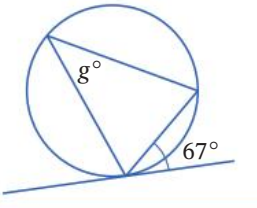
b



c



d



Foundation Standard Complex

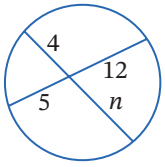
9780170453509

5 Find the value of n. R C

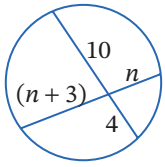
STAGE 5.3

EXAMPLE 8

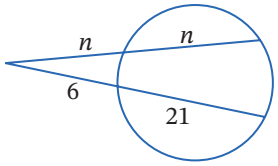
a



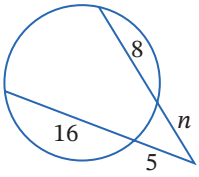
b



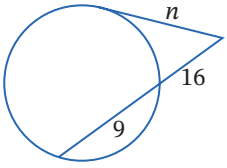
c



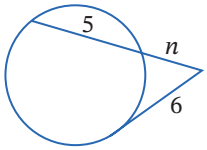
d



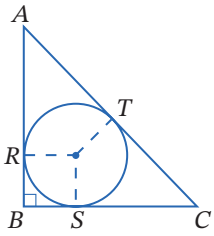
e



f

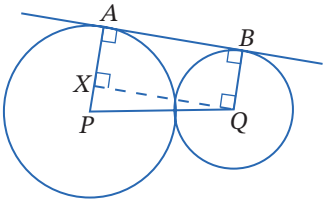


6 ABC is a right-angled triangle with an internal circle such that each side is a tangent to the circle. R, S and T are the respective points of contact of each of the tangents. If $SC = 30$ cm, $RB = 5$ cm and $AT = x$, find the value of x. R C



7 2 circles with centres P and Q and radii 18 cm and 8 cm respectively, touch each other externally as shown. Find the length of: R C

a XP **b** AB



16.04

Proofs using circle theorems#

16.05

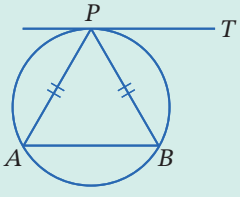
#NSW ONLY, NOT AUSTRALIAN CURRICULUM

STAGE 5.3

Example 9

PT is a tangent to a circle, and $PA = PB$.

Prove that $PT \parallel AB$.



Solution

$\angle TPB = \angle A$ (angle in alternate segment)

$\angle A = \angle B$ (equal angles of isosceles $\triangle PAB$)

$\therefore \angle TPB = \angle B$

$\therefore PT \parallel AB$ (alternate angles are equal)

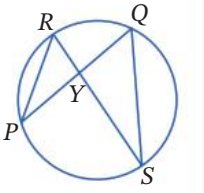
Foundation Standard Complex

EXERCISE 16.05 ANSWERS ON P. 654

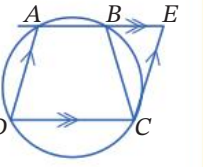
Proofs using circle theorems U F R C

EXAMPLE 9

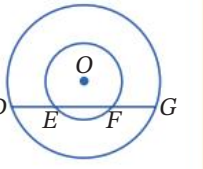
1 PQ and RS are 2 chords of a circle intersecting at the point Y. Prove that the triangles PYR and SYQ are similar and hence $PY \times YQ = RY \times YS$. R C



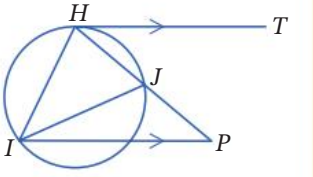
2 $ABCD$ is a cyclic quadrilateral. Side AB is produced to E so that $AECD$ is a parallelogram. Prove that $\triangle CBE$ is an isosceles triangle. R C



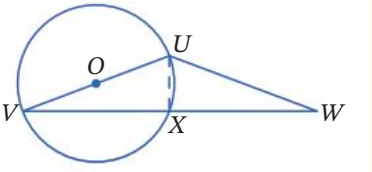
3 O is the centre of 2 concentric circles. $DEFG$ is a straight line. Prove that $DE = FG$. R C



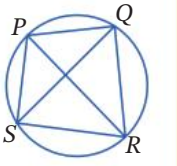
4 HT is a tangent and parallel to IP. HJP is a straight line. Prove that $\angle HIP = \angle HJI$. R C



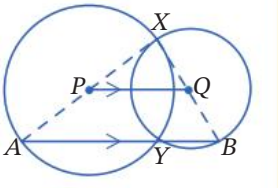
5 UVW is an isosceles triangle, where $UV = UW$. A circle with centre O is drawn on one of the equal sides. Prove that the circle bisects the base of the triangle. R C



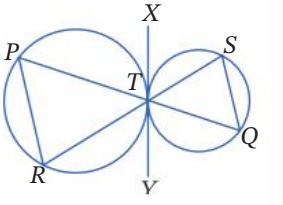
6 $PQRS$ is a cyclic quadrilateral. PR bisects $\angle QRS$. Prove that $\triangle SPQ$ is isosceles. R C



7 2 circles with centres P and Q intersect at X and Y. AB is parallel to PQ, the interval joining the centres of the circles. Prove that $PQ = \frac{1}{2}AB$. R C



8 2 circles touch externally at T. XY is the common tangent. PTQ and RTS are straight lines. Prove that $PR \parallel SQ$. R C



Foundation Standard Complex

9780170453509

ANSWERS

Chapter 1

SkillCheck

1 81, 25, 100, 16, 64

4 **a** $m^2 + 10m + 21$ **b** $y^2 - 3y - 4$
c $n^2 - 5n + 6$ **d** $6d^2 + 11d + 3$
e $4 - 17p - 15p^2$ **f** $3a^2 + 17af + 10f^2$
g $x^2 + 8x + 16$ **h** $y^2 - 6y + 9$
i $4k^2 + 4k + 1$ **j** $a^2 - 25$
k $t^2 - 49$ **l** $9m^2 - 16$

Exercise 1.01

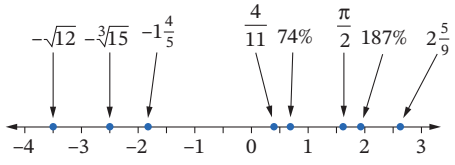
1 C **2** B

3 $\sqrt{32}, \sqrt{33}, \sqrt{4.9}, \sqrt{52}, \sqrt{200}$

4 **a** R **b** I **c** R **d** R
e R **f** R **g** R **h** R
i I **j** R **k** I **l** I

5 **a** $1\frac{4}{7}, \frac{\pi}{2}, \sqrt{2}$ **b** $2\frac{7}{9}, \sqrt[3]{20}, 2.\dot{6}$

6 **a** -1.8 **b** 0.7 **c** 0.4 **d** -3.5
e -2.5 **f** 2.6 **g** 1.6 **h** 1.9



7 1.4

8 Teacher to check, $\sqrt{5} \approx 2.24$, $\sqrt{10} \approx 3.16$, $\sqrt{17} \approx 4.12$

Exercise 1.02

1 **a** 2 **b** 5 **c** 27 **d** 250
e 0.09 **f** 28 **g** 45 **h** 50

2 **a** $5\sqrt{2}$ **b** $3\sqrt{3}$ **c** $2\sqrt{6}$ **d** $3\sqrt{6}$
e $9\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $10\sqrt{2}$
i $4\sqrt{6}$ **j** $3\sqrt{7}$ **k** $12\sqrt{2}$ **l** $6\sqrt{3}$
m $5\sqrt{3}$ **n** $7\sqrt{3}$ **o** $4\sqrt{2}$ **p** $11\sqrt{2}$
q $9\sqrt{2}$ **r** $7\sqrt{5}$ **s** $5\sqrt{5}$ **t** $16\sqrt{2}$

3 **a** $25\sqrt{2}$ **b** $6\sqrt{2}$ **c** $12\sqrt{3}$ **d** $56\sqrt{2}$
e $\sqrt{10}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{7}}{3}$ **h** $6\sqrt{6}$
i $18\sqrt{17}$ **j** $\frac{5\sqrt{5}}{2}$ **k** $3\sqrt{2}$ **l** $3\sqrt{3}$
m $40\sqrt{10}$ **n** $15\sqrt{3}$ **o** $14\sqrt{17}$ **p** $\frac{\sqrt{13}}{3}$

4 B **5** B

6 **a** false **b** false **c** true
d true **e** true **f** false

Exercise 1.03

1 **a** $7\sqrt{7}$ **b** $-5\sqrt{2}$ **c** $6\sqrt{5}$ **d** $4\sqrt{5}$
e 0 **f** $\sqrt{10}$ **g** $8\sqrt{15}$ **h** $-\sqrt{6}$
i $2\sqrt{3}$ **j** $10\sqrt{5}$ **k** $6\sqrt{10}$ **l** $-5\sqrt{3}$

2 **a** $5\sqrt{5} - 8$ **b** $13\sqrt{10} + 3\sqrt{2}$ **c** $5\sqrt{2} - 9\sqrt{3}$
d $7\sqrt{15} + 8\sqrt{2}$ **e** $-2\sqrt{5} - 3\sqrt{7}$ **f** $2\sqrt{6} - 8\sqrt{3}$
g $13\sqrt{11} - \sqrt{3}$ **h** $11\sqrt{7} - 6\sqrt{13}$ **i** $-6\sqrt{7}$
j $-3\sqrt{5}$

3 **a** D **b** A

4 **a** $6\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-2\sqrt{5}$
d $-\sqrt{7}$ **e** $5\sqrt{6}$ **f** $7\sqrt{5}$
g $-\sqrt{10}$ **h** $8\sqrt{11}$ **i** $6\sqrt{2}$
j $8\sqrt{3}$ **k** $3\sqrt{2}$ **l** $9\sqrt{2}$
m $11\sqrt{3}$ **n** $-\sqrt{5}$ **o** $-6\sqrt{3}$
p $30\sqrt{3}$ **q** $5\sqrt{7}$ **r** $41\sqrt{2}$
s $5\sqrt{6}$ **t** $29\sqrt{2}$ **u** $-15\sqrt{3}$
v 0 **w** $6\sqrt{2} + 2\sqrt{3}$ **x** $12\sqrt{3} + 3\sqrt{6}$
y $3\sqrt{2} - 6\sqrt{5}$ **z** $4\sqrt{6}$

Exercise 1.04

1 **a** $\sqrt{14}$ **b** $-\sqrt{35}$ **c** $4\sqrt{3}$ **d** 6
e $-5\sqrt{2}$ **f** 45 **g** $15\sqrt{30}$ **h** $-10\sqrt{21}$
i 140 **j** $-30\sqrt{2}$ **k** 36 **l** $-60\sqrt{2}$
m -112 **n** $24\sqrt{6}$ **o** 80 **p** $90\sqrt{6}$
q -396 **r** $160\sqrt{5}$ **s** $216\sqrt{2}$ **t** $-96\sqrt{6}$
u $36\sqrt{5}$ **v** $-60\sqrt{10}$ **w** $252\sqrt{3}$ **x** 144

2 **a** $\sqrt{5}$ **b** $-\sqrt{3}$ **c** $3\sqrt{6}$ **d** $2\sqrt{2}$
e $-\frac{\sqrt{7}}{2}$ **f** 21 **g** 1 **h** 8
i $5\sqrt{3}$ **j** $5\sqrt{2}$ **k** $4\sqrt{3}$ **l** $2\sqrt{6}$
m $2\sqrt{14}$ **n** $-\frac{\sqrt{2}}{4}$ **o** 1 **p** 10
q 4 **r** $-21\sqrt{2}$ **s** 12 **t** 2
u $\frac{2}{3}$

3 **a** 6 **b** 7 **c** 6
d $15y$ **e** x **f** $a\sqrt{a}$

4 C **5** A

6 **a** 2 **b** $4\sqrt{6}$ **c** $\sqrt{30}$
d $\frac{2}{45}$ **e** $14\sqrt{3}$ **f** 2

Mental skills 1

2 **a** 11 **b** 40 **c** 7 **d** 24 **e** 23
f 6 **g** 43 **h** 80 **i** 18 **j** 15
k 40 **l** 65 **m** 11 **n** 14 **o** 12
p 135

Exercise 1.05

1 **a** $\sqrt{15}+\sqrt{10}$ **b** $2\sqrt{3}-\sqrt{6}$ **c** $\sqrt{6}+\sqrt{14}$
d $3\sqrt{10}-5$ **e** $6+6\sqrt{6}$ **f** $\sqrt{55}-4\sqrt{11}$
g $42-8\sqrt{7}$ **h** $5\sqrt{5}+75$ **i** $24+3\sqrt{6}$

2 C

3 **a** $10+\sqrt{10}-6\sqrt{5}-3\sqrt{2}$ **b** $7+2\sqrt{7}-\sqrt{21}-2\sqrt{3}$
c $28\sqrt{6}+21+8\sqrt{2}+2\sqrt{3}$ **d** $20+\sqrt{10}$
e $109+10\sqrt{77}$ **f** $72-23\sqrt{6}$
g $16\sqrt{10}+54$ **h** $-16-\sqrt{35}$

4 C

5 **a** $8-2\sqrt{15}$ **b** $9+2\sqrt{14}$ **c** $9-4\sqrt{5}$
d $19+6\sqrt{10}$ **e** $77+30\sqrt{6}$ **f** $179-20\sqrt{7}$
g $38+12\sqrt{10}$ **h** $23+4\sqrt{15}$

6 **a** 1 **b** 22 **c** 8 **d** 2
e 1 **f** 166 **g** 13 **h** -43

7 C

8 **a** $88-30\sqrt{7}$ **b** $21\sqrt{2}-10$ **c** $5\sqrt{35}+29$
d $73+40\sqrt{3}$ **e** 29 **f** $92-12\sqrt{5}$

Exercise 1.06

1 B

2 **a** $\frac{\sqrt{2}}{2}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{3}$ **d** $\frac{3\sqrt{2}}{2}$
e $\frac{2\sqrt{7}}{7}$ **f** $\frac{\sqrt{2}}{6}$ **g** $\frac{\sqrt{3}}{6}$ **h** $\frac{\sqrt{7}}{28}$
i $\frac{7\sqrt{5}}{15}$ **j** $\frac{\sqrt{10}}{15}$ **k** $\frac{\sqrt{3}}{2}$ **l** $\frac{\sqrt{15}}{4}$

3 A **4** D

5 **a** $\frac{2-\sqrt{2}}{2}$ **b** $\frac{\sqrt{5}-5}{5}$
c $\frac{5\sqrt{2}+\sqrt{6}}{4}$ **d** $\frac{2\sqrt{3}-3\sqrt{2}}{18}$

6 **a** $\frac{2\sqrt{7}+7\sqrt{2}}{14}$ **b** $\frac{\sqrt{10}+5\sqrt{3}}{5}$ **c** $\frac{\sqrt{3}-\sqrt{2}}{2}$

Power plus

1 **a** $x=\sqrt{17}$ **b** $m=5, n=3\sqrt{2}$

2 **a** Yes, because you are multiplying by 1
b $\frac{3-\sqrt{2}}{7}$, yes

3 **a** $5-2\sqrt{3}$ **b** $2+\sqrt{3}$
c $\sqrt{5}-1$ **d** $\sqrt{7}+\sqrt{3}$

4 **a** $2+\sqrt{3}$ **b** $\frac{\sqrt{10}-\sqrt{2}}{4}$
c $\frac{\sqrt{7}+\sqrt{3}}{4}$ **d** $\frac{10-4\sqrt{3}}{13}$

5 $s=\frac{\sqrt{3}D}{3}$ **6** $\frac{\sqrt{2}}{2}$ cm

7 $(2+2\sqrt{3})$ mm **8** $\frac{4}{3}\sqrt{\frac{10\sqrt{3}}{3}}$ cm

Test yourself 1

1 A

2 **a** I **b** R **c** R **d** I
e R **f** R **g** R **h** I

3 **a** $6\sqrt{2}$ **b** $7\sqrt{2}$ **c** $5\sqrt{11}$ **d** $8\sqrt{2}$
e $15\sqrt{6}$ **f** $14\sqrt{7}$ **g** $48\sqrt{2}$ **h** $15\sqrt{5}$
i $28\sqrt{3}$ **j** $\sqrt{6}$ **k** $\frac{50\sqrt{5}}{3}$ **l** $2\sqrt{2}$
m $4\sqrt{11}$ **n** 6 **o** $6\sqrt{2}$

4 **a** $13\sqrt{2}$ **b** $5\sqrt{2}-7\sqrt{5}$ **c** $14\sqrt{2}+17\sqrt{3}$
d $32\sqrt{5}-9\sqrt{7}$ **e** $38\sqrt{2}-24\sqrt{3}$ **f** $8\sqrt{11}$

5 **a** $\sqrt{21}$ **b** $2\sqrt{10}$ **c** $4\sqrt{3}$ **d** $\sqrt{55}$
e $\sqrt{6}$ **f** $\sqrt{14}$ **g** $4\sqrt{6}$ **h** 1
i $\frac{5}{3}$ **j** $\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$ **k** $\frac{2}{3}$ **l** $\frac{7}{5}$

6 **a** $9\sqrt{2}-12$ **b** $\sqrt{10}-10\sqrt{5}$ **c** $7\sqrt{35}-27$
d $23-8\sqrt{7}$ **e** $77+10\sqrt{6}$ **f** 43
g 103 **h** $70+38\sqrt{5}$

7 **a** $\frac{\sqrt{10}}{10}$ **b** $\frac{3\sqrt{2}}{2}$ **c** $\frac{\sqrt{7}}{35}$
d $\frac{\sqrt{3}}{4}$ **e** $\frac{5\sqrt{6}}{6}$ **f** $\frac{2\sqrt{2}+1}{3}$

Chapter 2

SkillCheck

1 **a** \$7350 **b** \$4034.10 **c** \$8737.60
2 **a** \$5962.59 **b** \$33 433.46
c \$18 481.63 **d** \$64 937.10

Exercise 2.01

1 **a** \$874 **b** \$938.80 **c** \$734.40
2 Greta earns more per week by \$27.48.
3 **a** \$3461.86 **b** \$6923.73 **c** \$15 053.33
4 Job 1: \$1104.64; Job 2: \$1160; Job 2 by \$55.36
5 \$1107.40 **6** \$851.18 **7** \$780.86
8 A **9** \$13 312.50 **10** \$1394.40
11 \$2115 **12** 54 **13** \$63.95
14 **a** \$854 **b** \$700 **c** \$956.87 **d** \$625.55
15 **a** \$972.12 **b** \$680.48 **c** \$4568.96

Exercise 2.02

1 **a** $45 697 **b** $6398.53
2 **a** $114 719 **b** $29 943.03
3 **a** $90 904 **b** $21 131.48
4 C **5** $19 635.78 **6** $45 006.10
7 $696.42 **8** $623.52
9 **a** $462 **b** $1699.10 **c** 25.4%
10 **a** $468 **b** $1732.65 **c** 24.5%
11 **a** $2296 **b** $468 **c** $1634.73
12 **a** $2304.91 **b** $470 **c** $1543.71
13 Gross weekly income = $825.30;
Total deductions = $374.10; Net income = $451.20

Exercise 2.03

1 **a** $5040 **b** $1047.15 **c** $102.50
d $263.60 **e** $37.40 **f** $1192.32
2 **a** $87.50 **b** $5925.15 **c** $207 000
d $690 **e** $1404 **f** $723.04
3 A
4 **a** $11 200 **b** $1551.30
c $9392.50 **d** $10 695.31
5 **a** $1440 **b** $7440
6 4.5%
7 **a** $6400 **b** 5.82%
8 5.85% p.a. **9** approx. 2 years
10 30 weeks **11** 137 days **12** C
13 31 months **14** 2.6% p.a.
15 **a** $18.90 **b** $1063.90
16 B

Exercise 2.04

1 **a** Check with your teacher. Investment after 1st year = $24 150; Investment after 2nd year = $25 357.50
b Compound interest = $2357.50
2 **a** $16 153.36 **b** $1153.36
3 **a** $38 459.48 **b** $4359.48
4 **a** $5408, $408
b $30 245.29, $2445.29
c $11 113.20, $1513.20
d $41 905.55, $2405.55
e $19 337.39, $937.39
5 **a** $4791.80 **b** $1642.38
c $308.93 **d** $3913.84
e $6834.42

Mental skills 2

2 **a** 18 **b** $126 **c** 39 **d** $30.30
e $7.50 **f** 10.8 **g** $27 **h** 60
i $240 **j** $3.30 **k** 900 **l** $52.50
4 **a** 10 **b** 166 **c** $50 **d** $22
e 37.5 **f** $5.80 **g** 135 **h** $22.60
6 **a** 500 **b** $20 **c** 4.5 **d** $6.25
e $81 **f** $35 **g** 16.5 **h** 74.5
i $195 **j** $425 **k** $31.50 **l** 290
8 **a** 160 **b** $1.50 **c** 7.5 **d** $32.50
e $67.50 **f** $31.25 **g** 38 **h** 170

Exercise 2.05

1 B
2 **a** **i** $9754.75 **ii** $3254.75
b **i** $13 858.59 **ii** $3858.59
c **i** $12 634.81 **ii** $394.81
d **i** $43 949.46 **ii** $9349.46
e **i** $8427.39 **ii** $427.39
3 D **4** $1 301 018.83
5 B **6** A
7 **a** **i** $11 273.62 **ii** $1273.62
b **i** $52 751.13 **ii** $17 251.13
c **i** $9448.23 **ii** $548.23
d **i** $48 063.26 **ii** $6063.26
e **i** $17 829.56 **ii** $1329.56
f **i** $4990.24 **ii** $90.24
8 C
9 **a** $600 **b** $615 **c** Tegan by $15.
10 **a** D **b** A **c** C **d** B
11 **a** $17 807.64 **b** $124 less
12 **a** **i** $5746.85 **ii** $5793.89
iii $5817.79 **iv** $5833.87
b Monthly, because it earns the most interest.
13 D

Exercise 2.06

1 **a** $175.50 **b** $1579.50 **c** $328.14
d $1907.64 **e** $105.98 **f** $2083.14
2 **a** $1275 **b** $24 225 **c** $10 416.75
d $34 641.75 **e** $577.36 **f** $35 916.75
3 **a** $1379 **b** $2316.72 **c** $217.58
4 **a** $3420 **b** $720 **c** $1500 **d** 48%
5 **a** $2080 **b** $8320 **c** $13 200
d $4880 **e** 14.7%
6 **a** $104.40 **b** $1536 **c** $596.40 **d** 63.5%
7 **a** $1073.40 **b** $273.40 **c** 34.2%
8 **a** $2599 **b** $3576 **c** $677 **d** 10.4%
9 **a** $262.50 **b** 7.4%

Exercise 2.07

1 $933.89
2 **a** $20 429.69 **b** $29 560.31
3 **a** **i** $659.66 **ii** 60.0%
b **i** $17 406.69 **ii** 45.2%
c **i** $5073.42 **ii** 60.0%
d **i** $1024 **ii** 41.0%
e **i** $14 020.37 **ii** 51.0%
f **i** $1073.44 **ii** 37.0%
g **i** $403.03 **ii** 46.3%
h **i** $1782.95 **ii** 68.6%
4 **a** **i** 63% **ii** 25.00% **iii** 6.25%
b By guess-and-check, in approx. 1.5 years.

ANSWERS

5 **a** **i** $10 000 **ii** $8000 **iii** $4096
b 32.8%
6 **a** $17 970 **b** $7520
7 **a** $71 680 **b** $103 320
c Approx 5 years and 8 months.
d 13.4%
8 Yes, it will lose approximately 52% after 7 years.
9 **a** $1800 **b** 5 years **c** $798.67
d Yes, in the 30th year. **e** No

Power plus

1 4 years and 61 days **2** $4444.44
3 $12 838.71 **4** $63 367.50
5 $2276.87 **6** 790 000
7 **a** 18 years. **b** 18 years.
c No. The size of the interest rate and the number of compounding periods determine how quickly the principal takes to double in value.

Test yourself 2

1 $13 045.75 **2** $1349.18
3 **a** $879.45 **b** $1115.40
4 **a** $1052.51 **b** $736.76 **c** $4946.80
5 **a** $67 725 **b** $13 557.63
6 **a** $2400 **b** $245.31 **c** $45.90 **d** $238.19
7 **a** $5360.85 **b** $360.85
8 $36 282.78 **9** $12 107.19
10 $291.98 **11** $13 145.47
12 **a** $487.50 **b** $4387.50 **c** $1908.56
d $6296.06 **e** $174.89 **f** $6783.56
13 **a** $14 756 **b** $10 234 **c** 59.0%

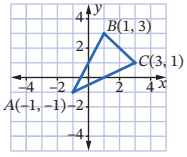
Chapter 3

SkillCheck

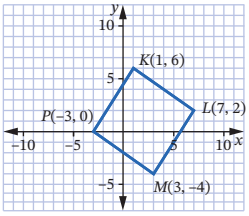
1 **a** (6, 1) **b** (−5, −4)
c 6 **d** 6
e $AC = BC = 4.5$ **f** isosceles
g $\frac{1}{3}$ **h** $-\frac{2}{3}$
2 **a** −2 **b** −8 **c** 5
d 4 **e** $-\frac{1}{4}$ **f** $2\sqrt{17}$

Exercise 3.01

1 B **2** C **3** A
4 **a** $\frac{1}{3}$ **b** −2 **c** $\frac{7}{3}$
5 **a** **i** 2.2 **ii** (6, 2.5) **iii** $-\frac{1}{2}$
b **i** 10.8 **ii** (3.5, 3) **iii** $\frac{2}{3}$
c **i** 7.1 **ii** (−2.5, −0.5) **iii** −1
d **i** 7.6 **ii** (0.5, −7.5) **iii** $-\frac{3}{7}$
e **i** 10.2 **ii** (−6, −3) **iii** −5
f **i** 5.7 **ii** (5, 0) **iii** 1
6 **a** $\sqrt{89}$ **b** $\sqrt{194}$ **c** $\sqrt{82}$
7 k: $m = \frac{1}{5}$; l: $m = -\frac{1}{2}$
8 B
9 **a**



b $AB = AC = \sqrt{20}$ or $2\sqrt{5}$, $BC = \sqrt{8}$ or $2\sqrt{2}$
c $AB = AC = \sqrt{20}$ or $2\sqrt{5}$
d isosceles
e 11.8
10 **a**



b square
c $m_{KL} = -\frac{2}{3}$, $m_{PM} = -\frac{2}{3}$
d $m_{KP} = \frac{3}{2}$, $m_{LM} = \frac{3}{2}$
e the gradients are equal, they are parallel
f $KL = LM = PM = KP = \sqrt{52}$ or $2\sqrt{13}$
g 28.8 **h** 52 sq. units
11 **a** $P(-2, 1)$, $Q(1, 3)$
b $PQ = 3.6$, $AC = 7.2$, $AC = 2 \times PQ$
c $m_{PQ} = \frac{2}{3}$, $m_{AC} = \frac{2}{3}$; the gradients are equal.
12 **a** 72° **b** 27° **c** 45° **d** 68°
e 117° **f** 37° **g** 174° **h** 146°
13 **a** 1.73 **b** −0.40 **c** 0.90 **d** −14.30
e 0.14 **f** −1 **g** −0.05 **h** 0

Exercise 3.02

1 **a** neither **b** perpendicular **c** parallel
d neither **e** parallel **f** neither
2 **a** 4 **b** −2 **c** $\frac{1}{3}$ **d** −0.2
3 **a** −1 **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $-\frac{2}{5}$
4 D **5** B **6** A
7 **a** $m_{AB} = -\frac{4}{3}$, $m_{CD} = -\frac{4}{3}$; $\therefore AB \parallel CD$
b $m_{PQ} = \frac{3}{4}$, $m_{CD} = -\frac{4}{3}$; $\therefore PQ \perp CD$
8 **a** $\frac{1}{3}$ **b** −3

9780170453509

Exercise 3.03

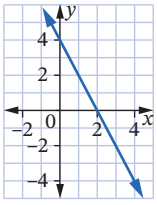
1 Teacher to check graphs.

a i $\frac{1}{3}$ ii -1 **b** i -2.5 ii 5

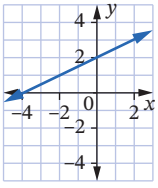
c i 4 ii 4 **d** i -1 ii -2

e i 0 ii 0 **f** i -6 ii 3

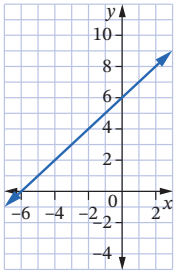
2 **a** $2, 4$



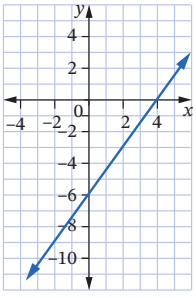
b $-4, 2$



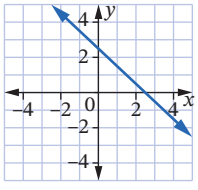
c $-6, 6$



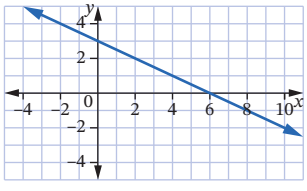
d $4, -6$



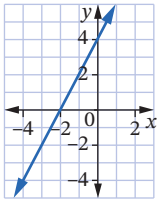
e $2.5, 2.5$



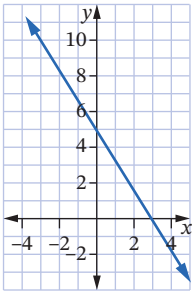
f $6, 3$



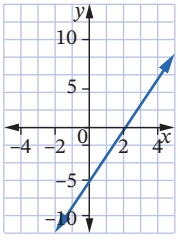
g $-2, 4$



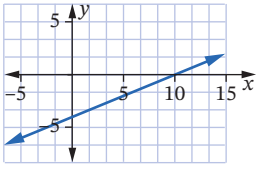
h $3, 5$



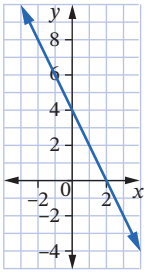
i $2, -6$



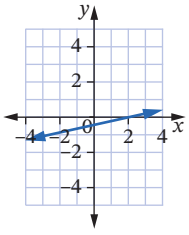
j $10, -4$



k $2, 4$



l $2, -\frac{1}{2}$

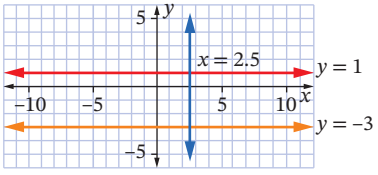


3 **a** no **b** yes **c** yes
d yes **e** no **f** no

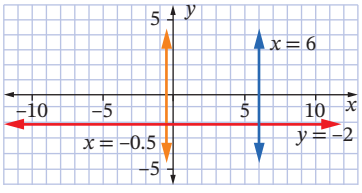
4 C

5 **a** $x = -4$ **b** $x = 1$ **c** $y = 5$ **d** $y = -3$

6 **a**



b



7 **a** $y = 2$ **b** $x = 4$ **c** $x = -1$ **d** $y = -2$
e $y = 3$ **f** $x = -1$ **g** $y = 6$ **h** $x = -1$

8 A **9** C

10 **a** x-axis **b** y-axis

Exercise 3.04

1 **a** $m = 3, c = -2$ **b** $m = -2, c = 7$
c $m = 1, c = 4$ **d** $m = -1, c = 9$
e $m = \frac{3}{4}, c = 6$ **f** $m = 1, c = 0$

ANSWERS

g $m = \frac{1}{2}, c = -11$ **h** $m = \frac{2}{3}, c = 6$

i $m = -\frac{1}{3}, c = -8$ **j** $m = 2, c = -6$

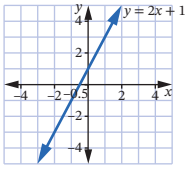
k $m = -3, c = 11$ **l** $m = 1, c = -\frac{7}{2}$

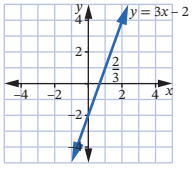
2 **a** $y = 2x + 1$ **b** $y = \frac{3}{4}x + 2$

c $y = -7x + 5$ **d** $y = -\frac{2}{5}x + 3$

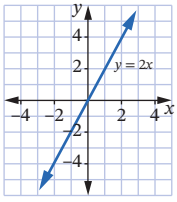
e $y = -2x - 3$ **f** $y = -3x + \frac{1}{2}$

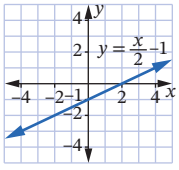
3 **a** $m = 2, c = 1$ **b** $m = 3, c = -2$



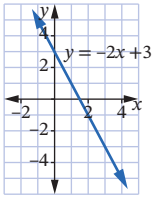


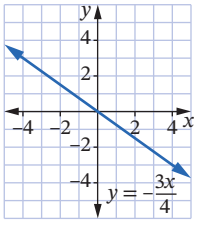
c $m = 2, c = 0$ **d** $m = \frac{1}{2}, c = -1$



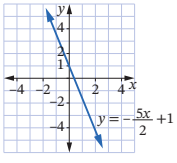


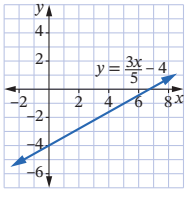
e $m = -2, c = 3$ **f** $m = -\frac{3}{4}, c = 0$





g $m = -\frac{5}{2}, c = 1$ **h** $m = \frac{3}{5}, c = -4$





4 $y = 2x$

5 **a** C **b** B **c** D **d** A

6 **a** C **b** B, D **c** B

d C, D **e** A, B **f** D

7 **a** $y = 4x + 3, y = 4x - 6$

b $3x - y + 7 = 0, y = 3x - 2$

Mental skills 3

2 **a** 8 hours 30 min **b** 5 hours 40 min

c 3 hours 25 min **d** 8 hours 15 min

e 11 hours 25 min **f** 1 hour 40 min

g 5 hours 10 min **h** 5 hours 45 min

i 7 hours 55 min **j** 7 hours 40 min

Exercise 3.05

1 **a** $x - y + 2 = 0$ **b** $3x - y - 1 = 0$

c $5x - y + 8 = 0$ **d** $x + 2y - 3 = 0$

e $x - 2y - 6 = 0$ **f** $8x - y + 2 = 0$

g $6x - y - 3 = 0$ **h** $x - 2y - 6 = 0$

i $3x - 5y + 10 = 0$

2 **a** $m = -2, c = 6$ **b** $m = 4, c = -5$

c $m = \frac{3}{2}, c = 2$ **d** $m = -2, c = 1$

e $m = -2, c = -5$ **f** $m = -\frac{4}{3}, c = 4$

3 B **4** B

5 **a** $4x - 12y + 3 = 0$ **b** $5x + 8y - 12 = 0$

c $15x - 20y - 8 = 0$ **d** $2x + 4y + 3 = 0$

e $15x - 10y + 4 = 0$ **f** $27x + 30y + 14 = 0$

Exercise 3.06

1 **a** $2x - y + 1 = 0$ **b** $x + y + 2 = 0$

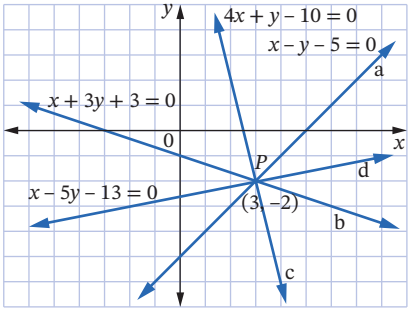
c $4x - y - 20 = 0$ **d** $2x - 3y - 4 = 0$

e $x + 5y + 38 = 0$ **f** $3x + y - 4 = 0$

g $4x + y + 1 = 0$ **h** $3x - 4y + 10 = 0$

i $2x + y + 10 = 0$

2



3 **a** $x - y - 4 = 0$ **b** $4x - 5y + 18 = 0$

c $5x - 6y + 23 = 0$ **d** $8x + 3y - 10 = 0$

e $3x + 2y - 6 = 0$ **f** $5x - 3y - 1 = 0$

g $6x + 11y + 38 = 0$ **h** $x + y - 3 = 0$

i $4x - 3y - 11 = 0$

4 $k: x + 2y - 7 = 0, l: 3x - y + 7 = 0$

5 $4x + y - 20 = 0$ **6** $5x - 7y + 42 = 0$

7 $2x - 3y + 18 = 0$ **8** $3x + 5y - 30 = 0$

9 **a** $2x - y + 1 = 0$ **b** same

10 **a** $m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

b $x - y - 4 = 0$

c same

Exercise 3.07

1 **a** $y = 2x + 5$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -3x + 6$

d $y = -x + 3$ **e** $y = \frac{1}{2}x + 3$ **f** $y = -3x - 3$

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2 **a** $y=\frac{1}{2}x+2$ **b** $y=x$ **c** $y=-\frac{1}{2}x+5$

d $y=-\frac{1}{2}x+3$ **e** $y=-3x-3$ **f** $y=-x-2$

g $y=3x-10$ **h** $y=\frac{2}{5}x+2$ **i** $y=2x-3$

Exercise 3.08

1 **a** $y=2x+4$ **b** $y=3x+6$ **c** $y=-\frac{1}{2}x+\frac{11}{2}$

d $y=2x-12$ **e** $y=-5x-13$ **f** $y=\frac{1}{2}x-10$

2 **a** $y=-2x-2$ **b** $y=\frac{1}{5}x-\frac{1}{5}$ **c** $y=-\frac{1}{3}x+\frac{4}{3}$

d $y=-3x-3$ **e** $y=x+6$ **f** $y=-\frac{1}{3}x-\frac{31}{3}$

3 **a** $m=2$ **b** $M(0, 2)$

c $-\frac{1}{2}$ **d** $y=-\frac{1}{2}x+2$

4 **a** $y=\frac{1}{3}x+1$ **b** -3 **c** $y=-3x+11$

5 **a** $y=-\frac{4}{5}x+8$ **b** $A(10, 0)$ **c** $\frac{5}{4}$

d $y=\frac{5}{4}x-\frac{25}{2}$ **e** $(0, -12.5)$

Exercise 3.09

1 **a** **i** $5x+2y-18=0$

ii $3x-4y-16=0$

iii $(0, 9)$ **iv** $(0, -4)$ **v** 26 units^2

b **i** $x-5y+20=0$

ii $x+2y+6=0$

iii $(0, 4)$ **iv** $(0, -3)$ **v** 35 units^2

2 **a** $5x-2y-25=0$ **b** $5x+7y-25=0$

c $w=5$ **d** $t=-10\frac{2}{5}$

3 **a** $DE=EF=FG=DG=5$ units

b For DE and GF, $m=0$

For DG and EF, $m=-\frac{4}{3}$

c Diagonal DF, $m=-\frac{1}{2}$

Diagonal EG, $m=2$

Since $-\frac{1}{2}\times 2=-1$ it is true that $DF\perp EG$

d Midpoint of $DF=(0, 0)$

Midpoint of $EG=(0, 0)$

The diagonals bisect each other because their midpoints are the same.

e Opposite sides are equal and parallel, adjacent sides are equal, diagonals bisect each other at right angles

4 **b** $6\sqrt{5}$ units **c** $(-1, -1)$

d No, since $m_{PR}\times m_{QS}\neq -1$

e Rectangle, diagonals are equal and bisect each other but not at right angles

5 **b** $CE=\sqrt{130}$ units, $DF=\sqrt{130}$ units

c $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$

d $m_{CE}=\frac{11}{3}, m_{DF}=-\frac{3}{11}$

$\therefore CE\perp DF$ because $\frac{11}{3}\times -\frac{3}{11}=-1$

e Square, diagonals are equal and bisect each other at right angles

6 **a** $BC=DE=\sqrt{61}$ units, $CD=BE=\sqrt{65}$ units

b $m_{BC}=\frac{5}{6}, m_{CD}=-\frac{4}{7}, m_{DE}=\frac{5}{6}, m_{BE}=-\frac{4}{7}$

c Midpoint of $BD\equiv\left(1\frac{1}{2}, 2\frac{1}{2}\right)$

Midpoint of $CE\equiv\left(1\frac{1}{2}, 2\frac{1}{2}\right)$

d Parallelogram, opposite sides are parallel and equal

7 **a** $AC=BD=\sqrt{104}$ units

b Midpoint of $AC\equiv(1, 2)$, midpoint of $BD\equiv(1, 2)$

$m_{AC}=-5, m_{BD}=\frac{1}{5}, \therefore AC\perp BD$

c The diagonals are equal and bisect each other at right angles.

8 Midpoint of $KM=$ Midpoint of $LN\equiv\left(2\frac{1}{2}, \frac{1}{2}\right)$

$m_{KM}\times m_{LN}=1\times(-1)=-1$

9 Teacher to check.

10 **a** $m_{JK}=-\frac{1}{3}, m_{LM}=-\frac{1}{3}, m_{KL}=-\frac{5}{2}, m_{JM}=-\frac{5}{2}$

b Parallelogram because opposite sides are parallel.

11 **a** $X\left(3, \frac{1}{2}\right)$ $Y\left(-1\frac{1}{2}, 3\frac{1}{2}\right)$

b $m_{XY}=-\frac{2}{3}, m_{CB}=-\frac{2}{3}. \therefore XY \parallel CB$

c $XY=\frac{1}{2}\sqrt{117}, CB=\sqrt{117}. \therefore CB=2XY$

12 **a** $m_{WN}=-1$ and $m_{CT}=1$

$m_{WN}\times m_{CT}=-1$ $\therefore WN\perp CT$

$M_{WN}\left(-1\frac{1}{2}, -\frac{1}{2}\right)$ and $M_{CT}\left(-1\frac{1}{2}, -\frac{1}{2}\right)$

$\therefore$ Diagonals bisect at right angles

b rhombus

13 Trapezium

14 $ST=WX=\sqrt{37}$ units

$TW=SX=2\sqrt{37}$ units

$XS\perp ST$ because $m_{XS}\times m_{ST}=-\frac{1}{6}\times 6=-1$

$\therefore STWX$ is a rectangle because opposite sides are equal and angles are right angles.

Power plus

1 **a** $\frac{2}{3}$ **b** $y=\frac{2}{3}x-2$ **c** $y=4$

2 $k=5$ **3** $B(2, -1)$ **4** $X(-2, 3)$

5 Teacher to check, see worked solutions.

6 **a** $-\frac{3}{2}$ **b** $3x+2y+2=0$ or $y=-\frac{3x}{2}-1$

7 $(-3, 5)$ or $(7, 9)$ or $(1, 3)$

ANSWERS

Test yourself 3

1 **a** 12.6 **b** $M(-1, 4)$ **c** $\frac{1}{3}$

2 **a** $HJ = JK = KL = HL = \sqrt{58}$

b $m_{HJ} = \frac{3}{7}, m_{JK} = \frac{7}{3}, m_{KL} = \frac{3}{7}, m_{HL} = \frac{7}{3}$

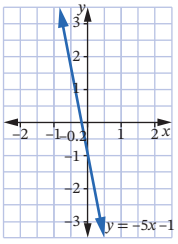
c $HK = \sqrt{200}$ or $10\sqrt{2}$, $JL = \sqrt{32}$ or $4\sqrt{2}$

d rhombus

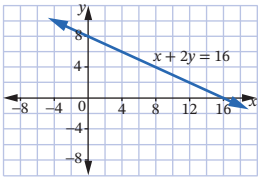
3 **a** 72° **b** 51° **c** 135° **d** 146°

4 **a** $-\frac{1}{2}$ **b** 2

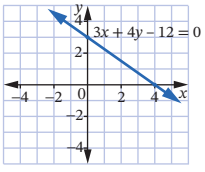
5 **a**



b



c



6 C **7** D

8 **a** $m = 2, c = -10$ **b** $m = 4, c = 3$

c $m = -\frac{3}{8}, c = \frac{1}{2}$

9 **a** C **b** B **c** A **d** D

10 **a** $3x - y + 5 = 0$ **b** $2x - 5y - 50 = 0$

c $x - 3y - 6 = 0$

11 **a** $m = 1, c = 2$ **b** $m = \frac{1}{4}, c = 1$

c $m = -3, c = 9$

12 **a** $3x + y - 20 = 0$ **b** $2x - 3y + 26 = 0$

13 **a** $3x - 5y - 20 = 0$ **b** $x + y + 3 = 0$

14 **a** $y = 2x + 3$ **b** $y = -\frac{x}{2} - 4$

The product of their gradients, 2 and $-\frac{1}{2}$, is –1, so they are perpendicular.

15 **a** $y = 3x - 6$ **b** $y = -2x$

16 $8x + 3y - 95 = 0$

17 $PN = LM = NM = PL = \sqrt{34}$ units

$m_{PN} \times m_{PL} = \frac{3}{5} \times \left(-\frac{5}{3}\right) = -1 \quad \therefore PN \perp PL$

$\therefore LMNP$ is a square because all sides are equal and $PN \perp PL$

Chapter 4

SkillCheck

1 **a** 696 cm² **b** 336.96 cm² **c** 900 cm²

d 814.2 cm² **e** 54.9 cm² **f** 1.14 cm²

2 **a** 35.3 cm² **b** 40.8 m² **c** 1.7 m²

Exercise 4.01

1 **a** 282 m² **b** 298 cm² **c** 2720 mm²

d 204 m² **e** 1288 mm² **f** 165 m²

2 A

3 **a** 80 m², $4400 **b** 171.4 m²

4 **a** 1036 cm² **b** 1020 mm² **c** 204 m²

d 390 cm² **e** 672 cm² **f** 5672 mm²

5 B

6 **a** 60 336 cm² **b** Need 7 m², cost $175

Exercise 4.02

1 **a** 31.7 m² **b** 12 370.0 cm²

c 805.8 cm² **d** 41.7 m²

2 **a** 49 m² **b** 4623 cm²

3 **a** 102π cm² **b** $\frac{165\pi}{16}$ or 10.3125π m²

c $\frac{39\pi}{200}$ or 0.195π m² or 1950π cm²

4 **a** 1009 m² **b** 3054 cm² **c** 1355 cm²

d 7 cm² **e** 905 cm² **f** 39 m²

g 17 m² **h** 5 m² **i** 1253 cm²

5 587.2 cm² **6** D

7 **a** 64.4 m² **b** 8 L

8 **a** 27.3 m² **b** 56.7 m²

c Cost = 28 × $18.50 + 57 × $21.75 = $1757.75

Exercise 4.03

1 **a** 432 cm² **b** 2150 cm² **c** 173.2 cm²

2 **a** 275 m² **b** 564 mm² **c** 472 cm²

3 B

4 **a** 166.4 m² **b** 3456 mm² **c** 743.1 cm²

5 **a** 843 cm² **b** 1592 cm² **c** 3116 cm²

6 85 854 m² **7** 1869 m²

8 **a** 1344 mm² **b** 80 cm² **c** 343.4 m²

9 **a** 42 m **b** $A = 735$ m²

c 35 m **d** 28 m

Exercise 4.04

1 **a** 101 cm² **b** 628 cm² **c** 2419 cm²

2 **a** 392.7 mm² **b** 62.8 m² **c** 192.4 cm²

3 **a** 1695.0 cm² **b** 2.0 m² **c** 614.3 cm²

4 **a** 90π m² **b** 224π mm² **c** 450π cm²

5 **a** 2827.4 mm² **b** 380.1 m² **c** 366.4 cm²

6 **a** 432π m² **b** 192π cm² **c** 768π mm²

7 B

8 **a** 314 m² **b** 628 m² **c** 628 m² **d** 402 m²

9 **a** 5.11×10^8 km² **b** 149 000 000 km²

c 5.17% **10** 5525 cm²

11 **a** 30.16 cm **b** 4.80 cm

c 8 cm **d** 193.02 cm²

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12 **a** 21.9 mm **b** 25.2 mm **c** 30.9 mm
13 **a** 6.9 cm **b** 85.3 cm **c** 85 cm

Mental skills 4

Exact answers shown

2 **a** 331 **b** 157 **c** 1587 **d** 255
e 421 **f** 203 **g** 413 **h** 734
i 6723 **j** 15 744 **k** 276 **l** $72\frac{3}{7}$

4 **a** 28.231 **b** 14.187 **c** 177.4967 **d** 416.752
e 2.4156 **f** 5.0237 **g** 21.75 **h** 3.69
i 5.81 **j** 23.9121

Exercise 4.05

1 **a** 446.96 cm^2 **b** 49 270 cm^2 **c** 864 cm^2
d 11 064 cm^2 **e** 45 160 cm^2 **f** 40 270 cm^2
2 464 cm^2
3 **a** 352 cm^2 **b** 76 cm^2
4 **a** 9721.7 cm^2 **b** 14 031.4 cm^2 **c** 14 778.1 cm^2
d 2858.8 cm^2 **e** 2793.5 cm^2 **f** 394.7 cm^2
5 B **6** 2953 cm^2
7 **a** 26.14 m^2 **b** 19 m^2
8 1028.32 cm^2 **9** A
10 19 858.0 cm^2
11 **a** 857.7 cm^2 **b** 412.3 cm^2 **c** 1042.0 cm^2
d 5969.0 cm^2 **e** 250.6 cm^2 **f** 628.3 cm^2
g 282.7 cm^2 **h** 652.9 cm^2 **i** 501.6 cm^2
j 3769.9 cm^2 **k** 1148.8 cm^2 **l** 3017.7 cm^2
m 6615.4 cm^2 **n** 3908.4 cm^2 **o** 7940 cm^2

Exercise 4.06

1 **a** 1399.6 cm^3 **b** 46.6 m^3 **c** 56 160 cm^3
d 17 066 cm^3 **e** 33 931.8 cm^3 **f** 192.4m^3
g 1.2 m^3 **h** 21 756 cm^3 **i** 208.8 m^3
2 **a** 251.3 cm^3 **b** 320 cm^3 **c** 21.5%
3 350.4 m^3
4 **a** 118 800 L **b** 113 040 L
5 2 500 000 L
6 **a** 182.83 m^3 **b** $21.94 per day **7** 1415.7 cm^3
8 **a** 4825.49 cm^3 **b** 5026.55 cm^3 **c** 1989.38 cm^3
d 6375 cm^3 **e** 5301.44 cm^3 **f** 3084.96 cm^3
g 536.19 cm^3 **h** 1884.96 cm^3 **i** 167.33 cm^3
j 12 900 cm^3 **k** 167.55 cm^3 **l** 794.12 cm^3

Exercise 4.07

1 **a** 192 cm^3 **b** 200 cm^3 **c** 120 cm^3
d 336 m^3 **e** 1200 cm^3 **f** 80 m^3
2 **a** **i** 24 cm **ii** 3200 cm^3
b **i** 20 m **ii** 19 200 m^3
c **i** 40 mm **ii** 4320 mm^3
d **i** 60 mm **ii** 28 160 mm^3
e **i** 7.7 m **ii** 133.1 m^3
f **i** 84 cm **ii** 564 480 cm^3
3 D
4 **a** 42 412 mm^3 **b** 3 m^3 **c** 2618 cm^3
d 616 cm^3 **e** 393 cm^3 **f** 2545 mm^3
5 **a** **i** 13.3 cm **ii** 680.7 cm^3
b **i** 3.9 m **ii** 19.9 m^3
c **i** 11.3 cm **ii** 189.6 cm^3
d **i** 3.5 m **ii** 2.4 m^3
e **i** 244.6 m **ii** 296 162.1 m^3
f **i** 71.9 cm **ii** 129 638.4 cm^3
6 B
7 **a** 14 137.2 cm^3 **b** 696.9 m^3
c 659.6 cm^3 **d** 3619.1 m^3
e 1072.3 cm^3 **f** 8578.6 mm^3
8 1.09×10^{12} km^3
9 **a** 33.75 m^3 **b** 19 t
10 22.5 m **11** 14.1 cm
12 5.7 m **13** 2.8 cm
14 A

Exercise 4.08

1 **a** 59 m^3 **b** 59 kL
2 **a** 343 cm^3 **b** 240 cm^3 **c** 1152 cm^3
d 6100 cm^3 **e** 6048 cm^3 **f** 2500 cm^3
3 **a** **i** 31 416 cm^3 **ii** 31.416 L
b **i** 616 cm^3 **ii** 0.616 L
c **i** 264 cm^3 **ii** 0.264 L
4 28.27 kL
5 **a** 12 balls **b** 60 balls **c** 31 416 cm^3 **d** 48%
6 **a** 1963 cm^3 **b** 0.55 cm^3/s
7 **a** 250 m^3 **b** 210 kL **c** $415.80
8 **a** 1696 m^3 **b** 1018 t

Exercise 4.09

1 **a** 9 : 1 **b** 9 : 25 **c** 81 : 25 **d** 4 : 9
2 **a** 3 : 5 **b** 1 : 10 **c** 8 : 5 **d** 4 : 9
3 **a** 3.5 **b** 18 **c** 48 **d** 36.75
4 54 cm^2 **5** 44.1 cm^2
6 7.5 cm **7** 154 cm
8 The area is quadrupled ($\times 4$).
9 The sides are decreased by a factor of 3.
10 The area has increased by a factor of 6.25.
11 The sides have decreased by a factor of $\frac{1}{10}$.

Exercise 4.10

1 **a** **i** 9 : 25 **ii** 125 : 27
b **i** 4 : 9 **ii** 27 : 8
c **i** 16 : 25 **ii** 125 : 64
d **i** 4 : 25 **ii** 125 : 8
2 **a** 9 : 10 **b** 729 : 1000
3 **a** 5 : 7 **b** 25 : 49
4 76 800 mm^3 **5** 75.6 mL
6 2531.25 g or 2.531 kg **7** 78 L
8 **a** 2.25 **b** 3.375
9 There has been a $\frac{27}{64}$ decrease in the volume.

ANSWERS

Power plus

Teacher to check.

Test yourself 4

1 **a** 1.08 m² **b** 3150 mm² **c** 5236 cm²
d 277.6 m² **e** 216 cm² **f** 482 mm²

2 **a** 354 371.7 cm² **b** 4863.2 cm² **c** 17.4 m²
d 14 294.2 cm² **e** 5871.2 cm² **f** 4427.8 cm²

3 **a** 960 cm² **b** 7776 cm² **c** 1356 cm²

4 **a** 704 m² **b** 4524 m² **c** 2488 m²
d 452 m² **e** 681 m² **f** 5890 m²

5 **a** 3180 cm² **b** 1268 cm² **c** 395 cm²
d 3318 cm² **e** 1728 cm² **f** 3436 cm²

6 **a** 36 816 m³ **b** 20 160 m³ **c** 10 016 m³

7 **a** 322.67 m³ **b** 540 cm³ **c** 1568 mm³
d 1340.41 cm³ **e** 10 262.54 mm³ **f** 904.78 m³

8 **a** 360 498 mm³ **b** 145 125 mm³ **c** 455 cm³
d 229 m³ **e** 3054 cm³ **f** 18 096 m³

9 **a** 234.375 cm² **b** $6\frac{1}{4}$

10 **a** 250 cm³ **b** 36 : 49

Practice set 1

1 **a** 31 743.4 cm² **b** 348 m² **c** 2110 cm²

2 143°

3 **a** \$1505.94 **b** \$1054.16 **c** \$7077.92

4 **a** \$575 **b** \$1164.38 **c** \$1495

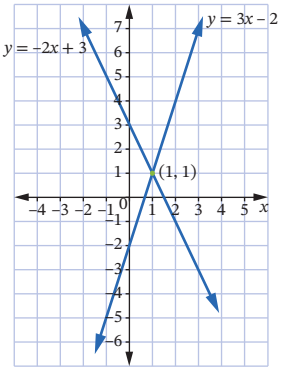
5 **a** $-\frac{4}{3}$ **b** $\frac{3}{4}$

6 **a** 46.4 m² **b** 8972.4 cm² **c** 3239.8 cm²

7 **a** $13\sqrt{2}$ **b** $-4\sqrt{11}$
c $6\sqrt{6}-6\sqrt{3}$ **d** $45\sqrt{2}-9\sqrt{5}$

8 **a** \$71 904 **b** \$14 915.80

9



10 **a** 2024 cm² **b** 134.0 m² **c** 13 824 cm²

11 **a** 183 m² **b** 510 m² **c** 66 m²

12 $2x + 3y + 7 = 0$ **13** B

14 **a** \$52 042.84 **b** \$52 142.72 **c** \$52 165.50

15 $3x - 4y + 24 = 0$

16 gradient 2.5, y-intercept 1.5

17 **a** \$768 **b** \$6912 **c** \$4008.96
d \$10 920.96 **e** \$227.52 **f** \$11 688.96

18 **a** \$26 237.44 **b** \$13 752.56 **c** 65.6%

19 **a** 35 m³ **b** 69 266 cm³ **c** 60 m³

20 2.45% **21** $5x + 4y - 7 = 0$ **22** $5x - 2y - 30 = 0$

23 **a** 340.2 m² **b** 14 940 cm²

24 **a** $\frac{\sqrt{10}}{10}$ **b** $\frac{5\sqrt{6}}{6}$ **c** $\frac{4\sqrt{5}+5}{15}$ **d** $\frac{6\sqrt{2}-3}{9}$

25 $PR = QS = \sqrt{68}$, midpoint of PR and QS is (−1, 3), gradient $PR = 4$, gradient $QS = -\frac{1}{4}$

$\therefore PQRS$ is a square since the diagonals are equal and bisect each other at right angles.

26 **a** 6426 cm³ **b** 59 837.7 cm³ **c** 98 960.2 cm³

27 $8x - 5y - 41 = 0$ **28** 1000 cm²

29 **a** 218 **b** $55 - 14\sqrt{6}$ **c** $2\sqrt{15} + 54$

30 40 cm

Chapter 5

SkillCheck

1 **a** g^9 **b** r^6 **c** d^{15} **d** k^2
e h^{10} **f** m^4 **g** a **h** 1
i $6e^7$ **j** $3n^4$ **k** $1000w^9$ **l** 25
m v^5w^5 **n** $\frac{m^3}{n^3}$ **o** $\frac{1}{y}$ **p** $\frac{1}{p^2}$

2 **a** $\frac{19a}{20}$ **b** $\frac{p}{6}$ **c** $\frac{5}{3}$ **d** $\frac{x}{7y}$

Exercise 5.01

1 **a** $24m^{14}$ **b** $\frac{6k^8}{5}$ **c** $-y^{10}$ **d** $9q^6$
e $4a^6d$ **f** $21x^7y^{12}$ **g** $-5e^{10}g^4$ **h** $-3wy^7$
i $64c^{21}$ **j** $-32p^5$ **k** $\frac{3k^2m}{2}$ **l** $54u^4v^3w^{10}$

2 **a** $l^{18}m^{30}$ **b** $\frac{n^3}{8}$ **c** $\frac{w^{20}}{m^{15}}$ **d** 1
e $64k^2y^{10}$ **f** $81p^8q^{12}r^{16}$ **g** 4 **h** $-125d^9y^{15}$
i $\frac{16b^4}{81d^4}$ **j** −9 **k** 12 **l** $-\frac{27k^{12}}{1000}$

3 **a** 1 **b** 5 **c** −64 **d** 64
e 36 **f** 1 **g** $\frac{1}{8}$ **h** 5
i 1 **j** 49 **k** 1 **l** 1
m $\frac{1}{15}$ **n** $\frac{1}{16}$ **o** 2 **p** 25

4 D

5 **a** $\frac{1}{8}$ **b** $\frac{1}{1000000}$ **c** $\frac{1}{8}$ **d** $\frac{1}{36}$ **e** $\frac{1}{81}$

6 **a** $\frac{1}{5^4}$ **b** $\frac{1}{8^2}$ **c** $\frac{1}{w^6}$ **d** $\frac{1}{m^5}$
e $\frac{4}{y^3}$ **f** $\frac{1}{a^2d^2}$ **g** $\frac{1}{32x^5}$ **h** $\frac{3m}{p^2}$
i $\frac{10}{g^2h^3}$ **j** $\frac{1}{256u^4}$ **k** $\frac{2a^5}{q^2}$ **l** $\frac{4p^2}{x^3}$
m $\frac{10c^3}{a^5}$ **n** $\frac{-1}{27r^{18}}$ **o** $\frac{3}{4q^2p}$ **p** $\frac{4}{125k^9}$

7 C

8 **a** $\frac{4d^2}{25}$ **b** $\frac{5}{y^2}$ **c** $\frac{1}{125m^3}$ **d** $\frac{x^2y^3}{w^2}$

e $\frac{9}{16y^6}$ **f** $\frac{7k}{a}$ **g** $\frac{8}{9a^2b^4}$ **h** $\frac{5k^2}{6c}$

9 **a** 16 **b** $\frac{27}{125}$ **c** $\frac{81}{625}$ **d** $\frac{25}{169}$

e $\frac{m^3}{64}$ **f** $\frac{49}{9a^2}$ **g** $-\frac{y^5}{32}$ **h** $\frac{25}{64q^4}$

i $-\frac{w^9}{64}$ **j** $\frac{d^{10}}{k^4}$ **k** $-\frac{243d^{15}}{100000g^{20}}$ **l** $\frac{625}{2401h^{24}}$

10 **a** $5000x^{28}y^6$ **b** $256k^{44}m^4$ **c** $2r^8q^{-14} = \frac{2r^8}{q^{14}}$

d $\frac{2}{9w^7m}$ **e** $\frac{256x^{16}}{a^4}$ **f** $\frac{4}{9g^4n^8}$

g $-128p^{-3}h^{21} = \frac{-128h^{21}}{p^3}$ **h** $-8d^5n^4$

i $-1000c^{21}e^{18}$

Exercise 5.02

1 **a** 8 **b** −3 **c** 25 **d** 10

e 2 **f** −0.2 **g** 0.1 **h** −2

i −2 **j** −9 **k** 2 **l** 5

2 **a** $\sqrt[3]{7}$ **b** $\sqrt{15}$ **c** $\sqrt[4]{d}$ **d** $\sqrt{y}$

e $\sqrt[6]{4x}$ **f** $\sqrt{10m^3}$ **g** $\sqrt[10]{2a^7}$ **h** $\sqrt[3]{12m}$

3 C

4 **a** $80^{\frac{1}{5}}$ **b** $40^{\frac{1}{5}}$ **c** $20^{\frac{1}{2}}$ **d** $c^{\frac{1}{4}}$

e $b^{\frac{1}{2}}$ **f** $(xy)^{\frac{1}{3}}$ **g** $(25w)^{\frac{1}{5}}$ **h** $(4y)^{\frac{1}{8}}$

5 **a** 8 **b** 3125 **c** $-\frac{1}{243}$ **d** 4

e $-\frac{1}{8}$ **f** $\frac{1}{2}$ **g** 100 000 **h** $\frac{1}{30}$

i 32 **j** $\frac{1}{2187}$ **k** $\frac{1}{1296}$ **l** $\frac{1}{9}$

6 B

7 **a** −6.30 **b** 6.08 **c** 0.14 **d** −2.02

e 0.10 **f** 4.90 **g** 0.01 **h** 3.01

8 **a** $k^{\frac{3}{4}}$ **b** $m^{-\frac{1}{2}}$ **c** $x^{\frac{7}{3}}$ **d** $y^{\frac{3}{2}}$

e $d^{-\frac{2}{5}}$ **f** $\left(2a^3\right)^{\frac{1}{2}}$ **g** $(4p)^{-\frac{1}{5}}$ **h** $(vw)^{-\frac{1}{4}}$

9 D

10 **a** $3125m^{20}$ **b** $4y^4$ **c** $64x^3w^9$ **d** $\frac{1}{243c^{10}}$

e $\frac{1}{16k^{12}}$ **f** $\frac{1}{4p^6}$ **g** $\frac{1}{100n^2w^3}$ **h** $81x^8y^4$

i $36p^{\frac{8}{3}}n^3$ **j** $\frac{1}{125y^6d^3}$ **k** 4 **l** $36a^4b^6$

Exercise 5.03

1 **a** $\frac{5n}{14}$ **b** $\frac{17c}{10}$ **c** $\frac{43r}{14}$ **d** $\frac{-19y}{24}$

e $\frac{13t}{9}$ **f** $\frac{-y}{16}$ **g** $\frac{13t}{36}$ **h** $\frac{17a}{30}$

i $\frac{3a+4}{k}$ **j** $\frac{5d}{3y}$ **k** $\frac{1}{x}$ **l** $\frac{7p}{6}$

m $\frac{5r+3q}{qr}$ **n** $\frac{7}{4h}$ **o** $\frac{29}{12b}$ **p** $\frac{27p-8n}{6np}$

2 **a** $\frac{4w+7n}{28}$ **b** $\frac{21x+40u}{56}$ **c** $\frac{4a-9h}{30}$ **d** $\frac{3d-2r}{48}$

e $\frac{36k-5q}{45}$ **f** $\frac{25m+24r}{60}$ **g** $\frac{15h+8}{10}$ **h** $\frac{35u-12a}{28}$

i $\frac{9a+5c}{45}$ **j** $\frac{15q-22w}{36}$ **k** $\frac{55k+24}{88}$ **l** $\frac{23n}{18}$

3 C **4** B **5** A

6 **a** $\frac{9m+2}{20}$ **b** $\frac{14p+10}{15}$ **c** $\frac{11y+5}{12}$ **d** $\frac{10x-17}{21}$

e $\frac{41h+50}{18}$ **f** $\frac{-13k+51}{70}$ **g** $\frac{a-3}{6}$ **h** $\frac{32d-69}{42}$

i $\frac{14y+49}{36}$ **j** $\frac{27-37w}{70}$ **k** $\frac{33-7e}{6}$ **l** $\frac{-23-34x}{44}$

Exercise 5.04

1 **a** $\frac{5d}{42}$ **b** $\frac{xw}{24}$ **c** $\frac{15}{kb}$ **d** $\frac{20}{3hm}$

e $\frac{3}{wy}$ **f** $\frac{5a}{9c}$ **g** $\frac{3x}{4y}$ **h** $\frac{2}{3}$

i $\frac{16h^2}{3gp}$ **j** $\frac{2}{9}$ **k** $\frac{1}{6hq}$ **l** $\frac{12a^2}{k^2}$

2 **a** $\frac{7m}{3g}$ **b** $\frac{y}{20}$ **c** $\frac{8}{25d^2}$ **d** $\frac{24}{5}$

e $\frac{y}{3}$ **f** $\frac{27}{5h}$ **g** $\frac{3c}{20e}$ **h** $\frac{15}{c}$

i $\frac{12}{7kx}$ **j** $\frac{25d}{3}$ **k** $\frac{2}{9ep}$ **l** $\frac{21g}{4r}$

3 D **4** B

5 **a** $\frac{4w}{3y}$ **b** $\frac{8d}{25k}$ **c** $\frac{9n}{2}$ **d** $\frac{14a}{f}$

e $\frac{2c}{5}$ **f** $\frac{8}{3m}$ **g** $\frac{2ab^3}{9d}$ **h** $\frac{54hy}{k}$

i $\frac{4}{3d}$ **j** $\frac{5p}{9h}$ **k** $\frac{9acyh}{w^2}$ **l** uy

Mental skills 5

2 **a** 176 **b** 363 **c** 261 **d** 405

e 682 **f** 707 **g** 1818 **h** 3564

i 152 **j** 540 **k** 2142 **l** 588

m 288 **n** 693 **o** 3939 **p** 852

Exercise 5.05

1 **a** $5d + 55$ **b** $-3r - 30$ **c** $7x - 63y$

d $-4a + 20w$ **e** $-2 + p^2$ **f** $-20e^3 - 30e$

g $6y + 42y^2$ **h** $12x^2y^2 - 4xy$ **i** $16rq^2 - 8r^2q$

j $12ab^2 - 21a^2b$ **k** $-6h^2 + 18h^3$ **l** $-25x^3 - 20xy$

m $-3 - 8a$ **n** $-6m^3 + 8m^2n$ **o** $15g + 35g^3$

p $-5e + 12$

2 C

3 **a** Yes **b** No **c** Yes

ANSWERS

4 **a** $3k^2 - 20k$ **b** $-23h + 7h^2$
c $3w^3 - 15w$ **d** $49x^3 - 10x^4$
e $2 + 21d$ **f** $12n - 26n^2$
g $4y^2 - 6y + 5$ **h** $5 - 13a - 4a^2$
i $16 + 50w$ **j** $20y^3 - 8y^2 + 8$
k $-2v^2 - v - 6$ **l** $8 - 9a + a^2$
m $10c^2 - 30c + 20$ **n** $2m^2 + 18m^3$
o $20x + 26xy - 60y$

5 **a** $5(3y - 4)$ **b** $7(3 + 5w)$
c $p(2 + p)$ **d** $10y(3 - 2y)$
e $12d(3d + 2)$ **f** $7k(4k - 3)$
g $(c - 5)(8 - c)$ **h** $(3 + 2m)(m + 7)$
i $-q(q + 36)$ **j** $-4x(2 - 3x)$
k $(3b + 5)(b - 2)$ **l** $-4cd(3d - 2)$
m $-hn(n - h)$ **n** $-3g(5g + 6)$
o $6q(8q - 9)$

6 B

7 **a** $4my(2my - 3)$ **b** $9bc(4ab + 3)$
c $12mn(2m - 9n)$ **d** $5g(4dg - 7a)$
e $8wy^2(5y + 3w)$ **f** $25gh(3g^2h - 5)$
g $p(1 - 8p - 4p^2)$ or $-p(4p^2 + 8p - 1)$
h $3mn(2n + 1 + 16m)$
i $8pg(4p^2 + g - 1)$ **j** $3a^2(6a^3 - 4 + 5a^2)$
k $7mh^2(4m^2 - 3)$ **l** $3w(5kp - 8p^2 - 3k)$

Exercise 5.06

1 B

2 **a** $3m^2 + 22m + 7$ **b** $28y^2 + 41y + 15$
c $4k^2 + 25k + 6$ **d** $10h^2 + 11h - 6$
e $2v^2 - 25v + 72$ **f** $20a^2 - 7a - 6$
g $55 - 59e + 12e^2$ **h** $40 - 43g - 6g^2$
i $-8y^2 + 46y - 63$ **j** $-12a^2 + 6 - a$
k $84 - 83k + 20k^2$ **l** $14w^2 + 43w - 90$

3 **a** $14y$ **b** h^2 **c** m^2
d $+ 22u$ **e** $+ 30k$ **f** $- 48xw, 9w^2$

4 **a** $h^2 + 14h + 49$ **b** $p^2 - 10p + 25$
c $w^2 - 12w + 36$ **d** $16 - 8c + c^2$
e $121 + 22y + y^2$ **f** $d^2 - 2dw + w^2$
g $100 - 20x + x^2$ **h** $d^2 + 2dq + q^2$
i $25u^2 + 10u + 1$ **j** $9b^2 + 42b + 49$
k $49e^2 - 154e + 121$ **l** $4 - 20h + 25h^2$
m $64x^2 - 48xy + 9y^2$ **n** $36m^2 + 60mc + 25c^2$
o $g^2 + 24gw + 144w^2$ **p** $4 + \frac{8}{a} + \frac{4}{a^2}$
q $\frac{36}{m^2} - 12 + m^2$ **r** $\frac{9}{x^2} + 18 + 9x^2$

5 D

6 **a** $k^2 - 49$ **b** $x^2 - 64$ **c** $g^2 - 1$
d $144 - w^2$ **e** $16 - d^2$ **f** $49b^2 - 25$
g $9h^2 - 49$ **h** $100e^2 - 1$ **i** $121 - 16y^2$
j $1 - 25p^2$ **k** $9a^2 - 49c^2$ **l** $81h^2 - 25m^2$
m $\frac{16}{m^2} - m^2$ **n** $9g^2 - \frac{1}{g^2}$ **o** $\frac{16a^2}{9} - 4$

7 B

8 **a** $9t^2 - 6td + d^2$ **b** $49k^2 + 42k + 9$
c $81 - 16g^2$ **d** $4a^2 - 25$
e $a^2b^2 - 64$ **f** $20n^2 + 9pn - 20p^2$
g $100x^2 - 140xy + 49y^2$ **h** $4 - 121h^2m^2$
i $x^2y^2 - \frac{1}{y^2}$

9 **a** $2y + 1 + 8y^2$ **b** $12w + 70 - 2w^2$
c $-3a^2 + 2ab$ **d** $-p^2 - 65p + 220$
e $-12y$ **f** $4x^2 + 25$
g $-m^2 + 8m + 4$ **h** $17 - 35d + 2d^2$

10 C

11 **a** $(3x - 10)(2x + 15)$ **b** $6x^2 + 25x - 150$
c $750 - 6x^2 - 25x$

12 **a** $3k(k + 5) = 3k^2 + 15k$ **b** $(3k - 8), (k + 1)$
c $(3k - 8)(k + 1)$ **d** $3k^2 - 5k - 8$
e $20k + 8$

13 **a** $10d^2 + d - 21$ **b** $\frac{33}{2}k^2 + 2k - 6$
c $\frac{3\pi(25a^2 + 40a + 16)}{4}$ or $\frac{75\pi a^2 + 120\pi a + 48\pi}{4}$

Exercise 5.07

1 **a** $(3p + 2q)(x + y)$ **b** $(h + k)(2w - 3u)$
c $(3k + 4g)(5m + 2n)$ **d** $(x - 2a)(4y + 7a)$
e $(7a + 4)(2k - 5f)$ **f** $(d + y)(c - h)$
g $4(m + t)(a + e)$ **h** $3(y + 4)(k - 2b)$
i $(3c + 5w)(4a - 7m)$ **j** $(3k - 4e)(7h + 2p)$
k $3(2y - 1)(5y - q)$ **l** $(4h + 3k)(8g - m)$
m $(2g + 5h)(3n - 2c)$ **n** $(w - p)(p + 1)(p - 1)$
o $(m + a)(d - 3 - x)$ **p** $(k + d)(x + y + e)$

2 **a** $(d + 4)(d - 4)$ **b** $(x + 5)(x - 5)$
c $(y + 13)(y - 13)$ **d** $(p + 9)(p - 9)$
e $(5 - h)(5 + h)$ **f** $(11 - a)(11 + a)$
g $(2r + 3d)(2r - 3d)$ **h** $(5g + 2e)(5g - 2e)$
i $(12 - 7m)(12 + 7m)$ **j** $(9y + 4k)(9y - 4k)$
k $(1 + 2d)(1 - 2d)$ **l** $(m + 5n)(m - 5n)$
m $(5q + 3b)(5q - 3b)$ **n** $(8b + u)(8b - u)$
o $(12x + 1)(12x - 1)$ **p** $(6k + w)(6k - w)$
q $(g + 4p)(g - 4p)$ **r** $(2e + 11d)(2e - 11d)$
s $(mp - 5)(mp + 5)$ **t** $(a + qw)(a - qw)$
u $\left(x + \frac{1}{4}\right)\left(x - \frac{1}{4}\right)$ **v** $\left(\frac{5}{3} + h\right)\left(\frac{5}{3} - h\right)$
w $\left(\frac{5}{2} + 3c\right)\left(\frac{5}{2} - 3c\right)$ **x** $\left(4v - \frac{3}{2}\right)\left(4v + \frac{3}{2}\right)$

3 **a** $4(m + 2p)(m - 2p)$ **b** $3(y + 3)(y - 3)$
c $w(w + 6)(w - 6)$ **d** $q(1 - 8q)(1 + 8q)$
e $12(1 + 2u)(1 - 2u)$ **f** $r(h + r)(h - r)$
g $5(5ab + 2v)(5ab - 2v)$
h $3(x^2 + 3p)(x^2 - 3p)$
i $6gp(3g + p)(3g - p)$ **j** $7c(2 - 3w)(2 + 3w)$
k $36e^3(2e + 1)(2e - 1)$
l $9\left(d - \frac{1}{2}\right)\left(d + \frac{1}{2}\right)$ or $\frac{9}{4}(2d - 1)(2d + 1)$
m $25\left(2k - \frac{1}{2}\right)\left(2k + \frac{1}{2}\right)$ or $\frac{25}{4}(4k - 1)(4k + 1)$
n $2(7a + w)(7a - w)$
o $\left(c - \frac{4}{3}\right)\left(c + \frac{4}{3}\right)$ or $\frac{1}{9}(3c - 4)(3c + 4)$
p $27m(2mn - 1)(2mn + 1)$

9780170453509

4 **a** $\left(\frac{1}{5}-x\right)\left(\frac{1}{5}+x\right)$ or $\frac{1}{25}(1-5x)(1+5x)$

b $\left(\frac{d}{3}-\frac{w}{7}\right)\left(\frac{d}{3}+\frac{w}{7}\right)$ **c** $3\left(p-\frac{4}{p}\right)\left(p+\frac{4}{p}\right)$

d $4\left(\frac{4b}{5}-\frac{y}{9}\right)\left(\frac{4b}{5}+\frac{y}{9}\right)$ **e** $(u-2)(u+2)(u^2+4)$

f $(m-2)(m+8)$ **g** $x(x-2y)$

h $2a(a-3)(a+3)(a^2+9)$ **i** $4px$

j $\frac{1}{9}\left(2c-\frac{e}{2}\right)\left(2c+\frac{e}{2}\right)$ **k** $(g+2k)(3g-2k)$

l $5w(w-2)(w+2)(w^2+4)$

Exercise 5.08

1 **a** $-3, -8$ **b** $10, -4$ **c** $3, 5$ **d** $-12, 4$

2 **a** $(x+4)(x+8)$ **b** $(w+7)(w+2)$

c $(y+3)(y+9)$ **d** $(k+4)(k+9)$

e $(a+10)(a+9)$ **f** $(g+3)(g+7)$

g $(m-4)(m-7)$ **h** $(p-3)(p-5)$

i $(c-6)(c-9)$ **j** $(u-4)(u-1)$

k $(h-6)(h-8)$ **l** $(e-4)(e-4)$

m $(d+7)(d-4)$ **n** $(q+10)(q-8)$

o $(n+9)(n-5)$ **p** $(x-7)(x+1)$

q $(g-16)(g+5)$ **r** $(w-12)(w+8)$

s $(r-14)(r+2)$ **t** $(b-9)(b+6)$

u $(m-6)(m+2)$ **v** $(y-7)(y-7)$

w $(d-6)(d+5)$ **x** $(e+15)(e-6)$

Exercise 5.09

1 **a** $2(k+3)(k+5)$ **b** $3(q+5)(q-6)$

c $5(x-4)(x-7)$ **d** $6(y+3)(y-1)$

e $n(n-9)(n-7)$

f $-(a-6)(a+4)$ or $(6-a)(a+4)$

g $-(p-8)(p+4)$ **h** $-x^2(x-3)(x+7)$

i $-4(h-5)(h-1)$

2 **a** $(3d+5)(2d+3)$ **b** $(4m+3)(2m+1)$

c $(2y+5)(y+1)$ **d** $(2w+1)(w+15)$

e $(3k+4)(2k+5)$ **f** $(3u+7)(u+5)$

g $(4x+9)(x+3)$ **h** $(5e+4)(e+7)$

i $(3h+1)(4h+3)$

3 **a** $(4k-3)(k-2)$ **b** $(3w-1)(2w-5)$

c $(5p-3)(p-4)$ **d** $(2g-7)(3g-7)$

e $(3x-2)(x-10)$ **f** $(3m-2)(7m-4)$

g $(5r-2)(4r-3)$ **h** $(2a+1)(5a-6)$

i $(5u+3)(4u-7)$ **j** $(2p+1)(p-9)$

k $(3y-8)(2y+5)$ **l** $(4b+1)(3b-5)$

m $(6q+5)(2q-3)$ **n** $(4h-3)(h+4)$

o $(5n-2)(2n+7)$

4 **a** $(4w-7)^2$ **b** $(11h+8)^2$ **c** $(3p-15)^2$

5 **a** $3(2m+5)(m-4)$ **b** $2x(4x-1)(x-2)$

c $-(5e-1)(e+6)$ **d** $-3(2w+7)(3w-2)$

e $-4(3k+1)(4k-1)$ **f** $-(5n-4)(3n+2)$

g $6(4d+3)(2d+5)$ **h** $-3(4h+1)(2h-3)$

i $-(7p-3)(4p+1)$

6 A

7 **a** $(7d-1)(4d-1)$ **b** $-2(5y+3)(2y-1)$

c $(10w-3)(3w+5)$ **d** $(4p+3)(p+5)$

e $-(2a-3)(5a-4)$ **f** $(9r+20)(r+1)$

g $(7k+3y)(4k+y)$ **h** $-3p(p-4)(p+6)$

i $-(6m-1)(3m+4)$ **j** $-2g(6g-1)(3g+5)$

k $(11u-3)(4u+1)$ **l** $(8h-3m)(4h+5m)$

Exercise 5.10

1 **a** $15(2y-1)(2y+1)$ **b** $(5m-2)(3m-7)$

c $(k-7)^2$ **d** $-(2a-3)(a-4)$

e $(3g+4c)(2y+3p)$ **f** $-(4x-3)(x+5)$

g $(8q-1)^2$ **h** $9(3r-2w)(3r+2w)$

i $(bn-p)(3n+2d)$ **j** $(3k+4)(2k+5)$

k $(6a+7)(7a+4)$ **l** $4(1-5h)(1+5h)$

m $(9c-7)(4c+9)$ **n** $-(7d-2)(5d+8)$

o $(5u+2)^2$ **p** $(3y+4)(x-2w)$

q $(b-1)^2(b+1)$ **r** $(7c-4h)(4c+h)$

s $(5n+7)(3n-8)$ **t** $ap(a-1)(a+1)$

u $2(8r-k)(r-9k)$

2 **a** $3(4c+5)(4c-5)$ **b** $2(q+4)(4q-5)$

c $-x^2(2x+5)(x-3)$ **d** $(9p+4r)(3p-4r)$

e $3(3w-7)(w+5)$ **f** $n^2(n-2)(n+2)$

g $y(y-1)^2(y+1)$ **h** $(12u+7)(3u+7)$

i $(11k-9)^2$ **j** $(c-3d)(3c-d)$

k $2(2y+3d)(5y+3)$ **l** $ah(h-6-h^2)$

m $9(2b+3)(b-3)$ **n** $8g(2-gy)(2+gy)$

o $-2(6e-1)(3e+4)$ **p** $3p(3q+1)(3q-1)$

q $-15h(h+2)(h-1)$ **r** $(w-1)(w^3-w-1)$

s $a^3(a-16)(a+16)$ **t** $(m-k)(m+k+9)$

u $5(3a-1)(2a+5)$

3 D

Exercise 5.11

1 **a** $5x+3y$ **b** $\frac{1}{2r-3}$ **c** $k-2d$ **d** $a-2$

e $\frac{3}{g-h}$ **f** $\frac{p+2}{3}$ **g** $\frac{w-7}{4(w+7)}$ **h** -1

i $\frac{c+6}{6}$ **j** $\frac{h+p}{e+d}$ **k** $\frac{k+8}{2(k+3)}$ **l** $\frac{3y-2q}{y-2q}$

m $\frac{r-6}{r+8}$ **n** $\frac{8-b}{4b+7}$ **o** $\frac{6-p}{m+8}$ **p** $-\frac{2a+5}{3a+2}$

q $-\frac{n(n-4)}{n-1}$ or $\frac{n(n-4)}{1-n}$ **r** $\frac{xy+2}{y-3m}$

2 **a** $\frac{3k+11}{k(k-1)(k+1)}$ **b** $\frac{11x-21}{x(x+4)(x-7)}$

c $\frac{5m+10}{(m+3)(2m+1)(m-2)}$ **d** $\frac{5h-43}{3(h+8)(h-8)}$

e $\frac{13p-28}{p(p+4)(p-4)}$ **f** $\frac{-a^2+9a-20}{(a-5)(a+5)(a+5)}$

g $\frac{5w-3}{(w-4)(w-5)(w+6)}$ **h** $\frac{-5d-7}{2(d-4)(d+1)}$

i $\frac{72m^2-25m-20}{8m(4-5m)(4+5m)}$ **j** $\frac{6u^2-47u+10}{2(u+7)(u-7)(u-2)}$

k $\frac{3c^2-3c-7}{(5c-4)(2c+3)(c-7)}$ **l** $\frac{13n-19}{20n(n-3)(n+3)}$

ANSWERS

3 **a** $\frac{r}{15}$ **b** $\frac{y+3}{2}$ **c** $\frac{5w^2}{w-6}$

d $\frac{2(a-1)}{9}$ **e** $\frac{5(n-5)}{2(n+7)(n+2)}$ **f** $\frac{(x-1)(x-4)}{3x}$

g $\frac{12}{(y-1)(5y+4k)}$ **h** $\frac{3(m-2)(m+2)}{5}$ **i** $\frac{-2(1+3m)}{(m+4)(m+2)}$

j $\frac{c^2(c-3)}{6(c-8)}$ **k** $\frac{-12(1+e)}{5(e+6)}$ **l** $\frac{2(p+7)}{(2p+1)(p+6)}$

m $\frac{(d+3)(d+6)}{5(d-1)(d+2)}$ **n** $\frac{(h+2)(h+1)}{4(h+4)}$

Power plus

1 **a** $x^3 + 15x^2 + 75x + 125$

b $y^3 - 6y^2 + 12y - 8$

c $a^3 + 3a^2b + 3ab^2 + b^3$

d $27d^3 + 270d^2 + 900d + 1000$

2 **a** 441 **b** 2025 **c** 841

d 3481 **e** 10 404 **f** 9604

3 899

4 **a** 399 **b** 2499 **c** 8099 **d** 6396

5 **a** $3y^2 + 12y + 14$ **b** $3x^2 + 9$

c $34n^2 - 34$ **d** $-4b^2$

6 **a** $(p - 2q)(p + 2q)(p^2 + 4q^2)$

b $(w - 2)(w + 2)(w^2 + 4)(w^4 + 16)$

c $-8mn$

7 **a** $a^3 - b^3$

b **i** $(x - y)(x^2 + xy + y^2)$

ii $(2m - 3p)(4m^2 + 6mp + 9p^2)$

8 **a** $a^3 + b^3$

b **i** $(d + w)(d^2 - dw + w^2)$

ii $(u + 4)(u^2 - 4uh + 16h^2)$

Test yourself 5

1 **a** $6v^9w^{11}$ **b** $8r^7h^6$ **c** $25x^2y^4$

d 1 **e** $\frac{1}{4k}$ **f** $\frac{8p^3}{27}$

g $\frac{1}{16m^2}$ **h** $\frac{4}{m^2}$ **i** $625b^{24}y^{12}$

j $512t^{14}u^{16}$ **k** $5c^4d^4$ **l** $\frac{6a}{5b}$

2 **a** 100 000 **b** $\frac{25}{9}$ **c** $\frac{343}{64g^3}$ **d** $\frac{4d^2}{81}$

3 **a** 20 **b** −243 **c** $\frac{1}{8}$ **d** 4

4 **a** $64a^6$ **b** $243m^{15}$ **c** $\frac{1}{2p^2}$ **d** $\frac{1}{25x^2y^4}$

5 **a** $-\frac{7r}{20}$ **b** $\frac{19g}{6}$ **c** $-\frac{5x}{24}$ **d** $\frac{21-8m}{28}$

6 **a** $\frac{19k-12}{15}$ **b** $\frac{5w+21}{36}$ **c** $\frac{p+20}{12}$ **d** $\frac{5y+51}{88}$

7 **a** $\frac{3m^2}{10}$ **b** $\frac{8e}{7}$ **c** $\frac{b}{2a}$ **d** $\frac{16w^2}{9}$

8 **a** $56p^2q - 40pq^2$ **b** $-15h^3 - 35h^2$

c $4eg^2 - 30e^2g$ **d** $93 - 22n$

e $8x^3 + 11x^4$ **f** $21 - 21y - 20y^2$

9 **a** $8ar(2r + 3)$ **b** $-6(4p - 3q)$

c $(5x - 1)(2 - 3x)$ **d** $15xy^2(1 - 2x^2y)$

e $6p(t^2 + 2pt - 8p^2)$ **f** $4r^2m^3(8m + 3r^2)$

g $25x^3y^3(2x - 3y)$ **h** $-8p^3q^3(1 - 6q^3)$

i $(n^2 + 6)(n - 1)$

10 **a** $b^2 + 13b + 30$ **b** $d^2 - 2d - 80$

c $15w - 54 - w^2$ **d** $20x^2 + 13x - 21$

e $49y^2 - 9$ **f** $25p^2 - 80p + 64$

11 **a** $n^2 - 81$ **b** $9y^2 + 12yd + 4d^2$

c $16n^2 - 121$

12 **a** $(t + u)(3p + 2q)$ **b** $(2a + 3c)(2b - 3d)$

c $(b + 10)(b - 10)$ **d** $(5 - 4y)(5 + 4y)$

e $5(2x + 5)(2x - 5)$ **f** $3r(r + 3)(r - 3)$

13 **a** $(y + 5)^2$ **b** $(q - 20)(q - 1)$

c $(n + 11)(n - 3)$ **d** $(a - 7)(a - 4)$

e $(m - 12)(m + 7)$ **f** $(p + 9)(p - 6)$

14 **a** $(3w + 2)(w + 1)$ **b** $(2y + 3)(y - 3)$

c $-5(b + 4)(b - 3)$ **d** $(3p - 2)(p + 4)$

e $2(3x - 1)(2x - 7)$ **f** $(3n - 2)(2n - 3)$

g $(8y - 3)(5y + 8)$ **h** $(4c - 7)(12c + 5)$

i $(5e - 1)(2e - 1)$

15 **a** $5(q - 3)(q + 3)$ **b** $4(5x - 3)(x - 2)$

c $(c - 1)^2(c + 1)$ **d** $-(12k - 1)(k + 4)$

e $(3m + 8)(2m - 5)$ **f** $a(a - 1)(a + 1)^2$

16 **a** $\frac{4d-7}{2}$ **b** $\frac{4}{7r}$

c $\frac{u+4}{2(u+8)}$ **d** $\frac{4h-7}{(5-h)(5+h)(h+8)}$

e $\frac{6p+6}{(p-3)(p+3)^2}$ **f** $\frac{6(m-2)}{7}$

g $\frac{3y-12-y^2}{y(y+4)(y-4)}$ **h** 4

i $\frac{3(b+2)}{b(b-3)}$ **j** $\frac{6a^2-44a-6}{(a+2)(a+3)(a-7)}$

k $\frac{(1+2q)(q-1)}{2q(q+2)}$ **l** $\frac{12r(r-5)}{(r+3)(r+7)}$

Chapter 6

SkillCheck

1 **a** **i** 10 **ii** 16.5 **iii** 15 **iv** 15

b **i** 13 **ii** 1.8 **iii** 2.5 **iv** 3

c **i** 48 **ii** 34.3 **iii** 34.5 **iv** 24, 35

d **i** 5 **ii** 2.2 **iii** 2 **iv** 2

2 **a** **i** 31 **ii** 33.3 **iii** 62

b 78

c **i** median = 30, mean = 28.3, range = 25

ii The outlier has increased the median (by 1), the mean (by 5) and the range (by 37).

Exercise 6.01

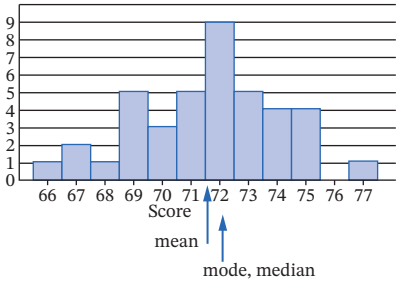
1 **a** **i** symmetrical

ii clustering at 9, no outliers

b **i** bimodal
ii clustering in the 30s and 60s, no outliers
c **i** positively skewed
ii clustering at 1, 8 is an outlier
d **i** negatively skewed
ii clustering at 23–24, no outliers
e **i** positively skewed
ii clustering at 13, 23 is an outlier
f **i** symmetrical
ii clustering at 4 and 5, no outliers
g **i** positively skewed
ii clustering at 130–150, no outliers
h **i** bimodal
ii clustering at 50s and 100s, 136 is an outlier

2 **a**

Score	Frequency
66	1
67	2
68	1
69	5
70	3
71	5
72	9
73	5
74	4
75	4
76	0
77	1



b There are no outliers.
c negatively skewed
d In golf scoring, the lower the score, the better. So in a distribution of golf scores, you'd expect fewer golfers to have lower scores.
e clustering at 72
f mode = 72, $\bar{x} = 71.6$, median = 72

3 **a** 46 **b** 1–9 hours (stem of 0)
c no outliers **d** positively skewed
e Most students spend limited time on their computers, and have other commitments and do other activities such as sport. Only a few students spend many hours on the computer during the week.
f Mode = 1, $\bar{x} = 13.9$, median = 11

4 **a**

Stem	Leaf
12	3
13	
14	2 3
15	0 1 3 3 3 5 5
16	0 0 1 2 2 2 3 4 5 5 7 8 9
17	0 0 1 2 3
18	2

b Symmetrical
c 123 is an outlier
d Clustering occurs in the 160s
e mode = 162, median = 162, $\bar{x} = 160.2$

5 **a** Bimodal
b No
c 29.3 **d** 28.4 **e** 28.45 **f** 8.7
g Yes, because there aren't any outliers.

6 B

Exercise 6.02

1 **a** 5, 6.5, 8 **b** 18, 20, 26.5 **c** 32, 34.5, 38
2 **a** range = 7, IQR = 3
b range = 22, IQR = 8.5
c range = 16, IQR = 6
3 **a** 7.5 **b** 3
4 **a** 283 **b** 128
5 **a** 3 **b** 2.5 **c** 17.5
d 19 **e** 21.5 **f** 1.5
6 **a** 34 **b** 13
c **i** 68, 72, 72, 75, 77, 78, 79, 80 **ii** 50%
d 75%
7 **a** **i** 28 **ii** 9.5
b The interquartile range, as it is not affected by the value of 35.
c 48, 48, 48, 49, 51, 53, 55; 54%

Exercise 6.03

1 **a** 2.66 **b** 2.63 **c** 1.19 **d** 1.33 **e** 2.01
2 **a** 7 **b** **i** 2.64 **ii** 2.28
c Decreases the standard deviation
3 **a** C **b** A
4 **a** 1.99 **b** 13.43
5 **a** $\bar{x} = 165.89$, $\sigma = 8.37$
b **i** less than 157 or greater than 175
ii between 157 and 174
6 **a** $\bar{x} = 11.37$, $\sigma = 0.43$
b **i** less than 10.9, greater than 11.8
ii between 10.9 and 11.8
7 C

Exercise 6.04

1 **a** Men: $\bar{x} = 71.40$, $\sigma = 6.77$
Women: $\bar{x} = 77.53$, $\sigma = 6.96$
b Yes, the mean of women's pulse rates is much higher. The standard deviation for women is slightly higher.

2 **a** Dominant hand: $\bar{x} = 0.40$, $\sigma = 0.11$;
Non-dominant hand: $\bar{x} = 0.52$, $\sigma = 0.51$

b Yes, the mean reaction time and standard deviation of the dominant hand are much lower than the mean and standard deviation of the non-dominant hand.

c **i** 0.61 and 0.75 **ii** $\bar{x} = 0.37$, $\sigma = 0.05$

iii Removing the outliers has reduced the mean from 0.40 to 0.37 and more than halved the standard deviation.

d $\bar{x} = 0.40$, $\sigma = 0.06$

e The removal of the outlier from the dominant hand had the greater effect on the mean and standard deviation as the outlier of 2.60 was a more extreme value than the outliers for the dominant hand.

3 **a** Western Tigers: $\bar{x} = 122.92$, $\sigma = 26.98$;
Barrington City: $\bar{x} = 120.92$, $\sigma = 23.62$

b The Barrington City team is slightly more consistent as the standard deviation is 23.62 compared with 26.98 for Western Tigers.

4 **a** Vatha: $\bar{x} = 13.76$, $\sigma = 0.55$;
Ana: $\bar{x} = 14.14$, $\sigma = 0.66$

b Vatha is more consistent as the standard deviation for her times is significantly lower than the standard deviation for Ana's times.

5 B

6 **a** Maths: **i** range = 47 **ii** IQR = 14 **iii** $\sigma = 10.97$
Science: **i** range = 45 **ii** IQR = 19 **iii** $\sigma = 13.16$

b Maths: $\bar{x} = 67.82$; Science: $\bar{x} = 61.25$

c The students performed better in Maths as the mean was 67.82 compared to 61.25 for Science. The marks for Maths were also more consistent as the IQR and standard deviation were both lower than those of Science.

7 **a** Roosters: **i** range = 48 **ii** IQR = 20
iii $\bar{x} = 26.67$ **iv** $\sigma = 12.55$
Dragons: **i** range = 32 **ii** IQR = 8
iii $\bar{x} = 15.88$ **iv** $\sigma = 7.36$

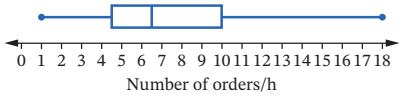
b The range, IQR and the standard deviation for the Dragons are significantly lower than those of the Roosters, which show that the Dragons are more consistent in the number of points they scored per match.

However the mean of the Roosters is significantly greater than the mean of the Dragons, which would indicate they are a better team as they were able to score many more points per match.

Exercise 6.05

1 **a** 1, 4.5, 6.5, 10, 18

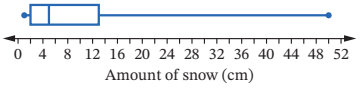
b



2 **a** 26

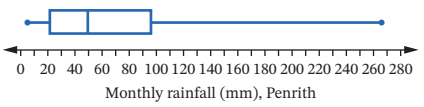
b 1, 2, 5, 13, 50

c



3 **a** 261 **b** 5, 21.5, 49.5, 96, 266

c

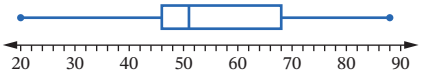


4 **a** 27.5 h **b** 26 h **c** 30 h **d** 4 h **e** 50%

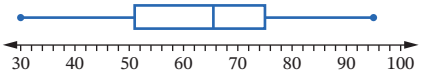
5 **a** 26 **b** 21 **c** 14 **d** **i** 25% **ii** 75%

6 **a** 6, 10, 19, 23, 29 **b** 13
c **i** 14 **ii** 7 **iii** 7 **iv** 21

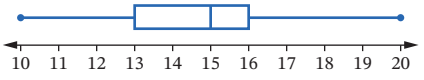
7 **a** 20, 46, 51, 68, 88



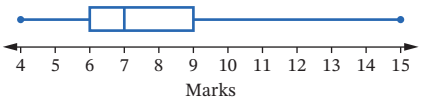
b 30, 51, 65.5, 75, 95



c 10, 13, 15, 16, 20

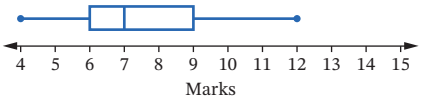


8 **a** 4, 6, 7, 9, 15



b Dot plot is positively skewed. The length of the box plot from the median to the highest value is greater than the length from the median to the lowest value.

c 15 **d** 4, 6, 7, 9, 12



e **i** The box plots are the same up to Q_3.

ii The whisker from Q_3 is reduced without the outlier.

Exercise 6.06

1 **a** **i** Year 10: 3.5; Year 8: 8

ii Year 10: 7.5; Year 8: 8.5

iii Year 10: 1; Year 8: 2

b **i** 25% **ii** 75% **c** **i** 10 **ii** 0

2 **a** **i** 32 **ii** 34

b Swifts: 59.5, Thunderbirds: 51.5

c Swifts: 8, Thunderbirds: 11

d The range for the Swifts is slightly smaller (32) and the IQR = 8 for the Swifts is less than the IQR = 11 for the Thunderbirds indicating that the Swifts are more consistent in their performance.

e The position of the Swifts box plot shows that they scored more points in games, they have a higher median of 59.5 than the Thunderbirds who had a median of 51.5 so the Swifts performed better in the season.

3 **a** 10K: 9; 10N: 10 **b** 10K: 6.5; 10N: 5.5

c 10K: 3; 10N: 4

d 10K – lower range and IQR.

e 75%

4 C

5 **a** Brisbane: 26.9, 9.3, 4.7 Sydney: 23.5, 8.5, 4.9
Melbourne: 21.4, 13, 8.6 Hobart: 18.6, 11.2, 7

b Melbourne – it has the highest range and IQR.

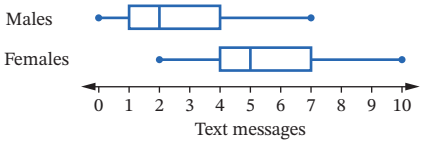
c Brisbane, more than half of the mean monthly temperatures are higher than most of the mean monthly temperatures of the other cities.

d Sydney's median temperature is significantly higher than Melbourne's, so Sydney is the warmer city.

e Sydney has the smaller range and IQR of mean monthly temperatures so it is more consistent.

6 **a** Males: 0, 1, 2, 4, 7; Females: 2, 4, 5, 7, 10

b

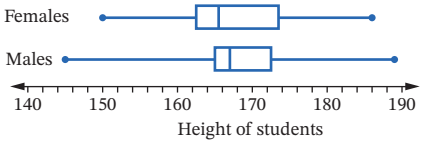


c Males: 3 Females: 3

d Males: 7 Females: 8

e Both are positively skewed, the interquartile range is the same, and the range of females is 1 more that of the males. Females do receive more text messages as the box plot shows that 75% of females receive more messages than 75% of males.

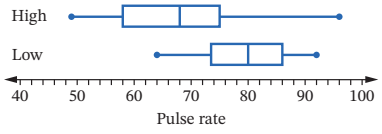
7 **a** Males: 145, 165, 167, 172.5, 189;
Females: 150, 162.5, 165.5, 173.5, 186



b Male: range = 44; IQR = 7.5
Female: range = 36; IQR = 11

c Male students have a greater range (44 compared to 36), but a smaller interquartile range (7.5 compared to 12).

8 **a** Low: 64, 73.5, 80, 86, 92; High: 49, 58, 68, 75, 96

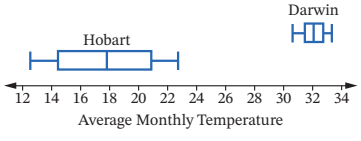


b Low: 28, 12.5; High: 47, 17

c The range and interquartile range of the High Frequency group are both greater than the Low Frequency group.

d The High Frequency group.

9 **a** Hobart: 12.5, 14.4, 17.8, 20.85, 22.7;
Darwin: 30.6, 31.45, 32.05, 32.7, 33.3



b Hobart: 10.2°, 6.45° Darwin: 2.7°, 1.25°

c Darwin average monthly temperatures were more consistent than Hobart, since its range (2.7) was much smaller than Hobart's (10.2), while Hobart's IQR was much larger (6.45) than Darwin's (1.25).

10 **a** Simone **b** Simone: 12; Amal: 10

c Amal, smaller range. **d** Simone: 10; Amal: 9

e Simone: 4; Amal: 5

f Not enough information given to make a valid decision. The interquartile range and range only differ by 1.

g 25% **h** 25%

Mental skills 6

2 **a** 160 **b** 70 **c** 240 **d** 900 **e** 2600
f 900 **g** 140 **h** 300 **i** 180 **j** 770
k 18 **l** 34 **m** 46 **n** 26 **o** 18
p 12 **q** 40 **r** 8 **s** 14 **t** 24

Exercise 6.07

1 **a** Boys: $34.58; Girls: $31.78

b Boys: $33.50; Girls: $28

c Boys: Range = 72, IQR = 25;
Girls: Range = 69, IQR = 30.5

d **i** Boys are positively skewed slightly, girls are positively skewed.

ii There are no outliers, clustering occurs for the boys in the 20–30s and for the girls in the 10–20s.

e Boys generally carry more cash – they have a higher mean than the girls and the shape of the data for girls is more positively skewed.

2 **a** 21 games **b** **i** 34 **ii** 51

c Scorpions: $\bar{x} = 1.6$ goals; Vale United: $\bar{x} = 2.4$

d Scorpions 5, Vale United 6

e The shape of both teams' results is positively skewed. Clustering for Scorpions occurs at 1 and 2 and for Vale United it occurs at 2.

f Vale United performed better as its mean was 2.4 goals/game compared to Scorpions 1.6 goals/game.

3 **a** Sydney: $\bar{x} = 26.2$, median = 26.5, mode = 28
Perth: $\bar{x} = 34.3$, median = 35, mode = 38

b Sydney: Range = 9°, IQR = 3
Perth: Range = 16°, IQR = 8

c The temperatures for Sydney and Perth are both negatively skewed, there are no outliers. Sydney's temperatures are clustered from 26 to 28, while Perth's have no distinct cluster.

d Sydney's temperatures are lower than Perth's, as evidenced by the significantly lower mean, median and mode. The range and interquartile range for Perth are greater than the range and interquartile range for Sydney, indicating greater spread.

4 **a** 30

b Quiz 1: $\bar{x} = 5.6$, mode = 6;
Quiz 2: $\bar{x} = 6.3$, mode = 7

c Quiz 1: 6; Quiz 2: 7

d Quiz 1: **i** Range = 7 Quiz 2: **i** Range = 8
ii IQR = 2 **ii** IQR = 2

e Quiz 1: Results are symmetrical with clustering 5–6, no outliers.
Quiz 2: Results are bimodal with clustering at 5 and 7–8, no outliers.

f Scores for Quiz 2 are better than Quiz 1, as the mean of Quiz 2 is higher than the mean of Quiz 1.The spread for both quizzes are similar as there is only a difference of 1 between the ranges and the IQRs are equal.

5 **a** 39

b **i** mode = 2 **ii** median = 2
iii range = 6 **iv** IQR = 1.5

c positively skewed, no outliers

d 50%

e **i** By the highest columns.
ii By the short length of the box when compared to the whole length of the box plot.

f **i** The shape of the distribution, the frequency for each household size and the mode. The mean can also be calculated from the histogram.
ii The shape of the distribution, the median and the quartiles Q_1 and Q_2.

6 **a** **i** 5 **ii** 16

b **i** mode = 22 **ii** range = 18
iii IQR = 24 – 16 = 8

c Negatively skewed.
i The tail of the dot plot goes to the left.
ii The length of the box plot from the lowest score to the median is longer than from the median to the highest score.

d **i** dot plot **ii** box plot
iii dot plot **iv** box plot

7 **a** Sunbeam Valley: range = 24, median = 71, IQR = 8
Bentley's Beach: range = 30, median = 73, IQR = 15

b Sunbeam Valley: negatively skewed (slight)
Bentley's Beach: positively skewed

c Sunbeam Valley's speeds are clustered in the 70s.

d 25%

e Bentley's Beach – higher median, positively skewed. 25% of drivers drive faster than all drivers in Sunbeam Valley. This may be due to more main roads with higher speed limits.

8 **a** 36

b Lamissa: mode = 7, median = 7
Anneka: mode = 7, median = 6

c Lamissa: range = 8, IQR = 8 – 6 = 2
Anneka: range = 9, IQR = 7 – 4 = 3

d Lamissa's distribution of scores is negatively skewed with clustering at 7. Anneke's distribution is negatively skewed with clustering at 6 and 7.

e **i** 25% **ii** 50%

f Lamissa is the better archer. Her median score is higher than Anneke's. According to the box plot, roughly 25% of Lamissa's scores are less than 6 compared to Anneke's 50%, and 50% of Lamissa's scores are equal to or better than 75% of Anneke's. Lamissa's range and IQR are also slightly smaller than Anneke's, which suggests she is the more consistent archer.

9 **a** The range (47) is too large.

b Women: 31 Men: 37

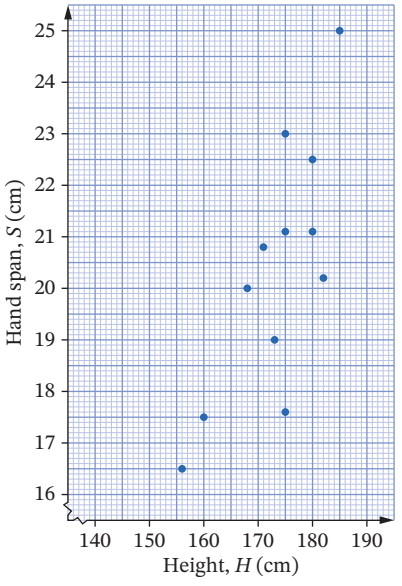
c Women: Range = 38, IQR = 40 – 24 = 16
Men: Range = 47, IQR = 46 – 25 = 21

d Distribution for women is positively skewed. Distribution for men is symmetrical.

e Men have the greater spread in the number of sit-ups completed, as the range and IQR are both greater than those for women.

Exercise 6.08

1 **a**



b linear

c As the heights of students increase, their hand spans tend to increase.

2 **a** weak negative relationship

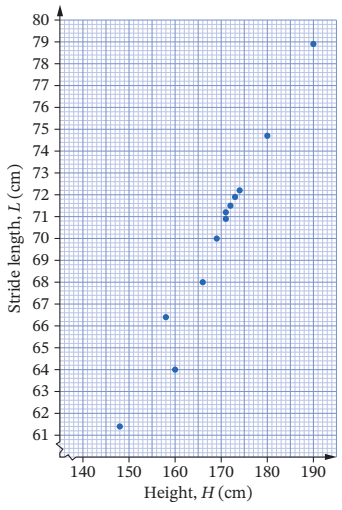
b no relationship

c strong positive relationship

3 Weak positive

4 **a** Stride length depends on a person's height as the taller the person, the longer their legs are.

b



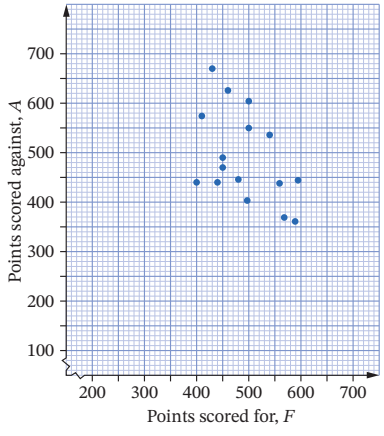
c linear

d Students' stride length increases with height

e strong positive relationship

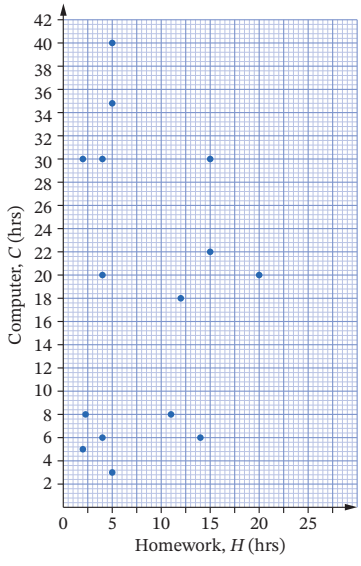
f 72.5 to 73 cm

5 a



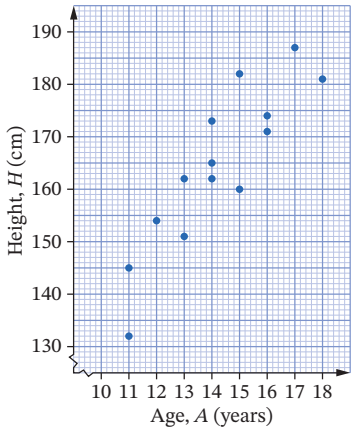
b Yes **c** Weak negative relationship

6 a



b no relationship

7 a



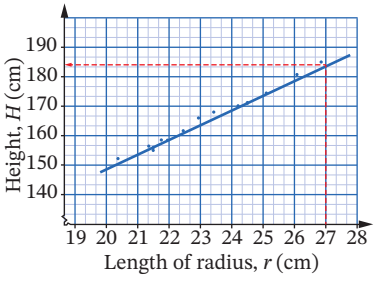
b Height, because its value depends on age (not the other way around).

c weak positive relationship

Exercise 6.09

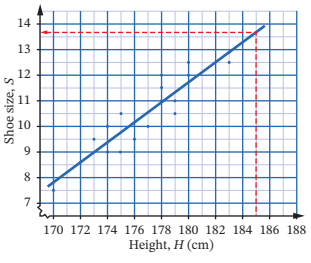
Answers will vary as equations of lines will vary.

1 a



b $H = 5r + 48.5$ **c** 173.5 cm **d** 184 cm

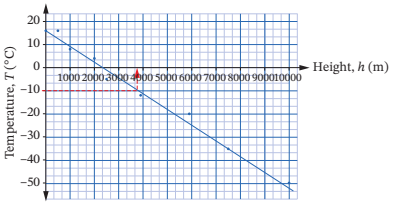
2 a



b $S = 0.39H - 59$ **c** 8

d 12 **e** 13.5

3 a

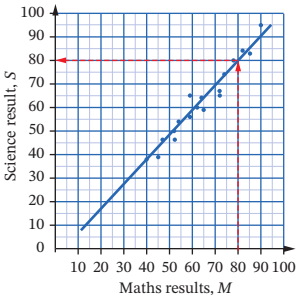


b $T = -0.0068h + 16$ **c** 5.8°C

d 3800 m

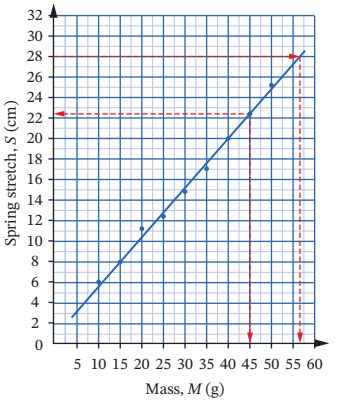
ANSWERS

4 **a**



b 80 **c** 95

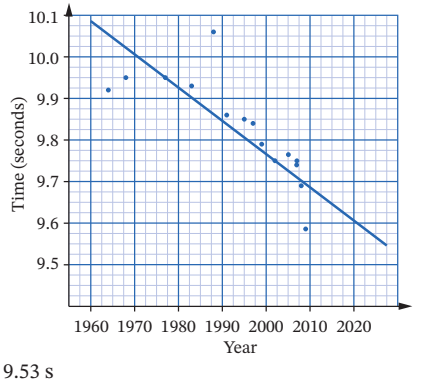
5 **a**



b 22.4 cm **c** 56 g

d Yes, because a spring has an elastic limit, which is the point at which a spring will not return to its original length as a result of the Mass attached to the spring being too heavy.

6 **a**



b 9.53 s

c There is a limit to how fast a person can run.

Exercise 6.10

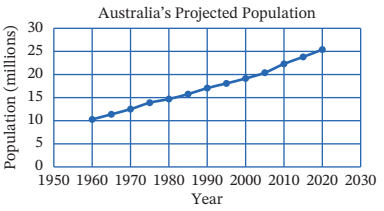
1 **a** **i** 25 **ii** 42 **iii** 15

b December, more customers due to summer and Christmas holiday season.

c June, fewer customers due to winter, busy end-of-financial year season.

d Number of people employed peaks in December, then falls, only to increase in March, April (the Easter holiday period). It then falls again to a low in June, July and then slowly the number of people employed rises to a peak in December. From 2017 to 2020, the number of people employed is showing a slow increase.

2 **a**



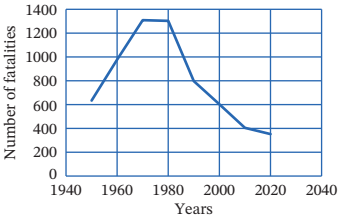
b 2005–2010

c Australia's population increased at 1.1–1.2 million every 5 years up to 1975. The population growth then slowed down for 5 years. From 1980, the population grew at a steady rate of just over a million people every 5 years, but in 2005–2010, the population then increased by 2 million for this 5-year period and this decreased slightly for 2010–2020.

d **i** approx. 28 million

ii approx. 35 million

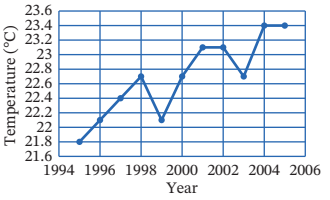
3 **a**



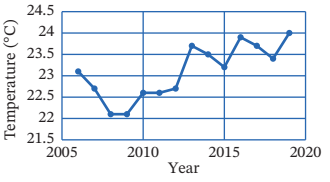
b From 1960, road fatalities rose at a steady rate, reaching a peak of approximately 1300 in 1970–1980. Road fatalities then fell at a rapid rate between 1980–1990 and continued to fall at a slower rate until 2020.

c Improved safety in cars, with seat belts being compulsory, then drink driving laws introduced and regular police presence on the roads and demerit points.

4 **a** **i**



ii

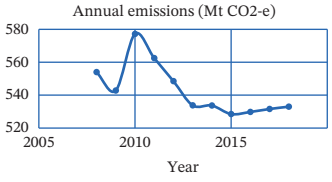


b **i** Starting at 21.8° in 1995, the temperature has seen a series of increases of less than 0.5° up to 1998, followed by a decrease of less than 1° to 1999. The increase from 1999 to 2001 is about 1°, remaining the same for 2002 then dropping by about 0.4° in 2003, increasing by about 0.7° in 2004 and remaining at 23.4° in 2005.

ii Starting at 23.1° in 2006, the temperature falls to 22.7° in 2008 and then increases to a high of 23.7° in 2013 before falling to 23.2° in 2015. From 2015, the temperature rose, to 23.9° then dropped to 23.4° in 2018 before increasing to 24° in 2019.

c Both time periods have 3 periods of big increases with smaller decreases in between. The lengths of these periods of increase and decrease are different between the 2 time periods. The range of annual temperatures for both periods is under 2°, but the minimum and maximum temperatures for 2006–2019 are about 0.6° higher than for 1995–2005.

5 a



b There was an increase of 35 Mt of Carbon emissions from 2009 to 2010, but then there was a decrease of 45 Mt overall, to 2018.

c Carbon emissions stabilised.

d More environmentally-friendly policies and practices in Australia.

e **i**, **ii** Teacher to check and discuss.

6 a 3 800 000

b 20 400 000

c 325 000 persons per year

d 30 000 000

7 a Gradual increase in passenger movements with peaks in October and troughs in February.

b **i** approx. 4.7 million **ii** approx. 5.0 million **iii** approx. 5.0 million **iv** approx. 5.1 million

c approx. 8.5%

Exercise 6.11

1 a Just surveying 300 people between 9 a.m. and 11 p.m. in shopping centres only targets a narrow group of people in certain areas.

b The sample needs to be more random and over a large area, not just in shopping centres. A telephone survey should produce more accurate feedback.

2 The report does not say what conditions are needed for the hot water system to work effectively. The temperature in Queensland is much warmer than in NSW and Victoria. Consequently, with the cooler climate in NSW and Victoria, especially in winter, the heat pump system may not provide the savings that people in Queensland obtain.

3 a **i** The price of petrol has shown little increase from December to February.

ii The price of petrol has shown marked rises and falls over the period from December to February.

b Both graphs could be improved by starting the vertical scale at 0 cents/litre.

4 a That there is a marked difference between the fuel consumption of the different cars.

b **i** 0.2 L/100 km **ii** 1 L/100 km **iii** 0.2 L/100 km

c Begin the scale on the vertical axis with 0 and use a scale of 1 cm = 0.5 L/100 km instead of 1 cm = 0.2 L/100 km.

5 Yes, as there is no option for a customer to rate the app as unsatisfactory or poor.

6 a An example of a biased question could be: Which of these colours do you prefer – red, black, silver, blue?

b Apart from surveying people, they need to look at the sales figures of all cars. This will give information about the most popular car colour.

7, 8 Teacher to check.

Exercise 6.12

1 a approx. 180 **b** approx. 220%

2 Teacher to check

3 a 75%, –2% **b** 9.2%

c–e Teacher to check

4 a Tasmania **b** 30%

c Highest: Darwin – long distance to travel to, business travellers typically need to stay overnight

Lowest: Hobart – more focused on local activities, tourism and agriculture

d Business activity is predominantly held in larger cities of Sydney and Melbourne, which are closer, whereas it is much further (in distance and time) to travel to Perth and Hobart.

e Adelaide: compared to other capital cities not a major tourist destination, also less accessible

ACT: travellers for business and government (work-related)

5, 6 Teacher to check

Power plus

1 a –1 and 1

b There is no relationship between the variables.

c **i** 1 **ii** – 0.2 **iii** –0.8

Other answers possible, teacher to check

2 **b**, **d**, **f**

3 a $\bar{x} = 13.35$, median = 14, mode = 14

b range = 10, IQR = 15 – 12.5 = 2.5

c The mean, median, and mode will increase by 4, the range and the interquartile range remain unchanged.

Test yourself 6

1 a **i** negatively skewed

ii clustering at 7 and 8, no outliers

b **i** negatively skewed

ii clustering at 16 and 17, 10 is an outlier

c **i** positively skewed

ii clustering in the 40s and 50s, no outliers

2 a 6.5 **b** 6 **c** 2.5 **d** 12.5 **e** 2

3 a $\bar{x} = 0.40$, $\sigma = 0.08$

b range = 0.33, IQR = 0.08

c The interquartile range is the better measure as the standard deviation is affected by the outlier 0.62.

4 a Girls: $\bar{x} = 67.73$, $\sigma = 16.08$

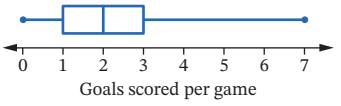
Boys: $\bar{x} = 61.67$, $\sigma = 12.35$

b The girls performed better than the boys as their mean mark was about 6 more than the mean mark of the boys. However the boys marks had less spread than the girls.

5 a range = 7, IQR = 3 – 1 = 2

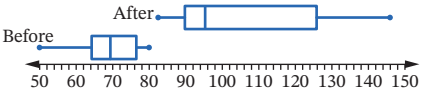
b 0, 1, 2, 3, 7

c



6 a Before: 50, 64, 69, 76, 80;
After: 82, 89, 95.5, 126, 146

b



c i range = 30, IQR = 12

ii range = 64, IQR = 37

d The pulse rates for after exercise are significantly higher. In fact, all the rates for after exercise are above all the rates for before exercise. The median pulse for after exercise is 95.5 compared to the median pulse of 69 for before exercise. The range and interquartile range are also greater for the after exercise pulse rate.

7 a i Both **ii** Stem-and-leaf plot

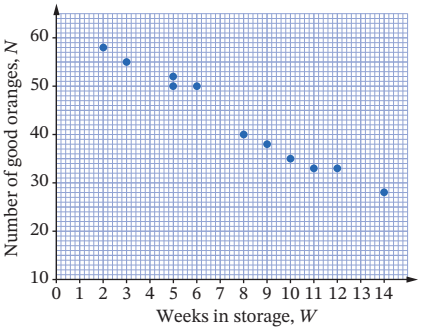
b The range (126 – 70 = 56) is too large

c i median = 92 **ii** IQR = 99.5 – 84 = 15.5

d 50%

8 a Weeks in storage: this determines how many oranges stay good.

b

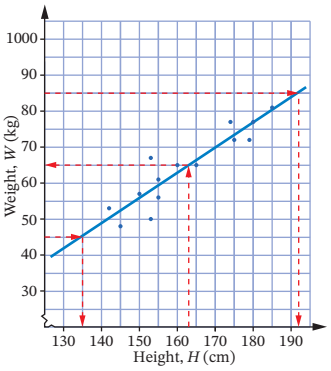


c linear

d The longer the oranges remain in storage, the less good oranges are in the box.

e strong negative correlation

9 a

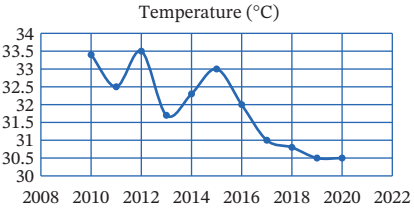


b $W = 0.7H - 49$ **c** 70 kg **d** 65 kg

e i 192 cm **ii** 135 cm

10 a Year

b



c For the last 10 years, the mean maximum temperatures for Perth, after starting at 33.4°, have dropped to 32.5°in 2011 and then after an increase dropped again to 31.7° in 2013, rising to 33° in 2015 and then dropping consistently to a finishing temperature of 30.5° in 2020. This shows there has been some change, approx. 3° overall, in the temperature for the month of January over the last 10 years.

11 a That the product is healthy.

b There is no data given on the actual fat content in the product. This should also be stated in terms of a percentage of daily requirement of fat or mg of fat.

Chapter 7

SkillCheck

1 a $a = -12$ **b** $x = 6$ **c** $x = 8$

2 a $(k + 4)(k + 1)$ **b** $(y - 8)(y - 2)$

c $(m - 8)(m + 7)$ **d** $(u + 13)(u - 5)$

e $(w - 7)(w - 3)$ **f** $(x - 6)(x + 4)$

Exercise 7.01

1 a $y = 15$ **b** $a = 9$ **c** $m = 2$

d $k = 65$ **e** $n = -35$ **f** $y = -7$

g $x = 31$ **h** $y = 46$ **i** $m = 18$

j $x = -29$ **k** $x = 24$ **l** $m = 10$

m $n = -\frac{1}{4}$ **n** $n = -\frac{3}{5}$ **o** $d = 3\frac{3}{4}$

9780170453509

2 **a** $k=1\frac{7}{8}$ **b** $w=1\frac{1}{3}$ **c** $x=-1\frac{1}{3}$

d $x=3$ **e** $y=3$ **f** $a=-8\frac{3}{5}$

g $p=9\frac{2}{3}$ **h** $y=3$ **i** $y=-\frac{5}{6}$

j $w=10$ **k** $w=50$ **l** $w=9\frac{3}{5}$

m $a=\frac{6}{11}$ **n** $y=60$ **o** $a=1\frac{11}{13}$

p $m=3\frac{1}{3}$ **q** $h=1\frac{22}{23}$ **r** $y=-6\frac{5}{6}$

3 **a** C **b** A

4 **a** $x=\frac{7}{15}$ **b** $p=\frac{109}{7}$ **c** $x=\frac{133}{7}$ **d** $x=\frac{76}{7}$

e $x=\frac{92}{9}$ **f** $y=\frac{53}{5}$ **g** $a=-41$ **h** $a=\frac{107}{14}$

i $a=\frac{75}{14}$

Exercise 7.02

1 **a** $m=\pm12$ **b** $x=\pm20$

c $y=\pm\sqrt{35}$ **d** $k=\pm13$

e $y=\pm1$ **f** $w=\pm\sqrt{24}=\pm2\sqrt{6}$

g $x=\pm2$ **h** $t=\pm4$

i $a=\pm4$ **j** $k=\pm\sqrt{45}=\pm3\sqrt{5}$

k $w=\pm10$ **l** $d=\pm12$

m $k=\pm\sqrt{14}$ **n** $w=\pm5$

o $x=\pm\frac{1}{2}$ **p** $m=\pm\sqrt{40}=\pm2\sqrt{10}$

q $y=\pm1$ **r** $p=\pm3$

s $k=\pm\sqrt{8}=\pm2\sqrt{2}$ **t** $y=\pm10$

2 **a** $m=\pm2$ **b** $a=\pm9$ **c** $m\approx\pm5.3$

d $m\approx\pm1.9$ **e** $k\approx\pm0.6$ **f** $x\approx\pm7.6$

g $k\approx\pm9.8$ **h** $k\approx\pm9.5$ **i** $y\approx\pm0.3$

j $a\approx\pm9.2$ **k** $y\approx\pm6.2$ **l** $w\approx\pm7.1$

3 **a** $x=-2,-1$ **b** $y=-4,-1$ **c** $y=-4,-12$

d $x=-4,3$ **e** $x=-3,1$ **f** $x=-8,5$

4 **a** $x=6,-5$ **b** $x=4$ **c** $x=11,-6$

d $d=0,2$ **e** $x=5,-2$ **f** $n=0,-4$

g $k=0,7$ **h** $y=0,5$ **i** $v=0,12$

5 You cannot take the square root of a negative number.

6 **a**, **c**, **f**: cannot find the square root of a negative number.

7 **a** A **b** C

Exercise 7.03

1 **a** $x=1$ **b** $m=5$ **c** $a=11$

d $u=-2$ **e** $y=-9$ **f** $n=\sqrt[3]{20}$

g $h=\sqrt[3]{11}$ **h** $k=\sqrt[3]{-48}$ **i** $m=\sqrt[3]{-15}$

j $m=4$ **k** $x=\sqrt[3]{\frac{-81}{4}}$ **l** $x=\sqrt[3]{-40}$

2 **a** $w=2.5$ **b** $m=2.5$ **c** $m=6$

d $t=3.2$ **e** $x=3$ **f** $x=-5.5$

g $x=-2.4$ **h** $x=0.8$ **i** $a=5.5$

j $a=0.4$ **k** $x=-2.3$ **l** $t=-4.4$

3 **a** yes **b** when c is positive

c when c is negative **d** no

Exercise 7.04

1 18 mm, 36 mm, 36 mm **2** Child: $21, adult: $48

3 57 mm, 19 mm **4** 13 cm, 29 cm

5 61, 62, 63 **6** Tyson is 27, Charlotte is 3

7 26 **8** 4

9 Vatha is 22, Chris is 14 **10** 213, 214, 215, 216

11 94, 96, 98 **12** 117

13 6 **14** Scott is 11, Kait is 34

15 25°, 50°, 105° **16** 72 L when full

17 8 teachers, 120 students

Mental skills 7

2 **a** 3.5 **b** 2.4 **c** 0.12 **d** 0.36

e 0.8 **f** 0.027 **g** 0.2 **h** 8.8

i 0.24 **j** 0.012 **k** 1.8 **l** 0.028

4 **a** 66.3 **b** 6630 **c** 6.63 **d** 0.663

e 6.63 **f** 663 **g** 0.663 **h** 663

i 6630 **j** 66.3 **k** 0.663 **l** 0.0663

Exercise 7.05

1 **a** 52 cm **b** 17 m

2 **a** 36 km/h **b** 86.4 km/h **c** 180 km/h

3 30.$\dot{5}$ m/s

4 **a** 27°C **b** 0°C **c** 100°C **d** 39°C

5 43

6 **a** 11.2 **b** 9 **c** 17.3

7 **a** 15.1 m **b** 31.8 cm

8 **a** 21.0 **b** 105.8 kg

9 **a** 137.3 cm^3 **b** 4.9 m

10 **a** 93 km/h **b** 436 km **c** 7 h

11 **a** $950 **b** 24 km

12 **a** 73.9 m^2 **b** $h=13.2$ cm

Exercise 7.06

1 **a** $y=\frac{5-x}{2}$ **b** $y=\frac{k-m}{p}$

c $y=\frac{P-8}{k}$ **d** $y=\frac{5m}{3}$

e $y=\frac{K-D}{M}$ **f** $y=\frac{4d-5}{8}$

g $y=\frac{2c+k}{a}$ **h** $y=\frac{20m}{3}-3$ or $y=\frac{20m-9}{3}$

i $y=\pm\sqrt{\frac{w-5}{x}}$ **j** $y=kx^2$

k $y=\frac{5n-d}{n}$ or $y=5-\frac{d}{n}$

l $y=cT^2-k$

2 **a** $b=\pm\sqrt{c^2-a^2}$ **b** $a=\frac{2(s-ut)}{t^2}$

c $a=\frac{v-u}{t}$ **d** $r=\sqrt[3]{\frac{3V}{4\pi}}$

ANSWERS

e $R=\pm\sqrt{\dfrac{A+\pi r^2}{\pi}}$ **f** $l=\dfrac{A-\pi r^2}{\pi r}$

g $n=\dfrac{S+360}{180}$ or $\dfrac{S}{180}+2$ **h** $r=\dfrac{sx}{x-s}$

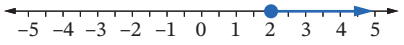
i $b=\pm\sqrt{x^2+4ac}$ **j** $x=\dfrac{5-y}{4}$

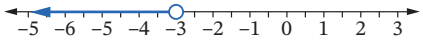
k $A=\dfrac{mn}{5-2m}$ **l** $p=\dfrac{a}{a-S}$

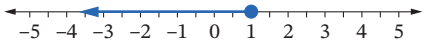
m $a=\dfrac{b(X+Y)}{Y-X}$ **n** $x=\dfrac{5-2a}{5}$ or $1-\dfrac{2a}{5}$

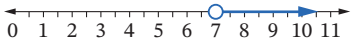
o $b=\dfrac{u(y-1)}{x-ay}$

Exercise 7.07

1 **a**


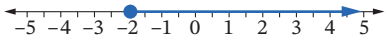
b


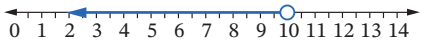
c


d


e


f


g


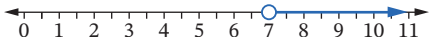
h


2 **a** $x<4$ **b** $x\geq 2$ **c** $x>-6$ **d** $x\leq 1$

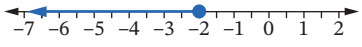
3 B

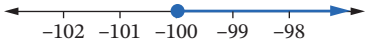
4 **a** $x\geq 1$ **b** $x<4$ **c** $x>6$
d $x\leq 2$ **e** $x>-6$ **f** $x\leq -1$
g $x\geq -4$ **h** $x\geq 25$ **i** $x<0$

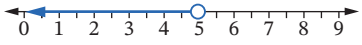
Exercise 7.08

1 **a** $x>7$


b $y\geq 4$

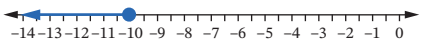

c $m\leq -2$


d $x\geq -100$


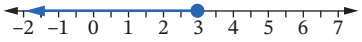
e $x<5$


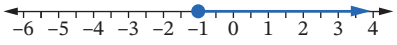
f $y>-4$

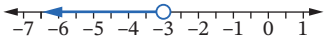

g $a\geq \frac{1}{2}$

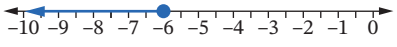

h $w\leq -10$


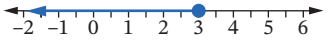
i $a\geq 5$

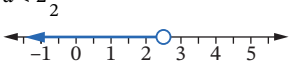

j $a\leq 3$


k $a\geq -1$


l $w<-3$


m $a\leq -6$


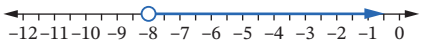
n $y\leq 3$


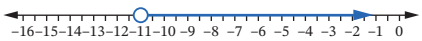
o $a<2\frac{1}{2}$


2 C

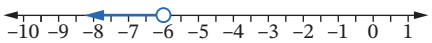
3 **a** $x\geq 1$ **b** $m\leq 6$ **c** $y\leq -8$ **d** $x<6$
e $y\geq -2$ **f** $a\leq 2\frac{4}{5}$ **g** $x\leq 5$ **h** $w>-3$
i $a<4$ **j** $m\leq 3\frac{1}{2}$ **k** $m\geq -2\frac{1}{2}$ **l** $x<35$

4 **a** $x\geq 3$


b $y>-8$


c $k>-11$


d $m\leq 0$


e $p<-6$


f $t\leq -4$


5 **a** $x>-3$ **b** $k\leq -12$ **c** $t<-2\frac{2}{5}$
d $x\geq 12$ **e** $w<-1$ **f** $y\geq -2$
g $m<4$ **h** $a>1$ **i** $d<-5\frac{1}{2}$
j $w<-11$ **k** $x\geq -\frac{2}{7}$ **l** $p<-4$

Exercise 7.09

1 **a** 2 **b** 3 **c** 2 **d** 4
e 5 **f** 3 **g** 3 **h** 2
i 6 **j** 8 **k** 6 **l** 3

2 **a** $\log_5 25 = 2$ **b** $\log_4 64 = 3$
c $\log_{10} 10\,000 = 4$
d $\log_{25} 5 = \frac{1}{2}$ **e** $\log_2 \frac{1}{16} = -4$ **f** $\log_3 \frac{1}{9} = -2$
g $\log_8 4 = \frac{2}{3}$ **h** $\log_{10} 0.01 = -2$ **i** $\log_4 \sqrt{2} = \frac{1}{4}$
j $\log_{16} 4 = \frac{1}{2}$ **k** $\log_9 27 = \frac{3}{2}$ **l** $\log_6 \frac{1}{\sqrt{6}} = -\frac{1}{2}$

3 **a** $125 = 5^3$ **b** $10 = 10^1$ **c** $27 = \sqrt{3}^6$
d $8\sqrt{2} = 2^{3.5}$ **e** $64 = 2^6$ **f** $\frac{1}{81} = 3^{-4}$
g $\frac{1}{125} = 5^{-3}$ **h** $\sqrt{2} = 8^{\frac{1}{6}}$ **i** $10 = 100^{\frac{1}{2}}$
j $5\sqrt{5} = 5^{\frac{3}{2}}$ **k** $2 = 8^{\frac{1}{3}}$ **l** $\frac{1}{100} = 100^{-1}$

4 Because a base raised to any power always gives a positive number.

Exercise 7.10

1 **a** 7 **b** 3 **c** 2 **d** −1 **e** $\frac{1}{2}$
f −2 **g** −4 **h** −4 **i** 1 **j** 2
k 2 **l** 2 **m** 1 **n** 2

2 **a** $\log_x 30$ **b** $\log_x 5$ **c** $\log_x 8$
d $\log_x 2$ **e** $\log_x 40$ **f** $\log_x 10$
g $\log_x \frac{1}{4}$ **h** $\log_x \frac{1}{5}$ **i** $\log_x 12$

3 **a** 1.2042 **b** 2.6021 **c** 3.6021 **d** 0.301 05
e −0.3979 **f** 2.2042 **g** 0.3979 **h** 0.801 05

4 **a** 1 **b** 3 **c** 3 **d** 2 **e** −1
f 0.5 **g** 3 **h** 2 **i** −1 **j** −4

5 **a** 5 **b** 3 **c** 1
d 7 **e** 0 **f** $\frac{3}{2}\log_a x$ or $\log_a\left(\sqrt{x}\right)^3$

Exercise 7.11

1 **a** $k = 9$ **b** $m = 7$ **c** $d = 10$
d $x = 2.5$ **e** $y = -4.5$ **f** $a = 3.5$
g $k = 1.5$ **h** $n = -1.5$ **i** $d = 2.75$

2 **a** $x = 1.425$ **b** $x = 2.227$ **c** $x = 2.519$
d $x = -0.943$ **e** $x = 0.428$ **f** $x = -0.661$
g $x = 7.555$ **h** $x = -0.107$ **i** $x = -1.121$
j $x = 1.011$ **k** $y = 0.975$ **l** $k = -2.069$

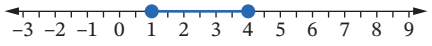
3 **a** $x = 2$ **b** $x = \frac{5}{3}$ **c** $x = \frac{5}{4}$ **d** $x = -\frac{1}{2}$
e $x = \frac{7}{2}$ **f** $x = -\frac{13}{6}$ **g** $x = \frac{5}{4}$ **h** $x = -2$

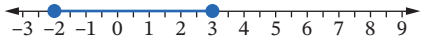
4 **a** $x = 8$ **b** $x = 1000$ **c** $x = \frac{1}{25}$ **d** $x = \frac{1}{64}$
e $x = 3$ **f** $x = 2$ **g** $x = \frac{1}{1000}$ **h** $x = 16\sqrt{2}$
i $x = \frac{1}{\sqrt{10}}$ **j** $x = \frac{1}{8}$ **k** $x = 128$ **l** $x = \frac{1}{25}$
m $x = 2$ **n** $x = \frac{1}{5}$ **o** $x = \frac{1}{2}$ **p** $x = 0.1$
q $x = 16$ **r** $x = 2$ **s** $x = 3.915$ **t** $x = 23.04$

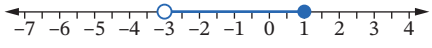
5 11.89 ≈ 12 years **6** 22.43 ≈ 23 months
7 **a** $A = 106$ g **b** $t = 20$ days **c** $t = 58$ days

Power plus

1 **a** $x = \frac{11}{13}$ **b** $x = \frac{1}{3}$ **c** $x = -8$

2 **a**


b


c


3 The solution to $D = \frac{1}{2}n(n-3) = 100$ is not a positive integer.

4 $p = 4, q = 3$

5 **a** $a = 7$ **b** $x = 20$ **c** $m = 2$ **d** $h = 5$

Test yourself 7

1 **a** $w = 6$ **b** $y = 8\frac{3}{4}$ **c** $a = -3$
d $m = 3$ **e** $s = 4$ **f** $x = 1\frac{2}{3}$

2 **a** $m = \frac{25}{3}$ **b** $y = \frac{5}{7}$ **c** $m = \frac{3}{2}$

3 **a** $y = \pm 2$ **b** $p = \pm 10$ **c** $x = \pm\sqrt{10}$
d $m = \pm 1$ **e** $w = \pm\sqrt{50}$ $\left(\text{or } \pm 5\sqrt{2}\right)$
f $x = -7, -1$ **g** $h = 9, -1$ **h** $u = 7, -11$
i $k = 0, -5$ **j** $m = 0, 2$ **k** $b = -10$
l $w = 9, 0$

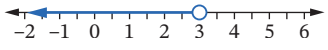
4 **a** $u = 1.9$ **b** $m = 2.9$ **c** $x = 1.4$

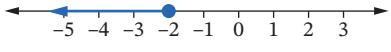
5 Grace: 13, Jamiela: 16 **6** 120 m

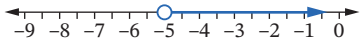
7 **a** 160 mm^3 **b** 300 m^2

8 **a** $a = \frac{y-b}{x}$ **b** $mP^2 = a$ **c** $a = \frac{1-M}{M+1}$

9 **a** $x \geq 0$


b $x < 3$


c $x \leq -2$


d $x > -5$


10 **a** $y \geq 16$ **b** $y \leq -7\frac{1}{2}$ **c** $a > -5$
d $x > -3$ **e** $a < -16$ **f** $x \leq -3$

11 **a** $6^3 = 216$ **b** $2^{-4} = \frac{1}{16}$ **c** $7^{\frac{3}{2}} = 7\sqrt{7}$

12 **a** 1 **b** 0 **c** 3

13 **a** 0.9542 **b** 2.4771 **c** 0.5229 **d** 0.9771

14 **a** $x = 1.490$ **b** $x = -0.943$
c $x = 0.236$ **d** $x = 1.420$

Practice set 2

1 **a** **i** negatively skewed
ii no outliers, clusters at 7 and 8
b **i** positively skewed
ii outlier 98, clustering in 40s and 50s

2 **a** $b = -7$ **b** $x = 10$ **c** $x = 49$

3 **a** $81n^4m^8$ **b** $4a^4b^3$ **c** $\frac{n^2}{9m^8}$

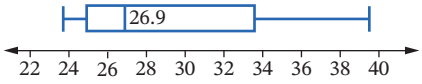
4 **a** $\frac{6a-20}{15}$ **b** $\frac{5x+36}{24}$ **c** $\frac{7r-6}{10}$

5 **a** range = 33, IQR = 7
b Interquartile range, because it is not affected by the outlier of 112.

6 **a** $x = \pm 6\sqrt{3}$ **b** $n = 0$ or 64 **c** $u = -15$ or 4

7 **a** $\frac{9d}{2}$ **b** $\frac{2m}{3}$ **c** $\frac{pv}{3}$

8 **a** range 15.8, IQR = 8.7
b 23.7, 24.9, 26.9, 33.6, 39.5
c April's maximum temperature first fortnight 2020



d 39.5°C

9 **a** weak negative **b** strong negative
c weak positive

10 **a** 4 **b** 16 **c** $\frac{1}{12}$ **d** $\frac{1}{1000}$

11 $y = -5.8$

12 **a** $53gh - 45gh^2$ **b** $7y^4 + 2y^3$

13 **a** Girls: mean = 62.7, standard deviation = 16.1
Boys: mean = 66.9, standard deviation = 12.2
b The boys performed better as their mean is higher.

14 **a** $x^2 + 14x + 49$ **b** $25m^2 - 20mn + 4n^2$
c $9n^2 - 100$

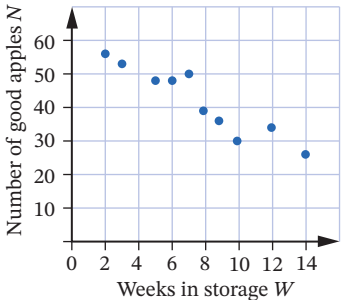
15 20, 21, 22

16 **a** **i** stem-and-leaf plot **ii** stem-and-leaf plot
b 41 **c** 18

17 **a** $(y - 16)(y - 2)$ **b** $(n + 8)(n - 6)$ **c** $(a - 9)(a + 8)$

18 **a** The independent variable is W, the weeks in storage. The number of weeks in storage is set first after which time the number of good apples is counted.

b



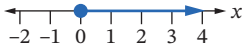
c The number of good apples decreases the longer the apples are in storage.

d There is a strong and negative relationship between the variables W and N

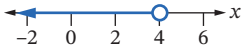
19 **a** $(5n + 2)(n + 1)$ **b** $(2a + 3)(a - 5)$
c $(4x - 3)(2x + 5)$

20 $n = 3t^2$

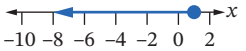
21 **a**



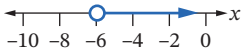
b



c



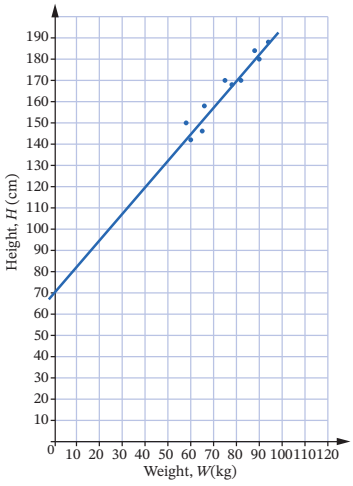
d



22 **a** $n \geq 5$ **b** $k < -7$ **c** $y \geq -1\frac{1}{2}$ **d** $x \leq 2\frac{1}{2}$

23 **a** $\frac{d-4}{d-1}$ **b** 16 **c** $\frac{3(y+1)}{y(y-3)}$

24 **a**



b $H = 1.2W + 70$; other answers possible.
c 48 kg **d** 170 cm

25 **a** 1 **b** 0 **c** 3

26 **a** $x = \frac{3}{4}$ **b** $x \approx 3.453$

Chapter 8

SkillCheck

1 **a** −1 **b** 29 **c** −3 **d** 69

2 **a** 4 **b** $-\frac{1}{2}$ **c** $\frac{8}{5}$ **d** −3.2

3 **a** 625 **b** 3125 **c** 1 **d** $\frac{1}{25}$

Exercise 8.01

1 **a** 190, $D = 190T$
b **i** 3.8 km **ii** 8.55 km
c 1 h 5 min

2 **a** 26.2, $E = 26.2h$ **b** $183.40 **c** 5.5 h

3 **a** $I = \frac{16D}{425}$ **b** \$33.88 **c** \$67.76

4 A

5 $b = 2.5a$

6 **a**

h	c
1	\$7.50
2	\$15
3	\$22.50

b $c = 7.5$ h **c** \$45 **d** 11

e 7.5. It is the same.

7 C

8 **a** $F = 0.006m$ **b** 15 L/100 km

9 A

10 **a** 22.8 kg **b** 84.1 kg

Exercise 8.02

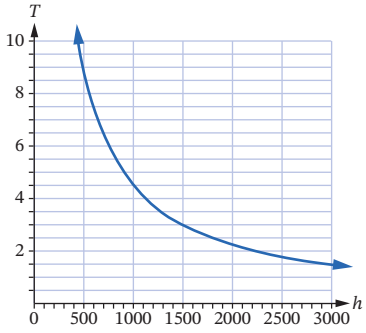
1 **a** 920, $T = \frac{920}{s}$ **b** 10 h 13 min **c** 92 km/h

2 **a** 4500, $T = \frac{4500}{h}$

b **i** 15°C **ii** 1.8°C

c **i** 562.5 m **ii** 200 m

d



3 **a** $N = \frac{112}{S}$ **b** 101 **c** 2.0 m² **d** 28

4 C **5** B **6** $\frac{14}{15}$

7 **a** 8 min **b** 4 people

8 **a** $b = \frac{8}{a}$ **b** $b = \frac{100}{a}$

9 **a** $F = \frac{112}{L}$ **b** 6 beats/sec **c** 25 cm

10 **a** $y = \frac{1}{16}$ **b** $x = 1\frac{1}{4}$

11 **a** 2.5 h **b** 5 friends

Exercise 8.03

1 **a** **i** £22 **ii** £49

b **i** \$33 **ii** \$108

c **i** £22 – £20 = £2 **ii** \$120 – \$108 = \$12

2 **a** **i** 2 km **ii** 20 km **iii** 34 km

b **i** 50 furlongs **ii** 125 furlongs **iii** 180 furlongs

c 60 km **d** 500 furlongs

3 **a** **i** ¥16 000 **ii** ¥63 000 **iii** ¥78 000

b **i** \$250 **ii** \$760 **iii** \$920

4 **a** –18°C **b** 10°C **c** 26°C

d 32°F **e** 14°F **f** 86°F

5 **a** 4.9 ha **b** 2 ha **c** 10.8 acres **d** 12.2 acres

e **i** 32 000 m² **ii** 3.2 ha **iii** 7.8 acres

6 **a** **i** ₱520 **ii** ₱210 **iii** ₱310

b \$10.20 **c** \$5.80 **d** ₱4200

Exercise 8.04

1 **a** The cyclist leaves the starting point, travels at a speed of 20 km/h for 1 hour; stops for 1 hour; then continues for another hour at a speed of 10 km/h. At D, the cyclist stops for half an hour, then cycles back towards the starting point at a speed of 30 km/h for 1 hour.

b No, as the gradients of the intervals are all different.

c The cyclist is moving back towards the starting point.

2 **a** Kate 133 m/min, Camila 114 m/min

b Kate, in 9 minutes **c** 1.5 min

d 680 m **e** 120 m

f The graph shows the distance they move down the slope and this increases as more time passes.

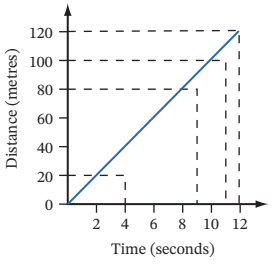
3 **a** The person starts the journey fast (the graph is steep), then slows down (the graph becomes less steep) before increasing speed again (the graph becomes steeper).

b The person is travelling or returning towards a specific place. Initially the person's speed is fast, then he slows down and stops (the graph is horizontal).

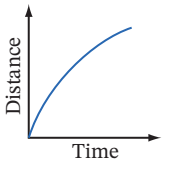
c The person starts the journey at a high speed and then gradually slows down to a stop.

4 **a** H **b** D **c** A–B **d** F **e** E **f** C

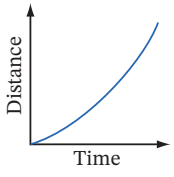
5



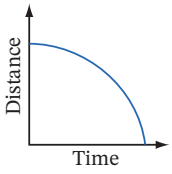
6 **a**



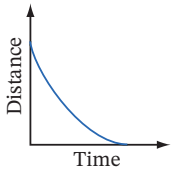
b



c



d



ANSWERS

7 **a** **I** C **II** B **III** A

b I is the steepest (has the greatest gradient) and must be the fastest (Jade).

II is the least steep (the smallest gradient) and must be the slowest (Cameron).

III The slope of this graph is between the other two (Kiet).

c Jade stopped to talk to a friend. (Other answers possible.)

d This person speeds up slightly and maintains speed for a while, slowing down gradually to a stop.

8 **a** C **b** D **c** E **d** F **e** B **f** A

Exercise 8.05

1 **a** Temperature increases to maximum on first day then cools down to the initial temperature. Then second day temperature increases to slightly higher maximum than first day and then returns to similar minimum temperature as for previous day.

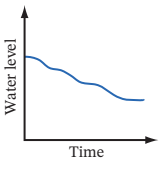
b This graph could indicate 4 high tides and 4 low tides in a reasonably regular pattern.

c Amount of petrol decreases at constant rate. Petrol tank is filled, then petrol is used at a constant rate.

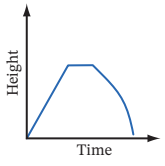
d Height of person increases slowly at first then more quickly (possibly teenage years/growth spurt indicated by steeper graph) then slows down as final height reached.

2 **a** B **b** A **c** C **d** B **e** A

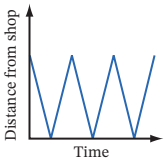
3 **a**



b



c



4 **a** H **b** A **c** C **d** F **e** I **f** D **g** B **h** E **i** G

5 **a** A **b** B **c** B **d** A

6 **a** B **b** H **c** A **d** F **e** C

7 **a** C **b** E **c** F **d** D **e** A **f** B

8 **a** E **b** F **c** B **d** C **e** A **f** D

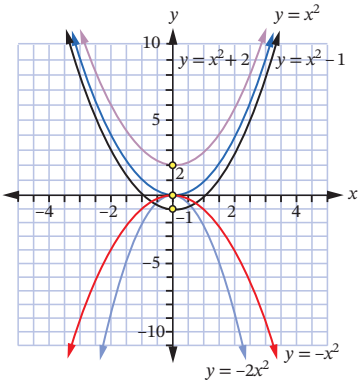
Mental skills 8

2 **a** 2, 5 **b** 3 **c** 2, 3, 6 **d** 3, 5 **e** 2 **f** 2, 3, 5, 6 **g** 5 **h** 3

3 **a** 4, 9 **b** 4, 10 **c** 9 **d** 10 **e** 4, 9 **f** none of these **g** 9 **h** 4, 9, 10

Exercise 8.06

1 **a**



b **i** $y = x^2, y = x^2 + 2, y = x^2 - 1$

ii $y = -x^2, y = -2x^2$

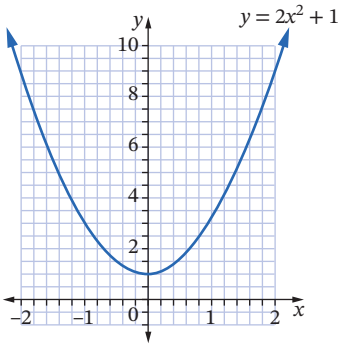
iii $y = x^2, y = -x^2, y = -2x^2$

2 A **3** C

4 **a** F **b** I **c** A **d** K **e** J **f** C **g** B **h** L **i** H **j** E **k** G **l** D

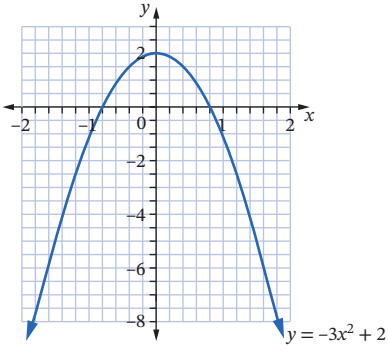
5 **a** $y = -x^2$ **b** $y = x^2$ **c** $y = -x^2 - \frac{1}{4}$ **d** $y = -x^2 - 9$ **e** $y = \frac{1}{2} - x^2$ **f** $y = x^2 + 9$

6 **a**



b (0, 1) **c** concave up **d** $y = 1$

7 **a**



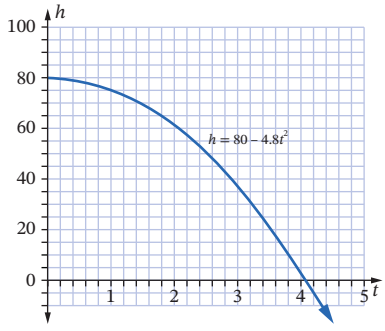
b (0, 2) **c** $x = 0$ **d** $y = 2$

8 A

9 **a** **i** narrower **ii** up **iii** (0, 3)
b **i** wider **ii** up **iii** (0, 1)
c **i** narrower **ii** down **iii** (0, −5)
d **i** wider **ii** down **iii** (0, −12)

10 **a** $x = \pm 4$ **b** $x = \pm 11$

11 **a**

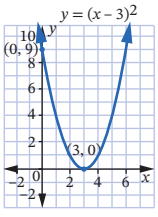


b 80 m **c** 36.8 m **d** 4.1 s **e** 3.95 s

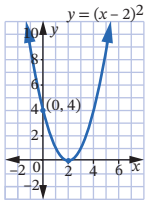
12 **a** B **b** G **c** D **d** J
e E **f** I **g** A **h** K
i H **j** L **k** F **l** C

Exercise 8.07

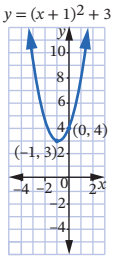
1 **a**



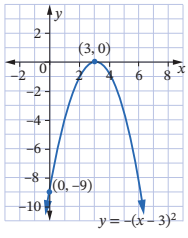
b



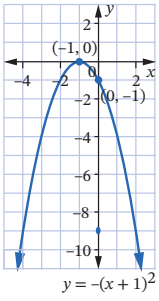
c



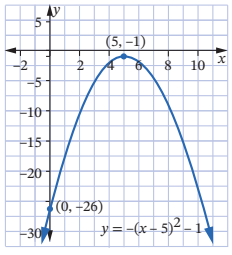
d



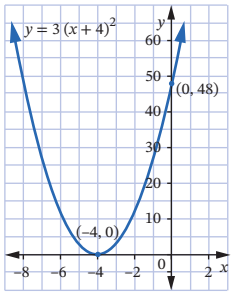
e



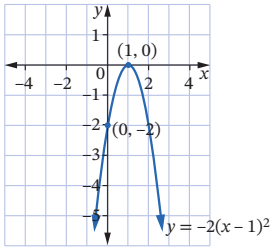
f



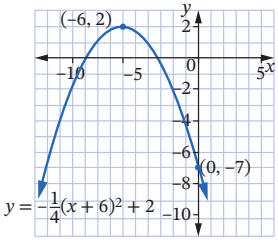
g



h



i

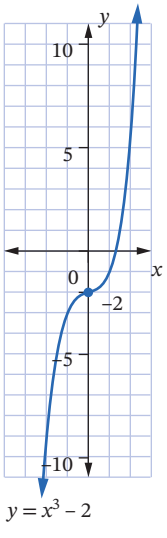


2 B

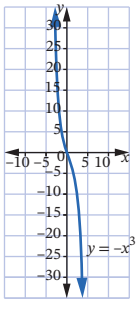
3 **a** D **b** G **c** A **d** E
e B **f** H **g** F **h** C

Exercise 8.08

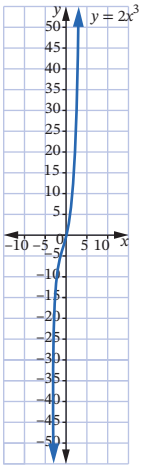
1 **a**



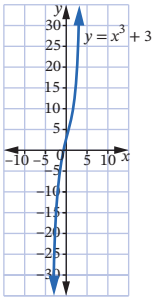
b



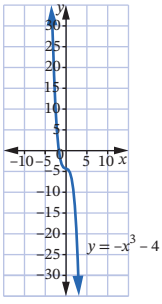
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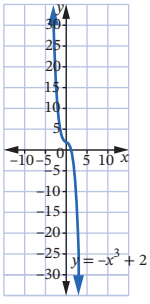
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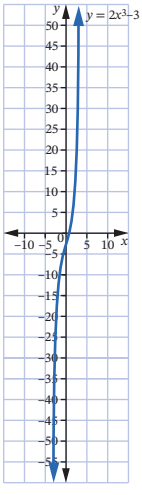
e



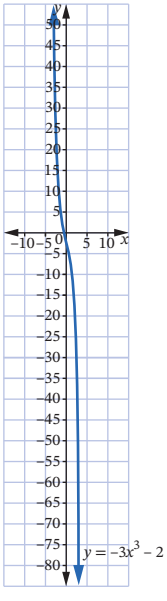
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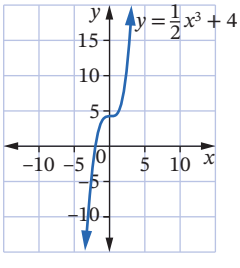
g



h



i

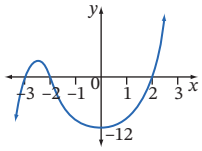


2 **a** A **b** E **c** G **d** C **e** F
f D **g** H **h** I **i** B

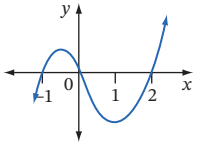
3 **a** $y = 2x^3 - 3$ **b** $y = -4x^3 + 1$

Exercise 8.09

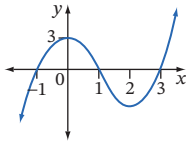
1 **a**



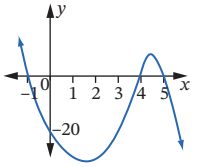
b



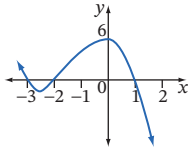
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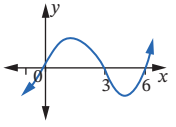
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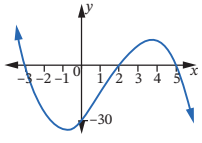
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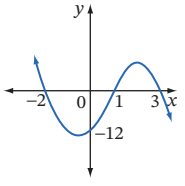
f



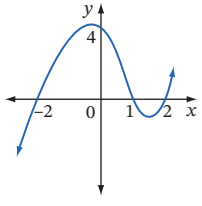
g



h



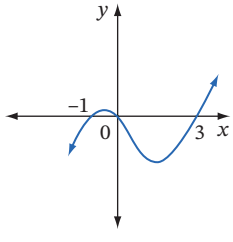
i



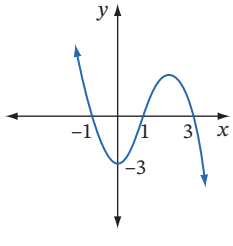
2 B **3** A

4 **a** Substitute $x = 0$. **b** Substitute $y = 0$ and solve.

5 **a** x-intercepts are -1, 0, 3 and y-intercept is 0.

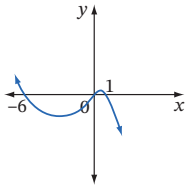


b x-intercepts are -1, 1, 3 and y-intercept is -3.

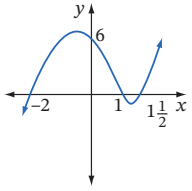


9780170453509

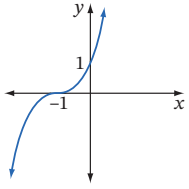
c x-intercepts are -6, 0, 1 and y-intercept is 0.



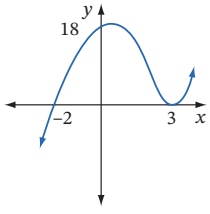
d x-intercepts are -2, 1 and $1\frac{1}{2}$ and y-intercept is 6.



e x-intercept is -1 and y-intercept is 1.



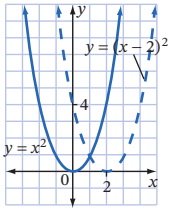
f x-intercepts are -2 and 3 and y-intercept is 18.



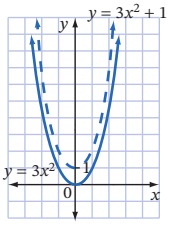
Exercise 8.10

1 **a** E **b** H **c** I **d** C **e** G
f A **g** F **h** D **i** B

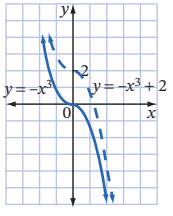
2 **a**



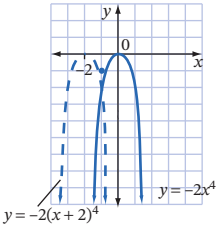
b



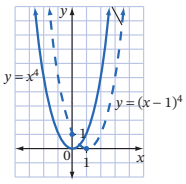
c



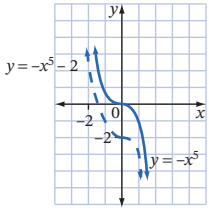
d



e



f



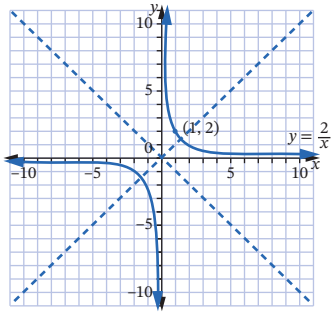
3 **a** Move up 4 units. **b** Move right 5 units.
c Move left 3 units. **d** Move up 4 units.
e Move right 3 units. **f** Move left 2 units.

Exercise 8.11

1 **a**

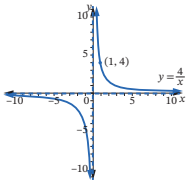
x	−3	−2	−1	0	1	2	3
y	$-\frac{2}{3}$	−1	−2	–	2	1	$\frac{2}{3}$

b, c

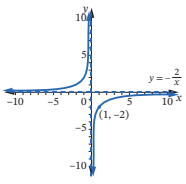


d $y = x, y = -x$

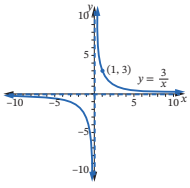
2 **a**



b



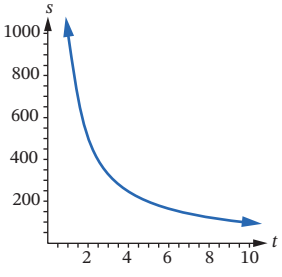
c



3 **a**

t	1	2	3	4	5	6	7	8	9	10
s	1000	500	333	250	200	167	143	125	111	100

b

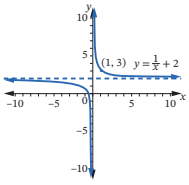


c The time taken is always positive and it is impossible to travel with zero time. Also, you cannot divide by zero.

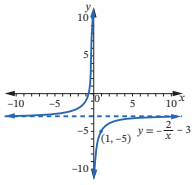
d Yes, when $t = 2$ h, $s = 500$ km/h and when $t = 4$ h, $s = 250$ km/h.

4 **a** $k = 3$ **b** $y = \frac{3}{x}$

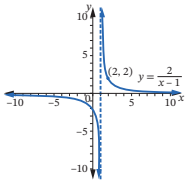
5 **a**



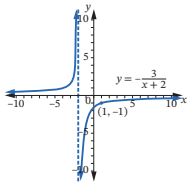
b



c



d



6 **a** $c = 1, k = -6$ **b** $y = -\frac{6}{x} + 1$

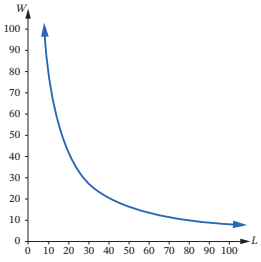
7 **a**

L	10	20	30	40	50	60	70	80	90	100
W	80	40	27	20	16	13	11	10	9	8

b $WL = 800$ or $W = \frac{800}{L}$

c If the length or width equals zero, the block of land doesn't exist.

d



e As the length increases, the width decreases. The graph flattens out and gets closer to the horizontal axis, but never touches it (an asymptote).

f As the length decreases, the width increases. The graph is steeper and gets closer to the vertical axis, but never touches it (an asymptote).

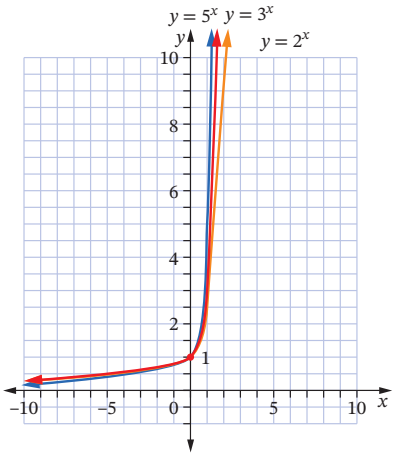
8 A

9 **a** $x = 0, y = 0$ **b** $x = 0, y = c$

c $x = b, y = 0$ **d** $x = b, y = c$

Exercise 8.12

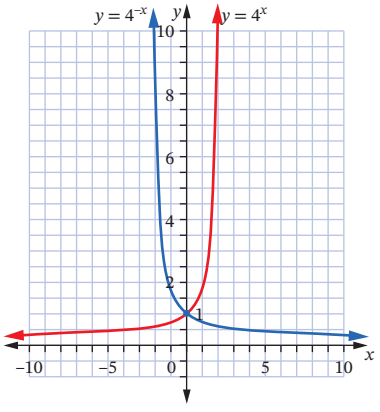
1 **a**



b 1

c becomes steeper, in 1st quadrant

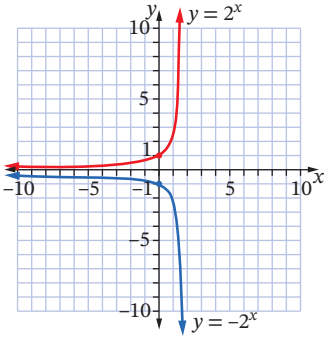
2 **a**



b **i** $y = 4^{-x}$ **ii** $y = a^{-x}$

3 B

4 **a**

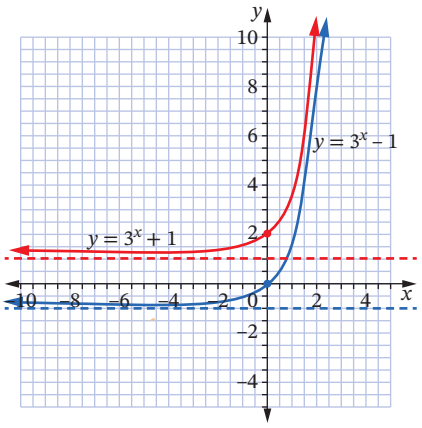


b one is the reflection of the other in the x-axis

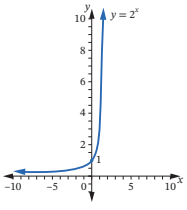
c $y = -a^x$

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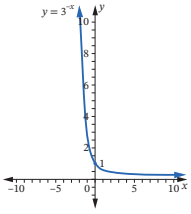
5 Same shape, shifted down 2 units.



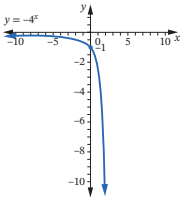
6 **a**



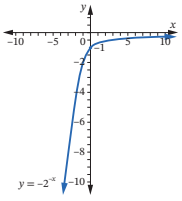
b



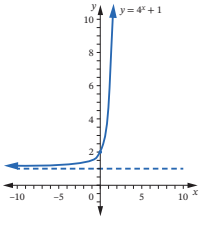
c



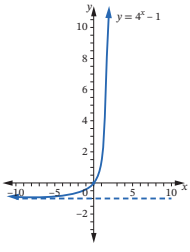
d



e



f



7 $y = 4^{-x}$

Exercise 8.13

1 **a** centre (0, 0), $r = 2$ **b** centre (0, 0), $r = 6$
c centre (0, 0), $r = 8$ **d** centre (0, 0), $r = 10$
e centre (0, 0), $r = 9$ **f** centre (0, 0), $r = 5$

2 D

3 **a** LHS $= 8^2 + 6^2$
$= 100$
b LHS $= 5^2 + 9^2$
$= 106$
c outside

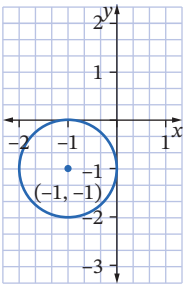
4 **a** inside **b** on **c** outside
d inside **e** outside

5 **a** centre (−2, 4), $r = 7$
b centre (3, 1), $r = 1$
c centre (9, 12), $r = 15$
d centre (0, −3), $r = 2$
e centre (−6, −1), $r = \sqrt{10}$
f centre (−5, −8), $r = 4$
g centre (0, 0), $r = 6\sqrt{2}$
h centre (−2, 1), $r = 5\sqrt{2}$
i centre (2, 0), $r = 8$
j centre (4, −3), $r = 2.5$
k centre (3, 4), $r = 9$
l centre (0, −1), $r = \frac{1}{3}$

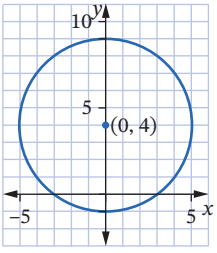
6 B

7 **a** $(x - 1)^2 + (y + 2)^2 = 9$
b $(x - 10)^2 + (y + 11)^2 = 4$
c $(x + 3)^2 + (y - 2)^2 = 100$
d $x^2 + (y + 1)^2 = 1$
e $(x + 6)^2 + (y - 2)^2 = 5$
f $(x + 1)^2 + (y - 5)^2 = 8$

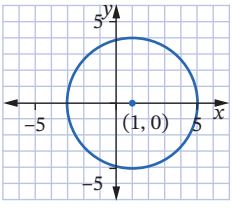
8 **a**



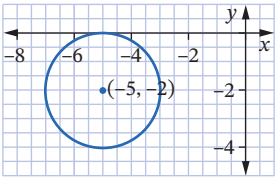
b



c



d



9 **a** 1, 1 **b** 9, 3 **c** 16, 4 **d** 4, 2
e $\frac{49}{4}, \frac{7}{2}$ **f** $\frac{9}{4}, \frac{3}{2}$ **g** $\frac{1}{4}, \frac{1}{2}$ **h** $\frac{25}{4}, \frac{5}{2}$

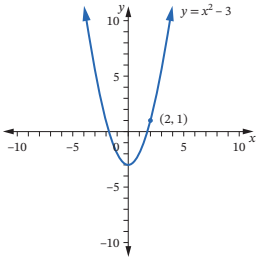
10 **a** $(-3, 1), r = 5$ **b** $(4, 2), r = 7$ **c** $(-2, 5), r = 6$
d $(-10, 6), r = 1$ **e** $(2, -4), r = 5$ **f** $(6, -3), r = 4$
g $(-3, 10), r = 9$ **h** $(4, -1), r = 2\sqrt{3}$

Exercise 8.14

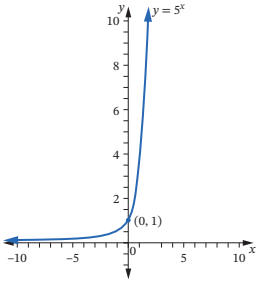
1 **a** P **b** L **c** E **d** L **e** C **f** L
g P **h** L **i** P **j** E **k** P **l** C

2 **a** G **b** J **c** H **d** D **e** A
f F **g** C **h** E **i** B **j** I

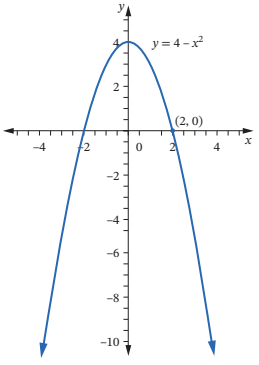
3 **a**



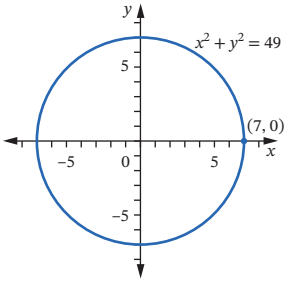
b



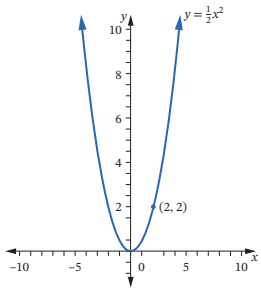
c



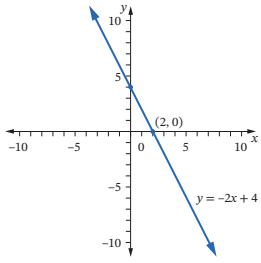
d



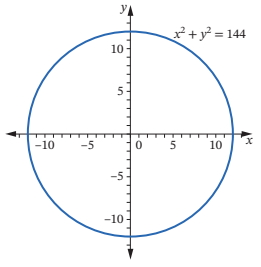
e



f



g



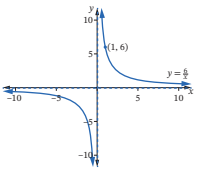
4 **a** 1 **b** 3 **c** −6 **d** 1

5 **a** H **b** C **c** P **d** H
e Q **f** C **g** E **h** H

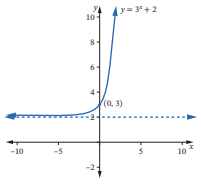
6 **a** D **b** E **c** C **d** A
e H **f** B **g** F **h** G

7 **a** **i** exponential **ii** 1 **iii** $y = 0$
b **i** exponential **ii** 2 **iii** $y = 1$
c **i** exponential **ii** −2 **iii** $y = -3$
d **i** hyperbola **ii** none **iii** $x = 0, y = 0$
e **i** hyperbola **ii** $-\frac{2}{3}$ **iii** $x = 3, y = 0$
f **i** hyperbola **ii** none **iii** $x = 0, y = 2$

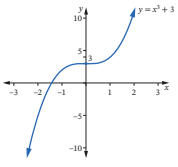
8 **a**



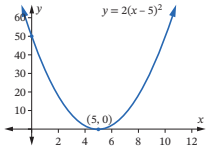
b



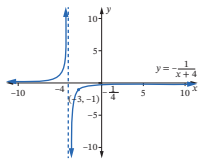
c



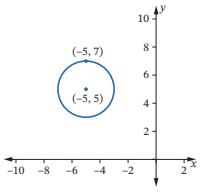
d



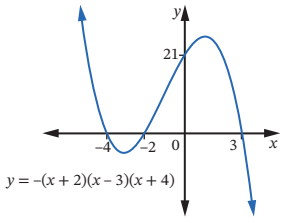
e



f

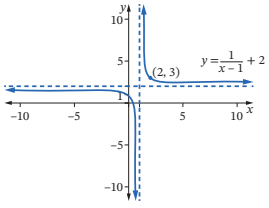


g

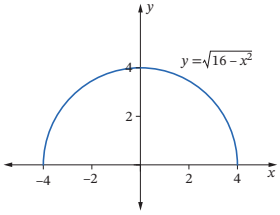


Power plus

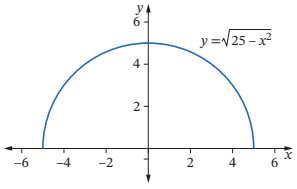
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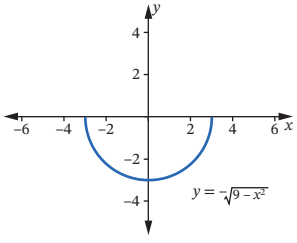
2 a centre (0, 0) and $r = 4$



b centre (0, 0) and $r = 5$



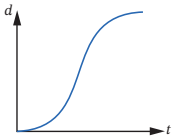
c centre (0, 0) and $r = 3$



Test yourself 8

1 $H = 310.5$ **2** 10°C **3 a** £39 **b** $A101

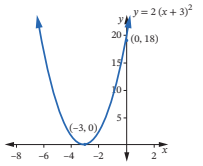
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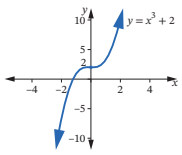
5 a C **b** A **c** B

6 a C **b** F **c** A **d** E **e** D **f** B

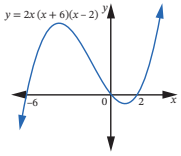
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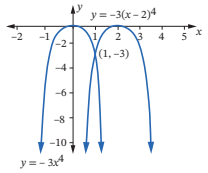
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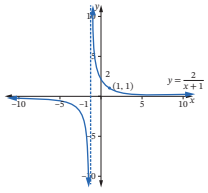
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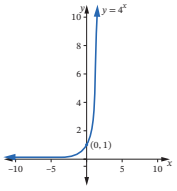
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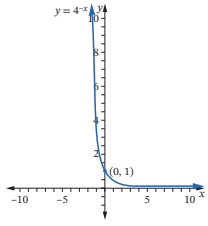
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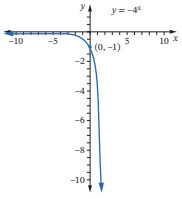
12 a



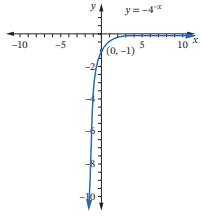
b



c



d



13 a centre (0, 0), $r = 10$ **b** centre (0, 0), $r = 6$
c centre (0, 0), $r = 7$ **d** centre (−5, 6), $r = 9$
e centre (10, 0), $r = 15$ **f** centre (−7, −10), $r = 4\sqrt{5}$

14 a D **b** C **c** B **d** J **e** E **f** H
g L **h** G **i** I **j** A **k** K **l** F

ANSWERS

Chapter 9

SkillCheck

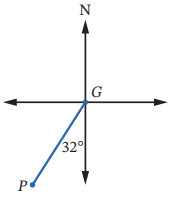
1 **a** 64° **b** 26° **c** 12° **d** 86°
2 **a** 0.8480 **b** 0.7760 **c** 0.1539 **d** 64.9839
e 13.9884 **f** 13.7044 **g** 0.0336 **h** 8.8133
3 **a** 45°48′ **b** 33°11′ **c** 5°21′ **d** 84°23′
4 **a** 64°37′ **b** 69°41′ **c** 28°8′

Exercise 9.01

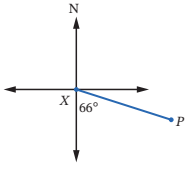
1 **a** 64.7 cm **b** 14.2 cm **c** 54.5 cm
d 18.5 cm **e** 5.1 cm **f** 17.4 cm
g 48.8 cm **h** 59.0 cm **i** 17.5 cm
2 C
3 **a** 38°41′ **b** 56°7′ **c** 42°56′
d 52°57′ **e** 64°37′ **f** 45°1′
4 **a** 73° **b** 5.7 m
5 A **6** 13°53′
7 114 m **8** 0.63 m or 63 cm
9 **a** 51°20′ **b** $\frac{5}{4}$
10 2.6 km **11** 24.6 m **12** 1.5 km
13 **a** 12.4 m **b** 11.9 m
14 701 m **15** 63°
16 12 km **17** 95.6 m
18 864 m above sea level
19 149.1 m **20** 129.6 m

Exercise 9.02

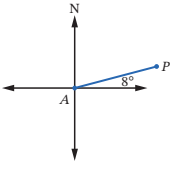
1 **a** 118° **b** 205° **c** 311°
d 043° **e** 142° **f** 252°
g 285° **h** 224° **i** 059°
2 **a** 000° **b** 090° **c** 180° **d** 270°
e 343° **f** 018° **g** 125° **h** 292°
i 234° **j** 169° **k** 048°
3 **a** SSE **b** 157.5°
4 **a** 225° **b** 090° **c** 045° **d** 112.5°
e 292.5° **f** 270° **g** 135° **h** 247.5°
5 **a**



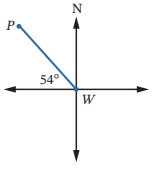
b



c



d



6 320° **7** 280°
8 **a** 90° **b** 157.5° **c** 157.5° **d** 67.5°
9 ESE
10 **a** 4.89 km **b** 038°
11 **a** 738 km **b** 251°
12 **a** 39 km **b** 332.8°
13 155.4 km **14** 015°
15 **a** 1281 km **b** 024°27′
16 326° 11′

Exercise 9.03

1 **a** $4\sqrt{13}$ cm **b** 15.6 cm **c** 23°
2 **a** $\sqrt{800}$ cm or $20\sqrt{2}$ cm **b** 34.64 cm
c 35°16′
3 **a** 20.40 cm **b** 79° **4** 34°
5 **a** 9.1 cm **b** 28°
6 **a** 53 m **b** 113 m **7** 37.5 m
8 **a** 50° **b** 868 m
9 **a** 85 m, 50 m **b** 99 m apart
10 **a** $\angle WHF = 52°$, $\angle WFH = 38°$ and $\angle HWF = 90°$
b 49.1 m
11 **a** 285° **b** 7°
12 **a** 26°, 054° **b** 9120 m
13 **a** 4.5 km **b** 210° 38′

Exercise 9.04

1 **a** 43° **b** 16° **c** 87.45°
d 34.8° **e** 51° 43′ **f** 24° 34′
g 25° **h** 58° **i** 29°
2 **a** 0 **b** 1 **c** 1 **d** 2
3 $\cos\beta = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\beta = \frac{12}{13}$
4 $\sin F = \frac{40}{41}$, $\sin E = \frac{9}{41}$, $\cos F = \frac{9}{41}$
5 $\cos Y = \frac{\sqrt{5}}{3}$, $\sin Y = \frac{2}{3}$, $\sin X = \frac{\sqrt{5}}{3}$
6 $\cos\phi = \frac{\sqrt{5}}{4}$, $\sin\phi = \frac{\sqrt{11}}{4}$, $\cos\theta = \frac{\sqrt{11}}{4}$
7 **a** 8 **b** 12 **c** $3\sqrt{2}$
d $\frac{20}{\sqrt{3}}$ or $\frac{20\sqrt{3}}{3}$ **e** 45° **f** 30°
g 30°
8 $35\sqrt{3}$ m
9 **a** $8\left(1-\frac{1}{\sqrt{3}}\right)$ or $\frac{8(3-\sqrt{3})}{3}$ **b** $5(\sqrt{3}-1)$

Mental skills 9

2 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{7}$ **d** $\frac{1}{2}$
e $\frac{1}{4}$ **f** $\frac{1}{6}$ **g** $\frac{5}{6}$ **h** $\frac{2}{5}$
i 5 : 9 **j** 5 : 9 **k** 9 : 20 **l** 4 : 5
m 9 : 7 **n** 4 : 3 **o** $\frac{3}{5}$ **p** $\frac{4}{35}$
3 **a** $\frac{17}{40}$ **b** $\frac{2}{3}$ **c** $\frac{16}{25}$ **d** $\frac{1}{4}$ **e** $\frac{5}{24}$ **f** $\frac{2}{25}$

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Exercise 9.05

1 **a** $\tan A = \frac{60}{91}$ **b** $\tan Y = 1.\dot{3}$ **c** $\tan X = \frac{2}{3}$
d $\tan P = \frac{9}{40}$ **e** $\tan Q = \frac{\sqrt{40}}{3}$ **f** $\sin X = \frac{11}{61}$
g $\cos X = \frac{7}{25}$ **h** $\sin X = \frac{2}{3}$

2 **a** P **b** P **c** N **d** N
e P **f** N **g** N **h** P

3 **a** –0.89 **b** –0.19 **c** –0.77 **d** –0.11
e 0.51 **f** –0.58 **g** 0.05 **h** –0.42
i –0.78 **j** –0.87 **k** 0.18 **l** 0.28

4 **a, b** Teacher to check.
c The graph has a wave shape that repeats itself after 360°.
Maximum $y = 1$ at $\theta = 90°$; Minimum $y = -1$ at $\theta = 270°$.
d No
e Yes, centre of symmetry at (180°, 0).
f **i** $0° < \theta < 180°$ (1st and 2nd quadrants)
ii $180° < \theta < 360°$ (3rd and 4th quadrants)

5 **a, b** Teacher to check.
c The graph has a wave shape that repeats itself after 360°.
Maximum $y = 1$ at $\theta = 0°$ and $\theta = 360°$; Minimum $y = -1$ at $\theta = 180°$
d Yes, axis of symmetry $\theta = 180°$
e No
f **i** $0° < \theta < 90°$ and $270° < \theta < 360°$ (1st and 4th quadrants)
ii $90° < \theta < 270°$ (2nd and 3rd quadrants)
g Similarities: Both graphs have the same wave shapes that run between $y = -1$ and $y = 1$ and repeat themselves after 360°.
Differences: The graphs have different x- and θ-intercepts

6 **a** 10° **b** 70° **c** 50°
d 83° **e** 65° **f** 12°

7 **a** –cos 38° **b** sin 75° **c** –cos 25°
d –tan 78° **e** –cos 7.3° **f** sin 64.5°
g –cos 40°25′ **h** –tan 9.2° **i** sin 59°25′
j –tan 19°50′ **k** sin 84.5° **l** –tan 40.5°

8 **a** $\frac{1}{2}$ **b** –1 **c** $\frac{1}{\sqrt{2}}$ **d** $-\frac{1}{2}$ **e** $-\frac{1}{\sqrt{3}}$
f $\frac{\sqrt{3}}{2}$ **g** $-\frac{\sqrt{3}}{2}$ **h** $-\sqrt{3}$ **i** 1 **j** $-\frac{1}{\sqrt{2}}$

9 **a, b** Teacher to check.
c The tan graph is broken into 3 sections and repeats itself after 180°. It has asymptotes at 90° and 270°.
d No **e** centre of symmetry at (180°, 0).
f **i** $0° < \theta < 90°$ and $180° < \theta < 270°$ (1st and 3rd quadrants).
ii $90° < \theta < 180°$ and $270° < \theta < 360°$ (2nd and 4th quadrants).

Exercise 9.06

1 **a** 57°, 123° **b** 143° **c** 110°
d 130° **e** 7°, 173° **f** 135°
g 100° **h** 25°, 155° **i** 114°
j 33°, 147° **k** 105° **l** 118°

2 **a** 145° 9′ **b** 131° 57′ **c** 159° 26′
d 173° 48′ **e** 152° 58′ **f** 115° 51′
g no solution **h** 126° 52′ **i** no solution
j 163° 18′ **k** 126° 52′ **l** 154° 37′

3 **a** 137° **b** 136° **c** 61°, 119°
d 69° **e** 42°, 138° **f** 143°
g 45° **h** 60° **i** 45°, 135°

Exercise 9.07

1 **a** 18.4 **b** 21.1 **c** 105.0

2 **a** $a = 20.51$ **b** $b = 11.91$ **c** $c = 12.58$
d $d = 4.10$ **e** $e = 30.85$ **f** $f = 3.55$
g $k = 5.99$ cm **h** $w = 29.17$ m **i** $p = 8.29$ m

3 C **4** 79 m **5** 25 m

6 **b** 1042 cm

7 **a** 110° **b** 131.6 m **8** 561 km

9 **d** 124.7 m **10** **b** 595 m **11** 1104 km

Exercise 9.08

1 **a** 27° **b** 37° **c** 54°

2 **a** 44.5° **b** 46.6° **c** 32.0°
d 67.3° **e** 18.8° **f** 31.8°

3 **a** 149° 7′ **b** 129° 0′ **c** 142° 8′
d 135° 33′ **e** 129° 29′ **f** 162° 13′

4 B **5** 39°

6 **a** 46° or 134° **b** 39°
c 43° or 137° **d** 43° or 137°

7 **a** 21° or 75° **b** 41° **c** 84°

8 C **9** 128°

Exercise 9.09

1 **a** 5.6 **b** 13.1 **c** 35.8

2 **a** $a = 8.30$ **b** $c = 54.52$ **c** $e = 88.41$
d $b = 16.33$ **e** $d = 19.44$ **f** $f = 40.72$

3 A **4** 0.6 m **5** C

6 **a** Teacher to check.
b $\angle XYN = 180° - 130° = 50°$ (co-interior angles on parallel lines)
$\angle XYZ = 50° + 25° = 75°$
c 4.4 km

7 47 km

8 **a** 0 **b** $c^2 = a^2 + b^2$
c With $\cos 90° = 0$, the cosine rule reverts to Pythagoras' theorem.

Exercise 9.10

1 **a** 70° **b** 33° **c** 109° **d** 131°

2 **a** 112° **b** 108° **c** 121°
d 23° **e** 60° **f** 83°

3 D **4** 20.8° **5** 64° 40′

6 99° **7** 289°

Exercise 9.11

1 **a** 413.4 m² **b** 463.1 cm² **c** 326.9 mm²
d 132.9 mm² **e** 320.4 cm² **f** 0.1 m²

2 **a** 97.4 m² **b** 463.6 m² **c** 246.2 m²
d 227.6 m² **e** 93.5 m² **f** 152.2 m²

3 **a** 225 m **b** 2770 m²

4 **a** 130° **b** 766.04 m²

5 **a** 418.9 cm² **b** 173.2 cm² **c** 245.7 cm²

6 **a** 112° **b** 37 cm² **c** 740 cm³

7 111.7 cm

Exercise 9.12

1 **a** 10.2 **b** 16.1 **c** 17.1
d 13.1 **e** 3.9 **f** 18.2

2 **a** 32° **b** 142° **c** 29°
d 55° **e** 37° **f** 125°

3 **a** $\angle ATB = 55° - 32°$ (exterior angle of a triangle)
b 108.50 m **c** 89 m

4 **a** 1 **b** **i** 15.4 **ii** 15.4
c The results are the same. The sine rule $\frac{d}{\sin 90°} = \frac{12.8}{\sin 56°}$ becomes $d = \frac{12.8}{\sin 56°}$ (since $\sin 90° = 1$), which is the same result when using the sine ratio.

5 7.5 km **6** 486 km

7 **a** $AF = 67.2$ m, $BF = 46.9$ m **b** 48.3 m

8 783 m

Power plus

1 **a** $\sqrt{3}-1$ **b** $\frac{1}{2}(\sqrt{3}-1)$

2 **a** 45° **b** 60° **c** 30°
d 150° **e** 120° **f** 135°
g 60°, 120° **h** 30°, 150° **i** 45°, 135°

3 Proofs: see worked solutions.

4 $\sqrt{2}$ **5** 45°, 135°

Test yourself 9

1 **a** 10.9 m **b** 4.4 m **c** 11.5 cm

2 **a** 64° 59′ **b** 48° 59′ **c** 57° 12′

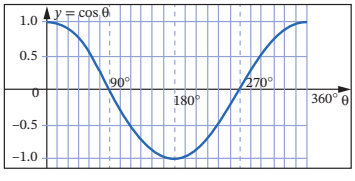
3 13° **4** **a** 231° **b** 051°

5 **a** 2250 km **b** 153°

6 **a** 33 cm **b** 65°

7 **a** 24 **b** $48\sqrt{3}$ **c** $\frac{48}{\sqrt{2}} = 24\sqrt{2}$

8



9 **a** 57° **b** 87° **c** 70°

10 **a** 143° 8′ **b** 64° 59′
c 12° 22′, 167° 38′

11 **a** 0.4 **b** 14.8 **c** 136.4

12 **a** 81° 54′ or 98° 6′ **b** 77° 24′ or 102° 36′
c 49° 37′

13 **a** 6.8 **b** 112.1 **c** 7.6

14 **a** 96° **b** 56° **c** 125°

15 **a** 165 cm² **b** 286 mm² **c** 30 m²

16 15.5 cm **17** 28.0 m

Chapter 10

SkillCheck

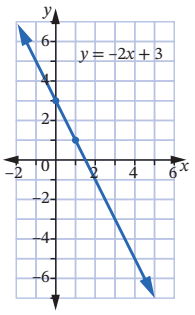
1 **a** 5 **b** 13 **c** 6 **d** −1

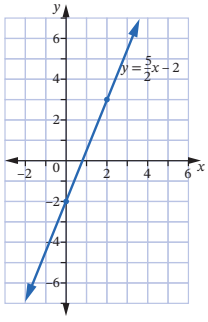
2 **a** −11 **b** 1 **c** 7 **d** $5\frac{1}{2}$

3 (−2, 3) lies on *a*, *d*

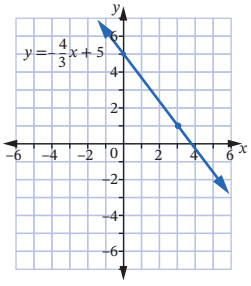
4 **a** For $y = 2x + 1$, when $x = 2$, $y = 2 \times 2 + 1 = 5$
$\therefore$ (2, 5) lies on $y = 2x + 1$
For $x + y = 7$, when $x = 2$, $y = 5$, $2 + 5 = 7$
$\therefore$ (2, 5) lies on $x + y = 7$
b (2, 5)

5 **a** $m = -2$, $c = 3$ **b** $m = \frac{5}{2}$, $c = -2$





c $m = -\frac{4}{3}$, $c = 5$



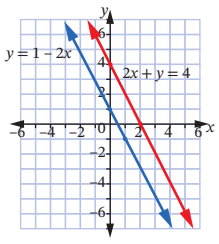
Exercise 10.01

1 **a** $x = 3, y = -1$ **b** $x = 2, y = 1$
c $x = -1, y = -5$

2 **a** $x = 1, y = 2$ **b** $x = -5, y = -9$
c $x = 1, y = 2$ **d** $x = -\frac{1}{2}, y = 2\frac{1}{2}$
e $x = -2, y = -9$ **f** $x = 5, y = -4$
g $x = \frac{1}{2}, y = 6\frac{1}{2}$ **h** $x = 3, y = 2$
i $x = 5, y = 1\frac{1}{2}$ **j** $x = 5, y = 8$
k $x = 1\frac{1}{2}, y = 2\frac{1}{2}$ **l** $x = 4, y = 0$

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3 **a**



b The lines are parallel.

Exercise 10.02

1 **a** $d = -3, k = 2$ **b** $x = 5, w = 4$

c $g = 2, h = -\frac{2}{5}$ **d** $n = 3\frac{1}{4}, p = -1$

e $q = 5, r = -4$ **f** $k = -4\frac{3}{5}, x = 5.$

2 **a** $d = -14, k = 6$ **b** $a = 1, c = 1$

c $h = 3, y = 4$ **d** $e = 3, x = \frac{1}{3}$

e $q = 3, w = 6\frac{1}{2}$ **f** $c = 3, p = 4$

3 **a** $q = -3, w = 3$ **b** $m = -9, x = 7$

c $d = 23, h = -7$ **d** $g = -1, n = 3$

e $h = 0, m = 2$ **f** $e = -4, y = 3$

g $q = 1, w = -4$ **h** $a = \frac{1}{2}, d = \frac{1}{2}$

i $k = 5, p = -2$ **j** $a = -2, f = -2$

k $c = -64, r = -38$ **l** $x = -4, y = -3$

m $x = 8, y = -1$ **n** $g = 4\frac{1}{2}, h = 2\frac{1}{2}$

o $k = -2\frac{1}{2}, w = 3\frac{1}{2}$ **p** $x = 4\frac{13}{29}, y = -\frac{2}{29}$

q $p = 4\frac{1}{9}, m = -7\frac{8}{9}$ **r** $d = -\frac{12}{43}, k = 1\frac{9}{43}$

Exercise 10.03

1 **a** $x = 2, y = 5$ **b** $x = \frac{3}{5}, y = 3\frac{4}{5}$ **c** $x = 7, y = 2$

d $x = 2, y = -2$

2 **a** $x = 9, y = 21$ **b** $x = 5, y = 3$ **c** $x = -\frac{1}{4}, y = 2$

d $x = -3, y = 1$ **e** $x = 2, y = 2$ **f** $x = -7, y = 4$

g $x = 7, y = 3$ **h** $x = 3, y = 2\frac{1}{2}$ **i** $x = 2\frac{2}{3}, y = 1$

j $x = -3\frac{1}{5}, y = -4\frac{3}{5}$ **k** $y = 2\frac{1}{7}, d = 2\frac{5}{7}$

l $a = 1\frac{35}{41}, w = 1\frac{21}{41}$

Exercise 10.04

1 285 **2** 680

3 **a** Teacher to check **b** 396

4 55 **5** Tayyab 36, Sejuti 12

6 16 **7** black 37, colour 23

8 Children: \$15.50, Adult: \$21.50

9 Supreme 32, Vegetarian 13

10 Strawberries \$3.95; Blueberries \$3.50

11 **a** Teacher to check.

b 20-cent coins: 485, 50-cent coins: 368

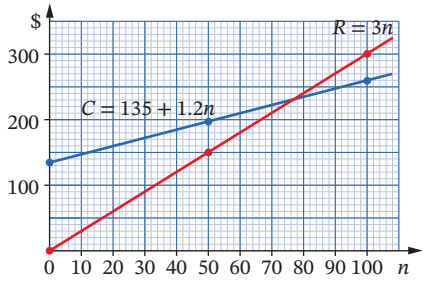
12 **a** Teacher to check.

b $C = 135 + 1.2n$ $R = 3n$

n	0	50	100
C	135	195	255

n	0	50	100
R	0	150	300

c



d $n = 75$

Mental skills 10

2 **a** \$408 **b** 99 **c** 200 **d** \$404

e 672 **f** \$81 **g** \$517 **h** 225

i 560 **j** \$84 **k** \$350 **l** 84

4 **a** 330 **b** \$240 **c** 1600 **d** \$425

e \$225 **f** \$60 **g** \$63 **h** 76

i \$68 **j** \$3762 **k** \$374 **l** \$100

Power plus

1 **a** $x = 1\frac{1}{2}, y = -5\frac{1}{2}, w = 4\frac{1}{2}$

b $a = 1\frac{7}{13}, c = 4\frac{3}{13}, d = 8\frac{11}{13}$

c $p = -11\frac{3}{13}, m = 18\frac{11}{13}, n = -13\frac{4}{13}$

2 **a** Teacher to check.

b $ae - bd = 0$ and a fraction cannot have denominator 0.

c $x = 3, y = -1$

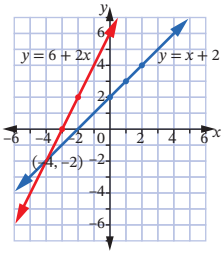
d Teacher to check.

i $x = 2, y = -2$ **ii** $x = 28, y = 16$

iii $x = \frac{1}{11}, y = 2\frac{20}{33}$

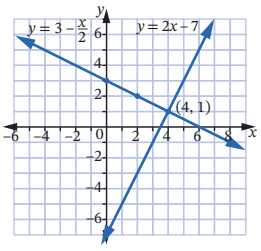
Test yourself 10

1 **a**



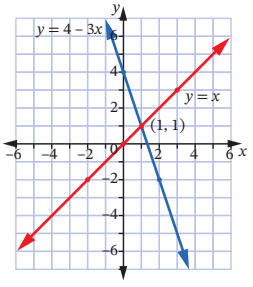
$x = -4, y = -2$

b



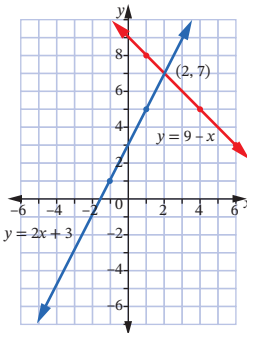
$x = 4, y = 1$

c



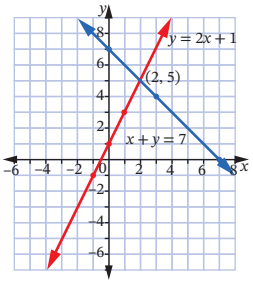
$x = 1, y = 1$

d



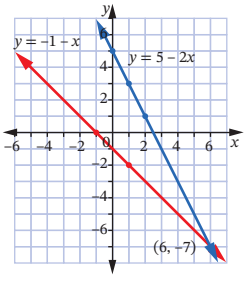
$x = 2, y = 7$

e



$x = 2, y = 5$

f



$x = 6, y = -7$

2 **a** $m = 5, c = -9\frac{1}{2}$ **b** $x = 2, y = \frac{1}{3}$

c $a = 1, d = 1$ **d** $x = 6, y = 15$

e $x = -5, y = -2$ **f** $d = -3, w = -10$

3 **a** $x = 2, y = 11$ **b** $m = 1, p = 3$

c $h = 10, t = 4$ **d** $a = 3, c = \frac{1}{2}$

e $x = 1, y = 1$ **f** $p = 12, q = -4$

4 **a** 1600 adults, 900 children

b 18 children, 12 adults

c \$38

d 28 cheesecakes, 47 mudcakes

e 120 boys, 93 girls

Chapter 11

SkillCheck

1 **a** $x = \pm 5$ **b** $m = \pm 2$ **c** $x = \pm 9$

d $k = \pm\frac{1}{2}$ **e** $u = \pm 3\frac{1}{2}$ **f** $w = \pm 3$

2 **a** $(4 - m)(4 + m)$ **b** $(d - 11)(d + 11)$

c $2y(7 - y)$ **d** $5p(2p + 5)$

e $5(x - 8)(x + 8)$ **f** $2(3w - 5)(3w + 5)$

g $(k + 4)(k + 1)$ **h** $(y - 8)(y - 2)$

i $(m - 8)(m + 7)$ **j** $(u + 13)(u - 5)$

k $(w - 7)(w - 3)$ **l** $(x - 6)(x + 4)$

3 **a** $y = -2$ **b** $y = -10$

c $y = 5$ **d** $y = -8\frac{3}{4}$

4 **a** $(3a + 1)(a + 3)$ **b** $(5x + 2)(x - 3)$

c $(2y - 5)(3y + 8)$ **d** $(3t - 1)(5t + 4)$

e $(5v + 3)(v - 7)$ **f** $(2y + 5)(4y + 7)$

g $(3h - 4)(5h - 1)$ **h** $(4p - 3)(3p + 5)$

i $(4d + 5)^2$

Exercise 11.01

1 **a** $m = -7$ or -3 **b** $d = 3$ or 7 **c** $y = -5$ or 3

d $k = 0$ or 3 **e** $t = -7$ or 0 **f** $p = 0$ or 3

g $w = 0$ or $\frac{2}{3}$ **h** $n = -\frac{1}{2}$ or 3 **i** $a = \frac{1}{2}$ or $\frac{3}{5}$

j $x = -\frac{1}{3}$ or $-1\frac{1}{2}$ **k** $c = 2\frac{1}{2}$ **l** $f = \frac{1}{2}$

m $c = -\frac{1}{3}$ or $-\frac{1}{4}$ **n** $h = -1$ or $\frac{1}{2}$ **o** $e = \frac{5}{7}$ or 1

2 C

3 **a** $y = -2$ or $-1\frac{1}{2}$ **b** $g = -1$ or $-1\frac{1}{2}$ **c** $d = -1$ or $-\frac{2}{3}$

d $t = -2\frac{1}{5}$ or -1 **e** $p = 1\frac{1}{2}$ or 4 **f** $x = \frac{2}{5}$ or $1\frac{1}{2}$

g $y = \frac{3}{4}$ or $-\frac{1}{2}$ **h** $a = -\frac{1}{2}$ or $1\frac{1}{3}$ **i** $w = -1\frac{1}{4}$ or 3

j $c = 1$ or $-1\frac{2}{5}$ **k** $e = \frac{1}{4}$ or $-1\frac{1}{2}$ **l** $q = -3$ or $1\frac{2}{3}$

m $g = 2\frac{1}{2}$ **n** $m = -\frac{2}{3}$ or $\frac{5}{6}$ **o** $w = -4$ or $1\frac{1}{3}$

p $y = -3$ or 4 **q** $f = 6$ **r** $h = \frac{3}{4}$ or -1

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4 **a** $x = -2\frac{1}{2}$ or 3 **b** $t = -2\frac{1}{2}$ or $\frac{1}{2}$ **c** $u = -\frac{1}{8}$ or -5

d $m = \frac{1}{7}$ or 1 **e** $p = -4$ or 7 **f** $e = -1$ or 5

g $t = 1\frac{1}{2}$ or 5 **h** $d = -2\frac{1}{3}$ or $\frac{1}{3}$ **i** $h = \pm 5$

j $f = 0$ or $\frac{1}{2}$ **k** $w = \frac{1}{6}$ or 3 **l** $a = -2$ or $\frac{1}{3}$

5 8

Exercise 11.02

1 A

2 **a** $x^2 + 2x + 1 = (x + 1)^2$

b $p^2 - 6p + 9 = (p - 3)^2$

c $m^2 - 8m + 16 = (m - 4)^2$

d $k^2 + 4k + 4 = (k + 2)^2$

e $y^2 + 7y + \frac{49}{4} = \left(y + \frac{7}{2}\right)^2$

f $w^2 - 3w + \frac{9}{4} = \left(w - \frac{3}{2}\right)^2$

g $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

h $h^2 - 5h + \frac{25}{4} = \left(h - \frac{5}{2}\right)^2$

i $a^2 + \frac{7}{2}a + \frac{49}{16} = \left(a + \frac{7}{4}\right)^2$

j $v^2 - \frac{5}{3}v + \frac{25}{36} = \left(v - \frac{5}{6}\right)^2$

3 D

4 **a** $d = -3 + \sqrt{7}, -3 - \sqrt{7}$ **b** $x = 5 + \sqrt{5}, 5 - \sqrt{5}$

c $p = -1 + \sqrt{10}, -1 - \sqrt{10}$ **d** $y = 1 + \sqrt{2}, 1 - \sqrt{2}$

e $m = \frac{1 + 2\sqrt{5}}{2}, \frac{1 - 2\sqrt{5}}{2}$ **f** $t = \frac{-2 + 3\sqrt{3}}{3}, \frac{-2 - 3\sqrt{3}}{3}$

g $c = \frac{-2 + \sqrt{42}}{2}, \frac{-2 - \sqrt{42}}{2}$ **h** $w = \frac{6 + \sqrt{82}}{2}, \frac{6 - \sqrt{82}}{2}$

i $n = \frac{-2 + \sqrt{7}}{3}, \frac{-2 - \sqrt{7}}{3}$ **j** $e = \frac{3 + \sqrt{71}}{2}, \frac{3 - \sqrt{71}}{2}$

k $d = 2 + \sqrt{5}, 2 - \sqrt{5}$ **l** $x = \frac{3 + 4\sqrt{2}}{4}, \frac{3 - 4\sqrt{2}}{4}$

5 **a** $h = -1 \pm \sqrt{6}$ **b** $r = 1 \pm \sqrt{2}$ **c** $m = -3 \pm \sqrt{7}$

d $w = -6, 10$ **e** $a = 5 \pm \sqrt{30}$ **f** $y = -4 \pm \sqrt{19}$

g $p = -6 \pm \sqrt{41}$ **h** $x = 2 \pm \sqrt{2}$ **i** $u = -7, -2$

j $d = \frac{-1 \pm \sqrt{29}}{2}$ **k** $c = \frac{9 \pm \sqrt{73}}{2}$ **l** $e = \frac{-5 \pm \sqrt{17}}{2}$

m $y = \frac{3 \pm \sqrt{41}}{2}$ **n** $b = \frac{1 \pm \sqrt{21}}{2}$ **o** $q = \frac{3 \pm \sqrt{5}}{2}$

p $g = \frac{-7 \pm \sqrt{73}}{4}$ **q** $x = -3\frac{1}{2}, 1$ **r** $f = \frac{-2 \pm \sqrt{22}}{3}$

6 **a** $x \approx -11.20$ or -0.80 **b** $m \approx -0.43$ or 16.43

c $g \approx -4.65$ or 0.65 **d** $h \approx 1.27$ or -2.77

e $w \approx 1.27$ or -0.47 **f** $y \approx 1.14$ or -1.47

g $p = -2$ or $1\frac{1}{3}$ **h** $e \approx 1.13$ or -0.88

i $n = 1$ or -2.5

Exercise 11.03

1 **a** $x = -3 \pm \sqrt{7}$ **b** $m = \frac{5 \pm \sqrt{37}}{2}$ **c** $w = 4 \pm \sqrt{13}$

d $k = \frac{-3 \pm \sqrt{29}}{2}$ **e** $y = 2 \pm \sqrt{5}$ **f** $p = \frac{-1 \pm \sqrt{21}}{2}$

g $u = \frac{7 \pm \sqrt{61}}{2}$ **h** $a = \frac{-3 \pm \sqrt{65}}{4}$ **i** $q = \frac{1}{5}, 1$

j $c = \frac{-1 \pm \sqrt{7}}{3}$ **k** $e = \frac{5 \pm \sqrt{57}}{8}$ **l** $x = \frac{-4 \pm \sqrt{10}}{3}$

m $d = \frac{2 \pm \sqrt{14}}{2}$ **n** $a = \frac{5 \pm \sqrt{31}}{3}$ **o** $t = -2\frac{1}{2}, 1$

p $y = -\frac{2}{3}, -2$ **q** $k = \frac{5}{6}, 1$ **r** $n = \frac{5 \pm \sqrt{113}}{4}$

2 A

3 **a** $y = \frac{9 \pm \sqrt{141}}{10}$ **b** $m = \frac{-1 \pm \sqrt{22}}{3}$ **c** $x = \frac{3 \pm \sqrt{41}}{8}$

d $k = -2 \pm \sqrt{5}$ **e** $m = \frac{3 \pm \sqrt{21}}{6}$ **f** $g = \frac{-1 \pm \sqrt{6}}{5}$

g $h = \frac{9 \pm \sqrt{17}}{4}$ **h** $w = \frac{1 \pm \sqrt{7}}{3}$ **i** $p = \frac{2 \pm \sqrt{7}}{3}$

j $u = \frac{-2 \pm \sqrt{14}}{5}$ **k** $a = \frac{-2 \pm \sqrt{58}}{6}$ **l** $y = \frac{-3 \pm \sqrt{89}}{4}$

4 B

5 **a** $k = 8.89, 0.11$ **b** $c = 1.41, -1.41$

c $m = 2.65, -2.65$ **d** $n = 3.19, 0.31$

e $p = 0.85, -2.35$ **f** $w = 0.30, -1.13$

g $x = 2.39, 0.28$ **h** $h = 3.83, -1.83$

i $x = 1.62, -0.62$ **j** $a = 4, 9$

k $v = 1.48, -1.48$ **l** $c = 2.31, 0.69$

m $t = 8.09, -3.09$ **n** $x = 4.27, 7.73$

o $d = 3.31, -1.81$

Exercise 11.04

1 **a** $x = 0$ or $\frac{3}{2}$ **b** $m = \pm 6$ **c** $w = \frac{5 \pm \sqrt{37}}{6}$

d $u = 4$ or -3 **e** $k = \frac{3}{5}$ or $-\frac{7}{2}$ **f** $a = \pm\frac{4}{5}$

g $x = 9$ or -7 **h** $n = 3 \pm \sqrt{19}$ **i** $a = -3 \pm \sqrt{5}$

j $d = -3$ or 1 **k** $t = 0$ or 4 **l** $y = \frac{3 \pm \sqrt{11}}{2}$

m $x = -10$ or 3 **n** $a = \frac{2}{3}$ or $-\frac{3}{2}$ **o** $x = \frac{1 \pm \sqrt{10}}{6}$

ANSWERS

2 **a** $y = 0$ or 5 **b** $a = \pm 12$
c $u = 5.32, -1.32$ **d** $x = 3$
e $x = 9, -2$ **f** $x = -0.53, -9.47$
g $a = 2.14, -1.64$ **h** $w = 2.77, -1.27$
i $m = 1.05, -0.65$ **j** $k = -1.10, 4.10$
k $x = \frac{9}{2}$ or -4 **l** $h = -6, -1$

Exercise 11.05

1 **a** $y = \pm 2, \pm 2\sqrt{2}$ **b** $m = \pm 4$ **c** $w = \pm\sqrt{2}, \pm\sqrt{6}$
d $m = 1, 2$ **e** $k = -1, 3$ **f** $w = 2, -\sqrt[3]{4}$
g $x = -1, \frac{1}{3}$ **h** $y = -\frac{4}{5}, 1\frac{2}{5}$ **i** $a = -1\frac{1}{2}, -\frac{1}{2}$
j $p = \sqrt[3]{5}, -1$ **k** $g = \frac{1}{2}, 2$ **l** $c = -2, \frac{1}{5}$

2 **a** $m \approx \pm 1.2, \pm 2.8$ **b** $x \approx \pm 0.9, \pm 1.4$
c $a \approx -0.9, 1.3$ **d** $y \approx -1.4, 1.3$
e $w \approx -1.2, 1.9$ **f** $e \approx \pm 0.5$
g $w = \pm 5$ **h** $x = -2, 1.3$ **i** $k = \pm 2$
j $m = \pm 1.4$ **k** $y = \pm 1.2$ **l** $v = -2, 1.4$

Mental skills 11

2 **a** \$100, \$50 **b** \$1200, \$900 **c** \$160, \$560
d \$500, \$1500 **e** \$2700, \$1800 **f** \$900, \$2100
g \$3000, \$600 **h** \$600, \$1000 **i** \$550, \$440
j \$800, \$3200 **k** \$2100, \$2800 **l** \$2000, \$1200

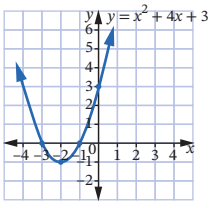
Exercise 11.06

1 12 m by 8 m **2** 40 m by 35 m
3 42 m by 24 m **4** Length 58 m, width 38 m
5 Length 13 m **6** 15 m by 12 m
7 0.52 s, 3.08 s
8 **a** 4000 m **b** 2000 m **c** 24.5 s
9 **a** 34 m **b** 3.9 s
c **i** 0.5 s and 1.9 s **ii** 3.5 s
10 8 or –9 **11** 24, 25 or –24, –25
12 35 or –34 **13** 27 and 15
14 **a** \$9200 **b** \$5550 **c** 471
15 49 m²

Exercise 11.07

1 **a** **i**

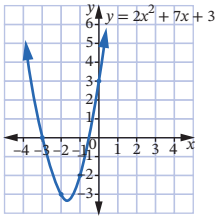
x	–3	–2	–1	0	1	2	3
y	0	–1	0	3	8	15	24



ii –3 and –1 **iii** 3
iv $x = -3$ and -1, same as x-intercepts

b **i**

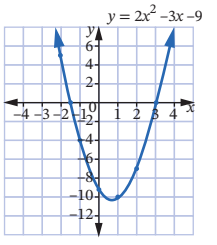
x	–3	–2	–1	0	1	2	3
y	0	–3	–2	3	12	25	42



ii -3 and $-\frac{1}{2}$ **iii** 3
iv $x = -3$ and $-\frac{1}{2}$, same as x-intercepts

c **i**

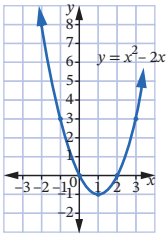
x	–3	–2	–1	0	1	2	3
y	18	5	–4	–9	–10	–7	0



ii $-1\frac{1}{2}$ and 3 **iii** –9
iv $x = -1\frac{1}{2}$ and 3, same as x-intercepts

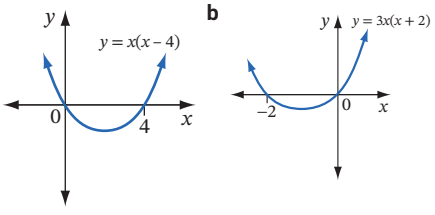
d **i**

x	–3	–2	–1	0	1	2	3
y	15	8	3	0	–1	0	3



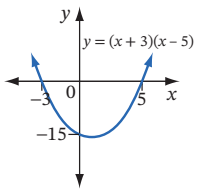
ii 0 and 2 **iii** 0
iv $x = 0$ and 2, same as x-intercepts

2 **a** –5 **b** 3 **c** 0
3 C **4** B
5 **a** **b**

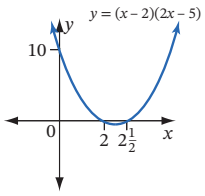


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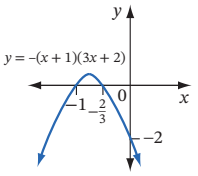
c



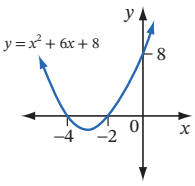
d



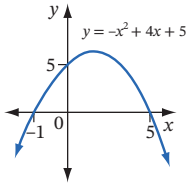
e



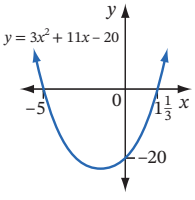
f



g



h



Exercise 11.08

1 **a** $x = 3$ **b** $x = 0$ **c** $x = 2\frac{1}{2}$

d $x = -1$ **e** $x = 1\frac{1}{2}$ **f** $x = 3$

2 **a** **i** $x = 3$ **ii** $(3, -1)$

b **i** $x = 5$ **ii** $(5, 16)$

c **i** $x = 1$ **ii** $(1, 9)$

d **i** $x = 4$ **ii** $(4, 25)$

e **i** $x = \frac{1}{2}$ **ii** $(\frac{1}{2}, -24\frac{3}{4})$

f **i** $x = -4$ **ii** $(-4, -80)$

g **i** $x = 4$ **ii** $(4, 48)$

h **i** $x = -\frac{1}{4}$ **ii** $\left(-\frac{1}{4}, -1\frac{1}{4}\right)$

i **i** $x = -\frac{1}{6}$ **ii** $\left(-\frac{1}{6}, 1\frac{1}{4}\right)$

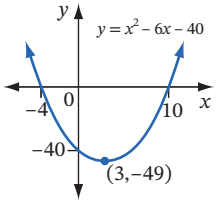
3 **a** **i** $-4, 10$

ii -40

iii $x = 3$

iv $(3, -49)$

v concave up



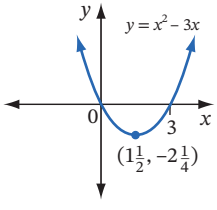
b **i** $0, 3$

ii 0

iii $x = 1\frac{1}{2}$

iv $\left(1\frac{1}{2}, -2\frac{1}{4}\right)$

v concave up



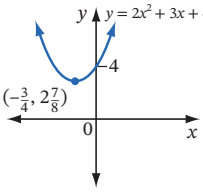
c **i** No x-intercepts

ii 4

iii $x = -\frac{3}{4}$

iv $\left(-\frac{3}{4}, 2\frac{7}{8}\right)$

v concave up



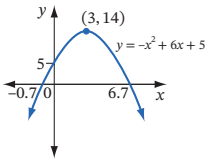
d **i** $-0.7, 6.7$

ii 5

iii $x = 3$

iv $(3, 14)$

v concave down



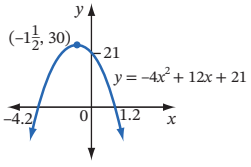
e **i** $-4.2, 1.2$

ii 21

iii $x = -1\frac{1}{2}$

iv $\left(-1\frac{1}{2}, 30\right)$

v concave down



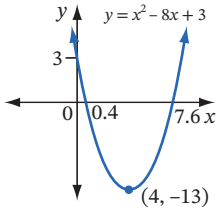
f **i** $0.4, 7.6$

ii 3

iii $x = 4$

iv $(4, -13)$

v concave up



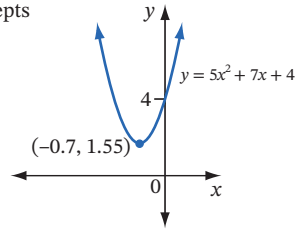
g **i** No x-intercepts

ii 4

iii $x = -0.7$

iv $(-0.7, 1.55)$

v concave up



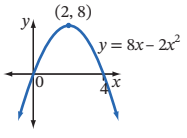
h **i** $0, 4$

ii 0

iii $x = 2$

iv $(2, 8)$

v concave down



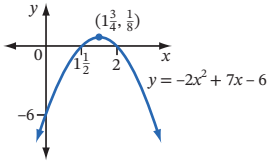
i **i** $1\frac{1}{2}, 2$

ii -6

iii $x = 1\frac{3}{4}$

iv $\left(1\frac{3}{4}, \frac{1}{8}\right)$

v concave down



4 A

5 **a** $(-2, 3)$ **b** $(-5, -2)$ **c** $(3, 1)$

d $(1, 5)$ **e** $(-7, -2)$ **f** $(10, -8)$

6 **a** $y = (x + 2)^2 - 3, (-2, -3)$

b $y = \left(x + \frac{5}{2}\right)^2 - 10\frac{1}{4}, \left(-\frac{5}{2}, -10\frac{1}{4}\right)$

c $y = -(x-1)^2 + 6$, (1, 6)
d $y = (x-3)^2 - 9$, (3, −9)
e $y = (x-1)^2 + 1$, (1, 1)
f $y = 2\left(x - \frac{5}{4}\right)^2 - \frac{9}{8}, \left(1\frac{1}{4}, -1\frac{1}{8}\right)$

Exercise 11.09

1 **a** (3, 9), (2, 4) **b** (0, 0), (4, 32)
c (4, 26), (−2, 14) **d** $(-1, 5), \left(1\frac{1}{5}, 7\frac{1}{5}\right)$
e $(-2, 18), \left(\frac{1}{3}, 11\right)$ **f** (−2, −22), (5, 20)
g (7, 39) **h** (2, 0), (−1, 3)

2 **a** (1, −2), (2, −1) **b** (0, 1), (1, 0)
c (0, 3), (3, 0) **d** (0, 5), (−5, 0)
e $(1, 7), \left(-1\frac{2}{5}, -5\right)$ **f** $(3, 5), \left(-2\frac{1}{2}, -6\right)$
g $(-1, 3), \left(-1\frac{1}{2}, 2\right)$

3 Teacher to check. The graphs of the equations do not cross.

Power plus

1 **a** Proof: see worked solutions.
b $x = \frac{4}{3}, 1, \frac{7 \pm \sqrt{37}}{6}$

2 **a** $x = 5$ or 3 **b** $x = 0$ or −1 **c** $x = -3, -2$
d $x = 0$ **e** $x = 0$ **f** $x = 3$
g $x = 0, 2$

3 **a** Proof: see worked solutions.
b **i** $x^2 - 9x + 14 = 0$ **ii** $x^2 + 3x - 40 = 0$
iii $2x^2 + 7x - 4 = 0$ **iv** $x^2 - 3 = 0$
v $x^2 - 6x + 7 = 0$

4 Intersects twice

5 72 cm × 24 cm

Test yourself 11

1 **a** $m = 2, -\frac{3}{2}$ **b** $x = 0, 5$ **c** $p = -6, 8$
d $y = 2, 6$ **e** $w = -\frac{4}{3}, -5$ **f** $k = \frac{3}{4}, -\frac{1}{2}$
g $n = -3, 5$ **h** $d = -4\frac{1}{2}, 1\frac{1}{2}$ **i** $x = -1\frac{1}{2}, 2\frac{1}{2}$

2 **a** $y = -2 \pm \sqrt{11}$ **b** $p = \frac{3 \pm \sqrt{33}}{4}$ **c** $w = \frac{-1 \pm \sqrt{6}}{5}$

3 **a** $m = \frac{-5 \pm \sqrt{37}}{2}$ **b** $d = \frac{7 \pm \sqrt{137}}{4}$ **c** $k = \frac{1 \pm \sqrt{253}}{6}$

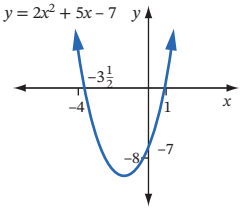
4 **a** $x \approx 0.30, -3.30$ **b** $h \approx 1.55, -0.80$
c $w \approx 5.37, -0.37$

5 **a** $x = 0$ or 3 **b** $h = 11$ or −1 **c** $a = 4$ or −1
d $x = \frac{-7 \pm \sqrt{69}}{10}$ **e** $y = 3 \pm 2\sqrt{6}$ **f** $t = \frac{-5 \pm \sqrt{345}}{20}$

6 **a** $m = \pm 2, \pm\sqrt{2}$ **b** $y = \pm 2$ **c** $x = -1$ or 2

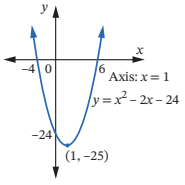
7 **a** 32 and 34 **b** 14 m × 22 m

8 $y = 2x^2 + 5x - 7$

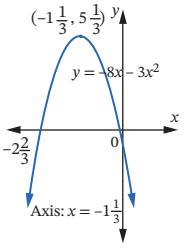


9 **a** (−5, 1) **b** (7, 4) **c** (−3, −9)

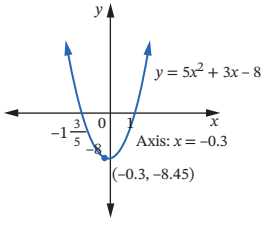
10 **a**



b



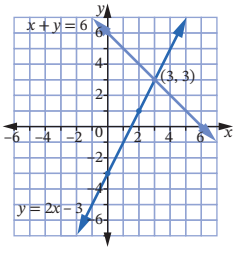
c



11 **a** $(1\frac{1}{2}, 4\frac{1}{2}), (-1, 2)$ **b** (−5, 13), (1,1)
c (0, 4), (4, 0) **d** (4, −2), (2, −4)

Practice set 3

1



$x = 3, y = 3$

2 **a** $m = 6.8$ cm **b** $k = 22.7$ m **c** $d = 11.3$ m

3 **a** $t = \frac{99.96}{S}$ **b** 9.52 m/s **c** 9.70 s

4 **a** $x = 1$ **b** (1, 2)

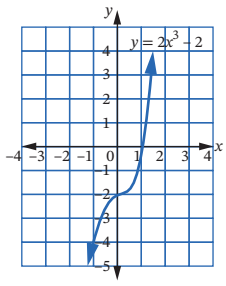
5 **a** $m = 4, d = \frac{2}{5}$ **b** $g = -6\frac{2}{5}, w = -13\frac{3}{5}$
c $f = 3, y = 3$

6 **a** 29° **b** 45° **c** 43°

7 $x = \frac{2 \pm \sqrt{7}}{3}$

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8



9 **a** 340 mm² **b** 81 m² **c** 140 cm²

10 **a** $m = -\frac{1}{4}$ or -1 **b** $y = \frac{4}{5}$ or $-\frac{1}{2}$ **c** $w = \frac{5}{2}$ or -1

11 **a** 32.5 **b** 5.1

12 **a** $A + C = 395$, $20A + 15C = 6700$

b 240 children

13 C

14 **a** 106° 36′ **b** $\alpha = 40°\ 30'$

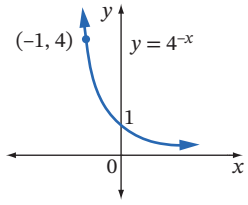
15 Centre $(5, -3)$, radius 6 **16** 36 km

17 **a** $x = 2\frac{1}{2}, y = 5\frac{1}{2}$ **b** $k = 2, h = -\frac{1}{2}$

c $m = 6\frac{2}{7}, w = 3\frac{3}{7}$

18 52°, 128°

19



20 **a** 301 km **b** 114°

21 **a** $P = 1.75T$ **b** 262.5 **c** 68.6

22 **a** $\frac{1}{\sqrt{2}}$ **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{\sqrt{3}}{2}$

23 $x = \pm 3, \pm 2$

24 **a** $\theta = 37°\ 56'$ **b** 41° 33′ **c** 68° 31′

25 **a** $4w^2 - 5w - 1071 = 0$ **b** 17 m × 63 m

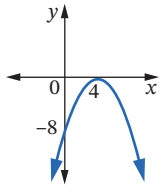
26 (4, 1) and (−1, −4) **27** C

28 **a** 10.9 m **b** 10° 8′

29 D **30** 113°

31 **a** $\frac{-5 \pm \sqrt{17}}{2}$ **b** $3 \pm 2\sqrt{3}$ **c** $\frac{-7 \pm \sqrt{73}}{6}$

32



Chapter 12

SkillCheck

1 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{5}{6}$

2 **a** 0 **b** 1

3 B **4** 0.15

Exercise 12.01

1 **a** **i** $\frac{1}{5} = 0.2$ **ii** $\frac{19}{50} = 0.38$

iii $\frac{33}{100} = 0.33$ **iv** $\frac{9}{100} = 0.09$

b **i** $\frac{1}{4} = 0.25$ **ii** $\frac{7}{20} = 0.35$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{1}{10} = 0.1$

c Yes

d Expected frequency = 40. The expected frequency compares very favourably with the observed frequency of 42.

2 **a** 50 **b** Teacher to check.

c **i** Teacher to check. **ii** $\frac{1}{2}$

d Teacher to check.

3 **a** 600

b **i** $\frac{281}{600} \approx 0.468$ **ii** $\frac{322}{600} \approx 0.537$

iii $\frac{227}{600} \approx 0.378$ **iv** $\frac{522}{600} \approx 0.87$

c **i** 0.5 **ii** 0.5 **iii** 0.33 **iv** 0.83

d The probabilities are similar.

4 **a** Teacher to check.

b **i** $\frac{1}{2} = 0.5$ **ii** $\frac{1}{5} = 0.2$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{7}{10} = 0.7$

c Teacher to check.

5 **a** **i** $\frac{3}{10} = 0.3$ **ii** $\frac{3}{25} = 0.12$

iii $\frac{12}{25} = 0.48$ **iv** $\frac{1}{10} = 0.1$

b **i** 0.33 **ii** 0.17 **iii** 0.33 **iv** 0.17

c No, the experimental probability of yellow was higher and the experimental probability of green was lower.

d Expected frequency of not yellow is 33. This is more than the observed frequency of 26.

6 **a** 200

b **i** $\frac{4}{200} = 0.02$ **ii** $\frac{27}{200} = 0.135$

iii $\frac{13}{200} = 0.065$ **iv** $\frac{86}{200} = 0.43$

v $\frac{87}{200} = 0.435$ **vi** $\frac{59}{200} = 0.295$

vii $\frac{51}{200} = 0.255$

7 **a** 200

b **i** $\frac{27}{200} = 0.135$ **ii** $\frac{62}{200} = 0.31$

iii $\frac{80}{200} = 0.4$ **iv** $\frac{21}{200} = 0.105$

v $\frac{1}{200} = 0.005$

c Ferry, light rail (tram)

d Teacher to check.

8 **a** $\frac{1}{6} \approx 0.17$ **b** 16 or 17 times

c, **d**, **e** Teacher to check.

9 Teacher to check. The probability of winning would be different for each horse, due to methods of training, strength of horse and conditions of the racetrack, so it is not a simple probability of 1 in 10.

Exercise 12.02

1 C

2 **a** **i** $\frac{11}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{1}{5}$ **iv** $\frac{6}{25}$ **v** $\frac{19}{25}$

b $\frac{3}{10}$

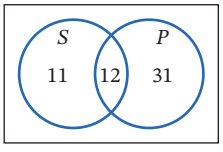
3 **a** 156

b **i** $\frac{11}{52}$ **ii** $\frac{7}{52}$ **iii** $\frac{7}{78}$ **iv** $\frac{22}{39}$ **v** $\frac{19}{78}$ **vi** $\frac{1}{26}$

c $\frac{7}{31}$

4 **a** 135 **b** $\frac{56}{135}$ **c** $\frac{1}{5}$ **d** $\frac{17}{52}$

5 **a**



b **i** 1 **ii** $\frac{11}{54}$ **iii** $\frac{31}{54}$ **iv** $\frac{7}{9}$

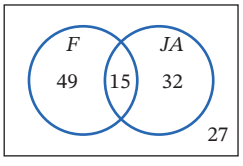
6 **a** 92

b **i** $\frac{1}{46}$ **ii** $\frac{15}{92}$ **iii** $\frac{5}{92}$ **iv** $\frac{17}{23}$

c **i** $\frac{21}{29}$ **ii** $\frac{5}{29}$

7 **a** 123

b



c 81

d **i** $\frac{49}{123}$ **ii** $\frac{32}{123}$ **iii** $\frac{9}{41}$ **iv** $\frac{27}{41}$

8 **a** 200

b **i** $\frac{79}{200}$ **ii** $\frac{51}{100}$ **iii** $\frac{77}{200}$

iv $\frac{121}{200}$ **v** $\frac{81}{100}$ **vi** $\frac{3}{50}$

c $\frac{29}{40}$

d No, because all the people surveyed indicated a day on which they preferred to shop.

9 **a** 196

b **i** $\frac{7}{196}$ **ii** $\frac{13}{98}$ **iii** $\frac{15}{49}$ **iv** $\frac{34}{49}$

c **i** $\frac{6}{59}$ **ii** $\frac{43}{59}$

Exercise 12.03

1 **a** 150

b **i** $\frac{4}{25}$ **ii** $\frac{53}{150}$ **iii** $\frac{28}{75}$

c 63%

2 **a** 128 **b** **i** 68 **ii** 60

c $\frac{17}{32}$ **d** $\frac{3}{32}$

3 **a** 97

b **i** 32.0% **ii** 25.8% **iii** 11.3%

c **i** 21.2% **ii** 44.4%

d The percentage of females in the opposition is just more than double that of females in the government.

4 **a** 150

b **i** 0.5 **ii** 0.04 **iii** 0.43 **iv** 0.23

c $\frac{22}{75} = 0.293$

5 **a** 160

b **i** $\frac{7}{20} = 0.35$ **ii** $\frac{11}{160} \approx 0.069$ **iii** $\frac{9}{80} \approx 0.113$

c $\frac{35}{82} \approx 0.43$

6 **a** 200 **b** 55%

c **i** 49.5% **ii** 45% **iii** 36% **iv** 31.5%

d $\frac{72}{110} \approx 65.5\%$

7 **a** 878

b **i** $\frac{679}{878} = 0.773$ **ii** $\frac{545}{878} = 0.621$

iii $\frac{67}{439} = 0.153$ **iv** $\frac{21}{439} = 0.048$

Exercise 12.04

1 **a**

		Girls					
		Be	**Ca**	**Em**	**M**	**R**	**S**
Boys	**Be**	Ben, Be	Ben, Ca	Ben, Em	Ben, M	Ben, R	Ben, S
	C	C, Be	C, Ca	C, Em	C, M	C, R	C, S
	Ew	Ew, Be	Ew, Ca	Ew, Em	Ew, M	Ew, R	Ew, S
	W	W, Be	W, Ca	W, Em	W, M	W, R	W, S

b **i** $\frac{1}{24}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$

2 **a**

	H	***T***
H	HH	HT
T	TH	TT

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

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3 a

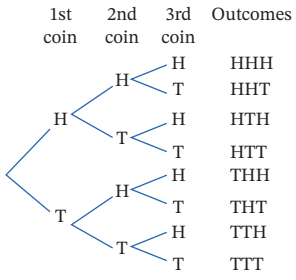
2nd die \ 1st die	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

b 36

c i $\frac{1}{36}$ **ii** $\frac{11}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{4}$

v $\frac{11}{36}$ **vi** $\frac{1}{4}$ **vii** $\frac{1}{2}$ **viii** $\frac{5}{12}$

4 a

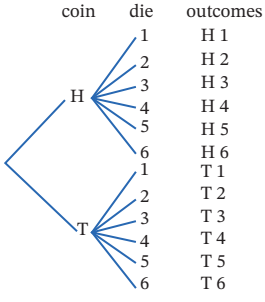


b 8

c i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{1}{2}$

d i $\frac{7}{8}$ **ii** $\frac{7}{8}$ **e i** 75 **ii** 25

5



6 a

2nd die \ 1st die	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4

b 24

c i $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$ **v** 0

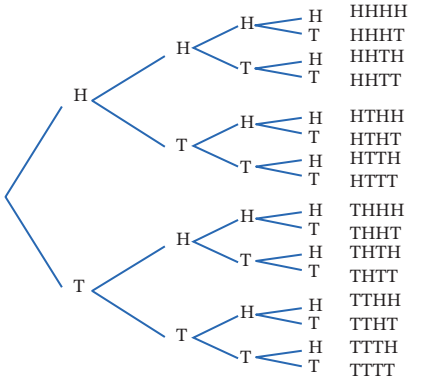
7 a

2nd die \ 1st die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b i $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

v 0 **vi** $\frac{5}{12}$ **vii** $\frac{7}{12}$ **viii** $\frac{5}{12}$

8 a

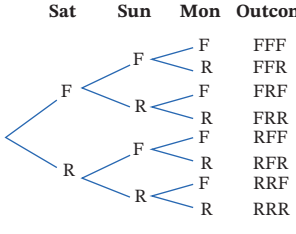


b i $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{8}$

iv $\frac{15}{16}$ **v** $\frac{1}{16}$ **vi** $\frac{5}{16}$

c i 63 **ii** 375 **iii** 938

9 a



b i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

Mental skills 12

2 a \$700 **b** \$800 **c** 400 **d** 700

e \$300 **f** \$400 **g** 250 **h** \$300

i \$300 **j** 500

Exercise 12.05

1 a C = Cate, A = Amal, G = Gemma, J = Josie, E = Evangeline, R = Rukshana

Vice-captain \ Captain	C	A	G	J	E	R
C	-	AC	GC	JC	EC	RC
A	CA	-	GA	JA	EA	RA
G	CG	AG	-	JG	EG	RG
J	CJ	AJ	GJ	-	EJ	RJ
E	CE	AE	GE	JE	-	RE
R	CR	AR	GR	JR	ER	-

30 possible pairings

b $\frac{1}{3}$ **c** $\frac{1}{6}$

2 a i

	A	B	C	D	E
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

ii

	A	B	C	D	E
A		AB	AC	AD	AE
B	BA		BC	BD	BE
C	CA	CB		CD	CE
D	DA	DB	DC		DE
E	EA	EB	EC	ED	

b i $\frac{1}{5}$ **ii** $\frac{4}{25}$ **iii** $\frac{12}{25}$

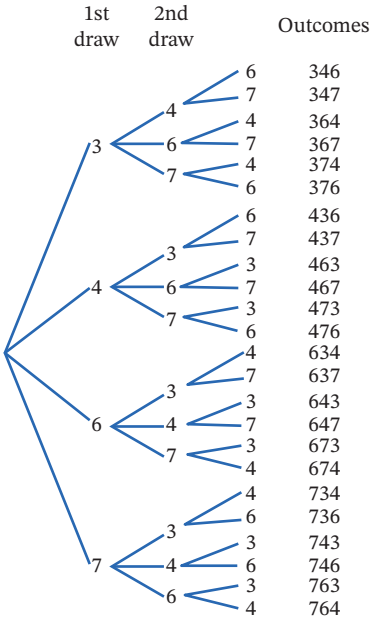
c i $\frac{1}{10}$ **ii** $\frac{3}{5}$ **iii** $\frac{2}{5}$ **iv** $\frac{4}{5}$

3 a

1st course \ 2nd course	B	H	P	S	T
C	CB	CH	CP	CS	CT
F	FB	FH	FP	FS	FT
Y	YB	YH	YP	YS	YT

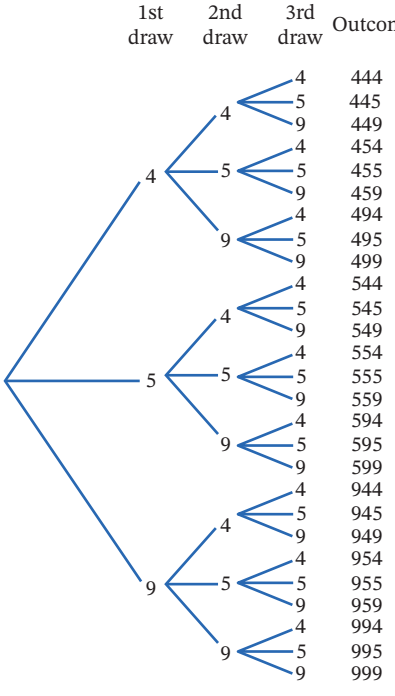
b i $\frac{1}{3}$ **ii** $\frac{4}{15}$ **iii** $\frac{1}{15}$

4 a



b i $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

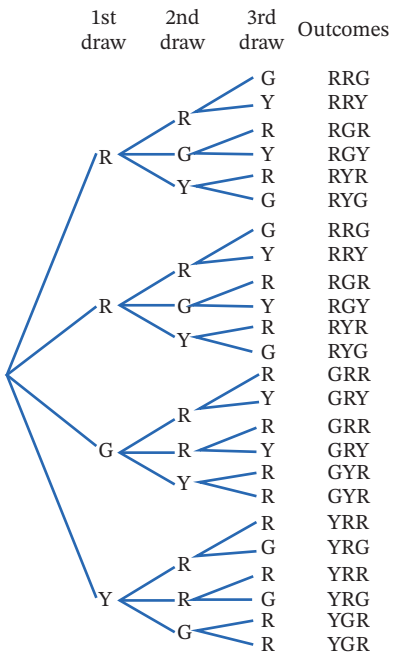
5 a



b i $\frac{1}{9}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{9}$

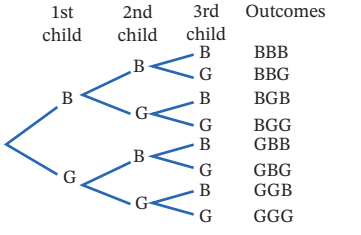
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6 a



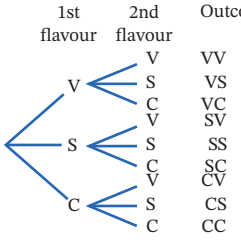
b i $\frac{1}{2}$ **ii** $\frac{1}{12}$ **iii** 1

7 a

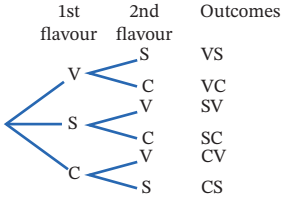


b i $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{8}$

8 a i



ii



b i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{4}{9}$ **iv** $\frac{5}{9}$

c i 0 **ii** 1 **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

Exercise 12.06

1 a independent **b** independent **c** dependent
d independent **e** dependent **f** independent
g independent

2 Dependent, as the balls are not replaced when drawn.

3 a independent **b** $\frac{1}{2}$

4 a i $\frac{1}{3}$ **ii** $\frac{1}{4}$

b 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 1G, 2G, 3G, 4G, 5G, 6G, 1B, 2B, 3B, 4B, 5B, 6B, 1R, 2R, 3R, 4R, 5R, 6R,

c $\frac{1}{12}$ **d** Yes, $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

e independent

5 a i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **b** $\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$

6 a $\frac{5}{9}$ **b** $\frac{4}{8}=\frac{1}{2}$

c Dependent, as the first draw changes the contents of the bag.

7 a i $\frac{5}{8}$ **ii** $\frac{4}{7}$ **b i** $\frac{3}{8}$ **ii** $\frac{5}{7}$

c i $\frac{5}{8}$ **ii** $\frac{3}{7}$ **d i** $\frac{3}{8}$ **ii** $\frac{2}{7}$

8 $\frac{1}{2}$

9 a $\frac{1}{12}$ **b** $\frac{1}{36}$ **c** $\frac{1}{3}$ **d** $\frac{1}{6}$

10 a $\frac{13}{25}$ **b** $\frac{7}{25}$

11 a 0.15 **b** 0.1 **c** 0.3

12 a 0.147 **b** 0.343 **c** 0.027 **d** 0.189

Exercise 12.07

1 a $\frac{2}{6}=\frac{1}{3}$ **b** $\frac{3}{6}=\frac{1}{2}$ **2** $\frac{2}{9}$

3 a $\frac{4}{11}$ **b** $\frac{7}{11}$

4 a $\frac{2}{11}$ **b** $\frac{4}{11}$ **c** $\frac{4}{11}$ **d** $\frac{5}{11}$

5 $\frac{1}{3}$ **6** $\frac{1}{6}$

7 a

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b i $\frac{1}{2}$ **ii** 1 **c** $\frac{2}{11}$

ANSWERS

d i $\frac{9}{27}=\frac{1}{3}$ ii $\frac{6}{27}=\frac{2}{9}$ **e** $\frac{1}{6}$

8 **a** i $\frac{1}{9}$ ii $\frac{8}{9}$ **b** $\frac{2}{8}=\frac{1}{4}$ **c** 6

9 $\frac{1}{40}$ **10** $\frac{10}{49}$ **11** $\frac{1}{13}$ **12** $\frac{1}{6}$

13 **a**

		1st die					
		1	2	3	4	5	6
2nd die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b i $\frac{1}{6}$ ii $\frac{1}{18}$ iii $\frac{1}{6}$

c i $\frac{1}{2}$ ii 0 **d** i $\frac{1}{11}$ ii $\frac{4}{11}$

e i $\frac{6}{18}=\frac{1}{3}$ ii 1

Power plus

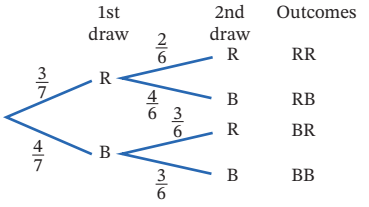
1 **a** 320

b i $\frac{7}{40}=0.175$ ii $\frac{3}{32}=0.094$

iii $\frac{57}{320}=0.178$ iv $\frac{3}{20}=0.15$

c $\frac{16}{45}$ **d** $\frac{45}{107}$

2 **a**



b i $\frac{1}{7}$ ii $\frac{2}{7}$ iii $\frac{4}{7}$ iv $\frac{6}{7}$

3 **a** i $\frac{14}{30}=\frac{7}{15}$ ii $\frac{20}{30}=\frac{2}{3}$ iii $\frac{4}{30}=\frac{2}{15}$

iv $\frac{4}{20}=\frac{1}{5}$ v $\frac{4}{14}=\frac{2}{7}$

b i $\frac{\frac{2}{15}}{\frac{2}{3}}=\frac{1}{5}$ ii Yes

c $P(B|A)=\frac{2}{7}, \frac{P(A \text{ and } B)}{P(A)}=\frac{\frac{2}{15}}{\frac{7}{15}}=\frac{2}{7}$

Test yourself 12

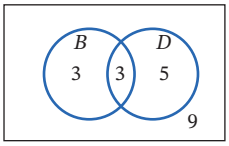
1 **a** i 0.353 ii 0.427 iii 0.087 iv 0.513

b i $\frac{13}{15}=0.867$ ii $\frac{2}{15}=0.133$

c Different – at least one head occurring excludes zero heads occurring, which is the same as three tails occurring. The events are complementary.

2 **a** 90 **b** $\frac{19}{45}$ **c** $\frac{2}{9}$ **d** $\frac{11}{49}$

3 **a**



b i $\frac{3}{20}$ ii $\frac{2}{5}$ iii $\frac{3}{20}$ iv $\frac{3}{5}$

c $\frac{9}{20}$

4 **a** 200

b i 0.305 ii 0.11 iii 0.225 iv 0.425

c $\frac{25}{110}\approx 0.227$

d i $\frac{47}{69}\approx 0.681$ ii $\frac{22}{69}\approx 0.319$

5 D

6 **a**

		1st die			
		1	2	3	4
2nd die	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4

b 16

c i $\frac{1}{2}$ ii $\frac{1}{4}$ iii $\frac{7}{16}$

iv $\frac{1}{4}$ v $\frac{1}{4}$ vi $\frac{1}{2}$

7 **a** i RR, RB, RG, RY, RBla, BR, BB, BG, BY, BBla, GR, GB, GG, GY, GBla, YR, YB, YG, YY, YBla, BlaR, BlaB, BlaG, BlaY, BlaBla

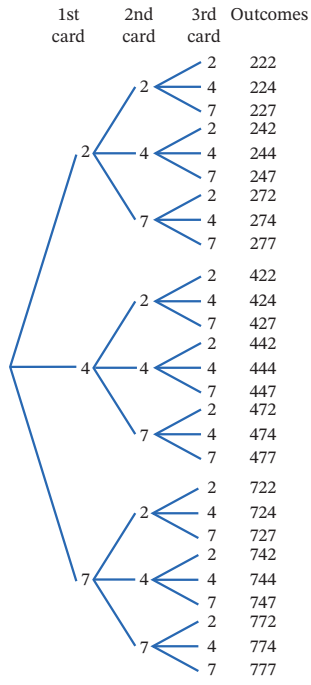
ii RB, RG, RY, RBla, BR, BG, BY, BBla, GR, GB, GY, GBla, YR, YB, YG, YBla, BlaR, BlaB, BlaG, BlaY

b i $\frac{1}{25}$ ii $\frac{1}{5}$ iii $\frac{2}{25}$ iv $\frac{9}{25}$

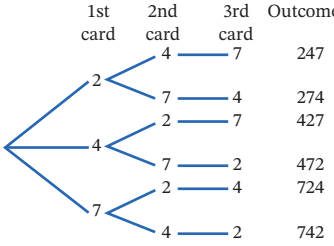
c i $\frac{1}{10}$ ii $\frac{3}{5}$

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8 a i



ii

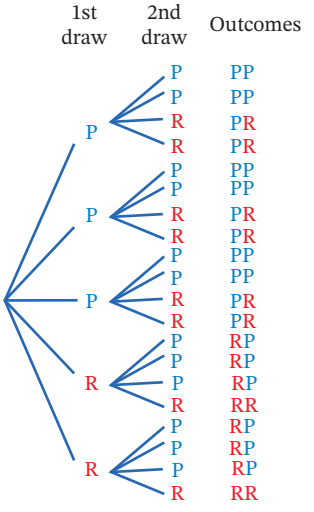


b i $\frac{2}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{2}{9}$

c i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$

9 a independent **b** dependent **c** dependent
d independent **e** dependent **f** independent

10 a



b i $\frac{1}{2}$ **ii** $\frac{1}{2}$ **c i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

d i $\frac{1}{4}$ **ii** $\frac{3}{4}$

11 a

		1st die			
		1	2	3	4
2nd die	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b i $\frac{1}{2}$ **ii** 1 **c** $\frac{2}{7}$

d i $\frac{1}{3}$ **ii** $\frac{1}{3}$ **e** $\frac{1}{4}$

Chapter 13

SkillCheck

1 a $h = 71$ **b** $p = 105$ **c** $x = 126$
d $m = 68$ **e** $a = 24$ **f** $w = 36$
g $k = 90, w = 30$ **h** $r = 83$ **i** $p = 26, r = 98$
j $p = 52$ **k** $y = 42$ **l** $d = 54$

2 a $m = 65$ (angles on a straight line), $n = 65$ (alternate angles on || lines), $p = 50$ (angle sum of $\triangle XWY$)

b Isosceles ($m = n = 65$)

Exercise 13.01

1 a 1800° **b** 1440° **c** 1260°
d 3240° **e** 2340°

2 a $e = 112$ **b** $m = 135$
c $x = 83, y = 97$ **d** $w = 108, h = 72$
e $k = 203$ **f** $a = 32$

3 a 6 **b** 21 **c** 13 **d** 30 **e** 9

4 a 16 **b** 157.5°

5 a 144° **b** 135° **c** 120° **d** 150°

6 C

7 a 30 **b** 45 **c** 25

8 a 72° **b** 30° **c** 20° **d** 60°

9 a 140° **b** 162° **c** 144° **d** 168°

10 a 24 **b** 5 **c** 18 **d** 9 **e** 72

11 C

12 a 8 **b** 10 **c** 15 **d** 180 **e** 24 **f** 12

Exercise 13.02

Congruent triangles proofs: see worked solutions.

Exercise 13.03

1 $\angle A = \angle C$ and $\angle B = \angle D$

Now $\angle A + \angle C + \angle B + \angle D = 360°$ (angle sum of a quadrilateral)

$\therefore 2\angle A + 2\angle B = 360°$ ($\angle C = \angle A, \angle D = \angle B$)

$\therefore \angle A + \angle B = 180°$

$\therefore AD \parallel BC$ (co-interior angles are supplementary)

ANSWERS

Also, from $\angle A + \angle C + \angle B + \angle D = 360°$ (angle sum of a quadrilateral)

$\therefore 2\angle A + 2\angle D = 360°$ ($\angle C = \angle A$, $\angle B = \angle D$)

$\therefore \angle A + \angle D = 180°$

$\therefore AB \parallel DC$ (co-interior angles are supplementary)

$\therefore ABCD$ is a parallelogram (opposite sides are parallel).

2 In $\triangle LMP$ and $\triangle NPM$

$LM = NP$ (given)

PM is common

$\angle LMP = \angle NPM$ (alternate angles, $LM \parallel NP$)

$\therefore \triangle LMP \equiv \triangle NPM$ (SAS)

$\angle LPM = \angle NMP$ (matching angles of congruent triangles)

$\therefore LP \parallel NM$ (alternate angles are equal)

$\therefore LMNP$ is a parallelogram (opposite sides are parallel).

3 In $\triangle DXH$ and $\triangle GXE$

$\angle DXH = \angle GXE$ (vertically opposite angles)

$HX = EX$ (given)

$DX = GX$ (given)

$\therefore \triangle DXH \equiv \triangle GXE$ (SAS)

$\angle HDX = \angle EGX$ (matching angles of congruent triangles)

$\therefore HD \parallel EG$ (alternate angles are equal)

Similarly, $\angle GHX = \angle DEX$ (matching angles of congruent triangles HXG and EXD)

$\therefore HG \parallel ED$ (alternate angles are equal)

$\therefore DEGH$ is a parallelogram (opposite sides are parallel).

4 $\triangle FHC \equiv \triangle FHE \equiv \triangle DHE \equiv \triangle DHC$ (SAS)

$\therefore FC = FE = DE = DC$ (matching sides of congruent triangles)

Also, $\angle CFH = \angle EDH$ and $\angle CDH = \angle EFH$ (matching angles of congruent triangles)

$\therefore CDEF$ is a rhombus (opposite sides are parallel and all sides are equal).

5-16 See worked solutions for proofs for remaining questions.

Exercise 13.04

Proofs: see worked solutions.

Mental skills 13

2 **a** 12:25 p.m. **b** 1:10 a.m. **c** 10:50 p.m.
d 10:55 p.m. **e** 06:10 **f** 00:10
g 9:10 a.m. **h** 3:15 a.m. **i** 11:00
j 23:05 **k** 12:20 a.m. **l** 11:35 a.m.

3 **a** 6:05 p.m. **b** 6:40 a.m. **c** 12:10 p.m. **d** 2:50 a.m.
e 12:45 **f** 03:55 **g** 10:50 p.m. **h** 12:15 p.m.
i 15:45 **j** 04:00

Exercise 13.05

See worked solutions for full proofs.

Exercise 13.06

1 **a** $w = 22.4$ **b** $m = 10$
c $p = 20, h = 21$ **d** $x = 18$
e $a = 12.8, w = 7.5$ **f** $g = 11\frac{1}{9}, q = 18$
g $y = 26\frac{2}{3}, b = 9\frac{3}{5}$ or 9.6
h $u = 12\frac{4}{5}$ or 12.8, $t = 6\frac{7}{8}$ or 6.875

2 $h = 13\frac{5}{7}$ **3** $x = 8\frac{8}{9}$ **4** $w = 16$ cm
5 12 m **6** B **7** $h = 2.408$ m
8 D **9** 2.24 m

Exercise 13.07

1 **a** 2 pairs of angles are equal (AA).
b All 3 pairs of matching sides are in the same ratio, $\frac{9}{18} = \frac{11}{22} = \frac{15.5}{31} = \frac{1}{2}$ (SSS).
c 2 pairs of matching sides are in the same ratio $\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$ and the included angles are equal (SAS).
d 2 pairs of angles are equal (AA).
e All 3 pairs of matching sides are in the same ratio $\frac{9}{12} = \frac{9}{12} = \frac{14.25}{19} = \frac{3}{4}$ (SSS).
f In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{12}{15} = \frac{20.8}{26} = \frac{4}{5}$ (RHS).
g 2 pairs of angles are equal (AA).
h All 3 pairs of matching sides are in the same ratio $\frac{18}{14.4} = \frac{27.5}{22} = \frac{20}{16} = \frac{5}{4}$ (SSS).
i All 3 pairs of matching sides are in the same ratio $\frac{6}{8} = \frac{8}{10\frac{2}{3}} = \frac{10}{13\frac{1}{3}} = \frac{3}{4}$ (SSS).
j 2 pairs of matching sides are in the same ratio $\frac{26}{18.2} = \frac{30}{21} = \frac{10}{7}$, and the included angles are equal (SAS).

2 **a** B and C (SAS) **b** A and C (SSS)
c B and D (RHS)

3 **a** $\triangle UWY \,|||\, \triangle HEK$ (SAS)
b $\triangle DML \,|||\, \triangle TPA$ (RHS)
c $\triangle ABC \,|||\, \triangle QTP$ (AA)
d $\triangle GHN \,|||\, \triangle WVS$ (SSS)

Exercise 13.08

See worked solutions for complete proofs.

1 **a** $\frac{TC}{PM} = \frac{18}{10.8} = \frac{5}{3}$

$\frac{TB}{PB} = \frac{25}{15} = \frac{5}{3}$

$\angle C = \angle M = 90°$

b $\angle V = \angle L = 22°$
$\angle W = \angle Q = 123°$

c $\frac{AB}{WM} = \frac{17}{25.5} = \frac{2}{3}$
$\frac{AC}{TM} = \frac{12}{18} = \frac{2}{3}$
$\frac{BC}{WM} = \frac{16}{24} = \frac{2}{3}$

d $\frac{EV}{DN} = \frac{21}{12} = \frac{7}{4}$
$\frac{VH}{NL} = \frac{35}{20} = \frac{7}{4}$
$\angle V = \angle N = 90°$

2 In $\triangle ADE$ and $\triangle ABC$
$\frac{AD}{AB} = \frac{1}{2}$ (D is the midpoint of AB)
$\frac{AE}{AC} = \frac{1}{2}$ (E is the midpoint of AC)
$\angle A$ is common

3 $\angle AFB = \angle FED$ (corresponding angles, $BF \parallel CE$)
$\angle ABF = \angle ACE$ (corresponding angles, $BF \parallel CE$)
$\angle ACE = \angle FDE$ (corresponding angles, $AC \parallel FD$)
$\therefore \angle ABF = \angle FDE$

4 $\angle WXY = \angle TXW = 90°$ (given)
$\angle YWX = 90° - \angle WYX$ (angle sum of $\triangle WXY$)
$\angle XTW = 90° - \angle WXY$ (angle sum of $\triangle WTY$)
$= \angle YWX$

5 $\frac{ND}{NQ} = \frac{8}{16} = \frac{1}{2}$
$\frac{NL}{NR} = \frac{10}{20} = \frac{1}{2}$
$\angle N$ is common

6 $\frac{HW}{WX} = \frac{18}{12} = \frac{3}{2}$
$\frac{XW}{YX} = \frac{12}{8} = \frac{3}{2}$
$\angle HWX = \angle YXW$ (alternate angles, $HW \parallel YX$)

7 $\angle NML = \angle KLP$ (alternate angles, $NM \parallel LK$)
$\angle LNM = \angle PKL$ (opposite angles of a parallelogram)

8 **a** $\angle FLN = \angle FDE$ (corresponding angles, $LN \parallel DE$)
$\angle F$ is common

b $d = 9$

9 **a** $\angle EAC = \angle DBC = 90°$ (given)
$\angle C$ is common

b $y = 5\frac{5}{11}$

10 **a** $\angle YRT = \angle WUT = 90°$ (given)
$\angle YTR = \angle WTU$ (vertically opposite angles)

b $g = 15$

11 **a** $\angle NMP = \angle PCB = 90°$ (given)
$\angle MNP = \angle CPB$ (corresponding angles, $TN \parallel CP$)

b $w = 7.5$

12 **a** $\angle TYN = \angle MNY$ (alternate angles, $TY \parallel MN$)
$\angle TNY = \angle YMN$ (given)

b $h = 12$

13 **a** $\angle BUH = \angle XBD$ (given)
$\angle UBH = \angle DXB$ (alternate angles, $BU \parallel DX$)

b $y = 18$

14 **a** $\angle GMX = \angle LKX$ (alternate angles, $MG \parallel LH$)
$\angle MGX = \angle KLX$ (alternate angles, $MG \parallel LH$)

b $x = 16$

15 **a** $\angle CML = \angle TEL = 90°$ (given)
$\angle L$ is common

b $TL = 9$ cm

16 **a** $\angle PUT = \angle KPB$ (alternate angles, $PK \parallel TU$)
$\angle PTU = \angle KBP = 90°$ (given)

b $PB = 4$ cm

Power plus

See worked solutions for full proofs.

1 **a** $\angle ACB = 90°$ (by Pythagoras' theorem in $\triangle ABC$)
$\angle CDB = 90°$ (given)
$\therefore \angle ACB = \angle CDB$
$\angle B$ is common
$\angle ACB = 90°$ (by Pythagoras' theorem in $\triangle ABC$)
$\angle CDA = 90°$ (given)
$\therefore \angle ABC = \angle CDA$
$\angle A$ is common

b $CD = 4\frac{8}{13} \approx 4.62$

Test yourself 13

1 **a** 2340° **b** 3960° **c** 1800° **d** 8280°

2 156° **3** 36

4 **a** 36 **b** 15 **c** 8 **d** 24

5 B

6 Proof: see worked solutions.

7 $\angle W = \angle X$ (given)
$\angle WZY = \angle XZY = 90°$ ($YZ \perp WX$)
YZ is common

8 Prove $\triangle NMA \equiv \triangle PQB$ (SAS).
$\therefore \angle CAB = \angle CBA$
$\therefore AC = BC$ (sides opposite the equal angles in isosceles $\triangle CBA$)
Also, $NA = PB$ (matching sides of congruent triangles)
$\therefore NC = NA + AC = PB + BC = PC$

9 **a** $\frac{4}{5}$ **b** $\frac{5}{3}$

10 Yes,

a $\frac{27}{18} = \frac{15}{10} = \frac{3}{2}$

b Yes, $\frac{22}{27.5} = \frac{16}{20} = \frac{12}{15} = \frac{9}{11.25} = \frac{4}{5} = 0.8$

11 **a** $k = 12\frac{6}{7}$ **b** $d = 6$

c $y = 4\frac{1}{5}$ or 4.2 **d** $x = 5\frac{1}{2}$ or 5.5

12 $11\frac{3}{7}$

13 **a** SAS **b** RHS **c** AA

14 $\angle SAB = \angle SRT$ (corresponding angles, $AB \parallel RT$)
$\angle S$ is common, $x = 4.8$ cm, $y = 9.6$ cm

15 Proof: see worked solutions.

Practice set 4

1 40

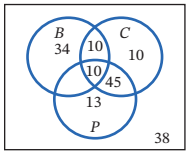
2 **a** 80

b **i** $\frac{13}{80}$ **ii** $\frac{36}{80} = \frac{9}{20}$ **iii** $\frac{42}{80} = \frac{21}{40}$ **iv** $\frac{56}{80} = \frac{7}{10}$

c $\frac{1}{6} = 0.1\dot{6}$, which is lower than the experimental probability of $\frac{17}{80} = 0.2125$

3 **a** 78 **b** **i** $\frac{25}{39}$ **ii** $\frac{14}{39}$ **c** $\frac{35}{78}$

4 **a**



b **i** $\frac{13}{160}$ **ii** $\frac{19}{80}$ **iii** $\frac{9}{32}$
iv $\frac{109}{160}$ **v** $\frac{57}{160}$ **c** $\frac{5}{27}$

5 $\angle BAC = \angle DCA$ (alternate angles, $AB \parallel CD$)
$\angle BCA = \angle DAC$ (alternate angles, $AD \parallel CB$)
AC is common

6 independent

7 **a** **i** 0.39 **ii** 0.43 **iii** 0.18
b **i** 0.33 **ii** 0.42 **iii** 0.25
c drawing a black marble
d 88, which is close to the observed frequency of 85

8 **a** 36 **b** 170°

9 $OC = OD$ (equal radii)
$\angle CEO = \angle DFO = 90°$ ($CE \perp AB$ and $DF \perp AB$)
$\angle COE = \angle DOF$ (vertically opposite angles)

10 $LM = LN$ (given)
$MP = NP$ (P is the midpoint of MN)
LP is common
$\therefore \angle LPM = \angle LPN$ (matching angles of congruent triangles)
But $\angle LPM + \angle LPN = 180°$ (angles on a straight line)
$\therefore \angle LPM = \angle LPN = 90°$

11 **a** 135 **b** $\frac{40}{135} = \frac{8}{27}$
c **i** $\frac{67}{135}$ **ii** $\frac{60}{135} = \frac{4}{9}$ **iii** $\frac{72}{135} = \frac{8}{15}$

12 **a** $h = 5\frac{1}{3}$ **b** $p = 35$

13 **a** Teacher to check
b **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$ **v** $\frac{1}{4}$

14 **a** 3600° **b** 900° **c** 5400° **d** 3060°

15 36 **16** $y = 10.6$

17 **a**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b $\frac{5}{11}$ **c** **i** 1 **ii** 0 **d** $\frac{1}{2}$ **e** 1

18 **a** $YX = VW$ and $YX \parallel VW$ (opposite sides of a rectangle)
$\angle YXT = \angle WVT$ (alternate angles, $YX \parallel VW$)
$\angle YTX = \angle WTV$ (vertically opposite angles)
b $\therefore YT = TW$ and $XT = TV$ (matching sides in congruent triangles)
The diagonals of a rectangle bisect each other.

19 **a** $PT = RQ$ (given)
$RT = PQ$ (given)
PR is common
b Proof: see worked solutions.

20 **a** $\angle ABW = \angle CDW$ (alternate angles, $AB \parallel CD$)
$\angle BAW = \angle DCW$ (alternate angles, $AB \parallel CD$)
b $CW = 4\frac{5}{7}$

21 Proof: see worked solutions.

General practice

1 $7\sqrt{2}$ **2** $\frac{3\sqrt{10}}{10}$

3 $98 + 24\sqrt{10}$ **4** \$15 700

5 gradient $-\frac{4}{5}$, y-intercept 2

6 **a** $6x(x + 2)$ **b** $(a - 10)(a + 4)$
c $25(1 + 2y)(1 - 2y)$ **d** $-2(2p + 1)(p - 3)$

7 $x = \frac{-7 \pm \sqrt{41}}{2}$ **8** $\bar{x} = 50.3$, $\sigma = 12.2$

9 **a** 360 498 mm^3 **b** 145 125 mm^3

10 $\theta = 141° 15'$

11 **a** 210 **b** **i** $\frac{1}{6}$ **ii** $\frac{143}{210}$
c 32% **d** $\frac{29}{36}$

12 **a** $56x^{12}$ **b** $\frac{x^5}{4}$ **c** $\frac{1}{9y^2}$ **d** $\frac{n}{m^7}$

13 **a** $y = 6\frac{1}{4}$ **b** $k = 3\frac{3}{7}$ **c** $x = \pm 3$

14 $d = 19.1$ **15** B

16 **a** -1 **b** 4 **c** $\frac{1}{2}$

17 **a** 5 **b** 1 **c** -3

18 $AB = DE = 10$ cm (given)
$CB = FE = 12$ cm (given)
$\angle A = \angle D = 90°$ (given)

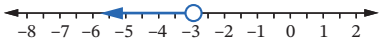
19 **a** $y \geq -1$



b $x < 1\frac{1}{2}$



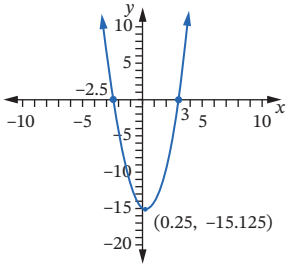
c $x < -3$



20 $2x + 3y + 5 = 0$ **21** $x = 2, y = 2$

22 $x^2 + y^2 = 64$

23 x-intercepts at $-2\frac{1}{2}$ and 3, y-intercept at −15, axis of symmetry $x = \frac{1}{4}$, vertex $\left(\frac{1}{4}, -15\frac{1}{8}\right)$



24 **a** 7.2 m³ **b** 78.5 cm³ **c** 5747.0 cm³

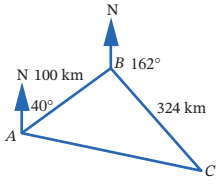
25 \$1607.41

26 **a** **i** $\sqrt{13}, m = \frac{2}{3}$ **ii** $2\sqrt{10}, m = -\frac{1}{3}$

iii $\sqrt{13}, m = \frac{2}{3}$ **iv** $2\sqrt{10}, m = -\frac{1}{3}$

b parallelogram (opposite sides equal and parallel)

27 **a**

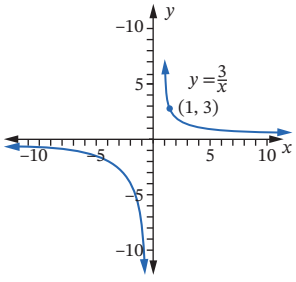


b $\angle NBA = 180° - 40° = 140°$ (co-interior angles on parallel lines)

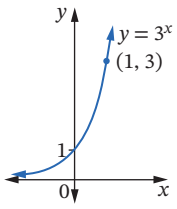
$\angle ABC = 360° - 140° - 162° = 58°$ (angles at a point)

c 284 km **d** 144° 37′

28 **a**

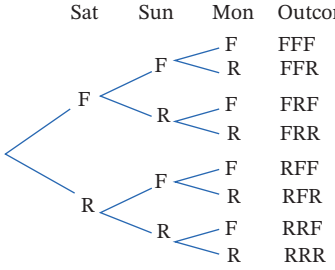


b



29 $h = 21, p = 20$

30 **a** F = fine, R = rain

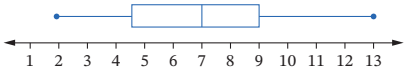


b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

31 19.2 min **32** $x = 2, y = 1$

33 Value \$1174.66, depreciation \$3775.34

34 Lowest value = 2, $Q_1 = 4.5$, Median = 7, $Q_3 = 9$, Highest value = 13

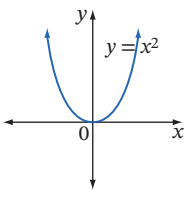


35 **a** 12.1 m² **b** 22.6 m²

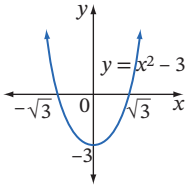
Chapter 14

SkillCheck

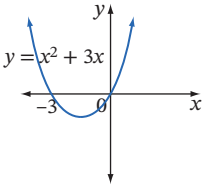
1 **a**



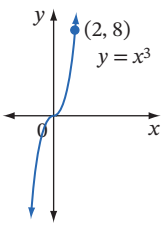
b



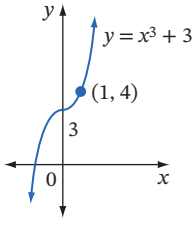
c



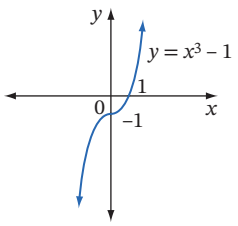
2 **a**



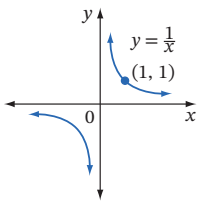
b



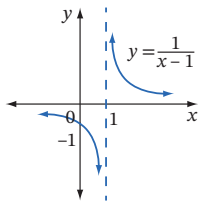
c



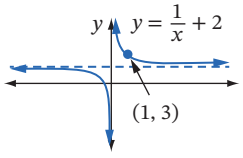
3 a



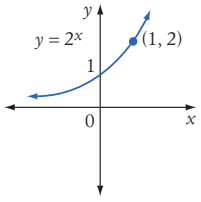
b



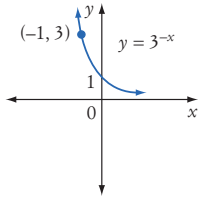
c



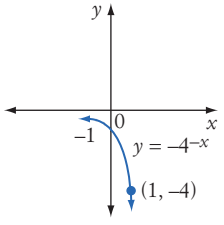
4 a



b



c



3 a $x = \frac{y+1}{2}$ **b** $x = \frac{3}{y} - 1$

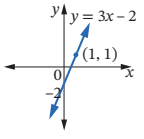
c $x = \pm\sqrt{y-4}$ **d** $x = y^2 + 2$

Exercise 14.01

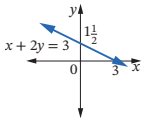
1 a Yes **b** No **c** Yes **d** Yes
e Yes **f** Yes **g** No **h** Yes

2 a Yes **b** Yes **c** No **d** No
e No **f** Yes **g** No **h** Yes
i Yes **j** No **k** Yes **l** Yes
m Yes **n** No **o** Yes **p** Yes

3 a i

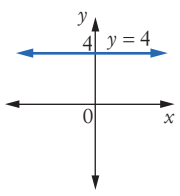


ii



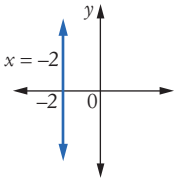
$m = -\frac{1}{2}$

iii



$m = 0$

iv



Gradient is undefined.

b **i**, **ii** and **iii**

c **i** For all values of m.

ii If m is undefined (a vertical line).

Exercise 14.02

1 a i 6 **ii** −2 **iii** 0
b i −2 **ii** 2 **iii** 1
c i 24 **ii** −4 **iii** 0
d i $\frac{1}{7}$ **ii** −1 **iii** 1
e i ±3 **ii** ±1 **iii** $\pm\sqrt{3}$
f i $1\frac{2}{5}$ **ii** $\frac{1}{3}$ **iii** −1
g i $1\frac{2}{5}$ **ii** $\frac{1}{3}$ **iii** −1
h i −2 **ii** 6 **iii** 1
i i 60 **ii** 12 **iii** 21

2 a 0 **b** −28 **c** $9k - 9k^2$

3 a 16 **b** −10 **c** $6d - 20$
d 12 **e** $2\frac{5}{6}$

4 a 4 **b** $-\frac{1}{2}$ **c** −2 **d** $x = 2\frac{1}{2}$

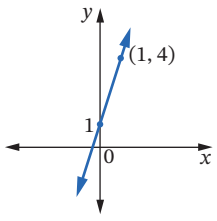
5 a 5 **b** 0 **c** 14 **d** $t = -2, 1\frac{1}{2}$

6 a i 11 **ii** 19 **iii** 10
b i $9 - 2y$ **ii** −2 **c** 16

7 a $-3 \le x \le 3$ (this means $x \ge -3$ and $x \le 3$);
$0 \le y \le 3$ (this means $y \ge 0$ and $y \le 3$)
b $-4 \le x \le 0; 0 \le y \le 4$ **c** $x \ge 0; y \le 5$
d $x \ge -1; y \ge 0$ **e** $x \ge -1; y \ge -3$
f all x; $y = -3$ **g** all x; $y > 0$
h all x; $y = 3$ **i** all x; $y > -2$
j all x; $-1 \le y \le 1$ **k** all x; $y \ge -4$
l all x; $0 < y \le 4$

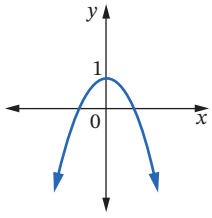
9780170453509

8 **a**



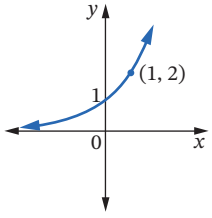
i All x **ii** All y

b



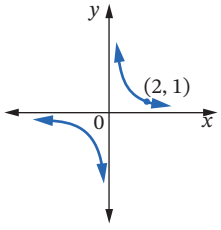
i All x **ii** $y \leq 1$

c



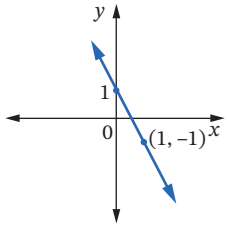
i All x **ii** $y > 0$

d



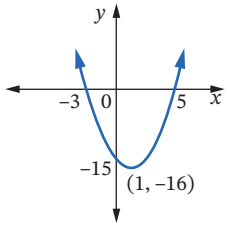
i $x \neq 0$ **ii** $y \neq 0$

e



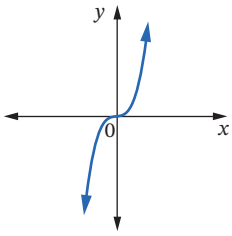
i All x **ii** All y

f



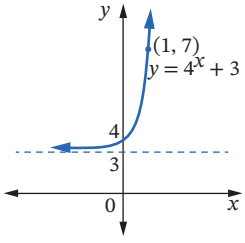
i All x **ii** $y \geq -16$

g



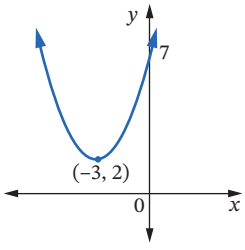
i All x **ii** All y

h



i All x **ii** $y > 3$

i



i All x **ii** $y \geq 2$

9 **a** **i** 11 **ii** −5 **iii** 6

b Teacher to check **c** $m = 3, -1$

10 **a** 3 **b** 12

c $\sqrt{-1}$ has no value **d** $x = -\frac{1}{2}$

11 **a** 8 **b** 11 **c** 40

d $3k^4 - 2k^2 + 3$ **e** $3k^2 + 2k + 3$ **f** $-4k$

12 **a** $8\frac{1}{8}, 4\frac{1}{4}, 2^{a+1} + \frac{1}{2^{a+1}}$

b Yes, they both equal $2^x + 2^{-x}$

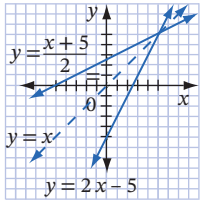
13 **a** **i** $\frac{1}{3}$ **ii** $-6\frac{1}{3}$ **iii** $\frac{1}{3}$

b no **c** yes

Exercise 14.03

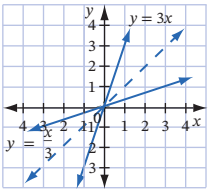
1 **a** **i** $f^{-1}(x) = \frac{x+5}{2}$

ii



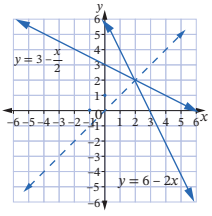
b **i** $f^{-1}(x) = 3x$

ii



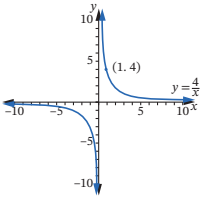
c **i** $f^{-1}(x) = 6 - 2x$

ii



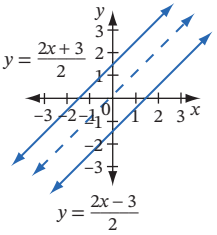
d **i** $f^{-1}(x) = f(x) = \frac{4}{x}$

ii



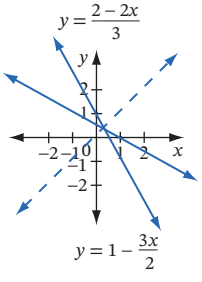
e **i** $f^{-1}(x) = \frac{2x+3}{2}$

ii



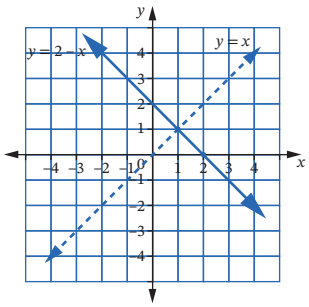
f **i** $f^{-1}(x) = \frac{2-2x}{3}$

ii



2 **a** Proof: see worked solutions.

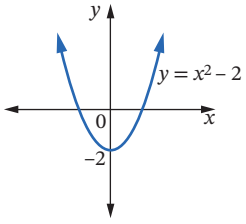
b



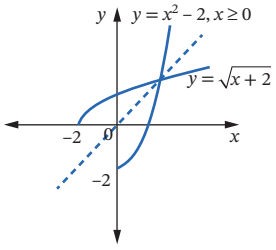
The graph of $f(x) = 2 - x$ is itself symmetrical about the line $y = x$.

3 **b**, **c** and **h**

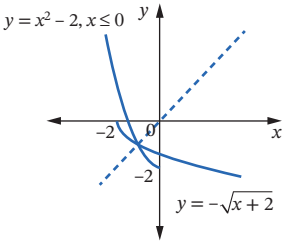
4 **a**



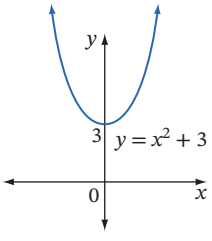
b No **c** $y = \sqrt{x+2}$



d $y = -\sqrt{x+2}$



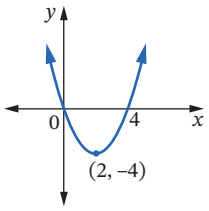
5 **a**



b No **c** $x \geq 0$ or $x \leq 0$

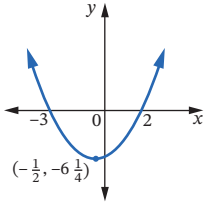
9780170453509

6 **a**



b $x \geq 2$ or $x \leq 2$

7 **a**



b No

c $x \geq -\frac{1}{2}$ or $x \leq -\frac{1}{2}$

Exercise 14.04

1 $x = 2^y$

$\therefore \log_2 x = \log_2 2^y$

$\therefore y = \log_2 x$

2 **a** Domain: all x; range: $y > 0$

b Domain: $x > 0$; range: all y

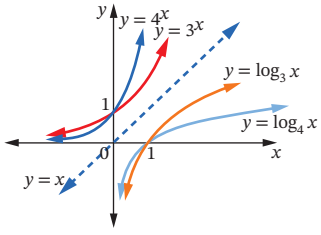
c They have interchanged.

3 Both graphs are increasing.

For $y = 2^x$, y-intercept = 1, no x-intercept.

For $y = \log_2 x$, x-intercept = 1, no y-intercept.

4



a They are all increasing graphs and have a y-intercept of 1.

For $x > 0$, $y = 4^x$ is steeper than $y = 3^x$, which is steeper than $y = 2^x$.

For $x < 0$, $y = 4^x$ is closer to the x-axis than $y = 3^x$, which is closer to the x-axis than $y = 2^x$.

b They are all increasing graphs and have an x-intercept of 1.

For $x > 1$, $y = \log_2 x$ is steeper than $y = \log_3 x$, which is steeper than $y = \log_4 x$.

For $x < 0$, $y = \log_4 x$ is closer to the y-axis than $y = \log_3 x$, which is closer to the y-axis than $y = \log_2 x$.

5 **a** 2^x **b** 4^x

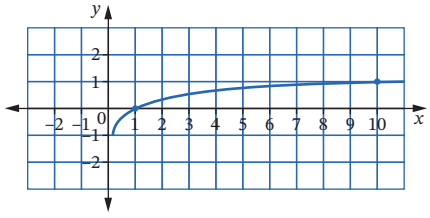
6 **a** $\log_4 x$ **b** $\log_2 x$

7 **a** 1.3010 **b** 2.7973 **c** 3.7345

d 0.9138 **e** 0.3979 **f** –0.1192

8

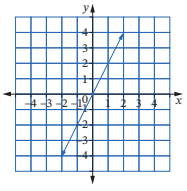
x	0.5	1	2	5	8	10
y	–0.3	0	0.3	0.7	0.9	1



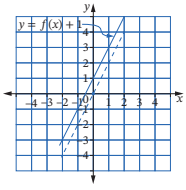
9 **a** 0.5 **b** 0.8 **c** 0.9 **d** –0.3

Exercise 14.05

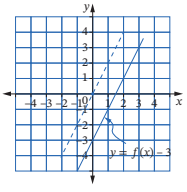
1 **a**



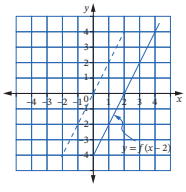
b



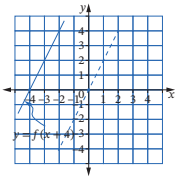
c



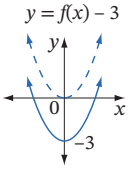
d



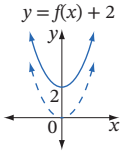
e



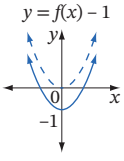
2 **a**



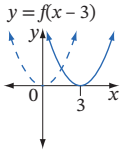
b



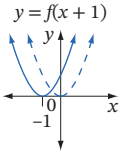
c



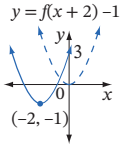
d



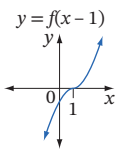
e



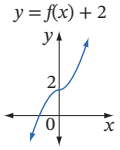
f



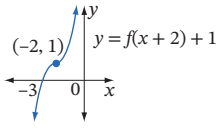
3 **a**



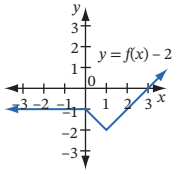
b



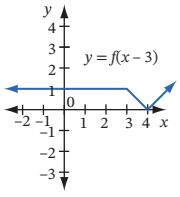
c



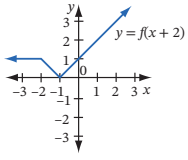
4 **a**



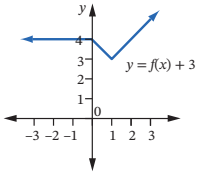
b



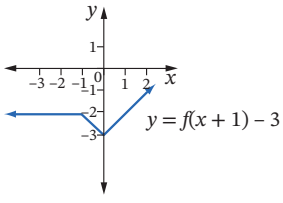
c



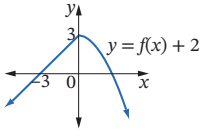
d



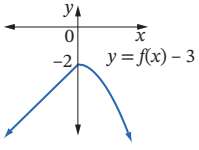
e



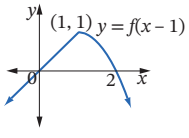
5 **a**



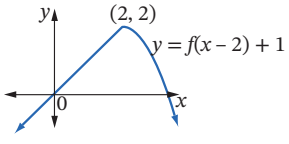
b



c



d



Chapter 15

Exercise 15.01

1 **a** Yes, not monic **b** No
c Yes, not monic **d** Yes, not monic
e No **f** Yes, monic
g Yes, not monic **h** No
i Yes, monic **j** No
k Yes, monic **l** Yes, not monic

2 **a** **i** 5 **ii** 9 **iii** 1
b **i** 5 **ii** −6 **iii** 3
c **i** 2 **ii** 11 **iii** −10
d **i** 1 **ii** −6 **iii** 0
e **i** 5 **ii** 7 **iii** 3
f **i** 0 **ii** 9 **iii** 9
g **i** 6 **ii** 1 **iii** −11
h **i** 1 **ii** $-\frac{5}{4}$ **iii** 22
i i 3 **ii** $\frac{1}{3}$ **iii** 0

3 **a** −1 **b** 17 **c** −7 **d** $-\frac{5}{8}$ **e** $7\sqrt{2}-5$

4 **a** 1 **b** 2 **c** 14 **d** −3
e $\frac{47}{64}$ **f** $13-5\sqrt{3}$ **g** 56 **h** −7

Exercise 15.02

1 **a** $9x^3 + 8x^2 + 6x - 2$ **b** $-x^3 + 4x + 2$
c $-4x^2 - 3x - 2$ **d** $5x^4 - 3x^3 - 5x^2 + 4$
e $x^4 - x^3 + x^2 - 5x$
f $2x^5 + 10x^4 + 2x^3 - 4x^2 + 4x + 22$
g $7x^6 + x^5 + x^3 + x^2 + 3x - 2$
h $8x^4 + 15x^2 + 4x - 7$ **i** $-4x^4 - 8x^3 + 5x^2 + x - 2$
j $6x^3 - 2x^2 - 6x + 1$

2 **a** $x^2 + 11x - 1$ **b** $-x^2 - 3x - 5$
c $x^2 + 3x + 5$ **d** $2x^2 + 26x - 5$

3 **a** $2x^2 - 15x + 9$ **b** $x^2 + 4x - 15$
c $-3x^2 + 11x + 6$ **d** $-x^2 + 7x + 3$
e $3x^2 - 15x + 3$ **f** $3x^2 - 7x - 15$

4 **a** $x^2 - 7x + 6$ **b** $24 - 21\sqrt{2}$ **c** 1, 6

Exercise 15.03

1 **a** $3x^4 + 2x^2 - 3x^2 - 2x$
b $32x^3 + 80x^2 - 22x$
c $45x^4 - 13x^3 + 3x^2 - 2x$
d $18x^4 + 6x^3 - 7x^2 - 11x - 6$
e $-21x^7 + 11x^6 + 2x^5 - 3x^4 - x^3 + 2x^2$
f $8x^5 + 47x^4 + 41x^2 + 28x - 12$

2 $4x^3 + 25x^2 - 13x - 6$

3 **a** $-2x^4 + 11x^3 + 12x^2 - 66x$
b $-4x^3 + 9x^2 + 24x - 54$
c $8x^3 - 62x^2 + 99x$

Exercise 15.04

1 **a** $x^2 + 7x + 4 = (x + 2)(x + 5) - 6$
b $x^2 - 6x + 2 = (x - 3)(x - 3) - 7$
c $4x^2 + 3x + 10 = (x - 1)(4x + 7) + 17$
d $8x^2 + 9x + 11 = (2x + 1)\left(4x + 2\frac{1}{2}\right) + 8\frac{1}{2}$
e $x^3 + 6x^2 + 5x - 4 = (x - 3)(x^2 + 9x + 32) + 92$
f $4x^3 + 2x^2 + x = (x + 4)(4x^2 - 14x + 57) - 228$

9780170453509

g $2x^3 - x^2 + 5x + 3 = (x + 6)(2x^2 - 13x + 83) - 495$

h $3x^3 - x^2 + 11 = (x + 2)(3x^2 - 7x + 14) - 17$

i $x^5 - x^4 + 8x^3 + 2x^2 - x - 1 =$
$(x + 1)(x^4 - 2x^3 + 10x^2 - 8x + 7) - 8$

j $x^4 - x^2 - 10 = (x + 3)(x^3 - 3x^2 + 8x - 24) + 62$

2 **a** $(3x - 1), R = 3$ **b** $(x + 7), R = 14$

c $(3x^3 + 14x^2 - 2x + 21), R = 42$

d $(4x + 6), R = 17$

3 **a** $(2x - 1)(3x + 2)$ **b** $(2x - 1)(x^2 + x + 1)$

c $(2x - 1)(4x + 7)$ **d** $(2x - 1)(3x^2 + 2x + 1)$

e $(2x - 1)(x^3 - 3x^2 - 4x + 2)$

f $(2x - 1)(x^3 - x + 3)$

g $(2x - 1)(3x^2 + 1)$ **h** $(2x - 1)(4 - 3x - x^5)$

Exercise 15.05

1 **a** 5 **b** −181 **c** −1 **d** 179

e −7 **f** 1709 **g** −85 **h** 29

2 **a** 54 **b** 2 **c** 14 **d** −2 **e** −12

f 174 **g** 0 **h** 6 **i** 115 **j** −16 993

Exercise 15.06

1 **a** B, C **b** B, C **c** A

d A, B, C **e** A, B

2 Teacher to check.

3 **a** $x(x + 2)(x + 4)$ **b** $x(x - 2)(x + 1)$

c $(x - 1)(x + 1)(x + 2)$ **d** $(x - 2)(2x - 1)(x + 4)$

e $(x - 1)(x - 2)(x - 3)$ **f** $(x - 2)(x + 8)(x - 5)$

g $(x - 6)(x + 1)(3x - 1)$ **h** $(x - 2)(3x + 1)(2x - 1)$

i $x^2(2x - 1)(x - 2)$

j $(x + 1)(x - 2)(x - 3)(2x + 5)$

4 **a** $x = -4, \frac{1}{2}, 3$ **b** $x = -4$ **c** $x = -2, \frac{5}{2}, 3$

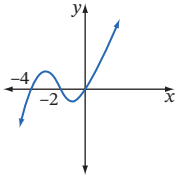
d $x = \pm 5$ **e** $x = \pm 4, -3, 2$ **f** $x = -7, 0, 2$

g $x = -3, -2, 3$ **h** $x = -2, -1, 4$ **i** $x = -4, -1, 5$

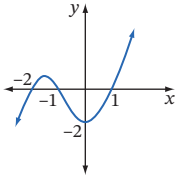
j $x = -\frac{1}{2}, 1, 3$ **k** $x = \frac{1}{4}, \frac{2}{3}, 1$ **l** $x = 3$

Exercise 15.07

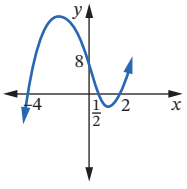
1 **a**



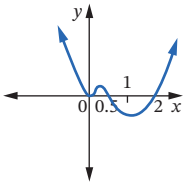
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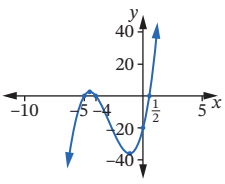
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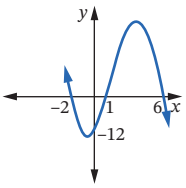
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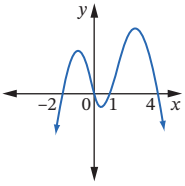
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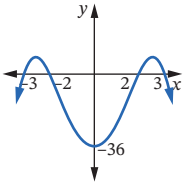
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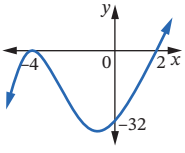
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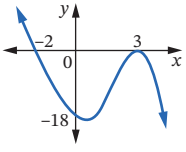
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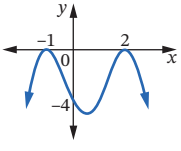
2 **a**



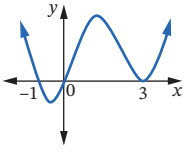
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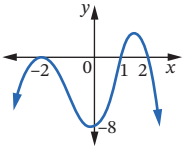
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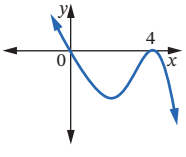
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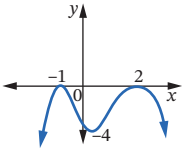
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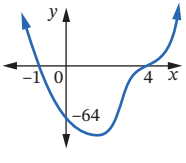
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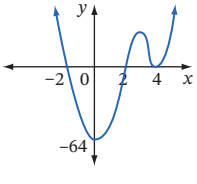
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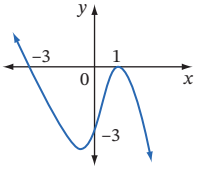
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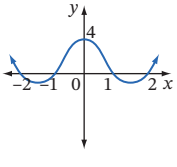
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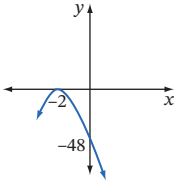
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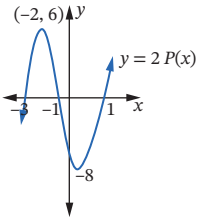


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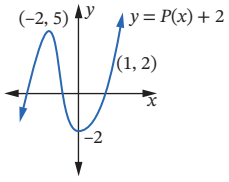


Exercise 15.08

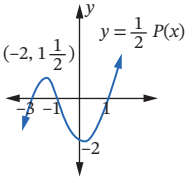
1 **a**



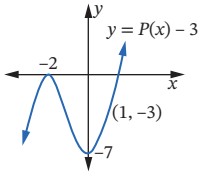
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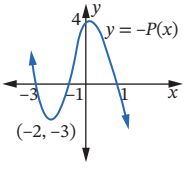
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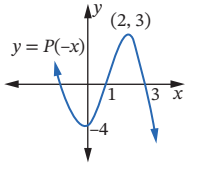
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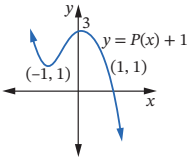
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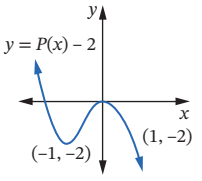
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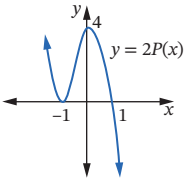
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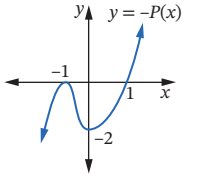
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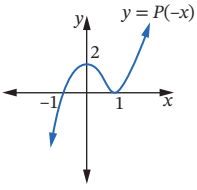
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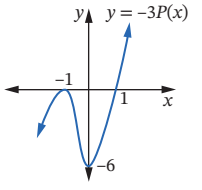
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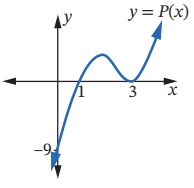
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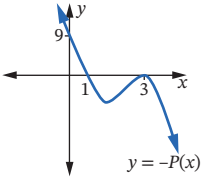
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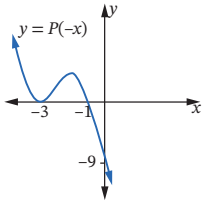
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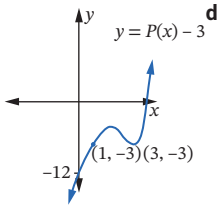
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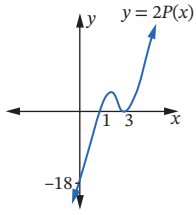
b



c



d



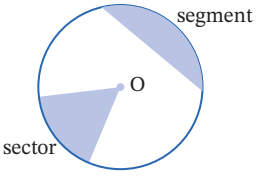
4 **a** shift down 2 units
b shift up 1 unit
c stretch vertically by a factor of 2
d reflect in x-axis and shift up 3 units
e reflect in x-axis and stretch vertically by a factor of 3
f reflect in x-axis and shift up 2 units
g stretch vertically by a factor of 2 and shift down 5 units
h reflect in x-axis, stretch vertically by a factor of 3 and shift up 4 units
i compress vertically by a factor of $\frac{1}{2}$ and shift up 4 units

Chapter 16

Exercise 16.01

1 **a** radius **b** chord
c circumference **d** tangent

2 **a**



b Sector is drawn from centre of circle and bounded by 2 radii and arc. Segment is bounded by chord and arc.

3 $d = 2r$

4 D

5 **a** diameter **b** segment
c sector **d** arc

6 **a** radius **b** quadrant
c tangent **d** diameter
e chord **f** arc
g sector **h** circumference
i segment

Exercise 16.02

1 **a** Proof by SAS: see worked solutions

2 **a**, **b** Proof by RHS: see worked solutions

3 **a** Proof by SSS: see worked solutions

4 **c** The centre of the circle

5 **a**, **b** Proof by SAS: see worked solutions

6 5.74 cm **7** 60 cm

8 **a** $UC = 4.5$ m (the perpendicular from the centre bisects the chord and the chords are equal as they are the same distance from the centre)

b $DE = 12$ m (chords of equal length subtend equal angles at the centre of a circle)

c $\angle UVO = 58°$ (chords of equal length subtend equal angles; angle sum of isosceles triangle)

d $PQ = 30$ mm (Pythagoras' theorem: the line from the centre is the perpendicular bisector of the chord)

e $OM = 21$ cm (same reason as **d**)

f $OD = 18\sqrt{2}$ (same reason as **d**)

9 52 cm **10** 18.4 km

11 $MN = 4$ cm **12** 77 cm

13 34 cm, 20 cm

14 **a** 30 cm **b** 840 cm^2

15 **a** 52 cm **b** 1920 cm^2

Exercise 16.03

1 Reason: angle at the centre is twice the angle on the circumference

a 45 **b** 112 **c** 120 **d** 232

e 40 **f** 74 **g** 63 **h** 104

i 90 (angle in a semicircle)

2 Reason: angles in the same segment (or angles standing on the same arc)

a 48 **b** 36 **c** 30

d 35 **e** 74 **f** 90

3 **a** reflex $\angle ROT = 216$ (angles at a point), $\angle S = 108$ (angle at the centre is twice the angle at the circumference)

b $x = 43$ (angles opposite equal sides in an isosceles triangle), $y = 86$ (exterior angle of triangle, or angle at the centre is twice the angle at the circumference)

c $n = 37$ (angles in the same segment), $m = 74$ (angle at the centre is twice the angle at the circumference)

d $p = 37$ (angles in the same segment)

e $w = 50$ (angle at the centre is twice the angle at the circumference)

f $h = 113$ (angle at the centre is twice the angle at the circumference, co-interior angles on parallel lines)

4 **a** $m = 75$ (opposite angles of cyclic quadrilateral)

b $p = 88$ (opposite angles of cyclic quadrilateral); $q = 121$ (opposite angles of cyclic quadrilateral)

c $x = y = 90$ (angles in a semicircle)

5 **a** $n = 106$ (exterior angle of cyclic quadrilateral)

b $w = 60$ (angle in an equilateral triangle; exterior angle of cyclic quadrilateral)

c $x = 84$ (exterior angle of cyclic quadrilateral), $y = 110$ (exterior angle of cyclic quadrilateral)

6 Reason: angle in a semicircle, angle sum of a triangle

a 23 **b** 9 **c** 45

d 63 (opposite angles of a cyclic quadrilateral)

Reason: exterior angle of a cyclic quadrilateral

e 75 **f** 88

7 **a** $x = 75$ (angle at the centre is twice the angle at the circumference)

$y = 33$ (angles in the same segment)

$z = 72$ (angle sum of a triangle)

b $x = 108$ (angle at the centre is twice the angle at the circumference)

$y = 126$ (opposite angles of a cyclic quadrilateral)

$z = 252$ (angles at a point, or angle at the centre is twice the angle at the circumference)

c $x = 70$ (angles on a straight line)

$y = 110$ (exterior angle of a cyclic quadrilateral)

$z = 70$ (angles on a straight line)

d $x = 96$ (angle at the centre is twice the angle at the circumference)

$y = 42$ (angles opposite equal sides of an isosceles $\triangle$, angle sum of a triangle)

$z = 264$ (angles at a point)

e $x = 140$ (angles opposite equal sides of an isosceles $\triangle$, angle sum of a triangle)

$y = 70$ (angle at the centre is twice the angle at the circumference)

$z = 35$ (angle sum of an isosceles $\triangle$, and by subtraction)

f $x = 62$ (angle in a semicircle, angle sum of a triangle)

$y = 118$ (opposite angles of a cyclic quadrilateral)

$z = 31$ (angle sum of an isosceles $\triangle$)

8 C ($WXYZ$) is a cyclic quadrilateral because opposite angles are supplementary:
$\angle W + \angle Y = 180°$ and $\angle X = \angle Z = 180°$

Exercise 16.04

1 **a** $\angle YMP$

b $OP \perp AB$ (angle between a tangent and radius)

c Proof by angle sum of an isosceles triangle: see worked solutions

d Proof by angle at centre is twice angle at circumference: see worked solutions

2 **a** angle between a tangent and radius

b Proof by angles on a straight line: see worked solutions

3 **a** Proof by equiangular triangles: see worked solutions

4 **a** $a = 56$ (the angle between the radius and the tangent is a right angle)

b $b = 21$ (radius is perpendicular to a tangent, and Pythagoras' theorem)

c $c = 134$ (a tangent is perpendicular to the radius; angle sum of a quadrilateral)

d $g = 67$ (angle in alternate segment)

5 **a** 15 **b** 5 **c** 9 **d** 7 **e** 20 **f** 4

6 $x = 7$ cm

7 **a** 10 cm **b** 24 cm

Exercise 16.05

1 $\left.\begin{matrix}\angle R = \angle Q \\ \angle P = \angle S\end{matrix}\right\}$ (angles at the circumference standing on the same arc)

$\therefore \triangle PYR \,|||\, \triangle SYQ$ (equiangular or AA)

$\therefore \frac{PY}{SY} = \frac{RY}{QY}$ (matching sides in similar triangles)

$\therefore PY \times YQ = RY \times YS$

2 $\angle ADC = \angle BEC$ (opposite angles of a parallelogram)

$\angle ADC = \angle CBE$ (exterior angle of a cyclic quadrilateral)

$\therefore \angle BEC = \angle CBE$

$\therefore \triangle CBE$ is isosceles (2 equal angles)

3 Draw a perpendicular from O to meet DG at P.
Since the perpendicular from the centre to a chord bisects the chord:

$DP = GP$ and $EP = FP$.

$\therefore DE = DP - EP$

$= GP - FP$

$= FG$

4 $\angle THJ = \angle HIJ$ (angle in alternate segment)

$\angle THJ = \angle HPI$ (alternate angles, $HT \parallel IP$)

$\therefore \angle HIJ = \angle HPI$

In $\triangle HIP$ and $\triangle HJI$:

$\angle HIJ = \angle HPI$ (proved above)

$\angle IHJ = \angle IHP$ (common angles)

$\therefore \triangle HIP \,|||\, \triangle HJI$ (equiangular)

$\therefore \angle HIP = \angle HJI$ (third pair of equal angles in similar triangles)

5 In $\triangle UVX$ and $\triangle UWX$:

$\angle UXV = 90° = \angle UXW$ (angle in a semicircle, angles on a straight line)

$UV = UW$ (given)

UX is common.

$\therefore \triangle UVX \equiv \triangle UWX$ (RHS)

$\therefore VX = VW$ (matching sides in congruent triangles)

$\therefore$ Circle bisects base of triangle.

6 Let $\angle QRP = x$

$\therefore \angle SRP = x$ (PR bisects $\angle QRS$)

$\therefore \angle PSQ = x = \angle QRP$ (angles at the circumference standing on the same arc)

$\angle PQS = x = \angle SRP$ (angles at the circumference standing on the same arc)

$\therefore \angle PQS = \angle PSQ = x$

$\therefore \triangle SPQ$ is isosceles (2 equal angles).

7 Join APX and BQX.

In $\triangle XPQ$ and $\triangle XAB$:

$\angle X$ is common

$\angle XPQ = \angle XAB$ (corresponding angles, $PQ \parallel AB$)

$\therefore \triangle XPQ \,|||\, \triangle XAB$ (equiangular or AA)

$\therefore \frac{XP}{XA} = \frac{PQ}{AB}$ (matching sides in similar triangles)

but $XP = \frac{1}{2}XA$ (radius is half the diameter)

$\therefore \frac{\frac{1}{2}XA}{XA} = \frac{PQ}{AB}$

$\therefore \frac{1}{2} = \frac{PQ}{AB}$

$\therefore PQ = \frac{1}{2}AB$

8 Let $\angle PTX = a$

$\therefore \angle R = a$ (angle in alternate segment)

Now $\angle QTY = a$ (vertically opposite angles)

$\therefore \angle S = a$ (angle in alternate segment)

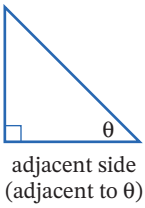
$\therefore \angle R = \angle S = a$

$\therefore PR \parallel SQ$ (alternate angles are equal).

GLOSSARY AND INDEX

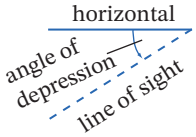
A

adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle. (p. 348)

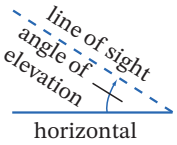


allowable (tax) deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**. (p. 31)

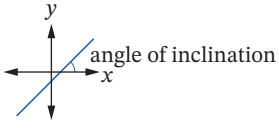
angle of depression The angle of looking **down**, measured from the horizontal. (p. 349)



angle of elevation The angle of looking **up**, measured from the horizontal. (p. 349)

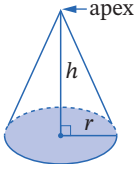


angle of inclination The angle that a line makes with the x-axis in the positive direction. (p. 64)

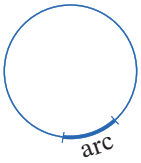


annual leave loading (or **holiday loading**) Extra payment to a worker during annual leave based on 17.5% of 4 weeks' pay. (p. 27)

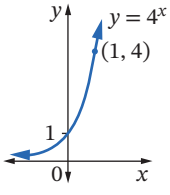
apex The pointed top of a **pyramid** or **cone.** (p. 109)



arc Part of the circumference of a circle. (p. 570)



asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve. (pp. 326, 331)



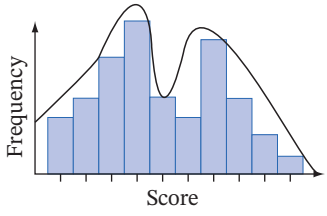
average See **mean**. (p. 196)

B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base. (p. 151)

bearing The angle used to show the direction of one location from a given point. (p.354) See also **compass bearing** and **three-figure bearing**.

bimodal distribution A statistical distribution that has 2 peaks. (p. 187)

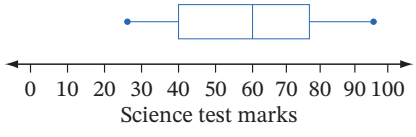


binomial expression An algebraic expression with 2 terms, for example, $x + 9$, $2y - 12$. (p. 165) See also **trinomial**.

bisect To cut in half. (p. 500)

bivariate data Data that measures 2 variables, such as a person's height and arm span, represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis. (p. 221)

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the 'box' contains the middle 50% of values while the 'whiskers' extend to the 2 extremes. (p. 204)

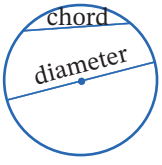


C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). (p. 136) See also **volume**.

chance experiment An activity or process that involves chance, for example, rolling a die or tossing a coin. (p. 465)

chord An interval joining 2 points on a circle. (p.570).



cluster A group of data values that are bunched or close together. (p. 187)

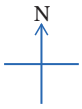
coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2. (pp. 171, 308, 402)

9780170453509

collinear points Points that lie on a straight line. (p. 96)

compass bearing A bearing that refers to one of the 16 points of a mariner's compass; for example, north-northwest (NNW). (p. 354) See also **bearing** and **three-figure bearing**.

compass rose A cross-shaped diagram that shows the direction of north. (p. 355) See also **compass**.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

compound event A chance event that is a combination of 2 or more simple events, for example, 'female or left-handed'. (pp. 453, 458)

compound interest Interest that is calculated as a percentage of the original principal and the accumulated interest. (p. 38) See also **simple interest**.

conditional probability The probability that an event occurs given that another event occurs. (p.480)

cone A solid shape with a circular base, a curved surface and an apex. (p. 109)

congruence test One of 4 tests for proving that 2 triangles are congruent: SSS, SAS, AAS and RHS. (p. 497)

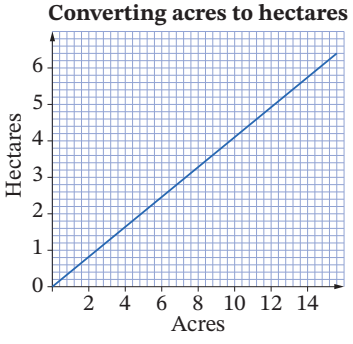
congruent Identical, exactly the same. The symbol '≡' means 'is congruent to' or 'is identical to'. (p. 497)

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10. (p. 256)

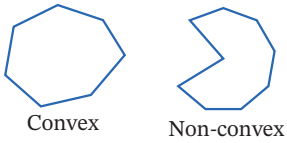
constant term The term in an algebraic expression that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6. (pp. 171, 552)

converse A rule or statement turned back-to-front; the reverse statement. (p. 573)

conversion graph A line graph for converting between different units or currencies, for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0). (p. 293)



convex polygon A polygon whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180°. (p. 493)



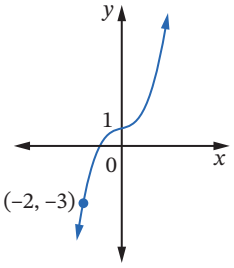
cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 348) See also **sine** and **tangent**.

cosine rule A rule that relates the 3 sides and one of the angles of any triangle: $c^2 = a^2 + b^2 - 2ab\cos C$. (p. 382)

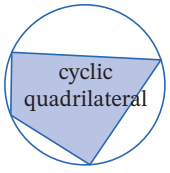
cubic curve The graph of a cubic equation such as $y = \frac{1}{2}x^3 + 1$. (p. 318)



cubic equation An equation in which the highest power of the variable is 3, that is, a variable cubed, for example, $x^3 = 12$. (pp. 254, 318)

curved surface area The area of the curved surface of a solid such as a cylinder, cone or sphere. (p. 143)

cyclic quadrilateral A quadrilateral bounded by a circle, whose vertices lie on the circle's circumference. (p. 580)



D

data Information, a collection of facts. (p.187)

degree of a polynomial The highest power in a polynomial. For example the degree of $8x^3 + 4x - 7$ is 3. (p. 552)

dependent event An event whose outcome (and probability) depends upon the outcome of another event; for example, the colour of the second marble drawn from a bag depends on the colour of the first marble drawn. (p. 476)

dependent variable A variable that depends on another variable for its value. For example, if y depends on x, then the dependent variable is y and the **independent variable** is x. (pp. 223, 536)

depreciation The decrease in the value of items over time due to ageing or use. (p. 50)

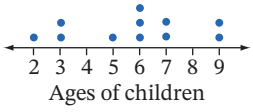
difference of 2 squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$. (p. 166)

direct proportion (or **direct variation**) A relationship between 2 variables of the form $y = kx$, where k is a constant called the **constant of proportionality**. For example, if $y = 8.5x$, then y is directly proportional to x. (p. 286)

distance–time graph A line graph illustrating a journey, showing distance travelled against time. Also called a **travel graph.** (p. 296)

domain The set of permissible x-values for the function $y = f(x)$. (p. 540)

dot plot A graph that uses dots above a number line to show the frequencies of data values. (p. 187)



double time Overtime pay that is calculated at 2 times the normal pay rate. (p. 26)

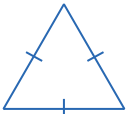
E

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables. (p. 401)

equation A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 318)

equiangular All angles equal. (pp. 560, 568)

equilateral triangle A triangle with all 3 sides equal (and all angles 60°). (p. 511)



event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 452)

exact ratio The sine, cosine and tangent of the special angles 30°, 45° and 60°, which can be expressed as exact fractions or surds rather than decimal approximations. (p. 362)

expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials. (p. 453)

experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula $P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$ (p. 452)

exponential curve The graph of an exponential equation $y = a^x$. (p. 331) See **asymptote** for a diagram.

exponential equation or **exponential function** An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, for example, $y = 4^x$. (pp. 274, 331, 546)

extrapolation Making a prediction from a set of bivariate data about values that are beyond the range of the data, on a scatterplot. Different from **interpolation.** (p. 225)

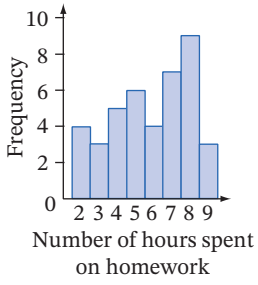
F

factor theorem The rule that if $(x - a)$ is a factor of the polynomial $P(x)$, then $P(a) = 0$. (p. 559)

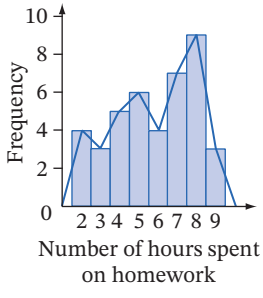
five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value; used to draw a **box plot**. (p. 204)

formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 258)

frequency histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings. (p. 187)



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the column tops of a histogram. The graph looks like a mountain. (p. 200)



frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx. (p. 187)

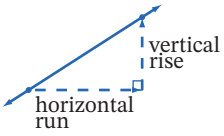
function A rule or relationship between 2 variables where, for each value of the independent variable (input value), x, there is only one value of the dependent variable (output value), y. (p. 536)

function notation A way of writing a function using the form $y = f(x)$. (p. 539)

G

general form of a linear equation The equation of a straight line $ax + by + c = 0$, where a, b and c are integers and a is positive. (p. 80)

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$. (p. 61)



gradient–intercept form of a linear equation The equation of a straight line $y = mx + c$, where m is the **gradient** and c is the y-intercept. (p. 76)

gross pay Pay received before tax and other deductions are taken out. (p. 54)

H

hemisphere Half a **sphere**. (p. 114)

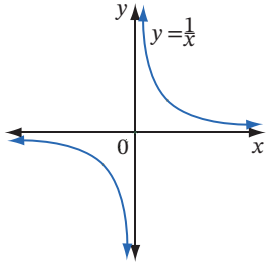


highest common factor (HCF) or **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$. (p. 162)

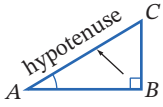
hire-purchase See **term payments**. (p. 46)

histogram See **frequency histogram.** (p. 187)

hyperbola The graph of $y = \frac{k}{x}$, where k is a constant, a curve with 2 branches. (p. 326)



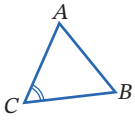
hypotenuse The longest side of a right-angled triangle, opposite the right angle. (p. 348)



I

image A transformed shape after it has been enlarged or reduced. (p. 491)

included angle The angle between 2 given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$. (pp. 387, 497)



income tax A tax paid to the government based on the size of a person's income. (p. 31)

independent event An event whose outcome (and probability) does not depend upon the outcome of another event; for example, the number rolled on the second die does not depend on the number rolled on the first die. (p. 476)

independent variable A variable whose value does not depend on another variable. For example, if y depends on x, then the **dependent variable** is y and the independent variable is x. (pp. 223, 536)

index (Plural: **indices**, pronounced 'in-da-sees') See **power**. (p. 152)

index law An algebraic rule for simplifying expressions involving powers of the same base, for example, $a^m \times a^n = a^{m+n}$. (p. 152)

inequality A mathematical statement that 2 quantities are not equal, involving algebraic expressions and an inequality sign ($>$, $\geq$, $<$, or $\leq$), for example, $-3 > -10$ or $2x - 7 \leq 15$. (p. 263)

instalment See **repayment**. (p. 46)

intercept (in geometry) The distance between the points when a line crosses 2 lines or a curve at 2 points. (p. 586)

interpolation Making a prediction from a set of bivariate data about values that are within the range of the data, on a scatterplot. Different from **extrapolation.** (p. 225)

interquartile range (IQR) The difference between the upper quartile and lower quartiles,

$$\text{IQR} = Q_3 - Q_1$$

representing the middle 50% of values. (pp. 192, 222)

interval A section of a line with a definite length, such as AB shown. (p. 61)

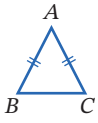


inverse function The reverse of a function $f(x)$, written $f^{-1}(x)$, which 'undoes' the original function, found by interchanging the dependent and independent variables (x and y). (p. 543)

inverse proportion (or **inverse variation**) A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x. (p. 289)

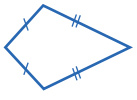
irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. (p. 4) See also **rational number** and **real number**.

isosceles triangle A triangle with 2 equal sides (and 2 equal angles opposite those sides). (p. 499)



K

kite A quadrilateral with 2 pairs of equal adjacent sides. (p. 500)



L

leading coefficient of a polynomial The coefficient of the **leading term**. For example, the leading coefficient of $8x^3 + 4x - 7$ is 8. (p. 552)

9780170453509

leading term of a polynomial The term with the highest power. For example, the leading term of $8x^3 + 4x - 7$ is $8x^3$. (p. 552)

LHS The left-hand side (of an equation). (p. 251)

line of best fit A straight line drawn through the points on a scatterplot that best describes the bivariate data. (p. 225)

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (p. 72)

logarithm The power of a number, to a given base. For example, $\log_{10}1000 = 3$, meaning that the logarithm of 1000 to base 10 is 3, because $1000 = 10^3$. (p. 269)

logarithmic equation An equation involving a logarithm such as $\log_5 x = -3$, which can be solved by rewriting the equation in index form. (p. 275)

logarithmic function A function of the form $y = \log_a x$, whose inverse is the exponential function. (p. 546)

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the values by the number of values. (p. 220)

measure of central tendency or **measure of location** An average, middle or typical value of a set of data. The 3 measures of central tendency are the **mean**, **median** and **mode**. (p. 217)

measure of spread A statistical value that describes how the values in a data set are spread, for example, **range**, **interquartile range**, or **standard deviation**. (p. 192)

median The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 191)

midpoint The point in the middle of an interval or halfway between 2 given points. (p. 61)

minute (symbol ′) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60'$. (p. 349)

mode The most common or frequent value(s) in a set of data. (p. 217)

monic polynomial A polynomial that has a **leading coefficient** of 1, for example, $P(x) = x^4 - 7x^2 + x + 8$. (p. 552)

mutually exclusive events Events or categories that have no items in common. (p. 458)

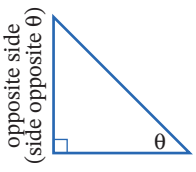
N

negatively skewed See **skewed distribution**. (p. 187)

net pay Pay received after deductions from gross pay; 'take-home' pay. (p. 31)

O

opposite side In a right-angled triangle, the side directly facing the given angle. (p. 348)



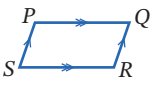
outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 452)

outlier An extreme data value that is much different to the other values in a set. (p. 222)

P

parabola A U-shaped curve that is the graph of a quadratic equation such as $y = x^2$. (pp. 307, 432)

parallelogram A quadrilateral in which the opposite sides are parallel. (p. 500)



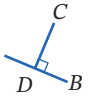
PAYG (Pay As You Go) tax Income tax deducted from your pay each payday by your employer. (p. 31)

perfect square A square number or an algebraic expression that represents one; for example, 64, $(x + 9)^2$, $(a - b)^2$. (p. 184)

per annum (p.a) Per year. (p. 38)

perpendicular bisector An interval or line that bisects another interval at right angles. (p. 573)

perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means 'AB is perpendicular to CD'. (pp. 68, 571)



point-gradient form of a linear equation The equation of a line with gradient m and that passes through the point (x_1, y_1), which is $y - y_1 = m(x - x_1)$. (p. 83)

polygon Any flat shape made up of straight sides. (pp. 112, 493)



polynomial An algebraic expression involving powers of x that are positive integers. For example $P(x) = 8x^3 + 4x - 7$. (p. 552)

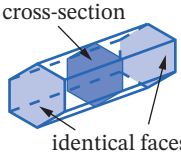
population In statistics, all of the items being studied, the entire group. (p. 234)

positively skewed See **skewed distribution**. (p. 187)

power (or **index**) The number of times a base is multiplied by itself. In 2^5, the power is 5. Also called the *exponent*. (p. 302)

principal An amount of money invested or borrowed, on which interest is calculated. (p. 34)

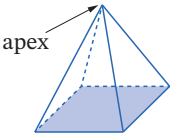
prism A solid shape with identical cross-sections and straight sides. (p. 112)



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (p. 452)

pronumeral Another name for **variable**. (p. 258)

pyramid A solid with a polygon for a base and triangular faces that meet at a point called the **apex.** (p. 109)



square pyramid

Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle. (p. 570)



quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared; for example, $3x^2 - 6 = 69$ or an equation such as $y = 3x^2 - 6$, whose graph is a **parabola**. (pp. 250, 307, 416)

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$. (p. 171)

quadratic formula The formula for solving quadratic equations of the form $ax^2 + bx + c = 0$, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (p. 422)

quadratic trinomial See **trinomial**. (p. 171)

quadrilateral test A property of a quadrilateral that proves that it is a particular type of quadrilateral, for example, if opposite angles are equal, then it *must* be a parallelogram. (p. 491)

quarterly Occurring regularly 4 times a year, that is, every 3 months. (p. 44)

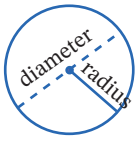
quartic polynomial A polynomial with degree 4, for example, $P(x) = -6x^4 + x^3 - 2x^2 + 9x$. (p. 552)

quartile The values Q_1, Q_2, Q_3 that divide a set of data into 4 equal parts. The 1st quartile Q_1 is the lower quartile, the 2nd quartile Q_2 is the **median**, the 3rd quartile Q_3 is the upper quartile. (p. 191)

quotient The 'whole' part of the answer when a polynomial is divided by another polynomial. (p. 556)

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**. (pp. 106, 578)



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 454)

range In a set of data, the difference between the highest and lowest values. (p. 192)

range (of a function) The set of permissible y-values for the function $y = f(x)$. (p. 540)

rational number A number that can be written as a fraction in the form $\dfrac{a}{b}$, where a and b are integers and $b \neq 0$. (p. 4) See also **irrational number** and **real number**.

rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd). (p.19)

real number A **rational** or **irrational number** that can be ordered on a number line. (p. 5)

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\dfrac{1}{5}$ and the reciprocal of $\dfrac{2}{3}$ is $\dfrac{3}{2}$. (p. 177)

rectangle A quadrilateral with 4 right angles. (p. 500)



regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has 5 equal sides and 5 equal angles. (p. 493)



relative frequency The number of times an event or data value occurred, written as a fraction of the total number of events or data values. (p. 452) See also **experimental probability**.

remainder The 'left-over' part of the answer when a polynomial is divided by another polynomial. (p. 556)

remainder theorem The rule that if a polynomial $P(x)$ is divided by the linear expression $(x - a)$ then the remainder is $P(a)$. (p. 558)

repayment (or **instalment**) The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan. (p.46)

rhombus A quadrilateral with 4 equal sides. (p. 500)



RHS The right-hand side (of an equation). (p. 251) See also **similarity test**. (p.515)

rise Short for 'vertical rise', this is the change in vertical position between 2 points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. (p. 61) See **gradient**.

root of an equation A value of x that is a solution to the polynomial equation $P(x) = 0$. A root of $P(x) = 0$ is the same as a **zero** of $P(x)$. (p. 563)

run Short for 'horizontal run', this is the change in horizontal position between 2 points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. (p. 61) See **gradient**.

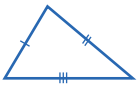
S

sample In statistics, a group of people or items selected from a population for study. (p. 234)

sample space In a probability situation, the set of all possible outcomes. (p. 465)

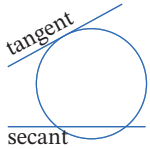
scale factor The amount by which a shape has been enlarged or reduced, equal to $\dfrac{\text{image length}}{\text{original length}}$. (p. 491)

scalene triangle A triangle with no equal sides.



scatterplot A graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern. (p. 221)

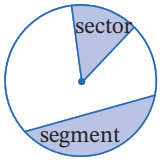
secant A line that intersects a curve at 2 points. See also **tangent.** (p. 570)



second (″) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$. (p. 349)

sector A region of a circle cut off by 2 radii. (p. 570)

segment A region of a circle cut off by a chord. (p. 570)



shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, positively **skewed** or negatively **skewed**. (p. 187)

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits; for example, 31 487 000 has 5 significant figures. (p. 107)

similar To have the same shape but not necessarily the same size, an enlargement or reduction. The symbol '|||' means 'is similar to'. (p. 512)

similarity test One of 4 tests for proving that 2 triangles are similar. (p. 515)

simple interest Interest that is calculated as a percentage of the original principal. (p. 34) See also **compound interest**.

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$. (p. 399)

sine A ratio in a right-angled triangle:

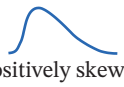
$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 348) See also **cosine** and **tangent**.

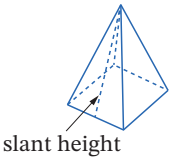
sine rule A rule that relates the sides of any triangle to the sine of their opposite angles: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. (p. 375)

skewed distribution A distribution in which most of the values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. (p. 187) See also **symmetrical distribution**.


Negatively skewed

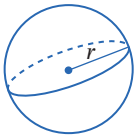

Positively skewed

slant height The height of a pyramid or cone from its **apex** (top) to its base along a side face rather than its perpendicular height. (p. 109)



solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true. (p. 560)

sphere A ball shape, a solid that is completely round. (p. 114)



square A quadrilateral with 4 equal sides and 4 right angles. (p. 500)



standard deviation (symbol σ) A measure of spread that depends on every value in the data set and their mean. (p. 196)

stem-and-leaf plot A 'number graph' that lists all the data values, in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test scores, from 42 to 82. (p.187)

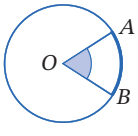
Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key, 5|8 stands for 58

subject of a formula The variable for which a formula is written, the variable on the left-hand side of a formula. The subject of the formula $A = \frac{1}{2}bh$ is A. (p. 258)

substitution method A method of solving **simultaneous equations** that involves substituting one equation into another equation. (pp. 401, 404)

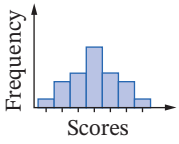
subtend an angle To sit opposite an angle and 'hold' its arms. In the diagram, the arc AB subtends the angle at O, or angle O is subtended by the arc AB. (p. 572)



surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (p. 4)

surface area The total area of all the faces of a solid shape. (p. 102)

symmetrical distribution A distribution in which all values are distributed equally on both sides of the centre, its shape having line symmetry. (p. 187) See also **skewed distribution**.



T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. (p. 348) See also **sine** and **cosine**.

tangent A line that touches a curve at one point but does not cross it. See also **secant** for diagram. (pp. 570, 584)

tax deduction See **allowable deduction**. (p. 31)

taxable income The part of a person's income that is taxed, equal to yearly income minus allowable deductions. (p. 31)

term payments Paying for an expensive item through a loan in which regular instalments are made over time. Also called **hire-purchase**. (p. 46)

test for a quadrilateral See **quadrilateral test.** (p. 491)

theoretical probability Probability calculated using the formula:

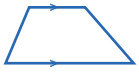
$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$ (p. 452)

three-figure bearing (or **true bearing**) A bearing that uses 3-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. (p. 354) See also **bearing** or **compass bearing**.

three-step experiment (or **three-stage experiment**) A chance experiment with 3 steps or stages, such as tossing 3 coins together. (p. 465)

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate. (p. 26)

trapezium A quadrilateral with one pair of opposite sides parallel. (p. 500)



travel graph See **distance-time graph.** (p. 325)

tree diagram A diagram of branches for listing all of the possible outcomes of a multi-step chance experiment. (p. 466)

trial One go or run of a repeated probability experiment; for example, one roll of a die. (p. 452)

trinomial An algebraic expression with 3 terms, for example, $3x + 2y - 5$. In a **quadratic** trinomial such as $x^2 + 4x + 6$, the highest power of the variable is 2. See also **binomial** and **quadratic expression.** (p. 171)

two-step experiment (or **two-stage experiment**) A chance experiment with 2 steps or stages, such as rolling a pair of dice. (p. 465)

two-way table A table that shows the number of items belonging to overlapping categories. (p. 462)

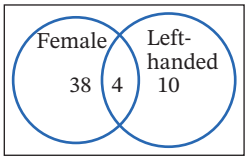
	Can swim	Cannot swim
Boys	13	2
Girls	9	3

V

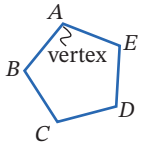
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**. (p. 258)

variation See **direct proportion**. (p. 286)

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories. (p. 457)



vertex (plural: **vertices)** A corner of a shape, angle or curve. (p. 308)



vertical line test A test applied to a graph to determine whether it is the graph of a function. (p. 537)

volume The amount of space taken up by a solid object, measured in cubic units. (p. 136)

X

***x*-intercept** The x-value at which a graph cuts the x-axis. (p. 72)

Y

***y*-intercept** The y-value at which a line cuts the y-axis. (p. 73)

Z

zero of a polynomial A value of x that makes the value of the polynomial $P(x)$ equal to 0. (p. 563)

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